The Pearson Guide t
AIEEE Physic

Ravi Raj Dudeja

ALWAYS LEARNING

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The Pearson Guide to

OBJECTIVE PHYSICS

FOR AIEEE

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OBJECTIVE PHYSICS

FOR AIEEE

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PREFACE

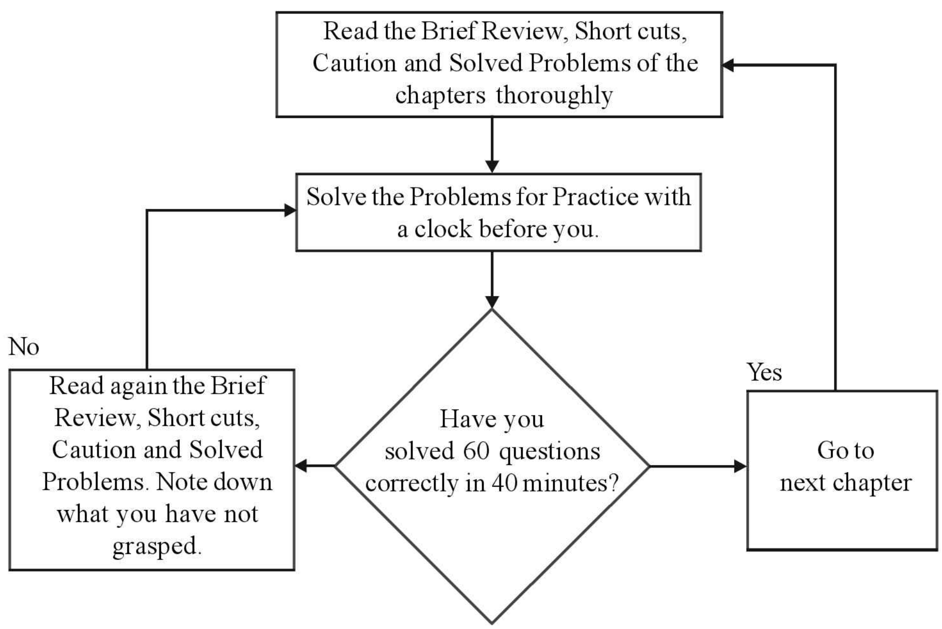
To get admission in a premier institute of technology is the dream of each student opting for science. Admission in these institutes is based on clearing with a good rank in the All India Engineering Entrance Examination.

To score a good rank, aspirants have to answer questions ranging in difficulty levels in the shortest possible time. Therefore, one has to learn techniques to solve these questions. This has been kept in mind while presenting this book so that a candidate should master the art of solving at the fastest rate such that almost all problems are solved with accuracy.

The unique features of this book are:

- Definitions, concepts, formulae, etc. form the brief review about each topic.
- Shortcuts and points to note provide ready-to-use formulae and other shortest possible techniques to solve the problems.
- Caution helps students avoid committing common mistakes.
- Self tests and questions from competitive examinations with answers and explanations are given at the end of each section.
- Model test papers with their explanations are given at the end of the book.

The ideal method to prepare for the examination is as follows:



Adoption of the technique in the flow chart ensures 100 per cent success. Work hard, hard work is the key to success.

With best wishes for a great future.

Ravi Raj Dudeja

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1

Units, Dimensions and Errors

BRIEF REVIEW

In physics, we deal with observations, measurement and description of natural phenomenon related to *matter* and *energy*. The natural phenomena may be classified as mechanics, properties of matter, sound, thermodynamics, light, electricity, atomic physics, nuclear physics, particle physics, semiconductors, superconductors and so on.

Physical Quantities

These are quantities that are used to describe the laws of physics. Physical quantities may be divided into six categories.

1. ***Constant or ratio*** Such quantities have only magnitude, for example, refractive index, dielectric constant and specific gravity. Such quantities have no units.
2. ***Scalars*** These quantities have magnitude and unit. Some of them may have direction also but vector laws are not applied. Examples are charge, mass, distance, speed and current.
3. ***Vectors*** These quantities possess magnitude, unit and direction. They also follow triangle law of addition. For example, velocity, force, momentum, torque and displacement.
4. ***Phasors*** These possess magnitude (amplitude) and phase. They follow triangle law. SHM, waves, AC voltage and AC current are phasors.
5. ***Tensors*** Such quantities do not have any specified direction but have different values in different directions. For example, moment of inertia. In anisotropic media even density, refractive index, dielectric constant, electric conductivity, stress, strain and so on become tensor. A physical quantity which has only one component is called a scalar or a tensor of zero rank. If it has more than one component but less than or equal to four, it is called a vector or tensor of rank 1. If the components are greater than four, it is termed as tensor of a higher rank.
6. ***Conversion factors*** Some physical quantities convert into another when multiplied by a factor. For example, in a wave $y = y_0 \sin(\omega t - kx)$, k is a conversion factor. When k is multiplied by displacement or path difference it generates phase or phase difference. In frequency modulation $kE_m f_c = \delta$, k converts voltage into angle and is termed as a conversion factor. Many other conversion factors can be thought of.

In general, a physical quantity = magnitude × unit. If the unit changes, the magnitude will also change. We apply $n_1u_1 = n_2u_2$.

Physical quantities may be divided into ***fundamental*** and ***derived*** quantities.

Fundamental Quantities

The quantities that do not depend upon any other quantity are called fundamental or absolute or basic quantities. Initially, only three fundamental quantities—length, mass and time—were considered. With the development of science, four more physical quantities were added. These are temperature, electric current, luminous intensity and amount of a substance.

Derived Quantities

The physical quantities derived from fundamental quantities are called derived quantities like velocity, acceleration, force and momentum.

Units

The fixed and definite quantity taken as standard of reference with which other quantities of the same kind are measured is defined as a unit.

Fundamental Units

The units of fundamental quantities are called fundamental units. For example, units of length, mass and time or those of fundamental physical quantities, are called fundamental units. Table 1.1 lists all fundamental and supplementary units (SI) and their symbols.

Derived Units

Units of derived physical quantities are called derived units. For example, units of velocity, density, force, momentum and volume.

Initially, three systems of units, namely, CGS, FPS and MKS based on three fundamental quantities, length, mass and time, came into existence. In 1970, in a world conference a consensus evolved and a standard international system of units was developed. It is more popularly known as the SI system. In addition to seven fundamental units, two supplementary units were also included, namely, plane angle or angle (unit radian abbreviated as rad) and solid angle (unit stredian abbreviated str). SI units since 1978 are observed throughout the world.

Practical Units

Apart from fundamental units, supplementary units and derived units, we come across some practical units like light year, horse power, energy unit (1unit = 3.6×10^6J), Curie (Ci), Ratherford (R) etc.

While writing a unit, the following convention is adopted:

(a) Unit named after a person starts with a capital letter. For example, Newton is written as N, Curie as Ci and Rutherford as R.

(b) Fundamental units are written with small letters, for example, metre as m and kilogram as kg.

(c) The symbols are not expressed in plural form. For example, 50 metres will be written as 50 m.

(d) Punctuation mark such as fullstop are not used after the symbol of unit. For example, 1 Millilitre = 1 ml or 1 cc (not m.l. or c.c.)

Table 1.1 Fundamental Physical Quantities

Physical quantity	*SI unit*	*Dimensional symbol*	*Unit symbol*
Length	metre	L	m
Mass	kilogram	M	kg
Time	second	T	s
Temperature	Kelvin	K	K
Electric current	Ampere	A	A
Luminous intensity	Candela	I	Cd
Amount of substance	mole	mol	mol
Supplementary units			
Angle	radian	—	rad
Solid angle	stredian	—	str

Standards of Length

The most common unit is metre (m), Foot is also used sometimes. In 1889 the standard metre was defined as the distance between two fixed marks engraved on a platinum-iridium bar preserved at constant temperature of 73.16 K and constant pressure of 1 bar in the international bureau of weights and measures at Serves near Paris in France. All other meters are calibrated with it to an accuracy of 0.1ppm.

In 1960 the standard metre was defined in terms of wavelength of Kr-86 and is called atomic standard of length. 1 m is the distance covered by 1650763.73 wavelength of orange red light of Kr-86 in vacuum. An accuracy $1:10^9$ parts can be obtained with it.

In 1983 metre was defined as the length of path travelled by light in vacuum in $\frac{1}{299,792,458}$ th second.

Some other important units of length are

$1Å = 10^{-10}$m, 1x-ray unit (1 XU) = 10^{-13}m

1 yard = 3 foot = 0.9144 m, 1" (inch) = 2.54 cm

1 astronomical unit (1 AU) = 1.49×10^{11}m,

1 light year (1 ly) = 9.46×10^{15}m,

1 Parsec (1 pc) = 3.08×10^{16}m = 3.26 ly

Standard of Mass

Originally, 1 kg mass was defined as the mass of 1litre (10^3 cc) of water at 4°C, nowadays standard kg is the mass of platinum-iridium cylinder stored in a special vault in the International Bureau of Standards at Serves (France). The accuracy of this standard is 1 in 10^8 parts.

To measure atomic masses, the unit amu (or u) is used.

$$1u = \frac{1}{12}\text{th of mass of } {}^{12}_{6}C \text{ atom}$$

or $$1u = \frac{1}{12} \times \left(\frac{12}{6.023 \times 10^{26}}\right) kg = 1.67 \times 10^{-27}\ kg$$

In FPS system pound (lb) is the unit of mass, sometimes ***slug*** is also used.

1 lb = 0.453592737 kg, 1 slug = 32.2 lb = 14.59 kg.

Note: In astrophysics, we sometimes come across Chandrashekhar limit. 1 Chandrashekhar limit = 1.4 times mass of the sun = 2.8×10^{30} kg. Chandrashekhar showed that if the mass of an object becomes 1.4 times the mass of the sun, under gravitational collapse it turns out to be white dwarf.

Standards of Time

Initially, it was defined on the basis of solar or lunar motion.

$$1 \text{ mean solar day} = \frac{1 \text{ year}}{365.25 \text{ days}}$$

and $$1\ s = \frac{1 \text{ year}}{365.25 \times 24 \times 60 \times 60}$$

But because of tidal friction, the length of a day is increasing at a rate of 7 μs per year. Therefore, in 1965 the atomic standard was defined. According to this standard, 1s is the interval of 91192631770 vibrations of radiation corresponding to the transition between two specific hyperfine levels in ^{133}Cs (cesium) clock which will go wrong by 1s in 3000 years. Hydrogen maser promises a producing error of 1*s* in 33,000,000 years.

Note that (*i*) time can never flow back, i.e., negative time does not exist and (*ii*) at a given instant of time, a particle cannot be present in more than one position in space.

Table 1.2 describes prefixes used for multiple and submultiples of metric quantities.

Dimensions and Dimensional Formulae

All physical quantities can be expressed in terms of seven fundamental quantities. The powers to which these fundamental physical quantities be raised are termed as dimensions. For example, force = MLT^{-2}, i.e., force has dimensions of mass, length and time as 1, 1 and –2 and $[MLT^{-2}]$ is the dimensional formula for force. Dimensional formulae for work, torque and resistance are $[M^2LT^{-2}]$, $[ML^2T^{-2}]$ and $[ML^2T^{-3}A^{-2}]$ respectively.

Table 1.2 Prefixes Used Before Units

atto	a	10^{-18}	deca	da or D	1d
femto	f	10^{-15}	hecto	h	10^2
pico	p	10^{-12}	kilo	k	10^3
nano	n	10^{-9}	mega	M	10^6
micro	m	10^{-6}	Giga	G	10^9
milli	m	10^{-3}	terra	T	10^{12}
centi	c	10^{-2}	peta	p	10^{15}
deci	d	10^{-1}	exa	E	10^{18}

Applications of Dimensional Formulae

(a) ***To check the correctness of a given physical relation.*** It is based on the principle of homogeneity, that is, the dimensions on two sides be same for a given relation. For example,

$F = 6\pi\eta rv$

dimensions of LHS $F = MLT^{-2}$

RHS $6\pi\eta rv = [ML^{-1}T^{-1}]\,[L][LT^{-1}] = MLT^{-2}$

LHS and RHS have identical dimensions; therefore, the relation is dimensionally correct.

If the dimensions on two sides differ, the relation is incorrect.

(b) ***To derive a relation.*** We illustrate with an example. The amount of liquid flowing per second through a tube of radius *r* depends upon radius *r* of the tube, coefficient of viscosity and pressure gradient. Derive the expression.

$$\frac{dV}{dt} \propto \eta^a r^b \left(\frac{p}{l}\right)^c$$

or $$\frac{dV}{dt} = k\eta^a r^b \left(\frac{p}{l}\right)^c$$

where *k* is a dimensionless constant.

Then $[M^0L^3T^{-1}] = [ML^{-1}T^{-1}]^a\ [L]^b\ [ML^{-2}T^{-2}]^c = M^{a+c}\ L^{-a+b-2c}T^{-a-2c}$

Comparing powers of *M*, *L* and *T* on both sides

$$0 = a + c$$
$$3 = -a + b - 2c$$
$$-1 = -a - 2c$$

Solving, we get $a = -1, b = 4, c = 1$

Thus $$\frac{dV}{dt} = k\frac{r^4 \frac{p}{l}}{\eta}$$

The value of *k* is determined experimentally.

Hence, $$\frac{dV}{dt} = \frac{\pi r^4 P}{8\eta l};\ k = \pi/8$$

(c) ***To convert the value of a physical quantity from one system of units to another system of units.*** We illustrate with an example. Let us convert 1J (SI) to erg (CGS) system energy $E = ML^2T^{-2}$

$$n_2 = n_1\left[\frac{M_1}{M_2}\right]\left[\frac{L_1}{L_2}\right]^2\left[\frac{T_1}{T_2}\right]^{-2}$$

$$= 1\left[\frac{10^3}{1}\right]\left[\frac{10^2}{1}\right]^2\left[\frac{1}{1}\right]^{-2} = 10^7 \text{erg.}$$

Limitations of Dimensional Analysis

1. Dimensional analysis cannot be applied to derive a relation involving sum of products or product of sums from a relation like $s = ut + \frac{1}{2}at^2$. It can derive a relation of a single product term. If $s = ut + \frac{1}{2}at^2$ is to be derived using dimensional analysis then $s = ut$ and $s = \frac{1}{2}at^2$ will be derived separately and then added from your knowledge.

2. Dimensional analysis cannot be applied to derive a relation involving more than three unknowns. However, we can join two variables to make one, for example, we used pressure gradient $\left(\frac{p}{l}\right)$ and not p and l separately. Note that we can check the correctness of a relation involving any number of variables or terms.
3. If two quantities have same dimensions then such a relation involving these quantities cannot be derived. However, we can check the correctness of the relation.
4. Numerical constants, trignometric ratios and other dimensionless ratio cannot be derived.
5. For a given dimension, physical quantity is not unique. For example, Torque and energy have the same dimensional formula $[ML^2T^{-2}]$.

Significant Figures

These give the accuracy with which a physical quantity is expressed. The number of digits which are known reliably or about which we have confidence in measurement, plus the first digit that is uncertain, are termed as significant figures. For instance, length of a table is 137.2 cm. This has 4 significant figures and 2 (after decimal) is uncertain. It is worth mentioning that significant figure of a physical quantity depends upon the least count of the instrument with which it is being measured.

Rules for Determining Significant Figures

1. All the nonzero digits are significant, for example, 187.25 has 5 significant digits.
2. All the zeros occurring between two nonzero digits are significant. For instance, 105.003 has 6 significant digits.
3. The zeros occurring between the decimal point and the non-zero digits are not significant provided integral part is zero, i.e, in 0.0023 there are only two significant point figures.
4. All zeros to the right of a non-zero digit in number written without decimal are not significant. For example, 32500 has only 3 significant figures.

Exception This rule does not work when we record the values on actual measurement basis. For example, distance between two places is 121m has 4 significant digits.

5. All zeros occurring to the right of non-zero digit in a number written with a decimal point are significant. For example, 2.3200 has 5 significant figures.

Note: The number of significant figures does not vary with the choice of units. For example, length of a rod is 76 cm. If we the represent it as 0.76 m or 0.00076 km significant figures remain 2.

In exponent form, exponent figure does not contribute to significant figure. Thus, 7.24×10^7 has only 3 significant digits.

Rules for Rounding Off

1. If the digit to be dropped is less than 5, the preceding digit be kept unchanged. For example, 3.92 may be expressed 3.9.
2. If the digit to be dropped is greater than 5, the preceding digit be increased by 1. For example, 5.87 be rounded off as 5.9.
3. If the digit to be dropped is 5 followed by digits other than zero, then the preceding digit be increased by 1. For example, 14.454 be rounded off as 14.5 to the first decimal place.

Resolution, Accuracy and Precision of an Instrument

Resolution Stands for least count or the minimum reading which an instrument can read.

Accuracy An instrument is said to be accurate if the physical quantity measured by it resembles very closely its true value.

Precision An instrument is said to have high degree of precision if the measured value remains unchanged, howsoever, large number of times it may have been repeated.

Errors in Measurement Deviation between measured and actual (or mean) value of a physical quantity is called error. For example, if a_m is measured value of a physical quantity and its true value is a_t then error = $\Delta a = |a_m - a_t|$.

Errors may be divided into two types: systematic and random errors.

Systematic Error

Errors arising due to the system of measurement or the errors made due to parts involved in the system of measurement are called systematic errors. Since the system involves instrument, observer and environment, systematic error is of three types, namely, instrumentation error, personal error (error made by observer due to carelessness or eye defect) and environmental error. Instrumental error equal to least count of instrument is unavoidable and hence is always accounted for.

Random Error (or Statistical Error)

The error which creeps in due to a large number of events or large quantity termed as random error. Consider an example. The probability of tossing a coin is 1/2. If we toss a coin 1000 times or 1000 coins are tossed simultaneously then we will hardly ever get 500 heads and 500 tails. Thus we find even random errors cannot be removed.

Methods to Express Error

(a) ***Absolute error*** The deviation of true value from measured value or deviation of the value from its mean value (of all observations) is called absolute error.

Thus, $\Delta x_i = |x_i - x_m|$ where x_m is mean value and x_i is the ith component of the observation.

(b) ***Relative error*** $\frac{\Delta x}{x_m}$ or $\frac{\Delta x}{x}$, i.e., ratio of mean absolute error to the true value is called relative error.

(c) ***Percentage error*** equals to relative error $\times 100$

$$= \frac{\Delta x}{x} \times 100$$

Propagation or Combination of Errors

1. When $x = a + b$ maximum possible % error

$$= \frac{\Delta x}{x} \times 100 = \frac{\Delta a + \Delta b}{a+b} \times 100$$

2. When $x = a - b$ maximum possible % error

$$= \frac{\Delta x}{x} \times 100 = \frac{\Delta a + \Delta b}{a-b} \times 100$$

3. When $x = ab$ or $x = a/b$ maximum possible % error =

$$\frac{\Delta x}{x} \times 100 = \left(\frac{\Delta a}{a} + \frac{\Delta b}{b}\right) \times 100$$

4. When $x = \frac{a^n b^m}{y^p z^k}$ maximum possible % error

$$= \frac{\Delta x}{x} \times 100 = \left(\frac{n\Delta a}{a} + \frac{m\Delta b}{b} + \frac{p\Delta y}{y} + \frac{k\Delta z}{\Delta z}\right) \times 100$$

Least count of vernier callipers also called vernier constant (VC)

VC = (1MSD – 1VSD) (value of 1MSD). If n VSD coincide with $(n - 1)$ MSD then $VC = \left(1 - \frac{n-1}{n}\right)$ (value of 1MSD)

Least Count of Screw Gauge or Spherometer

$$= \frac{\text{pitch}}{\text{number of divisions on circular scale}}$$

and $\text{pitch} = \frac{\text{number of divisions moved on linear scale}}{\text{number of rotations given}}$

• Short Cuts and Points to Note

1. Remember all possible formulae connecting the physical quantity and see whose dimensions are known to you. Use that formula, for example,

$$C = \frac{\varepsilon_0 A}{d},\ C = \frac{Q}{V},\ E = \frac{Q^2}{2C}$$

if you use $C = \frac{Q^2}{E} = \frac{(AT)^2}{ML^2T^{-2}} = M^{-1}L^2T^4A^2$ then the dimension of C is easily calculated.

2. Remember all the rules to find significant digits mentioned in ***Brief Review***.
3. Remember the least count of instrument, sometimes it is not given in problem; remember the rules to find the propagation of error and its calculation.

(*a*) If $x = a + b$ then $\frac{\Delta x}{x} = \frac{\Delta a + \Delta b}{a + b}$

(*b*) If $x = a - b$ then $\frac{\Delta x}{x} = \frac{\Delta a - \Delta b}{a - b}$

(*c*) If $x = ab$ or $x = a/b$ then $\frac{\Delta x}{x} = \left(\frac{\Delta a}{a} + \frac{\Delta b}{b}\right) \times 100$

(*d*) If $x = \frac{a^n b^m}{z^p y^k}$

then $\frac{\Delta x}{x} = \left(\frac{n\Delta a}{a} + \frac{m\Delta b}{b} + \frac{p\Delta z}{z} + \frac{k\Delta y}{y}\right)$

4. Absolute error $\Delta x = x\left(\frac{n\Delta a}{a} + \frac{m\Delta b}{b} + \frac{p\Delta z}{z} + \frac{k\Delta y}{y}\right)$

or in general $\Delta x = x$ (relative error)
5. Note that as we find maximum possible error, due to each variable is added.
6. Dimensional analysis always works to check the correctness of a relation dimensionally. In other cases, it has its limitations as mentioned in ***Brief Review***.
7. If in a vernier callipers n VSD coincide with $(n - 1)$ MSD then vernier constant or its least count is $VC = \left(1 - \frac{n-1}{n}\right)$(value of 1MSD) or $\frac{1}{n}$ (value of MSD).
8. Least count of screw gauge or spherometer

$$= \frac{\text{pitch}}{\text{number of divisions on circular scale}} \quad \text{and}$$

$$\text{pitch} = \frac{\text{number of divisions moved on linear scale}}{\text{number of rotations given}}$$

= Linear distance moved in one rotation.
9. Random error = $\sqrt{n}$ where n = number of events or n = number of quantities.
10. Radius of curvature using spherometer $R = \frac{l^2}{6h} + \frac{h}{2}$

• Caution

1. Not recalling the relation whose other dimensions are known in order to find the dimensional formula of a given physical quantity.

⇒ Assume we have to find dimensions of resistance we write $R = \frac{V}{I}$. To write dimensions of R we need to know dimensions of V. If we use power $P = I^2R$ or

$R = \frac{P}{I^2} = \frac{ML^2T^{-3}}{A^2}$, then $R = [ML^2T^{-3}A^{-2}]$ can be written very easily.

Even to find V using $V = IR$ is not a good method. Rather, use qV = energy E. Then

$$V = \frac{E}{q} = \frac{ML^2T^{-2}}{AT} = ML^2T^{-3}A^{-1}$$

2. Not remembering rules to find significant figures.

⇒ Must remember these rules with exceptions given in text.

3. Assuming that in $x = a - b$ or $x = \frac{a}{b}$ while finding error, the error part of b should be subtracted from error part of a.

⇒ Since we always list maximum possible error, therefore, errors of each part must be added.

4. Assuming only scalars and vectors to be physical quantities.

⇒ Physical quantities are even constants or ratios like refractive index, specific gravity etc. Phasors which have amplitude and phase and follow triangle law of addition— SHM, waves, AC potential and current etc. are phasors. Moment of inertia, strain, stress, refractive index of anisotropic medium are tensors.

5. Assuming error is made by observer only due to his/her carelessness.

⇒ This type of error is personal error. Instrumental and environmental errors also occur. Random or statistical errors are also present.

6. Assuming accuracy or precision of an instrument to be synonyms.

⇒ Accuracy means the measured quantity by an instrument is very close to true value of the physical quantity. The more close the measured quantity to true value, the more accurate the instrument is.

If we repeat the measurement large number of times and each time we get the same reading then instrument is said to possess a high degree of precision.

SOLVED PROBLEMS

1. Which of the following does not have the same dimension? **[IIT Screening 2005]**

(a) Electric flux, electric field, electric dipole moment
(b) Pressure, stress, Young's modulus
(c) Electromotive force, potential difference, electric voltage
(d) Heat, potential energy, work done.

Solution (a) Electric flux = Electric field × area, dipole moment = charge × length.

2. The ratio of the dimension of Planck's constant and that of moment of inertia is the dimension of

(a) time (b) frequency
(c) angular momentum (d) velocity

[CBSE PMT 2005]

Solution (b) $hf = I\alpha$ or $\frac{h}{I} = \frac{\alpha}{f} = \frac{T}{T^2} = \frac{1}{T}$

3. Out of the following pairs, which one does not have the same dimensions?

[AIEEE 2005]

(a) Angular momentum and Planck's constant
(b) Impulse and momentum
(c) Moment of inertia and moment of force
(d) Work and torque.

Solution (c) MOI $I = Mr^2 = [ML^2]$, moment of force = torque $= I\alpha = r \times F = ML^2T^{-2}$

4. Parsec is the unit of

(a) time (b) distance
(c) frequency (d) angular acceleration.

[AIIMS 2005]

Solution (b) astronomical unit of distance

5. A. Specific gravity of a fluid is a dimensionless quantity

R. It is the ratio of density of fluid to density of water.

[AIIMS 2005]

(a) Both A and R are correct and R is the correct explanation of A.
(b) Both A and R are correct but R is not the correct explanation of A
(c) A is correct but R is false
(d) Both A and R are false.

Solution (a)

6. A force F is given by $F = at + bt^2$ where t is time, what are the dimension of a and b?

[BHU 2005]

(a) MLT^{-1}, MLT^0, (b) MLT^{-3}, ML^2T^4,
(c) MLT^{-4}, MLT^{-1} (d) $ML^{-3}T, MLT^{-4}$

Solution (d) $a = \frac{F}{t} = MLT^{-3}$, $b = \frac{F}{t^2} = MLT^{-4}$

7. Which of the following is not a unit of Young's modulus?

[CET Karnataka 2005]

(a) Nm^{-1} (b) Nm^{-2}
(c) dyne cm^{-2} (d) mega pascal

Solution (a)

8. If M is mass of the earth and R its radius, the ratio of the gravitational acceleration and the gravitational constant is

[CET Karnataka 2005]

(a) $\frac{R^2}{M}$ (b) $\frac{M}{R^2}$
(c) MR^2 (d) $\frac{M}{R}$

Solution (b) $g = \frac{GM}{R^2}$ or $\frac{g}{G} = \frac{M}{R^2}$

9. A hypothetical experiment conducted to find Young's modulus $Y = \frac{\cos\theta T^x \tau}{l^3}$ where τ is torque and l is length then find x.

[CBSE PMT 2005 Mains]

Solution $Y = ML^{-1}T^{-2} = \frac{T^x(ML^2T^{-2})}{L^3}$

$= ML^{-1}T^{-2}T^x \quad \therefore x = 0$

10. Find the dimensions of $\frac{B^2}{\mu_0}$

(a) ML^2T^{-2} (b) $ML^{-1}T^{-1}$
(c) $ML^{-2}T^{-2}$ (d) $ML^{-1}T^{-2}$

Solution (d) $\frac{B^2}{\mu_0}$ = energy density

$= \frac{ML^2T^{-2}}{L^3} = ML^{-1}T^{-2}$

11. While printing a book a printer made certain mistakes in the following relation. Find the correct relation.

(a) $y = A \sin \omega\theta$ (b) $y = A \sin (\omega x + \theta)$
(c) $y = A \sin (\omega t + \theta)x$ (d) $y = \frac{A}{x} \sin \omega t + \theta$

Solution (c)

12. The velocity of surface waves depends upon surface tension, coefficient of viscosity and density. The relation is

(a) $\frac{s^2}{\rho\eta}$ (b) $\frac{s}{\eta}$
(c) $\frac{\eta\rho}{s^2}$ (d) $\frac{\rho}{\eta}$

Solution (b) $[M^0LT^{-1}] = [MT^{-2}]^a\, [ML^{-1}T^{-1}]^b\, [ML^{-3}]^c$
$= [M^{a+b+c}\, L^{-b-3c}\, T^{-2a-b-3c}]$

on solving $a = 1, b = -1, c = 0, v = \frac{s}{\eta}$

13. Which of the following pair has different dimensions?

(a) Electric pressure, energy density
(b) Intensity, $\in_0 E_0^2 c$
(c) Reynold's number and time constant
(d) Work, Torque

Solution (c)

14. A chocolate cookie is a circular disk of diameter 8.50 $\pm$ 0.02 cm and thickness 0.050 $\pm$ 0.005 cm. The average volume in cm^3 is

(a) 2.83 $\pm$ 0.3 (b) 2.38 $\pm$ 0.27
(c) 11.35 $\pm$ 1.2 (d) 9.31 $\pm$ 1.12

Solution (a) $V = \pi r^2 l \quad \frac{dV}{V} = \frac{2dr}{r} + \frac{dl}{l}$

$V = \frac{22}{7} \times \frac{8.5}{2} \times \frac{8.5}{2} \times 0.5$

$dV = 2.83\left(\frac{2 \times .02}{8.5} + \frac{.005}{.050}\right) = 2.83\left(\frac{1}{21} + \frac{1}{10}\right)$

$= 0.296\ cm^3$

Thus $V = 2.83 \pm 0.3\ cm^3$

15. The fastest commercial airline service is 1450 mi/h. Find the speed in kmh^{-1} and ms^{-1}.

(a) 1938 kmh^{-1}, 618.3 ms^{-1}
(b) 2030 kmh^{-1}, 623.1 ms^{-1}
(c) 2334 kmh^{-1}, 647.5 ms^{-1}
(d) None

Solution (c) $1450 \times 1.61 = 2334 kmh^{-1}$ 1 mile = 1.61km

$2334 \times \frac{5}{18}\ ms^{-1} = 647.5\ ms^{-1}$

16. Two capacitors $C_1 = 5.2\mu F \pm 0.1\mu F$ and $c_2 = 12.2\mu F$ are joined (i) In series (ii) In parallel.

Find the net capacitance in these two cases.

(a) 2.8%, 1.23% (b) 3.6%, 1.31%
(c) 3.4%, 1.3% (d) 3.9%, 1.15%

Solution (d) In parallel $c = c_1 + c_2$ and

$\frac{\Delta c}{c} \times 100 = \frac{\Delta c_1 + \Delta c_2}{c_1 + c_2} \times 100 = \frac{0.2x100}{17.4}.$

In series $c = \frac{c_1 c_2}{c_1 + c_2}$ and

$\frac{\Delta c}{c} \times 100 = \left(\frac{\Delta c_1}{c_1} + \frac{\Delta c_2}{c_2} + \frac{\Delta c_1 + \Delta c_2}{c_1 + c_2}\right)100$

$= \left(\frac{0.1}{5.2} + \frac{0.1}{12.2}\right) \times 100 + 1.15 = 3.9\%$

17. V^{-1} stands for

(a) electric flux (b) electric pressure
(c) electric field density (d) capacitance

Solution (d) $C = Q/V$

18. A spherometer has 20 threads per cm. Its circular scale has 100 divisions. Find the least count of spherometer.

(a) 5 μm (b) 50 μm
(c) 0.5 μm (d) 0.5 μm

Solution (a)

$$Least\ count = \frac{pitch}{number\ of\ division\ on\ circular\ scale}$$
$$= \frac{0.1}{2} = 0.5 \times 10^{-3}\ \text{cm}$$

TYPICAL PROBLEMS

19. Vernier scale of Vernier callipers has 50 divisions which coincide with 49 main scale divisions. Find the Vernier constant. Given: there are 20 main scale divisions cm^{-1}.

(a) 100 μm (b) 1000 μm
(c) 10 μm (d) none of these

Solution (c) $VC = \frac{1}{50} \times$(value of 1 MSD)

$$= \frac{1}{50} \times \frac{1}{20} = 0.001\ \text{cm}$$

20. The legs of a spherometer are 5 cm apart. There are 10 division cm^{-1} on linear scale and circular scale has 100 divisions. The height h of a convex mirror measured is 2 MSD + 37 circular scale divisions. Find radius of curvature of convex mirror.

(a) 20.003 cm (b) 18.408 cm
(c) 17.399 cm (d) 17.983 cm

Solution (b) Least count $= \frac{1/10}{100} = 10^{-3}$ cm

$h = 2 \times (0.1) + 37\,(10^{-3}) = 0.237$ cm

$$R = \frac{l^2}{6h} + \frac{h}{2} = \frac{25}{1.362} + 0.118 = 18.408\ \text{cm}$$

21. A student measured the length of a pendulum 1.351 m and time for 30 vibrations is 2 minutes 10 sec using his wrist watch. Find the percent error in g committed.

(a) 1.72% (b) 1.813%
(c) 1.63% (d) 1.513%

Solution (c) Percent error in g

$$= \frac{\Delta g}{g} \times 100 = \left(\frac{\Delta l}{l} + \frac{2\Delta T}{T}\right) \times 100$$
$$= \left(\frac{.001}{1.351} + \frac{2 \times 1}{130}\right) \times 100 = 0.73 + 1.54 = 1.613\%$$

22. If force $F = \frac{Ke^{-br}}{r^2}$ varies with distance r. Then write the dimensions of K and b.

Solution Exponential must be dimensionless. Therefore b shall have dimension L^{-1} and

$K = Fr^2 = (MLT^{-2})\,(L^2) = ML^3T^{-2}$

23. What is the order of hair on your head?

(a) 10^6 (b) 10^7
(c) 10^8 (d) 10^5

Solution (d) 10^5

24. What is the dimensional formula for resistivity? How does resistivity of *Ge* varies with temperature?

Solution $\rho = \frac{RA}{l} = ML^3T^{-3}A^{-2}$. Since *Ge* is a semi-conductor, its resistivity falls with rise in temperature.

25. Given $X = \frac{a^n b^m}{p^r}$. The percent error in measurement of a, b and p is 1%, 0.5% and 0.75% respectively. If $n = 2$, $m = 2$ and $r = 4$ then percent error in x is

(a) 0 (b) 6%
(c) 5.25% (d) 0.75%

Solution (b) $\frac{dX}{X} \times 100 = \left(n\frac{da}{a} + m\frac{db}{b} + r\frac{dp}{p}\right) \times 100$

$= 2 \times 1 + 2(0.5) + 4(0.75) = 6\%$

26. In a system of units if force F, acceleration A and time T are taken as fundamental units, then the dimensional formula of energy is **[BHU 2005]**

(a) FA^2T (b) FAT^2
(c) F^2AT (d) FAT

Solution (b) Energy $= ML^2T^{-2} = [MLT^{-2}]^a\,[LT^{-2}]^b\,[T]^c$

$= M^a L^{a+b} T^{-2a-2b+c}$

$a = 1,\ a + b = 2$ $\therefore\ b = 1,$

$-2a - 2b + c = -2$ and $c = 2$

27. In a quartz oscillator L, C and R are analog of

(a) compliance, mass, viscous damping
(b) mass, viscous damping, compliance
(c) mass, compliance, viscous damping
(d) viscous damping, compliance, mass

Solution (c)

28. Choose the correct statement.

(a) 1 second = 10^8 shakes
(b) 1 year has less number of seconds than number of shakes in a second

(c) 1 year has more seconds than number of shakes in a second
(d) A century has more minutes than number of shakes in a second.

Solution (a), (b)

29. The dimensions $ML^{-1}T^{-2}$ may correspond to

(a) work (b) linear momentum
(c) pressure (d) energy density

Solution (c) and (d)

30. Choose incorrect statement/s.

(a) A dimensionally correct equation may be correct
(b) A dimensionally correct equation may be incorrect
(c) A dimensionally incorrect equation may be correct
(d) A dimensionally incorrect equation may be incorrect

Solution (c)

31. A unitless quantity

(a) never has nonzero dimensions
(b) always has nonzero dimensions
(c) may have a nonzero dimension
(d) does not exist.

Solution (a)

32. If force, length and time are fundamental quantities, then find the dimensions of

(a) density (b) pressure

Solution (a) $F = MLT^{-2}$ density $= ML^{-3} = FL^{-4}T^{2}$
(b) pressure $= F/A = FL^{-2}$

33. Which of the following are dimensionally correct?

(a) $h = \dfrac{2T\cos\theta}{\rho rg}$ (b) $v = \sqrt{\dfrac{p}{\rho}}$

(c) $\dfrac{dV}{dt} = \dfrac{\pi pr^4 t}{\delta\eta l}$ (d) $T = \sqrt{\dfrac{mgl}{I}}$

Solution (a), (b) and (c) are dimensionally correct.

34. $\int \dfrac{dx}{\sqrt{a^2 - x^2}} = \dfrac{1}{a}\sin^{-1}\dfrac{a}{x}$

(a) is dimensionally correct
(b) dimensionally incorrect
(c) such mathematical relations cannot be tested
(d) cannot say

Solution (b)

QUESTIONS FOR PRACTICE

1. Curie is the unit of

(a) decay constant (b) activity
(c) half-life (d) average life

2. SI unit of water equivalent of calorimeter is

(a) Kg (b) Kg^2
(b) Kg^3 (d) Kg^{-1}

3. The magnetic moment of electron is

(a) 9.27×10^{-24} Joule/Tesla
(b) 9.27×10^{-24} Tesla/Joule
(c) 9.27×10^{-23} Joule/Tesla
(d) 9.27×10^{-23}Tesla/Joule

4. One electrostatic unit (esu) of charge is equivalent to

(a) 3.3×10 Coulomb
(b) 3.3×10^{-9} Coulomb
(c) 3.3×10^{-10} Coulomb
(d) 3.3×10^{-11} Coulomb

5. The value of solar constant in SI system is

(a) 1340 watt/m^2 (b) 1340 watt/m^2
(c) 1340 m^2/watt (d) 1340 watt/m

6. The correct statement about Poisson's ratio is

(a) Its unit is N/m^2 (b) It is dimensionless
(c) Its unit is Newton
(d) Its dimensions are MLT^{-2}

7. The SI unit of gravitational potential is

(a) Joule/Kg (c) Joulex/Kg
(c) Kg/Joule (d) Joulex/Kg^2

8. One watt-hour is equivalent to

(a) 3.6×10^3 Joule (b) 3.6×10^{-3} JouIe
(c) 6.3×10^3 Joule (d) 6.3×10^{-3} Joule

9. An air bubble inside water oscillates due to some explosion with period T. If $T \alpha P^a d^b E^c$ then determine the values of a, b and c. Here P, d and E are the static pressure, density and total energy of explosion of water, respectively.

(a) $a = \dfrac{5}{6}, b = 1, c = \dfrac{1}{3}$
(b) $a = -\dfrac{5}{6}, b = \dfrac{1}{2}, c = \dfrac{1}{3}$
(c) $a = 1, b = 1, c = 1$
(d) $a = 0, b = 0, c = 1$

10. The dimensions of intensity of energy are

(a) ML^2T^{-1} (b) ML^2T^{-2}
(c) ML^3 (d) $ML^{-2}T^3$

11. The MKS unit of the quantity $\omega \eta r^4/2\,l$ is

(a) Nm Radian (b) N/m/Radian
(c) Radian/N/m (d) N/m/$Radian^2$

12. Oersted in the unit of

(a) intensity of magnetization
(b) magnetic moment
(c) magnetic induction
(d) magnetic flux

13. Lux is the unit of

(a) Luminous flux (b) Luminous intensity
(c) Density of illumination (d) Luminous efficiency

14. $M^{-1}L^{-2}T^{3}\,\theta^{1}$ are the dimensions of

(a) coefficient of thermal conductivity
(b) coefficient of viscosity
(c) modules of rigidity
(d) thermal resistance

15. Ampere-hour is the unit of

(a) power (b) energy
(c) quantity of electricity (d) strength of current

16. Which of the following is not the unit of time?

(a) Leap year (b) Lunar month
(c) Solar day (d) Parallactic second

17. The dimension of the ratio of angular momentum and linear momentum is

(a) L^0 (b) L^1
(c) L^2 (d) MLT

18. The SI unit of form factor is

(a) ampere (b) volt
(c) watt (d) none of the above

19. One micron is equivalent to

(a) 10^{-4}m (b) 10^{-6}m
(c) 10^{4}m (d) 10^{6}m

20. The velocity of a body falling under gravity is directly proportional to $g^a h^b$. If g and h are the acceleration due to gravity and height covered by the body, respectively, then determine the values of a and b.

(a) $\frac{1}{2}$ and $\frac{1}{2}$ (b) $-\frac{1}{2}$ and $-\frac{1}{2}$
(c) $\frac{1}{2}$ and $-\frac{1}{2}$ (d) $-\frac{1}{2}$ and $\frac{1}{2}$

21. The velocity of a particle depends upon time according to the relation $v = \alpha t + \beta/t + \gamma$. The dimensions of α, β and γ will be

(a) LT^{-2}, L, LT^{-1} (b) L, LT^{-1}, LT^{-2}
(c) LT^{-1}, L, LT^{-2} (d) $LT^{-1}, L, T^{-1}L$

22. If in the formula $X = 3YZ^2$, the dimensions of X and Z are those of the capacity and magnetic induction, then the dimensions of Y are

(a) $M^{-3}L^{-2}T^{8}A^{4}$ (b) $M^{-3}L^{-2}T^{4}A^{4}$
(c) $M^{-3}L^{-2}T^{-4}A^{-4}$ (d) $M^{-3}L^{-2}T^{-8}A^{-4}$

23. The mass of electron in MeV is

(a) 1.02 MeV/C^2 (b) 0.51 MeV/C^2
(c) 51 MeV/C^2 (d) 102 MeV/C^2

24. The ratio of nuclear magneton and Bohr magneton is

(a) m_e/m_p (b) m_p/m_e
(c) $m_e m_p$ (d) $2m_p/m_{ez}$

25. The value of Faraday number in SI unit is

(a) 9.65 Coulomb/Kg/equivalent
(b) 9.65 × 10^7 Coulomb/kg/equivalent
(c) 9.65 × 10^{-7} Coulomb/kg/equivalent
(d) 9.65 Coulomb/kg/equivalent

26. The velocity of ripples on water surface depends upon the wavelength λ, density of water d and acceleration due to gravity g. Which of the following relations is correct among these quantities?

(a) $V^2 \alpha g\lambda$ (b) $V^2\ \alpha\ 1/g\lambda$
(c) $V^2 \alpha\ \lambda/gd$ (d) $V^2\ \alpha\ g\lambda d$

27. The fundamental unit of the quantity of matter is

(a) Kg (b) mol
(c) gm (d) meter

28. In an experiment to determine acceleration due to gravity by simple pendulum, a student commits 1 % positive error in the measurement of length and 3% negative error in the measurement of time period. The percentage error in the value of g will be

(a) 7%. (b) 10%
(c) 4% (d) 3%

29. Which of the following pairs is not matched?

(a) Coefficient of self-induction-henry
(b) Magnetic flux-weber
(c) Electric flux-voltmeter.
(d) Electric capacity-farad-meter.

30. If the units of ML are doubled, then the unit of kinetic energy will become

(a) 8 times (b) 16 times
(c) 4 times (d) 2 times

31. The SI unit of $(1/2\pi\sqrt{LC}\ )$ is equivalent to that of

(a) time period (b) frequency
(c) wave length (d) wave number

32. Light year is the unit of

(a) distance (b) time
(c) speed (d) mass

33. The ratio of MKS units and CGS units of coefficient of viscosity is

(a) $\frac{\eta MKS}{\eta CGS} = 10$ (b) $\frac{\eta MKS}{\eta CGS} = 9.8$
(c) $\frac{\eta MKS}{\eta CGS} = 0.1$ (d) $\frac{\eta MKS}{\eta CGS} = \frac{1}{9.8}$

34. The dimension of the expression $\sqrt{1}\ /MB$ is

(a) L (b) T
(c) L^{-1} (d) T^{-1}

35. The SI unit of mobility of charges (μ) is

(a) Coulomb/ (s/Kg) (b) Coulomb/ (Kg/s)
(c) Coulomb/ (Kg/s^{-1}) (d) Coulomb/ (s^{-1}/Kg)

36. The dimensions of the coefficient of viscosity are

(a) $ML^{-1}T^{-1}$ (b) MLT
(c) $M^{-1}L^{-1}T^{-1}$ (d) $M^0L^0T^0$

37. If the error in the measurement of radius of a sphere is 1 %, then the error in the measurement of volume will be

(a) 8% (b) 5%
(c) 3% (d) 1%

38. The ratio of the atomic radius to nuclear radius is

(a) 10^4 (b) 10^{-4}
(c) 10^2 (d) 10^{-2}

39. One fermi is equivalent to

(a) 10^{-14} meter (b) 10^{14} meter
(c) 10^{-15} meter (d) 10^{15} meter

40. Debye is the unit of

(a) magnetic dipole moment
(b) electric dipole moment
(c) density
(d) RMS velocity

41. The units of the temperature coefficient of resistance are

(a) $\Omega\, K^{-1}$ (b) K^{-1}
(c) $\Omega\, K$ (d) $(\Omega\, K)^{-1}$

42. How many wave lengths of Kr^{86} are contained in one meter?

(a) 1553164.13 (b) 652189.63
(c) 2347127.23 (d) 1650763.73

43. A physical quantity is represented by the relation $Y = M^aL^bT^{-c}$. If the percentage errors in the measurement of M, L and T are α%, β% and γ% respectively, then the total error will be—

(a) $(\alpha a - \beta b + \gamma c)$% (b) $(\alpha a - \beta b - \gamma c)$%
(c) $(\alpha a + \beta b + \gamma c)$% (d) $(\alpha a + \beta b - \gamma c)$%

44. A science student takes 100 observations in an experiment. Second time he takes 500 observations in the same experiment. By doing so the possible error becomes

(a) 5 times (b) 1/5 times
(c) unchanged (d) none of these

45. If the error in the measurement of radius of a sphere is 1% then the error in the measurement of volume will be

(a) 1.1% (b) 3%
(c) 5% (d) 8%

46. Light year is the unit of

(a) speed (b) mass
(c) distance (d) time

47. Debye is the unit of

(a) density (b) rms velocity
(c) electric dipole moment
(d) magnetic dipole moment

48. The unit of surface energy per unit area may be expressed as

(a) Nm^{-2} (b) Nm^{-1}
(c) Nm (d) Nm^2

Answers to Questions for Practice

1.	(b)	**2.**	(a)	**3.**	(a)	**4.**	(c)	**5.**	(a)	**6.**	(b)	**7.**	(a)
8.	(a)	**9.**	(b)	**10.**	(c)	**11.**	(a)	**12.**	(a)	**13.**	(c)	**14.**	(d)
15.	(c)	**16.**	(d)	**17.**	(b)	**18.**	(d)	**19.**	(b)	**20.**	(a)	**21.**	(a)
22.	(a)	**23.**	(b)	**24.**	(a)	**25.**	(b)	**26.**	(a)	**27.**	(b)	**28.**	(a)
29.	(d)	**30.**	(a)	**31.**	(b)	**32.**	(a)	**33.**	(a)	**34.**	(b)	**35.**	(b)
36.	(a)	**37.**	(c)	**38.**	(a)	**39.**	(c)	**40.**	(a)	**41.**	(b)	**42.**	(d)
43.	(c)	**44.**	(b)	**45.**	(b)	**46.**	(c)	**47.**	(d)	**48.**	(b)		

2

Vectors

BRIEF REVIEW

Vector Directed segments (physical quantities having magnitude and direction) and which follow triangle law of addition are called vectors. For example force, acceleration, torque, momentum, angular momentum etc.

Properties of Vectors In addition to magnitude and unit (a) it has specified direction, (b) it obeys triangle law of addition, (c) their addition is commutative i.e.

$$\vec{A}+\vec{B}=\vec{B}+\vec{A}$$

(d) Their addition is associative $\left(\vec{A}+\vec{B}\right)+\vec{C} = \vec{A}+\left(\vec{B}+\vec{C}\right)$

Representation of Vectors Vectors may be represented in two forms: polar and cartesian.

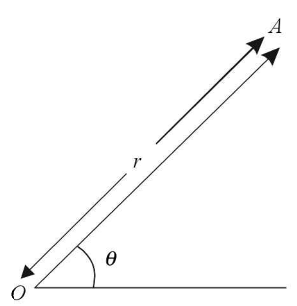


Fig. 2.1 Polar representation of a vector

Polar Form In this form $\overrightarrow{OA}=(r, \theta)$ where r is magnitude and θ is angle as shown in Fig 2.1

Cartesian Form In this form $\vec{A}=a_1\hat{i}+a_2\hat{j}+a_3\hat{k}$ where a_1, a_2 and a_3 are coefficients and $\hat{i}$, $\hat{j}$, $\hat{k}$ are unit Vectors along x, y and z directions, respectively, as illustrated in Fig 2.2

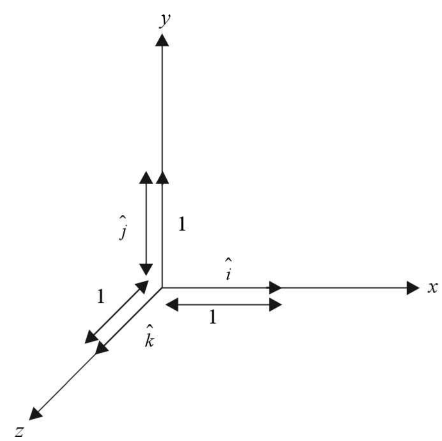


Fig. 2.2 Unit vector representation in rectangular coordinate system

Types of Vector In general vectors may be divided into three types:

1. Proper Vectors
2. Axial Vectors
3. Inertial or Pseudo Vectors.

Proper Vectors Displacement, force, momentum etc. are Proper Vectors.

Axial Vectors The vectors which act along axis of rotation are called axial vectors. For example, angular velocity, torque, angular momentum, angular acceleration are axial vectors.

Pseudo or Inertial Vectors The vectors used to make a non inertial frame of reference into inertial frame of reference are called pseudo or inertial. Vectors may further be subdivided as

(i) *Null Vector* It has zero magnitude and indeterminate direction.

(ii) *Unit Vector* Magnitude of unit vector is 1. It specifies direction only. Unit vector of a given vector is $\hat{a} = \frac{\vec{A}}{|\vec{A}|}$

i.e. vector divided by its magnitude represents unit vector.

(iii) *Like Vector or Parallel Vectors* If two vectors have the same direction but different magnitude then they are said to be parallel or like vectors. Fig 2.3(a) shows like vectors.

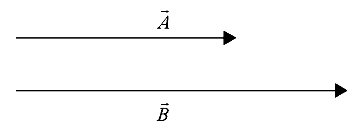


Fig. 2.3 (a) Like vectors

(iv) *Unlike Vectors* Two vectors having opposite directions and unequal magnitudes are called unlike vectors or parallel vectors in opposite sense. If their magnitudes are equal, they are called opposite vectors. Fig 2.3(b) shows unlike vectors.

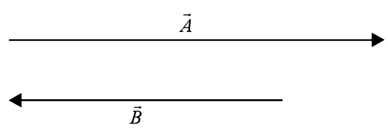


Fig. 2.3 (b) Unlike vectors

(v) *Equal Vectors* Two parallel vectors having equal magnitudes are called equal vectors.

(vi) *Co-initial Vectors* If vectors have a common initial point, they are known as co-initial vectors. In Fig. 2.4, $\overrightarrow{OA}$, $\overrightarrow{OB}$ and $\overrightarrow{OC}$ are co-initial vectors.

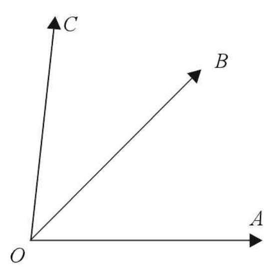


Fig. 2.4 Co-initial vectors

(vii) *Co-linear Vectors* Like, unlike, equal, opposite vectors may be grouped as co-linear vectors if they are either in the same line or parallel.

(viii) *Co-planar Vectors* Vectors lying in the same plane are termed as co-planar.

Resolution of a Vector Resolving a vector into its components is called resolution of a vector. Using triangle law one can write in Fig. 2.5 $\vec{A} = \vec{A}_x + \vec{A}_y$ or $\vec{A} = \vec{A}_x\hat{i} + \vec{A}_y\hat{j}$

or $\vec{A} = A\cos\theta\hat{i} + A\sin\theta\hat{j}$

$|\vec{A}| = \sqrt{A^2{}_x + A^2{}_y}$ and

$\tan\theta = \frac{A_y}{A_x}$ or $\theta = \tan^{-1}\left(\frac{A_y}{A_x}\right)$

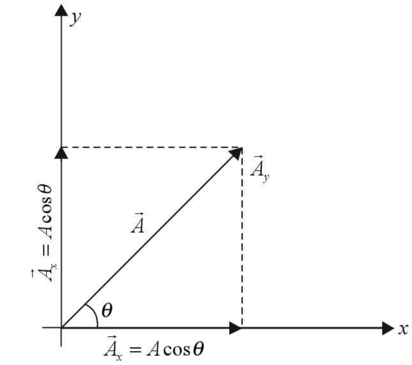


Fig. 2.5 Resolution of a vector

Laws of additions of vectors Vectors can be added using

(a) Triangle Law

(b) Parallelogram Law

(c) Polygon Law.

Triangle Law If two vectors acting on a body may be represented completely (in magnitude and direction) by two sides of a triangle taken in order, then their resultant is represented by third side of the triangle taken in opposite directions. In the fig 2.6 (a) $\overrightarrow{OP} + \overrightarrow{PQ} = \overrightarrow{OQ}$ or $\vec{A} + \vec{B} = \vec{R}$

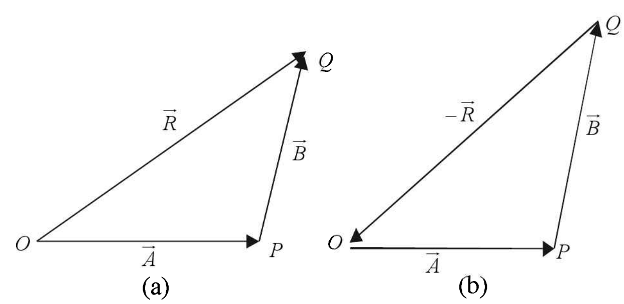


Fig. 2.6 Triangle law illustration

If three vectors acting on a body may completely be repesented by three sides of a triangle taken in order then the system is in equilibrium. In Fig. 2.6 (b)

$\overrightarrow{OP} + \overrightarrow{PQ} + \overrightarrow{QR} = \vec{A} + \vec{B} + (-\vec{R})$ or $\vec{R} - \vec{R} = 0$

Parallelogram Law If two vectors acting on a body may be represented completely by two adjacent sides of a parallelogram, then their resultant is represented by a diagonal passing through the common point. In Fig 2.7 from equal vector $\overrightarrow{PL} = \overrightarrow{OQ} = \vec{B}$ from Δ law $\vec{A} + \vec{B} = \vec{R}$

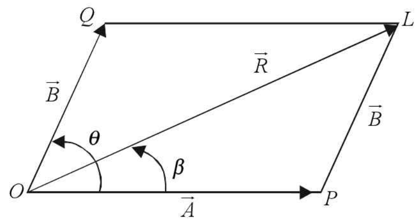


Fig. 2.7 Parallelogram law of vector illustration

$$|\vec{R}| = \sqrt{A^2 + B^2 + 2AB\sin\theta}$$

$$\tan\beta = \frac{B\sin\theta}{A + B\cos\theta}$$

Note: $A - B \le R \le A + B$ i.e. $R_{min} = A - B$ when $\theta = 180°$ and $R_{max} = A + B$ when $\theta = 0°$ $0 \le \theta \le 180°$. Remember θ cannot exceed 180°.

Note: Minimum number of coplanar vectors whose sum can be zero (or required for equilibrium)

= 2 (if vectors are equal and opposite)

= 3 if vectors are unequal or not opposite, minimum number of noncoplanar vectors whose sum can be zero = 4

Note: Subtraction of a vector is equivalent to addition of a negative vector.

Multiplication of Vectors Two types of multiplication is defined (a) dot product or scalar product, (b) Cross product or vector product.

Dot product or Scalar Product If the product of two vectors is a scalar, then this rule is applied $\vec{A}.\vec{B} = AB\cos\theta$

Note: $\vec{A}.\vec{B} = \vec{B}.\vec{A}$,i.e., scalar product is commutative $\vec{A}.(\vec{B} + \vec{C}) = \vec{A}.\vec{B} + \vec{A}.\vec{C}$, i.e., scalar product follows distributive law.

Rules: (i) $\hat{i}.\hat{i} = \hat{j}.\hat{j} = \hat{k}.\hat{k} = 1$

(ii) $\hat{i}.\hat{j} = \hat{j}.\hat{i} = \hat{j}.\hat{k} = \hat{k}.\hat{j} = \hat{i}.\hat{k} = \hat{k}.\hat{i} = 0$

Application of Dot Product

1. When the product of two vectors is a scalar. For example $W = \vec{F}.\vec{s}$, $P = \vec{F}.\vec{v}$ current $I = \int \vec{j}.\overrightarrow{ds}$,

   magnetic flux $\phi = \int B.ds$ etc.

2. To find an angle between two vectors $\theta = \cos^{-1}\left(\frac{\vec{A}.\vec{B}}{|A||B|}\right)$

3. If the dot product of two non zero vectors is zero, then-they are perpendicular to one another.

4. Find the component of a vector along a given direction. For instance, the component of $\vec{A}$ along $\vec{B}$ is $A\cos\theta = \frac{\vec{A}.\vec{B}}{B}$

Cross product or Vector product This product is used when the product of two vectors is a vector, i.e.,

$\vec{A} \times \vec{B} = AB\sin\theta\ \hat{n}$ where $\hat{n}$ is a unit vector perpendicular to both A and B. Apply right-handed screw rule to find the direction of $\hat{n}$ or $\vec{A} \times \vec{B}$. Vector product is noncommutative i.e. $\vec{A} \times \vec{B} \neq \vec{B} \times \vec{A}$ (magnitude will be equal but direction will be opposite). Vector product is distributive, i.e., $\vec{A} \times (\vec{B} + \vec{C}) = \vec{A} \times \vec{B} + \vec{A} \times \vec{C}$

Rules: (i) $\vec{A} \times \vec{A} = 0 = \hat{i} \times \hat{i} = \hat{j} \times \hat{j} = \hat{k} \times \hat{k}$

(ii) $\hat{i} \times \hat{j} = \hat{k} = -\hat{j} \times \hat{i}$, $\hat{j} \times \hat{k} = \hat{i} = -\hat{k} \times \hat{j}$,

$\hat{k} \times \hat{i} = \hat{j} = -\hat{i} \times \hat{k}$

Application of Vector Product

1. Cross product is used in rotational motion or product of two vectors is a vector. For example, Torque $\vec{\tau} = \vec{r} \times \vec{F}$, Poynting vector $\vec{P} = \vec{E} \times \vec{H} = \frac{1}{\mu_0}(\vec{E} \times \vec{B})$,

   Angular momentum $\vec{L} = \vec{r} \times \vec{p}$, $\vec{v} = \vec{\omega} \times \vec{r}$,

   $F = q(\vec{v} \times \vec{B})$

2. If the vector product of two nonzero vectors is zero, then they are parallel.

3. It can be used to find angle θ

$$\theta = \sin^{-1}\left[\frac{|\vec{A} \times \vec{B}|}{|\vec{A}||\vec{B}|}\right]$$

4. $|\vec{A} \times \vec{B}|$ represents area of a parallelogram whose sides are A and B. $\frac{1}{2}|\vec{D}_1 \times \vec{D}_2|$ represents area of a parallelogram where D_1 and D_2 are diagonals of the parallelogram.

   Since $\overrightarrow{D_1} = \vec{A} + \vec{B}$ and $\overrightarrow{D_2} = \vec{A} - \vec{B}$

$\therefore$ Area of a ||gm = $\frac{1}{2}\left|(\vec{A} + \vec{B}) \times (\vec{A} - \vec{B})\right|$

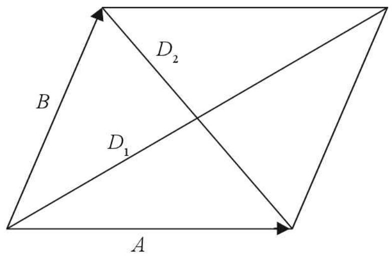


Fig. 2.8

5. $\frac{1}{2}|\vec{A}\times\vec{B}|$ represents area of a triangle when A and B are two sides of the triangle.

Relative Velocity Since absolute rest or absolute motion do not exist, therefore, every motion is a relative motion. Though, for convenience, we assume earth at rest and in common language measure the speed or velocity with respect to ground. But if two bodies A and B are moving with velocities V_A and V_B then relative velocity of A with respect B may be thought of velocity of A by bringing B to rest by applying equal and opposite velocity of B. Alternatively, Vector law may be applied from Fig. 2.9

$$\vec{V}_{AG} = \vec{V}_{AB} + \vec{V}_{BG} \quad \text{or}$$

$$\vec{V}_{AB} = \vec{V}_{AG} - \vec{V}_{BG} \quad \text{or}$$

$$\vec{V}_{AB} = \vec{V}_A - \vec{V}_B$$

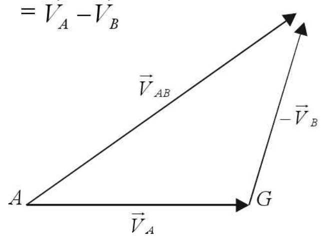


Fig. 2.9 Relative velocity illustration

The best way to solve the questions on relative velocity is to resolve it into x and y components, thus

$$\vec{V}_{AB} = (\vec{V}_{Ax}\hat{i} + \vec{V}_{Ay}\hat{j}) - (\vec{V}_{Bx}\hat{i} + \vec{V}_{By}\hat{j})$$

$$= \left(\vec{V}_{Ax} - \vec{V}_{Bx}\right)\hat{i} + \left(\vec{V}_{Ay} - \vec{V}_{By}\right)\hat{j}$$

Then $|V_{AB}| = \sqrt{(V_{Ax} - V_{Bx})^2 + (V_{Ay} - V_{By})^2}$ and

$\tan\beta = \dfrac{V_{Ay} - V_{By}}{V_{Ax} - V_{Bx}}$ with respect to x-axis or $\hat{i}$ direction

$\tan\beta' = \dfrac{V_{Ax} - V_{Bx}}{V_{Ay} - V_{By}}$ with respect to y-axis or $\hat{j}$ direction

- **Short Cuts and Points to Note**

1. Laws of Addition are Triangle law, parallelogram law, Polygon law.

   Resultant $|R| = \sqrt{A^2 + B^2 + 2AB\cos\theta}$

   $\tan\beta = \dfrac{A\sin\theta}{B + A\cos\theta}$ (see fig 2.10 carefully).

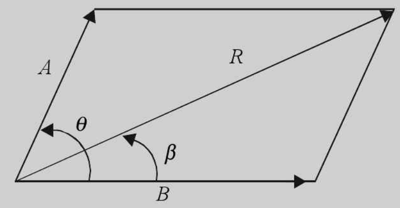


Fig. 2.10 Parallelogram law

2. Vector subtraction is identical to vector addition, i.e., $\vec{A} - \vec{B} = \vec{A} + (-\vec{B})$ change θ to $180 - \theta$. in equation of resultant.

3. The vectors are represented in $\hat{i}$, $\hat{j}$ form then Resultant $\vec{R}$ is given by

$$\vec{R} = \left(A_x\hat{i} + A_y\hat{j}\right) + \left(B_x\hat{i} + B_y\hat{j}\right)$$

$$= \left(A_x + B_x\right)\hat{i} + \left(A_y + B_y\right)\hat{j}$$

$$|\vec{R}| = \sqrt{\left(A_x + B_x\right)^2 + \left(A_y + B_y\right)^2} \quad \text{and}$$

$\tan\beta = \dfrac{A_y + B_y}{A_x + B_x}$ w.r.t x or $\hat{i}$ direction and

$\tan\beta' = \dfrac{A_x + B_x}{A_y + B_y}$ w.r.t y or $\hat{j}$ direction.

4. $R_{max} = A + B$ when $\theta = 0$, $R_{min} = A - B$ when $\theta = 180°$.

   Remember $0 \le \theta \le 180°$

5. If the system is in equilibrium, Lami's theorem may be applied. In Fig 2.11 $\dfrac{F_1}{\sin\alpha} = \dfrac{F_2}{\sin\beta} = \dfrac{F_3}{\sin\gamma}$

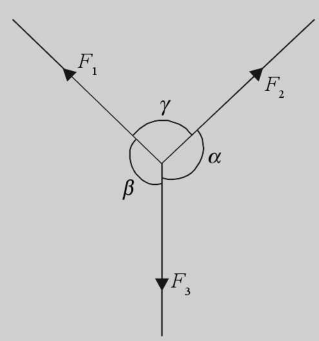


Fig. 2.11 Lami's Theorem

6. Equilibrium may be static or dynamic. However, in both cases $\sum \vec{F} = 0$ Linear equilibrium if $\sum \vec{F} = 0$. Rotational Equilibrium $\sum F = 0, \sum \tau = 0$ Equilibrium is stable if $\sum F = 0, \sum \tau = 0$ and PE is minimum. Unstable equilibrium means $\sum F = 0$, $\sum \tau = 0$ and PE is maximum. Neutral equilibrium means $\sum F = 0, \sum \tau = 0$ and PE is constant but not zero.

7. The best approach is to resolve the vectors into x and y components and then solve.

Use $\vec{v}_R = \vec{v}_A + \vec{v}_B$ if resultant or net velocity is to be found.

Use $\vec{v}_{AB} = \vec{v}_A - \vec{v}_B$ if relative velocity is to be found.

8. Magnitude of a vectors $V = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$

$|V| = \sqrt{a_x^2 + a_y^2 + a_z^2}$

9. If two non zero vectors are perpendicular, their dot product is zero.

10. If two vectors $\vec{A}$ and $\vec{B}$ are parallel, then $\vec{A} = k\vec{B}$ where k is a positive or negative real number. Moreover $\vec{A} \times \vec{B} = 0$

11. To find $\vec{A}$ and $\vec{B}$ use determinant method i.e.

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix}$$

$$\vec{A} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}, \ \vec{B} = b_x\hat{i} + b_y\hat{j} + b_z\hat{k}$$

$$\vec{A} \times \vec{B} = (a_y b_z - b_y a_z)\hat{i} - \hat{j}(a_x b_z - b_x a_z) + \hat{k}(a_x b_y - b_x a_y)$$

12. $\vec{A}.(\vec{A} \times \vec{B}) = \vec{B}.(\vec{A} \times \vec{B}) = 0$

13. $\vec{A} \times (\vec{B} \times \vec{C}) = (\vec{A}.\vec{C})\vec{B} - (\vec{A}.\vec{B})\vec{C}$

14. Vector division is not allowed.

15. Vector operator ∇ (nabla) is used to define

$$\nabla = \left[\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right]$$

(a) ∇V represents electric field i.e.

$$E = \nabla V = \left[\hat{i}\frac{\partial V}{\partial x} + \hat{j}\frac{\partial V}{\partial y} + \hat{k}\frac{\partial V}{\partial z}\right]$$

(b) $\nabla . E$ will give divergence of E

(c) $\nabla \times E$ will represent curl of E

16. Unit vector of a vector is $\hat{a} = \frac{\vec{A}}{|\vec{A}|}$

17. To cross a river along the shortest path, the swimmer/boat shall head at an obtuse angle to the flow of river provided $V_{swimmer} > V_{river}$.

18. To cross the river in the shortest time, the swimmer/boat shall head at right angle to the flow of river provided $V_{swimmer} > V_{river}$.

19. If $V_{river} > V_{swimmer}$ them to reach opposite end in minimum time the swimmer shall swim at an obtuse angle.

- **Caution**

1. Considering null vector has a specified direction.

⇒ Null vector has no specified direction.

2. Trying some other tools to prove two vectors are perpendicular.

⇒ For vectors most convenient method to prove two vectors perpendicular is $\vec{A}.\vec{B} = 0$

3. Not recognizing when relative velocity is to be found and when resultant velocity is to be determined.

⇒ If the word *appear* or *with respect to* has been used in problem, then find relative velocity.

If the word *actual* or *net* or *real* or *resultant* is used in problem, then find resultant velocity.

4. Not remembering vector laws of addition or not understanding its full meaning.

⇒ Parallelogram law or triangle law of addition leads to

$$|\vec{A} + \vec{B}| = |\vec{R}| = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

$$\tan\beta = \frac{A\sin\theta}{B + A\cos\theta}$$

$R_{max} = A + B$ if $\theta = 0$

$R_{min} = A - B$ if $\theta = 180°$

$$|\vec{A} - \vec{B}| = \sqrt{A^2 + B^2 - 2AB\cos\theta}$$

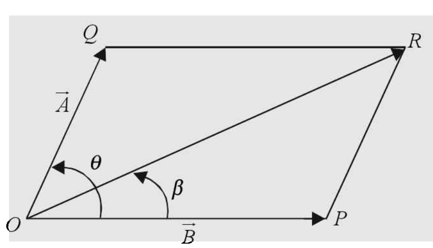


Fig. 2.12

5. Assuming $|\vec{A} + \vec{B}| = |\vec{A}| + |\vec{B}|$ or $|\vec{A} - \vec{B}| = |\vec{A}| - |\vec{B}|$

⇒ This will be true only if vectors are like vectors/parallel vectors. Otherwise apply triangle law.

6. Not resolving vectors (when vectors are more than two) and solving problem by conventional method.

⇒ Though problem can be solved using triangle law or parallelogram law but they make the problem unnecessarily lengthy and time consuming.

7. Not able to recognise direction in $\hat{i}$, $\hat{j}$ form.

⇒ If $\hat{j}$ is not vertical but on the earth's plane then right hand side is East (represented by $\hat{i}$); Left hand side is West (marked $-\hat{i}$); front of you is North (marked $\hat{j}$); your back represents South ($-\hat{j}$) as illustrated in Fig 2.13

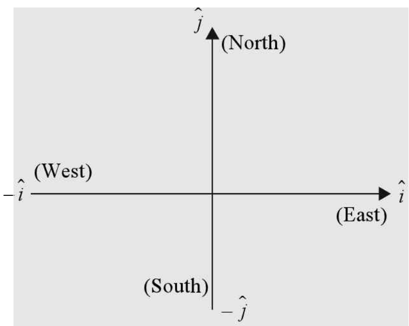


Fig. 2.13

$\hat{k}$ represents vertically up and ($-\hat{k}$) vertically down. Consider a specific problem:

$V_R = 3\hat{i} - 4\hat{j}$ lies in the 4th quadrant.

then $\tan\beta = \frac{4}{3}$ or 53° South of East.

It may also be written $\tan\beta' = \frac{3}{4}$ or 37° East of South.

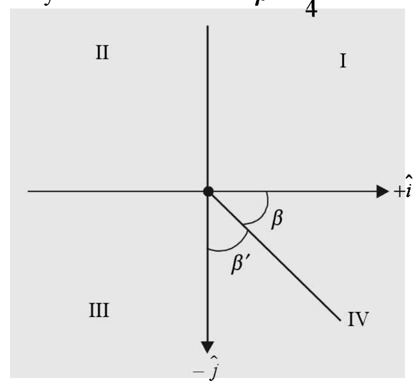


Fig. 2.14

8. Not recognizing the axes (in specific cases) about which the vectors be resolved.

⇒ If vectors are resolved about some specific axes in typical cases, problem becomes very simple. Recognition of such questions and axes is necessary.

9. Finding angle between vectors to prove vectors are parallel.

⇒ If $\vec{A} = k\vec{B}$ where k is a positive or negative number (integer or fraction) then $\vec{A}$ and $\vec{B}$ are parallel.

10. Assuming when a particle takes a u turn or 90° turn and continues to move with the same speed then ΔV (change in velocity) or average acceleration are zero.

⇒ Use vectors, i.e., write $\hat{i}$, $\hat{j}$, $\hat{k}$ etc. with initial and final velocities to find change in velocity or average acceleration.

SOLVED PROBLEMS

1. In CH_4 molecule, there are four $C - H$ bonds. If two adjacent bonds are in $\hat{i} + \hat{j} + \hat{k}$ and $\hat{i} - \hat{j} - \hat{k}$ direction, then find the angle between these bonds.

(a) $\sin^{-1}\left(\frac{-1}{3}\right)$ (b) $\cos^{-1}\left(\frac{1}{3}\right)$

(c) $\sin^{-1}\left(\frac{1}{3}\right)$ (d) $\cos^{-1}\left(\frac{-1}{3}\right)$

Solution (d) $\cos\theta = \frac{(\hat{i}+\hat{j}+\hat{k}).(\hat{i}-\hat{j}-\hat{k})}{\sqrt{3}.\sqrt{3}} = \frac{-1}{3}$ or

$\theta = \cos^{-1}\left(\frac{-1}{3}\right)$

2. Two vectors $\vec{A}$ and $\vec{B}$ have magnitude 3 each. $\vec{A} \times \vec{B} = -5\hat{k} + 2\hat{i}$. Find angle between A and B

(a) $\cos^{-1}\frac{\sqrt{29}}{9}$ (b) $\tan^{-1}\left(\frac{-5}{2}\right)$

(c) $\sin^{-1}\left(\frac{2}{5}\right)$ (d) $\sin^{-1}\left(\frac{\sqrt{29}}{9}\right)$

Solution (d) $\sin\theta = \frac{|\vec{A}\times\vec{B}|}{|\vec{A}||\vec{B}|} = \frac{\sqrt{5^2+2^2}}{9}$

$= \frac{\sqrt{29}}{9}$ or $\sin^{-1}\left(\frac{\sqrt{29}}{9}\right)$

3. A particle moving eastwards with 5ms^{-1}. In 10 s the velocity changes to 5ms^{-1} northwards. The average acceleration in this time is

[AIEEE 2005]

(a) $\frac{1}{\sqrt{2}}$ ms^{-2} towards Northeast

(b) $\frac{1}{2}$ ms^{-2} towards North

(c) $\frac{1}{\sqrt{2}}$ ms^{-2} towards Northwest

(d) Zero

Solution (c) $a_{av} = \frac{V_f - V_i}{t} = \frac{5\hat{j} - 5\hat{i}}{10}$ or $|a_{av}| = \frac{1}{\sqrt{2}}$ North west.

4. If a vector $2\hat{i} + 3\hat{j} + 8\hat{k}$ is perpendicular to the vector $4\hat{j} - 4\hat{i} + \alpha\hat{k}$ then the value of α is

[CBSE PMT 2005]

(a) $\frac{1}{2}$ (b) $\frac{-1}{2}$

(c) 1 (d) –1

Solution (b) $\left(2\hat{i}+3\hat{j}+8\hat{k}\right).\left(4\hat{j}-4\hat{i}+\alpha\hat{k}\right)=0$ or

$-8+12+8\alpha=0 \quad \alpha=\frac{-1}{2}$

5. If the angle between the vectors $\vec{A}$ and $\vec{B}$ is θ, the value of the product $\left(\vec{B}\times\vec{A}\right).\vec{A}$ equals

[CBSE PMT 2005]

(a) $BA^2 \sin\theta$ (b) $BA^2 \cos\theta \sin\theta$

(c) $BA^2 \cos\theta$ (d) zero

Solution (d)

6. A river is flowing from W to E with a speed 5m/min. A man can swim in still waters at a velocity 10m/min. In which direction should a man swim to take the shortest path to reach the south bank?

[BHU 2005]

(a) 30° East of South (b) 60° East of North

(c) South (d) 30° West of North

Solution (d) $v_s \sin\theta = v_{river}$ or

$\sin\theta = \frac{1}{2} \quad \theta = 30°$

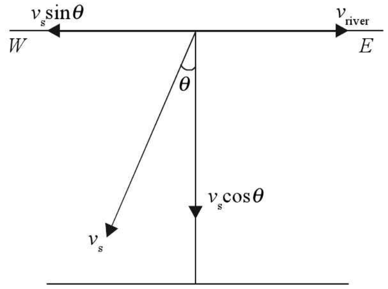


Fig. 2.15

7. Electrons in a TV tube move horizontally South to North. Vertical component of earth's magnetic field points down. The electron is deflected towards

(a) West (b) no deflection

(c) East (d) North to South

Solution (c) $F == q\left(\vec{v}\times\vec{B}\right) = -e\left(\hat{j}\times-\hat{k}\right) = \hat{i}\ e$

8. If $\vec{A}.\vec{B} = \vec{B}.\vec{C}$ then

(a) $\vec{A} = \vec{C}$ always (b) $\vec{A} \neq \vec{C}$ always

(c) $\vec{A}$ may not be equal to $\vec{C}$

(d) none of these

Solution (c)

9. $A = 3\hat{i} + 4\hat{j}$ Find a vector perpendicular to $\vec{A}$ in the plane of $\vec{A}$

(a) $\hat{k}$ (b) $-3\hat{i} - 4\hat{j}$

(c) $4\hat{j} - 3\hat{i}$ (d) $4\hat{i} - 3\hat{j}$

Solution (d), (a) is also perpendicualr to $\vec{A}$ but not in the same plane. Check using $\vec{A}.\vec{B} = 0$

10. Find a vector $\vec{x}$ which is perpendicular to both $\vec{A}$ and $\vec{B}$ but has magnitude equal to that of $\vec{B}$.

Rule: Inter change coeff. of $\hat{i}$ and $\hat{j}$ and change sign of one of the vectors.

$\vec{A} = 3\hat{i} - 2\hat{j} + \hat{k}$, $\vec{B} = 4\hat{i} + 3\hat{j} - 2\hat{k}$

(a) $\frac{1}{\sqrt{10}}\left(\hat{i}+10\hat{j}+17\hat{k}\right)$ (b) $\frac{1}{\sqrt{10}}\left(\hat{i}-10\hat{j}+17\hat{k}\right)$

(c) $\sqrt{\frac{29}{390}}\left(\hat{i}-10\hat{j}+17\hat{k}\right)$ (d) $\sqrt{\frac{29}{390}}\left(\hat{i}+10\hat{j}+17\hat{k}\right)$

Solution (d) $\vec{A}\times\vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -2 & 1 \\ 4 & 3 & -2 \end{vmatrix}$

$$\hat{n} = \frac{\vec{A}\times\vec{B}}{\left|\vec{A}\times\vec{B}\right|} = \frac{\hat{i}+10\hat{j}+17\hat{k}}{\sqrt{390}}$$

$$\vec{x} = \left|\vec{B}\right|\hat{n} = \frac{\sqrt{29}\left(\hat{i}+10\hat{j}+17\hat{k}\right)}{\sqrt{390}}$$

11. Rain is falling vertically with 3ms^{-1} and a man is moving due North with 4ms^{-1}. In which direction he should hold the umbrella to protect himself from rains?

(a) 37° North of vertical (b) 37° South of vertical

(c) 53° North of vertical (d) 53° South of vertical

Solution (c) $V_{rm} = V_r - V_m = -3\hat{k} - 4\hat{j}$;

$\tan\beta = \frac{4}{3} \Rightarrow \beta = 53°$ North of vertical

12. A man is moving on his bike with 54 kmh^{-1}. He takes a u-turn in 10 s and continues to move with the some velocity. Find average acceleration during this time.

(a) $3.0\,ms^{-2}$ (b) $1.5\,ms^{-2}$

(c) 0 (d) $-1.5\,ms^{-2}$

Solution (a) $a_{av} = \dfrac{-15\hat{i} - (15\hat{i})}{10} = -3\hat{i} = 3ms^{-2}$

13. A man starts from O moves 500 m turns by 60° and moves 500 m again turns by 60° and moves 500 m and so on. Find the displacement after (i) 5th turn, (ii) 3rd turn

(a) 500 m, 1000 m (b) 500 m, $500\sqrt{3}$ m

(c) 1000 m, $500\sqrt{3}$ m (d) none of these

Solution (a) A regular hexagon will be formed if we continue.

After 5th turn displacement = OE = 500m

After 3rd turn displacement = OC = 1000m (OC is diameter of the circle circumscribing regular hexagon).

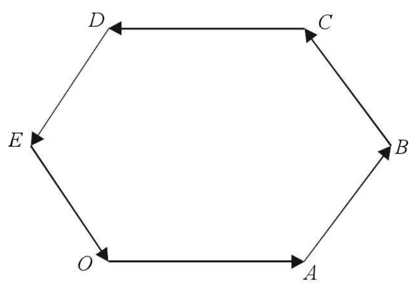


Fig. 2.16

14. If $\vec{B} = \lambda \vec{A}$ then $\dfrac{\vec{B}}{\vec{A}}$ = ...

(a) λ (b) $\dfrac{1}{\lambda}$

(c) $\dfrac{\lambda}{2}$ (d) Inderminate

Solution (d) $\because$ vector division is not allowed.

15. The acceleration of a particle as seen from two frames S_1 and S_2 has equal magnitude 5 ms^{-2}.

(a) The frames must be at rest with respect to each other.

(b) The frames may be moving with respect to each other but neither should be accelerated with respect to the other.

(c) The acceleration of frame S_2 with respect to S_1 be 0 or 10 ms^{-2}.

(d) The acceleration of S_2 with respect to S_1 lies between 0 and 10 ms^{-2}.

Solution (d) use Parallelogram law.

16. A man running on a horizontal road at 8 ms^{-1} finds rain falling vertically. If he increases his speed to 12 ms^{-1}, he finds that drops make 30° angle with the vertical. Find velocity of rain with respect to the road.

(a) $4\sqrt{7}$ ms^{-1} (b) $8\sqrt{2}$ ms^{-1}

(c) $7\sqrt{3}$ ms^{-1} (d) $8ms^{-1}$

Solution (a) $V_{rm} = (V_{rx} - V_m)\hat{i} + V_{ry}\hat{j}$

case (i) $\tan 90 = \dfrac{V_{ry}}{V_{rx} - V_m} = \dfrac{V_{ry}}{V_{rx} - 8}$ or $V_{rx} = 8ms^{-1}$

case (ii) $\tan 30 = \dfrac{V_{rx} - V_m}{V_{ry}} = \dfrac{8-12}{V_{ry}}$ or $V_{ry} = -4\sqrt{3}\ ms^{-1}$

$V_r = 8\hat{i} - 4\sqrt{3}\,\hat{j} = 4\sqrt{2^2+3} = 4\sqrt{7}\ ms^{-1}$

$\tan\theta = \dfrac{V_{ry}}{V_{rx}} = \dfrac{4\sqrt{3}}{8} = \dfrac{\sqrt{3}}{2}$ or

$\theta = \tan^{-1}\dfrac{\sqrt{3}}{2}$ with respect to road (horizontally).

17. In Fig. 2.17 (a) shown find the velocity of block m if both the rope ends are pulled with a velocity v.

(a) $2v\cos\theta$ (b) $\dfrac{v}{\cos\theta}$

(c) $\dfrac{v}{2\cos\theta}$ (d) $\dfrac{2v}{\cos\theta}$

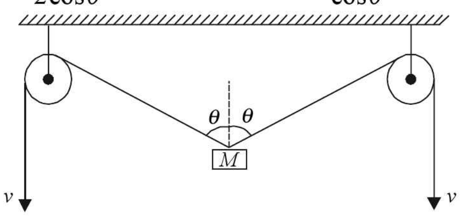


Fig. 2.17 (a)

Solution (b) $l^2 = x^2 + y^2$ or $2l\dfrac{dl}{dt} = 0 + 2y\dfrac{dy}{dt}$ ($\because x$ is constant its derivative is zero)

$\dfrac{dy}{dt} = \dfrac{dl/dt}{y/l} = \dfrac{v}{\cos\theta}$

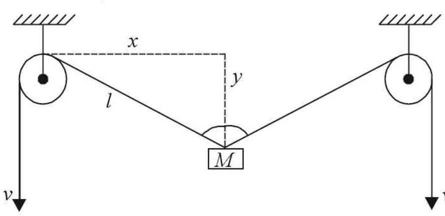


Fig. 2.17 (b)

18. Which of the following cannot be in equilibrium?

(a) 10N, 10N, 5N (b) 5N, 7N, 9N

(c) 8N, 4N, 13N (d) 9N, 6N, 5N

Solution (c) as $13N > 8 + 4$ [$\because R_{max} = A + B$]

19. $\vec{A} = 3\hat{i} + 4\hat{j} + 2\hat{k}$, $\vec{B} = 6\hat{i} - \hat{j} + 3\hat{k}$. Find a vector parallel to $\vec{A}$ whose magnitude is equal to that of $\vec{B}$.

(a) $\sqrt{\dfrac{46}{29}}(3\hat{i} + 4\hat{j} + 2\hat{k})$ (b) $\sqrt{\dfrac{46}{29}}(6\hat{i} - \hat{j} + 3\hat{k})$

(c) $\sqrt{\dfrac{29}{46}}(3\hat{i} + 4\hat{j} + 2\hat{k})$ (d) none

Solution (a)

$\vec{x} = \hat{A}\,|B| = \dfrac{(3\hat{i} + 4\hat{j} + 2\hat{k})\sqrt{36+1+9}}{\sqrt{9+16+4}}$

$$=\sqrt{\frac{46}{29}}\left(3\hat{i}+4\hat{j}+2\hat{k}\right)$$

20. $\vec{a}, \vec{b}, \vec{c}$ are three coplanar vectors. Find the vector sum.
$\vec{a}=4\hat{i}-\hat{j}$, $\vec{b}=-3\hat{i}+2\hat{j}$, $\vec{c}=-3\hat{j}$

(a) $\sqrt{5}, 297°$ (b) $\sqrt{5}, 63°$
(c) $\sqrt{3}, 297°$ (d) $\sqrt{3}, 63°$

Solution (a) $\vec{R}=\vec{a}+\vec{b}+\vec{c}=\hat{i}-2\hat{j}$

$|R| = \sqrt{5}$ and $\tan\theta=-2$ or $\theta=297°$

21. A block of mass m is connected to three springs, each of spring constant k as shown in Fig. 2.18. The block is pulled by x in the direction of C. Find resultant spring constant.

(a) k (b) $2k$
(c) $3k$ (d) $\frac{3k}{2}$

Solution (c) $F_{net}=-(kx+kx\cos 60+kx\cos 60)$

$=-2kx \therefore k_{eq}=2k$

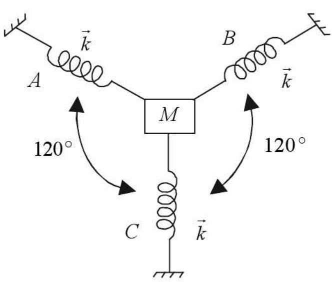


Fig. 2.18

22. A particle moves in the $x-y$ plane under the action of a force $\vec{F}$ such that the value of its linear momentum p at any instant is $p = 2(\cos t\,\hat{i} + \sin t\,\hat{j})$. The angle θ between F and p is

(a) 60° (b) 45°
(c) 30° (d) 90°

Solution (d) $F=\frac{dp}{dt}=2\left(-\sin t\hat{i}+\cos t\hat{j}\right)$

$\vec{F}.\vec{p} = 4\left(-\sin t\hat{i}+\cos t\hat{j}\right).\left(\cos t\hat{i}+\sin t\hat{j}\right)=0 \therefore$

$\theta = 90°$

23. Consider a collection of large number of particles, each moving with a speed v. The direction of velocity is randomly distributed in the collection. The magnitude of the relative velocity between a pair of particles averaged over all the pairs in the collection

(a) v (b) $\frac{2v}{\pi}$
(c) $\frac{\pi v}{4}$ (d) $\frac{4v}{\pi}$

Solution (d) $\vec{V}_{rel}=\vec{V}_A-\vec{V}_B$ or

$|V_{AB}| = \sqrt{v^2+v^2-2v^2\cos\theta}$
$= 2v\sin\frac{\theta}{2}$

$$(v_{rel})\text{average} = \frac{\int_0^{2\pi} v_{rel}\,d\theta}{\int_0^{2\pi} d\theta} = \frac{1}{2\pi}\int_0^{2\pi} 2v\sin\frac{\theta}{2}\,d\theta$$

$$= \frac{2v}{\pi}\left|-\cos\left(\frac{\theta}{2}\right)\right|_0^{2\pi} = \frac{4v}{\pi}$$

24. A steamer is moving due east with 36 km/h. To a man in the steamer the wind appears to blow at 18 km/h due north. Find the velocity of the wind.

(a) $5\sqrt{5}$ ms^{-1} $\tan^{-1}\frac{1}{2}$ North of east
(b) 5ms^{-1} $\tan^{-1}$ 2 North of east
(c) $5\sqrt{5}$ ms^{-1} $\tan^{-1}$ 2 North of east
(d) 5ms^{-1} $\tan^{-1}\frac{1}{2}$ North of east

Solution (a) $V_{ws}=V_w-V_s \Rightarrow \left(V_{wx}\hat{i}+V_{wy}\hat{j}\right)-10\hat{i}=5\hat{j}$ or

or $\left(V_{wx}\hat{i}+V_{wy}\hat{j}\right)=5\hat{j}+10\hat{i}$

$|V_w| = 5\sqrt{5}$ and $\tan\theta=\frac{1}{2}$ or $\theta=\tan^{-1}\frac{1}{2}$

i.e. wind is blowing at $5\sqrt{5}$ ms^{-1} $\tan^{-1}\frac{1}{2}$ North of east.

25. The position vector of a particle is $\vec{r} = a[\cos\omega t\,\hat{i} + \sin\omega t\,\hat{j}]$. The velocity of the particle is

(a) parallel to position vector
(b) directed towards origin
(c) directed away from origin
(d) perpendicular to position vector.

Solution (d) $\vec{v} = \frac{d\vec{r}}{dt} = a\omega[-\sin\omega t\,\hat{i}+\cos\omega t\,\hat{j}]$ and

$\vec{v}.\vec{r} = 0$

26. A force $6\hat{i}+3\hat{j}+\hat{k}$ Newton displaces a particles from A(0,3,2) to B(5,1,6). Find the work done.

(a) 10 J (b) 22 J
(c) 32 J (d) 41J

Solution (c) $\vec{d} = 5\hat{i}-2\hat{j}+4\hat{k}$

$W = \vec{F}.\vec{d} = \left(6\hat{i}+3\hat{j}+\hat{k}\right).\left(5\hat{i}-2\hat{j}+4\hat{k}\right)=32$ J.

27. Wind is blowing NE with $18\sqrt{2}$ km h^{-1} and steamer is heading due west with 18 km h^{-1}. In which direction is the flag on the mast fluttering?

(a) North West (b) North
(c) South West (d) South.

Solution (d) $\vec{V}_{Res}=\vec{V}_{steamer}+\vec{V}_{wind}=-5\hat{i}+5\hat{i}+5\hat{j}$

$=5\hat{j}$. The flag will flutter in a direction opposite to the direction of motion.

28. The resultant of two forces equal in magnitude is equal to either of two vectors in magnitude. Find the angle between the forces.

(a) 60° (b) 45°
(c) 90° (d) 120°

Solution (d) $F = \sqrt{F^2 + F^2 + 2F^2\cos\theta}$ or

$\cos\theta = \frac{-1}{2}$; $\theta = 120°$

29. A man goes 100 m North then 100 m East and then 20 m North and then $100\sqrt{2}$ m South West. Find the displacement.

(a) 20 m West (b) 20 m East
(c) 20 m North (d) 20 m South

Solution (c) $d = 100\,\hat{j} + 100\,\hat{i} + 20\,\hat{j} + (-100\,\hat{i} - 100\,\hat{j})$

$= 20\,\hat{j}$

TYPICAL PROBLEMS

30. A river flows 3 km h^{-1} and a man is capable of swimming 2 km h^{-1}. He wishes to cross it in minimum time. In which direction will he swim?

(a) $\sin^{-1}\left(\frac{2}{3}\right)$ (b) $\cos^{-1}\left(\frac{2}{3}\right)$

(c) $\tan^{-1}\left(\frac{2}{3}\right)$ (d) $\cot^{-1}\left(\frac{2}{3}\right)$

Solution (a) Let us assume he swims at an angle θ with the perpendicular as shown. If river is *lm* wide time taken to cross it

$t = \frac{l}{2\cos\theta}$, $v_x = 3 - 2\sin\theta$ horizontal distance covered along x direction during this period

$x = vt = (3 - 2\sin\theta)\frac{l}{2\cos\theta}$

for t to be min $\frac{dx}{d\theta} = 0$, or

$l\left[\frac{3}{2}\sec\theta\tan\theta - \sec^2\theta\right] = 0$ or

$\sin\theta = \frac{2}{3}$

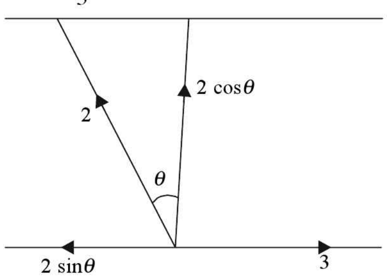


Fig. 2.19

31. A pilot is to flag an aircraft with velocity v due east. Wind is blowing due south with a velocity u. Find the time for a round journey A to B and back (A and B are l distance away).

(a) $\frac{l}{\sqrt{v^2 - u^2}}$ (b) $\frac{2l}{\sqrt{v^2 - u^2}}$

(c) $\frac{2l}{v}$ (d) $\frac{2l}{\sqrt{v^2 + u^2}}$

Solution (b) See Fig. 2.20 the velocity in the direction A to B will be $\sqrt{v^2 - u^2}$ $\therefore$ $t = \frac{l}{\sqrt{v^2 - u^2}}$ same time is needed to come back from B to A $\therefore$ $t_{net} = \frac{2l}{\sqrt{v^2 - u^2}}$

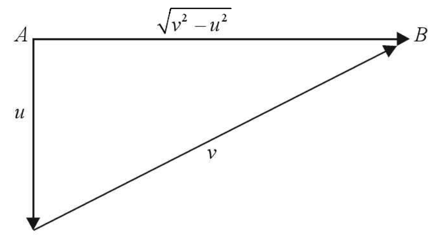


Fig. 2.20

32. When a mass m is rotated in a plane about a fixed point, its angular momentum is directed along

(a) the radius (b) tangent to the orbit
(c) the axis of rotation
(d) 45° to the axis of rotation

Solution (c) because angular momentum is an axial vector.

33. A pendulum hangs from the ceiling of a jeep moving with a speed v along a circle of radius R. Find the angle with the vertical made by the pendulum.

(a) 0 (b) $\tan^{-1}\left(\frac{v^2}{Rg}\right)$

(c) $\tan^{-1}\left(\frac{Rg}{v^2}\right)$ (d) none of these

Solution (b) $a_r = \frac{v^2}{R}$ $\tan\theta = \frac{\frac{v^2}{R}}{g} = \frac{v^2}{Rg}$

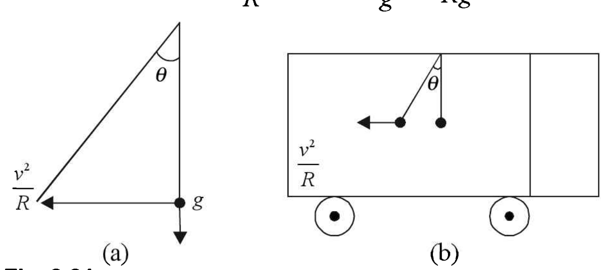


Fig. 2.21

34. If $|\vec{A}+\vec{B}| = |\vec{A}-\vec{B}|$ then angle between the vectors A and B is

(a) 0 (b) $\frac{\pi}{3}$

(c) $\frac{\pi}{2}$ (d) $\frac{\pi}{4}$

Solution (c) See Fig. 2.22

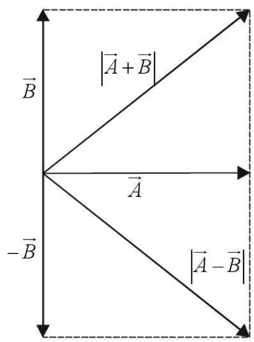


Fig. 2.22

35. Two identical pendulums are tied from the same rigid support. One is tied horizontally. The other is released when they are making the same angle θ with the vertical.

(a) $\tan^2\theta$ (b) $\cot^2\theta$
(c) 1 (d) $\sec^2\theta$

Solution (d) $T_1\cos\theta = mg$ or

$$T_1 = \frac{mg}{\cos\theta}$$

$$T_2 = mg\cos\theta$$

$$\therefore \frac{T_1}{T_2} = \frac{1}{\cos^2\theta}$$

$$= \sec^2\theta$$

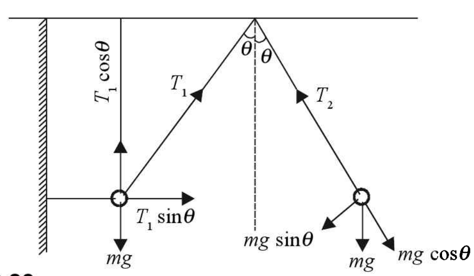


Fig. 2.23

36. Sixteen beads in a string are placed on a smooth incline as shown in equilibrium. The number of beads lying along the incline are

(a) 4 (b) 8
(c) 12 (d) none of these

Solution (c) Let n beads be hanging vertically. Then $(16-n)\,mg\sin\theta = n\,mg$. For $\sin\theta = \frac{1}{3}$, $n=4$ $\therefore$ $16-4$ = 12 beads lie along the plane.

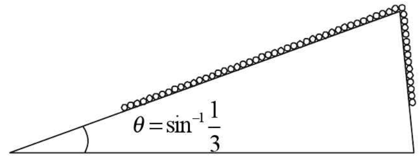


Fig. 2.24

37. Three particles A, B and C are situated at the verticles of an equilateral triangle of side l. Each of the particle starts moving with a constant velocity v such that A is always directed towards B, B towards C and C towards A. Find the time when they meet.

(a) $\frac{l}{\sqrt{3}v}$ (b) $\frac{2l}{v}$

(c) $\frac{2l}{3v}$ (d) none

Solution (c) We look into it as a problem of relative velocity and find v_{BA} in the direction of B.

$$t = \frac{l}{v_A - v_B\cos 120} = \frac{l}{v + v/2} = \frac{2l}{3v}$$

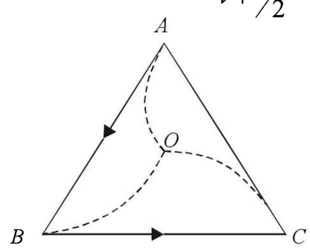


Fig. 2.25

38. Two particals are thrown horizontally in opposite directions from the same point from a height h with velocities 4 ms^{-1} and 3 ms^{-1}. Find the separation between them when their velocities are perpendicular.

(a) 0.15 s (b) 0.25 s
(c) 0.35 s (d) none of these

Solution (c) $\vec{V_1} = -4\hat{i} - gt\hat{j}$ and $\vec{V_2} = 3\hat{i} - gt\hat{j}$ are the velocities at any instant. For velocities to be perpendicular $\vec{V_1}\cdot\vec{V_2} = 0$

that is, $\vec{V_1}\cdot\vec{V_2} = 0$ or $t = \sqrt{\frac{12}{g^2}} = 0.35$ s

separation $= (u_1+u_2)t = 7(0.35) = 2.45$ m.

39. A ball is thrown with a velocity $6\hat{j}$ with an acceleration $6\hat{i}+2\hat{j}$. The velocity of the ball after 5 seconds is

(a) $30\hat{i}+10\hat{j}$ (b) $30\hat{i}+16\hat{j}$
(c) $10\hat{i}+24\hat{j}$ (d) none of these

Solution (b) using $v = u + at$ $v_x = 6(5) = 30$ ms^{-1}
$v_y = 6 + 2(5) = 16$ ms^{-1} $v = 30\hat{i}+16\hat{j}$

40. Ray AO in medium I emerges as OB in medium II then refractive index of medium II with respect to medium is

(a) $\frac{\overrightarrow{OA}.\overrightarrow{N_1O}}{\overrightarrow{OB}.\overrightarrow{N_2O}}$ (b) $\frac{|\overrightarrow{OA}\times\overrightarrow{N_1O}|}{|\overrightarrow{OB}\times\overrightarrow{N_2O}|}$

(c) $\frac{\overrightarrow{AO}.\overrightarrow{OP}}{\overrightarrow{OB}.\overrightarrow{OP}}$ (d) $\frac{\overrightarrow{OA}\times\overrightarrow{OP}}{\overrightarrow{OB}\times\overrightarrow{OP}}$

Assume OA, OB, N_1O, N_2O and OP radius of a circle.

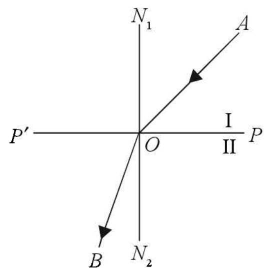


Fig. 2.26

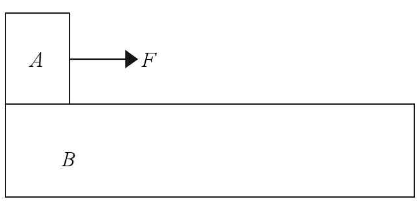
Solution (b)

41. Vector Laws

(a) vary if scale is changed
(b) vary if rotation of axes is performed
(c) vary if translation of coordinates is performed
(d) are invariant under translation and rotation of the coordinates.

Solution (d)

42. Block A is placed on B, whose mass is greater than that of A. Friction is present between the blocks while surface below B is smooth. Force F as shown increasing linearly with time, is applied at $t = 0$. The acceleration a_A and a_B of A and B, respectively, are plotted against time t. Choose the correct representation.



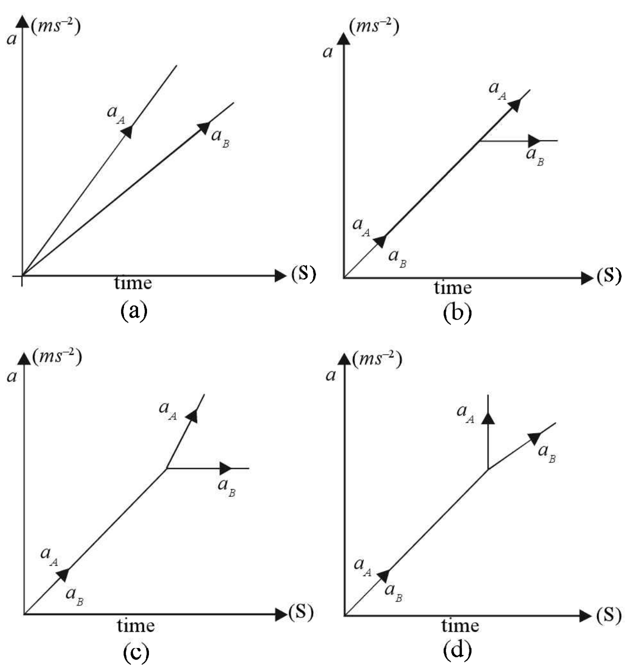


Fig. 2.27

Solution (c)

43. In Fig. 2.28 (a) mass of both the blocks are equal. v_A and v_B are instantaneous speed of A and B. Then

(a) B will never loose contact with the ground
(b) $|a_A| = |a_B|$
(c) $v_B = v_A \cos\theta$ (d) $v_B = \frac{v}{\cos\theta}$

Solution (a), (d) $BP + PA = B'P + PA'$ or

$l_1 + l_1 = (l_1 - x\cos\theta) + y$ .... (i)

differentiating (i) $0 = \frac{-dx}{dt}\cos\theta + \frac{dy}{dt} = 0$ or

$v_A = v_B \cos\theta$

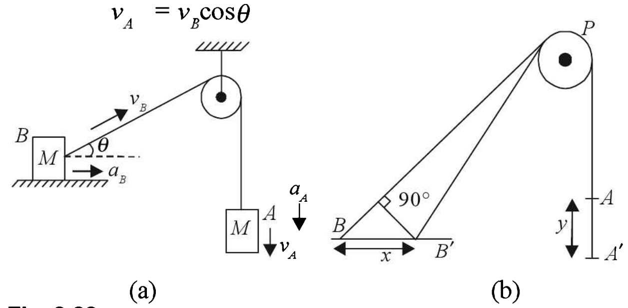


Fig. 2.28

44. The product of two vectors $\vec{A}$ and $\vec{B}$ may be

(a) $\geq AB$ (b) $\leq AB$
(c) $< AB$ (d) zero

Solution (b), (c), (d) $\because \vec{A}.\vec{B} = AB\cos\theta$ and $|\vec{A}\times\vec{B}| = AB\sin\theta$

45. $\vec{X} = \vec{A}.\vec{B}$ and $\vec{X} = \vec{C}.\vec{B}$ then

(a) $\vec{A} = \vec{C}$ always
(b) $\vec{A}$ may not be equal to $\vec{C}$
(c) $\vec{A}$ and $\vec{C}$ are parallel
(d) $\vec{A}$ and $\vec{C}$ are antiparallel.

Solution (b)

46. $\vec{A} + \vec{B} = \vec{C}$ Vectors A and B if rotated by θ in the same sense to form $\vec{A}'$ and $\vec{B}'$ then

(a) $\vec{A}' + \vec{B}' = \vec{C}$ (b) $\vec{A}' + \vec{B}' \neq \vec{C}$
(c) $\vec{A}'.\vec{B}' = \vec{A}.\vec{B}$ (d) $|\vec{A}'+\vec{B}'| = |\vec{C}|$

Solution (b), (c), (d)

47. $\vec{A}$ and $\vec{B}$ are two vectors such that $\vec{A} + \vec{B} = \vec{C}$ in a given coordinate sustem. The axes are rotated by θ. Then in new coordinate system

(a) $\vec{A} + \vec{B} = \vec{C}$
(b) $\vec{A} + \vec{B} \neq \vec{C}$
(c) $\vec{A}\times\vec{B}$ (old system) = $\vec{A}\times\vec{B}$ (new system)
(d) $\vec{A}$ and $\vec{B}$ (interchange in new system)

Solution (b)

48. A point moves according to the law $x = at$, $y = at(1 - \alpha t)$ where a and α are positive constants and t is time. Find the moment at which angle between velocity vector and acceleration vector is $\frac{\pi}{4}$.

Solution $\vec{v} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} = a\hat{i} + (a - 2a\alpha t)\hat{j}$

$$f = \frac{dv}{dt} = -2a\alpha\,\hat{j}$$

or

$$\cos\frac{\pi}{4} = \frac{-2a\alpha\hat{j}.\left[a\hat{i} + (a - 2a\alpha t)\hat{j}\right]}{2a\alpha\sqrt{a^2 + (a - 2a\alpha t)^2}}$$

$$\Rightarrow \frac{1}{\sqrt{2}} = \frac{-[a - 2a\alpha T]}{\sqrt{a^2 + (a - 2a\alpha t)^2}}$$

or $2(1 - 2\alpha t)^2 = 1 + (1 - 2\alpha t)^2$

or $(1 - 2\alpha t)^2 = 1$ or $t = t\frac{1}{\alpha}$

QUESTION FOR PRACTICE

1. Two particles are originally placed at P and Q distant d apart. At zero instant, they start moving such that velocity $\vec{v}$ of P is aimed towards Q and velocity $\vec{u}$ of Q is perpendicular to $\vec{v}$. The two projectiles meet at time $T =$

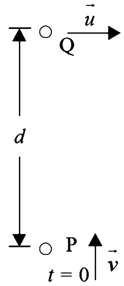


Fig. 2.29

(a) $\frac{(v^2 - u^2)d^2}{v^3 d}$ (b) $\frac{(v+u)d}{v^2}$

(c) $\frac{v(v-u)}{d}$ (d) $\frac{vd}{(v^2 - u^2)}$

2. A particle is moving in a circle of radius R in such a way that at any instant the a_r and a_t are equal. If the speed at $t = 0$ is v_0, the time taken to complete the first revolution is

(a) $\frac{R}{v_0}e^{-2\pi}$ (b) $v_0 R$

(c) $\frac{R}{v_0}$ (d) $\frac{R}{v_0}(1 - e^{-2\pi})$.

3. A boat which has a speed of 5 kmh^{-1} in still waters crosses a river of width 1 km along the shortest possible path in 15 minutes. The speed of the river in kmh^{-1} is

(a) 1 (b) 3

(c) 4 (d) $\sqrt{41}$

[Based on IIT]

4. A particle is moving in a plane with velocity given by $\vec{v} = \hat{i}u_0 + \hat{j}\,a\omega\cos\omega t$ if the particle is at origin at t = 0. Distance from origin at time $3\pi/2\omega$ is

(a) $\sqrt{a^2 + (3\pi u_0/2\omega)^2}$ (b) $\sqrt{a^2 + (2\pi u_0/\omega)^2}$

(c) $\left(\frac{\pi u_0}{\omega}\right)^2 + a^2$ (d) $\sqrt{a^2 + \left(\frac{2\pi u_0}{3\omega}\right)^2}$

[Based on Roorkee]

5. Mass A is released from the top of a frictionless inclined plane 18 m long and reaches the bottom 3 s later. At the instant when A is released, a second mass B is projected upwards along the plate from the bottom with a certain initial velocity.

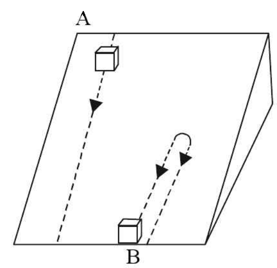


Fig. 2.30

Mass B travels a distance up the plane, stops and returns to the bottom so that it arrives simultaneously with A. The two masses do not collide. Initial velocity of A is

(a) 4 ms^{-1} (b) 5 ms^{-1}

(c) 6 ms^{-1} (d) 7ms^{-1}

[Based on IIT]

6. A particle is moving eastward with a velocity of 5 ms^{-1}. If in 10 s the velocity changes by 5 ms^{-1} northwards, what is the average acceleration in this time?

(a) $\frac{1}{\sqrt{2}}$ ms^{-1} NW (b) $\frac{1}{2}$ ms^{-1} EN

(c) $\sqrt{2}$ ms^{-1} NW (d) $2\sqrt{2}$ ms^{-1}NW.

[Based on IIT]

7. The driver of a truck travelling with a velocity v suddenly notices a brick wall in front of him at a distance d. To avoid crashing into the wall

(a) he should apply brakes
(b) he should take a circular turn without applying brakes
(c) both (a) and (b) alternately
(d) data is insufficient.

[Based on Roorkee]

8. To get a resultant displacement of 10 cm, two displacement vectors, one of magnitude 6 cm and another of 8 m, should be combined

(a) at an angle 60° (b) perpendicular to each
(c) parallel (d) anti-parallel

9. When mass is rotating in a plane about a fixed point, its angular momentum is directed along

(a) the axis of rotation
(b) line at an angle of 45° to the axis of rotation
(c) the radius
(d) the tangent to the orbit

10. Which of the following is a vector?

(a) Force (b) Mass
(c) Energy (d) Power

11. When two vectors $\vec{A}$ and $\vec{B}$ of magnitude a and b are added, the magnitude of the resultant vector is always

(a) greater than $(a + b)$
(b) not greater than $(a + b)$
(c) equal to $(a + b)$
(d) less than $(a + b)$

12. Identity the vector quantity.

(a) Heat (b) Angular momentum
(c) Time (d) Work

13. Which of the following quantities is a scalar?

(a) Magnetic moment
(b) Acceleration due to gravity
(c) Electric field
(d) Electrostatic potential

14. Which of the following quantities is a vector?

(a) Volume (b) Temperature
(c) Displacement (d) Density

15. The rectangular components of force 5 dyne are

(a) 3 and 4 dyne (b) 2.5 and 25 dyne
(c) 1 and 2 dyne (d) 2 and 3 dyne

16. Identify the scalar quantity.

(a) Work (b) Impulse
(c) Force (d) Acceleration

17. Moment of inertia is

(a) vector (b) scalar
(c) phasor (d) tensor

18. If the magnitude of vectors $\vec{A}$, $\vec{B}$ and $\vec{C}$ are 12 , 5 and 13 units, respectively, and $\vec{A} + \vec{B} = \vec{C}$, the angle between vectors $\vec{A}$ and $\vec{B}$ is

(a) $\pi/4$ (b) $\pi/2$
(c) π (d) 0

19. Angular displacement is

(a) a scalar (b) a vector
(c) neither (a) nor (b) (d) either (a) or (b)

20. A mosquito flies from the hole in a mosquito net top corner diametrically opposite. If the net is $3 \times 2 \times 2$ m then the displacement of the mosquito is

(a) $\sqrt{13}\ m$ (b) $\sqrt{17}\ m$
(c) $\sqrt{11}\ m$ (d) none of these

21. A man travels 1 mile due east, 5 mile due south, 2 mile due east and finally 9 miles due north. How far is the starting point?

(a) 3 miles (b) 5 miles
(c) 4 miles (d) between 5 and 9 miles

22. Two forces of magnitude 7 N and 5 N act on a p article at an angle θ to each other, θ can have any value. The minimum magnitude of the resultant force is

(a) 12 N (b) 8 N
(c) 2 N (d) 5 N

23. I started walking down a road to day-break facing the sun. After walking for some time, I turned to my left then I turned to the right once again. In which direction was I going?

(a) Northeast (b) South
(c) East (d) Northwest

24. If $\vec{A} = \vec{B} + \vec{C}$ and the magnitudes of $\vec{A}$, $\vec{B}$ and $\vec{C}$ are 5, 4 and 3 units, respectively. then the angle between $\vec{A}$ and $\vec{C}$ is

(a) $\pi/2$ (b) sin–1 (3/4
(c) $\cos^{-1}(3/5)$ (d) $\cos^{-1}(4/5)$

25. A boat which has a speed of 5 $kmhr^{-1}$ in still waters crosses a river of width 1 km along the shortest possible path in 15 minutes. The velocity of the water in $kmhr^{-1}$ is

(a) 4 (b) $\sqrt{41}$
(c) 1 (d) 3

26. If two waves of same frequency and same amplitude on superimposition produce a resultant wave of the same amplitude, the wave differs in phase by

(a) $\pi/5$ (b) $2\pi/3$
(c) $\pi/4$ (d) zero

27. If $\vec{n}$ is a unit vector in the direction of the vector $\vec{A}$, then

(a) $\vec{n} = |\vec{A}|/\vec{A}$ (b) $\vec{n} = \vec{n} \times \vec{n}$
(c) $\vec{n} = \vec{A}/|\vec{A}|$ (d) $\vec{n} = \vec{A}\,|\vec{A}|$

28. Two forces of 4 dyne and 3 dyne act upon a body. The resultant force on the body can only be

(a) between 3 and 4 dynes
(b) between 1 and 7 dynes
(c) more than 3 dynes
(d) more than 4 dynes

29. A river is flowing from west to east at a speed of 3 m/minute. A man on the south bank of the river, capable of swimming at 10 m in still waters wants to swim the river in the shortest time. He should swim in a direction

(a) 30° west of north
(b) 60° east of north
(c) 30° east of north
(d) due north

30. The resultant of two equal forces is double of either of the force. The angle between them is

(a) 0° (b) 60°
(c) 90° (d) 120°

31. An aeroplane is moving on a circular path with a speed 250 $kmhr^{-1}$. What is the change in velocity in half revolution?

(a) 0 (b) 125 $kmhr^{-1}$
(c) 250 $kmhr^{-1}$ (d) 500 $kmhr^{-1}$

32. A body constrained to move in y direction is subject to force given by $\vec{F} = (-2\vec{i} + 15\vec{j} + 6\vec{k})$ N. What is the work done by this force, in moving the body through a distance of 10m along y-axis?

(a) 20 J (b) 150 J
(c) 160 J (d) 190 J

33. I walked 4 miles turned to my left and walked 6 miles then turned to my right again and walked 4 mile. Which of the choice mentions the distance from the straight point to the place where *I* stopped?

(a) 10 mile (b) 14 mile
(c) 15 mile (d) 20 mile

34. A force $\vec{F} = 6\vec{i} - 8\vec{j} + 10\vec{k}$ newton produces an acceleration of 1 ms^{-2} in a body, the body would be

(a) 10 $\sqrt{2}$ kg (b) 6 $\sqrt{2}$ kg
(c) 20 kg (d) 200 kg

35. Maximum and minimum magnitudes of the resultant of two vectors of magnitudes P and Q are in the ratio 3 : 1. Which of the following relations is true?

(a) $PQ = 1$ (b) $P = 2Q$
(c) $P = Q$ (d) none of these

36. What is the projection of $\vec{P}$ on $\vec{Q}$?

(a) $\vec{Q}.\vec{P}$ (b) $\hat{P}.\hat{Q}$
(c) $\vec{P}.\vec{Q}$ (d) $\vec{P}.\hat{Q}$

37. Rain is falling vertically 4 ms^{-1}. A man is moving due east with 3 ms^{-1}. The direction in which he shall hold the umbrella with the vertical is

(a) 53° east of vertical
(b) 37° east of vertical
(c) 53° west of vertical
(d) 37° west of vertical

38. There are N co-planar vectors each of magnitude V. Each vector is inclined to the proceeding vector at angle $2\pi/N$. What is the magnitude of their resultant?

(a) zero (b) V/N
(c) V (d) NV

39. Which of the following operations between the two vectors can yield a vector perpendicular to either of them?

(a) Subtraction (b) Division
(c) Addition (d) Multiplication

40. Three vectors A, B and C satisfy the relation $\vec{A}.\vec{B} = 0$ and $A.\,C = 0$. The vector A is parallel to

(a) $\vec{B}.\vec{C}$ (b) $\vec{B}$
(c) $\vec{C}$ (d) $\vec{B}\times\vec{C}$

41. Angle between the vectors $(\vec{i}+\vec{j})$ and $(\vec{j}+\vec{k})$ is

(a) 60° (b) 90°
(c) 180° (d) 0°

42. Resultant of two vectors $\vec{P}$ and $\vec{Q}$ is inclined at 45° to either of them. What is the magnitude of the resultant?

(a) $\sqrt{P^2+Q^2}$ (b) $\sqrt{P^2-Q^2}$
(c) P + Q (d) P − Q

43. A steamer is heading due North with 20 ms^{-1}. The wind is blowing 10 ms^{-1}. The wind is blowing 10 ms^{-1} due west. In which direction the flag on the mast flutters?

(a) $\tan^{-1}\frac{1}{2}$ west of north
(b) $\tan^{-1}\frac{1}{2}$ east of north
(c) $\tan^{-1}\frac{1}{2}$ north of east
(d) $\tan^{-1}\frac{1}{2}$ north of west

44. What is the angle between $\hat{i} + \hat{j} + \hat{k}$ and $\hat{i}$?

(a) $\pi/3$ (b) $\pi/4$
(c) $\pi/6$ (d) none of these

45. What is the maximum number of components into which a vector can be split?

(a) 2 (b) 3
(c) 4 (d) more than 4

46. What is the maximum number of a rectangular components into which a vector can be split in its own plane?

(a) Two (b) Three
(c) Four (d) More than 4

47. A force of 6 kg and 8 kg can be applied together to produce the effect of a single force of

(a) 20 kg (b) 15 kg
(c) 11 kg (d) 1 kg

48. To a person going east in a car with a velocity of 25 $kmhr^{-1}$, a train appears to move towards north with a velocity of 25 $\sqrt{3}$ km/hr. The actual velocity

(a) 5 $kmhr^{-1}$ (b) 25 $kmhr^{-1}$
(c) 50 $kmhr^{-1}$ (d) 53 $kmhr^{-1}$

49. The area of a Δ formed with sides $5i + 3j - k$ and $3i + 2j - k$ is

(a) $\sqrt{6}$ (b) $\sqrt{3}$
(c) $\sqrt{\frac{3}{2}}$ (d) $\sqrt{\frac{5}{2}}$

50. At what angle should be the two forces $2p$ and $\sqrt{2}p$ act so that the resultant force is $p\sqrt{10}$

(a) 45° (b) 60°
(c) 90° (d) 120°

51. Two cars are moving. A along east with 10 ms^{-1}. At any instant it is 1500 m away from the crossing. *B* at the same instant is 1800 m away from the crossing and is moving towards the crossing with 15 ms^{-1}. When do they come closest?

(a) 109.3 s (b) 129.2 s
(c) 119.3 s (d) 99.3 s

52. What is the angle between $\vec{P}$ and the resultant of $(\vec{P}+\vec{Q})$ and $(\vec{P}-\vec{Q})$?

(a) $\frac{\tan^{-1}\left|(\vec{P}-\vec{Q})\right|}{|P+Q|}$ (b) $\tan^{-1}(Q/P)$
(c) tan–1 (P/Q) (d) zero

53. The length of seconds hand in a watch is 1 cm. The change in velocity of its tip in 15 seconds is

(a) zero (b) $\left(\frac{\pi}{15\sqrt{2}}\right)$ cms^{-1}
(c) $\left(\frac{\pi}{15}\right)$ cms^{-1} (d) $\left(\frac{\pi\sqrt{2}}{15}\right)$ cms^{-1}

54. Rain falling vertically downwards with a velocity of 3 kmh^{-1}. A person moves on a straight road with a velocity of 4 kmh^{-1}. Then the apparent velocity of the rain with respect to the person is

(a) 1 kmh^{-1} (b) 5 kmh^{-1}
(c) 4 kmh^{-1} (d) 3 kmh^{-1}

55. A large number of particles are moving towards each other with velocity *v* having directions of motion randomly distributed. What is the average relative velocity between any two particles averaged over all the pairs?

(a) $4v/\pi$ (b) $4\pi v$
(c) v (d) $\pi v/4$

56. The magnitudes of the *X* and *Y* components $\vec{P}$ are 7 and 6. The magnitudes of the *X* and *Y* components of $\vec{P} + \vec{Q}$ are 11 and 9, respectively. What is the magnitude of *Q*?

(a) 9 (b) 8
(c) 6 (d) 5

57. A swimmer can swim in still waters with speed *v* and the river flowing with velocity *v*/2. To cross the river in shortest time, he should swim making angle θ with the upstream. What is the ratio of the time taken to swim across in the shortest time to that in swimming across over shortest distance?

(a) sin θ (b) cos θ
(c) tan θ (d) cot θ

58. A vector of magnitude *a* is rotated through angle θ. What is the magnitude of the change in the vector?

(a) $2\alpha \sin\theta$ (b) $2\alpha \cos\theta$
(c) $2\alpha \sin(\theta/2)$ (d) $2\alpha \cos(\theta/2)$

59. Consider a vector $\vec{F} = 4\hat{i} - 3\hat{j}$. Another vector that is perpendicular to $\vec{F}$ is

(a) $4i + 3j$ (b) $7k$
(c) $3i - 4j$ (d) $6i$

60. A helicopter is flying south with a speed of 50 km h^{-1}. A train is moving with the same speed towards east. The relative velocity of the helicopter as seen by the passengers in the train will be 50 $\sqrt{2}$ km h^{-1} towards

(a) northwest (b) southwest
(c) northeast (d) southeast

61. A man is walking due east at the rate of 4 $kmhr^{-1}$ and the rain is falling at an angle of 30° east of vertical with a velocity of 6 $kmhr^{-1}$. The velocity of the rain relative to the man will be

(a) 5 km hr^{-1} (b) 7.118 km hr^{-1}
(c) 8.718 km hr^{-1} (d) 10 km hr^{-1}

62. A truck travelling due north at 20 ms^{-1} turns west and travels at the same speed. Then the change in velocity is

(a) 40 ms^{-1} northwest
(b) 20 $\sqrt{2}$ ms^{-1} northwest
(c) 20 $\sqrt{2}$ ms^{-1} southwest
(d) 40 ms^{-1} southwest

63. Given that *P* is a point on a wheel rolling on a horizontal ground. The radius of the wheel is *R*. Initially if the point *P* is in contact with the ground the wheel rolls through half the revolution. What is the displacement of point *P*?

(a) $R\sqrt{\pi^2+1}$ (b) $R\sqrt{\pi^2+4}$
(c) πR (d) $2\pi R$

64. A vector $\vec{F}_1$ is along x axis. If $\vec{F}_1 \cdot \vec{F}_2$ is zero $\vec{F}_2$ could be

(a) $(j+k)$ (b) $-(i+j)$

(c) $4(i+k)$ (d) $-4i$

65. A parallelogram is formed with $\vec{a}$ and $\vec{b}$ as the sides. Let $\vec{d}_1$ and $\vec{d}_2$ be the diagonals of the parallelogram then $a^2 + b^2 =$

(a) $\left(d_1^2+d_2^2\right)/2$ (b) $\left(d_1^2-d_2^2\right)/2$

(c) $d_1^2+d_2^2$ (d) $d_1^2-d_2^2$

66. If $|\vec{A}| = |\vec{B}|$, then what is the angle between $\vec{A} + \vec{B}$ and $\vec{A} - \vec{B}$

(a) 90° (b) 60°

(c) 30° (d) 0°

Answers to Question for Practice

1.	(d)	**2.**	(d)	**3.**	(b)	**4.**	(a)	**5.**	(c)	**6.**	(a)	**7.**	(a)
8.	(b)	**9.**	(a)	**10.**	(a)	**11.**	(b)	**12.**	(b)	**13.**	(d)	**14.**	(c)
15.	(a)	**16.**	(a)	**17.**	(d)	**18.**	(b)	**19.**	(d)	**20.**	(b)	**21.**	(b)
22.	(c)	**23.**	(c)	**24.**	(c)	**25.**	(d)	**26.**	(b)	**27.**	(c)	**28.**	(b)
29.	(d)	**30.**	(a)	**31.**	(d)	**32.**	(b)	**33.**	(a)	**34.**	(a)	**35.**	(b)
36.	(d)	**37.**	(b)	**38.**	(a)	**39.**	(d)	**40.**	(d)	**41.**	(a)	**42.**	(a)
43.	(b)	**44.**	(d)	**45.**	(d)	**46.**	(a)	**47.**	(c)	**48.**	(c)	**49.**	(c)
50.	(a)	**51.**	(b)	**52.**	(d)	**53.**	(b)	**54.**	(b)	**55.**	(d)	**56.**	(d)
57.	(a)	**58.**	(c)	**59.**	(b)	**60.**	(a)	**61.**	(c)	**62.**	(c)	**63.**	(b)
64.	(a)	**65.**	(a)	**66.**	(a)								

Explanation

1(d) Relative velocity of P and Q is $(v - u\cos\theta)$. The particles will meet when

$$\int_0^T (v-u\cos\theta)\,dt = d \text{ and } \int_0^T v\cos\theta\,dt = d$$

and $\int_0^T \cos\theta\,dt = \frac{uT}{v}$ or $vT - u\frac{uT}{v} = d$ or $(v^2 - u^2)\,T = vd$

or $T = \frac{vd}{v^2-u^2}$

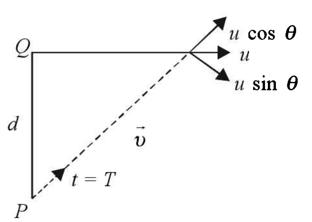


Fig. 2.31

2(d) Given, $a_r = a_t$ i,e., $R\omega^2 = R\alpha$ i,e., $R\omega^2 = R\frac{d\omega}{dt}$ or $\frac{d\omega}{dt} =$

ω^2 or $\frac{d\omega}{\omega^2} = dt$ i,e., $\int_{\omega_o}^{\omega} \frac{d\omega}{\omega^2} = \int_0^t dt$ i,e., ω

$= \frac{\omega_o}{1-\omega_{0'}}$

i,e., $\frac{d\theta}{dt} = \frac{\omega_0}{1-\cdot\omega_{0'}}$

i,e., $d\theta = \left(\frac{\omega_0}{1-\omega_{0'}}\right)dt$ again, $\int_0^{2\pi} d\theta = \int_0^T \left(\frac{\omega_0}{1-\omega_{0'}}\right)$

or $(1-\omega_0 T) = e^{-2\pi}$ or $T = \frac{1}{\omega_0}(1-e^{-2\pi})$

or $T = \frac{R}{v_0}(1-e^{-2\pi})$

3(b) Speed of boat, $v_b = 5$ kmh^{-1}

Speed of boat when water flows, $vr = \frac{1}{1/4} = 4$ kmh^{-1}

Resultant speed $v = \sqrt{vb^2 - vr^2} = \sqrt{5^2-4^2} = \sqrt{25-16}$

$= \sqrt{9}$

4(a) Comparing the given equation with $\vec{v} = \hat{i}\,v_x + \hat{j}\,v_y$,

we get $v_x = u_0$ and $\frac{dy}{dt} = a\omega\cos\omega t$ or $\frac{dx}{dt} = u_0$ and $\frac{dy}{dt} = a\omega\cos\omega t$. Integrating $x = \int u_0 dt$ and $y = \int a\omega \cos\omega t\,dt$ or $x = u_0 t + c_1$ and $y = a\sin\omega t + c_2$ At $t = 0$, $x = 0$ and $y = 0$ we get c_1 and c_2 as zero $\therefore$ $x = u_0 t$ and $y = a\sin\omega t$ but $t = 3\pi/2\omega$ $\therefore$ $x = u_0(3\pi/2\omega)$ and $y = -a$. Then distance from origin, $d = \sqrt{x^2+y^2}$

$= \sqrt{a^2 + (3\pi u_0/2\omega)^2}$

5(c) Here for A, $18 = 0 \times 3 + \frac{1}{2} a \times 3^2$ or $a = 4$ ms^{-2} for B, time taken to move up is given by, t1 = u/a ($\therefore$ the relation $v = u + at$ here becomes $0 = u - at_1$). Distance moved up is given by the relation $0 = u^2 - 2aS$ i.e., S $= u^2/2a$ $S = \frac{1}{2}at2^2$ and $t_2 = \sqrt{\frac{2S}{a}}$ or $t_2 = \sqrt{\frac{2}{a}\frac{u^2}{2a}} = \frac{u}{a}$

But $\frac{u}{a} = t_1$ hen $t_1 + t_2 = 3$ or $\frac{2u}{a} = 3$ or u = $\frac{3a}{2}$ or u = $\frac{3}{2} \times 4 = 6$ ms^{-1}

6(a) Change in velocity,

$\Delta\vec{v} = \vec{v}_2 + (-\vec{v}_1) = (5^2 + 5^2)^{1/2}$ NW $= 5\sqrt{2}$ ms^{-1} NW

Then $\vec{a}_{av} = \frac{\vec{\Delta}v}{\Delta t} = \frac{5\sqrt{2}}{10}$ NW $= \frac{1}{\sqrt{2}}$ ms^{-2} NW

7(a) For taking a circular turn to avoid. accident, the acceleration acquired will be $\frac{v^2}{d}$ Thus additional effort is required to meet this acceleration.

51(b) $x^2 = (1500 - 10t)^2 + (1800 - 15t)^2$, for x to be minimum, its first derivative should be zero. Thus $\frac{dx}{dt} = 0$

$= -20(1500 - 10t) - 30(1800 - 15t)$

or $t = 129.23$ s

53(b) $\Delta v = 2v \sin \theta/2 = 2 \times \frac{2\pi}{60} \times \frac{1}{\sqrt{2}}$

63(b)Displacement of $= \pi R\,\hat{i} + 2R\,\hat{j}$

Displacement of $P = \sqrt{(\pi R)^2 + (2R)^2} = R\sqrt{\pi^2 + 4}$

3

Motion in One and Two Dimensions

BRIEF REVIEW

Motion A body in motion keeps changing its position with respect to its surroundings with the passage of time. If the body does not change its position with respect to time it is said to be at rest.

Frame of Reference A set of coordinates x, y, z and t is said to be a frame of reference. Frame of reference may be **inertial** or **non-inertial. Inertial** frame of reference is one which is either fixed or moves with a uniform velocity in the same straight line. **Non-inertial** or **accelerated** frame of reference has an acceleration 'a'. Newton's laws are valid only in inertial frame. Pseudo or inertial vectors are to be applied to make the frame of reference inertial from non-inertial so that Newton's laws may be applied.

One-Dimensional Motion If the particle changes its position only in one of the x, y, or z directions with respect to time, then the motion is said to be one-dimensional. Since the particle moves along a straight line, the motion may also be termed as linear or rectilinear.

Speed The time rate of change of distance is called speed, that is, $v = \frac{dx}{dt}$ unit ms^{-1}.

Velocity The time rate of change of displacement is called velocity, that is, $\vec{v} = \frac{d\vec{x}}{dt}$. Unit ms^{-1}, $cm\ s^{-1}$ and $ft\ s^{-1}$ in SI, CGS and FPS system, respectively, $v = LT^{-1}$.

Displacement The shortest distance between initial and final position of the particle is called displacement.

Acceleration The time rate of change of velocity is called acceleration, $\vec{a} = \frac{d\vec{v}}{dt}$ units is ms^{-2}, $cm\ s^{-2}$ and $ft\ s^{-2}$ in SI, CGS and FPS system, respectively, $a = LT^{-2}$. Speed, velocity or acceleration may be of four types. We define here velocity and others can be anticipated in similar terms.

(a) ***Instantaneous Velocity*** The velocity, at a particular instant of time is called instantaneous velocity, for example, velocity, at 4.82 s may be expressed as $\vec{v} = \left.\frac{d\vec{x}}{dt}\right|_{t=4.82\,s}$

(b) ***Uniform Velocity*** If $\frac{dx}{dt}$ = constant throughout the motion and direction of motion does not vary throughout then such a velocity is called uniform velocity. Fig. 3.1 (a) shows displacement time graph and Fig. 3.1 (b) shows velocity time graph for a uniform velocity.

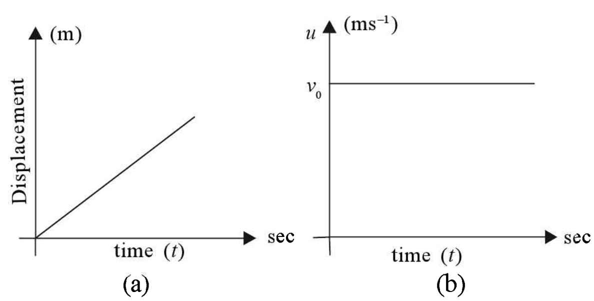


Fig. 3.1 Uniform velocity

(c) ***Variable Velocity*** If $\frac{dx}{dt}$ is not constant but varies at different intervals of time or $\frac{dx}{dt}$ is constant but direction varies or both vary, then such a velocity is said to be variable velocity. Fig. 3.2 (a) illustrates $x - t$ graph for a body moving with variable velocity and Fig. 3.2 (b) shows velocity V's time variation for a body moving with variable velocity.

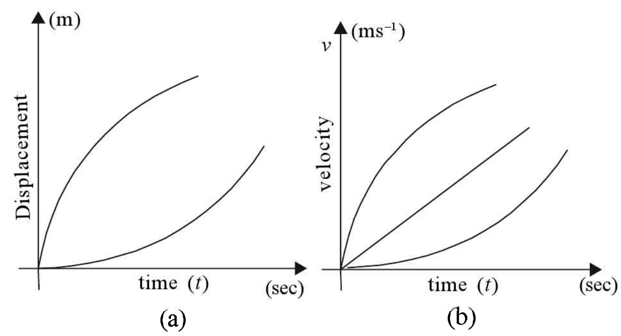


Fig. 3.2 Variable velocity

(d) ***Average Velocity*** It is that uniform velocity with which if the body would have moved it would have covered the same displacement as it does otherwise by moving with variable velocity. Thus

$$v_{av} = \frac{\text{total displacement covered}}{\text{total time taken}}.$$

Average Velocity in Different Cases

(i) Particles covering different displacement in different times Assume a particle covers s_1 displacement in t_1 and s_2 in time t_2 and so on then average velocity is

$$v_{av} = \frac{s_1 + s_2 + s_3 + \ldots}{t_1 + t_2 + t_3 + \ldots} = \frac{s_1 + s_2 + s_3 + \ldots}{\frac{s_1}{v_1} + \frac{s_2}{v_2} + \frac{s_3}{v_3} + \ldots}$$

Special case if $s_1 = s_2 = s$.

$$v_{av} = \frac{2s}{\frac{s}{v_1} + \frac{s}{v_2}} = \frac{2v_1v_2}{v_1 + v_2} \text{ (harmonic mean)}$$

(ii) Bodies moving with different velocity in different intervals of time A body moves with velocity v_1 in time t, v_2 in time t_2 and so on then v_{av} is given by

$$v_{av} = \frac{v_1t_1 + v_2t_2 + \ldots}{t_1 + t_2 + \ldots}$$

Special case if $t_1 = t_2 = t_3 = \ldots t_n = t$

Then $$v_{av} = \frac{v_1 + v_2 + \ldots + v_n}{n} \text{ (Arithmatic mean)}$$

Equations of Motion

(a) $v = u + at$ (b) $s = ut + \frac{1}{2}at^2$

(c) $v^2 - u^2 = 2as$ (d) $s_{nth} = u + \frac{s}{2}(2n - 1)$

The conditions under which these equations can be applied

1. Motion should be 1-dimensional.
2. Acceleration should be uniform.
3. Frame of reference should be inertial.

While drawing graphs compare your equation with the following and then draw (matching the equation) graphs

1. $y = mx + c$ straight line with positive intercept on y-axis.
   $y = mx$ straight line passing through origin.
   $y = mx - c$ straight line with negative intercept (on y-axis).

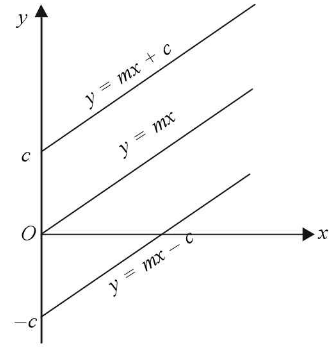


Fig. 3.3 Straight lines

2. $x^2 + y^2 = a^2$ circle with centre at origin.
   $(x - h)^2 + (y - k)^2 = r^2$ a circle with centre at (h, k).
3. $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ represents ellipse.
4. $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ shows hyperbola.
5. $y = \frac{1}{x}$ or $xy = k$ represents a rectangular hyperbola.
   See Fig. 3.4 (a)
6. $y = y_0e^{-\alpha x}$ and $y = y_0(1 - e^{-\alpha x})$ represents exponential. See Fig. 3.4 (b) and (c).

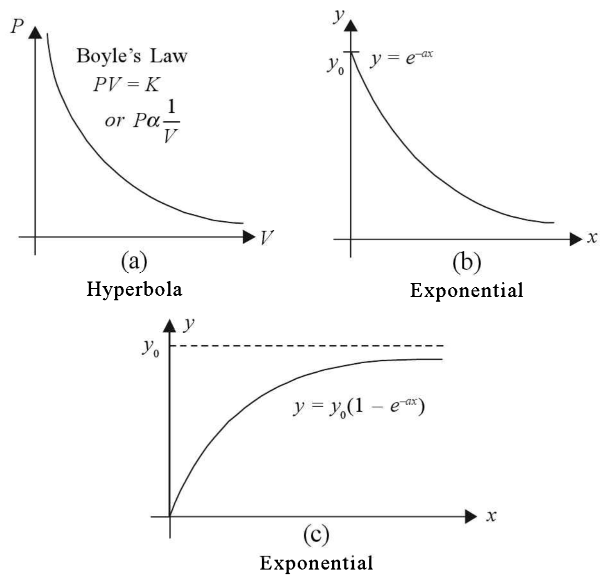


Fig. 3.4

While dealing with two-dimensional motion convert the problem into two one-dimensional motions. Separate v_x and v_y similarly a_x and a_y. Treat the motion in x- and y-directions.

Projectile A freely falling body having constant horizontal velocity may be termed as a projectile. In general, in one direction the motion be accelerated and in another direction the motion is uniform, then such a motion is called projectile motion. Fig. 3.5 shows acceleration in y-direction and uniform velocity in x-direction. Such bodies follow parabolic path.

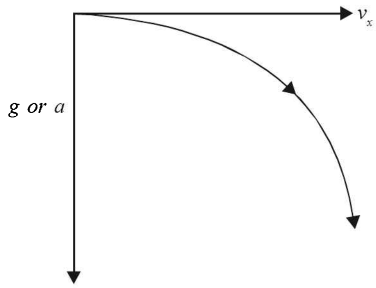


Fig. 3.5 Projectile motion

Oblique Projectile Motion Assume a projectile is fixed at an angle θ with horizontal, with a velocity u from point O as shown in Fig. 3.6. Resolve velocity along x and y-axis. Along y-axis g acts then **maximum height attained.**

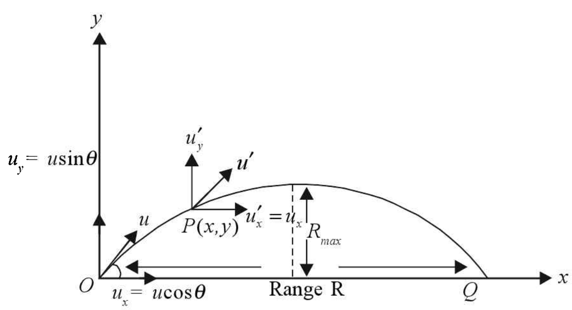


Fig. 3.6 Oblique Projectile motion

$$h_{max} = \frac{u^2 \sin^2\theta}{2g}.$$

Time of Flight $T = \dfrac{2u\sin\theta}{g}$.

Horizontal Range $R = \dfrac{u^2 \sin 2\theta}{g}$. Note that the range will be same if projected at complement angles, i.e, θ and $(90 - \theta)$ with same velocity.

Maximum Range $R_{max} = \dfrac{u^2}{g}$ when $\theta = 45°$

Trajectory $y = x\tan\theta - \dfrac{gx^2}{2u^2\cos^2\theta}$

or $y = x\tan\theta\left[1 - \dfrac{x}{R}\right]$ Path is parabolic.

Instantaneous velocity = $|v|$

$$= \sqrt{u_x^2 + v_y^2} + \sqrt{u_x^2 + (u_y - gt)^2}$$

$$= \sqrt{u^2 + g^2t^2 - 2ugt\sin\theta}$$

$$\tan\beta = \frac{u_y - gt}{u_x}.$$

Range and Time of Flight along an Inclined Plane

Consider an inclined plane of inclination α. Let a projectile be fixed at an angle θ with the horizontal or at an angle $(\theta - \alpha)$ with respect to incline plane as shown in Fig. 3.7.

The time of flight $T' = \dfrac{2u\sin(\theta - \alpha)}{g\cos\alpha}$

Range $R' = \dfrac{2u^2\sin(\theta - \alpha)\cos\theta}{g\cos^2\alpha}$

$$R = \frac{u^2}{g\cos^2\alpha}[\sin(2\theta - \alpha) - \sin\alpha]$$

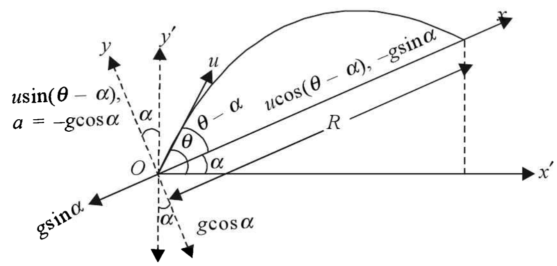


Fig. 3.7 Projectile motion along an incline

Range R' along the inclined is maximum if $2\theta - \alpha = \dfrac{\pi}{2}$ or $\theta - \alpha = \dfrac{\pi}{2} - \theta$. That is, R' is maximum when the direction of projection bisects the angle that the inclined plane makes with Oy' and $R'_{max} = \dfrac{u^2}{g\cos^2\alpha}.[1 - \sin\alpha]$

Note: In projectile motion along the plane acceleration acts along x and y axis both.

- **Short Cuts and Points to Note**

1. Slope of $x - t$ graph is velocity, slope of $v - t$ graph is acceleration.
2. Average velocity $v_{av} = \dfrac{\text{total displacement}}{\text{total time taken}}$

$$= \frac{x(t_2) - x(t_1)}{t_2 - t_1} = \frac{x_1 + x_2 + ...}{t_1 + t_2}.$$

$$= \frac{x_1 + x_2 + ...}{\frac{x_1}{v_1} + \frac{x_2}{v_2} + ...}$$

If a body covers equal displacement with different velocities $\dfrac{1}{v_{av}} = \dfrac{1}{n}\left[\dfrac{1}{v_1} + \dfrac{1}{v_2} + ... \dfrac{1}{v_n}\right]$.

If a body moves half distance with v_1 and other half with v_2 then $v_{av} = \dfrac{2v_1v_2}{v_1+v_2}$.

If a body moves with different velocities in equal intervals of time then

$$v_{av} = \frac{v_1+v_2+.....+v_n}{n} \text{ (arithmatic mean)}$$

3. Area under $v-t$ graph is displacement, area under $a-t$ graph is velocity.

4. When a body leaves a moving body it acquires its velocity but not acceleration.

5. Instantaneous velocity $v(t_1) = \left.\dfrac{dx}{dt}\right|_{t=t_1}$.

6. Apply $v = u + at$, $s = ut + \dfrac{1}{2}at^2$,

$v^2 - u^2 = 2as$, $s_{nth} = u + \dfrac{a}{2}(2n-1)$ when

(i) motion is one dimensional or made so if two or three dimensional [by resolving].

(ii) acceleration is uniform.

(iii) frame of reference is inertial.

7. If acceleration is variable then start with $\dfrac{dv}{dt} = f(t)$

and $v = \int f(t)\,dt$

8. If acceleration is variable and function of displacement or velocity. For example

$a = -kv^2$

then $\dfrac{dv}{dt} = \dfrac{dv}{dx}\cdot\dfrac{dx}{dt} = -kv^2$

or $\int \dfrac{dv}{v} = \int -k\,dx$.

9. Note carefully the graphs for $v = u + at$ as shown in Fig. 3.8 (a) and (b).

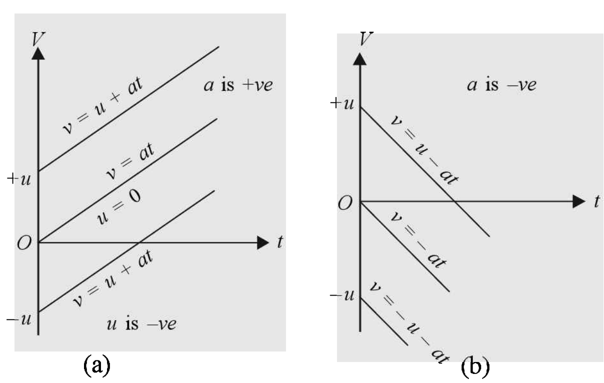


Fig. 3.8 Graphs for $v = u + at$

10. Note the graph for $s = at + \dfrac{1}{2}at^2$ carefully as shown in Fig. 3.9 (a), (b) and (c).

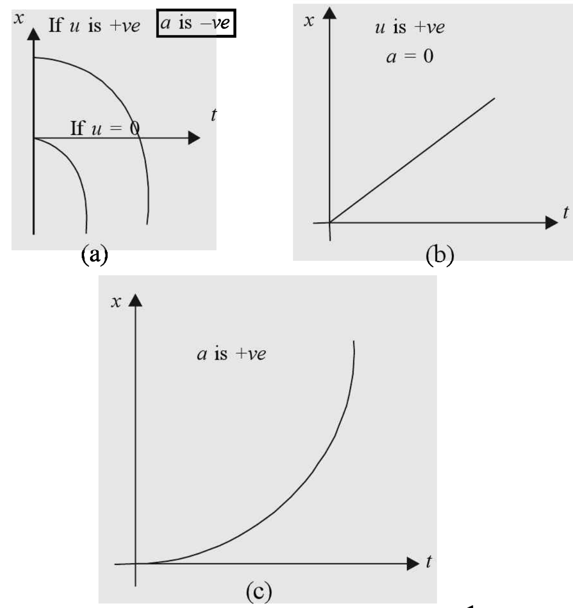


Fig. 3.9 Graphical representation of $s = ut + \dfrac{1}{2}at^2$

11. If a particle starts from rest with an acceleration α, after acquiring a maximum velocity the particle decelerates with β and finally comes to rest in time t, then

$v_{max} = \dfrac{\alpha\beta t}{\alpha+\beta}$ and distance covered $s = \dfrac{\alpha\beta t^2}{2(\alpha+\beta)}$.

12. Time cannot be negative in physics.

13. If in a projectile motion, direct formulae i are inapplicable, convert the problem into two one dimensional motions.

14. Average acceleration $v_{av} = \dfrac{v_f - v_i}{t}$

$$= \frac{(v_{fx}\hat{i} + v_{fy}\hat{j}) - (v_{ix}\hat{i} + v_{iy}\hat{j})}{t}$$

and direction $\beta = \tan^{-1}\left(\dfrac{v_{fx} - v_{fy}}{v_{fx} - v_{ix}}\right)$.

15. As far as possible apply vector laws to solve two dimensional problems if physical quantities involved are vectors.

16. Problems on relative velocity can even be solved using vector laws. Use $v_{AB} = v_A - v_B$

or $v_{AB} = (v_{Ax} - v_{Bx})\hat{i} + (v_{Ay} - v_{By})\hat{j}$; $\tan\beta = \dfrac{v_{Ay} - v_{By}}{v_{Ax} - v_{Bx}}$

with respect to x direction

$$|v_{AB}| = \sqrt{(v_{Ax} - v_{Bx})^2 + (v_{Ay} - v_{By})^2};$$

$\tan\beta' = \dfrac{v_{Ax} - v_{Bx}}{v_{Ay} - v_{By}}$ with respect to y-direction.

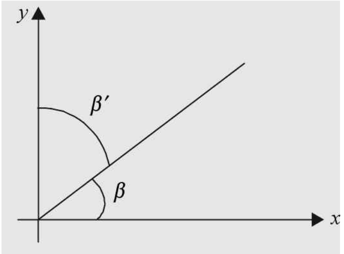


Fig. 3.10 Finding direction in two dimension motion

17. Whenever solving problems for inclined plane, consider axis along the plane as x-axis and perpendicular to it as y-axis. See Fig. 3.11.

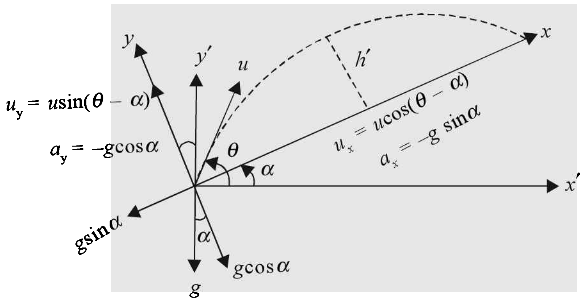


Fig. 3.11 Projectile motion along incline

Note: $\left.\begin{aligned} u_x &= u\cos(\theta-\alpha) \\ a_x &= -g\sin\alpha \end{aligned}\right]$ along the plane.

$\left.\begin{aligned} u_y &= u\sin(\theta-\alpha) \\ a_y &= -g\cos\alpha \end{aligned}\right]$ perpendicular to the plane

i.e. use accelerated motion along both x and y axis.

$$\text{Time of flight} = \frac{2u\sin(\theta-\alpha)}{g\cos\alpha} = \frac{2|u_y|}{|a_y|}$$

Note $T = \dfrac{2|u_y|}{|a_y|}$ is true everywhere.

$$h' = \frac{u^2\sin^2(\theta-\alpha)}{2g\cos\alpha} = \frac{u_y^2}{2|a_y|} \text{ is also true in all cases.}$$

18. To find radius of curvature of a projectile at any point $R = \dfrac{v^2}{a_r}$.

The velocity v and radial or normal acceleration at that point is used in the above relation.

If v and a_r cannot be determined then use

$$R = \frac{\left[1+\left(\dfrac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\dfrac{d^2y}{dx^2}}$$

19. Equation of trajectory means the relation between x and y. Try to establish relation between x and y by eliminating t.

20. To cross the river along the shortest path the swimmer shall strike at an obtuse angle to the flow of river so that resultant velocity v is along the normal as illustrated in Fig. 3.12. provided $v_{\text{swimmer}} > v_{\text{river}}$

From triangle law $v = \sqrt{v_s^2 - v_r^2}$ where v_s = velocity of swimmer and v_r = velocity or river.

If the width of the river is l then $t = \dfrac{l}{v} = \dfrac{R}{\sqrt{v_s^2 - v_r^2}}$.

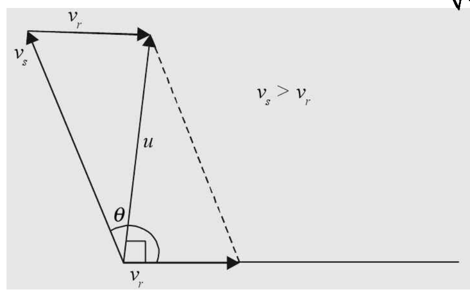


Fig. 3.12

21. To cross the river in the shortest time (when $v_{\text{swimmer}} > v_{\text{river}}$). Then the swimmer shall strike at right angle to the flow of the river and $t_{\min} = \dfrac{l}{v_{\text{swimmer}}}$.

22. A particle is projected from the top of an incline as shown in Fig. 3.13 $a_x = g\sin\alpha$ and $a_y = -g\cos\alpha$.

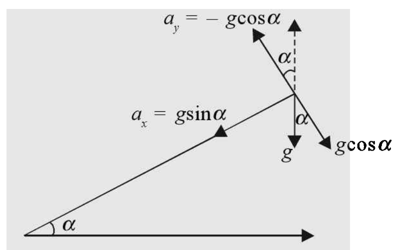


Fig. 3.13

23. Net acceleration in circular motion (Fig. 3.14)

$a_{\text{net}} = \sqrt{a_t^2 + a_r^2}$ where a_t is tangential acceleration and a_r is radial acceleration.

$\tan\beta = \dfrac{a_r}{a_t}$.

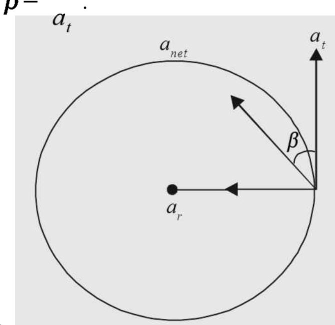


Fig. 3.14

24. If resultant of two motions is to be determined use

$$v_R = v_1 + v_2 = \left(v_{1x}\hat{i} + v_{1y}\hat{j}\right) + \left(v_{2x}\hat{i} + v_{2y}\hat{j}\right)$$

$$= (v_{1x}+v_{2x})\,\hat{i} + (v_{1y}+v_{2y})\,\hat{j}.$$

$$|v_R| = \sqrt{(v_{1x}+v_{2x})^2+(v_{1y}+v_{2y})^2} \text{ and}$$

$$\beta = \tan^{-1}\left(\frac{v_{1y}+v_{2y}}{v_{1x}+v_{2x}}\right) \text{ with respect to } x\text{-axis.}$$

25. If a particle is projected from the top of a tower or from a height h then consider the point of projection as origin. So that displacement is $-h$. Using $-h = u\sin\theta\, t - \frac{gt^2}{2}$. Find t and range $= x = (u\cos\theta)\,t$.

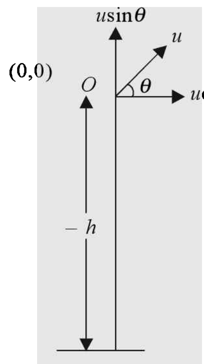


Fig. 3.15

26. If a person can throw a ball to maximum height h (vertically up) then the maximum horizontal distance up to which he can throw the ball is $2h$.

27. Once a particle is thrown in a gravitational field, it will return only after time of flight.

28. If a particle is thrown up, it will have same speed at the same height during ascent and during descent.

29. If $v_{\text{swimmer}} < v_{\text{river}}$ then one has to reach the directly opposite point on crossing the river. The drifted part on foot or by other means the minimise drift or minimum total time as per given problem. To minimize put first derivative zero.

30. If the frame of reference is noninertial, make it inertial by applying pseudo vectors before applying Newton's laws or equation of motion.

31. Projectile attains maximum range when θ (angle of projection) is 45°, on the same level. If projected from a height and the projectile reaches ground then angle is less than 45° and is determined using $\frac{dx}{d\theta} = 0$. where

$x = u_x$ (time of flight)

or $x = u_x$ (time spent in gravitational field).

32. Range will be same if a body is projected at θ or $(90 - \theta)$ (i.e. complimentary angle) with same velocity.

• Caution

1. In uniform motion $\vec{v} = \vec{v}_{av}$

⇒ Converse is, however, not true. That is if $\vec{v}_{av} = \vec{v}$, motion may or may not be uniform.

2. Applying $v = u + at$; $s = ut + \frac{1}{2}at^2$ etc. even when acceleration is not uniform.

⇒ When acceleration is not uniform and motion is not circular/rotational, use $\frac{dv}{dt} = a$ or $v\frac{dv}{dx} = a$ or $\frac{dx}{dt} = v$.

3. Applying $v = u + at$ etc. without modification when frame of reference is non-inertial.

⇒ For example, if the lift is moving up with an acceleration a then the effective acceleration for a body falling from the ceiling is $(g + a)$, i.e., apply vector algebra or relative acceleration.

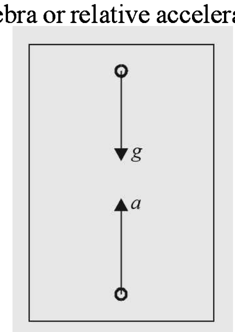


Fig. 3.16

4. Not differentiating between average and instantaneous velocities.

⇒ If a particle travels according to the equation $x = t^2 + 2t + 5$ where x is in metres and t in seconds. Then $v = \frac{dx}{dt}$ is instantaneous velocity. While $v_{av} = \frac{x(t_2) - x(t_1)}{t_2 - t_1}$ where t_2 and t_1 are final and initial times.

5. Applying direct equations of projectile when starting or terminating points are not the same vertical height or vertical displacement between initial and terminating point is non-zero.

⇒ Apply one-dimensional motion approach—one along x- and the other along y-axis with time of flight as combining factor.

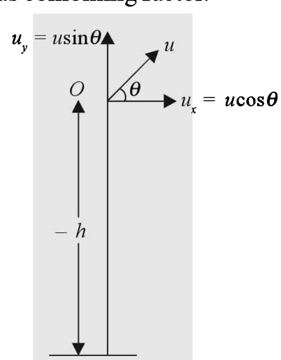


Fig. 3.17

6. Sticking to the origin at ground.

⇒ When the particle starts from a height h, consider it as origin so that the vertical displacement when it reaches the ground is h, i.e., use

$$-h = u\sin\theta\, t - \frac{gt^2}{2}$$

7. Not remembering common trignometric formulae.

⇒ Remember trigonometric relations like

$\sin 2\theta = 2\sin\theta\cos\theta$,

$\sin(180 - \theta) = \sin\theta$, $\sin(A + B) + \sin(A - B) = 2\sin A\cos B$ and $\sin A\sin B = \cos(A - B) - \cos(A + B)$

$\cos A\cos B = \frac{1}{2}[\cos(A + B) + \cos(A - B)]$.

8. Considering vertical distance given in problems in projectile motion as h_{max}.

⇒ It is not necessary that vertical distance given be h_{max}. If it is h_{max}, then velocity at this point is only horizontal velocity, i.e., vertical component of velocity is zero at the highest point. Otherwise use

$$y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$$

or $$y = x\tan\theta\left(1 - \frac{x}{R}\right).$$

Even in some cases these equations are not suitable, or make the problem lengthy then use

$y = u_y t - \frac{1}{2}gt^2$ and $x = u_x t$

9. Considering $x = u_x t$ along an inclined plane.

⇒ Along an inclined plane a_x is also present. Find out a_x and then apply $x = u_x t + \frac{1}{2}a_x t^2$.

10. Considering if the projectile strikes a wall or an obstacle its time of flight will change.

⇒ Time of flight remains fixed unless it is trapped somewhere.

SOLVED PROBLEMS

1. A ball is thrown up with a certain velocity so that it reaches a height h. Find the ratio of the times in which it is at $\frac{h}{3}$.

(a) $\frac{\sqrt{2}-1}{\sqrt{2}+1}$ (b) $\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$

(c) $\frac{\sqrt{3}-1}{\sqrt{3}+1}$ (d) $\frac{1}{3}$

Solution (b) $u^2 = 2gh$; $\frac{h}{3} = \sqrt{2gh}\,t - \frac{1}{2}gt^2$ or $gt^2 - 2\sqrt{2gh}\,t + \frac{2h}{3} = 0$.

$$t = \frac{2\sqrt{2gh} \pm \sqrt{8gh - (8gh)/3}}{2g} \quad \text{or}$$

$$\frac{t_1}{t_2} = \frac{2\sqrt{2gh} - 2\sqrt{2gh/3}(\sqrt{3}-1)}{2\sqrt{2gh} + 2\sqrt{2gh/3}(\sqrt{3}-1)}$$

$$= \frac{\sqrt{3}-(\sqrt{3}-1)}{\sqrt{3}+\sqrt{3}-1}$$

$$= \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$$

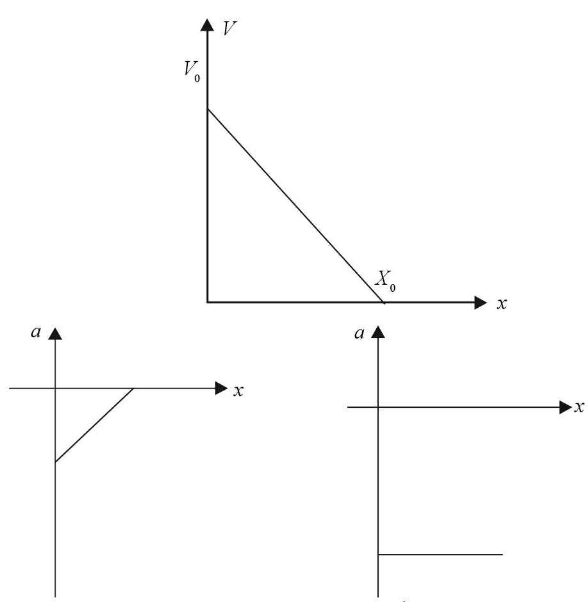
2. The displacement of a particle varies with time as $x = a e^{-\alpha t} + b e^{\beta t}$ where a, α, b, β are positive constants. The velocity of the particle

(a) will be independent of α and β
(b) drop to zero when $\alpha = \beta$
(c) go on decreasing with time
(d) go on increasing with time.

Solution (d) $\frac{dx}{dt} = -a\alpha e^{-\alpha t} + b\beta e^{\beta t}$ as t increases $\frac{-a\alpha}{e^{\alpha t}}$ decreases and $b\beta e^{\beta t}$ increases.

3. Convert given $v - x$ shown in Fig 3.18 to $a - x$ graph.

(IIT Screening 2005)



(a) (b)

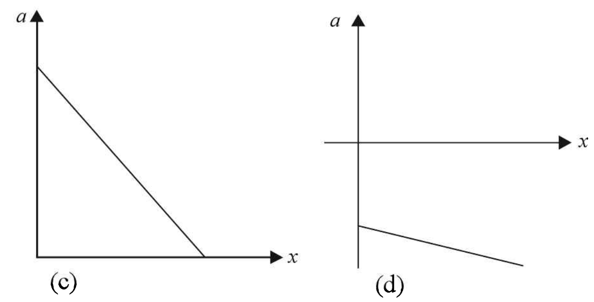


Fig. 3.18

Solution (a) equation of given curve is $v = \left(1 - \dfrac{x}{x_0}\right) v_0$

$$a = \frac{dv}{dt} = -\frac{v_0}{x_0}\frac{dx}{dt} = \frac{-v_0^2}{x_0}\left(1 - \frac{x}{x_0}\right).$$

4. The relation between time t and distance x is $t = ax^2 + bx$ where a and b are constant. The acceleration is

(a) $-2a\,bv^2$ (b) $2\,bv^3$
(c) $-2\,av^3$ (d) $2\,av^2$

(AIEEE 2005)

Solution (c) $t = ax^2 + bx$ or $\dfrac{dt}{dx} = 2ax + b$

or $$v = \frac{dx}{dt} = \frac{1}{2ax+b}.$$

$$\frac{dv}{dt} = \frac{-2a}{(2ax+b)^2}\frac{dx}{dt} = \frac{-2a}{(2ax+b)^3} = -2a\,v^3.$$

5. A car starting from rest accelerates at the rate f through a distance s, then continues at constant speed for time t and then decelerates at rate $\dfrac{f}{2}$ to come to rest. If the total distance covered is 15 s, then

(a) $s = \dfrac{ft^2}{72}$ (b) $s = \dfrac{ft^2}{4}$
(c) $s = \dfrac{ft^2}{6}$ (d) $s = \dfrac{ft^2}{2}$

(AIEEE 2005)

Solution (a) $s = v_0 t_1$ and $v_0 2t_1 = 2s$

Distance moved with uniform speed $(15 - 3)\,s = 12\,s$

$$v_0 = \sqrt{2sf}\ ;\quad 12\,s = v_0 t$$

$$12\,s = t\sqrt{2sf} \quad \text{or} \quad s = \frac{ft^2}{72}$$

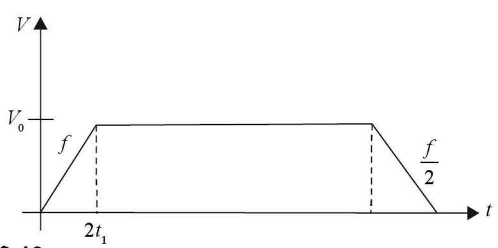


Fig. 3.19

6. A projectile can have the same range R for two angles of projection. If t_1 and t_2 are the times of flights in the two cases, then product of the time of flights is proportional to

(a) R^2 (b) $\dfrac{1}{R^2}$
(c) $\dfrac{1}{R}$ (d) R

(AIEEE 2005)

Solution (d) $t_1 = \dfrac{2u\sin\theta}{g}$, $t_2 = \dfrac{2u\cos\theta}{g}$ and

$$t_1 t_2 = \frac{2u^2\sin 2\theta}{g^2} = \frac{2R}{g}.$$

7. A particle is moving eastwards with a velocity 5 ms^{-1}. In 10 s, the velocity changes to 5 ms^{-1} northwards. The average acceleration in this time is

(a) $\dfrac{1}{\sqrt{2}}$ ms^{-2} *NE* (b) $\dfrac{1}{2}$ ms^{-2} *N*
(c) zero (d) $\dfrac{1}{\sqrt{2}}$ ms^{-2} *NW*

[AIEEE 2005]

Solution (d) $a_{av} = \dfrac{v_f - v_i}{t} = \dfrac{5i - 5\hat{i}}{10}$

$$= a = \frac{1}{\sqrt{2}}\ \text{ms}^{-2}\ \text{NW}.$$

8. A parachutist after bailing out falls 50 m without friction. When parachute opens, it decelerates at 2 ms^{-2}. He reaches the ground with a speed 3 ms^{-1}. At what height did he bail out?

(a) 91 m (b) 182 m
(c) 293 m (d) 111 m

Solution (c) $v^2 = 2gh = 2 \times 10 \times 50\ d$

$$= 50 + \left[\frac{3^2 - 2\times10\times50}{-2(2)}\right] = 293\ \text{m}.$$

9. In Fig. 3.20 the position time graph of a particle of mass 0.1 kg is shown. Find the impulse at $t = 2$ sec.

(a) 0.2 kg ms^{-1} (b) -0.2 kg ms^{-1}
(c) 0.1 kg ms^{-1} (d) -0.4 kg ms^{-1}

Solution (a) $dp = F.dt = m(v_f - 0) = 0.1\,(2) = 0.2$ kg ms^{-1}

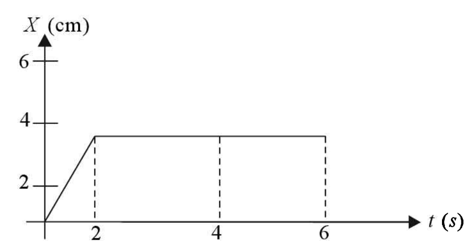


Fig. 3.20

10. When a ball is thrown up vertically with a velocity v_o it reaches a height h. If one wishes to triple the maximum height then the ball be thrown with a velocity

(a) $\sqrt{3}\, v_o$ (b) $3\, v_o$

(c) $9\, v_o$ (d) $\frac{3}{2}\, v_o$.

[AIIMS 2005]

Solution (a) $v^2 = 2gh$ or $v = \sqrt{2gh}$, i.e., $\frac{v_1}{v_2} = \sqrt{\frac{h_1}{h_2}}$

$\therefore v_2 = \sqrt{3}\, v_o$.

11. A car starts from rest, moves with an acceleration a and then decelerates at b for sometime to come to rest. If the total time taken is t, the maximum velocity is

(a) $\frac{abt}{a+b}$ (b) $\frac{a^2t}{a+b}$

(c) $\frac{at}{a+b}$ (d) $\frac{b^2t}{a+b}$

[BHU 2005]

Solution (a) $v = 0 + at_1$; $0 = at_1 - b\,(t - t_1)$

or $t_1 = \frac{bt}{a+b}$ $\therefore v_{max} = \frac{abt}{a+b}$.

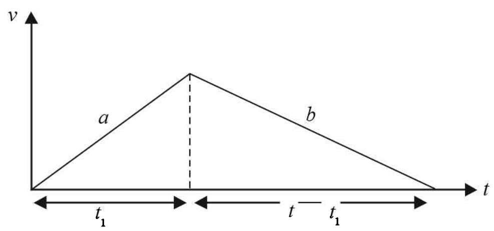


Fig. 3.21

12. From the top of a tower, two stones whose masses are in the ratio 1 : 2 are thrown, one straight up with an initial speed u and the second straight down with same speed u. Neglecting air resistance,

(a) the heavier stone hits the ground with a higher speed.
(b) the lighter stone hits the ground with a higher speed.
(c) both the stones will have same speed when they hit the ground.
(d) the speed cannot be determined with the given data.

Solution (c)

13. Two runners start simultaneously from the same point on a circular 200 m track in the same direction. Their speeds are 6.2 ms^{-1} and 5.5 ms^{-1}. How far from the starting point the faster will overcome the slower?

(a) 150 m away from the starting point
(b) 170 m away from the starting point
(c) 120 m away from the starting point
(d) none

Solution (b) $200 = (6.2 - 5.5)\, t$ or $t = 285.714$ s

$s = (6.2 \times 285.714) = 1770$ m (faster), $1770 - 8 \times 200 = 170$

Thus 170 m away from the starting point along the track in the direction of run.

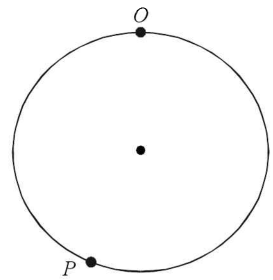


Fig. 3.22

14. A particle moves according to the equation $x = 2t^2 - 5t + 6$. Find (i) average velocity in the first 3 sec and (ii) velocity at $t = 3$ s.

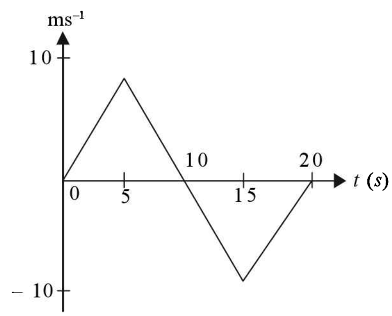
(a) 1 ms^{-1}, 7 ms^{-1} (b) 4 ms^{-1}, 3 ms^{-1}
(c) 2 ms^{-1}, 5 ms^{-1} (d) 3 ms^{-1}, 7 ms^{-1}

Solution (a) $x(3) = 2\,(3)^2 - 5\,(3) + 6 = 93x\,(0) = 6$

$$v_{av} = \frac{x(3) - x(0)}{3 - 0} = \frac{9 - 6}{3} = 1 \text{ ms}^{-1}$$

$$\left.\frac{dx}{dt}\right|_{t=3} = 4t - 5 = 4\,(3) - 5 = 7 \text{ ms}^{-1}$$

15. Plot acceleration time graph of the figure shown



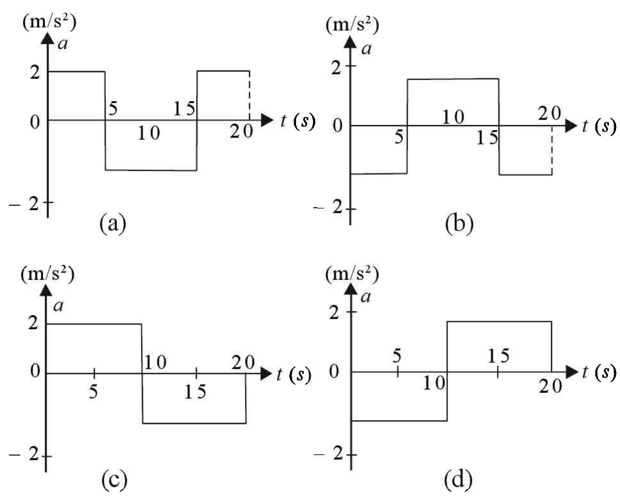


Fig. 3.23

Solution (a)

16. A girl after being angry throws her engagement ring from the top of a building 12 m high towards her boy friend with an initial horizontal speed of 5 ms^{-1}, speed with which the ring it touches the ground is

(a) 5 ms^{-1} (b) 14.3 ms^{-1}
(c) 1.5 ms^{-1} (d) 16.2 ms^{-1}

Solution (d) $v_y^2 = 2ay = 2 \times 10 \times 12$

$$v = \sqrt{25 + 240} = 16.2\ ms^{-1}.$$

17. The driver of a train A running at 25 ms^{-1} sights a train B on the same track with 15 ms^{-1}. The driver of train A applies brakes to produce a deceleration of 1.0 ms^{-2}. If the trains are 200 m apart, will the trains collide?

(a) yes (b) no
(c) collision just avoided (d) none of these

Solution (c) $v^2 - u^2 = 2as$ or $s = \dfrac{25^2 - 15^2}{2 \times 1} = 200$ m.

18. Two cars A and B are 5 m long each. Car A is at any instant just behind B. A and B are moving at 54 km/h and 36 km/h. Find the road distance covered by the car A to overtake B.

(a) 35 m (b) 30 m
(c) 32.5 m (d) 27.5 m

Solution (a) $v_{AB} = 15 - 10 = 5\ ms^{-1}$

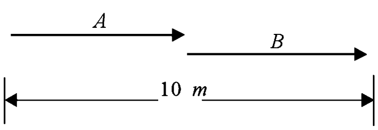


Fig. 3.24

$x_{AB} = 10$ m; $t = \dfrac{x_{AB}}{v_{AB}} = 2\ s$.

Road distance covered = $v_A\, t$ + length of car A

$= 15 \times 2 + 5 = 35$ m.

19. A flowerpot falls off a window sill and falls past the window below. It takes 0.5 s to pass through a 2.0 m high window. Find how high is the window sill from the top of the window?

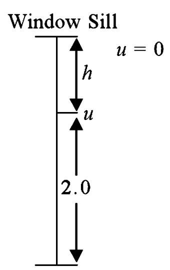


Fig. 3.25

(a) 10 cm (b) 7.5 cm
(c) 12.5 cm (d) 15 cm

Solution (c) $h = ut + \dfrac{1}{2}at^2$

or $2.0 = u(.5) + 5\left(\dfrac{1}{4}\right)$

or $u = 1.5\ ms^{-1}$.

Using $v^2 - u^2 = 2gh$

$$h = \frac{1.5^2}{2 \times 10} = \frac{2.25}{20} = 0.125\ m = 12.5\ cm.$$

20. A particle moves according to the law $a = -ky$. Find the velocity as a function of distance y, v_o is initial velocity.

(a) $v^2 = v_0^2 - ky^2$ (b) $v^2 = v_0^2 - 2ky$
(c) $v^2 = v_0^2 - 2ky^2$ (d) none

Solution (a) $a = \dfrac{dv}{dt} = \dfrac{dv}{dy} \cdot \dfrac{dy}{dt}$

or $\int_{v_o}^{v} v\,dv = \int_0^y -ky\,dy$ or $v_0^2 - v^2 = ky^2$.

21. A particle moves according to the equation $\dfrac{dv}{dt} = \alpha - \beta v$ where α and β are constants. Find velocity as a function of time. Assume body starts from rest.

(a) $v = \dfrac{\beta}{\alpha}(1 - e^{-\beta t})$ (b) $v = \dfrac{\alpha}{\beta}(1 - e^{-\beta t})$
(c) $v = \dfrac{\beta}{\alpha} e^{-\beta t}$ (d) $v - \dfrac{\alpha}{\beta} e^{-\beta t}$

Solution (b) $\int_0^v \dfrac{-\beta dv}{\alpha - \beta v} = -\beta \int_0^t dt$

or $\log_e \dfrac{(\alpha - \beta v)}{\alpha} = -\beta t$ or $v = \dfrac{\alpha}{\beta}(1 - e^{-\beta t})$

22. A boat moves relative to water with a velocity v and river is flowing with $2v$. At what angle the boat shall move with the stream to have minimum drift?

(a) 30° (b) 60°
(c) 90° (d) 120°

Solution (d) Let boat move at angle θ to the normal as shown in Fig. 3.26 then time to cross the river $= \dfrac{l}{v\cos\theta}$.

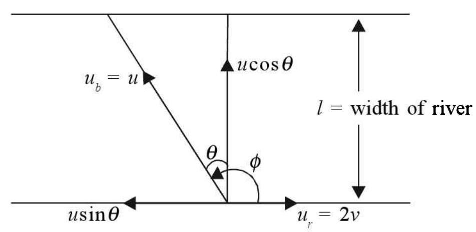


Fig. 3.26

drift $x = (2v - v\sin\theta)\dfrac{l}{v\cos\theta}$ for x to be minimum.

$\dfrac{dx}{d\theta} = 0 = 1(2\sec\theta\tan\theta - \sec^2\theta)$ or $\sin\theta = \dfrac{1}{2}$

or $\theta = 30°$ and $\phi = 90 + 30 = 120°$.

23. A car starts moving rectilinearly from rest with 5 ms^{-2} for sometime, then uniformly and finally decelerates at 5 ms^{-2} and comes to a stop. The total time of motion equals 25 s. How long does the car move uniformly? Given $V_{av} = 72$ km/h during motion.

(a) 5 s (b) 10 s
(c) 15 s (d) 20 s

Solution (c) Total distance covered = area under $v - t$ graph. From Fig. 3.27

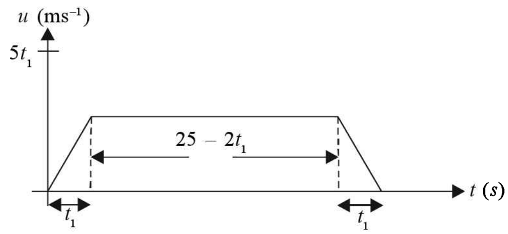


Fig. 3.27

$20 \times 25 = 5\,t_1^{\,2} + (25 - 2t_1)\,5t_1$

or $5t_1^{\,2} - 125\,t_1 + 500 = 0$

or $(t_1 - 5)\,(t_1 - 20) = 0$

$\Rightarrow$ $t_1 = 5\,s$ discard $t_1 = 20\,s$.

24. A ship moves along the equator to the east with a speed 30 km/h. Southeastern wind blows 60° to the east with 15 kmh⁻¹. Find the wind velocity relative to the ship.

(a) 39.7 kmh⁻¹, $\tan^{-1}\frac{1}{5}$ N of W

(b) 23.7 kmh⁻¹, $\tan^{-1}\frac{1}{3}$ N of W

(c) 37.5 kmh⁻¹, $\tan^{-1}\frac{1}{5}$ N of E

(d) none

Solution (a) $v_{ws} = v_w - v_s$

$$= (15\cos 60\,\hat{i} + 15\sin 60\,\hat{j}) - 30\,\hat{i}$$

$$|v| = \sqrt{(39.5)^2 + (7.5)^2} = 39.7\ \text{kmh}^{-1}$$

$$\tan\beta = \frac{7.5}{37.5} = \frac{1}{5}$$

$$\beta = \tan^{-1}\frac{1}{5}\ \text{North of West.}$$

25. A ball is thrown up with a velocity v_o and it returns to the spot of throw. Plot $v - t$ and $v - x$ graphs.

Solution $v = u + at$

and $a = -g$ is a straight line [Fig 3.28 (a)]

$v^2 - u^2 = 2\,ax$ is parabolic [Fig 3.28 (b)]

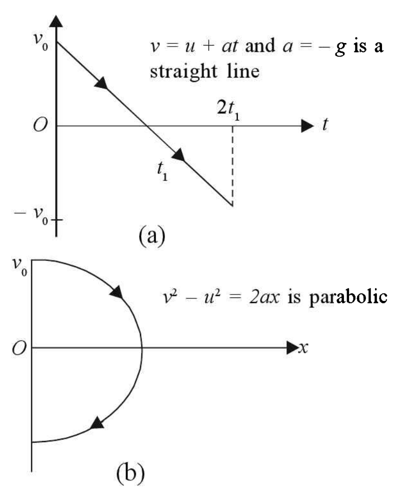


Fig. 3.28

26. From a tap 10 m high drops fall at regular intervals. When the first drop reaches the ground, the 5th drop is about to leave the tap. Find the separation between 2nd and 3rd drops.

(a) $\frac{35}{8}$ m (b) $\frac{31}{8}$ m

(c) $\frac{27}{8}$ m (d) none of these

Solution $\frac{1}{2}gt^2 = 10$ or $t = \sqrt{2}$ s

time interval $\Delta t = \frac{\sqrt{2}}{4} = \frac{1}{2\sqrt{2}}$ s.

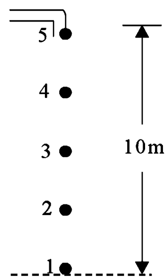


Fig. 3.29

$$x_2 - x_3 = \frac{1}{2}g\left[\left(\frac{3}{2\sqrt{2}}\right)^2 - \left(\frac{2}{2\sqrt{2}}\right)^2\right]$$

$$= 5\left[\frac{9}{8} - \frac{1}{2}\right] = \frac{25}{8}\ \text{m}$$

27. When a ball is h metre high from a point O, its velocity is v_o. When it is h m below O, its velocity is $2v$. Find the maximum height from O it will acquire

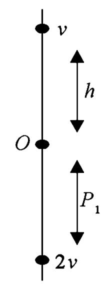


Fig. 3.30

(a) $\frac{2h}{3}$ (b) $\frac{5h}{3}$

(c) $\frac{3h}{2}$ (d) $2\,h$

Solution (b) $(2v)^2 - v^2 = 2g\,(2h)$

or $\frac{v^2}{2g} = \frac{2}{3}h$;

$$h_{max} = h + \frac{2h}{3} = \frac{5h}{3}$$

28. The first stage of the rocket launches a satellite to a height of 50 km and velocity attained is 6000 kmh⁻¹ at which point its fuel exhausted. How high the rocket will reach?

(a) 138.9 km (b) 188.9 km

(c) 88.9 km (d) 168.9 km

Solution (b) $h = \frac{v^2}{2g} + 50$ km

$$= \frac{(5000/3)^2}{20 \times 1000} + 50 = 188.9 \text{ km}.$$

29. A particle moves according to the equation $t = \sqrt{x} + 3$, when the particle comes to rest for the first time

(a) 3 s (b) 2.5 s
(c) 3.5 s (d) none of these

Solution (a) $x = (t - 3)^2$

$$v = \frac{dx}{dt} = -2(t-3) = 0 \text{ or } t = 3\ s.$$

30. A particle of mass m is projected with a velocity $6\hat{i} + 8\hat{j}$. Find the change in momentum when it just touches ground.

(a) 0 (b) 12m
(c) 16m (d) 20m

Solution (c) $\Delta p = m(v_f - v_i)$

$= m[(6\hat{i} - 8\hat{j}) - (6\hat{i} + 8\hat{j})] = -16 m\ \hat{j}$.

$|\Delta p| = 16\ m$

31. A particle is projected with a velocity $6\hat{i} + 8\hat{j}$, 3 m away from a vertical wall. After striking the wall it lands at ..... away from the wall.

(a) 3 m (b) 3.3 m
(c) 5.5 m (d) 6.6 m

Solution (d) $T = \frac{2u_y}{a_y} = \frac{2 \times 8}{10} = 1.6\ s$

$$t = \frac{3}{6} = 0.5 \text{ s}.$$

$$x = u_x(T - t) = 6(1.6 - 0.5) = 6.6 \text{ m}$$

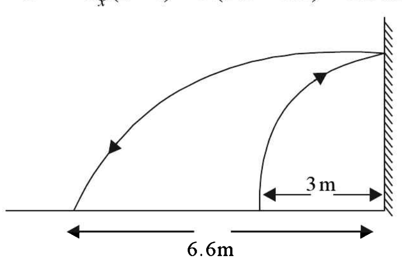


Fig. 3.31

32. The radius vector of a point A relative to the origin varies as $r = at\hat{i} + bt^2\hat{j}$ where a and b are positive constants. Find the equation of trajectory.

(a) $y = \frac{b}{a^2}x^2$ (b) $y^2 = \frac{b}{a^2}x$

(c) $y = \frac{a^2}{b}x^2$ (d) none of these

Solution (a) $x = at, y = bt^2$ or $y = b\left(\frac{x}{a}\right)^2$.

33. A particle moves in the xy plane as $v = a\hat{i} + bx\hat{j}$ where $\hat{i}$ and $\hat{j}$ are the unit vectors along x and y axis. The particle starts from origin at $t = 0$. Find the radius of curvature of the particle as a function of x.

(a) $\frac{a^2 + b^2x^2}{ba}$ (b) $\frac{a}{b}\left[1 + \left(\frac{bx}{a}\right)^2\right]^{\frac{3}{2}}$

(c) $\frac{b}{a}\left[1 + \left(\frac{ax}{b}\right)^2\right]^{\frac{3}{2}}$ (d) none of these

Solution (b) $\frac{dv}{dt} = a$ or $x = at$

$$\frac{dy}{dt} = bat \text{ or } y = \frac{bat^2}{2} \text{ or } y = \frac{bx^2}{2a}$$

$$\frac{dy}{dx} = \frac{b}{a}x \text{ and } \frac{d^2y}{dx^2} = \frac{b}{a}$$

$$R = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\frac{d^2y}{dx^2}} = \frac{\left[1 + \left(\frac{b}{a}x\right)^2\right]^{\frac{3}{2}}}{\frac{b}{a}}$$

$$= \frac{a}{b}\left[1 + \left(\frac{b}{a}x\right)^2\right]^{\frac{3}{2}}$$

34. A man riding on a flat car moving with 10 ms^{-1}. He attempts to throw a ball through a stationary hoop 5 m above his hand such that the ball moves horizontally through the hoop. He throws the ball with 12 ms^{-1} with respect to himself. Find the horizontal distance from where he throws the ball.

(a) 15 m (b) 14.2 m
(c) 16.7 m (d) 18.2 m

Solution (c) $h_{max} = 5 = \frac{u_y^2}{2g}$ $\therefore u_y = 10$

$$u_x = \sqrt{12^2 - 10^2} = \sqrt{44}.$$

$$v_x = 10 + \sqrt{44}\ ; \frac{T}{2} = \frac{u_y}{g} = 1\ s\ ;$$

$$x = v_x \cdot \frac{T}{2} = 10 + \sqrt{44} = 16.7 \text{ m}$$

35. A body standing on a long railroad car throws a ball straight upwards, the car is moving on the horizontal road with an acceleration 1 ms^{-2}. The vertical velocity given is 9.8 ms^{-1}. How far behind the boy the ball will fall on the railroad car?

(a) 1 m (b) $\frac{3}{2}$ m

(c) $\frac{7}{4}$ m (d) 2 m

Solution (d) $T = \frac{2u_y}{g} = 2 \times \frac{9.8}{9.8} = 2$ s ;

$$x = \frac{1}{2}a_x t^2 = \frac{1}{2}(1)(2)^2 = 2 \text{ m}.$$

36. Find the average velocity of a projectile between the instant it crosses one third the maximum height. It is projected with u making an angle θ with the vertical.

(a) $u \cos\theta$ (b) u
(c) $u \sin\theta$ (d) $u \tan\theta$

Solution (c) Note carefully the vertical velocities at the same height are in opposite directions and therefore their average sum = 0. It is horizontal velocity which is uniform and hence $v_{av} = u \sin \theta (= u_x)$.

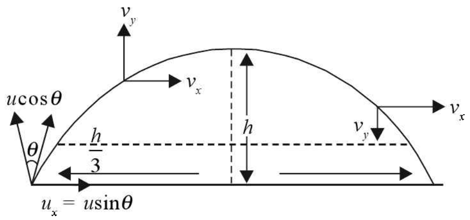


Fig. 3.32

37. A person is standing on a truck moving with 14.7 ms^{-1} on a horizontal road. He throws a ball so that it returns to him when the truck has moved 58.8 m. Find the speed of the ball and angle of projection as seen by a man standing on the road.

(a) 22.5 ms^{-1}, 53° (b) 24.5 ms^{-1}, 53°
(c) 19.6 ms^{-1}, vertical (d) none of these

Solution (b)

$$T = \frac{58.8}{14.7} = 4 \text{ s}$$

$$T = \frac{2u_y}{g} = 4 \therefore u_y = 19.6 \text{ ms}^{-1}$$

$$v = \sqrt{14.7^2 + 19.6^2}$$

$$= 24.5 \text{ ms}^{-1}$$

$$\tan \beta = \frac{v_y}{v_x} = \frac{19.6}{14.7} = \frac{4}{3} \text{ or } \beta = 53° \text{ wrt horizontal.}$$

38. Six persons are situated at the corners of a hexagon of side l. They move at a constant speed v. Each person maintains a direction towards the person at the next corner. When will the persons meet?

(a) $\frac{l}{v}$ (b) $\frac{2l}{3v}$
(c) $\frac{3l}{2v}$ (d) $\frac{2l}{v}$.

Solution (d) $t = \frac{l}{v_{AB}} = \frac{l}{v_A - v_B \text{ in the direction of A}}$

$$= \frac{l}{v - v\cos 60} = \frac{2l}{v}.$$

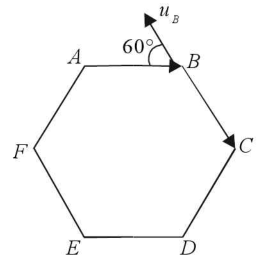


Fig. 3.33

39. The compass needle of the airplane shows it is heading due North and speedmeter indicates a velocity 240 km h^{-1}. Wind is blowing 100 km h^{-1} due east. Find the velocity of airplane with respect to earth.

(a) 260 ms^{-1}, 23° E of N (b) 260 ms^{-1}, 23° W of N
(c) 260 ms^{-1}, 32° E of N (d) none

Solution (a) $v_{AE} = 100\,\hat{i} + 240\,\hat{j}$

$$v_{AE} = \sqrt{(240)^2 + 100^2} = 260 \text{ ms}^{-1};$$

$$\phi = \tan^{-1}\left(\frac{100}{240}\right) = 23° \text{ E of N.}$$

40. In an exhibition, you win a prize if you toss a coin into a small dish placed. The dish is on a sheep 2.1 m away at a height h from the hand. The coin is tossed into the dish if its velocity is 6.4 ms^{-1} at an angle of 60°. Find h.

(a) 1.2 m (b) 1.35 m
(c) 1.5 m (d) 1.65 m

Solution (c) $y = x \tan \theta - \frac{gx^2}{2u^2 \cos^2 \theta}$

$$= 2.1 \tan 60 - \frac{9.8(2.1)^2}{2 \times 6.4^2 \times \left(\frac{1}{2}\right)^2}$$

$$= 2.1\sqrt{3} - \frac{4.9 \times 4.4}{10.24}$$

$$= 1.5 \text{ m}$$

TYPICAL PROBLEMS

41. A projectile is launched from a height h making an angle θ with the horizontal with speed v_o. Find the horizontal distance covered by it before striking the ground.

Solution $-h = v_o \sin \theta t - \frac{1}{2} g t^2$

or $g t^2 - 2 v_o \sin \theta t - 2h = 0.$

$$t = \frac{2v_o \sin\theta + \sqrt{4v_o^2 \sin^2\theta + 8gh}}{2g}$$

$$x = \frac{v_o \cos\theta}{2}\left[v_o \sin\theta + \sqrt{v_o^2 \sin^2\theta + 2gh}\right]$$

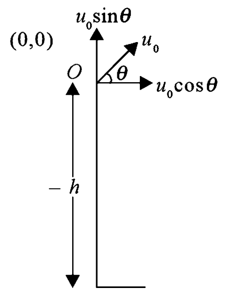


Fig. 3.34

42. A baseball is projected with a velocity v making an angle θ with the incline of indication α as shown in Fig. 3.35 (a). Find the condition that the ball hits the incline at right angle.

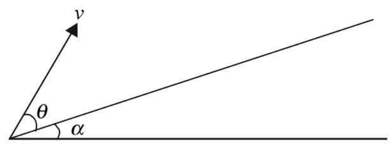


Fig. 3.35 (a)

(a) cot θ = tan θ (b) sin θ = cos α
(c) tan θ = sin α (d) cot θ = cos α

Solution (a) $T = \frac{2u_y}{|a_y|} = \frac{2v\sin\theta}{g\cos\alpha}$. It will hit vertically the incline if $v_x = 0$.

$0 = v \cos\theta T - g \sin\alpha T^2$

or $v\cos\theta\left(\frac{2v\sin\theta}{g\cos\alpha}\right) - \frac{g\sin\alpha}{2}\left(\frac{2v\sin\theta}{g\cos\alpha}\right)^2 = 0$

$\frac{2v^2\sin\theta}{g\cos\alpha}[\cos\theta\cos\alpha - \sin\alpha\sin\theta] = 0$

or cot θ = tan α

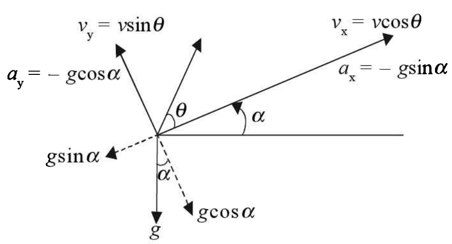


Fig. 3.35 (b)

43. An elevator is moving with 2.5 ms^{-1}. A bolt in the elevator ceiling 3 m above the elevator falls. How long does it take for the bolt to fall on the floor of elevator?

(a) 0.731 *s* (b) 0.762 *s*
(c) 0.782 *s* (d) 8.31 *s*

Solution (c) $\frac{1}{2} g t^2 = 3$ $u_{\text{initial}} = v_{\text{rel}} = 0$ because bolt will also get a velocity 2.5 ms^{-1}.

$t = \sqrt{.6} = 0.782\ s$

44. A point moves on the xy plane according to the law $x = a \sin \omega t$ and $y = a(1 - \cos\omega t)$ where a and ω are positive constants. Find the distance covered in time t_o.

(a) $a\,\omega\,t_o$ (b) $\sqrt{2a^2 + 2a^2\cos\omega t_o}$

(c) $2a\,\frac{\sin\omega t_o}{2}$ (d) $2a\,\frac{\cos\omega t_o}{2}$.

Solution $v_x = a\omega\cos\omega t$ and

$v_y = a\omega\sin\omega t$

or $v = a\omega\cos\text{cot}\,\hat{i} + a\omega\sin\omega t\,\hat{j}$

or $|v| = a\omega.$

$s = |v|\,t_o = a\omega t_o.$

45. A particle moves with a deceleration $\propto \sqrt{v}$. Initial velocity is v_o. Find the time after which it will stop.

(a) $\frac{\sqrt{v_o}}{k}$ (b) $\frac{\sqrt{v_o}}{2k}$

(c) $\frac{2\sqrt{v_o}}{k}$ (d) none of these

Solution (c) $\frac{dv}{dt} = -k\sqrt{v}$

or $\int_{v_o}^{0}\frac{dv}{\sqrt{v}} =$ or $t = \frac{2\sqrt{v_o}}{k}$.

46. A particle moves according to the equation $v = a\sqrt{x}$. Find the average velocity in the first s metres of the path.

(a) $\frac{\sqrt{s}}{a}$ (b) $\frac{\sqrt{s}}{2a}$

(c) $\frac{2a}{\sqrt{s}}$ (d) $\frac{2\sqrt{s}}{a}$

Solution (d) $\frac{dx}{dt} = a\sqrt{x}$

or $\int_0^s \frac{dx}{\sqrt{x}} = \int a\,dt$ or $t = \frac{2\sqrt{s}}{a}$.

$v_{av} = \frac{s}{t} = \frac{2\sqrt{s}}{a}$.

47. Particle A moves uniformly with velocity v so that vector v is continually aimed at point B which moves rectilinearly with a velocity $u < v$. At $t = 0$, v and u are perpendicular. Find the time when they converge. Assume A and B are separated by l at $t = 0$.

Solution A approaches B with a velocity $= v - u\cos\alpha$.

$\frac{dx}{dt} = v - u\cos a$

$\int_0^d dx = \int_0^t (v - u\cos\alpha)dt$

or $\frac{l - vt}{u} = \int -\cos\alpha\,dt$

$ut = \int v\cos\alpha\,dt$ or $ut = \frac{-v(l - vt)}{u}$

or $(v^2 - u^2) = lv$

or $t = \frac{lv}{v^2 - u^2}$.

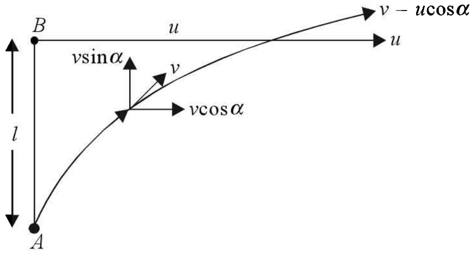


Fig. 3.36

48. From point A located on a highway, one has to get by a car as soon as possible to point B located in the field at a distance l from point D. If the car moves n times slower in the field, at what distance x from D one must turn off the highway.

Solution Let v be the velocity in the field and nv in the velocity on the highway.

Then $t_1 = \dfrac{AD-x}{nv}$ and $t_2 = \dfrac{\sqrt{l^2+x^2}}{v}$

For t to be minimum $\dfrac{d}{dx}(t_1+t_2)=0$

$$\frac{d}{dt}\left[\frac{1}{v}\left\{\left(\frac{AD-x}{n}\right)-\sqrt{l^2+x^2}\right\}\right]$$

$$=\frac{1}{n}-\frac{x}{\sqrt{l^2+x^2}}=0$$

or $l^2+x^2=n^2x^2$.

or $x=\dfrac{l}{\sqrt{n^2-1}}$

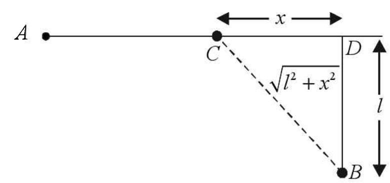


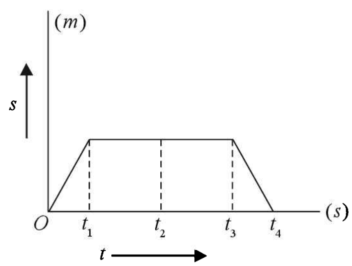
Fig. 3.37

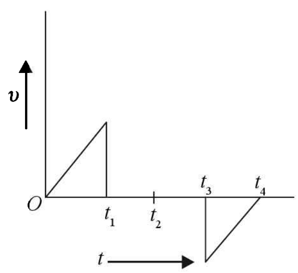
QUESTIONS FOR PRACTICE

1. A car moves in a semicircular track of radius 700 m. If it starts from one end of the track and stops at the other end, the displacement of car is

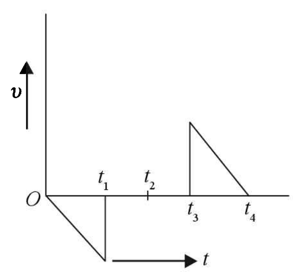
(a) 2200 m (b) 700 m
(b) 1400 m (d) none of these

2. Displacement-time graph of a body is shown below. Velocity-time graph of the motion of the body is

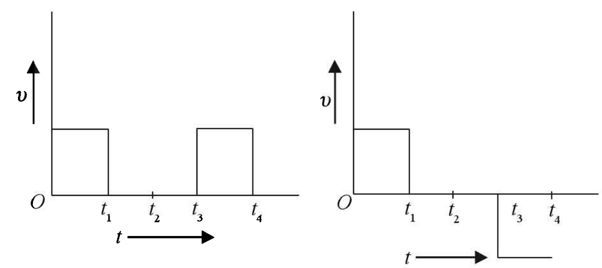




(a)



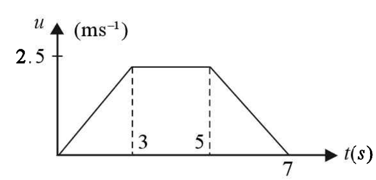
(b)

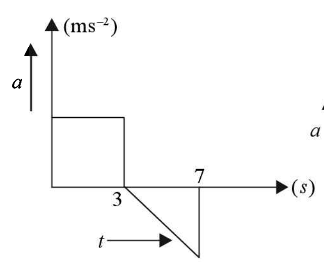


(c) (d)

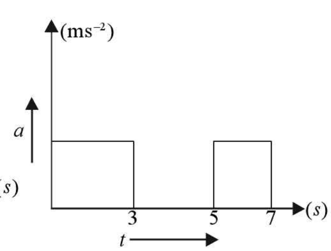
Fig. 3.38

3. For Figure of Q.3, the acceleration-time graph of the motion of the body is





(a)



(b)

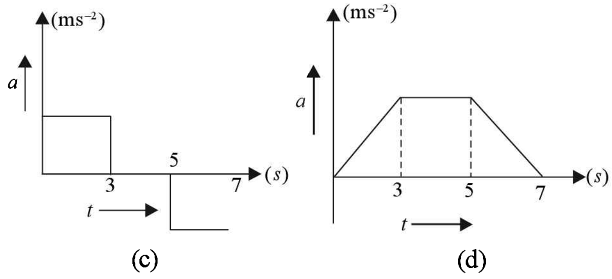


Fig. 3.39

4. A boy can throw a stone to maximum height of 50 m. To what maximum range can he throw this stone and to what height so that the maximum range is maintained?

(a) 100 m, 100 m (b) 100 m, 25 m
(c) 200 m, 50 m (d) 100 m, 50 m

5. A player throws a ball upwards with an initial speed of 29.4 ms^{-1}. The height to which the ball rises and the time taken to reach the player's hands are

(a) 22.05 m, 38 *s* (b) 44.1 m, 6 *s*
(c) 29.4 m, 6 *s* (d) 54.5 m, 9 *s*

6. It was known that a shell fired with a given velocity and at an angle of projection $\frac{5\pi}{36}$ radian can strike a target but a hill was found to obstruct its path. The angle of projection to hit the target should be

(a) Data is insufficient (b) $\frac{13\pi}{16}$ radian
(c) $\frac{10\pi}{36}$ radian (d) $\frac{23\pi}{26}$ radian

7. For the velocity-time curve shown below, the distance covered by a body from 5th to 7th second of its motion is ----- fraction of the total distance covered by it.

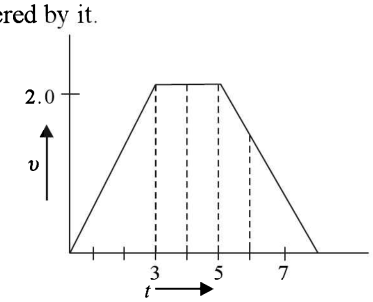


Fig. 3.40

(a) 2/9 (b) 9/2
(c) 1/2 (d) 2/3

8. A vehicle moves west with a speed of 50 ms^{-1} and then towards north with a speed of 50 ms^{-1} only. Total time taken by the body is 5s. What is the average acceleration of the body?

(a) 0 (b) 50 ms^{-2}
(c) 14 ms^{-2} (d) 20.4 ms^{-2}

9. A body is projected at an angle θ with the vertical with kinetic energy KE. What is the kinetic energy of the particle at the highest point?

(a) KE $\cos^2\theta$ (b) KE $\sin^2\theta$
(c) $\frac{KE}{2}$ (d) KE $\tan^2\theta$

10. A ball is thrown from the ground to clear a wall 3 m high at a distance of 6 m and falls 18 m away from the wall, the angle of projection of ball is

(a) $\tan^{-1}\frac{3}{2}$ (b) $\tan^{-1}\frac{2}{3}$
(c) $\tan^{-1}\frac{1}{2}$ (d) $\tan^{-1}\frac{3}{4}$

11. If the velocity of the motorcycle v is constant, then determine the velocity of the mass as a function of x. Given that ends P and R are coincident on Q when $x = 0$.

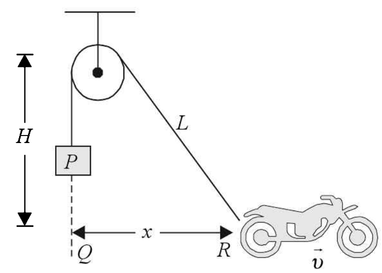


Fig. 3.41

(a) $\frac{xv}{\sqrt{H^2+x^2}}$ (b) $\frac{H^2+x^2}{xv}$
(c) $\sqrt{\frac{H^2+x^2}{vx}}$ (d) $\frac{H^2+x^2}{(vx)^2}$

12. Three points are located at the vertices of an equilateral triangle having each side as α. All the points move simultaneously with speed u such that first point continually heads for second, the second for the third and the third for the first. Time taken by the points to meet at the centre is

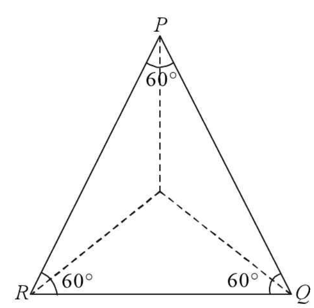


Fig. 3.42

(a) $\frac{\alpha}{3u}$ (b) $\frac{2\alpha}{3u}$
(c) $\frac{\alpha^2}{u^3}$ (d) $\frac{3\alpha}{2u}$

13. A wall clock has a 5 cm long minute hand. The average velocity of the tip of the hand reaching 0600 hrs. to 1830 hrs. is

(a) 2.2×10^{-14} cms^{-1} (b) 1.2×10^{-4} cms^{-1}
(c) 5.6×10^{-3} cms^{-1} (d) 3.2×10^{-3} cms^{-1}

14. A particle leaves the origin at $t = 0$ and moves in the positive x-axis direction. Velocity of the particle at any instant is given by $v = u\left(1 - \frac{t}{t'}\right)$. If $u = 10$ ms^{-1} and $t' = 5$s find the x coordinate of the particle at an instant of 10 s.

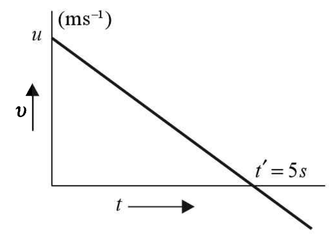


Fig. 3.43

(a) 0 (b) 10 m
(c) 20 m (d) –10 m

15. The acceleration of a particle is increasing linearly with time t as βt. If the particle starts from origin with initial velocity u, the distance travelled by it in t second is

(a) $ut + \frac{1}{2}\beta t^3$ (b) $ut + \frac{1}{2}\beta t^3$
(c) $ut + \frac{1}{3}\beta t^3$ (d) $ut + \frac{\beta t^3}{6}$

16. A drunkard takes a step of 1 m in 1 s. He takes 5 steps forward and 3 steps backward and so on. The time taken by him to fall in a pit 13 m away from the start is

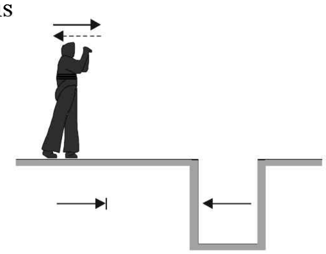


Fig. 3.44

(a) 26 s (b) 31 s
(c) 37 s (d) 41 s

17. A jeep moves at uniform speed of 60 kmh^{-1} on a straight road blocked by a wall. The jeep has to take a sharp perpendicular turn along the wall. A rocket flying at uniform speed of 100 kmh^{-1} starts from the wall towards the jeep when the jeep is 30 km away.

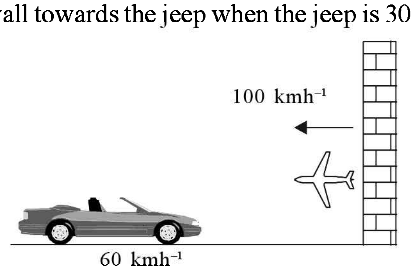


Fig. 3.45

The rocket reaches the windscreen and returns to wall. Total distance covered by the rocket is

(a) 100 km (b) 50 km
(c) 25 km (d) 75 km

18. A marble rolls down from top of a staircase with constant horizontal velocity u.

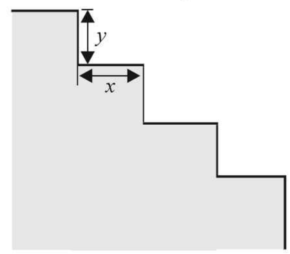


Fig. 3.46

If each step is y metre high and x metre wide, the marble just hits the edge of the nth step when n =

(a) $\frac{xu^2}{gy^2}$ (b) $\frac{u^2}{gy}$
(c) $\frac{2yu^2}{gx^2}$ (d) $\frac{4yu^2}{gx^2}$

19. A particle experiences a fixed acceleration for 6s after starting from rest. It covers a distance of s_1 in first two seconds, s_2 in the next 2 seconds and s_3 in the last 2 seconds then $s_3 : s_2 : s_1$ is

(a) 1 : 3 : 5 (b) 5 : 3 : 1
(c) 1 : 2 : 3 (d) 3 : 2 : 1

20. A football dropped from a height onto an elastic net, stretched horizontally much above the ground rebounds. The graph for the motion is

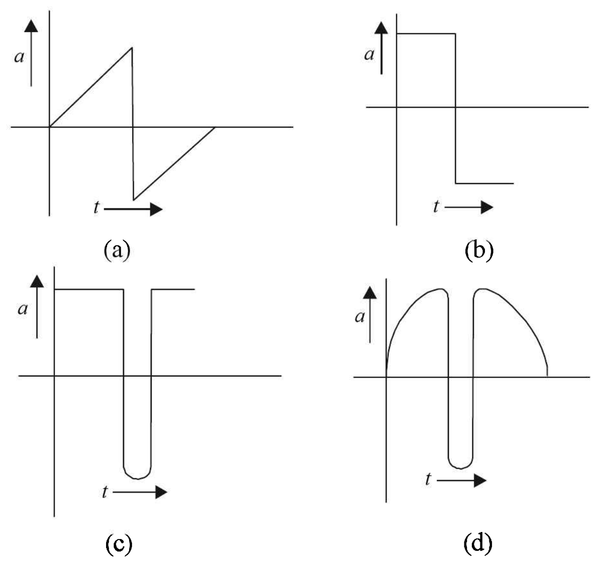


Fig. 3.47

21. A projectile is required to hit a target whose coordinates relative to horizontal and vertical axes through the point of projection are (α, β). If the gun velocity is $\sqrt{2g\alpha}$, it is impossible to hit the target if

(a) $\beta > \frac{3}{4}\alpha$ (b) $\beta \geq \frac{1}{4}\alpha$
(c) $\beta \leq \frac{3}{4}\alpha$ (d) $\beta \geq \frac{3}{4}\alpha$

22. A stone is allowed to fall from the top of a tower and cover half the height of the tower in the last second of its journey. The time taken by the stone to reach the foot of the tower is

(a) $(2-\sqrt{2})\,s$ (b) $4\,s$
(c) $(2+\sqrt{2})\,s$ (d) $(2\pm\sqrt{2})\,s$

23. A balloonist is ascending at a velocity of 12 ms^{-1} and acceleration 2 ms^{-2}. A packet is dropped from it when it is at a height of 65 m from the ground, it drops a packet. Time taken by the packet to reach the ground is

(a) $5\,s$ (b) $-5\,s$
(c) $7\,s$ (d) $\frac{13}{5}\,s$

24. Two shells are fired from a cannon with a speed u each at angles of α and β, respectively, to the horizontal. The time interval between the shots is T. They collide in mid-air after time t from the first shot. Which of the following conditions is not satisfied?

(a) $\alpha > \beta$
(b) $t\cos\alpha = (t-T)\cos\beta$
(c) $(t-T)\cos\alpha = t\cos\beta$
(d) $(u\sin\alpha)\,t - \frac{1}{2}gt^2 = (u\sin\beta)(t-T)\ \frac{1}{2}g(t-T)^2$

25. A river is flowing from west to east at a speed of 5 metres per minute. A man on the south bank of the river, capable of swimming at 10 metres per minute in still waters, wants to swim across the river in the shortest time. He should swim in a direction

(a) due north (b) 30° east of north
(c) 30° north of west (d) 60° east of north

26. In the arrangement shown in figure, the ends P and Q of an inextensible string move downwards with uniform speed u. Pulleys A and B are fixed. The mass M moves upwards with a speed.

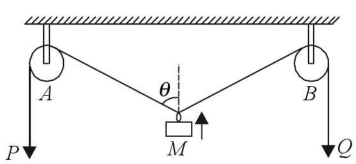


Fig. 3.48

(a) $2u\cos\theta$ (b) $u/\cos\theta$
(c) $2u/\cos\theta$ (d) $u\cos\theta$

27. Two extremely small blocks are lying on a smooth uniform rod of mass M and length L. Initially the blocks are lying at the centre. The whole system is rotating with an angular velocity ω_0 about an axis passing through the centre and perpendicular to the rod. When the blocks reach the ends of the rod, then the angular velocity of the rod will be

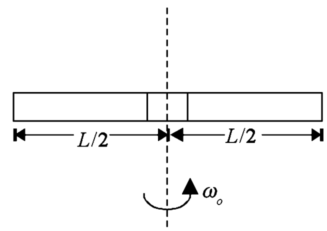


Fig. 3.49

(a) $\frac{M\omega_0}{M+2m}$ (b) $\frac{M\omega_0}{M+4m}$
(c) $\frac{M\omega_0}{M+6m}$ (d) $\frac{M\omega_0}{M+8m}$

28. A rocket is projected vertically upwards, whose time velocity graph is shown in. The maximum height reached by the rocket is

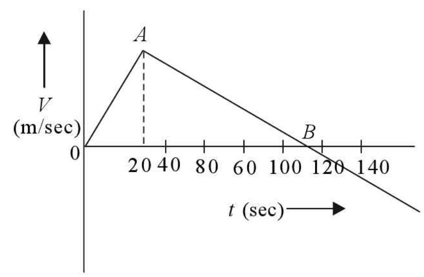


Fig. 3.50

(a) 1 km (b) 10 km
(c) 20 km (d) 60 km

29. Three blocks of mass m_1, m_2 and m_3 are lying in contact with each other on a horizontal frictionless plane as shown in the figure. If a horizontal force F is applied on m_1 then the force at the contact plane of m_1 and m_2 will be

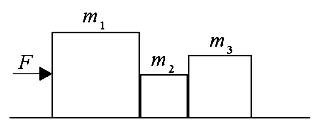


Fig. 3.51

(a) $\frac{F(m_2+m_3)}{(m_1+m_2+m_3)}$ (b) $\frac{m_1+F}{(m_1+m_2+m_3)}$
(c) m_1F (d) $\frac{F(m_1+m_2)}{(m_1+m_2+m_3)}$

30. A bullet is fired from a rifle. If the rifle recoils freely then the kinetic energy of the rifle will be

(a) equal to that of the bullet.
(b) less than that of the bullet.
(c) more than that of the bullet
(d) zero

31. A small disc is lying on the top of a hemispherical bowl of radius R. The minimum speed to be imparted to the disc so that it may leave the bowl without slipping is

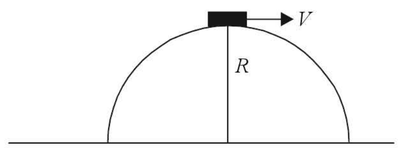


Fig. 3.52

(a) $v = \sqrt{g\frac{R}{2}}$ (b) $v = 2\sqrt{gR}$

(c) $v = \sqrt{gR}$ (d) $v = \sqrt{2gR}$

32. The quantity which remains constant for a body moving in a horizontal circle, is

(a) kinetic energy (b) acceleration
(c) force (d) velocity.

33. Force F varies with time in accordance with the following figure. The mean force will be

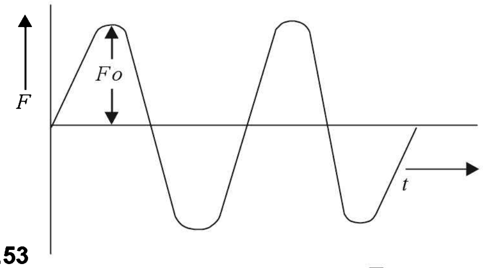


Fig. 3.53

(a) F_0 (b) $\frac{F_0}{2}$

(c) $2F_0$ (d) Zero

34. An object of mass 150 kg. is to be lowered with the help of a string whose breaking strength is 100 Kg/wt. What should be the minimum acceleration of the body so that the string may not break ?

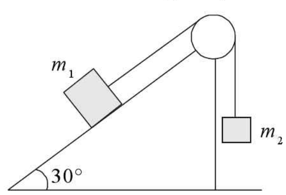


Fig. 3.54

(a) 2 m/s^2 (b) 4 m/s^2
(c) 3.33 m/s^2 (4) 4.5 m/s^2

35. Two blocks of mass 8 kg. and 5 kg are connected by a heavy rope of mass 3 kg. An upward force of 180 N is applied as shown in Fig. 3.55. The tension in the string at point P will be

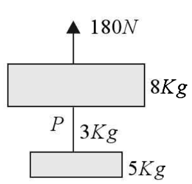


Fig. 3.55

(a) 60 N (b) 90 N
(c) 120 N (d) 150 N

36. A body is released from the top of a tower. The body covers a distance of 24.5 m in the last second of its motion. The height of tower is

(a) 59.8 m (b) 44.1 m
(c) 39.2 m (d) 49 m

37. A meter scale is suspended freely from one of its ends. Its another end is given a horizontal velocity v such that it completes one revolution in the vertical circle. The value of v is

(a) $\pi\sqrt{3}$ m/s. (b) $\pi\sqrt{6}$ m/s.
(c) $\pi\sqrt{2}$ m/s. (d) Π m/s.

38. A block slips with constant velocity on a plane inclined at an angle 9. The same block is pushed up the plane with an initial velocity v_0. The distance covered by the block before coming to rest is

(a) $\frac{v_0^2}{2g\sin\theta}$ (b) $\frac{v_0^2}{4g\sin\theta}$

(c) $\frac{v_0^2\sin^2\theta}{2g}$ (d) $\frac{v_0^2\sin^2\theta}{4g}$

39. A ball is dropped from a height of 19.6 m. The distance covered by it in the last second is

(a) 19.6 m (b) 14.7 m
(c) 4.8 m (d) 9.8 m

40. A particle is projected upwards. The times corresponding to height h while ascending and while descending are t_1 and t_2 respectively. The velocity of projection will be

(a) gt_1 (b) gt_2

(c) $gt\,(t_1 + t_2)$ (d) $\frac{g(t_1+t_2)}{2}$

41. A frictionless wire is fixed between A and B inside a sphere of radius R. A small ball slips along the wire. The time taken by the ball to slip from A to B will be

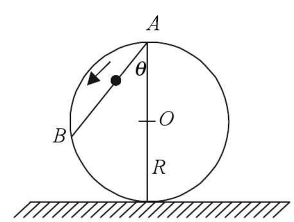


Fig. 3.56

(a) $\frac{2\sqrt{gr}}{g\cos\theta}$ (b) $\frac{2\sqrt{gR\cos\theta}}{g}$

(c) $2\sqrt{R/g}$ (d) $\frac{gR}{\sqrt{g\cos\theta}}$

42. Starting from rest, a body takes 4 seconds in slipping from top to bottom of an inclined plane. The time taken by the same body in covering one quarter distance on the same plane from rest will be

(a) 1 s (b) 2 s
(c) 4 s (d) 1.6 s

43. A 150 meter long train is moving towards north with a velocity of 10 m/s. A parrot is flying in the south direction parallel to tram at 5 m/s. The time taken by the parrot in crossing the train is

(a) 12 s (b) 8 s
(c) 15 s (d) 10 s

44. A uniform stationary sphere starts rolling down from the upper end of the surface as shown in the figure, and it reaches the lower right end $H = 27$ m and $h = 20$ m. The sphere will fall on the ground level at the following distance from A

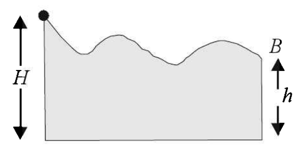


Fig. 3.57

(a) 10 m (b) 20 m
(c) 30 m (d) 40 m

45. A body of mass m starts moving with velocity V_0 at point A on a frictionless path as shown in the figure. The speed of the body at point B will be

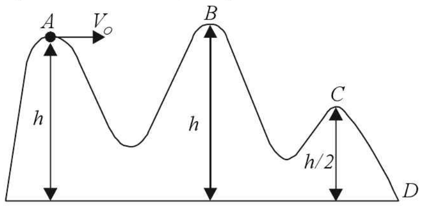


Fig. 3.58

(a) Zero (b) V_0
(c) $\frac{V_0}{2}$ (d) $2V_0$

46. In the above problem the speed at point C will be

(a) $2V_0$ (b) V_0
(c) $\frac{V_0}{2}$ (d) $\sqrt{v_0^2 + gh}$

47. A passenger train is moving with speed V_1, on rails. The driver of this train observes another goods train moving in the same direction with speed v_2 $(v_1 > v_2)$. If on applying brakes, the retardation produced is a, then the minimum distance covered by the passenger train so that it may not collide with the goods train will be

(a) $\frac{(v_1^2 - v_2^2)}{2a}$ (b) $\frac{(v_1 + v_2)}{2a}$
(c) $\frac{(v_1 - v_2)}{2a}$
(d) information is incomplete

48. Figure 3.86 represents a painter in a swing by the side of a building. When the painter pulls the string then the force applied on the surface is 450 N, whereas the weight of painter is 1000 N. If the weight of the swing is 250 N then the acceleration produced in the swing will be

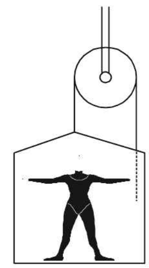


Fig. 3.59

(a) 4 m/s^2 (b) 2 m/s^2
(c) 5 m/s^2 (d) 6 m/s^2

49. The length of the arm of a nut-cracker is 15 cm. A force of 22.5 kg. weight is required to cut a nut without cracker. Where should the nut be placed on the cracker in order to cut it by a force of 2.25 kg/wt

(a) 1 cm from fulcrum (b) 1.5 cm from fulcrum
(c) 0.5 cm from fulcrum (d) 2.0 cm from fulcrum

50. Two balls A and B are simultaneously thrown. A is thrown from the ground level with a velocity of 20 ms^{-1} in the upward direction and B is thrown from a height of 40 m in the downward direction with the same velocity. Where will the two balls meet?

(a) 15 m (b) 25 m
(c) 35 m (d) 45 m

51. A body falls freely from the top of a tower. It covers 36% of the total height in the last second before striking the ground level. The height of the tower is

(a) 50 m (b) 75 m
(c) 100 m (d) 125 m

52. Two blocks are attached to the two ends of a string passing over a smooth pulley as shown in Fig. 3.60. The acceleration of the block will be (in m/s^2)

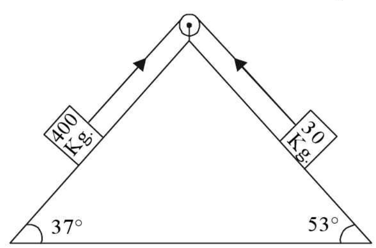


Fig. 3.60

(a) 0.33 (b) 1.32
(c) 1 (d) 0.66

53. In the above problem the tension in the string will be

(a) 456 N (b) 850 N
(c) 1000 N (d) 2000 N

54. A ball is thrown from a height of 12.5 m from the ground level in the horizontal direction. It falls at a horizontal distance of 200 m. The initial velocity of the ball is

(a) 40 m/s (b) 80 m/s
(c) 120m/s (d) 20 m/s

55. The distance traveled by a body in fourth second is twice the distance traveled in seconds. If the acceleration of the body is 3 m/s², then its initial velocity is

(a) $\frac{3}{2}$ m/s (b) $\frac{5}{2}$ m/s
(c) $\frac{7}{2}$ m/s (d) $\frac{9}{2}$ m/s

56. The diameter of the tap of a fire brigade pump is 5 cm. Water is thrown by this pump at a horizontal speed of 18 m/s on a wall. If water rebounds back from the wall, then the force exerted by water on wall will be

(a) 2.35×10^5 dyne (b) 5.76×10^6 dyne
(c) 6.36×10^7 dyne (d) 10^7 dyne

57. A 30 kg sphere, moving with a velocity 48 m/s, splits into two parts after explosion. The masses of these fragments are 18 kg and 12 kg. If the heavier fragment comes to rest after explosion, the velocity of second fragment will be

(a) 80 m/s (b) 100 m/s
(c) 110 m/s (d) 120 m/s

58. A bullet of mass 20 gm and moving with a velocity of 200 m/s strikes a sound and comes to rest after penetrating 3 cm inside it. The force exerted by the sand on the bullet will be

(a) 11.2×10^8 dyne (b) 15.7×10^8 dyne
(c) 13.3×10^8 dyne (d) 18.6×10^8 dyne

59. A bullet, moving with a velocity of 200 cm/s penetrates a wooden block and comes to rest after traversing 4 cm inside it. What velocity is needed for traversing a distance of 6 cm in the same block

(a) 104.3 cm/s (b) 136.2 cm/s
(c) 244.9 cm/s (d) 272.7 cm/s

60. The diameter of a solid disc is 0.5 m and its mass is 16 kg. What torque is required to increase its angular velocity from zero to 120 rotations/minute in 8 seconds?

(a) $\frac{\pi}{4}$ N/m (b) $\frac{\pi}{2}$ N/m
(c) Π N/m (d) Π N/m

61. In the above problem, at what rate is the work done by the toque at the end of eighth second?

(a) Π W (b) Π^2 W
(c) Π^3 W (d) Π^4 W

62. Two projectiles each of mass *m* are projected with same velocity *v* making an angle α and β from the same point in opposite directions. Find the change in their momentum at any instant.

(a) $2mv \sin(\alpha+\beta)$ (b) $2mv \sin\frac{\alpha+\beta}{2}$
(c) $2mv \cos(\alpha+\beta)$ (d) $2mv \cos\frac{(\alpha+\beta)}{2}$

63. An aircraft is flying at a height of 2800 m above the ground. The angle subtended by it in 10 s is 30°. Find the speed of the aircraft

(a) 150 ms^{-1} (b) 100 ms^{-1}
(c) 140 ms^{-1} (d) 125 ms^{-1}

64. A rifle with a muzzle velocity 1500 fts^{-1} shoots a bullet at a small target 150 ft away. How high above the target must the gun be aimed so that the bullet hits the target?

(a) 2.02 inch (b) 1.72 inch
(c) 1.82 inch (d) 1.92 inch

Answers to Questions for Practice

1.	(c)	**2.**	(d)	**3.**	(c)	**4.**	(b)	**5.**	(b)	**6.**	(b)	**7.**	(a)
8.	(c)	**9.**	(c)	**10.**	(b)	**11.**	(a)	**12.**	(b)	**13.**	(a)	**14.**	(a)
15.	(d)	**16.**	(c)	**17.**	(b)	**18.**	(c)	**19.**	(b)	**20.**	(c)	**21.**	(a)
22.	(d)	**23.**	(a)	**24.**	(c)	**25.**	(a)	**26.**	(b)	**27.**	(c)	**28.**	(d)
29.	(a)	**30.**	(b)	**31.**	(c)	**32.**	(a)	**33.**	(d)	**34.**	(c)	**35.**	(b)
36.	(b)	**37.**	(b)	**38.**	(b)	**39.**	(b)	**40.**	(d)	**41.**	(c)	**42.**	(b)
43.	(d)	**44.**	(b)	**45.**	(b)	**46.**	(d)	**47.**	(a)	**48.**	(b)	**49.**	(b)
50.	(a)	**51.**	(d)	**52.**	(b)	**53.**	(a)	**54.**	(a)	**55.**	(d)	**56.**	(c)
57.	(a)	**58.**	(c)	**59.**	(c)	**60.**	(c)	**61.**	(b)	**62.**	(d)	**63.**	(a)
64.	(d)												

Explanations

4.(b) For maximum height, the boy has to throw the stone at $\theta = 90°$, then

$$H = \frac{u^2}{2g} \text{ or } u^2 = 2gH$$

or $$u^2 = 2 \times 9.8 \times 50 = 980\ (\text{ms}^{-1})^2$$

Maximum range $R = \dfrac{u^2}{g} = \dfrac{980}{9.8} = 100\ \text{m}$

Greatest height for maximum range is given by

$$H = \frac{u^2}{2g}\sin^2 45 = \frac{u^2}{2g}\left(\frac{1}{\sqrt{2}}\right)^2 = \frac{u^2}{4g}.$$

$$= \frac{980}{4 \times 9.8} = 25\ \text{m}$$

5.(b) From $u = \sqrt{2gh}$ $\quad h = \dfrac{u^2}{2g} = 44.1\ \text{m}$

$$h' = ut + \frac{1}{2}gt^2 = 29.4\,t - \frac{1}{2}(9.8)\,t^2 = 0$$

$$\therefore t = 6\,s$$

Using $g = \dfrac{u}{t}$, where t is time taken to reach the highest point, we get

$$t = \frac{u}{g} = \frac{29.4}{9.8} = 3\,s$$

By symmetry, the time taken by the ball to reach from the highest point to the hands of the player is 3 s.

$\therefore$ total time $= 3 + 3 = 6\,s$.

6.(b) The other angle of projection for the same range is $90° - \theta$.

i.e. $\dfrac{\pi}{2} - \dfrac{5\pi}{36} = \dfrac{13\pi}{16}$ radian

8.(c) Change in velocity

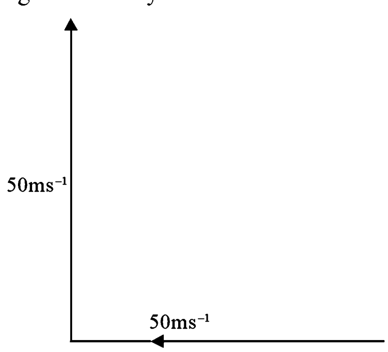


Fig. 3.61

$$= \sqrt{50^2 + 50^2} = \sqrt{5000} = 70.7\ \text{ms}^{-1}$$

$$\therefore \text{acceleration} = \frac{\text{change in velocity}}{\text{time}} = \frac{70.7}{5}$$

$= 14.14\ \text{ms}^{-2}$.

9. (c) Let v be the velocity of projection then $KE = \dfrac{1}{2}mv^2$, but velocity of body at highest point is $v_x = v\sin\theta$.

$\therefore$ kinetic energy of the body at the highest point is equal to

$$\frac{1}{2}m(v\sin\theta)^2 = \frac{1}{2}mv^2\sin^2\theta = KE\sin^2\theta$$

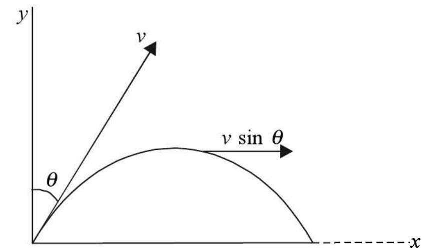


Fig. 3.62

for $\theta = 45°$,

$$K.E._{\text{top}} = KE\sin^2 45° = \frac{KE}{2}.$$

10. (b) Using $R = \dfrac{u^2\sin 2\theta}{g}$ we get

$$\frac{g}{u^2} = \frac{\sin 2\theta}{R}$$

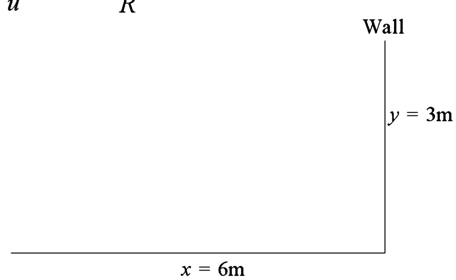


Fig. 3.63

Range, $R = 6 + 18 = 24\ \text{m}$

$$\therefore \frac{g}{u^2} = \frac{\sin 2\theta}{24}$$

As $y = x\tan\theta - \dfrac{gx^2}{2u^2\cos^2\theta}$

for $x = 6\ \text{m}, y = 3\ \text{m}$

and $\dfrac{g}{u^2} = \dfrac{\sin 2\theta}{24}$ we get

$$3 = 6\tan\theta - \frac{\sin 2\theta}{2 \times 24}\,\frac{6^2}{\cos^2\theta}$$

Using $\sin 2\theta = 2\sin\theta\cos\theta$, we get

$$\tan\theta = \frac{2}{3} \text{ or } \theta = \tan^{-1}(2/3)$$

11. (a) Here $L = \sqrt{H^2 + x^2}$ or $L^2 = H^2 + x^2$

Differentiating,

$$2L\left(\frac{dL}{dt}\right) = 0 + 2x\,\frac{dx}{dt}$$

But $\frac{dL}{dt}$ is velocity of the mass and $\frac{dx}{dt}$ is velocity of motorcycle.

$$\therefore Lv_m = xv$$

Or $$v_m = \frac{xv}{L} = \frac{xv}{\sqrt{H^2+x^2}}$$

12. (b) Net velocity $= u - (-u\cos 60°) = \frac{3}{2}u$

Integrating $\alpha = \int \frac{3}{2} u\, dt$ or $\alpha = \frac{3u}{2} t$

or $$t = \frac{2\alpha}{3u}.$$

13. (a) Displacement of minutes hand = 10 cm

Time difference between 0600 hrs to 1830 hrs is 12 hr and 30 minutes *i.e.* 45000 s

$$\therefore \text{Average velocity} = \frac{10}{45000}$$

$= 2.2 \times 10^{-4}\ \text{cms}^{-1}$

14. (a) $v = u\left(1 - \frac{t}{t'}\right)$ or $\frac{dx}{dt} = u\left(1-\frac{t}{t'}\right)$

Integrating, $x = u\left(t - \frac{t^2}{2t'}\right) + C$

at $t = 0, x = 0$ and $C = 0$

$$\therefore x = u\left(t - \frac{t^2}{2t'}\right) = 10\,t\left(1 - \frac{t}{10}\right)$$

Putting $t = 10$

$$x = 10 \times 10\left(1 - \frac{10}{10}\right) = 0$$

15. (d) Using the relation,

$$S = ut + \frac{k}{(n+1)(n+2)} t^{n+2}$$

for non-uniform accelerated motion, we get

$$S = ut + \frac{\beta}{(1+1)(1+2)} t^{(1+2)}$$

(There $a = \beta t$ so comparing it with $a = kt^n$ we get $n = 1$)

$$\therefore S = ut + \frac{\beta}{6} t^3.$$

16. (c) When he takes 8 steps, the displacement is (5 – 3) = 2m.

Time taken for 8 steps = 8 s

$$\therefore \text{Average velocity} = \frac{2}{8} = \frac{1}{4}\ \text{ms}^{-1}$$

In the last 5 steps the drunkard will not be able to come backward because he would fall in the pit.

$\therefore$ Total displacement required

$= 13 - 5 = 8$ m

$$\text{Time required} = \frac{8}{1/4} = 32\ \text{s}$$

17. (b) The time taken by jeep to cover a distance of 30 km

$$= \frac{\text{Distance}}{\text{Speed}} = \frac{30}{60} = \frac{1}{2}\ \text{hr.}$$

Total distance covered by rocket in this duration = speed × time.

$$= 100 \times \frac{1}{2} = 50\ \text{km}$$

18. (c) Total horizontal distance to be covered

$$= x \times n = nx$$

Total vertical distance to be covered

$$= y + n = ny$$

but $nx = ut$ ... (i)

and $$ny = \frac{1}{2} gt^2 \quad \text{...(ii)}$$

Substituting the value of t from (i) in (ii),

$$ny = \frac{1}{2} g \frac{n^2x^2}{u^2}$$

Or $$n = \frac{2yu^2}{gx^2}$$

19. (b) $S_1 = \frac{1}{2} a t^2 = \frac{1}{2} a \times 2^2 = 2a$ ... (i)

$S_1 + S_2 = \frac{1}{2} a \times (2+2)^2 = 8a$ ... (ii)

$S_1 + S_2 + S_3 = \frac{1}{2} a \times (2+2+2)^2 = 18\,a$ ... (iii)

Total time = 32 + time required to cover last 5 steps = 32 + 5 = 37 s.

20. (c) The football falls on the net with constant acceleration. Firstly, the net makes the acceleration drop but due to elasticity it again acclerates the football which ultimately gains constant acceleration.

22. (d) Using $\frac{h}{2} = \frac{g}{2}(2n-1)$ and $h = \frac{1}{2} gn^2$

(where n = total time and last moment of motion is nth second).

We get $n^2 - 4n + 2 = 0$

Solving, $n = \left(2 \pm \sqrt{2}\right)$ s

23. (a) Using $h = ut + \frac{1}{2} gt^2$ we get

$$-65 = 12\,t - \frac{1}{2} \times 10 \times t^2$$

Or $5t^2 - 12t - 65 = 0$

Solving $t = 5$ s or $-13/5$ s

Time cannot be negative, thus $t = 5$ s.

4

Circular Motion

BRIEF REVIEW

Circular motion may be divided into two types (i) motion in a horizontal circle (ii) motion in a vertical circle. In vertical circle acceleration due to gravity plays a role and hence speed at every point is different. We will deal with them separately.

Horizontal circular motion Acceleration is continuously required to change the direction even though if the speed is constant. Therefore, equations of motion used in translation cannot be applied. We define new variables and equations to describe motion.

(i) ***Angular displacement (θ)*** Change in angular position (initial to final) is called angular displacement as shown in Fig. 4.1. Unit is radian.

(ii) ***Angular velocity (ω)*** Time rate of change of angular displacement is called angular velocity. $\omega = \frac{d\theta}{dt}$. Average angular velocity $= \frac{2\pi}{T}$ where T is time period of revolution. If a revolution is not completed then $\omega_{av} = \frac{\text{angular displacement}}{\text{time taken}}$. Unit is rad s^{-1}.

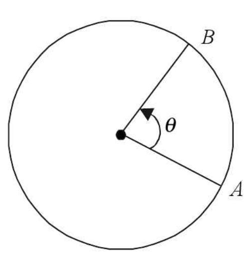


Fig. 4.1 Angular displacement

(iii) ***Angular acceleration (α)*** Time rate of change of angular velocity is called angular acceleration $\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$. If angular acceleration is uniform, then $\omega = \omega_0 + \alpha t$; $\theta = \omega_0 t + \frac{1}{2}\alpha t^2$ and $\omega^2 - \omega_0^2 = 2\alpha\theta$ can be applied. Here ω_0 is initial angular velocity, θ is angular displacement and t is time. If $\alpha = 0$, then particle moves in uniform circular motion.

Note that speed v remains unchanged.

Relation between v and ω; a and a_t

$v = r\omega$ where r is radius of the circle. Note from Fig. 4.2 that velocity is tangential.

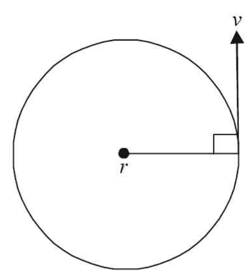


Fig. 4.2 Tangential or linear velocity

$a_t = r\alpha$ where a_t is tangential acceleration. Fig. 4.3 shows there are two accelerations, one along the radius called radial or centripetal or normal acceleration a_r. The other is tangential or along the tangent. Thus, net acceleration a_{net} is

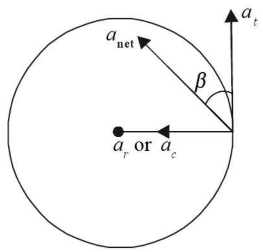


Fig. 4.3 Net acceleration illustration

$a_{net} = \sqrt{a_r^2 + a_t^2}$ and $\tan\beta = \dfrac{a_r}{a_t}$ with respect to tangential acceleration.

Centripetal force $F_c = \dfrac{mv^2}{r} = mr\,\omega^2$. It is a pseudo force and acts towards the centre.

Centrifugal force The inertial reaction required to take into account the acceleration of frame of reference is called centrifugal force and is equal to $-mr\,\omega^2$.

Motion in a vertical circle When a body moves in a vertical circle then at the highest point

$\dfrac{mv^2}{r} \geq$ mg or $v_{min} = \sqrt{rg}$ at the highest point. Minimum velocity, v_{min} at point P can be determined using the fact that the body has come down by $AL = AO + OL = r + r\cos\theta$ as shown in Fig. 4.4.

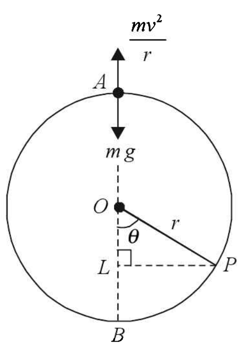


Fig. 4.4 Velocity at any point in a vertical circle

Thus $v^2_{min\,P} = v^2_{min\,top} + 2\,gh$

$= rg + 2g\,(r + r\cos\theta)$

or $v_{min,\,P} = \sqrt{3rg + 2rg\cos\theta}$

v_{min} at the lowest point or bottom is obtained by using $\theta = 0$, $\cos\theta = 1$.

or $v_{min,\,bottom} = \sqrt{5rg}$

Tension, if the string is used or normal reaction at any point P is obtained as T or $N = \dfrac{mv^2}{r} + mg\cos\theta$ (See Fig. 4.5) where v is velocity.

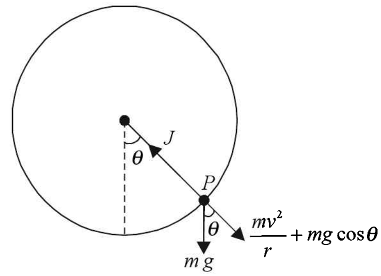


Fig. 4.5 Tension at any point P in a vertical circle

• Short Cuts and Points to Note

1. Circular motion will be uniform if $a_t = 0 = \alpha$, i.e., angular velocity ω remains uniform and radial acceleration $a_r = \dfrac{v^2}{r} = r\omega^2$ remains constant throughout the motion.

2. When α or a_t is present, ω varies with time. Assuming α = constant, apply

$$\omega = \omega_0 + \alpha t\,;\ \theta = \omega_0 t + \frac{1}{2}\,\omega t^2$$

or $\omega^2 - \omega_0^2 = 2\alpha\,\theta$ as the case may be. [Note $v = u + at$ etc. cannot be used]. Also note $v = r\omega$, $a_t = r\alpha$ and

$$\omega = \frac{d\theta}{dt} \text{ and } \alpha = \frac{d^2\theta}{dt^2}.$$

3. Since a_t and a_r are at right angles to one another. Therefore a_{net} is given by

$$a_{net} = \sqrt{a_t^2 + a_r^2} \text{ and } \tan\beta = \frac{a_r}{a_t} \text{ wrt. } a_t \text{ [See Fig. 4.3]}$$

4. If $a_t = 0$ or $\alpha = 0$, no work is done in circular motion, i.e., in uniform circular motion work done is zero. Note that centripetal force and displacement are perpendicular to one another. Therefore, centripetal force does no work. However, work will be done if $a_t \neq 0$ or $\alpha \neq 0$ since tangential force acts in the direction of displacement.

5. If the particle moves from A to B in uniform circular motion with a velocity v then change in velocity Δv $= 2\,v\sin\left(\dfrac{\theta}{2}\right)$ and average acceleration

$$a_{av} = \frac{2v\sin\left(\frac{\theta}{2}\right)}{\frac{r\theta}{v}} = \frac{2v^2\sin\left(\frac{\theta}{2}\right)}{r\theta}.$$

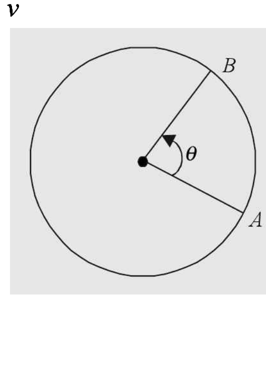


Fig. 4.6

6. Centripetal force $F_c = \dfrac{mv^2}{r} = mr\,\omega^2$.

7. Radius of curvature of a projectile at any instant $R = \dfrac{v^2}{a_r}$.

8. If a road is banked (the angle of banking = θ), then maximum speed a vehicle shall have while taking a turn is $\tan\theta = \dfrac{v^2}{rg}$.

   A cyclist will bend at $\theta = \tan^{-1}\left(\dfrac{v^2}{rg}\right)$ while taking a turn.

9. If friction is also present then on a banked road maximum and minimum velocity is determined as

$$v_{max} = \left[\frac{rg(\tan\theta + \mu)}{1 - \mu\tan\theta}\right]^{1/2} \quad \text{and}$$

$$v_{min} = \left[\frac{rg(\tan\theta - \mu)}{1 + \mu\tan\theta}\right]^{1/2}.$$

10. In vertical circular motion, minimum velocity at any point P is $v_{min,\,P} = \sqrt{3rg + 2rg\cos\theta}$. Minimum velocity at the highest point is $v_{min,\,top} = \sqrt{rg}$ and minimum velocity at the bottom $v_{min,\,bottom} = \sqrt{5rg}$.

11. In vertical circular motion $T_{bottom} - T_{top} = 6$ mg.

12. If the velocity at the top of a vertical circle is known, then velocity at the bottom is given by

    $v^2_{bottom} = v^2_{top} + 2\,(2r)\,g$. If the body comes down a distance h then

    $v^2_P = v^2_{top} + 2\,g\,h$.

13. If the velocity at the bottom of a vertical circle is given, then velocity at any point P (say at a height h from the bottom) is given by

    $v^2_P = v^2_{bottom} - 2\,gh$

    In this way velocity at the top will be $v^2_{top} = v^2_{bottom} - 4\,gr$.

14. Tension at any point $T = \dfrac{mv^2}{r} + mg\cos\theta$. If the particle is not tied to the string, then tension may be read as action or reaction.

15. The difference between rotational motion and circular motion is that in rotational motion different particles of the body move in different radii about a fixed axis within or outside the body. In circular motion all the particles move in the same radius.

16. If the distance between the wheels (side wheels) is a, and centre of mass of the truck/vehicle is at a height h then the maximum speed it can have without overturning is $v = \sqrt{\dfrac{gRa}{2h}}$.

• Caution

1. Considering no acceleration acts in a uniform circular motion as v and ω remain constant.

⇒ Radial or centripetal acceleration is required for changing direction continuously $\dfrac{v^2}{r} = r\omega^2$ is the radial acceleration acting in uniform circular motion.

2. Applying equations like $v = u + at$, $s = ut + ½\,at^2$ etc. In horizontal circular motion

⇒ $v = u + at$ etc. cannot be applied. Instead apply $\omega = \omega_0 + \alpha t$; $\theta = \omega_0 t + ½\,\alpha t^2$ and $\omega^2 = \omega_0^{\,2} + 2\,\alpha\,\theta$.

3. Assuming work must be done in circular motion as force is acting.

⇒ In uniform circular motion, no work is done as force and displacement are mutually perpendicular.

   However, work will be done in non-uniform circular motion.

4. Assuming minimum velocity to be given to the particle to complete vertical circle is $\sqrt{rg}$.

⇒ Minimum velocity to be given to the particle to complete vertical circle is $\sqrt{5rg}$ at the bottom.

5. Considering change in velocity in uniform circular motion is zero.

⇒ Change in speed is zero. If the particle moves by θ then $\Delta v = 2\,v\sin\left(\dfrac{\theta}{2}\right)$.

6. Assuming velocity at any point will be given by the equation $v = \sqrt{3rg + 2rg\cos\theta}$.

⇒ This is the minimum velocity at any point. If velocity is other than minimum, then use $v^2_{any\,point} = v^2_{top} + 2\,gh$. See Fig. 4.7 (a).

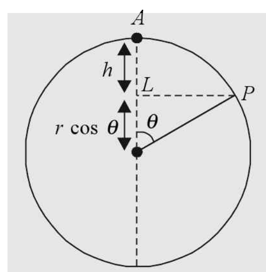


Fig. 4.7 (a)

$h = r - r\cos\theta \qquad v^2_{any\,point} = v^2_{top} + 2\,g\,r\,(1 - \cos\theta)$

If v_{bottom} is known then $v^2_{any\,point} = v^2_{bottom} - 2\,gh$

From Fig. 4.7 (b)

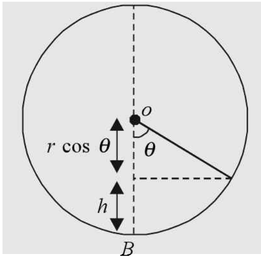


Fig. 4.7 (b)

$$v^2_{\text{any point}} = v^2_{\text{bottom}} - 2\,g\,r\,(1-\cos\theta)$$

7. If a car is accelerating on a circular road with accleration a then assuming a is the net acceleration.

$\Rightarrow$ Since $\frac{v^2}{r}$ also acts. Therefore, net acceleration at any instant will be $a_{\text{net}} = \sqrt{\left(\frac{v^2}{r}\right)^2 + a^2}$ $= \sqrt{(r\omega^2)^2 + a^2}$.

8. Considering like horizontal circle tension in vertical circle is also in $r\,\omega^2$ or $\frac{mv^2}{r}$.

$\Rightarrow$ In vertical circle $T = \frac{mv^2}{r} + mg\cos\theta$ where v is velocity at that point.

SOLVED PROBLEMS

1. A volunteer is rotated in a horizontal circle of radius 7 m. Find the period of rotation for which the acceleration is equal to 3g

(a) 2.61 s (b) 2.87 s
(c) 3.07 s (d) 3.31 s

Solution (c) $a_r = r\left(\frac{2\pi}{T}\right)^2 = 3\text{g}$ or $7\left(\frac{4\pi^2}{T^2}\right)$ $= 3\text{g}$ or $T = 3.07$ s.

2. A ferris wheel with radius 14 m is turning about a horizontal axis through its centre. The linear speed of the passenger on the rim is 7 ms^{-1}. Find the acceleration of a passenger at the highest point.

(a) 6.3 ms^{-2} downwards (b) 3.5 ms^{-2} upwards
(c) 13.3 ms^{-2} upward (d) none

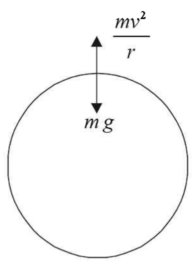


Fig. 4.8

Solution (b) $a = \frac{v^2}{r} = \frac{7^2}{14}$
$= 3.5$ ms^{-2} upward

3. An annular ring with inner and outer radii R_1 and R_2 is rolling without slipping with a uniform angular speed. The ratio of the forces experienced by the two situated on the inner and outer parts of the ring is

(a) $\frac{R_2}{R_1}$ (b) $\frac{R_1}{R_4}$
(c) 1 (d) $\frac{R_1}{R_2}$

Solution (d) $\frac{F_1}{F_2} = \frac{mR_1\omega^2}{mR_2\omega^2} = \frac{R_1}{R_2}$.

4. A given shaped glass tube having uniform area of cross-section is filled with water and is mounted on a rotatable shaft as shown in fig. If the tube is rotated with a constant angular velocity ω, then

[AIIMS 2005]

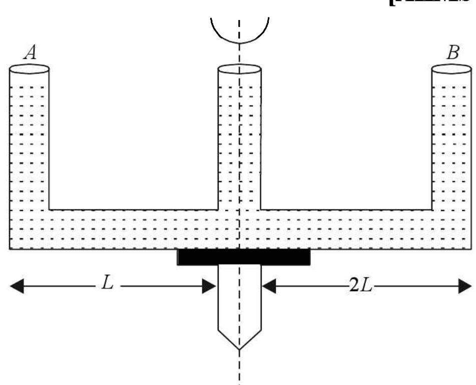


Fig. 4.9

(a) water level in A will rise and fall in B
(b) water level in both A and B will rise
(c) water level in B will rise and will fall in A
(d) water level remains same in both A and B

Solution (b) extra force $F = \frac{mr\omega^2}{2}$ and extra pressure

$$= \frac{m}{A} = \frac{mr\omega^2}{2A}.$$

Increases the level in the two.

5. A pendulum was kept horizontal and released. Find the acceleration of the pendulum when it makes an angle θ with the vertical.

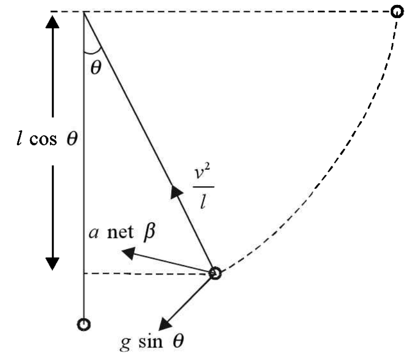


Fig. 4.10

(a) $g\sqrt{1+3\cos^2\theta}$ (b) $g\sqrt{1+3\sin^2\theta}$
(c) $g\sin\theta$ (d) $2g\cos\theta$

Solution (a) $\frac{mv^2}{2} = mgl\cos\theta$ or $\frac{v^2}{l} = 2g\cos\theta$

and $= \sqrt{a_t^2 + a_r^2} = \sqrt{(g\sin\theta)^2 + (2g\cos\theta)^2}$
$= g\sqrt{1+3\cos^2\theta}$

$\tan\beta = \frac{g\sin\theta}{\frac{v^2}{l}} = \frac{\tan\theta}{2}$

6. A circular track of radius 100 m is designed for an average speed 54 km/h. Find the angle of banking.

(a) $\tan^{-1}\left(\frac{3}{20}\right)$ (b) $\tan^{-1}\left(\frac{9}{40}\right)$
(c) $\tan^{-1}\left(\frac{3}{10}\right)$ (d) none of these

Solution (b) $\tan\theta = \frac{v^2}{rg} = \frac{15\times15}{100\times10} = \frac{9}{40}$,
$\theta = \tan^{-1}\left(\frac{9}{40}\right)$

7. A fighter plane is pulling out for a dive at 900 km/h in a vertical circle of radius 2 km. Its mass is 5000 kg. Find the force exerted by the air on it at the lowest point.
(a) 2.0625×10^4 N upward
(b) 2.0625×10^5 N downward
(c) 2.0625×10^5 N upward
(d) 2.0625×10^4 N downward

Solution (c) $\frac{mv^2}{r} + mg = \frac{5\times10^3\times(250)^2}{2\times10^3} + 5\times10^4$
$= 2.0625\times10^5$ N upward.

8. A particle of mass m rotates in a circle of radius a with a uniform angular speed ω. It is viewed from a frame rotating about z-axis with ω. The centrifugal force on the particles is

(a) $m\,\omega^2 a$ (b) $m\,\omega_0^2\,a$
(c) $m\left(\frac{\omega^2+\omega_0^2}{2}\right)$ (d) $m\,\omega\omega_0\,a$

Solution (b)

9. A motorcyclist is going on an overbridge of radius R. The driver drives with a constant speed. As the motor cycle is ascending on the overbridge, the normal force on it
(a) increases (b) decreases
(c) remains unchanged (d) fluctuates

Solution (a)

10. A particle is going in a spiral path as shown in Fig. 4.11 with constant speed. Then

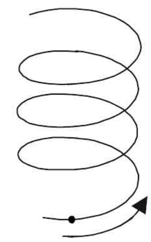


Fig. 4.11

(a) its velocity is constant
(b) its acceleration is constant
(c) magnitude of its acceleration is constant.
(d) The magnitude of acceleration decreases continuously

Solution (c)

11. A person applies a constant force $\vec{F}$ on a particle of mass m and finds that the particle moves in a circle of radius r with a uniform speed v when seen from an inertial frame of reference.
(a) This is not feasible
(b) Some other forces whose resultant varies in magnitude and direction also act on the particle
(c) The resultant of other forces is $\frac{mv^2}{r}$ towards the centre
(d) none of these

Solution (b)

12. Find the tension in the pendulum at the extreme position if amplitude is θ_0.

(a) $\frac{mv^2}{r}$ (b) $\frac{mv^2}{r} + mg\cos\theta_0$
(c) $mg\cos\theta_0$ (d) $\frac{mv^2}{r} - mg\cos\theta_0$

Solution (c) At the extreme position $v = 0$; therefore
$T = mg\cos\theta_0$.

13. Find the radius of curvature of a projectile at the highest point, fired with a velocity v, making an angle θ with the horizontal.

(a) $\frac{v^2\sin^2\theta}{g}$ (b) $\frac{v^2\sin^2\theta}{2g}$

(c) $\frac{v^2 \cos^2\theta}{2g}$ (d) $\frac{v^2\cos^2\theta}{g}$

Solution (d) $R = \frac{(v\cos\theta)^2}{g}\left(= \frac{v^2}{a}\right)$.

14. A track consists of two circular points ABC and CDE of equal radius 100 m and joined smoothly as shown. Each part subtends a right angle at its centre. A cyclist weighing 100 kg together with the cycle travels at a constant speed 1 km h^{-1} on the track. Find the normal force between the road and the cycle just before and just after the cycle crosses C.

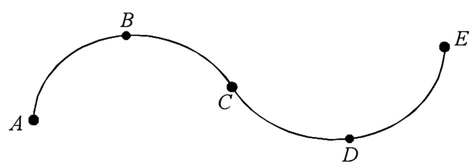


Fig. 4.12

(a) 682, 682 N (b) 732, 682 N
(c) 732, 732 N (d) 682, 732 N

Solution (d) just before crossing C

$$N = mg\cos 45 - \frac{mv^2}{r}$$
$$= 100\times 10\times .707 - \frac{100\times 5^2}{100}$$
$$= 682\text{N}$$

just after crossing C $\text{N} = mg\cos 45 + \frac{mv^2}{r}$
$$= 707 + 25 = 732\text{N}$$

15. An inclined plane ends into a vertical loop of radius R. A particle is released from a height $3R$. Can it loop the loop?

(a) Yes (b) No
(c) Cannot say (d) Yes if friction is present

Solution (a) To complete the loop minimum height be $\frac{5}{2}R$

16. A particle has velocity $\sqrt{3rg}$ at the highest point in a vertical circle. Find the ratio of tensions at the highest and lowest point.

(a) 1 : 6 (b) 1 : 4
(c) 1 : 3 (d) 1 : 2

Solution (b) $T_{\text{top}} = \frac{mv^2}{r} - mg$
$= 2\,mg$;

$$\frac{T_{\text{top}}}{T_{\text{bottom}}} = \frac{2mg}{2mg+6mg} = \frac{1}{4}.$$

17. A car is moving on a circular track of radius R. The road is banked at θ. μ is the coefficient of friction. Find the maximum speed the car can have.

(a) $\left[\frac{Rg(\sin\theta+\mu\cos\theta)}{\cos\theta+\mu\sin\theta}\right]^{1/2}$

(b) $\left[\frac{Rg(\cos\theta+\mu\sin\theta)}{\cos\theta-\mu\sin\theta}\right]^{1/2}$

(c) $\left[\frac{Rg(\sin\theta+\mu\cos\theta)}{\cos\theta-\mu\sin\theta}\right]^{1/2}$

(d) none

Solution (c) $v_{\max} = \left[\frac{Rg(\tan\theta+\mu)}{1-\mu\tan\theta}\right]^{1/2}$

$$= \left[\frac{Rg(\sin\theta+\mu\cos\theta)}{\cos\theta-\mu\sin\theta}\right]^{1/2}$$

19. A 1g coin is placed on an L.P. record 10 cm from the axis of rotation. The coin is not displaced. The minimum value of coefficient of friction is ____ if rotation speed is $33\frac{1}{3}$ rpm.

(a) 0.09 (b) 0.1
(c) 0.12 (d) none of these

Solution (c) $mr\,\omega^2 = \mu mg$

or $$m = \frac{0.1\times\left(\frac{100}{3}\times\frac{2\pi}{60}\right)^2}{10} = \frac{\pi^2}{81}.$$

TYPICAL PROBLEMS

20. A table with smooth horizontal surface is rotating at a speed ω about its axis. A groove is made on the surface along a radius and a particle is gently placed inside the groove at a distance a from the centre. Find the speed of the particle with respect to the table as its distance from the centre becomes l.

(a) $v = \omega l$ (b) $v = \omega(l-a)$
(c) $v = \frac{\omega(l+a)}{2}$ (d) $v = \omega\sqrt{l^2-a^2}$

Solution (d) $= \frac{F}{m} = \frac{m\omega^2 x}{m}$ or $\frac{dv}{dt} = \omega^2 x$

$$\frac{dv}{dx}\cdot\frac{dx}{dt} = \omega^2 x$$
$$vdv = \omega^2 xdx$$
$$\int_0^v vdv = \int_a^L \omega^2 xdx$$
$$v = \omega\sqrt{l^2-a^2}.$$

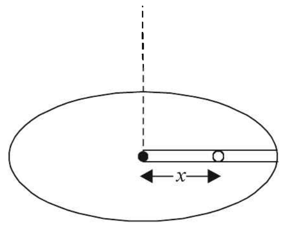


Fig. 4.13

21. A car moves on a horizontal track of radius r, the speed increasing constantly at rate $\frac{dv}{dt} = a$. The coefficient of friction between road and tyre is μ. Find the speed at which the car will skid.

(a) $[(\mu^2g^2 + a^2)r^2]^{1/4}$ (b) $\sqrt{\mu gr}$
(c) $[(\mu^2g^2 + a^2)r^2]^{1/4}$ (d) $\sqrt{ar}$

Solution (c) net acceleration is $\sqrt{a^2 + \left(\frac{v^2}{r}\right)^2} \geq \mu g$

or $\left(\frac{v^2}{r}\right)^2 = \mu^2g^2 - a^2$ or $v = [(\mu^2g^2 - a^2)\, r^2]^{1/4}$.

22. Two strings are tied 2 m apart on a rod and on the other end a mass 200 g is tied as shown in Fig. 4.14 (a). Each string is 1.25 m long. Find the tensions T_1 and T_2 if the rod is rotated with 60 rpm.

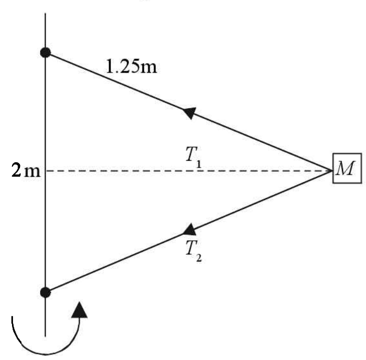


Fig. 4.14 (a)

(a) 6 N, 3 N (b) 6.25 N, 3.75 N
(c) 4.25 N, 5.75 N (d) 5.25 N, 4.75 N

Solution (b) Resolve T_1 and T_2.

Look into Fig. 4.14 (b) carefully

$T_1 \cos\theta + T_2 \cos\theta = \text{mr}\omega 2$

$T_1 \sin\theta = T_2 \sin\theta + mg$

or $(T_1 + T_2)(0.6)$

$$= 0.2 \times \left(\frac{3}{4}\right)(2\pi)^2$$

or $T_1 + T_2 = 10$

$$T_1 - T_2 = \frac{mg}{\sin\theta} = \frac{0.2 \times 10}{0.8}$$

$= 2.5$.

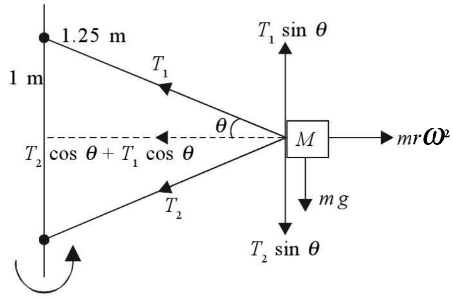


Fig. 4.14 (b)

solving $T_1 = 6.25\,N$ and $T_2 = 3.75\,N$

23. A car is moving on a horizontal circular road of radius 100 m with a uniform speed 10 ms^{-1}. It suddenly accelerates at 1 ms^{-2}. The acceleration is

(a) 1 ms^{-2} (b) $\sqrt{2}$ ms^{-2}
(c) 2 ms^{-2} (d) $\sqrt{5}$ ms^{-2}

Solution (b)

$$= a_{\text{net}}\sqrt{\left(\frac{v^2}{r}\right)^2 + a^2} = \sqrt{\left(\frac{10^2}{100}\right)^2 + 1^2} = \sqrt{2}\ \text{ms}^{-2}$$

24. A particle is projected with a velocity u making an angle θ with the horizontal. What is the radius of curvature of the parabola where the particle makes an angle $\theta/2$ with the horizontal?

Solution $v_x = u\cos\theta$; $v_y = u\sin\theta - gt$

$$\tan\left(\frac{\theta}{2}\right) = \frac{v_y}{v_x} = \frac{u\sin\theta - gt}{u\cos\theta}$$

or $u\sin\theta - gt = \text{u}\cos\theta\tan(\theta/2)$

$$v^2 = v_y^2 = u^2\cos^2\theta\left(1 + \tan^2\frac{\theta}{2}\right)$$

$$= \frac{u^2\cos^2\theta}{\cos^2(\theta/2)}$$

radius of curvature $r = \frac{v^2}{g} = \frac{u^2\cos^2\theta}{g\cos^2(\theta/2)}$.

25. A mass m is released from the top of a vertical circular track of radius r with a horizontal speed v_0. Find angle θ where it leaves the contact with circular track.

Solution $v^2 = v_0^2 + 2gh = v_0^2 + 2gr(1 - \cos\theta)$

$$\frac{mv^2}{r} = \frac{mv_0^2}{r} + 2mg(1 - \cos\theta)$$

Condition of leaving $\frac{mv^2}{r} = mg\cos\theta$

$$mg\cos\theta = \frac{mv_0^2}{r} + 2mg(1 - \cos\theta)$$

or $\cos\theta = \left[\frac{v_0^2}{3rg} + \frac{2}{3}\right]$

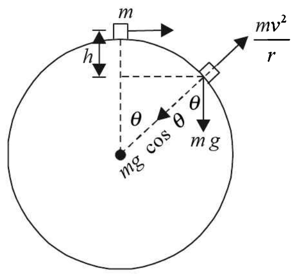


Fig. 4.15

26. A small object slides without friction from the height $H = 50$ cm and then loops the vertical loop of radius $r = 20$ cm from which a symmetrical section of angle 2α has been removed. Find angle α such that after losing contact at A and flying through air, the object will reach point B.

Solution $\frac{mv_A^2}{2} = mg\,[2.5r - r(1 + \cos\alpha)]$;

$h = 2.5r - r(1 + \cos\alpha)$

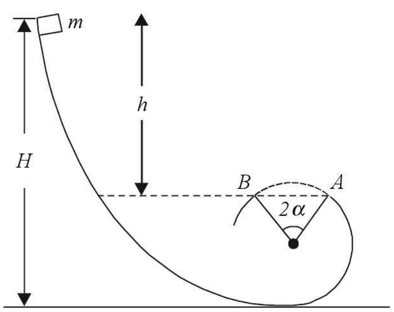


Fig. 4.16

$$v_A^2 = gr(3 - 2\cos\alpha)$$

Range $= AB = 2r\sin\alpha = \frac{v_A^2 \sin 2\alpha}{g}$ or $v_A^2 \cos\alpha = gr$

or $\frac{gr}{\cos\alpha} = gr[3 - 2\cos\alpha]$

or $2\cos^2\alpha - 3\cos\alpha + 1 = 0$

$(\cos\alpha - 1)(2\cos\alpha - 1) = 0$ or $\cos\alpha = 1$

or $\cos\alpha = 1/2$

$\alpha = 0$ or $\alpha = 60°$

as $\alpha \neq 0$ $\therefore \alpha = 60°$.

27. A pipe of length l contains a liquid of density ρ. Area of cross-section of the pipe is A. It is rotated about one end with an angular velocity ω after sealing both the ends. Find the force acting on the liquid.

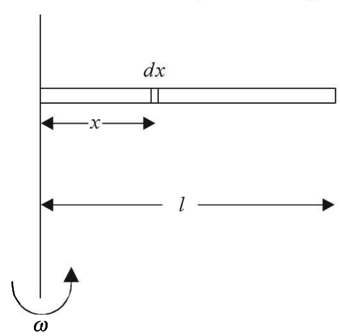


Fig. 4.17

Solution Consider a small element dx at a distance x from the axis of rotation. Mass of liquid in the element $dm = \rho A dx$; $dF = dm\, x\, \omega^2 = \omega^2 \rho A x\, dx$

$$E = \int_0^l \omega^2 \rho A x dx = \frac{A\rho\omega^2 l^2}{2}.$$

28. A spotlight is fixed 4 m from the vertical wall and is rotating at a rate 1 rads^{-1}. The spot moves horizontally on the wall. Find the speed of the spot on the wall when the spotlight makes an angle of 45° with the wall.

(a) 4 ms^{-1} (b) 6 ms^{-1}
(c) 8 ms^{-1} (d) none of these

Solution (c) s is the spot light shown in Fig. 4.18. and x is the distance moved by spot in time t.

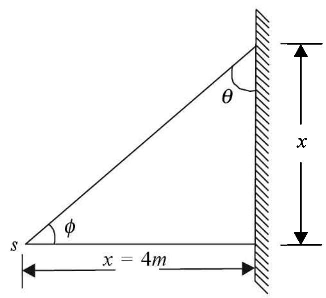


Fig. 4.18

From Fig. 4.20 $\frac{x}{y} = \tan\phi$

$x = y\tan\phi$

$v = \frac{dx}{dt} = y\sec^2\phi\,\frac{d\phi}{dt}$

and $\frac{d\phi}{dt} = \omega = 1$

$v = 4\,(2)\,1$ (when $\theta = 45°, \phi = 45°$) $= 8$ ms^{-1}.

29. A coca-cola bottle is whirled in a horizontal circle of radius r as shown in Fig. 4.19. Where will the gas stay.

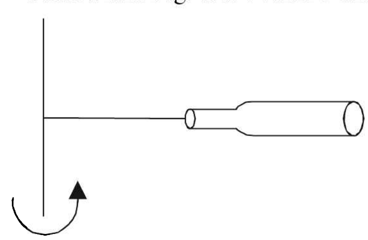


Fig. 4.19

(a) at the neck (b) in the middle
(c) at the bottom (d) anywhere

Solution (c) $F = m r \omega^2$ the smaller the mass, the more is the radius to maintain the same force

Note: This is the principle of cream separator from milk, centrifuge machines used in laboratories and even the Earth is built in the same way. Heaviest masses close to the axis and the lightest at the surface.

30. A particle moves from A to B in uniform circular motion with a velocity ω.

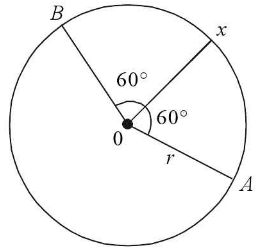


Fig. 4.20

(a) The average acceleration during A to B = Avg acceleration during A to X.

(b) $|a_{avg}(AX)| = |a_{avg}(XB)|$

(c) $\Delta V_{AX} = \Delta V_{AB} = 0$

(d) $|\Delta V_{AB}| = \sqrt{3}\ |\Delta V_{AX}|$.

Solution (b), (d) $|\Delta V_{AX}|$

$= 2V\sin 30 \quad |\Delta V_{AB}| = 2V\sin 60$

$\therefore \quad |\Delta V_{AB}| = \sqrt{3}\ \Delta V_{AX}$

31. A particle moves along a vertical circle of radius r with a velocity $\sqrt{8rg}$ at Y. If T_A, T_B, T_X, T_Y represent tension at A, B, X and Y, respectively, then

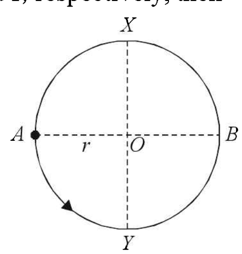


Fig. 4.21

(a) $T_A = T_B$ (b) $T_X - T_Y = 6$ mg

(c) $T_Y - T_X = 6$ mg (d) $T_Y > T_X \neq 6$ mg

Solution (a, c)

$$T_A = T_B = \frac{mv^2}{r}$$

32. A ring moves in a horizontal circle of radius r with a velocity ω in free space the tension is

(a) $2\ mr\ \omega^2$ (b) $mr\ \omega^2$

(c) $\frac{mr\omega^2}{2}$ (d) $\frac{mr\omega^2}{2\pi}$

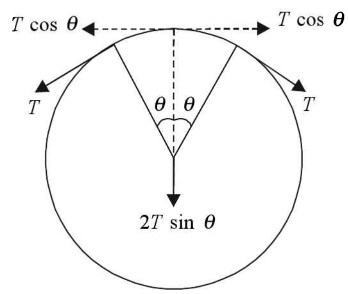


Fig. 4.22

Solution (d) $\frac{m}{2\pi r}(r\ \theta\ 2)\ \omega^2 r$

$= 2\,T\sin\theta$

$= 2\,T\,\theta$ assuming θ to be small.

$$T = \frac{mr\omega^2}{2\pi}.$$

QUESTIONS FOR PRACTICE

1. A body is moving with uniform speed v on a horizontal circle from A as shown in the Fig. 4.23. Change in velocity in the first quarter revolution is

(a) v^2 (b) $\sqrt{2}\ v$ southwest

(c) $\sqrt{2}$ northwest (d) $2v$ west.

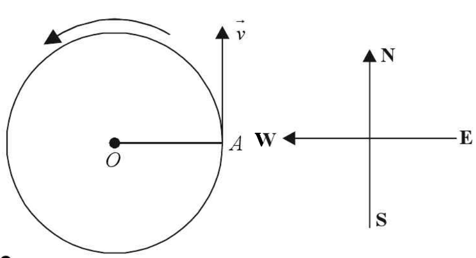


Fig. 4.23

2. Two particles A and B are moving on two different concentric circles with different velocities v_A and v_B then angular velocity of B relative to A as observed by A is given by:

(a) $\frac{v_B - v_A}{r_B - r_A}$ (b) $\frac{v_A}{r_A}$

(c) $\frac{v_A - v_B}{r_A - r_B}$ (d) $\frac{v_B + v_A}{r_B + r_A}$

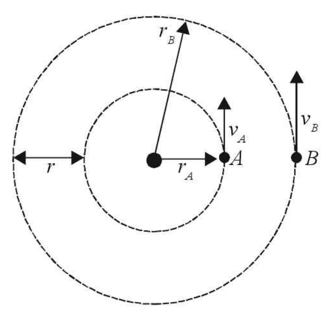


Fig. 4.24

3. A scooterist is approaching a circular turn of radius 80 m. He reduced his speed from 27 kmh^{-1} at constant rate of 0.5 ms^{-2}. His vector acceleration on the circular turn is

(a) $0.86\ ms^{-2}$, $54°$ (b) $0.68\ ms^{-2}$, $45°$
(c) $1.0\ ms^{-2}$, $45°$ (d) $0.5\ ms^{-2}$, $45°$

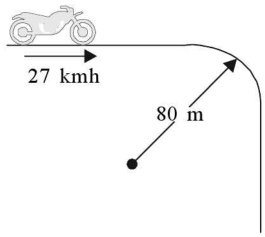


Fig. 4.25

4. A particle moves in a circle of radius 4 cm clockwise at constant speed of 2 cm s^{-1}. If $\hat{x}$ and $\hat{y}$ are unit acceleration vectors along x and y axes, respectively, the acceleration of the particle at the instant half way between PQ is given by

(a) $-4(\hat{x}-\hat{y})$ (b) $4(\hat{x}+\hat{y})$

(c) $\dfrac{-(\hat{x}+\hat{y})}{\sqrt{2}}$ (d) $\dfrac{\hat{x}-\hat{y}}{4}$

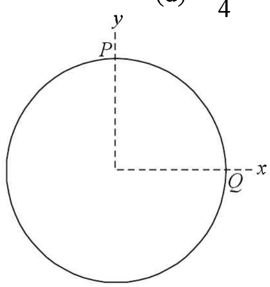


Fig. 4.26

5. A circular track of radius 300 m is banked at an angle $\dfrac{\pi}{12}$ radian. If the coefficient of friction between wheel of a vehicle and road is 0.2, the maximum safe speed of vehicle is

(a) $28\ ms^{-1}$ (b) $38\ ms^{-1}$
(c) $18\ ms^{-1}$ (d) $48\ ms^{-1}$

6. A small block slides with velocity $0.5\sqrt{gr}$ on the horizontal frictionless surface as shown in Fig. 4.27. The block leaves the surface at L. The angle α is

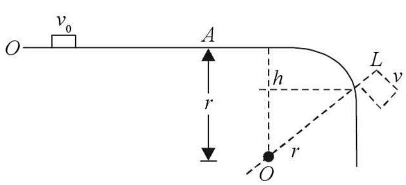


Fig. 4.27

(a) $\cos^{-1}\dfrac{3}{4}$ (b) $\cos^{-1}\dfrac{4}{3}$

(c) $\sin^{-1}\dfrac{3}{4}$ (d) $\sin^{-1}\dfrac{4}{3}$

7. A mass attached to a string of length L is revolved in a vertical circle with least velocity. The string breaks when the mass reaches at the highest point. The mass describes a parabolic path, then range of body w.r.t. the lowest point is

(a) L^2 (b) L
(c) $2L$ (d) $\sqrt{2}\ L$

8. A particle slides from rest from the highest point of a vertical circle of radius r, along a smooth chord. Time of descent of the particle along the chord is

(a) $\sqrt{\dfrac{r}{4g}}$ (b) $\sqrt{2rg}$

(c) $\sqrt{\dfrac{2r}{g}}$ (d) $\sqrt{\dfrac{4r}{g}}$

9. A pendulum is vibrating with an amplitude of $\dfrac{\pi}{2}$ radian. Value of α for which resultant acceleration of the bob directs along the horizontal is

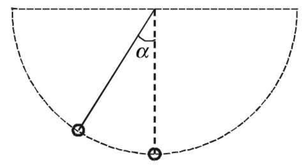


Fig. 4.28

(a) $\sin^{-1} 1/\sqrt{3}$ (b) $\cos^{-1} 1/\sqrt{3}$

(c) $\cos^{-1}\dfrac{\sqrt{3}}{2}$ (d) $\sin^{-1}\dfrac{\sqrt{3}}{2}$

10. A small block is placed at the top of a sphere. It slides on the smooth surface of the sphere. The angle made by the radius vector of the block with the horizontal when the block leaves the sphere is

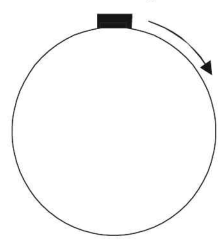


Fig. 4.29

(a) $24°$ (b) $36°$
(c) $42°$ (d) $15°$

11. A small ball describes a horizontal circle on the smooth inner surface of a cone. Speed of the ball at a height of 0.1 m above the vertex is

(a) $4\ ms^{-1}$ (b) $3\ ms^{-1}$
(c) $2\ ms^{-1}$ (d) $1ms^{-1}$

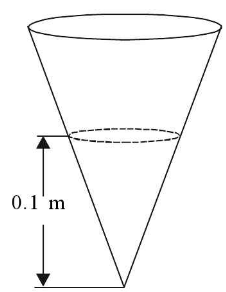


Fig. 4.30

12. Two balls P and Q are at opposite ends of the diameter of a frictionless horizontal circular groove. P is projected along the groove and at the end of T second, it strikes ball Q. Let difference in their final velocities be proportional to the initial velocity of ball P and coefficient of proportionally is e then second strike occurs at

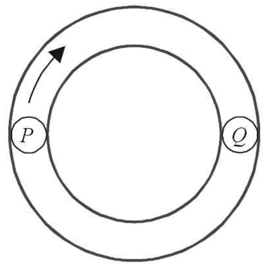


Fig. 4.31

(a) $2T/e$ (b) $e/2T$
(c) $2eT$ (d) $T/2e$.

13. A jeep runs around the curve of radius 0.3 km at a constant speed of 60 ms^{-1}. The resultant change in velocity, instantaneous acceleration and average acceleration over 60° arc are

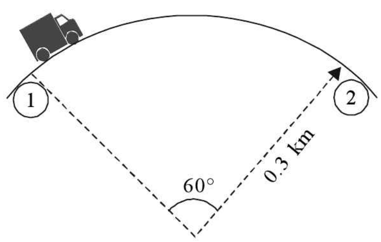


Fig. 4.32

(a) 30 ms^{-1}, 11.5 ms^{-2}, 12 ms^{-2}
(b) 60 ms^{-1}, 12 ms^{-2}, 11.5 ms^{-2}
(c) 60 ms^{-1}, 11.5 ms^{-2}, 12 ms^{-2}
(d) 40 ms^{-1}, 10 ms^{-2}, 8 ms^{-2}

14. A ball moves along an uneven horizontal road with fixed speed at all points on the road. The normal reaction of the road on the body at two different points A and B is

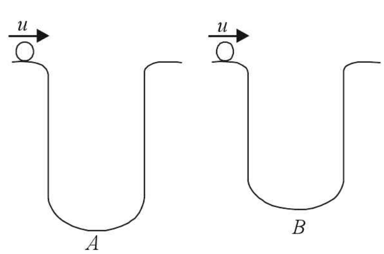


Fig. 4.33

(a) maximum at A (b) maximum at B
(c) minimum at B (d) same at A and B

15. A hollow vertical drum of radius r and height H has a small particle in contact with smooth inner surface of the upper rim at point P. The particle is given a horizontal speed u tangential to the rim. It leaves the lower rim at Q vertically below P. Taking n as an integer for number of revolutions, we get

(a) $n = \frac{2\pi r}{H}$ (b) $\frac{2\pi r}{\sqrt{2H/g}}$
(c) $n = \frac{2\pi r}{u\sqrt{2H/g}}$ (d) $n = \frac{u}{2\pi r}\sqrt{2H/g}$

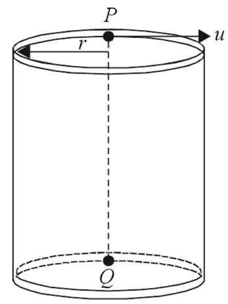


Fig. 4.34

16. A particle moves down the inclined surface and completes a vertical circle at the foot of the plane. Ratio of heights H and h is

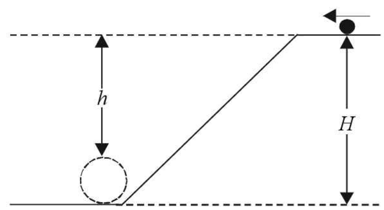


Fig. 4.35

(a) 5 (b) 4
(c) 2 (d) 3.

17. A motor car is travelling at 30 ms^{-1} on a circular road of radius 500 m. It is increasing speed at the rate of 2 ms^{-2}. The acceleration of car is

(a) 2 ms^{-2} (b) 2.7 ms^{-2}
(c) 3 ms^{-2} (d) 3.7 ms^{-2}

18. A string of length 1m is fixed at one end and carries a mass of 100 g at the other end. The string makes $2/\pi$ revolutions per second around the vertical axis through the fixed end. If angle of inclination of the string with the vertical is $\cos^{-1} 5/8$, the linear velocity of the mass is nearly

(a) 1 ms^{-1} (b) 2 ms^{-1}
(c) 3 ms^{-1} (d) 4 ms^{-1}

19. A car is moving on a circular horizontal track of radius 10 m with a constant speed of 10 ms^{-1}. A plumb bob is suspended from the roof of the car by a light rigid rod of length 10 m. The angle made by the rod with the track is

(a) zero (b) 30°
(c) 45° (d) 60°

20. A particle P is sliding down a frictionless hemispherical bowl. It passes the point A and $t = 0$. At this instant of time, the horizontal component of its velocity is v. A bead Q of the same mass as P is ejected from A at $t = 0$

along the horizontal string AB with speed v. Friction between the bead and the string may be neglected. Let t_P and t_Q be the respective times taken by P and Q to reach point B, then

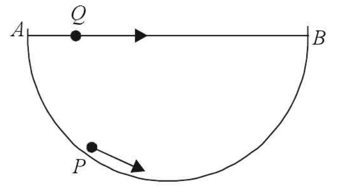


Fig. 4.36

(a) $t_P < t_Q$ (b) $t_P = t_Q$

(c) $t_P > t_Q$ (d) $\frac{t_P}{t_Q} = \frac{\text{length of arc ACB}}{\text{length of chord AB}}$.

21. A stone tied to a string of length L is whirled in a vertical circle with the other end of the string at the centre. At a certain instant of time the stone is at its lowest position and has a speed u. The magnitude of change in its velocity as it reaches a position, where the string is horizontal is

(a) $\sqrt{u^2 - 2gl}$ (b) $\sqrt{2gl}$

(c) $\sqrt{u^2 - gl}$ (d) $\sqrt{2(u^2 - gl)}$

22. If a particle goes from point A to point B in 1 s moving in a semicircle (see Fig. 4.37). The magnitude of the average velocity is

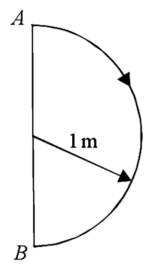


Fig. 4.37

(a) 3.14 ms^{-1} (b) 2 ms^{-1}

(c) 1 ms^{-1} (d) zero.

23. A small block is shot into each of the four tracks as shown below. Each of the track rises to the same height. The speed with which the block enters the track is the same in all cases. At the highest point of the track, the normal reaction is maximum in case of

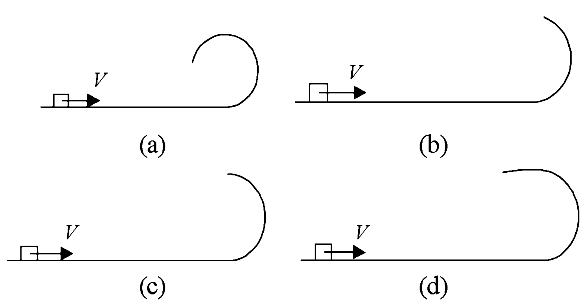


Fig. 4.38

24. A simple pendulum is oscillating without damping. When the displacement of the bob is less than maximum, its acceleration vector $\vec{a}$ is correctly shown in

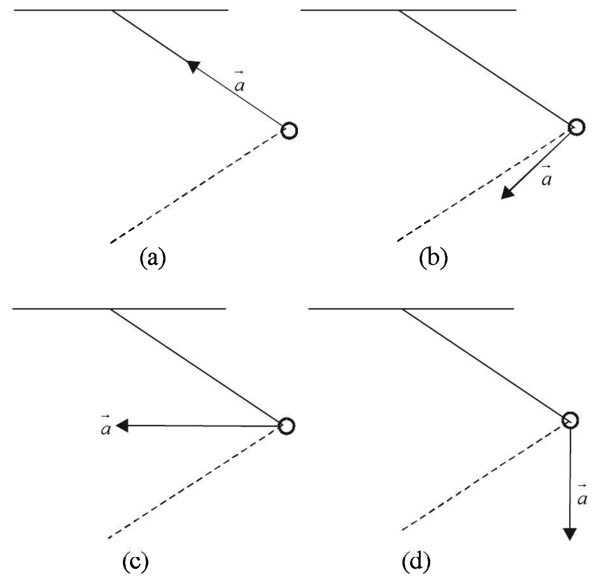


Fig. 4.39

[IIT Screening 2000]

25. A particle of mass m is tied to a light string and rotated with a speed v along a circular path of radius r. If T = tension in the string and mg = gravitational force on the particle then the actual forces acting on the particle are

(a) mg and T only
(b) mg, T and an additional force of mv^2/r directed inwards
(c) mg, T and an additional force of mv^2/r directed outwards
(d) only a force mv^2/r directed outwards

26. A particle of mass m is tied to a light string of length l and rotated along a vertical circular path. What should be the minimum speed at the highest point of its path so that the string does not become slack at any position?

(a) $\sqrt{2gl}$ (b) $\sqrt{gl}$

(c) zero (d) $\sqrt{gl/2}$

27. A simple pendulum has a string of length l and bob of mass m. When the bob is at its lowest position, it is given the minimum horizontal speed necessary for it to move in a circular path about the point of suspension. The tension in the string at the lowest position of the bob is

(a) 3 mg (b) 4 mg

(c) 5 mg (d) 6 mg

28. Water in a bucket is whirled in a vertical circle with a string to it. The water does not fall down even when the bucket is inverted at the top of its path. We conclude that in this position

(a) $mg = \frac{mv^2}{r}$
(b) mg is greater than $\frac{mv^2}{r}$
(c) mg is not greater than $\frac{mv^2}{r}$
(d) mg is not less than $\frac{mv^2}{r}$.

29. A stone of mass m tied to a string of length l is rotated in a circle with the other end of the string as the centre. The speed of the stone is v. If the string breaks, the stone will move

(a) towards the centre (b) away from the centre
(c) along a tangent (d) will stop

30. A coin placed on a rotating turntable just slips if it is placed at a distance of 4 cm from the centre. If the angular velocity of the turntable is doubled, it will just slip at a distance of

(a) 1 cm (b) 2 cm
(c) 12 cm (d) 8 cm

31. A motorcyle is going on an overbridge of radius R. The driver maintains a constant speed. As the motorcycle is ascending on the overbridge, the normal force on it.

(a) increases (b) decreases
(c) remains the same (d) fluctuates

32. Three identical cars A, B and C are moving at the same speed on three bridges. The car A goes on a plane bridge, B on a bridge convex upward and C goes on a bridge concave upward. Let F_A, F_B and F_C be the normal force exerted by the cars on the bridges when they are at the middle of bridges.

(a) F_A is maximum of the three forces.
(b) F_B is maximum of the three forces.
(c) F_C is maximum of the three forces.
(d) $F_A = F_B = F_C$.

33. A train A runs from east to west and another train B of the same mass runs from west to east at the same speed along the equator. A presses the track with a force F_1 and B presses the track with a force F_2

(a) $F_1 > F_2$ (b) $F_1 < F_2$
(c) $F_1 = F_2$
(d) Incomplete information to find the relation between F_1 and F_2

34. If the Earth stops rotating, the apparent value of g on its surface will

(a) increase everywhere
(b) decrease everywhere
(c) remain the same everywhere
(d) increase at some places and remain the same at some other places.

35. A rod of length L is pivoted at one end and is rotated with a uniform angular velocity in a horizontal plane. Let T_1 and T_2 be the tensions at the points $L/4$ and $3L/4$ away from the pivoted ends.

(a) $T_1 > T_2$ (b) $T_2 > T_1$
(c) $T_1 = T_2$
(d) The relation between T_1 and T_2 depends on whether the rod rotates clockwise or anticlockwise.

36. A simple pendulum having a bob of mass m is suspended from the ceiling of a car used in a stunt film shooting. The car moves up along an inclined cliff at a speed v and makes a jump to leave the cliff and lands some distance. Let R be the maximum height of the car from the top of the cliff. The tension in the string when the car is in air is

(a) mg (b) $mg - \frac{mv^2}{R}$
(c) $mg + \frac{mv^2}{R}$ (d) zero

37. Let θ denote the angular displacement of a simple pendulum oscillating in a vertical plane. If the mass of the bob is m, then tension in the string is $mg\cos\theta$

(a) always (b) never
(c) at the extreme position
(d) at the mean position.

38. An object follows a curved path. The following quantities may remain constant during the motion.

(a) speed (b) velocity
(c) acceleration
(d) magnitude of acceleration.

39. Assume that the Earth goes round the sun in a circular orbit with a constant speed of 30 km/s.

(a) The average velocity of the earth from 1st Jan, 90 to 30th June, 90 is zero.
(b) The average acceleration during the above period is 60 km/s².
(c) The average speed from 1st Jan, 90 to 31st Dec, 90 is zero.
(d) The instantaneous acceleration of the earth points towards the sun.

40. A particle P is going in a sprial path as shown in Fig. 4.40 with constant speed.

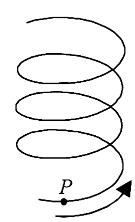


Fig. 4.40

(a) The velocity of the particle is constant.
(b) The acceleration of the particle is constant.
(c) The magnitude of acceleration is constant.
(d) The magnitude of acceleration is decreasing continuously.

41. A particle is moving along a circular path with uniform speed. Through what angle does its angular velocity change when it completes half of the circular path?

(a) 360° (b) 180°
(c) 45° (d) 0°

42. A fly wheel rotates at a constant speed of 3000 rpm. The angle described by the shaft in radian in one second is

(a) 3000 π (b) 100 π
(c) 50 π (d) 2 π

43. A scooter is going round a circular track with a speed of 30 ms^{-1}. The radius of the circular track is 50 m. The angular velocity of the scooter is

(a) 1000 rad s^{-1} (b) 0.6 rad s^{-1}
(c) 2.5 rad s^{-1} (d) 0.4 rad s^{-1}

44. A particle is moving along a circular path of radius 5 m and with uniform speed 5 ms^{-1}. What will be the average acceleration when the particle completes half revolution?

(a) 10 πms^{-2} (b) $10/\pi ms^{-2}$
(c) 10 ms^{-2} (d) zero

45. What determines the nature of the path followed by the particle?

(a) acceleration (b) speed
(c) velocity (d) none of these

46. A particle moves along a circular path of radius r with uniform speed v. The angle described by the particle in one second is given by

(a) vr^{-2} (b) $v^{-2}r$
(c) vr^{-1} (d) $v^{-1}r$

47. To enable a particle describe a circular path, what should be the angle between its velocity and acceleration?

(a) 180° (b) 90°
(c) 45° (d) 0°

48. A particle is describing the circular path of radius 10 m every 2 s. The average angular speed of the particle, during 4 s is

(a) 20 π rad s^{-1} (b) 4 π rad s^{-1}
(c) 2 π rad s^{-1} (d) none of these

49. The angular speed of the minute hand of the clock in degrees per second is

(a) 1.0 (b) 0.1
(c) 0.001 (d) none of these

50. The angular speed of a motor increases from 600 to 1200 rpm in 10 s. What is the angular acceleration of the motor?

(a) 60 rad s^{-2} (b) 2 π rad s^{-2}
(c) 60 π rad s^{-2} (d) 600 rad s^{-2}

51. A wheel is subjected to uniform angular acceleration about its axis. Initially its angular velocity is zero. In the first two seconds it rotates through angle θ_1. In the next two seconds it roatets through angle θ_2. What is the ratio θ_2/θ_1?

(a) 4 (b) 3
(c) 2 (d) 1

52. Two bullets are fired at angle θ and $(90 - \theta)$ to the horizontal with the same speed v. The ratio of their radii at the highest points is

(a) $\cos^2\theta$ (b) $\tan^2\theta : 1$
(c) 1 : 1 (d) $\cot^2\theta : 1$

53. A metal ring of mass m and radius r is placed on a smooth horizontal table and is set rotating about its axis so that each part of the ring moves with a speed v. The tension in the ring is

(a) $\dfrac{mv^2}{2\pi r}$ (b) $\dfrac{mv^2}{r}$
(c) $\dfrac{mv^2}{\pi r^2}$ (d) $\dfrac{mv^2}{2r}$

54. A particle moves in a circular path of radius r. In half the period of revolution its displacement and distance covered are

(a) $2r, \pi r$ (b) $r, \pi r$
(c) $2r, 2\pi r$ (d) $r\sqrt{2}, \pi r$

55. The angular acceleration of a particle moving along a circular path with uniform speed is

(a) zero (b) variable
(c) uniform but non-zero
(d) incomplete information

56. A bucket tied at the end of a 1.6 m long string is whirled in a vertical circle with a constant speed. What should be the minimum speed so that the water from the bucket does not spill out during rotation (g = 10 ms^2)?

(a) 9 ms^{-1} (b) 6.25 ms^{-1}
(c) 16 ms^{-1} (d) none of these

57. A stone of mass m tied to a string of length l is rotated in a circle with the other end of the string as the centre. The speed of the stone is v. If the string breaks, the stone will move

(a) along a tangent (b) towards the centre
(c) away from the centre (d) will stop

58. An automobile is turning around a circular road of radius r. The coefficieent of friction between the tyres and the road is μ. The velocity of the vehicle should not be more than

(a) $\sqrt{\dfrac{\mu g}{r}}$ (b) $\sqrt{\mu rg}$
(c) μrg (d) $\mu g/r$

59. The roadway bridge over a canal is in the form of a circular arc of radius 18 m. What is the greatest speed with which a motorcycle can cross the bridge without leaving the ground?

(a) 18.98 ms^{-1} (b) 18/9.8 ms^{-1}
(c) $\sqrt{9.8}$ ms^{-1} (d) $\sqrt{18 \times 9.8}$ ms^{-1}

60. An electric fan has blades of length 30 cm as measured from the axis of rotation. If the fan is rotating at 1200 rpm, the acceleration of a point on the tip of the blade is about

(a) 2370 ms^{-2} (b) 5055 ms^{-2}
(c) 1000 ms^{-2} (d) 4740 ms^{-2}

61. A bottle of soda water is held by the neck and swing briskly in a horizontal circle. Near which position of the bottle do the bubbles collect?

(a) in the middle of the bottle
(b) near the bottom
(c) bubbles uniformly distributed
(d) near the neck

62. Two cars c_1 and c_2 are going round concentric circles of radii r_1 and r_2. They complete the circular paths in the same time then $\frac{\text{speed of } c_1}{\text{speed of } c_2}$.

(a) r_2/r_1 (b) r_1/r_2
(c) 1 (d) none of these

63. A fan has 3 blades. The edges of two blades are 1 m apart. A coin of mass 2 *g* is placed on one of the blade edge. If the fan rotates with 600 rpm, the force experienced by the coin is nearly

(a) 3.1 N (b) 4.6 N
(c) 5.2 N (d) 2.8 N

64. A stone of mass 1 kg is tied to the end of a string 1 m long. It is whirled in a vertical circle. If the velocity of stone at the top be 4 ms^{-1}. What is the tension in the string at the lowest point? Take $g = 10\ ms^{-2}$

(a) 6 N (b) 66 N
(c) 5.2 N (d) 76 N

65. A body of mass 1 kg is rotating in a vertical circle of radius 1m. What will be the difference in its kinetic energy at the top and bottom of the circle? Take $g = 10\ ms^{-2}$

(a) 50 J (b) 30 J
(c) 20 J (d) 10 J

66. A thin uniform rod of length l is hinged at the lower end to the level floor and stands vertically. If the rod is allowed to fall, its upper end will strike the floor with a velocity

(a) $\sqrt{3gl}$ (b) $\sqrt{5gl}$
(c) $\sqrt{gl}$ (d) $\sqrt{2gl}$

67. A car is taking a turn on a level road. It may be thrown outwards because of the

(a) reaction of the ground
(b) frictional force
(c) weight
(d) lack of centripetal force

68. The velocity of a particle at highest point of the vertical circle is $\sqrt{3rg}$. The tension at the lowest point if mass of the particle is m is

(a) 2 mg (b) 4 mg
(c) 6 mg (d) 8 mg

69. A second is pendulum is suspended in a car that is travelling with a constant speed of 10 ms^{-1} round a circle of radius 10 m. If the pendulum undergoes small oscillations, the time period will be

(a) 2 s (b) 4 s
(c) less then 2 s
(d) greater than 2 s but less than 4 s

70. A hemispherical bowl of radius r is rotated about its axis of symmetry which is kept vertical. A small block is kept at a position where the radius makes an angle θ with the vertical. The block rotates with the bowl without any slipping. The friction coefficient between the block and the bowl is μ. The maximum speed for which the block will not slip

(a) $\left[\frac{g(\sin\theta - \mu\cos\theta)}{r\sin\theta(\cos\theta + \mu\sin\theta)}\right]^{1/2}$

(b) $\left[\frac{g(\sin\theta + \mu\cos\theta)}{r\sin\theta(\cos\theta + \mu\sin\theta)}\right]^{1/2}$

(c) $\left[\frac{g(\sin\theta + \mu\cos\theta)}{r\sin\theta(\cos\theta - \mu\sin\theta)}\right]^{1/2}$

(d) none of these

71. A car is travelling with linear velocity v on a circular road of radius r. If it is increasing its speed at the rate of 'a' ms^{-2}, then the resultant acceleration will be

(a) $\sqrt{\frac{v^4}{r^2} + a^2}$ (b) $\sqrt{\frac{v^4}{r^2} - a^2}$

(c) $\sqrt{\frac{v^2}{r^2} + a^2}$ (d) $\sqrt{\frac{v^2}{r^2} - a^2}$

72. A toy car is tied to the end of an unstretched string of length a. When revolved, the toy car moves in a horizontal circle of radius $2a$ with time period T. If it is now revolved in a horizontal circle of radius $3a$ with a period T' with the same force, then

(a) $T = \frac{\sqrt{3}}{2}T$ (b) $T = \sqrt{\frac{3}{2}}\,T$

(c) $T = T$ (d) $T = \frac{3}{2}T$

73. A proton goes round in a circular orbit of radius 0.01 m under a centripetal force of 4×10^{-3} N. What is the frequency of revolution of the proton?

(a) 2.5×10^{12} Hz (b) 4×10^{13} Hz
(c) $8 \times 10^{12}\ s^{-1}$ (d) $16 \times 10^{12}\ s^{-1}$

74. A block of mass m at the end of a string is whirled round in a vertical circle of radius r. The critical speed of the block at the top of its swing below which the string would slacken before block reaches the top is

(a) $\sqrt{2rg}$ (b) $\sqrt{3rg}$
(c) $\sqrt{rg}$ (d) $\sqrt{5rg}$

75. A tube of length l is filled completely with an incompressible liquid of mass m and closed at both the ends. The tube is then rotated in a horizontal plane about one of its ends with a uniform angular velocity ω. The force exerted by the liquid at the other end is

(a) $\frac{ml\omega^2}{3}$ (b) $ml\omega^2$
(c) $\frac{ml\omega^2}{2}$ (d) $\frac{ml\omega^2}{4}$

76. What is the work done in t seconds by a body of mass m moving in a circular path of radius r with a constant angular acceleration. Initially the body was at rest.

(a) $m\,\alpha^2R^2t^2$ (b) $2\pi mv^2t$
(c) $1/2\, m\alpha^2R^2t^2$ (d) zero

77. A string can withstand a tension of 25 N. What is the greatest speed at which a body of mass 1 kg can be whirled in a horizontal circle using 1m length of the string?

(a) 10 ms^{-1} (b) 7.5 ms^{-1}
(c) 5 ms^{-1} (d) 2.5 ms^{-1}

78. A car travels north with a uniform velocity. It goes over a piece of mud which sticks to the tyre. The particles of the mud are thrown

(a) vertically inward (b) vertically upward
(c) horizontally to south (d) horizontally to north

79. A car is moving with a speed of 30 ms^{-1} on a circular path of radius 500 m. Its speed is increasing at the rate of 2 ms^{-2}. The acceleration of the car is

(a) 2.7 ms^{-2} (b) 9.8 ms^{-2}
(c) 2 ms^{-2} (d) 1.8 ms^{-2}

80. If the overbidge is concave instead of being convex, the thrust on the road at the lowest position will be

(a) $mg - \frac{mv^2}{r}$ (b) $mg + \frac{mv^2}{r}$
(c) $\frac{v^2g}{r}$ (d) $\frac{m^2v^2g}{r}$

81. A body crosses the topmost point of a vertical circle with a critical speed. What will be the acceleration when the string is horizontal?

(a) $3g$ (b) $6g$
(c) g (d) $2g$

82. At a curved path of the road bed is raised a little on the side away from the centre of the curved path. The slope of the road bed is given by

(a) $\tan\theta = rg/v^2$ (b) $\tan\theta = r/gv^2$
(c) $\tan\theta = v^2/rg$ (d) $\tan\theta = v^2 g/r$

83. A truck has a width of 2.8 m. It is moving on a circular road of radius 48.6 m. The height of centre of mass is 1.2 m. The maximum speed so that it does not overturn will be nearly

(a) 24.3 ms–1 (b) 27.2 ms–1
(c) 13.4 ms–1 (d) 15.7ms–1

Answers to Questions for Practice

1.	(b)	2.	(a)	3.	(a)	4.	(c)	5.	(b)	6.	(a)	7.	(c)
8.	(d)	9.	(b)	10.	(c)	11.	(d)	12.	(a)	13.	(b)	14.	(a)
15.	(d)	16.	(a)	17.	(b)	18.	(c)	19.	(c)	20.	(a)	21.	(d)
22.	(b)	23.	(a)	24.	(c)	25.	(a)	26.	(b)	27.	(d)	28.	(c)
29.	(c)	30.	(a)	31.	(a)	32.	(c)	33.	(a)	34.	(d)	35.	(a)
36.	(d)	37.	(c)	38.	(a,d)	39.	(d)	40.	(c)	41.	(d)	42.	(b)
43.	(b)	44.	(b)	45.	(a)	46.	(c)	47.	(b)	48.	(d)	49.	(b)
50.	(b)	51.	(b)	52.	(d)	53.	(a)	54.	(a)	55.	(a)	56.	(a)
57.	(a)	58.	(b)	59.	(d)	60.	(d)	61.	(b)	62.	(b)	63.	(b)
64.	(b)	65.	(c)	66.	(a)	67.	(d)	68.	(d)	69.	(c)	70.	(c)
71.	(a)	72.	(b)	73.	(a)	74.	(c)	75.	(c)	76.	(c)	77.	(c)
78.	(b)	79.	(a)	80.	(b)	81.	(c)	82.	(c)	83.	(a)		

Explanations

1(b) For quarter revolution

$\Delta\vec{v} = v_2(-\hat{i}) - v_1\hat{j}$

or $\Delta\vec{v} = -v\hat{i} - v\hat{j}$

$\therefore\ \Delta\upsilon = \sqrt{\upsilon^2+\upsilon^2} = \sqrt{2}\,v$

Also $\alpha = \tan^{-1}\frac{\upsilon}{\upsilon} = 45°$

$\therefore\ \Delta\vec{\upsilon} = \sqrt{2}\,v$ south west.

2(a) Assuming the particles to be the closest, relative velocity

$\upsilon_r = |\vec{\upsilon}_B - \vec{\upsilon}_A| = \upsilon_B - \upsilon_A$

and relative position vector,

$r_r = |\vec{r}_B - \vec{r}_A| = r_B - r_A$

Using $\omega = \frac{v}{r}$, we get relative angular velocity.

$$\omega = \frac{v_r}{r_r} = \frac{v_B - v_A}{r_B - r_A}$$

3(a) Centripetal acceleration

$$a_r = \frac{v^2}{r} = \left(\frac{27\times1000}{3600}\right)^2 \Big/ 80$$

$$= \frac{7.5\times7.5}{80} = 0.703\ ms^{-2}$$

Net acceleration, $a = \sqrt{a_r^2 + a_t^2}$

where a_t = tangential acceleration = 0.5 ms^{-2}

$$\therefore \quad a = \sqrt{0.703^2 + 0.5^2} = 0.86\ ms^{-2} \quad \text{and}$$

$$\theta = \tan^{-1}\frac{a_r}{a_t} = \tan^{-1}\frac{0.703}{0.5} = 54°$$

4(c) Mid point acceleration,

$$a = \frac{v^2}{r} = \frac{4}{4} = 1\ ms^{-1} \text{ at } 45° \text{ with } x \text{ axis}$$

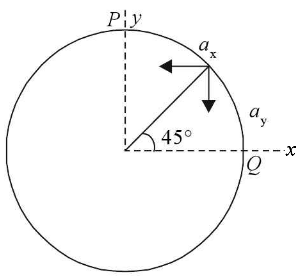


Fig. 4.41

$$\therefore \quad \vec{a}_x = -a\cos 45°\ \hat{x}$$

$$= -1\times\frac{1}{\sqrt{2}}\ \hat{x} = -\frac{1}{\sqrt{2}}\ \hat{x}$$

$$\vec{a}_y = -a\sin 45°\ \hat{y}$$

$$= -1\times\frac{1}{\sqrt{2}}\ \hat{y} = -\frac{1}{\sqrt{2}}\ \hat{y}$$

Thus $\hat{a} = \vec{a}_x + \vec{a}_y = -\frac{1}{\sqrt{2}}(\hat{x} + \hat{y})$

5(b) Maximum velocity,

$$v_{max} = \left[\left(\frac{\mu + \tan\theta}{1-\mu\tan\theta}\right)rg\right]^{1/2}$$

$$= \left[\left(\frac{0.2 + \tan 15°}{1-0.2\tan 15°}\right)300\times9.8\right]^{1/2}$$

$$\simeq 38\ ms^{-1}$$

6(a) Here $\cos\alpha = \frac{r-h}{r}$

or $\quad -h = r\cos\alpha - r$

or $\quad h = r(1-\cos\alpha)$

At L, $mg\cos\alpha = \frac{m}{r}v^2$

$$= \frac{m}{r}\left[v_0^2 + (\sqrt{2gh})^2\right]$$

or $\quad mg\cos\alpha = \frac{m}{r}\left(\frac{1}{4}gr + 2gh\right)$

$$= \frac{m}{r}\left(\frac{1}{4}gr + 2gr(1-\cos\alpha\right)$$

or $\quad \cos\alpha = \frac{1}{4} + 2 - 2\cos\alpha$

or $\quad 3\cos\alpha = \frac{9}{4}$ or $\cos\alpha = \frac{3}{4}$

or $\quad \alpha = \cos^{-1} 3/4$

7(c) Velocity at the highest point,

$$v_h = \sqrt{gL}$$

Height of the mass = $2L$

Time taken to fall = $\sqrt{\frac{2\times2L}{g}} = \sqrt{\frac{4L}{g}}$

Horizontal distance covered

$= v_h t = \sqrt{gL}\times\sqrt{4L/g}$

$= 2L$

8(d) Note $HH' = 2HO' = 2r\cos\alpha$

Using $s = 1/2\ at^2$, we get

$2r\cos\alpha = 1/2\ (g\cos\alpha)\ t^2$

or $\quad t^2 = \frac{4r}{g}$ or $t = \sqrt{\frac{4r}{g}}$

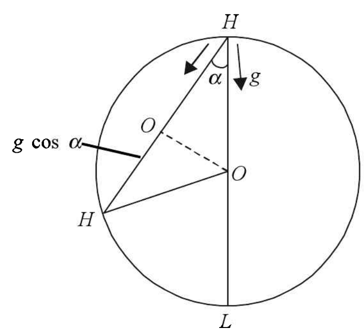


Fig. 4.42

9(b) Tangential acceleration, $a_t = g\sin\alpha$ and radial acceleration,

$$a_r = \frac{v^2}{l} = \frac{2gl\cos\alpha}{l}$$

$$= 2g\cos\alpha$$

Also $\tan\alpha = \frac{a_r}{a_t}$

$$= \frac{2g\cos\alpha}{g\sin\alpha} = \frac{2}{\tan\alpha}$$

or $\quad \tan^2\alpha = 2$ i.e., $\sec^2\alpha = 3$

or $\quad \frac{1}{\cos^2\alpha} = 3$ or $\cos\alpha = \frac{1}{\sqrt{3}}$

or $\quad \alpha = \cos^{-1}\frac{1}{\sqrt{3}}$

10(c)Using $\frac{mv^2}{R} = mg\sin\theta$

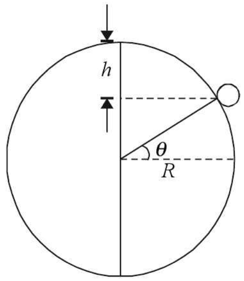


Fig. 4.43

where $v = \sqrt{2gh}$

and $\sin\theta = \frac{R-h}{R}$, we get

$$\frac{m\times 2gh}{R} = mg\sin\theta = mg\,\frac{R-h}{R}$$

or $h = R/3$ and $\sin\theta = 2/3$

or $\theta \simeq 42°$

11(d)Using $\frac{mv^2}{R}$ = cos θ and mg = R sin θ

we get $\tan\theta = \frac{gR}{v^2}$

or $\frac{R}{h} = \frac{gR}{v^2}$ i.e., $v = \sqrt{gh}$

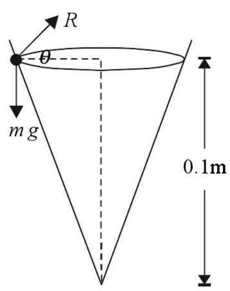


Fig. 4.44

or $v = \sqrt{10\times 0.1} = 1\ \text{ms}^{-1}$

12(a)Let u be the initial velocity of ball P, then $u = \frac{2\pi r}{2T}$ and difference in final velocities, $v' - v = eu$ (given)

Time for second strike $= \frac{2\pi r}{v'-v} = \frac{2\pi r}{eu}$

$$= \frac{2\pi r}{e\times\frac{2\pi r}{2T}} = \frac{2T}{e}$$

13(b)$\Delta\vec{v} = 2v\sin\left(\frac{\theta}{2}\right) = 2\times 60\sin 30 = 60\ \text{ms}^{-1}$

Instantaneous acceleration

$$= \frac{v^2}{r} = \frac{60^2}{0.3\times 1000} = 12\ \text{ms}^{-2}$$

Time taken to cover the arc

$$= t = \frac{\pi}{3}\times\frac{300}{60}$$

using $\Delta t = \frac{S}{v} = \frac{rd\theta}{v}$

∴ average acceleration $a = \frac{\Delta v}{t}$

$$= \frac{60}{\frac{\pi}{3}\times\frac{300}{60}} = 11.5\ \text{ms}^{-2}$$

14(a)Centripetal acceleration at A and B is $\frac{mu^2}{r_A}$ and $\frac{mu^2}{r_B}$ respectively. Clearly $r_B > r_A$. therefore centripetal acceleration at A is more. Thus, reaction at B is less than A because here the normal reaction depends on upwards force or acceleration.

15(d)For vertical motion

$H = \frac{1}{2}gt^2$ or $t = \sqrt{2H/g}$

For horizontal motion, distance covered is given by, $2\pi rn = ut$

or $2\pi rn = \sqrt{2H/g}$

or $n = \frac{u}{2\pi r}\sqrt{2H/g}$

16(a)Velocity at the bottom of the circle

$= \sqrt{5gr}$

Velocity at the top of the circle $= \sqrt{gr}$

Using P.E. = K.E., we get

$$mg\text{H} = \frac{1}{2}m(\sqrt{5gr})^2 = \frac{1}{2}m(5gr) \qquad \text{(i)}$$

$$\text{also } mgh = \frac{1}{2}m(\sqrt{gr})^2 = \frac{1}{2}mgr \qquad \text{(ii)}$$

Dividing (i) and (ii), we get

$$\frac{H}{h} = 5$$

17(b) Given $a_T = 2\ \text{ms}^{-2}$

and $a_r = \frac{v^2}{r} = \frac{30\times 30}{500} = 1.8\ \text{ms}^{-2}$

∴ $a = \sqrt{a_r^2 + a_t^2} = \sqrt{1.8^2 + 2^2}$

$= 2.7\ \text{ms}^{-2}$

18(c) Using $u = r\omega = r\times 2\pi f$

$= 1\sin\theta\times 2\pi f$, we get

(∵ r = l sin θ)

$$u = 1\times 0.78\times 2\pi\times\frac{2}{\pi}$$

$= 3.12\ \text{ms}^{-1}$

19(c) $\frac{mv^2}{R} = T\sin\theta$ and $mg = T\cos\theta$

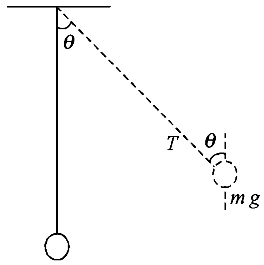


Fig. 4.45

$\therefore \quad \dfrac{v^2}{Rg} = \tan\theta$

Thus $\tan\theta = \dfrac{10^2}{10\times10} = 1$ or $\theta = 45°$

20(a) Horizontal displacement of both particles is equal.

For particle Q, the horizontal velocity is always equal to v.

For particle P, at the start the motion will be accelerated one, causing increase in velocity.

Thus $t_P < t_Q$

21(d) Here $v^2 - u^2 = -2gl$ ...(i)

$v^2 = u^2 - 2gl$

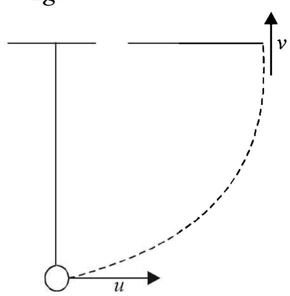


Fig. 4.46

Since the velocities are mutually perpendicular, change in velocity

$\Delta v = \sqrt{u^2 + v^2}$ ...(ii)

$= \sqrt{u^2 + u^2 - 2gl}$

(substituting the value of v^2 from (i))

or $\Delta v = \sqrt{2(u^2 - gl)}$

22(b) Here average velocity

$= \dfrac{\text{displacement } AB}{\text{time}}$

$= \dfrac{2}{1} = 2\ \text{ms}^{-1}$

23(a) Here, speed of the block at the highest points in all the four cases will be same. The normal reactions at these points will depend upon the centripetal acceleration experienced by the block.

Centripetal acceleration $= \dfrac{v^2}{R}$

Radius of curvature in the first case will be minimum, so centripetal acceleration and hence reaction will be maximum in this case.

24(c) Motion of a simple pendulum is an example of non uniform circular motion. The bob has some tangential as well as radial acceleration.

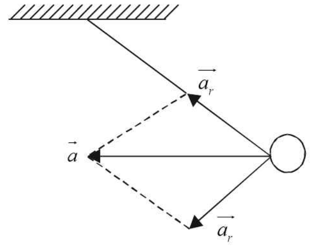


Fig. 4.47

Thus, total acceleration (A) will be vectorial sum of radial acceleration $(\vec{a}_R)$ and tangential acceleration $(\vec{a}_T)$.

25(a) The force mv^2/r directed outwards, called centrifugal force, is not a real force

At A, $mv_1^2/l = T_1 + mg$.

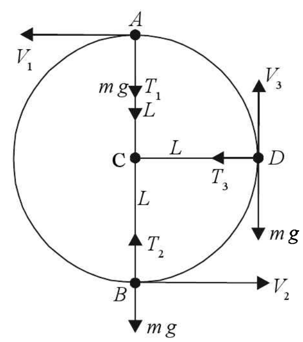


Fig. 4.48

26(b) For v_1 to be minimum, $T_1 = 0$.

or $v_1 = \sqrt{gl}$.

27(d) Applying the principle of conservation of energy between A and B.

$\frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2 = mg' \, 2l$

or $\quad \dfrac{mv_2^2}{l} = \dfrac{mv_1^2}{l} + 4mg$

At B, $\dfrac{mv_2^2}{l} = T_2 - mg$

or $\quad mg + 4mg = T_2 - mg$ or $T_2 = 6mg$.

48(d) $\omega = \dfrac{2\pi}{T} = \dfrac{2\pi}{2} = \pi\ \text{rads}^{-1}$.

52(d) $\dfrac{r_1}{r_2} = \dfrac{v_1^2/g}{v_2^2/g} = \dfrac{v^2\cos^2\theta}{v^2\sin^2\theta} = \cot^2\theta$.

56(a) $v = \sqrt{5rg} = \sqrt{5\times1.6\times10}\ 9\ \text{ms}^{-1}$.

59(d) $\dfrac{v^2}{r} = g$ or $v = \sqrt{rg}$.

72(b) $m2a\left(\dfrac{2\pi}{T}\right)^2 = m3a\int_0^l x\,dx = \because$ so $\dfrac{T'}{T} = \sqrt{\dfrac{3}{2}}$.

75(c)

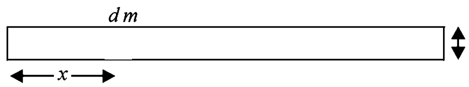


Fig. 4.49

$df = dm\, x\omega^2 = \rho A dx\, w^2$

where A is area of cross-section

$$F = \rho A \omega^2 \int_0^l x\, dx = \frac{\rho A k^2 \omega}{2} = \frac{ml\omega^2}{2} \qquad \{\because \rho A l = m\}$$

83(a) $mgl/2 = \dfrac{mv^2}{R}\ h.$

(overturning torque = restoring torque)

or $$v = \sqrt{\frac{gRl}{2h}}$$

$$= \sqrt{\frac{10\times 50\times 1.4}{1.2}} = 24.3\ \text{ms}^{-1}.$$

5

Newton's Laws of Motion

BRIEF REVIEW

Force is a pull or push which generates or tends to generate motion in a body at rest, stops or tends to stop a body in motion, increases or decreases the magnitude of velocity of the moving body, changes or tends to change the shape of the body.

Newton's First Law of Motion If a body is at rest it will remain at rest and a body in uniform motion will remain in the state of uniform motion unless it is compelled by some **external force** to change its state.

Inertia It is the inherent property of the body with which it cannot change by itself its state of rest or of uniform motion unless acted upon by an external force. Hence, Newton's first law of motion may also be called law of inertia.

Note that the term external force has been used in first law. It means there would be **internal force** also.

Internal Force If the force applying agent lies inside the system, force is internal. Internal force cannot provide motion. For example, if you are sitting in a car and you push the car, car does not move. If you come out of the car and apply the same force, car moves. When you were inside the car, the force applying agent was inside the car, hence, the force was internal and car did not move. When the force applying agent (you) had moved outside, the car moved.

The straight line along which force acts is called **Line of action of the force.**

In order to accelerate or decelerate a body an unbalanced force is required.

A system of bodies on which no external force acts is called a **closed system.** For example, two bodies moving towards each other due to their mutual electrostatic or gravitational force.

When many forces act on a body at the same point, they are called concurrent forces. The system of concurrent forces may be:

(a) Collinear, that is acting along the same straightline.

(b) Coplanar, that is in the same plane.

(c) Generally directed, but not in the same plane.

Mass In newtonian mechanics mass is considered to be a measure of inertia of a body and is considered independent of its velocity. It is a scalar quantity. Unit $\rightarrow$ kg (SI system).

Momentum The total quantity of motion contained in a body is called momentum. It is a vector quantity. Unit kg ms^{-1} (SI) $\vec{p} = m\vec{v}$.

If two different masses have same momentum, then the lighter one has more kinetic energy (also more velocity).

Newton's Second Law of Motion The time rate of change of momentum is directly proportional to force (external) applied on it and the change in momentum occurs in the direction of force

$$\vec{F} \propto \frac{dp}{dt}, \text{ or } \vec{F} = \frac{d\vec{p}}{dt} = \frac{md\vec{v}}{dt} = m\vec{a}$$

Newton considered mass to be constant. Unit of Force is Netwon (N) or kg Wt (kilogram weight) or kg f (kilogram force). 1 kg Wt = 1 kg f = 9.8 N.

If mass is varying and velocity constant $\vec{F} = v\dfrac{dm}{dt}$

$\vec{F} = \dfrac{dm dv}{dt}$ if both mass and velocity vary.

Impulse Product of force and time for which it acts is called impulse.

$F = \dfrac{dp}{dt}$ or $F \cdot dt = dp$ i.e. impulse = change in momentum

$F_{av} \cdot t = \Delta p$ is called impulse momentum theorem.

Newton's Third Law of Motion To every action there is an equal and opposite reaction, i.e., $\vec{F}_{AB} = -\vec{F}_{BA}$. Moreover, action and reaction act on different bodies. According to third law forces in nature occur in pairs. Single isolated force is not possible.

Note: In certain cases of electrostatics and in springs Newton's 3rd law fails.

Law of Conservation of Momentum If no external force acts then the total momentum of the system is conserved

$$\vec{F} = \frac{d\vec{p}}{dt} = 0 \text{ or } \vec{p} = \text{constant.}$$

Equilibrium

Translatory Equilibrium When several forces act on a body such that resultant force is zero, i.e., $\Sigma F = 0$, the body is said to be in translatory equilibrium. $\Sigma F = 0$ implies $\Sigma F_x = \Sigma F_y = \Sigma F_z = 0$. It means the body is in the state of rest (static equilibrium) or in uniform motion (dynamic equilibrium).

If the force is conservative then $F = \dfrac{du}{dr} = 0$ means potential energy u = maximum, minimum or constant.

Stable Equilibrium If on slight displacement from equilibrium position, body has the tendency to regain its original position. In such cases centre of Mass (COM) rises on slight displacement. Note that PE is minimum $\left(\dfrac{d^2u}{dr^2} = +ve\right)$ in stable equilibrium.

Unstable Equilibrium If on slight displacement from equilibrium position the body moves in the direction of displacement, the equilibrium is known to be unstable. The COM goes down on slight displacement. PE is maximum and $\dfrac{d^2u}{dr^2} = -ve$ for unstable equilibrium.

Neutral Equilibrium If the body remains at the displaced position after a slight displacement then such an equilibrium is neutral. The COM does not change and PE is constant but not zero.

Fig. 5.1 illustrates all the types of equilibrium: stable, unstable and neutral.

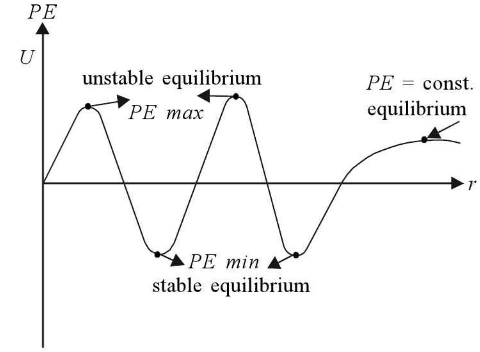


Fig. 5.1 Types of equilibrium explanations

Strings String is considered to be massless unless stated and hence, tension remains constant throughout the string.

String is assumed to be inextensible unless stated and hence, acceleration of any number of masses connected to it is always equal or same. If the pulley is massless and smooth, and string is also massless then tension at each point (or two sides of string) is constant as shown in Fig. 5.2.

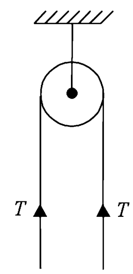


Fig. 5.2 Tension in string for a light and smooth pulley

If the string changes tension changes as illustrated in Fig. 5.3.

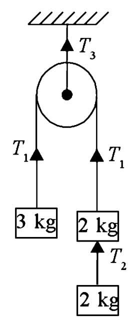


Fig. 5.3 Tension in different strings

T_1, T_2 and T_3 in Fig. 5.3 are different as string changes. In Fig. 5.3 $T_3 = 2T_1$

$T_2 = 2(g - a)$

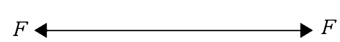


Fig. 5.4

If forces are equal and opposite on a massless string as shown in Fig. 5.4 then tension T is equal to either of the two forces, i.e., $T = F$.

The maximum tension which a string can bear is called its breaking strength. If the string has mass tension is different at each point as illustrated in Fig. 5.5.

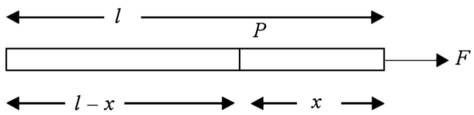


Fig. 5.5 Illustration of tension in a stringrod having mass

Mass per unit length $\lambda = \frac{M}{l}$. We have to find tension at P. Mass of $(l - x)$ part is $\frac{M(l-x)}{l}$.

Tension at $P = \frac{F}{M}\left(\frac{M(l-x)}{l}\right) = \frac{F(l-x)}{l}$.

Springs Springs are assumed massless unless stated. Restoring force is same every where, i.e. $F = -kx$

Springs can be stretched or compressed. Stretch or compression is taken positive.

Restoring force is linear as is clear from $F = -kx$. k is called spring constant or force constant.

$k \propto \frac{1}{l}$ (k also depends upon radius, length and material used).

In series $\frac{1}{k_{\text{effective}}} = \frac{1}{k_1} + \frac{1}{k_2} + ...$

In parallel $k_{\text{effective}} = k_1 + k_2 + ...$

If masses m_1 and m_2 connected to a spring as shown in Fig. 5.6 are oscillating or both masses move then find reduced mass μ $\frac{1}{\mu} = \frac{1}{m_1} + \frac{1}{m_2}$.

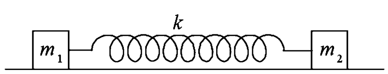


Fig. 5.6

If the spring has mass m_s, then $\frac{m_s}{3}$ is used to produce extension.

Pseudo Forces The hypothetical forces added while dealing with problems associated with non-inertial or accelerated frame of reference, so that Newton's laws may be applied are called psuedo forces or inertial forces. If a frame of reference is moving with an acceleration a_o, then force on a particle of mass m is ma_o. In the force equation a force $-ma_o$ will be added to make the frame of reference inertial.

Friction If we try to slide a body over a surface the motion is resisted by the bonding between the body and the surface. This resistance is represented by a single force called friction. The friction is parallel to the surface and opposite to the direction of intended motion. Remember static friction is a self adjusting force. If a body is at rest and not being pulled, force of friction is zero. If a pulling force is applied and the body does not move, friction still acts and is called static friction. The maximum value of static friction is called limiting friction. See Fig. 5.7. If we apply the force beyond limiting friction, the body begins to move and friction slightly decreases called kinetic friction.

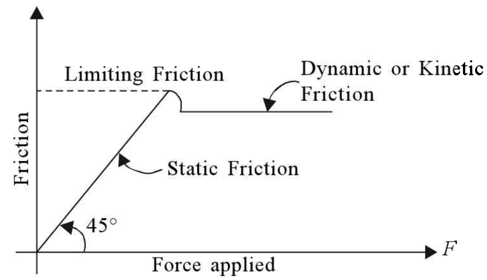


Fig. 5.7 Friction illustration

Limiting Friction $F_{fL} = \mu_s N$ where N is normal reaction. $\mu_s = \tan\theta$ where θ is the angle of limiting friction.

Note: $\mu_s > \mu_k > \mu_R$ where μ_s stands for coefficient of static friction, μ_k stands for coefficient of kinetic friction and μ_R stands for rolling friction.

Friction is independent of surface area of contact. However, it depends upon the nature of material of the surfaces in contact, their roughness, smoothness, inclination. Normally friction between too smooth bodies is more. If the bodies are made extra smooth by polishing the bonding force of cohesion or adhesion increases resulting in cold welding.

In practice $0 < \mu < 1$ but $\mu > 1$ is observed. For example; $\mu_s = 1$ for glass/glass, and, $\mu_s = 1.6$ for $Cu - Cu$.

Friction is a non-conservative force.

If force is applied and still the body is at rest then the force of the friction is equal to force applied.

Equation of motion for centre of mass (COM)

$$m\frac{dv_{\text{COM}}}{dt} = \sum F$$

- **Short Cuts and Points to Note**

1. Tension is a reaction force produced in a string or rod.
2. In a massless string (if not passing over a pulley) tension is equal at each point.
3. If pulley is massless and smooth and, string is massless and passing over a pulley as shown in Fig. 5.8, then

$$a = \frac{(m_2 - m_1)g}{m_1 + m_2}, T = \frac{2m_1m_2g}{m_1 + m_2}$$

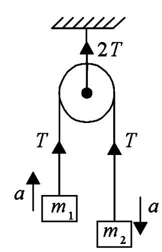


Fig. 5.8

4. If the string changes tension will change. Assume, in Fig. 5.9. Pulley is smooth and massless. String is also massless. Then,

$$a = \frac{[(m_1+m_3)-m_2]g}{m_1+m_2+m_3}, T = \frac{2(m_1+m_3)m_2 g}{m_1+m_2+m_3}$$

$$T' = m_3(g-a), T'' = 2T$$

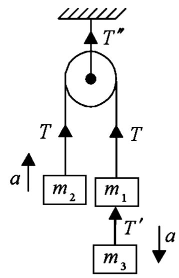


Fig. 5.9

5. If the pulley system of fig. 5.8 moves up with an acceleration a' then,

$$a = \frac{(m_2-m_1)(g+a')}{m_1+m_2} \quad \text{and}$$

$$T = \frac{2m_1m_2(g+a')}{m_1+m_2}$$

6. If the pulley system shown in Fig. 5.9 moves up with an acceleration a'. Then,

$$a = \frac{[(m_1+m_3)-m_2](g+a')}{m_1+m_3+m_2},$$

$$T = \frac{2(m_1+m_3)m_2(g+a')}{m_1+m_2+m_3},$$

$$T' = m_3(g+a'-a)$$

7. If $F > 2T$ in Fig 5.10 is applied on the pulley to move the system upwards.

Then, $a' = \dfrac{F-2T}{m_1+m_2}$; $a = \dfrac{(m_2-m_1)(g+a')}{m_1+m_2}$

$$T = \frac{2m_1m_2(g+a')}{m_1+m_2}$$

If $F < 2T$, then $a' = 0$

and $a = \dfrac{(m_2-m_1)(g)}{m_1+m_2}$

$$T = \frac{2m_1m_2g}{m_1+m_2}.$$

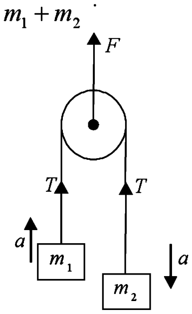


Fig. 5.10

8. If the springs are in parallel then their displacements are equal. For example, in Fig. 5.11 (a) and (b), the springs are in parallel, i.e., $k_{eff} = k_1 + k_2$

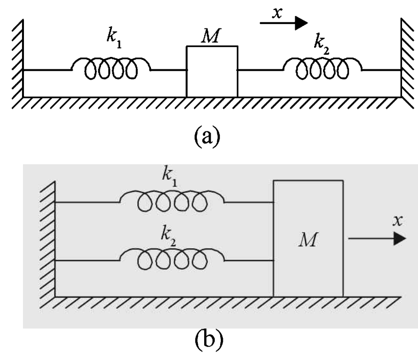


Fig. 5.11

9. If the springs are in series, as shown in Fig. 5.12 streches in spring are un-equal and $x = x_1 + x_2$

or $\dfrac{1}{k_{eff}} = \dfrac{1}{k_1} + \dfrac{1}{k_2}$.

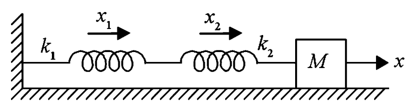


Fig. 5.12

10. If the spring is cut $k \propto \dfrac{1}{l}$. For example, if a spring of spring constant k is cut in the ratio 2 : 3 then shorter spring has $k' = \dfrac{5k}{2}$ and bigger one has spring constant $k'' = \dfrac{5k}{3}$.

11. In Fig. 5.13, if the block or pulley moves down by x, spring moves down by $2x$. Thus $T = F' = k(2x)$ and $F = 2T = k(4x)$.

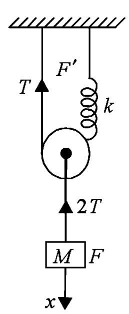


Fig. 5.13

In Fig. 5.14, if the block moves down by x then spring or pulley moves down by $\dfrac{x}{2}$. $F = T$, $F'' = 2T = k\left(\dfrac{x}{2}\right)$.

or $F = \dfrac{F''}{2} = k\left(\dfrac{x}{4}\right)$.

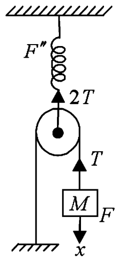


Fig. 5.14

12. As shown in Fig. 5.15, if the pulley moves forward by x then block moves forward by $2x$.

$\therefore\ a_{\text{block}} = 2\, a_{\text{pulley}}$; $\quad a_{\text{block}} = \dfrac{T}{m} = \dfrac{F}{2m}$

$a_{\text{pulley}} = \dfrac{F}{4m}$

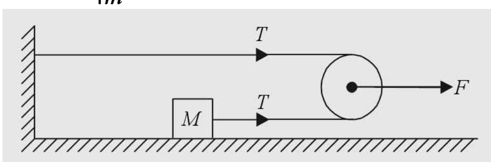


Fig. 5.15

13. Since force is a vector, apply vector algebra whenever there are two or more forces.

14. Draw free body diagram before you solve the problems. They make the problem very simple.

15. If force is applied on the body and body does not move, then, friction = force applied and not μN where N is normal reaction.

16. $\mu_s > \mu_k > \mu_R$. Barring few exception $\mu_s < 1$ and hence $\mu_k < 1$.

17. In conservative forces work done depends upon initial and final position. It is independent of the path followed. Net work done in a closed loop equals zero. Gravitational, electrostatic, magnetic forces are conservative. Friction is not conservative.

18. If there is no friction then acceleration down an incline is $a = g \sin \theta$ as shown in Fig. 5.16 (a).

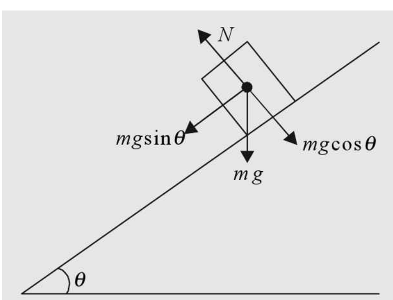


Fig. 5.16 (a)

19. If there is friction and coefficient of friction between the block and the incline is μ then,

$a = g \sin \theta - mg \cos \theta$ down the incline

or $F_{\text{down}} = \text{mg}\,(\sin \theta - \mu \cos \theta)$

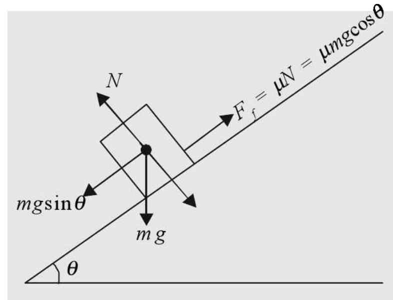


Fig. 5.16 (b)

20. If the block is to move up the incline with a constant velocity then $F_{\text{up}} = mg\,(\sin \theta + \mu \cos \theta)$ (See Fig. 5.17).

If it is to move up with an acceleration 'a' also then

$F_{\text{up}} = mg\,(\sin \theta + \mu \cos \theta) + ma$.

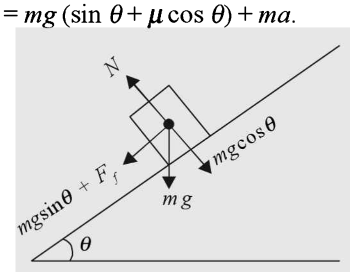


Fig. 5.17

21. On a horizontal plane deceleration due to friction is μg.

22. If a lift moves up with an acceleration a then effective or apparent weight is $m\,(g + a)$ as ma acting downward is pseudo force to be added to make frame of reference inertial.

Similarly if the pulley is moving down with an acceleration a then apparent weight of the body is $m\,(g - a)$.

23. If the force is a function of distance or velocity then use:

$$\frac{md^2x}{dt^2} = kx,\ \frac{mdv}{dt} = kx$$

$$\text{or}\quad \frac{mdv}{dt}\cdot\frac{dx}{dt} = kx \ \text{ or } \ \frac{mdv}{dx}\, v = kx.$$

24. It is always helpful to choose axis along the incline as x-axis and axis perpendicular to the incline as y-axis.

25. Remember frictional force and normal force are always perpendicular and F_f = Force applied if body remains stationary; $F_f = \mu_k N$ if the body is in motion.

26. Pulling at an angle decreases the kinetic friction as normal reaction decreases as illustrated in Fig. 5.18.

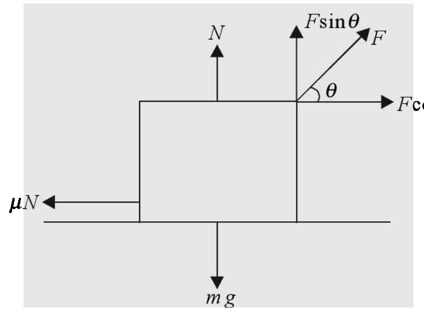


Fig. 5.18

$N = Mg - F\sin\theta$.

or, $F_f = \mu_k N = \mu_k (Mg - F\sin\theta)$.

27. If $\Sigma F_y = 0 = mg - k\,v_y$ then $v_y = \dfrac{mg}{k}$ is terminal velocity as in case of viscosity. $F = 6\,\pi\,\eta\,r\,v$ (Stoke's law) v is terminal velocity.

28. If a body/particle of mass m moves with a linear velocity v along the diameter of a turn table then an extra force is experienced by the body called coriolis force.

$F_{\text{coroilis}} = 2\,mv\omega$. Where ω is angular velocity of the turn table

• Caution

1. Applying Newton's law without caring about inertial/non-inertial frames.

⇒ In non-inertial frames of reference, first apply pseudo vectors to make the frame of reference inertial, only after that apply Newton's laws.

2. Considering action and reaction always act on different bodies.

⇒ In case of elastic bodies and springs, action and reaction act on same body. That is, in case of restoring force in a spring or deforming force in elastic bodies, action and reaction act on same body. These forces are therefore called internal forces.

3. Considering Newton's third law is always valid.

⇒ In certain cases of electrostatics Newton's third law fails.

4. Assuming friction always acts in a direction opposite to the motion.

⇒ If the friction causes motion then the friction acts in the direction of motion.

5. Considering force constant of a spring does not vary when spring is cut.

⇒ Spring constant $k \propto \dfrac{1}{l}$.

6. Assuming friction is always equal to μN.

⇒ If the body is moving, friction = $\mu_k N$. If the body is stationary then friction is equal to force applied.

7. Assuming if pulley is massless then tension in the string on two sides of the pulley is unequal as shown in Fig. 5.19.

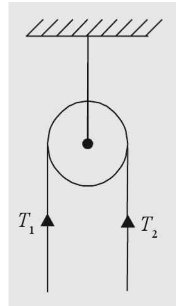


Fig. 5.19

⇒ If pulley is massless and smooth $T_1 = T_2 = T$

If pulley has mass then only T_1 and T_2 are unequal.

8. Not understanding constraints.

⇒ In problems like shown in Fig. 5.20, if the pulley moves forward by x, then thread $2x$ is used x below and x above which will be supplied by the block side as other is fixed. Therefore, block will move $2x$. Hence, $a_{\text{block}} = 2\,a_{\text{pulley}}$

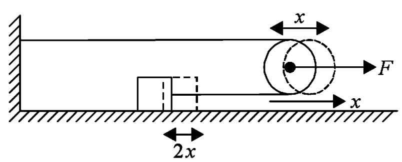


Fig. 5.20

9. Considering in equilibrium body must be at rest.

⇒ In static equilibrium body is at rest. In dynamic equilibrium, it moves with uniform velocity.

10. Assuming there is no tension if the rope is pulled by equal and opposite forces on two ends.

⇒ Tension is equal to either of the force applied.

11. Considering impulse always provides acceleration.

⇒ Sharp impulse only provides velocity.

12. Considering rough surfaces have more friction.

⇒ In general it may be true. But polished surfaces may offer more friction. For example, coefficient of friction between glass/wood is 0.23 and glass and glass is 1.0 and between $Cu - Cu$ is 1.6.

13. Considering horizontal plane as x-axis and therefore normal force N perpendicular to x-axis as shown in Fig. 5.21 (a).

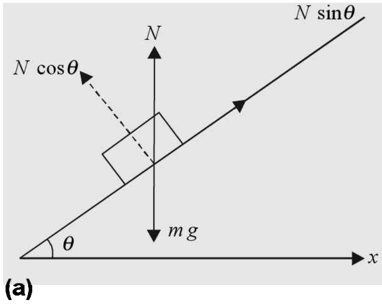


Fig. 5.21 (a)

⇒ Considering axis along the incline plane as x-axis and perpendicular to it as y-axis is more convenient way of solving problems as shown in Fig. 5.21 (b).

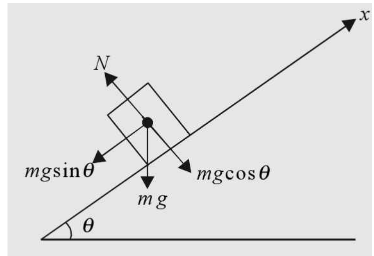


Fig. 5.21 (b)

SOLVED PROBLEMS

1. A smooth block is released at rest on a 45° incline and then slides a distance d. The time taken to slide is n times as much to slide on a rough incline plane than on a smooth incline. The coefficient of friction is

(a) $\mu_k = 1 - \frac{1}{n^2}$ (b) $\mu_k = \sqrt{1 - \frac{1}{n^2}}$

(c) $\mu_s = 1 - \frac{1}{n^2}$ (d) $\mu_s = \sqrt{1 - \frac{1}{n^2}}$.

[AIEEE 2005]

Solution (a)

case (i) Without friction $d = \frac{g}{2} \sin 45\, t^2$ ... (1)

case (ii) With friction $d = [\sin 45 - \mu_k \cos 45]\, t^2 n^2$ ... (2)

From (1) and (2) $\mu_k = 1 - \frac{1}{n^2}$.

2. The upper half of an inclined plane with inclination ϕ is perfectly smooth. While the lower half is rough. A body starting from rest at the top will again come to rest at the bottom if the coefficient of friction for the lower half is given by

(a) $2 \sin \phi$ (b) $2 \cos \phi$

(c) $2 \tan \phi$ (d) $\tan \phi$

[AIEEE 2005]

Solution (c) $mgs \sin \phi = \mu\, mg \cos \phi\, \frac{s}{2}$ or $\mu = 2 \tan \phi$.

3. A block is kept on a frictionless inclined surface with angle of inclination α. The incline is given an acceleration a to keep the block stationary. Then a is equal to

(a) $\frac{g}{\tan \alpha}$ (b) $g \operatorname{cosec} \alpha$

(c) g (d) $g \tan \alpha$

[AIEEE 2005]

Solution (d) $ma \cos \alpha = mg \sin \alpha$ or $a = g \tan \alpha$

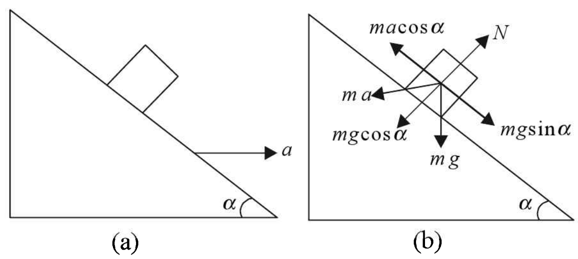


Fig. 5.22

4. A particle of mass 0.3 kg is subjected to a force $F = kx$ with $k = 15\ \text{N m}^{-1}$. What will be its initial acceleration if it is released from a point 20 cm away from the origin.

(a) $3\ \text{ms}^{-2}$ (b) $15\ \text{ms}^{-2}$

(c) $5\ \text{ms}^{-2}$ (d) $10\ \text{ms}^{-2}$

[AIEEE 2005]

Solution (d) $a = \frac{kx}{m} = \frac{15(0.2)}{0.3} = 10\ \text{ms}^{-2}$.

5. A. Frictional forces are conservative forces

R. Potential energy can be associated with frictional forces

(a) A and R both are true and R is correct explanation of A

(b) A and R are true but R is not correct explanation of A

(c) A is correct but R is wrong

(d) Both A and R are wrong

[AIIMS 2005]

Solution (d)

6. Which is true for rolling friction (μ_r), static friction (μ_s) and kinetic friction (μ_k);

(a) $\mu_s > \mu_k > \mu_r$ (b) $\mu_s < \mu_k < \mu_r$

(c) $\mu_s < \mu_k > \mu_r$ (d) $\mu_s > \mu_r > \mu_k$

[BHU 2005]

Solution (a)

7. Two weights w_1 and w_2 are suspended to the two ends of a string passing over a smooth pulley. If the pulley is pulled up with g then the tension in the string is

(a) $\frac{4w_1w_2}{w_1+w_2}$ (b) $\frac{2w_1w_2}{w_1+w_2}$

(c) $\frac{w_1w_2}{w_1+w_2}$ (d) $\frac{w_1+w_2}{2}$

[BHU PMT 2005]

Solution (a) See short cut rule 5 and put $a' = g$.

8. The adjacent figure is the part of a horizontally stretched net. Section AB is stretched by 10 N. The tensions in the section BC and BG are

(a) 10 N, 11 N (b) 10 N, 6 N

(c) 10 N, 10 N (d) cannot be determined

[CET Karnataka 2005]

Solution (c) Apply Lami's theorem

$$\frac{T_1}{\sin 120} = \frac{T_2}{\sin 120} = \frac{10}{\sin 120}$$

$$\therefore T_1 = T_2 = 10\text{ N}$$

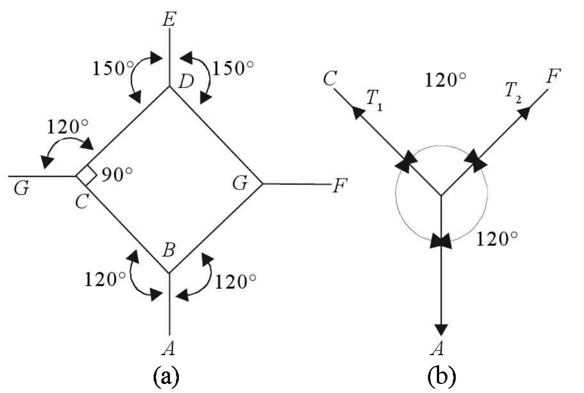


Fig. 5.23

9. In the figure shown, a cubical block is held stationary against a rough wall by applying force F then incorrect statement among the following is

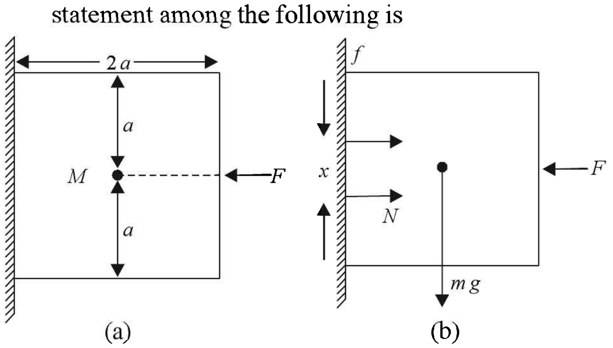


Fig. 5.24

(a) frictional force $f = Mg$
(b) $F = N$, N is normal reaction
(c) F does not apply any torque
(d) N does not apply any torque.

Solution (d) For equilibrium $f = Mg$ and $F = N$. for maintaining rotational equilibrium N will shift downward. Hence, torque due to friction about COM = Torque due to Normal reaction about COM.

10. A block of mass m is placed on a smooth wedge of inclination θ. The whole system is accelerated horizontally so that block does not slip on the wedge. The force exerted by the wedge on the block has a magnitude

(a) mg (b) $mg \cos\theta$

(c) $mg/\cos\theta$ (d) mg $\tan\theta$

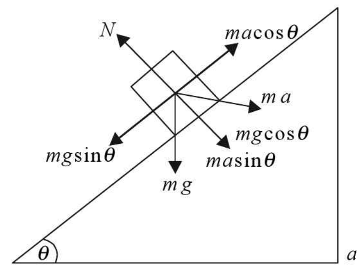


Fig. 5.25

Solution (c) $mg \sin\theta = ma \cos\theta$ or $a = g\tan\theta$

$N = ma \sin\theta + mg\cos\theta = m g \tan\theta \sin\theta + m g \cos\theta$

$$= \frac{mg}{\cos\theta}(\sin^2\theta + \cos^2\theta) = \frac{mg}{\cos\theta}$$

11. A person standing on the floor of an elevator drops a coin. The coin touches the floor of the elevator in time t, when the elevator is stationary and time t_2 when elevator is moving uniformly then

(a) $t_1 = t_2$ (b) $t_1 > t_2$ (c) $t_1 < t_1$ (d) $t_1 > t_2$ or $t_1 < t_2$ depends whether lift is moving up or down.

Solution (a) An object released from a moving body acquires its velocity also.

12. A boat of mass 300 kg moves according to the equation $x = 1.2\,t^2 - 0.2\,t^3$. When the force will become zero?

(a) 2 s (b) 1 s

(c) 6 s (d) 2.8 s

Solution (a) $\frac{dx}{dt} = 2.4\,t - 0.6\,t^2$ and

$$\frac{d^2x}{dt^2} = 2.4 - 1.2\,t = 0 \text{ or } t = 2\text{ s.}$$

13. A ball falls from a height h in a fluid which offers a resistance $f = -kv$. Find the terminal velocity if mass of the ball is m

(a) $\frac{mg}{k}$ (b) $\sqrt{\frac{mgh}{k}}$

(c) $\frac{mg - B}{k}$ (d) none of these

[B is Buoyant force]

Solution (a) $mg = kv$ or $v = \frac{mg}{k}$

14. Assuming coefficient of friction 0.25 between block and incline. Find the acceleration of each block in Fig. 5.26.

(a) $\frac{g}{5}$ (b) $\frac{g}{7}$

(c) $\frac{g}{6}$ (d) none of these

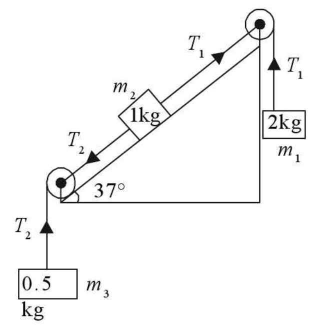


Fig. 5.26

Solution (a) $m_2 a = T_1 - T_2 - m_2 g \sin 37 - \mu m_2 g \cos 37$

or $a = \frac{T_1 - T_2}{m_2} - g \sin 37 - \frac{1}{4} g \cos 37$

or $T_1 - T_2 - 0.8g = a$ ...(2)

$T_2 - 0.5\, g = .5\, a$ ...(3)

$m_1 g - T_1 = m_1\, a$

or $2g - T_1 = 2a$ ...(1)

Adding (1), (2) and (3) $3.5a = 0.7g$

or $a = \frac{g}{5}$.

15. An 8 kg block of ice, released from rest at the top of a 1.5 m long smooth ramp, slides down and falls with a velocity 2.5 ms^{-1}. Find angle of the ramp with horizontal.

(a) 12° (b) 18°

(c) 15° (d) 30°

Solution (a) $v^2 = (2g \sin \theta)\, s$

or $\sin \theta = \frac{v^2}{2gs} = \frac{2.5 \times 2.5}{2 \times 10 \times 1.5} = \frac{0.5}{2.4} = 0.20$

or $\theta = 12°$.

16. A 60 kg boy stands on a scale in the elevator. The elevator starts moving and records 450 N. Find the acceleration of the elevator.

(a) 2.5 ms^{-2} upward

(b) 2.5 ms^{-2} downwards

(c) 2.5 ms^{-2} in either direction

(d) none of these

Solution (b) $450 = 60\,(g - a)$ or $60\, a = 150$ $a = 2.5$ ms^{-2} downwards.

17. A weight W is lifted by applying a force F as shown in Fig. Find F in terms of W. Assume constant velocity.

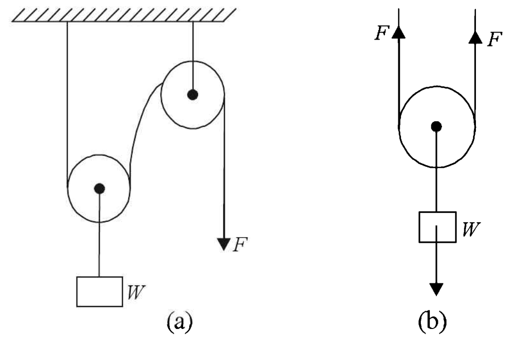


Fig. 5.27

(a) $F = W$ (b) $F = 2W$

(c) $F = \frac{W}{2}$ (d) none of these

Solution (c) $2F = W$ or $F = \frac{W}{2}$.

18. A window scrubber is used to brush up a vertical window as shown in figure. The brush weigh 12 N and coefficient of kinetic friction of 0.15. Calculate F

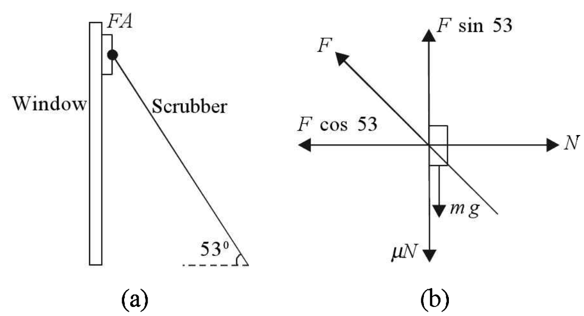


Fig. 5.28

(a) 15 N (b) 10.2 N

(c) 16.9 N (d) 18.1 N

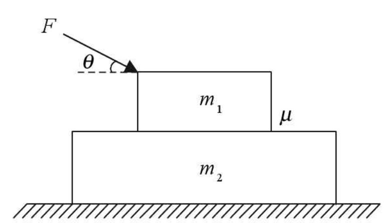
Solution (c) $\mu N + mg = F \sin 53$; $N = F \cos 53$

$$F = \frac{mg}{\sin 53 - \mu \cos 53}$$

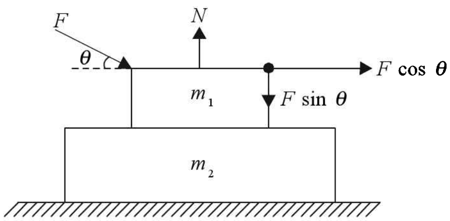
$$= \frac{12}{0.8 - 0.15 \times (0.6)}$$

$$= 16.9\, N.$$

19. Two blocks with masses m_1 and m_2 are stacked as shown in figure on a horizontal smooth surface. Coefficient of friction between the blocks is μ. A force F is applied at angle θ with the horizontal on block of mass m_1 as shown in figure. Find the maximum force F so that the blocks move together.



(a)



(b)

Fig. 5.29

Solution $N = (m_1 g + F \sin\theta)$; $F_f = \mu N$

$= \mu (m_1 g + F \sin\theta)$

$F \cos\theta - F_f = m_1 a$

$F \cos\theta - \mu (F \sin\theta + m_1 g) = m_1 a$ ... (1)

$F \cos\theta = (m_1 + m_2) a$ ... (2)

From (1) and (2) $F \cos\theta - \mu (F \sin\theta + m_1 g)$

$= \dfrac{m_1 (F \cos\theta)}{m_1 + m_2}$

or $m_2 F \cos\theta - \mu F \sin\theta (m_1 + m_2) - \mu m_1 (m_1 + m_2) g = 0$

or $F = \dfrac{\mu m_1 (m_1 + m_2) g}{m_2 \cos\theta - \mu (m_1 + m_2) \sin\theta}$.

20. A rock of mass m slides down with an initial velocity v_o. A retarding force $F = -k\, v^{\frac{1}{2}}$ acts on the body. The velocity at any instant is given by

(a) $v = v_o - \dfrac{kt}{m}$ (b) $v = v_o - \left(\dfrac{kt}{m}\right)^2$

(c) $\sqrt{v} = \sqrt{v_o} - \dfrac{kt}{2m}$ (d) none of these

Solution (c) $\dfrac{mdv}{dt} = -k\, v^{\frac{1}{2}}$

or $\displaystyle\int_{v_o}^{v} \frac{dv}{v^{1/2}} = -\int_{o}^{t} \frac{kdt}{m}$

or $v^{\frac{1}{2}} = v_o^{1/2} - \dfrac{kt}{m}$

or $\dfrac{v_o^{\frac{1}{2}} - v^{\frac{1}{2}}}{\frac{1}{2}} = \dfrac{kt}{m}$.

21. At $t = 0$, a force $F = kt$ is applied on a block making an angle α with the horizontal. Suppose surfaces to be smooth. Find the velocity of the body at the time of breaking off the plane.

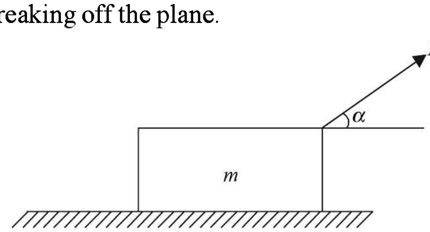


Fig. 5.30

(a) $\dfrac{mg \cos\alpha}{2a \sin\alpha}$ (b) $\dfrac{m^2 g^2 \cos\alpha}{2a \sin^2\alpha}$

(c) $\dfrac{mg^2 \cos\alpha}{2a \sin^2\alpha}$ (d) $\dfrac{mg^2 \cos^2\alpha}{2a \sin\alpha}$.

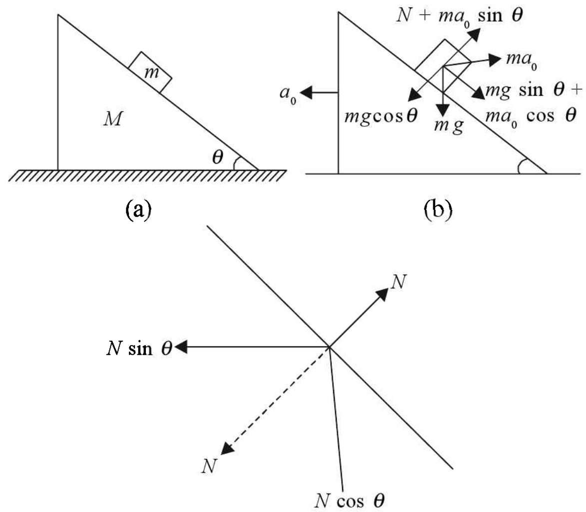
Solution (c) $at \cos\alpha = \dfrac{mdv}{dt}$ or $v = \dfrac{at^2 \cos\alpha}{2m}$

At the break off point $mg = at \sin\alpha$

or $t = \dfrac{mg}{a \sin\alpha}$.

$\therefore$ $v = \dfrac{a \cos\alpha}{2m} \left(\dfrac{mg}{a \sin\alpha}\right)^2 = \dfrac{mg^2 \cos\alpha}{2a \sin^2\alpha}$.

22. A block of mass m is placed on a wedge of mass m and inclination θ as shown in Fig. 5.31(a). All surfaces are smooth. Find the acceleration of wedge.



(c)

Fig. 5.31

(a) $\dfrac{mg \cos^2\theta}{M + m \sin^2\theta}$ (b) $\dfrac{mg \sin^2\theta}{M + m \cos^2\theta}$

(c) $\dfrac{mg \sin\theta \cos\theta}{M + m \sin^2\theta}$ (d) $\dfrac{mg \sin\theta \cos\theta}{M + m \cos^2\theta}$

Solution (c)

$ma = mg \sin\theta + ma_0 \sin\theta$

or $a = g \sin\theta + a_0 \cos\theta$ ... (1)

$N = mg \cos\theta - ma_0 \sin\theta$ ... (2)

$N \sin\theta = Ma_0$ ... (3)

From (2) and (3)

$\dfrac{Ma_0}{\sin\theta} = mg \cos\theta - ma_0 \sin\theta$

or $a_0 = \dfrac{mg \cos\theta \sin\theta}{M + m \sin^2\theta}$.

23. The tension T in the thread shown in figure is

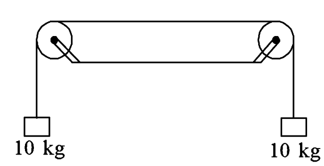


Fig. 5.32

(a) $10\,N$ (b) zero
(c) $98\,N$ (d) $196\,N$

Solution (c) $T = mg = 10\,(9.8) = 98\,N$

24. A light spring of spring constant k is cut into two equal halves. Each half is connected in parallel then net spring constant of the combination is

(a) $\frac{k}{4}$ (b) $\frac{k}{2}$
(c) $2k$ (d) $4k$ (e) k

Solution (d) $k_{eq} = 2k + 2k = 4k \left(\because k \propto \frac{1}{l}\right)$.

25. Momentum is closely related to

(a) force (b) impulse
(c) velocity (d) kinetic energy

[DCE 1997]

Solution (c)

26. A force $F = k\,t\,(\tau - t)$ acts on a particle of mass m, which is at rest at $t = 0$ where k is a constant. Find the momentum of the force when the action of the force is discontinued.

(a) $\frac{k\tau^3}{2}$ (b) $\frac{k\tau^3}{3}$
(c) $\frac{k\tau^3}{6}$ (d) $\frac{k\tau^3}{4}$.

Solution (c) $\Delta p = \int F dt = \int_0^\tau kt(\tau - t)dt$

$$= \frac{k\tau^3}{2} - \frac{k\tau^3}{3}$$

$$= \frac{k\tau^3}{6}.$$

TYPICAL PROBLEMS

27. A body of mass m rests on a horizontal plane with a friction coefficient μ. At $t = 0$, a horizontal force is applied ($F = a\,t$) where a is a constant vector. Find the distance traversed in first t sec.

(a) $\frac{a}{6m}\left(t - \frac{\mu mg}{a}\right)^3$ (b) $\frac{a}{6m}t^3$

(c) $\frac{a}{6m}t^3 - \frac{\mu g t^2}{2}$ (d) $\frac{a}{6m}t^3 - \frac{\mu g t^2}{3}$.

Solution (a) After the application of force body begins to move after a time t_0 such that

$$at_0 = \mu\, mg \text{ or } t_0 = \frac{\mu mg}{a} \quad \frac{dv}{dt} = \frac{a}{m}(t - t_0)$$

Put $t - t_0 = T$ or $v = \frac{a}{m}\int T dt = \frac{a}{m}\frac{T^2}{2}$.

$$\int dx = x = \frac{a}{m}\int \frac{T^2}{2}dt = \frac{a}{m}\frac{T^3}{6} = \frac{a}{6m}(t - t_0)^3.$$

28. A horizontal disc rotates with a constant angular velocity ω about a vertical axis passing through its centre. A small body m moves along a diameter with a velocity v. Find the force the disc exerts on the body when it is at a distance r located from the rotation axis.

(a) $mr\omega^2 + 2mv\,\omega$
(b) $mg + \sqrt{m^2r^2\omega^4 + (2mv\omega)^2}$
(c) $\sqrt{m^2g^2 + (2mv\omega)^2} + mr\,\omega^2$
(d) $m\sqrt{g^2 + r^2\omega^4 + (2v\omega)^2}$

Solution (d) The force mg is vertical, $2mv\omega$ perpendicular to vertical plane and $mr\,\omega^2$ outward along the diameter. The resultant force is

$$F = m\sqrt{g^2 + r^2\omega^4 + (2v\omega)^2}.$$

29. A bead A can slide freely along a smooth rod bent in the form of a half circle of Radius R. The system is set in rotation with a constant angular velocity ω about a vertical axis OO′. Find the angle θ corresponding to steady position of the bead.

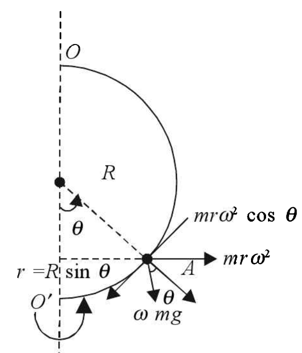


Fig. 5.33

(a) $\cos^{-1}\left(\frac{R\omega^2}{g}\right)$ (b) $\cos^{-1}\left(\frac{g}{R\omega^2}\right)$

(c) $\sin^{-1}\left(\frac{g}{R\omega^2}\right)$ (d) $\sin^{-1}\left(\frac{R\omega^2}{g}\right)$.

Solution (b) forces acting along the tangent to the radius $r = R\sin\theta$.

$mg\sin\theta - mr\,\omega^2\cos\theta = 0$

or $mg\sin\theta\left[1 - \frac{r\omega^2}{g}\cos\theta\right] = 0$ or $\theta = \cos^{-1}\left(\frac{g}{R\omega^2}\right)$.

30. A block of mass m is placed on a wedge of Mass M. Coefficient of friction between them is $\mu > \cot\theta$. The wedge is given an acceleration to its left. Find the maximum acceleration at which block appears stationary relative to wedge.

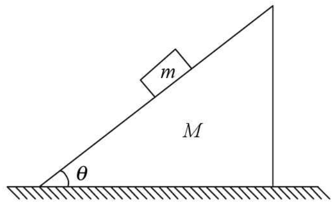


Fig. 5.34

(a) $\dfrac{g(\sin\theta - \mu\cos\theta)}{\cos\theta + \mu\sin\theta}$ (b) $\dfrac{g(\sin\theta + \mu\cos\theta)}{\cos\theta - \mu\sin\theta}$

(c) $\dfrac{g(\cos\theta + \mu\sin\theta)}{\sin\theta - \mu\cos\theta}$ (d) none

Solution (b)

$N = m\,(g\cos\theta + a\sin\theta)$

$ma\cos\theta = mg\sin\theta + \mu N$

$ma\cos\theta = mg\sin\theta + \mu\, mg\cos\theta + \mu\, ma\sin\theta$

$ma\,(\cos\theta\,\mu\sin\theta) = mg\sin\theta + mg\cos\theta)$

or $a = \dfrac{g(\sin\theta + \mu\cos\theta)}{\cos\theta - \mu\sin\theta}$

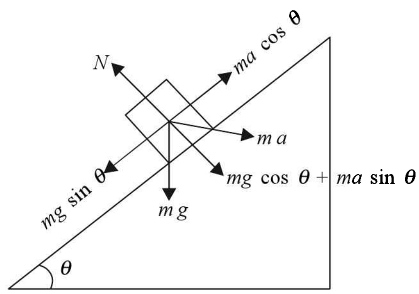


Fig. 5.35

31. In the arrangement shown in Fig. 5.36 pulleys are smooth and massless. Threads are massless and inextensible. Find acceleration of mass m_1.

(a) $\dfrac{[2m_1m_2 + m_0(m_1 - m_2)]g}{2m_1m_2 + m_0(m_1 + m_2)}$

(b) $\dfrac{[4m_1m_2 + m_0(m_1 - m_2)]g}{4m_1m_2 + m_0(m_1 + m_2)}$

(c) $\dfrac{[4m_1m_2 - m_0(m_1 - m_2)]g}{4m_1m_2 + m_0(m_1 + m_2)}$ (d) none

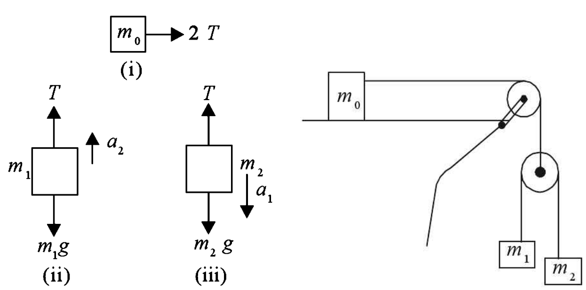


Fig. 5.36

Solution (b) $m_1 g - T = m_1 a_1$ [from Fig 5.36 (ii)]

$m_2 g - T = m_2 a_2$ [from Fig 5.36 (iii)]

$m_0 a = 2T$ [from Fig 5.36 (i)]

$a_1 + a_2 = 2a$

Solving we get $a_1 = \dfrac{[4m_1m_2 + m_0(m_1 - m_2)]g}{4m_1m_2 + m_0(m_1 + m_2)}$.

32. A train of 2000 tonne moves in the lattitude 60° North. Find the magnitude of the lateral force that the train exerts on the rails if it moves with a velocity 54 km h^{-1}.

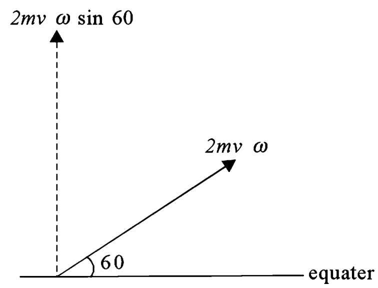


Fig. 5.37

(a) 2.4×10^3 N (b) 3.6×10^4 N
(c) 3.6×10^3 N (d) 2.4×10^4 N

Solution (c) $F = 2\,m\,v\,\omega \sin 60$

$= 2 \times 2 \times 10^6 \times \dfrac{15 \times 2\pi}{24 \times 60 \times 60} \times \dfrac{\sqrt{3}}{2}$.

$= 3.6 \times 10^3$ N.

33. A particle of mass m moves along the internal smooth surface of a vertical cylinder of radius r. Find the force with which the particle acts on the cylinder wall if at $t = 0$, its velocity is v_o and it makes an angle α with the horizontal.

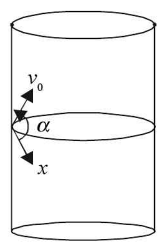


Fig. 5.38

(a) $\dfrac{mv_o^2}{R}$ (b) $\dfrac{mv_o^2}{R}\cos\alpha$

(c) $\dfrac{mv_o^2\sin\alpha}{R}$ (d) $\dfrac{mv_o^2\cos^2\alpha}{R}$

Solution (d) $F = \dfrac{mv_x^2}{R} = \dfrac{m(v_o\cos\alpha)^2}{R}$.

34. Find the magnitude and direction of force acting on a particle during its motion in a plane xy according to the law $x = a\sin\omega t$ and $y = b\cos\omega t$ where a, b and ω are constants.

Solution $\vec{r} = x\,\hat{i} + y\,\hat{j} = a\cos\omega t\,\hat{i} + b\sin\omega t\,\hat{j}$

$$v = \frac{dr}{dt} = -a\,\omega \sin \omega t\ \hat{i} + b\,\omega \cos \omega t\ \hat{j}$$

$$a = \frac{d^2r}{dt^2} = -a\,\omega^2 \cos \omega t\ \hat{i} - b\,\omega^2 \sin \omega t\ \hat{j}$$

$$= -\omega^2\,\vec{r}$$

$$\vec{F} = m\,\vec{a} = -m\,\omega^2\,\vec{r}$$

35. Two blocks one of mass $A = 1$ kg and $B = 2$kg. A force of $5\,N$ is applied on A [see Fig. 5.39]. Coefficient of friction between A and B is 0.2 and that of between B and horizontal surface is zero. Find (a) acceleration of A and B (b) The time taken for the front surface of A to coincide with that of B.

[CBSE PMT Mains 2005]

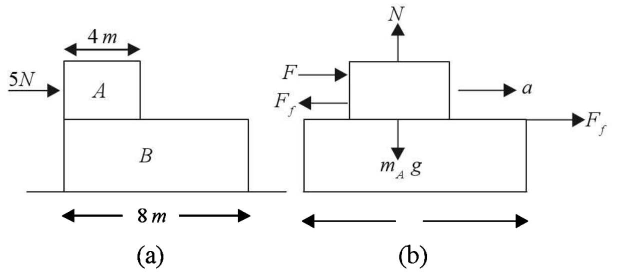


Fig. 5.39

Solution (a) $ma = F - \mu N$

or $1a_A = 5 - (.2)\times 10$

$= a_A = 3\ ms^{-2}$.

$$a_B = \frac{\mu N}{M_B} = 1\ \text{ms}^{-2}.$$

(b) $a_{rel} = a_A - a_B = 2\ \text{ms}^{-2}$.

$$s = \frac{1}{2}\,a_{rd}\,t_2 \quad t = \sqrt{\frac{4\times 2}{2}} = 2\ s.$$

36. As shown in Fig. 5.40 mass of bodies is equal to m each. If coefficient of friction between horizontal surface and mass is 0.2. Find the acceleration of the system

[CBSE PMT Mains 2005]

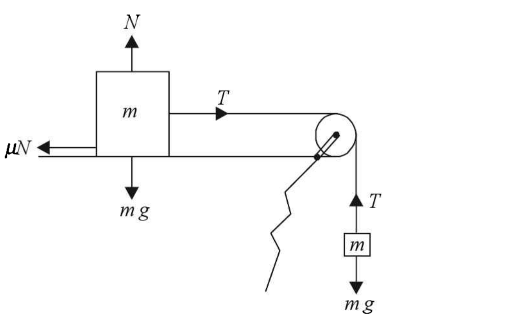


Fig. 5.40

Solution $mg - T = ma$ ...(1)

$T - \mu mg = ma$ ...(2)

Adding (1) and (2) and solving

$$\frac{g}{2}(1-\mu) = a$$

or $a = 0.4g = 4\ \text{ms}^{-2}$.

37. A particle is observed from the frames S_1 and S_2. The frame S_2 moves with respect to S_1 with an acceleration a. Let F_1 and F_2 be two pseudo forces acting on the particle when seen from S_1 and S_2 respectively. Which of the following are not possible.

(a) $F_1 = 0, F_2 \neq 0$ (b) $F_1 \neq 0, F_2 \neq 0$
(c) $F_1 \neq 0, F_2 = 0$ (d) $F_1 = 0, F_2 = 0$

Solution (d)

38. Figure 5.41 shows displacement of a particle going along the x-axis as a function of time. The force acting on the particle is zero in the region.

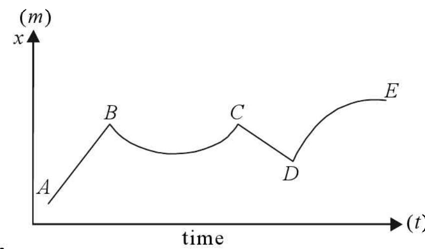


Fig. 5.41

(a) AB (b) BC
(c) CD (d) DE

Solution (a) and (c). In regions AB and CD

$$v = \frac{dx}{dt} = \text{constant}.$$

$$\therefore F = \frac{mdv}{dt} = 0.$$

39. A person says that he mesured acceleration of a particle to be non-zero while no force is acting on the particle. Then,

(a) he is a liar
(b) his clock might have run slow
(c) his meter scale might have been longer than the standard
(d) he might have used non-inertial frame of reference

Solution (d) Pseudo force will act in noninertial frame.

40. A man holds a thin stick at its two ends and bend it in an arc like a bow without a string. Which of the following figures correctly show the directions the force exerted by him on the stick. Neglect gravity.

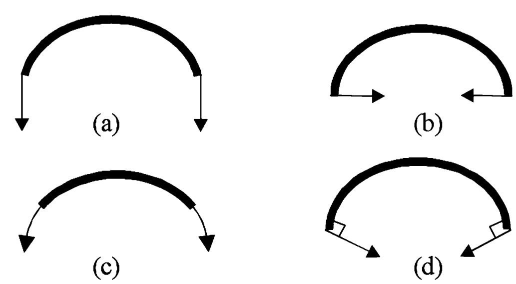


Fig. 5.42

Solution (b) Resultant of forces be zero.

QUESTIONS FOR PRACTICE

1. A toy train consists of three identical compartments A, B and C. It is being pulled by a constant force F along C. The ratio of the tension in the string connecting AB and BC is
   (a) 2 : 1 (b) 1 : 3
   (c) 1: 1 (d) 1 : 2

2. A block of mass M is pulled along a smooth horizontal surface with a rope of mass m. The acceleration of the block will be
   (a) F/(M + m) (b) F/(M – m)
   (c) F/M (d) F/m

3. A body of weight 50 N is dragged on a horizontal surface with a force of 28.2 N. The frictional force acting on the body and the normal reactional force will be

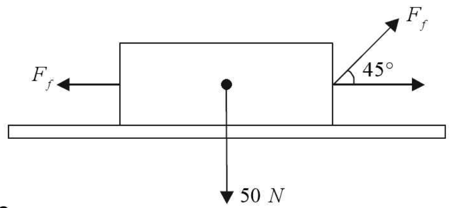


Fig. 5.43

   (a) 2N, 3N (b) 5N, 7N
   (c) 10 N, 15 N (d) 20 N, 30 N

4. Two blocks of mass 4 kg and 2 kg are placed in contact with each other on a frictionless horizontal surface. If we apply a push of 5N on the heavier mass, the force on the lighter mass will be
   (a) 2N (b) 4N
   (c) 5N (d) none of these

5. A jar containing water is placed in a train. The train accelerates from left to right. Which of the following shows the water level in a jar correctly?

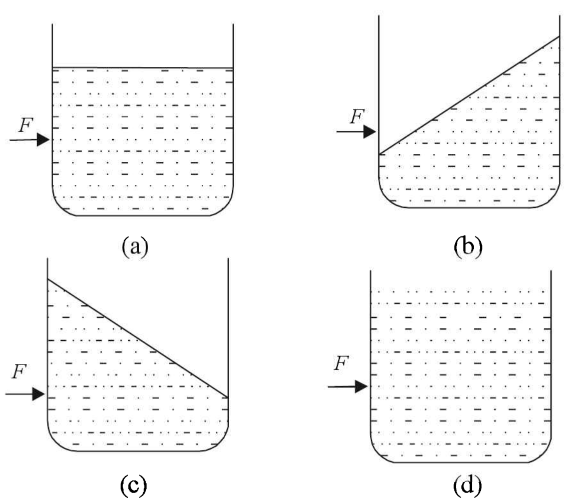


Fig. 5.44

6. A block of mass m is placed on a smooth inclined plane of inclination θ with the horizontal. The force exerted by the plane on the block has magnitude
   (a) $mg \tan \theta$ (b) $mg \cos \theta$
   (c) $mg/\cos \theta$ (d) mg

7. The work done in dragging a block of mass 5 kg on an inclined plane of height 2 m is 150 Joule. The work done against the frictional force will be
   (a) 200 Joule (b) 150 Joule
   (c) 100 Joule (d) 50 Joule

8. Two masses m and M are lying on a surface moving with acceleration a. Only the given supporting and moving surface has coefficient of friction as μ. The frictional forces for $\mu > a/g$ and $\mu < a/g$ are

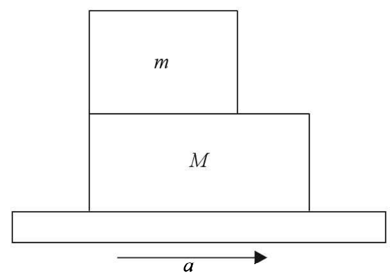


Fig. 5.45

   (a) ma, ma (b) ma, μmg
   (c) μmg, μmg (d) μmg, ma

9. A small sphere of mass m is attached to a spring of spring factor k and normal length l. If the sphere rotates with radius r at frequency v then tension in the spring is

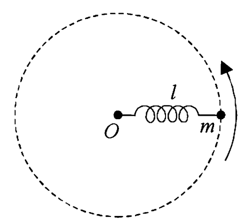


Fig. 5.46

   (a) $k^2 l$ (b) $k^2 (r - l)$
   (c) $mr(2\pi v)^2$ (d) kl

10. Two masses each equal to m are lying on x-axis at $(-a, 0)$ and $(+a, 0)$ respectively. They are connected by a light string. A force F is applied at the origin along y axis resulting into motion of masses towards each other. The acceleration of each mass when position of masses at any instant becomes $(-x, 0)$ and $(+x, 0)$ is given by
   (a) $\frac{F}{m}\frac{x}{\sqrt{a^2 - x^2}}$ (b) $\frac{F}{m}\frac{\sqrt{a^2 - x^2}}{x}$
   (c) $\frac{Fx}{2m\sqrt{a^2 - x^2}}$ (d) $\frac{F}{2m}\sqrt{\frac{a^2 - x^2}{x}}$

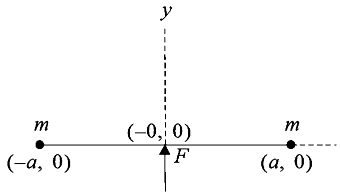


Fig. 5.47

11. A particle of mass m is suspended from a fixed point O by a string of length l. At $t = 0$, it is displaced from equilibrium position and released. The graph which shows the variation of tension T in string with time t is

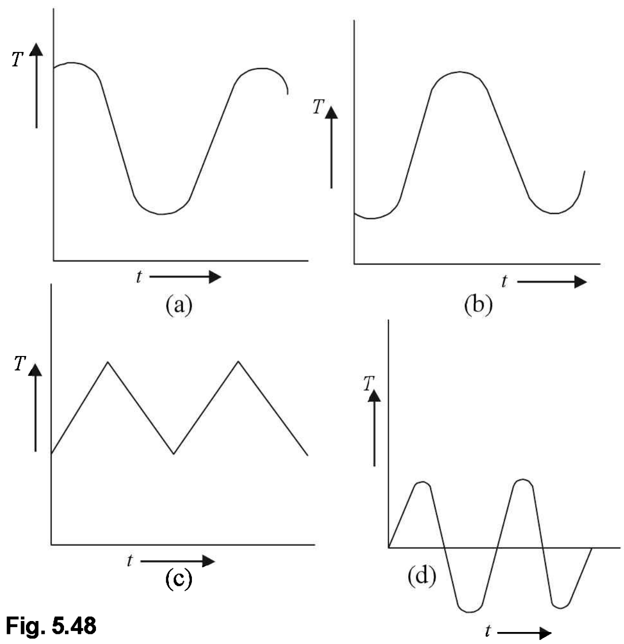


Fig. 5.48

12. Two blocks of masses m_1 and m_2 are connected to each other with the help of a spring. If pushing force is given to mass m_1 providing acceleration a to it, then acceleration of m_2 is

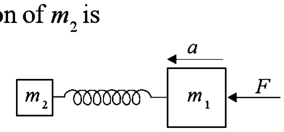


Fig. 5.49

(a) $\dfrac{m_1 a - F}{m_2}$ (b) $\dfrac{F - m_1 a}{m_2}$

(c) a (d) $\dfrac{F - m_1 a}{m_1}$

13. A stone weighing 1/2 kg is tied to a string 1/2 m long having withstand capacity of 20 kg. The stone is in horizontal circular motion over a frictionless table with a speed of 1.5 ms^{-1}. If tension in the string is equal to the breaking force of the spring, the speed attained is

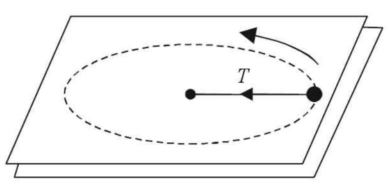


Fig. 5.50

(a) 14 ms^{-1} (b) 11 ms^{-1}
(c) 24 ms^{-1} (d) 17 ms^{-1}

14. A body takes n times, the time to slide down a rough inclined plane as it takes to slide down the same inclined plane when it is perfectly frictionless. The coefficient of kinetic friction between the body and the plane for an angle of inclination of 45° is given by μ

(a) $1 - \dfrac{1}{n}$ (b) $\dfrac{1}{n}$

(c) $\left(1-\dfrac{1}{n^2}\right)$ (d) $\left(\dfrac{1}{n^2}-1\right)$

15. Two blocks of masses 2 kg and 5 kg are at rest on ground. The masses are connected by a string passing over a frictionless pulley which is under the influence of a constant upward force $F = 50$ N. The accelerations of 5 kg and 2 kg masses are

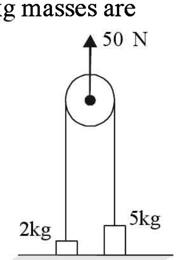


Fig. 5.51

(a) 0, 2.5 ms^{-2} (b) 0, 0
(c) 2.5 ms^{-2}, 2.5 ms^{-2} (d) 1 ms^{-2}, 2.5 ms^{-2}

16. A body starts to slide from P, down an inclined frictionless plane PQ having inclination α with horizontal and then ascends another smooth inclined plane QR with angle of inclination 2α. Neglecting impact at O

(a) $t_{PQ} = t_{QR}$ (b) $t_{PQ} < t_{QR}$
(c) $h' = 2h$ (d) $h' = h$

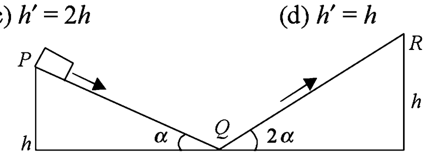


Fig. 5.52

17. A rod of length L is rotated in horizontal plane with constant angular velocity ω. A mass m is suspended by a light string of length L from the other end of the rod. If the angle made by vertical with the string is θ then angular speed, $\omega =$

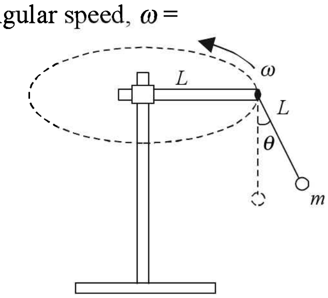


Fig. 5.53

(a) $\left[\dfrac{g\sin\theta}{L(1+\tan\theta)}\right]^{\frac{1}{2}}$ (b) $\left[\dfrac{L(1+\tan\theta)}{g\tan\theta}\right]^{\frac{1}{2}}$

(c) $\left[\dfrac{g\tan\theta}{L+\sin\theta}\right]^{\frac{1}{2}}$ (d) $\left[\dfrac{g\tan\theta}{L(1+\sin\theta)}\right]^{\frac{1}{2}}$

18. A stone of mass 1000 g tied to a light string of length 10/3 m is whirling in a vertical circle. If the ratio of the maximum tension to minimum tension is 4 and $g = 10$ ms^{-2}, then speed of stone at the highest point of circle is

(a) 20 ms^{-1} (b) $10/\sqrt{3}$ ms^{-1}
(c) $5\sqrt{3}$ ms^{-1} (d) 10 ms^{-1}

19. A man tries to remain in steady state by pushing his feet and hands against two parallel walls. Then for equilibrium

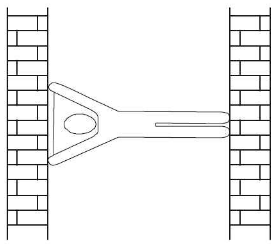


Fig. 5.54

(a) force of friction should be equal on the two walls.
(b) force exerted by him on both walls should be equal and the walls should not be frictionless.
(c) he should press his feet with greater force.
(d) coefficient of friction should be equal for both walls.

20. A block of mass 1 kg is connected by a light string passing over two smooth pulleys placed on a smooth horizontal surface as shown. Another block of 1 kg is connected to the other end of the string then acceleration of the system and tension in the string are

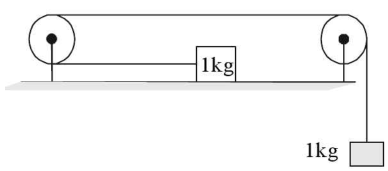


Fig. 5.55

(a) 5 ms^{-2}, 5 N (b) 1 ms^{-2}, 1 N
(c) 1 ms^{-2}, 5 N (d) 5 ms^{-2}, 10 N

21. A mass of 2 kg is placed on a trolley of 20 kg sliding on a smooth surface. The coefficient of friction between the mass and surface of trolley is 0.25. A horizontal force of 2N is applied to the mass. The acceleration of the system and the frictional force between the mass and surface of trolley are

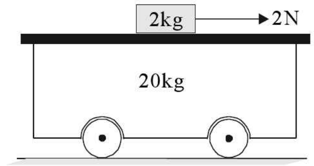


Fig. 5.56

(a) 1.8 ms^{-2}, 0.09 N (b) 0.9 ms^{-2}, 18 N
(c) 0.09 ms^{-2}, 1.8 N (d) 1 ms^{-2}, 2 N

22. Three blocks of masses 3 kg, 6 kg and 1 kg are connected by a string passing over two smooth pulleys attached at the two ends of a frictionless horizontal surface. The acceleration of 3 kg mass is

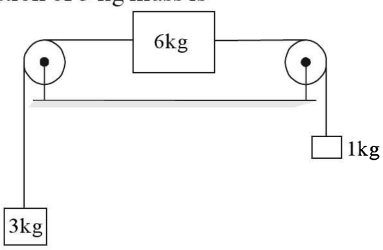


Fig. 5.57

(a) 1 ms^{-2} (b) 2 ms^{-2}
(c) 3 ms^{-2} (d) 4 ms^{-2}

23. A pearl of mass m is in a position to slide over a smooth wire. At the initial instant the pearl is in the middle of the wire. The wire moves linearly in a horizontal plane with an acceleration a in a direction having angle θ with the wire. The acceleration of the pearl with reference to wire is

(a) $g\sin\theta - a\cos\theta$ (b) $g\sin\theta - g\cos\theta$
(c) $g\sin\theta + a\cos\theta$ (d) $g\cos\theta + a\sin\theta$

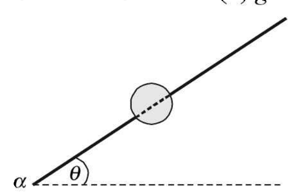


Fig. 5.58

24. A mass is resting on a rough plank. At initial instant a horizontal impulse is applied to the mass. If the velocity of mass at instant t is υ and displacement upto this instant is S then correct graph is

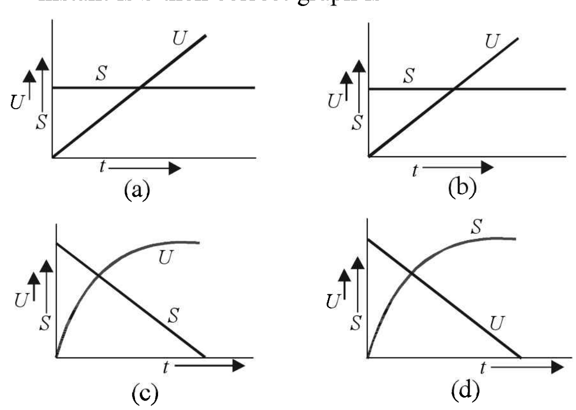


Fig. 5.59

25. A straight tube of length L contains incompressible liquid of mass M and the closed tube is whirled in horizontal plane about one of the ends. If ω is the uniform angular velocity, the force exerted by the liquid on the other end is

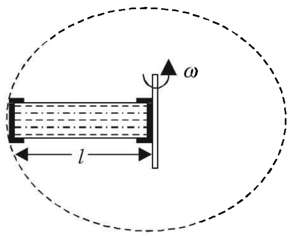


Fig. 5.60

(a) $\frac{ML\omega^2}{4}$ (b) $2ML\omega^2$

(c) $\frac{ML\omega^2}{4}$ (d) $ML\omega^2$

26. A light rope passes over a pulley. One section of the rope is held by a child and the other section by a man, then

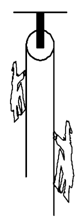


Fig. 5.61

(a) the man and the child have same vector acceleration.
(b) the man and the child have same magnitude of acceleration but in opposite direction.
(c) the man and the child have different magnitude of acceleration.
(d) the man and the child have accelerations which keep on interchanging with each other.

27. A trolley is under the action of a constant force F. The sand contained by it is poured out through a hole in the floor at the rate of m per second. If initial mass of sand and trolley was M and initial speed was u, then acceleration of trolley is given by

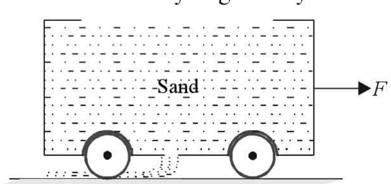


Fig. 5.62

(a) $\frac{F}{M - mt}$ (b) $\frac{F}{M + mt}$

(c) $\frac{F}{M - m}$ (d) $\frac{F}{M + m}$

28. A smooth track of incline of length l is joined smoothly with circular track of radius R. A mass of m kg is projected up from the bottom of the inclined plane. The minimum speed of the mass to reach the top of the track is given by, $v =$

(a) $[2g\,(l\cos\theta + R)\,(1 + \cos\theta)]^{1/2}$
(b) $(2gl\sin\theta + R)^{1/2}$
(c) $[2g\,\{1\sin\theta + R\,(1 - \cos\theta)\}]^{1/2}$
(d) $(2gl\cos\theta + R)^{1/2}$

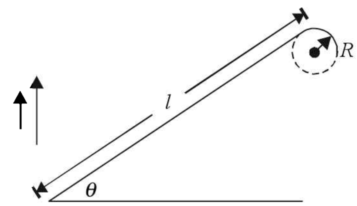


Fig. 5.63

29. A massless string of length l passes over a frictionless pulley with horizontal axis. Two monkeys hang from the ends of the string at the same distance $l/2$ from the pulley, the monkeys start climbing upwards simultaneously. First monkey climbs with a speed v relative to the string and the second with speed of $2v$. Both monkeys have got same masses. The time taken by the first and second monkeys in reaching the pulleys are respectively.

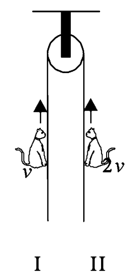


Fig. 5.64

(a) $\left(\frac{1}{v}\right), \left(\frac{1}{2v}\right)$ (b) $\sqrt{\frac{2l}{v}}, \sqrt{\frac{l}{v}}$

(c) $\left(\frac{l}{2v}\right)^{\frac{1}{2}}, \left(\frac{1}{v}\right)^{1/2}$ (d) $\left(\frac{1}{3v}\right), \left(\frac{1}{3v}\right)$

30. Neglecting the masses of the string and pulley and ignoring the friction in the system, we find that

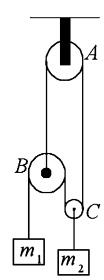


Fig. 5.65

(a) weights fall freely. Pulley B rotates clockwise and pulley A, C rotate anticlockwise.
(b) the two weights have different accelerations. Pulley C rotates clockwise and B, C rotate anticlockwise.

(c) acceleration of masses will be zero and the system will be at rest.
(d) acceleration of masses is equal to g. Pulley A and C rotate clockwise whereas B rotates anticlockwise.

31. A simple pendulum is vibrating with an angular amplitude of $\frac{\pi}{2}$. The value of α for which the resultant acceleration has a direction along the horizontal is

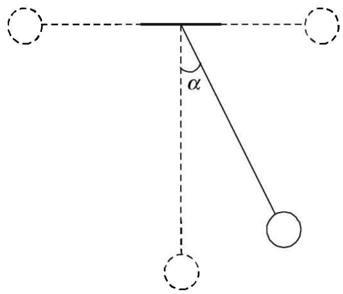


Fig. 5.66

(a) $\frac{\pi}{2}$ (b) 180°

(c) $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$ (d) $\cos^{-1}\left(\frac{1}{\sqrt{2}}\right)$

32. A body of mass m starting from rest slides down a frictionless inclined surface of gradient α fixed on the floor of a lift accelerating upward with acceleration a. Taking width of inclined plane as W, the time taken by body to slide from top to bottom of the plane is

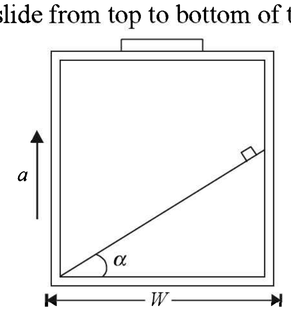


Fig. 5.67

(a) $\left(\frac{2W}{(g+a)\sin\alpha}\right)^{\frac{1}{2}}$ (b) $\left(\frac{4W}{(g-a)\sin\alpha}\right)^{\frac{1}{2}}$

(c) $\left(\frac{4W}{(g+a)\sin 2\alpha}\right)^{\frac{1}{2}}$ (d) $\left(\frac{W}{(g+a)\sin 2\alpha}\right)^{\frac{1}{2}}$

33. A very small mass m is fixed to one end of a massless spring of constant k and normal length l. The spring and the mass are rotated about the other end of the spring with angular speed ω. Neglect the effect of gravity. Extension in the spring is

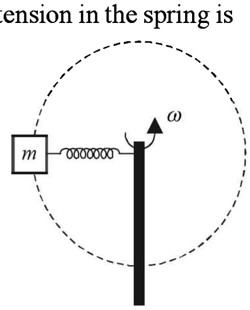


Fig. 5.68

(a) zero (b) $\frac{ml\omega^2}{k+m\omega^2}$

(c) $ml\omega^2$ (d) $\frac{m\omega^2 l}{k-m\omega^2}$

34. A rope is stretched between two boats at rest. A sailor in the first boat pulls the rope with a constant force of 100 N. First boat with the sailor has a mass of 250 kg whereas the mass of second boat is double of that mass. If the initial distance between the boats was 100 m, the time taken for two boats to meet each other is

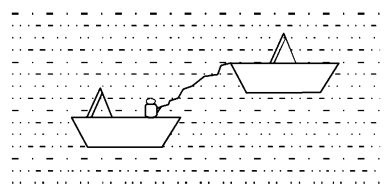


Fig. 5.69

(a) 13.8 s (b) 18.3 s

(c) 3.18 s (d) 31.8 s

35. A block of mass M is situated on a smooth horizontal table. A thread tied to the block passes through a hole in the table and carries a mass m at its other end. If the length of thread above the table is l and M is revolving in horizontal circle with angular speed ω on the table, then value of m so that it remains suspended at a constant height h is

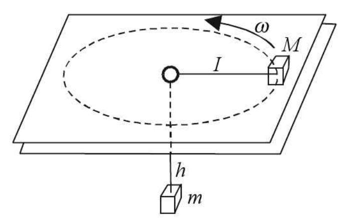


Fig. 5.70

(a) $Mgh\omega^2$ (b) $Mgl\omega^2$

(c) $\frac{Ml\omega^2}{g}$ (d) $Ml\omega^2$

36. A parachute of mass m starts coming down with a constant acceleration a. Determine the ballast mass to be released for the parachute to have an upward acceleration of same magnitude. Neglect air drag.

(a) $\frac{2ma}{a+g}$ (b) $\frac{ma}{a-g}$

(c) $\frac{ma}{a+g}$ (d) $\frac{2ma}{a-g}$

37. Block A is placed on block B (mass of B > mass of A). There is friction between the blocks but the ground is frictionless. A horizontal force F, increasing linearly with time, begins to act on A. Accelerations a_V and a_B of blocks A and B respectively is correctly plotted as

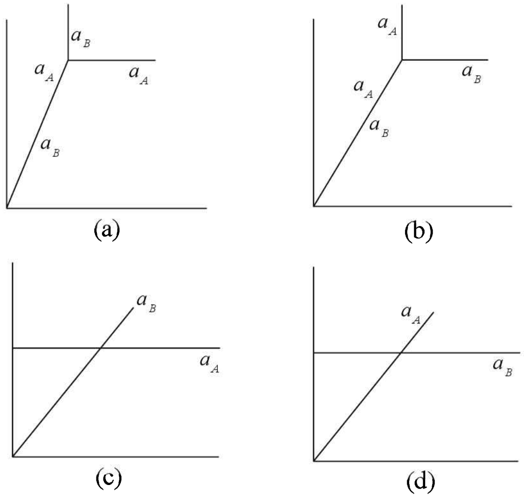


Fig. 5.71

38. A circular table has a radius of 1m and mass 20 kg. It has 4 legs of 1 m each fixed symmetrically on its circumference. The maximum weight which can be placed anywhere on this table without toppling it is

(a) 84.3 kg (b) 34.8 kg
(c) 48.3 kg (d) 43.8 kg

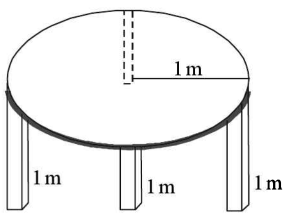


Fig. 5.72

39. A board is balanced on a rough horizontal semicircular log. Equilibrium is obtained with the help of addition of a weight to one of the ends of the board when the board makes an angle θ with the horizontal. Coefficient of friction between the log and the board is

(a) tan θ (b) cos θ
(c) cot θ (d) sin θ

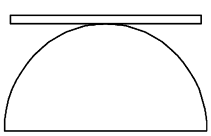


Fig. 5.73

40. Two similar planes of mass m each having failed engines are being pulled by a stronger plane in air.

At $t = 0$, they are travelling at uniform speed producing tension T_A in rope A. The stronger plane then accelerates with acceleration a. Tension in rope B just after the beginning of acceleration is

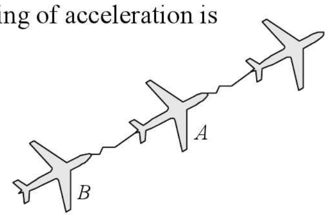


Fig. 5.74

(a) T_A (b) $T_A - ma$
(c) $2T_A + ma$ (d) $\frac{T_A}{2} + ma$

41. Velocity of a bullet changes from u to v after passing through a board of thickness d. Force of resistance is directly proportional to the velocity. Time of motion of bullet in the board is given by

(a) $\dfrac{d(u-v)}{uv \log_e \frac{u}{v}}$ (b) $\dfrac{du}{v \log_e \frac{u}{v}}$

(c) $\dfrac{dv}{u \log_e \frac{u}{v}}$ (d) $\dfrac{d(v-u)}{uv \log_e \frac{v}{u}}$

42. A rocket of mass m is fired vertically upward and after the fuel burning it weighs m'. Ejection of fuel gas is at a constant rate of m_0 per second with a constant velocity of u_{rel} relative to the rocket. Final speed of rocket after the complete burn out of fuel is given by $v =$

(a) $u_{rel} \log_e \frac{m}{m'}$ (b) $u_{rel} \log_e \frac{m_0}{m}$

(c) $-u_{rel} \log_e \frac{m_0}{m'}$ (d) $-u_{rel} \frac{dm}{m}$

43. A chain of length l is lying in a smooth horizontal tube such that a fraction of its length h hangs freely and the end touches the ground. At a certain moment the other end of chain is set free. The speed of this end of chain when it slips out of the tube is

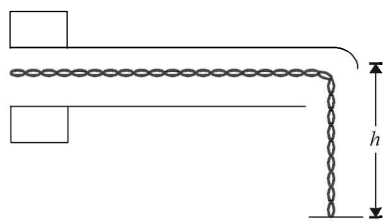


Fig. 5.75

(a) $\left[(2gh)\frac{dl}{dh}\right]^{1/2}$ (b) $\sqrt{gh}$

(c) $\sqrt{2gl}$ (d) $\left(2gh \log_e \frac{l}{h}\right)^{\frac{1}{2}}$

44. A block of mass M with semicircular track of radius R rests on a horizontal smooth surface. A cylinder of radius r slips on the track. If the cylinder is released from rest from top, the distance moved by block when cylinder reaches the bottom of the track is

(a) $R - r$ (b) $\dfrac{M(R-r)}{M+m}$

(c) $\dfrac{M}{M+m}(R-r)$ (d) $\dfrac{M}{M-m}r$

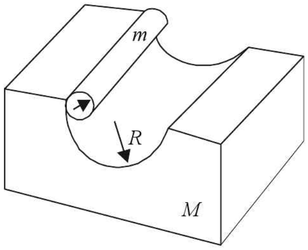


Fig. 5.76

45. A chain of length l is placed on a smooth spherical surface of radius r with one of its ends fixed at the top of the surface. Length of chain is assumed to be $l < \frac{\pi r}{2}$. Acceleration of each element of chain when upper end is released is

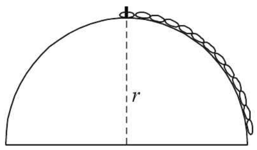


Fig. 5.77

(a) $\frac{lg}{r}\left(1-\cos\frac{r}{l}\right)$ (b) $\frac{rg}{l}\left(1-\cos\frac{l}{r}\right)$

(c) $\frac{lg}{r}\left(1-\sin\frac{l}{r}\right)$ (d) $\frac{rg}{l}\left(1-\sin\frac{l}{r}\right)$

46. A large free mass M and a small mass m are connected to a string such that m moves in horizontal circle. Length of string is l and θ is the angle this length makes with vertical. The frequency of rotation of mass m so that M remains at rest is

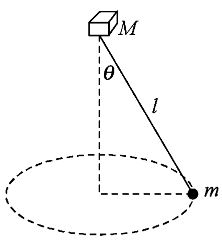


Fig. 5.78

(a) $2\pi\sqrt{\frac{ml}{Mg}}$ (b) $\frac{1}{2\pi}\sqrt{\frac{mg}{Ml}}$

(c) $\frac{1}{2\pi}\sqrt{\frac{ml}{Mg}}$ (d) $\frac{1}{2\pi}\sqrt{\frac{Mg}{ml}}$

47. Two blocks connected by a massless string slide down an inclined plane having angle of inclination as 37°. The masses of two blocks are 4 kg and 2 kg with μ as 0.75 and 0.25 respectively.

(a) The common acceleration of two masses is 1.3 ms^{-2} and tension in string is 5.3 N.
(b) Tension in the string is 14.9 N.
(c) Acceleration of the mass is 3 N.
(d) The acceleration of masses is 5.3 ms^{-2} and tension in the string is 1.3 N.

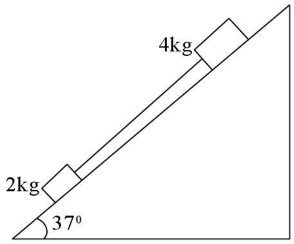


Fig. 5.79

48. Accelerations of the vehicle and mass m_2, when pulleys are light and all surfaces are frictionless, are

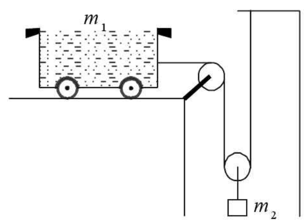


Fig. 5.80

(a) each $\frac{m_2 g}{4m_1+m_2}$ (b) $\frac{2m_2 g}{4m_1+m_2}, \frac{m_2 g}{4m_1+m_2}$

(c) each $\frac{2m_1 g}{4m_2+m_1}$

(d) $\frac{m_1 g}{4m_2+m_1}, \frac{2m_1 g}{4(m_2+m_1)}$

49. A block of mass m slides down an inclined right angled trough. If the coefficient of kinetic friction between the block and the trough is μ_k, acceleration of the block down the plane is

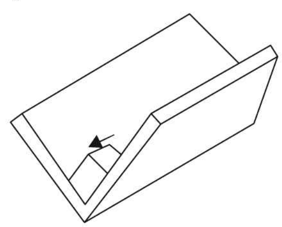


Fig. 5.81

(a) $g(\sin\theta - 2\mu_k\cos\theta)$ (b) $g(\sin\theta + 2\mu_k\cos\theta)$
(c) $g(\sin\theta + \sqrt{2}\,\mu_k\cos\theta)$ (d) $g(\sin\theta - \mu_k\cos\theta)$

50. A cylinder of radius $r = 1$ m and mass $m = 5\times 10^3$ kg is at rest on the edges of a structure as shown. Distance a is 0.5 m and $b = \frac{\sqrt{3}}{2}$ m. Reaction force on edges A and B are

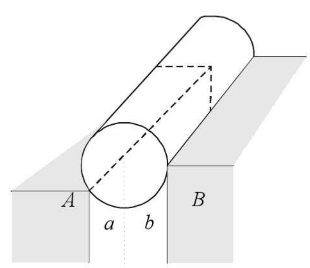


Fig. 5.82

(a) 24.6 kN, 24.6 kN (b) 42.6 kN, 42.6 kN
(c) 42.6 kN, 24.6 kN (d) 52.6 kN, 5.6 kN

51. Two blocks B_1 and B_2 of masses m_1 and m_2 respectively are connected with the help of a pulley and string as shown. Upper surface of vehicle is smooth but vertical surface is rough.

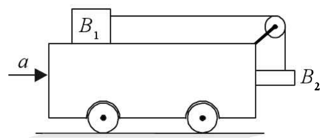


Fig. 5.83

Given $a = g/7$ and $m_1 = 7.5\, m_2$. Coefficient of friction between block B_2 and side of vehicle is

(a) 0.4 (b) 0.5
(c) 0.6 (d) 0.3

52. Sixteen beads in a string are placed on a smooth inclined plane of inclination $\sin^{-1}(1/3)$ such that some of them lie along the incline whereas the rest hang over the top of the plane. If acceleration at first bead is $g/2$, the arrangement of beads is that

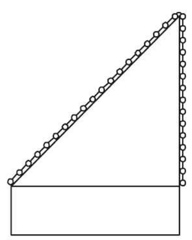


Fig. 5.84

(a) 12 hang vertically.
(b) 10 lie along inclined plane.
(c) 8 lie along inclined plane.
(d) 10 hang vertically.

53. A mass M is hung with a light inextensible string. Tension in horizontal part of string is

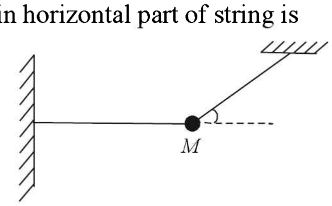


Fig. 5.85

(a) $\sqrt{3}\, Mg$ (b) $\sqrt{2}\, Mg$
(c) $\dfrac{Mg}{\sqrt{3}}$ (d) $\dfrac{Mg}{2}$

(IIT 1990)

54. A ship of mass 3×10^7 kg initially at rest is pulled by a force of 5×10^4 N through a distance of 3 m. Assuming that resistance due to water is negligible, the speed of ship is

(a) 0.2 ms^{-1} (b) 0.1 ms^{-1}
(c) 1 ms^{-1} (d) 2 ms^{-1}

(IIT 1990)

55. A bullet of mass M is fired with a velocity of 50 ms^{-1} at an angle θ with the horizontal. At the highest point of trajectory it collides with a bob of mass $3M$ suspended vertically by a massless string of length $\frac{10}{3}$ m and gets embedded into it. After the collision the string moves through an angle 120°, what is the angle of throw θ.

(a) $\cos^{-1}\frac{2}{5}$ (b) $\cos^{-1}\frac{3}{5}$
(c) $\cos^{-1}\frac{4}{5}$ (d) $\cos^{-1}\frac{1}{5}$

(IIT 1991)

56. A car is moving in a circular horizontal track of radius 10 m with a constant speed of 10 ms^{-1}. A plumb bob is suspended from the roof of car by a light rigid rod of length 1 m. The angle made by rod with the track is

(a) zero (b) 30°
(c) 45° (d) 60°

(IIT 1992)

57. A ball weighing 10g hits a hard surface vertically with a speed of 5 ms^{-1} and rebounds with the same speed. The ball remains in contact with the surface for 0.01 s. The average force exerted by the surface on ball is.

(a) 100 N (b) 10 N
(c) 1 N (d) 0.1 N

(Roorkee 1993)

58. Tension in rod of length L and mass M at a distance y from F_1 when the rod is acted on by two unequal forces F_1 and F_2 where $(F_2 < F_1)$ at its ends is

(a) $F_1(1 - y/L) + F_2(y/L)$ (b) $F_2(1 - y/L) + F_1(y/L)$
(c) $F_1(1 + y/L) + F_2(y/L)$ (d) $F_2(1 + y/L) + F_1(y/L)$

(IIT 1993)

59. The magnitude of force (in N) acting on a body varies with time t (in μs) as shown. AB, BC and CD are straight line segments. The magnitude of total impulse of force on the body from $t = 4\ \mu$s to $t = 16\ \mu$s is

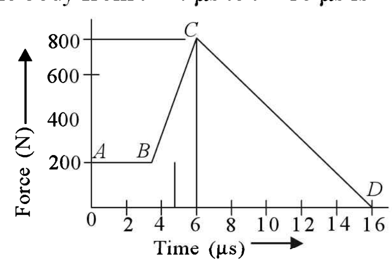


Fig. 5.86

(a) 6×10^{-3} Ns (b) 3×10^{-3} Ns
(c) 5×10^{-3} Ns (d) 6×10^{-3} Ns

(IIT 1994)

60. A smooth semicircular wire track of radius R is fixed in a vertical plane. One end of a massless spring of natural length $3R/4$ is attached to the lowest point O of the wire track. A small ring of mass m which can slide on the track is attached to the other end of the spring. The ring is held stationary at point P such that the spring makes an angle 60° with the vertical. Spring constant $K = mg/R$. The spring force is

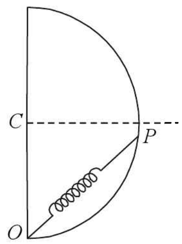


Fig. 5.87

(a) $\frac{mg}{3}$ (b) mg
(c) $\frac{mg}{2}$ (d) $\frac{mg}{4}$

(IIT 1996)

61. Block A of mass m and block B of mass $2m$ are placed on a fixed triangular wedge by means of massless, inextensible string and a frictionless pulley as shown. The wedge is inclined at 45° to horizontal on both sides. The coefficient of friction between block A and wedge is 2/3 and that between block B and wedge is 1/3. If system of A and B is released from rest then acceleration of A is

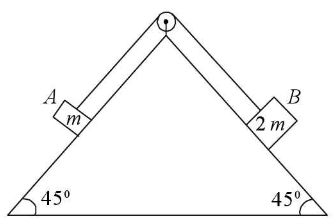


Fig. 5.88

(a) zero (b) 1 ms^{-2}
(c) 2 ms^{-2} (d) 3 ms^{-2}

(IIT 1997)

62. A large heavy box is sliding without friction down a smooth plane of inclination θ. From a point P on the bottom of the box, a particle is projected inside the box. The initial speed of particle with respect to the box is u and the direction of projection makes an angle α with the bottom as shown. Find the distance along the bottom of box between the point of projection P and point Q where the particle lands. (Assume that the particle does not hit any other surface of the box. Neglect air resistance)

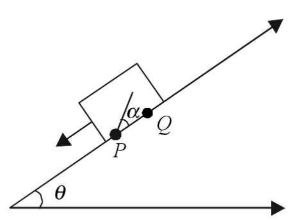


Fig. 5.89

(a) $\frac{u^2 \sin 2\alpha}{g}$ (b) $\frac{u^2 \sin^2 \alpha}{2g\cos\theta}$
(c) $\frac{u^2 \sin 2\alpha}{g\cos\theta}$ (d) $\frac{u^2 \sin \alpha}{g}$

(IIT 1998)

63. A spring of force constant K is cut into two pieces such that one piece is double the length of the other. Then the long piece will have a force constant of

(a) 2/3 K (b) 3/2 K
(c) 3K (c) 6K

(IIT 1999)

64. A cubical block of side L rests on a rough horizontal surface with coefficient of friction μ. A horizontal force F is applied on the block as shown. If the coefficient of friction is sufficiently high so that the block does not slide before toppling, the minimum force required to topple the block is

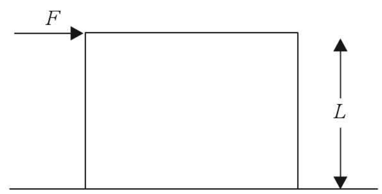


Fig. 5.90

(a) infinitesimal (b) $mg/4$
(c) $mg/2$ (d) $mg(1-\mu)$

(IIT Screening 2000)

65. An insect crawls up a hemispherical surface very slowly. The coefficient of friction between the surface and the insect is 1/3. If the line joining the centre of the hemispherical surface to the insect makes an angle α with the vertical, the maximum possible value of α is given by

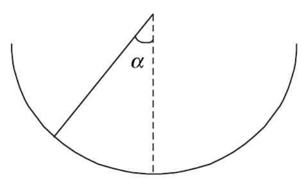


Fig. 5.91

(a) cot $\alpha = 3$ (b) tan $\alpha = 3$
(c) sec $\alpha = 3$ (d) cosec $\alpha = 3$

(IIT Screening 2001)

66. A string of negligible mass going over a clamped pulley of mass m supports a block of mass M as shown in the figure. The force on the pulley by the clamp is given by

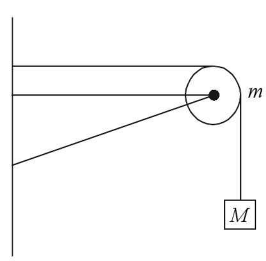


Fig. 5.92

(a) $\sqrt{2}\,Mg$ (b) $\sqrt{2}\,mg$

(c) $g\sqrt{(M+m)^2+m^2}$ (d) $g\sqrt{(M+m)^2+M^2}$

(IIT Screening 2001)

67. The pulleys and string shown in figure are smooth and of negligible mass. For the system to remain in equilibrium, the angle θ should be

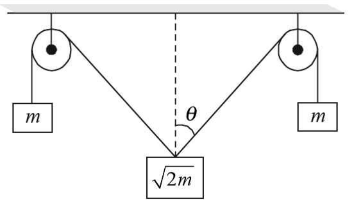


Fig. 5.93

(a) 0° (b) 30°
(c) 45° (d) 60°

(IIT Screening 2001)

68. An ideal spring with spring constant k is hung from the ceiling and a block of mass M is attached to its lower end. The mass is released with the spring *initially unstretched*. Then the maximum extension in the spring is

(a) 4 Mg/k (b) 2 Mg/k
(c) Mg/k (d) $Mg/2k$

(IIT Screening 2002)

69. A force F_1 acts on a particle so as to accelerate it from rest to a velocity v. The force F_1 is then replaced by F_2 which decelerates it to rest.

(a) F_1 must be equal to F_2 (b) F_1 may be equal to F_2
(c) F_1 must be unequal to F_2 (d) none of these

70. Two objects A and B are thrown upward simultaneously with the same speed. The mass of A is greater than the mass of B. Suppose the air exerts a constant and equal force of resistance on the two bodies

(a) the two bodies will reach the same height.
(b) A will go higher than B.
(c) B will go higher than A.
(d) any of the above three may happen depending on the speed with which the objects are thrown.

71. A smooth wedge A is fitted in a chamber hanging from a fixed ceiling near the earth's surface. A block B placed at the top of the wedge takes a time T to slide down the length of the wedge. If the block is placed at the top of the wedge and the cable supporting the chamber is broken at the same instant, the block will

(a) take a time longer than T to slide down the wedge.
(b) take a time shorter than T to slide down the wedge.
(c) remain at the top of the wedge.
(d) jump off the wedge.

72. In an imaginary atmosphere, the air exerts a small force F on any particle in the direction of the particle's motion. A particle of mass m projected upward takes a time t_1 in reaching the maximum height and t_2 in the return journey to the original point. Then

(a) $t_1 < t_2$ (b) $t_1 > t_2$
(c) $t_1 = t_2$
(d) the relation between t_1 and t_2 depends on the mass of the particle.

73. A person standing on the floor of an elevator drops a coin. The coin reaches the floor of the elevator in a time t_1 if the elevator is stationary and in time t_2 if it is moving uniformly. Then

(a) $t_1 = t_2$ (b) $t_1 < t_2$
(c) $t_1 > t_2$
(d) $t_1 < t_2$ or $t_1 > t_2$ depending on whether the lift is going up or down.

74. A free ^{238}U nucleus kept in a train emits an alpha particle. When the train is stationary, a nucleus decays and a passenger measures that the separation between the alpha particle and the recoiling nucleus becomes x at time t after the decay. If the decay takes place while the train is moving at a uniform velocity v, the distance between the alpha particle and the recoiling nucleus at a time t after the decay as measured by the passenger is

(a) $x + vt$ (b) $x - vt$
(c) x
(d) depends on the direction of the train.

75. Figure shows a heavy block kept on a frictionless surface and being pulled by two ropes of equal mass m. At $t = 0$, the force on the left rope is withdrawn but the force on the right end continues to act. Let F_1 and F_2 be the magnitudes of the forces by the right rope and the left rope on the block respectively.

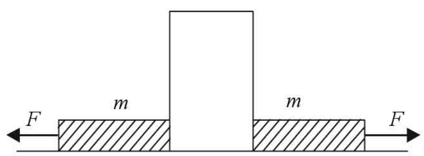


Fig. 5.94

(a) $F_1 = F_2 = F$ for $t < 0$
(b) $F_1 = F_2 = F + mg$ for $t < 0$
(c) $F_1 = F, F_2 = F$ for $t > 0$
(d) $F_1 < F, F_2 = F$ for $t > 0$.

76. A monkey of mass 20 kg is holding a vertical rope. The rope can break when a mass of 25 kg is suspended from it. What is the maximum acceleration with which the monkey can climb up along the rope?

(a) 7 ms^{-2} (b) 10 ms^{-2}
(c) 5 ms^{-2} (d) 2.5 ms^{-5}

77. A force of 5 Newton acts on a body of weight 9.8 Newton. What is the acceleration produced in ms^{-2}?

(a) 0.51 (b) 1.46
(c) 49.00 (d) 5.00

78. A body of mass m is released from the top of a rough inclined plane of length l. If the frictional force is f then the velocity of the body of the bottom in ms^{-1} will be

(a) $\sqrt{\frac{2}{m}(mgh-l)}$ (b) $2\,gh - fl$
(c) $\sqrt{\frac{2}{m}}\;gh$ (d) zero

79. A block of mass 2 kg is lying on a floor. The coefficient of static friction is 0.54. What will be the value of frictional force if the force is 2.8 N and $g = 10\ ms^{-2}$

(a) zero (b) 2 N
(c) 2.8 N (d) 8 N

80. A cube weighing 10 N is lying on a rough inclined plane of slope 3 in 5. The coefficient of friction between the plane and the cube is 0.6. The force necessary to move the cube up the plane will be

(a) 6.4 N (b) 10.8 N
(c) 21.6 N (d) 108 N

81. A block of metal is lying on the floor of a bus. The maximum acceleratin which can be given to the bus so that the block may remain at rest, will be

(a) μg^2 (b) $\mu^2 g$
(c) μg (d) μ/g

82. A body of weight w is lying at rest on a rough horizontal surface. If the angle of friction is θ, then the minimum force required to move the body along the surface will be

(a) $w\cos\theta$ (b) $w\tan\theta$
(c) $w\sin\theta$ (d) $w\cot\theta$

83. A block of mass 0.5 kg. rests against a wall exerting a horizontal force of 10 N on the wall. If the coefficient of friction between the wall and the block is 0.5 then the frictional force acting on the block will be

(a) 49.9 N (b) 9.8 N
(c) 4.90 N (d) 0.49 N

84. A rope of length l is pulled with a constant force f. T is the tension in the rope at a point distant x from the end where the force is applied. Then T is

(a) $f(l-x)/l$ (b) $fl/(l-x)$
(c) $\frac{(f-x)}{l-x}$ (d) $\frac{fl}{x}$

85. Two masses m_1 and m_2 are attached to a string which pass over a frictionless fixed pully. Given that $m_1 = 10$ kg and $m_2 = 6$ kg and $g = 10\ ms^{-2}$. What is the acceleration of the masses?

(a) $2.5\ ms^{-2}$ (b) $5\ ms^{-2}$
(c) $20\ ms^{-2}$ (d) $40\ ms^{-2}$

86. A block is lying on the table. What is the angle between the action of the block on the table and the reaction of the table on the block?

(a) 180° (b) 90°
(c) 45° (d) 0°

87. A parachutist of weight w strikes the ground with his legs fixed and comes to rest with an upward acceleration of magnitude 3 kg. Force exerted on him by ground during landing is

(a) $4\,w$ (b) $3\,w$
(c) $2\,w$ (d) w

88. The force that prevents the relative motion between the layers of a liquid is called

(a) static friction (b) sliding friction
(c) contact friction (d) none of these

89. Gravels are dropped on a conveyer belt at the rate of 0.5 kgs^{-1}. The extra force required in newtons to keep the belt moving at 2 $ms^{-1}$is

(a) 0.5 (b) 1
(c) 2 (d) 4

90. Starting from rest, a body slides down a 45° inclined plane in twice the time it takes to slide down the same distance in the absence of friction. The coefficient of friction between the body and the inclined plane is

(a) 0.25 (b) 0.33
(c) 0.75 (d) 0.80

91. When we walk once, we should take small steps to avoid slipping. This is because smaller steps ensure

(a) larger friction (b) smaller friction
(c) larger normal force (d) smaller normal force

92. A chain of length L and mass m is allowed to fall on a table such that the part falling on the table comes to rest instantaneously. The force acting on the table when l part of it has lied on the table is

(a) $\frac{3mlg}{L}$ (b) $\frac{2mlg}{L}$
(c) $\frac{mlg}{L}$ (d) $\frac{3mlg}{2L}$

93. Two balls of mass 1kg and 2 kg respectively are connected to the two ends of the spring. The two balls are pressed together and placed on a smooth table. When released, the lighter ball moves with an acceleration of 2 ms^{-2}. The acceleration of the heavier ball will be

(a) $0.2\ ms^{-2}$ (b) $1\ ms^{-2}$
(c) $2\ ms^{-2}$ (d) $4\ ms^{-2}$

94. A fireman wants to slide down a rope. The breaking load for the rope is 3/4th of the weight of the man. With what minimum acceleration should the fireman slide down? Acceleration due to gravity is g.

(a) zero (b) $\frac{g}{4}$
(c) $\frac{3g}{4}$ (d) $\frac{g}{2}$

95. A rain drop of mass 0.1g is falling with uniform speed of 10 cm^{-1}. What is the net weight of the drop?

(a) 10^{-2} N (b) 10^{-3} N
(c) 2×10^{-3} N (d) zero

96. A heavy unifrom bar is being carried by two men on their shoulders. The weight of the bar is w. If one man lets it fall from the end carried by him, what will be the weight experienced by the other?

(a) none of these (b) $w/4$
(c) $w/2$ (d) w

97. The coefficient of friction of an inclined plane is $1/\sqrt{3}$. If it is inclined at angle 30° with the horizontal, what will be the downward acceleration of the block placed on the inclined plane?

(a) 0 (b) $\sqrt{2}$ ms^{-2}
(c) $\sqrt{3}$ ms^{-2} (d) 3 ms^{-2}

98. A body is projected upwards with a kinetic energy of 100 J. Taking the friction of air into account, when it returns on earth, its kinetic energy will be

(a) more than 100 J (b) less than 100 J
(c) 100 J (d) none of these

99. Which of the following is a self adjusted force?

(a) Sliding friction (b) Static friction
(c) Limiting friction (d) Dynamic friction

100. A body is placed over an inclined plane of angle $\pi - \theta$. The angle between normal reaction and the weight of the body is

(a) equal to the angle of friction (b) more than θ
(c) less than θ (d) θ

101. The frictional force due to air on a body of mass 0.25 kg falling with an acceleration of 9.2 ms^{-1} will be

(a) 0.15 N (b) 1.5 N
(c) 15 N (d) zero

102. If a rough surface is polished beyond a certain limit than the magnitude of frictional force will

(a) nothing can be said
(b) some time increases and some time decreases
(c) increase
(d) decrease

103. A car is moving on a straight horizontal road with a speed of 72 kmh–1. If the coefficient of static friction between the tyre of the car and the road is 0.5, then the minimum distance, within which the car can be stopped will be

(a) 72 m (b) 40 m
(c) 30 m (d) 20 m

104. When we kick a stone, we get hurt. Due to which one of the following properties does it happens?

(a) Velocity (b) Momentum
(c) Inertia (d) Reaction

105. A cricket player catches a ball of mass 100 g and moving with a velocity of 25 ms^{-1}. If the ball is caught 0.1s, the force of the blow exerted on the hand of the player is

(a) 4 N (b) 40 N
(c) 25 N (d) 250 N

Answers to Questions for Practice

1.	(d)	**2.**	(a)	**3.**	(d)	**4.**	(d)	**5.**	(c)	**6.**	(b)	**7.**	(c)
8.	(b)	**9.**	(c)	**10.**	(c)	**11.**	(b)	**12.**	(b)	**13.**	(a)	**14.**	(c)
15.	(a)	**16.**	(d)	**17.**	(d)	**18.**	(d)	**19.**	(b)	**20.**	(a)	**21.**	(c)
22.	(b)	**23.**	(a)	**24.**	(d)	**25.**	(c)	**26.**	(b)	**27.**	(a)	**28.**	(c)
29.	(d)	**30.**	(a)	**31.**	(c)	**32.**	(c)	**33.**	(d)	**34.**	(b)	**35.**	(c)
36.	(a)	**37.**	(a)	**38.**	(c)	**39.**	(a)	**40.**	(d)	**41.**	(a)	**42.**	(a)
43.	(d)	**44.**	(b)	**45.**	(b)	**46.**	(d)	**47.**	(a)	**48.**	(b)	**49.**	(c)
50.	(c)	**51.**	(b)	**52.**	(d)	**53.**	(a)	**54.**	(b)	**55.**	(c)	**56.**	(c)
57.	(b)	**58.**	(a)	**59.**	(c)	**60.**	(d)	**61.**	(a)	**62.**	(c)	**63.**	(b)
64.	(c)	**65.**	(a)	**66.**	(d)	**67.**	(c)	**68.**	(b)	**69.**	(b)	**70.**	(b)
71.	(c)	**72.**	(b)	**73.**	(a)	**74.**	(c)	**75.**	(a)	**76.**	(d)	**77.**	(d)
78.	(a)	**79.**	(c)	**80.**	(b)	**81.**	(c)	**82.**	(b)	**83.**	(c)	**84.**	(a)
85.	(a)	**86.**	(a)	**87.**	(a)	**88.**	(d)	**89.**	(d)	**90.**	(c)	**91.**	(b)
92.	(a)	**93.**	(b)	**94.**	(b)	**95.**	(d)	**96.**	(a)	**97.**	(a)	**98.**	(b)
99.	(b)	**100.**	(d)	**101.**	(a)	**102.**	(c)	**103.**	(b)	**104.**	(d)	**105.**	(c)

Explanations

8. (b) The maximum force of friction,

$$f_{max} = \mu mg$$

or $$a_{max} = \frac{f_{max}}{m} = mg$$

or $$m = \frac{a_{max}}{g}$$

Taking a less than a_{max}, friction f is also less than f_{max}. Here, if the value of $\mu > a/g$ then $f \neq \mu mg$ rather $f = ma$.

9. (c) Tension in the spring is actually the centripetal force given by $mr\omega^2$ *i.e.*, $mr(2\pi\nu)^2$.

10. (c) Let F be applied at origin from figure 5.95

$$F = 2T\cos\theta$$

or $$T = \frac{F}{2\cos\theta}$$

Then force causing motion is given by T

$$T\sin\theta = \left(\frac{F}{2\cos\theta}\right)\sin\theta$$

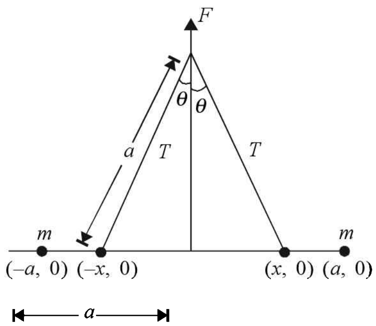


Fig. 5.95

$$= \frac{F}{2}\tan\theta = \frac{F}{2}\frac{x}{\sqrt{a^2-x^2}}$$

$$\therefore \text{acceleration} = \frac{F}{2m}\cdot\frac{x}{\sqrt{a^2-x^2}}$$

11. (b) The particle is displaced from mean position and then released, i.e., at $t = 0$, the tension is minimum because the particle is at the extreme position where the tension has to balance only the radial component of the weight of the particle. Tension is maximum at mean position because it has to provide the weight as well as centripetal force also.

12. (b) The force on mass m_1 is

$F_1 = m_1 a$ and force on m_2 is, $F_2 = m_2 a'$

but $F = F_1 + F_2 = m_1 a + m_2 a'$

$$\therefore a' = \frac{F - m_1 a}{m_2}$$

13. (a) Here, $20\times 9.8 = \dfrac{mv^2}{r} = 20\times 9.8$

or $$v = \sqrt{20\times 9.8} = 14\ \text{ms}^{-1}$$

14. (c) Let a be the acceleration down the rough plane and a' be the acceleration down the frictionless plane. Taking L as the length of the inclined plane, we get

$$a = g(\sin\theta - \mu\cos\theta)$$

$$= g\left(\frac{1}{\sqrt{2}} - \frac{\mu}{\sqrt{2}}\right) \quad (\because \theta = 45°)$$

and $$a' = g\sin\theta = g\frac{1}{\sqrt{2}}$$

Then $L = \dfrac{1}{2}at_1^2 = \dfrac{1}{2}a't_2^2$

or $$\frac{1}{2}g\left(\frac{1}{\sqrt{2}} - \frac{\mu}{\sqrt{2}}\right)t_1^2 = \frac{1}{2}\frac{g}{\sqrt{2}}t_2^2$$

But $t_1 = nt_2$ (given)

$$\therefore \frac{1}{2}g\left(\frac{1}{\sqrt{2}} - \frac{\mu}{\sqrt{2}}\right)n^2 t_2^2 = \frac{1}{2}\frac{g}{\sqrt{2}}t_2^2$$

or $$1 = (1-\mu)n^2 \text{ or } \mu = \left(1 - \frac{1}{n^2}\right)$$

15. (a) The masses will be lifted if the tension of the string is greater than the gravitational pull on masses.

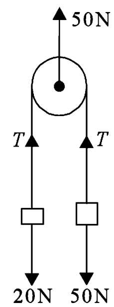


Fig. 5.96

Weight of 5 kg mass = 5 × 10 = 50 N and 2 kg mass = 2 × 10 = 20 N

From free body diagram

$50 - 2T = 0$ or $T = 25$ N

So 5 kg weight can not be lifted ($\because$ acceleration = 0) but 2 kg weight will be lifted.

$$\therefore 25 - 20 = 2a \text{ or } a = \frac{5}{2} = 2.5\ \text{ms}^{-2}$$

16. (d) Planes PQ and QR are frictionless and impact is neglected, so mechanical energy will conserve,

$\therefore h' = h$

17. (d) Radius of horizontal circle of ball

$= (L + L\sin\theta)$

$\therefore$ C.P. Acceleration $= (L + L\sin\theta)\,\omega^2$ $\quad(\because a = r\omega^2)$

Here $mg = T\cos\theta$ ... (i)

and $m\omega^2(L + L\sin\theta) = T\sin\theta$ ... (ii)

Dividing (ii) by (i)

$$\tan\theta = \frac{\omega^2(L + L\sin\theta)}{g}$$

or $$\omega^2 = \frac{g\tan\theta}{L(1+\sin\theta)}$$

18. (d) $$T_{\max} = \frac{mv_l^2}{L} + mg$$

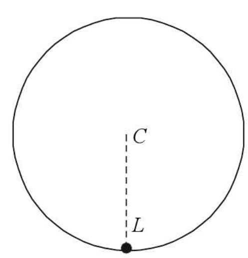


Fig. 5.97

and $T_{\min} = \dfrac{mv_h^2}{L} - mg$

Then $\dfrac{T_{\max}}{T_{\min}} = \dfrac{\dfrac{mv_l^2}{L} + mg}{\dfrac{mv_h^2}{L} - mg}$

$= \dfrac{v_l^2 + gL}{v_h^2 - gL}$ ... (i)

Using $v^2 - u^2 = 2\,aS$, we get

$v_h^2 - v_l^2 = -2g\,(2L) = -4gL$

or $v_l^2 = v_h^2 + 4gL$

Then from (i) $\dfrac{T_{\max}}{T_{\min}} = \dfrac{v_h^2 + 4gL + gL}{v_l^2 - gL}$

or $4 = \dfrac{v_h^2 + 5 \times 10 \times \dfrac{10}{3}}{v_h^2 - 10 \times \dfrac{10}{3}}$

or $3\,v_h^2 = 300 =$ or $v_h = 10\ \text{ms}^{-1}$

19. (b) For equilibrium, the forces exerted by both walls on the man should be equal so as the horizontal forces may balance but the vertical forces can be balanced even if the forces of friction on the two walls are unequal.

20. (a) Considering free body diagram,

$mg - T = ma$ (*for hanging mass*)

and $T = ma$

(for mass lying on surface)

Adding $mg = (m + m)\,a$

or $a = \dfrac{mg}{2m}$

$\therefore$ $a = \dfrac{g}{2} = \dfrac{10}{2} = 5\ \text{ms}^{-1}$

and $T = 1 \times 5 = 5\ \text{N}$

21. (c) Limiting frictional force $= \mu\, mg$

$= 0.25 \times 2 \times 10 = 5\ \text{N}$

So the block and trolley will not have relative motion for a force of 2N.

Here, $2 = (20 + 2)\,a$

or $a = \dfrac{2}{22} = \dfrac{1}{11}$

$= 0.09\ \text{ms}^{-2}$

Then frictional force

$= 20 \times 0.09 = 1.8\ \text{N}$

22. (b) Here $T_1 - T_2 = 6a$,

$T_2 - 1g = 1a$ and $3g - T_1 = 3a$

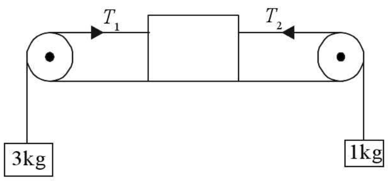


Fig. 5.98

Addition of the above three equations give

$10a = 3g - 1g = 2g$

or $a = \dfrac{2}{10}\,g = \dfrac{2}{10} \times 10 = 2\ \text{ms}^{-2}$

23. (a) Let a_x, a_y and a_r be the net leftward horizontal acceleration of bead, net downward vertical acceleration of bead and relative acceleration of bead with reference to rod respectively. Then

$a_y = a_r \cos\theta + a$

and $a_x = a_r \sin\theta$

Projecting forces vertically and horizontally

$mg - N\cos\theta = ma_r \sin\theta$ ... (i)

and $N\sin\theta = m\,(a_r \cos\theta + a)$ ... (ii)

From (i) and (ii)

$mg\sin\theta = ma_r + ma\cos\theta$

i.e., $a_r = g\sin\theta - a\cos\theta$

24. (d) Velocity of block at time t is given by $v = v_0 - \mu g t$, where v_0 is initial velocity and μ is the coefficient of friction. Thus, $v - t$ graph is a straight line having negative slope and a positive intercept on v axis. $s - t$ curve will be having a decreasing slope till it reduces to zero because velocity of block decreases continuously and remains positive till the block comes to rest position.

25. (c) Let there be a small element of length dl at a distance L from the end of rotational axis.

Mass of the element $dl = \dfrac{M}{L}\,dl$

Small radial force on this element

$= \left(\dfrac{M}{L}dl\right) l\omega^2$

$\therefore$ Total force $= \int_0^L \left(\dfrac{M}{L}dl\right) l\omega^2$

$= \dfrac{M}{L}\,\omega^2 \int_0^L l\,dl = \dfrac{ML\omega^2}{2}$

26. (b) Consider the man and the child to be simple masses hung from two ends of a string passing over pulley. Being a system, the man and child both have same magnitude of acceleration but opposite directions.

27. (a) Instantaneous acceleration,

$a = \dfrac{dv}{dt} = \dfrac{\text{Constant force}}{\text{Instantaneous mass}}$

or $a = \dfrac{F}{M - mt}$

($\because$ rate of fall of sand per second is m)

28. (c) Using $v^2 - u^2 = 2aS$ we get

$v^2 - u^2 = 2(-g)H$

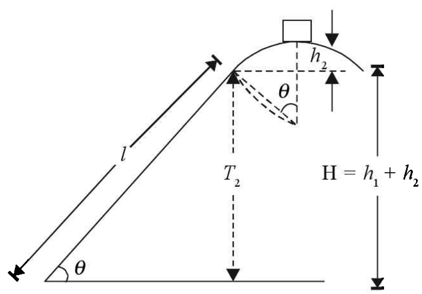


Fig. 5.99

i.e., $-u^2 = 2(-g)(h_1 + h_2)$

but $h_1 = l \sin\theta$

and $h_2 = R(1 - \cos\theta)$

$\therefore u^2 = 2g(l\sin\theta + R(1 - \cos\theta))$

or $u = [2g\{l\sin\theta + R(1 - \cos\theta)\}^{1/2}$

29. (d) Relative velocity of monkeys

$= v + 2v = 3v$

Total distance covered $= \dfrac{l}{2} + \dfrac{l}{2} = l$

$\therefore$ time taken by each monkey $= \dfrac{l}{3v}$

30. (a) Here $m_1 g - T = m_1 a_1$

(*taking a_1 as downward acceleration of m_1*)

$2T - m_2 g = m_2 a_2$

(taking a_2 as upward acceleration of m_2)

and $2T - T = 0$ ($\because$ *mass of pulley is zero*)

Thus, $T = 0$ $\therefore a_1 = a_2 = g$

Thus, the masses will have free fall.

Clearly pulley B rotates clockwise and the other pulleys in anticlockwise direction.

31. (c) Here $a_r = \dfrac{v^2}{l} = \dfrac{(\sqrt{2g(l\cos\alpha)})^2}{l}$

$= \dfrac{2gl\cos\alpha}{l} = 2g\cos\alpha$

and $\tan\alpha = \dfrac{a_r \sin 90°}{a_t + a_r\cos 90°} = \dfrac{2g\cos\alpha}{g\sin\alpha}$

($\because a_t = g\sin\alpha$)

$= \dfrac{2}{\tan\alpha}$

i.e., $\tan^2\alpha = 2$ or $\sec^2\alpha = 3$

or $\cos\alpha = \dfrac{1}{\sqrt{3}}$

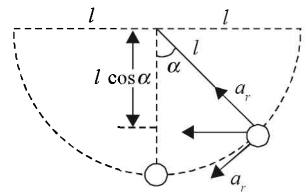


Fig. 5.100

or $\alpha = \cos^{-1}\left(\dfrac{1}{\sqrt{3}}\right)$

32. (c) $\cos\alpha = \dfrac{W}{OA}$ or $OA = \dfrac{W}{\cos\alpha}$

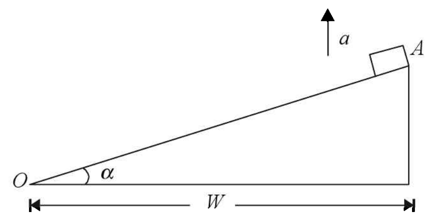


Fig. 5.101

Also $OA = \dfrac{1}{2}(g + a)\sin\alpha\, t^2$

$\left(\text{by using } S = ut + \dfrac{1}{2}at^2\right)$

$\therefore \dfrac{W}{\cos\alpha} = \dfrac{1}{2}(g + a)\sin\alpha\, t^2$

$$t = \left[\frac{2W}{\cos\alpha(g+a)\sin\alpha}\right]^{\frac{1}{2}}$$

$$= \left[\frac{4W}{(2\cos\alpha\sin\alpha)(g+a)}\right]^{1/2}$$

$$= \left[\frac{4W}{(g+a)\sin 2\alpha}\right]^{1/2}$$

33. (d) Here spring force = centripetal force

$\therefore kx = m(l + x)\omega^2$ (*where x is the extension in the length of the spring*)

i.e., $x = \dfrac{ml\omega^2}{k - m\omega^2}$

34. (b) The force of 100 N acts on both the boats

$\therefore 250\, a_1 = 100$ and $500\, a_2 = 100$

or $a_1 = 0.4 \text{ ms}^{-2}$

and $a_2 = 0.2 \text{ ms}^{-2}$

Then relative acceleration

$= a_1 + a_2 = 0.6 \text{ ms}^{-2}$

Using $S = ut + 1/2\, at^2$, we get

$100 = (1/2) \times 0.6 \times t^2$

or $t = 18.3$ s.

35. (c) The centripetal force is provided by the hanging weight *i.e.*,

$$Ml\,\omega^2 = mg$$

or $$m = \frac{Ml\omega^2}{g}$$

36. (a) If T is the upward thrust on parachute, then

$$ma = mg - T \quad \text{... (i)}$$

Let m' be the mass to be released, then

$$T - (m - m')\,g = (m - m')\,a \quad \text{... (ii)}$$

Adding (i) and (ii)

$$m'\,g = 2ma - m'\,a$$

or $$m' = \frac{2ma}{a+g}$$

37. (a) The two blocks move with same acceleration till force of friction between them is not reaching the limiting value. After reaching this value, acceleration of B becomes constant but acceleration of A continues to increase at faster rate.

38. (c) Let the weight W be placed on the nearer edge.

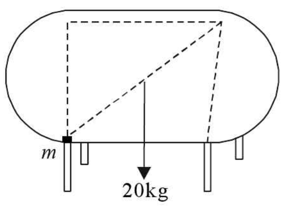


Fig. 5.102

Distance between two adjacent legs,

$$d = \sqrt{2} \times 1 = \sqrt{2} \times 1 = \sqrt{2}\ \text{m}$$

Then $mg \times 1 = (20\,g + mg) \times \dfrac{\sqrt{2}}{2}$

or $$mg = \frac{20g}{\sqrt{2}-1}$$

$$= 20\,(\sqrt{2}+1)\,g$$

or $$mg = 48.3\,g\,\text{kg}$$

or $$m = 48.3\,\text{kg}$$

39. (a) Let m_b be the mass of board and m be the mass placed at one end of the board.

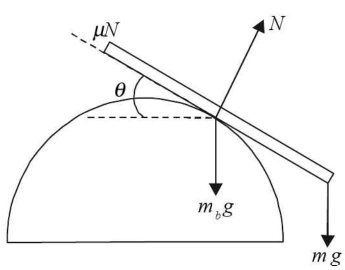


Fig. 5.103

Then for equilibrium

$\mu N \cos\theta = N \sin\theta$

i.e., $\mu = \tan\theta$

40. (d) Before accelerating, tension at rope B is $T = \dfrac{1}{2} \times$ tension at rope $A = \dfrac{T_A}{2}$.

Just after acceleration, total tension in rope B is given by $T_{\text{total}} = \dfrac{T_A}{2} + ma$

41. (a) Force of resistance, $f = -kv^2$

i.e., $m\dfrac{dv}{dt} = -kv^2$ i.e., $\dfrac{mdv}{v^2}$

i.e., $\displaystyle\int_u^v \frac{dv}{v^2} = \int_0^t \frac{-k}{m}dt$

i.e., $t = \left(\dfrac{u-v}{uv}\right)\dfrac{m}{k}$ ... (i)

Also $m\dfrac{dv}{dt}ds = -kv^2 ds$

i.e. $mv\,dv = -kv^2 ds$

i.e. $mdv = -kvds$

i.e. $\displaystyle\int_u^v m\frac{dv}{v} = -k\int_0^d ds$

i.e. $\log_e \dfrac{v}{u} = -\dfrac{cd}{m}$ i.e. $\log_e \dfrac{u}{v} = \dfrac{kd}{m}$

i.e. $d = \dfrac{m}{k}\log_e \dfrac{u}{v}$ ... (ii)

From (i) and (ii) $t = \dfrac{d(u-v)}{uv\left(\log_e \dfrac{u}{v}\right)}$

42. (a) $\dfrac{mdv}{dt} + u_{\text{rel}}\dfrac{dm}{dt} = 0$

or $\dfrac{mdv}{dt} = -u_{\text{rel}}\dfrac{dm}{dt}$

i.e., $dv = -u_{\text{rel}}\dfrac{dm}{m}$

i.e., $v = -u_{\text{rel}}\displaystyle\int_m^{m'}\frac{dm}{m} = -u_{\text{rel}}\log_e\frac{m'}{m}$

$= u_{\text{rel}}\log_e \dfrac{m}{m'}$

43. (d) For hanging part,

$mgh - T = mha$ ... (i)

and for part in tube,

$T = mxa$ ... (ii)

Adding the above equations, we get

$mgh = m\,(h + x)\,a$

or $a = \dfrac{gh}{h+x}$ or $\dfrac{dv}{dt} = \dfrac{gh}{h+x}$

or $\dfrac{dv}{dx}\dfrac{dx}{dt} = \dfrac{gh}{h+x}$ or $\dfrac{vdv}{dx} = \dfrac{gh}{h+x}$

or $vdv = \dfrac{gh}{h+x}\,dx$

$\therefore \int_0^v vdv = \int_0^{l-h} \dfrac{gh}{h+x}dx$

i.e., $v = \left[2gh\log_e \dfrac{1}{h}\right]^{1/2}$

44. (b) Let x be the distance moved by the block when cylinder moves from top to the bottom.

Here $(M+m)\,x = m\,(R-r)$

or $x = \dfrac{m(R-r)}{M+m}$

45. (b) Let mass per unit length of chain = $\dfrac{m}{l}$

Consider an element of chain of length dl subtending an angle of $d\theta$ at the centre of spherical surface.

Mass of element $dm = \dfrac{m}{l}\,dl = \dfrac{m}{l}\,rd\theta$

Force acting on element, $dF = dm\,g\sin\theta$

$= \dfrac{m}{l}\,rg\sin\theta\,d\theta$

$\therefore F = \dfrac{m}{l}\,rg\int_0^{\alpha}\sin\theta d\theta$

$= \dfrac{m}{l}\,rg\,(1-\cos\alpha)$

$= \dfrac{m}{l}\,rg\left(1-\cos\dfrac{l}{r}\right)$

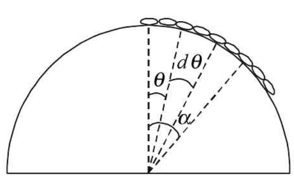


Fig. 5.104

Then $a = \dfrac{F}{m} = \dfrac{rg}{l}\left(1-\cos\dfrac{l}{r}\right)$

46. (d) Here $T = Mg$ and $T\cos\theta = Mg$

Also $T\sin\theta = m\omega^2 l\sin\theta$ ($\because$ *radius* = $l\sin\theta$)

i.e., $T = m\omega^2 l$

or $Mg = m\omega^2 l$ i.e., $\omega = \sqrt{\dfrac{Mg}{ml}}$

i.e., $2\pi v = \sqrt{\dfrac{Mg}{ml}}$ i.e., $v = \dfrac{1}{2\pi}\sqrt{\dfrac{Mg}{ml}}$

47. (a) *For mass* M_1

$T + M_1 g\sin 37 - \mu_1 M_1 g\cos 37 = M_1 a$ ... (i)

and for mass M_2

$M_2 g\sin 37 - T - \mu_2 M_2 g\cos 37 = M_2 a$ ... (ii)

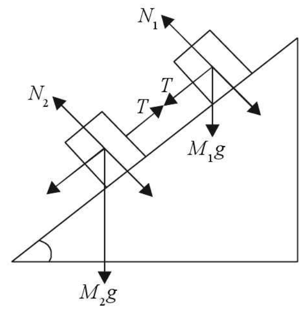


Fig. 5.105

Adding the substituting the given values, we get

$2g\times 0.6 + 4g\times 0.6 - 0.25\times 2g\times 0.8 - 0.75\times 4g\times 0.8 = (4+2)\,a$

$g\,(1.2+2.4-0.4-2.4)\,6a$

or $a = 1.3\ \text{ms}^{-2}$

Using (ii)

$2\times 9.8\times 0.6 - T - 0.25\times 2\times 9.8\times 0.8 = 2\times 1.3$

or $T = 11.76 - 3.98 - 2.6 = 5.29\ \text{N}$

48. (b) If mass m_1 travels s, m_2 travels by $s/2$.

$\therefore$ if acceleration of m_1 is a, then acceleration of mass m_2 is $\dfrac{1}{2}\,a$

Here $T = m_1 a$ and $m_2 g - 2T = m_2\,\dfrac{a}{2}$

Solving $a = \dfrac{2m_2 g}{4m_1 + m_2}$

49. (c) Note

(i) mg acts downward

(ii) frictional forces up the plane

(iii) reactions N as shown

Then $mg\sin\theta - 2\,\mu_k N = ma$ ... (i)

($\because$ *there are 2 surfaces*)

also $mg\cos\theta - \sqrt{2}\,N = 0$

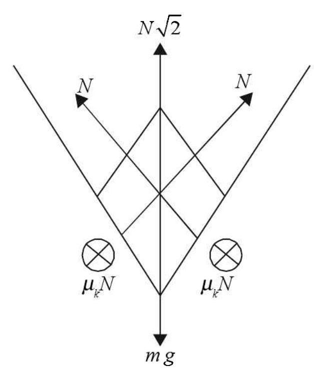


Fig. 5.106

From (i) and (ii)

$$mg\sin\theta - 2\mu_k \frac{mg\cos\theta}{\sqrt{2}} = ma$$

or $a = g(\sin\theta - \sqrt{2}\ \mu_k \cos\theta)$

50. (c) $\sin\alpha = \frac{a}{r} = \frac{0.5}{1} = \frac{1}{2}$

or $\alpha = 30°$

and $\sin\beta = \frac{\sqrt{3}}{2}\Big/1$

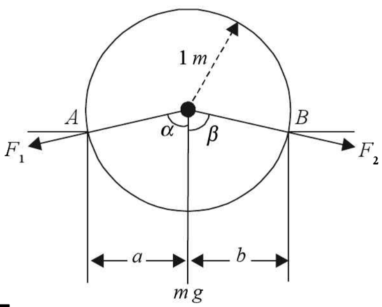


Fig. 5.107

or $\beta = 60°$

Then $F_1 = mg\cos\alpha$

$= 5 \times 10^3 \times 9.8 \times \cos 30 = 42.6\ k\text{N}$

and $F_2 = mg\cos\beta$

$= 5 \times 10^3 \times 9.8 \cos 60 = 24.5\ k\text{N}$.

51. (b) For equilibrium

$T = m_1 a$ ...(i)

and $T = m_2 g + \mu m_2 a$ ... (ii)

From (i) and (ii)

$m_1 a = m_2 g + \mu\, m_2 a$

or $\mu = \frac{m_1 a - m_2 g}{m_2 a}$

$$= \frac{m_1\left(\frac{g}{7}\right) - m_2 g}{m_2\left(\frac{g}{7}\right)} = \frac{m_1 - 7m_2}{m_2}$$

$$= \frac{7.5m_2 - 7m_2}{m_2} = 0.5$$

52. (d) If n balls each of mass m are hanging vertically

then, $nmg - T = nma = \frac{nmg}{2}$

or $T = \frac{nmg}{2}$ ... (i)

also $T - (16 - n)\, mg\sin\theta$

$= (16 - n)\, mg/2$

on $\frac{nmg}{2} - (16 - n)\, mg\,\frac{1}{3} = (16 - n)\,\frac{mg}{2}$

(*using (i)*)

or $n\,\frac{4}{3} = \frac{80}{6}$

or $n = \frac{80}{6} \times \frac{3}{4} = 10$

53. (A) Here $T_1 = T_2 \cos 30 = T_2\,\frac{\sqrt{3}}{2}$

i.e., $T_2 = \frac{2}{\sqrt{3}}\,T_1$

Also $Mg = T_2 \sin 30° = \frac{T_2}{2}$

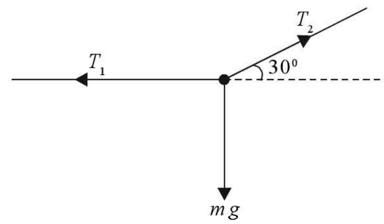


Fig. 5.108

i.e., $Mg = \frac{2}{\sqrt{3}}\,\frac{T_1}{2} = \frac{T_1}{\sqrt{3}}$

i.e., $T_1 = \sqrt{3}\ Mg$

54. (b) Using $F = ma$, we get

$$a = \frac{F}{m} = \frac{5\times10^4}{3\times10^7}$$

$$= \frac{5}{3} \times 10^{-3}$$

again using $2\,as = v^2 - u^2$, we get

$$2 \times \frac{5}{3} \times 10^{-3} \times 3 = v^2$$

55. (c) Velocity at highest point = $u\cos\theta$

Then $Mu\cos\theta = (M + 3M)\,v$

or $v = \frac{u\cos\theta}{4}$

and $4\,Mg\,h = \frac{1}{2}\,4M\left(\frac{u\cos\theta}{4}\right)^2$

Using the given values we get

or $\cos^2\theta = \frac{16}{25}$ or $\cos\theta = 4/5$

or $\theta = \cos^{-1} 4/5$

56. (c) $T\sin\theta = \frac{mv^2}{R}$

and $T\cos\theta = mg$

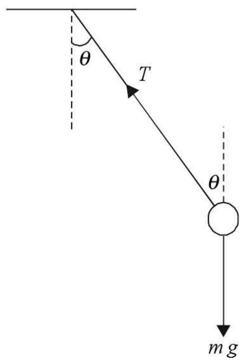


Fig. 5.109

$\therefore \quad \tan\theta = \frac{v^2}{Rg} = \frac{10}{10\times 10}$

or $\quad \tan\theta = 1$ or $\theta = 45°$

57. (b) Impulse = Ft = change in momentum

$= mv - (-mv)$

$= 2mv = 2\times 0.01\times 5 = 0.1$

$\therefore \quad F = \frac{0.1}{0.01} = 10\,\text{N}$

59. (c) Impulse = $\int F dt$ = area under graph

$\therefore$ Total impulse from $4\mu s$ to $16\mu s$

= Area $EBCD$

$= 1/2\,(200 + 800)^2 \times 10^{-6} + \frac{1}{2}\times 800\times 10\times 10^{-6}$

$= 5\times 10^{-3}$ Ns

60. (d) $CP = CO = R$

$\angle CPO = \angle POC = 60°$

Thus ΔOCP is an equilateral Δ

$\therefore \quad OP = R$

$\therefore$ Extension = R – natural length of spring

$= R - \frac{3R}{4} = \frac{R}{4}$

Thus spring force = $kx = \left(\frac{mg}{R}\right)\left(\frac{R}{4}\right) = \frac{mg}{4}$

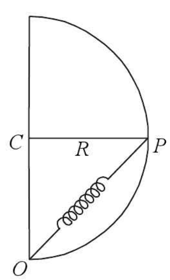


Fig. 5.110

61. (a) f_{max} for $A = \mu_1 (mg\cos 45°)$

$= \frac{2}{3}\frac{mg}{\sqrt{2}} = \frac{\sqrt{2}}{3}\,mg$

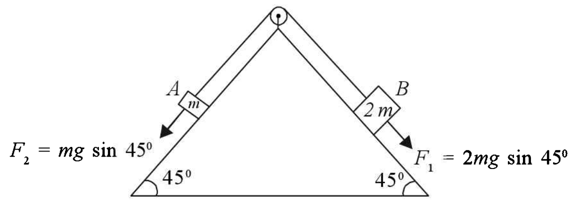


Fig. 5.111

Also f_{max} for $B = \mu_2\,(2mg\cos 45)$

$= 1/3\,(2mg\,\sqrt{2}) = \frac{\sqrt{2}}{3}\,mg$

Total frictional force

$= \frac{\sqrt{2}}{3}\,mg + \frac{\sqrt{2}}{3}\,mg = \frac{2\sqrt{2}}{3}\,mg$

But pulling force

$= F_1 - F_2 = \frac{2mg}{\sqrt{2}} - \frac{mg}{\sqrt{2}} = \frac{mg}{\sqrt{2}}$

$\therefore$ system can not accelerate.

62. (c) Acceleration of particle w.r.t. block

= Acceleration of particle – acceleration of block

$= (g\sin\theta\,\hat{i} + g\cos\theta\,\hat{j}) - g\sin\theta\,\hat{i}$

$= g\cos\theta\,\hat{j}$

Motion of particle with reference to block is parabolic

$\therefore PQ = \text{range} = \frac{u^2\sin 2\alpha}{g\cos\theta}$

63. (b) $l_1 = 2l_2 = \frac{2}{3}\,l$

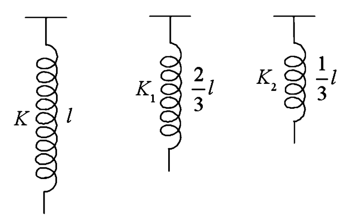


Fig. 5.112

Force constant $K \propto \frac{1}{\text{length of spring}}$

$\therefore K = \frac{3}{2}\,K.$

64. (c) Block will topple if

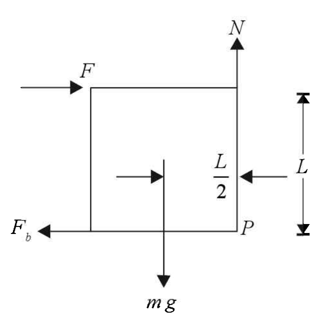


Fig. 5.113

$(F \times L) > (mg \times \frac{L}{2})$

or $F > \frac{mg}{2}$

$\therefore$ least force $= \frac{mg}{2}$

65. (a) For equilibrium

$\sum$ horizontal forces = 0

i.e., $N \sin \alpha = \mu N \cos \theta$

i.e., $\cot \alpha = 1/\mu$

or $\cot a = 3$

66. (d) Force applied by clamp = resultant of T and $(mg + T)$

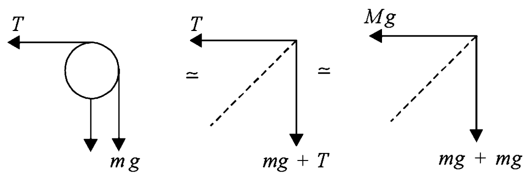


Fig. 5.114

But $T = Mg$

$\therefore$ Force applied by clamp

$= \sqrt{(Mg)^2 + (mg + Mg)^2}$

$= g\sqrt{M^2 + (M+m)^2}$

67. (c) Here $T = mg$ ... (i)

and $2T \cos \theta = (\sqrt{2}\, m)\, g$ ... (ii)

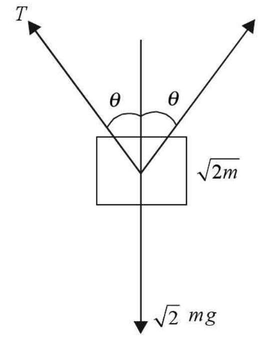


Fig. 5.115

Dividing (ii) by (i)

$\cos \theta = \frac{1}{\sqrt{2}}$ or $\theta = 45°$

68. (b) Here P.E. $= \frac{1}{2} kx^2$

i.e., $mgx = \frac{1}{2} kx^2$

or $x = \frac{2mg}{k}$

6

Work, Power and Energy

BRIEF REVIEW

Work The work is said to be done when a particle is displaced by the action of a force. It is a scalar quantity. Unit of work is Joule (SI) and CGS unit is erg. Practical unit of work (particularly in electric consumption) is kWh. 1 kWh = 3.6×10^6 J and 1 J = 10^7 ergs. Sometime eV is also used. 1 eV = 1.6×10^{-19} J. In problems of heat 1 calorie = 4.186 J

$$dW = \vec{F} \cdot \vec{ds}$$

$W = \vec{F} \cdot \vec{s} = Fs \cos \theta$ if force is constant throughout.

$W = \int F \cdot ds$ if force is variable

Work is positive or negative depending on the value of θ. For acute angles $\cos \theta$ is positive and hence, work is positive. For obtuse angle $\cos \theta$ may be negative making work negative. Positive work is parallel to displacement and negative work is opposite to displacement.

Work done in lifting a body up (against gravity) is positive and work done by the force of gravity (vertically downward motion) is negative.

No work will be done if the body is in static or dynamic equilibrium, i.e., $W = 0$ if $\Sigma F = 0$.

No work will be done if displacement is zero or force is perpendicular to the displacement. Thus, work done by centripetal force and work done by moving charged particle in a magnetic field is zero i.e., $F = q(\vec{v} \times \vec{B})$ will do no work. Work done depends upon the frame of reference. If frame of reference is changed displacement may vary and hence work done could be different in different frame of references.

In a conservative field work done is path independent

$$W = \Delta PE = \int F.ds .$$

In a force versus displacement curve, work done is area under the graph. The algebraic sum of the area is to be found out as illustrated in Fig. 6.1.

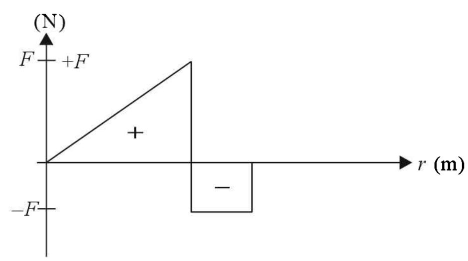


Fig. 6.1

$W = \int_{V_1}^{V_2} PdV$ Area under Pressure (P) and Volume (V) curve is work done.

$W = \Delta KE$, i.e., work = change in KE. This is also called work energy theorem.

For positive work $KE_{\text{final}} > KE_{\text{initial}}$. Work energy theorem is valid for all types of forces (internal or external; conservative or nonconservative).

In case of a spring $W = \frac{1}{2} kx^2$ where x is extension or compression in the spring.

$W = \frac{1}{2}$ stress × strain × volume in elastic bodies

Since work is independent of time. We define, **time rate of doing work is Power.**

$$P = \frac{dW}{dt} = \frac{d}{dt}(\vec{F}.d\vec{s}) = \vec{F}.\frac{d\vec{s}}{dt} = \vec{F}.\vec{v}$$

Power is a scalar quantity. Its SI unit is Watt (W) or J/s. Practical unit of power is HP or bhp (british horse power)

1 bhp = 746 W = 550 *ft – lb/s*

W $= \int P \, . \, dt$ or area under $P - t$ graph.

Note *KE* can never be negative while *PE* can be both negative or positive. Potential energy is defined only for conservative forces. It does not exist for nonconservative forces.

Elastic $PE = \frac{1}{2}kx^2$ and is taken positive in all cases.

Electric $PE = \frac{q_1 q_2}{4\pi \in_0 r}$ may be negative or positive.

Gravitational $PE = -\frac{GM_1M_2}{r}$ may be negative or positive.

Mechanical energy = *KE* + *PE* is conserved if internal forces are conservative and no work is done by nonconservative forces. If some of the internal forces are nonconservative mechanical energy of the system is not conserved.

Total energy = *KE* + *PE* + internal energy.

Internal energy is directly related to temperature. Larger the internal energy, higher is the temperature of the body.

Thermal energy is related to random motion of molecules while internal energy is related to motion as well as their configuration or arrangement.

$E = mc^2$ is mass energy relationship.

Quantization of energy Planck has shown that the radiations emitted by a black body are quantized. Quantum nature of energy is confirmed in atomic and subatomic world. Even light energy is quantized.

• Short Cuts and Points to Note

1. Work done $W = \vec{F}.\vec{s} = Fs\cos\theta$ if force F is constant

   $W = \int F.ds$ if force is variable

   $W = \Delta KE$ (work energy theorem)

   $W = \Delta PE$ (for conservative forces)

   $W = \frac{1}{2}kx^2$ in a spring

   $W = \frac{1}{2}$ stress × strain × volume (in elastic bodies)

   $W = \frac{F.x}{2}$ where x is extension produced in a spring or elastic bodies.

   $W = \int P\,dV$ where P is pressure and V is volume.

   $W = \int P\,.\,dt$ where P represents power

2. Power $P = \frac{dW}{dt} = \frac{dE}{dt}$

   $P = \vec{F}.\vec{v}$ if F is constant

   $P = \int \vec{F}.d\vec{v}$ if F is variable or v is variable.

3. Potential energy exists only for conservative forces. Nonconservative forces do not show *PE*. If a particle moves in a circle then binding energy = *KE* + *PE* = $\frac{1}{2}PE = -KE$. In a bound system like this *PE* is negative.

4. In conservative forces work done is independent of path followed. It depends only on the initial and final position. Total work done in a round trip is zero.

5. $PE = \frac{1}{2}kx^2$ (in a spring and is only positive)

   $PE = \frac{-GM_1M_2}{r}$ (in gravitational fields). It may be positive or negative

   $PE = mgh$ if h is small

   $PE = \frac{q_1q_2}{4\pi \in_0 r}$ (in electric fields). It may be positive or negative.

6. If a body is in static or dynamic equilibrium then $W = 0$.

7. If a force is always perpendicular to velocity then work done by this force is zero.

8. Mechanical energy = *KE* + *PE* is conserved if internal forces are conservative and do no work.

9. *KE* + *PE* is not conserved if nonconservative forces are present.

10. $KE = \frac{p^2}{2m}$ where p is momentum of the body.

11. If a lighter and a heavier body have equal *KE* then heavier body has more momentum.

12. If a lighter and heavier body have equal momentum then lighter body has more KE.
13. Area under Power time graph gives work.
14. ΔU = change in $PE = \int F \cdot dr$ for conservative forces.
15. If $\frac{dU}{dr} = 0$, body is said to be in equilibrium. Equilibrium is stable if U is minimum; unstable if U is maximum and neutral if U = constant.
16. If $\frac{l}{n}$ th part of the chain hangs then the work done to pull up the hanging chain is $\frac{mgl}{2n^2}$. [See Fig. 6.2]

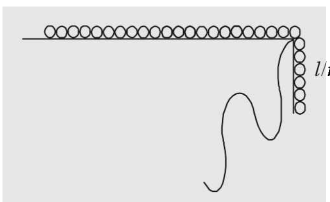


Fig. 6.2

17. If maximum displacement in a spring is to be found use $W = \frac{1}{2} kx^2$ if steady state displacement in a spring is to be found use $F = -kx$.
18. For a Rolling body $KE = \frac{1}{2} mv^2 + \frac{1}{2} I\omega^2$.
19. Equation of dynamics of a body with variable mass

$$F = \frac{mdv}{dt} + \frac{dm}{dt} v \text{ if reference frame is at rest}$$

$$F = \frac{mdv}{dt} + \frac{vdm}{dt} - \frac{udm}{dt} \text{ if reference frame is moving}$$

with a velocity u.

20. Change in total energy $\Delta E = W_{ext} + W_{int}$ (nonconservative).

- **Caution**

1. Applying $W = F \cdot s$ even if force is variable.

⇒ When force is variable use $W = \int F \cdot ds$ or $W = \Delta KE$ or $W = PE$ as it suits.

2. To find work done even when the force is perpendicular to velocity.

⇒ No work will be done when force is perpendicular to velocity. For example, no work is done by centripetal force in a circular motion. No work is done by the magnetic force $\vec{F} = q\ (\vec{v} \times \vec{B})$ as $\vec{F}$ and $\vec{v}$ are perpendicular.

3. Assuming when a body strikes another body connected to a spring, as shown in Fig. 6.3, it imparts its complete KE to the spring.

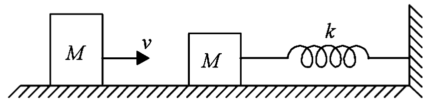


Fig. 6.3

⇒ If a body would have not been connected to the spring only then complete KE of the body would have been converted to PE stored in the spring. But if a body is connected then first conserve momentum. The KE of system after collision will be equal to PE stored in the spring.

4. To find maximum displacement in a spring applying the force equation i.e. $F = -kx$.

⇒ $F = -kx$ will provide steady state displacement. Maximum displacement is obtained when we equate energy i.e.

$$W = \frac{1}{2} kx^2.$$

5. Considering work done in a spring is always $\frac{1}{2} kx^2$.

⇒ If the force moves the block slowly and steadily then $W = \frac{1}{2} kx^2$ but if the movement of the block is very fast (or a force applied for a very short interval called sudden force/impulse) then work done by force F is $F \cdot x$

6. Conserve energy even when nonconservative force/s are present.

⇒ If nonconservative forces perform work then energy is not conserved.

7. Assuming even a rolling body has $KE = \frac{1}{2} mv^2$.

⇒ Rolling body possesses both linear KE and rotational KE. Total KE is sum of the two

$$KE_{Tot} = \frac{1}{2} mv^2 + \frac{1}{2} I\omega^2.$$

8. Assuming gravitational PE is only mgh.

⇒ The gravitational PE is mgh when distance from earth is not very large. If the distance is large employ

$$PE = -\frac{GMm}{R+h}.$$

If the attraction between two bodies is involved then $PE = \frac{-Gm_1m_2}{r}$.

9. In a system of mutual forces considering only KE due to one particle is equal to ΔPE.

⇒ Consider KE due to both the particles.

10. When a vehicle is moving up an incline and the efficiency of an engine is given then applying efficiency in a wrong manner. For example, a truck of mass 20 ton is moving up an incline of 1 : 10 with velocity 10 ms^{-1}. The friction is 500 N per ton. Efficiency is 80%. Find power. Then using

$P_{net} = 0.8\,(mg\sin\theta + F_f)\,v$.

$\Rightarrow$ Apply $P_{eff} = (mg\sin\theta + F_f)\,v$

and $P_{engine} \times (0.8) = P_{eff}$

or $P_{engine} = \dfrac{P_{eff}}{0.8} = \dfrac{(mg\sin\theta + F_f)v}{0.8}$.

SOLVED PROBLEMS

1. A spherical ball of mass 20 kg is stationary at the top of a hill of height 100 m. It rolls down a smooth surface to the ground and then climbs up another hill of height 30 m and finally rolls down to a horizontal base at a height of 20 m above the ground. The velocity attained by the ball is

[AIEEE 2005]

(a) 40 ms^{-1} (b) 20 ms^{-1}
(c) 10 ms^{-1} (d) $10\sqrt{30}$ ms^{-1}

Solution (a) $mgh = \frac{1}{2}mv^2$ or $v = \sqrt{2gh} = \sqrt{2\times10\times80}$ $= 40$ ms^{-1}.

2. The block of mass M moving on the frictionless horizontal surface collides with a spring of spring constant K and compresses it by L. The maximum momentum of the block after collision is

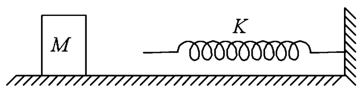


Fig. 6.4

(a) $\sqrt{MK}\,L$ (b) $\dfrac{KL^2}{2M}$
(c) Zero (d) $\dfrac{ML^2}{K}$

[AIEEE 2005]

Solution (a) $\frac{1}{2}KL^2 = \dfrac{p^2}{2M}$ or $p = \sqrt{MK}\,L$

3. If S is stress and Y is Young's modulus of material of a wire, the energy stored in the wire per unit volume is

(a) $2S^2Y$ (b) $\dfrac{S^2}{2Y}$
(c) $2Y/S^2$ (d) $S/2Y$

[AIEEE 2005]

Solution (b) $U = \frac{1}{2}$ stress $\times$ strain $= \frac{1}{2}\times S\times\dfrac{S}{Y} = \dfrac{S^2}{2Y}$.

4. A body of mass m is accelerated uniformly from rest to a speed v in a time T. The instantaneous power delivered to the body as a function of time is

(a) $\dfrac{mv^2}{T^2}t$ (b) $\dfrac{mv^2}{T^2}t^2$
(c) $\dfrac{mv^2}{2T^2}t$ (d) $\dfrac{mv^2}{2T^2}t^2$

[AIEEE 2005]

Solution (a) $P = ma\,v = ma^2t = m\left(\dfrac{v}{T}\right)^2 t$

5. A car is moving on a straight road with a speed 100 ms^{-1}. The distance at which car can be stopped is ..... $[\mu_k = 0.5]$

(a) 800 m (b) 1000 m
(c) 100 m (d) 400

[AIEEE 2005]

Solution (b) $D = \dfrac{u^2}{2g\mu_k} = \dfrac{100\times100}{2\times10\times0.5} = 1000$

6. A particle of mass 10 g is kept on the surface of a uniform sphere of mass 100 kg and radius 10 cm. Find the work to be done against the gravitational force between them to take the particle far away from them

(a) 13.34×10^{-10} J (b) 3.33×10^{-10} J
(c) 6.67×10^{-11} J (d) 6.67×10^{-10} J

[AIEEE 2005]

Solution (d) $W = \Delta U = \dfrac{GMm}{r}$

$= \dfrac{6.67\times10^{-11}\times100\times10^{-2}}{10^{-1}}$

$= 6.67\times10^{-10}$ J.

7. A force F acting on an object varies with distance x as shown in fig 6.5. The work done by the force in moving the object from $x = 0$ to $x = 6$ m is

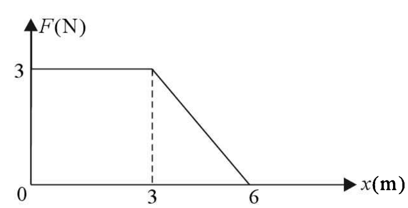


Fig. 6.5

(a) 18 J (b) 13.5 J
(c) 9 J (d) 4.5 J

[CBSE PMT 2005]

Solution (b) W = Area under $F - x$ graph.

8. A bomb of mass 30 kg at rest explodes into two pieces of masses 18 kg and 12 kg. The velocity of 18 kg mass is 6 ms^{-1}. The *KE* of other mass is

(a) 324 J (b) 486 J
(c) 256 J (d) 524 J

[CBSE PMT 2005]

Solution (b) $m_1 v_1 = m_2 v_2 \therefore v_2 = \frac{18 \times 6}{12} = 9\text{ ms}^{-1}$

$KE = \frac{1}{2} \times 12\,(9)^2 = 486\text{ J}$

9. A block of mass 10 kg is moving in x-direction with a constant speed 10 ms^{-1}. It is subjected to a retarding force $F = -0.1\,x$ J/m during its travel from $x = 20$ m to $x = 30$ m. Find the final *KE*.

(a) 475 J (b) 450 J
(c) 275 J (d) 250 J

[AIIMS 2005]

Solution (a) $KE_f = \int_{20}^{30} F\,.\,dx + KE_{\text{initial}}$

$= \int_{20}^{30} -(.1)x\,dx + \frac{1}{2} \times 10 \times 10^2$

$= \left[-\frac{30^2}{2} + \frac{20^2}{2}\right](.1) + 500. = 475\text{ J}$

10. Energy required to break a bond of DNA is approximately

(a) ~ 1 eV (b) ~ 0.1 eV
(c) ~ 0.01 eV (d) ~ 2.1 eV

[AIIMS 2005]

Solution (a)

11. A particle is moving with centripetal force $\frac{k}{r^2}$. Find the total energy associated.

[CBSE PMT Mains 2005]

Solution $\frac{mv^2}{r} = \frac{k}{r^2}$ or $mv^2 = \frac{k}{r}$

Total energy = $KE + PE = -KE = -\frac{k}{2r}$

12. A spring does not obey Hooke's law. Rather it follows, $F = kx - bx^2 + cx^3$. If the spring has natural length l and compressed length l' then find the work done.

Solution $W = \int_0^{l-l'} F\,.\,dx = \frac{k}{2}(l-l')^2 - \frac{b}{3}(l-l')^3 + \frac{c}{4}(l-l')^4$

13. A space shuttle of mass 86400 kg is revolving in a circular orbit of radius 6.66×10^6 m around the earth. It takes 90.1 minutes for the shuttle to complete one revolution. On a repair mission it moves 1 m closer to a disabled satellite every 3.0 s. Find the *KE* of shuttle relative to the satellite.

(a) 4800 J (b) 480 J
(c) 2.59×10^{12} J (d) 2.69×10^{11} J

Solution (a) $\frac{1}{2} m v^2 = \frac{1}{2} \times 86400 \times \left(\frac{1}{3}\right)^2 = 4800\text{ J}$

14. A particle of mass 6 kg moves according to the law $x = 0.2\,t^2 + 0.02\,t^3$. Find the work done by the force in first 4 s.

(a) 1.1231 J (b) 2.6428 J
(c) 2.1324 J (d) 1.6428 J

Solution (d) $W = \frac{1}{2} m \left(v_f^2 - v_i^2\right)$

$v = \frac{dx}{dt} = 0.4\,t - .06\,t^2$

$= \frac{1}{2} \times 6\left[(.74)^2 - 0\right] = 1.6428\text{ J}$

15. A moving electron has *KE* '*K*'. When a certain amount of work is done, it moves with one quarter of its velocity in opposite direction. Find the work in terms of *K*.

(a) $\frac{-15}{16}K$ (b) $\frac{-17}{16}K$
(c) $\frac{-5}{4}K$ (d) $\frac{-3}{4}K$

Solution (b) $\frac{-K}{(4)^2} = K + W$ or $W = \frac{-17}{16}K$

16. A brick of mass 1.8 kg is kept on a spring of spring constant K = 490 N m^{-1}. The spring is compressed so that after the release brick rises to 3.6 m. Find the compression in the spring.

(a) 0.21 m (b) 0.322 m
(c) 0.414 m (d) 0.514 m

Solution (d) $\frac{1}{2} kx^2 = mgh$ or

$x = \sqrt{\frac{2 \times 1.8 \times 10 \times 3.6}{490}} = \frac{3.6}{7} = 0.514\text{ m}$

17. 75 kW engine is generating full power. It is able to provide a 700 kg airplane a speed 2.5 ms^{-1}. Find the fraction of engine power used.

(a) $\frac{1}{100}$ (b) $\frac{3}{100}$
(c) $\frac{5}{200}$ (d) $\frac{7}{300}$

Solution (d) $\frac{P_{\text{used}}}{P_{\text{supplied}}} = \frac{F.V}{P_{\text{supplied}}} = \frac{700 \times 10 \times 2.5}{750 \times 10^3} = \frac{7}{300}$

18. In an ice rink a skator is moving at 3 ms^{-1} and encounters a rough patch that reduces her speed by 45% due to a friction force that is 25% of her weight. Find the length of the rough patch

(a) 1.56 m (b) 1.46 m
(c) 1.36 m (d) 1.26 m

Solution (d) $l = \dfrac{v_i^2 - v_f^2}{2\mu g} = \dfrac{3^2(1-(.55)^2)}{2\times 2.5}$

$= 1.8\,[.7] = 1.26$ m

19. A pump having efficiency 75% lifts 800 kg water per minute from a 14 m deep well and throws at a speed of 18 ms^{-1}. Find the power of the pump.

(a) 2060 W (b) 2490 W
(c) 3218 W (d) 1400 W

Solution (b) $P_{\text{eff}} = \dfrac{dm}{dt}\,gh = \dfrac{800}{60} \times 10 \times 14$

$= \dfrac{5600}{3}$ W,

$P_{\text{app}} = P_{\text{eff}} \times \dfrac{4}{3} = \dfrac{22400}{9} = 2488.88$ W

20. The heart takes and discharges 7500 l of blood in a day. Density of blood $= 1.05 \times 10^3$ kg m^{-3}. If on an average it takes to a height of 1.6 m. Find the power of the heart pump.

(a) 1.63 W (b) 1.36 W
(c) 1.96 W (d) 2.46 W

Solution (a) $P = \dfrac{dm}{dt}\,g\,h = \rho\,\dfrac{dV}{dt}\,gh$

$P = \dfrac{1.05\times10^3\times7500\times10^{-3}}{24\times60\times60} \times 10 \times 1.6$

$= 1.63$ W

21. In the system shown, find the speed with which 12 kg block weight hit the ground

(a) $2\sqrt{10}$ ms^{-1} (b) $4\sqrt{2}$ ms^{-1}
(c) $2\sqrt{5}$ ms^{-1} (d) 3 ms^{-1}

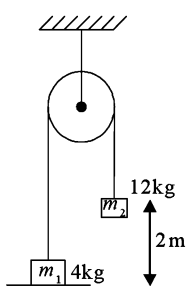


Fig. 6.6

Solution (c) $m_2gh - m_1gh = \dfrac{1}{2}\,m_1\,v_1^2 + \dfrac{1}{2}\,m_2\,v_2^2$

or $8 \times 10 \times 2 = \dfrac{1}{2} \times 16\,v^2$ or $v = 2\sqrt{5}$ ms^{-1}

22. A skier starts at the top of a snowball with negligible speed and skis straight down the side. At what point does he lose contact with the snowball.

(a) $\theta = \sin^{-1}\dfrac{2}{3}$ (b) $\theta = \cos^{-1}\dfrac{2}{3}$
(c) $\theta = \tan^{-1}\dfrac{2}{3}$ (d) none

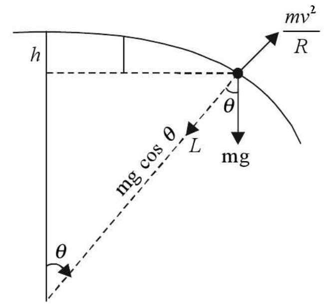


Fig. 6.7

Solution (b) $\dfrac{mv^2}{2} = mgh = mgR\,(1 - \cos\theta)$

or $\dfrac{mv^2}{R} = 2\,mg\,(1 - \cos\theta)$.

Condition of losing contact $\dfrac{mv^2}{R} = mg\cos\theta$

or $2\,mg\,(1-\cos\theta) = mg\cos\theta$ or $\theta = \cos^{-1}\dfrac{2}{3}$.

23. The force $F = C\,y^2\,\hat{j}$ with C as negative constant is

(a) conservative (b) restoring
(c) nonconservative (d) none

Solution (a) $W_{\text{tot}} = W_{AB} + W_{BC} + W_{CA} = 0 + \int_0^a -Cy^2 dy + \int_a^0 -Cy^2 dy = 0$

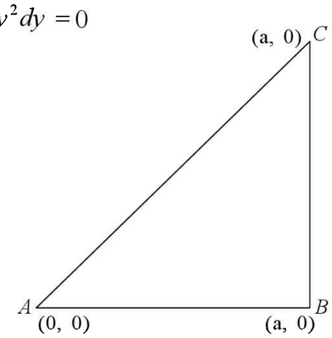


Fig. 6.8

Since the work done in a round trip is zero.

24. A particle has *PE vs x* curve as shown in Fig. 6.9. The unstable equilibrium occurs at

(a) A (b) B
(c) C (d) D

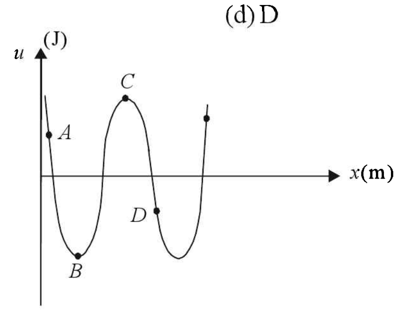


Fig. 6.9

Solution (c) $\Sigma F = 0$ and PE = maximum for unstable equilibrium.

TYPICAL PROBLEMS

25. A particle is released from the top of a quarter circle of radius 1.6 m. It stops at C, 3 m away from B. Find coefficient of friction which is present only on the horizontal surface.

(a) 0.533 (b) 0.333
(c) 0.433 (d) none

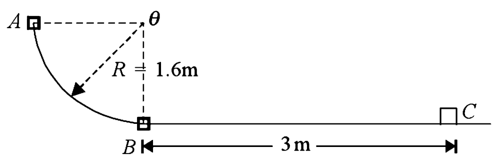


Fig. 6.10

Solution (a) $\mu = \frac{2gh}{2gl} = \frac{1.6}{3} = 0.533$

26. A 500 g ball is released from a height of 4m. Each time it makes contact with the ground it loses 25% of its energy. Find the KE it posses after 3rd hit.

(a) 15 J (b) 11.25 J
(c) 8.44 J (d) none

Solution (c) $KE = mgh\left(\frac{3}{4}\right)^3 = \frac{1}{2} \times 10 \times 4\left(\frac{3}{4}\times\frac{3}{4}\times\frac{3}{4}\right)$

$= \frac{270}{32}$ J.

27. The following data is obtained from a computer simulation for a patted baseball with mass 0.145 kg including air resistance. Find the work done by the air on the base ball as it moved from maximum height to back to its position

t (s)	x (m)	y (m)	v_x (ms^{-1})	v_y (ms^{-1})
0	0	0	30	40
3.05	70.2	53.6	18.6	0
6.59	124.4	0	12.0	–30

(a) 0 (b) 106.56 J
(c) 76 J (d) 213 J

Solution (b) $W = \frac{1}{2}\, m\left(v_{x_1}^2 + v_{y_1}^2 - v_{x_2}^2 - v_{y_2}^2\right)$

$= \frac{1}{2} \times 1.45\,(30^2 + 40^2 - 12^2 - 30^2) = 106.56$ J

28. A locomotive of mass m starts with a velocity $v = a\sqrt{x}$. Find the work done by all the forces acting on locomotive in first t sec.

(a) $\frac{ma^2t^2}{4}$ (b) $\frac{ma^4t^2}{4}$

(c) $\frac{ma^4t^2}{8}$ (d) $\frac{ma^4t^2}{2}$

Solution (c) $\frac{dv}{dt} = \frac{dv}{dx} \cdot \frac{dx}{dt} = \frac{a}{2\sqrt{x}}\,(a\sqrt{x})\ h = \frac{a^2}{2}$

$\therefore \quad F = \frac{ma^2}{2}$

$S = ut + \frac{1}{2}ft^2 = 0 + \frac{a^2}{4}t^2$

$W = F \,.\, s = \frac{ma^4t^2}{8}$.

29. The KE of a particle moving along a circle of radius R depends upon the distance covered x as $T = ax^2$. Find the force on the particle.

Solution $\frac{mv^2}{2} = ax^2$ or $v^2 = \frac{2ax^2}{m}$ ... (i)

differentiating (i) $2v\frac{dv}{dt} = \frac{4axv}{m}$

or acceleration $a_t = \frac{2ax}{m}$

Hence net force $= m\sqrt{a_r^2 + a_t^2}$

$= m\sqrt{\left(\frac{2ax^2}{mR}\right)^2 + \left(\frac{2ax}{m}\right)^2}$

$= 2ax\sqrt{1 + \left(x/R\right)^2}$

30. Two blocks of mass m_1 and m_2 are connected by a non deformed light spring resting on a horizontal table. The coefficient of friction between the blocks and table is μ. Find the minimum force applied on block 1 which will move the block 2 also. See Fig. 6.11.

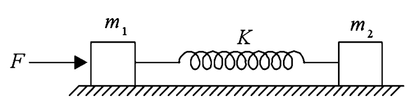


Fig. 6.11

Solution If x is the compression in the spring when block m_2 is just to move then

$Kx = \mu m_2 g$ ... (1) for min. force; $W = 0$

$F \,.\, x - \frac{1}{2}Kx^2 - \mu m_1 g x = 0$

or $\frac{Kx}{2} = F - \mu m_1 g$ ... (2)

From (1) and (2) $F = \mu g\left(m_1 + \frac{m_2}{2}\right)$.

31. A chain of mass m and length l rests on a rough table with part overhanging. The chain starts sliding down by itself if overhanging part is $\frac{l}{3}$. What will be the work performed by the friction forces acting on the chain by the moment it slides completely off the table.

(a) $\frac{Mgl}{3}$ (b) $\frac{2Mgl}{3}$

(c) $\frac{2Mgl}{9}$ (d) $\frac{Mgl}{9}$

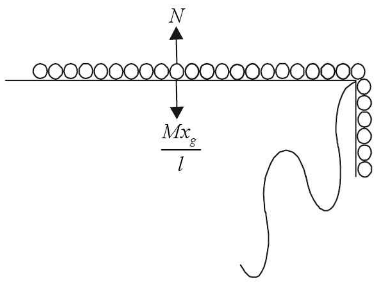


Fig. 6.12

Solution (d) $\mu \frac{2}{3} mg = \frac{m}{3} g$ or $\mu = \frac{1}{2}$

Assume at any instant the length of the chain on the table is x then force of friction $= \mu N = \frac{\mu M}{l} xg$

Work done against friction $\int_0^{2l/3} \mu \frac{M}{l} xg \cdot dx$

$$= \mu \frac{Mg}{l} \frac{x^2}{2} \Big|_0^{2l/3} = \frac{1}{2} \frac{M}{l} \frac{g}{2} \left(\frac{4l^2}{9}\right) = \frac{Mgl}{9}$$

32. In the system of two masses m_1 and m_2 tied through a light string passing over a smooth light pulley. Find the acceleration of COM. (Centre of mass).

(a) $\frac{(m_1 - m_2)}{m_1 + m_2} g$ (b) $\frac{(m_1 - m_2)^2}{m_1 + m_2} g$

(c) $\left(\frac{m_1 - m_2}{m_1 + m_2}\right)^2 \frac{g}{2}$ (d) $\left(\frac{m_1 - m_2}{m_1 + m_2}\right) \frac{g}{2}$

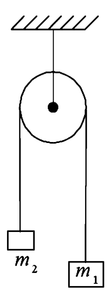


Fig. 6.13

Solution (b) $a_{COM} = \frac{m_1 a_1 + m_2 a_2}{m_1 + m_2}$ Note that $a_2 = -a_1$,

$\therefore \quad a_{COM} = \frac{(m_1 - m_2) a_2}{m_1 + m_2}$ and $a_1 = \frac{(m_1 - m_2) g}{m_1 + m_2}$

$$= \frac{(m_1 - m_2)^2 g}{(m_1 + m_2)^2}$$

33. Two blocks of masses m_1 and m_2 joined to a non deformed spring of length l_0 and stiffness K as shown in Fig. 6.14. If a force F is applied on block of mass m_2. Find the maximum separation between the blocks.

(a) $l_0 + \frac{m_1 F}{k(m_1 + m_2)}$ (b) $l_0 + \frac{m_2 F}{k(m_1 + m_2)}$

(c) $l_0 + \frac{2m_2 F}{m_1 + m_2}$ (d) $l_0 + \frac{2m_1 F}{m_1 + m_2}$

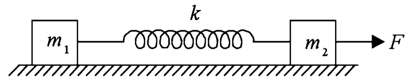


Fig. 6.14

Solution (d) x_1 and x_2 be the maximum displacement in m_1 and m_2 respectively.

$$a_{COM} = \frac{F}{m_1 + m_2}$$

$$\frac{1}{2} k (x_1 + x_2)^2 = \frac{m_1 F}{m_1 + m_2} x_1 + \left(F - \frac{m_2 F}{m_1 + m_2}\right) x^2$$

$$= \frac{m_1 F}{m_1 + m_2} (x_1 + x_2)$$

or $\quad x_1 + x_2 = \frac{2m_1 F}{k(m_1 + m_2)}$

Thus maximum separation $= l_0 + x_1 + x_2$

$$= l_0 + \frac{2m_1 F}{k(m_1 + m_2)}.$$

34. A particle of mass m is moving in a circular path of constant radius r such that radial acceleration $a_r = k^2 t^2 r$. Find the power delivered to the particle by the forces acting on it.

(a) $2\pi m k^2 r^2 t$ (b) $m k^2 r^2 t$

(c) $\frac{1}{3} m k^4 r^2 t^3$ (d) 0

Solution (b) $\frac{v^2}{r} = k^2 t^2 r$ or

$v = k t r \quad F = \frac{m dv}{dt} = m k r$ and

Power $P = \vec{F} \cdot \vec{v}$

$P = m k r (t k r) = m k^2 r^2 t$

35. In Fig. 6.15 pulley is light and smooth. Thread is massless. On applying force F, KE increases by 20 J in 1 s.

(a) Tension in the string is Mg.
(b) The tension in the string is F.
(c) Work done by the tension in 1 s is 20 J.
(d) The work done by the force of gravity is 20 J in 1 s.

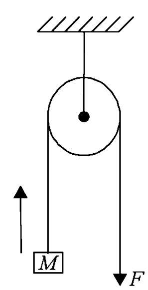


Fig. 6.15

Solution (b)

36. In a factory, 2000 kg metal is lifted by 12 m in 1 minute by a crane. The minimum horse power of the engine is

(a) 320 bhp (b) 32 bhp
(c) 5.3 bhp (d) 6.4 bhp

Solution (c) $P = \frac{dmgh}{dt} = \frac{2000}{60} \times 9.8 \times 12$

$$P\text{ (bhp)} = \frac{400 \times 9.8}{746}$$

$$= 5.3 \text{ bhp}$$

QUESTIONS FOR PRACTICE

1. A motorcycle of mass m resting on a frictionless road moves under the influence of a constant force F. The work done by this force in moving the motorcycle is given by $F^2t^2/2m$, where t is the time in seconds.

Ratio of instantaneous power to average power of the motorcycle in $t = T$ second is

(a) 1 : 1 (b) 2 : 1
(c) 3 : 2 (d) 1 : 2

2. Two ends of a uniform garland of mass m and length L are hanging vertically as shown. At instant t the end Z is released. If y is the distance moved by this end in time dt, change in momentum of an element of length dy is given as

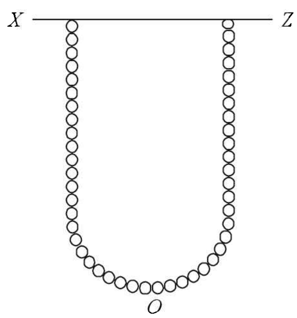


Fig. 6.16

(a) $\frac{m}{L}\, dy\, (2gy)^{\frac{1}{2}}$ (down ward)

(b) $\frac{m}{L}\, dy\, (2gy)^{\frac{1}{2}}$ (upward)

(c) $\frac{-m}{L}\, dy$ dy $\sqrt{gy}$ (upward)

(d) $\frac{m}{L}\, dy\, (gy)^{\frac{1}{2}}$ (upward)

3. A compressed spring of spring constant k releases a ball of mass m. If the height of spring is h and the spring is compressed though a distance x, the horizontal distance covered by ball to reach ground is

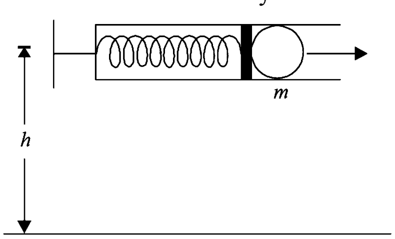


Fig. 6.17

(a) $x\sqrt{\frac{kh}{mg}}$ (b) $\frac{xkh}{mg}$

(c) $x\sqrt{\frac{2kh}{mg}}$ (d) $\frac{mg}{x\sqrt{kh}}$

4. A mass m starting from A reaches B of a frictionless track. On reaching A it pushes the track with a force equal to x times its weight, then applicable relation is

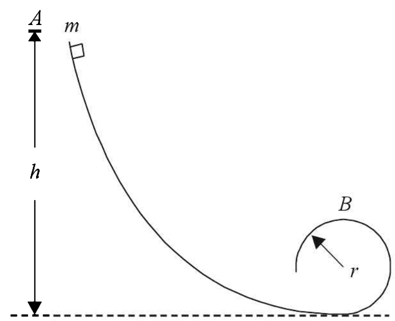


Fig. 6.18

(a) $h = \frac{(x+5)}{2} r$ (b) $h = \frac{x}{2} r$

(c) $h = r$ (d) $h = \left(\frac{x+1}{2}\right) r$

5. Coefficient of friction between a tool and grinding wheel is μ. Power developed in watt by the wheel of radius r running at n revolutions per second when tool is pressed to the wheel with F' *kgf* is

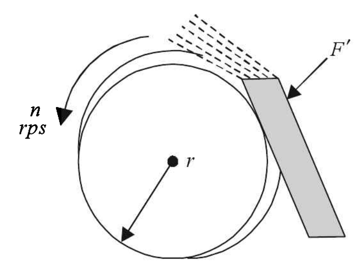


Fig. 6.19

(a) $\mu F' r (2\pi n)$ (b) $\mu F' gr (2\pi n)$
(c) $\mu F' r$ (d) $\mu F' g$

6. A minute particle resting at a frictionless surface is acted upon by a constant horizontal force. Neglecting frictional force the graph between work done on the particle w and speed of particle u is represented by

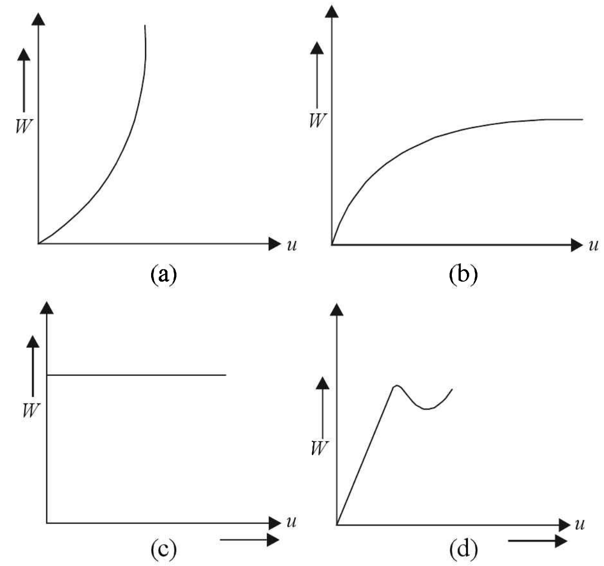


Fig. 6.20

7. A particle of mass m slides along curved – flat – curved track. The curved portions of the track are smooth. If the particle is released at the top of one of the curved portions the particle comes to rest at flat portion of length l and of coefficient of kinetic friction = μ_{kinetic} after covering length of

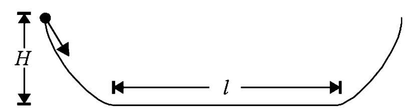


Fig. 6.21

(a) $l/3\mu_{\text{kinetic}}$ (b) $H/2\mu_{\text{kinetic}}$

(c) $l/6$ (d) $\dfrac{H}{\mu_{\text{kinetic}}}$

8. A light rigid rod of length L has a bob of mass M attached to one of its end just like a simple pendulum. Speed at the lowest point when it is inverted and released is

(a) $\sqrt{gL}$ (b) $\sqrt{2gL}$

(c) $2\sqrt{gL}$ (d) $\sqrt{5gL}$

9. Astronauts Mr. X and Mr. Y float in gravity zero space with no relative velocity to one another. Mr. Y throws a mass of 5 kg towards X with speed 2 ms^{-1}. If Mr. X catches it the changes in velocity of X and Y are

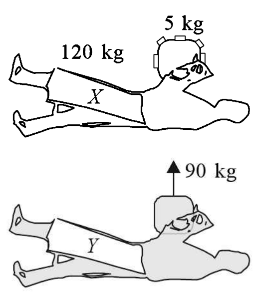


Fig. 6.22

(a) 0.21 ms^{-1}, 0.80 ms^{-1}
(b) 0.80 ms^{-1}, 0.21 ms^{-1}
(c) 0.12 ms^{-1}, 0.08 ms^{-1}
(d) 0.08 ms^{-1}, 0.12 ms^{-1}

10. A particle is dropped from a height h. A constant horizontal velocity is given to the particle. Taking g to be constant everywhere, kinetic energy E of the particle with reference to time t is correctly shown in

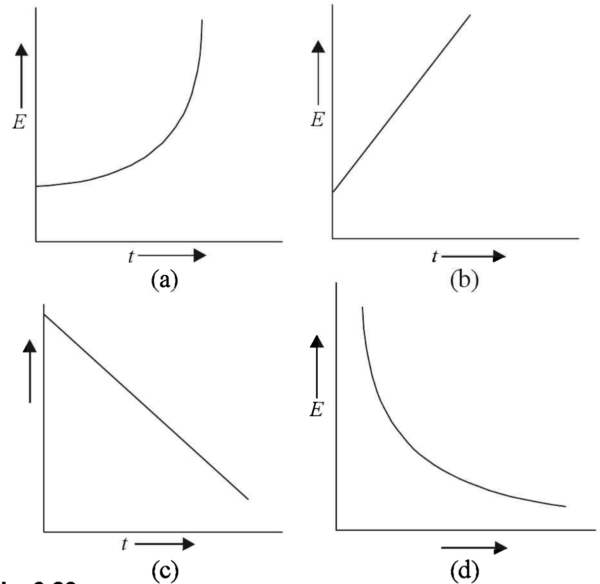


Fig. 6.23

11. An aeroplane is flying with a velocity U_p. The plane takes in volume V of air of mass μ per second to burn mass m of fuel every second. The exhaust relative to plane has a speed of U_E. bh.p. of engine of the aeroplane is

(a) $\dfrac{U_E^2(\mu+m)-U_P^2(\mu-m)}{776}$

(b) $\dfrac{U_E(\mu+m)-U_P(\mu-m)}{746}$

(c) $\dfrac{U_E U_P(\mu+m)-U_P^2\mu}{746}$

(d) $\dfrac{U_P^2(\mu+m)-U_P U_E(\mu-m)}{746}$

12. A vehicle of mass M is accelerated on a horizontal frictionless road under a force changing its velocity from u to v in distance S. A constant power P is given by the engine of the vehicle, then v =

(a) $\left(u^3+\dfrac{2PS}{M}\right)^{1/3}$ (b) $\left(\dfrac{PS}{M}+u^3\right)^{1/2}$

(c) $\left(\dfrac{PS}{M}-u^2\right)^{1/3}$ (d) $\left(\dfrac{3PS}{M}+u^3\right)^{1/3}$

13. Two frames, one stationary and the other moving, are initially coincident. Two observers in the two frames observe a body initially at rest in the coincident frame. A constant force F starts acting on the body along horizontal axis when moving frame starts separation from fixed frame. Work done 'W' as observed by stationary frame and W' as observed from moving frame are compared to each other as

(a) $W = W'$ (b) $W \neq W'$

(c) $W = \frac{1}{2}W'$ (d) $W = 2W'$

14. A proton (mass 1.67×10^{-27} kg) is accelerated along a straight line by 3.6×10^{15} ms^{-2}. If the length covered is 3.5 cm and initial speed of proton was 2.4×10^7 ms^{-1}, the gain in kinetic energy is

(a) 1.32 MeV (b) 1.23 MeV
(c) 2.13 MeV (d) 3.12 MeV

15. A mass m is thrown vertically upward into air with initial speed u. A constant force F due to air resistance acts on the mass during its travel. Taking into account the work done against air drag the maximum distance covered by the mass to reach the top is

(a) $\frac{u^2}{2g}$ (b) $\frac{u^2}{2g + (2F/m)}$

(c) $\frac{u^2}{2g + F/m}$ (d) $\frac{u^2}{g + F/m}$

16. A 20 g bullet passes through a plate of mass 1 kg and finally comes to rest inside another plate of mass 2980 g. It makes the plates move from rest to same velocity. The percentage loss in velocity of bullet between the plate is

(a) 0 (b) 50 %
(c) 75 % (d) 25 %

17. A particle of mass 1 kg has potential energy, P.E. $= 3x + 4y$. At $t = 0$, the particle is at rest at (6, 4). Work done by external force to displace the particle from rest to the point of crossing the x axis is

(a) 25 J (b) 20 J
(c) 15 J (d) 52 J

18. A body is lifted over route I and then route II such that force is always tangent to the path. Coefficient of friction is same for both the paths. Work done

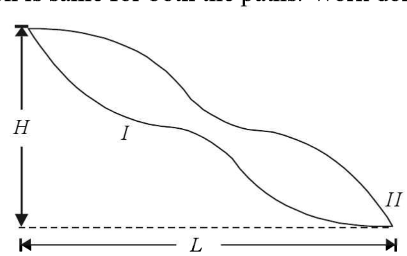


Fig. 6.24

(a) on both routes is same.
(b) on route I is more.
(c) on route II is more.
(d) on both routes is zero.

19. Interaction between atoms of a diatomic molecule is described by the relation P.E. $(x) = \frac{\alpha}{x^{12}} - \frac{\beta}{x^6}$, where P.E. is potential energy. If the two atoms enjoy stable equilibrium, the distance between them would be

(a) $\left(\frac{\alpha}{\beta}\right)^{1/6}$ (b) $\left(\frac{2\alpha}{\beta}\right)^{1/6}$

(c) $\left(\frac{2\beta}{\alpha}\right)^{1/6}$ (d) $\left(\frac{\beta}{\alpha}\right)^{1/6}$

20. A hammer of mass M falls from a height h repeatedly to drive a pile of mass m into the ground. The hammer makes the pile penetrate in the ground to a distance d in single blow. Opposition to penetration is given by

(a) $\frac{m^2gh}{M + md}$ (b) $\frac{m^2gh}{(M+m)d} + (M+m)g$

(c) $\frac{M^2gh}{M + md}$ (d) $\frac{m^2gh}{(m+M)d} - (M+m)g$

21. The height h from which a car of mass m has to fall to gain the kinetic energy equivalent to what it would have gained when moving with a horizontal velocity of $(u + v)$ is given by

(a) $\frac{v}{2g}$ (b) $\frac{v^2}{2g}$

(c) $\frac{(u+v)^2}{2g}$ (d) $\frac{(u+v)^2}{g}$

22. A gun requiring the support of shoulder shows intensive recoil when fired with

(a) the butt held tightly with the shoulder.
(b) the butt held loosely against the shoulder.
(c) the butt held loosely or tightly against the shoulder.
(d) a bullet of greater mass.

23. A mass m is allowed to fall on a pedestal fixed on the top of a vertical spring. If the height of the mass was H from the pedestal and the compression of the spring is d then the spring's force factor is given by

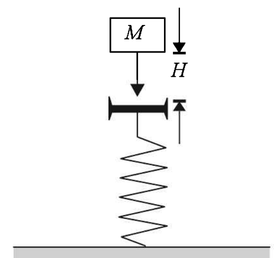


Fig. 6.25

(a) $Mg\frac{(H+d)}{d^2}$ (b) $2Mg\frac{(H-d)}{d^2}$

(c) $\frac{Mg}{2}\frac{H}{d^2}$ (d) $2Mg\frac{(H+d)}{d^2}$

24. How high can a man weighing m kg climb using the energy from a choclate producing 100 calories in him? Let his efficiency be η

(a) $\frac{420\eta}{mg}$ metre (b) $\frac{4.20\eta}{mg}$ metre

(c) $\frac{.\eta}{mg}$ metre (d) 42 η mg metre

25. A toy car moves up a ramp under the influence of force F plotted against displacement. The maximum height attained is given by

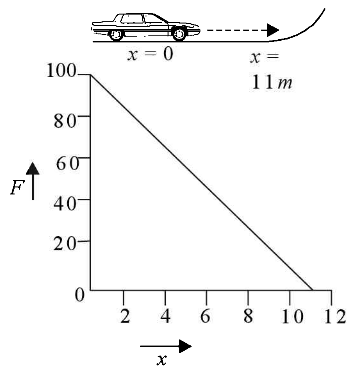


Fig. 6.26

(a) $y_{max} = 20$ m (b) $y_{max} = 15$ m
(c) $y_{max} = 10$ m (d) $y_{max} = 5$ m.

26. A block is placed on a plank which is placed on a horizontal plane. A massless but elastic string deviates by an angle θ with vertical when a force F is applied to the plank to shift it to the right making the block slide over it. If F_P and F_B are frictional forces between plank and plane and between block and plane respectively then work done by applied force is given by

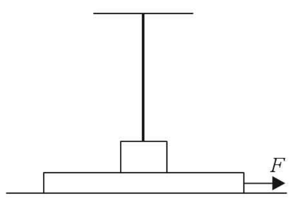


Fig. 6.27

(a) work done against friction acting on plank + energy in the elastic string - work done by friction acting on block.
(b) work done against frictions acting on plank and block + elastic energy.
(c) elastic energy of string.
(d) difference of work done against friction acting on plank and block.

27. A truck tows a jeep of mass 1200 kg at a constant speed of 10 ms^{-1} on a level road. The tension in the coupling is 1000 N. If they ascend an inclined plane of 1 in 6 with same velocity, the tension in coupling is

(a) 2000 N (b) 2960 N
(c) 1680 N (d) 1000 N

28. A screw jack of pitch 5 mm is used to lift the tyre of a vehicle of load 200 kg with the help of a handle of length 0.5 m. Neglecting the friction force between screw and nut of the jack, the least force required to raise the load is

(a) 1.2 N (b) 2.2 N
(c) 3.2 N (d) 4.2 N

29. A light rod of length L can revolve in a vertical circle around point O. The rod carries two equal masses of mass m each such that one mass is connected at the end of the rod and the second mass is fixed at the middle of the rod. u is the velocity imparted to the end P to deflect the rod to the horizontal position. Again mass m in the middle of the rod is removed and mass at end P is doubled. Now v is the velocity imparted to end P to deflect it to the horizontal position. Then v/u is

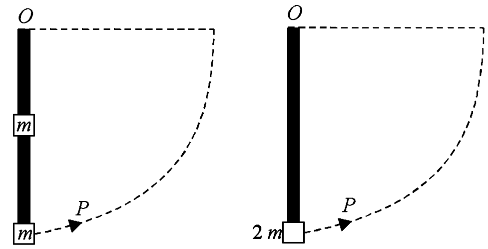


Fig. 6.28

(a) $(6/5)^{\frac{1}{2}}$ (b) 1
(c) 2.5 (d) $(5/6)^{\frac{1}{2}}$

30. A coconut of mass m falls from the tree through a vertical distance of s and could reach ground with a velocity of v ms^{-1} due to air resistance. Work done by air resistance is

(a) $-\frac{m}{2}(2gs - v^2)$ (b) $-\frac{1}{2}mv^2$
(c) mgs (d) $mv^2 + 2mgs$

31. A body of mass m is in vertical motion under the influence of gravity then

(a) work will be negative and kinetic energy increases during fall.
(b) kinetic energy decreases when body is projected up and work will be negative.
(c) kinetic energy increases when body is projected up and work will be positive.
(d) work will be positive and kinetic energy decreases during fall.

32. A smooth chain PQ of mass M rests against a $\frac{1}{4}$th circular and smooth surface of radius r. If released, its velocity to come over the horizontal part of the surface is

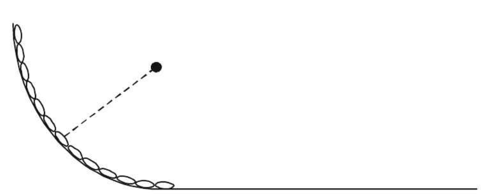


Fig. 6.29

(a) $\sqrt{2gr} \times \frac{1}{4}$ (b) $\sqrt{2gr\left(1-\frac{1}{\pi}\right)}$

(c) $\sqrt{2gr\left(1-\frac{2}{\pi}\right)}$ (d) $\sqrt{gr\left(1-\frac{2}{\pi}\right)}$

33. A block of mass m has initial velocity u having direction $+x$ axis. The block stops after covering distance S causing similar extension in the spring of spring constant K holding it. If μ is the kinetic friction between the block and the surface on which it was moving, the distances S is given by

(a) $\frac{1}{K}\ \mu^2 m^2 g^2$

(b) $\frac{1}{K}\ (mKu^2 - \mu^2 m^2 g^2)^{\frac{1}{2}}$

(c) $\frac{1}{K}\ (\mu^2 m^2 g^2 + mKu^2 + \mu mg)^{\frac{1}{2}}$

(d) $\frac{1}{K}\ (\mu^2 m^2 g^2 - mKu^2 + \mu mg)^{\frac{1}{2}}$

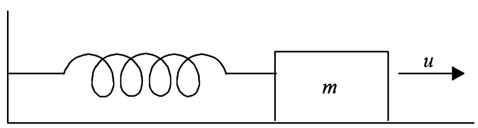


Fig. 6.30

34. A rigid cord and an elastic cord support two small spheres of same mass M. They are deflected from the mean position through an angle of 90°. When the spheres pass through the mean position the lengths of the two cords become same. If v_1 is the velocity of the sphere attached to rigid cord and v_2 is the velocity of the other sphere, then

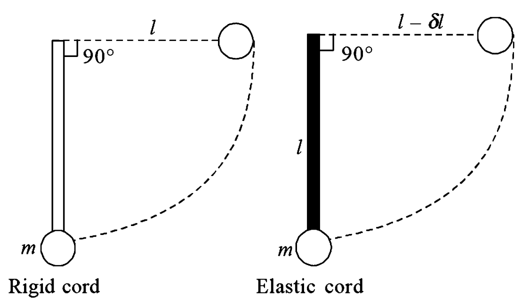


Fig. 6.31

(a) $v_1 = v_2$ (b) v_1 is more than v_2
(c) $v_1 < v_2$ (d) $v_1 = v_2 = 0$

35. A streamlined and almost symmetrical car of mass M has centre of gravity at a distance p from the rear wheel, q from the front wheel and h from the road. If the car has required power and friction, the maximum acceleration developed without tipping over towards back is

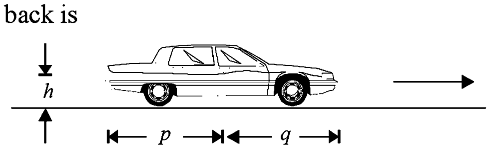


Fig. 6.32

(a) $\frac{qg}{h}$ (b) hgq

(c) $\frac{hMg}{q}$ (d) $\frac{pq}{hg}$

36. A pipe of uniform crossectional of area a has a right angled bend. Impure water of density ρ flows through the horizontal portion of the pipe. The distance covered by water per second at the bend to keep the pipe in equilibrium against an applied force F at the bend is

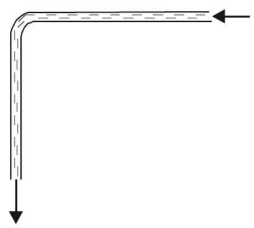


Fig. 6.33

(a) $\frac{\rho a F}{\sqrt{2}}$ (b) $\sqrt{2}\ \frac{\rho}{F}$

(c) $\left(\frac{\sqrt{2}F}{\rho a}\right)^{\frac{1}{2}}$ (d) $\left(\frac{F}{\sqrt{2}\rho a}\right)^{\frac{1}{2}}$

37. Spring of constant k fixed to its front wall. A body stretches this spring by distance x and in the mean time the compartment moves by a distance s. The work done by boy with reference to earth is

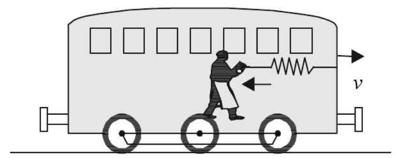


Fig. 6.34

(a) $\frac{1}{2}\ kx^2$ (b) $\frac{1}{2}\ (kx)\ (s+x)$

(c) $\frac{1}{2}\ kxs$ (d) $\frac{1}{2}\ kx\ (s+x+s)$

38. Six identical uniform rods PQ, QR, RS, ST, TU and UP each weighing W are freely joined at their ends to form a hexagon. The rod PQ is fixed in a horizontal position and middle points of PQ and ST are connected by a vertical string. The tension in string is

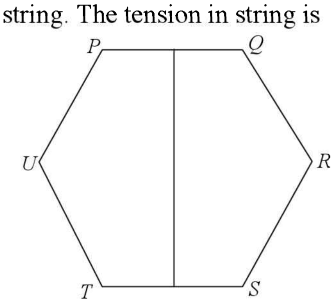


Fig. 6.35

(a) W (b) $3W$
(c) $2W$ (d) $4W$

39. N similar slabs of cubical shape of edge b are lying on ground. Density of material of slab is δ. Work done to arrange them one over the other is

(a) $(N^2 - 1)\, b^3 \rho g$ (b) $(N - 1)\, b^4 \rho g$

(c) $\frac{1}{2}\,(N^2 - N)\, b^4 \rho g$ (d) $(N^2 - N)\, b^4 \rho g$.

40. A floor wiper makes an angle $(90 - \theta)$ with horizontal floor. μ_s and μ_k are coefficients of static and kinetic frictions between the wiper and the floor. The wiper can not be used to wipe the floor for $\theta < \theta_0$ even if a large force directed along the handle aiming towards the centre is applied. Angle θ_0 is given by

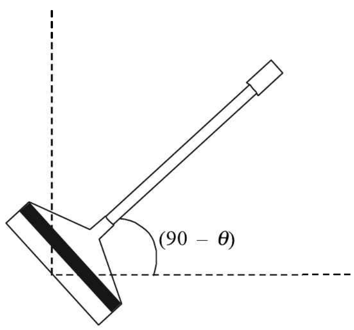


Fig. 6.36

(a) $\cos^{-1} \mu_k$ (b) $\tan^{-1} \mu_s$

(c) $\tan–1\, \mu_k$ (d) $\cot^{-1} \mu_s$.

41. For the equilibrium of the system of a toy shown the relation between force F_1 and F_2 when the members are arranged to form three identical rhombuses is

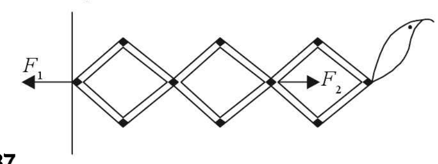


Fig. 6.37

(a) $F_1 = F_2$ (b) $F_2 = \frac{F_1}{3}$

(c) $F_2 = \frac{F_1}{2}$ (d) $F_1 = \frac{F_2}{3}$

42. An elastic cord of constant K and length L is hung from point A having a massless lock at the other end. A smooth ring of mass M falls from point A, the maximum elongation of cord is

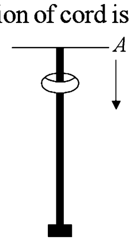


Fig. 6.38

(a) $\frac{Mg}{K}\left(1 + \frac{1 + 2KL}{Mg}\right)^{1/2}$

(b) $\frac{Mg}{K}\left(1 - \left(1 - \frac{2KL}{Mg}\right)^{1/2}\right)$

(c) $\frac{MgL}{K}$

(d) $\frac{Mg}{K}\left(1 + \left(1 + \frac{2KL}{Mg}\right)^{\frac{1}{2}}\right)$

43. In a two-step pulley arrangement meant for a load W, the ratio between force P and W in equilibrium position when radii of pulleys are r_1 and r_2

(a) $\frac{r_2}{r_1 - r_2}$ (b) $\frac{r_1 - r_2}{r_2}$

(c) $\frac{r_1}{r_2 - r_1}$ (d) $\frac{r_2 - r_1}{r_1}$

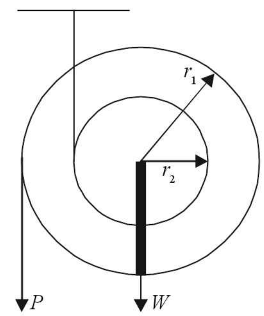


Fig. 6.39

44. A rope brake is fitted to a flywheel of diameter = 1m. The flywheel runs at 220 r. p. m. It is required to absorp 5.25 kW of brake power. Difference in the two pulls $(T - S)$ is

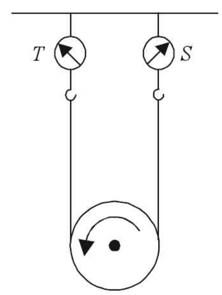


Fig. 6.40

(a) 456 N (b) 654 N

(c) 564 N (d) 465 N

45. A body of mass 2 kg is being dragged with a uniform velocity of 2 ms^{-1} on a horizontal plane. The coefficient of friction between the body and the surface is 0.2. Work done in 5s is

(a) 39.2 J (b) 9.32 J

(c) 23.9 J (d) 93.2 J

(Based on IIT 1980)

46. A small block of mass m is released from rest from point D and slides down DGF and reaches the point F with speed v_F. The coefficient of kinetic friction between block and both the surfaces DG and GF is μ, the velocity v_F is

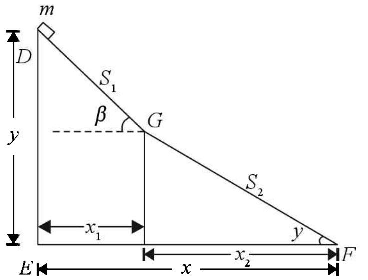


Fig. 6.41

(a) $\sqrt{2g(y-x)}$ (b) $\sqrt{2g(y-\mu x)}$

(c) $\sqrt{2gy}$ (d) $\sqrt{2g(y^2+x^2)}$

(Based on IIT 1980)

47. A body is moved along a straight line by a machine delivering constant power. The distance moved by the body in time t is proportional to

(a) $t^{1/2}$ (b) $t^{3/4}$

(c) $t^{3/2}$ (d) t^3

(IIT 1984)

48. A particle of mass 4 m which is at rest explodes into three fragments. Two of the fragments each of mass m are found to move with a speed v each in mutually perpendicular directions. The energy released in the process of explosion is

(a) $\frac{3}{2}mv^2$ (b) $3mv^2$

(c) $2mv^2$ (d) $\frac{1}{2}mv^2$

(Based on IIT 1987)

49. An engine of mass m is moving up a slope of inclination θ at a speed v. The coefficient of friction between engine and the rail is μ. If the engine has an efficiency η then the energy spent by engine in time t is

(a) $\eta\, mg(\sin\eta + \mu\cos\eta)\, vt$

(b) $\frac{mg(\mu\cos\theta)vt}{\eta}$

(c) $\frac{mg(\sin\theta+\mu\cos\theta)vt}{\eta}$

(d) $\frac{mg}{2}\left(\frac{\sin\theta}{\eta}\right)vt.$

(Based on Roorkee 1987)

50. A person decides to use his bath tub water to generate electric power to run a 40 W bulb. The bath tub is located at a height of h m from ground and it holds V litres of water. He installs a water driven wheel generator on ground. The rate at which water should drain from bath tub to light the bulb if efficiency of machine be 90% is

(a) $\frac{11.11}{\rho gh}$ (b) $44.44\,\rho gh$

(c) $\frac{44.44}{\rho gh}$ (d) $\frac{22.22}{\rho gh}$

(Based on Roorkee 1990)

51. Light from a discharge tube containing hydrogen atoms falls on the surface of a piece of sodium. The kinetic energy of the fastest photoelectrons emitted from sodium is 0.73 eV. The work function for sodium is 1.82 eV. The rocoil speed of emitting atom assuming it to be at rest before ionisation is (ionisation potential of hydrogen is 13.6 eV).

(a) $0.18\ \text{ms}^{-1}$ (b) $1.80\ \text{ms}^{-1}$

(c) $8.10\ \text{ms}^{-1}$ (d) $0.81\ \text{ms}^{-1}$

52. Two identical balls A and B of mass m kg are attached to two identical massless springs. The spring mass system is constrained to move inside a rigid pipe bent in the form of a circle as shown in figure. The pipe is fixed in a horizontal plane. The centres of the balls can move in a circle of radius r metre. Each spring has a natural length of $r\pi$ metre and spring constant K. Initially, both the balls are displaced by an angle θ radian with respect to diameter PQ of the circles and released from rest. The speed of ball A when A and B are at the two ends of dia PQ is

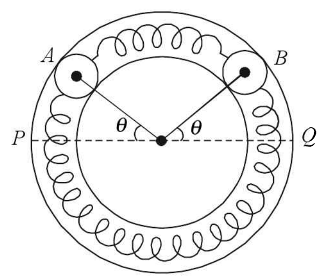


Fig. 6.42

(a) $R\theta\sqrt{\frac{m}{K}}$ (b) $2R\theta\sqrt{\frac{K}{m}}$

(c) $2R\theta\sqrt{\frac{m}{K}}$ (d) $2R\theta\sqrt{\frac{K}{m}}$

(Based on IIT 1993)

53. A particle of mass m is moving in a circular path of constant radius r such that is centripetal acceleration a_c is varying with time t as $a_c = k^2rt$, where k is a constant. The power delivered to the particle by force acting on it is

(a) $2\pi\, mk^2r^2$ (b) mk^2r^2t

(c) $\frac{mk^4r^2t^5}{3}$ (d) zero

(IIT 1994)

54. A cart is moving along x direction with a velocity of 4 ms^{-1}. A person on the cart throws a stone with a velocity of 6 ms^{-1} relative to himself. In the frame of reference of the cart the stone is thrown in $y - z$ plane making an angle of 30° with vertical z axis. At the highest point of its trajectory the stone hits an object of equal mass hung vertically from branch of a tree by means of a string of length L. The stone gets embedded in the object. The speed of combined mass immediately after the embedding with reference to an observer on the ground is

(a) 2.5 ms^{-1} (b) 1.5 ms^{-1}
(c) 5.2 ms^{-1} (d) 3.5 ms^{-1}.

(IIT 1997)

55. In Q. 55 the length l of string such that tension in string becomes zero when string becomes horizontal during subsequent motion of combined mass is

(a) 0.23 m (b) 0.32 m
(c) 0.13 m (d) 0.27 m

(IIT 1997)

56. A stone tied to a string of length L is whirled in a vertical circle with the other end of string at the centre. At a certain instant of time the stone is at its lowest position and has a speed u. The magnitude of change in velocity as it reaches a position where string is horizontal is

(a) $\sqrt{u^2 - 2gL}$ (b) $\sqrt{2gL}$
(c) $\sqrt{u^2 - gL}$ (d) $\sqrt{2(u^2 - gL)}$

(IIT 1998)

57. A force $F = -K(y\,\hat{i} + x\,\hat{j})$, (where K is a +ve constant) acts on a particle moving in xy plane starting from origin, the particle is taken along the positive x-axis to the point $(a, 0)$ and then parallel to y axis to the point $(a, 0)$. The total work done by force F on the particle is

(a) $-2Ka^2$ (b) $2Ka^2$
(c) $-Ka^2$ (d) Ka^2

(IIT 1998)

58. A particle free to move along x axis has potential energy given as $U_{(x)} = k(1 - \exp(-x^2))$ for $-a \le x \le +\infty$ where k is a positive constant of appropriate dimensions. Then

(a) at points away from the origin, the particle is in unstable equilibrium.
(b) for any finite non-zero value of x, there is a force directed away from the origin.
(c) if its total mechanical energy is $k/2$, it has its minimum kinetic energy at the origin.
(d) if its total mechanical energy is $k/2$, it has its maximum value at origin.

(Based on IIT 1999)

59. A particle which is constrained to move along the x-axis, is subjected to a force in the same direction which varies with the distance x of the particle from the origin as $F(x) = -kx + ax^3$. Here k and a are positive constants. For $x \ge 0$, the functional form of the potential energy U(x) of the particle is

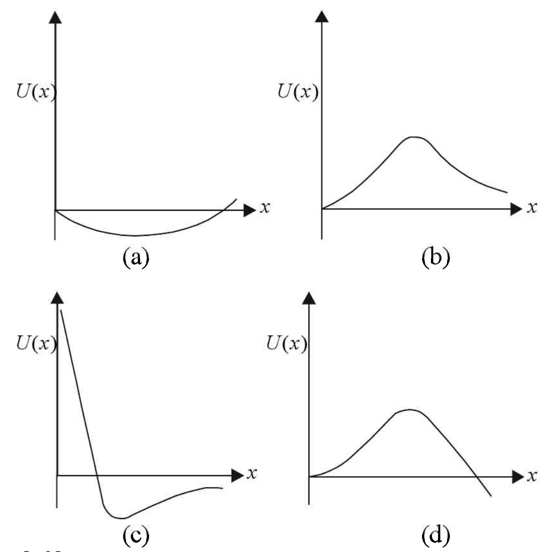


Fig. 6.43

(IIT Screening 2002)

60. The value of e for plastic bodies is

(a) 1 (b) zero
(c) 8 (d) arbitrary

61. A motor of 100 H.P. is moving with a constant velocity of 72 km/hour. The forward force exerted by the engine on the car is

(a) 3.73×10^3 N (b) 3.73×10^2 N
(c) 3.73×10^1 N (d) None of the above

62. Uniform constant retarding force is applied in order to stop a truck. If its speed is doubled then the distance travelled by it will be

(a) four times (b) double
(c) half (d) same

63. The kinetic energy of a man is half the kinetic energy of a boy of half of his mass. If the man increases his speed by 1 m/s then his kinetic energy becomes equal to that of the boy. The ratio of the velocity of the boy and that of the man is

(a) $\frac{2}{1}$ (b) $\frac{1}{2}$
(b) $\frac{3}{4}$ (d) $\frac{4}{3}$

64. In the above question the initial velocity of the man will be

(a) 3.571m/s (b) 2.415m/s
(c) 5.718m/s (d) 4.127m/s

65. Two elastic bodies P and Q having equal masses are moving along the same line with velocities of 16 m/s and 10 m/s respectively. Their velocities after the elastic collision will be in m/s

(a) 0 and 25 (b) 5 and 20
(c) 10 and 16 (d) 20 and 5

66. A squirrel of mass m is moving on a disc of mass M and radius R, in a circle of radius $R/2$ with angular velocity ω (Fig. 6.44). The angular frequency with which the disc will rotate in the opposite direction will be

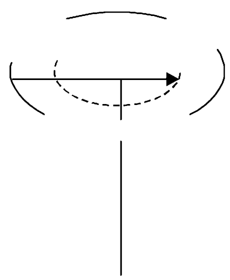


Fig. 6.44

(a) $\frac{m\omega}{2M}$ (b) $\frac{m\omega}{M}$

(c) $\frac{2m\omega}{M}$ (d) $\frac{2M\omega}{m}$

67. A ship of mass 3×10^7 Kg is initially at rest. It is being pulled by a force of 5×10^4 N through a distance $3m$. If the air resistance is negligible, then the speed of the ship will be

(a) 5 m/s (b) 1.5 m/s
(c) 60 m/s (d) 0.1 m/s

68. A metal ball does not rebound when struck on a wall, whereas a rubber ball of same mass when thrown with the same velocity on the wall rebounds. From this it is inferred that

(a) change in momentum is same in both.
(b) change in momentum in rubber ball is more.
(c) change in momemtum in metal ball is more.
(d) initial momemtum of metal ball is more than that of rubber ball.

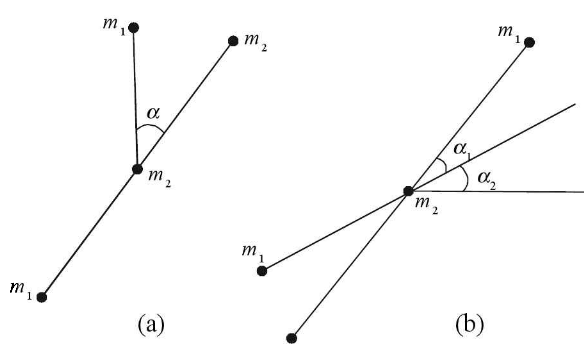
69. The unit of the co-efficient of restitution is

(a) m/s (b) s/m
(c) $m \times s$ (4) none of the above.

70. A particle moves in a potential region given by $U = 8x^2 - 4x + 400$ Joule. Its state of equilibrium will be

(a) $x = 25$ m (b) $x = 0.25$ m
(c) $x = 0.025$ m (d) $x = 2.5$ m

71. Keeping in view the law of conservation of momentum, which of the following figures is incorrect?



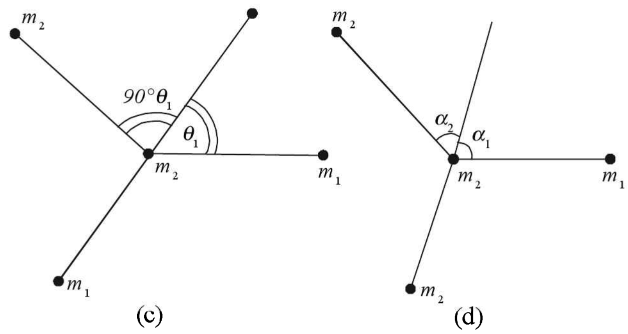


Fig. 6.45

72. The graph between $\sqrt{E_k}$ and 1/p is (E_k = kinetic energy and p = momentum)

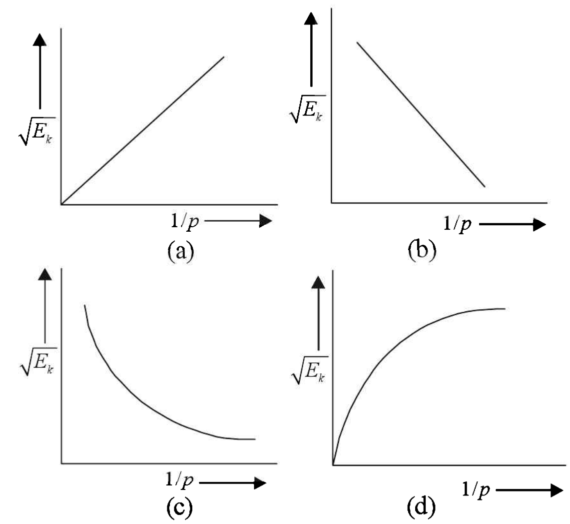


Fig. 6.46

73. The graph between U and X in the state of stable equilibrium will be

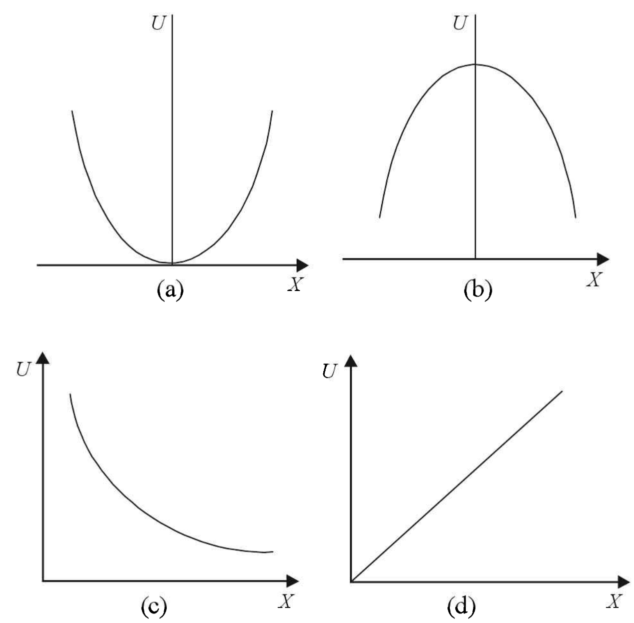


Fig. 6.47

74. Two masses $m_1 = 2$kg and $m_2 = 5$kg are moving on a directionless surface with velocities 10m/s and 3 m/s respectively m_2 is ahead of m_1. An ideal spring of spring constant $k = 1120$ N/m is attached on the back side of m_2. The maximum compression of the spring will be, if on collision the two bodies stick together

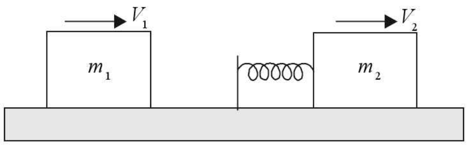


Fig. 6.48

(a) 0.51 m (b) 0.062 m
(c) 0.25 m (d) 0.72 m

75. A ball with velocity 9m/s collides with another similar stationary ball. After the collision the two balls move in directions making an angle of 30° with the initial direction. The ratio of the speeds of balls after the collision will be

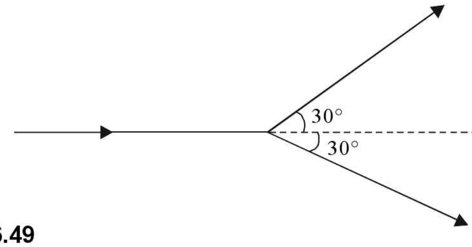


Fig. 6.49

(a) $\frac{v_1}{v_2} = 1$ (b) $\frac{v_1}{v_2} > 1$
(c) $\frac{v_1}{v_2} < 1$ (d) $\frac{v_1}{v_2} = 0$

Answers to Questions for Practice

1.	(b)	**2.**	(d)	**3.**	(c)	**4.**	(a)	**5.**	(b)	**6.**	(a)	**7.**	(d)
8.	(c)	**9.**	(d)	**10.**	(a)	**11.**	(c)	**12.**	(d)	**13.**	(b)	**14.**	(a)
15.	(b)	**16.**	(d)	**17.**	(a)	**18.**	(a)	**19.**	(b)	**20.**	(b)	**21.**	(c)
22.	(b)	**23.**	(d)	**24.**	(a)	**25.**	(c)	**26.**	(a)	**27.**	(b)	**28.**	(c)
29.	(d)	**30.**	(a)	**31.**	(a)	**32.**	(c)	**33.**	(d)	**34.**	(b)	**35.**	(a)
36.	(d)	**37.**	(a)	**38.**	(b)	**39.**	(c)	**40.**	(b)	**41.**	(d)	**42.**	(d)
43.	(a)	**44.**	(a)	**45.**	(a)	**46.**	(b)	**47.**	(a)	**48.**	(a)	**49.**	(c)
50.	(c)	**51.**	(d)	**52.**	(b)	**53.**	(b)	**54.**	(a)	**55.**	(b)	**56.**	(a)
57.	(c)	**58.**	(d)	**59.**	(d)	**60.**	(b)	**61.**	(a)	**62.**	(a)	**63.**	(a)
64.	(b)	**65.**	(c)	**66.**	(a)	**67.**	(d)	**68.**	(b)	**69.**	(d)	**70.**	(b)
71.	(a)	**72.**	(c)	**73.**	(a)	**74.**	(c)	**75.**	(a)				

Explanations

1(b) For instantaneous power

$$dP = \frac{dW}{dt} = \frac{dF^2t^2}{2m} = \frac{F^2}{2m} \cdot 2t$$

or $$dP = \frac{2F^2T}{2m}$$

But average power $= \frac{F^2(T^2-0)}{2mT} = \frac{F^2T}{2m}$

∴ Instantaneous power : average power = 2 : 1.

2(d) Let $XO = OZ = \frac{L}{2}$. When end Z goes down by y then velocity of bend

$$v = \sqrt{2g\frac{y}{2}}$$

because bend falls through $y/2$.

$$= -\left(\frac{m}{L}dy\right)v$$

$$= \frac{-m}{L}\, dy\, (gy)^{1/2} \quad \text{(downward)}$$

$$= \frac{m}{L}\, (gy)^{\frac{1}{2}}\, dy \quad \text{(upward)}$$

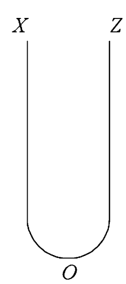


Fig. 6.50

3(c) Spring energy = kinetic energy

i.e. $\frac{1}{2}kx^2 = \frac{1}{2}mv^2$ or $v = x\sqrt{\frac{k}{m}}$

Time taken by ball to reach ground, t

$= \sqrt{\frac{2h}{g}}$ (from $S = ut + \frac{1}{2}at^2$)

$\therefore$ Horizontal distance covered = vt

$= x\sqrt{\frac{k}{m}}\sqrt{\frac{2h}{g}} = x\sqrt{\frac{2kh}{mg}}$

4(a) K.E. of block at B = P.E. at A – P.E. at B

i.e. $\frac{1}{2}mv^2 = mgh - mg^2r = mg(h-2r)$

i.e. $v^2 = 2g(h-2r)$ ....(i)

Also $\frac{mv^2}{r} - mg = x\,mg$

or $v^2 = (x+1)rg$ ....(ii)

Equating (i) and (ii)

$2g(h-2r) = (x+1)gr$

or $2gh = (x+1)gr + 4gr = (x+5)gr$

or $h = \left(\frac{x+5}{2}\right)r$

5(b) Using $P = Fv$, we get

$P = Fr\omega$ $(\because v = r\omega)$

Here $F = \mu N$

(where N is normal reaction)

$= \mu F' g$ $(\because 1\ kgf = g$ newton)

$\therefore$ $P = \mu F' grw = 2\pi\mu nF'$ gr

$(\because \omega = 2\pi r)$

6. (a) Work energy relationship *i.e.* $W = \frac{1}{2}mu^2$ indicates that the graph between w and u should be a *parabola* towards w axis.

7(d) use P.E. = Work done

$mgH = \mu_{\text{kinetic}}\, mgS$

or $S = \frac{H}{\mu_{\text{kinetic}}}$

The particle covers the length l or not, or covers it repeatedly is determined by the above relation.

8(c) P.E. with reference to lowest point = $Mg(2L)$

From conservation of energy,

$\frac{1}{2}Mv^2 = 2MgL$ *i.e.* $v = 2\sqrt{gL}$

9(d) Using conservation of momentum,

$(120+5)\,v_x = 5\times 2$

or $v_x = \frac{10}{125} = 0.08\ \text{ms}^{-1}$

Also $(90-5)\,v_y = 5\times 2$

or $v_y = \frac{10}{85} = 0.12\ \text{ms}^{-1}$

10. (a) Kinetic energy, $E = \frac{1}{2}mv^2$

here $v^2 = u^2 + (gt)^2$

($\because$ *vertical component of velocity is gt*)

$E = \frac{1}{2}m(u^2 + gt^2)$

Thus, curve should be a parabola of the type shown by (A).

11(c) The resultant force = forward thrust – backward thrust

and forward thrust = $U_E \times (\mu + m)$

Backward thrust = $U_P \times \mu$

$\therefore$ Resultant thrust,

$F = U_E(\mu+m) - U_P\mu$

Power = $F \times v$

$= \left[U_E\frac{(\mu+m) - U_P\mu}{746}\right]U_P$

$= \frac{U_EU_P(\mu+m) - U_P^2\mu}{746}$

12(d) Using $P = Fv = M\left(\frac{dv}{dt}\right)v$

i.e. $v^2\,dv = \frac{P}{M}\,vdt = \frac{P}{M}\,dS$

Integrating $\int_u^v v^2 dv = \frac{P}{M}\int_0^S dS$

$v^3 - u^3 = \frac{3PS}{M}$

or $v = \left[\frac{3PS}{M} + u^3\right]^{1/3}$

13(b) Let u be the speed of moving frame and horizontal distance covered at any instant from moving frame is x then distance from stationary frame is $(x + ut)$

Work done w.r.t. stationary frame W

$= F(x+ut) = Fx + Fut$

But Fx is the work done w.r.t. moving frame

$\therefore$ $W = W' + Fut$

i.e. $W \neq W'$

14(a) Using $v^2 - u^2 = 2aS$, we get

$v = (u^2 + 2aS)^{1/2}$

$= (2.4\times 10^7) + 2\times 3.6\times 10^{15}\times 0.035$

$= 2.88\times 10^7\ \text{ms}^{-1}$

Change in K.E. = $\frac{1}{2}m(v^2 - u^2)$

$= \frac{1}{2}(1.67\times 10^{-27})(2.88\times 10^7 - 2.4\times 10^7)$

$= 2.11\times 10^{-13}$ J

$= 1.32\times 10^6$ eV = 1.32 MeV

15(b) Loss in K.E. = Gain in P.E. + Work done against air drag

i.e. $\frac{1}{2}mu^2 = mgh + Fh = h(mg + F)$

or $h = \dfrac{u^2}{2(g+F/m)}$

$= \dfrac{u^2}{2g+2F/m}$

16(d)Using conservation of momentum

$0.02\,u = 0.02\,v + 1\,V_1$ ... (i)

(where V_1 *be the velocity of plate* of 1 kg)

and $0.02\,v = (2.98+0.02)\,V_1$ ... (ii)

($\because$ *plate of 3 kg has also same velocity i.e.* V_1)

or $V_1 = 0.02\,\dfrac{v}{3}$

Substituting

$\therefore$ $0.02\,v = 0.02\,v + 0.02\,\dfrac{v}{3}$

or $v = \dfrac{4v}{3}$ or $v = \dfrac{3u}{4}$

%age loss in velocity $= \dfrac{v-u}{u} \times 100$

$= \dfrac{v - \frac{3}{4}v}{v} \times 100 = 25\,\%$

17(a)The particle was initially at rest at (6, 4)

$\therefore$ $x = 6 + \dfrac{1}{2}a_x t^2$

and $y = 4 + \dfrac{1}{2}a_y t^2$ ... (i)

Acceleration

$\vec{a} = \dfrac{\vec{F}}{m} = -(3\,\hat{i} + 4\,\hat{j})$

$\left(\because F_x = \dfrac{-dP.E.}{dx} \text{ and } F_y = \dfrac{-dP.E.}{dx}\right)$

i.e. $a = (3^2+4^2)^{1/2} = 5\ \text{ms}^{-2}$ ... (ii)

Substituting the value of a from (ii) in (i)

$x = \left(6 - \dfrac{3}{2}t^2\right)$

and $y = \left(4 - 2t^2\right)$

Now $y = 0$ when particle crosses x – axis

$\therefore$ $t = \sqrt{2}$ s

Displacement $S = \dfrac{1}{2}at^2$

$= \dfrac{1}{2} \times 5 \times 2 = 5$ m

Then work done $= F.S. = 5 \times 5 = 25$ J

18(a)Work done, $W = mgh + \mu\, mgl$

which is a constant.

19(b)Potential energy is minimum if equilibrium is stable.

$\dfrac{d}{dx}$ [P.E. (x)] = 0

i.e. $\dfrac{d}{dx}\left[\dfrac{\alpha}{x^{12}} - \dfrac{\beta}{x^6}\right] = 0$

i.e. $\dfrac{-12\alpha}{x^{13}} + \dfrac{6\beta}{x^7} = 0$

i.e. $[-2\alpha + \beta x^6]\,\dfrac{6}{x^{13}} = 0$

i.e. $(-2\alpha + \beta x^6) = 0$

i.e. $x = \left(\dfrac{2\alpha}{\beta}\right)^{1/6}$

20(b)Using law of conservation of total momentum, we get

$(M+m)\,v = M\sqrt{2gh}$

or $v = \dfrac{M\sqrt{2gh}}{M+m}$

Let opposition to penetration be F then

Total work done = change in K.E.

i.e. $F = \dfrac{1}{2}(M+m)\,\dfrac{(M^2\,2gh)}{d(M+m)^2} + (M+m)\,g$

$= \dfrac{M^2 gh}{(M+m)d} + (M+m)\,g$

21(c)Loss in potential energy = Gain in kinetic energy

$mgh = \dfrac{1}{2}mv'^2$

$h = \dfrac{v'^2}{2g}\ \dfrac{(u+v)^2}{2g}$

($\because$ here velocity is given as $(u+v)$)

22(b) Let m be the mass of the bullet and M be the mass of the gun. If v is the velocity of bullet then

$MV = mv$ *i.e.*, $V = \dfrac{mv}{M}$

Thus, V is the velocity of recoil of gun when held loosely.

But when the gun is held tighly with the shoulder, the mass of the man supports the gun thus reducing the speed of gun. Hence, the gun experiences intensive recoil when fired with the butt held loosely with the shoulder.

23(d) Note $\dfrac{1}{2}kd^2 = mg\,(H+d)$

or $k = \dfrac{2mg(H+d)}{d^2}$

24(a) Efficiency $= \dfrac{\text{output}}{\text{input}}$

i.e. Output = efficiency $\times$ input

$= \eta \times 100$ calorie
$= \eta \times 100 \times 4.2$
$= 420\,\eta$ joule

or $mgh = 420\,\eta$

or $h = \dfrac{420\eta}{mg}$

25(c)Work done = mgh

$\frac{1}{2} \times$ base $\times$ height $= mgh$

(*area under the curve is work done*)

$\frac{1}{2} \times 10 \times 100 = 5 \times 10 \times h$

or $h = 10$ m

26(a)When plank is moved, the string is under tension. Work done by force F tries to neutralise the frictional force of plank and elongation of the string due to non sliding of the block due to its friction. Thus, work done by F is equivalent to option (a).

27(b)On ascending the plane, component of weight of tractor along the inclined plane will add to the given tension when the tractor and truck were moving on a level road *i.e.* new tension,

$T' = T + mg \sin\theta$

$= 1000 + 1200 \times 9.8 \times \frac{1}{6}$

$= 1000 + 1960 = 2960$ N

28(c)Work done in one rotation = work done against gravity

i.e. $F \times 2\pi \times \frac{1}{2} = 200 \times 10 \times 5 \times 10^{-3}$

($F \times 2\pi l = (mg) \times pitch$)

$F = \dfrac{10}{3.14} = 3.18$ N

29. Using conservation of energy

In *first case*, $\frac{1}{2}mu^2 + \frac{1}{2}m\left(\frac{u}{2}\right)^2$

$= mgL + mg\frac{L}{2}$

or $u = \left(\frac{12}{5}gL\right)$

In second case

$\frac{1}{2}(2)\,v^2 = 2\,mgL \quad v = (2gL)^{1/2}$

$\therefore \quad \dfrac{u}{v} = \dfrac{\left(\frac{12}{5}gL\right)^{1/2}}{[2gL]^{1/2}} = \left[\dfrac{6}{5}\right]^{\frac{1}{2}}$

or $v = \left[\dfrac{6}{5}\right]^{\frac{1}{2}}$

30(a)Work done = Change in K.E.

i.e. $W_g + W_a = \frac{1}{2}mv^2$

where W_g = mgs and W_a is the work done by air resistance

$\therefore \quad W_a = -mgs + \frac{1}{2}mv^2$

$= -\frac{m}{2}(-v^2 + 2gs) = -\frac{m}{2}(2gs - v^2)$

31(b)In case of vertical motion under gravity, when the body falls, force of gravity (a conservative force) is in the direction of motion of so K.E. increases but when the body is projected upward, force of gravity is opposite to motion so K.E. of body decreases. If kinetic energy of a body increases the work is taken as positive and vice versa.

32(c)Work done = ΔKE,

$Mg\left(r - \frac{2r}{\pi}\right) = \frac{1}{2}Mv^2$ ($\because$ of *c.g.* of the chain)

or $u = \sqrt{2gr\left(1 - \frac{2}{\pi}\right)}$

33(d)

$\frac{1}{2}mu^2 = -\mu mgS + \frac{1}{2}KS^2$

i.e. $S^2 + \dfrac{2\mu mgS}{K} - \dfrac{mu^2}{K} = 0$

Solving for μ,

$S = \frac{1}{K}(\mu^2 m^2 g^2 - mKu^2 - \mu mg)$

34. (b) For rigid cord,

$mgl = \frac{1}{2}mv_1^2$ ....(i)

Equating (i) and (ii)

$\frac{1}{2}mv_1^2 = \frac{1}{2}mv_2^2 + \frac{1}{2}k(\delta l)^2$

Clearly $v_1 > v_2$

35(a)For equilibrium

$Ma_{max} \times h = Mg \times p$

(q will not matter as per statement of the question because when the car is about to topple, the whole reaction is on the rear wheel as front wheel is no more in contact with the ground and here R is the reaction of the rear wheel)

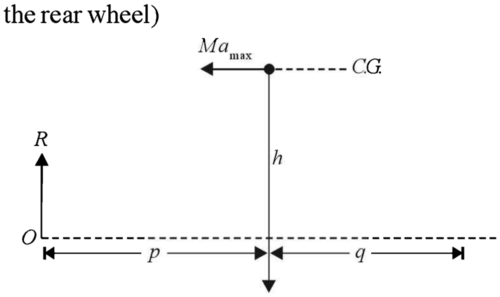


Fig. 6.51

$$a_{max} = \frac{qg}{h}$$

36(d)Force × Time = Change in momentum

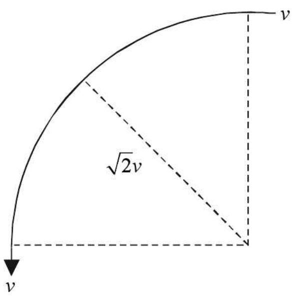


Fig. 6.52

or $F \times \Delta t = \Delta m \upsilon\sqrt{2}$ or $F = \frac{\Delta m}{\Delta t}\upsilon\sqrt{2}$

$= \upsilon\sqrt{2} \times \rho a \frac{\Delta l}{\Delta t}$

$(\because \Delta m = \rho a \Delta L)$

or $F = v\sqrt{2}\rho a v = v^2\sqrt{2}\rho a$

$$v = \left(\frac{F}{\sqrt{2}\rho a}\right)^{1/2}$$

37(a)Work done by man to stretch the spring

$= \left(\frac{1}{2}kx\right)(S + x)$

Work done by man on the floor

$= -\left(\frac{1}{2}kx\right)(S)$

∴ Total work done $= \frac{1}{2}kx^2$

38(b)Let a small displacement be given to the system in vertical plane in frame such that *ST* remains horizontal then let vertical displacement of centres of rods *UP* and *QR* be *y* then vertical displacement of centres of *UT* and *RS* will be 3*y* and that of *TS* will by 4*y*. Equating the total virtual work to zero we get

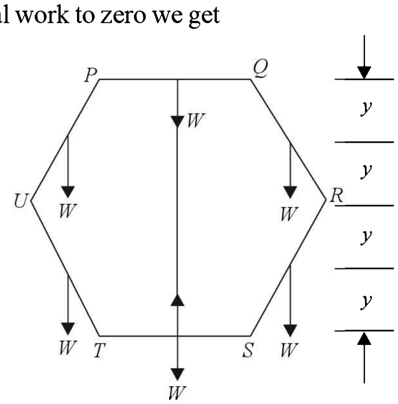


Fig. 6.53

$(W + W)\delta y + (W + W)3\delta y + W(4\delta y) - T(4\delta y) = 0$

(*where T is the tension in thread*)

or $2W + 6W + 4W = 4T$

or $T = 3W$

39(c)C.G. of first slab $= \frac{b}{2}$

Weight of each stab

= volume × density × g

$= b^3\rho g$

C.G. of column of slabs

$= \frac{\text{Total height of N slabs}}{2} = \frac{Nb}{2}$

Height of displacement of force

$= \left(\frac{Nb}{2} - \frac{b}{2}\right) = (N-1)\frac{b}{2}$

Work done $= N \times b^3 \rho G \times (N-1)\frac{b}{2}$

$= \frac{1}{2}(N^2 - N)b^4\rho g$

40(b)$mg + F\cos\theta = R$

Where m is mass of wiper, F is applied force towards centre and R is normal reaction of ground upward.

If motion is allowed on the floor,

$F\sin\theta - \mu N \geq 0$

i.e. $F\sin\theta - \mu(mg + F\cos\theta) \geq 0$

i.e. $\sin\theta - \mu\cos \geq \frac{\mu mg}{F}$

or $\tan\theta = \mu$

and in limiting case $\tan\theta_0 = \mu_s$

or $\theta_0 = \tan^{-1}\mu_s$

41(d)Let $AB = BC = CD = l$ unit and end A moves slightly towards right such that l is increased to $(l + \delta l)$.

Let the total work be zero, we get

$F_1(3\delta l) - F_2(\delta l) = 0$

$(\because$ displacement of $A = 3\delta l$

and that of C is $\delta l)$

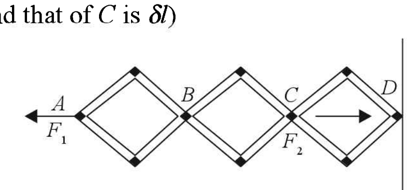


Fig. 6.54

or $F_1 = \frac{F_2}{3}$

42(d)Let the extension in cord be δL. Using conservation of energy, we get

$Mg(L + \delta L) = \frac{1}{2}K(\delta L)^2$

$K(\delta L) + 2 - 2MgL - 2Mg\delta L = 0$

or $K(\delta L)^2 - 2Mg\delta L - 2MgL = 0$

or $\delta L = \dfrac{2Mg \pm \sqrt{4M^2g^2 + 4K \times 2MgL}}{2K}$

(By solving the quadratic equation)

$i.e.\ \delta L = \dfrac{Mg}{K}\left(1 + \sqrt{1 + \dfrac{2KL}{Mg}}\right)$

($\because$ –ve sign is meaning less)

43(a)Let the pulley be given a small rotation θ in anticlockwise direction, the pulley and load will be lifted by $r_2\theta$ and P moves down by $(r_1\theta - r_2\theta)$.

Equating virtual work done to zero

$P(r_1\theta - r_2\theta) - Wr_2\theta = 0$

or $\dfrac{P}{W} = \dfrac{r_2}{r_1 - r_2}$

44(a)Here Brake power = $\dfrac{(T-S)(2\pi R)N}{60{,}000}$ kW

or $(T-S) = \dfrac{6000 \times \text{Brake power}}{2\pi RN}$

$= \dfrac{6000 \times 5.25}{2\pi \times 0.5 \times 220} = 455.8\ \text{N} \simeq 456\ \text{N}$

45(a)Normal reaction,

$R = mg = 2 \times 9.8$ N

Frictional force

$F = \mu R = 0.2 \times 2 \times 9.8$

$= 3.92$ N

Distance travelled

$5s = S$ i.e. $2 \times 5 = 10$ m

$\therefore$ Work done $= F \times S = 3.92 \times 10$

$= 39.2$ J

46(b)Here $mgy - \dfrac{1}{2} m v_F^2 = f_2 s_1 + f_2 s_2$

$i.e.\ \dfrac{1}{2} m v_F^2 = mgy - \mu mg \cos\beta\, s_1 - \mu mg \cos\gamma\, s^2$

or $\dfrac{1}{2} v_F^2 = g\left(y - \dfrac{\mu x_1 s_1}{s_1} - \dfrac{\mu x_2 s_2}{s_2}\right)$

or $v_F = \sqrt{2g(y - \mu x)}$

47(c)$P = Fv = c$

$\therefore \dfrac{mdv}{v}\ v = c$

$vdv = \dfrac{cdt}{m}$ or $\dfrac{v^2}{2} = \dfrac{ct}{m}$

(*after integration*)

$\left(\dfrac{ds}{dt}\right)^2 = \dfrac{2ct}{m}$ or $\left(\dfrac{ds}{dt}\right) = \sqrt{\dfrac{2c}{m}}\ t^{1/2}$

or $s \propto t^{3/2}$

48(a)Here momentum of third fragment is

$p^3 = \sqrt{p_1^2 + p_2^2}$

or $p^3 = \sqrt{(mv)^2 + (mv)^2}$

$= \sqrt{2}\ mv$

Final K.E. of the system

$= \dfrac{p_1^2}{2m} + \dfrac{p_2^2}{2m} + \dfrac{p_3^2}{2(2m)}$

$= \dfrac{3}{2} mv^2 + \dfrac{3}{2} mv^2 + \dfrac{3}{2} mv = \dfrac{3}{2} mv^2$

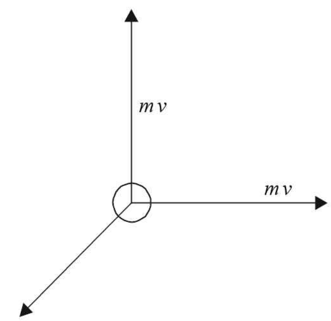


Fig. 6.55

Since initial K.E. = 0 therefore energy released = $\dfrac{3}{2} mv^2$.

49(c)Force exerted by engine = $mg \sin\theta + \mu mg \cos\theta =$ $mg(\sin\theta + \mu \cos\theta)$

Power exerted by engine = Fv

Work done by engine = Fvt

Efficiency of engine $\eta = \dfrac{\text{output}}{\text{input}}$

$= \dfrac{Fvt}{\text{input}}$

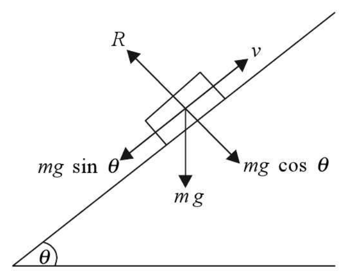


Fig. 6.56

$\therefore$ Input of engine

$= \dfrac{Fvt}{\eta} = \dfrac{mg(\sin\theta + \mu\cos\theta)vt}{\eta}$

50(c)Power $= P = \dfrac{dW}{dt} = \dfrac{d}{dt}(mhg)$

$= \dfrac{dV\rho gh}{dt}$

or $P = \dfrac{dV\rho gh}{dt} = \rho dh \dfrac{dV}{dt}$

Since $\eta = \frac{P_{\text{output}}}{P_{\text{input}}}$

$\therefore \quad P_{\text{output}} = \eta P_{\text{input}} = \eta \rho g h \frac{dV}{dt}$

or $\quad \frac{dV}{dt}\ \eta = \frac{P_{\text{output}}}{\eta \rho g h^2}$

$= \frac{40}{0.9}\ \rho g h = \frac{44.44}{\rho g h}$

51(d)Momentum of hydrogen atom = momentum of emitted photon

i.e. $mv = \frac{E}{c}$

or $\quad v = \frac{E}{mc}$

$= \frac{(0.73+1.82)1.6\times10^{-19}}{1.67\times10^{-27}\times3\times10^{8}}$

$= 0.81\ \text{ms}^{-1}$

52(b)In stretched position of spring,

system P.E. $= 2\times\frac{1}{2}\ k\times 2x^2 = 4kx^2$

In mean position, both balls have kinetic energy only;

$K.E. = 2\left[\frac{1}{2}mv^2\right] = mv^2$

but $P.E. = K.E.$

$\therefore \quad 4Kx^2 = mv^2$

or $\quad v = 2x\sqrt{\frac{K}{m}} = 2R\theta\sqrt{\frac{K}{m}}$

53(b)$a_c = k^2 rt^2$

i.e. $\frac{v^2}{r} = k^2 rt^2$ $\qquad$ or $v = krt$

Also $a_t = \frac{dV}{dt} = kr$

$\therefore \quad F = ma_t = mkr$

Then, power $= Fv = mkr\,(krt) = mk^2 r^2 t$

54(a)$\vec{V}_{\text{cart}} = 4\,\hat{i}$ $\qquad$ ... (i)

$\vec{V}_{\text{stone + cart}} = (6\sin 30)\,\hat{j} + (6\cos 30)\,\hat{k}$ $\qquad$ ... (ii)

$= (3\,\hat{j} + 3\sqrt{3}\,\hat{k})$

Then $V_{\text{stone}} = \text{(ii)} + \text{(i)} = (4\hat{i} + 3\,\hat{j} + 3\sqrt{3}\,\hat{k})$

Velocity of stone at highest point

$\vec{V}_{\text{stone + height}} = 4\hat{i} + 3\,\hat{j}$

[At highest point vertical component (*i.e.* *z* component) is zero]

or speed of stone at highest point

$V = \sqrt{4^2+3^2} = \sqrt{16+9} = 5\ \text{ms}^{-1}$

Using conservation of momentum

$mV = 2mV_{\text{combined}}$

or $\quad V_{\text{combined}} = \frac{V}{2} = \frac{5}{2} = 2.5\ \text{ms}^{-1}$

55(a)At horizontal position, tension is zero i.e. velocity of combined mass is zero.

$V^2_{\text{combined}} - 2gl = 0$

or $\quad l = \frac{V_{\text{combined}}}{2g} = \frac{2.5^2}{2\times9.8} = 0.32\ \text{m}$

56(a)Using conservation of energy

$\frac{1}{2}\ mv^2 + mgL = \frac{1}{2}\ mu^2$

or $\quad v = \sqrt{u^2 - 2gL}$

57(c)Work done = $\int F dt$

Then $W_1 = \int_0^a -(Kx\,\hat{j}).\,dx\,\hat{i} = 0$

and $\quad W_2 = \int_0^a -K(y\hat{i} + a\,\hat{j}).\,dy\,\hat{j}$

$= -Ka\int_0^a dy = -Ka^2$

$\therefore \quad W = W_1 + W_2 = -Ka^2$

58(d)The graph of $U(x)$ with x is as shown in Fig. 6.57.

Potential energy is zero at $x = 0$ and maximum at

As mechanical energy has fixed value *i.e.* $k/2$, the kinetic energy has to be maximum at $x = 0$ and maximum at $x = \pm\alpha$.

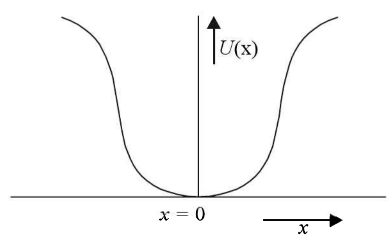


Fig. 6.57

59(d)Using $F = -\frac{dU}{dx}$ we get

$U = \frac{kx^2}{2} - \frac{ax^4}{4} + C$

$U = 0$ for $x = 0$

and $\quad x = \left(\frac{2k}{a}\right)^{1/2}$

which is satisfied by graph (d).

7

Conservation of Momentum

BRIEF REVIEW

Momentum Momentumor linear momentum $\vec{p} = m\vec{v}$. Unit kgms^{-1} or *Ns*. $p = [MLT^{-1}]$.

Linear momentum depends upon the frame of reference. For instance linear momentum of a body at rest in a moving train is zero with respect to a person sitting in the train while it is not zero with respect to ground.

Momentum is direction dependent. Thus two bodies having equal speed but different direction will have different momentums.

(a) $p \propto v$ if m is constant, *i.e.* for particles of equal mass, momentum will be maximum for a particle having largest velocity.

(b) $p \propto m$ if v is same, *i.e.* the heaviest particle will have maximum momentum if the particles have same velocity.

(c) If p = constant then $v \propto \frac{1}{m}$. , *i.e.*, for particles having same momentum the lightest particle will have maximum velocity and hence maximum *KE*.

$$\frac{p^2}{2m} = KE$$

$$\therefore p = \sqrt{2(KE)m}$$

(d) If p = const. $KE \propto \frac{1}{m}$ *i.e.* lightest particle will have maximum *KE* if the particles have equal momentum. See Fig. 7.1 (a).

(e) If *KE* of some particles is equal then the heaviest one will have maximum momentum i.e., $p \propto \sqrt{m}$ See Fig. 7.1 (b).

(f) If m is constant then $p \propto \sqrt{(KE)}$. See Fig. 7.1 (c)

Since $\vec{F} = \frac{d\vec{p}}{dt}$, the slope of $p - t$ curve will yield force and the area under F – t curve will give the change in momentum or impulse

Note $p = \frac{h}{\lambda}$ for a particle wave and $p = \frac{E}{c}$ for photons.

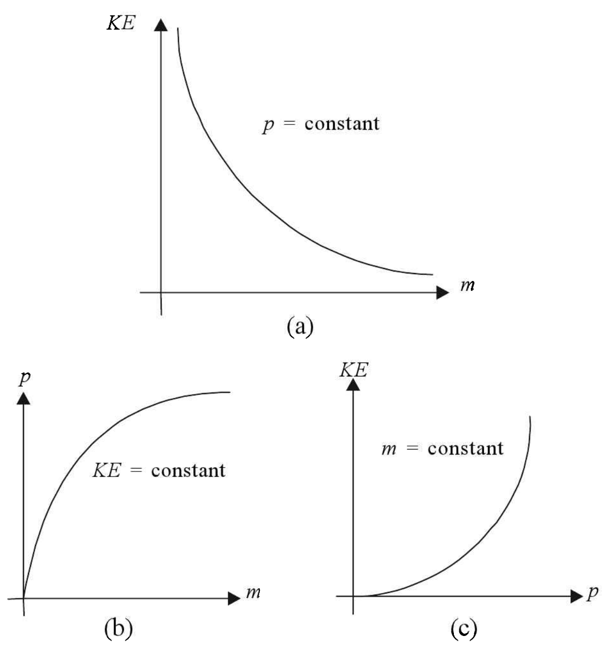


Fig. 7.1

Law of Conservation of Momentum

If $F_{ext} = 0$ then $\sum p_i$ = constant ,*i.e.* linear momentum of various particles may change but their vector sum remains unchanged.

Law of conservation of momentum is independent of frame of reference though momentum depends on the frame of reference.

Law of conservation of momentum is equivalent to Newton's third law of motion. $\vec{p}_1 + \vec{p}_2$ = const. or $\frac{d\vec{p}_1}{dt} + \frac{d\vec{p}_2}{dt} = 0$ or $F_1 + F_2 = 0$ or $F_2 = -F_1$, *i.e.* for every action there is an equal and opposite reaction.

Law of conservation of momentum is universal, i.e, it can be applied to microscopic as well as macroscopic particles. It holds good even in atomic and nuclear physics where classical physics fails.

Collision or Impact It is an isolated event in which a strong force acts for a short interval. The motion of colliding particles (at least one of them must) change abruptly. During collision particles may or may not come in physical contact. For examples, in collision between two balls, balls come in physical contact but in collision of charged particles like α–particle scattering there is no physical contact.

In collision, we consider the situation just before and just after impact. The duration of collision is negligibly small as compared to the time for which event is observed. During collision internal forces act on the colliding particles.

If the motion of the colliding particles before and after impact remains in the same straight line, the collision is said to be **direct** or **head on** or **one-dimensional** collision.

Note in One-dimensional collision velocity of COM of the colliding particles is in the same straight line.

If the two particles after collision donot maintain the same line of motion, the collision is said to be oblique. If in an oblique collision particles before and after collision remain in the same plane then collision is said to be two-dimensional, otherwise, it is three-dimensional.

Effect of external forces like friction, gravity is not considered in collision as duration of collision is very small. Average impulsive force responsible for collision is much greater than the external force acting on the system.

If charge on the interacting particles remains unchanged during collision, the process is termed as scattering. If the charge changes then reaction is the name given to such a process. However total charge remains conserved.

The impulsive force acting during collision is internal and hence, the total momentum of the system remains conserved.

If in a collision *KE* before and after collision are equal collision is said to be elastic. Collision between atomic and subatomic particles may be elastic.

If in a collision, colliding particle stick together or move with a common velocity after collision then such a collision is perfectly inelastic. For example, a bullet embedded in a wooden block after collision.

Most of the collision in our world (macroscopic) are imperfect or partially inelastic. For such collision we define coefficient of restitution (e).

$$e = \frac{\text{Velocity of Separation}}{\text{Velocity of approach}} = \frac{v_2 - v_1}{u_1 - u_2}$$

Note e = 0 means collision is perfectly inelastic and for $e = 1$, collision is perfectly elastic and for $0 < e < 1$ collision is partially inelastic.

One-dimensional collision (*Elastic*) $u_1 > u_2$

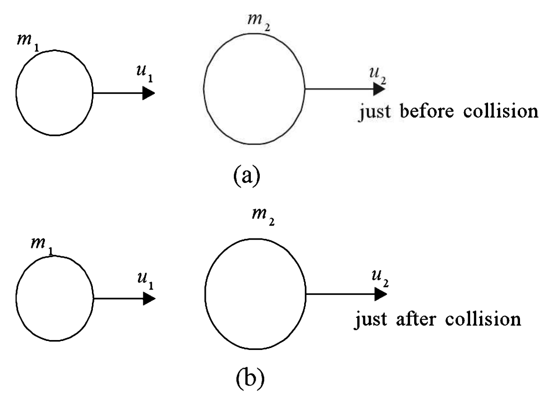


Fig. 7.2

Conserving momentum and *KE*, we can write

$$v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) u_1 + \frac{2m_2}{m_1 + m_2} u_2$$

$$v_2 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) u_2 + \frac{2m_2}{m_1 + m_2} u_1$$

Note

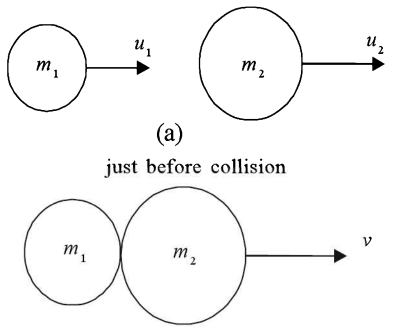
1. If $m_1 = m_2$ $v_1 = u_2$ and $v_2 = u_1$ *i.e.* velocities after collision interchange.
2. If target particle is at rest, *i.e.*, $u_2 = 0$ and $m_1 = m_2$, $v_1 = 0$, $v_2 = u_1$
3. If target is massive $m_2 >> m_1$ and is at rest $v_1 = u_1$ and $v_2 = 0$
4. If projectile is massive $v_1 = u_1$ and $v_2 = 2u_1 - u_2$. If target is at rest then $v_2 = 2u_1$.

Partially Inelastic Collision To solve problem conserve momentum

$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$ and exploit e = $\frac{v_2 - v_1}{u_1 - u_2}$

Remember in oblique collision coefficient of restitution e be employed only along common normal.

Perfectly inelastic collision



(b)
just after collision

Fig. 7.3

Oblique Collision Assuming two dimensional collision. Conserve momentum in x and y direction separately. If collision is elastic KE is also conserved. Remember KE is scalar.

Therefore, do not take its components along x and y direction. If two particles have equal mass and collision is oblique elastic then equal masses fly off at right angle to one another.

Motion of Two Masses Connected by a Spring

Assume spring is massless. The spring is compressed or stretched by x, so that m$_1$ is displaced by x_1 and m_2 by x_2 then $F_{ext} = 0$

$$\therefore \vec{p}_1 + \vec{p}_2 = 0 \text{ or } p_2 = p_1$$

$$|\vec{p}_1| = |\vec{p}_2| \therefore \frac{KE_1}{KE_2} = \frac{m_2}{m_1}$$, *i.e.*, lighter block moves faster or has more KE

or *COM* is at rest. Therefore

$m_1x_1 + m_2x_2 = 0$ and

$x = x_1 + x_2$

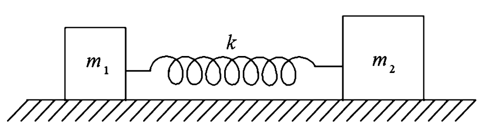


Fig. 7.4

Note KE of blocks is not constant.

Centre of Mass (COM) COM of a body is a point where the whole mass of the body may be assumed to be concentrated for dealing with its translatory motion. For a discrete system of particles co-ordinates of COM may be determined

$$\text{as } r_{COM} = \frac{m_1r_1 + m_2r_2 + \ldots + m_nr_n}{m_1 + m_2 + m_3 + \ldots + m_n} = \frac{\sum m_ir_i}{\sum m_i}$$

$$\text{or} \quad x_{COM} = \frac{\sum m_ix_i}{\sum m_i},$$

$$y_{COM} = \frac{\sum m_iy_i}{\sum m_i}$$

$$\text{and} \quad z_{COM} = \frac{\sum m_iz_i}{\sum m_i}$$

If the mass is uniformly distributed then take a mass element dm at positive $\vec{r}$ as a point mass and replace the summation by integration, i.e.,

$$r_{COM} = \frac{1}{m}\int r\,dm \text{ so that } x_{COM} = \frac{1}{m}\int x\,dm\ ;$$

$$y_{COM} = \frac{1}{m}\int y\,dm \text{ etc.}$$

There may or may not be any mass present at the COM. It may be within or outside the body.

For symmtrical bodies having uniform distribution of mass, it coincides with centre of symmetry or geometrical centre.

If the COM of the parts of a system is known, combined COM may be obtained by treating mass of the parts concentrated at their respective centre of masses.

If COM is the origin of a co-ordinate system the sum of moments of masses of the system about origin vanishes (=zero).

COM and centre of gravity in uniform gravitational field coincide but may be different for varying gravitational fields. For example, COM of a mountain may not coincide with its centre of gravity.

Motion of COM velocity of COM is given by

$$v_{COM} = \frac{m_1v_1 + m_2v_2 + \ldots\ldots}{m_1 + m_2 + \ldots\ldots} \quad \text{in a single}$$

direction.

If there are components of velocities then

$$v_{x,COM} = \frac{m_1v_{1x} + m_2v_{2x} + \ldots\ldots}{m_1 + m_2 + \ldots\ldots}$$

$$\text{and } v_{y,COM} = \frac{m_1v_{1y} + m_2v_{2y} + \ldots\ldots}{m_1 + m_2 + \ldots\ldots}$$

The velocity of particle in COM frame will be given by

$M\vec{v}_{com} = m_1\vec{v}_1 + m_2\vec{v}_2$

$$M\vec{v}_{COM} = \vec{p}_1 + \vec{p}_2 + \ldots = \sum \vec{p}_i = \vec{p}_{tot}.$$

Note if $\vec{v}_{COM} = 0$ then $\vec{p}_{tot} = 0$, that is, in the frame of reference of COM, the momentum of the system is zero. For this reason, COM frame is sometime termed as zero momentum frame.

$$M\frac{d\vec{v}_{COM}}{dt} = \frac{d\vec{p}_{tot}}{dt} = F_{ext} \text{ or } M\vec{a}_{COM} = F_{ext}$$ is the

equation of motion of COM. If $F_{ext} = 0$, $\vec{a}_{COM} = 0$ and $v_{COM} = \text{const}$.

• Short Cuts and Points to Note

1. Linear momentum is conserved whenever $F_{ext} = 0$. If force is mutual and a two-body system is considered then $\sum F = 0$ and momentum may be conserved. Note in such cases COM does not move. $\therefore v_{COM} = 0$; $\Delta x_{COM} = 0$
2. Elastic collision occur in atomic or subatomic particles. In real world, collisions are inelastic. Only collision between two ping-pong balls may be considered nearly elastic.
3. Impulse = $F.dt = dp$. Since during collision contact time is extremely small, we, therefore, assume that no external force or impulse has been imparted to the body.
4. Relativistic momentum $p_{rel} = \dfrac{mv}{\sqrt{1-\dfrac{v^2}{c^2}}}$
5. $p = \dfrac{h}{\lambda}$ (de-Broglie relation) used for matter waves and $p = \dfrac{E}{c}$ for photons.
6. In one dimensional elastic collision, coefficient of restitution = 1

$$v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) u_1 + \frac{2m_2}{m_1 + m_2} u_2 \text{ and}$$
$$v_2 = \left(\frac{m_2 - m_1}{m_1 + m_2}\right) u_2 + \frac{2m_1}{m_1 + m_2} u_1$$

where u_1 and u_2 are velocities just before collision and v_1 and v_2 are velocities just after collision.

If $m_1 = m_2$ velocities after collision interchange, i.e., $v_1 = u_2$ and $v_2 = u_1$

If $m_2 = \infty$ and $u_2 = 0$ then $v_1 = -u_1$ *i.e.* particle is just reflected back.

7. If collision is oblique (two dimensional) conserve momentum in x and y direction separately. If collision is elastic also then conserve KE also but do not take its components as KE is scalar.

If two equal masses collide obliquely and collision is elastic they will fly off at right angle to one another.

8. Collision is one dimensional or head on or direct if COM of the colliding particles move in the same straight line. See Fig. 7.5.

In Fig. 7.5 (a) the collision is one dimensional

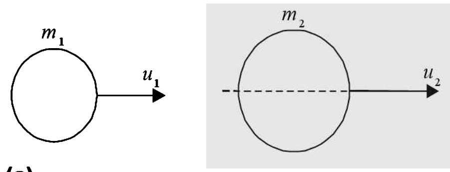


Fig. 7.5 (a)

In Fig. 7.5 (b) the collision is two dimensional

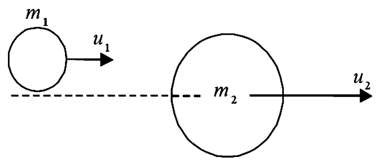


Fig. 7.5 (b)

9. In partially inelastic collision $0 < e < 1$, apply conservation of momentum

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

and $e = \dfrac{v_2 - v_1}{u_1 - u_2}$

10. In perfectly inelastic collision the colliding particles combine to move with a common velocity

$$v = \frac{m_1v_1 + m_2v_2}{m_1 + m_2};$$

Loss in $KE = \dfrac{m_1m_2}{2(m_1 + m_2)}(u_1 - u_2)^2$

11. Radioactivity is also considered to be a collision process. Therefore, momentum is conserved before and after emission.
12. In Rocket propulsion acceleration $a = \dfrac{v_g \dfrac{dm}{dt}}{M} - g$

where $M = M_0 - \dfrac{dm}{dt}t$. Upward force or upthurst (Net) $= Ma = v_g\dfrac{dm}{dt} - Mg$

only upthrust $= v_g\dfrac{dm}{dt}$. $\dfrac{dm}{dt}$ is rate of burning of fuel.

Velocity at any instant

$$v = v_0 + v_g \log_e \frac{M_0}{M_0 - \dfrac{dm}{dt}t} - gt$$

where v_0 is velocity at $t = 0$.

13. If a radioactive nucleus decays by γ-emission and energy of γ-rays is E, then momentum of γ-rays $p_\gamma = \dfrac{E}{c}$ where c is speed of light. Velocity of recoil of the nucleus $= \dfrac{E}{c \times m_{nucleus}}$. The KE of recoiling

nucleus $= \dfrac{p^2}{2m_{\text{nucleus}}} = \dfrac{E^2}{2c^2 m_{\text{nucleus}}}$

14. If a radioactive nucleus of mass number A decays by α–emission then conservation of momentum is as follows

$$0 = v_1(A-4) + v_2(4)$$

where v_1 and v_2 are velocities of recoiling nucleus and α-particle respectively.

15. Area under F-t graph is impulse or change in momentum.

16. If a particle strikes a wall and gets reflected then average force exerted by the wall is

$$F_{\text{av}} = \frac{\text{change in momentum}}{\text{time of contact}}.$$

17. If two point masses strike the rod as shown in Fig. 7.6 and after collision stick with the rod. In such case conserve linear momentum and angular momentum separately. Find angular velocity, find new COM about which the rod will rotate.

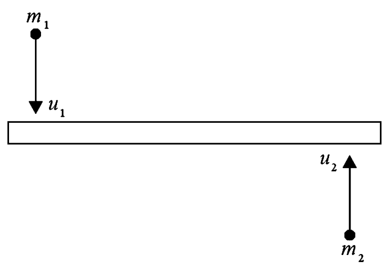


Fig. 7.6

• Caution

1. Assuming conservation of momentum can be applied only in collisions.

⇒ Conservation of linear momentum can be applied whenever $F_{\text{ext}} = 0$

2. Assuming in rotational motion linear momentum may not be conserved.

⇒ If a block moving with a velocity v collides with a disc then linear momentum is conserved as no external force acts. See Fig. 7.7.

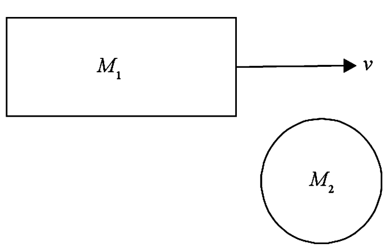


Fig. 7.7

3. Considering conservation of momentum can not be applied if F_{ext} is present.

⇒ In general law is true. But if force is mutual, and two body system is considered then $F_{\text{net}} = 0$ and we can apply conservation of momentum. (see Fig. 7.8)

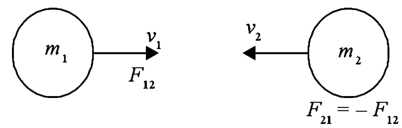


Fig. 7.8

If, however, single body system is considered, then force is external and conservation of momentum cannot be applied.

4. Considering that linear momentum is always conserved if collision occurs.

⇒ If a body strikes a hinged (target) rod, we have to conserve angular momentum instead of linear momentum. See Fig. 7.9

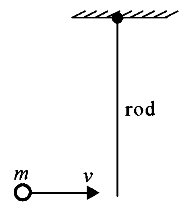


Fig. 7.9

5. Not using proper sign while conserving momentum.

⇒ Must apply proper sign. For example, if a projectile breaks up into two halves at the highest point. One of them retraces the path then, $mv \cos\theta = -\dfrac{m}{2} v \cos\theta + \dfrac{m}{2} v'$ *i.e.* momentum before breakup = momentum after breakup. See Fig. 7.10

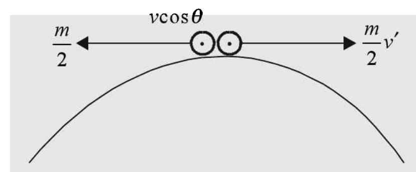


Fig. 7.10

6. Not taking into account acceleration due to gravity while finding net acceleration in a rocket propulsion.

⇒ use $a_{\text{net}} = \dfrac{v_g \dfrac{dm}{dt}}{M} - g$ where $M = M_0 - t\dfrac{dm}{dt}$

and $u = u_0 + v_g \log_e \dfrac{M_0}{M} - gt$ where u_0 is the velocity at $t = 0$

7. Assuming coefficient of restitution be applied in both the directions in oblique (two dimensional) collision.

⇒ Coefficient of restitution is applied along the common normal. For example, if motion is under gravity then along y – direction only.

8. Assuming when particles are moving in the same direction and collide, collision is one dimensional.

⇒ If the COM of the colliding particles are not in the same straight line as shown in Fig. 7.11, the collision is two dimensional or oblique as after collision particles move in different directions.

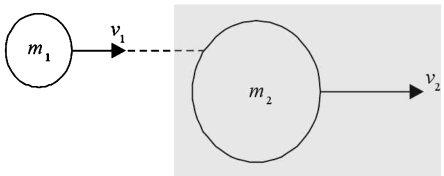


Fig. 7.11

9. Assuming if equal masses collide then their velocities are interchanged.

⇒ It is true only in one dimensional elastic collision. If collision is inelastic or two dimensional then it is not feasible. In oblique elastic collision, equal masses fly off at right angles to each other.

10. Resolving KE to x and y components in oblique collision.

⇒ KE is a scalar quantity and hence cannot be resolved.

11. Considering only linear momentum is conserved and not angular momentum during collision.

⇒ If a body rotates as a result of collision then both linear and angular momentum are conserved. In Select cases where the target is hinged only angular momentum is conserved.

12. Conserving momentum in a spaceship moving around the earth for a planet if its mass is varied.

⇒ Since orbital velocity is independent of mass of spaceship, therefore, do not apply conservation of momentum even if the mass is varied.

SOLVED PROBLEMS

1. A body A of mass M falling vertically downwards under gravity breaks into two parts; a body B of mass $\frac{M}{3}$ and a body of mass $\frac{2}{3}M$. The COM of bodies B and C taken together shifts compared to that of body A towards

(a) depends on height of breaking.
(b) does not shift.
(c) body C.
(d) body B.

[AIEEE 2005]

Solution (b) Since no horizontal force acts. Therefore, $v_{COM} = 0$ and $\Delta x_{COM} = 0$

2. A mass m moves with a velocity v and collides inelastically with another identical mass. After collision first mass moves with velocity $\frac{v}{\sqrt{3}}$ in a direction perpendicular to the initial direction of motion. Find the speed of second mass after collision.

(a) v (b) $\sqrt{3}v$

(c) $\frac{2v}{\sqrt{3}}$ (d) $\frac{v}{\sqrt{3}}$

[AIEEE 2005]

Solution (c) $mv = mv' \cos\theta$, $0 = \frac{mv}{\sqrt{3}} - mv' \sin\theta$

or $v'^2 [\cos^2\theta + \sin^2\theta] = v^2 + \frac{v^2}{3}$ or $v' = \frac{2v^2}{\sqrt{3}}$

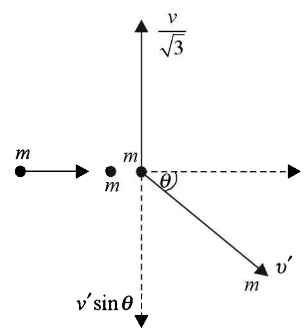


Fig. 7.12

3. A bomb of mass 30 kg at rest explodes into two pieces of masses 18 kg and 12 kg. The velocity of 18 kg mass is 6 ms^{-1}. The KE of the other mass is

(a) 324 J (b) 486 J
(c) 256 J (d) 524 J

[CBSE PMT 2005]

Solution (b) $k = \frac{p^2}{2m} = \frac{(18\times6)^2}{2\times12} = 486$ J

4. The PE of a particle of mass m is given by

$$V_{(x)} = \begin{cases} E_0 & 0 \le x \le 1 \\ 0 & x > 1 \end{cases}$$

λ_1 and λ_2 are the de-Broglie wavelengths of the particle, when $0 \le x \le 1$ and $x > 1$ respectively. If the total energy of the particle is $2E_0$, find $\frac{\lambda_1}{\lambda_2}$.

[I IT mains 2005]

Solution $KE = 2E_0 - E_0 = E_0$ for $0 \le x \le 1$

$$\lambda_1 = \frac{h}{\sqrt{2mE_0}}$$

$$KE = 2E_0 \text{ for } x > 1 \therefore \lambda_2 = \frac{h}{\sqrt{4mE_0}}$$

$$\therefore \frac{\lambda_1}{\lambda_2} = \sqrt{2}$$

5. Which of the following is not an inelastic collision.

(a) a man jumps on a cart
(b) a bullet imbedded in a block
(c) collision of two glass balls
(d) none of these.

[B.H.U PMT 2005]

Solution (c)

6. A stationary bomb explodes into two parts of masses 3 kg and 1 kg. The total *KE* of the two parts after explosion is 2400 J. The *KE* of the smaller part is

(a) 600 J (b) 1800 J
(c) 1200 J (d) 2160 J

Solution (b) $\frac{p^2}{2(1)} + \frac{p^2}{2(3)} = 2400$

or $p^2 = 2400 \times \frac{3}{2}$

and $\frac{p^2}{2m} = \frac{2400 \times 3}{4}$

7. In a perfectly elastic collision

(a) both *KE* and momentum are conserved.
(b) only *KE* is conserved.
(c) only momentum is conserved.
(d) neither *KE* nor momentum are conserved.

Solution (a)

8. A bullet of mass m and velocity υ is fired into a block of mass M and sticks to it. The final velocity of the system equals

(a) $\frac{M}{M+m}\upsilon$ (b) $\frac{M\upsilon}{m+M}$

(c) $\left(\frac{m+M}{M}\right)\upsilon$ (d) $\left(\frac{m+M}{m}\right)\upsilon$

Solution (b) $(m + m\upsilon') = m\upsilon$ or $\upsilon' = \frac{M\upsilon}{m+M}$

9. A mass m with velocity u strikes a wall normally returns with the same speed. The change in momentum is

(a) $-mu$ (b) mu
(c) zero (d) $-2mu$

Solution (d) $\Delta p = p_f - p_i = -mu\,(mu) = -2mu$

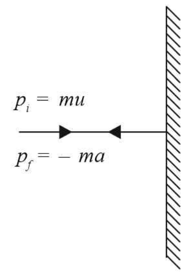


Fig. 7.13

10. Which one of the following is correct

(a) $E^2 = p^2c^2$ (b) $E^2 = p^2c$

(c) $E^2 = pc^2$ (d) $E^2 = \frac{p^2}{c^2}$

Solution (a) check dimensionally

11. If momentum decreases by 20%, KE will decrease by

(a) 40% (b) 18%
(c) 36% (d) 8%

Solution (c) $\Delta KE = \frac{m\upsilon^2}{2} - \frac{m}{2}(0.8\upsilon)^2 = 0.36\frac{m\upsilon^2}{2}$

$\therefore$ decrease is 36%

12. β-particles is emitted from the nucleus of mass number A with velocity u. Then, the recoil speed of the nucleus will be

(a) $\frac{\upsilon m_e}{A - m_e}$ (b) $\frac{\upsilon m_e}{A + m_e}$

(c) $\frac{4\upsilon}{A-4}$ (d) $\frac{\upsilon}{A}$

Solution (a) $\upsilon \,.\, m_e = (A - m_e)\,\upsilon'$ or $\upsilon' = \frac{\upsilon m_e}{A - m_e}$

13. Momentum is most closely related to

(a) force (b) impulse
(c) *KE* (d) power

Solution (b)

14. ${}^{210}_{84}P_O$ originally at rest emits α – particles of *KE* '*K*'. Find the *KE* of recoiling nucleus.

(a) $\frac{4}{214}K$ (b) $\frac{4}{206}K$

(c) $\frac{K}{206}$ (d) $\frac{K}{214}K.$

Solution (b) $206.\ \upsilon' = 4\upsilon$

KE of recoiling nucleus $= \frac{1}{2}(206)(\upsilon')^2$

$= \frac{1}{2}(206)\left(\frac{4\upsilon}{206}\right)^2$ or $K' = \frac{4}{206}K.$

15. On a muddy football field, 110 kg line backer tackles an 85 kg half back. Immediately before collision, the line backer is slipping with a velocity of 8.8 ms^{-1} North and the half back is sliding with a velocity 7.2 ms^{-1} east. What is the velocity at which two players move immediately after collision?

(a) 5.9 ms^{-1} (b) 6.8 ms^{-1}
(c) 8.0 ms^{-1} (d) 7.6 ms^{-1}

Solution (a) $(110+85)\,v = 110\,(8.8)\,\hat{j} + 85\,(7.2)\,\hat{i}$

or $|\vec{v}| = \dfrac{\sqrt{(11\times 88)^2+(85\times 7.2)^2}}{195} = 5.9\text{ ms}^{-1}$

$\tan\beta = \dfrac{153}{242}$ or $\beta = \tan^{-1}\dfrac{153}{244}$ North of east

16. A 250 g grass hooper moving due south at 20 cms^{-1} (at mid air) collides with 150 g grasshooper moving 60cms^{-1} due north. Find the decrease in *KE* if they move together after collision.

(a) 0.3 J (b) 3.0 J
(c) 0.03 J (d) 0.003 J

Solution (c) $\Delta KE = \dfrac{m_1m_2}{2(m_1+m_2)}(v_1-v_2)^2$

$= \dfrac{0.25\times 0.15(0.8)^2}{2(0.4)} = 0.03\text{ J}$

17. A 5 g bullet was fixed horizontal into a 1.2 kg wooden block resting on a wooden surface. The coefficient of kinetic friction between the block and surface is 0.2. The bullet remained embedded in the block. The block was found to slide 0.23 m along the surface before stopping. Find initial speed of bullet.

(a) 241 ms^{-1} (b) 229 ms^{-1}
(c) 221 ms^{-1} (d) 201 ms^{-1}

Solution (b) $v' = \dfrac{5v}{1205} = \dfrac{v}{241}$ and $v'^2 = 2\,as$

$\therefore \left(\dfrac{v}{241}\right)^2 = 2\times .2\times .23\times 10$

$v = \sqrt{241\times 241\times .92} = 229\text{ms}^{-1}$

18. A 0.15 kg glider moving to its right with 0.8 ms^{-1} collide head on with another glider of mass 0.3 kg moving with a speed 2.2 ms^{-1} to left. Find the velocity of 0.15 kg glider after collision. Assume collision is elastic.

(a) 2.2 ms^{-1} (b) 3.2 ms^{-1}
(c) 2.96 ms^{-1} to right (d) 2.96 ms^{-1} due left

Solution (b) $v_1 = \left(\dfrac{m_1-m_2}{m_1+m_2}\right)u_1 + \dfrac{2m_2}{m_1+m_2}u_2$

$v_2 = \dfrac{-0.15}{0.45}(0.8) + \dfrac{2\times 0.3}{0.45}(-2.2)$

$= -3.2\text{ ms}^{-1}$

19. In a particle accelerater, a beam of proton is emitted at 1.5×10^7ms^{-1}. The beam of some protons strike head on with gas atoms and rebound at 1.2×10^7 ms^{-1}. If the initial velocity of gas atom is negligible, find the mass of gas atom in terms of mass of proton.

(a) 3 m (b) 6 m
(c) m (d) 9 m
(e) ∞

Solution (d) $v_1 = \dfrac{(m_1-m_2)}{m_1+m_2}u_1 + \dfrac{2m_2u_2}{m_1+m_2}$

$= \dfrac{m-m_2}{m+m_2}(1.5\times 10^7) = 1.2\times 10^7$ or $m_2 = 9\text{m}$

20. A 70 kg astronaut floating in a 110 kg MMU experiences an acceleration of 0.029 ms^{-2} when he fires one of the MMU's thrusters, if the speed of escaping N_2 gas is 490 ms^{-1}, how much amount of gas is used in 5 s.

(a) 5.53 g (b) 5.53 kg
(c) 55.3 g (d) 55.3 g

Solution (d) $(70+110-\dfrac{dm}{dt}\,5)\,.029$

$= \dfrac{dm}{dt}\,490$

or $5\,\dfrac{dm}{dt} = \dfrac{180\times .029\times 5}{490}$

$= 0.0553\text{ kg} = 55.3\text{ g}.$

21. 3–identical pucks on a horizontal air table having repelling magnets are held together and then released simultaneously. Each has the same speed at any instant one puck moves west. What is the direction of other two

(a) 30° East of North, 30° East of South
(b) 60° East of N and 60° East of South
(c) 45° East of N, 45° East of South
(d) none of these

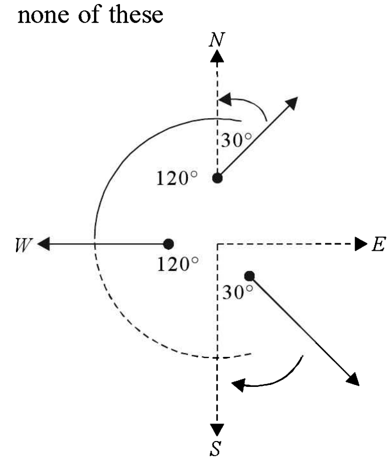


Fig. 7.14

Solution (a) It is possible if they are at 120° with each other.

∴ one shall be 30° East of North and other 30° East of South as illustrated in Fig.

22. A 60 kg woman stands up in a 120 kg canoe of length 5 m. She walks 1 m from one end to 1m from the other end. Ignoring water resistance, find the displacement of canoe.

(a) – 1 m (b) – 1.5 m
(c) – 2 m (d) none of these

Solution (a) $F_{ext} = 0$, $\therefore$ COM will remain unchanged.

Hence, $60 \times 3 = -180.\,x$ or $x = -1$ m

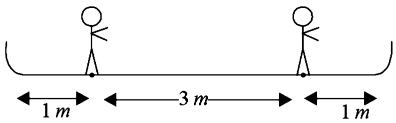


Fig. 7.15

23. A neutron decays to a proton and an electron. Find the fraction of energy gone to proton if total energy released is $k.m_p = 1836\, m_e$

(a) $\frac{1}{1836}$ (b) $\frac{1836}{1837}$
(c) $\frac{1835}{1837}$ (d) $\frac{1}{1837}$

Solution (d) $0 = 1836\, v_1 + v_2$ or $v_1 = \frac{v_2}{1836}$

$$\text{Total} \quad KE = \frac{1}{2}[1836\, v_1^2 + v_2^2] = \frac{1}{2}\left[\frac{v_2^2}{1836} + v_2^2\right]$$

$$\frac{K_P}{K_T} = \frac{\frac{1}{2}v_2^2 \times \frac{1}{1836}}{\frac{1}{2}v_2^2 \times \frac{1837}{1836}} = \frac{1}{1837}.$$

24. A monkey of mass m and a balloon of mass M are in equilibrium as shown in Figure. If the monkey reaches the top of balloon, by what distance balloon should descend.

(a) $-\frac{mL}{M}$ (b) $+\frac{mL}{M}$
(c) $-\frac{mL}{M+m}$ (d) $+\frac{mL}{M+m}$

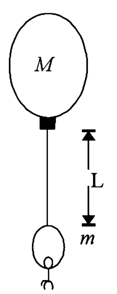


Fig. 7.16

Solution (a) $mL + My = 0$ or $y = -\frac{mL}{M}$

25. A beam of light of wavelength λ is incident on a mirror at an angle θ. Find the change in momentum of the photon after reflection.

(a) $2\frac{h}{\lambda}\sin\theta$ (b) $\frac{h}{\lambda}\cos\theta$
(c) $2\frac{h}{\lambda}\cos\theta$ (d) none of these

Solution (c) $\Delta p = 2p\cos\theta = 2\frac{h}{\lambda}\cos\theta$

26. Find the distance moved by wedge when the block just reaches ground in fig 7.17

(a) $\frac{Mh\cot\theta}{M-m}$ (b) $\frac{Mh\tan\theta}{M+m}$
(c) $\frac{Mh\tan\theta}{M-m}$ (d) $\frac{Mh\cot\theta}{M+m}$

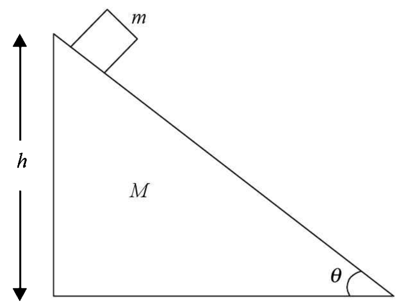


Fig. 7.17

Solution (d) $mh\cot\theta = -(M+m)x$

$$x = -\frac{Mh\cot\theta}{M+m}$$

27. A cavity of radius b is made in a disc of mass M, radius R, as shown in Fig. 7.18. Find the new COM

(a) $\frac{-b^2}{R+b}$ (b) $\frac{-b^2}{R-b}$
(c) $\frac{-R}{2R+b}$ (d) $\frac{-R}{3R+b}$

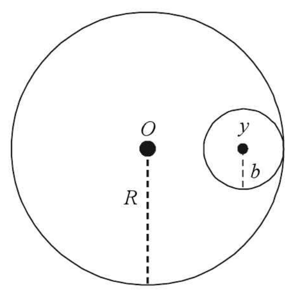


Fig. 7.18

Solution (b) mass removed m = $\frac{Mb^2}{R}$;

$$\bar{x} = \frac{M(0) - \frac{Mb^2}{R^2}(R-b)}{M - \frac{Mb^2}{R^2}} = \frac{-b^2}{(R+b)}$$

28. Find the maximum displacement of m_1 if each block is pulled by a force F as shown in Fig. 7.19

(a) $\frac{Fm_2}{m_1+m_2}$ (b) $\frac{Fm_1}{m_1+m_2}$
(c) $\frac{2Fm_2}{m_1+m_2}$ (d) $\frac{2Fm_1}{m_1+m_2}$

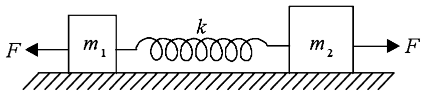


Fig. 7.19

Solution (c) $|x_1| + |x_2| = \frac{2F}{k}$; $m_1x_1 + m_2x_2 = 0$ or

$$|x_2| = \frac{m_1x_1}{m_2}$$

$$x_1 + \frac{m_1x_1}{m_2} = \frac{2F}{k} \quad \text{or}$$

$$x_1 = \frac{2Fm_2}{k(m_1+m_2)}$$

29. A body of mass m collides obliquely with another identical body at rest. Assuming collision to be elastic. The colliding bodies will move at ____

(a) 60° to one another (b) 90° to one another
(c) 120° to one another (d) 30° to one another

Solution (b)

30. A ball of mass m collides head on with another ball at rest. The KE of the system left is 50%. Find the coefficient of restitution.

(a) $\frac{1}{\sqrt{2}}$ (b) $\frac{\sqrt{2}}{3}$
(c) $\frac{1}{2}$ (d) zero

Solution (d) $mu + 0 = mv_1 + mv_2$ or

$$u = v_1 + v_2 \qquad \text{........ (1)}$$

$$e = \frac{v_2 - v_1}{u} \text{ or } eu = v_2 - v_1 \qquad \text{........ (2)}$$

From (1) and (2) $v_1 = \frac{(1-e)u}{2}$ and

$$v_1 = \frac{(1+e)u}{2} \left[\frac{1}{2}mv^2\right] \times \frac{1}{2}$$

$$= \frac{1}{2} m \left[\left(\frac{(1-e)u}{2}\right)^2 + \left(\frac{(1+e)u}{2}\right)^2\right] 1 = 1 + e^2 \text{ or } e = 0$$

TYPICAL PROBLEMS

31. A projectile is fired with a speed u at an angle θ above the horizontal field. The coefficient of restitution between the projectile and field is e. Find the position from the starting point when the projectile will land at its second collision.

(a) $\frac{e^2u^2 \sin 2\theta}{g}$ (b) $\frac{(1-e^2)u^2 \sin 2\theta}{g}$
(c) $\frac{(1-e)u^2 \sin\theta \cos\theta}{g}$ (d) $\frac{(1+e)u^2 \sin 2\theta}{g}$

Solution (d) Vertical velocity after first collision

$= eu \sin \theta$.

New Time of flight $T' = \frac{2eu\sin\theta}{g}$,

$R' = u_xT'$

$$x = R + R' = \frac{u^2 \sin 2\theta}{g} (1 + e)$$

32. In a γ-decay process, γ-rays of energy E is emitted. Find the decrease in internal energy of mass M (of nucleus).

(a) $\frac{E^2}{2Mc^2}$ (b) $E - \frac{E^2}{2Mc^2}$
(c) $E + \frac{E^2}{2Mc^2}$ (d) $E + \frac{E^2}{Mc^2}$

Solution (c) $\Delta E = E_r + E_{\text{nucleus}} = E + \frac{p^2}{2M} = E + \frac{E^2}{2Mc^2}$

33. A small block of superdense material has mass 2×10^{24} kg. It is at a height $h << R$. It falls towards earth. Find its speed when it is at a height $h/2$

(a) $\sqrt{\frac{2gh}{3}}$ (b) $\sqrt{\frac{3gh}{4}}$
(c) $\sqrt{\frac{3gh}{5}}$ (d) $\sqrt{\frac{gh}{2}}$

Solution (b) $\frac{M}{2} v_1^2 + \frac{1}{2} \times \frac{M}{3} v_2^2$

$$= \frac{GM\left(\frac{M}{3}\right)}{\left(\frac{h}{2}+R\right)} - \frac{GM\left(\frac{M}{3}\right)}{(h+R)}; \quad mv_1 = \frac{M}{3} v_2$$

$$\text{or} \quad v_2^2 \frac{2}{9} = \frac{GMh}{6R_2} \text{ or } v_2 = \sqrt{\frac{3gh}{4}}.$$

34. A system consists of two identical cubes each of mass m linked together by a massless spring of spring constant K. The spring is compressed by x connecting cubes by thread. Find minimum value of x for which lower cube will bounce up after the thread has been burnt.

(a) $\frac{2mg}{k}$ (b) $\frac{3mg}{k}$
(c) $\frac{3mg}{2k}$ (d) $\frac{mg}{2k}$

Solution (b) The elongation produced x' be such that $kx' = mg$. Apply energy conservation as at the maximum elongation $u_{block} = 0$.

$$\frac{1}{2}kx^2 = mg(x + x') + \frac{1}{2}kx'^2$$

$$x^2 - \frac{2mgx}{k} - 2\frac{mg}{k}x' - x'^2$$

or $$x^2 - \frac{2mg}{k}x - \frac{2mg}{k}\left(\frac{mg}{k}\right) - \left(\frac{mg}{k}\right)^2 = 0$$

or $$x = \frac{3mg}{k}$$

35. A cart loaded with sand moves along a horizontal plane due to a constant force F in the direction of velocity. Through a small hole in the bottom sand spills out at m kgs^{-1}. Find the velocity of the cart at any time t. Initial mass of the cart is m_0.

(a) $\frac{F}{m}\log_e\frac{m_0}{m_0 - mt}$ (b) $\frac{F}{m_0}\log_e\frac{m_0}{m_0 - mt}$

(c) $\frac{F}{m_0}t$ (d) $\frac{F}{m}t$

Solution (a) Mass of the cart at any instant is $m_0 - mt$

acceleration $a = \frac{dv}{dt} = \frac{F}{m_0 - mt}$

$$v = \int_0^t \frac{F}{m_0 - mt}dt = \frac{F}{m}\log_e\frac{m_0}{m_0 - mt}.$$

36. A block of mass m is given a velocity v. Find to what height the block will rise after breaking off from mass M. Assume all surface to be smooth.

(a) $\frac{mv^2}{2g(m+M)}$ (b) $\frac{mv^2}{2gM}$

(c) $\frac{v^2}{2g}$ (d) $\frac{Mv^2}{2g(m+M)}$

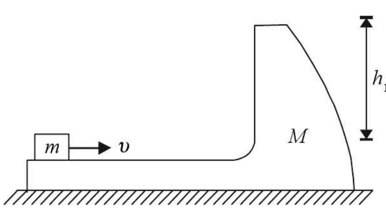


Fig. 7.20

Solution (d) $mv = (m + M)v_x$ or $v_x = \frac{mv}{m+M}$

Energy conservation gives

$$\frac{1}{2}mv^2 = \frac{1}{2}(m+M)v_x^2 + \frac{1}{2}m\,v_y'^2 + mgh_1$$

or $$\frac{1}{2}mv^2 = \frac{1}{2}(m+M)\left(\frac{mv}{M+m}\right)^2 + \frac{1}{2}m\,v_y'^2 + mgh_1$$

or $$v_y'^2 = v^2 - \frac{mv^2}{m+M} - 2gh$$

If h_2 is the height from break off point then $h_2 = \frac{v_y'^2}{2g}$

and $h = h_1 + h_2$

Thus $h = h_1 + h_2 = \frac{v^2}{2g} - \frac{mv^2}{2g(m+M)}$

37. Particle 1 collides a stationary particle 2 head on elastically. Find their mass ratio if after collision they move in opposite direction with equal velocities.

(a) 1 (b) 1/2

(c) 1/3 (d) 1/4

Solution (c) $$\frac{v_1}{v_2} = \frac{(m_1 - m_2)u_1 + 2m_2u_2}{(m_2 - m_1)u_2 + 2m_1u_1}$$

or $-1 = \frac{(m_1 - m_2)u}{2m_1u}$ or $\frac{m_1}{m_2} = \frac{1}{3}$.

38. Two persons each of mass m are standing at the two extremes of a railroad car of mass M resting on a smooth track. The person on the left jumps to the left with a velocity u. Thereafter second person jumps to right with same velocity u. Find the velocity of the train when both the persons have jumed off.

(a) $\frac{m^2u}{(m+M)m}$ towards left

(b) $\frac{m^2u}{M(m+M)}$ towards left

(c) $\frac{m^2u}{M(m+M)}$ towards right

(d) $\frac{m^2u}{m(m+M)}$ towards right

Solution (b) $mu = -(M + m)v$ or $v = \frac{-mu}{M+m}$

Negative sign means towords right

After the 2nd person jumps, conserving momentum (Now taking positive sign for right)

$$mu = Mv - Mv' \quad \text{or} \quad mu = \frac{M(mu)}{m+M} - Mv'$$

or $$v' = -\frac{m^2u}{M(m+M)}$$

Negative sign shows towards left.

39. A ball of mass m is dropped on to a floor from certain height, collision is perfectly elastic and the ball rebounds to the same height and again falls. Find the average force exerted by the ball on the floor during a long time interval.

(a) mg (b) > mg

(c) < mg (d) none of these

Solution (a) $\Delta p = m\left[\sqrt{2gh} - (-\sqrt{2gh})\right]$

$$= 2m\sqrt{2gh}\ ;\ F_{av} = \frac{\Delta p}{\Delta t} = \frac{n2m\sqrt{2gh}}{2n\sqrt{\frac{2h}{g}}}$$

40. A Rod of length l and mass 4 m is hinged at one end as shown in Fig. 7.21. A point mass m moving with a velocity v hits it on the other end and sticks. Find the angular velocity so formed. Assume the whole system is kept on a smooth horizontal table.

(a) $\frac{v}{5l}$ (b) $\frac{v}{3l}$

(c) $\frac{2v}{3l}$ (d) $\frac{3v}{7l}$

Solution (d) Conserve angular momentum

$$I\omega = mv(l)$$

$$(4\frac{ml^2}{3} + ml^2)\,\omega = mvl$$

or $$\omega = \frac{3v}{7l}$$

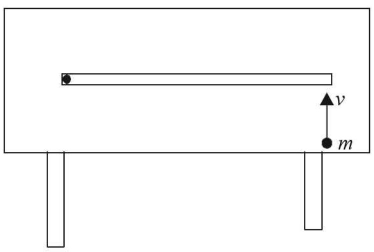


Fig. 7.21

41. Three spheres each of mass m are placed at the vertices of an equilateral Δ of side l. Each sphere is moving towards centroid with velocity v. After collision A comes to rest. B continues to move in the same direction with velocity v. Find the direction and velocity of C after collision.

(a) C move opposite to B.
(b) zero.
(c) C moves in the direction of B.
(d) C moves in its direction.

Solution (a) Net angular momentum just before impact = 0 (use Δ Law). Therefore, after collision net momentum should be zero. To make momentum zero, C shall move in a direction opposite to the direction of B and with a velocity v.

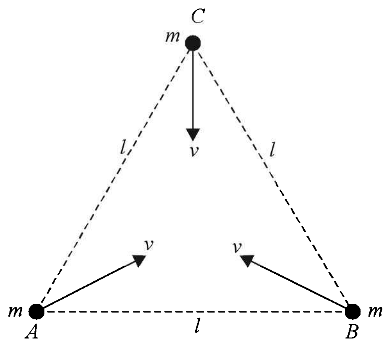


Fig. 7.22

QUESTIONS FOR PRACTICE

1. A rocket having fuel as its bulk is initially at rest. Neglecting the effect of gravity, when fuel is burning at a constant rate, acceleration a of the rocket with respect to time t is best represented by one of the graphs given below.

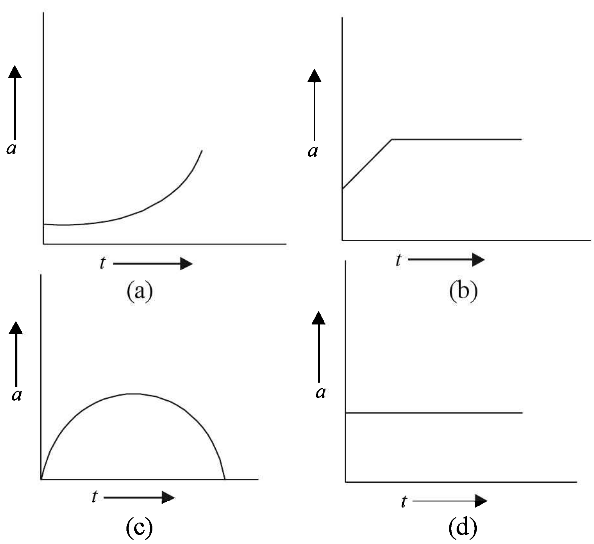


Fig. 7.23

2. Two girls of equal mass m jump off from the border line of a stationery carriage of mass M with same horizontal velocity u relative to the carriage. Neglecting the effect of friction

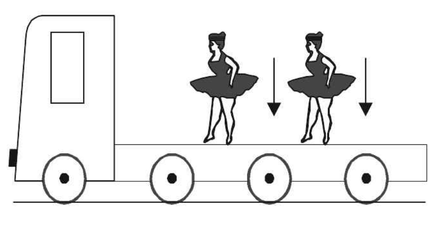


Fig. 7.24

(a) they will impart greater velocity to the carriage by jumping of simultaneously.
(b) they will impart greater velocity to the carriage by jumping one after the other.
(c) they will impart greater velocity to the carriage in whatever manner they jump off.
(d) insufficient data to reply.

3. In the shown system friction is meaningless. The carriage of mass M has constant initial velocity u along a straight horizontal track when at $t = 0$, it starts raining. The rain drops have a vertical velocity u' and result into addition of mass m per second to the carriage. The velocity of carraige after T second of start of rain is

(a) $\frac{Mu}{M+mt}$ (b) $\frac{Mu+mu'}{M+mt}$

(c) $\frac{(u+u')(M+m)}{M}$ (d) $\frac{M(u+u')}{mt}$

4. A boy of mass m kg boards a trolley of mass $2m$ moving with constant speed u along a horizontal track. Neglecting friction, if the boy jumps vertically up with reference to the trolley to catch hold of a branch of a tree, the speed of trolley after the boy has jumped off is

(a) u (b) $2u$

(c) $\frac{u}{2}$ (d) $\frac{3u}{4}$

5. Two similar bogies A and B of same mass M (empty bogie) move with constant velocities v_A and v_B towards each other on smooth parallel tracks. At an instant a boy of mass m from bogie A and a boy of same mass from bogie B exchange their positioin by jumping in a direction normal to the track, then bogie A stops while B keeps moving in the same direction with new velocity v_B. The initial velocities of bogie A and B are given by

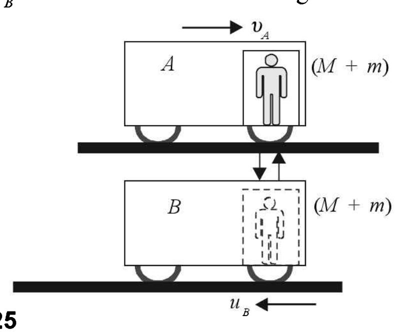


Fig. 7.25

(a) $\frac{M-m}{m}v_B$, $\frac{M-m}{M}v_B$

(b) $\frac{mv_B}{(M-m)}$, $\frac{Mv_B}{(M-m)}$

(c) $\frac{mv_B}{(M+m)}$, $\frac{Mv_B}{(M+m)}$

(d) $\frac{(M-m)v_B}{m}$, $\frac{(M-m)v_B}{M}$

6. A bomb of mass m is moving in x direction with velocity u when it separates into masses $\frac{1}{3}m$ and $\frac{2}{3}m$ moving horizontally in the same plane. If an additional energy of $4mu^2$ is generated, the relative speed of two masses is

(a) $3u$ (b) $4u$

(c) $6u$ (d) $8u$.

7. A rope ladder of length L is attached to a balloon of mass M. As the man of mass m climbs the ladder into the balloon basket the balloon comes downs by a vertical distance s. Then, increase in potential energy of man divided by increase in potential energy of balloon is

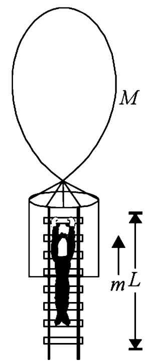


Fig. 7.26

(a) $\frac{L-s}{s}$ (b) $\frac{L}{s}$

(c) $\frac{s}{L-s}$ (d) $L-s$

8. A 0.5 kg block slides from A on horizontal track with an initial speed of 3 ms^{-1} towards a weightless horizontal spring of length 1m and force constant 2 Nm^{-1}. The part AB of the track is frictionless and the part BC has the coefficient of static and kinetic friction as 0.22 and 0.20 respectively. If the distance AB and BD are $2m$ and 2.14 m respectively, the total distance through which the block moves before it comes to rest completely is

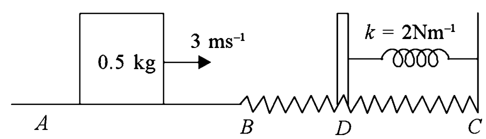


Fig. 7.27

(a) 2.5 m (b) 4.42 m

(c) 4.24 m (d) 2.44 m.

(Based on IIT 1983)

9. A shell is fired from a cannon with a velocity V *(m/s)* at an angle θ with the horizontal direction. At the highest point in its path it explodes into two pieces of equal mass. One of the pieces retraces its path to the cannon and the speed (in *m/s*) of the other piece immediately after the explosion is

(a) $3V\cos\theta$ (b) $2V\cos\theta$

(c) $\frac{3}{2}V\cos\theta$ (d) $\frac{\sqrt{3}}{2}V\cos\theta$

10. Two particles of masses m_1 and m_2 in projectile motion have velocities $\vec{v}_1$ and $\vec{v}_2$ respectively at time $t = 0$. They strike at time t_0 and their velocities become v_1' and v_2' at time $2t_0$ while still moving in air. The value of

$$\left|(m_1\overrightarrow{v_1'}+m_2\overrightarrow{v_2'})-(m_1\vec{v}_1+m_2\vec{v}_2)\right| \text{ is}$$

(a) zero (b) $(m_1+m_2)g\,t_0$

(c) $2(m_1+m_2)g\,t_0$ (d) $\frac{1}{2}(m_1+m_2)g\,t_0$

11. A light and a heavy body have equal kinetic energy. Which one has a greater momentum?

(a) The heavy body (b) The light body
(c) Both (d) Cannot be said

12. The linear momentum of a body is increased by 50%. Its kinetic energy will increase by

(a) 150% (b) 125%
(c) 100% (d) 50%

13. Two unequal masses are tied together with a compressed spring. When the cord is burnt with a match stick releasing the spring; the two masses fly apart with equal

(a) momentum (b) acceleration
(c) kinetic energy (d) speed

14. A particle at rest suddenly disintegrates into two particles of equal masses which start moving. The two fragments will

(a) move opposite with unequal speeds.
(b) move in any direction with any speed.
(c) move in same direction with equal speeds.
(d) move opposite with equal speed.

15. Two masses of 1g and 4 g are moving with kinetic energy in the ratio 4 : 1. What is the ratio of their linear mementum?

(a) 6 : 1 (b) 4 : 1
(c) 1 : 2 (d) 1 : 1

16. On a stationary sail boat air is blown from a fan attached to the boat. The boat

(a) moves in opposite direction in which air is blows.
(b) does not move.
(c) move in same direction in which air blows.
(d) spins around.

17. The spacecraft of mass M moving with a velocity v in free space explodes and breaks up into two pieces. If after explosion a piece of mass m comes to rest, the other piece of spacecraft will have a velocity

(a) $\frac{Mv}{(M-m)}$ (b) $\frac{Mv}{(M+m)}$

(c) $mV/(M-m)$ (d) $\frac{mv}{(M+m)}$

18. A shell explodes and many pieces fly off in different directions. The following is conserved

(a) kinetic energy (b) momentum
(c) both (d) none

19. A light and a heavy body have equal momenta. Which one has greater kinetic enerty?

(a) The heavy body (b) The light body
(c) Both (d) None

20. A bird on the floor of an air tight box which is being carried by a boy starts flying. The boy feels that the box is now

(a) lighter
(b) heavier
(c) shows no change in weight
(d) lighter in the beginning and then heavier

21. Two bodies of mass m_A and m_B have equal kinetic energy. The ratio of their momenta is

(a) $\sqrt{m_A} : \sqrt{m_B}$ (b) $m_A^2 : m_B^2$
(c) $m_A : m_B$ (d) $m_B : m_A$

22. A bullet weighing 50 gm leaves the gun with a velocity of 30 ms^{-1}. If the recoil speed imparted to the gun is 1ms^{-1}, the mass of the gun

(a) 1.5 kg (b) 15 kg
(c) 20 kg (d) 30 kg

23. If the kinetic energy of a body becomes four times of its initial value, then the new momentum will be

(a) double (b) 3 times
(c) 4 times (d) unchanged

24. A particle of mass m is moving in a horizontal circle of radius r with uniform speed v. When it moves from one point to a diametrically opposite point its

(a) KE changes by mv^2.
(b) KE changes by (1/4) mv^2.
(c) momentum does not change.
(d) momentum changes by 2 mv.

25. Kinetic energy of a body of mass m and momentum p is given by

(a) $p^2 m$ (b) $m^2/2p$
(c) mp (d) $p^2/2m$

26. When the velocity of a body is doubled

(a) kinetic energy is doubled.
(b) acceleration is doubled.
(c) momentum is doubled.
(d) potential energy is doubled.

27. A surface is hit elastically and normally by n balls per unit time. All the balls have the same mass m and move with the same velocity u. The force on the surface is

(a) 1/2 mnu^2 (b) mnu^2
(c) 2 mnu^2 (d) 2 mnu

28. A bomb of mass 9 kg explodes into two pieces of mass 3 kg and 6 kg. The velocity of mass 3 kg is 16ms^{-1}. The kinetic ennergy of mass 6 kg in joule is

(a) 768 (b) 96
(c) 384 (d) 192

29. A body of mass m at rest explodes into three pieces, two of which of mass $m/4$ each are thrown off in perpendicular directions with velocity of 3 ms^{-1} and 4 ms^{-1} respectively. The third piece will be thrown off with a velocity of

(a) 3 ms^{-1} (b) 2.5 ms^{-1}
(c) 2 ms^{-1} (d) 1.5 ms^{-1}

30. A body of mass 1 kg initially at rest explodes and breaks into three fregments of masses in the ratio

1 : 1 : 3. The two pieces of equal mass fly off perpendicular to each other with a speed of 15 ms^{-1}each. The speed of the heavier fregment is

(a) 45 ms^{-1} (b) 15 ms^{-1}
(c) 5 ms^{-1} (d) $5\sqrt{2}$ ms^{-1}

31. A block of mass 2 *m* moving with constant velocity $3\vec{v}$ collides with another block of mass *m* which is at rest and stick to it. The velocity of the compound block after the collision is

(a) $\vec{v}$ (b) $3\vec{v}/2$
(c) $2\vec{v}$ (d) $3\vec{v}$

32. When two bodies collide elestically then the quantity conserved is

(a) kinetic energy (b) mometum
(c) both (d) none

33. Two balls at the same temprature collide. What is conserved?

(a) Momentum (b) Kinetic energy
(c) Velocity (d) Temperature

34. Two balls each of mass 0.25 kg are moving towards each other in a straight line, one at 3 ms^{-1} and the other at 1 ms^{-1} collide. The balls stick together after the collision. The magnitude of the final velocity of the combined mass is

(a) 1/2 ms^{-1} (b) 1 ms^{-1}
(c) 2 ms^{-1} (d) 4 ms^{-1}

35. A bullet weighing 10 g and moving at 300 ms^{-1} strikes a 5 kg block of ice and drops dead. The ice block is sitting on frictionless level surface. The speed of the block, after the collision is

(a) 60 ms^{-1} (b) 160 ms^{-1}
(c) 16 ms^{-1} (d) 6 ms^{-1}

36. A body of mass *m* moving with a constant velocity *v* hits another body of the same mass moving with same velocity v but in the opposite direction, and sticks to it. The velocity of the compound body after collision is

(a) zero (b) *v*
(c) 2*v* (d) *v*/2

37. A bullet of mass '*a*' and velocity '*b*' is fired into a large block of wood of mass '*c*'. The final velocity of the system is

(a) $\frac{ab}{a+c}$ (b) $\frac{b}{a}$ (a + c)
(c) $\frac{cb}{(a+b)}$ (d) $\frac{b}{c}$ (a + b)

38. A mass m_1 moves with a great velocity. If it strikes another mass m_2 at rest in a head on collision it comes back along its path with a low speed after collision. Then

(a) $m_1 > m_2$ (b) $m_1 = m_2$
(c) $m_1 < m_2$
(d) there is relation between m_1 and m_2

39. Two solid rubber balls *A* and *B* having masses 200 g and 400 g respectively are moving in opposite directions with velocity of *A* equal to 0.3 ms^{-1}. After collision the two balls come to rest then the velocity of *B* is

(a) –0.15 ms^{-1} (b) 0.15 ms^{-1}
(c) 1.5 ms^{-1} (d) none of these

40. A body of mass m moving with a constant velocity *v* hits another body of the same mass at rest and sticks to it. The velocity to the compound body after collision is

(a) zero (b) *v*/2
(c) *v* (d) 2*v*

41. A body of mass 2 kg moving with a velocity of 3 ms^{-1} collides head on with a body of mass 1kg moving with a velocity of 4 ms^{-1} in opposite direction. After collision the two bodies stick together and move with a common velocity

(a) $\frac{2}{3}$ ms^{-1} (b) $\frac{3}{4}$ ms^{-1}
(c) $\frac{1}{4}$ ms^{-1} (d) $\frac{1}{3}$ ms^{-1}

42. A steel ball moving with a velocity $\vec{v}$ collides with an identical ball originally at rest. The velocity of the first ball after the collision is

(a) $\left(-\frac{1}{2}\right)\vec{v}$ (b) $-\vec{v}$
(c) $\vec{v}$ (d) zero

43. Two perfectly elastic particles *A* and *B* of equal masses travelling along the line joining them with velocity 15 ms^{-1} and 10 ms^{-1} respectively collide. Their velocities after the elastic collision will be (in ms^{-1}) respectively

(a) 20 and 5 (b) 10 and 15
(c) 5 and 20 (d) 0 and 25

44. A neutron travelling with a velocity *v* and kinetic energy *KE* collides elastically head on with the nucleus of an atom of mass number *A* at rest. The fraction of total energy retained by the neutron is

(a) $\left(\frac{A}{A+1}\right)$ (b) $\left(\frac{A}{A+1}\right)^2$
(c) $\left(\frac{A-1}{A+1}\right)$ (d) $\left(\frac{A-1}{A+1}\right)^2$

45. A massive ball moving with a speed *v* collides with a tiny ball having a very small mass then immediately after the impact, the second ball will move with a speed approximately equal to

(a) ∞ (b) *v*/2
(c) *v* (d) 2*v*

46. A sphere of mass *m* moving with a constant velocity *v* hits another stationary sphere of the same mass. If *e* is the coefficient of restitution, then the ratio of the velocities of the two spheres after collision will be

(a) $\left(\frac{e+1}{e-1}\right)$ (b) $\left(\frac{e-1}{e+1}\right)$
(c) $\left(\frac{1-e}{e+1}\right)$ (d) $\left(\frac{1+e}{e-1}\right)$

47. A system consists of mass M and m. The centre of mass of the system is
(a) nearer to m.
(b) at the position of large mass.
(c) nearer to M.
(d) in the middle.

48. A man weighing 80 kg is standing on a trolley weighing 320 kg. The trolly is resting on frictionless horizontal rails. If the man starts walking on the trolley along the rails at a speed $1 ms^{-1}$ then after 4 second his displacement relative to the ground will be
(a) 3.0 m (b) 3.2 m
(c) 4.8 m (d) 5 m

49. Two skaters A and B of mass 50 kg and 70 kg respectively stand facing each other 6 m apart. They then pull on a rope stretched between them. How far has each moved when they meet?
(a) A moves 3.5 m and B 2.5 m
(b) A moves 2 m and B 4 m
(c) both have moved 3 m
(d) both have moved 2.5 m

50. Two carts on horizontal straight rails are pushed apart by an explosion of a powder charge Q placed between the carts. Suppose the coefficient of friction of the two are equal and 200 kg cart travels a distance of 36 m and stops. The distance covered by the cart weighing 300 kg is
(a) 12 m (b) 16 m
(c) 24 m (d) 32 m

51. Two particles of mass m and 2 m are at a distance d apart. Under their mutual gravitational force they start moving towards each other. The acceleration of their centre of mass when they are $d/2$ apart is
(a) 89 m/d^2 (b) 44 m/d^2
(c) 29 m/d^2 (d) zero

52. In case of explosion of a bomb which of the following does not change?
(a) Chemical energy (b) Energy
(c) Kinetic energy (d) Mechanical energy

53. Which of the following is nonconservative force?
(a) Electric force (b) Elastic force
(c) Viscous force (d) Gravitational force

54. A ball of mass m moving with velocity v collides elastically with wall and rebounds. The change in momentum of the ball will be
(a) 4 mv (b) 2 mv
(c) mv (d) zero

55. A nucleus of mass number A originally at rest emits α particle with speed v. What will be the recoil speed of the daughter nucleus?
(a) $\frac{v}{A-4}$ (b) $\frac{v}{A+4}$
(c) $\frac{4v}{A-4}$ (d) $\frac{4v}{A+4}$

56. A ball is dropped from a height h. It rebounds from the ground a number of times. Given that the coefficient of restitution is e, to what height does it go after n^{th} rebounding?
(a) h/e^n (b) h/e^{2n}
(c) he^n (d) he^{2n}

57. A moving mass of 8 kg collides elastically with a stationary mass of 2 kg. If KE be the initial kinetic energy of the moving mass, the kinetic energy left with it after the collision will be
(a) 0.08 KE (b) 0.36 KE
(c) 0.64 KE (d) 0.80 KE

58. In the elastic collision of heavy vehicle moving with a velocity $10 ms^{-1}$ and a small stone at rest, the stone will fly away with a velocity equal to
(a) $40 ms^{-1}$ (b) $20 ms^{-1}$
(c) $10 ms^{-1}$ (d) $5 ms^{-1}$

59. A ball strikes against the floor and returns with double the velocity. In which type of collision is it possible?
(a) Inelastic (b) Perfectly inelastic
(c) Perfectly elastic (d) Not possible

60. A ball kept in a closed container moves in it making collision with the walls. The container is kept on a smooth surface. The velocity of the centre of mass of
(a) the ball remains fixed.
(b) the ball relative to container remains fixed.
(c) the container remains fixed.
(d) both container and ball remain fixed.

61. A ball A of mass m is moving with a velocity v along north. It collides with another ball B of same mass moving with velocity v along east. After the collision both balls stick together and move along northeast. The velocity of the combination is
(a) $v/\sqrt{2}$ (b) v
(c) $\sqrt{2}\ v$ (d) $2\ v$

62. A ball moving with velocity of $9 ms^{-1}$ collides with another similar stationary ball. After the collision both the balls move in directions making an angle of 30° with the initial direction. After the collision their speed will be
(a) $0.52 ms^{-1}$ (b) $2.6 ms^{-1}$
(c) $5.2 ms^{-1}$ (d) $52 ms^{-1}$

63. The physical quantiy which is conserved for all types of collision is
(a) KE (b) M
(c) $\vec{p}$ (d) L

64. Three particles α, β and γ of equal mass are moving with the same velocity v along the medians of an equilateral triangle. These particles collide at the centreoid G of a triangle. After collision α becomes stationary, β retraces its path with velocity v then the magnitude and direction of γ will be

(a) v and in the direction of β
(b) v and in the direction of γ
(c) v and opposite to β
(d) v and in the direction of α

65. Two particles each of mass m and travelling with velocities u_1 and u_2 collide perfectly inelastically. The loss of energy will be

(a) $\frac{1}{4} m (u_1 - u_2)^2$ (b) $\frac{1}{2} m (u_1 - u_2)^2$
(c) $2m (u_1 - u_2)^2$ (d) $m (u_1 - u_2)^2$

66. A particle of mass m_1 and moving with velocity v_1 collides head on with another stationary particle of mass m_2 elastically. If after the collision their velocities are v_1 and v_2 then under the condition $m_1 = m_2$ their values will be

(a) $v_1 = 0, v_2 = 0$ (b) $v_1 = u_1, v_2 = u_2$
(c) $v_1 = 0, v_2 = u_1$ (d) $v_2 = 0, v_1 = u_1$

67. A neutron, with kinetic energy KE and moving with velocity v, collides head on with a nucleus of mass number A perfectly elastically. The fraction of total energy possessed by the neutron transferred to nucleus is

(a) $\frac{4A}{(A-1)^2}$ (b) $\left(\frac{A-1}{A+1}\right)^2$
(c) $\left(\frac{A+1}{2A}\right)^2$ (d) $\left(\frac{A-1}{2A}\right)^2$

68. Two similar balls P and Q having velocities of 0.5 ms^{-1} and -0.3 ms^{-1} respectively collide elastically. The velocities of P and Q after the collision will respectively be

(a) 0.3 ms^{-1} and 0.5 ms^{-1}
(b) -0.5 ms^{-1} and 0.3 ms^{-1}
(c) 0.5 ms^{-1} and 0.3 ms^{-1}
(d) -0.3 ms^{-1} and 0.5 ms^{-1}

69. A ball falls from a height of 5 m and strikes the roof of a lift. If at the time of collision, lift is moving in the upward direction with a velocity of 1ms^{-1}. Then the velocity with which the ball rebounds after collision will be

(a) 13 ms^{-1} upwards (b) 12 ms^{-1} downwards
(c) 12 ms^{-1} upwards (d) 11 ms^{-1} downwards

70. A particle of mass m strikes a wall with a velocity v making an angle θ with the wall and rebounds. The change in momentum of the particle will be

(a) $-2m\vec{v}\cos\theta$ (b) 0
(c) $2m\vec{v}$ (d) $2m\vec{v}\sin\theta$

71. A ray of energy 14.2 MeV is emitted from a ^{60}Co nucleus. The recoil energy of the Co nucleus is nearly

(a) 3×10^{-16} J (b) 3×10^{-15} J
(c) 3×10^{-14} J (d) 3×10^{-13} J

72. Two masses m_1 and m_2 are connected to a spring of spring constant k at two ends. The spring is compressed by y and released. The distance moved by m_1 before it comes to a stop for the first time is

(a) $\frac{m_1 y}{m_1 + m2}$ (b) $\frac{m_2 y}{m_1 + m2}$
(c) $\frac{2m_1 y}{m_1 + m2}$ (d) $\frac{2m_2 y}{m_1 + m2}$

73. A radioactive nucleus decays by β emission. Both β and neutrino move mutually at right angels with momentum 6×10^{-21} kg ms^{-1} and 3×10^{-21} kg ms^{-1}. The direction of recoil of nucleus with respect to electron is

(a) $\tan^{-1}\left(\frac{1}{2}\right)$ (b) $\tan^{-1}(2)$
(c) $180 - \tan^{-1}\left(\frac{1}{2}\right)$ (d) $180 - \tan^{-1}(2)$

Answers to Questions for Practice

1.	(a)	**2.**	(b)	**3.**	(a)	**4.**	(a)	**5.**	(b)	**6.**	(c)	**7.**	(a)
8.	(c)	**9.**	(a)	**10.**	(c)	**11.**	(a)	**12.**	(b)	**13.**	(a)	**14.**	(d)
15.	(d)	**16.**	(b)	**17.**	(a)	**18.**	(b)	**19.**	(b)	**20.**	(c)	**21.**	(a)
22.	(a)	**23.**	(a)	**24.**	(d)	**25.**	(d)	**26.**	(c)	**27.**	(d)	**28.**	(d)
29.	(b)	**30.**	(d)	**31.**	(c)	**32.**	(c)	**33.**	(a)	**34.**	(b)	**35.**	(a)
36.	(a)	**37.**	(a)	**38.**	(c)	**39.**	(a)	**40.**	(b)	**41.**	(a)	**42.**	(d)
43.	(b)	**44.**	(d)	**45.**	(d)	**46.**	(c)	**47.**	(c)	**48.**	(b)	**49.**	(a)
50.	(c)	**51.**	(d)	**52.**	(b)	**53.**	(c)	**54.**	(b)	**55.**	(c)	**56.**	(d)
57.	(b)	**58.**	(b)	**59.**	(d)	**60.**	(d)	**61.**	(a)	**62.**	(c)	**63.**	(c)
64.	(c)	**65.**	(a)	**66.**	(c)	**67.**	(a)	**68.**	(d)	**69.**	(c)	**70.**	(d)
71.	(a)	**72.**	(d)	**73.**	(c)								

Explanations

1(a) Accleration of a rocket is given by

$$a = \frac{F}{(M_0 - mt)}$$

With the burning of fuel, mass of rocket decreases but engine keeps the force constant so acceleration is picked up in the manner shown in (A)

2(b) When boys jump one by one total velocity of the system will be more. (because from conservation of momentum, velocity of car in this case will be

$$u_{car} = u\left(\frac{1}{M+2m} + \frac{1}{M+m}\right)$$

instead of $mu\left(\frac{1}{M+2m} + \frac{1}{M+2m}\right)$

when they jump simultaneously.

3(a) From conservation of momentum

$$Mu = (M + mt)V$$

or $$V = \frac{Mu}{M+mt}$$

vertical down pour of rain does not effect the horizontal motion of carriage but addition of mass of water to mass of the carriage changes the velocity.

4(a) By conservation of momentum

$(m + 2m)u = mu + 2mV$

or $3\,mu = mu + 2mV$

or $V = u$ The vertical motion does not affect the horizontal motion. Hence the trolley moves with u.

5(b) Since bogie A stops after exchange of positions of boys, we get

$(M + mu)u_A - Mu_A - Mu_B = 0$

($\because$ boy in A carries away momentum and boy in B brings in $-ve$ momentum)

or $Mu_A = Mu_B$

For bogie B

$(M + m)\,u_B$ ....(i)

For bogie B

$(M + m)u_B - mu_B - mu_A = (M + m)v_B$

or $Mu_B - Mu_A = (M + m)v_B$ ....(ii)

From (i) and (ii)

$$u_A = \frac{mv_B}{M-m} \text{ and } u_B = \frac{mv_B}{M-m}$$

6(c) By conservation of momentum

$$mu = \frac{mu_1}{3} + \frac{2}{3}mu_2$$

or $3u = u_1 + 2u_2$ ....(i)

also additional energy

$$= \left(\frac{1}{2}\frac{mu_1^2}{6} + \frac{12}{2}\frac{2mu_2^2}{6}\right) - \frac{1}{2}mu^2$$

or $$4mu^2 = \frac{mu_1^2}{6} + \frac{2mu_2^2}{6} - \frac{1}{2}mu^2$$

or $24\,mu^2 = mu_1^2 + 2mu_2^2 - 3mu^2$

or $27\,mu^2 = mu_1^2 + 2mu_2^2$

Solving (i) and (ii)

$u_1 = 5\,u$ and $u_2 = -u$

$\therefore$ Relative velocity $= 5u - (-u) = 6u$

7(a) Work done by man,

$$mgL = mg\,(L - s) + mgs$$

Where $mg\,(L - s)$ is the increase in potential of energy of the man and (mgs) is the increase in potential energy of the balloon because the balloon would have been lifted up but for the climbing of the man.

$$\therefore \frac{\text{Increase in P.E. of man}}{\text{Increase in P.E. of balloon}} = \frac{mg(L-s)}{mgs} = \frac{L-s}{s}$$

8(c) At D. $K.E. = \frac{1}{2}mv^2 - \mu_k mg\,(BD)$

$= \frac{1}{2} \times 0.5 \times 3^2 - 0.2 \times 0.5 \times 10 \times 2.14$

$= 2.25 - 2.14 = 0.11$ J

Let the spring be pressed by distance x, then

$$0.11 = \frac{1}{2}kx^2 + \mu_R\,mgx$$
$$= \frac{1}{2} \times 2 \times x^2 + 0.2 \times 0.5 \times 10 \times x$$

or $x^2 + x - 0.11 = 0$ i.e. $x = 0.1$ m

Total distance covered

$= 2 + 2.14 + 0.1 = 4.24$ m.

9(a) By the principle of conservation of momentum, $mV\cos\theta = \frac{m}{2}V\cos\theta + \frac{m}{2}V''$

i.e. $\left(1 + \frac{1}{2}\right)mV\cos\theta = \frac{m}{2}V''$

i.e. $V'' = 3V\cos\theta$

10(c) $\left|(m_1\vec{v}_1' + m_2\vec{v}_2') - (m_1\vec{v}_1 + m_2\vec{v}_2)\right|$

= change in momentum

$= \int F dt = 2\,(m_1 + m_2)\,g\,t_0$

48(b) $80\,(4) = 400\,(\boldsymbol{x})$ or $x = \dfrac{4}{5}\, m$

Displacement $= 4 - \dfrac{4}{5} = 3.2\, m$.

62(c)

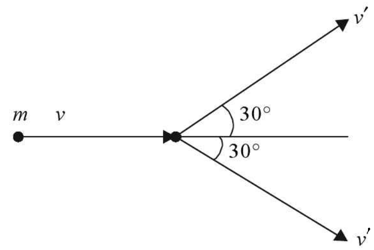


Fig. 7.28

$2\, m\, v' \cos 30 = mv$

$v' \sqrt{3} = 9$

or $v = \dfrac{9}{\sqrt{3}} = 5.2^{-1}$

71(a) $Pr = Pco' \; Pr = \dfrac{E}{c}$,

$$KE = \frac{p^2}{2m}$$

$$= \frac{E^2}{2c^2 m_{co}}$$

$$= \frac{\left(14\times1.6\times10^{-13}\right)^2}{2\times(3\times10^8\times60\times1.67\times10^{-27})}$$

$= 3 \times 10^{-16}$ J

8

Rotational Motion

BRIEF REVIEW

In Rotation motion we consider pure rotation and rolling. Rolling is basically rotational motion + linear motion.

Moment of Inertia (MOI) Moment of inertia is a tensor. It plays the same role in rotational motion as mass in the linear motion. Moment of inertia $I = \Sigma m_i r_i^2$.

$I = \int r^2 dm$ if mass is uniformly distributed; $I = Mk^2$ where M is total mass of the body and k is radius of gyration.

Radius of Gyration (k) It is the root mean square perpendicular distance of the body from axis of rotation.

$$k = \sqrt{\frac{m_1 r_1^2 + m_2 r_2^2 + ... + m_n r_n^2}{m_1 + m_2 + ... + m_n}}$$

$$= \frac{1}{M} \int r^2 dm$$

Given below are MOI of the bodies about an axis passing through their COM (centre of mass) and perpendicular to the plane of the body.

MOI of a Ring $I_{ring} = MR^2$

MOI of a disc (solid) $I_{disc} = \frac{MR^2}{2}$

MOI of an annular disc (Fig. 8.1)

$$I_{\text{Annular disc}} = \frac{M}{2}\left(R_1^2 + R_2^2\right).$$

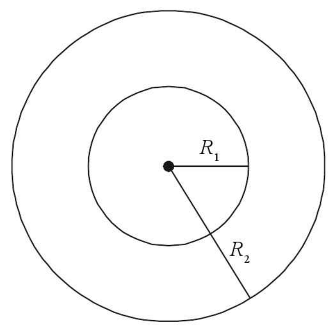


Fig. 8.1 Annular disc

MOI of a solid cylinder $I_{cylinder} = \frac{MR^2}{2}$

MOI of a hollow cylinder = MR^2 (if shell type, i.e., extremely thin walls).

$$= \frac{M}{2}\left(R_1^2 + R_2^2\right) \text{ (Fig. 8.2).}$$

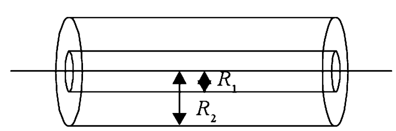


Fig. 8.2 Hollow cylinder

MOI of a spherical shell = $\frac{2}{3} MR^2$

MOI of a solid sphere = $\frac{2}{5} MR^2$

MOI of a hollow sphere = $\frac{2}{5} M \left(R_1^2 + R_2^2\right)$

MOI of a rod (cylindrical) = $\frac{M l^2}{12}$ (Fig. 8.3)

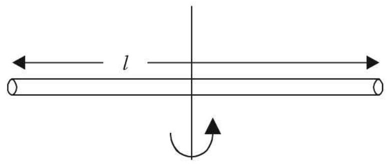


Fig. 8.3 MOI of a rod

MOI of a rod (rectangular) = $\frac{M(l^2 + b^2)}{12}$

MOI of a Lamina (rectangular) = $\frac{M}{12}(l^2 + b^2)$

MOI of a parallelopiped = $\frac{M}{12}(l^2 + b^2)$ (Fig. 8.4)

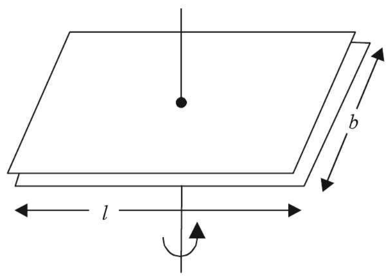


Fig. 8.4 Rectangular lamina

MOI of an elliptical disc = $\frac{M}{4}(a^2 + b^2)$ (Fig. 8.5)

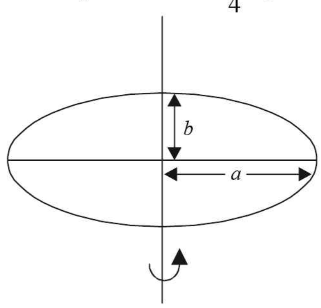


Fig. 8.5 Elliptical disc

MOI of a cone (Right circular cone) = $\frac{3}{10} MR^2$ (Fig. 8.6)

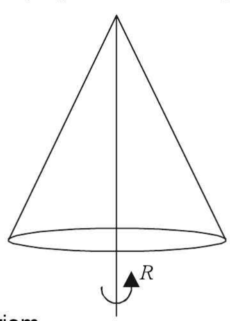


Fig. 8.6 MOI of prism

MOI of a prism or equilateral triangle = $\frac{M l^2}{6}$

MOI of a triangular lamina (about base)

$= \frac{M b^2}{6}$ (see Fig. 8.7)

MOI of a triangular lamina about perpendicular

$I_p = \frac{M p^2}{6}$.

MOI of a triangular lamina about hypotenuse

$I_h = \frac{mb^2 p^2}{6(p^2 + b^2)}$.

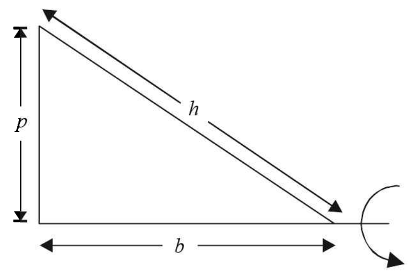


Fig. 8.7 MOI of a triangular lamina

MOI of a cone about XOX' = $\frac{3}{5} M \left(\frac{R^2}{4} + h^2\right)$ (Fig. 8.8)

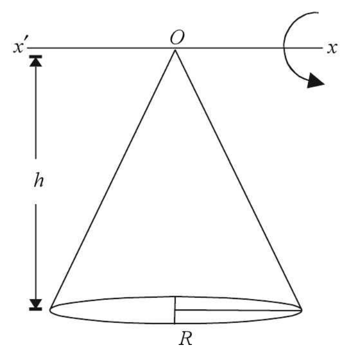


Fig. 8.8 MOI of a cone

MOI of a rod about one end = $\frac{Ml^2}{3}$ [Fig. 8.9]

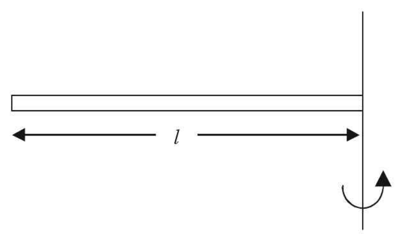


Fig. 8.9 MOI of a rod

Parallel axis theorem If MOI about an axis passing through COM of a body is known, the MOI of the body about an axis parallel to the axis passing through COM and at a distance x from it as illustrated in Fig. 8.10 is

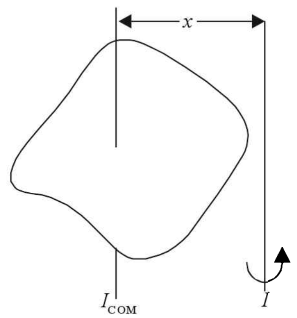


Fig. 8.10 Parallel axis theorem illustration

$I = I_{COM} + Mx^2$ where I_{COM} is the MOI about an axis passing through their COM.

Perpendicular Axis Theorem It can be applied only to plane lamina bodies. If x- and y- axes chosen in the plane of the body and z-axis be perpendicular to this plane, these being mutually perpendicular, then

$I_z = I_x + I_y$ where I_x and I_y are MOI about x-axis and y-axis respectively.

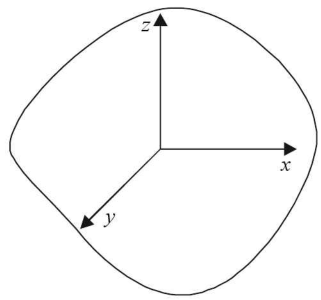


Fig. 8.11 Perpendicular axis theorem illustration

Angular velocity (instantaneous) $\omega = \dfrac{d\theta}{dt}$

Angular acceleration $\alpha = \dfrac{d\omega}{dt} = \dfrac{d^2\theta}{dt^2}$

Linear velocity $v = r\omega$; tangential acceleration $a_t = r\alpha$

$$\omega = \omega_0 + \alpha t \quad ; \theta = \omega_0 t + \frac{1}{2}\alpha t^2 ; \omega^2 = \omega_0^2 + 2\alpha\theta$$

Torque (τ) $\vec{\tau} = \vec{r} \times \vec{F} = I\alpha$

$|\vec{\tau}| = \Sigma$ Force $\times$ perpendicular from the axis of rotation.

$\vec{\tau} = \dfrac{dL}{dt}$ where L is angular momentum.

Note: Torque is moment of a force about a point. Though dimensions of torque are same as that of energy but it is not energy. Its unit is N–m. Dimensional formula is $[ML^2 T^{-2}]$.

If line of action of a force passes through its COM then such a force will not form torque.

Angular Momentum is moment of momentum (linear) about a point, i.e., $\vec{L} = \vec{r} \times \vec{p}$

$\vec{L} = I\omega$; $|\vec{L}| = \Sigma p \times$ (perpendicular distance from axis of rotation).

Note: If external torque is zero then angular momentum is conserved.

Dimensional formula of $L = [ML^2 T^{-1}]$. Its unit is kg m^2 s^{-1} and is same as that of Planck's constant h.

Angular impulse $J = \int_{t_1}^{t_2} \tau . dt = \Delta L = L_2 - L_1$

Rotational kinetic energy $= \dfrac{1}{2} I\omega^2$.

Note if a body only rotates about a fixed axis then it posseses only rotational KE. If, however, a body rolls then it possesses both rotational KE and linear KE, i.e. **Total KE**

$$= \textbf{Rotational KE} + \textbf{Linear KE} = \frac{1}{2} I\omega^2 + \frac{1}{2} mv^2$$

Work done $W = \int \vec{\tau} \, . \, d\vec{\theta}$; Rotational Power $P_{rot} = \vec{\tau} \, . \, \vec{\omega}$.

Acceleration of a body rolling down an incline plane: In Fig. 8.12

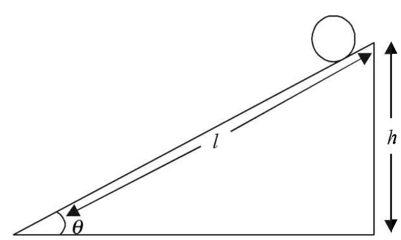


Fig. 8.12 Acceleration of a body rolling down an incline

$$a = \frac{g\sin\theta}{1 + \dfrac{I}{MR^2}} = \frac{g\sin\theta}{1 + \dfrac{k^2}{R^2}}$$

Velocity on reaching ground $v = a \, . \, t = \sqrt{\dfrac{2gh}{1 + \dfrac{k^2}{R^2}}}$

Time taken to reach the ground $t = \sqrt{\dfrac{2l\left(1 + \dfrac{k^2}{R^2}\right)}{g\sin\theta}}$

For a system to be in rotational equilibrium $\Sigma\vec{\tau} = 0$

For a system to be in linear equilibrium $\Sigma F = 0$

For total equilibrium (Rotational + Linear)

$\Sigma\tau = 0, \Sigma F = 0$.

Combined motion (Rotation + translation)

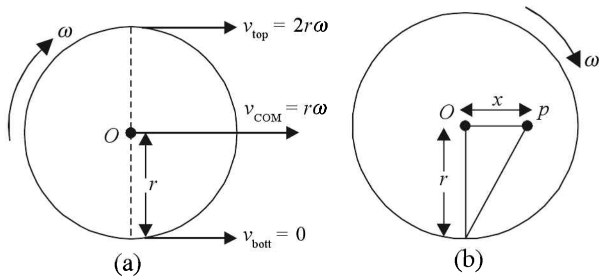


Fig. 8.13 Velocities of different points of a wheel

$$\vec{a}_{COM} = \frac{F_{ext}}{M} \text{ and } \alpha = \frac{\tau_{COM}^{ext}}{I_{COM}}$$

These equations together with initial conditions completely define the motion. τ_{COM}^{ext} is external torque about COM.

$$v_{COM} = r\omega$$

$$v_{bott} = 0\,;\ v_{top} = 2\,v_{COM} = 2r\omega \text{ [See Fig. 8.12 (a)]}$$

In Fig. 8.12 (b) in pure rolling $v_p = \omega\sqrt{r^2 + x^2}$.

Pure rolling $v_{COM} = r\omega$, the wheel completes 1 rotation and covers a distance $= 2\pi r$.

Rolling with forward slipping If the wheel moves a distance $> 2\pi r$ in one complete rotation then $v_{COM} > r\omega$ and motion is termed as rolling with forward slipping.

Rolling with backward slipping If the wheel moves a distance $< 2\pi r$ in one complete rotation then $v_{COM} < r\omega$ and motion is known as rolling with backward slipping.

Angular momentum of a body in combined rotation and translation:

$L = L_{COM} + M\left(\vec{r}_0 \times \vec{v}_0\right)$ where $M\left(\vec{r}_0 \times \vec{v}_0\right)$ is assumed to be the angular momentum as if mass is concentrated at COM and translating with v_0. In an accelerating wheel force of friction acts in the direction of motion. So that frictional torque acts in a direction to oppose the accelerating torque.

If the wheel is rolling with forward slipping then force of friction acts in a direction opposite to the motion of the wheel until pure rolling begins.

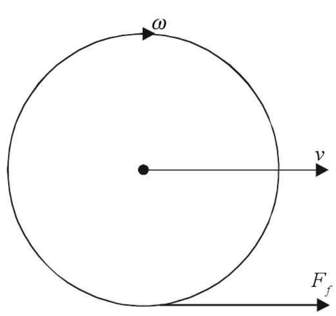


Fig. 8.14 Friction in accelerating wheel

Table 8.1. Equivalence between rotational and translation motion

Linear motion	Rotational motion
Displacement $= x$	Angular displacement $= \theta$
Linear velocity $v = \frac{dx}{dt}$	Angular velocity $\omega = \frac{d\theta}{dt}$
Acceleration $a = \frac{dv}{dt}$	Angular acceleration $\alpha = \frac{d\omega}{dt}$
mass $= m$	MOI $= I$
Linear momentum $p = mv$	Angular momentum $L = I\,\omega$
Force $F = ma$	Torque $\tau = I\,\alpha$
Impulse $I = \int F dt = \Delta p$	Rotational impulse $J = \int \tau\, dt$
Work $W = \int \vec{F}.d\vec{x}$	Work $W = \int \vec{\tau}.d\vec{\theta} = \Delta L$
KE $= \frac{1}{2} mv^2$	Rotational KE $= \frac{1}{2} I\,\omega^2$
Power $P = \vec{F}\,.\,\vec{v}$	Rotational Power $= \vec{\tau}\,.\,\vec{\omega}$

Three-dimensional rotation is understood from gyrostat.

A spinning top shows

(i) spinning (ii) precession (iii) nutation or wobbling.

Hipparchus in 135 BC found that due to precession of earth ($T_{Precession} = 27{,}725$ yrs) a change in the direction of the line of equinoxes occurs and phenomenon is called precession of equinoxes.

- **Short Cuts and Points to Note**

1. MOI of a ring about a diameter $= \frac{MR^2}{2}$ (about XY).

MOI of a ring about the tangent TT' parallel to diameter XY is $\frac{MR^2}{2} + MR^2$

$= \frac{3}{2} MR^2$ (See Fig. 8.15).

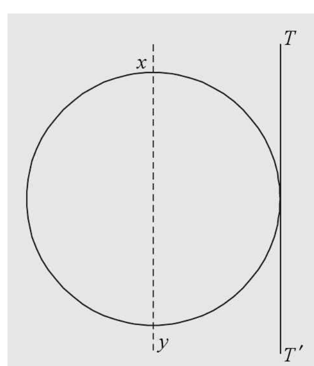


Fig. 8.15

2. MOI of the disc about one of the diameters = $\frac{MR^2}{4}$.
MOI of the disc about the tangent parallel to one of diameters = $\frac{5}{4}MR^2$
3. MOI of a ring about a tangent perpendicular to the plane of the ring = $2MR^2$ (See Fig. 8.16).

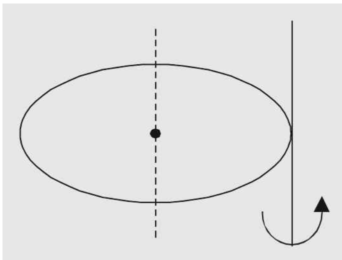


Fig. 8.16

MOI of a disc about a tangent perpendicular to the plane of the disc = $MR^2 + \frac{MR^2}{2} = \frac{3}{2}MR^2$
4. MOI is a tensor. Its value may vary with the direction. However, they are added like scalars.
5. MOI of hollow bodies is higher than MOI of solid bodies.
6. Acceleration of bodies rolling down an inclined plane is $a = \frac{g\sin\theta}{1+\frac{I}{MR^2}} = \frac{g\sin\theta}{1+\frac{k^2}{R^2}}$.
7. If bodies of equal radius roll down an inclined plane then a sphere will reach first (take minimum time) and ring will reach the last (take maximum time. Note, acceleration $\propto \frac{1}{I}$. Hence, more the MOI lesser will be the acceleration or higher is the time to roll down.

The minimum value of coefficient of friction required to roll down on incline plane is

$$\mu = \frac{\frac{I}{MR^2}\tan\theta}{1+\frac{I}{MR^2}}$$

8. While deciding about which axis MOI is maximum, consider $\Sigma\, m_i r_i^2$. The axis about which $\Sigma\, m_i r_i^2$ is large will have longer values of r for equal mass or heavier mass located farther.
9. Rotational KE = $\frac{1}{2}I\omega^2$ and total KE when a body is rolling is $KE_{Tot} = \frac{1}{2}I\omega^2 + \frac{1}{2}mv^2$.
10. x-component of the torque is

$$\hat{i}\,(F_z y - F_y z) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & y & z \\ 0 & F_y & F_z \end{vmatrix}.$$

Similarly, other components can be written y component is $-\hat{j}\,(F_z x - F_x z)$ and z component is $\hat{k}\,(F_y x - F_x y)$.
11. If mass of the pulley is m_p, thread is massless and pulley is smooth then

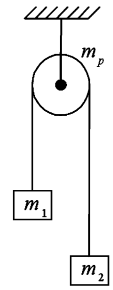


Fig. 8.17

$$a = \frac{(m_2 - m_1)g}{m_1 + m_2 + \frac{m_p}{2}} = \frac{(m_2 - m_1)g}{\left(m_1 + m_2 + \frac{I}{R^2}\right)} \text{ where}$$

I is MOI of the pulley and R its radius.
12. The system is in rotational equilibrium if $\Sigma\,\vec{\tau} = 0$ and it is also in linear equilibrium if $\Sigma F = 0$.
13. Apply conservation of angular momentum if $\vec{\tau}_{ext} = 0$.
14. A body rolling with forward slipping has friction in a direction opposite to its motion untill pure rolling begins. v_{COM} decreases and ω increases.
15. A body rolling with backward slipping has friction in the direction of motion so as to increase v_{COM} until $v_{COM} = r\omega$. That is v_{COM} increases and ω decreases.
16. When a body is accelerating with angular acceleration α, i.e., torque is applied then friction acts in the direction of motion as shown in Fig. 8.18 so that frictional torque opposes the accelerating torque.

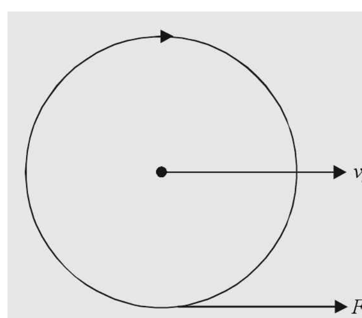


Fig. 8.18

17. In pure rolling $v_{COM} = r\omega$. Note, if a body is to roll down an incline plane then the incline must be rough. On smooth incline rolling cannot occur.

18. Though torque has dimensions of energy. It is not energy. Unit is *N–m*.

19. Velocity of precession $\omega_P = \frac{\tau_P}{L} = \frac{dL}{dt\,L}$

$\left\{\because \tau = \frac{dL}{dt}\right\}$ when ω_P increases body is about to fall.

20. Angular speed ω of a gyrostatic pendulum and time period $T = 2\pi\sqrt{\frac{l\cos\theta}{g}}$, and, $\omega = \sqrt{\frac{g}{l\cos\theta}}$.

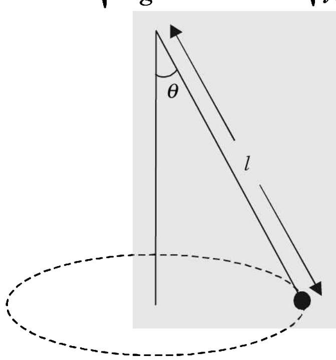


Fig. 8.19

21. If two rotating bodies having MOI I_1 and I_2 moving with speeds ω_1 and ω_2 join, then common angular

velocity $\omega = \frac{I_1\omega_1 + I_2\omega_2}{I_1 + I_2}$,

loss in KE $= \frac{I_1 I_2(\omega_1 - \omega_2)^2}{2(I_1 + I_2)}$.

• Caution

1. Adding MOI like vectors or taking its components while finding MOI of a composite body.

⇒ MOI is a tensor. It is added like scalar.

2. Considering acceleration of a body rolling down an incline as $g\sin\theta$.

⇒ Note $a = \frac{g\sin\theta}{1 + \frac{I}{MR^2}}$

3. Assuming in rotational motion $v = u + a\,t$, $s = u\,t + \frac{1}{2}\,a\,t^2$ etc can be applied.

⇒ Apply $\omega = \omega_0 + \alpha\,t$, $\theta = \omega_0\,t + \frac{1}{2}\,\alpha\,t^2$, $\omega^2 = \omega_0^{\,2} + 2\,\alpha\,\theta$

if there is pure rotation. If there is rolling apply both. For linear motion of COM apply $V_{COM} = u_{COM} + a_{COM}\,t$ and so on. During rolling i.e. combined motion both $\omega = \omega_0 + \alpha\,t$ etc. and $V_{COM} = u_{COM} + a_{COM}\,t$ are applied. To combine $a = r\alpha$ or $v = r\omega$ when pure rolling begins.

4. Considering frictional force stops rotation/rolling.

⇒ Rotational motion is stopped by frictional torque is not always true. If the body is rolling with forward slipping, friction acts in a direction to increase the rotational velocity or angular velocity.

Consider the case of a bicycle. The back wheel is paddled. The front wheel moves due to friction.

5. Considering solid bodies rotate more.

⇒ Hollow bodies have large MOI. They rotate more. This is the reason that all wheels are either made hollow or mass is concentrated at the rim.

6. Considering that a sphere can roll on a smooth inclined plane.

⇒ Minimum amount of coefficient of friction required is $\frac{2}{7}\tan\theta$, θ being angle of inclination.

7. Considering $v = r\omega$ in all cases of rolling.

⇒ $v = r\omega$ if there is pure rolling, i.e., rolling without slipping. In case of rolling with forward slipping $v > r\omega$ and in case of rolling with backward slipping $v < r\omega$.

8. When pulley is smooth but has mass, string is massless, considering mass of the pulley is redundant.

⇒ acceleration of blocks [see Fig. 8.20]

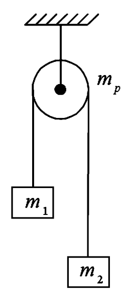


Fig. 8.20

$a = \frac{(m_2 - m_1)g}{\left(m_2 + m_1 + \frac{m_p}{2}\right)}$ where m_p is mass of the pulley. If MOI of the pulley is given then $a = \frac{(m_2 - m_1)g}{\left(m_2 + m_1 + \frac{I}{r^2}\right)}$

where I is MOI of the pulley.

9. Assuming perpendicular axis theorem (to find MOI) can be applied to any body.

⇒ Perpendicular axis theorem can be applied only to plane lamina. Parallel axis theorem is valid for all types of bodies.

10. Confused, what to do in rotational collision?

⇒ Conserve angular momentum.

SOLVED PROBLEMS

1. The moment of inertia of a uniform semicircular disc of mass M and radius R about a line perpendicular to the plane of disc and passing through the centre is

(a) $\frac{MR^2}{4}$ (b) $\frac{2}{5}MR^2$

(c) MR^2 (d) $\frac{MR^2}{2}$

[AIEEE 2005]

Solution (d) $2I = 2M\left(\frac{R^2}{2}\right)$ or $I = \frac{MR^2}{2}$.

2. A T shaped object with dimensions shown in figure, is lying on a smooth floor. A force F is applied at point P parallel to AB such that the object has only translational motion without rotation. Find location of P from C

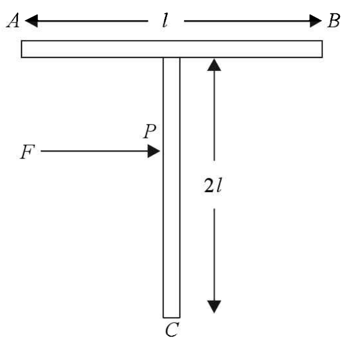


Fig. 8.21

(a) $\frac{2}{3}l$ (b) $\frac{3}{2}l$

(c) $\frac{4}{3}l$ (d) l

[AIEEE 2005]

Solution (c) P should be COM. Take C as origin.

$$x = \frac{2m(l) + m(2l)}{2m + m} = \frac{4l}{3}$$

3. A circular disc of radius $R/3$ is cut from a circular disc of radius R and mass $9M$ as shown. Then MOI of the remaining disc about O perpendicular to the disc is

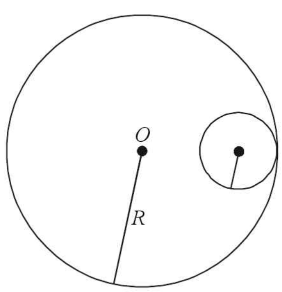


Fig. 8.22

(a) $4MR^2$ (b) $9MR^2$

(c) $\frac{37}{9}MR^2$ (d) $\frac{40}{9}MR^2$

[IIT Screening 2005]

Solution (a) $I = \frac{9MR^2}{2} - \left[\frac{M}{2}\left(\frac{R}{3}\right)^2 + M\left(\frac{2R}{3}\right)^2\right]$

$= 4MR^2$

mass of hole made $= M = \frac{9M}{\pi R^2}\left[\pi\left(\frac{R}{3}\right)^2\right]$

4. A sphere is rolling on a frictionless surface as shown in Fig 8.23 with a translational velocity v ms^{-1}. If it is to climb the inclined surface then v should be

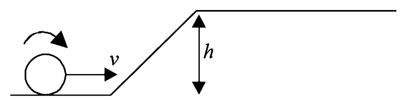


Fig. 8.23

(a) $\geq \sqrt{\frac{10}{7}gh}$ (b) $\geq \sqrt{2gh}$

(c) $2gh$ (d) $\frac{10}{7}gh$

[AIEEE 2005]

Solution (a) $\frac{1}{2}I\omega^2 + \frac{1}{2}mv^2 \geq mgh$

or $\frac{1}{2}\left(\frac{2}{5}mr^2\right)\omega^2 + \frac{1}{2}mv^2 \geq mgh$

or $v \geq \sqrt{\frac{10}{7}gh}$

5. A horizontal platform is rotating with uniform angular velocity around the vertical axis passing through its centre. At some instant a viscous fluid of mass m is dropped at the centre and is allowed to spread out and finally fall. The angular velocity during this period

(a) decreases continuously
(b) decreases initially and increases again
(c) remains unaltered
(d) increases continuously

[AIIMS 2005]

Solution (b) Using conservation of angular momentum.

6. A ladder is leaned against a smooth wall and allowed to slip on a frictionless floor. Which figure represents trace of its COM?

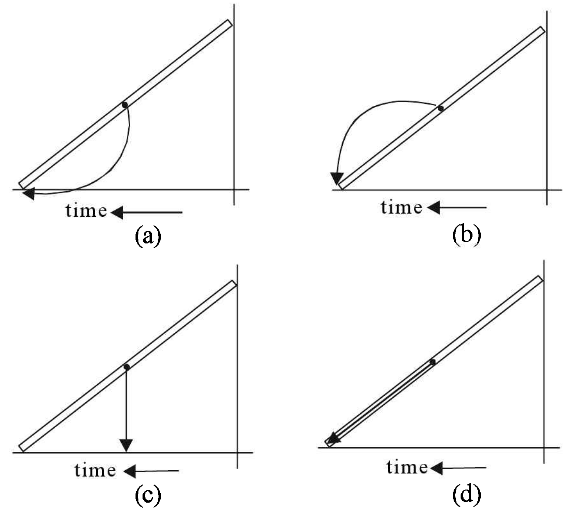


Fig. 8.24

[AIIMS 2005]

Solution (a)

7. The angular momentum of a rotating body changes from A_0 to $4A_0$ in 4 seconds. The Torque acting on the body is

(a) $\frac{3}{4}A_0$ (b) $4A_0$

(c) $3A_0$ (d) $\frac{3}{2}A_0$

[BHU PMT 2005]

Solution (a) $\tau = \frac{dL}{dt} = \frac{4A_0 - A_0}{4} = \frac{3A_0}{4}$

8. A wooden log of mass M and length L is hinged by a frictionless nail at O. A bullet of mass m strikes with velocity v and sticks to it, find the angular velocity of the system immediately after collision.

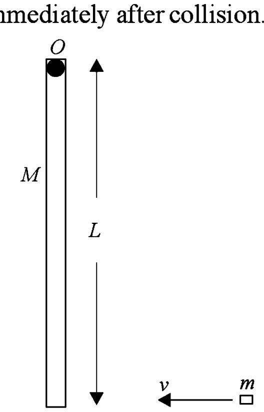


Fig. 8.25

[IIT Mains 2005]

Solution $(mv)L = \left(\frac{ML^2}{3} + mL^2\right)\omega$

or $\omega = \frac{3mv}{(M+3m)L}$.

9. A cylinder of mass m and radius R rolls down an incline plane of inclination θ. Find the linear acceleration of the axis of the cylinder.

[IIT Mains 2005]

Solution $a = \frac{g\sin\theta}{1+\frac{I}{MR^2}} = \frac{g\sin\theta}{1+\frac{1}{2}} = \frac{2}{3}g\sin\theta$

10. An electric motor exerts a constant torque 10 N–m on a grind stone mounted on a shaft. MOI of the grind stone about the shaft is 2 kg m^2. If the system starts from rest find the work done in 8s.

(a) 1600 J (b) 1200 J

(c) 800 J (d) 600 J

Solution (a) $\alpha = \frac{\tau}{I} = \frac{10}{2} = 5$ rad/s^2.

$\omega = \omega_0 + \alpha t = 5(8) = 40$ rad s^{-1}

$W = \Delta KE = \frac{1}{2}I\omega^2 - 0 = \frac{1}{2}\times 2\times(40)^2 = 1600$ J

11. The power output of an automobile engine is advertised to be 200 hp at 600 rpm. Find the corresponding torque

(a) 137 Nm (b) 237 Nm

(c) 337 Nm (d) 287 Nm

Solution (b) $\tau = \frac{P}{\omega} = \frac{200\times 746}{600\times\frac{2\pi}{60}} = 237$ N–m.

12. A cable is wrapped several times around a uniform solid cylinder that can rotate about its axis. The cylinder has diameter 12 cm and mass 50 kg. The cable is pulled with a force 9 N. Assuming cable unwinds without stretching or slipping, find its acceleration

(a) 0.3 ms^{-2} (b) 0.32 ms^{-2}

(c) 0.36 ms^{-2} (d) 0.4 ms^{-2}

Solution (c) $\tau = I\alpha = FR$ or $\alpha = \frac{FR}{I}$

$= \frac{2\times 9}{50\times(.06)} = 6$ rad s^{-2}

$a_t = R\alpha = 0.06\times 6 = 0.36$ ms^{-2}

13. A turbine fan in a jet engine has MOI 2.5 kg m^2 about its axis of rotation. Its angular velocity is $40t^2$. Find the net torque at any instant.

(a) $100t$ (b) $100t^2$

(c) $200t$ (d) $200t^2$

Solution (c) $\tau = \frac{dL}{dt} = \frac{d}{dt}(I\omega) = \frac{d}{dt}(2.5\times 40t^2)$

$= 200t$

14. A fly wheel of mass 2 kg and radius 20 cm has an angular speed 50 rad s^{-1} when a clutch plate of mass 4 kg, radius having an angular speed 200 rad s^{-1} is combined with it. Find the common speed of rotation.

(a) 125 rad s^{-1} (b) 150 rad s^{-1}
(c) 175 rad s^{-1} (d) 100 rad s^{-1}

Solution (d) $\omega = \dfrac{I_1\omega_1 + I_2\omega_2}{I_1 + I_2}$

$$= \frac{2(.2)^2(50) + 4(.1)^2(200)}{2(.2)^2 + 4(.1)^2}$$

$$= \frac{12}{0.12} = 100 \text{ rad s}^{-1}$$

15. A bicycle wheel has mass of the rim 1 kg and 50 spokes each of mass 5 g. If radius of the wheel is 40 cm then find the MOI of the wheel.

(a) 0.160 kg m^2 (b) 0.174 kg m^2
(c) 0.18 kg m^2 (d) 0.196 kg m^2

Solution (b) $I = MR^2 + 50\,\text{m}\,\dfrac{l^2}{3}$

$$= 1\,(0.4)^2 + \frac{50(5\times10^{-3})}{3}\left(\frac{0.4}{3}\right)^2$$

$$= 0.16\left[1 + \frac{0.25}{3}\right] = 0.174 \text{ Kgm}^2$$

16. A 2 kg rock has velocity 12 ms^{-1} when at point P as shown in Fig. 8.26. Find the angular momentum about point D.

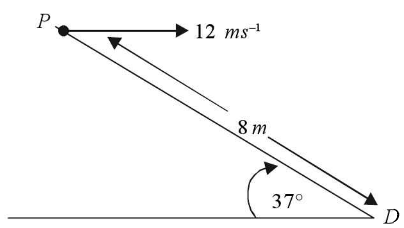


Fig. 8.26

(a) 115.2 kg m^2 s^{-1} (b) 125.2 kg m^2 s^{-1}
(c) 135 kg m^2 s^{-1} (d) none

Solution (a) $L = mv \times$ perpendicular distance

$$= 2 \times 12 \times 8 \times \frac{3}{5} = 115.2 \text{ kg m}^2\text{ s}^{-1}$$

17. A beam of length l lies on the $+x$ axis with its left end on the origin. A cable pulls the beam in y-direction with a force $F = F_0\left(1 - \dfrac{x}{l}\right)$. If the axis is fixed at $x = 0$ then find the torque.

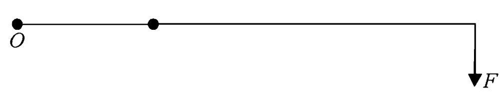


Fig. 8.27

(a) $-F_0 l$ (b) $\dfrac{F_0 l}{2}$
(c) $-\dfrac{F_0 l}{2}$ (d) none of these

Solution (b) Torque $d\tau = \int_0^l F\,x = \dfrac{F_0 l}{2}$

18. A solid cylinder of mass M and radius $2R$ is connected to a string through a frictionless yoke and axle. The string runs over a disk shaped pulley of mass M and radius R. The mass M is attached to the other end of the string. The cylinder rolls without slipping on the table top. Find the acceleration of the block after the system is released from rest.

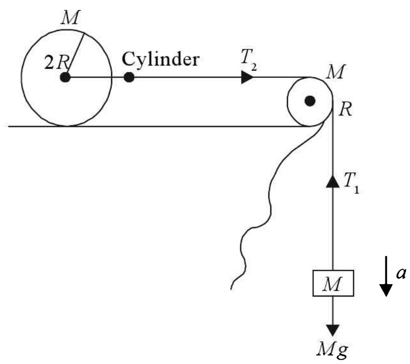


Fig. 8.28

(a) $\dfrac{2g}{5}$ (b) $\dfrac{g}{2}$
(c) $\dfrac{g}{3}$ (d) $\dfrac{g}{2}$

Solution (c) $2r\,\alpha_1 = r\,\alpha_2$ or $\alpha_2 = 2\,\alpha_1$

$$(T_2 - T_1)R = \left(\frac{MR^2}{2}\right)(2\alpha) \text{ or } T_2 - T_1 = Ma \quad ...(1)$$

$$Mg - T_1 = Ma \quad ....(2)$$

$$T_2(2R) = \frac{M(2R)^2}{2}(\alpha) \text{ or } T_2 = Ma \quad ....(3)$$

Adding (1), (2) and (3) $a = \dfrac{g}{3}$

19. A uniform disc of radius a has a hole of radius b at a distance c from the centre as shown. If the disc is free to rotate about a rod passing through the hole b, then find the MOI about the axis of rotation.

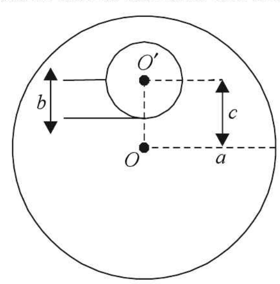


Fig. 8.29

(a) $\dfrac{M}{2}\left(a^2 + b^2 + \dfrac{2c^2a^2}{a^2 - b^2}\right)$

(b) $M\left(a^2+b^2+\frac{c^2a^2}{a^2-b^2}\right)$

(c) $\frac{M}{2}\left(a^2+b^2+\frac{c^2a^2}{a^2-b^2}\right)$ (d) none

Solution (a) Let ρ be the mass per unit area. Then MOI of the disc about O' $I = \pi\rho a^2\left(\frac{a^2}{2}\right) + \pi\rho a^2(c^2)$

$$-\pi b^2\rho\left(\frac{b^2}{2}\right) = \frac{\pi\rho a^2}{2}[a^2+2c^2] - \frac{\pi\rho b^4}{2}$$

$$= \frac{\pi\rho}{2}[a^2(a^2+2c^2)-b^4)$$

and $\rho = \frac{M}{\pi(a^2-b^2)}$

$$= \frac{M}{2}\left[\frac{a^4-b^4}{a^2-b^2}+\frac{2a^2c^2}{a^2-b^2}\right]$$

$$= \frac{M}{2}\left[a^2b^2+\frac{2c^2a^2}{a^2-b^2}\right]$$

20. Find the MOI of a uniform square plate of mass m and edge a about one of its diagonals.

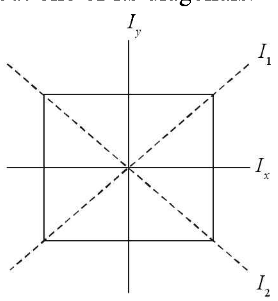


Fig. 8.30

(a) $\frac{Ma^2}{6}$ (b) $\frac{Ma^2}{3}$

(c) $\frac{Ma^2}{9}$ (d) $\frac{Ma^2}{12}$

Solution (d) $I_z = I_x + I_y = I_1 + I_2 = 2I_x = 2I_1$

or $I_1 = I_x = \frac{I_z}{2}$ $I_z = \frac{M}{12}(a^2+a^2) = \frac{Ma^2}{6}$

$\therefore I_1 = \frac{Ma^2}{12}$

21. Two spheres each of mass M and radius R are in contact as shown. Find the MOI if they are rotated about the common tangent.

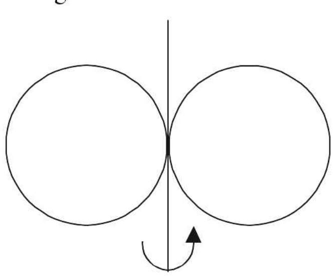


Fig. 8.31

(a) $\frac{7}{5}MR^2$ (b) $\frac{2}{3}MR^2$

(c) $\frac{4}{5}MR^2$ (d) $\frac{4}{3}MR^2$

Solution (b) $I = \left(\frac{2}{5}MR^2+MR^2\right)\times 2 = \frac{14}{5}MR^2$.

22. A boy of mass M stands on a platform of radius R capable to rotate freely about its axis. The moment of inertia of the platform is I. The system is at rest. The friend of the boy throws a ball of mass m with a velocity v horizontally. The boy on the platform catches it. Find the angular velocity of the system in the process.

(a) $\frac{mvR}{(M+m)R^2}$ (b) $\frac{mvR}{I+MR^2}$

(c) $\frac{mvR}{I+mR^2}$ (d) $\frac{mvR}{I+(M+m)R^2}$

Solution (d) $mvR = \left[I+(M+m)R^2\right]\omega$

or $\omega = \frac{mvR}{I+(M+m)R^2}$

23. A ball of steel rolls down an incline of inclination θ. Find the ratio of rotational KE to linear KE.

(a) $\frac{2}{7}$ (b) $\frac{2}{3}$

(c) $\frac{2}{5}$ (d) $\frac{5}{7}$

Solution (c) $\frac{\frac{1}{2}I\omega^2}{\frac{1}{2}mv^2} = \frac{2}{5}\frac{MR^2\omega^2}{Mv^2} = \frac{2}{5}$

24. The pulley shown in figure has MOI 0.5 kg m^2 and radius 10 cm. Assuming no friction anywhere, find the acceleration of 4 kg block

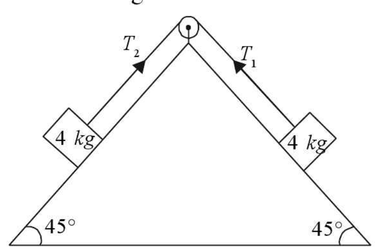


Fig. 8.32

(a) 1.11 ms^{-2} (b) 0.75 ms^{-2}

(c) 0.5 ms^{-2} (d) 0.25 ms^{-2}

Solution (d) $4g\cos 45 - T_1 = 4a$ ... (1)

$(T_1 - T_2)R = I\alpha$

or $T_1 - T_2 = \frac{I}{R^2}a$ ... (2)

$T_2 - 2g\cos 45 = 2a$ ... (3) Adding (1), (2) and (3)

$$2g\cos 45 = 6a + \frac{I}{R^2}\,a \text{ or } a = \frac{\sqrt{2}\times 10}{6+50} = 0.25\ \text{ms}^{-2}$$

25. A spherical shell of radius R is rolling down an incline of inclination θ without slipping. Find minimum value of coefficient of friction.

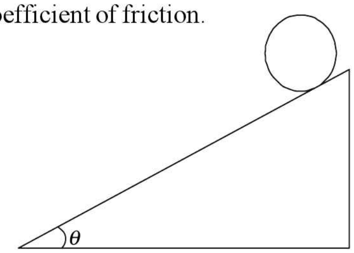


Fig. 8.33

(a) $\frac{2}{7}\tan\theta$ (b) $\frac{2}{5}\tan\theta$

(c) $\frac{2}{3}\tan\theta$ (d) none

Solution (b) $F_f\, r = I\,\alpha$

or $\mu mg\cos\theta = \frac{2}{3}Ma$

$$\mu mg\cos\theta = \frac{2}{3}M\,\frac{g\sin\theta}{1+\frac{2}{3}} \quad = \quad \mu = \frac{2}{5}\tan\theta$$

Shortcut $\mu = \dfrac{\frac{I}{MR^2}\tan\theta}{1+\frac{I}{MR^2}} = \frac{2}{5}\tan\theta$

TYPICAL PROBLEMS

26. A ball of radius r lies at the bottom of a vertical ring of radius R, find the minimum velocity to be given so that the ball completes the loop rolling without slipping.

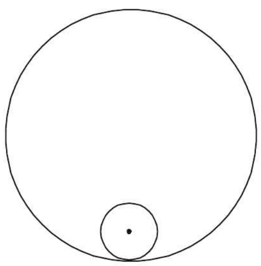


Fig. 8.34

(a) $\sqrt{5g(R-r)}$ (b) $\sqrt{\frac{27}{10}g(R-r)}$

(c) $\sqrt{\frac{27}{7}g(R-r)}$ (d) $\sqrt{\frac{27}{5}g(R-r)}$

Solution (c) $\frac{1}{2}mv_{\text{bott}}^2 + \frac{1}{2}I\omega_{\text{bott}}^2 = mg\,2(R-r) + \frac{1}{2}mv_{\text{top}}^2 + \frac{1}{2}I\omega_{\text{top}}^2$

$$mv_{\text{bott}}^2 + \frac{2}{5}mv_{\text{bott}}^2 = 4mg(R-r) + mv_{\text{top}}^2 + \frac{2}{5}mv_{\text{top}}^2$$

$$\frac{7}{5}mv_{\text{bott}}^2 = 4mg(R-r) + \frac{7}{5}mv_{\text{top}}^2 = 4mg(R-r) + \frac{7}{5}m[g(R-r)]$$

or $v_{\text{bott}} = \sqrt{\frac{27}{7}g(R-r)}$

27. The pulley shown in Fig. 8.35 has radius 20 cm and MOI 0.2 kg m^2. Spring used has force constant 50 N m^{-1}. The system is released from rest. Find the velocity of 1kg block when it has descended 10 cm.

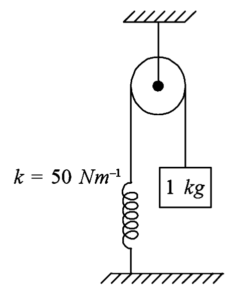


Fig. 8.35

(a) $\frac{1}{2}$ ms^{-1} (b) $\frac{1}{\sqrt{2}}$ ms^{-1}

(c) $\frac{1}{\sqrt{3}}$ ms^{-1} (d) none

Solution (a) $mgx = \frac{1}{2}kx^2 + \frac{1}{2}I\omega^2 + \frac{1}{2}mv^2$

$$1(10)(.1) = \frac{1}{2}\left[50(.1)^2 + (.2)\left(\frac{v}{.2}\right)^2 + 1(v^2)\right]$$

$$\Rightarrow 2 = 0.5 + 6v^2$$

or $v = \frac{1}{2}$ ms^{-1}

28. A thin spherical shell lying on a rough horizontal surface is hit by a cue in such a way that line of action passes through the centre of the shell. As a result shell starts moving with a linear speed v without any initial angular velocity. Find the linear velocity to the shell when it starts pure rolling.

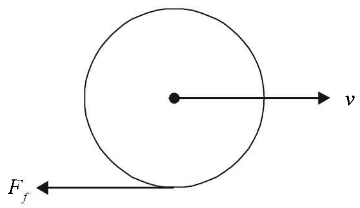


Fig. 8.36

(a) $\frac{3}{5}v$ (b) $\frac{2}{5}v$

(c) $\frac{4}{5}v$ (d) none

Solution (a) $v_f = v - \frac{F_f}{m}t$ ...(1)

$F_f r = I\alpha$

or $F_f r = \frac{2}{3}mr^2\alpha$

or $\alpha = \frac{3}{2}\frac{F_f}{mr}$ $\omega = 0 + \alpha t$

or $\omega = \frac{3}{2}\frac{F_f}{mr}t$

or $r\omega = v_f = \frac{3F_f}{2m}t$ ...(2)

From Eq (1) and (2) $v_f = \frac{3}{5}v$.

29. A uniform rod pivoted at upper end is released when it is making an angle of 60°. Find the radial force acting on a particle of mass *dm* at its tip when it makes an angle of 37° with the vertical.

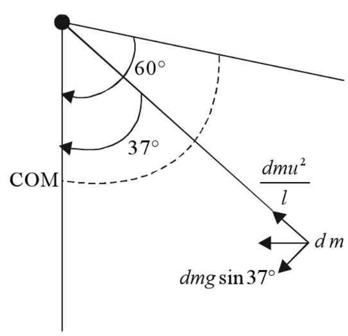


Fig. 8.37

(a) 0.6 *dm* g (b) 0.8 *dm* g

(c) 0.9 *dm* g (d) none

Solution (c) $\frac{1}{2}I\omega^2 = mg\frac{l}{2}(\cos 37 - \cos 60)$

or $\frac{Ml^2}{3}\omega^2 = mgl(.8 - .5)$

$\frac{mv^2}{l} = 0.9mg$ or $\frac{v^2}{l} = 0.9\,g$

$F_{rad} = dm\frac{v^2}{l} = 0.9\,(dm)\,g$

30. When a force 6 N is exerted at 30° to a wrench at a distance of 8 cm from a nut as shown in Fig 8.38, it is just able to loosen it. What force *F* is required to loosen the nut if applied 16 cm away to the wrench and normal to the wrench.

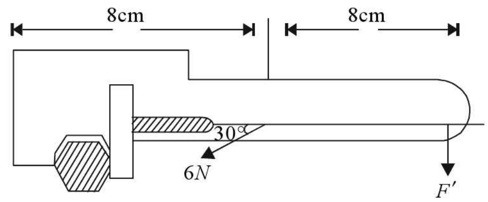


Fig. 8.38

(a) 3 N (b) $\sqrt{3}$ N

(c) 1.5 (d) none

Solution (c) $8 \times 6 \sin 30 = F \times 16$ or $F = 1.5$ N

31. Particles of mass 1 g, 2 g, 3 g, ..., 100 g are kept at the marks 1 cm, 2 cm, 3 cm, ... 100 cm respectively on a metre scale. Find the MOI of the system of particles about a perpendicular bisector of the metre scale.

Solution 0 g, 100 g; 1 g, 99 g; 2 g, 98 g; 3 g, 97 g; .... are equally spaced from the axis of rotation.

$\therefore I = (100)\,[1^2 + 2^2 + ... + 50^2]$

$= 0.1\,[1^2 + 2^2 + ... + 50^2] \times 10^{-4}$ kg m^2

$= 0.1 \times \frac{50 \times 51 \times 101}{6} \times 10^{-4}$

$= 0.43$ kg m^2 $\left\{\text{use}\sum n^2 = \frac{n(n+1)(2n+1)}{6}\right\}$.

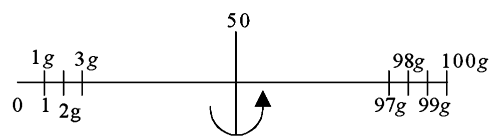


Fig. 8.39

32. A spring wrapped on a wheel of MOI 0.2 kg m^2 and radius 10 cm over a light pulley to support a block of mass 2 kg as shown in Fig. 8.40. Find the acceleration of the block.

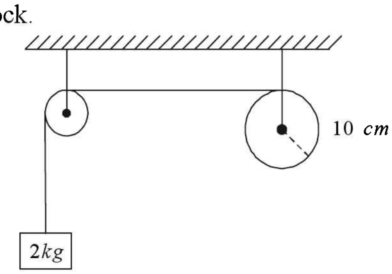


Fig. 8.40

(a) 0.89 ms^{-2} (b) 1.12 ms^{-2}

(c) 0.69 ms^{-2} (d) none

Solution (a) $2a = 2g - T$ ...(1)

$T.r = I\alpha$

or $\quad T = \dfrac{I}{r^2}\, a = \dfrac{0.2a}{(0.1)^2}$ ... (2)

From equations (1) and (2)

$$a = \frac{2g}{(2+20)} = 0.89 \text{ ms}^{-2}$$

33. A uniform rod of length $6a$ and mass 8 m lies on a smooth horizontal table. Two point masses m and 2 m moving in the same horizontal plane with speeds 2 v and v strike the rod as shown in the Fig. 8.41. Find the velocity of centre of mass and angular velocity about COM. Also find KE just after collision.

Solution Conserve momentum as external force is zero.

$$-2mv + m\,2v + 0 = (2m + m + 8m) \times v'$$

$v' = 0$ that is, velocity of COM is zero

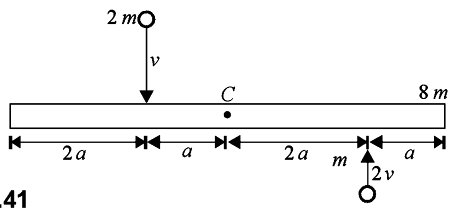


Fig. 8.41

$\tau_{ext} = 0$ $\therefore$ angular momentum is conserved

$$2mva + m(2v)(2a) = \left[2ma^2 + m(2a)^2 + \frac{8m(6a)^2}{12}\right]\omega$$

or $\quad \omega = \dfrac{v}{5a}$.

$$\text{KE after collision} = \frac{1}{2}I\omega^2 = \frac{1}{2}(30ma^2)\left(\frac{v}{5a}\right)^2 = \frac{3}{5}mv^2.$$

34. A uniform ball of radius r rolls without slipping down from the top of a sphere of radius R. The angular velocity of the ball when it breakes from the sphere is .... Assume initial velocity negligible.

Solution $\dfrac{mv^2}{(R+r)} = mg\cos\theta;$

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

$$mg(R+r)(1-\cos\theta) = \frac{1}{2}mv^2 + \frac{1}{5}mv^2 = \frac{7}{10}mv^2$$

$$\frac{10}{7}mg(1-\cos\theta) = mg\cos\theta$$

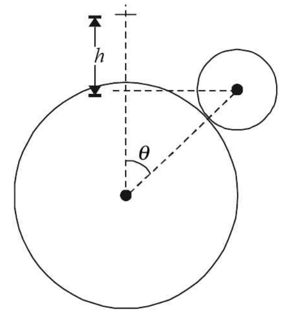


Fig. 8.42

$$mv^2 = \frac{10}{7}mg(R+r)(1-\cos\theta) \quad \frac{10}{7} = \frac{17}{7}\cos\theta$$

or $\quad \cos\theta = \dfrac{10}{17}$

$$v = \sqrt{g(R+r)\cos\theta} = \sqrt{\frac{10}{17}g(R+r)}$$

and $\quad \omega = \dfrac{v}{r} = \sqrt{\dfrac{10g(R+r)}{17r^2}}$

35. A uniform rod of mass m and length l is hinged at its upper end. It is released from a horizontal position. When it becomes vertical, what force does it exert on the hinge?

(a) $\dfrac{3}{2}$ mg (b) 2 mg

(c) $\dfrac{5}{2}$ mg (d) mg

Solution $N - mg = \dfrac{mv^2}{r}$ ...(1)

$$N = mg + \frac{m\left(\frac{l}{2}\omega\right)^2}{\frac{l}{2}} = mg + \frac{3}{2}mg = \frac{5}{2}mg$$

$$\frac{1}{2}I\omega^2 = mg\,\frac{l}{2}$$

$$\frac{1}{2}\frac{ml^2}{3}\omega^2 = mg\,\frac{l}{2}$$

or $\quad \dfrac{mv^2}{l} = 3\,mg$ ... (2)

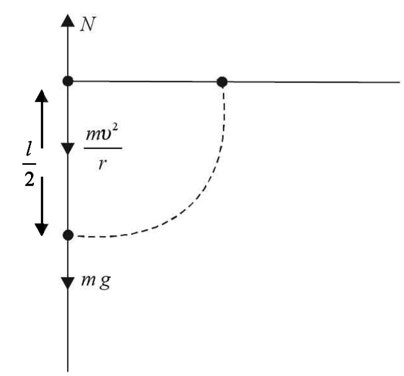


Fig. 8.43

QUESTIONS FOR PRACTICE

1. A square plate lies in the *xy* plane with its centre at the origin and its edges parallel to the *x* and *y* axes. Its moments of inertia about the *x*, *y* and *z* axes are I_x, I_y, and I_z respectively, and about a diagonal it is I_D.

(a) $I_x = I_y = \frac{1}{2} I_z$ (b) $I_x = I_y = 2I_z$

(c) $I_D = I_x$ (d) $I_D = I_z$

2. Four identical rods, each of mass *m* and length *l*, are joined to form a rigid square frame. The frame lies in the *xy* plane, with its centre at the origin and the sides parallel to the *x* and *y* axes. Its moment of inertia about

(a) the *x*-axis is $\frac{2}{3} ml^2$

(b) the *z*-axis is $\frac{4}{3} ml^2$

(c) an axis parallel to the z-axis and passing through a corner is $\frac{10}{3} ml^2$

(d) one side is $\frac{5}{2} ml^2$

3. *P* is the centre of mass of four point masses *A*, *B*, *C* and *D*, which are coplanar but not colinear.

(a) *P* may or may not coincide with one of the point masses.

(b) *P* must lie within the quadrilateral *ABCD*.

(c) *P* must lie within or on the edge of at least one of the triangles formed by taking *A*, *B*, *C* and *D* three at a time.

(d) *P* must lie on a line joining two of the points *A*, *B*, *C*, *D*.

4. When slightly different weights are placed on the two pans of a beam balance, the beam comes to rest at an angle with the horizontal. The beam is supported at a single point *P* by a pivot.

(a) The net torque about *P* due to the two weights is nonzero at the equilibrium position.

(b) The whole system does not continue to rotate about *P* because it has a large moment of inertia.

(c) The centre of mass of the system lies below *P*.

(d) The centre of mass of the system lies above *P*.

5. Two men support a uniform horizontal beam at its two ends. If one of them suddenly lets go, the force exerted by the beam on the other man will

(a) remain unaffected.

(b) increase.

(c) decrease.

(d) become unequal to the force exerted by him on the beam.

6. A sphere *S* rolls without slipping, moving with a constant speed on a plank *P*. The friction between the upper surface of *P* and the sphere is sufficient to prevent slipping, while the lower surface of *P* is smooth and rests on the ground. Initially, *P* is fixed to the ground by a pin *N*. If *N* is suddenly removed,

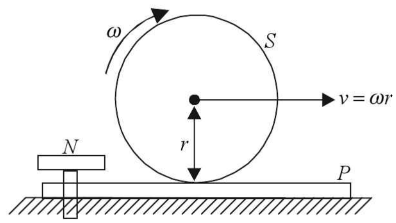


Fig. 8.44

(a) *S* will begin to slip on *P*.

(b) *P* will begin to move backwards.

(c) the speed of *S* will decrease and its angular velocity will increase.

(d) there will be no change in the motion of *S* and *P* will still be at rest.

7. A wheel of radius *r* rolls without slipping with a speed *v* on a horizontal road. When it is at a point *A* on the road, a small blob of mud separates from the wheel at its highest point and lands at point *B* on the road.

(a) $AB = v\sqrt{r/g}$ (b) $AB = 2v\sqrt{r/g}$

(c) $AB = 4v\sqrt{r/g}$

(d) If $v > \sqrt{4rg}$, the blob of mud will land on the wheel and not on the road.

8. The density of a rod gradually decreases from one end to the other. It is pivoted at an end so that it can move about a vertical axis through the pivot. A horizontal force *F* is applied on the free end in a direction perpendicular to the rod. The quantities, that do not depend on which end of the rod is pivoted, are

(a) angular acceleration.

(b) angular velocity when the rod completes one rotation.

(c) angular momentum when the rod completes one rotation.

(d) torque of the applied force.

9. Two uniform solid spheres having unequal masses and unequal radii are released from rest from the same height on a rough incline. If the spheres roll without slipping,

(a) the heavier sphere reaches the bottom first.

(b) the bigger sphere reaches the bottom first.

(c) the two spheres reach the bottom together.

(d) the information given is not sufficient to tell which sphere will reach the bottom first.

10. A hollow sphere and a solid sphere having same mass and same radii are rolled down a rough inclined plane.

(a) The hollow sphere reaches the bottom first.
(b) The solid sphere reaches the bottom with greater speed.
(c) The solid sphere reaches the bottom with greater kinetic energy.
(d) The two spheres will reach the bottom with same linear momentum.

11. A sphere cannot roll on

(a) a smooth horizontal surface.
(b) a smooth inclined surface.
(c) a rough horizontal surface.
(d) a rough inclined surface.

12. A sphere can roll on a surface inclined at an angle θ if the friction coefficient is more than $\frac{2}{7} g \sin\theta$. Suppose the friction coefficient is $\frac{1}{7} g \sin\theta$. If a sphere is released from rest on the inclined,

(a) it will stay at rest.
(b) it will make pure translational motion.
(c) it will translate and rotate about the centre.
(d) the angular momentum of the sphere about its centre will remain constant.

13. Figure (8.45) shows a smooth inclined plane fixed in a car accelerating on a horizontal road. The angle of incline θ is related to the acceleration a of the car as $a = g \tan \theta$. If the sphere is set in pure rolling on the incline

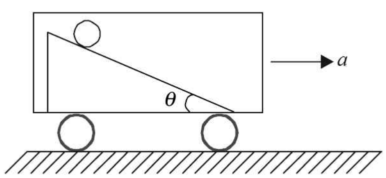


Fig. 8.45

(a) it will continue pure rolling.
(b) it will slip down the plane.
(c) its linear velocity will increase.
(d) its linear velocity will decrease.

14. Torque per unit moment of inertia is equivalent to

(a) angular velocity. (b) angular acceleration.
(c) radius of gyration. (d) inertia.

15. A circular disc starts slipping without rolling down an inclined plane then its velocity will be

(a) gh (b) $2gh$
(c) $\sqrt{gh}$ (d) $\sqrt{2gh}$

16. A spherical shell first rolls and then slips down an inclined plane. The ratio of its acceleration in two cases will be

(a) 5/3 (b) 3/5
(c) 15/13 (d) 13/15

17. The moment of inertia of a ring about its geometrical axis is I, then its moment of inertia about its diameter will be

(a) $2I$ (b) $I/2$
(c) I (d) $I/4$

18. A car is moving with a speed of 72 kmh^{-1}. The radius of its wheel is 50 cm. If its wheels come to rest after 20 rotations as a result of application of brakes, then the angular retardation produced in the car will be

(a) 23.5 rads^{-2} (b) 0.25 rads^{-2}
(c) 6.35 rads^{-2} (d) zero

19. The unit of moment of inertia is

(a) Joule/sec
(b) Joule/second/radian
(c) Joule/second^2/radian^2
(d) Joule/radian

20. A solid cylinder of mass 0.1 kg and radius 0.025 metre is rolling on a horizontal smooth table with uniform velocity of 0.1 ms^{-1}. Its total energy will be

(a) 7.5×10^{-2} Joule (b) 7.5×10^{-3} Joule
(c) 7.5×10^{-4} Joule (d) 0.07×10^{-4} Joule

21. A ring is rolling on an inclined plane. The ratio of the linear and rotational kinetic energies will be

(a) 2 : 1 (b) 1 : 2
(c) 1 : 1 (d) 4 : 1

22. The angular momentum and the moment of the inertia are respectively

(a) vector and tensor quantities.
(b) scalar and vector quantities.
(c) scalar and scalar quantities.
(d) vector and vector quantities.

23. In an arrangement four particles, each of mass 2 gm are situated at the coordinates point (3, 2, 0), (1, –1, 0), (0, 0, 0) and (–1, 1, 0). The moment of inertia of this arrangement about the Z-axis will be

(a) 8 units (b) 19 units
(c) 43 units (d) 34 units

24. The kinetic energy of rotation of a particle is 18 Joule. If the angular momentum vector coincides with the axis of rotation and the moment of inertia of the particle about this axis is 0.01 Kgm^2, then its angular momentum will be

(a) 0.06 J-sec (b) 0.6 J-sec
(c) 0.006 J-sec (d) zero

25. The relation between the linear velocity and angular velocity is

(a) $\vec{\omega} = \vec{r} \times \vec{v}$ (b) $\vec{v} = \vec{r} \times \vec{\omega}$
(c) $\vec{v} = \vec{\omega} \times \vec{r}$ (d) $\vec{\omega} = \vec{v} \times \vec{r}$

26. The moment of inertia of a body about a given axis of rotation depends upon

(a) the distribution of mass.
(b) distance of the body from the axis of rotation.
(c) shape of the body.
(d) all of the above.

27. A rigid body is rotating about a vertical axis at n rotations per minute. If the axis slowly becomes horizontal in t seconds and the body keeps on rotating at n rotations per minute then the torque acting on the body will be, if the moment of inertia of the body about axis of rotation is I.

(a) Zero (b) $\frac{2\tau nI}{60t}$
(c) $\frac{2\sqrt{2}\tau nI}{60t}$ (d) $\frac{4\tau nI}{60t}$

28. A particle is revolving in a circle of radius r. Its displacement after completing half the revolution will be

(a) πr (b) $2r$
(c) $2\pi r$ (d) $\frac{r}{2}$

29. The relation between angular momentum an angular velocity is

(a) $\vec{J} = \vec{r} \times \vec{\omega}$ (b) $\vec{J} = \vec{\omega} \times \vec{r}$
(c) $\vec{J} = \frac{1}{\vec{\omega}}$ (d) $\vec{J} = I\vec{\omega}$

30. Two metallic discs have same mass and same thickness but different densities. The moment of inertia about the geometrical axis will be more of the disc

(a) with lower density.
(b) with higher density.
(c) M.I. of both the discs will be same.
(d) nothing can be said.

31. Minimum time period in a compound pendulum is obtained when

(a) $1 = \pm \frac{K}{2}$ (c) $1 = \pm K$
(b) $1 = \pm \frac{K}{\sqrt{2}}$ (d) $1 = 0$

32. The moment of inertia of a diatomic molecule about an axis passing through its center of mass and perpendicular to the line joining the two atoms will be (μ = Reduced mass of the system)

(a) μr^2 (b) $\frac{\mu r^2}{2}$
(c) 0 (d) $\frac{3}{4}\mu r^2$

33. Which of the following quantities is zero about the center of mass of a body?

(a) Mass (b) Moment of mass
(c) Acceleration (d) Angular acceleration

34. A ring of mass 10 Kg and diameter 0.4 meter is rotating about its geometrical axis at 1200 rotations per minute. Its moment of inertia and angular momentum will be respectively

(a) 0.4 Kg/m^2 and 50.28 Joule/sec
(b) 50.28 Kg/m^2 and 0.4 Joule/sec
(c) 0.4 Joule/sec and 50.28 Kg/m^2
(d) 0.4 Kg/m^2 and zero

35. Two rotating bodies have same angular momentum but their moments of inertia are I_1 and I_2 respectively ($I_1 > I_2$). Which body will have higher kinetic energy of rotation?

(a) First
(b) Second
(c) Both will have same kinetic energy
(d) Not possible to predict

36. A chain couples and rotates two wheels in a bicycle. The radii of bigger and smaller wheels are 0.5m and 0.1 respectively. The bigger wheel rotates at the rate of 200 rotations per minute, then the rate of rotation of smaller wheel will be—

(a) 1000 rpm (b) $\frac{50}{3}$
(c) 200 rpm (d) 40 rpm

37. The moment of inertia of a fly-wheel is 4 Kg/m^2. A torque of 10 Newton-meter is applied on it. The angular acceleration produced will be—

(a) 25 radians/sec^2 (b) 0.25 radians/sec^2
(c) 2.5 radian/sec^2 (d) zero

38. The value of angular momentum of the earth rotating about its own axis is—

(a) 7×10^{33} Kg/m^2/sec. (b) 7×10^{33} Kg /m^2/sec.
(c) 0.7×10^{33} Kg/m^2/sec. (d) zero

39. The work done in rotating a body from angle θ_1 to angle
θ_2 will be—

(a) $\frac{\tau}{(\theta_1 - \theta_2)}$ (b) $\tau(\theta_2 - \theta_1)$
(c) Zero (d) $\frac{(\theta_1 - \theta_2)}{\tau}$

40. A girl sits near the edge of a rotating circular platform. If the girl moves from circumference towards the center of the platform then the angular velocity of the platform will

(a) decrease (b) increase
(c) remain same (d) becomes zero

41. The moment of inertia of a hollow sphere of mass 1Kg and inner and outer diameters 0.2 and 0.4 meter respectively about its diametric axis will be

(a) zero (b) 0.177 Kg/m^2
(c) 0.0177 Kg/m^2 (d) 177 Kg/m^2

42. A gramophone disc is rotating at 78 rotations per minute. Due to power cut, it comes to rest after 30 second. The angular retardation of the disc will be

(a) 0.27 radians/sec^2 (b) 0.127 radians/sec^2
(c) 12.7 radians/sec^2 (d) zero

43. A long thread is wrapped round a reel. If one end of thread is held in hand and the reel is allowed to fall under gravity, then the acceleration of the reel will be

(a) g (b) $\frac{3}{2}g$
(c) $\frac{3}{2}g$ (d) zero

44. The moment of inertia of a circular disc of mass 200 gm and radius 5cm about a tangential axis normal to the plane of the disc will be

(a) 750 g/cm^2 (b) 7500 g/cm^2
(c) 75 g/cm^2 (d) zero

45. A rod with rectangular cross-section oscillates about a horizontal axis passing through one of its ends and it behaves like a seconds pendulum. Its length will be

(a) 1.5 m (b) 1 m
(c) 3 m (d) 2 m

46. The ratio of kinetic energies of two spheres rolling with equal center of mass velocities is 2 : 1. If their radii are in the ratio 2 : 1, then the ratio of their masses will be

(a) 2 : 1 (b) 1 : 8
(c) 1 : 7 (d) $2\sqrt{2}$: 1

47. Out of the following bodies of same mass, which one will have maximum moment of inertia about an axis passing through its center of gravity and perpendicular to its place?

(a) Ring of radius r
(b) Disc of radius r
(c) Square frame of sides $2r$
(d) Square lamina of side $2r$

48. A particle is executing uniform circular motion with angular momentum J. If its kinetic energy is reduced to half and its angular frequency is doubled then its angular momentum becomes

(a) $2J$ (b) $4J$
(c) $\frac{J}{2}$ (d) $\frac{J}{4}$

49. The angle covered by a body in n^{th} second is

(a) $\omega_0 + \frac{\alpha}{2}(2n-1)$ (b) $\omega_0 - \frac{\alpha}{2}(2n-1)$
(c) $\omega_0 + \frac{\alpha}{2}(n-1)$ (d) $\omega_0 - \frac{\alpha}{2}(n-1)$

50. A particle of mass m is tied to the end of a string passing through a hollow tube. The particle is revolved with angular velocity ω. The force required to be applied at the lower end of the string in order to maintain dynamic equilibrium will be

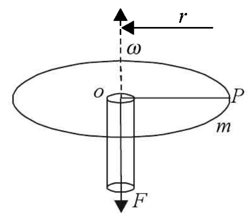


Fig. 8.46

(a) $m\omega^2 r$ (b) mg
(c) $\frac{m\omega^2 r}{2}$ (d) $m\omega^2 r + mg$

51. A Yo-Yo is a toy in the form of a disc with a concentric shaft. A string is wound on the shaft. If it is suspended from the free end, then the string unwinds and winds so that the Yo-Yo falls down and rises up again and again. The ratio of the tension in the string while descending and ascending is

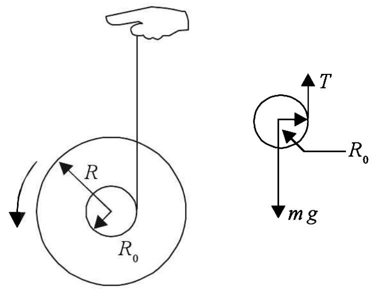


Fig. 8.47

(a) 1 : 2 (b) 1 : 1
(c) $R : R_0$ (d) $R_0 : R$

52. Two masses of 200 gm and 300 gm are attached to the 20 cm and 70 cm marks of a light meter scale respectively. The moment of inertia of this system about an axis passing through 50 cm mark will be

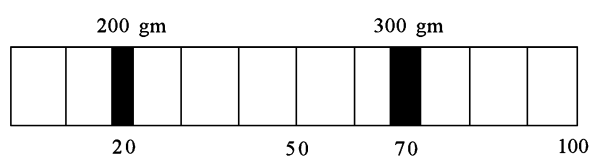


Fig. 8.48

(a) 0.3 Kg/m^2 (b) 0.03 Kg/m^2
(c) 0.15 Kg/m^2 (d) zero

53. The ratio of angular frequency and linear frequency is

(a) 2π (b) Π
(c) $\frac{1}{2\pi}$ (d) $\frac{\pi}{2}$

54. A mass M is moving with constant velocity parallel to x-axis. Its angular momentum about the origin

(a) is zero. (b) increases.
(c) decreases. (d) remains constant.

55. Which of the following bodies of same mass has maximum moment of inertia about its geometric axis?

(a) A bar pendulum (b) A solid sphere
(c) A circular ring (d) A circular disc

56. Which of the following relations is wrong ?

(a) $\vec{J} = \vec{r} \times \vec{P}$ (b) $\vec{a} = \vec{r} \times \vec{\alpha}$
(c) $\vec{v} = \vec{\omega} \times \vec{r}$ (d) $\tau = \frac{d\vec{J}}{dt}$

57. If the position vector of a particle is $\hat{r} = \left(3\hat{i} = 4\hat{j}\right)$ metre and its angular velocity is $\vec{\omega} = \left(\hat{j} = 2\hat{k}\right)$ rad/sec then its linear velocity is (in m/s)

(a) $-\left(8\vec{i} - 6\vec{j} + 3\vec{k}\right)$ (c) $\left(3\vec{i} - 6\vec{j} + 8\vec{k}\right)$
(b) $-\left(3\vec{i} - 6\vec{j} + 6\vec{k}\right)$ (d) $\left(6\vec{i} - 8\vec{j} + 3\vec{k}\right)$

58. Moon is revolving round the earth as well as it is rotating about its own axis. The ratio of its angular momenta in two cases will be—(orbital radius of moon $= 3.82 \times 10^8$ m and radius of moon $= 1.74 \times 10^6$ m)

(a) $1.22 \times 10^{5/4}$ (b) $1.22 \times 10^{5/3}$
(c) $1.22 \times 10^{5/2}$ (d) $1.22 \times 10^{5/1}$

59. The moment of inertia of a solid cylinder of mass M, length L and radius R about the diameter of one of its faces will be

(a) $M\left(\frac{L^2}{12} + \frac{R^2}{4}\right)$ (b) $M\left(\frac{L^2}{3} + \frac{R^2}{4}\right)$
(c) zero (d) $\frac{MR^2}{2}$

60. Equal torques are applied about a central axis on two rings of same mass and same thickness but made up of different materials. If ratio of their densities is 4 : 1 then the ratio of their angular acceleration will be

(a) 16 : 1 (b) 1 : 16
(c) 8 : 1 (d) 1 : 12

61. A circular hoop of mass M and radius R is suspended from a nail in the wall. Its moment of inertia about an axis along the nail will be

(a) zero (b) MR^2
(c) $2MR^2$ (d) $\frac{MR^2}{2}$

62. The direction of $\vec{\tau}$ is

(a) parallel to the plane of $\vec{r}$ and $\vec{F}$.
(b) perpendicular to the plane of $\vec{r}$ and $\vec{F}$.
(c) parallel to the plane of $\vec{r}$ and $\vec{P}$.
(d) perpendicular to the plane of $\vec{r}$ and $\vec{P}$.

63. The moment of inertia of a ring of mass 2 Kg about a tangential axis lying in its own plane is 3Kg/m². The radius of the ring is

(a) 1 m (b) 3 cm
(c) 3 mm (d) 6 m

64. A fly-wheel of moment of inertia 0.4 Kg/m² and radius 0.2 m is free to rotate about a central axis. If a string is wrapped around it and it is pulled with a force of 10 newton then its angular velocity after four seconds will be

(a) 5 radians/sec (b) 20 radians/sec
(c) 10 radians/sec (d) 0.8 radians/sec

65. The equation of motion of a compound pendulum is

(a) $\frac{d^2x}{dt^2} + \omega^2 x = 0$ (b) $\frac{d^2\theta}{dt^2} + \omega^2\theta = 0$
(c) $F = -Kx$ (d) $\frac{d\theta}{dt^2} + \omega^2\theta = 0$

66. A block of mass 12 kg is attached to a string wrapped around a wheel of radius 10 cm. The acceleration of the block moving down an inclined plane is measured at 2 m/s². The tension in the string is

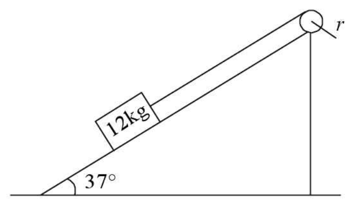


Fig. 8.49

(a) 24.5 Newton (b) 68.7 Newton
(c) 23.4 Newton (d) 46.8 Newton

67. In the above problem the moment of inertia of the wheel is

(a) 0.23 Kg/m/s² (b) 0.46 Kg/m/s²
(c) 0.92 Kg/m/s² (d) 0.69 Kg/m/s²

68. In Q. No. 41, the angular speed of the wheel after the 3 seconds after start will be (in radians/sec)

(a) 10 (b) 20
(c) 40 (d) 60

69. A uniform solid cylinder of mass M and radius R rotates about a frictionless horizontal axle. Two similar masses suspended with the help of two ropes wrapped around the cylinder. If the system is released from test then the tension in each rope will be

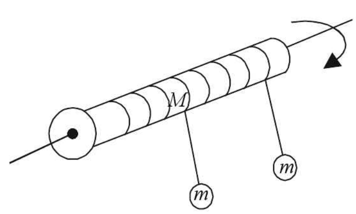


Fig. 8.50

(a) $\frac{Mmg}{(M+m)}$ (b) $\frac{Mmg}{(M+2m)}$

(c) $\frac{Mmg}{(M+3m)}$ (d) $\frac{Mmg}{(M+4m)}$

70. In the above problem the acceleration of each mass will be

(a) $\frac{4mg}{(M+2m)}$ (b) $\frac{4mg}{(M+4m)}$

(c) $\frac{2mg}{(M+m)}$ (d) $\frac{2mg}{(M+2m)}$

71. In the Q. No. 44, the angular velocity of the cylinder, after the masses fall down through distance h, will be

(a) $\frac{1}{R}\sqrt{8mgh/(M+4m)}$ (b) $\frac{1}{R}\sqrt{8mgh/(M+m)}$

(c) $\frac{1}{R}\sqrt{mgh/(M+m)}$ (d) $\frac{1}{R}\sqrt{8mgh/(M+2m)}$

72. A massless string is wrapped round a disc of mass M and radius R. Another end is tied to a mass m which is initially at height h from ground level as shown in the figure. If the mass is released then its velocity while touching the ground level will be

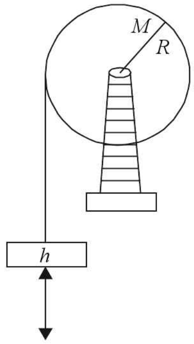


Fig. 8.51

(a) $\sqrt{2gh}$ (b) $\sqrt{2gh\frac{M}{m}}$

(c) $\sqrt{2gh\ \ m/M}$ (d) $\sqrt{4mgh/2m+M}$

73. The centres of four spheres each of mass m and diameter $2a$ are at the four corners of a square of side b. The moment of inertia of the system about one side of the square will be

(a) $\frac{2}{5}m\left[4a^2+5b^2\right]$ (b) $\frac{2}{5}m\left[5a^2+4b^2\right]$

(c) $\frac{2}{5}m\left[a^2+b^2\right]$ (d) $m\left[\frac{8}{5}a^2+b^2\right]$

74. In the above question the moment of inertia of the system about the diagonal of square will be

(a) $\frac{2}{5}m\left[4a^2+5b^2\right]$ (b) $\frac{2}{5}m\left[5a^2+4b^2\right]$

(c) $\frac{2}{5}m\left[a^2+b^2\right]$ (d) $m\left[\frac{8}{5}a^2+b^2\right]$

75. In Q. No. 98 the moment of inertia of the system about an axis passing through one corner of the square and perpendicular to its plane will be

(a) $\frac{4}{5}m\left[2a^2+5b^2\right]$ (b) $\frac{5}{4}m\left[a^2+2b^2\right]$

(c) $\frac{2}{5}m\left[3a^2+4b^2\right]$ (d) $\frac{3}{4}m\left[2a^2+4b^2\right]$

76. A solid cylinder of mass 2 Kg and radius 0.2 m is rotating about its own axis without friction with angular velocity 3 rad/s. A particle of mass 0.5 Kg and moving with a velocity of 5 m/s strikes the cylinder and sticks to it as shown in. The angular momentum of the cylinder before collision will be

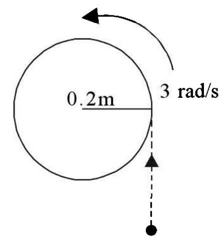


Fig. 8.52

(a) 0.12 Joule/s (b) 12 Joule/s
(c) 1.2 Joule/s (d) 1.12 Joule/s

77. In the above question the angular velocity of the system after the particle sticks to it will be

(a) 0.3 radians/sec. (b) 5.3 radians/sec
(c) 10.3 radians/sec (d) 8.3 radians/sec

78. In Q. No. 51, the energy of the system in the beginning is

(a) 1.43 J (b) 2.43 J
(c) 3.43 J (d) 8.3 J

79. The rotational kinetic energy of two bodies of moments of inertia 9 Kg/m^2 and 1 Kg/m^2 are same. The ratio of their angular momenta is

(a) 3 : 1 (b) 1 : 3
(c) 9 : 1 (d) 1 : 9

80. The moment of inertia of a circular disc about its own axis is 4 Kg/m^2. Its moment of inertia about the diameter will be

(a) 4 Kg/m^2 (b) 2 Kg/m^2
(c) zero (d) 8 Kg/m^2

81. A hollow cylinder is rolling on an inclined plane, inclined at an angle of 30° to the horizontal. It's speed after travelling a distance of 10 m will be

(a) 49 m/sec (b) 0.7 m/sec
(c) 7 m/sec (d) zero.

82. The moment of inetia of a spherical shell about a tangential axis is

(a) $\frac{2}{5}MR^2$ (b) $\frac{7}{5}MR^2$

(c) $\frac{2}{3}MR^2$ (d) $\frac{5}{3}MR^2$

83. A body with moment of inertia 3 Kg/m² is at rest. A torque of 6 newton/metre applied on it rotates the body for 20 second. The angular displacement of the body is

(a) 800 radians (b) 600 radians
(c) 400 radians (d) 200 radians

84. The ratio of the angular velocities of the hour hand and minute hand of a watch is

(a) 1 : 1 (b) 1 : 12
(c) 43200 : 1 (d) 720 : 1

85. The second equation of motion in rotatory motion is

(a) $S = ut + \frac{at^2}{2}$ (b) $\theta = \omega_1 t + \frac{\alpha t^2}{2}$
(c) $\omega_2^2 = \omega_1^2 + 2\alpha\theta$ (d) $\omega_2 = \omega_1 + \alpha t$

86. Which of the following pairs do not match?

(a) Rotational power/Joule.
(b) Torque/Newton meter.
(c) Angular displacement/Radian.
(d) Angular acceleration/Radian/sec.

87. A stone tied to one end of the string is revolved round a rod in such a way that the string winds over the rod and gets shortened. In this process which of the following quantities remains constant ?

(a) Mass (b) Momentum
(c) Angular momentum (d) Kinetic energy

88. A solid sphere of mass 0.1 Kg and radius 2 cm rolls down an inclined plane 1.4 m in length (slope 1 in 10). Starting from rest its final velocity will be

(a) 1.4 m/sec (b) 0.14 m/sec
(c) 14 m/sec (d) 0.7 m/sec

89. Four particles each of mass m are lying symmetrically on the rim of a disc of mass M and radius R. The moment of inertia of this system about an axis passing through one of the particles and perpendicular to the plane of the disc is

(a) $16\,m R^2$ (b) $(3\,M + 16\,m)\,\frac{R^2}{2}$
(c) $(3\,m + 16\,M)\,\frac{R^2}{2}$ (d) zero

90. A cockroach of mass m is moving on the rim of a disc with velocity V in the anticlockwise direction. The moment of inertia of the disc about its own axis is I and it is rotating in the clockwise direction with angular speed ω. If the cockroach stops moving then the angular speed of the disc will be

(a) $\frac{I\omega}{I + mR^2}$ (b) $\frac{I\omega + mVR}{1 + mR^2}$
(c) $\frac{I\omega - mVR}{I + mR^2}$ (d) $\frac{I\omega - mVR}{I}$

91. If the force applied on a particle is zero then the quantities which are conserved are

(a) only momentum.
(b) only angular momentum.
(c) momentum and angular momentum.
(d) only potential energy.

92. A body starts rolling down an inclined plane of length L and height h. This body reaches the bottom of the plane in time t. The relation between L and t is

(a) $t \propto L$ (b) $\frac{t \infty 1}{L}$
(c) $t \propto L^2$ (d) $\frac{t \infty 1}{L^2}$

93. In hydrogen atom an electron revolves in a circular path. If the radius of its path is 0.53 A° and it makes 7 × 10^{15} revolutions per second, its angular momentum about proton is

(a) 11.2×10^{-35} Joule/sec (b) 11.2×10^{-34} Joule/sec
(c) 11.2×10^{-33} Joule/sec (d) 11.2×10^{-33} Joule/sec

94. The expressions for the tangential and centripetal accelerations are

(a) $r\alpha$ and $\omega^2 r$ (b) $\omega^2 r$ and $r\alpha$
(c) ωr and $r\alpha$ (4) $r\alpha$ and ωr

95. The unit of J/P is

(a) meter/sec (b) meter
(c) Joule (d) Joule/sec

96. If the tangential and centripetal accelerations are tangents and along the centre, respectively, then the resultant acceleration (a) will be

(a) $a = a_t + a_c$ (b) $a = \sqrt{a_t^2 + a_c^2}$
(c) $a = a_t - a_c$ (d) $a = a_c - a_t$

97. The dimensions of $\tau/\propto$ are

(a) ML^{-2} (b) ML^2
(c) M^2L^2 (d) $M^{-2}L^{-2}$

98. The block of mass M is initially moving to the right without friction with speed V_1. It passes over the cylinder to the dashed position, when it first makes contact with the cylinder, it slips on the cylinder but the friction is large enough so that slipping ceases before M losses contact with the cylinder. The final velocity of the V_2 of the block will be, if the radius of the cylinder is R and its moment of inertia is 1 and initially it is at rest.

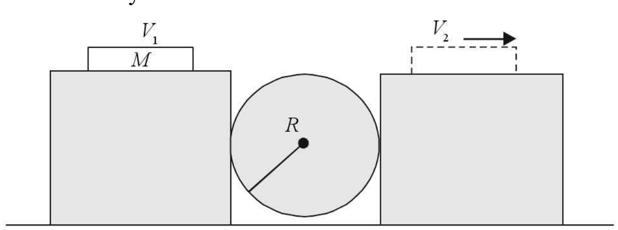


Fig. 8.53

(a) $V_2' = \dfrac{MV_1'}{M+\dfrac{m}{2}}$ (b) $V_2' = \dfrac{mV_1'}{M+\dfrac{M}{2}}$

(c) $V_2' = V_1'$ (d) $V_2 = \dfrac{V_1 I}{MR^2}$

99. A solid sphere rests on a horizontal surface, A horizontal impulse is applied at height h from the centre (Fig. 8.54).

The sphere starts rotating just after the application of impulse. The ratio ω_3 will be

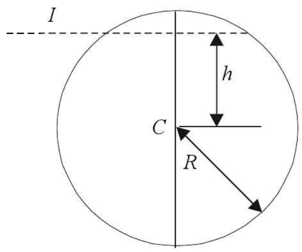


Fig. 8.54

(a) $\dfrac{1}{2}$ (b) $\dfrac{2}{5}$

(c) $\dfrac{1}{4}$ (d) $\dfrac{1}{5}$

100. A solid cube of side l is made to oscillate about a horizontal axis passing through one of its edges. Its time period will be

(a) $2\pi\sqrt{\dfrac{2\sqrt{2}}{3}\dfrac{l}{g}}$ (b) $2\pi\sqrt{\dfrac{2}{3}\dfrac{l}{g}}$

(c) $2\pi\sqrt{\dfrac{\sqrt{3}}{2}\dfrac{l}{g}}$ (d) $2\pi\sqrt{\dfrac{2}{\sqrt{3}}\dfrac{l}{g}}$

101. The radius of gyration of a plane lamina of mass M, length L and breadth B about an axis passing through its center of gravity and perpendicular to its plane will be

(a) $\sqrt{(L^2+B^2)/12}$ (b) $\sqrt{(L^2+B^2)/8}$

(c) $\sqrt{(L^2+B^2)/2}$ (d) $\sqrt{(L^3+B^3)/12}$

102. Two spheres each of mass 1Kg are attached to the two ends of a rod of mass 1Kg and length 1m. The moment of inertia of the system about an axis passing through its centre of gravity and perpendicular to the rod will be

(a) 1 Kgm^2 (b) $\dfrac{1}{2}$ Kgm^2

(c) $\dfrac{7}{12}$ Kgm^2 (d) $\dfrac{12}{7}$ Kgm^2

103. The diameter of a solid disc is 0.5m and its mass is 16 kg. What torque will increase its angular velocity from zero to 120 rotations/minute in 8 seconds ?

(a) $\dfrac{\pi}{4}$ N/m (b) $\dfrac{\pi}{2}$ N/m

(c) $\dfrac{\pi}{3}$ N/m (d) π N/m

104. In the above problem at what rate is work done by the torque at the end of 8th second ?

(a) π Watt (b) π^2 Watt

(c) π^3 Watt (d) π^4 Watt

105. Two point masses are lying on a smooth uniform mass M and length L. Initially the masses are in the middle of the rod. The system is rotating about an axis passing through the center and perpendicular to the rod with angular velocities ω. No external force acts on the system. When the masses reach the ends of the rod, then the angular velocity of the system will be

(a) $\dfrac{M\omega_0}{M+2m}$ (b) $\dfrac{M\omega_0}{M+4m}$

(c) $\dfrac{M\omega_0}{M+6m}$ s (d) $\dfrac{M\omega_0}{M+8m}$

Answers to Questions for Practice

1.	(a)	**2.**	(b)	**3.**	(b)	**4.**	(a)	**5.**	(c)	**6.**	(d)	**7.**	(c)
8.	(d)	**9.**	(c)	**10.**	(b)	**11.**	(b)	**12.**	(c)	**13.**	(a)	**14.**	(b)
15.	(d)	**16.**	(c)	**17.**	(a)	**18.**	(a)	**19.**	(a)	**20.**	(b)	**21.**	(c)
22.	(c)	**23.**	(d)	**24.**	(c)	**25.**	(c)	**26.**	(d)	**27.**	(c)	**28.**	(b)
29.	(d)	**30.**	(a)	**31.**	(b)	**32.**	(a)	**33.**	(b)	**34.**	(a)	**35.**	(b)
36.	(a)	**37.**	(c)	**38.**	(a)	**39.**	(b)	**40.**	(b)	**41.**	(c)	**42.**	(a)
43.	(b)	**44.**	(b)	**45.**	(a)	**46.**	(a)	**47.**	(c)	**48.**	(d)	**49.**	(a)
50.	(a)	**51.**	(b)	**52.**	(b)	**53.**	(a)	**54.**	(d)	**55.**	(c)	**56.**	(b)
57.	(a)	**58.**	(d)	**59.**	(b)	**60.**	(a)	**61.**	(c)	**62.**	(b)	**63.**	(a)
64.	(b)	**65.**	(b)	**66.**	(d)	**67.**	(a)	**68.**	(d)	**69.**	(d)	**70.**	(a)
71.	(a)	**72.**	(a)	**73.**	(b)	**74.**	(a)	**75.**	(d)	**76.**	(a)	**77.**	(d)
78.	(a)	**79.**	(a)	**80.**	(c)	**81.**	(d)	**82.**	(a)	**83.**	(b)	**84.**	(c)
85.	(d)	**86.**	(c)	**87.**	(b)	**88.**	(b)	**89.**	(d)	**90.**	(c)	**91.**	(a)
92.	(b)	**93.**	(c)	**94.**	(c)	**95.**	(b)	**96.**	(b)	**97.**	(b)	**98.**	(a)
99.	(b)	**100.**	(a)	**101.**	(a)	**102.**	(c)	**103.**	(a)	**104.**	(b)	**105.**	(c)

Explanations

5(c) When the beam is supported at A and B, the force exerted by each man = $mg/2$.

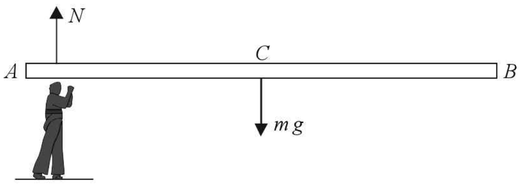


Fig. 8.55

When the support at B is withdrawn, taking torque about A,

$\tau = (mg)\, l/2 = I\alpha = (ml^2/3)\,\alpha$

or $\alpha = 3g/2l$.

The instantaneous linear acceleration of the centre of mass is

$a_{CM} = (\alpha)(AC) = (3g/2l)\, l/2 = 3g/4$.

Let N = force exerted on the beam at A.

$\therefore \quad mg - N = ma_{CM} = m(3g/4)$ or $N = \dfrac{1}{4}\, mg$.

6(d) In rolling without slipping, at constant speed, there is no force of friction between the surfaces.

Therefore, removing the pin causes no change to the system.

7(c) At the point of leaving the wheel, the blob of mud is at a height $2r$ above the road and has a horizontal velocity $2v$.

Let t = time of travel from D to B. Then, $2r = \dfrac{1}{2}\, gt^2$

or $\quad t = 2\sqrt{r/g}$ and $AB = (2v)\, t$.

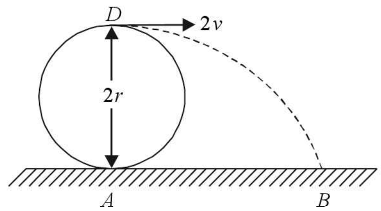


Fig. 8.56

9

Gravitation

BRIEF REVIEW

Gravitational force is first natural force The modern science came to notice. It started from planetory motion and then took the shape as we know today.

Newton's Law of Gravitation Newton in 1665 formulated $F \propto m_1 m_2$

$$F \propto \frac{1}{r^2} \text{ or } F = \frac{Gm_1m_2}{r^2}$$

Where $G = 6.67 \times 10^{-11}$ N m^2 kg^{-2} and is called universal gravitational constant. The value of G was first measured by Cavendish in 1736. The value of G measured for small distances ($r < 200$ m) is less by about 1% and perhaps gives an indication of a fifth natural force. Note, gravitational field is independent of the nature of medium between the masses.

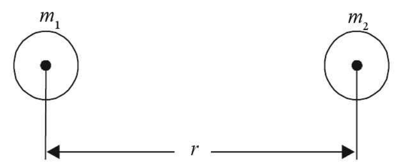


Fig. 9.1 Gravitational force between two masses

Gravitational Field Intensity Gravitational force per unit mass is called gravitational field intensity. Gravitational field intensity of earth is 'g'.

$$E_g = \frac{F}{m} = \frac{GM}{r^2} \text{ and } g = \frac{GM_e}{R_e^2}$$

Gravitational field intensity due to a ring at any point on the axial line as illustrated in Fig. 9.2 is $E_g = \frac{GMx}{(x^2 + R^2)^{3/2}}$

E_g is maximum if $x = R/\sqrt{2}$.

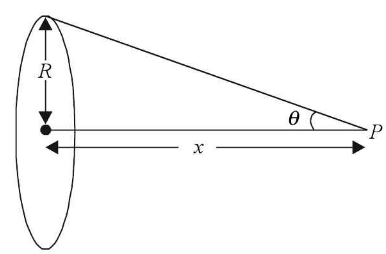


Fig. 9.2 Gravitational field intensity at a point on the axial line of ring.

Gravitational field due to a disc at any point on the axial line

$$E_g = \frac{2GM}{R^2}\left[1 - \frac{x}{\sqrt{x^2 + R^2}}\right] = \frac{2GM}{R^2}[1 - \cos\theta]$$ in terms of angle θ.

Gravitational field intensity due to a shell

$$E_{g\text{ inside}} = 0, E_{g\text{ surface}} = \frac{GM}{R^2}$$

$$E_{g\text{ out}} = \frac{GM}{x^2} \quad x > R$$

See Fig. 9.3.

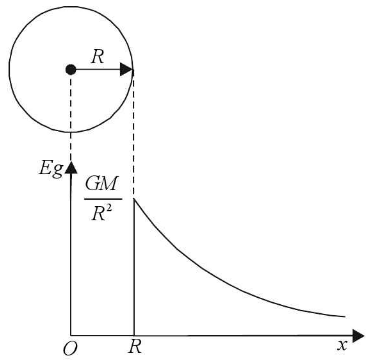


Fig. 9.3 Gravitational field intensity due to a shell

Gravitational field intensity due to a solid sphere

$$E_{g\ \text{inside}} = \frac{GMx}{R^3} \quad x < R$$

$$E_{g\ \text{surface}} = \frac{GM}{R^2} \quad x = R$$

$$E_{g\ \text{outside}} = \frac{GM}{R^2} \quad x > R$$

See Fig. 9.4.

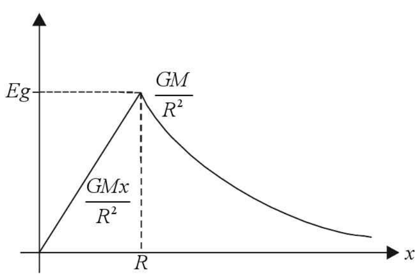


Fig. 9.4 Gravitational field intensity due to a solid sphere

Gravitational Potential (V_g) The amount of work done to bring a unit mass from infinity to that point under the influence of gravitational field of given mass M without changing the velocity $V_g = \frac{-GM}{r} = \int_{\infty}^{r} E_g \, .dx$

Gravitational potential due to a ring at any point on the axial line

$$V_g = \frac{-GM}{\sqrt{x^2 + R^2}}$$

Gravitational potential due to a shell

$$V_{\text{inside}} = V_{\text{surface}} = \frac{-GM}{R} \quad x \le R$$

$$V_{\text{outside}} = \frac{-GM}{x} \quad x > R$$

See Fig. 9.5.

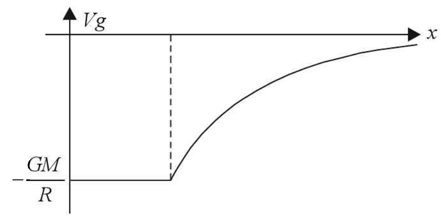


Fig. 9.5 Gravitational potential due to a shell

Gravitational potential due to a solid sphere

$$V_{\text{inside}} = \frac{-GM}{2R^3}\left[3R^2 - x^2\right] \quad x < R$$

$$V_{\text{surface}} = \frac{-GM}{R} \quad x = R$$

$$V_{\text{outside}} = \frac{-GM}{x} \quad x > R$$

See Fig. 9.6

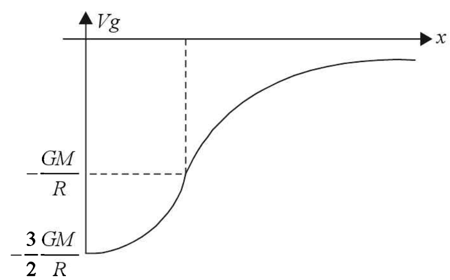


Fig. 9.6 Gravitational potential due to a solid sphere

Gravitational Potential Energy It is the amount of work done to bring a mass m from infinity to that point under the influence of gravitational field of a given mass M without changing the velocity $u_g = \frac{-GMm}{r}$. Note, that $u_g = mV_g$.

Work done $W = \Delta u_g$

Variation of 'g' due to height $g' = g\left(1 - \frac{2h}{R}\right)$ if $h << R$

$$g' = \frac{g}{\left(1 + h/R\right)^2} \text{ if } h \text{ is comparable to } R$$

Variation of 'g' due to depth $g' = g\left(1 - \frac{x}{R}\right)$ where x is depth

$g' = 0$ if $x = R$ i.e. at the centre of the earth.

Variation of 'g' with rotation of the earth lattitude

$g' = g\left(1 - \frac{R\omega^2}{g}\cos^2\lambda\right)$, i.e., g is maximum at the poles (where $\lambda = 90^\circ$) and minimum at the equator (where $\lambda = 0$).

Orbital velocity $v_o = \sqrt{\frac{GM}{r}}$ Orbital velocity v_o is the velocity with which a planet or a satellite moves in its orbit of radius r.

Escape velocity Escape velocity is the minimum velocity given to a body so that it escapes (from the surface of the earth/planet) from its gravitational field. $v_e = \sqrt{\frac{2GM}{r}}$.

Note $v_e = \sqrt{2}\ v_o$.

$$\text{Time period } T = \frac{2\pi r}{v_o} \text{ or } T^2 = \frac{4\pi^2 r^3}{GM}$$

$$KE = \frac{1}{2} m v_o^2 = \frac{GMm}{2r}\ ;\ PE = \frac{-GMm}{r}$$

$$\text{Total energy or Binding energy} = KE + PE = \frac{-GMm}{2r}.$$

Kepler's Laws

First law The planets revolve around the sun in the elliptical orbits with sun at one of the focus as illustrated in Fig. 9.7 (a).

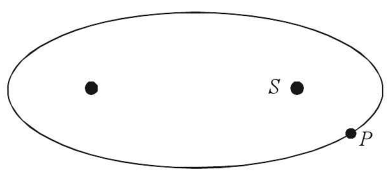


Fig. 9.7 (a) Kepler's 1st law illustration

Second law A line from the sun to the planet sweeps equal area in equal intervals of time as shown in Fig. 9.7 (b). This law is based on conservation of angular momentum.

From Kepler's 2nd law one can easily derive

$$\frac{v_1}{v_2} = \frac{r_2}{r_1} = \frac{v_{\text{Perihelion}}}{v_{\text{aphelion}}} = \frac{r_{\text{aphelion}}}{r_{\text{Perihelion}}} \qquad \text{Fig. 9.7 (b)}$$

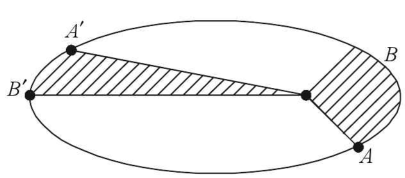
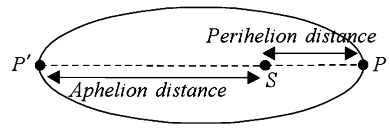


Fig. 9.7 (b) Kepler's 2nd law illustration

i.e., when the planet is closer to the sun it moves faster.

Closest distance of a planet from the sun is called **perihelion distance**. **Farthest** distance of the planet from the sun is called **aphelion distance**.



Closest distance of a planet from the sun is called perihelion distance. Farthest distance of the planet from the sun is called Aphelion distance.

Fig. 9.7 (c) Perihelion and aphelion distance illustration

Third law The square of the time period of a planet is proportional to the cube of the semimajor axis, i.e., $T^2 \propto r^3$.

If e is the eccentricity of an elliptical orbit then

r is semi-major axis.

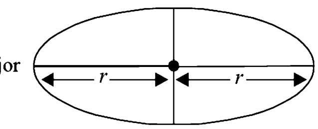


Fig. 9.7 (d) Kepler's 3rd law illustration

$\frac{r_{\text{Aphelion}}}{r_{\text{Perihelion}}} = \frac{1+e}{1-e}$. $r_{\text{Aphelion}} + r_{\text{Perihelion}} = 2r$ r being semimajor axis.

Schwarzschild radius $R_S = \frac{2GM}{c^2}$ where c is speed of light with radius R_S.

Event horizon The surface of the sphere with radius R_s surrounding a blackhole is called **event horizon.** Since, light cannot escape from with in this sphere, we can not see events occuring inside.

Weightlessness in a satellite $\frac{GMm}{r^2} - N = \left(\frac{GM}{r^2}\right) m$

or $N = 0$ where N is normal contact force exerted by the surface. That is in a satellite surface does not exert any force on the body. Hence, apparent weight of the body is zero.

• Short Cuts and Points to Note

1. Gravitational force is only attractive force and $F = \frac{Gm_1m_2}{r^2}$. The force is conservative. If $r \leq 10^{-8}$ m then intermolecular forces dominate. Gravitational force is the weakest of the known natural forces.

   This force does not depend on the nature of medium present in between the two masses.
2. Gravitational field intensity $E_g = \frac{GM}{r^2}$ is force per unit mass. Gravitation field intensity of the earth = $g = \frac{GM}{R^2}$.
3. Aryabhatt in 5th century AD first described that the earth revolves around the sun and the moon revolves around the earth.
4. The moon takes 27.3 days to complete one revolution around the earth. The mean radius of the orbit is 3.85×10^5 km.

5. The value of G was measured by Cavendish for the first time. The value of G is about 1 % less when distance < 200 m, indicating the possibility of a 5th natural force.

6. Gravitational field intensity due to a ring along the axial line is $E_g = \frac{GMx}{(R^2 + x^2)^{3/2}}$. E_g will be maximum if $x = \frac{R}{\sqrt{2}}$. $E_{centre} = 0$.

7. Gravitational field intensity due to a disc

$$E = \frac{2GM}{R^2}\left[1 - \frac{x}{\sqrt{x^2 + R^2}}\right]$$

$$= \frac{2GM}{R^2}[1 - \cos\theta].$$

$$E_{centre} = \frac{2GM}{R^2}.$$ See Fig. 9.8.

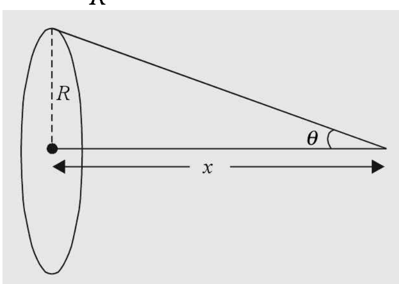


Fig. 9.8

8. Gravitational field intensity due to a shell

$E_{g,\,inside} = 0 \quad x < R$

$E_{g,\,surface} = \frac{GM}{R^2} \quad x = R$

$E_{g,\,outside} = \frac{GM}{x^2} \quad x > R$

as illustrated in Fig. 9.9

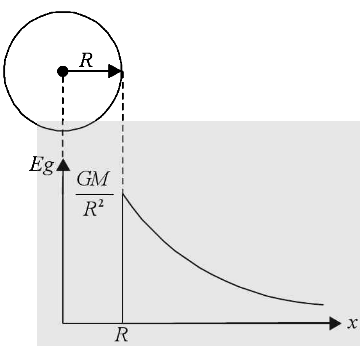


Fig. 9.9

9. Gravitational field intensity due to a solid sphere

$E_{g,\,inside} = \frac{GMx}{R^3} \quad x < R$

$E_{g,\,surface} = \frac{GM}{R^2}$ for $x = R$;

$E_{g,\,outside} = \frac{GM}{x^2} \quad x > R$. See Fig. 9.10.

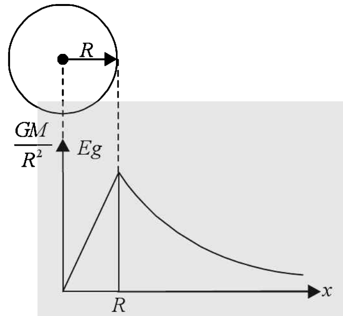


Fig. 9.10

10. Gravitational Potential $V_g = \frac{-GM}{r}$ due to a point mass at any point P distance r from the point mass. Negative sign shows force is attractive.

11. Gravitational Potential due to a ring at any point P on the axial line $V_g = \frac{-GM}{\sqrt{R^2 + x^2}}$.

12. Gravitational Potential due to a shell

$V_{inside} = V_{surface} = \frac{-GM}{R}$.

$V_{outside} = \frac{-GM}{x} \quad x > R$ See Fig. 9.11.

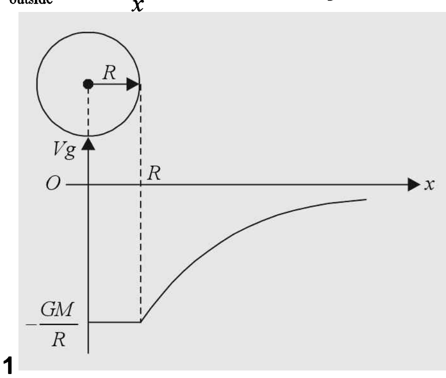


Fig. 9.11

13. Gravitational Potential due to a solid sphere

$V_{inside} = \frac{-GM}{2R^3}[3R^2 - x^2] \quad x < R$

$V_{surface} = \frac{-GM}{R}$ and $V_{out} = \frac{-GM}{x} \quad x > R$

See Fig. 9.12.

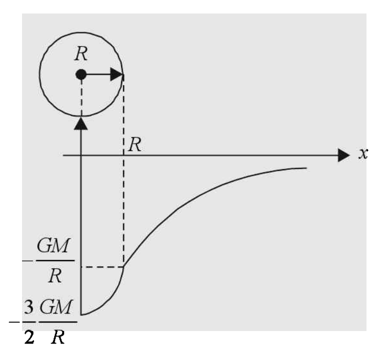


Fig. 9.12

14. Gravitational potential energy $U_g = \frac{-GMm}{r}$ $= m V_g$. Negative sign indicates force is attractive. Work done to raise a body of mass m to a height $= nR$ where R is radius of the earth is $\left(W = \frac{mgR}{1+\frac{1}{n}} \right)$

n could be an integer or fraction.

15. W = Δ PE because gravitational force is conservative, $F = \frac{-dU}{dr}$. At equilibrium $\frac{dU}{dr} = 0$.

16. Gravitational field intensity $g = \frac{GM}{R^2}$ (due to the earth) is valid upto 10 km above the surface of the earth. With height or depth g decreases.

17. Variation of g with height $g' = g\left(1 - \frac{2h}{R}\right)$ if $h < \frac{R}{10}$.

$$g' = \frac{g}{\left(1+\frac{h}{R}\right)^2} \text{ if } h > \frac{R}{10}.$$

Note, g, never becomes zero with height. Therefore in space we come across the term microgravity and not weightlessness. ($g \to 0$ only if $h \to \infty$).

18. Variation of g with lattitude

$g' = g\left[1 - \frac{R\omega^2 \cos^2 \lambda}{g}\right]$. At poles $\lambda = 90°$, $g' = g$ and is maximum .

At equator $g' = g\left(1 - \frac{R\omega^2}{g}\right)$ $\because \lambda = 0$. g is minimum at equator. If earth rotates (spins) at a rate 17 times the present value the weight of a body at equator will become zero.

19. Variation of g with depth $g' = g\left(1 - \frac{x}{R}\right)$. If $x = R$, i.e., at the centre of the earth $g' = 0$. The body will becomes weightless at the centre of the earth.

20. **Kepler's first law:** The planets revolve around the sun in elliptical orbits with sun at one of the foci.

Second law: The line joining the sun and the planet sweeps equal area in equal interval of time or the areal velocity is constant. The law is based on conservation of angular momentum and leads to

$\frac{v_1}{v_2} = \frac{r_2}{r_1}$ or $\frac{v_{\text{Perihelion}}}{v_{\text{aphelion}}} = \frac{r_{\text{aphelion}}}{r_{\text{Perihelion}}}$. See Fig (9.13) if e is eccentricity of the orbit then

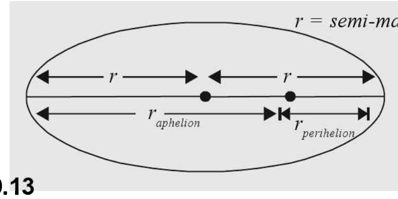


Fig. 9.13

$$\frac{r_{\text{aphelion}}}{r_{\text{Perihelion}}} = \frac{1+e}{1-e};\; r_{\text{aphelion}} + r_{\text{Perihelion}} = 2r$$

- If $e < 1$ and $v > v_{\text{escape}}$ (or total energy $KE + PE > 0$). The path of the satellite is hyperbolic and it escapes.
- If $e < 1$, total energy is negative (< 0) or $v < v_{\text{escape}}$ the satellite moves in an elliptical path.
- If $e = 0$, total energy is negative, i.e. $v < v_{\text{escape}}$, the satellite moves in a circular path.
- If $e = 0$, total energy is zero or $v = v_e$, the satellite will acquire parabolic path.

Third law: $T^2 \propto r^3$ where r is semimajor axis.

21. Orbital velocity $= v_o = \sqrt{\frac{GM}{r}}$ where r is radius of the orbit. $r = R_e + h$ for a satellite.

$\frac{v_{o1}}{v_{o2}} = \sqrt{\frac{R_e + h_2}{R_e + h_1}}$ if $v_o < v < v_e$ then the path is elliptical.

22. Escape velocity $v_e = \sqrt{\frac{2GM}{r}} = \sqrt{2}\, v_o$ i.e. if the velocity of a satellite revolving around earth (or that of a planet revolving around the sun) is increased by 41.4% then it will escape away. If $v > v_e$ satellite takes hyperbolic path and escapes from the gravitational field of the earth then

23. Time period of revolution $T^2 \propto r^3$.

$$T = \frac{2\pi r}{v} = \frac{2\pi r^{3/2}}{\sqrt{GM}} \text{ or } T^2 = \frac{4\pi^2}{GM} r^3$$

24. Total energy or binding energy of a body revolving around the earth/planet or the sun is BE (= $E_{\text{Tot.}}$)

$$= KE + PE = \frac{GMm}{2r} - \frac{GMm}{r} = \frac{-GMm}{2r}$$

i.e., $KE + PE = -KE = \frac{PE}{2}$ or $PE = -2\,KE$.

25. The path of the projectiles thrown to lower height is parabolic and thrown to larger heights is elliptical.

26. Geostationary or communication satellites have circular orbit. They are situated at a height 36000 km above the surface of the earth (r = 42400 km from the centre of the earth). Minimum number of communication satellites to cover whole globe is 3 as one satellite covers nearly 41% area. Maximum number of communication satellites = 180 which can be operative at a time (at a slot of 2° each).

27. Schwarzchild radius It is the distance surrounding a blackhole where even the light cannot escape $R_S = \frac{2GM}{c^2}$. The surface of the sphere surrounding black hole upto a radius R_S is called event horizon. We cannot see events occuring in this region.

28. Coroilis force = 2 $mv\omega$. When a body of mass m moves along a diameter with a velocity v on a turn table rotating with angular speed ω the coroilis force is experienced.

29. Prigee is the shortest distance of a satellite from the earth.

Apogee is the farthest distance of a satellite from the earth.

• Caution

1. Not remembering that gravitational field intensity depends upon shape, geometry and distance.

⇒ $E_g = 0$ at the centre of a ring $E_g = \frac{GMx}{(r^2+x^2)^{3/2}}$ at any point on the axial line.

$E_g = 0$ inside the shell (only due to shell). The presence of other body will cause E_g.

If a part of the body is cut, E_g will vary not only due to the fact that mass has varied but also due to the fact that shape has varied.

2. Assuming gravitational potential is only a function of distance.

⇒ Like gravitational field, gravitational potential also depends upon the shape and geometry. Gravitational field inside the shell is zero but gravitational potential inside the shell is non zero and remains constantly equal to the gravitational potential at the surface.

3. Assuming work done in gravitation is $F \cdot d$

⇒ Work done = $\int F.dx = \Delta PE = PE_{\text{final}} - PE_{\text{initial}}$. Do not apply $W = F \cdot d$ as force is variable.

4. Assuming g varies with height as $g' = g\left(1-\frac{2h}{R}\right)$

⇒ $g' = g\left(1-\frac{2h}{R}\right)$ is valid only if $h \le \frac{R}{10}$ otherwise use $g' = \frac{g}{\left(1+\frac{h}{R}\right)^2}$.

5. Assuming when the earth is closer to the sun only then summer is experienced.

⇒ Though in principle it appears correct but in case of the earth, the solar radiations are incident oblique. During winter earth is closer to the sun. That is why winter is of short duration.

6. Assuming $g = 0$ at the equator or g to be constant over the surface of the earth.

⇒ g is maximum at poles and minimum at the equator. Note, the variation is small and occurs due to outward radial force because of rotation of the earth.

7. Assuming any star will die as black hole.

⇒ Only those stars whose mass > 5 times the mass of the sun end as black hole.

8. Assuming in the relation of orbital velocity $v_o = \sqrt{\frac{GM}{r}}$, M is the mass of the satellite.

⇒ M is the mass of the earth/planet. Remember orbital velocity and escape velocity both are independent of the mass of the satellite being put into the orbit or to escape.

9. Assuming gravitational Binding energy is nothing but gravitational PE.

⇒ Binding energy $= KE + PE = -KE = \frac{1}{2} PE = \frac{-GMm}{2r}$.

10. Assuming Kepler's laws can be applied to planets only.

⇒ Keplers laws can be applied to planets and satellites (artificial or natural).

11. Not remembering relations relating eccentricity e and r (semimajor axis).

⇒ $\frac{r_{\text{aphelion}}}{r_{\text{Perihelion}}} = \frac{1+e}{1-e}$ and $2r = r_{\text{aphelion}} + r_{\text{Perihelion}}$

12. Assuming shielding effect in gravitational force also.

⇒ Gravitational force does not depend upon medium. Therefore, no medium can shield or block gravitational field.

13. Considering centripetal or centrifugal force cannot be applied on the earth.

⇒ Particles on the poles or on the axis of rotation do not have any such forces. At all other lattitudes apparent weight = $m(g - a_r) = m(g - R\,\omega^2 \cos^2 \lambda)$.

14. Assuming gravitational field inside the shell is zero always irrespective of presence of other masses outside the shell.

⇒ Gravitational field due to the shell is only zero. Refer to fig. 9.14. m is in a shell of mass M and radius R. m_1 is a mass distant x from m then force experienced by

m is $\frac{Gmm_1}{x^2}$.

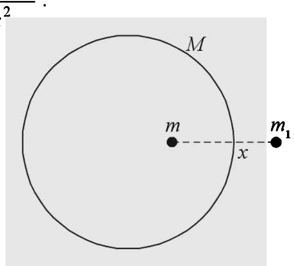


Fig. 9.14

15. Considering that escape velocity depends upon direction.

⇒ Theoretically it is independent of direction. However, practically a little dependence is observed.

16. Considering that gravitational PE is always negative.

⇒ It depends upon the reference used. So far we have assumed PE at infinity is zero. If we assume PE at the surface of the earth is zero then PE elsewhere will be +ve.

SOLVED PROBLEMS

1. For a satellite moving in an orbit around the earth, the ratio of KE to PE is

(a) $\frac{1}{2}$ (b) $\frac{1}{\sqrt{2}}$

(c) 2 (d) $\sqrt{2}$

[CBSE PMT 2005]

Solution (a) $KE = -\frac{1}{2}PE$ in a conservative bound system of forces.

2. Imagine a planet having the same density as that of the earth but radius is three times the radius of the earth. If acceleration due to gravity on the surface of the earth is g and that of the said planet is g' then

(a) $g' = \frac{g}{9}$ (b) $g' = 9g$

(c) $g' = \frac{g}{27}$ (d) $g' = 3g$

[CBSE PMT 2005]

Solution (d) $g = \frac{GM}{R^2} = \frac{G\frac{4}{3}\pi R^3 \rho}{R^2} = G\frac{4}{3}\pi R\rho$

$\because R_{Planet} = 3R$ Hence $g' = 3g$

3. Average density of the earth

(a) does not depend on g.
(b) is a complex function of g.
(c) is directly proportional to g.
(d) is inversely proportional to g.

[AIEEE 2005]

Solution (c) $g = G\frac{4}{3}\pi R\rho$

4. The change in the value of g at a height h above the surface of the earth is the same as at a depth d below the surface of the earth. When both h and d are much smaller than the radius of earth, then which one of the following is true?

(a) $a = h/2$ (b) $d = 3h/2$

(c) $d = 2h$ (d) $h = d$

[AIEEE 2005]

Solution (c) $g' = g\left(1 - \frac{2h}{R}\right) = g\left(1 - \frac{d}{R}\right) \therefore d = 2h$

5. A particle of mass 10 g is kept on the surface of a uniform sphere of mass 100 kg and radius 10 cm. Find the work to be done against the gravitational force between them to take the particle far away from the sphere.

(a) 13.34×10^{-10} J (b) 3.33×10^{-10} J

(c) 6.67×10^{-9} J (d) 6.67×10^{-10} J

[AIEEE 2005]

Solution (d) $W = \Delta PE = \frac{-GMm}{\infty} - \left(\frac{-GMm}{R}\right)$

$= \frac{6.67\times10^{-11}\times100\times10\times10^{-3}}{.1} = 6.67 \times 10^{-10}$ J

6. The condition for a uniform spherical mass m of radius r to be a black hole is [G = gravitational constant, g = acceleration due to gravity].

(a) $\left[\frac{2GM}{r}\right]^{1/2} \le c$ (b) $\left[\frac{2gm}{r}\right]^{1/2} = c$

(c) $\left[\frac{2GM}{r}\right]^{1/2} \ge c$ (d) $\left[\frac{gm}{r}\right]^{1/2} \ge c$

[AIIMS 2005]

Solution (c) $\left[\frac{2GM}{r}\right]^{1/2} \ge c$

7. Two planets are revolving around the earth with velocities v_1, v_2 and in radii r_1 and r_2 ($r_1 > r_2$) respectively. Then

(a) $v_1 = v_2$ (b) $v_1 > v_2$

(c) $v_1 < v_2$ (d) $\frac{v_1}{r_1} = \frac{v_2}{r_2}$

Solution (c) $v_o = \sqrt{\dfrac{GM}{r}}$ $\therefore$ $\dfrac{v_1}{v_2}$

$= \sqrt{\dfrac{r_2}{r_1}}$ $\because r_1 > r_2 \therefore v_2 > v_1$

8. Earth is revolving around the sun if the distance of the earth from the sun is reduced to $\dfrac{1}{4}$th of the present distance then the present length of the day is reduced by

(a) $\dfrac{1}{4}$ (b) $\dfrac{1}{2}$

(c) $\dfrac{1}{8}$ (d) $\dfrac{1}{6}$

Solution (c) $T^2 \propto r^3$ $\dfrac{T_1}{T_2} = \left(\dfrac{r/4}{r}\right)^{3/2} = \dfrac{1}{8}$

9. Helios-*B* spacecraft had a speed of 71 km/s when it was 4.3×10^7 km from the sun. Its orbit is

(a) circular. (b) helical.
(c) elliptical. (d) parabolic.

Solution (c) $v_o = \sqrt{\dfrac{GM}{R}} = \sqrt{\dfrac{6.67\times10^{-11}\times2\times10^{30}}{4.3\times10^7\times10^3}}$

$\simeq$ 56 km/ssince $v > v_o$, but $< v_e$, therefore, orbit is elliptical.

10. Find the weight of an object at neptune which weighs 19.6 N on the earth. Mass of Neptune = 10^{26} kg, radius $R = 2.5 \times 10^4$ km and rotates once around its axis in 16 *h*.

(a) 19.6 N (b) 20.0 N
(c) 20.4 N (d) 20.8 N

Solution (d) Weight $W = \dfrac{GMm}{R^2}$

$= \dfrac{6.67\times10^{-11}\times10^{26}\times2}{(2.5\times10^7)^2} = 20.8$ N.

11. An earth's satellite moves in a circular orbit with an orbital speed 6280 ms^{-1}. Find the time of revolution.

(a) 130 min (b) 145 min
(c) 155 min (d) 175 min

Solution (d) $v_o = \sqrt{\dfrac{GM}{r}}$ or $r = \dfrac{GM}{v_o^2}$ and

$T = \dfrac{2\pi r^{3/2}}{\sqrt{GM}} = \dfrac{2\pi GM}{v_o^3}$.

$T = \dfrac{2\pi\times6.67\times10^{-11}\times6\times10^{24}}{(6280)^2\times6280\times60} = 175$ min.

12. A mass m_1 is placed at the centre of a shell of radius R, m_2 is placed at a distance R from the surface and is immersed in an oil of dielectric constant 10. Find the force on m_1

(a) zero (b) $\dfrac{G(M+m_2)m_1}{10R^2}$

(c) $\dfrac{Gm_1m_2}{4R^2}$ (d) $\dfrac{Gm_1(4M+m_2)}{4R^2}$

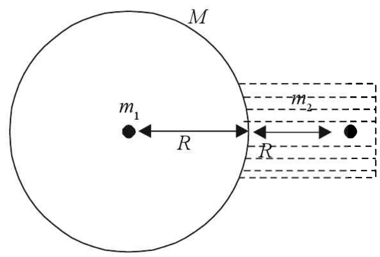


Fig. 9.15

Solution (c) $F = \dfrac{Gm_1m_2}{(2R)^2}$ shell exerts no force as its gravitational field inside the shell is zero. Oil does not play any role.

13. A 75 kg astronaut is repairing Hubble telescope at a height of 600 km above the surface of the earth. Find his weight there.

(a) 740 N (b) 700 N
(c) 650 N (d) 610 N

Solution (d) $mg' = mg\left(1 - \dfrac{2h}{R}\right)$

$= 75 \times 10\,(1 - \cdot185) = 610$ N

14. 5 kg and 10 kg spheres are 1 m apart. Where the gravitational field intensity be zero 5 kg block?

(a) 0.4 m (b) 0.3 m
(c) 0.25 m (d) 0.35 m

Solution (a) $\dfrac{GM_1}{x^2} = \dfrac{GM_2}{(1-x)^2}$ or $\dfrac{1-x}{x} = \dfrac{\sqrt{2}}{1}$

or $x = \dfrac{1}{\sqrt{2}+1} = 0.4$ m

15. A ring has mass M, radius R. A point mass m is placed at a distance x on the axial line as shown. Find x so that force experienced is maximum.

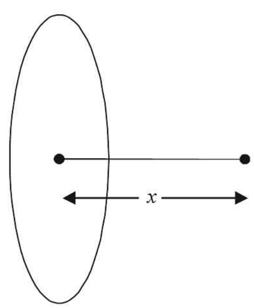


Fig. 9.16

(a) $R/3$∈ (b) $R/2$

(c) $R/\sqrt{2}$ (d) $R/\sqrt{3}$

Solution (c) $\frac{d}{dx}\left[\frac{GMmx}{(x^2+R^2)^{3/2}}\right] = 0$ or $x = R/\sqrt{2}$

16. How much work will be done to take a space craft orbiting at 200 km above the surface of earth to an orbit at 4000 km above the surface of the earth. Assume circular orbit. Mass of spacecraft equal to 2000 kg.

(a) 0.83×10^{10} J (b) 1.23×10^{10} J
(c) 1.53×10^{10} J (d) 1.83×10^{10} J

Solution (d) $W = \Delta PE + \Delta KE = PE_{final} - PE_{initial} + KE_{final} - KE_{initial}$

$= KE_{initial} - KE_{final} = \frac{GMm}{10^3 \times}\left[\frac{1}{8400} - \frac{1}{10400}\right]$

$= \frac{6.67\times10^{-11}\times6\times10^{24}\times2000}{10^3} \times \left[\frac{2000}{8400\times10400}\right]$

$= \frac{40\times10^{14}}{21\times10^4} = 1.83 \times 10^{10}$ J

17. A comet travels around the sun in elliptical orbit. Its mass is 10^8 kg when 2.5×10^{11} m away its speed is 2×10^4 ms^{-1}. Find the change in KE when it has reached 5×10^{10} m away from the sun.

(a) 38×10^8 J (b) 48×10^8 J
(c) 58×10^8 J (d) 56×10^8 J

Solution (b) $v_1 r_1 = v_2 r_2$ or

$v_2 = \frac{2\times10^4\times2.5\times10^{11}}{5\times10^{10}} = 10^5$

$\Delta KE = \frac{1}{2} \times 10^8[(10^5)^2 - 4 \times 10^8]$

$= 48 \times 10^8$ J

18. Gravitational field in a region is given by $(3\hat{i} + 2\hat{j})$ N kg^{-1}. Find the work done by the gravitational field when a particle of mass m moves from one point (x_1, y_1) to another (x_2, y_2) on the line $2y + 3x = 5$

(a) zero (b) $9(x_2 - x_1) + 4(y_2 - y_1)$
(c) $9(x_1 - x_1) + 4(y_1 - y_2)$ (d) none

Solution (a) m_1 (slope of the line) $= \frac{-3}{2}$

Slope of gravitational field $m_2 = \frac{\text{Coeff. of } j}{\text{Coeff. of } i} = \frac{2}{3}$

$\because m_1 m_2 = -1$ i.e. line and field are perpendicular.

Hence, work done = 0

19. A body is fired from the surface of the earth. It goes to a maximum height R (radius of the earth) from the surface of the earth. Find the initial velocity given.

(a) 6.9 km s^{-1} (b) 7.4 km s^{-1}
(c) 7.9 km s^{-1} (d) 8.4 km s^{-1}

Solution (c) $\frac{1}{2} mv^2 = GMm\left[\frac{1}{R} - \frac{1}{2R}\right]$

or $v = \sqrt{\frac{GM}{R}}$

$= \sqrt{\frac{6.67\times10^{-11}\times6\times10^{24}}{6400\times10^3}} = 7.9$ km s^{-1}.

20. Find the height above the surface of the earth where weight becomes half.

(a) $\frac{R}{2}$ (b) $(\sqrt{2} - 1)R$
(c) $\frac{R}{(\sqrt{2}+1)}$ (d) $\frac{R}{\sqrt{2}}$

Solution (b) $\frac{1}{2} = \frac{1}{\left(1+\frac{h}{R}\right)^2}$ or $h = (\sqrt{2} - 1)R$

21. A pendulum clock which keeps correct time at the surface of the earth is taken into a mine then

(a) it keeps correct time. (b) it gains time.
(c) it loses time. (d) none of these.

Solution (c) $T = 2\pi\sqrt{\frac{l}{g}}$ as g decreases, T increases. $\therefore$ it loses time.

22. Find the velocity of the earth at which it should rotate so that weight of a body becomes zero at the equator.

(a) 1.25 rad s^{-1} (b) 1.25×10^{-1} rad s^{-1}
(c) 1.25×10^{-2} rad s^{-1} (d) 1.25×10^{-3} rad s^{-1}

Solution (d) $g' = g\left(1 - \frac{R\omega^2}{g}\right) = 0$ or $\omega = \sqrt{\frac{g}{R}}$

$= \sqrt{\frac{10}{6400\times10^3}} = \frac{1}{800} = 1.25 \times 10^{-3}$ rad s^{-1}

23. The radius of a planet is R_1 and a satellite revolves around it in a radius R_2. Time period of revolution is T. Find the acceleration due to gravity.

(a) $\frac{4\pi^2 R_2^3}{R_1^2 T^2}$ (b) $\frac{4\pi^2 R_2^2}{R_1 T^2}$
(c) $\frac{2\pi^2 R_2^3}{R_1 T^2}$ (d) $\frac{4\pi^2 R_2}{T^2}$

Solution (a) $T = \frac{2\pi R_2^{3/2}}{\sqrt{GM}}$ or $GM = \frac{4\pi^2 R_2^3}{T^2}$

and $g = \frac{GM}{R_1^2} = \frac{4\pi^2 R_2^3}{R_1^2 T^2}$.

24. A pendulum having a bob of mass m is hanging in a ship sailing along the equator from east to west. When the ship is stationary with respect to water, the tension in the string is T_o. Find the difference between tensions when the ship is sailing with a velocity v.

(a) $mv\omega$ (b) $2\,mv\omega$

(c) $\frac{mv\omega}{2}$ (d) $\sqrt{2}\,mv\omega$

Solution (b) $2\,mv\omega$ Additional force is coroilis force which acts perpendicular to the plane of motion.

25. A solid sphere of mass m and radius r is placed inside a hollow thin spherical shell of mass M and radius R as shown in Fig. 9.17. A particle of mass m' is placed on the line joining the two centres at a distance x from the point of contact of the sphere and shell. Find the magnitude of force (gravitational) on this particle when $2r < x < 2R$.

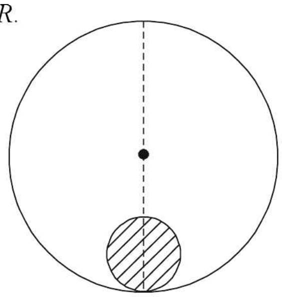


Fig. 9.17

(a) $\frac{Gmm'}{x^2}$ (b) $\frac{Gmm'}{(x-r)^2}$

(c) $\frac{Gmm'}{x^2} + \frac{GmM}{x^2}$ (d) $\frac{Gmm'}{(x-r)^2} + \frac{GMm'}{(x-R)^2}$

Solution (b) $\frac{Gmm'}{(x-r)^2}$

26. A satellite is in a circular orbit of radius r. At some point it is given an impulse along its direction of motion causing its velocity to increase n times. It now goes into an elliptical orbit with the planet at the centre of the ellipse. The maximum possible value of n could be

(a) $\sqrt{2}$ (b) 2

(c) $\sqrt{2}+1$ (d) $\frac{1}{\sqrt{2}-1}$

Solution (a) For satellite to revolve around the planet $v_o \le v_e$

and $v_e = \sqrt{2}\,v_o \quad \therefore n \le \sqrt{2}$

27. A dense sphere of mass M is placed at the centre of a circle of radius R. Find the work done when a particle of mass m is brought from A to B along a circle as shown in Fig. 9.18.

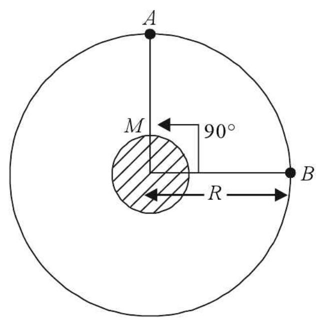


Fig. 9.18

(a) zero (b) $\frac{GMm}{R}$

(c) $-\frac{GMm}{R}$ (d) $\frac{2GMm}{R}$

Solution (a) $W = \Delta PE = 0$

TYPICAL PROBLEMS

28. A, B, C and D are four masses each of mass m lying on the vertices of a square of side 'a'. They always move along a common circle with velocity v. Find v so that they always remain on the vertices of the square.

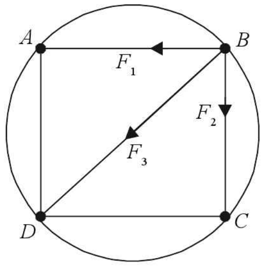


Fig. 9.19

(a) $\sqrt{\frac{GM(2\sqrt{2}+1)}{2\sqrt{2}a}}$ (b) $\sqrt{\frac{GM(\sqrt{2}+1)}{2a}}$

(c) $\sqrt{\frac{GM\sqrt{2}(2+1)}{a}}$ (d) none

$|F_1| = |F_2| \;\therefore\; |\vec{F}_1 + \vec{F}_2| = \sqrt{2}\,F_1$ (See Fig Q.3)

Solution (a) $F_{net} = \sqrt{2}\,F_1 + F_3 = \frac{mv^2}{R}$

$$\sqrt{2}\,a = 2R \text{ or } R = \frac{a}{\sqrt{2}}$$

$$\sqrt{2}\,\frac{Gm^2}{a^2} + \frac{Gm^2}{2a^2} = \frac{mv^2}{a}\sqrt{2}$$

or $$v = \sqrt{\frac{GM(2\sqrt{2}+1)}{2\sqrt{2}a}}$$

29. A planet of mass m moves along an ellipse so that perihelion and aphelion distances are r_1 and r_2. Find the angular momentum of the planet.

(a) $m\sqrt{\dfrac{GM_S r_1 r_2}{r_1 + r_2}}$ (b) $m\sqrt{\dfrac{2GM_S r_1 r_2}{r_1 + r_2}}$

(c) $m\sqrt{GM_S(r_1 + r_2)}$ (d) $m\sqrt{2GM_S(r_1 + r_2)}$

Solution (b) $m v_1 r_1 = m v_2 r_2$ or $v_1^2 = v_2^2\left(\dfrac{r_2}{r_1}\right)^2$

Using energy conservation $\dfrac{-GmM_S}{r_1} + \dfrac{mv_1^2}{2}$

$$= \frac{-GmM_S}{r_2} + \frac{mv_2^2}{2}$$

or $$\frac{-GM_S}{r_1} + \frac{-GM_S}{r_1} = \frac{v_2^2}{2} - \frac{v_2^2}{2}\left(\frac{r_2}{r_1}\right)^2$$

or $$v_2 = \sqrt{\frac{2GM_S r_1}{r_2(r_1 + r_2)}}$$

and $$L = m v_2 r_2 = m\sqrt{\frac{2GM_S r_1 r_2}{r_1 + r_2}}$$

30. At what height over the earth's pole the freefall acceleration decreases by 1%

(a) 64 km (b) 16 km

(c) 8 km (d) 32 km

Solution (d) $g' = g\left(1 - \dfrac{2h}{R}\right)$ $\dfrac{2h}{R} = \dfrac{1}{100}$ or $h = 32$ km

31. A satellite of moon revolves around it in a radius n times the radius of moon (R). Due to cosmic dust it experiences a resistance $F = \alpha v^2$. Find how long it will stay in the orbit.

(a) $\dfrac{m}{\alpha\sqrt{\dfrac{GM}{R}}}\sqrt{n}$ (b) $\dfrac{m}{\alpha}\sqrt{\dfrac{R}{am}}(\sqrt{n} - 1)$

(c) $\dfrac{m}{\alpha}\dfrac{(\sqrt{n} - 1)}{v}$ (d) $\dfrac{m}{\alpha}\dfrac{v_i}{v_f^2}$

Solution (b) $m\dfrac{dv}{dt} = \alpha v^2 dt$ or $\dfrac{dv}{v^2} = \dfrac{\alpha dt}{m}$

and $$v = \sqrt{\frac{GM}{r}}.$$

We know $v_i = \sqrt{\dfrac{GM}{nR}}$ and $v_f = \sqrt{\dfrac{GM}{R}}$

$$\therefore \int_{v_i}^{v_f} \frac{dV'}{V'^2} = \int \frac{\alpha dt}{m}$$

or $$t = \frac{m}{\alpha}\left[\frac{1}{v_i} - \frac{1}{v_f}\right] = \frac{m}{\alpha\sqrt{\dfrac{GM}{R}}}[\sqrt{n} - 1]$$

32. A particle is projected with a velocity 15 km s^{-1}. Find its velocity in the space far off from the earth.

(a) 3.8 km s^{-1} (b) 7.6 km s^{-1}

(c) 10 km s^{-1} (d) 11.2 km s^{-1}

Solution (c) $v_f^2 = v_i^2 - v_e^2 = 15^2 - (11.2)^2$ or $v_f = 10$ km s^{-1}

33. Find the minimum velocity to be imparted to a body so that it escapes the solar system

(a) $\sqrt{\dfrac{2GM_E}{R_E} + \dfrac{2GM_S}{R_{SE}}}$

(b) $\sqrt{\dfrac{2GM_E}{R_E} + (\sqrt{2} - 1)^2\dfrac{GM_S}{R_{SE}}}$

(c) $\sqrt{(\sqrt{2} - 1)^2\left[\dfrac{GM_E}{R_E} + \dfrac{GM_S}{R_{SE}}\right]}$ (d) none

Solution (b) $\dfrac{1}{2} m v_3^2 = \dfrac{1}{2} m v_1^2 + \dfrac{1}{2} m (\Delta v)^2$

or $$v_3 = \sqrt{v_1^2 + (\sqrt{2} - 1)^2 v_2^2}$$

where $v_1 = \sqrt{\dfrac{2GM_E}{R_E}}$ and $v_2 = \sqrt{\dfrac{2GM_S}{R_{SE}}}$

R_{SE} is the orbital distance of earth from the sun.

R_E is the radius of the earth.

34. A tunnel is dug along a chord of the earth at a perpendicular distance $\dfrac{R}{3}$ from the earth's centre. Assume wall of the tunnel is frictionless. Find the force exerted by the wall on mass m at a distance x from the centre of the tunnel.

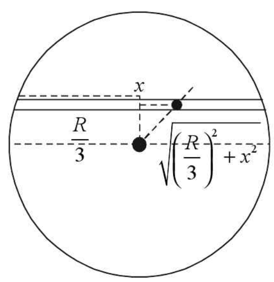


Fig. 9.20

(a) $\dfrac{mg\sqrt{\dfrac{R^2}{9} + x^2}}{R}$ (b) $\dfrac{mg\,x}{\sqrt{R^2/a + x^2}}$

(c) $\dfrac{mg}{3}$ (d) $\dfrac{mg\,x}{R}$

Solution (c) $F = mg' = mg\left[1 - \dfrac{\sqrt{\dfrac{R^3}{9} + x^2}}{9}\right]$

$$= \frac{mg\sqrt{R^2/9 + x^2}}{R} \cdot \frac{R/3}{\sqrt{R^2/9 + x^2}} = \frac{mg}{3}$$

35. A body weighs 1 kg by a spring balance at the north pole. What will it weigh at the equator?

(a) 0.977 kg (b) 0.967 kg
(c) 0.987 kg (d) 0.997 kg

Solution (d) $mg' = mg\left[1 - \frac{R\omega^2}{g}\right]$

$$= 1\left[1 - \frac{6400\times10^3\times(2\pi)^2}{10\times(3600\times24)^2}\right] = 0.997\text{ kg}$$

36. Let V_G and E_G denote gravitational potential and field respectively, then it is possible to have

(a) $V_G = 0, E_G = 0$ (b) $V_G \neq 0, E_g = 0$
(c) $V_G = 0, E_g \neq 0$ (d) $V_G \neq 0, E_G \neq 0$

Solution (a, b, d)

37. Which of the following quantities remain constant in a planetary system when seen from the surface of the sun.

(a) *KE* (b) angular speed
(c) speed (d) angular momentum
(e) binding energy

Solution (d) & (e)

38. The gravitational potential (V_G) and gravitational field (E_g) are plotted against distance *r* from the centre of a uniform spherical shell. Consider the following statement

(A) The plot of V_G against *r* is discontinuous.
(B) The plot of E_G against *r* is discontinuous.
(a) both A and B are correct.
(b) both A and B are wrong.
(c) A is correct but B is wrong.
(d) B is correct but A is wrong.

Solution (d)

39. Two satellites *X* and *Y* move round the earth in the same orbit. The mass of *B* is twice that of *A*. Then

(a) $v_A = v_B$ (b) $KE_A = KE_B$
(c) $(KE + PE)_A = (KE + PE)_B$ (d) $PE_A = PE_B$

Solution (a)

40. Find the work done to take a particle of mass *m* from surface of the earth to a height equal to 2*R*.

(a) $2\,mgR$ (b) $\frac{mgR}{2}$
(c) $3\,mgR$ (d) $\frac{2mgR}{3}$

Solution (d) $W = \Delta PE = GMm\left[\frac{1}{R} - \frac{1}{3R}\right]$

$$= \frac{2GMm}{3R} = \frac{2}{3}\,gm\,R.$$

41. Find the height at which the weight will be same as at the same depth from the surface of the earth.

(a) $\frac{R}{2}$ (b) $\sqrt{5}\,R - R$
(c) $\frac{\sqrt{5}R - R}{2}$ (d) $\frac{\sqrt{3}R - R}{2}$

Solution (c) $\frac{g}{\left(1 + \frac{x}{R}\right)^2} = g\left(1 - \frac{x}{R}\right)$

or $\left(1 - \frac{x}{R}\right)\left(1 + \frac{x^2}{R^2} + \frac{2x}{R}\right) = 1$

or $\frac{x^3}{R^3} + \frac{x^2}{R^2} - \frac{x}{R} = 0 = \frac{x}{R}\left(\frac{x^2}{R^2} + \frac{x}{R} - 1\right)$

or $\frac{x}{R} = \frac{-1 \pm \sqrt{1+4}}{2}$ or $x = \frac{\sqrt{5}R - R}{2}$

42. Velocity of a satellite is $v_o < v < v_e$ then

(a) its orbit is open.
(b) its orbit is closed and circular.
(c) its orbit is closed and parabolic.
(d) its orbit is closed and elliptical.
(e) none.

Solution (d)

QUESTIONS FOR PRACTICE

1. The rotation of the earth about its axis speeds up such that a man on the equator becomes weightless. In such a situation, what would be the duration of one day?

(a) $2\pi\sqrt{R/g}$ (b) $\frac{1}{2\pi}\sqrt{R/g}$
(c) $2\pi\sqrt{Rg}$ (d) $\frac{1}{2\pi}\sqrt{Rg}$

2. Two identical trains *A* and *B* move with equal speeds on parallel tracks along the equator. *A* moves from east to west and *B*, from west to east. Which train will exert greater force on the tracks?

(a) *A* (b) *B*
(c) They will exert equal force.
(d) The mass and the speed of each train must be known to reach a conclusion.

3. Let ω be the angular velocity of the earth's rotation about its axis. Assume that the acceleration due to gravity on the earth's surface has the same value at the equator and the poles. An object weighed by a spring balance gives the same reading at the equator as at a height h above the poles ($h << R$). The value of h is

(a) $\dfrac{\omega^2 R^2}{g}$ (b) $\dfrac{\omega^2 R^2}{2g}$

(c) $\dfrac{2\omega^2 R^2}{g}$ (d) $\dfrac{\sqrt{Rg}}{\omega}$

4. Use the assumptions of the previous question. An object weighed by a spring balance at the equator gives the same reading as a reading taken at a depth d below the earth's surface at a pole ($d << R$). The value of d is

(a) $\dfrac{\omega^2 R^2}{g}$ (b) $\dfrac{\omega^2 R^2}{2g}$

(c) $\dfrac{2\omega^2 R^2}{g}$ (d) $\dfrac{\sqrt{Rg}}{\omega}$

5. The escape velocity for a planet is v_e. A particle starts from rest at a large distance from the planet, reaches the planet only under gravitational attraction, and passes through a smooth tunnel through its centre. Its speed at the centre of the planet will be

(a) v_e (b) $1.5\, v_e$

(c) $\sqrt{1.5}\ v_e$ (d) $2\, v_e$

6. The escape velocity for a planet is v_e. A particle is projected from its surface with a speed v. For this particle to move as a satellite around the planet,

(a) $\dfrac{v_e}{2} < v < v_e$ (b) $\dfrac{v_e}{\sqrt{2}} < v < v_e$

(c) $v_e < v < \sqrt{2}\ v_e$ (d) $\dfrac{v_e}{\sqrt{2}} < v < \dfrac{v_e}{2}$

7. Two small satellites move in circular orbits around the earth, at distances r and $r + \Delta r$ from the centre of the earth. Their time period of rotation are T and $T + \Delta T$. ($\Delta r << r, \Delta T << T$)

(a) $\Delta T = \dfrac{3}{2}\, T\, \dfrac{\Delta r}{r}$ (b) $\Delta T = -\dfrac{3}{2}\, T\, \dfrac{\Delta r}{r}$

(c) $\Delta T = \dfrac{2}{3}\, T\, \dfrac{\Delta r}{r}$ (d) $\Delta T = T\, \dfrac{\Delta r}{r}$

8. Let S be an imaginary closed surface enclosing mass m. Let $d\vec{S}$ be an element of area on S, the direction of $d\vec{S}$ being outward from S. Let $\vec{E}$ be the gravitational intensity at $d\vec{S}$. We define $\phi = \oint_S \vec{E} \cdot d\vec{S}$, the integration being carried out over the entire surface S.

(a) $\phi = -Gm$ (b) $\phi = -4\pi Gm$

(c) $\phi = -\dfrac{Gm}{4\pi}$

(d) No relation of the type (a), (b) or (c) can exist.

9. Take the effect of bulging of earth and its rotation in account. Consider the following statements.

(A) There are points outside the earth where the value of g is equal to its value at the equator.
(B) There are points outside the earth where the value of g is equal to its value at the poles.

(a) Both A and B are correct.
(b) A is correct but B is wrong.
(c) B is correct but A is wrong.
(d) Both A and B are wrong.

10. The time period of an earth-satellite in circular orbit is independent of

(a) the mass of the satellite.
(b) radius of the orbit.
(c) none of them.
(d) both of them.

11. The magnitude of gravitational potential energy of the moon-earth system is U with zero potential energy at infinite separation. The kinetic energy of the moon with respect to the earth is K.

(a) $U < K$ (b) $U > K$
(c) $U = K$ (d) none

12. Figure 9.21 shows the elliptical path of a planet about the sun. The two shaded parts have equal area. If t_1 and t_2 be the time taken by the planet to go from a to b and from c to d respectively,

(a) $t_1 > t_2$ (b) $t_1 = t_2$
(c) $t_1 > t_2$
(d) insufficient information to deduce the relation between t_1 and t_2.

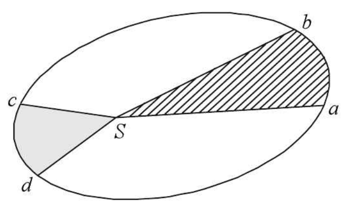


Fig. 9.21

13. A person sitting in a chair in a satellite feels weightless because

(a) the earth does not attract the objects in a satellite.
(b) the normal force by the chair on the person balances the earth's attraction.
(c) the normal force is zero.
(d) the person in satellite is not accelerated.

14. A body is suspended from a spring balance kept in a satellite. The reading of the balance is W_1 then the satellite goes in an orbit of radius R and is W_2 when it goes in an orbit of radius $2\,R$.

(a) $W_1 = W_2$ (b) $W_1 < W_2$
(c) $W_1 > W_2$ (d) $W_1 \neq W_2$

15. The kinetic energy needed to project a body of mass m from the earth's surface to infinity is

(a) $\frac{1}{4}\,mgR$ (b) $\frac{1}{2}\,mgR$
(c) mgR (d) $2\,mgR$

16. A particle is kept at rest at a distance R (earth's radius) above the earth's surface. The minimum speed with which it should be projected so that it does not return is

(a) $\sqrt{\frac{GM}{4R}}$ (b) $\sqrt{\frac{GM}{2R}}$
(c) $\sqrt{\frac{GM}{R}}$ (d) $\sqrt{\frac{2GM}{R}}$

17. A satellite is orbiting the earth close to its surface. A particle is to be projected from the satellite to just escape from the earth. The escape speed from the earth is v_e. Its speed with respect to the satellite

(a) will be less than v_e.
(b) will be more than v_e.
(c) will be equal to v_e.
(d) will depend on direction of projection.

18. A uniform spherical shell gradually shrinks maintaining its shape. The gravitational potential at the centre

(a) increases (b) decreases
(c) remains constant (d) oscillates

19. Two satellites A and B move round the earth in the same orbit. The mass of B is twice the mass of A.

(a) Speeds of A and B are equal.
(b) The potential energy of earth + A is same as that of earth + B.
(c) The kinetic energy of A and B are equal.
(d) The total energy of earth + A is same as that of earth + B.

20. Which of the following quantities remain constant in a planetory motion (consider elliptical orbits) as seen from the sun?

(a) Speed (b) Angular speed
(c) Kinetic energy (d) Angular momentum

21. The velocity of a satellite in a parking orbit is—

(a) 8 Km/s (b) 3.1 Km/s
(c) 2.35 Km/s (d) Zero

22. The distances of two satellites P and Q from earth are in the ratio 3 : 1. The ratio of their total energy will be

(a) 3 : 1 (b) 1 : 3
(c) 1 : 1 (d) $\frac{1}{3}$: 1

23. Two satellites P and Q of same mass are revolving near the earth surface in the equatorial plane. The satellite P moves in the direction of rotation of earth whereas Q moves in the opposite direction. The ratio of their kinetic energies with respect to a frame attached to earth will be

(a) $\left(\frac{8363}{7437}\right)^2$ (b) $\left(\frac{7437}{8363}\right)^2$
(c) $\frac{8363}{7437}$ (d) $\frac{7437}{8363}$

24. The semi-major axes of the orbits of Mercury and Mars in the astronomical units are 0.387 and 1.524 respectively.

If the time period of Mercury is 0.241 year, then the time period of Mars will be

(a) 0.9 Year (b) 0.19 Year
(c) 1.9 Year (d) 2.9 Years

25. If the orbital speed of moon is increased by 41.4% then moon will

(a) leave its orbit and will escape out.
(b) fall on earth.
(c) attract all bodies on earth towards it.
(d) have time period equal to 27 days.

26. Two artificial satellites P and Q are revolving round the earth in circular orbits. If the ratio of their radii is 1 : 4 and ratio of their masses is 3 : 1 then the ratio of their time periods will be

(a) $\frac{1}{8}$ (b) 8
(c) 4 (d) 3

27. Three particles of equal mass m are situated at the vertices of an equilateral triangle of side 1. The work done in increasing the side of the triangle to 21 will be

(a) $\frac{3G_2m}{21}$ (b) $\frac{Gm^2}{21}$
(c) $\frac{3Gm^2}{21}$ (d) $\frac{3Gm^2}{1}$

28. A space ship is released in a circular orbit near earth surface. How much additional velocity will have to be given to the ship in order to escape out of this orbit.

(a) 3.28 m/s (b) 3.28×10^3 m/s
(b) 3.28×10^7 m/s (d) 3.28×10^{-3} m/s

29. The centripetal force acting on a satellite revolving round the earth is F. The gravitational force on that planet is also F. The resultant force on the satellite is

(a) Zero (b) F
(c) $2F$ (d) $\frac{F}{2}$

30. If the force inside earth surface varies as r^x then the value of x will be (r = distance of body from centre of earth)

(a) $x = -1$ (b) $x = -2$
(c) $x = 1$ (d) $x = 2$

31. An artificial satellite is revolving round the earth. The radius of its circular orbit is half the orbital radius of moon. The time taken by this satellite in completing one revolution will be

(a) 2 lunar months. (b) $2^{-2/3}$ lunar months.
(c) $2^{-3/2}$ lunar months. (d) 1/2 lunar months.

32. The value of acceleration due to gravity at height h from earth surface will become half its value on the surface if (R = radius of earth)

(a) $h = R$ (b) $h = 2R$
(c) $h = \sqrt{2} - 1$ (d) $h = (\sqrt{2} + 1)$

33. If Ve is the escape velocity of a body from a planet of mass M and radius R. Then, the velocity of satellite revolving at height h from the surface of planet will be

(a) $v = v_e\sqrt{R/(R+h)}$ (b) $v = v_e\sqrt{2R/(R+h)}$
(c) $v = v_e\sqrt{(R+h)/R}$ (d) $v = v_e\sqrt{R/2(R+h)}$

34. The orbital radius of moon around the earth is 3.8×10^8 meter and its time period is 27.3 days. The centripetal acceleration of moon will be

(a) -2.4×10^{-3} m/s^2 (b) 11.2 m/s^2
(c) 2.7×10^{-3} m/s^2 (d) 9.8 m/s^2

35. The change in the value of acceleration of earth towards sun, when the moon comes from the position of solar eclipse to the position on the other side of earth in line with sun is (mass of moon = 7.36×10^{22} Kg., the orbital radius of moon 3.82×10^8 m)

(a) 6.73×10^{-2} m/s^2 (b) 6.73×10^{-3} m/s^2
(c) 6.73×10^{-4} m/s^2 (d) 6.73×10^{-5} m/s^2

36. A projectile is fired from the surface of earth with initial velocity of 10 Km/sec If the radius of earth is 6400 Km. how high will it go from earth surface

(a) 2500 Km (b) 2500 Km
(c) 2.5×10^4 Km (d) 2.5×10^6

37. A satellite is launched in a circular orbit of radius R and another satellite is launched in circular orbit of radius 1.01 R. The time period of second satellite is different from that of the first satellite by

(a) 1.5% increased (b) 1% decreased
(c) 1% increased (d) 1.5% decreased

38. How far must a particle be on the line joining earth to sun, in order that the gravitational pull on it due to sun is counterbalanced by that due to earth. (Given orbital radius of earth is 10^8 *Km* and $Ms = 3.24 \times 10^5 ME$)

(a) 64×10^5 Km (b) 1.75×10^2 Km
(c) 1.75×10^9 Km (d) 6400 Km

39. A satellite is projected with a velocity $\sqrt{1.5}$ times its orbital velocity just above earth atmosphere. The initial velocity of the satellite is parallel to the surface. The maximum distance of the satellite from earth will be

(a) $2R$ (b) $8R$
(c) $4R$ (d) $3R$

40. The gravitational potential difference between the surface of a planet and a point 20 m above the surface is 2 Joule/Kg. If the gravitational field is uniform then the work done in carrying a 5 Kg body to a height of 4 m above the surface is

(a) 2 Joule (b) 20 Joule
(c) 40 Joule (d) 10 Joule

41. A sky laboratory of mass 2×10^3 Kg is raised from a circular orbit of radius $2R$ to a circular orbit of radius $3R$. The work done is approximately.

(a) 1×10^{16} Joule (b) 2×10^{10}
(c) 1×10^6 Joule (d) 3×10^{10} Joule

42. A planet revolves round the sun. Its velocity at the nearest point, distant d_1 from sun, is v_1. The velocity of the planet at the farthest point distant d_2 from sun will be.

(a) $\frac{d_1^2 v_1}{d_2^2}$ (b) $\frac{d_2 v_1}{d_1}$
(c) $\frac{d_1 v_1}{d_2}$ (d) $\frac{d_2^2 v_1}{d_1^2}$

43. If the change in the value of g at height h above earth surface is the same as that at depth x (x or $h < Re$), then

(a) $x = h^2$ (b) $x = h$
(c) $x = \frac{h}{2}$ (d) $x = 2h$

44. The escape velocity on a planet with radius double that of earth and mean density equal to that of earth will be (escape velocity on earth = 11.2 Km/s).

(a) 11 Km/s (b) 22 Km/s
(c) 5.5 Km/s (d) 15.5Km/s

45. The motion of a planet around sun in an elliptical orbit is shown in the following figure. Sun is situated on one focus. The shaded areas are equal. If the planet takes time t_1 and t_2 in moving from A to B and from C to D respectively then

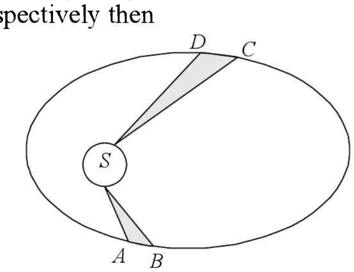


Fig. 9.22

(a) $t_1 > t_2$ (b) $t_1 < t_2$
(c) $t_1 = t_2$
(d) information incomplete

46. The acceleration due to gravity at a place is g m/s^2 A lead sphere of density d Kg/m^3 is dropped into a liquid column of density p Kg/m^3. If $d > p$ then the sphere will fall down with

(a) g acceleration.
(b) without acceleration.
(c) acceleration.
(d) with an acceleration $g\,(p/d)$.

47. A tunnel is dug along a diameter of earth. The force on a particle of mass of distant x from the centre in this tunnel will be

(a) $\frac{GM_e m}{R^3 x}$ (b) $\frac{GM_e m R^3}{x}$

(c) $\frac{GM_e mx}{R^2}$ (d) $\frac{GM_e mx}{R^3}$

48. A balloon filled with hydrogen gas is carried from earth on moon. Then the balloon will

(a) neither fall nor rise.
(b) fall with acceleration less than g.
(c) fall with acceleration g.
(d) rise with acceleration g.

49. An artificial satellite is revolving close to earth. Its orbital velocity mainly depends upon

(a) the mass of earth (b) the radius of earth
(c) the orbital radius (d) the mass of satellite

50. The potential energy of a rocket of mass 100 kg at height 10^7 m from earth surface is 4×10^9 joule. The weight of the rocket at height 10^9 will be

(a) 4×10^{-2} N (b) 4×10^{-3} N
(c) 8×10^{-2} N (d) 8×10^{-3} N

51. The venus appears more shining than other stars because

(a) it is heavier than other stars.
(b) its density is more than that of other stars.
(c) it is nearer the earth than other stars.
(d) there is no atmosphere on it.

52. A communication satellite is carried from one orbit to another orbit, with radius double that of the first. Its time period in the new orbit will be

(a) $24\sqrt{2}$ Hours (b) $48\sqrt{2}$ Hours
(c) 24 Hours (d) 48 Hours

53. The initial concept about the origin of universe was given by

(a) Bondi (b) Hawllacks
(c) Narlikar (d) Hubli

54. The value of G for two bodies in vacuum is 6.67×10^{-11} N/m²/Kg² Its value in a dense medium of density 10^{10} gm/cm³ will be

(a) 6.67×10^{-11} N/m²/Kg (b) 6.67×10^{-31} N/m²/Kg
(c) 6.67×10^{-21} N/m²/Kg (d) 6.67×10^{-10} N/m²/Kg

55. The length of the day from today when the sun is directly overhead till tomorrow again when the sun is directly overhead can be determined by the

(a) rotation of earth about its own axis.
(b) revolution of earth around sun.
(c) inclination of axis of rotation of earth from the plane of revolution.
(d) rotation of earth about its own axis as well as its revolution around sun.

56. Two satellites of mass m_1 and m_2 ($m_1 > m_2$) revolve round the earth in circular orbits of radii r_1 and r_2 respectively ($r_1 > r_2$) Their speeds are related as

(a) $v_1 = v_2$ (b) $v_1 < v_2$

(c) $v_1 > v_2$ (d) $\frac{v_1}{r_1} = v_2 r_2$

57. Presuming earth to be a uniform sphere, a scientist A goes deep inside a mine and another scientist B goes high in a balloon above earth surface. The intensity of gravitational field

(a) decreases when measured by A and increases when measured by B.
(b) decreases when measured by B and increases when measured by A.
(c) decreases when measured by both.
(d) remains constant when measured by both.

58. A person jumps from the fifth storey of a building with load on his head. The weight experienced by him before reaching the earth will be

(a) Zero (b) g Kg/wt
(c) $m\,(g + a)$ (d) mg

59. Two artificial satellites of unequal masses are revolving in a circular orbit around the earth with a constant speed. Their time periods

(a) will be different.
(b) will be same.
(c) will depend on their masses.
(d) will depend upon the place of their projection.

60. The mass of earth is 80 times that of moon. Their diameters are 12800 Km and 3200 Km respectively. The value of g on moon will be, if its value on earth is 980 cm/s²

(a) 98 cm/s² (b) 196 cm/s²
(c) 100 cm/s² (d) 294 cm/s²

61. A bomb blasts on moon. Its sound will be heard on earth after

(a) 3.7 minutes
(b) 10 minutes
(c) 138 minutes
(d) sound will never be heard

Answers to Questions for Practice

1.	(a)	**2.**	(a)	**3.**	(b)	**4.**	(a)	**5.**	(c)	**6.**	(b)	**7.**	(a)
8.	(b)	**9.**	(b)	**10.**	(a)	**11.**	(b)	**12.**	(b)	**13.**	(c)	**14.**	(a)
15.	(c)	**16.**	(c)	**17.**	(d)	**18.**	(b)	**19.**	(a)	**20.**	(d)	**21.**	(b)
22.	(b)	**23.**	(a)	**24.**	(c)	**25.**	(a)	**26.**	(a)	**27.**	(c)	**28.**	(b)
29.	(b)	**30.**	(c)	**31.**	(c)	**32.**	(c)	**33.**	(d)	**34.**	(c)	**35.**	(d)
36.	(c)	**37.**	(a)	**38.**	(b)	**39.**	(d)	**40.**	(a)	**41.**	(b)	**42.**	(c)
43.	(d)	**44.**	(b)	**45.**	(c)	**46.**	(b)	**47.**	(d)	**48.**	(b)	**49.**	(b)
50.	(a)	**51.**	(c)	**52.**	(b)	**53.**	(d)	**54.**	(a)	**55.**	(b)	**56.**	(b)
57.	(c)	**58.**	(a)	**59.**	(b)	**60.**	(b)	**61.**	(d)				

Explanations

1(a) Let ω = angular velocity of the earth about its axis.

$$mg - N = m\omega^2 R$$

for $N = 0$, $\omega^2 = g/R$.

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{R/g}.$$

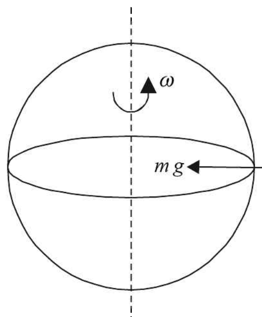


Fig. 9.23

2(a) Let v = speed of each train relative to the earth's surface,

v_E = speed of earth's surface relative to the earth's axis,

v_A, v_B = speeds of A and B relative to the earth's axis.

Then, $v_A = v_E - v$, $v_B = v_B + v$.

$$N_A = mg - m\left(\frac{v_A^2}{R}\right), N_B = mg - m\left(\frac{v_B^2}{R}\right).$$

$\therefore$ $N_A > N_B$.

3(b) Apparent weight at the equator $= mg - m\omega^2 R$

Weight at a height h above the pole $= mg\left(1 - \frac{2h}{R}\right)$.

Putting $mg - m\omega^2 R = mg\left(1 - \frac{2h}{R}\right)$.

or $\omega^2 R = \frac{2gh}{R}$ or $h = \frac{\omega^2 R^2}{2g}$

5(c) Taking the potential at a large distance from the planet as zero, the potential at the centre of the planet $= -\frac{3GM}{2R}$.

$$\therefore \quad \frac{1}{2} mv^2 = m\left[0 - \left(-\frac{3GM}{2R}\right)\right]$$

or $v^2 = \frac{3GM}{R} = 3Rg = \frac{3}{2}(2RG) = \frac{3}{2} v_e^2$

or $v = \sqrt{1.5}\, v_e$.

6(b) For a satellite orbiting very close to the earth's surface, the orbital velocity $= \sqrt{Rg}$ ($\because$ $mg = mv^2/R$). This is equal to the velocity of projection and is the minimum velocity required to go into orbit. Also, the satellite would escape completely and not go into orbit for $v \geq v_e$.

$\therefore$ $v_e/\sqrt{2} < v < v_e$.

7(a) $T^2 \propto r^3$ or $T^2 = cr^3$

$\therefore$ $2T\Delta T = 3cr^2 \Delta r$.

Dividing, $\frac{2T\Delta T}{T^2} = \frac{3cr^2 \Delta r}{cr^3}$.

8(b) Follow the method used to prove Gauss's law.

$$E = G . \frac{m}{r^2}$$

$$\vec{E} . d\vec{S} = EdS\cos(180° - \theta) = -EdS\cos\theta$$

$$\phi = \oint_s \vec{E} . d\vec{S} = \oint_s -G\frac{m}{r^2} dS\cos\theta$$

$$= -Gm . \oint_s \frac{dS\cos\theta}{r^2} = -Gm . \oint_s d\omega$$

$$= -4\pi Gm.$$

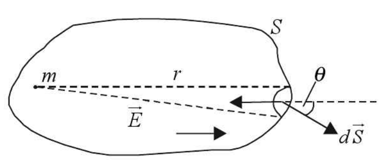


Fig. 9.24

10

Interatomic Forces and Elasticity

BRIEF REVIEW

We know atoms and molecules constitute matter. An atom consists of nucleus and forces operating between different nucleons are responsible for the structure of nucleus. Electromagnetic forces operate between a pair of electrons and between electrons and nucleus. These forces form the structure of a molecule. These interatomic or molecular forces are responsible for the structure of the material.

Interatomic and inter molecular forces The forces between two atoms can be typically represented by the potential energy curve shown in Fig. 10.1.

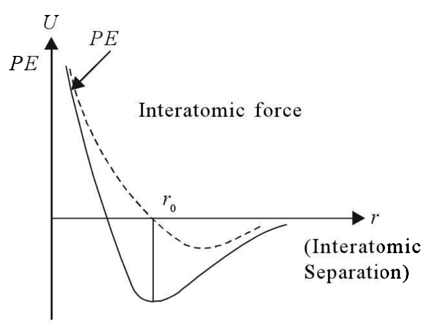


Fig. 10.1 Interatomic forces vs interatomic spacing

The zero of PE is taken when atoms are widely separated, *i.e.*, $(r \to \infty)$. The minimum PE indicates equilibrium position $(r = r_0)$. Interatomic force is a result of attractive and repulsive forces. When $r < r_0$, repulsive force dominates, and for $r > r_0$, attractive force dominates. At $r = r_0$, the repulsive and attractive forces are balanced. Therefore, at $r = r_0$, $\frac{du}{dr} = 0 = F$

Attractive forces $\propto \frac{1}{r^7}$ and repulsive forces are proportional to $\frac{1}{r^{13}}$, *i.e.*, $U = \frac{A}{r^{12}} - \frac{B}{r^6}$.

$$F = \frac{-du}{dr}$$

Repulsive force contribution is $\frac{12A}{r^{13}}$ and attractive force contribution is $\frac{-6B}{r^7}$

Force between identical molecules is termed as cohesive force and force between dissimilar molecules is called adhesive force.

The range of molecular force is of the order of 10^{-10} m. A sphere around a molecule having a radius equal to the range of molecular force is called sphere of molecular action of that molecule.

Bonds The electromagnetic interaction between electron and nuclei results in bonds. Some important examples are ionic or electrovalent, covalent bonding and metallic bonding.

In solids, separation between atom is of the order of r_0. In liquids, atomic separation is slightly greater than r_0 and in liquids $r >> r_0$.

Due to stong, interatomic forces atoms or ions, in solids, stay in their equilibrium position. If these equilibrium positions have a very regular 3-dimensional arrangement, such solids are called **Crystals**. The position occupied by the molecules or the ions are called **lattice points**. Crystalline solids are of 4 types depending upon the nature of bonding between the ionic units.

Molecules are formed due to covalent bonding between the atoms. Bonding may be **polar** or **nonpolar**. If the centre of negative charges coincides with centre of positive charge then molecule is **nonpolar**. Molecules of H_2, O_2, Cl_2 and so on are nonpolar. Otherwise molecules are said to be polar. For example, H_2O (water) is polar compound. Bond between polar compound is called **dipole – dipole bond** and bond between **nonpolar** molecules is called Vanderwaal's bond. Molecular solids are usually soft and have low melting points. They are poor conductors of electricity. Lattice, in **Ionic solids,** are occupied by positive and negative ions. Electrostatic forces between the ions bond the solids. Since these attractive forces are very strong, these materials are usually hard and have high melting points. They are poor conductor of electricity. However, in molten or aqueous solution form they conduct electricity.

In **covalent solids,** atoms are arranged in the crystalline form. The neighbouring atoms are bonded by shared electrons. Large solid structures are possible. Silicon, diamond are examples. These are quite hard and poor conductor of electricity.

In **metallic solids,** positive ions are situated at the Lattice sites. The ions are formed by detatching one or more electron from the constituent atoms. These electrons are highly **mobile** and move throughout the solid just like a gas. They are good conductors of electricity.

Amorphous or Glassy solids do not exhibit short range ordering instead of long range ordering. They only show local ordering. Four or five molecules are bonded together to form a structure. Such independent units are randomly arranged just like liquids. However, intermolecular force in amorphous solid form are much stronger than those in liquids. This presents the amorphous solid to flow like a liquid. These solids do not have a well defined melting point.

Homogeneous bodies having identical properties in all directions are called **isotropic**. Heterogeneous bodies having different properties in different directions are called **anistropic**.

Elasticity A body is said to be elastic if on releasing deforming force it regains its original shape. If it retains its new size or shape after removal of deforming force, it is said to be plastic. Fig. 10.2 and 10.3 show stress vs strain diagrams for a brittle and ductile material respectively.

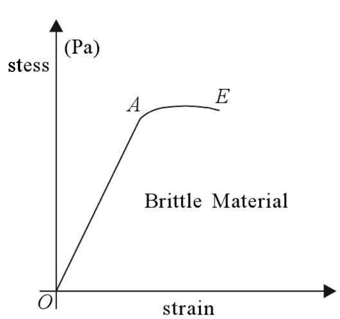


Fig. 10.2(a) Stress vs strain for plastic body

Hooke's law under elastic limit stress ∝ strain

or $\frac{\text{stress}}{\text{strain}}$ = Elastic modulus.

The stress at yield point b in Fig. 10.2 (b) is called elastic limit. The elastic behaviour ends at this point. Beyond c plastic deformation occurs. d is fracture point. Plastic deformation is irreversible. The stress required to cause actual fracture of a material is called the **breaking stress** or the **ultimate stress** or the **tensile stress**. More the cd part more ductile is the material.

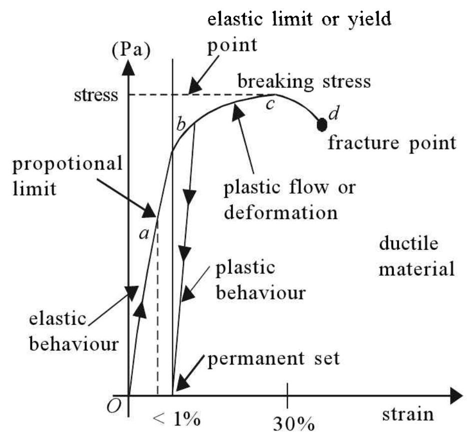


Fig. 10.2(b) Stress vs strain for an elastic body

working stress = $\frac{\text{Breaking stress}}{\text{Safety factor}}$

Young's molecules Y = $\frac{\text{tensile stress}}{\text{tensile strain}} = \frac{F_{\perp}/A}{\Delta l / l}$

$$= \frac{Fl}{A\Delta l} = \frac{mgl}{A\Delta l}$$

Bulk moleculus B = $\frac{\text{volumetric stress}}{\text{volumetric strain}} = -\frac{P}{\Delta V / V}$

$$= -\frac{PV}{\Delta V}$$

$B_{isothrmal} = P$ and $B_{adiabatic}$

$$= \gamma P \text{ where } \gamma = \frac{C_P}{C_V} \text{ in gases/air}$$

Compressibility $C = \frac{1}{B}$ (reciprocal of Bulk modulus).

Shear modulus $\eta = \frac{\text{Shear or tangential stress}}{\text{Shear strain}}$

$$= \frac{F_{11}/A}{x/h}$$

(See in Fig. 10.3) or $\eta = \frac{F_{11}h}{Ax} = \frac{F}{A\tan\theta}$

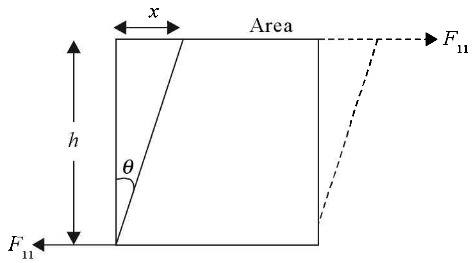


Fig. 10.3 Shear modulus illustration

Poisson's ratio $\sigma = \dfrac{\text{lateral strain}}{\text{longitudinal strain}} = \dfrac{-\frac{\Delta r}{r}}{\frac{\Delta l}{l}}$. For most of the materials σ lies between 0.18 to 0.25. Though theoretically σ may lie between –1 to 0.5. So far no material with negative poisson ratio has been found. Negative Poisson ratio would mean that radius increases on applying stress along with the length.

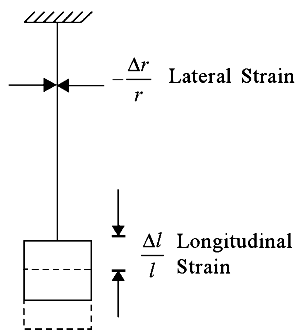


Fig. 10.4 Lateral and longitudinal strain

Relation between Y, B, η and σ

$$B = \frac{Y}{3(1-2\sigma)};$$

$$\frac{Y}{\eta} = 2(1+\sigma);$$

$$Y = \frac{9\eta B}{\eta + 3B}$$

$$\frac{\eta}{3B} = \frac{1-2\sigma}{2(1+\sigma)} \quad \text{and} \quad \sigma = \frac{3B-2\eta}{2(\eta+3B)}$$

Torsional rigidity (c) Torsional couple $G = CQ$,

Torsional rigidity $C = \dfrac{\pi\eta r^4}{2l}$ and $\dfrac{T}{\phi} = \eta$ where T is tangential stress. $\phi = \dfrac{x\theta}{l}$

Elastic potential energy $U = \dfrac{AY}{2l}(\Delta l)^2$ in a stretched wire. $U = \dfrac{1}{2}$ Force × Stretch

$$U = \frac{1}{2}\text{ stress} \times \text{strain} \times \text{volume} = \frac{1}{2} C\,\theta^2$$

Elastic potential energy per unit volume

$$u = \frac{1}{2}\text{ stress} \times \text{strain}$$

Note: An impurity with higher elasticity increases the elasticity of material if added and an impurity of less elasticity decreases the net elasticity of the material

Note: Elasticity of the material decreases on heating. Elasticity of Invar steel remains unaffected with temperature. Thermal stress = $Y\,\alpha\,\Delta\,\theta$, Thermal strain = $\alpha\,\Delta\,\theta$ and $B_{solid} > B_{liquid} > B_{gas}$.

- **Short Cuts and Points to Note**

1. Note that the potential energy curve at lowest point shows small variations are harmonic in nature. Therefore, for a small compression or elongation, the rigid body is elastic. That is, it acquires original shape after removal of deforming force (provided elongation or compression is small)
2. Repulsion is strong when the interatomic distance $< r_0$, the equilibrium distance and attraction is strong when $r > r_0$. These forces may be termed as restoring forces.
3. $PE\ U = \dfrac{A}{r^{12}} - \dfrac{B}{r^6}$ or $F = \dfrac{-du}{dr} = \dfrac{12A}{r^{13}} - \dfrac{6B}{r^7}$.

   At $r = r_0$, $F = 0$

   Thus $r_0 = \left(\dfrac{2A}{B}\right)^{1/6}$
4. A sphere around a molecule having a radius equal to the range of molecular force is called sphere of molecular action of that molecule.
5. Substances may be molecular, ionic, covalent or metallic if they are crystalline. Amorphous solids are glassy or super saturated supercooled fluids with strong inter molecular forces. Ionic solids have quite high melting point and are poor conductors in solid form and good conductors in molten or aqueous solution form.

   Polar molecules have dipole-dipole bond and the bond between nonpolar molecules is Vander waal's.
6. Brittle substances cannot bear more strain while ductile substances can bear large strain.
7. Working stress = $\dfrac{\text{Breaking stress}}{\text{Safety factor}}$. It is an important parameter while designing bridge, building / gas cylinders etc.
8. Youngs modulus is measured using Searle's apparatus. $Y = \dfrac{F_{\perp} l}{A\Delta l} = \dfrac{mgl}{A\Delta l}$

9. Bulk modulus $B = \frac{PV}{\Delta V}$, $B_{\text{isothermal}} = P$ and $B_{\text{adiabatic}} = \gamma P$ where $\gamma = \frac{C_P}{C_V}$. Note: pressure is a scalar while stress is a tensor.

10. Compressibility $C = \frac{1}{B}$

11. Shear modulus $\eta = \frac{F_{11} h}{Ax} = \frac{F_{11}}{A \tan\theta}$

12. Poisson ratio $\sigma = \frac{-\Delta r/r}{-\Delta l/l}$

13. $\frac{Y}{\eta} = 2(1+\sigma)$; $B = \frac{Y}{3(1-2\sigma)}$; $Y = \frac{9\eta B}{\eta + 3B}$, $\frac{\eta}{3B} = \frac{1-2\sigma}{2(1+\sigma)}$, $\sigma = \frac{3B - 2\eta}{2(\eta + 3B)}$.

14. Torsional couple $G = C\theta$ where C is torsional rigidity

$G = C = \frac{\pi \eta r^4}{2l}$ if $\theta = 1$.

$\eta = \frac{T}{\phi}$ where $\phi = \frac{x\theta}{l}$ and T is tangential stress.

15. Elastic energy $U = \frac{1}{2}$ stress × strain × volume

$= \frac{1}{2} F.\Delta l = \frac{AY\Delta l^2}{2l}$

Elastic energy per unit volume $u = \frac{1}{2}$ stress × strain.

16. Elastic fatigue If the deforming force applied on the body is rapidly changed, it temporarily loses elastic property. This is called elastic fatigue.

17. Elastic relaxation The property by virtue of which body does not regain its original shape immidiately after removal of the deforming force. It takes sometime to regain its original shape.

18. Bending moment is Torque necessary to bend a beam is called bending moment $C = \frac{YI_G}{R}$ where I_G is geometric MOI of bent beam and R is radius of curvature. Flexural rigidity $YI_G = CR$. For a rod of breadth b and thickness d, $I_G = \frac{bd^3}{12}$. For a rod of circular cross – section r, $I_G = \frac{\pi r^4}{4}$; $I_G = \frac{\pi(r_2^4 - r_1^4)}{4}$ if the rod is hollow.

19. Cantilever The beam clamped at one end and loaded at another as shown in Fig 10.5 is called cantilever.

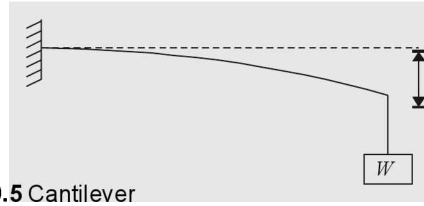


Fig. 10.5 Cantilever

$\delta = \frac{Wl^3}{3YI_G}$ where I_G is geometric MOI as defined in point 18.

20. Depression produced in a beam supported at two ends and loaded in the middle as shown in the Fig 10.6 is

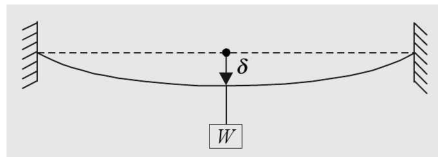


Fig. 10.6

$\delta = \frac{Wl^3}{4bd^3Y}$ for a circular cross-section $\delta = \frac{Wl^3}{12\pi r^4 Y}$.

21. Elastic relaxation time is the time delay in regaining the original shape after removal of deforming forces. Elastic relaxation time for gold, silver and phosphor bronze is negligible. For quartz it is minimum. Therefore, quartz fibre is nearly perfectly elastic. Putty is perfectly plastic (Putty – gypsum).

22. Young's modulu is practically equal to stress which will double the length of wire.

23. Resilience is used to denote the work done in straining a body within the elastic limit Resilience $= \frac{P^2}{2Y} = \frac{S^2}{2Y}$ P or S being stress

The greatest strain energy which can be stored in a body without permanent strain in called **proof resilience**.

- **Caution**

1. Assuming molecular forces are only attractive.

⇒ If $r > r_0$ net force is attractive. If $r < r_0$ the net force is repulsive. It forms the basis of repulsive force which is restoring in nature.

2. Considering all types of bonds are equally strong.

⇒ Strong bonds like ionic and covalant make the material hard and give it high melting point.

3. Considering ionic solids are conductors of electricity.

⇒ Ionic solids are poor conductors of electricity. They become good conductors in molten or in aqueous solution.

4. Assuming bulk modulus and compressibility are identical.

⇒ Compressibility is reciprocal of bulk modulus i.e. $c = \frac{1}{B}$

5. Considering that shear strain can be applied only on Cuboidal solids.

⇒ It can be applied to any shape of the body. During torsional oscillation, shear strain is applied to the wire/string supporting the disc/rod and so on or shear may produce tension.

6. Not remembering formulae between γ, σ, β and η.

⇒ Note $\beta = \frac{Y}{3(1-2\sigma)}$, $\frac{Y}{\eta} = 2(1-\sigma)$, $Y = \frac{a\eta\beta}{\eta+3\beta}$,

Torsional rigidity $C = \frac{k\eta r^4}{2l}$

Torsional couple (Torque) $G = C\theta$. If tangential stress is T then $\frac{T}{4} = \eta$ where ϕ is shear angle. $\phi = \frac{x\theta}{l}$ where θ is angle of twist.

7. Confusing with elastic potential energy and elastic potential energy per unit volume

⇒ Elastic potential energy per unit volume $u = \frac{1}{2}$ stress × strain

Elastic potential energy $U = \frac{1}{2}$ stress × strain × volume $= \frac{1}{2} F \times \Delta l = \frac{AY(\Delta l)^2}{2l}$.

8. Confusing Young's modulus with breaking stress

⇒ The unit of elasticity (all modulus of elasticity) is Pa or Nm^{-2}, i.e., of pressure or stress as strain is dimensionless. When a stress greater than breaking stress is applied the rigid body under investigation fractures. While determining Young's modulus we work in the area that strain < 1%.

Hence, the values of Young's modulus of a material will be greater than those of breaking stress.

9. Not able to relate breaking stress with working stress.

⇒ Working stress = $\frac{\text{Breaking stress}}{\text{Safety limit}}$

10. Assuming no effect of temperature on γ, β or η.

⇒ γ, β or η decrease with rise in temperature. Also note

$Y \alpha \frac{1}{\Delta l}, B \alpha \frac{1}{\Delta v}, \eta \alpha \frac{1}{\phi}$.

11. Assuming gases donot possess any modulus of elasticity.

⇒ Gases possess Bulk modulus of elasticity only. $B_{isothermal} = P$ and $B_{Adiabatic} = \gamma P$. That is why only longitudinal waves can travel through gases. For transverse waves to propagate medium must possess shear modulus of elasticity.

SOLVED PROBLEMS

1. A vertical steel post of diameter 25 cm and length 2.5 m supports a weight of 8000 kg. Find the change in length produced $Y = 2x\ 10^{11}\ Pa$.

(a) 2.1 cm (b) 0.21 cm
(c) 0.21 mm (d) 0.021 mm

Solution (c) $\Delta l = \frac{Fl}{AY} = \frac{4 \times 8000 \times 10 \times 2.5}{\pi(.25)^2 \times 2 \times 10^{11}}$

$= \frac{4 \times 16 \times 10^{-5}}{\pi} = 0.21$ mm.

2. Which of the following is an amorphous solid

(a) glass (b) diamond
(c) salt (d) sugar

[AIIMS 2005]

Solution (a)

3. For a constant hydraulic stress on an object, the fractional change in the object's volume $\left(\frac{\Delta V}{V}\right)$ and its bulk modulus (B) are related as

(a) $\frac{\Delta V}{V} \propto B$ (b) $\frac{\Delta V}{V} \propto B^{-1}$

(c) $\frac{\Delta V}{V} \propto B^2$ (d) $\frac{\Delta V}{V} \propto B^{-2}$

[AIIMS 2005]

Solution (b) $B = \frac{P}{\frac{\Delta v}{v}}$

4. Which one of the following is not a unit of Young's modulus?

(a) $N m^{-1}$ (b) $N m^{-2}$
(c) mega pascal (d) dyne Cm^{-2}

[Karnataka 2005]

Solution (a)

5. If S is the stress and Y is Young's modulus of the material of a wire, the energy stored in the wire per unit volume is

(a) $2S^2Y$ (b) $\frac{S^2}{2Y}$
(c) $\frac{2Y}{S^2}$ (d) $\frac{S}{2Y}$

[AIEEE 2005]

Solution (b) $u = \frac{1}{2}$ stress × strain $= \frac{1}{2} S\left(\frac{S}{Y}\right) = \frac{S^2}{2Y}$.

6. Outside a house 1 km from ground 0 of a 100 kiloton nuclear bomb explosion, the pressure will rapidly rise to as high as 2.8 atm while the pressure inside the house is 1 atm. If the area outside the house is 50 m². The resulting net force exerted by the air in front of the house is.

(a) 9.1×10^6N (b) 2.1×10^7N
(c) 1.41×10^7N (d) 1.63×10^7N

Solution (a) $F = \Delta P A = 1.8 \times 1.01 \times 10^5 \times 50$
$= 9.1 \times 10^6$ N

7. A petite young woman of 50 kg distributes her weight equally over her high-heeled shoes. Each heel has an area of 0.75 cm². Find the pressure exerted by each heel.

(a) 6.66×10^6Pa (b) 3.33×10^6Pa
(c) 1.67×10^6Pa (d) 2.23×10^6Pa

Solution (b) $P = \frac{250}{.75} \times 10^4 = 3.33 \times 10^6$Pa

8. A new alloy was found to break when a force of 90.8 N applied perpendicular to each end. The diameter of the wire is 1.84 mm. Find the breaking stress.

(a) 2.47×10^7 Pa (b) 1.43×10^7 Pa
(c) 3.41×10^7 Pa (d) 4.41×10^7 Pa

Solution (c) Breaking stress $= \frac{F}{A} = \frac{90.8\times10^6}{\pi\times(.92)^2}$
$= 3.41 \times 10^7$Pa

9. A steel wire with cross section 3 cm² has elastic limit 2.4×10^8 Pa. Find the maximum upward acceleration that can be given to a 1200 kg elevator supported by this cable if the stress is not to exceed 1/3 rd of the elastic limit.

(a) 9 ms^{-2} (b) 10 ms^{-2}
(c) 11 ms^{-2} (d) 12 ms^{-2}

Solution (b) $1200 \times (g + a) = \frac{1}{3} \times 2.4 \times 10^8 \times 3 \times 10^{-4}$
or $a = 10$ ms^{-2}.

10. Find the density of water 2 km deep in a sea. Bulk modulus = 2×10^9 Pa.

(a) 10^3 kgm^{-3} (b) 1010 kgm^{-3}
(c) 1100 kgm^{-3} (d) 1040 kgm^{-3}

Solution (b) $\frac{\Delta V}{V} = \frac{P}{B} = \frac{2\times10^3\times10^3\times10}{2\times10^9} = .01$

$\frac{\Delta V}{V} = \frac{\Delta\delta}{\delta}$ or $\Delta\delta = 10$ kg/m³.

∴ density of water = 1010 kg m^{-3}

11. A wire 1 m long has cross-section 1 mm² and Y = 1.2×10^{11} Pa. Find the work done in stretching it by 2 mm.

(a) 2.4 J (b) 0.24 J
(c) 0.024 J (d) 1.2 J

Solution (b) $W = \frac{YA(\Delta l)}{2l} = \frac{1.2\times10^{11}\times10^{-6}\times4\times10^6}{2}$
$= 0.24$ J.

12. Which apparatus are used to measure Y of a wire and shear modulus of the wire?

(a) Searle's apparatus, Ewing extensometer.
(b) Searle's apparatus, Maxwell needle method.
(c) Searle's apparatus, Torsion pendulum.
(d) Kundt's apparatus, Searle's apparatus.

Solution (b) and (c).

13. Proof resilience is related to

(a) PE stored in an elastic body.
(b) stiffness of a beam.
(c) elastic fatigue.
(d) elastic relaxation.

Solution (a)

14. Each wire in Fig. 10.7 has cross-section 5×10^{-3} cm² and $Y = 2 \times 10^{11}$ Pa. P, Q and R have mass 3 kg each find the strain developed in a A. Assume surface to be smooth.

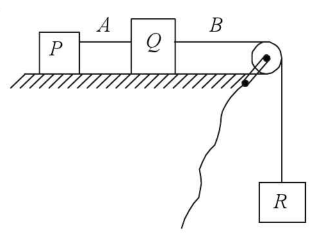


Fig. 10.7

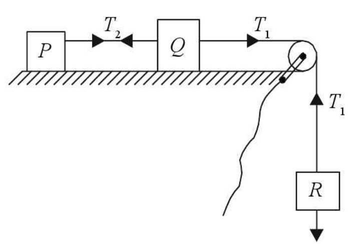


Fig. 10.8

(a) 10^{-5} (b) 10^{-6}
(c) 10^{-4} (d) none

Solution (c) $3g - T_1 = 3a$ –(1)

$T_1 - T_2 = 3a$ – (2)

$T_2 = 3a$ – (3)

Adding (1), (2) and (3)

$a = \frac{g}{3}$; $T_2 = 3a = g$

$$\frac{\Delta l}{l} = \frac{F/A}{Y} = \frac{T}{AY} = \frac{g}{5\times10^{-3}\times2\times10^{11}\times10^{-4}} = 10^{-4}.$$

15. A uniform heavy rod of length L, weight W and cross-sectional area A is hanging from a fixed support. Young's modulus of the material is Y. Find the elongation of the rod.

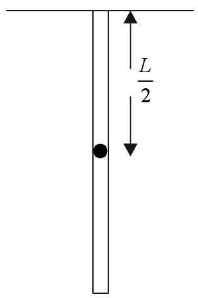


Fig. 10.9

(a) $\frac{WL}{AY}$ (b) $\frac{WL}{2AY}$

(c) $\frac{WL}{4AY}$ (d) $\frac{WL}{3AY}$

Solution (b) weight acts at COM.

$\therefore$ effective length $= \frac{L}{2}$. Hence $\frac{\Delta l}{L/2} = \frac{W}{AY}$

or $\Delta l = \frac{WL}{2AY}$

Alternate method

$$T = (L-x)\frac{W}{L}\text{ ; elongation} = \frac{Tdx}{AY}$$

$$= \frac{(L-x)Wdx}{LAY}\text{ ; Total elongation}$$

$$= \frac{W}{LAY}\int_0^L (L-x)dx$$

$$= \frac{WL}{2AY}.$$

16. The length of a wire is l_1 when tension is T_1 and l_2 when tension is T_2. The natural length of the wire is

(a) $\frac{l_1 + l_2}{2}$ (b) $\sqrt{l_1 l_2}$

(c) $\frac{l_1T_2 - l_2T_1}{T_2 - T_1}$ (d) $\frac{l_1T_2 + l_2T_1}{T_2 + T_1}$

(e) $\frac{l_1T_1 - l_2T_2}{T_1 - T_2}$

Solution (c) $\frac{T_1}{T_2} = \frac{l_1 - l}{l_2 - l}$ or $T_1(l_2 - l) = T_2(l_1 - l)$ or

$$l = \frac{l_1T_2 - l_2T_1}{T_2 - T_1}.$$

17. A steel wire of length L and area of cross-section A shrinks by Δl during night. Find the tension developed at night if Young's modulus is Y and wire is clamped at both ends.

(a) $\frac{AYL}{\Delta l}$ (b) AYL

(c) $AY\Delta l$ (d) $\frac{AY\Delta l}{L}$

Solution (d) $\frac{\Delta l}{L} = \frac{F}{AY}$ or $F = \frac{AY\Delta l}{L}$

TYPICAL PROBLEMS

18. A sphere of mass M kg is suspended by a metal wire of length L and diameter d. When in equilibrium, there is a gap of Δl between the sphere and the floor. The sphere is gently pushed aside so that it makes an angle θ with the vertical. Find θ_{max} so that sphere fails to rub the Floor. Young's modulus of the wire is Y.

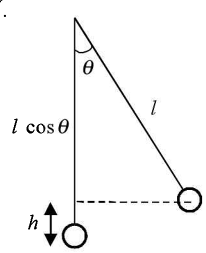


Fig. 10.10

(a) $\sin^{-1}\left(1 - \frac{Y\pi d^2\Delta l}{8MgL}\right)$ (b) $\tan^{-1}\left(1 - \frac{Y\pi d^2\Delta l}{8MgL}\right)$

(c) $\cos^{-1}\left(1 - \frac{Y\pi d^2\Delta l}{8MgL}\right)$ (d) none

Solution (c) $Y = \frac{Fl}{A\Delta l} = \frac{2Mg(1-\cos\theta)L}{\pi\frac{d^2}{4}\Delta l}$

or $1 - \cos\theta = \frac{Y\pi d^2\Delta l}{8Mgl}$ or $\cos\theta = 1 - \frac{Y\pi d^2\Delta l}{8Mgl}$

$$\frac{mv^2}{2} = mgl(1-\cos\theta)$$

or $\frac{mv^2}{l} = 2mg\,(1 - \cos\theta)$

$$\theta = \cos^{-1}\left(1 - \frac{Y\pi d^2 \Delta l}{8MgL}\right)$$

19. A wire of length L is clamped at two ends so that it lies horizontally and without tension. A weight W is suspended from the middle point of the wire. The vertical depression is____ Young's modulus is Y.

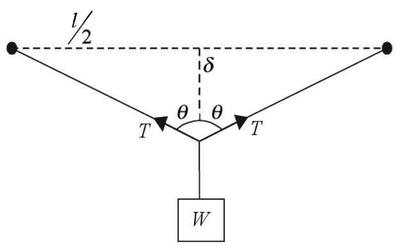


Fig. 10.11

(a) $\sqrt{\frac{2Tl^2}{4AY} + \frac{T^2l^2}{4A^2Y^2}}$ (b) $\sqrt{\frac{2Tl^2}{4AY} - \frac{T^2l^2}{4A^2Y^2}}$

(c) $\sqrt{\frac{2Tl^2}{4AY}}$ (d) $\frac{Tl}{2AY}$

Solution (a) $2T\cos\theta = W$

or $T = \frac{W}{2\cos\theta}$

$$\Delta l = \frac{Tl}{2AY} \quad \delta = \sqrt{\left(\frac{l}{2} + \Delta l\right)^2 - \frac{l^2}{4}}$$

or $\delta = \sqrt{\left(\frac{l}{2} + \frac{Tl}{2AY}\right)^2 - \frac{l^2}{4}} = \sqrt{\frac{2Tl^2}{4AY} + \frac{T^2l^2}{4A^2Y^2}}$.

20. A copper wire of cross- section A is under a tension T. Find the decrease in the cross-section area. Young's modulus is Y and Poisson's ratio is σ.

(a) $\frac{\sigma T}{2AY}$ (b) $\frac{\sigma T}{AY}$

(c) $\frac{2\sigma T}{AY}$ (d) $\frac{4\sigma T}{AY}$

Solution $\frac{\Delta r}{r} = \sigma \frac{\Delta l}{l}$ and $\frac{\Delta l}{l} = \frac{T}{AY}$

$$\frac{\Delta A}{A} = \frac{2\Delta r}{r} = \frac{2\sigma T}{AY}.$$

21. A steel plate has face area 4 cm^2 and thickness 0.5 cm is fixed rigidly at the lower surface. A tangential force of 10 N is applied on the upper surface. Find the lateral displacement of the upper surface with respect to the lower surface. Rigidity modulus of steel = 8.4×10^{10} Nm^{-2}

(a) 1.5 μm (b) 1.5 A°

(c) 1.5 ηm (d) 1.5 pm

Solution (c) $x = \frac{Fl}{A\eta} = \frac{10\times(.5)\times10^{-2}}{4\times10^{-4}\times8.4\times10^{10}}$

$= 1.5 \times 10^{-9}$m.

22. Two identical rods of identical dimensions and Young's modulii Y_1 and Y_2 are joined end to end. The equivalent young's modulus for the composite rod is

(a) $\frac{Y_1 + Y_2}{2}$ (b) $\frac{2Y_1Y_2}{Y_1 + Y_2}$

(c) $Y_1 + Y_2$ (d) $\frac{Y_1Y_2}{Y_1 + Y_2}$

Solution (b) $\Delta l_1 + \Delta l_2 = \Delta l$ or $\frac{Fl}{Y_1A} + \frac{Fl}{Y_2A} = \frac{F2l}{YA}$

or $Y = \frac{2Y_1Y_2}{Y_1 + Y_2}$.

23. A bar of cross-section A is subjected to equal and opposite tensile forces F at its ends. Consider a plane through the bar making an angle θ with a plane at right angles to the bar. Then shearing stress will be maximum if θ

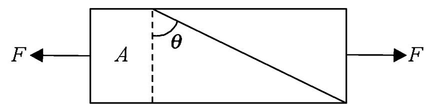


Fig. 10.12

(a) 0° (b) 30°

(c) 45° (d) 60°

(e) 90°

Solution (c) Shear stress = $\frac{F\sin\theta}{A/\cos\theta} = \frac{F\sin 2\theta}{2A}$

Shear stress will be maximum if $\sin 2\theta = 1$ or $2\theta = 90°$ i.e. $\theta = 45°$.

24. A stone of mass **m** tied to one end of a thread of length l. The diameter of the thread is d and it is suspended vertically. The stone is now rotated in a horizontal plane and makes an angle θ with the vertical. Find the increase in length of the wire. Youngs modulus of the wire is Y.

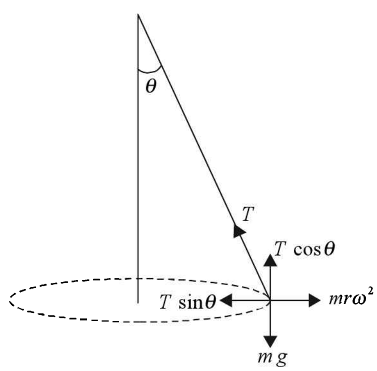


Fig. 10.13

(a) $\frac{4mgl}{\pi d^2 Y\cos\theta}$ (b) $\frac{4mgl}{\pi d^2 Y\sin\theta}$

(c) $\frac{4mgl}{\pi d^2 Y}$ (d) $\frac{4mgl}{\pi d^2 Y\sec\theta}$

Solution (a) $T\cos\theta = mg$.

$$Y = \frac{TL}{A\Delta L} \text{ or } \Delta L = \frac{TL}{AY} = \frac{mgL}{AY\cos\theta}$$

$$= \frac{4mgl}{\pi d^2 Y \cos\theta}.$$

25. Assuming that shear stress of base of a mountain is equal to force per unit area to its weight, calculate the maximum possible height of a mountain on the earth if breaking stress of a typical rock is 3×10^8 N m^{-2} and specific gravity is 3.

(a) 10 km (b) 8 km
(c) 7 km (d) 6 km

Solution (a) $p = \frac{W}{A} = h\rho g \le$ Breaking stress

or $h \le \frac{\text{Breaking stress}}{\rho g} = \frac{3\times10^8}{3\times10^3\times10}$

$h \le 10$ km.

26. In gases which wave can not propagate

(a) standing wave (b) longitudinal wave
(c) transverse wave (d) none

Solution (c) For transverse waves to propagate medium must possess shear modulus η and gases do not possess η.

27. Poisson's ratio cannot exceed

(a) 0.25 (b) 1.0
(c) 0.75 (d) 0.5

Solution (d) $B = \frac{Y}{3(1-2\sigma)}$ if $B = \infty$ $1 - 2\sigma \to 0$ or

$\sigma_{max} = \frac{1}{2}$.

28. Bulk modulus of water is 2×10^9 Pa. Air is ____ times more compressible than water.

(a) 200 (b) 2×10^3
(c) 2×10^5 (d) 2×10^4

Solution (d) $= \frac{B_{water}}{B_{air}} = \frac{C_{air}}{C_{water}} = \frac{2\times10^9}{1\times10^5}$

$= 2 \times 10^4 \{\because C = \frac{1}{B}\}$

29. Find the volume density of elastic energy of fresh water at a depth of 1000 m

(a) 2.5 kJm^{-3} (b) 25 kJm^{-3}
(c) 0.25 kJm^{-3} (d) none

Solution (b) $\frac{dW}{V} = \frac{1}{2} P \frac{\Delta V}{V} = \frac{1}{2} P \left(\frac{P}{B}\right)$

$= \frac{(\rho g h)^2}{2\times2\times10^9} = \frac{(10^3\times10\times10^3)^2}{2\times2\times10^9} = 2.5 \times 10^4$ J/m^3.

QUESTIONS FOR PRACTICE

1. A spherical ball is compressed by 0.01% when a pressure of 100 atmosphere is applied on it. Its bulk modulus of elasticity in dyne/cm^2 will be approximately

(a) 10^{12} (b) 10^{14}
(c) 10^6 (d) 10^{24}

2. A stress of 2 Kg/mm^2 is applied on a wire. If $Y = 10^{12}$ dyne/cm^2 then the percentage increase in its length will be

(a) 0.196% (b) 19.6%
(c) 1.96% (d) 0.0196%

3. The shearing strain is measured by

(a) angle of twist. (b) shearing stress.
(c) angle of shear. (d) modulus of rigidity.

4. The expression for the determination of Poisson's ratio for rubber is

(a) $\sigma = \frac{1}{2}\left[1 - \frac{dV}{AdL}\right]$ (b) $\sigma = \frac{1}{2}\left[1 + \frac{dV}{AdL}\right]$

(c) $\sigma = \frac{1}{2}\frac{dV}{AdL}$ (d) $\sigma = \frac{dV}{AdL}$

5. The elastic relaxation time is minimum for

(a) glass. (b) quartz.
(c) rubber. (d) clay.

6. The correct relation between interatomic force constant K, Young modulus Y and interatomic distance r_0 is

(a) $K = Yr_0$ (b) $K = \frac{r_0}{Y}$
(c) $K = \frac{Y}{r_0}$ (d) $K = r_0^2 Y^2$

7. The shearing strain is equivalent to

(a) tensile strain + compression strain.
(b) tensile strain – compression strain.
(c) compression strain ÷ tensile strain.
(d) tensile strain × compression strain.

8. The value of angle of twist at the clamped end of a rod is

(a) 90° (b) 180°
(c) 60° (d) 0°

9. The angle of shear is related to the radius of cylinderial shell as

(a) directly proportional.
(b) inversely proportional.
(c) directly proportional to square root.
(d) inversely proportional to square root.

10. Which of the following shafts is stronger

(a) solid. (b) hollow.
(c) cylindrical. (d) circular.

11. The end of a wire of length 0.5 m and radius 10^{-3} m is twisted through 0.80 radian. The shearing strain at the surface of wire will be

(a) 1.6×10^{-3} (b) 1.6×10^{3}
(c) 16×10^{3} (d) 16×10^{6}

12. The breaking stress of a material is 10^9 pascal. If the density of material is 3×10^3 Kg/m^3. The minimum length of the wire for which it breaks under its own weight will be

(a) 3.4 m (b) 3.4×10^4 m
(c) 3.4×10^5 m (d) 3.4×10^3 m

13. The reciprocal of bulk modulus of elasticity is equal to

(a) T (b) η
(c) σ (d) compressibility

14. The modulus of elasticity at constant temperature is

(a) γP (b) $\frac{P}{\gamma}$
(c) P (d) $\frac{P}{V}$

15. If the stress produced in a material is P and strain is S then according to Hooke's law, the modulus of elasticity is

(a) $K = \frac{P}{S^2}$ (b) $K = \frac{S}{P}$
(c) $K = PS$ (d) $K = \frac{P}{S}$

16. Equal weights are suspended from the wires of same material and same lengths but with radii in the ratio 1 : 2. The ratio of extensions produced in them will be

(a) 4 : 1 (b) 1 : 4
(c) 1 : 2 (d) 2 : 1

17. The Hooke's law defines

(a) modulus of elasticity. (b) stress.
(c) strain. (d) elastic limit.

18. A wire is stretched through 1 mm by certain load. The extension produced in the wire of same material with double the length and radius will be

(a) 4 mm (b) 3 mm
(c) 1 mm (d) 0.5 mm.

19. A wire of length L and cross-sectional area A is constructed of a material whose Young's modulus of elasticity is Y. The energy density stored in the wire when stretched by x meter will be

(a) $Y x^2$ (b) $\frac{Yx^2}{L}$
(c) $\frac{Yx^2}{2L^2}$ (d) $\frac{Yx^2}{L^2}$

20. Two identical wires of different materials have Young's modulii of elasticity as 22×10^{10} N/m^2 and 11×10^{10} N/m^2 respectively. If these are stretched by equal loads then the ratio of extensions produced in them will be

(a) 2 : 1 (b) 1 : 2
(c) 4 : 1 (d) 1 : 4

21. If the tensile force is suddenly removed from a wire then its temperature will

(a) decrease (b) increase
(c) become zero (d) remain constant

22. The limit upto which the stress is directly proportional to strain is called

(a) elastic limit (b) elastic fatigue
(c) elastic relaxation (d) breaking limit

23. For which of the following is the modulus of rigidity highest?

(a) Glass (b) Quartz
(c) Rubber (d) Water

24. Longitudinal strain can be produced in

(a) glass (b) water
(c) honey (d) hydrogen gas

25. Out of the following whose elasticity is independent of temperature?

(a) steel (b) copper
(c) Invar steel (d) glass

26. The modulus of elasticity of a material does not depend upon

(a) shape (b) temperature
(c) nature of material (d) impurities mixed

27. The ratio of change in dimension at right angles to applied force to the initial dimension is defined as

(a) Y (b) η
(c) β (d) K

28. Which of the following is not dimension less?

(a) Poisson ratio (b) Shearing strain
(c) Longitudinal strain (d) Volume stress

29. If a wire is stretched by applying force at one of its ends, then the elastic potential energy density in terms of Young's modulus Y and linear strain α will be

(a) $\frac{\alpha Y^2}{2}$ (b) $\frac{Y\alpha}{2}$
(c) $\frac{\alpha^2 Y}{2}$ (d) $2\alpha^2 Y$

30. A solid sphere of radius R and bulk modulus of elasticity K is kept in a liquid inside a cylindrical container. A massless piston of cross-sectional area A floats on liquid surface. A mass M is put on the piston in order to compress the liquid. The fractional change in the radius of the sphere will be

(a) $\frac{3Mg}{KA}$ (b) $\frac{3Mg}{2KA}$
(c) $\frac{Mg}{KA}$ (d) $\frac{Mg}{3KA}$

31. Two identical wires of copper and steel are joined and a force is applied on them so that the combined length increases by 1 cm. In both the wires there will be
(a) equal stress and equal strain.
(b) unequal stress and unequal strain.
(c) equal stress and equal strain.
(d) equal strain and unequal stress.

32. Which of the following pairs is not correct ?
(a) Strain-dimensionless
(b) Stress-N/m^2
(c) Modulus of elasticity-N/m^2
(d) Poisson's ratio-N/m^2

33. The Young's modulus for steel is $Y = 2 \times 10^{11}$ N/m^2. If the inter-atomic distance is 3.2 A the interatomic force constant in N/A will be
(a) 6.4×10^9 (b) 6.4×10^{-9}
(c) 3.2×10^9 (d) 3.2×10^{-9}

34. In the load-extension graph for a wire, the elastic limit lies between the points

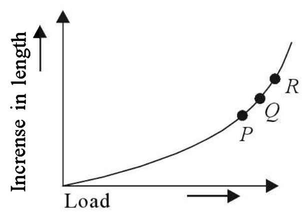


Fig. 10.14

(a) Q and P (b) Q and P
(c) P and R (d) Q and R

35. For the Hooke's law to hold good the inter molecular distance, as compared to the equilibrium distance, must be
(a) very much less (b) much more
(c) approximately same (d) zero

36. A 10-meter long thick rubber pipe is suspended from one of its ends. The extension produced in the pipe under its own weight will be (Y = 5 × 10^6 N/m^2 and density of rubber = 1500 Kg/m^3)
(a) 1.5 meter (b) 0.15 meter
(c) 0.015 meter (d) 0.0015 meter

37. A wire of radius 1 mm is bent in the form of a circle of radius 10 cm. The bending moment will be
(Y = 2×10^{11} N/m^2)
(a) 3.14 N/m (c) 6.28 N/m
(c) 1.57 N/m (d) 15.7 N/m

38. The depression produced at the end of a 50 cm long cantilever on applying a load is 15 mm. The depression produced at a distance of 30 cm from the rigid end will be
(a) 3.24 mm (b) 1.62 mm
(c) 6.48 mm (d) 12.96 mm

39. A solid cylindrical rod of radius 3 mm gets depressed under the influence of a load through 8 mm. The depression produced in an identical hollow rod with outer and inner radii of 4 mm and 2 mm respectively, will be
(a) 2.7 mm (b) 1.9 mm
(c) 3.2 mm (d) 7.7 mm

40. The breaking stress for a copper wire is 2.2×10^8 N/m^2. The maximum length of the copper wire which when suspended vertically for which the wire will not break under its own weight, will be (Density of copper = 8.8×10^3 Kg/m^3)
(a) 25000 m (b) 2500 m
(c) 250 m (d) 25m.

41. The length of a wire gets doubled on applying a stress of 20×10^8 N/m^2. The Young's modulus of elasticity for the wire in N/m^2, will be
(a) 40×10^8 (b) 5×10^8
(c) 10×10^8 (d) 20×10^8

42. Two wires of same length and same radius (one of copper and another of steel) are welded to form a long wire. An extension of 3 cm is produced in it on applying a load at one of its ends. If the Young's modulus of steel is twice that of copper, then the extension in the steel wire will be
(a) 1 cm (b) 2 cm
(c) 1.5 cm (d) 2.5 cm.

43. The coefficient of linear expansion of copper is one and half times that of iron. Identical rods of copper and iron are heated through same temperature range. The ratio of forces developed in them will be (the Young's modulus for copper and iron is the same)
(a) $\frac{2}{3}$ (b) $\frac{4}{9}$
(c) $\frac{9}{4}$ (d) $\frac{3}{2}$

44. A steel girder can bear a load of 20 tons. If the thickness of girder is doubled, then for the same depression it can bear a load of
(a) 40 ton (b) 80 ton
(c) 160 ton (d) 5 ton

45. A ball of radius R and with bulk modulus of elasticity K is kept in a liquid inside a cylindrical container. It is pressed by putting a mass m on a massless piston of cross-sectional area A, then decrease in the radius of bail will be
(a) $\frac{Mg}{3KR}$ (b) $\frac{Mg}{3KA}$
(c) $\frac{Mg}{KA}$ (d) $\frac{MgK}{3AR}$

46. The hardest material out of the following is
(a) diamond (b) steel
(c) aluminium (d) glass

47. The density of water at 4 C is
(a) minimum (b) maximum
(c) 10^3 Kg/m^3 (d) 1Kg/m^3

48. The ratio of kinetic energy to potential energy for solids is

(a) $\frac{Ek}{U} < 1$ (b) $\frac{Ek}{U} > 1$

(c) $Ek = U$ (d) $Ek > U$

49. In polar molecules the positive and negative charge centres

(a) coincide
(b) do not coincide
(c) sometimes coincide and sometimes do not
(d) nothing can be predicted

50. The total number of atoms per unit cell is equal to

(a) $N_b + \frac{N_f}{2} + \frac{N_c}{8}$ (b) $N_b + \frac{N_f}{2} + \frac{N_c}{8}$

(c) $N_b + \frac{N_c}{8}$ (d) $N_b + \frac{N_f}{2} + \frac{N_c}{4}$

51. The distance between two atoms in contact is known as

(a) lattice parameter
(b) packing factor
(c) atomic radius
(d) number of constituent particles

52. The ionic bond is absent in

(a) NaC_1 (b) CsC_1
(c) LiF (d) H_2O

53. The hydrogen bond is absent in

(a) H_2O (b) HF
(c) NH_3 (d) Ge

54. The electrical conductivity of materials bound by ionic binding is

(a) very low (b) very high
(c) infinity (d) zero

55. The molecular range for materials is

(a) 10^{-12}m (b) 10^{-10}m
(c) 10^{-11}m (d) 10^{-9}m

56. Out of the following which solid is best semi-conductor

(a) van der Walls bond (b) ionic bond
(c) metallic bond (d) covalent bond

57. Two electrons each of two atoms *P* and *Q* form a compound *PQ*. This is an example of

(a) polar molecule
(b) non-polar molecule
(c) polar covalent bond
(d) non-polar covalent bond

58. Which type of binding is present in semiconductors

(a) Van der Walls (b) metallic
(c) covalent (d) ionic

59. The lattice of a covalent crystal is composed of

(a) atoms (b) molecules
(c) ions (d) compounds

60. The thermal conductivity of silver and copper is high because they have following binding

(a) metallic (b) ionic
(c) covalent (d) Van der Waals

61. The distance between two neutral atoms in equilibrium is

(a) $\sqrt{\frac{2A}{B}}$ (b) $\left(\frac{2A}{B}\right)^{1/6}$

(c) $\frac{2A}{B}$ (d) $\left(\frac{2A}{B}\right)^{-(1/2)}$

62. The points of maximum and minimum attraction in the curve between potential energy (*U*) and distance (*r*) of a diatomic molecules are respectively

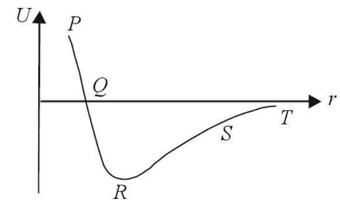


Fig. 10.15

(a) S and T (b) T and S
(c) R and S (d) S and R

63. In the above *U*-*r* curve, at what point the attractive or repulsive force is zero

(a) *P* (b) *Q*
(c) *R* (d) *S*

64. The correct curve for a stable diatomic molecule is

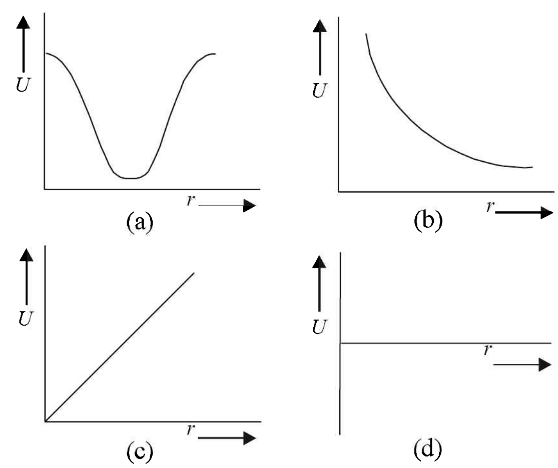


Fig. 10.16

65. The melting point of a material with low binding energy is

(a) high (b) low
(c) infinity (d) negative

66. The total spin of two electrons forming a covalent bond is

(a) Zero (b) ± 1/2
(c) 1 (d) ± 3/2

67. The strongest binding out of the following is

(a) van der Waals (b) ionic
(c) covalent (d) metallic

68. The correct relation between the intermolecular distances r_s, r_1 and r_g in solids. liquids and gases respectively, is

(a) $r_s < r_1 < r_g$ (b) $r_s > r_1 > r_g$
(c) $r_s = r_1 = r_g$ (d) $r_s > r_1 < r_g$

69. Which bond is present in *Ne* ?

(a) Ionic (b) Covalent
(c) Metallic (d) Van der Waals

70. The correct relation between the intensities of inter-molecular attraction I_s, I_1 and I_g in solids, liquids and gases respectively, is

(a) $I_s < I_1 < I_g$ (b) $I_s > I_1 > I_g$
(c) $I_s = I_1 = I_g$ (d) $I_s > I_1 > I_g$

71. The volume of an fcc crystal having *N* atoms is (a = lattice parameter)

(a) $\frac{Na^3}{4}$ (b) $\frac{Na^3}{2}$
(c) Na^3 (d) $2\,Na^2$

72. The number of atoms per unit cell in gol d is

(a) 3 (b) 4
(c) 2 (d) 1

73. The interaction abundantly found in *Cu*, *Ag* and *Au* is

(a) attraction (b) repulsion
(c) ion-ion (d) electron-electron

74. Ice is an example of the following crystal

(a) covalent (b) ionic
(c) Vander Waals (d) hydrogen binding

75. The lattices of *Na* and *Al* are bcc and fcc respectively. Presuming them to be closed packed, their packing factors are respectively

(a) 0.68 and 0.34 (b) 0.68 and 0.74
(c) 0.34 and 0.34 (d) 0.52 and 0.52

76. The formula expressing the force $F(r)$ acting between two neutral molecules is

(a) $F_{(r)} = \frac{A}{r^{13}} - \frac{B}{r^7}$ (b) $F_{(r)} = \frac{A}{r^{12}} - \frac{B}{r^6}$

(c) $F_{(r)} = \frac{A}{r^7} + \frac{B}{r^{13}}$ (d) $F_{(r)} = \frac{A}{r^6} + \frac{B}{r^{12}}$

77. The fourth state of matter is

(a) solid (b) fluid
(c) liquid (d) plasma

78. In order to separate out two molecules from each other, the potential energy between them must be

(a) positive (b) negative
(c) minimum (d) zero

79. The relation connecting the energy *U* and distance *r* between dipole and induced dipole is

(a) $U \propto r$ (b) $U \propto r^2$
(c) $U \propto r^{-6}$ (d) $U \propto r^6$

80. The binding for solids with high cohesive energy will be

(a) only metallic
(b) only Van der Waals
(c) metallic or Van der Walls
(d) covalent

81. The bond present in CH_4 is

(a) covalent (b) ionic
(c) hydrogen (d) Van der Waals

Answers to Questions for Practice

1. (a)	**2.** (d)	**3.** (c)	**4.** (a)	**5.** (b)	**6.** (a)	**7.** (a)
8. (d)	**9.** (a)	**10.** (b)	**11.** (a)	**12.** (b)	**13.** (d)	**14.** (c)
15. (d)	**16.** (a)	**17.** (a)	**18.** (d)	**19.** (c)	**20.** (b)	**21.** (b)
22. (a)	**23.** (b)	**24.** (a)	**25.** (c)	**26.** (a)	**27.** (c)	**28.** (d)
29. (c)	**30.** (d)	**31.** (c)	**32.** (d)	**33.** (b)	**34.** (b)	**35.** (c)
36. (b)	**37.** (c)	**38.** (c)	**39.** (a)	**40.** (b)	**41.** (d)	**42.** (a)
43. (d)	**44.** (c)	**45.** (b)	**46.** (a)	**47.** (b)	**48.** (a)	**49.** (b)
50. (a)	**51.** (a)	**52.** (d)	**53.** (d)	**54.** (a)	**55.** (d)	**56.** (d)
57. (c)	**58.** (c)	**59.** (a)	**60.** (a)	**61.** (b)	**62.** (a)	**63.** (c)
64. (d)	**65.** (b)	**66.** (a)	**67.** (b)	**68.** (a)	**69.** (d)	**70.** (b)
71. (a)	**72.** (b)	**73.** (c)	**74.** (d)	**75.** (b)	**76.** (a)	**77.** (d)
78. (d)	**79.** (c)	**80.** (d)	**81.** (d)			

11

Hydrodynamics and Properties of Fluids

BRIEF REVIEW

Anything that can flow is called a fluid. Therefore, **liquids** and **gases** fall in this category. A perfect liquid is compressible and its shearing stress not maintainable.

Streamlined, Steady State Flow By steady state or stationary flow we mean that at any place in a fluid, the velocity never changes.

Streamline It is a curve tangent to which at any point gives the direction of fluid velocity at that point.

Equation of Continuity Volume leaving per second = Volume entering per second

or $A_2 v_2 = A_1 v_1$

Where A_1 and A_2 are area of cross-section of a pipe at enterance and leaving points and v_1 and v_2 are velocities at the respective points as shown in Fig. 11.1 (a). Fig. 11.1 (b) shows variation of velocity with area of cross-section.

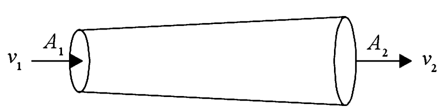


Fig. 11.1 (a) Equation of continuity illustration

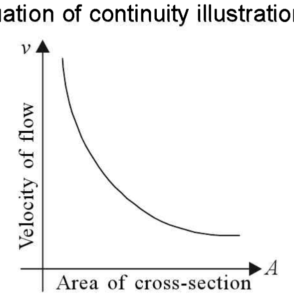


Fig. 11.1 (b) Velocity vs area of cross-section

Bernoulli's Theorem It states that total energy of a (flowing) liquid is constant. That is,

$$KE + PE + \text{Pressure head energy} = \text{Constant}$$

$$\frac{1}{2} m v^2 + mgh + P\Delta V = \text{constant}$$

or $$\frac{1}{2} \rho v^2 + \rho gh + P = \text{Constant}$$

In a horizontal pipe $\frac{1}{2} \rho v^2 + P = \text{Constant}$

Torricelli's Theorem According to this theorem velocity of efflux

$$v_{\text{efflux}} = \sqrt{2g(H-h)}$$. See Fig. 11.2

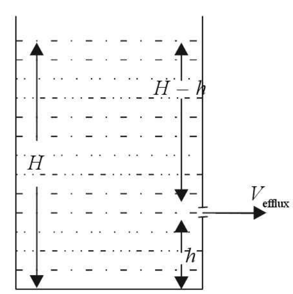


Fig. 11.2 Velocity of efflux from an open vessel

Velocity of efflux from a closed vessel having Pressure P inside (See Fig. 11.3).

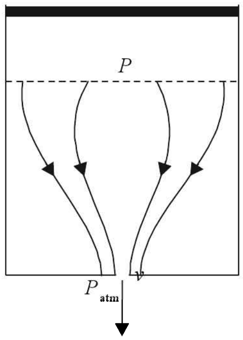


Fig. 11.3 Velocity of efflux from a closed vessel

$$P = P_{atm} + \frac{1}{2}\rho v^2 \quad \text{where } P_{atm} \text{ is atmospheric pressure.}$$

or $$v = \sqrt{\frac{2(P - P_{atm})}{\rho}}.$$

Dynamic Lift or Magnus Effect When a ball is spinning in a fluid as shown in Fig. 11.4, the resultant velocity at the top (above the ball) increases and resultant velocity below the ball decreases. If v_1 and v_2 are velocities of liquid and spinning ball respectively. The ball experiences an upward thrust, Such a phenomenon is called *dynamic lift* or *Magnus effect*.

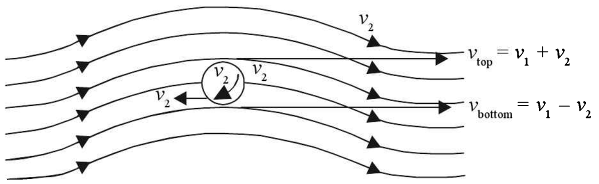


Fig. 11.4 Dynamic lift illustration

Venturimeter

$$P_1 - P_2 = H\rho g \qquad \text{(See Fig. 11.5)}$$

$$\frac{P_1}{\rho g} + \frac{v_1^2}{2g} = \frac{P_2}{\rho g} + \frac{v_2^2}{2g}.$$

or $$\frac{P_1 - P_2}{\rho g} = \frac{v_2^2 - v_1^2}{2g}$$

or $$\frac{H\rho g}{\rho g} = \frac{V^2}{2g}\left[\frac{1}{a^2} - \frac{1}{A^2}\right]$$ or

$$V = aA\sqrt{\frac{2Hg}{A^2 - a^2}}$$

and gives volume flowing per second.

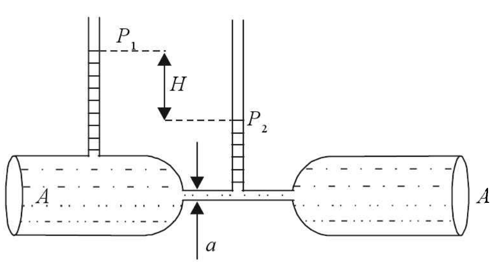


Fig. 11.5 Venturimeter

Surface Tension The property of the liquid with which the surface behaves as a stretched membrane and can support small objects placed on its surface.

Surface tension = Surface energy

Surface energy is defined as work done due to surface tension per unit area. Dimensional formula of surface tension $[MT^{-2}]$ Force per unit length. Consider two molecules of a liquid X on the surface and Y inside as illustrated in Fig. 11.6. Y is completely balanced due to forces exerted by other molecules of the liquid. X has unbalanced forces. The Figure clearly demontrates why the surface of a liquid behaves as a stretched membrane. The molecular force between the molecules of a liquid is cohesive force. Due to surface tension a liquid would try to acquire minimum surface area with maximum volume. Therefore, the drops acquire spherical shape.

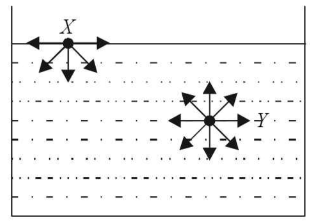


Fig. 11.6 Surface tension illustration

Excess pressure in a drop/bubble $\Delta P = \frac{2T}{r}$ if the bubble has one surface like air bubble. Where T is surface tension and r is radius.

If a bubble has two surfaces, like soap bubble, then excess pressure is $\Delta P = \frac{4T}{r}$. See Fig. 11.7.

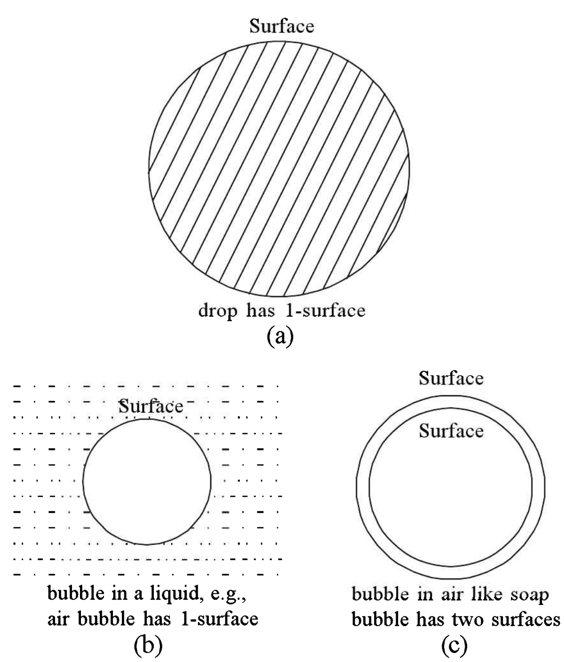


Fig. 11.7

Angle of contact is the angle between the tangent to the liquid vapour interface and liquid solid interface. In other words it is the angle between the tangent to the meniscus (at the point of contact) and wall of the container. As illustrated in Fig. 11.8 convex upward meniscus makes obtuse angle of contact, and, concave upward meniscus makes acute angle of contact.

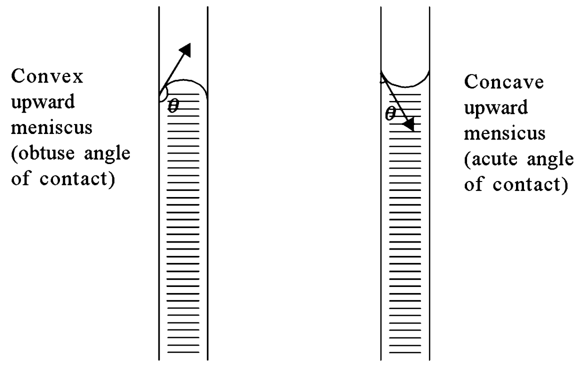


Fig. 11.8

Liquids like water, alcohol, ether, CCl_4 (Carbon tetrachloride) xylene, glycerine and acetic acid have angle of contact zero or nearly zero with glass. Meniscus may be concave upward or convex upward.

Table 11.1. Comparison of concave upward and convex upward meniscus

Concave upward meniscus	Convex upward meniscus
1. Angle of contact is acute.	Angle of contact is obtuse
2. Adhesive force between the liquid molecules and molecules of glass (wall of the container) is greater than cohesive force between liquid molecules.	Cohesive force between liquid molecules is greater than adhesive force between the molecules of liquid and wall of the container.
3. The liquid wets the walls of the container	The liquid does not wet the walls of the container

Hg has angle of contact 138° with the glass and angle of contact of water with chromium is 160°.

Ascent of liquid in a capillary tube

$$h = \frac{2T\cos\theta}{r\rho g}$$ if meniscus is not taken into account.

$$h = \frac{2T\cos\theta}{r\rho g} - \frac{r}{3}$$ if meniscus is also taken into account.

$$h = \frac{2T}{r\rho g} \text{ if } \theta = 0$$

Note that if angle of contact θ is acute or meniscus is concave liquid rises in the capillary. If the angle of contact is obtuse or meniscus is convex upward then liquid dips as cos θ will be negative as shown in Fig. 11.9 (b).

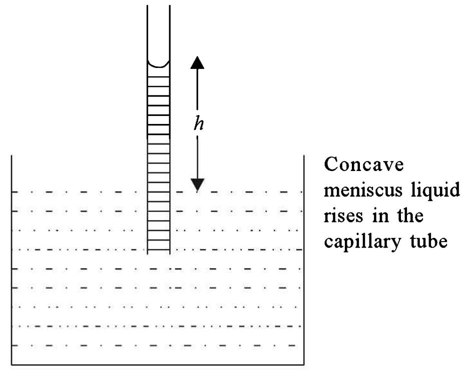


Fig. 11.9 (a)

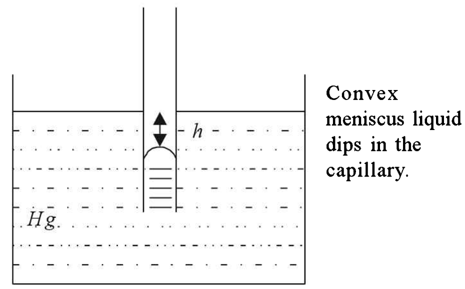


Fig. 11.9 (b)

If the liquid rises in a capillary and capillary is of insufficient height then the excess liquid will collect in the form of a drop at the top as shown in Fig. 11.10 but does not fall. That is no overflow will take place. If overflow would have taken place liquid would have risen again. Thus, a prepetual motion would have begun and such a motion is disallowed.

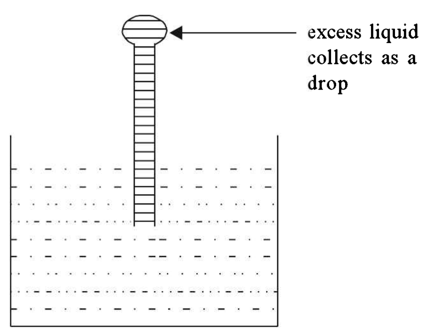


Fig. 11.10

In **Jaeger's method** surface tension is estimated by $T = \frac{r}{2}(H\sigma - h\rho)g$ where r is radius of the capillary, σ is density of Xylol and ρ is the density of liquid under investigation. h is the depth of the capillary below the surface and H is the difference in two levels of the U tube. Fig. 11.11 illustrates Jaeger's method.

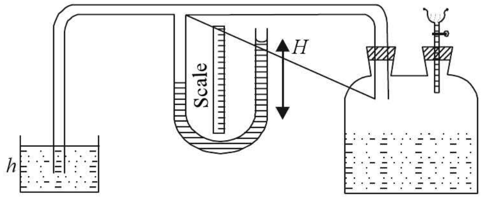


Fig. 11.11 Jaeger method illustration

Quinke's method If a big drop is placed on a clean glass plate then angle of contact θ is determined by (see Fig. 11.12).

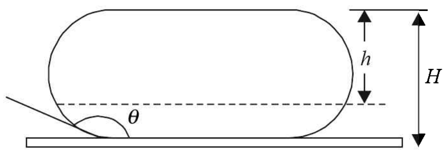


Fig. 11.12 Angle of contact measurement using Quinke's method

$\cos\theta = 1 - \frac{h^2}{H^2}$. The method is applicable for liquids making obtuse angle of contact or which do not wet the wall of the container.

Velocity of a simple harmonic wave on the surface of a liquid $v = \sqrt{\frac{\lambda g}{2\pi}}$. It is valid only if the amplitude of circular vibration is very small as compared to the wavelength λ. If the amplitude is large then $v = \sqrt{\frac{\lambda}{2\pi}\left(g + \frac{4\pi^2 T}{\rho\lambda^2}\right)}$.

Energy *E* required to split a big drop of radius R into n small drops each of radius *r*

$$E = 4\pi r^2 n^{2/3} [n^{1/3} - 1]\, T = 4\pi R^2 \left[n^{\frac{1}{3}} - 1\right] T$$

where $R = n^{1/3}\, r$

Same amount of energy will be released when n drops each of radius r coalesce to form a big drop of radius R.

Viscosity The property of a fluid to oppose relative motion between its layers is called viscosity. This property can be observed when the flow is steady or the liquid moves with a constant velocity. The flow may be called laminar. The opposition is due to intermolecular forces (cohesive force). Fig. 11.13 shows the velocity of the layers decreases in a direction perpendicular to the flow.

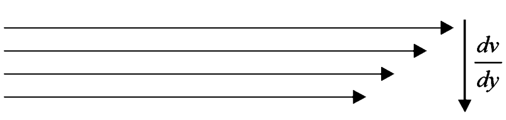


Fig. 11.13 Viscosity shows velocity gradient

Therefore, shearing stress $\frac{F}{A} \propto$ velocity gradient $\frac{dv}{dy}$ (Rate of change of strain).

or $F = -\eta A \frac{dv}{dy}$ where η is coefficient of viscosity.

Dimensions of η $[ML^{-1}T^{-1}]$ SI unit Poiseuille (Pl) and CGS unit is Poise

1 Pl = 10 poise.

Since the coefficient of viscosity is the ratio of shearing stress to the rate of change of strain, it may be regarded as **transient** or **fugitive rigidity.** Maxwell regarded viscosity as the limiting case of elastic solid.

Note viscosity of liquids is greater than that of gases. For example, viscosity of water is 0.01 poise while that of air is 200 μ poise.

The motion profile of a liquid in a capillary is parabolic as shown in Fig. 11.14.

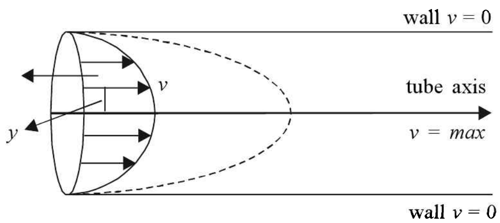


Fig. 11.14 Motion profile of a liquid in a capillary (Note parabolic profile)

$$v = \frac{p}{4\eta L}(r^2 - y^2)$$

Velocity of flow is maximum when $y = 0$ (along the axis of tube) and is $v_{max} = \frac{pr^2}{4\eta L}$

r is radius of the tube, L is length of tube and p is pressure difference.

Velocity of flow is zero at the walls when $y = r$, $v = 0$.

Critical velocity The velocity at which **steady** or **laminar** flow changes to **turbulant** or **eddy** flow is called **critical velocity.**

Reynolds number $R = \frac{\rho v_c D}{\eta}$ Reynolds number R is dimensionless.

or $v_c = \frac{R\eta}{\rho D}$.

It has been found if $R < 2000$, flow is steady and if $R > 3000$ flow is turbulant. For water $R < 2000$ corresponds to $v < 20$ cm s^{-1} at 20° C.

$$R = \frac{\rho v_c D}{\eta} = \frac{\frac{1}{2}\rho v^2}{\frac{\eta v}{r}} = \frac{\text{inertial force}}{\text{viscous drag}}$$ D is diameter of the tube.

Kinematic viscosity is $\frac{\eta}{\rho}$. Its unit is **stokes**.

Stoke's formula The viscous force opposing the motion of a sphere is $F = 6\pi\,\eta\, rv$ when a sphere travels through a fluid as illustrated in Fig. 11.15 (a). Velocity u is called terminal velocity.

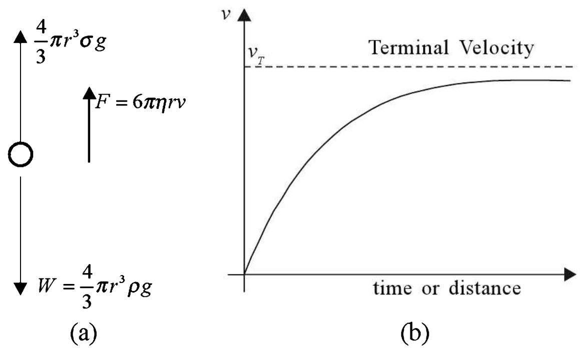


Fig. 11.15

Terminal velocity $v_T = \frac{2r^2(\rho-\sigma)g}{9\eta}$ where ρ is density of sphere or drop and σ is density of fluid (medium).

Note $v_T \propto r^2$ and $v_T \propto$ density ρ.

Poiseuille's equation The amount of liquid flowing per second, through a tube of radius r is given by

$$\frac{dV}{dt} = \frac{\pi P r^4}{8\eta l}$$ where $\frac{P}{l}$ is pressure gradient and $\frac{P}{l} = \frac{\rho g(h_1 - h_2)}{l}$. See Fig. 11.16 (a). Fig. 11.16 (b) shows how rate of flow varies with pressure head.

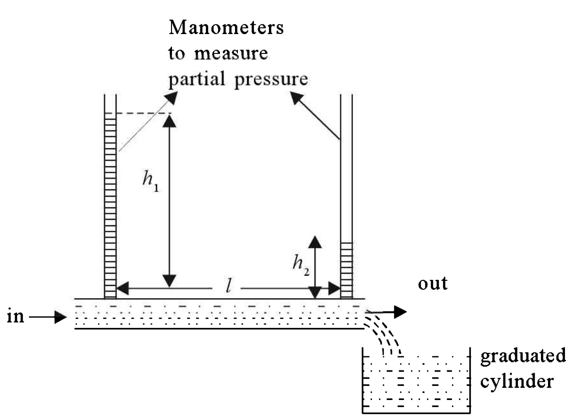


Fig. 11.16 (a)

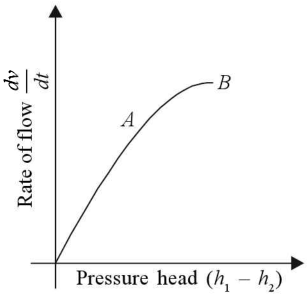


Fig. 11.16 (b)

Variation of viscosity with temperature in liquids

$h\,\rho^{-1/2} = A\,e^{C\rho/T}$ where A and C are constants, ρ is density and T is temperature in Kelvin., i.e., viscosity of the liquids decreases with rise in temperature.

For gases $\eta = \eta_0\, a\, T^{1/2}$, i.e., in gases viscosity increases with rise in temperature.

In gases, coefficient of viscosity $\eta = \frac{1}{3}\lambda\rho C$ where λ is mean free path, ρ is density and C is *rms* velocity of the gas. Searles apparatus is used to measure the viscosity of gases.

Viscosities of two liquids can be compared using viscometer. Ostwald viscometer is quite common.

$$\frac{\eta_1}{\eta_2} = \frac{\rho_1 t_1}{\rho_2 t_2}$$ where ρ_1 and ρ_1 are densities and t_1 and t_2 are times the two liquids take to vacate the viscometer.

Density: $\rho = \frac{dm}{dV}$ units kg m^{-3} [SI] and g cm^{-3} [CGS].

Relative density or specific gravity

$= \frac{\text{density of a substance}}{\text{density of water at 4°C}}$. It is dimensionless and represents density of substance in magnitude of CGS units, since in CGS units $\rho_{water} = 1$ g cm^{-3}. Variation of density with temperature $\rho = \rho_0(1 - \gamma\Delta\theta)$.

If ρ_1 is density of mass m_1 and ρ_2 density of mass m_2 then density of the combination is

$$\rho = \frac{m_1 + m_2}{\frac{m_1}{\rho_1} + \frac{m_2}{\rho_2}} = \frac{\Sigma m_i}{\Sigma \frac{m_i}{\rho_i}}$$

$$\rho = \frac{2\rho_1\rho_2}{\rho_1 + \rho_2} \quad \text{If } m_1 = m_2 \qquad \text{(harmonic mean)}$$

$$\rho = \frac{\rho_1 + \rho_2}{2} \quad \text{if } V_1 = V_2 \qquad \text{(Arithmatic mean)}$$

density increases with rise in pressure $\rho = \rho_0\left(1 + \frac{\Delta P}{B}\right) = \rho_0\frac{V_0}{V}$.

Pressure $(P)\ P = \frac{dF}{dS}$ $P = P_{atm} + \rho gh$ at a depth h below the surface of a liquid.

$P - P_{atm} = \rho gh$ is called gauge pressure or partial pressure. Dimensions of pressure is $[ML^{-1}T^{-2}]$

1 atm = 760 torr = $1.01 \times 10^5\ Pa$ and 1 torr = 1 mm of Hg.

Pressure is a scalar quantity as its direction is always normal to the area. Pressure is independent of amount of liquid, shape of the container or cross-sectional area. It depends only on depth below the surface, nature of the liquid (ρ) and acceleration due to gravity. Barometers are used to measure atmospheric pressure. Fortin's barometer is most common. Manometers measure partial pressure. Pirani and ionization gauge is used to measure vacuum.

Pascal law If an external pressure is applied to an enclosed fluid it is transmitted undiminished to every position of the fluid and to the walls of the container. That is at any point $P = P_{atm} + \rho gh$ if we apply external pressure ΔP then

$P' = (P_{atm} + \Delta P) + \rho gh$

Archimedes' principle and buoyancy When a body is immersed in a fluid wholly or partly, its weight decreases equal to the weight of the fluid displaced by the body. The upward thrust is called buoyancy and acts vertically upward (opposite to the weight) of the body through the centre of gravity of the displaced fluid (called centre of buoyancy).

Upthrust $F_{up} = lA\sigma g$ where σ is the density of fluid and lA is the volume displaced. Note, the situation of Fig. 11.17 carefully. See how the reading of base balance and hanging spring balance varies. When the suspension is independent of vessel, the base balance reading increases by upthrust or F_{up}. When the base of the vessel also holds the rigid support from where block is hanged via spring balance, then reading of the base balance is $b + mg$.

Newtons laws can be applied

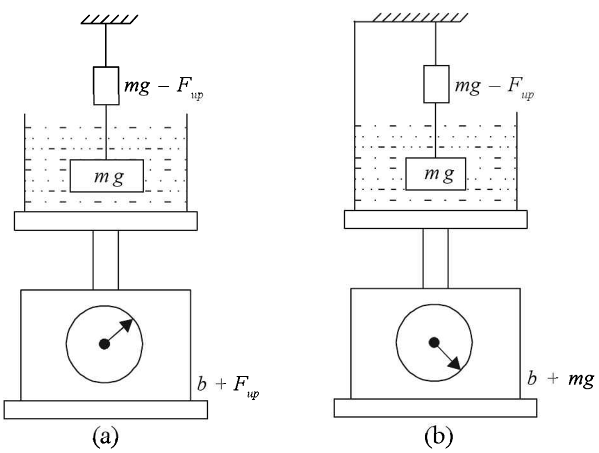


Fig. 11.17

Note: b is weight of vessel + liquid and mg is the weight of block.

Floatation A body will float if the weight of the body mg ≤ upthrust or buoyant force,

i.e., if $\rho_{body} \le \rho_{liquid}$

See Fig. 11.18

If $\rho_{body} < \rho_{liquid}$ it floats

If $\rho_{body} = \rho_{liquid}$ it just floats.

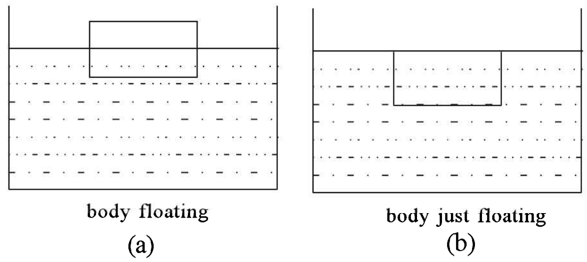


Fig. 11.18

• Short Cuts and Points to Note

1. In a steady flow of incompressible and nonviscous fluid, volume entering per second, i.e., $A_1 v_1 = A_2 v_2$ or Av = constant and is called equation of continuity.

   If you note carefully the water falling down from a tap narrows its cross-section as it falls (as velocity increases) as illustrated in Fig. 11.19 (a).

   Fig. 11.19 (b) shows the graph between v and A.

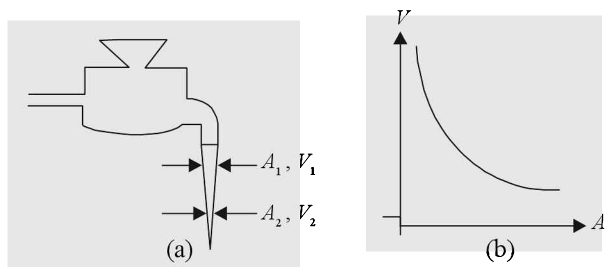


Fig. 11.19

2. Bernoulli's principle is based on conservation of energy. Sum of *KE*, *PE* and pressure head energy is constant, that is, $\frac{1}{2}\rho v^2 + \rho gh + P$ = constant

   or $\frac{1}{2}\rho v_1^2 + \rho g h_1 + P_1 = \frac{1}{2}\rho v_2^2 + \rho g h_2 + P_2$.

   In a horizontal pipe $\frac{1}{2}\rho v^2 + P$ = Constant

   Note: In a horizontal pipe liquid flows due to pressure difference.

3. During a hurricane or wind tornado when the high speed wind flows over a straw (or tin) roof pressure outside P_{out} becomes low and pressure inside becomes large due to Bernoulli's principle and the roof is lifted up and then blown off by the wind. See Fig. 11.20.

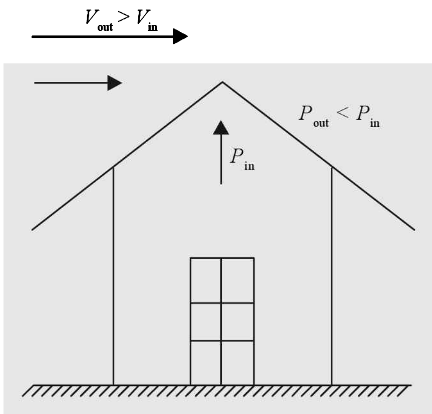


Fig. 11.20

Two boats moving in the same direction come closer as water/wind passing in between is quite faster and creates a low pressure region.

A spinning ball shows a dynamic lift or Magnus effect as resultant velocity above the ball increases and below the ball decreases. Hence, a high pressure is created below the ball and it is lifted up. See Fig. 11.21.

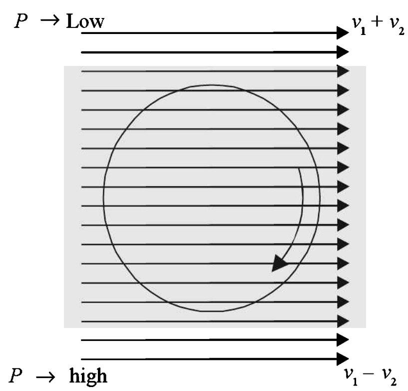


Fig. 11.21

In airplanes same principle is applied. Due to special shape of the wings, high speed wind passes over its wings when it runs on the highway creating a high pressure beneath the wings, which helps the airplane to be lifted up.

4. Efflux velocity in an open tank $v_{efflux} = \sqrt{2g(H-h)}$. The liquid coming out will cover maximum horizontal distance if $H - h = h$ or $h = \frac{H}{2}$ i.e. hole is in the middle. If area of the hole is large then v^2_{efflux} $= \dfrac{2g(H-h)}{1-\left(\frac{A_0}{A}\right)^2}$. See Fig. 11.22.

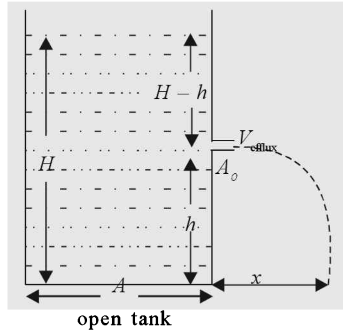


Fig. 11.22

5. Velocity of efflux in a closed tank is

$$v_{efflux} = \sqrt{\frac{2(P_{inside} - P_{atm})}{\rho}}.$$

6. In venturimeter, volume flowing per second

$$\frac{dV}{dt} = aA\sqrt{\frac{2gH}{A^2 - a^2}}.$$

7. Surface tension is the resultant molecular force per unit length at the surface.

   Surface tension = Surface energy = Work done due to surface tension per unit area.

   Surface tension depends only on the nature of liquid and is independent of the area of surface or length of line considered.

8. Surface tension is a scalar. It has a unique direction and need not be specified.

9. Surface tension decreases with rise in temperature. It becomes zero at critical temperature (where interface between vapour and liquid disappears). See Fig. 11.23.

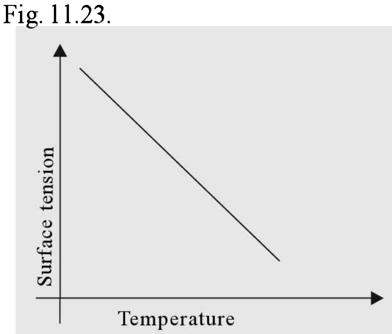


Fig. 11.23

10. Surface tension decreases with impurities. In case of soluble impurities surface tension may increase or decrease depending upon the nature of impurity. Highly soluble impurities like salt increase surface tension. Sparingly soluble impurities like soap decrease the surface tension.

11. Excess pressure in a bubble/drop having one surface

$$\Delta P = \frac{2T}{r}.$$

Excess pressure in soap bubble (bubble having two surfaces) $\Delta P = \frac{4T}{r}$.

12. Pressure just above concave meniscus in a capillary say at A in Fig. 11.24 (a)

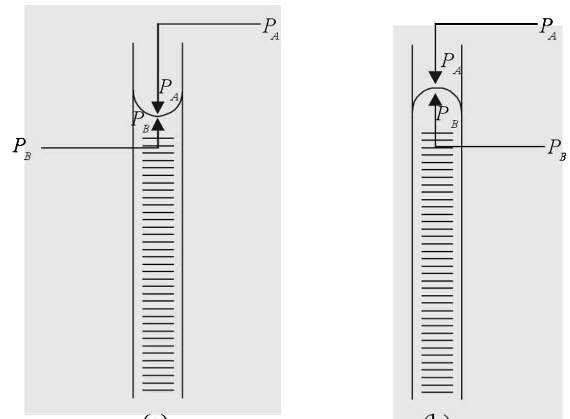


Fig. 11.24 (a) (b)

$$P_A = P_B + \frac{2T}{r} \text{ where } P_A = \text{Atmospheric Pressure.}$$

or $$P_B = P_{atm} - \frac{2T}{r} = P_A - \frac{2T}{r}.$$

In convex upward meniscus just below the meniscus [Fig. 11.24 (b)]

$$P_B = P_A + \frac{2T}{r} \text{ or } P_B = P_{atm} + \frac{2T}{r}.$$

13. Excess pressure in cylindrical surfaces $\Delta P = \frac{T}{r}$.

If there are two surfaces then $\Delta P = T\left[\frac{1}{r_1} - \frac{1}{r_2}\right]$.

See Fig. 11.25.

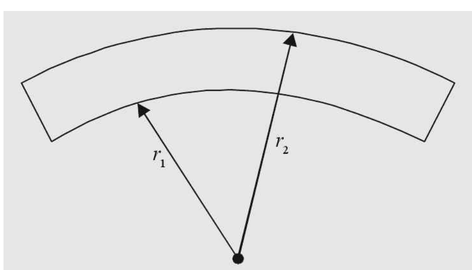


Fig. 11.25

14. Height to which a liquid rises in a capillary $h = \frac{2T\cos\theta}{r\rho g}$.

If effect of meniscus is also taken into account then

$$h = \frac{2T\cos\theta}{r\rho g} - \frac{r}{3}.$$

Note: if θ is acute, liquid rises. If θ is obtuse, liquid will dip.

Note: $h \propto \frac{1}{r}$ or hr = constant. Fig. 11.26 illustrates the variation of h with r.

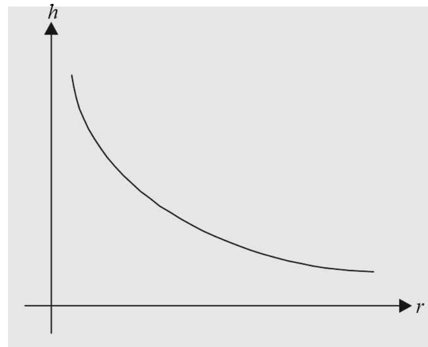


Fig. 11.26

15. If angle of contact is acute such a liquid will wet the walls of the container. It will make concave upward meniscus.

If angle of contact is obtuse, the meniscus will be convex upward and the liquid does not wet the walls of the container.

A water proof cement applied on a water leak point increases the angle of contact from acute to obtuse.

16. In a capillary tube of insufficient height, the excess liquid will collect in the form of a drop at the top of capillary but the liquid will not overflow.

17. When a big drop of radius R splits to n drops of radius r each or if n drops each of radius r coalesce to form a big drop the energy required or released is

$$\Delta E = 4\pi r^2 n^{2/3}(n^{1/3} - 1) \text{ or } \Delta E = 4\pi R^2 T(n^{1/3} - 1).$$

Increase in the temperature of big drop when n drops coalesce $\Delta\theta = \frac{3T}{J}\left[\frac{1}{r} - \frac{1}{R}\right]$.

18. Liquids having angle of contact obtuse, their angle of contact can be measured using Quinke's method.

$$\cos\phi = \left[1 - \frac{H^2}{h^2}\right].$$

See Fig. 11.27.

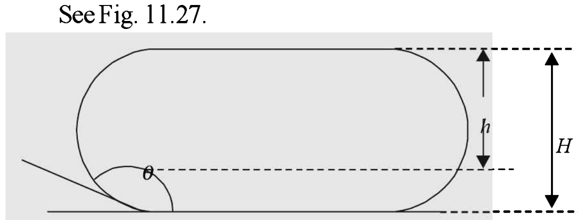


Fig. 11.27

Note: Such liquids do not wet the walls of the container.

19. Viscosity is fluid friction or intermolecular friction between various layers of a liquid. SI unit is Poiseuille (Pl) 1 Pl = 10 poise.

Poise is CGS unit of coefficient of viscosity.

Dimensional formula $[ML^{-1}T^{-1}]$ shearing stress $\frac{F}{A}$

$\propto$ Rate of change of strain $\left(\frac{dv}{dy}\right)$.

or $F = \eta A \frac{dv}{dy}$. Viscosity is also called fugitive rigidity.

20. Terminal velocity $v_T = \frac{2r^2(\rho - \sigma)g}{9\eta}$. Note $v_T \propto r^2$.

More the density of fluid in which a drop or spherical body is falling, less would be the terminal velocity. Terminal velocity is based on Stoke's law

$F = 6\pi\eta r v$.

21. Critical velocity is the maximum velocity up to which flow remains laminar/steady.

$v_C = \frac{\eta R}{\rho D}$ where R is Reynolds number, D is diameter of tube. R is dimensionless.

If $R < 2000$, flow is steady. If $R > 3000$, flow is turbulant.

22. Kinematic viscosity $\eta_k = \frac{\eta}{\rho}$ unit is stoke.

23. Poiseuille's formula $\frac{dV}{dt} = \frac{\pi P r^4}{8\eta l}$ may be compared to Ohm's law.

$i_{\text{flow}} = \frac{V_{\text{flow}}}{R_{\text{flow}}}$

$i_{\text{flow}} = \frac{dV}{dt}$ rate of liquid flow

$V_{\text{flow}} = P$ (Pressure difference determines the direction of flow)

Thus, $R_{\text{flow}} = \frac{8\eta l}{\pi r^4}$. Add resistances R_{flow} in series/ Parallel like electrical resistances.

24. Viscosity of liquids fall with the rise in temperature while that of gases increases with rise in temperature.

25. Viscometer can be used to compare viscosity of two liquids $\frac{\eta_1}{\eta_2} = \frac{\rho_1 t_1}{\rho_2 t_2}$.

26. Pressure at a depth h below the surface of a liquid, $P = P_{\text{atm}} + \rho g h$.

27. Pascal law: If the pressure is changed at a point in an enclosed fluid it is transmitted to entire liquid without being diminished in magnitude. On this principle are based hydraulic lift, hydraulic press or hydraulic brakes.

In hydraulic lift, weight lifted $W = F\frac{A_2}{A_1}$ where F is force applied and A_1 and A_2 are areas of cross-section of force and weight sides as shown in Fig. 11.28.

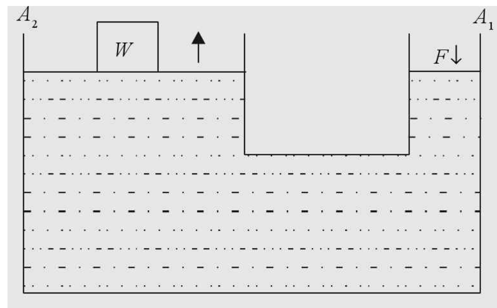


Fig. 11.28

28. Archimedes principle: Weight lost by a body when weighed in a fluid is equal to $V\sigma g$ where V is volume of the immersed part of the body or volume of fluid displaced, σ is density of fluid.

$$\frac{\sigma_{\text{liquid}}}{\rho_{\text{body}}} = \frac{\text{density of liquid}}{\text{density of body}}$$

$$= \frac{\text{Loss in weight in a liquid}}{\text{Weight of the body}}.$$

$$\text{Specific gravity} = \frac{\rho_{\text{body}}}{\rho_{\text{water}} \text{ at } 4°\text{C}}$$

$$= \frac{\text{Weight of body in air}}{\text{Loss in weight in water}}$$

29. To find volume of cavity: Volume of material of the body $V = \frac{\text{weight in air}}{\text{density of the body}}$.

Outer volume of the body

$$V_{\text{outer}} = \frac{\text{weight lost in water}}{\text{density of water}} = \frac{W_{\text{air}} - W_{water}}{\rho_{\text{water}}}.$$

Volume of cavity = Outer volume – volume of the material of the body = $(V_{\text{outer}} - V)$.

30. Impurity determination Let m be total mass of the body. Density of material is ρ_1 and that of impurity is ρ_2. Let x be the amount of impurity.

Then $\frac{m-x}{\rho_1} + \frac{x}{\rho_2} = V$ (total volume of the body)

Note: In CGS system weight lost by a body in water (in grams) = Volume of the body in CC.

31. A body will float only if $\rho_{body} \le \rho_{liquid}$.

32. Small fog like particles act like rigid bodies due to excess pressure in it $\left(\Delta P \propto \frac{1}{r}\right)$ and cause invisibility.

33. If two bubbles coalesce in isothermal conditions then r = $\sqrt{r_1^2 + r_2^2}$.

If the bubbles coalesce adiabatically $r = \frac{r_1 r_2}{r_2 - r_1}$.

34. Cohesive force $F_C \propto \frac{1}{r^8}$.

35. If a bubble rises from a depth to surface, its volume/radius increases as pressure decreases.

Apply $P_1 V_1 = P_2 V_2$

- **Caution**

1. Assuming when two bubbles coalesce pressure inside will increase.

⇒ As radius will increase pressure inside will decrease because $P \propto \frac{1}{r}$.

Therefore, use $P_{net} = P_1 - P_2$ or $\frac{2T}{R} = \frac{2T}{r_1} - \frac{2T}{r_2}$

or $\frac{1}{R} = \frac{1}{r_1} - \frac{1}{r_2}$.

⇒ If two bubble coalesce isothermally then

$P_1 V_1 + P_2 V_2 = P V$

or $r_1^2 + r_2^2 = R^2$

2. Assuming in Fig. 11.29 when valve V is opened smaller bubble will become bigger and bigger will become smaller.

⇒ Bigger bubble will grow more fat and smaller will become more small as $\Delta P \propto \frac{1}{r}$. Therefore, pressure in smaller bubble is more and air will flow from smaller towards bigger.

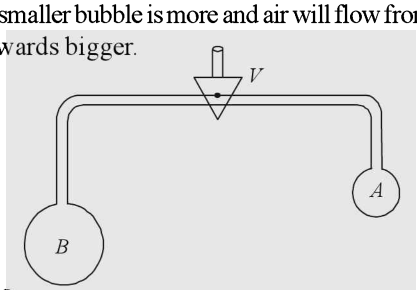


Fig. 11.29

3. Assuming if a capillary tube is of insufficient height, the excess liquid will overflow.

⇒ Excess liquid will collect as a drop at the top of capillary. Overflow is not permitted (due to inhabitance of prepetual motion).

4. Assuming Archimedes laws different from Newton's laws of motion.

⇒ Archimedes law is based on Newton's third law. Weight lost = buoyant force or upthrust is the reaction of the force applied on liquid.

5. Assuming melting of ice in a liquid will raise the level of the liquid.

⇒ The level does not vary as on melting the volume of ice decreases. However, if the ice had a metal piece placed over it or inside the ice then water level will fall as the ice melts completely.

6. Assuming if a body sinks in one liquid, it will sink in other liquid also.

⇒ Floatation depends on density. If effective density of the body is less than or equal to that of the liquid, then it will float.

7. Assuming the density of a body will remain unaltered.

⇒ If the body develops a cavity or had a cavity, its density varies. For example, density of ship (made of steel) < density of water. So it floats.

8. Considering water proof material fills the pores to stop seepage.

⇒ It changes angle of contact from acute to obtuse and hence, it will not wet the surface.

9. Assuming $\rho g h$ as normal pressure exerted by the liquid.

⇒ Normal pressure of the liquid is different from gravitational pressure $\rho g h$.

10. Considering that flowing liquid offers no resistance

⇒ $R_{flow} = \frac{8\eta l}{\pi r^4}$. In series $R = R_1 + R_2$ when capillary tube are joined in series.

$\frac{1}{R_{Parallel}} = \frac{1}{R_1} + \frac{1}{R_2}$ when capillary tubes are joined in parallel.

In series $\frac{dV}{dt} = \frac{\pi(P_1 - P_2)r_1^4}{8\eta l_1} = \frac{\pi(P_2 - P_3)r_2^4}{8\eta l_2}$ as $\frac{dV}{dt}$ remains same.

In Parallel $\frac{dV}{dt} = \frac{dV_1}{dt} + \frac{dV_2}{dt}$ and $\frac{1}{R} = \frac{\pi}{8\eta l}\left(r_1^4 + r_2^4\right)$ if the lengths and pressures at two ends are equal.

11. Assuming flowing liquid only possesses *KE* and *PE*.

⇒ Liquids along with *KE* and *PE* possess pressure head energy.

12. Considering velocity of efflux depends on the density of the liquid.

⇒ $v_{\text{efflux}} = \sqrt{2g(H-h)}$ in open tank.

$v_{\text{efflux}} = \sqrt{\dfrac{2(P_{\text{inside}} - P_{\text{atm}})}{\rho}}$ in a closed tank.

Note: In open tank efflux velocity is independent of density of liquid. Only in closed tanks. v_{efflux} depend on density and $v_{\text{efflux}} \propto \dfrac{1}{\sqrt{\rho}}$.

SOLVED PROBLEMS

1. A *u* tube is rotated about one of the limbs with an angular velocity ω. Find the difference in height *H* of the liquid level. Density of the liquid is ρ. Diameter of the tube $d << L$.

[IIT Mains 2005]

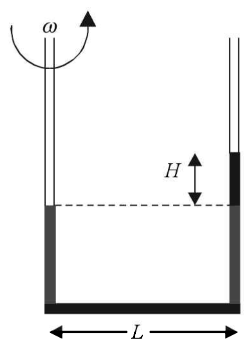


Fig. 11.30

Solution $dF = dm\, x\, \omega^2 = A\, \rho\, x\, \omega^2\, dx$

or $F = A\rho\, \omega^2 \int_0^L x dx = \dfrac{A\rho L^2\omega^2}{2}$;

Pressure $P = \dfrac{F}{A} = \dfrac{\rho L^2 \omega^2}{2} = \rho g H.$

or $H = \dfrac{L^2\omega^2}{2g}$

2. Ratio of area of the hole to beaker is 0.1. Height of the liquid in the beaker is 3m, and a hole is at a height of 52.5 cm from the bottom of the beaker. Find the square of the velocity of the liquid coming from the hole.

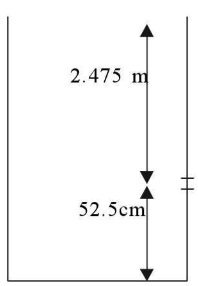


Fig. 11.31

(a) 52 $(\text{ms}^{-1})^2$ (b) 50.5 $(\text{ms}^{-1})^2$
(c) 51 $(\text{ms}^{-1})^2$ (d) 42 $(\text{ms}^{-1})^2$

[IIT Screening 2005]

Solution (a) $v^2_{\text{efflux}} = \dfrac{2g(H-h)}{1-\left(\dfrac{A_0}{A}\right)^2}$

$= \dfrac{2\times10\times2.475}{1-.01}$

$= 50\,\text{ms}^{-2}$

3. A 20 cm long capillary tube is dipped in water. The water rises up to 8 cm. If the entire arrangement put in a freely falling elevator the length of water column in the capillary tube will be

(a) 8 cm (b) 10 cm
(c) 4 cm (d) 20 cm

[AIEEE 2005]

Solution (d) As the system becomes weightless. Therefore, liquid will rise to full height.

4. A candle of diameter *d* is floating in a cylindrical container of diameter *D* ($D >> d$) as shown in Fig 11.32. If it is burning at a rate 2 cm h^{-1} then the top of the candle will

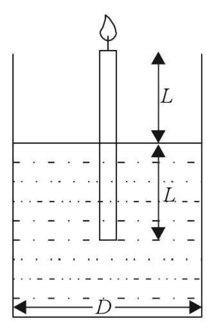


Fig. 11.32

(a) remain at the same height.
(b) fall at the rate of 1 cm h^{-1}.
(c) fall at the rate of 2 cm h^{-1}.
(d) go up at the rate of 1 cm h^{-1}.

[AIIMS 2005]

Solution (b) density of candle is half the density of water. Therefore, it will always remain half in water. Hence, if it burns 2 cm h^{-1}, the candle will come up by 1 cm from the water.

5. *A*. For Reynold number $R > 2000$, the flow of the fluid is turbulant.

R Inertial forces are dominant compared to viscous forces at such high Reynolds number.

(a) Both *A* and *R* are correct and *R* is correct explanation of *A*.

(b) *A* and *R* are correct but *R* is not correct explanation of *A*.

(c) *A* is true but *R* is false.

(d) both *A* and *R* are false.

[AIIMS 2005]

Solution (a) $\because R = \dfrac{\text{Inertial force}}{\text{Viscous drag}}$

6. An electrical short cuts off all power to a submersible when 30 m below the surface of ocean. The crew must push out a hatch of area 0.75 m² and weight 300 N on the bottom to escape. If the pressure inside is 1 atm. What downward force it must exert on the hatch to open it?

(a) 1.53×10^5 N (b) 2.03×10^5 N
(c) 2.23×10^5 N (d) 3.03×10^5 N

Solution (c) $\Delta P = 30 \times 10^3 \times 10 = 3 \times 10^5$ Pa.

$F = 3 \times .75 \times 10^5 + 300\text{ N} = 2.28 \times 10^5$ N

7. A tapered pressurised tank for a rocket contains 0.25 m³ of kerosene, with mass 205 kg. The pressure at the top of the kerosene is 2.01×10^5 Pa. The kerosene exerts a force of 16.4 kN on bottom having area 0.07 m². Find the depth of the kerosene.

(a) 2.1 m (b) 0.29 m
(c) .414 m (d) 4.14 m

Solution (d) $F = (P_0 + \rho g h)A$

$$\left(2.01\times10^5 + \frac{205}{.25}\times10\times h\right).07 = 16.4\times10^3$$

or $h = 4.14$ m

8. A cubical block of wood of side 10 cm floats at the interface between oil and water with its lower surface 1.5 cm below the interface. The density of the oil is 790 kg m⁻³. What is the density of the block?

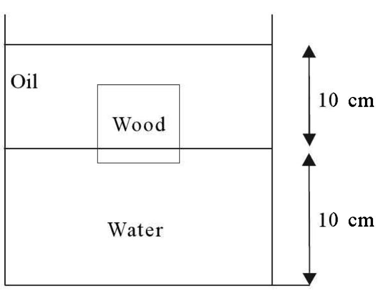


Fig. 11.33

(a) 821.5 kg m⁻³ (b) 800.5 kg m⁻³
(c) 820 kg m⁻³ (d) none

Solution (a) $M = 1.5 \times 100 + 8.5 \times 100 \times .79$

$= 821.5$ g

$$\rho = \frac{821.5\times10^{-3}}{10^3\times10^{-6}} = 821.5 \text{ kg m}^{-3}$$

9. A 45 kg woman is standing on an ice slab without getting her feet wet. What is the minimum volume of the slab?

(a) 0.562 m³ (b) 0.5 m³
(c) 0.812 m³ (d) none of these

Solution (a) $0.92\text{ V} + 45000 = 1.0 \times \text{V}$

or $.08\text{ V} = 45000$ cc

or $V = \dfrac{45000}{8} \times 10^2$

$= 562.25 \times 10^3$ cc

or 0.562 m³

10. A plastic sphere is held below the surface of a lake by tying a thread at the bottom of the lake. The tension in the thread is 900 N. The sphere has volume 0.65 m³. Find the mass of the sphere.

(a) 250 kg (b) 340 kg
(c) 470 kg (d) 560 kg

Solution (d) $T = V(\sigma - \rho) g = B - mg$ or $mg = B - T$

$mg = 0.65 \times 10^3 \times 10 - 900 = 5600\ N$ or $M = 560$ kg

11. A shower head has 20 circular opening each of radius 1 mm. Shower head is connected to a pipe of radius 0.8 cm. The speed of water in the pipe is 3 ms⁻¹. Find the speed of water as it exits from the shower head openings.

(a) 3.6 ms⁻¹ (b) 5.6 ms⁻¹
(c) 7.6 ms⁻¹ (d) 9.6 ms⁻¹

Solution (d) $A_1 v_1 = A_2 v_2$

or $\pi(1)^2 \times 20\ v = \pi\ 8^2 \times 3$

or $v = 9.6$ ms⁻¹

12. A sealed tank containing water to a height 11 m also contains air at 3 atm. Water flows out from the bottom of a tank through a small hole. Find efflux velocity.

(a) 18.1 ms⁻¹ (b) 24.2 ms⁻¹
(c) 26.4 ms⁻¹ (d) 28.6 ms⁻¹

Solution (d) $v_{\text{efflux}} = \sqrt{\dfrac{2P_{\text{in}}}{\rho}}$

$$= \sqrt{\frac{2(3\times10^5 + 11\times10\times10^3)}{10^3}}$$

$= \sqrt{820} = 28.6$ ms⁻¹

13. Air streams horizontally past a small airplane's wings with a speed 70 ms⁻¹ over the top surface and 60 ms⁻¹

past the bottom surface. Plane has mass 1340 kg and wing area 16.2 m^2. Find the net vertical force. Given density of air 1.2 kg m^{-3}.

(a) 500 N upward (b) 500 N downward
(c) 765 N upwards (d) none of these

Solution (b) $F_{net} = \frac{1}{2}\rho\left(v_1^2 - v_2^2\right) g - mg = \frac{1}{2} \times 1.2\,(70^2 - 60^2) \times 1.8 - 1340 \times 9.8 = -500$ N

14. The upper edge of a gate in a dam runs along the water surface. The gate is 2 m high and 4 m wide and is hinged along the horizontal line through its top. Find the torque due to the force from water.

(a) 10.67×10^4 *N-m* (b) 5.33×10^4 *N-m*
(c) 2.66×10^4 Nm (d) none

Solution (a) Consider a strip of (4 $m \times dy$) at a height y. Force on the strip

$$dF = \rho g (2-y)\,4dy;$$

Torque $d\tau = \rho g (2-y)^2\, 4\, dy$

$$\tau = \rho g 4\left[4y + \frac{y^3}{3} - 2y^2\right]$$

$$= 10^3 \times 10 \times 4\left[8 + \frac{8}{3} - 8\right]$$

$$= 10.67 \times 10^4 \; N\text{-}m.$$

15. An astronaut is standing at the north pole of a newly discovered planet of radius R. He holds a container full of liquid having mass m and volume V. At the surface of the liquid pressure is p_0 and at a depth d below the surface, pressure is p. From this information determine the mass of the planet.

(a) $\frac{(p+p_0)VR^2}{Gdm}$ (b) $\frac{(p+p_0)R^2V}{2Gdm}$

(c) $\frac{(p-p_0)R^2V}{2Gdm}$ (d) $\frac{(p-p_0)R^2V}{Gdm}$

Solution (d) $\rho g d = (p - p_0)$

or $$g = \frac{(p-p_0)}{d\left(\frac{m}{V}\right)}$$

or $$\frac{GM}{R^2} = \frac{(p-p_0)V}{dm}.$$

or $$M = \frac{(p-p_0)VR^2}{Gdm}$$

16. If the density of earth varies as $\rho(r) = A - Br$. Then find the value of $g(r)$ inside the earth.

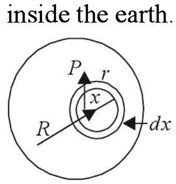


Fig. 11.34

(a) $\frac{4}{3}\pi G R\left(A - \frac{3}{4}BR\right)$ (b) $\frac{4}{3}\pi G R\left(A - \frac{3}{4}Br\right)$

(c) $4\pi G\left(Ar - \frac{Br^2}{2}\right)$ (d) none

Solution (b) $g = \frac{GM}{R^2} = \int_0^r \frac{G}{r^2}\rho\,(4\pi x^2)\,dx$

$$= \int_0^r \frac{a(A-Bx)}{r^2}(4\pi x^2\,dx)$$

$$= \frac{4\pi GR}{3}\left(A - \frac{3}{4}Br\right).$$

17. A U tube open at both ends contains Hg. 15 cm of water is added in left arm. Find the difference in the height of two levels.

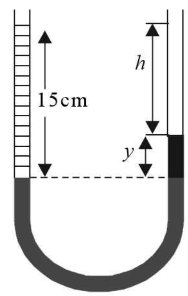


Fig. 11.35

(a) 1.1 cm (b) 13.9 cm
(c) 7 cm (d) 8 cm

Solution (b) $13.6 \times y \times Ag = 1 \times 15 \times A \times g$

$$y = \frac{15}{13.6} \text{ and}$$

$$h = 15 - \frac{15}{13.6}$$

$$= 13.9\text{cm}$$

18. An aluminium hunk is covered with gold shell to form an ingot. The ingot weighs 45 N in air and 39 N in water. Find the weight of the gold in the shell.

(a) 12 N (b) 11.5 N
(c) 33.5 N (d) 36 N

Solution (c) $6 = \frac{45-x}{2.7} + \frac{x}{19.3}$

or $x = 33.5$ N.

19. A mould has density of metal ρ_m. Weight of the casting W and buoyant force B when it is completely immersed in water. Find the volume of cavity.

(a) $\frac{B}{\rho_{water}} - \frac{W}{\rho_m}$ (b) $\frac{B-W}{\rho_m g}$

(c) $\frac{W-B}{\rho_m g}$ (d) $\frac{B}{(g\rho_{water})} - \frac{W}{\rho_m g}$

Solution (d) Outer volume $= \dfrac{B}{\rho_{\text{water}} g}$; Volume of metal $= \dfrac{W}{\rho_{\text{metal}} g}$; Volume of cavity $= \dfrac{B}{\rho_{\text{water}} g} - \dfrac{W}{\rho_m g}$.

20. A cubical block of density ρ is immersed in a liquid of density σ ($\sigma > \rho$). The side of the block is L. What fraction is immersed in the liquid?

(a) $1 - \dfrac{\rho}{\sigma}$ (b) $\dfrac{\sigma}{\rho} - 1$

(c) $\dfrac{\rho}{\sigma}$ (d) none of these

Solution (c) $L \rho A g = h \sigma A g$ or $h = \dfrac{L\rho}{\sigma}$ or $\dfrac{h}{L} = \dfrac{\rho}{\sigma}$.

21. A cylindrical container of an incompressible liquid rotates with ω as shown. The density of liquid is ρ. The liquid acquires parabolic shape. Find the height of the liquid. Radius of container is r.

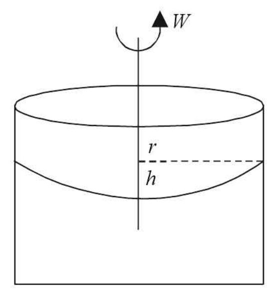


Fig. 11.36

Solution The force experienced by element dx at distance x from the centre $dF = A\, dx\, \rho\, \omega^2 x$ and

$$dP = \frac{dF}{A} = \rho\, \omega^2 x\, dx$$

$$P = \int_0^r \rho \omega^2 x dx = h \rho g \text{ or } h = \frac{r^2\omega^2}{2g}.$$

Note: This technique is employed to make parabolic telescopic mirrors. Liquid glass is rotated and allowed to solidify while rotating.

22. Water flows steadily from an open tank as shown. Find x where the water lands. Assume area of cross-section of hole is much less than area of cross-section of tank.

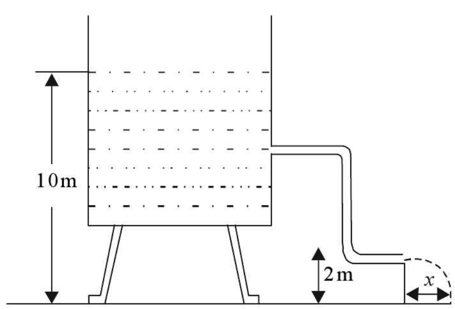


Fig. 11.37

Solution $v = \sqrt{2gh} = \sqrt{2 \times 10 \times 8} = 4\sqrt{10}$ ms^{-1}

$$\frac{1}{2} g t^2 = 2 \text{ or } t = \sqrt{.4} = .63 s$$

$$x = v_x t = 4\sqrt{10} \times (.63)$$

$$= 2.52\,(3.162) = 7.97 \text{ m}$$

23. A beaker is exactly full of water with an ice piece floating. The ice piece has a lead piece in it. When ice melts then

(a) level remains unchanged.

(b) water overflows.

(c) level falls. (d) none of these.

Solution (c) Since the average density of ice with lead piece increases, it displaces more water.

24. Drops of liquid of density d are floating half immersed in a liquid of density ρ. If the surface tension of liquid is T then the radius of the drop is

(a) $\sqrt{\dfrac{3T}{g(3d-\rho)}}$ (b) $\sqrt{\dfrac{6T}{g(2d-\rho)}}$

(c) $\sqrt{\dfrac{3T}{g(2d-\rho)}}$ (d) $\sqrt{\dfrac{3T}{g(4d-3\rho)}}$

Solution (c) Weight of the drop = forces due to surface tension + buoyant force

$$\frac{4}{3}\pi r^3 d g = 2\pi r T + \frac{1}{2} \times \frac{4}{3}\pi r^3 \rho g$$

or $$r = \sqrt{\frac{3T}{g(2d-\rho)}}$$

25. A block of mass m just floats in a liquid. It is pushed down and released then

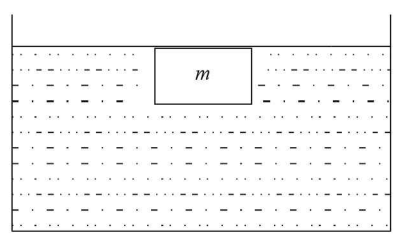


Fig. 11.38

(a) it will oscillate.

(b) it will rise to original position and stay there.

(c) it will sink.

(d) it will rise to another position and stay there.

[CEEE Delhi 1999]

Solution (c) The additional push gives an unbalanced force and according to Newton's first law it will continue to move down and sink.

26. A wooden block with a coin placed on its top, floats in water as shown in Fig. 11.39. The distance l and h are illustrated. If the coin falls into the water then

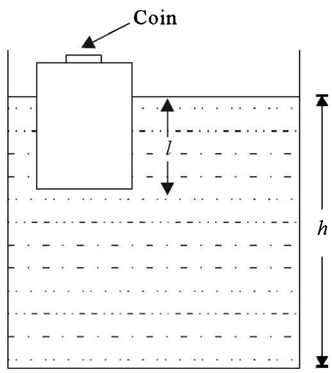


Fig. 11.39

(a) l decreases and h increases.
(b) l increases and h decreases.
(c) both l and h increase.
(d) both l and h decrease.

[IIT Screening 2002]

Solution (d) When the coin falls, it occupies less volume, therefore, h decreases, l decreases as density of wood decreases.

27. A piece of metal floats on Hg. The coefficient of expansion of metal and Hg are γ_1 and γ_2 respectively. If the temperature of both Hg and metal are increased by an amount ΔT, by what factor the fraction of the volume of metal submerged in mercury changes?

(a) $(\gamma_2-\gamma_1)\Delta T$ (b) $\left(\dfrac{\gamma_2+\gamma_1}{2}\right)\Delta T$

(c) $\dfrac{2\gamma_1\gamma_2}{\gamma_1+\gamma_2}\Delta T$ (c) $\dfrac{\gamma_1\gamma_2}{\gamma_1+\gamma_2}\Delta T$

Solution (a) $f_{in}=\dfrac{V_{in}}{V}=\dfrac{\rho}{\sigma}$ where ρ is density of metal and σ is density of Hg.

$$\frac{\Delta f}{f} = \frac{f'_{in}-f_{in}}{f_{in}} = \frac{f'_{in}}{f_{in}}-1$$

$$= \frac{\dfrac{\rho}{\sigma}\left(\dfrac{1+\gamma_2\Delta T}{1+\gamma_1\Delta T}\right)}{\dfrac{\rho}{\sigma}}-1 = (\gamma_2-\gamma_1)\Delta T$$

(using Binomial theorem)

28. A tank has a hole of area of cross-section A. Water from a pipe of inner cross-section A is entering the tank with a velocity v. To what height the tank will be filled?

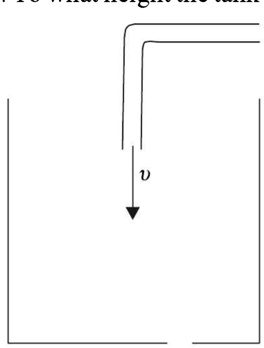


Fig. 11.40

(a) zero (b) $\dfrac{v^2}{2g}$

(c) $\dfrac{P_{atm}}{\rho g}$ (d) $\dfrac{2P_{atm}}{\rho g}$

Solution (b) $v_{efflux}=\sqrt{2gh}$ = tank will fill to a height h until.

$$v_{efflux} = v \text{ or } h=\frac{v^2}{2g}.$$

29. A liquid of density ρ comes out with a velocity v from a horizontal tube of area of cross-section A. The reaction force exerted by the liquid on the tube is F. Then

(a) $F\propto v$ (b) $F\propto v^2$

(c) $F\propto A$ (d) $F\propto\dfrac{1}{\rho}$

Solution (b) and (c)

$$F = v\frac{dm}{dt}=v\left(\rho A\frac{dx}{dt}\right)=\rho A v^2$$

30. An ideal liquid flows through a tube of uniform cross-section shown in Fig. 11.41. The liquid has velocities v_A and v_B and pressure P_A and P_B at points A and B respectively. Then

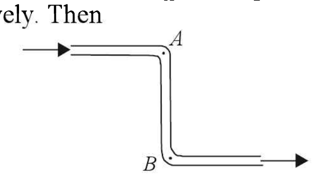


Fig. 11.41

(a) $v_A=v_B$ (b) $v_B>v_A$

(c) $P_A=P_B$ (d) $P_B>P_A$

Solution (a) and (d) $v_A=v_B$

and $P_B=P_A+\rho g h$

31. A vertical glass capillary tube open at both ends contains some H_2O. Which of the following shapes for water is correct?

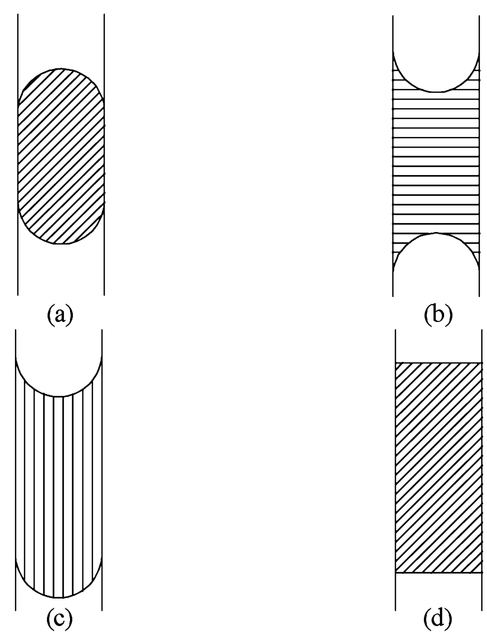


Fig. 11.42

Solution (c) The force on two liquid states must provide a net upward force due to surface tension to balance the weight of H_2O.

32. The weight of an empty balloon on a spring balance is W_1. The weight becomes W_2 when the balloon is filled with air. Let the weight of air itself be ω. Neglect thickness of the balloon when it is filled with air and also neglect difference of air densities inside and outside the balloon. Then

(a) $W_2 = W_1$ (b) $W_2 = W_1 + \omega$
(c) $W_2 < W_1 + \omega$ (d) $W_2 > W_1$

Solution (a), (c) The buoyant force on the balloon = weight of air inside.

33. In a streamline flow

(a) the speed of the particle remains the same throughout.
(b) the velocity of the particle remains same throughout.
(c) the KE of all the particles arriving at a given point are the same.
(d) the momenta of all the particles arriving at a given point are the same.

Solution (c) and (d)

34. A liquid is contained in a vertical tube of semi circular cross-section. The contact angle is zero. The force of surface tension on the flat part to curved part is

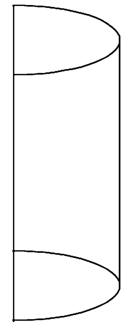


Fig. 11.43

(a) 1 : 1 (b) 1 : 2
(c) π : 2 (d) 2 : π

Solution (d) $F = T \cdot l$ $\dfrac{F_{\text{Flat}}}{F_{\text{Curved}}} = \dfrac{T(2R)}{T(\pi R)} = \dfrac{2}{\pi}$

35. The velocity of raindrop having radius 1 mm is 20 cm s^{-1}. The velocity of raindrops of size 3 mm is
(a) 60 cm s^{-1} (b) 120 cm s^{-1}
(c) 180 cm s^{-1} (d) 20 cm s^{-1}
(e) none

Solution (c) $v \propto r^2$

TYPICAL PROBLEMS

36. A wire forming a loop is dipped into soap solution and taken out so that a film of soap solution is formed. A thread of length 6.28 cm is gently put on the film and film is pricked with a needle inside the loop.

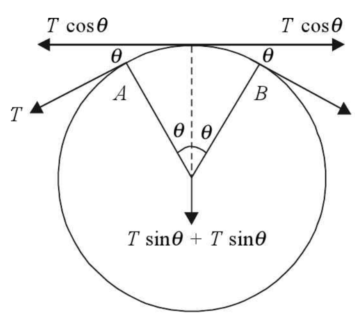


Fig. 11.44

Find the tension in the thread if surface tension of soap solution is 0.03 N m^{-1}.

(a) 18.84×10^{-4} N (b) 6×10^{-14} N
(c) 3×10^{-4} N (d) 9.42×10^{-4} N

Solution (c) $2T \sin\theta = S\,2r\,\theta$ where S is surface tension and $2\,r\,\theta$ is length of thread AB.

Since θ is small $\therefore$ $\sin\theta = \theta$

$2T\theta = S\,2r\,\theta$

or $T = Sr = .03 \times 10^{-2} = 3 \times 10^{-4}$ N

37. Water flows through a tube of radius 1 cm at a speed of 6 cm s^{-1}. Find Reynolds number. Is the flow steady?

(a) 6000, No (b) 3000, No
(c) 600, Yes (d) 1200, Yes

Solution (d) $R = \dfrac{\rho v D}{\eta} = \dfrac{1 \times 6 \times 2}{0.01} = 1200.$

Since R < 2000, Flow is steady.

38. Find the surface energy of water kept in a cylindrical vessel of radius 6 cm. Surface tension of H_2O $= 75 \times 10^{-3}$ N m^{-1}.

(a) 17×10^{-4} J (b) 8.5×10^{-4} J
(c) 4.2×10^{-4} J (d) 8.5×10^{-2} ergs.

Solution (b) $\pi r^2 (T) = \dfrac{22}{7} \times 6^2 \times 75 \times 10^{-3} \times 10^{-4}$

$= 8.5 \times 10^{-4}$ J.

39. A u tube containing a liquid is accelerated horizontally with an acceleration a_0. The separation between the vertical arms is l. Find the difference in the heights of the two arms.

(a) $\frac{gl}{a_0}$ (b) zero

(c) $\frac{gl}{a_0}$ (d) $\left(\frac{a_0 - g}{g}\right) l$

(e) none

Solution (c) $ma_0 = (\rho g h) A$ or $(\rho A l)\, a_0 = \rho g h A$

or $h = \frac{l a_0}{g}$.

40. Find the pressure at A immediately below the meniscus.

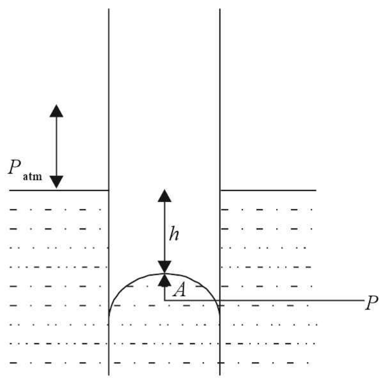


Fig. 11.45

(a) $P_{atm} + \rho g h$ (b) $P_{atm} - \rho g h$

(c) $P_{atm} + \rho g h + \frac{2T}{r}$ (d) $P_{atm} + \frac{2T}{r}$

Solution (d)

41. Find the excess pressure on one side of a soap film of surface tension T over that on the other side r_1 and r_2 are radii of two surfaces.

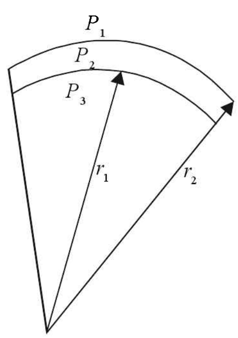


Fig. 11.46

QUESTIONS FOR PRACTICE

1. Figure shows a capillary tube of radius r dipped into water. If the atmospheric pressure is P_0, the pressure at point A (just below the meniscus) is

(a) P_0 (b) $P_0 + \frac{2S}{r}$

(c) $P_0 - \frac{2S}{r}$ (d) $P_0 - \frac{4S}{r}$

where S is surface tension

(a) $2T\left[\frac{1}{r_1} - \frac{1}{r_2}\right]$ (b) $2T\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$

(c) $T\left[\frac{1}{r_1} - \frac{1}{r_2}\right]$ (d) $T\left[\frac{1}{r_1} + \frac{1}{r_2}\right]$

Solution (a) $P_3 - P_2 = \frac{2T}{r_1}$ ....(1)

$P_2 - P_1 = \frac{2T}{r_2}$ ....(2)

Adding (1) & (2) $P_3 - P_1 = 2T\left[\frac{1}{r_1} - \frac{1}{r_2}\right]$.

42. Two spherical bubbles coalesce. V is the consequent change in volume and S is the total change in surface area then

(a) $3PV + 4ST = 0$ (b) $4PV + 3ST = 0$

(c) $2PV + 3ST = 0$ (d) $3PV + 2ST = 0$

Solution (a) $P_1 V_1 + P_2 V_2 = P_3 V_3$

or $\left(P_0 + \frac{4T}{r_1}\right)\left(\frac{4}{3}\pi r_1^3\right) + \left(P_0 + \frac{4T}{r_2}\right)\left(\frac{4}{3}\pi r_2^3\right)$

$= \left(P_0 + \frac{4T}{r}\right)\left(\frac{4}{3}\pi r^3\right)$.

or $4T(r_1^2 + r_2^2 - r^2) = -P_0 (r_1^3 + r_2^3 - r^3)$

$4T\frac{S}{4\pi} + \frac{3}{4\pi} P_0 V = 0$

or $4TS + 3P_0 V = 0$

43. A U tube is made of two capillary tubes of diameter 0.5 mm and 1.0 mm respectively. It contains H_2O. (Surface tension 75 dynes/cm). Find the difference in two levels.

(a) 2.4 cm (b) 2.6 cm

(c) 2.8 cm (d) 3.0 cm

Solution (d) $h = \frac{2T}{\rho r g}$

$\Delta h = \frac{2T}{\rho g}\left[\frac{1}{r_1} - \frac{1}{r_2}\right]$

$= \frac{2 \times 75}{1 \times 980}\left[\frac{1}{.025} - \frac{1}{.05}\right] = 3$ cm

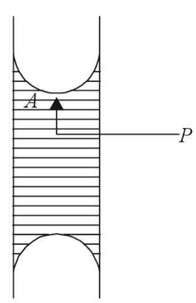


Fig. 11.47

2. The excess pressure inside a soap bubble is twice the excess pressure inside a second soap bubble. The volume of the first bubble is n times the volume of the second where n is

(a) 4 (b) 2
(c) 8 (d) 0.125

3. Which of the following graphs may represent the relation between the capillary rise h and the radius r of the capillary?

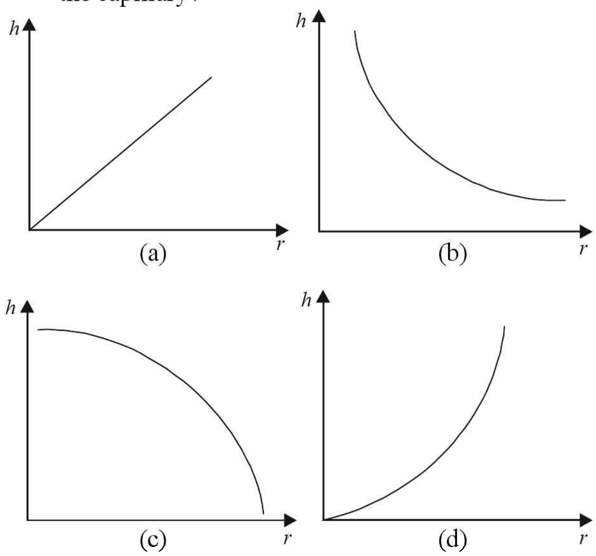


Fig. 11.48

4. Water rises in a vertical capillary tube upto a length of 10 cm. If the tube is inclined at 45°, the length of water risen in the tube will be

(a) 10 cm (b) $10\sqrt{2}$ cm
(c) $10/\sqrt{2}$ (d) none of these

5. A 20 cm long capillary tube is dipped in water. The water rises up to 8 cm. If the entire arrangement is put in a freely falling elevator, the length of water column in the capillary tube will be

(a) 8 cm (b) 6 cm
(c) 10 cm (d) 20 cm

6. Viscosity is a property of

(a) liquids only.
(b) solids only.
(c) solids and liquids only.
(d) liquids and gases only.

7. The force of viscosity is

(a) electromagnetic. (b) gravitational.
(c) nuclear. (d) weak.

8. The viscous force acting between two layers of a liquid is given by $\frac{F}{A} = -\eta \frac{dv}{dz}$. This F/A may be called

(a) pressure. (b) longitudinal stress.
(c) tangential stress. (d) volume stress.

9. A raindrop falls near the surface of the earth with almost uniform velocity because

(a) its weight is negligible.
(b) the force of surface tension balances its weight.
(c) the force of viscosity of air balances its weight.
(d) the drops are charged and atmospheric electric field balances its weight.

10. A piece of wood is taken deep inside a long column of water and released. It will move up

(a) with a constant upward acceleration.
(b) with a decreasing upward acceleration.
(c) with a deceleration.
(d) with a uniform velocity.

11. A solid sphere falls with a terminal velocity of 20 m/s in air. If it is allowed to fall in vaccum,

(a) terminal velocity will be 20 m/s.
(b) terminal velocity will be less than 20 m/s.
(c) terminal velocity will be more than 20 m/s.
(d) there will be no terminal velocity.

12. A spherical ball is dropped in a long column of a viscous liquid. The speed of the ball as a function of time may be best represented by the graph

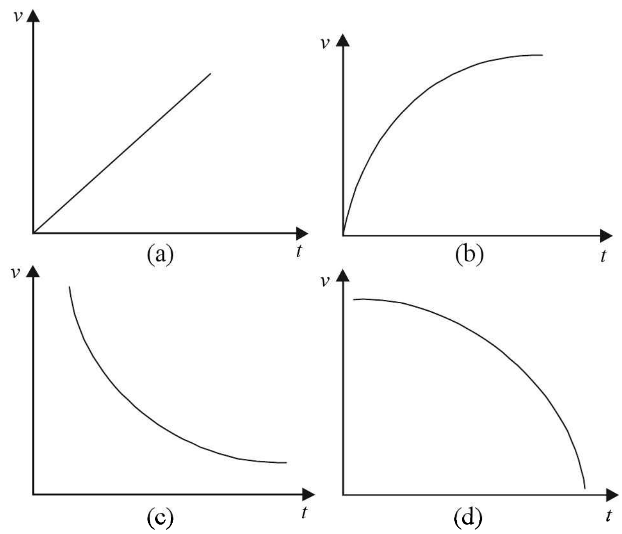


Fig. 11.49

13. What will be the critical velocity of water in a tube of radius 1 m, if the coefficient of viscosity of water is 1793 poise and Reynold's number is 1000.

(a) 17.93 cm/s (b) 17.93×10^2 cm/s
(c) 17.93×10^4 cm/s (d) 17.93×10^3 cm/s

14. Two different liquids are flowing in two tubes of equal radius. The ratio of coefficients of viscosity of liquids is 52 : 49 and the ratio of their densities is 15.6 : 1, then the ratio of their critical velocities will be

(a) 0.068 (b) 0.68
(c) 6.8 (d) 68

15. If the radius of narrow hole in the bottom of a rocket is 2 cm and the pressure difference between inside and outside the chamber is 5 atmospheres, then the reactional force in dynes acting on the rocket will be

(a) 1.27×10^4 (b) 1.27×10^{-6}
(c) 1.21×10^8 (d) 1.27×10^{10}

16. The difference of two liquid levels in a manometer is 10 cm and its density is 0.8 gm/cm^3. If the density of air is 1.3×10^3 gm/cm^3 then the velocity of air will be (in cm/sec)

(a) 347 (b) 34.7
(c) 3470 (d) 0.347

17. The unit of the coefficient of viscosity in S.I. system is

(a) m/kg/s (b) m/s/kg^2
(c) kg/m/s^2 (d) kg/m/s

18. Two water pipes of diameter 2 cm and 4 cm are connected to main water source. The rate of water flow through 2 cm pipe as compared to that through 4 cm pipe will be

(a) one fourth. (b) double.
(c) half. (d) four times.

19. The mass of a lead ball is M. It falls down in a viscous liquid with terminal velocity V. The terminal velocity of another lead ball of mass $8M$ in the same liquid will be

(a) $64\,V$ (b) $4\,V$
(c) $8\,V$ (d) V

20. A small spherical solid ball is falling down in a viscous liquid. Its velocity in the viscous liquid is best represented by the curve

(a) D (b) C
(c) B (d) A

21. The flow of water in a horizontal pipe is streamline. At a point in the tube where its cross-sectional area is 10 cm^2, the velocity of water and pressure are 1 m/s and 2×10^2 pascal, respectively. The pressure of water at a different point, where the cross-sectional area is 5 cm^2, will be

(a) 2000 Pascal. (b) 1000 Pascal
(c) 250 Pascal. (d) 500 Pascal.

22. The Bernoulli's theorem is based on

(a) conservation of energy.
(b) conservation of momentum.
(c) conservation of mass.
(d) conservation of charge.

23. An incompressible liquid is continuously flowing through a cylindrical pipe whose radius is $2\,R$ at point A. The radius at point B, in the direction of flow, is R. If the velocity of liquid at point A is V then its velocity at point B will be

(a) V (b) $4\,V$
(c) $2\,V$ (d) $\frac{V}{2}$

24. A water tank is filled with water upto a height H. A hole is made in the tank wall at a depth D from the surface of water. The distance X from the lower end of wall where the water stream from tank strikes the ground is

(a) $2\sqrt{gD}$ (b) $2\sqrt{D(H+D)}$
(c) $2\sqrt{D(H-D)}$ (d) $\sqrt{D}$

25. The rate of flow of a liquid through a capillary tube under and constant pressure head is Q. If the diameter of the tube is reduced to half and its length is doubled, then the new rate of flow of liquid will be

(a) $\frac{Q}{4}$ (b) $\frac{Q}{8}$
(c) $16Q$ (d) $\frac{Q}{32}$

26. Three vertical tubes X, Y and Z are connected to a horizontal pipe as shown in figure 11.50. At the points of joint, the radii of the pipe are 2 cm, 1 cm and 2 cm respectively. Water is flowing in this pipe. The water level in the vertical tubes will be

(a) upto same height in X and Z.
(b) upto same height in all the three.
(c) upto same height in X and Y.
(d) upto maximum height in X.

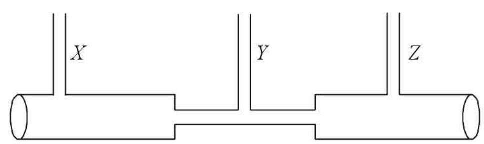


Fig. 11.50

27. A small sphere of mass M and density is dropped in a vessel filled with glycerine. If the density of glycerine is D_2 then the viscous force acting on the ball will he in Newton.

(a) MD_1D_2 (b) $Mg\left[1-\frac{D_2}{D_1}\right]$
(c) $\frac{MD_1g}{D_2}$ (d) $\frac{M}{g}(D_1+D_2)$

28. Water is flowing with a velocity of 2 m/s in a horizontal pipe with cross-sectional area decreasing from 2×10^{-2} m^2 to 0.01 m^2 at pressure 4×10^4 pascal. The pressure at smaller cross-section in pascal will be

(a) 32 (b) 3.4
(c) 3.4×10^4 (d) 3.4×10^5

29. A pitot's tube is attached at one of the wings of an aeroplane in order to determine its velocity with respect to air. If the difference of two liquid levels in manometer is 10cm and density of liquid is 0.8 gm/cm^3, then the velocity of plane with respect to air will be (given density of air = 1.293×10^{-3} gm/cm^3).

(a) 34.82 cm/s (b) 3.48 cm/s
(c) 348.2 cm/s (d) 3482 cm/s

30. The coefficient of viscosity of a liquid does not depend upon

(a) the density of liquid. (b) temperature of liquid.
(c) pressure of liquid. (d) nature of liquid.

31. When the velocity of a liquid is greater than its critical velocity then the flow of liquid will be

(a) streamline. (b) turbulent.
(c) sometimes streamline and sometimes turbulent.
(d) none of the above.

32. The cause of viscosity in gases is

(a) cohesive force. (b) adhesive force.
(b) diffusion. (d) conductivity.

33. The cause of viscosity of liquids is

(a) diffusion. (b) adhesive force.
(c) gravitational force. (d) cohesive force.

34. The dimensions of velocity gradient are

(a) T^{-1} (b) T
(c) T_2 (d) T°

35. The correct formula of critical velocity (V_c) is

(a) $V_c = \frac{k\eta d}{r}$ (b) $V_c = \frac{k\eta}{dr}$
(c) $V_c = \frac{dr}{K\eta}$ (d) $V_c = \frac{r\eta}{dk}$

36. The viscous force acting on a body falling under gravity in a viscous fluid will be

(a) $\frac{6\pi V}{\eta r}$ (b) $\frac{6\pi\eta r}{V}$
(c) $6\pi\eta r V$ (d) $\frac{rV}{6\pi\eta}$

37. The equation of continuity is

(a) aV^{-1} = constant. (b) a^2V = constant.
(c) $\frac{V}{a}$ = constant. (d) aV = constant.

38. The viscosity of an ideal fluid is

(a) zero. (b) infinity.
(c) one. (d) 0.5.

39. The relative velocity of two consecutive layers is 8 cm/s. If the perpendicular distance between the layers is 0.1 cm, then the velocity gradient will be

(a) 8 sec^{-1} (b) 80 sec^{-1}
(c) 0.8 sec^{-1} (d) 0.08 sec^{-1}

40. One end of a horizontal pipe is closed with the help of a valve and the reading of a barometer attached to the pipe is 3×10^5 pascal. When the value in the pipe is opened then the reading of barometer falls to 10^5 pascal. The velocity of water flowing through the pipe will be in m/s

(a) 0.2 (b) 2
(c) 20 (d) 200

41. There is a small hole of diameter 2 mm in the wall of a water tank at a depth of 10 m below free water surface. The velocity of efflux of water from the hole will be

(a) 0.14 m/s (b) 1.4 m/s
(c) 0.014 m/s (d) 14 m/s

42. In the above problem the rate of flow of water in m^3/s will be

(a) 4.4×10^{-5} (b) 4.4×10^{-4}
(c) 4.4×10^{-3} (d) 4.4×10^{-2}

43. The diameter of ball y is double that of x. The ratio of their terminal velocities inside water will be

(a) 1 : 4 (b) 4 : 1
(c) 1 : 2 (d) 2 : 1

44. The terminal velocity (V_c) of a small sphere falling under gravity in a viscous liquid is related to its radius as

(a) $V_c \propto r$ (b) $\frac{V_c\alpha_1}{r}$
(c) $V_c \propto r^2$ (d) $V_c \propto r^3$

45. One poise is equivalent to

(a) 0.001 pascal second (b) 0.0001 pascal second
(c) 0.01 pascal second (d) 0.1 pascal second

46. The velocity of kerosene oil in a horizontal pipe is 5 m/s. If g = 10 m/s^2 then the velocity head of oil will be

(a) 1.25 m (b) 12.5 m
(c) 0.125 m (d) 125 m

47. A layer of glycerine of thickness 1 mm is enclosed between a big plate and another plane of area 10^2 m^2. If the coefficient of viscosity of glycerine is 1 kg/m/s, then the force in Newton required to move the plate with a velocity of 0.07 m/s will be

(a) 7 (b) 0.7
(c) 70 (d) 0.07

48. The velocity of a liquid coming out of a hole in the tank wall is

(a) more if the hole is near the upper end.
(b) more if the hole is at the centre of the wall.
(c) more if the hole is near the bottom.
(d) velocity of flow does not depend upon the position of hole.

49. The formula for the resistance of a fluid is

(a) $R = \frac{\pi r^4}{8\eta l}$ (b) $R = \frac{8\eta l}{\pi r^2}$
(c) $R = \frac{8\eta l}{\pi r^3}$ (d) $R = \frac{8\eta l}{\pi r^4}$

50. A plate of area 100 cm^2 is lying on the upper surface of a 2 mm thick oil film. If the coefficient of viscosity of oil is 15.5 poise then the horizontal force required to move the plate with a velocity of 3 cm/s will be

(a) 0.2325 Newton (b) 2.325 Newton
(c) 23.25 Newton (d) 232.5 Newton

51. Out of the following the maximum viscosity is of

(a) oxygen. (b) glycerine.
(c) mercury. (d) water.

52. The Magnus effect is equivalent to

(a) electric field. (b) magnetic field.
(c) Bernouli's theorem.
(d) magnetic effect of current.

53. The cause of floating of clouds in atmosphere is—

(a) low viscosity. (b) low temperature.
(c) low pressure. (d) low density.

54. The velocity efflux of a liquid is

(a) $\sqrt{2}\,gh$ (b) $2\,gh$
(c) gh (d) $\sqrt{g}\,h$

55. For which type of liquid is the value of Reynold's number low?

(a) Low density (b) High viscosity
(c) Low velocity (d) All of the above

56. When a solid ball of volume V is dropped into a viscous liquid, then a viscous force F acts on it. If another ball of volume 2 V of the same material is dropped in the same liquid then the viscous force experienced by it will be

(a) $2\,nF$ (b) $\frac{nF}{2}$
(c) $2\,F$ (d) $\frac{F}{2}$

57. According to Beynouli's equation the expression which remains constant is

(a) $P+\frac{\rho v^2}{2}$ (b) $P+\frac{\rho v^2}{2}-\rho gh$
(c) $P+\rho gh$ (d) $P+\rho gh+\frac{\rho v^2}{2}$

58. Water is flowing with a velocity of 3 m/s in a pipe of diameter 4 cm. This water enters another tube of diameter 2 cm. The velocity of water in this tube is

(a) 12 m/s (b) 6 m/s
(c) 3 m/s (d) 1.5 m/s

59. The height of water level in a tank is H. The range of water stream coming out of a hole at depth $H/4$ from upper water level will be.

(a) $\frac{\sqrt{3}H}{2}$ (b) $\frac{2H}{\sqrt{3}}$
(c) $\frac{H}{\sqrt{3}}$ (d) $\sqrt{3}H$

60. A wooden block of mass 8 kg is tied to a string attached to the bottom of a tank. The block is completely inside the water. Relative density of wood is 0.8. Taking g = 10 m/s^2, what is the tension in the string?

(a) 100 N (b) 80 N
(c) 50 N (d) 20 N

61. An air bubble of radius r rises from the bottom of a tube of depth H. When it reaches the surface, its radius becomes $3r$. What is the atmospheric pressure in terms of the height of water column?

(a) $\frac{H}{2}$ (b) $\frac{H}{4}$
(c) $\frac{H}{8}$ (d) $\frac{H}{16}$

62. A balloon of mass $1g$ has $100g$ of water in it. If it is completely immersed in water, its mass will be:

(a) $0.5g$ (b) $1.0g$
(c) $2.0g$ (d) $101g$

63. A wooden ball of density p is immersed in a liquid of density σ to a depth H and then released. The height h above the surface to which the ball rises will be:

(a) $h=H$ (b) $h=\frac{\sigma}{\rho}H$
(c) $h=\frac{\sigma-\rho}{\rho}H$ (d) $h=\frac{\sigma}{\rho}H$

64. A sphere of solid material of relative density 9 has a concentric spherical cavity and just floats in water. If the radius of the sphere be R, then the radius of the cavity (r) will be related to R as:

(a) $r^3=\frac{8}{9}R^3$ (b) $r^3=\frac{2}{3}R^3$
(c) $r^3=\frac{\sqrt{8}}{3}R^3$ (d) $r^3=\sqrt{\frac{2}{3}}R^3$

65. Given that for a gas B_i = isothermal bulk modulus, B_a = adiabatic bulk modulus, p = pressure and y = ratio of the specific heat at constant pressure to that at constant volume. Which of the following relations is correct ?

(a) $B_a = p$ (b) $B_i = p$
(c) $B_i = \gamma B_a$ (d) None of the above.

66. A bar is subjected to equal and opposite forces as shown in the figure. PQ is a plane making angle θ with the cross–section of the bar. If the area of cross–section be 'a', then what is the tensile stress on PQ ?

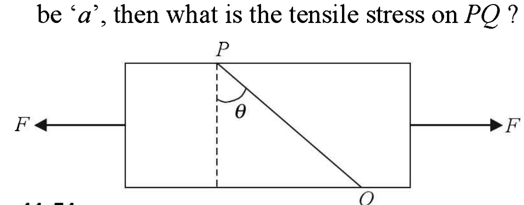


Fig. 11.51

(a) $\frac{F}{a}$ (b) $\frac{F\cos\theta}{a}$
(c) $\frac{F\cos^2\theta}{a}$ (d) $\frac{a}{F\cos\theta}$

Answers to Questions for Practice

1.	(c)	**2.**	(d)	**3.**	(b)	**4.**	(b)	**5.**	(d)	**6.**	(d)	**7.**	(a)
8.	(c)	**9.**	(c)	**10.**	(b)	**11.**	(d)	**12.**	(b)	**13.**	(d)	**14.**	(a)
15.	(b)	**16.**	(c)	**17.**	(d)	**18.**	(d)	**19.**	(b)	**20.**	(b)	**21.**	(d)
22.	(a)	**23.**	(b)	**24.**	(c)	**25.**	(d)	**26.**	(a)	**27.**	(b)	**28.**	(c)
29.	(d)	**30.**	(a)	**31.**	(b)	**32.**	(c)	**33.**	(d)	**34.**	(a)	**35.**	(b)
36.	(c)	**37.**	(d)	**38.**	(a)	**39.**	(b)	**40.**	(c)	**41.**	(d)	**42.**	(a)
43.	(b)	**44.**	(c)	**45.**	(d)	**46.**	(a)	**47.**	(b)	**48.**	(c)	**49.**	(d)
50.	(a)	**51.**	(b)	**52.**	(c)	**53.**	(d)	**54.**	(a)	**55.**	(b)	**56.**	(c)
57.	(d)	**58.**	(a)	**59.**	(a)	**60.**	(d)	**61.**	(c)	**62.**	(b)	**63.**	(c)
64.	(a)	**65.**	(b)	**66.**	(c)								

SELF TEST 1

1. $\sqrt{\dfrac{m}{F/x}}$ has dimensions of

(a) T (b) T^2

(c) LT^{-1} (d) TL^{-1}

(e) none

2. $ML^{-1}T^{-2}$ the dimension of

(a) $\vec{r} \times \vec{F}$ (b) $\frac{1}{2}\varepsilon_0 E^2$

(c) MB (d) $\vec{p} \times \vec{E}$

3. The dimensional formula for Bulk modulus is the same as for

(a) torque (b) $\dfrac{B^2}{2\mu_0}$

(c) coefficient of viscosity (d) Reynolds number

4. Poynting vector is closely related to

(a) energy density (b) intensity

(c) power (d) solar constant

5. A vernier callipers has m division per cm on a main scale and it has in all n divisions on a vernier scale which coincide with p divisions of main scale. Then vernier constant is

(a) $\dfrac{n-p}{mn}$ (b) $\dfrac{p}{mn}$

(c) $\dfrac{n}{mp}$ (d) $\dfrac{n-p}{mp}$

(e) $\dfrac{n-p}{np}$

6. A particle moves according to the equation $t = \alpha x^2 + \beta x + \gamma$ its acceleration is

(a) $-2\alpha V'$ (b) $-2\alpha V^2$

(c) $-2\alpha V^3$ (d) none

7. A vector $\vec{X} = 2\hat{i} - 3\hat{j}$. The vector perpendicular to $\vec{X}$ lying in the same plane is

(a) $3\hat{k}$ (b) $2\hat{i} - \hat{j}$

(c) $3\hat{i} + 2\hat{j}$ (d) $3\hat{i} - 2\hat{j}$

8. Find the resultant of the three forces acting on a body.

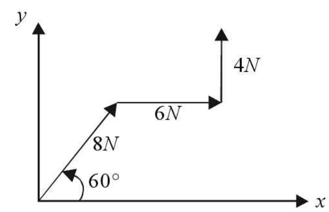


Fig. 1

(a) 14 N (b) 13.3 N

(c) 12.9 N (d) 13.1 N

9. $\vec{A} \cdot (\vec{A} \times \vec{B}) =$

(a) $\vec{A} \times (\vec{A} \cdot \vec{B})$ (b) $\vec{B} \times (\vec{A} \cdot \vec{B})$

(c) $\vec{A} \times \vec{A}$ (d) $\vec{A} \cdot \vec{A} + \vec{A} \cdot \vec{B}$

10. If a ball is thrown to follow a trajectory $y = \dfrac{-x^2}{2} + x$ then find the maximum height it will attain.

(a) 1/2 m (b) 1/4 m

(c) 1/3 m (d) 1/16 m

11. A particle moves according to the equation $x = at^2 + bt$. Find the time after which the particle comes to a rest for the first time.

(a) $\dfrac{-b}{2a}$ (b) $\dfrac{b}{2a}$

(c) $\dfrac{b}{a}$ (d) $\dfrac{-b}{a}$

(e) none

12. A particle moves according to the equation $x = 5t^2 + 2t + 6 = 0$. The average velocity between first 3 seconds and $t = 2$ s to $t = 5$ s is

(a) 13 ms^{-1}, 23 ms^{-1} (b) 13 ms^{-1}, 33 ms^{-1}

(c) 18 ms^{-1}, 33 ms^{-1} (d) 32 ms^{-1}, 52 ms^{-1}

(e) none

13. Find the area bounded by the curve $y = e^{-x}$ between x and y axis.

(a) 1 (b) 1.3

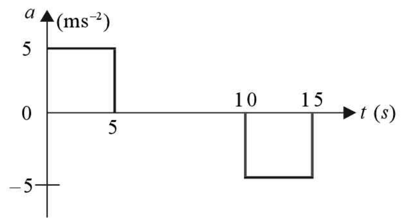
(c) 1.73 (d) 1.43

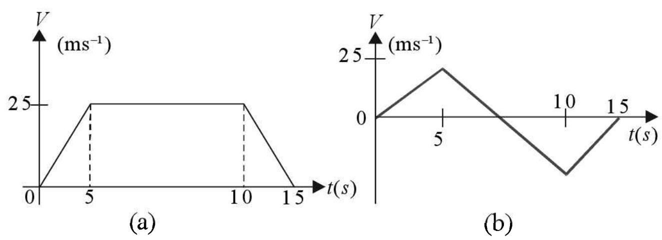
14. A car is moving at a constant speed 60 kmh^{-1} along a straight road which heads towards a large vertical wall and makes a sharp 90° turn by the side of the wall when the car at an instant is 30 km away from the wall, a bird begins to fly with 120 kmh^{-1} between car and wall until the car takes 90° turn. Find the distance moved by the bird.

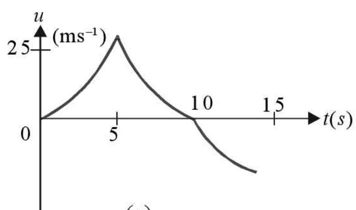
(a) 30 km (b) 50 km

(c) 60 km (d) 45 km

15. For the given $a-t$ graph the $V-t$ graph is







(c) (d) none

Fig. 2

16. A man moves due east with 18 kmh^{-1} and turns to north in 5 s and continues to move with the same speed. Find average acceleration.

(a) 2 ms^{-2} *NW* (b) $\sqrt{2}$ ms^{-2} *NW*

(c) 0 (d) $\sqrt{2}$ ms^{-2} *NE*

17. A river is 500 m wide. It is flowing 3 kmh^{-1}. A man capable of swimming 6 kmh^{-1} in still water is standing at the North coast and is to reach just the opposite in the south coast. The direction in which he should swim is

(a) 120° with the flow of river
(b) 120° opposite to the flow of river
(c) along south
(d) none of these

18. A ball is thrown with a velocity $6\hat{i} + 8\hat{j}$. How long will it remain in air? Take $g = 10\ ms^{-2}$.

(a) 0.8 *s* (b) 2.0 *s*

(c) 1.6 *s* (d) 1.8 *s*

(e) none

19. A tap is 10 m high and water drops are falling at regular intervals. When the 5th drop is about to leave the tap first drop touches the ground. Find the separation between 2nd and 3rd drop.

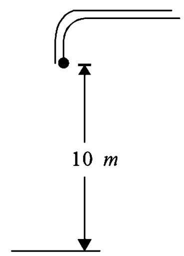


Fig. 3

(a) 3.125 m (b) 4.251 m

(c) 2.725 m (d) 3.725 m

20. A boy can throw a ball to a maximum horizontal distance 50 m. How high can he throw?

(a) 50 m (b) 35 m

(c) 25 m (d) 40 m

(e) none

21. Six particles are situated at the corners of a regular hexagon of side *l*. They move at a constant speed V. Each particle maintains a direction towards the particle at the next corner. The time the particles will take to meet each other is

(a) $\frac{l}{V}$ (b) $\frac{2l}{3V}$

(c) $\frac{2l}{V}$ (d) $\frac{l}{2V}$

(e) none

22. A person standing on a truck moving with a uniform velocity 14.7 ms^{-1} on a horizontal road throws a ball in such a way that it returns to him after 4 s. Find the speed and angle of projection as seen by a man on the road.

(a) 19.6 ms^{-1}, vertical
(b) 24.5 ms^{-1}, vertical
(c) 19.6 ms^{-1}, 53° with the road
(d) 24.5 ms^{-1}, 53° with the road

23. A gun is fired with a speed 100 ms^{-1} horizontally so that it misses the target 10 m away by...cm.

(a) 5 cm (b) 7.25 cm

(c) 6.25 cm (d) 8.125 cm

(e) none

24. An elevator is descending with uniform acceleration. To measure acceleration, a boy drops a coin from 6ft above the base of elevator. The coin strikes the floor after 1 s. The magnitude of acceleration is

(a) 12 fts^{-1} (b) 18 fts^{-1}

(c) 20 fts^{-1} (d) none

25. A bullet travelling with 16 ms^{-1} penetrates a tree trunk and comes to rest in 0.4 m. The time taken during retardation is

(a) 0.1 s (b) 0.05 s
(c) 0.25 s (d) 0.025 s
(e) none

26. A and B are standing in such a way that wind blows in a direction perpendicular to the line joining A and B. A beats the drum and B hears it. What is the time in which sound reaches to B? Assume $AB = x$ velocity of sound V and velocity of wind V_ω.

(a) $\dfrac{x}{\sqrt{V^2 - V_\omega^2}}$ (b) $\dfrac{x}{\sqrt{V^2 + V_\omega^2}}$

(c) $\dfrac{x}{V + V_\omega}$ (d) $\dfrac{x}{V - V_\omega}$

(e) none

27. A ball is thrown up with a velocity u. The $u - t$ graph representing the motion is

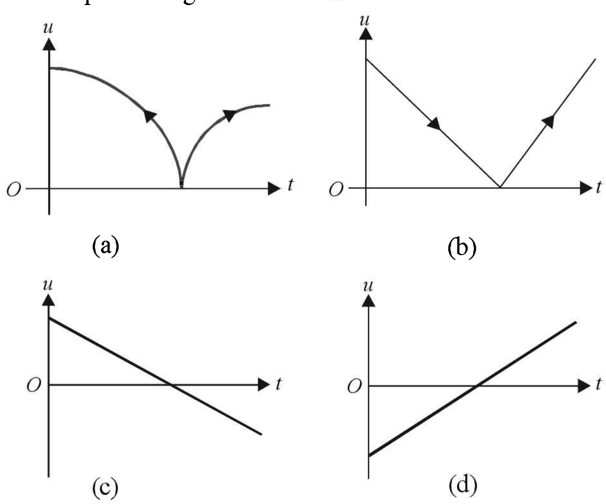


Fig. 4

28. A projectile is fired with a velocity u making an angle 30° so that it covers a maximum distance x_1. In order it covers the same horizontal distance in different time with same velocity it should be projected at

(a) 45° (b) 60°
(c) 30° (d) 75°

29. Action and reaction

(a) act on two different objects
(b) have equal magnitude
(c) have opposite direction
(d) have zero resultant
(e) all (a), (b), (c) and (d)

30. The weight of a 120 kg body in a geostationary satellite is

(a) 17 N (b) 27 N
(c) 37 N (d) 57 N

31. A rope tied between two poles sags due to its own weight. The tension at the lowest point is

Fig. 5

(a) $\dfrac{Mg}{2\sin\theta}$ (b) $\dfrac{Mg}{2\tan\theta}$

(c) $\dfrac{Mg}{\tan\theta}$ (d) $\dfrac{Mg}{2\cos\theta}$

32. The acceleration of 3 kg block in the figure shown is ___. Given coefficient of friction is 0.2.

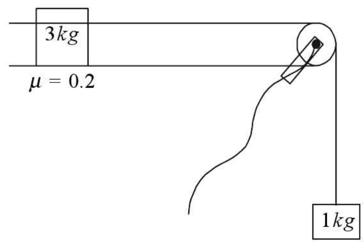


Fig. 6

(a) 1 ms^{-2} (b) 1.3 ms^{-1}
(c) 2.1 ms^{-2} (d) 1.7 ms^{-2}

33. A block of 2 kg is placed on a 37° wedge. The force required to move up with a uniform speed of 5 ms^{-1} is____. Given coefficient of friction between two surfaces is 0.2

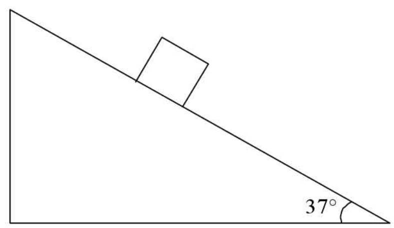


Fig. 7

(a) 11.6 N (b) 12.4 N
(c) 13.8 N (d) 15.2 N

34. Find the minimum accleration that must be given to the wedge so that block of mass m kept on the wedge falls vertical.

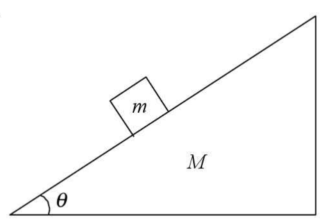


Fig. 8

(a) $g\tan\theta$ (b) $g\cos\theta$
(c) $g\cot\theta$ (d) $g\sin\theta$

35. Find the acceleration of 2 kg and 3 kg block when a force of 50 N is applied (see figure)

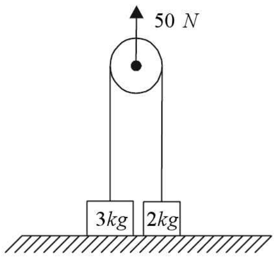


Fig. 9

(a) 0, 0 (b) 2.5 ms^{-2}, 2.5 ms^{-2}

(c) 2.5 ms^{-2}, 0 (d) 0, 2.5 ms^{-2}

36. The relation between acceleration of M_1 and M_2 is

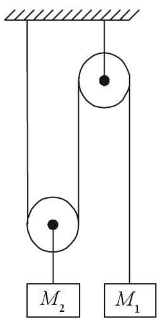


Fig. 10

(a) 1 (b) –1

(c) 2 (d) –2

37. A jeep is moving with an acceleration a. A pendulum is hanging from the ceiling. Find the angle made by the pendulum with the vertical.

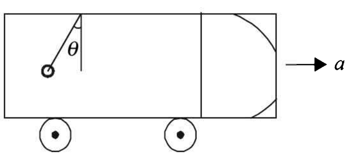


Fig. 11

(a) 0 (b) $\sin^{-1}\dfrac{a}{g}$

(c) $\cos^{-1}\dfrac{a}{g}$ (d) $\tan^{-1}\dfrac{a}{g}$

38. A man sitting in a car pushes the car by a force F. Assume the car is in neutral state. Mass of the car is M. The acceleration produced is

(a) F/M

(b) zero

(c) slightly less than F/M

(d) cannot be determined as friction is not known

39. An elevator is moving up with 5 ms^{-1}. A boy in the elevator throws up a ball with 4 ms^{-1}. The ball returns in the hands of the boy after

(a) 0.8 s (b) 1.8 s

(c) 0.9 s (d) $0.8 < t < 0.9\ s$

40. Two block of mass m_1 and m_2 are connected to a light string passing over a smooth light pulley as shown in the figure. If the system remains in equilibrium then normal reaction exerted by incline on the body is

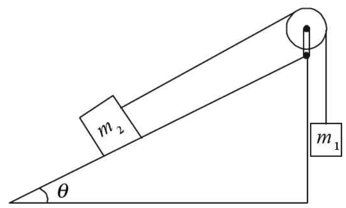


Fig. 12

(a) $m_1 g\sqrt{1-\left(\dfrac{m_1}{m_2}\right)^2}$ (b) $m_2 g\sqrt{1-\left(\dfrac{m_1}{m_2}\right)^2}$

(c) $m_1 g\sqrt{1-\left(\dfrac{m_2}{m_1}\right)^2}$ (d) $m_2 g\sqrt{1-\left(\dfrac{m_2}{m_1}\right)^2}$

41. Assume each surface is smooth, pulley light and smooth, and string massless. In the given figure, acceleration of the pulley is

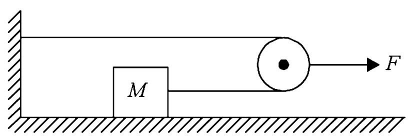


Fig. 13

(a) $\dfrac{F}{2M}$ (b) $\dfrac{F}{M}$

(c) $\dfrac{F}{4M}$ (d) ∞

(e) zero

42. The block of mass M is displaced by x and released. The maximum acceleration could be

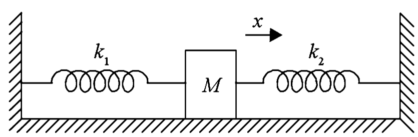


Fig. 14

(a) $\dfrac{(k_1+k_2)x}{M}$ (b) $\dfrac{(k_1-k_2)x}{M}$

(c) $\dfrac{2(k_1+k_2)x}{M}$ (d) $\dfrac{2(k_1-k_2)x}{M}$

43. A light string of length l is passing over a smooth pulley. One end of the string is attached to a block of mass M and on the other end is a monkey of same mass M. The monkey starts climbing up the pulley. The time in which monkey reaches the top is

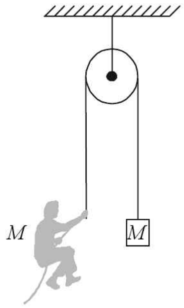


Fig. 15

(a) l/V' (b) $2\,l/V'$
(c) $l/2\,V'$ (d) $l/3\,V'$

44. In a simple machine two unequal masses 300 *g* and 600 *g* are hanging through a string. The system is released from rest. The larger mass is stopped after 2.0 s the system is set into motion. Find the time elapsed before the string gets tight again.

(a) 1/3 s (b) 2/3 s
(c) 1 s (d) none

45. The reading of the spring balance in the figure is ____. Assume elevator moving up with an acceleration $g/5$

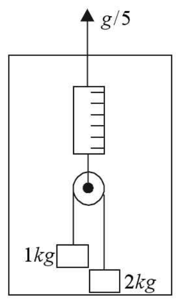


Fig. 16

(a) 3.2 kg wt (b) 32 kg wt
(c) 1.6 kg wt (d) 16 kg wt
(e) none

46. The horizontal surface below the bigger block is smooth. The coefficient of friction between the blocks is μ. The maximum force which can be applied to keep the smaller blocks at rest with respect to the bigger block is

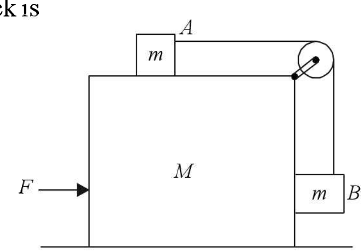


Fig. 17

(a) $\left(\frac{1-\mu}{1+\mu}\right)(M+2m)\,g$ (b) $\left(\frac{1+\mu}{1-\mu}\right)(M+m)\,g$

(c) $\left(\frac{1+\mu}{1-\mu}\right)(M+2\,m)\,g$ (d) $(1+\mu)(M+2\,m)\,g$

47. The friction coefficient between the board and the floor in the figure is μ. The maximum force that the man can exert on the rope so that the board does not slip on the floor is

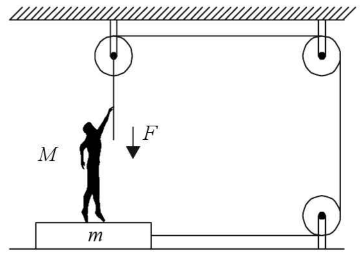


Fig. 18

(a) $\mu(M+m)\,g$ (b) $\frac{\mu(M+m)g}{1-\mu}$

(c) $\mu(M-m)\,g$ (d) $\frac{\mu(M+m)g}{1+\mu}$

48. If $\mu_s = 0.8$, then acceleration of the block down the incline is

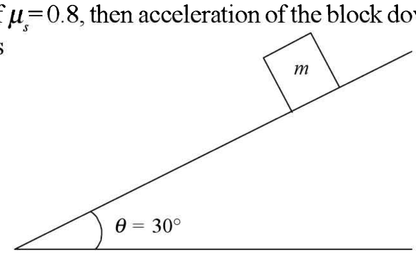


Fig. 19

(a) finite but not zero (b) zero
(c) negative (d) none

49. Identical cars *A* and *B* are connected through a light string as shown in the figure passing over a smooth pulley. The graph which best represents the acceleration of the cars is

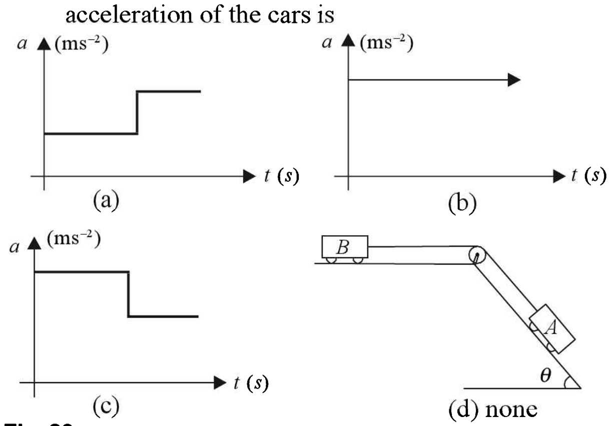


Fig. 20

50. A truck of mass *M* is moving *a* with an acceleration *a*. The maximum value of *a* so that block **m** does not fall is ____ . Given coefficient of friction is μ .

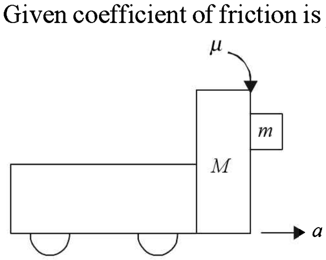


Fig. 21

(a) g/μ (b) $\mu\,g$
(c) $(\mu+1)\,g$ (d) $(1-\mu)\,g$

Answers

1.	(a)	**2.**	(b)	**3.**	(b)	**4.**	(b)	**5.**	(a)	**6.**	(c)	**7.**	(c)
8.	(b)	**9.**	(c)	**10.**	(a)	**11.**	(a)	**12.**	(b)	**13.**	(a)	**14.**	(c)
15.	(a)	**16.**	(b)	**17.**	(a)	**18.**	(c)	**19.**	(a)	**20.**	(c)	**21.**	(c)
22.	(d)	**23.**	(a)	**24.**	(c)	**25.**	(b)	**26.**	(a)	**27.**	(c)	**28.**	(b)
29.	(e)	**30.**	(b)	**31.**	(b)	**32.**	(a)	**33.**	(d)	**34.**	(c)	**35.**	(c)
36.	(d)	**37.**	(d)	**38.**	(b)	**39.**	(a)	**40.**	(b)	**41.**	(c)	**42.**	(a)
43.	(a)	**44.**	(b)	**45.**	(a)	**46.**	(c)	**47.**	(d)	**48.**	(b)	**49.**	(a)
50.	(a)												

Explanations

1(a) $\sqrt{\dfrac{m}{F/x}} = \sqrt{\dfrac{M}{\dfrac{MLT^{-2}}{L}}} = T$

2(b) $ML^{-1}T^{-2}$ represent energy density.

3(b) Bulk modulus, pressure and energy density have same dimensional formula

4(b) Intensity $= \dfrac{\text{Power}}{\text{Area}} = MT^{-3} = \dfrac{1}{\mu_0}\ (E \times B)$

5(a) $Vc = \left(1 - \dfrac{p}{n}\right) \times \dfrac{1}{m} = \dfrac{n-p}{mn}$

6(c) $\dfrac{dt}{dx} = 2\alpha x + \beta$

or $V = \dfrac{dt}{dx} = \dfrac{1}{2\alpha x + \beta}$; $\dfrac{dV}{dt} \cdot \dfrac{dx}{dt}$

$= \dfrac{-2\alpha}{(2\alpha x + \beta)^3} = -2\alpha V^3$

7(c) Two vectors are $\perp$ if $A.\,B = 0$

3 $\hat{k}$ does not lie in the same plane

8(b) $(8\sin 60 + 4)\ \hat{j} + (8\cos 60 + 6)\ \hat{i}$

$= 10\hat{i} + 4\left(1+\sqrt{3}\right)\hat{j}$

$= \sqrt{10^2 + 16(1+\sqrt{3})^2} = 13.3\,N$

9(c) $\vec{A}\cdot(\vec{A} \times \vec{B}) = 0 = \vec{A} \times \vec{A}$

10(a) $\dfrac{dy}{dx} = 1 - x = 0$ or $x = 1$

and $y = -1/2 + 1 = 1/2$ m

11(a) $V = \dfrac{dx}{dt} = 2\text{ at} + \text{b} = 0$ or $\text{t} = \dfrac{-b}{2a}$

12(b) $V_{av} = \dfrac{x(5) - x(0)}{3-0} = \dfrac{5(3)^2 - 2(3) + 6 - 6}{3}$

$= 13\ \text{ms}^{-1}$

$V_{as}\ (2-5\,s) = \dfrac{V(5) - V(2)}{5-2}$

$= \dfrac{5(5)^2 - 2(5) - 5(2)^2 + 2(2)}{3} = 33\ \text{ms}^{-1}$

13(a) $A = \int_0^\infty y\ dx = \int_0^\infty e^{-x}\ dx = -e^{-x}\Big|_0^\infty = 1$

14(c) time taken by car to reach the wall $= \dfrac{30}{60}$ 1/2 *hr*

distance covered by bird $= \dfrac{1}{2} \times 120 = 60\ km$

15(a)

16(b) $\dfrac{5\hat{j} - 5\hat{i}}{5}$ a_{av} or $a_{aV} = \sqrt{2}\ \text{ms}^{-2}\ NW$

17(a) $6\sin\theta = 3$ or $\theta = 30°$ ie. 120° with the direction of flow

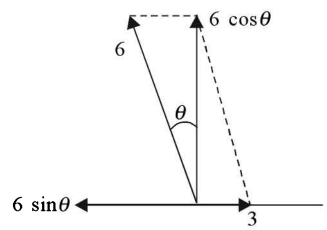


Fig. 22

18(c) $T = \dfrac{2V_y}{g} = \dfrac{2(8)}{10} = 1.6\,s$

19(a) $\dfrac{1}{2}gt^2 = 10$ or $t = \sqrt{2}\ s$;

$T = \dfrac{t}{4} = \dfrac{\sqrt{2}}{4}$; $\Delta y = \dfrac{1}{2}g\left[\left(\dfrac{3\sqrt{2}}{4}\right)^2 - \left(\dfrac{2\sqrt{2}}{4}\right)^2\right]$

$\Delta y = 5\left(\dfrac{9}{8} - \dfrac{1}{2}\right) = \dfrac{25}{8} = 3.125\,m$

20(c) height $= \dfrac{1}{2}$ maximum range.

21(c) $t = \frac{d}{V_{AB}}$ $t = \frac{l}{V - V\cos 60} = \frac{l}{V/2} = \frac{2l}{V}$

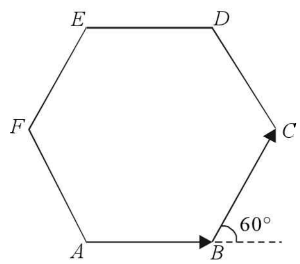


Fig. 23

22(d) $U_x = 14.7\ \text{ms}^{-1}; U_y = 9.8 \times 2 = 19.6\ \text{ms}^{-1}$

$u = \sqrt{19.6^2 + 14.7^2} = 24.5\ \text{ms}^{-1}$

$\tan\theta = \frac{19.6}{14.7} = \frac{4}{3}$ or $\theta = 53°$

23(a) $t = \frac{x}{V} = \frac{10}{100} = 0.1 s$ and $y = \frac{1}{2} g(t)^2 = 5(.1)^2$ $= 5\ cm$

24(c) $\frac{1}{2}(g-a)t^2 = 6$ or $\frac{1}{2}(32-a)(1)^2 = 6$ or $a = 20\ fts^{-2}$

25(b) $a = \frac{v^2}{2s} = \frac{16 \times 16}{2 \times .4} = 320\ \text{ms}^{-2}$

$t = \frac{v}{a} = \frac{16}{320} = \frac{1}{20} = 0.05\ s$

26(a) $t = \frac{x}{\sqrt{V^2 - V_\omega^2}}$

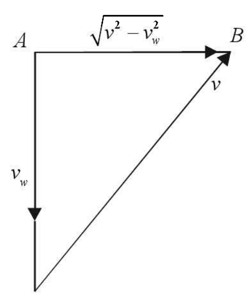


Fig. 24

27(c)

28(b) at complementary angles.

29(e)

30(b) $mg' = \frac{120 \times g}{\left(1 + \frac{42400}{6400}\right)^2} = 27\ N$

31(b) $2T\sin\theta = Mg$ and $T\cos\theta = \frac{Mg}{2\tan\theta}$ is tension at the bottom

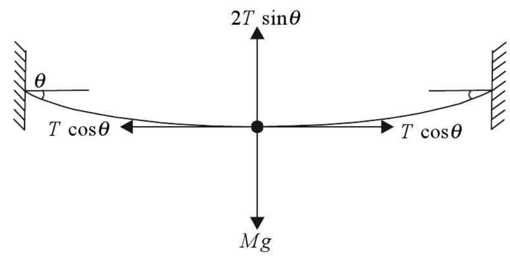


Fig. 25

32(a) $1g - T = 1a$ ...(1)

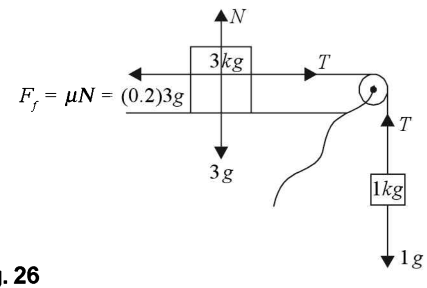


Fig. 26

$T - (.2)(3g) = 3a$ ...(2)

Adding (1) and (2) $a = 1\ \text{ms}^{-2}$

33(d) $N = 2g\cos 37$ and $F_{ap} = 2g[.6 + (.2).8] = 15.2$

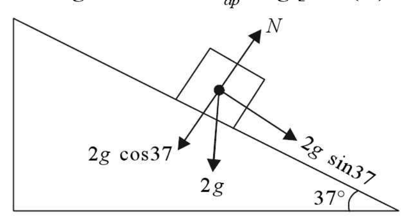


Fig. 27

34(c) For the block to fall vertical $ma\sin\theta = mg\cos\theta$

or $a = g\cot\theta$

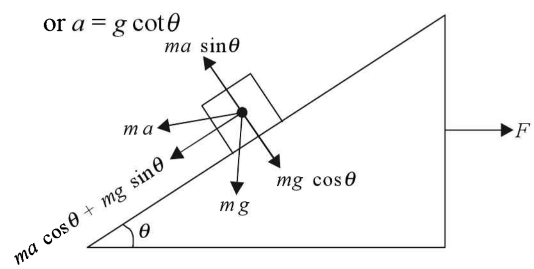


Fig. 28

35(c) $T - Mg = Ma$

$25 - 20 = 2a$

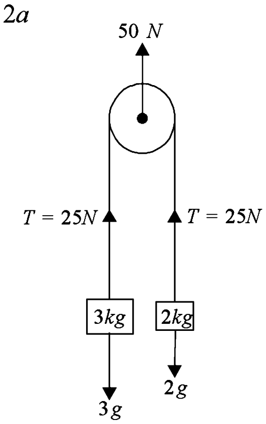


Fig. 29

or $a = 2.5\,\text{ms}^{-2}$

acceleration of 3 kg block is 0 as T is less then $3g$

36(d) $a_1 = -2a_2$ as M_1 moves downward while M_2 moves upward.

37(d) $\tan\theta = \dfrac{ma}{mg} = \dfrac{a}{g}$ or $\tan^{-1}(a/g)$

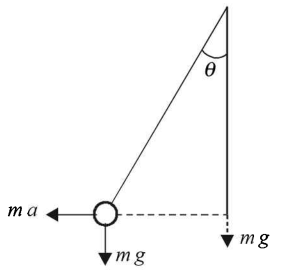


Fig. 30

38(b) as the force is internal car will not move

39(a) $y = ut + \dfrac{1}{2}gt^2 = 0$ or $t = \dfrac{2u}{g} = \dfrac{2\times 4}{10} = 0.8s$

40(b) $m_1 g = T = m_2 g\sin\theta$

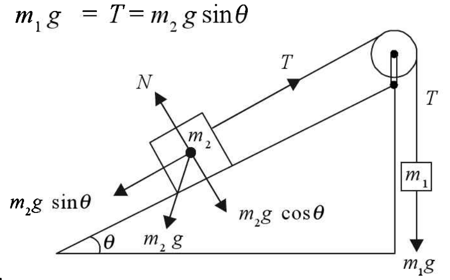


Fig. 31

or $\sin\theta = \dfrac{m_1}{m_2}$; $N = m_2\, g\cos\theta$

$$= m_2 g\sqrt{1-\left(\frac{m_1}{m_2}\right)^2}$$

41(c) $2T = F$ or $T = F/2$

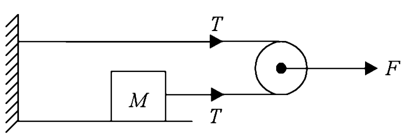


Fig. 32

$$T = Ma_{\text{Black}} \text{ or } a_{\text{black}} = \frac{F}{2M}$$

$$a_{\text{pulley}} = \frac{a_{\text{black}}}{2} = F/(4m)$$

42(a) $F_{max} = (k_1 + k_2)\,x = Ma_{max}$

or $a_{max} = \dfrac{(k_1+k_2)x}{M}$

43(a) equal masses remain at equal height.

44(b) $a = \dfrac{.6g-.3g}{06+.3} = \dfrac{g}{3}$; $V = at = \dfrac{2g}{3}$ $(= gt^1)$

$\therefore\ t' = \dfrac{2}{3}s$

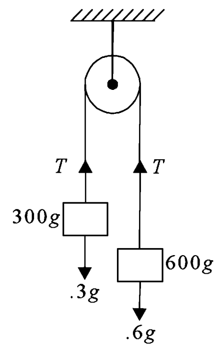


Fig. 33

45(a) $T = \dfrac{2m_1m_2}{m_1+m_2}(g+a)$ and reading of spring balance $= 2T$

$$= 2\,T = \frac{4m_1m_2}{m_1+m_2}(g+a)\ \frac{4\times2\times1}{3}\left(\frac{6g}{5}\right)$$

$= 3.2$ kgwt

46(c) $T + \mu mg = \text{ma}$

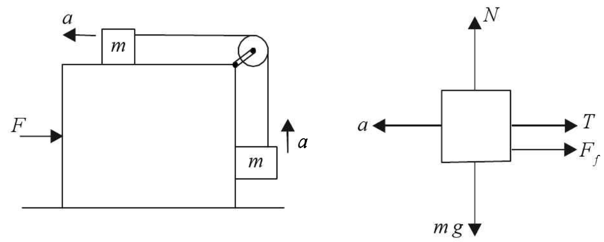


Fig. 34

for vertical equilibrium $T = \mu ma + ma$

or $F = (2m+M)\dfrac{(1+\mu)}{1-\mu}g$

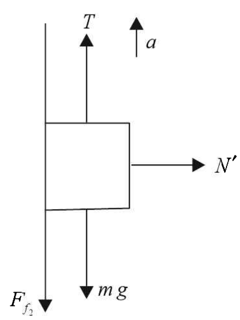


Fig. 35

47(d) $N = (M+m)\,g - T$

$\mu N = T = F$

$T = F = \mu\,[M+m)\,g - T]$

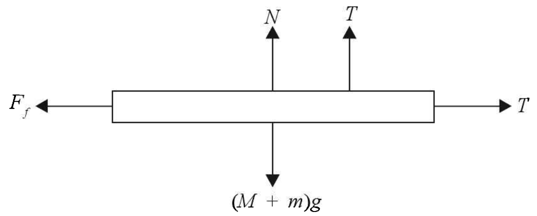


Fig. 36

or $\quad T = F = \dfrac{\mu(M+m)g}{1+\mu}$

48(b)

49(a) when both cars will come on incline $a = g\sin\theta$. Now it is $(g\sin\theta)/2$

50(a) $\mu\, ma = mg$

or $\quad a = \dfrac{g}{\mu}$

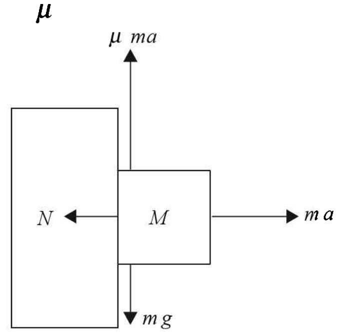


Fig. 37

SELF TEST 2

1. A coin placed on a rotating turntable just slips if it is placed at a distance of 8 cm from the centre. If its speed is doubled, it will just begin to slip at a distance of

(a) 1 cm (b) 2 cm
(c) 4 cm (d) 3.2 cm
(e) none

2. A particle is going along a spiral path with constant speed then

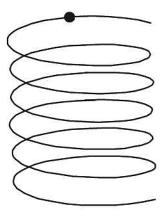


Fig. 1

(a) its velocity is constant
(b) its acceleration is constant
(c) its acceleration is decreasing
(d) the magnitude of its acceleration is constant.

3. A park has radius 10 m. A car goes round it at a speed 18 kmh^{-1}. The angle of banking shall be

(a) $\tan^{-1} 4$ (b) $\tan^{-1} 1/2$
(c) $\tan^{-1} 1/4$ (d) $\tan^{-1} 2$
(e) none

4. A small particle is placed over a sphere of radius R. It leaves the sphere at ____ from top if it starts sliding

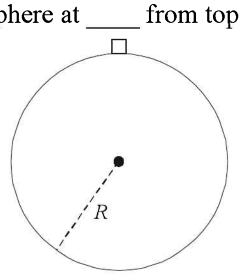


Fig. 2

(a) $\dfrac{R}{3}$ (b) $\dfrac{2R}{3}$
(c) R (d) none

5. The speed of a particle at the top of a vertical circle is $\sqrt{3rg}$. The ratio of tension at the highest to lowest point is

(a) 1 (b) 1/2
(c) 1/3 (d) 1/4
(e) none

6. A spring of natural length l is fixed at one end and a mass M is attached at the other end. It is rotated about a fixed axis with angular velocity ω. Then extension in the spring is ____ k is spring constant.

(a) $\dfrac{Ml\omega^2}{k - M\omega^2}$ (b) $\dfrac{Ml\omega^2}{k + M\omega^2}$
(c) $\dfrac{2Ml\omega^2}{k + M\omega^2}$ (d) $\dfrac{2Ml\omega^2}{k - M\omega^2}$

7. An inclined plane ends into a vertical loop of radius R. A point mass is released from a height $4R$ from the inclined plane. Find the ratio of action at the highest to the lowest point.

(a) 1/2 (b) 1/4
(c) 1/3 (d) 1/5
(e) 2/5

8. A car starts from rest with an acceleration $a = \dfrac{dv}{dt}$ on a circular road of radius R. Find the maximum velocity just before skidding if coefficient of friction between road and car is μ .

(a) $[(a^2 - \mu^2 g^2) R^2]^{1/4}$ (b) $[(\mu^2 g^2 - a^2) R^2]^{1/4}$
(c) $[(\mu^2 g^2 + a^2) R^2]^{1/4}$ (d) none

9. A rod of length l is clamped to axis of rotation. A string of length l is tied at one end and a mass m at the other as shown. Find angular velocity such that the string makes an angle θ with vertical

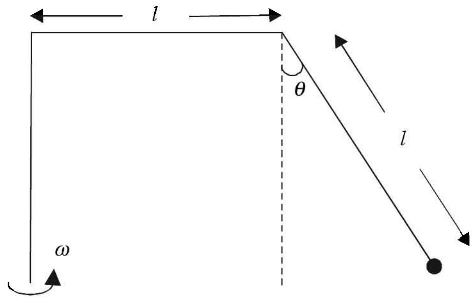


Fig. 3

(a) $\sqrt{\dfrac{g\tan\theta}{l-l\sin\theta}}$ (b) $\sqrt{\dfrac{g\tan\theta}{l+l\sin\theta}}$

(c) $\sqrt{\dfrac{g\tan\theta}{l+l\cos\theta}}$ (d) $\sqrt{\dfrac{g\tan\theta}{l-l\cos\theta}}$

10. A boy of mass m is inside a hollow cylinder rotating with ω as shown. The coefficient of friction is μ. The minimum value of ω that the boy does not fall is

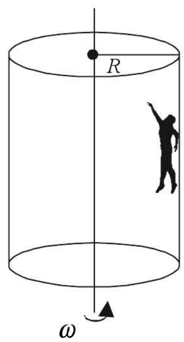


Fig. 4

(a) $\sqrt{g/\mu R}$ (b) $\sqrt{\mu g/R}$

(c) $\sqrt{(\mu+1)g/R}$ (d) $\sqrt{g/(\mu+1)R}$

11. A hemispherical bowl has radius R. It is set rotating about its axis. A block of mass m is placed on the surface of the hemisphere. The radius through the block makes an angle θ with the vertical. The angular speed with which bowl is rotating is ____. Assume no friction

(a) $\sqrt{\dfrac{g}{R\tan\theta}}$ (b) $\sqrt{\dfrac{g}{R\cos\theta}}$

(c) $\sqrt{\dfrac{g}{R\sin\theta}}$ (d) $\sqrt{\dfrac{g}{R\theta}}$

12. A closed pipe has liquid of density ρ filled in it. The pipe rotates with ω about an edge. The area of cross section of pipe is A. Find force exerted by the liquid

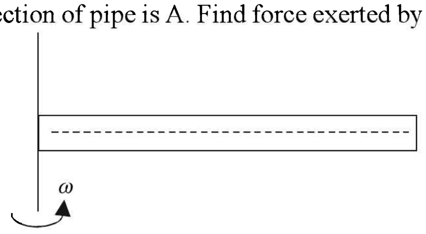


Fig. 5

(a) $A\rho\,\omega^2 l^2$ (b) $A\rho\dfrac{\omega^2 l^2}{2}$

(c) $2A\rho\,\omega^2 l^2$ (d) $\dfrac{A\rho\omega^2 l^2}{3}$

13. A 20 N push is given to a block of 2 kg parallel to incline. The acceleration found is 10 ms^{-2}. If the block startes from rest, find the work done in the last second.

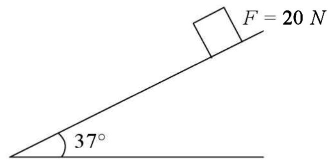


Fig. 6

(a) 400 J (b) 100 J

(c) 300 J (d) 200 J

14. A projectile is fired with 60 ms^{-1} making angle of 30° with upward horizontal from a 40 m height. Speed with which it hits the ground is

(a) 66.6 ms^{-1} (b) 67.6 ms^{-1}

(c) 68.6 ms^{-1} (d) none

15. 20 kg water per sec. is lifted by a pump to a height 80 m. Its efficiency is 75%. The power of the pump in bhp is

(a) 7.62 bhp (b) 16.7 bhp

(c) 2.72 bhp (d) 27.2 bhp

16. The speed of a bullet decreases by 1/20 th of its original speed when it passes through a plank. The maximum number of planks it can pass through is

(a) 10 (b) 11

(c) 9 (d) 20

(e) 19

17. Consider the situation shown in the figure. Block of mass M is displaced by x and released. The speed when block is at $x/2$ is

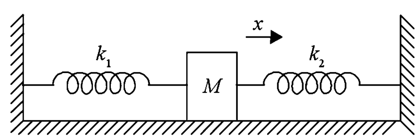


Fig. 7

(a) $\sqrt{\dfrac{3(k_1+k_2)x^2}{M}}$ (b) $\sqrt{\dfrac{3(k_1+k_2)x^2}{2M}}$

(c) $\sqrt{\dfrac{x^2(k_1+k_2)}{M}}$ (d) $\sqrt{\dfrac{3(k_1+k_2)x^2}{4M}}$

18. A rod of length l, mass m is hinged at one end. Two springs of spring constant k_1 and k_2 are connected to it as shown. The rod is displaced by a small angle θ and released. The angular frequency of oscillation is

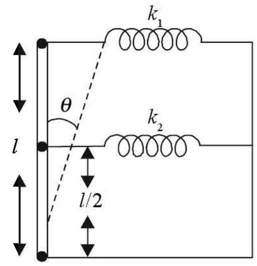


Fig. 8

(a) $\sqrt{\frac{3(k_1+k_2)}{M}}$ (b) $\sqrt{\frac{3\left(k_1+\frac{k_2}{2}\right)}{M}}$

(c) $\sqrt{\frac{3\left(k_1+\frac{k_2}{4}\right)}{M}}$ (d) none

19. A block of mass M moving with velocity v hits another block at rest connected to a spring of spring constant k. The maximum compression of the spring is

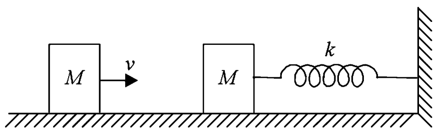


Fig. 9

(a) $\sqrt{\frac{mv^2}{2k}}$ (b) $\sqrt{\frac{mv^2}{k}}$

(c) $\sqrt{\frac{2mv^2}{3k}}$ (d) none

20. A balloon of mass M is at a height. H. A string of length l is hanging. A monkey of mass m at the bottom of the string as shown is trying to reach the balloon. What will be the shift in COM when the monkey reaches balloon?

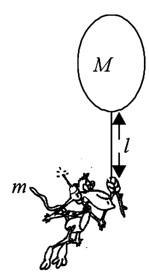


Fig. 10

(a) $\frac{-Ml}{M+m}$ (b) $\frac{Ml}{M+m}$

(c) $\frac{ml}{M+m}$ (d) none

21. A ball falls from a height h on an incline of inclination θ. After the fall the ball rebounds and hits the incline after a distance ____ along the incline.

(a) zero (b) 8 h sinθ

(c) 4 h sinθ (d) 2 h sinθ

22. A rocket has mass 5000 kg including 4000 kg of fuel. The fuel is burning at 10 kg s^{-1} The gas is emerging at a rate 3 km s^{-1}. The velocity of the rocket after 5 min is

(a) 2.76 k ms^{-1} (b) 3 k ms^{-1}

(c) 2.93 k ms^{-1} (d) 3.21k m s^{-1}

23. A bullet of mass m moving with velocity u hits a wooden block of mass M hanging from the ceiling. How high will the block rise if the bullet gets embedded?

(a) $\frac{mu^2}{(M+m)g}$ (b) $\frac{m^2u^2}{(M+m)2g}$

(c) $\frac{2m^2u^2}{(M+m)^2g}$ (d) $\frac{m^2u^2}{2(M+m)^2g}$

24. A door is 6 ft high and 3 ft wide. It weighs 12 kg. The handle is 2.8 ft away from the hinge. What minimum force is needed to close the door?

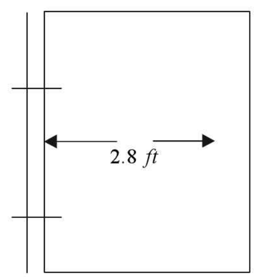


Fig. 11

(a) 60.43 N (b) 56.73 N

(c) 71.23 N (d) 64.3 N

25. Three discs (identical) each of mass m, radius R are placed as shown. The MOI about an axis shown in the figure is

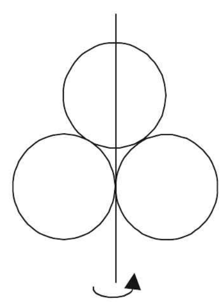


Fig. 12

(a) $\frac{7}{2}MR^2$ (b) $\frac{9}{4}MR^2$

(c) $\frac{10}{4}MR^2$ (d) $\frac{11}{4}MR^2$

26. Three identical rods each of mass M, length l are joined to form an equilateral Δ ABC. Find the MOI about BC as shown.

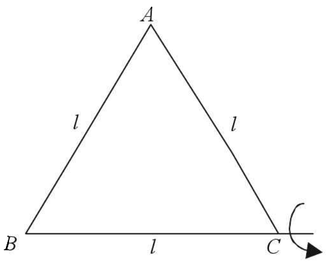


Fig. 13

(a) $\frac{2}{3} Ml^2$ (b) $\frac{Ml^2}{4}$

(c) $\frac{Ml^2}{2}$ (d) none

27. Four spheres each of mass m and radius r are fixed at the vertices of a square (skelton) of side a. It is rotated about side AB. The MOI is

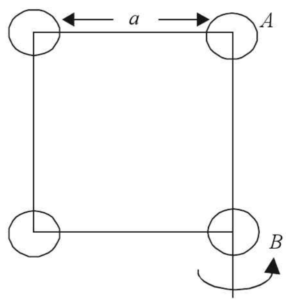


Fig. 14

(a) $\frac{8}{5} mr^2$ (b) $\frac{8}{5} mr^2 + ma^2$

(c) $\frac{8}{5} mr^2 + 2\, ma^2$ (d) $\frac{8}{5} mr^2 + 4\, ma^2$

28. A disc has MOI about its axis. One quarter of it is cut. The MOI of the remaining disc is

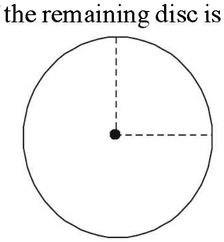


Fig. 15

(a) $\frac{3}{4} I$ (b) $\frac{5}{4} I$

(c) $\frac{3}{4} I < I < \frac{5}{4} I$ (d) none

29. A block at rest explodes to M, M and $3M$. The blocks of mass M each fly off at right angles with velocities 8 ms^{-1} and 6 ms^{-1}. The velocity of $3M$ is

(a) $\frac{10}{3}$ ms^{-1}, 37° with 6 ms^{-1}

(b) $\frac{10}{3}$ ms^{-1}, 53° with 6 ms^{-1}

(c) $\frac{10}{3}$ ms^{-1}, 127° with 6 ms^{-1}

(d) $\frac{10}{3}$ ms^{-1}, 143° with 6 ms^{-1}

30. A dog of 30 kg moves from one end to the other on a halted boat in the stream. The boat is 6 m long and weighs 120 kg. The displacement of the boat is

(a) –1.2 m (b) + 1.2 m

(c) zero (d) 4.8 m

(e) none

31. A body of mass m makes an oblique elastic collision with another body of equal mass at rest. After collision they move at an angle θ with each other. θ is

(a) 0 (b) 45°

(c) 90° (d) 180°

(e) any angle

32. A block of mass m lies at the top of a wedge of inclination α as shown. The wedge has mass M and lies on a smooth horizontal surface. When the block reaches ground, the wedge has moved by

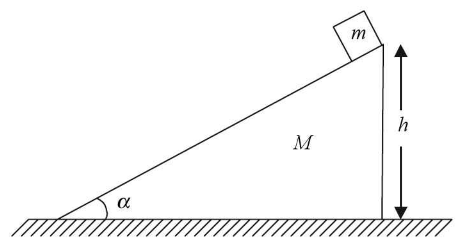


Fig. 16

(a) $\frac{mh \tan\alpha}{M+m}$ (b) $\frac{mh \sec\alpha}{M+m}$

(c) $\frac{mh \operatorname{cosec}\alpha}{M+m}$ (d) $\frac{mh \cot\alpha}{M+m}$

33. In a partially inelastic collision

(a) $e < 0$ (b) $0 < e < 1$

(c) $e = 1$ (d) $e > 1$

34. In an elastic collision

(a) both KE and momentum are conserved

(b) KE and momentum just before collision are equal to just after collision respectively

(c) only momentum is conserved

(d) only momentum just before collision = momentum just after collision.

35. A concave mirror has radius 1m. A small steel ball is placed 1cm high from the pole. It rolls back and forth. Coefficient of friction is 0.01. The distance moved by it before it comes to rest is

(a) 10 cm (b) 1 m

(c) 21 cm (d) none of these

36. Points A, B and C are marked as shown on a wheel rolling on a smooth surface with angular velocity ω. Then

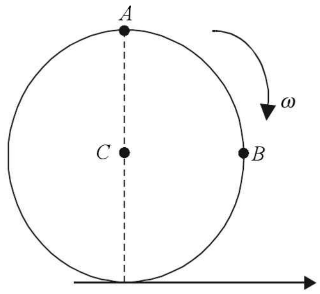


Fig. 17

(a) $V_A > V_B > V_C$ (b) $V_A = V_B = V_C$

(c) $V_A > V_C = V_B$ (d) none

37. Find the correct statement.

(a) In rolling with forward slipping particle covers a distance = $2\pi r$ in one complete rotation

(b) In rolling with backward slipping $V > r\omega$

(c) In pure rolling the particle covers a distance > $2\pi r$ in one complete rotation

(d) In rolling with backward slipping particle covers a distance < $2\pi r$ in one complete rotation

38. A wheel has a mark on the rim. In half rotation the displacement of the mark is

(a) πr (b) $2r$

(c) $r\sqrt{14}$ (d) $r\sqrt{12}$

(e) none

39. Two satellites are at a height R and $3R$ from the surface of the earth, the ratio of their orbital velocity is

(a) $\sqrt{2}$ (b) $\frac{1}{\sqrt{2}}$

(c) 2 (d) $\sqrt{3}$

(e) none

40. If the moon is to escape out of the gravitational field of the earth, its velocity should be increased by

(a) 1% (b) 14.4%

(c) 41.4% (d) 21.2%

41. The atmosphere does not exist on moon because

(a) its radius is small

(b) its escape velocity is less than rms velocity of gases

(c) its mass is small

(d) earth attracts the gases

42. A rod is kept vertical on a smooth surface. It is allowed to fall. Then

(a) COM follows an elliptical path

(b) COM follows a circular path

(c) the upper edge follows an elliptical path

(d) the upper edge follows a circular path

43. A body is weighed with a spring balance on the equator. If the balance is to read zero, the speed of the earth will be

(a) 7 times the present speed

(b) 17 times the present speed

(c) 21 times the present speed

(d) 14 times the present speed.

44. A bubble of radius 2 mm is taken 70 m deep in water. The radius of the bubble becomes

(a) 2 mm (b) 1.6 mm

(c) 1.4 mm (d) 1.0 mm

45. A metal piece is kept on ice top as shown. If the metal piece falls into water what happens to level h_1 and h_2?

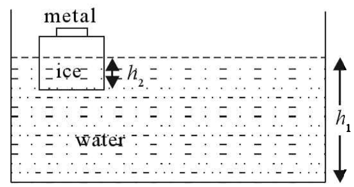


Fig. 18

(a) h_1 increases, h_2 decreases

(b) h_2 increases and h_1 decreases

(c) both h_1 and h_2 increase

(d) both h_1 and h_2 decrease

46. Pressure at point A just below the meniscus is...

T is surface tension and r is radius of the cavity.

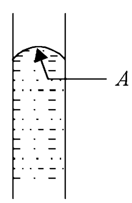


Fig. 19

(a) P_{atm} (b) $P_{atm} + \frac{2T}{r}$

(c) $P_{atm} - \frac{2T}{r}$ (d) $P_{atm} + \frac{4T}{r}$

(e) none

47. If two soap bubbles coalesce to form a big bubble under isothermal conditions, then new radius is

(a) $\frac{1}{R} = \frac{1}{R_1} - \frac{1}{R_2}$ (b) $R = \sqrt{R_1^2 - R_2^2}$

(c) $R = \sqrt{R_1^2 + R_2^2}$ (d) $R = \frac{R_1 R_2}{R_1 + R_2}$

48. The bond which is strongest is

(a) ionic .bond (b) covalent bond

(c) metal bond (d) hydrogen bond

49. If density changes by 0.10% at a depth of 2 km in an ocean then bulk modulus of water is

(a) $2 \times 10^8 Pa$ (b) $2 \times 10^9 Pa$

(c) $2 \times 10^{10} Pa$ (d) $2 \times 10^{11} Pa$

50. A 10 cm long wire is brought 20 cm up in a liquid of surface tension 40 dyne cm^{-1}. Find the work done against surface tension.

(a) 0.8 *m* J (b) 1.6 *m* J

(c) 1.2 *m* J (d) 3.2 *m* J

(e) none

51. In the system shown, extension produced is 0.5 cm. Find Young's modulus and energy stored.

Area of crosssection is 0.2 mm^2

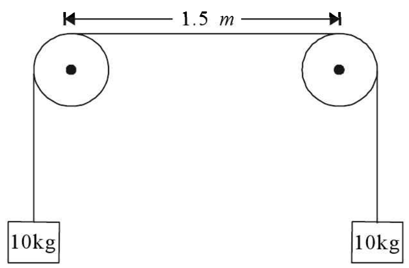


Fig. 20

(a) $1.5 \times 10^{11} Pa, 0.25$ J (b) $2 \times 10^{11} Pa, 0.5$ J

(c) $1.5 \times 10^{11} Pa, 0.5$ J (d) none

52. Energy stored per unit volume in an elastic body having Y as young's modulus and σ as stress is

(a) $\frac{Y^2}{2\sigma}$ (b) $\frac{Y\sigma}{2}$

(c) $\frac{\sigma^2}{2Y}$ (d) $\frac{Y\sigma^2}{2}$

53. If a soap bubble of radius R expands to double its radius. Then work done is ____. Assume T as surface tension

(a) $12\,\pi T R^2$ (b) $16\,\pi T R^2$

(c) $20\,\pi TR^2$ (d) $24\,\pi TR^2$

54. Viscosity of gases ____ with rise in temperature and that of liquid____ with rise in temperature.

(a) increases, decreases (b) decreases, increases

(c) decreases, decreases (d) increases, increases

55. Surf (a detergent) has been added in water. The surface tension

(a) increases (b) decreases

(c) remains unchanged (d) none

Answers

1.	(b)	**2.**	(d)	**3.**	(c)	**4.**	(a)	**5.**	(d)	**6.**	(a)	**7.**	(c)
8.	(b)	**9.**	(b)	**10.**	(a)	**11.**	(b)	**12.**	(b)	**13.**	(c)	**14.**	(a)
15.	(d)	**16.**	(a)	**17.**	(d)	**18.**	(c)	**19.**	(a)	**20.**	(c)	**21.**	(b)
22.	(a)	**23.**	(d)	**24.**	(d)	**25.**	(d)	**26.**	(c)	**27.**	(c)	**28.**	(a)
29.	(d)	**30.**	(a)	**31.**	(c)	**32.**	(d)	**33.**	(b)	**34.**	(b)	**35.**	(b)
36.	(a)	**37.**	(d)	**38.**	(c)	**39.**	(a)	**40.**	(c)	**41.**	(b)	**42.**	(c)
43.	(b)	**44.**	(d)	**45.**	(c)	**46.**	(b)	**47.**	(c)	**48.**	(a)	**49.**	(c)
50.	(a)	**51.**	(a)	**52.**	(c)	**53.**	(d)	**54.**	(a)	**55.**	(b)		

Explanations

1(b) $F = mr\omega^2$ or $r_1\,\omega_1^2 = r_2\,\omega_2^2\; r_2 = \frac{r_1\omega_1^2}{\omega_2^2}$

$= 8 \times \frac{1}{2^2} = 2\,cm$

2(d)

3(c) $\tan\theta = \frac{V^2}{rg} = \frac{25}{100} = \frac{1}{4}$

4(a) $h = AB = AO - OB = R\,(1-\cos\theta)\;\frac{mv^2}{2} = mgh$

or $\frac{mv^2}{2}$ = mg $(1-\cos\theta)\,R$ or $\frac{mv^2}{R} = 2$ mg $(1-\cos\theta)$

mg $\cos\theta = 2$ mg $(1-\cos\theta)$

or $\cos\theta = 2/3$ or $h = R/3$

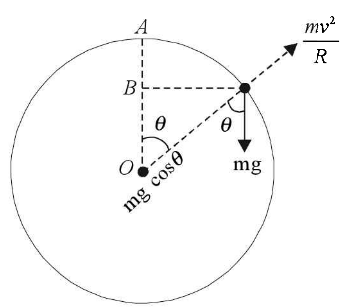


Fig. 21

5(d) $T_{top} = \frac{mv^2}{r} - mg = \frac{m(3rg)}{r} - mg = 2\,mg$;

$$T_{\text{bottom}} = T_{\text{top}} + 6\,\text{mg}$$

$$\frac{T_{top}}{T_{bottom}} = \frac{2mg}{8mg} = 1/4$$

6(a) $m\,(l+x)\,\omega^2 = kx$

or $$x = \frac{Ml\omega^2}{k - M\omega^2}$$

7(c)

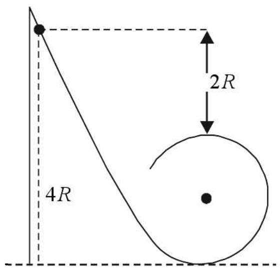


Fig. 22

$$\frac{mV_{top}^2}{2} = \text{mg}\,(2R) \text{ or } \frac{mV_{top}^2}{R} = 4\,\text{mg}$$

$$T_{\text{top}} = \frac{mV_{top}^2}{R} - \text{mg} = 3\text{ mg}\,;\, T_{\text{bottom}} = T_{\text{top}} + 6\text{ mg}$$

$$\frac{T_{top}}{T_{bottom}} = 1/3$$

8(b) $\left(\dfrac{mV^2}{R}\right)^2 + (ma)^2 = (\mu\,\text{mg})^2$ or $V = \left\{R^2\left(\mu^2 g^2 - a^2\right)\right\}^{1/4}$

9(b) $T\cos\theta = \text{mg}$

$T\sin\theta = m\,(l + l\sin\theta\,)\,\omega^2$

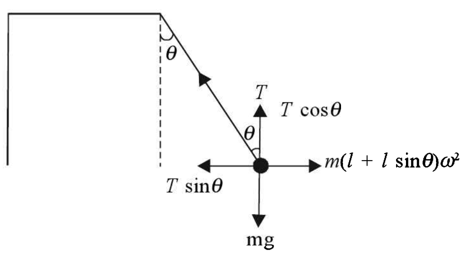


Fig. 23

dividing (II) by (I) $\tan\theta = \dfrac{(l + l\sin\theta)}{g}\,\omega^2$

or $$\omega = \sqrt{\frac{g\tan\theta}{l + l\sin\theta}}$$

10(a)

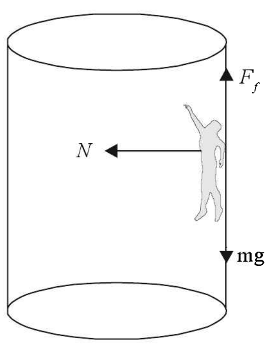


Fig. 24

$$F_f = \mu N = mg;\text{ or; } \mu\, m\, R\omega^2 = \text{mg or } \omega = \sqrt{\frac{g}{\mu R}}$$

11(b)

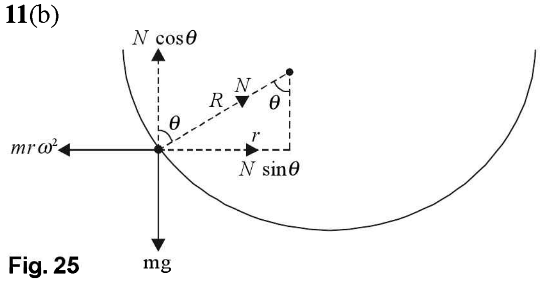


Fig. 25

$N\cos\theta = \text{mg}$

$N\sin\theta = mr\omega^2 = mR\sin\theta\;\omega^2$

or $$\omega = \sqrt{\frac{g}{R\cos\theta}}$$

12(b) consider a small element dx, then $dm = A\,\rho\,dx$

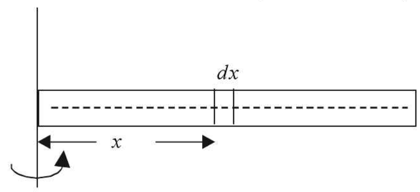


Fig. 26

$$dF = dm\,x\,\omega^2 \text{ or } F = \int_0^e A^e \rho\omega^2 x\,dx$$

$$= A\rho\,\frac{\omega^2 l^2}{2}$$

13(c) $S\,n\text{th} = u + \dfrac{a}{2}\,(2n-1) = 0 + 5\,[2\,(2) - 1] = 15m$

$W = 20 \times 15 = 300\text{ J}$

14(a) $\dfrac{mv^2}{2} = \dfrac{mu^2}{2} + mgh$

or $v = \sqrt{u^2 + 2gh} = \sqrt{60^2 + 2\times10\times40}$

$= 66.67\text{ ms}^{-1}$

15(d) Power in $bhp = \dfrac{4}{3}\times\dfrac{\frac{dm}{dt}gh}{746}$

$$= \frac{4}{3}\times\frac{20\times10\times80}{746} = 26.24\text{ bhp}$$

16(a) $\dfrac{1}{2}\,mv^2\left[1 - \left(\dfrac{19}{20}\right)^2\right] = F.\,S$

or $\dfrac{1}{2}\,mv^2 = \dfrac{400}{39}\,F.s \text{ or } n = \dfrac{400}{39} = 10\text{ planks}$

17(d) $\dfrac{1}{2}\,(k_1 + k_2)\,x^2 = \dfrac{1}{2}(k_1 + k_2)\,(x/2)^2 + \dfrac{1}{2}\,Mv^2$

or $$v = \sqrt{\frac{3(k_1 + k_2)x^2}{4M}}$$

18(c) $\dfrac{Ml^2\alpha}{3} = -\left(k_1 + \dfrac{k_2}{4}\right)l^2\,\theta$

or $\omega = \sqrt{\dfrac{3\left(k_1 + \dfrac{k_2}{4}\right)}{M}}$

19(a) $Mv = 2MV'$ or $V' = v/2$; $\dfrac{1}{2}kx^2 = \dfrac{1}{2}(2M)\left(\dfrac{v}{2}\right)^2$

or $x = \sqrt{\dfrac{Mv^2}{2k}}$

20(c) $= \overline{y_1} = \dfrac{-ml + M(0)}{m+M}$;

$\overline{y_2} = 0$ Then $\Delta y = \dfrac{ml}{m+M}$

21(b) $V^2 = 2gh; T = \dfrac{2V\cos\theta}{g\cos\theta} = \dfrac{2V}{g}$;

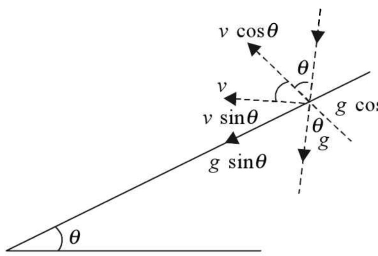


Fig. 27

$x = V\sin\theta(T) + \dfrac{1}{2}g\sin\theta(T)^2$

$= V\sin\theta\left(\dfrac{2V}{g}\right) + \dfrac{1}{2}g\sin\theta\left(\dfrac{2V}{g}\right)^2 = 8h\sin\theta$

22(a) $V = u_g 2.303\log\dfrac{M_o}{M_o - rt} = 3\times10^3\times2.30\log\dfrac{5}{2}$

$= 6.9\times10^3(.3980) = 2.76\,\text{kms}^{-1}$

23(d) $v = \dfrac{mu}{M+m}$; $h = {v^2}/{2g} = \dfrac{m^2u^2}{2(M+m)^2 g}$

24(d) $F(2.8) = 12g(1.5)$ or $F = \dfrac{180}{2.8} = 64.28\,N$

25(d) $I = I_1 + I_2 + I_3 = \dfrac{5MR^2}{4} + \dfrac{5}{4}MR^2 + \dfrac{MR^2}{4}$

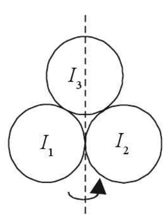


Fig. 28

$= \dfrac{11}{4}MR^2$

26(c) $I = I_1 + I_2 + I_3$

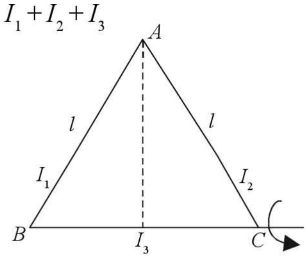


Fig. 29

$= \dfrac{M(\ell\sin60)^2}{3} + \dfrac{M(l\sin60)^2}{3} + 0$

$= \dfrac{Ml^2}{4} + \dfrac{Ml^2}{4} = \dfrac{Ml^2}{2}$

27(c) $4\left(\dfrac{2}{5}mr^2\right) + 2(ma^2)$

28(a) $\dfrac{3}{4}$ I $\because$ new mass in 3/4 of original mass

29(c) $6m\sin\theta = 8\,\text{m}\cos\theta$ or $= 53°$

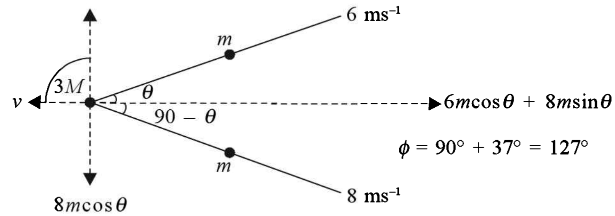


Fig. 30

$3mv = 8(.8)m + 6(.6)m$ or $V = 10/3\,\text{ms}^{-1}$

30(a) $30(6) = -(30+120)x$ or $x = -1.2\,m$

31(c) equal masses in oblique elastic collision fly off at right angle to one another

32(d) $m(h\cot\alpha) = (M+m)x$

or $x = \dfrac{mh\cot\alpha}{M+m}$

33(b)

34(b)

35(b) $V^2 = 2gh = 2(\mu g)s$

or $S = \dfrac{h}{\mu} = \dfrac{10^{-2}}{.01} = 1m$

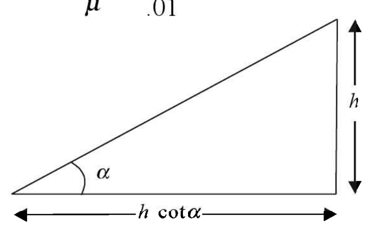


Fig. 31

36(a) $V_A = 2V_c; V_B = \sqrt{2}V_c$

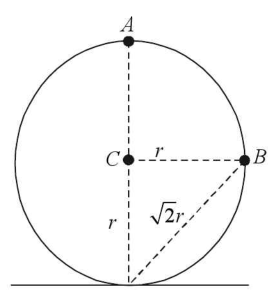


Fig. 32

$V_A = 2r\omega$; $V_c = r\omega$

$V_B = (\sqrt{2}r)\omega$

37(d)

38(c) $d = \sqrt{(2r)^2 + (\pi r)^2} = r\sqrt{14}$

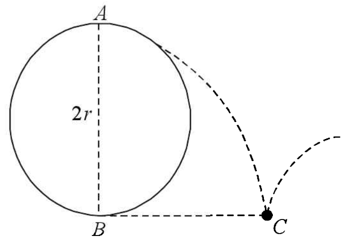


Fig. 33

39(a) $\frac{V_{01}}{V_{02}} = \frac{\sqrt{GM/(R+R)}}{\sqrt{\frac{GM}{R+3R}}} = \sqrt{2}$

40(c) $V_e = \sqrt{2}\, V_o$

$\Delta v = 0.414\, V_o$ or 41.4%

41(b)

42(c) COM follows vertical straight line.

43(b) $g' = g\left(1 - \frac{R\omega^2}{g}\right) = 0$

or $\omega_{\text{new}} = \sqrt{\frac{g}{R}}$; $\frac{\omega_{new}}{\omega_{present}} = \frac{\sqrt{\frac{10}{6400\times 10^3}}}{\frac{2\pi}{24\times 3600}} = 17$

44(d) $P_1 V_1 = P_2 V_2$

or $\frac{4}{3}\pi\,(2)^3 \times 10^5$

$= \frac{4}{3}\pi\, r^3\,(70^{10} \times 10^3 + 10^5)$

or $r = 1mm$

45(c),

46(b)

47(c) $P_1 V_1 + P_2 V_2 = PV$

or $\frac{4T}{R_1}\left(\frac{4}{3}\pi R_1^3\right) + \frac{4T}{R_2}\left(\frac{4}{3}\pi R_2^3\right) = \frac{4T}{R}\left(\frac{4}{3}\pi R^3\right)$

or $R = \sqrt{R_1^2 + R_2^2}$

48(a)

49(c) $\therefore \frac{\Delta\rho}{\rho} = \frac{\Delta V}{V}$, therefore

$B = \frac{P}{\frac{\Delta V}{V}} = \frac{(10^5 + 10^3 \times 2 \times 10^3 \times 10)\times 10^2}{.1}$

$= 2 \times 10^{10}\, Pa$

50(b) $W = 2T\,(l.\,x) = 2 \times 40 \times 10^{-3} \times .2 \times .1 = 1.6\, mJ$

51(a) $Y = \frac{mgL}{A\Delta l} = \frac{10\times 10\times 1.5}{0.2\times 10^{-6}\times 5\times 10^{-3}}$

$= 1.5 \times 10^{11} Pa$

$W = \frac{1}{2} F.\, dl = \frac{10\times 10\times 5\times 10^{-3}}{2} = 0.25$ J

52(c) $\frac{W}{vol} = \frac{1}{2}$ stress × strain $= \frac{\sigma^2}{2Y}$ ($\because$ strain $= \frac{\text{stress}}{Y}$)

53(d) $W = 2T$ (change in area) $= 2T\,[4\pi\,(4R^2) - 4\pi R^2]$

$= 24\,\pi T R^2$

54(a)

55(b)

SELF TEST 3

1. The diagram shows the variation of $\frac{1}{v}$ (where v is velocity of the particle) with respect to time. At time $t = 3s$, using the details given in the graph, the instantaneous acceleration will be equal to

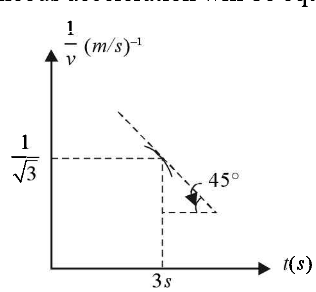


Fig. 1

(a) $-2\ ms^{-2}$ (b) $+3\ ms^{-2}$

(c) $+5\ ms^{-2}$ (d) $-6\ ms^{-2}$

2. A small electric car has a maximum constant acceleration of 1 ms^{-2}, a maximum constant deceleration of 2 ms^{-1} and a maximum speed of 72 kmh^{-1}. The amount of time it would take to drive this car one kilometer starting from rest and finishing at rest is

(a) 15 s (b) 50 s

(c) 35 s (d) 65 s

3. A car accelerates uniformely from 80 km/hr at $t = 0$ to 120 km/hr at $t = 10s$. Which graph in Figure best describes the motion of the car?

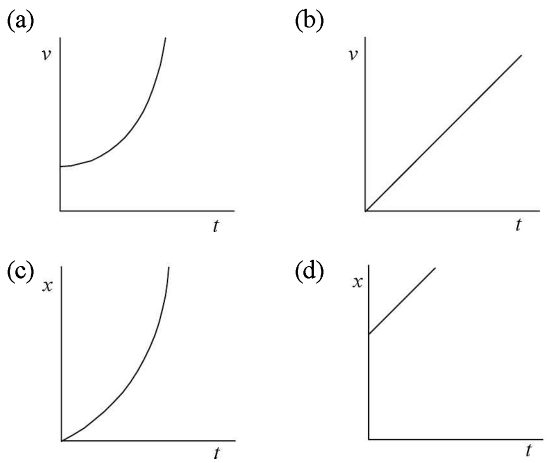


Fig. 2

4. A projectile is launched with a speed of 10 ms^{-1} at an angle 45° with the horizontal from a sloping surface of inclination 30°. The range R is (Take g = 10 ms^{-2})

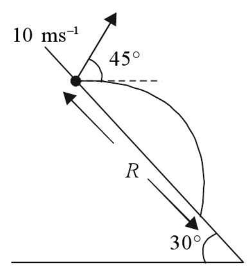


Fig. 3

(a) 4.9 m (b) 18.2 m
(c) 9.1 m (d) 2.45 m

5. The nearest star, Proxima Centauri is 4.22 light years from the earth. Ignoring relative motion between the solar system and Proxima Centauri, suppose that a spacecraft accelerates from the vicinity of the earth at 0.01g (g is acceleration due to gravity of earth) until it reaches one-tenth the speed of light, coasts until time decelerate, then decelerates at 0.01g until it comes to rest in the vicinity of Proxima Centauri. The time that the trip takes is (1 light year = 9.5 × 10^{15} m, 1 year = 3 × 10^{7} sec) nearly

(a) 27.2 solar year (b) 4.22 solar year
(c) 54.4 solar year (d) 44 solar year

6. As the driver of an automobile travels north at 25 km/hr in a parking lot, he observes a truck approaching from the northwest. After he reduces his speed to 15 km/hr and turns so that he is travelling in a northwest direction, the truck appears to be approaching from the west. Assuming that the velocity of the truck is constant during the period of observation, the magnitude and direction of the velocity of the truck is

(a) $\frac{25}{\sqrt{2}}$ kmh^{-1} at 37° north of east

(b) $25\sqrt{2}$ kmh^{-1} at 37° north of east

(c) $\frac{25}{\sqrt{2}}$ kmh^{-1} at 37° east of north

(d) $25\sqrt{2}$ kmh^{-1} at 37° east of north

7. A ball is dropped onto a step at point A and rebounds with a velocity v_o at an angle of 45° with the vertical. The value of v_o knowing that just before the ball bounces at point B its velocity v_B forms an angle of 30° with the vertical is

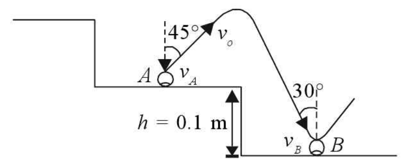


Fig. 4

(a) 0.7 ms^{-1} (b) 1.4 ms^{-1}
(c) 2.1 ms^{-1} (d) 2.8 ms^{-1}

8. Coal is discharged from the tailgate A of a dump truck with an initial velocity v_A = 2 m/s at 60° with horizontal as shown in Fig. 5. The radius of curvature of the trajectory described by the coal at point A is

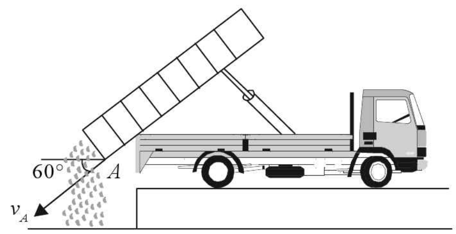


Fig. 5

(a) 0.4 m (b) 0.8 m
(c) 1.2 m (d) 1.6 m

9. A gun shoots three times as high when its angle of elevation is α as when it is β. The ratio of the horizontal distances travelled respectively in the two cases is

(a) 1 : 3 (b) 1 : 1
(c) 2 : 1 (d) 3.5 : 1

10. A projectile is projected with speed u making an angle 2θ with the horizontal. The speed when its direction of motion makes an angle θ with the horizontal is

(a) $(u \cos 2\theta)/2$ (b) $u \cos \theta$
(c) $u(2 \cos\theta - \sec\theta)$ (d) $u(\cos\theta - \sec\theta)$

11. The car is travelling at 57.6 km/hr on an unbanked circular curve of radius $\frac{160}{3}$ m. If the coefficient of friction between the road and the car is 0.8, the largest tangential deceleration the driver can achieve by applying the brakes is

(a) 8 ms^{-2} (b) 4.8 ms^{-2}
(c) 6.4 ms^{-2} (d) 16 ms^{-2}

12. The net thrust (nozzle thrust T minus air resistance R) required for the airplane to climb at an angle θ with the horizontal with an acceleration 'a' in the direction of flight is n times the weight of the jet airplane where n is

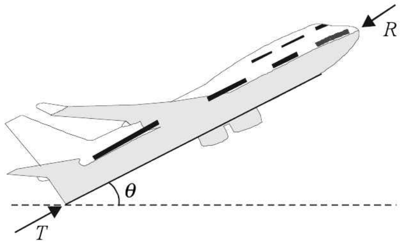


Fig. 6

(a) $\sin \theta$ (b) $\frac{a}{g}$

(c) $\frac{g}{a}$ (d) $\sin \theta + \frac{a}{g}$

13. A car of 400 kg is hoised up the incline using the cable and motor M. For a short time, the force in the cable is $F = 3200t^2$ N where t is in second. If the car has an initial velocity $v_1 = 2$ m/s at $s = 0$ and $t = 0$, the distance it moves up the plane when $t = 2s$ is (Take $g = 10$ m/s²)

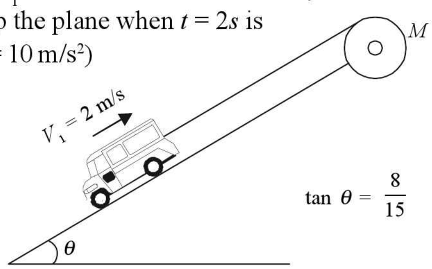


Fig. 7

(a) 4 m (b) 1 m

(c) 5.25 m (d) 2 m

14. An insect tries to climb out of a hemispherical bowl of radius 0.6 m. The coefficient of static friction between insect and bowl is 0.4. If the bowl is spun about a vertical axis with an angular velocity ω, then

(a) the insect will be always at rest with in the bowl

(b) the insect will be able to get out of the bowl if ω is greater than limiting angular velocity

(c) the insect will be able to get out of the bowl only if friction is absent

(d) None of the above

15. The 50 kg block shown in Figure moves down the smooth slope defined by the equation $y = 0.05x^2 - 5$. The normal force exerted on the ground by the block at the instant it arrives at point A where its velocity is 10 m/s is

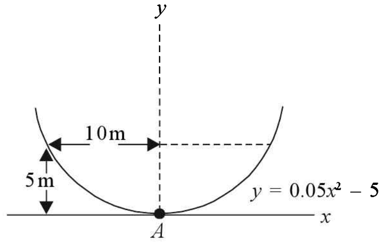


Fig. 8

(a) zero (b) 990 N

(c) 490 N (d) 500 N

16. A 10 N block is released from rest at A and slides down along the smooth cylindrical surface. The attached spring has a stiffness, $k = 30$ Nm⁻¹. If it does not allow the block to leave the surface until $\theta = 60°$, its unstretched length is

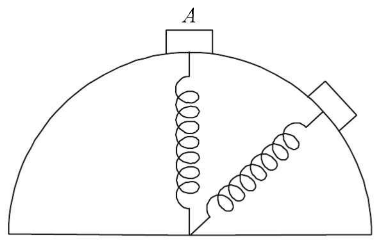


Fig. 9

(a) 0.166 m (b) 0.5 m

(c) 0.333 m (d) 0.666 m

17. A car enters a section of curved road in the horizontal plane and slows down at uniform rate from a speed of 25 ms⁻¹ at A to a speed 20 m/s as it passes C. The radius of curvature ρ of the road at A is 400 m and at C is 80 m. The distance between A and C along the road is 200 m. For the motion of car, point B is the inflection point where the curvature changes its direction. Which of the following statement is wrong?

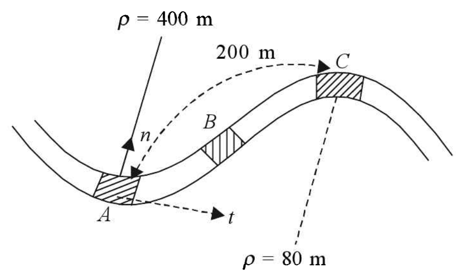


Fig. 10

(a) Tangential components of acceleration at A, B and C are equal

(b) Tangential acceleration at any point depends upon the radius of curvature.

(c) Normal component of acceleration varies from one point to another due to variation of radius of curvature of the road.

(d) Normal force at B is zero since point B is inflection point where the curvature changes direction.

18. A block of mass 5 kg rests on a smooth plane which can turn about the y-axis. The length of cord l is 0.5 m. The angular velocity necessary to cause the weight to float just above the plane is (Take $g = 10$ m/s²)

(a) $\sqrt{20}$ rads⁻¹ (b) $\sqrt{3}$ rads⁻¹

(c) $\sqrt{40}$ rads⁻¹ (d) $\sqrt{\frac{20}{\sqrt{3}}}$ rads⁻¹

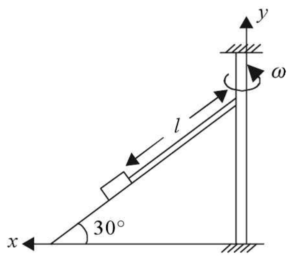


Fig. 11

19. The bent water pipe of constant cross-section is rotating about the vertical axis AB with the constant angular velocity $\omega = 1.5$ rads^{-1}. If the speed of water in portion AB of the pipe is 4 m/s (constant), the magnitude of the acceleration of a water particle just before it exits the pipe at C is

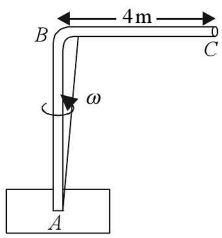


Fig. 12

(a) 9 m/s^2 (b) 12 m/s^2
(c) 15 m/s^2 (d) 9.8 m/s^2

20. A man standing in an elevator that is moving with a constant acceleration; holds a 3 kg block B between two other blocks in such a way that the motion of B relative to A and C is impending assuming that the coefficients of friction between the surfaces are $\mu_s = 0.3$ and $\mu_k = 0.25$. The acceleration of the elevator if it is moving upward and each of the forces exerted by the man on blocks A and C has a horizontal component equal to twice the weight of B is

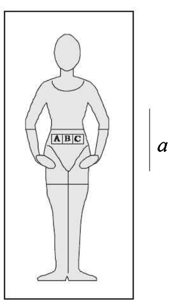


Fig. 13

(a) 9.8 ms^{-2} (b) 0.98 ms^{-2}
(c) 1.96 ms^{-2} (d) 19.6 ms^{-2}

21. The bob of a 0.2 m pendulum describes an arc of circle in a verticle plane. If the tension in the cord is $\sqrt{3}$ times the weight of the bob when the cord makes an angle 30° with the vertical, the acceleration of the bob in that position is

(a) 9.8 ms^{-2} (b) 0.98 ms^{-2}
(c) 1.96 ms^{-2} (d) 19.6 ms^{-2}

22. A block *B* is resting on a horizontal plate in the *x-y* plane and the coefficient of friction between the block and the plate is μ. The plate begins to move in the *x*-direction and its velocity is $v = bt^2$, *t* being the time and '*b*' being a constant. At what time will the block start sliding on the plate?

(a) $\frac{\mu b}{g}$ (b) $\frac{\mu g b}{2}$
(c) $\frac{\mu g}{b}$ (d) $\frac{\mu g}{2b}$

23. Which one of the following statements is correct?

(a) Gravitational force has an inverse-square dependence on distance, whereas electromagnetic, weak and strong forces are the short range forces.
(b) Gravitational and electromagnetic forces have the inverse-sqaure dependence on distance, whereas weak and strong forces are the short range forces.
(c) Gravitational, electromagnetic forces have the inverse-square dependence on distance, whereas strong force is the short range force.
(d) Gravitational, electromagnetic, weak and strong forces have the inverse-square dependence on distance.

24. The mass of rocket is *M* and the total mass of the rocket and the fuel is M_o. The average exhaust velocity of gases ejected form rocket motors is u and the final velocity attained by the rocket after using up all fuel is *u*. the final velocity *v* is proportional to

(a) In $\left(\frac{M_o}{M}\right)$ (b) $\frac{M_o}{M}$
(c) $(M_o - M)$ (d) ln $\left(\frac{M}{M_o}\right)$

25. A body is moving along *OX* with a momentum of 10 kg m/s. A force of 0.2 *N* acts on it in the direction *OY* (at right angles to *OX*) for two minutes. If the intial kinetic energy of the body is 2 *J*, its final kinetic energy (after the action of the force) is

(a) 10.5 *J* (b) 13.5 *J*
(c) 19.5 *J* (d) 6.8 *J*

26. A light inextensible string carrying masses m_1 and m_2 at its ends passes over a light smooth pulley. The breaking tension of the string is $(m_1 + m_2)g$. When the system is released, the least possible acceleration of the masses is

(a) $\frac{3\sqrt{3}}{4}g$ (b) $\frac{\sqrt{3}}{3}g$
(c) $\frac{\sqrt{3}}{2}g$ (d) $\frac{2\sqrt{3}}{3}g$

27. The Figure shows how the force F required to push an arrow slowly through a bale of hay varies with the distance of penetration *x*. Assuming that the *F-x* plot is independent of the speed of penetration, the exit speed of a 58 g arrow if its speed at entry is 100 m/s is

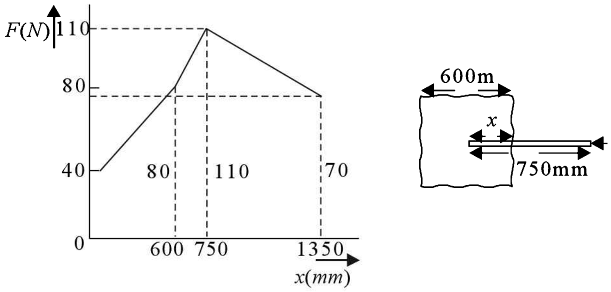


Fig. 14

(a) 100 ms^{-1} (b) zero
(c) 60 ms^{-1} (d) 80 ms^{-1}

28. A hypothetical vehicle is moving at speed V_0 in Figure. On this vehicle two bodies each of mass m are sliding along a horizontal rod at a speed v relative to the rod. This rod is rotating at angular speed 10 rad/sec relative to the vehicle. The kinetic energy of the two bodies relative to ground (XYZ) when they are at a distance r from point A is

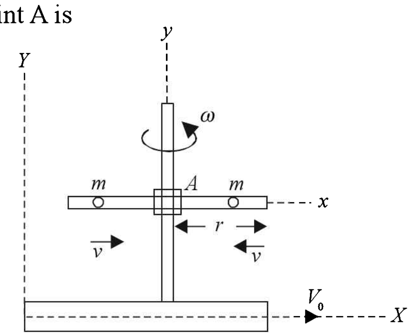


Fig. 15

(a) $m[v^2 + (\omega r)^2]$ (b) mV_0^2
(c) $mV_0^2 + mv^2$ (d) $m[V_0^2 + v^2 + (\omega r)^2]$

29. An external torque T of 65 N-m is applied to a solid cylinder B which has mass of 30 kg and a radius of 0.2 m. The cylinder rolls without slipping. Block A, having a mass of 20 kg is dragged up the 30° incline. The dynamic coefficient of friction μ_d between block A and incline is 0.25. The connections at C and D are frictionless. The velocity of the system after moving a distance of 2 m is (Take $g = 10\ ms^{-2}$)

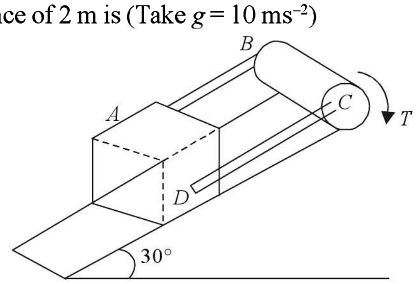


Fig. 16

(a) 1 ms^{-1} (b) $\sqrt{2}$ ms^{-1}
(c) $\sqrt{3}$ ms^{-1} (d) 2 ms^{-1}

30. When t = 0 a body of mass 2 kg is at rest at a point A, position vector $2\hat{i} + 3\hat{j}$. The body is subjected to a constant force F = $(6\hat{i} + 2\hat{j})$ N causing it to accelerate and 3 seconds later, the body passes through the point B. The kinetic energy of the body as it passes through B is

(a) zero (b) $(\sqrt{40})(\sqrt{13})J$
(c) 90 J (d) 45 J

31. The two diagrams show the situation before and after a collision between two spheres A and B of equal radii moving along the same straight line on a smooth horizontal surface. The coefficient of restitution e is

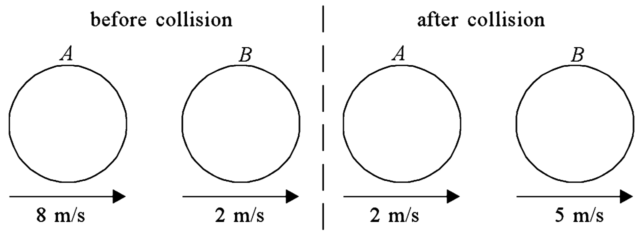


Fig. 17

(a) $\frac{2}{3}$ (b) $\frac{1}{2}$
(c) $\frac{1}{3}$ (d) $\frac{1}{4}$

32. A car of mass 1500 kg travels along a level road with maximum speed 40 ms^{-1} against a constant resistance 500 N to its motion. If the engine of the car works at the same rate and the resistances are unchanged, the maximum speed of the car when ascending an incline of $\sin^{-1}\left(\frac{1}{49}\right)$ is

(a) 40 ms^{-1} (b) 25 ms^{-1}
(c) 20 ms^{-1} (d) 10 ms^{-1}

33. The ball is released from position A and drops 10 m to the incline. If the coefficient of restitution in the impact is $e = \frac{\sqrt{3}}{2}$; the time taken by the ball to strike the plane again is

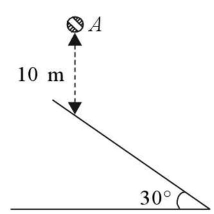


Fig. 18

(a) 1.5 s (b) 2 s
(c) 2.5 s (d) 3 s

34. A 10^3 kg body is being lifted with a constant velocity of 3 m/s. The power required is (Take $g = 10\ ms^{-2}$)

(a) 10 kW (b) 20 kW
(c) 30 kW (d) 40 kW

35. The power required if the 10^3 kg body has a velocity of 3 m/s upward and is accelerated 1.5 ms^{-2} upward is (Take $g = 10\ ms^{-2}$)

(a) 25.5 kW (b) 30 kW
(c) 34.5 kW (d) 11.5 kW

36. A ball is dropped from rest, it bounces from the floor repeatdly. The coefficient of restitution is 0.5 and the speed just before the first bounce is 5 ms^{-1}. The total time taken by the ball to come to rest is

(a) 2 *s* (b) 1 *s*
(c) 0.5 *s* (d) 0.25 *s*

37. A satellite is moving in a circular orbit round the earth with a diameter of the orbit 2R. At a certain point a rocket fixed to the satellite is fired such that it increases the velocity of the satellite tangentially. The resulting orbit would be

(a) same as before
(b) circular orbit with diameter greater than 2R
(c) elliptical orbit with minor axis 2R
(d) elliptical orbit with major axis 2R

38. Let R_s and R_m be the distances of the geostationary satellite and moon from the centre of the earth. Then R_m/R_s is approximately

(a) $(29)^{\frac{1}{2}}$ (b) $(29)^{\frac{2}{3}}$
(c) 29 (d) $(29)^{\frac{3}{2}}$

39. A non-uniform rod AB of weight W is supported horizontally in a vertical plane by two light strings PA and QB as shown in the Figure. G is the centre of gravity of the rod. If PA and QB make angles 30° and 60° respectively with the vertical, the ratio $\frac{AB}{GB}$ is

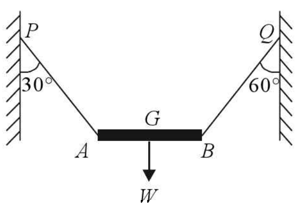


Fig. 19

(a) $\frac{1}{2}$ (b) $\sqrt{3}$
(c) $\frac{1}{3}$ (d) $\frac{1}{\sqrt{3}}$

40. A uniform rod of length *L* and mass *M* lies on a frictionless table and is free to move on the table. A particle of mass *m* moving with velocity *u* collides elastically with the rod at a distance $\frac{L}{3}$ from its centre and comes to rest immediately after collision. The ratio $\frac{M}{m}$ is

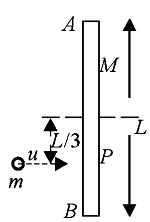


Fig. 20

(a) 3 : 7 (b) 1 : 3
(c) 3 : 1 (d) 7 : 3

41. A homogeneous sphere of radius *r* rotating with angular velocity ω_0 about a horizontal diameter is gently placed on a table whose coefficient of friction is *m*. There will be slipping at the point of contact for a time *t*. The number of revolutions made by the sphere on the table till that time is

(a) $\frac{\omega_0^2}{2\pi\mu g}$ (b) $\frac{r\omega_0^2}{2\pi\mu g}$
(c) $\frac{9\omega_0^2}{2\pi(49\mu g)}$ (d) $\frac{9r\omega_0^2}{2\pi(49\mu g)}$

42. Two smooth cylindrical bars weighing *W* N each lie next to each other in contact. A similar third bar is placed over the two bars as shown in Figure. Neglecting friction, the horizontal force on each lower bar necessary to keep them together is

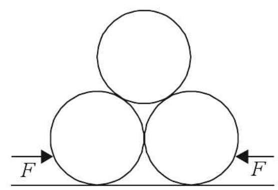


Fig. 21

(a) $\frac{W}{2}$ (b) W
(c) $\frac{W}{\sqrt{3}}$ (d) $\frac{W}{2\sqrt{3}}$

43. A uniform thin rod of mass 3m and length 2*a* is free to rotate in a horizontal plane about a smooth fixed vertical axis which passes through the midpoint O of the rod. Two smooth small rings, each of mass *m* are free to slide on the rod. At time *t* = 0, the rings are on opposite sides of O and are at distance $\frac{a}{2}$ from O. The rod is then given and initial angular velocity $2\sqrt{\frac{g}{a}}$ the rings being initially at rest relative to the rod. The rings are about to slip off the rod, its angular velocity is

(a) $\sqrt{\frac{2g}{a}}$ (b) $\sqrt{\frac{g}{2a}}$
(c) $\sqrt{\frac{a}{g}}$ (d) $\sqrt{\frac{g}{a}}$

44. The homogeneous thin plate as shown in Figure is of uniform thickness and has density 15 gm^{-3}, its moment of inertia about the *y*-axis is nearly

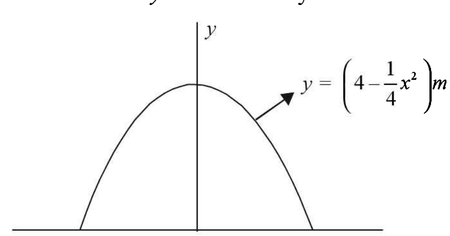


Fig. 22

(a) 1 kg m^2 (b) 2 kg m^2

(c) 2.5 kg m^2 (d) 5 kg m^2

45. Which of the following statements is wrong?

(a) In the northen hemisphere, winds converging on a centre of low pressure tend to rotate about it in the counterclockwise direction.

(b) In the southern hemisphere, winds converging on a centre of low pressure tend to rotate about it in the anticlockwise direction.

(c) During Tsunami produced due to earthquake (Dec. 26, 2004) the tidal waves in the southern hemisphere moving north are deflected to the left.

(d) All these above effects are due to gravitational attraction of earth.

46. Which of the following statements is wrong?

(a) Instantaneous centre is a point on a rigid body whose velocity is zero at a given instant.

(b) The instantaneous centre may not be a point of the rigid body.

(c) For a rolling disc on a fixed surface the centre of disc is instantaneous centre.

(d) By identifying the instantaneous centre of a rigid body in plane motion, the velocities of its points is proportional to their distances from the instantaneous centre.

47. The 30 kg slender bar has an initial angular velocity $\omega_0 = 4$ rad/s in the vertical position, when the spring is unstretched. Elastic constant of spring $k = 3000$ Nm^{-1}. The angular velocity of the bar as it strikes the horizontal surface is

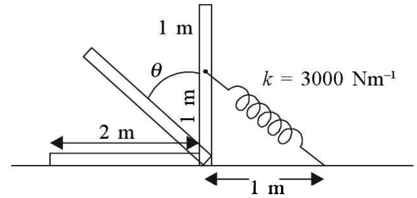


Fig. 23

(a) 4 $rads^{-1}$ (b) greater than 4 $rads^{-1}$

(c) less than 4 $rads^{-1}$ (d) zero

48. A homogeneous bar AB of mass 20 kg is falling with a velocity of 10 ms^{-1} downward and no angular velocity when the pin at C strikes the fixed bearings at D with plastic impact. The angular velocity of the bar immediately after impact is

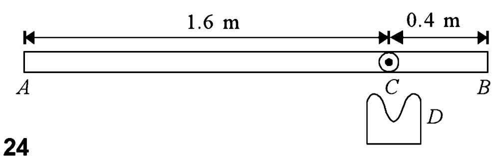


Fig. 24

(a) zero

(b) 8.65 rad/s in the anticlockwise direction

(c) 8.65 rad/s in the clockwise direction

(d) 4.3 rad/s in the anticlockwise direction

Answers

1.	(b)	**2.**	(d)	**3.**	(c)	**4.**	(b)	**5.**	(c)	**6.**	(a)	**7.**	(b)
8.	(b)	**9.**	(b)	**10.**	(c)	**11.**	(c)	**12.**	(d)	**13.**	(c)	**14.**	(b)
15.	(b)	**16.**	(c)	**17.**	(b)	**18.**	(c)	**19.**	(c)	**20.**	(c)	**21.**	(a)
22.	(d)	**23.**	(a)	**24.**	(a)	**25.**	(b)	**26.**	(c)	**27.**	(d)	**28.**	(d)
29.	(b)	**30.**	(c)	**31.**	(b)	**32.**	(b)	**33.**	(c)	**34.**	(c)	**35.**	(c)
36.	(b)	**37.**	(c)	**38.**	(b)	**39.**	(c)	**40.**	(d)	**41.**	(d)	**42.**	(d)
43.	(d)	**44.**	(a)	**45.**	(d)	**46.**	(c)	**47.**	(c)	**48.**	(b)		

Explanations

1.(b) $\frac{d}{dt}\left(\frac{1}{v}\right) = -\tan 45° = \frac{1}{v^2}\frac{dv}{dt} = -1$ or $\frac{dv}{dt} = v^2$.

At $t = 3s$, $\frac{1}{v} = \frac{1}{\sqrt{3}}\frac{dv}{dt} = 3$ m/s^2

2.(d) Given A = 1 km = 10^3 m, $\alpha = 1$ m/s^2, $\beta = 2$ m/s^2,

$v_{max} = 72$ km/hr = 20 m/s

Total time taken, $T = \frac{1}{v_{max}} + \frac{v_{max}}{2}\left(\frac{1}{\alpha}+\frac{1}{\beta}\right)$

$= \frac{10^3}{20} + \frac{20}{2}\left(\frac{1}{1}+\frac{1}{2}\right) = 5 + 10 \times \frac{3}{2} = 65$ sec

3.(c)

4.(b) Range down the sloping surface, $R = \frac{2u^2\sin(\alpha+\beta)\cos\alpha}{g\cos^2\beta}$

Given $\beta = 30°$, $\alpha = 45°$, $u = 10$ m/s

$$R = \frac{u^2[\sin(2\alpha+\beta)\sin\beta]}{g\cos^2\beta}$$

$$= \frac{10^2[\sin 120° + \sin 30°]}{10\cos^2 30°}$$

$$= \frac{10\left[\frac{\sqrt{3}}{2}+\frac{1}{2}\right]}{\left(\frac{\sqrt{3}}{2}\right)^2} = \frac{20(\sqrt{3}+1)}{3} = 18.2 \text{ m}$$

5.(c) Time taken, $T = \frac{\ell}{v} + \frac{v}{2}\left(\frac{1}{\alpha}+\frac{1}{\beta}\right)$

Given $\alpha = \beta = 0.01g = \frac{10}{100} = 0.1\text{ m/s}^2$,

$$v = \frac{1}{10}c = 3 \times 10^7\text{ m/s}$$

$$\ell = 4.22\text{ light year} = 4.22 \times 9.5 \times 10^{15}\text{ m}$$

$$T = \frac{4.22 \times 9.5 \times 10^{15}}{3 \times 10^7} + \frac{3 \times 10^7}{2}\left(\frac{1}{0.1}+\frac{1}{0.1}\right)$$

$$= \frac{40}{3} \times 10^8 + 3 \times 10^6$$

$$= \frac{49}{3} \times 10^8\text{ sec} = \frac{49 \times 10^8}{3 \times 3 \times 10^7} \simeq 54.4\text{ solar year}$$

6.(a) **Case I**

Let v be the actual velocity of truck making an angle θ with last.

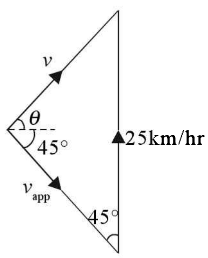


Fig. 25

Velocity of automobile $v_1 = 25$ km/hr (toward north). Let v_{app} be the apparent velocity of truck with respect to the driver of the automobile.

Using Lami's theorem,

$$\frac{25}{\sin(\theta + 45°)} = \frac{v}{\sin 45°} \quad \text{... (i)}$$

Case II

$$\frac{15}{\sin\theta} = \frac{v}{\sin 45°} \quad \text{... (ii)}$$

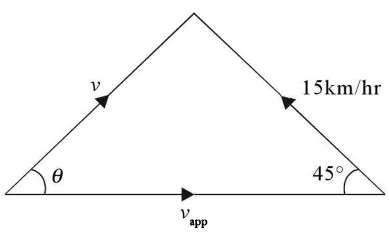


Fig. 26

From equations (i) and (ii),

$$5 \sin\theta = \frac{25}{\sin(\theta + 45°)} = \frac{v}{\sin 45°}$$

$$\tan\theta = \frac{25}{\sin(\theta + 45°)} = \frac{v}{\sin 45°}$$

$$\theta = 37° \Rightarrow \sin\theta = 0.6$$

From equation (ii),

Velocity of truck, $v = \frac{15 \sin 45°}{\sin 37°}$

$$= \frac{15 \times 15}{3 \times \sqrt{2}} = \frac{25}{\sqrt{2}}\text{ km/hr at 37° north of east}$$

7.(b) At A, the horizontal component of velocity of ball,

$(v_B)_x = v_o \cos 45°$

The horizontal velocity is unaffected by gravity.

At B, the vertical velocity

$$(v_B)_y = \sqrt{(v_o \sin 45°)^2 - 2g(-h)}$$

$$\tan\beta = \frac{(v_B)_x}{(v_B)_y} = \frac{v_o \cos 45°}{\sqrt{(v_o \sin 45°)^2 + 2gh}}$$

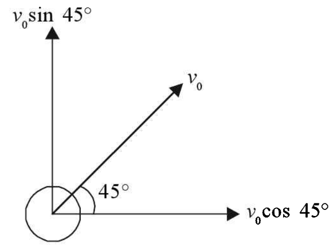
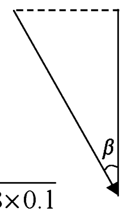


Fig. 27



Given $\beta = 30°$, $\tan\beta = \frac{1}{\sqrt{3}}$, $h = 0.1$ m

$$3 v_0^2 \cos^2 45° = v_0^2 \sin^2 45° + 2gh$$

$$\frac{2v_0^2}{2} = 2gh \Rightarrow v_o = \sqrt{2gh} = \sqrt{2 \times 9.8 \times 0.1}$$

$$= \sqrt{1.96} = 1.4\text{ m/s}$$

8.(b) At A, normal acceleration, $a_N = g \cos 60°$

$$\frac{v_A^2}{\rho_A} = g\cos 60° = g \cos 60°$$

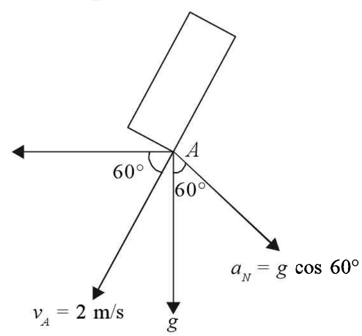


Fig. 28

Radius of curvature of the trajectory described by the coal at point A,

$$\rho_A = \frac{v_A^2}{g\cos 60°} = \frac{2 \times 2}{10 \times 1/2} = \frac{8}{10} = 0.8\text{ m}$$

9.(b) $\frac{H_1}{R_1} = \frac{\tan\beta}{4}$

$$\frac{H_2}{R_2} = \frac{\tan\alpha}{4} \Rightarrow \frac{\tan\beta}{\tan\alpha} = \frac{H_1}{R_1} \cdot \frac{R_2}{H_2} = \frac{R_2}{3R_1} \quad \text{... (i)}$$

$$\frac{H_1}{H_2} = \frac{\sin^2\beta}{\sin^2\alpha} = \frac{1}{3}$$

$$\Rightarrow \quad \frac{\sin\beta}{\sin\alpha} = \frac{1}{\sqrt{3}} \quad \text{...(ii)}$$

For $\frac{R_1}{R_2} = \frac{1}{3}$, $\alpha = \beta = 45°$ (It is not possible). Similarly (C) and (D) not possible

From eqns (i) and (ii) the only choice $R_1 = R_2$ i.e., $\beta = 30°$; $\alpha = 60°$

10.(c) Horizontal velocity is not affected by gravity

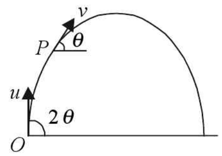


Fig. 29

$u \cos 2\theta = v \cos \theta$

$$\Rightarrow \quad v = \frac{u\cos 2\theta}{\cos\theta} = \frac{v(2\cos^2\theta - 1)}{\cos\theta} = u(2\cos\theta - \sec\theta)$$

11.(c) $\sqrt{\left(\frac{v^2}{r}\right)^2 + (a_t^2)^2} = \mu g$

$v = 57.6$ km/hr $= 57.6 \times \frac{5}{18}$ m/s $= 16$ m/s, $r = \frac{160}{3}$ m

Tangential acceleration, $a_t = \sqrt{\mu^2 g^2 - \left(\frac{v^2}{r}\right)^2}$

$$= \sqrt{(0.8\times10)^2 - \left(\frac{16^2}{\frac{160}{3}}\right)^2}$$

$$= \sqrt{8^2 - \left(\frac{16\times16}{\frac{160}{3}}\right)^2} = \sqrt{8^2 - 4.8^2} = 6.4 \text{ m/s}^2$$

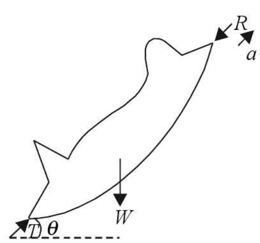
12.(d) $T - R = F$

$F - \text{mg}\sin\theta = \text{ma}$

$F = W\sin\theta + \frac{W}{g}a$

$F = W\left(\sin\theta + \frac{a}{g}\right) = nW$

where $n = \sin\theta + \frac{a}{g}$



13.(c)Net force on car $= ma = F - \text{mg}\sin\theta$

$$\frac{dv}{dt} = \frac{3200t^2}{400} - \text{g}\sin\theta$$

On integration, $v = \frac{8}{3}t^3 - \text{g}\sin\theta\, t + c$

At $t = 0$, $v_1 = 2$ m/s, $c = 2$ m/s

$$\frac{dx}{dt} = v = \frac{8t^3}{3} - \text{g}\sin\theta\, t + 2$$

On integration, $x = \frac{2t^4}{3} - \text{g}\sin\theta\,\frac{dv}{dt} + 2t$

$\tan\theta = \frac{8}{15}$, $\sin\theta = \frac{8}{17}$

At $t = 2$ sec,

$$x = \frac{2\times16}{3} - \frac{10\times8}{17} \times \frac{4}{2} + 2\times2$$

$= 10.66 - 9.41 + 4 = 5.25$ m

14.(b) The range of angular speed for which the insect will not slip

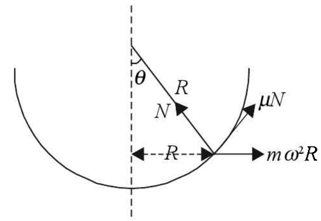


Fig. 30

$$\left[\frac{g(\sin\theta - \mu\cos\theta)}{R\sin\theta(\cos\theta + \mu\sin\theta)}\right]^{\frac{1}{2}} \le \omega \le \left[\frac{g(\sin\theta + \mu\cos\theta)}{R\sin\theta(\cos\theta - \mu\sin\theta)}\right]^{\frac{1}{2}}$$

The bug just be able to get out of the bowl when $\theta = \frac{\pi}{2}$,

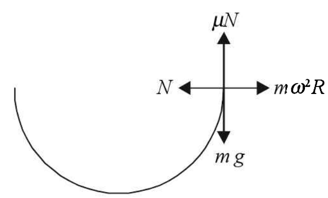


Fig. 31

$\text{m}\omega^2 R = N$, $\text{mg} = \mu N$

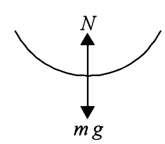
$$\omega_{min} = \sqrt{\frac{g}{R\mu}} = \sqrt{\frac{9.8}{0.6\times0.4}}$$

$\omega > \omega_{min}$ the insect will be able to get out of the bowl.

15.(b) $y = 0.05x^2 - 5$; $\frac{dy}{dx} = 0.1x$; $\frac{d^2y}{dx^2} = 0.1$

Radius of curvature, $\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\frac{d^2y}{dx^2}}$

$$= \frac{\left[1 + (0.1x)^2\right]^{\frac{3}{2}}}{0.1}$$



At A, $x = 0 \Rightarrow \rho = \frac{1}{0.1} = 10$ m

$N - \text{mg} = \frac{mv^2}{\rho}$

$N = \frac{mv^2}{\rho} + \text{mg} = \frac{50\times10^2}{10} + 50\times9.8$

$= 500 + 490 = 990\,N$

16.(c)At A let x_1 be the extension of spring, then

$N_1 = mg + kx_1$ ... (i)

A P, let x_2 be the extension of spring, then

$$mg\cos 60° - N_2 + kx_2 = \frac{mv^2}{R}$$ ... (ii)

The block will leave the surface if $N_2 = 0$

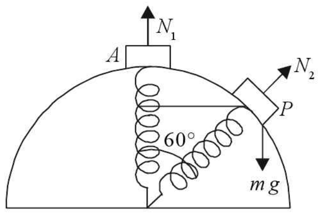


Fig. 32

$$\frac{1}{2}mv^2 = mgR(1-\cos 60°) = \frac{mgR}{2}$$ ... (iii)

$\Rightarrow$ $v = \sqrt{gR}$

Substituting in equation (ii),

$$\frac{mg}{2} + kx_2 = mg \qquad (x_1 = x_2)$$ ... (iv)

$$x_2 = \frac{mg}{2k} = \frac{10}{2\times 30} = \frac{1}{6}\text{m} = 0.166\text{ m}$$

Unstretched length, $x_0 = R - x_2 = 0.5\text{ m} - 0.166\text{ m}$ $= 0.333\text{ m}$

17.(b) The tangential components of acceleration at A, B and C are equal

$$a_t = \frac{v_c^2 - v_A^2}{2\Delta s} = \frac{20^2 - 25^2}{2\times 200} = -\frac{225}{400} = -\frac{9}{16}\text{m/s}^2$$

Normal acceleration at B = $(a_N)_B = 0$

$$(a_N)_A = \frac{v_A^2}{v_A} = \frac{25^2}{400} = \frac{625}{400} = \frac{25}{16}\text{m/s}^2$$

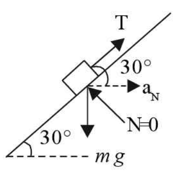
$$(a_N)_C = \frac{v_C^2}{v_C} = \frac{20^2}{80} = \frac{400}{80} = 5\text{ m/s}^2$$

18.(c) $T\cos 30° = ma_N$

$T\sin 30° = mg$

$a_N = g\cot 30°$

$\omega^2\ell\cos 30° = g\cot 30° = g\times\sqrt{3}$



$$\omega = \sqrt{\frac{2g}{\ell}} = \sqrt{\frac{20}{0.5}} = \sqrt{40}\text{ rad/sec}$$

19.(c) Force exerted by the flow of water = $m\omega^2 r$

Radial acceleration of water particle just before it exits the pipe at C, $a_r = \omega^2 r = 1.5^2 \times 4 = 9\text{ m/s}^2$

Corioils acceleration $a_C = 2\vec{\omega}\times\vec{v} = 2\times 1.5\times 4 = 12\text{ m/s}^2$

Acceleration of a water particle just before it exits the pipe at C,

$$\vec{a} = \vec{a}_r + \vec{a}_c$$

$$= \left(-9\hat{e}_r + 12\hat{e}_c\right)\text{m/s}^2$$

$$|\vec{a}| = 15\text{ m/s}^2$$

20.(c) Let F_A and F_B be the horizontal component of forces exerted by the man on blocks A and C respectively. Given $F_A = 2m_Bg$

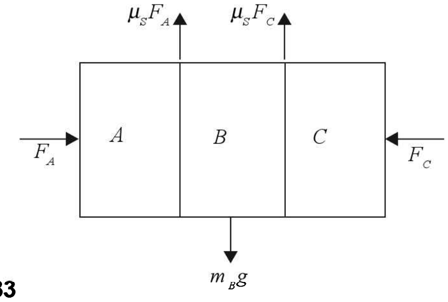


Fig. 33

For the vertical equilibrium of block B,

$\mu_s(F_A + F_C) = m_B(g + a)$

For the horizontal equilibrium, $F_A = F_C$

$2\mu_s F_A = m_B(g+a) \Rightarrow \mu_s(4m_Bg) = m_B(g+a)$

Given $\mu_s = 0.3, m_B = 3$ kg

Acceleration of the elevator, $a = 4\mu_s g - g = (4\times 0.3 - 1)$ $9.8 = 0.2\times 9.8 = 1.96\text{ m/s}^2$

21.(a) Weight of the bob, $W = mg$

Tension in the cord, $T = \sqrt{3}$ mg

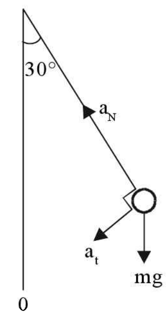


Fig. 34

Tangential force

$\Sigma F_1 = m a_1 = mg\sin 30°$

Tangential acceleration, $a_t = g\sin 30° = 4.9\text{ m/s}^2$

Normal force, $\Sigma F_N = T - mg\cos 30° = ma_N$

$$= \sqrt{3}\,mg - mg\frac{\sqrt{3}}{2}$$

$$= mg\frac{\sqrt{3}}{2} = ma_N$$

Normal acceleration, $a_N = \frac{\sqrt{3}}{2}g = 4.9\sqrt{3}\text{ m/s}^2$

Acceleration of the bob in that position,

$$a = \sqrt{a_t^2 + a_N^2} = \sqrt{(4.9)^2 + \left(\sqrt{3}\times 4.9\right)^2} = 9.8\text{ m/s}^2$$

22.(d) $v = bt^2$

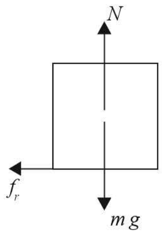


Fig. 35

The block will start sliding in the plate, $ma_t = f_r a_t = \mu g$

Tangential acceleration, $a_t = \frac{dv}{dt} = 2bt = \mu g$

$$t = \frac{\mu g}{2b}$$

23.(a)

24.(a) The change in the velocity of the rocket

$$v_f - v_i = u\ ln\left(\frac{M_0}{M}\right) \quad [v_i = 0]$$

$$v \propto ln\left(\frac{M_0}{M}\right)$$

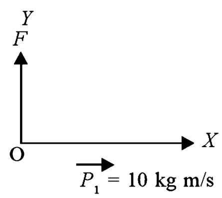
25.(b) Initial K.E. = 2J = $\frac{p_1^2}{2m} = \frac{10^2}{2m} \Rightarrow m = 25$ kg

Change in momentum along Y-axis

$p_2 = F_t$

$= (0.2\text{ N})(2 \times 60\text{s})$

$= 24$ kg m/s



Final momentum = $\sqrt{p_1^2 + p_2^2}$

$= \sqrt{10^2 + 24^2} = 26$ kg m/s

Final K.E. energy = $\frac{(26)^2}{2 \times 25} = 13.52$ J

26.(c) $T = \frac{2m_1m_2g}{m_1 + m_2} = \frac{1}{8}(m_1 + m_2)g$

$(m_1 + m_2)^2 = 16\, m_1m_2 \Rightarrow m_1 + m_2 = 4\sqrt{m_1m_2}$

$(m_1 - m_2)^2 = 12\, m_1m_2 \Rightarrow m_1 - m_2 = \sqrt{12}\sqrt{m_1m_2}$

Acceleration = $\frac{(m_1 - m_2)g}{(m_1 + m_2)}$

$= \frac{\sqrt{12}}{4}g = \frac{\sqrt{3}g}{2}$

27.(d) Area under F-x graph

$= \sqrt{m_1m_2}\ (80 + 40)600 \times 10^{-3} + \frac{1}{2}(80 + 110) \times 10^{-3} +$

$\frac{1}{2}(110 + 70)600 \times 10^{-3} = 104.25$ J

Area under F-x graph = work done against resistive force

= change in kinetic energy

$$104.25 = \frac{1}{2}m\left(v_i^2 - v_f^2\right)$$

$$v_f^2 = v_i^2 - \frac{2 \times 104.25}{58 \times 10^{-3}}$$

$= 100^2 - 36 \times 10^2 = 6400$

Exit speed of arrow = $v_f = 80$ m/s

28.(d) The centre of mass corresponds to point A and is thus moving at a speed V_0 relative to the ground

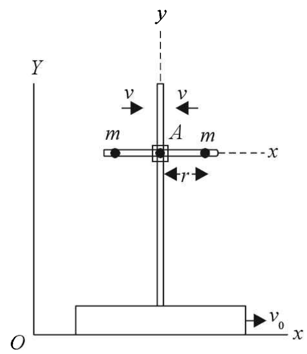


Fig. 36

Kinetic energy for some reference can be considered to be composed of two parts (1) th kinetic energy of the total mass moving relative to that reference with the velocity of the mass centre and (2) the kinetic energy of the motion of the particles relative to the mass centre.

$$\text{K.E.} = \frac{1}{2}Mv_c^2 + \frac{1}{2}\sum_{i=1}^{n} m_i\rho_{ci}^2$$

Here $v_c = V_0, M = m + m$

$$(\text{K.E.})\text{cm} = \frac{1}{2}(2m)V_0^2 = mV_0^2$$

$$\frac{1}{2}\sum_{i=1}^{n} m_i\rho_{ci}^2 = \frac{2 \times m}{2}\left[r^2 + (\omega r)^2\right] = m\left[r^2 + (\omega r)^2\right]$$

Total kinetic energy of the two masses relative to the ground = $mV_0^2 + m\left[v^2 + (\omega r)^2\right]$

29.(b) By the conservation of energy,

work done = change in potential energy + change in kinetic energy

$$T\theta - \mu N_A d = [(m_A + m_B)g\, d \sin 30°] + \left[\frac{1}{2}(m_A + m_B)v^2 + \frac{1}{2}I_B\omega_B^2 - 0\right]$$

where $\theta = \frac{d}{r} = \frac{2\ m}{0.2\ m} = 10, T = 65\ Nm$

$m_A = 20$ kg, $m_B = 30$ kg, $I_B = \frac{m_B R^2}{2}, R\omega = v,$

$N_A = m_A g \cos 30°$

$$65 \times \left(\frac{2}{0.2}\right) - (0.25)\left(20 \times 10 \times \frac{\sqrt{3}}{2}\right) \times 2$$

$$= \left[(20 + 30)10 \times 2 \times \frac{1}{2}\right] + \frac{1}{2}(20 + 30)v^2 + \frac{1}{4} \times 30v^2$$

$650 - 86.6 = 500 + 25v^2 + 7.5v^2$

$63.4 = 32.5v^2$

$v = \sqrt{1.95} \simeq \sqrt{2}$ ms

30.(c) Acceleration of the body, a = $\frac{F}{m} = \frac{(6\hat{i}+2\hat{j})}{2} = 3\hat{i}+\hat{j}$

Velocity at 3 sec, v = at = $3(3\hat{i}+\hat{j})$

$|v| = (9^2+3^2)^{\frac{1}{2}}$

K.E. of the body when t = 3 seconds = $\frac{1}{2}mv^2$

$= \frac{1}{2}\times 2\times 90 = 90J$

31.(b) Newton's law gives e = $\frac{(v_2 - v_1)}{(u_2 - u_1)}$

Given $u_2 = 8$ m/s, $v_1 = 2$ m/s, $v_2 = 5$ m/s

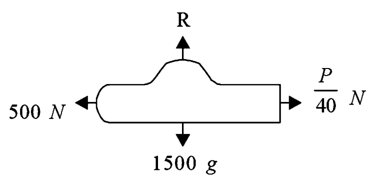
$e = \frac{(5-2)}{(2-8)} = \frac{3}{6} = \frac{1}{2}$

32.(b)Let P be power of the engine.

At maximum speed there is no acceleration.

$\frac{P}{40} = 500$

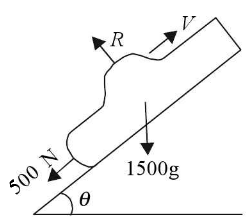
$P = 2\times 10^4 W$



On the incline, let the maximum speed be v.

At maximum speed there is no acceleration.

Fig. 37



$\frac{P}{v} = 500 + 1500\,g\sin\theta = 500 + 1500\times 9.8\times\frac{1}{49} = 800$

$v = \frac{2\times 10^4}{800} = 25$ m/s

33.(c)Velocity of the ball when it strikes the plane

$u = \sqrt{2gh} = \sqrt{2\times 9.8\times 10}$

Componet of the velocity on the line of impact before impact = $u_y = -14\cos 30° = -7\sqrt{3}$ m/s

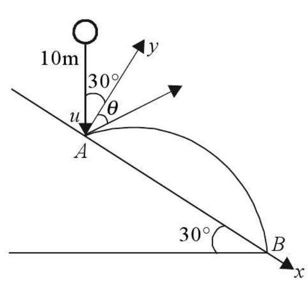


Fig. 38

Velocity of ball impact.

$v_y - 0 = [e(0-(-u))]$

$v_y = eu_y = \frac{\sqrt{3}}{2}\times 7\sqrt{3} = 10.5$ m/s

The motion normal to the plane (i.e., from A to B)

$0 = v_y\,T - \frac{1}{2}g\cos 30°\,T^2$

time of flight, T = $\frac{2v_y}{g\cos 30°} = \frac{2\times 10.5}{9.8\frac{\sqrt{3}}{2}}$

$= 2.47$ sec $\simeq 2.5$ sec

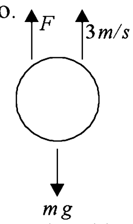
34.(c) When the body is lifted with constant velocity, v = 3 m/s, the net force acting on the body is zero.

i.e., F. = mg

Power required, $P_1 = \vec{F}\cdot\vec{v}$

= mg. $v = 10^3\times 10\times 3$

$= 3\times 10^4$ J/s = 30 kW



35.(c) When the body has a velocity of 3 m/s upward and is accelerated 1.5 m/s² upward, the net force

F = mg = ma

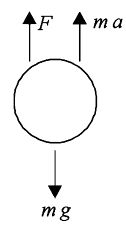
Fa = m(g + a)

$= 10^3(10+1.5) = 11.5\times 10^3$

$P_2 = \vec{F}\cdot\vec{v}$

Power required, $P_2 = 11.5\times 10^3\times 3$

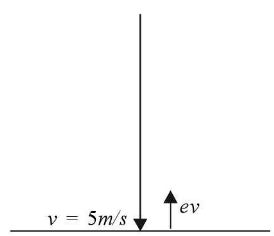
$= 34.5\times 10^3$ watt = 34.5 kw



36.(c) e = 0.5, v = 5m/s

Total time taken by the ball to come to rest

Fig. 39



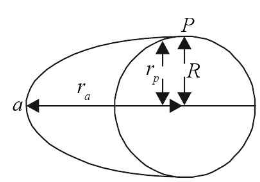
$= \frac{2v}{g}e\,(1 + e + e^2 + ...)$

$= \frac{2v}{g}e\left(\frac{1}{1-e}\right) = \frac{2\times 5}{10}0.5\left(\frac{1}{1-0.5}\right) = 1$s

37. $r_a = a(1+e)$

$r_p = a(1-e)$

$\frac{v_p}{v_a} = \frac{1+e}{1-e}$



38.(b) $\frac{GMm}{R_m^2} = m\,\omega_m^2 R_m$

$\omega_m^2 = \frac{GM}{R_m^3}$

$\omega_s^2 = \frac{GM}{R_s^3}$

$\left(\frac{\omega_m}{\omega_s}\right)^2 = \left(\frac{R_s}{R_m}\right)^3 \Rightarrow \frac{R_m}{R_s} = \left(\frac{T_m}{T_s}\right)^{\frac{2}{3}}$

$= (29)^{\frac{2}{3}} \quad \begin{bmatrix} T_s = 1\text{day} \\ T_m = 29\text{days} \end{bmatrix}$

39.(c) Let T, and T_2 be the tension in PA and QB respectively for horizontal equilibrium.

$T_1 \sin 30° = T_2 \sin 60°$

For rotational equilibrium about C,

$(AG)\, T_1 \cos 30° = (GB)\, T_2 \cos 60°$

$\frac{AG}{GB} = \frac{T_2}{T_1} \times \frac{\cos 60°}{\cos 30°} = \frac{\tan 30°}{\tan 60°} \frac{1}{\sqrt{3}} \times \frac{1}{\sqrt{3}} = \frac{1}{3}$

40.(d) mu = Mv (conservation of linear momentum) ...(i)

$\text{mu} = \frac{L}{3} = I\omega$ (conservation of angular momntum)

$= \frac{ML^2\omega}{12}$

From equations (1) and (2), $\frac{v}{\omega} = \frac{L}{4}$ ...(ii)

By the conservation of energy, $\frac{1}{2}\text{mu}^2 =$

$\frac{1}{2}\left(\frac{ML^2}{12}\right)\omega^2 + \frac{1}{2}Mv^2$ ...(iii)

Solving, $\frac{M}{m} = \frac{7}{3}$

41.(d) For translational motion, $ma = \mu R = \mu mg$

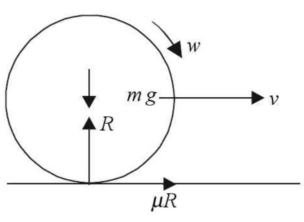


Fig. 40

$\Rightarrow \quad v = at = \mu gt$ ...(i)

For rotational motion, $\omega = \omega_0 - \alpha t$ ...(ii)

$\frac{2}{5}mr^2 \alpha = \mu mgr$

$\alpha = \frac{5\mu g}{2r}$ ...(iii)

$\therefore \quad \omega = \omega_0 - \frac{5\mu gt}{2r}$

When $v = \omega r$, slipings ceases,

$\text{ugt} = rw_o - \frac{5\mu gt}{2}$

$t = \frac{2rw_o}{7\mu g} \Rightarrow w = w_o - \frac{5}{7}w_o = \frac{2w_o}{7}$

Number of revolutions made = $\frac{w_{av}t}{2\pi}$

$= \frac{\left(w_o + \frac{2w_o}{7}\right)}{2} \frac{t}{2\pi}$

$= \frac{9w_o}{28\pi} \times \frac{2rw_o}{7\mu g} = \frac{9rw_o^2}{2\pi(49\mu g)}$

42.(d) For horizontal equilibrium,

$F_1 = F_2 = F$

Let A, B, C respectively the centres of gravity of the three bars. Let f be the normal contact force between two bars.

For vertical equilibrium of A,

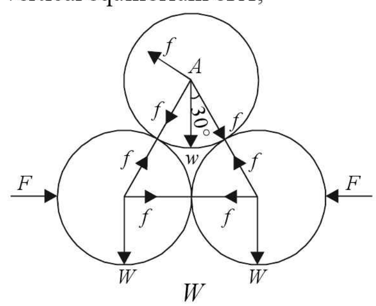


Fig. 41

$2f \cos 30°\, W \Rightarrow f = \frac{W}{\sqrt{3}}$

For equilibrium of B,

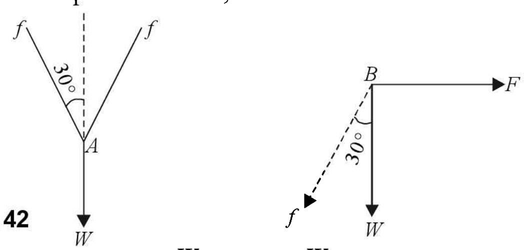


Fig. 42

$F = f \sin 30° = \frac{W}{\sqrt{3}} \times \frac{1}{2} = \frac{W}{2\sqrt{3}}$

43. (d) By the conservation of angular momentum,

$L_i = L_f$

$\Rightarrow I_i w_i = I_f w_f$

$\left[3m\frac{(2a)^2}{12} + 2m\left(\frac{a}{2}\right)^2\right] 2\sqrt{\frac{g}{a}}\ [ma^2 + 2ma^2]\, w_f$

$\frac{3ma^2}{2} \times 2\sqrt{\frac{g}{a}} = 3ma^2 w_t$

$\Rightarrow w_t = \sqrt{\frac{g}{a}}$

44.(a) Consider a small strip ABCD.

Area of this element $dA = y\,dx$

Mass of this element $(dm) = pydx$

Moment of inertia of this element about y-axis dI

$= (dm)x^2$

$$I = \int dI = \int p y dx x^2 = \int_{-4}^{4} p\left(4 - \frac{x^2}{4}\right) x^2\, dx$$

$$\left[\because y = 4 - \frac{x^2}{4} \text{ and } y = 0 \text{ when } x = \pm 4\right]$$

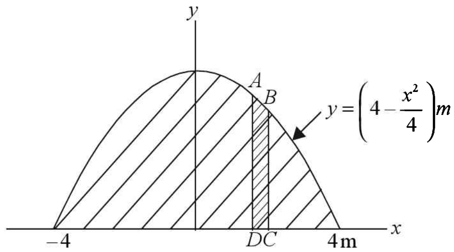


Fig. 43

$$= p\left[4\frac{x^3}{3} - \frac{x^5}{20}\right]_{-4}^{4} = 2p\left[4\frac{(4)^3}{3} - \frac{(4)^5}{20}\right]$$

$$= 2p\left(\frac{256}{3} - \frac{512}{10}\right) = 2p\,\frac{[2560 - 1536]}{30} = \frac{1024}{15}p$$

Given $p = 15\ g/m^2$

$$I = \frac{1024}{15} \times 15 \times 10^{-3} \simeq 1 \text{kg}\, m^2$$

45.(d)

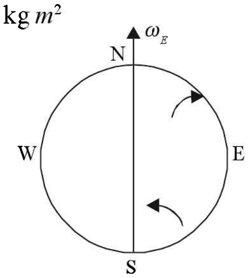


Fig. 44

All these effects are due to coriois force due to rotation of earth. The earth's angular velocity vector wE points north. When an object in the southem hemisphere that is moving at a tangent to the earth's surface travels north, the coriolis acceleration $-\vec{w}_E \times \vec{v}_{rel}$ points w west and tends to cause the objects to turn to the left.

46.(c)

47.(c) By the conservation of energy,

$$\frac{1}{2}\left(\frac{ml^2}{3}\right)w_o^2 = \frac{1}{2}\left(\frac{ml^2}{3}\right)w^2 + \frac{1}{2}k\left[\left(2\frac{l}{2} - \sqrt{2}\frac{l}{2}\right)^2\right]$$

$-\text{mg}\,\frac{l}{2}$ Given $w_o = 4$ rad/s, $m = 30$ kg, $l = 2\ m$,

$k = 3000$ N/m

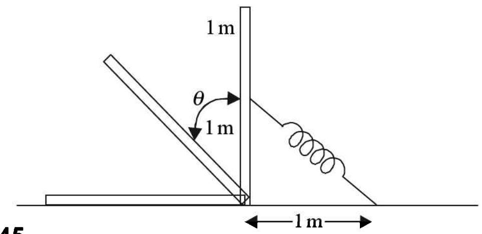


Fig. 45

Unstretched length of spring $= \sqrt{2}\left(\frac{l}{2}\right)$

Extension, $x = 2\left(\frac{l}{2}\right) - \sqrt{2}\left(\frac{l}{2}\right)$

$$w_o^2 - w^2 = \left[\frac{1}{8}kl^2(2-\sqrt{2})^2 - mg\frac{l}{2}\right]\frac{6}{ml^2}$$

$= 75\,(2 - \sqrt{2})^2 - 15 > 0$

i.e., $w_o > w$.

48.(b) A free body diagram of the bar during the impact period is shown in figure. Since the time of the impact is very small, the angular impulse of the weight with respect to the fixed bearing at C may be neglected during the impact period and the angular momentum of the body about an axis through C is conserved.

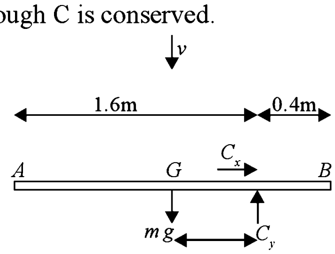


Fig. 46

Given m = 20 kg, v = 10 m/s, GC = 0.6m

mv (GC) $= I_C w$

where $I_C = I_G + mx^2$

$$= \left(\frac{ml^2}{12} + mx^2\right) = \left(\frac{20\times 2^2}{12} + 20\times 0.6^2\right)$$

$= 6.66 + 7.2 = 13.866$

$$\omega = \frac{20\times 10\times 0.6}{\left[\frac{20\times 2^2}{12} + 20\times(0.6)^2\right]}$$

$= 8.65$ rad/s in the anticlock wise direction.

QUESTIONS FROM COMPETITIVE EXAMINATIONS

AIEEE 2002

1. A ball whose kinetic energy is E, is projected at an angle of 45° to the horizontal. The kinetic energy of the ball at the highest point of its flight will be:
(a) E (b) $\frac{E}{\sqrt{2}}$
(c) $\frac{E}{2}$ (d) zero

2. From a building two balls A and B are thrown such that A is thrown upwards and B downwards
(a) $v_B > v_A$ (b) $v_A = v_B$
(c) $v_A > v_B$
(d) their velocities depend on their masses

3. If a body loses half of its velocity on penetrating 3 cm in a wooden block, then how much will to penetrate more before coming to rest?
(a) 1 cm (b) 2 cm
(c) 3 cm (d) 4 cm

4. If suddenly the gravitational force of attraction between earth and a satellite revolving around it becomes zero, then the satellite will
(a) continue to move in its orbit with same velocity
(b) move tangentially to the original orbit with same velocity
(c) become stationary in its orbit
(d) move towards the earth

5. Energy required to move a body of mass m from an orbit of radius $2R$ to $3R$ is:
(a) $GMm/12R^2$ (b) $GMm/3R^2$
(c) $GMm/8R$ (d) $GMm/6R^2$

6. Two identical particles move towards each other with velocity $2v$ and v respectively. The velocity of centre of mass is
(a) v (b) $v/3$
(c) $v/2$ (d) zero

7. The escape velocity of a body depends upon mass as:
(a) m^0 (b) m^1
(c) m^2 (d) m^3

8. Identify the pair whose dimensions are equal.
(a) Torque and work (b) Stress and energy
(c) Force and stress (d) Force and work

9. The kinetic energy needed to project a body of mass m from the earth's surface (radius R) to infinity is:
(a) $mgR/2$ (b) $2mgR$
(c) mgR (d) $mgR/4$

10. Initial angular velocity of a circular disc of mass M is ω_1. Then two small spheres of mass m are attached gently to two diametrically opposite points on the edge of the disc. What is the final angular velocity of the disc?
(a) $\left(\frac{M+m}{M}\right)\omega_1$ (b) $\left(\frac{M+m}{m}\right)\omega_1$
(c) $\left(\frac{M}{M+4m}\right)\omega_1$ (d) $\left(\frac{M}{M+2m}\right)\omega_1$

11. The minimum velocity (in ms^{-1}) with which a car driver must traverse a flat curve of radius 150 m and coefficient of friction of 0.6 to avoid skidding is:
(a) 60 (b) 30
(c) 15 (d) 25

12. A cylinder of height 20 m is completely filled with water. The velocity of efflux of water (in ms^{-1}) through a small hole on the side wall of the cylinder near its bottom is
(a) 10 (b) 20
(c) 25.5 (d) 5

13. A spring of force constant 800 N/m has an extension of 5 cm. The work done in extending it form 5 cm to 15 cm is:
(a) 16 J (b) 8 J
(c) 32 J (d) 24 J

14. A lift is moving down with acceleration a. A man in the lift drops a ball inside the lift. The acceleration of the ball as observed by the man in the lift and a man standing stationary on the ground are respectively:
(a) g, g (b) $g, g-a$
(c) $g-a, g$ (d) a, g

15. A solid sphere, a hollow sphere and a ring are released from top of an inclined plane (frictionless) so that they slide down the plane. Then maximum acceleration down the plane is for (for rolling):
(a) solid sphere (b) hollow sphere
(c) ring (d) all same

16. Moment of inertia of a circular wire of mass M and radius R about its diameter is:
(a) $\frac{MR^2}{2}$ (b) MR^2
(c) $2MR^2$ (d) $\frac{MR^2}{4}$

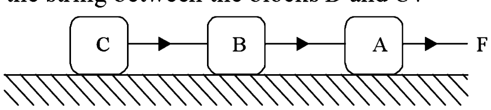
17. When forces F_1, F_2, F_3 are acting on a particle of mass m such that F_2 and F_3 are mutually perpendicular, then the particle remains stationary. If the force F_1 is now removed then the acceleration of the particle is:
(a) F_1/m (b) F_2F_3/mF_1
(c) $(F_2 - F_3)/m$ (d) F_2/m

18. Two forces are such that the sum of their magnitudes is 18 N and their resultant is perpendicular to the smaller force. Then the magnitudes of the forces are:
(a) 12 N, 6 N (b) 13 N, 5 N
(c) 10 N, 8 N (d) 16 N, 2 N

19. Speed of two identical cars are u and $4u$ at a specific instant. The ratio of the respective distances at which the two cars are stopped from that instant in sametime is
(a) 1 : 1 (b) 1 : 4
(c) 1 : 8 (d) 1 : 16

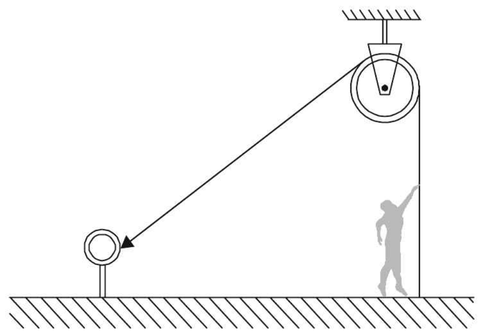
20. A light string passing over a smooth light pulley connects two blocks of masses m_1 and m_2 (vertically). If the acceleration of the system is $g/8$, then the ratio of the masses is:
(a) 8 : 1 (b) 9 : 7
(c) 4 : 3 (d) 5 : 3

21. The identical blocks of masses $m = 2$ kg are drawn by a force $F = 10.2\ N$ with an acceleration of 0.6 ms^{-2} on a frictionless surface, then what is the tension (in N) in the string between the blocks B and C?



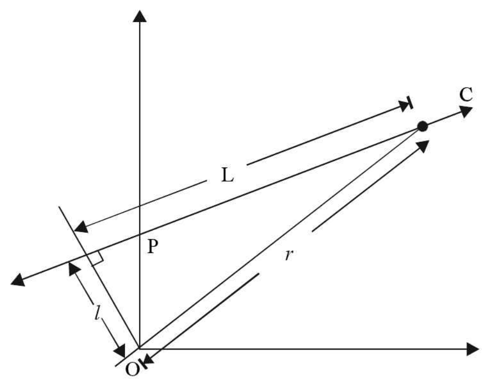
(a) 9.2 (b) 7.8
(c) 4 (d) 9.8

22. One end of massless rope, which passes over massless and frictionless pulley P is tied to a hook c while the other end is free. Maximum tension that the rope can bear is 360 N. With what value of minimum safe acceleration (in ms^{-2}) can a man of 60 kg climb down the rope?



(a) 16 (b) 6
(c) 4 (d) 8

23. A particle of mass m moves along line PC with velocity v as shown. What is the angular momentum of the particle about P?



(a) mvL (b) mvl
(c) mvr (d) zero

Answers

1.	(c)	**2.**	(b)	**3.**	(a)	**4.**	(c)	**5.**	(d)	**6.**	(c)	**7.**	(a)
8.	(a)	**9.**	(c)	**10.**	(c)	**11.**	(b)	**12.**	(b)	**13.**	(b)	**14.**	(c)
15.	(d)	**16.**	(b)	**17.**	(a)	**18.**	(b)	**19.**	(b)	**20.**	(b)	**21.**	(b)
22.	(c)	**23.**	(d)										

Explanations

1(c) At the hihgest point of its flight, vertical component of velocity is zero and only horizontal component is left which is

$u_x = u\cos\theta$

Given: $\theta = 45°$

$$u_x = u\cos 45° = \frac{u}{\sqrt{2}}$$

Hence, at the highest point kinetic energy

$$E' = \frac{1}{2} mu_x^2$$

$$= \frac{1}{2}m\left(\frac{u}{\sqrt{2}}\right)^2 = \frac{1}{2}m\left(\frac{u^2}{2}\right)$$

$$= \frac{E}{2} \qquad \left(\because \frac{1}{2}mu^2 = E\right)$$

2(b) From conservation of energy, potential energy at height h = K.E. at ground Therefore, at height h, P.E. of ball A
P.E. $= m_A gh$

K.E. at ground $= \frac{1}{2} m_A v_A^2$

So, $m_A gh = \frac{1}{2} m_A v_A^2$

$v_A = \sqrt{2gh}$

Similarly, $v_B = \sqrt{2gh}$

Therefore, $v_A = v_B$

3(a) Let initial velocity of body at point A is v. AB is 3 cm.
From $v^2 = u^2 - 2as$

$$\left(\frac{v}{2}\right)^2 = v^2 - 2a \times 3$$

$$a = \frac{v^2}{8}$$

Let on penetrating 3 cm in a wooden block, the body moves x distance from B to C.
So, for B to C

$u = \frac{v}{2}$, $v = 0$

$s = x$, $a = \frac{v^2}{8}$ (deceleration)

$\therefore$ $(0)^2 = \left(\frac{v}{2}\right)^2 - 2.\frac{v^2}{8}.x$

$x = 1$

4(c) When gravitational force becomes zero, then centripetal force on satellite becomes zero and therefore, the satellite will become stationary in its orbit.

5(d) Gravitational potential energy of body will be

$$E = -\frac{GM_e m}{r}$$

where M_e = mass of earth,
m = mass of the body,
R = radius of earth
AT $r = 2R$,

$$E_1 = -\frac{GM_e m}{(2R)}$$

AT $r = 3R$,

$$E_2 = -\frac{GM_e m}{(3R)}$$

Energy required to move a body of mass m from an orbit of radius $2R$ to $3R$ is

$$\Delta E = \frac{GM_e m}{R}\left[\frac{1}{2} - \frac{1}{3}\right]$$

$$= \frac{GM_e m}{6R}$$

6(c) By the conservation of linear momentum

$(m + m)v' = m \,.\, 2v - mv$

$2mv' = mv$

$v' = \frac{v}{2}$

7(a) Escape velocity $= \sqrt{2gR_e}$ So, escape velocity is independent of ***m*** So, answer is m^0

8(a) The dimensions of torque and work is $[MT^2T^{-2}]$.

9(c) The kinetic energy required to project a body of mass m from eath's surface to infinity is known as escape energy. Therefore,

$$K.E. = \frac{GM_e m}{R}$$

$$= mgR \qquad \left(\because gR = \frac{GM_e}{R}\right)$$

10(c) Conservation of angular momentum gives

$$\frac{1}{2} MR^2 \omega_1 = \left(\frac{1}{2}MR^2 + 2MR^2\right)\omega_2$$

$$\Rightarrow \frac{1}{2} MR^2 \omega_1 = \frac{1}{2} R2\,(M + 4m)\,\omega_2$$

$$\therefore \omega_2 = \left(\frac{M}{M + 4m}\right)\omega_1$$

11(b) Using the relation

$$\frac{mv^2}{r} = \mu R, R = mg$$

$$\frac{mv^2}{r} = \mu R \qquad \text{or } v^2 = \mu rg$$

or $v^2 = 0.6 \times 150 \times 10$

$\Rightarrow$ $v = 30$ m/s

12(b) As, P.E. = K.E.

$$mgh = \frac{1}{2} mv^2$$

$$v = \sqrt{2gh}$$

$$= \sqrt{2 \times 10 \times 20} \qquad [\text{Hence : } g = 10 \text{ m/s}^2]$$

$= 20$ m/s

13(b) The work is stored as the P.E. of the body and is given by,

$$U = \int_{x_1}^{x_2} F_{external} dx$$

or $$U = \int_{x_1}^{x_2} kx\, dx$$

$$= \frac{1}{2} k\,(x_2^2 - x_1^2)$$

$$= \frac{800}{2} = [(0.15)^2 - (0.05)^2]$$

$[k = 800 \text{ (given)}]$
$= 400[0.2 \times 0.1]$
$= 8$ joule

14(c) Apparent weight of ball

$w' = w - R$

$R = ma$ acts upward $= ma$

$w' = mg - ma = m(g - a)$

Hence, apperent acceleration in thelift is $g - a$. Now if the man is standing stationary on the ground, then the apparent acceleration of the falling ball is g.

15(d) The acceleration of a body of mass m on an inclined plane

$$a = \frac{g\sin\theta}{\left(1 + \frac{K^2}{R^2}\right)}$$

where K = radius of gyration.

Since, the inclined plane is frictionless, then there will be no rolling and the mass will only slide down. In this situation

$K = R$

Hence, acceleration $a = g\sin\theta$ is same for solid sphere, hollow sphere and ring.

16(b) We know that M.I. of a circular wire of mass M and radius R about its diameter is MR^2.

17(a) The formula for force is given by

$F_1 = ma$

Acceleration of the particle

$$a = \frac{F_1}{m}$$

21. As the string is inextensible, both masses have the same acceleration a. Also, the pulley is massless and frictionless, hence, the tension at both ends of the string is the same. Suppose the mass m_2 is greater than masss m_1, so, the heavier mass m_2 is accelerated downward s and the lighter mass m_1 is accelerated upwards.

22. To climb on the rope, the downward force should be equal to or less than upward forces.

Therefore, $W = T + ma$

a is acceleration of mass m

$T = 360$ N,

$m = 60$ kg

$mg = T + ma$ [Let $g = 10$ m/s^2]

The maximum safe acceleration at which a man of 60 kg weight can climb is

$60 \times 10 = 360 + 60 \times a$

$a = 4$ m/s

23. Let a particle A of mass m whose position vector is $\vec{x}$ w.r.t. the point P in an inertial reference frame.

From the formula

$\vec{p} = m\vec{v}$

Then, angular momentum

$\vec{J} = \vec{x} \times \vec{p}$

θ is the angle between x and p.

Since, $\theta = 0°$

Hence, $j = xp\sin\theta = 0$

AIEEE 2003

1. A spring balance is attached to the ceiling of a lift. A man hangs his bag on the spring and the spring reads 49 N, when the lift is stationary. If the lift moves downwards with an acceleration of 5 m/s^2, the reading of the spring balance will be:

(a) 24 N (b) 74 N
(c) 15 N (d) 49 N

2. Dimensions of $\frac{1}{\mu_0\varepsilon_0}$, where symbols have their usual meaning, are:

(a) $[L^{-1}T]$ (b) $[L^2T^2]$
(c) $[L^2T^{-2}]$ (d) $[LT^{-1}]$

3. A circular disc X of radius R is made from an iron plate of thickness t, and another disc Y of radius $4R$ is made from an iron plate of thickness $t/4$. Then the relation between the moment of inertia I_X and I_Y is:

(a) $I_Y = 32I_X$ (b) $I_Y = 16I_X$
(c) $I_Y = I_X$ (d) $I_Y = 64I_X$

4. The time period of a satellite of earth is 5 hours. If the separation between the earth and the satellite is increased to 4 times the previous value, the new time period will become:

(a) 10 hour (b) 80 hour
(c) 40 hour (d) 20 hour

5. A particle performing uniform circular motion has angular momentum L. If its angular frequency is doubled and its kinetic energy halved, then the new angular momentum is:

(a) $L/4$ (b) $2L$
(c) $4L$ (d) $L/2$

6. Two spherical bodies of mass M and $5M$ and radii R and $2R$ respectively are released in free space with initial separation between their centres equal to $12R$. If they attract each other due to gravitational force only, then the distance covered by the smaller body just before collision is:

(a) $2.5R$ (b) $4.5R$
(c) $7.5R$ (d) $1.5R$

7. A car moving with a speed of 50 km/hr, can be stopped by brakes after at least 6m. If the same car is moving at a speed of 100 km/hr, the minimum stopping distance is:

(a) 12m (b) 18m
(c) 24m (d) 6m

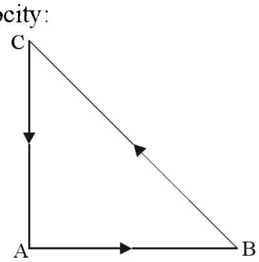
8. A boy playing on the roof of a 10 m high building throws a ball with a speed of 10 m/s at an angle of 30° with the horizontal. How far from the throwing point will the ball be at the height of 10 m from the ground?
[g = 10 m/s², sin 30° = 1/2, cos 30° = $\sqrt{3}/2$]

(a) 5.20 m (b) 4.33 m
(c) 2.60 m (d) 8.66 m

9. The physical quantities not having same dimensions are:

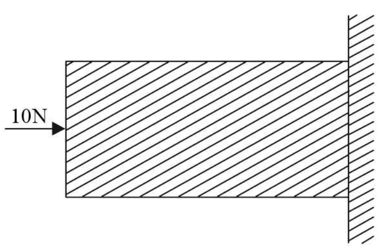
(a) torque and work
(b) momentum and Planck's constant
(c) stress and Young's modulus
(d) speed and $(\mu_0\varepsilon_0)^{-1/2}$

10. Three froces start acting simultaneoulsy on a particle moving with velocity $\vec{v}$. These forces are represented in magnitude and direction by the three sides of a triangle *ABC* (as shown). The particle will now move with velocity:



(a) less than $\vec{v}$
(b) greater than $\vec{v}$
(c) |*v*| in the direction of largest force *BC*
(d) $\vec{v}$, remaining unchanged

11. A horizontal force of 10 N is necessary to just hold a block stationary against a wall. The coefficient of friction between the block and the wall is 0.2. The weight of the block is:



(a) 20 N (b) 50 N
(c) 100 N (d) 2 N

12. A marble block of mass 2 kg lying on ice when given a velocity of 6 m/s is stopped by friction in 10 s. Then the coefficient of friction is:

(a) 0.02 (b) 0.03
(c) 0.06 (d) 0.01

13. Consider the following two statements:

A. Linear momentum of a system of particles is zero. Then:

B. Kinetic energy of a system of particles is zero. Then:

(a) *A* does not imply *B* and *B* does not imply *A*
(b) *A* implies *B* but *B* does not imply *A*
(c) *A* does not imply *B* but *B* implies *A*
(d) *A* implies *B* and *B* implies *A*

14. A block of mass *M* is pulled along a horizontal frictionless surface by a rope of mass *m*. If a force *P* is applied at the free end of the rope, the force exerted by the rope on the block is:

(a) $\frac{Pm}{M+m}$ (b) $\frac{Pm}{M-m}$
(c) P (d) $\frac{Pm}{M+m}$

15. A light spring balance hangs from the hook of the other light spring balance and block of mass *M* kg hangs from the former one. Then the true statement about the scale reading is:

(a) both the scales read *M* kg each
(b) the scale of the lower one reads *M* kg and of the upper one zero
(c) the reading of the two scales can be anything but the sum of the reading will be *M kg*
(d) both the scales read *M/2* kg

16. A wire suspended vertically from one of its ends is stretched by attaching a weight of 200 N to the lower end. The weight stretched the wire by 1 mm. Then the elastic energy stored in the wire is:

(a) 0.2 J (b) 10 J
(c) 20 J (d) 0.1 J

17. The escape velocity for a body projected vertically upwards from the surface of earth is 11 km/s. If the body is projected at an angle of 45° with the vertical, the escape velocity will be:

(a) 11 $\sqrt{2}$ km/s (b) 22 km/s
(c) 11 km/s (d) 11/ $\sqrt{2}$ m/s

18. Two particles *A* and *B* of equal masses are suspended from two massless springs of spring constant k_1 and k_2, respectively. If the maximum velocities, during oscillations are equal, the ratio of amplitudes of *A* and *B* is:

(a) $\sqrt{k_1/k_2}$ (b) k_2/k_1
(c) $\sqrt{k_2/k_1}$ (d) k_2/k_1

19. Let $\vec{F}$ be the force acting on a particle having position vector $\vec{r}$ and $\vec{\tau}$ be the torque of this force about the origin. Then:

(a) $\vec{r}.\vec{F} = 0$ and $\vec{F}.\vec{\tau} = 0$ (b) $\vec{r}.\vec{\tau} \neq 0$ and $\vec{F}.\vec{\tau} = 0$
(c) $\vec{r}.\vec{\tau} \neq 0$ and $\vec{F}.\vec{\tau} = 0$ (d) $\vec{r}.\vec{\tau} = 0$ and $\vec{F}.\vec{\tau} = 0$

20. The coordinates of a moving particle at any time are given by $x = \alpha t^3$ and $y = \beta t^3$. The speed of the particle at time t is given by:

(a) $3t\sqrt{\alpha^2+\beta^2}$ (b) $3t^2\sqrt{\alpha^2+\beta^2}$

(c) $t^2\sqrt{\alpha^2+\beta^2}$ (d) $\sqrt{\alpha^2+\beta^2}$

21. A spring of spring constant 5×10^3 N/m is stretched initially by 5 cm from the unstretched position. Then the work required to stretch it further by another 5 cm is:

(a) 12.05 N-m (b) 18.75 N-m
(c) 25.00 N-m (d) 6.25 N-m

22. A body is moved along a straight line by a machine delivering a constant power. The distance moved by the body in time t is proportional to:

(a) $t^{3/4}$ (b) $t^{3/2}$
(c) $t^{1/4}$ (d) $t^{1/2}$

23. A rocket with a lift-off mass 3.5×10^4 kg is blasted upwards with an initial acceleration of 10 m/s^2. Then the initial thrust of the blast is:

(a) 3.5×10^5 N (b) 7.0×10^5 N
(c) 14.0×10^5 N (d) 1.75×10^5 N

Answers

1.	(a)	**2.**	(c)	**3.**	(d)	**4.**	(c)	**5.**	(a)	**6.**	(c)	**7.**	(c)
8.	(d)	**9.**	(b)	**10.**	(d)	**11.**	(d)	**12.**	(c)	**13.**	(c)	**14.**	(d)
15.	(a)	**16.**	(a)	**17.**	(c)	**18.**	(c)	**19.**	(d)	**20.**	(b)	**21.**	(b)
22.	(b)	**23.**	(a)										

Explanations

1(a) In stationary position,

$mg = 49$

$m = \dfrac{49}{9.8} = 5$ kg

When lift moves downwards.

$mg - T = ma$

Reading of balance

$T = mg - ma$

$= 5\,(9.8 - 5)$

$= 5 \times 4.8$

$= 24.0$ N

2(c) As we know that formula of velocity is

$v = \dfrac{1}{\sqrt{\mu_0 \varepsilon_0}}$

$\therefore\ v^2 = \dfrac{1}{\mu_0 \varepsilon_0} = [LT^{-1}]^2$

$\therefore\ \dfrac{1}{\mu_0 \varepsilon_0} = [L^2T^{-2}]$

3(d) Mass of disc (X), $m_X = \pi R^2 t\,\rho$

where ρ = density of material of disc

$\therefore\ I_X = \dfrac{1}{2} I_X R^2 = \dfrac{1}{2}\pi R^2 t\,\rho\,R^2$

$I_X = \dfrac{1}{2}\pi\,\rho\,tR^4$

Again mass of disc (Y)

$m_Y = \pi(4R)^2\,\dfrac{t}{4} = 4\pi R^2 t\,\rho$

and $I_Y = \dfrac{1}{2} m_Y (4R)^2 = \dfrac{1}{2} 4\pi R^2 t\,\rho\,.\,16R^2$

$\Rightarrow\ I_Y = 32\pi t\,\rho\,R^4$

$\therefore\ \dfrac{I_Y}{I_X} = \dfrac{32\pi t\rho R^4}{\frac{1}{2}\pi\rho t R^4}$

$\Rightarrow\ = 64$

$\therefore\ I_Y = 64\,I_X$

4(c) According to Kepler's law

$T^2 \propto r^3$

or $5^2 \propto r^3$

$(T)^2 \propto (4r)^3$

From equations (i) and (ii)

$\dfrac{25}{(T)^2} = \dfrac{r^3}{64r^3}$

$T' = \sqrt{1600}$

$T' = 40$ hour

5(a) Angular momentum

$L = I\omega$

Kinetic energy

$K = \dfrac{1}{2}I\omega^2 = \dfrac{1}{2}L\omega$ [from eq. (i)]

$\therefore\ L = \dfrac{2K}{\omega}$

Now $L' = \dfrac{2\left(\dfrac{K}{2}\right)}{2\omega}$

$\Rightarrow \quad L' = \dfrac{L}{4}$

6(c) Let at 'O' there will be a collision. If smaller sphere moves x distance to reach at O, then bigger sphere will move a distance of $(9R - x)$.

$$F = \frac{GM \times 5M}{(12R - x)^2}$$

$$a_{small} = \frac{F}{M} = \frac{G \times 5M}{(12R - x)^2}$$

$$a_{big} = \frac{F}{5M} = \frac{GM}{(12R - x)^2}$$

$$x = \frac{1}{2} a_{small} t^2 = \frac{1}{2} \frac{G \times 5M}{(12R - x)^2} t^2 \text{ ...(i)}$$

$$(9R - x) = \frac{1}{2} a_{big} t^2 = \frac{1}{2} \frac{GM}{(12R - x)^2} t^2 \text{ ...(ii)}$$

Thus, dividing eq. (i) by eq. (ii), we get

$\therefore \quad \dfrac{x}{9R - x} = 5$

$\Rightarrow \quad x = 45R - 5x$

$\Rightarrow \quad 6x = 45R$

$\Rightarrow \quad x = 7.5R$

7(c) $v^2 = u^2 + 2as$

$0 = (50 \times 5/18)^2 + 2a \times 6$

$a = -16 m/s^2$

$\therefore$ Again $v^2 = u^2 + 2as$

$0 = (100 \times 5/18)^2 - 16 \times s$

$$s = \frac{(100 \times 5)^2}{18 \times 18 \times 32} = 24.1 \approx 24 \text{ m}$$

8(d) $R = \dfrac{u^2 \sin 2\theta}{g}$

$$= \frac{10 \times 10 \times \sin 60°}{10} = 10 \times \frac{\sqrt{3}}{2}$$

$= 5 \times 1.732 = 8.66$ m

9(b) h = Planck's constant = J-s = $[ML^2T^{-1}]$

p = momentum = kg m/s = $[MLT^{-1}]$

10(d) Resultant force is zero. as three force acting on triangle force in same order. Hence by Newton's 2nd law

$\left(\vec{F} = m\dfrac{d\vec{v}}{dt}\right)$ particle $(\vec{v})$ will be same.

11(d) Force, $F = \mu R = Mg$

weight of block = $\mu R = 0.2 \times 10 = 2N$

12(c) $F = ma$

$\mu mg = ma$

$\Rightarrow \quad \mu = \dfrac{a}{g}$

Now, $v = u + at$ or $0 = 6 + 10a$

or $\quad \dfrac{0 - 6}{10} = a = -0.6$

$\therefore \quad \mu = \dfrac{0.6}{10} = 0.06$

14(d) Since, force is applied at the free end, so,

$P = (M + m)a$

Force exerted by the rope is.

$$F = Ma = \frac{PM}{M + m}$$

15(a) Both scales read M kg.

16(a) Elastic energy stored in hte wire is

$$U = \frac{1}{2} \text{stress} \times \text{strain} \times \text{volume}$$

$$= \frac{1}{2} \frac{F}{A} \times \frac{\Delta l}{L} \times AL$$

$$= \frac{1}{2} F \Delta l$$

$$= \frac{1}{2} \times 200 \times 1 \times 10^{-3} = 0.1\text{J}$$

17(c) The escape velocity is independent of angle of projection, hence, it will remain same i.e., 11km/sec.

18(c) $v_{max} = a\,\omega = a\dfrac{2\pi}{T}$

$$= \frac{2\pi a}{2\pi\sqrt{\dfrac{m}{k}}} = a\sqrt{\frac{k}{m}}$$

Hence, $\dfrac{v_{max_1}}{v_{max_2}} = \dfrac{a_1}{a_2}\sqrt{\dfrac{k_1}{k_2}}$

$\because \quad v_{max_1} = v_{max_2}$ (given)

$$\frac{a_1}{a_2} = \sqrt{\frac{k_1}{k_2}}$$

19(d) $\vec{\tau} = \vec{r} \times \vec{F}$ implies that r, F and τ all are mutually perpendicular to each other.

$\therefore \quad \vec{\tau}.\vec{r} = 0, \ \vec{F}.\vec{\tau} = 0$

20(b) $x = \alpha t^3, y = \beta t^2$

$$v_x = \frac{dx}{dt} = 3\,\alpha\, t^2$$

$$v_y = \frac{dy}{dt} = 3\,\beta\, t^2$$

Resultant velocity

$v = \sqrt{v_x^2 + v_y^2}$

$= \sqrt{9\alpha^2 t^4 + 9\beta^2 t^4}$

$= 3t^2 \sqrt{\alpha^2 + \beta^2}$

21(b) $k = 5 \times 10^3$ N/m

$x = 5$ cm

$W_1 = \frac{1}{2} k \times x_1^2 = \frac{1}{2} \times 5 \times 10^3 \times (5 \times 10^{-2})^2$

$= 6.25$ J

$W_2 = k(x_1 + x_2)^2$

$= \frac{1}{2} \times 5 \times 10^3 (5 \times 10^{-2} + 5 \times 10^{-2})^2$

$= 25$ J

22(b) $v^2 = u^2 + 2ax$

$v^2 = 2ax \qquad \Rightarrow a = \frac{v^2}{2x}$

$P = Fv = ma.v$

$= m.\frac{v^2}{2x}.v = \frac{mv^3}{2x}$

$\therefore \quad v^3 \propto x \qquad (\because P = \text{constant})$

$v \propto x^{\frac{1}{3}}$

$\frac{dx}{dt} \propto x^{\frac{1}{3}}$

$\Rightarrow \quad \int x^{\frac{1}{3}} dx \propto \int dt$

$\frac{3}{2} x^{\frac{2}{3}} \propto t$

$\therefore \quad x \propto t^{\frac{3}{2}}$

23(a) Initial thrust of the blast $= ma$

$= 3.5 \times 10^4 \times 10$

$= 3.5 \times 10^5$ N

AIEEE 2004

1. Which one of the following represents the correct dimensions of the coefficient of viscosity?

(a) $[ML^{-1}T^{-2}]$ (b) $[MLT^{-1}]$

(c) $[ML^{-1}T^{-1}]$ (d) $[ML^{-2}T^{-2}]$

2. A particle moves in a straight line with retardation proportional to its displacement. Its loss of kinetic energy for any displacement x is proportional to:

(a) x^2 (b) e^x

(c) x (d) $\log_e x$

3. A ball is released from the top of a tower of height h metres. It takes T seconds to reach the ground. What is the position of the ball in $T/3$ second?

(a) $h/9$ metre form the ground

(b) $7h/9$ metre from the ground

(c) $8h/9$ metre from the ground

(d) $17h/18$ metre from the ground

4. If $\vec{A} \times \vec{B} = \vec{B} \times \vec{A}$, then the angle between A and B is:

(a) π (b) $\pi/3$

(c) $\pi/2$ (d) $\pi/4$

5. A projectile can have the same range R for two angles of projectile. If T_1 and T_2 be the time of flights in the two cases, then the product of the two times of flights is proportional to:

(a) $1/R^2$ (b) $1/R$

(c) R (d) R^2

6. Which of the following statements is false for a particle moving in a circle with a constant angular speed?

(a) The velocity vector is tangent to the circle

(b) The acceleration vector is tangent to the circle

(c) The acceleration vector points to the centre of the circle

(d) The velocity and acceleration vectors are perpendicular to each other

7. An automobile travelling with a speed of 60 km/h, can brake to stop within a distance of 20 m. If the car is going twice as fast, i.e. 120 km/h, the stopping distance will be:

(a) 20 m (b) 40 m

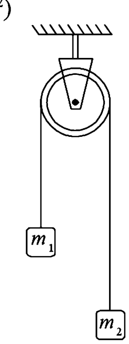
(c) 60 m (d) 80 m

8. A machine gun fires a bullet of mass 40 g with a velocity 1200 ms^{-1}. The man holding it, can exert a maximum force of 144 N on the gun. How many bullets can he fire per second at the most?

(a) One (b) Four

(c) Two (d) Three

9. Two masses $m_1 = 5$ kg and $m_2 = 4.8$ kg tied to a string are hanging over a light frictionless pulley. What is the acceleration of the masses when lift is free to move? ($g = 9.8$ m/s^2)



(a) 0.2 m/s² (b) 9.8 m/s²
(c) 5 m/s² (d) 4.8 m/s²

10. A uniform chain of length 2 m is kept on a table such that a length of 60 cm hangs freely from the edge of the table. The total mass of the chain is 4 kg. What is the work done in pulling the entire chain on the table?
(a) 7.2 J (b) 3.6 J
(c) 120 J (d) 1200 J

11. A block rests on a rough inclined plane making an angle of 30° with the horizontal. The coefficient of static friction between the block and the plane is 0.8. If the frictional force on the block is 10 N, the mass of the block (in kg) is (take g = 10 m/s²):
(a) 2.0 (b) 4.0
(c) 1.6 (d) 2.5

12. A force $\vec{F} = (5\vec{i} + 3\vec{j} + 2\vec{k})$ N is applied over a particle which displaces it from its origin to the point $r = (2\vec{i} - \vec{j})$ *m*. The work done on the particle in joules is:
(a) –7 (b) +7
(c) + 10 (d) + 13

13. A body of mass *m* accelerates uniformly from rest to v_1 in time t_1. The instantaneous power delivered to the body as a function of time *t* is:
(a) $\frac{mv_1t}{t_1}$ (b) $\frac{mv_1^2t}{t_1^2}$
(c) $\frac{mv_1t^2}{t_1}$ (d) $\frac{mv_1^2t}{t_1}$

14. A particle is acted upon by a force of constant magnitude which is always perpendicular to the velocity of the particle. The motion of the particle takes place in a plane, it follows that:
(a) its velocity is constant
(b) its acceleration is constant
(c) its kinetic energy is constant
(d) it moves in a straight line

15. A solid sphere is rotating in free space. If the radius of the sphere is increased keeping mass same which one of the following will not be affected?
(a) Moment of inertia
(b) Angular momentum
(c) Angular velocity
(d) Rotational kinetic energy

16. A ball is thrown from a point with a speed v_0 at an angle of projection θ. From the same point and at the same instant, a person starts running with a constant speed $v_0/2$ to catch the ball. Will the person be able to catch the ball? If yes, what should be the angle of projection?
(a) Yes, 60° (b) Yes, 30°
(c) No (d) Yes, 45°

17. One solid sphere *A* and another hollow sphere *B* are of same mass and same outer radii. The moment of inertia about their diameters are respectively I_A and I_B such that:
(a) $I_A = I_B$ (b) $I_A > I_B$
(c) $I_A < I_B$ (d) $I_A/I_B = d_A/d_B$

18. A satellite of mass m revolves around the earth of radius *R* at a height *x* from its surface. If g is the acceleration due to gravity on the surface of the earth, the orbital speed of the satellite is:
(a) gx (b) $\frac{gR}{R-x}$
(c) $\frac{gR^2}{R+x}$ (d) $\left(\frac{gR^2}{R+x}\right)^{1/2}$

19. The time period of an earth satellite in circular orbit is independent of:
(a) the mass of the satellite
(b) radius of its orbit
(c) both the mass and radius of the orbit
(d) neither the mass of the satellite nor the radius of its orbit

20. If *g* is the acceleration due to gravity on the earth'surface, the gain in the potential energy of an object of mass m raised from the surface of the earth to a height equal to the radius *R* of the earth, is:
(a) $2mgR$ (b) $\frac{1}{2}mgR$
(c) $\frac{1}{4}mgR$ (d) mgR

21. Suppose the gravitational force varies inversely as the n^{th} power of distance. Then the time period of a planet in circular orbit of radius *R* around the sun will be proportional to:
(a) $R^{\left(\frac{n+1}{2}\right)}$ (b) $R^{\left(\frac{n-1}{2}\right)}$
(c) R^n (d) $R^{\left(\frac{n-2}{2}\right)}$

22. A wire fixed at the upper end stretches by length *l* by applying a force *F*. The work done in stretching is:
(a) $F/2l$ (b) Fl
(c) $2Fl$ (d) $Fl/2$

23. Spherical balls of radius *R* are falling in a viscous fluid of viscosity η with a velocity *v*. The retarding viscous force acting on the spherical ball is:
(a) directly proportional to radius but inversely proportional to *v*
(b) directly proportional to both radius *R* and velocity *v*
(c) inversely proportional to both radius R and velocity *v*
(d) inversely proportional to R but directly proportional to velocity *v*

24. If two soap bubbles of different radii are connected by a tube:

(a) air flows from the bigger bubble to the smaller bubble till the sizes become equal

(b) air flows from bigger bubble to the smaller bubble till the sizes are interchanged

(c) air flows from the smaller bubble to the bigger

(d) there is no flow of air

Answers

1.	(c)	**2.**	(a)	**3.**	(c)	**4.**	(a)	**5.**	(c)	**6.**	(b)	**7.**	(d)
8.	(d)	**9.**	(a)	**10.**	(b)	**11.**	(a)	**12.**	(b)	**13.**	(b)	**14.**	(c)
15.	(b)	**16.**	(a)	**17.**	(c)	**18.**	(d)	**19.**	(a)	**20.**	(b)	**21.**	(a)
22.	(d)	**23.**	(b)	**24.**	(c)								

Explanations

1(c) From Newton's formula

$$\eta = \frac{F}{A(\Delta v_x / \Delta z)}$$

$\therefore$ Dimensions of

$$\eta = \frac{\text{dimensions of force}}{\text{dimensions of area} \times \text{dimensions}}$$

$$= \frac{[\text{MLT}^{-2}]}{[\text{L}^2][T^{-1}]} = [\text{ML}^{-1}\,\text{T}^{-1}]$$

2(a) As given in question, retardation (negative acceleration) a $\propto x$

$\Rightarrow$ $a = kx$

where k is a proportionality constant

$\Rightarrow$ $\frac{dv}{dt} = kx$

$\Rightarrow$ $\frac{dv}{dx}.\frac{dx}{dt} = kx$

$\Rightarrow$ $v\frac{dv}{dx} = kx \left(\because\ v = \frac{dx}{dt}\right)$

$\Rightarrow$ $v\,dv = kx\,dx$

Integrating, we get

$$\int_{v_i}^{v_f} v\,dv = \int_0^x kx\,dx$$

where v_i and v_f respectively are initial and final velocities of particle.

$$\left(\frac{v^2}{2}\right)_{v_i}^{v_f} = k\left(\frac{x^2}{2}\right)_0^x$$

$\Rightarrow$ $\frac{v_f^2}{2} - \frac{v_i^2}{2} = k\,\frac{x^2}{2}$

$\Rightarrow$ $\frac{1}{2}mv^2f - \frac{1}{2}m\,v_i^2 = \frac{1}{2}mkx^2$

$\Rightarrow$ $\text{K.E.}_{\text{final}} - \text{K.E.}_{\text{initial}} = \frac{1}{2}mkx^2$

Hence, loss in kinetic energy $\propto x^2$

3(c) Second law of motion gives

$$s = ut + \frac{1}{2}gt^2$$

or $h = 0 + \frac{1}{2}gt^2$

$\therefore$ $T = \sqrt{\left(\frac{2h}{g}\right)}$

At $t = \frac{T}{3}$ sec,

$$s = 0 + \frac{1}{2}g\left(\frac{T}{3}\right)^2$$

$\Rightarrow$ $s = 0 + \frac{1}{2}g.\frac{T^2}{9}$

$\Rightarrow$ $s = \frac{g}{18} \times \frac{2h}{g} \left(\because T = \sqrt{\frac{2h}{g}}\right)$

$\therefore$ $s = \frac{h}{9}$ m

Hence, the position of ball from the ground

$$= \text{h} - \frac{h}{9} = \frac{8h}{9}\text{ m}$$

4(a) $(\vec{A} \times \vec{B}) = (\vec{B} \times \vec{A})$

$\Rightarrow$ $(\vec{A} \times \vec{B}) - (\vec{B} \times \vec{A}) = \vec{0}$

$(\vec{A} \times \vec{B}) + (\vec{A} \times \vec{B}) = \vec{0}$

$[\because (\vec{B}\times\vec{A}) = -(\vec{A}\times\vec{B})]$

$\Rightarrow$ $2(\vec{A}\times\vec{B}) = \vec{0}$

$\Rightarrow$ $2AB\sin\theta = 0$

$\Rightarrow$ $\sin\theta = 0$

$[\because |\vec{A}| = A \neq 0, |\vec{B}| = B \neq 0]$

$\therefore$ $\theta = 0$ or π

5(c) We know in advance that range of projectile is same for complementary angles i.e. for θ and $(90° - \theta)$.

$\therefore$ $T_1 = \frac{2u\sin\theta}{g}$

$T_2 = \frac{2u\sin(90° - \theta)}{g} = \frac{2u\cos\theta}{g}$

and $R = \frac{u^2\sin 2\theta}{g}$

Therefore, $T_1T_2 = \frac{2u^2(2\sin\theta)}{g} \times \frac{2u\cos\theta}{g}$

$= \frac{2u^2(2\sin 2\theta\cos\theta)}{g} = \frac{2u^2(\sin 2\theta)}{g} = \frac{2R}{g}$

$\therefore$ $T_1T_2 \propto R$

6(b) For a particle moving in a circle with constant angular speed, velocity vector is always tangent to the circle and acceleration vector always points towards the centre of cirlce or is always along radius of the circle. Since, tangential vector is perpendicular to radial vector, therefore, velocity vector will be perpendicular to the acceleration vector is tangent to the case acceleration vector. But in no case acceleration vector is tangent to the circle.

7(d) Third equation of motion gives

$v^2 = u^2 + 2as$

$\Rightarrow$ $s \propto u^2$ $\quad (\because v = 0)$

where a = retardation of body in both the cses

$\therefore$ $\frac{s_1}{s_2} = \frac{u_1^2}{u_2^2}$ ...(i)

Here, $s_1 = 20$ m, $u_1 = 60$ km/h, $u_2 = 120$ km/h

Putting the given values in eq. (i), we get

$\frac{20}{s_2} = \left(\frac{60}{120}\right)^2$

$\Rightarrow$ $s_2 = 20 \times \left(\frac{120}{60}\right)^2 = 20 \times 4 = 80\text{m}$

8(d) The force exerted by machine gun on man's hand in firing a bullet

= change in momentum per second on a bullet or rate of change of momentum

$= \left(\frac{40}{1000}\right)\times 120 = 48\text{ N}$

The force exerted by man on machine gun $= \frac{144}{48} = 3$

9(a) On releasing, the motion of the system will be according to figure.

$m_1g - T = m_1a$ ...(i)

and $T - m_2g = m_1a$ ...(ii)

On solving;

$a = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)g$ ...(iii)

Here, $m_1 = 5$ kg, $m_2 = 4.8$ kg,

$g = 9.8$ m/s^2

$\therefore$ $a = \left(\frac{5 - 4.8}{5 + 4.8}\right)\times 9.8$

$= \frac{0.2}{9.8}\times 9.8 = 0.2$ m/s^2

10(b) Mass per unit length

$= \frac{M}{L} = \frac{4}{2} = 2$ kg/m

The mass of 0.6 m of chain

$= 0.6 \times 2 = 1.2$ kg

The centre of mass of hanging part $= \frac{0.6 + 0}{2} = 0.3$ m

Hence, work done in pulling the chain on the table

$W = mgh$

$= 1.2 \times 10 \times 0.3 = 3.6$ J

11(a) Let the mass of block be m.

Frictional force in rest position

$F = mg\sin 30°$

$10 = m \times 10 \times \frac{1}{2}$

$\therefore$ $m = \frac{2\times 10}{10} = 2$ kg

12(b) Work the is displacing the particle

$W = \vec{F}.\vec{r}$

$= (5\vec{i} + 3\vec{j} + 2\vec{k}).(2\vec{i} - 2\vec{j})$

$= 5 \times 2 + 3 \times (-1) + 2 \times 0 = 10 - 3 = 7$ J

13(b) Let the constant acceleration of body of mass m is a.

From equation of motion

$v_1 = 0 + at_1$

$\Rightarrow$ $a = \frac{v_1}{t_1}$ ...(i)

At an instant t, the velocity v of the body

$v = 0 + at$

$v = \frac{v_1}{t_1}t$ ...(ii)

Therefore, instantaneous power

$P = Fv$

$= mav$ $\quad (\because F = ma)$

$= m\left(\frac{v_1}{t_1}\right)\times\left(\frac{v_1}{t_1}.t\right)$

[from equations (i) and (ii)]

$$= \frac{mv_1^2 t}{t_1^2}$$

14(c) When a force of constant magnitude acts on velocity of particle perpendicularly, then there is no change in the kinetic energy of particle. Hence, kinetic energy remains constant.

15(b) In free space, neither acceleration due to gravity nor external torque act on the rotating solid sphere. Therefore, taking the same mass of inertia, rotational kinetic energy and angular velocity will change but according to law of conservation of momentum, angular momentum will not change.

16(a) Man will catch the ball if the horizontal component of velocity becomes equal to the constant speed of man *i.e.*

$$v_0 \cos\theta = \frac{v_0}{2}$$

$$\Rightarrow \quad \cos\theta = \frac{1}{2}$$

$$\Rightarrow \quad \cos\theta = \cos 60°$$

$$\therefore \quad \theta = 60°$$

17(c) Let same mass and same outer radii of solid sphere and hollow sphere are M and R respectively.

The moment of inertia of solid sphere A about its diameter

$$I_A = \frac{2}{5}MR^2 \qquad \text{...(i)}$$

Similarly the moment of inertia of hollow sphere (spherical shell) B about its diameter

$$I_B = \frac{2}{3}MR^2 \qquad \text{...(iii)}$$

It is clear from eqs. (i) and (ii), $I_A < I_B$

18(d) The gravitational force exerted on satellite at a height x

is $F_G = \frac{GM_e m}{(R+x)^2}$ where M_e = mass of earth

Since, gravitational force provides the necessary centripetal force, so,

$$\Rightarrow \quad \frac{GM_e m}{(R+x)^2} = \frac{mv_0^2}{(R+x)}$$

$$\Rightarrow \quad \frac{gR^2 m}{(R+x)} = mv_0^2 \left(\because g\frac{GM_e}{R^2}\right)$$

$$\Rightarrow \quad v_0 = \sqrt{\left[\frac{gR^2}{(R+x)}\right]}$$

$$= \left[\frac{gR^2}{(R+x)}\right]^{\frac{1}{2}}$$

19(a) Time period of satellite $T = 2\pi\sqrt{\frac{(R+h)^3}{GM_e}}$

where $R + h$ = orbital radius of satellite,

M_e = mass of earth. Thus, time period does not depend on mass of satellite.

20(b) Gravitational potential energy of body on earth's surface $U = -\frac{GM_e m}{R}$

At a height h from earth's surface, its value is

$$U_h = -\frac{GM_e m}{R}$$

$$= -\frac{GM_e m}{R} \qquad (\because h = R)$$

where M_e = mass of earth,

m = mass of body

R = radius of earth

$\therefore$ Gain in potential energy

$$= U_h - U$$

$$= -\frac{GM_e m}{2R} - \left(-\frac{GM_e m}{R}\right)$$

$$= -\frac{GM_e m}{2R} + \frac{GM_e m}{R}$$

$$= \frac{GM_e m}{2R} = \frac{gR^2 m}{2R} \left(\because g = \frac{GM_e}{R^2}\right)$$

$$= \frac{1}{2}mgR$$

21(a) The necessary centripetal force required for a planet to move round the sun

= gravitational force exerted on it

i.e. $$\frac{mv^2}{R} = \frac{GM_e m}{R^n}$$

$$v = \left(\frac{GM_e}{R^{n-1}}\right)^{\frac{1}{2}}$$

Now, $$T = \frac{2\pi R}{v} = 2\pi R \times \left(\frac{R^{n-1}}{GM_e}\right)^{\frac{1}{2}}$$

$$= 2\pi\left(\frac{R^2 \times R^{n-1}}{GM_e}\right)^{\frac{1}{2}} = 2\pi\left(\frac{R^{\frac{(n+1)}{2}}}{(GM_e)^{\frac{1}{2}}}\right)$$

$$\therefore \quad T \propto R^{(n+1)/2}$$

22(d) Work done in stretching the wire

$$= \text{potential energy stored} = \frac{1}{2} \times \frac{F}{A} \times \frac{l}{L} \times AL$$

$$= \frac{1}{2}Fl$$

23(b) Retarding force acting on a ball falling into a viscous fluid

$F = 6\pi\eta Rv$

where R = radius of ball,

v = velocity of ball,

and η = coefficient of viscosity

$\therefore$ $F \propto R$ and $F \propto v$

Or in words, retarding force is proportional to both R and v.

24(c) The excess pressure inside the soap bubble is inversely proportional to radius of soap bubble i.e. $P \propto 1/r$, r being the radius of smaller bubble is greater than that inside a bigger bubble. Thus, if these two bubbles are connected by a tybe, air will flow from smaller bubble to bigger bubble and the bigger bubble grows at the expense of the smaller one.

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1. A projectile can have the same range 'R' for two angles of projection. If 't_1' and 't_2' be the times of flights in the two cases, then the product of the two times of flights is proportional to:

(a) R^2 (b) $\frac{1}{R^2}$

(c) $\frac{1}{R}$ (d) R

2. An annular ring with inner and outer radii R_1 and R_2 is rolling without slipping with a uniform angular speed. The ratio of the forces experienced by the two particles situated on the inner and outer parts of the ring, $\frac{F_1}{F_2}$ is:

(a) $\frac{R_2}{R_1}$ (b) $\left(\frac{R_1}{R_2}\right)^2$

(c) 1 (d) $\frac{R_1}{R_2}$

3. A smooth block is released at rest on a 45° incline and then slides a distance 'd'. The time taken to slide is 'n' times as much to slide on rough incline than on a smooth incline. The coefficient of friction is:

(a) $\mu_k = 1 - \frac{1}{n^2}$ (b) $\mu_k = \sqrt{1 - \frac{1}{n^2}}$

(c) $\mu_s = 1 - \frac{1}{n^2}$ (d) $\mu_s = \sqrt{1 - \frac{1}{n^2}}$

4. The upper half of an inclined plane with inclination ϕ is perfectly smooth, while the lower half is rough. A body starting from rest at the top will again come to rest at the bottom, if the coefficient of friction for the lower half is given by:

(a) 2 sin ϕ (b) 2 cos ϕ

(c) 2 tan ϕ (d) tan ϕ

5. A bullet fired into a fixed target loses half of its velocity after penetrating 3 cm. How much further it will penetrate before coming to rest, assuming that is faces constant resistance to motion?

(a) 3.0 cm (b) 2.0 cm

(c) 1.5 cm (d) 1.0 cm

6. Out of the following pairs, which one does not have identical dimensions?

(a) Angular momentum and Planck's constant

(b) Impulse and momentum

(c) Moment of inertia and moment of a force

(d) Work and torque

7. The relation between time t and distance $\boldsymbol{x}$ is $t = ax^2 + bx$, where a and b are constants. The acceleration is:

(a) $-2abv^2$ (b) $2bv^3$

(c) $-2av^3$ (d) $2av^2$

8. A car, starting from rest, accelerates at the rate f through a distance S, then continues at constant speed for time t and then decelerates at the rate $\frac{f}{2}$ to come to rest. If the total distance travelled is 15 S, then:

(a) $S = ft$ (b) $S = \frac{1}{6}ft^2$

(c) $S = \frac{1}{2}ft^2$ (d) $S = \frac{1}{4}ft^2$

9. A particle is moving eastwards with a velocity of $5 ms^{-1}$. In 10 seconds the velocity changes to $5 ms^{-1}$ northwards. The average acceleration in this time is:

(a) $\frac{1}{\sqrt{2}} ms^{-2}$ towards north-east

(b) $\frac{1}{2} ms^{-2}$ towards north

(c) zero

(d) $\frac{1}{2} ms^{-2}$ towards north-west

10. A parachutist after bailing out falls 50 m without friction. When parachute opens, it decelerates at 2 m/s^2. He reaches the ground with a speed of 3 m/s. At what height, did he bail out?

(a) 91 m (b) 182 m

(c) 293 m (d) 111 m

11. A block is kept on a frictionless inclined surface with angle of inclination 'α'. The incline is given an acceleration 'a' to keep the block stationary. Then 'a' is equal to:

(a) g/tan α (b) g cosec α

(c) g (d) g tan α

12. A spherical ball of mass 20 kg is stationary at the top of a hill of height 100 m. It rolls down a smooth surface to the ground, then climbs up another hill of height 30 m and finally rolls down to a horizontal base at a height of 20 m above the ground. The velocity attained by the ball is:

(a) 40 m/s (b) 20 m/s
(c) 10 m/s (d) $10\sqrt{3}$ m/s

13. A body A of mass M while falling vertically downwards under gravity breaks into two parts; a body B of mass $\frac{1}{3}M$ and, a body C of mass $\frac{2}{3}M$. The centre of mass of bodies B and C taken together shifts compared to that of body A towards:

(a) depends on height of breaking
(b) does not shift
(c) body C
(d) body B

14. The moment of inertia of uniform semicircular disc of mass M and radius r about a line perpendicular to the plane of the disc through the centre is:

(a) $\frac{1}{4}Mr^2$ (b) $\frac{2}{5}Mr^2$
(c) Mr^2 (d) $\frac{1}{2}Mr^2$

15. A particle of mass 0.3 kg is subjected to a force $F = kx$ with $k = 15$ N/m. What will be its initial acceleration, if it is released from a point 20 cm away from the origin?

(a) 3 m/s^2 (b) 15 m/s^2
(c) 5 m/s^2 (d) 10 m/s^2

16. The block of mass M moving on the frictionless horizontal surface collides with the spring of spring constant k and compresses it by length L. The maximum momentum of the block after collision is:

(a) $\sqrt{MkL}$ (b) $\frac{kL^2}{2M}$
(c) zero (d) $\frac{ML^2}{k}$

17. A mass 'm' moves with a velocity 'v' and collides inelastically with another identical mass. After collision the 1st mass moves with velocity in a direction perpendicular to the initial direction of motion. Find the speed of the 2nd mass after collision:

(a) v (b) $\sqrt{3}v$
(c) $\frac{2}{\sqrt{3}}v$ (d) $\frac{v}{\sqrt{3}}$

18. A 20 cm long capillary tube is dipped in water. The water rises upto 8 cm. If the entire arrangement is put in a freely failling elevator, the length of water column in the capillary tube will be:

(a) 8 cm (b) 10 cm
(c) 4 cm (d) 20 cm

19. If 'S' is stress and 'Y' is Young's modulus of material of a wire, the energy stored in the wire per unit volume is:

(a) $2S^2Y$ (b) $\frac{S^2}{2Y}$
(c) $\frac{2Y}{S^2}$ (d) $\frac{S}{2Y}$

20. Average density of the earth:

(a) does not depend on g
(b) is a complex function of g
(c) is directly proportional to g
(d) is inversely proportional to g

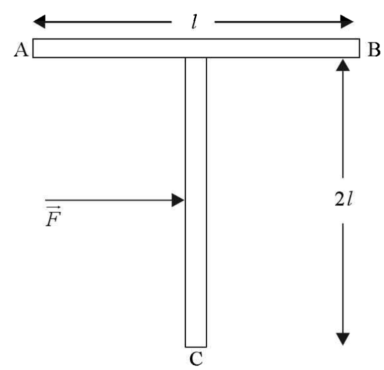
21. A body of mass m is accelerated uniformly from rest to a speed v in a time T. The instantaneous power delivered to the body as a function of time, is given by:

(a) $\frac{mv^2}{T^2}.t$ (b) $\frac{mv^2}{T^3}.t^2$
(c) $\frac{1}{2}\frac{mv^2}{T^2}.t$ (d) $\frac{1}{2}\frac{mv^2}{T^3}.t^2$

22. Consider a car moving on a straight road with a speed of 100 m/s. The distance at which car can be stopped, is: [$\mu_k = 0.5$]

(a) 800 m (b) 1000 m
(c) 100 m (d) 400 m

23. A 'T' shaped object with dimensions shown in the figure, is lying on a smooth floor. A force 'F' is applied at the point P parallel to AB, such that the object has only the translational motion without rotation. Find the location of P with respect to C:



(a) $\frac{2}{3}l$ (b) $\frac{3}{2}l$
(c) $\frac{4}{3}l$ (d) l

24. The change in the value of 'g' at a height 'h' above the surface of the earth is the same as at a depth 'd' below the surface of earth. When both 'd' and 'h' are much smaller than the radius of earth, then, which one of the following is correct?

(a) $d = \frac{h}{2}$ (b) $d = \frac{3h}{2}$
(c) $d = 2h$ (d) $d = h$

25. A particle of mass 10 g is kept on the surface of a uniform sphere of mass 100 kg and radius 10 cm. Find the work to be done against the gravitational force between them, to take the particle far away from the sphere (you may take $G = 6.67 \times 10^{-11}$ Nm²/kg²):
(a) 13.34×10^{-10} J (b) 3.33×10^{-10} J
(c) 6.67×10^{-9} J (d) 6.67×10^{-10} J

Answers

1.	(d)	**2.**	(d)	**3.**	(a)	**4.**	(c)	**5.**	(d)	**6.**	(c)	**7.**	(c)
8.	(d)	**9.**	(a)	**10.**	(c)	**11.**	(d)	**12.**	(a)	**13.**	(b)	**14.**	(d)
15.	(d)	**16.**	(a)	**17.**	(c)	**18.**	(d)	**19.**	(b)	**20.**	(c)	**21.**	(a)
22.	(b)	**23.**	(c)	**24.**	(c)	**25.**	(d)						

Explanations

1(d) A projectile can have same range if angles of projection are complementary i.e.,θ and (90° – θ). Thus, in both cases:

$$t_1 = \frac{2u\sin\theta}{g} \quad ...(i)$$

$$t_2 = \frac{2u\sin(90^\circ - \theta)}{g}$$

$$= \frac{2u\cos\theta}{g} \quad ...(ii)$$

From Eq. (i) and (ii)

$$t_1 t_2 = \frac{4u^2\sin\theta\cos\theta}{g^2}$$

$$t_1 t_2 = \frac{2u^2\sin 2\theta}{g^2}$$

$$= \frac{2}{g}\frac{u^2\sin 2\theta}{g^2}$$

$$t_1 t_2 = \frac{2R}{g} \quad \left(\because R = \frac{u^2\sin 2\theta}{g}\right)$$

Hence, $t_1 t_2 \propto R$

2(d) $ma_1 = \frac{mv_1^2}{R_1}$

and $ma_2 = \frac{mv_2^2}{R_2}$

and $\frac{F_1}{F_2} = \frac{ma_1}{ma_2}$

$$= \frac{mR_1^2\omega^2}{R_1} \cdot \frac{R_2}{mR_2^2\omega^2}$$

$\therefore$ $\frac{F_1}{F_2} = \frac{R_1}{R_2}$

3(a) When friction is absent

$a_1 = g\sin\theta$

$s_1 = \frac{1}{2}a_1 t_1^2$

When friction is present

$a_2 = g\sin\theta - \mu g\cos\theta$

$s_2 = \frac{1}{2}a_2 t_2^2$

From Eq. (i) and (ii)

$$\frac{1}{2}a_1 t_1^2 = \frac{1}{2}a_2 t_2^2$$

or $a_1 t_1^2 = a_2 (nt_1)^2 \quad (\because t_2 = nt_1)$

or $a_1 = n^2 a_2$

or $\frac{a_2}{a_1} = \frac{g\sin\theta - \mu_k g\cos\theta}{g\sin\theta}$

or $\frac{g\sin 45^\circ - \mu_k g\cos 45^\circ}{g\sin 45^\circ} = \frac{1}{n^2}$

or $1 - \mu_k = \frac{1}{n^2}$

or $\mu_k = 1 - \frac{1}{n^2}$

4(c) According to work-energy theorem,
(Initial and final speeds are zero)
Work done by friction + work done by gravity = 0

$-(\mu\, mg\cos\phi) + mgl\sin\phi = 0$

or $\frac{\mu}{2}\cos\phi = \sin\phi$

$\therefore$ $\mu = 2\tan\phi$

5(d) According to work-energy theorem,

$W = \Delta K$

Case I: $-F \times 3 = \frac{1}{2} m\left(\frac{v_0}{2}\right)^2 - \frac{1}{2} mv_0^2$

where F is resistive force and v_0 is initial speed.

$\therefore \quad -F(3-s) = K_f - K_i$

$= -\frac{1}{2}mv_0^2$

or $\quad -\frac{1}{8}mv_0^2\ (3-s) = -\frac{1}{2}mv_0^2$

or $\quad \frac{1}{4}(3-s) = 1$

or $\quad \frac{3}{4} + \frac{s}{4} = 1$

$\therefore \quad s = 1$ cm

6(c) $I = mr^2$

$\therefore \quad$ [I] [ML^2]

and $\vec{\tau}$ = moment of force = $\vec{r} \times \vec{F}$

$\therefore \quad [\vec{\tau}] = [MLT^{-2}] = [ML^2T^{-2}]$

7(c) Given $t = ax^2 + bx$

Differentiating w.r.t. 't'

$$\frac{dt}{dt} = 2ax\frac{dx}{dt} + b\frac{dx}{dt}$$

$$v = \frac{dx}{dt} = \frac{1}{(2ax+b)}$$

Again differentiating. w.r.t. 't'

$$\frac{d^2x}{dt^2} = \frac{d(2ax+b)^{-1}}{d(2ax+b)} .2a\frac{dx}{dt}$$

$\therefore \quad f = \frac{d^2x}{dt^2}$

$= \frac{-1}{(2ax+b)^2} \cdot \frac{2a}{(2ax+b)}$

or $\quad f = \frac{-2a}{(2ax+b)^3}$

$\therefore \quad f = -2av^3$

8(d) $v = ft_1$

and the final velocity of

OA = initial velocity of BC

$ft_1 = \frac{f}{2}t_2$

$\therefore\ t_2 = 2\,t_1$

In graph

$S_1 = \frac{1}{2}\ ft_1^2 \qquad$ ...(i)

Given, $S_1 = S$

$S_2 = (ft_1)t$

$S_3 = \frac{1}{2}\frac{f}{2}.(2t_1)^2$

Thus, $S_1 + S_2 + S_3 = 15\ S$

$S + (ft_1)t + ft_1^2 = 15S$

$S + (ft_1) + 2S = 15S \left(S = \frac{1}{2}ft_1^2\right)$

$(ft_1)t = 12S$

From Eq. (i) and (ii), we have

$$\frac{12S}{S} = \frac{(ft_1)t}{\frac{1}{2}(ft_1)t_1}$$

$\therefore \quad t_1 = \frac{t}{6}$

From Eq. (i), we get

$\therefore \quad S = \frac{1}{2}f(t_1)^2$

$\therefore \quad S = \frac{1}{2}f(t/6)^2 = \frac{1}{72}ft^2$

Hence, none of the given options is correct.

9(a) $\vec{v_1} = -5\,\hat{i}$

$\vec{v_2} = 5\,\hat{j}$

$\Delta\,\vec{v} = 5\,\hat{j} - 5\,\hat{i}$

$|\Delta\vec{v}| = 5\sqrt{2}$

$a = \frac{|\vec{v}|}{t} = \frac{5\sqrt{2}}{10} = \frac{1}{\sqrt{2}}$ ms^{-2}

For direction, $\tan\,\alpha = -\frac{5}{5} = -1$

$\therefore \quad$ Average acceleration is $\frac{1}{\sqrt{2}}$ ms^{-2} towards north-east.

10(c) Parachute bailing out at A.

Velocity at A

$v = \sqrt{2gh} = \sqrt{2\times9.8\times50} = \sqrt{980}$ m/s

The velocity at ground v_1 = 3 m/s (given)

Acceleration = – 2 m/s^2 (given)

$\therefore \quad H - h = \frac{v^2 - v_1^2}{2\times2} = \frac{980\times9}{4} = \frac{971}{4} = 242.75$

$\therefore \quad H = 242.75 + h$

$= 242.75 + 50 \approx 293\,m$

11(d) In the frame of wedge, the force diagram of block is shown in figure. From force diagram of wedge,

12(a) According to conservation of energy,

$mgH = \frac{1}{2}mv^2 + mgh_2$

or $\quad mg\,(H - h_2) = \frac{1}{2}mv^2$

or $\quad v = \sqrt{2g(100-20)}$

$v = \sqrt{2\times10\times80} = 40$ m/s

13(b) Before breaking, the centre of mass of system is moving under gravity. Thus, acceleration of the centre of mass is gravitational acceleration. During breaking, internal forces come into play which are not responsible for the acceleration of the centre of mass.
This indicates that, the acceleration of centre of mass remains the same (equal to gravitational acceleration). Thus, the centre of mass of system continues its original path.

14(d) The mass of complete (circular) disc is
$M+M=2M$
The moment of inertia of disc is
Let the moment of inertia of semicircular disc is I_1.
The disc may be assumed as combination of two semi-circular parts.
Thus, $I_1 = I - I_1$

$\therefore \quad I_1 = \frac{1}{2} = \frac{Mr^2}{2}$

15(d) Given: $m = 0.3$ kg, $x = 20$ cm, and
$k = 15$ N/m
$F = -kx$
and $F = ma$
$\therefore \quad ma = -kx$

or $\quad a = -\frac{15}{0.3} \times 200 + 10^{-2}$

$a = -\frac{15}{3} \times 2 = 10$ m/s

$\therefore$ Initial acceleration $a = 10$ m/s

16(a) According to conservation of energy

$$\frac{1}{2}kL^2 = \frac{1}{2}Mv^2$$

$\Rightarrow \quad kL^2 = \frac{(Mv)^2}{M}$

$MkL^2 = p^2 \qquad (p = mv)$

$\therefore \quad p = L\sqrt{Mk}$

17(c) In x-direction
$mu_1 + 0 = 0 + mv_x$
$\Rightarrow \quad mv = mv_x$
$\Rightarrow \quad v_x = v$
In y-direction

$$0 + 0 = m\left(\frac{v}{\sqrt{3}}\right) - mv_y$$

$\therefore$ Velocity of second mass after collision

$$v' = \sqrt{\left(\frac{v}{\sqrt{3}}\right)^2 + v^2} = \sqrt{\frac{4}{3}v^2}$$

$$v' = \frac{2}{\sqrt{3}}v \qquad \therefore$$

18(d) Water fills the tube entirely in gravity less condition.

19(b) Energy stored in wire

$= \frac{1}{2}$ Stress × Strain × Volume

and Young's modulus $= \frac{\text{Stress}}{\text{Strain}}$

$\Rightarrow \quad$ Strain $= \frac{S}{Y}$

$$\frac{\text{Energy stored in wire}}{\text{Volume}} = \frac{1}{2}S \times \frac{S}{Y} = \frac{S^2}{2Y}$$

20(c) $g = \frac{GM}{R^2}$; $M = \left(\frac{4}{3}\pi R^3\right)\rho$

$\therefore \quad g = \frac{4G}{3}\frac{\pi R^3}{R^2}.\rho$

$\Rightarrow \quad g = \left(\frac{4G\pi R}{3}\right)\rho$

(ρ = average density)

$\Rightarrow \quad g \propto \rho$ or $\rho \propto g$

21(a) $F = ma = \frac{mv}{T} \quad \left(\therefore a = \frac{v-0}{T}\right)$

Instantaneous power $= Fv = mav = \frac{mv}{T}.at = \frac{mv}{T}.\frac{v}{T}.$

$= \frac{mv^2}{T^2}$.t

22(b) $s = \frac{v2}{2\mu_k g} = \frac{100\times100}{2\times0.5\times10} = \frac{100\times100}{5\times2} = 1000$ m

23(c) For pure translatory motion, net torque about centre of mass should be zero.
Thus, $\vec{F}$ is applied at centre of mass of system.

$$P = \frac{0\times l\times l.2l}{l+2l} = \frac{2l2}{3l} = \frac{2l2}{3l}$$

$$\therefore \quad PC = \left(l - \frac{2l}{3} + l\right) = \left(2l - \frac{2l}{3}\right) = \frac{4l}{3}$$

24(c) $g_h = g\left(1 - \frac{2h}{R}\right) \qquad$ ...(i)

$g_d = g\left(1 - \frac{d}{R}\right) \qquad$ ...(ii)

From Eq. (i) and (ii),

$$g\left(1 - \frac{2h}{R}\right) = g\left(1 - \frac{d}{R}\right)$$

$\Rightarrow \quad 2h = d$

25(d) $U_i = -\frac{GMm}{r}$

$$U_i = -\frac{6.67\times10^{-11}\times100\times10^{-2}}{0.1}$$

$$U_i = -\frac{6.67\times10^{-11}}{0.1}$$

$= -6.67 \times 10^{-10}$ J

We know

$\therefore \quad W = \Delta U$

$= U_f - U_i \qquad (\because U_f = 0)$

$\therefore \quad W = U_i = 6.67 \times 10^{-10}$ J

AIEEE 2006

1. A particle located at $x = 0$ at time t = 0, starts moving along the positive x-direction with a velocity 'v' that varies as $v = \alpha\sqrt{x}$. The displacement of the particle varies with time as:

(a) t^2 (b) t
(c) $t^{1/2}$ (d) t^3

2. A bomb of mass 16 kg at rest explodes into two pieces of masses 4 kg and 12 kg. The velocity of the 12 kg mass is 4 ms^{-1}. The kinetic energy of the other mass is:

(a) 144 J (b) 288 J
(c) 192 J (d) 96 J

3. A mass of M kg is suspended by a weightless string. The horizontal force that is required to displace it until the string makes an angle of 45° with the initial vertical direction is:

(a) $Mg(\sqrt{2}+1)$ (b) $Mg\sqrt{2}$
(c) $\frac{Mg}{\sqrt{2}}$ (d) $Mg(\sqrt{2}-1)$

4. A particle of mass 100g is thrown vertically upwards with a speed of 5 m/s. The work done by the force of gravity during the time the particle goes up is:

(a) – 0.5 J (b) – 1.25 J
(c) 1.25 J (d) 0.5 J

5. Which of the following units denotes the dimensions $[M^2/Q^2]$, where Q denotes the electric charge?

(a) Wb/m^2 (b) henry (H)
(c) H/m^2 (d) weber (Wb)

6. A ball of mass 0.2 kg is thrown vertically upwards by applying a force by hand. If the hand moves 0.2 m while applying the force and the ball goes upto 2 m height further, find the magnitude of the force. Consider g = 10 m/s^2:

(a) 4 N (b) 16 N
(c) 20 N (d) 22 N

7. Consider a two particle system with particles having masses m_1 and m_2. If the first particle is pushed towards the centre of mass through a distance d, by what distance should the second particle be moved, so as to keep the centre of mass at the same position?

(a) $\frac{m_2}{m_1}d$ (b) $\frac{m_1}{m_1+m_2}d$
(c) $\frac{m_1}{m_2}d$ (d) d

8. A player caught a cricket ball of mass 150 g moving at a rate of 20 m/s. If the catching procss is completed in 0.1s, the force of the blow exerted by the ball on the hand of the player is equal to:

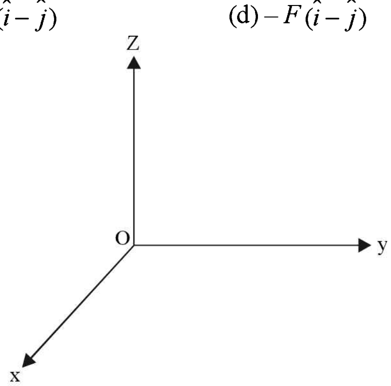
(a) 150 N (b) 3 N
(c) 30 N (d) 300 N

9. Four point masses, each of value m, are placed at the corners of a square $ABCD$ of side l. The moment of inertia of this system about an axis passing through A and parallel to BD is:

(a) $2\,ml^2$ (b) $\sqrt{3}\,ml^2$
(c) $3\,ml^2$ (d) ml^2

10. A force of $-F\hat{k}$ acts on O, the origin of the coordinate system. The torque about the point (1, – 1), is:

(a) $F(\hat{i}-\hat{j})$ (b) $-F(\hat{i}-\hat{j})$
(c) $F(\hat{i}-\hat{j})$ (d) $-F(\hat{i}-\hat{j})$



11. The potential energy of a 1 kg particle free to move along the x-axis is given by $V(x) = \left(\frac{x^4}{4}-\frac{x^2}{2}\right)J$. The total mechanical energy of the particle is 2 J. Then, the maximum speed (in m/s) is:

(a) $3/\sqrt{2}$ (b) $\sqrt{2}$
(c) $1/\sqrt{2}$ (d) 2

12. A thin circular ring of mass m and radius R is rotating about its axis with a constant angular velocity ω. Two objects each of mass M are attached gently to the opposite ends of a diameter of the ring. The ring now rotates with an angular velocity ω'.

(a) $\dfrac{\omega(m+2M)}{m}$ (b) $\dfrac{\omega(m-2M)}{(m+2M)}$

(c) $\dfrac{\omega m}{(m+M)}$ (d) $\dfrac{\omega m}{(m+2M)}$

13. A wire elongates by l mm when a load W is hanged from it. If the wire goes over a pulley and two weights W each are hung at the two ends, the elongation of the wire will be (in mm):

(a) l (b) $2l$

(c) zero (d) $l/2$

14. If the terminal speed of a shpere of gold (density = 19.5 kg/m³) is 0.2 m/s in a viscous liquid (density = 1.5 kg/m³) of the same size in the same liquid.

(a) 0.4 m/s (b) 0.133 m/s

(c) 0.1 m/s (d) 0.2 m/s

Answers

1.	(a)	**2.**	(b)	**3.**	(d)	**4.**	(b)	**5.**	(b)	**6.**	(d)	**7.**	(c)
8.	(c)	**9.**	(c)	**10.**	(c)	**11.**	(a)	**12.**	(d)	**13.**	(a)	**14.**	(c)

Explanations

1(a) $v = a\sqrt{x}$

$$\frac{dx}{dt} = a\sqrt{x}$$

$$\frac{dx}{\sqrt{x}} = a\,dt$$

Perform integration

$$\int_0^x \frac{dx}{\sqrt{x}} = \int_0^t \alpha\,dt$$

[∴ at $t = 0$, $x = 0$ and let at any itme t, particle is at x]

$$\Rightarrow \left.\frac{x^{\frac{1}{2}}}{\frac{1}{2}}\right|_0^x = \alpha t \Rightarrow x^{\frac{1}{2}} = \frac{\alpha}{2}t \Rightarrow x = \frac{\alpha^2}{4} \times t^2\, x \propto t^2$$

2(b) Here momentum of the system is remaining conserved as no external force is acting on the bomb (system). Initial momentum (before explosion) = Final momentum (after explosion)

Let velocity of 4 kg mass is v m/s. From momentum conservation we can say that its direction is opposite to velocity of 12 kg mass.

From

$\Rightarrow$ $0 = 12 \times 4 - 4 \times v \Rightarrow v = 12$ m/s

$$\text{KE of 4 kg mass} = \frac{4\times(12)^2}{2} = 288 \text{ J}$$

3(d) Here, the constant horizontal force required to take the body from position 1 to position 2 can be calculated theorem. Let us assume that body is taken slowly so that its speed doesn't change, then

$\Delta K = 0$

$= W_F + W_{Mg} + W_{tension}$

[symbols have their usual meanings]

$W_F = F \times l \sin 45°$

$W_{Mg} = M_g(l - l\cos 45°)$, $W_{tension} = 0$

$F = Mg(\sqrt{2} - 1)$

4(b) The height (h) traversed by particle while going up is $v = 0$

$$h = \frac{u^2}{2g} = \frac{25}{2\times 9.8}$$

Work done by gravity force = $m\,\vec{g}.\vec{h}$

$$= 0.1 \times g \times \frac{25}{2\times 9.8}\cos 180°$$

[Angle between force and displacement is 180°]

$$\therefore \quad W = -0.1 \times \frac{25}{2} = -1.25 \text{ J}$$

5(b) Magnetic energy $= \dfrac{1}{2}Li^2 = \dfrac{LQ^2}{2t^2}$

[$L \rightarrow$ inductance $i \rightarrow$ current]

Energy has the dimensions $[ML^2T^{-2}]$.

Square the dimensions, we have

$$[ML^2T^{-2}] = (\text{henry}) \times \frac{Q^2}{T^2}$$

$$\Rightarrow \quad [\text{henry}] = \frac{[ML^2]}{[Q^2]}$$

6(d) The situation is shown in figure. At initial time, the ball is at P, then under the action of a force (exerted by hand) from p to A and then from A to B let acceleration of ball during PA is a m/s² [assumed to be constant] in upward direction and velocity of ball at A is v m/s.

Then for PA, $v^2 = 0^2 + 2a \times 0.2$

For AB, $0 = v2 - 2 \times g \times 2$

$\Rightarrow$ $v^2 = 2g \times 2$

From above equation,

$a = 10\,g = 100\ \text{m/s}^2$

Then for PA, FBD of ball is

$F - mg = ma$ [F is the force exerted by hand on ball]

$\Rightarrow$ $F = m\,(g + a) = 0.2\,(11g)$

$= 22\,\text{N}$

Alternate solution:

Using work-energy theorem

$W_{mg} + W_F = 0$

$\Rightarrow$ $-m \times 22 + F \times 0.2 = 0 \Rightarrow F = 22\ \text{N}$

7(c) To keep the COM at the same position, velocity of COM is zero, so

$$\frac{m_1 \vec{v_1} + m_2 m \vec{v_2}}{m_1 + m_2} = 0$$

[where $\vec{v_1}$ and $\vec{v_2}$ are velocity of particles 1 and 2 respectively].

$$m_1 = \frac{d\vec{r_1}}{dt} + m_2 \frac{d\vec{r_2}}{dt}$$

$$[\because \vec{v_1} = \frac{d\vec{r_1}}{dt} \text{ and } \vec{v_2} = \frac{d\vec{r_2}}{dt}]$$

$m_, d\vec{r_1} + m_2 d\vec{r_2} = 0$ [$\vec{r_1}$ and $\vec{r_2}$ represent the change in displacement of particles]

Let 2nd particle has been displaced by distance x.

$\Rightarrow$ $m_1(d) + m_2(x) = 0 \Rightarrow x = -\dfrac{m_1 d}{m_2}$

$-ve$ sign shows that both the particles have to move in opposite directions.

So, $\dfrac{m_1 d}{m_2}$ is the distance moved by 2nd particle to keep COM at the same position.

8(c) This is the question based on impulse-momentum theorem $|\vec{F}.\Delta t| = |$change in momentum$|$

$\Rightarrow$ $F \times 0.1 = |\vec{P}_f - \vec{P}_i|$

As the ball will stop after catching;

$P_i = mv_i - 0.15 \times 20 = 3, p_f = 0$

$\Rightarrow$ $F \times 0.1 = 3 \Rightarrow F = 30\ \text{N}$

9(c) The situation is shown in figure.

$I\,xx' = m \times DP^2 + m \times BQ^2 + m \times CA^2$

$$= m \times 2 \times \left(\frac{\sqrt{2}l}{2}\right)^2 + m \times (\sqrt{2}\,l)^2 = 3\,ml^2$$

10(c) $\vec{\tau} = \vec{r} \times \vec{F}$

$\vec{\tau} = (\vec{i} - \vec{j}) \times (-F\hat{k})$

$= F[(-\vec{i} \times \hat{k}) + (\hat{j} \times \hat{k})] = F[\hat{j} + \hat{i}] = F[\hat{i} + \hat{j}]$

11(a) $V(x) = \left(\dfrac{x^4}{4} - \dfrac{x^2}{2}\right)$

For minimum value of V, $\dfrac{dV}{dx} = 0$

$\Rightarrow$ $\dfrac{4x^3}{4} - \dfrac{2x}{2} = 0 \Rightarrow x = 0, x = \pm 1$

So, $V_{min.}(x = \pm 1) = \dfrac{1}{4} - \dfrac{1}{2} = \dfrac{-1}{4}\,J$

$K_{max.} + V_{min.}$ = Total mechanical energy.

$\therefore$ $K_{max.} = \left(\dfrac{1}{4}\right) + 2\,K_{max.} = \dfrac{9}{4}$

$\Rightarrow$ $\dfrac{mv^2}{2} = \dfrac{9}{4} \Rightarrow v = \dfrac{3}{\sqrt{2}}\ \text{m/s}$

12(d) As no external torque is acting about the axis, angular momentum of system remains conserved.

$\therefore$ $I_1\omega = I_2\omega'$

$\Rightarrow$ $mR^2\omega = (mR^2 + 2MR^2)\,\omega'$

$\Rightarrow$ $\omega' = \left(\dfrac{m}{m + 2M}\right)\omega$

13(a) Let us consider the length of wire as L and cross-sectional area A, the material of wire has Young's modulus as Y.

Then for 1st case $Y = \dfrac{W/A}{l/L}$

For 2nd case, $Y = \dfrac{W/A}{2l'/L}$

$\therefore$ $l' = \dfrac{l}{2}$

So, total elongation of both sides = $2l' = l$

14(c) Terminal speed of spherical body in a viscous liquid is given by

$$v_T = \frac{2r2(\rho - \sigma)g}{9\eta}$$

where ρ = density of substance of body

σ = density of liquid

From given data

$$\frac{v_T(Ag)}{v_T(Gold)} = \frac{\rho\,Ag - \sigma_l}{\rho\,Gold - \sigma_l}$$

$\Rightarrow$ $v_T(Ag) = \dfrac{10.5 - 1.5}{19.5 - 1.5} \times 0.2 = \dfrac{9}{18} \times 0.2 = 0.1\ \text{m/s}$

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2 Simple Harmonic Motion and Waves

12

Simple Harmonic Motion

BRIEF REVIEW

Periodic motion If a moving body repeats its motion after regular intervals of time, the motion is said to be **harmonic or periodic**. The time interval after which it repeats the motion is called **time period**. If the body moves **to and fro** on the same path, the motion is called **oscillatary**. In simple harmonic motion the particle moves in a straight line or along the angle and the acceleration of the particle is always directed towards a fixed point on the line. This fixed point is called mean position or centre of oscillation. The acceleration in SHM is given by

$a = -\omega^2 x$ or $F = -m\omega^2 x$ or $F = -kx$

where $k = m\omega^2$ is called force constant or spring constant.

The force which brings the particle back towards the equilibrium or mean position is called restoring force. Such a motion is also called isochronous.

SHM may be assumed as a **projection of uniform circular motion along a diameter**

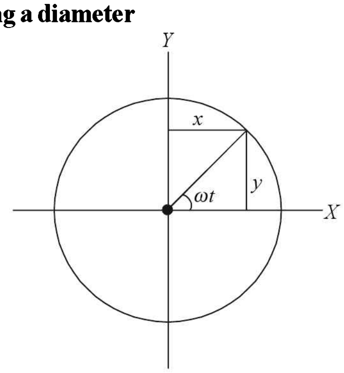


Fig. 12.1

$x = r\cos\omega t$; $y = r\sin\omega t$; $a = -\omega^2 x$

or $\dfrac{d^2x}{dt^2} = -\omega^2 x$. This differential equation gives the solution

$x = x_0 \sin\omega t$ (if the particle starts from mean position)

$x = x_0\cos\omega t$ (if the particle starts from extreme position)

$x = x_0 \sin(\omega t \pm \phi)\}$ (if the particle starts in between mean and extreme position)

$x = x_0\cos(\omega t \pm \phi)$.

The solution of differential equation in exponential form is $x = x_0\, e^{\pm(\omega t \pm \phi)}$.

Here x is instantaneous displacement, x_0 is amplitude (maximum displacement), ϕ is initial phase angle or epoch or angle of repose and, ω is angular frequency.

Linear frequency $f = \dfrac{1}{T} = \dfrac{\omega}{2\pi}$; T being time period.

Velocity of the particle executing SHM

Assume $x = x_0 \sin\omega t$. then $v = \dfrac{dx}{dt} = x_0\omega\cos\omega t$

$v = x_0\omega\sqrt{1-\sin^2\omega t} = \omega\sqrt{x_0^2 - x^2}$

$v_{max} = x_0\omega$; $v_{min} = 0$ at extreme position

Fig. 12.2 (a) shows graph between velocity and displacement and Fig. 12.2 (b) shows the graph between velocity and time.

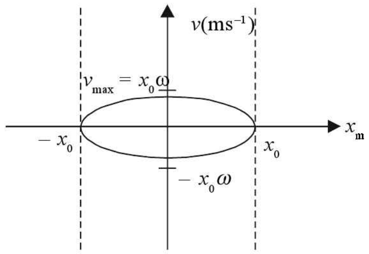


Fig. 12.2 (a) Velocity — displacement graph

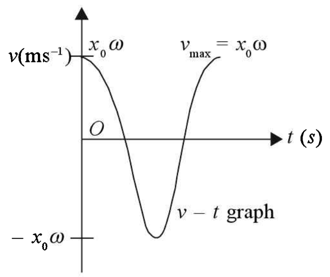


Fig. 12.2 (b) Velocity — time graph

Fig. 12.3 (a) and (b) shows graph between acceleration and displacement and acceleration and time.

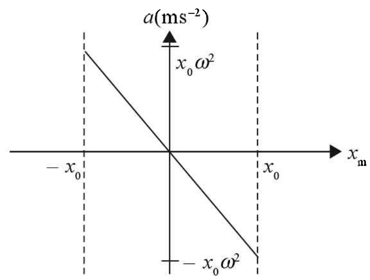


Fig. 12.3 (a) Acceleration — displacement graph

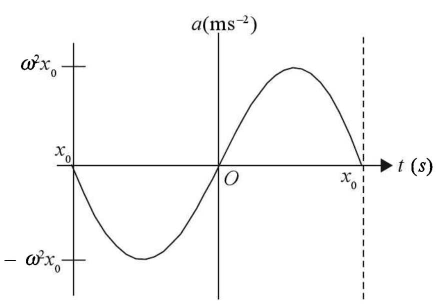


Fig. 12.3 (b) Acceleration — time graph

Note the graph between velocity and acceleration is an ellipse.

Note velocity leads the displacement by $\frac{\pi}{2}$ but velocity lags the acceleration by $\frac{\pi}{2}$

$$a_{max} = x_0\,\omega^2$$
$$v = x_0\omega\cos\omega t$$
$$\frac{dv}{dt} = -x_0\omega^2\sqrt{1-\cos^2\omega t} \text{ or } a = -\omega^2 x,\ a_{max} = \omega^2 x_0$$

$$a = -\omega\sqrt{(x_0\omega)^2-(x_0\omega\cos\omega t)}$$
$$a = -\omega\sqrt{v_0^2 - v^2}$$

or $$\frac{a^2}{\omega^2 v_0^2} + \frac{v^2}{v_0^2} = 1.$$

Note velocity is manimum at mean position and acceleration is zero at mean position. Velocity is zero at extreme position and acceleration is maximum at extreme position. Kinetic energy (KE) of a particle executing SHM = $\frac{1}{2}\,m\omega^2(x_0^2 - x^2)$

Potential energy (PE) of a particle executing SHM = $\frac{1}{2}\,m\omega^2 x^2$

Total energy = $KE + PE = \frac{1}{2}\,m\omega^2 x^2$

Note KE is maximum at mean position and zero at extreme postion. PE is zero at mean position and maximum at extreme position. See Fig. 12.4.

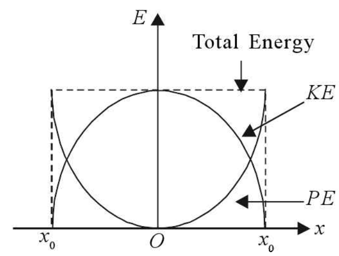


Fig. 12.4 KE, PE and total energy depiction

In SHM, velocity displacement curve is an ellipse. see Fig. 12.5 (a)

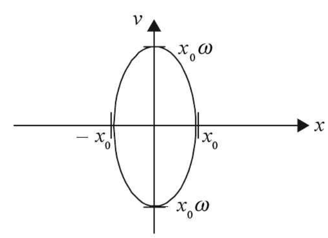


Fig. 12.5 (a) Velocity displacement graph

$$x = x_0\sin\omega t\,;$$
$$v = x_0\omega\cos\omega t$$

or $$\frac{x}{x_0} = \sin\omega t \qquad \text{—(1);}$$

$$\frac{v}{x_0\omega} = \cos\omega t \qquad \text{—(2)}$$

Square and add (1) and (2)

$$\frac{x^2}{x_0^2} + \frac{v^2}{x_0^2\omega^2} = 1$$

acceleration – velocity relationship in SHM is an ellipse

$$a = -\omega^2 x_0\sin\omega t\,;$$
$$v = x_0\omega\cos\omega t$$

or $$\frac{a^2}{\omega^4 x_0^2} + \frac{v^2}{x_0^2\omega^2}$$ see Fig. 12.5 (b)

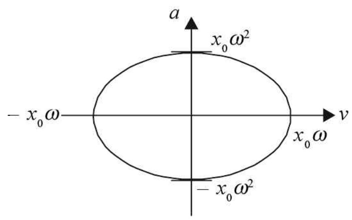


Fig. 12.5 (b) Acceleration — velocity graph

If a tunnel is dug in the earth diametrically or along a chord irrespective of its position or angle then $T = 2\pi\sqrt{\dfrac{R}{g}}$ = 84 min 36 s for a particle released in the tunnel. See Fig. 12.6

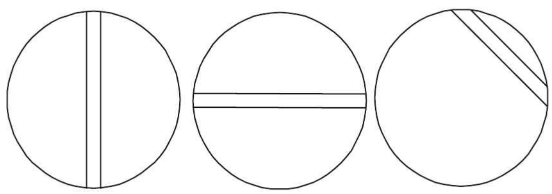


Fig. 12.6 SHM in tunnel in the earth

If a point charge q is tunnelled in a uniformly charged sphere having charge Q and radius R then

$$T = 2\pi\sqrt{\frac{4\pi\varepsilon_0 R^3 m}{Qq}}$$

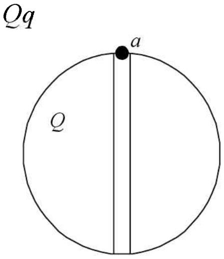


Fig. 12.7

Angular SHM A body free to rotate about a given axis can make angular oscillations when it is slightly pushed aside and released. The angular oscillations are called **angular SHM**.

(a) there is a mean position where the resultant torque on the body is zero ($\theta = 0$).

(b) the body is displaced through an angle from the mean position, a resultant torque $\propto \theta$ (angular displacement) acts.

(c) the nature of the torque (clockwise or anticlockwise) is to bring the body towards mean position.

$$\tau = -k\,\theta \text{ i.e.}$$

$$\alpha I = -k\,\theta \text{ or } \alpha = -\frac{k}{I}\,\theta$$

$$\alpha = -\omega^2\,\theta \qquad \text{or}$$

$$\omega = \sqrt{\frac{k}{I}} \text{ or } T = 2\pi\sqrt{\frac{I}{k}}.$$

Solution of the equation $\alpha = -\omega^2\,\theta$ is

$\theta = \theta_0 \sin \omega t$ if the particle starts from mean position

$\theta = \theta_0 \cos \omega t$ if the particle starts from extreme position

$\theta = \theta_0 \sin(\omega t \pm \phi)$ if the particle starts from in between mean and extreme.

$$\theta = \theta_0 \cos(\omega t \pm \phi) \quad \Omega = \frac{d\theta}{dt} = \theta_0\,\omega \cos \omega t$$

$$\text{or } \frac{d\theta}{dt} = \omega\sqrt{\theta_0^2 - \theta^2}$$

Pendulums may be of 5 types: **simple pendulum**, **spring pendulum**, **conical pendulum**, **physical or compound** and **torsional pendulum**. Note the time period of each of them.

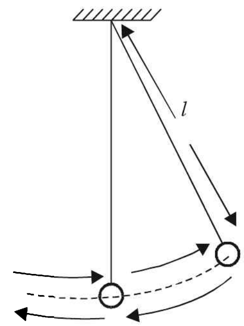


Fig. 12.8 (a) Simple Pendulum

$$T = 2\pi\sqrt{\frac{l}{g}} \text{ if } \theta \text{ is small}$$

$$T = 2\pi\sqrt{\frac{l}{g}}\left[1 + \frac{\theta_0^2}{16}\right] \text{ if } \theta \text{ is finite and } \theta = \theta_0$$

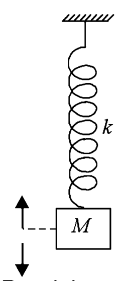


Fig. 12.8 (b) Spring Pendulum

$$T = 2\pi\sqrt{\frac{M}{k}}$$

Note no effect of 'g' on spring pendulum.

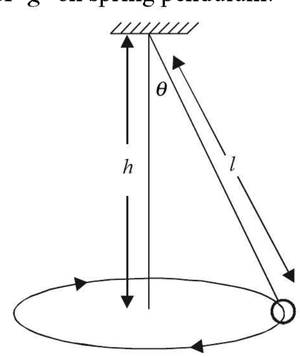


Fig. 12.8 (c) Conical Pendulum

$$T = 2\pi\sqrt{\frac{h}{g}}$$

or $T = 2\pi\sqrt{\dfrac{L\cos\theta}{g}}$

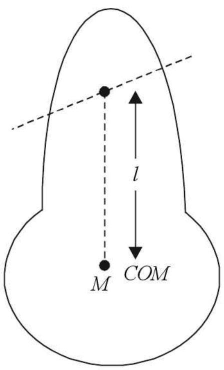


Fig. 12.8 (d) Physical Pendulum

$$T = 2\pi\sqrt{\frac{I}{mgl}}$$

or $T = 2\pi\sqrt{\dfrac{k^2+l^2}{lg}} = 2\pi\sqrt{\dfrac{(k+l)^2-2kl}{lg}}$

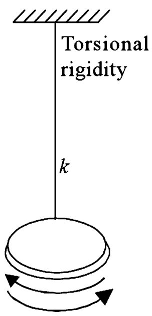


Fig. 12.8 (e) Torsional Pendulum

$$T = 2\pi\sqrt{\frac{I}{k}}$$

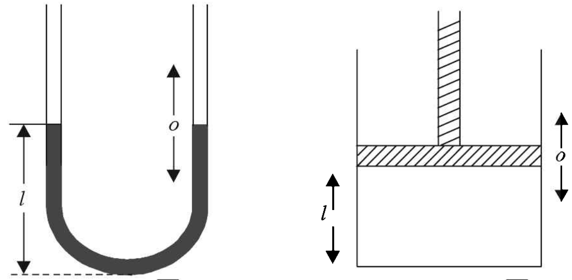
Note in physical pendulums T is maximum if $l = 0$ or $l = \infty$ and T is minimum if $k = l$.

Seconds pendulum: If the time period of a simple pendulum is 2s, it is called seconds pendulum.

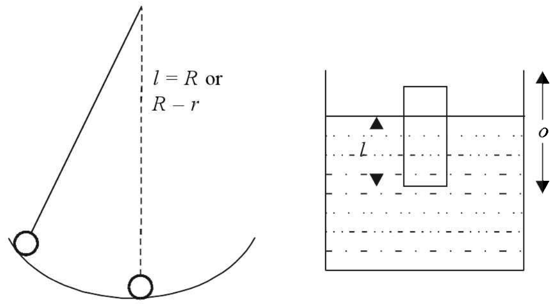
Longest time period (for $T = 2\pi\sqrt{\dfrac{1}{g\left(\dfrac{1}{l}+\dfrac{1}{R}\right)}}$ if $l \to \infty$ $T = 2\pi\sqrt{\dfrac{R}{g}}$ = 84 min. 36 s. for an infinitely long simple pendulum) where R is radius of the earth

If $l = R$, the radius of the earth then $T = 2\pi\sqrt{\dfrac{R}{2g}}$ = 60 min or 1h.

SHM under gravity If SHM occurs due to restoring force provided by weight or acceleration due to gravity then $T = 2\pi\sqrt{\dfrac{l}{g}}$. Some of the examples of this type are motion of a liquid in a U-tube vertical cylinder/piston. Motion of a ball in a concave mirror/ bowl and a floating cylinder as illustrated in Fig. 12.9.



(a) $T = 2\pi\sqrt{\dfrac{l}{g}}$ (b) $T = 2\pi\sqrt{\dfrac{l}{g}}$



(c) $T = 2\pi\sqrt{\dfrac{R}{g}}$ if ball does not roll but slips. $T = 2\pi\sqrt{\dfrac{7(R-r)}{5g}}$ if the ball rolls. (d) $T = 2\pi\sqrt{\dfrac{l}{g}}$

Fig. 12.9

Effect of temperature on time period of simple pendulum

$\dfrac{T}{T_0} = \left[1+\dfrac{\alpha\Delta\theta}{2}\right]$ where α is linear expansion coefficient and $\Delta\theta$ is rise in temperature. If temperature falls take $\Delta\theta$ negative.

or $\Delta T = T_0\,\dfrac{\alpha\Delta\theta}{2}$

If the upthrust of the liquid is taken into account Then time period $T = 2\pi\sqrt{\dfrac{l}{g(1-\frac{\sigma}{\delta}}}$ and $a = g' = g\left(1-\dfrac{\sigma}{\delta}\right)$ where σ is density of liquid and δ is density of the body. Damping of liquid is assumed negligible.

If the suspended wire stretches due to elasticity then time period $T' = 2\pi\sqrt{\dfrac{l}{g}}\left[1+\dfrac{Mg}{2\pi r^2 Y}\right]$ or $\Delta T = 2\pi\sqrt{\dfrac{l}{g}}\,\dfrac{Mg}{2\pi r^2 Y}$

or $\Delta T = T\,\dfrac{Mg}{2\pi r^2 Y}$ where $T = 2\pi\sqrt{\dfrac{l}{g}}$ and Y is young's modulus.

If a carriage (lift) is moving up with an acceleration '*a*' carrying a pendulum then $T = 2\pi\sqrt{\dfrac{l}{g+a}}$

If the carriage (lift) moves down with an acceleration '*a*' carrying the pendulum then

$$T = 2\pi\sqrt{\frac{l}{(g-a)}}$$

If the carriage moves horizontally (e.g. a car) with an acceleration '*a*' then $T = 2\pi\sqrt{\frac{l}{\sqrt{g^2+a^2}}}$

If the carriage is in circular motion of radius R with uniform speed v then

$$T = 2\pi\sqrt{\frac{l}{\sqrt{g^2+\left(\frac{v^2}{r}\right)^2}}}$$

If the bob of a pendulum is charged and is placed in a uniform electric field [charge q on the bob is assumed + ve in Fig. 12.10 (a) and 12.10 (b)]

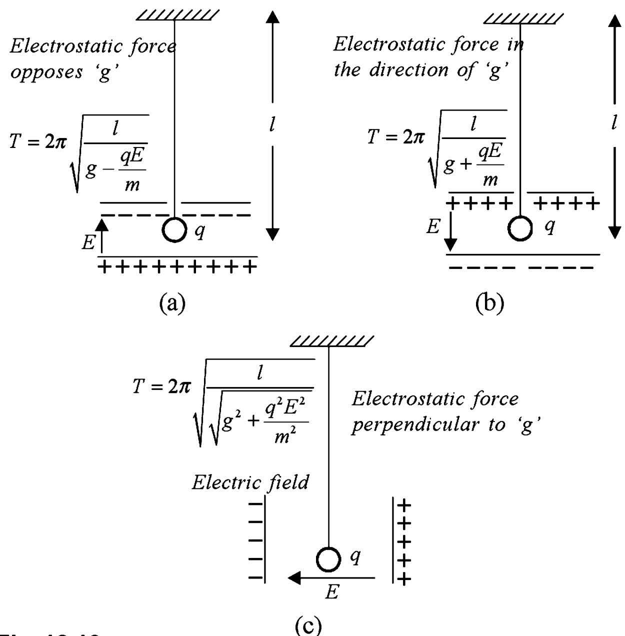
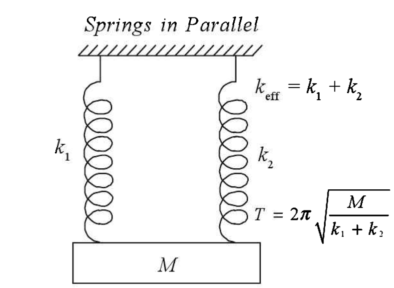


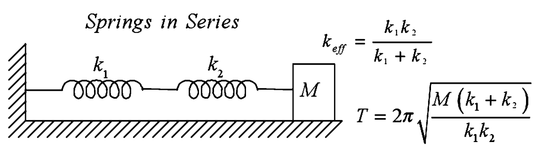
Fig. 12.10

For Spring System:

(a)



(b)



(c)

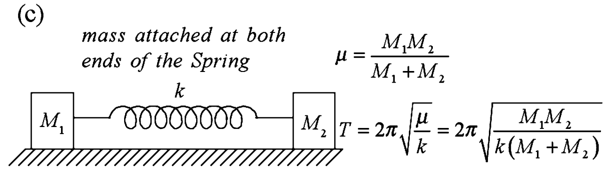


Fig. 12.11

Spring Pulley System:

(a)

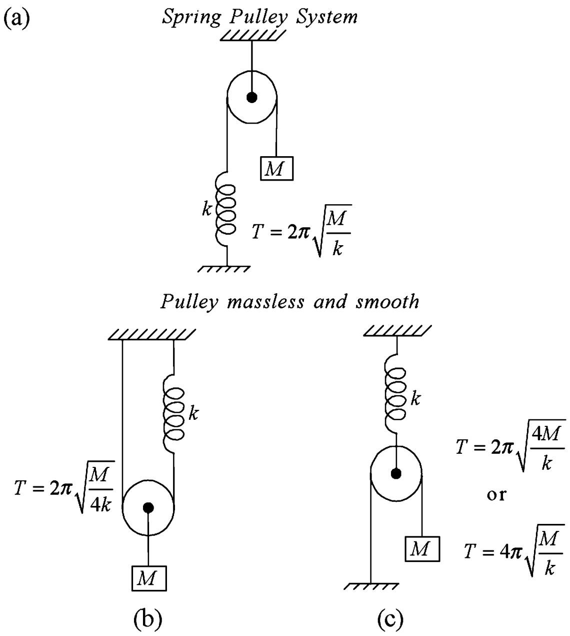


Fig. 12.12

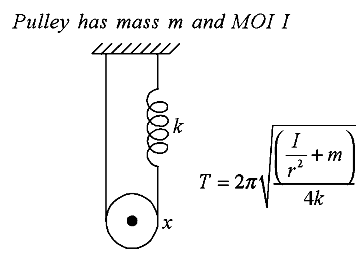


Fig. 12.13

Composition of two SHMs in same direction

$x_1 = x_{01} \sin \omega t$;

$x_2 = x_{02} \sin (\omega t + \theta)$

$x = x_0 \sin (\omega t + \phi) = x_1 + x_2 = x_{01} \sin \omega t + x_{02} \sin (\omega t + \theta)$

$x_0 = \sqrt{x_{01}{}^2 + x_{02}^2 + 2x_{01}x_{02}\cos\theta}$ and $\tan \phi$

$= \dfrac{x_{02}\sin\theta}{x_{01} + x_{02}\cos\theta}$

Note SHMs can be added like vectors. Result is same as parallelogram Law.

Composition of two perpendicular directions give rise to Lissajous figures.

$$x = x_0 \sin \omega t \text{ or } \sin \omega t = \frac{x}{x_0} \text{ and } \cos \omega t = \sqrt{1-\frac{x^2}{x_0^2}}$$

$$y = y_0 \sin(\omega t + \phi) = y_0 \sin \omega t \cos \phi + y_0 \cos \omega t$$
$\sin \phi$

$$y = y_0 \frac{x}{x_0} \cos \phi + y_0 \sqrt{1-\frac{x^2}{x_0^2}} \sin \phi$$

or $$\left(\frac{y}{y_0} - \frac{x}{x_0}\cos\phi\right)^2 = \left(1-\frac{x^2}{x_0^2}\right)\sin^2\phi$$

or $$\frac{y^2}{y_0^2} + \frac{x^2}{x_0^2} - \frac{2xy}{x_0 y_0}\cos\phi = \sin^2\phi$$

If $\phi = 0$ $\left(\frac{y}{y_0} - \frac{x}{x_0}\right)^2 = 0$ or $y = \frac{y_0}{x_0}x$, see Fig. 12.14 (a)

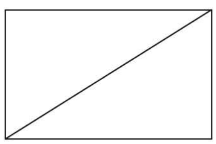


Fig. 12.14 (a)

If $0 < \phi < \frac{\pi}{2}$ for example $\phi = \frac{\pi}{4}$, oblique ellipse as shown in Fig. 12.14 (b) is obtained.

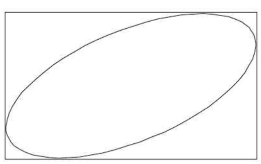


Fig. 12.14 (b)

If $\phi = \frac{\pi}{2}$, ellipse is obtained and if $x_0 = y_0$ the circle is obtained. See Fig. 12.15 (a) and (b)

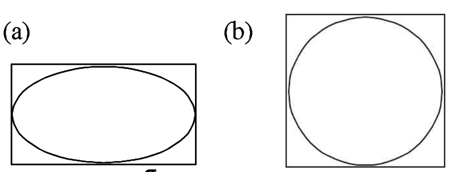


Fig. 12.15

If ϕ = 180° or π-radian then a straight line is obtained.

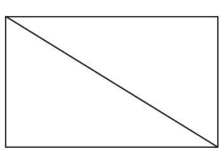


Fig. 12.16

Lissajous figures If the frequency of SHM in x-and y-direction are different then in Fig. 12.17 (a)

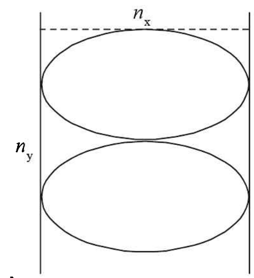


Fig. 12.17 (a)

$$\frac{\omega_x}{\omega_y} = \frac{n_y}{n_x}$$

$$= \frac{\text{number of times it touches } y\text{-axis}}{\text{number of times it touches } x\text{-axis}}$$

$= \frac{2}{1}$ and in Fig 12.17 (b)

$$\frac{\omega_x}{\omega_y} = \frac{2.5}{1}$$

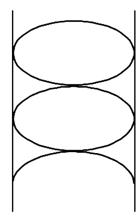


Fig. 12.17 (b)

Types of oscillations Oscillations may be of four types

(a) free or natural or fundamental frequency.
(b) forced. (c) resonant.
(d) damped.

Free or natural oscillations depend upon dimensions and nature of the material (elastic constant).

If a periodic force of frequency other than the natural frequency of the material is applied then **forced oscillations result.**

For example if $y = y_0 \sin \omega t$ was the equation of SHM of a particle and a periodic force $p \sin \omega_1 t$ is applied ($\omega \neq \omega_1$) then $y = y_0 \sin \omega t + p \sin \omega_1 t$. The resultant frequency is different from natural frequency of oscillation

Resonant oscillation are a special kind of forced oscillation in which frequency of the source = frequency of the applied force, i.e., $y = y_0 \sin \omega t + p \sin \omega t = (y_0 + p) \sin \omega t$. That is amplitude increases or intensity increases with resonance.

In damped oscillations amplitude of vibrations falls with time as shown in Fig. 12.18.

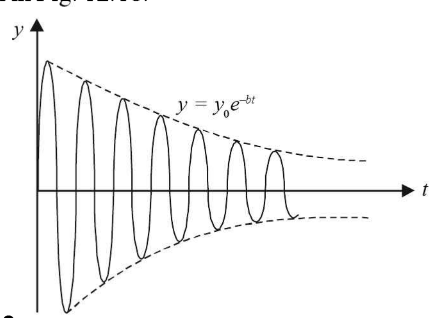


Fig. 12.18

Amplitude at any instant is given by $= y = y_0 e^{-bt}$ where y_0 is amplitude of first vibration and y is amplitude at time t and b is damping coefficient.

Damped harmonic motion

$$\frac{md^2x}{dt^2} + r\frac{dx}{dt} + kx = 0$$

or $$\frac{d^2x}{dt^2} + \frac{r}{m}\frac{dx}{dt} + \frac{k}{m}x = 0$$

or $$\frac{d^2x}{dt^2} + 2b\frac{dx}{dt} + \omega^2 x = 0$$

where $b = \frac{r}{2m}$ is called damping coefficient.

$$x = \frac{x_0}{2}e^{-bt}\left[\left(1+\frac{b}{\sqrt{b^2-\omega^2}}\right)e^{t\sqrt{b^2-\omega^2}} + \left(1-\frac{b}{\sqrt{b^2-\omega^2}}\right)e^{-t\sqrt{b^2-\omega^2}}\right]$$

gives amplitude at any instant.

If $\frac{r}{2m} > \sqrt{\frac{k}{m}}$ or $b > \omega$ motion is over damped and non-oscillatory

If $\frac{r}{2m} = \sqrt{\frac{k}{m}}$ or $b = \omega$ motion is critically damped and $x = x_0 e^{-bt}$

If $\frac{r}{2m} < \sqrt{\frac{k}{m}}$ $b < \omega$ damped oscillatory motion with

time period $T = \frac{2\pi}{\sqrt{\omega^2 - b^2}} = \frac{2\pi}{\sqrt{\frac{k}{m} - \frac{r^2}{4m^2}}}$

If $r = 0$ motion is undamped and $T = 2\pi\sqrt{\frac{m}{k}}$.

- **Short Cuts and Points to Note**

1. Periodic motion may also be termed as isochronous. Fourier theorem can be employed to express a complex perodic function as series of sine and cosine functions. That is, if $f(T)$ is a complex function of time then $f(T) = a_0 + \sum_{n=1}^{\infty} a_n \sin n\omega t + \sum_{n=1}^{\infty} b_n \cos n\omega t$.

2. SHM may be divided into two types

(a) Linear (b) Angular.

In Linear SHM

$a = -\omega^2 x$ or $F = -kx$.

In angular SHM

$\alpha = -\omega^2\theta$

or $\tau = -C\,\theta$. Note in both cases acceleration is proportional to displacement.

3. Solution to equation $\frac{d^2x}{dt^2} = -\omega^2 x$ are (i) $x = x_0 \sin\omega t$ if motion starts form mean position at $t = 0$ (ii) $x = x_0 \cos\omega t$ if motion starts from extreme position at $t = 0$

$$\left.\begin{array}{l} x = x_0 \sin(\omega t \pm \phi) \\ x = x_0 \cos(\omega t \pm \phi)\end{array}\right\} \text{if the motion starts in between mean and extreme at } t = 0$$

We may also represent $x = x_0\, e^{i\omega t \pm \phi}$ as SHM in exponential form.

4. If $x = x_0 \sin \omega t$ then $v = \frac{dx}{dt} = x_0\omega \cos \omega t$ use these equations when time is given in the problems.

If $x = x_0 \cos \omega t$ then $v = \frac{dx}{dt} = -x_0\omega \sin \omega t$

If displacement is given then $v = \omega\sqrt{x_0^2 - x^2}$.

Note $v_{max} = x_0\omega$ at mean position. $v = 0$ at extreme position. Velocity displacement graph is ellipse.

5. Use $a = -\omega^2 x$ if displacement is given. Acceleration displacement graph is a straight line with obtuse angle slope.

Use $a = -\omega^2 x_0 \sin \omega t$ if time is known and particle starts from mean position at $t = 0$

Use $a = -\omega^2 x_0 \cos \omega t$ if time is known and particle starts from extreme position at $t = 0$.

$a_{max} = -\omega^2 x_0 \cos \omega t$ at extreme position. $a_{min} = 0$ at mean position. acceleration – velocity graph is an ellipse.

6. $KE = \frac{1}{2} m\omega^2 (x_0^2 - x^2)$ KE is maximum at mean position $KE_{max} = \frac{1}{2} m\omega^2 x_0^2$ KE is minimum at mean position $KE_{min} = 0$.

7. The frequency of KE or PE is twice the frequency of SHM.

8. PE $= \frac{1}{2} m\omega^2 x$. PE is maximum at extreme position when $x = x_0$. $PE_{max} = \frac{1}{2} m\omega^2 x_0$. PE is minimum at mean position. From this equation it appears $PE = 0$ if $x = 0$, i.e., at mean position. But it is not necessary that PE at mean position be zero. For example, if the bob of a pendulum is at a height h at $x = 0$, i.e, pendulum has some PE at mean position. Thus, general equation of $PE = \frac{1}{2} m\omega^2 x +$ positional PE.

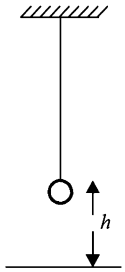


Fig. 12.19

9. If a tunnel is dug in earth diametrically or along a chord then $T = 2\pi\sqrt{\frac{R}{g}}$ along the tunnel.

However, if the ball is released from a height h along the tunnel as shown in Fig 12.20 then

$$T = 4\sqrt{\frac{2h}{g}} + 4\sqrt{\frac{R}{g}}\sin^{-1}\left[\frac{h}{R+2h}\right].$$

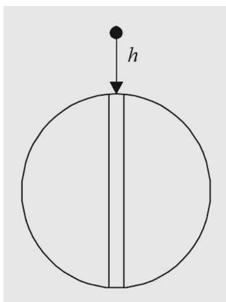


Fig. 12.20

10. If a charged paritcle having charge q is released in a tunnel in a charged solid sphere of charge Q and radius R then

$$T = 2\pi\sqrt{\frac{4\pi\varepsilon_0 R^3 m}{Qq}}.$$

11. If a dipole of moment p is suspended in a uniform electric field then time period for small oscillation is

$$T = 2\pi\sqrt{\frac{I}{pE}}$$ where I is moment of inertia.

12. If a magnetic dipole of moment M is suspended in a magnetic field of induction B then time period

$$T = 2\pi\sqrt{\frac{I}{MB}}$$

13. For a simple pendulum

$$T = 2\pi\sqrt{\frac{l}{g}}$$ if θ is small

for a pendulum with finite angle $\theta = \theta_0$

$$T = 2\pi\sqrt{\frac{l}{g}}\left(1+\frac{\theta_0^2}{16}\right)$$

If the bob of the pendulum has radius r

$$T = 2\pi\sqrt{\frac{l^2+\frac{2}{5}r^2}{lg}}$$

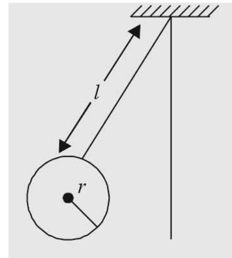


Fig. 12.21

Time period of a long pendulum

$$T = 2\pi\sqrt{\frac{1}{\left(\frac{1}{l}+\frac{1}{R}\right)g}}$$

if $l \to \infty$

$$T = 2\pi\sqrt{\frac{R}{g}} = 84 \text{ min } 36\text{s}.$$

14. For a physical or compound pendulum

$$T = 2\pi\sqrt{\frac{I}{mgl}} = \sqrt{\frac{k^2+l^2}{gl}}$$

where l is distance of axis rotation from COM.

where k is radius of gyration.

The plot time period versus displacement from axis of rotation in a bar pendulum (or a compound pendulum) is shown in fig. 12.22.

Note T is maximum if $l = \infty$ or $l = 0$

$T_{max} = \infty$

T is minimum if $l = k$

$$T_{min} = 2\pi\sqrt{\frac{2kl}{lg}}.$$

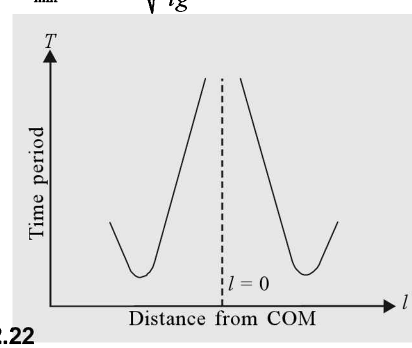


Fig. 12.22

15. For a conical pendulum

$$T = 2\pi\sqrt{\frac{l\cos\theta}{g}} = 2\pi\sqrt{\frac{h}{g}}.$$ See Fig. 12.23.

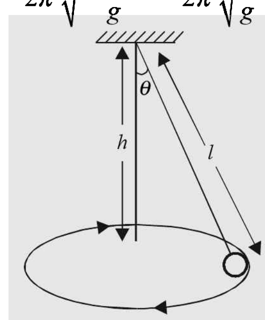


Fig. 12.23

16. For a torsional pendulum

$$T = 2\pi\sqrt{\frac{I}{K}} \text{ where } K \text{ is torsional rigidity.}$$

17. In a spring pendulum

$$T = 2\pi\sqrt{\frac{M}{R}} \text{ if spring has mass } M_s \text{ then}$$

$$T = 2\pi\sqrt{\frac{M+\frac{M_s}{3}}{k}}$$

If springs are in parallel, use $k_{\text{eff}} = k_1 + k_2 + \ldots$

If springs are in series, use $\frac{1}{k_{\text{eff}}} = \frac{1}{k_1} + \frac{1}{k_2} + \ldots$

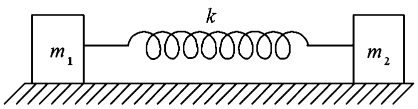


Fig. 12.24

If masses M_1 and M_2 are attached to two ends of a spring of spring constant k. The spring is compressed by x and released to oscillate. Use reduced mass in such cases.

$$\frac{1}{\mu} = \frac{1}{M_1} + \frac{1}{M_2}$$

or $$\mu = \frac{M_1 M_2}{M_1 + M_2}$$

$\therefore$ $$T = 2\pi\sqrt{\frac{\mu}{k}}$$

$$= 2\pi\sqrt{\frac{M_1 M_2}{(M_1+M_2)k}}.$$

18. In a spring pulley system:

Fig. 12.25 and Fig. 12.26

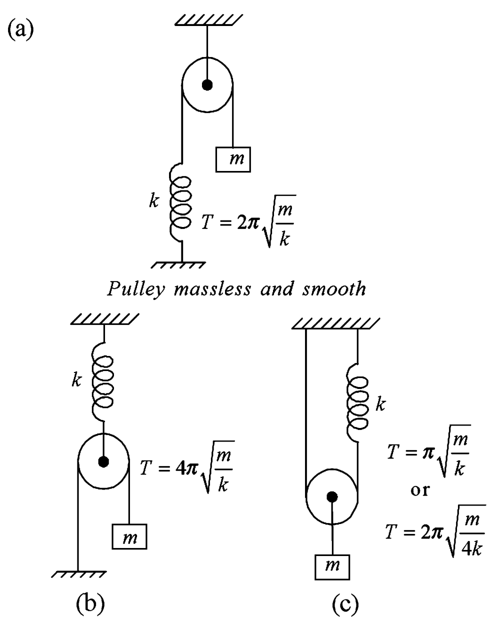


Fig. 12.25

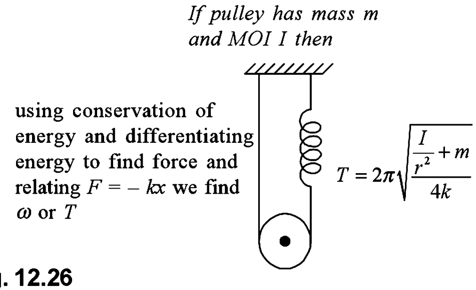


Fig. 12.26

19. SHM of liquid in U-tube:

In Fig. 12.27 (a)

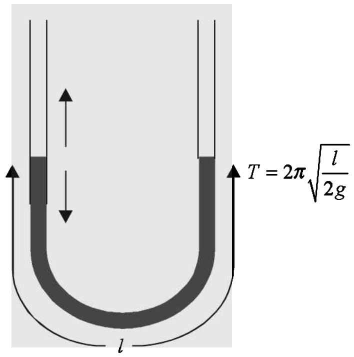


Fig. 12.27 (a)

If one side of liquid has length as shown in Fig. 12.27 (b) then

$$T = 2\pi\sqrt{\frac{l}{g}}$$

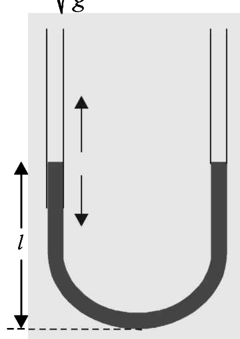


Fig. 12.27 (b)

20. SHM in cylinder – piston In a vertical cylinder

$$T = 2\pi\sqrt{\frac{l}{g}} \text{ as shown in Fig. 12.28.}$$

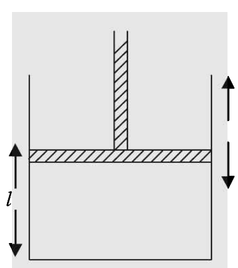


Fig. 12.28 (a)

In a horizontal cylinder/piston system having a gas of bulk modulus B or pressure P. Volume V_0

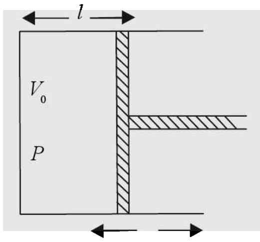


Fig. 12.28 (b)

$$T = 2\pi\sqrt{\frac{MV_0}{A^2B}} = 2\pi\sqrt{\frac{MV_0}{A^2\gamma P}}$$

In (adiabatic conditions)

$$\gamma = \frac{C_P}{C_V}$$

$$T = 2\pi\sqrt{\frac{MV_0}{A^2P}} = 2\pi\sqrt{\frac{Ml}{AP}}$$

In (isothermal conditions).

21. If pendulum is in a carraige moving vertically up or down with an acceleration a, then $T = 2\pi\sqrt{\frac{l}{g \pm a}}$ use $+ve$ sign for upward motion and use $-ve$ sign for downward motion.

22. If the carriage is accelerated horizontally with an acceleration 'a' than $T = 2\pi\sqrt{\frac{l}{(g^2+a^2)^{1/2}}}$

23. If the carriage moves in a circle of radius r with velocity v, i.e., it is in a merry-go-round then

$$T = 2\pi\sqrt{\frac{l}{\left(g^2+\left(\frac{v^2}{r}\right)^2\right)^{\frac{1}{2}}}}$$

24. If the sphere of radius r slides on a concave mirror of radius R then

$$T = 2\pi\sqrt{\frac{(R-r)}{g}}$$

25. If the sphere of radius r rolls on a concave mirror of radius R then

$$T = 2\pi\sqrt{\frac{7(R-r)}{5g}}$$. See Fig. 12.29.

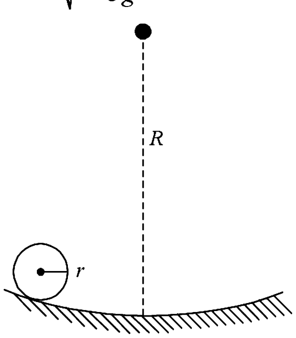


Fig. 12.29

26. If the bob of a pendulum is immersed in a liquid (non viscous) $T = 2\pi\sqrt{\frac{l}{g\left(1-\frac{\sigma}{\rho}\right)}}$

27. SHM in the same direction but with phase difference are added like vectors. Thus, if $x_1 = x_{01}$ sin ωt and $x_2 = x_{02}$ sin $(\omega t + \theta)$ are added then resultant is an SHM

$x = x_0 \sin(\omega t + \phi)$. So that

$$x_0 = \sqrt{x_{01}^2 + x_{02}^2 + 2x_{01}x_{02}\cos\theta} \text{ and}$$

$$\tan\phi = \frac{x_{02}\sin\theta}{x_{01} + x_{02}\cos\theta}.$$

28. If two SHMs are at right angles i.e. along x and y directions and a phase shift of 0° or 180° exists between them they form a straight line otherwise an ellipse is formed. If $\phi = 90°$ or $\frac{\pi}{2}$ radian and $x_0 = y_0$ (amplitudes are equal) then a circle results.

29. If the frequency of SHMs in x and y directions are different, Lissajous figures are formed

$$\frac{\omega_x}{\omega_y} = \frac{n_y}{n_x} = \frac{\text{number of times it touches } y\text{-axis}}{\text{number of times it touches } x\text{-axis}}$$

phase difference ϕ can be found from oblique ellipse as shown in Fig. 12.30.

$$\phi = \sin^{-1}\frac{a}{b}$$

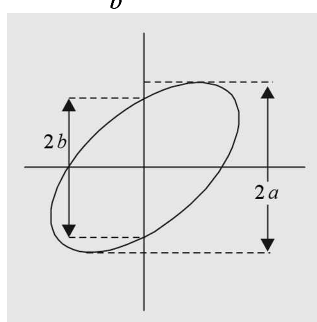


Fig. 12.30

30. Oscillation are of four types: free, forced, resonant and damped. Resonant oscillations are a special kind of forced oscillations in which frequency of force = frequency of source. In damped oscillation amplitude at any instant is obtained using $x_0' = x_0 e^{-bt}$ where x_0 is amplitude of first oscillation.

31. Quality factor $Q = 2\pi \frac{\text{Average energy stored}}{\text{energy loss in one cycle}}$

$= \omega_0\tau$ where τ is relaxation time. Relaxation time

$= \frac{m}{b}$ (for energy). Relaxation time for velocity $=$

$\frac{2m}{b}$.

32. If $b = \frac{r}{2m} > \omega \left[= \sqrt{\frac{k}{m}}\right]$ motion is non oscillatory and overdamped

If $b = \omega$ motion is critically damped.

If $b < \omega$ damped oscillatory motion occurs.

If $r = 0$ undamped oscillations result.

• Caution

1. Considering every vibratory motion as SHM.

⇒ Only those vibratory motions in which $a = -\omega^2 x$ or $\alpha = -\omega^2 \theta$ are SHMs. Note in SHMs amplitude x_0 or θ_0 are extremely small. Force is directed towards mean or equilibrium position.

2. Assuming that decreasing amplitude with time in simple pendulum also decreases time.

⇒ Time period remains unchanged.

3. Considering amplitude synonym of span of SHM.

⇒ Span of SHM = twice the amplitude.

4. Considering ϕ as net phase in $x = x_0 \sin(\omega t + \phi)$

⇒ ϕ is initial phase at $t = 0$. It is also called angle of repose or epoch. Note ϕ should be small if motion is to be SHM.

5. Considering that the time periods in case of spring pendulums on different planes like the one on inclined plane, other on vertical plane and yet another on horizontal plane are different.

⇒ Time period in spring pendulum is independent of 'g'.

6. Considering total length as length of pendulum in a compound or physical pendulum.

⇒ length from COM to point of suspension is to be used.

7. Assuming at half the amplitude time will be $\frac{1}{8}$ th of the total time period.

⇒ $t = \frac{T}{12}$ if the particle starts from mean position,

and, $t = \frac{T}{6}$ if the particle starts from extreme position.

8. Assuming spring constant remains invariant when spring is cut.

⇒ Spring constant $k \propto \frac{1}{l}$.

9. Considering total energy = $KE + PE$.

⇒ Total energy = $KE + PE$ + resting energy (positional PE)

10. Assuming Average PE = Average KE always.

⇒ Average KE = Average PE = $\frac{1}{4} m \omega^2 x_0^2$ (with respect to time averaged over time period).

Average KE = 2 Average PE with respect to position.

Average KE with respect to position = $\frac{1}{3} m\omega^2 x_0^2$

and Average PE with respect to position = $\frac{1}{6} m\omega^2 x_0^2$.

11. Considering motion in V-tube is alike motion in U-tube.

⇒ In V-tube shown in Fig. 12.31

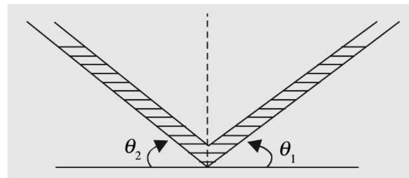


Fig. 12.31

$$T = 2\pi \sqrt{\frac{m}{A\rho g(\sin\theta_1 + \sin\theta_2)}}$$ where ρ is density of the liquid.

12. Considering that time period of a simple pendulum depends upon mass or amplitude as time period of a cylindrical bob having a hole at the base varies when the sand/water leaks through it.

⇒ $T = 2\pi\sqrt{\frac{l}{g}}$ time period varies with length l or g.

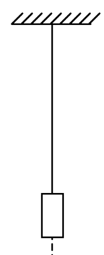


Fig. 12.32

When the sand/water is being vacated distance from COM (length) varies hence, T varies.

13. Considering mass of the spring does not affect time period.

⇒ If mass of the spring is M_s then net mass is $m + \frac{M_s}{3}$

and $T = 2\pi\sqrt{\frac{\frac{M_s}{3} + m}{k}}$.

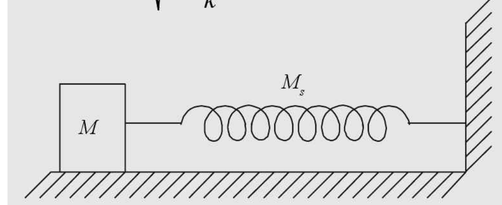


Fig. 12.33

14. Considering we cannot find frequency of oscillation using energy conservation.

⇒ In SHM total energy is conserved. Therefore, $\frac{dE}{dt} = 0$.

15. Considering that only one restoring force can act in SHM.

⇒ There can be more than one restoring force.

16. Not remembering trignometric relations.

⇒ Remember trignometric relations for better understanding of SHM and waves.

SOLVED PROBLEMS

1. The circular motion of a particle with constant speed is

(a) periodic but not SHM.
(b) SHM but not periodic.
(c) periodic and SHM.
(d) neither periodic nor SHM.

[CBSE PMT 2005]

Solution (a)

2. Two SHMs are represented by $y_1 = 0.1 \sin(100\,\pi t + \frac{\pi}{3})$ and $y_2 = 0.1 \cos \pi t$. The phase difference of the velocity of particle 1 with respect to the particle 2 is

(a) $\frac{-\pi}{6}$ (b) $\frac{\pi}{3}$
(c) $\frac{-\pi}{3}$ (d) $\frac{+\pi}{6}$

[AIEEE 2005]

Solution (a) $\phi = \frac{\pi}{3} - \frac{\pi}{2} = \frac{-\pi}{6}$

$\because \cos \pi t = \sin\left(\frac{\pi}{2} + \pi t\right)$.

3. Which of the following functions represent a simple harmonic motion?

(a) $\sin \omega t - \cos \omega t$ (b) $\sin^2 \omega t$
(c) $\sin \omega t + \sin^2 \omega t$ (d) $\sin \omega t - \sin^2 \omega t$

[AIIMS 2005]

Solution (a) $\because \frac{d^2y}{dt^2} \propto y$.

4. A small body attached to one end of a vertically hanging spring is performing SHM about its mean position with angular frequency ω and amplitude a. If at height y from the mean position the body gets detatched from the spring. Calculate the value of y so that height obtained by the mass is maximum. The body does not interact with the spring during its subsequent motion after detachment.

[IIT Mains 2005]

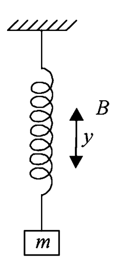


Fig. 12.34

Solution The point B should be such that $PE = 0$ at B so that KE is maximum and it can rise to a maximum height.

$\therefore \quad y = \frac{mg}{k} = \frac{g}{\omega^2} < a$ (amplitude)

5. From the displacement time graph of an oscillating particle. Find the maximum velocity of the particle.

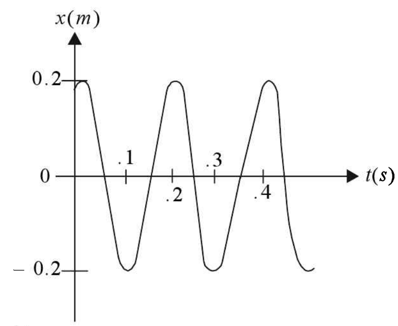


Fig. 12.35

(a) $2\ \text{ms}^{-1}$ (b) $\pi\ \text{ms}^{-1}$
(c) $2\pi\ \text{ms}^{-1}$ (d) $\frac{\pi}{2}\ \text{ms}^{-1}$

Solution (c) $v_{max} = x_0\,\omega = 0.2\left(\frac{2\pi}{0.2}\right) = 2\pi \text{ms}^{-1}$

6. A machine part is undergoing SHM with a frequency of 5 Hz and amplitude 1.8 cm. How long does it take the part to go from $x = 0$ to $x = -1.8$ cm?

(a) $\frac{1}{20}$ s (b) $\frac{1}{15}$ s
(c) $\frac{1}{10}$ s (d) $\frac{1}{30}$ s

Solution (a) $t = \frac{T}{4} = \frac{1}{5 \times 4} = \frac{1}{20}$ s.

7. A 42.5 kg chair is attached to a spring. It takes 1.3 s to make one complete oscillation when the chair is empty. When a lady is sitting on the chair with her feet off the floor, the chair now takes 1.84 s for one cycle. The mass of the lady is

(a) 35.5 kg (b) 40.5 kg
(c) 42.5 kg (d) 45 kg

Solution (b) $T = 2\pi\sqrt{\frac{M}{k}}$

or $\left(\frac{T_2}{T_1}\right)^2 = \frac{M+M_{\text{Lady}}}{M}$

$\Rightarrow \left(\frac{1.84}{1.3}\right)^2 = (1.40)^2 = 1.96 = 1 + \frac{M_{\text{lady}}}{42.5}$.

$M_{\text{lady}} = 42.5 \times 0.96 = 40.5\,\text{kg}$

8. Find the velocity when $KE = PE$ of the body undergoing SHM. Amplitude $= x_0$ and angular frequency is ω. How many times in a cycle $KE = PE$.

(a) $\frac{\omega x_0}{\sqrt{2}}, 2$ (b) $\omega x_0, 2$

(c) $\frac{\omega x_0}{\sqrt{2}}, 4$ (d) $\omega x_0, 4$

Solution (c) $\frac{1}{2} m\,\omega^2 (x_0^2 - x^2) = \frac{1}{2} m\,\omega^2 x^2$

or $x = \frac{x_0}{\sqrt{2}}$

$V = \omega \sqrt{x_0^2 - x^2} = \frac{\omega x_0}{\sqrt{2}}$.

This will be achieved 4 times in a cycle.

9. In the Vander Waals interaction

$$U = U_0\left[\left(\frac{R_0}{r}\right)^{12} - 2\left(\frac{R_0}{r}\right)^{6}\right].$$

A small displacement x is given from equilibrium position $r = R_0$. Find the approximate PE function.

(a) $\frac{36U_0}{R_0^2}x^2 - U_0$ (b) $\frac{24U_0}{R_0}x - U_0$

(c) $\frac{96U_0}{R_0^2} - U_0$ (d) none of these

Solution (a) $U = U_0\left[\left(\frac{R_0}{R_0+x}\right)^{12} - 2\left[\frac{R_0}{R_0+x}\right]^{6}\right]$

$$= U_0\left[\frac{R_0^{12}}{R_0^{12}}\left[\frac{1}{1+\frac{x}{R_0}}\right]^{12} - 2\left[\frac{1}{1+\frac{x}{R_0}}\right]^{6}\right]$$

$$= U_0\left[\left(1+\frac{x}{R_0}\right)^{-12} - 2\left[1+\frac{x}{R_0}\right]^{-6}\right]$$

$$= U_0\left[\left[1-\frac{12x}{R_0}+\frac{66x^2}{R_0^2}\right] - 2\left(1-\frac{6x}{R_0}+\frac{15x^2}{R_0^2}\right)\right]$$

$$= \frac{36U_0}{R_0^2}x^2 - U_0$$

10. A hydrogen atom has mass 1.68×10^{-27} kg. When attached to a certain massive molecule it oscillates with a frequency 10^{14} Hz and with an amplitude 10^{-9} cm. Find the force acting on the hydrogen atom.

(a) 2.21×10^{-9} N (b) 3.31×10^{-9} N

(c) 4.42×10^{-9} N (d) 6.63×10^{-9} N

Solution (d) $\omega^2 = \frac{k}{m}$ or $4\pi^2 f^2 m = k$

or $F = kx_0 = 4\pi^2 f^2 m\, x_0$

or $F = 4 \times \pi^2 \times 10^{28} \times 1.68 \times 10^{-27} \times 10^{-11}$

$= 6.63 \times 10^{-9}$ N.

11. An unhappy mouse of mass m_0, moving on the end of a spring of spring constant p is acted upon by a damping force $F_x = -b\,v_x$. For what value of b the motion is critically damped.

(a) $b = \sqrt{\frac{p}{m_0}}$ (b) $b = 2\sqrt{pm_0}$

(c) $b = \sqrt{\frac{p^2}{2m_0}}$ (d) $b = \sqrt{\frac{p}{2m_0}}$

Solution (b) $\frac{b}{2m_0} = \sqrt{\frac{p}{m_0}}$

or $b = 2\sqrt{pm_0}$.

12. Using equation $x = A\,e^{\frac{-bt}{2m}}\,t\cos(\omega' t + \phi)$ and assuming $\phi = 0$ at $t = 0$, find the expression for acceleration at $t = 0$

(a) $A\left[\frac{k}{m} - \frac{b^2}{4m^2}\right]$ (b) $A\left[\frac{k}{m} + \frac{b^2}{4m^2}\right]$

(c) $A\left[\frac{b^2}{4m^2} - \frac{k}{m}\right]$ (d) $A\left[\frac{b}{2m} - \frac{k}{m}\right]$

Solution (c) $\frac{dx}{dt} = \frac{-b}{2m} A\, e^{\frac{-bt}{2m}} \cos(\omega' t) - A\, e^{\frac{-bt}{2m}}\,\omega' \sin(\omega' t)$.

$$\frac{d^2x}{dt^2} = -\frac{-Ab}{2m}\left[\frac{-b}{2m}e^{\frac{-bt}{2m}}\cos(\omega' t) - e^{\frac{-bt}{2m}}\omega'\sin(\omega' t)\right]$$

$$+ \frac{bA}{2m}\,\omega' e^{\frac{bt}{2m}}\sin(\omega' t+\phi) - A\,\omega'^2\, e^{\frac{-bt}{2m}}\cos(\omega' t+\phi)\;\left.\frac{d^2x}{dt^2}\right|_{t=0}$$

$$= -A\,\omega'^2 + A\,\frac{b^2}{4m^2} = A\left[\frac{b^2}{4m^2} - \frac{k}{m}\right].$$

13. A glider is oscillating in SHM on an air track with an amplitude A. You slow it so that its amplitude becomes half. Find the total mechanical energy in terms of previous value.

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$

(c) $\frac{1}{\sqrt{5}}$ (d) $\frac{1}{4}$

Solution (d) initial total energy = $\frac{1}{2} m\,\omega^2 A^2$;

final total energy = $\frac{1}{2} m\,\omega^2 \left(\frac{A}{2}\right)^2$.

$\therefore$ Total mechanical energy becomes ¼ th.

14. A block of mass M is connected to one end of a spring of spring constant k. The other end is connected to the wall. Another block of mass m is placed on M. The coefficient of static friction is μ. Find the maximum amplitude of oscillation so that block of mass m does not slip on the lower block.

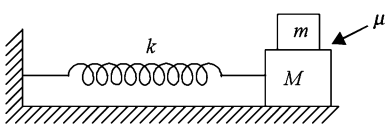


Fig. 12.36

(a) $\frac{\mu mg}{k}$ (b) $\frac{\mu Mg}{k}$

(c) $\frac{\mu(M+m)g}{k}$ (d) $\mu\left(\frac{\mu Mg}{k(M+m)}\right)$

Solution (c) $\mu\,(M+m)\,g = k\,x_0$

or $x_0 = \frac{\mu(M+m)g}{k}$.

15. A hanging wire is 185 cm long having a bob of 1.25 kg. It shows a time period of 1.42 s on a Planet Newtonia. If the circumference of Newtonia is 51400 km, find the mass of the planet.

(a) 3.5×10^{25} kg (b) 9.08×10^{24} kg

(c) 2.6×10^{25} kg (d) 3.14×10^{24} kg

Solution (a) $T = 2\pi\sqrt{\frac{l}{g}}$ or $g = \frac{4\pi^2 l}{T^2}$ and

$$\frac{GM}{R^2} = \frac{4\pi^2 l}{T^2}$$

or $$M = \frac{4\pi^2 lR^2}{GT^2}.$$

$$M = \frac{4\times 10\times 1.85\times (8185)^2\times 10^6}{6.67\times 10^{-11}\times (1.42)^2}$$

$$= 3.5 \times 10^{25}\ \text{kg}$$

16. A uniform rod of length l mass m is fixed at the centre. A spring of spring constant k is connected to rod and wall as shown in Fig. 12.37. The rod is displaced by small angle θ and released. Find time period of oscillation.

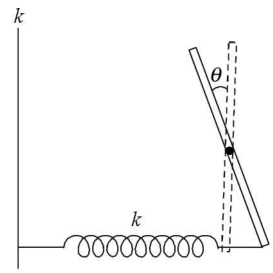


Fig. 12.37

(a) $2\pi\sqrt{\frac{m}{k}}$ (b) $2\pi\sqrt{\frac{m}{2k}}$

(c) $2\pi\sqrt{\frac{3m}{k}}$ (d) $2\pi\sqrt{\frac{m}{3k}}$.

Solution (d) $\tau = I\,\alpha = -(kx)\,\frac{l}{2}$

or $$\frac{ml^2}{12}\,\alpha = -\left(k\frac{l}{2}\theta\right)\frac{l}{2}.$$

or $$\alpha = -\frac{3k}{m}\,\theta$$

or $$\omega = \sqrt{\frac{3k}{m}}$$

or $$T = 2\pi\sqrt{\frac{m}{3k}}$$

17. A uniform rod of length L oscillates through small angles about a point x from its centre for what value of L its angular frequency will be maximum.

(a) $\frac{L}{12}$ (b) $\frac{L}{3}$

(c) $\frac{L}{\sqrt{8}}$ (d) $\frac{L}{\sqrt{12}}$

Solution (d) ω will be maximum if T is minimum. T is minimum if $x = k$ the radius of gyration

or $$x = \frac{L}{\sqrt{12}}.$$

18. Two solid cylinders connected with a short light rod about common axis have radius R and total mass M rest on a horizontal table top connected to a spring of spring constant k as shown. The cylinders are pulled to the left by x and released. There is sufficient friction for the cylinders to roll. Find time period of oscillation.

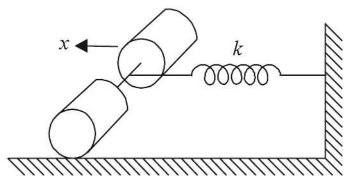


Fig. 12.38

(a) $2\pi\sqrt{\frac{M}{k}}$ (b) $2\pi\sqrt{\frac{M}{2k}}$

(c) $2\pi\sqrt{\frac{3M}{2k}}$ (d) $2\pi\sqrt{\frac{M}{3k}}$

Solution (c) $\tau = \left(\frac{MR^2}{2} + MR^2\right)\alpha = -kx \,.\, R$

or $a = R\,\alpha = -\frac{2kx}{3M}$

or $\omega = \sqrt{\frac{2k}{3M}}$

or $T = 2\pi\sqrt{\frac{3M}{2k}}$

19. A particle moves according to the equation $x = a\sin^2(\omega t - \frac{\pi}{4})$. Find the amplitude and frequency of oscillations.

(a) a, ω (b) $\frac{a}{2}$, ω

(c) $\frac{a}{2}$, $\frac{\omega}{2}$ (d) $\frac{a}{2}$, 2ω

Solution (d) $x = a\sin^2(\omega t - \frac{\pi}{4})$

$= \frac{a}{2}\,[1 - \cos 2\,(\omega t - \frac{\pi}{4})]$

20. The superposition of two SHMs of the same direction results in the oscillation of a point according to the law $x = x_0 \cos(2.1\,t)\cos 50\,t$. Find the angular frequencies of the constituent oscillations and period with which they beat.

(a) 52.1 s^{-1}, 47.9 s^{-1}, 0.2 s (b) 50 s^{-1}, 2.1 s^{-1}, 0.22 s

(c) 52.1 s^{-1}, 47.9 s^{-1}, 1.5 s (d) none

Solution (c) $x_0 \cos(2.1\,t)\cos 50\,t$

$= \frac{x_0}{2}\,[\cos 52.1\,t + \cos 47.9\,t]$.

$f = \frac{\omega_1 - \omega_2}{2\pi}$

or $T = \frac{2\pi}{\omega_2 - \omega_1} = \frac{2\times\pi}{4.2} \simeq 1.5\text{ s}$

21. A point moves in the plane xy according to the law $x = a\sin\omega t$, $y = b\cos\omega t$. The particle has a trajectory ____.

(a) hyperbolic (b) elliptic

(c) circular (d) straight line

Solution (b) $\frac{x}{a} = \sin\omega t$ .... (1)

$\frac{y}{b} = \cos\omega t$ .... (2)

square & add (1) and (2)

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ which shows ellipse.

TYPICAL PROBLEMS

22. Determine the period of oscillation of 200 g of Hg into a bent tube whose right arm forms an angle 30° with the vertical as shown. The cross-sectional area of the tube is 0.5 cm^2. Neglect viscosity.

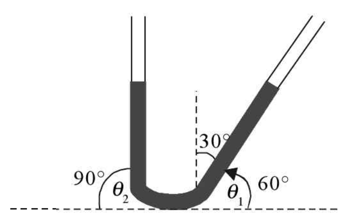


Fig. 12.39

(a) 0.68 s (b) 0.74 s

(c) 0.8 s (d) 0.88 s

Solution (c) $T = 2\pi\sqrt{\frac{M}{A\rho g(\sin\theta_1 + \sin\theta_2)}}$

$= 2\pi\sqrt{\frac{.2}{.5\times10^{-4}\times13.6\times10^{3}\times10\left(1+\frac{\sqrt{3}}{2}\right)}}$.

$= 0.8\text{s}$

23. In the system shown, a long uniform rod is attached at one end of a spring constant k and the other end is hinged. It is displaced slightly and allowed to oscillate. The time period of oscillation is

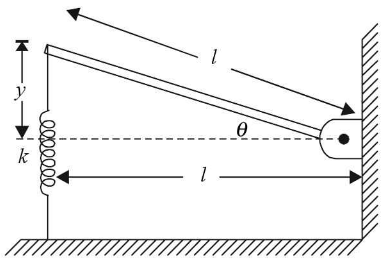


Fig. 12.40

(a) $2\pi\sqrt{\dfrac{M}{k}}$ (b) $2\pi\sqrt{\dfrac{M}{2k}}$

(c) $2\pi\sqrt{\dfrac{M}{3k}}$ (d) none of these

Solution (c) $\tau = kyl = kl^2\theta$

$$\tau = \frac{Ml^2}{3}\alpha \quad \text{therefore,}$$

$$\frac{Ml^2}{3}\alpha = -kl^2\theta$$

or $$\alpha = \frac{-3kl^2}{Ml^2}\theta \quad \omega^2 = \frac{3k}{M}$$

$$T = 2\pi\sqrt{\frac{M}{3k}}$$

24. A solid ball of mass m is allowed to fall from a height h to a pan suspended with a spring of spring constant k. Assume the ball does not rebound and pan is massless, then amplitude of the oscillation is

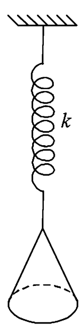


Fig. 12.41

(a) $\dfrac{mg}{k}$ (b) $\dfrac{mg}{k}+\left(\dfrac{2hk}{mg}\right)^{1/2}$

(c) $mg\sqrt{1+\dfrac{1+2hk}{mg}}$ (d) $\dfrac{mg}{k}\left[1+\sqrt{1+\dfrac{2hk}{mg}}\right]$

Solution (d) mg $(h + x) = \dfrac{1}{2}kx^2$

or $$x^2 - \left(\frac{2mg}{k}\right)x - \frac{2mgh}{k} = 0$$

or $$x = \frac{mg}{k} + \frac{mg}{k}\sqrt{1+\frac{2hk}{mg}}$$

25. A partical executes SHM of frequency f. The frequency of its kinetic energy is

(a) f (b) $\dfrac{f}{2}$

(c) $2f$ (d) zero

Solution (c) KE $= \dfrac{1}{2}m\omega^2\left(x_0^2 - x^2\right)$

$$= \frac{1}{2}m\omega^2\left(x_0^2 - x_0^2\sin^2\omega\right)$$

$$= \frac{1}{2}m\omega^2 x_0^2\left(\frac{1+\cos^2\omega}{2}\right)$$

Note the frequency is 2ω or $2f$

26. For the particle executing SHM the displacement x is given by $x = A\cos\omega t$. Identify which represents variation of potential energy as a function of time t and displacement x

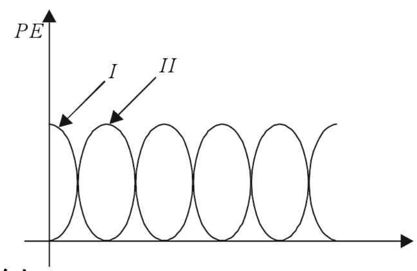


Fig. 12.42 (a)

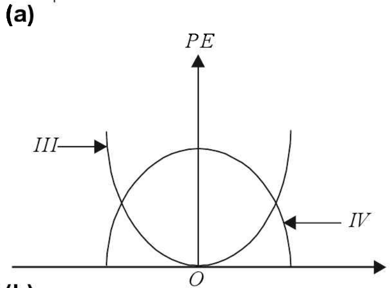


Fig. 12.42 (b)

(a) I, III (b) II, IV

(c) II, III (d) I, IV

[IIT Screening 2003]

Solution (a) $x = A\cos\omega t$

$$PE = \frac{1}{2}m\omega^2x^2 = \frac{1}{2}m\omega^2A^2\cos^2\omega t$$

that is, at t = 0 potential energy is maximum and potnetial energy $\alpha\, x^2$ So choice is I and III.

27. The earing of a lady is 5 cm long. She sits in a merry-go-round moving at 4 ms^{-1} in a circle of radius 2 m. The time period of oscillations is nearly

(a) 0.6 s (b) 0.4 s

(c) 0.8 s (d) none of these

Solution (b) $T = 2\pi\sqrt{\dfrac{l}{\sqrt{g^2+\left(\dfrac{v^2}{R}\right)^2}}}$

$$= 2\pi\sqrt{\frac{l}{\sqrt{g^2+\left(\frac{v^2}{R}\right)^2}}} \cong 0.4 \text{ s (nearly)}$$

28. A rod of length l and mass m is hanged from one edge. The time period of small oscillations is

(a) $2\pi\sqrt{\frac{l}{3g}}$ (b) $2\pi\sqrt{\frac{l}{g}}$

(c) $2\pi\sqrt{\frac{2l}{3g}}$ (d) none of these

Solution (c) $T = 2\pi\sqrt{\frac{I}{mgl}}$

$$= 2\pi\sqrt{\frac{\frac{Ml^2}{12}+\frac{Ml^2}{4}}{Mgl/2}}$$

$$= 2\pi\sqrt{\frac{2l}{3g}}$$

29. A, B, C are identical springs each of spring constant k as shown in Figure. Mass M is displaced slightly along C and released. Find the time period of small oscillation.

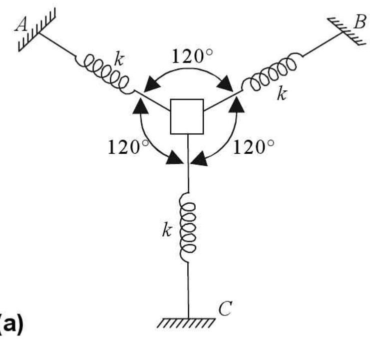


Fig. 12.43 (a)

(a) $T = 2\pi\sqrt{\frac{m}{k}}$ (b) $T = 2\pi\sqrt{\frac{m}{2k}}$

(c) $T = 2\pi\sqrt{\frac{m}{2k}}$ (d) $T = 2\pi\sqrt{\frac{3m}{2k}}$

Solution (c) Net force as shown in Fig. 12.43 (b) is

$$F = -(kx + kx\cos 60 + kx\cos 60) = -2kx$$

$$ma = -2kx$$

or $$a = \frac{-2k}{m}x$$

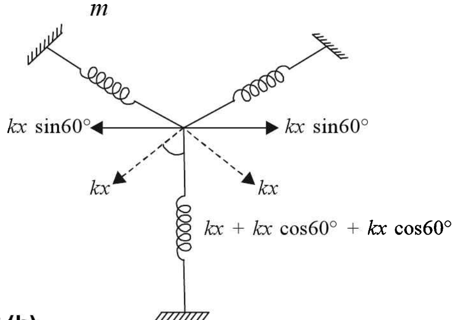


Fig. 12.43 (b)

$$\omega = \sqrt{\frac{2k}{m}}$$

or $$T = 2\pi\sqrt{\frac{m}{2k}}$$

30. A particle is executing SHM $x = 3\cos\omega t + 4\sin\omega t$. Find the phase shift and amplitude.

(a) 50°, 5 units (b) 37°, 3.5 units

(c) 53°, 3.5 units (d) 37°, 5 units

Solution (d) $x = x_0\sin(\omega t + \phi) = x_0\sin\omega t\cos\phi + x_0\cos\omega t\sin\phi = 3\cos\omega t + 4\sin\omega t$

Comparing we get $x_0\cos\phi = 4$ .... (1)

and $x_0\sin\phi = 3$ .... (2)

dividing (2) by (1) $\tan\phi = 3/4$ or $\phi = 37°$. Squaring and adding (1) and (2) we get $x_0 = 5$.

31. When two sinusoids oscillating in different frequencies are fed to x and y plates of a cathode ray oscilloscope the pattern shown in Fig. 12.44 is observed.

$\frac{\omega_x}{\omega_y}$ = ________

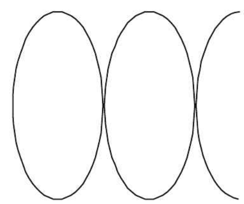


Fig. 12.44

(a) $\frac{5}{2}$ (b) $\frac{2}{5}$

(c) $\frac{1}{3}$ (d) 3

Solution (b) $\frac{\omega_x}{\omega_y} = \frac{n_y}{n_x} = \frac{1}{2.5}$

32. Two particles are moving in uniform circular motion in opposite direction with same angular velocity. Radius of the circle is R. Their resultant motion is equivalent to

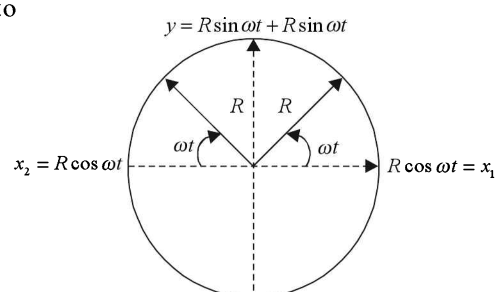


Fig. 12.45

(a) angular SHM of angular frequency 2ω.
(b) linear SHM of angular frequency 2ω.
(c) linear SHM of amplitude $2R$.
(d) angular SHM of amplitude 2 radian.

Solution (c) Resolving $x_1 = R\cos\omega t$; $x_2 = -R\cos\omega t$

$$y_1 = R\sin\omega t;\ y_2 = R\sin\omega t.$$

$$x = x_1 + x_2 = 0$$

$$y = y_1 + y_2 = 2R\sin\omega t$$

QUESTIONS FOR PRACTICE

1. Two simple pendulums of lengths 1 m and 16 m respectively are both given small displacements in the same direction at the same instant. They will again be in phase after the shorter pendulum has completed n oscillations where n is
(a) 1/3 (b) 1/4
(c) 4 (d) 5

2. A simple spring has length l and force constant k. It is cut into two springs of length l_1 and l_2 such that $l_1 = nl_2$ (n = an integer) the force constant of spring of length l_1 is
(a) $k(1 + n)$ (b) $k/(n + 1)$
(c) $k(1 + n)$ (d) k

3. The kinetic energy and potential energy of a particle executing SHM will be equal when displacement (amplitude = a) is
(a) $a\sqrt{2/3}$ (b) $a/2$
(c) $a/\sqrt{2}$ (d) $a\sqrt{2}$

4. If a particle performs SHM with a frequecy v, then its K.E. will oscillate with a frequency
(a) zero (b) $v/2$
(c) v (d) $2v$

5. The dispalacement y in centimeters is given in terms of time t s by the equation: $y = 3\sin 314\, t + 4\cos 314\, t$, then the amplitude of SHM is
(a) 3 cm (b) 4 cm
(c) 5 cm (d) 7 cm

6. A body of mass m is suspended from rubber and with force constant k. The maximum distance over which the body can be pulled down for the body's oscillation to remain harmonic is
(a) $mg/2k$ (b) $2k/mg$
(c) $2\ mg/k$ (d) mg/k

7. Length of second's pendulum is decreased by 1% then the gain or loss in time per day will be nearly
(a) 0.44 s (b) 4.4 s
(c) 44 s (d) 440 s

8. Two pendulums oscillate with a constant phase difference of 90°, and same amplitude. The maximum velocity of one is v. The maximum velocity of other will be
(a) $\sqrt{2v}$ (b) $v\sqrt{2}$
(c) v (d) $2v$

9. What is the length of second's pendulum where g is 980 cms^{-2}?
(a) 102.4 cm (b) 99.2 cm
(c) 88 cm (d) 78 cm

10. The displacement of particle in SHM in one time period is
(a) zero (b) a
(c) $2a$ (d) $4a$

11. For a particle executing SHM having amplitude 'a' the speed of the particle is one half of its maximum speed when its displacement from the mean position is
(a) $a/2$ (b) a
(c) $a\sqrt{3/2}$ (d) $2a$

12. A spring pendulum is suspended from the top of a car. If the car accelerates on a horizontal road, the frequency of oscillation will
(a) be zero. (b) remain same.
(c) increase. (d) decrease.

13. The length of the seconds pendulum on the surface of earth is 1m. Its length on the surface of earth is
(a) 1/36 m (b) 1/6 m
(c) 6m (d) 36 m

14. The phase angle between the projections of uniform circular motion on two mutually perpendicular diameter is
(a) π (b) $3\pi/4$
(c) $\pi/2$ (d) zero

15. When particle oscillates simple harmonically its potential energy varies periodically. If frequency of the particle is n, the frequency of the potential energy is
(a) $n/2$ (b) n
(c) $2n$ (d) $4n$

16. A simple pendulum performs SHM about $x = 0$ with an amplitude A and time period T. The speed of the pendulum at $x = A/2$ will be
(a) $3\pi 2A/T$ (b) $\pi A\dfrac{\sqrt{3}}{T}$
(c) $\pi\dfrac{A}{T}$ (d) $\pi A\dfrac{\sqrt{3}}{2T}$

17. The potential energy of a particle with displacement x is $U(x)$. The motion is simple harmonic. If k is a positive constant then
(a) $U = kx$ (b) $U = k$
(c) $U = -kx^2/2$ (d) $U = kx^2$

18. A bob is suspended by a string of length l. The minimum horizontal velocity imparted to the ball for reaching it to the height of suspension is

(a) $\sqrt{l/g}$ (b) $\sqrt{2gl}$
(c) $\sqrt{g/l}$ (d) $2\sqrt{gl}$

19. The angle between the instantaneous velocity and the acceleration of a particle executing SHM is

(a) zero or π (b) $\pi/2$
(c) zero (d) π

20. The frequency of SHM is 100 Hz. Its time period is

(a) 0.01 s (b) 0.1 s
(c) 1s (d) 100 s

21. The displacement y of a particle executing periodic motion is given by $y = 4\cos^2(t/2)\sin(100t)$.

This expression may be considered to be a result of the super position of

(a) 2 (b) 3
(c) 4 (d) 5

[IIT 92]

22. Two bodies M and N of equal masses are suspended from two separate massless spring of spring constants k_1 and k_2 respectively. If the two bodies oscillate vertically such that their maximum velocities are equal the ratio of the amplitude of M to that of N is

(a) k_2/k_1 (b) $\sqrt{k_2/k_1}$
(c) k_1/k_2 (d) $\sqrt{k_1/k_2}$

[IIT 88]

23. If E is the total energy of a particle executing SHM and 'A' is the amplitude of the vibratory motion, the E and 'A' are related as

(a) $E \propto A^2$ (b) $E \propto 1/A^2$
(c) $E \propto A$ (d) $E \propto 1/A$

24. A particle excites SHM with a time period of 2 s and amplitude 5 cm. Maximum magnitude of its velocity is

(a) $10\,\pi$ cm s^{-1} (b) $20\,\pi$ cm s^{-1}
(c) $2.5\,\pi$ cm s^{-1} (d) $5\,\pi$ cm s^{-1}

25. The dimensional formula for amplitude of SHM is

(a) MLT (b) $M^oL^oT^o$
(c) M^oLT^o (d) MLT^o

26. A simple pendulum is attached to the roof of a lift. Its time period of oscillation, when the lift is stationary is 0.5 s. Its frequency of oscillation when the lift falls freely will be

(a) infinite. (b) zero.
(c) 2 Hz. (d) 20 Hz.

27. A spring of force constant k is cut into two equal parts, which are then joined parallel to each other. The force constant of the combination will be

(a) $4k$ (b) $2k$
(c) k (d) $k/2$

28. A particle is placed on a plank undergoing SHM of frequency $3/\pi$ Hz. The maximum amplitude of the plank so that the particle does not leave the plank will be

(a) $\frac{5}{18}$ m (b) $\frac{5}{8}$ m
(c) $\frac{2}{9}$ m (d) none of these

29. The intensities of two notes are equal. If frequency of one note is one-fourth that of the other then the ratio of their amplitudes is

(a) 16 (b) 4
(c) 2 (d) 1

30. A person wearing a wrist watch that keeps correct time at the equator goes to N-pole. His watch will

(a) keep correct time. (b) gain time.
(c) loose time. (d) cannot say.

31. Which of the following is not essential for the free oscillation of a mass attached to a spring?

(a) Elasticity (b) Gravity
(c) Inertia (d) Restoring force

32. A pendulum suspended from the ceiling of the train beats seconds when the train is at rest. What will be the time period of the pendulum if the train accelerates at 10 ms^{-2}? Take g = 10 ms^{-2}.

(a) $(2/\sqrt{2})$ s (b) 2 s
(c) $2\sqrt{2}$ s (d) none of these

33. Which of the following quantities connected with SHM do not vary periodically?

(a) Total energy (b) Velocity
(c) Displacement (d) Acceleration

34. A mass m is suspended from a spring of force constant k. The angular frequency of oscillation of the spring will be

(a) k/m (b) $\sqrt{m/k}$
(c) m/k (d) $\sqrt{k/m}$

35. What is the number of degrees of freedom of an oscillating simple pendulum?

(a) more than three (b) 3
(c) 2 (d) 1

36. The graph between restoring force and time in case of SHM is a

(a) parabola. (b) sine curve.
(c) straight line. (d) circle.

37. In SHM, which of the following quantities does not vary as per nature of the sine curve?

(a) acceleration (b) time period
(c) displacement (d) velocity

38. Two particles P and Q describe SHM of same amplitude a and frequency v along the same straight line. The maximum distance between the two particles is $a\sqrt{2}$. The initial phase difference between the particle is

(a) $\pi/3$ (b) $\pi/2$
(c) $\pi/6$ (d) zero

39. A particle is moving on a circle with uniform speed. Its motion is

(a) aperiodic motion.
(b) periodic and SHM.
(c) periodic but not SHM.
(d) none of these.

40. The mass and radius of a planet are double that of the earth. The time period of a pendulum on that planet which is a seconds pendulum on earth, will be

(a) $\sqrt{\frac{1}{\sqrt{2}}}$ s (b) 0.5 s
(c) $2\sqrt{2}$ s (d) 2 s

41. The work done by a simple pendulum in one completed oscillation is

(a) equal to *Ek*. (b) equal to *U*.
(c) zero. (d) equal to $U + Ek$.

42. A particle is moving such that its acceleration is represented by the equation $a = -bx$, where x is its displacement from mean position and b is a constant. Its time period will be

(a) $2\pi/\sqrt{b}$ (b) $2\pi/b$
(c) $2\pi\sqrt{b}$ (d) $2\sqrt{\frac{\pi}{b}}$

43. The displacement of a particle executing SHM is half its amplitude. The fraction of its kinetic energy will be

(a) 2/3 (b) 3/4
(c) 1/3 (d) 1/2

44. The phase difference between the velocity and displacement of a particle executing SHM is

(a) $\pi/2$ radian (b) π radian
(c) 2π radian (d) zero

45. The ratio of the maximum velocity and maximum displacement of a particle executing SHM is equal to

(a) n (b) g
(c) T (d) ω

46. The physical quantity conserved in simple harmonic motion is

(a) time period (b) total energy
(c) potential energy (d) kinetic energy

47. The bob of a simple pendulum consists of a sphere filled with mercury. If a small quantity of mercury is taken out, then the period of pendulum will

(a) become erroneous (b) decrease
(c) increase (d) remain unchanged

48. The time period of a second's pendulum is 2 s. The mass of the spherical bob is 50 g and is empty. If it is replaced by another solid bob of same radius but mass 100 g then its time period will be

(a) 8 s (b) 4 s
(c) 2 s (d) 1 s

49. The amplitude and time period of simple harmonic oscillator are a and T respectively. The time taken by it in displacing from $x = 0$ to $x = a/2$ will be

(a) T (b) $T/2$
(c) $T/4$ (d) $T/6$

50. The time period of the hour hand of a watch is

(a) 24 h (b) 12 h
(c) 1 h (d) 1 min.

51. A simple pendulum is released when $\theta = \pi/6$. The time period of oscillation is

(a) $2\pi\sqrt{\frac{l}{g}}$ (b) $2\pi\sqrt{\frac{l}{g}}\left(\frac{293}{288}\right)$
(c) $2\pi\sqrt{\frac{l}{g}}\left(\frac{288}{293}\right)$ (d) none of these

52. A mass m is suspended from a spring. Its frequency of oscillation is f. The spring is cut into two equal halves and the same mass is suspended from one of the two pieces of the spring. The frequency of oscillation of the mass will be

(a) $\sqrt{2}f$ (b) $2f$
(c) $f/2$ (d) f

53. Which of the following characteristics must remain constant for undamped oscillations of the particle?

(a) acceleration (b) phase
(c) amplitude (d) velocity

54. Identical springs of spring constant K are connected in series and parallel combinations. A mass m is suspended from them. The ratio of their frequencies of vertical oscillations will be

(a) 1 : 4 (b) 1: 2
(c) 4 : 1 (d) 2 : 1

55. The maximum displacement of a particle exectuting SHM from its mean position is 2 cm and its time period is 1 s. The equation of its displacement will be

(a) $x = 2\sin 4\pi t$ (b) $x = 2\sin 2\pi t$
(c) $x = \sin 2\pi t$ (d) $x = 4\sin 2\pi t$

56. The maximum value of the time period of a simple pendulum is

(a) 84.6 min (b) 1 year
(c) 1 day (d) 12 h

57. If a simple pendulum of length l has maximum angular displacement θ, then the maximum kinetic energy of the bob of mass m is

(a) $\frac{1}{2}m(l/g)$ (b) $mgl(1 - \cos\theta)$
(c) $(mgl\sin\theta)/2$ (d) $mg/2l$

58. A simple pendulum is carried at a depth of 1 km below sea level. It becomes

(a) slow. (b) unchanged.
(c) fast. (d) none of these.

59. The ratio of the kinetic energy E_k and potential energy of a particle executing SHM, at a distance x from mean position will be

(a) $\frac{x^2}{x^2-a^2}$ (b) $\frac{x^2-a^2}{x^2}$
(c) $\frac{x^2}{a^2-x^2}$ (d) $\frac{a^2-x^2}{x^2}$

60. The curve between the acceleration and velocity of a particle executing SHM is a an

(a) ellipse. (b) circle.
(c) parabola. (d) straight line.

61. On increasing the length of a second's pendulum by Z%, its time period will

(a) increase by 2 Z%. (b) decrease by Z%.
(c) decrease by 2 Z%. (d) increase by 0.5 Z%.

62. A clock purchased in 1942 loses 1 min in 1 day. Its time period must have become

(a) extremely small. (b) extremely large.
(c) shorter. (d) longer.

63. The displacement of a particle executing SHM at any instant t is $x = 0.01 \sin 100(t + 0.05)$ then its time period will be

(a) 0.06 s (b) 0.2 s
(c) 0.1 s (d) 0.02 s

64. The equation of displacement of a particle executing SHM is $x = 0.40 \cos(2000t + 18)$. The frequency of the particle is

(a) 10^3 Hz (b) 20 Hz
(c) 2×10^3 Hz (d) $\frac{10^3}{\pi}$ Hz

65. The displacement of a particle executing SHM at any instant t is $x = a \sin \omega t$. The acceleration of the particle at $t = \frac{T}{4}$ will be

(a) $a\omega$ (b) $a\omega^2$
(c) $-a\omega$ (d) $-a\omega^2$

66. Which of the following quantities is doubled on doubling the amplitude of a harmonic oscillator

(a) E_t (b) E_k
(c) v_{max} (d) U

67. The amplitude of a particle executing SHM is 2 cm and its frequency is 50 cycles/s. Its maximum acceleration will be

(a) 1972 ms^{-2} (b) 19.72 ms^{-2}
(c) 1.972 ms^{-2} (d) zero

68. The work done in stretching a spring of force constant k from y_1 to y_2 from mean position will be

(a) $\frac{k}{2}\left(y_2^2-y_1^2\right)$ (b) $\frac{k}{2}\left(y_1^2-y_2^2\right)$
(c) $k(y_2-y_1)$ (d) zero

69. A watch becomes fast by 5 minutes in a day. In the watch makers shop it keeps correct time. It is due to

(a) natural vibrations. (b) forced vibrations.
(c) damped vibrations. (d) none of these.

70. The potential energy of a simple pendulum in its resting position is 10 J and its mean kinetic energy is 5 J. Its total energy at any instant will be

(a) 5 J (b) 10 J
(c) 15 J (d) 20 J

71. A simple pendulum is hanging from the ceiling of a train. If the train gets accelerated with a uniform acceleraion a, then its direction from the vertical wall be

(a) $\tan^{-1}(a/g)$ (b) $\sin^{-1}(a/g)$
(c) $\cot^{-1}(a/g)$ (d) $\cos^{-1}(a/g)$

72. The ratio of the kinetic energy of a particle executing SHM at its mean position to its potential energy at extreme position is

(a) = 1 (b) = $1/g$
(c) > 1 (d) < 1

73. The time period of a torsional pendulum is

(a) $T = \pi\sqrt{C/I}$ (b) $T = 2\pi\sqrt{I/C}$
(c) $T = 2\pi\sqrt{C/I}$ (d) $T = \pi\sqrt{I/C}$

74. The average kinetic energy of a simple harmonic oscillator with respect to position will be

(a) $\frac{ka^2}{6}$ (b) $\frac{ka^2}{4}$
(c) $\frac{ka^2}{3}$ (d) $\frac{ka^2}{2}$

75. The time period of a simple pendulum of infinite length is

(a) $T = 2\pi\sqrt{\frac{R_e}{g}}$ (b) infinity
(c) $T = \pi\sqrt{\frac{R_e}{g}}$ (d) zero

76. If the amplitude of oscillation of a simple pendulum is increased by 30%, then the percentage change in its time period will be

(a) 90% (b) 60%
(c) 30% (d) zero

77. If at any instant of time the displacement of a harmonic oscillator is 0.02 m and its acceleration is 2 ms^{-2}, its angular frequency at that instant will be

(a) 0.1 $rads^{-1}$ (b) 1 $rads^{-1}$
(c) 10 $rads^{-1}$ (d) 100 $rads^{-1}$

78. The potential energy of a simple harmonic oscillator is given by $U(x) = \alpha + \beta x + \gamma x^2 + \delta x^3$ the term representing the SHM will be

(a) α (b) βx
(c) γx^2 (d) δx^3

79. The phase shift of the resultant $y = 3 \sin \omega t + 4 \cos \omega t$ is

(a) 37° (b) 53°
(c) 90° (d) none of these

80. Which physical quantity oscillates with the same frequency as that of SHM for a harmonic oscillator?

(a) Acceleration (b) E_{total}
(c) KE (d) PE

81. Two particles execute SHM along a straight line with same amplitude and same frequency. Everytime they meet in opposite directions at a displacement equal to half the amplitude. The difference between them is

(a) $2\pi/3$ (b) π
(c) $\pi/3$ (d) $\pi/2$

82. The work done in displacing a harmonic oscillator through an angle ϕ from its mean position is

(a) $mgl(1 - \cos \phi)$ (b) mgl
(c) $mgl \cos \phi$ (d) zero

83. The time period of a second's pendulum on the surface of moon will be

(a) $\frac{2}{\sqrt{6}}$ s (b) $6\sqrt{2}$ s
(c) $2\sqrt{6}$ s (d) 2 s

84. The equation of simple harmonic motion of a particle is $\frac{d^2x}{dt^2} + 0.2\frac{dx}{dt} + 36x = 0$. Its time period is approximately

(a) $\pi/6$ s (b) $\pi/4$ s
(c) $\pi/3$ s (d) $\pi/2$ s

85. A spring of spring constant k is cut into two pieces such that their lengths are in the ratio 1 : 2, then the force constant of the bigger piece will be

(a) $3/2\ k$ (b) $k/2$
(c) $2k$ (d) $3k$

86. A body of mass 100 gm is suspended from a spring of force constant 50 N/m. The maximum acceleration produced in the spring is

(a) $g/2$ (b) g
(c) $g/3$ (d) $g/4$

87. A second's pendulum is kept in a satellite revolving at a height $3R_E$ from earth's surface. Its time period will be

(a) $2\sqrt{3}$ s (b) 1 s
(c) zero (d) infinity

88. The time period of a particle executing SHM is $\frac{2\pi}{\omega}$ and its velocity at a distance b from mean position is $\sqrt{3}\ b\omega$. Its amplitude is

(a) b (b) $2b$
(c) $3b$ (d) $4b$

89. The time period of a second's pendulum on the surface of moon is found to be 5 s. The acceleration due to gravity on the surface of moon is

(a) 3.2 ms^{-2} (b) 1.6 ms^{-2}
(c) 0.8 ms^{-2} (d) 0.6 ms^{-2}

90. The displacement of a particle executing SHM at any instant t is $x = 7 \cos 0.5\ \pi t$. The time taken by the particle in reaching from mean position to extreme position will be

(a) 4 s (b) 2 s
(c) 1 s (d) 1/2 s

91. A body of mass M and charge q is suspended from a spring. When slightly displaced, it oscillates with period T. If a uniform electric field acts vertically downwards, then the new time period will be

(a) $T' = T$ (b) $T' < T$
(c) $T' > T$ (d) cannot be predicted

92. A body of mass M is situated in a potential field $U(x) = U_0(1 - \cos dx)$, where U_0 and d are constants. The time period of small oscillations will be

(a) $2\pi\sqrt{MU_0d^2}$ (b) $2\pi\sqrt{\frac{M}{U_0d^2}}$
(c) $2\pi\sqrt{\frac{U_0d^2}{M}}$ (d) $2\pi\sqrt{\frac{U_0}{Md^2}}$

93. Restoring force in the SHM is

(a) conservative. (b) nonconservative.
(c) frictional. (d) centripetal.

94. What will be the percentage change in the time period of a simple pendulum if its length is increased by 5%?

(a) 1/9% (b) 2.5%
(c) 5.2% (d) 9%

95. Which of the following quantities is non-zero at the mean position for a particle executing SHM?

(a) Force (b) Acceleration
(c) Velocity (d) Displacement

96. A bar magnet is oscillating in the earth's magnetic field with time period T. If its mass is increased four times then its time period will be

(a) $T/2$ (b) T
(c) $2T$ (d) $4T$

97. A 5 kg. weight is suspended from a spring. The spring stretches by 2 cm/kg. If the spring is stretched and released, its time period will be

(a) 0.0628 s (b) 0.628 s
(c) 6.28 s (d) 62.8 s

98. The ratio of total energy of a harmonic oscillator to its average potential energy with respect to position will be

(a) 1 : 2 (b) 3 : 1
(c) 1 : 3 (d) 2 : 11

99. The height of liquid column in a *U*-tube is 0.3 m. If the liquid in one of the limbs is depressed and then released, then the time period of liquid column will be

(a) 0.11 s (b) 1.1 s
(c) 2 s (d) 19 s

100. The displacement of a particle executing simple harmonic motion at any instant t is $x = 3\cos 2\pi t$. The acceleration produced in the particle will be

(a) $-12\,\pi^2 \sin 2\pi t$ (b) $-12\,\pi^2\cos 2\pi t$
(c) $12\,\pi\cos 2\pi t$ (d) zero

101. A simple pendulum of mass m and length l is oscillating along a circular arc of angle ϕ. Another bob of mass m is lying at the extreme position of the arc. The momentum imparted by the moving bob to the stationary bob will be

(a) $mlv \cos \phi$ (b) $mv \sin \phi$
(c) mlv/ϕ (d) zero

102. A block of 4 kg produces an extension of 0.16 m in a spring. The block is replaced by a body of mass 0.50 kg. If the spring is stretched and then released the time period of motion will be

(a) 28.3 s (b) 2.83 s
(c) 0.283 s (d) 0.0283 s

103. The resultant of $y = y_0 \sin \omega t$ and $x = x_0 \sin \omega t$ will be a/an

(a) ellipse. (b) circle.
(c) figure of 8. (d) straight line.

104. Two particles are executing SHM. Their displacements at any instant of time t are $x = a \sin(\omega t - \phi)$ and $y = b \cos(\omega t - \phi)$. The phase difference between them will be

(a) 0° (b) ϕ
(c) 90° (d) 180°

105. The equation of displacement of a harmonic oscillator is $x = 3 \sin \omega t + 4 \cos \omega t$. The amplitude of the particles will be

(a) 1 (b) 5
(c) 7 (d) 12

106. The maximum velocity of a harmonic oscillator is d and its maximum acceleration is β. Its time period will be

(a) $\dfrac{\pi\beta}{d}$ (b) $2\pi d\beta$
(c) $\dfrac{2\pi d}{\beta}$ (d) $\dfrac{2\pi\beta}{d}$

107. A particle of mass 0.1 kg is executing simple harmonic motion of amplitude 1 m and time period 0.2 s. The maximum force on the particle will be

(a) 0.99 N (b) 9.9 N
(c) 9 N (d) 99 N

108. The bob of a simple pendulum of mass m is oscillating with angular amplitude 40°. The tension in the string at the instant when its angular displacement is 20° will be

(a) $mg \cos 20°$ (b) $> mg \cos 20°$
(c) $< mg \cos 20°$ (d) mg

109. A simple pendulum is suspended from the celling of a lift. When the lift is at rest, its time period is T. With what acceleration should lift be accelerated upwards in order to reduce its time period to $T/2$?

(a) $-3g$ (b) g
(c) $2g$ (d) $3g$

110. The kinetic energy of a harmonic oscillator is $K = K_0 \cos^2 \omega t$. The maximum potential energy of the particle is

(a) zero (b) $\dfrac{K_0}{2}$
(c) K_0 (d) $2K$

111. A body is lying on a piston which is executing vertical SHM. Its time period is 2 s. For what value of amplitude, the body will leave the piston

(a) = 1 m (b) 0.248 m
(c) 0.428 m (d) 0.842 m

112. The time period of a simple pendulum as seen by an astronaut in a spaceship is

(a) 84.6 min (b) 2 s
(c) ∞ (d) 0

113. Infinite springs with force constant k, 2k, 4k 8k ____ respectively are connected in series. The effective force constant of the spring will be

(a) $k/2$ (b) k
(c) $2\,k$ (d) $2048\,k$

114. A particle is executing SHM along a straight line 8 cm long. While passing through mean position its velocity is 16 cms^{-1}. Its time period will be

(a) 0.0157 s (b) 0.157 s
(c) 1.57 s (d) 15.7 s

115. The time period of the second's hand of watch is

(a) 1 s (b) 1 min
(c) 1 h (d) 12 h

Answers to Questions for Practice

1.	(c)	**2.**	(c)	**3.**	(c)	**4.**	(d)	**5.**	(c)	**6.**	(d)	**7.**	(d)
8.	(c)	**9.**	(b)	**10.**	(a)	**11.**	(c)	**12.**	(b)	**13.**	(b)	**14.**	(c)
15.	(c)	**16.**	(b)	**17.**	(d)	**18.**	(b)	**19.**	(b)	**20.**	(a)	**21.**	(b)
22.	(b)	**23.**	(a)	**24.**	(d)	**25.**	(c)	**26.**	(b)	**27.**	(a)	**28.**	(a)
29.	(b)	**30.**	(a)	**31.**	(b)	**32.**	(d)	**33.**	(a)	**34.**	(d)	**35.**	(c)
36.	(b)	**37.**	(b)	**38.**	(b)	**39.**	(c)	**40.**	(c)	**41.**	(c)	**42.**	(a)
43.	(b)	**44.**	(a)	**45.**	(d)	**46.**	(b)	**47.**	(c)	**48.**	(c)	**49.**	(d)
50.	(b)	**51.**	(b)	**52.**	(a)	**53.**	(c)	**54.**	(b)	**55.**	(b)	**56.**	(a)
57.	(b)	**58.**	(a)	**59.**	(d)	**60.**	(a)	**61.**	(d)	**62.**	(c)	**63.**	(a)
64.	(d)	**65.**	(d)	**66.**	(c)	**67.**	(a)	**68.**	(a)	**69.**	(b)	**70.**	(b)
71.	(a)	**72.**	(a)	**73.**	(b)	**74.**	(c)	**75.**	(a)	**76.**	(d)	**77.**	(c)
78.	(c)	**79.**	(b)	**80.**	(a)	**81.**	(a)	**82.**	(a)	**83.**	(c)	**84.**	(c)
85.	(a)	**86.**	(b)	**87.**	(d)	**88.**	(b)	**89.**	(b)	**90.**	(c)	**91.**	(b)
92.	(b)	**93.**	(a)	**94.**	(b)	**95.**	(c)	**96.**	(c)	**97.**	(c)	**98.**	(b)
99.	(b)	**100.**	(b)	**101.**	(d)	**102.**	(c)	**103.**	(d)	**104.**	(c)	**105.**	(b)
106.	(c)	**107.**	(d)	**108.**	(b)	**109.**	(d)	**110.**	(c)	**111.**	(a)	**112.**	(c)
113.	(a)	**114.**	(c)	**115.**	(b)								

Explanations

8. (c) $\Delta T = \frac{T\Delta l}{2l} = \frac{86400}{2 \times 100}$

28. (a) $x_0\omega^2 = g$ or $x_0 = \frac{10}{\left(2\pi \times \frac{3}{\pi}\right)^2} = \frac{5}{18}$.

54. (b) $T = 2\pi\sqrt{\frac{l}{g}}\left(1 + \frac{\theta_0^2}{16}\right) = 2\pi\sqrt{\frac{l}{g}}\left(1 + \frac{(\pi/6)^2}{16}\right)$

$$= 2\pi\sqrt{\frac{l}{g}}\left(1 + \frac{10}{36 \times 16}\right)$$

58. (a) $T = \sqrt{\frac{1}{g\left(1 - \frac{\sigma}{\rho}\right)}}$

71. (a) Total energy = Resting energy + KE + PE

$= 10\,\text{J} + 5\text{J} + 5\text{J}$

$= 201.$

84. (c) $\omega^2 = 36$ or $\omega = 6$

or $\frac{2\pi}{T} = 6$

or $T = \frac{\pi}{3}$.

92. (b) $U = U_0(1 - \cos dx)$

$$F = -\frac{dU}{dx}$$

$$= -U_0 d \sin dx$$

or $a = \frac{-U_0 d^2}{M}x$

or $\omega = \sqrt{\frac{U_0 d^2}{M}}$.

106. (c) $v_{max} = x_0\omega$ $a_{max} = x_0\omega^2$

$\therefore$ $\omega = \frac{a_{max}}{v_{max}} = \frac{\beta}{d}$

or $T = \frac{2\pi}{\beta} = \frac{\beta}{d}$

or $T = \frac{2\pi d}{\beta}$.

13

Wave Motion and Wave in a String

BRIEF REVIEW

Three types of waves may be defined — mechanical, electromagnetic and matter waves as illustrated in Fig. 13.1. Here we are concerned with mechanical waves only.

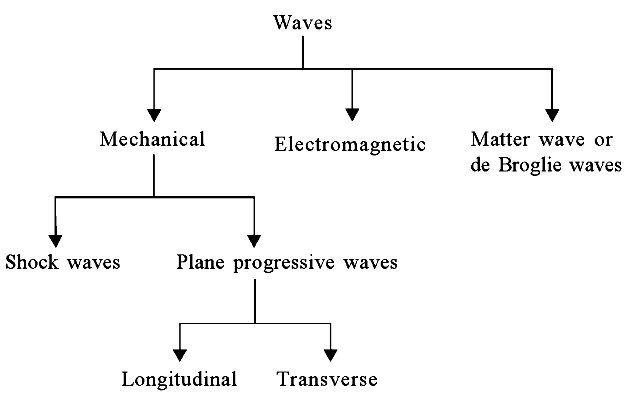


Fig. 13.1 Waves description

A wave is a disturbance which propagates energy from one place to the other without transporting matter. It is spread over a region without clear-cut boundaries. It is not localized.

Diffraction It is a convincing proof of wave nature. It differentiates between particle nature and wave nature.

Mechanical waves require an elastic medium to propagate. Therefore, mechanical waves are also called **elastic waves.** Waves like electromagnetic and matter waves do not require any medium to propagate.

Shock waves are a kind of pulse propagation and are mathematically expressed as

$$y = \frac{a}{b+(x \pm vt)^2}.$$

Shock waves are produced during earthquakes, volcanic eruptions, bomb blasts and during a sonic boom.

$y = y_0\ (\omega t - kx)$ is the wave propagating along positive x direction..

Plane progressive wave is given by

$$y = y_0 \sin(\omega t - kx)$$

where k is called propagation constant or wave number, ω is called angular frequency, y_0 amplitude and y instantaneous displacement. Such a wave is called a displacement wave.

$K = \frac{2\pi}{\lambda}$ where λ is wavelength, $(\omega t - kx)$ is the phase at any instant. When path difference $\Delta x = \lambda$, then phase shift $\Delta\phi = 2\pi$. In general $k\Delta x = \Delta\phi$.

A wave can have two types of velocities.

Wave velocity or phase velocity and group velocity or particle velocity.

Wave velocity $v = \frac{dx}{dt} = \frac{\omega}{k} = f\lambda$

In a dispersive medium, wave travels with a group velocity

$$v_{\text{group}} = v - \lambda \frac{dv}{d\lambda}$$

This is the case for electromagnetic waves. For example, in water and glass and so on different wavelength travel with different velocities.

Particle velocity $v_{\text{particle}} = \frac{dy}{dt} = -\frac{dx}{dt} \times \frac{dy}{dx} = -v(\text{slope})$

$= -$ wave velocity $\times$ slope at that point

A plane progressive wave mechanical or electromagnetic may be expressed in one of the following forms

$$y = y_0 \sin(\omega t - kx)$$

$$y = y_0 \sin\omega\left(t - \frac{x}{v}\right)$$

$$y = y_0 \sin 2\pi\left(ft - \frac{x}{\lambda}\right)$$

$$y = y_0 \sin 2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right)$$

$$y = y_0 \sin k(vt - x)$$

- If the sign between t and x is negative then the wave propagates in positive x direction.
- When a wave passes from one medium to the other its frequency does not change.

Velocity of wave in a string $v = \sqrt{\frac{T}{\mu}}$

where T is tension in the string and π is mass/length of the string.

Frequency of Wave in a String

If vibrating in p-loops (transverse),

fundamental frequency $f = \frac{p}{2l}\sqrt{\frac{T}{\mu}}$.

Longitudinal waves in a string have p-loops

$$f = \frac{p}{l}\sqrt{\frac{T}{\mu}}.$$

Average power transmitted along the string

$$P_{\text{average}} = \frac{1}{2}\frac{\omega^2 x_0^2 F}{v}$$

$$= 2\pi^2 \mu\, x_0^2 f^2 v$$

Average intensity

$$I_{\text{average}} = 2\pi^2 \rho\, x_0^2 f^2 v$$

Interference of Waves in the Same Direction

If $y_1 = y_{01}\sin(kx - \omega t)$ and

$y_2 = y_{02}\sin(\omega t - kx + \phi)$

then $y = y_1 + y_2$ and $y = y_0 \sin(\omega t - kx + \delta)$

Apply vector laws

$$y_0 = \sqrt{y_{01}^2 + y_{02}^2 + 2y_{01}y_{02}\cos\phi}$$

$$\tan\delta = \frac{y_{02}\sin\phi}{y_{01} + y_{02}\cos\phi}.$$

y_0 will be maximum when $\cos\phi = 1$ or $\phi = 0$ or $2n\pi$ where $n = 0, 1, 2 \ldots$

y_0 will be minimum when $\cos\phi = -1$

$\cos\phi = (2n+1)\pi$

when amplitude y_0 is maximum the **constructive interference** is said to take place as sound intensity will be maximum and the path difference is $n\lambda$.

$y_{0(\max)} = y_{01} + y_{02}$;

$y_{0\min} = y_{01} - y_{02}$

Amplitude is minimum when phase difference is an odd multiple of π or path difference is an odd multiple of half the wave length.

$$\frac{I_{\max}}{I_{\min}} = \frac{(y_{01} + y_{02})^2}{(y_{01} - y_{02})^2}$$

when intensity is minimum **destructive interference** occurs.

If the reflection occurs from a denser medium, it introduces a phase shift of π radian or 180^0 between incident and reflected wave as illustrated in Figure 13.2

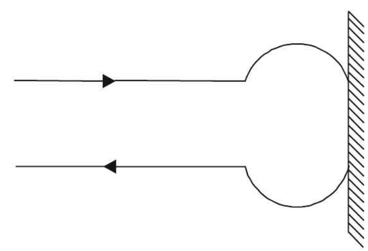
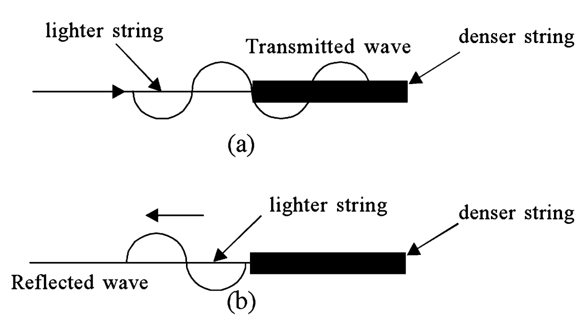


Fig. 13.2 Reflection from the wall

Note that if the wave is travelling in a string which is a combination of two mediums then, if the wave travels from the lighter to the denser string it is reflected out of phase (or 180^0) from the junction but is transmitted in phase.

If a wave propagates from denser to lighter string phase is shifted neither for the reflected nor for the transmitted wave as illustrated in figure 13.3.



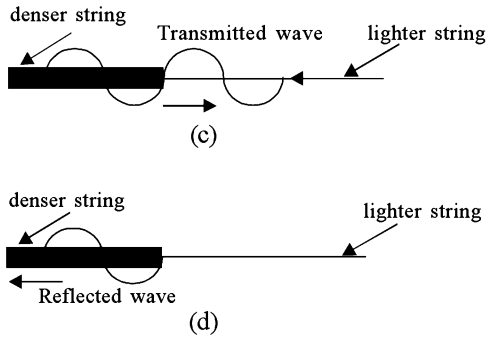


Fig. 13.3 Reflection and transmission from denser and rarer medium

Standing waves or stationary waves are produced when two waves having the same amplitude and same frequency superpose while travelling in opposite directions.

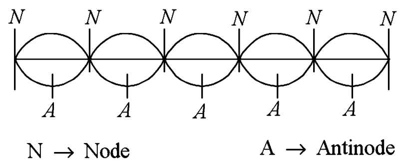


Fig. 13.4 Standing wave illustration

that is, $y_1 = y_0 \sin(kx - \omega t)$

and $y_2 = y_0 \sin(kx - \omega t)$ superpose.

Thus, $y = y_1 + y_2 = (2y_0 \sin kx) \cos \omega t$.

Normally a wave and its reflected wave superpose to produce standing waves. The points of maximum amplitude are called antinodes and minimum amplitude are called nodes. Separation between two consecutive nodes or antinodes is $\lambda/2$. Separation between a node and an antinode is $\lambda/4$.

Notes:

1. In a travelling wave the disturbance produced in a region propagates with a definite velocity and in a *standing wave* it is confined to the region where it is produced.
2. In a travelling wave the motion of all the particles is similar. In a *standing wave* different particles move with different amplitude.
3. In a *standing wave particles at node* always remain at *rest*. In a travelling wave there is no such particle which remains at rest always.
4. In a *standing wave* all the *particles cross* their *mean positions together*. In a travelling wave there is no instant when all the particles are at the mean position.
5. In a *standing wave* all the *particles* move in *phase*. In a travelling wave the phases of neighbouring particles are always different.
6. In a travelling wave energy from one region of space is transferred to the other region of space. In a *standing wave* the energy is always *confined* in that region.

Standing wave ratio (SWR) $\dfrac{y_{0\max}}{y_{0\min}} = \dfrac{y_{01} + y_{02}}{y_{01} - y_{02}}$

For a progressive wave, SWR = 1 (as $y_{02} = 0$). For standing wave SWR = ∞

In standing waves $\dfrac{d^2y}{dx^2} = \dfrac{1}{v^2}\dfrac{d^2y}{dt^2}$

The amplitude of the wave $y_0 = 2y_0 \sin kx$ is a periodic function of position (and not of time as in beats). If a loop vibrates in a single loop, the mode is fundamental. There are two nodes and one antinode and frequency is f. If there are n loops we say the string is vibrating with nth harmonic or $(n - 1)$th overtone and there will be n antinodes and $n + 1$ nodes. Frequency in this case will be nf.

That is, in a string fixed at both ends all integral multiples of fundamental frequency are allowed and

$$f = \frac{n}{2l}\sqrt{\frac{T}{\mu}}.$$

Vibrations of Strings Fixed at One End

Note that at the open end an antinode will be formed and at the fixed end a node will be formed.

- Only odd multiple of frequencies are allowed.
- n_{th} harmonic = $(n - 1)$th overtone.
- Fundamental frequency is also called note or first harmonic.
- Octave is the tone whose frequency is double the fundamental frequency

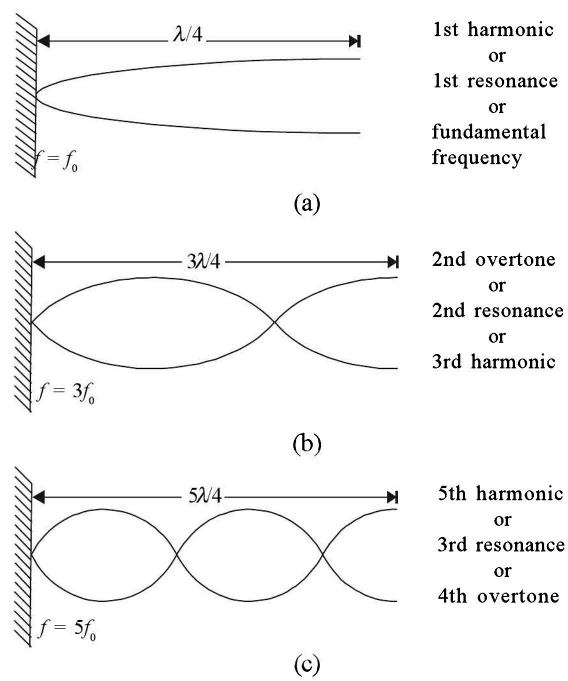


Fig. 13.5 Standing wave in a string fixed at one end and open at other

Melde's Experiment

$$f = \frac{p}{l}\sqrt{\frac{T}{\mu}}\ ;\ p = \text{number of loops}$$

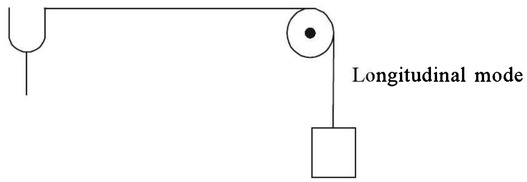


Fig. 13.6 (a)

$$f = \frac{p}{2l}\sqrt{\frac{T}{\mu}} \quad \text{where } P = \text{number of loops}$$

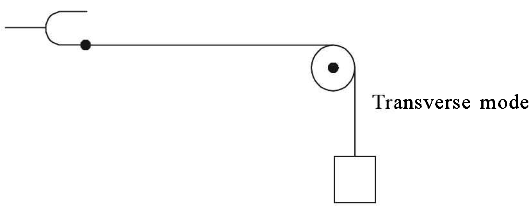


Fig. 13.6 (b) Melde's experiment

$$p_{\text{longitudinal}} = \frac{p_{transverse}}{2}$$

Velocity of a wave on the surface of a liquid is

$$v_s = \sqrt{\frac{g\lambda}{2\pi} + \frac{2\pi T}{\lambda\rho}}$$

where T is surface tension and ρ is density of the liquid.

Velocity of Torsional waves in a rod is

$$V_T = \sqrt{\frac{\eta}{\rho}}$$

• Short Cuts and Points to Note

1. To produce longitudinal waves the medium should possess bulk modulus of elasticity.
2. To produce transverse wave the medium must possess shear modulus of elasticity.
3. $y = y_0\sin(\omega t - kx + \phi)$ is the equation of a plane progressive wave in positive x direction. ϕ is initial phase angle or epoch. Normally, we write a simplified equation, $y = y_0\sin(\omega t - kx)$. A more general equation of wave is $y = y_0 e^{\pm i(\omega t - kx)}$
4. $k = \frac{2\pi}{\lambda}$ is propagation vector or wave number.
5. Wave velocity is $\frac{dx}{dt} = v = \frac{\omega}{k} = f\lambda$ and may be called phase velocity. In a dispersive medium waves travel with group velocity v_g given by

   $v_g = v - \frac{\lambda dv}{d\lambda}$. In dispersive mediums waves of different wavelengths travel with different velocity.
6. Particle velocity $= \frac{dy}{dt} = \frac{-dx}{dt} \times \frac{dy}{dx} = -v_{\text{wave}}$ (slope) Maximum particle velocity $= y_0\omega$
7. Frequency of the wave does not vary when a wave passes from one medium to the other.
8. Power (average) transmitted along the string is

   $$p_{av} = \frac{\omega^2 x_0^2 F}{2v} = 2\pi^2 \mu x_0^2 f^2 v$$

   Intensity $= 2\pi^2 \rho x_0^2 f^2 v$ where μ = mass/length and ρ = density of the medium.
9. Interference of waves travelling in the same direction is obtained using vector laws

   $$y_0 = \sqrt{y_{01}^2 + y_{02}^2 + 2y_{01}y_{02}\cos\phi} \quad \text{and}$$

   $$\tan\delta = \frac{y_{02}\sin\phi}{y_{01} + y_{02}\cos\phi}$$

   if $y_1 = y_{01}\sin(kx - \omega t)$ and

   $y_2 = y_{02}\sin(\omega t - kx + \phi)$ interfere.
10. $$\frac{I_{max}}{I_{min}} = \frac{(y_{01} + y_{02})^2}{(y_{01} - y_{02})^2}$$

    Maximum intensity is obtained when phase shift is zero or path difference is an integral multiple of wavelength. Minimum intensity or destructive interference occurs when phase shift is an odd integral multiple of π radian or odd integral multiple of half the wavelength.
11. Reflection from a denser medium causes a phase shift of 180^0 and reflection from rarer or lighter medium occurs without change of phase in the string.
12. Standing waves result when two waves having same amplitude and same frequency travelling in opposite directions superpose.
13. $y = 2y_0\sin kx \cos \omega t$ represents a stationary wave in strings. Note that ($2y_0\sin kx$) shows amplitude and is a function of distance.

    At certain places, amplitude is maximum (antinodes) and at other places amplitude is zero (nodes). Separation between consecutive nodes or antinodes is $\lambda/2$. Distance between a node and consecutive antinode is $\lambda/4$.

14. $\frac{v^2 d^2 y}{dx^2} = \frac{d^2 y}{dt^2}$ satisfies the standing wave equation.

15. $y = 2y_0 \cos kx \sin \omega t$ represents stationary wave equation which fits in closed or open pipes.

16. $f = \frac{1}{2l}\sqrt{\frac{T}{\mu}}$ is used for fundamental mode in strings vibrating with transverse stationary waves.

17. In a sonometer, transverse stationary waves are produced.

$$f = \frac{n}{2l}\sqrt{\frac{T}{\mu}}$$ can be used for finding frequency if the string is vibrating in n loops.

18. In Melde's experiment if the transverse mode is used

$$f = \frac{p}{2l}\sqrt{\frac{T}{\mu}}$$ and

$$f = \frac{p}{l}\sqrt{\frac{T}{\mu}}$$ for longitudinal mode.

19. Only transverse waves can be polarised. Longitudinal waves cannot be polarised.

20. Sound waves being pressure waves are longitudinal.

21. If the disturbance produced is always along a fixed direction. The wave is linearly polarised in that direction. For example, $y = y_0 \sin(\omega t - kx)$ is linearly polarised in y direction. Linearly polarised waves are also called plane polarised waves.

22. If each particle of a string moves in a small circle as the wave passes through it then the wave is circularly polarised. If each particle moves in an ellipse it is elliptically polarised and if each particle is randomly displaced, it is unpolarised.

23. A circularly polarised or unpolarised wave passing through a slit does not show change in intensity as the slit is rotated in its plane. But the transmitted wave becomes linearly polarised in the direction parallel to the slit.

24. Number of nodes and antinodes in a tuning fork when vibrating in fundamental mode as illustrated in figure. Antinodes are A_1 and A_2 and A_3. A_3 is longitudinal antinode while A_1 and A_2 are transverse antinodes. If the tuning fork vibrates in nth harmonic then number of nodes = $2n$ and number of antinodes = $(2n + 1)$

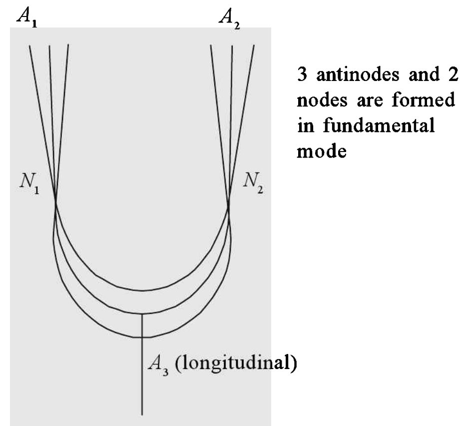


Fig. 13.7 Nodes and Antinodes in a tuning fork

- **Caution**

1. Considering that all waves require a medium.

⇒ only mechanical waves require an elastic medium

2. Considering that both longitudinal and transverse waves can be produced in any medium.

⇒ Transverse waves require medium that shall possess shear modulus of elasticity. Therefore, transverse waves cannot be produced in gases.

3. Considering that waves could be only longitudinal or transverse.

⇒ Waves could be a combination of both. For example, ripples in water, seismic waves during earthquakes.

4. Considering that medium is also transported along with energy during propagation of a wave.

⇒ Only energy is transported and not the medium during propagation of the wave.

5. Considering only functions like $y_0 \sin(\omega t - kx)$ or $y_0 \cos(\omega t - kx)$ can represent a wave.

⇒ Functions like

$$y = y_0 \sin(\omega t - kx), y = y_0 \cos(\omega t - kx) \quad \text{and}$$

$$y = y_0 + \sum_{n=1}^{\infty} a_n \sin n\omega t + \sum_{n=1}^{\infty} b_n \cos n\omega t$$

also represent waves which are complex periodic waves. However, plane progressive harmonic waves can be represented as

$$y = y_{01} \sin(\omega t - kx) + y_{02} \sin(\omega t - kx)$$

$$y = y_0 \sin(\omega t - kx) \text{ or } y = y_0 \cos(\omega t - kx).$$

6. Confusion about the formula to be applied in strings to calculate frequency

⇒ use $f = \frac{1}{2l}\sqrt{\frac{T}{\mu}}$ when vibrating in fundamental transverse mode and

$$f = \frac{1}{l}\sqrt{\frac{T}{\mu}}$$ when vibrating in fundamental longitudinal mode.

7. Confusion about the modes of production of transverse and longitudinal wave.

Fig. 13.8

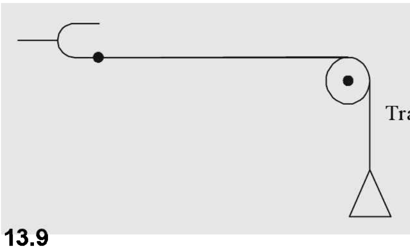


Fig. 13.9

8. Considering that velocity of a wave is same at every point in the string.

⇒ Velocity of wave in vertical strings is different at different points.

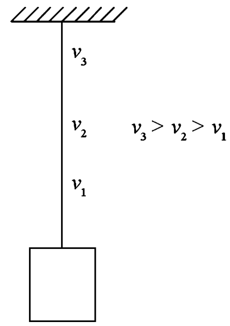


Fig. 13.10

SOLVED PROBLEMS

1. The displacement wave in a string is $y = (3 \text{ cm}) \sin 6.28 (0.5x - 50t)$ where x is in centimetres and t in seconds. The wavelength and velocity of the wave is

(a) 2 cm, 100 cms^{-1} (b) 10 cm, 50 cm^{-1}
(c) 20 cm, 2 ms^{-1} (d) 2 m, 100 ms^{-1}

Solution (a) $k = \dfrac{2\pi}{\lambda}$

or $\lambda = \dfrac{2\pi}{k} = \dfrac{6.28}{6.28(0.5)} = 2\text{ cm}$

$v = \dfrac{\omega}{k} = \dfrac{50\times 6.28}{0.5\times 6.28} = 100 \text{ cms}^{-1}$

2. The equation of a wave is $10 \sin(6.28x - 314t)$ where x is in centimetres and t is in seconds. The maximum velocity of the particle is

(a) 62.8 cm s^{-1} (b) 3140 ms^{-1}
(c) 50 cms^{-1} (d) 31.4 m s^{-1}

Solution (d) $y_{max} = \omega y_0 = 314\,(10)$ cm/s or 31.4 ms^{-1}.

3. The speed of a transverse wave travelling on a wire having a length 50 cm and mass 50 g is 80 ms^{-1}. The area of cross-section of the wire is 1 mm^2 and its Young's modulus is 16×10^{11} Nm^{-2}. Find the extension of the wire over natural length.

(a) 2 cm (b) 2 mm
(c) 0.2 mm (d) 0.02 mm

Solution (d) $v = \sqrt{\dfrac{T}{\mu}}$

or $T = v^2\mu = (80)^2\left(\dfrac{5}{0.5}\times 10^{-3}\right) = 64\text{ N}$

and $Y = \dfrac{Fl}{A\,\Delta l}$

or $\Delta l = \dfrac{Fl}{AY} = \dfrac{64\times 0.5}{10^{-6}\times 16\times 10^{11}} = 2\times 10^{-5}\text{m.}$

4. Which of the following wave is progressing in the y direction?

(a) $x = x_0\cos(\omega t - ky)$ (b) $y = y_0\cos(\omega t - ky)$
(c) $y = y_0\cos kx \sin \omega t$ (d) $y = y_0 \sin kx \cos \omega t$

Solution (a) The wave $x = x_0\cos(\omega t - ky)$ travels along y direction.

5. Velocity of sound in air is 332 m/s. Its velocity in vacuum is

(a) > 332 ms^{-1} (b) 3×10^8ms^{-1}
(c) 332 ms^{-1} (d) none of these

Solution (d) None of these as velocity is zero as sound waves require medium.

6. A cork floating in a calm lake is executing SHM of frequency f. When a boat passes close to the cork then the

(a) frequency becomes greater than f.
(b) frequency becomes less than f.
(c) frequency remains constant.
(d) none of these.

Solution (c) Frequency remains constant and velocity will vary, that is, wavelength will vary.

7. Two waves of equal amplitude x_0 and equal frequency travel in the same direction in a medium. The amplitude of the resultant wave is

(a) 0 (b) x_0
(c) $2x_0$ (d) between 0 and $2x_0$

Solution (d) use $x_0' = \sqrt{x_{01}^2 + x_{02}^2 + 2x_{01}x_{02}\cos\theta}$

$\because$ $\theta = 0$ and $x_{01} = x_{02} = x_0$; $x_0' = 2x_0$.

8. The fundamental frequency of a string is proportional to

(a) inverse of the length. (b) the diameter.
(c) tension. (d) density.

Solution (a) $f \propto \frac{1}{l}$

9. A uniform rope of length 12 m and mass 6 kg hangs vertically from a rigid support. A block of mass 2 kg is attached to the free end of the rope. A transverse pulse of wavelength 0.06 m is produced at the lower end of the rope. What is the wavelength of the pulse when it reaches the top of the rope.

(a) 0.06 m (b) 0.12 m
(c) 0.09 m (d) none of these

[IIT 1984]

Solution (d) $v = \frac{T}{\mu}$

$$\frac{v_{top}}{v_{bottom}} = \sqrt{\frac{T_T}{T_B}} = \sqrt{\frac{(6+2)g}{2g}} = 2$$

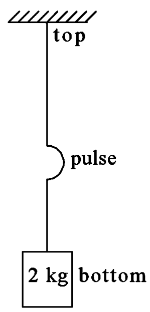


Fig. 13.11

$$\frac{f\lambda_{\text{Top}}}{f\lambda_{\text{Bottom}}} = 2 \text{ as frequency does not change}$$

$\therefore$ $\lambda_{\text{top}} = \lambda_{\text{bott}} \times 2 = 0.12$ m.

10. A uniform rope of mass 0.1 kg and length 2.45 m hangs from a ceiling. The speed of transverse waves in the rope at a point 0.5 m from the lower end is

(a) 2.21 ms^{-1} (b) 4.21 ms^{-1}
(c) 7.21 ms^{-1} (d) 3.31 ms^{-1}

[Roorkee 1991]

Solution (a) $T = \frac{M}{L}(x)g$

and $$v = \sqrt{\frac{\frac{M}{L}(x)g}{M/L}} = \sqrt{gx} = \sqrt{9.8 \times 0.5} = 2.21 \text{ m/s}$$

11. The equations of motion of two waves propagating in the same direction is given by

$y_1 = A\sin(\omega t - kx)$

and $y_2 = A\sin(\omega t - kx - \theta)$.

The amplitude of the medium particle will be

(a) $\sqrt{2}\, A\cos\theta$ (b) $2A\cos\theta$
(c) $\sqrt{2}\, A\cos\theta/2$ (d) $2A\cos\theta/2$

[BHU 2003]

Solution (d) $y_0 = \sqrt{A^2 + A^2 + 2A(\cos\theta)}$

$= A\sqrt{2(1+\cos\theta)} = 2A\cos\theta/2.$

12. The displacement y of a wave travelling in x direction is given by

$$y = 10^{-1}\sin\left(600t - 2x + \frac{\pi}{3}\right)\text{m}$$

Where x is expressed in metres and t in seconds. The speed of the wavemotion in metre per second is

(a) 600 (b) 1200
(c) 200 (d) 300

Solution (d) $v = \frac{\omega}{k} = \frac{600}{2} = 300\ ms^{-1}$.

13. A steel wire of linear mass density 9.8 g/m is stretched with a tension of 10 kg. It is kept between poles of an electromagnet and it vibrates in resonance when carrying an arc of frequency n. The frequency n is

(a) 100 Hz (b) 200 Hz
(c) 25 Hz (d) 50 Hz

Solution (d) $f = \frac{1}{2l}\sqrt{\frac{T}{\mu}} = \frac{1}{2}\sqrt{\frac{10 \times 9.8}{9.8 \times 10^{-3}}} = \frac{10^2}{2} = 50$ Hz.

14. The equation of a progressive wave is

$y = 8\sin\left[\pi\left(\frac{t}{10} - \frac{x}{4}\right) + \frac{\pi}{3}\right]$. The wavelength of the wave is

(a) 8 m (b) 4 m
(c) 2 m (d) 10 m

[CET Maharashtra 2002]

Solution (a) $\frac{2\pi}{\lambda} = k$ or $\lambda = \frac{2\pi}{k} = \frac{2\pi}{\pi/4} = 8$

15. The equation of a stationary wave is $y = \sin\frac{\pi x}{3}\cos 10\pi t$ where x and y are in centimetres and t in seconds. The separation between two consecutive nodes is

(a) 1.5 cm (b) 6.0 cm
(c) 3.0 cm (d) 18 cm

[DPMT 2002]

Solution (c) $\lambda = \frac{2\pi}{k} = \frac{2\pi}{\pi/3} = 6$ cm.

Separation between two consecutive nodes $= \lambda/2 = 3$ cm.

16. If the amplitude of velocity of a particle acted by a force $F = F_0\cos\omega t$ along x-axis is given by

$$v_0 = \frac{1}{(a\omega^2 - b\omega + c)^{1/2}} \text{ where } b^2 > 4ac.$$

The frequency of resonance is:

(a) $\omega = b/a$ (b) $b/2a$
(c) a/b (d) $a/2b$

Solution (b) For resonance $v_0 \to \infty$ (max)

$\therefore$ $(a\omega^2 - b\omega + c)^{1/2}$ should be minimum

or $\dfrac{d}{d\omega}(a\omega^2 - b\omega + c) = 0$

or $2a\omega - b = 0$ or $\omega = \dfrac{b}{2a}$

17. An observer on the sea shore observes 54 waves reaching the coast per minute. If the wavelength is 10 m. The velocity is

(a) 9 ms^{-1} (b) 54 ms^{-1}
(c) 18 ms^{-1} (d) 36 ms^{-1}

Solution (a) $f = \dfrac{54}{60} = \dfrac{9}{10}$ Hz

$v = f\lambda = \dfrac{9}{10} \times 10 = 9\ ms^{-1}$

18. A light pointer fixed to one prong of a tuning fork touches a vertical smoked plate. The fork is set to vibration and the plate is allowed to fall freely. Eight complete waves are counted when the plate falls through 10 cm. The frequency of the tuning fork is

(a) 112 Hz (b) 14 Hz
(c) 28 Hz (d) 56 Hz

[IIT 1996]

Solution (d) $t = \sqrt{\dfrac{2h}{g}} = \sqrt{\dfrac{2\times 0.1}{9.8}} = \dfrac{1}{7}\ s.$

$f = \dfrac{\text{number of waves}}{\text{time}} = \dfrac{8}{1/7} = 56$ Hz.

19. A progressive wave of frequency 500 Hz is travelling with a velocity 360 ms^{-1}. How far are two points 60^0 out of phase?

(a) 0.06 m (b) 0.12 m
(c) 0.18 m (d) 0.24 m

Solution (b) $\lambda = \dfrac{v}{f} = \dfrac{360}{500} = 0.72$ m

$\Delta\phi = \dfrac{2\pi}{\lambda}(\Delta x)$

or $\Delta x = \dfrac{\Delta\phi\lambda}{2\pi} = \dfrac{\pi/3(0.72)}{2\pi} = 0.12$ m

20. Two blocks each having a mass 3.2 kg are connected by a wire CD and the system is suspended from the ceiling by another wire AB as shown in figure. The linear mass density of AB is 10 gm^{-1} and that of the CD is 8 gm^{-1}. The speed of the transverse wave pulse produced in AB and CD is

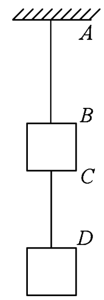


Fig. 13.12

(a) 80 ms^{-1}, 40 ms^{-1} (b) 40 ms^{-1}, 80 ms^{-1}
(b) 80 ms^{-1}, 63 ms^{-1} (d) none of these

Solution (c) $v = \sqrt{\dfrac{T}{\mu}} \Rightarrow v_{AB} = \sqrt{\dfrac{6.4\times 10}{10\times 10^{-3}}} = 80\ ms^{-1}$

$v_{CD} = \sqrt{\dfrac{3.2\times 10}{8\times 10^{-3}}} = 63\ ms^{-1}$

21. A transverse wave described by $y = 0.02 \sin(x + 30t)$ propagates on a stretehed string of linear density 12 gm^{-1}. The tension in the string is

(a) 2.16 N (b) 1.08 N
(c) 0.108 N (d) 0.0108 N

Solution (c) $v = \dfrac{\omega}{k} = \dfrac{30}{1} = 30\ ms^{-1}$

$T = v^2\mu = (30)^2 \times 12 \times 10^{-3} = 0.108$ N.

22. A circular loop of the string rotates about its axis on a frictionless horizontal plane at a uniform rate so that the tangential speed of any particle of the string is v. If a small transverse disturbance is produced at a point of the loop, the speed (relative to the string) at which the disturbance will travel is

(a) v (b) $\dfrac{v}{2}$
(c) $2v$ (d) $\dfrac{v}{4}$

Solution (a) v

TYPICAL PROBLEMS

23. In the figure the string has a mass 4.5 g. How much time will it take for a transverse disturbance produced at the floor to reach the pulley? (Take $g = 10\ ms^{-2}$)

(a) 0.04 s (b) 0.2 s
(c) 0.4 s (d) 0.02 s

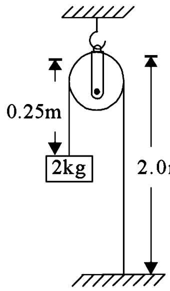


Fig. 13.13

Solution (d) $v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{20}{2\times10^{-3}}} = 100\ \text{ms}^{-1}$

$\mu = \frac{4.5}{2.25} = 2\ \text{gm}^{-1}$

$s = ut - \frac{1}{2}gt^2$

$2 = 100t - 5t^2$

$r^2 - 20t + 0.4 = 0$

$t = \frac{+20 \pm \sqrt{400 - 1.6}}{2} = 0.02\ \text{s}$

24. Two wires are kept tight between the same pair of supports. The tensions in the wires are in the ratio 2:1. The radii are in the ratio 3 : 1 and the densities are in the ratio 1 : 2. The ratio of their fundamental frequencies are

(a) 1/2 (b) 2/3
(c) 3/4 (d) 4/9

Solution (d) $\frac{f_1}{f_2} = \frac{\frac{1}{2l}\sqrt{\frac{T_1}{\mu_1}}}{\frac{1}{2l}\sqrt{\frac{T_2}{\mu_2}}} = \sqrt{\frac{T_1\rho_2 A_2}{T_2\rho_1 A_1}}$

$= \sqrt{\frac{T_1\rho_2\pi r_2^2}{T_2\rho_1\pi r_1^2}} = \frac{2}{3}$

25. The equation of a standing wave produced on a string fixed at both the ends is $y = 0.4\sin(0.314x)\cos(600\pi t)$. The smallest length of the string would be—— (where x is in cm, t is in seconds)

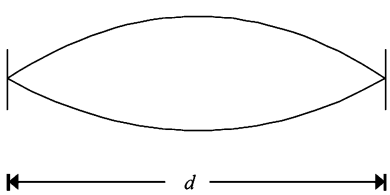


Fig. 13.14

(a) 20 cm (b) 40 cm
(c) 10 cm (d) none of these

Solution (c) $\lambda = \frac{2\pi}{k}$

$\lambda = \frac{2\pi}{0.314} = 20\ \text{cm}$

Smallest length $d = \frac{\lambda}{2} = 10$ cm.

26. A uniform rope of mass 0.1 kg and length 2.45 m hangs from a ceiling. The time taken by the transverse wave to travel through the full length of the rope is

(a) 2.0 s (b) 1.2 s
(c) 1.0 s (d) 2.2 s

Solution (c) $T = \left(\frac{M}{L}x\right)$ g and $v = \sqrt{\frac{\frac{Mx}{L}g}{\mu/L}} = \sqrt{gx}$

$v = \frac{dx}{dt} = \sqrt{gx}$ or $dt = \frac{dx}{\sqrt{gx}}$

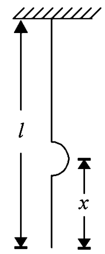


Fig. 13.15

or $t = \frac{1}{\sqrt{g}}\int_0^L x^{-1/2}dx$

or $t = 2\sqrt{\frac{L}{g}} = 2\sqrt{\frac{2.45}{9.8}} = 1$ s.

27. An aluminium wire of length 60 cm is joined to a steel wire of length 80 cm and stretched between two fixed supports. The tension produced is 40 N. Cross-sectional area is 1 mm^2 (steel) and 3 mm^2 (aluminium). Minimum frequency of the tuning fork which can produce standing waves with the joint as a node is

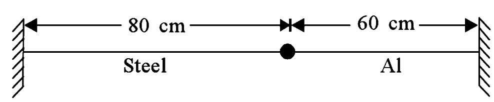


Fig. 13.16

(density of Al = 2.6 g cc^{-1} and density of steel = 7.8 g cc^{-1})

(a) 90 Hz (b) 145 Hz
(c) 180 Hz (d) 250 Hz

Solution (c) $f = \frac{n}{2l}\sqrt{\frac{T}{\mu}} = \frac{n}{2l}\sqrt{\frac{T}{A\rho}}$

since frequency will remain same

$\therefore \quad \frac{n}{2l_1}\sqrt{\frac{T}{A_1\rho_1}} = \frac{p}{2l_2}\sqrt{\frac{T}{A_2\rho_2}}$

or $\frac{n}{p} = \frac{l_1}{l_2}$ that is, $\frac{n}{p} = \frac{4}{3}$

$$f = \frac{3}{2(0.6)}\sqrt{\frac{40}{10^{-6}\times 2.6\times 10^{3}\times 3}}$$

$$= \frac{1}{0.4}\sqrt{\frac{40\times 10^{3}}{2.6\times 3}}$$

$$= \frac{100}{0.4}\sqrt{\frac{2}{3.9}} = 180\text{ Hz.}$$

28. A 200 Hz wave with amplitude 1 mm travels on a long string of linear mass density 6 gm^{-1} kept under a tension of 60 N. The average power transmitted across a given point in the string is

(a) 0.53 W (b) 0.83 W
(c) 0.47 W (d) 0.89 W

Solution $p_{\text{average}} = 2\pi^2\mu\, x_0^2 f^2 v$

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{60}{6\times 10^{-3}}} = 100\text{ ms}^{-1}$$

$$= 2\times\pi^2(6\times 10^{-3})(10^{-6})(2\times 10^{2})^2\times 100 = 0.47\text{ W.}$$

29. The ends of a stretched string of length L are fixed at $x = 0$ amd $x = L$. In one experiment, the displacement of the wire is $y_1 = A\sin\frac{\pi x}{L}\cos\omega t$ and energy E_1 and in another experiment, its displacement is $y_2 = A\sin\frac{2\pi x}{L}\sin 2\omega t$ and energy E_2 then

(a) $E_2 = E_1$ (b) $E_2 = 2E_1$
(c) $E_2 = 4E_1$ (d) $E_2 = 16E_1$

[IIT Screening 2002]

Solution (c) $\sin E\ \alpha\ y_0^2 f2$ and $y_0 = A$ in each case but $f_1 = f.f_2 = 2f$, therefore, $E_2 = 4E_1$.

30. A sonometer wire resonates. with a given tuning fork forms a standing wave with 5 antinodes between two bridges when a 9 kg weight is suspended from the wire. When the mass is replaced by a mass M, the wire resonates with the same tuning fork forming 3 antinodes for the same position of wedges. The value of M is

(a) 2.25 kg (b) 5 kg
(c) 12.5 kg (d) $\frac{1}{25}$ kg

[IIT Screening 2002]

Solution (a) $f = \frac{n_1}{2l}\sqrt{\frac{T_1}{\mu}} = \frac{n_2}{2l}\sqrt{\frac{T_2}{\mu}}\sqrt{\frac{T_1}{T_2}} = \frac{n_2}{n_1}$

or $\frac{T_1}{T_2} = \frac{n_2^2}{n_1^1} = \left(\frac{4}{2}\right)^2 = 4\,M = \frac{9}{4} = 2.25\text{ kg.}$

QUESTIONS FOR PRACTICE

1. What happens when a sound wave is reflected from the boundary of a denser medium? The compression of the incident wave is returned as a

(a) rarefaction. (b) crest.
(c) trough. (d) compression.

2. The velocity of sound in vacuum is

(a) 332 ms^{-1} (b) 288 ms^{-1}
(c) 330 ms^{-1} (d) zero

3. The frequency of a man's voice is 300 Hz. If velocity of sound waves is 336 ms^{-1}, the wavelength of the sound is

(a) 1.12 m (b) 300 × 336 m
(c) 330/336 m (d) none of these

4. The angle between particle velocity and wave velocity in transverse waves is

(a) π (b) $\pi/2$
(c) $\pi/4$ (d) zero

5. Longitudinal waves cannot travel through

(a) liquids. (b) gases.
(c) vacuum. (d) solid.

6. In case of the electromagnetic waves the angle between the electric and magnetic field vectors is

(a) π (b) $\pi/2$
(c) zero (d) none of these

7. A wave of length 2 m is superposed on its reflected wave to form a stationary wave. A node is located at $x = 3$ m, the next node will be located at $x =$

(a) 4 m (b) 3.75 m
(c) 3.50 m (d) 3.25 m

8. Velocity of sound in the atmosphere of a planet is 500 ms^{-1}. The minimum distance between the source of sound and the obstacle for an echo heard is

(a) 17 m (b) 50 m
(c) 25 m (d) 20 m

9. If F is restoring force, k is force constant and y is displacement, which of the following expressions represent the equation of simple harmonic motion?

(a) $F = -ky$ (b) $F = \sqrt{ky}$
(c) $F = ky$ (d) none of these

10. The number of beats produced per second by two tuning forks when sounded together is 4. One of them has a frequency of 250 Hz. The frequency of the other cannot be less than

(a) 254 Hz (b) 252 Hz
(c) 248 Hz (d) 246 Hz

11. A bomb explodes on the moon. How long will it take for the sound to reach the earth?

(a) 1 day (b) 1000 s
(c) 10 s (d) none of these

12. Two simple harmonic waves having the same amplitude and frequency with zero phase difference superimpose at right angles to each other. The resultant motion will be

(a) linear. (b) eliptical.
(c) circular. (d) none of these.

13. A property of the progressive wave that does not depend upon other characteristics mentioned below is

(a) wavelength. (b) amplitude.
(c) frequency. (d) wave velocity.

14. Two waves of same frequency but amplitudes equal to a and $2a$ travelling in the same direction superimpose out of phase. The resultant amplitude will be

(a) $\sqrt{a^2 + 2a^2}$ (b) $3a$
(c) $2a$ (d) a

15. The oscillators that can be described in terms of sine or cosine functions are called

(a) simple harmonic. (b) natural.
(c) sympathetic. (d) free.

16. The distance between two consecutive antinodes is 0.5 m. The distance travelled by the wave in half the time period is

(a) 2 m (b) 1 m
(c) 0.5 m (d) 0.25 m

17. Which of the following expressions is that of a simple harmonic progressive wave?

(a) $A \sin (\omega t - kx)$ (b) $A \sin \omega t$
(c) $A \sin \omega t \cos kx$ (d) $A \cos kx$

18. A wave of frequency 400 Hz has a velocity of 320 ms^{-1}. The distance between the particles differing in phase by 90^0 is

(a) 80 cm (b) 60 cm
(c) 40 cm (d) 20 cm

19. The ratio of intensities of two waves is 1 : 16. The ratio of their amplitudes is

(a) 16/17 (b) 1/16
(c) 1/4 (d) 1/2

20. Two waves each of loudness L superimpose to produce beats. The maximum loudness of the beats will be

(a) $4L$ (b) $2L$
(c) L (d) none of these

21. Two waves of intensities I and $4I$ superimpose. The minimum and maximum intensities will respectively be

(a) $I, 9I$ (b) $3I, 5I$
(c) $I, 5I$ (d) none of these

22. The velocity of sound in oxygen at NTP is v. The velocity of sound in hydrogen at NTP will be

(a) $2\sqrt{2}\,v$ (b) $2v$
(c) $4v$ (d) none of these

23. The isothermal elasticity of a medium is E_i and the adiabatic elasticity in E_a. The velocity of the sound in the medium is proportional to

(a) $\sqrt{E_i}$ (b) E_a
(c) $\sqrt{E_a}$ (d) E_i

24. The velocity of sound in air is v and the root mean square velocity of the molecules is c. Then v/c =

(a) $\gamma/3$ (b) $\gamma/\sqrt{3}$
(c) $\frac{\sqrt{\gamma}}{3}$ (d) $\sqrt{\frac{\gamma}{3}}$

25. The velocity of sound at 0^0C is 332 ms^{-1}. At what temperature will it be 664 ms^{-1}?

(a) 273^0C (b) 546^0C
(c) 819^0C (d) 1092^0C

26. Velocity of hydrogen at NTP is v. What will be the velocity of sound in a mixture of hydrogen and oxygen in the ratio 4 : 1 at NTP is

(a) v (b) $2v$
(c) $v/2$ (d) $v/4$

27. A sound wave is travelling in a medium in which the velocity is v. It is incident on the second medium in which the velocity of the wave is $2v$. What should be the minimum angle of incidence on the first medium, so that the wave fails to cross the surface of separation of the two media?

(a) 60^0 (b) 45^0
(c) 30^0 (d) 15^0

28. Beats are produced because of the superposition of two progressive notes. Maximum loudness at the waxing is n times the loudness of either notes. What is the value of n?

(a) 4 (b) 2
(c) $\sqrt{2}$ (d) 1

29. The first resonance length in a closed organ pipe is 50 cm. Then the second resonance length will be

(a) 50 cm (b) 100 cm
(c) 150 cm (d) 200 cm

30. Which type of oscillations give rise to resonance?

(a) damped (b) free
(c) forced (d) all of these

31. At what temperature the speed of sound in air will be 1.5 times its value at 27^0C in air?

(a) 102^0C (b) 204^0K
(c) 204^0C (d) 402^0C

32. The ratio of speeds of sound in hydrogen gas and oxyen gas at same temperature will be
(a) 8 : 1 (b) 4 : 1
(c) 1 : 8 (d) 1 : 4

33. The distance between a node and an anti-node is
(a) 2λ (b) λ
(c) $\lambda/2$ (d) $\lambda/4$

34. The speed of a supersonic wave, as compared to that of sound is
(a) less. (b) more.
(c) equal. (d) 1/10.

35. The increase in the speed of sound, on increasing the temperature of the medium by 10^0C, will be
(a) 600 ms^{-1} (b) 6 ms^{-1}
(c) 0.61 ms^{-1} (d) 60 ms^{-1}

36. The velocity of sound in air is 350 ms^{-1}. The fundamental frequency of an open pipe of length 50 cm will be
(a) 700 s^{-1} (b)350 s^{-1}
(c) 175 s^{-1} (d)50 s^{-1}

37. The fundamental frequency of an open organ pipe is n. The pipe is vertically immersed in water such that half of its length is submerged. The fundamental frequency of air column in this position will be
(a) $n/3$ (b) $n/2$
(c) n (d) $2n$

38. If the ratio of amplitudes of two waves at any point in the medium is 1 : 3, then the ratio of maximum and minimum intensities because of their superposition will be
(a) 2 : 1 (b) 3 : 1
(c) 4 : 1 (d) 2 : 3

39. The phase difference between the particles vibrating between two consecutive nodes is
(a) zero (b) $\pi/2$
(c) π (d) 2π

40. The frequency of an open organ pipe is n. If one end is closed then its fundamental frequency will be
(a) $n/2$ (b) $3n/4$
(c) n (d) $2n$

41. Two sound waves of equal intensity I superimpose at point P in 90^0 out of phase. The resultant intensity at point P will be
(a) $4I$ (b) $\sqrt{2}\,I$
(c) $2I$ (d) I

42. The equation of a wave propagating in a string is $y = 2 \cos \pi(100t - x)$. Its wave length will be
(a) 2 cm (b) 5 cm
(c) 2 m (d) 50 cm

43. On vibrating a tuning fork of frequency 256 Hz with another fork A, six beats per second are heard. On loading A, again six beats per second are heard. The frequency of A will be
(a) 244 Hz (b) 250 Hz
(c) 262 Hz (c) 268 Hz

44. The ratio of frequencies in a stretched string is
(a) 1 : 2 : 3 (b) 1 : 3 : 5
(c) 2 : 4 : 6 (d) 3 : 2 : 1

45. The property of a medium necessary for wave propagation is its
(a) elasticity. (b) low resistance.
(c) inertia. (d) all of above.

46. The ratio (v) of velocities of sound in dry air and humid air is
(a) $v < 1$ (b) $v > 1$
(c) $v = 1$ (d) zero

47. The waves propagating on water surface are
(a) ultrasonic. (b) longitudinal.
(c) unaudible. (d) transverse.

48. A tuning fork produces four beats per second with 49 cm and 50 cm lengths of a stretched wire of a sonometer. The frequency of the fork is
(a) 196 Hz (b) 296 Hz
(c) 396 Hz (d) 693 Hz

49. In Melde's experiment, eight loops are formed with a tension of 0.75 N. If the tension is increased to four times then the number of loops produced will be
(a) 2 (b) 4
(c) 8 (d) 16

50. The third harmonic in an open organ pipe is known as
(a) fundamental frequency
(b) second overtone
(c) third overtone
(d) first overtone

51. The correct graph between the frequency n and square root of density (ρ) of a wire, keeping its length, radius and tension constant, is

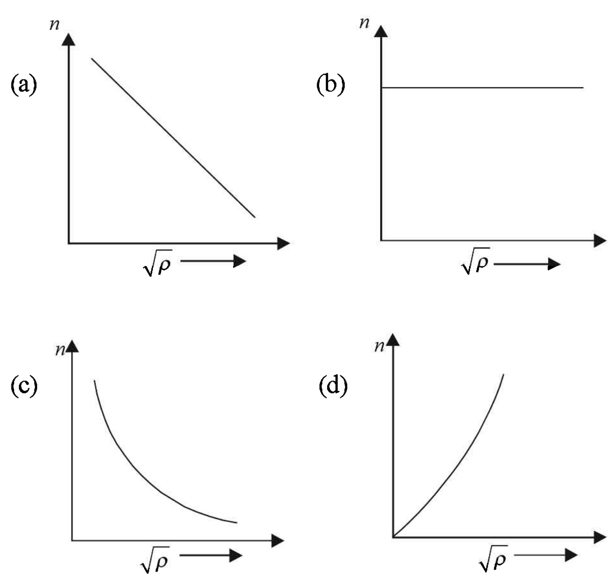


Fig. 13.17

52. P is the junction of two wires A and B. B is made of steel and is thicker while A is made of aluminium and is thinner as shown. If a wave pulse as shown in the figure approaches P, the reflected and transmitted waves from P are respectively

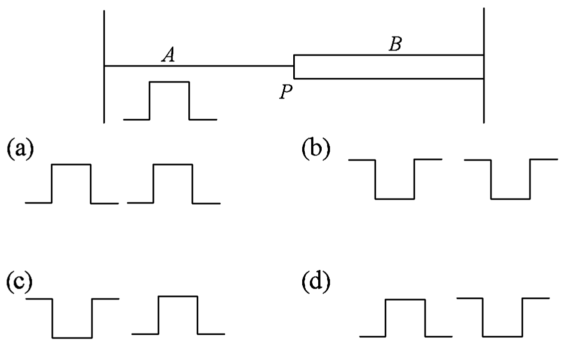


Fig. 13.18

53. In Kundt's tube, when waves of frequency 10^3 Hz are produced the distance between five consecutive nodes is 82.5 cm. The speed of sound in gas filled in the tube will be

(a) 660 ms^{-1} (b) 330 ms^{-1}
(c) 230 ms^{-1} (d) 100 ms^{-1}

54. A resonance tube is resonated with tuning fork of frequency 256 Hz. If the length of resonating air columms are 32 cm and 100 cm, then end correction will be

(a) 1 cm (b) 2 cm
(c) 4 cm (d) 6 cm

55. On decreasing the temperature the frequency of an organ pipe becomes

(a) less. (b) more.
(c) equal. (d) infinity.

56. When a sound wave is reflected from a rigid support then its path changes by

(a) $\lambda/2$ (b) λ
(c) π (d) 2π

57. The vibration in the stem of tuning fork are

(a) transverse. (b) longitudinal.
(c) both. (d) none of these.

58. The ratio of the velocity of body and velocity of sound is known as

(a) laplace number. (b) positive integer.
(c) stable number. (d) mach number.

59. For constructive interference, the path difference between two waves must be

(a) $(2n+1)\,\lambda/2$ (b) $(2n+1)\lambda$
(c) $n\dfrac{\lambda}{2}$ (d) $n\lambda$

60. The frequencies of two sound sources are 256 Hz and 260 Hz respectively. The beat frequencies produced by them will be

(a) 0.025 s^{-1} (b) 2.5 s^{-1}
(c) 4.00 s^{-1} (d) 25 s^{-1}

61. On sounding a string and a tuning fork simultaneously, six beats per second are produced if the length of the string is 95 cm or 100 cm. The frequency of the fork is

(a) 117 Hz (b) 234 Hz
(c) 432 Hz (d) 702 Hz

62. The resultant amplitude, when two waves of same frequency but with amplitudes a_1 and a_2 superimpose with a phase difference of $\pi/2$ will be

(a) $a_1^2 + a_2^2$ (b) $\sqrt{a_1^2 + a_2^2}$
(c) $a_1 - a_2$ (d) $a_1 + a_2$

63. In a stationary wave the distance between consecutive antinodes is 25 cm. If the wave velocity is 300 ms^{-1} then the frequency of wave will be

(a) 150 Hz (b) 300 Hz
(c) 600 Hz (d) 750 Hz

64. The intensity of sound after passing through a slab decreases by 20%. On passing through two such slabs, the intensity will decrease by

(a) 30% (b) 36%
(c) 40% (d) 50%

65. Two waves travel in the mutually opposite directions in a medium. When superimposed, the phenomenon observed is

(a) stationary waves. (b) harmonic nodes.
(c) beats. (d) resonance.

66. When two plane progressive waves travelling in same direction superpose over each other, the velocity of resultant wave will

(a) increase. (b) remain unchanged.
(c) be zero. (d) decrease.

67. The fundamental frequency in a stretched string is 100 Hz. To double the frequency, the tension in it must be changed to

(a) $T_2 = 2T_1$ (b) $T_2 = 4T_1$
(c) $T_2 = T_1$ (d) $T_2 = \dfrac{T_1}{4}$

68. The frequency of an open pipe is 300 Hz. The first overtone of this pipe is the same as the second overtone of a closed pipe. The length of the closed organ pipe is

(a) 11 cm (b) 21 cm
(c) 42 cm (d) 84 cm

69. In the equation of the motion of a particle $y = 0.5 \sin(0.3t + 0.1)$, the initial phase of motion is

(a) $(0.3t + 0.1)$ (b) 0.3
(c) $0.3t$ (d) 0.1

70. Two waves of same amplitude and same frequency reach a point in a medium simultaneously. The phase difference between them for resultant amplitude to be zero, will be

(a) 4π (b) 2π
(c) π (d) 0^0

71. Under similar conditions of temperature and pressure, the velocity of sound is maximum in

(a) CO_2 (b) H_2
(c) N_2 (d) O_2

72. The sound box of a sonometer increases

(a) speed of sound.
(b) sound intensity.
(c) wavelength of sound.
(d) sound frequency.

73. The loudness of sound depends on

(a) amplitude. (b) wavelength.
(c) pitch. (d) speed.

74. The ratio of intensities of two sound waves is 4 : 9. The ratio of their amplitudes will

(a) 9 : 4 (b) 4 : 9
(c) 2 : 3 (d) 3 : 2

75. The speed of sound in air is 320 ms^{-1}. The length of a closed pipe is 1 m. Neglecting the end correction, the resonant frequency for the pipe will be

(a) 80 Hz (b) 240 Hz
(c) 320 Hz (d) 400 Hz

76. On increasing the distance between the source of sound and observer thrice, the intensity of sound becomes

(a) 3 (b) 1/3
(c) 9 (d) 1/9

77. The velocity of sound in air at room temperature is 340 m/s and density of air is 1.2 kgm^{-1}. The value of atmospheric pressure in terms of the height of mercury column in metre will be

(a) 0.75 (b) 7.5
(c) 75 (d) 750

78. The best source of sound in order to obtain a pure note is

(a) harmonium. (b) tuning fork.
(c) flute. (d) sonometer.

79. Sound waves are propagating in a medium. The moduli of isothermal and adiabatic elasticity of the medium are E_T and E_s respectively. The velocity of sound wave is proportional to

(a) $\sqrt{E_T}$ (b) $\sqrt{E_s}$

(c) E_T (d) $\frac{E_S}{E_T}$

80. For a pulse moving in a heavy string the junctions of the string behaves as a

(a) perfectly rigid end (b) free end
(c) partially rigid end (d) rigid end

81. The intensity of a soundwave in an elastic medium falls by 10% on travelling a distance of 1 m. If the initial intensity of the wave is 100% then on travelling a distance of 3 m in that medium the intensity will become

(a) 81% (b) 70%
(c) 72.9% (d) 60%

82. A 75 cm long wire is reduced by 0.5 cm then it produces three beats per second with the fork. The frequency of the fork is

(a) 0.47 Hz (b) 4.47 Hz
(c) 47.7 Hz (d) 447.0 Hz

83. When two identical wires on a sonometer are kept under same tension their fundamental frequency is 500 Hz. In order to produce five beats per second the percentage change in the tension of one of the wires will be

(a) 2% (b) 4%
(c) 6% (d)8%

84. The minimum distance between the sound and the reflecting surface, in order to hear an echo, must be

(a) 0.65 m (b) 1.65 m
(c) 16.5 m (d) 165 m

85. The total mass of a sonometer wire remains constant. On increasing the distance between two bridges to four times, its frequency will become

(a) 0.25 times (b) 0.5 times
(c) 4 times (d) 2 times

86. The lowest pitch out of the following sources is that of a

(a) man. (b) boy.
(c) lion. (d) mosquito.

87. If the frequency of a sound wave is doubled then the velocity of sound will be

(a) zero. (b) half.
(c) double. (d) unchanged.

88. If the wavelength of a wave is decreased by 20% then its frequency will become

(a) 20% less. (b) 25% more.
(c) 20% more. (d) 25% less.

89. If the energy density and velocity of a wave are u and c respectively then the energy propagating per second per unit area will be

(a) u/c. (b) c^2u.
(c) uc. (d) c/u.

90. In which of the following is the energy loss maximum?

(a) sonometer (b) tuning fork
(c) thin tube (d) broader pipe

91. What is produced at a rigid reflecting plane for a displacement wave?

(a) beats. (b) node and antinode.
(c) antinode. (d) node.

92. If the equations of two sound waves are $y_1 = 5 \sin 252 \pi t$ and $y_2 = 5 \sin 280\pi t$ respectively, then the number of beats heard per second will be
(a) none (b) 6
(c) 12 (d) 34

93. If the density of materials of two strings of same length, tension and area of cross-section are 2 kgm^{-3} and 4 kgm^{-3} respectively then the ratio of their frequencies will be
(a) $1 : \sqrt{2}$ (b) 2 : 1
(c) 1 : 2 (d) $\sqrt{2} : 1$

94. The time taken by a particle to travel between a trough and a crest in a transverse wave is
(a) T (b) $3T/4$
(c) $T/2$ (d) $T/4$

95. The second overtone of a closed organ pipe P_1 and the third overtone of an open pipe P_2 are in unison with a tuning fork. The ratio of the length of P_1 and P_2 will be
(a) 3 : 8 (b) 1 : 3
(c) 8 : 3 (d) 3 : 1

96. Two open pipes, whose lengths are 50 cm and 51 cm respectively, produce five beat per second when sounded together. The fundamental frequencies in Hertz will be
(a) 65 and 60 (b) 90 and 95
(c) 95 and 90 (d) 255 and 250

97. The mass of a 4m long string is 0.01 kg. It is stretched by a force of 400 N. The velocity of the transverse wave propagating in the string is
(a) 100 ms^{-1} (b) 200 ms^{-1}
(c) 300 ms^{-1} (d) 400 ms^{-1}

98. The length, mass and tension of a string are 1000 cm, 0.01 kdg and 10 N respectively, the speed of transverse waves in the string will be
(a) 10^2 ms^{-1} (b) 10^4 ms^{-1}
(c) 10^6 ms^{-1} (d) 10^8 ms^{-1}

99. In Melde's experiment, the string vibrates in seven segments under tension of 9 gram weight. If string is to be vibrated in three segmets then the tension required will be
(a) 1.4 gm-wt (b) 13 gm-wt
(c) 49 gm-wt (d) 61 gm-wt

100. The length of strings of a cello is 0.8 m. In order to change the pitch in frequency ratio 5/4, their length should be decreased by
(a) 0.08 m (b) 0.2 m
(c) 0.13 m (d) 0.16 m

101. Two waves of same frequency but of amplitudes a and $2a$ respectively superimpose over each other. The intensity at a point where the phase difference is $3\pi/2$, will be
(a) a (b) $3a^2$
(c) $5a^2$ (d) $9a^1$

102. A resonance tube of diameter 2 cm is vibrating in unison with a tuning fork of frequency 512 Hz. When the length of air column above water level is 13.5 cm, the wavelength of the note will be
(a) 31.0 cm (b) 32.2 cm
(c) 54 cm (d) 62.0 cm

103. If the slope is s, wave velocity is v_w and particle velocity is v_p then
(a) $v_p = svw$ (b) $v_p = \frac{v_w}{s}$
(c) $v_p = sv_w$ (d) $v_p = \frac{-v_w}{s}$

104. A wave $y = 10 \sin (200\pi t - 0.5x)$ travels in space. The separation between two rarefactions is
(a) 10 cm (b) 2π cm
(c) 2 cm (d) 4π cm

105. The density of air at NTP is 1.293 kgm^{-3} and density of mercury at 0°C is 13.6×10^3 kgm^{-3}. If $C_p = 0.2417$ calkg^{-1}°C^{-1} and $C_V = 0.1715$, the speed of sound in air at 100°C will be (g = 9.8 Nkg^{-1})
(a) 260 ms^{-1} (b) 332 ms^{-1}
(c) 350.2 ms^{-1} (d) 388.4 ms^{-1}

106. Two organ pipes are of same size. Hydrogen and oxygen gases are filled in them. Taking the elasticity of two gases to be same, the ratio of their fundamental frequencies will be
(a) 1 : 4 (b) 4 : 1
(c) 8 : 1 (d) 16 : 1

107. A sitar wire vibrates with frequency of 330 vibrations per second. If its length is increased three times and tension is increased four times then the frequency of the wire will be
(a) 110 Hz (b) 220 Hz
(c) 330 Hz (d) 440 Hz

108. A u-tube of uniform cross-section is kept in a vertical position. A liquid of mass m and density d is filled in one of its limbs. This liquid will oscillate in this tube with time period T given by
(a) $T = 2\pi\sqrt{\frac{m}{gda}}$ (b) $T = 2\pi\sqrt{\frac{ma}{gd}}$
(c) $T = 2\pi\sqrt{\frac{m}{g}}$ (d) $T = 2\pi\sqrt{\frac{m}{2agd}}$

109. The ends of two wires of radii r and $2r$ are mutually joined. The compound wire is used in a sonometer. The joint is kept at the centre between two bridges and tension T is produced into it. If the joint is a node, then the ratio of the number of loops produced in two wires will be

(a) 1 : 4 (b) 1 : 5
(c) 3 : 1 (d) 1 : 2

110. The length of a string is 1 m, tension in it is 40 N and mass of the string is 0.1 kg. Then the velocity of transverse waves produced in the string will be

(a) 400 ms^{-1} (b) 180 ms^{-1}
(c) 80 ms^{-1} (d) 20 ms^{-1}

111. The equation $y = 0.15 \sin 5x \cos 300t$ represents a stationary wave. The wavelength of this stationary wave will be

(a) 0.628 m (b) 2.512 m
(c) 1.256 m (d) zero

112. A resonance tube of length 1 m is resonated with a tuning fork of frequency 700 Hz. If the velocity of sound in air is 330 ms^{-1} then the number of harmonics produced in the tube will be

(a) 1 (b) 2
(c) 3 (d) 4

113. A man is standing between two cliffs. If he claps his hands once, a series of echoes at the intervals of one second are heard. If the speed of sound is 340 ms^{-1} the distance between the cliffs is

(a) 170 m (b) 680 m
(c) 340 m (d) 510 m

114. A wire of mass 4 kg, length 4 m is hanged vertically. A weight of 2 kg is attached at the bottom. The ratio of velocity top to bottom is

(a) $\sqrt{3}$ (b) $\sqrt{2}$
(c) 3 (d) 2

115. A transverse wave is given by $y = a \sin 2\pi (ft - x/\lambda)$. The maximum particle velocity is four times the wave velocity, when

(a) $\lambda = 2\pi a$ (b) $\lambda = \pi a$
(c) $\lambda = \pi a/4$ (d) $\lambda = \pi a/2$

116. A student sees a jet plane flying from east to west. When the jet is seen just above his head the sound of Jet appears to reach him making angle 60° with the horizontal from east. If the velocity of sound is C, then that of the jet plane is

(a) $\frac{2}{\sqrt{3}}C$ (b) $\frac{C}{2}$
(c) $\frac{\sqrt{3}}{2}$ (d) $2C$

117. A ship sends a longitudinal wave towards the bottom of the sea. The wave returns from the bottom of the sea after 2.0 s. The bulk modulus of the sea vater is $2.2 \times 10^9 Nm^{-2}$ and the density of water is 1.1 g cm^{-3}. The depth of the sea is about

(a) 2200 m (b) 2000 m
(c) 1400 m (d) 1100 m

118. A jet aeroplane is flying at the speed of sound. When the sound of the jet appears to be coming verically downwards, the angle of sight of the aeroplane with the horizontal cannot be

(a) 60° (b) 40°
(c) 30° (d) 25°

119. A vibrating tuning fork is placed close to another of equal frequency. The other also starts vibrating. This happens because of

(a) interference.
(b) resonance.
(c) superposition.
(d) formation of stationary waves.

120. The distance between two consecutive nodes on a stretched string is 10 cm. It is in resonance with tuning fork of frequency 256 Hz. What is the velocity of the progressive wave in the string?

(a) 6. 40 ms^{-1} (b) 51.20 ms^{-1}
(c) 25.60 ms^{-1} (d) 12.50 ms^{-1}

121. An organ pipe P_1 closed at one end vibrating in its first harmonic and pipe P_2 open at both ends vibrating in its third harmonic are in resonance with the same tuning fork. The ratio of their lengths is

(a) 3/8 (b) 3/4
(c) 1/8 (d) 1/6

122. In an open organ pipe the harmonics that are missing are

(a) depends upon length of the pipe.
(b) none.
(c) even.
(d) odd.

123. The string of a sonometer is divided into two parts with the help of wedge. The total length of the string is 1 m and the two parts differ in length by 2 mm. When sounded together, they produce two beats. The frequencies of the notes emitted by the two parts are

(a) 499 and 497 (b) 501 and 503
(c) 501 and 499 (d) none of these

124. The frequency of the first overtone of an open organ pipe is f, then that of the closed organ pipe will be

(a) $3f/4$ (b) f
(c) $f/4$ (d) $f/2$

125. A tube closed at one end and containing air produces fundamental note of frequency of 256 Hz. If the tube is open at both ends, the fundamental frequency will be

(a) 512 Hz (b) 384 Hz
(c) 128 Hz (d) 64 Hz

126. A sonometer wire is to be divided in to three segments having fundamental frequencies in the ratio 1 : 2 : 3 What should be the ratio of lengths?

(a) 4 : 2 : 1 (b) 4 : 3 : 1
(c) 6 : 3 : 1 (d) 3 : 2 : 1

127. If we add 8 kg load to the hanger of a sonometer. the fundamental frequency becomes three times of its initial value. The initial load in the hanger was

(a) 4 kg-wt (b) 2 kg-wt
(c) 1 kg-wt (d) 0.5 kg-wt

128. Two identical organ pipes are producing fundamental notes of frequency 200 Hz at 15°C. If the temperature of one pipe is raised to 27°C, the number of beats produced will be

(a) 8 (b) 6
(c) 4 (d) 2

129. A rod 70 cm long is clamped from middle. The velocity of sound in the material of the rod is 3500 ms^{-1}. The frequency of fundamental note produced by it is

(a) 3500 Hz (b) 2500 Hz
(c) 1250 Hz (d) 700 Hz

130. The tension in a wire is decreased by 19%. The percentage decrease in frequency will be

(a) 10% (b) 19%
(c) 0.19% (d) none of these

131. An open organ pipe of length 50 cm vibrates in unison with a tuning fork of frequency *f*. The diameter of the pipe is 3 cm. What is the wavelength of the note produced?

(a) 204 cm (b) 102 cm
(c) 96 cm (d) 52 cm

132. Two waves of same amplitude superimpose to produce two beats per second. What is the ratio of maximum loudness to that of one of the waves?

(a) 2 (b) 4
(c) 8 (d) ∞

133. What is the phase difference between the particles of a string on the two sides of an antinode?

(a) 180° (b) 90°
(c) 45° (d) 0°

134. A progressive wave of frequency 500 Hz is travelling at a speed of 360 ms^{-1}. How far are the two points on it having a phase difference of 60°?

(a) 0.12 m (b) 0.36 m
(c) 0.24 m (d) 0.18 m

135. The standing waves set upon a string are given by

$$y = 4\sin\left(\frac{\pi x}{12}\right)\cos(52\pi t).$$

If *x* and *y* are in centimetres and *t* is in seconds, what is the amplitude of the particle at $x = 2$ cm?

(a) 12 cm (b) 4 cm
(c) 2 cm (d) 1 cm

136. A boat at anchor is rocked by a wave whose crests are 100 m apart and whose velocity is 0.25 ms^{-1}. These waves reach the boat once every

(a) 2500 s (b) 1500 s
(c) 4.00 s (d) 0.25 s

137. If the amplitude of waves at a distance 10 cm from a point source is *A*, the amplitude at a distance 40 cm will be

(a) $A/2$ (b) $A/4$
(c) A (d) $2A$

138. A wave has amplitude of velocity

$v_0 = \dfrac{1}{4\omega^2 - 11\omega + 6}$. The resonant frequency is

(a) $\omega = 2s^{-1}$ (b) $\omega = \frac{3}{4}\ s^{-1}$
(c) zero (d) $\omega = \frac{11}{8}\ s^{-1}$

139. The distance between two consecutive crests in a wave train produced in a string is 5 cm. If two complete waves pass through any point per second. The velocity of the wave is

(a) 15 cms^{-1}. (b) 10 cms^{-1}.
(c) 2.5 cms^{-1}. (d) 5 cms^{-1}.

140. A tuning fork of frequency 290 Hz appear to be vibrating with a frequency of 10 Hz when observed in a flashing illumination. The frequency of the flashing light is

(a) 10 Hz (b) 270 Hz
(c) 280 Hz (d) 290 Hz

141. For a resonance tube the air columns for the first and the second resonance differ in length by 31.5 cm. The wavelength of the sound wave is (in cm)

(a) 126.0 (b) 63.0
(b) 31.5 (d) 252.0

142. Transverse waves are being produced in a stretched string whose equation is $y = 0.021\sin(x + 30t)$. If the linear density of string is 1.3×10^{-4} kgm^{-1} then the tension produced in string will be

(a) 10 N (b) 0.5 N
(c) 0.117 N (d) 0.107 N

143. Two waves of frequencies 1 Hz and 3 Hz are moving in a medium with same velocity. The ratio of intensities at a point where amplitudes of two waves are equal will be

(a) 1 : 1 (b) 1 : 4
(c) 1 : 2 (d) 1 : 9

144. Power of 10 W is emitted by a loudspeaker. The sound intensity radiated by it at a distance of 3 m is 2 W/m^2. If the intensity of loudspeaker is doubled the intensity at 6 m will be

(a) 1 W/m^2 (b) 4 W/m^2
(c) 0.5 W/m^2 (d) 2 W/m^2

145. A wave represented by $y = 100\sin(ax + bt)$ is reflected from a dense plane at the origin. If 36% of energy is lost and rest of the energy is reflected then the equation of the reflected wave will be

(a) $y = -80\sin(ax + bt)$ (b) $y = -8.1\sin(ax + bt)$
(c) $y = -10\sin(ax + bt)$ (d) $y = -8.1\sin(ax + bt)$

146. If one of the arms of a tuning fork is broken then its frequency of vibration is

(a) not effected. (b) less than before.
(c) as before. (d) more than before.

147. If the frequency of the wave is 100 Hz then the particles of the medium cross the mean position in 1 s will be
(a) 50 times. (b) 100 times.
(c) 200 times. (d) 400 times.

148. Which of the following laws of strings is not correct?
(a) $n \propto \frac{1}{\sqrt{m}}$ (b) $n \propto l$
(c) $n \propto \sqrt{T}$ (d) $n \propto \frac{1}{l}$

149. The wavelength of a wave in a medium is 0.5 m. Due to this wave the phase difference between two particles of the medium is $\pi/5$. The minimum distance between these points is
(a) 5 km. (b) 5 m.
(c) 5 mm. (d) 5 cm.

150. In strings, the position of antinodes are obtained at
(a) $\lambda, 2\lambda, 3\lambda$ (b) $0, \lambda/2, \lambda$
(c) $2\lambda, 4, 6\lambda$ (d) $\lambda/4, 3\lambda/4, 5\lambda/4$

Answers to Questions for Practice

1.	(d)	**2.**	(d)	**3.**	(a)	**4.**	(b)	**5.**	(c)	**6.**	(b)	**7.**	(a)
8.	(c)	**9.**	(a)	**10.**	(d)	**11.**	(d)	**12.**	(a)	**13.**	(b)	**14.**	(d)
15.	(a)	**16.**	(c)	**17.**	(a)	**18.**	(d)	**19.**	(c)	**20.**	(a)	**21.**	(a)
22.	(c)	**23.**	(c)	**24.**	(d)	**25.**	(c)	**26.**	(c)	**27.**	(c)	**28.**	(a)
29.	(c)	**30.**	(c)	**31.**	(d)	**32.**	(b)	**33.**	(d)	**34.**	(b)	**35.**	(b)
36.	(b)	**37.**	(c)	**38.**	(c)	**39.**	(a)	**40.**	(a)	**41.**	(b)	**42.**	(c)
43.	(c)	**44.**	(a)	**45.**	(d)	**46.**	(a)	**47.**	(d)	**48.**	(c)	**49.**	(b)
50.	(b)	**51.**	(c)	**52.**	(c)	**53.**	(b)	**54.**	(b)	**55.**	(a)	**56.**	(a)
57.	(b)	**58.**	(d)	**59.**	(d)	**60.**	(c)	**61.**	(b)	**62.**	(b)	**63.**	(c)
64.	(b)	**65.**	(a)	**66.**	(b)	**67.**	(b)	**68.**	(c)	**69.**	(d)	**70.**	(c)
71.	(b)	**72.**	(b)	**73.**	(a)	**74.**	(c)	**75.**	(a)	**76.**	(d)	**77.**	(a)
78.	(b)	**79.**	(b)	**80.**	(a)	**81.**	(c)	**82.**	(d)	**83.**	(a)	**84.**	(c)
85.	(a)	**86.**	(c)	**87.**	(d)	**88.**	(b)	**89.**	(c)	**90.**	(d)	**91.**	(d)
92.	(a)	**93.**	(d)	**94.**	(c)	**95.**	(a)	**96.**	(d)	**97.**	(d)	**98.**	(a)
99.	(c)	**100.**	(d)	**101.**	(c)	**102.**	(c)	**103.**	(c)	**104.**	(d)	**105.**	(d)
106.	(b)	**107.**	(b)	**108.**	(c)	**109.**	(d)	**110.**	(d)	**111.**	(c)	**112.**	(b)
113.	(c)	**114.**	(a)	**115.**	(d)	**116.**	(b)	**117.**	(c)	**118.**	(a)	**119.**	(b)
120.	(b)	**121.**	(d)	**122.**	(b)	**123.**	(c)	**124.**	(a)	**125.**	(a)	**126.**	(c)
127.	(c)	**128.**	(c)	**129.**	(b)	**130.**	(a)	**131.**	(b)	**132.**	(b)	**133.**	(a)
134.	(a)	**135.**	(c)	**136.**	(c)	**137.**	(b)	**138.**	(d)	**139.**	(b)	**140.**	(c)
141.	(b)	**142.**	(c)	**143.**	(d)	**144.**	(a)	**145.**	(a)	**146.**	(a)	**147.**	(c)
148.	(b)	**149.**	(d)	**150.**	(d)								

Explanations

27. (c) $\frac{\sin i}{\sin r} = \frac{\text{velocity in medium 2}}{\text{velocity in medium 1}} = \frac{v}{2v}$

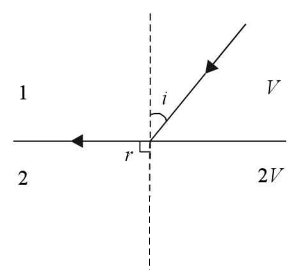


Fig. 13.19

$$\frac{\sin i}{\sin 90^\circ} = \frac{1}{2} \text{ or } i = 30^\circ.$$

48. (c) $\frac{v}{98} - \frac{v}{100} = 8$ or $v = 4 \times 98$ m/s $f_2 = 400$ Hz, $f_1 = 392$ Hz

$\therefore$ Using fork which produces 4 beats with these wires has frequency 396 Hz.

53. (b) $\frac{5\lambda}{2} = 82.5$ cm

$\lambda = 33$ cm and $v = f\lambda = 330$ ms^{-1}.

54. (b) $\frac{\lambda}{4} = 32 + 0.3d$

$$\frac{3\lambda}{4} = 100 + 0.3d$$

$$\frac{\lambda}{2} = 68 \text{ cm}$$

$$\frac{\lambda}{4} = 34 \text{ cm or } 0.3d = 2 \text{ cm.}$$

109. (d) $\left.\frac{dy}{dt}\right|_{max} = A\pi f$ and wave velocity $= f\lambda$;

$A\pi f = 4f\lambda$ or $\lambda = \frac{A\pi}{4}$

116. (b) $v = c\cos 60° = \frac{c}{2}$

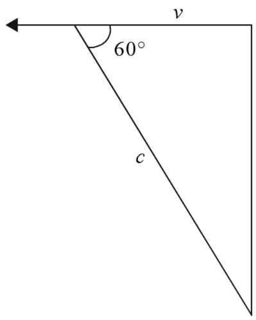


Fig. 13.20

117. (c) $v = \sqrt{\frac{\beta}{\rho}} = \sqrt{\frac{2.2\times10^9}{1.1\times10^3}} = 10^3\sqrt{2}\ \text{ms}^{-1}$

$x = \frac{vt}{2} = \frac{10^3\sqrt{2}\times2}{2} = 1400$ m.

123. (c) $\frac{v}{49.9} - \frac{v}{5.1} = 2$ or $.2v = 2\times50\times49.9$

or $v = 50.1\times499$

$f_1 = \frac{50.1\times499}{50.1} = 499$ Hz $f_2 = 501$ Hz

128. (c) $\Delta f = \frac{\Delta v}{\lambda}$ $\Delta f = 0.6\times12 = 7.2\ \text{ms}^{-1}$

$\lambda = \frac{340}{200} = 1.7 \therefore \Delta f = 4$ (nearly.)

130. (a) $f \propto \sqrt{2}\ f_{new} = \sqrt{0.81T} = 0.9\sqrt{T}$, that is,

or decrease of 10%.

144. (a) $\frac{k\frac{10}{3^2}}{k\frac{20}{6^2}} = \frac{2}{x}$

or $x = 1\ \text{Wm}^{-2}$.0000

14

Sound Waves

BRIEF REVIEW

Pressure waves given by $p = p_0 \sin(\omega t - kx)$ are termed as sound waves. Sound waves are longitudinal in nature and consist of alternate compressions and rarefactions. To human ear only the frequency range 20-20000 Hz is audible. These limits are subjective and may vary slightly from person to person

If $y = y_0 \sin(\omega t - kx)$ is displacement wave, then change in volume

$$dV = Ady = \frac{Ady}{dx}\, dx = A\, y_0(-k)\cos(\omega t - kx)\, dx.$$

Volumetric strain

$$\frac{dV}{V} = \frac{Ay_0\left(-\frac{\omega}{v}\right)\cos(\omega t - kx)dx}{Adx} \qquad \left\{ \because \frac{\omega}{k} = v \quad \therefore k = \frac{\omega}{v} \right.$$

$$p = -\frac{B\partial V}{V} = \frac{B.y_0\omega}{v}\cos(\omega t - kx)$$

where B is bulk modulus.

Also note that there exists a phase shift of 90° between displacement and pressure wave

General formula $v = \sqrt{\frac{E}{\rho}}$ where E is elastic constant

Speed of the Sound Wave

Newton's formula $v = \sqrt{\frac{P}{\rho}}$

Newton considered the change to be isothermal.

Laplace's correction Laplace considered adiabatic change and derived

$$v = \sqrt{\frac{\gamma P}{\rho}} \text{ (It gives correct results) where}$$

$$\gamma = \frac{C_P}{C_V}$$

$$v = \sqrt{\frac{\gamma RT}{M}} \qquad v = \sqrt{\frac{B}{\rho}}$$

and B is bulk modulus where M is molecular mass of the gas.

In solids we may write

$$v = \sqrt{\frac{Y}{\rho}} \text{ were } Y \text{ is Young's modulus}$$

For transverse waves in solids (bulk material)

$$v = \sqrt{\frac{B + \eta/3}{\rho}} \text{ where } \eta \text{ is shear modulus.}$$

Effect of Temperature

$$v = \sqrt{T} \text{ where } T \text{ is temperature in Kelvin.}$$

$$\frac{v}{v_0} = \sqrt{\frac{T}{273}} = \sqrt{1 + \frac{t}{273}}$$

where t is temperature in celsius (°C)

when temperature rises by 1°C velocity of sound increases by 0.61 ms^{-1}.

Intensity $I = 2\pi^2 \rho y_0^2 f^2 v$

$$I = \frac{2\pi^2 B y_0^2 f^2}{v} = \frac{P_0^2 v}{2B} = \frac{P_0^2}{2\rho v}$$

Intensity $I \propto \frac{1}{r^2}$ (for an isotropic source)

$I \propto \frac{1}{r}$ (for cylindrical source)

where r is the distance between the source and observer.

Effect of Pressure Velocity of sound is not affected by pressure.

Effect of density $v \propto \frac{1}{\sqrt{\rho}}$ where ρ is density.

With increase in humidity the density of air decreases and, hence, speed of sound increases.

Appearance of sound in human air is characterised by three parameters — pitch, loudness and quality.

Pitch is related to frequency. Higher the pitch sweeter is the sound. Children and Ladies speak at higher pitch as compared to men, therefore, their sound appears sweeter. Higher the frequency higher is the pitch.

Loudness is correlated with sound level. Human ear can hear a minimum intensity

$I_0 = 10^{-12}\ \text{W/m}^2$

whispering $10 \rightarrow$ dB

normal talk $\rightarrow$ 60dB

sound level in dB $SL = 10 \log_{10}\left(\frac{I}{I_0}\right)$

Even at 80 dB (heard continuously for sometime) headache begins. At 130 dB person may become temporarily insane.

Quality No source of sound generates a single frequency. For example, even a tuning fork marked 288 Hz will not produce only fundamental frequency of 288 Hz but also produces along with it, integral multiple of frequencies like $2 \times 288 = 576$ Hz, $3 \times 288 = 864$ Hz and so on and so forth. The difference in sound of a **tabla** and **mridung** being played at same frequencies is due to difference in number of harmonics produced and their amplitudes.

Remember that speech ends upto 3 kHz. Rest frequency range upto 20 kHz are only higher harmonics and are used in music. The higher harmonics are particularly pleasant to the ear. A noise has frequencies that do not bear any well-defined relationship among themselves.

Interference of sound waves If $P_1 = P_{01}\sin(\omega t - kx)$ and $P_2 = P_{02}\sin(\omega t - kx + \delta)$ interfere we assume the sources are coherent (say two tuning forks of same frequency) then

$$P = P_1 + P_2 = P_0 \sin(\omega t - kx + \phi)$$

$$P_0 = \sqrt{p_{01}^2 + p_{02}^2 + 2p_{01}p_{02}\cos\delta} \quad \text{and}$$

$$\tan\phi = \frac{p_{02}\sin\delta}{p_{01} + p_{02}\cos\delta}$$

If phase difference $\delta = k\Delta x = 0$ or $2n\pi$ then, intensity will be maximum and constructive interference results. Path difference in such cases is an intergral multiple of wavelength.

If phase difference $\delta = k\Delta x = (2n + 1)\pi$ then, intensity will be minimum and destructive interference results. In such cases path difference $\Delta x = (2n + 1)\frac{\lambda}{2}$

$$\frac{I_{max}}{I_{min}} = \frac{(p_{01} + p_{02})^2}{(p_{01} - p_{02})^2} = \frac{(y_{01} + y_{02})^2}{(y_{01} - y_{02})^2}$$

Quinke's tube is used to demonstrate interference of sound.

If path difference is Δx then phase difference

$$\delta = k\Delta x = \frac{2\pi\Delta x}{\lambda}.$$

Reflection of sound wave can cause: (a) echo (b) longitudinal standing waves.

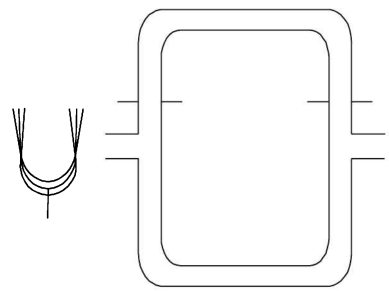


Fig. 14.1 Interference in Quinke tube

Echo is produced when the reflected wave (sound) is heard again by the producer or by others also.

Echo is produced when a minimum distance between the source and the reflector is 16.6 m as demonstrated in the Fig. 14.2. Distance covered by sound to reach the producer is

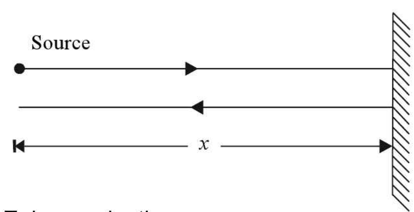


Fig. 14.2 Echo production

$$2x = vt$$

$$x = \frac{vt}{2} = \frac{332}{2} \times \frac{1}{10} = 16.6\ \text{m} \simeq 55\ \text{ft}.$$

We take $t = \frac{1}{10}$ s because this is the minimum time between two syllables being heard clearly.

Echo can be heard in a smaller room provided it is empty and windows and doors are closed.

To find the distance in echo production use $x = \frac{vt}{2}$.

Standing Waves

(a) Standing waves in closed pipes are with reference to diplacement waves. For pressure waves position of nodes and antinodes will interchange. Same is true for Fig. 14.4, that is, open pipes.

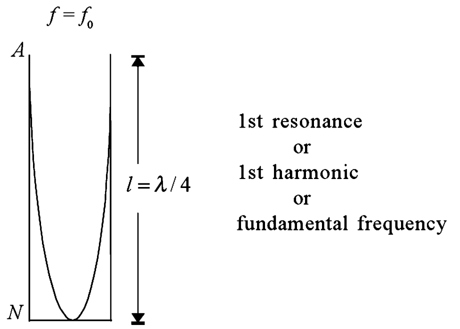


Fig. 14.3 (a)

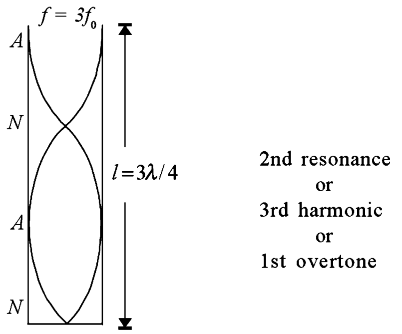


Fig. 14.3 (b)

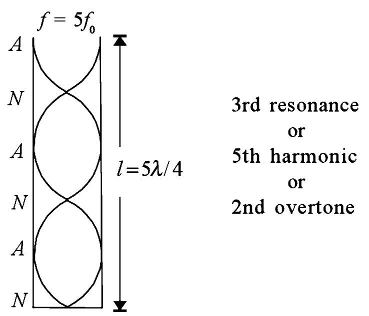


Fig. 14.3 (c)

In closed pipes resonance occurs at

$$l = \frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}, \ldots, \text{that is, at odd multiple of } \lambda/4.$$

Only odd integral multiples of fundamental frequencies f_0, $3f_0$, $5f_0$ and so on are allowed.

Note that at the open end an antinode occurs and at closed end a node occurs.

(b) Open pipes

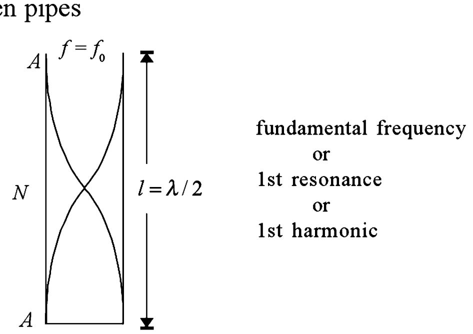


Fig. 14.4 (a)

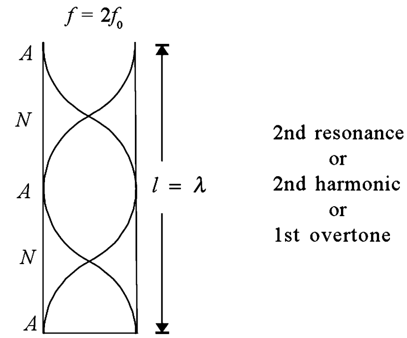


Fig. 14.4 (b)

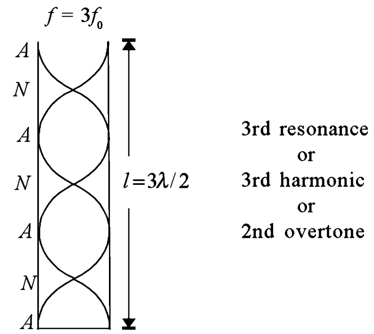


Fig. 14.4 (c)

In open pipes resonance occurs at

$$l = \frac{\lambda}{2}, \lambda, \frac{3\lambda}{2}, \ldots, \text{that is, all integral multiples of } \lambda/2.$$

All integral multiple of fundamental frequencies f_0, $2f_0$, $3f_0$ .... are allowed or all harmonics are allowed

End correction $l_1 + 0.3\,d = \lambda/4$ for first resonance, where, d is diameter of the pipe for second resonance.

$$l_2 + 0.3\,d = \frac{3\lambda}{4}$$

$$(l_2 - l_1) = \frac{\lambda}{2}$$

$$v = 2(l_2 - l_1)f\ (= f\lambda).$$

In Kundt's Tube heaps of lycopodium powder/sand are collected at nodes

$\therefore$ separation between two heaps is equal to $\frac{\lambda}{2}$.

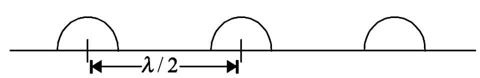


Fig. 14.5 Kundt's tube method

Beats Periodic increase and fall in the intensity of sound is called beats. Beats are produced when two sources of sound of nearly same frequency are sounded together. Beat frequency

$n = |f_2 - f_1|$

$n \leq 10 \text{ Hz}$

Beats can also be produced by superposition of tones. We illustrate it by an example. Assume two sources of sound of frequencies 200 Hz and 404 Hz are sounded together [as $f_2 - f_1 >> 10$ no beat should have been heard]. Then 4 beats/s are heard. It is because of the fact that

404 – 2(200) = 4 beats/s are produced.

That is, fundamental frequency of 404 Hz superposes with 2nd harmonic of 200 Hz wave to produce 4 beats/s.

Note: Beat is interference in the time regime while generally known interference is superposition in distance or space regime.

Refraction of Sound

As solids are most elastic and gases are least elastic

or $E_{\text{solid}} > E_{\text{liquid}} > E_{\text{gas}}$

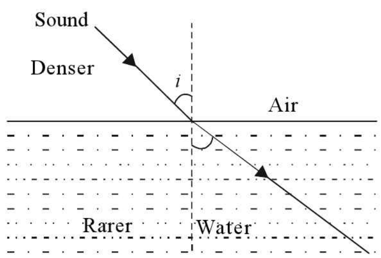


Fig. 14.6 (a) Refraction of sound

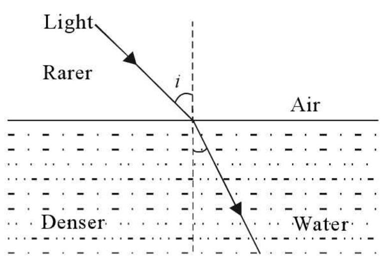


Fig. 14.6 (b) Refraction of light

Therefore, velocities are maximum in solids.

$v_{\text{solid}} > v_{\text{liquid}} > v_{\text{gas}}$.

There could be few exceptions. For example, in vulcanized rubber velocity of sound is less than that of gases. In alcohol also velocity of sound is less than that of gases. Velocity of sound is virtually independent of frequency variations.

For light, water is denser as the speed of light decreases. In case of sound water is rarer as speed of sound increases as illustrated in Fig. 14.6.

However, $\dfrac{\sin i}{\sin r} = \dfrac{v_1}{v_2}$ is valid even for sound.

Diffraction Bending of waves from an obstacle or an opening is called diffraction. Diffraction is a characteristic property of wave motion. All types of waves are diffracted. Wave nature of even electrons was demonstrated by Davisson and Germer by diffraction of electrons. Diffraction of sound is more pronounced as the wavelength is large. Therefore, it can be diffracted from any material object.

Doppler Effect When there is relative motion between the source and the listener the apparent frequency changes. This change in apparent frequency because of relative motion is called Doppler effect.

Fig. 14.7 Doppler effect

Let v be the velocity of sound, v_S velocity of the source, v_L velocity of the listener then

$$f_{\text{app}} = \frac{v - v_L}{v - v_S} f$$

where f_{app} is the apparent frequency heard by the listener and frequency f is the frequency of the source.

The above formula is written keeping in view the positive and negative sign to be assigned for v_S and v_L as shown in Fig. 14.7.

If any of the two is at rest that particular velocity becomes zero in the above formula. Thus, this formula may be applied to all cases.

If the source or listener moves with a velocity greater than velocity of sound then Doppler effect cannot be applied.

When the source of sound goes past the observer (stationary) the change in frequency is

$$\Delta f = \frac{2 v v_S f}{v^2 - v_S^2}$$

If the observer goes past a stationary source then change in frequency

$$\Delta f = \frac{2 v_L}{v_S} f.$$

Doppler effect in light is $\dfrac{\Delta\lambda}{\lambda} = \dfrac{\Delta f}{f} = \dfrac{v}{c}$

Reverberation time $T = \dfrac{0.17V}{A}$

where V is total volume and $A = \sum a_i s_i$ where a_i is absorption coefficient for surface area s_i.

• Short Cuts and Points to Note

1. $\dfrac{v_{\text{sound}}}{v_{\text{rms}}(\text{of a gas})} = \sqrt{\dfrac{\gamma}{3}}$ where $\gamma = \dfrac{C_p}{C_v}$.

2. Speed of sound

$$v = \sqrt{\frac{\gamma P}{\rho}} = \sqrt{\frac{\gamma RT}{M}}$$

where M is molecular mass of the gas and ρ is density of the gas.

In solids $v = \sqrt{\frac{Y}{\rho}}$ if rod or string or long rail where Y is Young's modulus.

$v = \sqrt{\frac{B}{\rho}}$ in bulk of material. Where B is bulk modulus.

$v = \sqrt{\frac{B + \eta/3}{\rho}}$ in bulk of material for transverse mechanical waves where η is shear modulus.

3. $v = \frac{\omega}{k} = f\lambda$.

4. There is a phase shift of 90° between pressure and displacement wave.

5. $p = p_0 \sin(\omega t - kx)$ is the pressure wave or sound wave.

$p_0 = \frac{By_0\omega}{v}$ where, B is bulk modulus. y_0 is amplitude of displacement wave.

6. Effect of temperature $v \propto \sqrt{T}$

or $\frac{v}{v_0} = \sqrt{\frac{T\,(\text{K})}{273}} = \sqrt{1 + \frac{t^{\circ}\text{C}}{273}}$

$v_0 = 330\ \text{ms}^{-1}$ at 0°C

7. Velocity of sound in a medium is independent of wavelength or frequency. Frequency of a tuning fork in falling plate method is $f = m\sqrt{\frac{g}{d_2 - d_1}}$

where m is complete number of waves used and d_2 and d_1 are consecutive distance for m number of waves. In stroboscopic method $f = mp$ where m is number of holes on the plate and p is angular frequency in revolution per second.

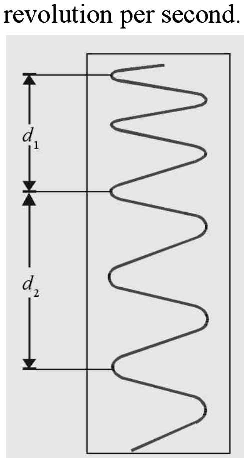


Fig. 14.8 Velocity of sound by falling plate method

8. Velocity of sound is independent of pressure. But it varies with density $v \propto \frac{1}{\sqrt{\rho}}$. Velocity of sound is maximum in rainy season.

9. Intensity of sound $I = 2\pi^2 \rho y_0^2 f^2 v$

$$I = 2\pi^2 \rho y_0^2 f^2 v = \frac{2\pi^2 B y_0^2 f^2}{v} = \frac{p_0^2 v}{2B} = \frac{p_0^2}{2\rho v}.$$

10. Pitch is related to frequency. Higher the pitch, higher is the frequency. Children and ladies speak at higher pitch compared to men. Higher frequency or higher pitch sound is more sweet.

11. Loudness is correlated with sound level. Minimum intensity that is audible to human ear is $10^{-12}\ \text{Wm}^{-2}$

Sound level in dB $SL = 10 \log_{10}\left(\frac{I}{I_0}\right)$.

$$10 \log \frac{I_2}{I_0} = 10 \log \frac{I_1}{I_0} - 10 \log \frac{I_1}{I_2}$$

80 dB sound level can cause headache if heard continuously for some time.

Sound level ≥ 130 dB may make a person temporarily insane. Maximum tolerable sound is 120 dB. Normal talking level is 60 dB.

12. Quality of sound is related to number of harmonics produced and their amplitude by a source. It is due to quality of sound that we can recognize a person by his/her voice. Even an instrument being played can be judged.

13. Interference of sound in time frame (regime) produces beats, that is, if two sources having frequencies nearly equal superpose then periodic increase and fall in the intensity of sound is heard. This is called beats.

Beat frequency $n = |f_1 - f_2| \le 10$ if they are to be heard.

If a tuning fork is vaned or filed its frequency slightly increases and if a tuning fork is loaded or waxed its frequency slightly decreases. More than 10 beats/s cannot be heard.

Tuning fork frequency $f = \frac{1}{2l}\sqrt{\frac{T}{\mu}}$.

14. Superposition of wave in space (x) is termed as interference and gives intensity of sound varying with distance. Sound intensity is maximum if phase shift is an integral multiple of 2π or path difference is $n\lambda$. Intensity is minimum when phase difference is $(2n + 1)\pi$ or path difference is $(2n + 1)\frac{\lambda}{2}$.

$$\frac{I_{max}}{I_{min}} = \left(\frac{p_{01} + p_{02}}{p_{01} - p_{02}}\right)^2 = \left(\frac{y_{01} + y_{02}}{y_{01} - y_{02}}\right)^2$$

15. Quinke's tube is used to study interference of sound.

16. Reflection of sound from a general obstacle may result in an echo. For echo to be produced separation between source and obstacle should be 16.6 m or 55 ft. Though echo can be produced because of multiple reflection in a closed and empty room. Felts, cushion and curtains and so on are absorbers of sound. A window (or opening) is the best absorber of sound. Human beings also absorb sound

17. Reflection of sound wave in organ pipes produce standing waves. A flute may be used both as closed and an open pipe. If all the holes are closed it acts like closed pipe. By closing different holes, we can vary the length of the pipe and hence frequency varies. In closed pipes resonance occurs at

$l = \frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}, \ldots$ or $l = (2n-1)\lambda/4$

where $n = 1, 2, 3, \ldots$

Only odd integral multiple of frequencies are allowed in closed pipes, i.e., $f_0, 3f_0, 5f_0, \ldots$ are allowed

18. In open pipes all integral multiple of fundamental frequency are allowed, i.e., $f_0, 2f_0, 3f_0, \ldots$ are allowed. Resonance occurs when

$l = \frac{\lambda}{2}, \lambda, \frac{3\lambda}{2}, \ldots$, that is, $l = \frac{n\lambda}{2}$ $(n = 1, 2, 3, \ldots)$

19. End correction is required in resonance tube experiment.

$l_1 + 0.3\,d = \frac{\lambda}{4}$ for first resonance.

and $l_2 + 0.3\,d = \frac{3\lambda}{4}$ for second resonance.

$v = 2f(l_2 - l_1)$.

20. Separation between two consecutive nodes or antinodes is $\frac{\lambda}{2}$ and separation between a node and an antinode is $\frac{\lambda}{4}$.

21. Refraction of sound occurs when sound wave travels from one medium to another. Normally velocity of sound follows the trend

$v_{solid} > v_{liquid} > v_{gas}$.

For light, glass or water is denser than air. But for sound, glass or water is rarer than air as velocity of sound is more in these materials.

$\mu = \frac{\sin i}{\sin r} = \frac{v_1}{v_2}$, that is, Snell's law is valid.

22. Diffraction of sound is more pronounced than light because wavelength of sound is large. The diffraction occurs from any obstacle or a hole. Diffraction is a specific characteristic of wave.

23. Doppler effect is the apparent change in frequency of sound appearing to the listener because of motion between source and listener.

$f_{app} = \left(\frac{v - v_L}{v - v_S}\right) f$ can be applied

If wind of velocity v_w blows in the direction of sound then change v to $v + v_w$ or $v - v_w$ depending upon wind is blowing in same or opposite direction. If wind is in the direction of sound then.

$$f_{app} = \frac{(v + v_w - v_L) f}{(v + v_w - v_s)}.$$

24. If the source or listener move with a speed greater than the speed of sound then Doppler effect cannot be applied.

25. Music is formed only with vowels. Octave (1 : 2) majortone (8 : 9), minortone (9 : 10) and semitone (15 : 16).

26. Mach number = $\frac{\text{Velocity of a body}}{\text{Velocity of sound}} = \frac{v_{body}}{330}$

27. Number of nodes = $2n$ and number of antinodes = $2n + 1$ when a tuning fork vibrates in nth harmonic.

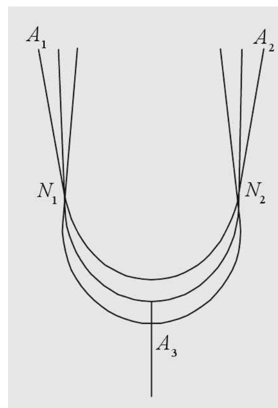


Fig. 14.9

- **Caution**

1. Not applying end correction in resonant pipes.

⇒ Apply an end correction equal to 0.3 d where d is diameter of the pipe using

$\lambda/4 = l_1 + 0.3\,d$ and $\frac{3\lambda}{4} = l_2 + 0.3\,d$ use $\frac{\lambda}{2} = l_2 - l_1$.

2. Confusing formulae for open and closed pipes.

⇒ In closed pipes only odd integral multiple of fundamental frequency are allowed.

length of the pipe $l = \frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}, \ldots$

for 1st, 2nd, 3rd, harmonic.

In open pipes all harmonics are allowed and length of the pipe is

$l = \frac{\lambda}{2}, \lambda, \frac{3\lambda}{2}, \ldots$

3. Considering that a vibrating source always produces sound.

⇒ Sound frequency lies between 20 and 20000 Hz. Frequencies less than 20 Hz are called infrasonic and are not audible to human ear. Frequencies > 20 kHz are inaudible and termed as ultrasonic. Bodies/waves having velocity > 330 ms^{-1} (velocity of second) are termed as supersonic.

4. Considering intensity of sound and loudness as identical terms.

⇒ Loudness is related to level of sound. It is measured in dB.

Sound level $SL = 10 \log_e \frac{I}{I_0}$

where $I_0 = 10^{-12}$ Wm^{-2} is the minimum intensity audible to human ear. Pressure variation upto 10^{-10} Nm^{-2} can be detected.

5. Considering that a source/musical instrument of same frequency will have same number of harmonics.

⇒ Number of harmonics and their amplitudes are different and form quality of sound.

6. Assuming that frequency, wavelength and velocity all change when a wave passes from one medium to another.

⇒ Frequency does not vary. Also note that unlike light waves, sound waves with different wavelengths pass through a medium with same velocity.

7. Considering that doppler effect can always be applied if there is a relative motion between source and listener.

⇒ You cannot apply Doppler effect if the velocity of source/listener is larger than speed of sound.

8. Comfusing between wave number and velocity amplitude and acceleration amplitude.

⇒ Wave number or propagation constant

$k = \frac{2\pi}{\lambda}$.

Velocity amplitude $v_0 = \frac{2\pi y_0}{T} = \omega y_0$

Acceleration amplitude

$a_0 = \frac{2\pi^2 y_0}{T^2} = \frac{\omega^2 y_0}{2}$ where $T \rightarrow$ time period

9. Considering that there is no exception in the rule $v_{solid} > v_{liquid} > v_{gas}$.

⇒ In vulcanised rubber the velocity of sound < velocity of sound in hydrogen. In alcohol also velocity of sound < velocity of sound in hydrogen.

10. Not remembering value of γ for monoatomic, diatomic or polyatomic gases

⇒ Values of $\gamma = \frac{5}{3}$ for monoatomic, $\gamma = 1.4$ for diatomic

and $\gamma = \frac{4}{3}$ for polyatomic gases.

Use $v = \sqrt{\frac{\gamma P}{\rho}} = \sqrt{\frac{\gamma RT}{M}}$

11. Confusing how to use the formula

$v = \sqrt{\frac{\gamma RT}{M}} = \sqrt{\frac{\gamma P}{\rho}}$ in a mixture of gases.

⇒ Use $\gamma_{av} = \frac{n_1\gamma_1 + n_2\gamma_2}{n_1 + n_2}$;

$M_{av} = \frac{n_1 M_1 + n_2 M_2}{n_1 + n_2}$ where n_1 and n_2 are number of moles of gas 1 and gas 2 respectively.

12. Considering that mediums which are denser for light are also denser for sound waves from refractive index point of view.

⇒ Velocity of sound waves $v_{solid} > v_{liquid} > v_{gas}$. Therefore, gases appear to have higher refractive index than liquids or solids.

However, $\frac{v_1}{v_2} = \frac{\sin i}{\sin r} = \mu$ is valid.

SOLVED PROBLEMS

1. Sound signal is sent through a composite tube as shown in Fig. 14.10. The radius of the semicircle is r. Speed of sound in air is v. The source of sound is capable to generate frequencies in the range f_1 to f_2 ($f_2 > f_1$). If n is an integer then frequency for maximum intensity is given by

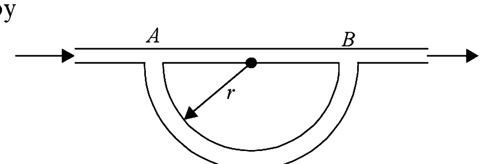


Fig. 14.10

(a) $\frac{nv}{r}$ (b) $\frac{nv}{r(\pi - 2)}$

(c) $\frac{nv}{\pi r}$ (d) $\frac{nv}{(r-2)\pi}$

Solution (b) Path difference $\pi r - 2r = n\lambda$

or $r(\pi - 2) = \frac{nv}{f}$ thus $f = \frac{nv}{r(\pi - 2)}$.

2. Two tuning forks when sounded together produce 6 beats/s. The first fork has the frequency 3% higher than a standard one and the second has the frequency 2% less than the standard fork. The frequencies for the forks are

(a) 126.3, 120.3 Hz (b) 162.7, 156.7 Hz
(c) 136.2, 130.2 Hz (d) 123.6, 117.6 Hz

Solution (d) $\left(f+\frac{3f}{100}\right)-\left(f-\frac{2f}{100}\right)=6$

$\Rightarrow \quad 5f = 600$ or $f = 120$ Hz

$f+\frac{3f}{100} = 123.6$ Hz,

$f-\frac{2f}{100} = 120-\frac{2\times 120}{100} = 117.6$ Hz.

3. The dimensions of an auditorium is $100 \times 40 \times 10$ m^3. It has 1000 m^2 curtains of absorption coefficient 0.2 m^{-2} 2000 m^2 of carpets of absorption coefficient 0.7 m^{-2}. If 1000 men of absorption coefficient 0.9 per person are sitting in the hall, then reverberation time is

(a) 2.7 s (b) 7.2 s
(c) 3.5 s (d) 3.7 s

Solution (a) $T = \frac{0.17V}{A} = \frac{0.17V}{\sum a_i s_i}$

$= \frac{0.17(100\times 40\times 10)}{0.2(1000)+0.7(2000)+0.9(1000)}$

$= \frac{0.17\times 4000\times 10}{2500} = 2.72$ s

4. A and B are two wave trains shown in the Fig. 14.11, the ratio of intensity of A to B is

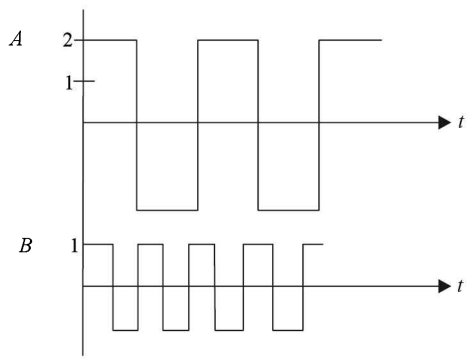


Fig. 14.11

(a) 1 (b) 2
(c) 4 (d) 8

Solution (a) $I \propto x_0^2$ and $I \propto f^2$

In A $x_{0A} = 2x_0$ and $f_{0A} = f_0$

In B $x_{0B} = x_0$ and $f_{0B} = 2f_0$ $\therefore \frac{I_A}{I_B} = 1$.

5. The fundamental frequency of a closed organ pipe is same as the first overtone frequency of the open pipe. The length of open pipe is 50 cm. The length of closed pipe is

(a) 25 cm (b) 100 cm
(c) 200 cm (d) 125 cm

Solution (d) $f_{0(\text{open})} = \frac{330}{2l} = 330$ Hz

$f_1 = 2\times 330 = 660$ Hz

$f_{0(\text{closed})} = \frac{330}{4l} = 660$

or $\quad 8l = 1$ m, that is, $l = 12.5$ cm.

6. Sound waves from a tuning fork F reach a point P by two separate routes FAP and FBP. FBP is 12 cm larger than FAP. There is silence at P. If the separation becomes 24 cm, the sound becomes maximum at P and at 36 cm there is again silence and so on. The least frequency of tuning fork is

(a) 1357 Hz (b) 1735 Hz
(c) 1375 Hz (d) 1400 Hz

Solution (c) $\frac{\lambda}{2} = 12$ cm or $\lambda = 24$ cm

$f = \frac{330}{0.24} = 1375$ Hz.

7. A sound source emits sound waves in a uniform medium. If energy density is E and maximum speed of the particles of the medium is v_{max}, the plot between E and v_{max} is best represented by

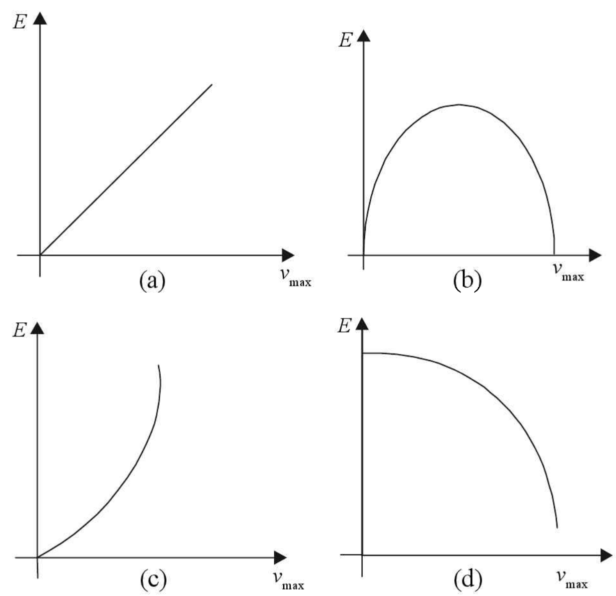


Fig. 14.12

Solution (c) Energy density $= \frac{I}{v} = 2\pi^2 \rho f^2 x_0^2$

$v_{max} = \omega x_0 = 2\pi f x_0$

that is, $E \propto v^2_{max}$.

8. A sound source rotates anticlock wise with an angular velocity ω. Radius of the circle is R. A person is at P. The maximum frequency is heard when position of the source is at

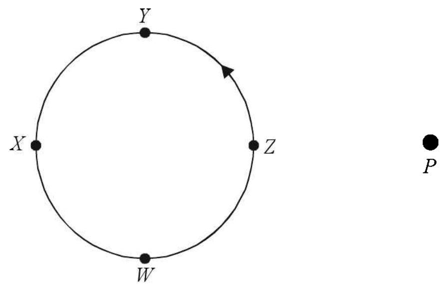


Fig. 14.13 (a)

(a) Y (b) X
(c) Z (d) W

Solution (d) Note from Fig. 14.13(b) that velocity at W is towards the listener. Hence according to Doppler's effect maximum frequency is heard at W.

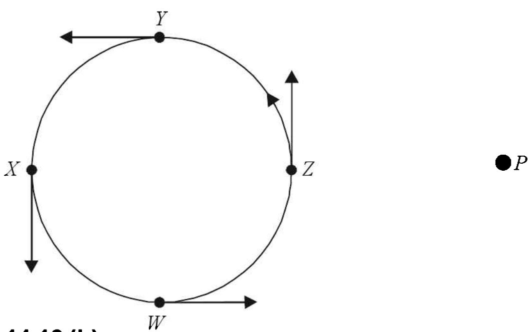


Fig. 14.13 (b)

9. The intensity level 1 m away from a source is 60 dB. Threshold intensity of hearing is 10^{-12}Wm^{-2}. If there is no loss of sound power in air then intensity level at 2000 cm from the source is

(a) 45 dB (b) 34 dB
(c) 35 dB (d) 64 dB

Solution (b) $I \propto \frac{1}{r^2}$ and $\Delta I = I_1 - I_2$

$$10 \log \frac{I_2}{I_0} = 10 \log \frac{I_1}{I_0} - 10 \log \frac{I_1}{I_2}$$

and $10 \log \frac{I_1}{I_2} = 10 \log 400 = 26.02$ dB

$\therefore$ intensity level at 2000 cm away is

$60 - 26 = 34$ dB

10. Three tuning forks of frequency 400 Hz, 401 Hz and 402 Hz are sounded simultaneously. The number of beats heard per second are

(a) 1 (b) 2
(c) 3 (d) none of these

[IIT 1992]

Solution (a) See from Fig. 14.14 that 2 Hz and 1 Hz are sounded together giving 1 beats/s.

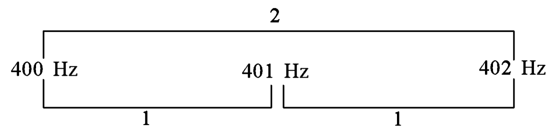


Fig. 14.14

11. Two open pipes of length 50 cm and 51 cm produce 6 beats when sounded together, find the speed of sound.

(a) 330 ms^{-1} (b) 316 ms^{-1}
(c) 306 ms^{-1} (d) 360 ms^{-1}

Solution (c) $f_1 - f_2 = 6$ or $\frac{v}{2l_1} - \frac{v}{2l_2} = 6$

$$\frac{v}{2(0.5)} - \frac{v}{2(0.51)} = 6 \text{ or } v = 306 \text{ ms}^{-1}.$$

12. If fundamental frequency of an open pipe is f_0, its fundamental frequency when it is half-filled with water is

(a) f_0 (b) $\frac{\lambda}{4}$
(c) $2f_0$ (d) none of these

[CBSE 1998]

Solution (a) See the situation shown in the Fig. 14.15(b). When the pipe is half-filled with water it becomes a closed pipe and the length.

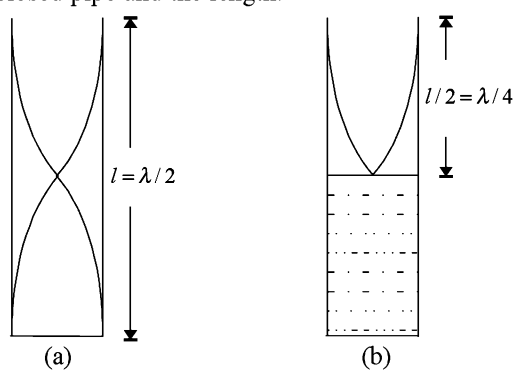


Fig. 14.15

$$\frac{l}{2} = \frac{\lambda}{4} \text{ or } \lambda = 2l$$

same wavelength existed in open pipe. Therefore, frequency remains unchanged as $f = \frac{v}{\lambda}$.

13. In the experiment for determination of the speed of sound in air using resonance tube method. The length of air column that resonates with fundamental mode with a tuning fork is 0.1 m. When its length is changed to 0.35 m it resonates in first over tone. The end correction is

(a) 0.012 m (b) 0.025 m
(c) 0.05 m (d) 0.0024 m

Solution (b) $l_1 + 0.3\, d = \frac{\lambda}{4}$, $l_2 + 0.3\, d = \frac{3\lambda}{4}$;

$$\frac{\lambda}{2} = l_2 - l_1 = 0.25 \text{ m or } \frac{\lambda}{4} = 0.125 \text{ m}$$

$$0.3\, d = \frac{\lambda}{4} - l_1 = 0.025 \text{ m}$$

14. An observer moves towards a stationary source of sound with one-fifth of the speed of sound. The wavelength and frequency of the source emitted are λ and f respectively. The apparent frequency and wavelength recorded by the observer are

(a) $0.85f, 0.8\lambda$ (b) $1.2f, 1.2\lambda$
(c) $1.2f, \lambda$ (d) $f, 1.2\lambda$

[CBSE 2003]

Solution (c) $f_{app} = \dfrac{v+v/5}{v} f = 1.2f$ wavelength remains unchanged.

15. An air column closed at one end and open at the other end resonates with a tuning fork when 45 and 99 cm of length. The wavelength of the sound in air column is

(a) 36 cm (b) 54 cm
(c) 108 cm (d) 180 cm

Solution (c) $\dfrac{\lambda}{2} = 99 - 45 = 54$ cm

or $\lambda = 108$ cm

[DPMT 2002]

16. The frequency of a tuning fork is 384 Hz and velocity of sound in air is 352 ms^{-1}. How far sound has travelled when fork completes 36 vibration?

(a) 33 m (b) 16.5 m
(c) 11 m (d) 22 m

[DPMT 2002]

Solution (a) $x = v.t = 352 \times \dfrac{36}{384} = 33$ m.

17. A sound source is falling under gravity. At some time $t = 0$ the detector lies vertically below source at a height H as shown in Fig. 14.16. If v is velocity of sound and f_0 is frequency of the source then the apparent frequency recorded after $t = 2$ second is

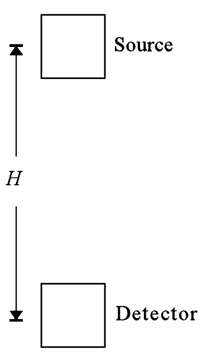


Fig. 14.16

(a) f_0 (b) $f_0 \dfrac{(v+2g)}{v}$

(c) $f_0 \dfrac{(v+2g)}{v}$ (d) $f_0 \left(\dfrac{v}{v-2g}\right)$

Solution (d) $v_s = 0 + g(2) = 2g$

and $f_{app} = f_0 \dfrac{v}{v - v_S} = f_0 \left(\dfrac{v}{v-2g}\right)$.

18. An open pipe is suddenly closed at one end. As a result the frequency of third harmonic of the closed pipe is found to be higher by 100 Hz. The fundamental frequency of open pipe is

(a) 200 Hz (b) 30 Hz
(c) 240 Hz (d) 480 Hz

[IIT 1996]

Solution (a) $f_{0(\text{closed})} = \dfrac{v}{\lambda} = \dfrac{v}{4l}$

third harmonic of closed pipe $= 3f_{0(\text{closed})} = \dfrac{3v}{4l}$

$\dfrac{3v}{4l} - \dfrac{v}{2l} = 100$ or $\dfrac{v}{4l} = 100$ and

$f_{0(\text{open})} = \dfrac{v}{2l}$

$\dfrac{v}{2l} = 200.$

19. As a wave propagates

(a) the wave intensity remains constant for a plane wave.
(b) the wave intensity decreases as the inverse of the distance from source for a spherical wave.
(c) the wave intensity falls as the inverse square of the distance from a spherical wave.
(d) total intensity of the spherical wave over the spherical surface centred at the source remains constant at all times.

Solution (a), (c) and (d).

20. Two monatomic ideal gases 1 and 2 of molecular masses m_1 and m_2 respectively are enclosed in separate containers kept at the same temperature. The ratio of the speed of sound in gas 1 to gas 2 is given by

(a) $\dfrac{m_1}{m_2}$ (b) $\sqrt{\dfrac{m_1}{m_2}}$

(c) $\dfrac{m_2}{m_1}$ (d) $\sqrt{\dfrac{m_2}{m_1}}$

[IIT 2000]

Solution (d) As $v = \sqrt{\dfrac{\gamma RT}{M}}$ $\therefore \dfrac{v_1}{v_2} = \sqrt{\dfrac{m_2}{m_1}}$.

21. A siren placed at a railway platform is emitting sound of frequency 5 kHz. A passenger sitting in a moving train A records a frequency of 5.5 kHz when the train approaches the siren. During his return journey in a different train B he records the frequency of 6 kHz while approaching the same siren. The ratio of velocity of train B to train A is

(a) $\dfrac{242}{252}$ (b) $\dfrac{5}{6}$

(c) 2 (d) $\dfrac{11}{6}$

[IIT Screening 2002]

Solution (c) $\left(\frac{v+v_{L1}}{v}\right)5=5.5,\ \left(\frac{v+v_{L2}}{v}\right)5=6$

or $\frac{v_{L1}}{v}=0.5$ or $\frac{v_{L2}}{v}=1$ or $\frac{v_{L2}}{v_{L1}}=2$

22. A piezo electric quartz crystal of thickness 0.005 m is vibrating in resonate conditions. Calculate the fundamental frequency f_0 for quartz.

$Y=8\times10^{10}\text{Nm}^{-2}$ and $\rho=2.65\times10^3\text{ kgm}^{-3}$

(a) 5.5 MHz (b) 55 MHz
(c) 0.55 MHz (d) 5.5 kHz

Solution (c) $v=\sqrt{\frac{Y}{\rho}}=\sqrt{\frac{8\times10^{10}}{2.69\times10^3}}$

$=5.5\times10^3\text{ms}^{-1}$;

$f=\frac{v}{\lambda}=\frac{5.5\times10^3}{2\times0.005}=5.5\times10^5\text{Hz}.$

23. Calculate the ratio of speed of sound wave in Neon to that in H_2O vapours at any temperature.

(a) $\frac{9}{8}$ (b) $\frac{3}{2\sqrt{2}}$

(c) $\frac{3}{2}$ (d) $\frac{8}{9}$

[Roorkee 1992]

Solution (b) $\frac{v_{Ne}}{v_{H_2O}}=\sqrt{\frac{\gamma_{Ne}M_{H_2O}}{M_{Ne}\gamma_{H_2O}}}$

$=\sqrt{\frac{5/3\times18}{4/3\times20}}=\sqrt{\frac{9}{8}}=\frac{3}{2\sqrt{2}}.$

24. Find the speed of sound in a mixture of 1 mole of He and 2 mole of O_2 at 27°C.

(a) 480 ms^{-1} (b) 621 ms^{-1}
(c) 401 ms^{-1} (d) 601 ms^{-1}

[IIT 1995]

Solution (c) $M_{mix}=\frac{n_1M_1+n_2M_2}{n_1+n_2}$

$=\frac{1\times4+2\times32}{1+2}=\frac{68}{3}$

$C_{V(\text{mixtue})}=\frac{n_1C_{V1}+n_2C_{V2}}{n_1+n_2}$

$=\frac{\left(1\times\frac{3}{2}+2\times\frac{5}{2}\right)R}{1+2}$

$=\frac{13}{6}R$

$C_{P(\text{mix})}=C_V+R=\frac{19}{6}R$ or $\frac{C_P}{C_V}=\frac{19}{13}R$

$v=\sqrt{\frac{\frac{19}{13}\times\frac{8.31\times300}{\frac{68}{3}\times10^{-3}}}{}}=400.9\text{ ms}^{-1}.$

25. The velocity of sound is v_S in air. If density of air is increased twice then the new velocity of sound will be

(a) v_S (b) $\frac{v_S}{\sqrt{2}}$

(c) $\sqrt{2}\,v_S$ (d) $\frac{3}{2}v_S$

[BHU 2003]

Solution (b) $v=\sqrt{\frac{\gamma P}{\rho}}$, that is,

$\frac{v_S'}{v_S}=\sqrt{\frac{\rho}{2\rho}}$

$\Rightarrow v_s'=\frac{v_s}{\sqrt{2}}.$

26. Two radio stations broadcast their programmes at the same amplitude A and at slightly different frequencies ω_1 and ω_2 respectively where $\omega_2-\omega_1=1$ kHz. A detector receives the signals from the two stations simultaneously. It can only detect signals of intensity > $2A^2$. Find the interval between successive maxima of the intensity of the signal received by the detector.

(a) 2×10^{-3}s (b) 4×10^{-3}s
(c) 1.5×10^{-3}s (d) 10^{-3}s

Solution (d) $y_1=A\sin2\pi\omega_1t$ and $y_2=A\sin2\pi\omega_2t$

$y=y_1+y_2=A\sin2\pi\omega_1t+A\sin2\pi\omega_2t$

$=2A\sin2\pi\frac{(\omega_2+\omega_1)}{2}t\cos2\pi\frac{(\omega_2+\omega_1)}{2}t$

$A'=2A\cos2\pi\frac{(\omega_2+\omega_1)}{2}t$

$=2A\cos\pi(\omega_2-\omega_1)t$

$I\propto A'^2=4A^2\cos^2\pi(\omega_2-\omega_1)t$

For I to be maximum $\cos\pi(\omega_2-\omega_1)t=\pm1$

or $\pi(\omega_2-\omega_1)t=0,\pi,2\pi,\ldots$

$T=t_2-t_1=\frac{1}{\omega_2-\omega_1}=10^{-3}\text{s}.$

27. Which of the following will pair up to produce stationary wave?

(a) $Z_1=A\cos(kx-\omega t)$ (b) $Z_2=A\cos(kx+\omega t)$
(c) $Z_3=A\cos(kx-\omega t)$ (d) $Z_4=A\cos(kx+\omega t)$

(a) 1 and 2 (b) 2 and 3
(c) 3 and 4 (d) 1 and 3

[IIT 1993]

Solution (a) The waves must be travelling in opposite directions and have same amplitude and same frequency.

28. A quartz crystal is used to produce ultrasonic. The frequency will be inversely related to

(a) Young's modulus. (b) thickness.
(c) density. (d) length.

Solution (b) $f\propto1/t$.

TYPICAL PROBLEMS

29. Two successive resonance frequencies in an open organ pipe are 1944 and 2592 Hz. Find the length of the tube. The speed of sound in air is 324 ms^{-1}

(a) 25 cm (b) 50 cm
(c) 12.5 cm (d) none of these

Solution (d) $f_0 = 2592 - 1944 = 648$ Hz

$$\lambda = \frac{v}{f} = \frac{324}{648} = \frac{1}{2} \text{ m or } l = \frac{\lambda}{2} = 25 \text{ cm.}$$

30. A cylindrical metal tube has a length of 50 cm and is open at both ends. Find the frequencies between 1 kHz to 2 kHz at which the air column in the tube resonates. The temperature on that day is 20° C.

(a) 1020, 11360, 1700 Hz (b) 1026, 1368, 1710 Hz
(c) 1328, 1660, 1922 Hz (d) none of these

Solution (b) $v(T) = 330\sqrt{1 + \frac{20}{273}} = 330\sqrt{\frac{293}{273}}$

$$= 342 \text{ ms}^{-1} \Rightarrow f = \frac{v}{\lambda} = \frac{342}{1} = 342 \text{ Hz.}$$

wavelengths allowed between 1000 Hz and 2000 Hz are 1026 Hz, 1368 Hz, 1710 Hz.

31. A tuning fork produces 4 beats per second with another tuning fork of frequency 256 Hz. The first one is now loaded with a little wax and number of beats heard are 6 per second. The original frequency of the tuning fork is

(a) 252 Hz (b) 260 Hz
(c) 250 Hz (d) 262 Hz

Solution (a) $f = 256 \pm 4$ Hz.

On loading the first one the number of beats increase. Therefore, the frequency of the tuning fork must be 252 Hz. As it will decrease further on loading and number of beats/s increase.

32. Two stereo speakers are separated by a distance of 2.4 m. A person stands at a distance of 3.2 m as shown directly in front of one of the speakers. Find the frequencies in audible range for which the listener will hear a minimum sound intensity.
Speed of the sound in air is 320 ms^{-1}.

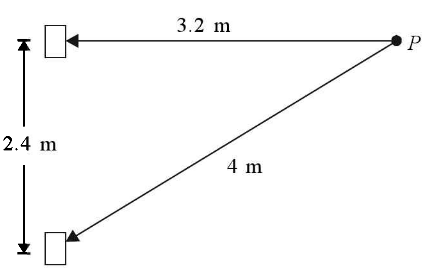


Fig. 14.17

(a) $160(2n+1)$ (b) $320(2n+1)$
(c) $200(2n+1)$ (d) $100(2n+1)$

Solution (c) $\sqrt{3.2^2 + 2.4^2} = 4$ m

Path difference $= 0.8 \text{ m} = (2n+1)\frac{\lambda}{2}$

$$\lambda = \frac{1.6}{(2n+1)} \text{ using } f = \frac{v}{\lambda} = \frac{320}{1.6}(2n+1)$$

$$= 200(2n+1)\text{Hz.}$$

$$n = 1, 2, 3, \ldots 49 \text{ are allowed.}$$

33. A bullet passes past a person at a speed 220 ms^{-1}. Find the fractional change in the frequency of the whistling sound heard by the person as the bullet crosses the person. Speed of sound = 330 ms^{-1}.

(a) 0.67 (b) 0.8
(c) 1.2 (d) 3.0

Solution (b) Limiting cases when it is just at the verge of crossing and when it has just crossed are taken.

$$f_1 = \frac{v}{v + v_S} f = 0.6f \text{ and}$$

$$f_2 = \frac{v}{v + v_S} f = 3f$$

$$f_{\text{net}} = \frac{f_1 + f_2}{2} = \frac{3.6f}{2} = 1.8f$$

$$\Delta f = 0.8f \text{ or } \frac{\Delta f}{f} = 0.8.$$

34. A source of sound emitting 1200 Hz note travels along a straight line at a speed of 170 ms^{-1}. A detector is placed at a distance 200 m from the line of motion of the source. The frequency of the sound received by the detector when it is closest is (velocity of sound is 340 ms^{-1})

(a) 1600 Hz (b) 800 Hz
(c) 2400 Hz (d) none of these

Solution (a) $f_1 = \frac{v}{v + v_S} f = \frac{340}{510} \times 1200 = 800$ Hz;

$$f_2 = \frac{v}{v - v_S} f = \frac{340}{170} \times 1200 = 2400 \text{ Hz}$$

$$f = \frac{f_1 + f_2}{2} = \frac{800 + 2400}{2} = 1600 \text{ Hz.}$$

35. A driver of a car approaching a vertical wall notices that the frequency of his car horn has changed from 440 to 480 Hz when it gets reflected from the wall. Find the speed of the car if the speed of the sound is 330 ms^{-1}

(a) 16.3 ms^{-1} (b) 15.3 ms^{-1}
(c) 14.3 m/s (d) none of these

Solution (c) Let the velocity of car be u

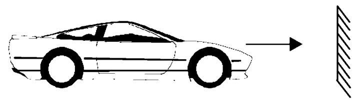


Fig. 14.18

Then $480 = \dfrac{v+u}{v-u} 440$

or $\dfrac{48}{44} = \dfrac{v+u}{v-u}$ or $u = \dfrac{330}{23} = 14.3 \text{ ms}^{-1}$.

36. Two sources of sound S_1 and S_2 vibrate at the same frequency and are in phase. The intensity of sound detected at a point P is I_0. If $\theta = 45°$, what will be the intensity of sound reaching P if one of the sources is switched off. What will be the intensity if $\theta = 60°$

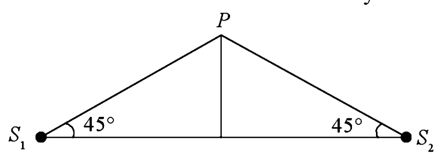


Fig. 14.19

(a) $\dfrac{I_0}{4}, \dfrac{I_0}{8}$ (b) $\dfrac{I_0}{4}, \dfrac{I_0}{2\sqrt{2}}$

(c) $\dfrac{I_0}{4}, \dfrac{I_0}{4}$ (d) $\dfrac{I_0}{4\sqrt{2}}, \dfrac{I_0}{8}$

Solution (c) Since the waves reach in phase $I_0 = 4I$

$I_{max} \alpha (y_{01} + y_{02})^2 = (2y_{02})^2$

$= 4\, y_{02}^2 = 4I$

$\therefore \quad I = \dfrac{I_0}{4}$ It is independent of θ.

37. An electric train in Japan runs with a speed 1.3 Mach. It is approaching a station and blows a whistle of frequency 800 Hz. The frequency of the whistle heard by a stationary observer on the platform is

(a) 800 Hz (b) 1600 Hz

(c) 1040 Hz (d) insufficient data.

Solution (a) Since the velocity of source > velocity of sound, Doppler effect is inapplicable.

38. A person P is 600 m away from the station when train is approaching station with 72 km/h, it blows a whistle of frequency 800 Hz when 800 m away from the station. Find the frequency heard by the person. Speed of sound $= 340 \text{ ms}^{-1}$

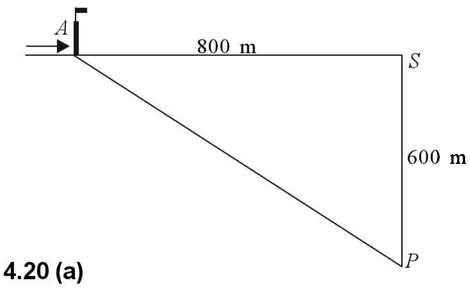


Fig. 14.20 (a)

(a) 800 Hz (b) 839.5 Hz

(c) 829.5 Hz (d) 843.5 Hz

Solution (b) $f_{app} = \dfrac{v}{v - v_S \cos\theta} f$

$= \dfrac{340}{340 - 16} \times 800 = 839.5 \text{ Hz}$

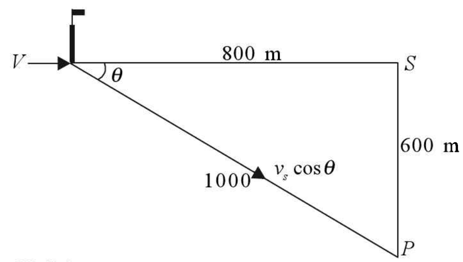


Fig. 14.20 (b)

39. Phenomenon of beats can take place

(a) for longitudinal waves only.

(b) for transverse waves only.

(c) for both longitudinal and transverse.

(d) for sound waves only.

Solution (c)

40. In the absence of teacher a class of 50 students make a noise level of 50 dB. 50 more students enter the class. Assuming each student on an average produces same intensity of sound then the noise level increases by

(a) 50 dB (b) 25 dB

(c) 8.33 dB (d) 3 dB

Solution (d) $\Delta S_L = S_{L2} - S_4$

$= 10 \log \dfrac{2I}{I_0} - 10 \log \dfrac{I}{I_0}$

$= 10 \log_2 = 3\text{dB}$

QUESTIONS FOR PRACTICE

1. When both source and listener move in the same direction with a velocity equal to half the velocity of sound, the change in frequency of the sound as detected by the listener is

(a) 50% (b) 25%

(c) zero (d) none of these

2. The wavelength of the sound produced by a source is 0.8 m. If the source moves towards the stationary listener at 32 ms^{-1}, what will be apparent wavelength of the sound? The velocity of sound is 320 ms^{-1}

(a) 0.80 m (b) 0.72 m
(c) 0.40 m (d) 0.32 m

3. The wavelength of light received from a galaxy is 10% greater than that received from an identical source on the earth. The velocity of the galaxy relative to the earth is

(a) 3×10^6 ms^{-1} (b) 3×10^5 ms^{-1}
(c) 3×10^8 ms^{-1} (d) 3×10^7 ms^{-1}

4. The velocity of sound in air is 330 ms^{-1}. To increase the apparent frequency of the sound by 50%, the source should move towards the stationary source with a velocity equal to

(a) 110 ms^{-1} (b) 105 ms^{-1}
(c) 220 ms^{-1} (d) 330 ms^{-1}

5. A source of sound moves towards a stationary listener with the velocity of sound. If the actual frequency of the sound produced by the source be f, then change in frequency will be

(a) f (b) $f/2$
(c) $f/4$ (d) none of these

6. A train is approaching the platform with a speed of 4 ms^{-1}. Another train is leaving the platform with the same speed. The velocity of sound is 320 ms^{-1}. If both the trains sound their whistles at frequency 230 Hz, the number of beats heard per second will be

(a) 10 (b) 8
(c) 7 (d) 6

7. A man runs towards a source of sound at 10 ms^{-1}. The frequency of the sound produced by the source is 400 Hz and heard is 410 Hz. The speed of the sound perceived by the man will be

(a) 330 ms^{-1} (b) 400 ms^{-1}
(c) 350 ms^{-1} (d) 340 ms^{-1}

8. A pendulum vibrates with a time period of 1 s. The sound produced by it is in the

(a) audible range. (b) infrasonic range.
(c) ultrasonic range. (d) supersonic range.

9. Which of the following characteristics successively increases in the musical scale?

(a) Quality (b) Pitch
(c) Loudness (d) None of these

10. To change the quality of sound produced by an instrument, we need to vary the

(a) number of wavertones. (b) pitch.
(c) loudness. (d) amplitude.

11. Two sound waves given by $y_1 = 5 \sin(300\,\pi t)$ and $y_2 = 4 \sin(302\,\pi t)$ superimpose. The ratio of the maximum to minimum intensity of the sound waves will be

(a) 302/300 (b) 81
(c) 9 (d) 5/4

12. The pressure of air increases by 100 mm of Hg the temperature decreases by 1^0C. What will be the change in the speed of sound in air?

(a) 61 ms^{-1} (b) 61 mm^{-1}
(c) 61 cms^{-1} (d) none of the above

13. A sound wave propagating in air may be treated either as a displacement wave or a pressure wave. What is the phase difference between the displacement and pressure wave?

(a) 180° (b) 90°
(c) 45° (d) zero

14. If A is the amplitude of sound wave after covering a distance r, then

(a) $A \propto 1/r$ (b) $A \propto r^2$
(c) $A \propto 1/r^2$ (d) $A \propto r$

15. If the loundness changes from 30 dB to 60 dB, what is the ratio of the intensities in two cases?

(a) 10,000 (b) 1000
(c) 100 (d) 10

16. The power of a loud speaker is increased from 20 W to 400 W. What is the power increase as compared to the original value?

(a) 13 dB (b) 7 dB
(c) 4 dB (d) 2 dB

17. What is the ratio of the speed of sound in neon and water vapour at the same temperature. It is nearest to

(a) 2.5 (b) 2
(c) 1.5 (d) 1

18. The wavelength of a sound wave is reduced by 50%. Then the percentage change in its frequency will be

(a) 100% (b) 200%
(c) 400% (d) 800%

19. A resonance tube of length 1 m is resonated with a tuning fork of frequency 300 Hz. If the velocity of sound in air is 300 ms^{-1} then the number of harmonics produced in the tube will be

(a) 1 (b) 2
(c) 3 (d) 4

20. The velocity of sound in dry air at 0^0C and at 74 cm of Hg preassure is 332 ms^{-1}, then the velocity of sound at 50^0C and 77.5 cm of Hg pressure in ms^{-1} will be

(a) 322.7 (b) 347.1
(c) 352.4 (d) 361.1

21. The frequency of sound in oxygen at room temperature is v. The frequency of sound in a mixture of oxygen and hydrogen at the same temperature will be

(a) uncertain. (b) equal to v.
(c) less than v. (d) more than v.

22. Beats are the result of

(a) constructive and destructive interference in time.
(b) constructive and destructive interference in space.

(c) destructive interference in space.
(d) constructive interference in space.

23. In Kundt's tube experiment wavelength in the metallic rod and air are 80 cm and 16 cm respectively. If the velocity of sound in air is 300 ms^{-1} then the velocity of sound in rod will be
(a) 80 ms^{-1} (b) 3.75 ms^{-1}
(c) 240 ms^{-1} (d) 1500 ms^{-1}

24. The velocity of sound in air is 332 ms^{-1}. The length of a closed pipe whose frequency of second overtone is 332 Hz, will be
(a) 0.51 m (b) 0.75 m
(c) 1.25 m (d) 1.75 m

25. A closed organ pipe, of length 1.2 m and filled with a gas, is resonated in its fundamental mode with a fork. Another open pipe of same length but filled with air resonates with the same fork. The room temperature is 40°C. If the speed of sound in air at 40°C is 360 ms^{-1}, then the speed of sound in gas at 40°C will be
(a) 341.5 ms^{-1} (b) 637 ms^{-1}
(c) 633 ms^{-1} (d) 720 ms^{-1}

26. The velocity of sound in dry air at 0°C and 74 cm pressure is 332 ms^{-1} then the velocity of sound at 50°C and 77.5 cm pressure in ms^{-1} will be
(a) 322.7 (b) 347.1
(c) 352.4 (d) 361.1

27. In Kundt's tube experiment the metallic rod executes
(a) transverse vibration.
(b) longitudinal vibrations.
(c) both.
(d) none of these.

28. Five beats per second are produced on vibrating two closed organ pipes simultaneously. If the ratio of their lengths is 21/20, then their frequencies will be
(a) 105 Hz and 100 Hz. (b) 105 Hz and 110 Hz.
(c) 100 Hz and 105 Hz. (d) 110 Hz and 105 Hz.

29. If the adiabatic constant for helium and hydrogen gases at the same temperature are 5/3 and 7/5 respectively, then the ratio of velocity of sound in these gases will be
(a) 42 : 5 (b) 5 : $\sqrt{42}$
(c) $\sqrt{42}$: 5 (d) 5 : 42

30. A source of sound is emitting sound waves in all directions in an absorptionless medium. This source is at distance of 4 m and 16 m from points x and y respectively. The ratio of amplitudes of waves at points x and y will be
(a) 2 : 4 (b) 4 : 1
(c) 4 : 2 (d) 1 : 4

31. Two open pipes of length L are vibrated simultaneously. If length of one of the pipes is reduced by y, then the mumber of beats heard per second will nearly be (if the velocity of sound is v and $y < L$)
(a) $\frac{vy}{2L}$ (b) $\frac{2L^2}{Vy}$
(c) $\frac{vy}{2L^2}$ (d) $\frac{vy}{L^2}$

32. Two waves of wavelength 1.00 m and 1.01 m produce 10 beats in 3 s in a gas. The speed of sound in the gas will be
(a) 316.6 ms^{-1} (b) 336.6 ms^{-1}
(c) 356.6 ms^{-1} (d) 396.6 ms^{-1}

33. A source of sound of frequency 90 Hz is moving towards an observer with a velocity one-tenth the velocity of sound. The frequency heard by the observer will be
(a) 50 Hz (b) 100 Hz
(c) 200 Hz (d) 300 Hz

34. A source of sound of frequency 512 Hz is moving towards a wall with velocity v equal to that of second. An observer is standing between the source and the wall, then he will listen
(a) no beats s^{-1} (b) 3 beats s^{-1}
(c) 6 beats s^{-1} (d) 12 beats s^{-1}

35. An engine blowing whistle, is approaching a stationary observer with a velocity of 110 ms^{-1}. The ratio of frequencies heard by the observer while engine approaching and receding away from him will be (if $v = 330\ ms^{-1}$)
(a) 1 : 4 (b) 4 : 1
(c) 2 : 1 (d) 1 : 2

36. Earth is moving towards a stationary star with a velocity 100 kms^{-1}. If the wavelength of light emitted by the star is 5000 Å, then the apparent change in wavelength obseved by the observer on earth will be
(a) 0.67 Å (b) 1.67 Å
(c) 16.7 Å (d) 167 Å

37. An observer measures speed of light to be C when he is stationary with respect to the source. If the observer moves with velocity v towards the source then the velocity of light observed will be
(a) $c - v$ (b) $c + v$
(c) $\sqrt{1-v^2/c^2}$ (d) c

38. A whistle is revolved with high speed in a horizontal circle of radius R. To an observer at the centre of the circle the frequency of the whistle will appear to be
(a) decreasing. (b) increasing.
(c) both. (d) constant.

39. A source of sound is emitting a waves of wavelength 40 cm in air. If this source starts moving towards east with a velocity one-fourth the velocity of sound then the apparent wavelength of sound in a direction opposite to that of source will be
(a) 20 cm (b) 50 cm
(c) 80 cm (d) 100 cm

40. A siren is producing sound of frequency 930 Hz. This siren is moving away from an observer towards a wall with velocity of 20 ms^{-1}. The frequency of sound directly coming from the siren will be

(a) 882 Hz (b) 1000 Hz
(c) 930 Hz (d) 977 Hz

41. The apparent wavelength of light from a star moving away from earth is observed to be 0.01% more than its real wavelength. The velocity of star is

(a) 120 kms^{-1} (b) 90 kms^{-1}
(c) 60 kms^{-1} (d) 30 kms^{-1}

42. A star is receding away from earth with a velocity of 10^5 ms^{-1}. If the wavelength of its spectral line is 5700 Å then Doppler shift will be

(a) 0.2 Å (b) 1.9 Å
(c) 20 Å (d) 200 Å

43. The wavelength of H_α line in hydrogen spectrum was found 6563 Å in the laboratory. If the wavelength of same line in the spectrum of a milky way is observed to be 6586 Å then the recessional velocity of the milky way will be

(a) 0.105×10^6 ms^{-1} (b) 1.05×10^6 ms^{-1}
(c) 10.5 ms^{-1} (d) none of these

44. If a soldier jumps from an aeroplane moving with a constant horizontal velocity, then the ratio of the frequency of aeroplane sound heard by him f' and real frequency f will be

(a) 1 : 4 (b) 2 : 1
(c) 1 : 2 (d) 1 : 1

45. A rocket is receding away from earth with velocity 0.2 c. The rocket emits signal of frequency 4×10^7 Hz. The apparent frequency of the signal produced by the rocket observed by the observer on earth will be

(a) 3×10^6 Hz (b) 4×10^6 Hz
(c) 3.2×10^7 Hz (d) 5×10^7 Hz

46. A spectral line is obtained from a gas discharge tube at 5000 Å. If the rms velocity of gas molecules is 10^5 ms^{-1} then the width of spectral line will be

(a) 3.3 Å (b) 4.8 Å
(c) 7.2 Å (d) 9.1 Å

47. When an observer is approaching a stationary source with a velocity v_0 then the apparent change in frequency observed by him will be

(a) $\frac{v}{v+v_0} n$ (b) $\frac{v}{v_0} n$
(c) $\frac{v+v_0}{v} n$ (d) $\frac{v_0}{v} n$

48. A car is moving towards a person. The person observes a change of 2.5% in the frequency of its horn. If the velocity of sound is 320 ms^{-1} then the velocity of car is

(a) 6 ms^{-1} (b) 8 ms^{-1}
(c) 7.5 ms^{-1} (d) 800 ms^{-1}

49. A whistle produces 256 waves per second. If the velocity of sound towards the observer and its magnitude is one-third the velocity of sound in air then the number of waves received by the observer per second will be

(a) 192 (b) 200
(c) 300 (d) 384

50. When a source moves away from observer then apparent change in freuency is Δn_1. When an observer approaches the stationary source with same velocity v then change in frequency is Δn_2 then

(a) $\Delta n_1 = \Delta n_2$ (b) $\Delta n_1 > \Delta n_2$
(c) $\Delta n_1 < \Delta n_2$ (d) none of these

51. A SONAR inside sea works at 40 kHz. A submarine is approaching it with a velocity 360 Kmh^{-1}. If the speed of sound in water is 1450 ms^{-1} then the apparent frequency of waves after reflection from submarine will be

(a) 11.5 kHz (b) 36.8 kHz
(c) 45.9 kHz (d) 98.6 kHz

52. When a source of sound approaches a stationary observer with velocity v_s then the apparent frequency observed by the observer will be (v = velocity of sound)

(a) $\frac{v_s}{v-v_s} n$ (b) $\frac{v-v_s}{v} n$
(c) $\frac{v+v_s}{v_s} n$ (d) $\frac{v_s}{v-v_s} n$

53. A supersonic jet is moving with a velocity twice that of sound, the angle of conical wave front produced by the jet will be

(a) 120° (b) 90°
(c) 60° (d) 30°

54. The frequency of radar waves is 7.8×10^9 Hz. The frequency of these waves after reflection from aeroplane is observed to have increased by 2.7×10^4 Hz. The velocity of aeroplane in $kmhr^{-1}$ will be

(a) 1.872×10^3 (b) 2.6×10^3
(c) 3.1×10^3 (d) 7.398×10^3

55. A boy blowing a whistle, is running away from a wall towards an observer with a speed of 1 ms^{-1}. The frequency of whistle is 680 Hz. The number of beats heard per second by the observer will be (given v = 340 ms^{-1})

(a) zero (b) 2
(c) 4 (d) 8

56. An ultrasonic scanner is used in a hospital to detect tumour in tissue. The working frequency of the scanner is 4.2 mega Hz. The velocity of sound in the tissue is 1.7 kms^{-1}. The wavelength of sound in the tissue is nearest to

(a) 4×10^{-3} m (b) 8×10^{-3} m
(c) 4×10^{-4} m (d) 8×10^{-4} m

57. In a Kundt's tube experiment the heaps of lycopodium powder are collected at 20 cm separations. The frequency of tuning fork used is

(a) 660 Hz (b) 825 Hz
(c) 775 Hz (d) 915 Hz

58. When a sound wave of frequency 300 Hz passes through a medium, the maximum displacement of a particle of the medium is 0.1 cm. The maximum velocity of the particle is equal to

(a) 30 cms^{-1} (b) $30\pi cms^{-1}$
(c) 60 cms^{-1} (d) $60\pi cms^{-1}$

59. Which of the following is mechanical wave?

(a) Light waves (b) Sound waves
(c) X-rays (d) Radio waves

60. Which of the following properties of sound is not affected by change in the temperature of air?

(a) Wavelength (b) Intensity
(c) Amplitude (d) Frequency

61. A sound wave is represented by $y = a \sin(1000\pi t - 3x)$. The distance between two points having a phase difference of $\pi/3$ is

(a) $5\pi/18$ (b) $2\pi/9$
(c) $\pi/18$ (d) $\pi/9$

62. If a tuning fork sends a wave 5 sin $\left(600\omega t - \frac{\pi}{0.6}x\right)$ then the amplitude of the intensity heard is

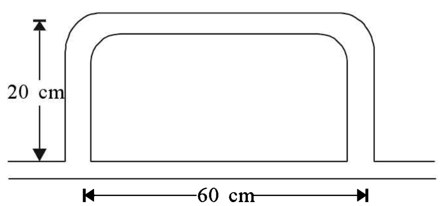


Fig. 14.21

(a) 5 (b) $5\sqrt{2}$
(c) $5\sqrt{3}$ (d) none of these

63. Surface waves strike the rock with their crests 160 m apart. The velocity of the waves is 40 ms^{-1}. The time interval between two crests striking the rock is

(a) 8 s (b) 4 s
(c) 2 s (d) 1 s

64. Sound waves in air differ from the electromagnetic waves in that they cannot be

(a) diffracted. (b) polarised.
(c) reflected. (d) refracted.

65. When a stone is dropped on the surface of still water the waves produced are

(a) stationary. (b) transverse.
(c) longitudinal. (d) none of these.

66. Velocity of sound in a gas is proportional to

(a) square root of isothermal elasticity.
(b) isothermal elasticity.
(c) square root of adiabatic elasticity.
(d) adiabatic elasticity.

67. The velocity of sound in a gas is 300 ms^{-1}. The root mean square velocity of the molecules is of the order of

(a) 4 ms^{-1} (b) 40 ms^{-1}
(c) 400 ms^{-1} (d) 4000 ms^{-1}

68. A man standing unsymmetrically between two parallel cliffs, claps his hands and starts hearing a series of echoes at intervals of 1 s. If the speed of sound in air is 340 ms^{-1}, then the distance between the two parallel cliffs, is

(a) 170 m (b) 340 m
(c) 510 m (d) 680 m

69. Compressional wave impulses are sent to the bottom of sea from a ship and the echo is heard after 4 s. If bulk modulus of water is 2×10^9 Nm^{-2} and the mean temperature is 4°C, then depth of the sea is

(a) 2000×10^3 m (b) 2828 m
(c) 1414 m (d) 707 m

70. Which of the following cannot produce ultrasonics?

(a) Galton whistle. (b) Quartz crystal.
(c) Magnetostriction effect. (d) Quincke's tube.

71. Velocity of sound in oxygen at NTP is v. The velocity of sound in helium at NTP should be

(a) $2v$ (b) $4v$
(c) $2\sqrt{2}\,v$ (d) none of these

72. A stone is dropped into a well and the sound of impact of stone with the water is heard after 2.056 s of the release of stone from the top. If the acceleration due to gravity is 980 cms^{-2} and the velocity of sound in air is 350 ms^{-1}, then the depth of the well is

(a) 7 m (b) 19.6 m
(c) 700 m (d) 1960 m

73. When you speak to your friend, which of the following quantities have a unique value in the sound produced?

(a) Amplitude (b) Wave velocity
(c) Frequency (d) Wavelength

74. Bullet train in Japan travels with 1.2 Mach. A man standing on the platform will hear the frequency as if the train blows a whistle of 800 Hz while approaching the station.

(a) 800 Hz (b) 960 Hz
(c) 1960 Hz (d) none of these

75. A rope of length / and mass m hangs freely from the ceiling. The velocity of transverse wave as a function of position x along the rope is proportional to

(a) $1/\sqrt{x}$ (b) $\sqrt{x}$
(c) x (d) x^0

76. A sound wave of freequency 500 Hz covers a distance of 1000 m in 5 s between points x and y. Then the number of waves between x and y are

(a) 5000 (b) 2500
(c) 100 (d) 500

77. A string has a mass per unit length of 10^{-6} kgcm^{-1}. The equation of simple harmonic wave produced in it is $y = 0.2 \sin(2x + 80t)$. The tension in the string is

(a) 0.0016 N (b) 0.16 N
(c) 16 N (d) 1.6 N

78. The velocity of sound in air is 332 ms^{-1}. If the air consists of nitrogen and oxygen in the ratio 4 : 1, the velocity of sound in oxygen is

(a) 278 ms^{-1} (b) 315 ms^{-1}
(c) 372 ms^{-1} (d) 418 ms^{-1}

79. A jet aeroplane is flying at supersonic speed. When the sound of the jet appears to be coming vertically downwards, the angle of sight of the aeroplane with the horizontal cannot be

(a) 60° (b) 40°
(c) 30° (d) 25°

80. The first loud sound is heard in a resonance tube when the air column is 26.2 cm and second load sound is heard when the air column is 80 cm. The diameter of the tube is

(a) 0.6 cm (b) 1.2 cm
(c) 2.0 cm (d) 1.6 cm

81. A supersonic jet produces waves in air. The wave front is

(a) ellipsoidal. (b) conical.
(c) spherical. (d) paraboloidal.

82. The train is approaching station with 72 kmh^{-1}. When 1 km away it blows a whistle of frequency 600 Hz. The frequency heard by the person as shown in figure is (v_{sound} = 350 ms^{-1})

(a) 612 Hz (b) 625 Hz
(c) 632 Hz (d) none of these

83. A person places his ear at the end of a long steel pipe. He hears two distinct sounds at an interval of 0.5 s when another person hammers at the other end of the pipe. If the speed of the sound in metal and air are 3630 ms^{-1} and 330 ms^{-1} respectively, then the distance between the two persons is

(a) 90.75 m (b) 181.5 m
(c) 363 m (d) 1650 m

84. The wave produced in a sonometer is

(a) longitudinal. (b) timesavers.
(c) transverse stationary. (d) longitudinal stationary.

85. A tube closed at one end and containing air produces, when excited the fundamental note of frequency 512 Hz. If the tube is open at both ends, the fundamental frequency that can be excited is (in Hz)

(a) 128 (b) 256
(c) 512 (d) 1024

86. The number of beats produced per second by two tuning forks when sounded together is 4. One of them has a frequency of 250 Hz. The frequency of the other cannot be more than

(a) 252 Hz (b) 254 Hz
(c) 246 Hz (d) 248 Hz

87. At what sound level headache begins?

(a) 120 dB (b) 100 dB
(c) 80 dB (d) 60^0 dB

88. A tuning fork is vibrating in the 3rd harmonic. Number of antinodes produced in it are

(a) 3 (b) 5
(c) 7 (d) 6

89. Which type of wave is produced in the stem of tuning fork?

(a) Transverse (b) Transverse stationary
(c) Longitudinal (d) Longitudinal stationary

90. An organ pipe P_1 closed at one end and vibrating in its first overtone and another pipe P_2 open at both ends vibrating in its third overtone are in resonance with a given tuning fork. The ratio of the lengths of P_1 to that of P_2 is

(a) 3/8 (b) 1/3
(c) 1/2 (d) 8/3

91. Which type of wave is produced in a resonance tube?

(a) Longitudinal (b) Transverse
(c) Transverse stationary (d) Longitudinal stationary

92. The intensity level due to two waves of the same frequency in a given medium are 1 dB and 4 dB. The ratio of their amplitudes is

(a) 1 : 10^4 (b) 1 : 4
(c) 1 : 2 (d) 1 : 10^2

93. The velocity of sound in air is 330 ms^{-1}. The fundamental frequency of an organ pipe open at both ends and length 0.3 m will be

(a) 200 Hz (b) 275 Hz
(c) 300 Hz (d) 550 Hz

94. The relation between the objective measurement of intensity of sound, I and the subjective sensory response called loudness L is given by

(a) $L = K \log I^2$ (b) $I = K \log L$
(c) $L = I$ (d) $L = K \log I$

95. 20 tuning forks are so arranged in series that each fork gives 4 beats per second with the previous one. The frequency of the 20th fork is three times that of the first. What is the frequency of the first tuning fork?

(a) 60 Hz (b) 57 Hz
(c) 40 Hz (d) 38 Hz

96. The displacement y of a particle executing periodic motion is given by $y = 4\cos^2(t)\sin(1000\,t)$. This expression may be considered to be a result of the superposition of waves

(a) 5 (b) 4
(c) 3 (d) 2

97. As an empty vessel is filled with water, its frequency
(a) increases. (b) decreases.
(c) remains unchanged. (d) none of these.

98. Combination of notes that produced jarring effect on the ear is called
(a) discord. (b) harmony.
(c) noise. (d) melody.

99. A pure sine wave is called
(a) chord. (b) melody.
(c) tone. (d) overtone.

100. A tuning fork of frequency f resonates with a closed organ pipe when the length of the shortest air columns are a and b respectively. The speed of sound in air is
(a) $2f(b+a)$ (b) $\frac{f(v-a)}{2}$
(c) $2f(b-a)$ (d) $\frac{f(v+a)}{2}$

101. Most of the human ears cannot hear sound of intensity less than
(a) 10^{-12} Wm^{-2} (b) 10^{-6} Wm^{-2}
(c) 10^{-3} Wm^{-2} (d) 1 Wm^{-2}

102. Two waves having the intensities in the ratio 9 : 1 produce interference. The ratio of maximum to minimum intensity is equal to
(a) 4 : 1 (b) 9 : 1
(c) 2 : 1 (d) 10 : 8

103. The expression $y = a \sin bx \sin \omega t$ represents a stationary wave. The distance between the consecutive nodes is
(a) $1/b$ (b) $\pi/2b$
(c) $2\pi/b$ (d) π/b

104. A pendulum vibrates with a time period of 1 s. The sound produced by it is in the range of
(a) supersonic range. (b) ultrasonic range.
(c) audible range. (d) infrasonic range.

105. In a good auditorium the reverberation time is
(a) 0.17 V/A (b) > 0.17 V/A
(c) < 0.17 V/A (d) none of these

106. A sound is said to be of rich quality when it contains
(a) a note of high amplitude.
(b) only the fundamental frequency.
(c) a note of high frequency.
(d) many harmonics.

107. The persistence of sound in a hall is called
(a) reverberation. (b) resonance.
(c) acoustics. (d) articulation.

108. The wavelength of light received from milky way is 0.4% higher than that from the same source on earth. The velocity of milky way with respect to earth will be
(a) 0.2×10^6 ms^{-1} (b) 1.2×10^6 ms^{-1}
(c) 2×10^6 ms^{-1} (d) 5×10^6 ms^{-1}

109. On a quiet day, when a whistle crosses you then its pitch decreases in the ratio 4/5. If the temperature on that day is 20° C, then the speed of whistle will be (given the velocity of sound at 0^0 = 332 ms^1)
(a) 48.2 ms^{-1} (b) 68.2 ms^{-1}
(c) 86 ms^{-1} (d) 48.2 ms^{-1}

110. The pitch of a sound source appears to be 20% decreased to an observer. The velocity of source with respect to the observer will be
(a) 0.825 ms^{-1} (b) 8.25 ms^{-1}
(c) 82.5 ms^{-1} (d) 330 ms^{-1}

111. The separation between a node and an antinode in the stationery wave $y = 20 \sin(2x) \cos 400\pi t$ is
(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{4}$
(c) $\frac{\pi}{6}$ (d) $\frac{\pi}{8}$

112. $P = (10^5 \pm 14pa) \sin(600\pi t - \pi x)$ is a pressure wave of amplitude 10 cm. The bulk modulus is
(a) $\frac{10^6}{\pi}$ Pa (b) $\frac{10^2}{\pi}$ Pa
(c) $\frac{140}{\pi}$ Pa (d) $\frac{14}{\pi}$ Pa

113. Ultrasonics produced in quartz is due to
(a) magneto striction effect.
(b) pyroelectric effect.
(c) piezoelectric effect.
(d) none of these.

114. If wave length changes by $\Delta\lambda$ then change in propagation vector is
(a) Δk (b) $2\pi\Delta\lambda$
(c) $\frac{2\pi}{\lambda}\Delta\lambda$ (d) $\frac{2\pi}{\lambda^2}\Delta\lambda$

115. The pressure amplitude in a sound wave is tripled and frequency halved, the intensity of the sound is increasesd by a factor of
(a) 1.25 (b) 3.25
(c) 9.25 (d) 2.25

116. How many times more intense is a 90 db sound than a 40 db sound
(a) 10^5 (b) 500
(c) 50 (d) 5

117. Beats are produced by two waves $y_1 = a \sin 200\pi t$ and $y_2 = a \sin 208\pi t$. The number of beats heard per second is

(a) 8 (b) 4
(c) 1 (d) 0

118. For a resonance tube the air columns for the first and the second resonance are 24.5 and 75 cm. The wavelength of the sound wave is

(a) 202 cm (b) 50.5 cm
(c) 101 cm (d) 98 cm

119. The speed of sound in air is 350 ms^{-1}. The fundamental frequency of an open pipe 50 cm long will be

(a) 50 Hz (b) 175 Hz
(c) 350 Hz (d) 700 Hz

120. A sound source is moving towards a stationary observer with one-tenth of the speed of sound. The ratio of apparent to real frequency is

(a) $(9/10)^2$ (b) $11/10)^2$
(c) 10/9 (d) 11/10

121. The frequency of the sound of a car horn as received by an observer towards whom the car is moving differs from the frequency of the horn by 2.0%. Assuming that the velocity of sound of air is 350 ms^{-1}, the velocity of car is

(a) 6.0 ms^{-1} (b) 7.5 ms^{-1}
(c) 7 ms^{-1} (d) 8.5 ms^{-1}

122. Fidelity refers to

(a) reproduction of original sound.
(b) reproduction of original image.
(c) reproduction of music.
(d) reproduction of a CD from original copy.

123. On the diatonic scale the frequency of the note Re is 81. The frequency of Ga will be

(a) 93 (b) 90
(c) 87 (d) 84

124. The frequency of the key note on the equally tempered scale is 24. The frequency of the highest note in it will be

(a) 72 (b) 60
(c) 48 (d) 36

125. Which analysis is applied to convert a complex sound into notes

(a) millman theorem. (b) de morgan laws.
(c) lissajous theorem. (d) fourier theorem.

126. Which of the following can be used to determine the velocity of sound in solids, liquids as well as in gases?

(a) Resonance tube (b) Kundt's tube
(c) Sonometer (d) Organ pipe

127. Bass control in a stereo system increases the power of

(a) low frequencies.
(b) high frequencies.
(c) medium frequencies.
(d) ultrasonics.

128. Two waves each of loudness L superimpose to produce beats. The maximum loudness of the beats will be

(a) $4L$ (b) L
(c) $2L$ (d) none of these

129. Three sources of sounds of equal intensities have frequencies 251, 252 and 253 Hz respectively. When sounded together, the number of beats heard per second will be

(a) 3 (b) 2
(c) 1 (d) none of these

130. Sound waves in air cannot be polarised because they are

(a) progressive. (b) stationary.
(c) transverse. (d) longitudinal.

131. Which of the following is essential for interference of sound waves?

(a) Constant phase difference
(b) Same frequency
(c) Same amplitude
(d) None of these

132. The number of beats produced per second by two tuning forks when sounded together is 4. One of them has a frequency of 250 Hz and if waxed, number of beats become 6. The frequency of the other tuning fork is

(a) 254 Hz (b) 252 Hz
(c) 248 Hz (d) 246 Hz

133. What happens when two sound waves of frequencies differing by more than 10 Hz reach our ear simultaneously?

(a) Beats are not produced.
(b) The waves destroy each other's effect.
(c) Interference of sound does not take place.
(d) Beats are produced but cannot be heard.

134. Velocity of sound in the atmosphere of a planet is 600 ms^{-1}. The minimum distance between the source of sound and the obstacle to hear echo should be

(a) 60 m (b) 25 m
(c) 30 m (d) 17 m

135. Radio waves of wavelength λ are sent from a RADAR towards an aeroplane. If the aeroplane is moving towards the RADAR station, the wavelength of the radiowaves received after reflection from the aeroplane will be

(a) λ (b) $> \lambda$
(c) $< \lambda$
(d) more or less than λ, depending on the speed of aeroplane

Answers to Questions for Practice

1.	(c)	**2.**	(a)	**3.**	(d)	**4.**	(a)	**5.**	(d)	**6.**	(c)	**7.**	(b)
8.	(b)	**9.**	(b)	**10.**	(a)	**11.**	(b)	**12.**	(c)	**13.**	(b)	**14.**	(a)
15.	(b)	**16.**	(a)	**17.**	(d)	**18.**	(a)	**19.**	(b)	**20.**	(d)	**21.**	(d)
22.	(a)	**23.**	(d)	**24.**	(b)	**25.**	(d)	**26.**	(d)	**27.**	(b)	**28.**	(c)
29.	(b)	**30.**	(b)	**31.**	(c)	**32.**	(b)	**33.**	(b)	**34.**	(a)	**35.**	(c)
36.	(b)	**37.**	(d)	**38.**	(d)	**39.**	(b)	**40.**	(a)	**41.**	(d)	**42.**	(b)
43.	(b)	**44.**	(d)	**45.**	(c)	**46.**	(a)	**47.**	(d)	**48.**	(b)	**49.**	(d)
50.	(b)	**51.**	(c)	**52.**	(d)	**53.**	(c)	**54.**	(a)	**55.**	(c)	**56.**	(c)
57.	(b)	**58.**	(d)	**59.**	(b)	**60.**	(d)	**61.**	(d)	**62.**	(a)	**63.**	(b)
64.	(b)	**65.**	(b)	**66.**	(c)	**67.**	(c)	**68.**	(c)	**69.**	(b)	**70.**	(d)
71.	(c)	**72.**	(b)	**73.**	(b)	**74.**	(a)	**75.**	(b)	**76.**	(b)	**77.**	(b)
78.	(b)	**79.**	(a)	**80.**	(c)	**81.**	(b)	**82.**	(c)	**83.**	(b)	**84.**	(c)
85.	(d)	**86.**	(b)	**87.**	(c)	**88.**	(c)	**89.**	(c)	**90.**	(a)	**91.**	(d)
92.	(d)	**93.**	(d)	**94.**	(d)	**95.**	(d)	**96.**	(c)	**97.**	(a)	**98.**	(a)
99.	(c)	**100.**	(c)	**101.**	(a)	**102.**	(a)	**103.**	(d)	**104.**	(d)	**105.**	(a)
106.	(d)	**107.**	(a)	**108.**	(b)	**109.**	(c)	**110.**	(c)	**111.**	(a)	**112.**	(c)
113.	(c)	**114.**	(d)	**115.**	(d)	**116.**	(a)	**117.**	(b)	**118.**	(c)	**119.**	(c)
120.	(c)	**121.**	(c)	**122.**	(a)	**123.**	(b)	**124.**	(b)	**125.**	(d)	**126.**	(b)
127.	(a)	**128.**	(a)	**129.**	(c)	**130.**	(d)	**131.**	(d)	**132.**	(a)	**133.**	(d)
134.	(c)	**135.**	(c)										

Explanations

48. (b) $\frac{\Delta f}{f} = \frac{v}{V}$ or $v = 320 \times \frac{2.5}{100} = 8\ \text{ms}^{-1}$.

58. (d) $\frac{\lambda}{2} = 0.1$ cm, $v = f\lambda = 60\,\pi\ \text{cms}^{-1}$.

62. (a) $\Delta x = 0.4$ m,

$$\phi = k\Delta x = \frac{2\pi}{3} = \sqrt{5^2 + 5^2 + 2\times 5\cos(2\pi/3)}$$

$= 5.$

67. (c) $v_{rms} = v_{sound}\sqrt{\frac{3}{\gamma}}$.

68. (c) $x = \frac{vt}{2} = 340 \times \frac{3}{2} = 510$ m.

88. (c) Number of antinodes $= (2n + 1) = 2\,(3 + 1) = 7$.

112. (c) $P_0 = \frac{By_0\omega}{v}$ or $B = \frac{140}{\pi}$.

114. (d) $k = \frac{2\pi}{\lambda}$ differentiating $dk = \frac{-2\pi}{\lambda^2}\,d\lambda$

123. (b) $f\,\frac{9}{10} = 81$ or $f = 90$ Hz.

SELF TEST 1

1. In a stationary wave
   (a) all the particles of the medium vibrate in phase
   (b) all the antinodes vibrate in phase
   (c) all the particles between consecutive nodes vibrate in phase.
   (d) all the particles have the same energy

2. Both the strings shown in figure are made of same material and same thickness. The pulleys are light and smooth. The speed of transverse waves in AB and CD are V_1 and V_2 respectively, then

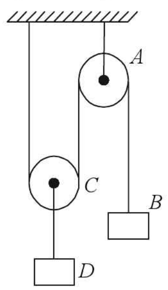


Fig. 1

   (a) $\frac{V_1}{V_2} = \sqrt{2}$ (b) $\frac{V_1}{V_2} = 1$
   (c) $\frac{V_1}{V_2} = \frac{1}{\sqrt{2}}$ (d) $\frac{V_1}{V_2} = 2$
   (e) none

3. The velocity of sound in air is 332 ms^{-1}. Velocity in vacuum is
   (a) 332 ms^{-1} (b) > 332 ms^{-1}
   (c) < 332 ms^{-1} (d) none

4. Longitudinal wave cannot
   (a) transmit energy
   (b) have a unique wavelength
   (c) have a unique velocity
   (d) be polarized.

5. Which of the following pairs can form stationary waves?
   (1) $z = A \sin(\omega t - kx)$ (2) $y = A' \sin(\omega t - kx)$
   (3) $y = A \cos(\omega t + kx)$ (4) $y = A' \sin(\omega t + kx)$
   (5) $y = A \cos(\omega t - kx)$
   (a) only 2 and 4 (b) only 1 and 3
   (c) 2 and 4; 3 and 5 (d) 1, 3; 2, 4 and 3,5

6. 2 kg weight is hanging over the pulley shown. The string connecting the weight passing over the pulley has mass 5g. Find the time the transverse wave produced at the bottom (near floor) to reach the top of pulley.

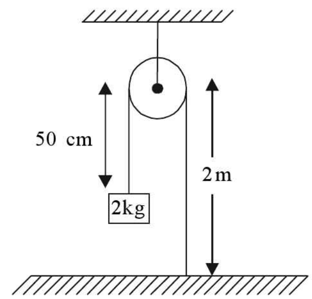


Fig. 2

   (a) 0.02 s (b) 0.03 s
   (c) 0.04 s (d) 0.01s

7. A transverse propagating in a string of linear mass density 1.2×10^{-4} kg m^{-1} is given by $y = (0.2m) \sin[1.0x + 30t]$. The tension in the string is
   (a) 0.108 N (b) 0.09 N
   (c) 0.12 N (d) none

8. The speed of a tansverse wave in a stretched string of length 1 m is 60 ms^{-1}. The fundamental frequency of vibration is–
   (a) 60 Hz (b) 120 Hz
   (c) 30 Hz (d) none

9. The correct relation between wave velocity and particle velocity is
   (a) particle velocity = $\frac{\text{wave velocity}}{\text{strain}}$
   (b) particle velocity = wave velocity × strain
   (c) particle velocity = wave velocity
   (d) particle velocity = wave velocity

10. Select the correct relation for a simple harmonic wave.

(a) $\frac{d^2y}{dt^2} = V\frac{d^2x}{dy^2}$ (b) $V^2 = \frac{d^2y}{dx^2} / \frac{d^2y}{dt^2}$

(c) $\frac{d^2y/dt^2}{d^2y/dx^2} = V^2$ (d) $\frac{d^2y}{dt^2} = V^2$

11. The velocity of sound increases by approximately____ when temperature rises by 1°C.

(a) 61 cms^{-1} (b) 0.61 ft s^{-1}

(c) 0.061 m s^{-1} (d) 61 mms^{-1}

12. The bulk modulus of sea water is 2.25×10^{10} dynes cm^{-2}. The velocity of sound in it is

(a) 150 ms^{-1} (b) 1.5 kms^{-1}

(c) 0.15 kms^{-1} (d) 15 kms^{-1}

13. A disc has n holes. When it is rotated at m revolutions s^{-1}, a bright mark on the prong appears stationary. The frequency of the tuning fork is

(a) m (b) $\frac{m}{n}$

(c) mn (d) none

14. A smoked plate is allowed to fall with stylus of the tuning fork touching the plate. If 3 complete waves conscutively are made in 10 *cm* and 20 *cm* then frequency of the tuning fork is

(a) 20 Hz (b) $\frac{20}{3}$ Hz

(c) $\frac{10}{3}$ Hz (d) 30 Hz

(e) none

15. In cine films

(a) sound is recorded separately

(b) the sound is recorded proportional to opacity or transparency of the film

(c) the sound band is 2.5 mm on a standard 35 mm film

(d) the sound band is 2.5 mm above and below the 35 mm film

16. Ohm's law in sound states that

(a) ear can resolve a complex note into its harmonic components

(b) loudness depends upon intensity or rate of flow of energy to the ear

(c) sound energy is absorbed to different extents by different objects

(d) dissonance depends upon formation of beats

17. The equation of standing wave produced on a string fixed on both sides is $y = (0.4 \text{ cm}) \sin [(.314 \text{ cm}^{-1}) x] \cos 200 \pi t$. The smallest length of the string could be

(a) 20 cm (b) 15 cm

(c) 10 cm (d) 5 cm

18. For a displacement wave $y = y_0 \sin (\omega t - kx)$. The pressure wave has amplitude $P_0 =$_____. Assume B as bulk modulus

(a) $P_0 = B\omega y_0$ (b) Bky_0

(c) $\frac{By_0\omega}{V}$ (d) $\frac{B\omega}{y_0}$

19. The velocity of sound in water is _____ than that of air and speed of light in water is ____ than that of air.

(a) greater, smaller (b) smaller, smaller

(c) greater , greater (d) smaller, greater

20. The length of a closed pipe is 25 cm. The frequencies allowed between 200 Hz to 2 kHz are ____ $V_{sound} = 340$ ms^{-1}

(a) 340, 680, 1020, 1360, 1700 Hz

(b) 340, 1020, 1700 Hz

(c) 340, 680, 1360 Hz

(d) none of these

21. The second resonance in an open pipe shows pressure wave

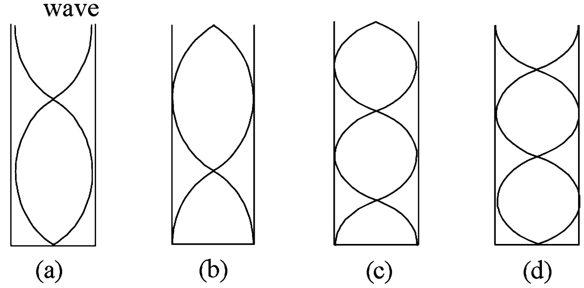


Fig. 3

22. The ratio of intensity of wave A to wave B is

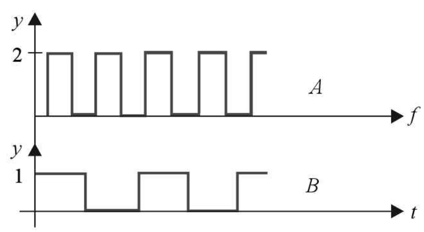


Fig. 4

(a) 1 (b) 1/4

(c) 4 (d) 1/16

(e) 16

23. An open pipe of length l has frequency f_o (fundamental) . The pipe is half filled with water. The fundamental frequency will be

(a) f_o (b) $2f_o$

(c) $f_o/2$ (d) $4f_o$

24. Two open pipes of length 50 cm and 51cm produce 8 beats s^{-1}. Find the speed of sound.

(a) 344 ms^{-1} (b) 374 ms^{-1}

(c) 404 ms^{-1} (d) 364 ms^{-1}

25. A resonance tube has length 1.25 m. Find the maximum number of resonance which can be obtained with a tuning fork of 500 Hz. $V_{sound} = 350\ ms^{-1}$.

(a) 3 (b) 2
(c) 1 (d) 4
(e) none

26. A kundt's tube shows heaps of sand at 40 cm and 65 cm mark, respectively. Find the frequency of the tuning fork used if the day's temperature is 30°C.

(a) 500 Hz (b) 700 Hz
(c) 650 Hz (d) 1000 Hz

27. The phase difference between points 2.5 m apart of the wave $y = (.4m) \sin [(.628\ m^{-1})]\ x - 600\ \pi t]$

(a) $\pi/4^{rad}$ (b) $\pi/3^{red}$
(c) $\pi/2^{red}$ (d) $2\pi/3^{rad}$
(e) none

28. 30 tuning forks are so kept that each gives 3 beats with preceding or succeeding tuning fork. The frequency of last tuning fork is octave of first. The frequency of 3rd fork is

(a) 87 Hz (b) 90 Hz
(c) 84 Hz (d) 93Hz
(e) none

29. The sound level 10 m away from a source is 60 db. The sound level at 50 m away is

(a) 12 db (b) 24 db
(c) 36 db (d) 46 db

30. Pitch is related to

(a) frequency (b) amplitude
(c) wavelength (d) loudness

31. $y = y_0 \sin \omega t$ and $x = x_0 \sin 2\ \omega t$ are two *SHMs*. Their Lissajous figure will be

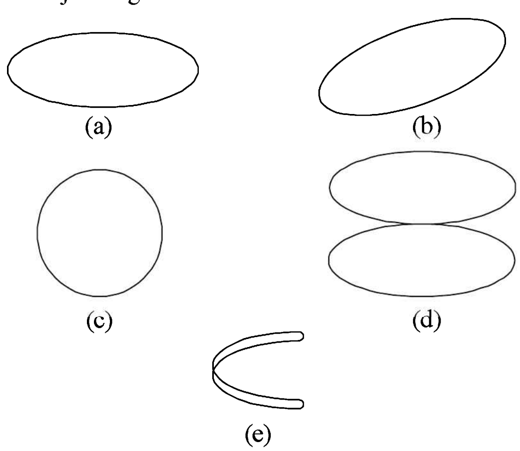


Fig. 5

32. Two waves $y_1 = 6 \sin (\omega t - kx)$ and $y_2 = 8 \sin (\omega t - kx)$ interfere. The maximum to minimum intensity ratio is

(a) 7 (b) 49
(c) 25 (d) 3.8
(e) none

33. Two successive resonant frequencies in a resonant pipe are 1944 and 2592 Hz. Find the length of the tube $V_s = 324\ ms^{-1}$

(a) 25 cm (b) 50 cm
(c) 40 cm (d) none

34. A tuning fork produces 4 beats s^{-1} with another tuning fork of frequency 256 Hz. The first is loaded with wax and number of beats heard are 6. The frequency of first tuning fork is

(a) 250 Hz (b) 262 hz
(c) 252 Hz (d) 260 Hz

35. A kundt's tube has 1m long Cu rod clamped at 25 cm from one of the ends. It contains air $V_s = 340\ ms^{-1}$. The heaps of lycopodium powder appear at a separation of 5 cm. The speed of sound in copper is

(a) 340.0 ms^{-1} (b) 3400 ms^{-1}
(c) 1700 ms^{-1} (d) 6800 ms^{-1}

36. An electronically driven loudspeaker is placed near the open end of a resonance column apparatus. The length of air colunm in the tube is 80 cm. The frequency of loudspeaker can be varied from 20 Hz to 2 *k*Hz. $V_s = 320\ ms^{-1}$. The frequencies at which the air colunm will resonate are

(a) $100\ (2n + 1)$ where n = 0, 1____ , 9
(b) $100\ n$ where n = 2, 4, 6,____ 20
(c) $200\ (2n + 1)$ where n = 1, 2, 3, 4
(d) $200\ n$ where n = 1, 2,____ 10

37. If the room temperature changes from T to $T + \Delta T$, then in an organ pipe f changes to $f + \Delta f$ such that

(a) $\frac{\Delta f}{f} = \frac{\Delta T}{T}$ (b) $\frac{\Delta f}{f} = \frac{\Delta T}{2T}$
(c) $\frac{\Delta f}{f} = \frac{\Delta T}{3T}$ (d) $\frac{\Delta f}{f} = \frac{2\Delta T}{T}$

38. Two trains are travelling towards each other at a speed of 90 kmh^{-1}. If one of the trains sounds a whistle at 500 Hz the apparent frequency heard in the other train is ____. $V_s = 350\ ms^{-1}$

(a) 577 Hz (b) 538 Hz
(c) 535.7 Hz (d) 557 Hz

39. A car moving at 108 kmh^{-1} finds another car in front of it in the same direction at 72 kmh^{-1}. The first car sounds a horn at 800 Hz. The frequency heard by the driver of front car is

(a) 835 Hz (b) 825 Hz
(c) 843 Hz (d) 850 Hz

40. A source of sound, emiting a 1200 Hz note, travels along a straight line at a speed 170 ms^{-1}. A detector is placed at a distance 200 m from the line of motion. The frequency of sound detected by detector at the instant when the source gets closest to it is ____. $V_s = 340\ ms^{-1}$

(a) 2400 Hz (b) 800 Hz
(c) 1600 Hz (d) none

41. In South Korea a magnetic train is moving at 1.2 Mach. When 1 km from station it sounds a whistle of 600 Hz. V_s = 340 ms^{-1}. Find the frequency heard by a person at the station.

(a) 600 Hz (b) > 600 Hz
(c) < 600 Hz (d) none

42. A piano wire A vibrates at a fundamental frequency of 600 Hz. A second identical wire B produces 6 beats per sec when the tension in A is slightly increased. Find T_A/T_B.

(a) 1.01 (b) 1.03
(c) 1.04 (d) 1.02

43. A person is at P. The sound source is rotating with an angular velocity ω in anticlockwise direction. At what point, P hears the maximum frequency if the sound source emits a continuous sound of frequency f_o.

(a) A (b) D
(c) B (d) C

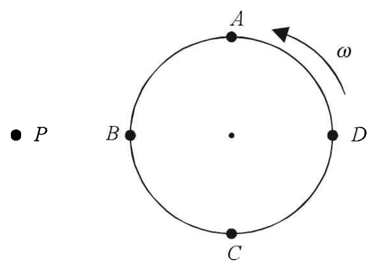


Fig. 6

44. Source S and detector D are distance D apart. Source emits a wavelength of $d/2$ found to be in phase with direct wave received from the source. By what minimum distance reflector be shifted so that direct and reflected waves reach out of phase.

(a) 0.09 d (b) 0.10 d
(c) 0.11d (d) 0.13 d

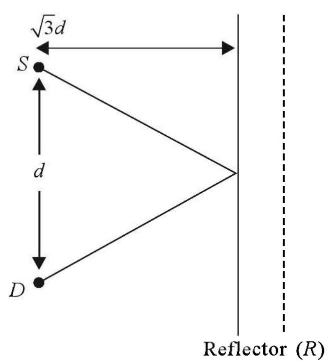


Fig. 7

45. Sources S_1, S_2 and S_3 of equal intensity are placed in a straight line with $S_1S_2 = S_2S_3$. At a point P far away from the sources. Wave coming from S_2 is 120° ahead of phase with respect to S_1 and wave coming from S_3 120° ahead of phase from S_2. The resultant intensity at P is

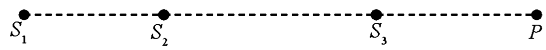


Fig. 8

(a) $\sqrt{3}\, I$ (b) $2I$
(c) zero (d) none

46. A spring of spring constant k is attached to masses m_1 and m_2 at its ends. The frequency of oscillations is ____ if the spring is compressed and released.

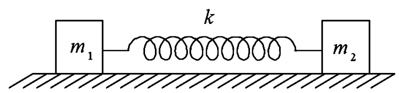


Fig. 9

(a) $f = \dfrac{1}{2\pi}\sqrt{\dfrac{k(m_1+m_2)}{m_1m_2}}$ (b) $2\pi\sqrt{\dfrac{k(m_1+m_2)}{m_1m_2}}$

(c) $\dfrac{1}{2\pi}\sqrt{\dfrac{k}{m_1+m_2}}$ (d) $2\pi\sqrt{\dfrac{k}{m_1+m_2}}$

47. Epoch is the

(a) phase angle at any time t of an SHM or wave
(b) phase difference between two SHMs or waves
(c) phase angle at $t = 0$ of an SHM or a wave
(d) phase difference between two SHMs or waves at $t = 0$

48. Two SHMs are added like ____ and two waves are added like____.

(a) vector, vector (b) scalar, scalar
(c) vector, scalar (d) scalar, vector

49. The relation between velocity and acceleration of a particle executing SHM is

(a) parabola (b) straight line
(c) hyperbola (d) ellipse

50. A lady wearing an earing of length 5 cm sits in a merry-go-round of radius 2 m moving with a speed 4 ms^{-1}. The time period of vibration of her earring is

(a) 0.30 s (b) 0.32 s
(c) 0.37 s (d) 0.35 s

51. The maximum tension of an oscillating pendulum is twice the minimum tension. The angular magnitude is

(a) $\cos^{-1} 1/2$ (b) 90°
(c) $\cos^{-1} 2/3$ (d) $\cos^{-1} 3/4$

52. Find the time period of small oscillation for the figure

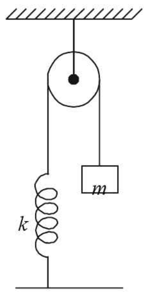


Fig. 10

(a) $T = 2\pi\sqrt{\frac{m}{k}}$ (b) $2\pi\sqrt{\frac{2m}{k}}$

(c) $2\pi\sqrt{\frac{m}{2k}}$ (d) $\pi\sqrt{\frac{m}{k}}$

53. A closed circular wire hung on a nail undergoes small oscillation of 2°. Time period of oscillation is 2 *s*. The radius of the wire is

(a) 40 cm (b) 50 cm

(c) 60 cm (d) 1 m

54. A pendulum of length *l* hangs from the ceiling of a car. The car is moving with uniform acceleration *a*. The time period of oscillation is

(a) $T = 2\pi\sqrt{\frac{l}{g+a}}$ (b) $T = 2\pi\sqrt{\frac{l}{g-a}}$

(c) $T = 2\pi\sqrt{\frac{l}{\sqrt{g^2-a^2}}}$ (d) $2\pi\sqrt{\frac{l}{\sqrt{g^2+a^2}}} = T$

55. A pendulum has a small hole in its bob. The bob is hollow spherical and filled with sand. As the sand comes out the time period of oscillation

(a) remains fixed

(b) first increases then decrecases

(c) first decreases then increases

(d) increases continuously

Answers

1.	(c)	**2.**	(c)	**3.**	(d)	**4.**	(d)	**5.**	(c)	**6.**	(a)	**7.**	(a)
8.	(c)	**9.**	(b)	**10.**	(c)	**11.**	(a)	**12.**	(b)	**13.**	(c)	**14.**	(d)
15.	(b,c)	**16.**	(a)	**17.**	(c)	**18.**	(b,c)	**19.**	(a)	**20.**	(b)	**21.**	(b)
22.	(e)	**23.**	(a)	**24.**	(c)	**25.**	(d)	**26.**	(b)	**27.**	(c)	**28.**	(d)
29.	(d)	**30.**	(a)	**31.**	(d)	**32.**	(b)	**33.**	(a)	**34.**	(c)	**35.**	(b)
36.	(a)	**37.**	(d)	**38.**	(a)	**39.**	(b)	**40.**	(c)	**41.**	(a)	**42.**	(d)
43.	(a)	**44.**	(c)	**45.**	(d)	**46.**	(a)	**47.**	(c)	**48.**	(a)	**49.**	(d)
50.	(c)	**51.**	(d)	**52.**	(a)	**53.**	(b)	**54.**	(d)	**55.**	(b)		

Explanations

1(c)

2(c) $\frac{V_1}{V_2} = \sqrt{\frac{T_{AB}}{T_{CD}}} = \frac{1}{\sqrt{2}}$

3(d) Sound cannot travel in vacuum.

4(d)

5(c) amplitude and frequency are equal and they travel in opposite direction

6(a) $V = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{20}{\frac{5\times10^{-3}}{2.5}}} = 100\ \text{ms}^{-1}$

$$t = \frac{2}{100} = 0.02\text{S}$$

7(a) $V = \sqrt{\frac{T}{\mu}}$ or $T = V^2\mu = \left(\frac{\omega}{k}\right)^2\mu$

$$= \left(\frac{30}{1}\right)^2 (1.2\times10^{-4}) = 0.108\,N$$

8(c) $f = \frac{V}{2l} = \frac{60}{2} = 30\ \text{Hz}$

9(b)

10(c)

11(a) $\Delta V = V_0\left[1+\frac{1}{273}\right]^{½} - V_0 = \frac{332}{546} = 0.61\ \text{ms}^{-1}$

12(b) $V = \sqrt{\frac{K}{\rho}}$ $f = \sqrt{\frac{2.25\times10^9}{10^3}} = 1500\ \text{ms}^{-1}$

13(c)

14(d) $f = m\sqrt{\frac{g}{d_2 - d_1}} = 3\sqrt{\frac{10}{.10}} = 30\ \text{Hz}$

15(b,c)

16(a)

17(c) $k = .314\,cm^{-1} = \frac{2\pi}{\lambda}$ or $l = \frac{\lambda}{2} = \frac{\pi}{.314} = 10\,cm$

18(b, c)

19(a)

20(b) $f = \frac{V}{4l} = \frac{340}{1} = 340\,Hz$.

frequencies allowed are 340 Hz, 1020 Hz, 1700 Hz.

21(b)

22(e) $\frac{I_A}{I_B} = \frac{(2y_0)^2(2f)^2}{y_0^2(f)^2} = 16$

23(a) f_o, open= $\frac{V}{\lambda} = \frac{V}{2l}$,

f_o, closed = $\frac{V}{4l} = \frac{V}{4(l/2)} = \frac{V}{2l} = f_o$, open

24(c) $\frac{V}{2(0.5)} - \frac{V}{2(0.51)} = 8$ or $0.2\,V = 8.08$ or $V = 404\,ms^{-1}$

25 (d) $\lambda = \frac{350}{500} = 0.7\,m$, $\lambda/4 = \frac{70}{4}$

= 17.5 *cm*; resonance will occur at 17.5, 52.5, 87.5, 122.5 cm, i.e, in all 4 resonace can occur

26(b) $V_s = 332 + 30\,(.61) = 350\,ms^{-1}$

$f = \frac{V}{\lambda} = \frac{V}{2l} = \frac{350}{2(.25)} = 700\,Hz$

27(c) $\phi = kx = .628\,(2.5) = \frac{2\pi}{10} \times 2.5 = \pi/2$ rad.

28(d) $3\,(29) + f_o = 2f_o$ or $f_o = 87$ Hz $f_3 = 87 + 2\,(3) = 93$ Hz

29(d) $SL = 10 \log \frac{I}{I_o}$; $I = \frac{I_{max}}{r^2}$; $SL_2 = SL_1 - 10 \log 25$

$= 60 - 10\,(1.398) = 46$ dB

30(a)

31(d)

32(b) $\frac{I_{max}}{I_{min}} = \left(\frac{y_{01} + y_{02}}{y_{01} - y_{02}}\right)^2 = \left(\frac{14}{2}\right)^2 = 49$

33(a) $2f_0 = 648$ or $f_0 = 324$ Hz, $l = \lambda/4 = \frac{V}{4f_0} = \frac{1}{4}\,m$

34(c) $f_1 = f_2 \pm 4$ or $f_1 = 256 \pm 4$

On waxing the frequency decreasing. Therefore, number of beats increases

35(b) $\lambda_{air} = 0.1\,m$; $f = \frac{340}{0.1} = 3400\,Hz$

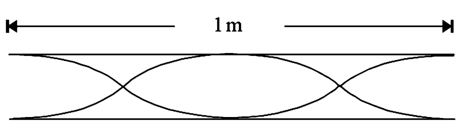


Fig. 11

$\lambda_{cu} = 1\,m$; $= V_{cu} = f\lambda_{cu} = 3400 \times 1 = 3400\,ms^{-1}$

36(a) $f_o = \frac{320}{4(.8)} = 100\,Hz$, $f = 100\,(2n+1)$

where n = 0, 1, ___, 9

37(b) $f_o = \frac{V}{4l}$, $f_o + \Delta f_o = \frac{V_0}{4l}\sqrt{1 + \frac{\Delta T}{T}}$

or $\frac{\Delta f_o}{f_o} = \frac{\Delta T}{2T}$ (use Binomial theorem)

38(a) $f_{ap} = 500\,\frac{(350+25)}{350-25} = 575\,Hz$

39(b) $f_{ap} = \frac{800 \times 330}{320} = 825\,Hz$

40(c) $f_1 = 1200 \times \frac{340}{340-370} = 2400\,Hz$

$f_2 = 1200 \times \frac{340}{340+170} = 800\,Hz$

$f = \frac{f_1 + f_2}{2} = 1600\,Hz$

41(a) *If* $V > V_s$ doppler effect is inapplicable

42(d) $\sqrt{\frac{T_A}{T_B}} = \frac{606}{600} = \frac{101}{100}$ or $\frac{T_A}{T_B} = \left(1 + \frac{1}{100}\right)^2 = 1.02$

43(a) Note that at point *A* the velocity of the source is towards the person *P*.

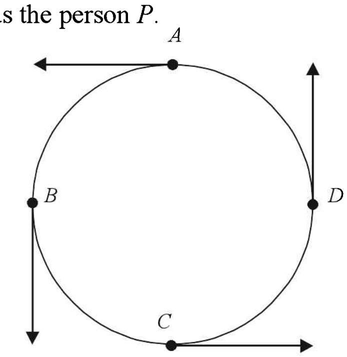


Fig. 12

44(c) $3d - d = \frac{nd}{2}$ or n = 4

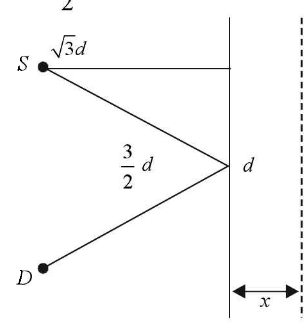


Fig. 13

$2\sqrt{\left(\sqrt{3}d + x\right)^2 + \frac{d^2}{4}} - d = (2n+1)\,\frac{d}{4}$

on solving we get $x = .13\,d$

45(d) $I + (I\cos 120 + I\cos 120) = 0$

46(a) $$-kx = \frac{m_1 m_2}{m_1 + m_2}\, a$$

or $$a = \frac{-k(m_1 + m_2)}{m_1 m_2}\, x$$

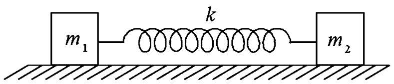


Fig. 14

$$\omega = \sqrt{\frac{k(m_1 + m_2)}{m_1 m_2}}$$

or $$f = \frac{1}{2\pi}\sqrt{\frac{k(m_1 + m_2)}{m_1 m_2}}$$

47(c)

48(a)

49(d)

50(c) $$T = 2\pi \sqrt{\frac{l}{\sqrt{g^2 + \left(\frac{V^2}{r}\right)^2}}}$$

$$= 2\pi \sqrt{\frac{5\times10^{-2}}{\sqrt{10^2 + 64}}}$$

51(a) $mg + 2\,mg\,(1 - \cos\theta) = 2\,mg\cos\theta$

$\cos\theta = 3/4$ or $\theta = \cos^{-1} 3/4$

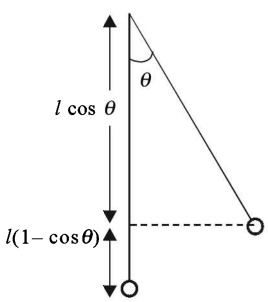


Fig. 15

52(a) $$T = 2\pi\sqrt{\frac{m}{k}}$$

53(b) $$T = 2\pi\sqrt{\frac{I}{mgl}}$$

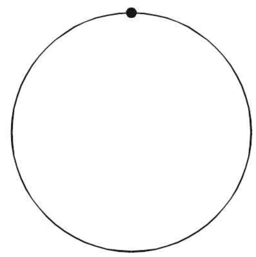


Fig. 16

or $$2 = 2\pi\sqrt{\frac{2MR^2}{MgR}}$$

or $$\left(\frac{1}{\pi}\right)^2 = \frac{2R}{g};$$

$$R = \frac{1m}{2} \text{ or } 50\,cm$$

54(d) $$T = 2\pi\sqrt{\frac{l}{a_{net}}} = 2\pi\sqrt{\frac{l}{\sqrt{a^2 + g^2}}}$$

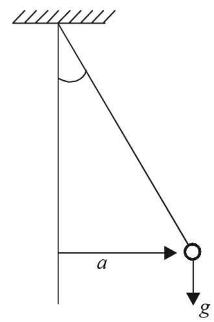


Fig. 17

55(b) As the sand comes out *COM* shifts downwards. Therefore, time period increases. When the bob is vacated more than half COM shifts upwards and l decreases. So time period decreases.

SELF TEST 2

1. At a place, the length of a pendulum is adjusted, so that the period is one second. When the pendulum is placed in an elevator undergoing uniform acceleration it is found to be 1.025 s. The acceleration of the elevator is ($g = 9.8$ m/s^2)

   (a) 0.49 m/s^2 ↑ (b) 0.49 m/s^2 ↓
   (c) 9.8 m/s^2 ↑ (d) 9.8 m/s^2 ↑

2. A uniform thin rod has a mass 1 kg and carries a mass 2.5 kg at B. The rod is hinged at A and is maintained in the horizontal position by a spring having a spring constant 18 kNm^{-1} at C as shown in figure. The angular frequency of oscillation is nearly

   (a) 10 rad/s (b) 20 rad/s
   (c) 40 rad/s (d) 80 rad/s

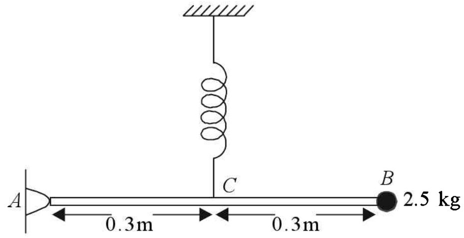


Fig. 1

3. Springs of spring constant k, $3k$, $9k$, $27k$, ..., $315k$ are connected in series. A mass M is attached to the last spring and the system is allowed to make oscillation. The time period will be

(a) $2\pi\sqrt{\dfrac{M}{k}}$ (b) $2\pi\sqrt{\dfrac{3M}{2k}}$

(c) $2\pi\sqrt{\dfrac{2M}{3k}}$ (d) $2\pi\sqrt{\dfrac{M}{k}}$

4. The circular sector of mass m is cut from steel plate of uniform thickness and mounted in a bearing at its centre O so that it can swing freely in the vertical plane. The period τ for small oscillation about the position $\theta = \dfrac{\pi}{2}$ is

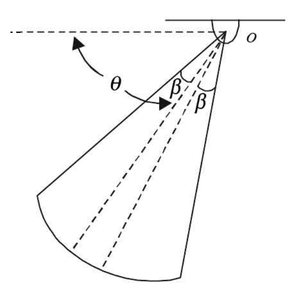


Fig. 2

(a) $2\pi\sqrt{\dfrac{r}{g}}$ (b) $2\pi\sqrt{\dfrac{r\beta}{g\sin\beta}}$

(c) $2\pi\sqrt{\dfrac{3r}{g}}$ (d) $\pi\sqrt{\dfrac{3r\beta}{g\sin\beta}}$

5. A circular disc of mass M lies on a smooth horizontal table. A particle of mass m resting on the disc is attached to the centre of the disc by a spring of spring constant k. Initially the disc and the particle are at rest and spring is stretched through a length x when the system is released. Time period of oscillation is

(a) $2\pi\sqrt{\dfrac{M}{k}}$ (b) $2\pi\sqrt{\dfrac{Mm}{(M+m)k}}$

(c) $2\pi\sqrt{\dfrac{m}{k}}$ (d) $2\pi\sqrt{\dfrac{M+m}{k}}$

6. Mercury of mass m and density ρ is poured into fixed bent tube whose right arm is inclined at α to the horizontal and left arm β to the vertical. If the cross-sectional area is A throughout, the period of oscillation of mercury column (neglecting the effect due to viscosity of mercury) is

(a) $2\pi\sqrt{\dfrac{m}{A\rho g}}$ (b) $2\pi\sqrt{\dfrac{m}{A\rho g(\cos\alpha+\cos\beta)}}$

(c) $2\pi\sqrt{\dfrac{A\rho g}{m}}$ (d) $2\pi\sqrt{\dfrac{m}{A\rho g(\sin\alpha+\cos\beta)}}$

7. For the system shown each spring has a stiffness of 175 N/m. The mass of the pulleys may be neglected. The period of vertical oscillation is

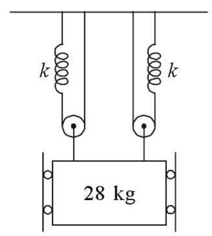


Fig. 3

(a) $\sqrt{2}\,s$ (b) $\pi\sqrt{2}\,s$

(c) $\dfrac{\pi}{5}\,s$ (d) $\dfrac{\pi}{5}\sqrt{2}\,s$

8. The spoked wheel of radius r, mass m and centriodal radius of gyration k attached to a spring of spring constant K as shown in figure rolls without slipping on the incline. The natural frequency of oscillation is

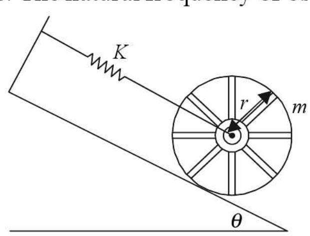


Fig. 4

(a) $\sqrt{\dfrac{K}{m}}$ (b) $\sqrt{\dfrac{m}{K}}$

(c) $\sqrt{\dfrac{K}{m\left(1+\dfrac{k^2}{r^2}\right)}}$ (d) $\sqrt{\dfrac{m\left(1+\dfrac{k^2}{r^2}\right)}{K}}$

9. During the design of the spring support system for the 4000 kg weighing platform; it is decided that the frequnecy of free vertical vibration in the unloaded condition shall not exceed 3 cycles per sec. The maximum acceptable spring constant k for each of three identical spring is

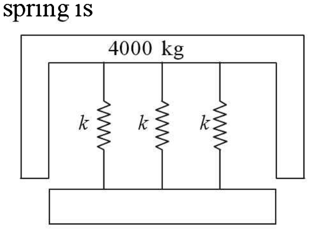


Fig. 5

(a) 12×10^4 N/m (b) 24×10^4 N/m
(c) 36×10^4 N/m (d) 48×10^4 N/m

10. A uniform spring of spring constant k is cut into two pieces whose lengths are in the ratio 1 : 2. The ratio of frequencies of oscillations in the vertical direction when mass m each is suspended from the smaller and the larger pieces of the spring is

(a) $\frac{1}{\sqrt{2}}$ (b) $\sqrt{2}$
(c) $\frac{1}{2}$ (d) 2

11. A mass m attached to a spring is oscillating in water. If the spring constant is k and restoring force Rx. Which one of the following is correct?

(a) $ma = Rx + kx$ (b) $ma = Rx - kx$
(c) $ma = -Rx + kx$ (d) $ma = -Rx - kx$

12. A load of mass M is free to move on smooth horizontal rails. A bob of mass m is suspended from the load vertically. The period of small oscillations of m in vertical planes parallel to and perpendicular to the rails are T_1 and T_2 respectively. The ratio $\frac{T_1}{T_2}$ is

(a) $\sqrt{\frac{Mm}{M^2 - m^2}}$ (b) $\frac{M+m}{m}$
(c) $\frac{Mm}{M^2 - m^2}$ (d) $\sqrt{\frac{M}{M+m}}$

13. The ratio of densities of nitrogen and oxygen is 14 : 16. At what temperature the speed of sound will be the same which is in nitrogen at 15°C?

(a) 56.1°C (b) 4.0°C
(c) 38°C (d) 64°C

14. Simple harmonic wave is represented by $y(x, t) = a_0 \sin 2\pi\left(vt - \frac{x}{\lambda}\right)$. If the maximum particle velocity is three times the wave velocity, what is the value of λ?

(a) $\frac{\pi a_0}{3}$ (b) $\frac{2\pi a_0}{3}$
(c) πa_0 (d) $\frac{\pi a_0}{2}$

15. Calculate the number of beats heard per second when three sources of equal intensity with frequencies n, $(n - 1)$ and $(n + 1)$ are sounded together.

(a) 1 (b) 2
(c) 3 (d) 4

16. Two perfectly identical wires kept under tension are in unison. When the tension in wire is increased by 1% then on sounding them together 3 beats are heard in 2 seconds. Find the frequency of each wire.

(a) 300 Hz (b) 400 Hz
(c) 256 Hz (d) 288 Hz

17. An organ pipe closed at one end vibrating in its first overtone and another pipe open at both ends vibrating in third overtone are in unison with a given fork. If the length of closed pipe is 30 cm, what is the length of open pipe?

(a) 80 cm (b) 100 cm
(c) 60 cm (d) 70 cm

18. The radius, density and tension of string A are twice the radius, density and tension of another string B. If the lengths are equal, what is the ratio of their fundamental frequencies?

(a) 2 (b) 1
(c) $\frac{1}{2}$ (d) $\frac{1}{4}$

19. A hollow metallic tube of length L and closed at one end produces resonance with a tuning fork of frequency f. The entire tube is then heated so that at equilibrium temperature the length changes by ℓ. If the change in velocity V of sound is v, then the resonance will now be produced by tuning fork of frequency f =

(a) $4(L - \ell)$ (b) $\frac{V+v}{4(L+\ell)}$
(c) $\frac{V+v}{4(L-\ell)}$ (d) $\frac{V-v}{4(L-\ell)}$

20. Two trains cross each other travelling in opposite directions at 72 km/hr. One engine sounds whistle of frequency 1088 Hz. The apparent frequencies heard by observer in the other train before corssing is 1224 Hz and if the speed of sound in air is 340 m/sec. What is the apparent frequency after they cross each other?

(a) 967 Hz (b) 867 Hz
(c) 758 Hz (d) 1000 Hz

21. When a transverse plane wave traverses a medium individual particles execute periodic motion given by equation $y = 4\sin\frac{\pi}{2}\left(2t + \frac{x}{8}\right)$. The phase difference for two position of same particle which are occupied by time intervals 0.4 second apart is

(a) 72° (b) 135°
(c) 45° (d) 108°

22. Sound travels in air at 0°C with a speed of 332 m/sec. The speed of sound at 819°C and 4 atmosphere pressure is

(a) 332 m/s (b) 664 m/s
(c) 996 m/s (d) 1328 m/s

23. A person standing on a main road detects a drop of 10% in the pitch of the horn of a motor car as it crosses him with a high speed. If the velocity of sound in air at that time is 330 m/sec. The speed of the car is

(a) 17.4 m/s (b) 8.7 m/s
(c) 34.8 m/s (d) 69.6 m/s

24. The ratio of fundamental frequencies of two identical strings after one of them was stretched by 2%, while the other was stretched by 4% is (The tension in the string can be taken to be proportional to extension.)

(a) 2 (b) 0.5
(c) 1.4 (d) 0.7

25. A metal rod of length 100 cm is clamped at two points A and B as shown in the figure. Distance of the clamps from nearer ends are 5 cm and 15 cm respectively. The minimum frequency of natural longitudinal oscillation of the rod is (given that Young's modulus of elasticity and density of the rod are $Y = 1.6 \times 10^{11}$ N/m² and ρ = 2500 kg/m³ respectively.)

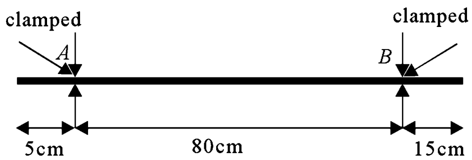


Fig. 6

(a) 20 kHz (b) 40 kHz
(c) 60 kHz (d) 80 kHz

26. A rocket, moving away from earth with a speed of 0.01c, gives a signal of frequency 33.33 MHz where c is the velocity of electromagnetic waves. The apparent frequency observed on earth is

(a) 32.67 MHz (b) 33 MHz
(c) 33.67 MHz (d) 34 MHz

27. A train of sound waves is propagated along a wide pipe and it is reflected form an open end. If the amplitude of the waves is 0.002 cm, the frequency 1000 Hz and the wavelength 40 cm, the amplitude of vibration at a point 10 cm from open end inside the pipe will be

(a) zero (b) 0.002 cm
(c) 0.001 cm (d) 0.0005 cm

28. A small source of sound moves on a circle as shown in figure and an observer is sitting at O. Let n_1, n_2, n_3 be the frequencies heard when the source is at A, B and C respectively. Then

(a) $n_1 > n_2 > n_3$ (b) $n_1 = n_2 > n_3$
(c) $n_1 = n_2 = n_3$ (d) $n_1 > n_2 = n_3$

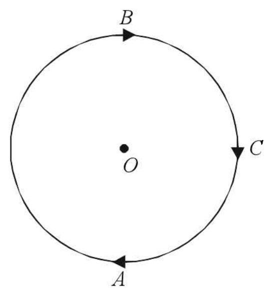


Fig. 7

Answers

1.	(b)	**2.**	(c)	**3.**	(b)	**4.**	(d)	**5.**	(b)	**6.**	(d)	**7.**	(d)
8.	(c)	**9.**	(d)	**10.**	(a)	**11.**	(d)	**12.**	(d)	**13.**	(a)	**14.**	(b)
15.	(a)	**16.**	(a)	**17.**	(a)	**18.**	(c)	**19.**	(b)	**20.**	(a)	**21.**	(a)
22.	(b)	**23.**	(a)	**24.**	(c)	**25.**	(b)	**26.**	(b)	**27.**	(a)	**28.**	(c)

Explanations

1(b) $T_0 = 1\text{s} = 2\pi\sqrt{\dfrac{l}{g}}$

$T = 2\pi\sqrt{\dfrac{l}{g+a}} = 1.025\text{s}$

or $\sqrt{\dfrac{g}{g+a}} = \dfrac{1.025}{1}$ or $a = 0.49\text{ms}^{-2}\downarrow$

2(c) Let x_0 be the extension in spring in equilibrium. In equilibrium

$mgl = kx_0\dfrac{l}{z}$

$mgl - k(x + x_0) = I_A \propto$

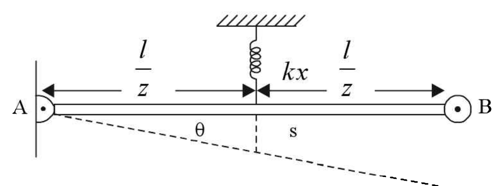


Fig. 8

Where $I_A = ml^2 + \dfrac{m_{rod}l^2}{3} = 2.5\,(.6)^2 + \dfrac{1\times(.6)^2}{3} = 1.02\text{ kgm}^2$

or $I_A \propto = -kx\dfrac{l}{2}$

$\propto = \dfrac{-kl}{I_{A^2}}x$

comparing with $\propto = -w^2x$

$$w = \sqrt{\frac{kl}{2I_A}} = \sqrt{\frac{18\times10^3\times.3}{1.02}} = 40 \text{ rod s}^{-1}$$

3(b) If k_s is effective spring constant,

$$\frac{1}{k_s} = \frac{1}{k} + \frac{1}{3k} + \frac{1}{9k} + \ldots + \frac{1}{315k}$$

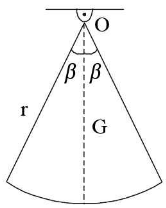
$$\frac{1}{k_s} = \frac{\frac{1}{k}}{1-\frac{1}{3}} = \frac{3}{2k}$$

$$k_s = \frac{2k}{3} \Rightarrow \mathbf{T} = 2\pi\sqrt{\frac{M}{\frac{2k}{3}}} = 2\pi\sqrt{\frac{3M}{2k}}$$

4(d) This is a physical pendulum, with time period

$$\tau = 2\pi\sqrt{\frac{I}{mgd}} \text{ where d = OG}$$

For circular sector, the centre of mass lies at a distance $\overline{X}$ from



O where $\overline{X} = \frac{2r \sin\beta}{3\beta}$

$$I_0 = \frac{mr^2}{2}$$

$$\tau = 2\pi\sqrt{\frac{mr^2\times3\beta}{2\times mg\times2r\sin\beta}} = \pi\sqrt{\frac{3r\beta}{g\sin\beta}}$$

5(b) Since there is no external force on the system, the centre of mass remains fixed in space.

Let the particle m moves towards right through a distance and x and the disc M moves towards left through a distance X before the spring acquires the natural length x_0.

$x + X = x_0$

Here x and X will be amplitude of particle m and disc M respectively.

$mx = Mx$ i.e., $x = \frac{Mx_0}{M+m}$

Tension of spring, $T = kx_0 = k\left(\frac{M+m}{M}\right)x$

Linear acceleration, $\alpha = \frac{T}{m} = \frac{k(M+m)}{mM}x$

Time period, $T = 2\pi\sqrt{\frac{mM}{(M+m)k}}$

6(d) If x be the displacement of mercury column from its equilibrium level on one side then restoring force

$F = -A\,x\,(\sin\alpha + \cos\beta)\,\rho\,g$

$$\omega^2 = \frac{A(\sin\alpha+\cos\beta)\rho g}{m}$$

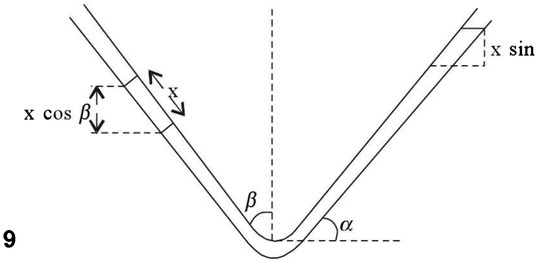


Fig. 9

$$T = 2\pi\sqrt{\frac{m}{A(\sin\alpha+\cos\beta)\rho g}}$$

7(d) Potential energy of the oscillation is concerned with restoring conservative forces only. Here weight of the block is treated as a constant force which is restoring in nature.

When the block is displaced slightly through a distance x, the spring is stretched by a distance 2x.

By the conservation of energy,

$\frac{1}{2}mv^2 + 2\times\frac{1}{2}k(2x)^2 = \text{constant}$, where $A = \frac{dv}{dt}$ and $v = \frac{dx}{dt}$

$\therefore mvA + 8kxv = 0$

Acceleration, $A = -\frac{8kx}{m} = -\omega^2 x$ where $\omega = \sqrt{\frac{8k}{m}}$

Time period, $T = 2\pi\sqrt{\frac{m}{8k}}$

Given m = 28kg, k = 175 N/m

$$\therefore T = 2\pi\sqrt{\frac{28}{8\times175}} = \pi\sqrt{\frac{14}{175}} = \frac{\pi}{5}\sqrt{2}\text{ sec}$$

8(c) By the conservation of energy,

$$\frac{1}{2}Kx^2 + \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = \text{constant}$$

where I is moment of energy of spoked wheel

$I = mk^2$

Since the wheel rolls without slipping, $v = \omega r$

$$\frac{1}{2}Kx^2 + \frac{1}{2}mv^2 + \frac{1}{2}mk^2\frac{v^2}{r^2} = \text{constant}$$

$$Kxv + mvA + \frac{mk^2}{r^2}vA = 0$$

Acceleration, $A = \frac{-Kx}{m+\frac{mk^2}{r^2}} = \omega^2 x$

Nautral frequency of oscillation, $\omega_n = \sqrt{\frac{k}{m\left(1+\frac{k^2}{r^2}\right)}}$

9(d) $v_n = \dfrac{1}{2\pi}\sqrt{\dfrac{k_{eff}}{m}}$

Given $k_{eff} = 3k$, $m = 4000$ kg, $v_n = 3$ Hz, $\pi^2 \simeq 10$

$= \dfrac{1}{2\pi}\sqrt{\dfrac{3k}{4000}}$

$k = \dfrac{4\pi^2 \times v_n^2 \times 4000}{3} = \dfrac{40\times 3^2 \times 4000}{3} = 48 \times 10^4$ N/m

10(a) When a spring of spring constant k and length l is cut in to two pieces of length ℓ_1 and ℓ_2 such that $\ell_2 = n\,\ell_1$ where n is whole number, the spring constant of length $\ell_1 = k_1 = \dfrac{k(n+1)}{n}$ and of length $\ell_2 = k_2 = (n+1)\,k$. In this problem $n = \dfrac{\ell_2}{\ell_1} = 2$

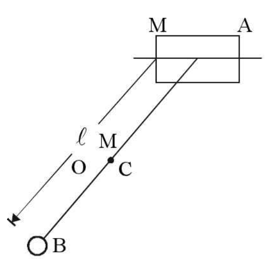
$k_1 = \dfrac{3k}{2}$; $k_2 = 3k$

For smaller one $v_1 = \dfrac{1}{2\pi}\sqrt{\dfrac{k_1}{m}}$

longer one $v_2 = \dfrac{1}{2\pi}\sqrt{\dfrac{k_2}{m}}$ or $\dfrac{v_1}{v_2} = \sqrt{\dfrac{k_1}{k_2}} = \sqrt{\dfrac{1}{2}}$

11(d)

12(d) $T_1 = 2\pi\sqrt{\dfrac{\ell_1}{g}}$



$\ell_1 = \ell - oA = \ell - \dfrac{ml}{M+m}$

$= \dfrac{Ml}{M+m}$

$T_2 = 2\pi\sqrt{\dfrac{\ell}{g}}$ or $\dfrac{T_1}{T_2} = \sqrt{\dfrac{M}{m+M}}$

13(a) If M is the molecular weight of the gas and T be the absolute temperature, then speed of sound in a gas,

$v = \sqrt{\dfrac{\gamma RT}{M}}\ \left\{\because \dfrac{P}{d} = \dfrac{RT}{M}\right\}$

Velocity of sound in oxygen at t°C = $\sqrt{\dfrac{\gamma R(27+t)}{m_o}}$

Velocity of sound in nitrogen at 15°C = $\sqrt{\dfrac{\gamma R(273+15)}{M_N}}$

According to the given problem,

$\sqrt{\dfrac{\gamma R(273+t)}{M_O}} = \sqrt{\dfrac{\gamma R(273+15)}{M_N}}$

or $\dfrac{M_O}{M_N} = \dfrac{273+t}{288}$

$\dfrac{M_O}{M_N} = \dfrac{16}{14}$

Solving, we get t = 56.1°C

14(b) Maximum particle velocity = $\omega a_0 = 2\pi n a_0$

Wave velocity = $n\lambda$

Given $2\pi n a_0 = 3n\lambda$

$\therefore \lambda = \dfrac{2\pi a_0}{3}$

15(a)

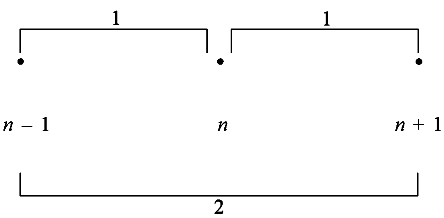


Fig. 10

The system is alike 1Hz and 2 Hz sounded together

$\therefore$ $\Delta f = 2 - 1 = 1$ Hz

16(a) $n = \dfrac{1}{2\ell}\sqrt{\dfrac{T}{m}}$ ... (i)

When the tension in one is increased by 1%, the new tension is $\left(T + \dfrac{T}{100}\right) = 1.01$ T. Since now it produces 3 beats in 2 seconds i.e., 1.5 beats in 1 second, its frequency should be $(n + 1.5)$

$n + 1.5 = \dfrac{1}{2\ell}\sqrt{\dfrac{101T}{m}}$

$= 1.005 \times \dfrac{1}{2\ell}\sqrt{\dfrac{T}{m}}$ ... (ii)

Dividing (ii) by (i), we get

$\dfrac{n+1.5}{n} = 1.005$

or $n = 300$

17(a) If ℓ_1 and ℓ_2 are the lengths of closed and open organ pipes and n_1 and n_2 are the frequencies, then $n_1 = 3.\dfrac{v}{4\ell_1}$ and $n_2 = 4.\dfrac{v}{2\ell_2}$

For resonance, $\dfrac{3v}{4\ell_1} = \dfrac{4v}{2\ell_2}$

or $\dfrac{\ell_1}{\ell_2} = \dfrac{3}{8}$

Given $\ell_1 = 30$ cm $\Rightarrow \ell_2 = 80$ cm

18(c) The frequency $n = \dfrac{1}{2\ell}\sqrt{\dfrac{T}{\pi r^2 d}}$

$$n_A = \frac{1}{2\ell}\sqrt{\frac{2T}{\pi(2r)^2(2d)}}\ ;\ n_B = \frac{1}{2\ell}\sqrt{\frac{T}{\pi r^2 d}}$$

$$\frac{n_A}{n_B} = \sqrt{\frac{1}{4}} = \frac{1}{2}$$

or $n_A : n_B = 1 : 2$

19(b) In the case of a closed pipe before heating the resonance frequency $= \frac{V}{4L}$

After heating the speed of sound becomes $(V + v)$ and the length of the tube becomes $(L + \ell)$.

Hence the frequency of fork in resonance with the tube after heating $= \frac{V+v}{4(L+\ell)}$

20(a) $v_s = v_o = 72$ km/hr $= 20$ m/s

Apparent frequency according to Doppler's principle is given by

$$n' = \frac{n(c-v_o)}{(c-v_s)} = \frac{n[340-(-20)]}{(340-20)} = \frac{n\times 360}{320}$$

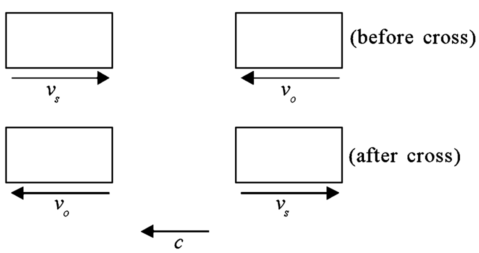


Fig. 11

$$n'' = \frac{n(340-20)}{[340-(-20)]} = \frac{n\times 320}{360}$$

$$\frac{n''}{n'} = \left(\frac{320}{360}\right)^2 = \left(\frac{8}{9}\right)^2$$

$$n'' = n'\left(\frac{8}{9}\right)^2 = 1224 \times \frac{72}{81} = 967 \text{ Hz}$$

21(a) $\omega = \frac{2\pi}{T} = \pi$

$\phi = \omega t = \pi(0.4)$

$= 0.4\,\pi$ rad

$= 0.4(180) = 72°$

22(b) The speed of sound id directly proportional to the square root of the absolute temperature of the gas. Let v_0 and v_{819} be the speed of sound in air at 0°C and 819°C respectively

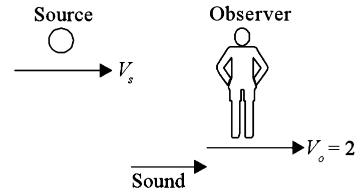
$$\frac{v_{819}}{v_0} = \sqrt{\frac{273+819}{273+0}} = \sqrt{\frac{1092}{273}} = \sqrt{4} = 2$$

$\therefore$ $v_{819} = 2 \times v_0 = 2 \times 332 = 664$ m/s

These is no effect of the change of pressure on the speed of sound.

23(a) By Doppler effect

$$n' = n\left(\frac{v-v_o}{v-v_s}\right)$$



When the car is approaching the person

$$= n \cdot \frac{330}{330-v_s} \quad \text{... (i)}$$

After the car crosses the person (opposite to the direction of sound)

$$n'' = n \cdot \frac{v}{v+v_s} = n \cdot \frac{330}{330+v_s} \quad \text{... (ii)}$$

According to the problem

$$\frac{n'}{n''} = \frac{100}{90} = \frac{10}{9} = \frac{330+v_s}{330-v_s}$$

Solving, $v_s = 17.4$ m/sec.

24(c) If original length is ℓ, then the elongation

$\ell_1 = 1.02\,\ell$ and $\ell_2 = 1.04\,\ell$ ... (i)

Tension $\propto$ elongation

$$\frac{T_1}{T_2} = \frac{n_1}{n_2} = \frac{2}{4} = 0.5 \quad \text{... (ii)}$$

The new frequencies are

$$n_1 = \frac{1}{2(1.02\ell)}\sqrt{\frac{T_1(1.02\ell)}{M}} \quad \text{... (iii)}$$

$$n_2 = \frac{1}{2(1.04\ell)}\sqrt{\frac{T_2(1.04\ell)}{M}} \quad \text{... (iv)}$$

Dividing (iv) by (iii), we get

$$\frac{n_1}{n_2} = \sqrt{\frac{1.04\,T_2}{1.02\,T_1}}$$

$$= \sqrt{\frac{1.04}{1.02}\times\frac{1}{0.5}} = 1.428$$

25(b) Let left, middle and right parts are oscillating in l^{th}, m^{th} and n^{th} harmonics of their fundamental modes and v be the speed of the wave in the rod.

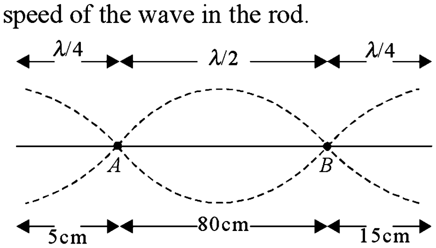


Fig. 12

$$\text{So } \frac{lv}{4\times 5\text{ cm}} = \frac{mv}{2\times 80\text{ cm}} = \frac{nv}{4\times 15\text{ cm}},\ \frac{l}{1} = \frac{m}{8} = \frac{n}{3},$$

For minimum frequency, $l = 1$, $m = 8$ and $n = 3$

And the minimum frequency $f = \frac{v}{0.2}$

$$= \sqrt{\frac{1.6\times10^{11}}{2500}}\left(\frac{1}{0.2}\right) = 40\,\text{kHz}.$$

26(b) $\frac{\Delta f}{f} = -\frac{v}{C}$

$$= -\frac{0.01C}{C} = -0.01$$

$$\Delta f = f_{app} - f = -0.01f$$

$$f_{app} = 0.99f$$

$$= 0.99\times\left(\frac{100}{3}\text{MHz}\right)0.33\times100\,\text{MHz}$$

$$= 33\,\text{MHz}$$

27(a)

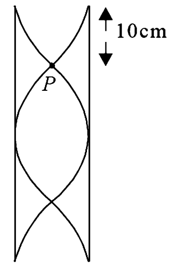


Fig. 13

$\lambda = 40\text{ cm} \Rightarrow \frac{\lambda}{4} = 10\,\text{cm}$

Node is formed at a point 10 cm from the open end inside the pipe i.e., Amplitude at P = zero

28(c)

QUESTIONS FROM COMPETITIVE EXAMINATIONS

AIEEE 2002

1. Tube A has both ends open while tube B has one end closed, otherwise they are identical. The ratio of fundamental frequency of tubes A and B is:

(a) 1 : 2 (b) 1 : 4
(c) 2 : 1 (d) 4 : 1

2. A tuning fork arrangement (pair) produces 4 beat/sec with one fork of frequency 288 cps. A little wax is placed on the unknown fork and it then produces 2 beats/sec. The frequency of the unknown fork is:

(a) 286 cps (b) 292 cps
(c) 294 cps (d) 288 cps

3. A wave $y = a \sin(\omega t - kx)$ on a string meets with another wave producing a node at $x = 0$. Then the equation of the unknown wave is:

(a) $y = a \sin(\omega t - kx)$
(b) $y = -a \sin(\omega t - kx)$
(c) $y = a \cos(\omega t - kx)$
(d) $y = -a \cos(\omega t - kx)$

4. If a spring has time period T, and is cut into n equal parts, then the time period of each part will be:

(a) $T\sqrt{n}$ (b) $T/\sqrt{n}$
(c) nT (d) T

5. Length of a string tied to two rigid supports is 40 cm. Maximum length (wavelength in cm) of a stationary wave produced on it, is:

(a) 20 (b) 80
(c) 40 (d) 120

6. When temperature increases, the frequency of a tuning fork:

(a) increases (b) decreases
(c) remains same
(d) increases or decreases depending on the material

7. A particle is executing SHM then at mean position:

(a) kinetic energy is minimum, potential energy is maximum
(b) both kinetic and potential energies are maximum
(c) kinetic energy is maximum, potential energy is minimum
(d) both kinetic and potential energies are minimum

8. A child swinging on a swing in sitting position, stands up, then the time period of the swing will:

(a) increase (b) decrease
(c) remian same
(d) increase if the child is long and decrease if the child is short

Answers

1.	(c)	**2.**	(b)	**3.**	(b)	**4.**	(b)	**5.**	(b)	**6.**	(b)	**7.**	(c)
8.	(b)												

Explanations

1.(c) For open tube

$$n_A = \frac{v}{2l} \quad ...(1)$$

Since, tube B is closed at one end, therefore, its fundamental frequency
For closed tube

$$n_B = \frac{v}{4l} \quad ...(2)$$

From eqs. (1) and (2), we get

$$\frac{n_A}{n_B} = \frac{v/2l}{v/4l} = \frac{2}{1}$$

$f = 288 \pm 4$ i.e. 284 Hz or 292 Hz
On placing a little wax on unknown tuning fork, its frequency decreases but now the number of beats produced per second is 2, i.e., the frequency difference now decreases. It is possible only when before placing the wax, the frequency of given tuning fork is 292 Hz. Hence, the frequency of unknown tuning fork = 292 Hz

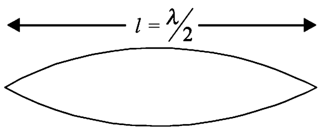
3.(b) $Y = 2a \sin kx \cos \omega t$ satisfies node at $x = 0$

which is formed when the given waves super poses with (b)

4.(b) $T = 2\pi\sqrt{\frac{k}{m}}$

$T' = 2\pi\sqrt{\frac{nk}{m}}$

5.(b) See fig



So that, $\frac{\lambda}{2} = l$

$\lambda = 2 \times 40 = 80$ cm

6.(b)

7.(c) $KE = \frac{1}{2}mw^2(x_0^2 - x^2)$

At $x = 0$ KE is maximum

$P.E. = \left(\frac{1}{2}m\omega^2x^2\right)$

at $x = 0$, P.E. = 0 (minimum)

8.(b) As the child stands up, the effective length of pendulum decreases due to the reason that the centre of gravity rises up. Hence T will decrease

AIEEE 2003

1. A mass M is suspended from a spring of negligible mass. The spring is pulled a little and then released so that the mass executes SHM of time period T. If the mass is increased by m, the time period becomes $5T/3$. Then the ratio of $\frac{m}{M}$ is:

(a) 3/2 (b) 25/9
(c) 16/9 (d) 5/3

2. The lenght of a simple pendulum executing simple harmonic motion is increased by 21%. The perecentage increase in the time period of the pendulum of increase length is:

(a) 11% (b) 21%
(c) 42% (d) 10%

3. The displacement y of a wave travelling in the x-direction is given by $y = 10^4\sin\left(600t - 2x + \frac{\pi}{3}\right)$ metre, where x is expressed in metres and t in seconds. The speed of the wave-motion, in ms^{-1} is:

(a) 300 (b) 600
(c) 1200 (d) 200

4. The displacement of a particle varies according to the relation $x = 4(\cos\pi t + \sin\pi t)$. The amplitude of the particle is:

(a) –4 (b) 4
(c) $4\sqrt{2}$ (d) 8

5. A metal wire of linear mass density of 9.8 g/m is stretched with a tension of 10 kg-wt between two rigid supports 1 metre apart. The wire passes at its middle point between the poles of a permanent magnet and it vibrates in resonance when carrying an alternating current of frequency n. The frequency n of the alternating source is:

(a) 50 Hz (b) 100 Hz
(c) 200 Hz (d) 25 Hz

6. A tuning fork of known frequency 256 Hz makes 5 beats per second with the vibrating string of a piano. The beat frequency decreases to 2 beat per second when the tension in the piano string is slightly increased. The frequency of the piano string before increasing the tension was:

(a) (256 + 2) Hz (b) (256 – 2) Hz
(c) (256 – 5) Hz (d) (256 + 5) Hz

7. A body executes simple harmonic motion. The potential energy (P.E.), the kinetic energy (K.E.) and total energy (T.E.) are measured as function of displacement x. Which of the following statements is true?

(a) K.E. is maximum when $x = 0$
(b) T.E. is zero when $x = 0$
(c) K.E. is maximum when x is maximum
(d) P.E. is maximum when $x = 0$

Answers

1. (c) **2.** (d) **3.** (a) **4.** (c) **5.** (a) **6.** (c) **7.** (a)

Explanations

1.(c) $T = 2\pi\sqrt{\frac{M}{k}}$ or $\frac{T'}{T} = \sqrt{\frac{M+m}{M}}$

$\therefore \quad \frac{5}{3} = \sqrt{\frac{M+m}{M}}$

$\frac{25}{9} = \frac{M+m}{M}$ or $\frac{m}{M} = \frac{16}{9}$

2.(d) $T = 2\pi\sqrt{\frac{l}{g}}$

$\frac{dT}{T} = \frac{1}{2}\frac{dl}{l} = \frac{1}{2} \times 21\% = 10.5\,\%$

3.(a) The given equation of wave

$y = 10^{-4} \sin\left(600t - 2x + \frac{\pi}{3}\right)$ ...(1)

Standard equation of wave
$y = a \sin(\omega t - kx + \phi)$

Now comparing equations (1) and (2), we get,
$\omega = 600$ and $k = 2$

so, velocity of wave $= \frac{\omega}{k} = \frac{600}{2} = 300$ m/s

4.(c) Amplitude $x_0 = \sqrt{4^2 + 4^2} = 4\sqrt{2}$

5.(a) $T = 10 \times 9.8\text{N} = 98\text{N}$,
$m = 9.8 \times 10^{-3}$ kg/m

Frequency $f = \frac{1}{2l}\sqrt{\left(\frac{T}{m}\right)}$

$= \frac{1}{2\times1}\sqrt{\left(\frac{98}{9.8\times10^{-3}}\right)} = 50$ Hz

6.(c) f_2 = frequency of paino = (256 ± 5) Hz
when tension is increased, the beat frequency decreases to 2 beats. This is possible if $f_2 = 251$ Hz as $f \propto \sqrt{T}$

7.(a) At $x = 0$, kinetic energy is maximum and potential energy is minimum.

AIEEE 2004

1. The bob of a simple pendulum executes simple harmonic motion in water with a period t, while the period of oscillation of the bob is t_0 in air. Neglecting frictional force of water and given that the density of the bob is (4/3) × 1000 kg/m³. What relationship between t and t_0 is true?

(a) $t = t_0$ (b) $t = t_0/2$
(c) $t = 2t_0$ (d) $t = 4t_0$

2. A particle at the end of spring executes simple harmonic motion with a period t_1, while the corresponding period for another spring is t_2. If the period of oscillation with the two spring in series is T, then:

(a) $T = t_1 + t_2$ (b) $T^2 = t_1^2 + t_2^2$
(c) $T^{-1} = t_1^{-1} + t_2^{-1}$ (d) $T^{-2} = t_1^{-2} + t_2^{-2}$

3. The total energy of a particle, executing simple harmonic motion is:

(a) $\propto x$ (b) $\propto x^2$
(c) indenpendent of x (d) $\propto x^{1/2}$

where x is the displacement from the mean position.

4. The displacement y of a particle in a medium can be expressed as: $y = 10^{-6} \sin\left(100t + 20x + \frac{\pi}{4}\right)m$, where t is in second and x in metre. The speed of the wave is:

(a) 2000 m/s (b) 5 m/s
(c) 20 m/s (d) 5π m/s

5. A particle of mass m is attached to a spring (of spring constant k) and has a natural angular frequency ω_0. An external force $F(t)$ proportional to cos ω t ($\omega \neq \omega_0$) is applied to the oscillator. The time displacement of the oscillator will be proportional to:

(a) $\frac{m}{\omega_0^2 - \omega^2}$ (b) $\frac{1}{m(\omega_0^2 - \omega^2)}$
(c) $\frac{1}{m(\omega_0^2 + \omega^2)}$ (d) $\frac{m}{\omega_0^2 + \omega^2}$

6. In forced oscillation of a particle, the amplitude is maximum for a frequency ω_1 of the force, while the energy is maximum for a frequency ω_2 of the force, then:

(a) $\omega_1 = \omega_2$ (b) $\omega_1 > \omega_2$
(c) $\omega_1 < \omega_2$ when damping is small and $\omega_1 > \omega_2$ when damping is large
(d) $\omega_1 < \omega_2$

Answers

1. (a)	**2.** (b)	**3.** (c)	**4.** (b)	**5.** (c)	**6.** (a)

Explanations

1.(a) The time period of simple pendulum in air

$$t_o = 2\pi\sqrt{\left(\frac{l}{g}\right)} \quad ...(i)$$

Let ρ be the density to bob and ρ^1 density of water.

$$\therefore \quad t = 2\pi\sqrt{\left[\frac{l}{\left(1-\frac{\rho'}{\rho}\right)g}\right]} \quad ...(2)$$

from (1) and (2)

$$\text{Thus, } \frac{t}{t_0} = \sqrt{\frac{\rho}{\rho-\rho_1}} = \sqrt{\frac{\frac{4}{3}}{\frac{4}{3}-1}} = 2 \text{ or } t = 2\,t_o$$

2.(b) $t_1 = 2\pi\sqrt{\left(\frac{m}{k_1}\right)}$ ...(1)

For second spring

$$t_2 = 2\pi\sqrt{\left(\frac{m}{k_2}\right)} \quad ...(2)$$

$$T = 2\pi\sqrt{\left[\frac{m(k_1+k_2)}{k_1k_2}\right]}$$

$$\Rightarrow \quad T^2 = \frac{4\pi^2 m(k_1+k_2)}{k_1k_2} \text{I} \quad ...(3)$$

from (1), (2) and (3)

$\therefore \quad t_1^2 + t_2^2 = T^2$ [from equation (iii)]

3.(c) Total energy remains const.

4.(b) $y = 10^{-6} \sin\left(100t + 20x + \frac{\pi}{4}\right)$ ...(1)

$$\therefore \quad v = \frac{\omega}{k} = \frac{100}{20} = 5 \text{ m/s}$$

5.(c) Initial angular velocity of particle = ω_0
and at any instant t, angular velocity = ω
Therefore, for a displacement x, the resultant acceleration

$f = (\omega_0^2 - \omega^2)x$ ...(i)

External force $F = m(\omega_0^2 - \omega^2)x$

$$\text{or} \quad x = \frac{F}{m(\omega_0^2 - \omega^2)}$$

$$\text{or} \quad x \propto \frac{1}{m(\omega_0^2 - \omega^2)}$$

6.(a) This is possible only at resonance.
Hence $\omega_1 = \omega_2$

AIEEE 2005

1. The function $\sin^2(\omega t)$ represents:

(a) a periodic, but not simple harmonic, motion with a period $2\pi/\omega$
(b) a periodic, but not simple harmonic, motion with a period π/ω
(c) a simple harmonic motion with a period $2\pi/\omega$
(d) a simple harmonic motion with a period π/ω

2. When two tuning forks (fork 1 and fork 2) are sounded simultaneously, 4 beats per second are heard. A tape is attached on the prong of the fork 2. When the tuning forks are sounded again, 6 beats per second are heard. If the frequency of fork 1 is 200 Hz, then what was the original frequency of fork 2?

(a) 200 Hz (b) 202 Hz
(c) 196 Hz (d) 204 Hz

3. If a simple harmonic motion is represented by $\frac{d^2x}{dt^2} + \alpha x = 0$, its time period is:

(a) $\frac{2\pi}{\alpha}$ (b) $\frac{2\pi}{\sqrt{\alpha}}$
(c) $2\pi\alpha$ (d) $2\pi\sqrt{\alpha}$

4. The bob of a simple pendulum is a spherical hollow ball filled with water. A plugged hole near the bottom of the oscillating bob gets suddenly unplugged. During observation, till water is coming out the time period of oscillation would:

(a) first increase and then decrease to the original value
(b) first decrease and then increase to the original value
(c) remain unchanged
(d) increase towards a saturation value

5. An observer moves towards a stationary source of sound, with a velocity one-fifth of the velocity of sound. What is the percentage increase in the apparent frequency?

(a) 40% (b) 10%
(c) 30% (d) 20%

Answers

1. (b) 2. (c) 3. (b) 4. (a) 5. (d)

Explanations

1.(b) Hence, $y = \sin^2 \omega t$

for SHM $y \propto \sin \omega t$

Hence, function is not SHM, but periodic

From the y-t graph, time period is π/ω

2.(c) $t_2 = 200 \pm 4 = 196$ or 204 Hz

Since, on attaching the tape on the prong of fork 2, its frequency is decrease and beats increase

$\therefore \quad f_2 = 196$ Hz

3.(b) $\dfrac{d^2x}{dt^2} = -\alpha x$

comparing it with $\dfrac{d^2x}{dt^2} = -\omega^2 x$

$$\sqrt{\alpha} = \omega = \frac{2\pi}{T}$$

or $T = \dfrac{2\pi}{\sqrt{\alpha}}$

4.(a) As the COM moves down, length increases until the bob is half filled. Then COM moves up and length of pendulum decreases. Hence time period first increases and then decreases

5.(d) $f' = f_o \dfrac{v + v_L}{v}$

$= f_o \left(\dfrac{6}{5}\right)$

$= f_o(1.2)$

or 20% increase

AIEEE 2006

1. A whistle producing sound waves of frequencies 9500 Hz and above is approaching a stationary person with speed v ms^{-1}. The velocity of sound in air is 300 ms^{-1}. If the person can hear frequencies upto a maximum of 10,000 Hz, the maximum value of v upto which he can hear the whistle is:

(a) $15\sqrt{2}$ ms^{-1} (b) $15/\sqrt{2}$

(c) 15 ms^{-1} (d) 30 ms^{-1}

2. The maximum velocity of a particle, executing simple harmonic motion with an amplitude 7mm, is 4.4 m/s. The period of oscillation is:

(a) 0.01 s (b) 10 s

(c) $T^{-1} = t_1^{-1} + t_2^{-1}$ (d) 100 s

3. Starting from the origin a body oscillates simple harmonically with a period of 2 s. After what time its kinetic energy be 75% of the total energy?

(a) $\dfrac{1}{6}s$ (b) $\dfrac{1}{4}s$

(c) $\dfrac{1}{3}s$ (d) $\dfrac{1}{12}s$

4. A string is stretched between fixed points separated by 75.0 cm. It is observed to have resonant frequencies of 420 Hz and 315 Hz. There are no other resonant frequencies between these two. Then, the lowest resonant frequency for this string is:

(a) 105 Hz (b) 1.05 Hz

(c) 1005 Hz (d) 10.5 Hz

5. A coin is placed on a horizontal platform which undergoes vertical simple harmonic motion of angular frequency ω. The amplitude of oscillation is gradually inceased. The coin will have contract with the platform for the first time:

(a) at the mean position of the platform

(b) for an amplitude of g/ω^2

(c) for an amplitude of $g2/\omega^2$

(d) at the height position of the platform

Answers

1. (c) **2.** (a) **3.** (a) **4.** (a) **5.** (b)

1.(c) $1000 = 9500\left(\frac{300}{300 - v_{source}}\right)$

or $1 - \frac{v_s}{300} = .95$

or $v_s = 300 \times .05 = 15\ ms^{-1}$

2.(a) $v_{max} = x_0\omega = x_0\frac{2\pi}{T}$

$\therefore\quad 4.4 = (7 \times 10^{-3}) \times \frac{2\pi}{T}$

or $\quad T = \frac{7\times10^{-3}}{4.4} \times \frac{2\times22}{7}$

3.(a) $KE = (KE_{max}) \times \frac{75}{100}$

or $\quad \frac{m\omega^2 x_0^2}{2}\cos^2\omega t = \frac{m\omega^2 x_0^2}{2} \times \frac{3}{4}$

or $\quad \cos\omega t = \pm\frac{\sqrt{3}}{2}$

or $\quad \omega t = \frac{\pi}{6} \Rightarrow \frac{2\pi}{T}\times t = \frac{\pi}{6}$ or $t = \frac{T}{12} = \frac{1}{6}s$

4.(a) $315 = \frac{nv}{2L}$ ...(1)

$420 = \frac{(n+1)v}{2L}$ ...(2)

or $\frac{315}{420} = \frac{n}{n+1} \Rightarrow n = 3$

5.(b) $g = y_0\omega^2$

or $y_0 = \frac{g}{\omega^2}$

AIIMS 2005

1. Which of the following functions represents a simple harmonic oscillation?

(a) $\sin\omega t - \cos\omega t$ (b) $\sin^2\omega t$
(c) $\sin\omega t + \cos 2\omega t$ (d) $\sin\omega t - \cos 2\omega t$.

2. If the length of a pendulum is made 9 times and mass of the bob is made 4 times then the value of time period becomes

(a) $3T$ (b) $\frac{3}{2}T$
(c) $4T$ (d) $2T$.

3. There are 26 tuning forks arranged in the decreasing order of their frequencies. Each tuning fork gives 3 beats with the next. The first one is octave of the last. What is the frequency of 18th tuning fork?

(a) 100 (b) 99
(c) 96 (d) 103.

4. A pendulum is undergoing SHM with frequency f. What is the frequency of its kinetic energy?

(a) $\frac{f}{2}$ (b) $2f$
(c) $3f$ (d) $4f$

5. A person is observing two trains one coming towards him and other leaving with the same velocity $4\ \frac{m}{s}$. If their whistling frequencies are 240 Hz each then the number of beats per sec heard by the person will be (if velocity of sound is $320\ \frac{m}{s}$)

(a) 3 (b) 6
(c) 9 (d) zero.

6. The equation of stationary wave along a stretched string is given by $y = 5\sin\frac{\pi x}{3}\cos 40\pi t$, where x and y are in cm and t in second. The separation between two adjacent nodes is

(a) 1.5 cm (b) 3 cm
(c) 6 cm (d) 4 cm.

7. Two waves having sinusoidal waveforms have different wavelengths and different amplitude. They will be having

(a) same pitch and different intensity
(b) same quality and different intensity
(c) different quality and different intensity
(d) same quality and different pitch.

Answers

1. (a) **2.** (a) **3.** (b) **4.** (b) **5.** (b) **6.** (b) **7.** (a)

Explanations

1. (a)

2. (a) $T = 2\pi\sqrt{\dfrac{l}{g}}$

Time period of pendulum is independent of mass of the bob. So after increasing its length 9 times its period will become $T' = 3T$

3. (b) $2f_L = f_L + 25 \times 3$

or $f_L = 75; f_1 = 2f_L = 150$ Hz

$f_{18} = 150 - 17 \times 3 = 99$ Hz

4. (b) Since in each complete oscillation of a body, kinetic energy of oscillations varies from zero to maximum and maximum to zero, two times.

5. (b) The frequency observed by observer from the train approaching him is

$$f' = \frac{v}{v+v_s} f = \frac{320}{320+4} \times 240 = 237 \text{ Hz.}$$

The beats heard per sec by him = 243 – 237 = 6.

6. (b) Given equation of stationary wave

$$y = 5 \sin \frac{\pi x}{3} \cos 40\,\pi t$$

Comparing with standard equation of stationary wave, $y = A \sin kx \cos \omega t$

$$k = \frac{\pi}{3} = \frac{2\pi}{\lambda} \text{ or } \lambda = 6 \text{ cm.}$$

The separation between two adjacent nodes = $\dfrac{\lambda}{2} = 3$ cm.

7. (a) The pitch depends upon the frequency of the source. As the two waves have different amplitude therefore they having different intensity. While quality depends on number of harmonics/overtone produced and their relative intensity. Assuming that their frequency are the same.

Karnataka Cet 2005

1. A particle on the trough of a wave at any instant will come to the mean position after a time (T= time period)

   (a) $T/2$ (b) $T/4$
   (c) T (d) $2T$

2. The disc of a siren containing 60 holes rotates at a constant speed of 360 rpm. The emitted sound is in unison with a tuning fork of frequency

   (a) 10 Hz (b) 360 Hz
   (c) 216 Hz (d) 6 Hz

3. In an experiment with sonometer a tuning fork of frequency 256 Hz resonates with a length of 25 cm and another tuning fork resonates with a length of 16 cm. Tension of the string remaining constant the frequency of the second tuning fork is

   (a) 163.84 Hz (b) 400 Hz
   (c) 320 Hz (d) 204.8 Hz

4. The apparent frequency of a note is 200 Hz when a listener is moving with a velocity of 40 ms^{-1} towards a stationary source. When he moves away from the same source with the same speed, the apparent frequency of the same note is 160 Hz. The velocity of sound in air in *m/s* is

   (a) 340 (b) 330
   (c) 360 (d) 320

Answers

1.	(b)	**2.**	(b)	**3.**	(b)	**4.**	(c)

Explanations

1. (b) The particle will come after a time $\dfrac{T}{4}$ to its mean position.

2. (b) Frequency = (360/60) × 60 = 360 Hz.

3. (b) In case of sonometer frequency is given by

$f = \dfrac{n}{2l}\sqrt{\dfrac{T}{\mu}}$, where μ is the mass per unit length.

$$\therefore \frac{f_2}{f_1} = \frac{l_1}{l_2} \Rightarrow \upsilon_2 \frac{25}{16} \times 256 = 400 \text{ Hz.}$$

4. (c) When listener is moving towards the source,

$$f_1 = \frac{v+v_0}{v} f \Rightarrow 200 = \frac{v+40}{v} f \qquad ...(1)$$

$$f_2 = \frac{v - v_0}{v} f$$

$$\Rightarrow \quad 160 = \frac{v - 40}{v} f \quad ...(2)$$

Dividing equations (1) and (2), $\frac{200}{160} = \frac{v+40}{v-40} = \frac{5}{4}$

$5v - 200 = 4v + 160$

or $v = 360 \text{ ms}^{-1}$.

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1. A massless rod of legth l is hung from the ceiling with the help of two identical wires attached at its ends. A block is hung on the rod at a distance x from the left end. In the case, the frequency of the 1st harmonic of the wire on the left end is equal to the frequency of the 2nd harmonic of the wire on the right. The value of x is.

(a) $\frac{l}{2}$ (b) $\frac{l}{3}$

(c) $\frac{l}{4}$ (d) $\frac{l}{5}$

Solution 1.(d)

1. (d) $y = \sin\omega t - \cos\omega t$

$$\frac{dy}{dt} = \omega\cos\omega t + \omega\sin\omega t$$

$$\frac{dy}{dt} = -\omega^2\sin\omega t + \omega^2\cos\omega t$$

$= -\omega^2(\sin\omega t - \cos\omega t)$.

$a = -\omega^2 y$. $\therefore a \propto -y$.

$f_1 = 2f_2$

$\sqrt{T_1} = 2\sqrt{T_2}$

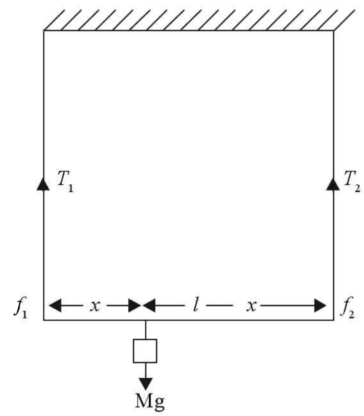
$T_1 = 4T_2$

$T_1(x) = T_2(l - x)$

$4T_2(x) = T_2(l - x)$

$5x = l$

$x = \frac{l}{5}$



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3 Heat and Thermodynamics

15

Kinetic Theory of Gases, Calorimetry and Specific Heat of Gases

BRIEF REVIEW

Zeroeth Law of Thermodynamics If two bodies A and B are in thermal equilibrium and B and C are also in thermal equilibrium then A and C are also in thermal equilibrium.

Note that thermal equilibrium deals with equal temperature. Temperature is measured using a thermometer. An ideal thermometer shall have infinite temperature range. Since no thermometer is ideal, therefore we have large number of thermometers.

Thermometer	Principle	Temperature Range
1. Mercury thermometer	Linear expansion	– 35° C to + 500° C (With compressed N_2)
2. Constant pressure gas thermometer	$\Delta V \propto \Delta T$	0 K to 500 K
3. Constant volume gas thermometer	$\Delta P \propto \Delta T$	0 K to 500 K
4. Platinum resistance thermometer	$\Delta R \propto \Delta T$	500 K to 2300 K
5. Thermocouple thermometer	emf $\varepsilon = \alpha T + \beta T^2$	500 K to 2300 K
6. Pyrometer	Radiation theory	> 2000 K

In mercury thermometer

$\Delta T = \dfrac{\Delta l}{l_0 \alpha}$ is the formula employed, α is linear expansion coefficient.

In constant pressure gas thermometer,

$$\Delta T = \frac{\Delta V}{V_0 \gamma} = \frac{\Delta V \times 273}{V_0}$$

$$\gamma = \frac{1}{273} \text{ for ideal gases.}$$

In constant volume gas thermometer,

$$\Delta T = \frac{\Delta P}{P_{\text{triple point}}\gamma} = \frac{273 \Delta P}{P_{\text{triple point}}}$$

In platinum resistance thermometer,

$$\Delta T = \frac{\Delta R}{R\alpha}$$

α is thermal coefficient of resistance.

In thermocouple thermometer scale is nonlinear. Temperature is either matched with a standard curve supplied by the manufacturer or a digital display is provided. α and β depend upon the materials used to form thermocouple.

$$\text{emf } \varepsilon = \alpha T + \frac{\beta T^2}{2}$$

Pyrometer uses Stefan's Law

Intensity $E = \sigma T^4$

Relation between different temperature scales

$$\frac{C}{100} = \frac{F - 32}{180} = \frac{R}{80} = \frac{R_a - 460}{212}$$

$C \to {}^\circ C$ (degree Celsius)

$F \to {}^\circ F$ (degree Fahrenheit)

$R \to$ Reamuer

$R_a \to$ Rankine

Triple point of water 273.16 K or 0.16° C. Temperature of human body is 37° C or 98.4° F

$-40°C = -40°F$; $574.25 K + 574.25°F$

Barometric formula $P = P_0 e^{-Mgh/RT}$ where M is molar mass, h is height, P_0 is pressure at $h = 0$

$$R = \rho vl/h$$

l being characteristic length

Production and measurement of very low temperatures is called **cryogenics** while measurement of very high temperatures is called **pyrometry.**

Ideal gas equation $PV = nRT$

n = number of moles

$R = 8.314 \text{ J mol}^{-1} \text{K}^{-1}$

Ideal gas is a gas which always obeys the equation $PV = nRT$.

Force exerted by the gas

$$F = \frac{m}{3L}\sum v^2$$

$$\text{Pressure } P = \frac{F}{L^2} = \frac{1}{3}\rho v_{\text{rms}}^2$$

$$PV = \frac{1}{3} M v_{\text{rms}}^2$$

$$PV = \frac{1}{2} N m v_{\text{rms}}^2$$

where N = total number of molecules/atoms

$$v_{\text{rms}} = \sqrt{\frac{3P}{\rho}} = \sqrt{\frac{3PV}{M}} = \sqrt{\frac{3RT}{M}}$$

$M \to$ molar mass

Number of collisions exerted by a gas/unit area of the wall surface per unit time $N = \dfrac{n v_{\text{average}}}{4}$

where $v_{\text{average}} = \sqrt{\dfrac{8kT}{\pi m}} = \sqrt{\dfrac{8RT}{\pi M}}$ m is mass of a single molecule.

Relative number of gas molecules traversing distance x without collision is $N = N_0 e^{-x/\lambda}$ where $\lambda = \dfrac{1}{\sqrt{2}\pi d^2 n}$ is mean free path, n = number of molecules per unit volume and d is effective diameter of the molecule.

$$\text{Kinetic energy } K = \frac{1}{2} M v^2; \quad \frac{P}{P_{\text{triple point}}} = \frac{T}{273.16}$$

Boyle's law $P \propto \dfrac{1}{V}$ or PV = constant

Charles's law of pressure

$$P \propto v_{\text{rms}}^2 \text{ or } p \propto T; \; v_{\text{rms}}^2 \propto T$$

Avogadro's law At the same temperature and pressure, equal volumes of all gases contain equal number of molecules.

Graham's Law of Diffusion When two gases at the same pressure and temperature are allowed to diffuse into each other, the rate of diffusion of each gas is inversely proportional to the square root of the density of the gas.

$$\frac{r_1}{r_2} = \sqrt{\frac{\rho_2}{\rho_1}}$$

$$\text{Diffusion coefficient } D = \frac{1}{3}\lambda v_{\text{average}}$$

Dalton's law of partial pressure The pressure exerted by a mixture of several gases equals the sum of the pressures exerted by each gas occupying the same volume as that of the mixture.

$$P = P_1 + P_2 + \ldots$$

$$v_{\text{average}} = \sqrt{\frac{8kT}{\pi m}} = \sqrt{\frac{8RT}{\pi M}}$$

$$v_{\text{most probable}} = \sqrt{\frac{2kT}{m}} = \sqrt{\frac{2RT}{M}}$$

Maxwell's speed distribution

$$dN = 4\pi N \left[\frac{m}{2\pi kT}\right]^{3/2} v^2 e^{\frac{-mv^2}{2kT}} dv$$

Van der Waals' equation of state

$$\left(P + \frac{a}{V^2}\right)(V - b) = RT \text{ (for one mole)}$$

or $$\left(P + \frac{n^2 a}{V^2}\right)(V - nb) = nRT \text{ for } n \text{ moles}$$

$$V_{\text{critical}} = 3b, \; P_{\text{critical}} = \frac{a}{27b^2},$$

$$T_{\text{critical}} = \frac{8a}{27Rb} = \frac{8}{27} T_B$$

where T_B is Boyle's temperature and $T_B = \dfrac{a}{Rb}$

Relative humidity

$$= \frac{\text{Vapour pressure of air}}{\text{Saturated vapour pressure at the same temperature}}$$

$$= \frac{\text{Saturated vapour pressure at dew point}}{\text{Saturated vapour pressure at air temperature}}$$

The amount of heat needed to raise the temperature of 1g of water by 1°C (14.5°C to 15.5°C) at a pressure of 1 atm is called a calorie.

Specific heat capacity (c) It is the amount of heat required to raise the temperature of a unit mass of substance by 1°C.

$$c = \frac{\Delta Q}{m\Delta\theta} \text{ where } \Delta\theta \text{ is rise in temperature.}$$

Latent heat It is the amount of heat required to change the state of unit mass of a substance from liquid to vapour or solid to liquid without changing the temperature. It is of two types: latent heat of fusion (solid $\rightarrow$ liquid) and latent heat of vaporisation (liquid $\rightarrow$ vapour).

Latent heat of fusion of ice (water) = 80 calg^{-1}

$$L = \frac{\Delta Q}{M}$$

Latent heat of vaporisation of water = 537 calg^{-1} practically taken to be 540 calg^{-1}.

Heat of sublimation Amount of heat required to convert a solid (unit mass) directly to vapour (gas).

Hoar frost is reverse of sublimation. Freezing of clouds is an example.

Reglation is the melting of ice under pressure and its resolidification when pressure is removed.

Variation of melting point in solids and boiling point in liquids (T_k) is given by

$$\frac{dP}{dt} = \frac{JL}{T_K\left(V_{\text{final}} - V_{\text{initial}}\right)}$$

Mechanical equivalent of heat 1 calorie = 4.186 J (practically taken to be 4.2 J).

Law of calorimetry If no heat is wasted to the surroundings then heat gained = heat lost when two bodies are in contact.

Thermal capacity or water equivalent $W = mc$

Specific heat of gases is of two types: specific heat of gases at constant volume (S_V) and specific heat of gases at constant pressure (S_P).

$$S_V = \left[\frac{\Delta Q}{m\Delta T}\right]_{\text{constant volume}}$$

and molar specific heat at constant volume

$$C_v = \left(\frac{\Delta Q}{n\Delta T}\right)_{\text{constant volume}}$$

$$S_P = \left[\frac{\Delta Q}{m\Delta T}\right]_{\text{constant pressure}}$$

and molar specific heat at constant pressure is

$$C_P = \left(\frac{\Delta Q}{n\Delta T}\right)_{\text{constant pressure}}$$

Mayer formula $C_P - C_V = R$

Note that C_V (adiabatic process) = 0

C_V (isothermal process) = ∞

Law of equipartition of energy Each degree of freedom in a gas, on an average, contributes equal energy (average) $\frac{1}{2}kT$/ molecule or $\frac{1}{2}RT$ per mole.

For a monatomic gas

$$C_V = \frac{3}{2}R;\ C_P = \frac{5}{2}R \text{ and } \gamma = \frac{C_P}{C_V} = \frac{5}{3} = 1.67$$

Number of degrees of freedom = 3 (all translational).

For a diatomic gas

$$C_V = \frac{5}{2}R,\ C_P = \frac{7}{2}R \text{ and } \gamma = 7/5 = 1.4$$

Number of degrees of freedom = 5 (3 translation + 2 rotation), assuming molecules do not vibrate.

There are certain gases which do vibrate. In such cases, number of degrees of freedom = 7.

$$C_V = \frac{7}{2}R,\ C_P = \frac{9}{2}R,\ \frac{C_P}{C_v} = \frac{9}{7} = 1.29$$

For polyatomic gases number of degrees of freedom = 6 (3 translation, 2 rotation and 1 vibration)

$$C_V = 3R,\ C_P = 4R\ \gamma = \frac{C_P}{C_v} = \frac{4}{3}$$

If all the degrees of freedom (translational, rotational and vibrational) are excited then for an N-atomic molecule (volume or network), number of degrees of freedom = $(6N-3)$. For an N–atomic linear molecule, number of degrees of freedom = $(6N-5)$

Dulong Petit's law Specific heat of solids at constant volume = $3R$ or 6 calmol^{-1} °C^{-1}. It is valid at high temperatures. It is based on the fact that there are 3 N vibrational states for N molecules.

For a mixture of gases

$$\gamma_{\text{mix}} = \frac{C_{P\text{mix}}}{C_{V\text{mix}}}$$

$$C_{V\text{mix}} = \frac{n_1C_{V1} + n_2C_{V2}}{n_1 + n_2};\ C_{P\text{mix}} = C_{V\text{mix}} + R$$

Specific heat of H_2 is maximum [3.5 calg^{-1} °C^{-1}]. This is followed by water [1 calg^{-1} °C^{-1}]. It is minimum for Radon and Actinium [0.22 calg^{-1} °C^{-1}].

At low temperature, specific heat $C \propto T^3$ (in superconducting range) and at high temperature $C \propto T$.

- **Short Cuts and Points to Note**

1. No gas in real practice is ideal. However, gases like H_2, N_2, O_2 and He may be considered ideal as these cannot be liquified easily. An ideal gas will follow $PV = nRT$ strictly. No molecular forces are present in an ideal gas.

   Therefore $\frac{\partial U}{\partial V} = 0$

2. Vapours above critical temperature are termed as gases, that is, a gas cannot be liquified. Critical temperature is that temperature above which a gas cannot be liquified.

3. Volume correction $b = 4N_A\left[\frac{4}{3}\pi\left(\frac{d}{2}\right)^3\right]$

$= 4N_A\left[\frac{4}{3}\pi r^3\right]$

4. In linear expansion $\Delta L = \alpha L \Delta T$

In superficial (area of the surface) expansion $\Delta S = \beta S \Delta T$

In cubical expansion $\Delta V = \gamma V \Delta T$

Note that $\beta = 2\alpha$

$\gamma = 3\alpha$

Variation of density with temperature

$\rho(T) = \rho_0(1 - \gamma\Delta T)$

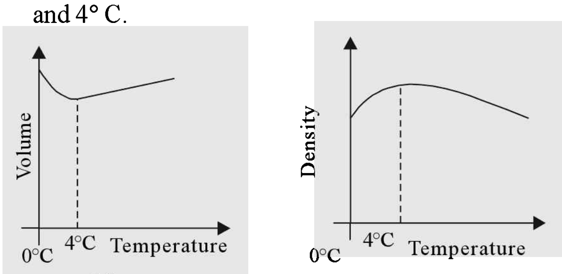
Thermal stress $= Y\alpha\Delta T$

where Y is Young's modulus.

Force = stress × area $= Y\alpha\Delta TA$

A pendulum clock becomes slower in summer and faster in winter. Change in time $\Delta T = \frac{1}{2}\alpha\Delta\theta(T)$

5. Water shows an anomalous behaviour between 0° and 4° C.



(a) (b)

Fig. 15.1 Anomalous behaviour of water

It is due to anomalous behaviour of water that aquatic animals survive even when the upper surface of water in a lake freezes. High density water remains at 4°C.

6. When a liquid expands, since it is contained in a container, we actually measure apparent expansion coefficient γ_{app}. If γ_r is real expansion coefficient then $\gamma_r = \gamma_a + 3\alpha_{container}$

If $\gamma_r > \gamma_{container}$ then $\gamma_{app} > 0$ (level of the liquid rises)

If $\gamma_r < \gamma_{container}$ then $\gamma_{app} < 0$ (level of the liquid falls on heating)

if $\gamma_r = \gamma_{container}$ then $\gamma_{app} = 0$ (level remains unchanged)

$\gamma = \frac{1}{273}$ per degree Celsius for gases

7. Substances like ice do not expand on heating (melting). Rather they expand on cooling in a specific range.

8. 1.0 g of steam at 0° C melts 8 g of ice at 0° C.

9. Thermal capacity = water equivalent = mc (units JK^{-1} or cal °C^{-1}). Heat required when a substance changes state without changing temperature

$\Delta Q = mL$

L is called latent heat.

According to calorimetry heat gained by a substance = heat lost by the other when the two are in contact. It is assumed that no heat is wasted to surroundings.

Specific heat of water = 1 cal g^{-1} °C^{-1}

Specific heat of ice = 0.5 cal g^{-1} °C^{-1}

Heat gained by $-\theta_1$ °C_{ice}

$= m_{ice}C_{ice}\theta_1 + m_{ice}L + m_{ice}C_W\Delta\theta$

Heat gained by 0 °C_{ice}

$= m_{ice}L + m_{ice}C_W\Delta\theta$

Heat lost by water $= m_W C_W \Delta\theta$

10. $v_{rms} = \sqrt{\frac{3P}{\rho}} = \sqrt{\frac{3PV}{M}} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3KT}{m}}$

where M is molar mass and m is mass of a molecule/atom.

R is gas constant and k is Boltzmann's constant.

$R = 8.314 JK^{-1} mol^{-1}$

$k = 1.386 \times 10^{-23}$ erg °C^{-1}

Average velocity $v_{average} = \sqrt{\frac{8RT}{\pi M}} = \sqrt{\frac{8kT}{\pi m}}$

Most probable velocity

$v_{most\ probable} = \sqrt{\frac{2RT}{M}} = \sqrt{\frac{2kT}{m}}$

11. Molar specific heat of gases $C_v = \frac{3}{2}$ R, $C_P = \frac{5}{2}R$,

$\gamma = \frac{5}{3}$ for monoatomic gases

For diatomic gases $C_V = \frac{5}{2}R$, $C_P = \frac{7}{2}R$, $\gamma = \frac{7}{5}$

For polyatomic gases $C_V = 3R$, $C_P = 4R$, $\gamma = \frac{4}{3}$

12. Dulong Petit's law for solids is valid at high temperatures

$C_V = 3R = 6$ cal $mol^{-1}K^{-1}$

13. Specific heat of lighter elements is higher than that of heavier elements and vice versa. Specific heat of the same substance in different states (solid, liquid and vapour) is different. For example, specific heat of water is 1 calg^{-1} °C^{-1}, that of ice is 0.5 calg^{-1} °C^{-1}.

14. With rise in temperature, the weight of a body will increase due to decreasing upthrust.

15. Area of volume expansion of an isotropic body is independent of size and shape of the hole/cavity inside it.

16. For ionic solids linear expansion coefficient is about 10 times more than that of non-ionic substances.

17. Boiling point of a liquid rises with pressure while melting point falls with increasing pressure. Melting points of wax and sulphur (expand on melting) rise with increase in pressure. Impurities also increase boiling point and lower melting point. For example, ice + salt forms freezing mixture.

18. Saturated vapours do not obey gas laws. However, they obey Dalton's law of partial pressure.

19. $\dfrac{v_{\text{rms}}}{v_{\text{sound}}} = \sqrt{\dfrac{3}{\gamma}} = \sqrt{\dfrac{3f}{f+2}}$ where f is number of degrees of freedom and f = (3 N – I) where N is number of particles having I independent relations.

20. KE of monoatomic gas = $\dfrac{3}{2}$ RT (1 mole)

KE of diatomic gas = $\dfrac{5}{2}$ RT (1 mole)

KE of polyatomic gas = 3 RT (one mole)

KE = $\dfrac{m}{2}(v_{\text{rms}})^2$; KE per unit volume

$\dfrac{\text{KE}}{\text{vol.}} = \dfrac{\rho}{2}(v_{\text{rms}})^2$ $\rho \to$ density of the gas

Mean free path $\lambda = \dfrac{1}{\sqrt{2}\pi d^2 n}$ where $d \to$ diameter

- **Caution**

1. Considering all gases have same specific heat.

⇒ Monoatomic gases have

$C_V = \dfrac{3}{2}R,\ C_P = \dfrac{5}{2}R$

Diatomic gases have

$C_V = \dfrac{5}{2}R,\ C_P = \dfrac{7}{2}R$ (no vibration)

Polyatomic gases have $C_V = 3R$, $C_P = 4R$.

2. Considering $nC_V\Delta T$ is change in external energy.

⇒ $nC_V\Delta T$ forms change in internal energy.

3. Considering substances only expand on heating

⇒ Materials like silicon, germanium selenium and cobalt and so on have negative expansion coefficient. Water has negative expansion coefficient between 0 and 4°C.

4. Considering that freezing point of ice = melting point of ice = 0 °C

⇒ Freezing point of ice is –4 °C.

5. Considering specific heat of ice, water and steam are equal.

⇒ Specific heat of ice = 0.5 calg^{-1}°C^{-1} and specific heat of water = 1 calg^{-1} °C^{-1}. Specific heat of steam = 0.75 cal g^{-1} °C^{-1}

6. Confusing molar specific heat and specific heat to be equal.

⇒ Molar specific heat C = molar mass × specific heat = Mc

7. Considering that specific heat is +ve and finite.

⇒ Specific heat can be positive, zero, infinite or negative

Specific heat of saturated vapours is negative.

In adiabatic process specific heat is zero and in isothermal process it is infinite.

8. Considering that increase in length of the pendulum increases time period, and therefore, the clock becomes fast.

⇒ As time interval increases, it will take less oscillation and hence it will become slow.

9. Considering that boiling point and freezing point of water are standard.

⇒ Melting point and boiling point vary with pressure and impurities.

10. Confusing the values of R and k.

⇒ R = 8.314 $JK^{-1}mol^{-1}$

or R = 2 cal

$k = \dfrac{R}{\text{Avogadro number}} = 1.38 \times 10^{-23}\ JK^{-1}$

11. Confusing SI and CGS units.

⇒ It is advised to do questions on calorimetry in CGS as calculations become simple. If the final answer is in Joules, then convert calories into Joules by multiplying the result by 4.2.

SOLVED PROBLEMS

1. A vessel contains 1 mole of O_2 (molar mass 32 g) at a temperature T. The pressure is P. An identical vessel containing 1 mole of He gas (molar mass 4) at a temperature $2T$ has a pressure

(a) P (b) $\dfrac{P}{8}$

(c) $2P$ (d) $8P$

Solution (c) $PV = nRT$

∴ $P_{\text{He}} = 2P$ as temperature of He is doubled.

2. 70 calorie of heat is required to raise the temperature of a diatomic gas at constant pressure from 30 to 35°C. The amount of heat required (in calorie) to raise the temperature of the same gas through the same range (30 to 35°C) at constant volume is

(a) 30 (b) 60
(c) 50 (d) 70

Solution (c) $\frac{C_P}{C_V} = \gamma = 1.4$

$$\frac{(\Delta Q)_P}{(\Delta Q)_V} = \frac{nC_P\Delta T}{nC_V\Delta T}$$

$$= \frac{C_P}{C_V} = 1.4$$

$$\therefore \quad (\Delta Q)_V = \frac{(\Delta Q)_P}{1.4} = 50 \text{ cal}$$

3. A vessel contains 1 mole of O_2 and 1 mole of He. The value of γ for the mixture is

(a) 1.4 (b) 1.50
(c) 1.53 (d) none of these

Solution (b) $C_{V\text{mix}} = \frac{\frac{3}{2}R + \frac{5}{2}R}{2} = 2R$

$$C_{P\text{mix}} = 2R + R = 3R$$

$$\gamma_{\text{mix}} = \frac{C_P}{C_V} = \frac{3}{2}$$

4. Steam at 100° C is passed into a calorimeter of water equivalent 10 g containing 94 cc of H_2O and 10 g of ice at 0° C. The temperature of the calorimeter and contents rise by 5° C. The amount of steam passed is

(a) 1 g (b) 2 g
(c) 3 g (d) 4 g

Solution (b) Let m_S be the amount of steam in grams.

Then $m_s L + m_S C_w (100 - 5)$

$= W_{cal}(5-0) + 10 \times 80 + (104)(5-0)$

$m_S(540) + m_S(95) = 10(5) + 800 + 420$

$635\, m_S = 1270$ or $m_S = 2$g.

5. 10 g of ice is added in 40 g of water at 15°C. The temperature of the mixture is

(a) 0 (b) 3°C
(c) 12°C (d) 8°C

Solution (a) Heat required to melt the ice $= mL$

$= 10 \times 80$

$= 800$ cal.

Maximum heat which can be supplied by hot water $= mC\Delta T = 40 \times 1 \times 15 = 600$ cal

As heat supplied < heat required to melt ice,

∴ temperature of the mixture will be 0° C

(as whole of the ice will not melt).

6. A and B are made up of an isotropic medium. Both A and B are of equal volume. Body B has cavity as shown in Fig 15.2 (b). Which of the following statements is true?

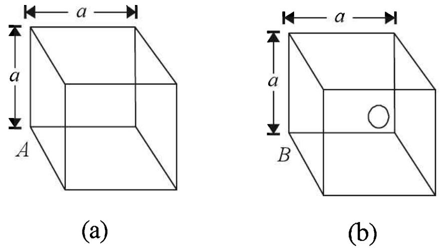


Fig. 15.2

(a) Expansion in volume of A > expansion in B
(b) Expansion in volume of B > expansion in A
(c) Expansion in A = expansion in B
(d) None of these

Solution (c) Thermal expansion of isotropic bodies is independent of shape, size and availability of hole/cavity.

7. If an annular disc of radii r_1 and r_2 is heated, then

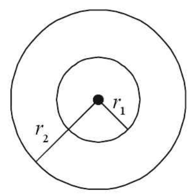


Fig. 15.3

(a) r_1 increases, r_2 decreases
(b) r_2 increases, r_1 decreases
(c) both r_1 and r_2 increase
(d) r_2 increases, r_1 remains unchanged.

Solution (c) The hole also expands.

8. An isotropic solid has linear expansion (coefficient of α_x, α_y and α_z for three rectangular axes in a solid. The coefficient of cubical expansion is

(a) $\alpha_x\alpha_y\alpha_z$ (b) $\frac{\alpha_x}{\alpha_y + \alpha_z}$
(c) $\alpha_x + \alpha_y + \alpha_z$ (d) $\alpha_x^2 + \alpha_y^2 + \alpha_z^2$

Solution (c) $V(T) = V_0(1 + \gamma\Delta T)$ .... (1)

$V(T) = L_x(1 + \alpha_x\Delta T)\,L_y(1 + \alpha_y\Delta T) \times L_z(1 + \alpha_z\Delta T)$

$= LxLyL_z(1 + \alpha_x\Delta T + \alpha_y\Delta T + \alpha_z\Delta T)$ .... (2)

Neglecting square and higher power terms of α_x, α_y, α_z

or $\alpha_x\alpha_y$,..., and $\alpha_x\alpha_y\alpha_z$.

Comparing (1) and (2) we get

$\gamma = \alpha_x + \alpha_y + \alpha_z$.

9. Three rods of equal length l are joined to form an equilateral ΔABC. O is the midpoint of BC. Distance OA remains same for small change in temperature. If the coefficient of linear expansion for AB and AC is α_2 and for BC is α_1, then

(a) $\alpha_2 = 3\alpha_1$ (b) $\alpha_z = 4\alpha_1$
(c) $\alpha_1 = 3\alpha_2$ (d) $\alpha_1 = 4\alpha_2$

Solution (d) $AO^2 = AB^2 - BO^2 = l^2 - \left(\frac{l}{2}\right)^2$

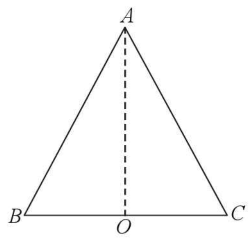


Fig. 15.4

$$l^2 - \left(\frac{l}{2}\right)^2 = l^2(1+\alpha_2\Delta T)^2 - \left(\frac{l}{2}\right)^2 (1+\alpha_1\Delta T)^2$$

$$l^2 - \frac{l^2}{4} = l^2(1+2\alpha_2\Delta T) - \frac{l^2}{4}(1-2\alpha_1\Delta T)$$

(Apply binomial theorem)

or $\alpha_2\Delta T - \frac{\alpha_1}{4}\Delta T = 0$

or $\alpha_1 = 4\alpha_2$

10. Which of the substances A, B or C has the highest specific heat? The temperature *vs* time graph is shown.

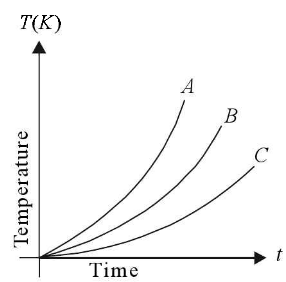


Fig. 15.5

(a) A (b) B

(c) C

(d) All have equal specific heat

Solution (c) Substances having more specific heat take longer time to get heated to a higher temperature and longer time to get cooled.

11. A thin wire of length l when heated to a certain temperature increases its length by 1%. A sheet of the same material of area $2l \times l$ is heated to the same temperature. The percentage increase in area will be

(a) 3% (b) 2.5%

(c) 2% (d) 1.5%

Solution (c) Because $\beta = 2\alpha$.

12. 10^{23} molecules of a gas strike a target of area 1 m^2 at angle 45° to normal and rebound elastically with speed 1 kms^{-1}. The impulse normal to wall per molecule is

(a) 4.7×10^{-24} kg ms^{-1} (b) 7.4×10^{-24} kg ms^{-1}

(c) 3.32×10^{-24} kg ms^{-1} (d) 2.33 kg ms^{-1}

Given: mass of a molecule = 3.32×10^{-27} kg

Solution (a) Change in momentum = 2 mv cos 45

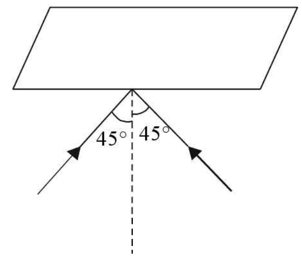


Fig. 15.6

$$= 2 \times 3.32 \times 10^{-27} \times 10^3 \times \frac{1}{\sqrt{2}}$$

$$= 4.7 \times 10^{-24}\ \text{kg ms}^{-1}.$$

13. A cylinder has an alloy piston at a temperature of 20° C. There is all round clearance of 0.5 mm between piston and cylinder wall when the internal diameter of the cylinder is exactly 10 cm. The temperature at which it will exactly fit into the cylinder is

(a) 220° C (b) 250° C

(c) 270° C (d) 290° C

Given: expansion coefficient of alloy is 1.6×10^{-5}/°C and expansion coefficient of cylinder is 1.2×10^{-5}/°C.

Solution (c) Total clearance = 0.05 mm × 2 = 0.1 mm

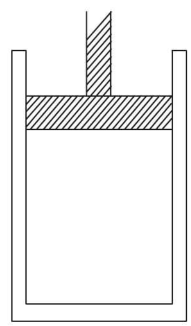


Fig. 15.7

$d(\alpha_2 - \alpha_1)\Delta T = 0.1$ mm = 0.01 cm

$10\,(0.4 \times 10^{-5})\,\Delta T = 0.01$ or $\Delta T = 250$

or $T = 250 + 20 = 270$° C

14. The temperature of an ideal gas is increased from 120 K to 480 K. If at 120 K the rms velocity of the gas molecules is v, at 480 K it becomes

(a) $4v$ (b) $2v$

(c) $\frac{v}{2}$ (d) $\frac{v}{4}$

[IIT 1996]

Solution (b) $\frac{v_1}{v_2} = \frac{\sqrt{\frac{3RT}{M}}}{\sqrt{\frac{3RT_2}{M}}} = \sqrt{\frac{T_1}{T_2}}\quad \therefore v_2 = 2v$

15. The average energy and the rms speed of molecules in a sample of oxygen gas at 300 K are 6.21×10^{-21} J and

484 ms^{-1} respectively. The corresponding values of 600 K are nearly

(a) 12.42×10^{-21} J, 968 ms^{-1}
(b) 8.78×10^{-21} J, 684 ms^{-1}
(c) 6.21×10^{-21} J, 968 ms^{-1}
(d) 12.42×10^{-21} J, 684 ms^{-1} **[IIT 1997]**

Solution (d) Average energy $\propto T$

and $v_{rms} \propto \sqrt{T}$

$\therefore \quad v_{rms2} = \sqrt{2}\, v_{rms1}$

16. A block of ice at $-10°$ C is slowly heated and converted to steam at 100° C. Which of the following curves represents the phenomenon qualitatively?

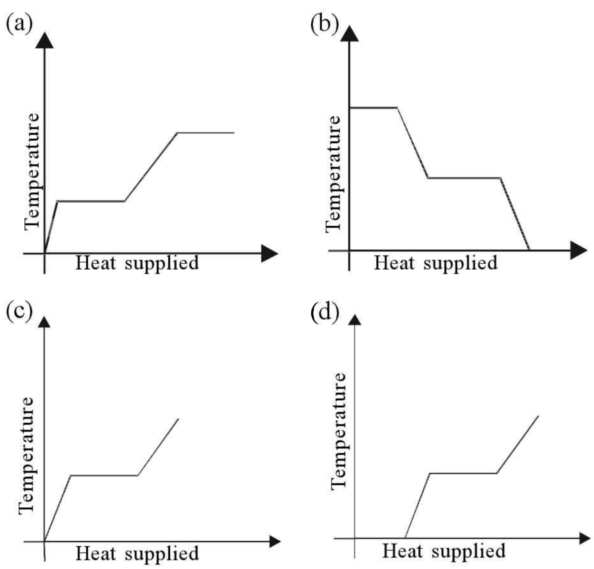


Fig. 15.8

Solution (a) Initially, on heating temperature rises from $-10°$ C to 0° C. Then ice melts and temperature does not rise. After the whole ice has melted, temperature begins to rise until it reaches 100° C. Then it becomes constant, as at the boiling point temperature will not rise.

17. An ideal gas is taken through a cycle $A \to B \to C \to A$ as shown in Fig. 15.9 If the net heat supplied in the cycle is 5 J, then work done by the gas in the process $C \to A$ is

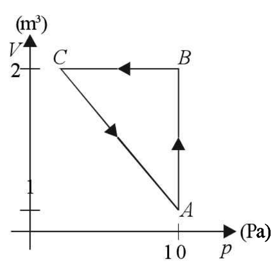


Fig. 15.9

(a) -5 J (b) -10 J
(c) -15 J (d) -20 J

[IIT Screening 2002]

Solution (a) Work done = area under the curve 10 J; $5 = W_{CA} + 10$ or $W_{CA} = -5$ J

18. Two gases having same pressure P and volume V are mixed at a temperature T. If the mixture is at a temperature T and occupies the same volume V then pressure of the mixture would be

(a) P (b) $2P$
(c) $\frac{P}{2}$ (d) $3P$

[VMMC 2003]

Solution (b) If the temperature is constant, then

$$P_1V_1 = P_2V_2$$

$$\therefore \quad P_2 = 2P.$$

19. A and B are two gases. $\frac{T_A}{M_A} = 4\frac{T_B}{M_B}$ where T is the temperature and M is the molecular mass. If C_A and C_B are rms speeds, then $\frac{C_A}{C_B}$ will be

(a) 2 (b) 4
(c) 0.5 (d) 0.25

[BHU 2003]

Solution (a) $\frac{C_A}{C_B} = \frac{\sqrt{\frac{3RT_A}{M_A}}}{\sqrt{\frac{3RT_B}{M_B}}} = 2$

20. The P-T diagram for an ideal gas is shown in Fig. 15.10(a). Where AC is an adiabatic process. The corresponding PV diagram is

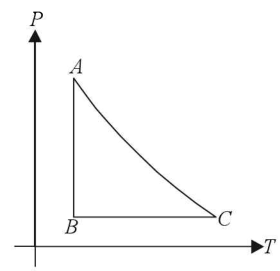


Fig. 15.10 (a)

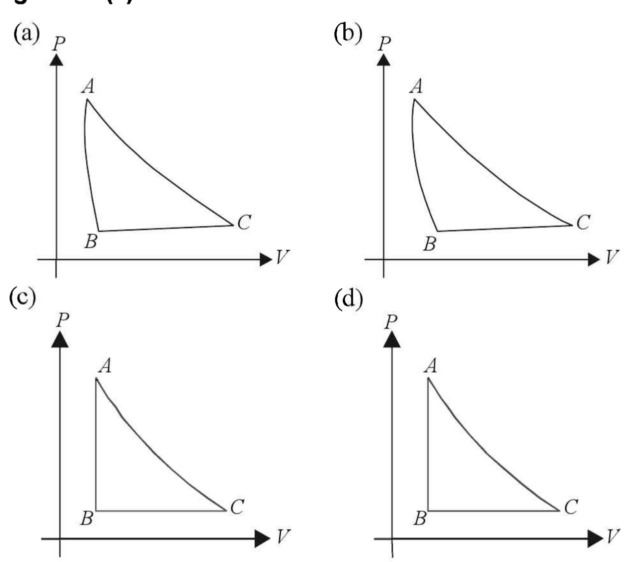


Fig. 15.10 (b)

[IIT Screening 2003]

Solution (b) Process A to B is isothermal. Then $P \propto \frac{1}{V}$. Process $B \to C$ is isobaric and $C \to A$ adiabatic. Slope of adiabatic > slope of isothermal.

21. The temperature of the sun, if pressure is 1.4×10^9 atm, density is 1.4 gcm^{-3} and average molecular weight is 2, will be

[Given $R = 8.4$ J mol^{-1} k^{-1}]

(a) 1.2×10^7 K (b) 2.4×10^7 K
(c) 0.4×10^7 K (d) 0.2×10^7 K

Solution (b) $PV = nRT$ $n = \frac{m}{M}$ and $\rho = \frac{m}{V}$

or $$T = \frac{PV}{nR} = \frac{PM}{\rho R}$$

$$= \frac{1.4\times10^9 \times 1.01\times10^5 \times 2\times10^{-3}}{1.4\times10^3 \times 8.4}$$

$= 2.4 \times 10^7$ K

22. A glass tube scaled at both ends is 1 m long. It lies horizontally with the middle 10 cm containing Hg. The two ends of the tube, equal in length, contain air at 27°C and pressure 76 cm of Hg. The temperature at one end is kept 0° C and at the other end it is 127° C. Neglect the change in length of Hg column. Then the change in length on two sides is

(a) 12.3 cm (b) 10.311 cm
(c) 9.9 cm (d) 8.49 cm

Solution (d) Initially $l = 45$ cm ($2l + 10 = 100$ cm)

$P_1 = P_2 = P$ (say) .... (1)

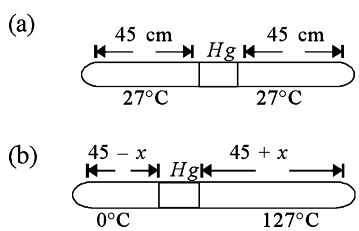


Fig. 15.11

Applying gas law at end A,

$$\frac{45AP}{300} = \frac{(45-x)AP_1}{273} \quad .... (2)$$

At end B, $$\frac{45AP}{300} = \frac{(45+x)AP_2}{400} \quad .... (3)$$

From (1), (2) and (3)

$$\frac{(45-x)}{273} = \frac{45+x}{400} = 8.49 \text{ cm}$$

23. A thin tube of uniform cross-section is sealed at both ends. It lies horizontally. The middle 5 cm contains Hg and two equal ends contain air at the same pressure P_0. When the tube is held at an angle of 60° with the vertical, the length of the air column above and below the Hg are 46 cm and 44.5 cm. Calculate pressure P_0 in cm of Hg. Assume temperature of the system to be constant.

(a) 55 cm of Hg (b) 65 cm of Hg
(c) 70.4 cm of Hg (d) 75.4 cm of Hg

[IIT 1986]

Solution (d) $2L + 5 = 46 + 5 + 44.5 \Rightarrow L = 45.25$ cm

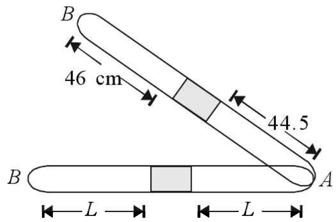


Fig. 15.12

In case (ii) $P_A = P_B + 5 \cos 60$

or $P_A - P_B = 2.5$ cm of Hg

$P_A(44.5) = P_B(46)$

or $$\left(\frac{46}{44.5} - 1\right) P_B = 2.5$$

or $$P_B = \frac{2.5 \times 44.5}{1.5}$$

$P_0(44.25) = P_B \times 46$

Thus $$P_0 = P_B \frac{46}{45.25} = \frac{2.5}{1.5} \times \frac{44.5 \times 46}{45.25}$$

$= 75.4$ cm of Hg

24. Find the amount of work done to increase the temperature of one mole of an ideal gas by 30° C if it is expanding under the condition $V \propto T^{2/3}$.

(a) 166.2 J (b) 136.2 J
(c) 126.2 J (c) none of these

[MNR 1994]

Solution (a) $PV = RT$ for 1 mole

$$W = \int P dV = \int \frac{RT}{V} dV$$

$$V = CT^{2/3}$$

$$\therefore \quad dV = \frac{2}{3} C T^{-2/3} dT$$

or $$\frac{dV}{V} = \frac{2}{3} \frac{dT}{T}$$

$$\therefore \quad W = \int_{T_1}^{T_2} RT \left(\frac{2}{3}\right) \frac{dT}{T}$$

$$= \frac{2}{3} R(T_2 - T_1) = 166.2 \text{ J}$$

25. At what temperature the average translational KE of the molecules of a gas will become equal to the KE of an electron accelerated from rest through 1 V potential difference?

(a) 10^4 K (b) 2.34×10^4 K
(c) 7.73×10^3 K (d) none of these

Solution (c) $\frac{3}{2}KT = 1eV = 1.6 \times 10^{-19}$ J

or $T = \frac{2\times1.6\times10^{-19}}{3\times1.38\times10^{-23}} = 7730$ K

26. 3 mole of H_2 is mixed with 1 mole of Ne. The specific heat at constant pressure is

(a) $\frac{9}{4}R$ (b) $\frac{13}{4}R$

(c) $\frac{9R}{2}$ (d) $\frac{13R}{2}$

Solution (b) $C_{V\text{mix}} = \frac{n_1C_{V1}+n_2C_{V2}}{n_1+n_2}$

$= \frac{\frac{5}{2}R\times3+\frac{3}{2}R\times1}{4}$

$= \frac{18R}{8} = \frac{9R}{4}$

$C_{P\text{mix}} = C_{V\text{mix}}+R = \frac{13R}{4}$

27. The specific heat of Ar at constant volume is 0.075 kcal $kg^{-1}K^{-1}$. Calculate the atomic weight (R = 2 cal $mol^{-1}K^{-1}$).

(a) 40 (b) 40.4

(c) 40.2 (d) 40.8

Solution (a) $M = \frac{C_V}{S_V} = \frac{\frac{3\times2}{2}}{0.075} = 40$

$\because$ $C_V = \frac{3}{2}R$

28. Two rods, one of A1 and other of steel, having initial lengths l_1 and l_2 are connected together to form a single rod of length $l_1 + l_2$. The coefficient of linear expansion of aluminium and steel are α_A and α_S respectively. If the length of each rod increases by same amount when the temperature is raised by t^0C then find the relation $\frac{l_1}{l_1+l_2}$.

(a) $\frac{\alpha_A}{\alpha_A+\alpha_S}$ (b) $\frac{\alpha_S}{\alpha_A}$

(c) $\frac{\alpha_A}{\alpha_S}$ (d) $\frac{\alpha_S}{\alpha_A+\alpha_S}$

[IIT Screening 2003]

Solution (d) $\frac{l_2}{l_1} = \frac{\alpha_A}{\alpha_S}$

or $1+\frac{l_2}{l_1} = 1+\frac{\alpha_A}{\alpha_S}$

or $\frac{l_1}{l_1+l_2} = \frac{\alpha_S}{\alpha_A+\alpha_S}$

29. 2 kg of ice at –20° C is mixed with 5 kg of H_2O at 20° C in an insulating vessel having negligible heat capacity. Calculate the final mass of water left in the container. **Given:** specific heats of water and ice are 1 kcal kg^{-1} °C^{-1} and 0.5 kcal kg^{-1} °C^{-1} and latent heat of fusion of ice is 80 k cal kg^{-1}.

(a) 7 kg (b) 6 kg

(c) 4 kg (d) 2 kg

[IIT Screening 2003]

Solution (b) $m'_{\text{ice}}L + m_{\text{ice}}20\,(0.5) = m_W(1)\,(20)$

$m'_{\text{ice}}80 = 5\times20 - 2\times(20)\times0.5 = 80$

$\therefore$ $m'_{\text{ice}} = 1$ kg

Hence total amount of water = 6 kg.

30. An electrically heating coil is placed in a calorimeter containing 360 g of H_2O at 10° C. The coil consumes energy at the rate of 90 W. The water equivalent of calorimeter and the coil is 40 g. The temperature of water after 10 minutes will be

(a) 42.14°C (b) 32.14°C

(c) 22.14°C (d) 52.14°C

Solution (a) $Q = P\,.\,t = (m+W)\,C\,\Delta T$

$\frac{90\times600}{4.2} = (360+40)\Delta T$

or $\Delta T = 32.14$°C

or $T = 10+32.14 = 42.14$°C

31. A bimetallic strip is formed out of two identical strips, one of Cu and the other of brass. The coefficients of linear expansion of the two metals are α_C and α_B. If on heating the temperature of the strip goes up by ΔT and the strip bends to form an arc of radius R, then R is

(a) proportional to ΔT

(b) inversely proportional to ΔT

(c) proportional to $|\alpha_B - \alpha_C|$

(d) inversely proportional to $|\alpha_B - \alpha_C|$

[IIT 1999]

Solution (b), (d) $l_B = l_0(1+\alpha_B\Delta T)$;

$l_C = l_0(1+\alpha_C\Delta T)$

$l_C = R\theta;\ l_B + (R+d)\theta$

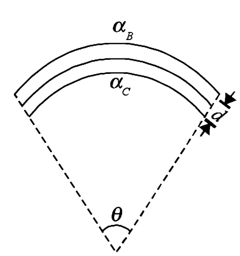


Fig. 15.13

Thus $\frac{R+d}{R} = \frac{1+\alpha_B\Delta T}{1+\alpha_C\Delta T}$

or $1+\frac{d}{R} = 1+(\alpha_B-\alpha_C)\Delta T$

(Thus correct choices are (b) and (d)

TYPICAL PROBLEMS

32. A Cu rod and a steel rod maintain a difference in their lengths constant = 10 cm at all temperatures. If their coefficients of expansion are $1.6 \times 10^{-5} K^{-1}$ and $1.2 \times 10^{-5} K^{-1}$, then the length of the Cu rod is

(a) 40 cm (b) 30 cm
(c) 32 cm (d) 24 cm

Solution (b) Let l_1 be the length of Cu and l_2 that of steel $(l_2 - l_1) = (l_2 - l_1) + (l_2\alpha_2 - l_1\alpha_1)\,\Delta T$

or $l_2\alpha_2 - l_1\alpha_1 = 0$

that is, $\frac{l_2}{l_1} = \frac{\alpha_1}{\alpha_2}$

$$\frac{l_2}{l_1} - 1 = \frac{\alpha_1}{\alpha_2} - 1$$

$$\frac{l_2 - l_1}{l_1} = \frac{\alpha_1 - \alpha_2}{\alpha_2} \text{ or } \frac{10}{l_1} = \frac{0.4}{1.2}$$

or $l_1 = 30$ cm.

33. Fig 15.14 (a) shows a $V - T$ diagram for an ideal gas. Convert it to PV diagram.

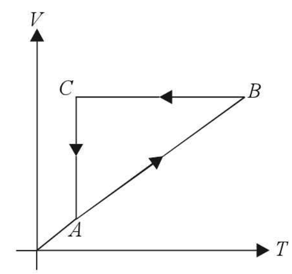


Fig. 15.14 (a)

Solution In process AB, $V \propto T$. That is, P is constant. In BC volume is constant.

$\therefore$ $P \propto T$

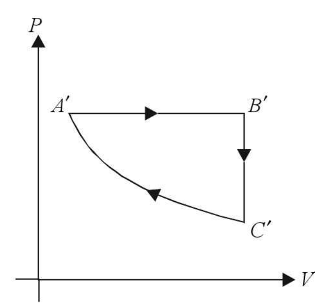


Fig. 15.14 (b)

In C to A, T is constant.

$\therefore$ $P \propto \frac{1}{V}$ (Boyle's law).

34. A mercury thermometer reads 80° C when the mercury is at 5.2 cm mark and 60° C when the mercury is at 3.9 cm mark. Find the temperature when the mercury level is at 2.6 cm mark.

Solution $\frac{l_1 - l_2}{l_1 - l_3} = \frac{\alpha l_0 (T_1 - T_2)}{\alpha l_0 (T_1 - T_3)} = \frac{T_1 - T_2}{T_1 - T_3}$

or $\frac{5.2 - 3.9}{5.2 - 2.6} = \frac{80 - 60}{80 - T_3}$

$1.3\,(80 - T_3) = 2.6\,(20)$ or $T_3 = 40°$ C

35. A thermally insulated vessel contains two liquids at temperature T_1 and T_2 respectively and specific heats C_1 and C_2 separated by a non-conducting partition. The partition is removed and the difference between the initial temperature of one of the liquids and the temperature T established in the vessel turns out to be equal to half the difference between the initial temperature of the liquids. Determine the ratio $\frac{m_1}{m_2}$ (masses of the liquids).

(a) $\frac{C_1}{C_2}$ (b) $\frac{C_2}{C_1}$
(c) $\frac{C_2 + C_1}{C_2 - C_1}$ (d) $\frac{C_2 - C_1}{C_2 + C_1}$

[Olympiad 1994]

Solution (b) $T - T_2 = T_1 - T = \frac{(T_1 - T_2)}{2}$ (given T is temperature of the mixture)

Apply heat lost = heat gained

$m_1 C_1 (T_1 - T) = m_2 C_2 (T - T_2)$

$$\frac{m_1}{m_2} = \frac{C_2 (T - T_2)}{C_1 (T_1 - T)} = \frac{C_2}{C_1}$$

36. The saturated vapour pressure on a planet is 760 mm of Hg. Determine the vapour density.

[Olympiad 1995]

Solution This vapour pressure can be obtained only at 100°C or 373 K, that is, at boiling point of water.

Using $PV = nRT$ $n = \frac{m}{M}$ and $\rho = \frac{m}{V}$

$$\frac{PM}{\rho} = RT$$

$$\therefore \quad \rho = \frac{PM}{RT} = \frac{10^5 \times 18 \times 10^{-3}}{8.31 \times 373} = 0.58 \text{ kg/m}^3$$

37. A vertical cylinder piston system has cross-section S. It contains 1 mole of an ideal monoatomic gas under a piston of mass M. At a certain instant a heater is switched on which transmits a heat q per unit time to the cylinder. Find the velocity v of the piston under the condition that pressure under the piston is constant and the system is thermally insulated.

[Olympiad 1996]

Solution Gas pressure $= P_0 + \frac{Mg}{S}$, where M is mass of the piston.

$$\text{As } C_v = \frac{3}{2}R$$

$$\therefore \quad \Delta U = \frac{3}{2}R\Delta T = \frac{3}{2}P\Delta V$$

$$Q = P\Delta V + \Delta U = P\Delta V + \frac{3}{2}P\Delta V = \frac{5}{2}P\Delta V$$

$$\Delta V = Sdx$$

$$\text{or} \quad Q = q\,.\,dt = \frac{5}{2}PSdx$$

$$\text{or} \quad \frac{dx}{dt} = \frac{2q}{5PS} = \frac{2q}{5\left(P_0 + \frac{Mg}{S}\right)S}$$

38. An Indian pitcher has 10 kg of water. Water cools by means of evaporation through the pores. Find the time in which the temperature of water falls by 5° C. The rate of vaporisation is 5g min^{-1}

(a) 20 min 10 s (b) 18 min 26 s
(c) 14 min 12 s (d) none of these

Solution (b) Let $\frac{dm}{dt}$ be the rate of evaporation and $M = M_0 - t\frac{dm}{dt}$ is the amount of liquid left in the pitcher at any instant.

$$MC\,d\theta = \left(\frac{dm}{dt}t\right)L$$

$$\left(M_0 - \frac{dm}{dt}.t\right)Cd\theta = Lt\frac{dm}{dt}$$

$$\text{or} \quad \left(10000 - \frac{5}{60}.t\right)1 \times 5 = \frac{5}{60}t(540)$$

$$\text{or} \quad 50000 = 45t + \frac{5}{12}t$$

$$\text{or} \quad t = \frac{50,000 \times 12}{545} = 18 \text{ min } 26 \text{ s}$$

39. One mole of an ideal gas undergoes a process $P = \frac{P_0}{1+\left(\frac{V}{V_0}\right)^2}$, where P_0 and V_0 are constants.

Find the temperature of the gas when $V = V_0$.

(a) $\frac{P_0V_0}{R}$ (b) $\frac{2P_0V_0}{3R}$
(c) $\frac{2P_0V_0}{R}$ (d) $\frac{P_0V_0}{2R}$

Solution (d) Since the gas is ideal, therefore $P_0V_0 = RT$

when $V = V_0$, $P = \frac{P_0}{2}$

$$\therefore \quad \frac{P_0}{2}(V_0) = RT$$

$$T = \frac{P_0V_0}{2R}.$$

40. A barometer tube 90 cm long contains some air above mercury. The reading is 74.8 cm when true atmospheric pressure is 76 cm and temperature is 30° C. If the reading is observed to be 75.4 cm on a day when temperature is 10° C, then find the true pressure.

(a) 76.07 cm (b) 75.6 cm
(c) 76.57 cm (d) 77.123 cm

Solution (c) Let A be the area of the cross-section

$$V_1 = (90 - 74.8)A = 15.2A \text{ cm}^3$$

$$P_1 = 76 - 74.8 = 1.2 \text{ cm of Hg}$$

$$P_2 = (P - 75.4) \text{ cm of Hg}$$

$$V_2 = (90 - 75.4)A = 14.6A \text{ cm}^3$$

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2} = \frac{1.2 \times 15.2 \times 283}{303 \times 14.6} = 75.4 + 1.17$$

$$= 76.57 \text{ cm of Hg}$$

41. A copper cube of mass 200 g slides down on a rough inclined plane of inclination 37° at a constant speed. Assume mechanical energy goes into the copper block as thermal energy. Find the increase in temperature of the block as it slides down through 60 cm. Specific heat capacity of copper is 420 Jkg^{-1}K^{-1}

(a) .086 °C (b) 0.86 °C
(c) 0.0086 °C (d) none

Solution (c) $E = \text{mgh} = 0.2 \times 10 \times (0.36) = 0.72$ J

$$E = mC\Delta\theta = 0.72$$

$$\Delta\theta = \frac{0.72}{420 \times 0.2} = 8.6 \times 10^{-3}\,°\text{C}$$

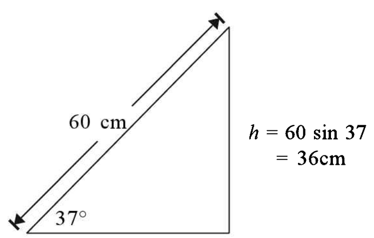


Fig. 15.15

42. A 5 g piece of ice at –20° C is added into 50 g of water at 20° C. Find the temperature of the mixture.

(a) 10 °C (b) 12.6 °C
(c) 13.1 °C (d) 14.2 °C

Solution (a) Let the temperature of mixture be θ. Heat lost by hot water = heat gained by ice.

$$m_WC_W(20 - \theta) = mC_{ice}\Delta\theta + mL + mC_W\theta$$

$$50(1)(20 - \theta) = 5 \times 20(0.5) + 5 \times 80 + 5 \times 1 \times \theta$$

$$55\,\theta = 550$$

$$\text{or} \quad \theta = 10°\text{C}$$

QUESTIONS FOR PRACTICE

1. The internal energy of 1 mol of an ideal gas depends on

(a) only volume (b) only temperature
(c) only pressure
(d) temperature and pressure

2. The temperature of a gas is increased from 27° C to 127° C. The ratio of its mean kinetic energies will be

(a) 10/9 (b) 9/16
(c) 4/3 (d) 3/4

3. A vessel of volume 4 litres contains a mixture of 8 g of O_2, 14 g of N_2 and 22 g of CO_2 at 27° C. The pressure exerted by the mixture is

(a) 5×10^6 N/m² (b) 6×10^3 N/m²
(c) 10 atmosphere (d) 7.79×10^5 N/m²

4. Equal masses of N_2 and O_2 gases are filled in vessels A and B. The volume of vessel B is double of A. The ratio of pressure in vessel A and B will be

(a) 16 : 7 (b) 16 : 14
(c) 32 : 7 (d) 32 : 28

5. The mean kinetic energy of a gas molecule at 27° C is 6.21×10^{-21} Joule. Its value at 227° C will be

(a) 12.35×10^{-21} Joule (b) 11.35×10^{-21} Joule
(c) 10.35×10^{-21} Joule (d) 9.35×10^{-21} Joule

6. The value of γ of triatomic gas (linear arrangement) molecules is

(a) 5/3 (b) 7/5
(c) 8/6 (c) 9/7

7. The correct relation between V_{rms}, V_{av} and V_{mp} is

(a) $V_{rms} > V_{mp} > V_{av}$ (b) $V_{rms} < V_{av} < V_{mp}$
(c) $V_{rms} > V_{av} > V_{mp}$ (d) $V_{rms} < V_{av} > V_{mp}$

8. One mol of a monoatomic gas is mixed with one mol of a diatomic gas. The molar specific heat of mixture at constant volume will be

(a) $R/2$ (b) R
(c) $2R$ (d) $3R$

9. The value of C_V for 1 mol of polyatomic gas is (f = number of degrees of freedom)

(a) $\frac{fR}{2T}$ (b) $\frac{fR}{2}$

(c) $\frac{fRT}{2}$ (d) $2fRT$

10. The pressure of a gas in a container is 10^{-11} pascal at 27° C. The number of molecules per unit volume of vessel will be

(a) 6×10^{23} cm^{-3} (b) 2.68×10^{19} cm^{-3}
(c) 2.5×10^6 cm^{-3} (d) 2400 cm^{-3}

11. The value of γ for gas X is 1.66, then X is

(a) Ne (b) O_3
(c) N_2 (d) H_2

12. The mass of O_2 molecules is 16 times that of H_2 molecules. The rms velocity of O_2 molecules at room temperature is C_{rms}. The rms velocity of H_2 molecules at the same temperature will be

(a) $16\,C_{rms}$ (b) $4\,C_{rms}$
(c) $\frac{C_{rms}}{4}$ (d) $\frac{C_{rms}}{16}$

13. The amount of heat required to increase the temperature of 1 mol of a triatomic gas (non-linear) at constant volume is n times the amout of heat required for 1 mol of monatomic gas. The value of n will be

(a) 1 (b) 1.3
(c) 2 (d) 2.5

14. At what temperature will the mean molecular energy of a perfect gas be one-third of its value at 27° C?

(a) 10° C (b) 10^1 K
(c) 10^2 K (d) 10^3 K

15. In the gas equation $PV = RT$, V is the volume of

(a) 1 mol of gas (b) 1 g of gas
(c) gas (d) 1 litre of gas

16. The mean kinetic energy of gas molecules is zero at

(a) 0° C (b) –273° C
(c) 100 K (d) 100° C

17. The speed of sound in a gas is V. If the rms velocity of gas molecules is C_{rms} then the value of $\frac{V}{C_{rms}}$ will be.

(a) $\frac{3}{\gamma}$ (b) $\frac{\gamma}{3}$
(c) $\sqrt{\frac{3}{\gamma}}$ (d) $\sqrt{\frac{\gamma}{3}}$

18. The temperature at which the rms speed of gas molecules becomes double its value at 0° C is

(a) 819° C (b) 760° C
(c) 273° C (d) 224° C

19. One mol of a gas at NTP is suddenly expanded to three times its initial volume. If C_V = 2R, the ratio of initial to final pressure of gas will be

(a) 5 (b) 4
(c) 3 (d) 2

20. The internal energy of a monoatomic ideal gas is
(a) only kinetic (b) only potential
(c) partly kinetic and partly potential
(d) none

21. If the number of gas molecules in a cubical vessel is increased from N to $3N$, then its pressure and total energy will become
(a) four times (b) three times
(c) double (d) half

22. Which of the following curves is not correct at constant temperature?

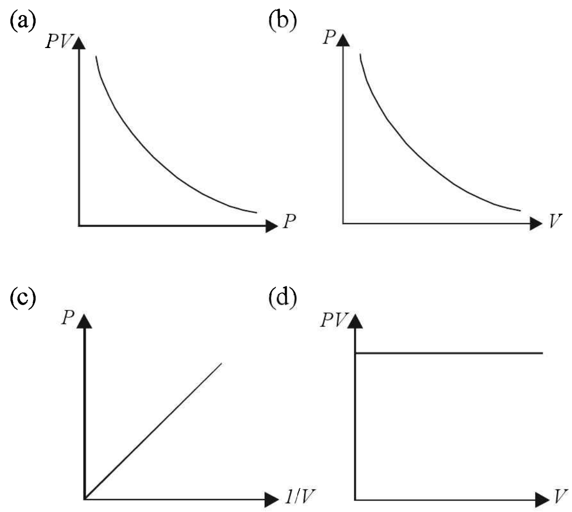


Fig. 15.16

23. The Graham's law of diffusion is

(a) $\frac{C_1}{C_2} = \frac{d_2}{d_1}$ (b) $\frac{C_1^2}{C_2^2} = \sqrt{\frac{d_2}{d_1}}$

(c) $\frac{C_1}{C_2} = \sqrt{\frac{d_2}{d_1}}$ (d) $\frac{C_1}{C_2} = \sqrt{\frac{d_1}{d_2}}$

24. The correct curve between V/T and $1/V$ for a gas at constant pressure is

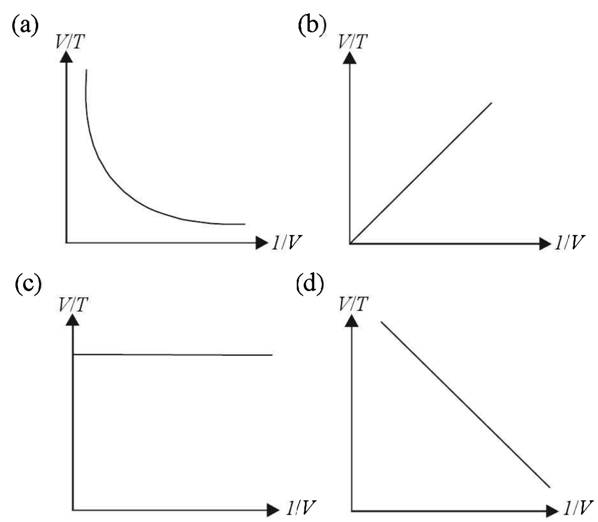


Fig. 15.17

25. If the mass of molecules of a gas in a closed vessel is halved and the speed doubled, then the ratio of initial to final pressure will be
(a) 4 : 1 (b) 1 : 4
(c) 1 : 2 (d) 2 : 1

26. If the total number of molecules in a gas is N then the number of molecules moving in negative X-direction will be
(a) N/6 (b) N/4
(c) N/3 (d) N

27. The temperature below which a gas can be liquified by increasing its pressure is known as
(a) zero (b) neutral temperature
(c) critical temperature (d) Boyle temperature

28. The expression for mean free path is

(a) $\lambda = \frac{KT}{\sqrt{2}\pi d^2 P}$ (b) $\lambda = \frac{\pi dP}{kT}$

(c) $\lambda = \frac{\pi d^2 P}{kT}$ (d) $\lambda = \frac{kT}{\pi dP}$

29. If the absolute temperature of a gas is tripled, then the rms velocity of gas molecules will become
(a) 1/3 (b) $\sqrt{3}$ times
(c) 3 times (d) 9 times

30. The correct expression for pressure exerted by a gas on wall of a container is

(a) $P = \frac{mn}{3l^3}\sqrt{c^2}$ (b) $P = \frac{mnc^2}{3l^3}$

(c) $P = \frac{3l^2c^2}{mn}$ (d) $P = \frac{mc^2}{3l^2}$

31. A gas is filled in a container at any temperature and at pressure 76 cm of Hg. If at the same temperature the mass of gas is increased by 50% then the resultant pressure will be
(a) 38 cm of Hg (b) 76 cm of Hg
(c) 114 cm of Hg (d) 152 cm of Hg

32. If the critical temperature of a gas is 100 K then its Boyle temperature will be
(a) 33.3 K (b) 103 K
(c) 337 K (d) 500 K

33. The critical temperature for CO_2 is
(a) 31 K (b) 31.1 K
(c) 31.1° C (d) 31.3 F

34. At what temperature will the linear kinetic energy of a gas molecule be equal to that of an electron accelerated through a potential difference of 10 volt?
(a) 273 K (b) 19×10^3 K
(c) 38.65×10^3 K (d) 11.3×10^3 K

35. Which of the following expressions is not correct for rms velocity?

(a) $\sqrt{\frac{3\rho}{P}}$ (b) $\sqrt{\frac{3RT}{M}}$

(c) $\sqrt{\frac{3PV}{M}}$ (d) $\sqrt{\frac{2E_k}{M}}$

36. The correct graph between PV and P of one mol of gas at constant temperature will be

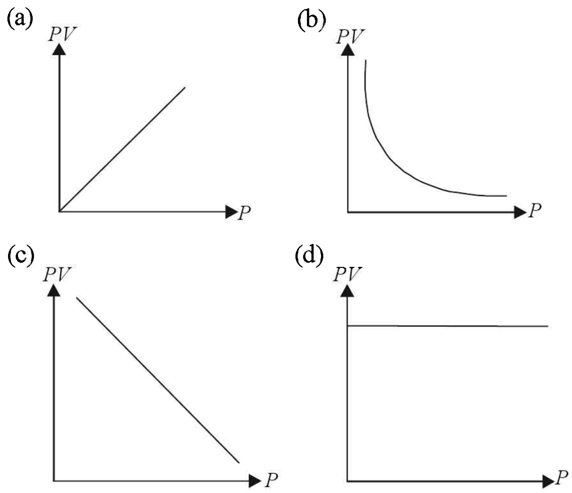


Fig. 15.18

37. How many times is the forbidden volume compared to actual volume of gas molecules?
(a) Double (b) 3 times
(c) 4 times (d) 8 times

38. At what temperature will the kinetic energy of gas molecules be double of its value at 27° C?
(a) 54° C (b) 108° C
(c) 300° C (d) 327° C

39. If the pressure of a gas is increased then its mean free path becomes
(a) zero (b) less
(c) more (d) ∞

40. On which of the following is the kinetic theory of gases not applicable?
(a) On free electron gas (b) On bound electrons
(c) On water vapour (d) On smoke particles

41. The mean molecular energy of a gas at 300 K will be
(a) 2.6×10^{-20} Joule (b) 6.2×10^{-21} Joule
(c) 6.2×10^{-20} Joule (d) 6.2×10^{20} Joule

42. In outer space there are 10 molecules per cm^3 on an average and the temperature there is 3 K. The average pressure of this light gas is
(a) 10^5 Nm^{-2} (b) 5×10^{-14} Nm^{-2}
(c) 0.4×10^{-16} Nm^{-2} (d) 4.14×10^{-16} Nm^{-2}

43. The velocities of three molecules A, B and C of a gas are 1, $\sqrt{3}$ and $\sqrt{5}$ ms^{-1}. The value of their rms velocity will be
(a) 1.73 ms^{-1} (b) 2 ms^{-1}
(c) 4 ms^{-1} (d) 9 ms^{-1}

44. If the mean kinetic energy per unit volume of a gas is n times its pressure, then the value of n is
(a) 4.5 (b) 3.5
(c) 2.5 (d) 1.5

45. The volume of 0.1 mol of gas at NTP is
(a) 0.22 litre (b) 2.24 litre
(c) 1 Litre (d) 22.4 litre

46. When a molecule moving with velocity u collides normally with the wall of the container, then the change in its velocity and momentum will be
(a) $-u$ and mu (b) $2u$ and $2\,mu$
(c) $2u$ and mu (d) u and $-mu$

47. Heat required to melt 1 g of ice is 80 cal. A man melts 60 g of ice by chewing in one minute. His power is
(a) 1.33 W (b) 0.75 W
(c) 336 W (d) 4800 W

48. Taking the unit of work as Joule and the unit of amount of heat as K cal, the magnitude of Joule's mechanical equivalence of heat is
(a) 1 (b) 4.2×10^7
(c) 4.2 (d) 4.2×10^3

49. The temperature of 5 moles of a gas which was held at constant volume was changed from 100° to 120° C. The change in the internal energy of the gas was found to be 80 Joule. The total heat capacity of the gas at constant volume will be equal to
(a) 0.4 JK^{-1} (b) 4 Jk^{-1}
(c) 0.8 JK^{-1} (d) 8 JK^{-1}

50. The rms speed of He gas atom is 5/7 of the rms speed of H_2 gas molecules. If the temperature of H_2 gas be 0° C, then the temperature of He will be approximately
(a) 273° C (b) 100 K
(c) 0° C (d) 0 K

51. If ice at –10° C is added to 40g of water at 15° C, the temperature of the mixture is
(a) 3.75° C (b) 0° C
(c) 3° C (d) –2° C

52. A cylinder contains 2 kg of air at a pressure of 10^5 Pa. If 2 kg more air is pumped into it, keeping the temperature constant, the pressure will be
(a) 10^{10} Pa (b) 2×10^5 Pa
(c) 10^5 Pa (d) 0.5×10^5 Pa

53. 10 g of steam passes over an ice block. What amount of ice will melt?
(a) 8 g (b) 18 g
(c) 45 g (d) 80 g

54. When a gas filled in a closed vessel is heated through 1° C, its pressure increases by 0.4%. The initial temperature of the gas was
(a) 25° C (b) 250° C
(c) 250 K (d) 2500 K

55. An inverted vessel (bell) lying at the bottom of a lake 50.6 m deep has 50 cc of air trapped in it. The bell is brought to the surface of lake. The volume of the trapped air will now be

(a) 200 cc (b) 250 cc
(c) 300 cc (d) 350 cc

56. A mixture of two gases X and Y is enclosed at constant temperature. The relative molecular mass of X, which is diatomic, is 8 times that of Y, which is monatomic. The ratio v_{rms} of Y molecules to that of molecules of X is
(a) 8 (b) 4
(c) $2\sqrt{2}$ (d) 2

57. 50 g of ice at –5° C is added to 200 g of water at 40° C in a calorimeter. The water equivalent of calorimeter is 50 g of water. The temperature of the mixture is
(a) 9.58° C (b) 12.58° C
(c) 19.58° C (d) 29.58° C

58. The triple point of water is
(a) 273 K (b) 0 K
(c) 273.16 K (d) 0° C

59. Compared to a burn due to water at 100^0 C, a burn due to steam at 100° C is
(a) more dangerous (b) less dangerous
(c) equally dangerous (d) none of these

60. The pressure of a gas kept in an isothermal container is 200 KPa. If half of the gas is removed from it, the pressure will be
(a) 800 K Pa (b) 400 K Pa
(c) 200 K Pa (d) 100 K Pa

61. According to the Boltzmann's law of equipartition of energy, the energy per degree of freedom and at a temperature TK is
(a) (3/2) KT (b) (2/3) KT
(c) KT (d) 1/2 KT

62. One mole of a gas at a pressure 2 Pa and temperature 27° C is heated till both pressure and volume are doubled. What is the temperature of the gas?
(a) 1200 K (b) 900 K
(c) 600 K (d) 300 K

63. What is number of degrees of freedom of an ideal diatomic molecule at ordinary temperature?
(a) 7 (b) 6
(c) 5 (d) 3

64. To raise the temperature of 100 g of ice at 0° C to 10° C by a heater of 420 W, the time required is
(a) 90 min (b) 90 seconds
(c) 21.2 min (d) 21.2 seconds

65. A man is climbing up a spiral type staircase. His degrees of freedom are
(a) 1 (b) 2
(c) 3 (d) more than 3

66. The law of equipartition of energy was given by
(a) Claussius (b) Maxwell
(c) Boltzmann (d) Carnot

67. The law of equipartition of energy is applicable to the system whose constituents are
(a) in random motion
(b) in orderly motion
(c) at rest
(d) moving with constant speed

68. A system consists of N particles, which have independent K relations among one another. The number of degrees of freedom of the system is given by
(a) 3 NK (b) 3 N/K
(c) 3 N/K (d) 3 N – K

69. The dimension of universal gas constant R are
(a) $M^2L^2T^{-2}$ (b) $ML^2T^{-2}\theta^{-1}$
(c) $M^2L^2T^{-2}\theta^{-2}$ (d) $MLT^{-2}\theta^{-2}$

70. The rms velocity of air at NTP will be ——— if density of air is 1.29 kg/m^3.
(a) 0.485 ms^{-1} (b) 0.485×10^2 ms^{-1}
(c) 4.85×10^2 ms^{-1} (d) 10^4 ms^{-1}

71. The temperature, pressure and volume of two gases X and Y are T, P and V respectively. When the gases are mixed then the volume and temperature of the mixture becomes V and T respectively. The pressure and mass of the mixture will be
(a) P and M (b) P and 2 M
(c) 2P and 2 M (d) 2P and

72. The specific heat of a monoatomic gas at constant volume is 0.075 kcal kg^{-1}K^{-1}. Its atomic weight will be
(a) 10 (b) 30
(c) 40 (d) 90

73. The specific heat of ——— is negative.
(a) CO_2 (b) Ne
(c) saturated vapours (d) none

74. The first excited state of hydrogen atom is higher to its ground energy level by 10.2 eV. The temperature necessary to excite hydrogen atom to first excited state will be
(a) 0.88 K (b) 7.88×10^2 K
(c) 7.88×10^3 K (d) 7.88×10^4 K

75. The diameter of oxygen molecules is 2.94×10^{-10} m. The van der Waals gas constant in m^3/mol will be
(a) 3.2 (b) 32
(c) 32×10^{-6} (d) 32×10^{-3}

76. The correct relation connecting the universal gas constant (R), Avogadro number N_A and Boltzmann constant (K) is
(a) $R = NK^2$ (b) $K = NR$
(c) $N = RK$ (d) $R = NK$

77. The rms velocities of molecules of two gases of equal volume are 2 ms^{-1} and 3 ms^{-1} respectively. The ratio of their pressure will be
(a) 3 : 2 (b) 2 : 3
(c) 9 : 4 (d) 4 : 9

78. N balls, each of mass M kg, strike per second per unit area a surface normally with velocity V. The pressure

exerted by them on the surface will be
(a) $2MNV^2$ (b) $2MNV$
(c) MNV^2 (d) MNV

79. The most probable velocity for monoatomic gas is
(a) $\sqrt{\frac{3kT}{m}}$ (b) $\sqrt{\frac{8kT}{\pi m}}$
(c) $\sqrt{\frac{2kT}{m}}$ (d) zero

80. The specific heat at constant volume of mixture of N_2 and He (N_2 : He :: 3 : 2) will be
(a) 1.7 R (b) 1.5 R
(c) 1.9 R (d) 2.1 R

81. The behaviour of the gases, which can be easily liquified, is like that of the
(a) triatomic gases (b) ideal gases
(c) van der Waals gases (d) all of the above

82. The direction of flow of heat between two bodies is determined by
(a) internal energy (b) kinetic energy
(c) total energy (d) none of these

83. The mass of a gas molecule is 4×10^{-30} kg. If 10^{23} molecules strike per second at 4 m^2 area with a velocity 10^7 ms^{-1}, the pressure exerted on the surface will be (in pascal)
(a) 1 (b) 2
(c) 3 (d) 4

84. The speed of a molecule of gas in a cubical vessel of side 5 m is 15 ms^{-1}. This molecule is constantly colliding with the walls of the container. The collision frequency will be
(a) 0.2 per second (b) 1.5 per second
(c) 2.5 per second (d) 5 per second

85. At what temperature does the mean kinetic energy of hydrogen atoms increase to such an extent that they will escape out of the gravitational field of earth forever?
(a) 10075° K (b) 10000° K
(c) 12075° K (d) 20000° K

86. A bottle is inverted and dipped into a tank. At the bottom of the tank, 3/5th part of the bottle is filled with water. The depth of the tank is
(a) 20 m (b) 15 m
(c) 10 m (d) 5 m

87. The rms speed of hydrogen molecules at any temperature is 1996 m/s. The rms speed of colloidal particles with gram molecular weight 32×10^5 $gmol^{-1}$ will be
(a) 5.40 ms^{-1} (b) 4.99 ms^{-1}
(c) 2.70 ms^{-1} (d) 1.35 ms^{-1}

88. An ideal gas (C_V = 3/2 R) is maintained in a vessel of volume 83×10^{-4} m^3 at pressure 1.6×10^6 Nm^{-2} and temperature 300 K. If 2.49×10^4 Joule heat is given to this vessel, then its final temperature will be
(a) 600 K (b) 625 K
(c) 650 K (d) 675 K

89. A container contains 1 mol of nitrogen gas at temperature 17° C and pressure 2 atmospheres. If the radius of nitrogen molecule is 1 A° then the volume of gas will be
(a) 46.52 litre (b) 23.76 litre
(c) 1.89 litre (d) 5.42 litre

90. Equal mass of helium and oxygen are filled in identical containers. The ratio of pressure exerted by them will be
(a) 1 : 8 (b) 1 : 16
(c) 8 : 1 (d) 16 : 11

91. An enclosure of volume 3 litre contains 16 g of oxygen, 7 g of nitrogen and 11 g of carbondioxide at 27° C. The pressure exerted by the mixture is approximately
(a) 9 atmosphere (b) 8.3 atmosphere
(c) 3 atmosphere (d) 1 atmosphere

92. An astronaut carries with him a cylinder of capacity 10 litre filled with nitrogen gas at temperature 27° C and pressure 50 atmosphere. He makes a hole of area 1 cm^2 in it. In how much time will it be emptied?
(a) 575 s (b) 1.312 s
(c) 0.513 s (d) 0.387 s

93. The mass of 0.5 litre of hydrogen gas at NTP will be approximately
(a) 0.0892 g (b) 0.045 g
(c) 2 g (d) 22.4 g

94. The amount of heat required to heat 1 mol of a monoatomic gas from 200° C to 250° C will be —— if the heat required to heat the diatomic gas from 200° C to 300° C is Q.
(a) $2Q/3$ (b) $3Q/5$
(c) $3Q/1$ (d) $2Q/5$

95. The velocity of three molecules are $3V$, $4V$ and $5V$ respectively. Their rms speed will be
(a) $3/15V$ (b) $\sqrt{\frac{3}{50}}V$
(c) $50/3V$ (d) $\sqrt{\frac{50}{3}}V$

96. The pressure of a gas filled in a closed vessel increases by 0.4%. When temperature is increases by 1° C the initial temperature of the gas is
(a) 250° C (b) 250 K
(c) 250° F (d) 2500° C

97. The rms speed of smoke particles of mass 500×10^{-19} kg at NTP will be
(a) 15×10^{-3} ms^{-1} (b) 11.5×10^{-3} ms^{-1}
(c) 1.5×10^{-3} ms^{-1} (d) 0.15 cms–1

98. At what temperature will hydrogen molecules escape from earth surface?
(a) 10^1 K (b) 10^2 K
(c) 10^3 K (d) 10^4 K

99. The total kinetic energy of 8 litres of helium molecules at 5 atmosphere pressure will be

(a) 6078 erg (b) 6078 Joule
(c) 607 erg (d) 607 Joule

100. The mass of hydrogen molecules is 3.32×10^{-27} kg. If 10^{23} hydrogen molecules strike per second at 2 cm^2 area of a rigid wall at an angle of 45° from the normal and are rebound back with speed of 1000 m/s, then the pressure exerted on the wall is

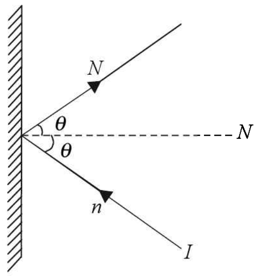


Fig. 15.19

(a) 23.4×10^3 Pascal (b) 2.34×10^3 Pascal
(c) 0.23×10^3 Pascal (d) 2 Pascal

101. If 10 g of water is evaporated from 1 m^2 field in one second then heat radiation incident/(m^2 – s) is

(a) 5400 cal/(m^2 – s) (b) 5400 J/(m^2 – s)
(c) 540 cal/(m^2 – s) (d) 540 J/(m^2 – s)

102. The molar heat capacity of an ideal gas

(a) cannot be negative
(b) must be equal to either C_v or C_p
(c) must lie in the range $C_v \le C \le C_p$
(d) may have any value between $-\infty$ and $+\infty$

103. The specific heat capacity of a body depends on

(a) the heat given (b) the temperature raised
(c) the mass of the body (d) the material of the body

104. Which of the following pairs represent units of the same physical quantity?

(a) Kelvin and joule (b) Kelvin and calorie
(c) Newton and calorie (d) Joule and calorie

105. Two bodies at different temperatues are mixed in a calorimeter. Which of the following quantities remains conserved?

(a) Sum of the temperatures of the two bodies
(b) Total heat of the two bodies
(c) Total internal energy of the two bodies
(d) Internal energy of each body

106. Water equivalent of a body is measured in

(a) kg (b) calorie
(c) kelvin (d) m^3

107. The ratio of specific heat capacity to molar heat capacity of a body

(a) is a universal constant
(b) depends on the mass of the body
(c) depends on the molecular weight of the body
(d) is dimensionless

108. Heat and work are equivalent. This means,

(a) when we supply heat to a body we do work on it.
(b) when we do work on a body we supply heat to it.
(c) the temperature of a body can be increased by doing work on it.
(d) a body kept at rest may be set into motion along a line by supplying heat to it.

109. The mechanical equivalent of heat

(a) has the same dimension as heat
(b) has the same dimension as work
(c) has the same dimension as energy
(d) is dimensionless

110. When a hot liquid is mixed with a cold liquid, the temperature of the mixture

(a) first decreases and then becomes constant
(b) first increases and then becomes constant
(c) continuosly increases
(d) is undefined for some time and then becomes nearly constant

111. Consider the quantity $\frac{MkT}{pV}$ of an ideal gas where M is the mass of the gas. It depends on the

(a) temperature of the gas (b) volume of the gas
(c) pressure of the gas (d) nature of the gas

Answers to Questions for Practice

1.	(b)	**2.**	(d)	**3.**	(d)	**4.**	(a)	**5.**	(c)	**6.**	(d)	**7.**	(c)
8.	(c)	**9.**	(b)	**10.**	(d)	**11.**	(a)	**12.**	(b)	**13.**	(c)	**14.**	(c)
15.	(a)	**16.**	(b)	**17.**	(d)	**18.**	(a)	**19.**	(a)	**20.**	(a)	**21.**	(b)
22.	(a)	**23.**	(c)	**24.**	(c)	**25.**	(c)	**26.**	(a)	**27.**	(c)	**28.**	(a)
29.	(b)	**30.**	(b)	**31.**	(c)	**32.**	(c)	**33.**	(c)	**34.**	(d)	**35.**	(a)
36.	(d)	**37.**	(c)	**38.**	(d)	**39.**	(b)	**40.**	(b)	**41.**	(b)	**42.**	(d)
43.	(a)	**44.**	(d)	**45.**	(b)	**46.**	(b)	**47.**	(c)	**48.**	(d)	**49.**	(b)
50.	(c)	**51.**	(b)	**52.**	(b)	**53.**	(d)	**54.**	(c)	**55.**	(c)	**56.**	(c)
57.	(c)	**58.**	(c)	**59.**	(a)	**60.**	(d)	**61.**	(d)	**62.**	(a)	**63.**	(c)
64.	(b)	**65.**	(c)	**66.**	(a)	**67.**	(a)	**68.**	(d)	**69.**	(b)	**70.**	(c)
71.	(c)	**72.**	(c)	**73.**	(c)	**74.**	(d)	**75.**	(c)	**76.**	(d)	**77.**	(d)
78.	(b)	**79.**	(c)	**80.**	(d)	**81.**	(c)	**82.**	(d)	**83.**	(b)	**84.**	(b)
85.	(a)	**86.**	(b)	**87.**	(b)	**88.**	(d)	**89.**	(c)	**90.**	(c)	**91.**	(b)
92.	(d)	**93.**	(b)	**94.**	(c)	**95.**	(d)	**96.**	(b)	**97.**	(a)	**98.**	(d)
99.	(b)	**100.**	(b)	**101.**	(a)	**102.**	(d)	**103.**	(d)	**104.**	(d)	**105.**	(c)
106.	(a)	**107.**	(c)	**108.**	(c)	**109.**	(d)	**110.**	(d)	**111.**	(d)		

Explanations

10. $10^5\,\text{Pa} \equiv 6.023\times10^{23}$

$10^{-11}\,\text{Pa} \equiv 6.023\times10^{7}$

$22400\text{ cc} \Rightarrow 6.023\times10^{7}$

$1\text{ cc} \Rightarrow \frac{6.023\times10^{7}}{22400} = 2400$

19. $P_1V_1^{\gamma} = P_2V_2^{\gamma}$

$$\text{P}_2 = P_1\left(\frac{V_1}{V_2}\right)^{\gamma} = 1\left(\frac{1}{3}\right)^{3/2}$$

$$\frac{P_1}{P_2} = 3\sqrt{3} = 5.1.$$

32. $T_c = \frac{8}{27}T_B$

or $T_B = \frac{100\times27}{8} = 337\,\text{K}.$

42. $P = \frac{nKT}{V} = \frac{10\times1.38\times10^{-23}\times3}{10^{-6}}$

$= 4.14\times10^{-6}\,\text{Nm}^{-2}.$

43. $V_{\text{rms}} = \sqrt{\frac{1^2+\left(\sqrt{3}\right)^2+\left(\sqrt{5}\right)^2}{3}}.$

54. $\frac{dP}{P} = \frac{dT}{T}$ or $T = \frac{dT}{\frac{dP}{P}} = \frac{1}{\frac{0.4}{100}} = 250\,\text{K}.$

72. $\frac{3/2\times2}{0.075} = 40.$

75. $b = 4N\times\frac{4}{3}\pi\frac{d^3}{8}$

$$= \frac{4\times6.02\times10^{23}\times3.14\times2.94^3\times10^{-30}}{3\times8}$$

$= 32\times10^{-6}.$

80. $\frac{3\times5/2R+2\times3/2R}{3+2} = \frac{21}{5\times2}R = 2.1\,\text{R}.$

92. $t = \frac{d}{V_{\text{rms}}} = \frac{2V}{AV_{\text{rms}}}$

$$= \frac{2V}{A\sqrt{\frac{3RT}{M}}}$$

$$= \frac{2\times10^{-2}}{10^{-4}\sqrt{\frac{3\times8.3\times300}{28\times10^{-3}}}}.$$

16

Thermodynamics

BRIEF REVIEW

There are 4 Laws of Thermodynamics. Zeroeth, First Law, Second Law and Third Law. Zeroeth law deals with thermal equilibrium.

First law of Thermodynamics Consider an ideal gas in a cylinder fitted with a piston. Assume that the piston is fixed at its position and the walls of the cylinder are kept at a higher temperature than that of the gas. The gas molecules strike the wall and rebound. The average KE of a wall molecule is higher than the average KE of a gas molecule. On collision the gas molecule receives some energy from the wall molecule. This increasesd KE is shared by other gas molecules also. In this way total internal energy of the gas increases.

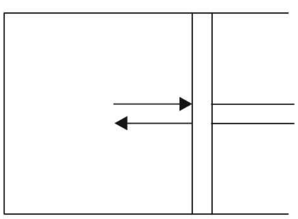


Fig. 16.1 Internal energy illustration

Now consider that the walls of the cylinder are also at the same temperature as that of the gas. As the gas molecules collide with the piston coming towards it, the speed of the molecule increases on collision (Assuming elastic collision $v_2 = v_1 + 2u$). This way internal energy of the molecules increases as the piston is pushed in. Thus we see that energy transfer and work go together. If ΔQ is the heat supplied and ΔW is the work done, then the internal energy of the gas must increase by $\Delta Q - \Delta W$.

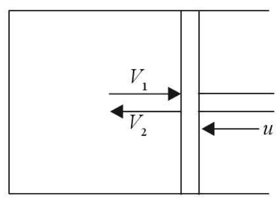


Fig. 16.2 Illustration of internal energy and work done

Hence $\Delta U = \Delta Q - \Delta W$

or $\Delta Q = \Delta U + \Delta W$

is called the first law of thermodynamics.

Work done by a gas = $P\Delta V$ or $W = \int_{V_1}^{V_2} PdV$

The first law denies the possibility of creating or destroying energy.

Thermal Processes In general thermal processes may be of three types: (a) reversible, (b) irreversible and (c) cyclic. A reversible process means if a process takes up the path *AB* (Fig. 16.3) then on reversing the conditions it comes back by *BA*. A thermal process however cannot be reversible. It could be reversible if the change is extremely small (infinitesimally small).

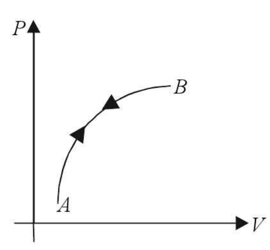


Fig. 16.3 Reversible process

In an irreversible process one will not reach back to A if the process AB has occurred.

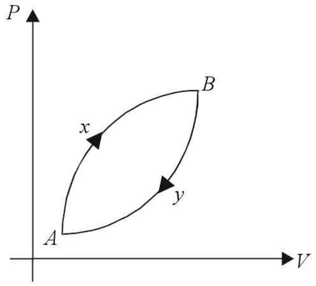


Fig. 16.4 Cyclic process

In a cyclic process, if the process takes the path AxB, it returns via ByA (Fig. 16.4).

Thermal processes may be cyclic or irreversible. Change in internal energy in a cyclic process is zero.

Hence $\Delta Q = \Delta W$

We can divide these processes as

(a) isobaric	(b) isochoric
(c) isothermal	(d) adiabatic
(e) throttling	(f) polytropic

In isobaric process pressure remains constant and work done

$$W = P\Delta V = P(V_2 - V_1)$$

$$\therefore \quad dQ = dU + pdV$$

In isochoric process volume remains constant. Therefore $dV = 0$

Hence work done is zero

$$\therefore \quad \Delta Q = \Delta U$$

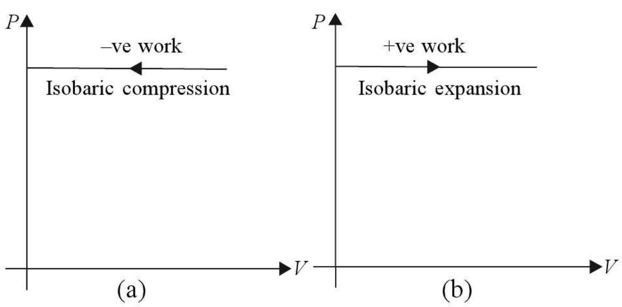


Fig. 16.5 (a) Isobaric compression and (b) expansion

In isothermal process the temperature remains constant. Melting and boiling are examples. Specific heat in isothermal process is ∞.

Work done,

$$W = \int pdV = n\mathrm{R}\mathrm{T}\int_{V_1}^{V_2}\frac{\mathrm{dV}}{\mathrm{V}} = nRT\log_e\frac{V_2}{V_1}$$

$$= 2.303\, nRT\log\frac{V_2}{V_1} = 2.303\, nRT\,\log_e\frac{P_1}{P_2}$$

Isothermal elasticity = P (Bulk modulus)

In an adiabatic process heat is neither allowed to enter nor allowed to escape the system. Specific heat in an adiabatic process is zero.

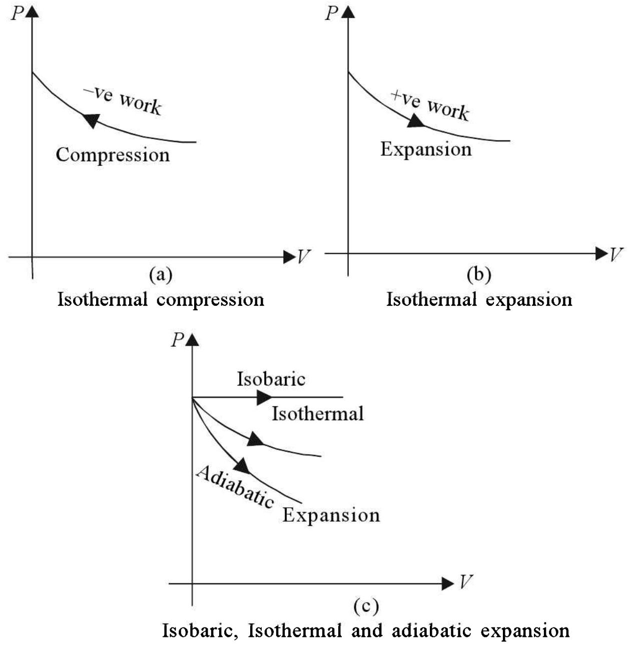


Fig. 16.6

Since $dQ = 0$

$$\therefore \quad dU = -pdV$$

In an adiabatic process

(i) PV^{γ} = constant
(ii) $P^{1-\gamma}T^{\gamma}$ = constant
(iii) $TV^{\gamma-1}$ = constant

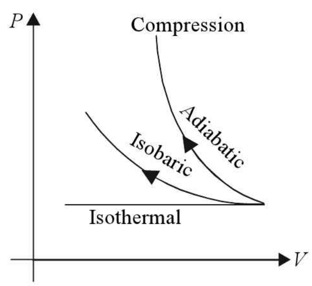


Fig. 16.7 Work done in various cases

Work done in an adiabatic process,

$$W = \frac{P_1V_1 - P_2V_2}{\gamma - 1} = \frac{nR(T_1 - T_2)}{\gamma - 1} \quad \text{where } \gamma = \frac{C_P}{C_V}$$

Adiabatic elasticity (Bulk modulus) = γP

Second law of thermodynamics The second law denies the possibility of utilisation of heat out of a single body. The definitions of the second law of thermodynamics are:

(a) It is impossible to construct an engine which, operating in a cycle, will produce no effect other than the extraction of heat from a reservoir and the performance of an equivalent amout of work. (Kelvin Planck statement)

(b) Heat cannot flow itself from a colder to a hotter body.

(c) It is impossible to have a process in which the entropy of an isolated system is decreased.

Adiabatic → Thermally insulating

Diathermic → Thermally conducting

Heat Engine A heat engine takes a heat Q_1 from the furnace and rejects Q_2 to the heat sink and does a work $W = Q_1 - Q_2$

Thus efficiency of an engine $\eta = 1 - \frac{Q_2}{Q_1} = 1 - \frac{T_2}{T_1}$

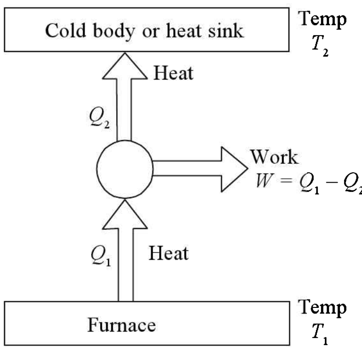


Fig. 16.8 Carnot engine

Entropy $dS = \frac{dQ}{T}$ or $S_2 - S_1 = \int \frac{dQ}{T}$

Note that T is not differentiable. Entropy is a measure of randomness or disorder in a system.

Clausius inequality $\oint \frac{dQ}{T} \le 0$

or $\Delta S \ge \int \frac{dQ}{T}$ or $dQ = TdS \ge dU + pdV$

Relation between entropy and statistical weight **Ω** (thermodynamic property)

$S = K \log_e \Omega$

where k is Boltzmann's constant.

Amount of heat required to form a unit area of the liquid surface layer during the isothermal increase of its surface $H = -T\frac{d\sigma}{dT}$ where σ is surface tension.

Carnot Engine The french scientist (auto engineer) NL Sadi Carnot in 1824 suggested an idealised engine called Carnot engine. It has a cylinder piston system. The walls and the piston are completely adiabatic (insulating) and the base is diathermic (thermally conducting). It contains an ideal gas. It undergoes isothermal expansion, adiabatic expansion, isothermal compression and adiabatic compression to complete the cycle. PV and ST plots for a Carnot cycle are shown in Fig 16.9. Carnot's engine is a reversible engine.

Carnot's theorem All reversible engines operating between the same two temperatures have equal efficiency and no engine operating between the same two temperatures can have an efficiency greater than this.

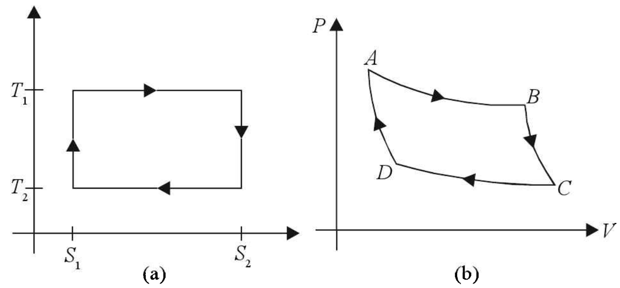


Fig. 16.9 Carnot cycle

According to Carnot's theorem, maximum efficiency

$$\eta = 1 - \frac{T_2}{T_1} = 1 - \frac{Q_2}{Q_1}$$

Since T_2 cannot be zero (as 0 K cannot be obtained), therefore, efficiency cannot be 1.

Refrigerator or heat pump A heat engine takes heat from a hot body, converts part of it into work and rejects to cold body. The reverse operation is done by a refrigerator (or heat pump). It takes an amount Q_2 of heat from a cold body, an amount of work W is done on it by the surrounding and a total heat $Q_1 = Q_2 + W$ is supplied to hot body as illustrated in Fig. 16.10.

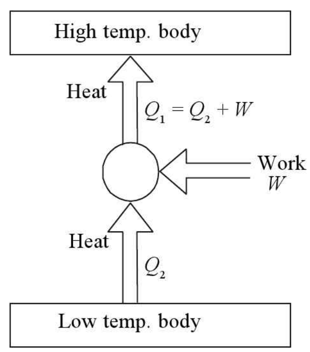


Fig. 16.10 Refrigerator based on carnot cycle

$$\frac{Q_1}{Q_2} = \frac{T_1}{T_2}$$

$$\frac{Q_2 + W}{Q_2} = \frac{T_1}{T_2}$$

or $$W = Q_2\left(\frac{T_1}{T_2} - 1\right)$$

This leads to another statement of second law:

It is not possible to design a refrigerator which works in a cyclic process and whose only result is to transfer heat from a colder body to a hotter body. This is the Claussius statement of the second law of thermodynamics.

Coefficient of performance,

$$K = \frac{\text{heat extracted}}{\text{work done}} \frac{Q_2}{W} = \frac{Q_2}{Q_1 - Q_2}$$

In a perfect refrigerator $K = \infty$

that is, $Q_1 = Q_2$ or $W = 0$

• Short Cuts and Points to Note

1. According to the first law of thermodynamics, total energy is conserved, that is, the first law denies the possibility of creating or destroying energy. Thus

$$\Delta Q = \Delta U + W$$

or $$dQ = dU + PdV$$

2. Processes may be reversible, irreversible or cyclic. Thermal processes cannot be completely reversible.

3. Thermal processes may be isochoric, isobaric, isothermal, adiabatic, polytropic and throttling.

In an isochoric process, $\Delta V = 0$

$\therefore$ $W = 0$, Specific heat $= C_V$

$$\Delta Q = \Delta U = nc_V\Delta T$$

In an isobaric process, work done

$$W = P\Delta V = P(V_2 - V_1);\ \text{specific heat} = C_P$$

$$W = nR(T_2 - T_1)$$

$$dQ = dU + PdV$$

or $$nC_P\Delta T = nC_V\Delta T + nR\Delta T$$

In an isothermal process. work done

$$W = nRT \log_e \frac{V_2}{V_1} = nRT \log_e \frac{P_1}{P_2},$$

Specific heat $= \infty$

In an adiabatic process, work done

$$W = \frac{nR(T_1 - T_2)}{\gamma - 1} = \frac{P_1V_1 - P_2V_2}{\gamma - 1},$$

Specific heat = 0

$$PV^\gamma = \text{constant},\ P^{1-\gamma}T^\gamma = \text{constant}$$

and $$TV^{\gamma-1} = \text{constant}$$

4. In a throttling process a fluid, originally at high pressure, seeps through a porous wall or needle-like narrow opening into a region of constant lower pressure. Work done

$$W = P_2V_2 - P_1V_1$$

Since the process is adiabatic, therefore

$$\Delta U = U_2 - U_1 = -(P_2V_2 - P_1V_1)$$

The sum $U + PV$ is called enthalpy. Throttling process plays an important role in refrigeration.

5. The slope of an adiabatic process is higher than isothermal change.

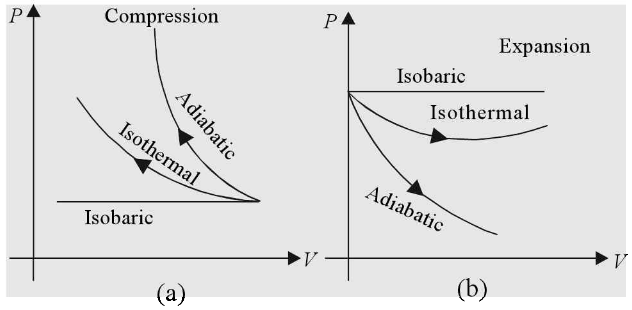


Fig. 16.11

Note- During expansion $W_{\text{isobaric}} > W_{\text{isothermal}} > W_{\text{adiabatic}}$ during compression $W_{\text{adiabatic}} > W_{\text{isothermal}} > W_{\text{isobaric}}$

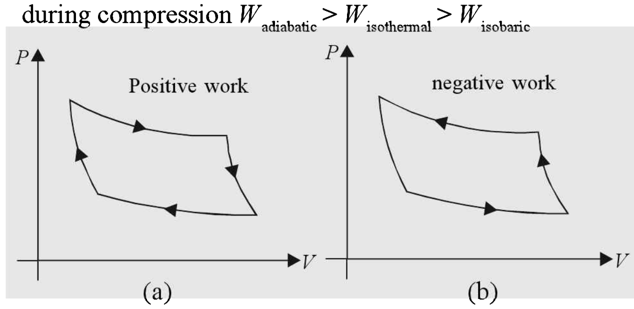


Fig. 16.12

6. Work done is positive if the arrow is clockwise and negative if the arrow is anticlockwise in a PV diagram as illusttrated in Fig 16.11 (a) and (b)

7. *Second law of thermodynamics*: The various definitions are

(a) It is impossible to construct an engine which, operating in a cycle, will produce no effect other than the extraction of heat from a reservoir and the performance of an equivalent amount of work (Kelvin Planck Statement)

(b) Heat cannot flow by itself from a colder to a hotter body.

(c) It is impossible to have a process in which the entropy of an isolated system is decreased.

(d) It is not possible to design a refrigerator which works in a cyclic process and whose only result is to transfer heat from a colder body to a hotter body. (Claussius statement)

8. Entropy is a measure of randomness or disorder in a system.

$$dS = \frac{dQ}{T}$$

Note that T is not a differentiable quantity.

9. Thermal equilibrium: If two systems have the same temperature they are said to be in thermal equilibrium.

10. Thermodynamic equilibrium is when there is the state of thermal, mechanical and chemical equilibrium.

Mechanical equilibrium means $\Sigma F = 0, \Sigma\tau = 0$ (torque)

Chemical equilibrium means the concentration of reactants and products remains constant.

11. Thermodynamic variables P, V, T, and so on which form the equation of state are called thermodynamic variables.

12. Heat engine takes up heat from a hotter body, converts it partly into work and rejects rest of the energy to a cold body (heat sink). Efficiency of a heat engine (Carnot)

$$\eta = \frac{W}{Q_1} = \frac{Q_1 - Q_2}{Q_1} = 1 - \frac{Q_2}{Q_1}$$

13. Carnot suggested that even an idealied engine cannot have efficiency 1. He considered a cyclic process consisting of four processes:

(a) isothermal expansion
(b) adiabatic expansion
(3) isothermal compression
(4) adiabatic compression

Efficiency of Carnot engine

$$\eta = 1 - \frac{Q_2}{Q_1} = 1 - \frac{T_2}{T_1}$$

$T_2 \rightarrow$ temperature of heat sink (colder body)
$T_1 \rightarrow$ temperature of furnace or hot body
Since T_2 cannot be 0 K, therefore efficiency cannot be 1.

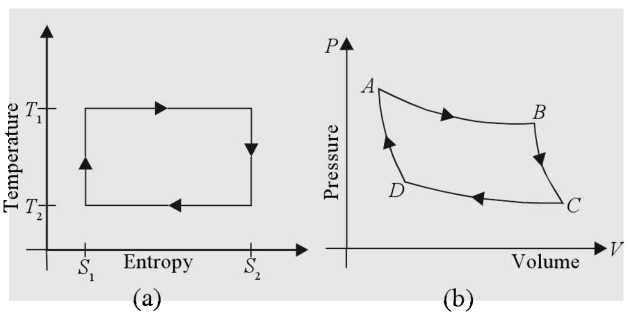


Fig. 16.13

14. Refrigerator or heat pump is reciprocal of heat engine

$$\frac{Q_1}{Q_2} = \frac{T_1}{T_2} \text{ or } \frac{Q_2 + W}{Q_2} = \frac{T_1}{T_2}$$

or $$W = Q_2\left(\frac{T_1}{T_2} - 1\right)$$

Performance coefficient $K = \frac{Q_2}{W} = \frac{Q_2}{Q_1 - Q_2}$

15. In perfect refrigerator K = ∞, that is $Q_1 = Q_2$

Area under $P - V$ graph is work done.

16. Molar heat capacity in a polytropic process PV^k = constant

$$C = \frac{R}{\gamma - 1} - \frac{R}{k - 1} = \frac{(k - \gamma)R}{(k-1)(\gamma - 1)}$$

17. For a van der Waals gas $U = C_V T - \frac{a}{V_M}$ for one mole

- **Caution**

1. Not understanding the difference between C_p and C_V

$\Rightarrow$ Specific heat at constant volume C_V forms internal energy
$\Delta U = nC_V \Delta T$
When volume is constant, work done
$W = PdV = 0 \therefore dQ = dU$
When C_p is being used $dQ = nC_p \Delta T$ and work done
$W = PdV = nRdT$ and $dQ = dU + PdV$

2. Not understanding whether work is positive or negative.

$\Rightarrow$ When there is expansion of the gas or when the piston moves in the forward direction then work is positive. When there is compression or when the piston moves in a backward direction, work done is negative. Alternatively, if the arrow is clockwise, work done by the gas is positive (+ve). If the arrow is anticlockwise, work is done on the gas, and is negative (–ve) in a PV diagram.

3. Not remembering the three relations of adiabatic process.

$\Rightarrow$ In an adiabatic process
(a) PV^γ = constant or $P_1V_1^\gamma = P_2V_2^\gamma$
(b) $P^{1-\gamma}T^\gamma$ = constant or $P_1^{1-\gamma}T_1^\gamma = P_2^{1-\gamma}T_2^\gamma$
(c) $TV^{\gamma-1}$ = constant or $T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$

4. Confusing when to take the process as adiabatic if not mentioned directly in the problem.

$\Rightarrow$ When the change is sudden or abrupt, the process is adiabatic.

5. Confusing between adiabatic and isothermal expansion or compression.

$\Rightarrow$ In isothermal expansion or compression the slope is not large while in adiabatic expansion or compression the slope is large (see Fig 16.14).

Note : during expansion $W_{isobaric} > W_{isothermal} > W_{adiabatic}$
during compression $W_{adiabatic} > W_{isothermal} > W_{isobaric}$

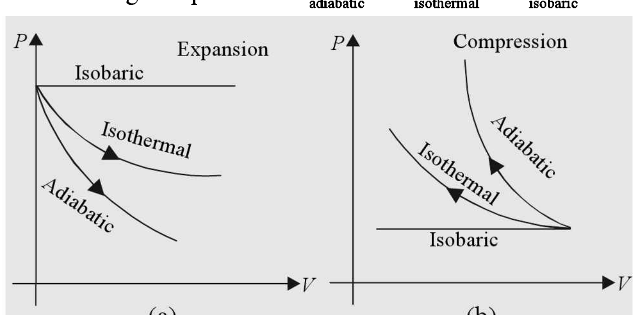


Fig. 16.14

Note: During expansion more work is done by an isothermal process and during compression more work is done on an adiabatic process.

6. Not remembering the work done in various processes.

⇒ Work done in isochoric process, $W_{\text{isochoric}} = 0$

$$W_{\text{isobaric}} = P\,\Delta V = P\,(V_2 - V_1) = nR(\Delta T)$$
$$= nR(T_2 - T_1)$$
$$W_{\text{isothermal}} = 2.303\, nRT \log_{10} \frac{V_2}{V_1}$$
$$= 2.303\, nRT \log_{10} \frac{P_1}{P_2}$$
$$W_{\text{adiabatic}} = \frac{P_1V_1 - P_2V_2}{\gamma - 1} = \frac{nR\,(T_1 - T_2)}{\gamma - 1}$$

7. Considering that efficiency of an engine can be 1 (ideally or theoretically).

⇒ Efficiency of an engine cannot be 1. It is always < 1.

According to Carnot's theorem $\eta = 1 - \frac{T_2}{T_1}$

$\eta \to 1$ if $T_2 \to 0$. As 0 K or absolute 0 cannot be achieved, therefore, $\eta \neq 1$.

8. Considering that total heat energy can be converted into mechanical work just like mechanical work which can be completely converted to heat.

⇒ Mechanical work can be converted to heat. But the whole of the heat cannot be converted into work.

9. Considering all engines give efficiency like Carnot engine.

⇒ Carnot is a theoretical idealised engine. Practically heat engines give efficiency much less than that given by Carnot engine.

10. Confusing between first law and second law of thermodynamics.

⇒ The first law is based on conservation of energy. The second law states that no heat can flow by itself from a cold body to a hot body.

11. Not recalling a polytropic process.

⇒ In a polytropic process PV^k = constant and k is different from γ. Molar specific heat in polytropic processis $C = \frac{R}{\gamma - 1} - \frac{R}{k - 1}$ However, $C_V = \frac{R}{\gamma - 1}$

12. Thinking that temperature may be taken in ^{0}C.

⇒ Use temperature in Kelvin (K)

SOLVED PROBLEMS

1. Calculate the work done by the gas in the diagram shown.

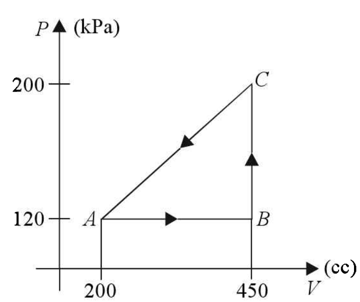


Fig. 16.15

(a) 30 J (b) 20 J
(c) –20 J (d) –10 J

Solution (d) Work done = Area under the $P - V$ curve

$$W = (80\,\text{kPa})\,(250 \times 10^{-6}) \times \frac{1}{2} = 10\,\text{J}$$

Since the arrow is anticlockwise,

∴ Work done = –10 J

2. Variation of molar specific heat of a metal with temperature is best depicted by

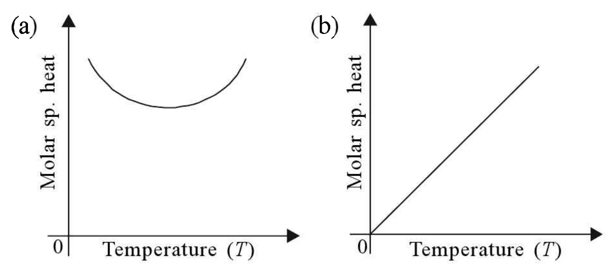


Fig. 16.16 (a)

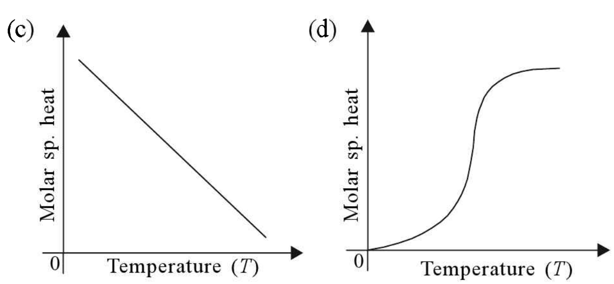


Fig. 16.16 (b)

Solution (d) For $T \to 0$, that is, at low temperature, molar specific heat $\propto T^3$ and at high temperature it becomes constant = $3R$.

3. 1 g of H_2O changes from liquid to vapour phase at constant pressure at 1 atm. The volume increases from 1 cc to 1671 cc. The heat of vaporisation at this pressure is 540 cal/g. The increase in internal energy of water is

(a) 2099 J (b) 3000 J
(c) 992 J (d) 2122 J

Solution (a) $W = P(dV) = 1.01 \times 10^5 (1671 - 1) \times 10^{-6}$
$= 167$ J
$\Delta Q = \Delta U + \Delta W$
or $\Delta U = \Delta Q - \Delta W$
$= mL - 167 = 540 \times 4.2 - 167 = 2099$ J

4. A gas mixture consists of 2 moles of oxygen and 4 moles of Ar at temperature T. Neglecting all vibrational modes, the total internal energy of the system is

(a) $4RT$ (b) $15RT$
(c) $9RT$ (d) $11RT$

[IIT 1999]

Solution (d) $u = n\dfrac{F}{2}RT = 2 \times \dfrac{5}{2}RT + 4 \times \dfrac{3}{2}RT = 11RT$

5. A tyre pumped to a pressure 3.375 atm at 27° C suddenly bursts. What is the final temperature (γ = 1.5)?

(a) 27° C (b) –27° C
(c) 0° C (d) –73° C

Solution (d) $T_1^\gamma P_1^{1-\gamma} = T_2^\gamma P_2^{1-\gamma}$

or $$\left(\frac{T_1}{T_2}\right)^\gamma = \left(\frac{P_1}{P_2}\right)^{\gamma-1} = \left(\frac{300}{T_2}\right)^{3/2} = \left(\frac{3.375}{1}\right)^{3/2-1}$$

or $$T_2 = \frac{300}{(3.375)^{1/3}} = 200\text{ K} = -73°\text{ C}$$

6. A sound wave passing through air at NTP produces a pressure of 0.001 dyne/cm² during a compression. The corresponding change in temperature (given γ = 1.5 and assume gas to be ideal) is

(a) 8.97×10^{-4} K (b) 8.97×10^{-6} K
(c) 8.97×10^{-8} K (d) none of these

Solution (c) $T^\gamma P^{1-\gamma}$ = constant. Differentiating
$\gamma T^{\gamma-1} dT P^{1-\gamma} + T^\gamma (1-\gamma) P^{-\gamma} dP = 0$

or $$dT = \frac{(\gamma-1)T}{\gamma P} dP$$

or $$dT = \left(\frac{1.5-1}{1.5}\right)\left(\frac{273}{76 \times 13.6 \times 981} \times 0.001\right)$$
$= 8.97 \times 10^{-8}$ K

7. When a system is taken from state 1 to 2 along the path 1a2 it absorbs 50 cal of heat and work done is 20 cal. Along the path 1b2, Q = 36 cal. What is the work done along 1b2?

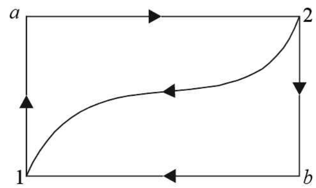


Fig. 16.17

(a) 56 cal (b) 66 cal
(c) 16 cal (d) 6 cal

Solution (d) $dQ = du + dW$ or $Q = (u_2 - u_1) + W$
$W = Q_{1b2} - (u_2 - u_1)$ or $Q_{1a2} - W = u_2 - u_1$
$= 36 - 30 = 6$ cal
or $u_2 - u_1 = 50 - 20 = 30$ cal

8. 1 g mole of an ideal gas at STP is subjected to a reversible adiabatic expansion to double its volume. Find the change in internal energy (γ = 1.4).

(a) 1169.5 J (b) 769.5 J
(c) 1369.5 J (d) 969.5 J

Solution (c) Use $T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$

or $$T_2 = \frac{T_1 V_1^{\gamma-1}}{V_2^{\gamma-1}} = \frac{273}{(2)^{0.4}} = 207\text{ K}$$

Change in internal energy

$$\Delta u = \frac{R}{(\gamma-1)}(T_1 - T_2)$$
$$= \frac{8.31(273-207)}{1.4-1} = 1369.5\text{ J}$$

9. A gram mole of a gas at 127° C expands isothermally until its volume is doubled. Find the amount of work done.

(a) 238 cal (b) 548 cal
(c) 548 J (d) 238 J

Solution (b) $W = 2.303\text{ RT}\log\left(\dfrac{V_2}{V_1}\right)$
$= 2.303 \times 8.311 \times 400 \times \log 2$
$= 2310.1\text{ J} = 548$ cal.

10. Find the work required to compress adiabatically 1 g of air initially at NTP to half its volume. Density of air at NTP = 0.001129 gcm⁻³ and $\dfrac{C_P}{C_V} = 1.4$.

(a) 62.64 J (b) 32.64 J
(c) –32.64 J (d) –62.64 J

Solution (d) $T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$

or $$T_2 = T_1\left(\frac{V_1}{V_2}\right)^{\gamma-1} = 273\,(2)^{0.4} = 360\text{ K}$$

$$V = \frac{1g}{0.00129}\text{ cc}$$

We find R for 1 g of air using $PV = RT$

$$R = \frac{76\times13.6\times981}{273\times0.00129} = 2.88\times10^6$$

$$W = \frac{R}{\gamma-1}(T_1 - T_2) = \frac{2.88\times10^6}{0.4}\ 9273-360)$$

$= 62.64\times10^7$ erg.

$= -62.64$ J

11. A carnot engine has the same efficiency between (i) 100 K and 500 K and (ii) T and 900 K. Find T.

(a) 200 K (b) 190 K
(c) 180 K (d) none of these

Solution (c) $\eta = 1 - \frac{T_2}{T_1}$ or $\frac{T_2}{T_1} = \frac{T_2'}{T_1'}$ or $\frac{100}{500} = \frac{T}{900}$

12. A reversible engine takes in heat from a reservoir of heat at 527° C and gives heat to the sink at 127° C. How many calorie/s shall it take from the reservoir to do a work of 750 W?

(a) 257 $cals^{-1}$ (b) 357 $cals^{-1}$
(c) 1500 $cals^{-1}$ (d) none of these

Solution (b) $\eta = 1 - \frac{T_2}{T_1} = 1 - \frac{400}{800} = \frac{1}{2} = \frac{W}{Q_1}$

or $Q_1 = 2W = \frac{2\times750}{4.2} = 357.1\ cals^{-1}$.

13. A Carnot engine has efficiency 40% (heat sink 27° C). To increase efficiency by 10%, the temperature be increasesd by

(a) 15.7 K (b) 25.7 K
(c) 50.7 K (d) 35.7 K

Solution (d) $\eta = 40\% = \frac{2}{3}$ $\eta = 1 - \frac{T_2}{T_1}$ or $\frac{T_2}{T_1} = \frac{3}{5}$

$\therefore$ $T_1 = 300\times\frac{5}{3} = 500$ K

new efficiency $= 40 + 40\times\frac{10}{100} = 44\%; 0.44$

$= 1 - \frac{300}{T_1}$

or $T'_1 = 535.7$ K

$\therefore$ Temperature of heat source be raised by 35.7 K

14. Two engines are working in such a way that sink of one is source of the other. Their efficiencies are equal. Find the temperature of the sink of first if its source temperature is 927° C and temperature of sink of the second is 27° C.

(a) 327 K (b) 327° C
(c) 600° C (d) none of these

Solution (b) $\eta = 1 - \frac{T_2}{T_1} = 1 - \frac{T_3}{T_2}$ or $T_2^2 = T_1T_3$

or $T_2 = \sqrt{1200\times300} = 600\text{ K} = 327°\text{ C}$

15. An ideal gas expands according to the law $PV^{3/2}$ = constant. We conclude

(a) The adiabatic exponent of the gas $K = 1.5$
(b) The molar heat capacity $C = C_V - 2R$
(c) Temperature increases during the process
(d) Such a process is not feasible

Solution (b) Molar heat capacity

$$C = C_V + \frac{R}{1-K} = C_V + \frac{R}{1-\frac{3}{2}} = C_V - 2R$$

16. The ratio of work done by an ideal diatomic gas to the heat supplied by the gas in an isobaric process is

(a) $\frac{5}{7}$ (b) $\frac{3}{5}$
(c) $\frac{2}{7}$ (d) $\frac{5}{3}$

Solution (c) $\Delta U = nC_V\Delta T = n\frac{5}{2}R\Delta T$

$\Delta Q = nC_P\Delta T = n\frac{7}{2}R\Delta T$

$W = \Delta Q - \Delta U = \frac{n7}{2}R\Delta T = nR\Delta T$

$\frac{W}{Q} = \frac{2}{7}$

17. A monoatomic gas is supplied heat Q very slowly keeping the pressure constant. The work done by the gas is

(a) $\frac{2}{5}Q$ (b) $\frac{3}{5}Q$
(c) $\frac{Q}{5}$ (d) $\frac{2}{3}Q$

Solution (a) For monoatomic gas

$\frac{\Delta U}{Q} = \frac{3}{5}$ or $\Delta U = \frac{3}{5}Q$

From the first law of thermodynamics

$Q = \Delta U + W$

$\therefore$ $W = \frac{2}{5}Q$

18. Which of the following parameters does not characterize the thermodynamic state of matter?

(a) work (b) pressure
(c) temperature (d) volume

[AIEEE 2003]

Solution (a) P, V and T are thermodynamic variables.

19. A Carnot engine takes 3×10^6 cal of heat from a reservoir at 627° C, and gives it to a sink at 27° C. The work done by the engine is

(a) 8.4×10^6 J (b) 16.8×10^6 J
(c) zero (d) 4.2×10^6 J

Solution (a) $\eta = 1 - \frac{T_2}{T_1} = 1 - \frac{300}{900} = \frac{W}{Q_1}$

or $W = \frac{2}{3} Q = 2 \times 10^6 \text{ cal} = 8.4 \times 10^6 \text{ J}$

20. An ideal gas heat engine operated between 227° C to 127° C in a Carnot cycle. It absorbs 6 K cal at the higher temperature. The amount of heat (in kcal) converted to work is equal to

(a) 1.2 (b) 4.8
(c) 3.5 (d) 1.6

[CBSE 2003]

Solution (a) $\eta = 1 - \frac{T_2}{T_1} = 1 - \frac{400}{500} = \frac{W}{Q_1}$

or $W = \frac{Q_1}{5} = 1.2 \text{ k cal}$

21. The efficiency of a Carnot engine operating between reservoirs maintained at 27° C and –123° C is

(a) 0.75 (b) 0.4
(c) 0.25 (d) 0.5

[DPMT 2002]

Solution (d) $\eta = 1 - \frac{T_2}{T_1} = 1 - \frac{150}{300} = \frac{1}{2}$

22. Calculate the change in entropy when 1 g of ice at 0° C is heated to form water at 40° C

(a) 0.28 cal/° C (b) 1.411 cal/° C
(c) 0.41 cal/° C (d) none of these

Solution (c) $\Delta S = \Delta S_1 + \Delta S_2$

To melt ice + to rise the temperature

$$= \frac{mL}{T} + mC \int_{T_1}^{T_2} \frac{\Delta T}{T}$$

$$= \frac{1 \times 80}{273} + 1 \times 1 \times 2.303 \log \frac{313}{273}$$

$= 0.28 + 0.1366 = 0.42 \text{ cal °C}^{-1}$

23. Calculate the change in entropy of n moles of a perfect gas when its temperature changes from T_1 to T_2 while its volume changes from V_1 to V_2

Solution $dQ = TdS = du + PdV$

or $$dS = \frac{nC_V dT}{T} + \frac{PdV}{T}$$

Since $PV = nRT$, therefore,

$$\frac{P}{T} = \frac{nR}{V}$$

Thus $dS = nC_V \frac{dT}{T} + n\text{R} \frac{dV}{V}$

Integrating $S_2 - S_1 = nC_V \log_e \frac{T_2}{T_1} + n\text{R} \log_e \frac{V_2}{V_1}$.

TYPICAL PROBLEMS

24. The energy required to break one bond in DNA is approximetely

(a) ~ 1ev (b) ~ 0.1ev
(c) ~ 0.0 1ev (d) ~ 2.1ev **[AIIMS 2005]**

Solution (a)

25. In isothermol process which of the following is not true

(a) temp remains const
(b) Internal energy does not change
(c) no heat enters or leaves the system
(d) none

[BHU 2005]

Solution (c)

26. In a heat engine sink is fitted at temp 27°C and a heat of 100 kcal is taken from source at temp 677°C work done in 10^6 J is

(a) 0.28 (b) 2.8
(c) 28 (d) 0.028 **[BHU 2005]**

Solution (a) $\frac{W}{Q} = \frac{900-300}{900} = \frac{2}{3}$ or

$$W = \frac{4.2 \times 10^5}{3} \times 2 = 2.8 \times 10^5 = .28 \times 10^6 J$$

27. An ideal monoatomic gas is taken around the cycle ABCDA as shown in PV diagram. The work done during the cycle is given by

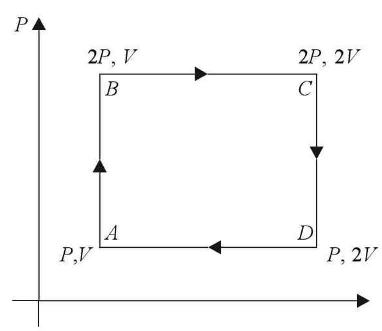


Fig. 16.18

(a) ½ PV (b) PV
(c) 2 PV (d) 4 PV

Solution (b) Area under the graph is work done.

$\therefore$ W = (2 P–P) (2 V–V) = PV

28. The wanelength of the radiation emitted by a body depends upon
(a) The nature of surface
(b) The area of the surface
(c) The temperature of the surface
(d) all the above

[CET Karnataka 2005]

Solution (c) We know $\lambda_m T = b$ (Wein's displacement law)

29. Which of the following is incorrect regarding the first law of thermodynamics
(a) It is not applicable to any cycle process
(b) It is a restatement of principnle of conservation of energy
(c) It introduces concept of internal energy
(d) It introduces concept of entropy

[AIEEE 2005]

Solution (a, d)

30. A gaseous mixture consists of 16 g He and 16 g O_2. The ratio of $\frac{C_p}{C_v}$ of the mixture is

(a) 1.59 (b) 1.62
(c) 1.4 (d) 1.54

[AIEEE 2005]

Solution (b)

$$C_v = \frac{n_1C_{v1}+n_2C_{v2}}{n_1+n_2} = \frac{4\left(\frac{3}{2}R\right)+\frac{1}{2}\left(\frac{5}{2}R\right)}{4+\frac{1}{2}} = \frac{29R}{18}$$

$$C_p = C_v + R = \frac{47R}{18};\ r = \frac{C_p}{C_v} = \frac{47}{29} = 1.62$$

31. The temperature entropy diagram of a reversible engine cycle is shown in

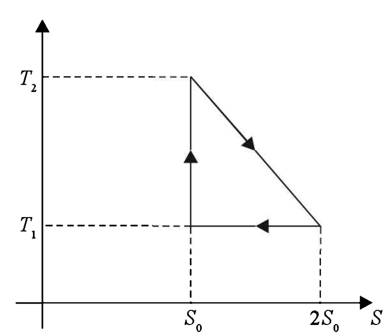


Fig. 16.19

Its efficiency is

(a) $\frac{1}{2}$ (b) $\frac{1}{4}$

(c) $\frac{1}{3}$ (d) $\frac{2}{3}$

[AIEEE 2005]

Solution (c) $\eta = \frac{W}{Q_1} = \frac{\frac{S_oT_o}{2}}{\frac{3}{2}S_oT_o} = \frac{1}{3}$

32. The figure shows a system of two concentric spheres of radii r_1 and r_2 and kept at temp T_1 and T_2 respectively. The radial rate of flow of heat in a substance between the two concentric sphere is proportional to

(a) $\frac{r_2 - r_1}{r_1r_2}$ (b) $\log_e\left(\frac{r_2}{r_1}\right)$

(c) $\frac{r_1r_2}{r_2 - r_1}$ (d) $\log_e(r_2 - r_1)$

[AIEEE 2005]

Solution (c) $\frac{dq}{dt} = (T_1 - T_2)\frac{4\pi r_1r_2K}{(r_2 - r_1)}$

33. A system goes from A to B via two processes I an II as shown in Figure. If ΔU_1 and ΔU_2 are the changes in internal energies in the processes I and II respectively then
(a) $\Delta U_\backslash = vU_2$
(b) the relation between ΔU_1, and ΔU_2 can not be established
(c) $\Delta U_2 > \Delta U_1$ (d) $\Delta U_1 > \Delta U_2$

Solution (a) Since internal energy is a state function.

34. A refrigerator, whose coefficient of performance $K = 5$, extracts heat from the cooling compartment at the rate of 250 J/cycle. What is the work done per cycle to operate the refrigerator? How much heat is discharged per cycle to the room which acts as the high temperature reservoir?

(a) 50 J (b) 200 J
(c) 300 J (d) none

Solution (c) $W = \frac{Q_2}{K} = \frac{250}{5} = 50$ J

$Q_1 = Q_2 + W = 300$ J

QUESTIONS FOR PRACTICE

1. The amount of work done by the gas system in increasing the volume of 10 mols of an ideal gas from one litre to 20 litres at 0^0 C will be

(a) zero (b) 3.49 Joule
(c) 3.49×10^4 Joule (d) 6.79×10^4 Joule

2. The minimum number of thermodynamic parameters required to specify the state of gas system is

(a) 1 (b) 2
(c) 3 (d) ∞

3. If C_P and C_V are the molar specific heats of a gas at constant pressure and volume respectively then the ratio of adiabatic and isothermal moduliii of elasticity will be

(a) $\frac{C_P - C_V}{C_P}$ (b) $C_P C_V$
(c) $\frac{C_V}{C_P}$ (d) $\frac{C_P}{C_V}$

4. The internal energy of a compressed real gas, as compared to that of the normal gas at the same temperature, is

(a) less (b) more
(c) sometimes less, sometimes more
(d) none of these

5. A system is given 400 calories of heat and 1000 Joule of work is done by the system, then the change in internal energy of the system will be

(a) –860 Joule (b) 680 erg
(c) 680 Joule (d) 860 Joule

6. In a certain process 500 calories of heat is given to a system and the system does 100 Joule of work. The increase in internal energy of the system is

(a) 40 calorie (b) 82 calorie
(c) 1993 Joule (d) 2193 Joule

7. 11 g of carbondioxide is heated at constant pressure from 27^0 C to 227^0 C. The amount of heat transferred to carbondioxide will be

(a) 2200 calorie (b) 350 calorie
(c) 220 calorie (d) 110 calorie

8. The specific heat of a gas at constant pressure as compared to that at constant volume is

(a) less (b) equal
(c) more (d) constant

9. An air bubble of volume 15 cm^3 is formed at a depth of 50 m in a lake. If the temperature of the bubble while rising remains constant then the volume of bubble at the surface will be ($g = 10$ ms^{-2} and atmospheric pressure = 1.0×10^5 Pa)

(a) 100 cm^3 (b) 90 cm^3
(c) 80 cm^2 (d) 40 cm^2

10. The ratio of the slopes of adiabatic and isothermal curves is

(a) γ^{-2} (b) $1/\gamma$
(c) γ^3 (d) γ

11. Equal volumes of monoatomic and diatomic gases of same initial temperature and pressure are mixed. The ratio of the specific heats of the mixture (C_P/C_V) will be

(a) 1.53 (b) 1.52
(c) 1.5 (d) 1

12. For a thermodynamic process $\delta Q = -50$ calorie and $W = -20$ calorie. If the initial internal energy is – 30 calorie then, final internal energy will be

(a) – 100 calorie (b) – 60 calorie
(c) 100 calorie (d) 191.20 calorie

13. The change in internal energy of two mols of a gas during adiabatic expansion is found to be –100 Joule. The work done during the process is

(a) –100 Joule (b) 0
(c) 100 Joule (d) 200 Joule

14. Out of the following, the indicator diagram is

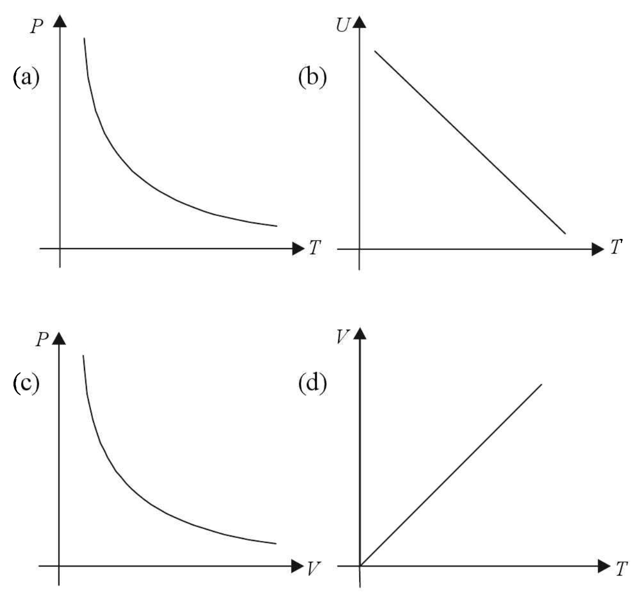


Fig. 16.20

15. The amount of heat required to raise the temperature of 100 g water from 20° C to 40° C will be

(a) zero (b) 100 calorie
(c) 2000 calorie (d) 4000 calorie

16. A liquid boils at such a temperature at which the saturated vapour pressure, as compared to atmospheric pressure, is

(a) one-third (b) equal
(c) half (d) double

17. The initial pressure of a gas is P. It is kept in an insulated container and suddenly its volume is reduced to one-third. Its final pressure will be

(a) $-3^{\gamma}P$ (b) $\frac{P}{(3)^{\gamma}}$
(c) $P/3$ (d) $3P$

18. The work done in Fig. 16.23 is

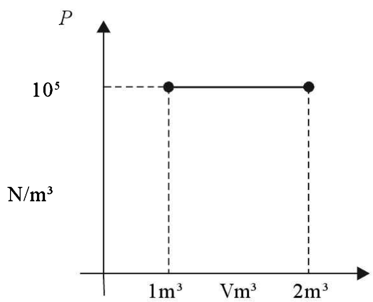


Fig. 16.21

(a) 3×10^5 J (b) 2×10^5 J
(c) 10^5 J (d) zero

19. 1 g of ice at 0° C is converted to steam at 100° C. The amount of heat required will be

(a) 12000 calorie (b) 756 calorie
(c) 716 calorie (d) 430 calorie

20. The heat capacity of a meterial depends upon

(a) density of matter
(b) specific heat of matter
(c) temperature of matter
(d) structure of matter

21. The isothermal bulk modulus of elasticity of a gas is 1.5×10^5 Nm^{-2}. Its adiabatic bulk modulus of elasticity will be (if $\gamma = 1.4$)

(a) 3×10^5 Nm^{-2} (b) 2.1×10^5 Nm^{-2}
(c) 1.5×10^5 Nm^{-2} (d) ∞

22. In changing the state of a system from state A to state B adiabatically the work done on the system is 322 Joule. If 100 calories of heat are given to the system in bringing it from state A to state B, then the work done on the system in this process will be

(a) 15.9 Joule (b) 38.2 Joule
(c) 98 Joule (d) 15.9 calorie

23. The indicator diagrams representing maximum and minimum amounts of work done are respectively

(a) a and b (b) b and c
(c) b and d (d) c and d

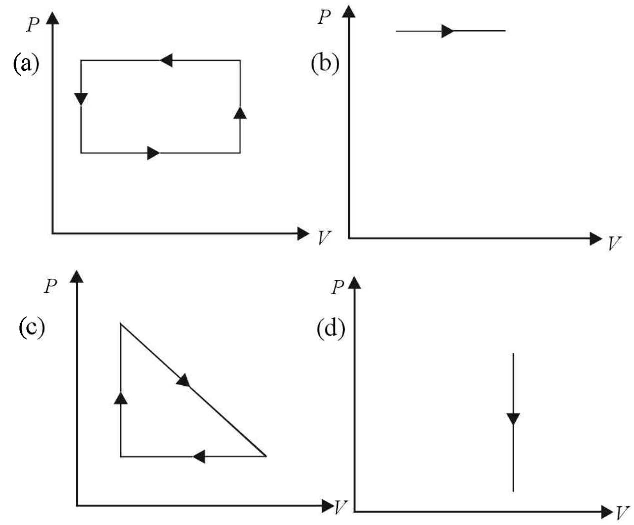


Fig. 16.22

24. Two samples of a gas A and B, initially at same temperature and pressure, are compressed to half their initial volume, A isothermally and B adiabatically. The final pressure in the two cases is related as

(a) $A = B$ (b) $A > B$
(c) $A < B$ (d) $A^2 = B$

25. A piece of ice at 0^0 C is dropped into water at 0^0 C. Then ice will

(a) melt (b) be converted to water
(c) not melt (d) partially melt

26. Four curves A, B, C and D are drawn for given mass a gas (Fig. 16.23). The curves which represent adiabatic and isothermal expansion are respectively

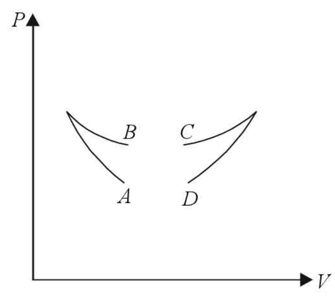


Fig. 16.23

(a) A and B (b) C and D
(c) B and A (d) D and C

27. How much work can be done by 250 calories of heat?

(a) zero (b) 1045 erg
(c) 1045 watt (d) 1050 Joule

28. In the gas equation $PV = RT$, V represents the volume of

(a) 1 mol of gas (b) 1 g of gas
(c) 1 litre of gas (d) any mass of gas

29. If, in defining the specific heat, temperature is represented in °F instead of °C then the value of specific heat will

(a) be converted to heat capacity
(b) remain unchanged
(c) decrease (d) increase

30. The specific heat of an ideal gas varies as

(a) T^3 (b) T^2
(c) T^1 (d) T^0

31. When an ideal diatomic gas is heated at constant pressure then what fraction of heat given is used to increase internal energy of gas?

(a) 2/5 (b) 3/5
(c) 3/7 (d) 5/7

32. When the temperature of a gas in a vessel is increased by 1° C then its pressure is increased by 0.5%. The initial temperature is

(a) 100 K (b) 200 K
(c) 273 K (d) 300 K

33. The internal energy of air in a room of volume 50 m³ at atmospheric pressure will be

(a) 2.5×10^7 erg (b) 2.5×10^7 Joule
(c) 5.25×10^7 Joule (d) 1.25×10^7 Joule

34. One mol of helium is heated at 0° C and constant pressure. How much heat is required to increase its volume threefold?

(a) 2730 calorie (b) 273 calorie
(c) 27.30 calorie (d) 2.730 calorie

35. The pressure and volume of a gas are P and V respectively. If it is compressed suddenly to 1/32 of its initial volume then its final pressure will be

(a) $P/128$ (b) $P/32$
(c) $128P$ (d) $32P$

36. The net amount of work done in the following indicator diagram is

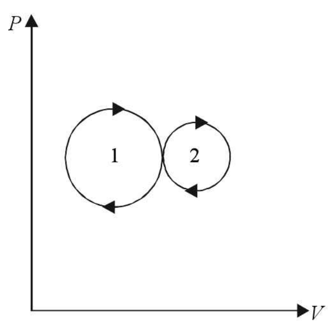


Fig. 16.24

(a) zero (b) positive
(c) negative (d) infinite

37. The volume of a gas is reduced to 1/4 of its initial volume adiabatically at 27° C. The final temperature of the gas (if $\gamma = 1.4$) will be

(a) $27 \times (4)^{0.4}$ K (b) $300 \times (1/4)^{0.4}$ K
(c) $100 \times (4)^{0.4}$ K (d) $300 \times (4)^{0.4}$ K

38. The concept of temperature is related to

(a) zeroeth law of thermodynamics
(b) first law of thermodynamics
(c) second law of thermodynamics
(d) third law of thermodynamics

39. When a liquid is heated retaining its liquid state, then its molecules gain

(a) kinetic energy (b) potential energy
(c) heat energy
(d) both kinetic and potential energy

40. A system absorbs 10^3 calories of heat and the system does 1677.3 Joule work. The internal energy of the system increases by 2515 Joule. The value of J is

(a) 4.19 Joule/cal (b) 4.18 cal/Joule
(c) 42 Joule/cal (d) 420 Joule/cal

41. The relation between P and T for monoatomic gas during adiabatic process is $P \propto T^C$. The value of c is

(a) 3/5 (b) 2/5
(c) 5/3 (d) 5/2

42. A player gets 400 kilocalories energy daily from the food. His power will be

(a) zero (b) 1.93 watt
(c) 19.3 watt (d) 193.5 watt

43. The thermodynamic scale of temperature was given by

(a) Dewar (b) Fahrenheit
(c) Kelvin (d) Carnot

44. 1 m³ of a gas is compressed suddenly at atmospheric pressure and temperature 27° C such that its temperature becomes 627° C. The final pressure of the gas (if γ= 1.5) will be

(a) 2.7×10^5 Nm^{-2} (b) 7.2×10^5 Nm^{-2}
(c) 27×10^5 Nm–2 (d) 27×10^6 Nm^{-2}

45. The volume of 1 m³ of gas is doubled at atmospheric pressure. The work done at constant pressure will be

(a) zero (b) 10^5 calorie
(c) 10^5 Joule (d) 10^5 erg

46. If the volume of a gas is decreased by 10% during isothermal process then its pressure will

(a) decrease by 10% (b) increase by 10%
(c) decrease by 11.11% (d) increase by 11.11%

47. At the boiling point of water the saturated vapour pressure will be (in mm of Hg)

(a) 750 (b) 760
(c) 850 (d) 860

48. The ratio of the latent heat of steam to latent heat of ice is

(a) 4/9 (b) 9/4
(c) 4/27 (d) 27/4

49. The maximum efficiency of an engine operating between the temperature 400° C and 60° C is

(a) 55% (b) 75%
(c) 95% (d) none of these

50. A Carnot engine works between ice point and steam point. Its efficiency will be

(a) 85.42% (b) 71.23%
(c) 53.36% (d) 26.81%

51. If the temperature of the sink is absolute zero, the efficiency of the heat engine should be

(a) 100% (b) 50%
(c) zero (d) none of these

52. When the temperature difference between the source and the sink increases, the efficiency of the heat engine will

(a) increase (b) decrease
(c) is not affected
(d) may increase or decrease depending upon the nature of the working substance

53. A Carnot engine can be 100% efficient if its sink is at

(a) 0 K (b) 0° C
(c) 0° F (d) 273 K

54. Which of the following is the best container for gas during adiabatic process?

(a) Wood vessel (b) Thermos flask
(c) Copper vessel (d) Glass vessel

55. In which of the following processes does the system always returns to the original thermodynamic state?

(a) Isobaric (b) Cyclic
(c) Isothermal (d) Adiabatic

56. A Carnot engine has an efficiency of 50% when its sink is at a temperature of 27° C. The temperature of the source is

(a) 300° C (b) 327° C
(c) 373° C (d) 273° C

57. Figure 16.25 shows four indicator diagrams. In which case is the work done maximum?

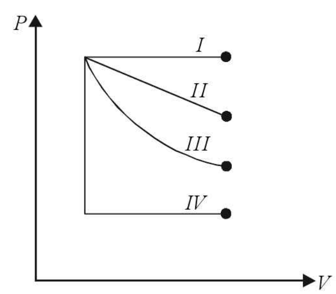


Fig. 16.25

(a) IV (b) II
(c) III (d) I

58. One mole of a monoatomic gas and one mole of a diatomic gas are mixed together. What is the molar specific heat at constant volume for the mixture?

(a) $5/2\,R$ (b) $2\,R$
(c) $3/2\,R$ (d) $3\,R$

59. What is the value of dp/p for adiabatic expansion of the gas?

(a) $\gamma dV/V$ (b) $-dV/V$
(c) dV/V (d) $-\gamma dV/V$

60. Which of the following has higher efficiency? An engine working between the temperatures

(a) 40 K and 20 K (b) 60 K and 40 K
(c) 80 K and 60 K (d) 100 K and 80 K

61. The temperature of the source of a Carnot heat engine is 0° C and that of sink is – 39° C. The efficiency of the heat engine is

(a) 39% (b) 14.3%
(c) zero (d) none of these

62. Work done during isothermal expansion depends on change in

(a) volume (b) pressure
(c) both (a) and (b) (d) none of these

63. For an engine operating between the temperatures t_1 °C and t_2 °C, the efficiency will be

(a) $\frac{t_1 - t_2}{t_1}$ (b) $\frac{t_1 - t_2}{t_1 + 273}$

(c) $\frac{t_2 + 273}{t_1 + 273}$ (d) $\frac{t_2}{t_1}$

64. For 100% efficiency of a Carnot engine the temperature of the source should be

(a) 273° C (b) 0° C
(c) –273° C (d) none of these

65. When 1 mole of a monoatomic gas expands at constant pressure the ratio of the heat supplied that increases the internal energy of the gas and that used in expansion is

(a) 2/3 (b) 3/2
(c) 0 (d) ∞

66. The efficiency of the heat engine working between 327° C and 27° C is to be increased by 10%. The temperature of the source should be increased by

(a) 52° C (b) 67° C
(c) 37° C (d) 77° C

67. A Carnot engine operates with a source at 500 K and sink at 375 K. If the engine consumes 600 K cal of heat in one cycle, the heat rejected to the sink per cycle is

(a) 550 K cal (b) 450 K cal
(c) 350 K cal (d) 250 K cal

68. The change in which of the following solely determines the work done by a gas during adiabatic process?

(a) Temperature (b) Pressure
(c) Volume (d) None of these

69. A Carnot engine, whose source is at 400 K, takes 200 cal of heat and reflects 150 cal to the sink. What is temperature of the sink?

(a) 300 K (b) 400 K
(c) 800 K (d) none of these

70. A gas at pressure 6×10^5 Nm^{-2} and volume 1 m^3 expands to 3 m^3 and its pressure falls to 4×10^5 Nm^{-2}. Given that the indicator diagram is a straight line, the work done on the system is

(a) 12×10^5 J (b) 6×10^5 J
(c) 4×10^5 J (d) 3×10^5 J

71. During an adiabatic expansion of 5 moles of gas, the internal energy decreases by 75 J. The work done during the process is

(a) –75 J (b) zero
(c) 15 J (d) 75 J

72. A monoatomic gas expands isobarically. The percentage of heat supplied that increases the thermal energy and that involved in doing work for expansion is

(a) 40 : 60 (b) 60 : 40
(c) 50 : 50 (d) none of these

73. How many dead centres are there in one cycle of steam engine?

(a) 4 (b) 3
(c) 2 (d) 1

74. For adiabatic expansion of a monoatomic perfect gas, the volume increases by 2.4%. What is the percentage decrease in pressure?

(a) 2.4% (b) 4.0%
(c) 4.8% (d) 7.1%

75. Figure 16.26 represents two processes a and b for a given sample of gas. Let ΔQ_1 and ΔQ_2 be the heat absorbed by the systems in the two cases respectively. Which of the following relations is correct?

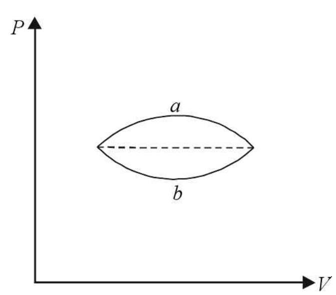


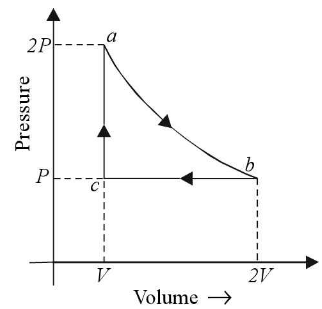
Fig. 16.26

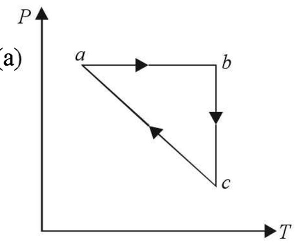
(a) $\Delta Q_1 = \Delta Q_2$ (b) $\Delta Q_1 > \Delta Q_2$
(c) $\Delta Q_1 \leq \Delta Q_2$ (d) $\Delta Q_1 < \Delta Q_2$

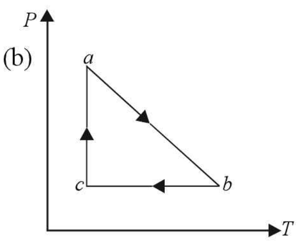
76. A cylinder contains helium at 2.5 atmosphere pressure. Another identical cylinder contains argon at 1.5 atmosphere pressure at the same temperature. If both the gases are filled in any one of the cylinders, the pressure of the mixture will be

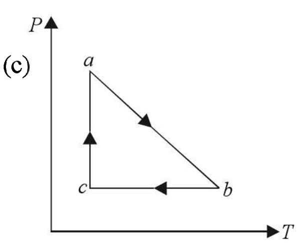
(a) 1.5 atm (b) 2.5 atm
(c) 4 atm (d) none of these

77. Figure 16.27 shows a cyclic process $abca$ for one mole of an ideal gas. If ab is isothermal process, then which of the following is the $P - T$ diagram for the cyclic process?



(a)


(b)


(c)


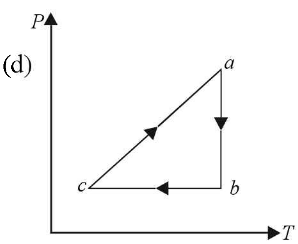
(d)


Fig. 16.27

78. A gas is enclosed in a vessel of volume 1000 cc at a pressure of 72.6 cm of Hg. It is being evacuated with the help of a piston pump, which expels 10% gas in each stroke. The pressure after the second stroke will be nearest to

(a) 60 cm (b) 55 cm
(c) 66 cm (d) 50 cm

79. An ideal gas heat engine operates in Carnot cycle between 272° C and 127° C. It absorbs 6.0×10^4 cal at the higher temperature. The amout of heat converted into work is equal to

(a) 1.2×10^4 cal (b) 1.6×10^4 cal
(c) 3.5×10^4 cal (d) 4.8×10^4 cal

80. In free expansion of a gas the internal energy of the system

(a) increases (b) decreases
(c) is unchanged (d) changed

81. A Carnot engine whose sink is at a temperature of 300 K has an efficiency of 40%. By how much should the temperature of the source be increased so as to increase the efficiency to 60%?

(a) 250 K (b) 275 K
(c) 325 K (d) 380 K

82. During an adiabatic expansion of 2 moles of a gas, the change in internal energy was found to be equal to – 200 J. The work done during the process will be equal to

(a) –100 Joule (b) zero
(c) 100 Joule (d) 200 Joule

83. Which of the following represents an isothermal expansion?

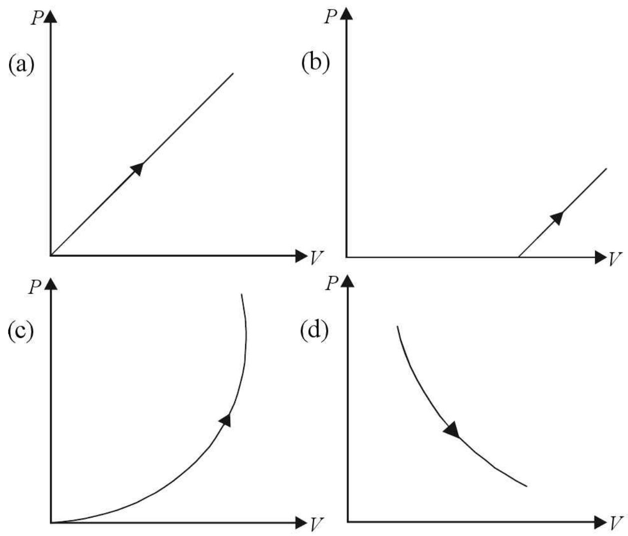


Fig. 16.28

84. Two perfect gases having masses m_1 and m_2 at temperatures T_1 and T_2 are mixed without any loss of internal kinetic energy of the molecules. The molecular weights of the gases are M_1 and M_2. What is the final temperature of the mixture?

(a) $\dfrac{\frac{M_1}{m_1}T_1 + \frac{M_2}{m_2}T_2}{\frac{M_1}{m_1} + \frac{M_2}{m_2}}$ (b) $\dfrac{\frac{m_1}{M_1}T_1 + \frac{m_2}{M_2}T_2}{\frac{m_1}{M_1} + \frac{m_2}{M_2}}$

(c) $\dfrac{M_1T_1 + M_2T_2}{M_1 + M_2}$ (d) $\dfrac{m_1T_1 + m_2T_2}{m_1 + m_2}$

85. In the following $V - T$ diagram, what is the relation between P_1 and P_2?

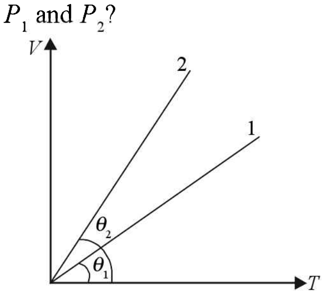


Fig. 16.29

(a) $P_2 = P_1$ (b) $P_2 > P_1$
(c) $P_2 < P_1$ (d) cannot be predicted

86. In a thermodynamic process pressure of a fixed mass of gas is changed in such a manner that the gas releases 30 Joule of heat and 18 Joule of work was done on the gas. If the initial internal energy of the gas was 60 Joule, then the final internal energy will be

(a) 96 Joule (b) 72 Joule
(c) 48 Joule (d) 32 Joule

87. A Carnot engine working between 300 K and 600 K has a work output of 800 J per cycle. The amount of heat energy supplied to the engine from the source in each cycle is

(a) 6400 J (b) 3200 J
(c) 1600 J (d) 800 J

88. Find the change in internal energy of the system when a system absorbs 2 kilocalorie of heat and at the same time does 500 Joule of work.

(a) 8200 J (b) 7900 J
(c) 6400 J (d) 5600 J

89. If 1 g of water at 40° C is converted to steam at 100° C, the change in entropy is

(a) $2.303 \log \dfrac{373}{313}$ cal °C^{-1}

(b) $\dfrac{600}{373}$ cal °C^{-1}

(c) $\dfrac{600}{313}$ cal °C^{-1}

(d) $\dfrac{540}{373} + \log \dfrac{373}{313}$ cal °C^{-1}

90. If the value of $R = 2/5\, C_V$ for a gas, then the atomicity of the gas will be

(a) monoatomic (b) diatomic
(c) polyatomic (d) any of these

91. 10 g of ice at 0° C melts. The entropy then

(a) decreases by 2.93 cal ^{0}C^{-1}
(b) increases by 2.93 cal ^{0}C^{-1}
(c) remains unchanged
(d) none of these

92. Which of the curve shows adiabatic compression?

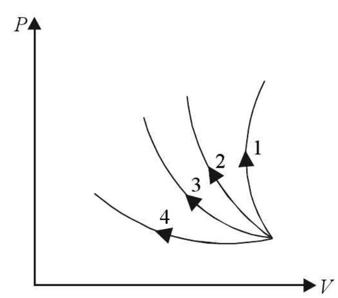
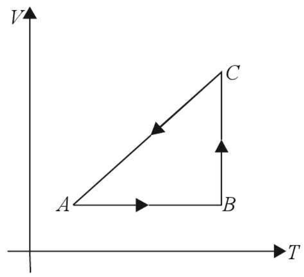
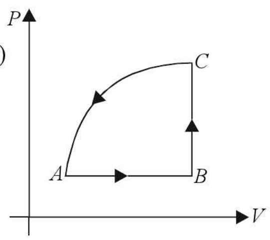


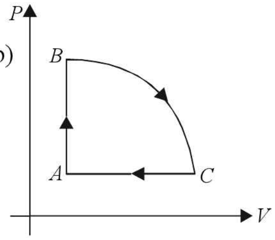
Fig. 16.30

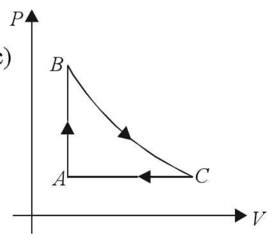
(a) 4 (b) 2
(c) 3 (d) 1

93. Figure 16.31 shows a cyclic process. Which of the following is its PV conversion?



(a)


(b)


(c)


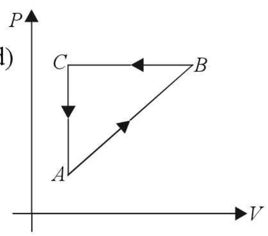
(d)


Fig. 16.31

94. The value of C_p/C_V for the mixture of 2 mols of oxygen and 5 mols of ozone is

(a) 1.34 (b) 1.41
(c) 1.51 (d) 1.67

95. When the temperature of an iron sphere of mass 1 kg falls from 30° C to 25° C, then 550 calorie of heat is released. The heat capacity of the iron sphere will be (in cal °C^{-1})

(a) 440 (b) 330
(c) 220 (d) 110

96. A hail at 0° C falls from a height of 1 km on an insulated surface and its whole kinetic energy is converted into heat. What fraction of it will melt?

(a) $1/33 \times 10^{-4}$ (b) 1/33
(c) 1/8 (d) whole of it will melt

97. A 50 g piece of iron at 100°C is dropped into 100 g water at 20° C. The temperature of the mixture is 25.5 °C. The specific heat of iron (in calg–1 °C^{-1}) is

(a) 0.148 (b) 0.082
(c) 0.267 (d) 0.341

98. If the radii of two copper spheres are in the ratio 1 : 3 and their temperatures are in the ratio 9 : 1 then the ratio of the heat contents in them will be

(a) 1 : 3 (b) 1 : 4
(c) 2 : 1 (d) 4 : 1

99. A reversible heat engine converts 1/6th of heat which it absorbs from source into work. When the temperature of sink is reduced by 62 °C, its efficiency is doubled. The temperature of the source is

(a) 372 K (b) 272 K
(c) 172 K (d) 72 K

100. An ideal gas is filled in a container of volume 8.3×10^3 m^3 at 300 K temperature and 2.0×10 Nm^{-2} pressure. If it is given an additional energy 2.5×10^9 J, then its final temperature will be

(a) 600 K (b) 625 K
(c) 650 K (d) 675 K

101. For which process is the relation $dQ = dU$ true?

(a) Isobaric (b) Isochoric
(c) Isothermal (d) Adiabatic

102. The molar specific heat of an ideal gas at constant pressure and volume are C_P and C_V respectively. The value of C_V is

(a) R (b) γR
(c) $\frac{R}{\gamma - 1}$ (d) $\frac{\gamma R}{\gamma - 1}$

103. Heating of a wheel on applying brakes is due to the relation

(a) $P \propto \frac{1}{V}$ (b) $P \propto T$
(c) $W \propto Q$ (d) $V \propto T$

104. A bicycle is moving at a speed of 36 kmh^{-1}. Brakes are applied. It stops in 4 m. If mass of the bicycle is 40 kg then temperature of the wheel risen is [specific heat of wheel 0.25 calg^{-1} °C^{-1}, mass of the = wheel = 5 kg]

(a) 0.19° C (b) 0.47° C
(c) 4.7° C (d) 1.9° C

105. Out of the following whose specific heat is maximum?

(a) Lead (b) Brass
(c) Glass (d) Iron

106. The correct value of temperature on Kelvin scale corresponding to 0° C is

(a) 0 K (b) 273.15 K
(c) 273.2 K (d) 273 K

107. For Boyle's law to hold good, the necessary condition is

(a) isothermal (b) adiabatic
(c) isobaric (d) isochoric

108. The mechanicl equivalent of heat (J) is a

(a) conversion factor (b) constant
(c) physical quantity (d) none of these

109. The internal energy of an isolated system

(a) keeps on changing (b) remains constant
(c) zero (d) none of these

110. The number of specific heats for a gas system is

(a) 1 (b) 2
(c) three (d) infinite

111. The internal energy of a piece of lead when beaten by a hammer will

(a) increase
(b) decrease
(c) remain constant
(d) sometimes increase and sometimes decrease

112. 20 g of water at 20° C is contained in a calorimeter of water equivalentof 10 g of water at 50° C is mixed into it. The temperature of the mixture will be

(a) 15.2° C (b) 27.5° C
(c) 35.2° C (d) 43.7° C

113. How much heat is required to heat 2 moles of a monoatomic ideal gas from 0° C to 100° C if no mechanical work is done during heating? [The specific heat of gas at constant pressure is 2.5 R, where R is the universal gas constant.]

(a) 378.6 cal (b) 417.1 cal
(c) 596.4 cal (d) 782 cal

114. Two steam engines A and B have their sources respectively at 700 K and 650 K and their sinks at 350 K and 300 K. Then

(a) A is more efficient than B
(b) B is more efficient than A
(c) both are equally efficient
(d) depends on the fuels used in A and B

115. A Carnot engine works between 600 K and 300 K. In each cycle of operations, the engine draws 1000 Joule of energy from the source at 600 K. The efficiency of the engine is

(a) 90% (b) 70%
(c) 50% (d) 20%

116. During isothermal expansion at 800 K, the working substance of a Carnot engine releases 480 calories of heat. If the sink is at 300 K then the work done by the working substance during isothermal expansion will be

(a) 480 cal (b) 300 cal
(c) 270 cal (d) 190 cal

117. A heat engine operates between 2100 K and 700 K. Its actual efficiency is 40%. What percentage of its maximum possible efficiency is this?

(a) 66.67% (b) 60%
(c) 40% (d) 33.33%

118. An ideal heat engine exhausting heat at 77° C is to have 30% efficiency. It must take heat at

(a) 673° C (b) 327° C
(c) 227° C (d) 127° C

119. The temperature, which is the same in ^{0}C and ^{0}F, is

(a) –20 (b) 20
(c) –40 (d) 40

120. Isobaric bulk modulus of elasticity is

(a) ∞ (b) zero
(c) P (d) $\frac{C_P}{C_V}$

121. A Carnot engine has same efficiency between (i) 100 K and 500 K and (ii) T_K and 900 K. The value of T is

(a) 360 K (b) 270 k
(c) 180 k (d) 90 K

122. Two systems are in thermal equilibrium. The quantity which is common for them is

(a) heat (b) momentum
(c) specific heat (d) temperature

123. The amount of heat necessary to raise the temperature of 0.2 mol of N_2 at constant pressure from 37° C to 337° C will be

(a) 764 Joule (b) 1764 erg
(c) 1764 calorie (d) 1764 Joule

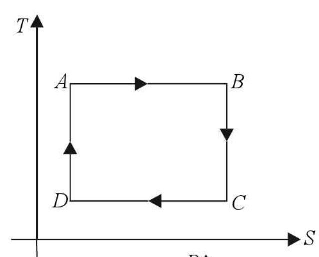
124. Air is filled in a motor car tube at 27° C temperature and 2 atmosphere pressure. If the tube suddenly bursts then the final temperature will be [given $(1/2)^{2/7} = 0.82$]

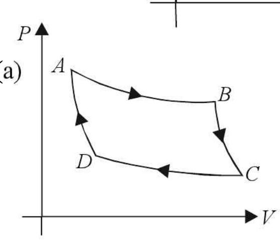
(a) 642 K (b) 563 K
(c) 300 K (d) 246 K

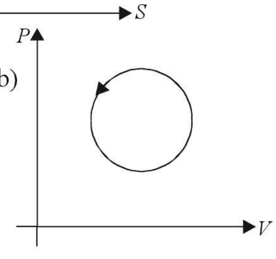
125. A reversible engine takes heat from a reservoir at 527° C and gives out to perform useful mechanical work at the rate of 750 watt. The efficiency of the engine is

(a) 70% (b) 50%
(c) 30% (d) 10%

126. The temperature *vs* entropy diagram is shown in Fig. 16.32. Its PV equivalent diagram is



(a)


(b)


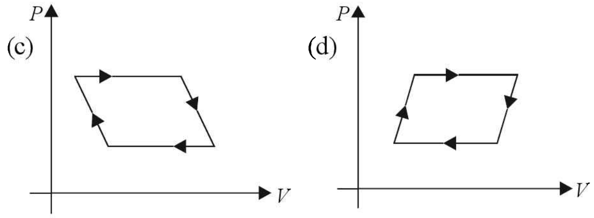


Fig. 16.32

127. Pure water, cooled to –15° C, is kept in an insulated flask. Some ice is dropped into the flask. The fraction of water frozen into ice is (specific heat of ice = 0.5 calg–1 °C^{-1})

(a) 6/29 (b) 3/35
(c) 6/35 (d) 2/35

128. A block of ice of mass 50 kg is pushed out on a horizontal plane with a velocity of 5 m/s. Dur to friction it comes to rest after covering a distance of 25 m. How much ice will melt?

(a) 100 g (b) 1000 g
(c) 1.86 g (d) 0.86 g

129. The height of a water spring is 50 m. The difference of temperature at the top and bottom of the spring will be

(a) 0.117° C (b) 1.17° C
(c) 0.437° C (d) 11.7° C

130. The radiator of a car contains 20 litre water. If the motor supplies 2 × 10^5 calorie heat to it, then rise in its temperature will be

(a) 1000° C (b) 100° C
(c) 20° C (d) 10° C

Answers to Questions for Practice

1. (d)	**2.** (b)	**3.** (d)	**4.** (a)	**5.** (c)	**6.** (c)	**7.** (b)
8. (c)	**9.** (b)	**10.** (d)	**11.** (c)	**12.** (b)	**13.** (c)	**14.** (c)
15. (c)	**16.** (b)	**17.** (a)	**18.** (c)	**19.** (c)	**20.** (b)	**21.** (b)
22. (c)	**23.** (c)	**24.** (c)	**25.** (c)	**26.** (a)	**27.** (d)	**28.** (a)
29. (c)	**30.** (d)	**31.** (d)	**32.** (b)	**33.** (d)	**34.** (a)	**35.** (c)
36. (a)	**37.** (d)	**38.** (a)	**39.** (a)	**40.** (a)	**41.** (d)	**42.** (c)
43. (c)	**44.** (c)	**45.** (c)	**46.** (b)	**47.** (b)	**48.** (d)	**49.** (d)
50. (d)	**51.** (a)	**52.** (a)	**53.** (a)	**54.** (b)	**55.** (b)	**56.** (b)
57. (d)	**58.** (b)	**59.** (d)	**60.** (a)	**61.** (b)	**62.** (c)	**63.** (b)
64. (c)	**65.** (b)	**66.** (b)	**67.** (b)	**68.** (a)	**69.** (a)	**70.** (c)
71. (d)	**72.** (b)	**73.** (c)	**74.** (b)	**75.** (b)	**76.** (c)	**77.** (d)
78. (a)	**79.** (b)	**80.** (c)	**81.** (a)	**82.** (d)	**83.** (d)	**84.** (b)
85. (c)	**86.** (c)	**87.** (c)	**88.** (b)	**89.** (d)	**90.** (b)	**91.** (b)
92. (b)	**93.** (c)	**94.** (a)	**95.** (d)	**96.** (b)	**97.** (a)	**98.** (a)
99. (a)	**100.** (d)	**101.** (b)	**102.** (c)	**103.** (c)	**104.** (a)	**105.** (a)
106. (b)	**107.** (a)	**108.** (a)	**109.** (b)	**110.** (b)	**111.** (a)	**112.** (b)
113. (c)	**114.** (b)	**115.** (c)	**116.** (b)	**117.** (b)	**118.** (c)	**119.** (c)
120. (b)	**121.** (c)	**122.** (d)	**123.** (d)	**124.** (d)	**125.** (b)	**126.** (a)
127. (c)	**128.** (c)	**129.** (a)	**130.** (d)			

Explanations

7.(b) $n = \frac{1}{44} = \frac{1}{4}$

$\Delta Q = nc_p\Delta T = \frac{1}{4} \times \frac{7}{2} R \times 200 = 350$ cal.

12.(b) $\Delta Q = \Delta U + W$ or $\Delta U = -50 + 20 = -30$ cal

$U_f = u_i + \Delta u = -30 - 30 = -60$ cal.

21.(b) $P_{adiabatic} = \gamma P_{iso} = 1.4\,(1.5 \times 10^5) = 2.1 \times 10^5$ Nm^{-2}.

32.(b) $\frac{\Delta P}{P} = \frac{\Delta T}{T} = \frac{0.5}{100}$ or $\frac{1}{T} = \frac{0.5}{100}$ or $T = 200$ K.

33.(d) $U = \frac{5}{2}nRT = \frac{5}{2} \times \frac{50}{22.4\times10^{-3}} \times 8.3 \times 273$

$= 1.25 \times 10^7$ J.

37.(d) $T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$ or $T_2 = 300\,(4)^{0.4}$ K.

44.(c) $P_1^{1-\gamma}\,T_1^{\gamma} = P_2^{1-\gamma}\,T_2^{\gamma}$

or $P_2 = P_1\left(\frac{T_1}{T_2}\right)^{\frac{\gamma}{1-\gamma}} = 10^5(3)^3$.

66.(b) $\eta = 1 - \frac{300}{600}$. $\eta_{new} = 0.55 = 1 - \frac{300}{T}$ or $T = \frac{300}{0.45}$.

69.(a) $\frac{Q_1 - Q_2}{Q_1} = 1 - \frac{T_2}{T_1}\ \frac{50}{200} = 1 - \frac{T_2}{400}$

or $T_2 = 300$ K.

70.(c) $W = (P_1 - P_2)(V_2 - V_1) = 2 \times 10^5 \times 2 = 4 \times 10^5 J$

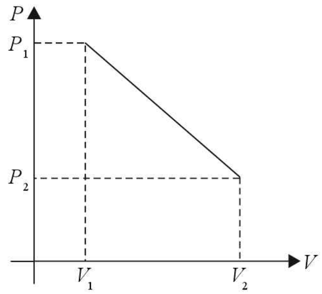


Fig. 16.33

78.(a) $P_1V_1 = P_2V_2P_2 = 72.6\,(.81)$ as volume after two strokes will be 810 cc.

85.(c) Conclude using $\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$.

89.(d) $S = \frac{dQ}{T}$ $\Delta S = mc \int \frac{dT}{T} + \frac{mL}{T}$

$$= 1 \times 1 \int_{313}^{373} \frac{dT}{T} + \frac{540}{373}.$$

90.(b) $C_V = \frac{5}{2} R$ ∴ the gas is diatomic.

96.(b) $\frac{m'}{m} = \frac{gh}{JL} = \frac{980 \times 10^5}{4.2 \times 10^7 \times 80} = \frac{1}{33}$.

127. (c) $M \times 1 \times 15 = m\,80 + m(0.5)(15)$

$$\frac{m}{M} = \frac{15}{87.5} = \frac{6}{35}.$$

17

Heat Transfer Processes

BRIEF REVIEW

Heat energy can be transferred in three distinguished methods: conduction, convection and radiation. Conduction usually occurs in solids, convection in fluids. Radiation does not require any medium.

Thermal conduction Let A area of cross-section of a conductor, l its length, K thermal conductivity, T_1 and T_2 temperatures at two ends, then rate of transfer of heat $\frac{dQ}{dt}$ or thermal current is given by

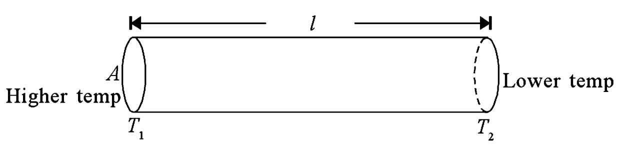


Fig. 17.1 Heat transfer by conduction in a rod

$$\frac{dQ}{dt} = \frac{KA(T_1 - T_2)}{l} = \frac{-KAdT}{dx}$$

Note that temperature gradient $\frac{dT}{dx}$ is negative.

Comparing it with Ohm's law in electricity $i = V/R$

$$i_{\text{thermal}} = \frac{dQ}{dt},\ V_{\text{thermal}} = T_1 - T_2 \text{ and } R_{\text{thermal}} = \frac{l}{KA}$$

Laws of resistances in case of thermal resistances in series and parallel are alike their counterparts in electricity. It is believed that current carriers (free electrons) are perhaps heat carriers also as all electrical conductors are also thermal conductors. In general, metals are better thermal conductors than liquids and gases as metals have large number of free electrons.

Thermometric Conductivity (*D*) It is the ratio of thermal conductivity to thermal capacity per unit volume. Thus **thermometric conductivity** or **diffusivity** is

$$D = \frac{K}{\rho C} \text{ where K} \rightarrow \text{thermal conductivity}$$

ρ →density

C →specific heat

Thermal Conductivity K of gases $K = \frac{1}{3} v_{av} \lambda \rho C_V$

$= D\rho C_V$ where $D = \frac{1}{3} v_{av} \lambda$ is diffusion coefficient and

$$\lambda = \frac{1}{\sqrt{2}\pi d^2 n} \text{ is mean free path.}$$

d →effective diameter of a molecule

n →number of molecules/ volume

Wiedemann–Franz law Wiedemann and Franz have shown that at a given temperature T, the ratio of thermal conducitivity (K) to electrical conductivity (σ) is constant. That is,

$$\frac{K}{\sigma T} = \text{constant}$$

Ingen–Housz's experiment Ingen Housz showed that if a number of identical rods of different metals are coated with wax and one of their ends is put in boiling water, then in steady state, the square of length of the bar over which wax melts is directly proportional to the thermal conductivity of the metal. That is,

$$\frac{K}{L^2} = \text{constant}$$

Thermal resistances in series

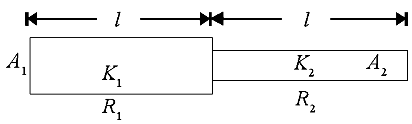


Fig. 17.2 Thermal resistances in series

$$R = R_1 + R_2$$

$$\frac{l_1 + l_2}{K_{eff} A_{eff}} = \frac{l_1}{K_1 A_1} + \frac{l_2}{K_2 A_2}$$

$$\text{if } A_1 = A_2 \text{ and } l_1 = l_2$$

then $$\frac{2}{K_{eff}} = \frac{1}{K_1} + \frac{1}{K_2}$$

Thermal resistances in parallel

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$\frac{K_{eff} A_{eff}}{l} = \frac{K_1 A_1}{l} + \frac{K_2 A_2}{l}$$

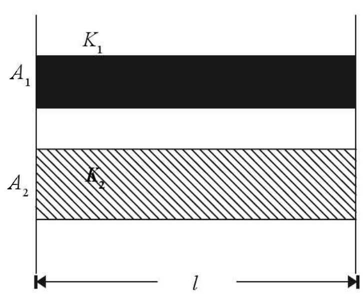


Fig. 17.3 Thermal resistances in parallel

or $K_{eff} A_{eff} = K_1 A_1 + K_2 A_2$

If the area of cross-section is equal then

$A_{eff} = A_1 + A_2 = 2A$

and $$K_{eff} = \frac{K_1 + K_2}{2}$$

Growth of ice in a pond

$$dQ_1 = KA \frac{0 - (-\theta)}{y} dt = dQ_2 = mL = \rho A dy L$$

or $$\frac{dy}{dt} = \frac{K\theta}{\rho L} \times \frac{1}{y}$$

or $$t = \frac{\rho L}{K\theta} \frac{y^2}{2}$$

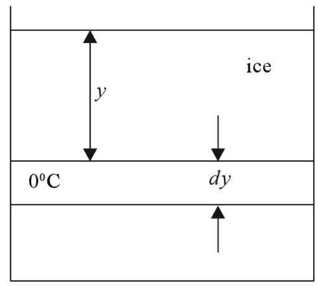


Fig. 17.4 Growth of ice in a lake

The ratio of times for thickness 0 to y : y to $2y$: $2y$ to $3y$: : 1 : 3 : 5

In a **shell** of radius r_1 and r_2

$$\frac{dQ}{dt} = K \frac{4\pi r_1 r_2}{(r_2 - r_1)} (\theta_2 - \theta_1) \text{ [See Fig. 17.5]}$$

Thickness of the shell $$(r_2 - r_1) = \frac{K 4\pi r_1 r_2}{\frac{dQ}{dt}}$$

where $r_1 - r_2 = r$ and $\theta_1 - \theta_2 = \theta$

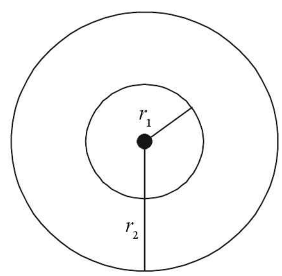


Fig. 17.5 Thermal conduction in a hollow sphere of radius r_1 and r_2

In a **cylinder** of length l, radii r_1 and r_2

$$\frac{dQ}{dt} = \frac{2\pi K l (\theta_1 - \theta_2)}{\log_e \frac{r_2}{r_1}} \text{ [See Fig. 17.6]}$$

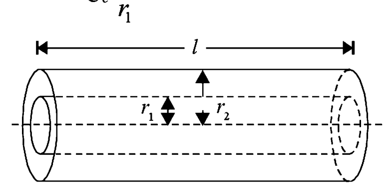


Fig. 17.6 Heat conduction in a hollow cyclinder

Convection It is the process in which heat is transferred from one place to the other by the actual movement of heated substance (usually fluid). Convection requires medium.

When a liquid is heated the particle of the liquid which gets heated moves upwards, delivers heat to the other particles by bodily movement and gets cooled. It comes down at the heating point, gets heated and goes back to its journey to distribute heat as illustrated in Fig. 17.7. This type of convection, which results from difference in densities, is called natural convection. However, if a heated fluid is forced to move by a blower fan or pump, it is called forced convection.

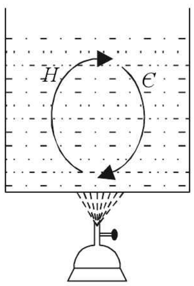


Fig. 17.7 Illustration of Convection

It is found that heat convection from an object is proportional to the temperature difference $\Delta\theta$ between the object and convective fluid and the contact area A. That is,

$$\left[\frac{dQ}{dt}\right]_{\text{Convection}} = hA\Delta\theta \text{ where } h \text{ is a constant.}$$

Lapse rate (the temperature falls by 6^0 C/km as we move up) is an example of natural convection. Blood circulation, which helps in maintaining the temperature of the body, is an example of forced convection.

Radiation The process by which heat is transferred directly from one body to another, without requiring a medium is called radiation. It is an electromagnetic radiation of wavelength 1 mm to 10^{-7} m. Velocity is equal to the speed of light. It can be detected by thermocouple, thermopile or radiometer, bolometer, and so on.

Black body A black body is capable of emitting or absorbing radiation of all possible wavelengths. Initially it was thought that a body which absorbs all possible radiation is only a black body. Later on emission characteristics were also included.

The blackest body on the earth is lamp black. It is 98% black.

Prevost's black body or Fery Platinum black body is shown in Fig 17.8

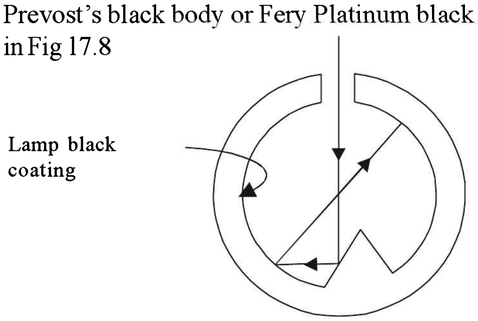


Fig. 17.8 Black body

Absorptive power The absorptive power of a substance is defined as the ratio of the radiant energy absorbed by it in a given time to the total radiant energy incident on it in the same time.

Spectral absorptive power

$$a = \int_0^\infty a_\lambda d\lambda \text{ unit } (\text{Wm}^{-2})$$

For a perfectly black body absorptive power is 1.

Emissive power (e) Radiant energy emitted per unit area of the surface. However, if we consider emissive power of a surface for a particular wavelength, it is called spectral emissive power. 'Spectral emissive power of a perfectly black body at that temperature.' Thus for a body having e and a as emissive and absorptive power

$$\frac{e}{a} = \frac{E}{A} = \frac{E}{1} = E$$

where E = emissive power of a black body

It implies a good absorber is also a good emitter (radiator).

Fraunhofer lines These are the dark lines in the spectrum of the sun and are explained on the basis of Kirchhoff's Law. White light emitted from core (photosphere) of the sun, when it passes through its atmosphere (chromosphere), radiations of those wavelengths will be absorbed by the gases present there, which they usually emit (in emission spectrum) resulting in dark lines in the spectrum of sun.

Stefan's law Radiant energy emitted per unit area per second (or emissive power or intensity) of a black body is directly proportional to the fourth power of temperature.

$E_R \propto T^4$ or $E_R = \sigma T^4$

If the body is not perfectly black then

$E_R = e\sigma T^4$ where $\sigma = 5.67 \times 10^{-8}$ Wm^{-2}K^{-4}

Energy radiated per second or radiant power

$$P_R = eA\sigma T^4 \text{ or } \int R\lambda d\lambda \propto T^4$$

Cooling by radiation If a body is at temperature T in an environment of temperature T_0 ($< T$), then body loses energy by emitting radiations at a rate

$P_1 = eA\sigma T^4$

and it receives energy by absorbing radiation at a rate

$P_2 = eA\sigma T_0^4$.

So the net rate of heat loss is $P = P_1 - P_2$

or $P = eA\sigma(T^4 - T_0^4)$

When a body cools by radiation, then rate of cooling depends upon the following factors:

(a) **Nature of the radiating surface, that is, emissivity.** Greater the emissivity faster will be the cooling.

(b) **Area of the radiating surface** More the surface area of the radiating surface, faster will be the cooling.

(c) **Mass of the radiating surface** Greater the mass of the radiating body, slower will be the cooling.

(d) **Specific heat of the radiating body** More the specific heat, slower will be the cooling.

(e) **Temperature of radiating body** Higher the temperature, faster will be the cooling.

(f) **Temperature of the surrounding** Lesser the temperature of the surrounding, faster is the cooling.

Newton's law of cooling The rate of cooling is proportional to the temperature difference between body and the surrounding provided the temperature difference is not very large from the surroundings. That is,

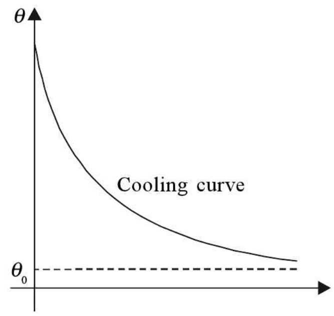


Fig. 17.9 Newton's law of cooling illustration

$$\frac{d\theta}{dt} = -K(\theta - \theta_0)$$

or $$\int_{\theta_1}^{\theta_2} \frac{d\theta}{\theta - \theta_0} = \int_0^t -K dt$$

$$\log_e \frac{\theta_2 - \theta_1}{\theta_1 - \theta_0} = -Kt$$

Fig. 17.9 shows the temperature vs time curve.

If time intervals are equal and successive then,

$$\frac{\theta_2 - \theta_0}{\theta_1 - \theta_0} = \frac{\theta_3 - \theta_0}{\theta_1 - \theta_0}$$

Assuming temperature at $t = 0$ is θ_1, after first interval it is θ_2 and after second equal interval of time it is θ_3.

Solar constant The sun is a perfectly black body as it emits all possible radiations ($e = 1$). Solar constant S is defined as the intensity of solar radiation at the surface of the earth. That is,

$$S = \frac{P_R}{4\pi r^2} = \frac{4\pi R^2 \sigma T^4}{4\pi r^2}$$

$$r = 1.5 \times 10^{11}\text{ km}$$

(distance between the sun and the earth)

$$R = 7 \times 10^8\text{ km (radius of the sun)}$$

$$S = \frac{2\text{cal}}{\text{cm}^2\text{ min}} = 1388\text{ W/m}^2$$

Wien's displacement law When a body is heated, it emits all possible radiations. However, intensity of different wavelengths is different. According to Wien's law the product of wavelength (corresponding to maximum intensity of radiation) and temperature of the body in Kelvin, is constant [See Fig 17.10]

$\lambda_m T$ = b (constant)

$b = 2.89 \times 10^{-3}$ m–K

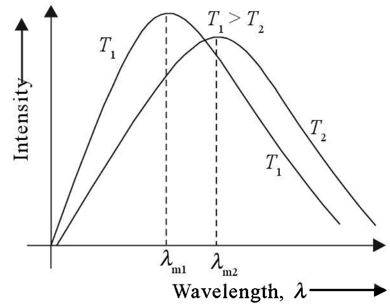


Fig. 17.10 Wein's displacement law illustration

Planck's law Planck assumed that electromagnetic radiations are not emitted or absorbed continuously but in discrete packets of energy called quanta of photons. The energy associated with each photon is

$$E = h\nu$$

where h is Planck's constant (=6.626 × 10^{-34} J – s). On the basis of quantum theory Planck showed that

$$E_R(\lambda) = \frac{2\pi hc^2}{\lambda^5} \frac{1}{\left[e^{\frac{hc}{\lambda kT}} - 1\right]}$$

- **Short Cuts and Points to Note**

1. Thermal conductivity

$$\frac{dQ}{dt} = \frac{KA(T_2 - T_1)}{l} = -KA\frac{dT}{dx}$$

$$i_{\text{thermal}} = \frac{dQ}{dt}, R_{\text{thermal}} = \frac{l}{KA}, V_{\text{thermal}} = (T_2 - T_1)$$

Laws of resistances in series and parallel and Kirchhoff's junction law are valid even in thermal conductance.

2. Thermometric conductivity (D)

$D = \frac{K}{\rho C}$ when K → thermal conductivity

ρ → density

C → specific heat

If σ is electrical conductivity then according to Weidemann–Franz Law

$$\frac{K}{\sigma T} = \text{constant}$$

3. Growth of ice in a pond

$$\frac{dy}{dt}=\frac{K\theta}{\rho L}\times\frac{1}{y}\text{ or } t=\frac{\rho L}{K\theta}\frac{y^2}{2}$$

where L is latent heat of fusion of ice

The ratio of times for thickness of ice 0 to y : y to $2y$: to $3y$: : 1 : 3 : 5.

$$\frac{dQ}{dt}=K\frac{4\pi r_1r_2}{(r_2-r_1)}(\theta_1-\theta_2)\text{ in a hollow sphere and } \theta$$

$$=\theta_1-\frac{b}{r}\frac{(r-a)}{(r_2-r_1)}(\theta_1-\theta_2)\text{ (temperature at any point distant } r \text{ from the centre)}$$

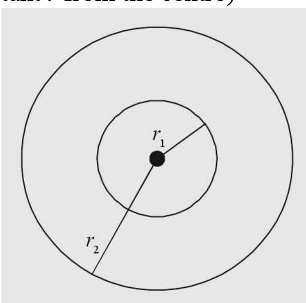


Fig. 17.11

In a cylinder of radius r_1 and r_2 and length l

$$\frac{dQ}{dt}=\frac{2\pi KJ(\theta_1-\theta_2)}{\log_e\frac{r_2}{r_1}}$$

and temperature at any point distant r from the centre

$$\theta=\theta_1-\frac{\log_e\frac{r}{r_1}}{\log_e\frac{r_2}{r_1}}(\theta_1-\theta_2)$$

4. Convection can be natural or forced.

It occurs in fluids $\left|\frac{dQ}{dt}\right|_{\text{convection}}=hAd\theta$

Lapse rate — Temperature falls by 6° C/km when we move up and this is an example of natural convection. Forced convection of blood by heart (as a pump) in body helps to maintain the temperature of body at 37° C.

5. Radiation: Heat transfer process without requiring medium. Heat radiations are em waves lying in *IR* and microwave range. Radiations exert pressure. A black body is capable of emitting or absorbing all possible radiations. Therefore, the sun is a black body. We can say that the radiations of a black body are white (having all wavelengths).

Radiations are detected using bolometer, thermopile, radiometer and so on.

6. Lamp black, the blackest body on earth, is 98% black.

7. Absorptive power

$$=\frac{\text{amount of radiation energy absorbed in a given time}}{\text{total radiation energy incident in that time}}$$

If spectral absorptivity is a_λ then total absorptivity

$$a=\int_0^\infty a_\lambda d\lambda$$

Emissive power radiant energy emitted per unit area of the surface

$$e=\int_0^\infty e_\lambda d\lambda$$

Emissivity (ε) It is the ratio of emissive power of a substance with respect to a perfectly black body. That is,

$$\varepsilon=\frac{e_{\text{body}}}{E_{\text{black body}}}$$

For a perfectly black body absorptivity or emissivity is 1.

8. Kirchhoff's Law: Ratio of emissivity to absorptivity of all bodies is fixed or constant and is equal to emissivity of a black body.

$$\frac{e}{a}=E\text{ (black body)}$$

If implies a good absorber is also a good emitter.

9. Stefan's Law: $E_R=\sigma T^4$ for a perfectly black body and $E_R=e\sigma T^4$ for a body not perfectly black (e being emissivity).

$\sigma=5.67\times10^{-8}$ W/m²

Radiant power $P_R=eA\sigma T^4$ or $\int E_R d\lambda\propto T^4$

Dimensional formula of $\sigma=[MT^{-3}\theta^{-4}]$

10. Newton's Law of Cooling

$$\frac{d\theta}{dt}\ \theta-\theta_0\text{ or }\int_{\theta_1}^{\theta_2}\frac{d\theta}{\theta-\theta_0}=-K\int_0^1 dt$$

$\log_e\frac{\theta_2-\theta_0}{\theta_1-\theta_0}=-Kt$ (Use this law if temperature at any time t is to be determined.)

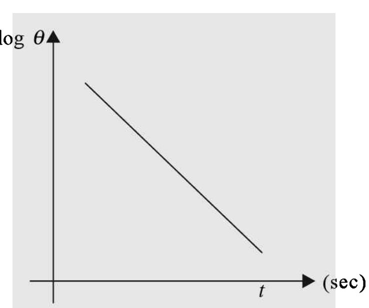


Fig. 17.12

Use $\dfrac{\theta_2-\theta_0}{\theta_1-\theta_0} = \dfrac{\theta_3-\theta_0}{\theta_2-\theta_0}$

in order to find temperature in successive equal intervals. θ_1 is temperature at $t = 0$, θ_2 after time t and θ_3 after a total time $2t$ or next interval of t. The law is valid if difference between temperature of the body and surroundings in not large (~30° C).

11. Solar constant $S = \dfrac{P_R}{4\pi r^2} = \dfrac{4\pi R^2\sigma T^4}{4\pi r^2} = 1388$ W/m². For practical purpose, we can assume solar constant (s) to be 1400 W/m².

Temperature of sun = 6000 K. Dimensional formula for $S = [MT^{-3}]$

12. Wein's displacement law: $\lambda_m T = b = 2.89 \times 10^{-3}$ m–K. Higher the temperature shorter is the wavelength of maximum intensity.

Use $\lambda_{m1}T_1 = \lambda_{m2}T_2$ if temperature or wavelength of a body is to be determined and that of the other is known; for example, for one star λ_{m1} and T_1 is known and λ_{m2} or T_2 is to be determined.

13. Planck's Law is based on quantum nature of radiations. It assumes that discrete energy packets called photons are emitted or absorbed.

$E = h\nu = h\dfrac{c}{\lambda}$ where $c \to$ speed of light

14. Intensity $I \propto \dfrac{1}{d^2}$ for a point source and amplitude $A \propto \dfrac{1}{\sqrt{d}}$ for a cylindrical source.

- **Caution**

1. When you forget to take total length in case of series and total area in case of parallel while finding effective thermal conductivity

⇒ In series $R = R_1 + R_2$

$$\frac{l_1+l_2}{K_{\text{eff}}A_{\text{eff}}} = \frac{l_1}{K_1A_1} + \frac{l_2}{K_2A_2}$$ and

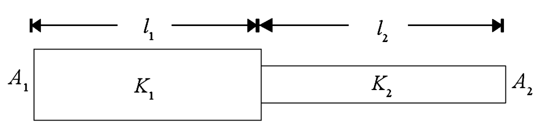


Fig. 17.13 (a)

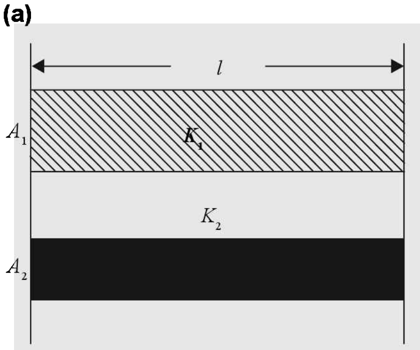


Fig. 17.13 (b)

In parallel $\dfrac{1}{R} = \dfrac{1}{R_1} + \dfrac{1}{R_2}$

$$\frac{K_{\text{eff}}(A_1+A_2)}{l} = \frac{K_1A_1}{l} + \frac{K_2A_2}{l}$$

or $K_{\text{eff}} = \dfrac{K_1A_1+K_2A_2}{A_1+A_2}$

2. Confusing how to find temperature of the junction (or at any other point) like the one shown in Fig 17.14.

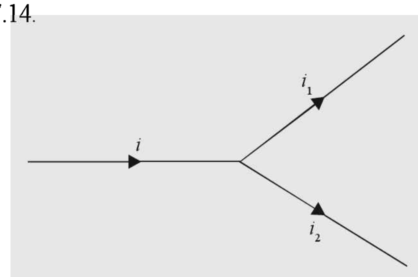


Fig. 17.14

⇒ Apply Kirchhoff's law, that is, $i = i_1 + i_2$ and so on

3. Considering freezing of lake as a convection process.

⇒ It is a conduction process and time taken to freeze a depth y is given by

$$t = \frac{\rho L}{K\theta}\frac{y^2}{2}$$

4. Considering that conduction does not occur in liquids or gases.

⇒ It does occur but it is quite small as compared to solids.

5. Considering any black colour object as a black body.

⇒ Black body is one which is capable of emitting or absorbing all possible radiations. Thus even the sun is a black body.

6. Considering Stefan's law can be applied only to perfectly black bodies.

⇒ True, but the modified form of Stefan's law ($E_R = e\sigma T^4$, where e is emissivity of a body) can be applied to any body.

7. Considering that only black coloured objects absorb radiations.

⇒ According to Kirchhoff's law a good absorber is also a good emitter. Hence they emit radiations too.

8. Confusing emissive power and emissivity.

⇒ Emissivity is emissive power of a body compared to a black body.

9. Not remembering value of σ or b (Wein's displacement Law)

⇒ $\sigma = 5.67 \times 10^{-8}$ W/m²; $b = 2.89 \times 10^{-3}$ m – K

10. Considering that Newton's law of cooling can be applied for any temperature difference between hot body and surrounding.

⇒ It is applicable only when difference between the temperature of the body and surroundings is not very large (~20 – 30° C).

SOLVED PROBLEMS

1. A man has a total surface area of 1.5 m². Find the total rate of radiation of energy from the body.

(a) 566 J (b) 682 J
(c) 732 J (d) 782 J

Solution (d) $P_R = \sigma A T^4$

$= 5.67 \times 10^{-8} \times 1.5 \times (310)^4 = 782$ J

2. Three identical rods A, B and C of equal lengths and equal diameters are joined in series as shown in Fig. 17.15. Their thermal conductivities are 2K, K and K/2 respectively. Calculate the temperature at two junction points.

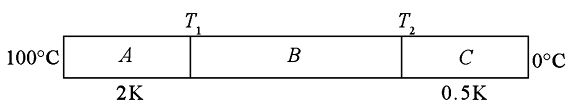


Fig. 17.15

(a) 85.7, 57.1° C (b) 80.85, 50.3° C
(c) 77.3, 48.3° C (d) 75.8, 49.3° C

[MNR 1993]

Solution (a) $i_{th\,1} = i_{th\,2} = i_{th\,3}$

$$\frac{(100 - T_1)2KA}{l} = \frac{(T_1 - T_2)KA}{l} = \frac{(T_2 - 0)KA}{2l}$$

$$2(100 - T_1) = (T_1 - T_2) = \frac{T_2}{2} \quad T_1 = \frac{3T_2}{2}$$

$$2(100 - T_1) = \frac{T_1}{3}$$

or $T_1 = \dfrac{600}{7} = 85.7^0$ C and $T_2 = \dfrac{2T_1}{3} = 57.1°$ C

3. Three rods of material x and three rods of material y are connected as shown in Fig. 17.16 (a). All the rods are of identical length and cross-section. If the end A is maintained at 60⁰ C and the junction E at 10⁰ C, find effective Thermal Resistance. Given length of each rod $= l$, area of cross-section $= A$, conductivity of x = K and conductivity of y = 2K.

(a) $\dfrac{4l}{3KA}$ (b) $\dfrac{7l}{3KA}$

(c) $\dfrac{4KA}{3l}$ (d) $\dfrac{7KA}{3l}$

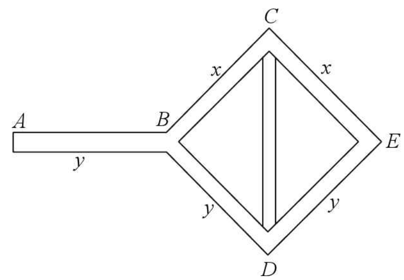


Fig. 17.16 (a)

Solution (b) $BCED$ part forms a Wheatstone bridge Therefore equivalent circuit is shown as Fig. 17.16 (b).

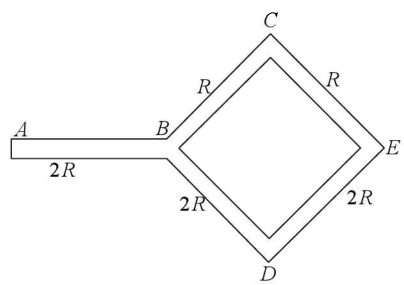


Fig. 17.16 (b)

Net resistance is

$$R_{eq} = R_{AB} + R_{BE},$$

$$R_{BE} = \frac{2R \times 4R}{2R + 4R} = \frac{4}{3}R = R + \frac{4}{3}R = \frac{7}{3}R = \frac{7l}{3KA}$$

4. Two spheres of same material have radii 1 m and 4 m and temperature 4000 K and 2000 K respectively. The ratio of energy radiated per second is

(a) 1 (b) 2
(c) 4 (d) none of these

[IIT 1988]

Solution (a) $\dfrac{P_1}{P_2} = \dfrac{e\sigma A_1 T_1^4}{e\sigma A_2 T_2^4} = \dfrac{(1)^2}{(4)^2}\dfrac{(4000)^2}{(2000)^2} = \dfrac{1}{1}$

5. The emissivity of a body of surface area 5 cm² and at temperature 727° C radiating 300 J of energy per minute is

(a) 0.48 (b) 0.38
(c) 0.28 (d) 0.18

Solution (d) $P = eA\sigma T^4$

$$e = \frac{P}{A\sigma T^4} = \frac{300/60}{5\times10^{-4}\times5.67\times10^{-8}(1000)^4}$$

6. A body cools in 7 minutes from 60° C to 40° C. The temperature after next 7 minutes will be ——. Given: temperature of surrounding is 10° C.

(a) 32° C (b) 38° C
(c) 22° C (d) none of these

Solution (d) $\frac{40-10}{60-10} = \frac{\theta_3 - 10}{40-10}$, that is, $\theta_3 = 28$

$$\left(\text{Using } \frac{\theta_2-\theta_0}{\theta_1-\theta_0} = \frac{\theta_3-\theta_0}{\theta_2-\theta_0}\right).$$

7. Bodies A and B have thermal emissivities of 0.01 and 0.81 respectively. The outer surface areas of the two bodies are same. The two bodies emit total radiant power at the same rate. Find the temperature of B if the temperature of A is 5802 K.

(a) 1634 K (b) 1734 K
(c) 1934 K (d) none of these

Solution (c) Given $P = e_A \sigma A T_A^4 = e_B \sigma A T_B^4$

or $$T_B = \left(\frac{e_A}{e_B}\right)^{1/4} T_A = \left(\frac{0.01}{0.81}\right)^{1/4} (5802)$$

$$T_B = \frac{5802}{3} = 1934$$

8. A 10 cm long copper rod is welded to 20 cm long steel rod each having cross-section A. If their thermal conductivities ar 386 $Jm^{-1}\,s^{-1}\,°C^{-1}$ and 46 $Jm^{-1}\,s^{-1}\,°C^{-1}$ the temperature of the junction will be ____, (Given: copper end is at 100° C and steel rod end is at 0° C.)

(a) 87.5° C (b) 79.3° C
(c) 75° C (d) 71.5° C

Solution (d) Let the junction temperature be T

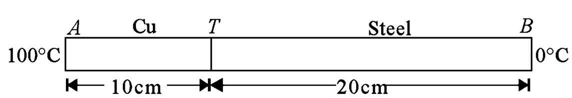


Fig. 17.17

$$\frac{100-T}{T} = \frac{23}{386} \text{ or } T = 71.63° \text{ C}$$

9. The temperature of a body falls from 40° C to 36° C in 5 minutes. The temperature of the body will become 32° C in

(a) less than 10 minutes (b) 10 minutes
(c) more than 10 minutes (d) none of these

Solution (c) $\frac{d\theta}{dt} \propto (\theta - \theta_0)$

As $(\theta - \theta_0)$ will decrease, it will take more time to cool.

10. A solid at temperature T_1 is kept in an evacuated chamber at temperature $T_2 > T_1$. The rate of growth of temperature is proportional to

(a) $T_2 - T_1$ (b) $T_2^2 - T_1^2$
(c) $T_2^3 - T_1^3$ (d) $T_2^4 - T_1^4$

Solution (d) $P_1 = Ae\sigma T_1^4$ (rate of loss of radiant energy)

$P_2 = Ae\sigma T_2^4$ (rate of gain of radiant energy)

Net rate of gain of radiant energy

$(P_2 - P_1) \propto (T_2^4 - T_1^4)$

11. Consider the radiations emitted by the human body. Which of the following statements is true?

(a) Radiations lie in ultraviolet region
(b) Radiations lie in infrared region
(c) Radiations are emitted only during the day
(d) Radiations are emitted during summer and absorbed during winter

[CBSE 2003]

Solution (b) Heat radiations lie in infrared and microwave regions.

12. Two bodies are at temperature 27° C and 927° C. The heat energy radiated by them will be in the ratio

(a) 1 : 256 (b) 1 : 64
(c) 1 : 4 (d) 1 : 16

[DPMT 2002]

Solution (b) $\frac{H_1}{H_2} = \frac{T_1^4}{T_2^4} = \left(\frac{300}{1200}\right)^4 = \frac{1}{64}$

13. The temperature of a black body increases from T to $2T$. The factor by which the rate of emission will increase is

(a) 2 (b) 4
(c) 8 (d) 16

[BHU 2003]

Solution (d) $\frac{P_1}{P_2} = \left(\frac{T}{2T}\right)^4 = \frac{1}{16}$

14. According to Newton's law of cooling, the rate of cooling of a body is proportional to $(\Delta\theta)^n$ where $\Delta\theta$ is the difference of temperature of the body and the surrounding and n is equal to

(a) 3 (b) 4
(c) 1 (d) 2

[AIEEE 2003]

Solution (c) $\frac{d\theta}{dt} \propto (\theta - \theta_0)$ or $\frac{d\theta}{dt} \propto \Delta\theta$

15. A black body at a temperature 77° C radiates heat at a rate of 10 $calcm^{-2}\,s^{-1}$. The rate at which this body would radiate heat in units of $calcm^{-2}\,s^{-1}$ at 427° C is closest to

(a) 40 (b) 160
(c) 200 (d) 400

[VMMC 2003]

Solution (b) $\frac{P_1}{P_2} = \left(\frac{350}{700}\right)^4$

$= \frac{10}{P_2}$ or $P_2 = 160$

16. Three rods made of the same material and having the same cross-section have been joined as shown in Fig. 17.18 (a). Each rod is of the same length. The left and right ends are kept at 0° C and 90° C respectively. The temperature of the junction of the three rods is

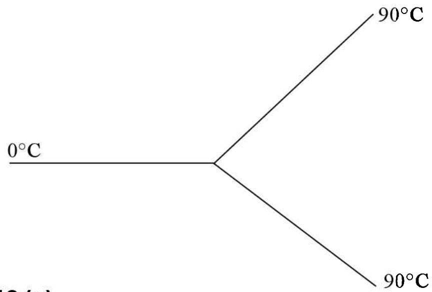


Fig. 17.18 (a)

(a) 45° C (b) 60° C
(c) 30° C (d) 20° C

[IIT Screening 2001]

Solution (b) $I = I_1 + I_2$

$$= \frac{KA(90-T)}{l} + \frac{(90-T)KA}{l}$$

$$= \frac{(T-0)KA}{l}$$

or $3T = 180°$ C or $T = 60°$ C

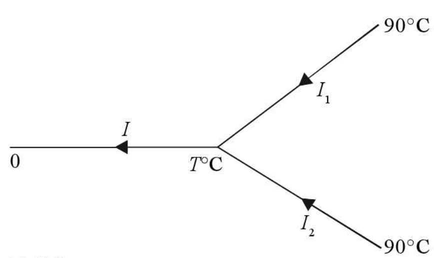


Fig. 17.18 (b)

17. An ideal black body at room temperature is thrown into a furnace. It is observed that

(a) initially it is the darkest body and at later times the brighest.
(b) it is darkest body at all times.
(c) it cannot be distinguished at all times.
(d) initially it is the darkest and at later times it cannot be distinguished.

[IIT Screening 2002]

Solution (a) According to Kirchhoff's law good absorbers also are good emitters.

18. The thermal conductivity of a rod depends on

(a) length (b) mass
(c) area of cross-section (d) material of the rod

Solution (d)

19. The rate of heat flow through a cross-section of the rod shown in Fig. 17.19 is ____. ($\theta_2 > \theta_1$ and thermal conductivity of the material of the rod is K.)

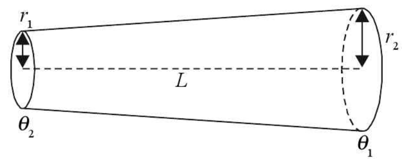


Fig. 17.19

(a) $\frac{K\pi r_1 r_2 (\theta_2 - \theta_1)}{L}$

(b) $\frac{K\pi (r_1 + r_2)^2 (\theta_2 - \theta_1)}{4L}$

(c) $\frac{K\pi (r_2 + r_1)^2 (\theta_2 - \theta_1)}{L}$

(d) $\frac{K\pi (r_2 + r_1)^2 (\theta_2 - \theta_1)}{2L}$

Solution (a) $r_{eff} = \sqrt{r_1 r_2}$

$$\frac{d\theta}{dt} = \frac{KA(\theta_2 - \theta_1)}{L}$$

$$= \frac{K\pi r_1 r_2 (\theta_2 - \theta_1)}{L}$$

20. Find the heat radiated per second by a body of surface area 12 cm^2 kept in thermal equilibrium in a room at temperature 20° C. The emissivity of the surface is 0.8 and $\sigma = 6 \times 10^{-8}$ Wm^{-2}K^{-4}.

(a) 4.2 J (b) 0.42 J
(c) 0.042 J (d) 42 J

Solution (b) $P = eA\sigma T^4$

$= 0.8 \times 12 \times 10^{-4} \times 6 \times 10^{-8} \times (293)^4$

$= 0.42$ J

21. Two copper spheres, one of large size and the other small, are heated to the same temperature. Which will cool first?

(a) Bigger
(b) Smaller
(c) Both in equal time
(d) Insufficient data to reply

Solution (b) Smaller sphere (cooling $\propto \frac{1}{\text{mass}}$)

TYPICAL PROBLEMS

22. The top of an insulated cylindrical container is covered by a disc having emissivity 0.6, conductivity 0.167 WK^{-1} m^{-1} and thickness 1 cm. The temperature is maintained by circulating oil.

(a) Find the radiation loss to the surroundings in $Jm^{-2}s^{-1}$ if temperature of the upper surface of the disc is 127° C and temperature of the surroundings is 27° C.

(b) Also find the temperature of the circulating oil. Neglect the heat loss due to convection.

Given: $\sigma = \frac{17}{3} \times 10^{-8}\, Wm^{-2} K^{-4}$

Solution (a) The rate of heat loss per unit area due to radiation $= e\sigma(T^4 - T_0^4) = 0.6\,[(400)^4 - (300)^4] \times \frac{17}{3} \times 10^{-8}$

$= 595\ Jm^{-2}s^{-1}$

(b) Suppose temperature of oil is θ. The rate of heat flow through conduction = rate of heat loss due to radiation.

$$\frac{0.167 \times A(\theta - 127)}{10^{-2}} = 595\,A$$

$$\theta = 162.3^0 C$$

23. A point source of heat of power P is placed at the centre of a spherical shell of mean radius R. The material of the shell has thermal conductivity K. Calculate the thickness of the shell if temperature difference between the outer and inner surface of the shell in steady state is T.

Solution Consider a concentric spherical shell of radius x and thickness dx as shown in Fig. 17.20. The radial rate of flow

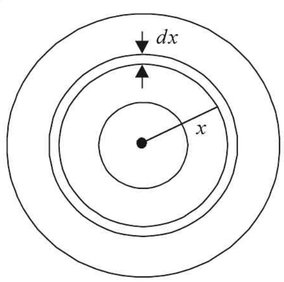


Fig. 17.20

$$H = \frac{dQ}{dt} = -KA\frac{d\theta}{dx}$$

$$= -K\,4\pi x^2 \frac{d\theta}{dx}$$

or $$\int_a^b \frac{dx}{x^2} = -\frac{4\pi K}{H}\int_{\theta_1}^{\theta_2} d\theta$$

or $$H = \frac{dQ}{dt} = \frac{K4\pi ab(\theta_1 - \theta_2)}{(b-a)}$$

In steady state no heat is absorbed,

$$\therefore H = P$$

$$\theta_1 - \theta_0 = T$$

and $$a = b = R$$

$$P = \frac{4\pi R^2 kT}{(b-a)}$$

or $$(b-a) = \frac{4\pi R^2 kT}{P}$$

24. A solid copper sphere (density ρ and specific heat C) of radius r at an initial temperature 200 K is suspended inside a chamber whose walls are at almost 0 K. What is the time required for temperature of the sphere to drop to 100 K?

[IIT 1991]

Solution According to Stefan's law, $P = eA\sigma T^4$

$$\frac{dQ}{dt} = \frac{-mcdT}{dt} = eA\sigma T^4$$

or $$\frac{-dT}{dt} = \frac{-eA\sigma T^4}{mc}$$

$$\frac{-dT}{dt} = \frac{e4\pi r^2\sigma T^4}{\frac{\rho 4\pi r^3 c}{3}}$$

or $$\frac{r\rho c}{3e\sigma}\int\frac{dT}{T^4} = \int_0^1 dt$$

or $$t = \frac{r\rho c}{9e\sigma}\left[\frac{1}{T^3}\right]_{200}^{100}$$

$$= \frac{7r\rho c \times 10^{-6}}{72e\sigma} \text{ seconds}$$

25. A liquid takes 5 minutes to cool from 80° C to 50° C. How much time will it take to cool from 60° C to 30° C? Temperature of the surrounding = 20° C

[MNR 1996]

Solution $-\frac{d\theta}{dt} = K(\theta - \theta_0)$ or $t = \frac{1}{K}\log\left[\frac{\theta_1 - \theta_2}{\theta_2 - \theta_0}\right]$

$$5 = \frac{1}{K}\log\left[\frac{80-20}{50-20}\right]$$

or $\quad \dfrac{5}{t} = \dfrac{\frac{1}{K}\log_e 2}{\frac{1}{K}\log 2^2}$

or $\quad t = 10$ minutes

26. A wire of length 1.0 m and radius 10^{-3} m is carrying a heavy current and it is assumed to radiate as a black body. At equilibrium its temperature is 900 K while that of the surroundings is 300 K. The resistivity of the material of the wire at 300 K is $\pi \times 10^{-8}$ ohm – m and its temperature coefficient of resistance is 7.8×10^{-30} C^{-1} Find the current in the wire. Given: $\sigma = 5.68 \times 10^{-8}$ $Wm^{-2}K^{-4}$.

[Roorkee 1994]

Solution $P = \sigma A\,(T^4 - T_0^4)$

$$= \sigma 2\pi r l\,(T^4 - T_0^4)$$
$$= 5.68 \times 10^{-8}\,(2\pi \times 10^{-3} \times 1)\,[(900)^4 - (300)^4]$$
$$= 73.6\pi \text{W}$$

$$P = I^2R = I^2\rho\,\frac{L}{A}$$
$$= I^2\,\frac{L}{A}\,\rho_0\,[1 + \alpha\Delta\theta]$$

$$73.6 = \frac{I^2 \times \pi^2 \times 10^{-8}\left[1 + 7.8\times10^{-3}\,(900 - 300)\right]}{\pi\left(10^{-3}\right)^2}$$

or $\quad I = 36A$

27. A space object has the shape of a sphere of radius R. Heat sources ensure that the heat evolution at a constant rate is distributed uniformly over its volume. The amount of heat liberated by a unit surface area is proportional to the fourth power of thermodynamic temperature. In what proportion would the temperature of the object change if its radius is decreasesd to half?

[Olympiad 1997]

Solution Heat liberated $\dfrac{dQ}{dt} \propto R^3$ and $\dfrac{dQ}{dt} \propto R^2T^4$

or $\quad T^4 \propto R$

Thus $\quad \dfrac{T_1^4}{T_2^4} = \dfrac{R}{R/2}$

or $\quad T_2 = \left(\dfrac{1}{2}\right)^4 T_1$

or $\quad T_2 = \dfrac{T_1}{1.19}$

that is, temperature decreases by a factor of 1.19.

28. The room temperature is +20° C when outside temperature is –20° C and room temperature is + 10° C when outside temperature is –40° C. Find the temperature of the radiator heating the room.

[Olympiad 1994]

Solution Applying Newton's law,

in case (1)

$K_1\,(T - T_{r1})\,K_2(T_{r2} - T_{out\,1})$

and in case (2)

$K_1\,(T - T_{r2})\,K_2(T_{r2} - T_{out\,2})$

Dividing these equations

$$\frac{T - T_{r1}}{T - T_{r2}} = \frac{T_{r2} - T_{out\,1}}{T_{r2} - T_{out\,2}}\;\frac{T - 20}{T - 10}$$

$$= \frac{20 - (-20)}{10 - (-40)}$$

or $\quad 10T = 600.$

or $\quad T = 60°$C

29. A body cools from 50°C to 45°C in 5 min. and 45°C to 40°C in 8 min. Find the temperature of the surrounding.

Solution Using $t = \dfrac{1}{K}\log\left[\dfrac{\theta_1 - \theta_0}{\theta_2 - \theta_0}\right]$

$$5 = \frac{1}{K}\log\left[\frac{50 - \theta_0}{45 - \theta_0}\right]$$

$$8 = \frac{1}{K}\log\left[\frac{45 - \theta_0}{40 - \theta_0}\right]$$

or $\quad \log\left[\dfrac{50 - \theta_0}{45 - \theta_0}\right]^{1.6} = \log\left[\dfrac{45 - \theta_0}{40 - \theta_0}\right]$

$$\left[\frac{50 - \theta_0}{45 - \theta_0}\right]^{1.6} = \left[\frac{45 - \theta_0}{40 - \theta_0}\right]$$

Solving for θ_0 we get $\theta_0 = 34°$ C

30. A heated body emits radiation which has maximum intensity near the frequency v_0. The emissivity of the material is 0.5. If the absolute temperature of the body is doubled, the maximum intensity of radiation will be ne0ar the frequency

(a) $2v_0$ (b) $\dfrac{v_0}{2}$

(c) $16v_0$ (d) $8v_0$

Solution (a) $\lambda_1 T_1 = \lambda_2 T_2$

$$\frac{c}{v_0}\,T_1 = \frac{c}{v}\,2T_1 \quad \text{or } v = 2v_0$$

QUESTIONS FOR PRACTICE

1. The temperature gradient in the earth's crust is 32° C per km and the mean conductivity of the rock is 0.008 CGS units. Considering the radius of the earth as 6000 km, the loss of heat by the earth everyday is about (in calories)

(a) 10^{16} (b) 10^{17}
(c) 10^{27} (d) 10^{37}

2. A sphere and a cube, both made of copper, have equal volumes and are blackened. These are allowed to cool at same temperature and in same atmosphere. The ratio of their rates of loss of heat will be

(a) $4/3\pi : 1$ (b) $(\pi/6)^{1/3} : 1$
(c) 1 : 1 (d) $(\pi/6)^{2/3} : 1$

3. The diameter of ball P is three times that of another ball Q of the same material. P and Q are heated to same temperature and allowed to cool up to same temperature. The relation between their rates of cooling will be

(a) $R_P = R_Q/3$ (b) $R_P = 3R_Q$
(c) $R_P = 9RQ$ (d) $R_P = R_Q$

4. A sphere, a cube and a thin circular disc, all made of the same material, have same mass. Their initial temperature is 3×10^3 °C. The body which cools rapidly is

(a) sphere (b) cube
(c) both sphere and cube (d) circular plate

5. The opposite faces of a cubical block of iron of cross-section 4 cm^2 are kept in contact with steam and melting ice. The quantity of ice melting at the end of 10 minutes will be (Given: $k_{iron} = 0.2$ CGS units)

(a) 700 g (b) 500 g
(c) 300 g (d) 100 g

6. A wall has two layers A and B, each made of different materials. Both the layers have same thickness. The thermal conductivity of A is twice that of B. Under thermal equilibrium, the temperature difference across the wall is 36^0 C. The temperature across the layer A is

(a) 6° C (b) 12° C
(c) 18° C (d) 24° C

7. The ratio of masses of two metal spheres A and B is 8 : 1. If their temperatures are 2000 K and 1000 K respectively, then the ratio of the rates of their energy emission will be

(a) 4 : 1 (b) 16 : 1
(c) 64 : 1 (d) 128 : 1

8. The temperature of a black body corresponding to which it will emit energy at the rate of 1 watt/cm^2 will be

(a) 650 K (b) 450 K
(c) 350 K (d) 250 K

9. Two rods of the same material have diameters in the ratio of 1 : 2 and lengths in the ratio of 2 : 1. If the temperature difference between their ends is the same, the ratio of heats conducted by them in a given time will be

(a) 1 : 4 (b) 1 : 8
(c) 4 : 1 (d) 8 : 1

10. If a piece of metal is heated to an absolute temperature T and then put in an enclosure at absolute temperature t, then the heat generated in the atmosphere will be proportional to

(a) $(T - t)^4$ (b) $T^4 - t^4$
(c) $T^2 - t^2$ (d) $T^3 - t^3$

11. If the pressure of a gas is doubled then its thermal conductivity will

(a) remain constant (b) decrease
(c) decrease exponentially (d) increase

12. The colour of a distant star in the sky is an indication of its

(a) size (b) temperature
(c) distance (d) frequency

13. Two bodies A and B are kept in an evacuated chamber at 27° C. The temperature of A and B are 327° C and 427° C respectively. The ratio of rates of loss of heat from A and B will be

(a) 0.23 (b) 0.52
(c) 1.52 (d) 2.52

14. The ratio of amplitudes of radiation emitted by a cylindrical source at distances $2r$ and $18r$ from its axis will be

(a) 1 : 3 (b) 2 : 1
(c) 1 : 1 (d) 3 : 1

15. A hot body will emit radiations more rapidly if its surface is

(a) black and polished (b) white and polished
(c) black and rough (d) white and rough

16. If the amount of heat incident upon a body is X calorie and it absorbs Y calorie out of it, then the coefficient of absorption will be

(a) $X + Y$ (b) XY
(c) Y/X (d) X/Y

17. There is a small hole in a hollow container. At what temperature should it be maintained in order that it emits one calorie of energy per second per meter^2?

(a) 10 K (b) 100 K
(c) 200 K (d) 500 K

18. In which part of the electromagnetic spectrum do the heat radiations lie?
(a) Visible (b) Violet
(c) Ultraviolet (d) Infrared

19. The correct relation between the intensity of radiations (I) and distance (d) from the point source is
(a) $I \propto 1/d$ (b) $I \propto d$
(c) $I \propto 1/d^2$ (d) $I \propto d^2$

20. When a yellow piece of glass is heated in a dark room, then it emits light of
(a) yellow colour (b) red colour
(c) blue colour (d) green colour

21. If the temperature of a black body is increased by 50% then the amount of radiation emitted by it will
(a) decrease by 50% (b) increase by 50%
(c) increase by 400% (d) decrease by 400%

22. A black body at temperature T radiates energy at the rate of E Wm^{-2}. In reducing its temperature to $T/2$ the rate of energy radiation will be
(a) $E/16$ (b) $E/4$
(c) $E/2$ (d) $< E$

23. A piece of red glass is heated till it starts shining in a dark room. The colour of this shining glass will be
(a) violet (b) orange
(c) green (d) red

24. The radiation force due to source of power P on a perfectly reflecting surface will be
(a) $\frac{P}{3c}$ (b) $\frac{2P}{c}$
(c) $\frac{P}{2c}$ (d) $\frac{P}{c}$

25. The value of solar constant is approximately
(a) 1340 watt/m^{-2} (b) 430 watt/m^2
(c) 340 watt m^2 (d) 1388 watt/m^{-2}

26. The material of prism used for obtaining spectrum of heat radiations is
(a) rock salt (b) quartz
(c) flint glass (d) crown glass

27. Fraunhoffer lines are the result of which of the following phenomenon of radiation?
(a) Scattering (b) Compression
(c) Emission (d) Absorption

28. On which of the following laws, does the constant volume thermometer work?
(a) Gay Lusac's law (b) Dalton's law
(c) Boyle's law (d) Charles's law

29. Two rods of equal length and diameter but of thermal conductivities 2 and 3 units respectively are joined in series. The thermal conductivity of the combination is
(a) 6 (b) 5
(c) 1 (d) none of these

30. Ice starts forming on the surface of a lake and takes 8 hours to form a layer of 1 cm thick. How much time will it take to increase the thickness of the layer to 2 cm?
(a) 8 hours (b) Less than 8 hours
(c) Between 8 and 16 hours (d) More than 16 hours

31. The amount of thermal radiations emitted from one square centimetre area of a black body in one second when at a temperature of 1000 K is
(a) 5.67 J (b) 56.7 J
(c) 567 J (d) 5670 J

32. How is the velocity of thermal radiations (v) related to the velocity of light (c)?
(a) $v < c$ (b) $v > c$
(c) $v = c$
(d) The relation depends upon the wavelength of the radiations

33. The Kirchhoff's law leads to the conclusion that the good radiators of thermal radiations are
(a) good absorbers (b) bad absorbers
(c) thermal insulators (d) none of these

34. The rate of loss of heat by radiation from a body at 400° C is R. The radiation from it when the temperature rises to 800° C is
(a) $16R$ (b) $4R$
(c) $2R$ (d) none of these

35. Given that p Joules of heat is incident on a body and out of it q Joules is reflected and transmitted by it, the absorption coefficeient of the body is
(a) $(q - p)/p$ (b) q/p
(c) $(p - q)/p$ (d) p/q

36. A graph is drawn between λ and $E\lambda$. The area A under the graph is related to the absolute temperature as
(a) $A \propto T^{-4}$ (b) $A \propto T^{-2}$
(c) $A \propto T^2$ (d) $A \propto T^4$

37. Corresponding to a given temperature, there is a wavelength λ_m, for which the intensity of heat radiations is
(a) maximum (b) constant
(c) zero (d) minimum

38. On which one of the factors does the nature of the thermal radiation depend inside an enclosure?
(a) Size of the enclosure
(b) Temperature
(c) Nature of the walls
(d) Colour of the walls

39. Thermal radiations are similar to
(a) α rays (b) X-rays
(c) cathode rays (d) none of these

40. A hot and a cold body are kept in vacuum separated from each other. Which of the following causes decrease in temperature of the hot body?

(a) Conduction (b) Radiations
(c) Convection
(d) The temperature of both the bodies remain unchanged

41. Who explained the Fraunhofer lines in the spectrum of solar radiations?
(a) Wein (b) Fraunhofer
(c) Stefan (d) Kirchhoff

42. The top of a lake is frozen. The air in contact with the surface of lake is at –15° C. Then the maximum temperature of the water in contact with the lower surface of ice will be
(a) –7.5° C (b) –4° C
(c) 0° C (d) 4° C

43. A slab consists of two parallel layers of two different materials of same thickness having thermal conductivities K_1 and K_2. The equivalent thermal conductivity of the slab is
(a) $\frac{K_1+K_2}{K_1K_2}$ (b) $\frac{2K_1K_2}{K_1+K_2}$
(c) $\frac{(K_1+K_2)}{2}$ (d) K_1+K_2

44. In the steady state, the two ends of a metre rod are at 30° C and 20° C. The temperature at the 60th cm is
(a) 22° C (b) 23° C
(c) 24° C (d) 25° C

45. Two stars *A* and *B* radiate maximum energy at 3600 Å and 4800 Å respectively. Then the ratio of absolute temperature of *A* and *B* is
(a) 256 : 81 (b) 81 : 256
(c) 4 : 3 (d) 3 : 4

46. A bucket full of hot water is kept in a room. It cools from 75° C to 70° C in T_1 minutes, from 70° C to 65° C in T_2 minutes and from 65° C to 60° C in T_3 minutes. Which of the following relation is correct?
(a) $T_1 > T_2 > T_3$ (b) $T_1 < T_2 < T_3$
(c) $T_1 < T_2 > T_3$ (d) $T_1 = T_2 = T_3$

47. The spectral energy distribution of the sun (temperature 6050 K) has a maximum at 4753 Å. The temperature of a star for which this maximum is at 9506 Å is
(a) 24200 K (b) 12100 K
(c) 6050 K (d) 3025 K

48. The temperature of a piece of metal is raised from 27° C to 51.2° C. The rate at which the metal radiates energy increases nearly
(a) 1.36 times (b) 2 times
(c) 4 times (d) 8 times

49. If K denotes coefficient of thermal conductivity, *d* the density and *C* the specific heat, the unit of *X*, where *X* = K/*dc*, will be
(a) cm sec (b) $cm^2\ sec^{-2}$
(c) $cm\ sec^{-2}$ (d) $cm^2\ sec^{-1}$

50. A 40 watt bulb converts 6% of its power to red light (wavelength 6500 Å). The number of red light photons emitted by the bulb per second is
(a) 100 (b) 4×10^{18}
(c) 8×10^{18} (d) 13×10^{18}

51. The energy emitted by a black body at 727° C is *E*. If the temperature of the body is increased to 227° C, the emitted energy will become
(a) 13 times (b) 2.27 times
(c) 1.9 times (d) 3.9 times

52. A metallic sphere cools from 50° C to 40° C in 300 seconds. If the room temperature is 20° C then its temperature in the next 5 minutes will be
(a) 30° C (b) 33.3° C
(c) 36° C (d) 38° C

53. Two stars *X* and *Y* emit yellow and blue lights. Out of these whose temperature will be more?
(a) That of *Y* (b) That of *X*
(c) That of both (d) Sometimes *X* and sometimes *Y*

54. A body cools from 62° C to 50° C in 10 minutes. If the temperature of the surroundings is 26° C then the temperature of the body after next 10 minutes will be
(a) 42° C (b) 44° C
(c) 46° C (d) 48° C

55. An ideal black body emits maximum intensity of radiation of wavelength 5000 Å at temperature 1227° C. If its temperature is increased by 10^3 °C then the maximum emission wavelength will be
(a) 5000 Å (b) 4000 Å
(c) 3500 Å (d) 3000 Å

56. The maximum emission wavelength at temperature 2000 *K* is 4 πm. The maximum wavelength corresponding to temperature 2400 K will be
(a) 0.66×10^{-6} m (b) 1 m
(c) 3.33×10^{-6} m (d) 10^{-6} m

57. If the rates of cooling of two bodies are same then for which body will the rate of fall of temperature be more? For the body whose thermal capacity is
(a) less (b) more
(c) infinity (d) any value

58. If the temperature of a body increases by 2% then energy radiated will increase by
(a) 2% (b) 8%
(c) 4% (d) 16%

59. If the rate of emission of heat radiation of an ideal black body is made sixteen times then its final temperature will become
(a) half (b) doubled
(c) 4 times (d) 8 times

60. Out of the following, which body is not an ideal black body?
(a) Wien's black body (b) Ferry's black body
(c) Coal (d) Sun

61. The ratio of wavelength corresponding to maximum emission of radiation is 1 : 195. The ratio of their temperature will be

(a) $(195)^2 : 1$ (b) 1 : 195
(c) $1 : (195)^2$ (d) 195 : 1

62. If the emissive and absorptive powers of a body are E and A respectively at temperature T then the emissive power of a black body will be

(a) E/A (b) EAT
(c) EA/T (d) A/E

63. When the temperature of a body is equal to that of the surroundings then the body appears

(a) in thermalequilibrium (b) red
(c) cold (d) hot

64. The hot water pipes used to heat up the room are painted

(a) white (b) yellow
(c) red (d) black

65. On increasing the temperature of a body, the frequency (f_{max}) corresponding to maximum emission of radiation will

(a) increase (b) decrease
(c) first decrease and then increase
(d) remain constant

66. The coefficient of transmission for an ideal black body is

(a) infinity (b) zero
(c) 1 (d) more than one

67. The temperature of a body is 3000 K. The wavelength correspoding to maximum emission of radiation will be

(a) 1 M (b) 1 Å
(c) 9.76×10^{-7} m (d) 48.8×10^{-7} m

68. The theoretical explanation of black body spectrum was given by

(a) Lumer and Pringsheim (b) Planck
(c) Stefan (d) Wien

69. The wavelength of yellow light is equal to the following mean of wavelengths of violet and red lights

(a) square of mean (b) arithmetic mean
(c) geometric mean (d) harmonic mean

70. The amount of radiations emitted by a black body depends on its

(a) size (b) mass
(c) temperature (d) density

71. If 40% of the radiations incident upon a body are absorbed and 30% are transmitted, then its coefficient of reflection will be

(a) 1 (b) 0.4
(c) 0.7 (d) 0.3

72. The amplitude of radiations from a cylindrical heat source is related to the distance as

(a) $A \propto 1/d^2$ (b) $A \propto \frac{1}{\sqrt{d}}$
(c) $A \propto d$ (d) $A \propto d^2$

73. The unit of Wien's constant is

(a) mK (b) $m^{-1}K$
(c) m^2K^{-1} (d) mK^{-1}

74. If the rate of emission of radiation by a body at temperature T(K) is E then the graph between log E and log T will be

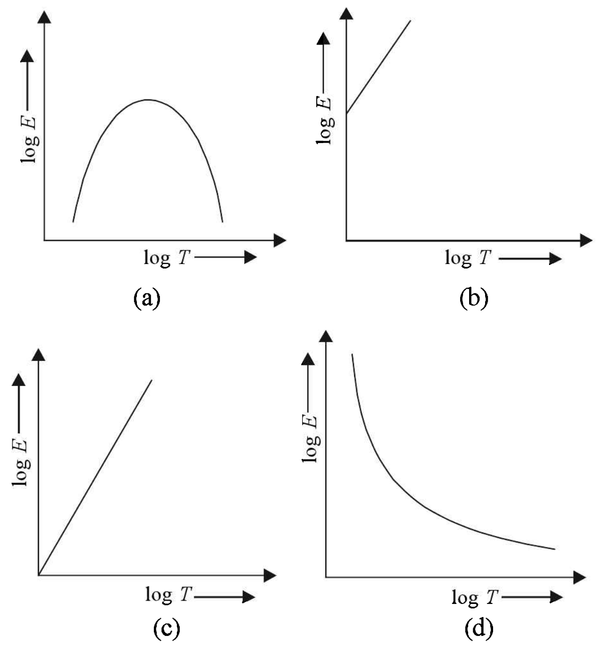


Fig. 17.21

75. Hot milk in a cup is to be drunk 5 minutes after pouring. When should sugar be mixed in it in order that it remains hot at the time of drinking?

(a) Just after pouring (b) At the time of drinking
(c) At any time (d) After some milk is taken

76. The unit of Newton's cooling constant is

(a) cal/s (b) cal/s° C
(c) cal/° C (d) °/C

77. A body in a laboratory takes 4 minutes to cool from 61°C to 59°C. If the laboratory temperature is 30°C then the time taken by it to cool from 51°C to 49°C will be

(a) 8 min (b) 6 min
(c) 5 min (d) 4 min

78. The frequency of ultraviolet rays is

(a) 10^{14} Hz (b) 10^4 Hz
(c) 10^8 Hz (d) 10^{10} Hz

79. In which part of the electromagnetic spectrum is the wavelength of 4000 Å situated?

(a) In invisible part (b) In visible part
(c) In ultraviolet part (d) In infrared part

80. The ratio of radii of two spheres of metal is 1 : 2. The ratio of the amounts of radiation emitted by them at same temperature will be

(a) 1 : 4 (b) 1 : 8
(c) 2 : 1 (d) 8 : 1

81. The solar heat and light reach the earth via

(a) radiation (b) conduction
(c) convection (d) all of these

82. Pervost's theory of heat exchange is not applicable at temperature

(a) 0° R (b) 0° C
(c) 0 K (d) 0° F

83. Two bodies A and B are kept in an evacuated chamber at 27° C. The temperature of A and B are 327° C and 427° C respectively. The ratio of rates of loss of heat from A and B will be

(a) 0.25 (b) 0.52
(c) 1.52 (d) 2.52

84. The time taken by the solar radiations to reach earth is approximately

(a) 8.3 hr (b) 1.3 min
(c) 8.3 min (d) 8.3 sec

85. An ideal black body emits radiations at the rate of 5.67 watt/cm^2. It temperature will be

(a) 10 K (b) 10^2 K
(c) 10^3 K (d) 10^4 K

86. On making the temperature of an ideal black body three times, its maximum intensity of radiation will become

(a) 9 times (b) 27 times
(c) 81 times (d) 243 times

87. The wavelength corresponding to maximum emission of solar radiations is

(a) 8000 Å (b) 4753 Å
(c) 457 Å (d) 753 Å

88. A TV centre tranmits 10 kilowatt of power at 150 MHz. The energy of a photon of electromagnetic wave is

(a) 6×10^{-7} J (b) 6×10^{-7} eV
(c) 6×10^{-7} J (d) 6×10^{-17} eV

89. A cup of tea cools from 80° C to 60° C in the first minute when the temperature of the surrounding is 30° C. The decrease from 60° C to 50° C will be in next

(a) 1 min (b) 50 sec
(c) 48 sec (d) 30 sec

90. The spectral emission power of a black body at 6000 K is maximum at 5000 Å. If the temperature is increased by 10% then decrease in the value of λ_m will be

(a) 10% (b) 7.5%
(c) 5.0% (d) 2.5%

91. If the temperature of an iron rod is doubled then the amount of radiation emitted by it, as compared to its initial value, becomes

(a) 1/2 (b) equal
(c) 4 times (d) 16 times

92. A body of surface area 5 cm^2 and temperature 727° C emits 300 joules of energy per minute. Its emissivity will be

(a) 0.18 (b) 0.28
(c) 0.81 (d) 1

93. In the Orion stellar system the shining of a star is 17×10^3 times that of the sun. If the temperature of the surface of the sun is 6×10^3 K then the temperature of this star will be

(a) 273 K (b) 652 K
(c) 6520 K (d) 68520 K

94. If a body emits 0.3 watt energy at 27° C then the amount of energy emitted by it at 627° C will be

(a) 2.42 watt (b) 0.242 watt
(c) 24.3 watt (d) 0.9 watt

95. The energy of a photon of wavelength 6000 Å in eV will be

(a) 1.06 eV (b) 0.206 eV
(c) 2.06 eV (d) 20 eV

96. A point source of 6 watt emits monochromatic light of wavelength 5000 Å. The number of photons striking normally per second per unit area of the surface distant 5 m from the source will be

(a) 4.82 (b) 4.82×10^{-4}
(c) 4.82×10^{-6} (d) 4.82×10^{16}

97. The surface area of a black body is 5×10^{-4} m^2 and its temperature is 727° C. The amount of radiation emitted by it per minute will be

(a) 0.17 J (b) 17 J
(c) 170 J (d) 1701 J

98. If the temperature of the sun is doubled then the maximum emission wavelength as compared to its initial value will be

(a) 1/4 (b) 1/2
(c) double (d) 4 times

99. A sphere of radius R, density D and specific heat S is heated to temperature θ and surrounding is at temperature θ_0. Its rate of fall of temperature will be proportional to

(a) DS/R (b) R/DS
(c) $1/RDS$ (d) RDS

100. If the maximum emission wavelength of radiations emitted by the moon and the sun are 10^{-4} m and 0.5×10^6 m respectively, then the ratio of temperature of the sun and the moon will be

(a) 1/100 (b) 200/1
(c) 1/200 (d) 100/1

101.

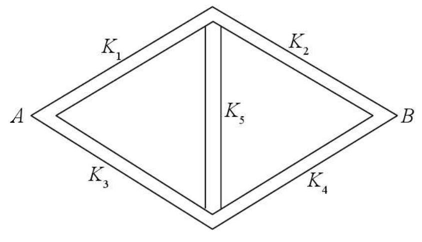


Fig. 17.22

Five rods of the same dimensions are arranged as shown. They have thermal conductivities k_1, k_2,k_3,k_4 and k_5. When points A and B are maintained at different temperatures, no heat flows through the central rod. It follows that

(a) $k_1 = k_4$ and $k_2 = k_3$ (b) $k_1/k_4 = k_2/k_3$
(c) $k_1k_4 = k_2k_3$ (d) $k_1k_2 = k_3k_4$

102. One end of the uniform rod of length 1 m is placed in boiling water while its other end is placed in melting ice. A point *P* on the rod is maintained at a constant temperature of 800°C. The mass of steam produced per second is equal to the mass of ice melted per second. If specific latent heat of steam is 7 times the specific latent heat of ice, the distance of P from the steam chamber must be

(a) $\frac{1}{9}$ m (b) $\frac{1}{8}$ m
(c) $\frac{1}{7}$ m (d) $\frac{1}{10}$ m

103. Three rods *A*, *B* and *C* have same dimension. Their thermal conductivities are k_A, k_B and k_C. They are placed individually, with their ends kept at the same temperature difference. The rate of heat flow through *C* is equal to the rate of combined heat flow through *A* and *B*. k_c must be equal to

(a) $k_A + k_B$ (b) $\dfrac{k_Ak_B}{k_A+k_B}$
(c) $\dfrac{1}{2}(k_A+k_B)$ (d) $2\left(\dfrac{k_Ak_B}{k_A+k_B}\right)$

104. In a 10-meter deep lake, the bottom is at a constant temperature of 4°C. The air temperature is constant at –4°C. The thermal conductivity of ice is 3 times that of water. Neglecting the expansion of water on freezing, the maximum thickness of ice will be

(a) 7.5 m (b) 6 m
(c) 5 m (d) 2.5 m

105.

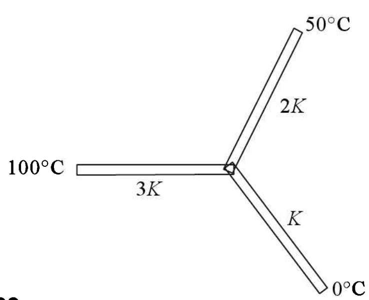


Fig. 17.23

Three rods of the same dimensions have thermal conductivities 3k, 2k and k. They are arranged as shown, with their ends at 100° C, 50°C and 0°C. The temperature of their junction is

(a) 75°C (b) $\frac{200}{3}$ °C
(c) 40°C (d) $\frac{100}{3}$ °C

106. Three rods *A*, *B* and *C* have the same dimensions. Their thermal conductivities are k_A, k_B and k_C respectively. *A* and *B* are placed end to end, with their free ends kept at a certain temperture difference. *C* is placed separately with its ends kept at the same temperature diffence. The two arrangements conduct heat at the same rate k_C equal to

(a) $k_A + k_B$ (b) $\dfrac{k_Ak_B}{k_A+k_B}$
(c) $\dfrac{1}{2}(k_A+k_B)$ (d) $2\left(\dfrac{k_Ak_B}{k_A+k_B}\right)$

107. *A* and *B* are two points on a uniform metal ring whose centre is *C*. The angle ACB = θ. A and B are maintained at two different constant temperatures. When θ = 180°, the rate of total heat flow from A to B is 1.2 W. When θ = 90°, this rate will be

(a) 0.6 W (b) 0.9 W
(c) 1.6 W (d) 1.8 W

108. A body with an initial temperature θ_1 is allowed to cool in surrounding which is at a constant temperature of $\theta_0 (\theta_0 < \theta_1)$. Assume that Newton's law of cooling is obeyed. Let *k* = constant. The temperature of the body after time t is best expressed by

(a) $(\theta_1 - \theta_0)e^{-kt}$ (b) $(\theta_1 - \theta_0)$ In (kt)
(c) $(\theta_0 + (\theta_1 - \theta_0)e^{-kt}$ (d) $\theta_1 e^{-kt} - \theta_0$

109. The temperature of an isolated black body falls from T_1 to T_2 in time *t*. Let *c* be constant.

(a) $t = c\left[\dfrac{1}{T_2} - \dfrac{1}{T_1}\right]$ (b) $t = c\left[\dfrac{1}{T_2^2} - \dfrac{1}{T_1^2}\right]$
(c) $t = c\left[\dfrac{1}{T_2^3} - \dfrac{1}{T_1^3}\right]$ (d) $t = c\left[\dfrac{1}{T_2^4} - \dfrac{1}{T_1^4}\right]$

110. A point source of heat of power P is placed at the centre of a spherical shell of mean radius R. The material of the shell has thermal conductivity k. If the temperature difference between the outer and inner surfaces of the shell is not to exceed T, the thickness of the shell should not be less than

(a) $\dfrac{4\pi kR^2T}{P}$ (b) $\dfrac{4\pi kR^2}{TP}$
(c) $\dfrac{4\pi R^2T}{kP}$ (d) $\dfrac{4\pi R^2P}{kT}$

111. A black body is at a temperature of 2880 K. The energy of radiation emitted by this object with wavelength between 499 nm and 500 nm is U_1, between 999 nm and 1000 nm is U_2 and between 1499 nm and 1500 nm is U_3. The Wien constant is $b = 2.88 \times 10^6$ nm K.

(a) $U_1 = 0$ (b) $U_3 = 0$

(c) $U_1 > U_2$ (c) $U_2 > U_1$

112. A planet is at an average distance d from the sun, and its average surface temperature is T. Assume that the planet receives energy only from the sun, and loses energy only through radiation from its surface. Neglect atmospheric effects. If $T \propto d^{-n}$, the value of n is

(a) 2 (b) 1

(c) $\frac{1}{2}$ (d) $\frac{1}{4}$

113. The power radiated by a black body is P, and it radiates maximum energy around the wavelength λ_B. If the temperature of the black body is now changed so that it radiates maximum energy around a wavelength $3\lambda_0/4$, the power radiated by it will increase by a factor of

(a) $\frac{4}{3}$ (b) $\frac{16}{9}$

(c) $\frac{64}{27}$ (d) $\frac{256}{81}$

114.

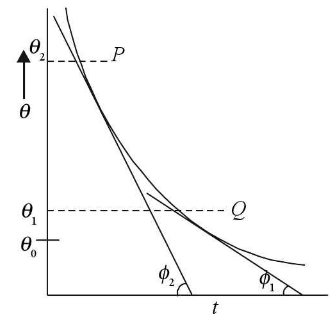


Fig. 17.24

A body cools in a surrounding which is at a constant temperature of θ. Assume that it obeys Newton's law of cooling. Its temperature θ is plotted against time t. Tangents are drawn to the curve at the points $P(\theta = \theta_1)$ and $Q(\theta = \theta_2)$. These tangents meet the time axis at angles of ϕ_2 and ϕ_1 as shown.

(a) $\frac{\tan\phi_2}{\tan\phi_1} = \frac{\theta_1 - \theta_0}{\theta_2 - \theta_0}$ (b) $\frac{\tan\phi_2}{\tan\phi_1} = \frac{\theta_2 - \theta_0}{\theta_1 - \theta_0}$

(c) $\frac{\tan\phi_1}{\tan\phi_2} = \frac{\theta_1}{\theta_2}$ (d) $\frac{\tan\phi_2}{\tan\phi_1} = \frac{\theta_2}{\theta_1}$

115. A system S receives heat continuously from an electrical heater of power 10 W. The temperature of S become constant at 50°C when the surrounding temperature is 20°C. After the heater is switched off, S cools from 35.1°C to 34.9°C in 1 minute. The heater capacity of S is

(a) 100 J/°C (b) 300 J/°C

(c) 750 J/°C (d) 1500 J/°C

Answers to Questions for Practice

1.	(b)	**2.**	(b)	**3.**	(c)	**4.**	(d)	**5.**	(c)	**6.**	(b)	**7.**	(c)
8.	(a)	**9.**	(b)	**10.**	(b)	**11.**	(a)	**12.**	(b)	**13.**	(b)	**14.**	(d)
15.	(c)	**16.**	(c)	**17.**	(b)	**18.**	(d)	**19.**	(c)	**20.**	(c)	**21.**	(c)
22.	(a)	**23.**	(c)	**24.**	(b)	**25.**	(d)	**26.**	(a)	**27.**	(d)	**28.**	(a)
29.	(d)	**30.**	(d)	**31.**	(a)	**32.**	(c)	**33.**	(a)	**34.**	(d)	**35.**	(c)
36.	(d)	**37.**	(a)	**38.**	(b)	**39.**	(b)	**40.**	(b)	**41.**	(d)	**42.**	(c)
43.	(c)	**44.**	(c)	**45.**	(c)	**46.**	(b)	**47.**	(d)	**48.**	(a)	**49.**	(d)
50.	(c)	**51.**	(b)	**52.**	(b)	**53.**	(a)	**54.**	(a)	**55.**	(d)	**56.**	(c)
57.	(a)	**58.**	(b)	**59.**	(b)	**60.**	(c)	**61.**	(d)	**62.**	(a)	**63.**	(a)
64.	(d)	**65.**	(a)	**66.**	(b)	**67.**	(c)	**68.**	(b)	**69.**	(b)	**70.**	(c)
71.	(d)	**72.**	(b)	**73.**	(a)	**74.**	(b)	**75.**	(a)	**76.**	(b)	**77.**	(b)
78.	(a)	**79.**	(b)	**80.**	(a)	**81.**	(a)	**82.**	(c)	**83.**	(b)	**84.**	(c)
85.	(c)	**86.**	(c)	**87.**	(b)	**88.**	(b)	**89.**	(c)	**90.**	(a)	**91.**	(d)
92.	(a)	**93.**	(d)	**94.**	(c)	**95.**	(c)	**96.**	(d)	**97.**	(d)	**98.**	(b)
99.	(c)	**100.**	(b)	**101.**	(c)	**102.**	(c)	**103.**	(a)	**104.**	(a)	**105.**	(b)
106.	(b)	**107.**	(c)	**108.**	(c)	**109.**	(c)	**110.**	(a)	**111.**	(d)	**112.**	(c)
113.	(d)	**114.**	(b)	**115.**	(d)								

Explanations

20.(c) One sees the complementary colour.

24.(b) $E = m_0c_2$ or $m_0c = \frac{E}{c}$

Change in momentum

$$\Delta p = 2m_0c = \frac{2E}{c}$$

$$F = \frac{dp}{dt} = \frac{2P}{c} \left(\because \frac{dE}{dt} = P \right).$$

29.(d) $\frac{2}{K_{eff}} = \frac{1}{K_1} + \frac{1}{K_2}$ or $K_{eff} = \frac{2\times2\times3}{2+3} = 2.4.$

30.(d)Use $t = \frac{\rho L}{k\theta}\frac{y^2}{2}$

34.(d)Since the temperature of surrounding is T_0 and

$R = \sigma(T^4 - T_0^4)$. ∴ it will not be 16 times.

95.(c)$E(ev) = \frac{12420}{6000} = 2.07.$

96.(d)$n = \frac{6\times10^{19}}{4\pi\times5^2\times2.84\times1.6}$.

101.(c)This is analogous to a balanced Wheatstone bridge

$R_1 = \frac{l}{k_1 A}$ etc., and $R_1R_4 = R_2R_3$ for balance.

102.(c)

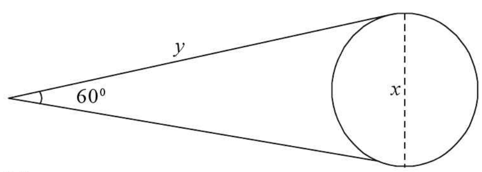


Fig. 17.25

Let Q_1 and Q_2 amounts of heat flow from P in any time t. Let m be the masses of steam formed and ice melted in time t. Let k and A be the thermal conductivity and the area of cross-section respectively of the rod.

$$Q_1 = kA\left(\frac{800-100}{x}\right)t = mL_{steam}$$

$$Q_2 = kA\left(\frac{800-0}{1-x}\right)t = mL_{ice}$$

Dividing, $\left(\frac{700}{x}\right)\left(\frac{1-x}{800}\right) = 7$

or $1-x = 8x$

or $x = \frac{1}{9}m.$

104.(a)The rate of heat flow is the same through water and ice in the steady state.

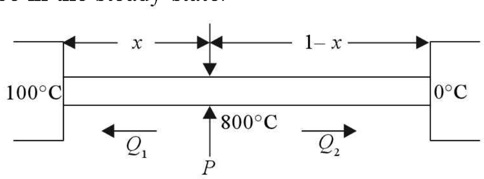


Fig. 17.26

$$I = kA\frac{4-0}{10-x} = 3kA\frac{0-(-4)}{x}$$

$x = (10–x)\,3$

or $x = 7.5$m

105.(b)Let θ = temperature of junction

$$R_1 = \frac{l}{3kA}, R_2 = \frac{l}{2kA}, R_3 = \frac{l}{kA}$$

Use Kirchhoff's first law to distribute current at the junction

$100 - \theta = R_1I_1,\ \theta = 50 = R_2I_2$

$\theta - 0 = R_3(I_1 - I_2)$

107.(c) Let R = total thermal resistance of the ring,

ΔT = difference in temperature between A and B

For θ = 180°, two sections of resitance $R/2$ each are in parallel. Equivalent resistance = $R/4$

Rate of total heat flow $I_1 = 1.2 = \frac{\Delta T}{R/4}$

or $0.3 = \frac{\Delta T}{R}$

For θ = 90°, two sections of resitance $R/4$ and $3R/4$ are in parallel.

Equivalent resistance = $\frac{(R/4)(3R/4)}{R/4 + 3R/4} = \frac{3R}{16}$

Rate of total heat flow

$$I_2 = \frac{\Delta T}{3R/16}W = \frac{16}{3}\left(\frac{\Delta T}{R}\right)W = \frac{16}{3}\times0.3\text{ W} = 1.6\text{ W}.$$

108.(c) $\frac{d\theta}{dt} = -k(\theta - \theta_0)$, where k = constant.

$$\int_{\theta_i}^{\theta}\frac{d\theta}{\theta - \theta_0} = -\int_0^t k.dt$$

or $[\text{in}(\theta - \theta_0)]_{\theta_i}^{\theta} = -kt$

or $\text{in}\,(\theta - \theta_0) - \text{In}\,(\theta_i - \theta_0) = -kt$

109.(c)$P = (4\pi r^2)(\sigma T^4) = ms\left(-\frac{dT}{dt}\right) = -\frac{4}{3}\pi r^3\rho s.\frac{dT}{dt}$

Here, $R = \frac{dT}{dt}$.

$P \propto r^2$ and $R \propto \frac{1}{r}$

$$\int_{T_1}^{T_2}\frac{dT}{T^4} = (\text{constant})\int_0^t dt$$

or $c\left[\frac{1}{T_2^3} - \frac{1}{T_2^3}\right] = t.$

110.(a)Area of spherical shell = $4\pi R^2$

Rate of heat flow $= P = k(4\pi R^2)\frac{T}{d}$, where d = thickness of shell.

111.(d)$\lambda_m T = b = 2.88 \times 10^6$ nm K

or $\lambda_m = \dfrac{2.88\times10^6 \text{nm}K}{2880K} = 1000$ nm.

The black body radiates maximum energy around λ_m.

$\therefore u_2$ is greater than u_1 or u_3.

Also, energy is radiated at all wavelengths.

$\therefore u_1, u_3 \neq 0$

112.(c)Let P = power radiated by the sun, R = radius of planet.

Energy received by planet = $\dfrac{P}{4\pi d^2}\times\pi R^2$

Energy radiated by planet = $(4\pi R^2)\sigma T^4$.

For thermal equilibrium, $\dfrac{P}{4\pi d^2}\times\pi R^2 = (4\pi R^2)\sigma T^4$.

or $T^4 \propto \dfrac{1}{d^2}$

or $T \propto \dfrac{1}{d^{1/2}}$

or $T \propto d^{-1/2}$.

113.(d)Let T_0 = initial temperature of the black body.

$\therefore \lambda_0 T_0 = b$ (constant)

Power radiated = $P_0 = c.T_0^4$ (c = constant)

Let T = new temperature of black body.

$\therefore \dfrac{3\lambda_0}{4}T = b = \lambda_0 T_0$

or $T = \dfrac{4T_0}{3}$.

Power radiated $= c \,.\, T^4 = (c\,T_0^4)\left(\dfrac{4}{3}\right)^4 = P_0\left(\dfrac{256}{81}\right)$.

114.(b) For θ-t plot, rate of cooling = $\dfrac{d\theta}{dt}$ = slope of the curve.

At P, $\dfrac{d\theta}{dt} = |\tan(180-\phi_1)| = \tan\phi_2 = k(\theta_2 - \theta_0)$,

where k = constant.

At Q, $\dfrac{d\theta}{dt} = |\tan(180-\phi_1)| = \tan\phi_1 = k(\theta_1 - \theta_0)$.

$\therefore \dfrac{\tan\phi_2}{\tan\phi_1} = \dfrac{\theta_2-\theta_0}{\theta_1-\theta_0}$.

115.(d)Rate of loss of heat $\propto$ difference in temperature with the surroundings.

At 50°C, $\dfrac{dQ}{dt} = k(50-20) = 10$, where k = constant

$\therefore k = \dfrac{1}{3}$

At an average temperature of 35°C,

$\dfrac{dQ}{dt} = \dfrac{1}{3}(35-20)$ J/s = 5 J/s.

Heat lost in 1 minute = $\dfrac{dQ}{dt}\times 60$ J = 5 × 60 J = 300 J = Q.

Fall in temperature = 0.2°C = $\Delta\theta$.

$Q = c\Delta\theta$.

Heat capacity = $c = \dfrac{Q}{\Delta\theta} = \dfrac{300J}{0.2°C}$ = 1500 J/°C.

SELF TEST 1

1. Which of the following is true for an identical size of a degree?

(a) Mercury scale and ideal gas scale

(b) Celsius scale and ideal gas scale

(c) Celsius scale and Mercury scale

(d) All of the above

2. The density of water at 0°C is 0.998 g cm^{-3} and at 4°C is 1 gcm^{-3}. The average coefficient of cubical expansion between 0 – 4°C is

(a) 2×10^{-4} °C^{-1} (b) 5×10^{-4} °C^{-1}

(c) -5×10^{-4} °C^{-1} (d) -2×10^{-4} °C^{-1}

3. A steel rod of length l is heated by ΔT. The strain developed is ____ Given Y is young's modulus. Assume it is placed on a smooth horizontal base.

(a) $\alpha \Delta T$ (b) $AY\alpha\ \Delta T$

(c) $Y\alpha \Delta T$ (d) zero

(e) none

4. A circular hole of diameter 2 cm is made in an Al sheet at 0°C. The coefficient of linear expansion for Al is 2.3×10^{-5} °C^{-1}. The diameter at 100°C will be

(a) 2.0046 cm (b) 2.046 cm

(c) 2.00046 cm (d) 2.005 cm

5. A steel ball initially at a pressure 10^5 Pa is heated from 20°C to 120°C keeping its volume constant $\alpha_{steel} = 1.2 \times 10^{-5}$°C^{-1} and bulk modulus of steel = 1.6×10^{11} Nm^{-2}. Find the final pressure

(a) 5.76×10^8 Pa (b) 5.76×10^5 Pa

(c) 5.76×10^6 Pa (d) 6.76×10^5 Pa

6. The state diagram P Vs V is shown. The process is

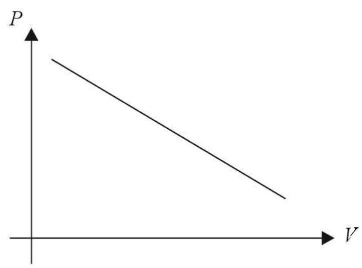


Fig. 1

(a) isobaric (b) adiabatic

(c) isothermal (d) polytropic

(e) throttling

7. The quantity $\dfrac{pV}{kT}$ represents

(a) mass of the gas

(b) KE of the gas

(c) number of moles of the gas

(d) number of molecules in the gas.

8. The rms speed of oxygen at room temperature is 500 ms^{-1}. The rms speed of H_2 at room temperature is

(a) 125 ms^{-1} (b) 8000 ms^{-1}

(c) 2000 ms^{-1} (d) 31.25 ms^{-1}

(e) none

9. A container has only one molecule of a gas in it as it has been evacuated. Then

(a) $V_{avg} > V_{rms}$ (b) $V_{rms} > V_{avg}$

(c) $V_{avg} = V_{rms}$ (d) V_{rms} cannot be defined

10. One mole of a gas undergoes a process $P = \dfrac{p_o}{1+\left(\dfrac{V}{V_0}\right)^2}$

where p_o and V_o are constants. The temperature of the gas when $V = V_o$

(a) $\dfrac{p_oV_o}{R_o}$ (b) $\dfrac{p_oV_o}{2R_o}$

(c) $\dfrac{2p_oV_o}{R_o}$ (d) $\dfrac{p_oV_o}{4R_o}$

11. 10 g of ice at 0°C is added to a container containing 20 g of water at 30°C. The temperature of the mixture is

(a) 20° C (b) 15° C

(c) 10° C (d) 0° C (e) none

12. 200 g of steam at 100°C is passed in a container containing 2.5 kg of ice at 0°C . The contents of H_2O are

(a) 1.6 kg (b) 1.7 kg
(c) 1.8 kg (d) none

13. 1.0 kg of ice at –10°C is added to 2.4 kg of H_2O at 30°C. The contents of mixture are

(a) 162. 5 *g* ice and 3.2375 kg water
(b) 162.5 *g* ice, 837.5 g water
(c) 837.5 ice, 2.5625 g H_2O
(d) none of these

14. The device which can measure high temperature more than 2000 *K* is

(a) resistance thermometer
(b) pyrometer
(c) thermocouple thermometer
(d) none

15. A cylinderical tube of length 30 cm is separated at 20 cm as shown. The partition is very weakly conducting and can freely slide along the tube. Ideal gas is filled along the two parts of the vessel.

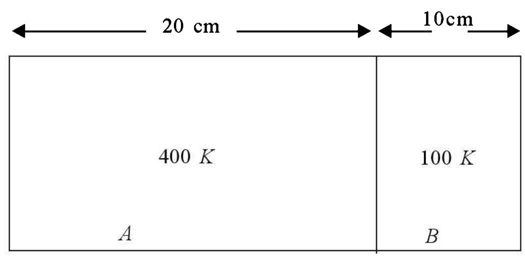


Fig. 2

In the beginning, the temperature in *A* and *B* are 400 *K* and 100 K respectively. Find the final equilibrium position.

(a) 10 cm from left end
(b) 15 cm from left end
(c) 12.25 cm from left end
(d) 12.25 cm from right end

16. The average translational *KE* of air molecules is 0.04. *eV*. The temperature of air is

(a) 278 *K* (b) 310 *K*
(c) 298 *K* (d) 330 *K*
(e) none

17. Estimate the number of collisions per second suffered by a molecule in a sample of hydrogen at *STP*. Mean free path = 1.38×10^{-5} *cm*

(a) 1.2×10^{10} (b) 1.2×10^{12}
(c) 1.2×10^{13} (d) 1.2×10^{11}

18. A vessel is partitioned by a fixed diathermic wall. Different ideal gases are filled in the two parts. The rms speed of the molecules in the left part equals the average speed of the molecules in the right part. The ratio of mass of a molecule in the left to that of in the right part is

(a) 1.18 (b) 1.31
(c) 1.28 (d) 1.38

19. Air is pumped into an automobile tyre's tube up to a pressure of 200 *kPa* in the morning ($T = 20$°C). During the day temperature rises to 40°C. The tube expands by 2%. The air pressure in the tube is

(a) 202 *kPa* (b) 204 *kPa*
(c) 208 *kPa* (d) 209 *kPa*

20. Convert the *PV* state diagram into *PT* diagram.

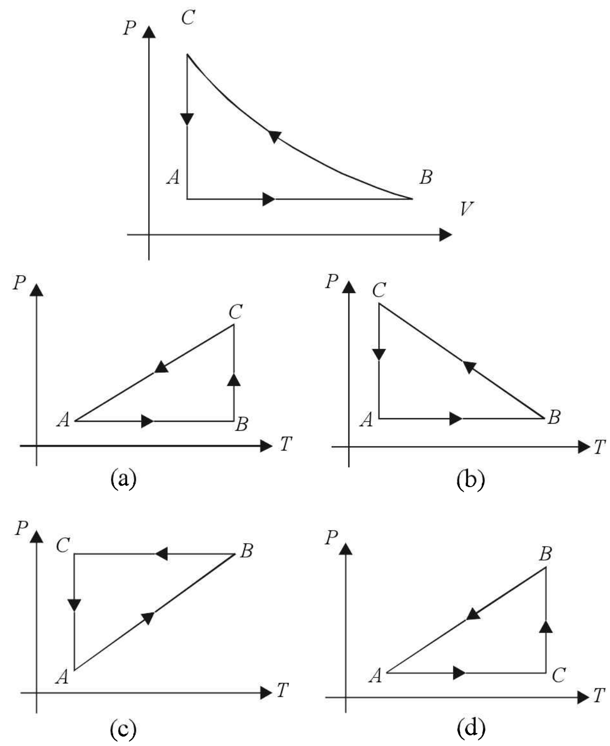


Fig. 3

21. A sample of an ideal gas has pressure P_o, volume V_o and temperature T_o. It is isothermally expanded to twice its original volume. Then compressed isobarically to V_o finally heated isochorically to achieve the same temperature T_o The heat absorbed in the process is

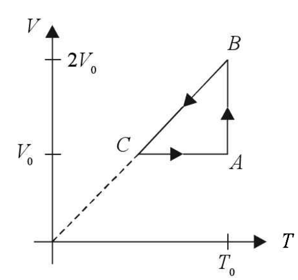


Fig. 4

(a) $1.193 P_o V_o$ (b) $0.193 p_o V_o$
(c) $0.693 P_o V_o$ (d) none of these

22. If Q_A and Q_B are the heat given in the process *A* and *B* respectively, then

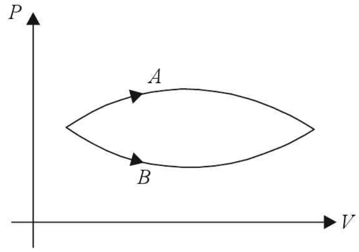


Fig. 5

(a) $Q_A = Q_B$ (b) $Q_A > Q_B$
(c) $QA < Q_B$ (d) $Q_A \leq Q_B$

23. In adiabatic, isothermal and isobaric expansion W_1, W_2 and W_3 are the work done, respectively, then

(a) $W_1 > W_2 > W_3$ (b) $W_2 > W_3 > W_1$
(c) $W_3 > W_2 > W_1$ (d) $W_3 > W_1 > W_2$

24. A gas is taken along the path AB as shown in figure. If 70 cal of heat is extracted from the gas in the process, the change in internal energy of the system is

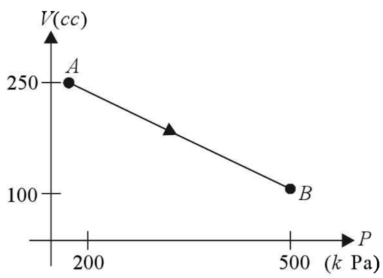


Fig. 6

(a) 292.6 J (b) 22.5 J
(c) 52.5 J (d) 241 J

25. The figure shows the variation in internal energy U with volume V of 2 mole at an ideal gas in a cyclic process *abcda*. The temperature of the gas at b and c are 500 K and 300 K respectively. The heat absorbed during the process is

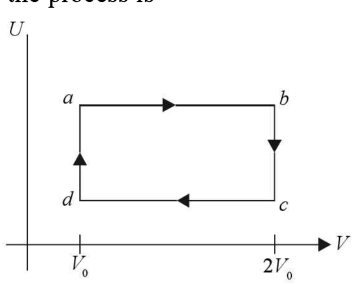


Fig. 7

(a) 2300 J (b) 3324 J
(c) 1024 J (d) none of these

26. Specific heat capacity at constant volume of gases in an adiabatic process is

(a) ∞ (b) finite but not zero
(c) zero (d) $0 < C_v < \infty$

27. The specific heat capacity for a mixture of gases at constant volume of He : N_2 in the ratio 1 : 2 is

(a) $\dfrac{12R}{6}$ (b) $\dfrac{13}{6}R$
(c) $\dfrac{11}{6}R$ (d) $\dfrac{14}{6}R$

28. 4 g of He and 16 g of O_2 are mixed. Find the $\dfrac{C_P}{C_V}$ = for the mixture.

(a) 17/11 (b) 19/13
(c) 3/2 (d) 16/11
(e) none

29. If $\dfrac{C_P}{C_V} = \gamma$ for a gas, then number of degrees of freedom it has is

(a) $\dfrac{\gamma+1}{2}$ (b) $\gamma + \frac{1}{2}$
(c) $\dfrac{2}{\gamma-1}$ (d) $\dfrac{4}{\gamma+1}$

30. A, B and C show the adiabatic processes for various gases. Recognize gas for process C

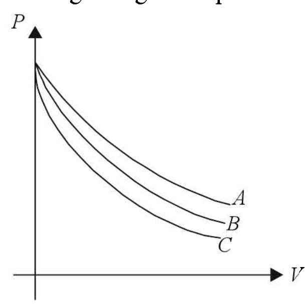


Fig. 8

(a) N_2 (b) CO_2
(c) NH_3 (d) He

31. The γ for an ideal gas is $\dfrac{7}{6}$. If the temperature of this gas is raised by 50°C, find the change in internal energy of 1mole of the gas when heated at constant pressure.

(a) 2493 J (b) 2944 J
(c) 450.5 J (d) none

32. Three samples 1, 2, 3 are allowed to cool. Which has the largest specific heat?

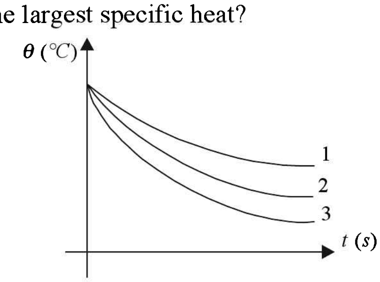


Fig. 9

(a) 1 (b) 2
(c) 3 (b) insufficient data to find

33. The volume of an ideal gas (γ = 1.5) is changed adiabatically from 4 to 3 *l*. Find the ratio of final pressure to initial pressure.

(a) $\frac{8}{3\sqrt{3}}$ (b) $\frac{6}{4}$

(c) $\frac{5}{3}$ (d) none

34. Half mole of an ideal gas (γ = 5/3) is taken through the cycle *abcda* as shown in figure. Take $R = \frac{25}{3}$ J mol^{-1} K^{-1}. Find the amount of heat supplied in the process *ab* and *bc*.

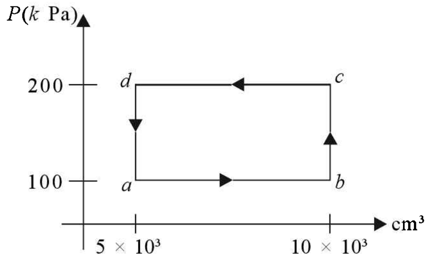


Fig. 10

(a) 1250 J, O (b) 1250 J, 1500 J

(c) 1250 J, 250 J (d) None

35. A rigid container of negligible heat capacity contains one mole of an ideal gas. The temperature of the gas increases by 1°C if 3 cal of heat is added to it. The gas may be

(a) He (b) N_2

(c) O_2 (d) CO_2

36. Four cylinders contain equal volume of Ar, H_2, CO_2 and NH_3 at *NTP*. The gas having minimum energy is

(a) H_2 (b) Ar

(c) CO_2 (d) NH_3

37. An adiabatic cylindrical tube fitted with an adiabatic separator has an ideal gas (γ= 1.5) in equal volume. Temperature and pressure are also equal. The separator is now shifted so that it divides the tube in the ratio 1: 3. Find the ratio of the temperatures in two parts.

(a) $\sqrt{2}$: 1 (b) $\sqrt{3}$: 1

(c) 2 : 1 (d) $\sqrt{5}$: 1

38. Standing waves of frequency *5kHz* are produced in a tube filled with O_2 at 300 *K*. The separation between the consecutive nodes is 3.3 cm. c_p/c_v for the gas is

(a) 1.39 (b) 1.31

(c) 1.66 (d) none

39. An ideal gas has density 1.7×10^{-3} $g\ cm^{-3}$ at a pressure of $1.5 \times 10^5 Pa$ is filled in kundt's tube. When the gas is resonated at a frequency 3 *kHz* nodes are formed at a separation of a 6 cm. The C_p of the gas is

(a) $\frac{3}{2}R$ (b) $\frac{5}{2}R$

(c) $3R$ (d) $\frac{7}{2}R$

40. Two rods of equal cross-section and equal length are added in parallel. The equivalent thermal conductivity k_{eq} will be ____ if individual thermal conductivities are k_1 and k_2 respectively.

(a) $K_1 + K_2$ (b) $\frac{K_1K_2}{K_1+K_2}$

(c) $\frac{K_1+K_2}{2}$ (d) $\frac{K_1K_2}{2CK_1+K_2}$

41. Find the thermal power from the unlisted part.

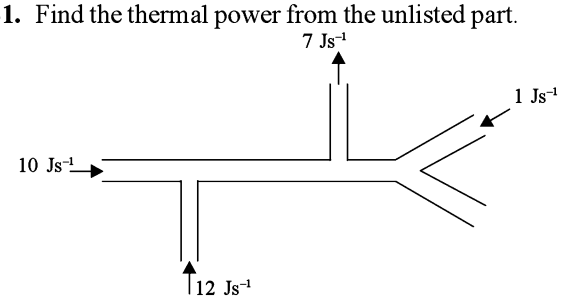


Fig. 11

(a) 12 Js^{-1} (b) 16 Js^{-1}

(c) 9 Js^{-1} (d) 14 Js^{-1}

42. Sun is _____ body.

(a) a nearly black (b) a parfectly black.

(c) not a black (d) a white

43. The ratio of emissive power to absorptive power is the same for all bodies at a given temperature represents

(a) Wein's law (b) Stefan's law

(c) Rayleigh Jean's law (d) Kirchhoff's law

44. The solar radiations have a maximum intensity around 490 nm. The surface temperature of the sun is

(a) 5814 *K* (b) 6130 *K*

(c) 6013 *K* (d) 5978 *K*

45. A black body of surface area 10 cm^2 is heated to 127° C. from 27°C. The initial rate of loss of heat from the body to the room is

(a) 0.99 W (b) 1.11 W

(c) 14. 2 W (d) none of these

46. A drum (insulating) contains water at 0°C. It is filled to a height 10 cm. It is kept in open field on a chilled day when the temperature is–θ°C. The time taken for the whole mass of water to freeze is ____. Thermal conductivity and latent heat of ice respectively are *K* and *L*

(a) $\frac{\rho Lh^2}{2k\theta}$ (b) $\frac{\rho Lh}{k\theta}$

(c) $\frac{\rho Lh^2}{3k\theta}$ (d) $\frac{\rho Lh^2}{3k\theta}$

47. The earth receives solar radiations at a rate of 8.2J cm^{-2} min^{-1}. The angle subtended by the sun on the earth is 0.53° and $\sigma = 5.67 \times 10^{-8}$ $Wm^{-2}K^{-4}$. The temperature of the sun is

(a) 5698 K (b) 5876 K

(c) 5794 K (d) none

48. A hot body is kept in a big room. Its temperature T is plotted against time t. The correct curve is

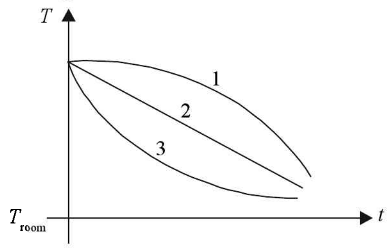


Fig. 12

(a) 1 (b) 2

(c) 3 (d) none

49. A body cools from 65°C to 60°C in 5 minutes. It will cool from 60°C to 55°C in

(a) 5 min. (b) more than 5 min

(c) less than 5 min

(d) more than or less than 5 min depending upon its mass

50. A carnot engine operates between 427°C and 127°C. The efficiency of the engine is nearly

(a) 43 % (b) 45 %

(c) 40 % (d) 49 %

Answers

1.	(c)	**2.**	(c)	**3.**	(d)	**4.**	(a)	**5.**	(a)	**6.**	(d)	**7.**	(d)
8.	(c)	**9.**	(c)	**10.**	(b)	**11.**	(d)	**12.**	(c)	**13.**	(a)	**14.**	(b)
15.	(a)	**16.**	(b)	**17.**	(a)	**18.**	(a)	**19.**	(d)	**20.**	(a)	**21.**	(b)
22.	(b)	**23.**	(c)	**24.**	(d)	**25.**	(a)	**26.**	(c)	**27.**	(b)	**28.**	(a)
29.	(c)	**30.**	(d)	**31.**	(a)	**32.**	(a)	**33.**	(a)	**34.**	(b)	**35.**	(a)
36.	(b)	**37.**	(b)	**38.**	(a)	**39.**	(d)	**40.**	(c)	**41.**	(b)	**42.**	(b)
43.	(d)	**44.**	(a)	**45.**	(a)	**46.**	(a)	**47.**	(c)	**48.**	(c)	**49.**	(b)
50.	(a)												

Explanations

1(c)

2(c) $\gamma = -\frac{.002}{1 \times 4} = -5 \times 10^{-4}\ °C^{-1}$

3(d) Since the rod is not fixed

4(d) $d = d_o(1 + \alpha\Delta T) = 2.0(1 + 2.3 \times 10^{-3})$

$= 2.0046$ cm

5(a) $P = \gamma B \Delta T = 3.6 \times 10^{-5} \times 1.6 \times 10^{11} \times 10^2$

$= 5.8 \times 10^8\ Pa$

6(d)

7(d)

8(c) $\frac{V_{H_2}}{V_{O_2}} = \sqrt{\frac{M_{O_2}}{M_{H_2}}}$ $\therefore V_{H_2} = 500 \times 4 = 2000\ ms^{-1}$

9(c)

10(b) $P = \frac{p_o}{1+1} = \frac{p_o}{2}$ when $V = V_o$ but $p_o V_o = RT$

or $T = \frac{p_o V_o}{2R}$

11(d) As the whole of the ice will not melt.

12(c) 1g of steam melts 8 g of ice therefore, 200 g of steam will, melt 1600 g of ice

$\therefore$ contents of $H_2O = 1600 + 200 = 1800\ g$

13(a) $1000 \times .5 \times 10 + m\,80 = 2400 \times 30$

or $m = \frac{6700}{8} = 837.5g$

162.5 g ice and 3.2375 $kg\ H_2O$

14(b)

15(a) Let the piston shift by x then

$$\frac{20 - x}{T} = \frac{20}{400} = \frac{1}{20}$$

$$\frac{10 + x}{T} = \frac{10}{100}$$

or $400 - 20x = T$ (1)

or $100 + 10x = T$ (2)

Solving (1) and (2) for x we get $x = 10$ cm

16(b) $\frac{3}{2} kT = .04 \times 1.6 \times 10^{-19}$

or $T = \frac{8\times1.6\times10^{-21}}{3\times1.38\times10^{-23}} = 310\,K$

17(a) $\frac{3}{2} P = n\lambda$ or $n = \frac{3\times10^{5}}{2\times1.38\times10^{-5}} = 1.2 \times 10^{10}$

18(a) $\sqrt{\frac{3RT}{M_1}} = \sqrt{\frac{8RT}{\pi M_2}}$ or $\frac{M_1}{M_2} = \frac{3\pi}{8} = 1.18$

19(d) $\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$ or $P_2 = \frac{200\times V}{293} \times \frac{313}{1.02V}$

$= 208.8\,kPa$

20(a) AB is Isobaric process, BC is isothermal process and CA is isochoric process

21(b) $H_{AB} = W_{AB} = NRT_o \log_e \frac{2V_0}{V_0} = P_o V_o \log_e 2 = 0.693 P_o V_o$

and pressure becomes $p_o/2$

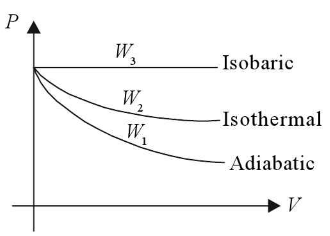


Fig. 13

$W_{BC} = \frac{P_o}{2}(V_o - 2V_o)$

$= \frac{-P_oV_o}{2}$

Heat supplied $= W = W_{AB} + W_{Bc}$

$= 0.193 P_o V_o$

22(b)

23(c) Obviously $W_3 > W_2 > W_1$

24(d) $W_{AB} = \frac{(250+100)\times10^{-6}\times300\times10^{3}}{2} = 52.5\,J$

$U = 70 \times 4.18 - 52.5 = 241\,J$

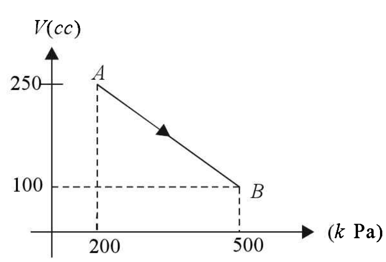


Fig. 14

25(a) $H = W_{\text{Isothermal}} = nR[T_1 - T_2] \log_e \frac{V_2}{V_1}$

$= 2 \times 8.31 \times 200 [\log_e 2]$

$= 3324(.693) = 2300\,J$

26(c) $C_v = \frac{\Delta Q}{n\Delta T} = 0 \because \Delta Q = 0$

27(b) $C_{V_{net}} = \frac{n_1C_{V1} + n_2C_{V2}}{n_1 + n_2}$

$= \frac{\left(1\times\frac{3}{2} + 2\times\frac{5}{2}\right)R}{1+2} = \frac{13}{6} R$

28(a) 4g He ≡ 1 mole; 16 g of O_2 ≡ ½mole

$C_{V_{net}} = \frac{(3/2 R) + 1/2 (5/2 R)}{1 + 1/2} = \frac{11}{6} R$

$C_P = C_V + R$ or $C_P = \frac{17}{6} R$ and $\gamma = \frac{C_P}{C_V} = \frac{17}{11}$

29(c) $\frac{n+2}{n} = \gamma$ or $n + 2 = n\gamma$ or $2 = n(\gamma - 1)$ or $n = \frac{2}{\gamma - 1}$

30(d) More the value of γ more the slope as PV^{γ} = const

31(a) $C_V = 6R$ and $\Delta Q = n\,C_V\,\Delta T$

$= 1 \times 6(8.31)(50) = 2493$ J

32(a) If specific heat is large it will take more time to get heated or cooled.

33(a) $P_1 V_1^{\gamma} = P_2 V_2^{\gamma}$

or $\left(\frac{P_1}{P_2}\right) = \left(\frac{V_2}{V_1}\right)^{\gamma} = \left(\frac{3}{4}\right)^{3/2}$

$= \frac{3}{4}\left(\frac{\sqrt{3}}{2}\right) = \left(\frac{3\sqrt{3}}{8}\right)$

or $\frac{P_2}{P_1} = \frac{8}{3\sqrt{3}}$

34(b) $Q_{ab} = n\,c_p \Delta T = \frac{1}{2} \times \frac{5}{2} R(240 - 120)$

$= \frac{5}{4} \times \frac{25}{3}(120) = 1250\,J$

$Q_{bc} = nC_V \Delta T = \frac{1}{2} \times \frac{3}{2} \times \frac{25}{3} \times 240 = 1500\,J$

$PV = nRT_a 100 \times 10^3 \times 10^{-3}$

$= \frac{1}{2} \times \frac{25}{3} \times T$

$T_a = 120\,K, T_b = 240\,K$

35(a) $\Delta Q = C_v\,\Delta T$ or $C_V = \frac{3cal}{1\times1} = \frac{3}{2} R$

36(b) Since volume at NTP are equal, therefore, number of moles are equal. Hence, the minimum for Ar as its C_v is the least.

37(b) $T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$ or $\frac{T_1}{T_2} = \left(\frac{3}{1}\right)^{0.5} = \frac{\sqrt{3}}{1}$

38 $V_s = \sqrt{\frac{\gamma RT}{M}}$ or $f\lambda = \sqrt{\frac{\gamma RT}{M}}$

or $5 \times 10^3 \times 6.6 \times 10^{-2} = \sqrt{\frac{\gamma \times 8.31 \times 300}{32 \times 10^{-3}}}$

or $\gamma = \frac{330 \times 32 \times 10^{-3}}{2500} \times 330 = 1.39$

39(d) $V = \lambda f = \sqrt{\frac{\gamma P}{\rho}}$

or $\gamma = \frac{\left(3 \times 10^3 \times 12 \times 10^{-2}\right)^2 \times 1.7}{1.5 \times 10^5} = 1.4$

$C_P = \frac{7}{2} R$

40(c) $\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} \Rightarrow \frac{K_{eq} 2A}{l} = \frac{K_1 A}{l} = \frac{K_2 A}{l}$

or $K_{eq} = \frac{K_1 + K_2}{2}$

41(b) Apply Kirchhoff's Junction law.

amount of heat entering s^{-1} = amount of heat leaving s^{-1} $10 + 12 + 1 = 7 + x$ or $x = 16\, J\, s^{-1}$

42(b)

43(d)

44(a) $\lambda T = b$ or $T = \frac{2.89 \times 10^{-3}}{490 \times 10^{-9}} = 5814\, K$

45(a) $U - U_0 = \sigma A\ (T^4 - T_0^4)$

$= 5.67 \times 10^{-8} \times 10 \times 10^{-4} \times (400^4 - 300^4) = 0.99\, W$

46(a) $t = \frac{\rho L h^2}{2K\theta}$

47(c) $\frac{D}{R} = 0.53 \times \frac{\pi}{180}$

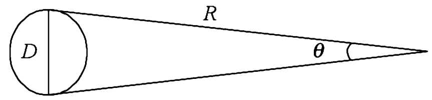


Fig. 15

$= 9.25 \times 10^{-3}$ radiations emitted s^{-1}

$= 4\pi \left(\frac{D}{2}\right)^2 \sigma T^4$

radiations falling /Area /sec $= \frac{\pi D^2 \sigma T^4}{4\pi R^2}$

or $\frac{\sigma T^4}{4} \left(\frac{D}{R}\right)^2 = 8.2\, J\, cm^{-2}\, min^{-1}$ solving $T = 5794\, K$

48(c)

49(b)

50(a) $n = 1 - \frac{T_2}{T_1} = 1 - \frac{400}{700} = \frac{3}{7}$ or 43% .

SELF TEST 2

1. The volume of 1 mole of an ideal gas at $S.T.P$ is

(a) $2.25\, m^3$ (b) $2.27 \times 10^{-3} m^3$

(c) $2.24 \times 10^{-2} m^3$ (d) $2.27 \times 10^{-4} m^3$

2. The possible values of specific heat of gas are

(a) only C_p and C_v (b) any value from negative to ∞

(c) only C_p (d) only C_v

3. The molar specific heat at constant volume Cv for a diatomic gas is

(a) $3/2\, R$ (b) $5/2\, R$

(c) $7/2\, R$ (d) $9/2\, R$

4. Ratio of the slope of adiabatic curve and isothermal curve at their point of intersection is (if $\gamma = Cp/Cv$)

(a) γ (b) $1/\gamma$

(c) $\gamma - 1$ (d) $\gamma + 1$

5. The speed of sound in gas is v the rms velocity of gas molecules is v_{rms} then the value of v/v_{rms} will be

(a) $\sqrt{\frac{\gamma}{3}}$ (b) $\sqrt{\frac{3}{\gamma}}$

(c) $\sqrt{\gamma}$ (d) $\sqrt{3}$

6. The degree of freedom for H_2 gas at temperature 1000 K

(a) 5 (b) 7

(c) 6 (d) 3

7. Cv for H_2 at 20 K is

(a) $3/2\, R$ (b) $5/2\, R$

(c) $9/2\, R$ (d) $2/3\, R$

8. The adiabatic and isothermal elasticities E_A and E_I are related as

(a) $\frac{E_A}{E_I} = \gamma$ (b) $\frac{E_I}{E_A} = \gamma$

(c) $\frac{E_A}{E_I} = 1 + \gamma$ (d) $\frac{E_I}{E_A} = 1 - \gamma$

9. A thermally insulated vessel containing O_2 moves with velocity 10 m/s the gas temperature increment resulting from the sudden stoppage of the vessel is

(a) $0.75 K$ (b) $7.5 K$
(c) $1.5 K$ (d) $15 K$

10. Amount of heat transferred to O_2 in the isobaric process for gas to perform the work 5 J

(a) 175 J (b) 17.5 J
(c) 17×10^5 J (d) 0.175 J

11. γ (Cp/Cv) in terms of degree of freedom f can be represented by

(a) $\frac{f+2}{f}$ (b) $\frac{f}{f+2}$
(c) $\frac{f^2}{f+2}$ (d) $\frac{f}{(f+2)^2}$

12. If pressure and volume of an ideal gas is P and V, then internal energy U is $(\gamma = Cp/Cv)$

(a) $PV(\gamma - 1)$ (b) PV/V
(c) $\frac{PV}{\gamma - 1}$ (d) $\frac{PV\gamma}{\gamma - 1}$

13. The PT relation for adiabatic expansion is

(a) $\frac{T^\gamma}{P^{\gamma-1}}$ = constant (b) $\frac{T^{\gamma-1}}{P^\gamma}$ = constant
(c) $\frac{P^\gamma}{T^{\gamma-1}}$ = constant (d) $P^\gamma T^{\gamma-1}$ = constant

14. Which of following gases has maximum rms speed at STP

(a) H_2 (b) N_2
(c) O_2 (d) CO_2

15. Relation between Cp and degree of freedom is

(a) $Cp = \left(\frac{f+2}{f}\right)R$ (b) $Cp = \frac{f}{2}R$
(c) $Cp = \frac{f+2}{2}R$ (d) $Cp = 2fR$

16. 2 moles of N_2 mixed with 3 moles of He then $\gamma(Cp/Cv)$ for gaseous mixture is

(a) 1.33 (b) 1.67
(c) 1.5 (d) 1.4

17. For a gas Sp and Sv are 1.04 J/gmK and 0.74 J/gmK molar mass of gas is

(a) 32 gm/mol (b) 28 gm/mol
(c) 14 gm/mol (d) 2 gm/mol

18. In Q. 17 degree of freedom for molecule is

(a) 5 (b) 7
(c) 2 (d) 3

19. A vessel of volume V = 5 litre contains a mixture of ideal gases He, H_2 and O_2 at temperataure 200 K, their quantities are 1, 2 and 3 mole, respectively. Pressure of mixture is

(a) 15 atm (b) 20 atm
(c) 10 atm (d) 5 atm

20. In Q. 19 mean molar mass of mixture is

(a) 15.3 g/mol (b) 17.3 g/mol
(c) 12.3 g/mol (d) 18.3 g/mol

21. If d is diameter of a molecule and n is the number of molecules per unit volume, then mean free path $\bar{\lambda}$ is given by

(a) $\bar{\lambda} = \frac{1}{2\pi d^2 n}$ (b) $\bar{\lambda} = \frac{1}{\pi d^2 n}$
(c) $\bar{\lambda} = \frac{1}{\sqrt{2}\pi d^2 n}$ (d) $\bar{\lambda} = \frac{d^2 n}{2\pi}$

22. The mean speed of H_2 molecule is 1690 ms^{-1}. The radius of H_2 molecule is 1.37×10^{-10}m, then the total cross-section is (take $n = 3 \times 10^{25}$ m^{-3})

(a) 2.36×10^{-20} m^2 (b) 1.2×10^{-10} m^2
(c) 23.6×10^{-20} m^2 (d) None of these

23. In Q. 22 collision frequency f is

(a) $1.2 \times 10^{10} s^{-1}$ (b) $1.2 \times 10^5 s^{-1}$
(c) $23.26 \times 10^8 s^{-1}$ (d) $1.4 \times 10^4 s^{-1}$

24. In Q. 22 mean free path is

(a) 1200 $A°$ (b) 1600 $A°$
(c) 1400 $A°$ (d) 1800 $A°$

25. In Q. 22 mean free time is

(a) 8.3×10^{-10} sec (b) 8.3×10^{-11} sec
(c) 8.3×10^{-6} sec (d) 8.3×10^{-5} sec

26. Critical temperature of helium if the critical pressure is 2.26 atm and cirtical density is 0.77 gm cm^{-3}

(a) $4.26 K$ (b) $2.26 K$
(c) $3.2 K$ (d) $2.8 K$

27. Vander wall's constant for helium if $Tc = 5.3 K$, $Pc = 2.25$ atm and $R = 8.31$ J/mol

(a) 3.59×10^{-3} Nm^4 mol^{-2} (b) 3.42×10^{-3} Nm^4 mol^{-2}
(c) 3 × 10–4 Nm^4 mol^{-2} (d) 3.21×10^{-6} Nm^4 mol^{-2}

28. Internal energy of a gas decreases when

(a) it gains heat (b) the change is cyclic
(c) the change is adiabatic (d) none of these

29. The value $\frac{P_C V_C}{RT_C}$ is

(a) 8/3 (b) 3/8
(c) 4/7 (d) 5/3

30. If the distance between two walls is L then collision frequency of a molecule moving with velocity V_n is given by

(a) $V_n/(2L)$ (b) $2L/V_n$
(c) $2LV_n$ (d) $2LV_n^2$

31. In a mixture there is 1 mole of H_2 and 1 mole of CO, Cp for mixture is

(a) $7R/2$ (b) $5R/2$
(c) $3R/2$ (d) $9R/2$

32. Relation between Cp, γ and R is given by

(a) $Cp = \frac{\gamma R}{\gamma - 1}$ (b) $Cp = \frac{R}{\gamma - 1}$
(c) $Cp = \frac{\gamma - 1}{R}$ (d) $Cp = \frac{\gamma^2}{R}$

33. Average velocity of molecule in a gas is

(a) $\sqrt{\frac{2KT}{m}}$ (b) $\sqrt{\frac{8}{\pi}\frac{KT}{m}}$
(c) $\sqrt{3\frac{KT}{m}}$ (d) none of these

34. Relation between gas constant R and avogadro number N is (k = Boltzmann constant)

(a) $R = K/N$ (b) $R = KN$
(c) $R = K^2N$ (d) $R = KN^3$

35. 1 mole of Ne is mixed with 2 mole of N_2. Cv for the mixture is

(a) $11R/6$ (b) $5R/6$
(c) $13R/6$ (d) $7R/6$

36. Degree of freedom for N atomic linear molecules is

(a) $6N - 5$ (b) $5N - 6$
(c) $3N - 5$ (d) $2N - 5$

37. Average kinetic energy of molecule along x-axis is

(a) $3/2\,KT$ (b) $1/2\,KT$
(c) KT (d) $2KT$

38. Kinetic energy associated with f degree of freedom is

(a) $fKT/2$ (b) fKT
(c) $3fKT$ (d) $fKT/3$

39. If 1 mole of O_2 at temperature 300 K is mixed with 2 moles of N_2 at temp 100 K, then temperature of mixture is

(a) 200 K (b) 300 K
(c) 250 K (d) 167 K

40. Degree of freedom for N atomic spherical molecule is

(a) $(6N - 6)$ (b) $(6N - 5)$
(c) $(3N - 6)$ (d) none of these

41. Work done in given cyclic process is

(a) -5π joules (b) -10π joules
(c) $+5\pi$ joules (d) $+10\pi$ joules

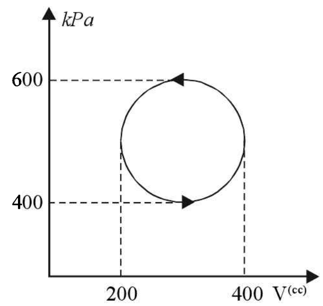


Fig. 1

42. For a polytropic process PV^{-1} = constant specific heat capacity is

(a) $C = Cv + R/2$ (b) $C = Cv - R/2$
(c) $C = Cv/2 + R/2$ (d) none of these

43. Relation between Cv, R and γ is

(a) $C_v = \gamma R/\gamma - 1$ (b) $C_v = R/\gamma - 1$
(c) $C_v = \gamma - 1/R$ (d) $C_v = \frac{(\gamma - 1)R}{\gamma}$

44. An amount Q of heat is added to a monatomic ideal gas in a process in which gas performs a work $Q/4$ on its surroundings. Molar heat capacity at constant pressure for the process is

(a) $3R$ (b) $4R$
(c) $2R$ (d) R

45. In Q. 44 Cv for the process is

(a) $3R$ (b) $2R$
(c) $4R$ (d) $6R$

46. Air (γ = 1.4) is pumped at 2 atm pressure in a balloon at temperature 20°C. If balloon suddenly bursts, the temperature of air coming out of balloon

(a) 230 K (b) 240 K
(c) 210 K (d) 240°C

47. A point is moving along curved path, it has

(a) only one degree of freedom
(b) only two degrees of freedom
(c) only there degrees of freedom
(d) only five degrees of freedom

48. If P is the number of particle constituting the system and C is the total number of constraints, then formula for degree of freedom is

(a) $f = 3C - p$ (b) $f = 3p - C$
(c) $f = 3p/C$ (d) $C/3p$

49. Average kinetic energy of air molecule at 300 K is

(a) 0.02 eV (b) 0.06 eV
(c) 0.03 eV (d) 0.05 eV

50. A cubical box of side 0.1 m contains 3×10^{22} molecules of O_2 at 300 K average pressure exerted by it on the walls of the cube is

(a) 13420 Nm^{-2} (b) 12420 Nm^{-2}
(c) 10000 Nm^{-2} (d) 20000 Nm^{-2}

Answers to Questions for Practice

1.	(c)	**2.**	(b)	**3.**	(b)	**4.**	(a)	**5.**	(a)	**6.**	(b)	**7.**	(a)
8.	(a)	**9.**	(a)	**10.**	(b)	**11.**	(a)	**12.**	(c)	**13.**	(a)	**14.**	(a)
15.	(c)	**16.**	(c)	**17.**	(b)	**18.**	(a)	**19.**	(b)	**20.**	(c)	**21.**	(c)
22.	(c)	**23.**	(a)	**24.**	(c)	**25.**	(b)	**26.**	(a)	**27.**	(a)	**28.**	(c)
29.	(b)	**30.**	(a)	**31.**	(a)	**32.**	(a)	**33.**	(b)	**34.**	(b)	**35.**	(c)
36.	(a)	**37.**	(b)	**38.**	(a)	**39.**	(d)	**40.**	(a)	**41.**	(b)	**42.**	(a)
43.	(b)	**44.**	(b)	**45.**	(a)	**46.**	(b)	**47.**	(b)	**48.**	(b)	**49.**	(b)
50.	(b)												

Explanations

1(c)

2(b)

3(b)

4(a)

5(a)

6(b) Note that the degree of freedom for H_2 at higher temperature increases.

7(a) Note that the degree of freedom for H_2 at lower temperature decreases.

8(a)

9(a) $n\,Cv\,\Delta T = 1/2\,Mv^2$

$$\frac{R}{\gamma-1}\,\Delta T = \frac{1}{2}\,Mv^2$$

$$\Rightarrow \quad \Delta T = \frac{(\gamma-1)}{2R}\,Mv^2 \text{ here M = 32g and } \gamma = 7/5$$

10(b) Amount of heat supplied

$$\Delta Q = \frac{\gamma \Delta W}{\gamma-1}$$

11(a)

12(c)

13(a)

14(a)

15(c)

16(c) $$\gamma_{\text{mix}} = \frac{n_1\gamma_1 + n_2\gamma_2}{n_1+n_2} = \frac{2\left(\frac{1}{5}\right)+3\left(\frac{5}{3}\right)}{5}$$

n_1 and n_2 are moles of N_2 and He γ_1 and γ_2 are their Cp/Cv respectively.

17(b) Molar mass $M = \dfrac{R}{S_P - S_V} = \dfrac{8.314}{0.3} \cong 28$

18(a)

19(b) $$P_{\text{mixture}} = \left(\frac{n_1+n_2+n_3}{V}\right)RT$$

$$= \left(\frac{6}{5\times10^{-3}}\right)\times 8.314\times 200 = 20 \text{ atm}$$

20(c) M_{mean} =

$$\frac{n_1 M_{He} + n_2 M_{H_2} + n_3 M_{O_2}}{n_1+n_2+n_3} = \frac{1\times4+2\times2+3\times32}{1+2+3}$$

$$= \frac{104}{3} = 17.3 \text{ g/mole}$$

21(c)

22(c) $\sigma = \pi d^2 = 4\pi\,(1.37)^2\times10^{-20}\,m^2$

$= 23.6\times10^{-20}\,m^2$

23(a) $f = \pi d^2 n\,v$

$= n\,v\,\sigma$

$= (3\times10^{25}\,\text{m}^{-3})\times(1.69\times10^3\text{ms}^{-1})\times(23.6\times10^{-20}m^2)$

$= 1.2\times10^{10}\,\text{s}^{-1}$

24(c) Mean free path $\lambda = \dfrac{1}{\sigma n}$

$$= \frac{1}{(3\times10^{25}m^{-3})\times(23.6\times10^{-20}m^2)} = 1400 \text{ A°}$$

25(b) $\tau = f^{-1} = 8.3\times10^{-11}$ sec

26(a)

27(a) Here $a = \dfrac{27R^2}{64}\times\dfrac{T_C}{P_C}$

28(c)

29(b)

30(a)

31(a) $$C_{p(\text{mix})} = \frac{n_1C_{P_1} + n_2C_{P_2}}{n_1+n_2}$$

32(a)

33(b)

34(b)

35(c) $C_{vmix} = \frac{n_{Ne}C_{Ne} + n_{N2}C_{N2}}{n_{Ne} + n_{N2}} = \frac{\left(1\times\frac{3}{2} + 2\times\frac{5}{2}\right)R}{1+2}$

36(a)

37(b)

38 (a)

39(d) Temperature of mixture

$$T = \frac{n_1T_1 + n_2T_2}{n_1 + n_2} = \frac{1\times300 + 2\times100}{1+2} = 167\,\text{K}$$

40(a)

41(b) Work done = area enclosed by cyclic process

$= \pi R \times R$

$= -\pi \times (100\times10^{-6}\,\text{m}^3) \times (100\times10^3\,Pa)$

$= -31.4\,\text{J}$

42(a) Specific heat in polytropic process PV^n = const is

$$C = Cv - \frac{R}{n-1} = Cv + \frac{R}{2}$$

43(b)

44(b) $nC_p\Delta T = Q$

$nR\ \Delta T = Q/4$

$C_p = 4R$

45(a) $nC_v\ \Delta T = 3\,Q/4$

$\Rightarrow \quad C_v = 3R$

46(b) $T_1^{\gamma}P_1^{1-\gamma} = T_2^{\gamma}P_2^{1-\gamma}$ or $T_2 = T_1\left(\frac{P_1}{P_2}\right)^{\frac{1-\gamma}{\gamma}}$

47(b)

48(b)

49(b)

50(b) $P = N/V\,KT$

here $N = \frac{5}{2}kT = \frac{5\times1.38\times10^{-23}\times300}{2\times1.6+10^{-19}}$

$$= \frac{3\times10^{+22}\times1.38\times10^{-23}\times300}{10^{-3}}$$

QUESTIONS FROM COMPETITIVE EXAMINATIONS

AIEEE 2002

1. Heat given to a body which raise its temperature by 1°C is:

(a) water equivalent (b) thermal capacity
(c) specific heat (d) temperature gradient

2. Infrared radiations are detected by:

(a) spectrometer (b) pyrometer
(c) nanometer (d) photometer

3. Which of the following is more close to a black body?

(a) Black board paint (b) Green leaves
(c) Black holes (d) Red roses

4. Which statement is incorrect?

(a) All reversible cycles have same efficiency
(b) Reversible cycle has more efficiency than an irreversible one
(c) Carnot cycle is a reversible one
(d) Carnot cycle has the maximum efficiency in all cycles

5. Cooking gas containers are kept in a lorry moving with uniform speed. The temperature of the gas molecules inside will:

(a) increase (b) decrease
(c) remain same
(d) decrease for same, while increase for others

6. If mass-energy equivalence is taken into account, when water is cooled to form ice, the mass of water should:

(a) increase (b) remain unchanged
(c) decrease
(d) first increase then decrease

7. At what temperature is the rms velocity of a hydrogen molecule equal to that of an oxgyen molecule at 47°C?
(a) 80 K (b) – 73 K
(c) 3 K (d) 20 K

8. Even Carnot engine cannot give 100% efficiency because we cannot:
(a) prevent radiation (b) find ideal sources
(c) reach absolute zero temperature
(d) eliminate friction

9. 1 mole of a gas with $\gamma = 7/5$ is mixed with 1 mole of a gas with $\gamma = 5/3$, then the value of γ for resulting mixture is:

(a) 7/5 (b) 2/5
(c) 24/16 (d) 12/7

10. Two sphere of the same material have radii 1m and 4 m and temperature 4000 K and 2000 K respectively. The ratio of the energy radiated per second by the first sphere to that the second is:

(a) 1 : 1 (b) 16 : 1
(c) 4 : 1 (d) 1 : 9

Answers

1.	(b)	**2.**	(b)	**3.**	(a)	**4.**	(a)	**5.**	(c)	**6.**	(a)	**7.**	(d)
8.	(c)	**9.**	(c)	**10.**	(a)								

Explanations

1.(b) As we know that thermal capacity of a substance is defined as the amount of heat required to raise its temperature by 1°C.

2.(b) Pyrometer is based on radiation theory.

3.(a)

4.(a) Efficiency of all reversible cycles depends upon temperature of source and sink which will be different.

5.(c) Temperature of a gas is determined by the total translational K.E. measured with respect to the centre of mass of the gas. Therefore, the motion of centre of mass of the gas does not affect the temperature. Hence, the temperature of gas will remain same.

6.(a) According to the mass-energy equivalence, mass and energy remain conserved. So, when water is cooled to form ice, water loses its energy so, change in energy increase the mass of water.

7.(d) $\sqrt{\dfrac{\gamma R(273+47)}{32\times10^{-3}}} = \sqrt{\dfrac{\gamma RT}{2\times10^{-3}}}$

or $T = 20\,k$

8.(c) The efficiency of Carnot's engine is,

$$\eta = 1 - \frac{T_2}{T_1} = 1 - \frac{Q_2}{Q_1}$$

for $\eta = 1$, Q_2 be zero

or T_2 be OK which is not possible according to 2nd law of thermodynamics.

9.(c) Using the relation

$$\frac{n_1 + n_2}{\gamma - 1} = \frac{n_1}{\gamma_1 - 1} + \frac{n_2}{\gamma_2 - 1}$$

$$\Rightarrow \quad \frac{1+1}{\gamma - 1} = \frac{1}{\left(\frac{5}{3} - 1\right)} + \frac{1}{\left(\frac{7}{5} - 1\right)}$$

$$\frac{2}{\gamma - 1} = \frac{3}{2} + \frac{5}{2}$$

$$\frac{2}{\gamma - 1} = 4$$

$$\gamma = \frac{3}{2} \qquad \therefore \gamma = \frac{24}{16}$$

10.(a) Energy radiated per second by a body which has surface area A at temperature T is given by Stefan's law,

$$E = \sigma A T^4$$

Therefore, $\frac{E_1}{E_2} = \left(\frac{r_1}{r_2}\right)^2 \left(\frac{T_1}{T_2}\right)^4 = \left(\frac{1}{4}\right)^2 \left(\frac{4000}{2000}\right)^4 = \frac{1}{1}$

AIEEE 2003

1. "Heat cannot by itself flow from a body at lower temperature to a body at higher temperature" is a statement or consequence of:

(a) second law of thermodynamics
(b) conservation of momentum
(c) conservation of mass
(d) first law of thermodynamics

2. During an adiabatic process, the pressure of a gas is found to be proportional to the cube of its absolute temperature. The ratio C_P/C_V for the gas is:

(a) 4/3 (b) 2
(c) 5/3 (d) 3/2

3. Which of the following parameters does not characterize the thermodynamic state of matter?

(a) Temperature (b) Pressure
(c) Work (d) Volume

4. A Carnot engine takes 3×10^6 cal of heat from a reservoir at 627°C and gives it to a sink at 27°C. The work done by the engine is:

(a) 4.2×10^6 (b) 8.4×10^6
(c) 16.8×10^6 (d) zero

5. The earth radiates in the infrared region of the spectrum. The spectrum is correctly given by:

(a) Rayleigh Jeans law (b) Planck's law of radiation
(c) Stefan's law of radiation
(d) Wien's law

6. According to Newton's law of cooling, the rate of cooling of a body is proportional to $(\Delta\theta)^n$ where $\Delta\theta$ is the difference of the temperature of the body and the surroundings, and n is equal to:

(a) two (b) three
(c) four (d) one

Answers

1. (a)	**2.** (d)	**3.** (c)	**4.** (b)	**5.** (a)	**6.** (d)

Explanations

1.(a) Heat cannot flow itself from a lower temperature to a body of higher temperature. This corresponds to seconds law of thermodynamics.

2.(d) Given: $P \propto T^3$ ...(i)

In a diabatic process

$T^{\gamma} P^{1-\gamma}$ = constant

$T^{(\gamma/\gamma-1)} \propto P$ ...(ii)

Comparing eqs. (i) and (ii), we get

$$\therefore \quad \frac{\gamma}{\gamma - 1} = 3$$

$$3\gamma - 3 = \gamma$$

or $2\gamma = 3$

or $\gamma = \frac{3}{2}$

3.(c) Work does not characterise the thermodynamic state of matter, it is a path function gives only relationship between two quantities.

4.(b) $T_1 = 627 + 273 = 900$ K

$Q_1 = 3 \times 10^6$ cal

$T_2 = 27 + 273 = 300$ K

$\therefore \quad \frac{Q_1}{T_1} = \frac{Q_2}{T_2}$

$Q_2 = \frac{T_2}{T_1} \times Q_1 = \frac{300}{900} \times 2 \times 10^6$

$= 1 \times 10^6$ cal

Work done $= Q_1 - Q_2$

$= 3 \times 10^6 - 1 \times 10^6$

$= 2 \times 10^6$ cal

$= 2 \times 4.2 \times 10^6 \text{ J} = 8.4 \times 10^6 \text{ J}$

5.(a)

6.(d) According to Newton's law of cooling

$$\frac{dQ}{dt} \propto \Delta\theta$$

Thus $n = 1$

AIEEE 2004

1. One mole of ideal monatomic gas ($\gamma = 5/3$) is mixed with one mole of diatomic gas ($\gamma = 7/5$). What is γ for the mixture? γ denotes the ratio of specific heat at constant pressure, to that at constant volume.

(a) 3/2 (b) 23/15
(c) 35/23 (d) 4/3

2. If the temperature of the sun were to increase from T to $2T$ and its radius from R to $2R$, then the ratio of the radiant energy received on earth to what it was previously, will be:

(a) 4 (b) 16
(c) 32 (d) 64

3. Which of the following statements is correct for any thermodynamic system?

(a) the internal energy changes in all processes
(b) Internal energy and entropy are state functions
(c) The change in entropy can never be zero
(d) The work done in an adiabatic process is always zero

4. Two thermally insulated vessels 1 and 2 are filled with air at temperatures (T_1, T_2), volume (V_1, V_2) and pressure (P_1, P_2) respectively. If the valve joining the two vessles is opened, the temperature inside the vessel at equilibrium will be:

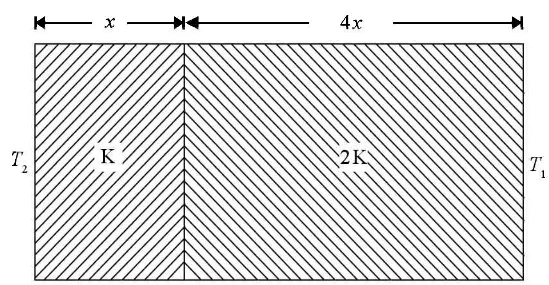
(a) $T_1 + T_2$ (b) $(T_1 + T_2)/2$

(c) $\frac{T_1T_2(P_1V_1 + P_2V_2)}{P_1V_1T_2 + P_2V_2T_1}$ (d) $\frac{T_1T_2(P_1V_1 + P_2V_2)}{P_1V_1T_1 + P_2V_2V_2}$

5. A radiation of energy E falls normally on a perfectly reflecting surface. The momentum transferred to the surface is:

(a) E/c (b) $2E/c$
(c) Ec (d) E/c^2

6. The temperature of the two outer surfaces of a composite slab, consisting of two materials having coefficients of thermal conductivity K and 2K and thickness x and 4x, respectively are T2 and T1 (T2 >T1). The rate of heat transfer through the slab, in steady state is $\left(\frac{A(T2 - T1)K}{x}\right)f$, with f equals to:



(a) 1 (b) 1/2
(c) 2/3 (d) 1/3

7. Time taken by a 836 W heater to heat one litre of water from 10°C to 40°C is:

(a) 50 s (b) 100 s
(c) 150 s (d) 200 s

Answers

1. (a)	**2.** (d)	**3.** (b)	**4.** (c)	**5.** (b)	**6.** (d)	**7.** (c)

Explanations

1.(a) $C_{V1} = \frac{3}{2}R$

$C_{V2} = \frac{5}{2}R$

$C_V = \frac{n_1C_{V1} + n_2C_{V2}}{n_1 + n_2}$

$= \frac{\frac{3}{2}R + \frac{5}{2}R}{2} = 2R$

$C_P = C_V + R = 3R$

$\gamma = \frac{C_P}{C_V} = \frac{3}{2}R$

2.(d) $\frac{E_2}{E_1} = \left(\frac{2R}{R}\right)^2\left(\frac{2T}{T}\right)^4 = 64$

3.(b) In thermodynamic system, entropy and internal energy are state functions.

4.(c) There will be no change in number of moles if the vessels are joined by valve. Therefore, from gas equation

$P_V = nRT$

$\Rightarrow \frac{P_1V_1}{RT_1} + \frac{P_2V_2}{RT_2} = \frac{P(V_1+V_2)}{RT}$

$\Rightarrow \frac{P_1V_1T_2 + P_2V_2T_1}{T_1T_2} = \frac{P(V_1+V_2)}{T}$

$\Rightarrow T = \frac{P(V_1+V_2)T_1T_2}{(P_1V_1T_2 + P_2V_2T_1)}$

Now, according to Boyle's law (pressure = constant)

$P_1V_1 + P_2V_2 = P(V_1+V_2)$

Hence, $T = \frac{(P_1V_1 + P_2V_2)T_1T_2}{(P_1V_1T_2 + P_2V_2T_1)}$

5.(b) On a perfectly reflecting surface

$\Delta p = \frac{2E}{c}$

6. Let the temperature of common inner slab (surface) be T° C.

Rate of heat flow will be equal

$H_1 = H_2$

$\therefore \frac{2KA(T - T_1)}{4x} = \frac{KA(T_2 - T)}{x}$

$\Rightarrow \frac{(T - T_1)}{2} = T_2 - T$

$\Rightarrow T - T_1 = 2T_2 - 2T$

$\Rightarrow T = \frac{2T_2 - T_1}{3}$

Hence, heat flow from composite slab is

$H = \frac{KA(T_2 - T)}{x}$ [from eq.] ...(i)

$= \frac{KA}{x}\left(T_2 - \frac{2T_2 - T_1}{3}\right) = \frac{KA}{3x}(T_2 - T_1)$ ...(ii)

Comparing (ii) with $H = \left[\frac{A(T_2 - T_1)K}{x}\right]f$

$\Rightarrow f = \frac{1}{3}$

7. Let time taken in boiling the water by the heater is t sec. Then $Q = ms\,\Delta T$

$\frac{836}{4.2}t = 1 \times 1000\,(40° - 10°)$

$\frac{836}{4.2}t = 1000 \times 30$

$t = \frac{1000 \times 30 \times 4.2}{836} = 150$ sec

AIEEE 2005

1. Which of the following is incorrect regarding the first law of thermodynamics?

(a) It is not applicable to any cyclic process

(b) It is a restatement of the principle of conservation of energy

(c) It introduces the concept of the internal energy

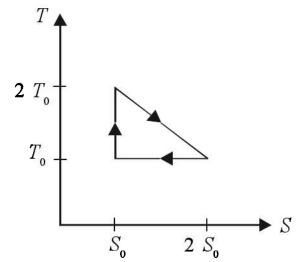
(d) It introduces the concept of the entropy

2. A gaseous mixture consists of 16 g of the helium and 16 g of oxygen. The ratio $\frac{C_P}{C_V}$ of the mixture is:

(a) 1.59 (b) 1.62

(c) 1.4 (d) 1.54

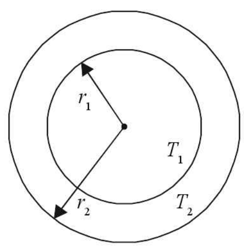
3. The temperature-entropy diagram of a reversible engine cycle is given in the figure. Its efficiency is:



(a) $\frac{1}{2}$ (b) $\frac{1}{4}$

(c) $\frac{1}{3}$ (d) $\frac{2}{3}$

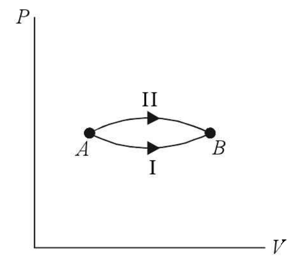
4. The figure shows a system of two concentric spheres of radii r_1 and r_2 and kept at temperatures T_1 and T_2, respectively. The radial rate of flow of heat in a subtance between the two concentric spheres is proportional to:



(a) $\frac{(r_2 - r_1)}{(r_1 r_2)}$ (b) $\ln\left(\frac{r_2}{r_1}\right)$

(c) $\frac{r_1 r_2}{(r_2 - r_1)}$ (d) $(r_2 - r_1)$

5. A system goes from A to B *via* two processes I and II as shown in figure. If Δu_1 and Δu_2 are the changes in internal energies in the processes I and II respectively, then:



(a) $\Delta u_1 < \Delta u_2$

(b) relation between Δu_1 and Δu_2 cannot be determined

(c) $\Delta u_2 > \Delta u_1$

(d) $\Delta u_2 = \Delta u_1$

Answers

1.	(a,d)	**2.**	(b)	**3.**	(c)	**4.**	(c)	**5.**	(d)

Explanations

1. (a,d) Statements (a) and (d) are wrong. Concept of entropy is associatd with second law of thermodynamics.

2. (b) $$C_v = \frac{n_1 C_{v_1} + n_2 C_{v_2}}{n_1 + n_2} = \frac{\frac{1}{2}\left(\frac{5}{2}R\right) + 4\left(\frac{3}{2}R\right)}{\frac{1}{2} + 4.0} = \frac{29R}{18}$$

$$C_P = C_V + R = \frac{47R}{18}$$

$$\gamma = \frac{C_P}{C_V} = \frac{47}{29} = 1.62$$

3. $$Q_1 = T_0 S_0 + \frac{1}{2} T_0 S_0$$

$$= \frac{3}{2} T_0 S_0$$

$$Q_2 = T_0 (2S_0 - S_0) = T_0 S_0$$

$$Q_3 = 0$$

$$\eta = \frac{W}{Q_1}$$

$$= \frac{Q_1 - Q_2}{Q_1} = 1 - \frac{Q_2}{Q_1}$$

$$= 1 - \frac{2}{3} = \frac{1}{3}$$

4. (c) $$\frac{dQ}{dT} = 4\text{pK}\, \frac{r_1 r_2 (T_1 - T_2)}{(r_2 - r_1)}$$

$$\therefore \frac{dQ}{dT} \propto \frac{r_1 r_2}{(r_2 - r_1)}$$

5. (d) The change in internal energy does not depend upon path followed by the process. It only depends on initial and final states.

Hence, $\Delta U_1 = \Delta U_2$.

AIEEE 2006

1. Assuming the sun to be a spherical body of radius R at a temperature of TK, evaluate the total radiant power, incident on earth, at a distace r from the sun:

(a) $4\ \pi r_0^2 R^2 \sigma T^4 / r^2$

(b) $\pi r_0^2 R^2 \sigma T^4 / r^2$

(c) $r_0^2 R^2 \sigma T^4 / 4r^2$

(d) $R^2 \sigma T^4 / r^2$

2. Two rigid boxes containing different ideal gases are placed on a table. Box A contains one mole of nitrogen at temperature T_0, while box B contains one mole of helium at temperature (7/3) T_0. The boxes are then put into thermal contact with each other, and heat flows between then until the gases reach a common final temperature (ignore the heat capacity of boxes). Then, the final temperature of the gases, T_f in terms of T_0 is:

(a) $T_f = \frac{3}{7} T_0$ (b) $T_f = \frac{7}{3} T_0$

(c) $T_f = \frac{3}{2} T_0$ (d) $T_f = \frac{5}{2} T_0$

3. The work of 146 kJ is performed in order to compress one kilo mole of a gas adiabatically and in this process the temperature of the gas increases by 7°C. The gas is: ($R = 8.37$ mol^{-1} K^{-1})

(a) diatomic
(b) triatomic
(c) a mixture of monotomic and diatomic
(d) monoatomic

Answers

1. (b) **2.** (c) **3.** (a)

Explanations

1.(b) From Stefan's law, the rate at which energy is radiated by sun at its surface is
[Sun is a perfectly black body as it emits radiations of all wavelength and so far it e = 1.]
The intensity of this power at earth's surface [under the assumption $r >> r_0$] is

$$I = \frac{P}{4\pi r^2}$$

$$= \frac{\sigma \times 4\pi R^2 T^2}{4\pi r^2}$$

$$= \frac{\sigma R^2 T^4}{r^2}$$

The area of earth which receives this energy is only one half of total surface area of earth, whose projection would be πr_0^2.

Total radiant power as received by earth

$$= \pi r_0^2 \times I$$

$$= \frac{\pi r_0^2 \times \sigma R^2 T^4}{r^2} = \frac{\pi r_0^2 R^2 \sigma T^4}{r^2}$$

2.(c) Here, change in internal energy of the system is zero. i.e., increase in internal energy of one equals decrease in internal energy of other.

$$\Delta U_A = 1 \times \frac{5R}{2}(T_f - T_0)$$

$$\Delta U_B = 1 \times \frac{3R}{2}\left(T_f - \frac{7}{3}T_0\right)$$

Now $\Delta U_A + \Delta U_B = 0$

$$\frac{5R}{2}(T_f - T_0) + \frac{3R}{2}\left(T_f - \frac{7}{3}T_0\right) = 0$$

$$\Rightarrow \quad 8T_f = 12T_0$$

$$\Rightarrow \quad T_f = \frac{12}{8}T_0$$

$$= \frac{3}{2}T_0$$

3.(a) For adiabatic process $dQ = 0$

So, $dU = -\Delta W$

$\Rightarrow$ $nC_v dT = +146 \times 10^3$ J

$$\Rightarrow \frac{nfR}{2} \times 7 = 146 \times 10^3$$

[$f \rightarrow$ Degree of freedom]

$$\Rightarrow \frac{10^3 \times f \times 8.3 \times 7}{2} = 146 \times 10^3 \text{ or } f = 5$$

AIIMS 2005

1. A perfect gas is found to obey the relation $PV^{\frac{3}{2}}$ = constant during an adiabatic process. If such a gas is initially at a temperature T, is compressed to half of its initial volume then its final temperature will be
   (a) $2T$ (b) $4T$
   (c) $(2)^{\frac{1}{2}}T$ (d) $2\,(2)^{\frac{1}{2}}$
2. In isothermal process, which of the following is not true?
   (a) Temperature remains constant
   (b) Internal energy does not change
   (c) No heat enters or leaves the system
   (d) None.
3. In heat engine sink is fitted at temperature 27°C and heat of 100 kcal is taken from source at temperature 677°C. Work done in $10^6 J$ is?
   (a) 0.28 (b) 2.8
   (c) 28 (d) 0.028.

Explanations

1. (c) $PV^{\frac{3}{2}}$ = constant

$$\frac{T}{V}V^{\frac{3}{2}} = \text{constant}$$

$$\therefore \quad TV^{\frac{1}{2}} = \text{constant.}$$

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\frac{1}{2}} = \left(\frac{2V}{V}\right)^{\frac{1}{2}} \Rightarrow T_2 = (2)^{\frac{1}{2}}T.$$

2. (c) In adiabatic system no heat enters or leaves the system.

3. (a) From second law of thermodynamics,

$$\frac{W}{Q_1} = 1 - \frac{T_2}{T_1} \quad \text{...(i)}$$

$$\text{Also, } \frac{Q_2}{Q_1} = \frac{T_2}{T_1} \text{ or, } Q_1 = \frac{Q_2 \times T_1}{T_2} \quad \text{...(ii)}$$

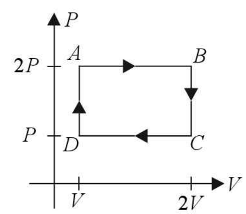
$$\text{Substituting in (i), } \frac{W \times T_2}{Q_2 \times T_1} = \frac{(T_1 - T_2)}{T_1}$$

$$\therefore \quad W = \frac{(T_1 - T_2)Q_2}{T_2} = \frac{(300 - 950)}{950} \times 100 \times 10^3 \times 4.2$$

[$\because$ 1 cal = 4.2 = – 287.3 × $10^3 J$ = – 0.287 × $10^6 J$.

KARNATAKA CET 2005

1. The wavelength of the radiation emitted by a body depends upon
   (a) the nature of the surface
   (b) the area of the surface
   (c) the temperature of the surface
   (d) all of the above factors.
2. An ideal monoatomic gas is taken around the cycle $ABCDA$ as shown in the PV diagram. The work done during the cycle is given by
   (a) $\frac{1}{2}PV$ (b) PV
   (c) $2\,PV$ (d) $4\,PV$



3. The ratio of velocity of sound in hydrogen and oxygen at STP is
   (a) 16 : 1 (b) 8 : 1
   (c) 4 : 1 (d) 2 : 1
4. Infrared radiation was discovered in 1800 by
   (a) William Wollaston (b) William Herschel
   (c) Wilhelm Roentgen (d) Thomas Young

Explanations

1. (c) According to Wien's displacement law, as the temperature of a black body increase, the maximum intensity of emission shifts (or is displaced) towards shorter wavelengths.

$\lambda_m T = b$ = constant

where λ_m is the wavelength at which maximum emission take place at absolute temperature T.

2. (b) Work done = area enclosed in the PV curve

= area of rectangle.

$= P \times V = PV$.

3.(c) $v = \sqrt{\frac{\gamma RT}{M}}$; g is the same as both hydrogen and oxygen are diatomic.

$$\therefore \quad \frac{v_H}{v_O} = \sqrt{\frac{Mo}{M_H}} = \sqrt{\frac{16}{1}} \Rightarrow v_H : v_O = 4 : 1.$$

4.(b) Infrared radiations were discovered in 1800 by William Herschel, a British musician and astronomer, when he observed that a thermometer placed just outside the visible spectrum of sunlight shows a greater increase in temperature than on placed in red region.

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4 Electrostatics and Electricity

18

Electrostatics

BRIEF REVIEW

Charge The fundamental property of matter with which it exerts coulomb force is called Charge. It is of two types, **Positive** and **Negative**. Like charges repel and unlike charges attract. A charged particle is capable to attract even an uncharged particle. GLE (Gold Leaf Electroscope) issued to detect charge. Charge is measured using electrometers. The unit of charge is **Coulomb**. The charge on electron or proton is termed as natural charge and is the minimum unit of charge which can be transferred from one body to another. $\frac{e}{m}$ is called specific charge. $e = 1.6 \times 10^{-19}$ C.

Charge is conserved (a) In an isolated system charge can neither be created nor destroyed (b) Total charge in the universe is constant (c) Charge can be created or destroyed but in equal and opposite pairs, for instance, a γ-ray can split to an electron and a positron, i.e.,

$$\text{If } E_\gamma (\geq 1.02 \text{ MeV}) \quad \gamma \rightarrow e^- + e^+$$

This process is called pair production. The electron and positron can combine to form γ-ray again. Such a process is termed as pair annihilation.

$$e^+ + e^- \longrightarrow \gamma (E_\gamma = 1.02 \text{ MeV})$$

Charge is quantised Charge on a body can be integral multiple of electronic charge. i.e. $Q = \pm ne$. If a body gains electrons, it is said to be negatively charged and if it loses electrons, it is said to be positively charged. Though there are particles called quarks which may have charge $\frac{e}{3}$ or $\frac{2e}{3}$, since these are generated during disintegration of nucleus (neutron, proton and so on) these cannot be transferred. Charge on an electron is 1.6×10^{-19} C.

$$1\ esu = \frac{1}{3 \times 10^9} \text{C}$$

and $$1\ emu = 10 \text{C}$$

A body can be charged by **rubbing**. For example, when glass rod and silk cloth are rubbed against each other, glass rod acquires positive charge and silk cloth, negative charge. We can also charge a body by **induction** and, by **physical contact** of an uncharged body with a charged body. A capacitor may be charged with a battery.

Coulomb's Law If two point charges q_1 and q_2 are distance r apart then force between two charges (see Fig. 18.1)

$$F \propto q_1 q_2$$

and $$F \propto \frac{1}{r^2}$$

i.e. $|F| = \frac{q_1 q_2}{4\pi\varepsilon_0 r^2}$ (in free space) where ε_0 is permittivity of free space.

$$|F| = \frac{q_1 q_2}{4\pi\varepsilon_0 \varepsilon_r r^2} = \frac{q_1 q_2}{4\pi\varepsilon_0 k r^2} \text{ (in a medium) where}$$

$\varepsilon_r = k = \frac{\varepsilon_m}{\varepsilon_0}$ is relative permittivity of the medium or dielectric constant of the medium.

Vector form of Coulomb law

$$\vec{F} = \frac{q_1 q_2 \vec{r}}{4\pi\varepsilon_0 r^3}$$

$$\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9 \text{Nm}^2\text{C}^{-2}$$

$$\varepsilon_0 = 8.85 \times 10^{-12} \text{C}^2\text{N}^{-1}\text{m}^{-2}$$

Note that ε_r *is dimensionless.*

Coulomb's law is valid if (i) charges are point charges or spherical charges (ii) distance r between the two charges ≥ 10^{-15} m.

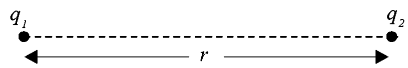


Fig. 18.1 Coulomb force

Dielectric is an insulator. It is of two types, polar and nonpolar. Polar dielectrics are those which have permanent dipole moment like water,

$$\varepsilon_r = 80\,(H_2O)$$

and $\quad \varepsilon_r = \infty$ (metals)

Electric field intensity or electric field strength is the force experienced by a unit positive charge at that point when placed in an electric field of the given charge. Its unit is N/C or Vm^{-1}.

$$|E| = \frac{Q}{4\pi\varepsilon_0 r^2} = \frac{|F|}{q}$$

In vector for m^2 $\vec{E} = \frac{Q\vec{r}}{4\pi\varepsilon_0 r^3} = \frac{\vec{F}}{q}$

Electric field vectors are of three types namely $\vec{E}$, $\vec{P}$ and $\vec{D}$

where $\vec{D} = \varepsilon_0\vec{E} + \vec{P}$

D is called electric displacement vector

$\vec{P} = \varepsilon_0 (K-1)\vec{E}$ is polarising vector. For vacuum $K = 1$ and $P = 0$

Electrets are the substances which do not follow

$$\vec{P} = \varepsilon_0 (K-1)\vec{E}$$

Further $E = \frac{-dv}{dx} = -\vec{\nabla} V$ (in 3-dimension)

where V is electric potential.

For equipotential surface $\boldsymbol{E = 0}$

Even work done to more from one point to another on equipotential surface = 0.

If charge is not a point charge then linear charge density $\lambda = \frac{Q}{l}$, surface charge density $\sigma = \frac{Q}{\text{Area}}$ or volume charge density $\rho = \frac{Q}{\text{Volume}}$ is determined. A small length dx (for linear charge density), a small area ds (for surface charge density) or a small volume dv (for volume charge density) is considered to find a point charge. Write the equation of electric field/force using the small element and integrate. Electric field and electric force obey superposition principle. Electric field/force is conservative.

Electric field lines or electric lines of force are imaginary lines originating from positive charge and terminating at negative charge, such that tangent at any point gives the direction of force. No two electric lines of force can intersect each other.

Electric Flux The lines of force passing through a given area in an electric field is called electric flux.

$\phi_E = \int \vec{E}.\vec{ds}$. If E and S are mutually perpendicular then $\phi_E = 0$. The unit of electric flux is Nm^2C^{-1} and dimensional formula is $[ML^3T^{-3}A^{-1}]$. It is a scalar quantity.

Electric Potential The amount of work done to bring unit positive charge from infinity to that point against the electric field of a given charge without changing its kinetic energy or velocity.

$$V = \int_{\infty}^{r} -E.dx = \frac{Q}{4\pi\varepsilon_0 r}$$

It is a scalar quantity and its unit is volt. 1 volt = $\frac{1J}{1C}$

Its dimensional formula is $[ML^3T^{-3}A^{-1}]$.

Potential Difference

$$\Delta V = V_2 - V_1$$

$$= \int_{r_1}^{r_2} -E.dr$$

$$= \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{r_2} - \frac{1}{r_1}\right]$$

Potential Energy

$$U = qV = \frac{Qq}{4\pi\varepsilon_0 r}$$

Equipotential Surface is the surface, where potential is equal at every point. For a point charge, a sphere will be equipotential surface with point charge at the centre of the sphere. Equipotential surface for a long line charge is a cylinder with line charge along its axis. Equipotential surface for a dipole is shown in Fig. 18.2.

The work done in carrying a charge from one point to another along an equipotential surface is zero.

The electric field lines are always perpendicular to the equipotential surface.

Every conductor (metal) is an equipotential surface and hence electric field lines will emerge perpendicular to it.

Electric field and surface charge density are maximum at pointed ends of a conductor.

Note $\oint \vec{E}.\vec{dl} = 0$

and $\int_a^b \vec{E}.\vec{dl} = V_a - V_b$

$$\vec{E} = -\vec{\nabla}V$$

$$= -\left(\hat{i}\frac{\partial V}{\partial x}+\hat{j}\frac{\partial V}{\partial y}+\hat{k}\frac{\partial V}{\partial z}\right)$$

$$= -\left(\hat{i}\frac{\partial}{\partial X}+\hat{j}\frac{\partial}{\partial Y}+\hat{k}\frac{\partial}{\partial Z}\right)V$$

i.e. $\vec{\nabla} = \hat{i}\frac{\partial}{\partial X}+\hat{j}\frac{\partial}{\partial Y}+\hat{k}\frac{\partial}{\partial Z}$ is called gradient operator and is written as grad or del.

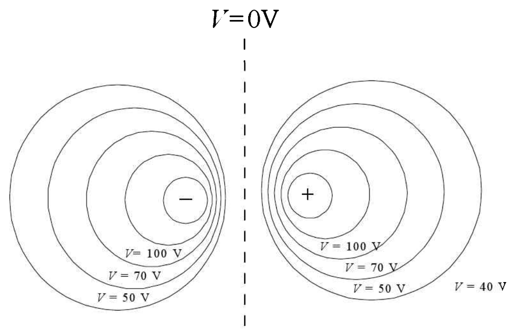


Fig. 18.2 Equipotential surface illustration for dipole

Electric field intensity due to a shell (spherical) having charge Q and radius R

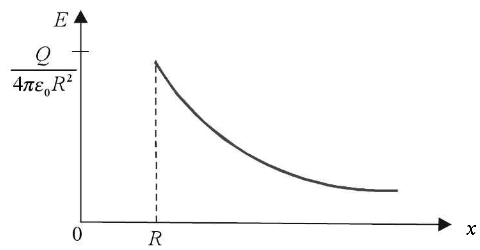


Fig. 18.3 Electric field due to shell

$$E_{\text{inside}} = 0 \qquad x < R$$

$$E_{\text{surface}} = \frac{Q}{4\pi\varepsilon_0 R^2} \qquad x = R$$

$$E_{\text{outside}} = \frac{Q}{4\pi\varepsilon_0 x^2} \qquad x > R$$

Electric potential due to a spherical shell (radius *R*, charge *Q*)

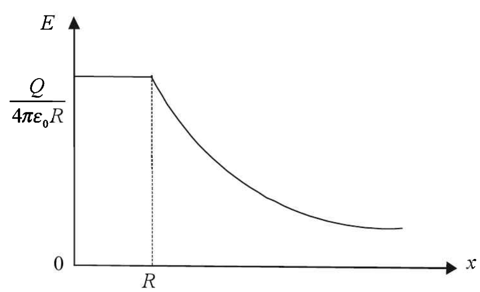


Fig. 18.4 Potential due to shell

$$V_{\text{inside}} = \frac{Q}{4\pi\varepsilon_0 R} = V_{\text{surface}} \qquad x \le R$$

$$V_{\text{out side}} = \frac{Q}{4\pi\varepsilon_\partial R} \qquad x > R$$

Electric field due to a finite line charge on perpendicular bisector

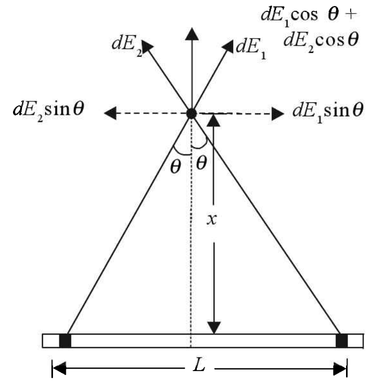


Fig. 18.5 Electric field due to a line charge along equatorial line

$$E = \frac{Q}{2\pi\varepsilon_0 x\sqrt{L^2+4a^2}}$$

Electric field intensity due to a ring of radius *R* at a distance *X* on the axial line

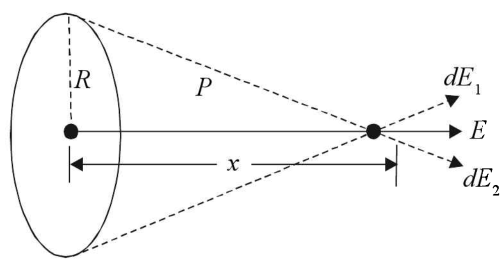


Fig. 18.6 Electric field due to ring

$$E = \frac{Qx}{4\pi\varepsilon_0\left(x^2+R^2\right)^{3/2}}$$

At the centre of the ring $E = 0$

Electric field is maximum at

$$x = \frac{R}{\sqrt{2}}$$

Electric Potential at any point *P* due to a ring on axial line

$$V = \frac{Q}{4\pi\varepsilon_0\sqrt{x^2+R^2}}$$

$$V\text{(centre of the ring)} = \frac{Q}{4\pi\varepsilon_0 R}$$

Electric field due to a disc of radius *R* having surface charge density σ at a point *P*, distant *x* on the axial line

$$E = \frac{\sigma}{2\varepsilon_0}\left[1-\frac{x}{\sqrt{x^2+R^2}}\right]$$

If $x \to 0$, i.e., at the centre of the disc, $E = \frac{\sigma}{2\varepsilon_0}$

Also $E = \frac{\sigma}{2\varepsilon_0}$ if $R \to \infty$, i.e., due to a long disc.

Electric potential *V* at any point *P* due to the disc along axial line

$$V = \frac{\sigma}{2\varepsilon_0}\left[\sqrt{x^2+R^2}-x\right]$$

Dipole Moment $\vec{p} = q(2l)$. The direction of electric dipole moment $\vec{p}$ is from negative towards positive charge as shown in Fig. 18.7.

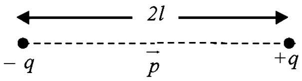


Fig. 18.7 Dipole

Electric field intensity due to a dipole

(a) Along axial line

$$E_{\text{axial}} = \frac{2px}{4\pi\varepsilon_0(x^2 - l^2)^2} \text{ (See Fig 18.8)}$$

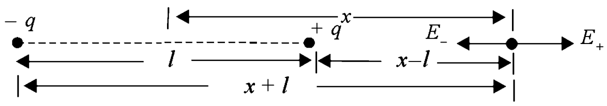


Fig. 18.8 Electric field due to a dipole along axial line

for a short dipole $x >> l$

$$E_{\text{axial}} = \frac{2p}{4\pi\varepsilon_0 x^3}$$

Note the direction of electric field is parallel to electric dipole moment.

Electric potential along axial line

$$V_{\text{axial}} = \frac{p}{4\pi\varepsilon_0(x^2 - l^2)}$$

$$V_{\text{axial}} = \frac{p}{4\pi\varepsilon_0 x^2} \quad \text{due to a short dipole.}$$

(b) Electric field along equatorial line

$$E_{\text{equatorial}} = \frac{p}{4\pi\varepsilon_0(x^2 + l^2)^{3/2}}$$

Note that the direction of electric field is antiparallel to dipole movement as shown in Fig. 18.9

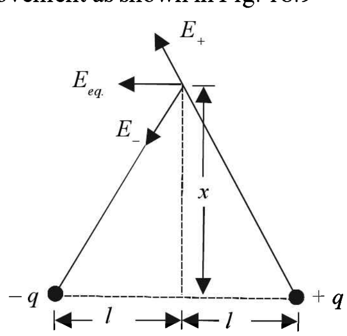


Fig. 18.9 Electric field along equational line

$$E_{\text{equatorial}} = \frac{p}{4\pi\varepsilon_0 x^3} \quad \text{due to a short dipole}$$

Electric potential at any point along equatorial line

$$V_{\text{equatorial}} = 0$$

(c) Electric field due to a short dipole at any point P

$$E_{\text{any point}} = \frac{p}{4\pi\varepsilon_0 x^3}\sqrt{3\cos^2\theta + 1} = \sqrt{E_x^2 + E_y^2}$$

and $\tan\beta = \dfrac{\tan\theta}{2}$

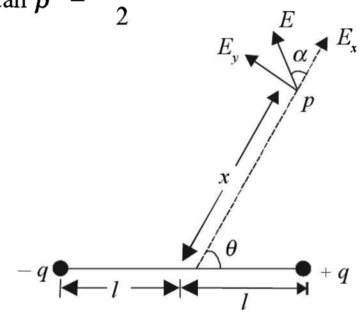


Fig.18.10 Electric field due to a dipole at any point

and $E_x = \dfrac{-\partial V}{\partial x}$

and $E_y = \dfrac{-\partial V}{\partial y} = \dfrac{-\partial V}{x\partial\theta}$

Special cases If $\theta = 0$, i.e., along axial line

$$E_{\text{axial}} = \frac{2p}{4\pi\varepsilon_0 x^3} \text{ due to a short dipole}$$

If $\theta = 90°$, i.e., along equaterial line $\cos 90 = 0$,

then $E_{\text{equaterial}} = \dfrac{p}{4\pi\varepsilon_0 x^3}$, due to a short dipole.

Electric potential due to a dipole at any point

$$V_{\text{any point}} = \frac{p\cos\theta}{4\pi\varepsilon_0(x^2 - l^2\cos^2)}$$

$$V_{\text{any point}} = \frac{p\cos\theta}{4\pi\varepsilon_0 x^2}, \text{ due to a short dipole.}$$

Torque experienced by a dipole when placed in a uniform electric field *E*

$\Sigma F = 0$, i.e., no linear motion is possible

$\vec{\tau} = \vec{p} \times \vec{E} = pE\sin\theta$

as illustrated in Fig. 18.11

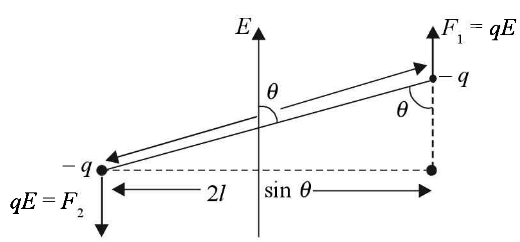


Fig. 18.11 Torque due to a dipole

If $\theta = 0$, $\tau = 0$, equilibrium is stable.

If $\theta = 90°$, $\tau = pE$ and is maximum.

If $\theta = 180°$, $\tau = 0$, equilibrium is unstable.

Work done $W = \int_{\theta_1}^{\theta_2} \tau \cdot d\theta = pE(\cos\theta_1 - \cos\theta_2)$

If $\theta_1 = 0, \theta_2 = 180°$ (i.e., dipole is reversed) when

$W = 2pE$

If $\theta_1 = 0, \theta_2 = 90°$ then

$W = pE$

Potential energy due to a dipole $U = -pE\cos\theta$

If electric field is non-uniform then $\Sigma F \neq 0$ and $\tau \neq 0$

$$\vec{F} = \vec{p} \times \frac{d\vec{E}}{dx}$$

Potential Energy (PE) It is the amount of work done to bring a charge q from infinity to that point against the electric field of a given charge Q without changing its KE.

$$\text{PE } U = \frac{qQ}{4\pi\varepsilon_0 r} = qV$$

Since the electrostatic force is conservative, therefore work done $W = \Delta\text{PE}$

$$\text{or} \quad W = U_f - U_i = \frac{Qq}{4\pi\varepsilon_0}\left[\frac{1}{r_{final}} - \frac{1}{r_{initial}}\right]$$

$$= q\left[V_{final} - V_{initial}\right]$$

Force on a charged surface The repulsive force acting on an element due to rest of the charged surface is called electric force on a charged conducting surface as illustrated in Fig. 18.12

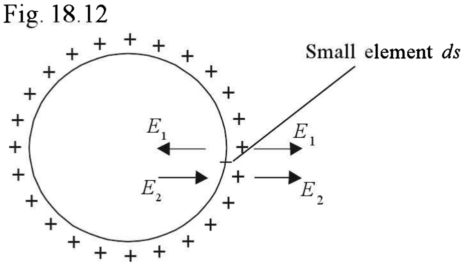


Fig. 18.12 Force on a charged surface

Outside $E = E_1 + E_2 = \frac{\sigma}{\varepsilon_0}$ (super position principle)

Inside $E = E_1 - E_2 = 0$

Note that electric field intensity due to a small element is equal to electric field intensity due to rest of the surface.

Hence $E = \frac{\sigma}{2\varepsilon_0}$ near a charged surface

and force $dF = \frac{\sigma^2}{2\varepsilon_0} ds$ or

$$F = \int \frac{\sigma^2}{2\varepsilon_0} ds$$

$$\text{Electric pressure } P = \frac{\text{Force}}{\text{Area}} = \frac{dF}{ds} = \frac{\sigma^2}{2\varepsilon_0}$$

In case of a soap bubble

$$P_{in} - P_{out} = P_{excess} = P_{ST} - P_{elect}$$

$$= \frac{4T}{r} - \frac{q^2}{2A^2\varepsilon_0}$$

$$= \frac{4T}{r} - \frac{q^2}{2(4\pi r^2)^2\varepsilon_0} = \frac{4T}{r} - \frac{q^2}{32\pi^2 r^4 \varepsilon_0}$$

In case of equilibrium $P_{in} = P_{out} \Rightarrow \frac{4T}{r} = \frac{q^2}{32\pi^2 r^4\varepsilon_0}$

Electric field intensity on soap bubble to maintain equilibrium

$$E = \sqrt{\frac{8T}{r\varepsilon_0}}$$

and electric potential to maintain equilibrium

$$V = \sqrt{\frac{8Tr}{\varepsilon_0}}$$

Energy associated with electric field

$$U = \frac{1}{2\varepsilon_0}\int \sigma^2 dV$$

$= \frac{\varepsilon_0}{2}\int E^2 dV$ = where V is volume of the whole field

Energy density $u = \frac{U}{V} = \frac{\varepsilon_0 E^2}{2} = \frac{\sigma^2}{2\varepsilon_0}$

Charged liquid drop If n identical drops each of radius r and charge q join to form a big drop of radius R and charge Q then

$$R = n^{1/3} r \,;\, Q_{big} = nq_{small}$$

$$E_{big} = n^{1/3} E_{small} \,;\, V_{big} = n^{2/3} V_{small}$$

$$\sigma_{big} = \sigma_{small} n^{1/3}$$

If a charged drop is in equilibrium in a given electric field then $qE = mg$ as shown in Fig. 18.13

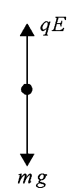


Fig.18.13 Equilibrium of charged particle

$$\text{or} \quad E = \frac{mg}{q}$$

Equilibrium is said to be stable if $\Sigma F = 0$ and PE U = minimum. This is feasible if at extreme ends charges are similar and in between (where equilibrium is found) charge is opposite in nature as shown in Fig. 18.14

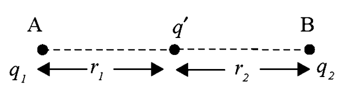


Fig. 18.14 Stable equilibrium

Thus for stable equilibrium $\Sigma F = 0$ at A, B or C

For charge q′ to be in equilibrium $\frac{q_1}{q_2} = \frac{r_1^2}{r_2^2}$ or $\frac{r_1}{r_2} = \sqrt{\frac{q_1}{q_2}}$

For q_2 to be in equilibrium $\frac{q'}{q_1} = \frac{r_2^2}{(r_1+r_2)^2}$ or $\sqrt{\frac{q'}{q_1}} = \frac{r_2^2}{(r_1+r_2)^2}$

A particle in stable equilibrium will execute SHM if disturbed slightly along x or y direction. However, if disturbance x is large, motion is oscillatory but not SHM.

For unstable equilibrium $\Sigma F = 0$ and $PE\ U$ = maximum.

This is possible if all charges are similar. Thus for q′ to be in equilibrium

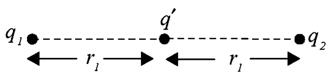


Fig. 18.15

$$\frac{q_1}{q_2} = \frac{r_1^2}{r_2^2} \text{ or } \sqrt{\frac{q_1}{q_2}} = \frac{r_1}{r_2}$$

Note in this case equilibrium cannot occur at q_1 and q_2. Moreover, particle will not execute *SHM* if slightly disturbed from its equilibrium position. Rather, it may move linearly.

Charged particle in motion

Force $F = qE$

$\therefore \quad ma = qE$ or

$$a = \frac{qE}{m}$$

Velocity v after travelling a distance d using $v^2 = 2ad$ is

$$v = \sqrt{\frac{2qEd}{m}}$$

Velocity after time t if it starts from rest $v = at = \frac{qEt}{m}$

Remember $\oint \vec{E} \cdot d\vec{s} = \frac{q}{\varepsilon_0}$

and $\oint \vec{D} \cdot d\vec{s} = q$

If a point/shell is grounded it means potential $V = 0$ but q may not be zero.

$$\frac{F_E}{F_G} = \frac{F_{electrostatic}}{F_{gravitational}} = 10^{39}$$

Unit of dipole moment is Debye in atomic scale

1 Debye = 3.3×10^{-33} C – m

For a single charge $E \propto r^{-2}$, $V \propto r^{-1}$

For a dipole $E \propto r^{-3}$; $V \propto r^{-2}$

For a quadrupole $E \propto r^{-4}$, $V \propto r^{-3}$

• Short Cuts and Points to Note

1. Coulomb force $\vec{F} = \frac{q_1 q_2 \vec{r}}{4\pi\varepsilon_0 r^3}$ or $|F| = \frac{q_1 q_2}{4\pi\varepsilon_0 r^2}$ is applicable in free space or vacuum only if (a) charges are point charges or spherical charges (b) separation between the charges > 10^{-15}m. If the charge is distributed, make a point charge by considering a small element and linear charge density λ (if charge is linear), surface charge density σ (if charge is spread on area) and volume charge density ρ (if charge is distributed throughout the volume).
2. Normally force is mutual i.e. $F_{12} = -F_{21}$. In certain cases Newton's 3rd law may not be valid. For example, if a charge q_1 is placed in the shell and q_2 lies outside at a distance r from q_1 as shown in Fig. 18.16

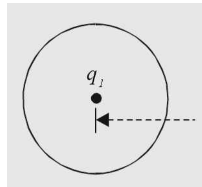


Fig. 18.16 Illustration of Newton's third law failure

then force due to q_1 on q_2 is non zero while force due to q_2 and q_1 is zero.

3. In a medium $F = \frac{q_1 q_2}{4\pi\varepsilon_0\varepsilon_r r^2} = \frac{q_1 q_2}{4\pi\varepsilon_0 k r^2}$

   where $\varepsilon_r = k$ is dielectric constant.
4. If there is more than one medium as shown in Fig. 18.17 where a dielectric slab of thickness t and dielectric constant k has been added in between two charges q_1 and q_2 separated by r. To solve such problems, find equivalent distance in vacuum. In the given problem equivalent distance in vacuum is $t\sqrt{k}$. Thus, net distance between the charges will be $\quad r - t + t\sqrt{k}$ or

   Force $\quad F = \frac{q_1 q_2}{4\pi\varepsilon_0 (r - t + t\sqrt{k})^2}$

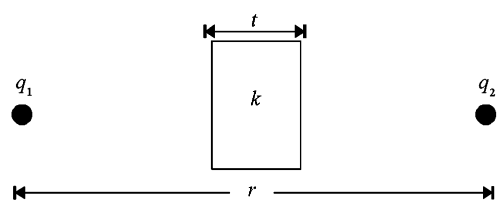


Fig. 18.17 Finding force between charges in more than one medium

Note that effective distance in vacuum for a dielectric of thickness t and dielectric constant k is

$t\sqrt{k}$ *i.e.* $t_{\text{eff}} = t\sqrt{k}$

5. The electric field intensity or electric force is a vector quantity. Therefore exploit vector algebra to solve the problems.
6. Electric field intensity due to a point charge Q at a distance r form it is $E = \frac{F}{q} = \frac{Q}{4\pi\varepsilon_0 r^2}$
7. Electric field intensity inside a hollow conducting body is zero irrespective of its shape.

   Gauss's Law $\oint E.ds = \frac{q}{\varepsilon_0}$

   Gauss's Law in differential form $\frac{\partial E}{\partial x} = \frac{\rho}{\varepsilon_0}$

8. Electric field intensity due to a shell (spherical) is

$E_{inside} = 0, E_{surface} = \frac{Q}{4\pi\varepsilon_0 R^2}$ and

$E_{outside} = \frac{Q}{4\pi\varepsilon_0 x^2}$

9. Electric field intensity due to a dipole $E \propto \frac{1}{x^3}$

$E_{axial} = \frac{2px}{4\pi\varepsilon_0 (x^2 - l^2)^2}$ and

$E_{axial} = \frac{2p}{4\pi\varepsilon_0 x^3}$ due to a short dipole.

E_{axial} is parallel to dipole moment.

$E_{equatorial} = \frac{p}{4\pi\varepsilon_0 (x^2 + l^2)^{\frac{3}{2}}}$ and for a short dipole

$E_{equartorial} = \frac{2p}{4\pi\varepsilon_0 x^3}$. $E_{equartorial}$ is antiparallel to dipole moment.

$E_{any\ point} = \frac{p}{4\pi\varepsilon_0 x^3}\sqrt{3\cos^2\theta + 1}$ and $\tan\alpha = \frac{\tan\theta}{2}$ gives the direction.

10. Electric field intensity due to a ring at any point on axial line.

$E_{ring} = \frac{Qx}{4\pi\varepsilon_0 (x^2 + R^2)^{3/2}}$. It is maximum when $x = \frac{R}{\sqrt{2}}$

$E_{ring} = 0$ at the centre of the ring.

11. Electric field due to a disc of radius R, along axial line is

$E_{disc} = \frac{\sigma}{2\varepsilon_0}\left[1 - \frac{x}{\sqrt{x^2 + R^2}}\right], E_{centre} = \frac{\sigma}{2\varepsilon_0}, E = \frac{\sigma}{2\varepsilon_0}$ if $R \to \infty$

12. Electric field due to a finite line charge at any point on its perpendicular bisector $E = \frac{Q}{2\pi\varepsilon_0 x\sqrt{L^2 + 4x^2}}$

13. If a dipole is suspended in a uniform Electric field then torque experienced by the dipole $\vec{\tau} = \vec{p} \times \vec{E}$ and $\sum F = 0$. Torque is maximum if $\theta = 90°$.

It is in stable equilibrium if $\theta = 0°$, and, it is in unstable equilibrium if $\theta = 180°$.

14. Work done by the dipole

$W = \int_{\theta_1}^{\theta_2} \tau . d\theta = pE(\cos\theta_1 - \cos\theta_2)$. Work done is maximum if angle of twist is 180°.

$pE(U) = -pE\cos\theta = -\vec{p}.\vec{E}$

15. If electric field is nonuniform then both torque and force act and force is given by $\vec{F} = \vec{p} \times \frac{d\vec{E}}{dx}$. Note that to balance a torque, a torque is needed and to balance a force, force is required. Hence force and torque are required to balance a dipole in a nonuniform field.

16. $V = \int -E.dl$ and $\oint E.dl = 0$ because electrostatic force is conservative .

17. If $|E_1| = |E_2|$ or $|F_1| = |F_2|$ then resolve the vector. We get magnitude and direction simultaneously. From Fig. 18.18

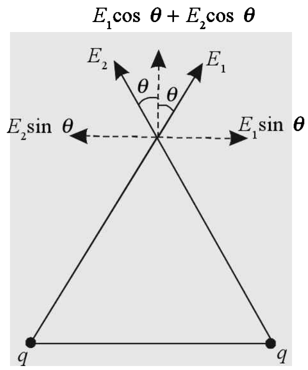


Fig. 18.18

$E = 2E_1\cos\theta$

18. $E = \frac{-dV}{dx}$ suggests E = 0, if V = maximum, V is minimum or V is constant.

19. It is possible to have $E = 0$ but $V \neq 0$ or vice versa. $E = 0, V \neq 0$ in a shell; $E \neq 0, V = 0$ along the equatorial line of a dipole. Moreover, if $Q = 0$ then $E = 0$ and $V = 0$.

20. A moving charge in a dielectric generates both electric and magnetic field. But current in a conductor generates only magnetic field. In a conductor $E_{inside} = 0$.

21. Electric potential $V = \int_{r_1}^{r_2} -E.dx = \frac{q}{4\pi\varepsilon_0 r}$

For a point charge potential difference

$\Delta V = \int_{r_1}^{r_2} -E.dx = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r_1} - \frac{1}{r_2}\right]$

For three dimensional electric field

$V = -\left[\int_\infty^x E_x.\partial x + \int_\infty^y E_y.\partial y + \int_\infty^z E_z.\partial z\right]$

22. Electric potential due to a shell

$V_{in} = V_{surface} = \frac{Q}{4\pi\varepsilon_0 R}, V_{out} = \frac{Q}{4\pi\varepsilon_0 x}$ $x > R$

23. Electric potential due to a dipole

$V_{axial} = \frac{p}{4\pi\varepsilon_0 (x^2 - l^2)}$;

$V_{\text{equatorial}} = 0$;

$V_{\text{any point}} = \dfrac{p\cos\theta}{4\pi\varepsilon_0 x^2}$ due to a short dipole.

24. Potential Energy $U = -\int_{\infty}^{r} F\cdot dx = \dfrac{Qq}{4\pi\varepsilon_0 r} = qV$

Work done = change in potential energy $W = \Delta PE$

$$= -\int_{r_1}^{r_2} F.dx = q\Delta V = \frac{Qq}{4\pi\varepsilon_0}\left[\frac{1}{r_2} - \frac{1}{r_1}\right] = q\,(V_2 - V_1)$$

25. For equipotential surface, work done $W = \Delta PE = 0$

In linear motion, however, gain in PE = loss in KE or vice versa.

26. Acceleration of charged particle in an electric field

E is $a = \dfrac{qE}{m}$. Apply equations

$v = v + at$, $v^2 - u^2 = 2\,as$ etc., if a is uniform.

If $a = \dfrac{qE}{m}$ is uniform and along y direction

Then $v_y = at = \dfrac{qEt}{m}$ and $u_{\text{net}} = \sqrt{u_x^2 + \left(\dfrac{qEt}{m}\right)^2}$ and

$\tan\theta = \dfrac{v_y}{v_x} = \dfrac{qEt}{mv_x}$

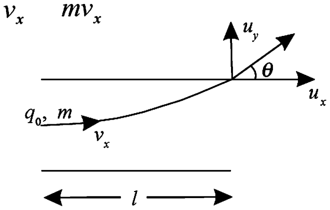


Fig. 18.19 Motion of a charged particle

For a charged particle projected in a limited electric field region $t = \dfrac{l}{v_x}$

27. If an opposite charge lies between two similar charges, equilibrium could be stable. If, however, all the charges are similar, equilibrium will be unstable.

28. If n drops, each of radius r and charge q, coalesce to form a big drop then $R_{\text{big}} = n^{1/3}r$, $Q_{\text{big}} = nq$,

$V_{\text{big}} = n^{2/3}V_{\text{small}}$,

$C_{\text{big}} = n^{1/3}C_{\text{small}}$ and $E_{\text{big}} = n^{1/3}E_{\text{small}}$

29. eV or electron volt is energy while volt is potential.

30. Two dipoles taken together having same charge and same separation between two charges as shown in Fig. 18.20 from a quadrupole. In a quadrupole $E \propto$

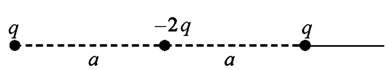


Fig. 18.20

$\dfrac{1}{r^4}$ and $V \propto \dfrac{1}{r^3}$

- **Caution**

1. Adding electric field intensity or force algebraically.

⇒ They are vectors and hence vector algebra be applied.

2. Considering potential is also a vector quantity.

⇒ $V = -\int \vec{E}\cdot d\vec{x}$ is a scalar. Therefore potential and PE be added alegebraically.

3. Not knowing how to get vector from scalars.

$E = -\dfrac{dV}{dr}$ as V is a scalar while E is electric field.

⇒ In one dimension $\vec{E} = -\dfrac{dV}{dx}\hat{i}$

In three dimensions $\vec{E} = -\vec{\nabla}V = -\left(\hat{i}\dfrac{\partial}{\partial x} + \hat{j}\dfrac{\partial}{\partial y} + \hat{k}\dfrac{\partial}{\partial z}\right)V$

4. Assuming electric field of one charge should affect the electric field of other charge when we have group of plates or group of charges.

⇒ We use superposition theorem. The effect of electric field of each plate or charge is considered individually on the given charge/test charge.

5. Ignoring the directions of velocities and accelerations. For instance, a charged particle is initially moving in x-direction with a velocity u but due to electric field, acceleration is developed in y-direction. Applying equation

$v = u + at$ or $s = ut + \dfrac{1}{2}at^2$ etc. is not correct.

⇒ Use $v = u\hat{i} + a_y t\hat{j}$ so that $|v| = \sqrt{u^2 + (a_y t)^2}$

and $\tan\beta = \left(\dfrac{a_y t}{u}\right)$

6. Assuming $E_{\text{inside}} = 0$ in all kinds of bodies.

⇒ $E_{\text{inside}} = 0$ in a cavity or hollow bodies.

$E_{\text{inside}} = \dfrac{Qx}{4\pi\varepsilon_0 R^3}$ in a uniformly charged sphere.

7. Assuming $V = 0$, if $E = 0$ because $V = -\int E.dx$

⇒ $V = 0$ if $E = 0$ and $Q = 0$, $V_{\text{inside}} = V_{\text{surface}} = \dfrac{Q}{4\pi\varepsilon_0 R}$ in a shell, though $E_{\text{inside}} = 0$. Interpreting $V = 0$ if $E = 0$ is superfluous. We come across cases when $V = 0$ but $E \neq 0$, for example, along equatorial line in a dipole.

8. Not recalling that work done on an equipotential surface is zero.

⇒ Since electrostatic force is conservative, $W = q\,(V_1 - V_2) = 0$ on equipotential surface.

Moreover, $\oint E.dl = 0$. However work may be done if charge moves from one equipotential surface to another equipotential surface.

9. Considering a small sphere or an end of a pin can hold a large charge.

⇒ Smaller the radius more is the surface charge density and hence very high electric field such that it surpasses the dielectric breakdown strength and hence charge leaks by means of carona discharge.

10. Considering equipotential surfaces can intersect.

⇒ Equipotential surfaces cannot intersect.

11. Considering a charged metal plate has uniformly distributed charge.

⇒ It has maximum charge density at the corners and minimum at flat portion.

12. Considering that a positively charged body has always positive potential.

⇒ It may have negative potential if placed in the electric field generated by strong negative charge.

13. The notion that similar charges only repel.

⇒ Though in principle it is correct but if one charge is very large as compared to other charge and they are place close to one another then they will attract. The reason being that there will be an induced charge (of opposite nature) in the body having a small charge.

14. Assuming work done is dependent on path followed.

⇒ Work done is independent of path followed as electrostatic force is conservative.

15. Considering that a charged particle must move along the electric field line.

⇒ Though $F = qE$ is the force present and acceleration is tangent to the field line, if particle was already in motion along some other direction then it will follow curved path.

16. Not knowing the directions of field lines and equipotential surfaces.

⇒ Field lines are always perpendicular to equipotential surfaces.

SOLVED PROBLEMS

1. An α-particle is travelling to its right with 1.5 kms^{-1}. What uniform magnetic field be applied so that it starts moving with same speed to its left after 2.65 μs?

(a) 2.35 N/C toward left (b) 235 NC^{-1} toward left
(c) 23.5 NC^{-1} toward left (d) 2.35 NC^{-1} toward left

Solution (c) $v = u + at$

$$-1.5_{km}\hat{i} = -1.5_{km}\hat{i} + a(2.65\times10^{-6}) \text{ or}$$

$$a = \frac{-3\times10^6\times10^3}{2.65}\hat{i} \text{ using } qE = ma$$

$$E = \frac{ma}{q} = -\frac{6.64\times10^{-27}\times3\times10^6\times10^3}{2\times1.6\times10^{-19}\times2.65}$$

$$= -23.5\,\hat{i}\ N/C$$

i.e. 23.5 N/C towards left.

2. A charge $+q$ is placed $(a, 0, 0)$ and another $+q$ charge is placed at $(-a, 0, 0)$

A charge $-q_1$ is placed at the origin. If it is slightly displaced along y axis. then

(a) it will move away

(b) it will oscillate but not SHM

(c) it will execute SHM

(d) it will stand at the displaced position.

Solution (c)

3. Assuming mass m of the charged particle, find time period of oscillation in Question 2.

(a) $2\pi\sqrt{\dfrac{q_1q}{4\pi\varepsilon_0 a^3 m}}$ (b) $2\pi\sqrt{\dfrac{4\pi\varepsilon_0 a^3 m}{q_1 q}}$

(c) $\dfrac{\pi}{2}\sqrt{\dfrac{q_1q}{4\pi\varepsilon_0 q^3 m}}$ (d) none of these

Solution (c) $E_0 = 2E_1\cos\theta = \dfrac{q}{4\pi\varepsilon_0(x^2+a^2)}\dfrac{x}{\sqrt{x^2+a^2}}$

$-q_1E = F = mf = \dfrac{-q_1qx}{4\pi\varepsilon_0(x^2+a^2)^{3/2}}$ as $x << a$, neglecting x^2 as compared to acceleration, $f = \dfrac{-q_1q_2x}{4\pi\varepsilon_0 a^3 m}$

Comparing it with $f = \omega^2 x$

$$\omega = \sqrt{\frac{q_1q}{4\pi\varepsilon_0 a^3 m}} \text{ or } T = 2\pi\sqrt{\frac{4\pi\varepsilon_0 a^3 m}{q_1q}}$$

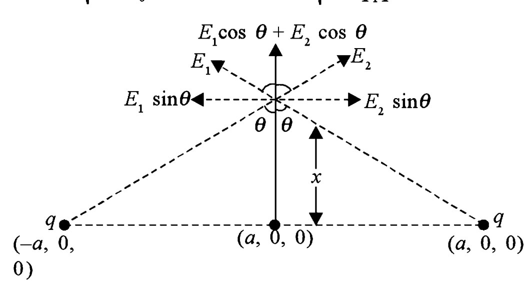


Fig. 18.21

4. As per diagram, a charge q is placed at the origin O. Work done by a charge $-Q$ in taking it from $A(0, a)$ to $B(a, 0)$ along the path AB **(CBSE 2005)**

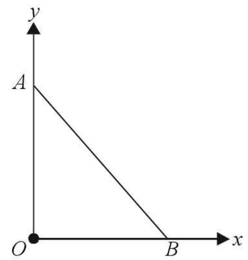


Fig. 18.22

(a) zero (b) $\sqrt{2}a\left(\dfrac{qQ}{4\pi\varepsilon_0 a^2}\right)$

(c) $\left(\dfrac{-qQ}{4\pi\varepsilon_0 a^2}\right)\sqrt{2}a$ (d) $\left(\dfrac{qQ}{4\pi\varepsilon_0 a^2}\right)\dfrac{a}{\sqrt{2}}$

Solution (a) A & B are at same potential

$\therefore W = 0$

5. Two charges q_1 and q_2 are placed 30 cm apart as shown in Fig. 18.23. Third charge q_3 is moved along the arc of a circle of radius 40 cm from C to D. The change in potential energy of the system is $\dfrac{q_3 k}{4\pi\varepsilon_0}$ where k is **(CBSE 2005)**

(a) $8q_1$ (b) $6q_1$
(c) $8q_2$ (d) $6q_2$

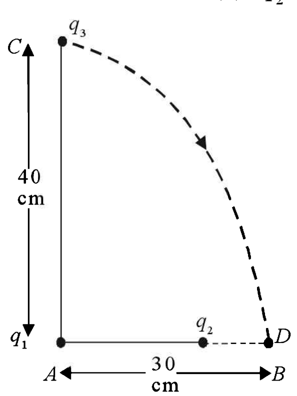


Fig. 18.23

Solution (c) $PE = \dfrac{q_2 q_3}{4\pi\varepsilon_0}\left[\dfrac{1}{.1} - \dfrac{1}{.5}\right] = \dfrac{8q_2 q_3}{4\pi\varepsilon_0}$

6. Two point charges $+8q$ and $-2q$ are located at $x = 0$ and $x = L$ respectively. The location of a point on the x-axis at which the net electric field due to these two point charges is zero **[AIEEE 2005]**

(a) $2L$ (b) $\dfrac{L}{4}$

(c) $8L$ (d) $4L$

Solution (a) $-\dfrac{2q}{4\pi\varepsilon_0(x-L)^2} + \dfrac{8q}{4\pi\varepsilon_0 x^2} = 0$ or $x = 2L$

7. Two thin wire rings each having a radius R are placed at distance d apart with their axes coinciding. The charges on the two rings are $+q$ and $-q$. The potential difference between the rings are **[AIEEE 2005]**

(a) $\dfrac{QR}{4\pi\varepsilon_0 d^2}$ (b) $\dfrac{Q}{2\pi\varepsilon_0}\left[\dfrac{1}{R} - \dfrac{1}{\sqrt{R^2+d^2}}\right]$

(c) $\dfrac{Q}{4\pi\varepsilon_0}\left[\dfrac{1}{R} - \dfrac{1}{\sqrt{R^2+d^2}}\right]$ (d) zero

Solution (b) $V_1 = \dfrac{Q}{4\pi\varepsilon_0}\left[\dfrac{1}{R} - \dfrac{1}{\sqrt{R^2+d^2}}\right]$ and

$V_2 = \dfrac{-Q}{4\pi\varepsilon_0}\left[\dfrac{1}{R} - \dfrac{1}{\sqrt{R^2+d^2}}\right]$

$\Delta V = V_1 - V_2$

8. A uniform electric field and a uniform magnetic field are acting along the same direction in a certain region. If an electron is projected along the direction of the fields with a certain velocity then **[AIEEE 2005]**

(a) its velocity decreases
(b) its velocity increases
(c) it will turn towards right of its motion
(d) it will turn towards left of direction of motion.

Solution (a) $F = -eE + e(\vec{v} \times \vec{B})$

$= -eE$ and $v = v_0 - \dfrac{eE}{m}t$

9. Four point positive charges of same magnitude Q are placed at the four corners of a rigid square frame as shown in Fig. 18.24

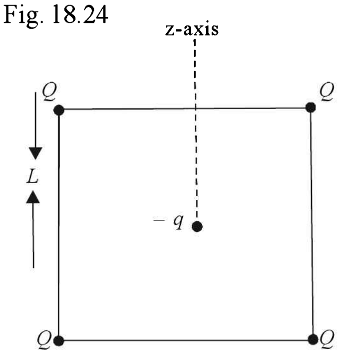


Fig. 18.24

The plane of the frame is perpendicular to z-axis. If a negative charge $-q$ is placed at a distance z away from the above frame ($z << L$) then

(a) negative charge oscillates along the z-axis
(b) it moves away from the frame
(c) it moves slowly towards the frame and stays in the plane of the frame.
(d) it passes through the frame only once.

[AIIMS 2005]

Solution (a) Because the resultant force acts as restoring force.

10. The work done in carrying a charge q once round a circle of radius r with a charge Q at the centre is

(a) $\dfrac{qQ}{4\pi\varepsilon_0 r}$ (b) $\dfrac{qQ}{4\pi\varepsilon_0^2 r^2}$

(c) $\dfrac{qQ}{4\pi\varepsilon_0 r^2}$ (d) none of these

(CET Karnataka 2005)

Solution (d) $W = 0$ $\because$ Electrostatic force is conservative.

11. Two small spheres each of mass m and charge q are tied from the same rigid support with the help of silk threads of length L. They make angle θ with the vertical as shown in the Fig. 18.25

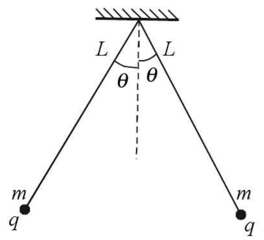


Fig. 18.25

If length L is decreased then angle θ with the vertical

(a) increases (b) decreases
(c) unaffected (d) cannot say

Solution (a) θ is related inversely to length L.

12. Two small spheres each of radius 1mm are kept 10 cm apart. Assuming each proton has a charge $+e$ and each electron has a charge 0.1% less than the $+e$ then find the force between the two spheres. Density of copper is 8.9 gcm^{-3} and atomic mass number is 63.5.

(a) 1.2×10^2N (b) 1.2×10^{-2}N
(c) 1.2×10^8N (d) 1.2×10^{14}N

Solution (d) mass of 1 mm radius sphere

$$= 8.9 \times \frac{4}{3}\pi(.1)^3$$

$$= 3.7 \times 10^{-2}\text{ g}$$

Charge on the sphere

$$\frac{3.7\times10^{-2}}{63.5} \times 6.023 \times 10^{23} \times 29 \times \frac{.1}{100} \times 1.6 \times 10^{-19}$$

$$= 1.61\, C$$

$$F = \frac{1.61\times1.61\times9\times10^9}{(0.1)^2}$$

$$= 2.34 \times 10^{12} N$$

13. Two thin rods of length L lie along x-axis, one between

$$x = \frac{a}{2} \text{ to } x = \frac{a}{2} + L$$

and the other between $x = -\dfrac{a}{2}$ to $x = -\dfrac{a}{2} - L$.

Each rod has positive charge Q distributed uniformly along the length. Find the magnitude of the force which one rod exerts on the other.

(a) $\dfrac{Q^2}{4\pi\varepsilon_0 L^2}\, log_e \dfrac{L+a}{L-a}$

(b) $\dfrac{Q^2}{4\pi\varepsilon_0 L^2}\, log_e \dfrac{(L+a)^2}{a(L-a)}$

(c) $\dfrac{Q^2}{4\pi\varepsilon_0 L^2}\, log_e \dfrac{(L+a)^2}{a(2L+a)}$

(d) $\dfrac{Q^2}{4\pi\varepsilon_0 L^2}\, log_e \dfrac{(L+a)^2}{L(2a+L)}$

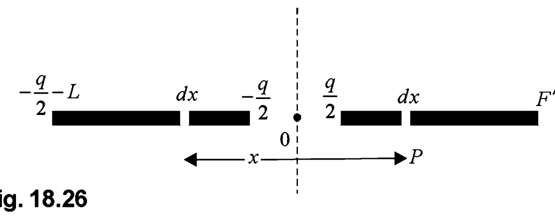


Fig. 18.26

Solution (c) Electric field at point P due to a small element dx of the rod on left side is $\int dE = \int \dfrac{Qdx}{L4\pi\varepsilon_0 x^2}$

$$= \frac{Q}{4\pi\varepsilon_0 L} \times \left[\frac{1}{x+\frac{a}{2}} - \frac{1}{x+\frac{a}{2}+L}\right]$$

Force exerted on a small element dx is

$$dF = \frac{QQ}{4\pi\varepsilon_0 L^2}\left[\frac{1}{x+\frac{a}{2}} - \frac{1}{x+\frac{a}{2}+L}\right]dx \quad \text{or}$$

$$F = \frac{Q^2}{4\pi\varepsilon_0 L^2}\left[\log_e x+\frac{a}{2}\Big|_{a/2}^{L+a/2} - \log_e x+\frac{a}{2}+L\Big|_{a/2}^{L+a/2}\right]$$

$$= \frac{Q^2}{4\pi\varepsilon_0 L^2}\left[\log_e \frac{L+a}{a} - \log\frac{2L+a}{L+a}\right]$$

$$= \frac{Q^2}{4\pi\varepsilon_0 L^2}\log_e\left[\frac{(L+a)^2}{a(2L+a)}\right]$$

14. An annular disc has inner and outer radius R_1 and R_2 respectively. Charge is uniformly distributed. Surface charge density is σ. Find the electric field at any point distant y along the axis of the disc.

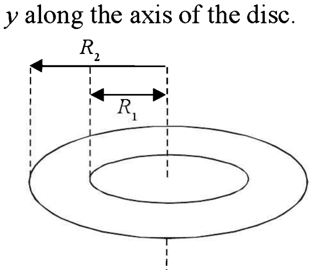


Fig. 18.27

(a) $\dfrac{\sigma}{2\varepsilon_0}$ (b) $\dfrac{\sigma y}{2\varepsilon_0(R_2 - R_1)}$

(c) $\dfrac{\sigma y}{2\varepsilon_0}\left[\dfrac{1}{\sqrt{R_1^2+y^2}} - \dfrac{1}{\sqrt{R_2^2+y^2}}\right]$

(d) $\dfrac{\sigma}{2\varepsilon_0}\log\dfrac{R_2+y}{R_1+y}$

Solution (c) Assume a hypothetical ring of radius x and thickness dx. Charge on the ring $dq = \sigma 2\pi x dx$. Electric field due to the ring at a point P distance y form the centre is

$$dE = \frac{dqy}{4\pi\varepsilon_0 (x^2+y^2)^{3/2}} \text{ or } dE = \frac{2\pi x dxy}{4\pi\varepsilon_0 (x^2+y^2)^{3/2}}$$

$$E = \frac{\sigma y 2\pi}{4\pi\varepsilon_0}\int_{R_1}^{R_2} \frac{xdx}{(x^2+y^2)^{3/2}} = \frac{\sigma y}{2\varepsilon_0}\left[\frac{-1}{\sqrt{x^2+y^2}}\right]_{R_1}^{R_2}$$

$$= \frac{\sigma y}{2\varepsilon_0}\left[\frac{1}{\sqrt{R_1^2+y^2}} - \frac{1}{\sqrt{R_2^2+y^2}}\right]$$

15. Find the minimum force between the two electrons of He nucleus. Assume radius of He nucleus = 6.8 A°.

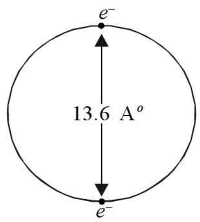


Fig. 18.28

(a) 12×10^{-10} N (b) 1.2×10^{-10} N
(c) 0.12×10^{-10} N (d) 0.012 N

Solution (b) Force will be minimum when electrons are diametrically opposite.

$$\therefore \quad F_{\min} = \frac{1.6 \times 1.6 \times 10^{-38} \times 9 \times 10^9}{(13.6)^2 \times 10^{-20}}$$

$$= 1.2 \times 10^{-10} \text{ N}$$

16. A line charge of length l and charge Q uniformly distributed over the whole length is placed a distance r from one edge from a point charge q as shown. Find the force on the point charge

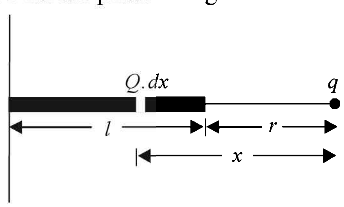


Fig. 18.29

(a) $\frac{qQ}{4\pi\varepsilon_0(r+l)}$ (b) $\frac{qQ}{4\pi\varepsilon_0\left(r+\frac{l}{2}\right)^2}$

(c) $\frac{qQ}{4\pi\varepsilon_0}\left[\frac{1}{r^2} - \frac{1}{(r+l)^2}\right]$ (d) none of these

Solution (a) Consider a small element dx of the line charge at a distance x from the point charge.

$$\text{Force } dF = q\frac{Qdx}{L 4\pi\varepsilon_0 x^2}$$

$$F = \frac{qQ}{4\pi\varepsilon_0 l}\int_r^{r+l}\frac{dx}{x^2} = \frac{qQ}{4\pi\varepsilon_0 l}\left[\frac{1}{r} - \frac{1}{r+l}\right]$$

$$= \frac{qQ}{4\pi\varepsilon_0 r(r+l)}$$

17. Two charges Q_1 and Q_2 are distance d apart. Two dielectrics of thickness t_1 and t_2 and dielectric constant k_1 and k_2 are introduced as shown. Find the force between the charges.

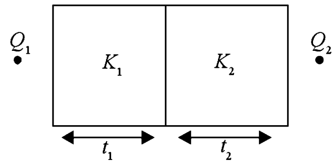


Fig. 18.30

(a) $\frac{Q_1Q_2}{4\pi\varepsilon_0\left[d-(t_1+t_2)+k_1t_1+k_2t_2\right]^2}$ (b) zero

(c) $\frac{Q_1Q_2}{4\pi\varepsilon_0\left[d+\sqrt{k_1}t_1+\sqrt{k_2}t_2\right]^2}$

(d) $\frac{Q_1Q_2}{4\pi\varepsilon_0\left[\sqrt{k_1}t_1+\sqrt{k_2}t_2+d-(t_1+t_2)\right]^2}$

Solution (d) effective distance in vacuum

$$= \sqrt{k_1}\,t_1 + \sqrt{k_2}\,t_2 + d - (t_1+t_2)$$

$$F = \frac{Q_1Q_2}{4\pi\varepsilon_0\left(\sqrt{k_1}t_1+\sqrt{k_2}t_2+d-(t_1+t_2)\right]^2}$$

18. When two charges are equal q each, force they exert on each other is F. When one of the charge is doubled, the $2q$ charge exerts a force $2F$ on charge q. The force exerted by q on $2q$ is

(a) F (b) $\frac{F}{2}$
(c) $\frac{F}{4}$ (d) $2F$
(e) $4F$

Solution (d) Force is mutual.

19. ABCD is a square frame of side l. The force at B if charges as shown in Fig. 18.31 (a) are placed at the corners of the square

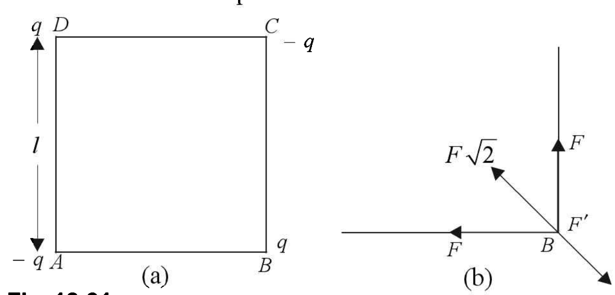


Fig. 18.31

(a) $\frac{q^2(2\sqrt{2}-1)}{4\pi\varepsilon_0 2l^2}$ (b) $\frac{q^2(2\sqrt{2}+1)}{4\pi\varepsilon_0 l^2}$

(c) $\frac{q^2(2\sqrt{2}-1)}{4\pi\varepsilon_0 l^2}$ (d) $\frac{q^2(2\sqrt{2}+1)}{4\pi\varepsilon_0 2l^2}$

Solution (a) As illustrated in Fig. 18.31 (b) the net force at B is

$$F\sqrt{2}-F'=\frac{q^2\sqrt{2}}{4\pi\varepsilon_0 l^2}-\frac{q2}{4\pi\varepsilon_0\left(\sqrt{2}l\right)^2}=\frac{q^2\left(2\sqrt{2}-1\right)}{4\pi\varepsilon_0 2l^2}$$

20. Charge Q is distributed uniformly on length l of a wire. It is bent in the form of a ring. Find the electric field at the centre of the ring.

(a) $\frac{Q\pi}{4\varepsilon_0 l^2}$ (b) $\frac{Q}{4\pi\varepsilon_0 l^2}$

(c) $\frac{Q}{2\pi\varepsilon_0 l^2}$ (d) $\frac{Q}{2\varepsilon_0 l^2}$

Solution (d) Consider two small elements of length dl each charge dq on each element. $dq=\frac{Qdl}{l}=\frac{Q(rd\theta)}{\pi r}$

$$=\frac{Qd\theta}{\pi}$$

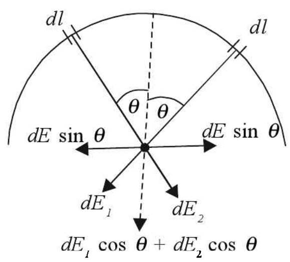


Fig. 18.32

$\because$ $|dE_1|=|dE_2|$, resolve dE_1 and dE_2, their sin components cancel out. $dE=2dE_1\cos\theta=\frac{2Qd\theta}{\pi 4\pi\varepsilon_0 r^2}\cos\theta$ and

$$E=\frac{2Q}{4\pi^2\varepsilon_0 r^2}\int_0^{1/2}\cos\theta d\theta \text{ or}$$

$$E=\frac{2Q}{4\pi^2\varepsilon_0\left(\frac{R}{\pi}\right)^2}[\sin 90-\sin 0]=\frac{Q}{2\varepsilon_0 l^2}$$

21. Two charged particles each of mass 5g and charge q are suspended as shown in Fig. 18.33. The system is taken in a satellite. The force between the charges is

(a) 23×10^{-3} N (b) 2.3×10^{-3} N
(c) 0.23×10^{-3} N (d) none of these

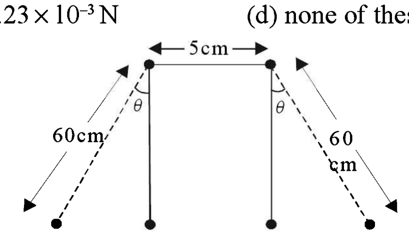


Fig. 18.33

Solution (a) $F=\frac{q^2}{4\pi\varepsilon_0 d^2}=\frac{2\times2\times10^{-12}\times9\times10^9}{(1.25)^2}$

$= 23\times10^{-3}$ N

{Here $d=(2l+5)$ cm = 125 cm as there is no gravity. Therefore electrostatic force will push them away.}

22. A child stands inside a large charged metal sphere. Will her hair stand on end ?

(a) Yes

(b) No

(d) Incomplete information

(d) No guess about her hair style

Solution (b) As electric field inside the shell is zero

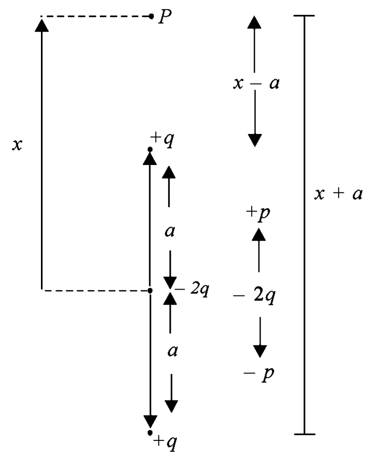


Fig. 18.34

23. Fig. 18.34 shows a quadrupole. Assuming $x >> a$ find the electric field at P where $p = qa$.

(a) $\frac{6pa^2}{4\pi\varepsilon_0 x^4}$ (b) $\frac{6pa}{4\pi\varepsilon_0 x^4}$

(c) $\frac{2pa}{4\pi\varepsilon_0 x^4}$ (d) $\frac{3pa}{4\pi\varepsilon_0 x^4}$

(e) $\frac{5pa}{4\pi\varepsilon_0 x^4}$

Solution (b) $E=\frac{q}{4\pi\varepsilon_0(x-a)^2}-\frac{2q}{4\pi\varepsilon_0 x^2}+\frac{q}{4\pi\varepsilon_0(x+a)^2}$

$$=\frac{q}{4\pi\varepsilon_0}\left[\frac{2x^2+2a^2}{\left(x^2-a^2\right)^2}-\frac{2}{x^2}\right]$$

$$=\frac{q}{4\pi\varepsilon_0}\left[\frac{6x^2a^2}{x^2\left(x^2-a^2\right)^2}\right]$$

$$=\frac{6pa}{4\pi\varepsilon_0 x^4}$$

24. An electron is projected with a velocity V_0 at an angle θ in the presence of an electric field E as shown in Fig. 18.35.

Find minimum value of d so that electron does not hit the plate.

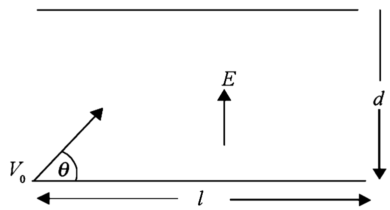


Fig. 18.35

(a) $d \geq \frac{mv_0^2}{2eE}$
(b) $\frac{mv_0^2 \cos^2\theta}{2eE}$
(c) $\frac{mv_0^2 \sin^2\theta}{2eE}$
(d) $\frac{mv_0^2 \tan^2\theta}{2eE}$

Solution (c) $a_y = \frac{eE}{m}$, $u_y = v_0 \sin\theta$

For the particle to fail to hit the plate $2a_y d \geq U_y^2$

or $v_0^2 \sin^2\theta = \frac{2eE}{m} d$

or $d = \frac{mv_0^2 \sin^2\theta}{2eE}$

25. Uniformly charged long cylinder has volume charge density ρ. Find the electric field at a distance $x < R$ from the axis of the cylinder.

(a) $\frac{\rho x}{\varepsilon_0}$
(b) $\frac{\rho x}{2\varepsilon_0}$
(c) $\frac{\rho x}{3\varepsilon_0}$
(d) $\frac{\rho x}{4\varepsilon_0}$

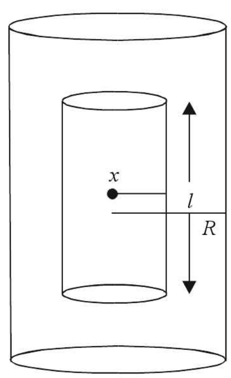


Fig. 18.36

Solution (b) Assume a hypothetical cylinder of radius x and length l. Apply Gauss's law $\oint E.ds = \frac{q_{in}}{\varepsilon_0}$ or

$\oint E.ds = \frac{\pi x^2 l \rho}{\varepsilon_0}$

$E(2\pi x l) = \frac{\pi x^2 l \rho}{\varepsilon_0} \Rightarrow E = \frac{\rho x}{2\varepsilon_0}$.

26. Two concentric shells carry charges q and Q. Their radius are r and R. The potential difference between the two is

(a) $\frac{q}{4\pi\varepsilon_0 R^2} - \frac{Q}{4\pi\varepsilon_0 R}$
(b) $\frac{R-q}{4\pi\varepsilon_0 R}$
(c) $\frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r} - \frac{1}{R}\right]$
(d) $\frac{(Q-q)}{4\pi\varepsilon_0}\left[\frac{1}{r} - \frac{1}{R}\right]$

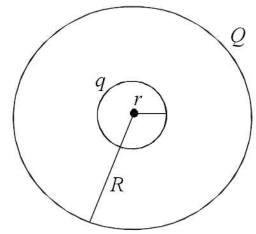


Fig. 18.37

Solution (c) $V_R = \frac{1}{4\pi\varepsilon_0}\left[\frac{R}{R} - \frac{q}{R}\right]$ and

$V_r = \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{r} - \frac{Q}{R}\right]$

$\Delta V = V_r - V_R$

$= \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r} - \frac{1}{R}\right]$

27. A sample of HCl is placed in an electric field of 2.5×10^4 NC^{-1}. The dipole moment of HCl is 3.4×10^{-30} C–m. Find the maximum torque that can act on a molecule.

(a) 7.6×10^{-26}Nm
(b) 4.3×10^{-26}Nm
(c) 6.5×10^{-26}Nm
(d) 8.5×10^{-26}Nm

Solution (d) $\tau_{max} = pE$

$= 3.4\times10^{-30}\times2.5\times10^4$

$= 8.5\times10^{-26}$Nm.

28. 12 J of work is to be done against an existing electric field to take a charge of 0.01C from A to B. Find The potential difference between B and A.

(a) 120 V
(b) 1200 V
(c) 1.2 V
(d) 12 V

Solution (b) $W = q\,\Delta V$

or $\Delta V = \frac{W}{q} = \frac{12}{0.01} = 1200$V

29. $E = 20\hat{i} + 30\hat{j}$ exists in space. If the potential at the origin is taken to be zero, find the potential at P (3, 2).

(a) –150V
(b) –100V
(c) +150V
(d) –120V
(e) 120V

Solution (d) $V = V_x + V_y$

$= \int_0^3 -E_x dx + \int_0^2 -E_y dy$

$= \int_0^3 -20dx + \int_0^2 -30dx = -60 - 60 = -120\,V$

30. A ring of radius R has charge Q. It is cut by dl. Find the electric field at the centre.

(a) zero
(b) $\frac{Qdl}{2\pi r^2\varepsilon_0}$
(c) $\frac{Qdl}{2\pi r^3\varepsilon_0}$
(d) $\frac{Qdl}{8\pi^2\varepsilon_0 r^3}$

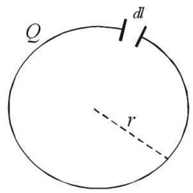


Fig. 18.38

Solution (d) $E = \dfrac{dq}{4\pi\varepsilon_0 r^2}$ and $dq = \dfrac{Qdl}{2\pi r}$.

Thus $E = \dfrac{Qdl}{8\pi^2\varepsilon_0 r^3}$

TYPICAL PROBLEMS

31. Electric potential existing in space is $V = K(x^2y + y^2z + xyz)$. Find the expression of electric field.

(a) zero

(b) $-K\,[(2xy + yz)\,\hat{i} + (x^2 + 2yz + xz)\,\hat{j} + (y^2 + xy)\,\hat{k}\,]$

(c) $-K\left[\left(\dfrac{x^3y}{3} + \dfrac{x^2yz}{2}\right)\hat{i} + \left(\dfrac{y^2z^2}{2} + \dfrac{x^2y^2z}{2}\right)\hat{k}\right]$

(d) $K\,[(2xy + yz)\,\hat{i} + (x^2 + 2yz + xz)\,\hat{j} + (y^2 + xy)\,\hat{k}\,]$

Solution (b)

$$E = -\left[\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial x} + \hat{K}\frac{\partial}{\partial x}\right][K(x^2y + y^2z + xyz)]$$

$$E = -K\,[(2xy + yz)\,\hat{i} + (x^2 + 2yz + xz)\,\hat{j}\,(y^2 + xy)\,\hat{K}\,]$$

32. Charges $-q$ and $+q$ are fixed at the ends of a light rod of length l. The rod is clamped at one end with axis of the dipole along the electric field. The rod is slightly displaced and then released, Neglecting gravity find the time period of small oscillations. Mass of charges is m each.

(a) $2\pi\sqrt{\dfrac{ml}{qE}}$ (b) $2\pi\sqrt{\dfrac{ml}{3qE}}$

(c) $2\pi\sqrt{\dfrac{ml}{2qE}}$ (d) $2\pi\sqrt{\dfrac{2ml}{qE}}$

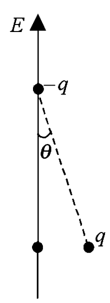


Fig. 18.39

Solution (a) Torque $p \times E = I\alpha$

Assume θ to be small $\therefore \sin\theta = \theta$

or $-pE\theta = ma^2x$

or $\alpha = -\dfrac{(ql)E}{ml^2}\,\theta$

$$\omega = \sqrt{\frac{qE}{ml}}$$

or $$T = 2\pi\sqrt{\frac{ml}{qE}}$$

33. Three charges are arranged as shown in Fig 18.40 (a). Find the net dipole moment.

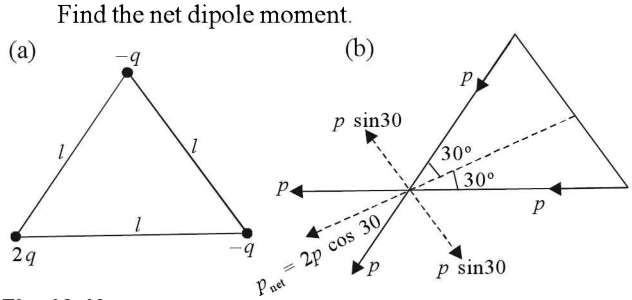


Fig. 18.40

(a) $2ql$ (b) $\sqrt{3}ql$

(c) $\dfrac{\sqrt{3}ql}{2}$ (d) $\dfrac{ql}{2}$

Solution (b) $p_{net} = 2p\cos 30 = ql\sqrt{3}$ [see Fig 18.40 (b)]

34. A block of mass m and charge q is placed on a smooth horizontal table which terminates in a vertical wall at a distance d from the block . A horizontal electric field E towards right is switched on. Assuming elastic collision, find time period of the resulting oscillatory motion.

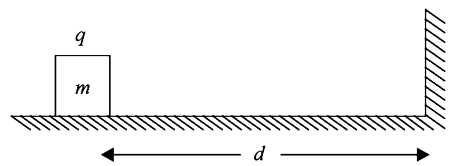


Fig. 18.41

(a) $\sqrt{\dfrac{8dm}{qE}}$ (b) $\sqrt{\dfrac{4dm}{qE}}$

(c) $\sqrt{\dfrac{2dm}{qE}}$ (d) none of these

Solution (a) $a = \dfrac{qE}{m}$ and $d = \dfrac{1}{2}at^2$

or $t = \sqrt{\dfrac{2d}{a}} = \sqrt{\dfrac{2dm}{qE}}$

Time period $T = 2t = 2\sqrt{\dfrac{2dm}{qE}}$

$= \sqrt{\dfrac{8dm}{qE}}$

35. A thin nonconducting ring of radius R has linear charge density $\lambda = \lambda_0 \cos\theta$ where λ_0 is a constant, θ is azimuthal angle. Find the electric field at the centre of the ring.

Solution Consider two small elements of length $dl = Rd\theta$ symmetrically at angle θ on both sides as shown in Fig. 18.42

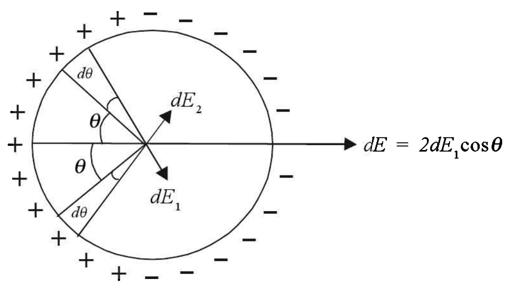


Fig. 18.42

Then $dE = 2dE_1\cos\theta = \dfrac{2dq}{4\pi\varepsilon_0 R^2}\cos\theta$

$$E = \int_0^{\pi} \frac{2\lambda_0 \cos\theta R d\theta \cos\theta}{4\pi\varepsilon_0 R^2} = \frac{2\lambda_0 R}{4\pi\varepsilon_0 R^2}\int_0^{\pi}\frac{(1+\cos 2\theta)}{2}d\theta$$

$$= \frac{2\lambda_0 R}{4\pi\varepsilon_0 R^2}\times\frac{\pi}{2} = \frac{\lambda_0}{4\varepsilon_0 R}$$

36. Two long parallel threads carry charge per unit length. The threads are separated by a distance l. Find the maximum field strength in the symmetry plane between them.

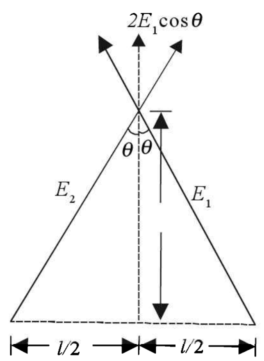


Fig. 18.43

Solution $E = 2E_1 \cos\theta = \dfrac{2\lambda\cos\theta}{2\pi\varepsilon_0\left(x^2+\dfrac{l^2}{4}\right)^{1/2}}$

$$= \frac{\lambda x}{\pi\varepsilon_0\left(x^2+\dfrac{l^2}{4}\right)} \qquad ....(1)$$

For maximum E, $\dfrac{dE}{dx} = 0$

$$= \frac{\lambda}{\pi\varepsilon_0}\left[\frac{1}{x^2+\dfrac{l^2}{4}} - \frac{x(2x)}{\left(x^2+\dfrac{l^2}{4}\right)^2}\right] = 0$$

or $x^2 = \dfrac{l^2}{4}$ or $x = \dfrac{l}{2}$

Substituting $x = \dfrac{l}{2}$ in eq. (1), $E = \dfrac{\lambda}{\pi\varepsilon_0 l}$

37. The electric field potential at a point in the space region depends only on x coordinates as $V = -ax^3 + b$. Find the space charge density $\rho(x)$.

Solution $E = \dfrac{-dV}{dx} = 3ax^2$. Using Gauss's law in differential form $\dfrac{\partial E}{\partial x} = \dfrac{\rho(x)}{\varepsilon_0} = 6\,ax$ or $\rho(x) = 6a\varepsilon_0 x$

38. In a Millikan's oil drop method, a charged drop of oil has mass m and charge q. It is made stationary by applying an electric field E. Find the strength of the electric field. If the result of Millikans experiment with different drops is 6.48×10^{-19}C, 12.82×10^{-19}C, 19.3×10^{-19}C, 8.02×10^{-19}C, 25.62×10^{-19}C, 16.04×10^{-19}C, find the elementary charge.

Solution $qE = mg \qquad E = {mg}/{q}$

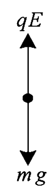


Fig. 18.44

The HCF of the data is 1.6×10^{-19}C

Hence elementary charge is 1.6×10^{-19}C.

39. A space is filled up with volume charge density $\rho = \rho_0\, e^{-\alpha r^3}$ where ρ_0 and α are positive constants. r is the distance from the centre of this system. Find the magnitude of electric field strength as a function of r.

Solution Differentiating from Gauss's Law is $\dfrac{\partial E}{\partial r} = \dfrac{\rho}{\varepsilon_0}$

or $\int \partial E = \int \dfrac{\rho}{\varepsilon_0} dr$

$$E = \int_0^r \frac{\rho_0 e^{-\alpha r^3}}{\varepsilon_0} dr$$

$$= \frac{\rho_0}{3\varepsilon_0 \alpha r^2}\left(1 - e^{-\alpha r^3}\right).$$

QUESTIONS FOR PRACTICE

1. Two identical rings of radii 0.1 m are placed co–axially at a distance 0.5 m apart. The charges on the rings are $2\mu C$ and $4\mu C$ respectively. The work done in transferring $5\mu C$ charge from the centre of one ring to that of the other will be nearest to

(a) 0.50 J (b) 0.75 J
(c) 1.00 J (d) 1.50 J

2. The electric field strength due to a ring of radius R at a distance x from its centre on the axis of ring carrying charge Q is given by $E = \frac{1}{4\pi\varepsilon_0}\frac{Qx}{(R^2+x^2)^{3/2}}$.

At what distance from the centre will the electric field be maximum ?

(a) $x = R$ (b) $x = \frac{R}{2}$
(c) $x = \frac{R}{\sqrt{2}}$ (d) $x = \frac{R}{\sqrt{2}}$

3. Positive charges of $2\mu C$ and $8\mu C$ are placed 15cm apart. At what distance from the smaller charge will the electric field due to them be zero ?

(a) 3 cm (b) 5 cm
(c) 7 cm (d) 10 cm.

4. A charge is distributed over two concentric hollow spheres of radii R and r, where $R > r$, such that the surface densities of charges are equal (σ). What is the potential at their common centre ?

(a) $\frac{\sigma}{\varepsilon_0}(R+r)$ (b) $\frac{\sigma}{\varepsilon_0}(R+r)$
(c) $\frac{\sigma}{\varepsilon_0}R$ (d) $\frac{\sigma}{\varepsilon_0}r$.

5. A charge q is distributed over two spheres of radii R and r such that their surface charge densities are equal. What is the ratio of the charges on the spheres ?

(a) $\frac{r}{R}$ (b) $\frac{r^2}{R^2}$
(c) $\frac{r^3}{R^3}$ (d) $\frac{r^4}{R^4}$

6. A charge q is distributed over two spheres of radii R and r such that their surface densities are equal. What is the ratio of their potentials ?

(a) $\frac{R}{r}$ (b) $\frac{R^2}{r^2}$
(c) $\frac{R^3}{r^3}$ (d) $\frac{R^4}{r^4}$

7. A ring of radius 6 cm is given a charge $10\mu C$. How much work will be done in transporting a charge of $6\mu C$ from its centre to a point 8 cm along its axis ?

(a) 63 m J (b) 84 m J
(c) 105 m J (d) 126 m J

8. Two conducting spheres of radii r_1 and r_2 are at the same potential. What is the ratio of the charges on them ?

(a) $\sqrt{\frac{r_1}{r_2}}$ (b) $\frac{r_1}{r_2}$
(c) $\frac{r_1^2}{r_2^2}$ (d) $\sqrt{r_1 r_2}$

9. Two conducting spheres of radii r_1 and r_2 are charged such that they have the same electric field on their surfaces. The ratio of the electric potential at their centres is

(a) $\sqrt{\frac{r_1}{r_2}}$ (b) $\frac{r_1}{r_2}$
(c) $\frac{r_1^2}{r_2^2}$ (d) none of the above.

10. An uncharged metallic hollow sphere is placed in uniform external electric field (Fig 18.45). The path of the electric field lines in and around the conductor is represented by

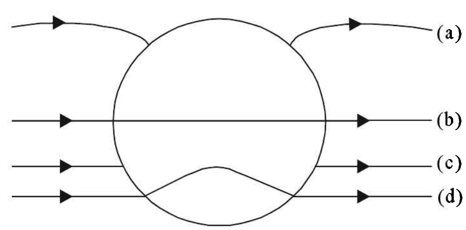


Fig. 18.45

11. How does the electric field (E) between the plates of a charged cylindrical capacitor vary with the distance (r) from the axis of the cylinder ?

(a) $E \propto \frac{1}{r^2}$ (b) $E \propto \frac{1}{r}$
(c) $E \propto r^2$ (d) $E \propto r$

12. The electric field line in the $X-Y$ plane is represented by $x^2 + y = 1$. A test charge q_0 is placed at a distance $x = 1$, $y = 1$. What will be the nature of the path followed by it ?

(a) Circle (b) Straight line
(c) Parabola (d) Cannot be predicted

13. A ring of radius R is carrying uniformly distributed charge $+Q$. A test charge $-q_0$ is placed on its axis at a distance $2R$ from the centre and released. The motion of the particle on the axis will be

(a) periodic (b) non periodic
(c) simple harmonic (d) random.

14. Five equal and similar charges are placed at the corners of a regular hexagon as shown in Fig. 18.46. What is the electric field and potential at the centre of the hexagon ?

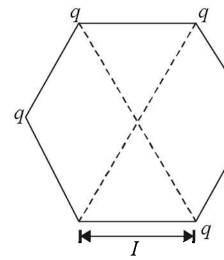


Fig. 18.46

(a) $\frac{5}{4\pi\varepsilon_0}\frac{q}{l}, \frac{5}{4\pi\varepsilon_0}\frac{q}{l^2}$ (b) $\frac{1}{4\pi\varepsilon_0}\frac{q}{l}, \frac{5}{4\pi\varepsilon_0}\frac{q}{l^2}$

(c) $\frac{5}{4\pi\varepsilon_0}\frac{q}{l}, \frac{1}{4\pi\varepsilon_0}\frac{q}{l^2}$ (d) $\frac{1}{4\pi\varepsilon_0}\frac{q}{l}, \frac{1}{4\pi\varepsilon_0}\frac{q}{l^2}$

15. Which of the following combinations of seven identical capacitors each of $2\mu F$ gives a capacitance of $10/11\mu F$?

(a) 5 in parallel with 2 in series
(b) 4 in parallel with 3 in series
(c) 3 in parallel with 4 in series
(d) 2 in parallel with 5 in series

16. The electric field at the centre of a uniformly charged ring is zero. What is the electric field at the centre of a half ring if the charge on it be Q and its radius be R?

(a) $\frac{1}{4\pi\varepsilon_0}\frac{Q}{\pi R^2}$ (b) $\frac{1}{4\pi\varepsilon_0}\frac{Q}{R^2}$

(c) $\frac{1}{4\pi\varepsilon_0}\frac{2Q}{\pi R^2}$ (d) $\frac{1}{4\pi\varepsilon_0}\frac{2Q}{R^2}$

17. What is the electric potential at the centre of a hemisphere of radius R and having surface charge density σ?

(a) $\frac{\sigma}{2\varepsilon_0}$ (b) $\frac{\sigma}{\varepsilon_0}$

(c) $\frac{\sigma}{\varepsilon_0}R$ (d) $\frac{\sigma}{2\varepsilon_0}R$

18. Fig. 18.47 shows four capacitors connected across a power supply of 310 V. What is the charge and potential difference across the $4\sigma F$ capacitor ?

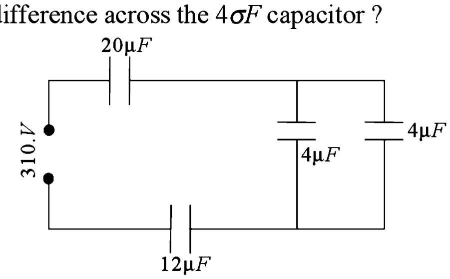


Fig. 18.47

(a) 1200 σF, 310 V (b) 600 σF, 310 V
(c) 600 σF, 150 V (d) 1200 σF, 150 V.

19. Five capacitors are connected to each other as shown in Fig. 18.48. What is potential drop and charge across $4\mu F$ capacitor ?

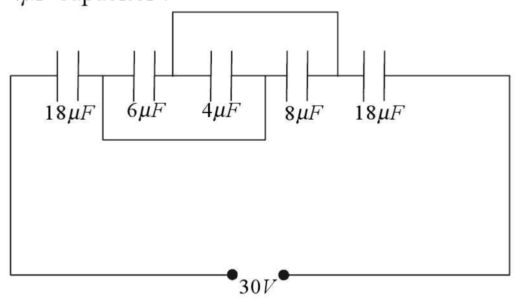


Fig. 18.48

(a) 6 V, 3 μC (b) 10 V, 30 μC
(c) 6 V, 40 μC (d) 10 V, 40 μC.

20. Five identical plates are connected across a battery as in Fig. 18.48. If the charge on plate 1 be +q, then the charges on the lates 2, 3, 4 and 5 are

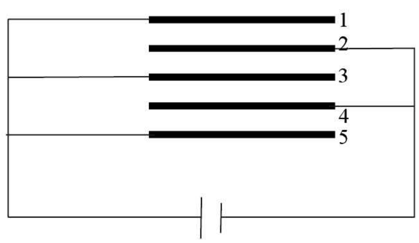


Fig. 18.49

(a) – q, + q, – q, + q (b) – 2q, + 2q, – 2q, + q
(c) – q, + 2q, – 2q, + q (d) none of the above.

21. Three capacitors are connected across a 45V power supply as shown in Fig. 18.50. What is the charge on the 6 μF capacitor?

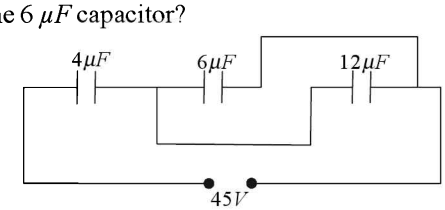


Fig. 18.50

(a) 60μC (b) 90μC
(c) 120μC (d) 180μC

22. Six identical capacitors each of $2\mu F$ are joined in parallel and each is charged to 10 V. They are then disconnected and joined in series so that positive plate of one is joined to the negative plate of the adjacent capacitor. What is the potential difference of the combination ?

(a) 10 V (b) 30 V
(c) 60 V (d) 120 V

23. A capacitor of capacitance $10\mu F$ is charged by connecting through a resistance of 20 Ω and a battery of 20 V. What is the energy supplied by the battery ?

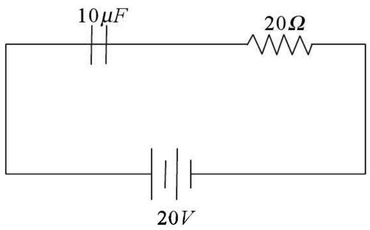


Fig. 18.51

(a) Less than 2 m J (b) 2 m J
(c) More than 2 m J (d) Cannot be predicted

24. Force acting on a test charge between the plates of a parallel capacitor is F. If one of the plates is removed, then the force on the same test charge will be

(a) zero (b) F
(c) $2F$ (d) $\frac{F}{2}$

25. Two point charges Q and $-3Q$ are placed certain distance apart. If the electric field at the location of Q be $\vec{E}$, then that at the location of $-3Q$ will be

(a) $3\vec{E}$ (b) $-3\vec{E}$
(c) $\frac{\vec{E}}{3}$ (d) $-\frac{\vec{E}}{3}$

26. The length of each side of a cubical closed surface is α. If charge q is situated on one of the vertices of the cube then the flux passing through each face of the cube will be

(a) $\frac{q}{6\varepsilon_0}$ (b) $\frac{q}{24\varepsilon_0}$
(c) $\frac{q}{8\varepsilon_0}$ (d) zero

27. A charged conductor has charge on its

(a) outside surface (b) surrounding
(c) middle point (d) inner surface

28. The laws of forces that govern the force between two electric charges were discovered by

(a) Faraday (b) Ampere
(c) Ohm (d) Coulumb

29. A charge Q is placed on to two opposite corners of a square. A charge q is placed at each of other two corners. Given that resultant electric force on Q is zero, then Q is equal to

(a) $\frac{(2\sqrt{2})}{q}$ (b) $\frac{-q}{(2\sqrt{2})}$
(c) $(2\sqrt{2})q$ (d) $(-2\sqrt{2})q$

30. Let us suppose that earth (radius 6400 km) had a net charge equivalent to one electron per m^2 of its surface area. Its potential in volts will be

(a) –1.2 (b) – 0.12
(c) 0.12 (d) 1.2

31. A charge Q is divided into two parts. The two charges kept at a distance apart have a maximum columbian repulsion. Then the ratio of Q and one of the parts is given by

(a) 1 : 4 (b) 1 : 2
(c) 2 : 1 (d) 4 : 1

32. In comparison with the electrostatic force between two electrons, the electrostatic force between two protons is

(a) zero (b) smaller
(c) same (d) greater

33. A positively charged ball hangs from a long silk thread. We put a positive test charge q_0 at a point and measure F/q_0, then it can be predicted that field E

(a) $>F/q_0$ (b) $<F/q_0$
(c) is equal to F/q_0 (d) none of these

34. An electron of mass m and charge e is accelerated from rest through a potential difference V in vacuum. Its final speed will be

(a) $\sqrt{\frac{2eV}{m}}$ (b) $\sqrt{\frac{eV}{m}}$
(c) $\frac{eV}{2m}$ (d) $\frac{eV}{m}$

35. A helium ion and a hydrogen ion are accelerated from rest through a potential difference of V to velocities v_{He} and v_H respectively. If helium has lost one electron, the ratio of v_{He}/v_H is

(a) 1/4 (b) 1/2
(c) 1 (d) $\sqrt{2}$

36. A charge q is placed at the centre of the line joining two equal charges Q. The system of the three charges will be equilibrium if q is equal to

(a) – (Q/4) (b) – (Q/2)
(c) (Q/2) (d) (Q/4)

[IIT 87]

37. A proton has a mass 1.67×10^{-27} kg and charge $+1.6 \times 10^{-19}\,C$. If the proton is being accelerated through a potential difference of one millions volts then the $K.E.$ is

(a) 1.6×10^{-25} J (b) 3.2×10^{-13} J
(c) 1.6×10^{-15} J (d) 1.6×10^{-13} J

38. An electron having charge $-e$ located at A, in the presence of point charge $+q$ located at O, is moved to the point B such that OAB forms an equilateral triangle. The work done in the process is equal to

(a) $-eq/AB$ (b) eq/AB
(c) q/AB (d) zero

39. The electric flux ϕ through a hemispherical surface of radius R, placed in a uniform electric field of intesity E parallel to the axis of its circular plane is

(a) (4/3) $\pi R^3 E$ (b) $2\pi R^2 E$
(c) $\pi R^2 E$ (d) $2\pi RE$

40. A charge of 6.76 μC in an electric field is acted upon by a force of 2.5 N. The potential gradient at this point is

(a) 3.71×10^5 Vm^{-1} (b) 3.71×10^{12} Vm^{-1}
(c) 3.71×10^{10} Vm^{-1} (d) 3.71×10^5 Vm^{-1}

41. Two identical thin rings each of radius R are coaxially placed at a distance R apart. If Q_1 and Q_2 are respectively the charges uniformly spread on the two rings, the work done in moving a charge q from the centre of one ring to the other is

(a) $\dfrac{q(Q_1/Q_2)(\sqrt{2}+1)}{\sqrt{2}4\pi\varepsilon_0 R}$ (b) $\dfrac{q(Q_1-Q_2)(\sqrt{2}-1)}{\sqrt{2}4\pi\varepsilon_0 R}$

(c) $\dfrac{q\sqrt{2}(Q_1+Q_2)}{4\pi\varepsilon_0 R}$ (d) zero

42. Two pith balls each of mass 1g and carrying a charge 1μC are attached to the ends of silk threads 1m long, the other ends of which are attached to some fixed point, in a gravity free space. The force between them is

(a) 9.8×10^{-12} N (b) 9.0×10^{-6} N
(c) 4.5×10^{-6} N (d) 2.25×10^{-3} N

43. Two equal metal balls are charged to 10 and 20 units of electricity. They are brought in contact with each other and then again separated to the original distance. The ratio of forces between the two balls before and after contact is

(a) 3 : 2 (b) 1 : 8
(c) 2 : 3 (d) 8 : 9

44. No current flows between two charged bodies when connected if they have same

(a) capacity (b) charge
(c) potential (d) none of the above

45. The dielectric strength of air is 3.0×10^6 NC^{-1}. The largest charge that a 0.30 cm radius metal sphere can hold without sparking is

(a) 9 nC (b) 8.2 nC
(c) 6 nC (d) 3 nC

46. A ring of radius R carries a uniformly distributed charge $+Q$. A point charge q is placed on the axis of the ring and released from rest. The force experienced by the particle varies with distance from the centre as

(a) $x/(R^2+x^2)^{3/2}$ (b) $1/\sqrt{x}$
(c) $1/x^3$ (d) $1/x^2$

47. An electron is accelerated through a potential difference of 500 volt. The velocity acquired by the electron is

(a) $(2/3) \times 10^7$ ms^{-1} (b) $(1/6) \times 10^7$ ms^{-1}
(c) $(1/3) \times 10^7$ ms^{-1} (d) none of the above

48. An electrical charge of 2 μC is placed at the point (1, 2, 3). At the point (2, 3, 4) the electric field and potential will be

(a) 6×10^3 NC^{-1} and 6×10^3 JC^{-1}
(b) 6000 NC^{-1} and 6000 $\sqrt{3}$ JC^{-1}
(c) 6×10^3 NC^{-1} and $3\sqrt{3}$ JC^{-1}
(d) none of the above

49. The charge per unit length for a very long straight wire is λ. The electric field at points near the wire (but outside it) and far from the ends varies with distance r as

(a) r (b) $1/r$
(c) $1/r^2$ (d) $1/r^3$

50. A charged particle of mass m and charge q is released from rest in an electric field of constant magnitude E. The $K.E.$ of the particle after time r is

(a) $\dfrac{Eq^2m}{2t^2}$ (b) $\dfrac{Eqm}{2t}$

(c) $\dfrac{2E^2t^2}{mq}$ (d) $\dfrac{E^2q^2t^2}{2m}$

51. A solid conducting sphere having a charge Q is surrounding by an uncharged concentric conducting hollow spherical shell. Let the potential difference between the surface of the solid sphere and that of the outer surface of the hollow shell be V. If the shell is now given a charge of $-3Q$ the new potential difference between the same two surfaces is

(a) $-2V$ (b) V
(c) $2V$ (d) $4V$

[IIT 1989]

52. A uniform electric field having a magnitude E_0 and direction along the positive x-axis exists. If the potential V is zero at $x = 0$, then its value at $x = +x$ will be

(a) $V(x) = x^2E_0$ (b) $V(x) = -x^2E_0$
(c) $V(x) = -xE_0$ (d) $V(x) = xE_0$

53. A given charge situated at a certain distance from an electric dipole in the end on position, experiences a force F. If the distance of the charge is doubled, the force acting on the charge will be

(a) $F/8$ (b) $F/4$
(c) $F/2$ (d) $2F$

54. Eight charged water drops each with a radius of 1 mm and a charge of 10^{-10} coulomb merge into a single drop. The potential of this single drop is

(a) 36 V (b) 1000 V
(c) 3600 V (d) 8000 V

55. $ABCD$ is a square of 1 metre side of a non-conducting material. Four metallic spheres of 4, 5, 8 and 10 cm diameters are placed at the four corners. All of them are connected by a fine metallic wire and charge of 540 units is imparted to the system. The potential at the centre of the square is

(a) $\frac{540\sqrt{2}}{400}$ (b) $\frac{540\sqrt{2}}{200}$

(c) $\frac{540\sqrt{2}}{100}$ (d) $\frac{540\sqrt{2}}{10}$

56. The electric potential at a point situated at a distance r on the axis of a short electric dipole of moment p will be $1/4(\pi\varepsilon_0)$ times

(a) p/r^3 (b) p/r^2

(c) p/r (d) none of the above

57. Two concentric thin, metallic spheres of radii R_1 and $R_2 (R_1 > R_2)$ bear charges Q_1 and Q_2 respectively. Then the potential at a radius r between R_1 and R_2 will be $1/4(\pi\varepsilon_0)$ times

(a) $\frac{Q_1+Q_2}{4}$ (b) $\frac{Q_1}{R_1}+\frac{Q_2}{r}$

(c) $\frac{Q_1}{R_1}+\frac{Q_2}{R_2}$ (d) $\frac{Q_1}{R_2}+\frac{Q_2}{R_1}$

58. An electric dipole of moment p is kept along an electric field E. The work done in rotating it from an equilibrium position by an angle θ is

(a) $PE(1-\cos\theta)$ (b) $PE(1-\sin\theta)$

(c) $PE\cos\theta$ (d) $PE\sin\theta$

59. A body has a charge of one coulomb. The number of excess (or lesser) electrons on it from its normal state will be

(a) ∞ (b) 1.6×10^{-19}

(c) 1.6×10^{19} (d) 6.25×10^{18}

Answers to Question for Practice

1.	(b)	**2.**	(c)	**3.**	(b)	**4.**	(a)	**5.**	(b)	**6.**	(a)	**7.**	(d)
8.	(b)	**9.**	(b)	**10.**	(a)	**11.**	(b)	**12.**	(d)	**13.**	(a)	**14.**	(c)
15.	(a)	**16.**	(c)	**17.**	(d)	**18.**	(c)	**19.**	(d)	**20.**	(b)	**21.**	(b)
22.	(c)	**23.**	(c)	**24.**	(d)	**25.**	(d)	**26.**	(b)	**27.**	(a)	**28.**	(d)
29.	(d)	**30.**	(b)	**31.**	(c)	**32.**	(c)	**33.**	(a)	**34.**	(a)	**35.**	(b)
36.	(a)	**37.**	(d)	**38.**	(d)	**39.**	(c)	**40.**	(d)	**41.**	(b)	**42.**	(d)
43.	(d)	**44.**	(c)	**45.**	(d)	**46.**	(a)	**47.**	(d)	**48.**	(b)	**49.**	(b)
50.	(d)	**51.**	(b)	**52.**	(c)	**53.**	(a)	**54.**	(c)	**55.**	(c)	**56.**	(b)
57.	(b)	**58.**	(a)	**59.**	(d)								

19

Gauss's Law

BRIEF REVIEW

Gauss's Law is used as an alternative to Coulomb's Law.

Electric flux $\oint E = \oint \vec{E}.\vec{ds}$. Note that Electric flux does not depend on the radius R of the sphere. It only depends upon the charge q enclosed in the sphere. According to Gauss's law the closed integral of electric field intensity is equal to $\frac{q}{\varepsilon_0}$ where q is charge enclosed in the closed surface. In other words, total flux through a closed surface enclosing a charge q is given by $\oint \vec{E}.\vec{ds} = \frac{q}{\varepsilon_0}$. Note the following points:

1. If E is at right angle to the surface area A at all points and has same magnitude at all points of the surface then $E_\perp = E$ and $\int E_\perp.ds = EA$.
2. If E is parallel to the surface on all points then $E_\perp = 0$ and hence $\int E_\perp.ds = 0$.
3. If $E = 0$ at all points on a surface then $\phi_E = 0$.
4. The surface need not be a real surface, it could be a hypothetical one.
5. Electric field in $\oint \vec{E}.\vec{ds}$ is complete electric field, it may be partly due to charge outside the surface and partly due to charge inside the surface. However, if there is no charge enclosed in the Gaussian surface $E_\perp$ will be zero and hence $\oint \vec{E}.\vec{ds} = 0$
6. While evaluating $\oint E.ds$, the field should lie on the surface and there should be enough symmetry to evaluate the integral.

Various forms of Gauss's Law

$$\phi_E = \oint E\cos\phi ds = \oint \vec{E}.\vec{ds} = \frac{q_{\text{enclosed}}}{\varepsilon_0}$$

Note that net number of field lines leaving a closed surface is proportional to the total charge enclosed by that surface.

The excess charge (other than the ions and free electrons that make up the neutral conductor) resides entirely on the surface and not in the interior of the material.

Electric field due to a long thread (line charge)

having linear charge density λ is $E = \frac{\lambda}{2\pi\varepsilon_0 y} = \frac{18\times10^9 \lambda}{y}$

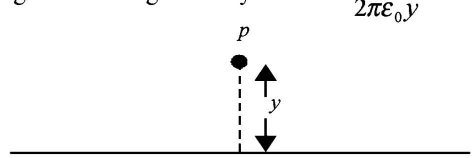


Fig. 19.1 Electric field due to a long line charge

Electric field due to a shell having radius *R* and charge Q

$$E_{\text{inside}} = 0, E_{\text{surface}} = \frac{Q}{4\pi\varepsilon_0 R^2}\ ; E_{\text{out}} = \frac{Q}{4\pi\varepsilon_0 x^2}\quad x > R$$

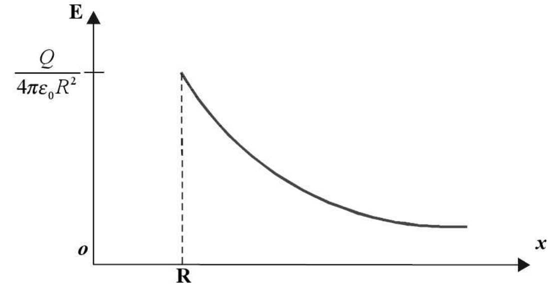


Fig. 19.2 Electric field due to a shell

Potential due to a shell

$$V_{\text{in}} = \frac{Q}{4\pi\varepsilon_0 R} = V_{\text{surface}}$$

$$V_{\text{out}} = \frac{Q}{4\pi\varepsilon_0 x}\ \text{for } x > R$$

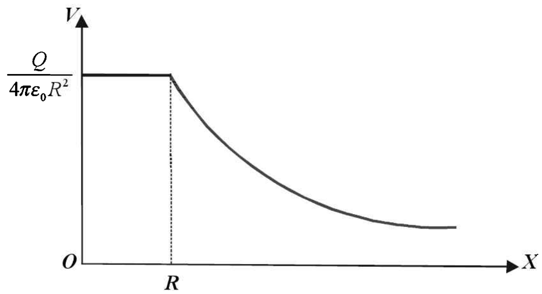


Fig. 19.3 Electric potential due to a shell

Electric field due to a sphere charged uniformly with charge Q

$$E_{\text{inside}} = \frac{Qx}{4\pi\varepsilon_0 R^3}\ \text{for } x < R$$

$$E_{\text{surface}} = \frac{Q}{4\pi\varepsilon_0 R^2}\ \text{for } x = R$$

$$E_{\text{outside}} = \frac{Q}{4\pi\varepsilon_0 x^2}\ \text{for } x > R$$

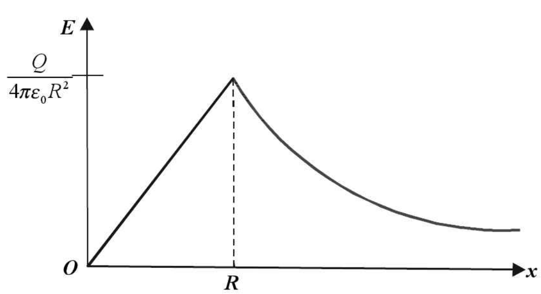


Fig. 19.4 Electric potential due to a sphere charged uniformly

$$V_{\text{out}} = \frac{Q}{4\pi\varepsilon_0 x}\ \text{for } x > R$$

$$V_{\text{inside}} = \int_R^x \frac{-Qx^2 dx}{4\pi\varepsilon_0 R^3} + \frac{Q}{4\pi\varepsilon_0 R}\ \text{for } x < R$$

$$V_{\text{surface}} = \frac{Q}{4\pi\varepsilon_0 R}\ \text{for } x = R.$$

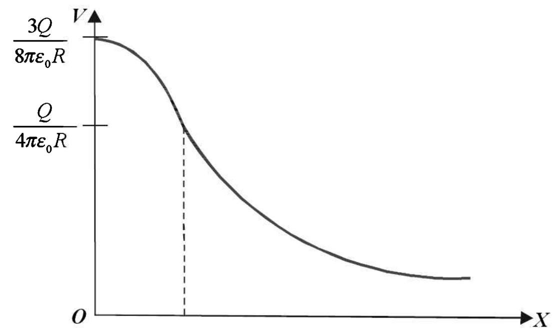


Fig. 19.5 Potential due to a uniformly charged sphere

Electric Field due to a thin plane Sheet (long) of charge density σ $E = \dfrac{\sigma}{2\varepsilon_0}$.

Electric field due to a charged surface having surface charge density σ $E = \dfrac{\sigma}{\varepsilon_0}$.

Electric field due to a conducting plate $E = \dfrac{\sigma}{2\varepsilon_0}$.

Electric field between two oppositely charged sheets at any point is $E_{\text{in}} = \dfrac{\sigma}{\varepsilon_0}$ $(= E_1 + E_2)$. Assuming equal surface charge density (for example in a capacitor) $E = \dfrac{\sigma}{\varepsilon_0}$. Electric field intensity is zero at any point outside the plates as $E_{\text{net}} = E_1 - E_2 = 0$, as shown in Fig. 19.6.

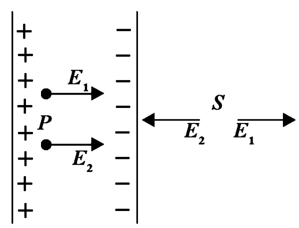


Fig. 19.6 Electric field due to charged plates

- **Short Cuts and Points to Note**

1. Electric flux through symmetrical surfaces placed in a uniform electric field is zero. For example, for a cylinder (solid or hollow) placed in a uniform electric field $\phi_E = 0$. Hence no charge is stored.

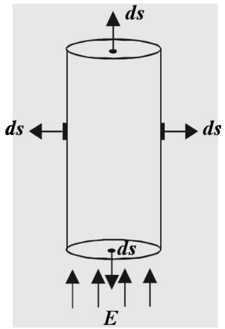


Fig. 19.7

2. $\oint \vec{E}.\vec{ds} = \frac{Q}{\varepsilon_0}$ where Q is charge enclosed in the surface. The electric field is perpendicular to the surface so that it is parallel to surface vector ds. If $\vec{E}$ and $\vec{ds}$ are not parallel take the dot product. i.e. $EA\cos\phi$ as the flux. Gauss's law in differential form $\frac{\partial E}{\partial r} = \frac{\rho}{\varepsilon_0}$.

3. Electric field intensity due to a long line charge at a distance y from one end as shown in Fig. 19.8 at P from end A is $\frac{\lambda\sqrt{2}}{4\pi\varepsilon_0 y}$ and is directed at 45° with the vertical.

Fig. 19.8

4. Electric field due to a uniformly charged sphere of radius R and charge Q is $E_{\text{inside}} = \frac{Qx}{4\pi\varepsilon_0 R^3}$ $x < R$

$$E_{\text{surface}} = \frac{Q}{4\pi\varepsilon_0 R^2} \quad x = R;$$

$$E_{\text{outside}} = \frac{Q}{4\pi\varepsilon_0 x^2} \quad x > R \text{ as shown in Fig. 19.9.}$$

Note that $E_{\text{centre}} = 0$.

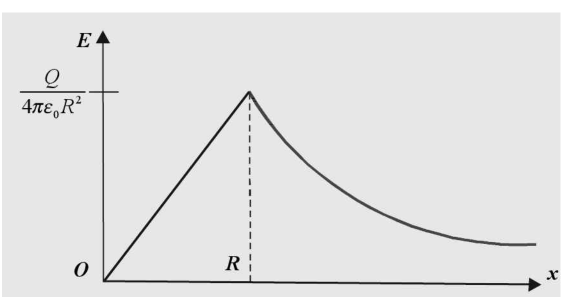


Fig. 19.9

5. Electric potential due to a uniformly charged sphere of radius R and charge Q

$$V_{\text{inside}} = \frac{Q}{4\pi\varepsilon_0 R} + \int_R^x -\frac{Qx^2}{4\pi\varepsilon_0 R^3}\,dx \qquad x < R$$

$$V_{\text{surface}} = \frac{Q}{4\pi\varepsilon_0 R} \qquad x = R.$$

Note that $V_{\text{centre}} = \frac{3Q}{8\pi\varepsilon_0 R}$ as shown in Fig 19.10.

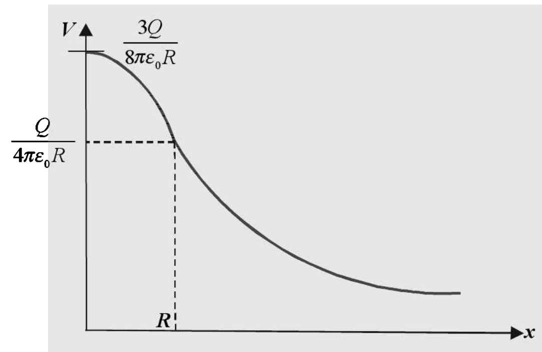


Fig. 19.10

6. Electrical shielding is achieved if a body is kept in a metallic shell or metallic enclosure irrespective of the shape of the enclosure. That is $E_{\text{in}} = 0$.

7. Electric field due to a thin sheet (thin sheet ≤ 200 A°) having linear charge density λ. The charge is distributed on both sides. Therefore $E = \frac{\sigma}{2\varepsilon_0}$.

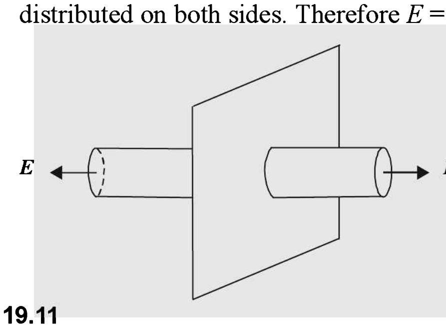


Fig. 19.11

8. Electric field due to a thick conducting sheet $E = \frac{\sigma}{\varepsilon_0}$

9. Work done in assembling the charged sphere of radius $R = \frac{3Q^2}{20\pi\varepsilon_0 R}$ = PE of the charged sphere.

10. Work done in assembling the charge on a shell $= \frac{Q^2}{8\pi\varepsilon_0 R}$ = *PE* of the charged shell spherical.

11. The electric field inside the capacitor sheets

$$E = \frac{\sigma}{2\varepsilon_0} + \frac{\sigma}{2\varepsilon_0} = \frac{\sigma}{\varepsilon_0} \text{ as shown in Fig 19.12.}$$

However electric field due to a single plate is $\frac{\sigma}{2\varepsilon_0}$.

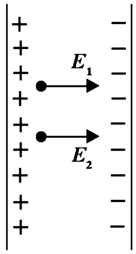


Fig. 19.12

12. Electric field in a long cylinder of radius R having charge per unit length λ

$$E_{\text{outside}} = \frac{\lambda}{2\pi\varepsilon_0 r} \text{ for } r > R$$

$$E_{\text{inside}} = 0 \text{ for } r < R.$$

13. Electric field due to long charged plates is uniform.

• Caution

1. Considering any electric field in $\oint E.ds$ will form flux.

⇒ $E_\perp$ which is parallel to surface vector $\vec{ds}$ will form flux. $E_{\parallel}$, which is parallel to the surface as shown in Fig 19.13, does not form any flux.

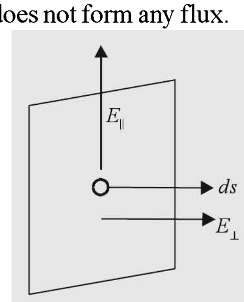


Fig. 19.13

2. Considering like a shell, electric field inside a charged sphere is also zero.

⇒ If the sphere has charge distributed only on its surface then $E_{\text{inside}} = 0$. If the charge is distributed throughout the volume then

$$E_{\text{inside}} = \frac{Qx}{4\pi\varepsilon_0 R^3} \text{ for } x < R.$$

3. Considering $E = 0$ if $V = 0$ or vice versa.

⇒ Inside a shell $E = 0$ but $V \neq 0$ Rather

$V_{\text{inside}} = \dfrac{Q}{4\pi\varepsilon_0 R} = V_{\text{surface}}$ and along the equatorial line of a dipole $V = 0$ but $E \neq 0$.

Note: $E = -\dfrac{dV}{dx}$ represents $E = 0$ if V is max, V is min or V = constant.

4. Considering $E_{\text{in}} = 0$ only in a shell (spherical).

⇒ $E_{\text{in}} = 0$ in any type of hollow metallic body. $E_{\text{in}} = 0$ even in a long metallic cylinder.

5. Considering $V_{\text{in}} = \dfrac{Q}{4\pi\varepsilon_0 R}$ in a charged sphere (charge Q, radius R).

⇒ $V_{\text{in}} = \dfrac{Q}{4\pi\varepsilon_0 R}$ inside a shell. If the charge is uniformly distributed throughout the volume then

$$V_{\text{in}} = \frac{Q}{4\pi\varepsilon_0 R} + \int_R^x \frac{-Qx}{4\pi\varepsilon_0 R^3}\, dx.$$

6. Not knowing the electric field lines direction in a metallic charged body.

⇒ Electric field lines are perpendicular to the surface because a metal body acts as an equipotential surface.

7. Considering equipotential surface has electric field intensity also equal at all points of the body.

⇒ Electric field is very large at pointed ends, sharp corners. i.e. $E \to \infty$ if $R \to 0$. In Fig 19.14 $E_B > E_C > E_A$. E is minimum at D.

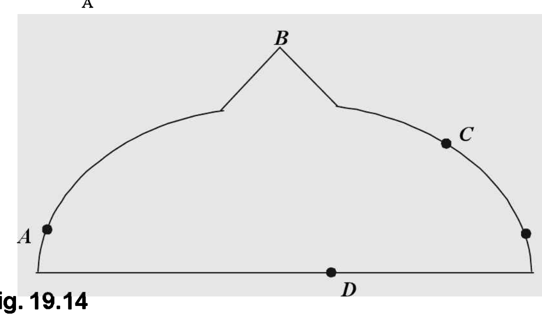


Fig. 19.14

SOLVED PROBLEMS

1. Three infinitely charged sheets are kept parallel to $x-y$ plane having charge densities as shown in Fig. 19.15. The electric field at P is

(IIT Screening 2005)

(a) $\dfrac{-4\sigma\hat{k}}{\varepsilon_0}$ (b) $\dfrac{4\sigma\hat{k}}{\varepsilon_0}$

(c) $\dfrac{-2\sigma}{\varepsilon_0}\hat{k}$ (d) $\dfrac{2\sigma}{\varepsilon_0}\hat{k}$

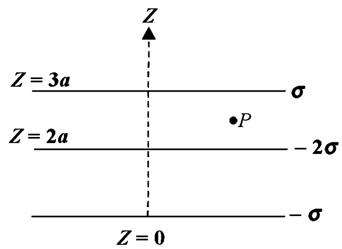


Fig. 19.15

Solution (c) $\vec{E} = \vec{E_1} + \vec{E_2} + \vec{E_3}$

$$= \frac{\sigma}{2\varepsilon_0}(-\hat{k}) + \left(\frac{2\sigma}{2\varepsilon_0}\right)(-\hat{k}) + \left(\frac{-\sigma}{2\varepsilon_0}\right)\hat{k} = \frac{-2\sigma\hat{k}}{\varepsilon_0}$$

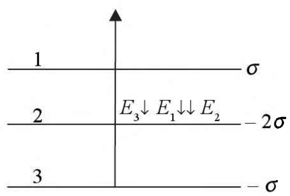


Fig. 19.16

2. A charged ball B hangs from a silk thread S which makes an angle θ with a large charged conducting sheet P as shown in Fig. 19.17 The surface charge density σ of the sheet is proportional to

(a) cos θ (b) cot θ
(c) sin θ (d) tan θ

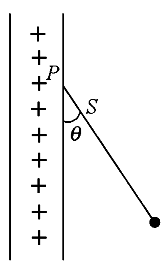


Fig. 19.17

Solution (d) Resolve T; $T\cos\theta = mg$ ....(1)

$$T\sin\theta = qE = q\frac{\sigma}{2\varepsilon_0} \quad ....(2)$$

Divide Eq (2) by (1)

$$\tan\theta = \frac{q\sigma}{2\varepsilon_0 g} \text{ i.e., } \sigma \propto \tan\theta$$

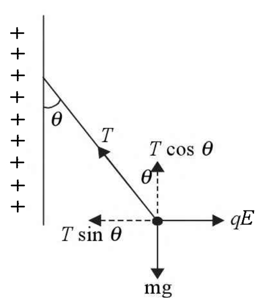


Fig. 19.18

(AIIMS 2005)

3. Two infinitely long parallel conduting plates having surface charge densities + σ and – σ respectively are separated by a small distance (Fig 19.19). The medium between the plates is vacuum. If ε_0 is the dielectric permittivity of vacuum then the electric field in the region between the plates is **(AIIMS 2005)**

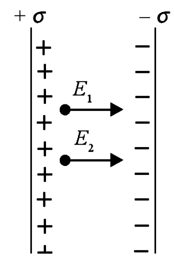


Fig. 19.19

(a) 0 Vm^{-1} (b) $\frac{\sigma}{2\varepsilon_0}$ Vm^{-1}

(c) $\frac{\sigma}{\varepsilon_0}$ Vm^{-1} (d) $\frac{2\sigma}{\varepsilon_0}$ Vm^{-1}

Solution (c) $E = E_1 + E_2 = \frac{\sigma}{2\varepsilon_0} + \frac{\sigma}{2\varepsilon_0} = \frac{\sigma}{\varepsilon_0}$.

4. A charge Q is distributed uniformly in a sphere (solid). Then the electric field at any point r where $r < R$ (R is radius of the sphere) varies as **(BHU 2005)**

(a) $r^{1/2}$ (b) r^{-1}
(c) r (d) r^{-2}

Solution (c) $E = \frac{Qr}{4\pi\varepsilon_0 R^3}$ i.e. $E \propto r$

5. An elliptical cavity is carved out in a perfect conductor. A positive charge q is placed at the centre of the cavity. The points A and B are shown in Fig. 19.20. Then

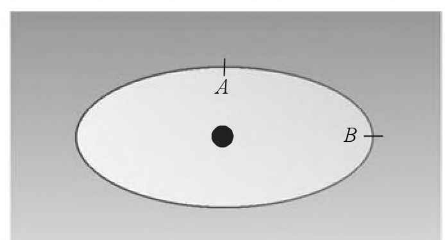


Fig. 19.20

(a) electric field near A in the cavity = electric field near B in the cavity.
(b) charge density at A = charge density at B
(c) potential at A = potential at B
(d) total electric field flux through the surface of the cavity = $\frac{q}{\varepsilon_0}$ **[IIT 1999]**

Solution (c) and (d) Because A and B lie on the same conductor, potential at each point is equal. For (d) Gauss law.

6. A solid conducting sphere having a charge Q is surrounded by an uncharged concentric conducting

hollow spherical shell. Let the potential difference between the surface of the solid sphere and that of the outer surface of the hollow shell be V. If the shell is now given a charge $-3Q$, the new potential difference between the same two surfaces is

(a) V (b) $2V$
(c) $4V$ (d) $-2V$.0

[IIT 1989, DCE 1995]

Solution (a) Because the potential difference between solid sphere and hollow shell depends on the radii of two spheres and charge q on the inner sphere. Since the two values have not changed, potential difference does not change.

7. A metallic solid sphere is placed in a uniform electric field. The lines of force follow the path(s) shown in Fig. 19.21 as

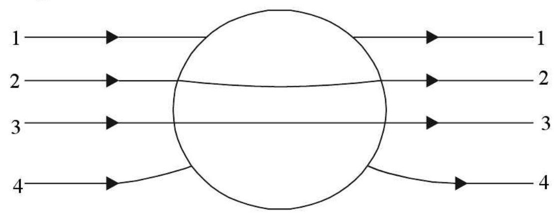


Fig. 19.21

(a) 1 (b) 2
(c) 3 (d) 4 **[IIT 1999]**

Solution (d) $E_{in} = 0$ and electric field lines are perpendicular to the equipotential surface.

8. Two conducting plates A and B are parallel. A is given a charge Q_1 and B is given a charge Q_2. The charge on inner side of B is

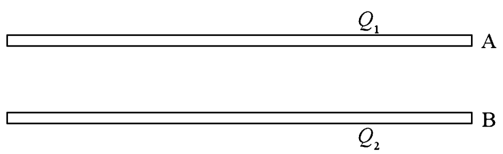


Fig. 19.22 (a)

(a) $\dfrac{Q_1 - Q_2}{2}$ (b) $\dfrac{(Q_1 - Q_2)}{2}$

(c) $\dfrac{(Q_1 + Q_2)}{2}$ (c) $\dfrac{-(Q_1 + Q_2)}{2}$

Solution (b) Electric field inside the conductor at point $P = 0$

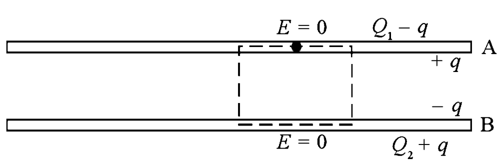


Fig. 19.22 (b)

$$\therefore \frac{(Q_1 - q)}{2A\varepsilon_0} - \frac{q}{2A\varepsilon_0} + \frac{q}{2A\varepsilon_0} - \frac{Q_2 + q}{2A\varepsilon_0} = 0$$

or $\quad Q_1 - q - (Q_2 + q) = 0$ or $q = \dfrac{Q_1 - Q_2}{2}$.

9. A hemisphere of radius r is placed in a uniform electric field of strength E. The electric flux through the hemisphere is

(a) $2E\pi r^2$ (b) $-E\pi r^2$
(c) $-2E\pi r^2$ (d) zero

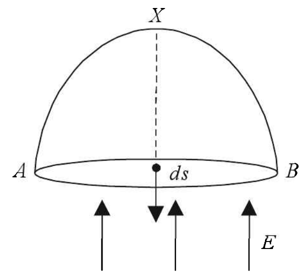


Fig. 19.23

Solution (b) $\phi = \oint E.ds = \int_A^B E.ds + \int_B^X E.ds + \int_X^A E.ds$

$= E\pi r^2 + E\pi r^2 - E\pi r^2 = -E\pi r^2$

Short cut: AXB is symmetrical surface $\therefore$ Electric flux due to this part is zero. However, electric flux due to AB part is $-E\pi R^2$

10. A positively charged sphere suspended with a silk thread is slowly pushed in a metal bucket. After its insertion the lid is closed. What will be the electric field intensity inside when the sphere has touched the bucket? σ is the surface charge density of sphere.

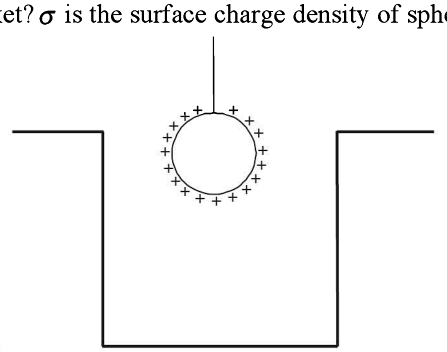


Fig. 19.24

(a) zero (b) $\dfrac{\sigma}{2\varepsilon_0}$

(c) $\dfrac{\sigma}{\varepsilon_0}$ (d) none of these

Solution (a) The charge of sphere will be taken by the bucket and appear on its outer surface only. $\therefore E_{in} = 0$

11. An electric field in a region is $800\sqrt{x}\,\hat{i}$. The charge contained in a cubical volume bounded by the surfaces $x = 0, x = a, y = 0, y = a, z = 0$ and $z = a$ is

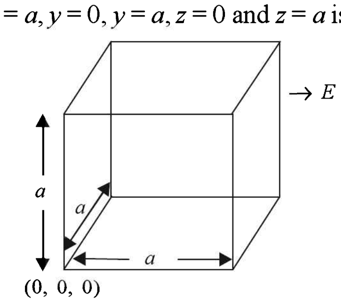


Fig. 19.25

(a) $\frac{800\sqrt{a}}{\frac{1}{2}}\varepsilon_0$ (b) $800\ a^{\frac{5}{2}}\varepsilon_0$
(c) $800\ a^2\varepsilon_0$ (d) $800\ \sqrt{a}\varepsilon_0$

Solution (b) $\phi = Ea^2$ and $Q = \phi\varepsilon_0 = Ea^2\varepsilon_0 = 800\ a^{\frac{5}{2}}\varepsilon_0$

12. A dipole is placed in a shell as shown. Find the electric flux emerging out of the shell and in a hypothetical sphere of radius r as shown in Fig. 19.26.

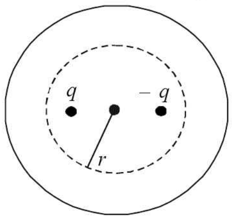


Fig. 19.26

(a) $\frac{2q}{\varepsilon_0}, 0$ (b) $\frac{q}{\varepsilon_0}, \frac{-q}{\varepsilon_0}$
(c) $\frac{q}{\varepsilon_0}, \frac{-q}{\varepsilon_0}$ (d) 0, 0

Solution (d)

13. Two concentric shells as shown enclose charge q and $-q$. The electric flux from the shell of radius R is

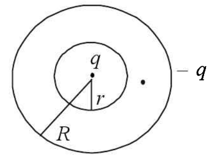


Fig. 19.27

(a) $\frac{-q}{\varepsilon_0}$ (b) $\frac{q}{\varepsilon_0}$
(c) $\frac{2q}{\varepsilon_0}$ (d) 0

Solution (d)

14. A charge Q is placed in a medium of dielectric constant K. The maximum number of lines of force are

(a) $\frac{Q}{\varepsilon_0}$ (b) $\frac{Q}{K}$
(c) $\frac{Q}{K\varepsilon_0}$ (d) 0

Solution (c)

TYPICAL PROBLEMS

15. An insulating sphere with radius a has a uniform charge density ρ. The sphere is not centered at the origin but at $\vec{r} = \vec{b}$. Find the electric field at any point inside the sphere.

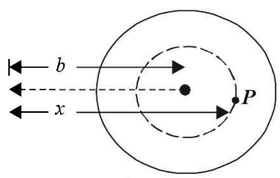


Fig. 19.28

Solution $Q_{\text{enclosed}} = \rho\frac{4}{3}\pi(x-b)^3$

$$\oint E.ds = \frac{\rho\frac{4}{3}(x-b)^3}{\varepsilon_0} \quad E\,4\pi(x-b)^2 = \frac{\rho\frac{4}{3}\pi(x-b)^3}{\varepsilon_0}$$

or $\vec{E} = \frac{\rho(\vec{x}-\vec{b})}{3\varepsilon_0}$

16. A charge Q is placed at the outer side of a cube. Find the flux passing through a face adjoining the charge

(a) $\frac{Q}{6\varepsilon_0}$ (b) $\frac{Q}{8\varepsilon_0}$
(c) $\frac{Q}{24\varepsilon_0}$ (d) $\frac{Q}{32\varepsilon_0}$

Solution (C) The charge will be completely covered by 8 cubes. 3 faces of each cube will share the flux. Hence 24 faces in all will share the flux

$$\therefore \oint E.ds = \frac{Q_{\text{inside}}}{\varepsilon_0} \text{ or } E(24\,a^2) = \frac{Q}{\varepsilon_0} \text{ or } Ea^2 - \frac{Q}{24\varepsilon_0}$$

17. An electron of 100eV is fired directly towards a metal plate having surface charge density $-2\times10^6\ \text{cm}^{-2}$. Find distance from where the electron be projected so that it just fails to strike the plate.

(a) 0.22 mm (b) 0.44 mm
(c) 0.66 mm (d) 0.33 mm

Solution (b) Gain in PE = loss in KE = $100e$

$$\frac{e\sigma}{\varepsilon_0}d \text{ or } d = \frac{100\times8.85\times10^{-12}}{2\times10^{-6}} = 0.44 \text{ mm}$$

18. A long conducting cylinder carrying a charge $+q$ is surrounded by a conducting cylindrical shell having charge $-2q$. The charge on the inner and outer side of conducting shell is

(a) $q, -q$ (b) $-q, -3q$
(c) $-q, +3q$ (d) $-q, -q$

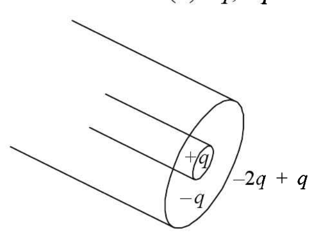


Fig. 19.29

Solution (d) Let $-q$ be the charge induced on the inner side then $-2q + q$ will be charge on outer side of the shell.

QUESTIONS FOR PRACTICE

1. A charge is kept at the centre of a shell. Shell has charge Q and radius R. The force on the central charge due to the shell is

(a) towards left (b) towards right
(c) upward (d) zero.

2. Fig. 19.30 (a) shows an imaginary cube of edge $L/2$. A uniformly charged rod of length L moves towards left at a small but constant speed v. At $t = 0$, the left end just touches the centre of the face of the cube opposite it. Which of the graphs shown in Fig. 19.30 (b) represents the flux of the electric field through the cube as the rod goes through it ?

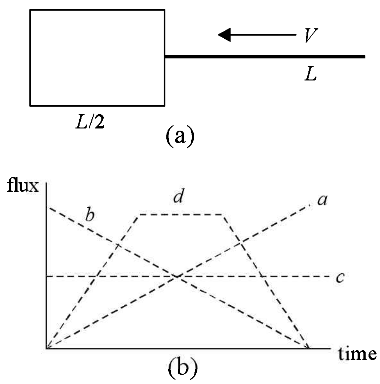


Fig. 19.30

3. Electric charges are distributed in a small volume. The flux of the electric field through a spherical surface of radius 10 cm surrounding the total charge is 25 V/m. The flux over a concentric sphere of radius 20 cm will be

(a) 25 V–m (b) 50 V–m
(c) 100 V–m (d) 200 V–m.

4. A charge q is placed at the centre of the open end of a cylindrical vessel (Fig. 19.31). The flux of the electric field through the surface of the vessel is

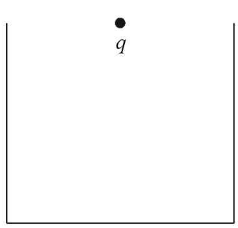


Fig. 19.31

(a) zero (b) $\frac{q}{\varepsilon_0}$
(c) $\frac{q}{2\varepsilon_0}$ (d) $\frac{2q}{\varepsilon_0}$.

5. Mark the correct options:

(a) Gauss's law is valid only for symmetrical charge distributions.
(b) Gauss's law is valid only for charges placed in vacuum.
(c) The electric field calculated by Gauss's law is the field due to the charges inside the Gaussian surface.
(d) The flux of the electric field through a closed surface due to all the charges is equal to the flux due to the charges enclosed by the surface.

6. A positive point charge Q is brought near an isolated metal cube.

(a) The cube becomes negatively charged.
(b) The cube becomes positively charged.
(c) The interior becomes positively charged and the surface becomes negatively charged.
(d) The interior remains charge free and the surface gets nonuniform charge distribution.

7. The electric field in a region is radially outwards and has a magnitude $E = Kr$. The charge contained in a sphere of radius a is

(a) $K4\pi\varepsilon_0 a^2$ (b) $K \frac{4}{3} \pi\varepsilon_0 a^3$
(c) $K4\pi\varepsilon_0 a^3$ (d) none of these

8. A charged particle having a charge -2×10^{-6} C is placed close to the non-conducting plate having a surface charge density 4×10^{-6} cm^{-2}. The force of attraction between the particle and the plate is nearly

(a) 0.9 N (b) 0.71 N
(c) 0.62 N (d) 0.45 N

9. A non-conducting sheet of large surface area and thickness d contains uniform charge density ρ. The electric field at a point P inside the plane at a distance x from the central plane $0 < x < d$

(a) $\frac{\rho x}{2\varepsilon_0}$ (b) $\frac{\rho d}{2\varepsilon_0}$
(c) $\frac{\rho x}{\varepsilon_0}$ (d) $\frac{\rho x}{2\varepsilon_0}$

10. A long cylinder contains uniformly distributed charge density ρ. The electric field at a point P inside the cylinder at a distance x from the axis is

(a) $\frac{\rho x}{\varepsilon_0}$ (b) $\frac{\rho x}{2\varepsilon_0}$
(c) $\frac{\rho x}{4\varepsilon_0}$ (d) none of these

11. A long cylindrical wire carries a linear density of 3×10^{-8} cm^{-1}. An electron revolves around it in a circular path under the influence of the attractive force. *KE* of the electron is

(a) 1.44×10^{-7} J (b) 2.88×10^{-17} J
(c) 4.32×10^{-17} J (d) 8.64×10^{-17} J

12. The electric field at a point 5 cm from a long line charge of density 2.5×10^{-6} cm^{-1} is

(a) 9×10^3 NC^{-1} (b) 9×10^4 NC^{-1}
(c) 9×10^5 NC^{-1} (d) 9×10^6 NC^{-1}

13. A charge q is uniformly distributed in the hollow sphere of radii r_1 and r_2 ($r_2 > r_1$). The electric field at a point P distance x from the centre for $r_1 < x < r_2$ is

(a) $\frac{Q(x)}{4\pi\varepsilon_0(r_2^3 - r_1^3)}$ (b) $\frac{Q(x^3 - r_1^3)}{4\pi\varepsilon_0(r_2^3 - r_1^3)}$

(c) $\frac{Q(x^3 - r_1^3)}{4\pi\varepsilon_0 x^2(r_2^3 - r_1^3)}$ (d) $\frac{Qr_1^3}{4\pi\varepsilon_0 x^2(r_3^3 - r_1^3)}$

14. The radius of gold nucleus is about 7×10^{-15} m ($Z = 79$). The electric field at the mid-point of the radius assuming charge is uniformly distributed is

(a) 1.16×10^{19} NC^{-1} (b) 1.16×10^{21} NC^{-1}
(c) 2.32×10^{21} NC^{-1} (d) 2.32×10^{19} NC^{-1}

15. A spherical volume has a uniformly distributed charge density 2×10^{-4} cm^{-3}. The electric field at a point inside the volume at a distance 4.0 cm from the centre is

(a) 3.15×10^5 NC^{-1} (b) 2.1×10^5 NC^{-1}
(c) 6.2×10^5 NC^{-1} (d) none of these

16. A charge Q is placed at the centre of a cube. The flux through the six surfaces of the cube is

(a) $\frac{Q}{\varepsilon_0}$ (b) $\frac{6Q}{\varepsilon_0}$

(c) $\frac{Q}{6\varepsilon_0}$ (d) $\frac{Q}{3\varepsilon_0}$

17. The electric field in a region is

$$E = \frac{5\times10^3\left(\text{NC}^{-1}\text{cm}^{-1}\right)x}{2}\hat{i}.$$

The charge contained inside a cubical volume bounded by the surfaces $x = 0$, $x = 1$, $y = 0$, $y = 1$, $z = 0$, $z = 1$ is (where x, y, z are in cm)

(a) 2.21×10^{-12} C (b) 4.42×10^{-12} C
(c) 2.21×10^{-8} C (d) 4.42×10^{-8} C

18. A charge Q is uniformly distributed over a rod of length l. Consider a hypothetical cube of edge l with the centre of the cube at one end of the rod. The minimum possible flux of the electric field through the entire surface of the cube is

(a) $\frac{Q}{\varepsilon_0}$ (b) $\frac{Q}{2\varepsilon_0}$

(c) $\frac{Q}{8\varepsilon_0}$ (d) $\frac{Q}{6\varepsilon_0}$

19. The electric field in a region is given by $\vec{E} = \frac{3}{5}E_0\hat{j}$ with $E_0 = 2 \times 10^3$ NC^{-1}. Find the flux of this field through a rectangular surface of area 0.2 m^2 parallel to the Y-Z plane.

(a) 320 Nm^{2}C^{-1} (b) 240 Nm^{2}C^{-1}
(c) 400 Nm^{2}C^{-1} (d) none of these

20. A positive point charge Q is brought near an isolated metal cube.

(a) The cube becomes negatively charged.
(b) The cube becomes positively charged.
(c) The interior becomes positively charged and the surface becomes negatively charged.
(d) The interior remains charge free and the surface gets non-uniform charge distribution.

21. Mark the correct options.

(a) Gauss's law is valid only for symmetrical charge distribution.
(b) Gauss's law is valid only for charges placed is vacuum.
(c) The electric field calculated by Gauss's law is the field due to the charges inside the Gaussian surface.
(d) The flux of the electric field through a closed surface due to all the charges is equal to the flux due to the charges enclosed by the surface.

22. Electric charges are distributed in a small volume. The flux of the electric field through a spherical surface of radius 10 cm surrounding the total charge is 25 V-m. The flux over concentric sphere of radius 20 cm will be

(a) 25 Vm (b) 50 Vm
(c) 100 Vm (d) 200 Vm

23. A metallic particle having no net charge is placed near a finite metal plate carrying a positive charge. The electric force on the particle will be

(a) towards the plate (b) away from the plate
(c) parallel to the plate (d) zero

Answers to Questions for Practice

1.	(b)	**2.**	(d)	**3.**	(a)	**4.**	(c)	**5.**	(d)	**6.**	(d)	**7.**	(c)
8.	(d)	**9.**	(c)	**10.**	(a)	**11.**	(c)	**12.**	(c)	**13.**	(c)	**14.**	(b)
15.	(a)	**16.**	(a)	**17.**	(a)	**18.**	(b)	**19.**	(b)	**20.**	(d)	**21.**	(d)
22.	(a)	**23.**	(a)										

20

Capacitor

BRIEF REVIEW

A capacitor is a device to store charge or electrostatic energy.

Capacitance It is the capacity of a capacitor to store charge. In a capacitor $Q \propto V$ or $Q = CV$; C is called the capacitance.

$$C = (M^{-1}L^{-2}T^{4}A^{2})$$

Fig. 20.1 (a) or (b) represent circuit symbol for a simple capacitor, Fig. 20.1 (c) represents electrolytic and Fig 20.1 (d) represents variable capacitor (tuner or trimmer).

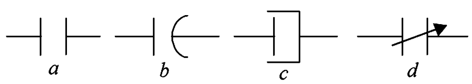


Fig. 20.1

From the point of view of the shape of capacitors, they are of three types: spherical, parallel plate and cylindrical.

Unit of capacitance is Farad $1F = \dfrac{1C}{1V}$

$1F$ is a very big unit. Therefore smaller units like μF, nF or $\mu\mu F$ (also called pF) are used very commonly.

Spherical Capacitors These are of two types:

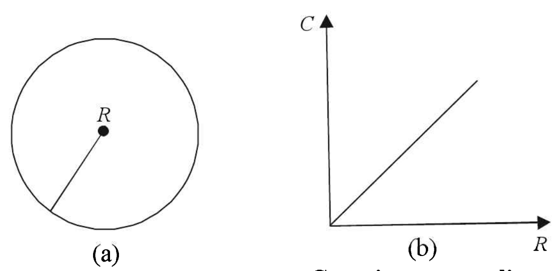
(a) Isolated spherical capacitors

(b) Concentric spherical capacitors

(a) Isolated spherical capacitor consists of a single sphere. Its capacitance $C = 4\pi\varepsilon_0 R$ i.e. $C \propto R$, where R is radius of the sphere. See Fig. 20.2.

(b) Two spherical shells (or inner one may be solid) form a concentric spherical capacitor as shown in Fig. 20.3. Note that normally outer sphere is grounded.

$$C = 4\pi\varepsilon_0 \frac{R_1 R_2}{R_2 - R_1}$$



(a) **Isolated capacitor**

(b) **Capacitance vs radius**

Fig. 20.2

If a dielectric of strength K is introduced between R_1 and R_2

$$C = 4\pi\varepsilon_0 K \frac{R_1 R_2}{R_2 - R_1}$$

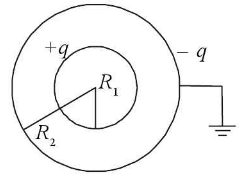


Fig. 20.3 Concentric shell capacitor

Parallel Plate Capacitor If two plates each of area A are separated by a distance d in vacuum as shown in Fig. 20.4

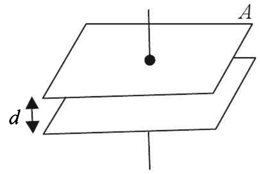


Fig. 20.4 Parallel plate capacitor

then $C = \dfrac{\varepsilon_0 A}{d}$

$C = \dfrac{k\varepsilon_0 A}{d}$ if a dielectric of strength k is completely filled in the gap.

$C = \dfrac{\varepsilon_0 A}{d - t\left(1 - \dfrac{1}{k}\right)}$ if the dielectric slab has thickness $t(t < d)$

Capacitance of a cylindrical capacitor shown in Fig. 20.5 is

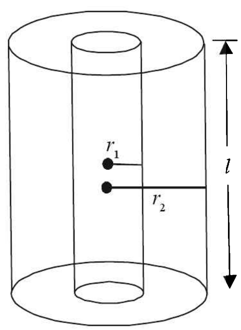


Fig. 20.5 Cylindrical capacitor

$C = \dfrac{2\pi\varepsilon_0 l}{\log_e \dfrac{r_2}{r_1}}$ and

capacitance per unit length $\dfrac{C}{l} = \dfrac{2\pi\varepsilon_0}{\log_e \dfrac{r_2}{r_1}}$

If the space between two cylinders is filled with a dielectric of strength k then $C = \dfrac{2\pi\varepsilon_0 kl}{\log_e \dfrac{r_2}{r_1}}$

Magnitude of induced charge in a dielectric of strength k is $Q_p = Q\left(1 - \dfrac{1}{k}\right)$

Force between the plates of a capacitor is attractive and its magnitude is $F = \dfrac{Q^2}{2A\varepsilon_0}$

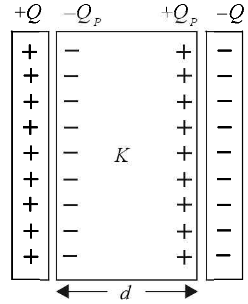


Fig. 20.6 Polarization illustration

Energy stored in a capacitor

$$U = \frac{1}{2}CV^2 = \frac{QV}{2} = \frac{Q^2}{2C}$$

If the charge is uniformly distributed throughout the volume then energy stored is $U = \dfrac{1}{2}\int V\rho dv$ where dv is volume element and V is potential difference . Volume density of electric field energy

$$u = \frac{ED}{2} = \frac{\varepsilon_0 E^2}{2} \text{ in free space}$$

volume density of electric field energy in a medium

$$u_{\text{med.}} = \frac{\varepsilon_0 KE^2}{2} = \frac{\varepsilon_0 \varepsilon_r E^2}{2}$$

The maximum capacitance of a tuner capacitor (used for tuning in radio) is $C = \dfrac{\varepsilon_0 A(n-1)}{d}$ where A is the area of each plate, n is total number of plates and d is separation between two successive plates. Normally a 11-plate tuner capacitor is available whose ratio of maximum to minimum capacity is 10 : 1.

Capacitor in series See Fig 20.7. In series, magnitude of the charge on each plate is equal but voltage across each capacitor is different.

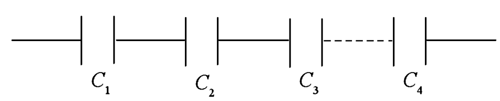


Fig. 20.7 Capacitors in Series

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \text{..........} + \frac{1}{C_n}$$

If n equal capacitors are in series then $C_{\text{eq}} = \dfrac{C}{n}$

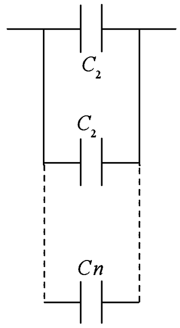


Fig. 20.8 Capacitors in Parallel

Capacitors in parallel If $C_1, C_2, C_3, ...C_n$ are connected in parallel as shown in Fig. 20.8 then

$$C_{eq} = C_1 + C_2 + C_3 ....+ C_n$$

Note that in parallel, charge on each capacitor is different while potential drop or voltage across each capacitor is equal.

If n equal capacitors are in parallel then $C_{eq} = nC$.

There are four methods to simplify capacitance networks

(a) Series parallel method
(b) Wheatstone bridge method
(c) Charge distribution method
(d) Star/delta method.

Wheatstone bridge cases Fig. 20.9 illustrates some common representations of Wheatstone bridge.

If $\frac{C_1}{C_2} = \frac{C_3}{C_4}$ then remove C_5 and simplify.

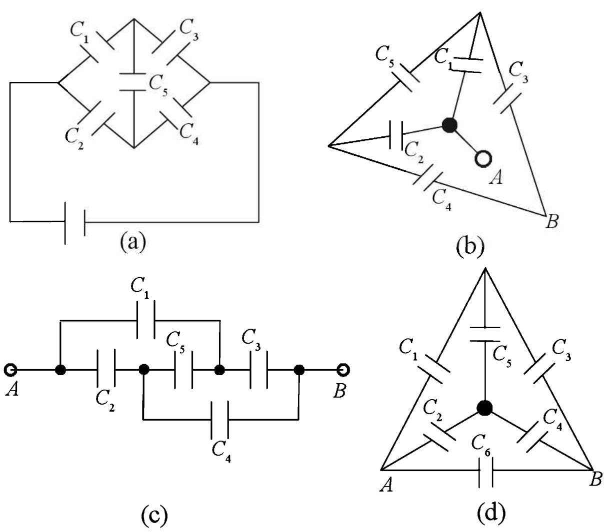


Fig. 20.9 Common representations of Wheatstone bridge

If in a Wheatstone bridge each capacitor is C then $C_{eq} = C$

Charge distribution method It can be applied in principle anywhere in tune with Kirchhoff's law but in symmetrical circuits it makes the problem very simple. In symmetrical circuits charge entering a branch = charge leaving the branch (identical) or mirror image branch.

If two capacitors C_1 and C_2 charged to V_1 and V_2 are joined together then common potential is

$$V_{common} = \frac{V_1C_1 + V_2C_2}{C_1 + C_2} = \frac{Q_1 + Q_2}{C_1 + C_2} \qquad \text{(Fig. 20.10)}$$

Charge on capacitors after joining $\frac{Q_1'}{Q_2'} = \frac{C_1}{C_2}$

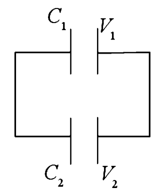


Fig. 20.10 Common potential

Loss in energy when two capacitors C_1 and C_2 charged to V_1 and V_2 are joined together as shown in Fig. 20.10 is

$$\Delta E = \frac{C_1C_2}{(C_1 + C_2)}(V_1 - V_2)$$

If dielectrics are added in the manner shown in Fig. 20.11 (a) then net capacitance is in a parallel combination of C_1, C_2 and C_3 as illustrated in Fig. 20.11 (b)

$$C_1 = \frac{\varepsilon_0 k_1 A/3}{d}, \qquad C_2 = \frac{\varepsilon_0 k_2 A/3}{d},$$

$$C_3 = \frac{\varepsilon_0 k_3 A/3}{d} \text{ and } \qquad C = C_1 + C_2 + C_3$$

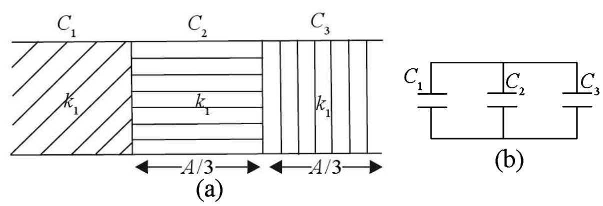


Fig. 20.11 Effect of dielectrics on capacitor

If dielectrics are arranged as shown in Fig. 20.12 (a) then C_{eq} is series combination of C_1, C_2 and C_3 as illustrated in Fig 20.12 (b)

$$C_1 = \frac{\varepsilon_0 K_1 A}{\frac{d}{3}}$$

$$C_2 = \frac{\varepsilon_0 K_2 A}{\frac{d}{3}}$$

$$C_3 = \frac{\varepsilon_0 K_3 A}{\frac{d}{3}}$$

$$C_{eq} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} = \frac{d}{3\varepsilon_0 A}\left[\frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3}\right]$$

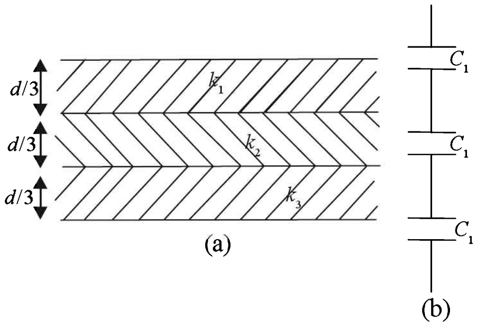


Fig. 20.12 Effect of dielectrics in a capacitor

If a dielectric slab in a capacitor is being introduced in the rigidly held plates connected across a battery of *emf* V'_0 then the force required to insert the slab is

$$F = \frac{1}{2}V'^2 \frac{dC}{dx}$$

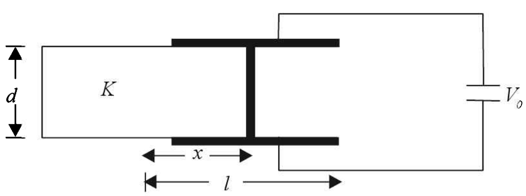


Fig. 20.13 Force required during introduction of dielectric in a capacitor

Charging of a capacitor or growth transient

When the switch is made ON at $t = 0$, current passes through capacitor for a very short time during its charging spree. The variation of charge/voltage across the capacitor is called charging transient. See Fig 20.14 (a) and (b)

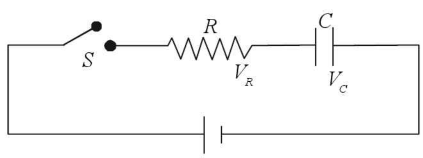


Fig. 20.14 (a) Charging of a capacitor

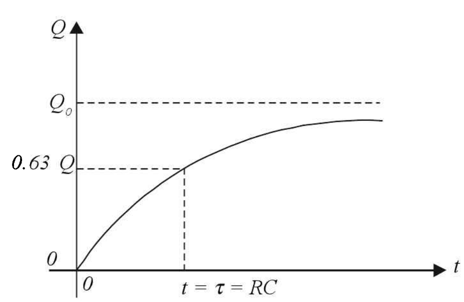


Fig. 20.14 (b) Charging transient

$$Q = Q_0(1 - e^{-t/RC}) \qquad \text{where } Q_0 = CV'_0$$

$$I = \frac{dQ}{dt} = \frac{Q_0}{RC} e^{-t/RC} \qquad \text{and } V'_R = IR$$

Time constant $\tau = RC$ is the time in which capacitor charges to 63% of its maximum value of charge.

Discharging of a capacitor (or decay transient)

When the capacitor has been charged for a long time. It is connected to a resistance R through a switch S as shown in Fig 20.15 (a). At $t = 0$, switch is closed and the capacitor starts discharging according to the equation.

$Q = Q_0\, e^{-t/RC}$ Fig 20.15 (b) shows discharging transient.

Time constant $\tau = RC$ is the time in which a capacitor discharges to 36% of its maximum value.

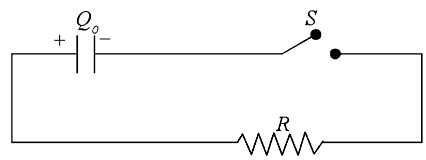


Fig. 20.15 (a) Discharging of a capacitor

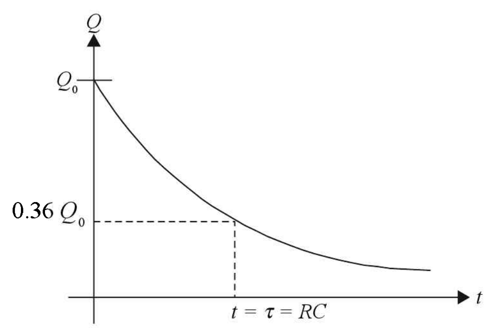


Fig. 20.15 (b) Discharging transient

Important functions of a capacitor

(i) In a timer (time setting in almost all automatic devices)

(ii) time base circuit in CRO (sawtooth generator).

(iii) filter circuits (low pass, high pass, band pass, band reject)

(iv) oscillators (LC oscillator $f_0 = \frac{1}{2\pi\sqrt{LC}}$) and RC oscillators

(v) tuner circuit in radio

(vi) as a trimmer in frequency setting with quartz oscillator

(vii) integrating and differentiating circuits

(viii) voltage multiplier

(ix) peak detector

(x) demodulator or detection

(xi) clamping circuits.

(xii) 0 – 90° phase shift producer in one –RC section and 0 –180° phase shift in 3-RC sections.

(xiii) in AC motor to enhance torque

(xiv) converts active power into wattless or passive power.

If n drops each of radius r and charge q combine to form a big drop of radius R then charge on big drop is

$$Q_{\text{big}} = nq$$

$$C_{\text{big}} = n^{1/3}\, C_{\text{small}};$$

$$V_{\text{big}} = n^{2/3}\, V_{\text{small}} \text{ and } R = n^{1/3} r$$

Capacitance of a transmission line as shown in Fig 20.16 is given by $C = \dfrac{\pi\varepsilon_0 Kl}{\log_e \dfrac{d}{r}}$

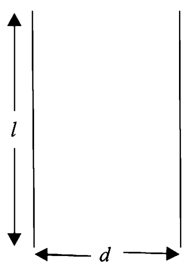


Fig. 20.16 Capacitance of transmission line

where K is dielectric constant of the material between two wires, r is radius of either wire.

Connecting wires offer stray capacitance and conducting wires or conducting points of a device offer parasitic capacitance.

• Short Cuts and Points to Note

1. If the outer shell is grounded as in Fig 20.17 (a)

then $C = \dfrac{\varepsilon_0 k r_1 r_2}{r_2 - r_1}$

If inner sphere is grounded as in Fig 20.17 (b) then

$C = \dfrac{\varepsilon_0 k r_1 r_2}{(r_2 - r_1)} + 4\pi\varepsilon_0 r_2$ because $\dfrac{\varepsilon_0 k r_1 r_2}{r_2 - r_1}$ and capacitance of isolated sphere (outer) $4\pi\varepsilon_0 r_2$ are in parallel.

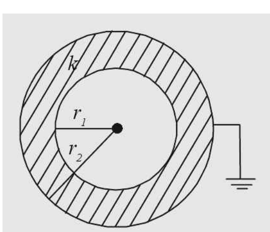


Fig. 20.17 (a)

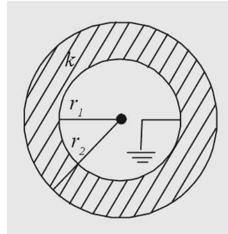


Fig. 20.17 (b)

2. The potential drops V_1 and V_2 across capacitors C_1 and C_2 are in the inverse ratio of their capacitances.

i.e. $V_1 = \dfrac{V_0 C_2}{C_1 + C_2}$ and $V_2 = \dfrac{V_0 C_1}{C_1 + C_2}$

Also $V_1 = V_0 - V_2$

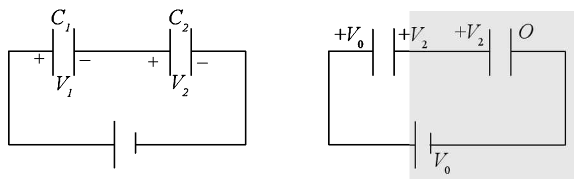


Fig. 20.18 Potential on each plate of capacitor

3. If two capacitors are in series then $C_{\text{eq}} = \dfrac{C_1 C_2}{C_1 + C_2}$. If n identical capacitors are in series $C_{\text{eq}} = \dfrac{C}{n}$ and if n identical capacitors are in parallel $C_{\text{eq}} = nC$.

4. In series, charge remains same and potential across the capacitors may be different. In parallel, potential difference across each capacitor is equal and charge on the capacitors may be different.

5. It two spheres of radius r_1 and r_2 are joined by a conducting wire or directly then common potential $V = \dfrac{Q_1 + Q_2}{4\pi\varepsilon_0 (r_1 + r_2)}$ (See Fig 20.19.).

and charge after joining $Q_1' = (Q_1 + Q_2)\dfrac{r_1}{r_1 + r_2}$

$Q_2' = (Q_1 + Q_2)\dfrac{r_2}{r_1 + r_2}$

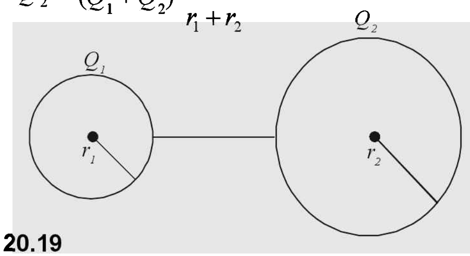


Fig. 20.19

6. To find potential drop across the capacitors in questions as shown is Fig 20.20 (a), convert it to equivalent circuit of Fig 20.20 (b) and then solve it using concept of point 2.

If negative terminal of one battery is connected to the positive terminal of other battery then $V_{net} = V_{01} + V_{02}$ as batteries are in series. It negative terminal of one battery is connected to negative of the other, then $V_{net} = V_{01} - V_{02}$ or $V_{02} - V_{01}$ depending upon which is greater. The net emf has direction of greater emf battery.

From Fig 20.20 (b) $V_1 = \dfrac{(V_{02}-V_{01})C_2}{C_1+C_2}$ and $V_2 = \dfrac{(V_{02}-V_{01})C_1}{C_1+C_2}$

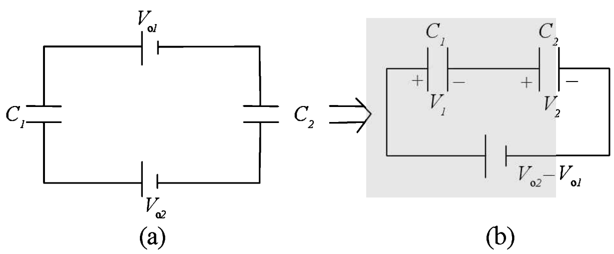


Fig. 20.20

7. If n identical plates each of area A are connected alternately and separation between two consecutive plates is d, then $C_{eq} = (n-1)\dfrac{A\varepsilon_0}{d}$. For example $C_{eq} = \dfrac{3A\varepsilon_0}{d}$ in Fig 20.21

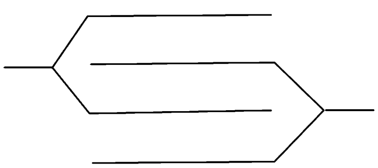


Fig. 20.21

8. If a metal plate of thickness t is introduced in between the plates of a capacitor separated by d then $C_{eq} = \dfrac{A\varepsilon_0}{(d-t)}$ (See fig 20.22)

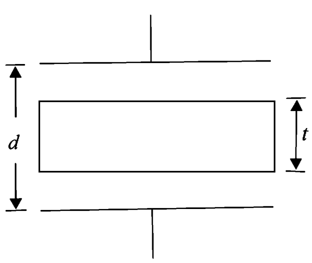


Fig. 20.22

9. If large number of identical capacitors of rating C/V are available and capacitor C^1/nV is to be designed then n capacitors are to be connected in series. Each row of n capacitors has $C_{eq} = \dfrac{C}{n}$. To make C^1, then $m = \dfrac{C^1}{C/n}$ rows of n capacitors in series will be required.

10. For the network shown in Fig 20.23

$$C_{eq} = C_{AB} = \frac{2C_1C_2 + C_1C_3 + C_2C_3}{C_1 + C_2 + 2C_3}$$

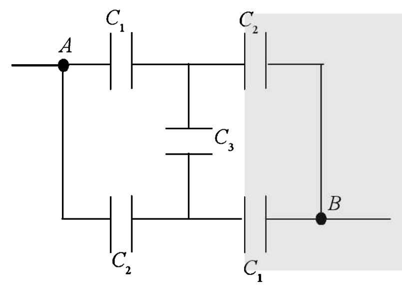


Fig. 20.23

11. If capacitor C is connected along each side of a skeleton cube then equivalent capacitance along the longest diagonal is $\dfrac{6C}{5}$. Equivalent capacitance along face diagonal is $\dfrac{4}{3}C$ and along one side is $\dfrac{12}{7}C$.

12. If one side of skeleton cube is open as shown in Fig 20.24 then $C_{AB} = \dfrac{5}{7}C$.

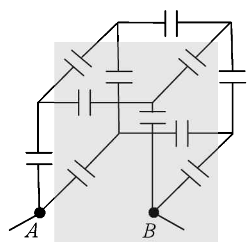


Fig. 20.24

13. If one or more shells of concentric shell system is/are grounded then net potential corresponding to grounded shells is zero. For example in Fig 20.25 $V_b = 0$

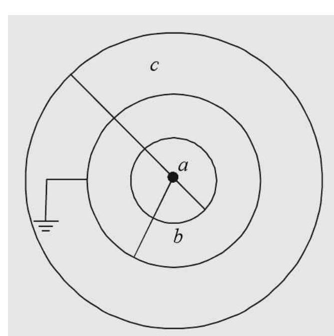


Fig. 20.25

14. If in a charged capacitor (battery not connected) a dielectric is added then charge remains the same, C increases, V decreases, electric field decreases and energy stored decreases each by a factor of dielectric constant.

15. If in a capacitor (connected to a battery) a dielectric is added, then V remains unchanged, C increases K–fold, electric field remains unchanged, energy stored in the capacitor increases K– fold, where K is dielectric constant.

16. If the battery is disconnected after the capacitor is charged and plates of capacitors are moved away then C decreases, Q remains unchanged, V increases, electric field remains unchanged and energy stored increases.

17. To find time of charging at a particular instant t, use $t = \tau \log_e \dfrac{Q_0}{Q_0 - Q}$ or $t = 2.303\, RC \log_{10} \dfrac{Q_0}{Q_0 - Q}$. Similarly discharge time is $t = 2.303\, RC \log_{10} \dfrac{Q_0}{Q}$.

18. A capacitor charges to 63.3% in one RC, 90% in 2.303 RC, 95% in 3RC, and, 99% in 5RC.

19. DC current does not pass through the capacitor except during charging or discharging for a short while. $\dfrac{dQ}{dt} = i = \dfrac{CdV}{dt}$. If V is constant, $i = 0$. i.e. DC current passes during transients only.

20. AC current passes through the capacitor. Displacement current and conduction current have a phase shift of 90°.

21. Since the plates of a capacitor are thin, if a charge Q is placed $Q/2$ appears on one side, $Q/2$ appears on other side and $-Q/2$ charge is induced on the inner side of plate as shown in Fig. 20.26 (b)

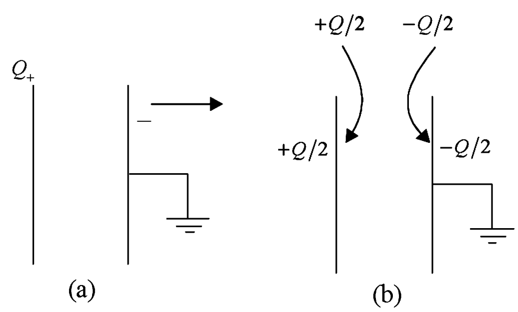


Fig. 20.26

22. When a thin metal sheet is introduced in between the space in a parallel plate capacitor then capacitance remains unchanged.

- **Caution**

1. Treating spherical charges of unequal radius like point charges when they are joined.

⇒ Charged spheres behave as capacitors and hence charge is distributed in accordance to capacitive laws. i.e. charge after joining the two spheres is proportional to the radius i.e. $Q_1' = \dfrac{(Q_1 + Q_2) r_1}{r_1 + r_2}$ and $Q_2' = \dfrac{(Q_1 + Q_2) r_2}{r_1 + r_2}$

2. Confusing whether on increasing or decreasing the distance between the plates of a capacitor (the battery removed after charging it) voltage remains constant or not.

⇒ When the battery is removed charge is conserved, i.e. charge remains constant and voltage increases or decreases depending upon the fact that separation between the plates is increased or decreased.

3. Confusing whether or not current passes through a capacitor.

⇒ DC Current does not pass through capacitor except during growth and decay transient. AC current passes through the capacitor.

4. Confusing that capacitors are added in series and parallel like resistors.

⇒ Capacitors in series are added according to the law $\dfrac{1}{C_{eq}} = \dfrac{1}{C_1} + \dfrac{1}{C_2} + \ldots$ and in parallel $C_{eq} = C_1 + C_2 + \ldots$.

5. Confusion in series and parallel cases.

⇒ Note that in series only one end of a capacitor is connected to one end of the other and in parallel both ends of the capacitors are joined with two ends of other capacitors as shown in Fig 20.27 (a) and 20.27 (b) respectively.

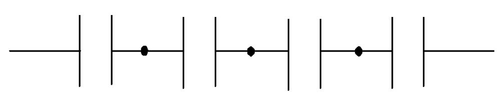


Fig. 20.27 (a)

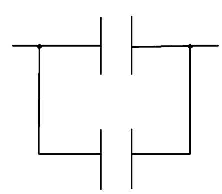


Fig. 20.27 (b)

6. Confusing in cases as shown in Fig 20.28 whether capacitors are in series or parallel.

⇒ Capacitors are in series in this case as battery is another element present in between and hence capacitors are not connected end to end.

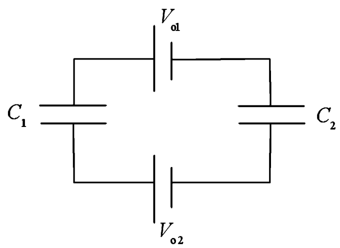


Fig. 20.28

7. Confusing whether new capacitors formed due to addition of dielectrics are is series or parallel.

⇒ Look into Fig. 20.29 (a) and Fig. 20.29 (b) carefully If dielectric divides the capacitors horizontally, they are in series and if the space between the capacitors is divided vertically then capacitors so formed are in parallel.

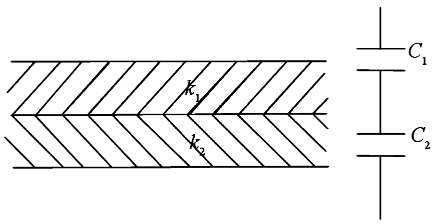


Fig. 20.29 (a)

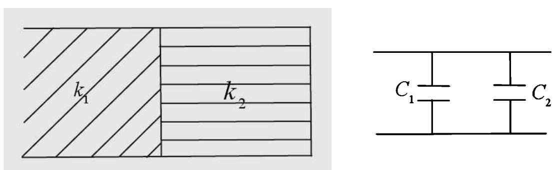


Fig. 20.29 (b)

8. Not understanding the effect of rating of the capacitor.

⇒ If a capacitor is marked 10 $\mu F/250V$ then it cannot hold a charge >2500 μc. If two capacitors of different rating are joined in series, then we cannot supply a charge greater than the rating value of smaller charge as charge remains same in series.

9. If more than two plates are connected at a point then difficulty in recognising series or parallel case.

⇒ Mark the plates 1, 2, 3,... etc. and reconstruct a simplified circuit so that you can easily recognise series and parallel case.

10. Confusion about Wheatstone bridge. Considering in the circuit shown in Fig 20.30 (a) AOPL, APML, AONP, LPBM, PONB, MPNB etc. as wheat stone bridge.

⇒ Note that AOPL, APML, etc. in Fig 20.30 (a) are not Wheatstone bridges. For a circuit to qualify as Wheatstone bridge $\frac{C_1}{C_2} = \frac{C_3}{C_4}$ then, remove C_5 as shown in Fig. 20.30 (b).

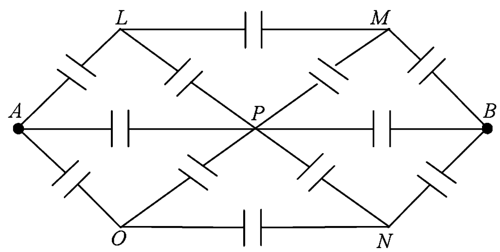


Fig. 20.30 (a)

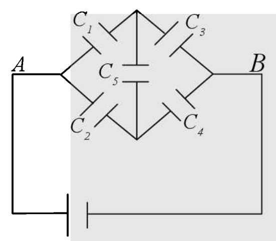


Fig. 20.30 (b)

SOLVED PROBLEMS

1. A capacitor has charge 50 μC. When the gap between the plates is filled with glass wool 120 μC charge flows through the battery. The dielectric constant of glass wool is

(a) 3.4 (b) 1.4
(c) 2.4 (d) none of these

Solution (a) $K = \frac{Q'}{Q} = \frac{120+50}{50} = 3.4$

2. A charge of 1 μC is given to one plate of a capacitor and a charge of 2 μC is given to the other plate of a 0.1 μF capacitor. Find the potential difference across the two plates of the capacitor.

(a) 5 V (b) 10 V
(c) 15 V (d) 30 V

Solution (a) $Q_{net} = 2 - 1 = 1\ \mu$C. charge = $\frac{1}{2}$ μC will appear on each side of the plate as illustrated in Fig. 20.31.

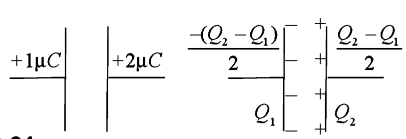


Fig. 20.31

$$\therefore V = \frac{Q_{net}/2}{C} = \frac{0.5}{0.1} = 5\,V$$

3. A large conducting plane has surface charge density 10^{-4} C/m^2. Find the electrostatic energy stored in a cubical volume of side 1cm in front of the plane.

(a) 1.4 J (b) 2.8 J
(c) 5.6 J (d) none of these

Solution (c) $U = \frac{1}{2}\varepsilon_0 E^2(\text{Vol.}) = \frac{1}{2}\varepsilon_0\left(\frac{\sigma}{\varepsilon_0}\right)^2(\text{Volume}) =$

$$\frac{\sigma^2}{2\varepsilon_0}(\text{Vol}) = \frac{(10^{-4})\times 10^{-6}}{2\times 8.85\times 10^{-12}} = \frac{100}{2\times 8.85} = 5.6\text{ J}$$

4. In the network shown $C_1 = C, C_2 = 2C, C_3 = 3C, C_4 = 4C$ find the ratio of charge C_2 to C_4.

(a) $4/7$ (b) $22/3$
(c) $7/4$ (d) $3/22$

(CBSE, 2005)

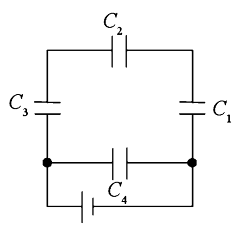


Fig. 20.32

Solution (d) C_2, C_1 and C_3 in series $\frac{1}{C_{eq}} = \frac{1}{C} + \frac{1}{2C} +$

$$\frac{1}{3C} = \frac{11}{6C} \text{ or } C_{eq} = \frac{6C}{11} = \frac{Q_2}{Q_4} = \frac{C_{eq}V_0}{C_4V_0} = \frac{6/11}{4} = \frac{3}{22}$$

5. A fully charged capacitor has a capacitance 'C'. It is discharged through a small coil of resistance wire embedded in a thermally insulated block of specific heat capacity *s* and mass *m*. If the temperature of the block is raised by ΔT, the potential difference V across the capacitance is

(a) $\sqrt{\frac{2mC\Delta T}{s}}$ (b) $\frac{mC\Delta T}{s}$

(c) $\frac{ms\Delta T}{C}$ (d) $\sqrt{\frac{2ms\Delta T}{C}}$

[AIEEE 2005]

Solution (d) $\frac{1}{2}CV^2 = ms\Delta T$ or $V = \sqrt{\frac{2ms\Delta T}{C}}$

6. A parallel plate capacitor is formed by stacking *n* equally spaced plates connected alternately. If capacitance between two adjacent plates is *C* then the resultant capacitance is

(a) $(n-1)C$ (b) $(n+1)C$
(c) C (d) nC

[AIEEE 2005]

Solution (a) $C_{eq} = (n-1)C$ (one less than the number of plates)

7. An air filled parallel plate capacitor has a capacity $2\,pF$. The separation between the plates is doubled and the inter space is filled with wax. If the capacity is increased to $6\,pF$, the dielectric constant of the wax is

(a) 2 (b) 4
(c) 3 (d) 6

[CET Karnataka 2005]

Solution (d) $K = \frac{C''}{C'} = \frac{6pF}{C/2} = \frac{6pF}{1pF} = 6$

8. If each capacitor has capacitance *C* in Fig. 20.33 (a) then find C_{AB}

(a) C (b) $C/2$
(c) $3C/2$ (d) none of these

Solution (c) Look into equivalent Fig. 20.33 (b) and (c). The dotted part is Wheatstone bridge with $C_{eq} = C$ then further equivalent circuit is shown in Fig. 20.33 (c).

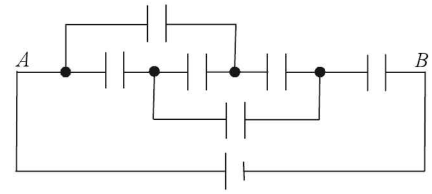


Fig. 20.33 (a)

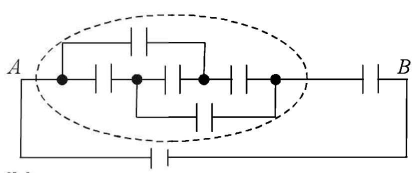


Fig. 20.33 (b)

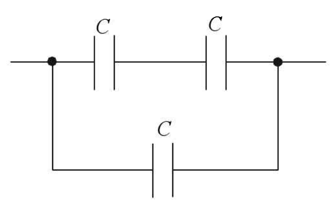


Fig. 20.33 (c)

9. Square plates of area $2a^2$ are filled with dielectric of strength k_1, k_2 and k_3 as shown in Fig 20.34. Find C_{eq}.

(a) $\frac{\varepsilon_0 a^2(k_1k_3 + k_2k_3)}{d(k_1+k_2+k_3)}$ (b) $\frac{2\varepsilon_0 a^2(k_1k_3 + k_2k_3)}{d(k_1+k_2+2k_3)}$

(c) $\frac{2\varepsilon_0 a^2(k_1k_3 + k_2k_3)}{d(2k_1+2k_2+k_3)}$ (d) none of these

Solution (b) The equivalent capacitance circuit is where

$$C_1 = \frac{\varepsilon_0 k_1 a^2}{d}, C_2 = \frac{\varepsilon_0 k_2 a^2}{d} \text{ and}$$

$$C_3 = \frac{2\varepsilon_0 k_3 a^2}{d} \quad \frac{1}{C_{eq}} = \frac{1}{C_1 + C_2} + \frac{1}{C_3}$$

$$= \frac{d}{\varepsilon_0 a^2 (k_1 + k_2)} + \frac{d}{\varepsilon_0 a^2 2k_3}$$

$$C_{eq} = \frac{2\varepsilon_0 a^2 (k_1 + k_2) k_3}{(k_1 + k_2 + 2k_3) d}$$

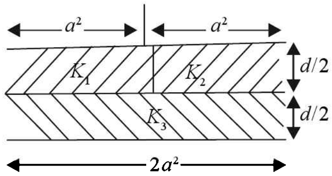


Fig. 20.34

10. Each capacitor has capacitance C in the Fig 20.35 (a). Find C_{AB}.

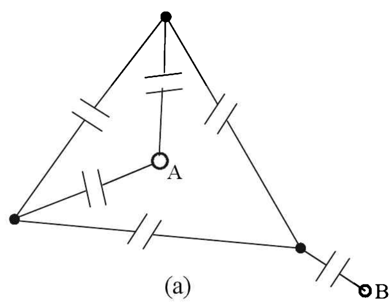


Fig. 20.35

(a) C (b) $2C$

(d) $C/2$ (d) $3C/2$

Solution (c) Note that dotted part in the circuit is a Wheatstone bridge with $C_{eq} = C$ $\therefore$ $C_{AB} = \frac{C}{2}$ from Fig 20.36 (c)

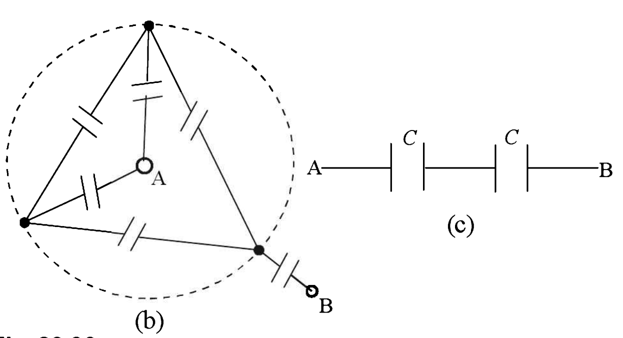


Fig. 20.36

11. Find C_{AB} in the infinite network shown in Fig 20.37 (a)

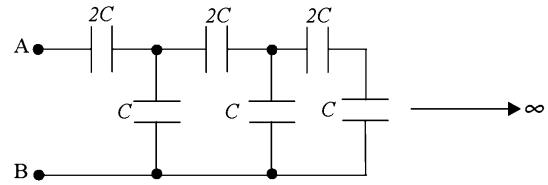


Fig. 20.37 (a)

(a) C (b) $-2C$

(c) $2C$ (d) $-C$

Solution (a) Let X be the equivalent capacitance. If one more network is added capacitance remains unchanged. Thus from equivalent circuit of Fig 20.37 (b)

$X = \frac{(C+X)2C}{X+3C}$ or $X^2 + 3CX = 2C^2 + 2CX$ or $X^2 + CX -$ $2C^2 = 0$ or $(X + 2C)(X - C) = 0$ or $X = C, X \neq -2C$

$\because$ capacitance is not negative.

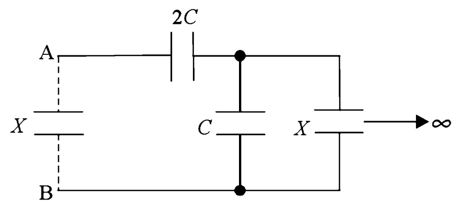


Fig. 20.37 (b)

12. Find C_{AB} if each capacitor is C in Fig 20.38 (a)

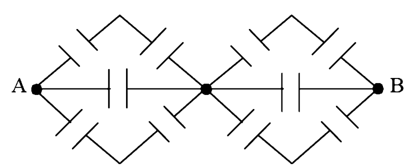


Fig. 20.38 (a)

(a) $3C$ (b) $2C$

(c) C (d) $C/2$

Solution (c) The equivalent circuit is shown in Fig 20.38 (b) and (c). From Fig 20.38 (c) $C_{eq} = \frac{2C}{2} = C$

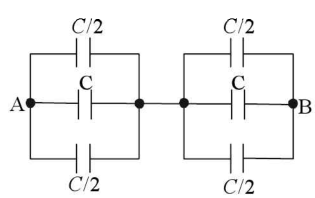


Fig. 20.38 (b)

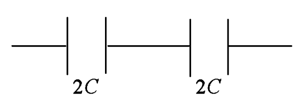


Fig. 20.38 (c)

13. In the circuit shown in Fig 20.39 charge on 1 μF and 3μF capacitors respectively is

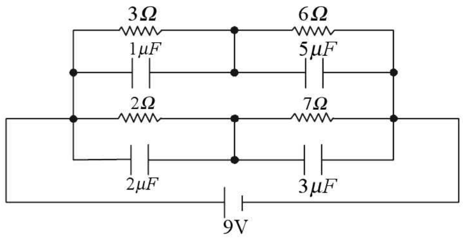


Fig. 20.39

(a) 7 μC, 3 μC (b) 3 μC, 3 μC

(c) 7 μC, 21 μC (d) 3 μC, 21 μC

Solution (d) $I = \dfrac{9V}{9\Omega} = 1A$

Potential drop across 1 μF capacitor = $1 \times 3 = 3V$

Potential drop across 3 μF capacitor = $1 \times 7 = 7V$

Charge on 1 μF capacitor $Q^1 = CV = 1 \times 3 = 3\ \mu C$

Similarly charge on 3μF capacitor $Q^2 = 7 \times 3 = 21\ \mu C$

14. Find the potential at D in Fig 20.40 taking potential at B to be zero.

(a) 24 V (b) 8 V

(c) 12 V (d) 16 V

Solution (d) $V^2 = \dfrac{C_1 V_0}{C_1 + C_2} = \dfrac{12 \times 24}{18} = 16\,V$

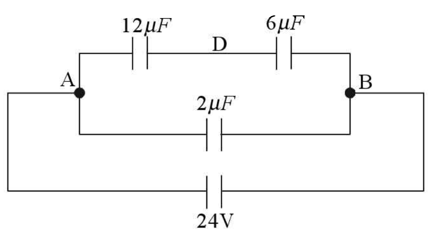


Fig. 20.40

15. Find potential drop across BD in the given Fig 20.41 (a).

(a) 5 V (b) 7.5 V

(c) 10 V (d) none of these

Solution (b) See equivalent circuit Fig 20.41 (b)

$$V_{BD} = \frac{C_1}{C_1 + C_2} V_0 = \frac{10 \times 15}{20} = 7.5\,V$$

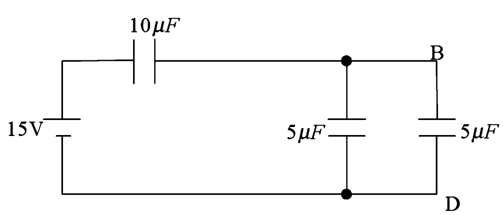


Fig. 20.41 (a)

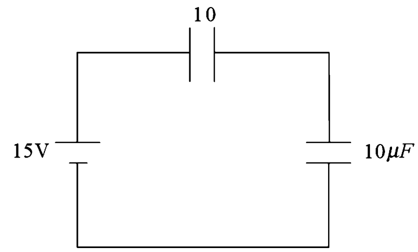


Fig. 20.41 (b)

16. If each capacitor is C find C_{AB} in the given circuit of Fig 20.42 (a)

(a) $3C/4$ (b) 4 C

(c) $C/4$ (d) $C/2$

Solution (a) equivalent circuit of Fig 20.42 (a) is

$$C_{AB} = \frac{C \times 3C}{C + 3C} = \frac{3C}{4} \text{ shown as Fig 20.42 (b)}$$

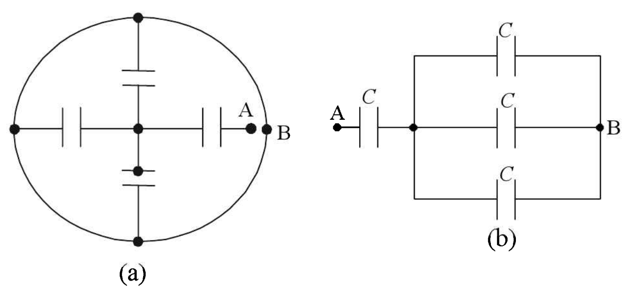


Fig. 20.42

17. Each capacitor in the circuit of Fig 20.43 is 4 μF. When the switch S is closed how much charge will flow through AB?

(a) 320 μC (b) 213 μC

(c) 107 μC (d) none of these

Solution (b) Case (i) switch is open $Q = \dfrac{8 \times 4}{12} \times 40$

$= \dfrac{320}{3}\ \mu C$

Case (ii) switch is closed: $Q = 8 \times 40 = 320\ \mu C$

Charge flowing through $AB = 320 - 320/3 = 640/3\ \mu C$

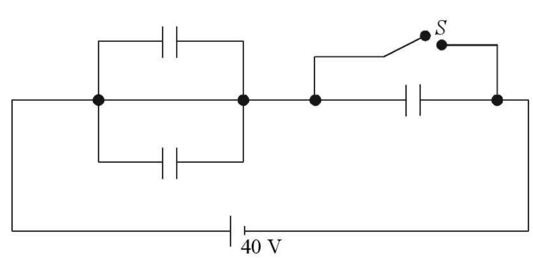


Fig. 20.43

18. A parallel plate capacitor has plate area 100cm^2 and separation between the plates is 1 cm. A glass plate ($k_1 = 6$) of thickness 6 mm and an ebonite plate ($k_2 = 4$) of thickness 4 mm are inserted. Find C_{eq}

(a) 4.085 pF (b) 40.85 pF

(c) .4085 nF (d) 40.85 nF

Solution (b) $C_{eq} = \dfrac{C_1C_2}{C_1+C_2} = \dfrac{\varepsilon_0 A\, k_1k_2}{(k_1d_2+k_2d_1)}$

$$= \frac{8.85\times10^{-12}\times10^{-2}\times6\times4}{(6\times6\times10^{-3}+4\times4\times10^{-3})}$$

$= 4.085\times10^{-11}F$

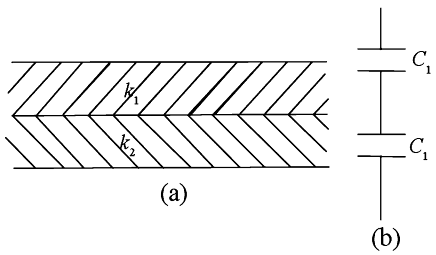


Fig. 20.44

19. A 5 μF capacitor is charged to 12 V. The positive plate of the capacitor is connected to the negative terminal of a 12V battery and vice versa. Find the heat developed in the connecting wires.

(a) 72 μJ (b) 720 μJ

(c) 1.44 mJ (d) 144 mJ

Solution (c) $E = \dfrac{1}{2}CV^2 = \dfrac{1}{2}\times5\times10^{-6}(24)^2 = 1.44\ mJ$

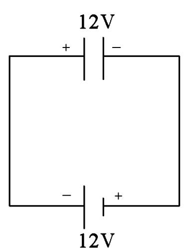


Fig. 20.45

20. Consider the assembly of 3 shells (conducting and concentric) of radii a, b and c as shown in Fig 20.46. Find the capacitance between A and B.

(a) $4\pi\varepsilon_0\left[\dfrac{ba}{b-a}+\dfrac{bc}{c-b}\right]$ (b) $\dfrac{4\pi\varepsilon_0(ba)(bc)}{b(b-a)+(c-b)c}$

(c) $\dfrac{4\pi\varepsilon_0 ca}{c-a}$ (d) none of these

Solution (c) Presence of a thin sheet between parallel plates does not affect the capacitance. Hence, $C = \dfrac{4\pi\varepsilon_0 ca}{c-a}$

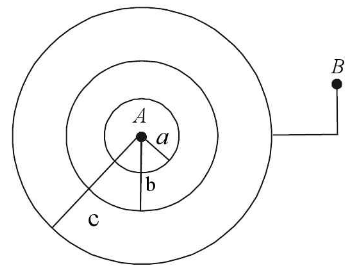


Fig. 20.46

21. A parallel plate capacitor with plate area 100 cm^2 and separation between the plate 5 mm is connected across a 24 V battery. The force of attraction between the plates is of the order of

(a) $10^{-6}N$ (b) $10^{-8}N$

(c) $10^{-4}N$ (d) $10^{-7}N$

Solution (a) $F = \dfrac{Q^2}{2A\varepsilon_0} = \dfrac{(CV)^2}{2A\varepsilon_0} = \dfrac{\left(\dfrac{A\varepsilon_0}{d}\right)^2 V^2}{2A\varepsilon_0}$

$$= \frac{A\varepsilon_0 V^2}{2d^2} = \frac{10^{-2}\times8.85\times10^{-12}\times24^2}{2\times25\times10^{-6}}$$

$= 1.08\times10^{-6}$N.

22. A capacitor 10 μF charged to 50 V is joined to another uncharged 50 μC capacitor. Find the loss in energy.

(a) 1.04×10^{-4}J (b) 4.01×10^{-4}J

(c) 6.25×10^{-4}J (d) 1.64×10^{-4}J

Solution (a) Energy loss $= \dfrac{C_1C_2}{2(C_1+C_2)}(V_1-V_2)^2$

$$= \frac{10\times50\times10^{-12}}{2(10+50)\times10^{-6}}(50-0)^2 = 1.04\times10^{-4}\text{J}$$

23. Two spheres of radius 5 cm and 10 cm, both charged to 120 μC, are joined by a metal wire and then metal wire is removed. What is the charge on each after removal of the wire?

(a) 120 μC, 120 μC (b) 80 μC, 160 μC

(c) 100 μC, 140 μC (d) None of these

Solution (b) $Q_1' = \dfrac{(Q_1+Q_2)r_1}{r_1+r_2}$

$$= \frac{240\times5}{15} = 80\ \mu\text{C}$$

$Q_2' = 240 - 80 = 160\ \mu$C

24. In the Fig 20.47 shown the potential drop across 3 μF capacitor when switch S is open and switch S is closed is

(a) 9 V, 8 V (b) 9 V, 9 V

(c) 6 V, 8 V (d) 12 V, 8 V

Solution (d) when switch is open 18 V is applied across 6 μF and 3 μF capacitor $V_1 = \dfrac{18\times 6}{6+3} = 12$ V

when the switch is closed potential drop across 8 Ω resistor is the potential drop across 3 μF capacitor i.e. 8 V.

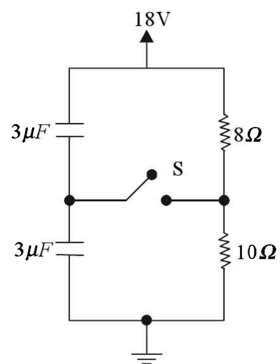


Fig. 20.47

25. Find the net capacitance between A and B in Fig 20.48.

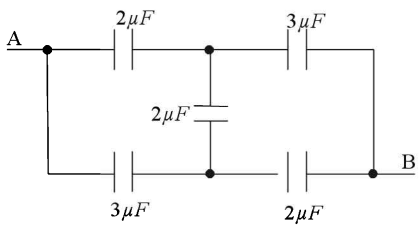


Fig. 20.48

(a) $\dfrac{5}{2}\mu$F (b) $\dfrac{2}{5}\mu F$

(c) $\dfrac{9}{22}\mu$F (d) $\dfrac{22}{9}\mu$F

Solution (d) $C_{AB} = \dfrac{2C_1C_2 + C_1C_3 + C_2C_3}{C_1 + C_2 + 2C_3}$

$$= \frac{2\times2\times3 + 2\times2 + 3\times2}{2+3+2\times2} = \frac{22}{9}\mu\text{F}$$

26. A 10 μF/400 V and a 4 μF/100 V capacitors are connected in series. Find the maximum potential which can be applied.

(a) 100 V (b) 500 V

(c) 400 V (d) 140 V

(e) None of these

Solution (d) In series charge remains same.

The maximum charge which can be applied is 400 μC (maximum rating of 4μF/ 100 V) capacitor.

Then potential which can be applied is

100 + 40 = 140 V.

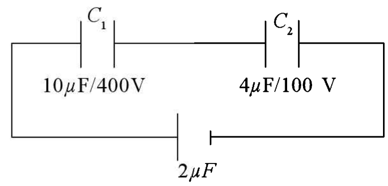


Fig. 20.49

27. Each side of a tetrahedral has a capacitor of capacitance C. Find the capacitance between a side.

(a) $\dfrac{C}{2}$ (b) $2C$

(c) C (d) $\dfrac{C}{3}$

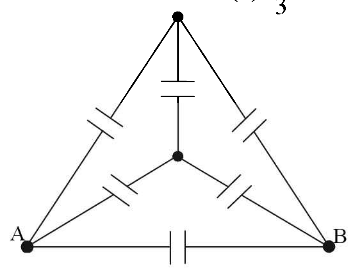


Fig. 20.50 (a)

Solution (b) The equivalent circuit of Fig 20.50 (a) is shown in Fig 20.50 (b) and Fig 20.50 (c) respectively.

$\therefore C_{AB} = 2C$

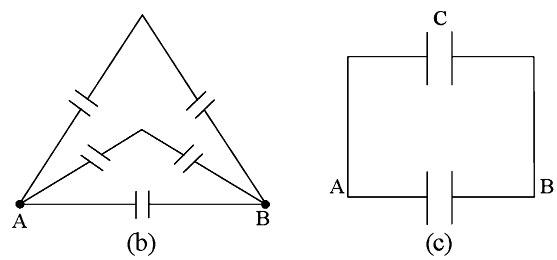


Fig. 20.50

28. The switch S is kept closed for a long time in Fig 20.51. It is opened at $t = 0$. Find the current in R_1 at $t = 1$ms.

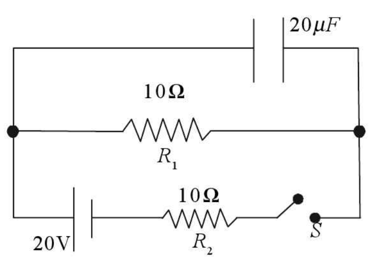


Fig. 20.51

(a) 11.2 mA (b) 12.4 mA

(c) 13.4 mA (d) 14.4 mA

Solution (c) $Q = Q_0\, e^{-t/RC}$, and $\dfrac{dQ}{dt} = i$

or $i = \dfrac{Q_0}{RC}\, e^{-t/RC}$

$= \dfrac{20\times20\times10^{-6}}{10\times20\times10^{-6}}\, e^{-5} = 2(.0067) = 13.4$ mA

TYPICAL PROBLEMS

29. Find the potential drop between points x and y in the given Fig. 20.52

(a) 25 V (b) 50 V

(c) 75 V (d) 57 V

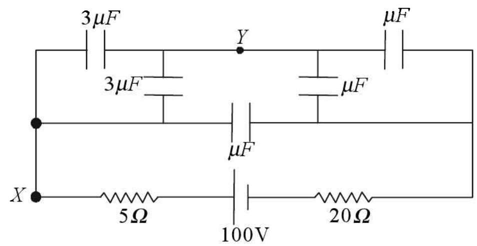


Fig. 20.52

Solution (a) Since no DC current passes through capacitor in steady state, no potential-drop occurs across resistors. Hence equivalent circuit is shown in Fig 20.53.

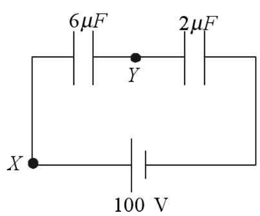


Fig. 20.53

$$V_{xy} = \frac{100\times2}{6+2} = 25\text{ V}$$

30. Each capacitor in Fig 20.54 has capacitance C. Find capacitance across AB.

(a) $\dfrac{15C}{8}$ (b) $\dfrac{15C}{7}$

(c) $\dfrac{3C}{2}$ (d) 2C

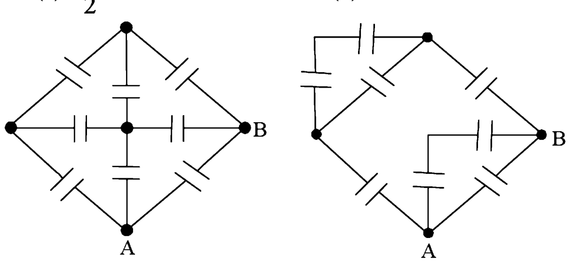


Fig. 20.54 (a) (b)

Solution (a) Passing through equivalent circuits of Fig. 20.54 (b), (c) and (d) we find $C_{AB} = \dfrac{3C}{8} + \dfrac{3C}{2} = \dfrac{15C}{8}$

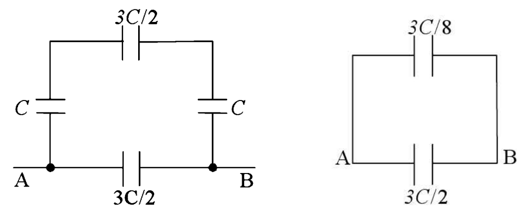


Fig. 20.54 (c) (d)

31. Find C_{AB} in the given circuit of Fig 20.55 (a). Assume each capacitor is C.

(a) $\dfrac{15C}{8}$ (b) $\dfrac{15C}{7}$

(c) $\dfrac{3C}{2}$ (d) 2C

Solution (b)

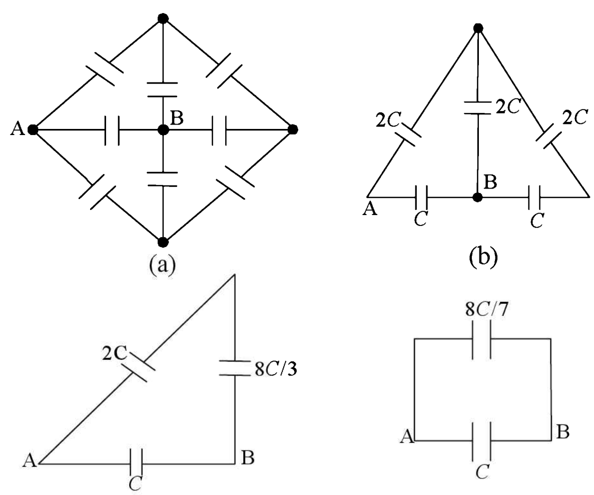


Fig. 20.55

(Olympiad 1999)

From equivalent circuit of Fig 20.55 (d)

$$C_{AB} = \frac{8C}{7} + C = \frac{15C}{7}$$

32. Each capacitor in the given circuit of Fig 20.56 has capacitance C. Find V_{AB}.

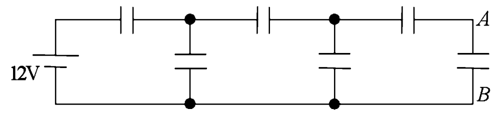


Fig. 20.56

(a) $\frac{65}{60}$ V (b) $\frac{60}{65}$ V

(c) $\frac{30}{35}$ V (d) none of these

Solution (b)

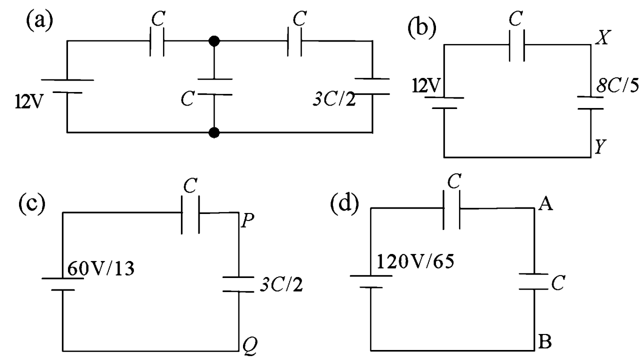


Fig. 20.57

From Fig 20.57 (b) $V_{XY} = \dfrac{12 \times C}{C + \dfrac{8C}{5}} = \dfrac{60}{13}$ V

From Fig 20.57 (c) $V_{PQ} = \dfrac{\dfrac{60}{13} \times C}{C + \dfrac{3C}{2}} = \dfrac{120}{65}$ V

From Fig 20.57 (d) $V_{AB} = \dfrac{\dfrac{120}{65} \times C}{C + C} = \dfrac{60}{65}$ V

33. If in the given Fig 20.58 switch S is closed, find the change in energy stored in the capacitors.

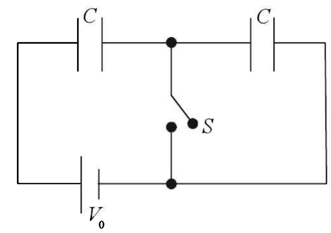


Fig. 20.58

(a) $\frac{CV_0^2}{2}$ (b) $\frac{CV_0^2}{4}$

(c) $\frac{CV_0^2}{8}$ (d) $\frac{3CV_0^2}{8}$

Solution (b) When switch is open each capacitor has voltage $\frac{V_0}{2}$.

$\therefore$ energy stored $E_1 = \frac{1}{2}C\left(\frac{V_0}{2}\right)^2 + \frac{1}{2}C\left(\frac{V_0}{2}\right)^2$

$= \frac{CV_0^2}{4}$

After the switch is closed capacitor is not connected to battery discharges. Energy is stored in the capacitor connected to the battery of emf V_0. Therefore energy stored after switch is made ON.

$E_2 = \frac{1}{2}CV_0^2 \quad \Delta E = E_2 - E_1$

$= \frac{CV_0^2}{4}$

QUESTIONS FOR PRACTICE

1. The gold leaf electroscope is charged so that its leaves somewhat diverge. If X-rays are incident on the electroscope then

(a) the divergence will decrease

(b) the divergence of leaves will remain unchanged

(c) the gold leaves will melt

(d) the divergence will increase

2. On removing the dielectric from a charged condenser, its energy

(a) increases (b) remains unchanged

(c) decreases (d) none of the above

3. A radioactive material is in the form of a sphere whose radius is 9×10^{-3} m. If 6.25×10^{12} β – particles are emitted per second by it then in how much time will a potential of 1volt be produced on the sphere if it is isolateed?

(a) 1.1 ms (b) 10^{-8}

(c) 1.0 μs (d) 11 μs

4. 64 water drops combine to form a bigger drop. If the charge and potential of a small drop are q and V respectively, then the charge on bigger drop will be

(a) 2 q (b) 4 q

(c) 16 q (d) 64 q

5. A condenser of capcity $0.2\mu F$ is charged to a potential of 600 V. The battery is now disconnected and the condenser of capacity $1\mu F$ is connected across it. The potential of the condenser will reduce to

(a) 600 V (b) 300 V
(c) 100 V (d) 120 V

6. Two condensers each of capacitance $2\mu\ F$ are connected in parallel and this combination is connected in series with a $12\mu F$ capacitor. The resultant capacity of the system will be

(a) $16\mu F$ (b) 13 μF
(c) 6 μF (d) 3 μF

7. The distance between the plates of a parallel plate air condenser is d. If a copper plate of same area but thickness $d/2$ is placed between the plates then the new capacitance will become

(a) doubled (b) half
(c) one fourth (d) remain unchanged

8. The energy stored in a condenser is in the from of

(a) potential energy (b) magnetic energy
(c) elastic energy (d) kinetic energy

9. A condenser of capcity 500 μF is charged at the rate of 50 $\mu F\ C/s$. The time taken for charging the condenser 10 V will be

(a) 10 s (b) 25 s
(c) 50 s (d) 100 s

10. Two condensers of capacity 4 μF and 6 μF are connected in series. A potential difference of 500 V is applied between the outer plates of the compound capacitor. The numerical value of charge on each condenser will be

(a) 1200 μC (b) 1200 C
(c) 6000 C (d) 6000 C

11. Two parallel wires are suspended in vacuum. When the potential difference between the wires is 30 V then the charge on the wires is 104 μC. The capacitance of the system of wires will be

(a) 3.48 μF (b) 5 μF
(c) $10.21\mu F$ (d) 50 μF

12. Two capacitors C_1 and C_2 are connected in parallel. If a charge Q is given to the combination, the charge gets charged. Then the ratio of charge on C_1 to charge on C_2 is

(a) $\frac{1}{C_1C_2}$ (b) C_1C_2
(c) C_2/C_1 (d) C_1/C_2

13. An uncharged parallel-plate capacitor having a dielectric of constant k is connected to a similar air filled capacitor charged to a potential V. The two share the charge and the common potential is V'. The dielectric constant k is

(a) $\frac{V-V'}{V'}$ (b) $\frac{V'-V}{V'}$
(c) $\frac{V'-V}{V'+V}$ (d) $\frac{V'-V}{V}$

14. The plates of a capacitor are charged to a potential difference of 100 V and are then connected across a resistor. The potential difference across the capacitor decays exponentially with time. After 1 second the potential difference between the plates of the capacitor is 80 V. The fraction of the stored energy which has been dissipated is

(a) 1/5 (b) 1/25
(c) 9/25 (d) 15/25

15. Two capacitors $6\mu\ F/200\mu\ F$ and $1\mu\ F/60\ V$ are connected in series. The maximum emf which can be applied is

(a) 260 V (b) 200 V
(c) 70 V (d) none of these

16. A 1 mm thick paper of dielectric constant 4 lies between the plates of a parallel-plate capacitor. It is charged to 100 volt. The intensity of electric field between the plates of the condenser will be

(a) 100 (b) 100000
(c) 400000 (d) 25000

17. The plates of a parallel-plate condenser are being moved away with velocity v. If the plate separation at any instant of time is d then the rate of change of capacitance with time is proportional to

(a) d^2 (b) d
(c) d^{-2} (d) d^{-1}

18. A condenser of capacitance C_1 is charged to V_0 volt. The energy stored in it is U_0. It is connected in parallel to another uncharged condenser of capacitance C_2. The energy loss in the process is

(a) $\frac{C_1C_2U_0}{2(C_1+C_2)}$ (b) $\left(\frac{C_1-C_2}{C_1+C_2}\right)^2 U_0$
(c) $\frac{C_1U_0}{C_1+C_2}$ (d) $\frac{C_2U_0}{C_1+C_2}$

19. A $4\mu\ F$ capacitor is charged to 50 V and another capacitor of 2 μF is charged to 100 V. The two condensers are connected such that their likely charged plates are connected together. The total energy of the system before and after joining will be in multiple of $10^{-2} J$.

(a) 3.0 and 2.67 (b) 2.67 and 3.0
(c) 1.5 and 1.33 (d) 1.33 and 1.5

20. A parallel-plate condenser of capacitance C is connected to a battery and is charged to potential V.

Another condenser of capacity 2 C is connected to another battery and is charged to potential 2 V. The charging batteries are removed and now the condensers are connected in parallel in such a way that the positive plate of one is connected to negative plate of another. The final energy of this system is

(a) 25 $CV^2/6$ (b) 9 $CV^2/2$

(c) 3 $CV^2/2$ (d) zero

21. Two condensers, each of capacitance 1 μF, are connected in parallel. These are charged by a DC source of 200 volt. The total energy of their charges in Joule will be

(a) 0.06 (b) 0.04

(c) 0.02 (d) 0.01

22. On increasing the plate separation of a charged condenser its energy

(a) remains unchanged (b) decreases

(c) increases (d) none of these

23. The force of attraction between the plates of a charged condenser is

(a) $q^2\,(2\varepsilon_0 A)$ (b) $q^2\,(2\varepsilon_0 A^2)$

(c) $q/\,(2\,\varepsilon_0 A^2)$ (d) none of these

24. A parallel plate condenser is connected to a battery of emf 4 volt. If a plate of dielectric constant 8 is inserted into it, then the potential difference on the condenser will be

(a) 32 V (b) 4 V

(c) 1/2 V (d) 2 V

25. A parallel plate condenser with plate separation d is charged with the help of a battery so that V_0 energy is stored in the system. The battery is now removed. A plate of dielectric constant k and thickness d is placed between the plates of condenser. The new energy of the system will be

(a) $V_0 k^{-2}$ (b) $k^2\,V_0$

(c) $V_0 k^{-1}$ (d) kV_0

26. The area of each plate of a parallel plate capacitor is 2 m^2. The space between the plates is filled with materials of dielectric constants 2, 3 and 6 and their thickness are 0.4 mm and 1.2 mm respectively. The capacitance of the capacitor will be

(a) $8.94 \times 10^{-4} F$ (b) $6.94 \times 10^{-7} F$

(c) $2.94 \times 10^{-8} F$ (d) $10^{-8} F$

27. The intensity of an electric field between the plates of a charged condenser of plate area A will be

(a) $A\,/\,(q\varepsilon_0)$ (b) $qA/\varepsilon 0$

(c) $q/\,(\varepsilon_0 A)$ (d) none of these

28. A battery of 100 V is connected to series combination of two identical parallel-plate condensers. If dielectric of constant 4 is slipped between the plates of second condenser, then the potential difference on the condensers will respectively become

(a) 80 V, 20 V (b) 75 V, 25 V

(c) 50 V, 80 V (d) 20 V, 80 V

29. The distance between the plates of a circular parallel plate condenser of diameter 40 mm, in order to make its capacitance equal to that of a metallic sphere of radius 1 m, will be

(a) 0.01 mm (b) 0.1 mm

(c) 1 mm (d) 10 mm

30. The minimum number of condensers each of capacitance of 2 μF, in order to obtain resultant capacitance of 5 μF will be

(a) 4 (b) 10

(c) 5 (d) 6

31. A 10 μF condenser is charged to a potential of 100 volt. It is now connected to another uncharged condenser. The common potential reached is 40 volt. The capacitance of second condenser is

(a) 2 μF (b) 10 μF

(c) 15 μF (d) 22 μF

32. When a thin mica sheet is placed between the plates of a condenser then the amount of charge, as compared to its previous value, on its plates will become

(a) unchanged (b) zero

(c) less (d) more

33. When dielectric medium of constant k is filled between the plates of a charged parallel-plate condenser, then the energy stored becomes, as compared to its previous value,

(a) k^{-2} times (b) k^{-3} times

(c) k^{-1} times (d) k times

34. A capacitor of capacitance C is connected to battery of emf V_0. Without removing the battery, a dielectric of strength ε_r is inserted between the parallel plates of the capacitor C, then the charge on the capacitor is

(a) CV_0 (b) $\varepsilon_r\,CV_0$

(c) $\dfrac{CV_0}{\varepsilon_r}$ (d) none of these

35. The capacitance of conducting metallic sphere will be 1 μF if its radius is nearly

(a) 1.12 cm (b) 10 cm

(c) 1.11 cm (d) 9 km

36. The potential difference between the plates of a condenser of capacitance 0.5 μF is 100 volt. It is connected to an uncharged condenser of capacity 0.2 μF by a copper wire. The loss of energy in this process will be

(a) 0 J (b) 0.5×10^{-3} J
(c) 0.7×10^{-3} J (d) 10^{-3} J

37. The electric energy density between the plates of charged condenser is
(a) $q/2\varepsilon_0 A^2$ (b) $q/2\varepsilon_0 A$
(c) $q^2/(2\varepsilon_0 A^2)$ (d) none of these

38. Farad is not equivalent to
(a) CV^2 (b) J/V^2
(c) Q^2/J (d) Q/V

39. The energy stored between the plates of a condenser in *not* represented by
(a) $U = \frac{CV^2}{2}$ (b) $U = 2$ qV
(c) $U = \frac{q^2}{2C}$ (d) $U = \frac{qV}{2}$

40. The capacitance of a spherical conductor of radius *r* is proportional to
(a) $1/r$ (b) r
(c) $1/r^2$ (d) r^2

41. The net charge on a condenser is
(a) infinity (b) $q/2$
(c) $2q$ (d) zero

42. Two charged conducting spheres are joined by a conducting wire then
(a) nothing will be conserved
(c) the total energy will be conserved
(c) the total charge will be conserved
(d) the total charge and energy will be conserved

43. The capacitance of a charged condenser is *C* and energy stored on account of charge on it is *U*, then the quantity of charge on the condenser will be
(a) $\sqrt{2UC}$ (b) $\sqrt{\frac{UC}{2}}$
(c) 2 *UC* (d) zero

44. A 100 μ *F* capacitor is charged to 200 volt. It is discharged through a 2 ohm resistance. The amount of heat generated will be
(a) 0.4 J (b) 0.2 J
(c) 2 J (d) 4 J

45. The capacitance of a condenser is 20 μ *F* and it is charged to a potential of 2000 V. The energy stored in it will be
(a) zero (b) 40 J
(c) 80 J (d) 120 J

46. If the diameter of earth is 128×10^2 km then its capacitance will be
(a) $711\mu F$ (b) $331\mu F$
(c) $211\mu F$ (d) $111\mu F$

47. A condenser is charged to a potential difference of 200 volts as a result of which it gains charge of 0.1 coulomb. When it is dischargd then the energy released will be
(a) 1 J (b) 2 J
(c) 10 J (d) 20 J

48. The capacitance of a parallel plate capacitor in air is 2 μF. If dielectric medium is placed between the plates then the potential difference reduces to 1/6 of the original value. The dielectric constant of the medium is
(a) 6 (b) 3
(c) 2.2 (d) 4.4

49. When two condensers of capacitance $1\mu F$ and $2\mu F$ are connected is series then the effective capacitance will be
(a) $\frac{2}{3}\mu F$ (b) $\frac{3}{2}\mu F$
(c) $3\mu F$ (d) $4\mu F$

50. What will be area of pieces of paper in order to make a paper condenser of capacitance 0.04 μF, if the dielectric constant of paper is 2.5 and its thickness is 0.025 mm?
(a) $1 m^2$ (b) $2 \times 10^{-3} m^2$
(c) $4.51 \times 10^{-3} m^2$ (d) $10^{-3} m^2$

51. Three condensers each of capacitance 2 *F*, are connected in series. The resultant capacitance will be
(a) 6 *F* (b) 5 *F*
(c) 2/3 *F* (d) 3/2 *F*

52. Which material sheet should be placed between the plates of a parallel plate condenser in order to increase its capacitance?
(a) mica (b) copper
(c) tin (d) iron

53. Three condensers of capacity 2 μF, 4 μF and 8 μF respectively, are first connected in series and then connected in parallel. The ratio of equivalent capacitances in two cases will be
(a) 7 : 3 (b) 49 : 4
(c) 3 : 7 (d) 4 : 49

54. A conducting hollow sphere of radius 0.1 m is given a charge of 10 μC. The electric potential on the surface of sphere will be
(a) zero (b) $3 \times 10^5 V$
(c) $9 \times 10^5 V$ (d) $9 \times 10^9 V$

55. The capacitance of parallel-plate capacitor is $4\mu F$. If a dielectric material of dielectric constant 16 is placed between the plates then the new capacitance will be
(a) 1/64 μ *F* (b) 0.25 μF
(c) 64 μ *F* (b) 40 μ *F*

56. The energy acquired by a charged particle of 4 μ *C* when it is accelerated through a potential difference of 8 volt will be

(a) 3.2×10^{-7} J (b) 3.2×10^{-5} J
(c) 2×10^{-6} J (d) 2×10^{-5} J

57. Two parallel-plate condensers of capacitance of 20 μF and 30 μF are charged to the potential of 30 V and 20 V respectively. If likely-charged plates are connected together then the common potential difference will be

(a) 10 V (b) 24 V
(c) 50 V (d) 100 V

58. 64 water drops having equal charges combine to form one bigger drop. The capacitance of bigger drop, as compared to that of smaller drop will be

(a) 4 times (b) 8 times
(c) 16 times (d) 64 times

59. Three capacitors C, C and 2C are arranged in different arrangements. The number of equivalent capacitances that can be fabricated are

(a) Four (b) Five
(c) Six (d) Seven

60. The equivalent capacitance between terminals is

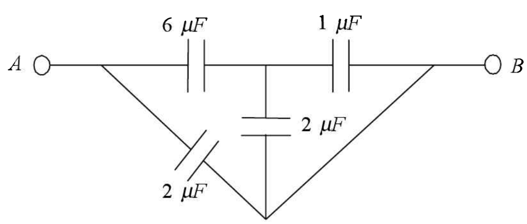


Fig. 20.59

(a) 2.4 μF (b) 3.2 μF
(c) 1.4 μF (d) 4.0 μF

61. A graph between energy (E) and potential (V) for a capacitor will be

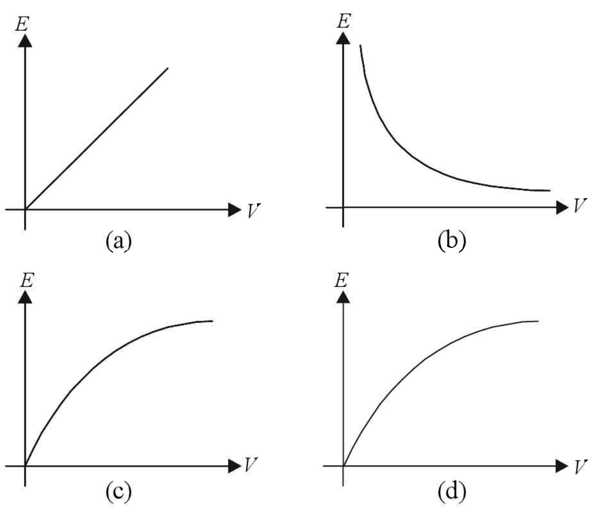


Fig. 20.60

62. The potential difference between A and B is 23 Volt. The p.d. in volts across the 3 μF capacitor is

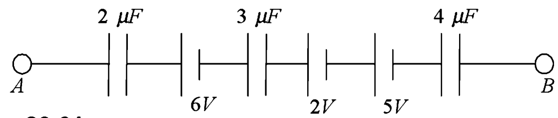


Fig. 20.61

(a) 4 V (b) 3 V
(c) 12 V (d) 13 V

63. Three identical parallel plates of equal dimension area have 36 πcm^2 each and separation between consecutive plates is 0.04 mm. The energy stored will be

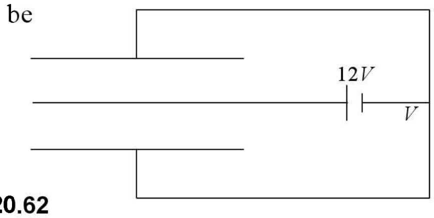


Fig. 20.62

(a) 0.36 μJ (b) 0.16 μJ
(c) .036 μJ (d) .061 μJ

64. A capacitor network is shown in figure. The potential across 2 μF capacitor will be

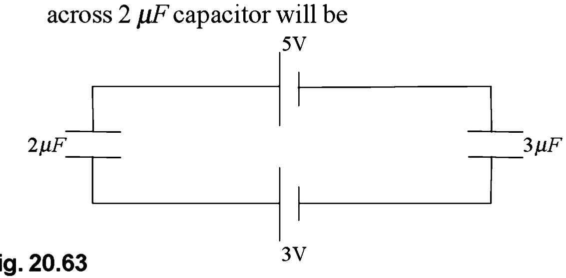


Fig. 20.63

(a) 1.2 Volt (b) 1.5 Volt
(c) 1.8 Volt (d) 2.4 Volt

65. A capacitor network is shown in figure. The equivalent capacitance between OA will be

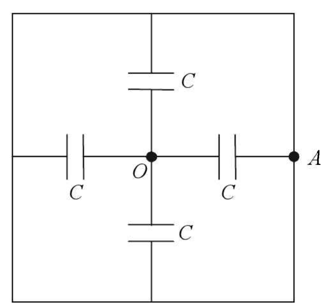


Fig. 20.64

(a) $\frac{C}{4}$ (b) $4C$
(c) $\frac{C}{2}$ (d) $2C$

66. If capacitance of capacitor is 1 μF. Its charge will be

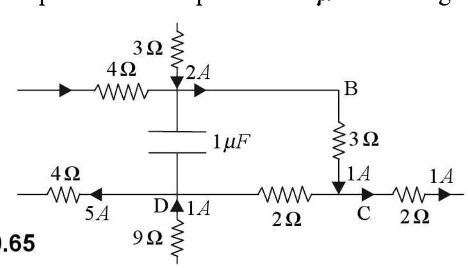


Fig. 20.65

(a) 13 μC (b) 21 μC

(c) 23 μC (d) 7 μC

67. A circuit network is shown in figure. The charge on capacitor will be

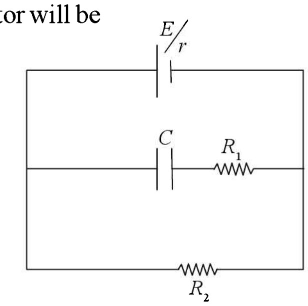


Fig. 20.66

(a) $\dfrac{ER_2}{(r+R_1)}$ (b) $\dfrac{ER_2}{(R_2+R_1+r)}$

(c) $\dfrac{ER_2}{(R_2+r)}$ (d) $\dfrac{ER_1}{(R_2+R_1)}$

68. The plates of a capacitor are charged with a battery so that the plates of capacitor have the p.d. equal to emf of the battery. The ratio of the work done by the battery and the energy stored in the capacitor is

(a) 1 : 1 (b) 1 : 2

(c) 2 : 1 (d) 1 : 4

69. The resistance of the capacitor when it is connected with battery will be

(a) zero (b) finite

(c) infinite (d) none

70. The equivalent capacitance of combination will be

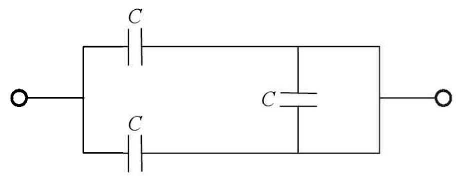


Fig. 20.67

(a) $2C$ (b) $\dfrac{3C}{2}$

(c) $\dfrac{2C}{2}$ (d) none

Answers to Questions for Practice

1. (a)	**2.** (a)	**3.** (c)	**4.** (d)	**5.** (c)	**6.** (d)	**7.** (a)
8. (a)	**9.** (d)	**10.** (a)	**11.** (a)	**12.** (d)	**13.** (a)	**14.** (c)
15. (c)	**16.** (d)	**17.** (c)	**18.** (d)	**19.** (c)	**20.** (c)	**21.** (b)
22. (c)	**23.** (a)	**24.** (b)	**25.** (c)	**26.** (c)	**27.** (c)	**28.** (a)
29. (b)	**30.** (a)	**31.** (c)	**32.** (a)	**33.** (c)	**34.** (b)	**35.** (d)
36. (c)	**37.** (c)	**38.** (a)	**39.** (b)	**40.** (b)	**41.** (d)	**42.** (c)
43. (a)	**44.** (c)	**45.** (b)	**46.** (a)	**47.** (c)	**48.** (a)	**49.** (a)
50. (c)	**51.** (c)	**52.** (a)	**53.** (d)	**54.** (c)	**55.** (c)	**56.** (b)
57. (b)	**58.** (a)	**59.** (c)	**60.** (d)	**61.** (c)	**62.** (a)	**63.** (a)
64. (a)	**65.** (b)	**66.** (c)	**67.** (c)	**68.** (c)	**69.** (c)	**70.** (a)

21

Electricity

BRIEF REVIEW

Electric Current The time rate of change of charge i. e. $\frac{dQ}{dt}$ is called current.

or $I = \frac{dQ}{dt}$.

The unit of current is Ampere (A). DC current is a scalar quantity. However, AC current is a phasor (vector).

Current may be divided into three types from the point of view of generation.

(a) **Drift current** When electric field is applied in a conductor, then current due to drift velocity flows. Such a current is called drift current and is given by $I = neAv_d$ where n is number electron density, e charge on an electron, v_d is drift velocity and A is area of cross-section.

Drift velocity (v_d) is the average directed velocity along the length of the conductor in the presence of applied electric field. It is given by $v_d = \frac{eE\tau}{m}$

where τ is relaxation time, E is applied electric field and m is mass of the electron.

Relaxation time The average time between two successive collisions of electrons is called relaxation time.

(b) **Diffusion current** Diffusion current occurs due to charge density gradient. Thermocouples and semiconductors show diffusion current.

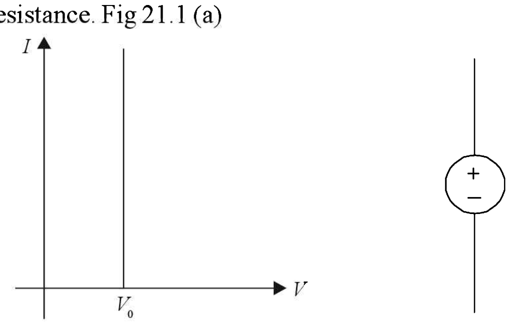
$I_{\text{diffusion}} = De\frac{dn}{dx}$ where D is diffusion constant and $e\frac{dn}{dx}$ is charge density gradient.

(c) **Displacement current** It is generated due to varying electric/magnetic flux.

$I_{\text{dispalcement}} = \varepsilon_0 \frac{d\phi_E}{dt}$ where ϕ_E is electric flux and ε_0 is permittivity of free space.

Cells and generators are common sources of electricity.

Ideal voltage source An ideal voltage source is one in which voltage does not vary irrespective of the value of current drawn. An **ideal voltage source** has zero internal resistance. Fig 21.1 (a)



(a) Characteristic

(b) Representation

Fig. 21.1 Ideal voltage source

Emf The maximum potential drop across a cell/device when no current is drawn. Emf is equivalent to open circuit voltage. Emf can be measured using ideal voltmeter. Since practically we do not have an ideal voltmeter, we use potentiometer to measure emf. Unit of emf is Volt (V), $1V = \frac{1J}{1C}$

An **ideal voltmeter** has infinite resistance. In present day technology VTVMs or electronic voltmeters are nearly ideal.

Current density $J = \frac{I}{A} = nev_d = \frac{ne^2\tau}{m}E$

Also $J = \sigma E$ where σ is conductivity. Thus $\sigma = \frac{ne^2\tau}{m}$

Note that current density is a vector. $\oint J \cdot ds = I$

Conductivity σ is reciprocal of resistivity ρ.

Thus $\sigma = \frac{1}{\rho}$ and hence $\rho = \frac{m}{ne^2\tau} = \frac{E}{J}$

Unit of conductivity is $(\text{ohm-m})^{-1}$

Resistivity of a substance is the resistance offered by a unit cube of the material., i.e., $\rho = R$ if $l = 1\text{m}$ and $A = 1\text{m}^2$

Its unit is ohm-m. Resistivity or specific resistance varies inversely with pressure. Moreover $\rho = \rho_0(1 + \alpha T)$

where α is thermal coefficient of resistance .

Resistance R of a conductor $\propto l$ (length of the conductor)

and $R \propto \frac{1}{A}$ where A is area of cross-section

$R = \rho\frac{l}{A}$ where ρ is the resistivity.

Resistance offers oppositon to the flow of current. Resistances are of three types ohmic, nonohmic and negative. Ohmic resistances follow Ohm's law $V = IR$ or $V \propto I$. Vacuum tubes and semiconductors are examples of nonohmic resistances. For such devices dynamic resistance or incremental resistance $r = \frac{dV}{dI}$ or $r = \frac{\Delta V}{\Delta I}$ is determined. A negtative resistance device shows inverse relation between V and I, i.e., $I \propto \frac{1}{V}$. Tunnel diode, tetrode and thyristors are examples of negative resistance devices.

Potential (V) Amount of work done to bring a unit positive charge from infinity to that point against the electric field of a given charge without changing velocity or kinetic energy is called potential. Its unit is volt (V). Practically we can measure only potential difference. Potential cannot be measured as infinity cannot be defined.

Potential difference is the difference of potentials between two points. Thus potential difference $V = V_1 - V_2$

Emf of a cell depends upon the nature of electrolyte and nature of electrodes. Table 21.1 shows the comparative study of different cells .

Table. 21.1 Comparative study of different cells

S.No.	Cell	Nature	Anode	Cathode	Emf
1.	Voltaic	Primary	Cu	Zn	1.1V
2.	Daniel	Primary	Cu	Zn (amalgamated)	1.1V
3.	Laclanche	Primary	C (graphite)	Zn (amalgamated)	1.35V
4.	Dry cell	Primary	C (graphite)	Zn (amalgamated)	1.5V
5.	Lead Acid accumulator	Secondary	PbO_2	Pb	2.2 when fully charged, 1.8V when and discharged
6.	Alkali accumulator	Secondary	Ni+NiO_2	FeO_2	1.35 when charged and 1.25 when Discharged

Resistance in conductors is caused by

(a) electron-electron collision

(b) collision between core and electron

(c) interaction between electrons and lattice vibration

(d) trap centres. $R = \frac{V}{I}$.

The device which offers resistance is called resistor.

Alloys have more trap centres and therefore their resistivity and hence resistance is higher as compared to metals forming them. Manganin is used to make standard resistances as its specific resistance is high and it varies very little with temperature. Alloy used in making rheostat is constantan. Nichrome is commonly used to make heaters used in press, geyser, room heaters etc. Manganin (84% Cu, Mn 12%, Ni 4%) constantan (Cu 60%, Ni 40%)

Silver is the best conductor followed by Cu, Au, Al, W(tungsten), steel, lead (Pb) and Hg. The best insulator is fused quartz with resistivity 75×10^{16} ohm-m.

Carbon resistors are colour coded. **First colour can not be black**. If there are four colour bands then $R = ab \times 10^C \pm d\%$ and $R \geq 10\Omega$ colours a, b, c and d, their values are listed in Table 21.2

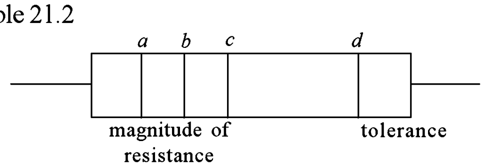


Fig. 21.2 Colour code in carbon resistors

Table. 21.2 Colour code of resistors

Black	Brown	Red	Orange	Yellow	Green	Blue	Violet	Grey	White
0	1	2	3	4	5	6	7	8	9

Table. 21.3 Tolerance in resistors

Gold	Silver	no fourth colour
5%	10%	20%

For example, Red Brown Orange Gold will stand for

$21 \times 10^3\,\Omega \pm 5\%$

and colour code for $10\Omega \pm 5\%$ will be $10 \times 10^0 \pm 5\%$

Brown Black Black Gold

If resistance is less than 10Ω then another scheme is used.

$R = ab \times 10^C$

where a and b are taken from Table 21.2 and C is taken from Table 21.4.

Note it has no fourth colour.

Table. 21.4 Third colour value for carbon resistors

Gold	Silver
–1	–2

< 10Ω

for instance 0.5Ω will have colour code

50×10^{-2} = Green Black Silver.

Every source of emf has internal resistance r.

Terminal voltage $V = \varepsilon - Ir$ (See Fig. 21.3)

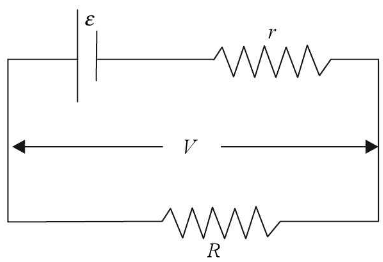


Fig. 21.3 Illustration of terminal voltage V

Where ε is emf. During charging of the battery or if the current is in opposite direction to normal direction of current from the cell or a battery, the terminal voltage is greater than emf.

Conductance (G) is reciprocal of resistance i.e. $G = \frac{1}{R}$.

Its unit is mho or $(\text{ohm})^{-1}$ or Siemen (S).

Superconductors have zero resistance. The highest critical temperature for a superconductor known till 2003 is (minus) 160°C. It is a complex oxide of Yttrium, Copper and Barium.

Cells in series If n identical cells are connected in series, each having emf E and internal resistance r, then current in an external resistance R is given by,

$$I = \frac{nE}{R + nr}$$

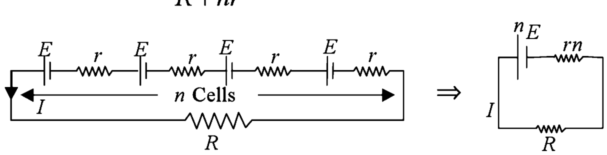


Fig. 21.4 Cells in series

Cells in parallel If n identical cells are connected in parallel, each having emf E and internal resistance r,

$$\text{then } I = \frac{E}{R + r/n}. \text{ (See Fig. 21.5)}$$

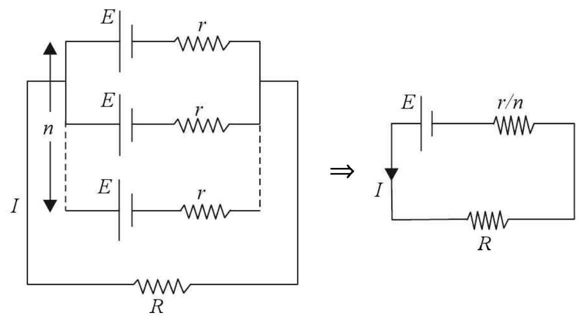


Fig. 21.5 Cells in parallel

Cells in mixed grouping m rows of n identical cells in series connected to an external resistance R.

$$\text{Then } I = \frac{nE}{\frac{nr}{m} + R}$$

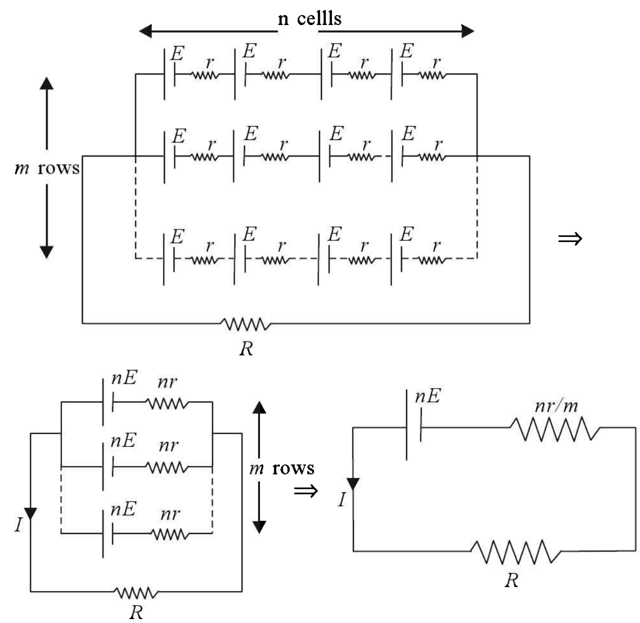


Fig. 21.6 Cells in mixed grouping

Maximum current is deliverd by a source when it is short circuited i.e. $R_{\text{external}} = 0$. Maximum power is delivered by a source under matched conditions i.e. $r_{\text{int}} = R_{\text{ext}}$

Law of Resistances

In series $R_S = R_1 + R_2 + \ldots + R_n$

If n equal resistances are in series then $R_S = nR$

$$\text{In parallel } \frac{1}{R_P} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots \frac{1}{R_n}$$

If n equal resistances are in parallel then $R_P = \frac{R}{n}$

For two resistances in parallel $R_P = \frac{R_1 R_2}{R_1 + R_2}$

Kirchhoff's Current Law (KCL) Algebraic sum of all the currents entering at any instant at a node (junction) is zero, or Sum of currents entering a junction at any instant = Sum of current leaving the junction at that instant. The law is based on conservation of charge.

Kirchhoff's Voltage Law (KVL) or Loop Law

Algebraic sum of all the potential drops in a closed circuit (or a loop) is zero. It is based on conservation of energy.

Wheatstone bridge The bridge is said to be balanced if $V_X = V_Y$ or $I_G = 0$. Under balanced condition $\frac{P}{Q} = \frac{R}{S}$. (See Fig. 21.7)

Fig. 21.7(a) Wheatstone bridge

Fig. 21.7(b), (c) are other representations of Wheatstone bridge.

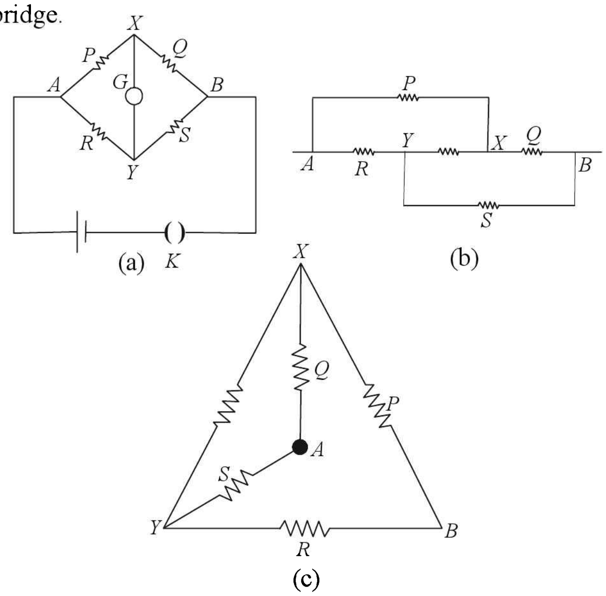


Fig. 21.7

Potentiometer The fall of potential along the length of a conductor of uniform area of cross-section and uniform density is proportional to its length when current I passes through it, provided physical conditions like temperature, pressure etc remain unchanged.

Potential gradient $k = \frac{V}{l}$. More the length or smaller the value of k, more sensitive is the potentiometer.

To find emf by comparison method:

$$\frac{\varepsilon_1}{\varepsilon_2} = \frac{l_1}{l_2}$$

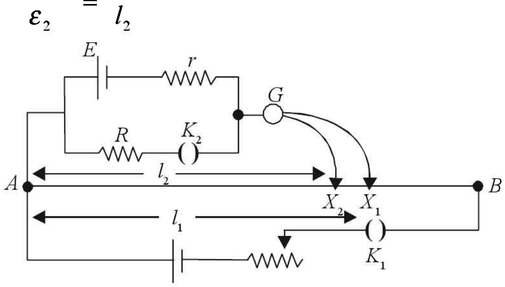


Fig. 21.8 To find internal resistance of a cell using potentiometer

To find internal resistance of a cell

$$r = R\left(\frac{l_1 - l_2}{l_1}\right)$$

where l_1 is the length when key k_2 is open and only k_1 is closed and null point is found while l_2 is the length of the potentiometer wire when k_2 is also inserted and null point determined.

Meter bridge or slide wire bridge

If balance point or null point is determined at X, then

$$\frac{P}{Q} = \frac{l}{100 - l}$$

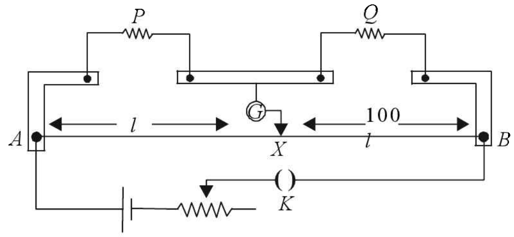


Fig. 21.9 Find unknown resistance using slide wire bridge

• Short Cuts and Points to Note

1. If n identical cells are connected in series and m of them are wrongly connected then $\varepsilon_{net} = n\varepsilon - 2m\varepsilon$ where ε is emf of each cell.
2. If a branch of a circuit contains capacitor then in steady state current through that branch is zero.
3. The current in a branch is zero if $V_1 = V_2$

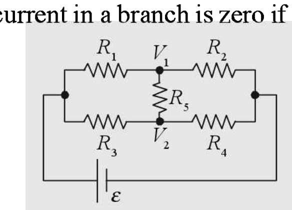


Fig. 21.10

4. If n identical cells are connected in order in a loop then potential drop across any two points is zero.
5. An ideal current source has infinite resistance. An ideal ammeter has zero resistance.
6. An ideal voltage source has zero resistance. An ideal voltmeter has infinite resistance.
7. Normally a voltmeter is connected in parallel. However, in order to find high resistance it may be connected in series as shown in Fig 21.11. If voltmeter has internal resistance R and it reads V then

$$V = \frac{V_0 R}{X + R}$$ and hence X can be determined.

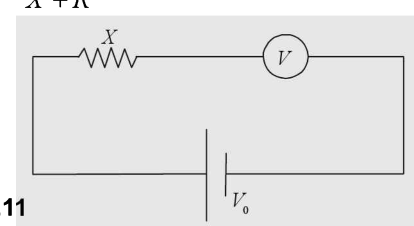


Fig. 21.11

8. In parallel, the net resistance is smaller than the smallest.
9. If two points in a circuit are short circuited then resistance across those points is zero irrespective of the resistance shown between those points.
10. Strictly speaking, resistance of metals vary nonlinearly with temperature $R(T) = R_0(1 + \alpha T + \beta T^2 + ...)$,
11. If two nonidentical cells are, in parallel, positive terminal connected to positive, then

$$E_{eq} = \frac{E_1 r_2 - E_2 r_1}{r_1 + r_2}$$

$$r_{eq} = \frac{r_1 r_2}{r_1 + r_2}$$

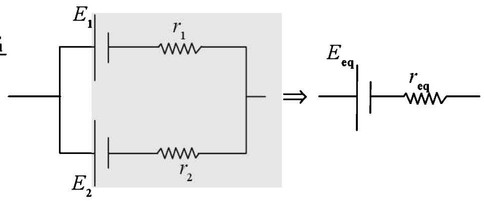


Fig. 21.12 (a)

If positive terminal of one cell/battery is connected to negative terminal of the other,

$$E_{eq} = \frac{E_1 r_2 + E_2 r_1}{r_1 + r_2}$$

$$r_{eq} = \frac{r_1 r_2}{r_1 + r_2}$$

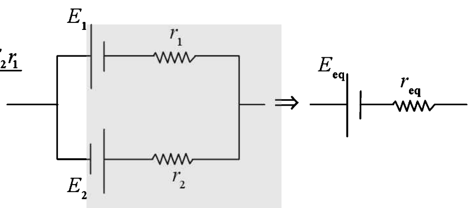


Fig. 21.12 (b)

12. To solve a resistive network, there could be four methods.
    (a) Series/parallel method (when clearly visible).
    (b) Wheatstone bridge method.
    (c) Current division method. Though it could be used for any circuit, it suits symmetrical circuits.
    (d) Star-delta method.

Star to delta conversion

From Fig. 21.13

$$R_{AB} = R_1 + R_2 + \frac{R_1 R_2}{R_3}$$

$$R_{AC} = R_1 + R_3 + \frac{R_1 R_3}{R_2}$$

$$R_{BC} = R_2 + R_3 + \frac{R_2 R_3}{R_1}$$

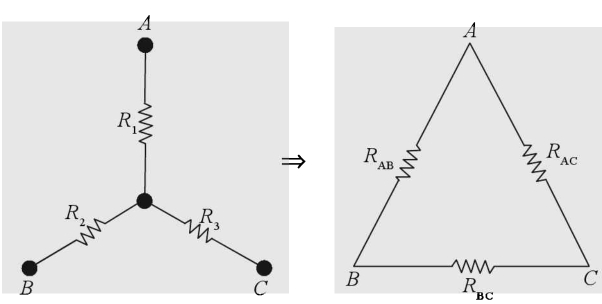


Fig. 21.13 Star to delta form

Delta to star conversion

From Fig. 21.14

$$R_A = \frac{R_1 R_2}{R_1 + R_2 + R_3}$$

$$R_B = \frac{R_1 R_3}{R_1 + R_2 + R_3}$$

$$R_C = \frac{R_2 R_3}{R_1 + R_2 + R_3}$$

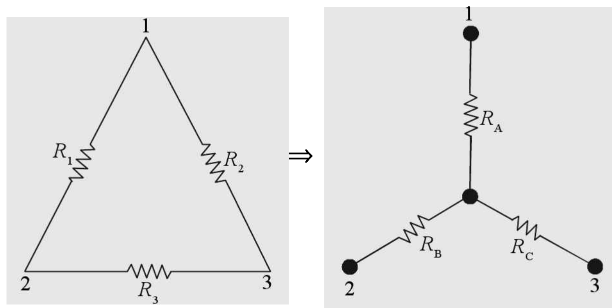


Fig. 21.14 Delta to star form

13. If current in its path meets positive terminal as shown in Fig. 21.15 then take E_1 positive in the Loop law or Kirchhoff's Voltage Law as it represents potential drop.

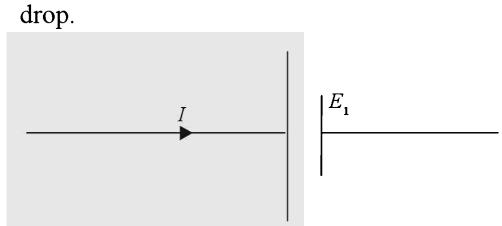


Fig. 21.15

14. If current enters the negative terminal of the battery then take $-E_2$ as it represents potential rise in the Loop law. (Fig. 21.16)

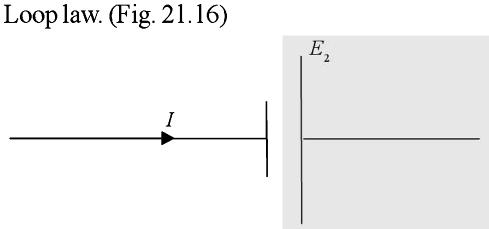


Fig. 21.16

15. To find I_1, I_2 and I_3 in the circuit shown in Fig. 21.17 find V_{AB} and then

$$I_1 = \frac{V_{AB}}{R_1},\ I_2 = \frac{V_{AB}}{R_2} \text{ and } I_3 = \frac{V_{AB}}{R_3}$$

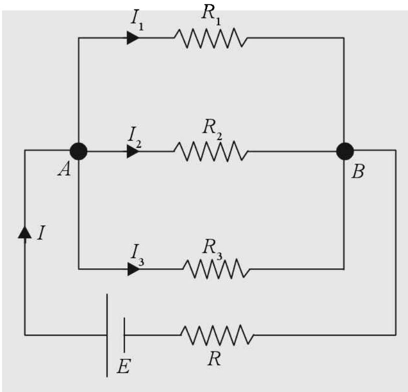


Fig. 21.17

16. If all the resistances in the Wheatstone bridge are identical or $R_1 = R_2 = R_3 = R_4 \neq R_5$ then

$R_{AB} = R_{eq} = R$. (See Fig. 21.18)

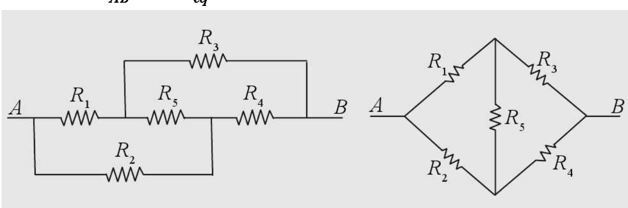


Fig. 21.18

17. Current division Rule

In Fig. 21. 19

$$I_1 = \frac{IR_2}{R_1+R_2}, I_2 = \frac{IR_1}{R_1+R_2}$$

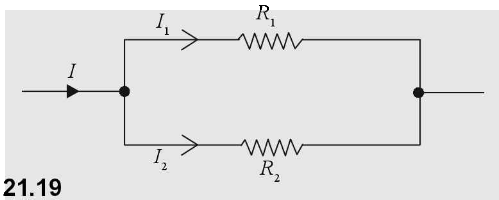


Fig. 21.19

18. Potential division Rule

In Fig. 21.20

$$V_1 = \frac{V_0R_1}{R_1+R_2+R_3}, V_2 = \frac{V_0R_2}{R_1+R_2+R_3},$$

$$V_3 = \frac{V_0R_3}{R_1+R_2+R_3},$$

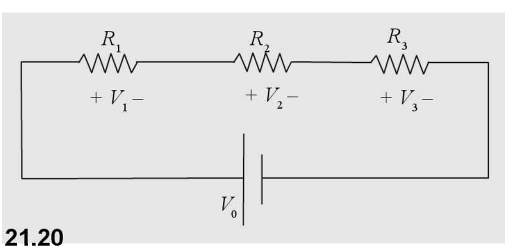


Fig. 21.20

19. To convert a galvanometer (or de Arsenol moment) into ammeter, a shunt (very small resistance) in parallel is connected. If I_g is full scale deflection through the galvanometer and R_g is its internal resistance then to convert it into an ammeter to measure I, a shunt S will be required in paralled such that $S = \frac{I_gR_g}{I - I_g}$

20. To convert a galvanometer into voltmeter to measure V volts a resistance R is to be connected in series given by

$R = \frac{V}{I_g} - R_g$ where I_g is full scale deflection current in galvanometer and R_g is the resistance of the galvanometer.

21. If a skelton cube is made with 12 equal resistances/ wires each having resistance R then net resistance across

(a) the longest diagonal is $\frac{5}{6}R$

(b) the face diagonal is $\frac{3}{4}R$

(c) One side is $\frac{7}{12}R$

(d) the open side as shown in Fig. 21.21 is $\frac{7}{5}R$

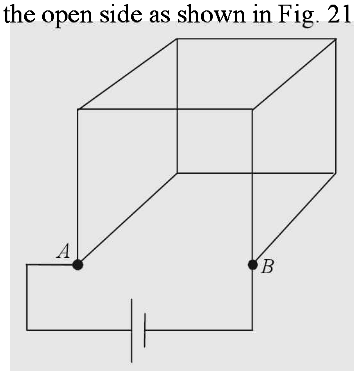


Fig. 21.21

22. Temperature can be determined using Wheatstone bridge arrangement with a vernier (small variable resistance) and Platinum resistance thermometer in one arm as shown in Fig. 21.22.

$$\frac{P}{R+\Delta R} = \frac{Q}{S(1+\alpha\Delta T)} \text{ If } P = Q \text{ then } R = S\alpha\Delta T$$

or $$\Delta T = \frac{\Delta R}{S\alpha}$$

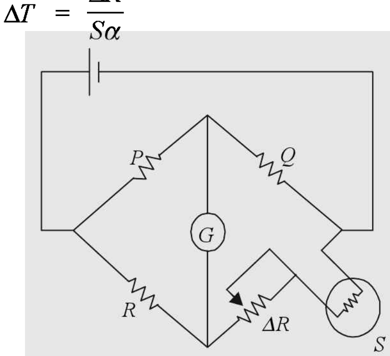


Fig. 21.22 Platinum resistance thermometer or a filament in hot bath

23. The equivalent resistance for the circuit shown in Fig. 21. 23 is

$$R_{AB} = \frac{R_1(R_1+3R_2)}{(R_2+3R_1)}$$

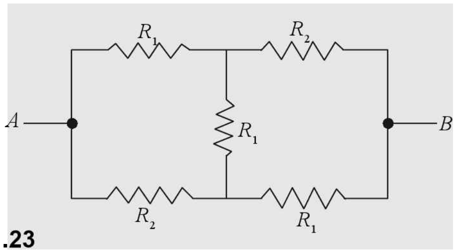


Fig. 21.23

The equivalent resistance for the circuit shown in Fig. 21.24 (a) is $R_{AB} = \dfrac{2R_1R_2 + R_3(R_1+R_2)}{2R_3 + R_1 + R_2}$

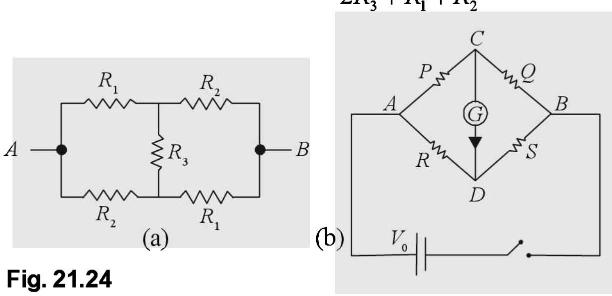


Fig. 21.24

Current through galvanometer in unbalanced wheatstone bridge of fig 21.24 (b)

$$I_g = \frac{V_0(RQ-PS)}{R_g(P+Q)(R+S)+PQ(R+S)+RS(P+Q)}$$

and $V_{CD} = V_C - V_D = \dfrac{V_0(QR-PS)}{(P+Q)(R+S)}$

24. Substances like Carbon (graphite), Ge, Si have negative thermal coefficient of resistance (α) i.e. their rersistivity and hence resistance falls with rise in temperature. Manganin has α = 0. Iron has very large value of α.

25. At high frequency applications like Radio, TV etc. carbon resistors are used. Wire wound resistors at high frequency behave like a resonant circuit and alter the value of resistance by offering impedance. The equivalent circuit of wire wound resistance is shown in Fig. 21.25 at high frequencies.

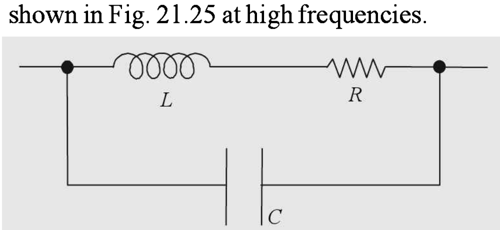


Fig. 21.25

26. If a resistive wire is stretched n times its resistance increases n^2 times, i.e., $R_{new} = n^2 R_{old}$.

27. Current is a 'through' variable and potential difference or voltage is an 'across' variable.

28. Terminal voltage $V = \varepsilon - Ir$ is the potential drop across the cell/source. Normally it is less than emf. However, when the direction of current is opposite as during charging, terminal voltage is greater than emf.

29. Galvanometer can measure small currents ~μA and small voltage ~ mV. That is why we can convert it to both voltmeter and ammeter.

30. DC current in steady state cannot pass through a capacitor.

31. Mean free path of electrons is $\lambda = v_d\tau = \dfrac{mv_d}{ne^2\rho}$

- **Caution**

1. Adding emfs to find net emf when negative terminal of one is connected to negative terminal of other battery or positive terminal of one battery connected to positive terminal of other battery/cell.

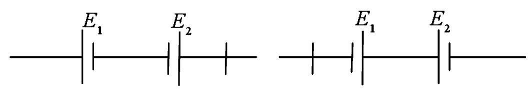


Fig. 21.26 (a)

⇒ In such cases use $E_1 - E_2$ or $E_2 - E_1$ keeping in mind which is greater on the direction of current chosen.

Emfs are added when positive terminal of one battery is connected to negative terminal of other in series. For example in Fig. 21.26 (b) $E_{net} = E_1 + E_1$

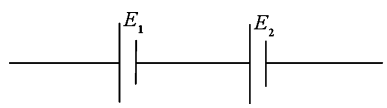


Fig. 21.26 (b)

2. Wrongly detecting Wheatstone bridge, for example, considering AXYO, XYBO, AOWZ and OBWZ as Wheatstone bridge in Fig. 21.27 (a).

⇒ If A and B are point of interest where equivalent resistance is to be determined and R_G is connected between XY terminals (other than the points of interest) and $\dfrac{P}{Q} = \dfrac{R}{S}$

only then R_G can be removed as no current will pass through R_G. Removing R_G suggests it acts like open circuit between X and Y.

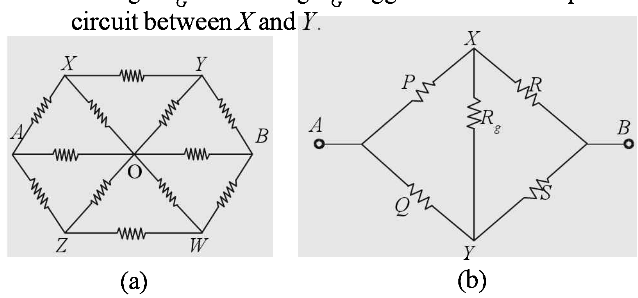


Fig. 21.27

3. Not applying current division properly when branching occurs at a point.

⇒ Stick to junction law in such cases and divide the current properly.

4. Not taking into account the resistance of voltmeter or ammeter when they are not ideal.

⇒ Remember voltmeter is connected in paralled. If its resistance is small it alters the resistance of the circuit drastically. Therefore, their resistances must be taken into account.

5. Not taking into account internal resistance of the cell.

⇒ When current in the circuit is flowing internal resistance of the cell must be taken into consideration. It alters the terminal voltage and even decreases the current in the circuit. For example in Fig. 21.28

$I = \frac{E}{R+r}$ and terminal voltage $V = E - Ir$

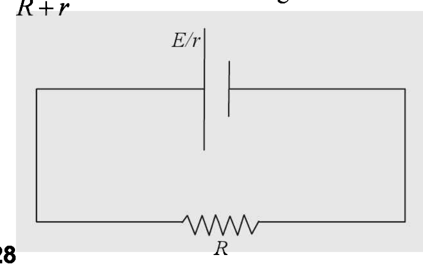


Fig. 21.28

6. Assuming DC current passes through capacitor in steady state.

⇒ $T_{dc} = 0$ in steady state through capacitor or branch containing a capacitor. However, during transient current passes through capacitor. You know $Q = CV$ or

$I = \frac{dQ}{dt} = C\frac{dV}{dt}$. i.e., if V is constant $\frac{dV}{dt} = 0$ and hence $I = 0$

7. Considering potentiometer or meterbridge has no resistance.

⇒ Must take into account the resistance of potentionmeter wire. To find potential gradient, find potential drop across the length of the wire.

The potential gradient

$$k = \frac{\text{potential drop across the length of the wire}}{\text{length of the wire}}$$

Note $k \neq \frac{\text{emf applied}}{\text{length of the potentiometer wire}}$

8. Considering resistivity varies with length of the wire or with area of cross-section.

⇒ Resistivity depends upon nature of the substance and is independent of the length and area of cross-section of the wire. However, it depends upon pressure and temperature.

Note: $\rho \propto T$ and $\rho \propto \frac{1}{P}$ in conductors.

9. Considering current through capacitor as zero, therefore no potential drop will occur across the capacitor.

⇒ Remember that potential drop across the capacitor may occur even though the current is zero.

10. Not understanding the meaning of a switch.

⇒ Potential drop across AB is –2V when switch is open and zero when switch is closed. Note that a closed switch is equivalent to short circuit or zero resistance.

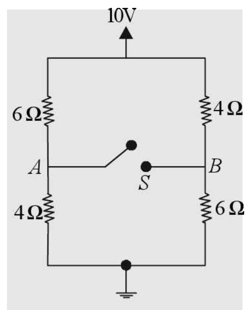


Fig. 21.29

11. Considering that current starts from positive terminal of the battery and is used up by the time it reaches negative terminal.

⇒ In fact current remains same at every point in a simple loop because it is based on conservation of charge. Charge entering per second is equal to charge leaving per second.

12. Considering that in a conductor, when current is increased, electron density increases.

⇒ If the conductor has uniform area of cross-section then increasing the current results in increasing the drift velocity.

13. In a conductor as shown in Fig. 21.30 considering that the drift velocity remains same everywhere.

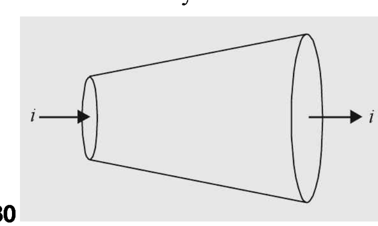


Fig. 21.30

⇒ Drift velocity is larger at smaller cross-sections than at higher cross-sections.

SOLVED PROBLEMS

1. Find the drift velocity in Cu wire if it has 1A current through 2 mm^2 cross-section. Free electron density is 8.5×10^{22} cm^{-3}.

(a) 0.36 mms^{-1} (b) 0.36 cms^{-1}

(c) 0.036 mms^{-1} (d) 0.036 cms^{-1}

Solution (c) $v_d = \dfrac{i}{neA}$

$$= \frac{1}{8.5\times10^{28}\times1.6\times10^{-19}\times2\times10^{-6}}$$

$$= \frac{1\times10^{-3}}{27.2} = 0.036 \text{ mms}^{-1}.$$

2. A 5 Ω constantan wire is bent to form a ring. Find the resistance across the diameter of the wire.

(a) 2.5Ω (b) 1.25 Ω

(c) 5 Ω (d) 0.625 Ω

Solution (b) $R \propto l$. Therefore 2.5 Ω resistances are in parallel across AB. Thus $R = \dfrac{2.5}{2} = 1.25\ \Omega$

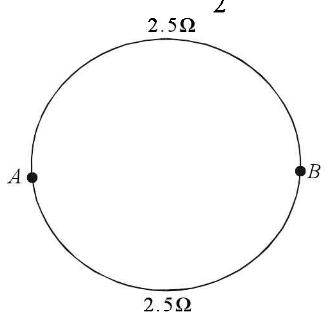


Fig. 21.31

3. Find the terminal voltage across E_1 and E_2 in Fig. 21.32 (a)

(a) 3.6 V, 7.8 V (b) 2.4 V, 7.8 V

(c) 3.6 V, 10 V (d) 2.4 V, 10.2 V

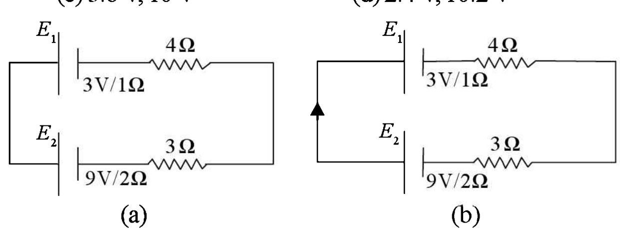


Fig. 21.32

Solution (a) Current in the circuit in Fig. 21.32 (b)

$$I = \frac{9-3}{10} = 0.6 \text{ A}$$

$$V_1 = E_1 + Ir_1 = 3 + 0.6 \times 1 = 3.6 \text{ V};$$

$$V_2 = E_2 - Ir_2$$

$$= 9 - 0.6 \times 2 = 7.8 \text{ V}$$

4. Find I_1 and I_2 in Fig. 21.33

(a) $2.0A, \frac{4}{3}A$ (b) $1A, \frac{2}{3}A$

(c) $2A, \frac{2}{3}A$ (d) $1A, \frac{1}{3}A$

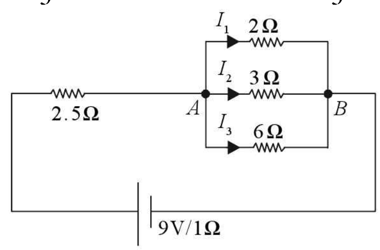


Fig. 21.33

Solution (c) $\dfrac{1}{R_{AB}} = \dfrac{1}{2} + \dfrac{1}{3} + \dfrac{1}{6} = \dfrac{3+2+1}{6}$ and

$$I = \frac{9}{1+1+2.5} = 2\text{A}$$

$$V_{AB} = \text{I}.R_{AB} = 2 \times 1 = 2\text{V}$$

$$I_1 = \frac{2V}{2\Omega} = 1\text{A}\ I_2 = \frac{2}{3}\text{A}$$

5. Find the value of R so that no deflection is noticed in the galvanometer when the switch S is closed or open.

(a) 4Ω (b) 8Ω

(c) 6Ω (d) none of these

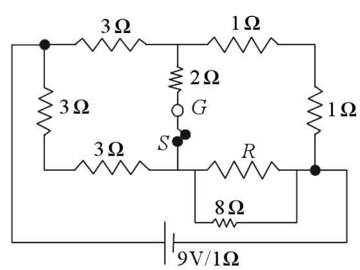


Fig. 21.34

Solution (b) It is a Wheatstone bridge case, therefore

$$\frac{3}{3+3} = \frac{1+1}{\frac{8R}{8+8}} \text{ or } \frac{8R}{8+R} = \Rightarrow R = 8\Omega$$

6. The ammeter reading in the circuit of Fig. 21.35 (a) is

(a) $\frac{15}{32}$ A (b) $\frac{14}{33}$ A

(c) $\frac{17}{33}$ A (d) $\frac{15}{31}$ A

Solution (a) $E_{eq} = \dfrac{3\times1+6\times2}{1+2} = 5\text{V};\ \text{req} = \dfrac{2\times1}{2+1} = \dfrac{2}{3}\Omega$

From equivalent circuit of Fig. 21.35 (b)

$$I = \frac{5}{10+2/3} = \frac{15}{32}\text{ A}$$

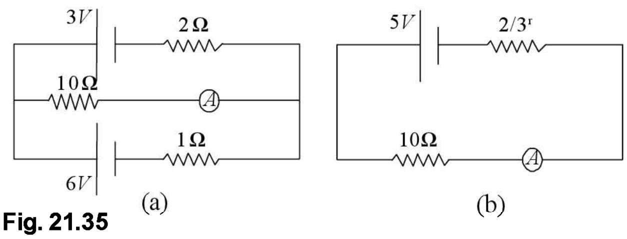


Fig. 21.35

7. Find the equivalent resistance about any branch of the base of the square pyramid shown. Assume resistance of each branch is R.

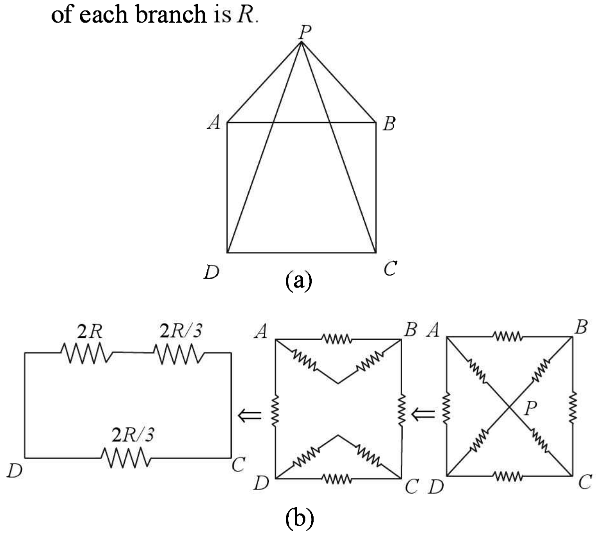


Fig. 21.36

(a) $\dfrac{7R}{15}$ (b) $\dfrac{8R}{15}$

(c) $\dfrac{R}{2}$ (d) none of these

Solution (b) From the eqnivalent circuits of Fig. 21.36 (b)

$$R_{eq} = \frac{\frac{8R}{3}\times\frac{2R}{3}}{\frac{8R}{3}+\frac{2R}{3}} = \frac{16R}{30} = \frac{8R}{15}$$

8. A hollow cylinder of radiis a and b is filled with a material of resistivity ρ. Find the current through ammeter.

(a) $\dfrac{E\pi l\left(b^2-a^2\right)}{\rho\lambda}$ (b) $\dfrac{E\pi l}{\left(\rho\log_\varepsilon \frac{b}{a}\right)}$

(c) $\dfrac{E2\pi l}{\rho\log_\varepsilon \frac{b}{a}}$ (d) $\dfrac{E2\pi l}{\rho\log_\varepsilon \frac{a}{b}}$

Solution (c) Assume a hypothetical cylinder of radius x and thickness dx then

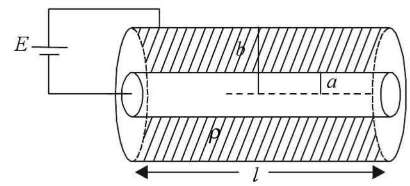


Fig. 21.37

$$\int dR = \int_a^b \frac{\rho dx}{2\pi xl}\quad R = \frac{\rho\log_e\frac{b}{a}}{2\pi l}\ \text{and}\ I = \frac{E}{R} = \frac{E2\pi l}{\rho\log_e\frac{b}{a}}$$

9. Find current I, in Fig. 21.38 (a)

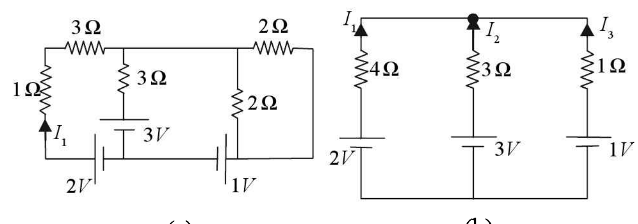


Fig. 21.38 (a) (b)

(a) $\dfrac{1}{19}A$ (b) $\dfrac{2}{19}A$

(c) $\dfrac{3}{19}A$ (d) none of these

Solution (b) Draw equivalent circuit as shown in Fig. 21.38 (b). Let node potential be V, then applying KCL (Junction law)

$$I_1 + I_2 + I_3 = 0$$

or $$\frac{2-V}{4} + \frac{3-V}{3} + \frac{1-V}{1} = 0$$

or $$6 - 3V + 12 - 4V + 12 - 12V = 0$$

or $$V = \frac{30}{19}\text{ Volt.}$$

$$I_1 = \frac{2-\frac{30}{19}}{4} = \frac{2}{19}\text{A}$$

10. Find the potential drop across 4μF capacitor and 6Ω resistor in Fig. 21.39

(a) 0,0 (b) 0, 3V

(c) 0, 2V (d) 2V, 0

Solution (d) $I = \dfrac{3}{2+4} = \dfrac{1}{2}A$

Potential drop across 4Ω resistor is $V = 4\times\dfrac{1}{2} = 2V$

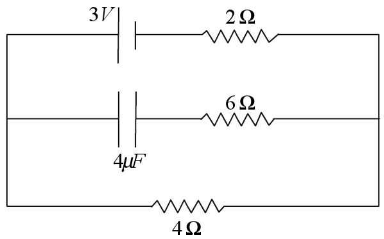


Fig. 21.39

The whole potential drop occurs across 4μF capacitor as current does not flow through the branch containing capacitor.

11. The temperature of a conductor is increased. The product of resistivity and conductivity

(a) increases (b) decreases

(c) remains constant

(d) may increase or decrease.

Solution (c)

12. Two non-ideal batteries are connected in parallel. Then

(A) the equivalent emf is less than either of the two emfs.

(B) the equivalent internal resistance is less than either of the two internal resistances.

(a) both A and B are correct

(b) only A is correct

(c) only B is correct

(d) both A and B are wrong

Solution (c)

13. A resistor connected to a battery is heated due to current through it. Which of the following quantity does not vary?

(a) Resistance (b) Drift velocity

(c) Resistivity

(d) Number of free electrons

Solution (d)

14. Find the electric field in the copper wire of area of cross-section 2mm^2 carrying a current of 1A. The resistivity of copper is $1.7 \times 10^{-8}\,\Omega$m.

(a) 4.25×10^{-3} V/m (b) 8.5×10^{-3}V/m

(c) 8.5 V/m (d) 8.5×10^{-3} V/m

Solution (b) $J = \sigma E$ or $E = \dfrac{J}{\sigma} = J\rho = \dfrac{I}{A}$

$$\rho = \frac{1 \times 1.7 \times 10^{-8}}{2 \times 10^{-6}} = 8.5 \times 10^{-3} \text{ V/m}$$

15. A high resistance voltmeter reads 1.52V when switch S is open and 1.48V when switch S is closed. The ammeter resistance is (Fig. 21.40)

(a) 0.2 Ω (b) 0.3 Ω

(c) 0.4 Ω (d) 0.8 Ω

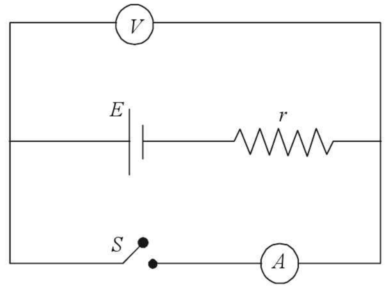


Fig. 21.40

Solution (c) $E = 1.52$V and $V = E - Ir$

$1.48 = 1.52 - 1\,(r)$ or $r = 0.4\Omega$

16. Find the resistance across $A\,B$ in Fig. 21.41 (a)

(a) R (b) $\dfrac{2}{3}R$

(c) $\dfrac{R}{3}$ (d) $\dfrac{4}{3}R$

(e) $3R$

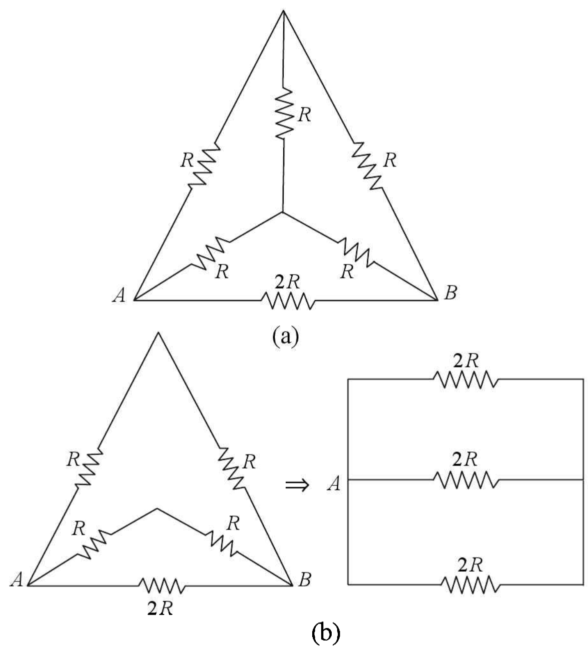


Fig. 21.41

Solution (b) Draw equivalent circuit of Fig. 21.41 (b).

then $R_{eq} = 2\,\dfrac{R}{3}$

17. When the current in a wire is 1A, the drift velocity is 1.2×10^{-4} ms^{-1}. The drift velocity when current become 5 A is

(a) 1.2×10^{-4} ms^{-1} (b) 3.6×10^{-4} ms^{-1}

(c) 6×10^{-4} ms^{-1} (d) 4.8×10^{-4} ms^{-1}

Solution (c) $I \propto v_d$ ∴ new drift velocity is 6×10^{-4} ms^{-1}.

18. An ideal voltmeter is connected in Fig. 21.42. The current in circuit is

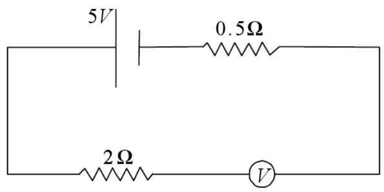


Fig. 21.42

(a) 2A (b) 2.5A

(c) ∝ (d) zero.

Solution (d) Ideal voltmeter has infinite resistance. Therefore current will be zero.

19. In Fig. 21.43 AB is 300 cm long wire having resistance 10Ω per meter. Rheostat is set at 20Ω. The balance point will be attained at

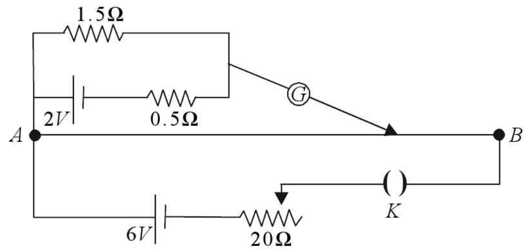


Fig. 21.43

(a) 1.0m (b) 1.25m

(c) 1.5m (d) cannot be determined.

Solution (b) $V_{AB} = \frac{6\times30}{50} = 3.6\text{V}$. Terminal voltage of

cell $= \frac{2\times1.5}{2} = 1.5\text{V}$

Using $V = kl \Rightarrow 1.5 = \frac{3.6}{300}\, l$ or $l = 125\text{cm}$

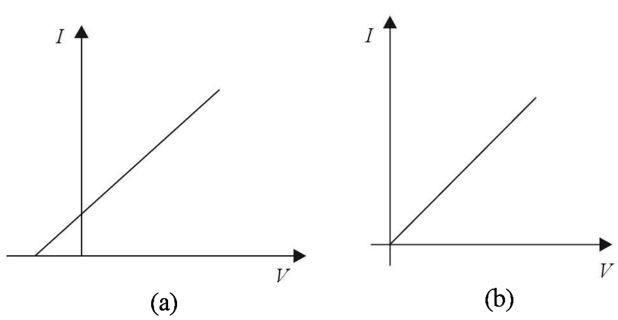
20. Ohm's law can be applied to

(a) ohmic devices (b) non-ohmic devices

(c) both (a) and (b) (d) none

Solution (c)

21. Which of the $V - I$ graph obeys Ohm's law?



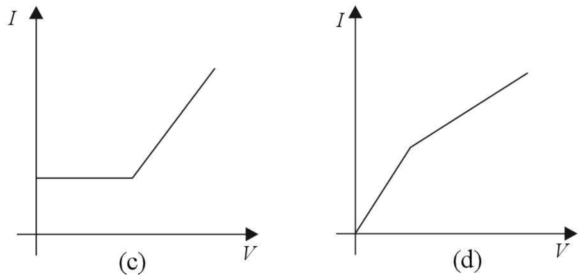


Fig. 21.44

Solution (b)

22. The $V - I$ graph of a conductor at two different temperatuses is shown in Fig. 21.45. The ratio of temperature $\frac{T_1}{T_2}$ is

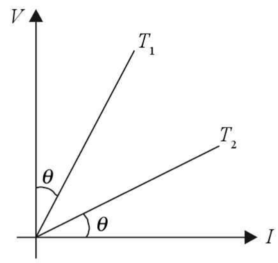


Fig. 21.45

(a) $\tan^2\theta$ (b) $\cot^2\theta$

(c) $\sec^2\theta$ (d) $\text{cosec}^2\theta$

Solution (b)

23. Two cells of emf's 1.25V and 0.75V having equal internal resistance are connected in parallel. The effective emf is

(a) 0.75V (b) 1.25V

(c) 2.0V (d) 1.0V

(e) 0.5 V

Solution (d) $E = \frac{1.25\times r + .75\times r}{2r}$

24. A 250 cm long wire has diameter 1mm. It is connected at the right gap of a slide wire bridge. When a 3π resistance is connected to left gap, the null point is obtained at 60cm. The specific resistance of the wire is

(a) $6.28\times10^{-6}\,\Omega\text{m}$ (b) $6.28\times10^{-5}\,\Omega\text{m}$

(c) $6.28\times10^{-8}\,\Omega\text{m}$ (d) $6.28\times10^{-7}\,\Omega\text{m}$

Solution (d) Using $\frac{R_1}{R_2} = \frac{l}{100-l} \Rightarrow \frac{3}{R_2} = \frac{60}{40}$

$\therefore \quad R_2 = 2\Omega$

$$\rho = \frac{R\times\pi r^2}{l} = \frac{2\times3.14\times(.5\times10^{-3})^2}{2.5}$$

$$= 6.28\times10^{-7}\,\Omega\text{m}$$

25. An ammeter reads 500 mA. When a shunt of 0.1 Ω is connected across the ammeter its reading drops to 50 mA. The resistance of the ammeter is

(a) 1 Ω (b) 1.1 Ω
(c) 0.9 Ω (d) none of These

Solution (c) In parallel voltage remains same

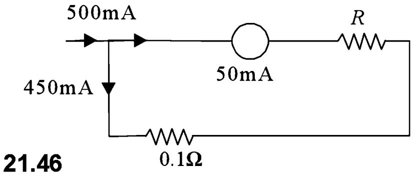


Fig. 21.46

$$50R = 450(0.1)$$
$$R = 0.9\,\Omega$$

TYPICAL PROBLEMS

26. Find the current in 100 Ω resistance in Fig. 21.47 (a).

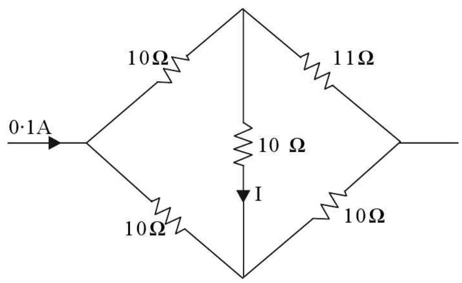


Fig. 21.47 (a)

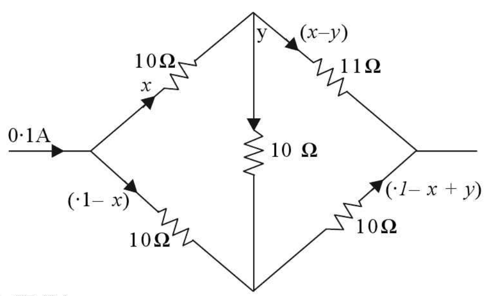


Fig. 21.47 (b)

(a) 0.01 A nearly (b) 0.001 A nearly
(c) 0.02 A nearly (d) 0.002 2A nearly.

Solution (b) Solve using Loop law $= 10x + 100y = 10\,(.1 - x)$

or $20x + 100y = 1$ ....(1)

$100y + 10\,(.1 - x + y) = 11\,(x - y)$

$21x - 121y = 1$ ....(2)

Solving eq. 1 & 2 for y we get $-y = 0.001\,A$

27. A plastic tube 25 m long and 4 cm in diameter is dipped into Ag solution deposting a silver layer 0.1 mm thick uniformly over its outer surface. Find the current if this coated tube is connected across a 12V battery. $\rho_{Ag} = 1.47 \times 10^{-8}\,\Omega\text{–m}$

(a) 4.14 A (b) 41.4 A
(c) 414 A (d) 414 mA

Solution (c) $R = \rho \dfrac{l}{\pi(r_2^2 - r_1^2)} = \dfrac{1.47\times10^{-8}\times25}{3.14(2.01^2 - 2^2)\times10^4}$

$$= \frac{1.47\times25\times10^{-4}}{3.14\times4.01\times(.01)}$$

$$I = \frac{12}{2.9\times10^{-2}} = 4.14\times10^2 = 414\text{ A}$$

28. Open circuit voltage of a source is 7.86 V and its short circuit current is 9.25 A. Find the current when an external resistance of 2.4 Ω is connected.

(a) 1.4 A (b) 1.82 A
(c) 2.01 A (d) 2.4 A

Solution (d) $r = \dfrac{7.86}{9.25} = 0.847\,\Omega$

$$I = \frac{7.86}{2.4 + .847} = 2.4\text{ A}$$

29. In the Fig. 21.48 find I, R, ε_1 and ε_2

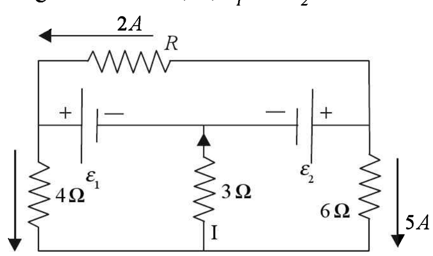


Fig. 21.48

(a) 8A, 9Ω, 54V, 36V (b) 8A, 9Ω, 36V, 54V
(c) 9A, 8Ω, 36V, 54V (d) none of these

Solution (b) $2R + \varepsilon_1 - \varepsilon_2 = 0$ ....(1)

$4(3) + 3I = \varepsilon_1$ ....(2)

$5(6) + 3I = \varepsilon_2$ ....(3)

$2R + (3) = 5(6)$ ....(4)

Solving eq (1), (2), (3) and (4), we get $R = 9\,\Omega$, $I = 8A$, $\varepsilon_1 = 36\text{V}$ and $\varepsilon_2 = 54\text{V}$

30. A current of 2A passes through a wire for 20 minutes. The number of electrons that crossed the cross-section in this period is

(a) 1.5×10^{21} (b) 1.5×10^{20}
(c) 1.5×10^{22} (d) 1.5×10^{23}

Solution (c) $n = \frac{It}{e} = \frac{2\times20\times60}{1.6\times10^{19}} = 15\times10^{21}$

31. To measure a small resistance ~ $10^{-5}\Omega$, one should use

(a) Wheatstone bridge (b) Postoffice Box

(c) Wein's bridge (d) Carrey Foster bridge

Solution (d)

32. The free electron gas theory explains conduction in

(a) metals only (b) semiconductors only

(c) insulators only (d) all of these

Solution (a)

33. Find current through 12 Ω resistor in Fig. 21.49 (a)

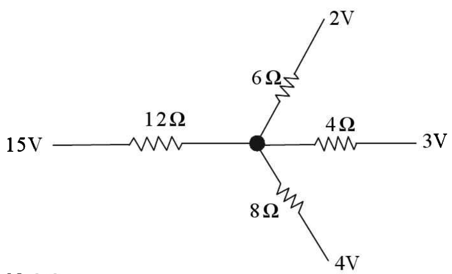


Fig. 21.49 (a)

(a) $\frac{49}{60}$ A (b) $\frac{41}{60}$ A

(c) $\frac{21}{40}$ A (d) $\frac{23}{40}$ A

Solution (a) Let V be the potential at P then applying *KCL* at junction *P*.

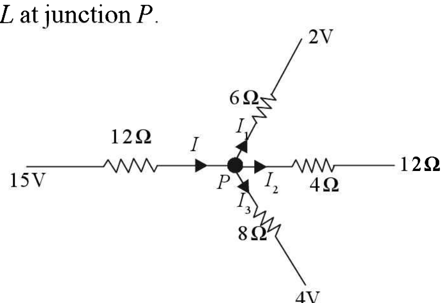


Fig. 21.49 (b)

$$I = I_1 + I_2 + I_3 \frac{15-V}{12}$$

$$= \frac{V-2}{6} + \frac{V-3}{4} + \frac{V-4}{8}$$

$$15-V = 2(V-2) + 3(V-3) + 1.5(V-4)\ 7.5V = 39$$

or $$V = \frac{39}{7.5} = 5.2\text{V}$$

and $$\text{I} = \frac{15-5.2}{12} = \frac{4.9}{6}\ A$$

34. To terminate the network shown in Fig. 21.50, the resistance required is

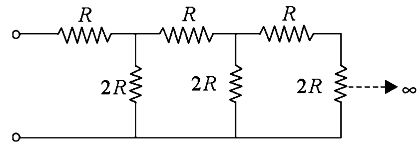


Fig. 21.50

(a) R (b) $2R$

(c) $3R$ (c) $\frac{R}{2}$

Solution (b) $2R$ (equivalent resistance).

35. The ammeter in Fig. 21.51 will read

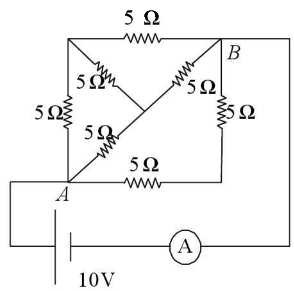


Fig. 21.51

(a) 3A (b) $\frac{10}{3}$ A

(c) 30A (d) $\frac{100}{3}$ A

Solution (a) $R_{AB} = \frac{10}{3}$ (use Wheatstone bridge)

$$I = \frac{10}{10/3} = 3\text{A}$$

36. Find x in the Fig. 21.52 so that galvanometer shows null deflection.

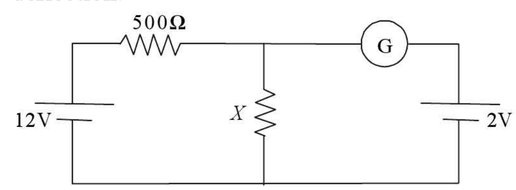


Fig. 21.52

(a) 100 Ω (b) 400 Ω

(c) 200 Ω (d) 250 Ω

[AIEEE 2005]

Solution (a) Potential drop across x should be 2 V.

$$\therefore \quad 2 = \frac{12x}{X+500}$$

or $$X = 100\Omega$$

37. When *Cu* and *Ge* are cooled to –150°C then resistance of *Cu*____and that of *Ge*____

(a) increases, increases (b) decreases, increases

(c) decreases, decreases (d) increases, decreases.

Solution (b) $\because$ metals have positive thermal coefficient of resistance while semiconductors have negative thermal coefficient of resistance.

38. Find current I and current through 6Ω resistance in Fig. 21.53 (a)

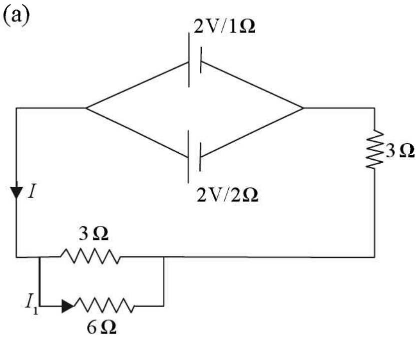


Fig. 21.53 (a)

(a) $\frac{6}{17}$A, $\frac{1}{17}$A (b) $\frac{3}{17}$A, $\frac{1}{17}$A

(c) $\frac{6}{17}$A, $\frac{2}{17}$A (d) none

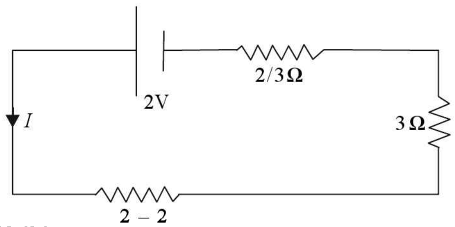


Fig. 21.53 (b)

Solution (c) $E_{eq} = \frac{2\times1+2\times2}{2+1} = 2V$; $r_{eq} = \frac{1\times2}{1+2} = \frac{2}{3}\Omega$

$$I = \frac{2}{5+2/3} = \frac{6}{17}\text{ A } I_1 = \frac{6}{17}\times\frac{3}{9} = \frac{2}{17}\text{ A}$$

39. An ideal voltmeter and an ideal ammeter are connected in Fig 21.54. The reading of the voltmeter is

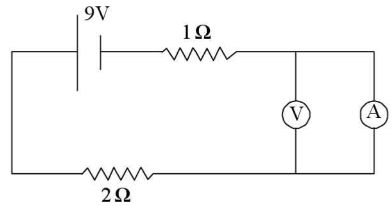


Fig. 21.54

(a) 9V (b) 6V

(c) 3V (d) zero

Solution (d) Since ideal ammeter has zero resistance and we are measuring potential drop across it, $V = 0$.

40. A voltmeter has a 25Ω coil and 575Ω in series. The coil takes 10mA for full scale deflection. The maximum potential difference which can be measured is

(a) 250mV (b) 5.75V

(c) 5.5V (d) 6.0V

Solution (d) $V_{max} = I_g(R_g + R) = 10\times10^{-3}(600) = 6V$

QUESTIONS FOR PRACTICE

1. Two cells of same emf are connected in series. Their internal resistances are r_1 and r_2 respectively and $r_1 > r_2$. When this combination is connected to an external resistance R then the potential difference between the terminals of first cell becomes zero. In this condition the value of R will be

(a) $\frac{r_1 - r_2}{2}$ (b) $\frac{r_1 + r_2}{2}$

(c) $r_1 - r_2$ (d) $r_1 + r_2$

2. The ratio of the drift velocity v_d and r.m.s. velocity of electrons is

(a) 10^{-10} (b) 10^{-5}

(c) 10^{-3} (d) 10^{-6}

3. The VI graph for a conductor at temperature T_1 and T_2 are shown in the Fig. 21.55 $(T_2 - T_1)$ will be proportional to

(a) cos 2θ (b) cot 2θ

(c) sin 2θ (d) tan 2θ

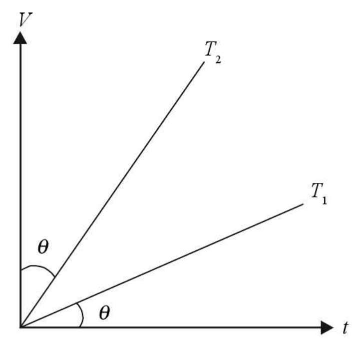


Fig. 21.55

4. In a meter bridge experiment, the known and unknown resistances are mutually interchanged to remove

(a) indicator error (b) end error

(c) contact error (d) thermoelectric error

5. The effective resistance Fig. 21.56 between points X and Y will be

(a) 4 Ω (b) 2 Ω

(c) 8 Ω (d) 16 Ω

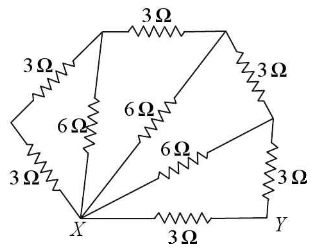


Fig. 21.56

6. The equivalent resistance between points X and Y in the following diagram will be

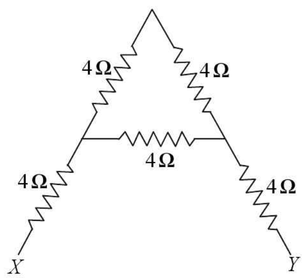


Fig. 21.57

(a) 10.6 Ω (b) 20 Ω

(c) 16 Ω (d) 8 Ω

7. Eleven resistances, each of value 2Ω, are connected as shown in the following diagram. The equivalent resistance between the points A and B will be

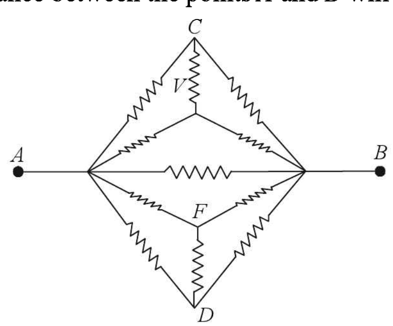


Fig. 21.58

(a) 2 Ω (b) $\frac{2}{3}$ Ω

(c) $\frac{3}{4}$ Ω (d) $\frac{4}{3}$ Ω

8. The equivalent resistance between the points A and B in the adjoining figure will be

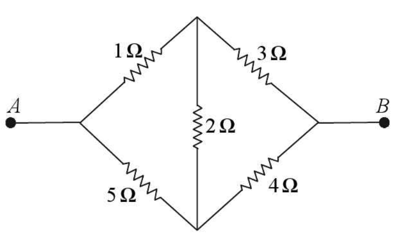


Fig. 21.59

(a) 2.96 Ω (b) 3.71 Ω

(c) 1.68 Ω (d) 5.12 Q

9. The value of current in the 60Ω resistance in the adjoining circuit diagram will be

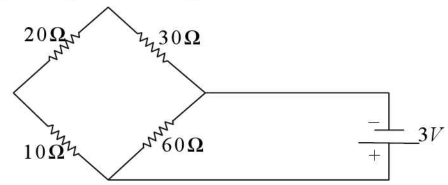


Fig. 21.60

(a) 0.1 A (b) 0.5 A

(c) 0.05 A (d) 0.01 A

10. The value of current in other resistances in the above question will be

(a) 0.1 A (b) 0.5 A

(c) 0.05 A (d) 0.01 A

11. The equivalent resistance between the points 1 and 7 in the adjoining circuit (Fig 21.61) will be

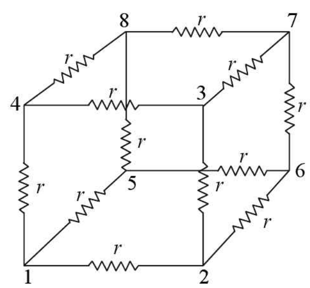


Fig. 21.61

(a) $\frac{7}{12}$ r (b) $\frac{5}{6}$ r

(c) $\frac{3}{4}$ r (d) $\frac{9}{4}$ r

12. In the above question, the effective resistance between the points 1 and 4 will be

(a) $\frac{7}{12}$ r (b) $\frac{5}{6}$ r

(c) $\frac{3}{4}$ r (d) $\frac{5}{7}$ r

13. In the circuit of question 11 the equivalent circuit between the points 1 and 3 will be

(a) $\frac{7}{12}$ r (b) $\frac{5}{6}$ r

(c) $\frac{3}{4}$ r (d) $\frac{3}{7}$ r

14. The equivalent resistance between A and B in the following circuit is

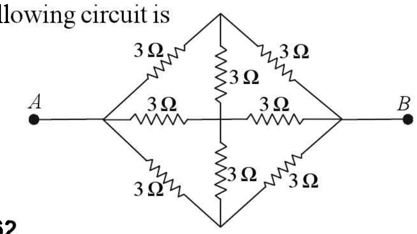


Fig. 21.62

(a) 1 Ω (b) 2 Ω
(c) 3 Ω (d) 4 Ω

15. An ammeter is always connected in series in a circuit because
(a) its resistance is very high
(b) its resistance is very low
(c) it does not draw current from the circuit
(d) its resistance is infinity

16. In order to convert a moving coil galvanometer into ammeter, the following will have to be connected
(a) high resistance in series
(b) low resistance in series
(c) high resistance in parallel
(d) low resistance in parallel

17. An ammeter can be converted into a voltmeter by connecting
(a) a low resistance in series
(b) a high resistance in series
(c) a low resistance in parallel
(d) a high resistance in parallel.

18. An ammeter of resistance 5 Ω can read 5 milli ampere current. If it is to be used to read voltage of 100 volt, then the resistance required to be connected in series with it will be
(a) 19995 Ω (b) 19,9995 Ω
(c) 199.995 Ω (d) 19999.95 Ω

19. The deflection in a moving coil galvanometer is
(a) directly proportional to the number of turns in the coil
(b) inversely proportional to the area of the coil
(c) inversely proportional to the current flowing in it
(d) directly proportional to the twisting couple per unit twist.

20. Whose resistance, out of the following, is maximum?
(a) Ammeter (b) Millimeter
(c) Microammeter (d) All of the above

21. The pole pieces of a horse-shoe magnet are made cylindrical so that the deflection of the coil is proportional to
(a) the current flowing in the coil
(b) $\dfrac{1}{\text{current flowing in the coil}}$
(c) the magnetic field
(d) the square of current flowing in the coil

22. The potentiometer is more appropriate for measuring potential difference than a voltmeter because
(a) the resistance of voltmeter is high
(b) the sensitivity of a potentiometer is higher than that of voltmeter
(c) the resistance of potentiometer wire is very low
(d) the potentiometer does not draw any current from the unknown source of emf.

23. The range of a voltmeter of resistance G Ω is V volt. The resistance required to be connected in series with it in order to convert it into a voltmeter of range nV volt, will be
(a) $(n-1)G$ (b) $\dfrac{G}{n}$
(c) nG (d) $\dfrac{G}{(n-1)}$

24. The deflection of a moving coil galvanometer reduces to half, on shunting it with a resistance of 60Ω. The resistance of galvanometer is
(a) 30Ω (b) 120Ω
(c) 60Ω (d) 15Ω

25. In the experiment of calibration of voltmeter, a 1.1 volt standard cell gets balanced at 440 cm length of the wire. The balancing length corresponding to a potential difference between the ends of a resistance comes out to be 190 cm. A voltmeter shows 0.5 volt for this potential difference. The error in the reading of voltmeter will be
(a) 0.025 Volt (b) 25 Volt
(c) 2 5 Volt (d) 0.25 Volt

26. The current flowing in the primary circuit of potentiometer is 2 A and the resistance of its wire is 0.2 Ω/m. If a one ohm standard coil gets balanced at 250 cm length of the wire, then the current flowing in the coil will be
(a) 0.5 A (b) 1A
(c) 1.5 A (d) 0.05 A

27. On comparing the emf.'s E_1 and E_2 $(E_1 > E_2)$ of two cells by a potentiometer, the balancing lengths come out to be l_1 and l_2 respectively, then
(a) $l_1 < l_2$ (b) $l_1 - l_2$
(c) $l_1 > l_2$ (d) none of the above

28. The internal resistance of a primary cell depends on
(a) the concentration of solution
(b) the current drawn from the cell
(c) the distance between the electrodes of the cell
(d) all of the above

29. The length of a potentiometer wire is 10 m. The distance between the null points on its wire corresponding to two cells comes out to be 60cm. If the difference of emf's of the cells is 0.4 volt then the potential gradient on potentiometer wire will be

(a) 0.67 V/m (b) 0.5 V/m
(c) 2.5 V/m (d) 0 V/m.

30. The two cells are connected in series, in a potentiometer experiment, in such a way so as to support each other and to oppose each other. The balancing lengths in two conditions are obtained as 150 cm and 50 cm respectively. The ratio of emf's of two cells will be

(a) 1 : 2 (b) 2 : 1
(c) 1 : 4 (d) 4 : 1

31. In the following circuit diagram (Fig. 21.63) if the ammeter reading is zero, then the voltmeter reading will be

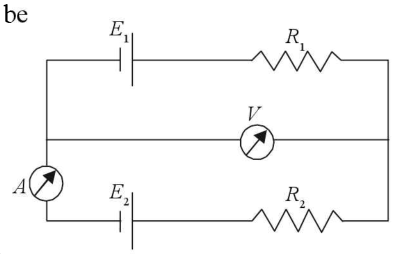


Fig. 21.63

(a) zero (b) $E_1 + E_2$
(c) E_1 (d) E_2

32. Which physical quantity cannot be determined with the help of potentiometer?

(a) I (b) V
(c) L (d) R

33. If I current is flowing in a potentiometer wire of length L and resistance R, then potential gradient will be

(a) $\frac{IR}{L}$ (b) IRL
(c) $\frac{RL}{I}$ (d) $\frac{IL}{R}$

34. In the adjoining diagram $R_1 = 10\Omega$, $R_2 = 20\Omega$, $R_3 = 40W$, $R_4 = 80\Omega$ and $VA = 5V$, $VB = 10V$, $VC = 20V$, $VD = 15V$. The current in the resistance R_1 will be

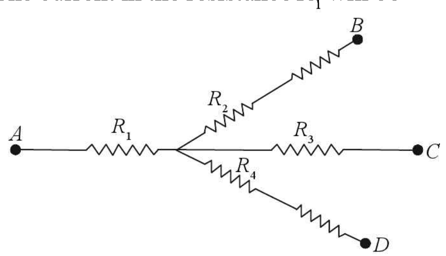


Fig. 21.64

(a) 0.4 A towards O (b) 0.4 A away from O
(c) 0.6 A towards O (d) 0.6 A away from O

35. In the circuit of Q.92, the current in R_2 will be

(a) 0.1 A towards O (b) 0.1 A away from O
(c) 0.05 A towards O (d) 0.05 A away from O

36. In the adjoining figure, the equivalent resistance between the points A and H will be

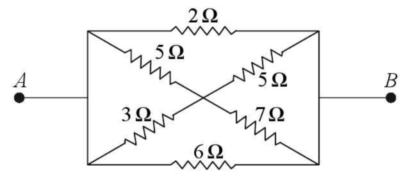


Fig. 21.65

(a) $\frac{7}{8}\ \Omega$ (b) $\frac{8}{7}\ \Omega$
(c) $\frac{9}{11}\ \Omega$ (d) $\frac{11}{9}\ \Omega$

37. In the following star circuit diagram, the equivalent resistance between the points A and H will be

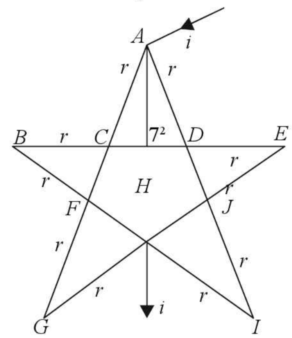


Fig. 21.66

(a) 1.944 r (b) 0.973 r
(c) 0.486 r (d) 0.243 r

38. In the adjoining circuit diagram each resistance is of 10 Ω. The current in the arm AD will be

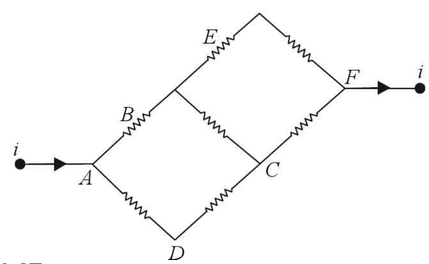


Fig. 21.67

(a) $\frac{2i}{5}$ (b) $\frac{3i}{5}$
(c) $\frac{4i}{5}$ (d) $\frac{i}{5}$

39. In the circuit of above question the current in the arm BC will be

(a) $\frac{2i}{5}$ (b) $\frac{3i}{5}$
(c) $\frac{4i}{5}$ (d) $\frac{i}{5}$

40. In Fig. 21.68 circuit of Wheatstone's bridge is represented. When the ratio arms P and Q are almost equal then the bridge gets balanced at $R = 400\ \Omega$. If P and Q are mutually interchanged then the bridge gets balanced at $R = 441\Omega$. The value of unknown resistance X will be

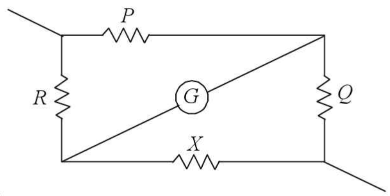


Fig. 21.68

(a) 402.49 Ω (b) 403 Ω

(c) 404 Ω (d) 420 Ω

41. In the above question, the ratio Q/P will be

(a) 1.05 : 1 (b) 1.005 : 1

(c) 1.5 : 1 (d) 1.25 : 1

42. A Wheatstone's bridge is constructed out of four resistances 10 Ω, 50 Ω, 100 Ω and 500 Ω. If a 25 volt battery is connected across 500 Ω resistance then current in 500 Ω resistance will be

(a) 5 mA (b) 0.5 A

(c) 5 A (d) 0.05 A

43. In the above problem current in rest of the resistances will be

(a) 8.8 A (b) 0.15 A

(c) 0.37 A (d) 3.5 A

44. In the following circuit if key *K* is pressed then the galvanometer reading becomes half. The resistance of galvanometer is

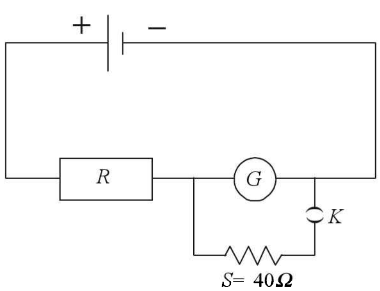


Fig. 21.69

(a) 20 Ω (b) 30 Ω

(c) 40 Ω (d) 50 Ω

45. If the central resistance is 15 Ω then the equivalent resistance between *A* and *B* will be

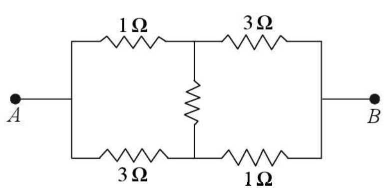


Fig. 21.70

(a) $\frac{3}{5}$ Ω (b) 4 Ω

(c) $\frac{33}{17}$ Ω (d) $\frac{1}{4}$ Ω

46. Each resistance in the following figure is of 3 Ω. The equivalent resistance between *A* and *B* will be

(a) 1.0 Ω (b) 2 Ω

(c) 3 Ω (d) 4 Ω

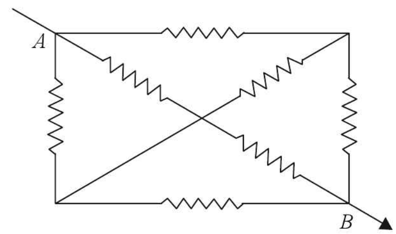


Fig. 21.71

47. The value of *i* in the following circuit diagram will be

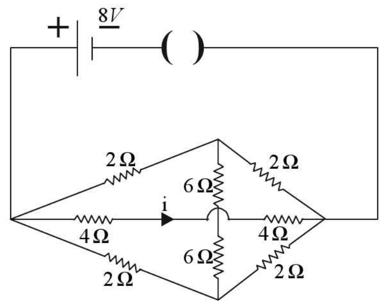


Fig. 21.72

(a) $\frac{3}{2}$ A (b) $\frac{3}{4}$ A

(c) $\frac{1}{2}$ A (d) 1 A

48. In the adjoining figure the potential difference between the points *A* and *B* will be

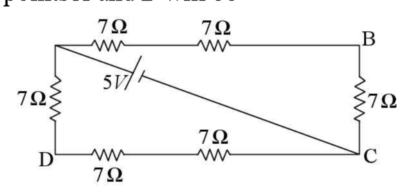


Fig. 21.73

(a) $\frac{10}{3}$ V (b) $\frac{4}{3}$ V

(c) $\frac{8}{9}$ V (d) $\frac{2}{3}$ V

49. In the above problem the potential difference between the points *A* and *D* will be

(a) $\frac{1}{3}$ V (b) $\frac{5}{3}$ V

(c) $\frac{7}{3}$ V (d) $\frac{10}{3}$ V

50. If Ohm's law is presumed to be valid, then drift velocity V_d and electric field *E* are related as

(a) $v_d \propto E^2$ (b) $v_d \propto \sqrt{E}$

(c) $v_d \propto E$ (d) $v_d \propto E_0$

51. In the circuit shown, the current in 2 Ω resistance will be

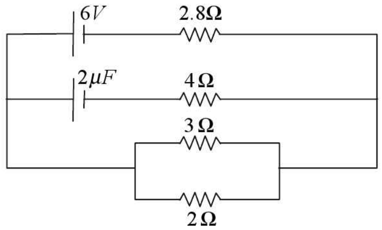


Fig. 21.74

(a) 1.25 A (b) 1.5 A
(c) 1.8 A (d) 0.9 A

52. In copper, each atom releases one electron. If a current of 1.1A is flowing in the copper wire of diameter 1mm then the drift velocity of electrons, will approximately be

(density of copper = 9×10^3 Kgm^{-3} and its atomic weight = 63).

(a) 10.3 mm/s (b) 0.1 mm/s
(c) 0.2 mm/s (d) 0.2 cm/s

53. If a copper wire is stretched to increase its length by 0.1% then percentage increase in its resistance will be

(a) 0.2% (b) 2%
(c) 1% (d) 0.1%

54. An electric cable contains a single copper wire of radius 9 mm. Its resistance is 5 Ω. This cable is replaced by six insulated copper wires, each of radius 3 mm. The resultant resistance of cable will be

(a) 7.5 Ω (b) 45 Ω
(c) 90 Ω (d) 270 Ω

55. A metallic wire of resistance 20 Ω is stretched such that its length becomes three times. The new resistance of the wire will be

(a) 6.67 Ω (b) 60.0 Ω
(c) 120 Ω (d) 180 Ω

56. The specific resistance of the material of a wire is ρ, its volume is 3 m^3 and its resistance is 3 Ω. The length of the wire will be

(a) $\frac{\sqrt{1}}{\rho}$ (b) $\frac{3}{\sqrt{\rho}}$
(c) $\frac{\sqrt{3}}{\rho}$ (d) $\frac{\rho}{\sqrt{3}}$

57. The dimensions of a rectangular parallelepiped are 1 cm × 1cm × 100 cm. If its specific resistance is 3×10^{-7} Ω × m then the resistance between its rectangular faces will be

(a) 3×10^{-9} Ω (b) 3×10^{-7} Ω
(c) 3×10^{-5} Ω (d) 3×10^{-3} Ω

58. The emf of a cell of negligible internal resistance is 2 V. It is connected to the series combination of 2 Ω, 3 Ω and 5 Ω resistances. The potential difference across 3 Ω resistance will be (in volt)

(a) 0.6 (b) $\frac{2}{3}$
(c) 3 (d) 6

59. Four similar wires, each of resistance 10 Ω, are used to construct a square. The effective resistance between two corners will be

(a) 10 Ω (b) 40 Ω
(c) 20 Ω (d) 7.5 Ω

60. Two resistances are connected in (a) series (b) in parallel. The effective resistances in two cases are 9 Ω and 2 Ω respectively. The value of resistances will be

(a) 2 Ω and 7 Ω (b) 3 Ω and 6 Ω
(c) 3 Ω and 9 Ω (d) 5 Ω and 4 Ω

61. The resistance of a uniform wire of length L and diameter d is R. The resistance of another wire of same material but of length $4L$ and diameter $2d$ will be

(a) $2R$ (b) R
(c) $\frac{R}{2}$ (d) $\frac{R}{4}$

62. Each resistance in the adjoining network is of 1Ω. The effective resistance between A and B will be

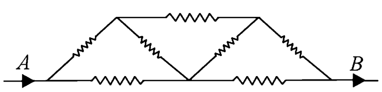


Fig. 21.75

(a) $\frac{4}{3}$ Ω (b) $\frac{3}{2}$ Ω
(c) 7 Ω (d) 8/7 Ω

63. The resistance between the points A and B in the adjoining figure will be

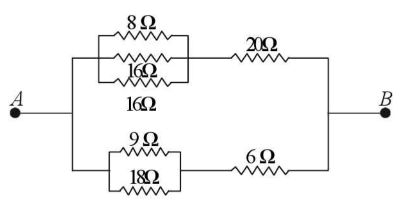


Fig. 21.76

(a) 6 Ω (b) 8 Ω
(c) 16 Ω (d) 24 Ω

64. A flash light cell of emf. 1.5 volt gives 15 ampere current when connected to an ammeter of resistance 0.04 Ω. The internal resistance of the cell will be

(a) 0.04 Ω (b) 0.06 Ω
(c) 0.10 Ω (d) 10 Ω

65. In the circuit shown, the value of resistance X in order that the potential difference between the points B and D is zero, will be

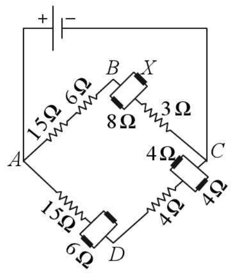


Fig. 21.77

(a) 4 Ω (b) 6 Ω
(c) 8 Ω (d) 9 Ω

66. If n, e, t and m are respectively the density, charge, relaxation time and mass of an electron then the resistance of wire of length / and cross-sectional area A, will be

(a) $\frac{ml}{ne^2\tau A}$ (b) $\frac{m\tau^2 A}{ne^2 l}$

(c) $\frac{ne^2\tau A}{ml}$ (d) $\frac{ne^2 A}{m\tau l}$

67. In the circuit shown the value of current given by the battery will be

(a) 1A (b) 2A
(c) 1.5A (d) 3A

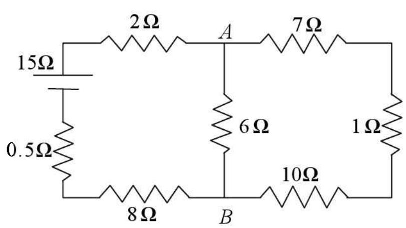


Fig. 21.78

68. In the circuit shown, if key K is open, then ammeter reading will be

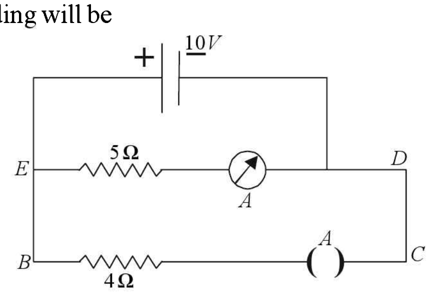


Fig. 21.79

(a) 50A (b) 2A

(c) 0.5A (d) $\frac{10}{9}$A

69. The resistance of an iron wire is 10 Ω and its temperature coefficient of resistance is 5 × 10^{-3}/°C. A current of 30mA is flowing in it at 20°C. Keeping potential difference across its ends constant, if its temperature is increased to 120° C then the current flowing in the wire will be (in mA)

(a) 20 (b) 35
(c) 10 (d) 40

70. A copper wire of length 1m and radius 1mm is connected in series with another wire of iron of length 2 m and radius 3 mm. A steady current is passed through this combination. The ratio of current densities in copper and iron wires will be

(a) 18 : 1 (b) 9 : 1
(c) 6 : 1 (d) 2 : 3

71. A battery of emf. 10V is connected to a network as shown in Fig. 21.80 The potential difference between the points A and B will be

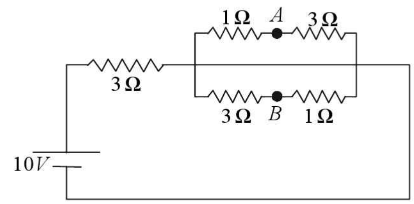


Fig. 21.80

(a) –2 V (b) 2 V

(c) 5 V (d) $\frac{20}{11}$ V

72. In the following circuit diagram, E = 4 V, r = 1 Ω and R = 45 Ω, then reading in the ammeter A will be

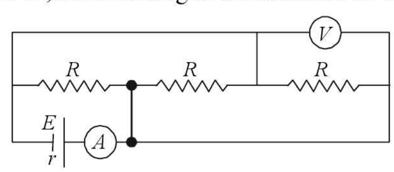


Fig. 21.81

(a) 1A (b) $\frac{1}{2}$ A

(c) $\frac{1}{8}$A (d) $\frac{1}{4}$A

73. In the above problem the voltmeter reading will be

(a) 4 V (b) 3 V

(c) 15 V (d) $3\frac{3}{4}$ V

74. A student connects four cells, each of internal resistance 1/4 Ω, in series. One of the cells is incorrectly connected because its terminals are reversed. The value of external resistance is 1Ω. If the emf of each cell is 1.5 volt then current in the circuit will be

(a) $\frac{4}{3}$ A (b) zero

(c) $\frac{3}{4}$A (d) 1.5 A

75. An aluminum rod and a copper rod are taken such that their lengths are same and their resistances are also same. The specific resistance of copper is half that of aluminum, but its density is three times that of aluminum. The ratio of the mass of aluminum rod and that of copper rod will be

(a) $\frac{1}{6}$ (b) $\frac{2}{3}$

(c) $\frac{1}{3}$ (d) 6

76. In the following circuit shown, if point B is earthed then potential at D will be

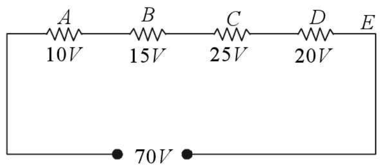


Fig. 21.82

(a) 40 V (b) – 40 V

(c) zero (d) 80 V

77. The potential drop across 4V battery in the following circuit will be

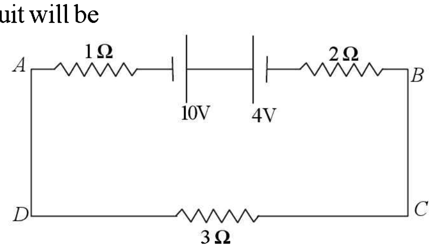


Fig. 21.83

(a) 2V (b) 5V

(c) 9V (d) 6V

78. In the following circuit, AB is a long resistance wire of 300 Ω. It is tapped at one third distance and is connected as shown in Fig. 21.84 The equivalent resistance between X and Y will be

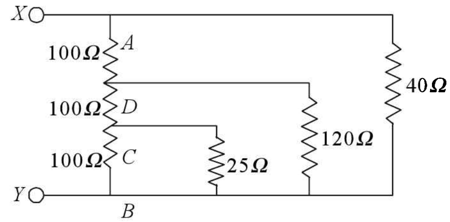


Fig. 21.84

(a) 20 Ω (b) 32 Ω

(c) 60 Ω (d) none of above

79. In the following circuit the resistance of wire AB is 10 Ω and its length is 1m. Rest of the quantities are given in the diagram. The potential gradient on the wire will be

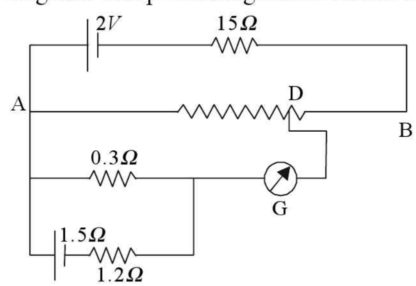


Fig. 21.85

(a) $0.08\ \frac{V}{m}$ (b) $0.008\ \frac{V}{m}$

(c) $0.8\ \frac{V}{m}$ (d) none of the above

80. In the above problem the length of wire AO, at null point, will be

(a) 37.5 cm (b) 3.75 cm

(c) 75 cm (d) 15 cm

Answers to Question for Practice

1.	(c)	**2.**	(a)	**3.**	(b)	**4.**	(b)	**5.**	(b)	**6.**	(a)	**7.**	(b)
8.	(a)	**9.**	(c)	**10.**	(c)	**11.**	(b)	**12.**	(a)	**13.**	(c)	**14.**	(b)
15.	(b)	**16.**	(d)	**17.**	(b)	**18.**	(a)	**19.**	(b)	**20.**	(c)	**21.**	(a)
22.	(d)	**23.**	(a)	**24.**	(c)	**25.**	(a)	**26.**	(d)	**27.**	(c)	**28.**	(d)
29.	(a)	**30.**	(b)	**31.**	(d)	**32.**	(c)	**33.**	(a)	**34.**	(b)	**35.**	(c)
36.	(b)	**37.**	(b)	**38.**	(a)	**39.**	(d)	**40.**	(d)	**41.**	(a)	**42.**	(d)
43.	(b)	**44.**	(c)	**45.**	(c)	**46.**	(b)	**47.**	(d)	**48.**	(a)	**49.**	(b)
50.	(c)	**51.**	(d)	**52.**	(b)	**53.**	(a)	**54.**	(a)	**55.**	(d)	**56.**	(b)
57.	(b)	**58.**	(a)	**59.**	(a)	**60.**	(b)	**61.**	(b)	**62.**	(d)	**63.**	(b)
64.	(b)	**65.**	(c)	**66.**	(a)	**67.**	(a)	**68.**	(b)	**69.**	(a)	**70.**	(b)
71.	(b)	**72.**	(d)	**73.**	(d)	**74.**	(d)	**75.**	(b)	**76.**	(b)	**77.**	(d)
78.	(b)	**79.**	(c)	**80.**	(a)								

22

Thermal and Chemical Effects of Current

BRIEF REVIEW

Thermal Effects

Joule's Law of Heating Heat produced in a conductor of resistance R when current I is passed through it for a time t is $H = I^2Rt$.

Heat produced due to Joule's law is independent of direction of current.

Seebeck Effect If two metal wires or strips A and B made of dissimilar metals are joined at the ends to form two junctions as shown in Fig. 22.1 then such a device is called thermocouple.

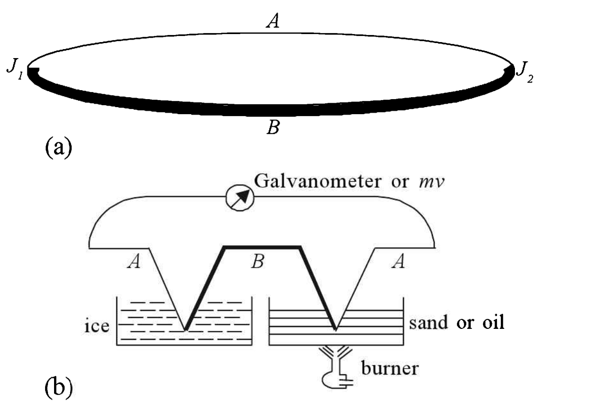


Fig. 22.1 Seebeck effect illustration

If two junctions of a thermocouple are kept at different temperatures, an electric current will be induced in the loop. This effect is called **seebeck effect** and emf so developed is known as **Seebeck emf** or **Thermo emf**. The magnitude and direction of the emf depends upon the metals used and the temperature difference between hot and cold junctions. The thermo emf induced is given by

$E = \alpha\theta + \dfrac{\beta\theta^2}{2}$ where θ is temperature difference between hot and cold junctions. Note that curve between E and θ is parabolic. Fig. 22.2 shows curve between emf and temperature difference θ, for various θ_C. Note that neutral temperature θ_N remains unchanged when temperature of cold junction θ_C is varied. However for each θ_C there is a particular θ_i.

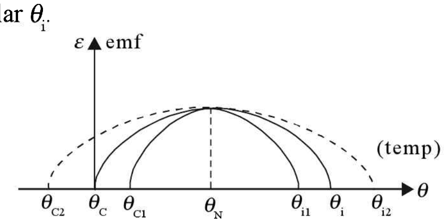


Fig. 22.2 Effect of temperature of cold junction

At neutral temperature θ_N, emf is maximum. Therefore at $\theta = \theta_N$

$$\frac{d\varepsilon}{d\theta} = 0 \text{ or } \theta_N = -\frac{\alpha}{\beta}.$$

Note that θ_N depends only on the nature of materials forming junctions. It does not depend upon θ_C, temperature of cold junction. However θ_i, the inversion temperature depends on both nature of the materials forming junctions and θ_C. At inversion temperature emf changes sign or direction of current reverses.

From Fig 22.2,

$$\theta_i - \theta_N = \theta_N - \theta_C$$

$$\text{or } \theta_N = \frac{\theta_i - \theta_C}{2}.$$

$$\text{If } \theta_C = 0 \text{ then } \theta_N = \frac{\theta_i}{2}$$

The graph between $\frac{d\varepsilon}{d\theta}$ and θ is a straight line with negative slope as shown in Fig. 22.3.

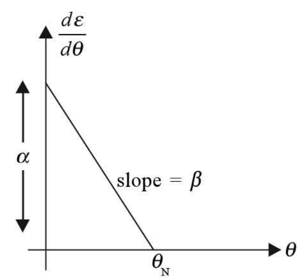


Fig. 22.3 Thermoelectric power $\frac{d\varepsilon}{d\theta}$ vs temperature θ

There is a series of metals called thermoelectric series. If the first and the last element of the series are used to form a thermocouple then emf induced is maximum. The series is

Sb, Fe, Zn, Ag, Au, Mo, Cr, Sn, Pb, Hg, Mn, Cu, Co, Ni, Bi.

If hot and cold junction are interchanged then direction of emf reverses.

The law of intermediate metals If three thermocouples formed with A, B ; B, C and A, C are used in same temperature conditions then $\varepsilon_{AC} = \varepsilon_{AB} + \varepsilon_{BC}$.

Note that $\varepsilon_{AB} = -\varepsilon_{BA}$.

Note that current induced in seebeck effect is diffusion current. Seebeck coeff $S = \frac{d\varepsilon}{d\theta}$

Peltier effect The converse of seebeck effect is peltier effect. If current is passed through a thermocouple or an external battery is applied across the two junctions of a thermocouple then one of the junctions becomes hot and other gets cold. The heat liberated or absorbed at one of the junctions is given by

$$\frac{\Delta H}{\Delta Q} = \pi_{AB}, \text{ the peltier emf}$$

where ΔQ is charge transferred.

Peltier coefficient is the amount of heat liberated or absorbed per second when 1 A of current is passed through a thermocouple. The hot and cold junction will interchange if the direction of current is reversed.

$$\pi = TS = T\frac{d\varepsilon}{d\theta}$$

Thomson Effect If two sections of a conductor are at different temperatures then emf is developed between these two sections. This effect is known as Thomson Effect. If dV is the potential difference between two sections of a conductor then Thomson coefficient σ is given by

$$\sigma = \frac{dV}{d\theta} = -T\frac{d^2\varepsilon}{d\theta^2} = -T\frac{dS}{d\theta}$$

If one part of a conductor is at different potential than the other or current is flowing, a temperature difference $d\theta$ is developed across the two sections.

Applications of Thermal Effects

(i) Electric power generation (thermopile) (ii) refrigeration (iii) Detection and heat radiation (iv) Measurement of temperature (thermocouple thermometer, thermistor thermometer and platinum resistance thermometer).

Power

$P = I^2R$ Use this formula when devices are in series.

$P = \frac{V^2}{R}$ This formula can be used when devices are in parallel.

$P = VI$ when current through the device and potential drop across it are known.

The SI unit of power is watt and practical unit of electrical consumption is 1 kwh or board of trade unit or simply unit. 1 unit = 3.6×10^6 J = 1 kwh

Chemical Effects

Voltameter or coulomb meter The vessel in which electrolysis is carried out. It is also called Electrolytic cell.

Electrolyte An ionic compound in aqueous solution or molten state is called an electrolyte.

Electrolysis On passing current through an electrolyte, chemical changes occurs in electrolyte and substances are liberated at the electrodes. This process is called electrolysis.

Faradays Laws

First law The mass of a substance liberated on an electrode is proportional to the current passed through it and proportional to the time for which current is passed.

i.e. $m \propto I$ and $m \propto t$ or $m = zIt$

where z is called ece or electrochemical equivalent. Since $It = Q$; $m = zQ$

$$z = \frac{\text{Chemical equivalent in gram or gram equivalent}}{\text{1 Faraday}}$$

Faraday (F): It is the amount of charge on 1 mole of electrons. 1F = 96485 C. Practically we use 1 F = 96500 C.

Second Law If same quantity of electricity (charge) is passed through different electrolytes, the masses of the substances deposited at the respective cathodes are directly propertional to the chemical equivalent,

i.e. $\frac{m_1}{m_2} = \frac{E_1}{E_2}$

Note 1 Faraday of charge will liberate 1 gram equivalent of any substance during electrolysis.

Electrolytic cell or voltameter may be used to measure the current more accurately than ammeters i.e. $I = \frac{m}{zt}$

Back emf It is the potential difference opposite to the external emf setup due to accumulation of ions around the insoluble electrodes. The back emf depends upon the nature of the electrodes and concentration of ions. In Fig 22.4 ε_b shows the back emf.

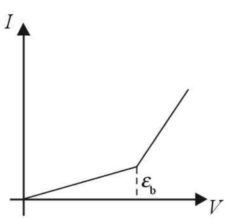


Fig. 22.4 Back emf illustration

If $V_{app} > \varepsilon_b$ the current increases and for $V_{app} \le \varepsilon_b$ resistances of the electrolytes do not follow Ohm's law. Back emf for water voltameter using platinum electrodes is 1.67 V.

On heating, the rate of electrolysis increases as the rate of decomposition of ions increases.

• Short Cuts and Points to Note

1. Joules heat energy $H = I^2Rt$
2. Power $P = \frac{V^2}{R}$. To find the resistance of an electrical device use power and voltage rating of the device. For instance an incandescent bulb rated 220 V/ 100 W will have a resistance $R = \frac{(220)^2}{100} = 484\ \Omega$.
3. Maximum power delivered by a cell/battery of internal resistance r is $\frac{\varepsilon^2}{4r}$ and is termed as available power. Condition to deliver maximum power is $R = r$.
4. If the devices are connected in series then a device with maximum power rating consumes minimum power and a device with lowest power rating consumes maximum power. Or, a device with higher resistance consumes more power. That is, in series, Power $P \propto R$ (Resistance).
5. In parallel, a device with higher power rating consumes more power or in parallel, $P \propto \frac{1}{R}$
6. Use power $P = I^2R$ in series and $P = \frac{V^2}{R}$ in parallel.
7. The effective power in a series combination is $$\frac{1}{P_{series}} = \frac{1}{P_1} + \frac{1}{P_2} + ...$$
8. The effective power in a parallel combination is $P_{parallel} = P_1 + P_2 + ...$
9. Power from a battery to load of resistance R is $P = \frac{\varepsilon^2 R}{(R+r)^2}$ and electric power supplied by the battery is $P_{supplied} = \varepsilon \times I$ $= \frac{\varepsilon^2}{R+r}$
10. Under matched conditions (when $R = r$), the power supplied $= \frac{\varepsilon^2}{2r}$ and power to load $= \frac{\varepsilon^2}{4r}$
11. Efficiency of a cell $\eta = \frac{R}{r+R}$
12. Efficiency of a secondary cell $$= \frac{\text{discharging capacity}}{\text{charging capcity}}$$
13. Maximum current through a battery or cell $$I_{max} = I_{short\ circuit} = \frac{\varepsilon}{r}$$ where ε is emf and r is internal resistance of the battery/cell.
14. Maximum or safe current in a fuse wire $I_{safe} \propto r^{3/2}$ where r is radius of the fuse wire. Note that I_{safe} is independent of length of the fuse wire.
15. A standard cell is one whose emf does not vary with temperature. Clarke cell and Weston cell are standard cells. Clarke cell has lesser thermal coefficient and its emf is consistent to a large extent.

    Clarke cell $\varepsilon = 1.4328\ V - 1.19 \times 10^{-6}\ V$ at 15°C

    Weston cell $\varepsilon = 1.0184\ V - 4.06 \times 10^{-3}\ V$ at 20°C.

16. During charging of a cell $I = \dfrac{\varepsilon_{\text{charger}} - \varepsilon_{\text{battery}}}{R+r}$

where R is external resistance and r is internal resistance of the battery.

17. The internal resistance of a fully charged secondary cell/battery is less than the discharged cell/battery.

18. Relation between Seebeck coefficient S, Peltier coeff π and Thomson coeff σ:

$$S = \frac{d\varepsilon}{d\theta} = \alpha + \beta\theta$$
$$S = \frac{d\varepsilon}{d\theta} = \frac{\pi}{\theta}$$
$$\sigma = -\theta\frac{dS}{d\theta} = -\theta\frac{d^2\varepsilon}{d\theta^2}$$

Seebeck coefficient is the resultant of Peltier and Thomson effect. $\sigma\Delta\theta$ is Thomson emf.

19. Thomson coeff of lead is zero.

20. In Peltier and Thomson effects heat produced is proportional to current while in Joules effect heat $\propto I^2$. Therefore heat produced in Peltier and Thomson effect depends upon the direction of current and Joule's heat is independent of direction of current.

21. The temperature scale in thermocouple thermometer is nonlinear.

22. The current in seebeck effect is diffusion current (generated due to charge-density gradient at hot junction).

23. If the temperature difference between the two junctions of thermocouples is equal, say θ, then

$$\varepsilon_{AC} = \varepsilon_{AB} + \varepsilon_{BC} \text{ and}$$
$$\varepsilon_{AB} = -\varepsilon_{BA}$$

24. In Peltier effect both drift and diffusion currents exist. Therefore heat produced is greater than Joule's heat. Joules heat is only due to drift current.

25. The capacity of accumulator is measured in ampere hour (Ah). The capacity depends upon number of plates used.

26. The specific gravity of lead accumulator should lie between 1.2 and 1.28. If it becomes less than 1.15 it is assumed permanently damaged.

27. Temperature of a glowing (fully) bulb is 2800 K to 3000 K.

28. Deposition at anode is called Anodisation.

• Caution

1. Assuming that a device with higher power rating consumes more power.

⇒ This is true in parallel connections. If the devices are connected in series then a device with the highest power rating will consume the least power.

2. Assuming maximum power is transferred when current is maximum, i.e. when short circuited.

⇒ Maximum power is transferred to a device under matched condition i.e. $R = r$. Under short circuit condition the source of energy will be damaged due to very large current.

3 Not knowing clearly which voltage (rating voltage of the device or applied voltage) be used to find resistance of the device.

⇒ To find resistance of a device rating voltage and rating power be used

$$\text{i.e. } R = \frac{V^2_{\text{rating}}}{P_{\text{rating}}}$$

4. Confused about the power relations.

⇒ Use $P = \dfrac{V^2}{R}$ in parallel only

Use $P = I^2R$ in series and $P = VI$ when current through the device and potential drop across it are known.

5. Considering in all cases where heat is produced, heat produced $\propto I^2$

⇒ Only in Joule's heat $H \propto I^2$

In Peltier and Thomson effect $H \propto I$

6. Considering heat produced is independent of directions of current applied.

⇒ This is true only in Joule's heat because $H \propto I^2$. But in Peltier and Thomson effect heat produced $H \propto I$. Therefore the heat produced depends upon the direction of current. If the direction of current is reversed, hot and cold junctions interchange.

7. Considering Ohm's law is valid in electrolytic cells.

⇒ If electrodes are soluble in electrolytes then Ohm's law can be applied. If electrodes are insoluble in electrolytes then Ohm's law cannot be applied . Rather, effect of back emf is observed as illustrated in Fig 22.5.

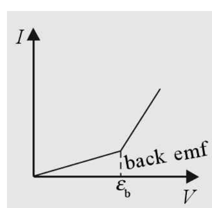


Fig. 22.5

It is due to accumulation of ions near the electrodes.

8. Considering conductivity of electrolytes is greater than that of metals as in electrolytes both –*ve* ion and +*ve* ions conduct electricity.

⇒ Conductivity of electrolytes is much less than that of metals because free electron density in metals is very large. Moreover, ions are heavy and hence their mobility is quite small as compared to that of electrons.

9. Considering in thermocouples $\varepsilon_{AB} = \varepsilon_{BA}$

⇒ Note that $\varepsilon_{AB} = -\varepsilon_{BA}$.

10. Considering any two metals can be used from the seebeck series to form a thermocouple.

⇒ For large emf generation, more the separation between the series metals more is the emf generated.

11. Considering current in seebeck effect is like drift current in conductors (generated due to application of battery).

⇒ Diffusion current, i.e., current due to charge density gradient at the hot junction is generated in seebeck effect or in a thermocouple.

12. Lack of clarity about current division in a complex circuit.

⇒ Apply current division wherever needed.

13. Confusion between equivalent weight (mass) and atomic weight (mass).

⇒ When 1 Faraday charge is passed (applied) then mass equal to equivalent weight is deposited on the cathode. If valency is 1 then atomic weights and equivalent weights are equal.

or Equivalent weight = $\dfrac{\text{atomic weight}}{\text{valency}}$

14. Considering potential drop is always less than emf.

⇒ If the battery is discharging, it is true. However, during charging current flows in opposite direction and $V = \varepsilon + Ir > \varepsilon$

SOLVED PROBLEMS

1. A 5 ampere fuse wire can withstand a maximum power of 1 watt in the circuit. The resistance of the fuse wire is

(a) 0.04 Ω (b) 0.2 Ω

(c) 5 Ω (d) 0.4 Ω

[CBSE 2005]

Solution (a) $I^2R = P$ or $R = \dfrac{1}{25}$

2. Two sources of equal emf are connected to external resistance R. The internal resistances of the two sources are R_1 and R_2 ($R_2 > R_1$). If the potential difference across the source having internal resistance R_2 is zero then

(a) $R = R_2 - R_1$ (b) $R = \dfrac{R_2(R_1 + R_2)}{R_2 - R_1}$

(c) $R = \dfrac{R_1R_2}{R_2 - R_1}$ (d) $R = \dfrac{R_1R_2}{R_1 + R_2}$

[AIEEE 2005]

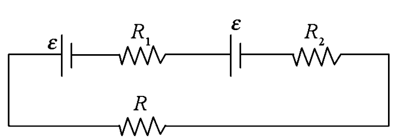


Fig. 22.6

Solution (a) $I = \dfrac{2\varepsilon}{R_1 + R_2 + R}$ If $IR_2 = \varepsilon$ then potential drop across the source of resistance R_2 will be zero.

$$\frac{2\varepsilon}{R_1 + R_2 + R}(R_2) = \varepsilon$$

or $2R_2 = R_1 + R_2 + R$

or $R = R_2 - R_1$

3. A heater coil is cut into two equal parts and only one part is now used in the heater. The heat generated will now be

[AIEEE 2005]

(a) four times (b) doubled

(c) halved (d) one fourth

Solution (b) ∵ $H = \dfrac{V_2}{R}t$. As R is halved H is doubled.

4. The resistance of a hot tungsten filament is about 10 times the cold resistance. What will be the resistance of 100 W/200V lamp when not in use?

[AIEEE 2005]

(a) 20 Ω (b) 40 Ω

(c) 200 Ω (d) 400 Ω

Solution (b) $R = \dfrac{(200)^2}{100} = 400\ \Omega$ when operating

$R\text{ (cold)} = \dfrac{400}{10} = 40\ \Omega$

5. Two voltameters, one of copper and another of silver are joined in parallel. When a total charge q flows equal amount of metals are deposited. If ece of Cu and Ag are

z_1 and z_2 respectively then charge flown through the silver voltameter is

(a) $\dfrac{q}{1+\dfrac{z_2}{z_1}}$ (b) $\dfrac{q}{1+\dfrac{z_1}{z_2}}$

(c) $q\dfrac{z_2}{z_1}$ (d) $q\dfrac{z_1}{z_2}$

Solution (a) $\because m = zq$

$$\therefore z_1q_1 = z_2q_2 \text{ or } q_2 = \frac{z_1q_1}{z_2} \text{ but } q = q_1 + q_2$$

$$\therefore q_2 = \frac{z_1}{z_2}(q_1 - q_2) \text{ or } q_2\left(1+\frac{z_1}{z_2}\right) = \frac{z_1}{z_2}q \text{ or}$$

$$q_2 = \frac{qz_1}{z_1+z_2} \text{ or } \frac{q}{1+\dfrac{z_2}{z_1}}$$

6. An 1800 W toaster, a 1.3 kW frying pan are plugged into the same 20 A/120 V line then

(a) fuse will not blow (b) fuse will blow

(c) supply will spark (d) fuse is just saved

Solution (b) $I_{\text{toaster}} = \dfrac{1800}{120} = 15$ A,

$$I_{\text{frying pan}} = \frac{1300}{120} = 10.83 \text{ A}$$

$$I_{\text{net}} = 25.83 \text{ A} > 20 \text{ A}$$

$\therefore$ fuse will blow

7. In the given circuit energy stored in capacitor is

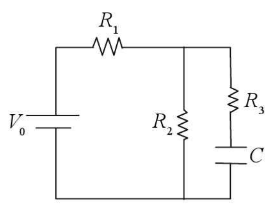


Fig. 22.7

(a) $\dfrac{CV_0^2}{2}$ (b) $\dfrac{C}{2}\left(\dfrac{V_0R_2}{R_1+R_2}\right)^2$

(c) $\dfrac{C}{2}\left(\dfrac{V_0R_3}{R_1+\dfrac{R_2R_3}{R_2+R_3}}\right)^2$ (d) $\dfrac{C}{2}\left(\dfrac{V_0R_1}{R_1+R_2}\right)^2$

Solution (b) The potential drop across capacitor is same as that across R_2.

Hence $E = \dfrac{C}{2}\left(\dfrac{V_0R_2}{R_1+R_2}\right)^2$

8. Two 32 W/100 V bulbs are connected (i) in series (ii) in parallel to 100 V supply. The power consumed by each in two cases is

(a) 16 W, 64 W (b) 8 W, 32 W

(c) 32 W, 32 W (d) 16 W, 32 W

Solution (b) $\dfrac{1}{P_{\text{series}}} = \dfrac{1}{32} + \dfrac{1}{32}$ or $P_{\text{series}} = 16$ watt by two bulbs. As bulbs are identical they consume 8 W each. In parallel they consume 32 W each.

9. The power consumed in 10 Ω resistor in the given circuit is

(a) 10 W (b) 8 W

(c) > 10 W (d) zero

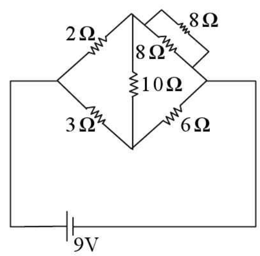


Fig. 22.8

Solution (d) As the bridge is balanced, no current will pass through 10 **Ω** resistance.

10. The wiring of a house has resistance 6 Ω. A 100 W/220 V bulb is glowing in the bathroom. A geyser of 1000 W/ 220 V is switched on. The drop in potential across the bulb is

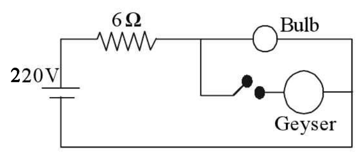


Fig. 22.9

(a) nil (b) 24 V

(c) 32 V (d) 12 V

Solution $R_{\text{bulb}} = \dfrac{220^2}{100}$ 484Ω

$R_{\text{Geyser}} = 48.4$ Ω

(i) When only bulb is ON

$$V_{\text{bulb}} = \frac{220 \times 484}{490} = 217.4 \text{ V}$$

(ii) When geyser is also switched ON

$$R_{\text{net}} = \frac{484 \times 48.4}{484 + 48.4} = 44\ \Omega;$$

$$V_{\text{bulb}} = \frac{220 \times 44}{50} = 193.6 \text{ V}$$

$$\Delta V = (217.4 - 193.6) \text{ V} \cong 24 \text{ V}$$

11. A capacitor C is charged to V_0 volts, a resistor R is connected across it. The heat produced in the resistor is

(a) CV_0^2 (b) $2CV_0^2$

(c) $CV_0^2/2$ (d) $4CV_0^2$

(e) $CV_0^2/4$

Solution (c) The energy stored in the capacitor will be converted to heat.

12. A silver, a copper and an iron wire of identical dimensions are connected in series with a voltage source. P_S, P_C and P_I are power consumed in three respectively then,

(a) $P_S > P_C > P_I$ (b) $P_C > P_S > P_I$

(c) $P_I > P_S > P_C$ (d) $P_I > P_C > P_S$

Solution (d) Larger the resistance, more is the power consumed in series.

13. When the switch S is made ON in Fig 22.10, the bulb which glows brightest is

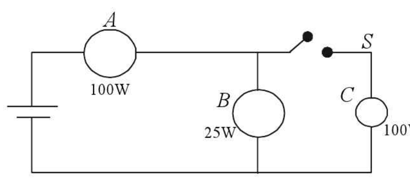


Fig. 22.10

(a) A (b) B

(c) C (d) A and C

Solution (a) Now the current will be divided amongst B and C. A gets maximum current.

14. A 10 A fuse wire is used in a mains line in a house. In another house a 15 A fuse wire is used. The ratio of their radii is

(a) $\sqrt{\frac{3}{2}}$ (b) $\sqrt{\frac{2}{3}}$

(c) $\left(\frac{2}{3}\right)^{3/2}$ (d) $\left(\frac{2}{3}\right)^{2/3}$

(e) none of these

Solution (d) $\frac{I_1}{I_2} = \left(\frac{r_1}{r_2}\right)^{3/2}$

or $\frac{r_1}{r_2} = \left(\frac{10}{15}\right)^{2/3} = \left(\frac{2}{3}\right)^{2/3}$

15. A 3 Ω resistor as shown in Fig 22.11 is dipped into a calorimeter containing H_2O. The thermal capacity of H_2O + calorimeter is 2000 J/K. If the circuit is active for 15 minutes find the rise in temperature of H_2O.

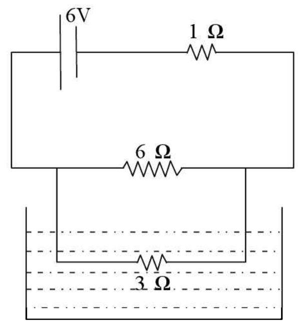


Fig. 22.11

(a) 2.4° C (b) 2.9° C

(c) 3.4° C (d) 1.9° C

Solution (a) $I = \frac{6}{R_{11}+1} = \frac{6}{2+1} = 2\text{ A}$

$$R_{11} = \frac{6\times3}{6+3} = 2\Omega$$

Current through 3 Ω resistor $I' = I\times\frac{6}{9} = \frac{4}{3}A$.

$$(mc)\Delta T = I'^2Rt$$

$$2000\times\Delta T = \left(\frac{4}{3}\right)^2\times3\times15\times60$$

$$\Delta T = 2.4°C$$

16. The potential difference across the terminals of a battery of emf 12 V and internal resistance $2r$ is 10 V when it is connected to an Ag voltameter. Find the Ag deposited in half an hour.

(a) 0.2 g (b) 0.1 g

(c) 0.02 g (d) 0.01 g

Solution (c) $I = \frac{12-10}{2} = 1\text{A}$

$$m = zIt = \frac{1.08}{96500}\times1\times30\times60 = 0.02\text{ g}$$

17. Find the charge required to liberate one atom of a divalent material in an electrolyte.

(a) 1.6×10^{-19} (b) 3.2×10^{-19}C

(c) 4.8×10^{-19}C (d) none of these

Solution (b) To neutralise divalent atom 3.2×10^{-19} C charge is required.

18. A plate of area 10 cm² is to be electroplated with Cu (density 9g/cc) to a thickness of 10 μm on both sides using a 12 V battery. Calculate the energy spent by the battery in this process. ECE of Cu is 3×10^{-17} kg/C

(a) 1.8 kJ (b) 3.6 kJ

(c) 5.4 kJ (d) 7.2 kJ

(e) none of these

Solution (d) $m = \rho\text{ (vol.)} = 9\times10\times20\times10^{-4}$ $= 18\times10^{-2}$ gram

$$m = zQ$$

$$Q = \frac{m}{z} = \frac{18\times10^{-2}}{3\times10^{-4}} = 600\text{ C}$$

Energy spent $E = VQ$

$$= 12\times600 = 7.2\text{ kJ}$$

19. A bulb of 100 W/200 V is connected to a 160 V supply. Find the power consumed.

(a) 32 W (b) 25 W

(c) 50 W (d) 64 W

Solution (d) $R = \frac{(200)^2}{100} = 400\,\Omega$

$$P_{\text{consumed}} = \frac{160\times160}{400} = 64\text{ W}$$

20. A kettle boils 1 kg of water in 16 min when one filament is used. If the second filament is used it takes 8 minutes. If both the filaments are used (i) in parallel (ii) in series time to boil the same quantity of H_2O is

(a) 6 min, 18 min (b) 16/3 min, 18 min

(c) 16/3 min, 24 min (d) 6 min, 24 min

Solution (c) In parallel $\frac{1}{t} = \frac{1}{t_1} + \frac{1}{t_2}$

or $t = \frac{16\times8}{16+8} = \frac{16}{3}$ min

In series $t = t_1 + t_2 = 24$ min

21. An electric bulb rated 220/100W will fuse if it consumes 121W. What voltage fluctuations can it withstand ?

(a) up to 230 V (b) up to 241 V

(c) up to 225 V (d) up to 232 V

Solution (d) $R = \frac{(220)^2}{100} = 484\,\Omega$

Case (ii) $V^2 = 484\times121$ or $V = 232\ V$

22. In an electrolysis experiment, after some time, the battery connection is reversed. Then

(a) the electrolysis will stop

(b) the rate of liberation of material at electrodes will increase

(c) the rate of liberation will decrease

(d) rate of liberation will remain unchanged

(e) more heat will be produced at the electrodes

Solution (d)

23. Find the thermo emf developed in a Cu-Ag thermo couple when the junctions are kept at 0° C and 40° C. α for Cu and Ag is 2.76 and 2.5 μV/°C and β for both Cu and Ag is 0.012 μV/(°C)^2

(a) 1.04 μV (b) 10.4 μV

(c) 210.4 μV (d) none of these

Solution (b) $\alpha_{\text{net}} = (2.76 - 2.50)\times10^{-6}$

$$\beta_{\text{net}} = 0$$

$$\therefore\ \varepsilon = \Delta\alpha\Delta\theta = 0.26\times40\,\mu\text{V}$$

$$= 10.4\,\mu\text{V}$$

24. Find the time required to liberate 1 litre of H_2 at STP in an electrolytic cell operating at 2A.

(a) 19 min (b) 24 min

(c) 29 min (d) 14.5 min

Solution (c) $\because$ 22.4 litre $\equiv 2g$

$$\therefore\ 1l = \frac{2}{22.4}g.\ \text{Using } m = zIt$$

$$\frac{2}{22.4} = \frac{1}{96500}\times5\times t \text{ or } t = 29\text{ min}$$

25. The power consumed by 6 Ω resistor in the given circuit of Fig. 22.12 is

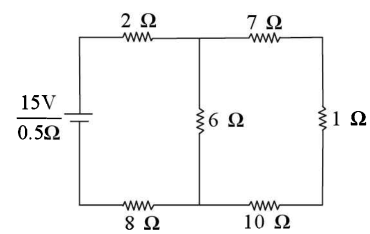


Fig. 22.12

(a) 4.611 W (b) 3.375 W

(c) 1.125 W (d) none of these

Solution (b) $I = \frac{15}{0.5+10+4.5} = 1\text{A}$

Using current division rule current in 6 Ω resistor

$$I' = \frac{18}{24}\times1 = \frac{3}{4}\text{ A}$$

$$p = I'^2R = \left(\frac{3}{4}\right)^2\times6 = 3.375\text{ W.}$$

26. The thermo emf of a thermocouple is 25 μV/°C at room temperature. A galvanometer of 40 Ω resistance capable of detecting current as low as 10^{-5} A is connected with the thermocouple. The smallest temperature difference that can be detected by the system is **[AIEEE 2003]**

(a) 12°C (b) 8°C

(c) 20°C (d) 16°C

Solution (d) $\Delta\theta = \frac{40\times10^{-5}}{25\times10^{-6}} = 16°\text{C}$

27. The negative Zn plate of a Daniel cell, sending a constant current through a circuit, decreases in mass by 0.13 gm in 30 minutes. If chemical equivalent of Zn and Cu are 32.5 and 31.5 respectively, the increase in mass of the copper plate is **[AIEEE 2003]**

(a) 0.141 g (b) 0.126 g
(c) 0.242g (d) 0.180g

Solution (b) $\frac{m_1}{m_2} = \frac{E_1}{E_2}$

$$m_{Cu} = \frac{31.5}{32.5}(.13) = 0.126\text{ g}.$$

28. In a copper voltameter, the mass deposited in 30 minutes is .... if the current as shown in Fig. 22.13.

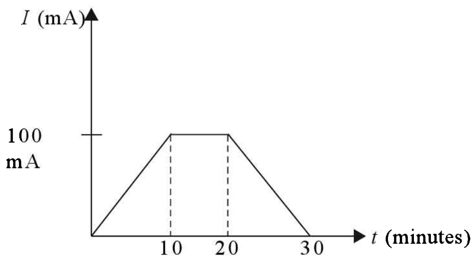


Fig. 22.13

(a) 0.078 g (b) 0.039 g
(c) 0.054 g (d) none of these

Solution (b)

$$Q = \text{area under graph}$$
$$= 0.1 \times 20 \times 60 = 120\text{ C}.$$
$$m = Z.Q = \frac{31.5}{96500} \times 120 = 0.039\text{ g}$$

29. In a thermocouple minimum current flows at

(a) neutral temperature
(b) inversion temperature
(c) half the neutral temperature
(d) $\frac{3}{2}$ of the neutral temperature

Solution (b)

It is clear from Fig. 22.14, that at T_i emf = 0.

∴ Current is also zero

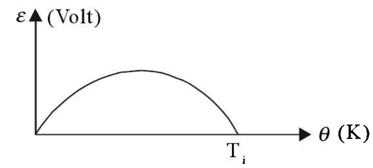


Fig. 22.14

30. Find the Peltier coefficient in thermocouple, if one junction is at 0°C and emf is given by $\varepsilon = \alpha\theta + \beta\theta^2$.

(a) $\theta(\alpha + 2\beta\theta)$
(b) $(273 - \theta)(\alpha + 2\beta)$
(c) $(\theta + 273)(\alpha + 2\beta)$
(d) $(\theta + 273)(\alpha + 2\beta\theta)$

Solution (d) $\pi = T\frac{\partial\varepsilon}{\partial\theta}$ $(\theta + 273)(\alpha + 2\beta\theta)$.

TYPICAL PROBLEMS

31. A wire of length L and three identical cells of negligible internal resistance are connected in series. Due to the current, the temperature of the wire is raised by ΔT in time t. N identical cells are now connected in series with a wire of length $2L$ of same material and cross-section. The temperature is raised by same amount ΔT in same time t. The value of N is

[IIT Screening 2001]

(a) 6 (b) 4
(c) 8 (d) 9

Solution (a) **Case (i)** $mC\Delta T = \frac{(3V_0)^2 t}{R}$

or $$\Delta T = \frac{9V_0^2 t}{mCR} \quad ...(1)$$

Case (ii) $2\,mC\Delta T = \frac{(NV_0)^2 t}{2R}$

or $$\Delta T = \frac{N^2V_0^2 t}{4mCR} \quad ...(2)$$

Equating (1) and (2) we get, $N^2 = 36$ or $N = 6$.

32. A servo stabilizer restricts voltage output to 220 V ± 1%. If an electric bulb 100 W/220 V is connected to it, find the maximum and minimum power consumed by it.

Solution $\frac{\Delta P}{P} = \frac{2\Delta V}{V}$

$$\Delta P = P\left(\frac{2\times1}{100}\right) = 100\left(\frac{2}{100}\right) = 2\text{ W}$$

P_{max} = 100 + 2 = 102 W;

P_{max} = 100 – 2 = 98 W.

33. The power released by the battery in the Fig 22.15 is

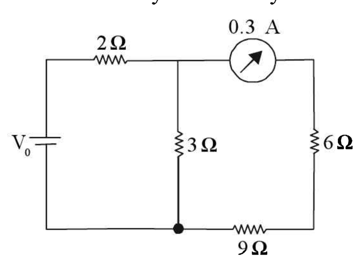


Fig. 22.15

Solution $\frac{I(3)}{18} = 0.3$ or $I = 1.8$ A

$P = I^2R = (1.8)^2(2.5 + 2) = 14.6$ W.

34. A fuse wire of radius 0.1 mm melts when a current of 10 A is passed through it. Find the current at which a fuse wire of 0.12 mm will melt.

Solution $\frac{I_1}{I_2} = \left(\frac{r_1}{r_2}\right)^{3/2}$ or

$I_2 = I_1\left(\frac{r_2}{r_1}\right)^{3/2} = 10\,(1.2)\,\sqrt{1.2} = 13.2$ A.

35. In a silver voltameter 2.16 g of silver is deposited in 10 minutes. The heat produced in 20 Ω resistor in the same period is nearly

(a) 198. 2 kJ (b) 123.2 kJ
(c) 171.2 kJ (d) 141.2 kJ

Solution (b) $m = zIt$ $I = \frac{m}{zt} = \frac{2.16 \times 96500}{8 \times 10 \times 60} = 3.22$ A

$H = I^2Rt = \left(\frac{9.65}{3}\right)^2 \times 20 \times 10 \times 60 = 123.2$ kJ

36. The coil of an electric bulb takes 40 W to start glowing. If power > 40 W is applied, 60% of the extra power is converted into light and rest into heat. Bulb is rated 100 W/220 V. Find the % drop in light intensity at a point if supply voltage changes from 220 V to 200 V.

(a) 16 % (b) 22 %
(c) 28 % (d) 34 %

Solution (c) $R = \frac{(220)^2}{100} = 484\ \Omega$

$P_{\text{consumed}} = \frac{(200)^2}{484}$

$= \frac{10^4}{121} = 82.6$ W

Case **(i)** Light energy 60 (.6)
= 36 W

Case **(ii)** 42.6(.6)
= 25.56

% change $= \frac{10.44 \times 100}{42.6} \cong 28\%$

QUESTIONS FOR PRACTICE

1. Faraday constant

(a) depends on the amount of the electrolyte
(b) depends on the current in the electrolyte
(c) is a universal constant
(d) depends on the amount of charge passed through the electrolyte.

2. An electrolysis experiment is stopped and the battery terminals are reversed.

(a) The electrolysis will stop.
(b) The rate of liberation of material at the electrodes will increase.
(c) The rate of liberation of material will remain the same.
(d) Heat will be produced at a greater rate.

3. Consider the following two statements.

(A) Free electron density is different in different metals.
(B) Free electron density in a metal depends on temperature.

Seebeck effect is caused

(a) due to both A and B
(b) due to A but not due to B
(c) due to B but not due to A
(d) neither due to A nor due to B.

4. The electrochemical equivalent of a material depends on

(a) the nature of the material
(b) the current through the electrolyte containing the material
(c) the amount of charge passed through the electrolyte
(d) the amount of this material present in the electrolyte.

5. According to Joule's law, if the potential difference across a conductor of resistivity ρ remains constant, then the heat produced in the conductor is proportional to

(a) $\frac{1}{\rho}$ (b) $\frac{1}{\sqrt{\rho}}$
(c) ρ^2 (d) ρ

6. Two electric bulbs, each designed to operate with a power of 500 W in a 220 V line, are connected in series with a 110 V line. The power consumed in each bulb is

(a) 125 W (b) 93.75 W
(c) 62.5 W (d) 31.25 W

7. A constant voltage is applied between the two ends of a uniform metallic wire, some heat is developed in it. If both the length and the radius of the wire are halved, the heat developed in the same duration will become

(a) four times (b) one fourth
(c) twice (d) half

8. A house wiring, supplied with a 220 V supply line, is protected by a 9 amp. fuse. The maximum number of 60 W bulbs in parallel that can be turned on is

(a) 44 (b) 33
(c) 22 (d) 11

9. A uniform wire connected across a supply produces heat H per second. If the wire is cut into n equal parts and all the parts are connected in parallel across the same supply, the heat produced per second will be

(a) $\frac{H}{n^2}$ (b) n^2H
(c) nH (d) $\frac{H}{n}$

10. An 800 W, 220 V kettle and three 100 W, 220 V bulbs are connected in parallel across a 220 V source. The current drawn from the source is

(a) 6.9 A (b) 5.5 A
(c) 5.0 A (d) 0.15 A

11. Two electric bulbs, one of 200 V – 40 W and other of 200 V – 100 W are connected in house wiring circuit.

(a) The resistance of 100 W bulb is more than that of 40 W bulb.
(b) The resistance of 40 W bulb is more than that of 100 W bulb.
(c) The resistances of bulbs are equal.
(d) Both the bulbs carry equal currents.

12. In Q. 16, if the bulbs are connected in series with a 200 V line, then

(a) the potential drop across two bulbs is the same
(b) the potential drop across 40 W bulb is greater than that across 100 W bulb
(c) the potential drop across 100 W bulb is greater than that across 40 W bulb
(d) none of these

13. Two heater coils separately take 10 min and 5 min to boil a certain amount of water. If both the coils are connected in series, the time taken will be

(a) 2.5 min (b) 3.33 min
(c) 7.5 min (d) 15 min

14. In Q. 13, if the coils are connected in parallel the time taken will be

(a) 6.66 min (b) 7.5 min
(c) 3.33 min (d) 2.5 min

15. Two electrical appliances are connected in parallel to a constant voltage supply. If the current in one appliance is 1% less than that in the second appliance, then the power of the first appliance will be less by

(a) 5% (b) 4%
(c) 2% (d) 1%

16. The parameter irrelevant for an electric fuse wire is

(a) its radius
(b) its specific resistance
(c) current flowing through it
(d) its length

17. In the circuit shown, the heat produced in 5 Ω resistor is 10 calorie/sec. The heat produced/sec in 4 Ω resistor will be

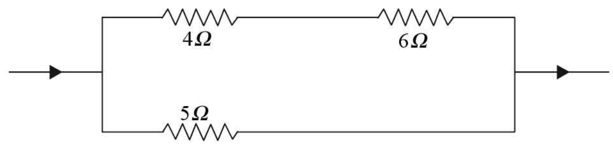


Fig. 22.16

(a) 1 cal (b) 2 cal
(c) 3 cal (d) 4 cal

18. The neutral temperature of a thermocouple is 275°C and the temperature of inversion is 600°C. The temperature of the cold junction is

(a) 50°C (b) 25°C
(c) –25°C (d) –50°C

19. Two cells, each of emf 2 V and internal resistance 1 Ω are given. The maximum heat that can be produced in a resistance of 0.5 Ω with the help of given two cells is

(a) 4.2 W (b) 2.9 W
(c) 2.0 W (d) 1 W

20. The thermo-emf of a copper-constant couple is 40μV per degree. The smallest temperature difference that can be detected with this couple and a galvanometer of 100 Ω resistance capable of measuring the minimum current of 1μA is

(a) 2.5°C (b) 2°C
(c) 1.5°C (d) 1°C

21. Two electric bulbs, each designed to operate with a power of 500 W in 220 V line, are connected in series in a 110 V line. The power generated by each bulb will be

(a) 11 W (b) 312.5 W
(c) 22 W (d) 31.25 W

22. A resistance coil of 60 Ω is immersed in 42 kg of water. What is the rise in temperature of water per minute if a steady current of 7 A is made to flow through the coil?

(a) 10.19°C (b) 9.15°C

(c) 1.0°C (d) 0.5°C

23. In the circuit shown below, the power supplied by the battery is

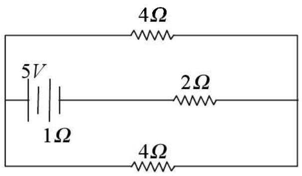


Fig. 22.17

(a) 5 W (b) 2.5 W

(c) 10 W (d) 25 W

24. When different parts of a metal are kept at different temperatures and current is passed through it, heat is either evolved or absorbed. This effect is called

(a) Thomson effect (b) Seebeck effect

(c) Peltier effect (d) none of these

25. In a metal with positive Thomson coefficient, current is passed from the lower temperature to higher temperature side, then

(a) heat will be evolved

(b) heat will be absorbed

(c) heat is neither absorbed nor evolved

(d) none of these

26. The emf in a thermoelectric circuit with one junction at 0 and the other at t°C is given by $E = at + bt^2$. The neutral temperature is

(a) $\frac{2b}{a}$ (b) $\frac{a}{b}$

(c) $-\frac{a}{b}$ (d) $-\frac{a}{2b}$

27. In a given thermocouple, the temperature of cold junction is 20°C while the neutral temperature is 270°C. The temperature of inversion is

(a) 590°C (b) 520°C

(c) 470°C (d) 420°C

28. The thermo-emf of a thermocouple is 40μV per degree temperature difference. A galvanometer of 50Ω resistance capable of detecting current as low as 10 A is connected with one such thermocouple. The smallest temperature difference that can be detected by such a thermocouple is

(a) 17.5°C (b) 12.5°C

(c) 8.5°C (d) 2.5°C

29. Amount of energy absorbed or evolved when one ampere of current passes for one second through a metal kept at a temperature difference of 1°C is called

(a) Thermo-emf

(b) Thermoelectric power

(c) Thomson coefficient

(d) Peltier coefficient

30. Two electroplating cells, one of silver and another of aluminum are connected in series. The ratio of the number of silver atoms to that of aluminum atoms deposited during time *t* will be

(a) 1 : 9 (b) 9 : 1

(c) 1 : 3 (d) 3 : 1

31. 10 A current deposits 10.8 gm silver in 900 s. The mass of copper deposited by 9A current in 1200s will be

(a) 12.7 gm (b) 10.8 gm

(c) 6.35 gm (d) 3.8 gm

32. A silver and a zinc voltameters are connected in series. A current is passed through them for time *t*. If W gm zinc is liberated, then the weight of silver deposited will be nearly

(a) 3.3 W (b) 2.4 W

(c) 1.7 W (d) 1.1 W

33. What is the approximate strength of current that will deposit 0.5 gm of silver on a spoon in 7.5 minutes? (Given e.c.e. of silver – 0.001118 gmC^{-1})

(a) 1 A (b) 0.005 A

(c) 0.5 A (d) 0.1 A

34. 1A of current flowing through a silver voltameter for 25 minutes deposits 1.5 gm of silver. The e.c.e. of silver will be (in gmC^{-1})

(a) 0.0001 (b) 0.001

(c) 0.01 (d) 0.1

35. An ammeter, suspected to give inaccurate reading, is connected in series with a silver voltameter. The ammeter indicates 0.54A. A steady current passed for one hour deposits 2.0124 gm of silver. If the e.c.e. of silver is 1.118×10^{-3} gmC^{-1}, then the error in ammeter reading is

(a) + 0.04 A (b) + 0.02 A

(c) – 0.03 A (d) – 0.01 A

36. Consider a thermocouple made of iron and constantan. If the thermo emf's of iron and constantan against platinum are + 800 and –1700 μV per 50°C difference of temperature, then the emf developed per °C difference of temperature between the junctions will be (in μV/°C)

(a) 200 (b) 150

(c) 100 (d) 50

37. A beam of 16 MeV deutrons from a cyclotron falls on a copper block. The beam is equivalent to a current of 15μA. At what rate do the deutrons strike the block?

(a) 9.4×10^9 (b) 9.4×10^7
(c) 9.4×10^{11} (d) 9.4×10^{13}

38. An electric bulb is operating at 110V. Power consumed by it, if it is rated $220V-100W$, will be

(a) 90 W (b) 75 W
(c) 50 W (d) 25 W

39. The same mass of copper is drawn into two wires 1mm and 2 mm thick. Two wires are connected in series and current is passed through them. Heat produced in the wires is in the ratio

(a) 2 : 1 (b) 1 : 16
(c) 4 : 1 (d) 16 : 1

40. The two head lamps of a car are in parallel. Together they consume 48 W with the help of a 6 V battery. The resistance of each bulb is

(a) 1.5 Ω (b) 4.0 Ω
(c) 0.67 Ω (d) 3.0 Ω

41. A coil of wire of resistance 50 Ω is embedded in a block of ice. If a potential difference of 210V is applied across the coil, the amount of ice melted per second will be

(a) 4.12 gm (b) 4.12 kg
(c) 3.68 kg (d) 2.625 gm

42. A steady potential difference of 1GQV produces heat at a constant rate in a resistor. The alternating voltage which will produce half the heating effect in the same resistor will be

(a) 100 V (b) 50 V
(c) 70.7 V (d) 141.4 V

43. Copper and silver voltameters are connected in series. If 0.8 gm of copper is deposited in copper voltameter, then the amount of silver deposited in silver voltameter will be (Chemical equivalents of copper and silver are 32 and 108 respectively).

(a) 0.27 kg (b) 2.7 kg
(c) 27 gm (d) 2.7 gm

44. In a copper voltameter, the mass deposited in 30s is m gram. If the current time graph is as shown in the figure, the e.c.e. of copper, in gm/coul, will be

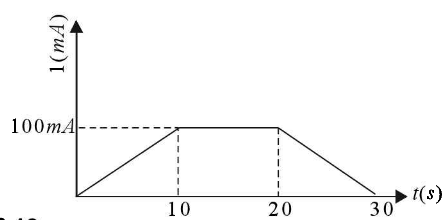


Fig. 22.18

(a) m (b) $\frac{m}{2}$
(c) 0.6 m (d) 0.1 m

45. If a current of 2A flowing for 1 hour liberates a certain amount of ions, then a current of 8A flowing for the same time will liberate ions

(a) sixteen times (b) six times
(c) eight times (d) four times

46. If 1A current is passed through CuSO4 solution for 10 s, the number of copper atoms deposited at the cathode will approximately be

(a) 3.1×10^{19} (b) 6.2×10^{19}
(c) 9.3×10^{19} (d) 1.6×10^{19}

47. How much electricity should be passed through a water voltameter in order to release 22.4 litre of hydrogen at N.T.P.?

(a) 1 coul (b) 22.4 coul
(c) 96500 coul (d) 1,93,000 coul

48. If N is the Avogadro number and e is the electronic charge then the Faraday constant F is equal to

(a) Ne (b) N^2e
(c) $\frac{N}{e}$ (d) $\frac{e}{N}$

49. A 3Ω resistor and a silver voltameter of resistance 2Ω are connected in series across a cell. How will the rate of deposition of silver be affected if a resistance of 2Ω is connected in parallel with the voltameter ?

(a) It will decrease by 25%
(b) It will increase by 25%
(c) It will increase by 37.5%
(d) It will decrease by 37.5%

50. In an electroplating experiment, m grams of silver are deposited when 4A current flows for 2 minutes. The amount (in grams) of silver deposited by 6A of current flowing for 40s will be

(a) 2 m (b) $\frac{m}{4}$
(c) $\frac{m}{2}$ (d) 4 m

51. E.C.E. of a substance is its mass liberated at an electrode by what amount of current in half a second

(a) 4 A (b) 3 A
(c) 2 A (d) 1 A

52. If 100 KWh of energy is consumed at 33V in a copper voltameter, the mass of copper liberated is (Given: e.c.e. of copper = 3.3×10^{-7} kg/C)

(a) 1.65 kg (b) 1.8 kg
(c) 3.3 kg (d) 3.6 kg

53. A current of 1.5A flows through a copper voltameter. The thickness of copper deposited on the electrode surface of area 50cm in 20 minutes will be (Density of copper = 9000kg/m³ and e.c.e, of copper = 0.00033 g/C)

(a) 2.6×10^{-5}m (b) 2.6×10^{-4} m

(c) 1.3×10^{-5}m (d) 1.3×10^{-4}m

54. A current of 16A flows through molten NaCl for 10 minutes. The amount of metallic sodium that appears at the negative electrode would be (Faraday constant $= 9.65 \times 10$ C)

(a) 11.5 gm (b) 2.3 gm

(c) 1.15 gm (d) 0.23 gm

55. For a given temperature difference, which of the following pairs will generate maximum thermo emf?

(a) Lead-nickel (b) Iron-copper

(c) Silver-gold (d) Antimony-bismuth

56. A 500W heating unit is designed to operate on a 115V line. If the line voltage drops to 110V line, the percentage drop in heat output will be

(a) 7.6% (b) 8.5%

(c) 8.1% (d) 10.2%

57. Which of the following statements is correct?

(a) Seebeck effect is irreversible.

(b) Thomson effect is localised at the junction.

(c) Joule heating effect is independent of the direction of flow of current in a conductor.

(d) Thomson effect is similar in origin to Peltier effect. They both are due to non-uniform distribution of electrons in a metal.

58. Which of the following statements is correct?

(a) Both Peltier and Joule effects are reversible.

(b) Both Peltier and Joule effects are irreversible.

(c) Joule effect is reversible whereas Peltier effect is irreversible.

(d) Joule effect is irreversible whereas Peltier effect is reversible.

59. A thermocouple is made from copper and iron. At the hot junction, the current

(a) flows from copper towards iron

(b) flows from iron towards copper

(c) flow decreases

(d) flow increases

60. The power of a heater is 500W at 800°C. What will be its power at 200°C it $\alpha = 4 \times 10^{-4}$ per °C?

(a) 611 W (b) 576 W

(c) 672 W (d) 484 W

61. A, B, C are three identical lamps connected as shown in the following figure. What changes occur in the brightness of the lamps when switch S is closed ?

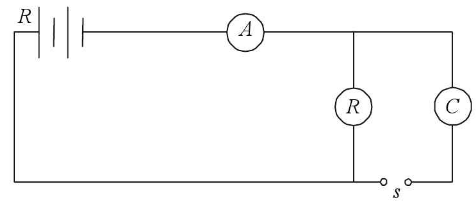


Fig. 22.19

(a) Brightness of A increases but that of B decreases.

(b) Brightness of A remains same but that of B decreases.

(c) Brightness of A increases but that of B remains same.

(d) Brightness of both A and B decreases.

62. In the following figure all the bulbs connected are identical. The bulb that lights most brilliantly is

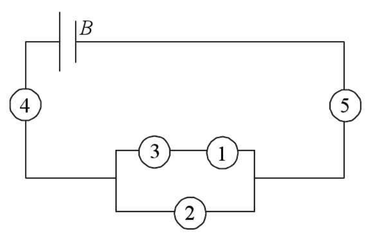


Fig. 22.20

(a) only 1 (b) only 2

(c) 1 and 5 (d) 2 and 4

63. The rate of heating of 10A AC current is the same as the rate of heating of a DC current of

(a) $\frac{10}{\sqrt{2}} A$ (b) $10\sqrt{2}\ A$

(c) $10A$ (d) $5A$

64. The resistance of a carbon filament at 0°C is 104Ω. It is connected in series with an iron wire. The temperature coefficients of resistivity of carbon and iron are -0.0003 and 0.0052 per °C respectively. What must be the resistance of iron wire so that the combined resistance does not change with temperature ?

(a) 1.8 Ω (b) 3.7 Ω

(c) 6.0 Ω (d) 15 Ω

65. Two metallic wires of same material and same length have different diameters When connected in series across a battery, the heat produced in thinner wire is Q_1 and that in thicker one is Q_2. The correct relation is

(a) $Q_1 = Q_2$ (b) $Q_1 < Q_2$

(c) $Q_1 > Q_2$

(d) will depend on the emf of the battery.

66. In the above problem, if the wires are connected in parallel, then

(a) $Q_1 - Q_2$ (b) $Q_1 < Q_2$
(c) $Q_1 > Q_2$
(d) will depend on the emf of the battery.

67. The power dissipated in a conductor of resistivity a is proportional to

(a) ρ^2 (b) ρ
(c) $\sqrt{\rho}$ (d) none of these

68. Charge carriers in a thermocouple are—

(a) + *ve* ions (b) –*ve* ions
(c) electrons (d) protons

69. The emf of the battery in a thermocouple is doubled.The rate of heat generation at one of the junctions will

(a) remain unchanged (b) become half
(c) become double (d) become four times

70. The unit of Peltier coefficient is

(a) JC^{-1} (b) JA^{-1}
(c) JV^{-1} (d) none of these

71. The unit of Thomson's coefficient is

(a) JC^{-1} (b) JA^{-1}
(c) JV^{-1} (d) none of these

72. Masses of three wires are in the ratio 1 : 3 : 5. Their lengths are in the ratio 5 : 3 : 1. When connected in series with a battery the ratio of heats produced in them will be

(a) 125 : 15 : 1 (b) 1 : 15 : 125
(c) 5 : 3 : 1 (d) 1 : 3 : 5

73. In the above problem, if the wires are connected in parallel to the battery, then the amounts of heat produced will be

(a) 125 : 15 : 1 (b) 1 : 15 : 125
(c) 5 : 3 : 1 (d) 1 : 3 : 5

74. Two identical batteries, each of emf 2V and internal resistance 1Ω, are available to produce heat in an external circuit. The maximum rate of production of heat in the external circuit will be

(a) 1 W (b) 8 W
(c) 4 W (d) 2 W

75. A heater boils 1 kg of water in lime t_1. Another heater boils it in lime t_2. If both are connected in series, the combination will boil same water in time

(a) $\frac{t_1 + t_2}{2}$ (b) $(t_1 + t_2)$
(c) $\frac{t_1 t_2}{t_1 + t_2}$ (d) $\frac{t_1 t_2}{t_1 - t_2}$

76. A tap supplies water at 22 °C. A man takes 1 litre per minute at 37°C from the geyser. The power of geyser is

(a) 2100 W (b) 1050 W
(c) 1575 W (d) 525 W

77. In the above problem, if the mains supply is 210V, then the current flowing in the geyser is

(a) 7.5 A (b) 2.5 A
(c) 5.0 A (d) 10 A

78. An electric heater operating at 220V boils 5 litres of water in 5 minutes. If it is used on 110V line, it will boil the same amount of water in

(a) 10 min (b) 20 mm
(c) 5 min (d) none of these

79. An immersion heater is rated 418 W. It should heat a litre of water from 10°C to 30°C in nearly

(a) 400 S (b) 200 S
(c) 144 S (d) 100 S

80. Two bulbs of equal wattage, one having carbon filament and the other having a tungsten filament, are connected in series to the mains. Then

(a) carbon filament bulb glows less
(b) carbon filament bulb glows more
(c) both bulbs glow equally
(d) tungsten filament bulb glows more

81. The water in an electric kettle begins to boil in ten minutes after being switched on. The wire used as the heating element is modified so that the water begins to boil in 7 minutes. Now,

(a) the length of the wire is increased
(b) the length of the wire is decreased
(c) another identical wire should be connected in series
(d) another identical wire should be connected in parallel

82. An electric heater and a fan are marked 1000 W, 220 V and 100 W, 220 V respectively. The resistance of fan is

(a) equal to that of heater
(b) greater than that of heater
(c) less than that of heater
(d) zero

83. You are given a resistance wire of length 100 cm and a battery of negligible internal resistance. In which of the following cases is the largest amount of heat generated?

(a) When the wire is divided in two parts and both the parts are connected to the battery in parallel.
(b) When the wire is connected to the battery directly.
(c) When the wire is divided into four parts and all the four parts are connected in parallel to the battery
(d) When only half of the wire is connected to the battery

84. When a current is passed in a conductor, 3°C rise in temperature is observed. If the strength of current is increased three times, then rise in temperature will approximately be

(a) 3°C (b) 9°C
(b) 27°C (d) 36°C

85. The temperature of hot junction of a thermocouple changes from 80°C to 100°C. The percentage change in thermoelectric power is

(a) 8% (b) 10%
(c) 20% (d) 25%

86. In a thermocouple minimum current flows at

(a) neutral temperature
(b) temperature of inversion
(c) twice the neutral temperature
(d) twice the temperature of inversion

87. A battery is connected to a thermocouple of copper and iron. The two junctions will be

(a) at the same temperature
(b) heated
(c) neither heated nor cooled
(d) undergoing thermoelectric effect leading to the heating of one junction and cooling of the other

88. The atomic mass number of zinc is 66 and its valency is 2. The mass of zinc liberated by 9.65A of current in 10 sec will be

(a) 33 μg (b) 33 mg
(c) 33 g (d) 33 kg

89. If 100 KWh of energy is consumed at 33V in a copper voltameter, then the mass of copper liberated will be (given e.c.c. of Cu = 0.33×10^{-6} kgC^{-1})

(a) 3.3 kg (b) 3.6 kg
(c) 1 kg (d) 1 mg

90. Which of the following is not the cause of low conductivity of electrolytes ?

(a) Low drift velocity of ions
(b) High resistance offered by the solution to the motion of ions
(c) Low number density of charge carriers
(d) Ionisation of salt

91. Current in an electrolyte is carried by

(a) only electrons (b) only –*ve* ions
(c) only + *ve* ions (d) both + *ve* and –*ve* ions

92. A box with two terminals is connected in series with a 2V battery, an ammeter and a switch. When the switch is closed, the needle of the ammeter moves quickly across the scale and then drops back to zero. The box contains

(a) a 50 Ω resistance
(b) a copper strip
(c) a diode
(d) a small length of fuse wire

Answers to Questions for Practice

1.	(c)	**2.**	(c)	**3.**	(a)	**4.**	(a)	**5.**	(a)	**6.**	(d)	**7.**	(d)
8.	(b)	**9.**	(b)	**10.**	(c)	**11.**	(b)	**12.**	(b)	**13.**	(d)	**14.**	(c)
15.	(d)	**16.**	(d)	**17.**	(b)	**18.**	(d)	**19.**	(c)	**20.**	(a)	**21.**	(d)
22.	(c)	**23.**	(a)	**24.**	(a)	**25.**	(a)	**26.**	(d)	**27.**	(b)	**28.**	(b)
29.	(c)	**30.**	(d)	**31.**	(d)	**32.**	(a)	**33.**	(a)	**34.**	(b)	**35.**	(a)
36.	(d)	**37.**	(d)	**38.**	(d)	**39.**	(d)	**40.**	(a)	**41.**	(d)	**42.**	(c)
43.	(d)	**44.**	(b)	**45.**	(d)	**46.**	(a)	**47.**	(d)	**48.**	(a)	**49.**	(b)
50.	(c)	**51.**	(c)	**52.**	(d)	**53.**	(c)	**54.**	(b)	**55.**	(d)	**56.**	(c)
57.	(c)	**58.**	(d)	**59.**	(a)	**60.**	(a)	**61.**	(a)	**62.**	(c)	**63.**	(c)
64.	(c)	**65.**	(c)	**66.**	(b)	**67.**	(d)	**68.**	(c)	**69.**	(c)	**70.**	(a)
71.	(d)	**72.**	(a)	**73.**	(b)	**74.**	(d)	**75.**	(b)	**76.**	(b)	**77.**	(c)
78.	(b)	**79.**	(b)	**80.**	(b)	**81.**	(b)	**82.**	(b)	**83.**	(c)	**84.**	(c)
85.	(d)	**86.**	(b)	**87.**	(d)	**88.**	(b)	**89.**	(b)	**90.**	(d)	**91.**	(b)
92.	(d)												

23

Magnetic Effects of Current

BRIEF REVIEW

Magnetic Force

A charged particle having charge q will experience a force $\vec{F} = q(\vec{v} \times \vec{B})$ if it enters a magnetic field B with a velocity v.

This principle may be employed in Television Receivers to deflect electrons. The SI unit of magnetic field is Wbm^{-2} or Tesla (T). The CGS unit is Gauss = Maxwell/cm^2

1 Gauss = $10^{-4}T$

If the charged particle is subjected to both electric and magnetic fields, the net force acting on the moving charged particle is given by Lorentz force

$$\vec{F} = q\vec{E} + q(\vec{v} \times \vec{B})$$

This method is employed in J.J. Thomson's experiment to find e/m.

If a charged particle always moves perpendicular to the magnetic field then it will describe a circle of radius R

such that $R = \dfrac{mv}{qB}$

Since it is a radial force, it only changes the direction and does not do any work. [See Fig. 23.1]

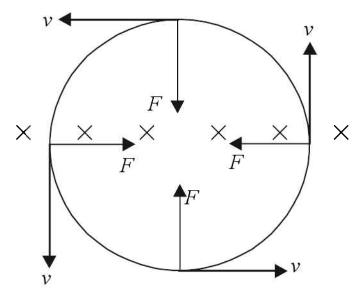


Fig. 23.1 Principle of cyclotron illustration

Time period of revolution $T = \dfrac{2\pi R}{v} = \dfrac{2\pi m}{qB}$

and cyclotron frequency $f_C = \dfrac{1}{T} = \dfrac{qB}{2\pi m}$

If the charged particle enters obliquely into a magnetic field B, the velocity can be resolved into two perpendicular components: one along the field and other perpendicular to the field. The perpendicular component describes a circle and parallel component causes linear motion. As a result the charged particle describes helix. See Fig. 23.2 (a) and (b).

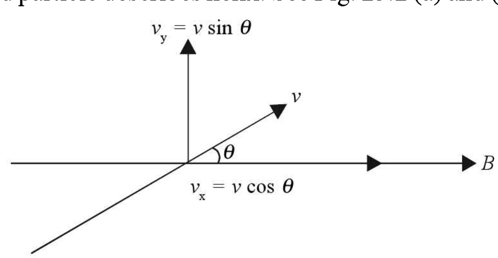


Fig. 23.2 (a) Oblique projection of a charged particle

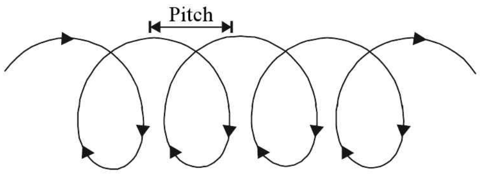


Fig. 23.2 (b) Helix

Pitch of helix The linear or horizontal distance moved in one complete rotation is called pitch of the helix.

$$\text{Pitch of the helix} = v_X . T = v_X\left(\frac{2\pi m}{qB}\right)$$

Magnetic force due to a current-carrying conductor when placed in a uniform magnetic field B is

$$\overrightarrow{dF} = I\,\overrightarrow{dl} \times \vec{B}$$

The direction of magnetic force is given by Fleming's Left hand rule.

Note $\vec{F} = I\vec{l} \times \vec{B}$ if the conductor is straight.

Otherwise $\vec{F} = \int I\left(\overrightarrow{dl} \times \vec{B}\right)$

Torque acting on a current-carrying loop when placed in a uniform magnetic field is $\tau = nI\left(\vec{A} \times \vec{B}\right)$

Where n is number of turns in the coil or loop and area vector A is perpendicular to the surface.

For a rectangular coil $A = lb$

and for a circular coil $A = \pi r^2$.

We can also write

$$\tau = nI\left(\vec{A} \times \vec{B}\right) = \vec{M} \times \vec{B} \text{ where } \vec{M} = nI\vec{A} \text{ is}$$

magnetic dipole moment.

Note that the coil will be in stable equilibrium if $\theta = 0$ and coil will be in unstable equilibrium if $\theta = 180°$. Torque is maximum if $\theta = 90°$.

If the magnetic field is non uniform then coil will experience torque as well as linear motion.

Biot Savart Law The magnetic field produced due to a current-carrying element of length dl at any point P is given

by $\overrightarrow{dB} = \mu_0 \dfrac{I\,\overrightarrow{dl} \times \vec{r}}{4\pi r^3}$ (See Fig. 23.3)

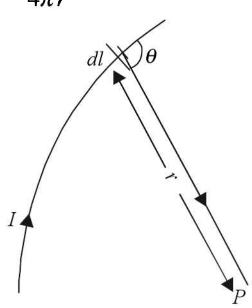


Fig. 23.3 Biot savart law

or $$|dB| = \frac{\mu_0 I dl \sin\theta}{4\pi r^2}$$

where μ_0 is permeability of free space and $\mu_0 = 4\pi \times 10^{-7}\ Wb$ (A–m)$^{-1}$ or Henry m^{-1}.

The direction of magnetic field is given by Right hand thumb rule as illustrated in Fig. 23.4

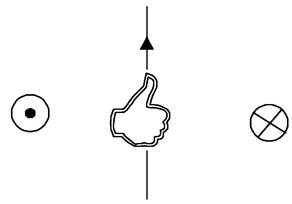


Fig. 23.4 (a)

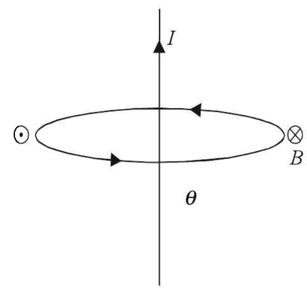


Fig. 23.4 (b) Illustration of direction of magnetic field

Magnetic field strength due to straight current-carrying conductor at a point *P*

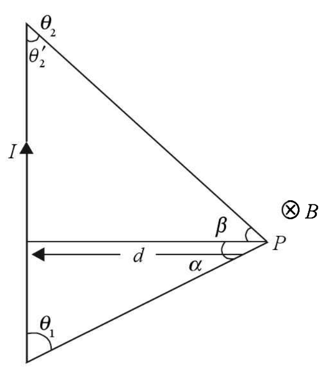


Fig. 23.5 Magnetic field due to finite straight conductor

From Fig. 23.5 magnetic field strength at P is

$$B = \frac{\mu_0 I}{4\pi d}(\cos\theta_1 - \cos\theta_2)$$

$$= \frac{\mu_0 I}{4\pi d}(\cos\theta_1 + \cos\theta_2')$$

$$= \frac{\mu_0 I}{4\pi d}(\sin\alpha + \sin\beta)$$

The direction of magnetic field is given by Right hand thumb rule. From Fig. 23.4(a), it is clear that magnetic field at P is perpendicular inwards the plane of paper and magnetic field at S is perpendicular outwards the plane of paper.

Magnetic field strength at point P on a perpendicular bisector is

$$B = \frac{\mu_0 I a}{2\pi d\sqrt{a^2 + 4d^2}}$$

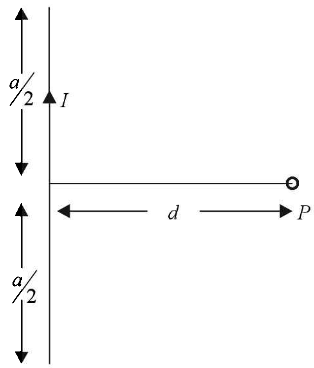


Fig. 23.6 Magnetic field at perpendicular bisector

Magnetic field strength at point p due to a long current-carrying conductor is $B = \frac{\mu_0 I}{2\pi d}$

Magnetic field strength at the center of a circular loop carrying current *I*

$$B = \frac{\mu_0 I}{2r}$$

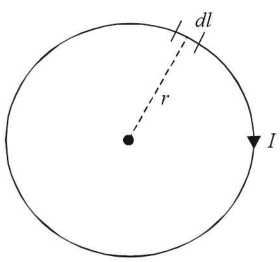


Fig. 23.7 Magnetic field due to a circular loop at its centre

The direction of magnetic field is perpendicular inwards the plane of loop if the current is clockwise and perpendicular outwards if the current is anti-clockwise.

Magnetic field strength due to a circular arc of radius *r* at the centre

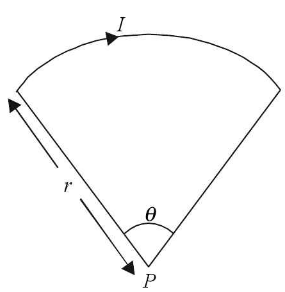


Fig. 23.8 Magnetic field due to a circular arc

See Fig. 23.8 and take θ in radian.

$$B = \frac{\mu_0 I}{2r}\left(\frac{\theta}{2\pi}\right)$$

Thus for a semicircular loop

$$B = \frac{\mu_0 I}{4r} \text{ as } \theta = \pi$$

Magnetic field strength at any point on axial line of circular ring carrying current *I*

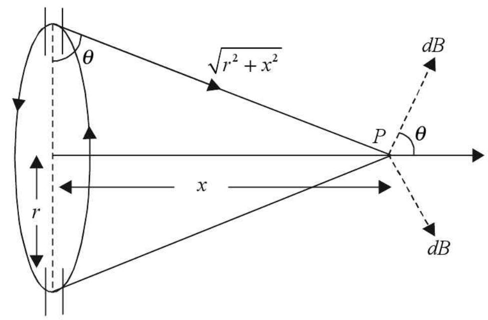


Fig. 23.9 Magnetic field due to a circular loop at any point

$$B = \frac{\mu_0 I r^2}{2\left(r^2 + x^2\right)^{3/2}}$$

where r is radius of the circular loop.

The direction of magnetic field is same as for circular coils.

Special cases Magnetic field at a very large distance from the centre i.e. $x >> r$

$$B = \frac{\mu_0 I r^2}{2x^3} = \frac{\mu_0 I\left(\pi r^2\right)}{2\pi x^3} = \frac{2\mu_0 M}{4\pi x^3}$$

That is, coil behaves as a magnetic dipole.

At the centre of the loop $x = 0$; $B = \frac{\mu_0 I}{2r}$

Ampere's Circuital law

$$\oint B.dl = \mu_0 I$$

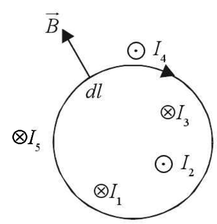


Fig. 23.10 Ampere circuital law illustration

See Fig. 23.10

If I_1 and I_3 are taken as positive and I_2 as negative then

$$I = I_1 + I_3 - I_2 \text{ and then}$$

$$\oint B.dl = \mu_0(I_1 + I_3 - I_2)$$

Note that any current outside the loop is not included in the right hand side in the current.

Note that $\oint B.dl = \mu_0 I$ can be applied even to a long conductor.

Magnetic field due to a long solenoid at the axis of the solenoid

$B = n\mu_0 I$ where n is number of turns per unit length

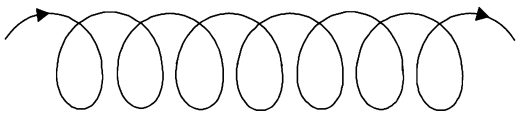


Fig. 23.11 Solenoid

Magnetic field outside the coil is zero.

Magnetic field at any point P in the solenoid as shown in Fig. 23.12 is

$$B_P = \frac{1}{2}\mu_0 nI[\cos\theta_1 - \cos\theta_2]$$

At point F or E $\quad B = \frac{1}{2}\mu_0 nI$ [see Fig. 23.12]

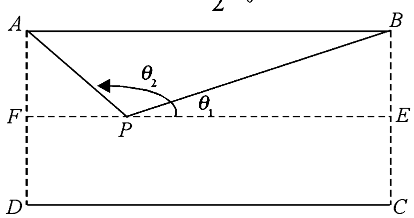


Fig. 23.12 Magnetic field inside the solenoid at any point

Magnetic field at any point P (acting tangentially) on a toroid

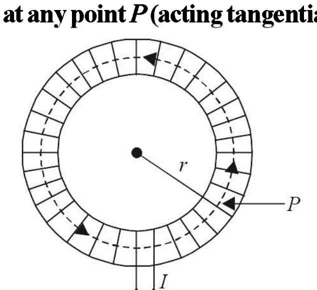


Fig. 23.13 Toroid

From Fig. 23.13 $B = \frac{\mu_0 NI}{2\pi r}$

where N is total number of turns.

Magnetic force between two long, parallel current-carrying conductors

If d is the separation between two long current-carrying conductors carrying currents I_1 and I_2 as shown in Fig. 23.14

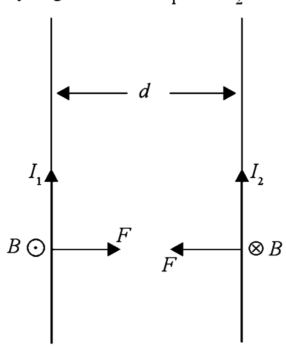


Fig. 23.14 Force between two long current-carrying conductors

Then $\frac{dF}{dl} = \frac{\mu_0 I_1 I_2}{2\pi d}$

Note that the force is attractive if the conductors carry current in the same direction. Force will be repulsive if they carry current in opposite directions.

Magnetic field due to a moving charge at any point P distant r from the charge.

$$B = \frac{\mu_0 qv\sin\theta}{4\pi r^2} \text{ or } \vec{B} = \frac{\mu_0 q(\vec{v}\times\vec{r})}{4\pi r^3}$$

Magnetic force between moving charges: If two charges q_1 and q_2 are moving with v_1 and v_2 parallel to each other at a separation r then

$$F_{mag} = \frac{\mu_0 q_1 q_2 v_1 v_2}{4\pi r^2}$$

Even the like charges moving in the same direction will repel as $F_{elect} > F_{mag}$

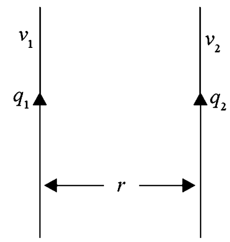


Fig. 23.15 Magnetic force between moving charges

Since electric force is repulsive and magnetic force is attractive, the net force is repulsive. Note the force will be attractive only if charges are unlike.

• Short Cuts and Points to Note

1. Cyclotron is normally used to accelerate positively charged particles though it can accelerate negatively charged particles except electrons. Electrons are accelerated using betatron.
2. The principle of mass spectrometer (to measure mass of atoms/molecules) is same as that of cyclotron.

   Here $\frac{m_1}{m_2} = \frac{r_1}{r_2}$ assuming they were monovalent/ divalent/ trivalent ions and enter the magnetic field with same velocity. Note that discovery of isotopes was made using mass spectrometer.
3. $\vec{F} = q(\vec{v}\times\vec{B})$

   or $\vec{dF} = i(\vec{dl}\times\vec{B})$ represent Ampere's force.
4. Magnetic flux $= \int B.ds$ is scalar
5. A cylindrical coil or a circular coil carrying current behaves like a bar magnet. A clockwise current generates S-pole and an anti-clockwise current will generate N-pole.

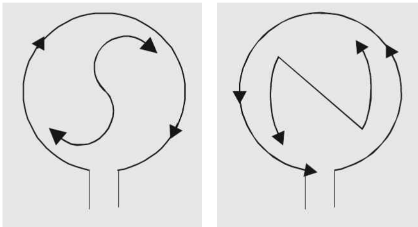


Fig. 23.16 Direction of magnetic field illustration

6. Magnetic field lines make closed loop. Unlike electric field, lines representing monopole cannot exist.

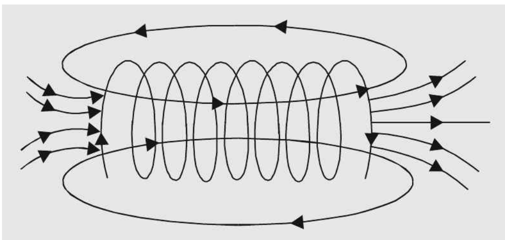


Fig. 23.17

7. When current passes through a spring it shrinks as all the rings in it carry current in the same direction and are attracted towards one another.
8. Momentum of a charged particle in a cyclotron

$p = Bqr = \sqrt{2(KE)m}$ where KE is kinetic energy of the particle.

9. In cyclotron when KE and magnetic fields for two charged particles are equal then,

$$\frac{r_1}{r_2} = \frac{q_2}{q_1}\sqrt{\frac{m_1}{m_2}}$$

If only magnetic field is same for the two charged particles then $\frac{r_2}{r_1} = \frac{q_2}{q_1}\sqrt{\frac{m_1(KE_1)}{m_2(KE_2)}}$

where KE_1 and KE_2 are kinetic energies for the two particles respectively.

10. No magnetic field occurs on a point P on the current carrying conductor or an any point S or R which lie on the extended part of the conductor as shown in Fig. 23.18

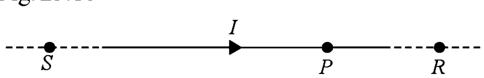


Fig. 23.18

i.e. $B_S = B_P = B_R = 0$

11. Magnetic field intensity at the centre of a loop made with a uniform cross-section wire and uniform density is zero irrespective of its shape provided current enters from a point and leaves from another point on the conductor as shown in Fig. 23.19.

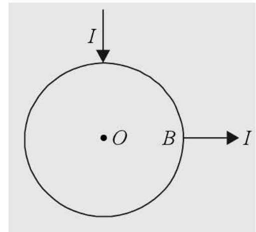


Fig. 23.19

12. If magnetic field and electric field are perpendicular to each other and a charged particle enters perpendicular to both electric and magnetic fields, if charged particle goes undeviated then,

$$E = Bv \text{ or } v = \frac{E}{B}$$

13. Magnetic field intensity in a thick current-carrying conductor at any point $x < r$ (inside the conductor) as illustrated in Fig. 23.20

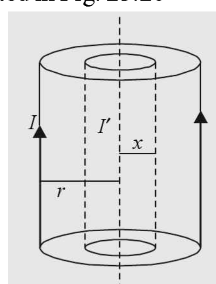


Fig. 23.20 (a)

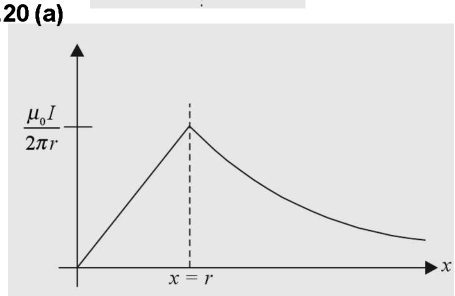


Fig. 23.20 (b)

$$B_{\text{inside}} = \frac{\mu_0 I x}{2\pi r^2} \quad \text{for } x < r$$

$$B_{\text{surface}} = \frac{\mu_0 I}{2\pi r} \quad \text{for } x = r$$

$$B_{\text{outside}} = \frac{\mu_0 I}{2\pi x} \quad \text{for } x > r$$

14. While finding magnetic force on the curved part of a conductor use displacement as length and then $F = IlB$ is valid where l is displacement between the extreme points of a curved part. For instance in Fig. 23.21 the force due to curved part is $2\,IRB$ as $l = 2R$ is displacement.

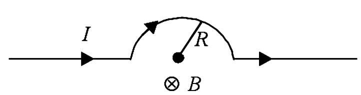


Fig. 23.21

15. Under the action of magnetic field alone, speed or KE of the charged particle remains unchanged.

16. Cyclotron principle can be used to detect leak in high vacuum system. For this purpose He^+ ions are used. A very small magnetic field is required which makes the leak detector a compact device.

17. Magnetic dipole moment $M = IA$. While finding torque on a coil carrying current due to a dipole if the (angle) is given between area vector $\vec{A}$ and magnetic field B (Area vector is normal to the plane of the surface of coil) then $\tau = IAB \sin \theta$. If angle between plane of the coil and magnetic field is known then $\tau = IAB \cos \theta$.

18. Potential energy $U = \int \tau . d\theta = -IAB \cos \theta = -\vec{M}.\vec{B}$.

19. Note that in parallel conductors carrying current you have $\dfrac{dF}{dl} = \dfrac{\mu_0 I_1 I_2}{2\pi d}$.

To find force, multiply with the length or integrate appropriately.

If the case is as shown in Fig. 23.22 (b) where the magnetic field varies at every point, assume an element. Find force on the element and then integrate.

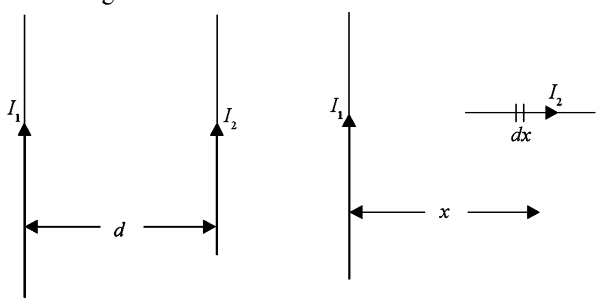


Fig. 23.22

20. $\vec{F} = q(\vec{v} \times \vec{B})$,

$d\vec{F} = I(dl \times B)$

For a straight conductor $F = Il \times B$ integrate otherwise.

21. $\oint E.dl = 0$ because force is conservative but $\oint B.dl \neq 0$. Rather $\oint B.dl = \mu_0 I$ as $B.dl$ is not related to work. Moreover, the current is enclosed in the loop.

22. A moving charge produces both electric and magnetic field $\vec{B} = \dfrac{\mu_0 q\vec{v} \times \vec{r}}{4\pi r^3}$

A stationary charge produces only electric field.

23. The magnetic field due to a short solenoid is illustrated in Fig. 23.23.

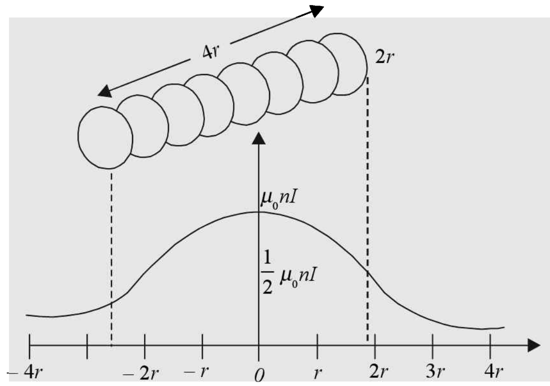


Fig. 23.23

Note that magnetic field is nearly half the value at the centre. r is radius of the coil.

24.

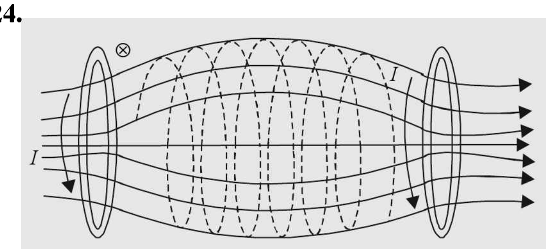


Fig. 23.24

Fig. 23.24 illustrates the method of retaining ionised gas particles having temperature $\sim 10^6$ K which would vaporise any material container. Such a system is termed as magnetic bottle.

25. SNOW rule can be applied if a current carrying conductor is placed over compass needle.

• Caution

1. Considering physical length of the conductor to be taken as length of the conductor in

$$F = \int idl \times B$$

⇒ Shortest distance between the end points be taken as the length.

2. Considering magnetic field exists on a (thin) current-carrying conductor or on its extended part.

⇒ Magnetic field does not exist on the conductor or on its extended part. Thus magnetic field at A, P and X is zero.

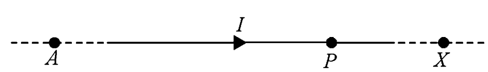


Fig. 23.25 ABC

3. Considering $\oint B.dl = 0$ on the lines of $\oint E.dl = 0$

⇒ Note that $\oint E.dl = 0$ because it represents work done in conservative force. $\oint B.dl$ does not define work and hence $\oint B.dl = \mu_0 I$

4. Confusing work is done by magnetic force.

⇒ Magnetic force only changes direction. Work done is zero. Hence speed and KE do not change.

5. Considering magnetic field to be zero only along the axis of a hollow cylindrical conductor carrying current.

⇒ Magnetic field is zero at all points inside a current-carrying hollow cylindrical conductor. However, magnetic field is zero along the axis of a solid cylindrical conductor carrying current.

∵ B_{inside} (Solid cylinder)

$$= \frac{\mu_0 I x}{2\pi r^2}$$

6. While finding direction of force with moving charged particle and applying Fleming's left hand rule.

⇒ We can apply Fleming's left hand rule if we take into account the appropriate direction of current a +*ve* or a –*ve* charge will form during motion. Conventional current direction be taken.

7. Not considering perpendicular distance due to straight conductor while finding magnetic field. For example as in Fig. 23.26 taking perpendicular distance *r* for *AB*.

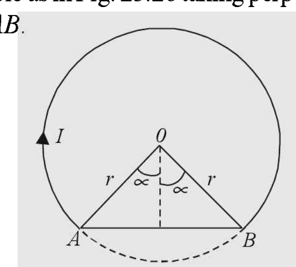


Fig. 23.26

⇒ Take perpendicular distance $r\cos\alpha$ if 2α is the angle made by *AB* at the centre.

8. Considering no force will act on current-carrying conductors placed transverse to a long current-carrying conductor as shown in Fig. 23.27.

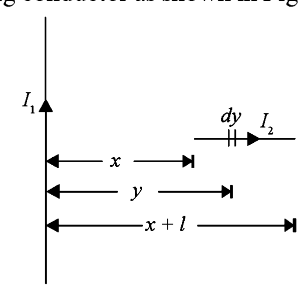


Fig. 23.27

⇒ In such a case magnetic field at every point will vary, therefore take an element *dy* at a distance *y*

then $$F = \frac{\mu_0 I_1 I_2}{2\pi}\int_x^{x+l}\frac{dy}{y} = \frac{\mu_0 I_1 I_2}{2\pi}\log_e\frac{x+l}{x}$$

9. Considering magnetic moment *M* shall depend upon the shape of the current-carrying conductor.

⇒ Magnetic moment $M = IA$ or nIA is independent of the shape of the conductor if their areas are equal and number of turns are also equal.

10. Considering that if plane of a coil is parallel to the magnetic field the net force acting on the coil is zero.

⇒ If the magnetic field is uniform the statement is correct. If magnetic field is not uniform then both torque and a net force are present.

11. Considering magnetic field is zero at the centre of a loop if current enters from a point and leaves at the other.

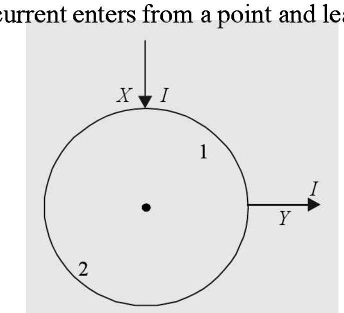


Fig. 23.28

⇒ This statement is true if the loop is made with a single wire of same material of uniform cross-section and uniform density. If however joining parts *X*1*Y* and *X*2*Y* are of different materials or one of them is thinner than the other (though they are of the same material), then magnetic field is non-zero at the centre.

12. Considering $B = \frac{\mu_0 I r^2}{2(r^2+x^2)^{3/2}}$ can be applied anywhere inside the coil.

⇒ $$B = \frac{\mu_0 I r^2}{2(r^2+x^2)^{3/2}}$$

$$= \frac{2\mu_0 M}{4\pi(r^2+x^2)^{3/2}}$$

can be applied along the axis only.

SOLVED PROBLEMS

1. A magnetic needle is kept in a non-uniform magnetic field. It experiences

(a) neither a force nor a torque

(b) a torque but not a force

(c) a force but not a torque

(d) a force and a torque. **[AIEEE 2005]**

Solution (d)

2. A charged particle of mass m and charge q travels on a circular path of radius r that is perpendicular to the magnetic field B. The time taken by the particle to complete one revolution is

(a) $\frac{2\pi q^2 B}{m}$ (b) $\frac{2\pi mq}{B}$

(c) $\frac{2\pi m}{qB}$ (d) $\frac{2\pi qB}{m}$

Solution (c)

3. Two concentric coils each of radius 2π cm are placed at right angles to each other. 3A and 4A are the currents flowing in them respectively. Find magnetic induction in Wb/m^2 at the centre of the coils.

[AIEEE 2005]

(a) 10^{-5} (b) 12×10^{-5}

(c) 7×10^{-5} (d) 5×10^{-5}

Solution (d) $B = \frac{\mu_0 I}{2r}$. Since the two coils are perpendicular, so are the magnetic inductions.

$$\therefore \quad B_{net} = \sqrt{B_1^2 + B_2^2} = \frac{\mu_0}{2r}\sqrt{I_1^2 + I_2^2}$$

$$= \frac{4\pi \times 10^{-7}}{2(2\pi \times 10^{-2})}\sqrt{3^2 + 4^2} = 5 \times 10^{-5} \text{ Wb/m}^2$$

4. Two thin long, parallel wires separated by a distance d carry a current i each in the same direction. They will

(a) repel each other with a force $\frac{\mu_0 i^2}{2\pi d}$

(b) attract each other with a force $\frac{\mu_0 i^2}{2\pi d}$

(c) repel each other with a force $\frac{\mu_0 i^2}{2\pi d^2}$

(d) attract each other with a force $\frac{\mu_0 i^2}{2\pi d^2}$

[AIEEE 2005]

Solution (b) $\frac{dF}{dl} = \frac{\mu_0 I_1 I_2}{2\pi d} = \frac{\mu_0 i^2}{2\pi d}$

Since currents are in same direction they attract each other.

5. A uniform electric field and a uniform magnetic field are acting along the same direction in a certain region. If an electron is projected along the direction of the fields with a certain velocity then

(a) its velocity will increase

(b) its velocity will decrease

(c) it will turn towards left of its motion

(d) it will turn towards right of its motion

[AIEEE 2005]

Solution (b) Since $F = q[\vec{E} + (\vec{v} \times \vec{B})]$

$F_{mag} = q(\vec{v} \times \vec{B}) = 0$ as $\vec{v}$ and $\vec{B}$ are parallel

Since electron is moving along the field, force qE is repulsive and hence it will slow down.

6. The Fig. 23.29 shows an infinitely long current-carrying wire out of the plane of paper (shown by ⊙). A current carrying loop ABCD is placed as shown. The loop

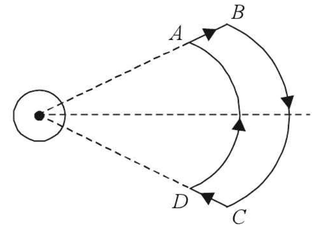


Fig. 23.29

(a) experiences no net force

(b) experiences no torque

(c) turns clockwise as seen by an observer at ⊙ position

(d) turns anticlockwise as seen by an observer at ⊙ position

[IIT 2006]

Solution (a), (c) Magnetic force due to AB and CD is zero ($\because \theta = 0°$ or $180°$). Magnetic force on BC is upward and on DA is downward. These two forces will tilt loop in clockwise direction when seen from ⊙ position.

7. A very long straight wire carries a current I. At the instant when a charge $+Q$ at point P has velocity $\vec{v}$, as shown in Fig 23.30, the force on charge is

[CBSE 2005]

(a) along oy (b) opposite to oy

(c) along ox (d) opposite to ox

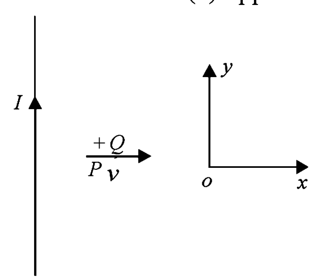


Fig. 23.30

Solution (a) The magnetic field at P is inwards due to a straight long conductor. Fleming's left hand rule gives the direction along oy.

8. An electron moves in a circular orbit with a uniform speed v. It produces a magnetic field B at the centre of the circle. The radius of the circle is proportional to

(a) $\sqrt{\frac{B}{v}}$ (b) $\frac{B}{v}$

(c) $\sqrt{\frac{v}{B}}$ (d) $\frac{v}{B}$

[CBSE 2005]

Solution (d) $r = \frac{mv}{qB}$ as $\frac{m}{q}$ is constant $\therefore r \propto \frac{v}{B}$

9. A circular loop of wire 4 cm in radius carries a current of 80 A. Find the energy density at the centre of the loop.

(a) $\pi\, J/m^3$ (b) $2\pi\, J/m^3$

(c) $0.1\pi\, Jm^3$ (d) $0.2\pi\, Jm^{-3}$

Solution (d) $B = \dfrac{\mu_0 i}{2r}$ and energy density $u = \dfrac{B^2}{2\mu_0}$

$$= \frac{\mu_0^2 i^2}{4r^2(2\mu_0)} = \frac{\mu_0 i^2}{8r^2}$$

$$u = \frac{4\pi\times10^{-7}\times(80)^2}{8(4\times10^{-2})^2} = \frac{4\pi\times8\times.8}{8\times16}$$

$= 0.2\pi = 0.628\, Jm^{-3}$

10. The adjacent Figure shows lines of a field. It cannot represent

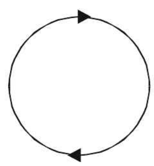


Fig. 23.31

(a) an electrostatic field (b) an induced electric field

(c) a gravitational field (d) a magnetostatic field

[IIT 2006]

Solution (a), (c) Electrostatic and gravitational field do not complete the loop.

11. An α–particle enters at the middle as shown in Fig. 23.32 with 10^5 ms^{-1}. In which direction will it bend?

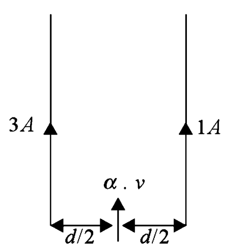


Fig. 23.32

(a) Towards 1 A wire (b) Towards 4 A wire

(c) Upwards the plane of wires

(d) Downwards the plane of wires

Solution (b)

12. A particle having mass m and charge q is released from the origin in a region in which electric field and magnetic fields are given by $B = -B_0\hat{j}$ and $E = E_0\hat{k}$. Find the speed of the particle as a function of its z coordinate.

(a) $\sqrt{\dfrac{qEz}{m}}$ (b) $\sqrt{\dfrac{2(qvB+qE)z}{m}}$

(c) $\sqrt{\dfrac{(-qvB+qE)2z}{m}}$ (d) $\sqrt{\dfrac{2qEz}{m}}$

Solution (d) $v^2 = 2az$ $a = \dfrac{qE}{m}$ or $v = \sqrt{\dfrac{2qEz}{m}}$

13. An electron has a speed $\sqrt{2}\times10^6$ ms^{-1}at A as shown in Fig. 23.33. Find the direction and magnitude of magnetic field so that electron reaches B following a semicircular path.

(a) 1.6×10^{-4}T ⊗ (b) 1.6×10^{-4} T ⊙

(c) 3.6×10^{-4}T ⊗ (d) none of these

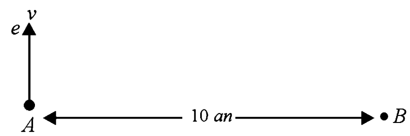


Fig. 23.33

Solution (a) $r = 5\text{cm} = \dfrac{mv}{qB}$

or $$B = \frac{mv}{er} = \frac{9\times10^{-31}\times\sqrt{2}\times10^6}{1.6\times10^{-19}\times5\times10^{-2}} = \frac{9\sqrt{2}\times10^{-4}}{8}$$

$= 1.6\times10^{-4}\,T$ direction perpendicular inwards the plane of paper.

14. An electron in the beam of a TV picture tube is accelerated by a potential difference 2 kV. Then it passes through a transverse magnetic field to produce a circular arc of radius 0.18m. Find the magnetic field.

(a) 6.38×10^{-4} T (b) 7.68×10^{-4} T

(c) 8.38×10^{-4} T (d) 8.98×10^{-4} T

Solution (c) $r = \dfrac{mv}{qB}$

or $$B = \frac{mv}{er} = \frac{\sqrt{2eVm}}{er} = \sqrt{\frac{2Vm}{er^2}}$$

or $$B = \sqrt{\frac{2\times10^3\times2\times9\times10^{-31}}{1.6\times10^{-19}\times10^{-4}}} = \sqrt{\frac{10^{-5}}{.4\times36}}$$

$$= \sqrt{\frac{10^{-5}}{14.4}} = 8.38\times10^{-4}\text{T}$$

15. Fig. 23.34 shows a thin 50 cm long rod resting on two metallic supports in a uniform magnetic field of 0.45 T. Find the maximum voltage which can be applied without breaking the circuit. Mass of the rod is 750 g. Take $g = 10$ ms^{-2}

(a) 83.3 V (b) 8.33 V

(c) 833 V (d) 0.833 V

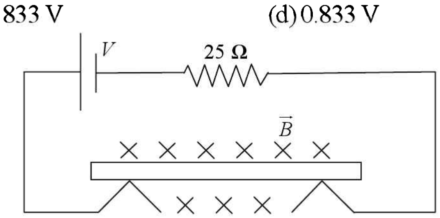


Fig. 23.34

Solution (c) mg = IlB or mg = $\dfrac{V}{R}lB$

or $$V = \frac{mgR}{lB} = \frac{3\times10\times25}{4\times.5\times.45} = 833\,V$$

16. A wire along x-axis carries a current 3.5 A. Find the force on a 1 cm section of the wire exerted by

$$B = 0.74\text{ T}\,\hat{j} - 0.36\text{ T}\,\hat{k}$$

(a) $2.59\hat{k} + 1.26\hat{j}$ (b) $1.26\hat{k} - 2.59\hat{j}$

(c) $-2.59\hat{k} - 1.26\hat{j}$ (d) $-1.26\hat{k} + 2.59\hat{j}$

Solution (a) $F = I(\vec{l} \times \vec{B})$

$= 3.5\,[10^{-2}\,\hat{i} \times (.74\,\hat{j} - 0.36\,\hat{k})]$

$= (2.59\hat{k} + 1.26\hat{j}) \times 10^{-2}$

17. An electron and a ${}^{7}_{3}Li$ nucleus enter a magnetic field with same velocity. Find the ratio of number of revolutions per second of the two.

(a) 2.44×10^3 (b) 4.24×10^3

(c) 3.24×10^3 (d) 5.42×10^3

Solution (b) $f = \dfrac{qB}{2\pi m}$

$$\therefore \quad \frac{f_e}{f_{Li}} = \frac{e/m_e}{3e/m_{Li}} = \frac{m_{Li}}{3m_e} = \frac{7\times1.6\times10^{-27}}{3\times9\times10^{-31}}$$

$$= \frac{11.2}{27} \times 10^4 = 4.24 \times 10^3$$

18. A thin uniform rod of negligible mass and length l is attached to the floor by a hinge P. The other end is connected to a spring of force constant k. Rod is in a uniform magnetic field B pointing inwards the plane of paper. A current I is passed through the rod. Find the torque acting on the rod due to magnetic force when the rod makes an angle 53° as shown in Fig. 23.35.

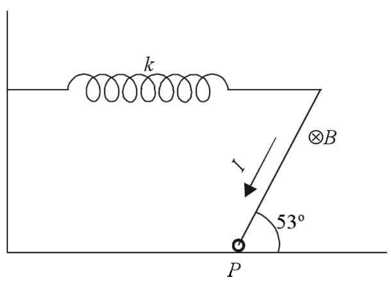


Fig. 23.35

(a) Il^2B (b) $\dfrac{Il^2}{2}B$

(c) $\dfrac{3Il^2B}{5}$ (c) $\dfrac{4}{5}Il^2B$

Solution (b) $F = kx = IlB$

$$\tau = \int d\tau = \int IBdl = \frac{Il^2B}{2}$$

19. Two long parallel wires are hung by 4 cm long cords from a common axis. The wires have a mass 0.0125 kg/m and carry equal currents in opposite direction. Find the current in each wire if the cords hang at 6° with the vertical as shown in Fig. 23.36 (a).

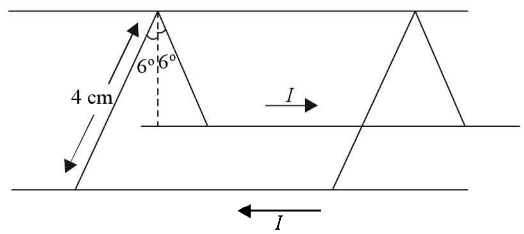


Fig. 23.36 (a)

(a) 11.2 A (b) 22.3 A

(c) 717 A (d) 71.7 A

Solution (b) $\dfrac{dF}{dl}\,rep = \dfrac{\mu_0 I^2}{2\pi(2d)} = T\sin 6°$

$$\frac{mg}{4} = T\cos 6° \qquad \tan 6° = \frac{\mu_0 I^2}{4\pi dmg}$$

$$I = \sqrt{\frac{\tan 6°(4\pi 4\sin 6°)mg\times10^{-2}}{\mu_0}}$$

$$= \sqrt{\frac{(.1)^2\times4\pi\times4\times.0125\times10\times10^{-2}}{4\pi\times10^{-7}}} = 22.3\text{ A}$$

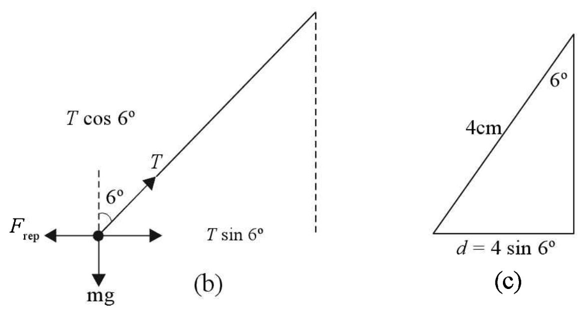


Fig. 23.36

$$\tan 6° = \sin 6° = \frac{6\times\pi}{180} = 0.1$$

20. An infinitely long wire carries a current i (see Fig. 23.37). Find magnetic field at p.

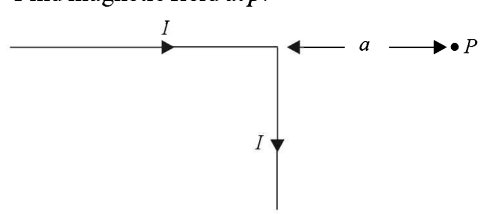


Fig. 23.37

(a) $\dfrac{\mu_0 I}{2\pi a}$ (b) $\dfrac{\sqrt{2}\mu_0 I}{4\pi a}$

(c) $\dfrac{\mu_0 I}{4\pi a}$ (d) $\dfrac{\mu_0 I}{4\sqrt{2}\pi a}$

Solution (c) $B = B_1 + B_2 = 0 + \dfrac{\mu_0 i}{4\pi a}[\sin 90 + \sin 0]$

$$= \frac{\mu_0 I}{4\pi a}$$

21. A wire bent as shown in Fig. 23.38 carries a current I. Find the magnetic field at P.

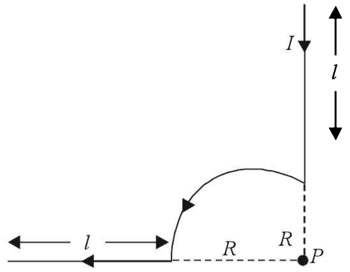


Fig. 23.38

(a) $\frac{\mu_0 I}{4R}$ (b) $\frac{3\mu_0 I}{2R}$

(c) $\frac{7\mu_0 I}{8R}$ (d) $\frac{\mu_0 I}{8R}$

Solution (d) $B = B_1 + B_2 + B_3 = 0 + \frac{\mu_0 I}{2R}\left(\frac{\pi/2}{2\pi}\right) + 0$

$= \frac{\mu_0 I}{8R}$

22. A wire bent as shown in Fig. 23.39 is oriented along yz plane. Find the magnetic field at and P along x and y directions.

(a) $\frac{\mu_0 I}{4a}, \frac{\mu_0 I}{2\pi x}$ (b) $\frac{\mu_0 I}{4a}, \frac{\mu_0 Ia}{2\pi\left(x^2+a^2\right)}$

(c) $\frac{\mu_0 I}{4a}, \frac{\mu_0 Ia}{2\pi a\sqrt{x^2+a^2}}$ (d) none of these

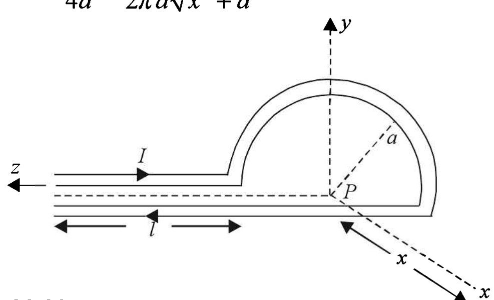


Fig. 23.39

Solution (c) $B_P = \frac{\mu_0 I}{4a}$ along $-x$

$$Bp_\perp = \frac{\mu_0 I(2a)}{2\pi x\sqrt{4a^2+4x^2}} \text{ along } y$$

[use magnetic field at perpendicular bisector]

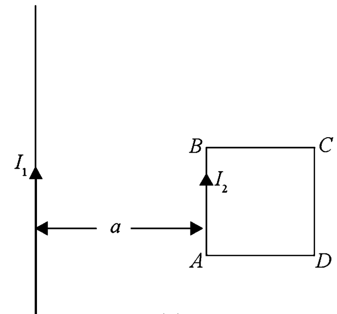
$$Bp_\perp = \frac{\mu_0 Ia}{2\pi x\sqrt{a^2+x^2}} \quad \text{along } y$$

[only contribution is from straight wire $-a$ to $+a$ along z axis]

23. A square loop of side a is placed at a distance a away from a long wire carrying a current I_1. If the loop carries a current I_2 as shown in Fig. 23.40 (a) then the nature of the force and its amount is

(a) $\frac{\mu_0 I_1 I_2}{2\pi a}$, attractive (b) $\frac{\mu_0 I_1 I_2}{4\pi}$, attractive

(c) $\frac{\mu_0 I_1 I_2}{4\pi}$, repulsive (d) $\frac{\mu_0 I_1 I_2}{4\pi a}$, repulsive



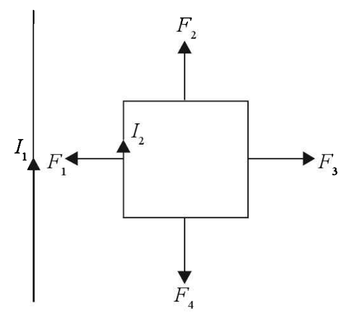
(a)

Fig. 23.40

[AFMC 1998, CEE Delhi 1997, 2000]

Solution (b) F_2 and F_1 cancel one another. F_1 is attractive F_3 is repulsive. But $F_1 > F_3$

∴ Force is attractive



(b)

Fig. 23.40 (b)

$$F_1 = \frac{\mu_0 I_1 I_2}{2\pi a}(a), F_3 = \frac{\mu_0 I_1 I_2}{4\pi a}(a)$$

$$F_{net} = F_1 - F_3 = \frac{\mu_0 I_1 I_2}{4\pi}$$

24. Fig. 23.41 shows a circular wire of radius r carrying a current i. The force of compression on the wire is

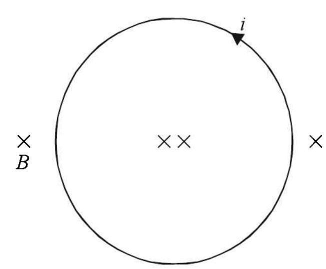


Fig. 23.41

(a) $2iaB$ (b) iaB

(c) $2\pi iaB$ (d) none of these

Solution (b) $dF = idlB$

$$F = \int idl\ B = iaB$$

25. A square coil of edge l having n turns carries a current i. It is placed on a smooth horizontal plate A magnetic field B parallel to one edge is applied. The total mass of the coil is M. The minimum value of B for which the coil will tip over is

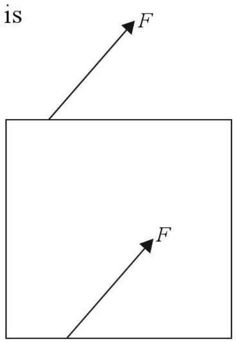


Fig. 23.42

(a) $\frac{Mg}{lin}$ (b) $\frac{Mg}{2lin}$

(c) $\frac{2Mg}{lin}$ (d) none of these

Solution (b) $F = lin\,B + lin\,B$

$Mg = 2\,lin\,B$ or $B = \frac{Mg}{2lin}$.

26. A particle of mass M and charge Q moving with a velocity $\vec{v}$ describes a circular path of radius R when subjected to a uniform transverse magnetic field of induction B. The work done by the field when the particle completes one full circle is

(a) zero (b) BQ2πR

(c) BQv(2πR) (d) $\left(\frac{Mv^2}{R}\right)(2\pi R)$

[AIEEE 2003]

Solution (a) As displacement is zero.

27. A particle of charge $q = 16 \times 10^{-18}\ C$ moving with $10\ ms^{-1}$ along x-axis enters a magnetic field of induction B along the y-axis and an electric field $10^4\ Vm^{-1}$ along negative z-direction. If the particle continues to move along x-axis then the strength of magnetic field is

(a) $10^5\ Wbm^{-2}$ (b) $10^{16}\ Wbm^{-2}$

(c) $10^{-3}\ Wbm^{-2}$ (d) $10^3\ Wbm^{-2}$

[AIEEE 2003]

Solution (d) $v = \frac{E}{B} \Rightarrow \frac{10^4}{B} = 10 \therefore B = 10^3\ Wbm^{-2}$

28. A conducting loop carrying a current I is placed in a uniform magnetic field pointing into the plane as shown in Fig. 23.43. The loop will have tendency to

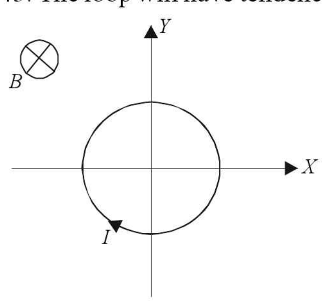


Fig. 23.43

(a) contract (b) expand

(c) move towards positive x-axis

(d) move towards negative x-axis

[IIT Screening 2003]

Solution (b) Using Fleming left hand rule you find that the force is acting outwards.

29. In a square loop made with a wire of uniform cross-section current I enters from point A and leaves from point B. The magnetic field strength B at the centre of the square is

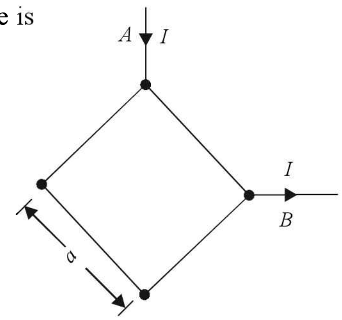


Fig. 23.44

(a) zero (b) $\frac{\mu_0 I 2\sqrt{2}}{4\pi a}$

(c) $\frac{4\sqrt{2}\mu_0 I}{4\pi a}$ (d) $\frac{2\sqrt{2}\mu_0 I}{4a}$

Solution (a) See shortcut (3).

30. In the Fig. 23.45 shown below each battery has emf = 5 V. Then the magnetic field at P is

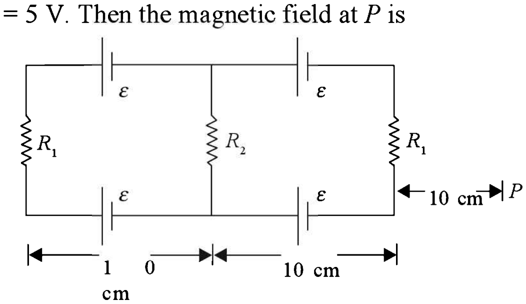


Fig. 23.45

(a) zero (b) $\frac{10\mu_0}{R_1(4\pi)(.2)}$

(c) $\frac{20\mu_0}{(R_1+R_2)(.8\pi)}$ (d) none of these

Solution (a) Because current in the loop is zero.

31. The magnetic field strength at O due to current I in the Fig. 23.46 is

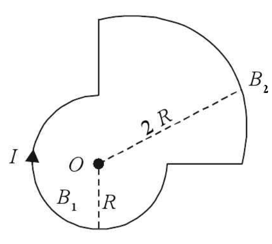


Fig. 23.46

(a) $\frac{7\mu_0 I}{16R}$ (b) $\frac{15\mu_0 I}{16R}$

(c) $\frac{11\mu_0 I}{32R}$ (d) $\frac{13\mu_0 I}{32R}$

Solution (a) $B = B_1 + B_2 = \frac{\mu_0 I}{2R}\left(\frac{3}{4}\right) + \frac{\mu_0 I}{4R}\left(\frac{1}{4}\right) = \frac{7\mu_0 I}{16R}$

32. Two long wires carrying current are kept crossed (not joined at O). The locus where magnetic field is zero is

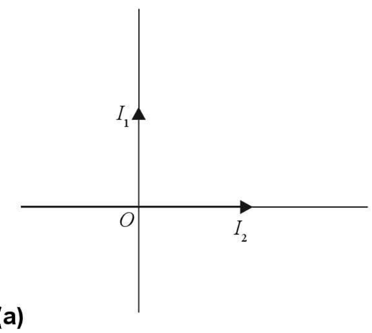


Fig. 23.47 (a)

(a) $I_1 = \frac{x}{y} I_2$ (b) $I_1 = \frac{y}{x} I_2$

(c) $I_1 = I_2$ (d) $I_1 = -I_2$

Solution (a) Magnetic field could be zero in 1st or 3rd quadrant.

$$\frac{\mu_0 I_1}{2\pi x} = \frac{\mu_0 I_2}{2\pi y}$$

or $$I_1 = \frac{x}{y} I_2.$$

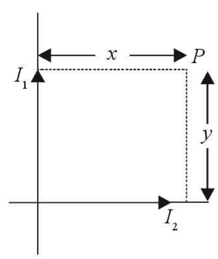


Fig. 23.47 (b)

33. A long solenoid has magnetic field strength of 3.14×10^{-2} T inside it when a current of 5A passes through it. The number of turns in 1 m of the solenoid is

(a) 1000 (b) 3000

(c) 5000 (d) 10000

Solution (c) $n = \frac{B}{\mu_0 I} = \frac{3.14 \times 10^{-2}}{4\pi \times 10^{-7} \times 5}$

$$= \frac{10^5}{20} = 5000.$$

34. A particle of mass m and charge q is projected into a region having a perpendicular magnetic field B. Find the angle of deviation as it comes out of the magnetic field if the width d of the region is very slightly less than $\frac{mv}{2qB}$

(a) 30° (b) 60°

(c) 90° (d) 45°

Solution (a) $\frac{mv\sin\theta}{qB} = \frac{mv}{2qB}$

i.e. $\sin\theta = \frac{1}{2}$ or $\theta = 30°$

35. Two metal strips each of length l as shown are kept b apart and connected to a battery of emf ε through a resistance R. A wire of mass m lies on it. Metal strips are smooth but floor has coeff of friction μ. Find how far the wire will land after leaving the metal strips after the switch is made ON in Fig. 23.48.

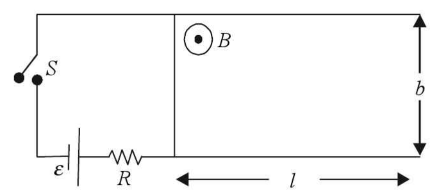


Fig. 23.48

(a) $\frac{\varepsilon b^2 B}{\mu R g m}$ (b) $\frac{\varepsilon l^2 B}{\mu R m g}$

(c) $\frac{\varepsilon l b B}{\mu R m g}$ (d) none of these

Solution (c) $I = \frac{\varepsilon}{R}$ $F = Ilb$ and $a = \frac{IlB}{m} = \frac{\varepsilon l B}{Rm}$

$V^2 = 2al = 2\mu g x$ if x is the distance moved on floor

or $$x = \frac{\varepsilon b B l}{\mu R m g}$$

TYPICAL PROBLEMS

36. A thin disc (or dielectric) having radius r and charge q distributed uniformly over the disc is rotated n rotations per second about its axis. Find the magnetic field at the centre of the disc.

(a) $\frac{\mu_0 q n}{a}$ (b) $\frac{\mu_0 q n}{2a}$

(c) $\frac{\mu_0 q n}{4a}$ (d) $\frac{3\mu_0 q n}{4a}$

Solution (a) Surface charge density $\sigma = \dfrac{q}{\pi a^2}$

Charge on the hypothetical ring $= \dfrac{q}{\pi a^2} 2\pi x dx$

$$dI = \frac{q}{T} = \frac{q}{1/n} = nq$$

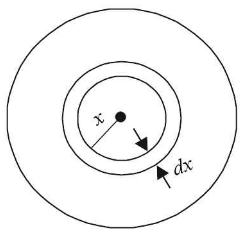


Fig. 23.49

Magnetic field due to the element

$$dB = \frac{\mu_0 dI}{2x} = \frac{\mu_0 2xdxqn}{a^2(2x)} = \frac{\mu_0 qndx}{a^2}$$

$$B = \int dB = \frac{\mu_0 qn}{a^2}\int_0^a dx = \frac{\mu_0 qn}{a^2}[x]_0^a = \frac{\mu_0 qn}{a}$$

37. Find the magnetic field intensity due to a thin wire carrying current I in the Fig. 23.50 (a)

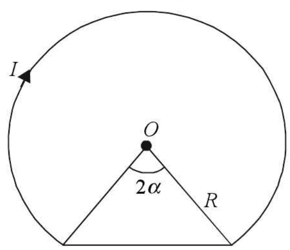


Fig. 23.50 (a)

(a) $\dfrac{\mu_0 i}{2\pi R}(\pi - \alpha + \tan\alpha)$ (b) $\dfrac{\mu_0 i}{2\pi R}(\pi - \alpha)$

(c) $\dfrac{\mu_0 i}{2\pi R}(\pi + \alpha)$ (d) $\dfrac{\mu_0}{2\pi R}(\pi + \alpha - \tan\alpha)$

Solution (a) $B_{\text{arc}} = \dfrac{\mu_0 I}{4\pi R}(2\pi - 2\alpha)$,

$$B_{\text{line}} = \frac{\mu_0 I(\sin\alpha + \sin\alpha)}{4\pi R\cos\alpha}$$

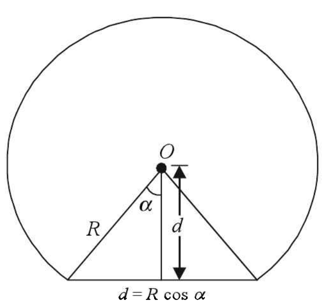


Fig. 23.50 (b)

$$B_{\text{net}} = \frac{\mu_0 I}{2\pi R}(\pi - \alpha + \tan\alpha).$$

38. Electrons emitted with negligible speed from an electron gun are accelerated through a potential difference V_0 along the x-axis. These electrons emerge from a narrow hole into a uniform magnetic field of strength B directed along x-axis, Some electrons emerging at slightly divergent angles as shown in the Fig. 23.51. These paraxial electrons are refocused on the x-axis at a distance

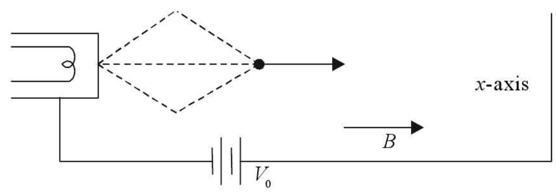


Fig. 23.51

(a) $\sqrt{\dfrac{8\pi^2 mV}{eB^2}}$ (b) $\sqrt{\dfrac{2\pi^2 mV}{eB}}$

(c) $\sqrt{\dfrac{4\pi^2 mV}{eB^2}}$ (d) $\sqrt{\dfrac{2\pi^2 mV}{eB^2}}$

Solution (a) $KE = \dfrac{1}{2}mv^2 \neq eV$ or $mv = \sqrt{2emV}$

The electron will be refocussed after travelling a distance = pitch of helix pitch

$$= \frac{2\pi mV}{qB} = \sqrt{\frac{4\pi^2 \times 2emV}{e^2B^2}} = \sqrt{\frac{8\pi^2 mV}{eB^2}}$$

39. The length of conductor ab carrying current I_2 is l. Find the force acting on it due to a long current carrying conductor as shown in Fig. 23.52 (a). The mid-point of wire ab is distance x apart from long wire.

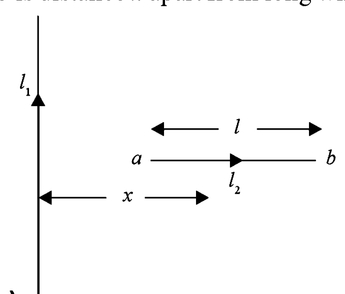


Fig. 23.52 (a)

Solution Consider a small element dy at a distance y from the long conductor. Force on this element

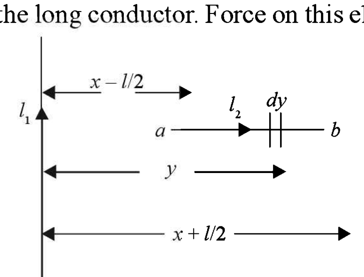


Fig. 23.52 (b)

$$dF = \frac{\mu_0 I dy}{2\pi y}$$

$$F = \frac{\mu_0 I}{2\pi}\int_{x-l/2}^{x+l/2}\frac{dy}{y} = \frac{\mu_0 I}{2\pi}\log_e\frac{x+l/2}{x-l/2}.$$

40. Find the magnetic field intensity at a point O. Assume linear parts to be long and the curved part has the radius R.

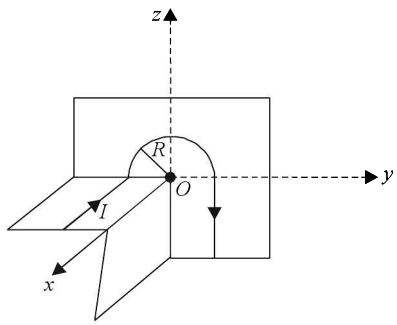


Fig. 23.53

Solution $B_1 = \dfrac{\mu_0 I}{4\pi R}$ along $-z$-axis due to horizontal part

$B_2 = \dfrac{\mu_0 I}{4R}$ along $-x$-axis due to semicircular part

$B_3 = \dfrac{\mu_0 I}{4\pi R}$ along $-x$-axis due to the vertical part

$B = B_1 + B_2 + B_3$

$= \dfrac{\mu_0 I(-\hat{k})}{4\pi R} + \dfrac{\mu_0 I(-\hat{i})}{4R} + \dfrac{\mu_0 I(-\hat{i})}{4\pi R}.$

41. Find the force acting on the conductor carrying current.

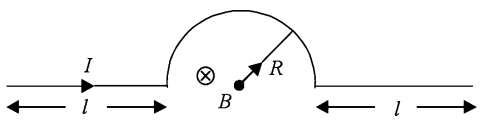


Fig. 23.54 (a)

(a) $\mu_0 I(2l + \mu R)B$ (b) $\mu_0 I(2l + R)B$
(c) $\mu_0 I(2l + 2R)B$ (d) none of these

Solution (c) $F_{net} = F + F + \int_0^{90} 2IF_1 \cos\theta$

$= IlB + IlB + \int_0^{90} 2I\, R\, d\theta \cos\theta$

$= 2IlB + 2IRB$

Short cut $F_{net} = F + F + IB$ (displacement length of curved part) $= IlB + IlB + IB(2R) = 2IlB + 2IRB$.

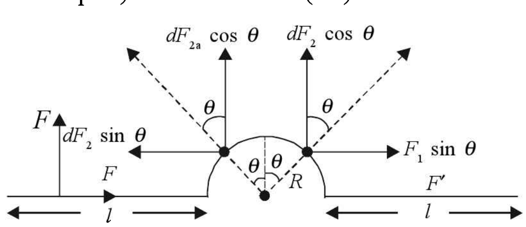


Fig. 23.54 (b)

42. Find the magnetic field strength B of an infinite plane carrying a current of linear density J (same at all points).

[Olympaid 1994]

Solution

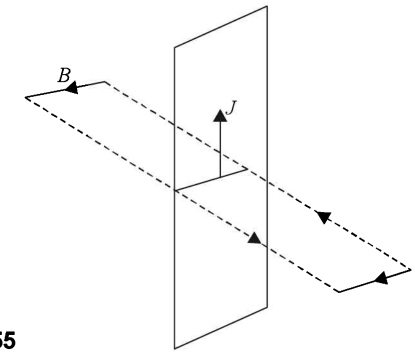


Fig. 23.55

$\oint B \cdot dl = \mu_0 J(l)\, B(2l) = \mu_0 Jl$ or $B = \dfrac{\mu_0 J}{2}$.

The magnetic field intensity is horizontal and parallel to the plane.

43. Two parallel long conductors carrying current I_1 and I_2 are shown in Fig. 23.56. Assuming magnetic field to be positive pointing for up the plane of paper and $I_1 = I_2$, which of the following graphs best represents the conditions?

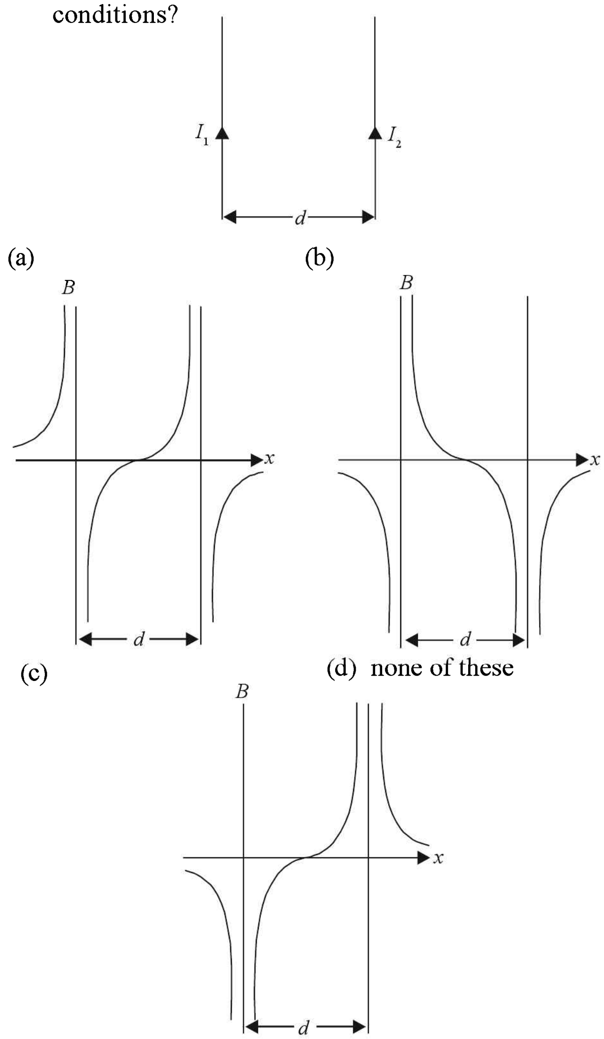


Fig. 23.56

[IIT Screening 2001]

Solution (a) Use $B = \dfrac{\mu_0 I}{4\pi d}$ and keep direction in mind.

QUESTIONS FOR PRACTICE

1. A 0.5 m long straight wire in which a current of 3.2 A is flowing is kept at right angle to a uniform magnetic field of 2.0 Tesla. The force acting on the wire will be
   (a) 2 N (b) 2.4 N
   (c) 1.2 N (d) 3

2. The radius of each coil of a Helmholtz galvanometer is 0.1 m and number of turns in each is 25. When a current is passed in it then the deflection of magnetic needle observed as 45°. If the horizontal component of earth's magnetic field is 0.314×10^{-4} Tesla then the value of current will be
   (a) 0.14 A (b) 0.28 A
   (c) 0.42 A (d) 0.07 A

3. An electron is revolving in a circular path of radius 2.0×10^{-10} m with a uniform speed of 3×10^{6} m/s. The magnetic induction at the centre of the circular path will be
   (a) 0.6 Tesla (b) 1.2 Tesla
   (c) 0.12 Tesla (d) zero

4. Two parallel straight conductors, in which current is flowing in the same direction, attract each other. The cause of it is
   (a) magnetic force between the two
   (b) electric force between the two
   (c) potential difference between the two
   (d) mutual induction between the two

5. A rectangular loop, carrying current i, is lying near a long straight conductor PQ as shown in the figure in such away that the wire is parallel to one of the sides of the loop and is in the plane of the loop. If constant current I is passed in the wire then the loop will

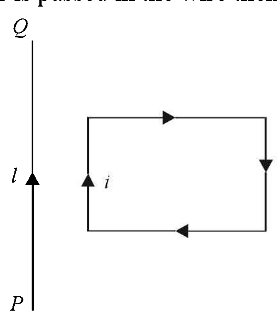


Fig. 23.57

   (a) move towards the wire
   (b) move away from the wire
   (c) remain stationary
   (d) rotate about an axis parallel to the wire

6. The distance between two thin long straight parallel conducing wires is b. On passing the same current i in them, the force per unit length between them will be
   (a) $\frac{\mu_0 i}{2\pi b}$ (b) $\frac{\mu_0 i^2}{2\pi}$
   (c) $\frac{\mu_0 i^2}{2\pi b}$ (d) zero

7. The wall of a straight tube of infinite length is thin. On passing current i through it, the value of magnetic induction inside the tube will be
   (a) $\frac{2i}{r}$ (b) $\frac{2i\mu_0}{r}$
   (c) $\frac{2r}{i}$ (d) Zero

8. A proton and an electron, with same momenta, enter a magnetic field in a direction at right angles to the lines of force. If the radii of their circular paths arc rp and re respectively then the value of rp:re will be
   (a) 1 : 1 (b) 1 : 2
   (c) 2 : 1 (d) 4 : 1

9. A magnetic needle placed in a non-uniform magnetic field experiences
   (a) only force (b) force and torque
   (c) only torque
   (d) neither force nor torque

10. A current i is flowing in a specific wire. It is turned into a circular coil of one turn. Then it is turned to make a coil of two turns and smaller radius. Now the magnetic induction at the centre for same current will be
    (a) half of its previous value
    (b) one fourth of its previous value
    (c) four times of its previous value
    (d) zero

11. Uniform electric and magnetic fields are directed along X-direction. An electron is projected in X-direction with a velocity v, then
    (a) magnitude of velocity of electron will increase
    (b) magnitude of velocity of electron will decrease
    (c) electron will turn towards right
    (d) electron will turn towards left

12. The force between two parallel conductors, each of length 50 m and distant 20 cm apart, is 1 newton. If the current in one conductor is double that in another one, then their values will respectively be

(a) 100 A and 200 A (b) 50 A and 400 A
(c) 10 A and 30 A (d) 5 A and 25 A

13. The magnetic induction due to a straight current-carrying conductor of infinite length at a distance d from it will be

(a) $\frac{\mu_0 i}{2d}$ (b) $\frac{\mu_0 i}{2\pi d}$

(c) zero (d) $\frac{\mu_0 i}{4\pi d}$

14. On applying a uniform magnetic field on a current-carrying coil the coil rotates in such a way that its plane

(a) becomes perpendicular to magnetic field
(b) becomes parallel to magnetic field
(c) makes an angle of 45° with the magnetic field
(d) makes any angle with the magnetic field

15. Which of the following quantities is not affected by a magnetic field?

(a) Stationary charge
(b) Moving charge
(c) Change in magnetic flux
(d) Current flowing in a conductor

16. The magnetic field inside a solenoid is

(a) infinite (b) zero
(c) uniform (d) non-uniform

17. A current of 10 A is flowing in a wire of length 1.5 m. When it is placed in a uniform magnetic field of 2 Tesla then a force of 15 N acts on it. The angle between the magnetic field and the direction of current flow will be

(a) 30° (b) 45°
(c) 60° (d) 90°

18. A wire is lying parallel to a square coil. Same current is flowing in same direction in both of them. The magnetic induction at any point P inside the coil will be

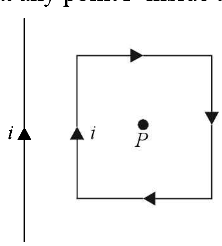


Fig. 23.58

(a) zero
(b) more than that produced by only coil
(c) less than that produced by only coil
(d) equal to that produced by only coil.

19. If the currents in two straight current-carrying conductors, disatant d apart, are i_1 and i_2 respectively in the same direction then they will

(a) rotate about a central axis
(b) attract each other
(c) repel each other
(d) neither attract nor repel each other

20. The correct curve between the magnetic induction (B) along the axis of a long solenoid due to current flow i in it and distance x from one end is

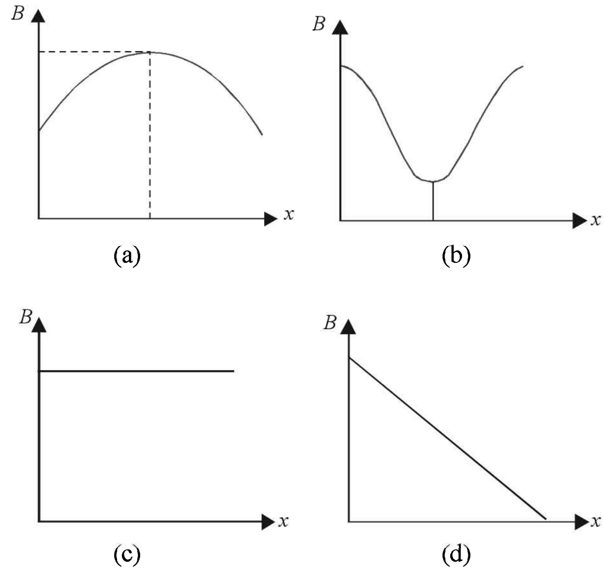


Fig. 23.59

21. Gauss is the unit of

(a) B (b) H
(c) M (d) 1

22. The correct expression for Lorentz force is

(a) $q[\vec{E}+(\vec{B}\times\vec{V})]$ (b) $q[\vec{E}+(\vec{V}\times\vec{B})]$

(c) $q(\vec{V}\times\vec{B})$ (d) $q\vec{E}$

23. The correct relation between B and M for a small current-carrying coil is

(a) $B=\frac{\mu_0 M}{2x^3}$ (b) $B=\frac{\mu_0 M}{x^3}$

(c) $B=\frac{\mu_0 M}{\pi x^3}$ (d) $B=\frac{\mu_0 M}{2\pi x^3}$

24. A proton, a deutron and an a-particle are moving with same momentum in a uniform magnetic field. The ratio of magnetic forces acting on them will be

(a) 1 : 1 : 2 (b) 1 : 2 : 3
(c) 2 : 1 : 1 (d) 1 : 1 : 1

25. An a- particle, a deutron and a proton are moving with same momentum in a uniform magnetic field. The ratio of their speeds will be

(a) 1 : 2 : 4 (b) 4 : 2 : 1
(c) 1 : 1 : 1 (d) 2 : 2 : 4

26. The value of B, at the point of inflexion in B-x curve is

(a) maximum (b) positive

(c) constant (d) negative

27. The resultant force on the current loop $PQRS$ due to a long current-carrying conductor will be, if the current flow in the loop is clockwise,

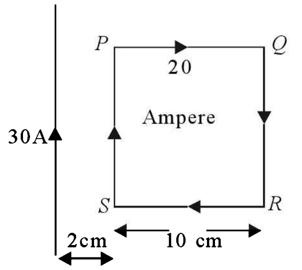


Fig. 23.60

(a) zero (b) $0.36 \times 10^{-3}\,N$

(c) $1.8 \times 10^{-3}\,N$ (d) $0.5 \times 10^{-3}\,N$

28. A small linear segment of an electric circuit is lying on x-axis extending from $x = -a/2$ to $x = a/2$ and a current i is flowing in it. The magnetic induction due to the segment at a point $x = a$ will be

(a) $\propto a$ (b) Zero

(c) $\propto a^2$ (d) $\alpha \frac{1}{a}$

29. The rays, which remain undeflected in a magnetic field, are

(a) $\propto$-rays (b) β-rays

(c) γ-rays (d) positive rays.

30. A current i is flowing in a straight conductor of length L. The magnetic induction at a point distant from its centre will be

(a) $\frac{4\mu_0 i}{\sqrt{5}\pi L}$ (b) $\frac{\mu_0 i}{\sqrt{2}L}$

(c) $\frac{\mu_0 i}{2\pi L}$ (d) zero

31. A current-carrying circular coil of magnetic moment M is situated in a magnetic field B. The work done in deflecting it from an angle 0^0 to θ^0 will be

(a) MB (b) $MB\,(1 - \cos\theta)$

(c) $-MB$ (d) $MB\,(1 - \sin\theta)$

32. Same current i is flowing in two straight parallel conducting wires situated a distance d apart. The magnetic induction at the centre between two wires will be

(a) zero (b) $\frac{\mu_0 i}{d}$

(c) $\frac{4\mu_0 i}{d}$ (d) $\frac{\mu_0 i}{2d}$

33. A uniform magnetic field B and a uniform electric field E act in a common region. An electron is entering this region of space. The correct arrangement for it to escape undeviated is

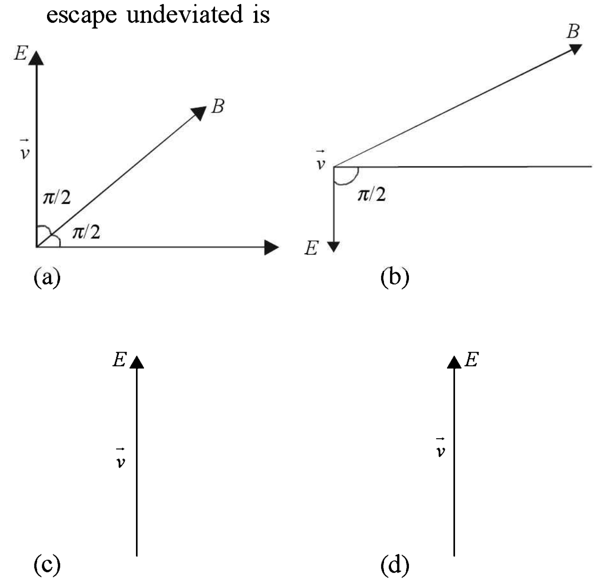


Fig. 23.61

34. A current of 30 amp is flowing in a conductor as shown in the figure. The magnetic induction at point O will be

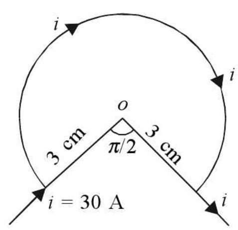


Fig. 23.62

(a) 1.5 Tesla (b) 4.71×10^{-4} Tesla

(c) zero (d) 0.15 Tesla

35. A current is flowing in a hexagonal coil of side a. The magnetic induction at the centre of the coil will be

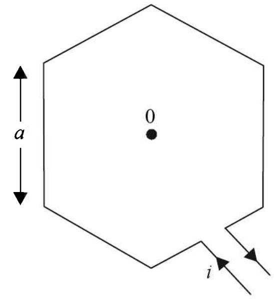


Fig. 23.63

(a) $\frac{3\sqrt{3}\mu_0 i}{\pi a}$ (b) $\frac{\mu_0 i}{3\sqrt{3}\pi a}$

(c) $\frac{\mu_0 i}{\sqrt{3}\pi a}$ (d) $\frac{\sqrt{3}\mu_0 i}{\pi a}$

36. A current i is flowing in an octagonal coil of side a. The magnetic induction at the centre of the coil will be

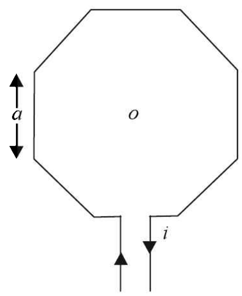


Fig. 23.64

(a) $\frac{5\mu_0 i}{4\pi a}$ (b) $\frac{5\sqrt{2}\mu_0 i}{\pi a}$

(c) $\frac{\mu_0 i}{\sqrt{5}\pi a}$ (d) $\frac{\sqrt{5}\mu_0 i}{2\pi a}$

37. Two similar coils of radius R and number of turns N are lying concentrically with their planes at right angles to each other. The currents flowing in them are I and $I\sqrt{3}$ respectively. Then the resultant magnetic induction at the centre will be (in Wb/m^2).

(a) $\frac{\mu_0 NI}{2R}$ (b) $\frac{\mu_0 NI}{R}$

(c) $\sqrt{3}\mu_0 \frac{NI}{2R}$ (d) $\sqrt{5}\frac{\mu_0 NI}{2R}$

38. A current of $10^{-3} A$ is flowing in a resistance of 1000 Ω. To measure potential difference accurately, a voltmeter should be used whose resistance is

(a) 0 Ω (b) 500 Ω

(c) 1000 Ω (d) >>1000 Ω

39. A galvanometer with resistance 100 Ω gives full scale deflection with a current of 10 mA. The value of shunt, in order to convert it into an ammeter of 10 ampere range, will be

(a) –10 Ω (b) 1 Ω

(c) 0.1 Ω (d) 0.01 Ω

40. An ammeter gives full scale deflection with a current of 1 amp. It is converted into an ammeter of range 10 amp. The ratio of the resistance of ammeter to the shunt resistance used will be

(a) 1 : 9 (b) 1 : 10

(c) 1 : 11 (d) 9 : 1

41. The value of shunt resistance, in order to pass 10% of the main current in the galvanometer of resistance 99 Ω, will be

(a) 9.9 Ω (b) 10 Ω

(c) 11 Ω (d) 9 Ω

42. A galvanomenter with resistance 5 Ω can read upto 5 mA. If this instrument is to be used to read upto 100 volt, then the value of resistance to be used in its series will be

(a) 19.9995 Ω (b) 199.995 Ω

(c) 1999.95 Ω (d) 19995 Ω

43. An ammeter of resistance 0.2 Ω and range 10 mA is to be used to read potential difference upto 1 volt. It will have to be connected to

(a) 99.8 W resistance in series

(b) 99.8 W resistance in parallel

(c) 0.1 W resistance in parallel

(d) 0.1 W resistance in series

44. The proper resistance to be connected in series with a voltmeter, in order to increase its range 10 times, will be

(a) nine times the resistance of voltmeter

(b) ten times the resistance of voltmeter

(c) eleven times the resistance of voltmeter

(d) one-tenth the resistance of voltmeter

45. The resistance required to be connected in parallel to an ammeter in order to increase its range 10 times, will be

(a) one-tenth the resistance of ammeter

(b) nine times the resistance of ammeter

(c) ten times the resistance of ammeter

(d) one-ninth the resistance of ammeter

46. A galvanometer of resistance 501 W gives full scale deflection with a current of 0.5 m A. The value of resistance to be connected in series with it, in order to convert it into a voltmeter of range 10 volt, will be

(a) 1,995 Ω (b) 2,000 Ω

(c) 19,950 Ω (d) 20,000 Ω

47. If only 1% of main current is to be passed through a galvanometer of resistance G, then the value of shunt resistance will be

(a) $\frac{G}{50}$ (b) $\frac{G}{49}$

(c) $\frac{G}{99}$ (d) 99G

48. A current of 10^{-7} ampere produces 50 division deflection in a galvanometer. Then its figure of merit will be

(a) 10^{-4} amp/div (b) 10^{-8} amp/div

(c) 10^{-10} amp/div (d) 2×10^{-9} amp/div

49. A voltmeter of 1000 Ω can read potential difference of 1.5 volt. What resistance will have to be connected in series with it, in order to measure potential difference upto 6 volt with the help of this voltmeter?

(a) 3000 Ω (b) 500 Ω

(c) 1,000 Ω (d) 10,000 Ω

50. The figures of merit of two galvanometers, whose resistances are 100 Ω and 20 Ω respectively, are 1 ′ 10^{-8} amp/div and 2 ′ 10^{-5} amp/div respectively. The galvanometer, whose voltage sensitivity is more, is

(a) nothing can be predicted (b) second
(c) both (d) first.

51. A resistance of 900 Ω is connected in series with a galvanometer of resistance 100 Ω. A potential difference of 1 volt produces 300 division deflection in the galvanometer. The value of figure of merit will be

(a) 10^{-2} A/div (b) 10^{-3} A/div
(c) 10^{-4} A/div (d) 10^{-5} A/div

52. A proton, a deutron and an α-particle are accelerated through the same potential difference and then they enter a uniform normal magnetic field. If the radius of circular path of proton is 8 cm then the radius of circular path of deutron will be

(a) 11.31 cm (b) 22 cm
(c) 5 cm (d) 2.5 cm

53. A proton and an α-particle enter a uniform magnetic field at right angles to it with same velocity. The time period of α particle as compared to that of proton, will be

(a) four times (b) two times
(c) half (d) one-fourth

54. A charged particle with charge q is moving in a uniform magnetic field. If this particle makes any angle with the magnetic field then its path will be

(a) circular (b) straight line
(c) helical (d) parabolic

55. A proton is moving with a velocity of 3×10^7 m/s in the direction of a uniform magnetic field of 0.5 Tesla. The force acting on proton is

(a) 2 N (b) 4 N
(c) 6 N (d) zero

56. The work done by a normal magnetic field in revolving a charged particle q in a circular path will be

(a) zero (b) $MB\,(1-\cos\theta)$
(c) MB (d) $-MB$

57. An electron is moving vertically downwards at any place. The direction of magnetic force acting on it due to horizontal component of earth's magnetic field will be

(a) towards east (b) towards west
(c) towards north (d) towards south

58. A positive charge is moving towards an observer. The direction of magnetic induction will be

(a) clockwise (b) anticlockwise
(c) towards right (d) towards left

59. Two parallel wires P and Q carry electric currents of 10 A and 2 A respectively in mutually opposite directions. The distance between the wires is 10 cm. If the wire P is of infinite length and wire Q is 2 m long, then the force acting Q will be

(a) $4 \times 10^{-5}\,N$ (b) 8×10^{-5} N
(c) $4 \times 10^{5}\,N$ (d) 0 N

60. A proton with kinetic energy 8 eV is moving in a uniform magnetic field. The kinetic energy of a deutron moving in the same path in the same magnetic field will be

(a) 2 eV (b) 4 eV
(c) 6 eV (d) 8 eV

61. Two wires carry currents of 100 A and 200 A respectively and they repel each other with a force of 0.4 N/m. The distance between them will be

(a) 1 m (b) 1 cm
(c) 50 cm (d) 25 cm

62. A current of 2 A is flowing in a wire of length 50 cm. If this wire is lying in a uniform magnetic field of 5×10^{-4} N/A—m making an angle of 60° with the field, then the force acting on the wire will be

(a) $4.33 \times 10^{-4}\,N$ (b) 4 N
(c) 4 dyne (d) Zero

63. In the following figure, three paths of a particle crossing a nucleus N are shown. The correct path is

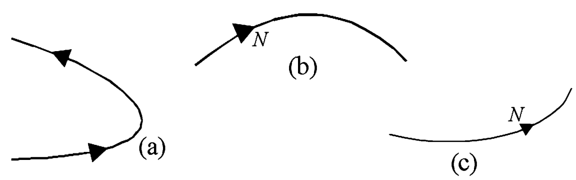


Fig. 23.65

(a) a and c (b) a and b
(c) a, b, and c (d) only a

64. The ratio of magnetic force (Fm) and electric force (Fe) acting on a moving charge is

(a) $\left(\frac{V}{C}\right)^2$ (b) $\left(\frac{C}{V}\right)^2$
(c) $\frac{V}{C}$ (d) $\frac{C}{V}$

65. A charge of 0.04 coulomb is moving in a magnetic field of 0.02 Tesla with a velocity 10m/s in a direction making an angle 30° with the direction of field. The force acting on it will be

(a) $4 \times 10^{-3}\,N$ (b) $2 \times 10^{-3}\,N$
(c) zero (d) $8 \times 10^{-3}\,N$

66. An electron is moving in a perpendicular magnetic field of strength 4×10^{-3} Tesla with a velocity of 4×10^{7} m/s. The radius of electron path will be

(a) 0.056 m (b) 0.056 m

(c) 56 m (d) 5.6 m

67. The magnetic moment of an electron with orbital angular momentum J will be

(a) $\frac{e\vec{J}}{m}$ (b) $\frac{e\vec{J}}{2m}$

(c) $\frac{2m}{e\vec{J}}$ (d) zero

68. The correct statement about magnetic moment is:

(a) It is a vector quantity. (b) Its unit is amp-m^2.

(c) Its dimensions are AL^2. (d) All of the above.

69. The use of Helmholtz coils is to produce

(a) uniform magnetic field

(b) non-uniform magnetic field

(c) varying magnetic field

(d) zero magnetic field.

70. The magnetic induction due to a straight current-carrying conductor of infinite length at a distance r from it proportional to

(a) i^{-1} (b) i

(c) i^{-2} (d) i^2

71. If a load is suspended from a spring and a direct current is passed through it then the spring gets

(a) stretched

(b) compressed

(c) sometimes stretched and sometimes compressed

(d) neither stretched nor compressed

72. The correct expression for Ampere's law is

(a) $\oint B.dl = \Sigma i$ (b) $\oint B.dl = \frac{1}{\Sigma i}$

(c) $\oint B.dl = \mu_0 \Sigma i$ (d) $\oint B.dl = \frac{\Sigma i}{\mu_0}$

73. The magnitude of magnetic induction for a current-carrying toroid of uniform cross-section is

(a) uniform over the whole cross-section

(b) maximum on the outer edge

(c) maximum on the inner edge

(d) maximum at the centre of cross-section

74. If a positively charged particle is moving as shown in the figure, then it will get deflected due to magnetic field towards

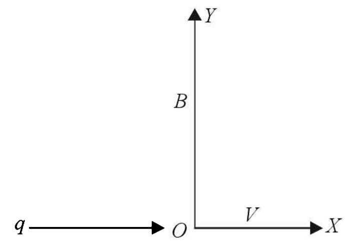


Fig. 23.66

(a) $+x$ direction (b) $+y$ direction

(b) $-x$ direction (d) $+z$ direction

75. A current-carrying loop lying in a magnetic field behaves like a

(a) magnetic dipole (b) magnetic pole

(b) magnetic material (d) non-magnetic material

76. The ratio of magnetic induction due to a bar magnet on its axial point and equatorial point will be

(a) 1 : 1 (b) 1 : 2

(c) 2 : 1 (d) 1 : 4

77. Two insulated wires of infinite length are lying mutually at right angles to each other as shown in the figure. Current of 2 A and 1.5 A respectively are flowing in them. The value of magnetic induction at point P will be

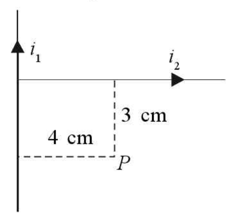


Fig. 23.67

(a) 2×10^{-3} N/A-m (b) 2×10^{-5} N/A-m

(c) Zero (d) 2×10^{-4} N/A-m

78. Two current-carrying parallel conductors are shown in the figure. The magnitude and nature of force acting between them per unit length will be

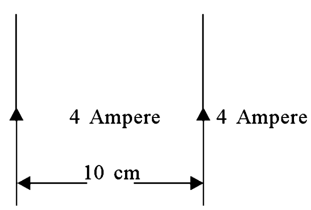


Fig. 23.68

(a) 8×10^{-8} N/m, attractive

(b) 3.2×10^{-5} N/m, repulsive

(c) 3.2×10^{-5} N/m, attractive

(d) 8×10^{-8} N/m, repulsive

Answers to Questions for Practice

1.	(c)	**2.**	(a)	**3.**	(b)	**4.**	(a)	**5.**	(a)	**6.**	(c)	**7.**	(d)
8.	(a)	**9.**	(b)	**10.**	(c)	**11.**	(b)	**12.**	(a)	**13.**	(b)	**14.**	(a)
15.	(a)	**16.**	(c)	**17.**	(a)	**18.**	(b)	**19.**	(b)	**20.**	(a)	**21.**	(a)
22.	(b)	**23.**	(a)	**24.**	(c)	**25.**	(a)	**26.**	(c)	**27.**	(a)	**28.**	(b)
29.	(c)	**30.**	(a)0	**31.**	(b)	**32.**	(a)	**33.**	(a)	**34.**	(b)	**35.**	(d)
36.	(a)	**37.**	(b)	**38.**	(d)	**39.**	(c)	**40.**	(d)	**41.**	(c)	**42.**	(d)
43.	(a)	**44.**	(a)	**45.**	(d)	**46.**	(c)	**47.**	(c)	**48.**	(d)	**49.**	(a)
50.	(d)	**51.**	(d)	**52.**	(a)	**53.**	(b)	**54.**	(c)	**55.**	(d)	**56.**	(a)
57.	(b)	**58.**	(b)	**59.**	(b)	**60.**	(b)	**61.**	(b)	**62.**	(a)	**63.**	(a)
64.	(a)	**65.**	(a)	**66.**	(b)	**67.**	(b)	**68.**	(d)	**69.**	(a)	**70.**	(b)
71.	(b)	**72.**	(c)	**73.**	(a)	**74.**	(d)	**75.**	(a)	**76.**	(c)	**77.**	(b)
78.	(c)												

24

Permanent Magnets

BRIEF REVIEW

Magnetic Dipole Moment (M) Magnetic dipole moment is defined as the product of pole strength (m) and length of the magnet (l) i, e,. $M = ml$ as illustrated in Fig. 24.1 Pole strength may sometimes be called magnetic charge. Magnetic poles are of two types N–pole and S–pole.

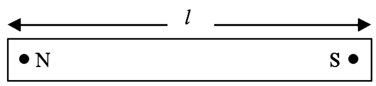


Fig. 24.1 Magnetic dipole

Also $M = IA$ where I is current and A is area. Note that unit of pole strength is Am while that of magnetic dipole moment is Am^2.

If l_g and l_m are geometric and magnetic lengths respectively then $\frac{l_m}{l_g}$ = 5/6 as illustrated in Fig. 24.2.

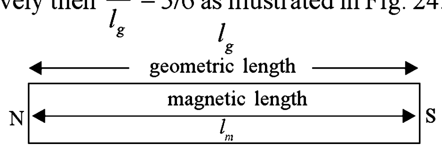


Fig. 24.2 Magnetic length

$\oint B.ds = 0 = \vec{\nabla}.\vec{B}$ and rejects the existence of monopole

Magnetic force $F = \frac{\mu_0 m_1 m_2}{4\pi x^2}$

Magnetic Field Strength (Magnetic induction)

$$\vec{B} = \frac{F}{m} = \frac{\mu_0 m}{4\pi x^2}$$ We define this using single pole

$$B = [M\,L^0\,T^{-2}\,A^{-1}]$$

Magnetic field due to a bar magnet along axial line (end-on position) at any point P

$$B = B_N - B_S$$ at

$$B = \frac{2\mu_0 M d}{4\pi(d^2 - l^2)^2}$$

$$B = \frac{2\mu_0 M}{4\pi d^2}$$ due to a short magnet.

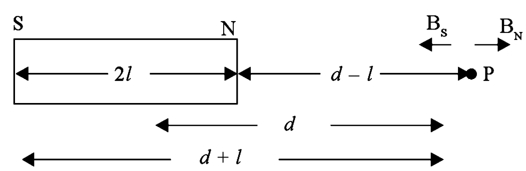


Fig. 24.3

Note that the direction of magnetic field is the one experienced by a unit north pole at that point i.e. unit north pole is taken as a standard test pole.

Magnetic field due to a bar magnet along equatorial line or on broadside on position

$$B = B_N \cos\theta + B_S \cos\theta$$
$$= 2\,B_N \cos\theta$$

$$B = \frac{\mu_0 M}{4\pi(d^2 + l^2)^{3/2}}$$ (along NS)

$$B = \frac{\mu_0 M}{4\pi d^3}$$ (along NS) due to a short bar magnet.

Unit of magnetic field strength or Magnetic induction (B) is Tesla (T) or $Wb\,m^{-2}$ which are SI units. Gauss is the CGS unit.

$$1\,\text{Gauss} = 10^{-4}\,T$$

Magnetic Potential (V)

$$B = -\frac{dV}{dx} \text{ or } V = -\int_{-\infty}^{r} B.dx = \frac{\mu_0 m}{4\pi r}$$

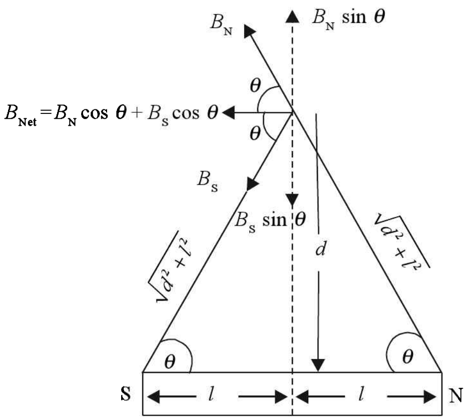


Fig. 24.4 Magnetic field due to a bar magnet on broadside on position

Magnetic Potential at any point due to a bar magnet

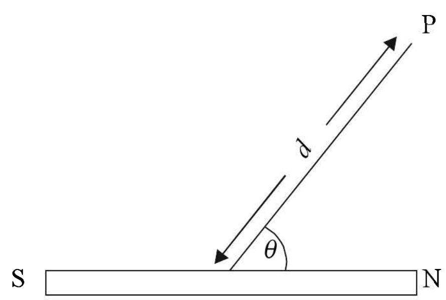


Fig. 24.5 Magnetic potential at any point

$$V = \frac{\mu_0 m\cos\theta}{4\pi(d^2 - l^2\cos^2\theta)}$$

$$V = \frac{\mu_0 M\cos\theta}{4\pi d^2} \text{ due to a short bar magnet.}$$

Special Cases

Along end on position $\theta = 0$, $V = \frac{\mu_0 M}{4\pi d^2}$ $\cos\theta = 1$

Along equatorial line or broad side on position $V = 0$

Magnetic field strength due to a bar magnet at any point P

$$B = \frac{\mu_0 M}{4\pi d^3}\sqrt{1+3\cos^2\theta}$$

$$\tan\beta = \frac{\tan\theta}{2}$$

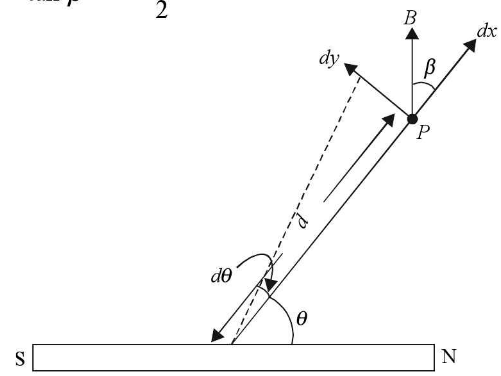


Fig. 24.6 Magnetic field at any point due to dipole

Magnetic Field Lines Magnetic field lines make a closed loop. They start from N-pole and end at S-pole outside the magnet, and S-pole to N-pole inside the magnet as illustrated in Fig. 24.7

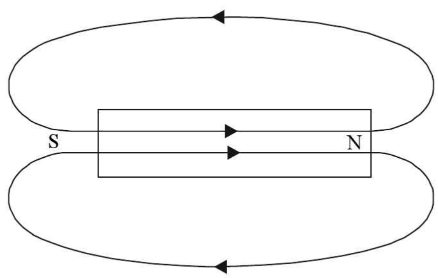


Fig. 24.7 Magnetic field lines

Magnetising Field $H = \frac{B}{\mu_o}$ (in vacuum). Its dimensional formula is $[M^0 L^{-1} T^0 A]$. The electric current enclosed in a closed path of unit length in a magnetic field is defined as magnetising field H, i.e.

$$H = \frac{I}{L}, \text{ i.e., when } L = 1m, H = I$$

Magnetic Flux Density or Magnetic Induction B

$$B = \frac{\phi}{A} \text{ or}$$

$$B = \phi \text{ if } A = 1\,m^2$$

Magnetic field line passing through unit normal area in a magnetic field is defined as magnetic induction. The direction in which a current-carrying conductor in a magnetic field experiences no force is the direction of magnetic induction.

Magnetic Moment of an electron due to its orbital motion Magnetic moment of electron is due to orbital angular momentum and spin angular momentum.

Orbital magnetic moment $\vec{M}_L = -\frac{e}{2m}\vec{L}$

$$= -n\left[\frac{eh}{4\pi m}\right]$$

$$\therefore \quad L = n\frac{h}{2\pi}$$

$$\mu_B = \frac{eh}{4\pi m}$$ is called Bohr magnetron.

$$\mu_B = 0.93 \times 10^{-23}\ \text{Am}^2$$

Spin magnetic moment $\vec{M}_S = -\frac{e}{m}\vec{S}$

i.e. $M_S = 2M_L$

Thus total magnetic moment

$$M_{Tot} = \vec{M}_L = \vec{M}_S = -\frac{e}{2m}\left[\vec{L} + 2\vec{S}\right]$$

The coefficient $\frac{e}{2m}$ is called gyro magnetic ratio.

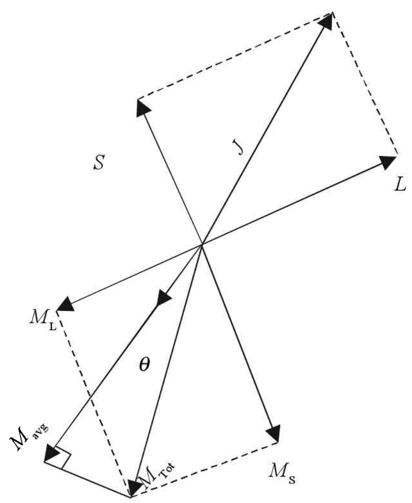


Fig. 24.8 Illustration of spin angular total and average magnetic moment

From Fig. 24.8 $M_{avg} = M_{Tot}.\cos\theta = g\left(\frac{-e}{2m}\right)J$

$$\text{where} \quad g = \frac{j(j+1)+s(s+1)-l(l+1)}{2j\left(j+1\right)}$$

For pure orbital motion $s = 0$, $j = l$ and $g = 1$, for pure spin motion $l = 0$, $j = s$ and $g = 2$. These values agree. $j = |l - s|$ if shell is less than half filled.

and $\quad j = |l + s|$ if shell is more than half filled.

$$\text{Torque } \frac{dL}{dt} = \tau = M \times B$$

$$= -\frac{e}{2m}\ L \times B = \omega_L$$

Thus ω_L represents Larmor's frequency and is given by

$$\omega_L = \frac{eB}{2m}$$

and describe precession as illustrated in Fig. 24.9.

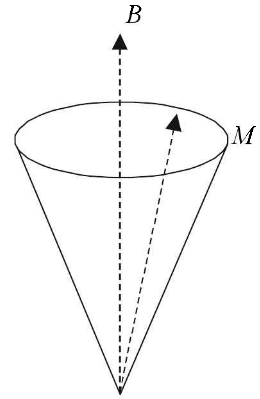


Fig. 24.9

About 90% of the magnetic moment is due to spin motion of electrons and about 10% due to orbital magnetic moment. If the magnet is cut along the length, pole strength decreases i.e. pole strength ∝ Area. Magnetic dipole moment of the earth is 8×10^{22} J/T. The magnetic axis makes 11.5° angle/geographic north with the axis of rotation of the earth. The point where the dipole axis cuts near N–pole and other near S–pole is termed as geomagnetic north pole and geomagnetic south pole respectively as illustrated in Fig. 24.10

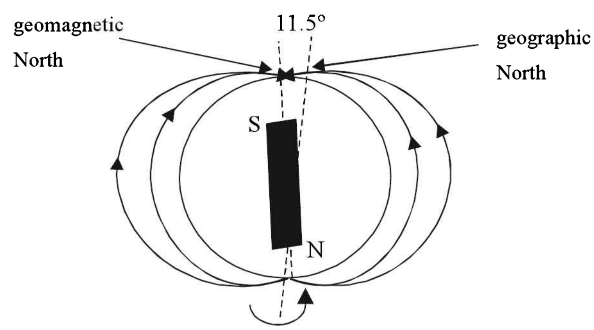


Fig. 24.10 Magnetic field of the earth

Earth's magnetic field changes both in magnitude and direction with passage of time. It is believed that the earth's magnetic field has reversed 171 times in the past 10^7 years. The latest reversal occurred 10,000 years ago.

Element of Earth's Magnetic Field Declination θ, dip (δ) and horizontal component of earth's magnetic field (B_H) are called elements of earth's field.

Declination θ Angle between the geographic meridian and magnetic meridian is called **declination θ**. The knowledge of declination fixes the vertical plane in which earths magnetic field lies as shown in Fig. 24.11.

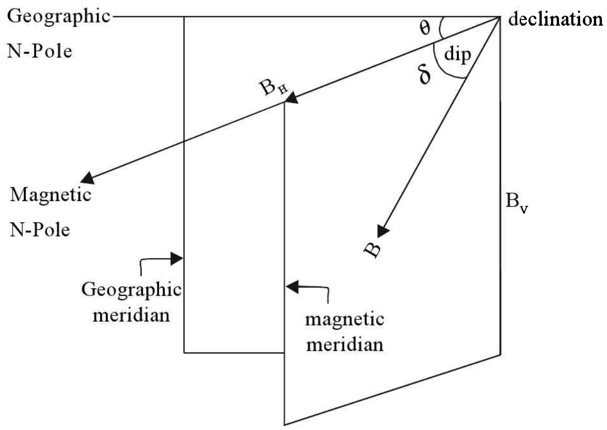


Fig. 24.11 Magnetic meridian, dip and declination

Dip (δ) The angle made by the earth's magnetic field with the horizontal direction in the magnetic meridian is called the **dip** or **inclination**.

In the northern hemisphere N-pole of compass needle (or magnetic needle) dips downwards and in the southern hemisphere S-pole of the compass needle dips downwards.

From Fig. 24.11

$$B_H = B\cos\delta,\ B_v = B\sin\delta \quad \text{and}$$

$$\tan\delta = \frac{B_v}{B_H}$$

Magnetic elements help to understand the location of a place. This is the reason that magnetic maps are used in navigation and aviation.

Isogonic lines are lines that join **same declination** in a magnetic map

Isoclinic lines join the **same dip** in magnetic maps.

Isodynamic lines join the same B_H (horizontal component of the earth's field) in magnetic maps.

A clinic line or magnetic equator is the line joining **zero dip**. At poles dip = 90°

Angle of dip is measured using dip circle. If dip circle is set in magnetic meridian then the angle read by dip circle is angle of dip.

If the dip circle is inclined at an angle θ with magnetic meridian and dip circle reads δ' called apparent angle of dip. True dip δ is then given by $\tan\delta = \tan\delta'\cos\theta$. If the angle θ, the dip circle makes with magnetic meridian is unknown, then rotate the dip circle by 90° after noting apparent dip δ'. At rotated position it reads dip δ'' The true dip δ is given by $\cot^2\delta = \cot^2\delta' + \cot^2\delta''$

Neutral Points The points where magnetic field due to a magnet is equal and opposite to the earth's horizontal field. Compass needle will stay in any direction at these points.

If N-pole of the magnet points to geomagnetic S-pole, the neutral points occur on end-on position or on the axial line.

If N-pole of the magnet points to geomagnetic S-pole, neutral points will lie on broadside on position or equatorial line.

Tangent Law If two fields are perpendicular and one of them is known, then other can be determined.

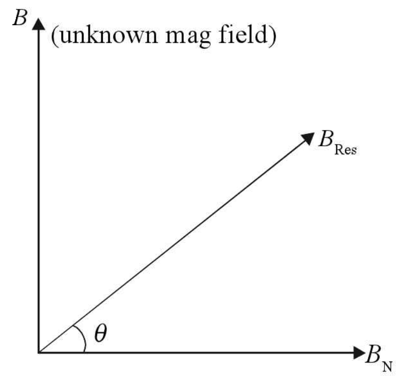


Fig. 24.12 Tangent law

From the Fig. 24.12,

$$B_H = B_{Res}\cos\theta \quad \text{and}$$
$$B = B_{Res}\sin\theta. \quad \text{Therefore}$$
$$B = B_H\tan\theta$$

Tangent Galvanometer It is based on tangent law. When a current I is passed through its coil having n turns

$$I = K\tan\theta \text{ where}$$

$$K = \frac{2rB_H}{\mu_o n}$$ where r is radius of the coil and K is reduction factor. Sensitivity of the tangent galvanometer is maximum if $\theta = 45°$.

Deflection Magnetometer

Tan A position Pointer and arms are along East-West and pointer coincides with zero-zero.

Magnetic needle points North-South as shown in Fig. 24.13(a).

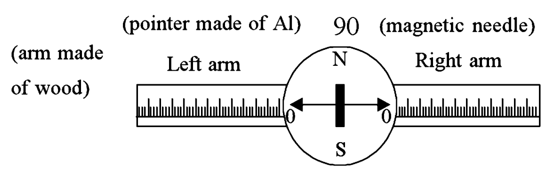


Fig. 24.13(a) Magnetometer set in Tan *A* position

$$\frac{M}{B_H} = \frac{4\pi(d^2 - l^2)^2}{2\mu_o d}\tan\theta$$

If magnetic dipole moment is same and θ is measured at two different places then $\frac{B_{H1}}{B_{H2}} = \frac{\tan\theta_2}{\tan\theta_1}$

If θ is kept same (45°) and two different magnetic dipoles (or bar magnets) are taken then

$$\frac{M_1}{M_2} = \frac{d_2(d_1^2 - l_1^2)^2}{d_1(d_2^2 - l_2^2)^2} \text{ and}$$

$$\frac{M_1}{M_2} = \left(\frac{d_1}{d_2}\right)^3 \text{ for a short dipole}$$

Tan B Position Arms and magnetic needle point North-South and pointer East-West and coincides with 0 – 0.

In tan B position

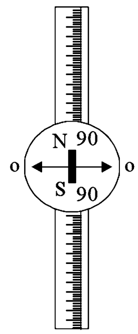


Fig. 24.13(b) Magnetometer set in tan *B* position

$$\frac{M}{B_H} = \frac{4\pi\left(d^2 + l^2\right)^{3/2} \tan\theta}{\mu_0} \quad \text{or}$$

$$\frac{M_1}{M_2} = \left(\frac{d_1^2 + l_1^2}{d_2^2 + l_2^2}\right)^{3/2}$$

$$\frac{M_1}{M_2} = \left(\frac{d_1}{d_2}\right)^3 \text{ For a short magnet}$$

Vibration Magnetometer Time period of oscillation

$$T = 2\pi\sqrt{\frac{I}{MB_H}}$$

If with same magnet time period is measured at two different places then

$$\frac{T_1^2}{T_2^2} = \frac{B_{H2}}{B_{H1}}$$ where I is Moment of Inertia. For a bar magnet

$$I = \frac{W(l^2 + b^2)}{12}$$ where W is mass of the bar magnet.

Comparison Method To compare the magnetic dipole moments of two magnets initially the pole of two magnets are aligned in what is called sum position as shown in Fig. 24.14 (a).

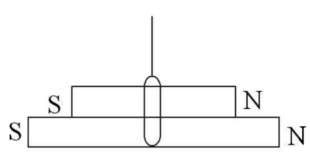


Fig. 24.14(a) Aligned poles or sum position

Let the time period of vibration be T_1. The poles are then misaligned (called difference position) as shown in Fig. 24.14B.

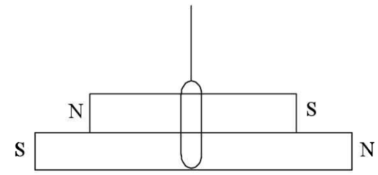


Fig. 24.14(b) Misaligned poles or difference position

Let the time period be T_2. Note $T_2 > T_1$. Then

$$\frac{M_1}{M_2} = \frac{T_2^2 + T_1^2}{T_2^2 - T_1^2}$$

- **Short Cuts and Points to Note**

1. Magnetic pole strength $\propto A$ (area of cross-section).
2. Magnetic field intensity due to a magnetic pole of pole strength *m* at a distance *d* from it is given by

$$B = \frac{\mu_0 m}{4\pi d^2} \text{ in SI system and}$$

$$B = \frac{m}{d^2} \text{ in CGS system.}$$

3. Magnetic field intensity at any point due to a magnetic dipole of moment *M* due to a short magnet is

$$B = \frac{\mu_0 M}{4\pi d^2}\sqrt{3\cos^2\theta + 1} \quad \text{and}$$

$$\tan\beta = \frac{\tan\theta}{2}$$

4. Magnetic field intensity due to a bar magnet along axial line is

$$B = \frac{2\mu_0 Mx}{4\pi(d^2 - l^2)^2} \quad \text{and}$$

$$B = \frac{2\mu_0 M}{4\pi d^3} \text{ due to a short magnet.}$$

5. Magnetic field intensity due to a bar magnet along equatorial line is

$$B = \frac{\mu_0 M}{4\pi(d^2 + l^2)^{3/2}} \quad \text{and}$$

$$B = \frac{\mu_0 M}{4\pi d^3} \text{ due to a short magnet.}$$

6. Torque experienced by a magnet suspended in a uniform magnetic field B

$$\vec{\tau} = \vec{M} \times \vec{B} = \text{MB} \sin\theta$$

7. Work done $W = \int_0^\theta \tau \cdot \theta = MB\,(1 - \cos\theta)$

8. Potential energy $U = -MB$ and change in PE = Work done. Thus,

$$W = \Delta U = U(\theta_2) - U(\theta_1)$$
$$= MB\,(\cos\theta_1 - \cos\theta_2)$$

9. If a magnetic dipole is suspended in two mutually perpendicular magnetic fields then it orients itself making an angle θ with the horizontal magnetic field B_H. Then $B = B_H \tan\theta$ and is called tangent law. Here B is magnetic field perpendicular to B_H.

10. If two magnets of dipole moment M_1 and M_2 are perpendicular then their resultant is

$$M_{net} = \sqrt{M_1^2 + M_2^2}$$

If $M_1 = M_2$ then M_{net}

$$= \sqrt{2}M.$$

11. Magnetic motive force

$$F_m = \oint H \cdot dl = \frac{1}{\mu_0} \oint B \cdot dl = \sum i \text{ where}$$

$\sum i$ is the total current enclosed in the loop. Unit of F_m is ampere turns.

12. Ampere circuital law

$$\oint B \cdot dl = \mu_o \sum i = \mu_o (I_1 + I_2 - I_3)$$

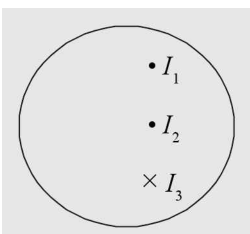


Fig. 24.15

13. In a tangent galvanometer $I = k \tan\theta$ where

$$k = \frac{2B_H r}{\mu_o n}$$

is the reduction factor. Sensitivity of the tangent galvanometer is maximum when $\theta = 45°$

Note: At $\theta = 45°$, $k = I$. The unit of reduction factor is current.

14. In deflection magnetometer in tan A position

$$\frac{B_{H_1}}{B_{H_2}} = \frac{\tan\theta_2}{\tan\theta_1} \text{ and}$$

$$\frac{M_1}{M_2} = \frac{d_2(d_1^2 - l_1^2)}{d_1(d_2^2 - l_2^2)^2}$$

$$= \frac{d_2(d_1^2 - l_1^2)}{d_1(d_2^2 - l_2^2)}$$

If dipoles are short then $\frac{M_1}{M_2} = \left(\frac{d_1}{d_2}\right)^3$

In tan B position $\frac{M_1}{M_2} = \frac{(d_1^2 + l_1^2)^{3/2}}{(d_2^2 + l_2^2)^{3/2}}$

For short dipoles $\frac{M_1}{M_2} = s\left(\frac{d_1}{d_2}\right)^3$

15. The cause of magnetism is elementary dipole i.e. orbital angular and spin angular momentum.

$$M_L = -\frac{e}{2m} L = -n\left[\frac{eh}{4\pi m}\right]$$

$$\because\ L = n M_S = -\frac{e}{m}, \text{ and } M_{Tot} = M_L + M_S$$

$$= s - \frac{e}{2m}[L + 2s] \text{ and } M_{avg} = g\left(\frac{-e}{2m}\right)J \quad \text{where}$$

$$g = \frac{j(j+1) + s(s+1) - l(l+1)}{2j(j+1)}$$ For pure orbital motion $s = 0$, $j = 1$ and $g = 1$. For pure spin motion $l = 0, j = s$ and $g = 2$. $j = |l - s|$ if shell is less than half filled, and, $j = |l + s|$ if shell is more than half filled.

16. In a vibration magnetometer $T = 2\pi\sqrt{\frac{I}{MB_H}}$ if a magnet is brought closer or a current carrying wire is brought closer such that the magnetic field is B' due to either of them. Then $T = 2\pi\sqrt{\frac{I}{M(B_H \pm B')}}$ depending upon the direction.

$$\frac{M_1}{M_2} = \frac{T_2^2 + T_1^2}{T_2^2 - T_1^2} \text{ where}$$

T_1 and T_2 are time periods in sum and difference positions.

17. At pole total intensity is 0.66 Oersted and at equator it is 0.33 Oersted $I_{pole} = 2\,I_{equator}$. Total intensity I of the earth's field is given by $I = I_0\sqrt{1 + 3\sin^2\lambda}$ where λ is latitude. Thus at equator $\lambda = 0$ and $I = I_0$; at poles $\lambda = 90°$ $I = 2I_0$

Note that in a limited region, the magnetic field lines due to earth's dipole are parallel. Hence we may consider the magnetic field of earth at a place to be uniform.

18. In order to shield a region from magnetic field, surround it in a hollow soft iron box.

19. If δ' is apparent dip at a place when dip circle is inclined at θ with the magnetic meridian then true dip δ is given by $\tan\,\delta = \tan\delta'\,\cos\theta$. If θ is unknown then, rotate the dip circle by 90° after noting δ' and read new apparent dip δ'' then

$\cot^2\,\delta\; = \cot^2\,\delta'\,\cot^2\,\delta''$

20. θ, δ and B_H are magnetic elements of earth. They can be used to find the location of a place during aviation or navigation.

21. $\oint B\cdot ds\; = 0$ rejects the possibility of monopole existing in magnetism.

22. Precession frequency or Larmor's frequency

$$\omega_L = \frac{eB}{2m}$$

23. $\frac{\mu_m}{L} = -\frac{e}{2m}$ is called gyromagnetic ratio.

Spin gyromagnetic ratio $\frac{\mu_m}{S} = -\frac{e}{m}$ is twice the orbital gyromagnetic ratio.

24. Splitting in Zeeman effect occurs in the magnetic field as the electron shift $l = 0$ to $l = \pm 1$ etc. giving ΔE (between splitted lines) $= 2\mu_B B$ where μ_B is Bohr magneton.

$E = \mu B\; ml$ where

$ml = [-(n-1), -(n-2), -0, -(n-2), (n-1)]$

- **Caution**

1. Considering that when a magnet is cut pole strength does not vary.

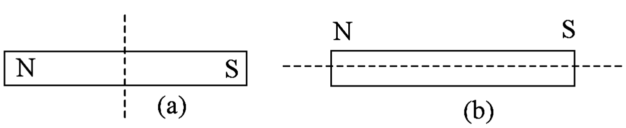


Fig. 24.16

⇒ If a magnet is cut in a vertical plane as shown in Fig. 24.16 (a) then pole strength does not vary. However if we cut the magnet along the horizontal plane as shown in Fig. 24.16 (b) then pole strength varies i.e. pole strength α A

2. Considering there is no difference between magnetic length and physical length of a magnet.

⇒ Since poles lie slightly inside, magnetic length is less than physical length (Magnetic length $l_{mag.} = 0.84\, l_{physical}$).

3. Assuming that dip circle reads angle of dip (true) in any orientation.

⇒ It reads true angle of dip only if dip circle is set into magnetic meridian. Otherwise true dip is given by $D = \tan\delta = \tan\,\delta'\,\cos\theta$ or $\cot^2\delta = \cot^2\delta' + \cot^2\delta''$

4. Considering magnetic intensity of earth is equal everywhere.

⇒ Magnetic intensity changes with latitudes. It is maximum at poles (0.66 Oersted) and minimum at magnetic equator (0.33 Oersted). $I = I_o\sqrt{1+3\sin^2\lambda}$ where λ is latitude.

5. Considering in a vibration magnetometer

$$\frac{M_1}{M_2} = \left(\frac{T_1}{T_2}\right)^2 \text{ where}$$

T_1 and T_2 are time periods in sum and difference positions respectively.

⇒ $$\frac{M_1}{M_2} = \frac{T_2^2+T_1^2}{T_2^2-T_1^2}$$

6. Not understanding tan A and tan B settings of deflection magnetometer.

⇒ In tan A position magnetic needle points N-S and pointer and arms point E-W and pointer coincides with zero-zero.

⇒ In tan B magnetic needle and arms point N-S, pointer E-W and coincides with zero-zero.

7. Assuming contribution of orbital angular momentum is greater than spin angular momentum in atomic or basic dipoles.

⇒ $M_S = 2M_L$ or $M_S = -\frac{e}{m}L$ and

⇒ $M_L = -\frac{e}{2m}L$ However $M_{Tot} = M_L + M_S$

8. Considering reduction factor is sensitivity of the tangent galvanometer.

⇒ Reduction factor $K = \frac{2rBH}{\mu_0 n}$ while current sensitivity is $\frac{d\theta}{dI}$, i.e., deflection per unit current.

9. Considering that the magnetic N–pole of the earth's dipole is at geographic N–pole of the earth.

⇒ Magnetic north pole of the earth's dipole is in southern hemisphere at geographic S–pole.

10. Considering magnetic field lines are like electric field lines.

⇒ Electric field lines do not make a complete loop. Hence monopole can exist. Magnetic field lines make a complete loop rejecting the existence of monopole.

SOLVED PROBLEMS

1. A small magnet is suspended freely by a vertical string. The horizontal and vertical components of earth's field is 20 μ T and 30 μ T respectively. Find the orientation in which magnet will stay in equilibrium.

(a) $\tan^{-1}\left(\frac{3}{2}\right)$ (b) $\tan^{-1}\left(\frac{2}{3}\right)$

(c) $\sin^{-1}\left(\frac{2}{3}\right)$ (d) $\cos^{-1}\left(\frac{2}{3}\right)$

Solution (a) $\tan\theta = \frac{30}{20}$

2. Two identical magnetic dipoles are placed as shown separated by distance *d*. The magnetic field midway between the dipoles is

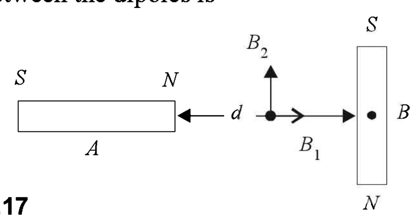


Fig. 24.17

(a) $\frac{\mu_0 M\sqrt{2}}{4\pi d^3}$ (b) $\frac{\mu_0 M\sqrt{5}}{4\pi d^3}$

(c) $\frac{2\mu_0 M\sqrt{5}}{\pi d^3}$ (d) none of these

Solution (c) $B = \sqrt{B_1^2 + B_2^2}$

$$= \frac{\mu_0 M}{4\pi\left(\frac{d}{2}\right)^2}\sqrt{1^2+2^2} = \frac{2\mu_0 M\sqrt{5}}{\pi d^3}$$

3. A 10 cm long magnet has pole strength 12Am. The magnetic field at a distance 20 cm from it is

(a) 2.4×10^{-5} T (b) 3.4×10^{-5} T

(c) 3.8×10^{-5} T (d) 3.0×10^{-5} T

Solution (b) $B = \frac{2\mu o Md}{4\pi\left(d^2 - l^2\right)^2} = \frac{2\times 4\pi\times 10^{-7}\times(\cdot 2)}{4\pi\left(0\cdot 2^2 - 0\cdot 05^2\right)^2}$

$= 3.4 \times 10^{-5}$ T

4. A bar magnet of dipole moment 10^4 JT^{-1} is free to rotate in a horizontal plane. A horizontal magnetic field 4×10^{-5}T exists in the space. Find the work done in rotating the magnet slowly from a direction parallel to the field to a direction 60° from the field.

(a) 0.1 J (b) 0.2 J

(c) 0.4 J (d) 0.5 J

Solution (b)

W = MB (cos 0 – cos 60)

= 1/2 MB

$= 1/2 \times 4 \times 10^{-5} \times 10^4$

= 0.2 J

5. A bar magnet of dipole moment M is bent to form a semi-circle. Find new dipole moment.

(a) $\frac{2M}{\pi}$ (b) $\frac{\pi M}{2}$

(c) $M\pi$ (d) 2M

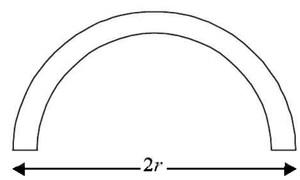


Fig. 24.18

Solution (a) $\pi r = l$ or $r = \frac{l}{\pi}$;

new dipole moment $M' = m(2r) = \frac{m(2l)}{\pi} = 2M/\pi$

6. Find the magnetic field due to a dipole of magnetic moment 1.2 Am² at a point 1m away from it and making an angle 60° with axis of the dipole

(a) 1.6×10^{-7} T, $\tan^{-1}\sqrt{3}$ (b) 1.6×10^{-7} T, 30°

(c) 1.6×10^{-7} T, $\tan^{-1}\frac{\sqrt{3}}{2}$ (d) none of these

Solution (c) $B = \frac{\mu_0 M}{4\pi r^3}\sqrt{3\cos^2\theta + 1}$

$$= \frac{1.2\times 10^{-7}\times 4\pi}{4\pi\times(1)^3}\sqrt{3(1/2)^2+1}$$

$$= 1.6\times 10^{-7}\text{ T}, \tan\beta = \frac{\tan\theta}{2} = \frac{\sqrt{3}}{2}$$

7. Horizontal component of earth's field is 3×10^{-5} T and of dip is 53°. Find mag field of earth at that place.

(a) 4×10^{-5} T (b) 3×10^{-5} T

(c) 5×10^{-5} T (d) 1×10^{-5} T

Solution (c) $B_H = B\cos\delta$ or

$$B = \frac{B_H}{\cos\delta} = \frac{3\times 10^{-5}}{315} = 5\times 10^{-5}\text{ T.}$$

8. At 45° to the magnetic meridian, apparent dip is 30°. Find true dip.

(a) $\tan^{-1}\frac{1}{\sqrt{2}}$ (b) $\tan^{-1}\sqrt{6}$

(c) $\tan^{-1}\sqrt{2}$ (d) $\tan^{-1}\frac{1}{\sqrt{6}}$

Solution (d) $\tan\delta = \tan\delta' \cos\theta = \tan 30 \cos 45$

$$= \frac{1}{\sqrt{3}} \times \frac{1}{\sqrt{2}} = \frac{1}{\sqrt{6}}$$

9. A tangent galvanometer has 28 turns and diameter of the coil 22 cm. When a current of 0.2 A is passed it gives a deflection of 45°. Evaluate horizontal component of the earth's field.

(a) 1.6×10^{-5} T (b) 2.6×10^{-5} T

(c) 1.6×10^{-6} T (d) 1.6×10^{-4} T

Solution (a) $i = k \tan\theta$ or $i = \frac{2rB_H}{\mu_o n}\tan\theta$ or

$$B_H = \frac{i\mu_o n}{2r\tan\theta} = \frac{0.2 \times 4\pi \times 10^{-7} \times 28}{2 \times 22 \times 10^{-2}}$$

$$= 1.6 \times 10^{-5}\ \text{T}$$

10. When two magnets are placed 15 cm and 20 cm away from a deflection magnetometer on two arms, no deflection is observed. The ratio of magnetic dipole moments is

(a) $\frac{3}{4}$ (b) $\frac{9}{16}$

(c) $\frac{27}{64}$ (d) $\frac{81}{256}$

Solution (c) $\frac{M_1}{M_2} = \left(\frac{d_1}{d_2}\right)^3 = \left(\frac{15}{20}\right)^3 = \left(\frac{3}{4}\right)^3 = \frac{27}{64}$

11. A compass needle oscillates 20 times per minute at a place where dip is 45° and 30 times per minute where the dip is 30°. Compare the total magnetic field of earth at the two places.

(a) 1.51 (b) 1.83

(c) 1.63 (d) 1.23

Solution (b) $\frac{B_{H_2}}{B_{H_1}} = \frac{T_1^2}{T_2^2} = \frac{B_2\cos\delta_2}{B_1\cos\delta_1} = \frac{T_1^2}{T_2^2}$

$$= \frac{B_2}{B_1} = \frac{T_1^2\cos\delta_1}{T_2^2\cos\delta_2}$$

$$= \left(\frac{3}{2}\right)^2 \frac{1/\sqrt{2}}{\sqrt{3}/2} = \frac{9}{4} \times \sqrt{\frac{2}{3}} = \frac{3\sqrt{3}}{2\sqrt{2}}$$

$$= 1.83$$

12. A bar magnet has pole strength 1 Am and is placed as shown in Fig. 24.19 (a). Find the magnetic field at p.

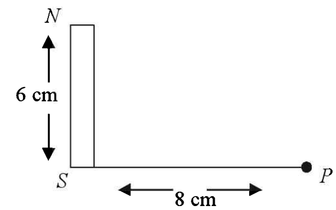


Fig. 24.19 (a)

(a) 11 μT (b) 14 μT

(c) 19 μT (d) 22 μT

Solution (d) $B = \sqrt{B_s^2 + B_N^2 + 2B_S B_N \cos 143}$

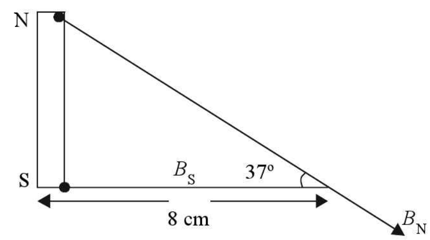


Fig. 24.19 (b)

$$= \sqrt{B_s^2 + B_N^2 + 2B_S B_N \cos(90 + 53)}$$

$$= \frac{\mu_0 M}{4\pi}\sqrt{\left(\frac{1}{8\times10^{-2}}\right)^4 + \frac{1}{(10\times10^{-2})^4} - \frac{2\times1}{(8\times10^{-2})^2} \times \frac{1}{(10\times10^{-2})^2}\sin 53}$$

$$= 0.22\times10^{-4}\ T = 22\ \mu T$$

13. A magnetic needle free to rotate in a fixed vertical plane stays at an angle 60° with the horizontal. If the dip at that place is 37°, find the angle of the fixed vertical plane with the meridian.

(a) $\theta = \cos^{-1}\frac{\sqrt{3}}{5}$ (b) $\theta = \cos^{-1}\frac{1}{\sqrt{3}}$

(c) $\theta = \cos^{-1}\left(\frac{\sqrt{3}}{4}\right)$ (d) $\theta = \cos^{-1}\frac{2}{3}$

Solution (c) $\tan\delta_1 = \frac{B_V}{B_H\cos\theta} = \frac{\tan\delta}{\cos\theta}$ or

$$\cos\theta = \frac{\tan\delta}{\tan\delta_1} = \frac{\tan 37}{\tan 60} = \frac{3/4}{\sqrt{3}} = \frac{\sqrt{3}}{4}$$

14. A dip circle shows an apparent dip of 60° at a place where the true dip is 45°. If the dip circle is rotated by 90° what will be the apparent dip?

(a) $\cos^{-1}\sqrt{\frac{2}{3}}$ (b) $\tan^{-1}\sqrt{\frac{2}{3}}$

(c) $\sin^{-1}\sqrt{\frac{2}{3}}$ (d) $\cot^{-1}\sqrt{\frac{2}{3}}$

Solution (d) $\cot^2\delta_2 = \cot^2\delta - \cot^2\delta_1$

$= \cot^2 45 - \cot^2 60 = 1 - \frac{1}{3}$

$= \frac{2}{3}$ $\delta_2 = \cot^{-1}\sqrt{\frac{2}{3}}$

15. A tangent galvanometer has a coil of 50 turns and a radius of 20 cm. The horizontal component of the earth's field is 3×10^{-5} T. Find the current through the coil.

(a) 0.19 A (b) 0.29 A

(c) 0.39 A (d) none of these

Solution (a) $\tan\theta\ B = \frac{n\mu_o I}{2\pi}$ $I = \frac{2rB}{n\mu_o}$

$$= \frac{2(.2)\times 3\times 10^{-5}}{50\times 4\pi\times 10^{-7}} = 0.19\text{ A}$$

16. A short magnet is executing oscillation in earth's horizontal magnetic field of 24μT. Time for 20 vibrations is 2 s. An upward electric current of 18A is established in a vertical wire placed 20 cm east of the magnet. The new time period is

(a) 0.1 s (b) 0.2 s

(c) 0.26 s (d) 0.16 s

Solution (b) $B = \frac{\mu_o I}{2\pi d} = \frac{4\pi\times 10^{-7}\times 18}{2\pi\times\cdot 2}$

= 18μT South to North.

$$\frac{T_{new}}{T} = \sqrt{\frac{B_H}{B_H - B}} = \sqrt{\frac{24}{24-18}} = 2$$ Thus

$T_{new} = 2(0.1) = 0.2$ s

17. Fig. 24.20(a) shows some of the equipotential surfaces of the magnetic scalar potential. The magnetic field at a point in the region is

(a) 2×10^{-5} T (b) 10^{-5} T

(c) 10^{-4} T (d) 2×10^{-4} T

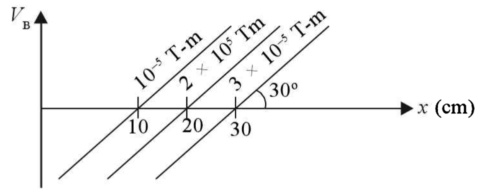


Fig. 24.20 (a)

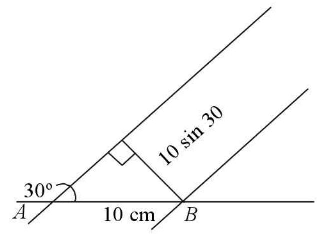


Fig. 24.20 (b)

Solution (d) $B = -\frac{dV_B}{dx}$

$$= \frac{2\times 10^{-5} - 1\times 10^{-5}}{(0\cdot 1)\sin 30} = 2\times 10^{-4}\text{ T.}$$

18. A magnetic dipole of moment 96 Am^2 is placed horizontally with the N-pole pointing south. The position of neutral point is...........if horizontal component of the earth's field is 24μT.

(a) 10 cm (b) 8 cm

(c) 20 cm (d) 20 mm

Solution (c) $B_H = \frac{2\mu_o M}{4\pi d^3}$ or $d^3 = \frac{2\mu_o M}{4\pi B_H}$

$$d^3 = \frac{2\times 4\pi\times 10^{-7}\times 0.96}{4\pi\times 24\times 10^{-6}}$$

$= 8\times 10^{-3}$ or $d = 0.2$ m

19. If the magnitude of the earth's magnetic field at the equator is 3.3×10^{-5} T then find the magnitude of the field at the poles.

(a) 3.3×10^{-5} T (b) 6.6×10^{-5} T

(c) 8.4×10^{-5} T (d) none the these

Solution (b) $B = B\sqrt{1+3\sin^2\lambda}$

$= 3.3\times 10^{-5}\times 2$

$= 6.6\times 10^{-5}$ T

TYPICAL PROBLEMS

20. A short magnet makes 20 oscillations per minute at a place where horizontal component of the earth's field is 25 μT. Another short magnet of dipole moment 1.6 Am^2 is placed 20 cm east of the oscillating magnet. Find the new frequency of oscillation if its N-pole is towards north.

(a) 20 (b) 22.3

(c) 18 (d) 54

Solution (c) $B = \frac{\mu_o M}{4\pi d^3} = \frac{4\pi\times10^{-7}\times1.6}{4\pi\times(\cdot2)^3} = 20\ \mu T$ in the opposite direction of B_H as illustrated in Fig. 24.21

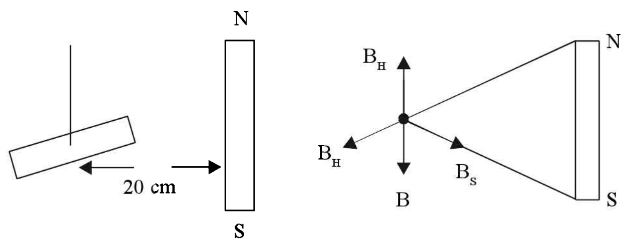


Fig. 24.21

$$\frac{f_1}{f_2} = \frac{T_2}{T_1} = \sqrt{\frac{B_H}{B_H - B}}.\ \text{Thus } f_2 = f_1\sqrt{\frac{B_H - B}{B_H}}$$

$$= 40\sqrt{\frac{25-20}{25}} = \frac{40}{2.23} = 18$$

21. To measure the magnetic moment of a bar magnet, one may not use

(a) a tangent galvanometer

(b) a deflection magnetometer if B_H is known

(c) a vibration magnetometer if B_H is known

(d) a deflection and vibration magnetometer. If B_H is not known.

Solution (a).

22. A combination of two bar magnets make 10 oscillations per second if their like poles are tied together and 2 oscillations per second when unlike poles are tied together The ratio of their magnetic moments is...... if induced magnetism is neglected.

(a) $\frac{3}{2}$ (b) $\frac{13}{12}$

(c) $\frac{8}{9}$ (d) $\frac{12}{11}$

Solution (b) $\frac{M_1}{M_2} = \frac{T_1^2 + T_2^2}{T_2^2 - T_1^2} = \frac{(1/2)^2 + (1/10)^2}{(1/2)^2 - (1/10)^2}$

$$= \frac{\cdot25 + \cdot01}{\cdot25 - \cdot01} = \frac{13}{12}$$

23. A small magnet vibrating horizontally in earth's magnetic field has time period of 4 s. When a magnet is brought closer to it, 50 vibrations take place in 160 s. Compare the magnetic field strengths of the magnet and earth.

Solution $T_1 = 4$ s and $T_2 = \frac{160}{50} = 3.2$ s

$$\frac{T_1^2}{T_2^2} = \frac{B' \pm B_H}{B_H}$$

$$\frac{B'}{B_H} = \frac{4^2}{(3.2)^2} \pm 1 \Rightarrow 9 : 16 \text{ or } 41 : 16$$

24. Consider the earth as a short magnet with its centre coinciding with the centre of the earth and dipole moment M. The angle of dip δ is related to latitude λ as

(a) $\tan\delta = \tan\lambda$ (b) $\tan\delta = 2\tan\lambda$

(c) $\tan\delta = \frac{\tan\lambda}{2}$ (d) $\tan\delta = \cot\lambda$

Solution (b) $B_r = \frac{2\mu_0 M}{4\pi r^3}\cos\theta,\ B_\theta = \frac{\mu_0 M\sin\theta}{4\pi r^3}$

as $\theta = 90 + \lambda$

$$B_r = \frac{2\mu_0 M}{4\pi r^3}\cos(90+\lambda) = \frac{-2\mu_0 M\sin\lambda}{4\pi r^3}$$

$$B\theta = \frac{\mu_0 M\sin(90+\lambda)}{4\pi r^3} = \frac{\mu_0 M\cos\lambda}{4\pi r^3}$$

$$\frac{B_r}{B_\theta} = \frac{B_r}{B_\theta} = -2\tan\lambda \text{ or } \tan\delta = 2\tan\lambda$$

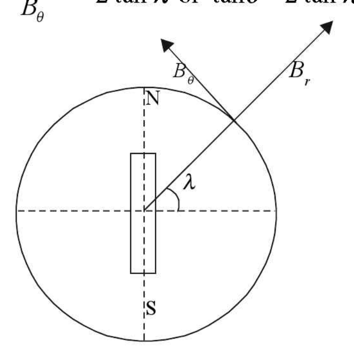


Fig. 24.22

25. A short magnet produces a deflection of 37° in a deflection magnetometer in tan$A$ position when placed 10 cm away. Find the ratio of earth's, field to magnetic dipole moment of the magnet.

Solution $B = \frac{\mu_0 M(2)}{4\pi d^3} = B_H \tan\theta$

$$\frac{B_H}{M} = \frac{2\mu_0}{4\pi d^3 \tan\theta} = \frac{2\times10^{-7}\times4}{(.1)^3\times3}$$

$$= \frac{8}{3}\times10^{-4}\ T^{-1}Am^{-2}.$$

26. A bar magnet has coercivity $4\times10^3\ Am^{-1}$. It is placed in a solenoid having 40 turns per cm and is 1.2 m long. The current carried by the solenoid to demagnetise the bar magnet is

(a) 6 A (b) 8 A
(c) 5.78 A (d) none of these

Solution (d) $\oint H.dl = ni$

$$4\times10^3 = \frac{40}{10^{-2}}i$$

or $i = 1A$.

QUESTIONS FOR PRACTICE

1. The intensity of magnetisation depends on magnetising field. For its low values

(a) $I \propto \frac{1}{H}$ (b) $I \propto H$
(c) $I \propto \frac{1}{H^2}$ (d) $I \propto H^2$

2. The value of earth's magnetic field is

(a) 5 G (b) 50 G
(c) 100 G (d) 500 G

3. The area of $I - H$ curve and area of $B - H$ curve are related as

(a) area of $I - H$ curce < area of $B - H$ curve
(b) area of $I - H$ curve > area of $B - H$ curve
(c) are of $I - H$ curve = area of $B - H$ curve
(d) none of these

4. A powerful magnet attracts

(a) all materials
(b) only ferromagnetic materials and their ores
(c) some materials and repels some other materials
(d) only iron and its ores but does not repel any material

5. At Curie temperature the ferromagnetic materials get converted into

(a) non-magnetic material
(b) paramagnetic material
(c) diamagetic material
(d) all of the above

6. The correct relation between B, H and I is

(a) $B = \mu_0 I \times H$ (b) $B = \frac{\mu_0 I}{H}$
(c) $B = \mu_0(I + H)$ (d) $B = \mu_0 I - H$

7. The value of Curie temperature for Ni is nearly

(a) 631 K (b) 770 K
(c) 890 K (d) 1100 K

8. Soft iron is used for the construction of electromagnets because for iron

(a) area of hysteresis loop is more
(b) coercive force is high
(c) retentivity is high
(d) magnetic saturation limit is high and corecivity is low

9. The magnetic moment of a magnet of mass 75 g is $9 \times 10^{-7}\ Am^2$. If the density of the material of magnet is 7.5×10^{-3} then intensity of magnetisation will be

(a) 90 Am^{-1} (b) 0.09 Am^{-1}
(c) 0.9 Am^{-1} (d) 9 Am^{-1}

10. 1 Weber is equivalent of

(a) 10^{-8} Maxwell (b) 10^{12} Maxwell
(c) 10^{8} Maxwell (d) 10^{4} Maxwell

11. The dipole moment of a coil of area A and number of turns N and carrying current i will be

(a) iA/N (b) Ni/A
(c) NiA (d) $N^2 i A$

12. In deflection magnetometer, to find dipole moment M of a magnet, angle of deflection should be

(a) 0° (b) 90°
(c) 45° (d) any angle

13. The magnetic susceptibility of a paramagnetic material at 73°C is 0.0075. Then its value at 173°C will be

(a) 0.0030 (b) 0.0075
(c) 0.0045 (d) 0.0015

14. The magnetic lines of forces due to the horizontal componant of earth's magnetic field will be

(a) circular (b) elliptical
(c) curved (d) horizontal and parallel

15. Two magnetic lines of force
(a) never cut each other
(b) cut near the poles
(c) cut according to the position of magnet
(d) cut at neutral point

16. The magnetic lines of force like electrical lines of force
(a) are closed (b) are not closed
(c) are open (d) are not open

17. The magnetic property inherent in all materials is
(a) paramagnetism (b) ferro magnetism
(c) diamagnetism (d) non-magnetism

18. The lines joining same dip are called
(a) aclinic (b) isogonic
(c) isodynamic (d) isoclinic

19. The resultant magnetic moment due to two current–carrying concentric coils of radius r, mutually perpendicular to each other will be
(a) $\sqrt{2}\ ir$ (b) $\sqrt{2}\ i\pi r^2$
(c) $2\pi r^2$ (d) $\sqrt{2}\ ir^2$

20. The value of magnetic susceptibility for paramagnetic substances is
(a) infinity (b) low positive
(c) low negative (d) zero

21. Which of the following does not form magnetic element?
(a) B_H (b) B_V
(c) dip (d) declination

22. The value of Bohr magneton is
(a) $e/4\pi m$ (b) $\dfrac{eh}{2\pi m}$
(c) $\dfrac{eh}{4\pi m}$ (d) $\dfrac{eh}{\pi m}$

23. Angle of dip at a place is 60° and earth's magnetic field is $8\mu T$. Its horizontal component B_H is
(a) $4\sqrt{3\mu}\ T$ (b) $4\ \mu T$
(c) $\dfrac{8}{\sqrt{3}}\ \mu T$ (d) $8\sqrt{3\mu}\ T$

24. A rod of ferromagnetic material with dimension $10 \times 0.5 \times 0.2\ cm^3$ is placed in a magnetic field of strength $0.5 \times 10^4\ Am^{-1}$ as a result of which a magnetic field of 5 Am^{-2} is produced in the rod. The value of magnetic induction will be
(a) 0.358 T (b) 0.54 T
(c) 6.28 T (d) 2.519 T

25. Which of the following behaves as a bar magnet?
(a) A long wire carrying current
(b) A circular coil carrying current
(c) None of (a) and (b)
(d) Both (a) and (b)

26. A current i is flowing in a conductor of length l. When it is bent in the form of a loop its magnetic moment will be
(a) $4\pi l^2 i$ (b) $il^2/4\pi$
(c) $4\pi/l^2 i$ (d) $l^2/4\pi$

27. A bar magnet of length l and dipole moment M is bent to form a semicircle. The new dipole moment is
(a) $\dfrac{2M}{\pi}$ (b) M
(c) $\dfrac{\pi M}{2}$ (d) none of these

28. A current of 1 A is flowing in a coil of 10 turns having radius 10 cm. Its magnetic moment will be
(a) 3140 Am^2 (b) 100 Am^2
(c) $\mu_0\ Am^2$ (d) 0.314 Am^2

29. The value of relative magnetic permeability (μ_r) for ferromagnetic materials is
(a) $\mu_r = 1$ (b) $\mu_r >> 1$
(c) $\mu_r < 1$ (d) $\mu_r > 1$

30. The magnetic susceptibility of a paramagnetic substance is $3 \times 10^{-4}\ Am^{-1}$. The intensity of magnetisation will be
(a) 24 Am^{-1} (b) $3 \times 108\ Am^{-1}$
(c) $12 \times 10^8\ Am^{-1}$ (d) 1.2 Am^{-1}

31. Which of the following expressions is applicable to the moving coil galvanometer?
(a) $\vec{F}_m = q(\vec{V} \times \vec{B})$ (b) $B = B_0 \tan\theta$
(c) $\vec{i} = \vec{M} \times \vec{B}$ (d) none of these

32. Points A and B are situated on an axis perpendicular to a 2 cm long bar magnet at large distances x and $3x$ from its centre on opposite sides. The ratio of the magnetic field at A and B will be approximately equal to
(a) 27 : 1 (b) 2 : 9
(c) 9 : 1 (d) 1 : 9

33. Magnetic susceptibility χ of a paramagnetic material changes with absolute temperature T as
(a) $\chi \propto T^{-1}$ (b) χ = constant
(c) $\chi \propto e^T$ (d) $\chi \propto T$

34. When N-pole points south of the earth the neutral points lie on

(a) axial line (b) equatorial line

(c) any of the lines

(d) both axial and equatorial lines

35. A coil (8 cm × 4 cm) carries a current 2 A and has 200 turns. Find the magnetic dipole moment.

(a) 12.8 Am^2 (b) 1.28 Am^2

(c) 6.4 Am^2 (d) 0.64 Am^2

36. At a place the horizontal component of eath's field is B_H. The time period of vibration of a magnet is T. If we take it to a place where earth's horzontal component is $B_H/2$ then the time period will be

(a) T (b) $\frac{T}{2}$

(c) $T\sqrt{2}$ (d) $2T$

37. The dipole moment of a magnet is M. Its time period of vibration is T. If the dipole moment of another magnet is 2 M with same physical dimensions as the former, the time period of vibration will be

(a) $\frac{T}{2}$ (b) $\frac{T}{\sqrt{2}}$

(c) $T\sqrt{2}$ (d) $2T$

38. The magnetic moment of a dipole is ____ if l is magnetic length and m pole strength.

(a) $M = 2ml$ (b) $M = ml$

(c) $\frac{2m}{l}$ (d) πml

39. The unit of pole strength is

(a) Am^2 (b) Am^{-1}

(c) Am (d) Am^{-2}

40. Dip circle is not set into magnetic meridian and the angle at which it is inclined to the magnetic meridian is unknown. δ' and δ'' are apparent dips at a place in which dip circle is kept in transverse positions. The dip is

(a) $\cot\delta = \cot\delta' + \cot\delta''$

(b) $\tan^2\delta' = \tan\delta' + \tan^2\delta''$

(c) $\cos^2\delta = \cos^2\delta' + \cos^2\delta''$

(d) $\cot^2\delta = \cot^2\delta' + \cot^2\delta''$

41. Which of the following is best suited to make the core of a transformer?

(a) Soft iron (b) Steel

(c) Alnico (d) None of these

42. A dip needle in a plane perpendicular to magnetic meridian will remain

(a) horizontal (b) vertical

(c) inclined at 45° with horizontal

(d) in any direction

43. The vertical component of eatrh's magetic field is zero at

(a) magnetic equator (b) magnetic poles

(c) geographical poles (d) everywhere

44. A galvanometer gives full scale deflection when the current passing through it is $1mA$. Its resistance is 100 Ω. Without connecting additional resistance in series with it, it can be used as a voltmeter of range

(a) 1.000 V (b) 0.010 V

(c) 0.001 V (d) 0.100 V

45. The angle between the magnetic meridian and the geographical meridian is known as

(a) magnetic pole stregth

(b) magnetic dip

(c) magnetic declination

(d) magnetic moment

46. Magnetic field strength due to a short bar magnet on its axial line at a distance x is B. What is its value at the same distance on the equatorial line?

(a) 4 B $B/2$

(c) B (d) 2 B

47. Angle of dip at the magnetic equator is

(a) 0° (b) 45°

(c) 90° (d) none of these

48. The angle of dip at a place where horizontal and vertical componets of earth's magnetic field are equal, is

(a) 90° (b) 30°

(c) 45° (d) 0°

49. When two magnets are placed 20 cm and 15 cm away on the two arms of a deflection magnetometer, it shows no deflection. The ratio of magnetic moments is

(a) $\frac{M_1}{M_2} = \frac{64}{27}$ (b) $\frac{M_1}{M_2} = \frac{4}{3}$

(c) $\frac{M_1}{M_2} = \frac{16}{9}$ (d) none of these

50. The time period of vibration of a magnetic dipole is T. On bringing a bar magnet closer along the axis of vibration, the new time period T' will be

(a) $T' > T$ (b) $T' < T$

(c) $T' = T$ (d) depends on pole faces

51. Gauss law in magnetism concludes that
(a) monopole does not exist
(b) magnetic flux can be determined
(c) $\nabla \times B = 0$ (d) $\nabla . B = 0$

52. Reduction factor in a tangent galvanometer is related to number of turns as
(a) n (b) n^2
(c) $n^{-1/2}$ (d) n^{-1}

53. Earth's magnetic field inside a closed iron box, as compared to that outside is
(a) same (b) less
(c) more (d) zero

54. Tan A position means
(a) magnetic needle and arms point north-south, pointer east-west and coincides 0-0
(b) magnetic needle and pointer point north-south and arms are along east-west
(c) magnetic needle points north-south, arms and pointer east-west and pointer coincides 0-0
(d) magnetic needle points north-south, arms and pointer east-west and pointer coincides 90 – 90°

55. Sensitivity of tangent galvanometer is maximum when deflection is
(a) 45° (b) 90°
(c) 30° (d) 04°

56. A current-carrying coil suspended freely in a uniform magnetic field be in stable equilibrium, if the angle between its magnetic dipole moment vector and the magnetic field is
(a) 180° (b) zero
(c) 45° (d) 90°

57. If a bar magnet of magnetic moment m is deflected through an angle θ in a uniform magnetic field of induction B, the work done in reversing the direction is
(a) $MB \sin\theta$ (b) MB
(c) $MB(1-\cos\theta)$ (d) $MB \cos\theta$

58. Isogonic lines on a magnetic map represent lines joining places with same
(a) B_H (b) dip
(c) zero dip (d) declination

59. The intensity of magnetic field at a distance d from an isolated pole of m units in air is
(a) md^2 (b) m/d
(c) md (d) m/d^2

60. The needle of the dip circle at a place stays at 30°. The dip circle is inclined at an angle of 30° with the magnetic meridian. The true dip at the place is
(a) $\tan^{-1}\sqrt{3}$ (b) $\tan^{-1}\frac{\sqrt{3}}{2}$
(c) $\tan^{-1}\frac{1}{2}$ (d) $\tan^{-1}\frac{3}{2}$

61. A person is facing magnetic north. An electron in front of him flies horizontally towards the north and deflects towards east. He is in/at the
(a) southern hemisphere (b) equator
(c) northern hemisphere (d) none of these

62. A bar magnet of magnetic dipole moment is cut into two halves by a horizontal plane. Which of the following statements is correct?
(a) Only pole strength becomes half
(b) Only magnetic dipole moment gets half
(c) Both dipole moment and pole strength become half
(d) Dipole moment remains unaltered.

63. A current-carrying loop suspended freely in a uniform magneticc field will experience
(a) torque only (b) force only
(c) neither torque nor force (d) both

64. A neutral point in a magnetic field is a point at which
(a) the resultant magnetic intensity is zero
(b) earth's field is zero
(c) the magnetic field is zero
(d) the magnetism is more

Answers to Questions for Practice

1.	(c)	**2.**	(d)	**3.**	(b)	**4.**	(b)	**5.**	(b)	**6.**	(c)	**7.**	(a)
8.	(d)	**9.**	(b)	**10.**	(c)	**11.**	(c)	**12.**	(c)	**13.**	(a)	**14.**	(d)
15.	(a)	**16.**	(a)	**17.**	(c)	**18.**	(d)	**19.**	(b)	**20.**	(b)	**21.**	(b)
22.	(c)	**23.**	(b)	**24.**	(c)	**25.**	(b)	**26.**	(b)	**27.**	(a)	**28.**	(d)
29.	(b)	**30.**	(d)	**31.**	(c)	**32.**	(a)	**33.**	(a)	**34.**	(a)	**35.**	(b)
36.	(c)	**37.**	(b)	**38.**	(b)	**39.**	(c)	**40.**	(d)	**41.**	(a)	**42.**	(b)
43.	(a)	**44.**	(d)	**45.**	(c)	**46.**	(b)	**47.**	(a)	**48.**	(c)	**49.**	(a)
50.	(b)	**51.**	(a)	**52.**	(d)	**53.**	(b)	**54.**	(c)	**55.**	(a)	**56.**	(b)
57.	(c)	**58.**	(d)	**59.**	(d)	**60.**	(c)	**61.**	(a)	**62.**	(c)	**63.**	(a)
64.	(a)												

25

Magnetic Properties of Substance

BRIEF REVIEW

The motion of electron in its orbit acts like a current loop. This provides magnetic dipole moment to an atom. Besides this, electron has a spin angular momentum which contributes about 90% of the dipole moment. Nucleus also has some magnetic moment but it is several thousand times less than the magnetic moment of an electron. Thus the resultant magnetic moment of an atom is the vector sum of all such magnetic moments. Thus

$$M = M_L + M_S = \frac{-e}{2m}\left[\vec{L} + 2\vec{S}\right]$$

The term $\frac{e}{2m}$ is called gyromagnetic ratio.

Note: $M_S = 2M_L$ where M_L and M_S are orbital magnetic moment and spin magnetic moment respectively.

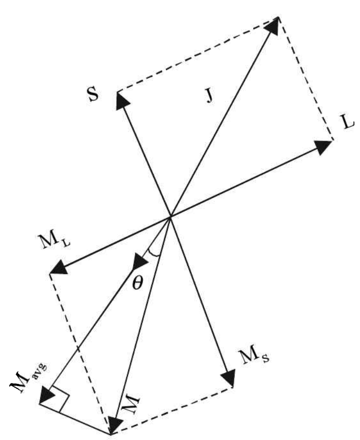


Fig. 25.1 Illustration M_L, M_S, M_{avg} and M_{Total}

$$M_{avg} = M\cos\theta = g\left(\frac{-e}{2m}\right)J \text{ as illustrated in Fig. 25.1}$$

where $$g = \left(\frac{J(J+1)+S(S+1)-l(l+1)}{2J(J+1)}\right)$$

For pure orbital motion $S = 0$, $J = l$ and $g = 1$. For pure spin motion $l = 0$, $J = s$ and $g = 2$. These values agree as explained earlier. $J = |l - s|$ if shell is less than half filled and $J = |l + s|$ if shell is more than half filled.

We know torque $\tau = \frac{dL}{dt} = \vec{M} \times \vec{B} = \frac{-e}{2m} L \times B = \omega_L L$

ω_L represents Larmors frequency and is given by $\omega_L = \frac{eB}{2m}$.

It describes precession as illustrated in Fig. 25.2

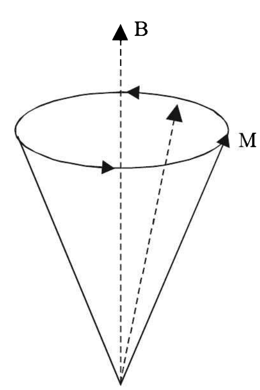


Fig. 25.2 Larmor's frequency illustration

$$\mu_B = \frac{e\hbar}{2m} = 9.3 \times 10^{-24}\,\text{J/T}$$

represents Bohr magneton.

$E = \mu_B B m_l$ where m_l is and integer given by $-l$, $-l+1, \ldots 0, \ldots l-1, l$

$\Delta E = \mu_B B$ and l is orbital quantum number.

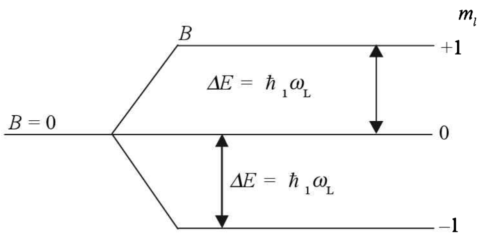


Fig. 25.3

Figure 25.3 illustrates splitting of an atomic level by a magnetic field (Zeeman effect) for $l = 1$

In case of spin $\Delta E = 2\mu_B B$ for $m_s = -1/2$ and $+1/2$

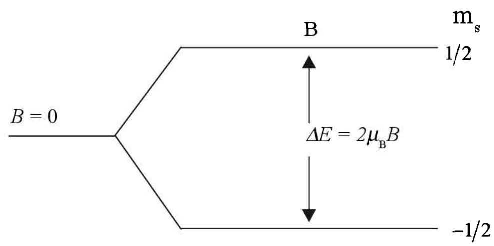


Fig. 25.4

Magnetization Vector $(I) = \frac{\text{Magnetic moment}}{\text{Volume}} = \frac{\vec{M}}{V}$

$$= \frac{2ml}{2lA} = \frac{m}{A} = \frac{\text{pole strength}}{\text{Area}} \text{ unit Am}^{-1}$$

Magnetizing Field Intensity $(H)\; H = \frac{B}{\mu_0}$ in vacuum or free space.

The magnetic induction inside an isotropic medium is given by $B = \mu_0 H + \mu_0 I$

$$= \mu_0 H\left(1 + \frac{I}{H}\right)$$

$$= \mu_0 H(1 + \chi)$$

Susceptibility $(\chi)\; \chi = \frac{I}{H}$

$\mu_r = 1 + \chi$ thus $B = \mu_0 \mu_r H = \mu_m H$

where $\mu_m = \mu_r \mu_0$ is permeability of the medium and

$\mu_r = \frac{\mu_m}{\mu_0} = 1 + \chi$ is relative permeability of the medium.

In vacuum or free space $H = \frac{Bo}{\mu_0}$ and $B_0 = \mu_0 ni$

$\therefore\; H = ni$

$$I = \frac{B}{\mu_0} - H = \frac{B}{\mu_0} - ni$$

Note that magnetic susceptibility has no physical relationship to electrical susceptibility.

Curie Law As the temperature rises the randomness of individual atomic magnetic moments increases. Therefore susceptibility of paramagnetic substances is inversely proportional to temperature.

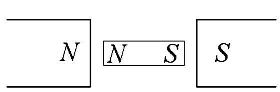
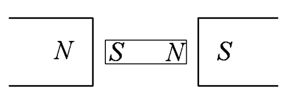
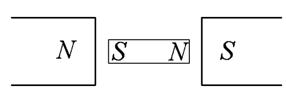
$\chi \propto 1/\text{T}$ or $\chi = \frac{C}{T}$. C is called Curie's constant.

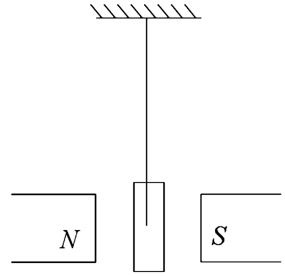
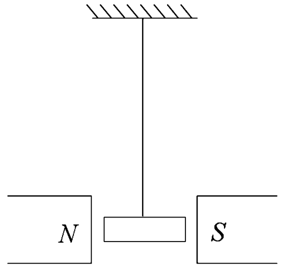
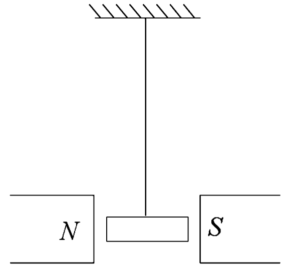
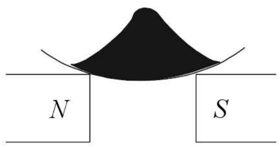
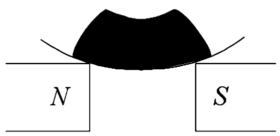
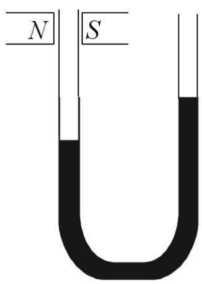
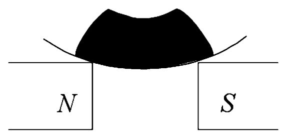
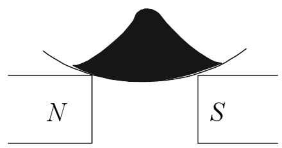
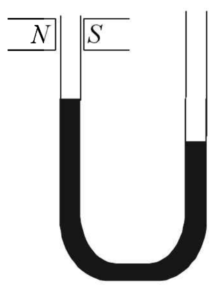
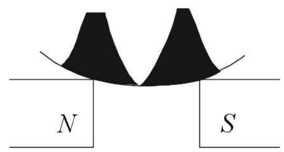
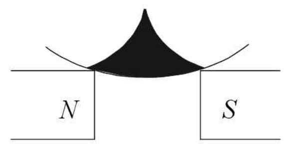
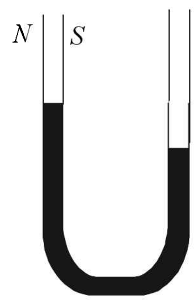
Ferromagnetic substances when heated beyond Curie temperature become paramagnetic. Thus for ferromagnetic substances $\chi = \frac{C}{T - T_C}$ where T_c is Curie temperature. Curie temperature for *Fe*, *Co*, *Ni* and Gadonium (*Gd*) are respectively 770°C, 1121°C, 358°C and 44°C. CGS unit of B is Gauss (G) $1G = 10^{-4}$T. CGS unit of H is Oersted.

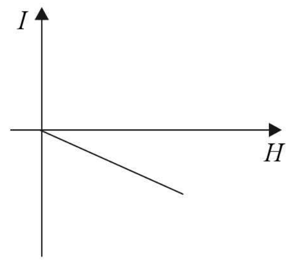
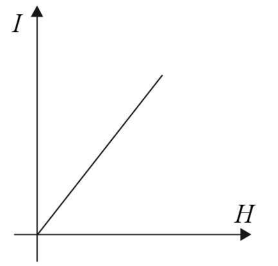
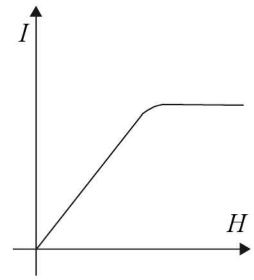
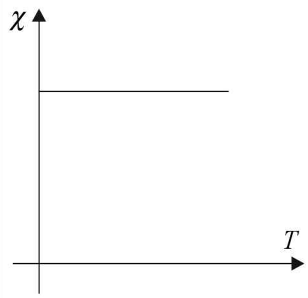
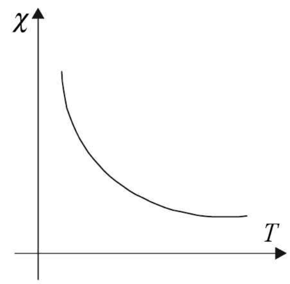
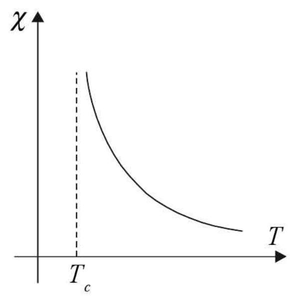
$$1 \text{ Oersted} = \frac{10}{\mu_0} = \frac{10^{-4}}{4\mu \times 10^{-7}} Am^{-1} = 80\, Am^{-1}$$

Neel's temperature (T_N) is that temperature at which an antiferromagnetic substance becomes paramagnetic.

Table. 25.1

Property	Diamagnetic	Paramagnetic	Ferromagnetic
Cause of magnetism	Orbital motion of electrons	Spin motion of electrons	Formation of domains
Behaviour in an external Magnetic field	Feebly repelled 	Feebly attracted 	Strongly attracted 
State of magnetisation	That is, weakly magnetised in the opposite direction to applied magnetic field	Weakly magnetised in the same direction. as the applied field	Strongly magnetised in the same direction as the applied field

Property	Diamagnetic	Paramagnetic	Ferromagnetic
Alignment of a freely suspended material	 Aligns at right angle to the field	 Aligns in the direction of field	 Aligns in the direction of field
Liquid or powder in a watch glass	 Weak field  Strong field  Level depressed in that limb	 Weak field  Strong field  Level slightly rises	 Weak field  Strong field  Level rises in that limb
Gas placed in magnetic field	Expands at right angle to to the field	Expands in the direction of the field	Expands in the direction of the field
Magnetic induction	$B < B_0$	$B > B_0$	$B >> B_0$
susceptibility (χ)	$\chi < 0$ (negative) χ does not depend upon temperature	$\chi \geq 0$ (positive but low) $\chi \propto \frac{1}{T}$ or $\chi = \frac{C}{T}$	$\chi >> 0\ (10^2 - 10^3)$ (positive and high) $\chi = -\frac{C}{T - T_C}$
Relative permeability μ_r	$\mu_r < 1$	$\mu_r > 1$	$\mu_r >> 1\ (10^2 - 10^3)$
Magnetising vector (*I*)	In opposite direction to H, has a very low value	In the direction of *H*, has a low value	In the direction of *H*, has a very high value

Property	Diamagnetic	Paramagnetic	Ferromagnetic
***I-H* curve**			
Magnetic Dipole Moment (*M*)	Very low and in opposite direction to *H*	Very low but in the direction of *H*	High in the direction of *H*
$\chi - T$ curve			
Examples	Cu, Ag, Au, Zn, Bi, Sb, NaCl, H_2O, air, Ne, He	Al, Mn, Pt, Na, $CuCl_2$ O_2 and crown glass	Fe, Co, Ni, Gd, Fe_3O_4
Nature of effect	Distortion	Orientation	Hysteresis

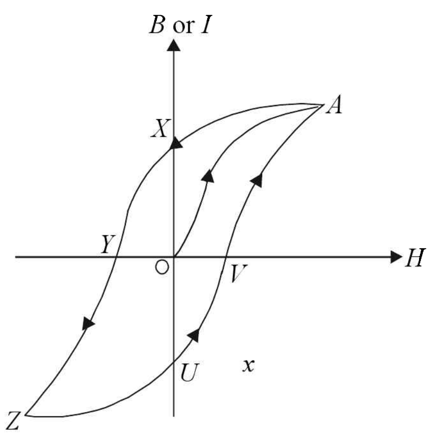


Fig. 25.5 Hysteresis curve of a ferromagnetic material

Hysteresis Fig 25.5 shows hysteresis curve. Area under hysteresis curve gives loss of energy per cycle. Hysteresis stands for not following original curve when conditions are reversed.

Magnetic Saturation The state of magnetic material in which value of I or B becomes maximum.

Retentivity When applied magnetising field is removed, the magnetism *B* or I that remains in the material is called retentivity or remnent magnetism. In the Fig. 25.5 *OX* = *OU* is retentivity.

Coercive force or coercivity The magnetising force or H applied in negative direction to make retentivity zero is called coercivity. In Fig. 25.5 *OY* = *OV* is coercivity.

To make permanent magnets Alnico cunife ($Cu + Ni + Fe$) cunico ($Cu + Ni + Co$) and carbon steel are used. Alnico is an alloy of *Fe*, *Al*, *Ni* and Co. Their coercivity is high and retentivity is relatively low (otherwise high).

Electromagnets are made from materials whose retentivity is high and coercivity is low, For example, soft iron. Such materials are also used to make core of transformers, motors, dynamo and so on. 4% Si is added in soft iron core to further reduce the coercivity. For transformer core permalloy is preferred. For high frequency transformers μ-metals or radio metals are used to make core.

• Short Cuts and Points to Note

1. Magnetic pole strength $\propto A$ (area of cross-section).
2. Atomic dipole moment is due to orbital and spin motion of electron. The unit of atomic dipole moment is Bohr magneton. $1\mu_B = 9.3 \times 10^{-24}$ J/T
3. The precession of electron in applied magnetic field causes Zeeman splitting. The energy difference between splitted lines is $2\ \hbar\ \omega_L$ or $\mu_B B$.
4. In vacuum $H = \dfrac{B}{\mu_0}$ and in a medium $H = \dfrac{B}{\mu_m}$ where

$\mu_m = \mu_r \mu_0$ and $\mu_r = 1 + \chi$ and $\chi = \dfrac{I}{H}$

Thus $\mu_m = \mu_0 (1 + \chi)$.

5. Mutual interaction force between two small magnets of magnetic moments M_1 and M_2 is $F = \dfrac{\mu_0 6 M_1 M_2}{4\pi r^4}$

6. Domains in ferromagnets have dimensions ~ 10^{-5}m. Each domain contains $10^{17} - 10^{21}$ atoms whose spins are aligned. When external magnetic field is applied either domains get aligned or domain boundaries grow.

7. The susceptibility for ferromagnets is quite high ~ 10^2 to 10^3 χ. For paramagnets it lies between 0 and 1. $(0 < \chi < 1)$ χ. For diamagnets it is negative *i.e.* $\chi < 0$ but close to zero.

8. μ_r for ferromagnets ~ 10^2 to 10^3

   μ_r for paramagnets > 1 and $1.0000 < \mu_r < 1.003$

   μ_r for diamagnets < 1 and $.99990 < \mu_r < .99999$

9. If $T > T_C$ (Curie temperature) ferromagnets become paramagnetic.

10. At Neel's temperature anti-ferromagnetic substance become paramagnetic.

11. A bar magnet attracts very strongly ferromagnets at poles but at the centre of the bar magnet attraction is nearly zero.

12. The tip of an alpin always acts like a magnet because it contains a single domain which is always aligned.

13. Materials suitable for permanent magnets shall possess high retentivity and high coercivity. Materials showing high retentivity and low coercivity are suitable for electromagnets, cores of motors, transformers etc.

14. For high frequency transformers used in radio, TV etc. μ-metal, radio metal or ferrites are used.

15. Super conductors are perfect diamagnets. They show Miessner effect. If superconductors are subjected to a magnetic field strength > critical field strength they become normal conductors.

16. Precessing frequency of proton in a magnetic field is $\omega_p = \dfrac{MB}{L}$ or $f_p = \dfrac{MB}{2\pi L}$ where M is magnetic dipole moment and L is quantised spin angular momentum.

17. Change in magnetic moment of a circulating electron if placed in a magnetic field of strength B is

$$\Delta M = \frac{1}{2} er^2 \Delta\omega \left(\frac{eB}{2m}\right) = \frac{e^2 r^2 B}{4m} \Delta\omega$$

Magnetic dipole moment $M = NiA$ and $M = ef\pi r^2$ for an electron.

18. **Slator's law** If A and B are two neighbouring atoms in its lattice, r is the radius of atom and r_{AB} separation between two atoms (interatomic distance) Then a substance is ferromagnetic if $\dfrac{r_{AB}}{r} > 3$

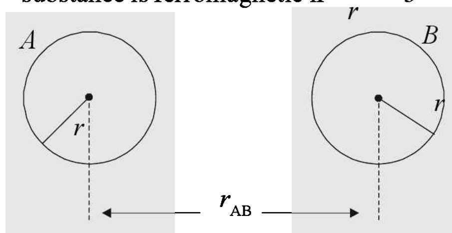


Fig. 25.6 Slator's rule illustration

19. In paramagnets $\chi = \dfrac{\mu_0 N \mu_B^2}{kT}$.

    This suggests $\chi \propto \dfrac{1}{T}$

20. The law of nature is diamagnetism. Pauli's exclusion principle and Fermi-Dirac statistics are based on it.

21. Ferromagnetism and anti-ferromagnetism can be understood by two sublattice model. The two sublattices have their spins oriented opposite to one another like the teeth of two combs fastened in each other as illustrated in Fig. 25.7(a)

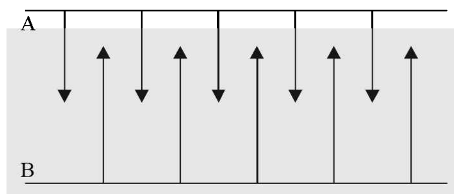


Fig. 25.7(a)

(a) Ferromagnetic substances are ferrites. Their net spin decreases.

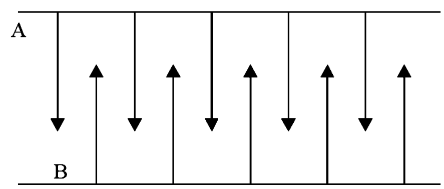


Fig. 25.7(b)

(b) Examples of anti-ferromagnetic substances: Mn F_2, MnO, FeO, CoO, NiO, MnTe, MnS, Cr_2O_3.
Their net spin = 0

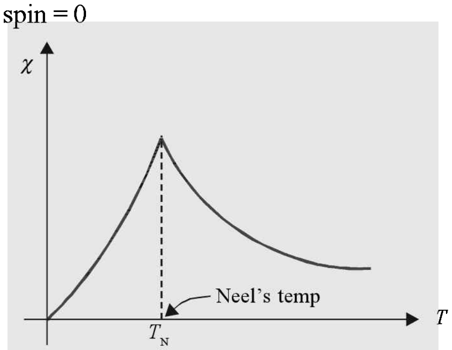


Fig. 25.7(c)

22. *NMR* (Nuclear magnetic resonance) is the analog of electron paramagnetic resonance $\omega_0 = (g_n e/2M_P)B_o$ where m is mass of proton and ω_0 is resonant frequency.

23. *FMR* (Ferromagnetic resonance) is analogous to the spin resonance and is the basis of many microwave devices. The larmor or resonant frequency is

$\omega_p = \frac{\mu_0 ge}{2m}(H_o - I_o)$ where H_o is applied external field and I_o is the saturation magnetisation.

24. The dynamic aspect of spin motion in a ferromagnet is spin waves. They represent collective excitations in spin systems. Exchange interaction is responsible for spin wave. Spin waves carry both energy and momentum. As the temperature is increased, energy is absorbed by the excitations of the spin waves, and the magnetisation also decreases.

25. Magnetic tapes and films are made of FeCoNi, $Gd_3Fe_9O_{12}$ and CrO_2.

- **Caution**

1. Considering diamagnetic and anti-ferromagnetic substances are identical.

⇒ In diamagnetic substances no net spin magnetisation occurs, for example in H_2, He, Ne, Cu, Ag, Au. In anti-ferromagnetic substances there are two types of substances whose spins are oriented in opposite directions. The magnitude of spins are equal so that they cancel each other. Anti-ferromagnetic substances, on heating beyond Neel's temperature become paramagnetic.

2. Not differentiating between magnetization vector and magnetising field intensity. Magnetisation

vector I = $\frac{\vec{M}}{V} = \frac{ml}{Al} = \frac{m}{A} = \frac{\text{Pole strength}}{\text{Area}}$ and

magnetising field intensity $\vec{H} = \frac{\vec{B}}{\mu_0} - I$.

3. Confusing magnetizing field intensity with magnetic induction.

⇒ H is magnetising field intensity and B is magnetic induction. In vacuum $\frac{B}{\mu_0} = H$. In a medium

$$H = \frac{B}{\mu_m}$$

4. Confusing Curie's temperature with Neel's temperature

⇒ At Curie temperature, a ferromagnetic substance changes to paramagnetic while at Neel's temperature an anti-ferromagnetic substance changes to paramagnetic.

5. Assuming that substances behave alike in strong and weak magnetic fields.

⇒ Their behaviours are opposite in strong and weak magnetic fields. See carefully the table comparing the properties of ferro, para and diamagnetic substances.

6. Assuming that magnetic induction increases in the medium as compared to free space in all substances.

⇒ In para and ferromagnetic substances it increases while in diamagnetic substances it decreases.

7. Considering there is no perfect diamagnet *i.e.* $\chi = -1$ cannot be achieved.

⇒ Super conductors are perfect diamagnets with $\chi = -1$.

8. Considering any ferromagnetic material may be suitable to make permanent magnets.

⇒ Substances having high coercivity and high retentivity are suitable to form permanent magnets. High carbon steel, Alnico (Al + Ni + Fe + Co), Cunife (Cu + Ni + Fe) and Cu Nico (Cu + Ni + Co) are commonly used materials.

9. Considering a soft iron can be used to form memory cores in computers.

⇒ Now a days, Mn-Mg ferrites with nearly rectangular hysteresis loop are employed to make memory cores of computers.

Metallic glasses are excellent ferromagnets. They possess high magnetic moments, very high permeability and zero magnetostriction. They are hard and corrosion resistant. They are, therefore, used as magnetic head recorders.

10. Considering no application of paramagnetic salts.

⇒ Paramagnetic salts are used to obtain very low temperature (< –272°C) by adiabatic demagnetisation. They are also used in solid state MASERS.

SOLVED PROBLEMS

1. Find the power loss due to hysteresis in a transformer core of volume $0.01\,m^3$ at $50\,Hz$. The area of the loop is $600\,J/m^3$

(a) 300 W (b) 320 W

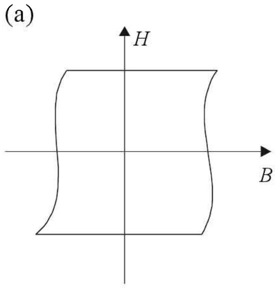
(c) 360 W (d) 400 W

Solution (a) $P_h = AfV$

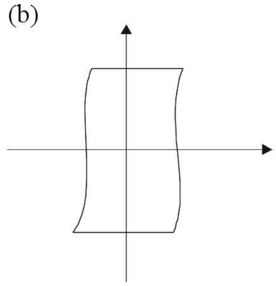
$= 600 \times 50 \times 0.01 = 300\text{W}$

2. The material suitable to make memory core in computers has hysteresis curve

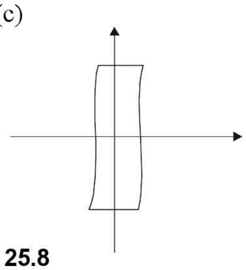
(a)



(b)



(c)



(d) none of these

Fig. 25.8

Solution (b)

3. Low temperature < -272^0 C can be achieved using adiabatic demagnetization of

(a) ferromagnetic salts (b) ferrimagnetic salts

(c) antiferromagnetic salts (d) paramagnetic salts.

Solution (d)

4. Which of the following shows paramagnetism beyond Neel's temperature?

(a) Fe (b) Gd_5 Si

(c) Mn F_2 (d) He

Solution (c) Anti-ferromagnetic substances show paramagnetism beyond Neel's temperature.

5. Self inductance of a coil is $10\,mH$. If a *Si* rod is inserted the self inductance

(a) increases (b) decreases

(c) remains unchanged (d) insufficient data to reply

Solution (b) Si is diamagnetic having $\mu_r < 1$ and

$$L' = \mu_0 \mu_r \frac{N^2 A}{l} \text{ decreases.}$$

6. The relative permeability of .... is zero.

(a) He (b) diamond

(c) graphite (d) super conductor

(e) Si

Solution (d) Only super conductors are perfect diamagnets.

7. When a core is added the self inductance of a coil becomes 50 mH from 10 mH. The substance is .... and its susceptibility is ....

(a) ferrite, 5 (b) ferrite, 4

(c) paramagnetic, 4 (d) ferromagnetic, 4

Solution (b) $\mu_r = \dfrac{L'}{L} = 5$ and $\chi = \mu_r - 1 = 4$

8. A cylindrical iron core has cross-section 5 cm^2 and is inserted in a long solenoid having 2000 turns m^{-1} and carrying a current $2A$. The magnetic field inside the core is 1.57 T. Neglecting the end effects, find the magnetisation I of the core.

(a) 1.0×10^7 Am^{-1} (b) 1.25×10^6 Am^{-1}

(c) 1.57×10^7 Am^{-1} (d) 2.5×10^6 Am^{-1}

Solution (b) $H = ni = 2000 \times 2 = 4000\ \text{Am}^{-1}$

$$B = \mu_0 (H + I)$$

or $$I = \frac{B}{\mu_0} - H = \frac{1.57}{4\pi \times 10^{-7}} - 4000 = 1.25 \times 10^6\ \text{Am}^{-1}$$

9. The magnetic energy density has the form

(a) $\dfrac{B \times H}{2}$ (b) $\dfrac{B.H}{2}$

(c) $\dfrac{\Delta B \times H}{2}$ (d) $\dfrac{\Delta B.H}{2}$

Solution (b) Magnetic energy density

$$\frac{B^2}{2\mu_0} = \frac{B \cdot \left(\dfrac{B}{\mu_0}\right)}{2} = \frac{B \cdot H}{2}.$$

10. The dipole moment associated with an iron atom in an iron bar is 1.8×10^{-23} Am^2. Assume the bar is 5 cm long, has cross-sectional area of 1 cm^2 and all iron atoms have their dipole moment aligned. The dipole moment of the bar is

(a) 5.6 Am^2 (b) 6.6 Am^2

(c) 7.6 Am^2 (d) none of these

Solution (c)

Net dipole moment = dipole moment on one atom × total number of atoms

$$= 1.8 \times 10^{-23} \times \frac{5\times1\times7.8\times6.023\times10^{23}}{56}$$

$= 7.6$ Am^2

11. The magnetic dipole moment of the earth is 6.4×10^{21} Am^2. What current could be set up in a single turn wire going around the earth at its magnetic equator?

(a) 20.48×10^{33} A (b) 10.24×10^{23} A
(c) 5×10^{11} A (d) 5×10^{7} A

Solution (d) $I\pi R^2 = M$

$$I = \frac{M}{\pi R^2} = \frac{6.4\times10^{21}}{3.14\times(6.4\times10^6)^2}$$

$= 5 \times 10^7$ A.

12. $\oint H \cdot dl = ni$ is valid for

(a) all currents (b) displacement current
(c) true current (d) none of these

[CPMT 1992]

Solution (c) H is independent of core material.

13. The area under the hysteresis loop is proportional to

(a) magnetic energy density
(b) thermal energy per unit volume
(c) electrical energy per unit volume
(d) mechanical energy per unit volume

Solution (b)

14. Gadolinium (Gd) belongs to....... magnetic substance.

(a) ferro (b) para
(c) antiferro (d) Ferri
(e) dia

Solution (a)

15. An electron has 10^8 rotations per second in its orbit of radius 0.53 A^0. The magnetic dipole moment associated with it. μ_B is Bohr magneton.

(a) $> 1\mu_B$ (b) $< 1\mu_B$
(c) $= \mu_B$ (d) cannot say

Solution (b)

$M = ef\pi r^2 = 1.6 \times 10^{-19} \times 10^8 \times \pi(.53 \times 10^{-10})^2$

$= 1.28 \times 10^{-31}$ Am^2

16. The material which shows the effect shown in Fig.25.9 when placed in a uniform magnetic field is called

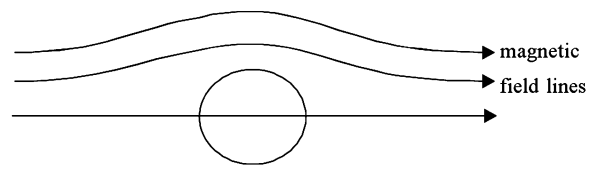


Fig. 25.9

(a) paramagnetic (b) diamagnetic
(c) ferromagnet (d) anti-ferromagnetic

Solution (b) diamagnetic materials show a feeble repulsion.

17. Which of the following represents magnetisation in domains of a single crystal of Ni in strong magnetic field?

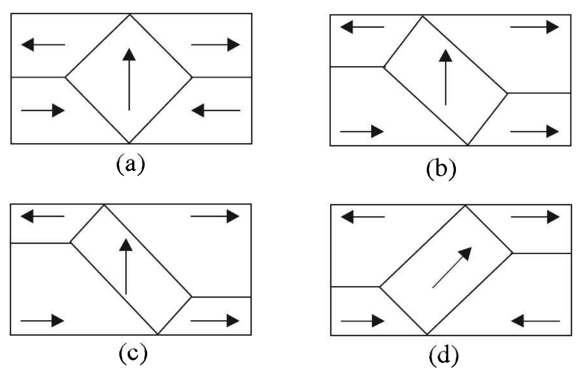


Fig. 25.10

Solution (c) In strong magnetic field, domains in the applied direction of magnetic field grow.

TYPICAL PROBLEMS

18. Assume that each iron atom has a permanent magnetic moment of $2\,\mu_B$ ($1\mu_B = 9.27 \times 10^{-24}$ Am^{-2}) The density of atoms in iron is 8.52×10^{28} m^{-3}. Find the maximum magnetisation I in a long cylinder. Also find maximum magnetic field B on the axis inside the cylinder.

Solution (i) $I = n\,\mu_B N$

$= 2 \times 9.27 \times 10^{-24} \times 8.52 \times 10^{28}$

$= 1.58 \times 10^6$ Am^{-1}

(ii) $B = \mu_0(H + I) = 4\pi \times 10^{-7}(0 + 1.58 \times 10^6) = 2$T

19. The coercive force for a certain permanent magnet is 4×10^4 Am^{-1}. This magnet is placed inside the solenoid of 40 turns cm^{-1} and a current is passed in the solenoid to demagnetise it completely. The current is

(a) 7.8×10^6A (b) 7.8×10^2A
(c) 10^3 A (d) 10A

Solution (d) $H = nI$

$4 \times 10^4 = 40 \times 10^2 I$ or $I = 10$A

20. Find the change in magnetic moment of an electron in H atom in a magnetic field of induction $2T$.

Solution $\Delta M = \pm \dfrac{e^2Br^2}{4m}$

$$= \frac{(1.6\times10^{-19})^2\times2\times(5.1\times10^{-11})^2}{(4\times9.1\times10^{-31})}$$

$= \pm\ 3.7 \times 10^{-29}$ Am^2

21. Find the precession frequency of a proton in a magnetic field of $0.5T$. given $M = 1.4 \times 10^{-26}$ Am^2 and

$L_p = 0.53 \times 10^{-34}$ kg m^2 s^{-1}

Solution $\omega_{precession} = \dfrac{MB}{2\pi L_p}$

$$= \frac{1.4\times10^{-26}\times0.5}{2\times3.14\times0.53\times10^{-34}}$$

$= 2.1 \times 10^7$ Hz.

22. The susceptibility of Mg at 300 K is 1.2×10^{-5}. At what temperature will it rise to 1.8×10^{-5}?

(a) 450 K (b) 300 K
(c) 200 K (d) 150 K
(e) none of these

Solution (c) using $\chi \propto \dfrac{1}{T}$ $\dfrac{\chi_1}{\chi_2} = \dfrac{T_2}{T_1}$ $T_2 = \dfrac{\chi_1 T_1}{\chi_2}$

$$= \frac{300\times1.2\times10^{-5}}{1.8\times10^{-5}} = 200\text{ K}$$

23. The coercive force for a certain permanent magnet is 4×10^4 Am^{-1}. This magnet is placed in a long solenoid having 20 turns per cm. What current be passed to completely demagnetise it?

(a) 10A (b) 20A
(c) 40A (d) 25A

Solution (b) $H = nI$

$\therefore$ $n = 20$ cm^{-1} = 2000 m^{-1}

$$I = \frac{4\times10^4}{2000} = 20A$$

24. An iron ring of mean radius 0.25 m and cross-sectional area 7.5×10^{-4} m^2 is wound uniformly with 1500 turns of wire. A current of 5A is set up in the wire. If relative permeability of iron is 1500 find the magnetic flux density of the ring.

(a) 3T (b) 4.5 T
(c) 6 T (d) 9 T
(e) none of these

Solution (d) $B = \dfrac{N}{2\pi r}\mu_0\mu_r I$

$$= \frac{1500\times4\pi\times10^{-7}\times1500\times5}{2\pi\times.25} = 9\text{T}$$

25. A paramagnetic salt has 2.25×10^{-4} atomic dipoles each having dipole moment 1.95×10^{-23} JT^{-1}. The sample is placed in a magnetic field of 0.39 T and cooled to a temperature 4.5 K. The degree of magnetic saturation achieved is 25%. Find the total magnetic dipole moment at 2.5 K and 0.48 T.

(a) 20.07 JT^{-1} (b) 24 JT^{-1}
(c) 27 JT^{-1} (d) 29.03 JT^{-1}

Solution (c) $\dfrac{M_1}{M_2} = \dfrac{B_1T_2}{T_1B_2}$ or $M_2 = \dfrac{M_1T_1B_2}{B_1T_2}$

$$= \frac{2.5\times10^{24}\times1.95\times10^{-23}\times\frac{1}{4}\times4.5\times.4}{.39\times2.5}$$

$= 27$ JT^{-1}.

QUESTIONS FOR PRACTICE

1. A ferromagnetic material is placed in an external magnetic field. The magnetic domains

(a) increase in size (b) decrease in size
(c) may increase or decrease in size
(d) have no relation with the field.

2. The permanent magnetic moment of the atoms of a material is not zero. The material

(a) must be paramagnetic (b) must be diamagnetic
(c) must be ferromagnetic (d) may be paramagnetic.

3. The magnetic susceptibility is negative for

(a) paramagnetic materials only
(b) diamagnetic materials only

(c) ferromagnetic materials only

(d) paramagnetic and ferromagnetic materials.

4. The dimensions of magnetic induction B are

(a) $M^{-1}A^{-1}T^{-1}$ (b) $MA^{-1}T^{-1}$

(c) $MA^{+1}T^{+2}$ (d) MA^2T^2

5. Tesla is the unit of

(a) H (b) μ

(c) K (d) B

6. The vertical component of earth's magnetic field is zero at

(a) magnetic meridian (b) magnetic poles

(c) geographical poles (d) none of the above

7. The value of Bohr magneton is

(a) $\frac{eh}{2\pi m}$ (b) $\frac{eh}{4\pi m}$

(c) $\frac{eh}{\pi m}$ (d) $\frac{e}{4\pi m}$

8. The liquid in the watch glass in the following figure is

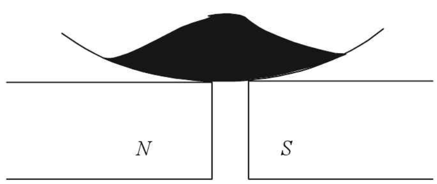


Fig. 25.11

(a) ferromagnetic (b) paramagnetic

(c) diamagnetic (d) any of these

9. Powerful permanent magnets are made of

(a) cobalt (b) aluminium

(c) alnico (d) cobalt-steel

10. A loop of area 0.5 m is placed in a magnetic field of strength 2 Tesla in direction making an angle of 60° with the field. The magnetic flux linked with the loop will be

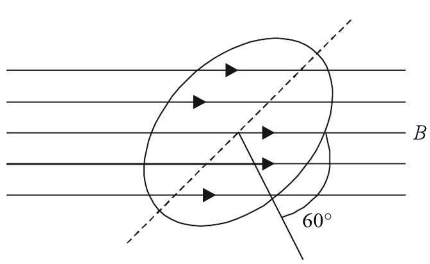


Fig. 25.12

(a) $\frac{1}{2}$ Weber (b) $\sqrt{\left(\frac{3}{2}\right)}$ Weber

(c) 2 Weber (d) $\sqrt{3}$ Weber

11. The correct relation between B, H and I is—

(a) $B = \mu_0(I + H)$ (b) $B = \mu_0 I - H$

(c) $B = \mu_0 \frac{I}{H}$ (d) $B = \mu_0 I \times \mu_0 H$

12. A powerful magnet attracts

(a) only iron and its ores and repels all the rest

(b) only iron and its ores but does not repel any material

(c) all materials

(d) some materials and repels some other materials

13. The magnetic needle at magnetic pole becomes

(a) horizontal

(b) vertical

(c) in any direction

(d) dipped at the angle of dip

14. The distance between the pole pieces of a horseshoe magnet is 4 cm and the pole strength of each pole is 40 cm. The magnetic potential between the pole pieces will be

(a) 20 units (b) 10 units

(c) 80 units (d) zero

15. Magnetic moment is a

(a) tensor (b) vector

(c) scalar (d) neutral quantity

16. The magnetic susceptibility of a paramagnetic substance is 3×10^{-4}. It is placed in a magnetising field of 4×10^3 amp/m. The intensity of magnetisation will be

(a) 3×10^8 A/m (b) 12×10^8 A/m

(c) 12 A/m (d) 24 A/m

17. Volt-second is the unit of

(a) B (b) f

(c) 1 (d) x

18. The magnetic force does not act on

(a) stationary proton (b) moving proton

(c) moving electron (d) moving ion

19. The volume susceptibility of a magnetic material is 30×10^{-4}. Its relative permeability will be

(a) 31×10^{-4} (b) 1.003

(c) 1.0003 (d) 29×10^{-4}

20. Which of the following statements is correct for ferromagnetic material?

(a) These become diamagnetic at Curie temperature

(b) These become paramagnetic at Curie temperature.

(c) Their magnetic susceptibility becomes zero at Curie temperature,

(d) Its magnetic properties are explained on the basis of electron principle.

21. The correct relation between magnetic susceptibility and relative permeability is

(a) $\chi = \mu_r - 1$ (b) $\chi = \mu_r + 1$

(c) $\chi = \mu + 1$ (d) $\chi = \mu - 1$

22. A current of 2 ampere is passed in a coil of radius 0.5 m and number of turns 20. The magnetic moment of the coil is

(a) 0.314 A/m^2 (b) 3.14 A/m^2

(c) 314 A/m^2 (d) 31.4 A/m^2

23. The value of magnetic susceptibility for super-conductors is

(a) zero (b) infinity

(c) + 1 (d) – 1

24. A current of 1 ampere is flowing in a coil of 10 turns with radius 10 cm. Its magnetic moment will be

(a) 0.314 A/m^2 (b) 3140 A/m^2

(c) 100 A/m^2 (d) μ_0 A/m^2

25. A material rod, when placed in a strong magnetic field, aligns itself at right angles to the magnetic field. The nature of material is

(a) diamagnetic (b) paramagnetic

(c) ferromagnetic (d) low ferromagentic

26. The relative permeability of air is

(a) zero (b) 1

(c) infinity (d μ_0

27. For permanent magnets the value of H is

(a) 1 (b) infinity

(c) zero (d) 2.5

28. The coercivity of a bar magnet is 100A/m. It is to be demagnetised by placing it inside a solenoid of length 100 cm and number of turns 50. The current flowing the solenoid will be

(a) 4 A (b) 2 A

(c) 1 A (d) zero

29. The horizontal component of earth's magnetic field is 5×10^{-5} Tesla. The magnetic flux linked with the coil of 10 turns and cross-sectional area 1m will be, if its plane is normal to the magnetic field

(a) 50 Weber (b) 5×10^{-2} Weber

(c) 5×10^{-3} Weber (d) 5×10^{-4} Weber

30. The magnetic field is produced by

(a) variations in electric field

(b) current carrying conductor

(c) a moving charge

(d) all of the above.

31. A current i is flowing in a conductor of length l. When it is bent in the form of a loop then its magnetic moment will be

(a) $\frac{l^2 i}{4\pi}$ (b) $\frac{l^2}{4\pi}$

(c) $\frac{4\pi}{l^2 i}$ (d) $4\pi l^2 i$

32. The slope of $B - H$ curve in magnetic saturation stage is

(a) zero (b) infinity

(c) μ_0 (d) $\frac{1}{\mu_0}$

33. The magnetic property inherent in all materials is

(a) ferromagnetism (b) diamagnetism

(c) paramagnetism (d) non-magnetism

34. When a big hole is made in a magnet, then its magnetic moment becomes

(a) more (b) less

(c) same (d) zero

35. In a ferromagnetic material the volume of each domain and number of atoms in it, are respectively

(a) 10^{-10} m^3 and 10^{20} (b) 10^{20} and 10^{-3} m^3

(c) 10^{-3} m^3 and 10^{-20} (d) 10^{-20} and 10^3 m^3

36. The cause of ferromagnetism is

(a) orbital motion of electrons

(b) spin motion of electrons

(c) permanent dipole moment

(d) neither spin motion nor orbital motion

37. A rod of ferromagnetic material with dimensions 10 cm $\times$ 0.5 cm $\times$ 0.2 cm is placed in a magnetic field of strength 0.5×10^4 amp/m as a result of which a magnetic moment of 5 amp/m^2 is produced in the rod. The value of magnetic induction will be

(a) 0.54 Tesla (b) 0.358 Tesla

(c) 2.519 Tesla (d) 6.28 Tesla

38. The ratio of intensities of magnetic field in the axial and equatorial positions of a magnet will be

(a) 1 : 4 (b) 4 : 1

(c) 1 : 2 (d) 2 : 1

39. Soft iron is used for the construction of electromagnets because for iron

(a) coercive force is high

(b) retentivity is high

(c) area of hysteresis loop is more

(d) magnetic saturation limit is high and coercivity is low

40. The magnetic moment of a bar magnet is 400 A/m^2. If it is cut into four equal parts then magnetic moment of each part will be

(a) 100 A/m² (b) 800 A/m²

(c) 200 A/m² (d) 1600 A/m²

41. A magnet is enclosed by an iron ring as shown in the figure. The magnetic lines of force will be

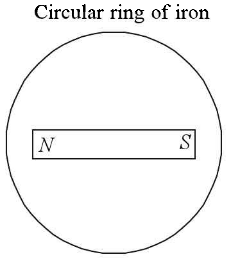


Fig. 25.13

(a) dense in air

(b) dense in ring

(c) uniformly distributed

(d) unaffected

42. The magnetic lines of force due to horizontal component of earth's magnetic field will be

(a) elliptical (b) circular

(c) horizontal and parallel (d) curved

43. The resultant magnetic moment due to two current (i) carrying concentric coils of radius *r*, mutually perpendicular to each other will be

(a) $\sqrt{2}i\pi r^2$ (b) $\sqrt{2}ir^2$

(c) $\sqrt{2}ir$ (d) $\sqrt{2}\pi r^2$

44. The magnetic susceptibility of a paramagnetic material at – 73°C is 0.0075. Then its value at – 173°C will be

(a) 0.0045 (b) 0.0030

(c) 0015 (d) 0.0075

45. The value of Curie temperature for *Ni* is nearly

(a) HOOK (b) 770 *K*

(c) 890 *K* (d) 631*K*

46. Paramagnetism is

(a) an orientation effect

(b) distortion effect

(c) both orientation and distortion effects

(d) neither orientation effect nor distortion effect

47. On placing a piece of ferromagnetic material of cross-sectional area 1cm² in a magnetic field of 200 Oersted, flux density of 3000 Gauss is produced in it. The values of relative permeability and magnetic susceptibility of the material will respectively be

(a) 11.9 and 150 (b) 150 and 11.9

(c) 50 and 11.9 (d) 15 and 14

48. To protect the machine of a watch from external magnetic field, its box should be made of

(a) paramagnetic material

(b) diamagnetic material

(c) ferromagnetic material

(d) non-magnetic material

49. The phenomenon of hysteresis represents the incapability of the material to

(a) magnetic saturation (b) low susceptibility

(c) negative susceptibility (d) retrace the path

50. The temperature, at which a ferromagnetic material gets converted into paramagnetic one, is known as

(a) curie temperature (b) boyle temperature

(c) critical temperature (d) neutral temperature

51. The dimensionless quantity, out of the following, is

(a) μ_r (b) χ

(c) $\dfrac{\mu}{\mu_0}$ (d) all of the above

52. The areas of cross-section of three magnets of same length are *A*, 2*A* and 6*A* respectively. The ratio of their magnetic moments will be

(a) 6 : 2 : 1 (b) 1 : 2 : 6

(c) 1 : 4: 36 (d) 36 : 4 : 1

53. If the radius of a circular coil is doubled and the current flowing in it is halved then the new magnetic moment will be if its initial magnetic moment is

(a) 8 units (b) 4 units

(c) 2 units (d) zero

Answers to Questions for Practice

1.	(c)	**2.**	(d)	**3.**	(b)	**4.**	(b)	**5.**	(d)	**6.**	(d)	**7.**	(b)
8.	(b)	**9.**	(c)	**10.**	(a)	**11.**	(a)	**12.**	(d)	**13.**	(b)	**14.**	(b)
15.	(b)	**16.**	(c)	**17.**	(b)	**18.**	(a)	**19.**	(b)	**20.**	(b)	**21.**	(a)
22.	(d)	**23.**	(d)	**24.**	(a)	**25.**	(a)	**26.**	(b)	**27.**	(c)	**28.**	(b)
29.	(d)	**30.**	(d)	**31.**	(a)	**32.**	(c)	**33.**	(d)	**34.**	(c)	**35.**	(a)
36.	(c)	**37.**	(d)	**38.**	(d)	**39.**	(d)	**40.**	(a)	**41.**	(b)	**42.**	(c)
43.	(a)	**44.**	(c)	**45.**	(d)	**46.**	(a)	**47.**	(d)	**48.**	(c)	**49.**	(d)
50.	(a)	**51.**	(d)	**52.**	(b)	**53.**	(a)						

26

Electromagnetic Induction

BRIEF REVIEW

Michael Faraday while doing experiments on magnets and coils showed that if a magnet is moved in or out of a coil then emf is induced across the coil. If the circuit is complete a current is induced such a current is called **Induced current** and the corresponding emf is called induced emf. Faraday formulated two laws.

First Law The emf/current is induced only for the period when magnetic flux is varying.

Second Law emf induced $\varepsilon = -\frac{d\phi_B}{dt}$ where flux

$\phi_B = \int \vec{B}.\vec{ds}$. Unit of flux is weber or Tm^2.

The current in the loop $= \frac{\varepsilon}{R}$ where R is resistance of the loop.

Lenz's Law The current is induced in a direction so as to oppose the change that has induced it. Thus $\varepsilon = -\frac{d\phi_B}{dt}$.

Lenz's law is based on conservation of energy.

The emf may be induced in two different basic processes: (*a*) motional emf and (*b*) induced electric field. In motional emf coil or conductor is varied with time but magnetic field remains fixed. In induced electric field coil remains fixed and magnetic field varies with time. There could be combination of the two also.

emf $\varepsilon = \oint \vec{E}.\vec{dl} = -\frac{d\phi_B}{dt}$ Note that to have an induced electric field the presence of conducting loop is not necessary. As long as $\vec{B}$ keeps varying, the induced electric field is present. If the loop is present free electrons start drifting and induced current results.

Note that $\oint E.dl \neq 0$, therefore electric field so generated is **nonconservative** and is different from electric field studied in electrostatics. Such an electric field is called **non electrostatic field**. The electric field lines so generated make closed loop like magnetic field lines. Also note that, however, like electrostatic field it gives force $\vec{F} = q\vec{E}$. The current so generated has a similarity to displacement current.

$\because \oint E \cdot dl = E(2\pi r)$. Thus $E = \frac{1}{2\pi r}\left|\frac{d\phi}{dt}\right|$

Self induction $\phi_B \propto i$ or $\phi_B = Li$ or $\varepsilon = -\frac{d\phi_B}{dt} = -L\frac{di}{dt}$.

If a coil has n turns, the flux through each turn is $\int \vec{B} \cdot \vec{ds}$. If this flux varies then $\varepsilon = -N\frac{d}{dt}\int \vec{B} \cdot \vec{ds}$

$L = \mu_0 n^2 Al = \frac{\mu_0 N^2 A}{l}$ where n is number of turns per unit length and N total number of turns, l length of the coil and A its area of cross section as shown in Fig. 26.1.

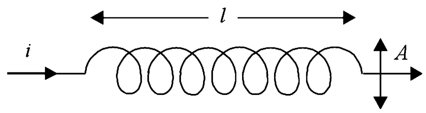


Fig. 26.1 Self inductance illustration

$L = \mu_r \mu_0 n^2 A l$ if a core having relative permeability μ_r is introduced. A coil or a solenoid of thick wire having negligible resistance may be considered as an **ideal inductor**. Unit of self induction is Henry (*H*).

Mutual induction $\varepsilon = -\frac{d\phi_B}{dt} = -M\frac{di}{dt}$. If two coils are placed close to each other and time varying current is passed through one (primary coil) then current is induced in the other (secondary coil) such a phenomenon is called mutual induction. M is mutual inductance of two coils having self inductance L_1 and L_2 (as illustrated in Fig. 26.2).

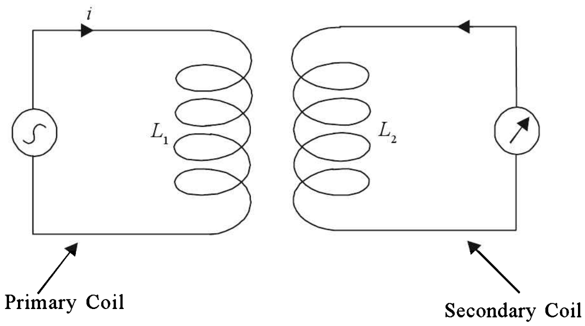


Fig. 26.2 Mutual inductance

Then $M = k\sqrt{L_1L_2}$ where k is coupling factor and $k \leq 1$. $k = 1$ if coils are wound one over the other.

If N_1 are number of turns per unit length in primary coil and N_2 are total number of turns in secondary, then in Fig. 26.3

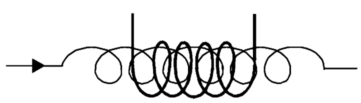


Fig. 26.3 Mutual inductance

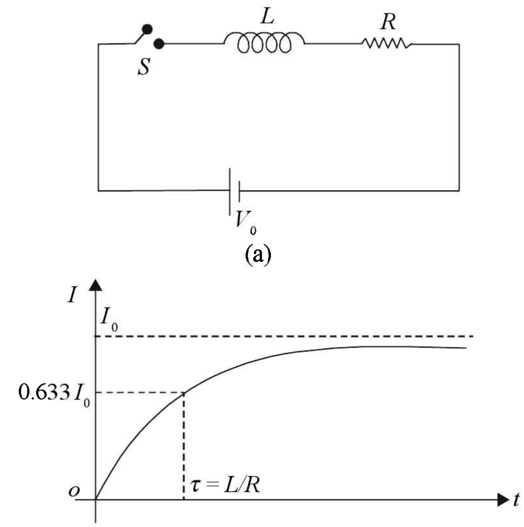
$M = \mu_0 n_1 N_2 A = \mu_0 n_1 n_2 A l$ where l is length of secondary coil. If a core of relative permeability μ_r is introduced then

$M = \mu_0 \mu_r n_1 n_2 A l$. Here $n_2 = \frac{N_2}{l}$ 0

Energy stored in an inductor $U = \frac{Li^2}{2}$ and energy is in the form of magnetic energy.

Growth of current in an *R* – *L* circuit $I(t) = I_0 [1 - e^{-t/\tau})$ where $\tau = \frac{L}{R}$ is the time constant of the circuit $I_0 = \frac{V_0}{R}$.

Time constant $\tau = \frac{L}{R}$ is the time in which the current rises to 63.3% of maximum current I_0 as illustrated in Fig. 26.4

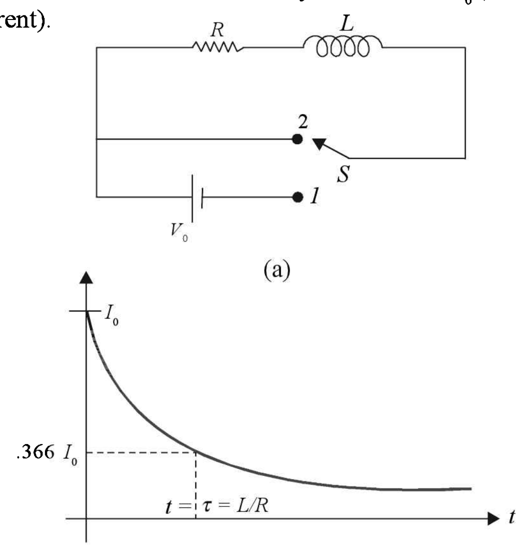


(b)

Fig. 26.4 Growth of current in an inductor

Decay transient After a long time at $t = 0$ the switch S is shifted from position 1 to position 2. Then $I(t) = I_0\, e^{-t/\tau}$

In one time constant current decays to 36.6% of I_0 (maximum current).

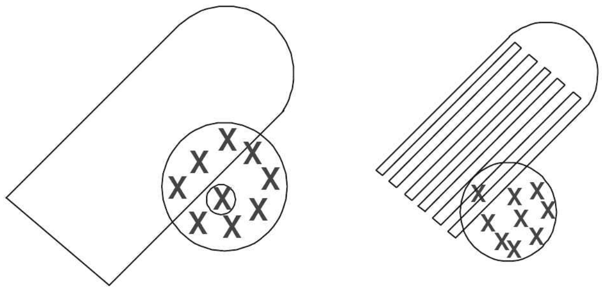


(b)

Fig. 26.5 Decay of current in an inductor

Energy density = $\frac{B^2}{2\mu_0}$ is the magnetic energy per unit volume.

Eddy current Assume a solid plate of metal entering a magnetic field. Consider a loop drawn on the plate, a part of which is in the magnetic field as shown in Fig. 26.6 (*a*). As the plate moves the magnetic flux through the area bounded by the loop changes and hence a current is induced. There may be number of such loops on the plate and hence currents are induced in **random directions**. Such currents are called **eddy currents**. Note that we do not have a definite conducting loop to guide the induced current. Because of eddy currents in the metal plate, thermal energy is produced. This energy comes at the cost of *KE* of the plate, *i.e.*, plates slow down.



(a) (b)

Fig. 26.6 Eddy current illustration

This effect is called **electromagnetic damping**. To reduce electromagnetic damping one can cut slots in the plate. This reduces the possible paths of the eddy current considerably.

AC generator $emf\ \varepsilon = N\omega BA_0 \sin\omega t$ where N is number of turns and A_0 is maximum area and ω is angular frequency. Note $V_P = N\omega BA_0$ is peak voltage. In *AC* generator slip rings are used.

In *DC* generator the scheme is same, however, in place of slip rings, split rings are used so that after each half cycle the direction of emf reverses as illustrated in Fig. 26.7 (b).

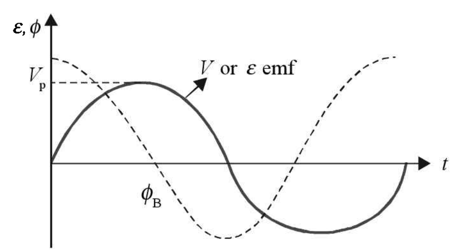


Fig. 26.7(a) Magnetic flux ϕ and voltage V in a AC generator

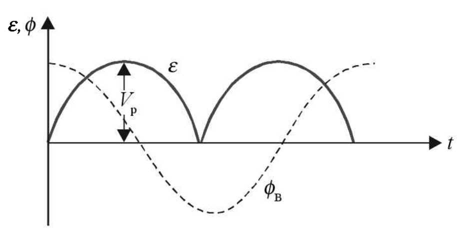


Fig. 26.7(b) Magnetic flux ϕ and voltage V in a DC generator

Displacement current $i_d = \dfrac{E d\phi_E}{dt} = -\dfrac{1}{R}\dfrac{d\phi_B}{dt}$

Slide wire generator See Fig. 26.8. Let R be the resistance of circuit (slide wire + U shaped conductor).

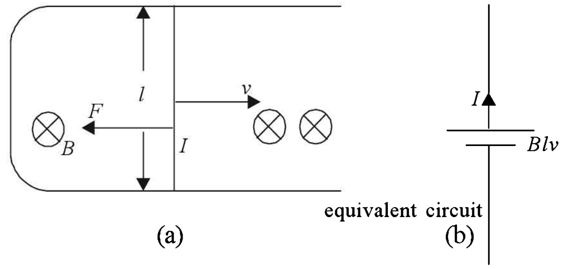


Fig. 26.8 Slide wire generator

$$emf\ \varepsilon = -Blv$$

$$\text{Current } |I| = \frac{Blv}{R}$$

$$\text{Power dissipated } P = I^2R = \frac{B^2l^2v^2}{R}$$

$$F = IlB = \frac{B^2l^2v}{R}$$

We may also write power $P = F.v = \dfrac{B^2l^2v^2}{R}$

Inductances are added in series or parallel like resistances *i.e.* $L_{series} = L_1 + L_2 + ...$

$$\frac{1}{L_{Parallel}} = \frac{1}{L_1} + \frac{1}{L_2} + ...$$

Hall Effect If i is the current in a strip of metal or semiconductor in the direction shown in Fig. 26.9 and B is the magnetic field then a Hall emf is developed in the transverse direction xy. The sign of emf will decide the nature of charge (positive or negative)

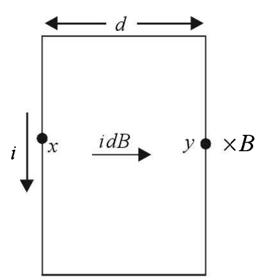


Fig. 26.9 Hall effect illustration

$$E_H = \frac{V_{xy}}{d},\ E_H = -v_d \times B\ ;\ E_H = \frac{JB}{ne}$$

$$\because\ v_d = \frac{J}{ne}$$

Poles of a coil can be found. If the current is clockwise the pole will be south, If the current is anti-clockwise, its pole will be north as shown in Fig. 26.10.

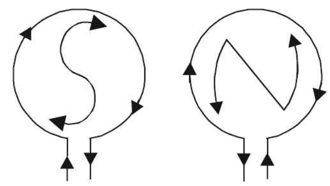


Fig. 26.10 Generator of magnetic pole

- **Short Cuts and Points to Note**

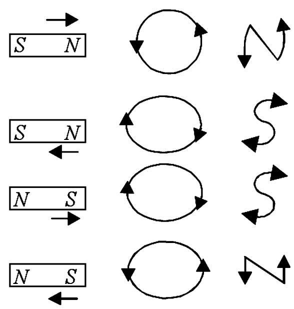


Fig. 26.11 Induced current in different cases

1. Time varying current or *emf* or voltage is *AC*. *AC* voltage and currents are phasors. Phasors are added like vectors. Therefore apply vector laws.
2. Note the directions of currents generated in the coil ring and magnetic pole when magnet moves in or out as shown in Fig. 26.11
3. If magnetic field changes with time and distance then the emf generated $= A\left[\frac{\partial B}{\partial t} + v\frac{\partial B}{\partial z}\right]$ where A is area and v is velocity.
4. When a rod conducting / non conducting of length l moves in a uniform magnetic field emf generated is Blv if B, l and v are mutually perpendicular (See Fig. 26.12)

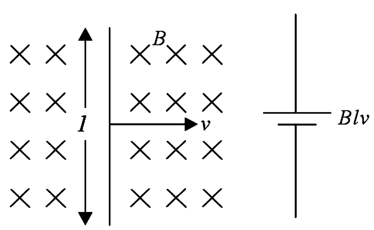


Fig. 26.12 Motional emf

If the velocity vector makes an angle θ with length or with magnetic field then *emf* induced = $Blv \sin\theta$ as illustrated in Fig. 26.13

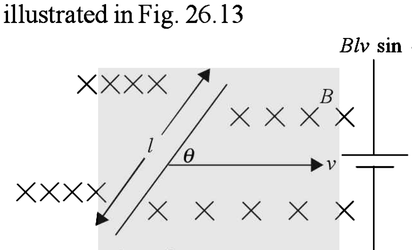


Fig. 26.13 Motional emf

If rod is conducting and a loop is made with conducting wire then current will also be induced and the direction of current will be given by Flemming's Right hand Rule.

Note: εmf can be generated in conducting or non conducting rod. For current to be induced conductor is a must and loop be completed.

5. If a rod OA clamped at O is rotated about O with an angular velocity ω in a uniform magnetic field of strength B then emf induced is

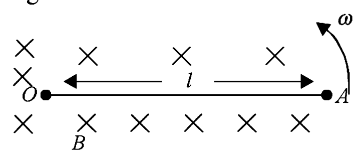


Fig. 26.14

$\varepsilon = \frac{Bl^2\omega}{2} = B\pi l^2 f$ where f is linear frequency. (See Fig. 26.14)

6. If R is the resistance of the circuit. then power consumed in moving the conductor in slide wire generator is $\frac{B^2l^2v^2}{R}$ on the lines $P = \vec{F}\cdot\vec{v}$ and $\vec{F} = IlB = \frac{B^2l^2v}{R}$
7. Fig. 26.15 (a), (b), (c) and (d) illustrate the effect of increasing and decreasing magnetic flux ϕ_B on induced emf and current

(a) $\phi_B > 0$, $\frac{d\phi_B}{dt} > 0$

∴ *εmf* ε and hence *I* are negative

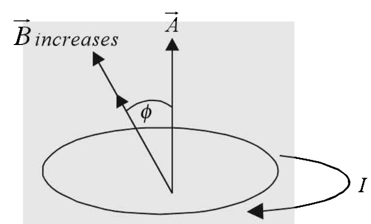


Fig. 26.15(a)

(b) $\phi_B > 0$, $\frac{d\phi_B}{dt} < 0$

∴ ε and hence *I* are positive

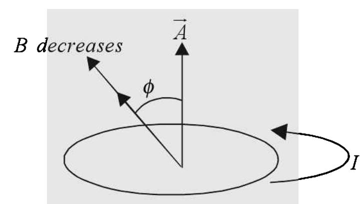


Fig. 26.15(b)

(c) $\phi_B < 0, \frac{d\phi_B}{dt} < 0$

$\therefore \varepsilon$ and hence I are positive

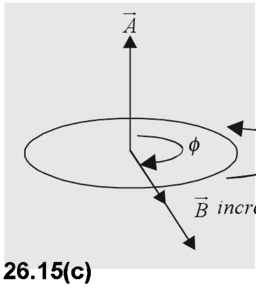


Fig. 26.15(c)

(d) $\vec{B}$ decreasing $\phi_B < 0, \frac{d\phi_B}{dt} > 0$

$\therefore$ ε and hence I are negative

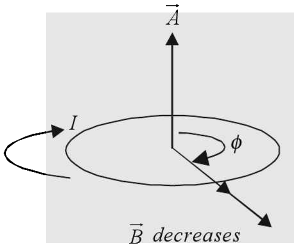


Fig. 26.15(d)

8. Motional *emf* $d\varepsilon = (\vec{v} \times \vec{B}) \cdot \vec{dl}$ and

$\varepsilon = \oint (\vec{v} \times \vec{B}) \cdot \vec{dl}$ If $\vec{B}$, $\vec{l}$ and $\vec{v}$ are mutually perpendicular then $\varepsilon = Blv$

9. If a disc of radius R rotates in a magnetic field B perpendicular to the plane of the rod with an angular velocity ω then *emf* $\varepsilon = \frac{BR^2\omega}{2}$

10. If in a ring magnetic flux is varying then electric field (non-electrostatic) is given by

$$\oint E \cdot dl = -\frac{d\phi_B}{dt} \quad \text{or}$$

$$E = \frac{1}{2\pi r}\left|\frac{d\phi_B}{dt}\right|$$ where r is radius of the ring.

11. Metal detectors work on the principle of eddy currents. The metal detector generates varying magnetic field. This induces eddy current in a conducting object carried, through the detector. The eddy current in turn produces a varying magnetic field B′. The detector's receiver coil receives this varying field and induces a current.

Eddy currents in action is Jupiter's moon *Io*. *Io* moves rapidly through Jupiters intense magnetic field and this sets up strong eddy currents within the interior of *Io*. These currents dissipate energy at a rate of 10^{12}W (= 1 Kiloton) in Io in every 4 *s*. This energy keeps interior of I_o hot and causes volcanic eruption.

12. If time varying current is passed in the inner coil. $B_{out} = 0$. However, magnetic flux per turn through the outer coil is B πr^2. If N are the number of turns in secondary (outer) coil then total flux (see Fig. 26.16)

$$\phi_B = NB\pi r^2 \text{ and } \frac{d\phi_B}{dt} = N\pi r^2 \frac{dB}{dt} = N\pi r^2 \mu_0 n \frac{di}{dt}$$

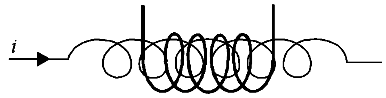


Fig. 26.16

Note that r is the radius of inner coil. Note that flux exists outside the solenoid and is equal to $B\ \pi r^2$ while magnetic field outside is zero.

13. Self inductance of a solenoid $L = \mu_0 \mu r\, n^2 Al = \mu_0 \mu r\, n^2 V$ where V is volume of the coil $V = Al$ and n is number of turns per unit length, l is length of the solenoid and A is area of cross-section.

14. Intrinsic energy of a current in a solenoid

$U = \frac{1}{2} L I^2$. The energy stored is in the form of magnetic energy.

15. If two coils have mutual inductance M and the currents in them are I_1 and I_2 then interaction energy of the two coils (see Fig. 26.17) is given by

$U = M I_1 I_2$

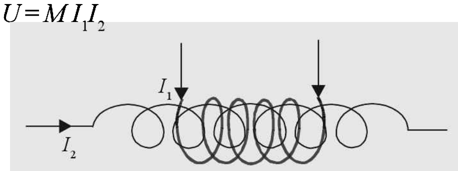


Fig. 26.17

16. Displacement current density $J_{displacement} = \frac{1}{R}\frac{\partial B}{\partial t}$

17. Volume density of magnetic field energy

$$= \frac{B^2}{2\mu_0\mu_r} = \frac{BH}{2}$$

18. Self inductance of a toroid $L = \frac{\mu_0 N^2 A}{2\pi r} = \frac{\mu_0 N^2 r}{2}$

19. Coupling factor $K = \frac{M}{\sqrt{L_1 L_2}}$

$$= \frac{\text{flux linked to secondary coil}}{\text{flux linked to primary coil}}$$

20. Self inductance of two co-axial cylinders per meter (Fig. 26.18) is

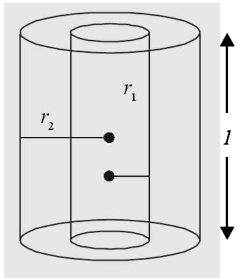


Fig. 26.18

$$L = \frac{\mu_0}{2\pi} \log_e \frac{r_2}{r_1} = \frac{2.303\mu_0}{2\pi} \log_{10} \frac{r_2}{r_1}$$

21. Mutual inductance between two concentric coils having radii r_p and r_s for primary and secondary coils as shown in Fig. 26.19 is

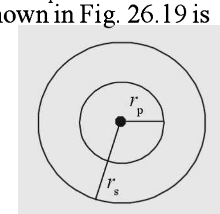


Fig. 26.19

$$M = \frac{\pi\mu_0 N_p N_s r_s^2}{2r_p}$$ where N_s and N_p are number of turns in secondary and primary coils respectively.

22. Inductance in series if mutual inductance is also present $L_{eff} = L_1 + L_2 + 2M$

23. Inductance in parallel if mutual inductance of the two coils is taken into account

$$L_{eff} = \frac{L_1 L_2 + M^2}{L_1 + L_2 + 2M}$$

24. In *RL* transients time for the current to grow 63% = τ (one time constant = L/R)

$t = 2.303\tau \log \frac{I_0}{I_0 - I}$ for growth of current

Time for the current to grow 90% of $I_{max} = 2.303\tau$

Time for the current to grow 95% of $I_{max} = 3\tau$

Time for the current to grow 99% of $I_{max} = 5\tau$

The same times are valid for decay also

$t = 2.303\tau \log \frac{I}{I_0}$ for decay transient.

25. When a magnet falls along the axis of a closed metal ring as shown in Fig. 26.20 its acceleration decreases (due to Lenz law) as it approaches the ring. If the ring is not complete, making open circuit, then acceleration will remain = *g* throughout as induced current will be absent. However emf is induced for decay transient.

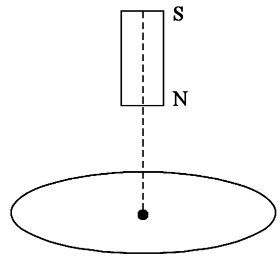


Fig. 26.20

26. Induced charge between time interval Δt is

$$\int i dt = \frac{-1}{R} \int d\phi = \frac{\phi_1 - \phi_2}{R}$$

27. In a generator εmf $\varepsilon = NAo\omega B \sin \omega t$ and peak voltage $V_p = NA_o B\omega$. Generator may also be called an Alternator.

- **Caution**

1. Considering *AC* voltage and currents are scalar like their *DC* counter parts.

⇒ *AC* voltage and currents are phasors. In EMI only *AC* voltage and currents are generated. Note phasors are added like vectors.

2. Assuming electric field generated in EMI (produced due to varying magnetic field) is conservative like electric field in electrostatics.

⇒ The induced electric field lines make a complete loop and $\oint E \cdot dl = \varepsilon \neq 0$. In electrostatics

$\oint E \cdot dl = 0$. Hence field is not conservative.

3. Considering there is no difference between current induced due to EMI and drift current.

⇒ Current induced due to varying magnetic field or varying magnetic flux is displacement current.

4. Assuming that angle between the coils plays no role in determining mutual inductance.

⇒ $M \propto \cos\theta$. If two coils are perpendicular as shown in Fig. 26.21 then $M = 0$. Therefore coupling factor $K = 0$ also.

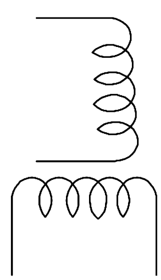


Fig. 26.21

5. While defining flux $B \cdot ds = d\phi$ considering area vector along the plane.

⇒ Area vector is perpendicular to the plane and angle between B and area vector ds be taken as illustrated in Fig. 26.22.

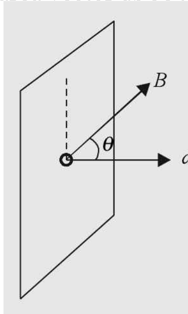


Fig. 26.22

6. Considering magnetic flux produces emf.

⇒ Change in magnetic flux produces emf. It may be produced in 3 ways (a) Area is fixed, B varies

i.e. $\varepsilon = -A \dfrac{dB}{dt}$

(b) B is fixed and area varies *i.e.* $\varepsilon = -B \dfrac{dA}{dt}$

(c) Both area and magnetic fields vary, *i.e.*

$$\varepsilon = -\frac{(B_2 - B_1)(A_2 - A_1)}{t}$$

7. Considering a wire or a cylinder does not possess any self inductance.

⇒ A wire or a cylinder has a very small self inductance. This property is used in the communication systems and rods / cylinders / wires act like antenna or tuner circuit. However $L = \dfrac{\mu_0 mr}{4\pi\rho l}$ where m is mass, r radius, ρ is density and l is length.

8. Not remembering which law be used to find direction of current.

⇒ Flemming's right hand rule which is mirror image of Flemming's left hand rule is used to find direction of current. Flemming's left hand rule is used to find direction of force in a given magnetic field.

9. Considering no emf will be generated in an incomplete ring as no current is induced in the ring when a magnet is falling along the axis of the ring or conductor is moving in the magnetic field.

⇒ emf will be induced. Since induced current is zero, no opposition is caused by the ring to the falling magnet. Hence acceleration = g throughout the motion.

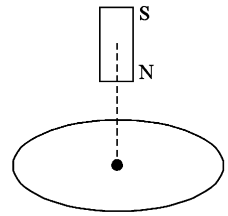


Fig. 26.23

10. If a magnet is falling along the axis of a long Cu cylinder then assuming that acceleration of the magnet < g as in case of a solenoid.

⇒ As the Cu cylinder has nearly zero resistance, it opposes the magnet fully and $a = g - g = 0$

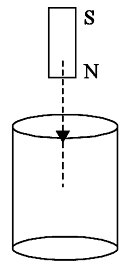


Fig. 26.24

11. Considering there is no effect of temperature when the magnet is falling along the axis of a metal ring.

⇒ If temperature increases, resistance inreases and current falls. Hence opposition to the motion of magnet decreases. Magnet falls faster.

12. Considering when a rod is rotating in a magnetic field emf $\varepsilon = Blv$ where $v = l\omega$

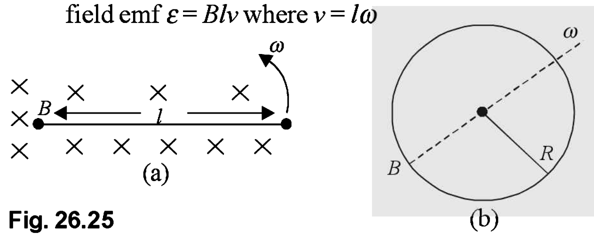


Fig. 26.25 (b)

i.e. $\varepsilon = Bl^2\omega$ similarly for a rotating disc $\varepsilon = BR^2\omega$

⇒ Note the velocity at each point is different, therefore,

for a rod $\varepsilon = \dfrac{Bl^2\omega}{2}$ and for a disc

$$\varepsilon = \int_0^R \omega Brdr = \frac{\omega BR^2}{2} = \int_0^l B\omega x dx$$

SOLVED PROBLEMS

1. A rectangular loop with a slide wire of length l is kept in a uniform magnetic field as shown in Fig. 26.26 (a) The resistance of slider is R. Neglecting self inductance of the loop find the current in the connector during its motion with a velocity v.

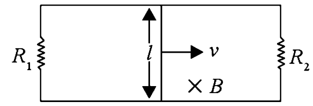


Fig. 26.26(a)

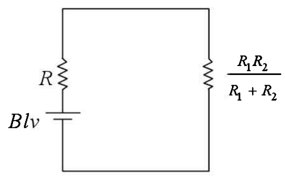


Fig. 26.26(b)

(a) $\dfrac{Blv}{R_1+R_2+R}$ (b) $\dfrac{Blv(R_1+R_2)}{R+(R_1+R_2)}$

(c) $\dfrac{Blv(R_1+R_2)}{RR_1+RR_2+R_1R_2}$ (d) $Blv\left(\dfrac{1}{R_1}+\dfrac{1}{R_2}+\dfrac{1}{R_3}\right)$

Solution (c) The equivalent circuit is shown in Fig. 26.26 (b)

obviously $I = \dfrac{Blv}{R+\dfrac{R_1R_2}{R_1+R_2}} = \dfrac{Blv(R_1+R_2)}{RR_1+RR_2+R_1R_2}$

2. A square wire frame of side a is placed a distance b away from a long straight conductor carrying current I. The frame has resistance R and self inductance L. The frame is rotated by 180^0 about OO′ as shown in Fig. 26.27. Find the electric charge flown through the frame.

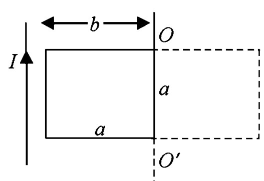


Fig. 26.27

(a) $\dfrac{2\mu_0 ia^2}{2\pi Rb}$ (b) $\dfrac{\mu_0 i}{2\pi R}\log_e\dfrac{b+a}{b-a}$

(c) $\dfrac{\mu_0 ia}{2\pi R}\log_e\dfrac{b+a}{b-a}$ (d) none of these

Solution (c) $i = \dfrac{1}{R}\left[\dfrac{d\phi}{dt}+L\dfrac{di}{dt}\right]$

$$q = \int idt = \frac{1}{R}[\Delta\phi+0] = \frac{\Delta\phi}{R} = \frac{1}{R}\int_{b-a}^{b+a} Badx$$

$$= \frac{1}{R}\int_{b-a}^{b+a}\frac{\mu_0 ia}{2\pi x}dx = \frac{\mu_0 ia}{2\pi R}\log_e\frac{b+a}{b-a}$$

3. One conducting U tube can slide inside the other as shown in Fig. 26.28 maintaining electrical contacts between the tubes. The magnetic field B is perpendicular to the plane of the Fig. 26.28. If each tube moves towards the other at a constant speed v then the induced emf in terms of B, l and v where l is the width of each tube, will be **[AIEEE 2005]**

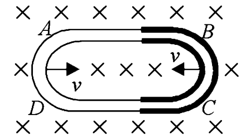


Fig. 26.28

(a) Blv (b) $-Blv$

(c) $2\,Blv$ (d) zero.

Solution (c) $\left|\dfrac{d\phi}{dt}\right| = 2\,Blv$

4. A coil of inductance 300 mH and resistance 2Ω is connected to a source of voltage 2V. The current reaches half of its steady state value in

(a) 0.1 s (b) 0.3 s

(c) 0.05 s (d) 0.05 s

[AIEEE 2005]

Solution (a) $I = I_0(I - e^{\frac{-Rt}{L}})$

$$t = \tau\log_e\left(\frac{I_0}{I_0-I}\right) = \frac{L}{R}\log_e 2 = 0.693\frac{L}{R}$$

$$= 0.693\times\frac{0.3}{2} \simeq 0.1\text{ s}$$

5. As a result of change in magnetic flux linked to the closed loop shown in Fig. 26.29, an emf V volt is induced in the loop. The work done in taking a charge Q coulomb once along the loop is **[CBSE PMT 2005]**

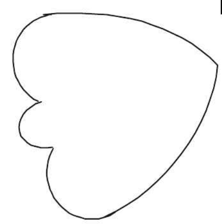


Fig. 26.29

(a) QV (b) 2QV

(c) QV/2 (d) zero

Solution (a) QV because induced electric field so generated is non conservative *i.e.* $\oint E\cdot dl = \text{V}$

6. A conducting ring of radius $1m$ is placed in a uniform magnetic field of 0.01 T oscillating with frequency 100 Hz with its plane at right angle to B. What will be the induced electric field?

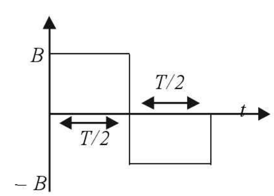


Fig. 26.30

(a) π V/m (b) 2 V/m

(c) 10 Vm^{-1} (d) 62 Vm^{-1}

[AIIMS 2005]

Solution (b) After every T/2 the field will change from B to $-B$ as illustrated in Fig. 26.30. $\varepsilon = 2BAf$

$$\oint E \cdot dl = \varepsilon$$

or $$E = \frac{\varepsilon}{2\pi R} = \frac{2B\pi R^2 f}{2\pi R}$$

$$= BRf = .01 \times 1 \times 200 = 2Vm^{-1}$$

7. A magnet is made to oscillate with a particular frequency passing through a coil as shown in Fig. 26.31. The time variation of the magnitude of emf generated across the coil during one cycle is

[CET Karnataka 2005]

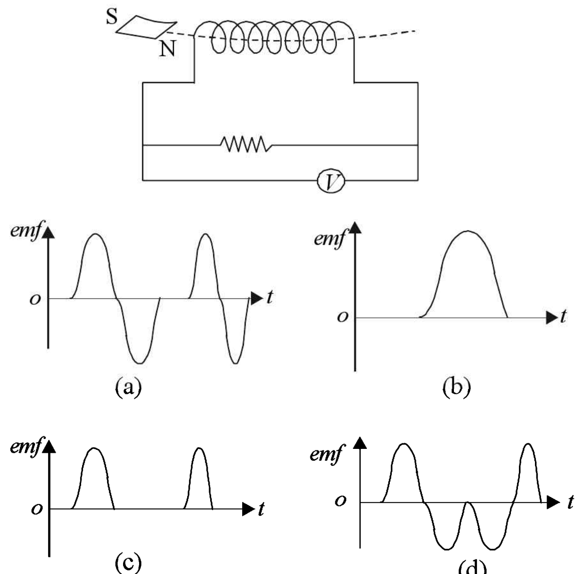


Fig. 26.31

Solution (a)

8. The induction coil works on the principle of

(a) self induction (b) mutual induction

(c) Amperes rule (d) Fleming's Right hand rule.

Solution (b)

9. The coil is wound on an iron core and looped back on itself so that core has two sets of closely wound coils carrying current in opposite directions. The self inductance is

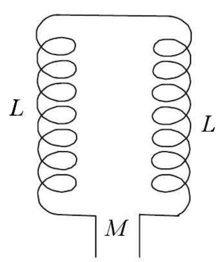


Fig. 26.32

(a) zero (b) 2L

(c) 2L + M (d) L + 2M

Solution (a) Left $= L_1 + L_2 - 2M = L + L - 2\sqrt{LL} = 0$

10. The magnetic flux in a coil is $\phi = 12t^2 + 5t + 1$. The emf induced in 5 s is (ϕ is in milliweber and t in s)

(a) 0 (b) 12.5V

(c) 0.15V (d) 0.125V

[CEE 1996]

Solution (d) $\dfrac{d\phi}{dt}$

$$= (24t + 5) \times 10^{-3}\Big|_{t=5}$$

$$= (24 \times 5 + 5) \times 10^{-3} = 0.125V$$

11. A magnet falls with its S-pole along the axis of a ring. The current generated is ...... and acceleration is ......

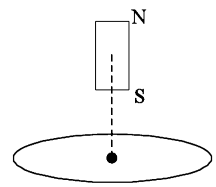


Fig. 26.33

(a) clockwise, > g (b) clockwise, < g

(c) anticlockwise, > g (d) anticlockwise, < g

(e) clockwise, = g

Solution (b) South pole should be formed by the current in the ring. ∴ current is clockwise and south will repel south, hence a < g

12. A long wire carries a current 5A. The energy stored in the magnetic field inside a volume 1mm^3 at a distance 10 cm from the wire is

(a) $\dfrac{\pi}{4} \times 10^{-13}$ J (b) $\dfrac{\pi}{2} \times 10^{-13}$ J

(c) $\pi \times 10^{-13}$ J (d) $\dfrac{\pi}{8} \times 10^{-13}$ J

Solution (d) u (energy per unit volume) $= \dfrac{B^2}{2\mu_0}$ and

energy $$U = \frac{B^2}{2\mu_0} \times vol.$$

$$U = \left(\frac{\mu_0 I}{2\pi d}\right)^2 \times \frac{1}{2\mu_0} \times vol.$$

$$= \frac{\mu_0 I^2}{8\pi^2 d^2} \times vol.$$

$$= \frac{4\pi \times 10^{-7} \times 25 \times 10^{-9}}{8 \times 10 (10^{-2})} = \frac{\pi}{8} \times 10^{-13} J$$

13. If a Bismuth rod is introduced in the air coil as shown then current in the coil

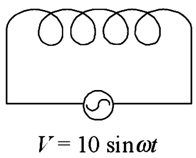


Fig. 26.34

(a) increases (b) remains unchanged

(c) decreases (d) none of these

Solution (a) L will decrease as Bi is diamagnetic

$\therefore \quad I = \dfrac{V}{X_L}$ will increase

14. The voltmater reading in the Fig. 26.35 is

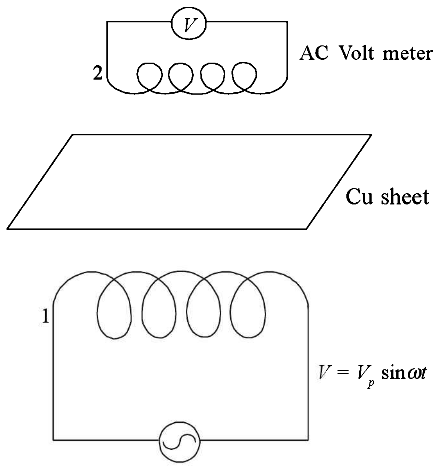


Fig. 26.35

(a) zero (b) $\dfrac{V_p}{\sqrt{2}}$

(c) $\dfrac{V_p}{2}$ (d) none of these

Solution (a) because Cu is diamagnetic, no magnetic flux will link to coil 2.

15. A satellite orbiting the earth at 400 km above the surface of the earth has a $2m$ long antenna oriented perpendicular to the earth's surface. At the equator the earth's magnetic field is $8 \times 10^{-5} T$ and is horizontal. Assuming the orbit to be circular, find emf induced across the ends of the antenna.

(a) 1.3 V (b) 1.2 V

(c) 1.0 V (d) 0.12 V

(e) 0.13 V

Solution (b) $v_0 = \sqrt{(R+h)g}$

emf $= Blv_0 = 8 \times 10^{-5} \times 2 \times 7.2 \times 10^3 = 1.2$ V

16. Assume a long solenoid is wound with 500 turns m^{-1} and current is increasing at 100 As^{-1}, the cross-section of the coil has area 4 cm^2. Find the induced electric field within the loop of radius 2 cm

(a) 2×10^{-4} Vm^{-1} (b) 4×10^{-4} Vm^{-1}

(c) 3×10^{-4} Vm^{-1} (d) none of these

Solution (a) emf $\varepsilon = -\mu_0 nA \dfrac{dI}{dt}$

$= 4\pi \times 10^{-7} \times 500 \times 4 \times 10^{-4} \times 100 = 25 \times 10^{-6}$ V

$\oint E \cdot dl = \varepsilon$ or $E = \dfrac{\varepsilon}{2\pi r}$

$= \dfrac{25\times10^{-6}}{2\pi\times2\times10^{-2}} = 2 \times 10^{-4}$ V

17. A long solenoid of radius 2 cm has 100 turns/cm and is surrounded by a 100 turn coil of radius 4 cm having a total resistance 20Ω. If current changes from $5A$ to $-5A$, find the charge through galvanometer.

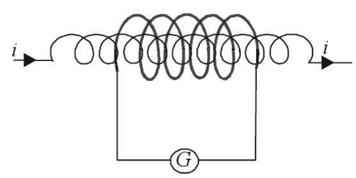


Fig. 26.36

(a) zero (b) 800 μc

(c) 400 μC (d) 600 μC

Solution (b) $\phi = B\pi r^2 \varepsilon = \dfrac{d\phi}{dt} = N\pi r^2 \dfrac{dB}{dt}$

$= N\pi r^2 \mu_0 n \dfrac{di}{dt}$

$I = \dfrac{\varepsilon}{R}$ and $\Delta Q = I\Delta t = \dfrac{N\pi r^2 \mu_0 n}{R} \Delta t$

$\Delta Q = \dfrac{100\times\pi\times\left(2\times10^{-2}\right)^2\times10^4\times4\pi\times10^{-7}\times10}{20}$

$= 8 \times 10^{-4} c = 800\ \mu C$

18. A rod of length l is moved with a velocity v in a magnetic field B as shown in Fig. 26.37. Sketch the equivalent electrical circuit.

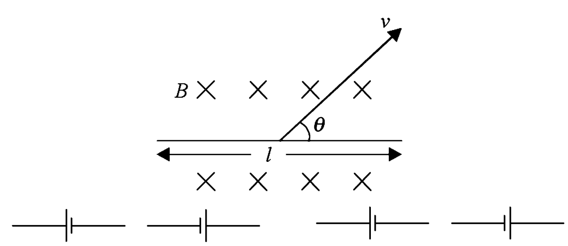


Fig. 26.37

(a) Blv (b) Blv

(c) $Blv \sin\theta$ (d) $Blv \sin\theta$

Solution (c) The positive charge of the rod shifts towards left due to $F = q(\vec{v} \times \vec{B})$.

19. Two conducting circular loops of radii R_1 and R_2 ($R_1 \gg R_2$) are placed in the same plane with their centres coinciding. Find the mutual inductance between them.

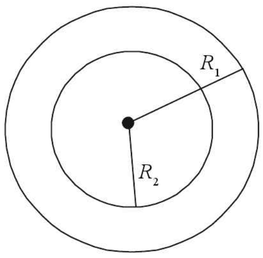


Fig. 26.38

(a) $\dfrac{\mu_0 \pi R_1^2}{R_2}$ (b) $\dfrac{\mu_0 \pi R_2^2}{R_1}$

(c) $\dfrac{\mu_0 \pi R_1^2}{2R_2}$ (d) $\dfrac{\mu_0 \pi R_2^2}{2R_1}$

Solution (d) Assume current i passes through outer loop. Then $B = \dfrac{\mu_0 i}{2R_1}$ and

$$\phi = \frac{\mu_0 i}{2R_1} \pi R_2^2 \text{ using } \phi = Mi, M = \frac{\mu_0 \pi R_2^2}{2R_1}$$

20. In a closed ring A and in an open ring B magnets are falling along the axis of the ring. The current generated in A and B have directions

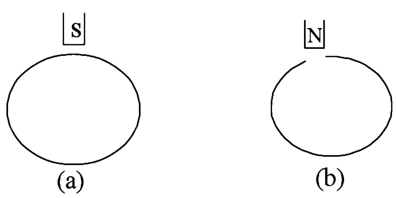


Fig. 26.39

(a) clockwise, anticlockwise

(b) anticlockwise, clockwise

(c) clockwise, zero

(d) anticlockwise, zero

(e) zero, zero

Solution (c) According to Lenz's law the current generated in A shall develop S pole to oppose the cause producing it. Therefore current is clockwise. In B the circuit is open. Therefore no current will flow.

21. A metallic wire bent into a right Δ abc moves with a uniform velocity v as shown in Fig. 26.40. B is the strength of uniform magnetic field perpendicular outwords the plane of triangle. The net emf is ...... and emf along ab is ....

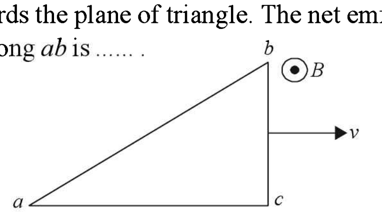


Fig. 26.40

(a) zero, zero

(b) zero, Bv(bc) with b positive

(c) zero, Bv(bc) with a positive

(d) Bv(bc) with c positive, zero

(e) Bv(bc) with b positive, zero.

Solution (c) Net emf and hence net current in a loop moved with uniform velocity is zero because ϕ = constant and

$$\frac{d\phi}{dt} = 0.$$

22. Two rail tracks are 1m apart and insulated from each other and insulated from ground. A millivoltmeter is connected across the railtracks. When a train travelling at 180 km/h passes through what will be the reading in millivoltmeter? Given: horizontal component of earth's field $\sqrt{3} \times 10^{-4}$ T and dip at the place 60°.

(a) 1.5 mV (b) 15 mV

(c) $\dfrac{15}{\sqrt{3}}$ mV (d) $\dfrac{1 \cdot 5}{\sqrt{3}}$ mV

(e) none of these

Solution (b) Vertical component of the magnetic field will be cut. $\varepsilon = B_v lv$ and B_v

$$= B_H \tan\delta = 3 \times 10^{-4} T$$
$$= 3 \times 10^{-4} \times 1 \times 50 = 15\, mV$$

23. A copper wire of length l is bent into a semicircle. It is moved with a velocity v in a region where magnetic field is uniform and perpendicular to the plane of the wire. If the strength of the field is B then emf induced is

(a) Blv (b) $B \dfrac{l}{\pi} v$

(c) $B \dfrac{2l}{\pi} v$ (d) none of these

Solution (c) $\pi r = l$

or $r = \dfrac{l}{\pi}$ $\varepsilon = B(2r)v = B\left(\dfrac{2l}{\pi}\right)v.$

24. A small circular ring is kept inside a larger loop connected to a switch and a battery as shown. The direction of induced current when the switch is made (i) ON (ii) OFF after it was ON for a long time is

(a) clockwise, anticlockwise

(b) clockwise, clockwise

(c) anticlockwise, clockwise

(d) anticlockwise, anticlockwise

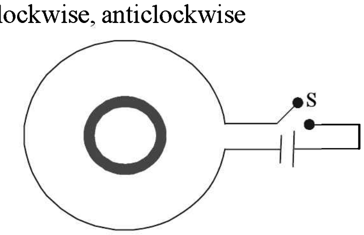


Fig. 26.41

Solution (a) Apply Lenz's law.

25. A square loop of Cu of side a enters a magnetic field spread from $-a$ to $+a$ as shown in Fig. 26.42. Plot induced emf as a function of x.

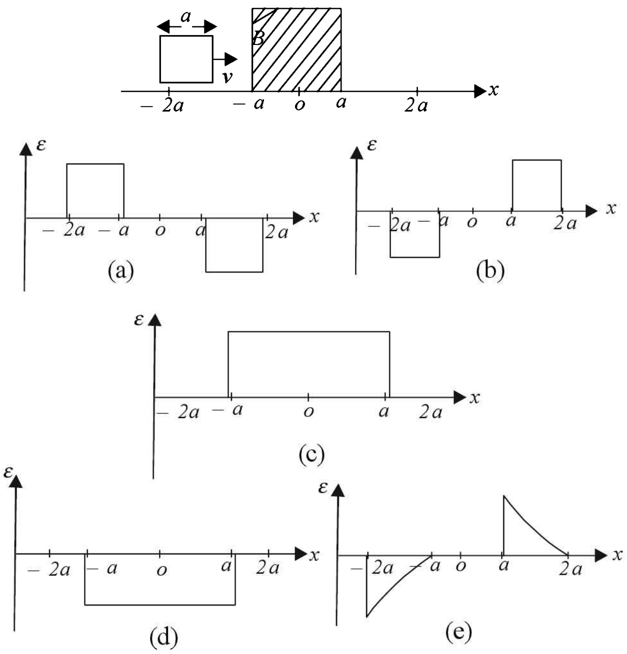


Fig. 26.42

Solution (b) $\because\ \varepsilon = -\dfrac{d\phi}{dx}$

26. The armature of a demonstrator generator consists of a flat square coil of side 4 cm and 200 turns. The coil rotates in a magnetic field of 0.75 T. The angular speed so that a maximum emf of 1.6 V is generated is .....

(a) $\dfrac{20}{3}$ rad/s (b) $\dfrac{20}{3}$ rotations/s

(c) $\dfrac{20}{3}$ rpm (d) none of these

Solution (a) $\varepsilon_{max} = NA_0B\omega$ or

$$\omega = \frac{\varepsilon_{max}}{NA_0B} = \frac{1.6}{200\times16\times10^{-4}(.75)} = \frac{20}{3}\ rad/s$$

27. The electric flux through a certain area of dielectric is $8.76\times10^3\,t^4$. The displacement current through the area is 12.9 pA at $t = 26.1$ ms. Find the dielectric constant of the material.

(a) 2×10^{-8} (b) 4×10^{-8}

(c) 8×10^{-8} (d) 2×10^{-7}

Solution (a) $i_D = \varepsilon\dfrac{d\phi_E}{dt}$ or

$$\varepsilon = \frac{i_D}{\frac{d\phi_E}{dt}}$$

$$\varepsilon = \frac{12.9\times10^{-9}}{4(8.76)\times10^3\times(26.1\times10^{-3})^3} \simeq 2\times10^{-8}$$

TYPICAL PROBLEMS

28. Magnetic flux during time interval τ varies through a stationary loop of resistance R as $\phi_B = at(\tau - t)$. Find the amount of heat generated during that time. Neglect the inductance of the loop.

(a) $\dfrac{a^2\tau^3}{R}$ (b) $\dfrac{a^2\tau^2}{2R}$

(c) $\dfrac{a^2\tau^3}{3R}$ (d) $\dfrac{a^2\tau^3}{4R}$

Solution (c) $i = \dfrac{d\phi}{dt}/R = \dfrac{a(\tau-2t)}{R}$

Heat produced $H = \int_0^\tau i^2R\,dt$

$$H = \int_0^\tau \frac{a^2(\tau-2t)^2}{R}\,dt = \frac{a^2\tau^3}{3R}$$

29. Find the steady state current through L_1 in the Fig. 26.43

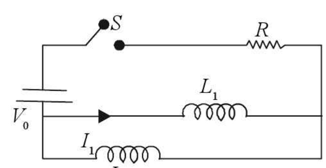


Fig. 26.43

(a) $\dfrac{V_0}{R}$ (b) $\dfrac{V_0L_1}{R(L_1+L_2)}$

(c) $\dfrac{V_0L_2}{R(L_1+L_2)}$ (d) none of these

Solution (c) $I_0 = \dfrac{V_0}{R}$ divide the current in L_1 and L_2 like resistors $I_1 = I_0\dfrac{L_2}{L_1+L_2}$

30. Two identical galvanometers are joined by connecting wires. One of them is placed on the table and the other is held in the hand. One in the hand is shaken violently so that it shows a deflection of 10 division. The reading in the other galvanometer (on the table) is

(a) zero (b) 10 division

(c) 5 division (d) insufficient data to reply.

Solution (b) The one shaken voilently acts as a generater and the other reads the emf generated.

31. A wire shaped as a semicircle of radius r rotates about the axis OO' with an angular velocity ω as shown in Fig. 26.44. Resistance of the circuit is R. Find the mean thermal power generated in the loop during a rotation period.

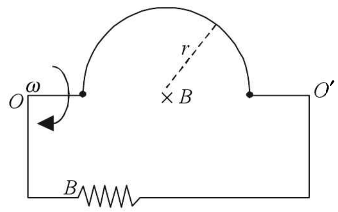


Fig. 26.44

Solution $A = \pi a^2 \cos\omega t$; $\phi = BA = B\pi a^2 \cos\omega t$

$$i = \frac{\varepsilon}{R} = -\frac{d\phi}{dt}/R = \frac{B\pi a^2\omega \sin\omega t}{R} \quad \text{and}$$

$$\langle P \rangle = \frac{1}{T}\int_0^T i^2 R\,dt = \frac{\left(B\pi a^2\omega\right)^2}{2R}$$

32. A long wire carries a current i. A rod of length l is moved with a velocity v in a direction parallel to the wire as shown in Fig. 26.45 (a). Find the motional emf induced in the rod.

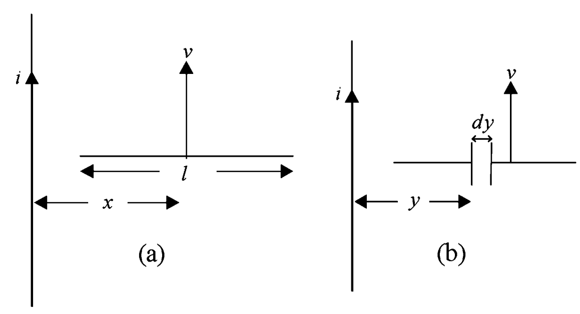


Fig. 26.45

(a) $\frac{\mu_0 i}{2\pi x} lv$ (b) $\frac{\mu_0 i}{2\pi} v \log_e \frac{x+l/2}{x-l/2}$

(c) $\frac{\mu_0 i}{2\pi} v\log_e \frac{x-l/2}{x+l/2}$ (d) $\frac{\mu_0 iv}{2\pi} \log_e \frac{l+x}{x}$

Solution (b) Consider small element dy at distance y from the long wire as shown in Fig 26.42 (b) $B = \frac{\mu_0 i}{2\pi y}$ and emf in element dy is

$$d\varepsilon = Bvdy. \text{ Thus } \varepsilon = \int_{x-l/2}^{x+l/2} \frac{\mu_0 iv}{2\pi}\frac{dy}{y}$$

$$= \frac{\mu_0 iv}{2\pi} \log_e \frac{x+l/2}{x-l/2}$$

33. A current in a 240 turn solenoid varies at 0.8 A/s. Find emf induced if the length of the solenoid is 12 cm and radius 2 cm

(a) 6.14×10^{-4} V (b) 6.4×10^{-3} V

(c) 3.07×10^{-3} V (d) 3.07×10^{-4} V

Solution (a) $\varepsilon = L\frac{di}{dt}$

$$= \frac{\mu_0 N^2\left(\pi r^2\right)}{l}\frac{di}{dt}$$

$$= \frac{4\pi\times10^{-7}\times(240)^2\times(\pi)\left(2\times10^{-2}\right)^2\times0.8}{12\times10^{-2}}$$

$= 6.14 \times 10^{-4}$ V

34. The magnetic field inside a $2mH$ inductor becomes 0.8 of its maximum value in $20\mu s$ when the inductor is joined to battery. Find resistance of the circuit.

(a) 160 Ω (b) 80 Ω

(c) 320 Ω (d) 240 Ω

(e) none of these

Solution (a) $i \propto B$. ∴ current also becomes 0.8 of its maximum value

$$t = \tau \log_e \frac{i_0}{i_0 - i}$$

or $$20\times10^{-6} = \frac{2\times10^{-3}}{R}\log_e 5$$

$R = 100 \times 2.303\,(.6990)$

$= 160\ \Omega$

35. A wire bent as a parabola $y = kx^2$ is located in a uniform magnetic field of induction B, the vector B being perpendicular to the plane xy. At $t = 0$, sliding wire starts sliding from the vertex O with a constant acceleration a linearly as shown in Fig. 26.46. Find the emf induced in the loop.

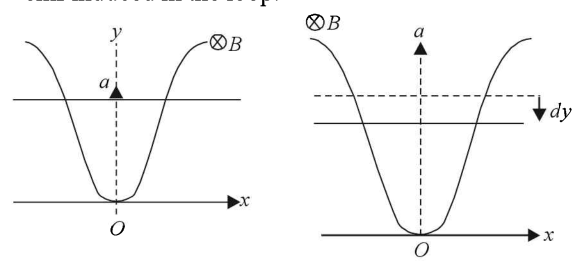


Fig. 26.46

(a) $By\sqrt{\frac{2a}{k}}$ (b) $By\sqrt{\frac{4a}{k}}$

(c) $By\sqrt{\frac{8a}{k}}$ (d) $By\sqrt{\frac{a}{k}}$

Solution (c) $d\phi = B \cdot dA = 2Bx\,dy$ and $y = kx^2$

$$\therefore \quad x = \sqrt{\frac{y}{k}}$$

$$\therefore \quad \text{emf}\,|\varepsilon| = \frac{d\phi}{dt} = 2B\sqrt{\frac{y}{k}}\,\frac{dy}{dt}$$

using $v^2 = 2as$

$$\frac{dy}{dt} = v = \sqrt{2ay}$$

$$\text{or} \quad |\varepsilon| = \frac{d\phi}{dt} = 2\text{B}\sqrt{\frac{y}{k}}\,\sqrt{2ay}$$

$$\text{or} \quad |\varepsilon| = \text{By}\sqrt{\frac{8a}{k}}$$

36. A long wire carrying current i is placed close to a u shaped conductor (of negligible resistance). A wire of length l as shown in Fig. 26.47 slides with a velocity v. Find the current induced in the loop as a function of distance x from the current carrying wire to slider.

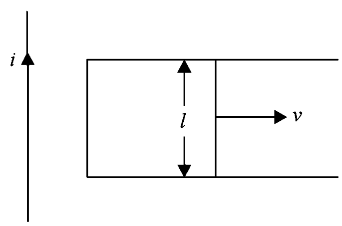


Fig. 26.47

(a) $\frac{\mu_0 ilu}{Rx}$ (b) $\frac{\mu_0 ilu}{2\pi Rx}$

(c) $\frac{\mu_0 ilu}{2Rx}$ (d) $\frac{\mu_0 ilu}{4\pi Rx}$

Solution (b) $B = \frac{\mu_0 i}{2\pi x}$ and $\varepsilon = Blv = \frac{\mu_0 ilv}{2\pi x}$ and

$$I = \frac{\varepsilon}{R} = \frac{\mu_0 ilu}{2\pi Rx}$$

37. A square frame with side a as shown in Fig. 26.48 is moved with a velocity v from a long straight wire carrying current I. Initial separation between straight long wire and square frame is x. Find the emf induced in the frame as a function of distance x.

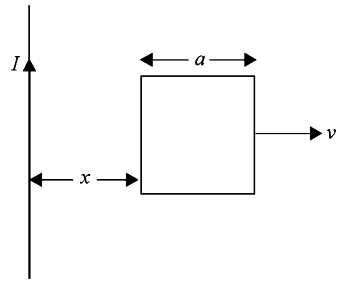


Fig. 26.48

(a) $\frac{\mu_0 Ia^2v}{2\pi x(x+a)}$ (b) $\frac{\mu_0 Iaxv}{2\pi x(x+a)}$

(c) $\frac{\mu_0 Ia^2v}{4\pi x(x+a)}$ (d) zero

Solution (a) $\varepsilon_1 = \frac{\mu_0 Iav}{2\pi x}$

$$\text{and} \quad \varepsilon_2 = \frac{\mu_0 Iav}{2\pi(x+a)}$$

$$\varepsilon_{\text{net}} = \varepsilon_1 - \varepsilon_2$$

$$= \frac{\mu_0 Iav}{2\pi}\left[\frac{1}{x} - \frac{1}{x+a}\right]$$

$$= \frac{\mu_0 Ia^2v}{2\pi x(x+a)}$$

QUESTIONS FOR PRACTICE

1. A cycle wheel with 64 spokes is rotating with N rotations per second at right angles to horizontal component of magnetic field. The induced emf generated between its axle and rim is E. If the number of spokes is reduced to 32 then the value of induced emf will be

(a) E (b) $2E$

(c) $\frac{E}{2}$ (d) $\frac{E}{4}$

2. When a current of 5 A flows in the primary coil then the flux linked with the secondary coil is 200 weber. The value of coefficient of mutual induction will be

(a) 1000 H (b) 40 H

(c) 195 Henry (d) 205 H

3. The direction of induced current in a coil or circuit is such that it opposes the very cause of its production. This law is given by

(a) Faraday (b) Kirchhoff

(c) Lenz (d) Ampere

4. The expression for induced charge in a coil is

(a) $q = \frac{N}{R}(\phi_1 - \phi_2)$ (b) $q = R(\phi_1 - \phi_2)$

(c) $q = \frac{NR}{(\phi_1 - \phi_2)}$ (d) $q = (\phi_1 - \phi_2)/NR$

5. When a conductor is rotated in a perpendicular magnetic field then its free electrons
   (a) move in the field direction
   (b) move at right angles to field direction
   (c) remain stationary
   (d) move opposite to field direction

6. The maximum value of induced emf in a coil rotating in magnetic field does not depend on
   (a) the resistance of coil
   (b) the number of turns in the coil
   (c) the area of the coil
   (d) rotational frequency of the coil

7. The coefficient of mutual induction between two coils is 4 H. If the current in the primary reduces from 5A to zero in 10^{-3} second then the induced emf in the secondary coil will be
   (a) $10^4\ V$ (b) $25 \times 10^3\ V$
   (c) $2 \times 10^4\ V$ (d) $15 \times 10^3\ V$

8. A transformer is used to
   (a) convert DC into AC
   (b) convert AC into DC
   (c) obtain the required DC voltage
   (d) obtain the required AC voltage

9. When a conducting ring is moved in a magnetic field then the total charge induced in it depends on
   (a) initial magnetic flux
   (b) final magnetic flux
   (c) the rate of change of magnetic flux
   (d) the total change in magnetic flux

10. When the number of turns per unit length in a solenoid is doubled then its coefficient of self induction will become
   (a) half (b) double
   (c) four times (d) unchanged

11. The number of turns in the primary and secondary coils of a transformer are 100 and 300 respectively. If the input power is 60 watt the output power will be
   (a) 3×10^3 Watt (b) 20 Watt
   (c) 60 Watt (d) 180 Watt

12. A magnetic field is directed normally downwards through a metallic frame as shown in the figure. On increasing the magnetic field

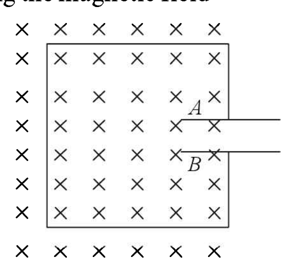


Fig. 26.49

   (a) plate B will be positively charged
   (b) plate A will be positively charged
   (c) none of the plates will be positively charged
   (d) all of the above

13. The coefficient of mutual induction between two coils is 1.25 H. If the rate of fall of current in the primary is 80 As^{-1}, then the induced emf in the secondary coil will be
   (a) 100 V (b) 64 V
   (c) 12.5 V (d) 0.016 V

14. Same current is flowing in two identical coaxial circular coils. On looking at the coils from a point at the centre between them, the current in one coil appears to be flowing in clockwise direction. If the two coils are displaced towards each other, then the value of current
   (a) in each coil will decrease
   (b) in each coil will increase
   (c) in each coil will remain unchanged
   (d) nothing can be predicted

15. A 1.2 m wide railway track is parallel to magnetic meridian. The vertical component of earth's magnetic field is 0.5 Gauss. When a train runs on the rails at a speed of 60 Km/hr, then the induced potential difference between the ends of its axle will be
   (a) $10^{-4}\ V$ (b) $2 \times 10^{-4}\ V$
   (c) $10^{-3}\ V$ (d) zero

16. The number of turns in an air core solenoid of length 25 cm and radius 4 cm is 100. Its self inductance will be
   (a) $5 \times 10^{-4}\ H$ (b) $2.5 \times 10^{-4}\ H$
   (c) $5.4 \times 10^{-3}\ H$ (d) $2.5 \times 10^{-3}\ H$

17. Two coils P and Q are lying a little distance apart coaxially. If a current I is suddenly set up in the coil P then the direction of current induced in coil Q will be—

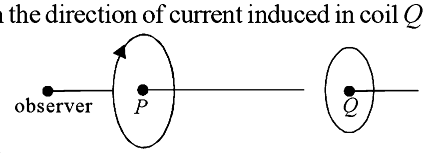


Fig. 26.50

   (a) clockwise (b) towards north
   (c) towards south (d) anticlockwise

18. A straight copper wire is moved in a uniform magnetic field such that it cuts the magnetic lines of force. Then
   (a) emf will not be induced
   (b) emf will be induced
   (c) sometimes emf will be induced and sometimes not
   (d) nothing can be predicted

19. A small straight conductor PQ is lying at right angles to an infinite current carrying conductor XY. If the conductor PQ is displaced on metallic rails parallel to the conductor XY then the direction of induced emf in PQ will be

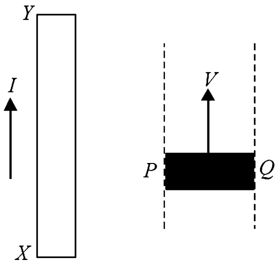


Fig. 26.51

(a) from Q to P (b) from P to Q
(c) vertically downwards (d) vertically upwards

20. A conducting rod PQ is moving parallel to X-axis in a uniform magnetic field directed in positive Y-direction. The end P of the rod will become

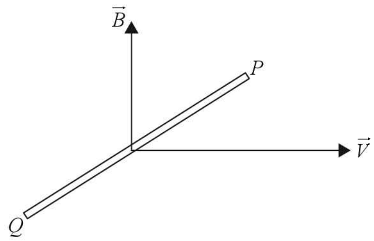


Fig. 26.52

(a) negative (b) positive
(c) neutral (d) sometimes negative

21. Two coils P and Q are lying parallels and very close to each other. Coil P is connected to an AC source whereas Q is connected to a sensitive galvanometer. On pressing key K

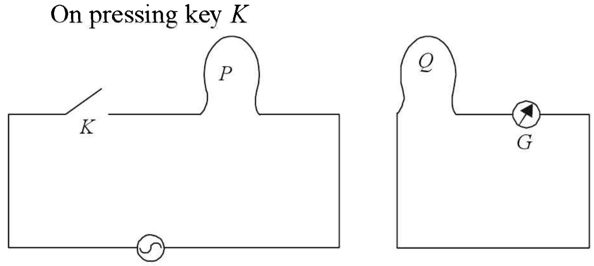


Fig. 26.53

(a) small variations are observed in the galvanomenter for applied 50 Hz voltage
(b) deflections in the galvanometer can be observed for applied voltage of 1 Hz to 2 Hz.
(c) no deflection in the galvanometer will be observed
(d) constant deflection will be observed in the galvanometer for 50 Hz supply voltage

22. The dimensions of RC are

(a) MLT^{-1} (b) $M^0L^0T^1$
(c) $M^0L^0T^{-1}$ (d) $M^{-1}L^{-1}T^{-1}$

23. A coil of insulating wire is connected to battery. If it is moved towards a galvanometer then its pointer gets deflected because

(a) the coil behaves like a magnet
(b) induced current is produced in the coil
(c) the number of turns in the galvanometer coil remains constant
(d) none of the above

24. The coefficient of self induction of a coil is given by

(a) $L = \left(-\frac{dI}{dt}\right)$ (b) $L = -\frac{edI}{dt}$

(c) $L = \frac{dI}{edt}$ (d) $L = \frac{dI}{dt}e^2$

25. When a coil of cross-sectional area A and number of turns N is rotated in a uniform magnetic field B with angular velocity ω, then the maximum emf induced in the coil will be

(a) BNA (b) $\frac{Ba\omega}{N}$
(c) $BNA\omega$ (d) zero

26. Two inductance coils, of same self inductance, are connected in parallel and the distance between them is large. The resultant self inductance of the coil will be

(a) $\frac{L}{4}$ (b) $2L$
(c) L (d) $\frac{L}{2}$

27. The resistance coils in a resistance box are made of double folded wire so that their

(a) self induction effect in nullified
(b) self inductance is maximum
(c) induced emf is maximum
(d) induced emf is zero

28. If a spark is produced on removing the load from an AC circuit then the element connected in the circuit is

(a) high resistance (b) high capacitance
(c) high inductance (d) high impedance.

29. The time constant in an L-R circuit is that time in which the value of current in the circuit becomes

(a) I_0 (b) $\frac{I_0}{2}$
(c) 63% I_0 (d) 37% I_0

30. The expression for the induced emf generated in a coil as a result of change in magnetic flux linked with it is

(a) $e = -A\frac{dB}{dt}$ (b) $e = -B\frac{dA}{dt}$

(c) $e = \frac{d}{dt}(\vec{A}.\vec{B})$ (d) $e = \frac{d}{dt}(\vec{A}x\vec{B})$

31. The turns ratio (r) for a step-up transformer is

(a) $r < 1$ (b) $r = 1$

(c) $r > 1$ (d) $r = 0$

32. Eddy currents can be minimised by

(a) using a laminated iron core

(b) moving the conductor rapidly

(c) moving the conductor slowly

(d) using a metallic core

33. The cause of production of eddy currents is

(a) the motion of a conductor in a varying magnetic field

(b) the motion of an insulator in a varying magnetic field

(c) current flowing in a conductor

(d) current flowing in an insulator

34. The correct relation between the impedance of secondary coil with that of primary coil is

(a) $Z_S = Z_P$ (b) $Z_S = Z_P \frac{N_S}{N_P}$

(c) $Z_S = Z_P \left(\frac{N_S}{N_P}\right)^2$ (d) $Z_S = Z_P \left(\frac{N_P}{N_S}\right)^2$

35. The long distance transmission of electrical energy is done at

(a) high potential and low current

(b) low potential and high current

(c) high potential and high current

(d) low potential and low current

36. Starter is used in

(a) high power electric motors

(b) low power electric motors

(c) transformers

(d) galvanometers

37. If the turns ratio of a transformer is 2 and the impedance of primary coil is 250 W then the impedance of secondary coil will be

(a) 1000 Ω (b) 500 Ω

(c) 250 Ω (d) 125 Ω

38. Number of turns in a generator coil is 10. The area of this coil is 4×10^{-2}m^2. This coil is rotating at the rate of 20 rotations/sec, about an axis lying in its own plane in a perpendicular magnetic field of 0.3 Tesla. The maximum emf induced in the coil will be

(a) 30.2 V (b) Zero

(c) 15.1 V (d) 60.4 V

39. The efficiency of a transformer is

(a) $\eta < 1$ (b) $\eta = 1$

(c) $\eta > 1$ (d) $\eta = 0$

40. Which of the remain constant in a transformer?

(a) current (b) potential

(c) power (d) frequency

41. If the current in the primary coil and number of turns in it are I_p and N_p respectively and the number of turns and current in the secondary are Ns and Is respectively then the value of $N_s : N_p$ will be

(a) $I_S : I_P$ (b) $I_P : I_S$

(c) $I_S^2 : I_P^2$ (d) $I_P^2 : I_S^2$

42. A metallic circular ring is suspended by a string and is kept in a vertical plane. When a magnet is approached towards the ring then it will

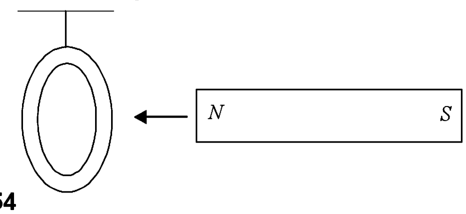


Fig. 26.54

(a) remain stationary

(b) get displaced away from the magnet

(c) get displaced towards the magnet

(d) nothing can be said

43. A current is flowing in a wire C as shown in the figure. The force on this conducting wire will be

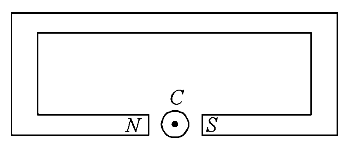


Fig. 26.55

(a) towards right (b) towards left

(c) downwards (d) upwards

44. The voltage in the primary and the secondary coils of a step up transformer are 200 V and 4 KV respectively. If the current in the primary is 1 ampere then the current in the secondary coil will be

(a) 50 mA (b) 500 mA

(c) 5A (d) 5 mA

45. If 2.2 kilowatt power is being transmitted at 44 KV on a 20 Ω line, then power loss will be

(a) 0.1 watt (b) 1.4 watt

(c) 100 watt (d) 0.05 watt

46. The rate of change of magnetic flux density through a circular coil of area 10 m and number of turns 100 is 10^3 Wb/m^2/s. The value of induced emf will be

(a) 10^{-2} V (b) 10^{-3} V

(c) 10 V (d) 10^3 V

47. The magnetic fields through two identical rings made of copper and wood are changing at the same rate. The induced electric field in copper ring will be

(a) more than that in the wooden ring

(b) less than that in the wooden ring

(c) finite and that in the wooden ring will be zero

(d) same as that in the wooden ring

48. The length of side of a square coil is 50 cm and number of turns in it is 100. If it is placed at right angles to such a magnetic field which is changing at the rate of 4 Tesla/s then induced emf in the coil will be

(a) 0.1 V (b) 1.0 V

(c) 10 V (d) 100 V

49. The number of turns in the primary and secondary coils of a step-down transformer are 200 and 50 respectively. If the power in the input is 100 watt at 1A then the output power and current will respectively be

(a) 100 W, 2A (b) 200 W, 2A

(c) 400 W, 4A (d) 100 W, 4A

50. A transformer changes 220 V to 22 V. If the currents in the primary and secondary coils are 10 A and 70 A respectively then its efficiency will be

(a) 100% (b) 90%

(c) 70% (d) 80%

51. If the magnetic field in the following figure is increased then

Fig. 26.56

(a) plate A of the condenser will get positively charged

(b) plate B of the condenser will get positively charged

(c) the condenser will not be charged

(d) both the plates will be charged alternately

52. An e.m.f. of 15 volt is applied in a circuit of inductance 5 henry and resistance 10 Ω. The ratio of currents flowing at $t = \infty$ and $t = 1$ second will be

(a) $\frac{e^{1/2}}{e^{1/2}-1}$ (b) $\frac{e^2}{e^2-1}$

(c) $1-e^{-1}$ (d) e^{-1}

53. Two circular conducting loops of radii R_1 and R_2 are lying concentrically in the same plane. If $R_1 > R_2$ then the mutual inductance (M) between them will be proportional to

(a) $\frac{R_1}{R_2}$ (b) $\frac{R_2}{R_1}$

(c) $\frac{R_1^2}{R_2}$ (d) $\frac{R_2^2}{R_1}$

54. A coil of area 80 cm^2 and number of turns 50 is rotating about an axis perpendicular to a magnetic field of 0.05 Tesla at 2000 rotations per minute. The maximum value of emf induced in it will be

(a) 200π volt (b) $\frac{10\pi}{3}$ volt

(c) $\frac{4\pi}{3}$ volt (d) $\frac{2}{3}$ volt

55. An athlete with 3 m long iron rod in hand runs towards east with a speed of 30 km/hour. The horizontal component of earth's magnetic field is 4×10^{-5} Wb/m^2. If he runs with the rod in horizontal and vertical positions then the induced emf generated in the rod in two cases will be

(a) zero in vertical position and 1×10^{-3} volt in horizontal position.

(b) 1×10 volt in vertical position and zero volt in horizontal position.

(c) zero in both positions

(d) 1×10^{-3} volt in both positions

56. A copper disc of radius 0.1m rotates about its axis in a uniform magnetic field of 0.1 Tesla at 10 rotations per second. The plane of the disc remains normal to the magnetic field. The induced emf along the radius of the disc will be

(a) $\frac{\pi}{10}$ volt (b) $\frac{2\pi}{10}$ volt

(c) $\frac{\pi}{100}$ volt (d) 2π volt

57. The distance between the ends of wings of an aeroplane is 50 m. It is flying in a horizontal plane at a speed of 360 Km/hour. The vertical component of earth's magnetic field at that place is 2.0×10^{-4} Wb/m^2, then the potential difference induced between the ends of the wings will be

(a) 0.1 volt (b) 1.0 volt

(c) 0.2 volt (d) 0.01 volt

58. The coefficients of self induction of two coils are L_1 = 8 mH and L_2 = 2 mH respectively. The current rises in the two coils at the same rate. The power given to the two coils at any instant is same. The ratio of currents flowing in the coils will be

(a) $\frac{i_1}{i_2} = \frac{1}{4}$ (b) $\frac{i_1}{i_2} = \frac{4}{1}$

(c) $\frac{i_1}{i_2} = \frac{3}{4}$ (d) $\frac{i_1}{i_2} = \frac{4}{3}$

59. In the above problem, the ratio of induced emf's in the coils will be

(a) $\frac{V_1}{V_2} = 4$ (b) $\frac{V_1}{V_2} = \frac{1}{4}$

(c) $\frac{V_1}{V_2} = \frac{1}{2}$ (d) $\frac{V_2}{V_1} = \frac{1}{2}$

60. In. Q. 91, the ratio of energies stored will be

(a) $\frac{W_1}{W_2} = 4$ (b) $\frac{W_1}{W_2} = \frac{1}{4}$

(c) $\frac{W_1}{W_2} = \frac{3}{4}$ (d) $\frac{W_1}{W_2} = \frac{4}{3}$

61. The number of turns in a coil of wire of fixed radius is 600 and its self inductance is 108 m H. The self inductance of a coil of 500 turns will be

(a) 74 mH (b) 75 mH

(c) 76 mH (d) 77 mH

62. Two coils X and Y are lying in a circuit. The change in current in X is 2 ampere and change in magnetic flux in Y is 0.4 weber. The coefficient of mutual induction between the coils will be

(a) 0.2 Henry (b) 5 Henry

(c) 0.8 Henry (d) 0.4 Henry

63. A millivoltmeter is connected in parallel to an axle of the train running with a speed of 180 Km/hour. If the vertical component of earth's magnetic field is 0.2×10^{-4} Wb/m^2 and the distance between the rails is 1m, then the reading of voltmeter will

(a) 10^{-2} volt (b) 10^{-4} volt

(c) 10^{-3} volt (d) 1 volt

64. The north pole of long horizontal bar magnet is carried towards a vertical conducting plane. The direction of induced current in the conducting plane will be

(a) horizontal (b) vertical

(c) clockwise (d) anticlockwise

65. If the voltage applied to a motor is 200 volt and back emf is 160 volt, then the efficiency of the motor will be

(a) 100% (b) 80%

(c) 50% (d) 25%

66. An inductance L and a resistance R are connected to a battery. After sometime the battery is removed but L and R remain connected in the closed circuit. The value of current will reduce to 37% of its maximum value in

(a) RL second (b) $\frac{R}{L}$ second

(c) $\frac{L}{R}$ second (d) $\frac{1}{LR}$ second

67. The magnetic flux linked with a coil is $\phi \leq 8t^2 + 3t + 5$ Weber. The induced emf in fourth second will be

(a) 16 V (b) 139 V

(c) 67 V (d) 145 V

68. The self inductance of a coil is 5 mH. If a current of 2 A is flowing in it then the magnetic flux produced in the coil will be

(a) 0.01 Weber (b) 10 Weber

(c) zero (d) 1 Weber

69. The magnetic flux in a coil of 100 turns increases by 12×10^3 Maxwell in 0.2 second due to the motion of a magnet. The emf induced in the coil will be—

(a) 6 V (b) 0.6 V

(c) 0.06 V (d) 60 V

70. A proton enters a perpendicular magnetic field of 20 Tesla. If the velocity of proton is 4×10^7 m/s then the force acting on it will be

(a) 1.28 Newton (b) 1.28×10^{-11} Newton

(c) 12.8 Newton (d) 12.8×10^{-11} Newton

71. A cylindrical bar magnet is lying along the axis of a circular coil. If the magnet is rotated about the axis of the coil then

(a) emf will be induced in the coil

(b) only induced current will be generated in the coil

(c) no current will be induced in the coil.

(d) both emf and current will be induced in the coil

72. The quantity in electricity which is equivalent to mass is

(a) L (b) C

(c) R (d) I

73. Eddy currents are not used in

(a) speedometer of vehicles

(b) induction furnace

(c) electromagnetic damping

(d) diffraction of X-rays

74. In the following figure the bulb will start lighting suddenly if

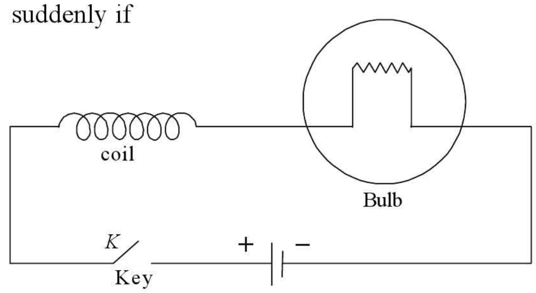


Fig. 26.57

(a) key is closed

(b) key is opened

(c) key is either closed or opened

(d) nothing is done

75. A coil of area A_0 is lying in such a magnetic field whose value changes from B_0 to $4B_0$ in t seconds. The induced emf in the coil will be

(a) $\frac{4B_0}{A_0 t}$ (b) $\frac{4B_0 A_0}{t}$

(c) $\frac{3B_0 A_0}{t}$ (d) $\frac{3B_0}{A_0 t}$

76. Electric current is flowing in same direction in two coaxial coils. On increasing distance between the two coils the value of current will

(a) decrease (b) increase

(c) remain unchanged (d) nothing can be said.

77. If the number of turns in a coil is N then the value of self inductance of the coil will become

(a) N times (b) N^2 times

(c) N^{-2} times (d) N° times

78. The value of mutual inductance can be increased by

(a) decreasing N (b) increasing N

(c) winding the coil on wooden frame

(d) winding the coil on china clay

79. The value of coefficient of mutual induction for the arrangement of two coils shown in the figure will be

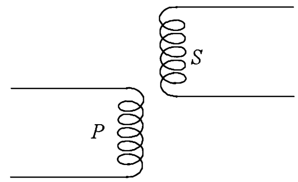


Fig. 26.58

(a) zero (b) maximum

(c) negative (d) positive

80. A conducting rod of length L is falling with velocity V in a uniform horizontal magnetic field B normal to the rod. The induced emf between the ends the rod will be

(a) $2\,BVl$ (b) zero

(c) BlV (d) $\frac{BVl}{2}$

81. A coil of area 0.01m^2 is lying in a perpendicular magnetic field of 0.1 Tesla. If a current of 10 A is passed in it then the maximum torque acting on the coil will be

(a) 0.01 N/m (b) 0.001 N/m

(c) 1.1 N/m (d) 0.8 N/m

Answers to Questions for Practice

1.	(a)	**2.**	(b)	**3.**	(c)	**4.**	(a)	**5.**	(b)	**6.**	(a)	**7.**	(c)
8.	(d)	**9.**	(c)	**10.**	(c)	**11.**	(c)	**12.**	(a)	**13.**	(a)	**14.**	(a)
15.	(c)	**16.**	(b)	**17.**	(a)	**18.**	(b)	**19.**	(c)	**20.**	(b)	**21.**	(b)
22.	(b)	**23.**	(b)	**24.**	(a)	**25.**	(c)	**26.**	(d)	**27.**	(a)	**28.**	(c)
29.	(c)	**30.**	(c)	**31.**	(c)	**32.**	(a)	**33.**	(a)	**34.**	(c)	**35.**	(a)
36.	(a)	**37.**	(a)	**38.**	(c)	**39.**	(a)	**40.**	(d)	**41.**	(b)	**42.**	(b)
43.	(d)	**44.**	(a)	**45.**	(d)	**46.**	(d)	**47.**	(d)	**48.**	(d)	**49.**	(d)
50.	(c)	**51.**	(b)	**52.**	(b)	**53.**	(d)	**54.**	(c)	**55.**	(c)	**56.**	(c)
57.	(b)	**58.**	(a)	**59.**	(a)	**60.**	(b)	**61.**	(b)	**62.**	(a)	**63.**	(c)
64.	(d)	**65.**	(b)	**66.**	(c)	**67.**	(c)	**68.**	(a)	**69.**	(c)	**70.**	(d)
71.	(c)	**72.**	(a)	**73.**	(d)	**74.**	(c)	**75.**	(c)	**76.**	(b)	**77.**	(b)
78.	(b)	**79.**	(a)	**80.**	(c)	**81.**	(a)						

27

Alternating Current

BRIEF REVIEW

A time dependent current $i(t)$ is termed as *AC* or alternating current. It is of four types:

(a) sinusoidal (b) complex periodic
(c) aperiodic (d) random.

Sinusoidal AC If the current or voltage varies in accordance with sine or cosine function or their combination, then such a current or voltage is termed as sinusoidal. For example,

$I = I_p \sin \omega t$, $I = I_p \sin(\omega t \pm \phi)$, $I = I_p \cos(\omega t \pm \phi)$ and $I = I_{p1} \sin \omega t + I_{p2} \cos \omega t$ etc. are sinusoidal *AC* currents. Note that in $I = I_p \sin(\omega t \pm \phi)$ where I is instantaneous value of current, I_p is its peak value, ω is angular frequency and ϕ is initial phase angle or epoch or angle of repose. $\omega = 2\pi f = \frac{2\pi}{T}$ or $f = \frac{1}{T}$ is linear frequency. T is time period. Fig. 27.1 shows sinusoidal variation of AC current.

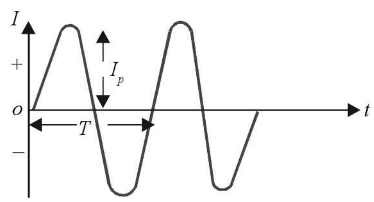


Fig. 27.1 Sinusoidal *AC* current

Complex periodic *AC* The voltage or current waveforms that are periodic but other than sine or cosine function such as rectangular, square wave, sawtooth, triangular wave form etc are complex periodic. These can be simplified using Fourier analysis. Thus the ***dc* value (or average value)**, **rms** value or peak value may be known using **Fourier analysis**.

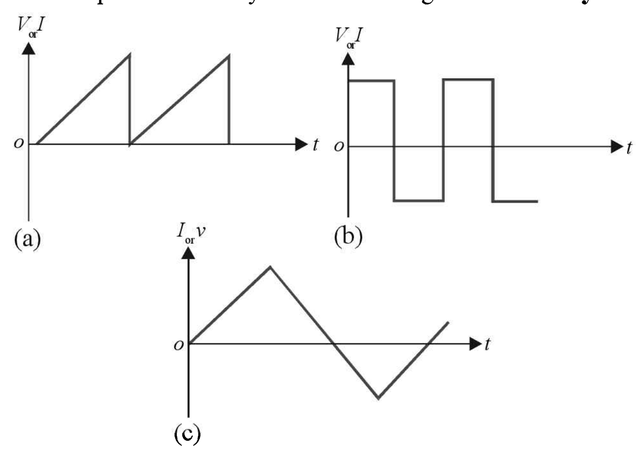


Fig. 27.2 Complex periodic *AC*

Fig. 27.2 shows examples of complex periodic wave.

Fourier theorem A complex periodic function can be expressed as a series of sine and cosine functions f (t) = A_o

$$+ \sum_{n=1}^{\infty} A_n \sin nwt + \sum_{n=1}^{\infty} B_n \cos nwt$$

Aperiodic *AC* The voltage or current wave form which is periodic but remains only positive or only negative and normally occurs for a short interval is called aperiodic wave. Pulse waveform is aperiodic as illustrated in Fig. 27.3.

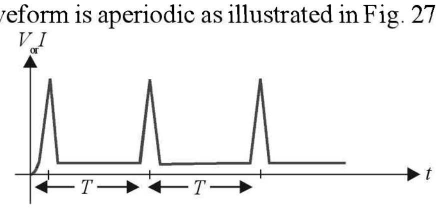


Fig. 27.3 Aperiodic *AC*

Random AC The voltage or current whose magnitude or time of occurrence is not well defined as shown in Fig. 27.4.

Four values of *AC* voltage or current may be defined as follows:

Fig. 27.4 Random *AC*

(a) peak voltage or peak current (V_p or I_p)

(b) mean or average voltage / current (V_{av} or I_{av})

(c) RMS voltage / current (V_{rms} or I_{rms})

(d) Peak-to-peak voltage / current (V_{pp} or I_{pp})

Look carefully into Fig. 27.5. The maximum voltage which one can have is called peak voltage V_p.

Note that $V_{pp} = 2\ V_p$

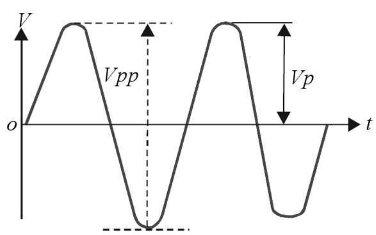


Fig. 27.5 Peak and peak to peak value illustration

AC can be measured using *AC* voltmeter / *AC* ammeter. *DC* meters cannot record *AC*. Moreover, *AC* meters measure RMS value of current or voltage. Two types of *AC* voltmeters are available: One which employs a rectifier which converts *AC* to *DC* and then *DC* meter records the voltage/current. The other type is based on heat produced is $\propto V^2$ or I^2 special type of meters are available which record peak voltage or mean voltage.

Mean or average voltage (V_{av}) Since the mean or average voltage over a complete cycle is zero, we define it for half the cycle.

$$\text{Thus } V_{av} = \frac{2}{T}\int_0^{T/2} V_p \sin \omega t\, dt$$

$$\text{Similarly } I_{av} = \frac{2}{T}\int_0^{T/2} I_p \sin \omega t\, dt$$

For sinusoidal voltage $V_{av} = 0.63\ V_p$,

similarly $I_{av} = 0.63\, I_p$.

RMS or Root Mean Square Voltage Also known as virtual or effective voltage, it is that value of *AC* voltage which will produce same amount of heat in a given resistance in a given time as is produced by *DC* voltage in the same resistance for the same time.

$$V_{rms}^2 = \frac{1}{T}\int_0^T V^2\, dt = \frac{1}{T}\int_0^T V_p^2 \sin^2\omega t\, dt = \frac{V_p^2}{2}$$

$$\text{or } V_{rms} = \frac{V_p}{\sqrt{2}} = 0.707\, V_p \text{ similarly}$$

$$I_{rms} = \frac{I_p}{\sqrt{2}} = 0.707\, I_p.$$

Reactance The resistance offered by an *AC* or reactive component (capacitor or inductor) when *AC* is applied is called reactance. It also introduces a phase shift of $\pi/2$ in voltage or current. Unit of Reactance is **Ohm.**

Capacitive reactance $X_c = \dfrac{1}{C\omega}$. When AC is applied across a capacitor the current leads the voltage wave form by $\pi/2$ radian or 90°. Fig. 27.6 shows the *V* and *I* phasor diagram in case of capacitor.

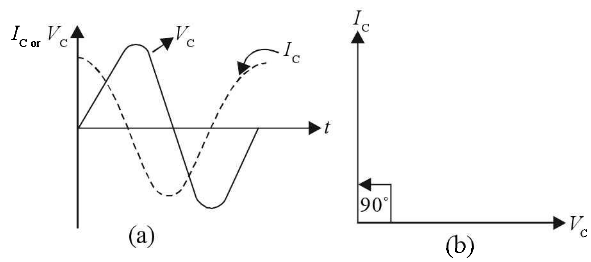


Fig. 27.6 Phasor diagram in case of a capacitor

Inductive reactance $X_L = L\omega$. Current lags the voltage waveform by 90° or $\pi/2$ radian when ac is applied across a pure Inductor. Fig. 27.7 illustrates the phasor diagram in case of inductor's *V* and *I*.

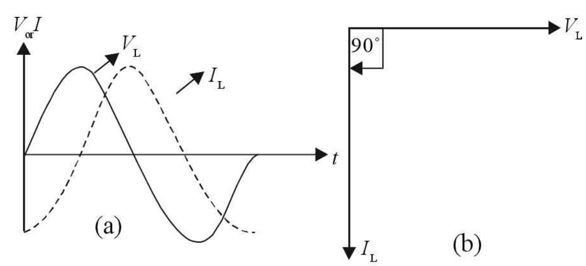


Fig. 27.7 Phasor diagram in case of an inductor

Note that capacitor and inductor act like filter capacitor.

Blocks *DC* and allows *AC* to pass $\because\ X_C = \dfrac{1}{C\omega}$ when $\omega \to 0$,

$X_C \to \infty$ and when $\omega \to \infty, X_C \to 0$

Inductor allows *DC* to pass and attenuates *AC*. $X_L = L$, when $\omega \to 0, X_L \to 0$ and when $\omega \to \infty, X_L \to \infty$

Reciprocal of reactance is called susceptance.

AC components offer phase shift between *V* and *I* along with reactance when *AC* is applied. *L* and *C* are *AC* components. *AC* components are also called reactive components.

DC components The circuit elements which do not offer any phase shift between V and I when *AC* is applied. Such elements behave alike in *AC* or *DC*. Resistor (R) is common example.

Impedance (Z) The net resistance offered in an *AC* circuit when both *AC* and *DC* circuit elements are present is called impedance. Unit is Ohm. There will be a phase shift between V and I such that $0 < \phi < 90^0$.

Admittance (Y) Reciprocal of impedance is called admittance. $Y = \frac{1}{Z}$ unit is ohm^{-1} or siemen (S).

Series RC circuit In series RC circuit, impedance

$$|Z| = \sqrt{R^2 + X_c^2} = \sqrt{R^2 + \frac{1}{C^2\omega^2}} \quad \tan\phi = \frac{1}{RC\omega}$$

and

$$i = \frac{V_p}{\sqrt{R^2 + \frac{1}{C^2\omega^2}}} \sin(\omega t + \tan^{-1}\frac{1}{RC\omega})$$

Note that current leads the voltage wave form by

$$\phi = \tan^{-1}\left(\frac{1}{RC\omega}\right)$$ as illustrated in Fig. 27.8.

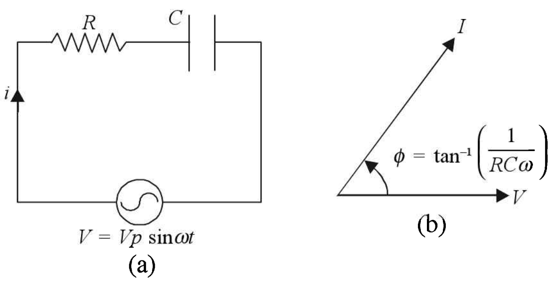


Fig. 27.8

Series RL circuit In series *RL* circuit current lags the voltage wave form by $\tan^{-1}\left(\frac{L\omega}{R}\right)$ as illustrated in Fig. 27.9 (*b*) Impedance *Z* of the circuit is

$$|Z| = \sqrt{R^2 + X_L^2}$$

$$= \sqrt{R^2 + L^2\omega^2};$$

$$\tan\phi = \frac{L\omega}{R}$$

$$i = \frac{V_p}{\sqrt{R^2 + L^2\omega^2}} \sin(\omega t - \tan^{-1}\frac{L\omega}{R})$$

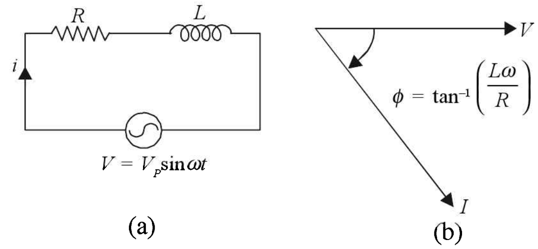


Fig. 27.9

Series RLC circuit It is also called resonant circuit.

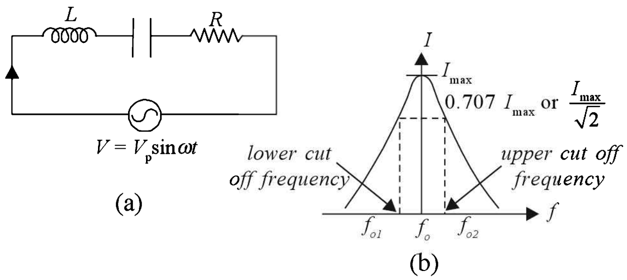


Fig. 27.10

$$|Z| = \sqrt{R^2 + (X_L - X_c)^2} = \sqrt{R^2 + \left(L\omega - \frac{1}{C\omega}\right)^2}$$

$$\tan\phi = \frac{X_L - X_c}{R} = \frac{X_c - X_L}{R}$$

$$i = \frac{V_p}{|Z|} \sin(\omega t + \phi)$$

Three causes arise

(i) when at a particular frequency $\omega_0, X_L = X_c$

$$\text{or } L\omega_0 = \frac{1}{Cw_0}, \text{ i.e., } \omega_0 = \frac{1}{\sqrt{LC}} \text{ or } f_0 = \frac{1}{2\pi\sqrt{LC}}$$

This frequency is called resonant frequency. At resonant frequency $Z = R$ *i.e.* impedance is pure resistance. No phase shift exists between V and I. Note that impedance is minimum at $f = f_0$ and hence current is maximum at f_0 as shown in Fig. 27.10 (*b*)

(ii) If $\omega < \omega_0$ or $f < f_0$, the impedance is capacitive as $X_c > X_L$ and hence current leads the voltage waveform.

(iii) If $\omega > \omega_0$ or $f > f_0$, the impedance is inductive as $X_L > X_c$ and hence current lags the voltage wave form.

***Q*-factor or quality factor** ($Q = \frac{L\omega}{r}$) where r is internal resistance of the coil $Q = \frac{L\omega}{r} = \frac{\omega}{\omega_{02} - \omega_{01}}$. If Q factor is large, resonance is sharp. It is clear from Fig. 27.11that if Q is low, resonance is poor, however band width is small.

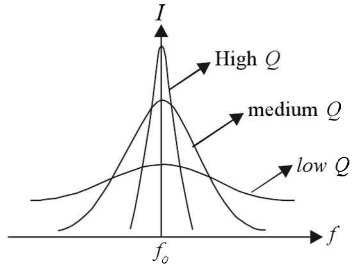


Fig. 27.11

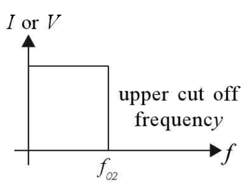
Bandwidth The band of allowed frequencies is called bandwidth. It is the difference between upper and lower cut off frequencies *i.e.* bandwidth $BW = \Delta f = f_{02} - f_{01}$ as shown in Fig 27.10 (b).

Note that a series LCR circuit may be used as a band pass filter.

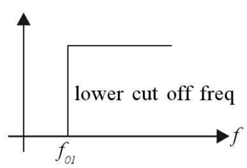
Cut off frequencies or – 3 dB frequencies These represent the frequencies at which power becomes half of the maximum or current becomes $\frac{I_{\max}}{\sqrt{2}}$.

Filters may be divided into three categories

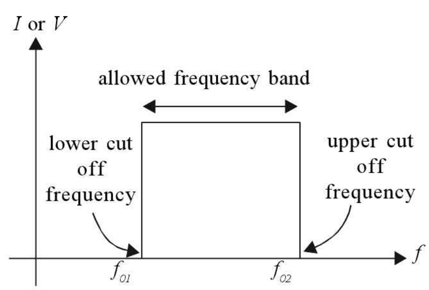
(a) Low-pass filter (b) High pass filter, and (c) Band-pass filters. An Ideal **low-pass filter** allows all frequencies less than a certain maximum *i.e.* $f < f_{02}$ are allowed as shown in Fig. 27.12 (*a*). A practical low pass-filter is shown in Fig. 27.13 (*b*). An ideal high pass filter allows all frequencies



(a) Ideal low-pass filter.



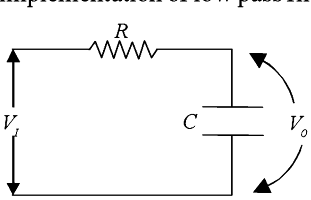
(b) Ideal high-pass filter.



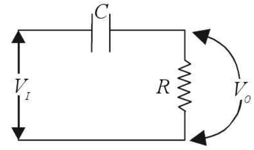
(c) Ideal band pass filter.

Fig. 27.12

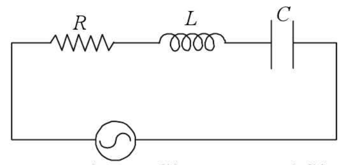
Greater than a minimum called lower cut off frequency to pass without attenuation as shown in Fig. 27.12 (*b*). Fig. 27.13 (a) shows implementation of low pass filter using *R* and C.



(a) Low-pass filter



(b) High-pass filter



(c) Band-pass filter or tuned filter

Fig. 27.13

An ideal **band pass filter** allows all frequencies lying between f_{01} and f_{02} to pass without attenuation. [Fig 27.13(*c*)]. A series LCR [Fig. 27.13 (c)] is a practical band pass filter implementation.

A parallel LC circuit acts as a band reject filter.

Power (*P*) $P = V_{\text{rms}} I_{\text{rms}} \cos \phi$; $\cos \phi$ is called power factor. Power companies try to supply at highest power factor ($\phi = 0$). ϕ is the phase shift between V and I.

$$P = V_{\text{rms}} I_{\text{rms}} \cos \phi = \frac{V_{rms}^2}{|Z|}$$

$$\cos \phi = \frac{V_p^2}{2|Z|} \cos \phi.$$

If $\phi = 90^0$, $P = 0$, such a power is called wattless power and energy meters cannot record it There could be two types of power active and reactive power. Only active power is read by energy meters. $P_{\text{active}} = V_{\text{rms}} I_{\text{rms}} \cos \phi$. Reactive power is not read by energy meters $P_{\text{reactive}} = V_{\text{rms}} I_{\text{rms}} \sin \phi$.

Transformer An ideal transformer is a loss-less element. The principle is mutual induction. It is used to transform the voltage and current levels in an ac circuit.

In an ideal transformer $V_1 I_1 = -V_2 I_2$, *i.e.,* $P_1 + P_2 = 0$

$$\frac{V_1}{V_2} = \frac{I_2}{I_1} = \frac{N_1}{N_2}$$

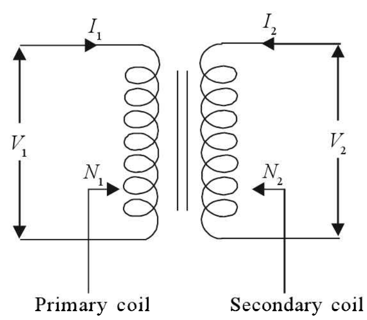


Fig. 27.14 Transformer

where N_1 and N_2 are number of turns in primary and secondary coils respectively.

Power transformers are of two types: step up transformer and step down transformer.

In step up transformers, $N_2 > N_1$

and hence $V_2 > V_1$ thus $I_2 < I_1$.

In a step down transformer $V_2 < V_1$, $N_2 < N_1$ but $I_2 > I_1$.

Step up transformers are used at the generator end in a power distribution system so that $I \to 0$, and hence power loss in transmission line $P = I^2R \to 0$ (due to heating produced in the transmission line). Step down transformer is used as the distribution end near a locality.

Efficiency of the transformer $\eta = \dfrac{P_{output}}{P_{input}} = \dfrac{V_{output}}{V_{input}}$

$$= \frac{V_2 - I_2 r}{V_2} \text{ see Fig. 27.15}$$

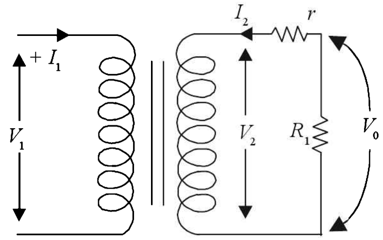


Fig. 27.15

Losses in transformers may be divided into two categories:

(a) Copper loss (due to resistance of Cu winding) [See Fig. 27.15]

(b) Magnetic losses (eddy current loss, flux linkage loss and hysteresis loss)

Eddy current loss is minimised using laminated core in form of E and I or Square Core. Flux linkage loss is prevented by winding one coil over the other. Hysteresis loss is minimised using soft iron core with 4% Si.

Generator Generators are of two types: *AC* generator and *DC* generator. The basic difference in construction is that in *AC* generators slip rings are used and in *DC* generators split rings are employed so that after each half cycle direction changes and same polarity is maintained.

To generate emf a coil is moved in a magnetic field and emf is generated $V = BA_0\omega N \sin \omega t$ where N is number of turns, B is uniform magnetic field. $A = A_0 \cos \omega t$ is area at any instant. And ω is angular frequency.

Tuned circuits or **Tank circuit** or **Oscillation circuits** Fig. 27.16 (*a*) is called tank circuit.

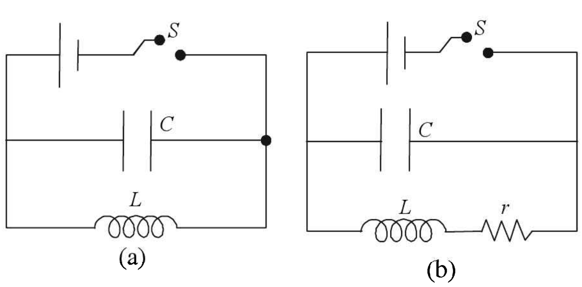


Fig. 27.16

$$f = \frac{1}{2\pi\sqrt{LC}}$$

or $\quad \omega = \dfrac{1}{\sqrt{LC}}$

Resonant frequency or frequency of oscillation In Fig. 27.16 (*b*) damped oscillations are produced and damped frequency $\omega' = \sqrt{\dfrac{1}{LC} - \dfrac{R^2}{4L^2}}$. These circuits in *AC* behave as band reject circuits. The current-frequency curve is shown in Fig. 27.17

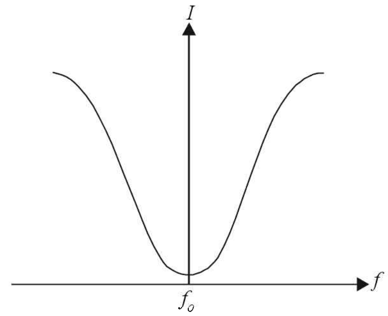


Fig. 27.17

• Short Cuts and Points to Note

1. $I_{av} = \dfrac{\int_0^T I dt}{\int_0^T dt} = \dfrac{1}{T}\int_0^T I dt$. If wave is sinusoidal or complex periodic, integrate for half cycle *i.e.*

$$I_{av} = \frac{2}{T}\int_0^{T/2} I dt.$$

For sinusoidal current $I_{av} = \dfrac{2I_p}{\pi} = 0.636 I_p$

2. $I_{rms} = \sqrt{\dfrac{1}{T}\int_0^T I^2 dt}$ $\quad I_{rms}$ is also called apparent or virtual or effective current.

For sinusoidal waves $I_{rms} = \dfrac{I_p}{\sqrt{2}} = 0.707 I_p$.

3. Peak-to-peak voltage $V_{pp} = 2V_p$ where V_p is peak voltage.

4. Form factor of *AC* $F = \dfrac{I_{rms}}{I_{av}}$ $\left(= \dfrac{\pi}{2\sqrt{2}}\right.$ for sinusoidal *AC*)

5. Capacitive reactance $X_C = \dfrac{1}{C\omega}$. Note that $X_C \propto \dfrac{1}{\omega}$ or $\dfrac{1}{f}$ *i.e.* capacitive reactance falls as frequency increases.

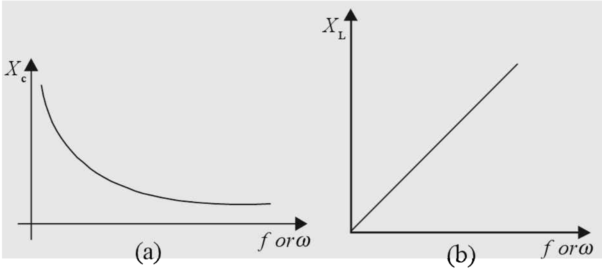


Fig. 27.18

Capacitor acts as a short circuit for very high frequencies $X_C \to 0$ if $\omega \to \infty$. Current leads the voltage by $\frac{\pi}{2}$ radian or 90^0 when AC is applied across a capacitor. Inductive reactance $X_L = L\omega$. Note that $X_L \propto \omega$ or f. Inductive reactance attenuates AC and allows DC to pass without attenuation. Current lags the voltage by 90^0 when AC is applied across pure inductor.

6. **In series RC circuit** $|Z| = \sqrt{R^2 + X_C^2}$

$= \sqrt{R^2 + \frac{1}{C^2\omega^2}}$; $\tan\phi = \frac{X_c}{R} = \frac{1}{RC\omega}$ current leads the voltage wave form by ϕ.

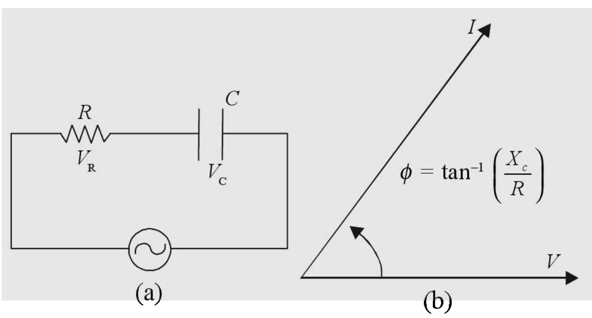


Fig. 27.19

$$V_c = IX_c;\; V_{rms} = \sqrt{V_R^2 + V_C^2}$$

or $$\frac{V_P^2}{2} = V_R^2 + V_C^2$$

$$I = \frac{V_P}{|Z|}\sin(\omega t + \phi)\;;\; \text{Power } P = \frac{V_{RMS}^2}{|Z|}$$

$$\cos\phi = \frac{V_P^2 R}{2|Z|^2} \quad \because \cos\phi = \frac{R}{|Z|}$$

7. **In series RL circuit** $|Z| = \sqrt{R^2 + X_L^2}$

$$= \sqrt{R^2 + L^2\omega^2}$$

$$\tan\phi = \frac{XL}{R} = \frac{L\omega}{R}.$$

Current lags the voltage wave form by ϕ.

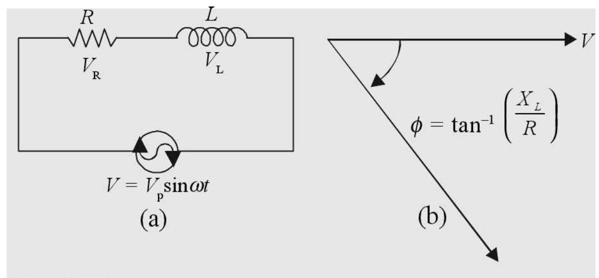


Fig. 27.20

$$V_L = IX_L;\; V_{app}(\text{rms}) = \sqrt{V_R^2 + V_L^2}$$

or $$\frac{V_P^2}{2} = V_R^2 + V_L^2$$

$$I = \frac{V_P}{|Z|}\sin(\omega t - \phi)$$

Power $$P = \frac{V_{rms}^2 \cos\phi}{|Z|} = \frac{V_P^2 R}{2|Z|^2}$$

8. **In series RLC circuit** $|Z| = \sqrt{R^2 + (X_L - X_c)^2}$

$$= \sqrt{R^2 + \left(L\omega - \frac{1}{C\omega}\right)^2}$$

$$\tan\phi = \left(\frac{\frac{1}{C\omega} - L\omega}{R}\right).$$

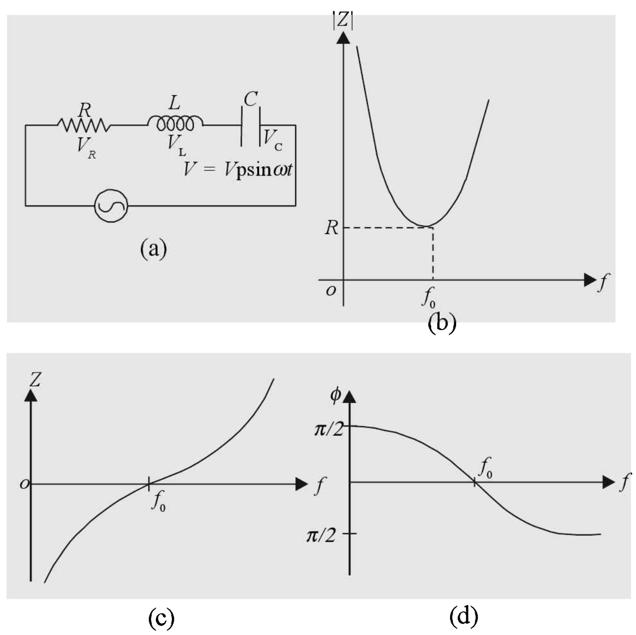


Fig. 27.21

If ϕ is positive impedance is capacitive and current leads the voltage wave form *i.e.* for $\omega < \omega_0$ impedance is capacitive. If $f = f_0$ or $\omega < \omega_0 = \frac{1}{\sqrt{LC}}$ then $\phi = 0$, impedance is pure resistive and is minimum. Current is maximum. These is no phase shift between V and I at resonance.

If $f > f_0$ or $\omega < \omega_0$, the impedance is inductive, ϕ is negative *i.e.* current lags the voltage wave form. Fig. 27.21 (*d*) shows variation of phase shift with frequency. Fig. 27.21 (*b*) and (*c*) represent variation of impedance with frequency.

$$\text{Power } P = \frac{V_P^2 R}{2|Z|^2} = \frac{V_P^2 R}{2\left[R^2 (X_L - X_c)^2\right]}$$

$$P_{(\text{resonance})} = \frac{V_P^2}{2R} \text{ is maximum.}$$

$$\text{Power at cut off frequencies} = \frac{V_P^2}{4R}$$

9. At cut off frequencies $Z = \sqrt{2}R$

$R^2 + (X_L - X_C)^2 = 2R^2$ or $L\omega - \frac{1}{C\omega} = R$

or $LC\omega^2 - RC\omega - 1 = 0$

or $\omega = \frac{RC \pm \sqrt{R^2C^2 + 4LC}}{2LC}$ represent cut off frequencies. Band width $\omega_{02} - \omega_{01} = \frac{\sqrt{R^2C^2 + 4LC}}{LC}$

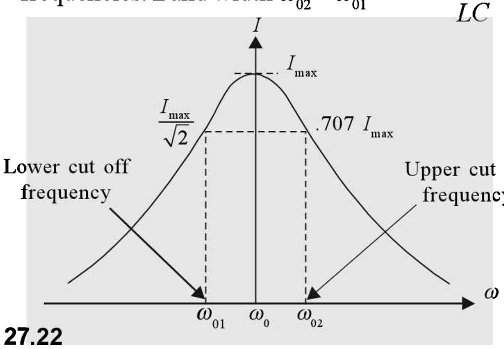


Fig. 27.22

Note that cut off frequencies are also called half power frequencies or –3*dB* frequencies.

10. ***Q* factor of a coil** $Q = \frac{L\omega}{r} = \frac{\omega_0}{\omega_{02} - \omega_{01}}$ Larger the value of Q, sharper is the resonance.

If $r \to 0$, $Q \to \infty$, $I \to \infty$ *i.e.* resonance catastrophe occurs and bandwidth $\to 0$.

11. Series RLC circuit acts as a band pass filter or tuned filter.

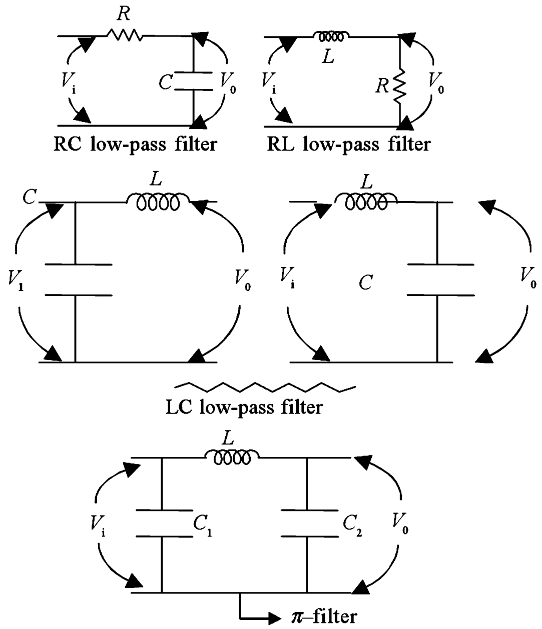


Fig. 27.23 Low-pass filter circuits

RC, *LC* or π low-pass filters are preferred.

RC high-pass filter is very commonly employed.

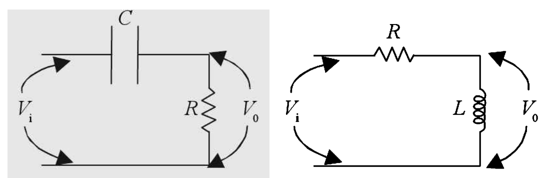


Fig. 27.24 High-pass filters

12. Active power $P = V_{rms} I_{rms} \cos\phi = \frac{V_P^2 R}{2|Z|^2}$. It is recorded by energy meters. Reactive power $P_{(\text{Reactive})} = V_{rms} I_{rms} \sin\phi$ is not recorded by energy meters.

13. $\angle i = -\angle Z$, *i.e.* –ve phase angle of impedance is the phase shift between V and I.

14. Kirchhoff's Laws can be applied to *AC* as well if stated as follows:

KCL or Junction Law: The algebraic sum of all the currents entering a node at any instant is zero.

KVL or Loop Law: The algebraic sum of all the potential drops in a loop at any instant is zero.

15. *AC* voltage and currents are phasors, So are impedance and reactances. Apply vector laws for analytical treatment.

16. In a transformer $\frac{V_1}{V_2} = -\frac{I_2}{I_1} = \frac{N_1}{N_2}$ Note: –ve sign shows phase reversal of current in secondary coil. Dot (.) on the coils of a transformer represents that their winding are made in the same direction *i.e.* either both clockwise or both anticlockwise so that a phase shift of 180^0 is developed between primary and secondary currents.

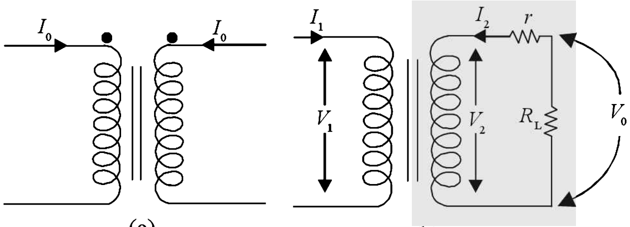


Fig. 27.25 Transformer

$$I_1 = \frac{V_1}{\left(\frac{N_1}{N_2}\right)R_L}.$$

$$\text{Efficiency } \eta\% = \frac{P_{output}}{P_{input}} \times 100$$

$$\text{From Fig. 27.25 (b) } \eta\% = \frac{V_{out\,put}}{V_2} \times 100 = \frac{V_2 - I_2 r}{V_2} \times 100$$

17. In parallel LCR circuit as shown in Fig. 27. 26

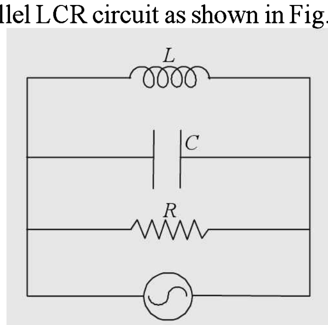


Fig. 27.26 Parallel LCR Circuit

$$|Y| = \frac{1}{|Z|} = \sqrt{\frac{1}{R^2} + \left(\frac{1}{X_C} - \frac{1}{X_L}\right)^2}$$

$$= \sqrt{\frac{1}{R^2} + \left(C\omega - \frac{1}{L\omega}\right)^2}$$

Parallel LCR circuit is called anti-resonant circuit or band reject circuit. Current is minimum at resonance.

18. In our country *AC* mains has frequency 50 Hz.

19. Fourier analysis can be used to find V_{av} or amplitude of fundamental frequency or other harmonics.

20. Maximum Power from an *AC* source is transferred to a load (ac) if $Z_L = Z_S$

or $R_L + jX_L = R_S - jX_S$

OR $R_L = R_S$

and $X_L = -X_S$, i.e., if source is inductive then load is capacitive and equivalent (so that X_L and X_S cancel out) or vice versa.

21. Current when $\phi = 90°$ is called wattless current. It is possible if pure capacitor or pure inductor is applied.

- **Caution**

1. Considering that impedances/reactances are added like resistors.

⇒ These quantities are phasors and vector algebra should be applied.

2. Considering $V_{app}(rms) = V_R + V_C$ in Fig 27.27 (a)

or $V_{app}(rms) = V_R + V_L$ in Fig. 27.27 (b)

or $V_{app}(rms) = V_R + V_L + V_C$ in Fig. 27.27 (c)

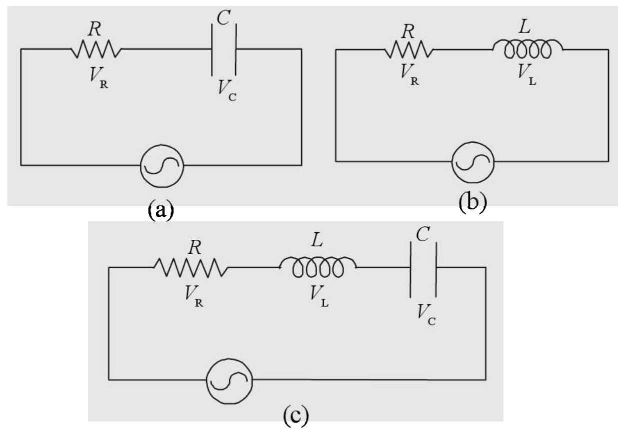


Fig. 27.27

⇒ Apply $V_{app}(rms) = \sqrt{V_R^2 + V_C^2}$ in Fig. 27.27 (a)

$V_{app}(rms) = \sqrt{V_R^2 + V_L^2}$ in Fig. 27.27 (b)

and $V_{app}(rms) = \sqrt{V_R'^2 + (V_C - V_L)^2}$ in Fig. 27.27 (c)

3. Considering transformers can step up or step down *DC* also.

⇒ Transformers can step up or step down only *AC* or time varying voltage/currents, as they are based on principles of mutual induction.

To step up or step down *DC* potential divider circuit or Rheostat can be used.

4. Considering $V_{rms} = \frac{V_P}{\sqrt{2}} = 0.707\ V_P$ for all types of *ACs*.

⇒ $V_{rms} = \frac{V_P}{\sqrt{2}}$ for sinusoidal *AC*.

For all others apply $V_{rms} = \sqrt{\frac{1}{T}\int_0^T V^2 dt}$

Similarly $V_{av} = \frac{2V_P}{\pi}$ or 0.636 V_P for sinusoidal AC.

For all other types use $V_{av} = \frac{1}{T}\int_0^T V dt$

or $\frac{2}{T}\int_0^{T/2} V dt$ depending upon whether they remain on one side of zero line or two sides of zero line.

5. Not remembering frequency of AC mains.

⇒ Frequency of AC mains is 50 Hz.

6. Considering $V = IX_L$ or $V = IX_C$ completely represents voltage across inductor or capacitor.

⇒ $V = IX_L$ or $V = IX_C$ only gives the magnitude (amplitude) of voltage and not instantaneous values.

7. Considering current cannot pass through capacitor as its plates are insulated.

⇒ AC current can pass as capacitor charges and discharges. There is an equal current i entering one plate and i leaving other plate and an equal displacement current between the plates.

Mathematically $Q = CV$ $\qquad \frac{dQ}{dt} = i = C\frac{dV}{dt}$

i.e. if V is varying with time, current can flow.

OR $X_C = \frac{1}{C\omega}$ if $\omega = 0, X_C \to \infty$ i.e. DC current cannot pass through capacitor but AC current can pass.

8. Considering in AC power consumed is same as in DC i.e. $P = V_{rms} I_{rms}$

⇒ Power consumed is $P = V_{rms} I_{rms} \cos\phi$. Power may be wattless if $\phi = 90°$

9. considering net impedance of circuit shown in Fig. 27.28

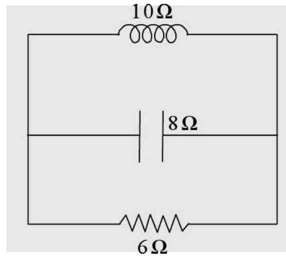


Fig. 27.28

$$\frac{1}{|Z|} = \frac{1}{10} + \frac{1}{8} + \frac{1}{6}$$

$$\Rightarrow \text{Apply} = \frac{1}{|Z|} = \sqrt{\frac{1}{6^2} + \left(\frac{1}{10} - \frac{1}{8}\right)^2}$$

10. Considering phase shift is fixed in series RLC circuit.

⇒ Phase shift varies with frequency as shown in Fig 27.29. It is $+ve$ if $f < f_0$ and 0° at $f = f_0$. Phase shift is negative when $f > f_0$

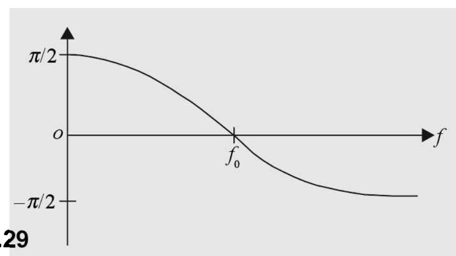


Fig. 27.29

SOLVED PROBLEMS

1. In a circuit L, C, R connected in series with an alternating voltage source of frequency f. The current leads the voltage by 45°. The value of C is

[CBSE PMT 2005]

(a) $\frac{1}{\pi f(2\pi fL - R)}$ (b) $\frac{1}{2\pi f(2\pi fL - R)}$

(c) $\frac{1}{\pi f(2\pi fL + R)}$ (d) $\frac{1}{2\pi f(2\pi fL + R)}$

Solution (d) $\tan 45 = \frac{\frac{1}{2\pi fC} - 2\pi fL}{R}$ or

$$C = \frac{1}{2\pi f(2\pi fL + R)}$$

2. The self inductance of a motor of an electric fan is 10 H. In order to impart maximum power at 50 Hz, it should be connected to a capacitance of

[AIEEE 2005]

(a) 4 μF (b) 8 μF
(c) 1 μF (d) 2 μF

Solution (c) Maximum power is transferred at resonance.

$$\therefore f_0 = \frac{1}{2\pi\sqrt{LC}}$$

or $C = \frac{1}{4\pi^2 f_0^2 L} = \frac{1}{4\times10\times(50)^2\times10}$

$= 10^{-6}\,F = 1\mu F$

3. A circuit has resistance of 12 Ω and an impedance of 15 Ω. The power factor of the circuit will be

[AIEEE 2005]

(a) 0.8 (b) 0.4
(c) 1.25 (d) 0.125

Solution (a) $\cos\phi = \frac{R}{|Z|} = \frac{12}{15} = 0.8$

4. The phase difference between alternating current and emf is $\frac{\pi}{2}$. Which one of the following cannot be the constituent of the circuit?

[AIEEE 2005]

(a) C alone (b) R, L
(c) L, C (d) L alone

Solution (b) $0 < \phi < 90°$ for a series RL circuit.

5. The circuit shown in Fig. 27.30 acts as a

[AIIMS 2005]

(a) tuned filter (b) low pass filter
(c) high pass filter (d) rectifier

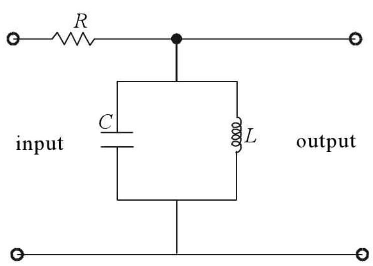


Fig. 27.30

Solution (a)

6. A 50 Hz, 20 V AC source is connected across RC series circuit as shown in Fig. 27.31 If the voltage across R is 12 V then voltage across capacitor is

[AIIMS 2005]

(a) 8 V (b) 16 V
(c) 10 V
(d) cannot be predicted as values of R and C are not given.

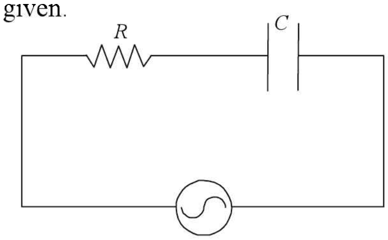


Fig. 27.31

Solution (b) $20^2 = 12^2 + V_C^2$ or $V_C = 16$ V

7. In an LCR circuit the capacitance is made $\frac{1}{4}$th then what should be the change in inductance that the circuit remains in resonance again?

[BHU 2005]

(a) 8 times (b) $\frac{1}{4}$ times
(c) 2 times (d) 4 times

Solution (d) $f_0 = \frac{1}{2\pi\sqrt{LC}}$ To keep frequency unchanged L be made 4 times.

8. Two inductors each equal to L are joined in parallel. The equivalent inductance is

[BHU 2005]

(a) zero (b) $2L$
(c) L (d) $\frac{L}{2}$

Solution (d) Inductances are added like resistors.

$$\frac{1}{L_{er}} = \frac{1}{L_1} + \frac{1}{L_2}$$

$$\therefore L_{er} = \frac{L}{2}$$

9. The turn ratio of a transformer is 2:3. If the current through primary is 3A, then current through load resistance is

[BHU 2005]

(a) 1A (b) 4.5 A
(c) 2 A (d) 1.5 A

Solution (b) $\frac{I_P}{I_S} = \frac{N_S}{N_P}$ or $\frac{3}{I_S} = \frac{2}{3}$

or $I_S = 4.5$ A

10. The square root of the product of inductance and capacitance has dimensions of

(a) length (b) mass
(c) time (d) dimensionless

[CET Karnataka 2005]

Solution (c) $f = \frac{1}{\sqrt{LC}}$ or $\sqrt{LC} = T$

11. In a series RLC circuit $R = 300\ \Omega$, L = 60 mH, $C = 0.5\ \mu F$, V = 50 V, $\omega = 10^4$ rad s^{-1}. Find the voltage across capacitor.

(a) 30 V (b) 20 V
(c) 60 V (d) 50 V

Solution (b) $I = \dfrac{V}{|Z|}$

$$= \frac{50}{\sqrt{300^2 + \left(60\times10^{-3}\times10^4 - \dfrac{1}{.5\times10^4\times10^{-6}}\right)^2}}$$

$$= \frac{50}{\sqrt{300^2+400^2}} = \frac{1}{10}\text{A}$$

$$V_C = IX_C = \frac{1}{10}\times 200 = 20\text{ V}$$

12. In series CR circuit excited by ac mains, $C = 10\ \mu$F, R = 300 Ω. Find power factor.

(a) $\dfrac{3}{\sqrt{17}}$ (b) $\dfrac{3}{\sqrt{16}}$

(c) $\dfrac{3}{\sqrt{18}}$ (d) $\dfrac{3}{\sqrt{19}}$

Solution (d) $\cos\phi = \dfrac{R}{|Z|} = \dfrac{R}{\sqrt{R^2 + \dfrac{1}{C^2\omega^2}}}$

$$= \frac{300}{\sqrt{300^2 + \left(\dfrac{1}{100\pi\times10^{-5}}\right)^2}}$$

$$= \frac{300}{100\sqrt{9+10}} = \frac{3}{\sqrt{19}}$$

13. In the given circuit what is the potential drop across resistance?

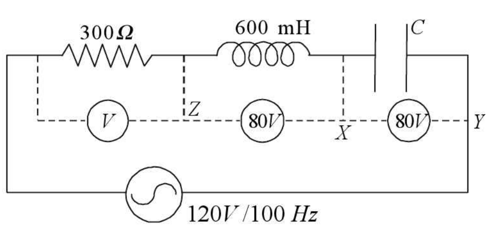


Fig. 27.32

(a) 40 V (b) 80 V

(c) 120 V (d) zero

Solution (c) $\because\ |V_L| = |V_C|$ It is possible at resonance.

Therefore $V_R = V_{app} = 120$ V

14. In Q.13 what is the potential drop across ZY?

(a) 160 V (b) $80\sqrt{2}$

(c) 80 V (d) zero

Solution (d) $V_{ZY} = V_C - V_L = 0$

15. In the circuit shown in Fig 27.33, for output voltage V_{AB} acts as

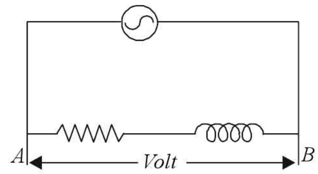


Fig. 27.33

(a) band-pass filter (b) low-pass filter

(c) high-pass filter (d) band reject filter

Solution (c)

16. For the circuit shown in Fig. 27.34 the rms current is

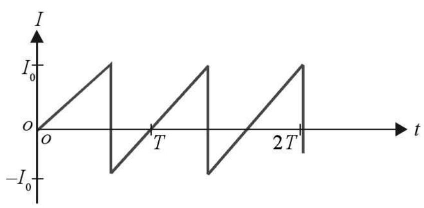


Fig. 27.34

(a) I_0 (b) $I_0/\sqrt{2}$

(c) $I_0/\sqrt{3}$ (d) $I_0/2$

Solution (c) $I = \dfrac{2I_0 t}{T}$ when $0 < t < T/2$

$$I = 2I_0(t-T)/T \text{ when } T/2 < t < T$$

$$I^2_{rms} = \frac{1}{T}\left[\int_0^{T/2} I^2 dt + \int_{T/2}^{T} I^2 dt\right]$$

$$= \frac{1}{T}\int_0^{T/2}\left(\frac{2I_0 t}{T}\right)^2 dt$$

$$+ \frac{1}{T}\int_{T/2}^{T}\left(\frac{2I_0 D}{T}(t-T)\right)^2 dt$$

$$I_{rms} = I_0/\sqrt{3}$$

17. The reactance of capacitor is 350 Ω, R = 180 Ω. Find X_L if in a series LCR circuit current leads the voltage by 53°.

(a) 120 Ω (b) 140 Ω

(c) 210 Ω (d) 110 Ω

Solution (d) $\tan 53 = \dfrac{X_C - X_L}{R} \Rightarrow \dfrac{4}{3} = \dfrac{350 - X_L}{180}$

or $X_L = 110\,\Omega$

18. A 400 Ω resistor and 6 μF capacitor are connected in parallel to a source of V_{rms} = 220 V and angular frequency 360 *rad/s*. Find the current in the resistor and capacitor.

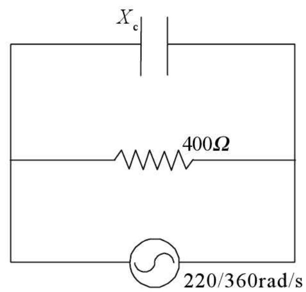


Fig. 27.35

(a) 0.55 A, 0.475 A (b) 0.55 A, 0.726 A

(c) 0.55 A, 0.176 A (d) none of these

Solution (a) $\dfrac{1}{X_C} = C\omega = 6 \times 10^{-6} \times 360 = 2.16 \times 10^{-3}$

$$\frac{1}{Z} = \sqrt{\frac{1}{R^2} + (C\omega)^2}$$

$$= \sqrt{\left(\frac{1}{400}\right)^2 + \left(2.16\times10^{-3}\right)^2}$$

$$I_{net} = 220 \times \frac{1}{Z} = 220 \times 10^{-3} \times 3.3 = 0.726;$$

$$I_R = \frac{220}{400} = 0.55\text{ A}$$

$$I_C = \sqrt{I_{net}^2 - I_R^2} = \sqrt{(.726)^2 - (.55)^2} = 0.475\text{ A}$$

Short cut $I_C = 220 \times \dfrac{1}{X_C} = 220 \times 2.16 \times 10^{-3} = 0.475$ A

$I_R = 220 \times 2.5 \times 10^{-3} = 0.55$ A

19. A 120 V *AC* line transformer is to supply 13000 V for a neon sign. To reduce shock hazard a 8.5 mA fuse is inserted. Find the maximum input power to the transformer.

(a) 120 W (b) 121 W

(c) 110 W (d) 104 W

Solution (c)

$P_{input} = P_{output} = 13000 \times 8.5 \times 10^{-3} = 110.5$ W

20. An LC circuit has L = 5 mH and C = 20 μF.

$V = 5 \times 10^{-3} \cos \omega t$ is supplied. ω is twice the resonant frequency. Find the maximum charge stored in the capacitor.

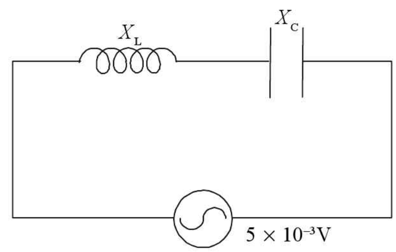


Fig. 27.36

(a) 66.6 nC (b) 11.3 nC

(c) 23.2 nC (d) 33.3 nC

Solution (d)

$$\omega = 2\,\omega_0 = 2 \times \frac{1}{\sqrt{LC}} = \frac{2\times10^4}{\sqrt{10}}$$

$$I = \frac{5\times10^{-3}}{\sqrt{(X_L - X_C)^2}} = \frac{5\times10^{-3}}{\sqrt{\left(10\sqrt{10} - 2.5\sqrt{10}\right)^2}}$$

$$= \frac{5\times10^{-3}}{7.5\sqrt{10}} = 2.1 \times 10^{-4}\text{ A}$$

$$V_C = IX_C \text{ and } Q = CV_C = I \times \frac{I}{C\omega} \times C$$

$$= \frac{I}{\omega} = \frac{2.1\times10^{-4}\times\sqrt{10}}{2\times10^4} = 33.3\text{ nC}$$

21. An AM radio operates at 550 kHz to 1650 kHz. If L is fixed and C is varied for tuning then minimum and maximum value of C is

(a) C, 3 C (b) C, 6 C

(c) C, 9 C (d) C, 12 C

Solution (c) $\dfrac{f_{max}}{f_{min}} = 3 \therefore \dfrac{\sqrt{LC_{max}}}{\sqrt{LC_{min}}} = 3$ or $\dfrac{C_{max}}{C_{min}} = 9$

22. In an ideal transformer turn ratio is 2 : 3. If input voltage is 100 V/60 Hz, then output voltage is

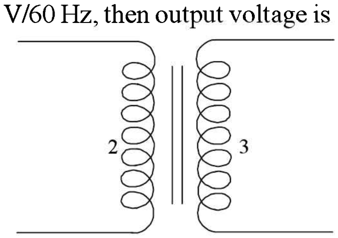


Fig. 27.37

(a) 150 V/90 Hz (b) 150 V/40 Hz
(c) 150 V/60 Hz (d) 66.6 V/60 Hz

Solution (c)

$$\frac{V_1}{V_2} = \frac{N_1}{N_2} \Rightarrow \frac{100}{V_2} = \frac{2}{3} \text{ and}$$

frequency does not change.

23. A centre tapped transformer is rated 12 – 0 – 12 V. The peak voltage obtained is

(a) 12 V (b) $\frac{12}{\sqrt{2}}$ V
(c) $12\sqrt{2}$ V (d) 24 V
(e) $24\sqrt{2}$ V

Solution (c)

$$V_{rms} = 12 \text{ V } V_P = V_{rms}\sqrt{2} = 12\sqrt{2} \text{ V}$$

24. $I = 6\cos wt + 8\sin wt$ is applied across a resistor of 40Ω. Find the potential difference across the resistor.

(a) 660 V (b) 80 V
(c) 330 V (d) 400 V

Solution (d) $I_{rms} = \sqrt{6^2+8^2} = 10$ A

$$V_R = I_{rms} \times R = 10 \times 40 = 400 \text{ V}$$

TYPICAL PROBLEMS

25. If 10 mV/ 1kHz input is given to a series RC circuit then output voltage across the capacitor is

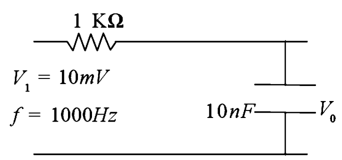


Fig. 27.38

(a) 8.5 mV (b) 10 mV (c) 7.5 mV
(d) 6.5 mV (e) none of these

Solution (a) $V_C = IX_C = \frac{V_{in}}{|Z|} X_C$

$$= \frac{10\times10^{-3}}{\sqrt{(10^3)^2 + \left(\frac{10^4}{2\pi}\right)^2}} \times \frac{1}{10^{-8}\times10^4\times2\pi}$$

$$= \frac{100}{10^3\times1.8} \times \frac{1}{2\pi} = 8.5 \text{ mV}$$

26. A 100 W/200 V bulb and an inductor are connected in series to a 220 V/50 Hz supply. Find the power consumed by the bulb.

(a) 100 W (b) 92 W
(c) 84 W (d) 74 W

Solution (d) $P = I^2R = \left(\frac{V}{|Z|}\right)^2 R$

$$= \left(\frac{220}{\sqrt{(400)^2+(100\pi)^2}}\right)^2 \times 400;$$

$$R = \frac{200^2}{100} = 400\ \Omega$$

$$= \frac{484\times10^2\times400}{16\times10^4\times10\times10^4} = \frac{1936}{26} = 74.46 \text{ W}$$

27. A parallel plate capacitor has area 20 cm^2 and separation between the plates is 0.1 mm. The dielectric break down strength is 3×10^6 V/m. The maximum rms voltage which can be safely applied is

(a) 210 V (b) 300 V
(c) 100 V < V < 300 V (d) < 200 V

Solution (a) $E = 3 \times 10^6 = \frac{V\sqrt{2}}{d}$

$$\text{or } V = \frac{3\times10^6\times10^{-4}}{\sqrt{2}} = 210 \text{ V}$$

28. A telegraph wire 200 km long has capacity 0.014 μF per km and resistance 2 Ω/km. If it carries an *AC* of frequency 5 kHz then what should be the inductance so that maximum power is delivered at the other end?

(a) 0.014 H (b) 140 mH
(c) 1.4 mH (d) none of these

Solution (a) $X_L = X_C$

$$\text{or } L = \frac{1}{C\omega^2} = \frac{1}{2.8\times10^{-6}\times(5\times10^3)^2}$$

$$= \frac{1}{2.8\times25} = \frac{1}{70} H.$$

29. When 4 V *DC* is connected across an inductor current is 0.2 A. When *AC* of 4 V is applied the current is 0.1A. Then self inductance of the coil is .... ω = 1000 $rads^{-1}$

(a) 20 mH (b) 40 mH
(c) $20\sqrt{3}$ mH (d) none of these

Solution (c) $Z = \dfrac{4}{0.1} = 40\,\Omega = \sqrt{R^2 + (L\omega)^2}$

and $R = \dfrac{4}{0.2} = 20\,\Omega$

$\therefore L\omega = 20\sqrt{3}\,\Omega$ or $L = 20\sqrt{3}$ mH

30. A capacitor has capacitance 0.5 nF. A choke of 5 μH is connected in series. An *em* wave of wave length λ is found to resonate with it. Find λ.

(a) 10πm (b) 20πm
(c) 30πm (d) 5πm
(e) none of these

Solution (c) $f = \dfrac{1}{2\pi\sqrt{LC}}$ and

$\lambda = \dfrac{C}{f} = 3 \times 10^8 \times 2\pi\sqrt{LC}$

$= 3 \times 10^8 \times 2\pi\sqrt{5\times10^{-6} 5\times10^{-10}} = 30\pi m$

31. The frequency of generator (*AC*) is measured using

(a) multimeter (b) AVO meter
(c) tachometer (d) speedometer

Solution (c)

32. An *AC* source of 100 V (*rms*) supplies a current of 10 A (*rms*) to a circuit. Then power consumed is

(a) = 1000 W (b) ≤ 1000 W
(c) ≥ 1000 W (d) < 1000 W only

Solution (b) $P = V_{rms} I_{rms} \cos\phi$ since $0 \le \cos\phi \le 1$

$\therefore p \le 1000$ W.

33. An inductor having some resistance is connected to an *AC* source. Which of the following quantities has zero average value over a complete cycle?

(a) current (b) induced emf
(c) Joule heat
(d) magnetic energy stored in the inductor

Solution (a) and (b)

34. A 60 W/120 V bulb is connected to a 240/60 Hz supply with an inductance in series. Find the value of inductance so that bulb gets correct voltage.

(a) $\dfrac{2.3}{\pi}$ H (b) $2\sqrt{3}$ H
(c) πH (d) $\dfrac{2\sqrt{3}}{\pi}$ H

Solution (d) $R = \dfrac{(120)^2}{60} = 240\,\Omega$ we require $i = 0.5$ A

or $|Z| = 480\,\Omega$

$X_L = \sqrt{480^2 - 240^2} = 240\sqrt{3}\,\Omega$

$L = \dfrac{240\sqrt{3}}{60\times 2\pi} = \dfrac{2\sqrt{3}}{\pi} H$

QUESTIONS FOR PRACTICE

1. A capacitor acts as an infinite resistance for

(a) *DC* (b) *AC*
(c) *DC* as well as *AC* (d) neither *AC* nor *DC*.

2. An inductor, a resistor and a capacitor are joined in series with an *AC* source. As the frequency of the source is slightly increased from a very low value, the reactance

(a) of the inductor increases
(b) of the resistor increases
(c) of the capacitor increases
(d) of the circuit increases

3. Which of the following plots may represent the reactance of a series *LC* combination?

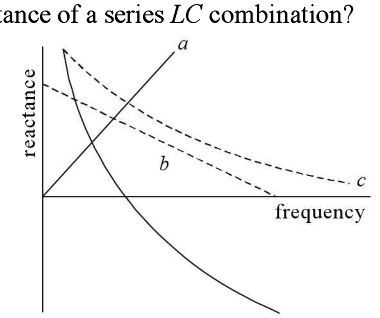


Fig. 27.39

4. The peak voltage in a 220 V, 50 Hz *AC* source is

(a) 220 V (b) about 160 V
(c) about 310 V (d) 440 V

5. An alternating current having peak value 14 *A* is used to heat a metal wire. To produce the same heating effect, a constant current *i* can be used where *i* is

(a) 14 *A* (b) about 20 *A*
(c) 7 *A* (d) about 10 *A*.

6. An *AC* source producing emf

$E = E_0[\cos(100\,\pi s^{-1})\,t + \varphi_1] + i_2\cos[(500\,\pi s^{-1})t + \varphi_2]$

is connected in series with a capacitor and a resistance, the steady state current in the circuit is found to be

(a) $i_1 > i_2$ (b) $i_1 = i_2$
(c) $i_1 < i_2$ (d) Insufficient information

7. A constant current of 2.8 A exists in a resistor. The *rms* current is

(a) 2.8 A (b) about 2 A
(c) 1.4 A
(d) undefined for a direct current

8. A series AC circuit has a resistance of 4 Ω and a reactance of 3 Ω. The impedance of the circuit is

(a) 5 Ω (b) 7 Ω
(c) 12/7 Ω (d) 7/12 Ω.

9. The magnetic field energy in an inductor changes from maximum value to minimum value in 5.0 ms when connected to an AC source. The frequency of the source is

(a) 20 Hz (b) 50 Hz
(c) 200 Hz (d) 500 Hz.

10. Transformers are used

(a) in DC circuits only (b) in AC circuits only
(c) in both DC and AC circuits
(d) neither in DC nor in AC circuits.

11. An AC source is rated 220 V, 50 Hz. The average voltge is calculated in a time interval of 0.01 s. It

(a) must be zero (b) may be zero
(c) is never zero (d) is $(220/\sqrt{2})$ V

12. An alternating current is given by
$i = i_1 \cos \omega t + i_2 \sin \omega t$.
The *rms* current is given by

(a) $\frac{i_1 + i_2}{\sqrt{2}}$ (b) $\frac{|i_1 + i_2|}{\sqrt{2}}$
(c) $\sqrt{\frac{i_1^2 + i_2^2}{2}}$ (d) $\sqrt{\frac{i_1^2 + i_2^2}{\sqrt{2}}}$

13. In an AC series circuit, the instantaneous current is zero when the instantaneous voltage is maximum. Connected to the source may be a

(a) pure inductor (b) pure capacitor
(c) pure resistor
(d) combination of a capacitor and an inductor

14. The AC voltage across a resistance can be measured using

(a) a potentiometer (b) a hot wire voltmeter
(c) A moving-coil galvanometer
(d) a moving-magnet galvanometer.

15.

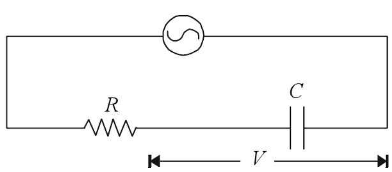


Fig. 27.40

A resistance R and a capacitor C are joined to a source of AC of constant emf and variable frequency. The potential difference across C is V. If the frequency of AC is gradually increased, V will

(a) increase (b) decrease
(c) remain constant
(d) first increase and then decrease

16. In an AC circuit, the reactance is equal to the resistance. The power factor of the circuit will be

(a) 1 (b) $\frac{1}{2}$
(c) $\frac{1}{\sqrt{2}}$ (d) zero

17. An inductance L, a capacitance C and a resistance R may be connected to an AC source of angular frequency ω, in three different combinations of RC, RL and LC in series. Assume that $\omega L = \frac{1}{\omega C}$. The power drawn by the three combinations are P_1, P_2, P_3 respectively. Then,

(a) $P_1 > P_2 > P_3$ (b) $P_1 = P_2 < P_1$
(c) $P_1 = P_2 > P_1$ (d) $P_1 = P_2 = P_3$.

18. An electrical heater and a capacitor are joined in series across a 220 V, 50 Hz AC supply. The potential difference across the heater is 90 V. The potential difference across the capacitor will be about

(a) 200 V (b) 130 V
(c) 110 V (d) 90 V

19. If the value of virtual voltages across L, C and R in an L-C-R circuit are V_L, V_c and V_R respectively then the source voltage will be

(a) $V = \sqrt{V_R^2 + (V_L - V_C)^2}$ (b) $V = V_L + V_C + V_R$
(c) $V = \sqrt{V_R^2 + (V_L^2 - V_C^2)}$ (d) $V = V_L - V_C + V_R$

20. The resultant reactance in an L-C-R circuit is

(a) $X_L + X_C$ (b) $X_L - X_C$
(c) $\sqrt{x_L^2 + x_C^2}$ (d) $\sqrt{x_L^2 - x_C^2}$

21. The unit of RC is

(a) second^{-1} (b) second^2
(c) second (d) second^3

22. The natural frequency of an L-C circuit is

(a) $\frac{1}{2\pi\sqrt{LC}}$ (b) $\frac{1}{2\pi}\sqrt{\frac{C}{L}}$
(c) $\frac{1}{2\pi}\sqrt{\frac{L}{C}}$ (d) $\sqrt{LC}$

23. The angular frequency of an AC source is 10 radian/sec. The reactance of $1\,\mu F$ capacitor will be—

(a) 10^4 Ω (b) 10^2 Ω
(c) 10^1 Ω (d) 10^5 Ω

24. If the values of inductance and frequency in an AC circuit are 2 henry and $10^3/2\pi$ *Hz* respectively then the value of inductive reactance will be

(a) $2 \times 10^3/\pi\,\Omega$ (b) $2 \times 10^2\,\Omega$
(3) $10^3\,\Omega$ (d) $2 \times 10^3\,\Omega$

25. If the reactance of a choke coil is X_L and its resistance is R, then

(a) $X_L = R$ (b) $X_L >> R$
(c) $X_L << R$ (d) $X_L = \infty$

26. If the phase difference between the emf and the current in an AC circuit is f then the RMS value of wattless current will be

(a) $I_{rms} \cos f$ (b) $I_{rms} \sin f$
(c) $I_{rms} \tan f/2$ (d) 0

27. An electric bulb of resistance 280 Ω is connected to 200 Volt supply line. The peak value of current flowing in the circuit will be

(a) nearly 1 amp (b) zero
(c) nearly 2 amp (d) nearly 4 amp

28. The correct curve representing the variation of capacitive reactance X_c with frequency f is

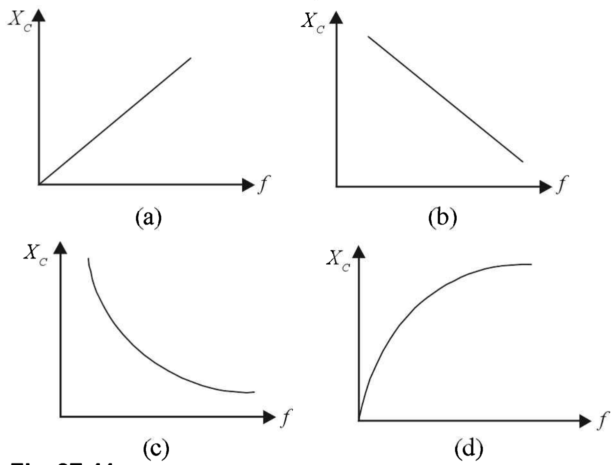


Fig. 27.41

29. If the equation of AC in an AC circuit is $I = 200 \cos(\omega t + \theta)$ ampere, then the effective value of current will be

(a) $\frac{200}{\sqrt{2}} A$ (b) $200\sqrt{2}\, A$
(c) $200 A$ (d) 0

30. The time constant of an L-R circuit is

(a) LR (b) $\frac{L}{R}$
(c) $\frac{R}{L}$ (d) $\frac{L^2}{R}$

31. Time constant of an R-C circuit is that time, in which on closing the circuit the value of charge q on condenser plates becomes

(a) $0.37\,q_0$ (b) $0.636\,q_0$
(c) $0.5q_0$ (d) q_0

32. The unit of $\sqrt{LC}$ is

(a) Henry (b) Farad
(c) Second (d) Ampere

33. In an L-C-R circuit the values of X_L, X_C and R are 300 Ω, 200 Ω and 100 Ω respectively. The total impedance of the circuit will be

(a) 600 Ω (b) 200 Ω
(c) 141 Ω (d) 310 Ω

34. When the frequency of applied emf in an L-C-R series circuit is less than the resonant frequency, then the nature of the circuit will be

(a) capacitive (b) resistive
(c) inductive (d) all of the above

35. The resonant frequency in an antiresonant circuit is

(a) $\frac{1}{2\pi\sqrt{LC}}$ (b) $\frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$

(c) $\frac{1}{2\pi}\sqrt{LC}$ (d) $\frac{1}{2\pi}\sqrt{\frac{C}{L}}$

36. The range of values of power factor is

(a) 0 to + 1 (b) 0 to – 1
(c) 0 to 8 (d) 2 to 8

37. How are the small bulbs, used for decoration purpose, connected ?

(a) In parallel (b) In series
(c) In mixed order (d) All of the above

38. To protect the electrical appliances the fuse wire in electric supply line is connected in

(a) series (b) parallel
(c) mixed order (d) none of the above

39. The unit of susceptance is

(a) ohm (b) ohm^{-1}
(c) ohm/cm (d) ohm/m

40. The frequency of AC supply line in houses in England is

(a) 40 Hz (b) 50 Hz
(c) 100 Hz (d) 200 Hz

41. The I-f curve for anti-resonant circuit is

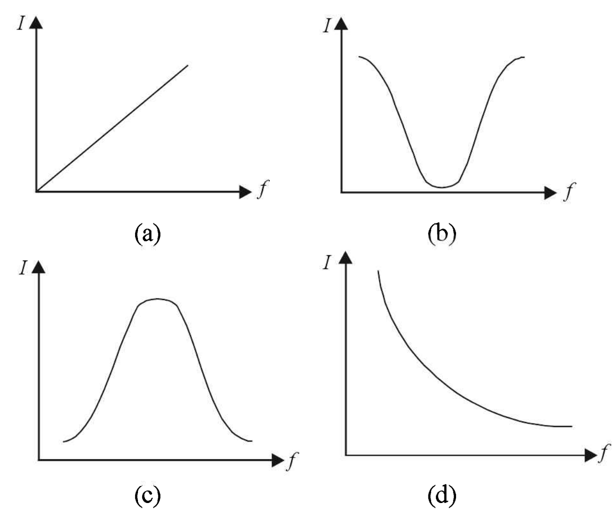


Fig. 27.42

42. The *RMS* value of effective current is

(a) $I_{rms}\cos\theta$ (b) $I_{rms}\sin\theta$

(c) $I_0\cos\theta$ (d) $I_0\sin\theta$

43. If *N* similar bulbs of *P* watt are connected in series then the power of combination will be

(a) PN (b) $\frac{P}{N^2}$

(c) $\frac{N}{P}$ (d) $\frac{P}{N}$

44. If the values of *L*, *C* and *R* in a series *L-C-R* circuit are 10 mH, 100 μF and 100 Ω respectively then the value of resonant frequency will be

(a) $\frac{10^3}{2\pi}$ Hz (b) 2×10^3 Hz

(c) $2\times\frac{10^3}{\pi}$ Hz (d) 10^3 Hz

45. If the current flowing in a choke coil of 2 H is decreasing at the rate of 5 amp/s, then induced emf across the ends of the coil will be

(a) 10^3 volt (b) 2.5 volt

(c) 10 volt (d) – 2.5 volt

46. The values of *XL*, *XC* and *R* in an AC circuit are 8 Ω, 6 Ω and 10 Ω respectively. The total impedance of the circuit will be

(a) 10.2 Ω (b) 12.2 Ω

(c) 10 Ω (d) 24.4 Ω

47. The electric supply line in houses works at 220 volt. The amplitude of emf will be

(a) 220 volt (b) 331 volt

(c) 110 volt (d) 440 volt

48. In the following circuit the values of *L*, *C*, *R* and E_0 are 0.01H, 10^{-5} *F*, 25 Ω and 220 volt respectively. The value of current flowing in the circuit at $f = 0$ and $f = \infty$ will respectively be

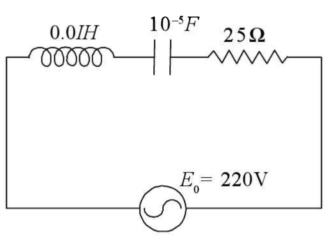


Fig. 27.43

(a) 8 *A* and 0 *A* (b) 0 *A* and 0 *A*

(c) 8 *A* and 8 *A* (d) 0 *A* and 8 *A*

49. If in an AC circuit *XL* = *XC* then the value of power factor will be

(a) 1 (b) 0

(c) infinity (d) $\frac{1}{2}$

50. An inductance coil of 1H and a condenser of capacity 1*pF* produce resonance. The resonant frequency will be

(a) $\frac{10^6}{\pi}$ Hz (b) $27\pi\times10^6$ Hz

(c) $\frac{2\pi}{10^6}$ Hz (d) $\frac{10^6}{2\pi}$ Hz

51. An alternating emf $E = 200\sqrt{2}\,\sin(100t)$ volt is connected to a condenser of capacity 0.1 μF through an AC ammeter. The reading of the ammeter will be

(a) 10 m*A* (b) 80 m*A*

(c) 40 m*A* (d) 2 m*A*

52. A pure capacitor is connected in an AC circuit. The power factor of the circuit will be

(a) 1 (b) infinity

(c) zero (d) 0.5

53. If the equations of alternating voltage and alternating current in an A.C. circuit are $E = E_0\sin\omega t$ volt and $I = I_0\sin(\omega t - \pi/2)$ ampere respectively. The power loss in the circuit will be

(a) zero (b) $\frac{E_0I_0}{\sqrt{2}}$

(c) $\frac{E_0I_0}{2}$ (d) $\frac{EI}{\sqrt{2}}$

54. The values of *L*, *C* and *R* in an *L-C-R* series circuit are 4 *mH*, 40 *pF* and 100 Ω respectively. The quality factor of the current is

(a) 10 (b) 100

(c) 1000 (d) 10,000

55. The phase difference between alternating emf and current in a purely capacitive circuit will be

(a) zero (b) π

(d) $-\frac{\pi}{2}$ (c) $\frac{\pi}{2}$

56. The power factor of wattless current is

(a) infinity (b) 1

(c) zero (d) $\frac{1}{2}$

57. A coil of self-inductance *L* and resistance *R* is connected to a cell of emf *E* volt. The value of current flowing in the circuit will be

(a) $\sqrt{\frac{EL}{R^2+L^2}}$ (b) $\sqrt{\frac{E}{R^2+L^2}}$

(c) $\frac{E}{L}$ (d) $\frac{E}{R}$

58. A bulb of 25 *W*, 220*V* and another bulb of 100*W*, 220*V* are connected in series with a supply line of 220*V*. Then

(a) both bulbs will glow with same brightness

(b) both bulbs will get fused

(c) 25 *W* bulb will glow more brightly

(d) 100 *W* bulb will glow more brightly

59. The readings of ammeter and voltmeter in an AC circuit are 10 A and 25 volt respectively. The power loss in the circuit will be

(a) more than 250 W (b) less than 150 W
(c) 250 W (d) 250 W or less than 250 W

60. The value of power factor in an AC circuit will be less if the value of R is

(a) more (b) less
(c) medium (d) infinity

61. The power factor of an L-R circuit is

(a) 1 (b) zero
(c) between 0 and 1 (d) infinity

62. The phase difference between the applied emf and the line current in an anti resonant circuit at resonance is

(a) $\frac{\pi}{2}$ radian (b) π radian
(c) $\frac{3\pi}{2}$ radian (c) zero

63. The value of R/Z is equal to

(a) $\cos\theta$ (b) $\sin\theta$
(c) $\tan\theta$ (d) θ

64. The ratio of apparent power to average power in an AC circuit is equal to

(a) $\cos\theta$ (b) $\frac{1}{\cos\theta}$
(c) form factor (d) $\frac{1}{\cos\theta}$

65. The correct curve between the uniform direct current and time is

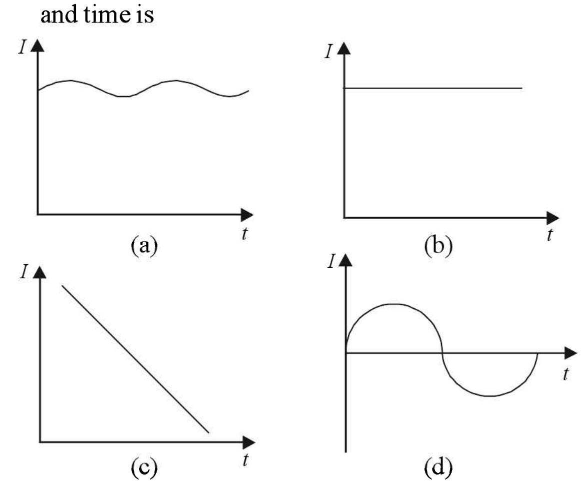


Fig. 27.44

66. In P-f curve the half power frequencies are those at which

(a) $P = \frac{P_{max}}{\sqrt{2}}$ (b) $P = \frac{P_{max}}{2}$
(c) $P = \frac{P_{max}}{4}$ (d) $P = P_{max}$

67. The correct phase diagram representing the relation between I_L, I_C and E in an anti-resonant L-C circuit is

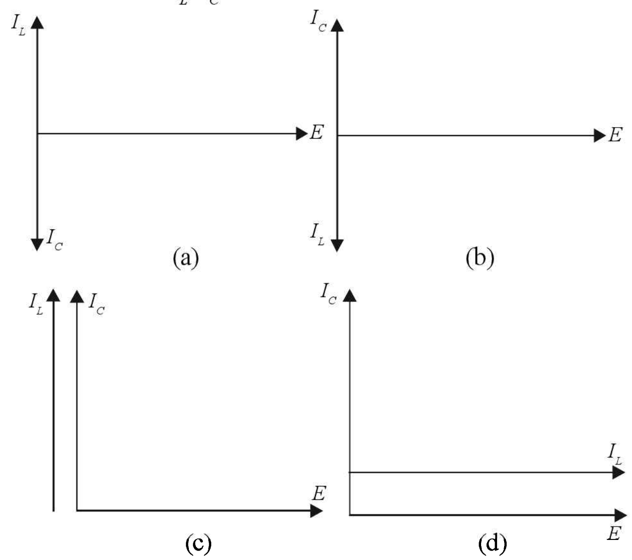


Fig. 27.45

68. The resonance point in $X_L - f$ and $X_C - f$ curves is

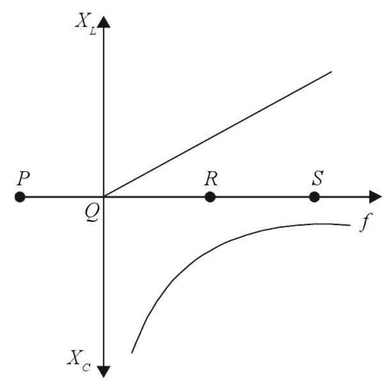


Fig. 27.46

(a) P (b) Q
(c) R (d) S

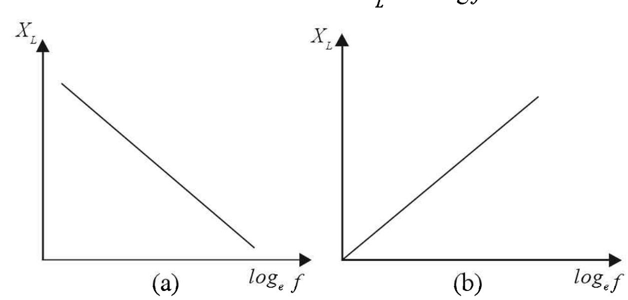
69. The unit of RC is

(a) second^{-1} (b) second^2
(c) second (d) second^3

70. On connecting a condenser in parallel to an electric fan connected to an AC circuit, the phase angle

(a) decreases (b) increases
(c) remains constant
(d) keeps on increasing and decreasing

71. The correct curve between X_L and $\log f$ is



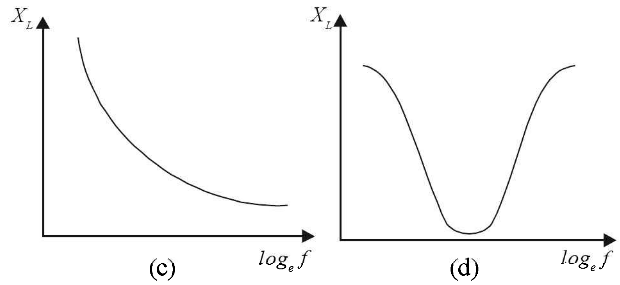


Fig. 27.47

72. The amplitude of effective current in an A.C. circuit is

(a) $I_0 \sin\theta$ (b) $I_0 \cos\theta$

(c) $I_{rms} \sin\theta$ (d) $I_{rms} \cos\theta$

73. The quality factor of an AC circuit is related to band width as

(a) inversely proportional

(b) directly proportional

(c) directly proportional to log

(d) inversely proportional to log

74. The impedance of a pure anti-resonant circuit at resonance is

(a) zero (b) infinity

(c) 1 (d) $\frac{1}{2}$

75. The sharpness of resonance in a series L-C-R resonant circuit, as the resistance of the circuit is increased, goes on

(a) increasing (b) decreasing

(c) tends from zero to infinity

(d) tends from infinity to zero

76. In the transmission and receiving circuits for radio waves, the series resonant circuits are used as

(a) selector circuits (b) rejecter circuits

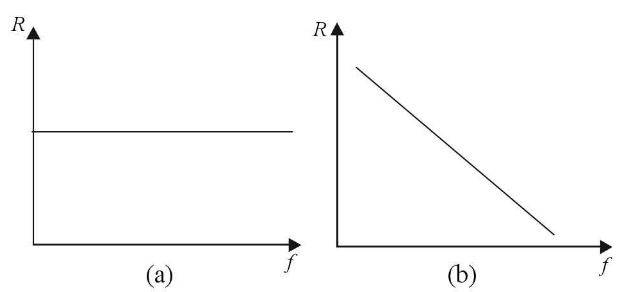
(c) rectifiers (d) oscillators

77. The value of admittance at resonance in antiresonant circuit

(a) $\sqrt{G^2 - S^2}$ (b) $G^2 + S^2$

(c) $\sqrt{G^2 + S^2}$ (d) $\frac{G^2}{S^2}$

78. The correct curve between the resistance of a conductor (R) and frequency (f) is



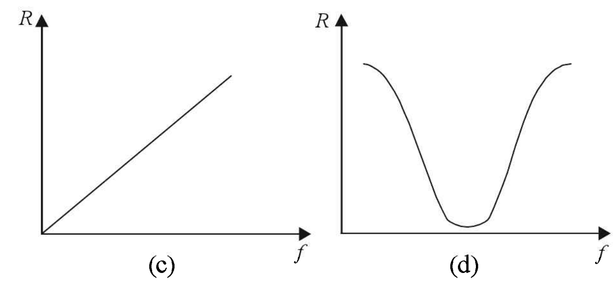


Fig. 27.48

79. Choke coil in an A-C circuit is used for

(a) decreasing AC (b) decreasing AV

(c) increasing AC (d) increasing AV

80. The inductive reactance of a coil is 2500 Ω. On increasing its self-inductance three times, the new inductive reactance will be

(a) 7500 Ω (b) 2500 Ω

(c) 1225 Ω (d) zero

81. In series L-C-R resonant circuit, to increase the resonant frequency

(a) L will have to be increased

(b) C will have to be increased

(c) LC will have to be decreased

(d) LC will have to be increased

82. An electric bulb is marked $100W$, $220V$. Its resistance is

(a) 484 Ω (b) 100 Ω

(c) 220 Ω (d) 48 Ω

83. The correct curve between inductive reactance (XL) and frequency (f) is

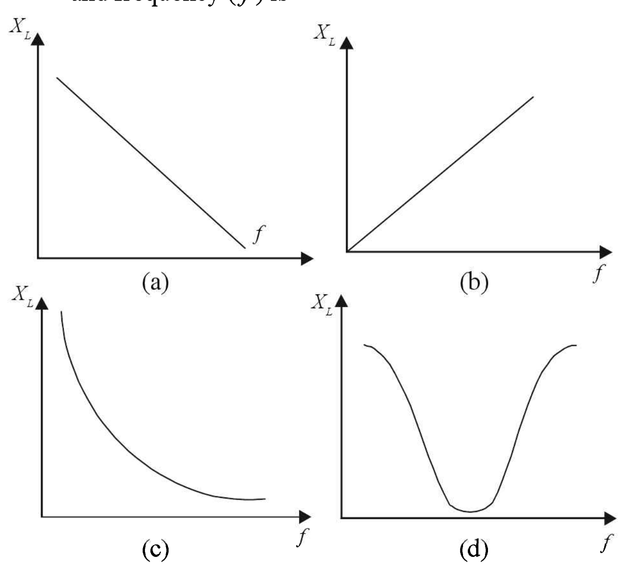


Fig. 27.49

84. An alternating current can be produced by

(a) transformer (b) generator

(c) turbine (d) electric motor

85. The capacitive reactance at 1600 Hz is 81 Ω. When the frequency is doubled then capacitive reactance will be

(a) 40.5 Ω (b) 81 Ω

(c) 162 Ω (d) zero

86. Energy in an inductance coil is stored in the form of

(a) magnetic energy (b) electrical energy

(c) heat energy (d) light energy

87. The incorrect statement about power factor is

(a) it is unit-less

(b) it depends on the nature of the components used

(c) its value can be anything between zero and 1

(d) $\cos\theta = \frac{Z}{R}$

88. If two bulbs of 60 watt and 40 watt are connected in parallel, then

(a) both bulbs with shine equally

(b) both bulbs will get fused

(c) 60 watt bulb will shine more brightly

(d) 40 watt bulb will shine more brightly

89. An alternating voltage source is connected in an AC circuit whose maximum value is 170 volt. The value of potential at a phase angle of 45° will be

(a) 120.56 Volt (b) 110.12 Volt

(c) 240 Volt (d) zero

90. The peak value of AC is $2\sqrt{2}$ ampere. Its apparent value will be

(a) 1 ampere (b) 2 ampere

(c) 4 ampere (d) zero

91. The correct curve between admittance (Y) and frequency (f) in an antiresonant circuit will be

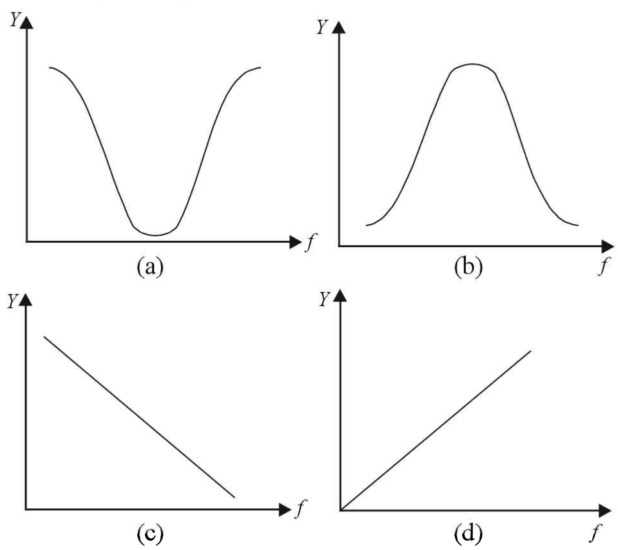


Fig. 27.50

92. The reactance of a condenser of capacity $50\mu F$ for an AC of frequency 2×10^3 Hertz will be

(a) 5 ohm (b) $\frac{2}{\pi}$ ohm

(c) $\frac{3}{\pi}$ ohm (d) – ohm

93. A voltmeter connected in an AC circuit reads 220 volt. IE represents

(a) peak voltage (b) $R.M.S.$ voltage

(c) average voltage (d) mean square voltage

94. In the equation of AC $I = I_0 \sin \omega t$, the current amplitude and frequency will respectively be

(a) $I_0, \frac{\omega}{2\pi}$ (b) $\frac{I_0}{2}, \frac{\omega}{2\pi}$

(c) $I_{rms}, \frac{\omega}{2\pi}$ (d) I_0, ω

95. The inductive reactance of a choke coil of $1/4\pi\, mH$ in an AC circuit of 50 Hz, will be

(a) 25 ohm (b) 0.25 ohm

(c) 0.025 ohm (d) 2.5 ohm

96. A pure resistance is connected as shown in the figure. The phase difference between the voltage applied and the current flowing in it will be

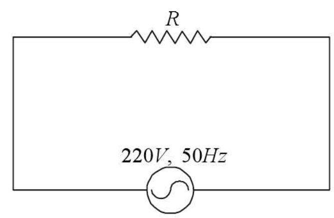


Fig. 27.51

(a) zero (b) $\frac{\pi}{2}$

(c) $-\frac{\pi}{2}$ (d) $\frac{\pi}{4}$

97. What will be the equation of alternating current of frequency 75 Hz if its RMS value is 20 ampere?

(a) $I = 20 \sin 150\pi t$. (b) $I = 20\sqrt{2} \sin(150\pi t)$

(c) $I = \frac{20}{\sqrt{2}} \sin(150\pi t)$ (d) $I = 20\sqrt{2} \sin(75\pi t)$

98. The time taken by an AC of frequency 50 Hz to complete one cycle will be

(a) 2 second (b) 0.2 second

(c) 0.02 second (d) 0.002 second

99. An R-C circuit is shown in the following diagram. The capacitive reactance and impedance will be

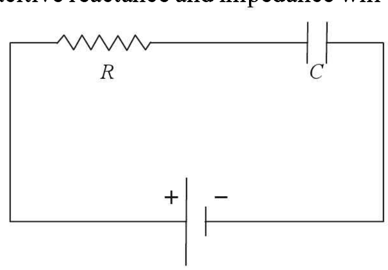


Fig. 27.52

(a) zero, zero (b) infinity, infinity

(c) $\frac{1}{\omega c}, \sqrt{R^2 + \frac{1}{C^2\omega^2}}$ (d) $\omega c, \sqrt{\frac{1}{\omega^2 C^2} + R^2}$

100. The value of alternating emf in the following circuit will be

(a) 220 volt (b) 140 volt

(c) 20 volt (d) 100 volt

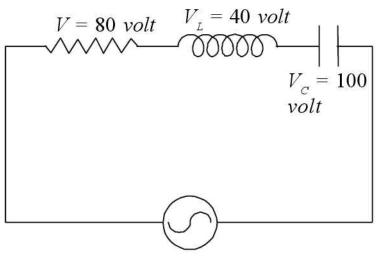


Fig. 27.53

101. The alternating emf applied and the current flowing in an *AC* circuit are represented by $E = E_0 \sin\omega t$ and $I = I_0 \sin(\omega t + \pi/2)$ respectively. The power loss in the circuit will be

(a) zero (b) $\frac{E_o I_o}{2}$

(c) $\frac{E_o I_o}{\sqrt{2}}$ (d) $\frac{E_o I_o}{4}$

102. A resistance of 10 Ω and an inductance of 100 *mH* are connected in series with an AC source of voltage $V = 100 \cos(100t)$ volt. The phase difference between the voltage applied and the current flowing in the circuit will be

(a) zero (b) $\frac{1}{2}$

(c) $\frac{\pi}{2}$ (d) π

103. When the values of inductance and capacitance in an *L-C* circuit are 0.5 *H* and 8μF respectively then current in the circuit is maximum. The angular frequency of alternating e.m.f. applied in the circuit will be

(a) 5×10^3 Radian/sec (b) 50 Radian/sec

(c) 5×10^2 Radian/sec (d) 5 Radian/sec

104. The frequency of applied A.V. is 2 KHz. Its time period will be

(a) 0.5×10^{-3} second (b) 5 second

(c) 0.5 second (d) 2 second

105. The value of induced e.m.f. in an *R-L* circuit at break, as compared to its value at make, will be

(a) less (b) more

(c) sometimes less and sometimes more

(d) nothing can be said

106. The time constant of an *R-C* circuit during discharge is that time in which charge on condenser plates, as compared to maximum charge (q_0), becomes

(a) 63.3% (b) 36.6%

(c) 50% (d) 25%

107. The unit of *I* ~~z~~ *q* is

(a) Newton (b) Dyne

(c) Joule (d) Joule/s

108. An *AC* voltmeter in an *L-C-R* circuit reads 30 volt across resistance, 80 volt across inductance and 40 volt across capacitance. The value of applied voltage will be

(a) 50 volt (b) 25 volt

(c) 150 volt (d) 70 volt

109. An experimentalist has a coil of 3 *mH*. He wants to make a circuit whose frequency is 106 Hz. The capacity of condenser used will be

(a) 0.44 *pf* (b) 8.44 *pf*

(c) 4.44 *pf* (d) zero

110. A coil of 10 *mH* and 10 Ω resistance is connected in parallel to a capacitance of 0.1μF. The impedance of the circuit at resonance will be

(a) $10^2\,\Omega$ (b) $10^4\,\Omega$

(c) $10^6\,\Omega$ (d) $10^8\,\Omega$

111. The self inductance of a coil is 1/2 henry. At what frequency will its inductive reactance be 3140 Ω

(a) 100 Hz (b) 10 Hz

(c) 1000 Hz (d) 10000 Hz

112. The capacitive reactance of a condenser of capacity 25μF for an AC of frequency 4000 Hz will be

(a) $\frac{5}{\pi}\,\Omega$ (b) $\frac{10}{\pi}\,\Omega$

(c) $5\pi\,\Omega$ (d) $\frac{\pi}{5}\,\Omega$

113. If the inductance of a coil is 1 henry then its effective resistance in a *D.C.* circuit will be

(a) ∞ (b) zero

(c) 1 Ω (d) 2 Ω

114. An *A.C.* ammeter can be used in

(a) only in DC circuit

(b) only in *AC* circuit

(c) both in *AC* and *DC* circuits

(d) neither in *AC* nor in *DC* circuits

115. The value of power factor, in an *AC* circuit, is zero in

(a) only inductive circuit (b) only resistive circuit

(c) an *L-R* circuit (d) an *R-C* circuit

116. The power loss in an *AC* circuit is $E_{rms} I_{rms}$, when in the circuit there is only

(a) *C* (b) *L*

(c) *R* (d) *L*, *C* and *R*

117. In the following circuit diagram, if the frequency of the source is doubled then the value of current flowing in R will become

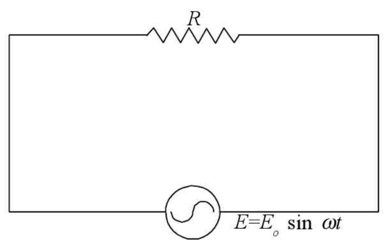


Fig. 27.54

(a) double (b) half
(c) remain unchanged (d) four times

118. The correct formula for the angular frequency ω_0 of an L-C resonant circuit is

(a) $\frac{1}{\sqrt{LC}}$ (b) $\sqrt{LC}$
(c) $\frac{1}{2\pi\sqrt{LC}}$ (d) $\frac{1}{2\pi LC}$

119. The specific resistance of fuse wire is

(a) high (b) low
(c) zero (d) infinity

120. The energy expended in 1 kW electric heater in 30 seconds will be

(a) $3\times I0^4$ joule (b) $3\times I0^4$ erg
(c) $3\times I0^4$ eV (d) zero

Answers to Questions for Practice

1.	(a)	**2.**	(a)	**3.**	(d)	**4.**	(c)	**5.**	(d)	**6.**	(c)	**7.**	(a)
8.	(a)	**9.**	(b)	**10.**	(b)	**11.**	(c)	**12.**	(c)	**13.**	(b)	**14.**	(b)
15.	(b)	**16.**	(c)	**17.**	(b)	**18.**	(a)	**19.**	(a)	**20.**	(b)	**21.**	(c)
22.	(a)	**23.**	(d)	**24.**	(d)	**25.**	(b)	**26.**	(a)	**27.**	(a)	**28.**	(c)
29.	(a)	**30.**	(b)	**31.**	(b)	**32.**	(c)	**33.**	(c)	**34.**	(a)	**35.**	(a)
36.	(a)	**37.**	(b)	**38.**	(a)	**39.**	(b)	**40.**	(b)	**41.**	(b)	**42.**	(a)
43.	(d)	**44.**	(a)	**45.**	(c)	**46.**	(a)	**47.**	(b)	**48.**	(b)	**49.**	(a)
50.	(d	**51.**	(d)	**52.**	(c)	**53.**	(a)	**54.**	(b)	**55.**	(c)	**56.**	(c)
57.	(d)	**58.**	(c)	**59.**	(d)	**60.**	(b)	**61.**	(c)	**62.**	(d)	**63.**	(a)
64.	(b)	**65.**	(b)	**66.**	(b)	**67.**	(b)	**68.**	(c)	**69.**	(c)	**70.**	(a)
71.	(b)	**72.**	(b)	**73.**	(a)	**74.**	(b)	**75.**	(b)	**76.**	(a)	**77.**	(c)
78.	(a)	**79.**	(a)	**80.**	(a)	**81.**	(c)	**82.**	(a)	**83.**	(b)	**84.**	(b)
85.	(a)	**86.**	(a)	**87.**	(d)	**88.**	(c)	**89.**	(a)	**90.**	(b)	**91.**	(b)
92.	(d)	**93.**	(b)	**94.**	(a)	**95.**	(c)	**96.**	(a)	**97.**	(b)	**98.**	(c)
99.	(b)	**100.**	(d)	**101.**	(a)	**102.**	(c)	**103.**	(c)	**104.**	(a)	**105.**	(b)
106.	(b)	**107.**	(c)	**108.**	(a)	**109.**	(b)	**110.**	(b)	**111.**	(c)	**112.**	(a)
113.	(b)	**114.**	(c)	**115.**	(a)	**116.**	(c)	**117.**	(c)	**118.**	(a)	**119.**	(a)
120.	(a)												

Explanations

20 (c) $\tan\varphi = \frac{X}{R} = 1$. Power factor $= \cos\varphi = \frac{1}{\sqrt{2}}$.

21 (b)The LC circuit draws no power. When $\omega L = \frac{1}{\omega C}$, the impedances of the RC and LR circuits are equal, and they draw the same power.

22 (a) The supply voltage E, the potential difference (V_R) across the resistance, and the potential difference (V_C) across the capacitor are related as

$$E^2 = V_R^2 + V_C^2 \;\therefore\; V_C = \sqrt{220^2 - (90)^2}$$
$$= 10^2\sqrt{4.03}$$

28

Electromagnetic Waves

BRIEF REVIEW

Assume electric field $E = E_0 \sin(\omega t - kx)$ and magnetic field $B = B_0 \sin(\omega t - kx)$ vary with distance x and time t in YZ plane. Such a combination of electric and magnetic fields in vacuum is known as an electromagnetic wave propagating along x direction in vacuum. Maxwell, in 1864, developed the theory of em waves.

Maxwell's equations Maxwell combined four equations connecting electric and magnetic fields. These are now popularly called Maxwell's equations.

$$\oint E \cdot ds = \frac{Q}{\varepsilon_0} \text{ Gauss law in electrostatics}$$

$$\oint E \cdot dl = \frac{-d\phi_{mag}}{dt} \text{ Faraday's law}$$

$$\oint B \cdot ds = 0 \text{ Gauss law in magnetism}$$

$$\oint B \cdot dl = \mu_0 i_C + \mu_0 i_d \text{ modified Ampere's law}$$

$$= \mu_0 i_c + \mu_0 \varepsilon_0 \frac{d\phi_E}{dt}$$

where $i_d = \varepsilon_0 \dfrac{d\phi_E}{dt}$ is displacement current.

In vacuum $i_c = 0$ $\therefore$ $\oint B \cdot dl = \mu_0 i_d = \mu_0 \varepsilon_0 \dfrac{d\phi_E}{dt}$

conduction current

or $i_d = \dfrac{\varepsilon_0 d\phi_E}{dt} = \dfrac{d(q_{in})}{dt}$ is displacement current.

Properties of Electromagnetic Waves

(a) Though em waves are generated due to variation of electric or magnetic fields, the waves themselves do not carry any charge.

(b) These waves are not deflected by electric or magnetic fields.

(c) These waves travel with speed of light c in vacuum.

(d) These waves can pass through vacuum as no medium is required for their propagation.

(e) They are only transverse in nature.

(f) They affect photographic plate (blackening it if wavelength < wavelength of red light).

(g) Their rest mass is zero but they possess momentum.

(h) They can be polarised.

Relation between *E* and *B*

$$E_0 = B_0 c \text{ and } c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$

where $\mu_0 = 4\pi \times 10^{-7}\, T\text{–}mA^{-1}$

$\varepsilon_0 = 8.85419 \times 10^{-12} C^2 N^{-1} m^{-2}$ wavelength $\lambda = \dfrac{c}{f}$

where f is frequency.

Refractive index $n = \sqrt{\mu_r \varepsilon_r}$ or $v = \dfrac{c}{\sqrt{\mu_r \varepsilon_r}}$

In a travelling electromagnetic wave $E\sqrt{\varepsilon_r\varepsilon_o} = H\sqrt{\mu_r\mu_0}$, momentum $p = \frac{U}{c}$ where $U = m_0c^2$ is energy.

When the wave reflects, momentum becomes –ve

Force $F = \frac{Power}{c}$ (in absorbing bodies) and $F = \frac{2Power}{c}$ (in reflecting bodies like mirror).

Intensity $I = \frac{1}{2}\varepsilon_0 E_0^2 c$ and

energy density $u = \frac{1}{2}\varepsilon_0 E_0^2 = \frac{B_0^2}{2\mu_0}$

$$u = \frac{1}{2}ED + \frac{BH}{2} = \frac{1}{2}\varepsilon_0 E^2 + \frac{B^2}{2\mu_0}$$

Flow density of electromagnetic energy or Poynting Vector $\vec{P} = \vec{E}\times\vec{H} = \frac{1}{\mu_0}\left(\vec{E}\times\vec{B}\right)$ this describes rate of energy flow per unit area in a plane electromagnetic wave. Unit of $\vec{P} = Wm^{-2}$ or $\left|\vec{P}\right| = \frac{E_0B_0}{2\mu_0}$ power per unit area.

Energy flow density of electric dipole radiation in a far field zone varies $\frac{\sin^2\theta}{r^2}$ where r is the distance from the dipole, and θ is the angle between the radius vector r and axis of the dipole.

Radiation power of a dipole with dipole moment $p(t)$ and of a charge q moving with an acceleration a is

$$P_{\text{radiation}} = \frac{1}{4\pi\varepsilon_0}\frac{2p^2}{3c^3} = \frac{1}{4\pi\varepsilon_0}\frac{2q^2a^2}{3c^3}$$

If h is the height of antenna then program can be received upto a radius of $r = \sqrt{2hR}$ where R is radius of the earth.

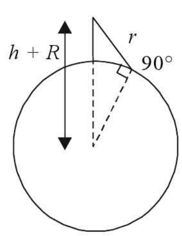


Fig. 28.1 Radius upto which transmission can be received

• Short Cuts and Points to Note

1. Electromagnetic waves are transverse waves produced due to variation of electric and magnetic fields held perpendicular to one another. Wave propagates perpendicular to both electric and magnetic fields.

   If $E_y = E_0\sin(\omega t - kx)$ varies in y– direction and $B_z = B_0\sin(\omega t - kx)$ varies in z–direction, then wave progresses in x– direction.

2. Maxwell equations in vacuum are

   $$\oint E\cdot dS = \frac{Q}{\varepsilon_0};\quad \oint B\cdot dS = 0$$

   $$\oint E\cdot dl = \frac{d\phi_{\text{magnetic}}}{dt};\quad \oint B\cdot dl = \mu_0\varepsilon_0\frac{d\phi_E}{dt} = \mu_0 i_d$$

   The electric field so generated is non-conservative. Note that in vacuum conduction current is zero. Therefore only displacement current remains.

3. Force experienced by electomagnetic wave

   $F = \frac{Power}{c}$ for a totally absorbing surface. $F = P_{\text{radiation}} A$ where $P_{\text{radiation}}$ is radiation pressure and A is area.

   $F = \frac{2Power}{c}$ for a perfectly reflecting surface.

4. Momentum $p = \frac{U}{c} = \frac{Energy}{c}$ for totally absorbing surface and $p = \frac{2U}{c}$ for totally reflecting surface

5. Average energy density

   $$u = \frac{U_{\text{average}}}{\text{Volume}} = \frac{1}{2}\varepsilon_0 E_0^2 = \frac{B_0^2}{2\mu_0}$$

6. Intensity $I = \frac{1}{2}\varepsilon_0 E_0^2 c$ and $E_0 = B_0 c$ and $c = \frac{1}{\sqrt{\mu_0\varepsilon_0}}$

   Note that refractive index $n = \sqrt{\mu_r\varepsilon_r}$ or $\varepsilon_r \propto n^2$

7. Phase velocity $v = \frac{c}{\sqrt{\mu_r\varepsilon_r}}$;

   $c = f\lambda = \frac{\omega}{k} = \frac{E_0}{B_0} = \frac{1}{\sqrt{\mu_0\varepsilon_0}}$ is independent of wavelength or frequency

8. Impedance of free space $z_0 = \frac{E}{H} = 377\Omega = \sqrt{\frac{\mu_0}{\varepsilon_0}}$

9. In a wave guide $v_{\text{phase}} = \frac{c}{\sqrt{1-\left(\frac{\lambda}{2a}\right)^2}}$

   $$v_{\text{group}} = c\sqrt{1-\left(\frac{\lambda}{2a}\right)^2}$$

   Guide wavelength $\lambda g = \frac{v_{phase}}{f} = \frac{v_{phase}\lambda}{c} = \frac{\lambda}{\sqrt{1-\left(\frac{\lambda}{2a}\right)^2}}$

   Cut off wavelength $\lambda_c = 2a$

10. Poynting vector $\vec{P} = \frac{1}{\mu_0}[\vec{E} \times \vec{B}]$. It describes power flowing per unit area and power = $\oint \vec{P} \cdot dA$

Intensity $I = <\vec{P}> = \frac{E_0 B_0}{2\mu_0} = \frac{B_0^2 c}{2\mu_0} = \frac{E_0^2}{2c\mu_0}$. It describes average power flowing per sec or intensity.

11. Another form of Maxwell's equation (differential form)

$$\vec{\nabla} \times \vec{E} = \frac{\partial \vec{B}}{\partial t} \; ; \; \vec{\nabla} \cdot \vec{B} = 0 \; ; \; \vec{\nabla} \times \vec{H} = j + \frac{\partial D}{\partial t} \; ;$$

$$\vec{\nabla} \cdot \vec{D} = \rho$$

12. Radius upto which transmission can be received from an antenna of height h $\quad r = \sqrt{2Rh}$

13. Doppler's effect in light $\frac{\Delta\lambda}{\lambda} = \frac{v}{c}$ or $\frac{\Delta f}{f} = \frac{v}{c}$

14. Relative velocity $v_{rel} = \dfrac{u_1 - u_2}{1 - \frac{u_1 u_2}{c^2}}$

15. Energy flow density of electric dipole radiation in a far field zone $\sim \frac{\sin^2\theta}{r^2}$

16. Radiation power of an electric dipole with momentum $p(t)$ and charge q moving with an acceleration a is

$$P_{radiation} = \frac{1}{4\pi\varepsilon_0} \frac{2\ddot{p}^2}{3c^2}$$

$$= \frac{1}{4\pi\varepsilon_0} \frac{2q^2a^2}{3c^3}$$

17. **Radiation Pressure**

$P_{radiation} = \frac{I}{c}$ if surface is absorbing

$P_{radiation} = \frac{2I}{c}$ if surface is totally reflecting, where I is intensity.

18. In standing em wave, nodal planes occur at $x = 0$, $\frac{\lambda}{2}, \lambda, \frac{3\lambda}{2}$,..... for electric field ($E\infty$) and antinodal planes for magnetic field (B = max). Antinodal planes of electric field (E= max) occur at $x = \frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}$ ... and are nodal planes for magnetic field ($B = 0$)

Note that standing em waves are formed in wave guides or cavities.

19. Visible spectrum

400–440 nm violet	440–480 nm blue
480–560 nm green	560–590 nm yellow
590–630 m orange	630–700 nm red.

em wave spectrum

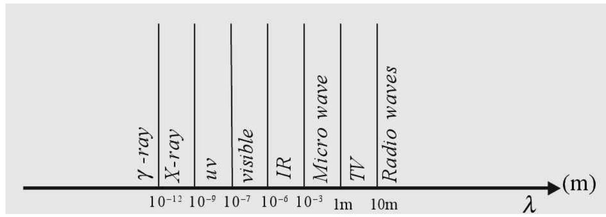


Fig. 28.2

• Caution

1. Assuming that since rest mass is zero, Therefore em wave cannot have any momentum.

⇒ Momentam = $\frac{Energy}{c} = \frac{U}{c}$ for absorbing surface

and $\frac{2U}{c}$ for reflecting surface.

2. Applying ordinary laws of relative velocity for photons or for particles moving with speed close to *c*.

⇒ Apply special theory of relativity in such cases.

3. Considering electromagnetic waves do not exert any force as they do not possess mass.

⇒ $F = \frac{Power}{c}$ for totally absorbing surface and $F = \frac{2Power}{c}$ for perfectly reflecting surface.

4. Assuming that even in a wave guide em wave travels with *c*.

⇒ In a wave guide they travel with group or phase velocities.

5. Not remembering the modification made by Maxwell in Ampere's Law

⇒ $\oint B.dl = \mu_0 i_c + \mu_0 \varepsilon_0 \frac{d\phi_E}{dt} = \mu_0 i_c + \mu_0 i_d$. The second term in the equation is a modification. It gives displacement current (in vacuum).

6. Confusing energy density and intensity.

⇒ Energy density is average energy per unit volume

$$u = \frac{1}{2}\varepsilon_0 E_0^2 = \frac{B_0^2}{2\mu_0}$$

while intensity is rate of flow of average energy per unit area. $I = \frac{1}{2}\varepsilon_0 E_0^2 c$

7. Confusing Poynting vector with intensity.

$\Rightarrow$ Poynting vector gives instantaneous rate of flow of energy per unit area carried by em waves. Since the frequency of em waves is quite large, it is very difficult to notice the variation with time. We therefore define intensity. Intensity gives rate of flow of average energy per unit area.

SOLVED PROBLEMS

1. A cube of edge a has its edges parallel to x,y and z-axis of rectangular co-ordinate system. A uniform electric field E is parallel to y-axis and a uniform magnetic field is parallel to x axis. The rate at which energy flows through each face of the cube is

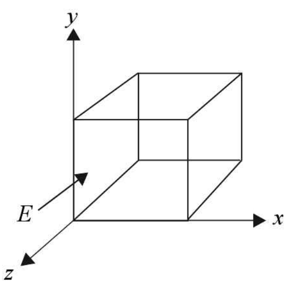


Fig. 28.3

(a) zero in all faces

(b) $\frac{a^2EB}{2\mu_0}$ parallel to xy plane face and zero in others

(c) $\frac{a^2EB}{\mu_0}$ parallel to xy plane face and zero in others

(d) $\frac{a^2EB}{2\mu_0}$ all faces

Solution (c) $\vec{p} = \frac{1}{\mu_0}\left[\vec{E}\times\vec{B}\right]$ gives the clue that energy flowing per second $= \frac{a^2EB}{\mu_0}$ in faces parallel to xy plane and zero in all others.

2. The amplitude of electric field in a parallel light beam of intensity 4 Wm^{-2} is

(a) 35.5 NC^{-1} (b) 45.5 NC^{-1}
(c) 49.5 NC^{-1} (d) 55.5 NC^{-1}

Solution (d) $I = \frac{1}{2}\varepsilon_0 E_0^2 c$ or $E_0 = \sqrt{\frac{2I}{\varepsilon_0 c}}$

$$= \sqrt{\frac{2\times 4}{8.85\times 10^{-12}\times 3\times 10^8}}$$

3. **(A) Assertion:** The energy E and momentum p of a photon are related as $p = \frac{E}{c}$.

(R) Reason: The photon behaves like a particle.

(a) Both A and R are correct and R is correct explanation of A.

(b) Both A and R are correct but R is not correct explanation of A.

(c) A is correct but R is wrong.

(d) Neither is correct.

[AIIMS 2005]

Solution (a)

4. **(A)** TV signals are received through sky wave propagation

(R) Ionosphere reflects em waves of frequencies > a critical frequency

(a) A and R are correct and R explains A.

(b) A and R are correct but R does not explain A.

(c) A is correct but R is wrong

(d) Neither is correct.

[AIIMS 2005]

Solution (d)

5. If $\lambda_v, \lambda_x, \lambda_m$ represent the wavelengths of vision, x-ray and microwaves respectively then

(a) $\lambda_m > \lambda_x > \lambda_v$ (b) $\lambda_m > \lambda_v > \lambda_x$
(c) $\lambda_v > \lambda_x > \lambda_m$ (d) $\lambda_v > \lambda_m > \lambda_x$

Solution (b)

6. Electrical conductivity of a semiconductor increases when em radiation of $\lambda < 2480$ nm is incident on it. The band gap in (ev) for the semiconductor is

(a) 1.1eV (b) 2.5 eV
(c) 0.7 eV (d) 0.5 eV

[AIEEE 2005]

Solution (d) $E_g = \frac{1240}{\lambda(nm)}$

$$= \frac{1240}{2480} = 0.5\text{ eV.}$$

7. Infrared radiation was discovered in 1860 by

(a) William Wallaston (b) William Herschel
(c) William Roentgen (d) Thomas Young

[CET Karnataka 2005]

Solution (b)

8. A radio wave of frequency 90 MHz (FM) enters a ferrite rod. In $\varepsilon_r = 10^3$ and $\mu_r = 10$ then the velocity and wavelength of ferrite are

(a) $3 \times 10^6\,ms^1, 3.33 \times 10^{-2}m$

(b) $3 \times 10^6 ms^{-1}, 3.33 \times 10^{-1} m$

(c) $3 \times 10^6 ms^{-1}, 3.33 \times 10^{-3} m$

(d) none of these

Solution (a) $v_{ferrite} = \dfrac{c}{\sqrt{\varepsilon_r \mu_r}} = \dfrac{3\times10^8}{\sqrt{10^3\times10}} = 3 \times 10^6\,ms^1$

$$\lambda_{ferrite} = \frac{v_{ferrite}}{f} = \frac{3\times10^6}{90\times10^6} = 3.33 \times 10^{-2}m$$

9. In a wave $E_0 = 100\ Vm^{-1}$. Find the Poynting vector magnitude.

(a) $13.25\ Wm^{-2}$ (b) $26.5\ Wm^{-2}$

(c) $18.25\ Wm^{-2}$ (d) $19.7Wm^{-2}$

Solution (b) $B = \dfrac{E}{c}$ $|\vec{p}| = \dfrac{EB}{\mu_0} = \dfrac{E^2}{c\mu_0}$

$$= \frac{10^4}{3\times10^8\times4\pi\times10^{-7}}$$

$= 26.5\ Wm^{-2}$

10. A radio station on the surface of the earth radiates 50kW. If transmitter radiates equally in all directions above the surface of earth find the amplitude of electric field detected 100 km away.

(a) $2.45\ Vm^{-1}$ (b) $2.45 \times 10^{-1}\,Vm^{-1}$

(c) $2.45 \times 10^{-2}\ Vm^{-1}$ (d) $2.45 \times 10^{-3}\,Vm^{-1}$

Solution (c) $I = \dfrac{P}{2\pi r^2}$ because emission is hemispherical and $I = \dfrac{\varepsilon_0 E_0^2 c}{2} = \dfrac{P}{2\pi r^2}$

$$E_0 = \sqrt{\frac{P}{\varepsilon_0 c\pi r^2}} = \sqrt{\frac{50\times10^3}{8.85\times10^{-12}\times3\times10^8\times10^{10}}}$$

$= 2.45 \times 10^{-2}\ Vm^{-1}$

11. Find the radiation pressure of solar radiation on the surface of earth. Solar constant is 1.4 $kW\ m^{-2}$

(a) 4.7×10^{-5} Pa (b) 4.7×10^{-6} Pa

(c) 2.37×10^{-6} Pa (d) 9.4×10^{-6} Pa

Solution (b) $P_{rad} = \dfrac{I}{c} = \dfrac{1.4\times10^3}{3\times10^8} = 4.7 \times 10^{-6}$Pa

12. An earth-orbiting satellite has solar energy collecting panel with total area 5 m^2. If solar radiations are perpendicular and completely absorbed find the average force associated with the radiation pressure.

(a) 2.33×10^{-5}N (b) 2.33×10^{-6}N

(c) 2.33×10^{-4}N (d) 2.33×10^{-7}N

Solution (a) Power $= I$ Area $= 1.4 \times 10^3 \times 5$

and $F = \dfrac{\text{Power}}{c} = \dfrac{1.4\times10^3\times5}{3\times10^8} = 2.33 \times 10^{-5}$N

13. Find the cut off wavelength in a waveguide of two parallel walls 1.5 cm apart. Where is $E = 0$ and where is E = maximum?

Solution $\lambda c = 2a = 2 \times 1.5 = 3$cm

$E = 0$ at walls as half wavelength points occur there at wall. $E = max$ in the middle of the walls as $\lambda/4$ planes occur there.

14. A radio transmitter transmits at 830 k Hz. At a certain distance from the transmitter magnetic field has amplitude 4.82×10^{-11}T. Find electric field and wavelength.

(a) $14.46 \times 10^{-3}\ NC^{-1}$, 35 m

(b) $14.46 \times 10^{-3}NC^{-1}$, 350 m

(c) $1.45 \times 10^{-3}NC^{-1}$, 35 m

(d) none of these

Solution (b) $E = Bc = 4.82 \times 10^{-11} \times 3 \times 10^8$

$= 14.46 \times 10^{-3}\ NC^{-1}$

$$\lambda = \frac{c}{f} = \frac{3\times10^8}{8.3\times10^5} = 3.5 \times 10^2 m.$$

15. An intense light source radiates uniformly in all directions. At a distance 5 m from the source, the radiation pressure on absorbing surface is 9×10^{-6} Pa. Find the total average power output.

(a) 8.5×10^5 W (b) 6.5×10^5 W

(c) 8.5×10^3 W (c) $6.5\ c\times 10^3$ W

Solution (a) $p_{rad} = \dfrac{I}{c}$ and Power $= I\,4\pi r^2$

$= p_{rad} \times C\ 4\pi r^2$

$= 9 \times 10^{-6} \times 3 \times 10^8 \times 12.56 \times 25$

$= 8.5 \times 10^5$ W

16. A bank of overhead arc lamps can produce a light intensity of 2500 Wm^{-2} in the 25 ft space stimulator facility at NASA. Find the average momentum density of a total absorbing surface.

(a) $8.33 \times 10^{-6}kgm^{-2}s^{-1}$ (b) $8.33 \times 10^{-14}kgm^{-2}s^{-1}$

(c) $2.78\ kgm^{-2}s^{-1}$ (d) $2.78 \times 10^{-14}kgm^{-2}s^{-1}$

Solution (d) $I = \dfrac{1}{2}\varepsilon_0 E_0^2 c$ and energy density

$= \frac{I}{c}$, momentum density $= \frac{I}{c^2}$

$(\because \frac{E}{c} = p)$ $\frac{2500}{9\times10^{16}} = 2.78 \times 10^{-14}$ kgm^{-2}s^{-1}

17. A standing em wave frequency 2.2×10^{10} Hz is produced in a certian material and nodal planes of magnetic field are 3.5 mm apart. Find wavelenth and speed of the wave in this material.

(a) 2.81×10^8ms^{-1} (b) 1.79×10^8ms^{-1}
(c) 3.08×10^8ms^{-1} (d) 1.54×10^8ms^{-1}

Solution (d) $\frac{\lambda}{2} = 3.5$ mm

$\lambda = 7.0$ mm

$v = f\lambda = 2.2 \times 10^{10} \times 0.7 \times 10^{-2}$

$= 1.54 \times 10^8$ ms^{-1}

18. Which of the following rays is emitted by a human body?

(a) x-rays (b) visible rays
(c) uv rays (d) IR rays
(e) none of these

Solution (d) IR because temperature of the body is 37°C and it emits heat radiation which falls in IR and microwave region.

19. Which of the following waves is used in Raman spectroscopy?

(a) uv (b) x-rays
(c) γ-rays (d) IR

Solution (d)

20. Which of the following relations is correct for em waves

(a) $\frac{\partial^2 E}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 E}{\partial t^2}$ (b) $\frac{\partial^2 E}{\partial x^2} = \left(\frac{\partial E}{\partial t}\right)^2$
(c) $\frac{\partial^2 E}{\partial x^2} = \frac{c^2 \partial^2 E}{\partial t^2}$ (d) $\frac{\partial^2 E}{\partial t^2} = c\frac{\partial^2 E}{\partial t^2}$

Solution (c)

TYPICAL PROBLEMS

21. A radiation of 200 W is incident on a surface which is 60% reflecting and 40% absorbing.

(a) 1.3×10^{-6} N (b) 1.07×10^{-6} N
(c) 1.07×10^{-7} N (d) 1.3×10^{-7} N

Solution (b) $F_{Tot} = F_{ref} + F_{abs} = \frac{1.2p}{c} + \frac{0.4p}{c} = \frac{1.6p}{c}$

$= \frac{1.6\times200}{3\times10^8} = 1.07 \times 10^{-6}$

22. The $<\vec{p}>$ for a standing wave is

(a) zero (b) $\frac{E_0B_0}{2\mu_0}$
(c) $\frac{\varepsilon_0 E_0^2}{2}$ (d) $\frac{E_0B_0}{\mu_0}$

Solution (a)

23. What amplitude of electric/magnetic field is required to be transmitted in a beam of cross-section area 100 m^2 so that it is comparable to electric power of 500 kV and $10^3 A$?

Solution $\frac{1}{2}\,\varepsilon_0 E_0^2\, c A = 5 \times 10^8$ or $\frac{1}{2}\,\varepsilon_0 E_0^2$

$= \frac{5\times10^8}{3\times10^8\times100} = \frac{5}{3} \times 10^{-2}$

$E_0 = \sqrt{\frac{10^{-1}}{3\times8.85\times10^{-2}}} = \frac{10^6}{16} = 6.125 \times 10^4$ N/C

$B_0 = \frac{E_0}{c} = \frac{6.125\times10^4}{3\times10^8} = 2.08 \times 10^{-4}$ T

24. A straight coaxial cable of negligible active resistance is receiving energy from a constant voltage source V. Current consumed is I. Find energy flux across the cross-section. Assume conductive sheath to be thin.

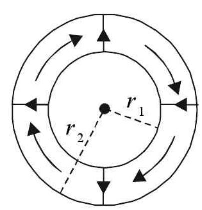


Fig. 28.4

Solution Let electric field from Gauss's law be $E_r = \frac{K}{r}$

and $V = \int_{r_2}^{r_1} -E_r dr = K \log_e \frac{r_2}{r_1}$

or $K = \frac{V}{\log_e \frac{r_2}{r_1}}$

and $E_r = \frac{V}{r \log_e \frac{r_2}{r_1}}$

$$H_0 = \frac{I}{2\pi r} \text{ using Ampere's circuital law}$$

Poynting vector $\vec{p}$ acts along z-axis and is non zero in the region ($r_1<r<r_2$). The total power flux is

$$= \int_{r_1}^{r_2} \frac{IV}{2\pi r^2 \log \frac{r_2}{r_1}} 2\pi r dr = IV$$

25. Consider that the space between the parallel plates of a capacitor has vacuum. The places are connected to a battery of emf and internal resistance R at $t=0$. If V is the potential difference between the plates, I_d is displacement current and we define $R_d = \frac{V}{i_d}$ then show that $R_d = R\,(e^{t/Rc} - 1)$.

Solution $R = \frac{V}{i_d} = \frac{V_0\left(1-e^{-t/Rc}\right)}{\frac{Q_0}{RC}\left(e^{-t/Rc}\right)} = R\,(e^{t/Rc} - 1)$ using

$$Q = Q_0(1 - e^{t/Rc})$$

$$\frac{dQ}{dt} = i = \frac{Q_0}{RC}\, e^{t/Rc}$$

$$i_d = i_c = \frac{Q_0}{RC}\, e^{t/Rc}$$

$$i_d = \frac{V_0}{R}\, e^{-t/Rc} \quad \because \frac{Q_0}{C} = V_0$$

QUESTIONS FOR PRACTICE

1. An electric field $\vec{E}$ and a magnetic field $\vec{B}$ exist in a region. The fields are not perpendicular to each other.

(a) This is not possible.

(b) No electromagnetic wave is passing through the region.

(c) An electromagnetic wave may be passing through the region.

(d) An electromagnetic wave is certainly passing through the region.

2. Consider the following two statements regarding a linearly polarised, plane electromagnetic wave:

(A) The electric field and the magnetic field have equal average values.

(B) The electric energy and the magnetic energy have equal average values.

(a) Both A and B are true.

(b) A is false but B is true.

(c) B is false but A is true.

(d) Both A and B are false.

3. A free electron is placed in the path of a plane electromagnetic wave. The electron will start moving

(a) along the electric field

(b) along the magnetic field

(c) along the direction of propagation of the wave

(d) in a plane containing the magnetic field and the direction of propagation

4. A plane electromagnetic wave is incident on a material surface. The wave delivers momentum p and energy E.

(a) $p=0, E\neq 0$. (b) $p\neq 0, E=0$.

(c) $p\neq 0, E\neq 0$. (d) $p=0, E=0$.

5. An electromagnetic wave going through vacuum is described by $E = E_0 \sin(kx - \omega t)$.

Which of the following is/are independent of the wavelength ?

(a) k (b) ω

(c) $\frac{\omega}{k}$ (d) $k\omega$.

6. Speed of electromagenetic waves is the same

(a) for all wavelengths (b) in all media

(c) for all intensities (d) for all frequencies

7. The energy contained in a small volume through which an electromagnetic wave is passing oscillates with

(a) zero frequency

(b) the frequency of the wave

(c) half the frequency of the wave

(d) double the frequency of the wave

8. If $\vec{E}$ and $\vec{B}$ are the electric and magnetic field vectors of electromagnetic waves then the direction of propagation of electromagnetic wave is along the direction of

(a) $\vec{E}$ (b) $\vec{B}$

(c) $\vec{E} \times \vec{B}$ (d) none of these

9. The charge on a parallel plate capacitor is varying as $q = q_0 \sin 2\partial nt$. The plates are very large and close together. Neglecting the edge effects, the displacement current through the capacitor is

(a) $\frac{q}{\varepsilon_0 A}$ (b) $\frac{q_0}{\varepsilon_0}$ sin 2*p nt*

(c) 2 *p* nq_0 cos 2*p nt* (d) $\frac{2\pi nq_0}{\varepsilon_0}$ cos 2*p nt*

10. The value of magnetic field between plates of capacitor, at distance of 1 m from centre where electric field varies by 10^3 V/m/s will be

(a) 5.56 *T* (b) 5.56 μT

(c) 5.56 *mT* (d) 55.6 *nT*

11. Electromagnetic waves do not transport

(a) energy (b) charge

(c) momentum (d) information

12. A capacitor is connected in an electric circuit. When key is pressed, the current in the circuit is

(a) zero (b) maximum

(c) any transient value

(d) depends on capacitor used

13. Displacement current is continuous

(a) when electric field is changing in the circuit

(b) when magnetic field is changing in the circuit

(c) in both types of fields

(d) through wires and resistance only

14. Instantaneous displacement current 1*A* in the space between the parallel plates of 1 μF capacitor can be established by changing the potential difference at the rate of

(a) 0.1 V/s (b) 1 V/s

(c) 106 V/s (c) 10^{-6} V/s

15. The magnetic field between the plates of a capacitor when $r > R$ is given by

(a) $\frac{\mu 0 I_D r}{2\pi R^2}$ (b) $\frac{\mu 0 I_D}{2\pi R}$

(c) $\frac{\mu 0 I_D}{2\pi r}$ (d) zero

16. The magnetic field between the plates of a capacitor is

$u_0 Ir$ given by $B = \frac{\mu_0 Ir}{2\pi R^2}$ when

(a) $r > R$ (b) $r \neq R$

(c) $r < R$ (d) $r = R$

17. The conduction current is the same as displacement current when the source is

(a) *AC* only (b) *DC* only

(c) both *AC* and *DC*

(d) neither for *AC* nor for *DC*

18. The wave function (in SI units) for an electromagnetic wave is given as

$\psi(x, t) = 10^3 \sin p\,(3 \times 10^6 x - 9 \times 10^{14} t)$. The speed of the wave is

(a) 9×10^{14} m/s (b) 3×10^8 m/s

(c) 3×10^6 m/s (d) 3×10^7 m/s

19. In the above problem, wavelength of the wave is

(a) 666 nm (b) 666 ?

(c) 666 μm (d) 6.66 nm

20. Maxwell's four equations are written as

(i) $\oint \vec{E}.\vec{ds} = \frac{q_0}{\varepsilon_0}$

(ii) $\oint \vec{B}.\vec{ds} = 0$

(iii) $\oint \vec{E}.\vec{dl} = \frac{d}{dt}\oint \vec{B}.\vec{ds}$

(iv) $\oint \vec{B}.\vec{ds} = \mu_0 \varepsilon_0 \frac{d}{dt}\oint \vec{E}.\vec{ds}$

The equations which have sources of field are

(a) (i), (ii), (iii) (b) (i), (ii)

(c) (i) and (iii) only (d) (i) and (iv) only

21. Out of the above four equations, the equations which do not contain source field are

(a) (i) and (ii) (b) (ii) only

(c) all of four (d) (iii) only

22. Out of the four Maxwell's equations above, which one shows non-existence of monopoles ?

(a) (i) and (iv) (b) (ii) only

(c) (iii) only (d) none of these

23. Which of the above Maxwell's equations shows that electric field lines do not form closed loops ?

(a) (i) only (b) (ii) only

(c) (iii) only (d) (iv) only

24. In an electromagnetic wave the average energy density is associated with

(a) electric field only

(b) magnetic field only

(c) equally with electric and magnetic fields

(d) average energy density zero

25. In an electromagnetic wave the average energy density associated with magnetic field will be

(a) $\frac{1}{2} LI^2$ (b) $\frac{B^2}{2\mu_0}$

(c) $\frac{1}{2} \mu_0 B^2$ (d) $\frac{1}{2}\frac{\mu_0}{B^2}$

26. In the above problem, the energy density associated with the electric field will be

(a) $\frac{1}{2}CV^2$ (b) $\frac{1}{2}\frac{q^2}{C}$

(c) $\frac{1}{2}\frac{\varepsilon^2}{E}$ (d) $\frac{1}{2}\varepsilon_0 E^2$

27. If there were no atmosphere, the average temperature on earth surface would be

(a) lower (b) higher

(c) same (d) 0°C

28. In which part of earth's atmosphere is the ozone layer present?

(a) Troposphere (b) Stratosphere

(c) Ionosphere (d) Mesosphere

29. Kenneley's Heaviside layer lies between

(a) 50 Km to 80 Km (b) 80 Km to 400 Km

(c) beyond 110 Km (d) beyond 250 Km

30. The ozone layer in earth's atmosphere is crucial for human survival because it

(a) has ions

(b) reflects radio signals

(c) reflects ultraviolet rays

(d) reflects infrared rays

31. The frequency from 3×10^9 Hz to 3×10^{10} Hz is

(a) high frequency band

(b) super high frequency band

(c) ultra high frequency band

(d) very high frequency band

32. The frequency from 3 to 30 MHz is known as

(a) audio band

(b) medium frequency band

(c) very high frequency band

(d) high frequency band

33. The AM range of radiowaves has frequency

(a) less than 30 MHz (b) more than 30 MHz

(c) less than 20000 Hz (d) more than 20000 Hz

34. The displacement current flows in the dielectric of a capacitor when the potential difference across its plates

(a) becomes zero

(b) has assumed a constant value

(c) is increasing with time

(d) is decreasing with time

35. Select wrong statement from the following: Electromagnetic waves

(a) are transverse

(b) travel with same speed in all media

(c) travel with the speed of light

(d) are produced by accelerating charge

36. The waves related to telecommunication are

(a) infrared (b) visible light

(c) microwaves (d) ultraviolet rays

37. Electromagnetic waves do not transport

(a) energy (b) charge

(c) momentum (d) information

38. The nature of electromagnetic wave is

(a) longitudinal (b) longitudinal stationary

(c) transverse (d) transverse stationary

39. Greenhouse effect keeps the earth surface

(a) cold at night (b) dusty and cold

(c) warm at night (d) moist

40. A parallel plate capacitor consists of two circular plates each of radius 12 cm and separated by 5.0 mm. The capacitor is being charged by an external source. The charging current is constant and is equal to 0.15 A. The rate of change of potential difference between the plates will be

(a) 8.173×10^7 V/s (b) 7.817×10^8 V/s

(c) 1.873×10^9 V/s (d) 3.781×10^{10} V/s

41. In the above problem, the displacement current is

(a) 15A (b) 1.5A

(c) 0.15A (d) 0.015A

42. The wave emitted by any atom or molecule must have some finite total length which is known as the coherence length. For sodium light, this length is 2.4 cm. The number of oscillations in this length will be

(Given: $\lambda = 5900$)

(a) 4.068×10^5 (b) 4.068×10^6

(c) 4.068×10^7 (d) 4.068×10^8

43. In the above problem, the coherence time will be

(a) 8×10^{-8} s (b) 8×10^{-9} s

(c) 8×10^{-10} s (d) 8×10^{-11} s

44. A parallel plate capacitor made of circular plates each of radius R = 6 cm has capacitance C = 100 pF. The capacitance is connected to a 230 V AC supply with an angular frequency of 300 rad/s. The *rms* value of conduction current will be

(a) 5.7 μA (b) 6.3 μA

(c) 9.6 μA (d) 6.9 μA

45. In the above problem, the displacement current will be

(a) 6.9 μA (b) 9.6 μA
(c) 6.3 μA (d) 5.7 μA

46. In Q. 44, the value of B at a point 3 cm from the axis between the plates will be
(a) 1.63×10^{-8} T (b) 1.63×10^{-9} T
(c) 1.63×10^{-10} T (d) 1.63×10^{-11} T

47. A plane electromagnetic wave of frequency 40 MHz travels in free space in the X-direction. At some point and at some instant, the electric field has its maximum value of 750 N/C in Y-direction. The wavelength of the wave is
(a) 3.5 m (b) 5.5 m
(c) 7.5 m (d) 9.5 m

48. In the above problem, the period of the wave will be
(a) 2.5 μs (b) 0.25 μs
(c) 0.025 μs (d) none of these

49. In Q. 47, the magnitude and direction of magnetic field will be
(a) 2.5 μT in X-direction (b) 2.5 μT in Y-direction
(c) 2.5 μT in Z-direction (d) none of these

50. In Q. 47, the angular frequency of emf wave will be (in rad/s)
(a) $8p \times 10^7$ (b) $4p \times 10^6$
(c) $2p \times 10^5$ (d) $\pi \times 10^4$

51. In Q. 47, the propagation constant of the wave will be
(a) 8.38 m^{-1} (b) 0.838 m^{-1}
(c) 4.19 m^{-1} (d) 0.419 m^{-1}

52. The sun delivers 10^3 W/m^2 of electromagnetic flux to the earth's surface. The total power that is incident on a roof of dimensions 8 m × 20 m, will be
(a) 6.4×10^3 W (b) 3.4×10^4 W
(c) 1.6×10^5 W (d) none of these

53. In the above problem, the radiation force on the roof will be
(a) 3.33×10^{-5} N (b) 5.33×10^{-4} N
(c) 7.33×10^{-3} N (d) 9.33×10^{-2} N

54. In Q. 52, the solar energy incident on the roof in 1 hour will be—
(a) 5.76×10^8 J (b) 5.76×10^7 J
(c) 5.76×10^6 J (d) 5.76×10^5 J

55. The sun radiates electromagnetic energy at the rate of 3.9×10^{26} W. Its radius is 6.96×10^8 m. The intensity of sunlight at the solar surface will be (in W/m^2)
(a) 1.4×10^4 (b) 2.8×10^5
(c) 4.2×10^6 (d) 5.6×10^7

56. In the above problem, if the distance from the sun to the earth is 1.5×10^{11} m, then the intensity of sunlight on earth's surface will be (in W/m^2)
(a) 1.38×10^3 (b) 2.76×10^4
(c) 5.52×10^5 (d) none of these

57. A laser beam can be focussed on an area equal to the square of its wavelength. A He-Ne laser radiates energy at the rate of 1mW and its wavelength is 632.8 nm. The intensity of focussed beam will be
(a) 1.5×10^{13} W/m^2 (b) 2.5×10^9 W/m^2
(c) 3.5×10^{17} W/m^2 (d) none of these

58. A flood light is covered with a filter that transmits red light. The electric field of the emerging beam is represented by a sinusoidal plane wave $E_x = 36 \sin(1.20 \times 10^7 z + 6 \times 10^{15} t)$ V/m. The average intensity of the beam will be
(a) 0.86 W/m^2 (b) 1.72 W/m^2
(c) 3.44 W/m^2 (d) 6.88 W/m^2

59. An electric field of 300 V/m is confined to a circular area 10 cm in diameter. If the field is increasing at the rate of 20 V/m/s, the magnitude of magnetic field at a point 15 cm from the centre of the circle will be
(a) $1.85 \times 10^{-15} T$ (b) $1.85 \times 10^{-16} T$
(c) $1.85 \times 10^{-17} T$ (d) $1.85 \times 10^{-18} T$

60. A lamp emits monochromatic green light uniformly in all directions. The lamp is 3% efficient in converting electrical power to electromagnetic waves and consumes 100 W of power. The amplitude of the electric field associated with the electromagnetic radiation at a distance of 10 m from the lamp will be
(a) 1.34 V/m (b) 2.68 V/m
(c) 5.36 V/m (d) 9.37 V/m

61. A plane electromagnetic wave of wave intensity 6W/m^2 strikes a small mirror of area 40 cm , held perpendicular to the approaching wave. The momentum transferred by the wave to the mirror each second will be
(a) 6.4×10^{-7} kg/m/s (b) 4.8×10^{-8} kg/m/s
(c) 3.2×10 kg/m/s (d) 1.6×10^{-10} kg/m/s

62. In the above problem, the radiation force on the mirror will be
(a) 6.4×10^{-7} N (b) 4.8×10^{-8} N
(c) 3.2×10^{-9} N (d) 1.6×10^{-10} N

Answers to Questions for Practice

1.	(c)	**2.**	(a)	**3.**	(a)	**4.**	(c)	**5.**	(c)	**6.**	(c)	**7.**	(d)
8.	(c)	**9.**	(c)	**10.**	(d)	**11.**	(b)	**12.**	(b)	**13.**	(a)	**14.**	(c)
15.	(c)	**16.**	(c)	**17.**	(b)	**18.**	(b)	**19.**	(a)	**20.**	(d)	**21.**	(b)
22.	(b)	**23.**	(a)	**24.**	(c)	**25.**	(b)	**26.**	(d)	**27.**	(a)	**28.**	(b)
29.	(c)	**30.**	(c)	**31.**	(b)	**32.**	(b)	**33.**	(a)	**34.**	(c)	**35.**	(b)
36.	(c)	**37.**	(a)	**38.**	(c)	**39.**	(c)	**40.**	(c)	**41.**	(c)	**42.**	(b)
43.	(d)	**44.**	(b)	**45.**	(a)	**46.**	(d)	**47.**	(c)	**48.**	(c)	**49.**	(c)
50.	(a)	**51.**	(b)	**52.**	(c)	**53.**	(b)	**54.**	(a)	**55.**	(d)	**56.**	(a)
57.	(b)	**58.**	(b)	**59.**	(d)	**60.**	(a)	**61.**	(d)	**62.**	(d)		

SELF TEST 1

1. The current through a wire is $i = i_o + \alpha t$ $\{i_o = 10\ A,$ $\alpha = 4As^{-1}\}$. The charge crossed through a section of wire in 10s is

(a) 500 C (b) 250 C

(c) 300 C (d) none of these

2. What length of a copper wire of cross-sectional area $0.01 mm^2$ is needed to form a resistance of 100 Ω? ($\rho = 1.7 \times 10^{-8} \Omega - m$)

(a) nearly 60 m (b) nearly 50 m

(c) nearly 30 m (d) nearly 600 m

3. A wire has length 2 m and resistance 5Ω. Find the electric field if it carries a current of 10 A.

(a) 5 Vm^{-1} (b) 10 Vm^{-1}

(c) 20 Vm^{-1} (d) 25 Vm^{-1}

(e) none

4. If a source of emf 10 V is applied across AB then power consumed is

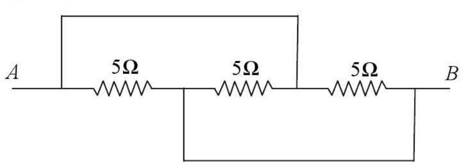


Fig. 1

(a) 20 W (b) 40 W

(c) 60 W (d) none

5. If a conical shape wire has resistivity ρ and structure as shown, then resistance of the wire is

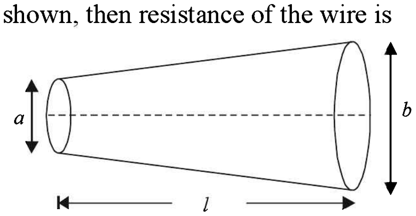


Fig. 2

(a) $\rho \dfrac{ab}{l}$ (b) $\rho \dfrac{l}{\pi ab}$

(c) $\rho \dfrac{2l}{\pi ab}$ (d) $\rho \dfrac{4l}{\pi ab}$

6. The potential drop across 6 μF and 3 μF capacitors is

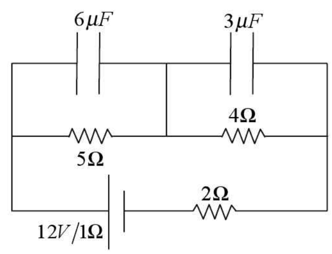


Fig. 3

(a) 5 V, 3 V (b) 5 V, 4 V

(c) 4 V, 4 V (d) 4 V, 8 V

7. Find the range of current read by ammeter

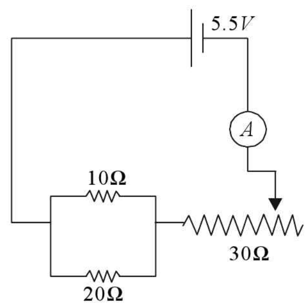


Fig. 4

(a) 0.15 to 0.825 A (b) 0.15 A to 0.652 A

(c) 0.15 A (d) 0.825 A

(e) none

8. Find the node potential Vp

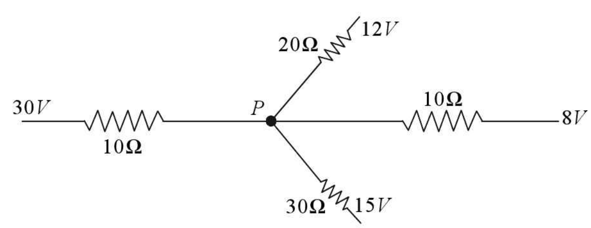


Fig. 5

(a) 16.3 V (b) 17.3 V

(c) 18.3 V (d 20.3 V

9. Each resistace is *r*, then R_{AB} =

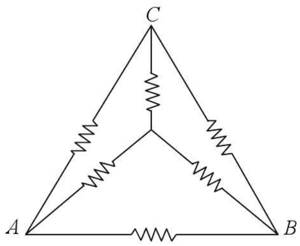


Fig. 6

(a) $\frac{3r}{2}$ (b) $3r$

(c) $2r$ (d) $\frac{r}{2}$

10. The equivalent potential difference between A and B is

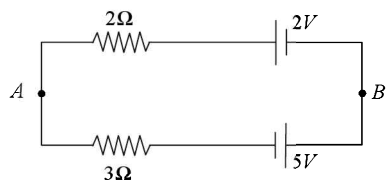


Fig. 7

(a) 0.8 V (b) –0.8 V

(c) 1.4 V (d) –1.4V

(e) –0.6 V

11. Find r_{ab} in the given circuit

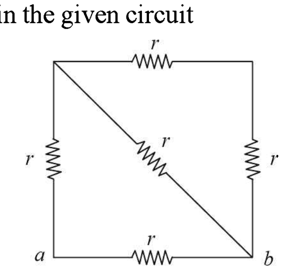


Fig. 8

(a) $\frac{5}{8}r$ (b) $\frac{8}{5}r$

(c) $\frac{5}{3}r$ (d) $\frac{3}{5}r$

(e) none

12. Each resistance shown is 3Ω , then current I is

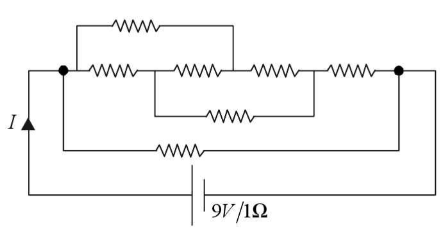


Fig. 9

(a) 4.3 A (b) 3.6 A

(c) 3.0 A (d) none

13. Under what condition a voltmeter is connected in series?

(a) To measure very low resistance

(b) To measure very high resistance

(c) To measure any resistance

(d) Voltmeter can never be connected in series

14. *xy* is 75 cm long wire. The balance point P is obtained at

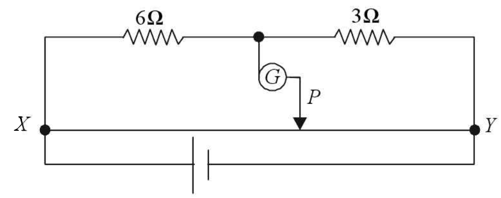


Fig. 10

(a) 50 cm from *x* (b) 50 cm from *y*

(c) 25 cm from *x* (d) none

15. Several resistors are connected in parallel, the net resistance is

(a) greater than lowest resistance

(b) less than lowest resistance

(c) equal to lowest resistance

(d) none of these

16. The current through 2*r* resistor is

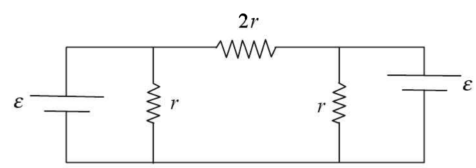


Fig. 11

(a) $\frac{2\varepsilon}{3r}$ (b) $\frac{\varepsilon}{2r}$

(c) $\frac{3\varepsilon}{2r}$ (d) none

17. Find V_{AB} in the given circuit.

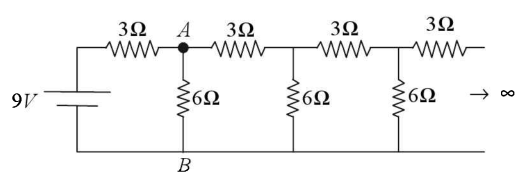


Fig. 12

(a) 6 V (b) 3 V

(c) 5.6 V (d) 4.5 V

18. Find R_{AB} in fig. 13.

(a) 3.32Ω (b) 2.23 Ω

(c) 2.33 Ω (d) none

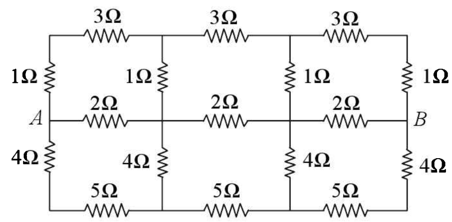


Fig. 13

19. An ideal ammeter has resistance

(a) zero (b) infinite
(c) large but finite (d) finite but not zero

20. A common base amplifier is a

(a) current dependent current source
(b) current dependent voltage source
(c) voltage dependent current source
(d) voltage dependent voltage source

21. A transformer is a

(a) voltage dependent voltage source
(b) voltage dependent current source
(c) current dependent voltage source
(d) current dependent current source

22. Ideal dependent current source is represented by

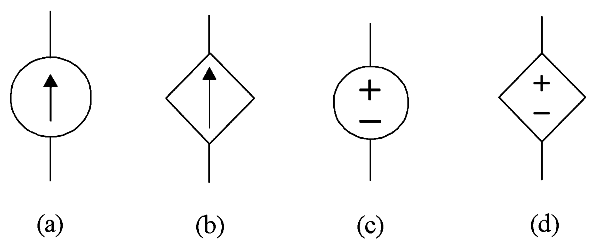


Fig. 14

23. Find the reading of the voltmeter.

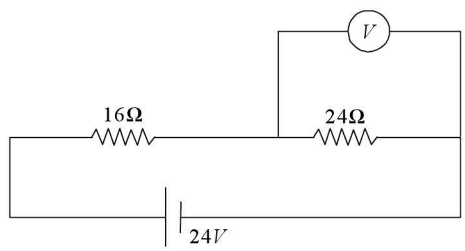


Fig. 15

(a) 9.6 V (b) 14.4 V
(c) 16 V (d) 15 V
(e) none

24. Find the potential drop across 6 V battery.

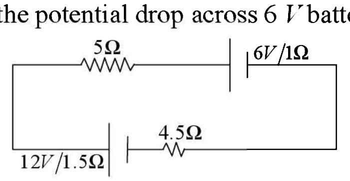


Fig. 16

(a) 6.5 V (b) 7.5 V
(c) 7.0 V (d) 5.5 V

25. A 12 V accumulator is being charged by a 24 V voltage source. The internal resistance of accumulator is 1.2Ω and external resistance is 4.8Ω. Then potential drop across the 12 V accumulator is

(a) 9.6 V (b) 14.4 V
(c) < 9.6 V (d) > 14.4 V
(e) none

26. Find V_{xy} in the given circuit.

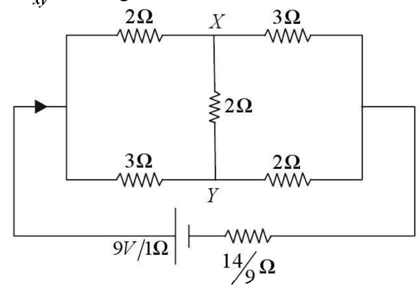


Fig. 17

(a) 0.6 V (b) 0.4 V
(c) 0.9 V (d) – 0.9V
(e) none

27. A current passes through a resistor. If k_1 and k_2 represent the average *KE* of an electron and metalion then

(a) $k_1 > k_2$ (b) $k_1 < k_2$
(c) $k_1 = k_2$ (d) none

28. The safe current of 5 A^{can} pass through a fuse of thickness 0.1 mm. What is the safe current in a fuse of thickness 0.2 mm?

(a) 7 A (b) 10 A
(c) 8.1 A (d) 6.3 A
(e) none

29. Currcent I is passing through a wire when connected to a battery. If the wire is stretched to double its length then current through the wire is

(a) I (b) I/2
(c) I/4 (d) I/8

30. In an electrolyte

(a) mobility of electron and ions is equal
(b) mobility of ions is greater than mobility of electrons
(c) mobility of electrons is greater than mobility of ions
(d) in a particular solution any one can have higher mobility

31. The length *AB* of the wire is 5 m and its resistance is 18Ω. Find the balance point from point *A*.

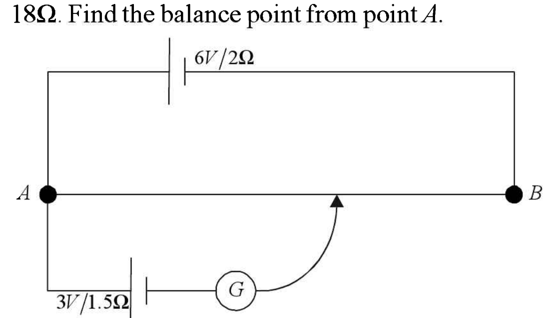


Fig. 18

(a) 2.5 m (b) 2.62 m
(c) 2.70 m (d) 2.78 m

32. The wire AB is 600 cm long and has a resistance $15r$. The balance point is obtained at

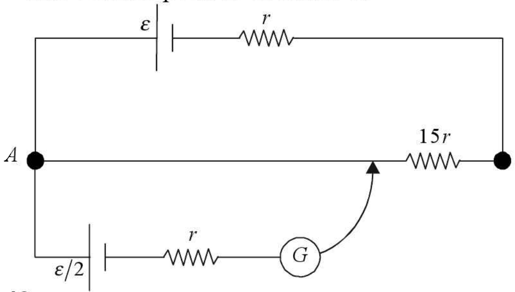


Fig. 19

(a) 320 cm from A (b) 320 cm from B
(c) 300 cm from A (d) none of these

33. AO is balanced at 280 cm and AB is balanced at 100 cm. Find the ratio of ε_1 to ε_2.

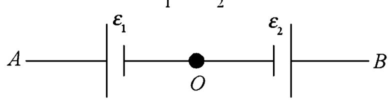


Fig. 20

(a) $\frac{9}{14}$ (b) $\frac{5}{14}$
(c) $\frac{14}{5}$ (d) $\frac{14}{9}$

34. A voltmeter of 5 kΩ/V is connected to an unknown resistance R in series. Find R if voltmeter reads 6 V.

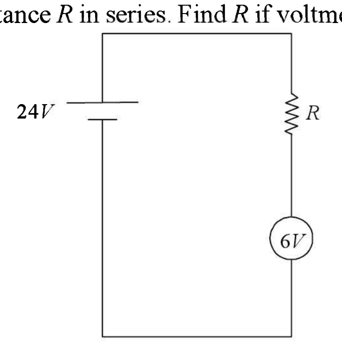


Fig. 21

(a) 15 $k\Omega$ (b) 30 $k\Omega$
(c) 60 $k\Omega$ (d) 90 $k\Omega$

35. A battery has internal resistor r and emf ε. Maximum current will be obtained when external resistance R

(a) $R = r$ (b) $R = o$
(c) $o < R < r$ (d) $R > r$

36. A battery has internal resistor r and emf ε. Maximum power is transferred to load resistance R if

(a) $R = r$ (b) $R > r$
(c) $0 < R < r$ (d) $R = 0$

37. A battery has emf ε and internal resistor r. Maximum power delivered by the battery is

(a) $\frac{\varepsilon^2}{2r}$ (b) $\frac{\varepsilon^2}{4r}$
(c) $\frac{\varepsilon^2}{r}$ (d) none of these

38. In which case more accurate value of resistance will be measured assuming resistance R is small?

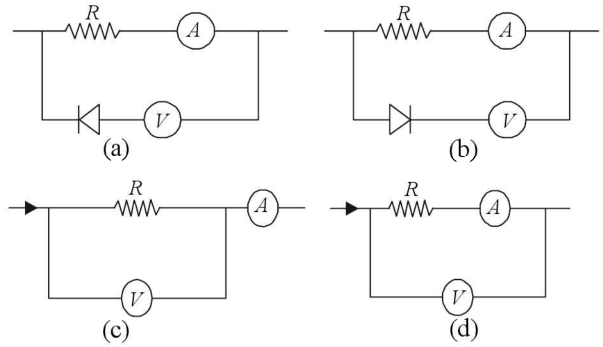


Fig. 22

39. A pn junction gives current according to equation I = $(10^{-6}A)\, e^{v/v_T}$ where V is applied voltage and $V_T = 0.25\ V$ and is connected to a $2V/2\Omega$ source The current through the pn junction is

(a) ∞ (b) $10^3 A$
(c) $1 A$ (d) cannot be determined

40. Cell 1 is not in operation and cell 2 is in operation then electric field inside the cell

(a) exists in cell 1 but not in cell 2
(b) exists in cell 2 but not in cell 2
(c) exists in cell 1 as well as in cell 2
(d) does not exist in either of the cells

41. A source of emf is connected to a thermocouple then

(a) One junction becomes hot and the other remains at room temperature
(b) One junction becomes cold and the other remains at room temperature.
(c) One junction gets hot and the other gets cold
(d) none of these

42. Each cell has emf ε and internal resistane r then current through resistor R is

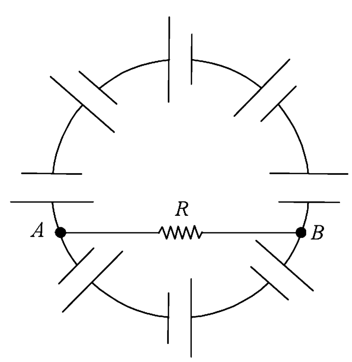


Fig. 23

(a) zero (b) $\frac{5\varepsilon - 3\varepsilon}{8r + R}$
(c) $\frac{(5\varepsilon - 3\varepsilon)(8r + R)}{8rR}$ (d) none of these

43. The curent through 6Ω resistor is

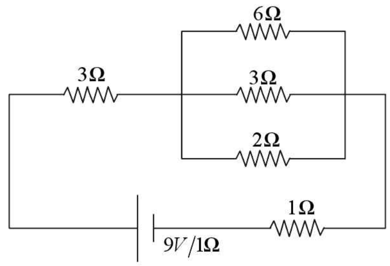


Fig. 24

(a) 0.5 A (b) 0.75 A

(c) 0.25 A (d) 0.5 A

44. When external resistance of 5Ω or 15Ω is connected to a battery of 9 V, power delivered is equal. The internal resistance of the battery is.

(a) 5.24Ω (b) 8.66 Ω

(c) 7.12Ω (d) 9.23Ω

45. A 100 W bulb and a 25 W bulb both rated at 220 V are connected to 110V source in parallel. The combined power is

(a) 62.5 W (b) 31.25 W

(c) 125 W (d) none

46. A 220 V/100 W and a 110 V/25 W bulb are in series. Find the maximum voltage applied so that 25 W bulb glows fully.

(a) 220 V (b) 185 V

(c) 198 V (d) 172 V

47. Find the power consumed in 6Ω resistor.

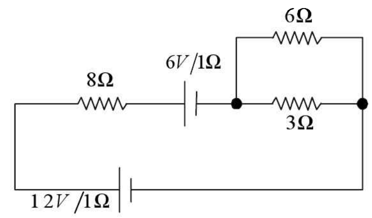


Fig. 25

(a) $\frac{1}{3}W$ (b) $\frac{1}{6}W$

(c) $\frac{1}{4}W$ (d) $\frac{1}{5}W$

(e) none

48. $V - I$ characteristics of a resistor at two different temperatures is plotted then

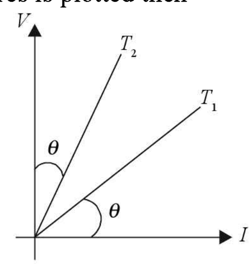


Fig. 26

(a) $T_2 > T_1$ (b) $T_1 > T_2$

(c) $T_1 = T_2$ (d) none

49. To measure a small resistance ~ 0.02Ω, the potentiometer wire should have a length

(a) 1m (b) 5 m

(c) 10 m (d) any length

50. A 100 w/220 V lamp is operated at 180V. The resistance of the bulb is

(a) 484Ω (b) > 484Ω

(c) < 484 Ω (d) none

Answers

1.	(c)	**2.**	(a)	**3.**	(d)	**4.**	(c)	**5.**	(d)	**6.**	(b)	**7.**	(a)
8.	(b)	**9.**	(d)	**10.**	(b)	**11.**	(a)	**12.**	(c)	**13.**	(b)	**14.**	(a)
15.	(b)	**16.**	(a)	**17.**	(d)	**18.**	(a)	**19.**	(a)	**20.**	(b)	**21.**	(a)
22.	(b)	**23.**	(b)	**24.**	(a)	**25.**	(a)	**26.**	(b)	**27.**	(a)	**28.**	(e)
29.	(c)	**30.**	(c)	**31.**	(d)	**32.**	(a)	**33.**	(d)	**34.**	(d)	**35.**	(b)
36.	(a)	**37.**	(a)	**38.**	(c)	**39.**	(c)	**40.**	(b)	**41.**	(c)	**42.**	(a)
43.	(c)	**44.**	(b)	**45.**	(b)	**46.**	(a)	**47.**	(b)	**48.**	(a)	**49.**	(c)
50.	(c)												

Explanations

1(c) $\int d\alpha = \int (i_o + \alpha t)\, dt = i_o t + \frac{\alpha t^2}{2}$

$= 10 \times 10 + \frac{4 \times 10^2}{2} = 300\, C$

2(a) $R = \rho \frac{l}{A} \Rightarrow 1.7 \times 10^{-8} \times \frac{l}{.01 \times 10^{-6}} = 100$

or $l = \frac{100}{1.7} \simeq 60\, m$

3(d) $V = IR = 50\,V$

$$E = \frac{50}{2} = 25\ \text{Vm}^{-1}$$

4(c) $$P = 3 \times \frac{V^2}{R} = \frac{3\times10^2}{5} = 60\,W$$

5(d) $$r = \sqrt{\frac{ab}{4}}\quad R = \rho\,\frac{l}{A} = \rho\,\frac{l(4)}{\pi ab}$$

6(b) Potential drop across 5Ω and potential drop across 4Ω is pot drop across 6 μF and 3μF

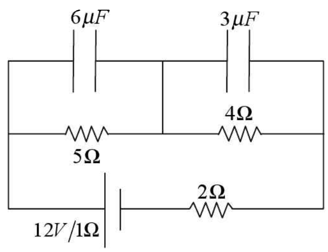


Fig. 27

7(a) $$R_1 = \frac{10\times20}{30} = \frac{20}{3}\ R_{\min} = \frac{20}{3};$$

$$R_{\max} = \frac{20}{3} + 30 = \frac{110}{3}$$

$$I_{\min} = \frac{5.5\times3}{110} = 0.15A,$$

$$I_{\max} = \frac{5.5\times3}{20} = 0.825A$$

8(b)

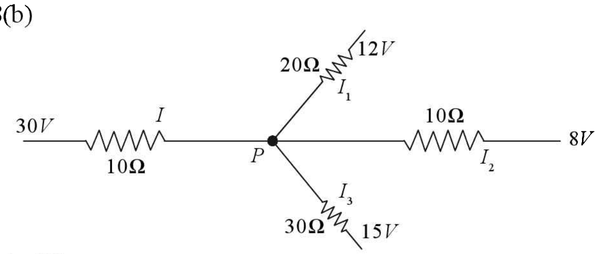


Fig. 28

Apply $KCL\ I = I_1 + I_2 + I_3$

$$\frac{30 - V_P}{10} = \frac{V_P - 12}{20} + \frac{V_P - 8}{10} + \frac{V_P - 15}{30}$$

$$180 - 6V_P = 3V_P - 36 + 6V_P - 48 + 2V_P - 30$$

$$17V_P = 294,\ V_P = 17.3\,V$$

9(d) Equivalent resistance is $= R_{AB} = \frac{r}{2}$

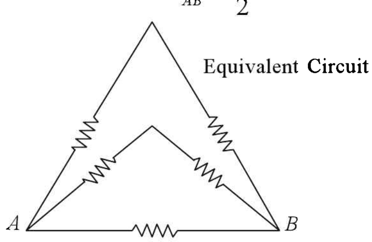


Fig. 29

10(b) $$V_{AB} = \frac{-5\times2+3\times2}{5} = -0.8\,V$$

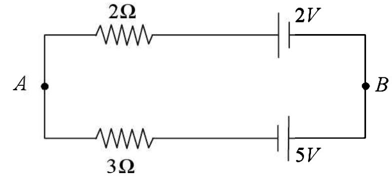


Fig. 30

11(a)

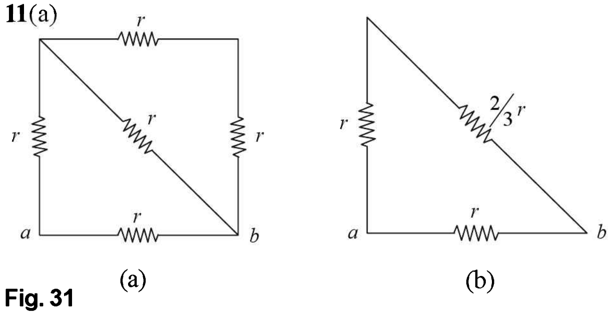


Fig. 31

$$\Rightarrow \frac{\frac{5r}{3}\times r}{\frac{5r}{3}+r} = \frac{5}{8}r$$

12(c) From equivalent circuit

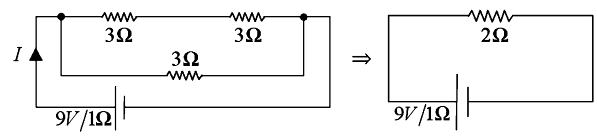


Fig. 32

$$I = 9/3 = 3\,A$$

13(b)

14(a) $$\frac{6}{3} = \frac{x}{75 - x} \Rightarrow 150 - 2x = x \text{ or } x = 50\ cm$$

15(b)

16(d)

17(d) $$\frac{x.6}{x+6} + 3 = x$$

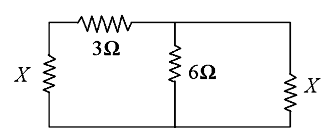


Fig. 33

or $6x + 3x + 18 = x^2 + 6x$

or $x^2 - 3x - 18 = 0$

$(x - 6)(x + 3) = 0$

$x = 6\Omega$

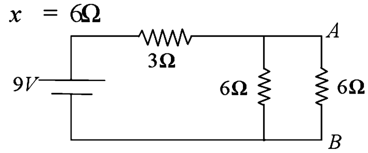


Fig. 34

$\therefore\ V_{AB} = 4.5\,V$

18(a) Equivalent circuit is

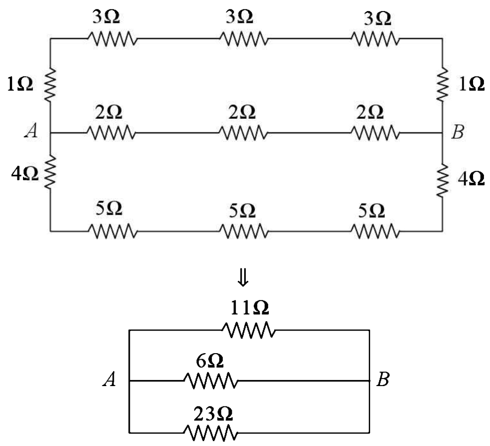


Fig. 35

19(a)

20(b)

21(a)

22(b)

23(b)

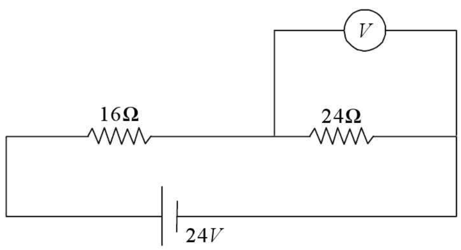


Fig. 36

$$V = 24 \times \frac{24}{40} = 14.4V$$

24(a) $I = \dfrac{12-6}{12} = 0.5A$

$V = 6 + Ir = 6 + 1\,(.5) = 6.5\,V$

25(b) $I = \dfrac{24-12}{1.2+4.8} = 2A$

$V = 12 + 2\,(1.2) = 14.4\,V$

26(b)

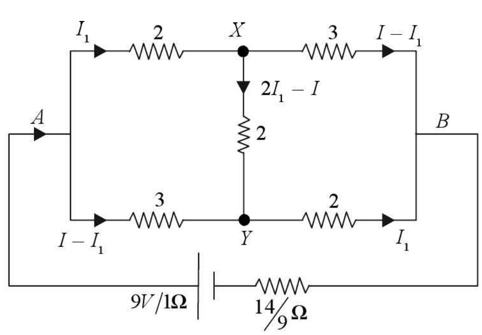


Fig. 37

$3\,(I-I_1) = 2\,(2I_1 - I) + 2I_1$

$9I_1 = 5I$

$I_1 = 5/9I\,A$

$IR_{AB} = 2I_1 + 3\,(I - I_1)$

$$R_{AB} = 2\left(\frac{5}{9}\right) + 3\left(\frac{4}{9}\right) = \frac{22}{9}\,\Omega, I = \frac{9}{1+\frac{14}{9}+\frac{22}{9}}$$

$= 1.8A$

$2I_1 - I = 0.2A$

27(a) $Vxy = +0.2 \times 2 = 0;\ 4V$

28(e) $I_2 = I_1\left(\dfrac{r_2}{r_1}\right)^{3/2} = 5\,(2)\ \sqrt{(2)} = 14.4\,A$

29(c) $R_{\text{new}} = 8R\ \therefore\ I_{\text{new}} = I/8$

30(c)

31(d)

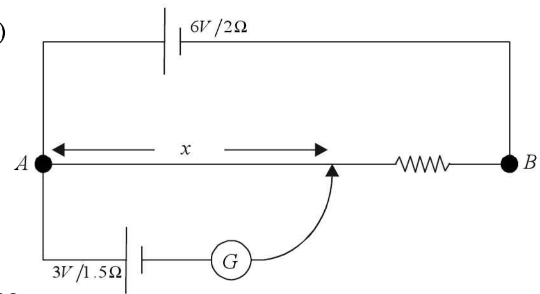


Fig. 38

$$V_{AB} = \frac{18\times 6}{20} = 5.4\,V,\ k = \frac{5.4}{5}\ kx = 3$$

$$\Rightarrow \quad x = \frac{3\times 5}{5.4} = 2.78\,m$$

32(a) $V_{AB}\ \dfrac{15\varepsilon}{16};\ k = \dfrac{V_{AB}}{600} = \dfrac{15\varepsilon}{16\times 600}$

$$\frac{\varepsilon}{2} = \frac{15\varepsilon x}{16\times 600} \text{ or } x = \frac{\varepsilon/2\ 16\times 600}{15\varepsilon} = 320\,cm$$

33(d) $\dfrac{\varepsilon_1 - \varepsilon_2}{\varepsilon_1} = \dfrac{100}{280} = \dfrac{5}{14}$

or $\dfrac{\varepsilon_2}{\varepsilon_1} = \dfrac{9}{14}$

or $\dfrac{\varepsilon_1}{\varepsilon_2} = \dfrac{14}{9}$

34(d) Potential drop across $R = 18V = \therefore\ \dfrac{R}{30k\Omega} = \dfrac{18}{6}$

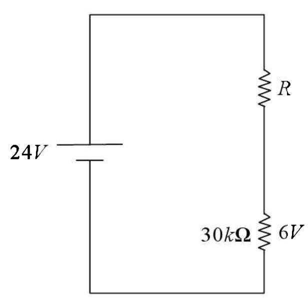


Fig. 39

or $R = 90k\Omega$

35(a) Short circuit current is the maximum current.

36(a)

37(a) $\therefore \ \dfrac{\varepsilon^2}{4r}$ is the power consumed is internal resistor and

$\varepsilon^2/4r$ is power given to load

38(c)

39(c) Current cannot exceed short circuit current.

40(b)

41(c)

42(a)

43(c)

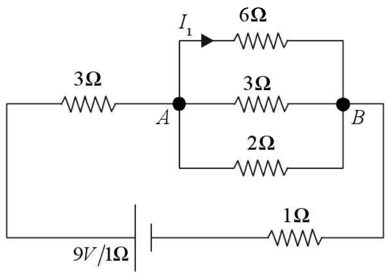


Fig. 40

$$I = 1.5A = \frac{9}{6} \quad V_{AB} = 1.5\,v$$

$$I_1 = \frac{1.5}{6} = 0.25A$$

44(b) $\dfrac{V^2(5)}{(5+r)^2} = \dfrac{V^2(15)}{(15+r)^2}$ or $(15+r)^2 = 3\,(5+r)^2$

or $\quad 15 + r = \sqrt{3}\,(5+r)$

or $\quad 0.732r = 15 - 5\sqrt{3} = 5\sqrt{3}\,(\sqrt{3}-1)$ or $r = 8.66r$

45(b) $R = \dfrac{220^2}{100} = 484\,\Omega\,; \ \dfrac{110^2}{484} = 25\,W$

similarly power of 25 W becomes 1/4 th $i.e$ 6.25W

46(a)

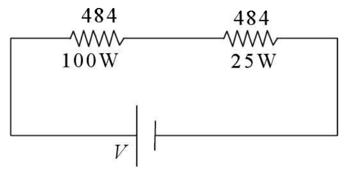


Fig. 41

$$\frac{v \times 484}{2 \times 484} = 110$$

or $\quad V = 220\,V$

47(b) $I = \dfrac{12-6}{12} = 1/2\,A$ and $I_1 = \dfrac{\frac{1}{2}\times 3}{6+3} = \dfrac{1}{6}A$

$$P = I_1^2\,R = \left(\frac{1}{6}\right)^2 6 = \frac{1}{6}W$$

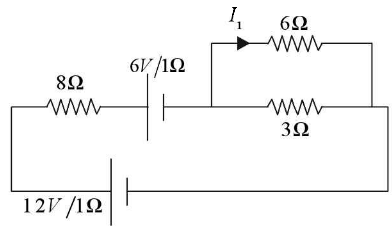


Fig. 42

48(a) $T_2 > T_1 \because R_2 > R_1$

49(c) More the length, the more the accuracy

50(c) 484Ω is the resistance when the bulb is fully glowing. When the Voltage is less, less heat is produced. Temperature is less, therefore, resistance is more than 484Ω

SELF TEST 2

1. A charged particle of mass m and charge q is thrown with a speed u against a uniform electric field E. How much distance will it travel before coming to rest momentarily?

(a) $\dfrac{mu^2}{qE}$ (b) $\dfrac{mu^2}{2qE}$

(c) $\dfrac{mu^2}{3qE}$ (d) none of these

2. A circular wire loop of radius r carries a total charge Q distributed uniformly on it. A small length dl is cut off. The electric field due to remaining wire is

(a) $\dfrac{Qdl}{8\pi^2 r^3 \varepsilon_0}$ (b) $\dfrac{Qdl}{8\pi^2 r^2 \varepsilon_0}$

(c) $\dfrac{Qdl}{16\pi^2 r^3 \varepsilon_0}$ (d) $\dfrac{Qdl}{16\pi^2 r^2 \varepsilon_0}$

3. The electric field of magnitude 5 N/C at a distance 40 cm from it is produced. The point charge is

(a) $8.2 \times 10^{-11}\,C$ (b) $8.6 \times 10^{-11}\,C$
(c) $8.9 \times 10^{-11}\,C$ (d) $9.3 \times 10^{-11}\,C$

4. Some equipotential surfaces are shown. The electric field at any point is

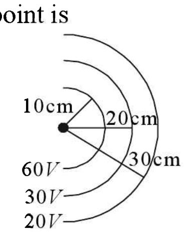


Fig. 1

(a) $\frac{6}{x^2}$ perpendicular to the plane of paper

(b) $\frac{600}{x^2}$ perpendicular to the plane of paper

(c) $\frac{6}{x^2}$ radially (d) $\frac{600}{x^2}$ radially

5. 12 J of work is done against an existing electric field to take the charge 10^{-2} C from A to B. Then $V_B - V_A$ is

(a) 1200 V (b) 120 V

(c) 12000 V (d) none of these

6. If an electric field is $E = 10x\,\hat{i}$ and considering potential at (10, 20) to be zero, find the potential at the origin.

(a) 1000 V (b) 100 V

(c) 375 V (d) 500 V

7. Three identical charges, each of charge q and mass m, are placed at the vertices of an equilateral Δ of side l. The electric field and potential at the centroid is

(a) zero, $\frac{3q}{4\pi\varepsilon_0 l}$ (b) $\frac{9q}{4\pi\varepsilon_0 l^2}, \frac{3\sqrt{3}q}{4\pi\varepsilon_0 l}$

(a) zero, $\frac{3\sqrt{3}q}{4\pi\varepsilon_0 l}$ (b) $\frac{\sqrt{3}q}{4\pi\varepsilon_0 l^2}, \frac{3\sqrt{3}q}{4\pi\varepsilon_0 l}$

8. Find the minimum force between the protons of a He nucleus. Radius of the orbit is $\sqrt{2}\ fm$

(a) 1.152 N (b) 11.52 N

(c) 0.1153 N (d) 115.2 N

9. In which case $E \neq 0, V = 0$

(a) along axial line of a dipole

(b) in a conducting shell

(c) along equatorial line in a dipole

(d) None of these

10. Eight identical charges are placed at the vertices of a cube of side 'l'. The net electric field at the centre is

(a) zero (b) non zero

(c) along xy plane (d) along yz plane

(e) none of these

11. A point charge $-q$ is brought from ∞ to the vicinity of a fixed charge $+Q$ then

(a) potential decreases as $-q$ approaches the fixed charge

(b) potential increases as $-q$ approaches the fixed charge

(c) potential remains fixed irrespective of position of $-q$

(d) none of these

12. Two charged particles 10 μC each are joined with threads of length 50 cm each and hanged from same point. Their masses are 10 milligram each. The system is taken in a satellite. The tension in the thread is

(a) 0.9 N (b) > 0.9 N

(c) < 0.9 N (d) none

13. Two charges are 1 cm apart. The minimum possible force between them is

(a) $2.3 \times 10^{-25}\,N$ (b) $2.3 \times 10^{-24}\,N$

(c) $2.3 \times 10^{-23}\,N$ (d) $2.3 \times 10^{-26}\,N$

14. The ratio of electric to gravitational force between the protons is of the order

(a) 10^{36} (b) 10^{38}

(c) 10^{39} (d) 10^{34}

15. In the fig shown, the direction of electric field is along perpendicular bisector of

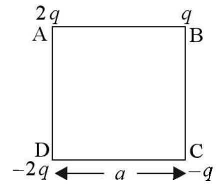


Fig. 2

(a) BC (b) CD

(c) AD (d) AB

16. Electric field at $x = 1$ m and $x = 5$ m in the graph between V and x is

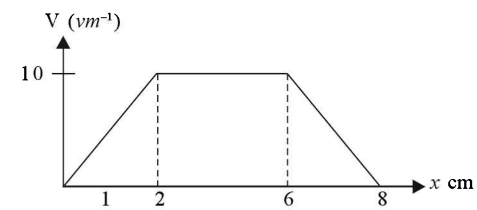


Fig. 3

(a) 5 Vm^{-1}, 0 (b) 0, 0

(c) $-5\ V$m^{-1}, $-2\ V$m^{-1} (d) $-5\ V$m^{-1}, 0

17. A drop of oil has charge $10e$ and kept suspended between two plates 2 cm apart having a potential difference of 20 V. Find the mass of the drop.

(a) 1.6×10^{-16} kg (b) 1.6×10^{-15} kg

(c) 1.6×10^{-9} kg (d) none

18. The dipole moment of the system of 3 charge is

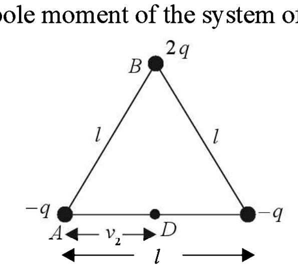


Fig. 4

(a) $2ql$ along BD (b) $\sqrt{3}\,ql$ along BD

(c) $2ql$ along DB (d) $\sqrt{3}\,ql$ along DB

(e) none of these

19. A charged particle of mass m charge q enters a uniform electric field E spread to a length l as shown with a velocity v from the middle point. Find the emergent velocity

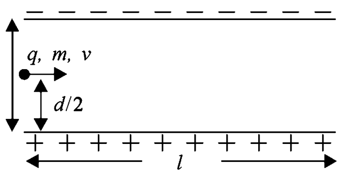


Fig. 5

(a) $\sqrt{v^2+\left(\frac{eEl}{mv}\right)^2}$ (b) $\sqrt{v^2+\frac{eEl}{mv}}$

(c) $\sqrt{v^2+\left(\frac{eEt}{m}\right)^2}$ (d) $v+\frac{eEl}{mv}$

(e) none of these

20. Two identical point charges each of charge q, mass m are hanged from a rigid support with strings of length l each. Assuming θ to be small, find charge q

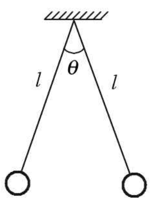


Fig. 6

(a) $\sqrt{4\pi\varepsilon_0 l^2\theta^3 mg}$ (b) $\sqrt{8\pi\varepsilon_0 l^2\theta^3 mg}$

(c) $\sqrt{2\pi\varepsilon_0 l^2\theta^3 mg}$ (d) none

21. A metal wire is bent in the form of a semicircle. It has charge Q distributed uniformly. A charge q is brought at point A, B or C if work done is W_A, W_B, W_C respectively then

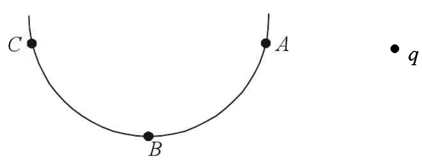


Fig. 7

(a) $W_A > W_B > W_C$ (b) $W_C > W_B > W_A$

(c) $W_A = W_B = W_C$ (d) $W_B > W_A > W_C$

22. HCl gas sample is placed in a uniform electric field $2.5 \times 10^{+4}$ N/C. The separation between H^+ and Cl^- ions is 2A°, find the maximum torque

(a) 8×10^{-25} N-m (b) 8×10^{-26} N-m

(c) 8×10^{-24} N-m (d) 80×10^{-25} N-m

23. How much work is done in assembling 3 charges q each at the vertices of equilateral Δ of side l?

(a) $\frac{q^2}{4\pi\varepsilon_0 l}$ (b) $\frac{3q^2}{8\pi\varepsilon_0 l}$

(c) $\frac{3q^2}{4\pi\varepsilon_0 l}$ (d) $\frac{3\sqrt{3}q^2}{4\pi\varepsilon_0 l}$

24. The KE of a charged particle decreases by 10 J as it moves from a point at 100 V to another point at 200 V. The charge on the particle is

(a) 0.05 C (b) 0.1 C

(c) 0.001 C (d) 0.5 C

25. The number of electrons in 1g of H_2O are

(a) 3.3×10^{24} (b) 3.3×10^{23}

(c) 3.3×10^{22} (d) none

26. A closed surface s' is constructed around a conducting wire the flux of electric field through the closed surface on closing the switch

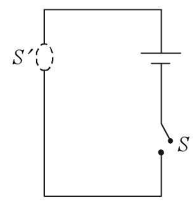


Fig. 8

(a) increases (b) decreases

(c) increases or decreases on the direction of current

(d) remains zero

27. A shell of Radius R is cut at $R/2$ as shown. A charge Q is placed between the two parts at centre as shown. The flux through upper part is

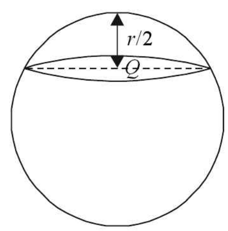


Fig. 9

(a) $\frac{Q}{2\varepsilon_0}$ (b) $< \frac{Q}{2\varepsilon_0}$

(c) $> \frac{Q}{2\varepsilon_0}$ (d) None

28. A cube is placed with one vertex at origin. The side of cube is a. the electric field is $E = 600\sqrt{x}\,\hat{i}$. The electric flux through the cube is

(a) $600\, a^2\sqrt{x}$ (b) $-600\, a^{\frac{5}{2}}$

(c) $600\, a^{\frac{5}{2}}$ (d) zero

29. A cube placed in a uniform electric field $E\hat{i}$. The flux through the cube i

(a) zero (b) positive

(c) negative

(d) depends upon on direction

30. A charge q_2 is placed distance R from the surface of a shell and q_1 is placed at the centre of the shell of radius R as shown then F_A and F_B are

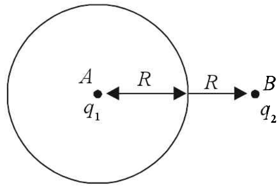


Fig. 10

(a) zero, zero (b) zero, $\frac{q_1q_2}{16\pi\varepsilon_0R^2}$

(a) $\frac{q_1q_2}{16\pi\varepsilon_0R^2}$, $\frac{q_1q_2}{16\pi\varepsilon_0R^2}$ (b) none

31. A cylinder of radius R and length l is placed in a uniform electric field. Find the flux through the cylinder

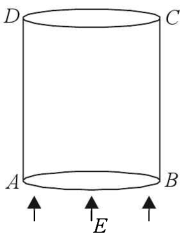


Fig. 11

(a) zero (b) $E(\pi r^2)$

(c) $-E\pi r^2$ (d) none

32. A long cylindrical volume consists of a uniformly distributed charge density ρ. The electric field at any point P inside the cylinder at a distance x from the centre is

(a) $\frac{\rho x}{2\varepsilon_0}$ (b) $\frac{\rho x}{\varepsilon_0}$

(c) $\frac{\rho x}{4\varepsilon_0}$ (d) $\frac{2\rho x}{\varepsilon_0}$

33. A long cylinderical wire carries a positive charge of linear density 2×10^{-8} cm^{-1}. An electron revolves around it. Find the KE of electron

(a) 180 J (b) $2.88 \times 10^{-19} J$

(c) $2.88 \times 10^{-17} J$ (d) $1.88 \times 10^{-17} J$

34. Square plates of Area A and separation between plates d are taken. Dielectrics of constants K_1, K_2 and K_3 as shown in figure are filled. the equivalent capacitance is

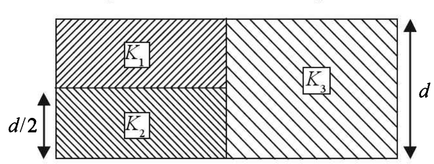


Fig. 12

(a) $\frac{A\varepsilon_0}{2d(k_1+k_2)}[k_1k_2 + k_2k_3 + 2k_1k_3]$

(b) $\frac{A\varepsilon_0}{2d(k_1+k_2)}[2k_1k_2 + k_2k_3 + k_1k_3]$

(c) $\frac{A\varepsilon_0}{2d(k_1+k_2)}[k_1k_2 + 2k_2k_3 + k_1k_3]$

(d) none

35. The equivalent capacitance of the network shown is

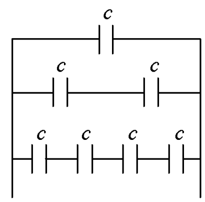


Fig. 13

(a) c (b) $\frac{c}{2}$

(c) $\frac{c}{3}$ (d) $2c$

(e) none

36. If each capacitor in the network is c then ceq between A and B is

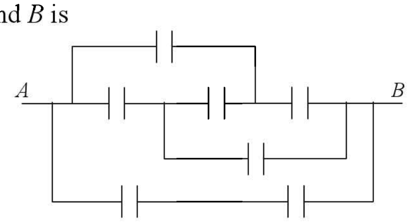


Fig. 14

(a) $\frac{c}{2}$ (b) c

(c) $\frac{3c}{2}$ (d) $2c$

37. In a parallel plate capacitor a thin metal sheet is placed in between. If initial capacitance is c, the new capacitance becomes

(a) c (b) $> c$

(c) $< c$ (d) none

38. In the given circuit V_{AB} = ________

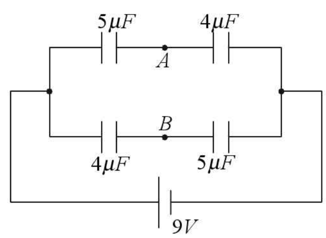


Fig. 15

(a) $1V$ (b) $-1V$

(c) $2V$ (d) $-2V$

39. Two spherical concentric shells of radius R_1 and R_2, innershell is grounded as shown. A dielectric of strength is filled between thetwo spheres. Outer surface is given a charge $+Q$. The equivalent capacitance is

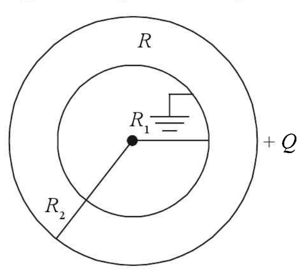


Fig. 16

(a) $4\pi\varepsilon_0\left[k\dfrac{R_1R_2}{R_2+R_1}+R_2\right]$ (b) $4\pi\varepsilon_0(R_1+R_2)k$

(c) $4\pi\varepsilon_0 k\left[\dfrac{R_1R_2}{R_2-R_1}\right]$ (d) $4\pi\varepsilon_0\left[k\dfrac{R_1R_2}{R_2-R_1}+R_2\right]$

40. Find equivalent capacitance between A and B. Assume each capacitance is c

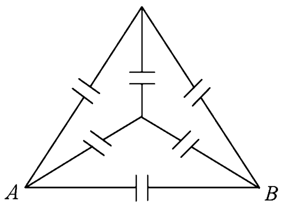


Fig. 17

(a) c (b) $\dfrac{c}{2}$

(c) $\dfrac{3c}{4}$ (d) $\dfrac{3c}{2}$

41. If each plate has area A and separation between successive plates is d from equivalent capacitance between A and B is

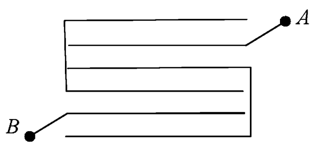


Fig. 18

(a) $\dfrac{A\varepsilon_0}{d}$ (b) $\dfrac{A\varepsilon_0}{2d/3}$

(c) $\dfrac{4A\varepsilon_0}{d}$ (d) $\dfrac{3A\varepsilon_0}{d}$

42. A charge – 20 μC is given to side GH of the negative plate of the capacitor. The charge on positive plate at CD side is

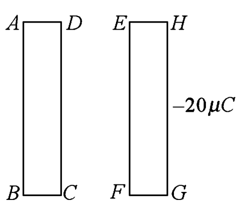


Fig. 19

(a) + 10 μc (b) + 20 μc

(c) – 10 μc (d) none of these

43. 3 plates A, B and C each having area a^2 and separation between successive plates is d. The equivalent capacitance is

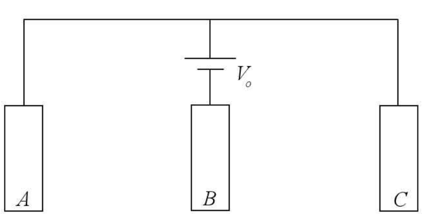


Fig. 20

(a) $\dfrac{A\varepsilon_0}{d}$ (b) $\dfrac{2A\varepsilon_0}{d}$

(c) $\dfrac{A\varepsilon_0}{2d}$ (d) $\dfrac{3A\varepsilon_0}{2d}$

44. Find the charge on 4 μf capacitor

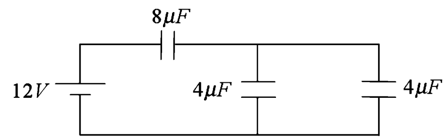


Fig. 21

(a) 24 μc (b) 12 μc

(c) 48 μc (d) none

45. Three concentric spherical shells A, B and C of radii a, b and c are taken. Charge on A and C are q and $-q$ respectively while B is earthed. The charge on the surface of B is

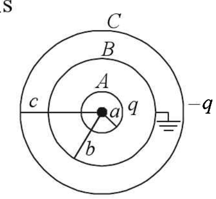


Fig. 22

(a) $\dfrac{b}{c}q$ (b) $\dfrac{c}{b}q$

(c) $\left(\dfrac{b-c}{b}\right)q$ (d) zero

46. The capacitor formed due to connecting wires is called

(a) parasitic capacitor (b) stray capacitor

(c) contact capacitor (d) trimmer

47. The capacitor used for fine tuning is known as

(a) tuner capacitor (b) parasitic

(c) trimmer (d) adjusting capacite

(e) varicap

48. The high value capacitors are made with _______ as electrolyte.

(a) oil (b) mica

(c) tantarium oxide (d) aluminium oxide

49. Find the maximum emf V of the battery in the circuit shown.

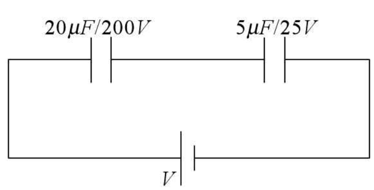


Fig. 23

(a) 200 V (b) 225 V

(c) 62.52 V (d) 31.25 V

50. The capacitor (Parallel Plate) is charged. Battery is removed. Separation between the plate is increased. The voltage across the capacitor

(a) decreases (b) increases

(c) remains unchanged (d) none

Answers

1.	(b)	**2.**	(a)	**3.**	(c)	**4.**	(c)	**5.**	(a)	**6.**	(d)	**7.**	(c)
8.	(d)	**9.**	(c)	**10.**	(a)	**11.**	(b)	**12.**	(a)	**13.**	(b)	**14.**	(a)
15.	(b)	**16.**	(d)	**17.**	(a)	**18.**	(d)	**19.**	(a)	**20.**	(c)	**21.**	(c)
22.	(a)	**23.**	(c)	**24.**	(b)	**25.**	(b)	**26.**	(d)	**27.**	(a)	**28.**	(c)
29.	(a)	**30.**	(b)	**31.**	(a)	**32.**	(a)	**33.**	(c)	**34.**	(b)	**35.**	(d)
36.	(c)	**37.**	(a)	**38.**	(a)	**39.**	(d)	**40.**	(d)	**41.**	(a)	**42.**	(a)
43.	(b)	**44.**	(a)	**45.**	(a)	**46.**	(b)	**47.**	(c)	**48.**	(c)	**49.**	(d)
50.	(b)												

Explanations

1(b) $\frac{1}{2}mv^2 = F.d$ or $\frac{1}{2}mu^2 = qE.d$ or $d = \frac{mu^2}{2qE}$

2(a) $dq = \frac{Qdl}{2\pi r}$

$$E = \frac{dq}{4\pi\varepsilon_0 r^2} = \frac{Qdl}{8\pi\varepsilon_0 r^3}$$

3(c) $5 = \frac{q\times10^9\times9}{(0.4)^2}$ or $q = \frac{5(0.4)^2}{9}\times10^{-9} = 8.89\times10^{-11}\,C$

4(c) $V = \frac{q}{4\pi\varepsilon_0 r}$ or $q = 60(0.1)\times4\pi\varepsilon_0$

$$E = \frac{q}{4\pi\varepsilon_0 x^2} = \frac{6}{x^2}$$

5(c) $W = q\Delta V$ or $\Delta V = \frac{12}{0.01} = 1200\,V$

6(d) $V(\text{origin}) = \int E\,dx = \frac{10x^2}{2} = \frac{10\times(10)^2}{2} = 500\,V$

7(c) $A_0 = \frac{l\sqrt{3}}{2}\times\frac{2}{3} = \frac{l}{\sqrt{3}}$ $\qquad E = 0\ (\because \text{of } \Delta \text{ law})$

$$V = \frac{3q}{4\pi\varepsilon_0 l/\sqrt{3}} = \frac{3\sqrt{3}q}{4\pi\varepsilon_0 l}$$

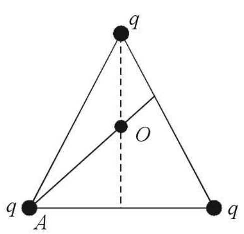


Fig. 24

8(d) $F = \frac{q^2}{4\pi\varepsilon_0\left(\sqrt{2}\times10^{-15}\right)^2} = \frac{\left(1.6\times10^{-19}\right)^2\times9\times10^9}{2\times10^{-30}}$

= 115.2 N

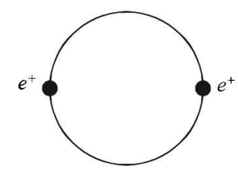


Fig. 25

9(c)

10(a)

11(b) Since potential is defined with unit positive charge. However, PE decreases.

12(a) In the absence of gravity angle between the threads is 180° due to mutual force

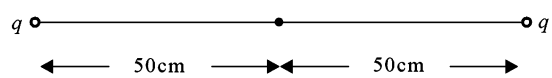


Fig. 26

$$T = \frac{q^2}{4\pi\varepsilon_0(1)^2} = 0.9\,N$$

13(b) $F = \frac{q^2}{4\pi\varepsilon_0 r^2} = \frac{\left(1.6\times10^{-19}\right)^2\times9\times10^9}{\left(10^{-2}\right)^2} = 23.04\times10^{-25}\,N$

14(a) $\dfrac{\frac{q^2}{4\pi\varepsilon_0}}{Gm^2} = \frac{\left(1.6\times10^{-19}\right)^2\times9\times10^9}{6.67\times10^{-11}\times\left(1.6\times10^{-27}\right)^2} = 1.34\times10^{36}$

15(b) See fig $E = \frac{6\sqrt{2}q}{4\pi t_0 a^2}$ along perpendicular bisector of CD.

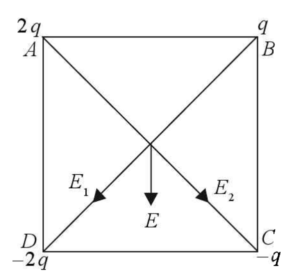


Fig. 27

16(d) $E = \frac{-dv}{dx}$ $E|_{x=1} = -\frac{10-0}{2-0} = -5\,Vm^{-1};\ E|_{x=5} = 0$

17(a) $mg = qE$ or $m = \frac{qE}{g} = \frac{10 \times 1.6 \times 10^{-19} \times 10^3}{10}$

$= 1.6 \times 10^{-16}$ kg

18(d) $p_{net} = \frac{\sqrt{3}}{2}(2p) = \sqrt{3}\,gl = [2p\cos 30]$ along DB

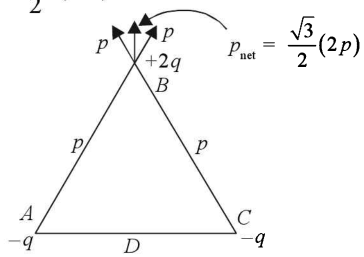


Fig. 28

19(a) The charged particle follows a projectile path

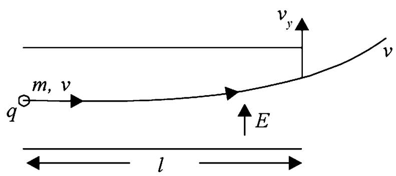


Fig. 29

$$t = \frac{l}{v};\ v_y = \frac{eE}{m}t = \frac{eEl}{mv}$$

$$v_f = \sqrt{v^2 + \left(-\frac{eEl}{mv}\right)^2}$$

$$\tan\beta = \frac{eEl}{m}$$

20(c) $T\sin\frac{\theta}{2} = \frac{q^2}{4\pi\varepsilon_0 (l\theta)^2}$ ... (i)

$T\cos\frac{\theta}{2} = mg$ ... (ii)

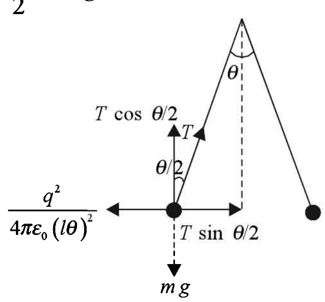


Fig. 30

dividing $\tan\frac{\theta}{2} = \frac{\theta}{2} = \frac{q^2}{4\pi\varepsilon_0 l^2\theta^2 mg}$ or $q = \sqrt{2\pi\varepsilon_0 l^2\theta^3 mg}$

21(c) ∵ wire is equipotential surface

22(a) $\tau_{max} = pE = 1.6 \times 10^{-19} \times 2 \times 10^{-10} \times 2.5 \times 10^4$

$= 8 \times 10^{-25}$ N-m

23(c)

24(b) $\Delta E = q\Delta V$ or $q = \frac{10}{100} = 0.1\,C$

25(b) 18 g of water has $e^- = 10 \times 6.023 \times 10^{23}$

1 g of H_2O has electrons $= \frac{10 \times 6.023 \times 10^{23}}{18} = 3.3 \times 10^{23}$

26(d)

27(a) in the upper part the flux is $\frac{Q}{2\varepsilon_0}$

28(c) $\phi = \int_{OABC} E.ds + \int_{DEFG} E.ds = 0 + 600\sqrt{a}\ (a^2)$

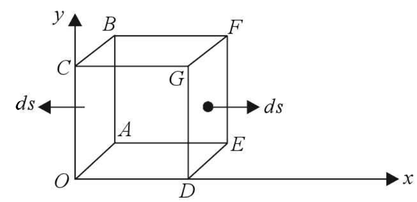


Fig. 31

29(a) Electric flux through a symmetric body is zero in a uniform field.

30(b) Electric field inside the shell is zero ∴ $F_A = 0$

$$F_B = \frac{q_1 q_2}{4\pi\varepsilon_0 (2R)^2}$$

31(a) Electric flux through a symmetric body in a uniform electric field is zero

32(a) $\oint E.ds = \frac{\rho\pi x^2 l}{\varepsilon_0}$

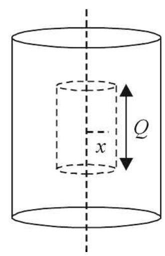


Fig. 32

$E(2\pi x l) = \frac{\rho\pi x^2 l}{\varepsilon_0}$ or $E = \frac{\rho x}{2\varepsilon_0}$

33(c) $\frac{mv^2}{x} = \frac{\lambda e}{2\pi\varepsilon_0 x}$ or $\frac{mv^2}{2} = \frac{\lambda e}{4\pi\varepsilon_0} = 2 \times 10^{-8} \times 9 \times 10^9$

$= 2.88 \times 10^{-17} J$

34(b)

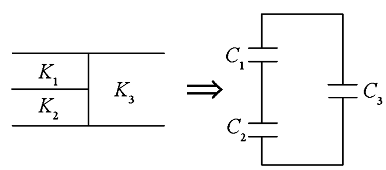


Fig. 33

$$\frac{C_1C_2}{C_1+C_2} + C_3 = \frac{k_1k_2A\varepsilon_0}{d(k_1+k_2)} + \frac{k_3A\varepsilon_0}{2d}$$

$$= \frac{A\varepsilon_0}{2d(k_1+k_2)}[2k_1k_2 + k_1k_3 + k_2k_3]$$

35(d) $C_{eq} = C + \frac{C}{4} + \frac{C}{4} + ... = \frac{C}{1-\frac{1}{2}} = 2C$

36(c) The network indotted boundary is Wheatstone bridge.

The equivatent circuit is shown $C_{eq} = C + \frac{C}{2} = 3\frac{C}{2}$

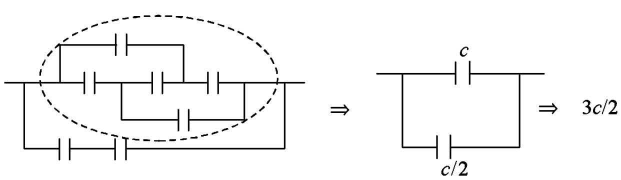


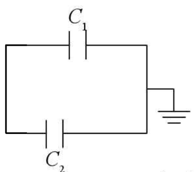
Fig. 34

37(a)

38(a)$V_A = \frac{5}{4+5} \times 9 = 5V$, $V_B = \frac{4 \times 9}{5+4} = 4V$

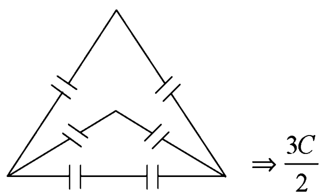
$V_{AB} = V_A - V_B = 1V$

39(d)



$\text{Ceq} = C_1 + C_2 = \frac{4\pi KRR_2\varepsilon_o}{(R_2 - R_1)} + 4\pi\ \varepsilon_0 R_2\ 1$

40(d) The equivalent circuit is



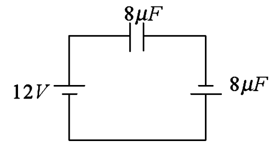
41(a) It is a wheat stone bridge

42(a)

43(b) The two capacitors are in parallel

44(a) $Q_{net} = 12 \times 4 = 48\,\mu\,C$

It is divided $4\,\mu$ F capacitors equally, that is, each has $24\,\mu$ C charge



$\text{ceq} = 4\,\mu\,F$

45(a)Potential at E line to various charged shrfaces is

$$\frac{q}{4\pi\varepsilon_o} + \frac{-q}{4\pi\varepsilon_o h} + \frac{q'}{4\pi\varepsilon_o h} \frac{-q'}{4\pi\varepsilon_o c} + \frac{q-q}{4\pi\varepsilon_o c} = 0$$

or $q' = \frac{b}{c} q$

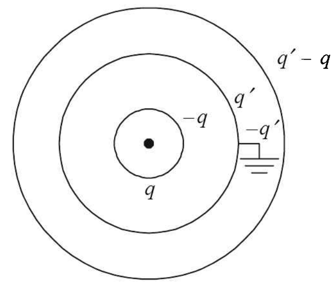


Fig. 35

46(b)

47(c)

48 (c)

49(d)The maximum charge which can be applied is $125\,\mu$ C because in series charge remains same. Therefore potential drop across $20\,\mu$ F capacitor is $\frac{125}{20}$ =6.25 V

$V = 6.25 + 25 = 31.25\ V$

50(b) charge remains same and C decreases. Therefore,

$V = \frac{Q}{C}$ increases

SELF TEST 3

1. Two conducting spherical shells or radii a and b are kept in contact and given charge "Q". The ratio of electric field intensity on the shells is

   (a) $\frac{a}{b}$ (b) $\frac{a^2}{b^2}$

   (c) $\frac{b}{a}$ (d) $\frac{b^2}{a^2}$

2. Five point charges each of value + q are placed at the five vertices of a regular hexagon of side l. The electric field intensity at the centre of the hexagon is

   (a) $\frac{1}{4\pi \in_o} \frac{q}{l^2}$ (b) $\frac{1}{4\pi \in_o} \frac{\sqrt{3}q}{l^2}$

   (c) $\frac{1}{4\pi \in_o} \frac{q}{l^2}\left(\sqrt{3}+1\right)$ (d) $\frac{1}{4\pi \in_o} \frac{5q}{l^2}$

3. An electric dipole of moment p_1 is placed on the equitorial plane of another dipole p_2. Assuming the distance between them to be much larger than their size, the force of electric interation between them is related to the distance r between them as

   (a) $\frac{1}{r^4}$ (b) $\frac{1}{r^4}$

   (c) $\frac{1}{r^4}$ (d) $\frac{1}{r^{\frac{3}{2}}}$

4. A solid sphere of radius R has charge Q distributed uniformly over its volume. If the electric field intensity at a distance $r = \frac{R}{3}$ from the centre of sphere is E, then electric field intensity at $r = \frac{2R}{3}$ from its centre is

(a) E (b) 2E
(c) 4E (d) None is correct

5. A small electric dipole is placed at the origin, with its moment vector along +x direction. Choose the incorrect statement

(a) $\vec{E}$ at all points on y axis is in –ve x direction
(b) potential at all points on y axis is zero
(c) $\vec{E}$ at all points on x axis is in –ve x direction
(d) electric potential at the origin is zero.

6. Two similar balls having charge +q each are attached to the ends of a spring of length "l". If the spring gets stretched by "x" due to repulsion between the charges then the force constant of spring is

(a) $\frac{1}{4\pi\epsilon_o}\frac{q^2}{(x+l)^3}$ (b) $\frac{1}{4\pi\epsilon_o}\frac{q^2}{(x+l)^2 x}$

(c) $\frac{1}{4\pi\epsilon_o}\frac{q^2}{x^3}$ (d) None is correct

7. Two charges are brought near
(a) The P.E. of system of charge increases
(b) The P.E. of system of charge decreases
(c) The P.E. of system of charge may increase
(d) The P.E. of system of charge remain constant.

8. Three charges +2q, –q, and –q are place at the corners of an equilanteral triangle.
(a) At the centre of triangle intensity and potential both are zero
(b) At the centre of triangle intensity and potential both are non zero.
(c) The system has net zero dipole moment.
(d) The system has net non zero dipole moment.

9. A hollow charged metal sphere is of radius R. The potential difference between its surface and at a point at a distance "3R" from its centre is V. The electric field intensity at a distance 3R from its centre is

(a) $\frac{V}{3R}$ (b) $\frac{V}{9R}$

(c) $\frac{V}{6R}$ (d) $\frac{V}{4R}$

10. A particle of mass 2 gm and of charge + 1 μ C is released at a distance of 1 m from a fixed charge of + 1 mC. The speed of the particle when it is at a distance of 10m from the fixed charge is

(a) 90 ms^{-1} (b) 45 ms^{-1}
(c) 180 ms^{-1} (d) 60 ms^{-1}

11. Two infinite long parallel wires having linear charge density λ_1 and λ_2 are separated by a distance R. The force due to one wire on unit length of the other wire is proportional to

(a) $\frac{\lambda_1\lambda_2}{R^2}$ (b) $\frac{\lambda_1\lambda_2}{R}$

(c) $\frac{\lambda^2{}_1\lambda_2}{R}$ (d) $\frac{\lambda_1\lambda_2}{R^3}$

12. A charge of + 1 μ C is moved a distance of 2m along a straight line 2x = y, in the space where uniform electric field of 10^6 is present along + ve Z direction. The work done on the charge is

(a) $\sqrt{3}$J (b) $\sqrt{2}$J
(c) 2 J (d) zero

13. Calculate the effective capacitance of the circuit shown below between M and N (all capacitors are in μ F)

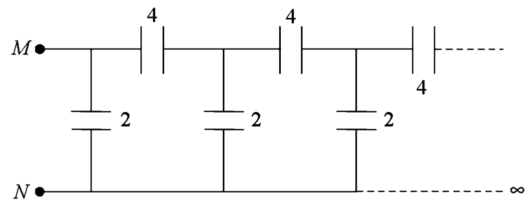


Fig. 1

(a) 2 μ F (b) 4 μ F
(c) $\sqrt{5\mu F}$ (d) 6 μ F

14. Calculate the potential difference across M and N in the circuit below

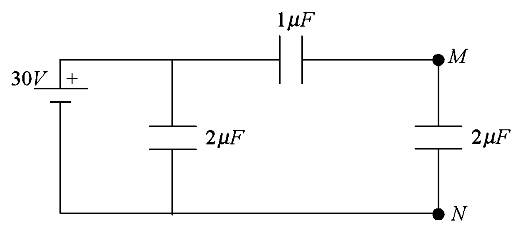


Fig. 2

(a) 10V (b) 15V
(c) 20V (d) 25V

15. A spherical capacitor of radii R_1 and R_2 ($R_1 < R_2$) is filled with two dielectric slabs of equal volume as shown. The effective capacitance is

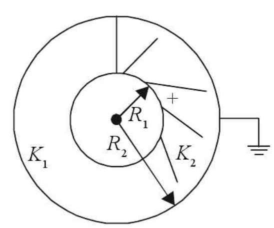


Fig. 3

(a) $4\pi\epsilon_0\left[\frac{k_1+k_2}{2}\right]\left(\frac{R_1R_2}{R_2-R_1}\right)$

(b) $4\pi\epsilon_0\left[\frac{2k_1+k_2}{2}\right]\left(\frac{R_1R_2}{R_2-R_1}\right)$

(c) $4\pi\in_0\left[\frac{2k_1+k_2}{k_1}\right]\left(\frac{R_1R_2}{R_2-R_1}\right)$

(d) None is correct

16. A capacitor of capacitance C is charged by a voltage source of V volts. The source is disconnected and is reconnected to the same charged capacitor with its polarities reversed. Calculate the total heat dissipated in the circuit after reconnecting the capacitor

(a) $\frac{1}{2}CV^2$ (b) CV^2
(c) $2CV^2$ (d) None is correct

17. An electric dipole is placed in a uniform electric field E, with its dipole moment vector "$\vec{p}$" making an angle θ with $\vec{E}$. For what value of θ is the dipole in equilibrium.

(a) 0° only (b) 180° only
(c) 0° and 180° both (d) 90°only

18. A parallel plate air capacitor of 1 μ F is filled four slabs of equal volume as shown. The capacitance after in introducing the slabs is

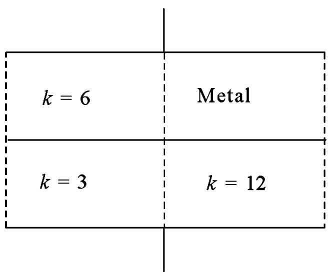


Fig. 4

(a) 9 μ F (b) 7 μ F
(c) 21 μ F (d) 14 μ F

19. A parallel plate air capacitor of capacitance C is connected to a constant voltage source of "V" volts. When the capacitor is fully charged, a dielectric slab of constant "K" is introduced between the plates of capacitor so as to fill the entire space. Calculate the heat produced in the charging circuit, when the slab is introduced

(a) $\frac{1}{2}(K-1)CV^2$ (b) $\frac{1}{2}KCV^2$
(c) $(K-1)CV^2$ (d) $\frac{1}{2}\left(\frac{K}{K-1}\right)CV^2$

20. 40 kW of electric energy is transmitted at 40kV, from generating station A. The supply reaching the station B is 39 k V. If the resistance of the transmission cable is 1000 ohm, calculate the power loss in the transmission cable.

(a) 1 KW (b) 39 KW
(c) 10 KW (d) none

21. Current flowing in a wire of resistance 2 Ω is expressed by I = $\sqrt{t}$ (I is in Amp and t is in seconds). Calculate the heat produced in the wire from t = 0 to t = 5 sec

(a) 10 J (b) 50 J
(c) 70 J (d) 25 J

22. Few identical resistances when connected in series dissipate 4W across a battery. When they are connected in parallel they dissipate 64 watt across the same battery. Calculate the power dissipated if only one such resistance is connected across the battery

(a) 4 W (b) 8 W
(c) 16 W (d) 32 W

23. The efficiency of a battery when connected to a resistance R is 60%. What will be its efficiency if external resistance is increased six times.

(a) 90% (b) 80%
(c) 70% (d) 60%

24. The heat liberated or absorbed per sec. at the junctions of thermo couple when current I is passed through it, is directly proportional to

(a) I (b) I^2
(c) $I^{1/2}$ (d) None is correct

25. In the circuit shown below each cell has e.m.f. 5V and internal resistance 1 Ω. The reading of volmeter is

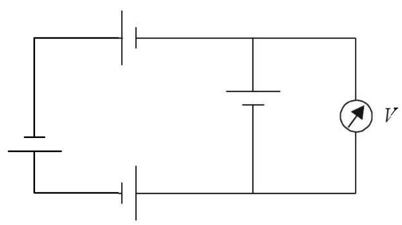


Fig. 5

(a) Zero (b) 2V
(c) 5V (d) 15V

26. In the circuit shown below ammeter reads 4A where as voltmeter reads 20 volts. The value of resistance R is

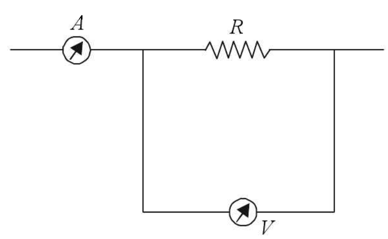


Fig. 6

(a) 5 Ω (b) less than 5 Ω
(c) more than 5 Ω (d) may be less than 5 Ω

27. Current in a conductor is given by I = 12t + 7 where I is in amperes and t is in seconds. Calculate the total charge passing through the conductor from t = 2 to t = 6 sec.

(a) 31 C (b) 356 C
(c) 220 C (d) 55 C

28. The effective resistance between A & B in the circuit shown below is (each resistance is of 1 Ω)

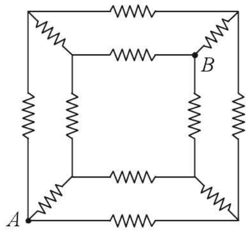


Fig. 7

(a) 1 Ω (b) 4 Ω

(c) $\frac{5}{6}\Omega$ (d) $\frac{6}{5}\Omega$

29. Two wires of same dimension but of resistivities K_1 & K_2 are connected in series. The equivalent resistivity of the combination is

(a) K_1+K_2 (b) $\frac{K_1+K_2}{2}$

(c) $\sqrt{K_1K_2}$ (d) None is correct

30. In the circuit shown below the value of current I is

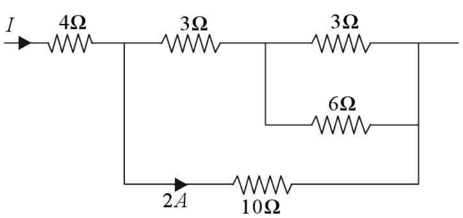


Fig. 8

(a) 3A (b) 5 A
(c) 6A (d) 10A

31. The locus of the points having same declination is called
(a) isogonic lines (b) agonic lines
(c) isoclinic lines (d) aclinic lines

32. A dip circle is placed at right angles to the magnetic meridian. In this plane the needle oscillates with a frequency n. If the horizontal component of earth's magnetic field is B_H, verticle component B_V, and B is the net magnetic field at that point, then *n* is proportional to
(a) B (b) B_H
(c) B_V (d) None is correct

33. The electric field of an e.m. wave is expressed by $E_y = \left(3000\frac{V}{m}\right)\sin(\omega t - kx)$. An electron is moving with a velocity of 2×10^7 ms^{-1} in y-direction, in the region of e.m. wave. Calculate the maximum magnetic force on the electron.
(a) Zero (b) $.6 \times 10^{-20}$N
(c) 4.5×10^{-10}N (d) 3.2×10^{-18}N

34. Relative permeability of a diamagnetic substance is
(a) < 1 (b) = 0
(c) > 1 (d) negative

35. em. radiatons of the energy range 10^6 eV are
(a) visible (b) ultraviolet
(c) X-rays (d) γ-rays

36. An electron is revolving in a circular orbit of radius R in Hydrogen atom with a speed V. The magnetic dipole moment created is
(a) Zero (b) evR

(c) $\frac{evR}{2}$ (d) $\frac{\mu_o ev}{2R}$

37. Figure shows a long conductor carrying current 'I' bent to form of a right angle

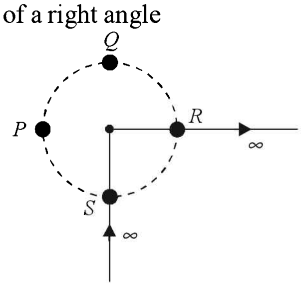


Fig. 9

(a) Magnetic field at P is zero
(b) Magnetic field at P is more than that of Q
(c) Magnetic field at R is more than that of Q
(d) Magnetic field at P, Q, R and S is same.

38. The integral of B.dl over the path shown in the Figure is (current I_1 is pointing in the plane of paper and I_2 is pointing out)

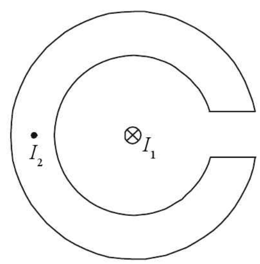


Fig. 10

(a) μ_o (I_1-I_2) (b) μ_o (I_1+I_2)

(c) μ_o I_2 (d) μ_o I_1

39. A proton and an Alpha particle moving with speeds of 2V and V respectively are projected perpendicular in a magnetic field. The ratio of the time periods of the particles in magnetic field is
(a) $1:\sqrt{2}$ (b) 1 : 2
(c) 2 : 1 (d) None

40. A magnetic field pointing in the paper is decreasing with time
(a) No electric field is produced
(b) Electric lines of forces produced are clockwise
(c) Electric lines of forces produced are anti-clockwise
(d) Electric lines of forces produced are straight

41. Magnetic flux associated with a coil of resistance 2 Ω is expressed as $\phi = t^2 - 2t$ (where ϕ is in weber and t is in second). The total charge which flows through the coil from $t = 0$ to $t = 2$ second is

(a) zero (b) 2C
(c) 4C (d) 1C

42. A circular coil of radius R carrying current I_1 is placed in the Y-Z plane with its centre coinciding to the origin. A straight conductor carrying current I_2 is placed along X-axis from x = 0 to x = ∞. The force of magnetic induction between the conductors is

(a) $\frac{\mu_o I_1 I_2}{2\pi R}$ (b) $\mu_o I_1 I_2$

(c) $\frac{\mu_o I_1 I_2}{2}$ (d) Zero

43. A bar magnet is dropped from some height so as pass through circular metallic ring placed horizontally above earth

(a) Induced e.m.f. in the coil is maximum when it just passes the centre of ring.
(b) The acceleration of the magnetic is zero when it passes the centre of ring
(c) The acceleration of the magnet is "g" when it passes the centre of ring
(d) The acceleration of the magnet is less than 'g' when it passes the centre.

44. To a coil of L = 1 H and R = 10Ω, a D.C. source of e.m.f. 100 V is connected at t = 0. The time taken by the current to be 10 A in the circuit is

(a) 1 sec (b) 2 sec
(c) 10 sec. (d) Infinite

45. The quantity $\frac{CR^2}{L}$ has the dimensions

(a) $M^1L^1T^2A^{-2}$ (b) $M^0L^0T^2A^0$
(c) $M^0LT^{-2}A^2$ (d) $M^0L^0T^0A^0$

46. A.C. represented by I = 10 sin 100π t is passed in an inductor of L = 2 H. The e.m.f. induced in the inductor at t = $\frac{1}{200}$ sec. is

(a) 2000 π volts (b) 1000 π volts
(c) 1000 $\sqrt{2\pi}$ volts (d) Zero

47. The turn ratio of a transformer is 10. If the current in the output coil is I A then the current in the input coil is

(a) 0.1A (b) 10A
(c) 1A (d) 2A

48. A circular current carrying coil is placed in an external magnetic field will experience a magnetic moment when

(a) the coil is not in equilibrium
(b) the coil is in stable equilibrium
(c) the coil is in unstable equilibrium
(d) the coil is in neutral equilibrium

49. A coil of area A having number of turns N, is revolving in a magnetic field B with a constant angular velocity ω about an axis perpendicular to magnetic field. The e.m.f. induced in the coil when its plane is parallel to magnetic field is

(a) NBAω (b) $\frac{NBA\omega}{2}$

(c) $\frac{NBA\omega}{\sqrt{2}}$ (d) Zero

50. For a L-C-R series circuit operating at resonant frequency

(a) The current is minimum
(b) The power factor of the circuit is zero
(c) The power factor of the circuit is one
(d) The power loss in the circuit is minimum.

Answers

1.	(c)	2.	(a)	3.	(c)	4.	(b)	5.	(c)	6.	(b)	7.	(c)
8.	(d)	9.	(c)	10.	(a)	11.	(b)	12.	(d)	13.	(b)	14.	(a)
15.	(a)	16.	(c)	17.	(c)	18.	(d)	19.	(a)	20.	(a)	21.	(d)
22.	(c)	23.	(a)	24.	(a)	25.	(a)	26.	(c)	27.	(c)	28.	(c)
29.	(b)	30.	(c)	31.	(a)	32.	(c)	33.	(d)	34.	(a)	35.	(d)
36.	(c)	37.	(d)	38.	(c)	39.	(b)	40.	(b)	41.	(a)	42.	(d)
43.	(c)	44.	(d)	45.	(d)	46.	(d)	47.	(b)	48.	(c)	49.	(a)
50.	(c)												

Explanations

1(c) The charge will be distributed on the shells in accordance of radii

i.e. $\frac{Q_a}{Q_b} = \frac{a}{b}$

Now $$\frac{E_a}{E_b} = \frac{\frac{1}{4\pi\epsilon_0}\frac{Q_a}{a^2}}{\frac{1}{4\pi\epsilon_0}\frac{Q_b}{b^2}} = \frac{b}{a}$$

2(a) The electric field of four charges is cancelled and that of fifth charge will exist, at whose opposite end the charge is absent.

3(c)

4(b) For a volumetrically charged sphere of radius R
$E \propto r$ for $(r < R)$

5(c)

6(b) in equilibrium. $Kx = \frac{1}{4\pi\epsilon_0}\frac{q^2}{(l+x)^2}$

7(c)

8(d)

9(c) $V = \frac{1}{4\pi\epsilon_0}\left[\frac{Q}{R} - \frac{Q}{3R}\right] = \frac{1}{4\pi\epsilon_0} Q\left[\frac{2}{3R}\right]$

or $Q = \frac{V \times 4\pi\epsilon_0 3R}{2}$

E at 3 R is

$E = \frac{1}{4\pi\epsilon_0}\frac{Q}{9R^2} = \frac{1}{4\pi\epsilon_0}\frac{V 4\pi\epsilon_0 3R}{2\times 9R^2} = \frac{V}{6R}$

10(a)The fall in P.E. is rise in K.E.

11(b)

12(d)since E is along Z direction so x-y plane is equipotential.

13(b)let the effective capacitance between M and N is "C". The circuit can be reduced to

$\frac{C4}{C+4} + 2 = C$

$6C + 8 = C^2 + 4C$

$C^2 - 2C - 8 = 0$

$(C-4)(C+2) = 0$

$C = 4\,\mu F$

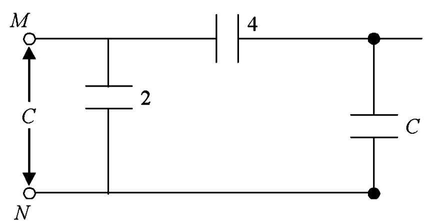


Fig. 11

14(a) $V_{MN} = \frac{30 \times 1}{2+1} = 10V$

15(a)

16(c)

17(c)

18(d)

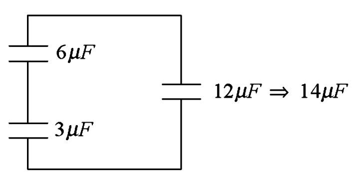


Fig. 12

19(a)The energy stored in the capacitor = The heat dissipated in the charging circuit during charging. Here initial energy of capacitor $E_i = \frac{1}{2}(k-1)CV^2$

20(a)here the current in the circuit = 1 amp.
$P = I^2R$

21(d) $H = \int I^2R\,dt = \int_0^5 2t\,dt = 25\,J$

22(c)let the number of resistance = n

$\frac{V^2}{nR} = 4$ and $\frac{V^2}{\frac{R}{n}} = 64$

$n^2 = 16$ or $\Rightarrow n = 4$

Now $\frac{V^2}{nR} = 4$ so $\frac{V^2}{R} = 16W$

it is the power in one resistance

23(a) $.6r = .4r$

$r = \frac{2}{3}R$

$\frac{\frac{ER}{R+r}}{\frac{E6R}{6R+r}} = \frac{.6}{X}$ or $\frac{6R+r}{6R+6r} = \frac{.6}{X}$

$\frac{20}{30} = \frac{.6}{X}$ $\quad \frac{6R + \frac{2}{3}R}{6R + 6\left(\frac{2}{3}R\right)} = \frac{.6}{X}$

or $X = 0.9$ i.e. $n = 90\%$

24(a)

25(a)The e.m.f. and potential drop in each cell is equal.

26(c)The current passing in R is less than 4Amp and $R = \frac{V}{I}$

27(c) $\frac{dQ}{dt} = 12t + 7$

$q = \int dq = \int_2^6 (12t+7)dt$

28(c)The circuit is a cube consisting of 12 resistance with points A and B at its diagonally opposite points.

29(b)

30(c)

31(a)

32(c)The needle oscillates only under the effect of B_v.

33(d)The magnetic field will in Z direction.

and $B_0 = \frac{E_0}{C} = \frac{300}{3\times10^8} = 10^{-6}$ Tesla

$F_{max} = B_0 qv$ sub 90˚

$= 10^{-6} \times 1.6 \times 10^{-19} \times 2 \times 10^7$

$= 3.2 \times 10^{-18}$ N

34(a)

35(d)

36(c)M = IA

$= \frac{ev}{2\pi R} \times \pi R^2 = \frac{evR}{2}$

37(d)

38(c)

39(b)$T = \frac{2\pi m}{qB} \Rightarrow T \alpha \frac{m}{q}$

$$\frac{T_p}{T_\alpha} = \frac{\frac{1}{1}}{\frac{4}{2}} = \frac{1}{2}$$

40(b)

41(a)Q = $\frac{\text{Total change in flux}}{\text{R}}$

42(d)F = IBl sin θ

here q = 0°

43(c)when the magnet just passes the centre of ring flux with ring is maximum and rate of change of flux is zero. So the induced e.m.f. and current is zero.

44(d)

45(d)

46(d)$E = -L\frac{aI}{dt} = 2000\pi \cos 100\pi t$

at $t = \frac{1}{200}$ sec, $E = 2000\pi \cos\frac{\pi}{2} = 0$

47(b)

48(c)

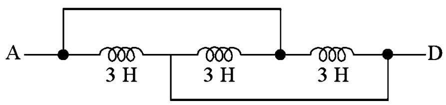
49(a)E = NBA ω. sin θ (Where θ is the angle between normal of coil and magnetic field)

50(c)

QUESTIONS FROM COMPETITIVE EXAMINATIONS

AIEEE 2002

1. The inductance between A and D is:



(a) 3.66 H (b) 9 H
(c) 0.66 H (d) 1 H

2. If an ammeter is to be used in place of a voltmeter, then we must connect with the ammeter a:
(a) low resistance in parallel
(b) high resistance in parallel
(c) high resistance in series
(d) low resistance in series

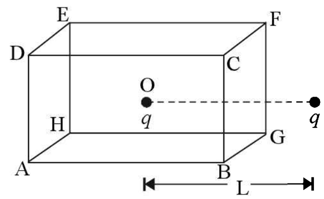
3. If in a circular coil A of radius R, current i is flowing and in another coil B of radius $2R$ a current $2i$ is flowing, then the ratio of the magnetic fields, B_A and B_B produced by them will be:
(a) 1 (b) 2
(c) $\frac{1}{2}$ (d) 4

4. A wire when connected to 220 V mains supply has power dissipation P_1. Now the wire is cut into two equal pieces which are connected in parallel to the same supply. Power dissipation in this case is P_2. Then $P_2 : P_1$ is
(a) 1 (b) 4
(c) 2 (d) 3

5. On moving a charge of 20 coulombs by 2 cm, 2 J of work is done, then the potential difference between the points is:
(a) 0.1 V (b) 8 V
(c) 2 V (d) 0.5 V

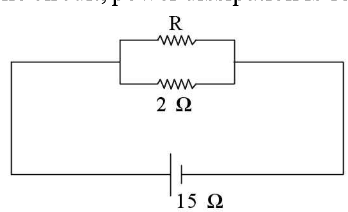
6. If an electron and a proton having same momenta enter perpendicularly to a magnetic field, then:
(a) curved path of electron and proton will be same (ignoring the sense of revolution)
(d) they will move undeflected
(c) curved path of electron is more curved than that proton
(d) path of proton is more curved

7. A charged particle q is placed at the centre O of cube of length $L(ABCDEFGH)$. Another same charge q is placed at a distance L from O. Then the electric flux through $ABCD$ is:



(a) $\frac{q}{4\varepsilon_0}$ (b) zero
(c) $\frac{q}{6\varepsilon_0}$ (d) $\frac{q}{3\varepsilon_0}$

8. If in the circuit, power dissipation is 150 W, then R is:



(a) 2 Ω (b) 6 Ω
(c) 5 Ω (d) 4 Ω

9. If a current is passed through a spring then the spring will:
(a) expand (b) compress
(c) remain same (d) none of these

10. Electromagnetic waves are transverse in nature is evident by:
(a) polarization (b) interference
(c) reflection (d) diffraction

11. If θ_i is the inversion temperature, θ_n is the neutral temperature, θ_c is the temperature of the cold junction then:
(a) $\theta_i + \theta_c = \theta_n$ (b) $\theta_i + \theta_c = 2\theta_n$
(c) $\frac{\theta_i + \theta_c}{2} = \theta_n$ (d) $\theta_i - \theta_c = 2\theta_n$

12. The power factor of an AC circuit having resistance R and inductance L (connected in series) and an angular velocity ω is:
(a) $\frac{R}{\omega L}$ (b) $\frac{R}{(R^2 + \omega L^2)^{\frac{1}{2}}}$
(c) $\frac{\omega L}{R}$ (d) $\frac{R}{(R^2 - \omega^2 L^2)^{\frac{1}{2}}}$

13. By increasing the temperature, the specific resistance of a conductor and a semiconductor:

(a) increases for both
(b) decrease for both
(c) increases, decreases respectively
(d) decreases increases respectively

14. If there are n capacitors in parallel connected to V volt source, then the energy stored is equal to:

(a) CV (b) $\frac{1}{2} nCV^2$
(c) CV^2 (d) $\frac{1}{2n} CV^2$

15. The mass of a product liberated on anode in an electrochemical cell depends on:

(a) $(It)^{\frac{1}{2}}$ (b) It
(c) $\frac{I}{t}$ (d) I^2t

(where t is the time period for which the current is passed)

16. The time period of a charged particle undergoing a circular motion in a uniform magnetic field is independent of its:

(a) speed (b) mass
(c) charge (d) magnetic induction

17. In a transformer, number of turns in the primary are 140 and that in the secondary are 280. If current in primary is 4A, then that in the secondary is:

(a) 4 A (b) 2 A
(c) 6 A (d) 10 A

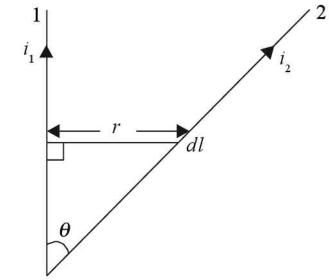
18. If a charge q is placed at the centre of the line joining two equal charge Q such that the system is in equilibrium then the value of q is:

(a) $\frac{Q}{2}$ (b) $-\frac{Q}{2}$
(c) $\frac{Q}{4}$ (d) $-\frac{Q}{4}$

19. Capacitance (in F) of a spherical conductor having radius 1 m, is:

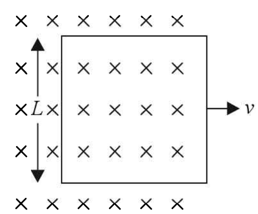
(a) 1.1×10^{-10} (b) 10^{-6}
(c) 9×10^{-9} (d) 10^{-3}

20. Wires 1 and 2 carrying current i_1 and i_2 respectively are inclined at an angle θ to each other. What is the force on a small element dl of wire 2 at a distance r from wire 1 (as shown in figure) due to the magnetic field of wire 1?



(a) $\frac{\mu_0}{2\pi r} i_1 i_2 dl \tan\theta$ (b) $\frac{\mu_0}{2\pi r} i_1 i_2 dl \sin\theta$
(c) $\frac{\mu_0}{2\pi r} i_1 i_2 dl \cos\theta$ (d) $\frac{\mu_0}{4\pi r} i_1 i_2 dl \sin\theta$

21. A conducting square loop of side L and resistance R moves in its plane with a uniform velocity perpendicular to one of its sides. A magnetic induction B constant in time and space, pointing perpendicular and into the plane at the loop exists everywhere with half the loop outside the field, as shown in figure. The induced emf is:



(a) zero (b) RvB
(c) $\frac{vBL}{R}$ (d) vBL

Answers

1.	(d)	**2.**	(c)	**3.**	(a)	**4.**	(b)	**5.**	(a)	**6.**	(a)	**7.**	(b)
8.	(b)	**9.**	(b)	**10.**	(a)	**11.**	(c)	**12.**	(b)	**13.**	(c)	**14.**	(b)
15.	(b)	**16.**	(a)	**17.**	(b)	**18.**	(d)	**19.**	(a)	**20.**	(c)	**21.**	(d)

Explanations

1(d) In parallel

$$\therefore \quad \frac{1}{L} = \frac{1}{3}+\frac{1}{3}+\frac{1}{3} \text{ or } L = 1H$$

2(c) A voltmeter is a high resistance device and is always connected in parallel with the circuit. While an ammeter is a low resistance device and is always connected in series with the circuit. so, to use voltmeter in place of ammeter a high resistance must be connected in series with the circuit.

3(a) Magnetic field in circular coil A is

$$\text{Similarly } B_A = \frac{\mu_0 Ni}{2R}$$

R is radius and i is current flowing in coil.

$$\text{Similarly, } B_B = \frac{\mu_0 N(2i)}{2.(2R)} = \frac{\mu_0 Ni}{2R}$$

$$\frac{B_A}{B_B} = \frac{1}{1} = 1$$

4(b) **In 1st case:**

Using the formula $P_l = \dfrac{V^2}{R}$

where R is resistance of wire, V is voltage across wire and P is power dissipation in wire and

In case: (ii)

$$R' = \frac{\frac{R}{2}.\frac{R}{2}}{\frac{R}{2}+\frac{R}{2}} = \frac{R}{4}$$

$$P_2 = \frac{V^2}{\frac{R}{4}}$$

$$\frac{P_2}{P_1} = \frac{4}{1}$$

5(a) W = q Δv

$$\Delta v = \frac{2}{20} = 0.1v$$

6(a) $r = \dfrac{mv}{qB}$

$\because$ mv is same for both $\therefore$ r is qual.

7(c) Due to charge inside the cube $\phi_{ABCD}\ \dfrac{q}{6\varepsilon_0}$

8(b) $P = \dfrac{V^2}{R_{\text{net}}}$

$$150 = \frac{(15)^2}{\frac{R.2}{2+R}}$$

$R = 6$ ohm

9(b) Due to flow of current in same direction in two adjacent sides, an attractive magnetic force will be produced due to which spring will get compressed.

10(a)

11(c)

12(b)

13(c)

14(b) Energy stored by any system of capacitors is

$$= \frac{1}{2} C_{\text{net}} V^2$$

$$C_{\text{net}} = nC$$

$$\therefore\ E_{\text{net}} = \frac{1}{2} nCV^2$$

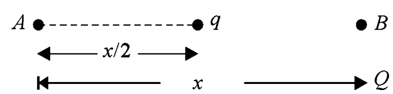
15(b)In electrochemical cell, the anode produces the charge q which depends on It.

16(c) $\because\ T = \dfrac{2\pi m}{qB}$

17(b) $\dfrac{N_P}{N_S} = \dfrac{I_S}{I_P}$

$\therefore\ I_S = 2\text{A}$

18(d)



$$\frac{-q}{Q} = \left(\frac{\frac{x}{2}}{x}\right)^2 \text{ or } q = \frac{-Q}{4}$$

19(a)Capacitance of spherical conductor

$$\subset = 4\pi\varepsilon_0 R = \frac{1}{9\times 10^9} = 1.1\times 10^{-10}\text{F}$$

20(c)Magnetic field on dl element due to current i_1 is

$$B = \frac{\mu_0}{2\pi}\frac{i_1}{r} \quad ...(i)$$

Unit force acting on dl element

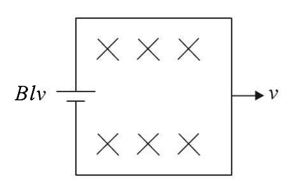
$$\frac{F}{dl} = i_2 B \sin(90° - \theta) \quad ...(ii)$$

$$\text{Thus, } \frac{F}{dl} = \frac{\mu_0}{2\pi}\frac{i_1 i_2}{r}\cos\theta$$

$$F = \frac{\mu_0 i_1 i_2 dl \cos\theta}{2\pi r}$$

21(d) E = Blv

See fig.



AIEEE 2003

1. A particle of mass M and charge Q moving with velocity $\vec{v}$ describe a circular path of radius R when subjected to a uniform transverse magnetic field of induction B. The work done by the field when the particle completes one full circle is:

(a) $\left(\frac{Mv^2}{R}\right)2\pi R$ (b) zero
(c) $BQ\,2\pi R$ (d) $BQv\,2\pi R$

2. A particle of charge -16×10^{-18} coulomb moving with velocity 10 ms^{-1} along the x-axis enters a region where a magnetic field of induction B is along the y-axis and an electric field of magnitude 10^4V/m is along the negative z-axis. If the charged particle continues moving along the x-axis, the magnitude of B is:

(a) 10^3 Wb/m^2 (b) 10^5 Wb/m^2
(c) 10^{16} Wb/m^2 (d) 10^{-3} Wb/m^2

3. A thin rectangular magnet suspended freely has a period of oscillation equal to T. Now it is broken into two equal halves (each having half of the original length) and one piece is made to oscillate freely in the same field. If its period of oscillation is T', the ratio $\frac{T'}{T}$ is:

(a) $\frac{1}{2}\sqrt{2}$ (b) $\frac{1}{2}$
(c) 2 (d) $\frac{1}{4}$

4. A magnetic needle lying parallel to a magnetic field requires W units of work to turn it through 60°. The torque needed to maintain the needle in this positioin will be:

(a) $\sqrt{3}\,W$ (b) W
(c) $\left(\frac{\sqrt{3}}{2}\right)W$ (d) $2W$

5. The magnetic lines of force inside a bar magnet:

(a) are from north-pole to sourth-pole of the magnet
(b) do not exist
(c) depend upon the area of cross-section of the bar magnet
(d) are from south-pole to north-pole of the magnet

6. Curie temperature is the temperature above which:

(a) a ferromagnetic material becomes paramagnetic
(b) a paramagnetic material becomes diamagnetic
(c) a ferromagnetic material becomes diamagnetic
(d) a paramagnetic material becomes ferromagnetic

7. The length of a wire of a potentiometer is 100 cm, and the emf of its standard and cell is E volt. It is employed to measure the emf of a battery whose internal resistance is 0.5 Ω. If the balance point is obtained at l = 30 cm from the positive end, the emf of the battery is:

(a) $\frac{30E}{100.5}$ (b) $\frac{30E}{100-0.5}$
(c) $\frac{30(E-0.5i)}{100}$, where i is the current in the potentiometer wire
(d) $\frac{30E}{100}$

8. A strip of copper and another of gernanium are cooled from room temperature to 80 K. The resistance of:

(a) each of these decreases
(b) copper strip increases and that of germanium decreases
(c) copper strip decreases and that of germanium increases
(d) each of these increases

9. The thermo-emf of a thermocouple is 25 μV/°C at room temperature. A galvanometer of 40 ohm resistance, capable of detecting current as low as 10^{-5} A, is connected with the thermocouple. The smallest temperature difference that can be detected by this system is:

(a) 16 °C (b) 12 °C
(c) 8 °C (d) 20 °C

10. The negative Zn pole of Daniell cell, sending a constant current through a circuit, decreases in mass by 0.13 g in 30 minutes. If the electrochemical equivalent of Zn and Cu are 32.5 and 31.5 respectively, the increase in the mass of the positive Cu pole in this time is:

(a) 0.180 g (b) 0.141 g
(c) 0.126 g (d) 0.242 g

11. The difference in the variation of resistance with temperature in a metal and a semiconductor arises essentially due to the difference in the:

(a) crystal structure
(b) variation of the number of charge carriers with temperature
(c) type of bonding
(d) variation of scattering mechanism with temperature

12. An ammeter reads upto 1 ampere. Its internal resistance is 0.81 ohm. To increase the range to 10 A, the value of the required shunt is:

(a) 0.03 Ω (b) 0.3 Ω
(c) 0.9 Ω (d) 0.09 Ω

13. If the electric flux entering and leaving an enclosed surface respectively is ϕ_1 and ϕ_2, the electric charge inside the surface will be:

(a) $(\phi_2 - \phi_1)\varepsilon_0$ (b) $(\phi_1 - \phi_2)\varepsilon_0$

(c) $(\phi_2 - \phi_1)\varepsilon_0$ (d) $(\phi_1 - \phi_2)\varepsilon_0$

14. Two coils are placed close to each other. The mutual inductance of the pair of coils depends upon:

(a) the rates at which currents are changing in the two coils

(b) relative position and orientation of the two coils

(c) the materials of the wires of the coils

(d) the currents in the two coils

15. When the current changes from $+2A$ to $-2A$ in 0.05 second, an emf of 8 V is induced in a coil. The coefficient of self-induction of the coil is:

(a) 0.2 H (b) 0.4 H

(c) 0.8 H (d) 0.1 H

16. In an oscillating LC circuit the maximum charge on the capacitor is Q. The charge on the capacitor when the energy is stored equally between the electric and magnetic fields is:

(a) $\frac{Q}{2}$ (b) $\frac{Q}{\sqrt{3}}$

(c) $\frac{Q}{\sqrt{2}}$ (d) Q

17. The core of any transformer is laminated so as to:

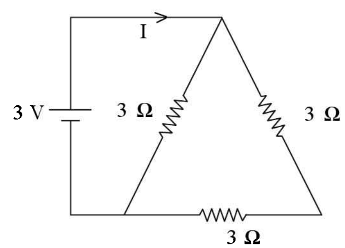
(a) reduce the energy loss due to eddy currents

(b) make it light weight

(c) make it robust and strong

(d) increases the secondary voltage

18. A 3 volts battery with negligible internal resistance is connected in a circuit as shown in the figure. The current I, in the circuit will be:



(a) $1\,A$ (b) $1.5\,A$

(c) $2\,A$ (d) $\frac{1}{3}\,A$

19. A sheet of aluminium foil of negligible thickness is introduced between the plates of a capacitor. The capacitance of the capacitors:

(a) decreases (b) remains unchanged

(c) becomes infinite (d) increases

20. A thin spherical conducting shell of radius R has a charge q. Another charge Q is placed at the centre of the shell. The electrostatic potential at a point P at a distance $\frac{R}{2}$ from the centre of the shell is:

(a) $\frac{2Q}{4\pi\varepsilon_0 R}$ (b) $\frac{2Q}{4\pi\varepsilon_0 R} + \frac{2q}{4\pi\varepsilon_0 R}$

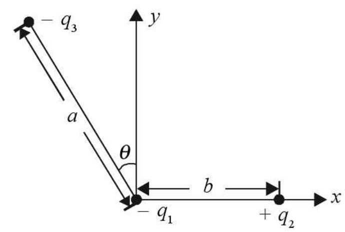
(c) $\frac{2Q}{4\pi\varepsilon_0 R} + \frac{q}{4\pi\varepsilon_0 R}$ (d) $\frac{(q+Q)}{4\pi\varepsilon_0 R} - \frac{2q}{R4\pi\varepsilon_0}$

21. The work done in placing a charge of 8×10^{-18} coulomb on a condenser of capacity 100 micro-farad is:

(a) 16×10^{-32} joule (b) 3.2×10^{-26} joule

(c) 4×10^{-10} joule (d) 32×10^{-32} joule

22. Three charges $-q_1$, $+q_2$ and $-q_3$ are placed as shown in the figure. The x-component of the force on $-q_1$ is proportional to:



(a) $\frac{q_2}{b^2} - \frac{q_3}{a^2}\cos\theta$ (b) $\frac{q_2}{b^2} + \frac{q_3}{a^2}\sin\theta$

(c) $\frac{q_2}{b^2} + \frac{q_3}{a^2}\cos\theta$ (d) $\frac{q_2}{b^2} - \frac{q_3}{a^2}\sin\theta$

23. A 220 volt, 1000 watt bulb is connected across a 110 volt mains supply. The power consumed will be:

(a) 750 watt (b) 500 watt

(c) 250 watt (d) 1000 watt

24. The length of a given cylindrical wire is increased by 100%. Due to the consequent decrease in diameter the change in the resistance of the wire will be:

(a) 200% (b) 100%

(c) 50% (d) 300%

Answers

1. (b)	**2.** (a)	**3.** (b)	**4.** (a)	**5.** (d)	**6.** (a)	**7.** (d)
8. (c)	**9.** (a)	**10.** (c)	**11.** (b)	**12.** (d)	**13.** (a)	**14.** (b)
15. (d)	**16.** (c)	**17.** (a)	**18.** (b)	**19.** (b)	**20.** (c)	**21.** (d)
22. (b)	**23.** (c)	**24.** (d)				

Explanations

1(b) When particle describes circular path in a magnetic field, its velocity is always perpendicular to the magnetic force.

Power $P = \vec{F} \cdot \vec{v} = Fv\cos\theta$

Here, $\theta = 90°$ $\therefore$ $P = 0$ or W = 0

2(a) qvB = qE

or $B = \frac{E}{v}$

$= \frac{10^4}{10} = 10^3$ T

3(b) $\frac{T_2}{T_1} = \sqrt{\frac{I_2 M_1 B}{M_2 I_1 B}}$

$= \sqrt{\frac{\left(\frac{m}{2}\right)\left(\frac{\ell}{2}\right)^{2M}}{ml^2 \frac{m}{2}}} = \frac{1}{2}$

4(a) $\frac{\tau}{w} = \frac{MB\sin\theta}{MB(1-\cos\theta)} = \frac{\frac{B}{2}}{1-\frac{1}{2}}$

$\tau = \sqrt{3}w$

5(d) Inside bar magnet, lines of force are from south to north.

6(a) Above Curie temperature, a ferromagnetic substance.

7(d) $\frac{V}{E} = \frac{l}{L}$

$V = \frac{30 \times E}{100}$

$= \frac{30}{100}E$

8(c) Germanium is semiconductor, whereas copper is conductor. In metals R $\propto \Delta$ T and in semiconductor

$R \propto \frac{1}{\Delta T}$

9(a) IR = 25 × 10^{-6} $\Delta\theta$

$\Delta\theta = \frac{10^{-5} \times 40}{25 \times 10^{-6}} = 16°C$

10(c) $\frac{m_{Zn}}{m_{Cu}} = \frac{z_{Zn}}{z_{Cu}}$

$\therefore$ $\frac{0.13}{m_{Cu}} = \frac{32.5}{31.5}$ or $m_{Cu} = 0.126$ g

11(b) The difference in the variation of resistance with temperature in metal and semiconductor is caused due to difference in the variation of the number of charge carriers with temperature.

12(d) $S = \frac{I_g G}{I - I_g} = \frac{1 \times 0.81}{10-1} = 0.09\,\Omega$

13(a) $\Delta\phi = \frac{q}{\varepsilon_0}$

or $\phi_2 - \phi_1 = \frac{q}{\varepsilon_0}$ or $q = (\phi_2 - \phi_1)\in_0$

14(b) Mutual inductance of the pair of coils depends on distance between two coils and geometry of two coils

15(d) $e = -L\frac{di}{dt} = -L\frac{(-2-2)}{0.05}$

$8 = L\frac{(4)}{0.05}$

$L = \frac{8 \times 0.05}{4} = 0.1\,H$

16(c) $\frac{1}{2}Li^2 = \frac{1}{2}\frac{q^2}{C}$ ...(i)

$\frac{1}{2}Li^2 + \frac{q^2}{2c} = \frac{Q^2}{2c}$ ...(ii)

From eq. (i) and (ii)

$\frac{q^2}{C} = \frac{Q^2}{2c}$

or $q = \frac{Q}{\sqrt{2}}$

17(a)

18(b) $\frac{3}{\frac{3\times 6}{3+6}} = \frac{3}{2}A$

19(b) $C = \frac{\varepsilon_0 A}{d-t}$,

As $t \to o$; $C \to \frac{\in_0 A}{d}$

Hence, capacitance of capacitor remains unchanged

20(c) $V = V_1 + V_2$

$= \frac{Q}{4\pi\in_0 \frac{R}{2}} + \frac{q}{4\pi\in_0 R}$

21(d) $W = \frac{1}{2}CV^2 = \frac{1}{2}\frac{q^2}{C}$

$= \frac{1}{2} \times \frac{(8\times10^{-18})^2}{100\times10^{-6}} = 32 \times 10^{-32}$ J

22(b) $\vec{F}_{12(x)} = \frac{q_1q_2}{4\pi\varepsilon_0 b^2}(-\hat{i})$

$\vec{F}_{13} = \frac{q_1q_3}{4\pi\varepsilon_0 a^2}(-\sin\theta\,\hat{i} + \cos\theta\,\hat{j})$

$\vec{F}_{13\,(x)} = \frac{q_1q_3}{4\pi\varepsilon_0 a^2}(-\sin\theta\,\hat{i})$

$\therefore$ Net $\vec{F}_x = \frac{Kq_1q_2}{b^2}(-\hat{i}) + \frac{Kq_1q_3}{a^2}\sin\theta(-\hat{i})$

$F_x = -Kq_1\left[\frac{q^2}{b^2} + \frac{q^3}{a^2}\sin\theta\right]$

$\therefore$ $F_x \propto \frac{q_2}{b^2} + \frac{q_3}{a^2}\sin\theta$

23(c) $R = \frac{V^2}{P} = \frac{(220)^2}{1000}$

$\therefore$ $P_{\text{consumed}} = \frac{V^2}{R} = \frac{110\times110}{220\times220} \times 1000$

$= 250$ watt

24(d) Given: $l' = l + 100\%\, l = 2l$

Initial volume = final volume

$A_1l_1 = A_2l_2 = A_2(2l_1)$

$A_2 = \frac{A_1}{2}$

$R^1 = \frac{\delta(2l_1)}{\frac{A_1}{2}}$

$= 4R$

Thus, $\Delta R = R' - R = 4R - R = 3R$

$\therefore$ $\%\Delta R = \frac{3R}{R} \times 100\% = 300\%$

AIEEE 2004

1. An electromagnetic wave of frequency $f = 3.0$ MHz passes from vacuum into a dielectric medium with permittivity $\varepsilon = 4.0$. Then

(a) wavelength is doubled and frequency remains unchanged

(b) wavelength is doubled and frequency becomes half

(c) wavelength is halved and frequency remains unchanged

(d) wavelength and frequency both remain unchanged

2. Two spherical conductors B and C having equal radii and carrying equal charges in them repel each other with a force F when kept apart at some distance. A third spherical conductor having same radius as that of B but uncharged, is brought in contact with B, then brought in contact with C and finally removed away from both. The new force of repulsion between B and C is:

(a) $\frac{F}{4}$ (b) $\frac{3F}{4}$

(c) $\frac{F}{8}$ (d) $\frac{3F}{8}$

3. A charged particle q is shot towards another charged particle Q which is fixed, with a speed v. It approaches Q upto a closest distance r and then returns. If q was given a speed $2v$, the closest distance of approach would be:

(a) r (b) $2r$

(c) $\frac{r}{2}$ (d) $\frac{r}{4}$

4. Four charges each equal to $-Q$ are placed at the four corners of a square and a charge q is at its centre. If the system is in equilibrium, the value of q is:

(a) $-\frac{Q}{4}(1+2\sqrt{2})$ (b) $\frac{Q}{4}(1+2\sqrt{2})$

(c) $-\frac{Q}{2}(1+2\sqrt{2})$ (d) $\frac{Q}{2}(1+2\sqrt{2})$

5. Alternating current can not be measured by D.C. ammeter because:

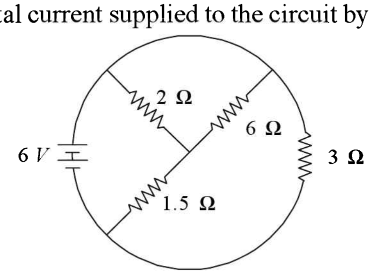
(a) A.C. cannot pass through D.C. ammeter

(b) A.C changes direction

(c) average value of current for complete cycle is zero

(d) D.C. ammeter will get damaged

6. The total current supplied to the circuit by the battery is:



(a) 1 A (b) 2 A
(c) 4 A (d) 6 A

7. The resistance of the series combination of two resistances is S. When they are joined in parallel, the total resistance is P. If $S = nP$, then the minimum possible value of n is:

(a) 4 (b) 3
(c) 2 (d) 1

8. An electric current is passed through a circuit containing two wires of the same material, connected in parallel. If the lengths and radii of the wires are in the ratio of $\frac{4}{3}$ and $\frac{2}{3}$, then the ratio of the currents passing through the wire will be:

(a) 3 (b) $\frac{1}{3}$
(c) $\frac{8}{9}$ (d) 2

9. In a metre bridge experiment, null point is obtained at 20 cm from one end of the wire when resistance X is balanced against another resistance Y. If $X < Y$ then where will be the new position of the null point from the same end, if one decides to balance a resistance of $4X$ against Y?

(a) 50 cm (b) 80 cm
(c) 40 cm (d) 70 cm

10. The thermo-emf of a thermocouple varies with the temperature θ of the hot junction as $E = a\theta + b\theta^2$ in volts where the ratio a/b is 700 °C. If the cold junction is kept at 0 °C, then the neutral temperature is:

(a) 700 °C (b) 350 °C
(c) 1400 °C
(d) no neutral temperature is possible for this themocouple

11. The electrochemical equivalent of metal is 3.3×10^{-7}kg per coulomb. The mass of the metal liberated at the cathode when a 3 A current is passed for 2 seconds, will be:

(a) 19.8×10^{-7}kg (b) 9.9×10^{-7}kg
(c) 66×10^{-7}kg (d) 1.1×10^{-7}kg

12. A current i ampere flows along an infinitely long straight thin walled tube, then the magnetic induction at any point inside the tube is:

(a) infinite (b) zero
(c) $\frac{\mu_0}{4\pi} \cdot \frac{2i}{r}$ tesla (d) $\frac{2i}{r}$ tesla

13. A long wire carries a steady current. It is bent into a circle of one turn and the magnetic field at the centre of the coil is B. It is then bent into a circular loop of n turns. The magnetic field at the centre of the coil will be:

(a) nB (b) n^2B
(c) $2nB$ (d) $2n^2B$

14. The magnetic field due to a current carrying circular loop of radius 3 cm at a point on the axis at a distance of 4 cm from the centre is 54 μT. What will be its value at the centre of the loop?

(a) 250 μT (b) 150 μT
(c) 125 μT (d) 75 μT

15. Two long conductors, separated by a distance d carry currents I_1 and I_2 in the same direction. They exert a force F on each other. Now the current in one of them is increased to two times and its direction is reversed. The distance is also increased to $3d$. Then new value of the force between them is:

(a) $-2F$ (b) $F/3$
(c) $-2F/3$ (d) $-F/3$

16. The length of a megnet is large compard to its width and breadth. The time period of its oscillation in a vibration magnetometer is $2s$. The magnet is cut along its length into three equal parts and three parts are then placed on each other with their like pole together. The time period of this combination will be:

(a) $2s$ (b) $\frac{2}{3s}$
(c) $2\sqrt{3}\, s$ (d) $\frac{2}{\sqrt{3}s}$

17. The materials suitable for making electromagnets should have:

(a) high retentivity and high coercivity
(b) low retentivity and low coercivity
(c) high retentivity and low coercivity
(d) low retentivity and high coercivity

18. In an LCR series A.C. circuit, the voltage across each of the components. L, C and R is 50 V. The voltage across the LC combination will be:

(a) 50 V (b) $50\sqrt{2}$ V
(c) 100 V (d) 0 V (zero)

19. A coil having n turns and resistance $R\ \Omega$ is connected with a galvanometer of resistance $4R\Omega$. This combination is moved in time t seconds from a magnetic field W_1 weber to W_2 weber. The induced current in the circuit is:

(a) $\frac{W_2 - W_1}{5Rnt}$ (b) $-\frac{n(W_2 - W_1)}{5Rt}$
(c) $-\frac{(W_2 - W_1)}{Rnt}$ (d) $-\frac{n(W_2 - W_1)}{Rt}$

20. In a uniform magnetic field of induction B, a wire in the form of semicircle of radius r rotates about the diameter of the circle with angular frequency ω. If the total resistance of the circuit is R, the mean power generated per period of rotation is:

(a) $\frac{B\pi r^2\omega}{2R}$ (b) $\frac{(B\pi r^2\omega)^2}{8R}$

(c) $\frac{(B\pi r\omega)^2}{2R}$ (d) $\frac{(B\pi r\omega^2)^2}{8R}$

21. In an LCR circuit, capacitance is changed from C to $2C$. For the resonant frequency to remain unchanged, the inductance should be changed from L to:

(a) $4L$ (b) $2L$

(c) $\frac{L}{2}$ (d) $\frac{L}{4}$

22. A metal conductor of length 1 m rotates vertically about one of its ends at angular velocity 5 radians per second. If the horizontal component of earth's magnetic field is $0.2 \times 10^{-4} T$, then the emf developed between the two ends of the conductor is:

(a) 5 μV (b) 50 μV

(c) 5 mV (d) 50 mV

23. A charged oil drop is suspended in uniform field of 3×10^4 V/m so that it neither falls nor rises. The charge on the drop will be: (take the mass of the charge $= 9.9 \times 10^{-15}$ kg and g = 10 m/s²)

(a) 3.3×10^{-18} C (b) 3.2×10^{-18} C

(c) 1.6×10^{-18} C (d) 4.8×10^{-18} C

Answers

1.	(c)	**2.**	(d)	**3.**	(d)	**4.**	(b)	**5.**	(c)	**6.**	(c)	**7.**	(a)
8.	(b)	**9.**	(a)	**10.**	(d)	**11.**	(a)	**12.**	(b)	**13.**	(b)	**14.**	(a)
15.	(c)	**16.**	(b)	**17.**	(c)	**18.**	(d)	**19.**	(b)	**20.**	(b)	**21.**	(c)
22.	(b)	**23.**	(a)										

Explanations

1(c) Ref. index $n = \sqrt{\in r} = 2$

$$\lambda' = \frac{\lambda}{\mu} = \frac{\lambda}{2}$$

2(d) $F = \frac{q^2}{4\pi \in_0 r^2}$

$$q_A = q_B = \frac{q_A + q_B}{2}$$

$$= \frac{0+q}{2} = \frac{q}{2}$$

Again when uncharged conductor A is brought in contact with C, then charge on each conductor

$$q_A = q_C = \frac{q_A + q_C}{2} = \frac{\frac{q}{2}+q}{2} = \frac{3q}{4}$$

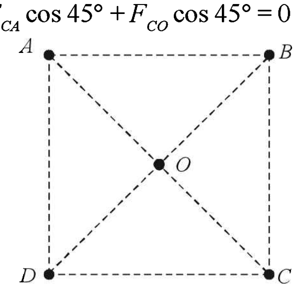
$$F' = \frac{\left(\frac{q}{2}\right)\left(\frac{3q}{4}\right)}{4\pi \in_0 r^2} = \frac{3}{8}\frac{q^2}{(4\pi \in_0 r^2)} = \frac{3}{8}F$$

3(d) $\frac{\frac{1}{2}mv^2}{\frac{1}{2}m(2v)^2} = \frac{\frac{kQq}{r}}{\frac{kQq}{r'}}$

$$\Rightarrow \frac{1}{4} = \frac{r'}{r} \Rightarrow r' = \frac{r}{4}$$

4(b) In steady state, equating the sum of x-components of force to zero *i.e*.

$$F_{CD} + F_{CA}\cos 45° + F_{CO}\cos 45° = 0$$



$$\Rightarrow \frac{1}{4\pi\varepsilon_0}\cdot\frac{(-Q)(-Q)}{a^2} + \frac{1}{4\pi\varepsilon_0}\frac{(-Q)(-Q)}{(\sqrt{2}a)^2}\times\frac{1}{\sqrt{2}} + \frac{1}{4\pi\varepsilon_0}\frac{(-Q)q}{\left(\frac{\sqrt{2}a}{2}\right)^2}\times\frac{1}{\sqrt{2}} = 0$$

$$\Rightarrow \frac{1}{4\pi\varepsilon_0}\frac{Q^2}{2a^2} + \frac{1}{4\pi\varepsilon_0}\frac{Q^2}{2a^2}\cdot\frac{1}{\sqrt{2}} - \frac{2Qq}{a^2}\times\frac{1}{\sqrt{2}} = 0$$

$$\Rightarrow Q + \frac{Q}{2\sqrt{2}} - \sqrt{2}q = 0 \Rightarrow 2\sqrt{2}\,Q + Q - 4q = 0$$

$\Rightarrow \; 4q = (2\sqrt{2}+1)\,Q$

$\Rightarrow \; q = (2\sqrt{2}+1)\,\dfrac{Q}{4}$

5(c) The full cycle of alternating current consists of two half cycles. For one half, current is positive and for second half, current is negative. Therefore, for an A.C. cycle, the net value of current average out to zero.

6(c) The equivalent of the given circuit can be found as 1.5 Ω

Hence, current supplied by the battery is

$i = \dfrac{V}{R} = \dfrac{6}{1.5}$

7(a) S = 2 R

$P = \dfrac{R}{2}$

S = x P

$n = 4$

8(b) Since, voltage remains same in parallel, so,

$i \propto \dfrac{1}{R}$

$\Rightarrow \; \dfrac{i_1}{i_2} = \dfrac{R_2}{R_1}$

$\dfrac{i_1}{i_2} = \dfrac{\frac{\rho l_2}{A_2}}{\frac{\rho l_1}{A_1}} \left(\because R = \dfrac{\rho l}{A}\right)$

$\Rightarrow \; \dfrac{i_1}{i_2} = \dfrac{l_2}{l_1} \times \left(\dfrac{r_1}{r_2}\right)^2 \; (\because A = \pi r^2)$

$\Rightarrow \; \dfrac{i_1}{i_2} = \dfrac{3}{4} \times \left(\dfrac{2}{3}\right)^2$

Hence, $\dfrac{i_1}{i_2} = \dfrac{1}{3}$

9(a) **case (i)**

$\therefore \; \dfrac{X}{Y} = \dfrac{20}{80}$ ...(i)

case (ii)

$\dfrac{4X}{Y} = \dfrac{l}{100-l} = \dfrac{4(20)}{80}$

or $\dfrac{l}{100l} = 1$

$l = 50$ cm

10(d) $E = a\theta + b\theta^2$ (given)

For neutral temperture (θ_n), $\dfrac{dE}{d\theta} = 0$

$\Rightarrow \; a + 2b\theta_n = 0$

$\Rightarrow \; \theta_n = -\dfrac{a}{2b}$

$\therefore \; \theta_n = -\dfrac{700}{2} \left(\because \dfrac{a}{b} = 700°C\right)$

$= -350°\,C < 0°C$

$V_n = .350°C$

Since θ_n can not be - vl

$\therefore$ no neutral temp is possible

11(a) Mass of substance liberated at cathode

$m = zit$

or $m = 3.3 \times 10^{-7} \times 3 \times 2$

$= 19.8 \times 10^{-7}$ kg

12(b) Magnetic induction inside the thin walled tube is zero.

13(b)The magnetic field at the centre of circular coil is

case (i)

$B = \dfrac{\mu_0 i}{2r}$

where r = radius of circle = $\dfrac{l}{2\pi}$ $(\because l = 2\pi r)$

$\therefore \; B = \dfrac{\mu_0 i}{2} \times \dfrac{2\pi}{l} = \dfrac{\mu_0 i\pi}{l}$

case (ii)

When wire of length l bents into a circular loop of n turns, then

$l = n\,2\pi r'$

$\Rightarrow \; r' = \dfrac{l}{n \times 2\pi}$

Thus, new magnetic field

$B' = \dfrac{\mu_0 ni}{2r'} = \dfrac{\mu_0 ni}{2} \times \dfrac{n \times 2\pi}{l}$

$= \dfrac{\mu_0 i\pi}{l} \times n^2 = n^2 B$

14(a)The magnetic field at a point on the axis of a circular loop at a distance x from the centre is

$B = \dfrac{\mu_0 i R^2}{2(R^2 + x^2)^{\frac{3}{2}}}$

or $54 = \dfrac{\mu_0 i \times (3)^2}{2(3^2 + 4^2)^{\frac{3}{2}}}$

or $54 = \dfrac{9\mu_0 i}{2(25)^{\frac{3}{2}}}$

$= \dfrac{9\mu_0 i}{2\times(5)^3}$

or $\mu_0 i = \dfrac{54\times2\times125}{9} = 1500$

B centre $= \dfrac{\mu_0 i}{2R}$

$= \dfrac{1500}{2\times3}$

$= 250\ \mu T$

15(c) $\dfrac{F_1}{F_2} = \dfrac{\dfrac{(\mu_0 I_1 I_2)}{2\pi d}}{\dfrac{-[\mu_0 I_1(2I_2)]}{3\pi d}}$

or $F_2 = \dfrac{2}{3}F_1$

16(b)The time period of oscillations of magnet

$T = 2\pi\sqrt{\left(\dfrac{I}{MH}\right)}$ ...(i)

where I = moment of inertia of magnet

$= \dfrac{mL^2}{12}$

(m, being the mass of magnet)

M = pole strength × L

When the three equal parts of magnet are placed on one another with their like poles together, then

$I' = \dfrac{1}{12}\left(\dfrac{m}{3}\right)\times\left(\dfrac{L}{3}\right)^2\times3$

$= \dfrac{1}{12}\dfrac{mL^2}{9}$

$= \dfrac{I}{9}$

and M' = pole strength × $\dfrac{L}{3}$ × 3

$= M$

Hence, $T' = 2\pi\sqrt{\left(\dfrac{\frac{I}{9}}{MH}\right)}$

$\Rightarrow\ T' = \dfrac{1}{3}\times T$

$T' = \dfrac{2}{3}$ sec

17(c) Electromagnets are made of material having high retentivity and low coercivity.

18(d)In an LCR series a.c. circuit, the voltage across inductor L leads the current by 90° and the voltage across capacitor C lags behind the current by 90°

Hence, the voltage across LC combination will be zero.

19(b) $e = -n\dfrac{d\phi}{dt} = -\dfrac{n(W_2 - W_1)}{t}$

$i = \dfrac{e}{5R} = -\dfrac{n(W_2 - W_1)}{t(5R)}$

20(d)The flux associated with coil of area A and magnetic induction B is

$\phi = BA\cos\theta = \dfrac{B\pi r^2}{2}\cos\omega t$

$e = -\dfrac{d\phi}{dt} = \dfrac{B\pi r^2 w}{2}\sin\omega t$

$P = \dfrac{e^2}{R} = \dfrac{B^2\pi^2 r^4\omega^2\sin^2\omega t}{4R}$

$P_{av} = \dfrac{B^2\pi^2 r^4\omega^2}{4R}\int_o^T \sin^2\omega t dt$

$P_{av} = \dfrac{B^2\pi^2 r^4\omega^2}{8R}$

21(c) use $L_1C_1 = L_2C_2$

$\because\ C_2 = 2C$

$\therefore\ L_2 = \dfrac{L}{2}$

22(b)The emf induced between ends of conductor

$e = \dfrac{1}{2}B\omega L^2$

$= \dfrac{1}{2}\times 0.2\times10^{-4}\times5\times(1)^2$

$= 50\ \mu v$

23(a)In steady state,

$qE = mg$

or $q = \dfrac{mg}{E}$

$= \dfrac{9.9\times10^{-15}\times10}{3\times10^4}$

$= 3.3\times10^{-18} C$

AIEEE 2005

1. A moving coil galvanometer has 150 equal divisions. Its current sensitivity is 10 divisions per milliampere and voltage sensitivity is 2 divisions per millivolt. In order that each division reads 1 volt, the resistance in ohms needed to be connected in series with the coil will be:

(a) 10^3 (b) 10^5

(c) 99995 (d) 9995

2. Two voltameters, one of copper and another of silver, are joined in parallel. When a total charge q flows through the voltameters, equal amount of metals are deposited. If the electrochemical equivalents of copper and silver are z_1 and z_2 respectively, the charge which flows through the silver voltameter is:

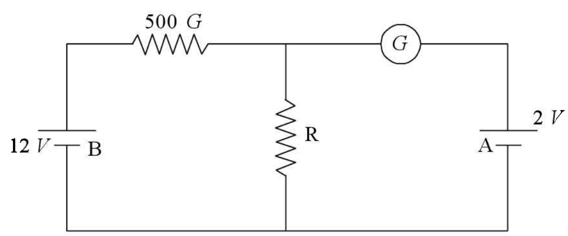
(a) $\dfrac{q}{1+\dfrac{z_1}{z_2}}$ (b) $\dfrac{q}{1+\dfrac{z_2}{z_1}}$

(c) $q\dfrac{z_1}{z_2}$ (d) $q\dfrac{z_2}{z_1}$

3. In the circuit, the galvanometer G shows zero deflection. If the batteries A and B have negligible internal resistance, the value of the resistor R will be:

(a) 2000Ω (b) 100Ω

(c) 500Ω (d) 1000Ω



4. Two sources of equal emf are connected to an external resistance R. Then internal resistances of the two sources are R_1 and R_2 ($R_2 > R_1$). If the potential difference across the source having internal resistance R_2, is zero, then:

(a) $R = \dfrac{R_2 \times (R_1 + R_2)}{(R_2 - R_1)}$ (b) $R = R_2 - R_1$

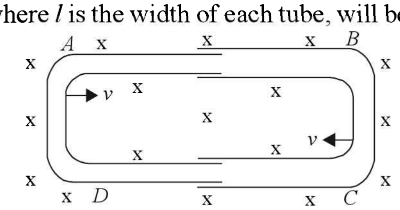
(c) $R = \dfrac{R_1 R_2}{(R_1 + R_2)}$ (d) $R = \dfrac{R_1 R_2}{(R_1 - R_2)}$

5. A fully charged capacitor has a capacitance 'C'. It is discharged through a small coil of resistance wire embedded in a thermally insulated block of specific heat capacity 's' and mass 'm'. If the temperature of the block is raised by 'ΔT', the potential difference 'V' across the capacitance is:

(a) $\dfrac{\sqrt{2mC\Delta T}}{s}$ (b) $\dfrac{mC\Delta T}{s}$

(c) $\dfrac{ms\Delta T}{C}$ (d) $\dfrac{\sqrt{2ms\Delta T}}{C}$

6. One conducing U tube can slide inside another as shown in figure, maintaining electrical contacts between the tubes. The magnetic field B is perpendicular to the plane of the figure. If each tube moves towards the other at a constant speed v, then the emf induced in the circuit in terms of B, l and v, where l is the width of each tube, will be:



(a) Blv (b) $-Blv$

(c) zero (d) $2Blv$

7. A heater coil is cut into two equal parts and only one part is now used in the heater. The heat generated will now be:

(a) doubled (b) four times

(c) one-fourth (d) halved

8. Two thin, long, parallel wires, separated by a distance 'd' carry a current of 'i' in the same direction. They will:

(a) attract each other with a force of $\dfrac{\mu_0 i^2}{(2\pi d)}$

(b) repel each other with a force of $\dfrac{\mu_0 i^2}{(2\pi d)}$

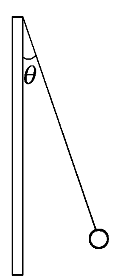
(c) attract each other with a force of $\dfrac{\mu_0 i^2}{(2\pi d^2)}$

(d) repel each other with a force of $\dfrac{\mu_0 i^2}{(2\pi d^2)}$

9. A charged ball B hangs from a silk thread S, which makes an angle θ with a large charged conducting sheet P, as shown in the figure. The surface charge density σ of the sheet is proportional to:

(a) $\cos\theta$ (b) $\cot\theta$

(c) $\sin\theta$ (d) $\tan\theta$



10. Two point charges + 8q and – 2q are located at $x = 0$ and $x = L$ respectively. The location of a point on the x-axis at which the net electric field due to these two point charges is zero is:

(a) $2L$ (b) $\frac{L}{4}$
(c) $8L$ (d) $4L$

11. Two thin wire rings each having a radius R are placed at a distance d apart with their axes coinciding. The charges on the two rings are $+q$ and $-q$. The potential difference between the centres of the two rings is:

(a) $\frac{qR}{4\pi\varepsilon_0 d^2}$ (b) $\frac{q}{2\pi\varepsilon_0}\left[\frac{1}{R}-\frac{1}{\sqrt{R^2+d^2}}\right]$
(c) zero (d) $\frac{q}{4\pi\varepsilon_0}\left[\frac{1}{R}-\frac{1}{\sqrt{R^2+d^2}}\right]$

12. A parallel plate capacitor is made by stacking n equally spaced plates connected alternatively. If the capacitance between any two adjacent plates is 'C', then the resultant capacitance is:

(a) $(n-1)C$ (b) $(n+1)C$
(c) C (d) nC

13. Two concentric coils each of radius equal to 2π cm are placed at right angles to each other. 3 ampere and 4 ampere are the currents flowing in each coil respectively. The magnetic induction in weber/m² at the centre of the coils will be ($\mu_0 = 4\pi \times 10^{-7}$ Wb/A.m):

(a) 12×10^{-5} (b) 10^{-5}
(c) 5×10^{-5} (d) 7×10^{-5}

14. A coil of inductance 300 mH and resistance 2Ω is connected to a source of voltage 2V. The current reaches half of its steady state value in:

(a) 0.05 s (b) 0.01 s
(c) 0.15 s (d) 0.3 s

15. The self inductance of the motor of an electric fan is 10 H. In order to impart maximum power at 50 Hz, it should be connected to a capacitance of:

(a) 4 μF (b) 8 μF
(c) 1 μF (d) 2 μF

16. An energy source will supply a constant current into the load, if its internal resistance is:

(a) equal to the resistance of the load
(b) very large as compared to the load resistance
(c) zero
(d) non-zero but less than the resistance of the load

17. A circuit has a resistance of 12 ohm and an impedance of 15 ohm. The power factor of the circuit will be:

(a) 0.8 (b) .0.4
(c) 1.25 (d) 0.125

18. The phase difference between the alternating current and emf is π/2. Which of the following cannot be the constituent of the circuit?

(a) C alone (b) R, L
(c) L, C (d) L alone

19. A uniform electric field and a uniform magnetic field are acting along the same direction in a certain region. If an electron is projected along the direction of the fields with a certain velocity, then:

(a) its velocity will decrease
(b) its velocity will increase
(c) it will turn towards right of direction of motion
(d) it will turn towards left of direction of motion

20. A charged particle of mass m and charge q travels on a circular path of radius r that is perpendicular to a magnetic field B. The time taken by the particle to complete one revolution:

(a) $\frac{2\pi mq}{B}$ (b) $\frac{2\pi q^2 B}{m}$
(c) $\frac{2\pi qB}{m}$ (d) $\frac{2\pi m}{qB}$

21. In a potentiometer experiment the balancing with a cell is at length 240 cm. On shunting the cell with a resistance of 2Ω, the balancing length becomes 120 cm. The internal resistance of the cell is:

(a) 1Ω (b) 0.5Ω
(c) 4Ω (d) 2Ω

22. The resistance of hot tungsten filament is about 10 times the cold resistance. What will be the resistance of 100Ω and 200 V lamp, when not in use?

(a) 40Ω (b) 20Ω
(c) 400Ω (d) 200Ω

23. A magnetic needle is kept in a non-uniform magnetic field. It experiences:

(a) a torque but not a force
(b) neither a force nor a torque
(c) a force and a torque
(d) a force but not a torque

Answers

1.	(d)	**2.**	(b)	**3.**	(b)	**4.**	(b)	**5.**	(d)	**6.**	(d)	**7.**	(a)
8.	(a)	**9.**	(d)	**10.**	(a)	**11.**	(b)	**12.**	(a)	**13.**	(c)	**14.**	(b)
15.	(c)	**16.**	(b)	**17.**	(a)	**18.**	(c)	**19.**	(a)	**20.**	(d)	**21.**	(d)
22.	(a)	**23.**	(c)										

Explanations

1(d) Voltage sensitivity

$= \dfrac{\text{Current sensitivity}}{\text{Resistance of galvanometer } (G)}$

$\therefore \quad 2 = \dfrac{10}{G}$

$\therefore \quad G = 5\Omega$

I_g = Full scale deflection current

$= \dfrac{150}{10}$ mA = 15 mA

Hence, $R = \dfrac{150}{150 + 10^{-3}} - 5 = 9995\ \Omega$

2(b) $\because$ M = $z_1 q_1 = z_2 q_2$

$\therefore \quad \dfrac{z_2}{z_1} = \dfrac{q_1}{q_2}$

Total charge $q = q_1 + q_2$

$\therefore \quad \dfrac{q}{q_2} = \dfrac{q_1}{q_2} + 1$

$\Rightarrow \quad q_2 = \dfrac{q}{\left(1 + \dfrac{q_1}{q_2}\right)}$

$\therefore \quad q_2 = \dfrac{q}{\left(1 + \dfrac{z_2}{z_1}\right)}$

3(b) $\dfrac{12R}{R + 500} = 2$

or $\quad 10R = 1000$

$R = 100\ \Omega$

4(b) $R_{eq} = R_1 + R_2 + R$

$\therefore \quad I = \dfrac{2E}{R_1 + R_2 + R}$

According to the question,

$V_A - V_B = E - I R_2$

$0 = E - I R_2$

$E = I R_2$

$E = \dfrac{2ER_2}{R_1 + R_2 + R}$

$\therefore \quad R_1 + R_2 + R = 2 R_2$

$R = R_2 - R_1$

5(d) $ms\ \Delta T = \left(\dfrac{1}{2}\right) CV^2$

$V = \dfrac{\sqrt{2ms\Delta T}}{C}$

6(d) Relative velocity = $v - (-v) = 2v = \dfrac{dl}{dt}$

e = $Bl\ (2v)$

7(a) $H_1 = \dfrac{V^2}{R} t$

$H_2 = \dfrac{\dfrac{V^2}{R} t}{2}$

$\therefore \quad \dfrac{H_2}{H_1} = 2$

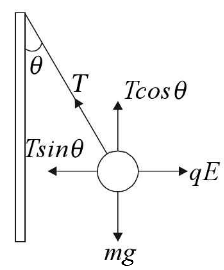
8(a) The force per unit length between the two wires is

$\dfrac{F}{l} = \dfrac{\mu_0}{4\pi} \cdot \dfrac{2i^2}{d}$

$= \dfrac{\mu_0 i^2}{2\pi d}$

The force will be attractive as current directions in both are same.

9(d) $E = \dfrac{\sigma}{2\varepsilon_0}$



$QE = \dfrac{Q\sigma}{2\varepsilon_0}$

Now $\quad T \cos\theta = mg$

and $\quad T \sin\theta = \dfrac{Q\sigma}{2\varepsilon_0 \varepsilon_a}$

Thus, $\tan\theta = \dfrac{Q\sigma}{2\varepsilon_0 mg}$

or $\quad \tan\theta \propto \sigma$

10(a) $\dfrac{8q}{4\pi \in_0 (x + L)^2} = \dfrac{2q}{4\pi \in_0 x^2}$

$\dfrac{2}{x + L} = \dfrac{1}{x}$

$2x = x + L$

or $\quad x = L$

Thus, at distance $2L$ from origin, net electirc field will be zero.

11(b) V_A = potential due to charge + q on ring A + potential due to charge – q on ring B

$V_A = \dfrac{q}{4\pi \in_0 R} - \dfrac{q}{4\pi \in_0 \sqrt{R^2 + d^2}}$

$V_B = \dfrac{-q}{4\pi \in_0 R} + \dfrac{q}{4\pi \in_0 \sqrt{R^2 + d^2}}$

$V_A - V_B = \dfrac{q}{2\pi \in_0} \left[\dfrac{1}{R} - \dfrac{1}{\sqrt{R^2 + d^2}}\right]$

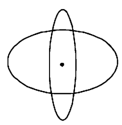
12(a) Capacitance of each plate

$$C = \frac{K\varepsilon_0 A}{d}$$

Capacitor consists of n plates, then there are $(n-1)$ combinations joined in parallel.

$\therefore \quad C_{AB} = (n-1)C$

13(c) $B = \sqrt{B_1^2 + B_2^2}$



$$= \frac{\mu_0}{2R}\sqrt{3^2+4^2}$$

$$= \frac{4\pi\times10^{-7}(5)}{2(2\pi\times10^{-2})} = 5\times10^{-5}\,\text{T}$$

14(b) $\frac{I_0}{2} = I_0\left(1-e^{\frac{-Rt}{L}}\right)$

$$\frac{1}{2} = \left(1-e^{\frac{-Rt}{L}}\right)$$

$$e^{-\frac{Rt}{L}} = \frac{1}{2}$$

$$\frac{Rt}{L} = \ln 2$$

$\therefore \quad t = \frac{L}{R}\ln 2 = \frac{300\times10^{-3}}{2}\times0.693 = 150\times0.693\times10^{-3}$

$= 0.010395$ sec $= 0.01$ sec

15(c) $50 = \frac{1}{2\pi\sqrt{LC}}$

$$C = \frac{1}{(100\pi)^2L} = \frac{1}{10^5\times10} = 1\mu F$$

16(b)

17(a) $\cos\phi = \frac{12}{15} = 0.8$

18(b)

19(a) When electron is projected in an electric field, then velocity of electron will decrease.

20(d) $T = \frac{2\pi m}{qB}$

21(d) The internal resistance of the cell

$$r = \left(\frac{l_1-l_2}{l_2}\right)R = \frac{240-120}{120}\times 2 = 2\,\Omega$$

22(a) $R_{\text{hot}} = \frac{V^2}{P}$

$$= \frac{200\times200}{100} = 400\,\Omega$$

$$R_{\text{cold}} = \frac{400}{10} = 40\,\Omega$$

23(c) Mangnetic needle is placed in non-uniform magnetic field. It experiences force and torque both due to unequal forces acting on poles.

AIEEE 2006

1. The kirchhoff's first law ($\sum i = 0$) and second law ($\sum iR = \sum E$), where the symbols have their usual meanings, are respectively based on:

(a) conservation of charge, conservation of momentum

(b) conservation of energy, conservation of charge

(c) conservation of mementum, conservation of charge

(d) conservation of charge, conservation of energy

2. Needles N_1, N_2 and N_3 are made of a ferromagnetic, a paramagnetic and a diamagnetic substance respectively. A magnet when brought close to them will:

(a) attract N_1 and N_2 strongly but repel N_3

(b) attract N_1 strongly, N_2 weakly and repel N_3 weakly

(c) attract N_1 strongly, but repel N_2 and N_3 weakly

(d) attract all three of them

3. A material 'B' has twice the specific resistance of 'A'. A circular wire made of 'B' has twice the diameter of a wire made of 'A'. Then for the two wires to have the same resistance, the ratio l_B/l_A of their respective lengths must be:

(a) 1 (b) $\frac{1}{2}$

(c) $\frac{1}{4}$ (d) 2

4. In a region, steady and uniform electric and magnetic fields are present. These two fields are parallel to each other. A charged particle is released from rest in this region. The path of the particle will be a:

(a) helix (b) straight line

(c) ellipse (d) circle

5. An electric dipole is placed at an angle of 30° to a non-uniform electric field. The dipole will experience:

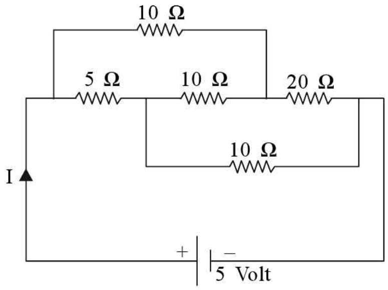
(a) a translational force only in the direction of the field

(b) a translational force only in a direction normal to the direction of the field

(c) a torque as well as a translational force

(d) a torque only

6. The current I drawn from the 5 volt source will be:



(a) $0.33A$ (b) $0.5A$
(c) $0.67A$ (d) $0.17A$

7. In a series resonant LCR circuit, the voltage across R is 100 volts and $R = 1k\Omega$ with $C = 2\mu F$. The resonant frequency ω is 200 rad/s. At resonance the voltage across L is:

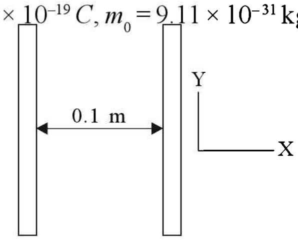
(a) 2.5×10^{-2} V (b) 40 V
(c) 250 V (d) 4×10^{-3} V

8. The resistance of a bulb filament is 100Ω at a temperature of 100 °C. If its temperature coefficient of resistance be 0.005 per °C, its resistance will become 200Ω at a temperature of:

(a) 300°C (b) 400°C
(c) 500°C (d) 200°C

9. Two insulating plates are both uniformly charged in such a way that the potential difference between them is $V_2 - V_1 = 20$ V. (*i.e.*, plate 2 is at a higher potential). The plates are separated by $d = 0.1$m and can be treated as infinitely large. An electron is released from rest on the inner surface of plate 1. What is its speed when it hits plate 2?

($e = 1.6 \times 10^{-19}\,C$, $m_0 = 9.11 \times 10^{-31}$ kg)



(a) 2.65×10^6 m/s (b) 7.02×10^{12} m/s
(c) 1.87×10^6 m/s (d) 32×10^{-19} m/s

10. In an AC generator, a coil with N turns, all of the same area A and total resistance R, rotates with frequency ω in a magnetic field B. The maximum value of emf generated in the coil is:

(a) $N.A.B.R.\omega$ (b) $N.A.B$
(c) $N.A.B.R$ (d) $N.A.B.\omega$

11. In a Wheatstone's bridge, three resistance P, Q and R are connected in the three arms and the fourth arm is formed by two resistances S_1 and S_2 connected in parallel. The condition for the bridge to be balanced will be:

(a) $\dfrac{P}{Q} = \dfrac{2R}{S_1 + S_2}$ (b) $\dfrac{P}{Q} = \dfrac{R(S_1 + S_2)}{S_1 S_2}$

(c) $\dfrac{P}{Q} = \dfrac{R(S_1 + S_2)}{2S_1 S_2}$ (d) $\dfrac{P}{Q} = \dfrac{R}{S_1 + S_2}$

12. The flux linked with a coil at any instant 't' is given by:

$$\phi = 10t^2 - 50t + 250$$

The induced emf at $t = 3s$ is:

(a) – 190 V (b) – 10 V
(c) 10 V (d) 190 V

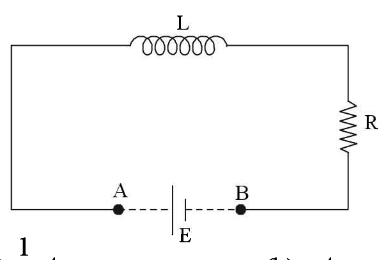
13. A long solenoid has 200 turns per cm and carries a current i. The magnetic field at its centre is 6.28×10^{-2} weber/m². Another long solenoid has 100 turns per cm and it carries a current $\dfrac{i}{3}$. The value of the magnetic field at its centre is:

(a) 1.05×10^{-2} weber/m² (b) 1.05×10^{-5} weber/m²
(c) 1.05×10^{-3} weber/m² (d) 1.05×10^{-4} weber/m²

14. An electric bulb is rated 220 volt – 100 watt. The power consumed by it when operated on 110 volt will be:

(a) 75 watt (b) 40 watt
(c) 25 watt (d) 50 watt

15. An inductor ($L = 100$ mH), a resistor ($R = 100\Omega$) and a battery ($E = 100$ V) are initially connected in series as shown in the figure. After a long time the battery is disconnected after short circuiting the points A and B. The current in the circuit 1 ms after the short circuit is:



(a) $\dfrac{1}{e}A$ (b) eA
(c) $0.1A$ (d) $1A$

Answers

1.	(d)	**2.**	(b)	**3.**	(d)	**4.**	(b)	**5.**	(c)	**6.**	(b)	**7.**	(c)
8.	(b)	**9.**	(a)	**10.**	(d)	**11.**	(b)	**12.**	(b)	**13.**	(a)	**14.**	(c)
15.	(a)												

Explanations

1(d)

2(b)

3(d) $R_A = R_B$

$$\frac{\rho_A \ell_A}{\pi r^2_A} = \frac{2\rho_A \ell_B}{\pi (2r_A)^2}$$

$$\therefore \quad \frac{l_B}{l_A} = \frac{2}{1}$$

4(b) $\because$ Fmag = 0

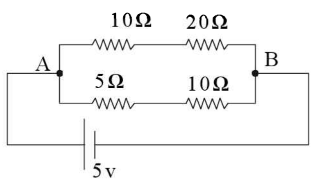
5(c) In a non-uniform electic field, the dipole may experience both non-zero torque as well as translational force.

For example as shown in figure,

$F_1 \neq F_2$ as $E_1 \neq E_2$

Torque would also be non-zero.

6(b) From equivalent circuit



$$I = \frac{V}{R} = \frac{5}{10} = 0.5\,A$$

7(c) At resonance, $\omega L = \dfrac{1}{\omega C}$

Current flowing through the circuit,

$$I = \frac{V_R}{R} = \frac{100}{1000} = 0.1\,A$$

So, voltage across L is given by

$V_L = IX_L = I\omega L$

but $\omega L = \dfrac{1}{\omega C}$

$$\therefore \quad V_L = \frac{1}{\omega C} = \frac{0.1}{200 \times 2 \times 10^{-6}} = 250\text{ V}$$

8(b) $R = R_0 = [1 + \alpha \Delta\theta]$

we have, $100 = R_0 [1 + 0.005 \times 100]$ ...(i)

and $200 = R_0 [1 + 0.005 \times \Delta T]$ ...(ii)

Dividing Eq. (ii) by Eq (i)

$$\frac{200}{100} = \frac{1 + 0.005\Delta T}{1 + 0.005 \times 100} \Rightarrow \Delta T = 400\,°\text{C}$$

9(a) $\dfrac{m_e v^2}{2} - 0 = e(V_2 - V_1)$

$$\therefore \quad \frac{9.11 \times 10^{-31}}{2} v^2 = 1.6 \times 10^{-19} \times 20$$

$$\Rightarrow \quad v = \sqrt{\frac{16 \times 10^{-19} \times 40}{9.11 \times 10^{-31}}} = 2.65 \times 10^6\text{m/s}$$

10(d) The emf generated would be maximum when flux (cutting) would be maximum *i.e.*, angle between area vector of coil and magnetic field is 0°. The emf generated is given by [as a function of time]

$e = NBA\omega \cos\omega t$

$\Rightarrow \quad e_{max} = NAB\omega$

11(b) For balanced Wheatstone's bridge

$$\frac{P}{Q} = \frac{R}{S} \text{ here } S = S_1 || S_2 = \frac{S_1 S_2}{S_1 + S_2}$$

$$\Rightarrow \quad \frac{P}{Q} = \frac{R(S_1 + S_2)}{S_1 S_2}$$

12(b) $\phi = 10t^2 - 50t + 250$

$$e = -\frac{d\phi}{dt}$$

$\therefore \quad e = -[10 \times 2t - 50]$

$\therefore \quad e|_{t=3s} = -[10 \times 6 - 50] = -10$ V

13(a) $\dfrac{B_1}{B_2} = \dfrac{\mu_0 n_1 i_1}{\mu_0 n_2 i_2}$

$$\frac{10^{-2} \times 6.28}{B_2} = \frac{200i}{100\frac{i}{3}}$$

$B_2 = 1.05 \times 10^{-2}$T

14(c) $\dfrac{100}{P_2} = \dfrac{\frac{220^2}{R}}{\frac{110^2}{R}}$

or $P_2 = 25$W

15(a) $I_0 = \dfrac{E}{R} = 1A$

$I = I_0 e^{-\frac{t}{\tau}}$ where $\tau = \dfrac{L}{R}$.

$I = 1 \times e^{-(1 \times 10-3)/(100 \times 10-3/100)}$

$= \left(\dfrac{1}{e}\right)A$

AIIMS 2005

1. A conducting ring of radius 1 meter is placed in an uniform magnetic field B of 0.01 tesla oscillating with frequency 100 Hz with its plane at right angle to B. What will be the induced electric field?

   (a) π volts/m (b) 2 volts/m
   (c) 10 volts/m (d) 62 volts/m.

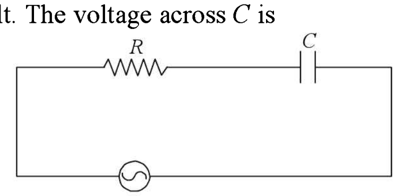
2. Two infinitely long parallel conducting plates having surface charge densities $+\sigma$ and $-\sigma$ respectively, are separated by a small distance. The medium between the plates is vacuum. If ε_0 is the dielectric permittivity of vacuum, then the electric field in the region between the plates is

   (a) 0 volt/meter (b) $\frac{\sigma}{2\varepsilon_0}$ volt/meter
   (c) $\frac{\sigma}{\varepsilon_0}$ volt/meter (d) $\frac{2\sigma}{\varepsilon_0}$ volt/meter

3. The magnetic moment (μ) of a revolving electron around the nucleus varies with principal quantum number n as

   (a) $\mu \propto n$ (b) $\mu \propto \frac{1}{n}$
   (c) $\mu \propto n^2$ (d) $\mu \propto \frac{1}{n^2}$.

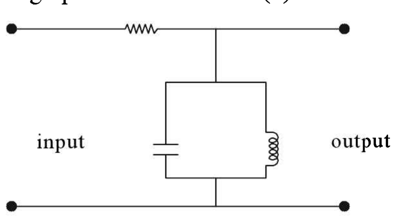
4. A 50 Hz a.c. source of 20 volts is connected across R and C as shown in figure. The voltage across R is 12 volt. The voltage across C is



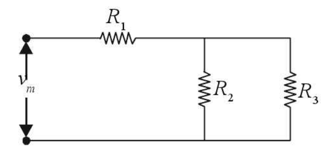
   (a) 8 V (b) 16 V
   (c) 10 V
   (d) not possible to determine unless values of R and C are given.

5. The circuit shown below acts as

   (a) tuned filter (b) low pass filter
   (c) high pass filter (d) rectifier.

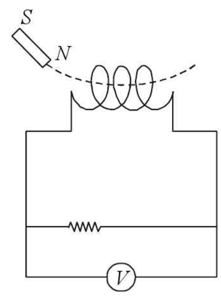


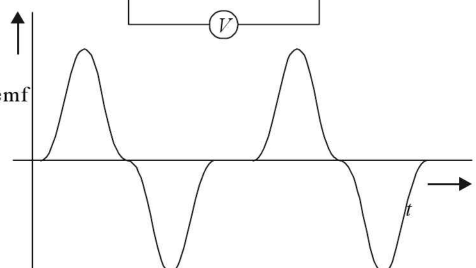
6. For ensuring dissipation of same energy in all three resistors (R_1, R_2, R_3) connected as shown in figure, their values must be related as.

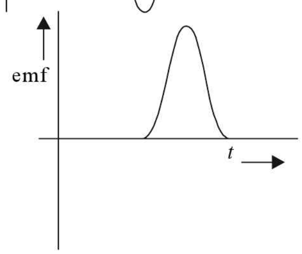


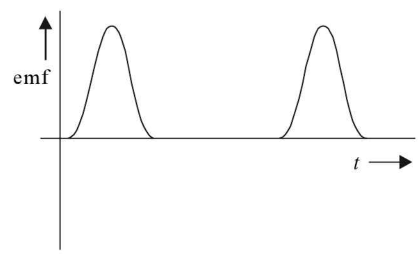
   (a) $R_1 = R_2 = R_3$ (b) $R_2 = R_3$ and $R_1 = 4R_2$
   (c) $R_2 = R_3$ and $R_1 = \frac{1}{4}R_2$ (d) $R_1 = R_2 + R_3$.

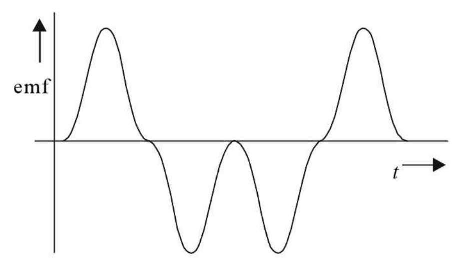
7. A magnet is made to oscillate with a particular frequency, passing through a coil as shown in figure. The time variation of the magnitude of e.m.f. generated across the coil during one cycle is



(a)


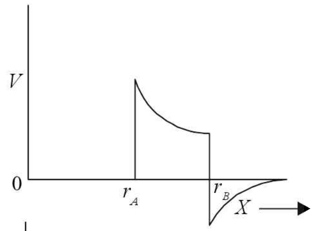
(b)


(c)


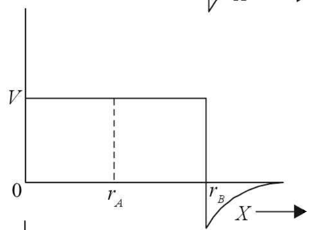
(d)


8. Two concentric conducting thin spherical shells A and B having radii r_A and r_B ($r_B > r_A$) are charged to Q_A and $-Q_B$ ($|Q_B| > |Q_A|$). The electrical field along a line, (passing through the centre is)

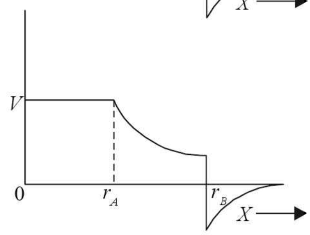
(a)



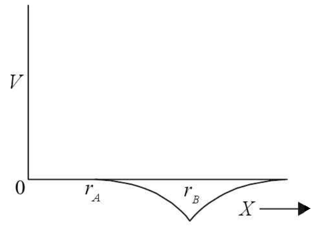
(b)



(c)



(d)



9. Four point +ve charges of same magnitude (Q) are placed at four corners of a rigid square frame as shown in figure. The plane of the frame is perpendicular to Z-axis. If a –ve point charge is placed at a distance z away from the above frame ($z << L$) then

(a) –ve charge oscillates along the Z-axis

(b) it moves away from the frame

(c) it moves slowly towards the frame and stays in the plane of the frame

(d) it passes through the frame only once.

Answers

1.	(b)	**2.**	(c)	**3.**	(a)	**4.**	(b)	**5.**	(a)	**6.**	(c)	**7.**	(a)
8.	(c)	**9.**	(a)										

Explanations

1(b) As a constant magnetic field conducting ring oscillates with a frequency of 100 Hz.

i.e. $T = \frac{1}{100}s$, for $\frac{T}{2}$, it goes up to B

∴ The corresponding frequency is 200 Hz.

$$\text{Induced emf} = \frac{\text{change in flux}}{\text{time}}$$

$$= \frac{2BA\cos\theta}{T} = 2BAf\cos\theta$$

$= \pi r^2 \times 2 \times 0.01 \times 200$ as $r = 1\ m^{-B}$

$= \pi r^2 \times 2 \times 0.01 \times 200 = 4\pi V.$

Non-electrostatic induced electric field along the circle,

$$E = \frac{1}{2\pi r} \times \left(\pi r^2 \frac{dB}{dt}\right) = \frac{e}{2\pi r} = \frac{4\pi}{2\pi r} = 2\frac{V}{m}.$$

2(c) Electric field between the plates is

$$= \frac{\sigma}{2\varepsilon_0} - \frac{(-\sigma)}{2\varepsilon_0}$$

$$= \frac{\sigma}{\varepsilon_0} \text{ volt/meter.}$$

3(a) $\vec{L} = n.\frac{h}{2\pi}$

$\mu_n = \frac{e}{2m}.n\frac{h}{2\pi}$ where $\frac{e\hbar}{2m}$ is the Bohr magnetism, the unit of magnetism.

4(b) $\omega = 50$ Hz, $V = 20\ V$, $V_R = 12\ V$

In case of RC circuit, $V = \sqrt{V_R^2 + V_C^2}$

or $(20)^2 = (12)^2 + (V_C)^2$

or $(V_C)^2 = 400 - 144 = 256 \Rightarrow V_C = 16$ V.

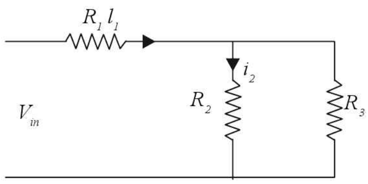
5(a)

6(c) As the voltage in R_2 and R_3 is same therefore, according to,

$$H = \frac{V^2}{R} \cdot t, R_2 = R_3$$

Also the energy in all resistance is same.

$\therefore \quad i_2^2 R_1 t = i_1^2 R_2 t$



$\because \quad i_2 = \frac{i_1}{2}$

$\therefore \quad R_2 = 4R_1$

Thus, $i_2 R_1 t = \frac{i^2}{4} R_2 t$ or, $R_1 = \frac{R_2}{4}$.

7(a) As the north pole approaches, a north pole is developed at that face i.e. the current flows anticlockwise. Finally when it completes the oscillation, when it is ready, no emf is present. Now south pole approaches the other side R.H.S. the current flows clockwise to repel the south pole. this means the current is anticlockwise at the L.H.S. as before. This is possible in (a). The breaks show when the pendulum is at the extreme and lmomentarily stationary.

8(c) $|Q_B| > |Q_A|$ (given). Inside the shell B, the potential is a constant and negative.

But V_A inside r_A is positive and greater than that of B because $r_A < r_B$. Upto r_B, one as hence positive potential. Beyond r_A, $V_A \propto \left(\frac{1}{r}\right)$, this is $> V_B$, one gets the course $\propto \frac{1}{r}$. At B, the negative potential is more therefore the potential line is still negative. (c) explains all these factors.

Note: *However the question asked is the fields and the diagrams given are for the potential. This is an error.*

9(a) The cos component of the force i.e.

$F_z \propto -z$

$\therefore$ particle executes SHM

BHU 2005

1. Two similar charges having mass m and $2m$ are placed in an electric field. The ratio of their kinetic energy is

(a) 4 : 1 (b) 1 : 1

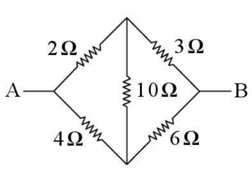
(c) 2 : 1 (d) 1 : 2

2. A charge Q is distributed uniformly in a sphere (solid). Then the electric field at any point r where $r < R$ (r is the radius of sphere) varies as

(a) $r^{\frac{1}{2}}$ (b) r^{-1}

(c) r (d) r^{-2}.

3. In the given circuit, what will be the equivalent resistance between the point A and B.



(a) $\frac{10}{3}\Omega$ (b) $\frac{20}{3}\Omega$

(c) $\frac{10}{5}\Omega$ (d) 5 Ω

4. If a charge particle enters perpendicularly in the uniform magnetic field then

(a) energy and momentum both remains constant

(b) energy remains constant but momentum changes

(c) both energy and momentum changes

(d) energy changes but momentum remains constant.

5. A uniform wire of length l and having resistance R is cut into n equal parts and all parts are connected in parallel then the equivalent resistance will be

(a) R (b) $\frac{R}{n}$

(c) $\frac{R}{n^2}$ (d) n^2R

6. In an LCR circuit, if the capacitance is made one-fourth, then what should be change in inductance, so that the circuit remains in resonance.

(a) 8 times (b) $\frac{1}{4}$ times

(c) 2 times (d) 4 times

7. Two bulbs rated 200 W, 220 V and 100 W, 220 V are connected in series. Combination is connected to 220 V supply. Power consumed by the circuit is?

(a) 80 W (b) 67 W

(c) 76 W (d) 65 W.

8. The turn ratio of a transformer is given as 2 : 3. If the current through the primary coil is 3 A, thus calculate the current through load resistance.

(a) 1 A (b) 4.5 A

(c) 2 A (d) 1.5 A

9. Two inductors each of inductance L are joined in parallel. What is their equivalent inductance?

(a) zero (b) 2 L

(c) $\frac{L}{2}$ (d) L.

Answers

1.	(c)	**2.**	(c)	**3.**	(a)	**4.**	(b)	**5.**	(c)	**6.**	(d)	**7.**	(b)
8.	(c)	**9.**	(c)										

Explanations

1(c) Force due to electric field on the particle of mass m is given by

$$F = qE \text{ or, } ma = qE \Rightarrow a = \frac{qE}{m}$$

$\therefore$ Velocity $(v) = u + at$

$$= \frac{qE}{m} t$$

$$\frac{K.E._1}{K.E._2} = \frac{\frac{1}{2} m_1 \left(\frac{q_1 E}{m_1}\right)^2}{\frac{1}{2} m^2 \left(\frac{q_2 E}{m_2}\right)^2} = \frac{m_2}{m_1}$$

$$= \frac{2m}{m} = 2:1$$

2(c) $E_{in} = \frac{Qr}{4\pi \in_0 R^3}$

3(a) This is an example of balanced Wheatstone bridge.

Thus, $R_{resultant} = \frac{(2+3)\times(4+6)}{2+3+4+6}$

$$= \frac{5\times10}{15} = \frac{10}{3}\Omega$$

4(b) When α +ve charge enters uniform magnetic field B acting in – OZ direction (– $\hat{k}$), the charge makes a circular trajectory.

$$qvB = \frac{mv^2}{R}$$

Speed V does not change but direction changes continuously

$\therefore$ momentum changes.

5(c) After cutting a wire in n equal parts, the resistance of single part becomes $\frac{R}{n}$.

When n wire of resistance $\frac{R}{n}$ are connected in parallel, then resultant resistance becomes $\frac{\frac{R}{n}}{n}$.

$$R_{resultant} = \frac{R}{n^2}.$$

6(d) Resonance of circuit = $\frac{1}{2\pi\sqrt{LC}}$

$$\therefore \frac{f'}{f} = \sqrt{\frac{LC}{L'C'}} \text{ or,}$$

$$1 = \sqrt{\frac{L}{L'}\frac{4C}{4C}} \left[\because C = \frac{1}{4}C\right]$$

$4LC = L'C$. $\therefore L' = 4L$.

7(b) $\frac{1}{P} = \frac{1}{P_1} + \frac{1}{P_2} = \frac{1}{200} + \frac{1}{300}$.

$\therefore P = 67\,W$

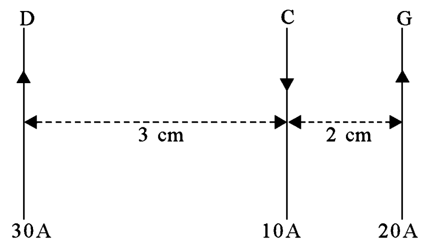
8(c) As $\frac{I_P}{I_S} = \frac{n_P}{n_S}$

i.e. $\frac{3}{I_S} = \frac{3}{2} \Rightarrow I_S = 2A$.

9. (c): $\frac{1}{L'} = \frac{1}{L} + \frac{1}{L} \Rightarrow L' = \frac{L}{2}$.

KARNATAKA CET 2005

1. Three long, straight parallel wires, carrying current, are arranged as shown in figure. The force experienced by a 25 cm length of wire C is



(a) 10^{-3} (b) 2.5×10^{-3}
(c) zero (d) 1.5×10^{-3}

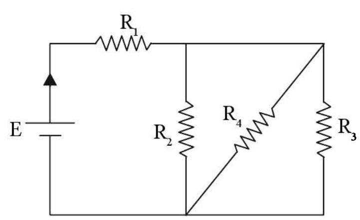
2. A 5.0 amp current is setup in an external circuit by a 6.0 volt storage battery for 6.0 minutes. The chemical energy of the battery is reduced by

(a) 1.08×10^4 J (b) 1.08×10^4 volt
(c) 1.8×10^4 J (d) 1.8×10^4 volt

3. The current in a simple series circuit is 5.0 amp. When an additional resistance of 2.0 ohms is inserted, the current drops to 4.0 amp. The original resistance of the circuit in ohms was

(a) 1.25 (b) 8.
(c) 10. (d) 20.

4. In the circuit given $E = 6.0\ V$, $R_1 = 100$ ohms, $R_2 = 50\ \Omega$ $R_3 = 50$ ohms, $R_4 = 75$ ohms. The equivalent resistance of the circuit, in ohms, is

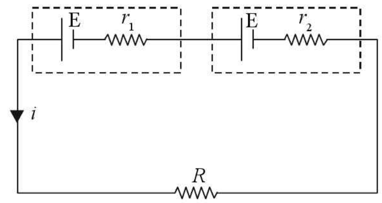


(a) 11.875 (b) 26.31
(c) 118.75 (d) none of these

5. Two resistances are connected in two gaps of a metre bridge. The balance point is 20 cm from the zero end. A resistance of 15 ohms is connected in series with the samller of the two. The null point shifts to 40 cm. The value of the smaller resistance in ohms is

(a) 3 (b) 6
(c) 9 (d) 12

6. If the potential difference across the internal resistance r_1 is equal to the *emf* E of the battery, then



(a) $R = r_1 + r_2$ (b) $R = r_1 + r_2$
(c) $R = r_1 - r_2$ (d) $R = r_2 / r_1$

7. By using only two resistance coils-singly, in series, or in parallel one should be able to obtain resistances of 3, 4, 12 and 16 ohms. The separate resistances of the coil are

(a) 3 and 4 (b) 4 and 12
(c) 12 and 16 (d) 16 and 3

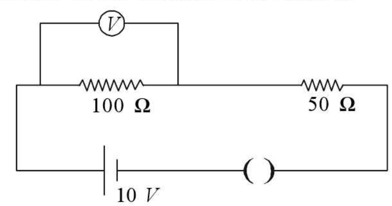
8. The electrons in the beam of a television tube move horizontally from south to north. The vertical component of the earth's magnetic field points down. The electron is deflected towards

(a) west (b) no deflection
(c) east (d) north to south

9. A tangent galvanometer has a reduction factor of 1A and it is placed with the plane if its coil perpendicular to the magnetic meridian. The deflection produced when a current of 1 A is passed through it is

(a) 60° (b) 45°
(c) 30° (d) none of these

10. In the given circuit, the voltmeter records 5 volts. The resistance of the voltmeter in ohms is



(a) 200 (b) 100
(c) 10 (d) 50

11. An electric field of 1500 V/m and a magnetic field of 0.40 weber/meter^2 act on a moving electron. The minimum uniform speed along a straight line the electron could have is

(a) 1.6×10^{15} m/s (b) 6×10^{-16} m/s
(c) 3.75×10^3 m/s (d) 3.75×10^2 m/s

12. In an ammeter 10% of main current is passing through the galvanometer. If the resistance of the galvanometer is G, than the shunt resistance, in ohms is

(a) $9G$ (b) $G/9$
(c) $90G$ (d) $G/90$

13. Among the following properties describing diamagnetism, which of the property is wrongly stated.

(a) diamagnetic material do not have permanent magnetic moment

(b) diamagnetism is explained in terms of diamagnetism induction

(c) diamagnetic materials have a small positive susceptibility

(d) the magnetic moment of individual electrons neutralize each other.

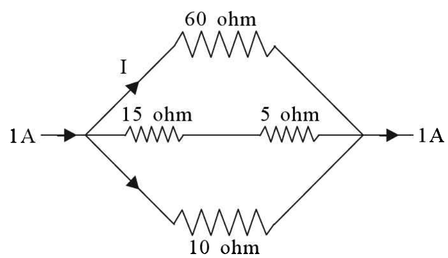
14. The induction coil works on the principle of

(a) self-induction (b) mutual induction

(c) Ampere's rule

(d) Fleming's right hand rule.

15. The magnitude of I in ampere unit is



(a) 0.1 (b) 0.3

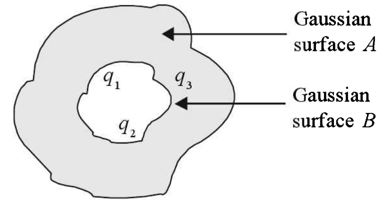
(c) 0.6 (d) none of these

16. Electron of mass m and charge q is travelling with a speed v along a circular path of radius r at right angles to a uniform magnetic field of intensity B. If the speed of the electron is doubled and the magnetic field is halved the resulting path would have a radius

(a) $2r$ (b) $4r$

(c) $r/4$ (d) $r/2$

17. The electric flux for Gaussian surface A that enclose the charged particles in free space is (given $q_1 = -14n$ C, $q_2 = 78.85\ nC$, $q_3 = -56\ nC$



(a) $10^3\ Nm^2C^{-1}$ (b) $10^3\ CN^{-1}\ m^{-2}$

(c) $6.32 \times 10^3\ Nm^2C^{-1}$ (d) $6.32 \times 10^3\ CN^{-1}\ m^{-2}$

18. Four metal conductors having different shapes

1. a sphere 2. cylindrical
3. pear 4. lightning conductor

are mounted on insulating stands and charged. The one which is best suited to retain the charges for a longer time is

(a) 1 (b) 2

(c) 3 (d) 4

19. The potential on a conductor depends on

(a) the amount of charge

(b) geometry and size of the conductor

(c) both (a) and (b)

(d) only on (a)

20. The work done in carrying a charge q once round a circle of radius r with charge Q at the centre is

(a) $\dfrac{qQ}{4\pi\varepsilon_0 r}$ (b) $\dfrac{qQ}{4\pi\varepsilon_0^2 r^3}$

(c) $\dfrac{qQ}{4\pi\varepsilon_0 r^3}$ (d) none of these

21. An air filled parallel plate condenser has a capacity of 2 pF. The separation of the plates is doubled and the interspace between the plates is filled with wax. If the capacity is increased to 6 pF, the dielectric constant of wax is

(a) 2 (b) 3

(c) 4 (d) 6

Answers

1.	(c)	**2.**	(a)	**3.**	(b)	**4.**	(c)	**5.**	(c)	**6.**	(c)	**7.**	(b)
8.	(c)	**9.**	(b)	**10.**	(b)	**11.**	(c)	**12.**	(b)	**13.**	(c)	**14.**	(b)
15.	(a)	**16.**	(b)	**17.**	(a)	**18.**	(a)	**19.**	(c)	**20.**	(d)	**21.**	(b)

Explanations

1(c) The magnetic field due to wire D at wire C is

$$B_D = \left(\frac{\mu_0}{4\pi}\right)\frac{21}{r}$$

$$= \frac{10^{-7} \times 2 \times 30}{0.03} = 2 \times 10^{-4}\ T.$$

which is directed into the page.

The magnetic field due to wire G at C is

$$B_G = \frac{10^{-2} \times 2 \times 30}{0.02} = 2 \times 10^{-4}\ T$$

which is directed out of the page.

Therefore, the field at the position of the wire C is

$B = B_D - B_G = 2 \times 10^{-4} - 2 \times 10^{4} = \text{zero}$.

Therefore on 25 cm of wire C is

$F = BIl \sin\theta = \text{zero}$.

2(a) The chemical energy reduced in battery

$= Vit = 6 \times 5 \times 6 \times 60\ \text{J} = 1.08 \times 10^4\ \text{J}$.

3(b) Let initially resistance in the circuit is $R\Omega$,

Thus, $V = I_1 R = 5R$ ...(i)

When addition resistance of 2.0 Ω is added in the circuit then

$V = 4(R+2)$ ....(ii)

As voltage across the resistance is same in both cases,

$\therefore\ 5R = 4R + 8 \Rightarrow R = 8\Omega$

4(c) In given circuit three resistances R_2, R_4 and R_3 are parallel.

$$\frac{1}{R} = \frac{1}{R_2} + \frac{1}{R_4} + \frac{1}{R_3}$$

$$= \frac{1}{50} + \frac{1}{50} + \frac{1}{75}$$

$$= \frac{75+75+50}{50 \times 75}$$

$$R = \frac{50 \times 75}{75+75+50} = \frac{50 \times 75}{200} = \frac{75}{4}\Omega = 18.75\Omega$$

This resistance is in series with R.

$\therefore\ R_{resultant} = R_1 + R = 100 + 18.75 = 118.75\Omega$

5(c) Let S be larger and R be smaller resistance connected in two gaps of meter bridge.

$$\therefore\ S = \left(\frac{100-l}{l}\right)R = \frac{100-20}{20}R = 4R \quad ...(i)$$

When 15 Ω resisitance is added to resistance R, then

$4R = \frac{6}{4}(R + 15)$ [using (i)]

or $16R = 6R + 90 \Rightarrow 10R = 90$

or $R = 9\,\Omega$.

6(c) From circuit, $V_1 + V_2 = iR$

$E - ir_1 + E - ir_2 = iR$. Also $ir_1 = E$ (given)

$\therefore\ ir_1 - ir_1 + ir_1 - ir_2 = iR \Rightarrow R = r_1 - r_2$.

Alternative: $i = \dfrac{2E}{r_1 + r_2 + R} = \dfrac{E}{r_1}$

As $ir_1 = E$ (given), $i = E/r_1$

$\Rightarrow\ 2r_1 = r_1 + r_2 + R \Rightarrow R = r_1 - r_2$.

7(b) When resistances 4 Ω and 12 Ω are connected in series $= 4 + 12 = 16\,\Omega$.

When these resistances are connected in parallel.

$$\frac{1}{R_P} = \frac{1}{4} + \frac{1}{12} \Rightarrow R_P = \frac{4 \times 12}{4+12} = \frac{4 \times 12}{16} = 3\Omega.$$

8(c) From Fleming's left hand rule the force n electron is towards the east means it is deflected towards east.

9(b) In tangent galvanometer, $I = K\tan\theta$ where K is reduction factor.

$I = 1A, K = 1A$.

$\therefore\ \tan\theta = 1 \Rightarrow \theta = 45°$.

10(b) $\dfrac{R_v 100^2}{R_v + 100} = 50$

$= R_v = R_v + 100$

or $R_v = 100\Omega$

11(c) $v = \dfrac{E}{B}$

$= \dfrac{1500}{.4}$

$= 3.75 \times 10^3\ ms^{-1}$

12(b) $S = \dfrac{I_g G}{I - I_g}$

As $I_g = \dfrac{1}{10}I$ (given)

$\backslash\ S = \dfrac{I}{10} \times \dfrac{G}{I - (I/10)} = \dfrac{G}{9}$.

13(c) Susceptibility of diamagnetic subtance is negative and it does not change with temperature.

14(b) Induction coil is a device used to produce high potential difference using a source of low potential difference. It is based on the principle of mutual induction.

15(a) $\dfrac{1}{R} = \dfrac{1}{10} + \dfrac{1}{20} + \dfrac{1}{60}$

$R = \dfrac{60}{6+3+1} = 6\Omega$

$v = 6v$

In parallel voltage remains same

$I = \dfrac{6}{60} = .1\text{A}$

16(b) In circular path, centripetal force of electron is equal to the force due to magnetic field.

$\therefore \quad \frac{mv^2}{r} = qvB \sin 90° \Rightarrow r = \frac{mv}{qB}$.

If the speed of the electron is doubled and the magnetic field is halved

then, $r' = \frac{m2v}{q(B/2)} = 2 \times 2 \times \frac{mv}{qB} = 4r$.

17(a) Flux is due to charges enclosed per ε_0

\ Total flux $= (-14 + 78.85 - 56)nC/\varepsilon_0$

$= \frac{8.85 \times 10^{-9}}{8.85 \times 10^{-12}} = 10^3$

18(a) Charge leaks from pointed ends.

19(c) $V = Q/C$

Q = the amount of charge

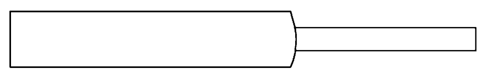
C = capacitance which depends on geometry and size of conductor.

20(d) The work done in carrying a charge round a closed path is zero, because the path is equipotential.

21(b) $\frac{C'}{C} = K = 3$

IIT-JEE 2006

1. A current I flows through the composite wire ABC, shown in the figure. The radius of cross-section of the portion AB of the wire is $2\,r$, while that portion BC is r. Then.



(a) current densities in AB and BC are the same

(b) potential difference across AB is twice the potential difference across BC

(c) heat dissipated in BC is 4 times that in AB

(d) heat dissipated in AB is 4 times that in BC

2. The values of time constants for the given circuits are

$(R_1 = 1\Omega, R_2 = 2\Omega\, C_1 = 2\mu F, C_2 = 4\mu F)$

(a) $4\,\mu, 18\mu s, \frac{8}{9}\mu s$ (b) $18\mu s, \frac{8}{9}\mu s, 4\mu s$

(c) $\frac{8}{4}\mu s, 4\mu s, 18\mu s$ (d) $4\mu s, \frac{8}{4}\mu s\ 18\mu s$

Answers

1. (c) **2.** (b)

Explanations

1(c). $R = \rho\frac{\ell}{A} = \rho\frac{\ell}{\pi r^2}$

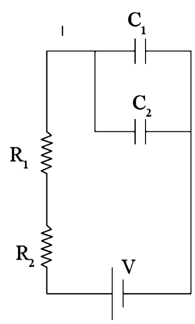
$R_{AB} : R_{BC} = \frac{1}{4} : 1 = 1 : 4$

$V_{AB} : V_{BC} = IR_{AB} : IR_{BC} = 1 : 4$

$P_{AB} : P_{BC} = I^2 R_{AB} : I^2 R_{AB}$

$= 1 : 4$

2.(b) $t_{eq} = R_{eq} . C_{eq}$

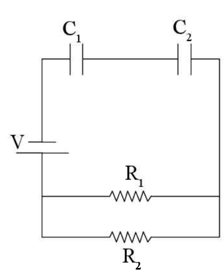


(i)

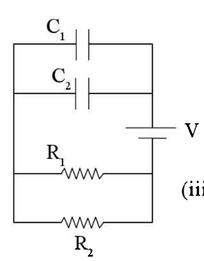
$\tau_1 = (2+1)(2+4) = 18\,\mu s$

$\tau_2 = \frac{2 \times 1}{2+1} \cdot \frac{2 \times 4}{2+4} = \frac{8}{9}\mu s$

$\tau_3 = \frac{2 \times 1}{2+1} \cdot (4+2) = 4\,\mu s$



(ii)



(iii)

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5 Optics

29

Ray Optics and Optical Instruments

BRIEF REVIEW

Reflection Rebounding of light from a polished surface like a mirror is called reflection.

Laws of Reflection

(*a*) Angle of incidence = angle of reflection

(*b*) Incident ray, normal and reflected ray are coplanar.

If mirror is rotated by θ, reflected ray moves by 2θ.

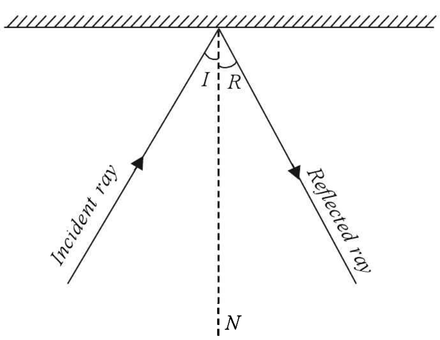


Fig. 29.1 Reflection from a polished surface

Diffusion Reflection from a rough surface (such as a wall) is called diffusion. A parallel beam will not emerge out parallel after suffering reflection because it meets different angles at the reflecting surface (see Fig. 29.2).

Characteristics of image formed with a plane mirror

(a) It is erect (b) It is virtual

(c) Size of image = Size of object

(d) Image distance = Object distance (measured from mirror).

(e) Lateral inversion (left appears right and right appears left).

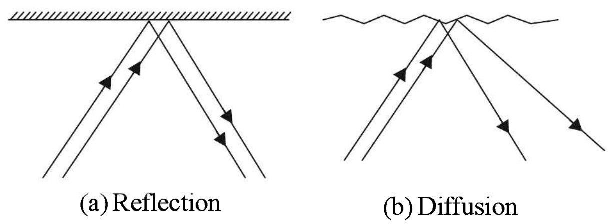


Fig. 29.2 Illustration of diffusion.

Number of Images If two mirrors are inclined at an angle θ the number of images formed for an object placed in front of them is given by

(a) number of images $n = \frac{360}{\theta}$ if $\frac{360}{\theta}$ is odd and object does not lie on angle bisector or is placed symmetrically.

$n = \frac{360}{\theta} - 1$ if $\frac{360}{\theta}$ is odd and object is placed on angle bisector.

(b) Number of images $n = \frac{360}{\theta} - 1$ if $\frac{360}{\theta}$ is even (object placed non-symmetric).

$n = \frac{360}{\theta}$ if $\frac{360}{\theta}$ is even (object placed symmetrically).

If two mirrors are parallel ($\theta = 0$) $n = \infty$.

A, H, I, M, O, ... U, V, X, Y etc. 11 letters show lateral symmetry.

If mirror is thick, second image (formed due to first reflection from polished surface) is the brightest.

When a ray is reflected from a plane mirror, angle of deviation $\delta = \pi - 2\theta$ as shown in Fig. 29.3.

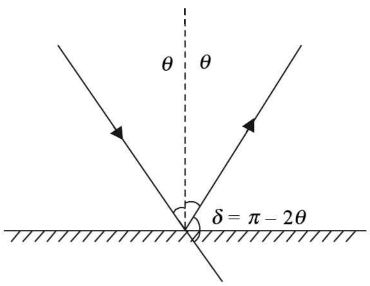


Fig. 29.3 Finding angle of deviation

Minimum height of a mirror so that a person can see his full image in the mirror is half the height of the mirror when standing at a distance = half the height away from the mirror.

Spherical mirrors are of two types: Convex and concave as shown in Fig. 29.4.

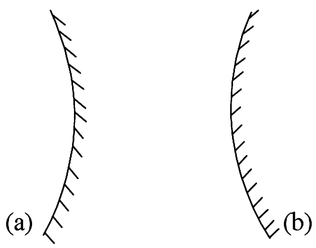


Fig. 29.4 (a) Concave and (b) Convex mirror

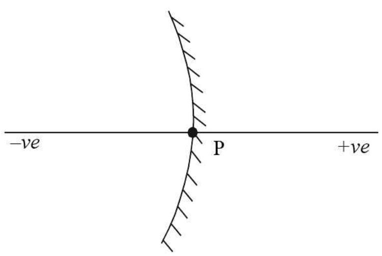


Fig. 29.5 Illustration of sign convention

Sign convention Consider pole P as origin. All distances to its left are negative and all distances to its right are positive.

Mirror formulae $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$ and $f = \frac{R}{2}$ where

v = image distance from pole to mirror

u = object distance from pole to mirror

f = focal length

R = radius of curvature.

Table 29.1

	Real Image	Virtual Image
1.	Rays actually converge to form image.	Rays appear to diverge from image.
2.	Image can be obtained on screen.	Image cannot be taken on screen.
3.	Image is inverted	Image is erect.
4.	Magnification is negative.	Magnification is positive.

Magnification M_{lat} (lateral) or linear magnification

$$M_{lat} = \frac{I}{O} = \frac{-v}{u} = \frac{v-f}{f} = \frac{f}{u-f}$$ See Fig. 29.6 (a)

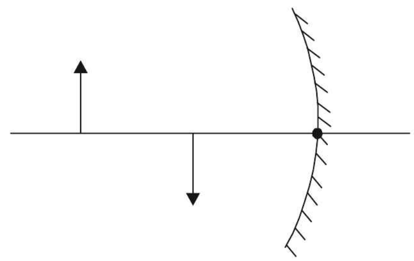


Fig. 29.6 (a) Lateral magnification

Magnification (axial) $M_{axial} = \frac{-v^2}{u^2}$ (used for small objects only). See Fig. 29.6 (b)

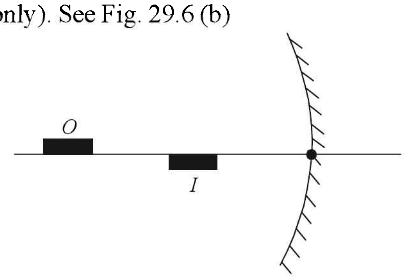


Fig. 29.6 (b) Axial magnification

Lens The part of an isotropic transparent medium bounded by at least one curved surface. Lenses are of two types. (a) convex (b) concave

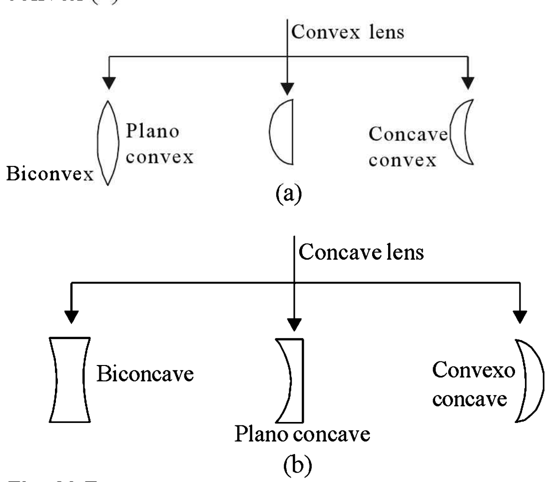


Fig. 29.7

Table 29.2 Image formation information for convex lens and concave mirrors

Position of object	Position of image and its nature
At ∞	At focus (real, inverted, diminished).
Away from $2f$	Between f and $2f$ (real, inverted and diminished).
At $2f$	At $2f$ (real, inverted and equal in size)
Between f and $2f$	Away from $2f$, (real, inverted and magnified).
At f	At ∞ (real, inverted and magnified)
Between pole and f	Behind the mirror (virtual, erect and magnified)
	In front of lens, i.e., on the same side of object.

Remember spherical mirrors have one principal focus while lenses have two principal focus one on each side as shown in Fig. 29.8.

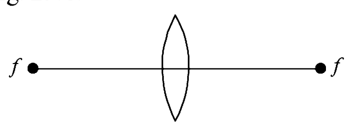


Fig. 29.8 Illustration of principal foci in a lens

Lens formulae for thin lenses

$$\frac{1}{f} = (\mu_2 - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right] \text{ (lens maker's formula) when}$$

surrounding medium is air or vacuum.

$$\frac{1}{f} = \left(\frac{\mu_2}{\mu_m} - 1\right)\left[\frac{1}{R_1} - \frac{1}{R_2}\right] \text{ if surrounding medium has}$$

refractive index μ_m.

Lens formula $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$ (in air or vacuum)

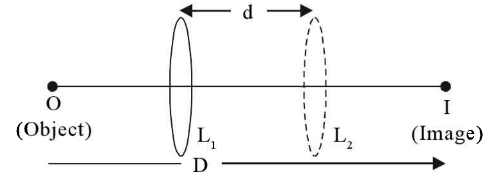


Fig. 29.9 Displacement method to find focal length of a convex lens.

Displacement method If object and image (or screen) are fixed at a distance D (> $4f$) [Fig. 29.9]. Lens is set at L_1 to form a magnified sharp image at I_0. Then lens is displaced by d again to form a sharp image at I (diminished), Then

$f = \frac{D^2 - d^2}{4d}$ and $O = \sqrt{I_1 I_2}$ where I_1 and I_2 are sizes of image in magnified and diminished position of lens L_1 and L_2 respectively O is size of object.

Lateral magnification

$$M_{\text{lateral}} = \frac{v}{u} = \frac{I}{O} \text{ for a convex lens.}$$

$$M_{\text{lateral}} = \frac{-v}{u} = \frac{I}{O} \text{ for a concave lens.}$$

$$M_{\text{lateral}} = \frac{f}{u+f} = \frac{f-v}{f}$$

Axial magnification

$$M_{\text{axial}} = \frac{-v^2}{u^2} \text{ (for small objects)}$$

If object and image are formed in different media then use

$$\frac{\mu_3}{f} = \frac{\mu_2 - \mu_1}{R_1} - \frac{\mu_2 - \mu_3}{R_2} \text{ to find focal length}$$

$$\frac{\mu_3}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R_1} - \frac{\mu_2 - \mu_3}{R_2} \text{ to find } v \text{ or } u$$

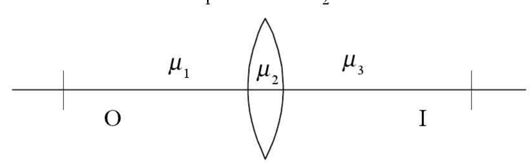


Fig. 29.10 Image formation when lens lies in two different media

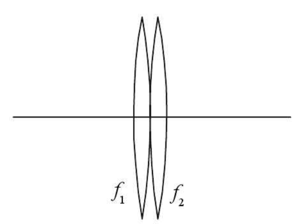


Fig. 29.11 Combination of two thin lenses

If two thin lenses are in contact as shown in Fig. 29.11

then $\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$

Newton's formula $x_1 x_2 = f_2$ [Fig. 29.12]

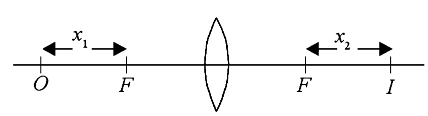


Fig. 29.12 Focal length using Newton's formula

If focal length on two sides is not equal then $f_1 f_2 = x_1 x_2$ (in case O and I are in different mediums)

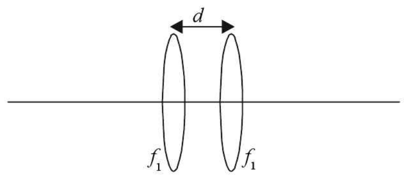


Fig. 29.13 Combination of lenses when at a distance d apart.

If two lenses are distance d apart as shown in Fig. 29.13 then their combined focal length $\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$

Focal length of a thick lens of thickness t

$$\frac{1}{f} = (\mu_2 - 1)\left[\frac{1}{R_1} - \frac{1}{R_2} - \frac{t(\mu_2 - 1)}{\mu_2 R_1 R_2}\right]$$

Cardinal points There are three sets of cardinal points.

(a) set of focal points

(b) principal points $x_1 x_2 = f_2$ (if lens is in air)

(c) nodal points.

Flare spots If strong light is used, more than one refraction occurs in a lens and hence more than one image is formed called flare spots. For nth flare spot

$$\frac{1}{f_n} = \frac{(n+1)\mu - 1}{f(\mu - 1)}$$

Power of the lens $P = \frac{1}{f(m)} = \frac{100}{f(cm)}$ The unit is dioptre (D).

Defects in lenses

(a) Spherical aberration (or monochromatic aberration): occurs as paraxial and marginal rays fail to meet at a point as illustrated in Fig. 29.14. Spherical aberration can be removed using optical stops or aplanatic lens. Astigmatism is cured by cylindrical lens.

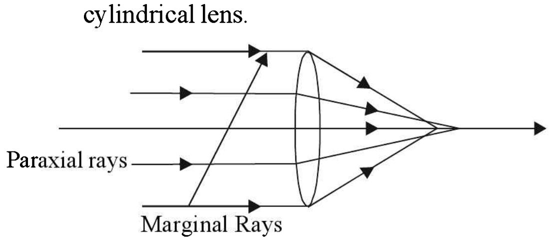


Fig. 29.14 Spherical aberration illustration

(b) Chromatic aberration: A white object when seen through a lens appears coloured. Such a defect is called chromatic aberration. Its removal is called achromatism. For achromatic aberration, a combination of a convex and a concave lens is needed such that $\frac{\omega_1}{f_1} + \frac{\omega_2}{f_2} = 0$ where ω_1 and ω_2 are dispersive powers of two lenses of focal length f_1 and f_2 respectively. Their combined focal length is

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$$. See Fig. 29.15 (a).

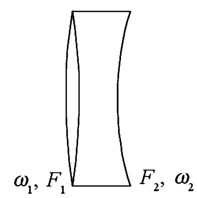


Fig. 29.15 (a) Achromat combination

Achromatic aberration can also be removed using two lenses of same kind separated by a small distance if

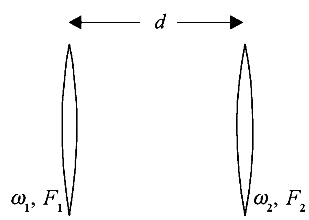


Fig. 29.15 (b) Achromatism using two convex lens

$$d = \frac{\omega_1 f_2 + \omega_2 f_1}{\omega_1 + \omega_2}$$ as illustrated in Fig. 29.15 (b).

Note: If $\omega_1 = \omega_2$ then $d = \frac{f_1 + f_2}{2}$.

If $d = f_1 - f_2$, spherical aberration is also removed.

Thus if $f_1 = 3f_2$ and $d = 2f_2$ then both the defects can be removed simultaneously. This approach is employed in Huygen's eye piece.

***Refraction** When **an oblique ray** of light enters from one medium to another (**optically different** or **dispersive medium**) then it changes its path. Such a phenomenon is called refraction. (See Fig. 29.16).

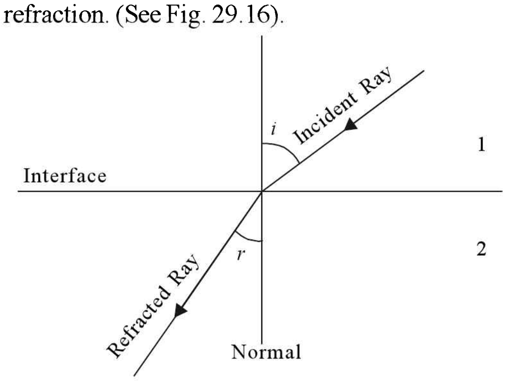


Fig. 29.16 Refraction in a dispersive medium

Note: **Most of the authors do not write the exact definition of refraction. This is the correct definition. Words broad, when added, make the definition correct.*

Note: It does not mean that if the ray is incident normal, it is not refracted.

Laws of refraction There are two laws of refraction.

(a) ${}^1\mu_2$ or $\mu = \dfrac{\sin i}{\sin r}$

(b) Incident ray, normal and refracted rays are coplanar.

$$\mu = \frac{\sin i}{\sin r} = \frac{c}{v} \text{ or } \frac{v_1}{v_2} = \frac{1}{\sin C} \text{ where } C \text{ is critical angle.}$$

$$\mu = \frac{\text{Real depth}}{\text{Apparant depth}}$$ (Apply this formula when incidence is normal)

$\mu = \dfrac{\lambda_1}{\lambda_2} = \tan\,\theta_P$ where θ_P is polarising angle and is equal to angle of incidence if angle between reflected and refracted rays is 90°.

$\mu = \dfrac{\sin \frac{A + D_m}{2}}{\sin \frac{A}{2}}$ in a prism. $\delta = (\mu - 1)\,\alpha$ where α is angle of prism and δ is angle of minimum deviation in a prism of small angle α (angle of prism).

$\mu = A + \dfrac{B}{\lambda^2} + \dfrac{C}{\lambda^4}$ is called Cauchy's principle.

${}^1\mu_2 = \dfrac{1}{{}^2\mu_1}$ (principle of reciprocity) ${}^1\mu_3 = {}^1\mu_2 \times {}^2\mu_3$

Fermat's principle When a ray of light passes from one point to another by any number of reflections or refractions, the path taken by the light is the one for which corresponding time taken is the least (or has shortest optical path).

Optical path length is μl if l is the distance travelled in a medium of refractive index μ.

Refraction through a curved surface

$\dfrac{\mu_2}{v} - \dfrac{\mu_1}{u} = \dfrac{\mu_2 - \mu_1}{R}$ (See Fig. 29.17)

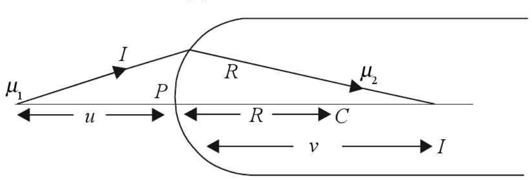


Fig. 29.17 Refraction through a curved surface

Note: That $\dfrac{\mu_2}{v} - \dfrac{\mu_1}{u} = \dfrac{\mu_2 - \mu_1}{R}$ can be applied for all curved surfaces with appropriate sign convention and remembering that μ_1 is the refractive index of the medium in which object lies.

Dispersion Splitting of a complex light into its constituent colours is called dispersion. For example, white light splits into seven colours when passed through a prism.

In a prism $i + e = A + D$ (See Fig. 29.18)

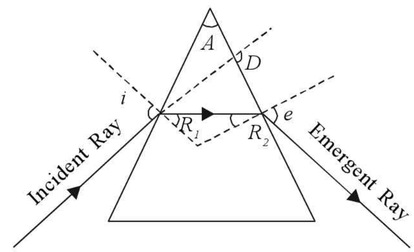


Fig. 29.18 Refraction through a prism

Fig. 29.19 shows graph between angle of deviation D and angle of incidence i. D_m is angle of minimum deviation.

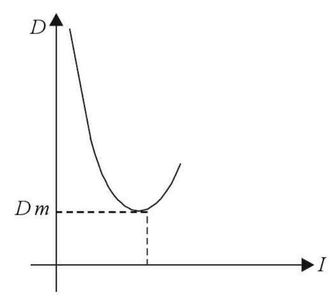


Fig. 29.19

At minimum deviation $i = e$ and $r_1 = r_2$. The ray through the prism is parallel to the base of the prism.

Under minimum deviation condition $\mu = \dfrac{\sin \frac{A + D_m}{2}}{\sin \frac{A}{2}}$

Dispersive power $\omega = \dfrac{\delta_v - \delta_r}{\delta} = \dfrac{\mu_v - \mu_r}{\mu - 1}$

where δ_v and δ_r are minimum deviations for violet and red colours, δ is mean deviation (for yellow colour). μ_v and μ_r are the refractive index for voilet and red colours and μ is the refractive index for yellow or mean colour.

Note: Use $\delta = \dfrac{\delta_v + \delta_r}{2}$ if δ is not given. Similarly use

$$\mu = \frac{\mu_v + \mu_r}{2} \text{ if } \mu \text{ is not given.}$$

$$\omega\delta = \delta_v - \delta_r \text{ is called angular dispersion.}$$

Rainbow Two types of rainbows are known: primary rainbow and secondary rainbow.

1. **Primary Rainbow** is formed when one total internal reflection (TIR) and two refractions occur from the suspended raindrops as illustrated in Fig. 29.20 (a). Violet colour on inner edge and red colour on outer edge are seen, as shown in Fig. 29.20 (c). Angles subtended with the direction of sun are 42° (red) and 40° (violet) above the horizon.

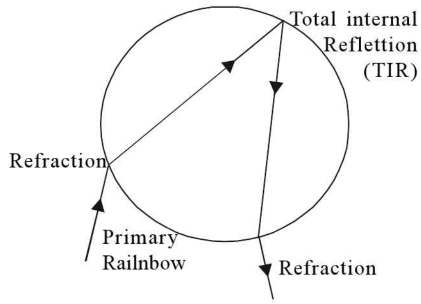


Fig. 29.20 (a) Primary rainbow formation

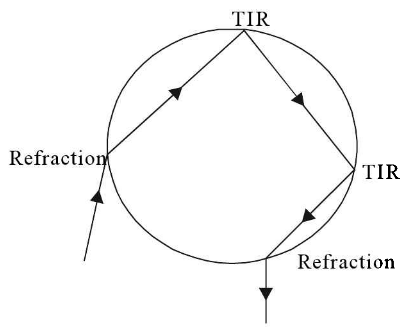


Fig. 29.20 (b) Secondary rainbow formation

2. **Secondary Rainbow** is formed due to two TIRs and two refractions from the raindrops suspended in air as shown in Fig. 29.20 (b). Inner edge has red colour and outer edge voilet, i.e., there is colour reversal from primary rainbow. It occurs due to an additional reflection which causes 180° phase shift. Angles are 51° for red and 54° for violet.

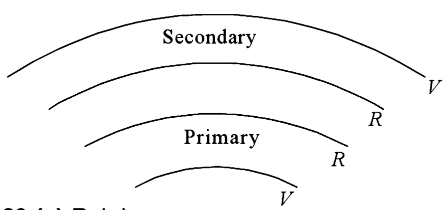


Fig. 29.20 (c) Rainbow

Deviation without dispersion See Fig. 29.21 (a)

Condition $\left(\delta_{v_1} - \delta_{r_1}\right) = \left(\delta_{v_2} - \delta_{r_2}\right)$

or $\left(\mu_{v_1} - \mu_{r_1}\right) \alpha_1 = \left(\mu_{v_2} - \mu_{r_2}\right) \alpha_2$

or $\delta_1 \omega_1 = \omega_2 \delta_2$

Dispersion without deviation See Fig. 29.21 (b). The mean colour should be parallel to incident ray.

$(\mu_1 - 1)\, \alpha_1 = (\mu_2 - 1)\, \alpha_2$

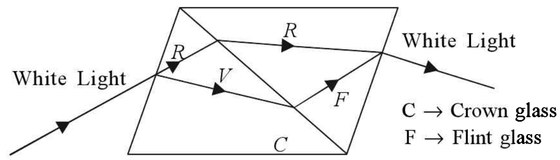


Fig. 29.21 (a) Deviation without dispersion

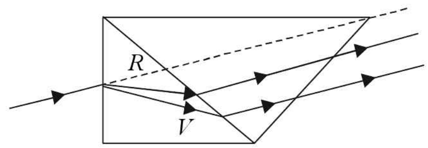


Fig. 29.21 (b) Dispersion without deviation

The prisms which produce dispersion without deviation are called direct vision prism and are employed in direct vision spectroscope. If more than two prisms are used the resolving power of the spectroscope is increased.

Defects in human eye

1. Myopia or shortsightedness
2. Hypermetropia or longsightedness
3. Presbyopia
4. Astigmatism
5. Colour blindness

1. **Myopic** eye is treated by concave lens. (Image is formed in front of the retina).
2. **Hypermetropic** eye is treated by convex lens. (Image is formed beyond the retina).
3. **Presbyopia:** Eye with this defect can neither see near objects nor far objects clearly. It is treated by bifocal lens (upper half concave and lower half convex).
4. **Astigmatism** is treated by specially prepared cylindrical lens.
5. **Colour blindness** Eye cannot differentiate between colours. Remedy is not available.

An alternative approach for correcting many defects of vision is to reshape the cornea. It is done using a procedure called **Laser assisted in situ Keratomileusis** or LASIK. An incision is made into the cornea and a flap of outer corneal tissue is folded back. A pulsed uv laser with a beam only 50 μm wide ($< \frac{1}{200}$th width of the hair) is then used to vaporise away microscopic area underlying the tissue. The flap is then folded back to the position where it conforms to the new shape carved by the laser.

Visual acuity or Resolving power of eye is $\frac{1}{60°}$ or 1 min.

Near point is 15 cm and least distance of distinct vision (normal near point) = 25 cm.

Eye pieces or occular Commonly used eyepieces are Huygen's and Ramsden. In Huygen eyepiece both the defects, spherical aberration and chromatic aberration, are removed

If $f_1 = 3f_2$ and $d = 2f_2$ then $d = \frac{f_1 + f_2}{2}$ removes chromatic aberration and $d = f_1 - f_2$ removes spherical aberration. The drawback in Huygen's eyepiece is that crosswire cannot be fitted. Therefore it can be used for qualitative work. Wherever quantitative (measurements) work is involved Ramsden's eyepiece is used. Ramsden eyepiece comprises of two lenses of equal focal length. $d = \frac{2}{3}f$. It is achromated for two selected colours. Spherical aberration is not removed completely. But crosswires can be connected.

Simple microscope or magnifier

Magnification $M = \left(1 + \frac{D}{f}\right)$.

Compound microscope

Magnification $M = \frac{v_o}{u_o}\left(1 + \frac{D}{f}\right) \simeq \frac{L}{f_o} \cdot \frac{D}{f_e}$ for normal adjustment where L is length of the microscope tube.

$$M = \frac{L}{f_o}\left(1 + \frac{D}{f_e}\right) \text{ for least distance vision.}$$

Length of microscope tube or separation between two lenses $L = v_o + u_e$

Resolving power of microscope R.P. $= \frac{\mu \sin\theta}{0.61\lambda}$ for self luminous points.

$$\text{R.P.} = \frac{2\mu \sin\theta}{\lambda} \text{ for non luminous points.}$$

Note: Resolving power can be increased if we immerse the objective in an oil and use uv light.

However, resolving power of electron microscope is maximum. Magnification is as high as 80,000. Limit of resolution $= \frac{1}{R.P.}$

Telescope (Astronomical) is of three types:

(a) Reflecting (b) Refracting

(c) Radio telescope.

Reflecting type is made with concave mirror. Focal length of concave mirror > 1 m (objective).

In refracting type telescope, objective has large focal length and large aperture $f \geq 1$ m, aperture ≥ 2 inch.

Magnification (Normal setting) $M_N = \frac{f_o}{f_e}$

and $L = f_o + f_e$.

Least distance of distinct vision setting

$$M_{LD} = -\frac{f_o}{f_e}\left(1 + \frac{f_e}{D}\right)$$

and $L = f_o + u_e$

Resolving power of telescope R P $= \frac{a}{1.22\lambda}$ where a is aperture.

Terrestrial Telescope

Magnification (Normal setting) $M_N = \frac{f_o}{f_e}$ and

$L = f_o + 4f_{er} + f_e$ where f_{er} is focal length of erecting lens.

Least distance setting $M_{LD} = \frac{-f_o}{f_e}\left(1 + \frac{f_e}{D}\right)$

and $L = f_o + 4f_{er} + u_e$.

Rayleigh's scattering $\propto \frac{1}{\lambda^4}$. That is why sky appears blue. Rising and setting sun appear red and danger signals are red in colour.

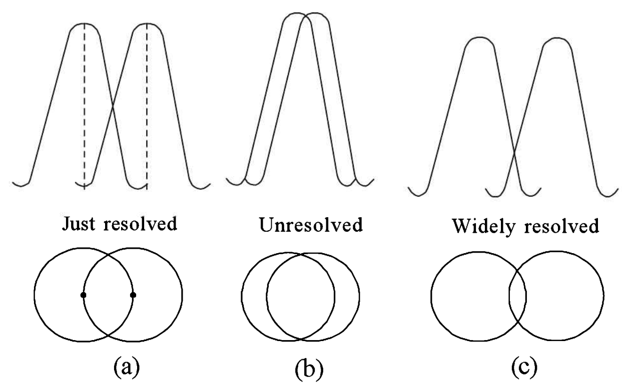


Fig. 29.22

Rayleigh's criterion for just resolution Two light sources close together are said to be just resolved if minima of one falls on the maxima of other as shown in Fig. 29.22(a).

- **Short Cuts and Points to Note**

1. If two mirrors are inclined at an angle θ $(0 \leq \theta \leq 90°)$ the number of images formed for an object placed in front of them is $\frac{360}{\theta}$ if $\frac{360}{\theta}$ is odd and number of

images formed = $\frac{360}{\theta} - 1$ if $\frac{360}{\theta}$ is even.

Number of images formed are even only if the object lies on angle bisector.

2. Use geometry to solve problems in optics. It is very helpful.
3. Second image is the brightest in a thick plane mirror.
4. Even virtual images can be photographed.
5. In spherical mirrors $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$ and $f = \frac{R}{2}$. Lateral magnification $M = \frac{-v}{u}$.
6. If a lens of focal length f (in air) and refractive index μ_L is immersed in a liquid of refractive index μ_m then new focal length f_{new} is given by

$$\frac{f_{new}}{f} = \frac{(\mu_L - 1)}{\left(\frac{\mu_L}{\mu_m} - 1\right)}$$

If $\mu_L = 1.5$ and $\mu_m = \frac{4}{3}$ (water) then $f_{new} = 4f$

If one side of the lens is in air and the other side in water as illustrated in Fig. 29.23 and $\mu_L = 1.5$,

$\mu_m = \frac{4}{3}$ then $f_{new} = 2f$

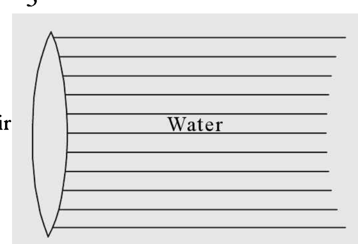


Fig. 29.23

7. If $\mu_L = 1.5$ and lens is equiconvex then $f = R$ and for a plano convex lens $f = 2R$
8. If a lens of refractive index μ_1 is immersed in a medium of refractive index μ_2 then if $\mu_1 > \mu_2$ lens behaves normal, i.e., a convex lens behaves as a convex lens and a concave lens behaves as a concave lens.

   If $\mu_1 = \mu_2$ the system acts as a slab i.e. rays pass undeviated. It ceases to act as a lens.

   If $\mu_1 < \mu_2$ lens behaves opposite, i.e., a convex lens behaves as a diverging lens and a concave lens acts as a converging lens.
9. In a system of three medias as shown in Fig. 29.24 focal length is determined using

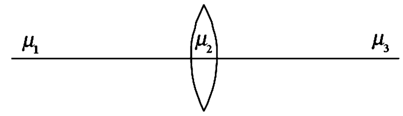


Fig. 29.24

$$\frac{\mu_3}{f} = \frac{\mu_2 - \mu_1}{R_1} - \frac{\mu_2 - \mu_3}{R_2}$$

To find v, apply $\frac{\mu_3}{v} - \frac{\mu_1}{u} = \frac{\mu_3 - \mu_1}{R_1} - \frac{\mu_2 - \mu_1}{R_2}$

10. If two thin lenses are joined then $\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$ [Fig. 29.25 (a)]

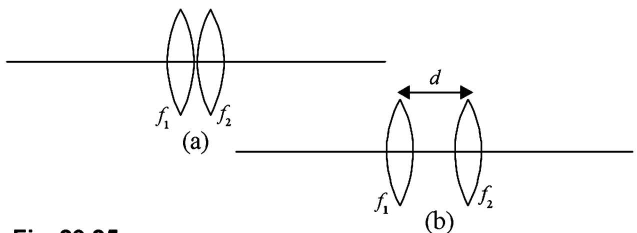


Fig. 29.25

If there is a separation d between the lenses,

then $\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$

Note: $\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$ can be used to find focal length of the combination. It cannot help to find v (image distance). While finding v, apply two lens theory, i.e. image formed by 1st acts as an object for the second.

11. In displacement method $f = \frac{D^2 - d^2}{4D}$. Note that $D \geq 4f$ and size of object $O = \sqrt{I_1 I_2}$
12. Power of the lens $P = \frac{1}{f(m)} = \frac{100}{f(\text{cm})}$ Unit is Diopter. If lenses are in contact, power is added i.e. $P_{net} = P_1 + P_2 + ...$
13. If the lens is silvered from one side use power to find new focal length.

Fig. 29.26

$$P = 2P_L + P_M \qquad \text{or}$$

$$\frac{1}{f_{new}} = \frac{2}{f} + \frac{2}{R} = \frac{2}{f} + \frac{1}{f(\mu-1)} \qquad \text{or}$$

$f_{new} = \dfrac{f(\mu-1)}{2\mu-1}$ for the case shown in Fig. 29.26, i.e., for equiconvex lens silvered on one side.

Note: If $\mu = 1.5$, then $f_{new} = f/4$.

14. Due to refraction the sun appears to rise a little earlier and appears to set a little later (about 3 minutes difference).

15. If the angle of prism is small, use $\delta = (\mu - 1)a$; otherwise use $\mu = \dfrac{\sin\left(\dfrac{A+D_m}{2}\right)}{\sin\left(\dfrac{A}{2}\right)}$. Constant deviation prisms are used in special type of spectrometers.

16. While finding position of spot when two or more medias are placed,

shift $\Delta y = \left(t_1 - \dfrac{t_1}{\mu_1}\right) + \left(t_2 - \dfrac{t_2}{\mu_2}\right)$ as shown in Fig. 29.27.

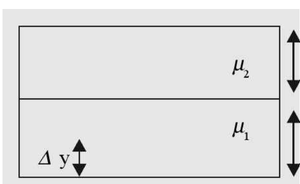


Fig. 29.27

17. Lateral shift in slab as shown in Fig. 29.28

$$y = \frac{t\sin(i-r)}{\cos r}$$

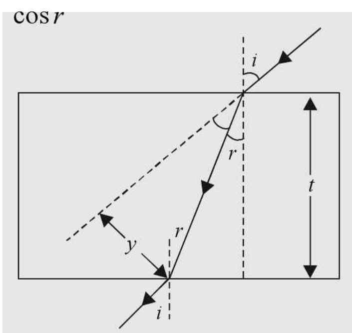


Fig. 29.28

18. A body disappears if its refractive index is equal to the reflective index of surrounding medium.

19. During refraction wavelength varies but frequency remains unchanged.

20. If a lens is partly covered, intensity or brightness of the image will decrease.

21. If a lens is cut horizontally, its focal length remains unchanged but if cut vertically focal length will change. For example, if an equiconvex lens is cut vertically then focal length of each planoconvex lens is $2f$. See Fig. 29.29.

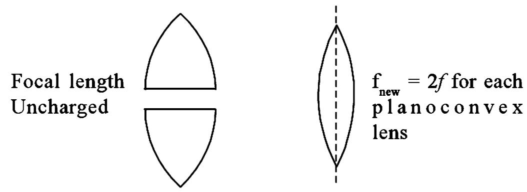


Fig. 29.29

22. If t is thickness and α is absorption coefficient then for incident light of intensity I_0, the emergent light intensity

$$I = I_0 e^{-\alpha t}$$

23. Apply $\dfrac{\mu_2}{v} - \dfrac{\mu_1}{u} = \dfrac{\mu_2 - \mu_1}{R}$ to find v and to find magnification in spherical surfaces shown in Fig. 29.30 use the formula

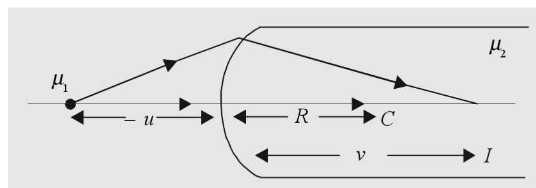


Fig. 29.30

$$M_{\text{lateral}} = \frac{\mu_1 v}{\mu_2 u}\ ; M_{\text{axial}} = \frac{\mu_1 v_1^2}{\mu_2 u^2}$$

Note: If the image is formed at the object side then image is virtual. If the image is formed on the other side then image is real.

24. Primary colours are red, blue and green. Complementary colours of primary colours are cyan for red; brown/orange for blue and purple or magenta for green.

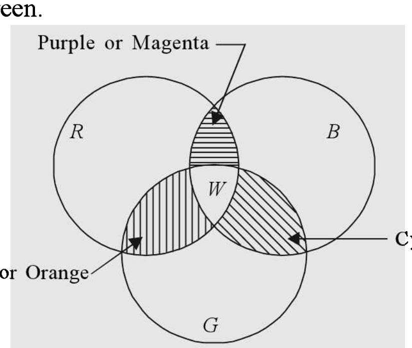


Fig. 29.31

25. The minimum distance between object and screen = $4f$ to form a real image in case of a lens.

26. For deviation without dispersion

$$\delta_{v_1} - \delta_{r_1} = \delta_{v_2} - \delta_{r_2}$$

or $\left(\mu_{v_1} - \mu_{r_1}\right)\alpha_1 = \left(\mu_{v_2} - \mu_{r_2}\right)\alpha_2$

27. To achieve dispersion without deviation $\delta_1 = \delta_2$
or $(\mu_1 - 1)\,\alpha_1 = (\mu_2 - 1)\,\alpha_2$

28. There are two types of defects in lens: spherical aberration and chromatic aberration. Spherical aberration is removed if $d = f_1 - f_2$

To achieve achromatism $\frac{\omega_1}{f_1} + \frac{\omega_2}{f_2} = 0$

if combination of a convex and a concave lens is used.

If two lenses of different material are kept d distance apart then to achieve achromatism $d = \frac{\omega_1 f_1 + \omega_2 f_2}{\omega_1 + \omega_2}$

$d = \frac{f_1 + f_2}{2}$ if lenses are made of same material.

29. f number in camera is given by

$$f \text{ number} = \frac{\text{Focal length}}{\text{Aperture diameter}} = \frac{f}{D}.$$

If one f number is known, the next number is obtained by multiplying by $\frac{1}{\sqrt{2}}$ i.e. $\frac{f}{2}, \frac{f}{2.8}, \frac{f}{4}, \frac{f}{5.6}, \frac{f}{8}, \frac{f}{11}, \frac{f}{16}$.

Thus if $\frac{f}{4}$ requires $\frac{1}{500}s$, then $\frac{f}{5.6}$ requires $\frac{1}{250}s$ and $\frac{f}{8}$ requires $\frac{1}{125}s$ for the same kind of exposure.

30. Zoom lens is based on $\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$ such that f can be varied 0 to ∞ by changing d.

31. In a pinhole camera $\frac{I}{O} = \frac{l_2}{l_1}$ (See Fig. 29.32)

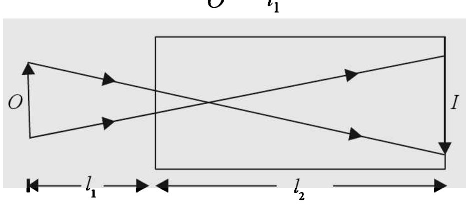


Fig. 29.32 Pinhole Camera

- **Caution**

1. Considering that real image cannot be formed with a plane mirror.

⇒ If the incident light beam is convergent, real image can be formed.

2. Considering that during refraction, ray must bend.

⇒ Rays incident normal do not bend and still refraction occurs. We apply $\mu = \frac{\text{Real depth}}{\text{Apparent depth}}$ in such cases.

3. Not differentiating between linear (lateral) and axial magnifications.

⇒ Lateral magnification is $M_{\text{lat}} = \frac{v}{u}$ and axial magnification for small objects is $\frac{-v^2}{u^2}$.

4. Considering that frequency varies during refraction.

⇒ Frequency does not vary during refraction. All other characteristics of wave like wavelength, velocity and amplitude vary.

5. Considering refraction is 100% or could be 100%.

⇒ Refraction can never be 100%. A fraction of light is always reflected from the interface of two media.

6. Considering that focal length found using

$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$ can be used to determine v also.

⇒ Individual lens analysis be employed to find v or u. Its use is limited to finding the focal length of the combination only.

7. Considering refraction through a slab can produce deviation.

⇒ Only lateral shift can be produced. The lateral shift is

$y = \frac{t\sin(i - r)}{\cos r}$. The lateral shift can be helpful in making images sharp without disturbing object lens or screen. The change in distance in such cases is equal to $\Delta l = \left(t - \frac{t}{\mu}\right)$.

8. Not being sure if colour is determined by wavelength or frequency.

⇒ Colour is determined by wavelength.

9. Assuming that achromat is made using two lenses, one convex and one concave, made of two different materials.

⇒ Two similar lenses (both convex) and made of same material (having equal dispersive power) can be used to achieve achromatism if $d = \frac{f_1 + f_2}{2}$.

If $d = f_1 - f_2$, then spherical aberration is also removed.

10. Considering that refractive index does not depend upon colour or wavelength.

⇒ Refractive index varies with colour or wavelength according to Cauchy's formula

$$\mu = A + \frac{B}{\lambda^2} + \frac{C}{\lambda^4}$$

11. Considering optical path length always greater than real path length.

⇒ Optical path length = μx. If $\mu > 1$, optical path length > real path length (say x). If $\mu = 1$, optical path length = x and if $\mu < 1$ optical path length < x.

12. Considering magnifying/resolving power of a microscope as fixed.

⇒ If we immerse the microscope lens/slide in an oil of refractive index μ, resolving power will increase. If uv light is selected, it will further increase.

13. Taking power $P = \frac{1}{f}$ and f in cm.

⇒ use $P = \frac{100}{f}$ if f is in cm.

14. Not knowing when the ray retraces its path.

⇒ The ray retraces its path only when it is incident normal on a reflecting surface.

15. Considering that in a camera, if f number increases exposure time decreases.

⇒ If f number increases exposure time increases, for example, if $\frac{f}{2}$ has exposure time $\frac{1}{1000}$ s, then $\frac{f}{2.8}$ has exposure time $\frac{1}{500}$ s and $\frac{f}{4}$ has exposure time $\frac{1}{250}$ s and so on.

SOLVED PROBLEMS

1. A light beam travels at a speed 1.94×10^8 ms^{-1} in quartz. The wavelength found in quartz is 355 nm. What would be the wavelength in air?

(a) 179 nm (b) 549 nm
(c) 355 nm (d) 707 nm

Solution (b) $\mu = \frac{c}{v} = \frac{\lambda}{\lambda'}$ or $\frac{3\times10^8}{1.94\times10^8} = \frac{\lambda}{355}$

or $\lambda = \frac{1065}{1.94} = 594$ nm.

2. In 11.5ns light travels 2.5 m in plastic find its refractive index.

(a) 1.38 (b) 1.48
(c) 1.18 (d) 1.58

Solution (a) $\mu = \frac{c}{v} = \frac{3\times10^8}{\frac{2.5}{11.5\times10^{-9}}} = \frac{3\times11.5\times10^{-1}}{2.5}$

$= 1.38$

3. Two mirrors are inclined at an angle θ. For an object placed in front of them, 11 images are noticed. Find the angle between the mirrors.

(a) 30° (b) 32.8°
(c) 16.4° (d) 15°

Solution (a) No. of images = 11 = $\frac{360}{\theta} - 1 \Rightarrow \theta = 30°$

4. A ray deviates at 90° after suffering reflection from a mirror. The angle of incidence is

(a) 90° (b) 30°
(c) 60° (d) 45° (e) none of these

Solution (d) $2\theta = 180 - \delta$ or $2\theta = 180 - 90 \Rightarrow \theta = 45°$

5. To what depth can a vessel be filled with water so that it appears half filled?

(a) $\frac{3}{4}h$ (b) $\frac{2}{3}h$
(c) $\frac{5}{7}h$ (d) $\frac{3}{5}h$

Solution (b) App. depth = $\frac{\text{Real depth}}{\mu}$

∴ Real depth = $\frac{4}{3}\left(\frac{h}{2}\right) = \frac{2}{3}h$

6. A room is 3m high and 5m long. A man is standing in front of one of the walls 1m from the wall. A mirror is to be installed on the wall. Find the height (minimum) of the mirror so that complete image of the wall behind him is seen.

(a) 1.5 m (b) 1 m

(c) 2 m (d) 0.5 m

Solution (d) ΔABM and ΔKLM are similar

$$\therefore \quad \frac{KL}{AB} = \frac{KM}{MB} \Rightarrow \text{KL} = \frac{3\times 1}{6} = \frac{1}{2}\text{ m}$$

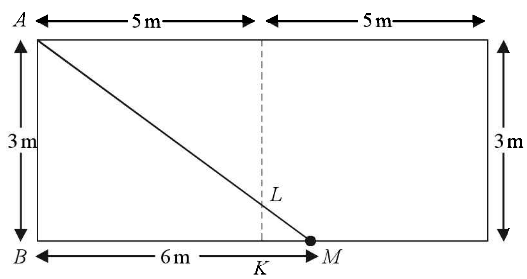


Fig. 29.33

7. A beam is incident parallel on the prism shown in Fig. 29.34 (a). Find the angle between emerging rays. $\mu_{\text{prism}} = 1.66$.

(a) 180° (b) 120°

(c) 135° (d) 40°

Solution (d) $\sin C = \dfrac{1}{1.66}$ or $C = 37°$

$\therefore$ ray is referacted out

$\sin r = \sin 25\,(1.66) = 0.4226\,(1.66) = .706$

$\therefore$ $r = 45°$

$\theta = 360 - 230 - 90 = 40°$

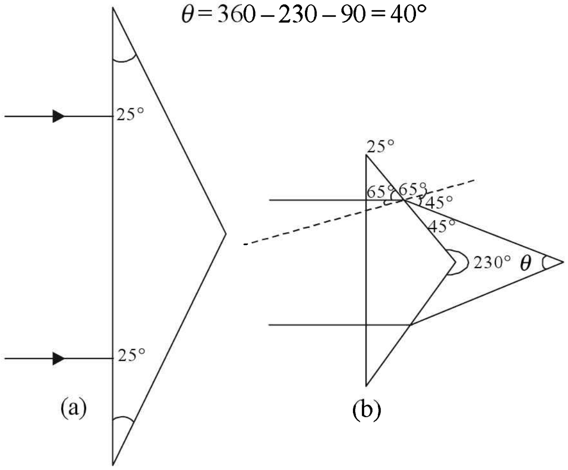


Fig. 29.34

8. A white light is incident at 20° on a material of silicate flint glass slab as shown. $\mu_{\text{violet}} = 1.66$ and $\mu_r = 1.6$. For what value of d will the separation be 1mm in red and violet rays.

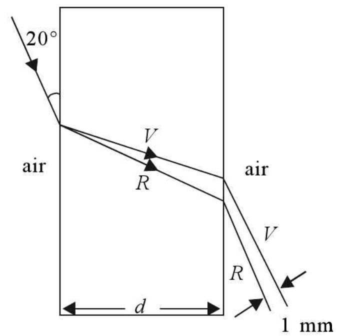


Fig. 29.35

(a) $\dfrac{5}{3}$ cm (b) $\dfrac{10}{3}$ cm

(c) 5 cm (d) $\dfrac{20}{3}$ cm

Solution (b) $\sin r_1 = \dfrac{\sin 70}{1.66} = \dfrac{.9397}{1.66}$ or $r_1 = 34°\,30'$

$$\sin r_2 = \frac{\sin 70}{1.6} = \frac{.9397}{1.6}$$

or $\quad r_2 = 36°$

Using $\quad y = \dfrac{t\sin(i-r)}{\cos r}$

$$y_1 - y_2 = d\left[\frac{\sin(i-r_1)}{\cos r_1} - \frac{\sin(i-r_2)}{\cos r_2}\right]$$

$$0.1 = d\left[\frac{\sin 35°30'}{\cos 34°30'} - \frac{\sin 34°}{\cos 36°}\right]$$

or $\quad 0.1 = d\left[\dfrac{0.5807}{0.8241} - \dfrac{0.5592}{0.8090}\right] = d[0.71 - 0.68]$

or $\quad d = \dfrac{0.1}{0.03} = \dfrac{10}{3}$ cm

9. A glass whose one end is hemispherical of radius 2 cm ($\mu_{\text{red}} = 1.52$) is kept in water ($\mu = 1.33$). The object is kept 8 cm in front of convex surface. Find the magnification.

(a) 2.33 (b) 1.33

(c) 2.66 (d) 1.76

Solution (a) $\dfrac{1.52}{v} - \dfrac{1.33}{-8} = \dfrac{1.52 - 1.33}{2}$

or $\quad v = -21.3$ cm;

$$M = -\frac{1.33(-21.3)}{1.52(8)} = 2.33$$

10. Refractive index of water in the situation shown in Fig. 29.36 is μ. Find the distance seen by the fish F of human eye E.

(a) $H+\dfrac{H}{2\mu}$ (b) $\dfrac{3H}{2\mu}$

(c) $\dfrac{H}{2}+H\mu$ (d) $\dfrac{3\mu H}{2}$

Solution (c) $\dfrac{H}{2}+\dfrac{H}{{}^{w}\mu_{a}}=\dfrac{H}{2}+\dfrac{H}{\frac{1}{\mu}}$

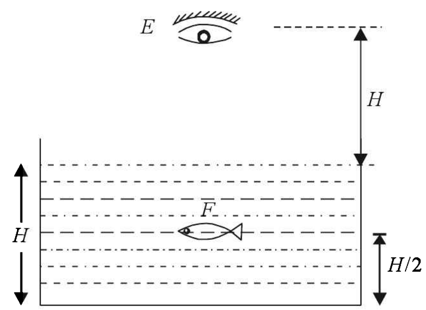


Fig. 29.36

11. In the previous question what is the distance of fish F seen by human eye E?

(a) $\mathrm{H}+\dfrac{H}{2\mu}$ (b) $\dfrac{H}{\mu}+\dfrac{H}{2}$

(c) $\dfrac{3H}{2\mu}$ (d) None of these

Solution (a) $H+\dfrac{H/2}{{}^{a}\mu_{w}}=H+\dfrac{H}{2\mu}$

12. A point object O is placed midway between the concave mirrors, distance d apart. What is the value of d for which object and images coincide? Each mirror has focal length F.

(a) $F, 2F$ (b) $2F, 3F$

(c) $F, 4F$ (d) $2F, 4F$

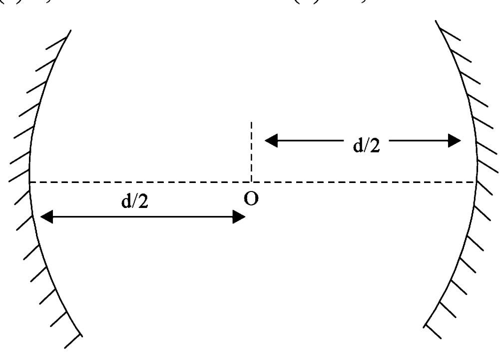


Fig. 29.37

Solution (d) When $\dfrac{d}{2}=F$, the rays from one mirror after reflection will reach parallel to the other mirror. Second mirror will refocus them at O. When object is at $2F$, image is formed at $2F$.

13. A cylindrical vessel of diameter 12 cm has 800 π cm^3 water. A cylindrical glass piece of diameter 8 cm and height 8 cm is placed in it as shown in Fig. 29.38. What is the position of image of bottom of the vessel seen through paraxial rays passing through the glass cylinder?

(a) 6.2 cm above the bottom

(b) 7.1 cm above the bottom

(c) 6.6 cm above the bottom

(d) none of these

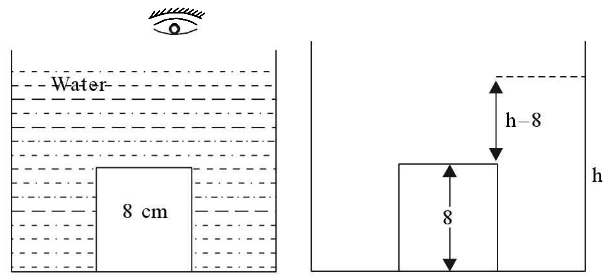


Fig. 29.38

Solution (b) $\pi\, 6^2\, h = 800\,\pi + \pi\, 4^2\,(8)$ or $h=\dfrac{928}{36}$

$= 25.8\,\text{cm}$

$$\Delta y = \left(17.8-\frac{17.8}{4/3}\right)+\left(8-\frac{8}{1.5}\right)$$

$= 4.45 + 2.67 = 7.1$ cm above the bottom.

14. Find angle of minimum deviation of an equilateral prism (μ = 1.732). Also find angle of incidence for this deviation.

(a) 60°, 75° (b) 60°, 55°

(c) 60°, 50° (d) 60°, 60°

Solution (d) $\mu=\dfrac{\sin\dfrac{A+D_m}{2}}{\sin\dfrac{A}{2}}$

or $\sqrt{3}\times\dfrac{1}{2}=\sin\dfrac{A+D_m}{2}\Rightarrow D_{\mathrm{m}}=60°$.

Using $2i = A + D_{\mathrm{m}}$, we get $i = 60°$.

15. An equiconvex lens is made from μ = 1.5 and r = 20 cm. It is 5 cm thick in the middle. Find the position of image far away from the lens.

(a) 10 cm (b) 9.2 cm

(c) 8.72 cm (d) 9.48 cm (e) 9.68 cm

Solution (b) Image is formed at focus.

$$\frac{1}{f} = (\mu-1)\left[\frac{2}{R}+\frac{(\mu-1)t}{\mu R^2}\right]$$
$$= \frac{1}{2}\left[\frac{2}{10}+\frac{2.5}{150}\right] = \frac{13}{120}$$

or $$f = \frac{120}{13} = 9.2$$

16. A slide projector is to project a (35 mm × 23 mm) slide on a 2m × 2m screen. Find the focal length of the lens used if screen is 10 m away from the lens.

(a) 15.1 cm (b) 17.2 cm

(c) 16.1 cm (d) 18.2 cm

Solution (b) $M = \frac{200}{3.5} = \frac{10}{u}$ or $u = \frac{7}{40}$ m; using

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f},$$ i.e.,

$$\frac{1}{f} = \frac{1}{10} + \frac{40}{7} \text{ or } f = 17.2 \text{ cm}$$

17. A paper weight is hemispherical with radius 3 cm. It is kept on a printed page, the printed letter will appear at a height _____ cm from the centre of the hemisphere when viewed vertically.

(a) 0 (b) 1 cm

(c) 2 cm (d) 1.21 cm

Solution (a) $\frac{1.5}{v} - \frac{1}{-3} = \frac{1.5-1}{-3}$ $v = -3$ cm.

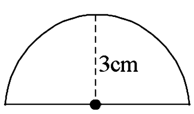


Fig. 29.39

18. A 1m long rod is half dipped in a swimming pool. The sunlight is incident at 45° on the rod. Find the length of the shadow on the bed of swimming pool.

(a) 73 cm (b) 78.25 cm

(c) 74.17 cm (d) 81.5 cm

Solution (d) Length of shadow $= x + 0.5$ m

$$x = 0.5 \tan r$$
$$\sin r = .707 \times \frac{3}{4} \text{ or}$$
$$r = 32° 12'$$

Thus $l = 50 (\tan 32° 12') + 50$ cm
$= 50 (.6297) + 50 = 81.5$cm

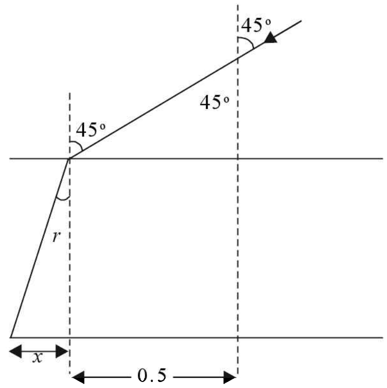


Fig. 29.40

19. A diverging lens of $f = 20$ cm and a converging mirror $f = 10$ cm are placed 5 cm apart coaxially. Where shall an object be placed so that object and its real image coincide?

(a) 60 cm away from lens (b) 15 cm away from lens

(c) 20 cm away from lens (d) 45 cm away from lens

Solution (a) If the rays are to retrace the path, light ray must fall normal on the mirror. Hence I' should be 20 cm from mirror and 15 cm from lens.

$$\frac{1}{-15} - \frac{1}{x} = \frac{1}{-20}$$

or $$\frac{1}{x} = \frac{1}{-15} + \frac{1}{20} = \frac{1}{-60}$$

$x = -60$ cm i.e. 60 cm away from lens.

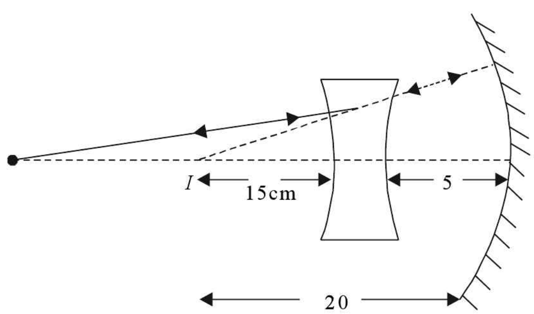


Fig. 29.41

20. A fish looking up through the water sees outside world contained in a circular horizon. The refractive index of water is $\frac{4}{3}$ and the fish is 12 cm below the surface. The radius of this circle in cm is

[AIEEE 2005]

(a) $36\sqrt{7}$ (b) $\frac{36}{\sqrt{7}}$

(c) $36\sqrt{5}$ (d) $4\sqrt{5}$

Solution (b) $\sin C = \frac{3}{4}$ and $\tan C = \frac{3}{\sqrt{7}} = \frac{r}{12}$

or $r = \frac{36}{\sqrt{7}}$ cm.

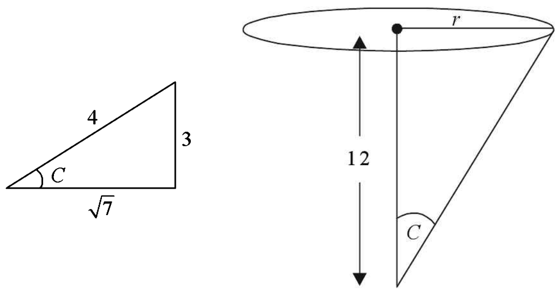


Fig. 29.42

21. Two point white dots are 1 mm apart on a black paper. They are viewed by eye of pupil of diameter 3 mm. Approximately what is the maximum distance upto which these dots can be resolved by the eye.

[AIEEE 2005]

(a) 5 m (b) 6 m

(c) 1 m (d) 4 m

Solution (a) $\frac{1.22\lambda}{3mm} = \frac{1mm}{d}$

or $d = \frac{3\times10^{-6}}{1.22\times5\times10^{-7}} = 5$ m

22. A thin glass (μ = 1.5) lens has power $-5D$ in air. Its power in a medium of refractive index 1.6 will be

(a) $\frac{5}{8}D$ (b) $\frac{25}{8}D$

(c) $-\frac{5}{8}D$ (d) $-\frac{25}{8}D$

Solution (a) $f_m = \frac{(\mu_L - 1)}{\left(\frac{\mu_L}{\mu_m} - 1\right)} f_a = \frac{\frac{1}{2}\times(-20)}{\frac{\frac{3}{2}}{1.6} - 1}$

$= \frac{\frac{1}{2}\times20\times3.2}{.2} = 160$ cm

$P = \frac{100}{160} = \frac{5}{8}$

23. The angular resolution of a telescope of 10 cm diameter at a wavelength of 5000 A° is of the order of

[CBSE 2005]

(a) 10^6 rad (b) 10^{-2} rad

(c) 10^{-4} rad (d) 10^{-6} rad

Solution (d) $\frac{\lambda}{d} = \frac{5\times10^{-7}}{10^{-1}} = 10^{-6}$

24. A tank of height 33.25 cm is completely filled with liquid (μ = 1.33). An object is placed at the bottom of the tank on the axis of concave mirror as shown in Fig. 29.43. Image of the object is formed at 25 cm below the surface of the liquid. Focal length of the mirror is

[IIT 2005]

(a) 10 cm (b) 15 cm
(c) 20 cm (d) 25 cm

Solution (c) Apparent depth = $\frac{33.25}{1.33} = 25$ cm

when object is at $2f$ image is formed at $2f$.

∴ $2f = 15 + 25 = 40$ cm and $f = 20$ cm

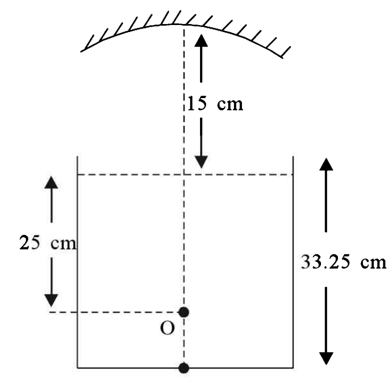


Fig. 29.43

25. A telescope has an objective lens of focal length 200 cm and an eyepiece with focal length 2 cm. It is used to see a 50 m tall building at a distance of 2 km. What is the height of the image of the building formed by the objective lens ?

[AIIMS 2005]

(a) 5 cm (b) 10 cm
(c) 1 cm (d) 2 cm

Solution (a) $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

or $\frac{1}{v} = \frac{1}{200} - \frac{1}{2000\times100}$

or $v = 200$ cm.

Using $\frac{v}{u} = \frac{I}{O}$

$$I = \frac{2}{2000} \times 50 = \frac{1}{20} \text{ m or } 5 \text{ cm.}$$

26. *A*. Resolving power of a telescope is more if the diameter of the objective lens is more.

R. Objective lens of large diameter collects more light.

[AIIMS 2005]

(a) both *A* and *R* are correct and *R* is correct explanation of *A*

(b) *A* and *R* both are correct but *R* is not correct explanation of *A*

(c) *A* is true but *R* is false

(d) both *A* and *R* are false

Solution (b) $RP = \frac{D}{1.22\lambda}$ $\therefore$ *A* is correct. Though *R* is correct but for larger resolution objects making small angle be distinguished or very close objects should be distinguished.

27. Focal number of the lens of a camera is 5*f* and that of another is 2.5*f*. The time of exposure for the second is......... if that for the first is $\frac{1}{200}s$

$\left(\text{Given } f = \frac{\text{focal length}}{\text{aperature}}\right)$

(a) $\frac{1}{200}$ s (b) $\frac{1}{800}$ s

(c) $\frac{1}{3200}$ s (d) $\frac{1}{6400}$ s

[BHU 2005]

Solution (b) *f* number decreases by 2 $\therefore$ time of exposure should decrease by (2^2).

$$\therefore \quad t_{new} = \frac{1}{4} \times \frac{1}{200} = \frac{1}{800} \text{ s.}$$

28. A convex lens of focal length 80 cm and a concave lens of focal length 50 cm are combined together. What will be their resultant power?

[BHU 2005]

(a) 0.65 D (b) –0.65 D

(c) 0.75 D (d) –0.75 D

Solution (d) $P_1 = 1.25\,D$ and $P_2 = -2\,D$

$P_{net} = P_1 + P_2 = -0.75\,D$

29. A plane slab is kept over various colour letters. The letter which appears least raised is

[BHU 2005]

(a) Red (b) Green

(c) Violet (d) Blue

Solution (a) $\mu = A + \frac{B}{\lambda^2}$

$\therefore$ μ is minimum for Red and App. depth $= \frac{\text{Real depth}}{\mu}$

30. A convex lens forms the full image of the object on a screen. If half of the lens is covered with an opaque object then

[BHU 2005]

(a) the image disappears

(b) half the image is seen

(c) full image of same intensity is seen

(d) full image of decreased intensity is seen.

Solution (d)

31. Time taken by light to pass through 4 mm thick glass slab of refractive index 1.5 will be

(a) 8×10^{-11} s (b) 2×10^{-11} s

(c) 8×10^{-8} s (d) 2×10^{-8} s

[BHU 2005]

Solution (b) $t = \frac{4\times10^{-3}\times1.5}{3\times10^{8}} = 2 \times 10^{-11}$ s

32. A lens acts as a converging lens in air and diverging lens in water. The refractive index of the lens is

[BHU 2005]

(a) = 1 (b) < 1.33

(c) > 1.33 (d) < 1

Solution (b)

33. A light passing through air has wavelength 6000 A°. Wavelength when same ray passes through a glass slab of refractive index 1.5 is

[BHU 2005]

(a) 4000 A° (b) 2000°

(c) 8000 A° (d) 1200 A°

Solution (a) $\lambda' = \frac{\lambda}{\mu} = \frac{6000}{1.5} = 4000$ A°

34. Which of the following is a wrong statement?

[CET Karnataka 2005]

(a) $D = \frac{1}{f}$ where f is focal length and D is optical power of lens.

(b) Power is in diopter when f is in meters.

(c) Power is in diopter and does not depend upon the system of unit used to measure f.

(d) D is positive for convergent lens and D is negative for divergent lens.

Solution (c)

$$\because \quad P = \frac{1}{f(m)} = \frac{100}{f(cm)}$$

[CET Karnataka 2005]

35. Identify the wrong description of the given figures.

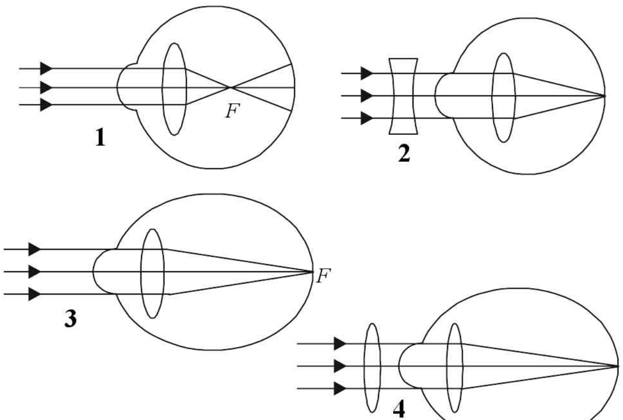


Fig. 29.44

(a) 1 represents far sightedness

(b) 2 is correction for short sightedness

(c) 3 represents far sightedness

(d) 4 represents correction for far sightedness

Solution (a)

36. Which mirror be used to obtain a parallel beam of light from a small lamp?

(a) plane mirror (b) convex mirror

(c) concave mirror (d) any of the above

[CET Karnataka, 2005]

Solution (c)

37. As shown in Fig. 29.45 $AB = AC$. Find the minimum value of refractive index μ for the given material.

(a) $\sqrt{2}$ (b) $\sqrt{3}$

(c) 1.5 (d) 1.6

Solution (a)

$$\mu = \frac{1}{\sin c} = \frac{1}{\sin 45} = \sqrt{2}$$

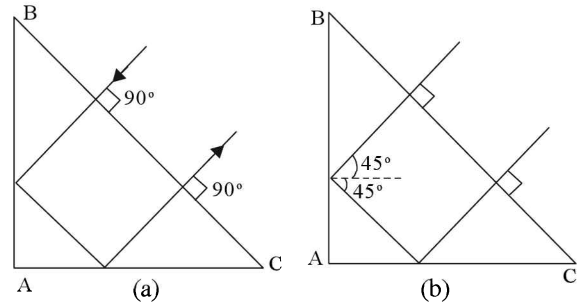


Fig. 29.45

38. The eyepiece of a refracting telescope has $f = 9$ cm. In the normal setting, separation between objective and eyepiece is 1.8 m. Find the magnification.

(a) 20 (b) 19

(c) 18 (d) 21

Solution (b) $f_0 = 1.8 - 0.09 = 1.71$ m

$$M = \frac{f_0}{f_e} = \frac{171}{9} = 19$$

39. The focal length of an $\frac{f}{4}$ camera lens is 300 mm. What is the aperture diameter of the lens?

(a) 75 mm (b) 650 mm

(c) 800 mm (d) 1200 mm

Solution (a) aperture $= \frac{300}{4} = 75$ mm

40. An object is placed at 15 cm in front of a convex lens of focal length 10 cm. Where shall we place a convex mirror of focal length 13 cm so that real image and object coincide?

(a) 6 cm from lens (b) 3 cm from lens

(c) 4 cm from lens (d) 2 cm from lens.

Solution (c) $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$; $\frac{1}{v} = \frac{1}{10} - \frac{1}{15}$

or $v = 30$ cm. In order the ray to retrace the path, the ray must be incident normal on the mirror. Hence distance of mirror from I′ should be equal to $R = 2f = 26$ cm or 4 cm from lens.

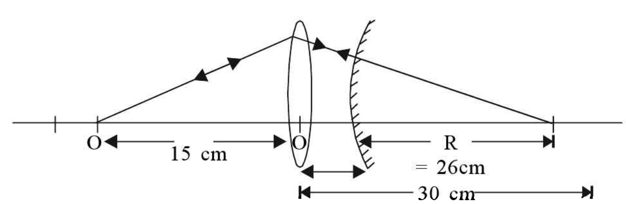


Fig. 29.46

41. In Fig. 29.47 a parallel beam emerges parallel. The relation between μ, μ_1 and μ_2 is

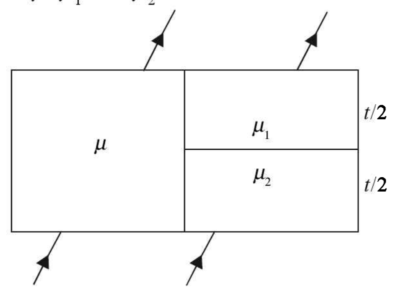


Fig. 29.47

(a) $\mu = \mu_1 + \mu_2$ (b) $\frac{1}{\mu} = \frac{1}{\mu_1} + \frac{1}{\mu_2}$

(c) $\mu = \frac{\mu_1 + \mu_2}{2}$ (d) $\frac{2}{\mu} = \frac{1}{\mu_1} + \frac{1}{\mu_2}$

Solution (d)

$$\frac{t}{\mu} = \frac{t/2}{\mu_1} + \frac{t/2}{\mu_2} \text{ or } \frac{2}{\mu} = \frac{1}{\mu_1} + \frac{1}{\mu_2}$$

TYPICAL PROBLEMS

42. A professor reads a greeting card on his 50th birthday with + 2.5 D glasses keeping the card 25 cm away. Ten years later he reads the greeting card with same glass keeping the card 50 cm away. What power glasses should he wear now?

(a) 2D (b) 0.5 D

(c) 2.25 D (d) 4.5 D

Solution (d) $\frac{1}{f'} = \frac{1}{25} - \frac{1}{50} = \frac{1}{50}$ or

$P = 2\text{D}$

$P_{net} = 2.5 + 2 = 4.5\text{D}$

43. A simple microscope is rated 5 × for a normal relaxed eye. What will be its magnifying power for a farsighted man whose near point is 40 cm ?

(a) 5 × (b) 3 ×

(c) 8 × (d) 13 ×

Solution (c) For a relaxed eye $M = \frac{D}{f}$ $\therefore f = 5$ cm

In case II $M = \frac{40}{5} = 8$

44. A particle is moving at a constant speed v from a large distance towards a concave mirror of radius R along the principal axis. Find the speed of the image as a function of the distance x of the particle from the mirror.

Solution Let y represent the image distance and x the object distance from the mirror. Then

$$\frac{1}{y} + \frac{1}{-x} = -\frac{2}{R}$$

or $$\frac{1}{y} = \frac{-2}{R} + \frac{1}{x} = \frac{-2x+R}{Rx}$$

or $$y = \frac{Rx}{R-2x} \quad \text{............... (1)}$$

Differentiating equation. (1)

$$\frac{dy}{dt} = \frac{R\frac{dx}{dt}}{(R-2x)} + \frac{2Rx\frac{dx}{dt}}{(2-2x)^2}$$

or $$\frac{dy}{dt} = \frac{[R(R-2x)+2Rx]\frac{dx}{dt}}{(R-2x)^2} = \frac{R^2 v}{(R-2x)^2}$$

45. When an equiconvex lens ($\mu_{lens} = 1.5$) is placed over a plane mirror as shown, then object needle and its image coincide at 15 cm. When a liquid of refractive index μ is filled in the gap between mirror and lens, the object needle and its image coincide at 40 cm. Find the ref. index μ of the liquid.

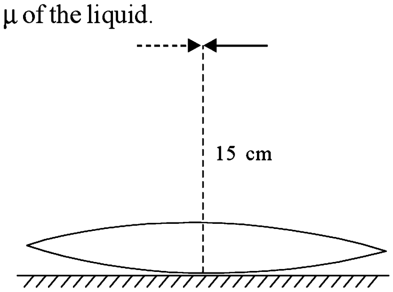


Fig. 29.48

(a) $\frac{8}{5}$ (b) $\frac{5}{8}$

(c) $\frac{13}{8}$ (d) $\frac{5}{13}$

(e) $\frac{13}{5}$

Solution (c) From case (i) we get $f_{lens} = 15$ cm

since $\mu_{lens} = 1.5$ $\therefore$ $f_{lens} = R_{lens} = 15$ cm

In case (ii) focal length of the combination = 40 cm [combination of lens + combination of (planoconcave) liquid lens].

$$\frac{1}{40} = \frac{1}{15} + \frac{1}{f_{\text{liquid lens}}}$$

or $$\frac{1}{f_{\text{liq.lens}}} = -\frac{1}{15} + \frac{1}{40}$$

$$f_{\text{liq. lens}} = -24 \text{ cm}$$

$$\frac{1}{f_{\text{liq.lens}}} = (\mu - 1)\left[\frac{1}{R} - \frac{1}{\infty}\right] \text{ or } \frac{1}{-24}$$

$$= (\mu - 1)\left[\frac{1}{-15}\right] \text{ or}$$

$$\mu = \frac{13}{8}$$

46. A particle executes SHM of amplitude 1 cm along principal axis of a convex lens of focal length 12 cm. The mean position of oscillation is at 20 cm from the lens. Find the amplitude of oscillation of the image of the particle.

(a) 2 cm (b) 2.6 cm
(c) 1 cm (d) 2.3 cm

Solution (d) $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

$$\frac{1}{v_1} = \frac{1}{12} - \frac{1}{21} = \frac{9}{21 \times 12}$$

or $$v_1 = 28 \text{ cm}$$

$$\frac{1}{v_2} = \frac{1}{12} - \frac{1}{19} = \frac{7}{19 \times 12}$$

or $$v_2 = \frac{19 \times 12}{7} = \frac{228}{7} = 32.6 \text{ cm}$$

$$\Delta x = 32.6 - 28 = 4.6 \text{ cm amplitude} = \frac{\Delta x}{2} = 2.3 \text{ cm}$$

47. Object and screen are fixed 90 cm apart. The lens is displaced. Two sharp images are obtained when lens is at L_1 and L_2 respectively such that $I_1 = 4I_2$. Find the focal length of the lens.

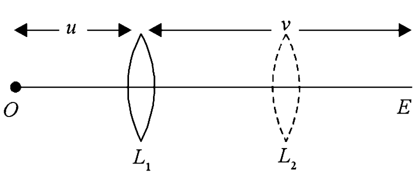


Fig. 29.49

(a) 18 cm (b) 15 cm
(c) 16 cm (d) 20 cm

Solution (d) $O = \sqrt{I_1 I_2} = 2I_2 = \frac{I_1}{2}$

$$\frac{v}{u} = 2$$

$$v = 2u$$

$$v + u = 90 \text{ cm}$$

$$3u = 90 \text{ cm or } u = 30 \text{ cm}$$

$$\frac{1}{f} + \frac{1}{v} + \frac{1}{u} = \frac{1}{60} + \frac{1}{30} = \frac{3}{60}$$

or $f = 20$ cm

48. You are looking at the rim from the vertical side to see the opposite edge at the bottom of a 16 cm high and 8 cm diameter vessel. A friend fills it with a liquid of refractive index μ so that a coin placed at the centre becomes visible. What is the value of μ?

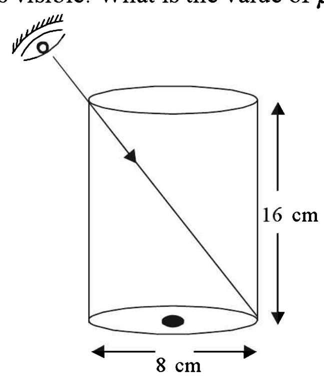


Fig. 29.50

(a) 1.34 (b) 1.6
(c) 1.73 (d) 1.84

Solution (d) $\tan i = \frac{1}{2}$ $\mu = \frac{\sin i}{\sin r} = \frac{1/\sqrt{5}}{1/\sqrt{17}}$

$$= \sqrt{\frac{17}{5}} = \sqrt{3.4} = 1.84$$

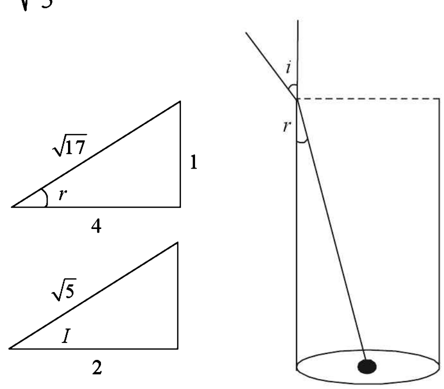


Fig. 29.51

49. At what angle should a ray of light be incident on the face of a prism of refracting angle 60° so that it just suffers total internal reflection at the other face. The refractive index of the prism is 1.524.

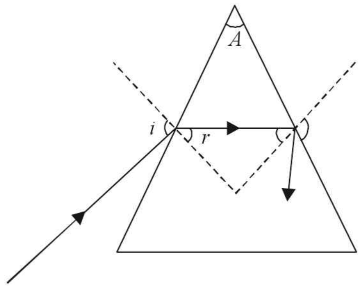


Fig. 29.52

(a) 30° (b) 40°
(c) 50° (d) 60°
(e) none of these

Solution (a) $\sin c = \frac{1}{\mu} = \frac{1}{1.524} = 0.65$ or $c = 40°30'$

$r = 19°\,30' = [180° - 120° - 40°\,30']$

$\mu = \frac{\sin i}{\sin r}$ or $\sin i = 1.524 \sin 19°\,30'$

$= 1.524\,(.33\,40) = 0.5$

or $i = 30°$

50. A crown glass prism of refracting angle 6° is to be used for deviation without dispersion with a flint glass of angle of prism α. Given: for crown glass $\mu_r = 1.513$ and $\mu_v = 1.523$, for flint glass $\mu_r = 1.645$ and $\mu_v = 1.665$. Find α.

(a) 3° (b) 4°
(c) 4.5° (d) 5°
(e) 6°

Solution (a) $(\mu_{v_1} - \mu_{r_1})\,\alpha_1 = (\mu_{v_2} - \mu_{r_2})\,\alpha_2$

or $\alpha_2 = \frac{(1.523 - 1.513)}{(1.665 - 1.645)}\,6° = 3°.$

QUESTIONS FOR PRACTICE

1. Two thin lenses, when in contact, produce a combined power + 10 dioptres. When they are 0.25 m apart, the power reduces to + 6 dioptres. The powers of the lenses in dioptres, are
   (a) 1 and 9 (b) 2 and 8
   (c) 4 and 6 (d) 5 each
2. The size of an object as perceived by an eye depends primarily on
   (a) actual size of thr object
   (b) distance of the object from the eye
   (c) aperture of the pupil
   (d) size of the image formed on the retina
3. A normal eye is not able to see objects closer than 25 cm because
   (a) the focal length of the eye is 25 cm
   (b) the distance of the retina from the eye-lens is 25 cm
   (c) the eye is not able to decrease the distance between the eye-lens and the retina beyond a limit
   (d) the eye is not able to decrease the focal length beyond a limit
4. An object is placed at a distance u from a simple microscope of focal length f. The angular magnification obtained depends
   (a) on f but not on u (b) on u but not on f
   (c) on f as well as u (d) neither on f nor on u.
5. The focal length of a normal eye-lens is about
   (a) 1 mm (b) 2 cm
   (c) 25 cm (d) 1 m.
6. The muscles of a normal eye are least strained when the eye is focused on an object
   (a) far away from the eye
   (b) very close to the eye
   (c) at about 25 cm from the eye
   (d) at about 1 m from the eye.
7. A person A can clearly see objects between 25 cm and 200 cm. Which of the following may represent the range of clear vision for a person B having muscles stronger than A, but all other parameters of eye identical to that of A?
   (a) 25 cm to 200 cm (b) 18 cm to 200 cm
   (c) 25 cm to 300 cm (d) 18 cm to 300 cm.
8. A man is looking at a small object placed at his near point. Without altering the position of his eye or the object, he puts a simple microscope of magnifying power 5 X before his eyes. The angular magnification achieved is
   (a) 5 (b) 2.5
   (c) 1 (d) 0.2

9. When objects at different distances are seen by the eye, which of the following remain constant?

(a) The focal length of the eye-lens.
(b) The object-distance from the eye-lens.
(c) The radii of curvature of the eye-lens.
(d) The image-distance from the eye_lens.

10. The maximum focal length of the eye-lens of a person is greater than its distance from the retina. The eye is

(a) always strained in looking at an object
(b) strained for objects at large distances only
(c) strained for objects at short distances only
(d) unstrained for all distances

11. To increase the angular magnification of a simple microscope, one should increase

(a) the focal length of the lens
(b) the power of the lens
(c) the aperture of the lens
(d) the object size

12. A thin biconvex lens of focal length f is used to form a circular image of the sun on a screen placed in its focal plane. The radius of the image formed on the screen is r. Then

(a) $\pi r^2 \propto f$ (b) $\pi r^2 \propto f^2$

(c) If the focal length of the lens is doubled keeping its aperture constant, the brightness of the image will increase.

(d) If half of the lens is covered the area of image will become $\frac{\pi r^2}{2}$.

[IIT 2006]

13.

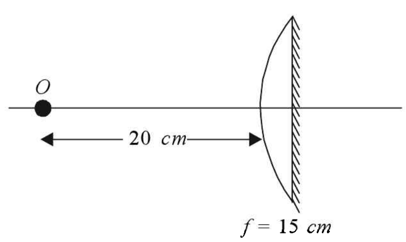


Fig. 29.53

An object is placed 20 cm in front of a plano convex lens of focal length 15 cm. The plane surface of the lnes is silvered. The image will be formed at a distance **[IIT 2006]**

(a) 60 cm to the left of the lens
(b) 60 cm to the right of the lens
(c) 12 cm to the left of the lens
(d) 12 cm to the right of the lens.

14. The distance of the eye-lens from the retina is x. For a normal eye, the maximum focal length of the eye-lens

(a) $= x$ (b) $< x$
(c) $> x$ (d) $= 2x$.

15. A point object P moves towards a convex mirror with a constant speed v, along its optic axis. The speed of the image

(a) is always $< v$
(b) may be $>$, $=$ or $< v$ depending on the position of P
(c) increases as P comes closer to the mirror
(d) decreases as P comes closer to the mirror

16. A battery-operated torch is adjusted to send an almost parallel beam of light. It produces an illuminance of 40 lux when the light falls on a wall 2 m away. The illuminance produced when it falls on a wall 4 m away is close to

(a) 40 lux (b) 20 lux
(c) 10 lux (d) 5 lux

17. Light from a point source falls on a screen. If the separation between the source and the screen is increased by 1%, the illuminance will decrease (nearly) by

(a) 0.5% (b) 1%
(c) 2% (d) 4%

18. Figure (29.54) shows a glowing mercury tube. The intensities at point A, B and C are related as

(a) $B > C > A$ (b) $A > C > B$
(c) $B = C > A$ (d) $B = C < A$.

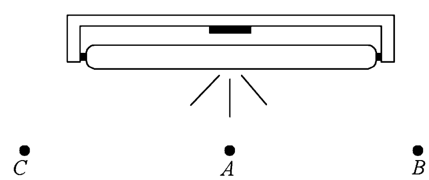


Fig. 29.54

19. The one parameter that determines the brightness of a light source sensed by an eye is

(a) energy of light entering the eye per second
(b) wavelength of the light
(c) total radiant flux entering the eye
(d) total luminous flux entering the eye

20. The intensity produced by a long cylindrical light source at a small distance r from the source is proportional to

(a) $\frac{1}{r^2}$ (b) $\frac{1}{r^3}$

(c) $\frac{1}{r}$ (d) none of these.

21. A point source of light moves in a straight line parallel to a plane table. Consider a small portion of the table directly below the line of movement of the source. The illuminance at this portion varies with its distance r from the source as

(a) $I \propto \frac{1}{r}$ (b) $I \propto \frac{1}{r^2}$

(c) $I \propto \frac{1}{r^3}$ (d) $I \propto \frac{1}{r^4}$

22. An electric bulb is hanging over a table at a height of 1 m above it. The illuminance on the table directly below the bulb is 40 lux. The illuminance at a point on the table 1 m away from the first point will be about

(a) 10 lux (b) 14 lux

(c) 20 lux (d) 28 lux

23. A photographic plate is placed directly in front of a small diffused source in the shape of a circular disc. It takes 12 s to get a good exposure. If the source is rotated by 60° about one of its diameters, the time needed to get the same exposure will be

(a) 6 s (b) 12 s

(c) 24 s (d) 48 s.

24. A photographic plate placed at a distance of 5 cm from a weak point source is exposed for 3 s. If the plate is kept at a distance of 10 cm from the source, the time needed for the same exposure is

(a) 3 s (b) 12 s

(c) 24 s (d) 48 s.

25. Three light sources A, B and C emit equal amount of radiant energy per unit time. The wavelengths emitted by the three sources are 450 nm, 555 nm and 700 nm respectively. The brightness sensed by an eye for the sources are X_A, X_B and X_C respectively. Then,

(a) $X_A > X_B, X_C > X_B$ (b) $X_A > X_B, X_B > X_C$

(c) $X_B > X_A, X_B > X_C$ (d) $X_B > X_A, X_C > X_B$

26. As the wavelength is increased from violet to red, the luminosity

(a) continuously increases

(b) continuously decreases

(c) increases, then decreases

(d) decreases, then increases.

27. The danger signals are made red because

(a) people may get frightened

(b) our eyes are most sensitive to red colour

(c) the red colour is scattered maximum

(d) the red colour is scattered minimum

28. The refracting angle of the prism is 4.5° and its refractive index is 1.52. The angle of minimum deviation will be

(a) 2° (b) 2.3°

(c) 4.5° (d) 1.5°

29. The angle of the prism for which there is no emergent ray will be, if its critical angle is i_c

(a) greater than i_c (b) less than i_c

(c) 2 i_c (d) greater than 2 i_c

30. A ray of light is incident on a glass slab of refractive index 1.52. If the reflected and refracted rays of light are mutually perpendicular to each other then the angle of incidence will be

(a) 90° (b) 60°

(c) 56°40' (d) 19°58'

31. A fish looks outside water. It is situated at a depth of 12 cm below water surface. If the refractive index of water is 4/3 then the radius of the circle through which it can see will be

(a) $12 \times \frac{3}{\sqrt{7}}$ cm (b) 12×3 cm

(c) $12 \times 3\sqrt{5}$ cm (d) $12 \times \frac{\sqrt{5}}{3}$

32. The cause of mirage in desert areas is

(a) the refractive index of atmosphere decreases with height

(b) the refractive index of atmosphere increases with height

(c) the refractive index of atmosphere does not change with height

(d) scattering

33. A glass plate inside a colourless liquid becomes invisible because

(a) the densities of both are same

(b) the refractive indices of both are same

(c) the colours of both are same

(d) liquid wets glass surface

34. An equilateral prism is lying on the prism table of a spectrometer in minimum deviation position. If the angle of incidence is 60° then the angle of deviation will be.

(a) 90° (b) 60°

(c) 45° (d) 30°

35. When a ray of light enters from one medium to another its velocity in second medium becomes double. The maximum value of angle of incidence so that total internal reflection may not take place will be

(a) 60° (b) 180°

(c) 90° (d) 30°

36. A beam of light is incident at point 1 on a screen. A plane glass plate of thickness t and refractive index n is placed in the path of light. The displacement of point will be

(a) $t\left(1-\frac{1}{n}\right)$ *nearer* (b) $t\left(1+\frac{1}{n}\right)$ *nearer*

(c) $t\left(1-\frac{1}{n}\right)$ *farther* (d) $t\left(1+\frac{1}{n}\right)$ *farther*

37. The relation between energy E and momentum p of a photon is

(a) $E = pc$ (b) $E = \frac{p}{c}$

(c) $p = Ec$ (d) $E = \frac{p^2}{c}$

38. The effective mass of photon of wavelength 40 Å will be

(a) 55.2×10^{-35} Kg (b) 55.2×10^{-33} gm

(c) 55.2×10^{-17} Kg (d) 55.2×10^{-38} Kg

39. The momentum of photon of frequency 10^{14} Hz will be

(a) 2.2×10^{-26} Kg/m/sec (b) 2.21×10^{-28} Kg/m/sec

(c) 10^{-28} Kg/m/sec (d) 0.21×10^{-2} Kg/m/sec

40. A ray of light takes 10^{-19} second to cross a glass slab of refractive index 1.5. The thickness of the slab will be

(a) 10 cm (b) 20 cm

(c) 30 cm (d) 40 cm

41. If the frequencies of an ultrasonic wave and an electromagnetic wave are same, then

(a) their wavelengths will be same

(b) wavelength of electromagnetic wave will be less than that of ultrasonic wave

(c) wavelength of electromagnetic wave will be more than that of ultrasonic wave

(d) the wavelengths of both will be nearly equal

42. The Poynting vector for an electromagnetic wave is

(a) $\vec{S} = \vec{E} \times \vec{H}$ (b) $\vec{S} = \vec{E} \times \vec{B}$

(c) $\vec{S} = \left(\vec{E} \times \vec{H}\right)/2$ (d) $\vec{S} = \left(\vec{E} \times \vec{B}\right)/2$

43. The total energy density for an electromagnetic wave in vacuum is

(a) $e_0 \frac{E^2}{3}$ (b) $e_0 E^2$

(c) $\frac{\varepsilon_0 E^2}{2}$ (d) $\frac{E^2}{\varepsilon_0}$

44. If radiations are incident obliquely on a perfectly reflecting surface then the pressure exerted by radiation on the surface will be

(a) $\frac{2}{3} e_0 E^2$ (b) $\frac{1}{3} e_0 E^2$

(c) $e_0 E^2$ (d) $\frac{\varepsilon_0 E^2}{4}$

45. Out of the following, whose velocity is equal to that of light?

(a) b-rays (b) Sound waves

(c) Ultrasonic waves (d) Thermal waves

46. The correct formula for intensity of electromagnetic wave is

(a) $I = <P>$ (b) $I = c<u>$

(c) $I = \frac{\varepsilon_0 E^2}{2}$ (d) $I = \frac{\varepsilon_0 E^2}{4}$

47. The hours in a clock are marked by points. When it is put in front of a mirror and seen in the mirror, then time noted is 8.20 The correct time is

(a) 4 : 40 (b) 8 : 20

(c) 2 : 40 (d) 3 : 40

48. The correct curve between the energy of photon (E) and its wavelength (l) is

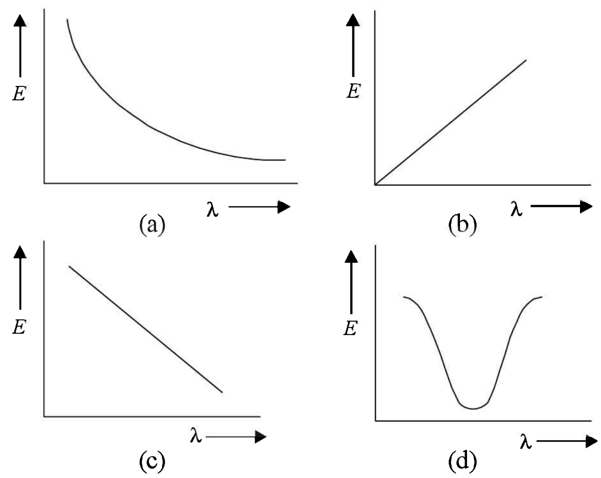


Fig. 29.55

49. All particles of a wave front vibrate

(a) in same phase (b) in opposite phase

(c) up and down (d) left and right

50. The unit of Poynting vector is

(a) Watt (b) Joule

(c) $\frac{\text{Watt}}{\text{m}^2}$ (d) $\frac{\text{Joule}}{\text{m}^2}$

51. For the propagation of electromagnetic waves

(a) medium is required (b) no medium is required

(c) E and B are in mutually opposite phase

(d) E and B are in the same phase

52. When a ray of light enters from air into water then its wavelength

(a) decreases (b) increases
(c) remains unchanged (d) becomes infinity

53. The value of $\frac{1}{\sqrt{\mu_0 \varepsilon_0}}$

(a) 3×10^6 m/s (b) 3×10^8 m/s
(c) 3×10^4 m/s (d) 332 m/s

54. The radius of a wavefront as the waves propagate

(a) decreases (b) increases
(c) becomes zero
(d) sometimes decreases and sometimes increases.

55. The ratio of E to B in electromagnetic waves is equal to

(a) c (b) $\frac{1}{c}$
(c) $4c$ (d) $\frac{1}{Z}$

56. The transverse nature of light waves is verified by

(a) reflection of light (b) polarisation of light
(c) refraction of light (d) interference of light

57. The velocity of light in a piece of matter is v. The thickness of the piece is t and its refractive index is μ. The distance travelled by light in air in time t/v is

(a) μt (b) μt^2
(c) μt^3 (d) μt^4

58. The spin of photon is

(a) $\hbar$ (b) $\frac{\hbar}{2}$
(c) $\frac{\hbar}{3}$ (d) none

59. The cause of shining in diamond is

(a) scattering of light
(b) refraction of light
(c) total internal reflection of light
(d) dispersion of light.

60. An optical fibre (m = 1.72) has coating of glass (m = 1.5). The critical angle for total internal reflection is

(a) $\sin^{-1}\left(\frac{75}{86}\right)$ (b) $\sin^{-1}\left(\frac{86}{75}\right)$
(c) $\sin^{-1}(0.8)$ (d) $\sin^{-1}(0.82)$.

61. Which of the following colours is scattered minimum?

(a) Violet (b) Blue
(c) Red (d) Yellow

62. The image in Figure shown Ts formed at

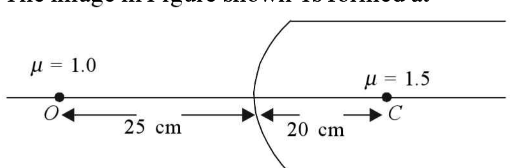


Fig. 29.56

(a) + 100 cm (b) – 100 cm
(c) + 80 cm (d) – 80 cm.

63. In the absence of atmosphere, the sky appears

(a) coloured (b) blue
(c) indigo (d) black

64. For total internal reflection the ray of light enters

(a) from rarer to denser medium
(b) from denser to rarer medium
(c) medium of same refractive index
(d) medium with same coefficient of reflection

65. The speeds of light in two media I and II are 2.2 × 10 m/s and 2.4×10^8 m/s respectively. The critical angle for light refracting from I to II medium will be

(a) sin $-\frac{112}{11}$ (b) sin $-\frac{111}{12}$
(c) sin $-\frac{121}{24}$ (d) sin $-\frac{124}{21}$

66. If n_{ag} = 3/2 and n_{aw} = 4/3, then the refractive index of glass with respect to water will be

(a) $\frac{9}{8}$ (b) $\frac{8}{9}$
(c) 4 (d) $\frac{1}{4}$

67. If the refractive index of water is 4/3 and that of glass is 5/3, then the critical angle of light entering from glass into water will be

(a) sin $-\frac{14}{5}$ (b) sin $-\frac{15}{4}$
(c) sin $-\frac{11}{2}$ (d) sin $-\frac{12}{1}$

68. A ray of light is incident on an equilateral prism in such a way that the angle of incidence is equal to the angle of emergence and each is equal to 3/4 of the prism angle. The angle of deviation for the ray of light is

(a) 45° (b) 30°
(c) 39° (d) 20°

69. An equilateral prism has μ = 1.732. The angle of incidence for minimum deviation is

(a) 60° (b) 30°
(c) 45° (d) none of these

70. A biconvex lens has focal length of 25 cm. The radius of curvature of one the surfaces is double of the other. Find the radii. Given μ_{lens} = 1.5

(a) 37.5 cm, 75 cm (b) 18.75 cm, 37.5 cm
(c) 7.5 cm, 15 cm (d) 15 cm, 30 cm

71. The sun appears elliptical before sun set because of

(a) refraction (b) reflection

(c) scattering

(d) sun contracts itself at that time.

72. Sunlight can undergo internal reflection if it enters from

(a) glass to air (b) air to glass

(c) air to water (d) water to glass.

73. The ratio of refractive indices of red and blue light in air will be

(a) $n_{12} < 1$ (b) $n_{12} > 1$

(c) $n_{12} = 1$ (d) $n_{12} = 8$

74. The refractive index of diamond is 2. The velocity of light (in cm/sec) in diamond will be

(a) 1.5×10^{10} (b) 2×10^{13}

(c) 6×10^{10} (d) 3×10^{10}

75. When a ray of light is made incident upon an isosceles right angle prism, then the following event takes place:

(a) reflection (b) total internal reflection

(c) refraction (d) dispersion

76. You have to design a compound microscope with objective lens of focal length 1 cm. The object should be placed at _____ distance from the lens.

(a) 8 mm (b) 11 mm

(c) 22 mm (d) 2 cm.

77. A 5 D lens forms an image four times the size of an object. The objects distance is

(a) 15 cm (b) 16 cm

(c) 18 cm (d) 12.5 cm.

78. A pencil dipped partially into water appears bent because of

(a) reflection at water surface

(b) diffraction at water surface

(c) refraction at water surface

(d) water is flowing

79. A particle is moving with a speed v along the principal axis towards a concave mirror of radius of curvature R. The speed of the image as observed is

(a) $\frac{R^2v}{(2x-R)^2}$ (b) $\frac{R^2v}{(x-R)^2}$

(c) $\frac{2R^2v}{(2x-R)^2}$ (d) $\frac{R^2v}{2(x-R)^2}$

80. The correct curve between refractive index n and wavelength A will be

(a) A (b) D

(c) B (d) C

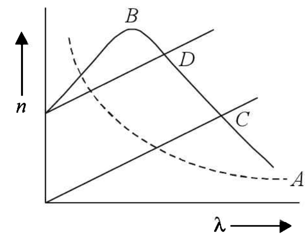


Fig. 29.57

81. A red flower when viewed through blue light appears

(a) red (b) blue

(c) black (d) violet

82. The frequency and wavelength of light in a material are 4×10^{14} Hz and 5×10^{-7} meter. The refractive index of material is

(a) 1.33 (b) 1.5

(c) 1 (d) 0.77

83. Electromagnetic flux of 1380 watt/m^2 is obtained on earth from the sun. The total power incident on 25 m × 50 m surface will be

(a) 1.725×10^6 watt (b) 3.45×10^6 watt

(c) 6.9×10^6 watt (d) 1.38×10^7 watt

84. An astronomical telescope with magnification 50 is to be designed in normal adjustment. Length of the tube is 102 cm. The powers of objective and eyepiece are respectively

(a) $2D, 50D$ (b) $1.5D, 20D$

(c) $1D, 40D$ (d) $1D, 50D$

85. The maximum value of E in an electromagnetic wave propagating in X-direction is 1000 Newton/Coulomb which is in Z-direction. The value of magnetic field at that point will be (in Wb/m^2)

(a) 7.5×10^{-6} in X-direction

(b) 3.33×10^{-6} in Y-direction

(c) 6×10^{-7} in Z-direction

(d) 10^{-5} in any other direction.

86. The maximum electric field at a distance of 11.2 m from a point source is 1.96 v/m. The maximum magnetic field will be (in nanotesla)

(a) 6.53 (b) 9.87

(c) 2.38 (d) 7.99

87. In Q. 86 the output power of the source will be

(a) 80.4 watt (b) 804 watt

(c) 0.804watt (d) 8.04 watt

88. The lens in the Fig. 29.58 is equiconvex ($\mu = 1.5$). The radius of curvature of the lens is

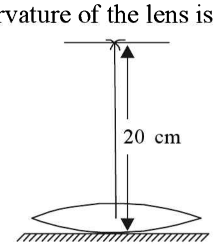


Fig. 29 .58

(a) 15 cm (b) 20 cm
(c) 40 cm (d) none

89. Assume you are sitting in sun for 2.5 hours. The area of your body exposed normally to sun rays is 1.3 m². The intensity of sun rays is 1.1 kilowatt/m². If your body completely absorbs the sun rays then the momentum transferred to your body will be (in Kg-m/s)

(a) 0.043 (b) 0.037
(c) 0.61 (d) –0.91

90. The value of electric field in an electromagnetic wave originating from a point source of light at a distance of 10 meter is $E = 500$ volt/m. The electric field at a distance of 5 meter will be

(a) 1000 Volt/meter (b) 200 Volt/meter
(c) 50 Volt/meter (d) 25 Volt/meter

91. If the relative permeability of a medium is μ_r and its dielectric constant is ε_r then the velocity of light in that medium will be

(a) $\frac{1}{\sqrt{\mu_r \varepsilon_r}}$ (b) $\frac{1}{\sqrt{\mu_r \varepsilon_r}}$
(c) $\sqrt{\mu_r \varepsilon_r / \mu_{\varepsilon_0}}$ (d) $\sqrt{\mu_0 \varepsilon_0 / \mu_r \varepsilon_r}$

92. The correct statement from the following is:

(a) Light exhibits particle nature in propagation and wave nature in mutual interaction with matter.
(b) Light exhibits both wave nature and particle nature in mutual interaction with matter.
(c) Light exhibits both wave and particle nature in propagation.
(d) Light exhibits wave nature in propagation and panicle nature in mutual interaction with matter.

93. A sphere ($\mu = 1.5$) has a small bubble 6 cm from the centre. Radius of the sphere is 10 cm. When seen normally from shorter side the bubble appears

(a) 3 cm below the surface
(b) 3 cm above the surface
(c) 7 cm inside the surface
(d) 4.6 cm below the surface

94. A magnifying glass has $f = 12$ cm. Where shall an object be placed to produce maximum angular magnification? Least distance of clear vision = 25 cm.

(a) 7.1 cm (b) 6.8 cm
(c) 8.4 cm (d) 8.1 cm

95. A plane mirror and a concave mirror are 50 cm apart. An object is 30 cm from a concave mirror such that image of the two coincide. The focal length concave mirror is

(a) 21 cm (b) 18 cm
(c) 15 cm (d) none of these

96. Photon is a

(a) Fermion (b) Boson
(c) Nucleon (d) Baryon

97. The chromatic aberration in Huygen's eyepiece is corrected using $f_1 = 3 f_2$ and separation between the lenses is

(a) f_2 (b) $\frac{3}{2} f_2$
(c) $1.2 f_2$ (d) $2 f_2$

98. The correct statement out of the following is:

(a) The nature of elecrtromagnetic radiations in travelling from one place of another is wave nature.
(b) The nature of electromagnetic radiations in mutual interaction with matter is photon.
(c) The main cause of microwaves being unfit for vision is the particle nature of electromagnetic waves.
(d) All of above.

99. The correct statement out of the following is:

(a) The wave theory and quantum theory both are valid for the whole electromagnetic spectrum.
(b) Wave theory is valid for long wavelength region and quantum theory is valid for short wavelength region.
(c) Wave theory is valid for short wavelength region whereas the quantum theory is valid for long wavelength region.
(d) Wave theory and quantum theory both are valid for short wavelength region.

100. If the velocity of light in glass is 2×10^8 m/s then its velocity in water will be, if $n_g = 1.5$ and $n_w = 4/3$,

(a) 3×10^8 m/s (b) 2.66×10^8 m/s
(c) 1.5×10^5 m/s (d) 2.25×10^8 m/s

101. If the velocity of light in water is 2.25×10^8 m/s then its velocity in carbondisulphide will be (n for carbon-disulphide = 1.63)

(a) 1.84×10^8 m/s (b) 2.25×10^8 m/s
(c) 2×10^8 m/s (d) 3×10^8 m/s

102. The refractive index of glass is 1.5 and velocity of light in vacuum is 3×10 m/s. Time taken by light in traveling 500 m in glass will be

(a) 1 μs (b) 1.5 μs
(c) 4.5 μs (d) 2.5 μs

103. A point object O is placed midway between the two converging mirrors of focal length f each. Find d so that object and image coincide

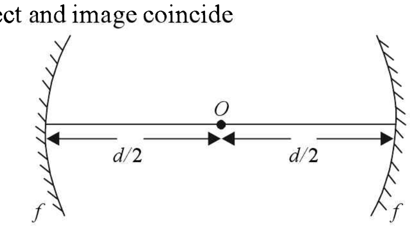


Fig. 29.59

(a) $2f, 4f$ (b) $2f, 3f$
(c) $4f$ (d) $2f, f$

104. A light wave of frequency 5×10 Hz passes through a medium of refractive index 2.4. Its wavelength in the medium will be

(a) 1×10^{-7} m (b) 4×10^{-7} m
(c) 3.3×10^{-7}m (d) 2.5×10^{-7}m

105. The effective mass of photon in microwave region, visible region and x-ray region is in the following order:

(a) microwave > x-ray > visible
(b) x-ray > microwave > visible
(c) microwave > visible > x-ray
(d) x-ray > visible > microwave

Answers to Questions for Practice

1.	(b)	**2.**	(d)	**3.**	(d)	**4.**	(c)	**5.**	(b)	**6.**	(a)	**7.**	(b)
8.	(c)	**9.**	(d)	**10.**	(a)	**11.**	(b)	**12.**	(b)	**13.**	(c)	**14.**	(a)
15.	(a)	**16.**	(a)	**17.**	(c)	**18.**	(d)	**19.**	(d)	**20.**	(c)	**21.**	(c)
22.	(b)	**23.**	(c)	**24.**	(b)	**25.**	(c)	**26.**	(c)	**27.**	(d)	**28.**	(b)
29.	(d)	**30.**	(c)	**31.**	(a)	**32.**	(b)	**33.**	(b)	**34.**	(b)	**35.**	(d)
36.	(a)	**37.**	(a)	**38.**	(a)	**39.**	(b)	**40.**	(b)	**41.**	(c)	**42.**	(a)
43.	(b	**44.**	(a)	**45.**	(d)	**46.**	(b)	**47.**	(a)	**48.**	(a)	**49.**	(a)
50.	(c)	**51.**	(b)	**52.**	(a)	**53.**	(b)	**54.**	(b)	**55.**	(a)	**56.**	(b)
57.	(a)	**58.**	(a)	**59.**	(c)	**60.**	(a)	**61.**	(c)	**62.**	(b)	**63.**	(d)
64.	(b)	**65.**	(b)	**66.**	(a)	**67.**	(a)	**68.**	(b)	**69.**	(a)	**70.**	(b)
71.	(a)	**72.**	(a)	**73.**	(a)	**74.**	(a)	**75.**	(b)	**76.**	(b)	**77.**	(a)
78.	(c)	**79.**	(a)	**80.**	(a)	**81.**	(b)	**82.**	(b)	**83.**	(a)	**84.**	(a)
85.	(b)	**86.**	(a)	**87.**	(c)	**88.**	(b)	**89.**	(a)	**90.**	(a)	**91.**	(b)
92.	(d)	**93.**	(a)	**94.**	(d)	**95.**	(a)	**96.**	(b)	**97.**	(d)	**98.**	(d)
99.	(b)	**100.**	(d)	**101.**	(a)	**102.**	(d)	**103.**	(a)	**104.**	(d)	**105.**	(d)

30

Wave Optics

BRIEF REVIEW

Interference If light waves emitted from two coherent sources superpose then it results in variation of intensity with distance. At certain places intensity is maximum and at other places intensity is minimum. This phenomenon is called interference.

Coherent Sources Two sources/wave trains are said to be coherent if there is a constant or zero phase difference between them. No two different sources (except lasers) could be coherent. Coherent sources are to be derived from a single source. Their state of polarisation remains same. Laser is considered highly coherent.

If coherent sources have phase shift ϕ then $\phi \neq f(t)$ and $\frac{d\phi}{dt} = 0$. Coherent sources can be obtained by division of wave front or by division of amplitude.

Coherent source obtained with

division of wave front	division of amplitude
(i) YDSE (young's double slit experiment)	(i) Newton's rings
(ii) Fresnel biprism	(ii) Thin films
(iii) Lloyd's mirror	(iii) Interferometer

Wave front is the locus of all adjacent parts at which the phase of vibration of a physical quantity associated with the wave is the same. That is, at any instant, all points on a wave front are at the same part of the cycle of their vibration. Wave fronts in general may be of three types: (a) Spherical (b) Cylindrical (c) Plane or Planar

Spherical wave fronts are generated from a **point source** or **circular slit.**
Cylindrical wave front results from a line source or rectangular slit.
Plane wave front is either of the two if the source is at infinity.

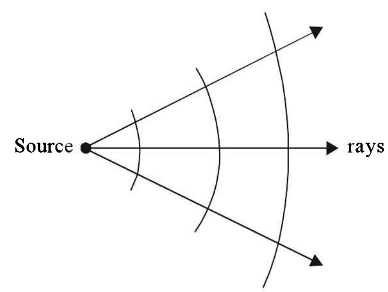


Fig. 30.1 (a) Wave front (Spherical)

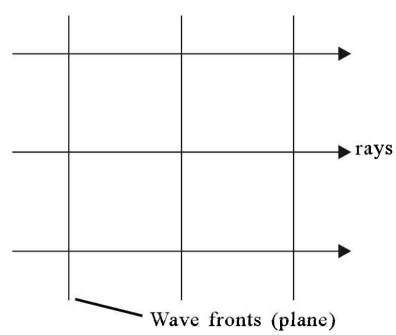


Fig. 30.1 (b) Illustrations of plane wave front

Constructive interference occurs when the coherent waves **superpose in phase** or the **path difference** is **integral** multiple of the **wavelength** or even multiple of half the wavelength. This type of interference is also called reinforcement as light intensity increases, i.e., bright fringes are formed. We may

call such points or curves as **antinodal**. See Fig 30.2 (a)
Destructive interference occurs when the coherent waves **superpose out of phase** or **path difference** is an odd multiple of half the wavelength. Dark fringes are formed. We may call such points or curves as **nodal** as illustrated in Fig 30.2 (b)

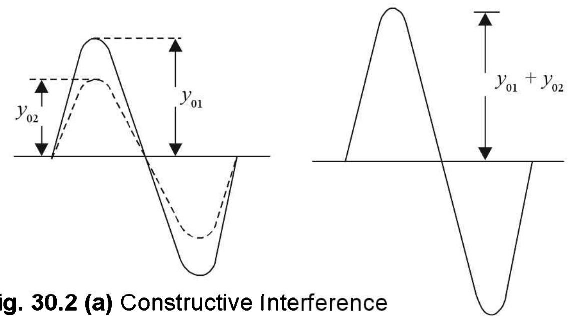


Fig. 30.2 (a) Constructive Interference

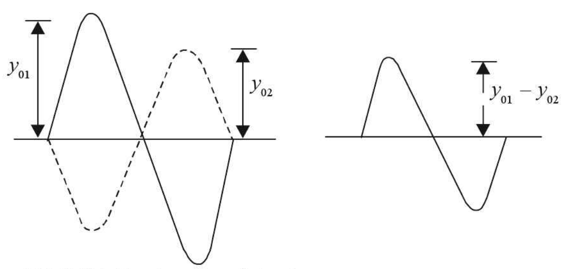


Fig. 30.2 (b) Destructive Interference

Path differences $\Delta x = n\lambda$ for constructive interference

Path difference $\Delta x = (2n+1)\dfrac{\lambda}{2}$ for destructive interference

$$\frac{I_{\text{bright}}}{I_{\text{dark}}} = \frac{I_{\max}}{I_{\min}} = \left(\frac{y_{01}+y_{02}}{y_{01}-y_{02}}\right)^2 = \left(\frac{\sqrt{I_1}+\sqrt{I_2}}{\sqrt{I_1}-\sqrt{I_2}}\right)^2$$

Conditions to obtain sustained interference

Necessary condition The two sources emitting waves must be coherent.

Desirable conditions (i) Sources should be monochromatic having same frequency. (ii) They shall have same amplitude (iii) They shall emit light continuously (iv) The separation between the two sources shall be small.

In YDSE

Fringe width $\beta = \dfrac{\lambda D}{d}$ (Difference between two successive dark or bright fringes, i.e., $\beta = x_n - x_{n-1} = \dfrac{\lambda D}{d}$

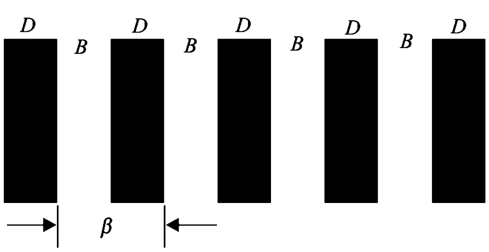


Fig. 30.3 Fringe pattern in YDSE

$$x_n = \frac{n\lambda D}{d} \text{ for nth bright fringe}$$

$$x_n = \frac{(2n-1)\lambda D}{2d} \text{ for nth dark fringe}$$

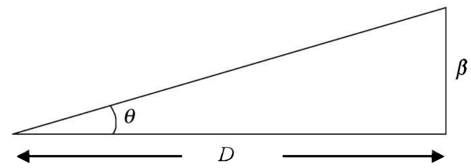


Fig. 30.4 Angular Fringe Width

Angular fringe width $\theta = \dfrac{\lambda}{d} = \dfrac{\beta}{D}$ (in radian)

$$= \frac{\lambda}{d} \times \frac{180}{\pi} \text{ (in degrees)}$$

Fringe Visibility $= \dfrac{I_{\max}-I_{\min}}{I_{\max}+I_{\min}} = \dfrac{\sqrt{2I_1I_2}}{I_1+I_2}$

Intensity at any point $I = 2y_0^2(1+\cos\delta) = 4I'\cos^2\left(\dfrac{\delta}{2}\right)$.

Assuming both sources emit waves of equal amplitude y_0 or equal intensity I'. δ is phase shift between two superposing waves

$$I = I_1 + I_2 + 2\sqrt{I_1}\sqrt{I_2}\cos\delta$$

$$= \left(y_{01}^2 + y_{02}^2 + 2y_{01}y_{02}\cos\delta\right)$$

if intensities or amplitude of superposing waves are unequal.

If YDSE is immersed in a liquid of refractive index μ then fringes shrink and hence fringe pattern shrinks.

$$\beta_{\text{new}} = \frac{\beta}{\mu} = \frac{\lambda D}{\mu d} \text{ or } x_{n\,(\text{new})} = \frac{x_n}{\mu} = \frac{n\lambda D}{\mu d} \text{ for } nth$$

bright fringe.

If a thin slice of thickness t and refractive index μ is inserted in front of one of the slits in YDSE, then central fringe shifts to a position where originally nth fringe was formed such that $(\mu-1)\,t = n\,\lambda$ or $\Delta x = \dfrac{D(\mu-1)t}{d}$

In Fresnel biprism both the sources S_1 and S_2 are virtual as shown in Fig. 30.5.

$D = a + b$

$d = 2a\,\delta = 2a(\mu-1)\,\alpha$

where α is angle of biprism.

$$\beta = \frac{\lambda D}{d} = \frac{\lambda(a+b)}{2a(\mu-1)\alpha}$$

$$x_n = \frac{n\lambda D}{d} = \frac{n\lambda(a+b)}{2a(\mu-1)\alpha} \text{ for } nth \text{ bright fringe.}$$

$$x_n = \frac{(2n-1)\lambda D}{2d} = \frac{(2n-1)\lambda(a+b)}{4\alpha(\mu-1)\alpha} \text{ for } n\text{th dark}$$

fringe.

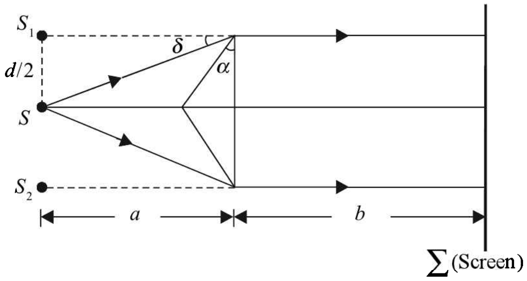


Fig. 30.5 Fringe pattern in fresnel biprism

If displacement method is used then $d = \sqrt{d_1 d_2}$

If Fresnel biprism is immersed in a liquid of refractive index μ', then

$$\beta_{new} = \frac{\frac{\lambda}{\mu'}(a+b)}{2a\left(\frac{\mu}{\mu'}-1\right)\alpha} = \frac{\lambda(a+b)}{2a(\mu-\mu')\alpha}$$

In Lloyd's Mirror: Condition of nth bright and dark fringe obtained in Lloyd's mirror gets reversed to what was obtained in YDSE; because of reflection an additional phase shift of π or an additional path difference $\frac{\lambda}{2}$ is achieved.

That is, $x_n = \frac{n\lambda D}{d}$ for nth dark fringe

and $x_n = \frac{(2n-1)\lambda D}{2d}$ for nth bright fringe.

In Lloyd's mirror one of the sources is real and other is virtual or image source.

Path difference $= 2\,\mu t \cos r = (2n+1)\,\frac{\lambda}{2}$ for nth bright fringe and $2\,\mu t \cos r = n\lambda$ for nth dark fringe. In reflected light

$$\left.\begin{aligned}\text{Path difference } 2\,\mu t\cos r &= n\lambda\\ 2\mu t\cos r &= (2n+1)\frac{\lambda}{2}\end{aligned}\right] \text{ for refracted or transmitted light}$$

Wedge Shaped Film

Fringe Width $\beta = \frac{\lambda}{2\theta}$, since

$$\theta = \frac{t}{x_n}, \text{ Therefore } \beta = \frac{\lambda x_n}{2t}$$

If plates are kept in a liquid of refractive index μ

$$\beta = \frac{\lambda}{2\mu\theta} = \frac{\lambda x_n}{2\mu t} \text{ or } 2\,\mu t = n\lambda$$

$t_{min} = \frac{\lambda}{2}$. It is due to interference that a soap bubble appears bright colour or oil drops spilled on road in rainy season appear of brilliant hue.

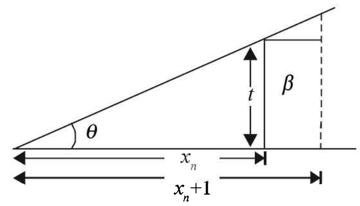


Fig. 30.6 Wedge shaped film

Time of coherence (t_c) is the time during which electric field vector is in the sinusoidal form. Its value is 10^{-10} *s*. **Coherence Length** $L_C = ct_c$. Note that if path difference $> L_C$, coherence nature is lost. Therefore we cannot keep distance between two slits or sources > 3 *cm*.

Diffraction The bending of wave from the obstacles of size of the order of wavelength is termed as diffraction. Planar or plane wave front is required for diffraction to take place. Diffraction is of two types (a) Fresunel's class of diffraction (b) Fraunhoffer class of diffraction.

Table. 30.1

	Fresnel class	Fraunhoffer class
1.	The source is at a finite distance.	The source is at infinite distance.
2.	No optical aid is required.	Optical aid in the form of collimating lens and focusing lens are required.
3.	Fringes are not sharp and well defined.	Fringes are sharp and well defined

Table. 30.2

	Interference	Diffraction
1.	Fringes are formed due to superposition of wave trains emitted from two coherent sources.	Fringes are formed due to superposition of bent rays due to superposition of secondary wavelets.
2.	Intensity of each fringe is equal	Intensity falls as the fringe order increases.
3.	Number of fringes is and quite large.	Number of fringes is finite (small).
4.	Fringe width is equal for each fringe.	Fringe width of primary and secondary maxima are different.

Huygen's Principle

1. Each point on the primary wavefront is a source of secondary wavelets.
2. Secondary wavelets move only in forward direction.
3. Secondary wavelets can superpose to produce disturbances.

4. Secondary wavelets as well as primary wavefronts move with c (speed of light).

Diffraction from a single slit

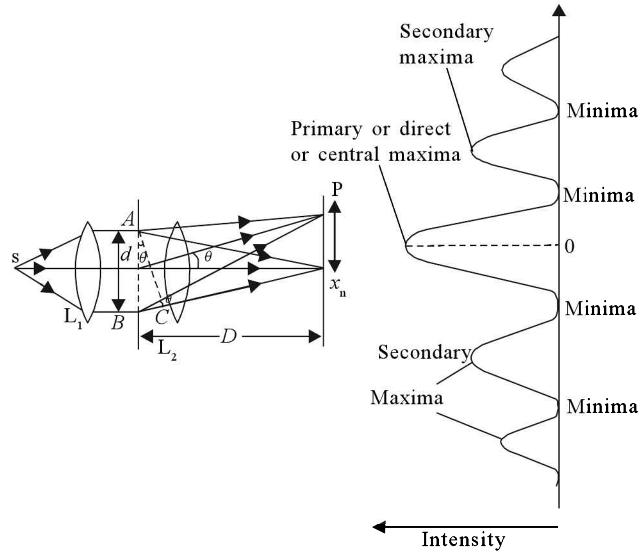


Fig. 30.7 Single slit diffraction

Path difference $= BC = AB \sin\theta$

$= d \sin\theta$

For minima

$d \sin\theta = n\lambda$

$\sin\theta = \tan\theta = \dfrac{x_n}{D}$. Thus $\dfrac{dx_n}{D} = n\lambda$

or $x_n = \dfrac{n\lambda D}{d}$ for nth minima.

Note $D = f$ (of focussing lens)

Fringe width $\beta_{primary} = \dfrac{2\lambda D}{d}$ and

$\beta_{secondary} = \dfrac{\lambda D}{D}$ (fringe width for secondary maxima is half the primary maxima)

Angular fringe width $\beta_{primary} = \dfrac{2\lambda}{d}$ (radian)

$= \dfrac{2\lambda}{d} \times \dfrac{180}{\pi}$ (degrees)

Angular fringe width $\beta_{secondry} = \dfrac{\lambda}{d}$ (radian)

$= \dfrac{\lambda}{d} \times \dfrac{180}{\pi}$ (degree)

If $\beta = \dfrac{\pi d \sin\theta}{\lambda}$ then $I = \dfrac{I_0 \sin^2\beta}{\beta^2}$

If aperture is circular then $\sin\theta = \dfrac{1.22\lambda}{r}$ where r is radius of aperture.

Radius of first dark ring $R = \dfrac{1.22\lambda D}{r} = \dfrac{1.22\lambda f}{r}$

Polarisation If plane of vibration is fixed then light will travel in a single direction. Such a state is called plane polarised light.

In the Fig. 30.8 electric field varies along y-axis and magnetic field along z-axis, wave travels along x-axis, plane of polarisation is $y - z$.

If $E_y = E_o \sin(\omega t - kx)$ is the electric field along y-axis and $B_z - B_o \sin(\omega t - kx)$ is the magnetic field active along z-axis then wave progresses in x-direction.

Only transverse waves can be polarised, longitudinal waves cannot be polarised. Plane polarised light can be achieved using

(a) reflection (b) refraction
(c) scattering (d) Nicol prism
(e) birefracting crystals.

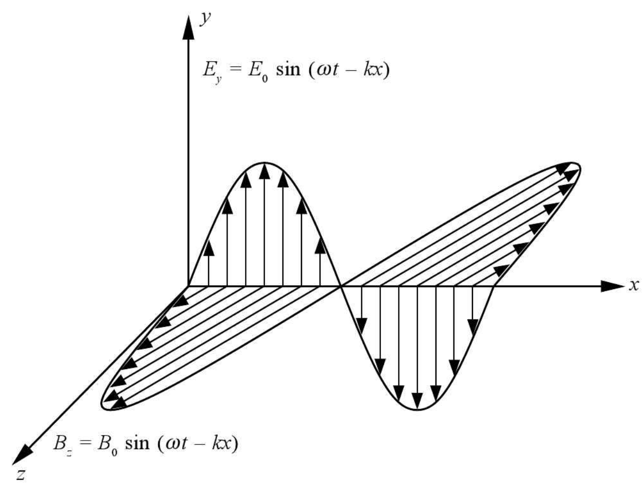


Fig. 30.8 Plane polarised light

Brewester's Law If light is incident on the interface of two media such that the angle between reflected and refracted radiations is 90° then reflected rays are completely polarised. Angle of incidence is called angle of polarisation (θ_p).

Then $\mu = \tan\theta_p$

Malus Law When the plane of polarisation is rotated by an angle θ then intensity of emergent light is given by $I = I_o \cos^2\theta$. I_o is intensity of incident polarised light. In birefracting analysis there are two rays – ordinary and extraordinary. The extraordinary ray does not follow law of refraction. If the **velocity** of **extraordinary ray** is **greater** than that of **ordinary ray** such crystals are called **negative crystals**. Examples of negative crystal are Iceland spar, tourmaline, sapphire, ruby, emerald and apatite. If the **ordinary ray** has **higher velocity** than the **extraordinary ray** then such crystals are called **positive crystals**. Examples of positive crystals are quartz, iron oxide.

If the amplitude of two waves are unequal and angle between the two is $\dfrac{\pi}{2}$ or path difference is $\dfrac{\lambda}{4}$ then an elliptically polarised wave front results, it could be elliptically polarised if amplitudes are equal but the angle between the two is $0 < \theta < \pi/2$.

- **Short Cuts and Points to Note**

1. Coherent sources are those in which wave trains have constant or zero phase difference. The coherent sources cannot be two separate sources except lasers. Normally they are derived from a single source either by division of wave front or by division of amplitude.
2. If two slits have unequal sizes (they act like intensity, the intensity of the resultant is

$$I = \left(\sqrt{I_1}\right)^2 + \left(\sqrt{I_2}\right)^2 + 2\sqrt{I_1}\sqrt{I_2}\cos\theta$$
$$= I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\theta$$
$$= k(S_1 + S_2 + 2\sqrt{S_1 S_2}\cos\theta)$$ where S_1 and S_2 are size of the slits.

3. Coherent length $l_{\text{coherence}} = \dfrac{\lambda^2}{\Delta\lambda}$. Coherence radius

$$\rho_{\text{coh}} = \frac{\lambda}{\phi}, \beta = \frac{\phi}{2}.$$

4. In YDSE maximum intensity occurs at $d\sin\theta = n\lambda$ and minimum intensity occurs at $d\sin\theta = (2n+1)\dfrac{\lambda}{2}$.
When interference from narrow slit is studied (slit width << λ) Then
$E(\theta) = E_m\cos\beta = 2E_o\cos\beta$ and
$I(\theta) = I_m\cos^2\beta = 4I_o\cos^2\beta$.
When slit is not so narrow then, position of nth bright fringe $x_n = \dfrac{n\lambda D}{d}$ fringe width $\beta = \dfrac{\lambda D}{d}$ and

angular fringe width $= \dfrac{\lambda}{d}$ (rad) $= \dfrac{\lambda}{d} \times \dfrac{180}{\pi}$ (degree)

$$x_n = \frac{(2n-1)\lambda D}{2d}$$ for nth dark fringe.

5. If the light reaching the point P is direct, or transmitted (not reflected) from two sources then P will be a bright fringe if the path difference $= n\lambda$.
On the other hand, if the light reaching P after reflection forms a bright fringe (at P) then path difference $= (2n+1)\dfrac{\lambda}{2}$ because reflection causes an additional path difference of $\dfrac{\lambda}{2}$ (or phase difference π radian).
6. If the interference occurs due to reflected light, central fringe (or ring in Newton's rings) will be dark.
If the interference occurs due to transmitted or direct light, central fringe will be bright.
7. If white light is used in YDSE, central fringe is white surrounded by coloured fringes in VIBGYOR order as illustrated in Fig. 30.9.

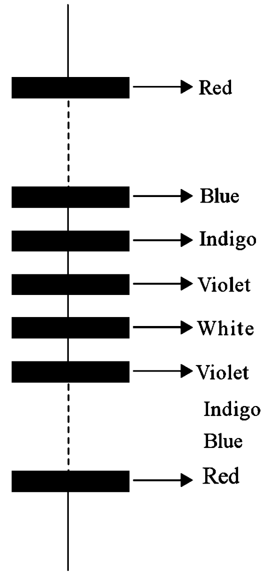


Fig. 30.9 Interference due to white light

8. Each fringe in YDSE has equal intensity while in diffraction intensity falls as the fringe order increases.
9. To locate the central fringe in YDSE, illuminate it with white light. The central fringe is white.
10. Fringes can be displaced by introducing a thin slice in front of one of the slits or in front of both the slits. If t is the thickness of the slice in front of one of the slits and μ its refractive index then $(\mu - 1)t = n\lambda$ describes the shift. (The central fringe now occupies the position which was previoulsy possessed by nth fringe) OR $\Delta x = \dfrac{D}{d}(\mu - 1)t$.
11. The fringes shrink by $\dfrac{1}{\mu}$ if YDSE is immersed in a liquid of refractive index $\mu > 1$.
12. Fresnel distance is the distance travelled by a beam without much broadening by diffraction. $z_F = \dfrac{a^2}{\lambda}$
Size of the Fresnel zone $a_F = \sqrt{\lambda z}$. Note that a is slit width.
13. In Fresuel biprism $d = 2a(\mu - 1)\alpha$ and $D = a + b$ so $x_n = \dfrac{n\lambda(a+b)}{2a(\mu-1)\alpha}$ for nth bright fringe. If displacement method is employed to find d then $d = \sqrt{d_1 d_2}$ where d_1 and d_2 are distance between images of virtual source in magnified and diminished cases.

14. In Newton's rings experiment, radius of nth ring is given by $r_n = \sqrt{n\lambda R}$ where R is radius of curvature of plano convex lens.

15. For nth dark fringe in thin films, $2\,\mu t \cos r = n\lambda$ for nth dark fringe in reflected light. For nth dark fringe in thin films in refracted or transmitted light $2\,\mu t =$

$(2n+1)\,\dfrac{\lambda}{2}$.

In wedge shaped films, for nth dark fringe $2\,\mu t = n\lambda$ if immersed in a liquid of refractive index μ.

In air $\mu = 1$, $t = \dfrac{n\lambda}{2}$ or $t_{min} = \dfrac{\lambda}{2}$

[Maximum number of fringes = 1,50,000 called Haidenger fringes]. Thickness of non reflective coating on a glass is $t = \dfrac{\lambda_{air}}{\mu 4}$ where μ is refractive index of coating.

16. In a zone plate $f_n = \dfrac{r_n^2}{(2p+1)n\lambda}$.

17. Diffraction occurs due to planar wave front. Fresnel diffraction is near field diffraction while Fraunhoffer diffraction is far-field diffraction.

18. Bragg's Law in diffraction of X-rays from crystals. $2d \sin\theta = n\lambda$

19. In diffraction grating if there are N slits/lines per inch then grating element $(a + b) = \dfrac{2.54}{N}$ and $(a + b)\sin\theta = n\lambda$ where n is order of the spectrum resolving power of grating is $\dfrac{\lambda}{d\lambda} = n$N.

20. If white light is used in single slit experiment, central fringe will be white followed by coloured fringes in VIBGYOR order.

21. Resolving power of a prism $\dfrac{td\mu}{d\lambda}$ where t is length of the base.

22. Only transverse waves can be polarised. Sound waves being longitudinal cannot be polarised.

23. The crystals in which **ordinary ray travels faster** than **extraordinary ray** or $\mu_{extraordinary} > \mu_{ordinary}$ are called **positive** crystals.
The crystals in which extraordinary ray travels faster than ordinary ray or $\mu_{ordinary} > \mu_{extraordinary}$ are called **negative** crystals.

24. The substances which rotate the plane of polarisation are called optically active. The substances which rotate the plane of polarisation to its **left** or **anti-clockwise** are called **Leveo rotatory** and the substances which rotate the plane of polarisation to its **right** or **clockwise** are called **dextrorotatory**.

25. According to Brewester's law $\mu = \tan\theta_p$ where θ_p is polarising angle (angle of incidence when angle between reflected and refracted rays is 90°).

26. The intensity of plane polarised light is $I_o/2$ if incident unpolarised light has intensity I_o. Malus law is $I = I_o \cos^2\theta$.

27. The sources like lasers are highly monochromatic and coherent.

28. Though sodium light gives a doublet, D_1 and D_2 lines of wavelength 589 $A°$ and 5896 $A°$. It may be considered monochromatic for most of the experiments.

29. For point sources or spherical wave fronts, intensity $I \propto \dfrac{I}{r^2}$. For cylindrical sources, amplitude $A \propto \dfrac{1}{\sqrt{r}}$, r being distance from the source.

30. If aperture is circular then radius of first dark ring

$R = \dfrac{1.22\lambda D}{d} = \dfrac{1.22\lambda f}{d}$ where f is focal length of focussing lens.

In single slit diffraction fringe width $\beta_{primary} = \dfrac{2\lambda D}{d}$

$\beta_{sec} = \dfrac{\lambda D}{d}$.

$d \sin\theta = n\lambda$ for nth minima.

• Caution

1. Considering path difference = $n\lambda$ for bright fringes in all cases.

⇒ Path difference = $n\lambda$ for bright fringes for transmitted or refracted light. If interference occurs due to reflected light, path difference = $n\lambda$ for dark fringe i.e destructive interference.

2. Considering slit width as amplitude of the wave.

⇒ Slit width acts like intensity. Therefore to find resultant intensity use

$$I = k\,(S_1 + S_2 + 2\sqrt{S_1 S_2}\cos\phi) \quad \text{and}$$

$$\frac{I_{max}}{I_{min}} = \left(\frac{\sqrt{S_1} + \sqrt{S_2}}{\sqrt{S_1} - \sqrt{S_2}}\right)^2$$

3. Applying same formula $x_n = \dfrac{n\lambda D}{d}$ even when sources are placed horizontally.

⇒ As illustrated in Fig. 30.10 path difference $S_1L = d \cos\theta$.
Use $d\cos\theta = n\lambda$ for nth bright fringe.

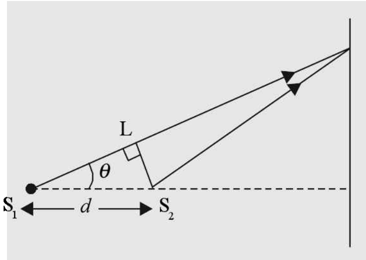


Fig. 30.10

4. Applying Malus Law even to unpolarised light when incident on a polariser.

⇒ Fig. 30.11 illustrates that if I_o was the intensity of unpolarised light then intensity of polarised light is $\frac{I_o}{2}$ after passing through first poloroid (P_1) and $\frac{I_o}{2}\cos^2\theta$ after passing through second poloroid P_2 inclined at an angle θ.

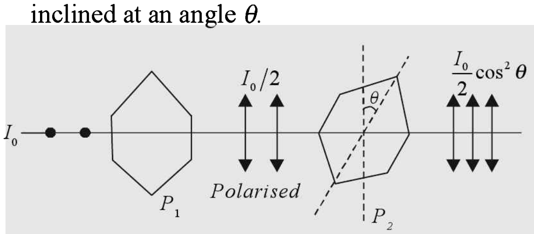


Fig. 30.11

Note that Malus law can be applied only to polarised light.

5. Considering any wavefront meeting an obstacle causes diffraction.

⇒ Planar wave front meeting an obstacle of the size of the order of wavelength will cause diffraction.

6. Considering interference and diffraction are alike.

⇒ For interference one needs coherent sources which can be derived from a single source by division of wave front or by division of amplitude. In diffraction bent rays or secondary wavelets superpose to form fringes.

7. Considering equal amplitudes of superposing waves is necessary for interference.

⇒ It may be a desirable condition. If amplitudes are unequal interference does occur and

$$\frac{I_{max}}{I_{min}} = \frac{I_{bright}}{I_{dark}} = \left(\frac{y_{01}+y_{02}}{y_{01}-y_{02}}\right)^2$$

Note: Dark fringe will not be completely dark if amplitudes y_{01} and y_{02} are not equal. However, intensity will be less at dark fringe positions as compared to bright fringe positions.

8. Considering only monochromatic light is needed for intereference or for diffraction to occur.

⇒ Interference and diffraction do occur with white light. In YDSE central fringe will be white surrounded by coloured fringes in VIBGYOR order. Same is the case for single slit diffraction experiment.

9. Not remembering the effect of refractive index of the medium (μ')

⇒ In YDSE fringes shrink by a factor of μ' while in Fresnel biprism the situation is not simple.

$$\beta_{new} = \frac{\lambda(a+b)}{2a(\mu-\mu')\alpha}.$$

10. Considering even two circular slits, illuminated with a source will give straight line fringes.

⇒ In such cases shape of fringes is hyperbola.

11. Assuming the intensity of primary maxima will increase in single slit experiment for increasing slit width.

⇒ Intensity of principal maxima is independent of slit width.

SOLVED PROBLEMS

1. In YDSE, an electron beam is used to obtain interference pattern. If speed of electrons is increased,

(a) no interference pattern will be observed
(b) distance between the consecutive fringes will increase
(c) distance between two consecutive fringes will decrease
(d) distance between two consecutive fringes remains same

[IIT Screening 2005]

Solution (c) $\lambda = \frac{h}{mv}$; if v increases, λ decreases.

Therefore $\beta = \frac{\lambda D}{d}$ will decreases.

2. In YDSE the angular position of a point on the central maxima whose intensity is $\frac{1}{4}$th of the maximum intensity.

(a) $\sin^{-1}\left(\frac{\lambda}{d}\right)$ (b) $\sin^{-1}\left(\frac{\lambda}{2d}\right)$
(c) $\sin^{-1}\left(\frac{\lambda}{3d}\right)$ (d) $\sin^{-1}\left(\frac{\lambda}{4d}\right)$

[IIT Screening 2005]

Solution (c) $\frac{2\cos\theta}{2} = 1$ $\frac{\cos\theta}{2} = \frac{1}{2}$ or $\phi = \frac{2\pi}{3}$

$\frac{2\pi}{\lambda} d\sin\theta = \frac{2\pi}{3}$ or $\phi = \sin^{-1}\left(\frac{\lambda}{3d}\right)$

3. A YDSE uses a monochromatic source. The shape of the fringe formed on the screen, is

(a) hyperbola (b) circle
(c) straight line (d) perabola

[AIEEE, 2005]

Solution (c)

4. When an unpolarised light of intensity, I_o is incident on a polarising sheet, the intensity of the light which does not get transmitted is

(a) $\frac{I_o}{2}$ (b) $\frac{I_o}{4}$
(c) zero (d) I_o

[AIEEE, 2005]

Solution (a)

5. The intensity of principal maxima in the single slit diffraction pattern is I_o? What will be its intensity when slit width is doubled?

(a) $2I_o$ (b) $4I_o$
(c) I_o (d) $\frac{I_o}{2}$

[AIEEE, 2005]

Solution (c)

6. Two waves of intensity I undergo interference. The maximum intensity obtained is

(a) $I/2$ (b) $2I$
(c) I (d) $4I$

[BHU, 2005]

Solution (d) $I_{max} = I + I + 2\sqrt{I}\ \sqrt{I}\ \cos\theta = 4I$. (for $\theta = 0$)

7. The wave theory in its original form was first postulated by

(a) Issac Newton (b) Thomas Young
(c) Christian Huygens (d) Augustin Jean Fresnel.

[Karnataka, 2005]

Solution (c)

8. Two coherent light beams of intensity I and $4I$ are superposed. The minimum and maximum possible intensities in the resulting beam are

(a) $9I$ and I (b) $9I$ and $3I$
(c) $5I$ and I (d) $5I$ and $3I$

[CET Karnataka, 2005]

Solution (a) $\frac{I_{max}}{I_{min}} = \left(\frac{\sqrt{4I}+\sqrt{I}}{\sqrt{4I}-\sqrt{I}}\right)^2 = \frac{9}{1}$

9. A single slit of width a is illuminated by violet light of wavelength 400 *nm* and width of the diffraction pattern is measured as y. Half of the slit is covered and illuminated with 600 *nm*. The width of the diffraction pattern will be

(a) $y/3$ (b) pattern vanishes and width is zero
(c) $3y$ (d) none of these.

[CET Karnataka, 2005]

Solution (c) $\beta = \frac{2\lambda D}{d}$ $\frac{y}{y'} = \frac{\frac{2\times 400D}{d}}{\frac{2\times 600D}{\frac{d}{2}}}$ or $y' = 3y$

10. When unpolarised light beam is incident in air into glass ($n = 1.5$ ` at polarising angle)

(a) reflected beam is 100% polarised
(b) reflected and refracted beam are partially polarised
(c) the reason for (a) is that almost all the light is reflected
(d) all the above

[CET Karnataka, 2005]

Solution (a)

11. Select the right option.

(a) Christian Huygens, a contemporary of Newton established the wave theory of light by assuming that light waves are transverse.
(b) Maxwell provided the compelling theoretical evidence that light is transverse in nature.
(c) Thomas Young experimentally proved the wave behaviour of light and Huygens assumption.
(d) All the statements given above correctly answer the question, what is light.

[CET Karnataka, 2005]

Solution (b)

12. In placing a thin sheet of mica of thickness 12×10^{-5} *cm* in the path of one of the interfering beams in YDSE, the central fringe shifts equal to a fringe width. Find the refractive index of mica. Given $\lambda = 600$ *nm*.

(a) 1.5 (b) 1.48
(c) 1.61 (d) 1.56

Solution (a) $\frac{\lambda D}{d} = (\mu - 1)\, t\, \frac{D}{d}$ or $\mu = \frac{\lambda}{t} + 1 = .5$

13. The waves emitted by a radio transmitter are

(a) linearly polarised (b) unpolarised
(c) monochromatic (d) elliptically polarised

Solution (a)

14. Dichorism means

(a) selective absorption of unpolarised light
(b) selective absorption of dispersed light
(c) selective absorption of scattered light
(d) selective absorption of one of the polarised components

Solution (d)

15. Two nicol prisms are kept perpendicular. One of them is illuminated with a light intensity (natural) I_o. Two more nicol prisms are introduced in between symmetrically. Find the light intensity emitted from the last nicol prism.

(a) $\frac{27I_o}{64}$ (b) $\frac{27I_o}{128}$

(c) $9\frac{I_0}{32}$ (d) $\frac{9I_o}{64}$

Solution (b) $I = \frac{I_o}{2}\cos^6 30 = \frac{I_o}{2}\left(\frac{\sqrt{3}}{2}\right)^6 = \frac{27I_o}{128}$

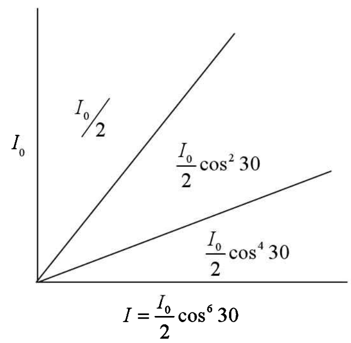


Fig. 30.12

16. The angle between reflected and refracted beams is 90° in the water air interface. The angle of incidence in water is

(a) 60° (b) 53°
(c) 30° (d) 37°

Solution (d) $\tan\theta = \frac{3}{4}$ $\theta = 37°$

17. In a birefracting crystal ordinary ray travels faster than extraordinary ray. The crystal is called

(a) positive crystal (b) negative crystal
(c) no such demarcation exists
(d) dextro rotatory (e) leveo rotatory.

Solution (a)

18. If in a birefracting crystal the magnitude of E_x and E_y are equal and phase angle between the two is 60° then the waves are

(a) linearly polarised (b) plane polarised
(c) circularly polarised (d) elliptically polarised

Solution (d)

19. Antinodal curves correspond to ______ interference.

(a) constructive (b) destructive
(c) where intensity is less than maximum but not completely zero
(d) none of these

Solution (a)

20. A radio station operating at a frequency 100 KHz has two vertical dipole antennas spaced 400 *m* apart oscillating in phase. In which directions is the intensity greatest?

(a) $0, \pm 30°, \pm 90°$ (b) $0, \pm 30°, \pm 60°$
(c) $0, \pm 45°, \pm 90°$ (d) $\pm 30°, \pm 60°, \pm 90°$

Solution (a) $\lambda = \frac{c}{f} = \frac{3\times10^8}{1.5\times10^6} = 200\,m$ $\sin\theta =$

$$\frac{n\lambda}{d} = \frac{n(200)}{400} = \frac{n}{2}, \text{ i.e., } \theta = 0, \pm 30°, \pm 90°.$$

21. In the above question where will minimum intensities be found?

(a) $\pm 14.5°, \pm 48.6°$ (b) $\pm 30°, \pm 45°$
(c) $\pm 14.5°, \pm 68.5°$ (d) $\pm 14.5°, \pm 79.6°$

Solution (a) $\sin\theta = \frac{(2n+1)}{4}$ $\theta = \sin^{-1}\frac{1}{4}$ or $\theta = \pm 14.5°$

$$\theta = \sin^{-1}\frac{3}{4} = \pm 48.6°$$

22. When exposed to sunlight, thin films of oil on water often exhibit brilliant colours due to the phenomenon of

(a) dispersion (b) interference
(c) diffraction (d) angular acceleration

Solution (b)

23. Two glass plates are 10 *cm* long. At one end a piece of paper 0.02 *mm* thick is placed to make a wedge as shown in Fig 30.13. Find the separation between the two fringes. Assume $\lambda = 500$ nm

(a) 1.25 nm (b) 1.5 nm
(c) 2.5 nm (d) none of these

Solution (a) $2t = n\lambda$; $\frac{t}{x} = \frac{h}{l}$

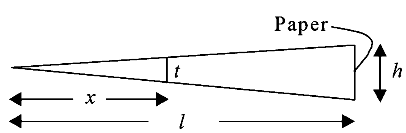


Fig. 30.13

$$\frac{2x_n h}{l} = n\lambda;\ x_n - x_{n-1} = \frac{\lambda l}{2h} = \frac{500\times10^{-9}\times.1}{2\times2\times10^{-5}}$$
$$= 1.25\times10^{-3}\,m$$

24. A commonly used lens coating material is MgF_2 with $n = 1.38$. Find the thickness of non-reflective coating one shall have for 550 *nm* light if it is applied to glass of $n = 1.52$.

(a) 400 nm (b) 200 nm
(c) 300 nm (d) 100 nm

Solution (d) 550 nm wavelength in MgF_2 will be

$$\lambda = \frac{\lambda_{air}}{n} = \frac{550}{1.38} = 400\text{ nm}$$

$$l = \frac{\lambda}{4} = 100\text{ nm}$$

TYPICAL PROBLEMS

25. In a single slit diffraction pattern, (a) find the intensity at a point where the total phase difference between the wavelets from top to bottom of the slit is 66 rad. (b) If this point is 7° away from the central maxima. Find the width of slit. Given: $\lambda = 600\ nm$.

Solution (a) $I = I_o \left[\dfrac{\sin(33\text{ rad})}{33\text{ rad}}\right]^2 = 9.2 \times 10^{-4}\, I_o$

(b) $a = \dfrac{B\lambda}{2\pi \sin\theta} = \dfrac{(66\text{ rad})600\times10^{-9}}{2\pi \sin 7°}$

$= 5.16 \times 10^{-5}\, m$ or 0.052 mm (nearly).

26. Consider the arrangement shown in Fig 30.14 (a). The distance D is large compared to d. Find minimum value of d so that there is a dark fringe at O. For the same value of d find x at which next bright fringe is formed.

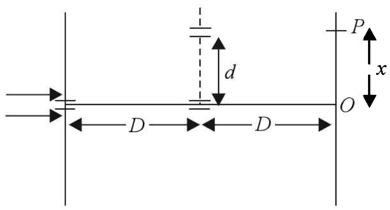


Fig. 30.14 (a)

Solution Path difference = AB + BO – 2D

$2\sqrt{(D^2 + d^2)} - 2D = \dfrac{\lambda}{2}$

or $2\sqrt{(D^2 + d^2)} = \dfrac{\lambda}{2} + 2D$

or $4(D^2 + d^2) = \dfrac{\lambda^2}{4} + 4D^2 + 2\lambda D$

Eliminate $\dfrac{\lambda^2}{4}$ as $\lambda << D$. or $d = \sqrt{\dfrac{\lambda D}{2}}$

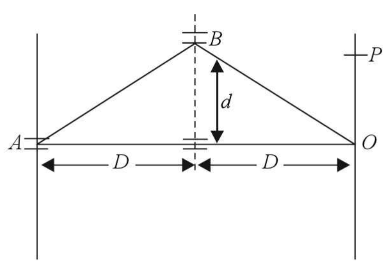


Fig. 30.14 (b)

Fig 30.14 (c) illustrates that if $PO = x = d$, path differnece will be zero and we will observe first maxima.

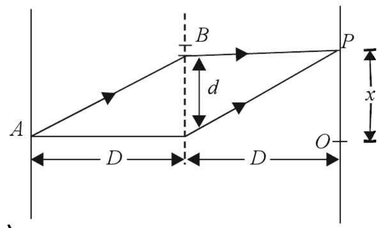


Fig. 30.14 (c)

27. A narrow slit S transmitting light of wavelength λ is placed a distance d above a large plane mirror as shown in Fig 30.15 (a). The light coming directly from the slit and that after reflection interfere converge at P on the screen placed at a distance D from the slit. What will be the intensity at a point just above O? What will be x for which first maxima occurs?

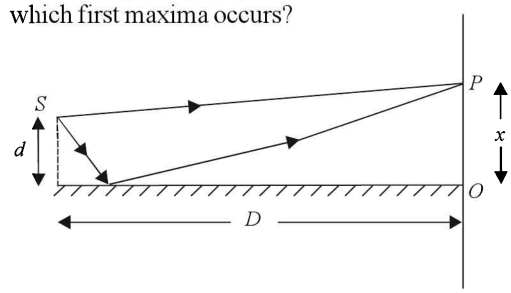


Fig. 30.15 (a)

Solution At just above O intensity is zero because reflection introduces an additional path difference of $\dfrac{\lambda}{2}$

$x_n = \dfrac{(2n-1)\lambda d}{2d}$

Put $n = 1$ and $d = 2d$ as image of s will be $2d$ apart as illustrated in Fig. 30.15 (b).

$x_1 = \dfrac{\lambda D}{2(2d)} = \dfrac{\lambda D}{4d}$

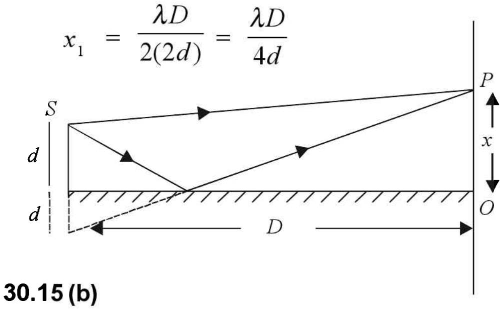


Fig. 30.15 (b)

28. Two trees are 1m apart. A person sees them from a distance of 1 km. Will he see the trees resolved?

(a) Yes (b) No
(c) May be resolved (d) None

[MNR 1996]

Solution (a) For trees to be resolved

$\frac{d}{D} \geq$ resolution power of eye $\frac{d}{D} = \frac{1}{10^3}$

$$RP = \left(\frac{1}{60}\right)^\circ = \frac{1}{60} \times \frac{\pi}{60} = \frac{3.14}{10.8} \times 10^{-3}$$

Since $\frac{d}{D}$ > resolution power of eye.

∴ Tree appear resolved.

29. Light of wavelength 560 nm goes through a pinhole of 0.2 mm and falls on a wall at a distance of 2 m. What is the radius of the central bright spot formed on the wall?

Solution $R = \frac{1.22\lambda D}{r}$

$$= \frac{1.22 \times 560 \times 2 \times 10^{-9}}{0.1 \times 10^{-9}} = 1.37 \text{ cm}$$

30. In a Lloyd's mirror experiment a light wave emitted directly by the source S interferes with reflected light from the mirror. The screen is 1m away form the source S. The size of fringe width is 0.25 mm. The source is moved 0.6 mm above the initial position, the fringe width decreases by 1.5 times. Find the wavelength of light.

[Olympiad 1998]

Solution $\beta = \frac{\lambda D}{d} = 0.25 \times 10^{-3}$ or $\lambda D = \frac{d}{4} \times 10^{-3}$

Case (ii) $\beta = \frac{\lambda D}{d + 1.2 \times 10^{-3}} = \frac{0.25 \times 10^{-3} d}{1.5} = \frac{10^{-3}}{6}$

or $\lambda D = \frac{10^{-3} d}{6} + \frac{1.2 \times 10^{-3} \times 10^{-3}}{6}$

$$\lambda = \frac{d \times 10^{-3}}{4D} = \frac{0.6 \times 10^{-3} \times 10^{-3}}{1.0} = 0.6\,\mu\text{m}$$

$$\frac{d}{4} = \frac{d}{6} + \frac{1.2 \times 10^{-3}}{6}$$

or $d = 2.4$ mm

31. A convex lens of diameter 8 cm is used to focus a parallel beam of light of wavelength 620 nm. Light is focussed at a distance 20 cm. from the lens. What would be the radius of central bright fringe?

Solution (a) $R = \frac{1.22\lambda D}{r} = \frac{1.22 \times 620 \times 10^{-9} \times 0.2}{4 \times 10^{-2}}$

$= 3.8 \times 10^{-6}$ m.

32. A glass plate (n = 1.53) that is 485μm thick and surrounded by air is illuminated by a beam of white light normal to the plate. (a) What wavelengths in the visible spectrum (400 to 700 nm) are intensified in the reflected beam? (b) What wavelengths are intensified in transmitted beam?

Solution (a) **In reflected light** $2\,\mu t = (2n+1)\,\frac{\lambda}{2}$

$$\lambda = \frac{4\mu t}{2n+1} = \frac{2970}{2n+1}\ nm = 594\ nm,\ 424\ nm$$

(b) **In transmitted light** $2\,\mu t = n\,\lambda$ or $\lambda = \frac{2\mu t}{n} = \frac{1485}{n}$

$= 495\ nm$

33. In case of linearly polarised light the magnitude of electric field vector

(a) varies periodically with time
(b) increases and decreases linearly with time
(c) does not change with time
(d) is parallel to the direction of propagation

Solution (a) ∵ $E = E_o \sin(\omega t - kx)$, it varies periodically with time.

QUESTIONS FOR PRACTICE

1. In a Young's double-slit experiment, the fringe width is β. If the entire arrangement is now placed inside a liquid of refractive index μ, the fringe width will become

(a) $\mu\beta$ (b) $\frac{\beta}{\mu}$
(c) $\frac{\beta}{\mu+1}$ (d) $\frac{\beta}{\mu-1}$

2. In a Young's double slit experiment, let S_1 and S_2 be the two slits, and C be the centre of the screen. If $\angle S_1CS_2 = \theta$ and λ is the wavelength, the fringe width will be

(a) $\frac{\lambda}{\theta}$ (b) $\lambda\theta$
(c) $\frac{2\lambda}{\theta}$ (d) $\frac{\lambda}{2\theta}$

3. When a drop of oil is spread on a water surface, it displays beautiful colours in daylight because of

(a) dispersion of light (b) reflection of light
(c) polarisation of light (d) interference of light.

4. In a Young's double slit experiment, let β be the fringe width, and let I_0 be the intensity at the central bright fringe. At a distance x from the central bright fringe, the intensity will be

(a) $I_0 \cos\left(\frac{x}{\beta}\right)$ (b) $I_0 \cos^2\left(\frac{x}{\beta}\right)$

(c) $I_0 \cos^2\left(\frac{\pi x}{\beta}\right)$ (d) $\left(\frac{I_0}{4}\right)\cos^2\left(\frac{\pi x}{\beta}\right)$.

5. Choose the correct statement.
   (a) Brewster's angle is independent of wavelength of light.
   (b) Brewster's angle is independent of nature of reflecting surface.
   (c) Brewster's angle is different for different wavelengths.
   (d) Brewster's angle depends on wavelength but not on the nature of reflecting surface.

6. A ray of light strikes a glass plate at an angle of 60°. If the reflected and refracted rays are perpendicular to each other the index of refraction of glass is
   (a) $\frac{1}{2}$ (b) $\frac{\sqrt{3}}{2}$
   (c) $\frac{3}{2}$ (d) 1.732

7. When unpolarised light is incident on a plane glass plate at Brewster's angle, then which of the following statements is correct?
   (a) Reflected and refracted rays are completely polarised with their planes of polarisation parallel to each other.
   (b) Reflected and refracted rays are completely polarised with their planes of polarization perpendicular to each other.
   (c) Reflected light is plane polarised but transmitted light is partially polarised.
   (d) Reflected light is partially polarised but refracted light is plane polarised.

8. A ray of light is incident on the surface of a glass plate of refractive index 1.55 at the polarising angle. The angle of refraction is
   (a) 0° (b) 147° 11'
   (c) 32°49' (d) 57°11'

9. A calcite crystal is placed over a dot on a piece of paper and rotated. On viewing through calcite, one will see
   (a) a single dot (b) two stationary dots
   (c) two rotating dots
   (d) one dot rotating about the other

10 From Brewster's law, it follows that the angle of polarisation depends upon
   (a) the wavelength of light
   (b) orientation of plane of polarisation
   (c) orientation of plane of vibration
   (d) none of these

11. Optically active substances are those which
   (a) produce polarised light
   (b) rotate the plane of polarisation of polarised light
   (c) produce double refraction
   (d) convert plane polarised light into circularly polarised light

12. Light transmitted by Nicol prism is
   (a) unpolarised (b) plane polarised
   (c) circularly polarised (d) elliptically polarised

13. A beam of light *AO* is incident on a glass slab (μ = 1.54) in a direction as shown in Fig 30.16. The reflected ray *OB* is passed through a Nicol prism. On rotating the Nicol prism we observe that

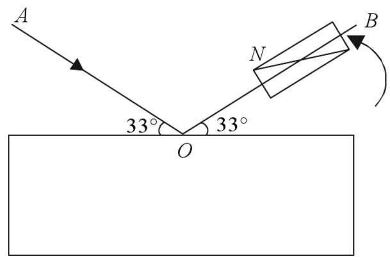


Fig. 30.16

   (a) the intensity is reduced to zero and remains zero
   (b) the intensity reduces somewhat and rises again
   (c) there is no change in intensity
   (d) the intensity gradually reduces to zero and then again increases

14. In the propagation of electromagnetic waves the angle between the direction of propagation and the plane of vibration is
   (a) π (b) $\frac{\pi}{2}$
   (c) $\frac{\pi}{4}$ (d) 0

15. An unpolarised beam of intensity $2a^2$ passes through a thin poloroid. Assuming zero absorption in the poloroid, the intensity of emergent plane polarised light will be
   (a) $2a^2$ (b) a^2
   (c) $\sqrt{2}a^2$ (d) $\frac{a^2}{\sqrt{2}}$

16. Two Nicols are oriented with their principal planes making an angle of 60°. The percentage of incident unpolarised light which passes through the system is
   (a) 50% (b) 100%
   (c) 12.5% (d) 37.5%

17. Unpolarised light falls on two polarising sheets placed one on top of the other. What must be the angle between the characteristic directions of the sheets if the intensity of the final transmitted light is one-third the maximum intensity of the first transmitted beam?
   (a) 75° (b) 55°
   (c) 35° (d) 15°

18. In the above problem if the final intensity is one third the intensity of incident beam, then the corresponding angle will be

(a) 75° (b) 55°
(c) 35° (d) 15°

19. Unpolarised light of intensity $32 Wm^{-2}$ passes through three polarisers such that the transmission axis of the last polariser is crossed with that of the first. The intensity of final emerging light is $3 Wm^{-2}$. The intensity of light transmitted by first polariser will be

(a) $32 Wm^{-2}$ (b) $16 Wm^{-2}$
(c) $8 Wm^{-2}$ (d) $4 Wm^{-2}$

20. In the above problem, the angle between the transmission axes of the first two polarisers will be

(a) 10° (b) 30°
(c) 45° (d) 60°

21. In YDSE find the missing wavelength in front of one of the slits

(a) $\frac{d^2}{2D}$ (b) $\frac{2d^2}{D}$
(c) $\frac{d^2}{3D}$ (d) $\frac{d^2}{4D}$

22. If YDSE is immersed in a liquid of refractive index μ then fringewidth β

(a) decreases by μ (b) increases by μ
(c) remains unchanged (d) none of these

23. In the visible region of the spectrum the rotation of the plane of polarisation is given by

$$\theta = a + \frac{b}{\lambda^2}$$

The optical rotation produced by a particular material is found to be 30° per mm at λ = 500Å and 50° per mm at λ = 4000 Å. The value of *constant a* will be

(a) $+\frac{50°}{9°}$ per mm (b) $-\frac{50°}{9°}$ per mm
(c) $+\frac{9°}{50°}$ per mm (d) $-\frac{9°}{50°}$ per mm

24. In the above problem, the value of constant b in degree $Å^2$ per mm, will be

(a) $\frac{8}{9}\times 10^9$ (b) $-\frac{8}{9}\times 10^9$
(c) $\frac{9}{8}\times 10^8$ (d) $-\frac{9}{8}\times 10^8$

25. In a diffraction (single slit experiment), slit is exposed by white light. The fringe surrounding the central fringe is

(a) red (b) yellow
(c) violet (d) green

26. A beam of natural light falls on a system of 6 polaroids, which are arranged in succession such that each polaroid is turned through 30° with respect to the preceding one. The percentage of incident intensity that passes through the system will be

(a) 100% (b) 50%
(c) 30% (d) 12%

27. A beam of unpolarised light is passed first through a tourmaline crystal A and then through another tourmaline crystal B oriented so that its principal plane is parallel to that of A. The intensity of final emergent light is I. The value of I is

(a) $\frac{I_0}{2}$ (b) $\frac{I_0}{4}$
(c) $\frac{I_0}{8}$ (d) none of these

28. In the above problem, if A is rotated by 45° in a plane perpendicular to the direction of incident ray, then intensity of emergent light will be

(a) $\frac{I}{8}$ (c) $\frac{I}{4}$
(c) $\frac{I}{2}$ (d) none of these

29. A beam of plane polarised light falls normally on a polariser of cross sectional area $3 \times 10^{-4} m^2$. The polariser rotates with an angular frequency of 31.4 rad/s. The energy of light passing through the polariser per revolution will be

(a) 10^{-4} Joule (b) 10^{-3} Joule
(c) 10^{-2} Joule (d) 10^{-1} Joule

30. In the above problem, the intensity of the emergent beam, if flux of energy of the incident ray is 10^{-3} W, will be (in W/m^2)

(a) $\frac{1}{3}$ (b) $\frac{2}{3}$
(c) $\frac{4}{3}$ (d) $\frac{5}{3}$

31. An unpolarised beam of light is incident on a group of four polarising sheets which are arranged in such a way that the characteristic direction of each polarising sheet makes an angle of 30° with that of the preceding sheet. The percentage of incident light transmitted by the first polariser will be

(a) 100% (b) 50%
(c) 25% (d) 12.5%

32. In the above problem, the percentage of incident light transmitted by the second polariser will be

(a) 12.5% (b) 25%
(c) 37.5% (d) 50%

33. In Q. 31, the percentage of incident light transmitted by the third polariser will be

(a) 11.5% (b) 17.125%

(c) 22.7% (d) 28.125%

34. In Q. 31, the percentage of incident light transmitted by the fourth polariser will be

(a) 21.1% (b) 28.125%

(c) 37.5% (d) 50%

35. In Fresnel's biprism experiment the amplitude of second coherent source is four times that of the first. The ratio of their intensities will be

(a) 4 : 1 (b) 1 : 4

(c) 16 : 1 (d) 1 : 16

36. In Young's double slit experiment the distance between two slits S_1 and S_2 is d. Interference pattern is obtained by these slits on a screen distant D from the slits. A dark fringes is produced at point P just in front of S_1, The wavelength of light used is

(a) $\lambda = \frac{D}{d^2}$ (b) $\lambda = \frac{d^2}{D}$

(c) $\lambda = \frac{D}{d}$ (d) $\lambda = \frac{d}{D}$

37. In Fresnel biprism experiment, when light of wavelength 6000 Å is used then 16th bright fringe is obtained at point P. If light of wavelength 4800 Å is used then the order of fringe obtained at point P will be

(a) 16th (b) 20th

(c) 18th (d) 24th

38. The maximum intensity produced by two coherent waves of intensity I_1 and I_2 will be

(a) $I_1 + I_2$ (b) $I_1^2 + I_2^2$

(c) $I_1 + I_2 + 2$ (d) zero

39. Two independent monochromatic sodium lamps can not produce interference because

(a) the frequencies of the two sources are different

(b) the phase difference between the two sources changes with respect to time

(c) the two sources become coherent

(d) the amplitudes of two sources are different

40. The path difference between two wave fronts emitted by coherent sources of wavelength 5460 Å is 2.1 micron. The phase difference between the wavefronts at that point is

(a) 7.692 (b) $7.692\,\pi$

(c) $\frac{7.692}{\pi}$ (d) $\frac{7.692}{3\pi}$

41. The path of difference between two interfering waves at a point on the screen is $\lambda/8$. The ratio of intensity at this point and that at the central fringe will be

(a) 0.853 (b) 8.53

(c) 85.3 (d) 853

42. The two coherent sources of equal intensity produce maximum intensity of 100 units at a point. If the intensity of one of the sources is reduced by 36% by reducing its width then the intensity of light at the same point will be

(a) 90 (b) 89

(c) 67 (d) 81

43. White light is incident on a soap film of thickness 15×10^{-5} cm and refractive index 1.33. Which wavelength is reflected maximum in the visible region?

(a) 26000 Å (b) 8866 Å

(c) 5320 Å (d) 3800 Å

44. If the whole biprism experiment is immersed in water then the fringe width becomes, if the refractive indices of biprism material and water are 1.5 and 1.33 respectively.

(a) 3 times (b) $\frac{3}{4}$ times

(c) $\frac{4}{3}$ times (d) $\frac{1}{3}$ times

45. In a biprism experiment fifth dark fringe is obtained at a point. If a thin transparent film is placed in the path of one of waves, then seventh bright fringe is obtained at the same point. The thickness of the film in terms of wavelength X and refractive index μ will be

(a) $\frac{1.5\lambda}{(\mu - 1)}$ (b) $1.5(\mu - 1)\lambda$

(c) $2.5(\mu - 1)\lambda$ (d) $\frac{2.5\lambda}{(\mu - 1)}$

46. Light of wavelength 7500 Å is incident on a thin glass plate ($\mu = 1.5$) so that the angle of refraction obtained is 30°. If the plate appears dark then the minimum thickness of plate will be

(a) $4000\sqrt{3}$ Å (b) $\frac{8000}{\sqrt{3}}$ Å

(c) $\frac{5000}{\sqrt{3}}$ Å (d) $1000\sqrt{3}$ Å

47. In Fresnel biprism experiment the refractive index for the biprism is $\mu = 3/2$ and fringe width obtained is 0.4 mm. If the whole apparatus is immersed as such in water then the fringe width will become ________. (refractive index of water is 4/3).

(a) 0.3 mm (b) 0.225 mm

(c) 0.4 mm (d) 1.2 mm

48. The distance between slit and biprism and that between biprism and screen each is 0.4 m. The obtuse angle of biprism is 179° and refractive index is 1.5. If the fringe width is 1.8×10^{-4} m then the distance between imaginary sources will be

(a) 8.7 mm (b) 4.36 mm
(c) 1.5 mm (d) 3.5 mm

49. In the above problem, the wavelength of light will be

(a) 7850 Å (b) 6930 Å
(c) 5890 Å (d) 3750 Å

50. In Young's double slit experiment one slit is covered with red filter and another slit is covered by green filter, the interference pattern will be

(a) red (b) green
(c) yellow (d) invisible

51. In biprism experiment, fringes are obtained by white light source. The fringe nearest the central fringe will be

(a) yellow (b) green
(c) violet (d) red

52. The distance between two coherent sources produced by a biprism is 1.0 mm. When distance between the source and the screen is 0.9 m the fringe width obtained is 0.12 mm. If the screen is placed at a distance of 1.8 m then fringe width will be

(a) 0.6 mm (b) 0.8 mm
(c) 0.9 mm (d) 0.24 mm

53. In Young's double slit experiment 62 fringes are visible in the field of view with sodium light (λ = 5893Å). If green light (λ = 5461Å) is used then the number of visible fringes will be

(a) 62 (b) 67
(c) 85 (d) 58

54. In Young's double slit experiment two light beams of wavelengths λ_1 = 6000 Å and λ_2 = 4800 Å are used. The distance between two slits is 2.5 mm. The distance between slits and the screen is 1.5 m. The distance between the central maxima obtained with two beams will be

(a) zero (b) 1.872 mm
(c) 2.872 mm (d) 2.652 mm

55. In the above problem the distance of seventh dark fringe for λ_2 from central maximum will be

(a) 1.652 mm (b) 1.872 mm
(c) 2.872 mm (d) 2.652 mm

56. Tenth fringe of wavelength 4000 Å coincides with 8th fringe of wavelength λ. Then λ is

(a) 50 nm (b) 555 nm
(c) 450 nm (d) none

57. Which of the following formula is incorrect in a biprism?

(a) $d = \sqrt{d_1 d_2}$ (b) $d = 2a(\mu - 1)\alpha$

(c) $d = \dfrac{D\lambda}{\beta}$ (d) $d = \dfrac{d_1^2}{d_2}$

58. The ratio of phase difference and path difference is

(a) 2p (b) $\dfrac{2\pi}{\lambda}$

(c) $\dfrac{\lambda}{2\pi}$ (d) $\dfrac{\pi}{\lambda}$

59. The correct relation between time interval ∂ and phase difference δ is

(a) $\partial = \dfrac{T}{2\pi}\delta$ (b) $\partial = \dfrac{2\pi}{T}\delta$

(c) $\partial = 2\pi\delta$ (d) $\partial = \dfrac{\delta}{2\pi}$

60. If the amplitude of two light waves are 1 and 2 units respectively then the average intensity will be

(a) 3 units (b) 1 units
(c) 5 units (d) $\sqrt{3}$ units

61. Interference event is observed

(a) only in transverse waves
(b) only in longitudinal waves
(c) in both types of waves
(d) none

62. The nature of light which is verified by the interference event is

(a) particle nature (b) wave nature
(c) dual nature (d) quantum nature

63. In the phenomenon of interference, energy is

(a) destroyed at bright fringes
(b) created at dark fringes
(c) conserved, but it is redistributed
(d) same at all points

64. For which colour is the fringe width minimum?

(a) Violet (b) Red
(c) Green (d) Yellow

65. How many colours comprise white light?

(a) Infinite (b) Seven
(c) Three (d) Fourteen

66. Monochromatic light is that light in which

(a) single wavelength is present
(b) various wavelengths are present
(c) red and violet light is present
(d) yellow and red light is present.

67. The refracting angle of biprism is

(a) 179° (b) 1°

(c) 1/2° (d) 90°

68. In biprism experiment the light source is

(a) extended (b) narrow

(c) multichromatic (d) all of above

69. A very thin film in reflected white light appears

(a) coloured (b) white

(c) black (d) red

70. The time of coherence is of the order of

(a) 10^{-4} s (b) 10^{-8} s

(c) 10^{-6} s (d) 10^{-2} s

71. If the frequency of light emitted by a source in an interference experiment is made four times then the fringe width will become

(a) four times (b) three times

(c) one fourth (d) half

72. In Young's double slit experiment if the maximum intensity of light is I_{max} then the intensity at path difference $\lambda/2$ will be

(a) I_{max} (b) $\frac{I_{max}}{2}$

(c) $\frac{I_{max}}{4}$ (d) Zero

73. The correct curve between fringe width β and distance between the slits (d) is

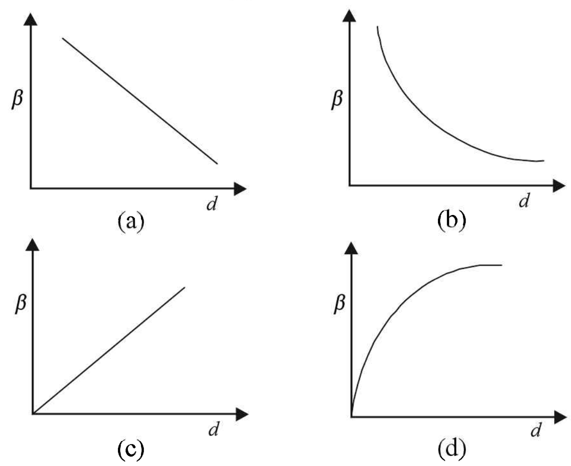


Fig. 30.17

74. Two coherent waves are represented by $y_1 = a_1 \cos \omega t$ and $y_2 = a_2 \sin \omega t$. The resultant intensity due to interference will be

(a) $(a_1 + a_2)$ (b) $(a_1 - a_2)$

(c) $(a_1^2 + a_2^2)$ (d) $(a_1^2 + a_2^2)$

75. Interference pattern can be produced by two identical sources. Here the identical sources mean that

(a) their size is same

(b) their wavelength is same

(c) the intensity of light emitted by them is same

(d) the amplitudes of light waves emitted by them are same

76. In biprism experiment when the slit and the eyepiece are set at 1cm and 100 cm, the width of 10 fringes is found to be 9.72 mm. If the distances between the images formed in the eyepiece in two positions of lens are 0.3 mm and 1.2 mm respectively then the wavelength of light used is

(a) 5832 Å (b) 5840 Å

(c) 5820 Å (d) 5700 Å

77. If in Young's double slit experiment, the distance between the slits is halved and the distance between slit and screen is doubled, then the fringe width will become

(a) half (b) double

(c) four times (d) unchanged

78. In coherent sources it is necessary that their

(a) amplitudes are same

(b) wavelengths are same

(c) frequencies are same

(d) initial phase remains constant

79. The intensity of central fringe in the interference pattern produced by two identical slits is I. When one of the slits is closed then the intensity at the same point is I_0. The relation between I and I_0 is

(a) $I = 4I_0$ (b) $I = 2I_0$

(c) $I = I_0$ (d) $I = \frac{I_0}{2}$

80. The fringe width for red colour as compared to that for violet colour is approximately

(a) three times (b) double

(c) four times (d) eight times

81. Light of wavelength 6.5×10^{-7} meter is made incident on two slits 1mm apart. The distance between third dark fringe and fifth bright fringe on a screen distant 1 m from the slits will be

(a) 0.35 mm (b) 0.65 mm

(c) 1.63 mm (d) 3.25 mm

82. The oil layer on the surface of water appears coloured due to interference. For this effect to be visible the thickness of oil layer will be

(a) 1 mm (b) 1 cm

(c) 100 Å (d) 1000 Å

83. In Young's double slit experiment the ratio of the slit widths is 1 : 4. The ratio of maximum and minimum intensities in the interference pattern will be

(a) 4 : 9 (b) 9 : 4
(c) 9 : 1 (d) 1 : 9

84. In Young's double slit experiment the ratio of maximum and minimum intensity in the interference experiment is 9. It means that the
(a) ratio of their amplitudes is 4
(b) ratio of their amplitudes is 2
(c) intensities due to two slits are 4 units and 1 unit respectively
(d) intensities due to two slits are 5 units and 4 units respectively

85. Intensity of light depends on
(a) amplitude (b) frequency
(c) wavelength (d) velocity

86. In Young's double slit experiment the source S and two slits A and B are lying in a horizontal plane. The slit A is above slit B. The fringes are obtained on a vertical screen K. The optical path from S to B is increased by putting a transparent material of higher refractive index. The path from S to A remains unchanged. As a result of this the fringe pattern moves some what
(a) upwards (b) downwards
(c) towards left horizontally
(d) towards right horizontally

87. In Fresnel's biprism experiment the coherent sources are obtained by
(a) interference (b) reflection
(c) refraction (d) total internal reflection

88. The colour of bright fringe nearest the central achromatic fringe in the interference pattern with white light will be
(a) violet (b) red
(c) green (d) yellow

89. In Young's double slit experiment the intensities of dark and bright fringes are I and 41 respectively. The ratio of amplitudes of sources is
(a) 4 : 1 (b) 1 : 3
(c) 3 : 1 (d) 1 : 2

90. When a thin film of thickness t is placed in the path of light wave emerging out of S_1 then increase in the length of optical path will be
(a) $(\mu - 1)t$ (b) $(\mu + 1)t$
(c) μt (d) $\frac{\mu}{t}$

91. A thin sheet of mica is placed in the path of S_2.The fringes will get shifted towards
(a) S_1 (b) S_2
(c) both sides
(d) first towards S_2 and then towards S_1

92. Two coherent waves of light will not produce constructive interference if the phase difference between them, is
(a) 0° (b) 360°
(c) 720° (d) 90°

93. In Young's double slit experiment, the interference pattern obtained with white light will be
(a) the central fringe bright and alternate bright and dark fringes
(b) the central fringe achromatic and coloured fringes for small path difference
(c) the central fringe dark
(d) the central fringe coloured

94. Two coherent sources with intensity ratio β produce interference. The fringe visibility will be
(a) $\frac{2\sqrt{\beta}}{1+\beta}$ (b) 2β
(c) $\frac{2}{(1+\beta)}$ (d) $\frac{\sqrt{\beta}}{1+\beta}$

95. In Fresnel's biprism experiment a mica sheet of refractive index 1.5 and thickness 6×10^{-6} m is placed in the path of one of interfering beams as a result of which the central fringe gets shifted through five fringe widths. The wavelength of light used is
(a) 6000 Å (b) 8000 Å
(c) 4000 Å (d) 2000 Å

96. What will be the distance between two slits which, when illuminated by light of wavelength 5000 Å, produce fringes of width 0.5 mm on a screen distant 1 meter from the slits?
(a) 10^{-2} meter (b) 10^{-3} meter
(c) 10^{-4} meter (d) 10^{-6} meter

97. If the ratio of maximum and minimum intensities in an interference pattern is 36 : 1 then the ratio of amplitudes of two interfering waves will be
(a) 5 : 7 (b) 7 : 4
(c) 4 : 7 (d) 7 : 5

98. Fringes are obtained with the help of a biprism in the focal plane of an eyepiece distant 1 meter from the slit. A convex lens produces images of the slit in two positions between biprism and eyepiece. The distances between two images of the slit in two positions are 4.05×10^{-3} m and 2.90×10^{-3} m respectively. The distance between the slits will be
(a) 3.43×10^{-3} m (b) 0.343 m
(c) 0.0343 m (d) 34.3 m

99. The device which produces highly coherent sources is
(a) Fresnel biprism (b) Young's double slit
(c) Laser (d) Lloyd's mirror

100. In Young's double slit experiment, if the sodium light is replaced by violet light of same intensity then in the interference pattern

(a) β will decrease (b) β will increase

(c) I will decrease (d) I will increase

101. The equations of waves emitted S_1, S_2, S_3 and S_4 are respectively $y_1 = 20 \sin(100\pi t)$, $y_2 = 20 \sin(200\pi t)$, $y_3 = 20 \cos(100 \pi t)$ and $y_4 = 20 \cos(100\pi t)$. The phenomenon of interference will be produced by

(a) y_1 and y_2 (b) y_2 and y_3

(c) y_1 and y_3

(d) Interference is not possible

102. In double slit experiment the distance between two slits is 0.6 mm and these are illuminated with light of wavelength 4800 Å. The angular width of dark fringe on the screen distant 120 cm from slits will be

(a) 8×10^{-4} Radian (b) 6×10^{-4} Radian

(c) 4×10^{-4} Radian (d) 16×10^{-4} Radian

103. In the above problem the ratio of intensities at the centre and at a distance of 1.2 mm from centre will be

(a) 1 : 2 (b) 1 : 1

(c) 4 : 1 (d) 1 : 4

104. Two coherent sources of wavelength 6.2 × 10 m produce interference. The path difference corresponding to 10th order maximum will be

(a) 6.2×10^{-6} m (b) 3.1×10^{-6} m

(c) 1.5×10^{-6} m (d) 12.4×10^{-6} m

105. In the above problem the path difference corresponding to the dark fringe between third and fourth maxima will be-

(a) 4.17×10^{-6} m (b) 2.17×10^{-6} m

(c) 6.17×10^{-6} m (d) 8.17×10^{-6} m

106. In Fresnel biprism experiment the distance between the source and the screen is 1m and that between the source and biprism is 10 cm. The wavelength of light used is 6000 Å. The fringe width obtained is 0.03 cm and the refracting angle of biprism is 1. The refractive index of the material of biprism is

(a) 1.531 (b) 1.573

(c) 1.621 (d) 1.732

107. A mica sheet of thickness 1.964 micron and refractive index 1.6 is placed in the path of one of the interfering waves. Now the mica sheet is removed and the distance between the slit and the screen is doubled. If this state the distance between two consecutive maxima or minima is equal to the displacement of fringe pattern on placing mica sheet, the wavelength of monochromatic light used is

(a) 5892 Å (b) 5269 Å

(c) 6271Å (d)3875 Å

108. The slits in Young's double slit experiment, are 0.5 mm apart and interference pattern is observed on a screen distant 100 cm from the slits. It is found that the 9th bright fringe is at a distance of 8.835 mm from the second dark fringe. The wavelength of light will be

(a) 7529 Å (b) 6253 Å

(c) 6779 Å (d) 5890 Å

109. In double slit experiment fringes are obtained using light of wavelength 4800 Å. One slit is covered with a thin glass film of refractive index 1.4 and another slit is covered by a film of same thickness but refractive index 1.7. By doing so the central fringe is shifted to fifth bright fringe in the original pattern. The thickness of glass film is

(a) 2×10^{-3} mm (b) 4×10^{-3} mm

(c) 6×10^{-3} mm (d) 8×10^{-3} mm

110. A glass plate of thickness 12×10^{-3} mm is placed in the path of one of the interfering beams in Young's double slit arrangement. Light of wavelenght 60000 Å is used in the arrangement. If the central band is displaced by a distance equal to the width of 10 bands then the refractive index of glass will be

(a) $\frac{5}{4}$ (b) $\frac{4}{3}$

(c) $\frac{3}{2}$ (d) $\frac{2}{1}$

111. In the above problem, what should be the thickness of a diamond plate of refractive index 2.5 which will restore the central band to its original position?

(a) 2×10^{-3} mm (b) 4×10^{-3} mm

(c) 8×10^{-3} mm (d) 6×10^{-3} mm

112. Light of wavelength 5880A is incident on a thin glass plate (μ = 1.5) such that the angle of refraction in the plate is 60°. The minimum thickness of the plate, so that it appears dark in the reflected light will be

(a) 3920 Å (b) 4372 Å

(c) 5840 Å (d) 6312 Å

113. The parallel rays of white light are made incident normally on an air film of uniform thickness. 250 fringes are seen in the transmitted light between 4000 Å and 6500 Å. Thickness of air film is

(a) 0.17mm (b) 0.15mm

(c) 0.13 mm (d) 0.11mm

114. White light is normally incident on a soap film. The thickness of the film is 5×10^{-7} meter and its refractive index is 1.33. Which wave length will be reflected maximum in the visible region?

(a) 26600 Å (b) 8860 Å

(c) 5320 Å (d) 3800 Å

115. In Young's double slit experiment the phase difference between the waves reaching the central fringe and third bright fringe will be

(a) zero (b) 2π

(c) 4π (d) 6π

116. The ratio of slit widths in Young's double slit experiment is 4 : 9. The ratio of maximum and minimum intensities will be

(a) 169 : 25 (b) 81 : 16

(c) 13 : 5 (d) 25 : 1

Answers to Questions for Practice

1.	(b)	**2.**	(a)	**3.**	(d)	**4.**	(c)	**5.**	(c)	**6.**	(d)	**7.**	(c)
8.	(c)	**9.**	(d)	**10.**	(a)	**11.**	(b)	**12.**	(b)	**13.**	(d)	**14.**	(d)
15.	(b)	**16.**	(c)	**17.**	(b)	**18.**	(c)	**19.**	(b)	**20.**	(b)	**21.**	(c)
22.	(c)	**23.**	(b)	**24.**	(a)	**25.**	(c)	**26.**	(d)	**27.**	(a)	**28.**	(c)
29.	(a)	**30.**	(d)	**31.**	(b)	**32.**	(c)	**33.**	(d)	**34.**	(a)	**35.**	(d)
36.	(b)	**37.**	(b)	**38.**	(c)	**39.**	(b)	**40.**	(b)	**41.**	(a)	**42.**	(d)
43.	(c)	**44.**	(a)	**45.**	(d)	**46.**	(c)	**47.**	(d)	**48.**	(d)	**49.**	(a)
50.	(d)	**51.**	(c)	**52.**	(d)	**53.**	(b)	**54.**	(a)	**55.**	(b)	**56.**	(a)
57.	(d)	**58.**	(b)	**59.**	(a)	**60.**	(c)	**61.**	(c)	**62.**	(b)	**63.**	(c)
64.	(a)	**65.**	(b)	**66.**	(a)	**67.**	(c)	**68.**	(b)	**69.**	(c)	**70.**	(b)
71.	(c)	**72.**	(d)	**73.**	(b)	**74.**	(c)	**75.**	(b)	**76.**	(a)	**77.**	(c)
78.	(d)	**79.**	(a)	**80.**	(b)	**81.**	(c)	**82.**	(d)	**83.**	(c)	**84.**	(c)
85.	(a)	**86.**	(b)	**87.**	(c)	**88.**	(a)	**89.**	(c)	**90.**	(a)	**91.**	(b)
92.	(d)	**93.**	(a)	**94.**	(b)	**95.**	(a)	**96.**	(a)	**97.**	(b)	**98.**	(d)
99.	(a)	**100.**	(c)	**101.**	(a)	**102.**	(d)	**103.**	(a)	**104.**	(b)	**105.**	(a)
106.	(b)	**107.**	(b)	**108.**	(a)	**109.**	(d)	**110.**	(d)	**111.**	(c)	**112.**	(b)
113.	(a)	**114.**	(c)	**115.**	(c)	**116.**	(d)						

31

Spectrum of Light and Photometry

BRIEF REVIEW

Spectrum A collection of dispersed light giving its wavelength composition is called a spectrum. For example, Hydrogen spectrum has Lyman series, Balmer series, Paschen series, Brackett series, P-fund series etc. and when a white light is incident on a prism a spectrum of different colours from red to violet is observed.

Pure and Impure Spectrum If each colour gives its sharp impression in the spectrum, then a well defined line spectrum is obtained. Such a spectrum is called pure spectrum. To achieve pure spectrum — (i) The beam of light incident on the dispersing element (prism or diffraction grating) should be parallel or collimated. (ii) The dispersed light should be focussed in such a way that all the rays of a particular wavelength are collected at a place.

Note: A spectrometer will satisfy the above requirements.

If the slit is wide, different points of the slit produce separate spectra which overlap each other. Thus colour impression gets diffused due to overlap resulting into an **impure spectrum.**

Kinds of Spectra Broadly speaking we can divide the spectrum into two types – **emission spectrum** and **absorption spectrum**.

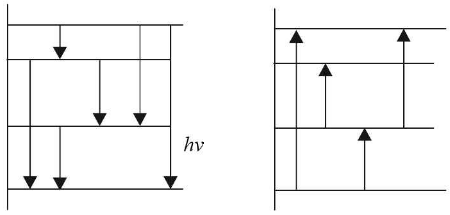
Light is emitted by an object when it is suitably excited by heating or by passing an electric discharge etc. If this light is passed through a dispersing element, **emission spectrum** is obtained. A lot of information about the source material can be obtained from the emission spectrum. Emission spectrum may be of three types:

Continuous Spectrum If the source is a hot solid such as bulb filament or liquid, the spectrum is continuous. Light emitted by a bulb, candle or red hot iron has continuously varying wavelengths. Even X-ray spectrum is continuous.

Line Spectrum When substances in its atomic state (gaseous or vapour state) de-excite, they produce bright colour lines. For example when common salt is thrown in a campfire, only a few colours appear in the form of isolated sharp parallel lines. Each line is the image of spectrograph slit deviated through an angle that depends upon the wavelength. A spectrum of this sort is called a line spectrum. For example sodium gives D_1 and D_2 doublet (589 and 589.6 nm). Hydrogen spectrum is well studied and so on.

Band Spectrum The molecular energy levels are generally grouped into several bunches, each bunch widely separated from the other but levels in a bunch are close to each other. The wavelengths emitted by such molecules are also grouped. Each group retains its identity (is separated from the other). The wavelengths in a group being close to each other and appear as continuous. The spectrum looks like a band of colours.

Absorption Spectrum When white light having all the wavelengths is passed through an absorbing material, the material may absorb certain wavelengths selectively (to get excited). These wavelengths will disappear when the transmitted light is dispersed (passed through a prism or grating). Dark lines or bands at the missing wavelengths appear on an otherwise bright continuous coloured background. Such a spectrum is called absorption spectrum. It is of two types line absorption spectrum and band absorption spectrum. When sunlight is dispersed certain sharply defined dark lines are seen. These lines are called **Fraunhoffer lines**. Fig. 31.1(a) and Fig. 31.1(b) illustrate emission and absorption process.



(a) emission (b) absorption

Fig. 31.1

Speed of light using Fizeau method

$c = \frac{2Dn\omega}{\pi} = 4\,Dnf$ where D is distance from the rotating wheel of the mirror ω is angular speed of rotation of the wheel when image is completely unseen for the first time and n is number of teeth in the wheel or number of rotations per second. $\omega = 2\pi f$ where f is linear frequency.

Foucault's method to find speed of light $c = \frac{4R^2\omega a}{S(R+b)}$ where R is the radius of concave mirror, a is distance between lens and source, b is distance between plane mirror and lens and S is shift in image.

Michelson method to find speed of light $c = \frac{DN\omega}{2\pi} = DfN$ where N is number of faces in a polygonal mirror, ω is angular speed of rotation and D is the distance travelled by light on reflections from polygonal mirror. $f = \frac{\omega}{2\pi}$ is the linear frequency.

Fresnel distance $Z_f = \frac{a^2}{\lambda}$ where a is slit width. Z_f describes the distance travelled by a beam without appreciable broadening of the beam.

Lambert's Cosine Law The surfaces which radiate according to the Lambert's Cosine Law are called perfectly diffused. $I = I_0 \cos\theta$.

Luminous flux Radiation emitted by a source has components corresponding to a wide range of wavelengths. Different component wavelengths have different energies and different brightness. The luminous flux is a quantity directly representing the total brightness producing capacity of the source. Its unit is lumen. Luminous flux of a source of $\frac{1}{685}$ W emitting monochromatic light of wavelength 555 *nm* is called 1 lumen. That is, a 1 watt source emitting a monochromatic light of wavelength 555 *nm* emits 685 lumen.

Relative Luminosity =

$$\frac{\text{Luminous flux of a source of given wavelength}}{\text{Luminous flux of a 555 nm source of same power}}$$

$$\text{Luminous efficiency} = \frac{\text{Total luminous flux}}{\text{Total radiant flux}} = \frac{\text{Luminous flux emitted}}{\text{Power input to the source}}$$

Luminous intensity or illuminating power (I) Luminous flux per unit solid angle is defined as luminous intensity. Its unit is candela (cd).

$I = \frac{dF}{d\Omega} = \frac{F}{4\pi}$ where F is luminous flux and Ω is solid angle.

1 Candela is the luminous intensity of a black body of surface area $\frac{1}{60}$ cm^2 placed at the freezing temperature of platinum at a pressure of 101.325 N m^{-2}.

Illuminance (E) is the luminous flux incident per unit area. E = $\frac{dF}{dA}$ units lumen m^{-2} or Lux. CGS unit is Phot.

Law of photometry A photometer is used to compare intensities of two sources $\frac{I_1}{I_2} = \left(\frac{d_1}{d_2}\right)^2$ where d_1 and d_2 are distances of the source from photometer.

- **Short Cuts and Points to Note**

1. In pure spectrum each colour has its sharp impression. Pure spectrum is achieved if
   (a) incident radiation (light beam) on dispersing element falls parallel.
   (b) light dispersed by it is collected in such a way that all the rays of a particular colour are collected at a plane.
   (c) slit width should be as small as possible.

   If the slit is wide, different parts of the slit produce different spectra which overlap resulting in impure spectrum.
2. Solid and liquid sources normally produce continuous spectrum. For example, a bulb, a candle, red hot iron etc. produce continuous spectrum. Even X-ray spectrum is continuous.
3. Line spectrum is obtained when atoms in their atomic state (gaseous or vapour form) de-excite; bright coloured lines are produced.

Band spectrum is produced due to de-excitation of molecules as their energy levels are grouped into several bunches. The wavelength in a bunch are close to each other.

Absorption spectrum is obtained when atoms/ molecules select particular wavelengths to get excited from the incident continuous light. These wavelengths are missing (form dark lines or bands) from the incident light when transmitted light is dispersed.

4. Speed of light is measured using:

Fizeau method $c = \dfrac{2Dn\omega}{\pi} = 4\,Dnf$

Foucault method $c = \dfrac{4R^2\omega a}{S(R+b)}$

Michelson method $c = \dfrac{D\omega N}{2\pi} = DfN$

5. Radiant flux = input power.

Luminosity of radiant flux produces a sensation of brightness in the eye.

6. Luminous flux of a 1 W source emitting wavelength 555 *nm* is 685 lumen.

7. Relative luminosity =

$$\frac{\text{Luminous flux of a source of given wavelength}}{\text{Luminous flux of a 555 } nm \text{ source of same power}}$$

8. Luminous efficiency $= \dfrac{\text{Total luminous flux}}{\text{Total radiant flux}}$

$$= \frac{\text{Luminous flux emitted}}{\text{Power input to the source}}$$

9. Luminous intensity or illuminating power is luminous flux per unit solid angle, unit is *cd.*

$$I = \frac{F}{4\pi} = \frac{dF}{d\Omega}.$$

10. Illuminance $E = \dfrac{dF}{dA} = \dfrac{I\cos\theta}{r^2}$ and follows inverse square law for a point source. In SI system it is measured in Lux. 1Lux = 1 *lm* m^{-2}

CGS unit is Phot.

11. Lamberts Cosine Law. An ideal source shall emit light radiations uniformly in all possible directions. But actual sources are extended and have a different luminous intensity in different directions given by $I = I_0 \cos\theta$.

12. **Law of Photometry** $\dfrac{I_1}{I_2} = \left(\dfrac{d_1}{d_2}\right)^2$

13. The total energy of radiations emitted per unit time is called total radiant flux. Unit is watt.

Note: It includes components of wavelengths beyond the visible region.

14. Luminosity of radiant flux measures the capacity to produce the sensation of brightness in the eye. A relative comparison of radiant flux of different relative intensities can be made by the curve shown in Fig. 31.2.

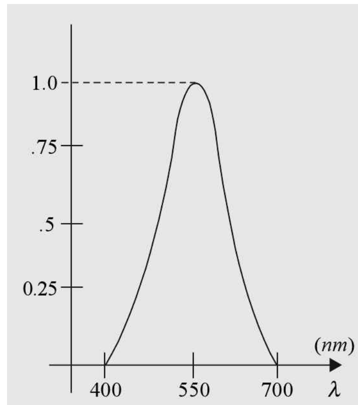


Fig. 31.2 Response of eye to different wavelengths of equal brightness

15. Inverse Square Law. Illuminance at a point distant *r* from the source is given by $E = \dfrac{I\cos\theta}{r^2}$.

16. **Photoluminescence.** Phenomenon of emission of visible light is called photoluminiscence. It is of two types **Florescence** and **Phosphorescence**. If light continues to emit for a **time > $10^8 s$** after removal of the source, the phenomenon is called phosphorescence otherwise it is florescence.

17. Persistence of vision: If our eye sees an object its impression does not fade out instantaneously. Rather, it takes $1/16$ second for image in the eye to completely fade out. This phenomenon is called persistence of vision. T.V., cinema make use of this fact.

18. Light is absorbed according to the law $I = I_0\, e^{-\alpha x}$ where x is the thickness it has passed and α is absorption coefficient.

• Caution

1. Using same luminosity, i.e., 685 lumen W^{-1} for all wavelengths.

⇒ 685 lumen W^{-1} is only for 555 nm. Multiply with relative luminosity for other wavelengths.

2. Assuming all types of light polarised or unpolarised vary as $\cos^2\theta$, i.e., $I = I_0 \cos^2\theta$.

⇒ Polarised light follow $I = I_0 \cos^2\theta$ (Malus Law) while unpolarised light for a diffused source follows Lamberts Law, i.e., $I = I_0 \cos\theta$.

3. Assume light intensity follows inverse square law.

⇒ For **diffused (point) source** it varies as $E = \dfrac{I_0 \cos\theta}{r^2}$

or amplitude of light reaches $\propto \dfrac{1}{r}$.

For **cylindrical source** intensity varies inversely with distance, i.e., $I \propto \dfrac{1}{r}$ or amplitude $a \propto \dfrac{1}{\sqrt{r}}$.

4. Not considering time for equally satisfying points in case of camera.

⇒ Use $E_1\, t_1 = E_2\, t_2$ or $\dfrac{I_1 t_1}{r_1^2} = \dfrac{I_2 t_2}{r_2^2}$ for equally satisfactory points.

5. Considering light is absorbed only in opaque medium.

⇒ Light is absorbed even in transparent medium. Coefficient of absorption depends upon the nature of material. Law of absorption is $I = I_0\, e^{-\alpha x}$

SOLVED PROBLEMS

1. A 100 W lamp is rated 0.8W per candela; the luminous flux of the source is nearly

(a) 1571.4 *lm* (b) 1603.4 *lm*
(c) 1501.3 *lm* (d) 1481.6 *lm*

Solution (a) $I = \dfrac{100}{0.8} = 125\ cd.$

Luminous flux $F = 4\pi I$

$$= 4 \times \frac{22}{7} \times 125 = 1571.4\ lm$$

2. A lamp placed 60 cm from a screen produces the same illumination as a standard 100 W lamp placed 90 cm away on the other side of the screen. The luminous intensity of the first lamp is

(a) 49.44 W (b) 44.44 W
(c) 54.44 W (d) 34.44 W

Solution (b) $P_1 = \dfrac{r_1^2}{r_2^2} P_2 = \dfrac{60\times60}{90\times90} \times 100$

$$= \frac{400}{9} = 44.44\ \text{W}$$

3. The time of exposure is 4 *s* to print a photograph from a negative when source is 1 m away. If the source is placed 0.5 m away the time of exposure will be

(a) 4 *s* (b) 2 *s*
(c) 1 *s* (d) 0.5 *s*

Solution (c) $\dfrac{I_1}{r_1^2} t_1 = \dfrac{I_2 t_2}{r_2^2}$ or

$$t_2 = \frac{t_1 r_2^2}{r_1^2} = \frac{10.51^2 \times 4}{1^2} = 1\ s$$

4. If relative luminosity is 0.65 then the luminous flux of a 10 W source is

(a) 12130.6 (b) 4125.5
(c) 4352.5 (d) 4452.5

Solution (d) $10 \times 685\ (0.65)$

5. The luminous intensity of a small plane source of light along the forward normal is 160 *cd*. Assuming the source to be perfectly diffused, the luminous flux emitted into a cone of solid angle 0.04 str around a line making 60° with forward normal is

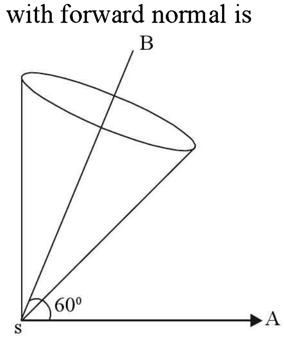


Fig. 31.3

(a) 3.2 *lm* (b) 0.8 *lm*
(c) 16 *lm* (d) 1.6 *lm*

Solution (a) $I = I_0 \cos 60 = 160 \times \dfrac{1}{2} = 80\ cd$

flux $= I\,\Delta\Omega$

$= 80 \times 0.04 = 3.2\ lm$

6. An illuminance of $2.56 \times 10^5\ lm/m^2$ is produced by the sun on the earth. The luminous flux of the sun is

(a) 5.625×10^{28} *lm* (b) 7.2×10^{28} *lm*
(c) 7.2×10^{27} *lm* (d) 5.625×10^{27} *lm*

Solution (b) $E = I/r^2$ and

flux $= 4\pi I = 4\pi E r^2$

$= 4 \times \frac{22}{7} \times 2.56 \times 10^5 (1.5 \times 10^{11})^2$

$= 7.2 \times 10^{28}$ *lm*.

7. Fig 31.4 shows a florescent lamp. The intensities at points A, B and C are related.

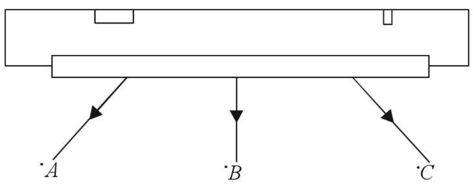


Fig. 31.4

(a) $I_A = I_B = I_C$ (b) $I_B > I_A > I_C$
(c) $I_A = I_C < I_B$ (d) $I_B = I_A > I_C$

Solution (c) Apply $I = \frac{I_0 \cos\theta}{r}$

8. A battery operated torch is adjusted to give parallel beam of light. It produces illuminance of 60 *lux* on a wall 2 m away. The illuminance produced 3 *m* away is

(a) 60 *lux* (b) $\frac{80}{3}$ *lux*
(c) 40 *lux* (d) none of these

Solution (a) $\frac{dF}{dA}$ is const.

9. A circular area of radius 1 cm is placed 2 m away from a point source. The source emits light uniformly in all directions. The source is on the line of the normal from the centre of the area. 2×10^{-3} *lm* flux is incident on the area. Find the total flux emitted by the source.

(a) 250 *lm* (b) 320 *lm*
(c) 285 *lm* (d) 355 *lm*

Solution (b) $\Delta\Omega = \frac{\pi \times 1 \times 10^{-4}}{2^2} = \frac{\pi}{4} \times 10^{-4}$ str.

Total flux $F = \frac{2 \times 10^{-3} \times 4\pi}{\frac{\pi}{4} \times 10^{-4}} = 320$ lm

10. A point source emitting uniformly in all directions is placed 0.5 m above a table top. The luminous flux of the source is 500 π lumen. Find the illuminance at a small surface area of the table top 0.8 m from the source.

(a) 125 *lux* (b) 122 *lux*
(c) 118 *lux* (c) 115 *lux*

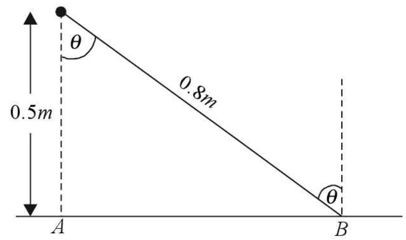


Fig. 31.5

Solution (b) $I = \frac{500\pi}{4\pi} = 125$ *cd*

$E = \frac{I \cos\theta}{r^2} = \frac{125 \times 5}{(.8)^2 \times 8}$

$= 122$ *lux*

11. The brightness producing power of a source
(a) does not depend on the power
(b) does not depend upon wavelength emitted
(c) depends on power
(d) none of these

Solution (c)

12. As the wavelength is increased from violet to red, the luminosity
(a) increases continuously
(b) decreases continuously
(c) first increases then decreases
(d) first decreases then increases

Solution (c)

13. The parameter that determines the brightness of a light source sensed by an eye is
(a) energy of light entering the eye per second
(b) wavelength of the light
(c) total radiant flux entering the eye
(d) total luminous flux entering the eye

Solution (d)

14. Light from a point source falls on a screen. If the separation between the source and the screen is increased by 1%, the illuminance will decrease

(a) 0.5% (b) 1%
(c) 2% (d) 4%

Solution (c) $E \propto \frac{1}{r^2}$

$$\frac{\Delta E}{E} = \frac{2\Delta r}{r} \Rightarrow 2\,(1\%) = 2\%$$

15. A room is illuminated by an extended source. The illuminance at a particular portion can be increased by

(a) moving the source

(b) rotating the source

(c) changing the colour of the source

(d) placing mirrors at proper positions

(e) all of these

Solution (e)

TYPICAL PROBLEMS

16. A photographic plate records sufficiently intense image when exposed with 10 W source for 12 *s*. How much time it will take if exposed with 12 W source

(a) 12 *s* (b) 10 *s*

(c) 8 *s* (d) 9 *s*

Solution (b) Energy be equal $P_1 t_1 = P_2 t_2$

or $t_2 = \frac{12\times10}{12} = 10\,s$

17. A source emits 31.4 W of radiant flux distributed uniformly in all directions. The luminous efficiency is 60 lumen/W. Find the luminous intensity of the source.

(a) 600 π *Cd* (b) 600 *Cd*

(c) 150 *Cd* (d) 100 *Cd*

Solution (c) $F = 31.4\times60\,lm$,

$$I = \frac{F}{4\pi} = \frac{31.4\times60}{4\pi} = 150\text{ Cd.}$$

18. Two light sources of 8 and 12 *Cd* are placed on the same side of the photometer screen at a distance of 40 cm from it. Where should a 80 *Cd* source be placed to balance the illuminance?

(a) 40 cm (b) 60 cm

(c) 20 cm (d) 80 cm

Solution (d) $\frac{(8+12)}{40^2} = \frac{80}{d^2}$

or d = 80 cm

19. A student is studying a book placed at the edge of a circular table of radius *R*. A point source of light is suspended directly above the centre of the table. What should be the height of the lamp so that maximum illuminance is produced at the position of the book?

(a) R (b) $R/2$

(c) $R/\sqrt{2}$ (d) $R/\sqrt{3}$

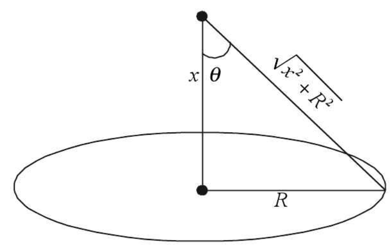


Fig. 31.6

Solution (c) $I = \frac{I_0\cos\theta}{(x^2+R^2)} = \frac{I_0 x}{(x^2+R^2)^{3/2}}$.

For I to be maximum $\frac{dI}{dx} = 0$ or

$$\frac{I_0}{(x^2+R^2)^{3/2}} - \frac{x(3/2)(2x)}{(x^2+R^2)^{5/2}} = 0 \quad \text{or}$$

$R^2 - 2x^2 = 0$ or

$x = R/\sqrt{2}$

20. An electric lamp and a candle produce equal illuminance on a screen when placed 80 cm and 20 cm from the screen respectivley. The lamp is now covered with a thin paper which transmit 49% of the luminous flux. By what distance the lamp be moved to balance the intensities at the screen again?

(a) 24 cm (b) 12 cm

(c) 18 cm (d) 56 cm

Solution (a) Case (i) $\frac{I_1}{I_2} = \left(\frac{80}{20}\right)^2$

Case (ii) $\frac{.49 I_1}{I_2} = \left(\frac{d}{20}\right)^2$ or

$d = 20\sqrt{.49(16)} = 20\,(2.8)$

$= 56$ cm

Lamp be moved by $80 - 56 = 24$ cm

21. In the Foucault's method, the distance from the rotating mirror to concave mirror is 20 m. Lens is placed mid way between the illuminated slit and the rotating mirror which are 2 m apart. If the rotating mirror makes 438 revolutions per second and the deflection of the image is 0.7 mm, calculate the velocity of light.

(a) $2.847 \times 10^8\ ms^{-1}$ (b) $2.927 \times 10^8\ ms^{-1}$

(c) $2.649 \times 10^8\ ms^{-1}$ (d) $2.992 \times 10^8\ ms^{-1}$.

Solution (d) $C = \dfrac{4\omega R^2 a}{S(R+b)} = \dfrac{4(2\pi \times 438) \times (20)^2 \times 1}{.7 \times 10^{-3}(20+1)}$

$= 2.992 \times 10^8\ ms^{-1}$

22. The illumination produced by A is balanced by B on the screen when B is 60 cm apart from the screen. A smoked glass plate is placed in front of A and to balance the illumination B is to move 15 cm further away. Find the transmission coefficient of the smoked glass.

(a) 0.36 (b) 0.64

(c) 0.49 (d) 0.51

Solution (b) $\alpha = \dfrac{I_2}{I_1} = \left(\dfrac{60}{75}\right)^2 = \left(\dfrac{4}{5}\right)^2 = \dfrac{16}{25} = 0.64$

23. Which of the following is the method used to measure the speed of light in laboratory

(a) Fizeau method (b) Roemer method

(c) Michelson method (d) Foucault's method

Solution (d)

24. In Michelson method distance travelled by light between two reflections is 4.8 km. The shape of rotating mirror is regular octagon. At what minimum speed (other than zero) is the image formed at the position where non rotating mirror forms it?

(a) 780 *rev/s* (b) 7800 *rev/s*

(c) 690 *rev/s* (d) 7500 *rev/s*

Solution (b) $C = Dfn\ \ f = \dfrac{C}{DN}$

$= \dfrac{3\times10^8}{4.8\times10^3} \times 8 = \dfrac{10^5}{12.6}$

QUESTIONS FOR PRACTICE

1. 1% of light of a source with luminous intensity 50 candela is incident on a circular surface of radius 10 cm. The average illuminance of the surface is

(a) 100 lux (b) 200 lux

(c) 300 lux (d) 400 lux

2. The illuminance of a surface is 10 lux. If the total area of the surface is 30 cm^2, then the luminous flux incident on it will be

(a) 3×10^{-4} lm (b) 3×10^{-3} lm

(c) 3×10^{-2} lm (d) 3×10^{-1} lm

3. The luminous efficiency of a lamp is 51mW^{-1} and its luminous intensity is 35Cd. The power of the lamp is

(a) 11W (b) 22W

(c) 44W (d) 88W

4. The luminous intensity of a light source is 500 Cd. The illuminance of a surface distant 10 m from it, will be if light falls normally on it

(a) 5 lux (b) 10 lux

(c) 20 lux (d) 40 lux

5. The luminous intensity of a light source is 300 Cd. The illuminance of a surface lying at a distance of 10 m from it will be if light falls normally on it

(a) 30 lux (b) 3 lux

(c) 0.3 lux (d) 0.03 lux

6. In the above problem, if light falls on the surface at an angle of 60°, then illuminance will be

(a) 12 lux (b) 6 lux

(c) 3 lux (d) 1.5 lux

7. A lamp is hanging at a height of 4 m above a table. The lamp is lowered by 1m. The percentage increase in illuminance will be

(a) 40% (b) 64%

(c) 78% (d) 92%

8. At what distance should a book be placed from a 50 Cd bulb so that the illuminance on the book becomes $2\,l\,m\,m^{-2}$

(a) 1m (b) 5m

(c) 10m (d) 50m

9. Two light sources with equal luminous intensity are lying at a distance of 1.2 m from each other. Where should a screen be placed between them such that the illuminance on one of its faces is four times that on the another face?

(a) 0.2m (b) 0.4m

(c) 0.8m (d) 1.6m

10. Two lamps of luminous intensity of 8 Cd and 32 Cd respectively are lying at a distance of 1.2 m from each other. Where should a screen be placed between two lamps such that its two faces are equally illuminated due to the two sources?

(a) 10cm from 8 Cd lamp (b) 10cm from 32 Cd lamp

(c) 40cm from 8 Cd lamp (d) 40 cm from 32 Cd lamp

11. In the above problem, another position of the screen where its one face will be equally illuminated by the two sources will be

(a) 60 cm from 8 Cd lamp

(b) 120 cm from 8Cd tamp

(c) 60 cm from 32Cd lamp

(d) 120 cm from 32 Cd lamp

12. A photoprint is required to be placed in front of 100 Cd lamp at a distance of 0.5 m for 25 sec for good impression. If it is to be placed in front of a 400 Cd lamp for 36 sec for the same impression then the distance of the print from the lamp will be

(a) 0.5m (b) 1.0m

(c) 1.2m (d) 1,5m

13. A lamp is hanging along the axis of a circular table of radius r. At what height should the lamp be placed above the table, so that the illuminance at the edge of the table is $\frac{3}{8}$ of that at its center?

(a) $\frac{r}{2}$ (b) $\frac{r}{\sqrt{2}}$

(c) $\frac{r}{3}$ (d) $\frac{r}{\sqrt{3}}$

14. Two small lamps A and B, the first placed 60cm to the left of a Bunsen photometer and the second placed 100 cm to the right of the same photometer, produce equal illuminance at the photometer. A large perfectly reflecting mirror is then placed 20 cm to the left of A with its reflecting surface normal to the axis of the bench. The illuminance on two sides of the photometer become unequal. Through what distance must the lamp B be moved in order to restore equality of illuminance on the photometer?

(a) 0.143 m (b) 0.234 m

(c) 0.369 m (d) 0.457 m

15. A photographic print was found to be satisfactory when the exposure was for 20 s at a distance of 0.6 m from a 40 W lamp. At what distance must the same paper be held from a 60W lamp in order that an exposure of 36 s may give the same result?

(a) 1.2 m (b) 0.98 m

(c) 0.72 m (d) 0.66 m

16. A screen receives 3 watt of radiant flux of wavelength 6000 Å. One lumen is equivalent to 1.5×10^{-3} watt of monochromatic light of wavelength 5550 Å. If relative luminosity for 6000Å is 0.685 while that for 5550Å is 1.00, then the luminous flux of the source is

(a) 4×10^3 lm (b) 3×10^3 lm

(c) 2×10^3 lm (d) 1.37×10^3 lm

17. A point source of 100 candela is held 5m above a sheet of blotting paper which reflects 75% of light incident upon it. The illuminance of blotting paper is

(a) 4 phot (b) 4 lux

(c) 3 phot (d) 3 lux

18. A fluorescent tube which is equivalent to a line source of 100 candela per meter is hung horizontally 5 meters above the table. The illuminance at a point vertically below the tube will be

(a) 8 lux (b) 8 phot

(c) 40 lux (d) 40 phot

19. A source of light emits a continuous stream of light energy which falls on a given area. Luminous intensity is defined as

(a) luminous energy emitted by the source per second.

(b) luminous flux emitted by the source per unit solid angle

(c) luminous flux falling per unit area of a given surface.

(d) luminous flux coming per unit area of an illuminated surface.

20. Inverse square law for illuminance is valid for

(a) isotropic point source

(b) cylindrical source

(c) search light

(d) all types of sources

21. The distance between a point source of light and a screen is doubled. The intensity will be

(a) four times the original value

(b) two times the original value

(c) half the original value

(d) one quarter of the original value

22. The unit of luminous efficiency of electric bulb is

(a) watt (b) lumen

(c) lumen/watt (d) lux

23. Candela is a unit of

(a) accoustic intensity (b) electric intensity

(c) luminous intensity (d) magnetic intensity

24. The luminous intensity of a 100 W unidirectional bulb is 100 candela. The total luminous flux emitted from the bulb will be

(a) 100π lumen (b) 200π lumen

(c) 300π lumen (d) 400π lumen

25. In the above problem the luminous efficiency of the bulb in lumen/watt will be

(a) 4π (b) 37π

(c) 2π (d) π

26. When sunlight falls normally on earth, a luminous flux of 1.57×10^5 lumen/m^2 is produced on earth. The distance of earth from sun is 1.5×10^8 Km. The luminous intensity of sun in candela will be

(a) 3.53×10^{27} (b) 3.53×10^{25}
(c) 3.53×10^{29} (d) 3.53×10^{21}

27. In the above problem, the luminous flux emitted by sun will be

(a) 4.43×10^{25} lm (b) 4.43×10^{26} lm
(c) 4.43×10^{27} lm (d) 4.43×10^{28} lm

28. An electric bulb of luminous intensity I is suspended at a height h from the center of the table having a circular surface diameter $2r$. The illuminance at the center of the circular disc will be

(a) $\frac{I}{r^2}$ (b) $\frac{I}{r}$
(c) $\frac{I}{h^2}$ (d) $\frac{I}{h}$

29. A lamp of 250 candle power is hanging at a distance of 6m from a wall. The illuminance at a point on the wall at a minimum distance from the lamp will be

(a) 9.64 lux (b) 4.69 lux
(c) 6.94 lux (d) none of these

30. In the above problem, the illuminance at a point on the wall at a distance 8 m below the previous point will be

(a) 2.5 lux (b) 1.5 lux
(c) 0.5 lux (d) none of these

31. If the distance of a surface from light source is doubled, then the illuminance will become

(a) $\frac{1}{2}$ times (b) 2 times
(c) $\frac{1}{4}$ times (d) 4 times

32. The luminous flux of a 60 W power bulb is 600 lumen. The luminous efficiency of the bulb will be

(a) 60 lmW^{-1} (b) 600 lmW^{-1}
(c) 0.1 lmW^{-1} (d) 10 lmW^{-1}

33. The luminous efficiency of a lamp is 5 lmW^{-1} and its luminous intensity is 30 candela. The power of the lamp will be

(a) 6π W (b) 12π W
(c) 24π W (d) 48π W

34. The luminous efficiency of a lamp is 8.8 lumen/watt and its luminous intensity is 700 Cd. The power of the lamp will be

(a) 10^1W (b) 10^2W
(c) 10^3W (d) 10^4W

35. The light from an electric bulb is normally incident on a small surface. If the surface is tilted to 60° from this position, then the illuminance of the surface will become

(a) half (b) one fourth
(c) double (d) four times

36. The illuminance of a surface distant 10 m from a light source is 10 lux. The luminous intensity of the source for normal incidence will be

(a) 10^1 Cd (b) 10^2 Cd
(c) 10^3 Cd (d) none of these

37. Light from a lamp is falling normally on a surface distant 10 m from the lamp and the luminous intensity on it is 10 lux. In order to increase the intensity 9 times, the surface will have to be placed at a distance of

(a) 10 m (b) $\frac{10}{3}$ m
(c) $\frac{10}{9}$ m (d) 10×9 m

38. The illuminance on screen distant 3 m from a 100 W lamp is 25 lm/m^2. Presuming normal incidence, the luminous intensity of the bulb will be

(a) 100 Cd (b) 25 Cd
(c) 225 Cd (d) none of these

39. In a grease spot photometer, light from a lamp with dirty chimney is exactly balanced by a point source distant 10 cm from the grease spot. On clearing the chimney, the point source is moved 2 cm to obtain balance again, The percentage of light absorbed by dirty chimney is nearly

(a) 56% (b) 44%
(c) 36% (d) 64%

40. A screen is illuminated by two point sources A and B. Another source C sends a parallel beam of light towards point P on the screen. The lines AP, BP and CP are 3m, 1.5 m and 1.5 m respectively. The radiant power of the sources A and B are 90 W and 180 W respectively. The beam coming from C is of intensity 20 W/m^2. The intensity at point P on the screen due to the source A will be

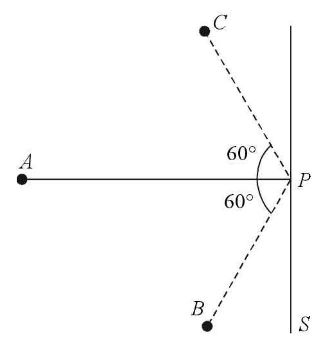


Fig. 31.7

(a) $\frac{1.25}{p}$ (b) $\frac{2.5}{p}$ W/m²

(c) $\frac{5}{p}$ W/m² (d) $\frac{10}{p}$ W/m²

41. In the above problem, the intensity at point P due to the source B will be

(a) $\frac{1.25}{p}$ W/m² (b) $\frac{2.5}{p}$ W/m²

(c) $\frac{5}{p}$ W/m² (d) $\frac{10}{p}$ W/m²

42. In Q. 40, the intensity at point P due to the source C will be

(a) 5 W/m² (b) 10 W/m²

(c) 15 W/m² (d) 20W/m²

43. In Q. 40, the total intensity at point P due to the source A, B and C will be

(a) 10W/m² (b) 20W/m²

(c) 14 W/m² (d) 28W/m²

44. A point source of light is situated at a distance of 5 cm from a screen. There is a hole of diameter 7 cm in the screen and light of luminous intensity 15.4 lumen passes through it. The solid angle subtended by the hole at the source will be (in str)

(a) 1.54 (b) 2.27

(c) 3.14 (d) 4.7

45. In the above problem, the luminous intensity of the source in the direction of hole will be

(a) 40 Cd (b) 30 Cd

(c) 20 Cd (d) 10 Cd

46. In Q45, the luminous flux emitted by the source, will be—

(a) 10p lumen (b) 20p lumen

(c) 30p lumen (d) 40p lumen

47. In the adjoining figure is shown a circular cross-section of tunnel. A bulb of 100W is lighting at the highest point S of the tunnel. The diameter of the tunnel is 4m. The illuminance at point Q will be

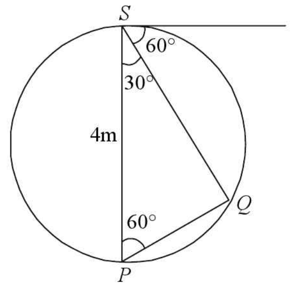


Fig. 31.8

(a) $\frac{25\sqrt{3}}{6} W/m^2$ (b) $\frac{25}{4} W/m^2$

(c) $\frac{25\sqrt{3}}{4} W/m^2$ (d) $\frac{25}{6} W/m^2$

48. In the above problem, the ratio of illuminance, at points Q and P will be

(a) $\frac{\sqrt{3}}{2}$ (b) $\frac{2}{\sqrt{3}}$

(c) $\frac{3}{2}$ (d) $\frac{2}{3}$

49. A lamp of luminous intensity 20 Cd is hanging at a height of 40 cm from the center of a square table of side 60 cm. The illuminance at the centre of the table will be

(a) 100 Lux (b) 125 Lux

(c) 150 Lux (d) none of these

50. In the above problem, the illuminance at the mid-point of the side of the table will be

(a) 125 Lux (b) 100 Lux

(c) 64 Lux (d) 32 Lux

51. In Q 50, the illuminance at the corner of the table will be

(a) 125 Lux (b) 64 Lux

(c) 72 Lux (d) 40.35 Lux

Answers to Questions for Practice

1.	(b)	**2.**	(c)	**3.**	(d)	**4.**	(a)	**5.**	(b)	**6.**	(d)	**7.**	(c)
8.	(b)	**9.**	(b)	**10.**	(c)	**11.**	(b)	**12.**	(c)	**13.**	(d)	**14.**	(a)
15.	(b)	**16.**	(d)	**17.**	(b)	**18.**	(c)	**19.**	(b)	**20.**	(d)	**21.**	(d)
22.	(c)	**23.**	(c)	**24.**	(d)	**25.**	(a)	**26.**	(a)	**27.**	(d)	**28.**	(c)
29.	(c)	**30.**	(b)	**31.**	(c)	**32.**	(d)	**33.**	(c)	**34.**	(c)	**35.**	(a)
36.	(c)	**37.**	(b)	**38.**	(c)	**39.**	(c)	**40.**	(b)	**41.**	(d)	**42.**	(b)
43.	(c)	**44.**	(a)	**45.**	(d)	**46.**	(d)	**47.**	(a)	**48.**	(b)	**49.**	(b)
50.	(c)	**51.**	(d)										

32

Photoelectric Effect and Dual Nature of Matter

BRIEF REVIEW

The energy of electro magnetic radiation is quantised. It is emitted and absorbed in particle like packages of definite energy, called **photon** or **quanta**. The energy of single photon is proportional to the frequency of radiation, i.e.,

$$E = hf = \frac{hc}{\lambda}. \text{ Where } h = 6.626 \times 10^{-34} \text{ Js}$$

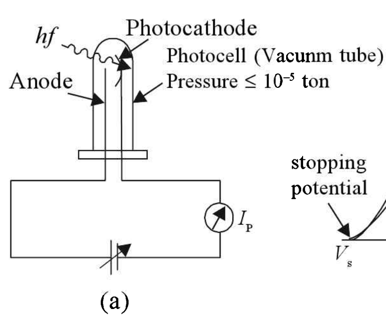
$$= 4.136 \times 10^{-15} \text{ eVs}$$

Even the internal energy of atoms is quantized. For a given atom, the energy cannot have just any value only **discrete** values called **energy levels** are allowed.

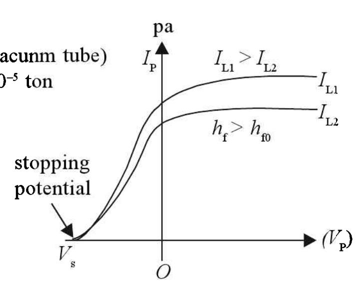
Properties of Photon

1. All photons in vacuum travel with speed of light.
2. Their velocity changes in the medium due to change in wavelength.
3. Rest mass of photon is zero, i.e., photon cannot exist at rest.
4. Each photon has definite energy $h\nu$ and definite momentum $p = \frac{h}{\lambda} = mc = \frac{E}{c}$.
5. A photon may collide with a material particle. Total energy and total momentum remain conserved in such a collision. The photon may get absorbed or new photon may be created (emitted). Thus the number of photons may not be conserved.
6. Increase in the intensity of light means increase in number of photons crossing a given area in a given time. The energy of photon remains the same.

Photoelectric effect describes the emission of electrons when light strikes a surface. When light of sufficiently high frequency (greater than a minimum frequency, called threshold frequency) is incident on a metal surface, **free electrons** are **ejected** from its **surface**. This phenomenon is called **photoelectric effect**. Electrons so emitted are called photo electrons. The experimental set up is shown in Fig. 32.1 (*a*). When photons are incident on a photo cathode, electrons are ejected provided they have energy greater than a certain minimum called work function. The anode at positive potential attracts these electrons and a current is seen.



(a)
Photo cell



(b)
Effect of intensity

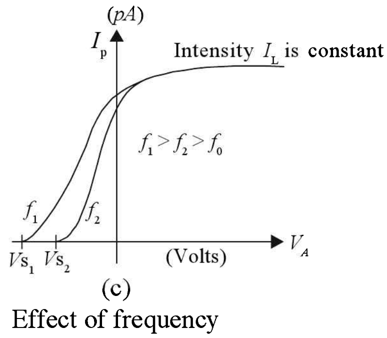


Fig. 32.1 Illustration of photoelectric effect

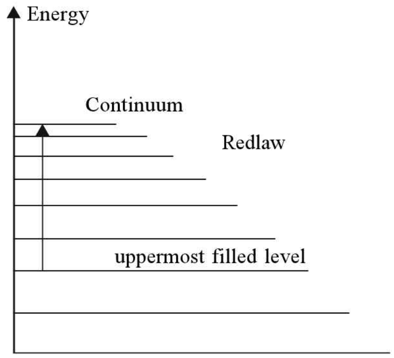


Fig. 32.2 Work function illustration

It is found experimentally that

(a) No emission occurs until the incident radiation has a frequency greater than a certain minimum called threshold frequency f_o irrespective of time of exposure.

(b) No emission occurs until the incident radiation has a frequency $f > f_o$ (threshold frequency) irrespective of intensity of incident radiation.

(c) If the frequency f of incident radiation is greater than f_o (threshold frequency) only then emission starts and photo current is observed.

(d) Keeping frequency of incident radiation $f > f_o$, if intensity of incident radiation is increased photo current increases as shown in Fig. 32.1 (b).

(e) If frequency of incident radiation, though $> f_o$, is further increased stopping potential increases as illustrated in Fig. 32.2 (c).

Einstein explained this theory in 1905. He was awarded Noble Prize in 1921 for his contribution to explain Photo-electric effect. According to Einstein's equation

$(KE)_{max} = hf - hf_o$ or

$(KE)_{max} = e\,V_s = hf - \phi$

i.e., maximum KE of photo electrons = Energy of incident radiation (photon) – work function.

Work Function (ϕ) Minimum energy given to the free electrons on the surface of the metal to be ejected is called work function. It is equivalent to ionisation energy. We may define work function as the minimum energy given to an electron present in the uppermost filled level to transit it to vacuum level or continuum as illustrated in Fig. 32.2.

Thus $\phi = hf_o$. Table 32.1 gives work functions of certain metals.

Table 32.1

Element	Work function (ev)
Al	4.3
C	5.0
Cu	4.7
Au	5.1
Ni	5.1
Ag	4.3
Si	4.8
Na	2.7

Stopping potential is that negative voltage given to the anode at which photo current stops (becomes zero). Obviously $eV_S = (KE)_{max}$

Thus $eV_s = hf - hf_o$ or $V_s = \frac{hf}{e} - \frac{hf_o}{e}$

or $V_s = \frac{hf}{e} - \frac{\phi}{e}$.

That is curve between stopping potential and frequency $f\left(\text{or } \frac{1}{\lambda}\right)$ is a straight line as illustrated in Fig. 32.3.

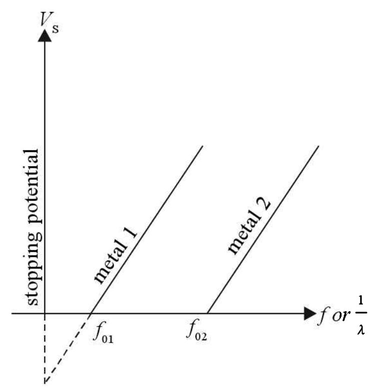


Fig. 32.3

The slope of the line is $\frac{h}{e}$ if curve is between V_S and f.

The slope of the line is $\frac{hc}{e}$ if curve is between V_S and $\frac{1}{\lambda}$.

Note that the curves between stopping potential V_S and frequency f for different metals are parallel and the slope is constant.

Matter waves The wavelength of matter wave is given by de-Broglie relation and confirms the dual nature of matter

$$\lambda = \frac{h}{p} = \frac{h}{mv}.$$

Experiments like Davison's and Germer's confirm the wave nature of electrons. de-Broglie was awarded Noble prize in 1929.

Classical mechanics works well for particles for size > 10^{-4} cm. For smaller particles Quantum mechanics should be applied. Quantum mechanics takes into account quantum nature or dual nature of particles like electrons, proton, neutrons and other subatomic particles.

- **Short Cuts and Points to Note**

1. Photoelectric effect can be explained completely by Einstein's equation
$(KE)_{max} = hf - \phi = hf - hf_o$
and $(KE)_{max} = eV_S$ where V_S is stopping potential. Until frequency of incident radiation $f > f_o$ (threshold frequency), no emission of photo electrons occurs.

2. Stopping potential does not depend upon intensity of incident radiations (or power rating of the source). OR maximum KE of ejected electrons is independent of intensity of incident radiation.

3. Stopping potential depends upon (i) frequency of incident radiation and (ii) nature of photo cathode (work function). More the frequency of incident radiation (or lesser the wavelength) more is the stopping potential. Lesser the work function more is the stopping potential.

4. Cesium (Cs) has least work function. Work function of any material can be decreased by oxide coating.

5. Stopping potential in volts = $(KE)_{max}$ in eV (That is, remove only e).

6. $\lambda\,(nm) = \dfrac{1240}{E(\text{eV})}$ for photons.

7. $\lambda = \dfrac{h}{p} = \dfrac{h}{\sqrt{2(KE)m}}$ for all other particles.

8. Slope of stopping potential vs frequency curve is $\dfrac{h}{e}$ and slope of stopping potential vs $\dfrac{1}{\lambda}$ is $\dfrac{hc}{e}$.

9. $\lambda\,(\text{A}°) = \dfrac{12.27}{\sqrt{V}}$ for electrons

$\lambda\,(\text{A}°) = \dfrac{0.286}{\sqrt{V}}$ for protons and neutrons

$\lambda\,(\text{A}°) = \dfrac{0.101}{\sqrt{V}}$ for α- particles

$\lambda = \dfrac{h}{mv_{rms}} = \dfrac{h}{\sqrt{3mkT}}$ for gas particles where k is Boltzmann's constant and T is temperature in kelvin.

10. Number of photons incident per second $N = \dfrac{I_P}{e}\dfrac{1}{\eta}$

$N = \dfrac{\text{Photo current}}{\text{Charge on an electron} \times \text{efficiency}}$.

Assuming each photon causes an electron emission.

Then $N = \dfrac{I_P}{e}$

Normal conversion efficiency is 1 – 2%

11. Force exerted by photons $F = \dfrac{dp}{dt} = \dfrac{P}{c}$ if surface is absorbing where P is power of the source, c is speed of light.

$F = \dfrac{2P}{c}$ if the surface is perfectly reflecting.

12. Momentum of photon = $p = \dfrac{E}{c} = \dfrac{h}{\lambda}$

13. Even when no light is incident or photocell is covered with a black cloth, a small amount of current is observed. It is in noticeable range if photomultiplier tube is used. Such a current is called dark current. Dark current originates because high energy radiations like γ-ray are able to penetrate and cause photoemission.

14. Photocells are of three types:
(a) Photo emissive [as shown in Fig. 32.1 (a)].
(b) Photo conductive or light dependent resistors (LDR).
(c) Photo voltaic or solar cells.

Note: Photoelectric effect was initially studied using Photo emissive cell which is a vacuum tube. Photo conductive and Photo voltaic cells are semiconductors.

15. Compton shift $\Delta\lambda = \dfrac{h}{m_o c}(1 - \cos\phi)$ where ϕ is the angle at which photon is scattered.

16. Number of photons emitted per second by a source $= \dfrac{\text{Power rating}}{hf}$.

17. According to Rayleigh Jeans Law $I(\lambda) = \dfrac{2\pi ckT}{\lambda}$

and Planck's law $I(\lambda) = \dfrac{2\pi hc^2}{\lambda^5\left(e^{\frac{hc}{\lambda kT}} - 1\right)}$ wavelength of maximum intensity

$\lambda_m = \dfrac{hc}{4.965kT}$ or $\lambda_m T = 2.89 \times 10^{-3}\, m - K$

- **Caution**

1. Considering all electrons ejected have $KE = hf - \phi$.

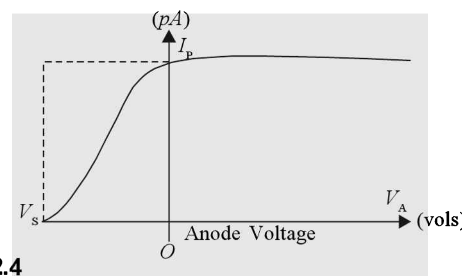


Fig. 32.4

⇒ The curve shown in Fig. 32.4 suggests only very few electrons have energy equal to KE_{max}. If all the electrons had energy equal to KE_{max} then the curve would have been as shown by dotted line.

2. Considering that same formula may be applied for photons and material particles to find wavelength.

⇒ $\lambda\,(nm) = \dfrac{1240}{E(eV)}$ is valid for photons like *uv* rays, *x*-rays, γ-rays light rays, *IR*, radiowaves etc.

3. Assuming that if a source has higher power rating then photons emitted from it have higher energy or higher frequency.

⇒ More power rating means more intensity or large number of photons emitted per second. Therefore with more power rating of the source, photo current will increase provided the frequency of the incident photons is greater than the threshold frequency.

4. Considering that when a photon enters a material particle it is always absorbed.

⇒ All photons incident on the material do not eject photo electrons. The efficiency of photo electron emission is never more than 10%. Normally it is around 1 – 2%.

5. Considering since energy is conserved, therefore number of photons is also conserved.

⇒ Photons may be absorbed or may cause emission of other photons. Hence number of photons is not conserved.

6. Considering the absorption process in matter for photons is only photoelectric emission.

⇒ Photons may be absorbed in four ways

(i) Adsorption (heating the material)
(ii) pair production
(iii) photoelectric effect
(iv) compton scattering.

7. Considering compton wavelength λ_C as compton shift

⇒ Compton wavelength $\lambda_C = \dfrac{h}{m_o c} = 0.024\,A°$

while $\Delta\lambda = \dfrac{h}{m_o c}\,(1 - \cos\phi) = 0.024\,(1 - \cos\phi)$

8. Assuming that even a moving photon has zero mass.

⇒ Mass of moving photon is $\dfrac{E}{c^2} = \dfrac{hf}{c^2}$ or $\dfrac{h}{c\lambda}$

9. Considering there is time lag between emission of electrons and incident light radiation.

⇒ Emission of photo electrons is instantaneous when exposed to radiation having frequency > threshold frequency ($\tau \le 10^{-10} S$).

10. Considering that the distance of source from photo cathode has no bearing.

⇒ As intensity is inversely proportional to square of the distance, therefore photo current is also inversely proportional to square of the distance, i.e., $I_P \propto \dfrac{1}{r^2}$.

SOLVED PROBLEMS

1. A radio station emits 10 kW power of 90.8 MHz. Find the number of photons emitted per second.

(a) $10^{28} \times 1.6$ (b) 1.6×10^{29}
(c) 1.6×10^{30} (d) 1.6×10^{32}

Solution (b) Number of photons emitted per sec

$$= \frac{10^4}{6.626\times10^{-34}\times90.8\times10^6}$$

$$= \frac{10^{32}}{621.64} = 1.6\times10^{29}\ \text{photon/s}.$$

2. A photosensitive metallic surface has work function hf_o. If photons of energy $2\,hf_o$ fall on this surface, the electrons come out with a maximum velocity $4\times10^6\ ms^{-1}$. When the photon energy is increased to $5\,hf_o$ then maximum velocity is

(a) $2\times10^7\ ms^{-1}$ (b) $2\times10^6\ ms^{-1}$
(c) $8\times10^6\ ms^{-1}$ (d) $8\times10^5\ ms^{-1}$

[CBSE 2005]

Solution (c) $\left(\dfrac{\frac{1}{2}mv_1^2}{\frac{1}{2}mv_2^2}\right) = \dfrac{hf_1 - \phi}{hf_2 - \phi}$ or

$$\frac{v_{1\max}}{v_{2\max}} = \sqrt{\frac{2hf_o - hf_o}{5hf_o - hf_o}} = \frac{1}{2}.$$

3. **A.** The energy *E* and momentum *p* of a photon are related as $p = \dfrac{E}{c}$

R. The photon behaves as a particle.

(a) *A* and *R* are both correct and *R* is correct explanation of *A*
(b) *A* and *R* are correct but *R* is not correct explanation of *A*
(c) *A* is correct but *R* is false
(d) both *A* and *R* are false

[AIIMS 2005]

Solution (a)

4. The light rays having photons of energy 1.8 eV are falling on a metal surface having a work function 1.2 eV. What is the stopping potential to be applied to stop the emitting electrons?

(a) 3 V (b) 1.2 V

(c) 0.6 V (d) 1.4 V

[BHU 2005]

Solution (c) $eV_s = hf - \phi$

or $V_s = 1.8 - 1.2 = 0.6\ V$.

5. A photon of energy 10.2 eV collides inelastically with H-atom in ground state. After a certain time interval of few μs another photon of energy 15 eV collides inelastically with the same H atom, the observation made by a suitable detector is

(a) 1 photon with energy 10.2 eV and an electron with 1.4 eV

(b) Two photons with 10.2 eV

(c) Two photons with 1.4 eV

(d) One photon with 3.4 eV and 1 electron with 1.4 eV

[IIT Screening 2005]

Solution (a) 10.2 eV photon will excite it to 2nd orbit which on de-excitation will emit 10.2 eV photon and 15 – 13.6 = 1.4 eV.

$\therefore$ electron emitted will have energy 1.4 eV.

6. From the Fig 32.5 describing photoelectric effect we may infer correctly that

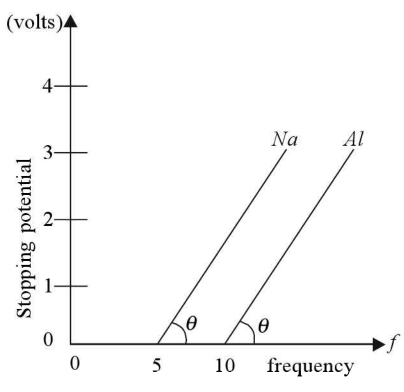


Fig. 32.5

(a) Na and Al both have same threshold frequency.

(b) Maximum KE for both the metals depend linearly on the frequency.

(c) The stopping potentials change differently for equal change in frequency.

(d) Al is a better photo sensitive material than Na.

Solution (b) $(KE)_{max} = hf - \phi$

7. A photo cell is illuminated by a small bright source placed 1 m away. When the same source of light is placed 0.5 m away, the number of electrons emitted by photo cathode would

(a) decrease by a factor of 4

(b) increase by a factor of 4

(c) decrease by a factor of 2

(d) increase by a factor of 2 **[AIEEE 2005]**

Solution (b) $\because\ I_p \propto \dfrac{1}{r^2}$

$\therefore$ current increases by a factor of 4.

8. If the KE of a free electron doubles then its de-Broglie wavelength changes by a factor

(a) $\dfrac{1}{2}$ (b) $\dfrac{1}{\sqrt{2}}$

(c) 2 (d) $\sqrt{2}$

[AIEEE 2005]

Solution (b) $\lambda = \dfrac{h}{p} = \dfrac{h}{\sqrt{2(KE)m}}$

9. A laser used to weld detached retinas emits light with a wavelength 652 nm in pulses that are of 20 ms duration. The average power during each pulse is 0.6 W. Find the energy in each pulse in eV and in a single photon.

(a) $7.5 \times 10^{15}\ eV, 1.9\ eV$ (b) $7.5 \times 10^{15}\ eV, 1.19\ eV$

(c) $7.5 \times 10^{16}\ eV, 0.19\ eV$ (d) $7.5 \times 10^{16}\ eV, 1.9\ eV$

Solution (d) $E(eV) = \dfrac{1240}{652} = 1.9\ eV$ (in a single photon)

Energy in 20 ms = $0.6 \times 20 \times 10^{-3} = 1.2 \times 10^{-2}$ J

$= \dfrac{1.2\times10^{-2}}{1.6\times10^{-19}}\ eV$

$= 7.5 \times 10^{16}\ eV$

10. Threshold wavelength for tungsten is 272 nm. Light of frequency 1.45×10^{15} Hz is incident. Find the stopping potential.

(a) 1.37 V (b) 1.47 V

(c) 1.57 V (d) 1.51V

Solution (b) $\phi = \dfrac{1240}{272} = 4.56\ eV$;

$$hf = \frac{6.625\times10^{-34}\times1.45\times10^{15}}{1.6\times10^{-19}}$$

$= 6.03\ eV$

$eV_s = hf - \phi$ or $V_s = 6.03 - 4.56 = 1.47\ V$

11. What should be the minimum work function of a metal so that visible light is able to cause emission? (Visible light = 400 – 700 nm)

(a) 1.77 eV (b) 1.87 eV

(c) 1.97 eV (d) none of these

Solution (a) $E\,(eV) = \dfrac{1240}{700} = 1.77\,eV$

12. A photon has momentum 9×10^{-28} $kg\,ms^{-1}$. What will be the stopping potential if photo cathode has work function 1.3 eV?

(a) 0.49 V (b) 1.4 V
(c) 0.59 V (d) 0.39 V

Solution (d) $E = pc = \dfrac{2.7\times10^{-19}\times3\times10^{8}}{1.6\times10^{-19}} = 1.69\,eV$

$V_s = 1.69 - 1.3 = 0.39\,V$

13. When the incident frequency is f_o, K is the $(KE)_{max}$ of the electrons emitted and ϕ is work function of the surface. If incident frequency is doubled new $(KE)_{max}$ will be

(a) $2K$ (b) $2K-\phi$
(c) $2K+\phi$ (d) $2K+2\phi$
(e) $2K-2\phi$

Solution (c) $K = hf - \phi$ or

$2K = h(2f) - 2\phi$

or $2K + \phi = 2hf - \phi$

14. Anode voltage is at +3V. Incident radiation has frequency 1.4×10^{15} Hz and work function of the photo cathode is 2.8 eV. Find the minimum and maximum KE of photo electrons in eV.

(a) 3, 6 (b) 0, 3
(c) 0, 6 (d) 2.8, 5.8

Solution (a) $hf = \dfrac{6.625\times10^{-34}\times1.4\times10^{15}}{1.6\times10^{-19}} = 5.8\,eV$;

$(KE)_{max} = hf - \phi = 5.8 - 2.8 = 3$

Since anode voltage is 3V, the electrons emitted with zero KE will acquire an energy = 3 eV and the electrons emitted with 3 eV will acquire 3 + 3 = 6 eV

∴ min KE = 3 eV and max KE = 6 eV.

15. A surface has work function 3.3 eV. Which of the following will cause emission ?

(a) 100 W incandascent lamp
(b) 40 W flouroscent lamp
(c) 20 W sodium lamp
(d) 20 W Hg lamp

Solution (d) Minimum wavelength of visible region

λ_{min} = 400 nm $E\,(eV)$

$= \dfrac{1242}{400} = = 3.1\,eV$.

∴ No visible light can cause emission. We require uv light. Only Hg lamp gives uv light.

16. A man wants current ~ mA. He should use

(a) photo multiplier tube
(b) photo cell and amplifier
(c) photo multiplier tube and amplifier
(d) photo cell and two stage amplifier.

Solution (c)

17. When a photo multiplier tube was used, the photo current recorded is 60 μA. The actual photo current is

(a) > 60 μA (b) = 60 μA
(c) < 60 μA (d) none of these

Solution (c) ∵ There will dark current also added in it. Actual current is 60 μA – dark current.

18. Tungsten has work function 4.8 eV. We wish to use tungsten as photo-cathode with a 600 nm wavelength. What shall we do?

(a) Coat tungsten with cesium
(b) Oxide coat tungsten
(c) Cu_2O_2 be coated on tungsten
(d) None of these

Solution (b)

19. Find the wavelength of 100 eV electron

(a) 1.227 A° (b) 1.72 A°
(c) 1.24 nm (d) 12.4 nm

Solution (a) $\lambda = \dfrac{12.27}{\sqrt{V}}\,A° = \dfrac{12.27}{\sqrt{100}} = 1.227\,A°$

20. Find the wavelength of 10 MeV α-particles

(a) 3 A° (b) 3 pm
(c) 3 fm (d) 30 fm

Solution (c)

$\lambda(A°) = \dfrac{0.101}{\sqrt{10^7}} = .03 \times 10^{-3}\,A° = 3 \times 10^{-15}\,m = 3\,fm$

21. The wavelength associated with 1 MeV proton is

(a) 28.6 pm (b) 2.86 pm
(c) 2.86 fm (d) 28.6 fm

Solution (d) $\lambda\,(A°) = \dfrac{0.286}{\sqrt{V}} = \dfrac{0.286}{\sqrt{10^6}}$

$= 2.86 \times 10^{-14}\,m = 28.6\,fm$

TYPICAL PROBLEMS

22. A photo multiplier has 12 plates including photo cathode and anode. Assume each dynode doubles the electrons. If a 10 W source of 300 nm is placed 2 m away then find the photo current shown by the photo multiplier tube. The dark current is 160 μA. Assume efficiency 10% and flux reaching the photo cathode is 10^{-4} of the original.

Solution $I_p \propto \dfrac{1}{r^2}$ $E(eV) = \dfrac{1240}{300} = 4.13\,eV$

∴ number of photons per second

$$N = \frac{P}{hf} = \frac{2.5\times10^{-4}}{4.13\times1.6\times10^{-19}} = 3.8\times10^{14}$$

Number of photo electrons emitted per second $Ne = 3.8 \times 10^{14} \times \frac{10}{100} = 3.8 \times \frac{10^{13}}{s^{-1}}$

Current shown by photo multiplier $I_p = 3.8 \times 10^{13} \times 1.6 \times 10^{-19} \times 2^{10} = 6.1\ mA$

Exact current = 6.1 mA – 160 μA = 5.94 mA.

23. A small metal plate of work function ϕ is kept at a distance d from a singly ionised, fixed ion. A monochromatic light beam is incident on the metal plate and photo electrons are emitted. Find the maximum wavelength of the light beam so that some of the electrons may go around the ion along a circle.

Solution $\left(\frac{1}{2}mv^2\right)_{max} = \frac{hc}{\lambda} - \phi$

for the electron to move around the ion (fixed)

$$\frac{1}{2}mv^2 = PE + KE$$

$$= \frac{-e^2}{4\pi\varepsilon_o d} + \frac{e^2}{8\pi\varepsilon_o d}$$

Thus $\frac{e^2}{8\pi\varepsilon_o d} = \frac{hc}{\lambda} - \phi$

or $\quad e^2 = \frac{hc8\pi\varepsilon_o d}{\lambda} - 8\pi\varepsilon_o \phi d$

or $\quad \lambda = \frac{8\pi\varepsilon_o hcd}{e^2 + 8\pi\varepsilon_o \phi d} = \frac{e^2}{8\pi\varepsilon_o d}$

24. The photo cathode and collector plate are kept 10 cm apart and connected through a galvanometer without a battery. A magnetic field B exists parallel to the plates. The work function of the emitter is 2.39 eV and the light incident on it has wavelength 400 to 600 nm. Find the minimum value of B so that galvanometer shows null deflection.

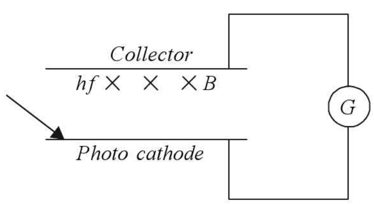


Fig. 32.6

Solution $(KE)_{max} = hf - \phi$

or $(KE)_{max} = 3.1 - 2.39 = 0.71\ eV$

$$r = \frac{mv}{qB} \quad \text{or}$$

$$B = \sqrt{\frac{2(KE)m}{qr}} \quad \text{or}$$

$$B = \sqrt{\frac{2(0.71 \times 1.6 \times 10^{-19}) \times 9 \times 10^{-31}}{1.6 \times 10^{-19} \times .1}}$$

$= 2.85 \times 10^{-5}$ T

25. The light of radiation 300 nm falls on a photocell operating in the saturation mode. The spectral sensitivity is 4.8 mA/W. Find the yield of photo electrons i.e., number of electrons produced per photon.

(a) 0.04 (b) 0.02
(c) 0.03 (d) 0.2

Solution (b) If N be the number of photons incident s^{-1} then power

$P = \frac{Nhc}{\lambda}$ Photo current $I_p = \eta Ne$

Photo current per watt = $\frac{I_P}{P} = \frac{\eta Ne\lambda}{Nhc}$ or

$$\eta \Rightarrow \frac{4.8 \times 10^{-3} \times 6.625 \times 10^{-34} \times 3 \times 10^8}{1.6 \times 10^{-19} \times 3 \times 10^{-7}}$$

$= 20 \times 10^{-3} = 0.02.$

26. Find the maximum KE of photo electrons emitted from the surface of lithium (ϕ = 2.39 eV) when exposed with $E = E_o(1 + \cos 6 \times 10^{14} t) \cos 3.6 \times 10^{15} t$.

(a) 0.37 eV (b) 0.1 eV
(c) 0.02 eV (d) 0.06 eV

Solution (a) For maximum KE, frequency should be maximum i.e.

$\omega = 4.2 \times 10^{15}\ s^{-1}$

$$(KE)_{max} = \frac{h\omega}{2\pi} - \phi$$

$$= \frac{6.625 \times 10^{-34} \times 4.2 \times 10^{+15}}{6.28 \times 1.6 \times 10^{-19}} - 2.39$$

$= 2.76 - 2.39 = 0.37\ eV$

27. Find the maximum potential a Cu ball (isolated) can have when irradiated with a wavelength λ = 140 nm. [ϕ_{Cu} = 4.47 eV]

(a) 4.47 V (b) 8.86 V
(c) 13.33 V (d) 4.39 V

Solution (d) $eV_S = \frac{hc}{\lambda} - \phi$

$$= \frac{1240}{140} - 4.47 = 8.86 - 4.47 = 4.39\ eV$$

$V_S = 4.39\ V$

28. The fringe width in a YDSE is 2 mm, distance between slits and screen 1.2 m and separation between the slits is 0.24 mm. The radiation of same source is also incident on a photo cathode of work function 2.2 eV. Find the stopping potential.

(a) 3.1 V (b) 2.2 V
(c) 0.9 V (d) 5.3 V

Solution (c) $\beta = \frac{\lambda D}{d}$ or

$$\lambda = \frac{\beta d}{D} = \frac{2 \times .24 \times 10^{-6}}{1.2} = 400\ nm.$$

$$e V_s = \frac{1240}{400} - 2.2 = 0.9\ eV \text{ or } V_s = 0.9\ V$$

29. The minimum energy required to dissociate $Ag\,Br$ bond in 0.6 eV. A photographic film is coated with a silver bromide layer. Find the maximum wavelength whose signature can be recorded on the film.
(a) 207 nm (b) 702 nm
(c) 207 $A°$ (d) 2070 nm

Solution (d) $\lambda = \frac{1242}{0.6} = 2070\ nm$

30. Find the correct statement.
(a) A free electron can absorb a photon completely.
(b) A free electron cannot absorb a photon completely.
(c) A free electron cannot exist.
(d) A free neutron can exist for a long time.

Solution (b) According to compton's scattering

$\Delta\lambda = \frac{h}{mc}(1 - \cos\phi)$ which shows a free electron cannot absorb a photon completely.

QUESTIONS FOR PRACTICE

1. In Millikan's oil drop experiment an oil drop of radius r and charge Q is held in equilibrium between the plates of a charged parallel plate capacitor when the potential difference is V. To keep a drop of radius $2r$ and with a charge $2Q$ in equilibrium between the plates, the potential difference required will be
(a) 8 V (b) 4 V
(c) 2 V (d) 1 V

2. What electric field can just support a water droplet 1.0×10^{-6} m in diameter carrying one electron charge?
(a) 3.21×10^4 V/m (b) 2.31×10^4 V/m
(c) 1.32×10^4 V/m (d) none of these

3. A charged particle is moving in a uniform magnetic field in a circular path. The energy of the particle is doubled. If the initial radius of the circular path was R, the radius of new circular path after the energy is doubled will be
(a) $\frac{R}{2}$ (b) $\sqrt{2}\,R$
(c) $2R$ (d) $\frac{R}{\sqrt{2}}$

4. An α-particle of mass 6.65×10^{-27} kg travels at right angles to a magnetic field of $0.2T$ with a speed of 6×10^5 m/s. The acceleration of α-particles will be
(a) 7.55×10^{11} ms^{-2} (b) 5.77×10^{11} ms^{-2}
(c) 7.55×10^{12} ms^{-2} (d) 5.77×10^{12} ms^{-2}

5. What voltage is needed to balance an oil drop carrying 5 electrons when located between the plates of a capacitor 5 mm apart? The mass of the drop is 3.12×10^{-16} kg.
(a) 15.5 V (b) 17.2 V
(c) 19.1 V (d) 21.7 V

6. You are sitting in a room in which uniform magnetic field is present in vertically downward direction. At the centre of the room an electron is projected in horizontal direction. It will be moving in a circular path with constant speed
(a) clockwise in vertical plane
(b) clockwise in horizontal plane
(c) anticlockwise in horizontal plane
(d) anticlockwise in vertical plane

7. A charged dust particle of radius 5 × 10 m is located in a horizontal electric field having an intensity of 6.28×10^5 V/m. The surrounding medium is air with coefficient of viscosity $\eta = 1.6 \times 10^{-5}$ Nsm^{-2}. If this particle moves with a uniform horizontal speed of 0.02 m/s, the number of electrons on it will be
(a) 10 (b) 20
(c) 30 (d) 40

8. Cathode rays of velocity 10^6 ms^{-1} describe an approximate circular path of radius 1m in an electric field of 400 Vcm^{-1}. If the velocity of cathode rays is doubled, the value of the electric field needed so that the rays describe the same circular path is
(a) 1200 Vcm^{-1} (b) 100 Vcm^{-1}
(c) 800 Vcm^{-1} (d) 1600 Vcm^{-1}

9. Electrons move at right angles to a magnetic field of $0.03T$ and enter with a velocity 9×10^7 m/s. The value of e/m will be— (Given radius of circular path = 1.764 cm)
(a) 1.7×10^{11} Ckg^{-1} (b) 2×10^{11} Ckg^{-1}
(c) 2.5×10^{11} Ckg^{-1} (d) none of these

10. A Cathode ray tube emits 1.875×10^3 electrons per second when heated. When 600 V is applied to the anode all the electrons emitted reach the anode. The maximum electron current will be

(a) 3 A (b) 3 μA
(c) 3 mA (d) none of these

11. The path of cathode rays in an electric field can be approximated to a circle of radius r. In order to double the radius of the circular path, we must

(a) reduce the electric field to half
(b) double the electric field
(c) increase electric field four times
(d) reduce electric field to one fourth

12. In a region of space cathode rays move along +ve Z-axis and a uniform magnetic field is applied along X-axis. If cathode rays pass undeviated, the direction of electric field will be along

(a) –ve X-axis (b) +ve Y-axis
(c) –ve Y-axis (d) +ve Z-axis

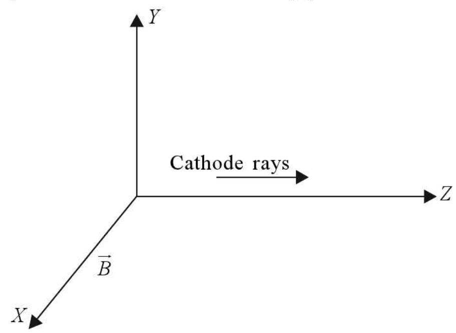


Fig. 32.7

13. A strong argument for particle nature of cathode rays is that they

(a) produce fluorescence
(b) travel through vacuum
(c) get deflected by electric and magnetic fields
(d) cast shadows

14. Cathode rays have velocities

(a) equal to that of light
(b) greater than that of light
(c) less than that of light
(d) greater than that of matter waves

15. Cathode rays are made to pass between the poles of a magnet. The effect of magnetic field is

(a) to increase their velocity
(b) to deflect them towards north pole
(c) to deflect them towards south pole
(d) to deflect them out of the plane of magnetic field

16. In Millikan's oil drop experiment, an oil drop is held stationary by a potential deference of 400V. If another drop of double the radius, but carrying the same charge, is to be held stationary, the potential difference required will be

(a) 800 V (b) 1600 V
(c) 3200 V (d) 400 V

17. In Millikan's oil drop experiment, a charged drop of mass –14 1.8 × 10 kg is stationary between the plates. The distance between the plates is 0.90cm and potential difference between them is 2.0 kV. The number of electrons on the drop is

(a) 500 (b) 50
(c) 5 (d) 0

18. Gases begin to conduct electricity at low pressures because

(a) at low pressure gases turn to plasma
(b) colliding electrons can acquire higher kinetic energy due to increased mean free path leading to ionisation of atoms
(c) atoms break up into electrons and protons at low pressures
(d) electrons in atoms can move freely at low pressures

19. In an experiment to determine e/m using Thomson's method, electrons from the cathode accelerate through a potential difference of 1.5 kV. The beam coming out of the anode enters crossed electric and magnetic field of strengths 2×10^4 V/m and $8.6 \times 10^{-4}\,T$ respectively, The value of e/m of electron will be

(a) 1.6×10^{11} C/kg (b) 1.7×10^{11} C/kg
(c) 1.8×10^{11} C/kg (d) 1.9×10^{11} C/kg

20. Cathode rays travelling from east to west into a region of electric field directed from north to south in the plane of the page. The deflection of cathode rays is towards

(a) east (b) west
(c) north (d) south

21. An electron and a proton are injected into a uniform magnetic field perpendicular to it with the same momentum. The radius of curvature is

(a) less for electron (b) more for electron
(c) same for both (d) none of these

22. In the above question if electron and proton have same kinetic energy then the radius of curvature is

(a) more for proton (b) more for electron
(c) same for both (d) none of these

23. In Q.21, if the two particles are injected into a uniform transverse electric field with same kinetic energy, then

(a) electron trajectory is more curved
(b) proton trajectory is more curved

(c) both trajectories are equally curved

(d) both trajectories are straight lines

24. In Q.21, if both particles are fired with same momentum into a transverse electric field, then

(a) electron trajectory is less curved

(b) proton trajectory is less curved

(c) both trajectories are equally curved

(d) both trajectories are straight lines

25. In a cyclotron the *AC* potential difference is V and frequency is *f*. The increase in energy of particle of charge *q* in one revolution will be

(a) q V (b) $2q$ V

(c) $q\ \frac{V}{2}$ (d) fq V

26. In the Millikan's oil drop experiment the oil drop is subjected to a horizontal electric field of 2 N/C and the drop moves with a constant velocity making an angle of 45° with the horizontal. If the weight of the drop is *W*, then the electric charge, in coulomb, on the drop is

(a) W (b) $\frac{W}{2}$

(c) $\frac{W}{4}$ (d) $\frac{W}{8}$

27. A charged oil drop falls with terminal velocity v_0 in the absence of electric field. An electric field *E* keeps it stationary. The drop acquires additional charge *q* and starts moving upwards with velocity v_0. The initial charge on the drop was

(a) $4\,q$ (b) $2\,q$

(c) q (d) $\frac{q}{2}$

28. A charged oil drop is held stationary in an electric field. The space surrounding the drop is exposed to a radioactive source and the drop moves with different terminal velocities v, $2v$, $3v$ etc. It is inferred that

(a) charge is conserved

(b) drop carries –ve charge

(c) charge is quantised

(d) drop carries +ve charge

29. An electron at rest is accelerated in a uniform electric field of strength *E*. It covers a distance *x*. The kinetic energy gained by the electron will be

(a) Eex (b) $\frac{Ex}{e}$

(c) $\frac{Ee}{x}$ (d) $\frac{ex}{E}$

30. In Thomson's experiment, the same H.T. supply provides potential to the anode, as also to the positive deflecting plate in the region of crossed fields. If the supply voltage is doubled, then the value of new magnetic field to keep the electron beam undetected will be

(a) $2\,B$ (b) B

(c) $\sqrt{B}$ (d) $\frac{B}{2}$

31. Cathode rays gain kinetic energy when accelerated by an electric field. If they are subjected to a uniform magnetic field, then their

(a) energy increases

(b) momentum increases

(c) energy and momentum decrease

(d) energy and momentum remain unaffected.

32. An oil drop with charge ne is held stationary between two plates with an external potential difference of 500 volt. If the size of the drop is doubled without any change of charge, the potential difference required to keep the drop stationary will be

(a) 500 V (b) 1000 V

(c) 2000 V (d) 4000 V

33. The value of threshold wavelength for photoelectric effect is 7000 Å. Which of the following radiations will not produce photoelectric effect?

(a) violet (b) ultraviolet

(c) infrared (d) yellow

34. The photoelectric effect was experimentally studied by

(a) Einstein (b) Lennard

(c) Hertz (d) Rutherford

35. The photoelectrons emitted from the surface of sodium metal are

(a) of speeds from zero to a certain maximum

(b) of same De-Broglie wavelength

(c) of same kinetic energy

(d) of same frequency

36. The necessary condition for photoelectric emission is

(a) $hv < hv_0$ (b) $hv > hv_0$

(c) $E_k < hv_0$ (d) $E_k > hv_0$

37. When light is made incident on a surface, then photoelectrons are emitted from it. The kinetic energy of photoelectrons

(a) depends on the wavelength of incident light

(b) is same

(c) is more than a certain minimum value

(d) none of these

38. The photoelectric effect was successfully explained by

(a) Hertz (b) Planck

(c) Millikan (d) Einstein

39. The value of stopping potential in the following diagram is

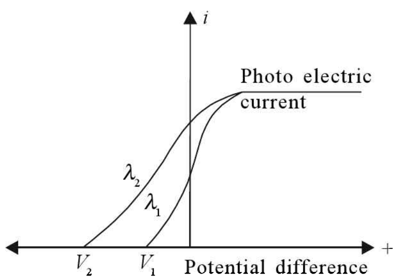


Fig. 32.8

(a) – 4V (b) – 3V

(c) – 2V (d) – 1V

40. If the energy of photon is $10eV$ and work function is $5eV$, then the value of stopping potential will be

(a) 15 V (b) 5 V

(c) 2 V (d) 50 V

41. The photoelectric equation is

(a) $hv = hv_0 - E_k$ (b) $kv = hv_0 + \frac{E_k}{V}$

(c) $hv = hv_0 + E_k$ (d) $hv = hv_0$

42. Light of frequency $2.5\, v_0$ is incident on a metal surface of threshold frequency $2v_0$. If its frequency is halved and intensity is made three times then the new value of photoelectric current will be

(a) zero (b) double

(c) four times (d) six times

43. The function of photoelectric cell is

(a) to convert electrical energy into light energy

(b) to convert light energy into electrical energy

(c) to convert mechanical energy into electrical energy

(d) to convert DC into AC

44. At stopping potential, the photoelectric current becomes

(a) minimum (b) maximum

(c) zero (d) infinity.

45. The curve between photoelectric current (i) and frequency (v) is

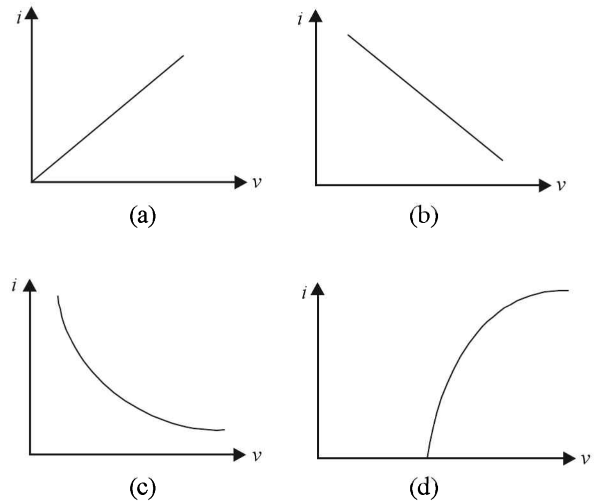


Fig. 32.9

46. Stopping potential depends on

(a) frequency of incident light

(b) intensity of incident light

(c) number of emitted electrons

(d) number of incident photons

47. Which conservation law is obeyed in Einstein's photoelectric equation ?

(a) charge (b) energy

(c) momentum (d) mass

48. The kinetic energy of photoelectrons depends on

(a) the sum of threshold frequency and frequency of incident light

(b) the ratio of threshold frequency and frequency of incident light

(c) the difference of threshold frequency and frequency of incident light

(d) the intensity of incident light

49. In the following diagram if $V_2 > V_1$ then

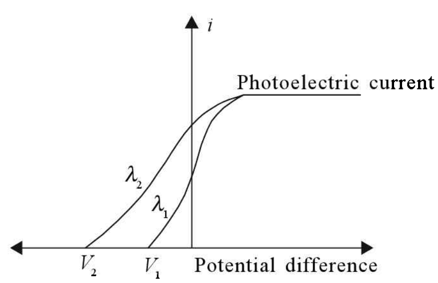


Fig. 32.10

(a) $\lambda_1 = \sqrt{\lambda_2}$ (b) $\lambda_1 < \lambda_2$

(c) $\lambda_1 = \lambda_2$ (d) $\lambda_1 > \lambda_2$

50. When photons of energy $h\nu$ are incident on the surface of photosensitive material of work function $h\nu_0$, then

(a) the kinetic energy of all emitted electrons is $h\nu_0$

(b) the kinetic energy of all emitted electrons is $h(\nu - \nu_0)$

(c) the kinetic energy of all fastest electrons is $h(\nu - \nu_0)$

(d) the kinetic energy of all emitted electrons is $h\nu$

51. The work function of cesium metal is $2ev$. It means that

(a) the energy necessary to emit electrons from metal surface is $2eV$

(b) the energy of electrons emitted from metallic surface is $2eV$

(c) the value of photoelectric current is $2eV$

(d) the value of threshold frequency is $2eV$

52. If the frequency of light incident on metal surface is doubled, then the kinetic energy of emitted electrons will become

(a) doubled (b) less than double

(c) more than double (d) nothing can be said

53. Two photons, each of energy $2.5eV$ are simultaneously incident on the metal surface. If the work function of the metal is $4.5eV$, then from the surface of metal

(a) two electrons will be emitted

(b) not even a single electron will be emitted

(c) one electron will be emitted

(d) more than two electrons will be emitted

54. The momentum of a photon of energy $1MeV$, in kg/m/s, will be

(a) 10^{-22} (b) 0.33×10^{6}

(c) 5×10^{-22} (d) 7×10^{-24}

55. The work function of a metal is $1eV$. On making light of wavelength 3000 Å incident on this metal, the velocity of photoelectrons emitted from it for photoelectric emission will be

(a) 2955 Å (b) 4200 Å

(c) 1100 Å (d) 3000 Å

56. On decreasing the intensity of incident light

(a) the photoelectric current will increase

(b) the number of photoelectrons emitted will increase

(c) the number of emitted electrons will decrease

(d) all of the above

57. An electron is accelerated through a potential difference of 1000 V. Its velocity will be

(a) 0.95×10^{7} m/s (b) 5.67×10^{7} m/s

(c) 1.89×10^{8} m/s (d) 3.78×10^{7} m/s

58. There are two light sources A and B. The intensity of source A is more than that of source B. The frequency of light emitted by source B is higher than that emitted by source A. The photoelectric current obtained will be more from source

(a) B (b) A

(c) same from A and B (d) none of these

59. The curve between current (i) and potential difference (v) for a photo cell will be

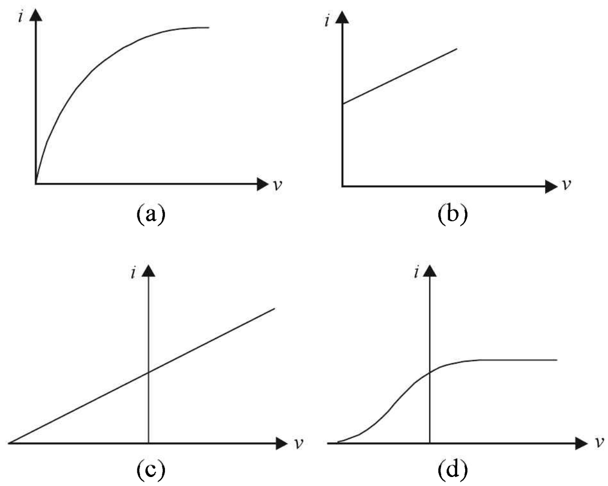


Fig. 32.11

60. A radio transmitter is working at frequency 880 *kHz* and power 10 *kW*. The number of photons emitted per second will be

(a) 1327×10^{34} (b) 0.075×10^{-34}

(c) 1.71×10^{31} (d) 13.27×10^{34}

61. When a photon of energy $7ev$ is made incident on a metal then the emitted electron is stopped by a stopping potential of -5.5 V. The work function of metal will be

(a) $-1.5\ eV$ (b) $1.5\ eV$

(c) $12.5\ eV$ (d) $37.5\ eV$

62. The work function of a metal is $2.5eV$. When photon of some proper energy is made incident on it, then an electron of $1.5\ eV$ is emitted. The energy of photon will be

(a) $4\ eV$ (b) $1\ eV$

(c) $1.5\ eV$ (d) $2.5\ eV$

63. If the energy of incident photon and work function of metal are $E\ eV$ and $\phi_0 eV$ respectively, then the maximum energy of emitted photoelectron will be

(a) $\frac{2}{m}[E - \phi_0]$ (b) $\sqrt{\frac{2}{m}(E - \phi_0)}$

(c) $\frac{m}{2}[E - \phi_0]$ (d) $2m\sqrt{(E - \phi_0)}$

64. When green light is made incident on a metal, photoelectrons are emitted by it but no photoelectrons are obtained by yellow light. If red light is made incident on that metal then

(a) no electrons will be emitted

(b) less electrons will be emitted

(c) more electrons will be emitted

(d) all of the above

65. On reducing the wavelength of light incident on a metal, the velocity of emitted photoelectrons will become

(a) zero (b) less

(c) more (d) remains unchanged

66. The threshold wavelength of lithium is 8000 Å. When light of wavelength 9000 Å is made incident on it, then the photoelectrons

(a) will not be emitted (b) will be emitted

(c) will sometimes be emitted and sometimes not

(d) nothing can be said

67. The work function of a metal is 1.5*eV*. Light of wavelength 6600 Å is made incident on it. The maximum kinetic energy of emitted photoelectrons will be

(a) 1.6×10^{-19} Joule (b) 0.6×10^{-19} Joule

(c) 1.6×10^{-13} Joule (d) 1.6×10^{19} Joule

68. Photoelectrons are emitted by making green light incident on a metallic surface. Which of the following lights can start photoelectric emission?

(a) yellow (b) orange

(c) blue (d) red

69. A photoelectric cell is illuminated by a small intense source distant 1m from it. When the same source is placed at a distance 2 m then the number of photoelectrons emitted from the cathode will be

(a) half (b) one fourth

(c) each carries one fourth of its previous energy

(d) each carries one fourth of its previous momentum,

70. In an experiment of photoelectric emission, for incident light of 4000 Å, the stopping potential is 2*V*. If the wavelength of incident light is made 3000 Å, then stopping potential will be

(a) less than 2 volt (b) more than 2 volt

(c) 2 volt (c) zero

71. If the intensity of incident light is made double, then the maximum number of emitted electrons will become

(a) double (b) four times

(c) eight times (d) half

72. The threshold frequency for a metal is 10 Hz. When light of wavelength 4000 Å is made incident on it, then

(a) photoelectrons will be emitted from it with zero speed.

(b) photoelectric emission will not be started.

(c) photoelectrons will be emitted with speed 10^5 m/s^{-1}

(d) photoelectrons will be emitted with speed 10^3 m/s^{-1}

73. The photoelectric currents at distances r_1 and r_2 of light source from photoelectric cell are I_1 and I_2 respectively. The value of I_1/I_2 will be

(a) $\frac{r_1}{r_2}$ (b) $\frac{r_2}{r_1}$

(c) $\left(\frac{r_1}{r_2}\right)^2$ (d) $\left(\frac{r_2}{r_1}\right)^2$

74. In a photoelectric cell, the cathode with work function W_1 is replaced by another one with work function W_2 ($W_2 > W_1$). If the current before this change is I_1 and that after the change is I_2 and other circumstances remain same and if $h\nu > W_2$, then

(a) $I_1 > I_2$ (b) $I_1 < I_2$

(c) $I_1 = I_2$ (d) $I_1 < I_2 < 2I_1$

75. The curve between the frequency (*f*) and stopping potential (*V*) in a photoelectric cell will be

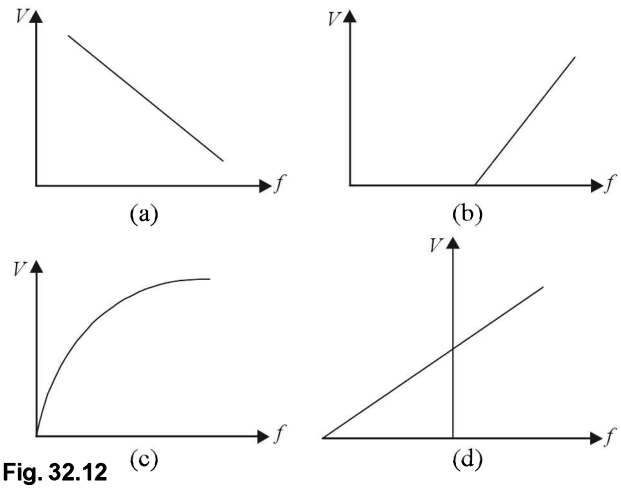


Fig. 32.12

76. Einstein was awarded Noble Prize for—

(a) the general theory of relativity

(b) the special theory of relativity

(c) the explanation of photoelectric effect

(d) the explanation of quantum theory

77. If the work function of a metal is ϕ, then its threshold wavelength will be

(a) $hc\phi_0$ (b) $\frac{c\phi_0}{h}$

(c) $\frac{h\phi_0}{c}$ (d) $\frac{hc}{\phi_0}$

78. The work function of a metal is *X eV*. When light of energy 2*X* is made incident on it then the maximum kinetic energy of emitted photoelectron will be

(a) 2 eV (b) 2 XeV

(c) XeV (d) 3 XeV

79. The photoelectric effect proves that light consists of

(a) photons (b) electrons

(c) electromagnetic waves (d) mechanical waves

80. The correct curve between the stopping potential (V) and intensity of incident light (I) is

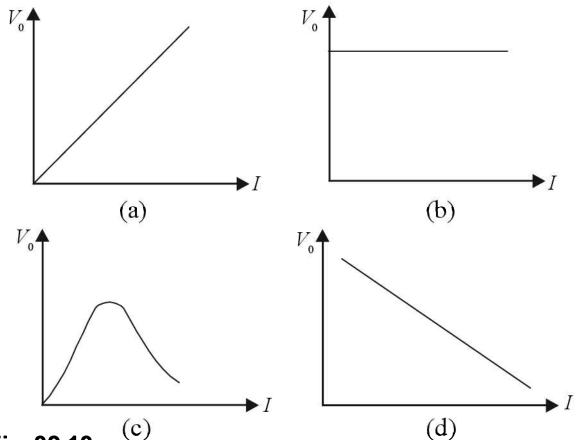


Fig. 32.13

81. Light of wavelength 5000 Å and intensity 3.96×10^{-3} W/cm^2 is incident on the surface of a photosensitive material. If 1% of incident photons only emit photoelectrons, then the number of electrons emitted per unit area from the surface will be

(a) 10^{16} (b) 10^{18}

(c) 10^{20} (d) 10^{22}

82. The work function of aluminum is 4.2eV. Light of wavelength 2000 Å is incident on it. The threshold frequency will be

(a) 10^{19} Hz (b) 10^{13} Hz

(c) 10^{15} Hz (d) 10^{18} Hz

83. In the above problem, the threshold wavelength will be

(a) 5868 Å (b) 2946 Å

(c) 3856 Å (d) 5000 Å

84. In Q. 82, the minimum energy of emitted electrons will be

(a) zero (b) 1.99 eV

(c) 4.2 eV (d) 6.19 eV

85. In Q.82, the maximum energy of emitted electrons will be

(a) zero (b) 1.99 eV

(c) 3.2 eV (d) 6.19 eV

86. In Q. 82, the stopping potential for fastest electrons will be

(a) zero (b) 1.99 volt

(c) 4.2 volt (d) 6.19 volt

87. In Q. 82, if the intensity of incident light is doubled, then the value of maximum energy will become

(a) double (b) four times

(c) one fourth (d) unchanged

88. If in a photoelectric cell, the wavelength of incident light is changed from 4000 Å to 3600 Å, then the change in stopping potential will be

(a) 0.34 volt (b) 0.57 volt

(c) 0.71 volt (d) 0.23 volt

89. A stopping potential of 0.82 volt is required to stop the photo electrons emitted from a metallic surface by light of wavelength 4000 Å. The stopping potential for wavelength 3000 Å will be

(a) 1.1 volt (b) 1.85 volt

(c) 2.03volt (d) 2.5 volt

Answers to Questions for Practice

1.	(b)	**2.**	(a)	**3.**	(b)	**4.**	(d)	**5.**	(c)	**6.**	(b)	**7.**	(c)
8.	(d)	**9.**	(a)	**10.**	(c)	**11.**	(b)	**12.**	(c)	**13.**	(a)	**14.**	(c)
15.	(d)	**16.**	(c)	**17.**	(c)	**18.**	(b)	**19.**	(c)	**20.**	(c)	**21.**	(c)
22.	(a)	**23.**	(c)	**24.**	(a)	**25.**	(b)	**26.**	(b)	**27.**	(d)	**28.**	(c)
29.	(a)	**30.**	(c)	**31.**	(d)	**32.**	(d)	**33.**	(c)	**34.**	(c)	**35.**	(a)
36.	(b)	**37.**	(c)	**38.**	(d)	**39.**	(a)	**40.**	(b)	**41.**	(a)	**42.**	(a)
43.	(b)	**44.**	(c)	**45.**	(d)	**46.**	(a)	**47.**	(b)	**48.**	(c)	**49.**	(d)
50.	(c)	**51.**	(a)	**52.**	(a)	**53.**	(b)	**54.**	(c)	**55.**	(c)	**56.**	(c)
57.	(c)	**58.**	(b)	**59.**	(d)	**60.**	(c)	**61.**	(b)	**62.**	(b)	**63.**	(b)
64.	(a)	**65.**	(c)	**66.**	(a)	**67.**	(b)	**68.**	(c)	**69.**	(b)	**70.**	(b)
71.	(a)	**72.**	(b)	**73.**	(d)	**74.**	(c)	**75.**	(b)	**76.**	(c)	**77.**	(d)
78.	(c)	**79.**	(a)	**80.**	(b)	**81.**	(b)	**82.**	(c)	**83.**	(b)	**84.**	(a)
85.	(b)	**86.**	(b)	**87.**	(d)	**88.**	(a)	**89.**	(b)				

SELF TEST 1

1. The angle between two plane mirrors is 68°. The number of images formed will be

(a) 5 (b) 6

(c) 5.3 (d) none of these

2. A lens is made of flint and crown glass layers as shown in Fig. 1. If an object is placed before it is 10 cm away, then number of images formed will be

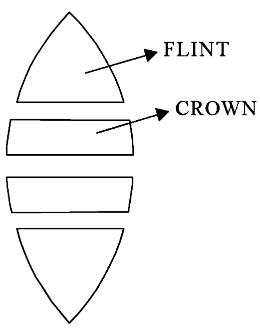


Fig. 1

(a) 1 (b) 2

(c) insufficient data to reply (d) 3

3. One side of a biconvex lens is polished as shown in Fig. 2. What would be the power of the combination? Assume P_1 is the power of lens and P_2 is power of the mirror.

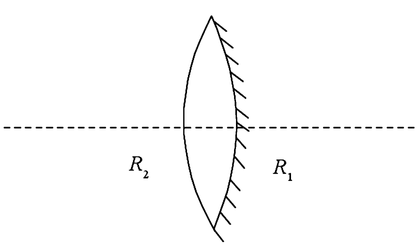


Fig. 2

(a) $P_1 + P_2$ (b)) $P_1 - P_2$

(c) $2P_1 + P_2$ (d) $2P_1 - P_2$

4. A biconvex lens is cut to two halves as shown in Fig. 3 and separated by a distance 2 cm. If the focal length of combined lens was 15 cm, find the focal length of the system.

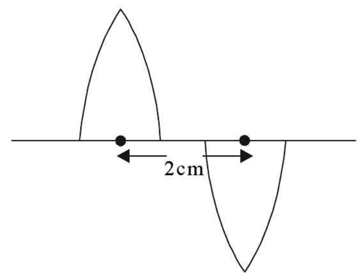


Fig. 3

(a) 8.0 cm (b) 7.5 cm

(c) 15 cm (d) 7.0 cm

5. Two thin lenses of power 2 D and – 1.5 D are combined. The focal length of the combined system is

(a) 0.5 m (b) 1.0 m

(c) 1.5 m (d) 2.0 m

6. When a light ray passes through one medium to another

(a) its wavelength changes

(b) its frequency changes

(c) both (a) and (b) change

(d) none of these

7. A white light is

(a) continuous source of light

(b) monochromatic source of light

(c) chromatic source of light

(d) none of these

8. Sodium light is

(a) continuous

(b) monochromatic

(c) nearly monochromatic

(d) none of these

9. In order to get ultraviolet light we use

(a) mercury lamp (b) sodium lamp

(c) both (a) and (b) (d) He-Ne laser

10. Ultraviolet light passes through
(a) glass (b) quartz
(c) lead-glass (d) none of these

11. Emission spectrum will always consist of
(a) bright lines (b) dark lines
(c) both bright and dark (d) none of these

12. Can a material be optically rarer though mechanically denser or vice versa?
(a) yes (b) no
(c) insufficient data to reply (d) none of these

13. Canada Balsam has refractive index equal to that of
(a) quartz (b) glass
(c) water (d) none of these

14. When light wave gets reflected its____varies.
(a) wavelength (b) frequency
(c) phase (d) amplitude

15. An object is placed before a lens-air assembly as shown in Fig. 4. The number of images formed on the other side of the assembly is

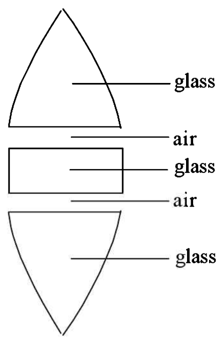


Fig. 4

(a) one (b) two
(c) three (d) none of these

16. A concave mirror of radius of curvature 40 cm is used. Few drops of a liquid are poured on its surface. The image of the needles as shown in Fig. 5 coincides at 30 cm from the pole of the mirror. Find the refractive index of the liquid.

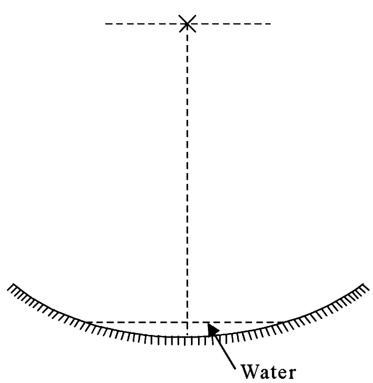


Fig. 5

(a) $\frac{3}{4}$ (b) 1.5
(c) 1.33 (d) $\frac{2}{3}$

17. A pond is 14 m deep. A bulb is located at the bottom of the pond. Find the maximum area of the surface of water illuminated.
(a) 100π (b) 200π
(c) 242π (d) 252π

18. An apochromat lens is made of
(a) crown and flint glass combination of lenses
(b) flourite and crown glass combination of lenses
(c) flourite and flint glass combination of lenses
(d) none of these

19. Constrigence means
(a) dispersive power
(b) reciprocal of dispersion
(c) reciprocal of refractive index
(d) reciprocal of deviation

20. Aplanatic lens removes spherical aberrations between
(a) real image and real object
(b) real image and virtual object
(c) real object and virtual image
(d) none of these

21. Visual acuity stands for
(a) the maximum distance upto which eye can see clearly
(b) the minimium distance from which eye starts seeing clearly
(c) increasing the resolving power of eye by using visual aids
(d) resolving power of the eye

22. When a piece of mica was introduced in front of one of the slits in a young's double slit experiment it was found that the central fringe shifts to a position earlier occupied by 3rd bright fringe. Find the thickness of mica sheet. Given refractive index of mica 1.6 and wavelength used = 5600 $A°$
(a) $4.1\mu m$ (b) $2.8\mu m$
(c) $3.2\mu m$ (d) $2.8\mu m$

23. Wavelength 6800 Å from the constellation virgo were observed to increase by 0.4%. The radial speed of constellation virgo with respect to earth is ____. Is it approaching or receding?
(a) 10^6 m/s, receding (b) 1.2×10^6 m/s, approaching
(c) 10^6 m/s, approaching (d) 1.2×10^6 m/s, receding

24. A plane mirror is fixed at a height h_2 from the bottom of a beaker containing a liquid of refractive index μ upto a height h_1. Find the position of the image of the bottom formed by the mirror.

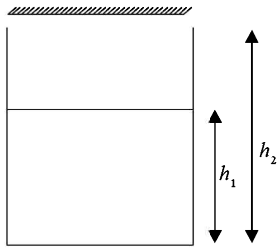


Fig. 6

(a) $h_2 - \dfrac{h_1}{\mu}$ behind the mirror

(b) $h_1 + \dfrac{h_2}{\mu}$ behind the mirror

(c) $h_2 + h_1 + \dfrac{h_1}{\mu}$ behind the mirror

(d) $h_2 - h_1 + \dfrac{h_1}{\mu}$ behind the mirror

25. Find the focal length of a convexo convave lens having radius of curvature R_1 and R_2 10 cm and 25 cm respectively.

(a) 40 cm (b) 25 cm

(c) 33.3 cm (d) 30 cm

26. A ray of light enters from face *ab* of a prism *abc* as shown in Fig. 7. If prism (made of glass) is immersed in air, find the maximum angle ϕ so that the ray is totally reflected from face *ac* [see Fig. 7]

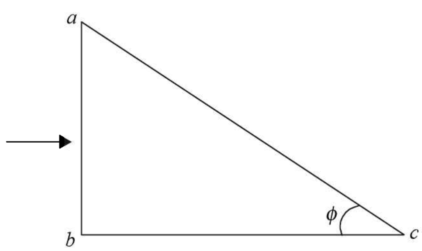


Fig. 7

(a) $\phi = \sin^{-1}\left(\dfrac{2}{3}\right)$ (b) $\cos^{-1}\left(\dfrac{2}{3}\right)$

(c) $\tan^{-1}\left(\dfrac{2}{3}\right)$ (d) $\cot^{-1}\left(\dfrac{2}{3}\right)$

27. A ray of light falls on the face *ab* of a square glass slab making an angle of 45°. What should be the refractive index of glass so that it is totally reflected from face *ad*?

(a) 1.5 (b) 1.33

(c) 1.23 (d) 1.42

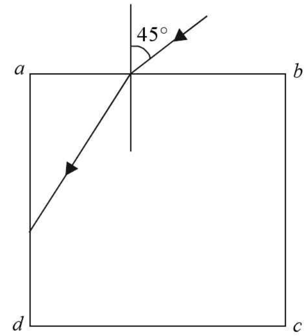


Fig. 8

28. If a light ray is partly reflected and partly refracted from an interface then

(a) refracted ray is polarised one

(b) reflected ray is polarised

(c) reflected ray is partly polarised

(d) refracted ray is partly polarised

29. What is the maximum permissible distance between two sources to achieve interference?

(a) 3 cm (b) 30 cm

(c) 300 cm (d) 3000 cm

30. A ray of wavelength λ passes obliquely in a medium of refractive index μ . The wavelength in the medium will be

(a) $\lambda\mu$ (b) $\dfrac{\lambda}{\mu}$

(c) $\dfrac{\lambda\mu}{\mu - 1}$ (d) $\dfrac{\lambda}{\mu - 1}$

31. A plane polarized wave, polarized in *xy* plane is given by

(a) $a = a_0 \sin(\omega t - kx)$ (b) $a = a_0 \sin(\omega t - ky)$

(c) $a = a_0 \sin(\omega t - kz)$ (d) any of (a), (b) or (c)

32. In optical instruments using a micrometer scale ____ eyepiece is used

(a) Hygen's (b) Ramsden's

(c) Kellner's (d) any of (a) , (b) or (c)

33. Ultramicroscopes are based on the principle of

(a) bright field illumination

(b) phase contrast

(c) darkfield illumination

(d) de-Broglie relation

34. If the lens is thick, the lens maker's formula will be given by (thickness = t)

(a) $\dfrac{1}{f} = (\mu - 1)\left[\dfrac{1}{R_1} - \dfrac{1}{R_2}\right]$

(b) $\dfrac{1}{f} = (\mu - 1)\left[\dfrac{1}{R_1} - \dfrac{1}{R_2} + \dfrac{(\mu - 1)t}{R_1 R_2}\right]$

(c) $\dfrac{1}{f} = (\mu - 1)\left[\dfrac{1}{R_1} - \dfrac{1}{R_2} + \dfrac{(\mu - 1)t}{\mu R_1 R_2}\right]$

(d) $\dfrac{1}{f} = (\mu - 1)\left[\dfrac{1}{R_1} - \dfrac{1}{R_2} + \dfrac{t}{\mu R_1 R_2}\right]$

35. The focal length of a lens made of glass in air is 20 cm. Find its focal length when it is immersed in water.

(a) 20 cm (b) 40 cm

(c) 80 cm (d) 50 cm

36. The Rayleigh's criterion of just resolution is

(a) the two patterns of equal intensity are said to be just resolved if maxima of one falls on the minima of the other

(b) the two patterns of unequal intensity are said to be just resolved if maxima of the one falls on the maxima of the other

(c) the two patterns of equal intensity are said to be resolved if minima of one falls on the minima of other.

(d) none of these

37. In zonal plate, the radii of different zones are proportional to

(a) natural numbers

(b) under root of natural numbers

(c) square of natural numbers

(d) none of these

38. Dispersive power of an instrument depends upon

(a) width of maxima

(b) width of minima

(c) angular separation of wavelengths

(d) distance between the slits

39. What is the suitable illumination level in an operation theatre?

(a) 10 – 50 foot candles (b) 50 – 100 foot candles

(c) 100 – 500 foot candles (d) none of these

40. For a lens system shown in Fig. 9. a point object placed in medium of refractive index μ_1 at a distance u, the image in medium (μ_3) will be formed at v given by

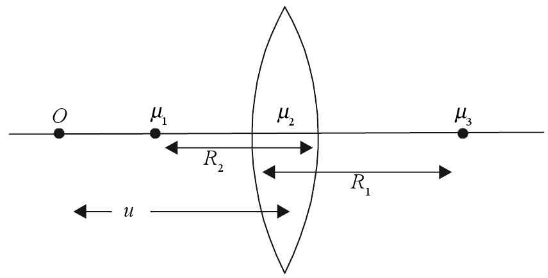


Fig. 9

(a) $\dfrac{\mu_3}{v} - \dfrac{\mu_1}{u} = \dfrac{\mu_2 - \mu_1}{R_1} - \dfrac{\mu_2 - \mu_3}{R_2}$

(b) $\dfrac{\mu_3}{v} - \dfrac{\mu_3}{u} = \dfrac{\mu_2 - \mu_3}{R_1}$

(c) $\dfrac{\mu_3}{v} - \dfrac{\mu_1}{u} = (\mu_2 - \mu_3)\left(\dfrac{1}{R_1} - \dfrac{1}{R_2}\right)$

(d) none of these

Answers to Questions for Practice

1.	(a)	**2.**	(b)	**3.**	(c)	**4.**	(a)	**5.**	(d)	**6.**	(a)	**7.**	(a)
8.	(c)	**9.**	(a)	**10.**	(b)	**11.**	(a)	**12.**	(a)	**13.**	(b)	**14.**	(c)
15.	(a)	**16.**	(c)	**17.**	(d)	**18.**	(c)	**19.**	(b)	**20.**	(c)	**21.**	(d)
22.	(b)	**23.**	(d)	**24.**	(d)	**25.**	(c)	**26.**	(b)	**27.**	(c)	**28.**	(c)
29.	(c)	**30.**	(b)	**31.**	(c)	**32.**	(b)	**33.**	(c)	**34.**	(c)	**35.**	(c)
36.	(a)	**37.**	(b)	**38.**	(c)	**39.**	(c)	**40.**	(a)				

Explanations

1(a) number of images formed

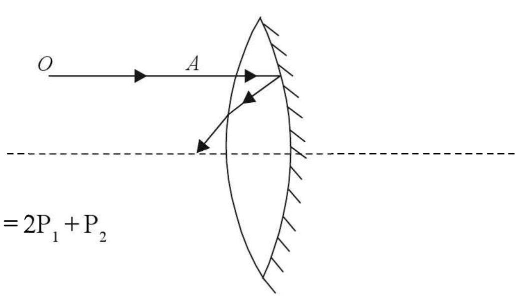
$$n = \frac{360}{\theta} \text{ if } \frac{360}{\theta} \text{ is odd:}$$

$$n = \frac{360}{\theta} - 1 \text{ if } \frac{360}{\theta} \text{ is even}$$

fractions are to be removed θ is the angle of inclination between two mirrors.

2(b) $\because \dfrac{1}{f} = (\mu - 1)\left(\dfrac{1}{R_1} - \dfrac{1}{R_2}\right)$, therefore this type of assembly will have two focal lengths and hence two images will be formed one due to crown glass and other due to flint glass as flint and crown glasses have different refractive index.

3(c) The ray which enters from the transparent side suffers refraction twice before the image is formed therefore power of lens comes two times.



$\therefore P = 2P_1 + P_2$

Fig. 10

4(a) $\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$ where f_1 and f_2 are focal lengths of two lenses and d is separation between the two

$$\frac{1}{f} = \frac{1}{15} + \frac{1}{15} - \frac{2}{225} \Rightarrow f = \frac{225}{28} = 8.0 \text{ cm}$$

5(d) Net Power

$$P = P_1 + P_2 \Rightarrow 2D + (-1.5D) = 0.5D; f = 2m$$

6(a)

7(a)

8(c) as it consists of a doublet of D_1 and D_2 lines of wavelength 5890 A° and 5896 A°.

9(a)

10(b) For this reason the envelope of mercury lamp is made of quartz and not of glass.

11(a) dark lines are given by absorption spectrum.

12(a) Yes, for example, polyethylene has refractive index 1.5 and is optically denser than water but its relative density is less than water.

13(b)

14(c) phase shift of 180° takes place provided the medium at interface has larger refractive index.

15(a) Only one image will be formed and the intensity of image will be diminished from the one if the lens had been of total glass (no air in between the pieces).

16(c) we can use $\mu = \frac{\text{Real depth}}{\text{Apparent depth}}$ as the incidence is normal and we know the image coincides at C (centre of curvature). Therefore, real depth = 40 *cm* and apparent depth = 30 *cm*.

17(d) $\mu = \frac{1}{\text{sinc}} = \frac{4}{3}$

or $\sin c = \frac{3}{4}$ Therefore $\tan c = \frac{3}{\sqrt{7}}$ $r = l \tan c$

$$= 14 \times \frac{3}{\sqrt{7}} = 6\sqrt{7} \text{ cm}$$

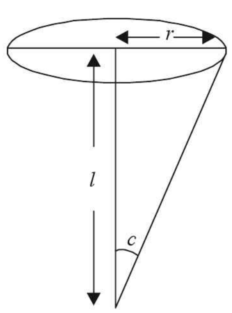


Fig. 11

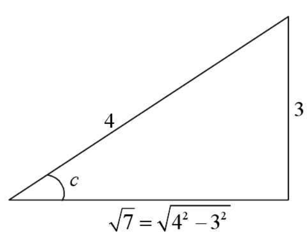


Fig. 12

r will be the radius of the circular area which will be illuminated

$$\text{Area} = \pi r^2 = \pi \left(6\sqrt{7}\right)^2 = 252\pi$$

18(c)

19(b)

20(c)

21(d) for a normal eye its value is 1 minute $\left(\frac{1^\circ}{60}\right)$

22(b) use the relation $(\mu - 1)\ t = n\lambda$ or $t = \frac{n\lambda}{(\mu - 1)}$

$$= \frac{3 \times 5600 \times 10^{-10}}{.6} = 2.8 \times 10^{-6} \text{m}$$

or $= 2.8\mu$m

23(d) Since the wavelength increases, the constellation is receding

$$\frac{\Delta\lambda}{\lambda} = \frac{v}{c} \text{ or } v = \frac{.4}{100} \times 3 \times 10^8$$

$$= 1.2 \times 10^6 \text{ms}^{-1}$$

24(d) Apparent depth = $\frac{h_1}{\mu}$ of bottom from

$$\text{mirror} = h_2 - \left(h_1 - \frac{h_1}{\mu}\right)$$

$$= h_2 - h_1 + \frac{h_1}{\mu}$$

25(c) $\frac{1}{f} = (\mu - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right] = 0.5\left[\frac{1}{10} - \frac{1}{25}\right]$

$$= \frac{0.5 \times 1.5}{25}\ f = \frac{25}{0.75} = 33.3\ cm$$

26(b) $i = 90 - \phi$ $\mu = \frac{1}{\sin i}$

or $\sin i = \frac{1}{\mu}$; $\sin(90 - \phi) = \frac{1}{1.5}$

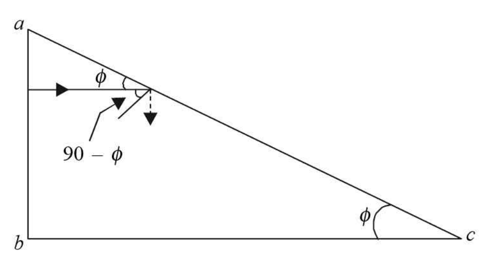


Fig. 13

$$\Rightarrow \cos\phi = \frac{2}{3} \text{ or } \phi = \cos^{-1}\left(\frac{2}{3}\right)$$

27(c) $\mu = \frac{\sin 45}{\sin r}$ Also $\mu = \frac{1}{\sin(90 - r)}$

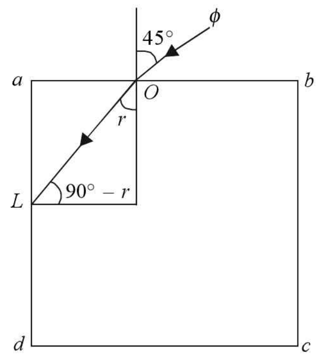


Fig. 14

$$\therefore \quad \frac{1}{\sin(90 - r)} = \frac{1}{\cos r} = \frac{\sin 45}{\sin r}$$

or $\tan r = \frac{1}{\sqrt{2}}$

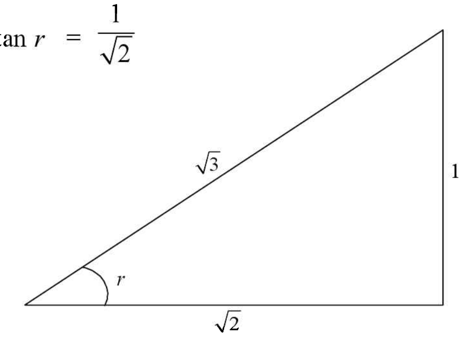


Fig. 15

$$\mu = \sec r = \frac{\sqrt{3}}{\sqrt{2}} = 1.225$$

28(c) Reflected ray is fully polarised if the angle of incidence is such that $\tan i = \mu$ otherwise it contains some % of polarised light

29(c) Theoretically, maximum permissible distance is 3m. Though practically distance should not be larger than 3 to 4 cm

We know that the life time of an excited state – 10^{-8} *s*. In this time the wave can travel 3m. Therefore, theoretical limit is 3m

30(b) $\mu = \frac{c}{v} = \frac{f\lambda}{f\lambda'}$

$\therefore \quad \lambda' = \frac{\lambda}{\mu}$

31(c) A plane polarized wave in *xy* plane will travel along *z*-direction.

32(b)

33(c)

34(c)

35(c) $\frac{f_{water}}{f_{air}} = \frac{(\mu_l - 1)}{\left(\frac{\mu_l}{\mu_m} - 1\right)} = \frac{1/2}{\frac{3/2}{4/3} - 1} = \frac{1/2}{1/8} = 4$

$\therefore f_{water} = 4 \times 20 = 80$ cm

36(a)

37(b) area is proportional to natural number but radius is proportional to $\sqrt{n}$ where *n* is a natural number

38(c)

39(c)

40(a)

SELF TEST 2

1. An object is placed 15 cm. in front of a convex lens of focal length 10 cm. Where should a convex mirror of focal length 12 cm be placed so that the image is formed at the object itself?

(a) 5 cm (b) 6 cm

(c) 30 cm (d) 18 cm

2. A convex lens is placed on a plane mirror then the object needle and its image coincide at 20 cm. If a liquid is filled within the empty space between lens and mirror then the object needle and image coincide at 30 cm. Find the focal length of liquid lens and refractive index of the liquid.

(a) – 40, 1.5 (b) – 60, 4/3

(c) – 60 , 3/2 (d) none of these

3. The size of image obtained on the screen with a convex lens at a certain distance is 25 cm. when the lens is moved away from the screen then the size of the image is 9 cm. Find the real size of the object.

(a) 14 cm (b) 12 cm

(c) 15 cm (d) 16 cm

4. A small object of length *l* is placed axially along the axis of a concave mirror of focal length *f* at a distance *u* from it. Find the size of the image.

(a) $l\left(\frac{f}{u-f}\right)^2$ (b) $l\left(\frac{u-f}{f}\right)$

(c) $l\left(\frac{f}{u-f}\right)$ (d) $l\left(\frac{u}{u-f}\right)^2$

5. A plano-convex lens has focal length 20 cm. Find its radius of curvature. Also find its focal length in water ($\mu_{lens} = 1.5$)

(a) 20 cm, 80 cm (b) 12 cm, 80 cm
(c) 10 cm, 80 cm (d) 16 cm, 60 cm

6. A wedge of glass plates as shown in Fig.1 is made by placing paper sheets of thickness 8 mm. If the maximum number of fringes obtained is 4×10^4, then the wavelength of the light used is

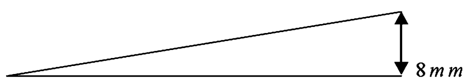


Fig. 1

(a) 500 nm (b) 400 nm
(c) 600 nm (d) 300 nm

7. The interference obtained with Newton's rings is an example of

(a) interference by division of amplitude
(b) interference by division of wave front
(c) interference by both (a) and (b)
(d) none of these

8. A wavelength λ produces an angular deviation '*d*' when passes through a prism. The angular deviation is ____ when a wave of wavelength 2λ is passed through the same prism.

(a) $d/2$ (b) $d/4$
(c) $d/8$ (d) $2d$

9. A mirror has thickness 9 cm. An object is placed 20 cm. in front of it. Find the position of image from the front surface.

(a) 29 cm (b) 20 cm
(c) 32 cm (d) 26 cm

10. A sphere of glass has radius 6 mm. A point object lies 1 mm from the centre. Find the position of image as seen from the smaller and larger distance side.

(a) 4.21 cm, 8.21 cm (b) 4.61cm, 8.1cm
(c) 4.61 cm, 7.64 cm (d) none of these

11. The maximum number of Hidenger fringes is ....

(a) 5×10^4 (b) 5×10^5
(c) 5×10^3 (d) 5×10^6

12. The equipment producing fringes of wavelength 500 nm is immersed in a liquid of refractive index 1.5, then

(a) fringe width increases by 33%
(b) fringe width decreases by 33%
(c) fringes are not produced
(d) fringes overlap with each other

13. A pin hole camera is 50 cm long. An object of height 10 cm is placed 2 m in front of it. The size of image is

(a) 10 cm (b) 5.0 cm
(c) 2.5 cm (d) 7.5 cm

14. A plastic sphere has a point object located in it. The sphere has radius 3 mm and refractive index 1.5. The image of the object appears at 1mm in side the sphere then the position of object is

(a) 1.231 mm inside (b) 1.222 mm outside
(c) 1.286 mm inside (d) none of these

15. An object is placed at a distance of 40 cm. in front of a combination of a thin converging lens of focal length 20 cm and a thin diverging lens focal length of 10 cm, 15 cm. apart along the common axis. Find the focal length of the combination and transverse magnification.

(a) $f = 40$ cm, $m = \frac{2}{3}$ (b) $f = 30$ cm, $m = \frac{1}{3}$
(c) $f = 30$ cm, $m = \frac{2}{3}$ (d) none of these

16. When seen normally through the flat surface, the greatest thickness of a plano-convex lens surface appears to be 2.435 cm, but when seen normally through the curved surface it appears to be 2.910 cm. The actual thickness is 3.66 cm. Find the refractive index of glass, and the focal length of the plano convex lens

(a) 1.4, 11.2 cm (b) 1.505, 7.52 cm
(c) 1.5, 12.1 cm (d) 1.505, 11.6 cm

17. Principal points are also called

(a) focal points (b) Gauss points
(c) nodal points (d) none of these

18. $\frac{df}{f}$ in a lens is equal to

(a) refractive index of the lens
(b) net curvature
(c) dispersive power
(d) longitudinal chromatic aberration

19. Triple aplanat is a lens

(a) consisting of 3 lenses
(b) two plano convex elements
(c) which is spherical
(d) none of these

20. Resolving power of a microscope is

(a) $\frac{0.61\lambda}{\mu \sin\theta}$ (b) $\frac{1.22\lambda}{2\mu \sin\theta}$
(c) $\frac{2\mu \sin\theta}{1.22\lambda}$ (d) none of these

21. The brightness of an image as seen through a terrestrial telescope is

(a) larger than the brightness of object

(b) less than the brightness of object

(c) equal to brightness of object

(d) none of these

22. Corresponding points mean

(a) waves which have constant phase relation

(b) cardinal points

(c) nodal points

(d) principal points

23. With white light in interference experiment

(a) only white band is seen

(b) central white band with rainbow coloured fringes is seen

(c) central white band with reverse rainbow coloured fringes is seen

(d) no interference seen

24. Under what condition can we obtain white fringes with white light?

(a) if width of red source is twice the width of violet source

(b) if width of violet source is twice the width of red source

(c) if it is impractical to talk about achromatic fringes

(d) if width of yellow source is twice the green source

25. If transverse magnification is m for a lens then axial magnification is

(a) m (b) 2 m

(c) m^2 (d) $2\,m^2$

26. The refractive index of a prism is $\sqrt{2}$ and angle of prism 30°. One of its refracting faces is polished. The incidence beam of light will return back for the angle of incidence

(a) 60° (b) 30°

(c) 45° (d) 0°

27. The sun rays strike the upper atmosphere of earth with intensity 1.38 kW/m^2. The peak value of electric field at that point will be

(a) 1.02 kV/m (b) 2.04 kV/m

(c) 4.05 kV/m (d) 8.16 kV/m

28. A beam of light of intensity 15 W/cm^2 is incident on totally refracting plane mirror of area 1.5 cm^2. Find the force acting on it

(a) $0.75 \times 10^{-8}\,N$ (b) $7.5 \times 10^{-8}\,N$

(c) $1.5 \times 10^{-7}\,N$ (d) $1.5 \times 10^{-6}\,N$

29. A telescope is formed using objective of focal length 60 cm and eye piece of focal length 5 cm. It is focussed on a distant object in such a way that parallel rays emerge from the eye piece. If the object subtends an angle of 2° at the objective, then angular width of image is

(a) 12° (b) 24°

(c) 3.6° (d) 4.8°

30. The focal length of an achromatic combination is 90 cm. If the dispersive power of the lenses are 0.024 and 0.036 respectively, then their focal length will be

(a) 30 and 60 cm (b) 45 and 90 cm.

(c) 15 and 45 cm (d) 30 and 45 cm.

31. The efficiency of a 40 watt flourescent light source emitting 2000 lumens of light is

(a) 12 lm/W (b) 20 lm/W

(c) 50 lm/W (d) insufficient data to reply

32. 2% of light emitted by a source of 500 cd falls normally on a circular surface of radius 10 cm. The average illuminance of the surface is

(a) 10^3 *lux* (b) 2×10^3 *lux*

(c) 4×10^3 *lux* (d) 6×10^3 *lux*

33. A small mirror of area s and mass m is suspended by means of a weightless thread in a vertical plane. When a beam of light intensity L falls normally on this mirror, the mirror is displaced so that the thread makes an angle θ with the vertical. Then θ is given by

(a) $\frac{2Ls}{cmg}$ (b) $\frac{2LC}{smg}$

(c) $\frac{2mg}{Lsc}$ (d) $\frac{mg}{2Lsc}$

34. If β is fringe width and d is distance between two slits in a young's double slit experiment then which of the curve represents the relation between the two?

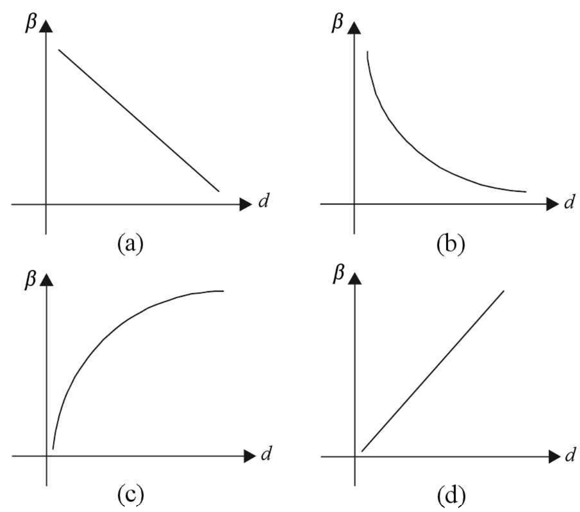


Fig. 2

35. The image of a hole (illuminated) is obtained 9 times on the screen. If distance between convex lens and hole is 40 cm then the focal length of the lens is

(a) 15 cm (b) 30 cm
(c) 45 cm (d) none of these

36. A thin prism of angle 4° 30′ (μ = 1.5) is combined with another prism for dispersion without deviation. If this prism has refractive index 1.75 then angle of the prism will be

(a) 2° (b) 2.5°
(c) 3° (d) 3.5°

37. A glass cube is placed on a white paper having spots of red, blue, yellow and green colour then which colour spot appears to be least raised?

(a) yellow (b) red
(c) blue (d) green

38. Light enters from rarer to denser medium at an angle of incidence *i* such that reflected and refracted rays are mutually perpendicular to one another. Then critical angle is

(a) $\sin^{-1}(\cot i)$ (b) $\sin^{-1}(\tan i)$
(c) $\sin^{-1}(\cos i)$ (d) $\cos^{-1}(\sin i)$

39. If in a prism angle of prism and angle of minimum deviation are equal, consider μ = 1.5 find angle of prism.

(a) 60° 48′ (b) 71° 24′
(c) 82° 48′ (d) 90°

40. A ray of light passes through a thickness 4.8 cm of mica (μ = 1.6) then time taken by the ray will be

(a) $1.6 \times 10^{-8}\,s$ (b) $1.6 \times 10^{-10}\,s$
(c) $2.56 \times 10^{-8}\,s$ (d) $2.56 \times 10^{-10}\,s$

41. What percentage of the time is increased when a ray of light passes through a layer of thickness 3 cm (μ = 1.4)?

(a) 20% (b) 33.3%
(c) 40% (d) none of these

42. A light is incident (λ = 600 nm) on a thin layer. The minimum thickness to cause interference is

(a) 2×10^{-7} m (b) 3×10^{-7} m
(c) 2×10^{-8} m (d) 3×10^{-6} m

43. White light is incident normally on a glass plate of thickness 0.5 μm (μ = 1.5). Which wavelengths are strongly reflected in the visible region?

(a) 683 nm, 529 nm (b) 600 nm, 429 nm
(c) 579 nm, 429 nm (d) none of these

44. A prism has base 5 cm thick and $\frac{d\mu}{d\lambda}$ = 4000. Find its resolving power and also the smallest wavelength which it can resolve.

(a) 800, 6 nm (b) 4000, 2 nm
(c) 6000, 0.2 nm (d) 20000, 0.2 nm

45. For what value of λ first secondary maxima coincides with the first minima of 690 nm wavelength?

(a) 230 nm (b) 460 nm
(c) 430 nm (d) 320 nm

46. Discontinuous spectrum is ____

(a) band spectrum (b) line spectrum
(c) prism spectrum (d) invisible spectrum

47. Number of fringes in a diffraction experiment are....

(a) infinite (b) finite and small
(c) dependent on wavelength
(d) dependent on slit width

48. Some substances become doubly refracting when placed in a strong electrostatic field. This phenomenon is called ....

(a) Faraday effect (b) Verdt's effect
(c) Kerr effect (d) Brewester's effect

49. In flourescence light radiations stop.... when exciting source is removed.

(a) instantly (b) $\le 10^{-8}\,s$
(c) $\ge 10^{8}\,s$ (d) none of these

50. Methyl alcohol is optically ____ and mechanically ____

(a) denser, rarer (b) rarer, denser
(c) rarer, rarer (d) denser, denser

Answers to Questions for Practice

1.	(b)	**2.**	(b)	**3.**	(c)	**4.**	(a)	**5.**	(c)	**6.**	(b)	**7.**	(a)
8.	(c)	**9.**	(c)	**10.**	(c)	**11.**	(b)	**12.**	(b)	**13.**	(c)	**14.**	(c)
15.	(a)	**16.**	(b)	**17.**	(b)	**18.**	(c)	**19.**	(a)	**20.**	(c)	**21.**	(c)
22.	(a)	**23.**	(b)	**24.**	(a)	**25.**	(c)	**26.**	(c)	**27.**	(a)	**28.**	(c)
29.	(b)	**30.**	(d)	**31.**	(c)	**32.**	(c)	**33.**	(a)	**34.**	(b)	**35.**	(b)
36.	(c)	**37.**	(b)	**38.**	(a)	**39.**	(c)	**40.**	(d)	**41.**	(c)	**42.**	(b)
43.	(b)	**44.**	(d)	**45.**	(b)	**46.**	(b)	**47.**	(b)	**48.**	(c)	**49.**	(b)
50.	(a)												

Explanations

1(b) If a convex mirror is not placed, i.e. with convex lens alone, the image will be formed at 30 cm away.

$$\left\{use\ \frac{1}{u} = \frac{1}{f} + \frac{1}{u}\right\}$$

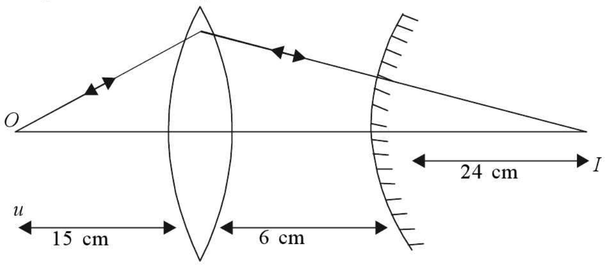


Fig. 3

If the ray falls normally on the convex mirror only then it will retrace its path to form the image at the object therefore, it should be placed $2f$ or 24 cm from I.

2(b) When the lens is placed on the mirror the image coincides with the object needle only if the object is placed at focus, therefore, $f_{\text{lens}} = 20$ cm;

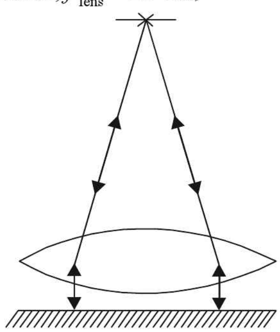


Fig. 4

$$f_{\text{combind}} = 30\,\text{cm}$$

$$\frac{1}{f_{\text{liqlens}}} = \frac{1}{30} - \frac{1}{20} = \frac{1}{-60} \Rightarrow f_{\text{liqlens}} = -60 \text{ cm}$$

$$\frac{1}{f_{\text{liqlens}}} = (\mu_{liq} - 1)\frac{1}{R}\ \frac{1}{-60} = (\mu_{liq} - 1)\frac{1}{-20}$$

$$\mu_{liq} - 1 = \frac{20}{60} = \frac{1}{3};\ \mu_{liq} = 1 + \frac{1}{3} = \frac{4}{3}$$

3(c) Size of the object $= \sqrt{I_1 I_2} = \sqrt{25 \times 9} = 15$ cm

4(a) $$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

or $$\frac{1}{v} = \frac{1}{u} = \left(\frac{u-f}{uf}\right) \text{ or m} = \frac{v}{u} = \left(\frac{f}{u-f}\right)$$

axial magnification $= m^2 = \left(\dfrac{f}{u-f}\right)^2$

$\therefore$ size of image $= l\left(\dfrac{f}{u-f}\right)^2$

5(c) $$\frac{1}{f} = (\mu - 1)\frac{1}{R} \Rightarrow \frac{1}{20} = \frac{0.5}{R} \Rightarrow R = 10\text{ cm}$$

$$\frac{f_{\text{water}}}{f_{\text{air}}} = \frac{\dfrac{\mu_g}{\mu_a} - 1}{\dfrac{\mu_g}{\mu_w} - 1} \Rightarrow \frac{f_{\text{water}}}{20} \frac{0.5}{\dfrac{9}{8} - 1} = 4$$

or $f_{\text{water}} = 80$ cm

6(b) $$\lambda = \frac{2t}{n} = \frac{2 \times 8 \times 10^{-3}}{4 \times 10^{-4}} = 4 \times 10^{-7} m \text{ or } 400\ nm.$$

7(a)

8(c) angular deviation $\propto \dfrac{1}{\lambda^3}$

$\therefore$ angular deviiation becomes $\dfrac{d}{8}$

9(c) Look into the figure, the silvered surface appears to be 6 cm away

$$\left(\text{Apparent depth} = \frac{\text{Real depth}}{\mu}\right) \Rightarrow \frac{9}{1.5} = 6 \text{ cm}$$

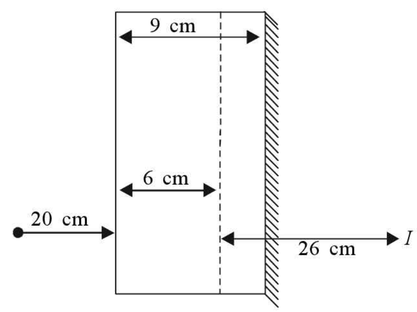


Fig. 5

Now object is 20 + 6 = 26 cm.

away from silver surface $\therefore$ image is formed 26 cm away from apparent silvered surface and 26 + 6 = 32 cm away from the front surface of the mirror.

10(c) $$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$ Case (i) from shorter distance side

$$\frac{1}{v} - \frac{1.5}{5} = \frac{-0.5}{6}$$

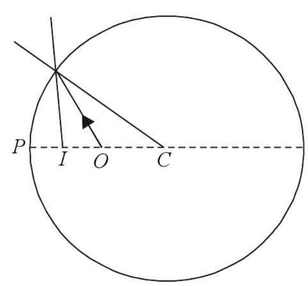


Fig. 6

or $$\frac{1}{v} = \frac{-1}{12} + \frac{1.5}{5} = \frac{-5+18}{60} = \frac{13}{60}$$

or $$v = \frac{60}{13} = 4.61 \text{ cm.}$$

Case (ii) from, larger distance side $\frac{1}{v} + \frac{1.5}{7} = \frac{+0.5}{6} \Rightarrow$

$$\frac{1}{v} = \frac{+1}{12} - \frac{1.5}{7} = \frac{-11}{84} \quad v = -7.64 \text{ cm.}$$

11(b)

12(b) When the equipment is immersed in a liquid, new wavelength becomes

$$\lambda' = \frac{\lambda}{1.5} = \frac{500}{1.5} = 333.33\, nm$$

fringe width ∝ wavelength

13(c) Size of image = length of camera (θ)

$$= 50 \times \frac{10}{200} = 2.5 \text{ cm}$$

14(c) $\frac{\mu_1}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R} \Rightarrow \frac{1}{1} - \frac{1.5}{u} = \frac{-0.5}{3}$

$$\Rightarrow \frac{1.5}{u} = \frac{+1}{6} + \frac{1}{1} = \frac{7}{6} \Rightarrow u = \frac{9}{7} = 1.286\, mm.$$

15(a) $$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2} = \frac{1}{20} - \frac{1}{10} + \frac{15}{200}$$

$$= \frac{5}{200} = \frac{1}{40}, f = 40 \text{ cm}$$

for converging lens $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

$$\frac{1}{v} = \frac{1}{20} - \frac{1}{40} = \frac{1}{40}$$

∴ $$m_1 = \frac{v}{u} = \frac{40}{40}$$

This image may be considered as a virtual object for concave lens

∴ distance = $40 - 15 = 25$ cm.

$$\frac{1}{v} = \frac{-1}{10} + \frac{1}{25} = \frac{-3}{50}$$

∴ $$m_2 = \frac{v}{u} = \frac{-50}{3} \times \frac{1}{25} = \frac{2}{3}$$

$$m = m_1 \times m_2 = 1 \times \frac{2}{3} = \frac{2}{3}$$

16(b) $$\mu = \frac{3.665}{2.435} = 1.505$$

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

$$\frac{1}{2.91} - \frac{1.5}{3.665} = \frac{-0.505}{R}, \frac{1}{2.91} = \frac{1}{2.435} = \frac{-0.505}{R}$$

or $R = 7.52$ cm.

17(b)

18(c)

19(a)

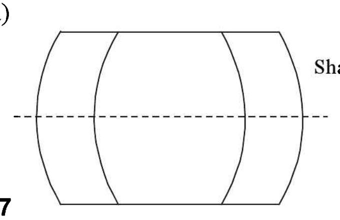


Fig. 7

20(c)

21(c)

22(a)

23(b)

24(a)

25(c) $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$ Differentiating it, we get

$$-\frac{dv}{v^2} + \frac{du}{u^2} = 0 \text{ or } \frac{dv}{du} = \frac{v^2}{u^2} \text{ axis magnification}$$

$$m' = \frac{dv}{du} = \frac{V^2}{u^2} = m^2$$

26(c) Any ray will return back if it falls normally

∴ $\angle r = 30°$ $(90° - 60°)$

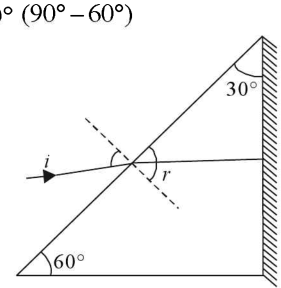


Fig. 8

$$\mu = \frac{\sin i}{\sin r} \Rightarrow \sqrt{2} = \frac{\sin i}{\sin 30}$$

or $$\sin i = \frac{1}{\sqrt{2}} \Rightarrow i = 45°$$

27(a) Intensity of light at a point is given by

$$I = \frac{E_m^2}{2\mu_0 c}$$

$$E_m = \sqrt{2\mu_0 I c} = \sqrt{2 \times 4\pi \times 10^{-7} \times 1.38 \times 10^3 \times 3 \times 10^8}$$

$$= 1.02 \times kV/m$$

28(c) According to Newton's law

$$F = \frac{dp}{dt} = \frac{2IA}{c} = \frac{2\times15\times1.5}{3\times10^8}$$

$$= 1.5\times10^{-7}\,N$$

29(b) Magnification

$$M = \frac{f_0}{f_e} = \frac{\beta}{\alpha} \Rightarrow = \frac{60}{5} = \frac{\beta}{2} \text{ or } \beta = 24°$$

30(d) $\frac{\omega_1}{f_1}+\frac{\omega_2}{f_2} = 0$ or $\frac{f_2}{f_1} = -\frac{\omega_2}{\omega_1}$

$$\Rightarrow \frac{1}{f_1} = \frac{0.36}{.024}\frac{1}{f_2} = -\frac{3}{2f_2}$$

$$\frac{1}{f_1}+\frac{1}{f_2} = \frac{1}{90}$$

$$-\frac{3}{2f_2}+\frac{1}{f_2} = \frac{1}{90} \Rightarrow \frac{-1}{2f_2} = \frac{1}{90} \text{ or } f_2 = -45\text{ cm}$$

$$\frac{1}{f_1} = \frac{-3}{2f_2} = +\frac{3}{90} = \frac{1}{30} \text{ or } f_1 = 30\text{ cm}$$

31(c) Efficiency of light source

$$= \frac{\text{luminous flux (lumen)}}{\text{input electric power}} = \frac{2000}{40} = 50$$

32(c) Total luminous flux $\phi_{Tot} = 4\pi I$ flux falling on the surface

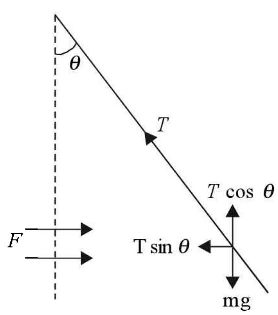
$$\phi = 2\% \text{ of } \phi_{Tot} = \frac{2}{100}\times4\pi I$$

Average illuminence

$$= \frac{\phi}{\pi r^2} = \frac{2\times4\pi\times500}{100\times\pi\times(.1)^2} = 4000\text{ lux}$$

33(a) Force exerted on mirror $F = \frac{2LS}{c}$

From equilibrium condition, $T\cos\theta = mg$



$$T\sin\theta = F = \frac{2LS}{c}$$

$$\tan\theta = \theta = \frac{2LS}{cmg}\text{, assuming } \theta \text{ to be small}$$

34(b) $\therefore \beta \propto \frac{1}{d}$

35(b) Transverse linear magnification

$$m = \sqrt{9} = 3$$

$$\therefore \quad \frac{v}{u} = 3$$

$$v = 3u$$

$$\because \quad u = 40\text{ cm} \therefore v = 120\text{ cm}$$

$$\frac{1}{f} = \frac{1}{v}+\frac{1}{u} = \frac{1}{120}+\frac{1}{40} = \frac{4}{120} = \frac{1}{30}$$

or $f = 30$ cm

36(c) Using $(\mu_1 - 1)\,\alpha_1 = (\mu_2 - 1)\,\alpha_2$

$$\alpha_2 = \frac{0.5\times4.5}{0.75} = 3°$$

37(b) $\mu = A + \frac{\beta}{\lambda^2}$ Wavelength of red is maximum and hence μ_r will be mimimum. The apparent depth will be maximum. Therefore, it will appear to be raised minimum.

38(a) $r = 90 - i,\ \mu = \frac{\sin i}{\sin r} = \frac{\sin i}{\sin(90-i)} = \frac{1}{\sin c}$

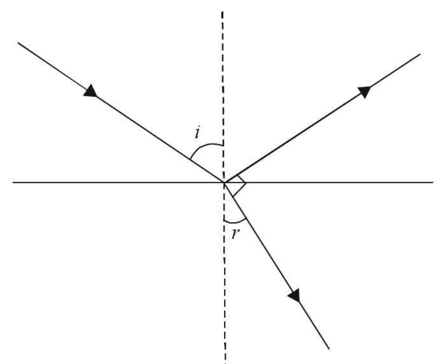


Fig. 10

or $\sin c = \cot i$ or $c = \sin^{-1}(\cot i)$

39(c) $\mu = \frac{\sin\left(\frac{A+D_m}{2}\right)}{\sin\frac{A}{2}} \Rightarrow 1.5 = \frac{\sin A}{\sin\frac{A}{2}} = \frac{2\sin\frac{A}{2}\cos\frac{A}{2}}{\sin\frac{A}{2}}$

or $\cos\frac{A}{2} = 0.75$

or $\frac{A}{2} = 41°24'$

or $A = 82°48'$

40(d) time taken $= \dfrac{\frac{t}{c}}{\mu} = \dfrac{t\mu}{c} = \dfrac{4.8\times10^{-2}\times1.6}{3\times10^{8}}$

$= 2.56 \times 10^{-10} s$

41(c) $t_{vac} = \dfrac{3\times10^{-2}}{3\times10^{8}} = 10^{-10}$ s, $t_{mat} = \dfrac{3\times10^{-2}\times1.4}{3\times10^{8}}$

$= 1.4 \times 10^{-10} s$

% increase $= \dfrac{0.4}{1} \times 100 = 40\%$

42(b) Minimum thickness to cause interference is $\lambda/2$

43(b) $\lambda = \dfrac{4\mu d}{2n+1} = \dfrac{4\times1.5\times0.5\times10^{-6}}{2n+1}$

Putting $\lambda = 400$ nm and $\lambda = 700$ nm, we get

$n = 3.25$ and 1.66, therefore, we can take in

$n = 2$ and 3

when $n = 2, \lambda = 600$nm and when

$n = 3, \lambda = 429$ nm

44(d) $R.P = \dfrac{d\mu}{d\lambda} t = 4000 \times 5 = 20000$

smallest wavelength to be resolved

$d\lambda = \dfrac{\lambda}{RP} = \dfrac{400}{2000} = 0.2$ nm

45(b) $\left(n+\dfrac{1}{2}\right)\lambda' = n\lambda$

$\lambda' = \dfrac{690}{3/2}$

$= 460$ nm

46(b)

47(b)

48(c)

49(b)

50(a)

QUESTIONS FROM COMPETITIVE EXAMINATIONS

AIEEE 2002

1. If two mirrors are kept at 60° to each other, then the number of images formed by them is:
   (a) 5 (b) 6
   (c) 7 (d) 8
2. Wavelength of light used in an optical instrument are λ_1 = 4000 Å and λ_2 = 5000 Å, then ratio of their respective resolving powers (corresponding to λ_1 and λ_2) is:
   (a) 16 : 25 (b) 9 : 1
   (c) 4 : 5 (d) 5 : 4
3. Which of the following is used in optical fibres?
   (a) Total internal reflection
   (b) Scattering
   (c) Diffraction
   (d) Refraction
4. An astronomical telescope has a large aperture to:
   (a) reduce spherical aberration
   (b) have high resolution
   (c) increase span of observation
   (d) have low dispersion

Answers

1. (a) **2.** (d) **3.** (a) **4.** (b)

Explanations

1. (a)Number of images $n = \frac{360°}{\theta} - 1$

where θ = angle between mirrors

Thus, θ = 60° (given)

So, number of images

$$n = \frac{360°}{60°} - 1 = 5$$

2. (d)Resolving power of an optical instrument is inversely proportional to λ i.e., R.P. $\propto \frac{1}{\lambda}$

$$\therefore \frac{\text{Resolving power at } \lambda_1}{\text{Resolving power at } \lambda_2} = \frac{\lambda_2}{\lambda_1} = \frac{5000}{4000} = 5 : 4$$

3. (a)

4. (b)

AIEEE 2003

1. To demonstrate the phenomenon of interference we require two sources which emit radiation of:
   (a) nearly the same frequency
   (b) the same frequency
   (c) different wavelength
   (d) the same frequency and having a definite phase relationship
2. The image formed by an objective of a compound microscope is:
   (a) virtual and diminished
   (b) real and diminished
   (c) real and enlarged
   (d) virtual and enlargd
3. To get three images of a single object, one should have two plane mirrors at an angle of:
   (a) 60° (b) 90°
   (c) 120° (d) 30°

Answers

1. (d) **2.** (c) **3.** (b)

1. (d)

2. (c)Objective of compound microscope is a convex lens. Convex lens forms real and enlarged image when an object is placed between its focus and lens.

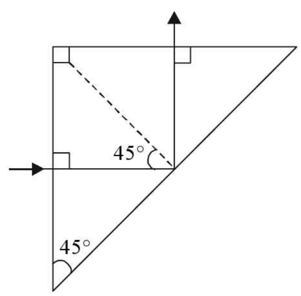
3. (d) $n = \frac{360°}{\theta} - 1$

$3 = \frac{360°}{\theta} - 1$

$\theta = 90°$

AIEEE 2004

1. A light ray is incident perpendicular to one face of a 90° prism and is totally internally reflected at the glass-air interface. If the angle of reflection is 45°, we conclude that the refractive index n:



(a) $n < \frac{1}{\sqrt{2}}$ (b) $n > \sqrt{2}$

(c) $n > \frac{1}{\sqrt{2}}$ (d) $n < \sqrt{2}$

2. A plano-convex lens of refractive index 1.5 and radius of curvature 30 cm is silvered at the curved surface. Now this lens has been used to form the image of an object. At what distance from this lens, an object be placed in order to have a real image of the size of the object?

(a) 20 cm (b) 30 cm

(c) 60 cm (d) 80 cm

3. The angle of incidence at which reflected light is totally polarized for reflection from air to glass (reflractive index n), is:

(a) $\sin^{-1}(n)$ (b) $\sin^{-1}(1/n)$

(c) $\tan^{-1}(1/n)$ (d) $\tan^{-1}(n)$

4. The maximum number of a possible interference maxima for slit-separation equal to twice the wavelength in Young's double-slit experiment, is:

(a) infinite (b) five

(c) three (d) zero

Answers

1. (b) **2.** (a) **3.** (d) **4.** (b)

Explanations

1. (b) For total internal reflection from glass-air interface, critical angle C must be less then angle of incidence.

i.e. $C < i$

or $C < 45°$ $(\because \angle i = 45°)$

But $n = \frac{1}{\sin C} \Rightarrow C = \sin^{-1}\left(\frac{1}{n}\right)$

or $\sin^{-1}\left(\frac{1}{n}\right) < 45°$

or $n > \frac{1}{\sin 45°}$ or $n > \sqrt{2}$

2.(a) $\frac{1}{f_{new}} = \frac{2}{f_{lens}} + \frac{2}{R}$

$= \frac{2}{60} + \frac{2}{30}$

$f_{new} = 10\text{ cm}$

when object is at $2f$ image is formed at $2f$.

3. (d) $n = \tan i_p$

or $i_p = \tan^{-1}(n)$

4. (b) For possible interference maxima on the screen, the condition is

$d \sin\theta = n\lambda$ ...(i)

Given: d = slit–width $= 2\lambda$

$\therefore$ $2\lambda \sin\theta = n\lambda$

$\Rightarrow$ $2\sin\theta = n$

The maximum value of $\sin\theta$ is 1, hence,

$n = 2 \times 1 = 2$

Thus, equation (i) must be satisfied by 5 integer values i.e. –2, –1, 0, 1, 2. Hence, the maximum number of possible interference maxima is 5.

AIEEE 2005

1. A Young's double slit experiment uses a monochromatic source. The shape of the interference fringes formed on a screen is:
(a) hyperbola (b) circle
(c) straight line (d) parabola

2. A fish looking up through the water sees the outside world, contained in a circular horizon. If the refractive index of water is $\frac{4}{3}$ and the fish is 12 cm below the water surface, the radius of this circle in cm is:
(a) $36\sqrt{7}$ (b) $\frac{36}{\sqrt{7}}$
(c) $36\sqrt{5}$ (d) $4\sqrt{5}$

3. Two point white dots are 1 mm apart on a black paper. They are viewed by eye of pupil diameter 3 mm. Approximately, what is the maximum distance at which these dots can be resolved by the eye? [Take wavelength of light = 500 mm]
(a) 5 m (b) 1 m
(c) 6 m (d) 3 m

4. A thin glass (refractive index 1.5) lens has optical power of $-5D$ in air. Its optical power in a liquid medium with refractive index 1.6 will be:
(a) $1D$ (b) $-1D$
(c) $25D$ (d) $-25D$

5. When an unpolarized light of intensity I_0 is incident on a polarizing sheet, the intensity of the light which does not get transmitted is:
(a) $\frac{1}{2}I_0$ (b) $\frac{1}{4}I_0$
(c) zero (d) I_0

6. If I_0 is the intensity of the principal maximum in the single slit diffraction pattern, then what will be its intensity when the slit width is doubled?
(a) $2I_0$ (b) $4I_0$
(c) I_0 (d) $\frac{I_0}{2}$

Answers

1.	(a)	2.	(b)	3.	(a)	4.	(a)	5.	(a)	6.	(c)

Explanations

1. (a)

2. (b) $\tan\theta_c = \frac{AB}{AO}$

$$AB = OA\tan\theta_c$$

or $$AB = \frac{OA}{\sqrt{\mu^2-1}} = \frac{12}{\sqrt{\left(\frac{4}{3}\right)^2-1}} = \frac{36}{\sqrt{7}}$$

3. (a) $\frac{y}{D} \geq 1.22\frac{\lambda}{d}$

$$D \geq \frac{yd}{1.22\lambda} = \frac{10^{-3}\times3\times10^{-3}}{1.22\times5\times10^{-7}}$$

$$= \frac{30}{6.1} = 5$$

$\therefore$ $D_{max} = 5m$

4. $$\frac{f_m}{f_{\text{air}}} = \frac{(\mu_L-1)}{\left(\frac{\mu_L}{\mu_m}-1\right)}$$

$$\frac{f_m}{-20} = \frac{V_2}{\frac{1.5}{1.6}-1} = -8$$

$$f_m = 1.6m$$

$$P_m = \frac{1.6}{1.6} = \mu$$

5.(a) Intensity of untransmitted light

$$= I_0 - \frac{I_0}{2} = \frac{I_0}{2}$$

($\because$ intensity of polarized light $= \frac{I_0}{2}$)

6. (c) $I = I_0\left(\frac{\sin\theta}{\theta}\right)^2$

and $\theta = \frac{\pi}{\lambda}\left(\frac{ay}{D}\right)$

For principal maximum $y = 0$

$\therefore$ $\theta = 0$

Hence, intensity will remain same.

AIEEE 2006

1. The refractive index of glass is 1.520 for red light and 1.525 for blue light. Let D_1 and D_2 be angles of minimum deviation for red and blue light respectively in a prism of this glass, then,:

(a) $D_1 < D_2$ (b) $D_1 = D_2$
(c) D_1 can be less than or greater than D_2 depending upon the angle of prism
(d) $D_1 > D_2$

Answer

1. (a)

Explanation

1. (d) $D = (\mu - 1)A$

For blue light μ is greater than that for red light, so $D_2 > D_1$.

AIIMS 2005

1. What should be the maximum acceptance angle at the air-core interface of an optical fibre if n_1 and n_2 are the refractive indices of the core and the cladding, respectively?

(a) $\sin^{-1}(n_2/n_1)$ (b) $\sin^{-1}\sqrt{n_1^2 - n_2^2}$
(c) $\left[\tan^{-1}\frac{n_2}{n_1}\right]$ (d) $\left[\tan^{-1}\frac{n_1}{n_2}\right]$

2. A telescope has an objective lens of focal length 200 cm and an eye piece with focal length 2 cm. If this telescope is used to see a 50 meter tall building at a distance of 2 km, what is the height of the image of hte building formed by the objective lens?

(a) 5 cm (b) 10 cm
(c) 1 cm (d) 2 cm

3. When exposed to sunlight, thin films of oil on water often exhibit brilliant colours due to the phenomenon of

(a) interference (b) diffraction
(c) dispersion (d) polarisation

4. The pressure exerted by an electromagnetic wave of intensity I (watt/m^2) on a nonreflecting surface is [c is the velocity of light]

(a) Ic (b) Ic^2
(c) I/c (d) I/c^2

5. In case of linearly polarised light, the magnitude of the electric field vector

(a) does not change with time
(b) varies periodically with time
(c) increases and decreases linearly with time
(d) is parallel to the direction of propogation.

Answers

1.	(b)	2.	(a)	3.	(a)	4.	(c)	5.	(b)

Explanations

1. (b) Core of acceptance angle $\theta = \sin^{-1}\sqrt{n_1^2 - n_2^2}$

2. (a) From the formula for convex lens,

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$\therefore \quad v = \frac{f \times u}{u - f} = \frac{200 \times 200 \times 10^3}{[200 \times 10^3 - 200]}$$

$$= \frac{200 \times 10^3}{999}$$

also, magnification, $m = \left|\frac{v}{u}\right| = \left|\frac{I}{O}\right|$

$$= \frac{200 \times 10^3}{999 \times 200 \times 10^3} = \frac{I}{50 \times 100}$$

$$I = \frac{5000}{999} \simeq 5\,\text{cm}.$$

3. (a) The colour in the oil film is formed due to interference os sunlight where the colour of the film will depend upon the thickness and the angle of inclination.

4. (c) Pressure = Force/area

$$I = \frac{\text{Energy}}{\text{area-time}} = \frac{F.S}{At} = \frac{Fc}{A} \qquad I/c = P.$$

5. (b) The magnitude of electric field vector varies periodically with time because it is the form of electromagnetic wave.

BHU 2005

1. If the f value of a lens of a camera is $5\,f$ and that of another is $2.5\,f$, what is the time of exposure for the second if for the first one is $\frac{1}{200}s$?

(a) $\frac{1}{200}$ (b) $\frac{1}{800}$

(c) $\frac{1}{6400}$ (d) $\frac{1}{3200}$

2. If a convex lens of focal length 80 cm and a concave lens of focal length 50 cm are combined together, what will be their resultant power?

(a) + 0.65 D (b) – 0.65 D

(c) + 0.75 D (d) – 0.75 D

3. A plane glass slab is kept over various colour letters, the letter which appears least raised is

(a) red (b) violet

(c) green (d) blue

4. Two waves of intensity I undergo interference. The maximum intersity obtained is

(a) $I/2$ (b) I

(c) $2I$ (d) $4I$

5. The time taken by light to pass through 4 mm thick glass slab of refractive index 1.5 will be (velocity of light in air = 3×10^8 m/s)

(a) 8×10^{-11} s (b) 2×10^{-11} s

(c) 8×10^{-8} s (d) 2×10^{-8} s

6. A lens acts as converging in air and diverging when immersed in water. Then refractive index of lens is

(a) equal to unity (b) below 1.33

(c) greater then 1.33 (d) less then unity

7. A light passing through air has wavelength 6000 Å. The wavelength when same ray passes through a glass slab of refractive index 1.5 will be

(a) 2000 Å (b) 4000 Å

(c) 8000 Å (d) 1200 Å

Answers

1. (b) **2.** (d) **3.** (a) **4.** (d) **5.** (b) **6.** (b) **7.** (b)

Explanations

1. (b) When the f value is decreased from 5 to 2.5, the time of exposure is reduced by $\left(\frac{1}{2}\right)^2$ i.e., $\frac{1}{4}$

$\therefore$ Time $= \frac{1}{800}s.$

2. (d) For convex lens, $f_1 = -50$ cm, $P_1 = \frac{1}{0.8}$ D = 1.25 D

For concave lens, $f_2 = -50$ cm, $P_2 = \frac{1}{0.5}$ D = – 2 D

$\therefore$ $P = P_1 + P_2 = +1.25 - 2.00 = -0.75$ D.

3. (a) $\mu = \frac{\text{real depth}}{\text{apparent depth}}$

Maximum apparent depth means minimum raised from surface. Red colour having minimum value of refractive index and thus maximum value of apparent depth.

4. (d) $I_{max} = (a_1 + a_2)^2$

$$= \left(\sqrt{I} + \sqrt{I}\right)^2 = \left(2\sqrt{I}\right)^2 = 4\,I$$

5. (b) Velocity of light in the slab is

$$v = \frac{c}{\mu} = \frac{3\times10^8}{1.5} = 2 \times 10^8 \text{ m/s}$$

Time taken by light to pass through glass slab

$$t = \frac{d}{v} = \frac{4\times10^{-3}}{2\times10^8} = 2 \times 10^{-11} \text{ s}$$

6. (b) Lens is converging in air, therefore $\mu > 1$ but lens is diverging in water, therefore $\mu < 1.33$

7. (b) Refractive index $= \frac{\lambda_{air}}{\lambda_{medium}}$

$$1.5 = \frac{6000\,\text{Å}}{\lambda_{medium}} \Rightarrow \lambda_{medium} = 4000\,\text{Å}$$

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6 Modern Physics

33

Bohr's Theory and Atomic Physics

BRIEF REVIEW

Thomson's Model: Thomson in 1898 stated :

(a) Atom as a whole is neutral, i.e., positive and negative charge are equal.

(b) The positive charge and the whole mass is distributed uniformly like a cake and electrons are embedded like cherries in the cake. Therefore, this model is also called plum pudding model.

(c) It cannot explain α-particle scattering and spectrum of an atom.

In 1903, Leonard suggested that atom is made up of tiny particles carrying negative and positive charges. These are termed as electrons and protons. Leonard, however, could not explain why heating of metals does not eject positively charged particles.

Rutherford's Model

Rutherford conducted α-particle scattering experiment. On the basis of which he proposed a model of an atom.

(a) The whole positive charge and whole mass of the atom is concentrated in a very small region called nucleus. The size of the nucleus is ~ 10^{-15} m or 1 *fm*.

(b) The electrons revolve around the nucleus in circular orbits. The size of an atom ~ 10^{-10} m. There exists a large empty space around the nucleus.

(c) Atoms are electrically neutral

Distance of closest approach $r = \dfrac{2Ze^2}{4\pi\varepsilon_o(KE)}$

Impact parameter $b = \dfrac{Ze^2 \cot\dfrac{\theta}{2}}{4\pi\varepsilon_o(KE)}$

The number of particles scattered through an angle θ is given by

$$N(\theta) \propto \frac{Z^2}{\sin^4\left(\dfrac{\theta}{2}\right)(KE)^2}$$

This model failed to explain as to why the revolving electrons do not lose energy and ultimately fall into the nucleus following a spiral path, i.e., stability of atom could not be explained with this model.

Bohr's Model

When Bohr proposed his model hydrogen spectrum was known.

Rydberg's emperical formula $\dfrac{1}{\lambda} = R\left[\dfrac{1}{n_1^2} - \dfrac{1}{n_2^2}\right]$ was also known. By certain assumptions, called Bohr's postulates, he could manage to explain Hydrogen spectrum.

(a) electrons move around the nucleus in circular orbits.

(b) The orbits are stable called stationary orbits. They have special values of their radii such that angular momentum is qunatized; i.e., $mvr = n\hbar$ where

$$\hbar = \frac{h}{2\pi}$$

(c) Energy is released when an electron makes a transition from higher to lower level as shown in Fig. 33.1(a) and energy is absorbed when an electron jumps from lower to higher orbit as shown in Fig. 33. 1(b)

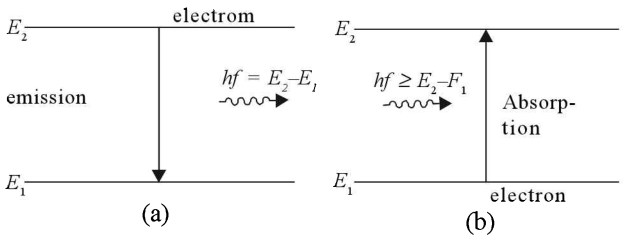


Fig. 33.1 Emission – Absorption of Radiation

The centripetal force is equal to the electrostatic force.

Radius of nth orbit $r_n = \dfrac{n^2 \varepsilon_o h^2}{\pi m Z e^2}$

Bohr radius (1st orbit of H atom) $r_1 = \dfrac{\varepsilon_o h^2}{\pi m e^2} = 0.53\,A°$

$r_n = n^2 r_1$ for Hydrogen and

$r_n = \dfrac{n^2 r_1}{Z}$ for Hydrogen like atoms.

Orbital speed $v_n = \dfrac{e^2}{2\varepsilon_o nh}$ for Hydrogen

Energy of nth orbit $= E_n = KE_n + PE_n = \dfrac{mZ^2e^4}{8\varepsilon_o^2 h^2 n^2}$

Note: If in place of m reduced mass be taken agreement is perfect. Reduced mass $m_r = \dfrac{m_p . m_e}{m_p + m_e}$ where m_p and m_e are mass of proton and electron respectively. Bohr's analysis are within 0.1% of measured values.

Sommerfeld Model

The electrons revolve around the nucleus in elliptical orbits. The mass of the electron varies with velocity relativistically

$$m = \frac{m_o}{\sqrt{1 - \frac{v^2}{c^2}}} .$$

Total angular momentum of an electron is the resultant of orbital angular momentum and radial angular momentum. These two angular momentum are separately quantized.

De-Broglie Theory

A standing wave on a string transmits no energy. Therefore, think of an electron as a standing wave fitted in a circle in one of the Bohr orbits. Only those circular orbits are possible whose circumference is an integral multiple of de-Broglie wavelength associated with the electron, i.e., $2\pi r = n\lambda$

since $\lambda = \dfrac{h}{mv}$ thus, $mvr = \dfrac{nh}{2\pi}$ which matches with Bohr's quantization theory and suggests strongly that wave character of electrons is important in atomic structure.

Representation of waves associated with orbital electrons in an atom is illustrated in Fig. 33.2 for $n = 1$, $n = 2$, $n = 3$ and $n = 4$.

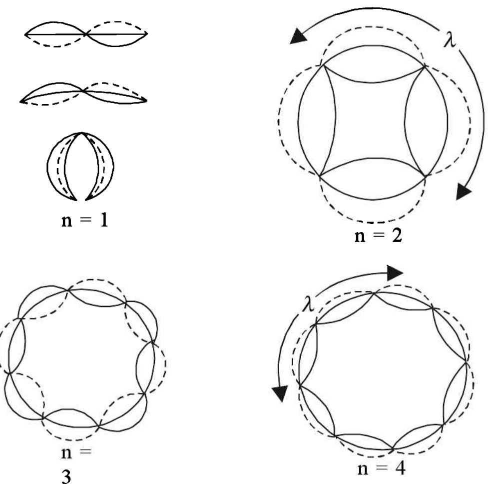


Fig. 33.2 de Broglie stationary waves in the orbit

Velocity of electron in the nth orbit $v_n = \dfrac{2\pi Z e^2}{4\pi\varepsilon_o nh}$

$$= \frac{c}{137}$$

$$= \frac{Z}{n} = 2.2 \times 10^6 \frac{Z}{n}\ ms^{-1}$$

$$\alpha = \frac{2\pi e^2}{4\pi\varepsilon_o ch} = \frac{1}{137}$$ is called **fine structure constant.**

Angular frequency of electron $\omega = \dfrac{8\pi^2 Z^2 e^4 m}{(4\pi\varepsilon_o)^2 n^3 h^3}$

$$= \frac{4.159 \times 10^6 Z^2}{n^3}\ \text{rad}\ s^{-1}$$

Electric current due to electron motion in nth orbit

$$I_n = \frac{4\pi Z^2 e^5 m}{n^3 h^3 (4\pi\varepsilon_o)^2} = \frac{1.06 Z^2}{n^3}\ mA$$

Magnetic induction produced in the nth orbit

$$B_n = \frac{\mu_o I_n}{2r_n}$$

$$= \frac{8\pi^4 Z^3 e^7 m^2}{n^5 h^5 (4\pi\varepsilon_o)^3} = \frac{1.2.58Z^3}{n^5} \text{ Tesla}$$

Magnetic moment produced in the nth orbit

$$M_n = \frac{e\hbar n}{2m} = \frac{ehn}{4\pi m} = 9.26 \times 10^{-24} n\, Am^2$$

$= n$ Bohr Magneton.

$$KE \text{ of electron} = \frac{e^2 Z^2}{8\pi\varepsilon_o r_n} = \frac{13.6 Z^2}{n^2}\, eV$$

$$PE \text{ of electron} = -2KE = \frac{-e^2 Z^2}{4\pi\varepsilon_o r_n} = -27.2\,\frac{Z^2}{n^2}\, eV$$

Binding energy of electron $BE = E_n = KE + PE$

$$= \frac{-e^2 Z^2}{8\pi\varepsilon_o r_n}$$

$$= -13.6\,\frac{Z^2}{n^2}\, eV$$

$$\text{Ionization potential } \frac{E_n}{e} = 13.6\,\frac{Z^2}{n^2} \text{ Volt}$$

$$\text{Rydberg constant } R = \frac{me^4}{8\varepsilon_o^2 ch^3} = 1.09737 \times 10^7\, m^{-1}$$

Ionization energy is the minimum energy required for an electron so that it loses its ground state and reaches vacuum level or continuum, or it is relieved from binding of the nucleus.

Excitation energy is the minimum energy required for an electron to jump to a higher energy state.

Hydrogen Spectrum

$$\text{Rydberg's emperical relation } \frac{1}{\lambda} = R\left[\frac{1}{n^2} - \frac{1}{m^2}\right]$$

Wave number $\bar{\nu} = \frac{1}{\lambda}$ defines the number of waves per unit length.

$\therefore$ Number of waves in a distance d is $N = \frac{d}{\lambda}$.

Lyman series

$$\frac{1}{\lambda} = R\left[\frac{1}{1^2} - \frac{1}{n^2}\right] \quad n = 2, 3, \ldots\ldots$$

$$\lambda_{max} = 1216 A°;\ \lambda_{min} = 912 A°.$$

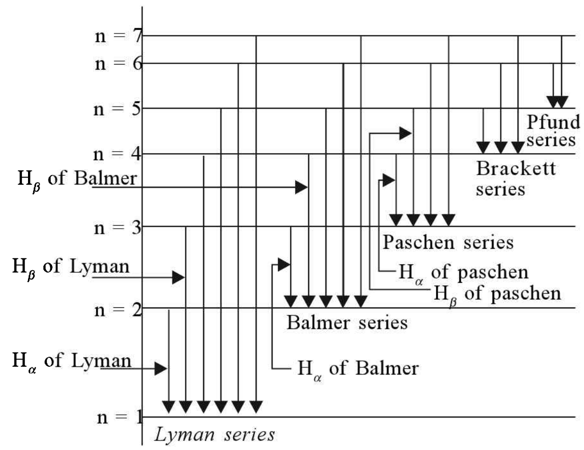


Fig. 33.3 Hydrogen spectrum

This series lies in *uv* region. It shows both emission and absorption spectrum.

Note: Transitions occur from higher energy states to ground state.

Balmar series In this series, transitions occur from higher energy states to $n = 2$ or 1st excited state.

$$\frac{1}{\lambda} = R\left[\frac{1}{2^2} - \frac{1}{n^2}\right] \quad n = 3, 4, 5, \ldots.$$

This series lies in visible region. It shows only emission spectrum.

$$\lambda_{max} = 656.3 \text{ nm}, \lambda_{min} = 364.6 \text{ nm}$$

Paschen series The transitions occur from higher energy levels D to $n = 3$

$$\frac{1}{\lambda} = R\left[\frac{1}{3^2} - \frac{1}{n^2}\right] \quad n = 4, 5, 6, \ldots.$$

This series lies in IR region with $\lambda_{max} = 1875.1$ nm and $\lambda_{min} = 810.7$ nm. Only emission spectrum is shown.

Brackett series The transitions from higher energy states to $n = 4$ result into Brackett series

$$\frac{1}{\lambda} = R\left[\frac{1}{4^2} - \frac{1}{n^2}\right] \quad n = 5, 6, \ldots.$$

The minimum and maximum wavelengths of the series are $\lambda_{max} = 4047.7$ nm $\lambda_{min} = 1457.2$ nm. It lies in deep IR region and shows only emission spectrum.

Pfund series The transition from higher levels to $n = 5$ result into Pfund series.

$$\frac{1}{\lambda} = R\left[\frac{1}{5^2} - \frac{1}{n^2}\right] \quad n = 6, 7, \ldots.$$

$\lambda_{max} = 7451.5$ nm and $\lambda_{min} = 2276.8$ nm. The series appears in deep IR region and shows only emission spectrum.

The number of spectral lines emitted $N = \frac{n(n-1)}{2}$ if electron lies in nth state

Schrödinger equation

$$\frac{-h^2}{8\pi^2 m}\left[\frac{\partial^2\psi}{\partial x^2}+\frac{\partial^2\psi}{\partial y^2}+\frac{\partial^2\psi}{\partial z^2}\right] - \frac{Ze^2\psi}{4\pi E_o r} = E\,\psi$$

Probability of finding a particle within a given volume dV is $P = |\psi(r, t)|^2 dV$

$= |\psi(x, y, z, t)|^2 dV$

$P(r) = |\psi(r, t)|^2$ is called probability distribution function.

$$P(r) = \frac{4}{r_B^3} r^2 e^{\frac{-2r}{r_B}} \text{ where } r_B \text{ is Bohr radius.}$$

$\nabla^2 \psi + \frac{2m}{\hbar^2}(E - u)\psi = 0$ is time independent Schrödinger equation. Where

$$\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{2}{r}\frac{\partial}{\partial r} + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2}{\partial\phi^2}$$

$i\hbar\frac{\partial\psi}{\partial t} - \frac{\hbar^2}{2m}\nabla^2\psi + u\,\psi$ is time dependent Schrödinger equation.

Quantum numbers Principal quantum number (n) $n = 1, 2, 3....$ describes major shell.

Orbital quantum number (l) $l = 0$ to $n - 1$ and represents number of subshells $l = 0 \rightarrow s\,(sharp)$;

$l = 1 \rightarrow p\,(principal)$; $l = 2 \rightarrow d\,(diffused)$;

$l = 3 \rightarrow f\,(fundamental)$ etc.

Magnetic quantum number (m) Its value varies from $-l$ to $+l$. That is, it gives total number of states $2l + 1$. For example, $l = 1$ means $m = -1, 0, 1$ gives a clue that there will be 3 p-orbitals.

Spin quantum number (s) It has two possible values

$$s = \pm\frac{1}{2} \text{ or } \pm\frac{\hbar}{2}$$

Total angular momentum $\vec{J} = \vec{l} + \vec{s}$ and varies from $-J$ to $+J$ or $j = l \pm s$

$j = |l - s|$ if the subshell is less than half-filled $j = |l + s|$ for remaining cases.

Pauli's exclusion principle No two electrons can occupy all the four quantum number equal. It describes in a subshell electron must be oriented in opposite spins and hence, favours diamagnetism as the law of nature.

Fermions The particles which follow Pauli's exclusion principle or Fermi-Dirac statistics are called Fermions. Electrons, neutrons, protons etc are Ferminos. They have spin $(2n + 1)\frac{\hbar}{2}$ or half odd multiple of $\hbar$.

Bosons The particles which follow Bose-Einstein statistics are called Bosons. Bosons have spin $n\,\hbar$ where n is an integer. Photons, Gravitons, Phonons excitons, cooper pair etc. are Bosons.

• Short Cuts and Points to Note

1. Bohr model could not explain Hydrogen spectrum completely what to talk of other atoms or hydrogen like atoms i.e. He^+, Li^{++} etc. 656.3 nm line in H-spectrum was found to split in 5 lines. No explanation for such a splitting is given in Bohr's model. Bohr arbitrarily assumed that orbits are stationary. He gave no reason as to why the moving charge do not lose energy. It could not even explain Zeeman and Stark effects.

According to Bohr's Theory

2. Radius of nth orbit $r_n = n^2\,(0.53)\,A°$ for Hydrogen.

   $r_n = \frac{n^2}{Z}(0.53)\,A°$ for hydrogen like atoms

3. Velocity of electron in nth orbit $v_n = \frac{2.2\times10^6}{n}\ ms^{-1}$ for hydrogen $v_n = \frac{2.2\times10^6 Z}{n}\ ms^{-1}$ for hydrogen like atoms.

4. Angular frequency $\omega_n = \frac{4.159\times10^6}{n^3}\ rads^{-1}$ for hydrogen.

   $\omega_n = \frac{4.159\times10^6 Z^2}{n^3}\ rads^{-1}$ for hydrogen like atoms.

5. Linear frequency $f = \frac{6.625\times10^5 Z^2}{n^3}\ s^{-1}$

6. Time period of revolution $T_n = \frac{1.5\times10^{-6} n^3}{Z^2}\ s$

7. Electric current due to an electron motion in the nth orbit $I_n = \frac{1.06Z^2}{n^3}\ mA$.

8. Magnetic induction $B_n = \frac{12.58Z^3}{n^5}$ Tesla reduce for electron revolving in nth orbit.

9. Magnetic moment $M_n = \frac{e\hbar}{2m} = \frac{eh}{4\pi m} = 9.26 \times 10^{-24}$ Am^2 for the first orbit of hydrogen and is called Bohr magneton. In nth orbit $M_n = nM = n \times 9.26 \times 10^{-24}$ Am^2.

10. Potential Energy, $PE = -2\,KE$

$$KE = \frac{13.6Z^2}{n^2}\ eV \quad PE = \frac{-27.2Z^2}{n^2}\ eV.$$

Binding energy $BE = KE + PE = -13.6\ \frac{Z^2}{n^2}\ eV.$

11. Ionization potential $= \frac{13.6Z^2}{n^2}$ Volt.

12. Rydberg constant $R = 1.09737 \times 10^7\ m^{-1}$

$Rhc = -13.6\ eV$ in terms of energy

13. Excitation potential $= -13.6\ Z^2 \left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$.

14. Lyman series in the Hydrogen spectrum shows both emission and absorption spectrum. All other series of Hydrogen spectrum show emission spectrum.

15. nth excited state means $(n + 1)$ th energy level or orbit.

16. Maximum number of spectral lines which could be emitted when electron is in the nth state $= \frac{n(n-1)}{2}$.

17. The energy level difference goes on decreasing as n increases.

18. Total number of elements for a given principal quantum number n is

$$= 2\left[n^2 + (n-1)^2 + \ldots\ldots 1^2\right]$$

For example, for $n = 3$.

Total number of elements $= 2\left[3^2 + 2^2 + 1^2\right] = 28$

19. Distance of closest approach in α-particle scattering $r = \frac{2Ze^2}{4\pi\varepsilon_o(KE)}$ and

impact parameter $b = \frac{Ze^2 \cot\left(\frac{\theta}{2}\right)}{4\pi\varepsilon_o(KE)}$.

Number of particles scattered at an angle θ

$$N(\theta) \propto \frac{Z^2}{\sin^4\left(\frac{\theta}{2}\right)(KE)^2}.$$

20. $\lambda\ (nm) = \frac{1240}{E(ev)} = \frac{1240}{E_2 - E_1}$ for absorption or emission of radiation.

21. If in Bohr's theory, instead of m (mass of electron), reduced mass $m_r = \frac{m_p m_e}{m_p + m_e}$ is taken ($m_p \to$ mass of proton, $m_e \to$ mass of electron) then a perfect result is obtained).

22. Probability of finding a particle in a given volume is given by

$$|\psi(r,t)|^2\ dV = \psi(r,t)\ \psi^*(r,t)\ dV$$

while $|\psi(r,t)|^2$ represents probability distribution.

23. Time dependent, Schrödinger equation

$$i\hbar\ \frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\ \nabla^2\ \psi + U\,\psi.$$

Time independent Schrödinger equation

$$\nabla^2\ \psi + \frac{2m}{2\hbar^2}\ (E-u)\ \psi = 0$$

where $\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{2}{r}\frac{\partial}{\partial r} + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2}{\partial\phi^2}$.

24. According to de-Broglie, electron may be considered to have stationary waves and only those orbits are allowed whose circumference $2\pi r = n\lambda$.

25. Angular momentum of an atom $M_L = \hbar\sqrt{L(L+1)}$

$$M_S = \hbar\sqrt{S(S+1)}$$

$$M_J = \hbar\sqrt{J(J+1)}$$

where $J = |L - S|$ if subshell is less than half-filled and $J = |L + S|$ otherwise.

26. Lende g factor

$$g = 1 + \frac{J(J+1) + S(S+1) - L(L+1)}{2J(J+1)}.$$

Magnetic moment of an atom $M = g\sqrt{J(J+1)}\ M_B$

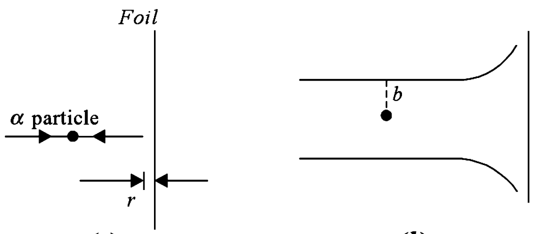
where M_B is Bohr magneton.

27. Zeeman splitting in a weak magnetic field $\Delta\omega = \dfrac{B(m_1 g_1 - m_2 g_2)M_B}{\hbar}$.

28. Uncertainty principle $\Delta x \,.\, \Delta p \geq \hbar$.

• Caution

1. Assuming there is no difference between closest distance of approach and impact parameter.



(a) *(b)*

Fig. 33.4 Illustration of distance of closest approach and impact parameter

⇒ Closest distance of approach is the minimum distance between α-particle and foil. The one which are reflected back by 180° reach closest as illustrated in Fig. 33. 4 (a).

$$r = \frac{2Ze^2}{4\pi\varepsilon_o(KE)}$$

Impact parameter is the radius of the incident cylindrical beam of α-particles as illustrated in Fig. 34.4 (b) α-particles scattered at different angle lie on different radius of the cylindrical beam.

2. Considering that Bohr's model can be applied to any atom.

⇒ It can be applied only to hydrogen like atoms/ions, even it does not satisfy hydrogen spectrum completely. Models based on 4 quantum numbers *n*, *l*, *m* and *s* give correct picture.

3. Assuming $|\psi(r,t)|^2$ is the probability of finding a particle.

⇒ $|\psi(r,t)|^2$ is the probability distribution function and $|\psi(r,t)|^2\ dV$ is the probability of finding the particle in a volume dV around the point (x, y, z) at time t.

4. Confusion between binding energy, ionization energy and excitation energy.

⇒ Binding energy is sum of *KE* + *PE* of the orbital electron. Ionization energy is the minimum energy required to remove an electron from its ground state to vacuum level (or ∞). Assuming electron is present in *n*th state then,

$E_{\text{ionization}} = E_\infty - E_n$.

Excitation energy is the amount of energy required to transit electron from lower level n_1 to higher level n_2, i.e., $E_{\text{excitation}} = E_{n2} - E_{n1}$

5. Not recognizing H_α, H_β lines of a series.

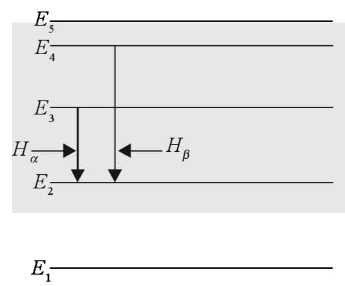


Fig. 33.5

⇒ Fig. 33.5 illustrates H_α and H_β lines of a given series (Balmar here). The transition $E_3 \rightarrow E_2$ is H_α line and transition $E_4 \rightarrow E_2$ gives H_β line of Balmar series.

6. Not remembering which series in hydrogen spectrum has its absorption spectrum.

⇒ Only Lyman series has both its emission and absorption spectrum. All other series have only emission spectrum.

7. Not understanding clearly the meaning of excited state.

⇒ *n*th excited state means (*n* + 1) th level.

8. Assuming only electrons are Fermions.

⇒ All particles which follow Pauli's exclusion principle or having spin half odd multiple of $\hbar$ are Fermious. Thus, even protons and neutrons are also fermious

9. Considering ionization energy/ionization potential and excitation energy/excitation potential are synonyms.

⇒ The minimum energy to ionize the atom is called ionization energy. The potential difference through which an electron be accelerated to acquire this energy is called ionization potential. Ionization energy of *H* is 13.6 eV and ionization potential is 13.6 V.

Energy required to take an electron from ground state to an excited state is called excitation energy. The potential through which an electron be accelerated to acquire this energy is called excitation potential. For example, excitation energy for transition $E_1 - E_2$ for hydrogen is 10.2 eV and excitation potential is 10.2 V.

10. Considering that spectral lines are just single lines.

⇒ They consist of fine lines. 656.3 nm line splits to 5 lines when seen under high resolution microscope. Zeeman and stark effects also show that the lines split when magnetic or electric field is applied.

11. Not remembering the relation between eV and Joule (J).

⇒ $1 \text{ eV} = 1.6 \times 10^{-19}$ J

12. Considering the spectral lines emitted such as in Hydrogen spectrum are due to a single atom.

⇒ Electric discharge is provided to the H gas in a discharge tube. Spectrum is the result of different atoms going through different transitions.

SOLVED PROBLEMS

1. The total energy of an electron in the first excited state of hydrogen is about –3.4 eV. Its KE in this state is

(a) 3.4 eV (b) 6.8 eV

(c) –3.4 eV (d) –6.8 eV

(CBSE PMT 2005)

Solution (a) $KE = -BE$.

2. Energy levels A, B and C of a certain atom correspond to increasing values of energy, i.e., $E_A < E_B < E_C$. If λ_1, λ_2 and λ_3 are wavelengths corresponding to transitions C to B, B to A and C to A respectively then

(a) $\lambda_3 = \lambda_1 + \lambda_2$ (b) $\lambda_3 = \dfrac{\lambda_1\lambda_2}{\lambda_1+\lambda_2}$

(c) $\lambda_1 + \lambda_2 + \lambda_3 = 0$ (d) $\lambda_3^2 = \lambda_1^2 + \lambda_2^2$

(CBSE PMT 2005)

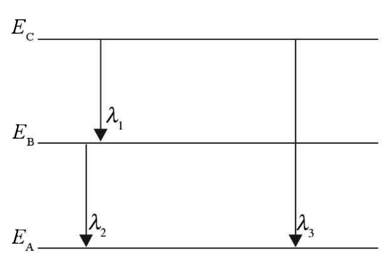


Fig. 33.6

Solution (b) $E_C - E_A = E_C - E_B + E_B - E_A$ (From fig).

$$\frac{hc}{\lambda_3} = \frac{hc}{\lambda_1} + \frac{hc}{\lambda_2} \text{ or } \frac{1}{\lambda_3} = \frac{1}{\lambda_1} + \frac{1}{\lambda_2}$$

3. The ground state energy of Hydrogen atom is –13.6 eV. What is the potential energy of the electron in this state.

(a) 0 eV (b) –27.2 eV

(c) 1 eV (d) 2 eV

[AIIMS 2005]

Solution (b) $PE = 2\,BE$

4. Radius of first Bohr orbit is r. The radius of 2nd Bohr orbit is

(a) $8r$ (b) $2r$

(c) $4r$ (d) $2\sqrt{2}r$

(BHU 2005)

Solution $r_n = n^2 r$ $\therefore r_2 = 4r$

5. The electron in a hydrogen atom makes a transition from n_1 to n_2 state. The time period of electron in n_1 is 8 times in n_2. The possible values of n_1 and n_2 are

(a) $n_1 = 8, n_2 = 1$ (b) $n_1 = 4, n_2 = 2$

(c) $n_1 = 2, n_2 = 4$ (d) $n_1 = 1, n_2 = 8$

(CET Karnataka 2005)

Solution (b) $T_n = \dfrac{1.5\times10^{-6}n^3}{Z^2}$ $\therefore n_1 = 4$ and $n_2 = 2$

6. Bohr's atom model assumes

(a) the nucleus is of infinite mass and is at rest.

(b) electron in a quantized orbit will not radiate energy.

(c) mass of the electron remains constant.

(d) all of these.

(CET Karnataka 2005)

Solution (d)

7. Identify the wrong statement in the following Coulomb's law correctly describes the electric force that

(a) binds the electrons of an atom to its nucleus.

(b) binds protons and neutrons in the nucleus of an atom.

(c) binds atoms together to form molecules.

(d) binds atoms and molecules to form solids.

(CET Karnataka 2005)

Solution (b) Nucleur force binds protons and neutrons.

8. The energy that should be added to an electron to reduce its de Broglie wavelength from 1 nm to 0.5 nm is

(a) four times the initial energy.

(b) equal to initial energy.

(c) twice the initial energy.

(d) thrice the initial energy.

[CET Karnataka 2005]

Solution (d) $\lambda = \frac{h}{p} = \frac{h}{\sqrt{2m(KE)}}$ $\therefore$ new energy should be 4 times and hence energy to be added is thrice the initial energy.

9. The diagram shows the energy levels for an electron in a certain atom. Which transition shown represents the emission of a photon with the most energy?

(a) *I* (b) *II*

(c) *III* (d) *IV*

[AIEEE 2005]

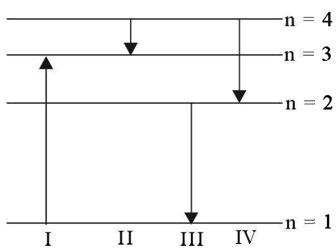


Fig. 33.7

Solution (c) *III* (*I* shows absorption).

10. Whenever a photon is emitted by hydrogen in Balmar series, it is followed by another photon in Lyman series. The wavelength of latter photon will be

(a) 102 nm (b) 112 nm

(c) 122 nm (d) none of these

Solution (c) $\lambda = \frac{1240}{E(eV)}$ (nm) $= \frac{1240}{10.2} = 122$ nm

11. A positive ion having only one electron ejects it if a photon of $\lambda \le 228\,A°$ is abosrbed by it. The ion is

(a) He^+ (b) Li^{++}

(c) Be^{+3} (d) B^{+4}

Solution (a) $E\,(eV) = \frac{1240}{22.8} = 54.38$

using $E = \frac{13.6Z^2}{1^2} = \Rightarrow Z = 2.$

12. Find the ratio of magnetic dipole moment to angular momentum in a hydrogen like atom.

(a) $\frac{e}{m}$ (b) $\frac{e}{2m}$

(c) $\frac{e}{3m}$ (d) $\frac{2e}{m}$

(e) $\frac{3e}{m}$

Solution (b) $\frac{M}{L} = \frac{iA}{mvr} = \frac{\frac{ev}{2\pi r} \times \pi r^2}{mvr} = \frac{e}{2m}$.

13. The average *KE* of molecules in a gas at temperature *T* is $\frac{3}{2}$ *kT*. Find the temperature at which the average KE of molecules equal to binding energy of its atoms.

(a) $1.05 \times 10^5\,K$ (b) $1.05 \times 10^4\,K$

(c) $1.05 \times 10^3\,K$ (d) none of these

Solution (a) $\frac{3}{2}\,kT = 13.6$ eV or $\frac{3}{2} \times \frac{1.38 \times 10^{-23}}{1.6 \times 10^{-19}} \times T$

$= 13.6$ or $T = 1.05 \times 10^5\,K$

14. Hydrogen atom in ground state takes up a photon of $\lambda = 50$ nm (*u v* light). Find the *KE* with which it is emitted.

(a) 14.4 eV (b) 12.2 eV

(c) 13.6 eV (d) 11.2 eV

Solution (d) $KE = \frac{1240}{\lambda} - BE \Rightarrow \left(\frac{1240}{50} - 13.6\right)$ eV

$= 11.2$ eV

15. A filter transmits only the radiation of wavelength ≥ 440 nm. Radiation from a hydrogen discharge tube goes through such a filter and is incident on a metal of work function 2.0 eV. Find the stopping potential which can stop

(a) 1.4 V (b) 1.3 V

(b) 0.85 V (d) 0.55 V

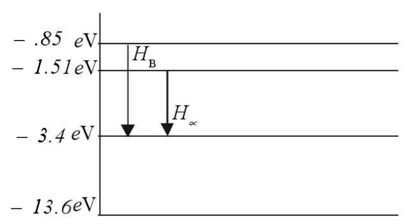


Fig. 33.8

Solution (d) H_β line has wavelength

$$\lambda = \frac{1240}{2.55} = 486.1 \text{ nm}$$

Apply $eV_s = hf - \phi$

$= 2.55 - 2.0 = 0.55$ eV

$V_s = 0.55$ V

16. Find the maximum work function a metal can have so that light from Balmer series can cause emission.

(a) 3.4 eV (b) 2.55 eV
(c) 1.89 eV (d) none of these

Solution (a) The maximum energy of photon emitted in Balmar series is 3.4 eV.

17. The electron is present in the –1.51 eV energy state. Find the angular momentum of the electron.

(a) $2\hbar$ (b) $\hbar$
(c) $3\hbar$ (d) $4\hbar$
(e) none of these

Solution Angular momentum $L = n\hbar \Rightarrow 3\hbar$

$\because$ –1.51 eV corresponds to $n = 3$

18. Which of the following products in a hydrogen atom is independent of principal quantum number n,

(a) vr (b) vn
(c) Er^2 (d) En

Solution (b)

19. The radius of shortest orbit in one electron system is 18 pm. It may be

(a) ${}^1_1H$ (b) ${}^2_1H$
(c) He^+ (d) Li^{++}

Solution (d) $r_n = \dfrac{n^2 r_B}{Z} \Rightarrow \dfrac{0.53A°}{Z} = 0.18A° \Rightarrow Z = 3.$

20. An atom initially at an energy level E = –6.52 eV. It absorbs a photon of wavelength 860 nm. What is the internal energy of atom after absorbing photon.

(a) 5.08 eV (b) 1.44 eV
(c) –1.44 eV (d) –5.08 eV

Solution (d) $E\,(ev) = \dfrac{1240}{860} = 1.44$ eV

$$E_{net} = -6.52 + 1.44 = -5.08 \text{ eV}$$

21. What is minimum frequency of the photon required to carry out transition $n = 2$ to $n = 3$

(a) 1.21×10^{15} Hz (b) 1.61×10^{15} Hz
(c) 1.21×10^{14} Hz (d) 1.61×10^{14} Hz

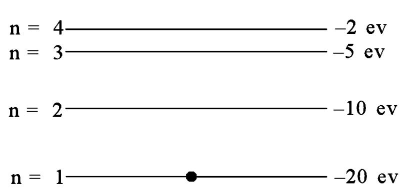


Fig. 33.9

Solution (a) $\lambda = \dfrac{1240}{5} = 248$ nm;

$$f = \frac{3 \times 10^8}{248 \times 10^{-9}}$$

$$= 1.21 \times 10^{15} \text{ Hz}$$

22. If A_n is the area enclosed in the nth orbit in a hydrogen atom then the graph log $\left(\dfrac{A_n}{A_1}\right)$ against log n

(a) will have slope 2 (straightline).
(b) will have slope 4 (straightline).
(c) will be a monotonically increasing nonlinear curve.
(d) will be a circle.

Solution (b) $r_n = n^2 r_1$

$\therefore$ log $\left(\dfrac{\pi r_n^2}{\pi r_1^2}\right)$ against log (n) has slope = 4.

23. The hydrogen atom emits a photon of 656.3 nm line. Find the momentum of the photon associated with it.

(a) 10^{-27} kg ms^{-1} (b) 10^{-23} kg ms^{-1}
(c) 10^{-25} kg ms^{-1} (d) none of these

Solution (a) $p = \dfrac{h}{\lambda} = \dfrac{6.625 \times 10^{-34}}{656.3 \times 10^{-9}}$

$$= 1.01 \times 10^{-27} \text{ kg ms}^{-1}.$$

24. A hydrogen atom emits uv radiation of wavelength 102.5 nm. Find the quantum numbers of the states involved.

(a) 3, 1 (b) 2, 1
(c) 4, 1 (d) 4, 2

Solution (a) $E\,(ev) = \dfrac{1240}{102.5} = 12.1;$

$$E_n = -13.6 + 12.1$$

$= -1.5$ eV i.e., transition is from $n = 3$ to $n = 1$

25. An electron with KE 6 eV is incident on a hydrogen atom in its ground state. The collision

(a) must be elastic.
(b) may be partially elastic.
(c) must be completely inelastic.
(d) may be partially inelastic.

Solution (a) Unless energy $\geq$ 10.2 eV electron from ground state of Hydrogen cannot be excited.

TYPICAL PROBLEMS

26. 1 *MeV* α-particle is scattered 60° by gold foil. Find aiming parameter.

Solution Aiming parameter = impact parameter

$$= \frac{Ze^2 \cot\frac{\theta}{2}}{4\pi\varepsilon_0 (KE)}$$

$$= \frac{79\times1.6\times10^{-19}\times1.6\times10^{-19}\times\sqrt{3}\times9\times10^9}{1.6\times10^{-13}}$$

$= 2.07 \times 10^{-13}$ m.

27. A particle of mass m moves around in a circular orbit in a centro symmetric potential field $u(r) = \frac{Kr^2}{2}$. Using Bohr's quantization rule, find the permissible energy levels.

Solution $F = \frac{d}{dr}u(r) = Kr = \frac{mv^2}{r}$ or $mv = \sqrt{Kr^2m}$ and $mvr = n\hbar$.

$\therefore r\sqrt{Kr^2m} = n\hbar$ or $r = \left(\frac{n\hbar}{\sqrt{Km}}\right)^{1/2}$ and

$$mv^2 = Kr^2 = K\left(\frac{n\hbar}{\sqrt{Km}}\right).$$

$$BE = KE + PE$$

$$= \frac{1}{2}K\left(\frac{n\hbar}{\sqrt{Km}}\right) + \frac{K}{2}\left(\frac{n\hbar}{\sqrt{Km}}\right) = n\hbar\sqrt{\frac{K}{m}}$$

28. A uniform magnetic field B exists in a region. An electron projected perpendicular to the field goes in a cricle. Assuming Bohr's quantization rule, calculate the radius of nth orbit and the minimum possible speed of electron.

Solution $r = \frac{mv}{eB}$ From Bohr's theory

$$mvr = n\hbar \text{ or } mv = \frac{n\hbar}{r}.$$

Thus $r = \frac{n\hbar}{reB}$ or $r = \sqrt{\frac{n\hbar}{eB}}$ and $v = \frac{n\hbar}{mr} = \frac{n\hbar}{m\sqrt{\frac{n\hbar}{eB}}}$

or $$v = \sqrt{\frac{n\hbar eB}{m^2}}$$

For minimum speed $n = 1$. Thus $v_{\min} = \sqrt{\frac{\hbar eB}{m^2}}$

29. Assume an imaginary world. Where angular momentum is quantized to even multiple $\hbar$. Find the longest possible wavelength emitted by Hydrogen in the visible spectrum.

Solution $mvr = 2n\hbar$

or $mv = \frac{2n\hbar}{r}$

$$mv^2 = \frac{m^2v^2}{n} = \frac{(2n\hbar)^2}{mr^2}$$

$$\frac{Ze^2}{4\pi\varepsilon_0 r^2} = \frac{mv^2}{r}$$

or $$\frac{Ze^2}{4\pi\varepsilon_o r^2} = \frac{(2n\hbar)^2}{mr^2(r)}$$

or $$r = \frac{(2n\hbar)^2 4\pi\varepsilon_o}{mZe^2}$$

$$BE = KE + PE = \frac{-Ze^2}{8\pi\varepsilon_o r} = \frac{-Z^2e^4m}{8\pi\varepsilon_o (2n\hbar)^2 4\pi\varepsilon_o}$$

$$= \frac{-Z^2e^4m}{32\varepsilon_o n^2h^2}$$

$BE = \frac{-3.4}{n^2}$ eV for Hydrogen. To find longest wavelength $h\upsilon = 3.4\left[1 - \frac{1}{4}\right]$

$$= 3.4 \times \frac{3}{4} = 2.55$$

$$\lambda(nm) = \frac{1240}{2.55} = 484 \text{ nm}$$

30. Consider a positronium atom consisting of a positron and an electron each having mass m (equal to mass of electron) in the orbit around its centre of mass. This structure lasts for 10^{-6} *s* only. Find the energy levels for this model.

Solution In Bohr, mass of proton was assumed infinite and the relation for binding energy is $E_n = \frac{me^4}{8\varepsilon_0^2 n^2 h^2}$

In the new system (positrinium atom) masses are equal. Therefore, we use reduced mass

$$m_{\text{reduced}} = \frac{m_1 m_2}{m_1 + m_2} = \frac{m}{2}.$$

Hence, $E_n = \frac{me^4}{\varepsilon^2{}_0 16 n^2 h^2}$.

QUESTIONS FOR PRACTICE

1. An electron with kinetic energy 5 eV is incident on a hydrogen atom in its ground state. The collision
(a) must be elastic.
(b) may be partially elastic.
(c) must be completely inelastic.
(d) may be completely inelastic.

2. Ionization energy of a hydrogen-like ion A is greater than that of another hydrogen-like ion B. Let r, u, E and L represent the radius of the orbit, speed of the electron, energy of the atom and orbital angular momentum of the electron respectively. In ground state
(a) $r_A > r_B$ (b) $u_A > u_B$
(c) $E_A > E_B$ (d) $L_A > L_B$.

3. When a hydrogen atom emits a photon of energy 12.1 eV, its orbital angular momentum changes by
(a) 1.05×10^{-34} Js (b) 2.11×10^{-34} Js
(c) 3.16×10^{-34} Js (d) 4.22×10^{-34} Js

4. Let f_1 be the frequency of the series limit of the Lyman series, f_2 be the frequency of the first line of the Lyman series, and f_3 be the frequency of the series limit of the Balmer series.
(a) $f_1 - f_2 = f_3$ (b) $f_2 - f_1 = f_3$
(c) $f_3 = \frac{1}{2}(f_1 + f_2)$ (d) $f_1 + f_2 = f_3$

5. An electron with kinetic energy = E eV collides with a hydrogen atom in the ground state. The collision will be elastic
(a) for all values of E. (b) for $E < 10.2$ eV.
(c) for $E < 13.6$ eV. (d) only for $E < 3.4$ eV.

6. When a hydrogen atom emits a photon in going from $m = 5$ to $n = 1$, its recoil speed is almost
(a) 10^{-4} m/s. (b) 2×10^{-2} m/s.
(c) 4 m/s. (d) 8×10^{2} m/s.

7. Let the potential emergy of a hydrogen atom in the ground state be zero. Then its energy in the first excited state will be
(a) 10.2 eV (b) 13.6 eV
(c) 23.8 eV (d) 27.2 eV

8. An electron is in an excited state in a hydrogen like atom. It has a total energy of –3.4 eV. The kinetic energy of the electron is E and its de Broglie wavelength is λ.
(a) $E = 6.8$ eV. $\lambda - 6.6 \times 10^{-10}$ m.
(b) $E = 3.4$ eV, $\lambda - 6.6 \times 10^{-10}$ m.
(c) $E = 3.4$ eV, $\lambda - 6.6 \times 10^{-11}$ m.
(d) $E = 6.8$ eV, $\lambda - 6.6 \times 10^{-11}$ m.

9. Which of the following subshells is represented by the quantum numbers $n = 4$ and $l = 1$?
(a) 4*s* (b) 4*f*
(c) 4*d* (d) 4*p*

10. If the transition of electron takes place from $n = 4$ state, then the maximum number of spectral lines obtained for transition to ground state will be
(a) 6 (b) 12
(c) 18 (d) 24

11. According to Bohr model, the diameter of first orbit of hydrogen atom will be
(a) 1 A° (b) 0.529 A°
(c) 2.25 A° (d) 0.725 A°

12. The ionisation potential of hydrogen atom is
(a) 12.97V (b) 10.2V
(c) 13.6V (d) 27.2V

13. α-particles are
(a) helium nucleii. (b) sodium nucleii.
(c) ionised nucleii. (d) hydrogen nucleii.

14. According to classical theory, the path of electron in Rutherford atom model is
(a) straight line (b) spiral
(c) circular (d) parabolic

15. A hydrogen atom is in the *d*-state. The values of *m* for this state are
(a) –1, 0, 1 (b) –3, –1, 0, 1, 3
(c) 2, 1, 0 (d) –2, –1, 0, 1, 2

16. The atomic number of silicon is 14. Its electronic configuration in the ground state will be
(a) $1s^2, 2s^2, 2p^6, 3s^2, 3p^2$ (b) $1s^2, 2s^2, 2p^6, 3s^1, 3p^3$
(c) $1s^2, 2s^2, 2p^8, 3s^2$ (d) $1s^2, 2s^2, 2p^6, 3s^4$

17. Which of the following radiations are not emitted by electron transitions in atoms ?
(a) Ultraviolet. (b) Infrared radiations.
(c) Visible rays. (d) α-rays.

18. The ionisation energy for excited hydrogen atom in eV will be
(a) 13.6. (b) less than 13.6.
(c) greater than 13.6. (d) 3.4 or less.

19. The angular momentum of electron is J. Its magnetic moment will be

(a) $\frac{mJ}{2e}$ (b) $\frac{eJ}{2m}$

(c) $\frac{2m}{eJ}$ (d) $\frac{emJ}{2}$

20. The energy emitted by a source is in the form of

(a) photons. (b) electrons.

(c) protons. (d) neutrons.

21. In basic metals the valence electron is

(a) d-electron. (b) s-electron.

(c) f-electron. (d) p-electron.

22. The ratio of the kinetic energy and the potential energy of electron in hydrogen atom will be

(a) 1 : 2 (b) –1 : 2

(c) 2 : 1 (d) –2 : 1

23. The energy necessary to remove the electron from n = 10 state in hydrogen atom will be

(a) 1.36 eV (b) 0.0136 eV

(c) 13.6 eV (d) 0.136 eV

24. The clarification of discrete energy levels in atom was first given experimentally by

(a) Thomson's experiment.

(b) Millikan's oil drop experiment.

(c) Frank-Hertz experiment.

(d) Rutherford scattering experiment.

25. Which of the following atom pair have the same structure?

(a) He, Ne+ (b) Li+, Na+

(c) N, C (d) B, Li

26. The unit of Planck's constant is equivalent to that of

(a) energy. (b) angular momentum.

(c) velocity. (d) force.

27. Hydrogen atom will be in its ground state, if its electron is in

(a) any energy level.

(b) the lowest energy state.

(c) the highest energy state.

(d) the intermediate state.

28. An atomic nucleus contains

(a) only electrons. (b) only protons.

(c) only neutrons.

(d) both protons and neutrons.

29. Isotopes are the atoms of the same element which contain equal number of

(a) nucleons. (b) neutrons.

(c) protons. (d) neutrons and protons.

30. An electron strikes with a H-atom in the ground state. The collision will be elastic for kinetic energy of electron

(a) < 10.2 eV (b) < 3.4 eV only

(c) < 13.6 eV (d) any value of KE

31. If the potential energy of a H-atom in the ground state be zero then its energy in the first excited state will be

(a) 10.2 eV (b) 13.6 eV

(c) 23.8 eV (d) 27.2 eV

32. On decreasing principal quantum number n, the values of r and v will

(a) decrease. (b) increase.

(c) r will increase but v will decrease.

(d) r will decrease but v will increase.

33. The value of principal quantum number for an ionised atom is

(a) 1 (b) 0

(c) 8 (d) 4

34. The values of n_1 and n_2 for P fund's series are

(a) $n_1 = 5, n_2 = 6, 7$ (b) $n_1 = 4, n_2 = 5, 6, 7....$

(c) $n_1 = 3, n_2 = 4, 5, 6....$ (d) $n_1 = 2, n_2 = 3, 4, 5....$

35. The time taken by a particle moving with velocity $c/10$ in crossing a nucleus will approximately be

(a) 10^{-12} second. (b) 10^{-8} second.

(c) 10^{-17} second. (d) 10^{-21} second.

36. When an electron jumps from n_1th orbit to n_2th orbit then the formula for energy radiated out is

(a) $E_1 - E_2 = hv$ (b) $E_2 - E_1 = hv$

(c) $hv = \frac{E_1 + E_2}{2}$ (d) $hv = \frac{E_1}{E_2}$

37. The energy of electron in an excited hydrogen atom is –3.4eV. Its angular momentum according to Bohr's theory will be

(a) $\frac{h}{\pi}$ (b) $\frac{h}{2\pi}$

(c) $\frac{3h}{2\pi}$ (d) $\frac{3}{2\pi h}$

38. The ionisation energy of second electron in helium atom is

(a) 100 eV (b) 27.2 eV

(c) 13.6 eV (d) 54.4 eV

39. In scattering experiment, by which force the α-particles get scattered ?

(a) Nuclear force. (b) Coulomb force.

(c) Both (1) and (2). (d) Gravitational force.

40. The possible values of principal quantum number can

(a) 1, 2, 3... 8. (b) 0, 1, 2... 8.

(c) only zero. (d) none of these.

41. The possible values of orbital quantum number are

(a) from 0 to $(n-1)$. (b) from 0 to n.

(c) from 0 to $(n+1)$. (d) all of the above.

42. The maximum possible values of magnetic orbital quantumumber (m_1) are

(a) $(2l+1)$. (b) $2l$.

(c) $(2l-1)$. (d) zero.

43. The possible values of m_1 are

(a) from -1 to $+1$. (b) from 0 to 8.

(c) from zero to $+1$. (d) none of these.

44. The maximum number of electrons in a shell is

(a) n^2 (b) $2n^2$

(c) $2n$ (d) n

45. The maximum number of electrons in sub-shell is

(a) $2(2l+1)$ (b) $2l+1$

(c) $2l$ (d) $2l-1$

46. The spin of electron, proton and neutron is

(a) $\hbar$ (b) $\frac{\hbar}{2}$

(c) zero (d) none of these

47. The spin neutrino is

(a) $\hbar$ (b) $\frac{\hbar}{2}$

(c) $\frac{\hbar}{3}$ (d) zero

48. According to Sommerfeld, an electron revolves round a nucleus in

(a) circular orbits. (b) elliptical orbits.

(c) hyperbolic orbits. (d) none of these.

49. The number of neutrons in sodium atom is

(a) 10 (b) 11

(c) 13 (d) 12

50. Which quantity remains same in isotones ?

(a) No. of protons. (b) No. of neutrons.

(c) Mass number. (d) All of the above.

51. Which quantity remains same in isobars?

(a) Number of neutrons. (b) Number of protons.

(c) Mass number. (d) All of the above.

52. The main defect of Bohr atom model is

(a) mixing of classical and quantum theories.

(b) exclusion of nuclear motion.

(c) failed to explain the fine structure of spectral lines.

(d) none of the above.

53. The maximum wavelength of Lyman series is—

(a) $\frac{4}{3R}$ (b) $\frac{1}{R^2}$

(c) $\frac{C}{R}$ (d) $\frac{1}{RC}$

54. The electron configuration of $_{11}Na^{23}$ is

(a) $1s^2, 2s^2, 2p^5, 3s^2$ (b) $1s^2, 2s^2, 2p^6, 3s^1$

(c) $1s^2, 2s^2, 2p^4, 3s^1, 3p^2$ (d) $1s^1, 2s^1, 2P^6, 3s^2, 3p^1$

55. The mass number of an atom is 24 and its atomic number is 11, then which of the following statements is correct ?

(a) Its nucleus contains 13 neutrons.

(b) Its nucleus contains 0 neutrons.

(c) Its nucleus contains 11 neutrons.

(d) Nothing can be said.

56. To get complete information about an electron in an atom the quantum numbers required are

(a) n, l, m_l (b) n, l

(c) n, l, m_l, s (d) n, l, s

57. The three unpaired electrons in last subshell of nitrogen are related to the following law

(a) Hunds law. (b) Auf-Bau law.

(c) $(n+l)$ law. (d) Pauli's exclusion law.

58. The isonutronic traid of nucleii is

(a) $_6C^{14}, _7N^{14}, _9F^{17}$ (b) $_6C^{14}, _7N^{14}, _9F^{19}$

(c) $_6C^{14}, _7N^{15}, _9F^{17}$ (d) $_6C^{12}, _7N^{14}, _9F^{19}$

59. For an atom $n = 4$. In this atom the electrons will be

(a) only s, p and d. (b) only s and d.

(c) only s and p. (d) only p and d.

60. According to classical theory the Rutherford atom is

(a) unstable. (b) stable.

(c) positive. (d) partially stable.

61. An α-particle passes through a potential difference of 2×10^6 volt and then it becomes incident on a silver foil. The charge number of silver is 47. The energy of incident particles will be (in Joules)

(a) 5×10^{-12} (b) 6.4×10^{-13}

(c) 5.8×10^{-14} (d) 9.1×10^{-15}

62. In the above problem, the kinetic energy of α-particles at a distance 5×10^{-14} m from the nucleus will be (in Joules)

(a) 6.4×10^{-13} (b) 4.3×10^{-13}

(c) 2.1×10^{-13} (d) 3.4×10^{-14}

63. In Q. No. 61 the distance of closest approach of the particle to the nucleus will be

(a) 6.4×10^{-13} m (b) 4.3×10^{-13} m

(c) 2.1×10^{-13} m (d) 3.4×10^{-14} m

64. The energy required to transfer an electron from second Bohr orbit to third Bohr orbit in a hydrogen like atom with nuclear charge Ze is 68.0 eV. The value of Z will be

(a) 3 (b) 4

(c) 5 (d) 6

65. In the above problem, the kinetic energy of electron in the first Bohr orbit will be

(a) 489.6 eV (b) – 489.6 eV

(c) 0.38 eV (d) – 0.38 eV

66. In Q. No. 64. the wavelength of radiation required to shift the electron from first orbit to infinite orbit will be

(a) 2.527 A° (b) 25.27 A°

(c) 252.7A° (d) 2527 A°

67. According to Bohr hypothesis, which of the following quantities is discrete ?

(a) Momentum. (b) Angular velocity.

(c) Potential energy. (d) Angular momentum.

68. Hydrogen atom does not emit X-rays, because

(a) its size is very small.

(b) it contains only single electron.

(c) energy levels in it are far apart.

(d) energy levels in it arc very close to each other.

69. The fine structure of hydrogen spectrum can be explained by

(a) the presence of neutrons in the nucleus.

(b) the finite size of nucleus.

(c) the orbital angular momentum of electrons.

(d) the spin angular momentum of electrons.

70. The limit of Balmer series is 3646 Å. The wavelength of first member of this series will be

(a) 6563 A°. (b) 3646 A°.

(c) 7200 A°. (d) 1000 A°.

71. The ratio of minimum wavelengths of Lyman and Balmer series will be

(a) 1.25 (b) 0.25

(c) 5 (d) 10

72. An atom absorbs 2eV energy and is excited to next energy state. The wavelength of light absorbed will be

(a) 2000 A°. (b) 4000 A°.

(c) 8000 A°. (d) 6206 A°.

73. The visible region of hydrogen spectrum was first studied by

(a) Lyman. (b) Balmer.

(c) Pfund. (d) Brackett.

74. If E_m and J_m are the magnitude of total energy and angular momentum of electron in the nth Bohr orbit respectively, then—

(a) $E \propto J_n^2$ (b) $E_n \propto \frac{1}{J_n^2}$

(c) $E \propto J_n$ (d) $E_n \propto \frac{1}{J_n}$

75. The energy required to excite an electron from $n = 2$ to $n = 3$ energy state is 47.2 eV. The charge number of the nucleus, around which the electron is revolving, will be

(a) 5 (b) 10

(c) 15 (d) 20

76. In the Bohr model of hydrogen atom, the ratio of the kinetic energy and total energy of electron in the nth quantum/state will be

(a) 1 (b) –1

(c) 2 (d) –12

77. An electron revolving in an orbit of radius 0.5 A in a hydrogen atom executes 10 revolutions per second. The magnetic moment of electron due to its orbital motion will be

(a) 1256×10^{-26} amp/m². (b) 653×10^{-26} amp/m².

(c) zero. (d) 256×10^{-26} amp/m².

78. How many revolutions does an electron complete in one second in the first orbit of hydrogen atom ?

(a) 6.57×10^{15} (b) 6.57×10^{13}

(c) 1000 (d) 6.57×10^{14}

79. With the increase in quantum number the energy difference between consecutive energy levels

(a) remains constant. (b) decreases.

(c) increases.

(d) sometimes increases sometimes decreases.

80. An electron makes transition from $n = 4$ to $n = 1$ state in a hydrogen atom. The maximum possible number of photons emitted will be

(a) 1 (b) 2

(c) 3 (d) 6

81. The energy of electron in the ground state of hydrogen atom is –13.6eV. The energy required for the transition from $n = 2$ to $n = 3$ will be

(a) 2eV (b) 4eV

(c) 1.89V (d) 2.89eV

82. If the radius of first Bohr orbit is r, then the radius of second orbit will be

(a) $2r$ (b) $\frac{r}{2}$

(c) $4r$ (d) $\sqrt{2}r$

83. A hydrogen atom is excited from $n = 1$ to $n = 3$ state. The amount of energy absorbed by the atom will be

(a) 12.1eV (b) 25eV

(c) 13.6eV (d) –13.6eV

84. The ratio of energies of first two excited states of hydrogen atom is

(a) 4 (b) $\frac{1}{4}$

(c) $\frac{4}{9}$ (d) $\frac{9}{4}$

85. The ratio of speed of electron in ground state of hydrogen atom to that of light is

(a) $\frac{1}{137}$ (b) $\frac{1}{207}$

(c) $\frac{2}{237}$ (d) $\frac{1}{237}$

86. The energy required to remove an electron from $n = 2$ state in hydrogen will be

(a) 13.6 eV (b) 3.4 eV

(c) 27.2 eV (d) 6.8 eV

87. The value of wavelength of radiation emitted due to transition of electrons from n = 4 to n = 2 state in hydrogen atom will be

(a) $\frac{5R}{36}$ (b) $\frac{16}{3R}$

(c) $\frac{36}{5R}$ (d) $\frac{3R}{16}$

88. The radius of first Bohr orbit in hydrogen atom is r_0, then the radius of first orbit in helium atom will be

(a) $2r_0$ (b) $4r_0$

(c) $\frac{r_0}{2}$ (d) r_0

89. There are seven orbitals in a subshell then the value of l for it will be

(a) $l = 4$ (b) $l = 3$

(c) $l = 2$ (d) $l = 1$

90. The excitation energy in the third orbit of hydrogen atom will be

(a) 1.5eV (b) 3.4eV

(c) 0.66eV (d) 0.85eV

91. The quantum numbers which are independent are

(a) n, l, m_s (b) n, l, m_l, m_s

(c) n, m_l (d) n, m_s

92. There are 42 protons and 53 neutrons in the atom of an element. This element is represented as

(a) ${}_{42}X^{95}$. (b) ${}_{42}X^{53}$.

(c) ${}_{53}X^{42}$. (d) ${}_{95}X^{42}$.

93. Which of the following pair is correct ?

(a) Rutherford-X-rays. (b) Roentgen-electron.

(c) Chadwick-neutron. (d) J-J-Thomson-photon.

94. A hydrogen atom in the ground state is excited by radiations of wavelength 975 Å. The energy state to which the atom is excited, is

(a) 1. (b) 2.

(c) 3. (d) 4.

95. In the above problem how many lines will be possible in emission spectrum?

(a) 2 (b) 4

(c) 6 (d) 8

96. In Q. 94 the longest wavelength will be

(a) 18835 A°. (b) 15622 A°.

(c) 19768 A°. (d) 11973 A°.

97. In Q. 94 the shortest wavelength will be

(a) 5887 A°. (b) 6568 A°.

(c) 3435 A°. (d) 973 A°.

98. In Q. 94 how many lines will be possible in absorption spectrum ?

(a) 2 (b) 3

(c) 4 (d) 5

99. The potential energy between electron and proton is given by $U = Ke^2/3r^3$ According to Bohr's theory, the energy in nth orbit of such a hypothetical atom will be propotional to

(a) n^6. (b) n^4.

(c) n^2. (d) n.

100. A hydrogen atom moving with velocity u collides inelastically with another hydrogen atom at rest. Both the atoms are in the ground state before collision. The minimum value of u, so that one of the atoms get excited, will be

(a) 3.12×10^6 m/s. (b) 9.36×10^5 m/s.

(c) 6.24×10^4 m/s. (d) 5×10^3 m/s.

101. An electron in H-atom makes a transition from $n = 3$ to $n = 1$. The recoil momentum of H-atom will be

(a) 6.45×10^{-27} N/m. (b) 6.8×10^{-27} N/m.

(c) 6.45×10^{-24} N/m. (d) 6.8×10^{-24} N/m.

102. In the above problem the recoil energy of H-atom will be

(a) 8.6×10^{-8} eV. (b) 8.6×10^{-6} eV.

(c) 7.78×10^{-8} eV. (d) 7.78×10^{-6} eV.

103. The ratio of radii of first orbit of hydrogen atom and the second orbit of singly ionised helium atom will be

(a) 1 : 2. (b) 4 : 1.

(c) 1 : 4. (d) 8 : 1.

104. If the ionisation potential of an atom is 122.4 V then its first excitation potential will be

(a) 71.8 V. (b) 91.8 V.

(c) 51.8 V. (d) 101.8 V.

105. In the above problem the second excitation potential of the atom will be

(a) 108.8 V. (b) 8.8 V.
(c) 208.8 V. (d) 308.8 V.

106. In Q. 104, the charge number of atom will be
(a) 1 (b) 2
(c) 4 (d) 3

107. In Q. 104, the recoil momentum of atom in Kg/m/sec will be
(a) 5.8×10^{-26}. (b) 1.8×10^{-26}.
(c) 2.8×10^{-26}. (d) 3.8×10^{-26}.

108. In Q. 104, the energy of recoil atom, in Joule, will be
(a) 0.44×10^{-25}. (b) 2.44×10^{-25}.
(c) 1.44×10^{-25}. (d) 3.44×10^{-25}.

109. Two electrons in an atom are moving in orbits of radii R and $9R$ respectively. The ratio of their frequencies will be
(a) 1 : 8 (b) 8 : 1
(c) 1 : 27 (d) 27 : 1

110. The wavelength of first line of Lyman series in hydrogen atom is 1216 Å. The wavelength of first line of Lyman series for 10 times ionised sodium atom will be
(a) 0.1 Å. (b) 1000 Å.
(c) 100 Å. (d) 10 Å.

111. The angular momentum of electron in hydrogen atom is proportional to
(a) $\sqrt{r}$. (b) $\frac{1}{r}$.
(c) r^2. (d) $\frac{1}{\sqrt{r}}$.

112. An electron revolves round a nucleus of charge +Ze. If the energy required to excite the electron from second to third Bohr orbit is 47.2eV, then the value of Z will be
(a) 5 (b) 4
(c) 3 (d) 1

113. In the above problem, the energy required to excite the electron from $n = 3$ to $n = 4$ state will be
(a) 16.53 eV. (b)13.6 eV.
(c) 1.51 eV. (d) none of the above.

114. An electron is confined in the region of width 1 Å. Estimate its KE
(a) 3.4 eV (b) 3.8 eV
(c) 13.6 eV (d) 10.2 eV

115. Na atom remains in lowest excited state energy for a time 1.6×10^{-8} s before it makes a transition to a ground state emitting a photon of wavelength 589 nm. The wave length spread corresponding to this line will be
(a) 10^{-4} nm (b) 10^{-5} nm
(c) 10^{-6} nm (d) 10^{-2} nm

116. The maximum wavelength of Brackett series of hydrogen atom will be
(a) 35,890 Å (b) 14,440 Å
(c) 62,160 Å (d) 40,400 Å

117. The minimum wavelength of Paschen series of hydrogen atom will be
(a) 18,700 Å (b) 970 Å
(c) 1,022 Å (d) 8,181 Å

Answers to Questions for Practice

1.	(a)	**2.**	(b)	**3.**	(b)	**4.**	(a)	**5.**	(b)	**6.**	(c)	**7.**	(c)
8.	(b)	**9.**	(d)	**10.**	(a)	**11.**	(a)	**12.**	(c)	**13.**	(a)	**14.**	(b)
15.	(d)	**16.**	(a)	**17.**	(d)	**18.**	(d)	**19.**	(b)	**20.**	(a)	**21.**	(b)
22.	(b)	**23.**	(d)	**24.**	(c)	**25.**	(b)	**26.**	(b)	**27.**	(b)	**28.**	(d)
29.	(c)	**30.**	(a)	**31.**	(c)	**32.**	(d)	**33.**	(c)	**34.**	(a)	**35.**	(d)
36.	(a)	**37.**	(a)	**38.**	(d)	**39.**	(b)	**40.**	(a)	**41.**	(a)	**42.**	(a)
43.	(a)	**44.**	(b)	**45.**	(a)	**46.**	(b)	**47.**	(b)	**48.**	(b)	**49.**	(d)
50.	(b)	**51.**	(c)	**52.**	(a)	**53.**	(a)	**54.**	(b)	**55.**	(a)	**56.**	(c)
57.	(a)	**58.**	(c)	**59.**	(a)	**60.**	(a)	**61.**	(b)	**62.**	(c)	**63.**	(d)
64.	(d)	**65.**	(a)	**66.**	(b)	**67.**	(d)	**68.**	(d)	**69.**	(d)	**70.**	(a)
71.	(b)	**72.**	(c)	**73.**	(b)	**74.**	(b)	**75.**	(a)	**76.**	(b)	**77.**	(a)
78.	(a)	**79.**	(b)	**80.**	(d)	**81.**	(c)	**82.**	(c)	**83.**	(a)	**84.**	(d)
85.	(a)	**86.**	(b)	**87.**	(b)	**88.**	(c)	**89.**	(b)	**90.**	(c)	**91.**	(d)
92.	(a)	**93.**	(c)	**94.**	(d)	**95.**	(c)	**96.**	(a)	**97.**	(d)	**98.**	(b)
99.	(a)	**100.**	(c)	**101.**	(a)	**102.**	(c)	**103.**	(a)	**104.**	(b)	**105.**	(a)
106.	(d)	**107.**	(a)	**108.**	(c)	**109.**	(d)	**110.**	(d)	**111.**	(a)	**112.**	(a)
113.	(a)	**114.**	(b)	**115.**	(a)	**116.**	(d)	**117.**	(d)				

Explanations

3. (b) Emission of a photon of 12.1 eV requires a transition from $n = 3$ to $n = 1$.

Change in orbital angular momentum

$$= \frac{h}{2n}(3-1) = \frac{h}{\pi}$$

4. (a) Series limit means the shortest possible wavelength (maximum photon energy), and first line means the longest possible wavelength (minimum photon energy) in the series.

$$f = C\left[\frac{1}{n^2} - \frac{1}{m^2}\right], \text{ where } C = \text{constant.}$$

For series limit of the Lyman series, $n = 1$, $m = \infty$,

$f_1 = C$.

For first line of the Lyman series, $n = 1$, $m = 2$,

$$f_2 = \frac{3C}{4}$$

For series limit of the Balmer series, $n = 2$, $m = \infty$,

$$f_3 = \frac{C}{4}$$

$\therefore \quad f_1 - f_2 = f_3$.

5. (b) Hydrogen atom in the ground state will only absorb energy greater than 10.2 eV. If this occurs, the collision will be inelastic. If there is no absorption of energy, the collision is elastic.

6. (c) Photon energy $= hv = 13.6\left(1 - \frac{1}{25}\right)\text{eV} \equiv 13 \text{ eV}$.

Photon momentum = momentum of hydrogen atom

$$= p = \frac{hf}{C}.$$

$$\therefore \quad mv = \frac{hf}{C}$$

or $\quad v = \frac{hf}{mc} = \frac{13 \times 1.6 \times 10^{-19}}{1.67 \times 10^{-27} \times 3 \times 10^8} \equiv 4 \text{ m/s.}$

8. (b) The potential energy $= -2 \times$ kinetic energy $= -2E$.

$\therefore$ total energy $= -2E + E = -E = -3.4$ eV

or $\quad E = 3.4$ eV.

Let p = momentum and m = mass of the electron.

$$\therefore \quad E = \frac{p^2}{2m} \text{ or } p = \sqrt{2mE.}$$

de Broglie wavelength, $\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mE}} - 6.6 \times 10^{-10}$ m.

114. (b) $\Delta p = \frac{\hbar}{\Delta x} = \frac{1.055 \times 10^{-34}}{10^{-10}} = 1.1 \times 10^{-24} \text{ kg ms}^{-1}$

$$KE = \frac{p^2}{2m} = \frac{(1.1 \times 10^{-24})^2}{2 \times 9.11 \times 10^{-31}} = 6.1 \times 10^{-19} \text{ J} = 3.8 \text{ eV}$$

115. (a) $\Delta E = \frac{\hbar}{\Delta t} = \frac{1.055 \times 10^{-34}}{1.6 \times 10^{-8}} = 4.1 \times 10^{-8}$eV

The fractional uncertainty in photon energy

$$= \frac{4.1 \times 10^{-8}}{2.105} = 1.95 \times 10^{-8} \qquad \left\{\frac{1240}{589} = 2.105\, eV\right\}$$

corresponding $\Delta\lambda = 1.95 \times 10^{-8} \times 589$

$= 0.000011\, nm$

34

X-rays

BRIEF REVIEW

Roentgen in 1895 discovered X-ray. X-ray is an *em* radiation whose energy is greater than uv rays and less than γ-rays i.e. in em spectrum X-ray lies between *uv* and γ-rays. The wave length of X-rays is of the order of A° (0.1 A° to 100 A°). The energy range of X-rays is 100 eV to 10^5 eV.

X-rays can be generated in two different ways: (a) the electrons are slowed down or stopped by the the target. Their *KE* is directly converted to continous spectrum of photons including X-rays. This process is called **bremsstrahlung** (German for '**braking radiation**'). (b) When a striking electron knocks out an inner electron of the target then the outer electron comes to take its place. The difference in energies of the two is released as X-ray called **characteristic X-ray.** This process is illustrated in Fig. 34.1

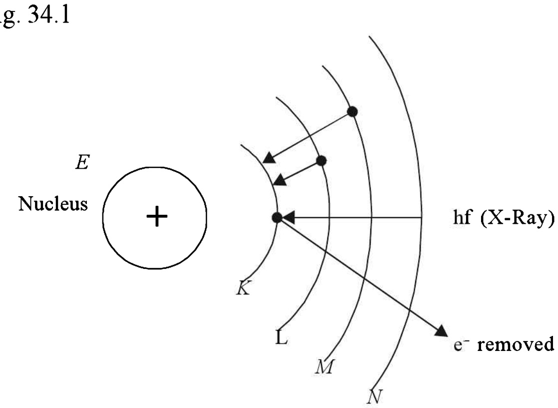


Fig. 34.1 Emission of characteristic X-ray

Fig. 34.2 (a) illustrates X-ray spectrum. The continuous background represents bremsstrahlung radiations and the peaks represent $K_\beta, K_\alpha, L_\beta, L_\alpha$ lines of characteristic X-rays. Fig. 34.2 (b) illustrate the origin of $K_\beta, K_\alpha, L_\beta, L_\alpha$ lines. Note that transition from $n = 2$ to $n = 1$ gives K_α and transition from $n = 3$ to $n = 1$ gives K_β and so on. λ_{min} or threshold wavelength shown in Fig. 34.2 (a) is given by

$$\lambda_{min} = \frac{1240 \times 10^{-9}}{V} \text{ m}$$

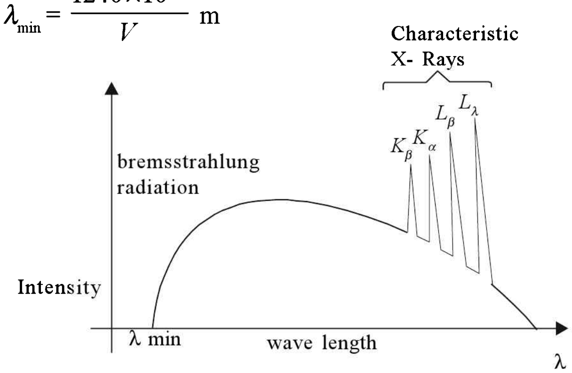


Fig. 34.2 (a) X-ray spectrum

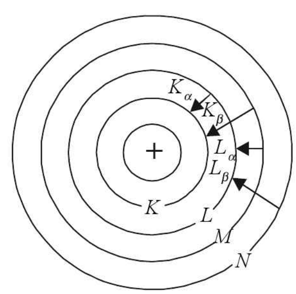


Fig. 34.2 (b) Illustration of characteristic X-rays.

Where V is potential difference between anode and cathode. λ_{min} may also be called cut off voltage.
Commercially X-rays are produced in a modified coolidge tube as shown in Fig. 34.3. The cathode K is heated by heater H and electrons are emitted by thermionic emission.
$I = Io\, A\, T^2\, e^{-\phi/kT}$

The electrons are accelerated by applying a high voltage (~ 10^4 V) between Anode and Cathode. Cathode K may be made of tungsten and anode A may be made of tungsten or molybdenum. The target shall have high melting point as enormous heat is produced when electrons strike the target. To absorb heat water is circulated. Target is making an angle of 45° so that X-rays move downwards as illustrated in Fig. 34.3. The pressure inside the tube is of the order of $10^{-4} - 10^{-6}$ torr. (1 torr = 1 mm of Hg). The Target is hollow and Wedge shaped. Hardly 1 to 2 % of incident electrons produce X-rays. Hardness of X-ray or penetrating power depends upon the accelerating potential of electrons (V) or the wavelength of X-rays. The penetrating power $\propto \frac{1}{\lambda}$. The intensity of X-rays will depend upon current through X-ray tube or the number of electrons incident per second.

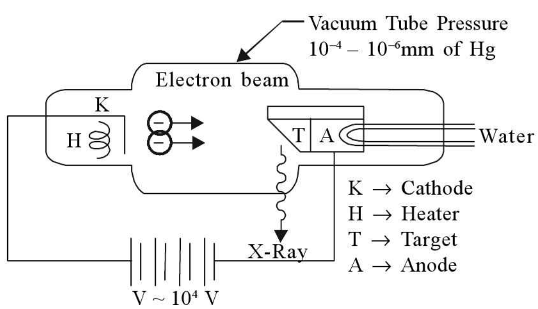


Fig. 34.3 Modern X-ray tube

AC can be applied to heater. Since X-rays are *em* radiations, they follow all the properties of *em* radiations. X-rays if incident on animate bodies or WBC (White blood cells), these get destroyed.

Compton Scattering When X-rays strike matter some of the radiation is scattered (analogous to diffusion deflection of visible light from rough surface). Compton found some of the scattered radiation have longer wavelength or small frequency than the incident. The change in wavelength depends upon the angle at which photons are scattered.

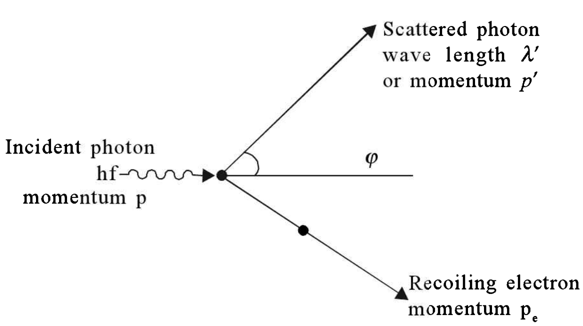


Fig. 34.4 Compton Scattering

$$\Delta\lambda = \frac{h}{mc}(1 - \cos\phi)$$

From Fig. 34.4 $\vec{p} = \vec{p}_e + \vec{p}'$

Absorption of X-rays $I = I_o e^{-\alpha x}$ where α is absorption coefficient.

$\alpha = \frac{0.693}{x_{1/2}}$ Note $\alpha \propto \lambda^3$ and $\alpha \propto Z^4$ where Z is atomic number and λ is wavelength. Lead is the best absorber of X-rays.
X-ray photography is shadow photography.
If potential difference between anode and cathode is increased, frequency of X-rays and intensity of X-rays both increase as shown in Fig. 34.5 (a). However, frequency of characterstic X-rays does not depend upon the applied accelerating potential difference, K_α, K_β, L_α and L_β are most studied characteristic X-rays. The characteristic X-ray energy is given by

$$hf = Rch\, Z^2\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right].$$

Fig. 34.2 (a) shows the characteristic X-ray superposed on continuous X-ray spectrum.

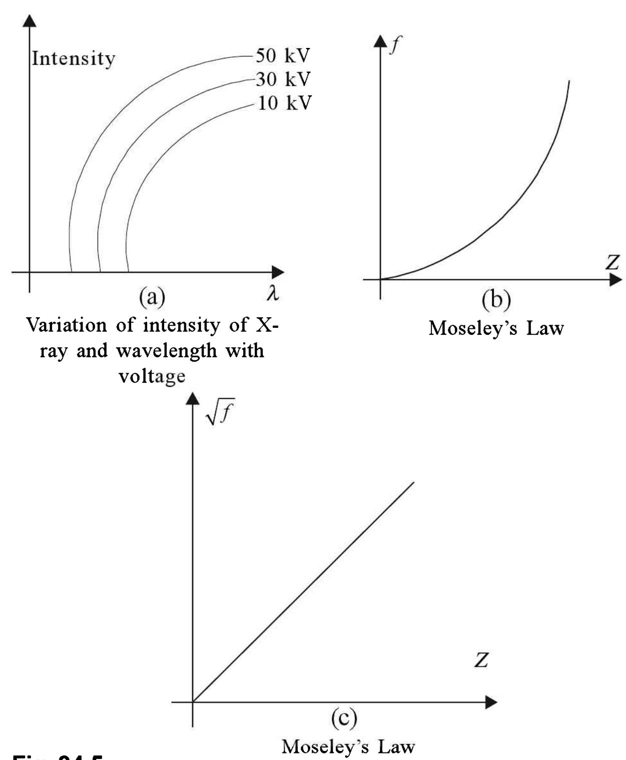


Fig. 34.5

Moseley's Law $\sqrt{f} = a(Z - b)$ where f is frequency and Z is atomic number. a and b are constant Fig. 34.5 (b) and (c) depict variation of characteristic X-ray frequency with atomic number Z. (Moseley's law).
Thus, according to Moseley's law, the basic properties of elements and their place in periodic table depends on their atomic numbers and not on atomic weights.

Lau's spot and Bragg's law confirm X-ray diffraction. Bragg's Law $2d \sin\theta = n\lambda$ is used to study crystallography.
The reverse phenomen of X-ray is photo electric effect.

Applications of X-rays

1. In medical science it is used as a diagnostic tool for fractures or cracks in bones/tooth decay. Ba SO_4 or such dyes are used to study any defect in intenstine, lungs and other organs.

   It can be used as a therapy to treat cancer.
2. It is used in food technology to increase the shelf life of food stuff. Microbes are killed when food is exposed to X-rays.
3. It is used in engineering to check cracks in machined parts particularly in those which are to be sent in space or used in space shuttles.
4. It is used in detectors at airport checkposts.
5. It is used in crystallography to study crystal structures.
6. Non distructive technique of elemental analysis. Every element has different characteristic X-ray and their intensity depends on the concentration of that element present in the salt.
7. In forensic applications, hair of every person has different elements. Even the twins formed from the same egg will have in their hair different % of the elements.
8. It is used in research laboratories.

• Short Cuts and Points to Note

1. X-rays may be divided into two categories: soft X-ray and hard X-ray. Soft X-rays are mainly used in medical science. (Frequency ~ 10^{16} Hz). Hard X-rays have wave length 0.1 A° to 10 A° and is mainly used in industry. (frequency ~ 10^{18} Hz)
2. Modified coolidge tube is used to produce X-rays. Highly accelerated electrons strike the target at low pressure of the order of 10^{-5} torr.
3. Continuous radiations are called 'Bremsstrahlung radiations' and minimum wavelength is given by

   $\lambda_{min} = \dfrac{1240\times10^{-9}}{V}$, where V is the potential difference applied between anode and cathode.
4. To find crystal structure Bragg's law $2d\sin\theta = n\lambda$ is used. Where d is inter-atomic distance. It is based on diffraction of X-rays from a crystal (as a diffraction grating).
5. Wavelength of characteristic X-ray depends upon atomic number Z. It does not depend upon applied potential. The energy of characteristic X-ray is given by

   $$E = hf = Rhc\,z^2\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right].$$
6. The characteristic X-rays also follow Moseley's law $\sqrt{f} = a(Z-b)$. The unit of a is $(\text{Hz})^{1/2}$ while b is dimensionless. The values of b are close to 1 for K-X-rays and $b = 7.4$ for L-X-rays.
   $f = 2.48\times10^{15}(Z-1)^2$.

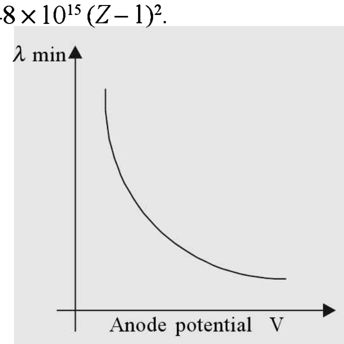


Fig. 34.6

   See variation of λ_{min} with anode potential in Fig. 34.6.
7. Hydrogen atoms cannot give X-rays.
8. X-rays cannot be reflected by ordinary targets. Therefore, these rays can not be used in RADARS.
9. X-ray ionize the gas through which they pass. They affect photographic plate also.
10. X-ray cause photo electric effect and compton scattering. In compton scattering $\Delta\lambda = \dfrac{h}{m_o c}(1 - \cos\phi)$

    $= 0.024\times10^{-10}(1-\cos\phi)$
11. X-rays are absorbed according to the law $I = I_o e^{-\alpha x}$ where α is the coefficient of absorption. $\alpha \propto \lambda^3$ and $\alpha \propto Z^4$. Thus, X-rays can penetrate and pass through low atomic number elements like Al, Wood, Plastics, human flesh etc.
12. Over exposure of X-rays is harmful. It may harm fetus during pregnancy if a pregnant woman is exposed to X-rays.
13. Absorption spectrum of X-ray are not the same as optical spectra. Reverse transition do not occur for $Z \geq 10$. Sudden jumps of absorption are found as shown in Fig. 35.7. If accelerating voltage is gradually increased. For example, in Fig. 34.7 K absorption edge for Mo is shown to occur at 20 keV.

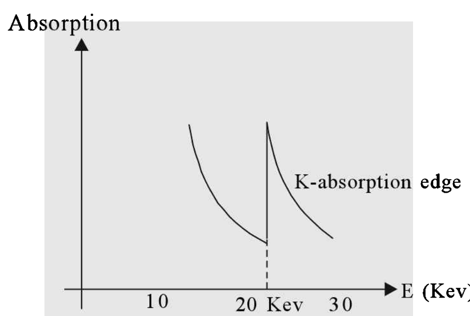


Fig. 34.7

- **Caution**

1. Considering X-rays can be reflected from any material surface like light rays.

⇒ X-rays pass through low atomic number metals like Al. This is the reason that X-rays cannot be used in RADAR.

2. Considering like optical spectra absorption spectra of X-ray are reversible.

⇒ With X-ray exposure all reverse transitions are not feasible particularly for $Z \geq 10$.

3. Assuming any X-ray will follow Moseley's law.

⇒ Only characteristic X-rays follows Moseley's law $f = 2.48 \times 10^{15} (Z-1)^2$ Hz.

4. Assuming $\lambda = \frac{hc}{eV} = \frac{1240}{V} \times 10^{-9}$ m may be applied to any X-ray.

⇒ It could be applied only to Bremsstrahlung radiation. Characteristic X-ray are target specific i.e. they depend upon the atomic number Z of the target used.

5. Considering the efficiency of X-ray production is quite high.

⇒ It is hardly 1%.

6. Confusing about maximum wavelength in Bragg's law

⇒ $2d \sin\theta = n\lambda$ when $n = 1$ $\lambda_{max} = 2d$
For diffraction to occur $\lambda < 2d$.

7. Assuming free electrons are bound in compton scattering.

⇒ The energy of X-rays is several k eV$_s$ and binding energy of electrons is few eV$_s$. Therefore, free electrons behave like unbounded electrons.

8. Confusing when anode potential is increased intensity of X-rays remains same.

⇒ When anode potential is increased intensity also increases along with frequency i.e., both frequency and intensity increase. Hence, minimum wavelength decreases.

SOLVED PROBLEMS

1. Find the minimum wavelength of X-ray produced if 10 kV potential difference is applied across the anode and cathode of the tube.

(a) 12.4 $A°$ (b) 12.4 nm
(c) 1.24 nm (d) 1.24 $A°$

Solution (d) $\lambda_{min} = \frac{1240}{10^4} \times 10^{-9} = .124$ nm $= 1.24\,A°$

2. A photon of frequency f under goes compton scattering from an electron at rest and scatters through an angle θ. The frequency of scattered photon is f' then

(a) $f' > f$ (b) $f' = f$
(c) $f' < f$ (d) none of these

Solution (c) Wavelength of scattered photon increases and hence, frequency decreases.

3. Protons are accelerated from rest by a potential difference 4 kV and strike a metal target. If a proton produces one photon on impact of minimum wavelength λ_1 and similarly an electron accelerated to 4 kV strikes the target and produces a minimum wavelength λ_2 then

(a) $\lambda_1 = \lambda_2$ (b) $\lambda_1 > \lambda_2$
(c) $\lambda_1 < \lambda_2$ (d) no such relation can be established

Solution (a)

4. The minimum wavelength X-ray produced in an X-ray tube operating at 18 kV is compton scattered at 45° (by a target). Find the wavelength of scattered X-ray.

(a) 68.8 pm (b) 68.08 pm
(c) 69.52 pm (d) none of these

Solution (c) $\lambda_{min} = \frac{1240\times10^{-9}}{18\times10^{3}} = 68.8 \times 10^{-12}$ m

$\Delta\lambda = 2.4 \times 10^{-12} (1 - \cos\phi)$
$= 2.4 \times 10^{-12} (1 - .7)$
$= .72 \times 10^{-12}$ m

λ'min $= (68.8 + .72) \times 10^{-12}$ m $= 69.52 \times 10^{-12}$ m

5. K_α wavelength of an unknown element is .0709 nm. Identify the element.

(a) Co (b) Cu
(c) Mn (d) Mo
(e) Sr

Solution (d) $f = \frac{c}{\lambda} = \frac{3\times10^8}{.0709\times10^{-9}} = 4.23 \times 10^{18}$ Hz

From Moseley's law $Z = 1 + \sqrt{\frac{f}{2.48\times10^5}}$ or

$Z = 1 + \sqrt{\frac{4.23\times10^{18}}{2.48\times10^{15}}} = 42.3$

$Z = 42$ corresponds to Mo.

6. The atomic number (Z) of an element whose K_α wavelength is λ is 11. The atomic number whose K_α wavelength is 4λ is equal to

(IIT Screening 2005)

(a) 6 (b) 11
(c) 44 (d) 4

Solution (a) $(Z-1)^2 \lambda = \text{const.}$

$\therefore\ 10^2 \lambda = (z-1)^2 4\lambda$ or

$z - 1 = 5$ or

$z = 6$

7. Who discovered X-rays?

(a) Roentgen (b) Marie curie
(c) Rutherford (d) all

[BHU 2005]

Solution (a)

8. Consider a photon of continous X-ray coming from a coolidge tube. Its energy comes from

(a) *KE* of the striking electron.
(b) *KE* of free electron of the target.
(c) *KE* of the ions of the target.
(d) an atomic transition in the target.

Solution (a)

9. Fig 34.8 shows intensity wavelength relation of X-rays in two different tubes *A* and *B* operating at V_A and V_B having targets of atomic number Z_A and Z_B then

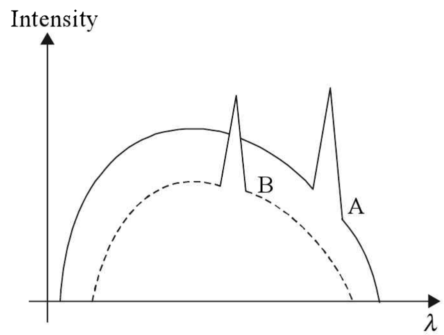


Fig. 34.8

(a) $V_A > V_B, Z_A > Z_b$ (b) $V_A > V_B, Z_A < Z_B$
(c) $V_A < V_B, Z_A > Z_B$ (d) $V_A < V_B, Z_A < Z_B$

Solution (b) λ_{min} of A < λ_{min} of *B*

$\therefore\ V_A > V_B;\ \lambda_{K_{\alpha B}} < \lambda_{K_{\alpha A}}$

$\therefore\ Z_B > Z_A$

10. 40% of the X-rays coming from a Coolidge tube can pass through 0.2 mm thick Al plate. The Anode voltage is increased. Now 40% of the X-rays will pass through Al foil of thickness.

(a) zero (b) < 0.2 mm
(c) = 0.2 mm (d) > 0.2 mm

Solution (d)

11. For harder X-rays

(a) wavelength is higher. (b) intensity is higher.
(c) frequency is higher. (d) all of these.

Solution (c)

12. Consider λ_1 and λ_2 are minimum wavelengths of continuous and characteristic X-rays then

(a) $\lambda_1 = \lambda_2$ (b) $\lambda_1 > \lambda_2$
(c) $\lambda_1 < \lambda_2$ (d) none of these

Solution (c)

13. An X-ray tube operating at 20 kV shows a current 1 *mA*. Assuming efficiency 1%. Find the number of X-ray photons emitted per second.

(a) 5.25×10^{13} (b) 6.25×10^{13}
(c) 6.25×10^{15} (d) 5.25×10^{15}

Solution (b) number of electrons/s = $\frac{I}{e}$ and no. of photons/s = $\frac{I}{100e}$

$$= \frac{10^{-3}}{1.6 \times 10^{-19} \times 100}$$

$$= 6.25 \times 10^{13}$$

14. The operating voltage is increased by 2%. By what % cut off wavelength of X-ray decrease in a Coolidge tube

(a) 1 % (b) 1.5 %
(c) 1.8 % (d) 2 %

Solution (d) $\lambda = \frac{1240 \times 10^{-9}}{V}$ or $\frac{d\lambda}{\lambda} = -\frac{dV}{V} = 2\%$

15. The wavelength of K_α and L_α X-rays of a material are 21.3 pm and 141 pm. Find the wavelength of K_β X-ray.

(a) 19 nm (b) 29 nm
(c) 26.1 nm (d) 38 nm

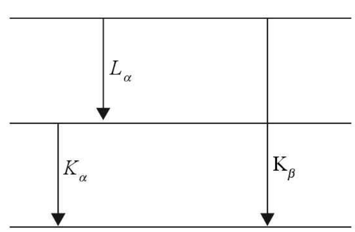


Fig. 34.9

Solution (a) See Fig 34.9 $\frac{hc}{\lambda_{K_\beta}} = \frac{hc}{\lambda_{K_\alpha}} + \frac{hc}{\lambda_{L_\alpha}}$

or $$\frac{1}{\lambda_{K_\beta}} = \frac{1}{\lambda_{K_\alpha}} + \frac{1}{\lambda_{L_\alpha}}$$

$$= \frac{1}{21.3} + \frac{1}{141} = \frac{1}{19}$$

16. A colour TV screen operates at 20 kV. The absorption coefficient of screen is 0.4 mm^{-1}. Find the mimimum thickness of the screen so that not more than 1% of X-rays come out.

(a) 2.5 mm (b) 5 mm

(c) 9.3 mm (d) 11.6 mm

Solution (d) $I = I_o e^{-\alpha x}$

or $x = \log_e\left(\dfrac{I_o/I}{\alpha}\right) = 2.303\ \dfrac{\log 10^2}{.4}$

$= 11.6\text{mm}$

17. Solid targets of different elements are bombarded by highly energetic electron beam. The frequency of the characteristic X-rays emitted from different targets varies with atomic number Z as

(a) $f \propto \sqrt{Z}$ (b) $f \propto Z^2$

(c) $f \propto Z$ (d) $f \propto Z^{3/2}$

Solution (b)

TYPICAL PROBLEMS

18. Assume 100 pm X-ray beam is passed through YDSE. Interference pattern is observed on a photographic plate placed 40 cm away from the slits. What should be the separation between the slits so that the separation between two successive maxima is 0.1 mm.

(a) 4 μm (b) 0.4 μm

(c) 4 nm (d) 40 μm

Solution (b) $\beta = \dfrac{\lambda D}{d}$ or $d = \dfrac{\lambda D}{\beta}$

$= \dfrac{10^{-10} \times .4}{.1 \times 10^{-3}} = 4 \times 10^{-7}\,\text{m}$

19. An X-ray tube operates at 40 kV. Suppose the electrons convert 70% its energy into a photon in each collision. Find the lowest 3 wavelength emitted.

Solution $\lambda_1 = \dfrac{1240 \times 10^{-9}}{.7(40) \times 10^3} = 44.3\,\text{pm},$

$\lambda_2 = \dfrac{1240 \times 10^{-9}}{0.21(40) \times 10^3} = 148\,\text{pm},$

$\lambda_3 = \dfrac{1240 \times 10^{-9}}{.063 \times 40 \times 10^3} = 493\,\text{pm}.$

20. Plot the graph between anode potential and $\dfrac{1}{\lambda_{min}}$. Also find its slope.

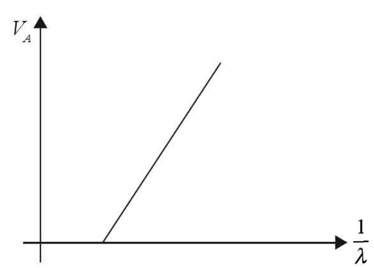


Fig. 34.10

Solution $\dfrac{hc}{\lambda} = e V_A$ Slope $= \dfrac{V_A}{\frac{1}{\lambda}} = \dfrac{hc}{e}$

21. The electric current in an X-ray tube operating at 40 kV is 5 mA. If efficiency is 1%. Find

(a) power emitted as X-ray.

(b) power converted to heat.

Solution $P_{\text{X-ray}} = 40 \times 5 \times \dfrac{1}{100} = 2\text{ W}$

$P_{\text{heat}} = 200 - 2 = 198\text{ W}$

22. A tissue paper soaked with polluted water showed K_α peaks at 78.9 pm, 146 pm, 158 pm and 198 pm. Find the elements it contained.

Solution $f = \dfrac{c}{\lambda} = 2.48 \times 10^{15}(Z-1)^2$

$Z = 1 + \sqrt{\dfrac{c}{\lambda \times 2.48 \times 10^{15}}}$

$= 1 + \sqrt{\dfrac{3 \times 10^8}{78.9 \times 10^{-12} \times 2.48 \times 10^{15}}} = 40$, i.e., Zr

$Z = 1 + \sqrt{\dfrac{3 \times 10^8}{146 \times 2.48 \times 10^{-3}}} = 30.1$, i.e., Zn

$Z = 1 + \sqrt{\dfrac{3 \times 10^8}{158 \times 2.48 \times 10^{-3}}} = 29$, i.e., Cu

$Z = 1 + \sqrt{\dfrac{3 \times 10^8}{198 \times 2.48 \times 10^{-3}}} = 26$, i.e., Fe

QUESTIONS FOR PRACTICE

1. If the potential difference applied to the tube is doubled and the separation between the filament and the target is also doubled the cut off wavelength
   (a) will remain uncharged.
   (b) will be doubled.
   (c) will be halved.
   (d) will become four times the original.
2. If the current in the circuit for heating the filament is increased, the cutoff wavelength
   (a) will increase. (b) will decrease.
   (c) will remain unchanged. (d) will change.
3. Frequencies of K_α X-rays of different materials are measured. Which one of the graphs in figure may represent the relation between the frequency f and the atomic numbe Z.

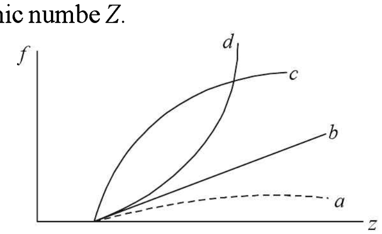


Fig. 34.11

4. The X-ray beam coming from an X-ray tube
   (a) is monochromatic.
   (b) has all wavelengths smaller than a certain maximum wavelength.
   (c) has all wavelengths greater than a certain minimum wavelength.
   (d) has all wavelengths lying between minimum and a maximum wavelength.
5. Figure shows the intensity–wavelength relations of X-rays coming from two different Coolidge tubes. The solid curve represents the relation for the tube A in which the potential difference between the target and the filament is V_A and the atomic number of the target material is Z_A. These quantities are V_B and Z_B for the other tube. Then
   (a) $V_A > V_B$, $Z_A > Z_B$ (b) $V_A > V_B$, $Z_A < Z_B$
   (c) $V_A < V_B$, $Z_A > Z_B$ (d) $V_A < V_B$, $Z_A < Z_B$

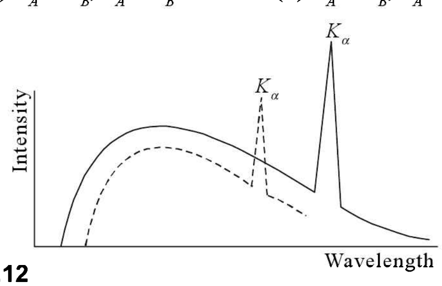


Fig. 34.12

6. 50% of the X-ray coming from a Coolidge tube is able to pass through a 0.1 mm thick aluminium foil. If the potential difference between the target and the filament is increased, the fraction of the X-ray passing through the same foil will be
   (a) 0% (b) < 50%
   (c) 50% (d) > 50%
7. Cut off wavelength of X-rays coming from a Coolidge tube depends on the
   (a) target material. (b) accelerating voltage.
   (c) separation between the target and the filament.
   (d) temperature of the filament.
8. X-ray from a coolidge tube is incident on a thin aluminium foil. The intensity of the X-ray transmitted by the foil is found to be I_0. The heating current is increased so as to increase the temperature of the filament. The intensity of the X-ray transmitted by the foil will be
   (a) zero (b) $< I_0$
   (c) I_0 (d) $> I_0$
9. Visible light passing through a circular hole forms a diffraction disc of radius 0.1 mm on a screen. If X-ray is passed through the same set up, the radius of the diffraction disc will be
   (a) zero (b) <0.1 mm
   (c) 0.1 mm (d) >0.1 mm
10. Moseley's law for characteristic X-ray is $\sqrt{f} = a(Z - b)$. In this,
    (a) both a and b are independent of the material.
    (b) a is independent but b depends on the material.
    (c) b is independent but a depends on the material.
    (d) both a and b depend on the material.
11. 50% of the X-ray coming from a Coolidge tube is able to pass through n 0.1 mm thick aluminium foil. The potential difference between the target and the filament is increased. The thickness of aluminium foil, which will allow 50% of the X-ray to pass through will be
    (a) zero (b) <0.1 mm
    (c) 0.1 mm (d) >0.1 mm
12. One of the following wavelengths is absent and the rest are present in the X-rays coming from a Coolidge tube. Which one is the absent wavelength?
    (a) 25 pm (b) 50 pm
    (c) 75 pm (d) 100 pm.
13. The K_α X-ray emission line of tungsten occurs at

$\lambda = 0.021$ nm. The energy difference between K and L levels in this atom is about

(a) 0.51 MeV (b) 1.2 MeV
(c) 59 keV (d) 13.6 eV

14. When a metal of atomic number Z is used as the target in a Coolidge tube, let f be the frequency of the K_α line. Corresponding values of Z and f are known for a number of metals. Which of the following plots will give a straight line?

(a) f against Z (b) $\frac{1}{f}$ against Z
(c) $\sqrt{f}$ against Z (d) f against $\sqrt{Z}$

15. In a Coolidge tube, the potential difference across the tube is 20 kV, and 10 mA current flows through the voltage supply. Only 0.5% of the energy carried by the electrons striking the target is converted into X-rays. The X-ray beam carries a power of

(a) 0.1 W (b) 1 W
(c) 2 W (d) 10 W

16. X-rays are absorbed maximum by

(a) lead. (b) paper.
(c) copper. (d) steel.

17. X-rays are not used in RADAR, because

(a) X-rays are not reflected by target.
(b) X-rays are completely absorbed by air.
(c) X-rays damage the target.
(d) all of the above.

18. In Coolidge tube, what fraction of incident energy is utilised in producing X-rays ?.

(a) 100% (b) 1%
(c) 50% (d) 25%

19. Water is circulated in Coolidge tube to

(a) cool the target.
(b) cool the cathode.
(c) cool both cathode and target.
(d) none of these.

20. If the incident electrons in Coolidge tube are accelerated through a potential of V volt, then the maximum frequency of continuous X-rays will be

(a) V (b) hV
(c) $\frac{eV}{h}$ (d) $\frac{h}{eV}$

21. What is the effect of electric and magnetic fields on X-rays?

(a) X-rays are deflected.
(b) X-rays are not deflected.
(c) X-rays are sometimes deflected and sometimes not.
(d) Nothing can be said.

22. The wavelength of continuous X-rays is proportional to

(a) intensity of incident electron beam.
(b) temperature of the target.
(c) intensity of X-rays.
(d) inversely to the energy of electrons striking the target.

23. If anode potential increases then

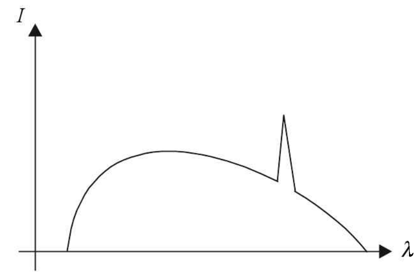


Fig. 34.13

(a) Bremsstrahlung radiation wavelength increases
(b) Bremsstrahlung radiation wavelength decreases
(c) Characteristic wavelength increases
(d) Characteristic wavelength decreases

24. The maximum frequency of X-rays produced by electrons accelerated through V volt is proportial to

(a) V (b) $\frac{1}{V}$
(c) V^2 (d) $\frac{1}{V^2}$

25. Which of the following wavelengths lies in X-ray region?

(a) 10000 Å (b) 1000 Å
(c) 1 Å (d) 10^{-2} Å

26. X-rays and γ-rays both are electromagnetic waves. Which of the following statements is correct?

(a) The wavelength of X-rays is greater than that of γ-rays.
(b) The wavelength of X-rays reduce is less than that of γ-rays.
(c) The frequency of γ-rays is less than that of X- rays.
(d) The frequency and wavelength of X-rays are more than those of γ-rays.

27. Hydrogen atom does not emit X-rays because

(a) its energy levels are very close to each other.
(b) the energy levels are far apart from each other.
(c) its size is very small.
(d) it contains only single electron.

28. Electrons of mass m and charge e are accelerated through a potential difference V and strike the target. The maximum speed of these electrons is

(a) $\frac{eV}{m}$ (b) $\frac{eV^2}{m}$

(c) $\sqrt{\frac{eV}{m}}$ (d) $\sqrt{\frac{2eV}{m}}$

29. Electrons of 10 KeV strike a tungsten target. The radiations emitted by it are

(a) visible light. (b) X-rays.

(c) infrared radiations. (d) radio waves.

30. When a beam of accelerated electrons strikes a target, then continuous spectrum of X-rays is obtained. The wavelength absent from the spectrum of X-rays emitted by an X-ray tube operated at 40 KV will be

(a) 1.5 Å (b) 0.5 Å

(c) 0.25 Å (d) 1.0 Å

31. In an X-ray tube if the electrons are accelerated through 140 KV then anode current obtained is 30 mA. If the whole energy of electrons is converted into heat then the rate of production of heat at anode will be

(a) 968 calorie. (b) 892 calorie.

(c) 1000 calorie. (d) 286 calorie.

32. The wavelength of limiting line of Lyman series is 911A. The atomic number of the element which emits minimum wavelength of 0.7A of X-rays will be

(a) 31 (b) 33

(c) 35 (d) 37

33. When X-rays of wavelength 0.5A pass through 7mm thick aluminum sheet, then their intensity reduces to one fourth. The coefficient of absorption of aluminum for these X-rays will be

(a) $0.198 mm^{-1}$ (b) $0.227 mm^{-1}$

(c) $0.752 mm^{-1}$ (d) $0.539 mm^{-1}$

34. In majority of crystals the value of lattice constant is of the order of 3A. The proper X-rays with which the crystal structure can be studied are

(a) 50 Å to 100 Å. (b) 10 Å to 50 Å.

(c) 5 Å to 10 Å. (d) 0.1 Å to 2.7 Å.

35. The lattice constant of a crystal is 2A. The maximum wavelength of X-rays which can be analysed by this crystal will be

(a) 1A (b) 2A

(c) 3A (d) 4A

36. X-rays cannot produce

(a) compton electron. (b) photoelectron.

(c) electron-positron pair. (d) all of the above.

37. The order of potential difference applied between cathode and anticathode in an X-ray tube will be

(a) 10^3V (b) 10^2V

(c) 10^4V (d) 10^1V

38. Which of the following properties is not exhibited by X-rays?

(a) Interference. (b) Diffraction.

(c) Polarisation.

(d) Deflection by electric field.

39. X-ray region is situated between

(a) visible and short radio wave regions.

(b) ultraviolet and visible regions.

(c) γ-rays and ultraviolet regions.

(d) short and long radio wave regions.

40. The coefficient of absorption for X-rays is related to the atomic number as

(a) $\mu \propto \frac{1}{Z^4}$ (b) $\mu \propto Z^3$

(c) $\mu \propto \frac{1}{Z^3}$ (d) $\mu \propto Z^4$

Answers to Questions for Practice

1.	(c)	**2.**	(c)	**3.**	(d)	**4.**	(d)	**5.**	(b)	**6.**	(b)	**7.**	(b)
8.	(d)	**9.**	(b)	**10.**	(a)	**11.**	(d)	**12.**	(a)	**13.**	(c)	**14.**	(c)
15.	(b)	**16.**	(a)	**17.**	(a)	**18.**	(b)	**19.**	(a)	**20.**	(c)	**21.**	(b)
22.	(d)	**23.**	(b)	**24.**	(a)	**25.**	(c)	**26.**	(a)	**27.**	(a)	**28.**	(d)
29.	(b)	**30.**	(c)	**31.**	(c)	**32.**	(d)	**33.**	(a)	**34.**	(d)	**35.**	(d)
36.	(c)	**37.**	(c)	**38.**	(d)	**39.**	(c)	**40.**	(d)				

Explanations

6. (b) $\because$ absorption coeff $\alpha \propto \lambda^3$

9. (b) $\because$ $r \propto \lambda$

14. From Moseley's law; $\sqrt{v} = a(Z - b)$, where a and b are constants.

15. Power drawn by the coolidge tube = $(20 \times 10^3 \text{ V})(10 \times 10^{-3} \text{ A}) = 200$ W.

Power of X-ray beam = $\frac{0.5}{100} \times 200 \text{ W} = 1$ W.

35

Nuclear Physics

BRIEF REVIEW

Nuclide A single nuclear specy having specific value of N and Z.

Mass of Proton $m_p = 1.672622 \times 10^{-24}$ kg $= 1.007276\, u$

$= 938.2732$ MeV/c^2

Mass of neutron $m_n = 1.674927 \times 10^{-27}$ kg $= 1.008665\, u$

$= 939.5696$ MeV/c^2

Mass of electron $m_e = 9.10938 \times 10^{-31}$ kg $= 0.00054858\, u$

$= 510.99$ KeV/c^2

$$1 \text{ amu} = 1\, u = \left(\frac{1}{12} \text{ of carbon}\right)$$

$$= 1.66053873 \times 10^{-27} \text{ kg}$$

Number of neutrons $= N$, Number of protons $= Z$ (atomic number)

Nucleon number = Mass number $A = N + Z$

Isotopes Nuclides having same atomic number (Z) but different number of neutrons N or different mass number A are termed as **isotopes**. For example, ${}^{12}_{6}\text{C}$, ${}^{14}_{6}\text{C}$; ${}^{35}_{17}\text{Cl}$, ${}^{37}_{17}\text{Cl}$ and ${}^{1}_{1}H$, ${}^{1}_{2}H$, ${}^{1}_{3}H$ are isotopes of carbon, chlorine and hydrogen, respectively.

Isotones The nuclides having same number of neutrons (N) are called isotones. They have different Z or A. Examples of isotones are ${}^{3}_{1}H$, ${}^{4}_{2}He$; ${}^{14}_{6}C$, ${}^{16}_{8}O$.

Isobars The nuclides having same mass number (A) but different atomic numbers (Z) are called isobars. For instance, ${}^{14}_{6}C$, ${}^{14}_{7}N$; ${}^{40}_{18}Ar$, ${}^{40}_{20}Ca$. Neutrons and protons together are called nucleons.

Nuclear radius $R = R_o A^{1/3}$ where $R_o = 1.1\, fm = 1.1 \times 10^{-15}$ m and has been determined experimentally.

Note that nuclear density is independent of mass number, i.e., all nuclei have the same nuclear density.

Nuclear spin like electrons, protons and neutrons are also $\frac{1}{2}$ spin particles. They can have spin odd half multiple of $\hbar$. The magnitude of spin angular momentum of a nucleon is $S = \sqrt{\frac{1}{2}\left(\frac{1}{2}+1\right)} = \sqrt{S(S+1)} = \sqrt{3/4}\ \hbar$. They follow Fermi-Dirac statistics or Pauli's exclusion principle. Hence they are called Fermions. Like electronic magnetic moments the nucleons also show magnetic moments. The unit like **Bohr magneton** in atoms is **nuclear magneton** (μ_n) for nucleons.

Nuclear magneton $\mu_n = \dfrac{e\hbar}{2m_p} = 5.05079 \times 10^{-27}$ J/T $= 3.15245 \times 10^{-8}$ eV/T.

The Z-component of the spin magnetic moment of the proton $|\mu_{SZ}|_{\text{proton}} = 2.7928\, \mu_n$.

The neutron has a corresponding magnitude $|\mu_{SZ}|_{\text{neutron}} = 1.9130\, \mu_n$.

Magnetic moment of proton and neutron is supposed to come from quarks. Protons and neutrons are not fundamental particles but made of quarks. Resonant signal can flip proton spin. Spin flip experiments are called **Nuclear Magnetic Resonance** (NMR). An elaboration of this basic idea leads to **Magnetic Resonance Imaging** (MRI).

Nuclear Force The force that binds protons and neutrons together in the nucleus, despite the electrical repulsion of protons. This is an exmaple of **strong interaction** and is termed as **Nuclear force**. Some of the characteristics of nuclear force are 1. It does not depend on charge. Binding is equal for proton and neutron. 2. It is a short range force extending upto 10 *fm* at the most. 3. Nuclear force is 50–60 times stronger than electromagnetic force. 4. Binding force favour binding of pairs of protons and neutrons of opposite spin and pairs of pairs, that is, a pair of proton and a pair of neutron, each pair having opposite spins. Hence, α-particle is an extremely stable nucleus.

Heisenberg in 1932 proposed exchange force theory. Yukawa extended this theory and found even mass of π-mesons. According to this theory proton does not remain proton forever and neutron does not remain neutron for ever. They go on changing. For instance,

$${}^1_0n \rightleftarrows {}^1_0\, n+\pi^0 ; \qquad {}^1_1p \rightleftarrows {}^1_0\, n+\pi^+ ;$$

$${}^1_1p \rightleftarrows {}^1_0p+\pi^0 ; \qquad {}^1_0n \rightleftarrows {}^1_1p+\pi^- .$$

Where π^0, π^+ and π^- are π-mesons having mass = 270 m_e. Later on, π-mesons were confirmed in cosmic rays. The heavy nuclides require more neutrons so that coulomb repulsion can be balanced. **Shell model** and **liquid drop model** represent the structure of nucleus.

Binding Energy $E_B = \left(Zm_H + Nm_n - {}^A_Z M\right)c^2$. The term in the bracket or E_B / C^2 is called **mass defect.** Binding energy per neuteon $\frac{E_B}{A}$ is defined as

$$\frac{E_B}{A} = \left(\frac{Zm_H}{A} + \frac{Nm_n}{A} - \frac{M}{A}\right) c^2$$

Mass excess Let A be the mass number of a nucleus. Let *Mu* (atomic mass units) be the mass of neutral atom, *Au* be the mass of nuclide in *amu* then excess mass

$$\text{Excess mass} = (Mu - Au) = (M - A)\ \frac{931.5}{c^2} \text{ in MeV/c}^2$$

Packing fraction $P = (M - A)/A$.

Magic numbers The nuclides having number of protons or number of neutrons 2, 8, 20, 28, 50, 82 or 126 are unusually stable. Nuclides with Z = 126 have not been observed in nature. There are nuclides in which Z and N are magic numbers including ${}^4_2He$, ${}^{16}_8O$, ${}^{40}_{20}Ca$, ${}^{48}_{20}Ca$.

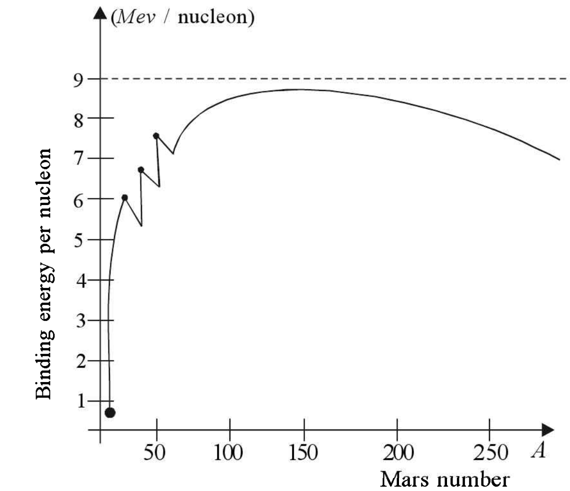


Fig. 35.1 Binding energy per nucleon vs Mass number.

Only four odd-odd nuclides are known to be stable ${}^2_1H$, ${}^6_3Li$, ${}^{10}_5B$ and ${}^{14}_7N$. The absence of other odd-odd nuclides show the influence of pairing.

Fig. 35.1 shows an approximate binding energy curve. Spikes show extra stable nuclides. Nuclides with binding energy > 7.6 MeV per nucleon are stable.

According to liquid drop model

1. Nuclear forces show saturation, i.e., an individual nucleon can interact with a few of its nearest neighbours. The effect gives a binding energy term as $C_1 A$.
2. The nucleons on the surface of the nucleus are less lightly bound. This effect leads to $-C_2\ A^{2/3}$ (proportional to surface area).
3. Each proton repels the remaining $(Z - 1)$ protons leading to an energy $-C_3 Z\,(Z-1)\,A^{-1/3}$
4. To make nucleus stable we shall have more neutrons therefore, a term $-C_4\,(N - Z)^2/A$ or $-C^1\,(A - 2Z)^2/\text{A}$ be added as correction term.
5. Finally, the nuclear force favours pairing. If both Z and N are even then this term be positive, if both Z and N are odd then this term be negative and zero other wise. The form of the term is $\pm\, C_5 A^{-4/3}$. Thus estimated binding energy E_B is sum of these five terms.

$$E_B = C_1 A - C_2 A^{2/3} - C_3\ \frac{Z(Z-1)}{A^{1/3}} - \text{C}_4\ \frac{(A-2Z)^2}{A} \pm C_5 A^{-4/3}.$$

The formula best fits if C_1 = 15.75 MeV, C_2 = 17.8 MeV, C_3 = 0.71 MeV C_4 = 23.69 MeV and C_5 = 39 MeV.

The semi emperical mass formula is

$${}^{A}_{Z}M = Zm_H + Nm_n - \frac{E_B}{c^2}$$

Stability Criterion According to a survey of periodic table, nuclides having $\frac{N}{Z} = 1$ or $\frac{N}{Z} = 1.6$ are stable. Amongst these, nuclides having even N or even Z are most stable. Nearly 90% of 2500 known nuclides are radioactive.

The heaviest stable nuclide is ${}^{209}_{83}Bi$. Lead $\left({}^{208}_{82}Pb\right)$ is the most stable heavy nuclide. All transuranic elements end into lead. The elements or nuclides which decay with time are called **radioactive nuclides**.

Radioactive decay Stable nuclides have definite atomic number and number of neutrons. Unstable nuclides decay by α or β emission. When the **recoiling nucleus** gets **de-excited γ-rays** are also produced.

Q-value of the reaction $Q = U_{\text{initial}} - U_{\text{final}} = (M_R - M_P)\,c^2$ where M_R is the mass of reactants and M_P is the mass of products. For the α-decay

$$Q = \left[m\left({}^{A}_{Z}X\right) - m\left({}^{A-4}_{Z-2}Y\right) - m\left({}^{4}_{2}He\right)\right]c^2.$$

α -rays A stream of α-particles coming out of a radioactive source is called α-rays.

α -decay: Gamow theory based on tunneling explains α-decay. In α-decay proton number decreases by 2 and mass number decreases by 4. The residual nucleus is, thus, different and is called daughter nucleus.

$$\underset{\text{Parent nucleus}}{{}^{A}_{Z}X} \rightarrow \underset{\text{daughter nucleus}}{{}^{A-4}_{Z-2}Y} \rightarrow + \underset{\alpha\text{–particle}}{{}^{4}_{2}He}$$

Conditions for α-decay

Mass number $A > 210$ and $\frac{N}{Z} > 1.6$

Three types of β-decays

(a) β^-(or electron emission)

(b) β^+ (positron emission) and,

(c) K-electron capture.

β-decays kept scientist puzzled for about 20 years. We consider radioactivity as a collison process. Momentum could not be conserved as emitted β-particles have different energies as illustrated in Fig. 35.2. It was then suggested, consider β-emission as a two particle emission. The second particle was soon detected as neutrino (ν). Neutrino is a fermion as it has spin quantum number $\pm\frac{1}{2}\hbar$. It is a mass-less particle or has **rest mass zero**.

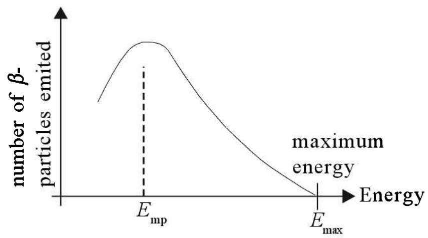


Fig. 35.2 Energy distribution of β-emission.

To understand β-emission we must have an idea of conservation rules.

Conservation Rules

1. Momentum is conserved.
2. Mass number is conserved.
3. Charge number is conserved.
4. Particle number is conserved.
5. Parity is conserved.

Particles and Antiparticles It is believed that particles live in positive sea and antiparticles live in negative sea separated by $2\,m_oc^2$ where m_0 is rest mass of the particle. See Fig 35.3. When particle and anti particle unite, an energy = $2\,mc^2$ is produced. It is believed that each particle has its antiparticle. For example, when electron

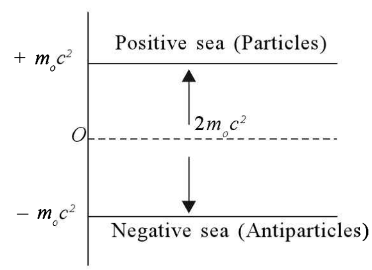


Fig. 35.3 Particle/Antiparticle illustration

and positron unite. γ-ray of energy 1.02 MeV is produced and the process is called **Pair annihilation.**

$$\underset{\text{electron}}{e^-} + \underset{\text{positron}}{e^+} \rightarrow \underset{\text{rays}}{\gamma}\ (E_\gamma = 1.02\,\text{MeV}).$$

Each particle is assigned a particle number +1 and each antiparticle is assigned particle number –1.

β-decay

$${}^{A}_{Z}X \rightarrow {}^{A}_{Z+1}Y + {}^{0}_{-1}\beta + \bar{\upsilon}.$$

Antinutrino ($\bar{\upsilon}$) is assumed to be emitted during β^- decay to conserve particle number

$$A = A + 1 - 1$$

Note that daughter nucleus has atomic number one larger than parent nucleus while mass number A remains unchanged. It is assumed that a neutron in the nucleus decays to a proton by the following process to facilitate β -emission.

$${}^{1}_{0}n \rightarrow {}^{1}_{1}p + {}^{0}_{-1}\beta + \underset{\text{antinutrino}}{\bar{\upsilon}}$$

Remember the electrons emitted from nucleus are called β^--particles.

Condition for β^-decay to occur $\frac{N}{Z} > 1$ for low Z-nuclides. Or $\frac{N}{Z} > 1.6$ for high Z-nuclides.

Note that **positron** is an **antiparticle** of **electron.**

β^+ (positron) emission

$$ {}^{A}_{Z}X \rightarrow {}^{A}_{Z-1}Y + \underset{\text{positron}}{{}^{0}_{+1}\beta} + \underset{\text{nutrino}}{\nu}. $$

See carefully that during β^+-emission charge number decreases by 1 and mass number remains unchanged. It is assumed that a proton changes to a neutron in order to achieve $\frac{N}{Z} = 1$ for light nuclides or $\frac{N}{Z} = 1.6$ for heavy nuclides.

$$ {}^{1}_{1}p \rightarrow {}^{1}_{0}n + \underset{\text{positron}}{{}^{0}_{+1}e} + \underset{\text{nutrino}}{\nu} $$

Condition for β^+-decay $\frac{N}{Z} < 1$ or 1.6 for light and heavy nuclides respectively.

K-electron capture If electron from K-shell is captured by the nuclide, the process is called K-electron capture. The resulting daughter nuclide will have atomic number one less than the parent like β^+-emission. The only difference in **β^+-emission** and **K-electron** capture is that in latter case X-ray is emitted (atomic process) while in β^+-emission γ-ray is emitted (nuclear process).

$$ {}^{A}_{Z}X + \underset{\text{K-electron capture}}{{}^{0}_{-1}e} \rightarrow + {}^{A}_{Z-1}Y + \underset{\text{nutrino}}{\nu} \text{ (+ X-ray)} $$

γ-emission The daughter nucleus after α-decay or after β^-- or β^+-decay gets excited. It de-excites after a fraction of second and emits γ-rays. Neither mass number nor atomic number changes during γ-emission. In naturally occuring radioactive substances γ-emission follows α- or β-emission.

However, artificial radioactive samples can decay only by γ-emission also.

Law of Radioactivity $\frac{dN}{dt} = -\lambda N$ where λ is decay constant or distintegration constant.

$$ \int_{N_0}^{N} \frac{dN}{N} = \int_{0}^{t} -\lambda dt \Rightarrow N = N_0 e^{-\lambda t} $$

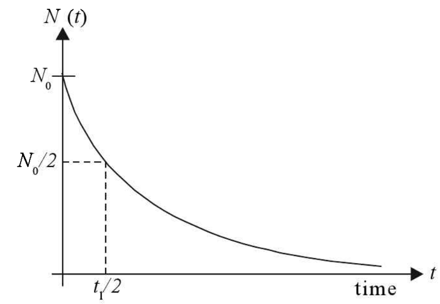


Fig. 35.4 Rodioactive decay process

Fig. 35.4 shows how number of nuclides left undecayed vary with time. The quantity $\frac{-dN}{dt}$ gives the number of decay per second and is called **activity.** Thus,

$$ \frac{-dN}{dt} = \lambda N = A \text{ (Activity) and } A = A_0 e^{-\lambda t} $$

The SI unit of activity is Becquerel (Bq). 1 Bq = 1 dps (decay per second). The practical unit of activity are Curie and Rutherford.

1 Curie (Ci) = 3.7×10^{10} dps

1 Rutherford (R) = 10^6 dps

Activity per unit mass is called specific activity. α-, β-, γ-, neutrons and X-rays break molecular bonds and create ions, hence, the term **ionizing radiations** is used for them. They can destroy tissue cells, cause alteration in genetic material and destruction of the components in bone marrow that produce red blood cells.

Radiation dose SI unit of absorbed dose is Joule/kg and is called Gray (Gy)

1 Gy = 1 J/kg Another unit is rad. 1 rad = 0.01 J/kg = 0.01 Gy

Equal amounts of different radiation cause different effects. Therefore, **relative biological effectiveness** (RBE), also called quality factor (QF) of each specific radiation is defined by a numerical factor. X-rays with 200 keV of energy are defined to have an RBE unity. SI unit of biological equivalent dose is called Sievert (Sv)

1 Sv = (RBE) × [absorbed dose (Gy)]

The permitted dose is 50 μ Sv per annum.

A dose of 5 Sv or more causes a death in few days. Table 35.1 lists RBE for various radiations. Another common unit is Roentgen equivalent for Man (rem), 1 rem = .01Sv.

Table. 35.1

Radiation	RBE (Sv/Gy)
X-ray and γ-ray	1
Electrons	1 – 1.5
Slow neutrons	3 – 5
Protons	10
α-particles	20
Heavy ions	20

Half-life $t_{1/2} = \dfrac{0.693}{\lambda}$. The time in which activity reduces to the half the present value is called half life.

Average life $t_{av} = \dfrac{1}{\lambda} = 1.44\, t_{1/2}$

Properties of α-rays

1. It is a stream of He nuclides.
2. Since they have two unit positive charge. They are deflected by electric and magnetic fields.
3. Their ionizing power is very large (maximum amongst α, β, and γ).
4. Their penetrating power is minimum. They can travel few cm in air.
5. They produce scintillation on striking with floroscent material like barium platinocyanide.
6. They affect photographic plates.

Properties of β^--rays

1. It is a stream of electrons.
2. They are deflected by electric and magnetic fields.
3. Their ionizing power is less than that of α- but greater than that of γ-particles.
4. Their penetrating power is more than that of α- but less than that of γ-rays.
5. They produce scientillation on striking a fluorescent screen.
6. They affect photographic plates.

Note: β^+ rays possess same properties as β^- rays except they are positively charged and will deflect in a direction opposite to β^- in the applied magnetic or electric field.

Properties of γ -rays

1. γ-rays are *em* radiations (no charge, rest mass zero), move with speed of light.
2. They are not deflected by electric or magnetic fields.
3. γ-rays have maximum penetrating power amongst α, β and γ.
4. The iozining power of γ-rays is minimum amongst α, β and γ.
5. They affect photographic plates.

Nuclear fission is a decay process in which an unstable nucleus splits into two fragments of comparable mass.

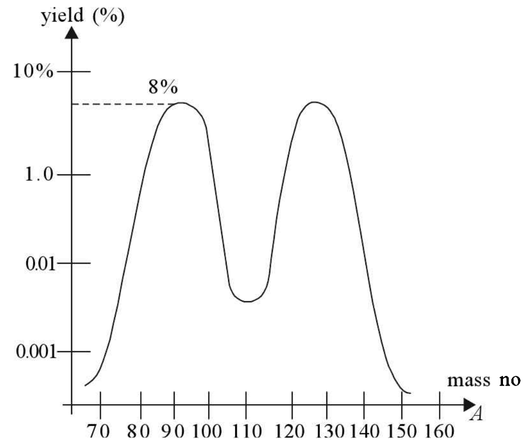


Fig. 35.5 Yield of fragments in a nuclear fission

$$^{235}_{92}U + {}^{1}_{0}n \rightarrow {}^{236}_{92}U$$

$$^{236}_{92}U \rightarrow {}^{137}_{53}I + {}^{97}_{39}Y + 2\,{}^{1}_{0}n$$

$$^{97}_{39}Y \xrightarrow{\beta^-} {}^{97}_{40}Zr \xrightarrow{\beta^-} {}^{97}_{41}Nb \xrightarrow{\beta^-} {}^{97}_{42}Mo$$

$$^{137}_{53}I \xrightarrow{\beta^-} {}^{137}_{54}Xe \xrightarrow[\text{delayed neutron}]{{}^{1}_{0}n} {}^{136}_{54}Xe;$$

$$^{236}_{92}U \rightarrow {}^{144}_{56}Ba + {}^{89}_{36}Kr + 3\,{}^{1}_{0}n$$

$$^{236}_{92}U \rightarrow {}^{140}_{56}Ba + {}^{94}_{36}Kr + 2\,{}^{1}_{0}n;$$

$$^{236}_{92}U \rightarrow {}^{140}_{54}Xe + {}^{94}_{38}Sr + 2\,{}^{1}_{0}n$$

Large number of reactions are feasible. Some of the most prominent are listed. Fig. 35.5 shows percentage yield Vs mass number of fission products of $^{236}_{92}U$. Note that some reactions give 2 neutrons and others emit 3 neutrons per reaction. Thus, on an average 2.47 neutrons per reaction are emitted. Most of the fragments have mass number 90 to 100 and 135 to 145. The delayed neutron helps a lot in controlling fission rate.

About 200 MeV energy per reaction is released in each fission. Neutrons take away about 5 MeV energy in each reaction. As the fragments further decay an additional 15–20 MeV energy is released.

Nuclear fission may be explained with liquid drop model as illustrated in Fig. 35.6.

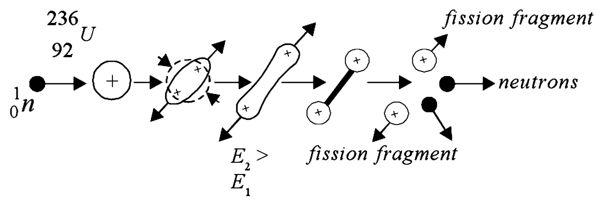


Fig. 35.6 Liquid drop model for nuclear fission

Initially assume the nuclide ${}^{236}_{92}U$ in the state E_1. It gains energy by itself for a short interval according to Hiesenberg's uncertainty principle $\Delta E \,.\, \Delta t \approx \hbar$ and reaches a higher energy state E_2. The shape gets distorted due to internal vibrations and becomes like a dumb bell and finally breaks up into two nuclides releasing energy $E_1 - E_3$ as illustrated in Fig. 35.6 and 35.7.

Table 35.2 shows fission probabilities of various substances. Note ${}^{240}Pu$ is 1.5 times more efficient than ${}^{236}U$. This is why it is the most desirable fissionable material.

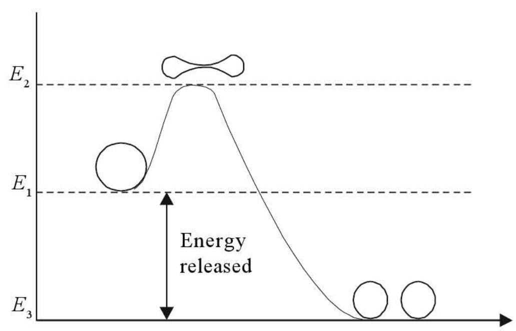


Fig. 35.7 Energy transfer depiction during fission

Table. 35.2 Fission Probability

Nuclide	Fission Probability ralative to ${}^{236}U_{92}$
${}^{236}U$	1 (assumed arbitrarily)
${}^{238}U$	$< 10^{-3}$
${}^{240}Pu$	1.5
^{244}**Am**	$< 2 \times 10^{-4}$

Critical mass The minimum mass of fissionable material required to carry out fission reaction. It is 10 kg for ${}^{236}U$.

Critical Reaction One neutron per reaction used to carry out further chain reaction while other neutrons are absorbed.

Moderator slows down the neutrons.

Thermal/Slow neutrons Neutrons having energy of the order of room temperature (0.02 eV) are termed as slow or thermal neutrons. The normal nuclear reactors use ${}^{236}U$ or ${}^{235}U$ while **breeder** reactors use ${}^{238}U$ and produce nuclear fuel which is more efficient than consumed.

$${}^{238}_{92}U + {}^{1}_{0}n \rightarrow {}^{239}_{92}U \xrightarrow{\beta^-} {}^{239}_{93}Np \xrightarrow{\beta^-} {}^{239}_{94}Pu$$

Since Pu is 1.5 times more efficient than ${}^{235}U$. Thus, a breeder reactor converts a non-fissionable material to a one which is rather more efficient.

Enriched Uranium Increasing the proportion of ${}^{235}U$ from its natural value of 0.7% to 3% or more by isotope separation processing using p^2 centrifuge is termed as enriched uranium. The fission of ${}^{235}U$ is triggered by the absorption of slow neutrons.

Nuclear fusion occurs when two light nuclide unite or fuse together to form a heavy nucleus.

To carry out fusion, the temperature should be of the order of 10^7 K

$${}^{1}_{1}H + {}^{1}_{1}H \rightarrow {}^{2}_{1}H + {}^{0}_{+1}e + \upsilon$$

$${}^{2}_{1}H + {}^{1}_{1}H \rightarrow {}^{3}_{1}H + {}^{0}_{+1}e + \upsilon \text{ or } {}^{3}_{2}He$$

$${}^{3}_{1}H + {}^{1}_{1}H \rightarrow {}^{4}_{2}He$$

$$\text{or } {}^{3}_{2}He + {}^{1}_{1}H \rightarrow {}^{4}_{2}He + {}^{0}_{+1}e + \upsilon$$

The overall energy released is nearly 27 MeV

The overall reaction is

$$4\,{}^{1}_{1}H + {}^{4}_{2}He + 2\,{}^{0}_{+1}e + 2\upsilon$$

Such a reaction is called thermal nuclear fusion reaction. In stars where the temperature is 10^8 K, another cycle called proton carbon cycle takes place.

$$4\,{}^{1}_{1}H + 2\,{}^{4}_{2}He \rightarrow {}^{12}_{6}C + 2\,{}^{0}_{+1}e + 2\upsilon$$

$${}^{1}_{1}H + {}^{12}_{6}C \rightarrow {}^{13}_{7}N + \gamma;$$

$${}^{13}_{7}N \rightarrow {}^{13}_{6}C + {}^{0}_{+1}e + \upsilon$$

The process continues until A = 56 (Iron). The element heavier than iron can be produced by neutron absorption and subsequent β decay.

$${}^{1}_{1}H + {}^{13}_{6}C \rightarrow {}^{14}_{7}N + \gamma;$$

$${}^{1}_{1}H + {}^{14}_{7}N \rightarrow {}^{15}_{8}O + \gamma$$

$${}^{15}_{8}O \rightarrow {}^{15}_{7}N + {}^{0}_{+1}e + \upsilon$$

$${}^{1}_{1}H + {}^{15}_{7}N \rightarrow {}^{16}_{8}O \text{ or } {}^{12}_{6}C + {}^{4}_{2}He.$$

Nuclear fusion in laboratory Lawson criterion $n\,\tau \geq 10^{14}\, s\ \text{cm}^{-3}$ where n is density of fusing particles and τ is time of confinement. The quantity $n\tau$ is called Lawson number. Lawson showed that in order to achieve energy output > energy input $n\tau \geq 10^{14}\ \text{cm}^{-3} - s$.

Four forces and their mediating particles are listed in table 35.3.

Table 35.3 Four fundamental forces

Interaction	Relative Strength	Range	Mediating particle Name	Mass	Charge	Spin
Strong	1	Short $(\sim 10^{-15} m)$	Gluon	0	0	1
Electromagnatic	$\frac{1}{137}$	Long $\left(\propto \frac{1}{r^2}\right)$	Photon	0	0	1
Weak	10^{-9}	Short $(\sim 10^{-3} fm)$	$w\pm$	80.4 Gev/c²	$\pm e$	0
			$z°$	91.2 Gev/c²	0	1
Gravitational	10^{-38}	long $\left(\propto \frac{1}{r^2}\right)$	graviton	0	0	2

Leptons: The leptons do not show strong interaction. They include 6 particles and their antiparticles electrons, positron sand their neutrinos v_e and $\overline{v}_e$, muons and their neutrinos, tau and their neutrinos.

Hardons are the strong interacting particles. Each hardon has its antiparticle. There are two subclasses of hardons: **mesons** and **baryons.** Baryons include nucleons and hyperons like λ, Σ, Ξ and Ω. Hardons are made from quarks.

Quarks quark are of 6 types, *u, d, s, c, b* and *t*. Fig. 36.8 illustrates how hardons are made with quarks.

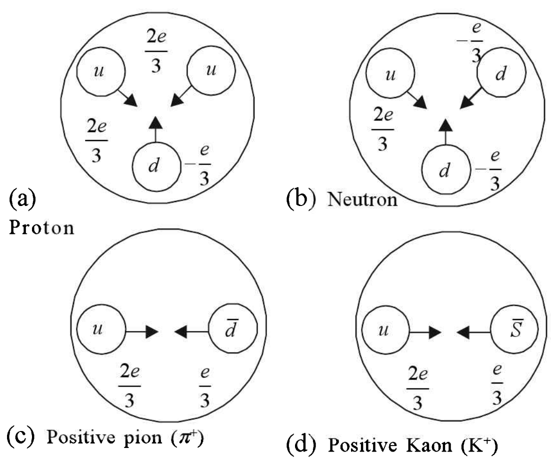


Fig. 35.8 Illustration of Hardons formation using quarks

The standard model includes 3 families of particles.

1. Six Leptons which have no strong interaction.
2. The six quarks from which all hardons are made.
3. The particles which mediate the various interactions. These mediators are gluons for strong interaction among quarks, photons for electromagnetic interaction; $W^{\pm}$, Z^0 for weak interaction (during β-decay), and the gravitons for gravitational interaction.

• Short Cuts and Points to Note

1. The particles inside the nucleus are called nucleons. They are mesons and baryon. Baryons are made of quarks.
2. The radius of the nucleus is given by $R = R_0 A^{1/3}$ where $R_0 = 1.1 fm$ $(1 fm = 10^{-15}$ m).
3. The nucleus density is independent of mass number A.
4. For nuclides to be stable $\frac{N}{Z} = 1$ for light nuclides and $\frac{N}{Z} = 1.6$ for heavy nuclides. Nuclides having number of protons or number of neutrons 2, 8, 20, 28, 50, 82 or 126 are unusually stable and termed as magic numbers. If binding energy per nucleon is greater than 7.6 MeV/nucleon, the nuclides are stable.
5. Binding energy $E_B = \left(Zm_H + Nm_n - {}_Z^A M\right)c^2$. The term in the bracket is called mass defect

$$\frac{E_B}{A} = \left[\frac{Zm_H + Nm_n - {}_Z^A M}{A}\right]c^2$$ is binding energy per nucleon.

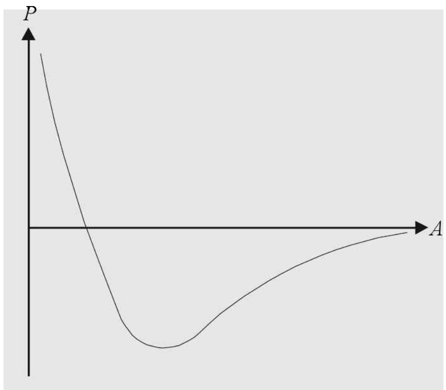


Fig. 35.9

6. Packing fraction $P = \dfrac{\left({}^{A}_{Z}M - A\right)}{A}$ where ${}^{A}_{Z}M$ is atomic mass and A is mass number. More negative value of P indicates more stable the nuclide is. Fig. 35.9 shows the variation of Packing fraction with mass number A.

7. Protons and neutrons both possess spin which is half integral multiple of $\hbar$. Thus, making them fermions. They show magnetic moments

   $|\mu_{SZ}|_{\text{proton}} = 2.7928\,\mu_n$ and

   $|\mu_{SZ}|_{\text{neutron}} = 1.9130\,\mu_n$. Where μ_n is nuclear magneton

   $\mu_n = \dfrac{e\hbar}{2m_p}$. Interaction energy = $\vec{\mu}\,.\,\vec{B}$ when placed in a magnetic field.

8. Nuclear force binds neutrons and protons both in the nucleus. It is independent of charge. This is short range force. It depends upon distance and spin. Binding force favours binding of pairs of protons and pairs of neutrons of opposite spin and pairs of pairs. That is, a pair of proton and a pair of neutron each having opposite spins. Hence α-particle $\left({}^{4}_{2}He\right)$ is an extremely stable nuclide.

9. Nuclear fission is modelled as liquid drop model. Nuclear fission of ${}^{235}_{92}U$ generates 200 MeV per reaction. 5 MeV is the energy taken by neutrons. 2.47 neutrons per fission reaction are emitted on an average. To carry out controlled reaction 1 neutron per reaction or nearly 40% of neutrons emitted are required.

   Critical mass of the fuel is 10 kg for ${}^{235}U$. Enriched uranium means increasing the % of ${}^{235}U$ from 0.7% (naturally occuring) to 3% or more by isotope separation process. Uncontrolled chain reaction is used in nuclear bombs.

10. In breeder reactor, more efficient fuel is produced than consumed. These reactors convert a nonfissionable material ${}^{238}_{92}U$ into ${}^{239}_{94}Pu$.

11. Slow neutrons or thermal neutrons are used to carry out chain reaction.

12. Neutron reproduction factor =

    $$\frac{\text{rate of production of neutrons}}{\text{rate of loss of neutrons}} = \frac{\text{number of neutrons produced}}{\text{neutrons absorbed + neutron leakage}}$$

    Heavy water (D_2 O) is used as a moderator.

13. A fusion reaction $4\,{}^{1}_{1}H \rightarrow {}^{4}_{2}He + 2\,{}^{0}_{+1}e + 2\,\nu$ generates 27 MeV energy per reaction. Such thermal nuclear fission reaction are possible at a temperature $10^7\,K$. If temperature is $10^8\,K$ carbon cycle is feasible. Such fusion reactions can occur upto Fe (A = 56). After that higher Z elements are formed due to neutron absorption and subsequent β-emission.

14. $\dfrac{-dN}{dt} = \lambda N$ or $N = N_o\,e^{-\lambda t}$.

    $\lambda N = A = \dfrac{-dN}{dt}$ is called activity.

15. $t_{1/2}$ (half life) $= \dfrac{0.693}{\lambda}$, $t_{av} = \dfrac{1}{\lambda} = 1.44\,t_{1/2}$.

16. **Law of successive transformation**

    $$\frac{dN_2}{dt} = \lambda_1 N_1 - \lambda_2 N_2$$

    $$N_2 = \frac{\lambda_1}{\lambda_2 - \lambda_1} N_1(o)\left[e^{-\lambda_1 t} - e^{-\lambda_2 t}\right] + N_2(o)\,e^{-\lambda_2 t}$$

    $$= \frac{\lambda_1}{\lambda_2 - \lambda_1} N_1(o)\,e^{-\lambda_1 t} + \left[N_2(o) - \frac{N_1(o)\lambda_1}{\lambda_2 - \lambda_1}\right]e^{-\lambda_2 t}$$

Special cases for radioactive equilibrium

a) Extremely long lived parent ($\lambda_1 << \lambda_2$)

$$N_2 = N_1(o)\,\frac{\lambda_1}{\lambda_2}\left(1 - e^{-\lambda_2 t}\right) \text{ or } N_2(eq) = \frac{\lambda_1}{\lambda_2} N_1(o)$$

b) Relatively long lived parent ($\lambda_1 < \lambda_2$)

$$N_2 = N_1(o)\,e^{-\lambda_1 t}\,\frac{\lambda_1}{\lambda_2 - \lambda_1} - N_1\,\frac{\lambda_1}{\lambda_2 - \lambda_1}$$

$$\frac{A_1}{A_2} = \frac{\lambda_1 N_1}{\lambda_2 N_2} = \frac{\lambda_2 - \lambda_1}{\lambda_2} = \frac{T_1 - T_2}{T_1}$$ where T_1 and T_2 are average life times.

c) Relatively shortlived parent ($\lambda_1 > \lambda_2$)

$$\frac{A_2}{A_1} = \frac{\lambda_2 N_2}{\lambda_1 N_1} = \frac{\lambda_2}{\lambda_1 - \lambda_2}\left[e^{(\lambda_1 - \lambda_2)t} - 1\right]$$

If $t >> T_1$ then $N_2 = N_1(o)\, e^{-\lambda_2 t}$

d) Daughter and parent of nearly equal half life

$$\frac{A_2}{A_1} = \frac{t}{T_2}$$ i.e. linearly proportional to time

17. Number of nuclides left after n half lives $N = \frac{N_o}{2^n}$.

Similarly, mass of parent left after n half lives is $m = \frac{m_o}{2^n}$.

18. α-decay is explained using Gamow's theory of tunneling while β-decay is explained using neutrino hypothesis.

19. Relation between range and energy of α-particles

$R = 0.318\ E^{3/2}$

Geiger Nuttal law $\log \lambda = A + B \log R$

Range of α-particles is 2.6 cm to 8.6 cm in air.

20. α-particles are detected using scintillation counter. β and γ radiations are detected using GM (Geiger-Muller) counter.

21. Radioactivity is a nuclear process as it is not associated with atomic electrons.

22. If $E_\gamma > 1.02$ MeV then pair production may occur.

$\gamma \rightarrow e^- +$ and e^+ and $e^+ + e^- \rightarrow \gamma$

electron and positron unite to form γ-ray ($E_\gamma = 1.02$ MeV), this process is called pair annihilation.

23. Absorption of α, β and γ result in ionization. γ-ray may also be absorbed causing photoelectric effect, compton scattering or pair production.

24. Leptons do not show strong interactions. e^-, e^+, ν_e, $\bar{\nu}_e$; $\mu^-, \mu^+, \nu_\mu, \bar{\nu}_\mu$ and $\tau, \bar{\tau}, \upsilon_\tau, \bar{\upsilon}_\tau$ are 6 pairs of leptons (particle + antiparticle).

25. Lambda (λ), Sigma (Σ), ksi (Ξ) and Omega (Ω) along with nucleons (neutron and proton) are Baryons. λ, Σ, Ξ, Ω are called hyperons.

Hardons can be divided into two types mesons and Baryons. K-meson, π-meson are examples of mesons. All Hardons are formed from quarks.

26. Gluons, photons, $W^\pm$, Z^0 and gravitons are respective mediatory particles for strong interaction, electromagnetic interaction, weak interaction and gravitational interaction respectively.

27. Fermions are spin half particles. Examples of fermions include electron, proton, neutron, neutrino etc. They follow Fermi Dirac statistics. They follow Pauli's exclusion principle.

28. Bosons are integral spin particles. Examples of Bosons are photon, graviton, pion, kaon phonons, excitons, magnons, η-meson, cooper pair etc. Bosons follow Bose–Einstein statistics.

• Caution

1. Difficulty in remembering which type of neutrons cause fission.

⇒ Only thermal or slow neutrons having energy ~ 0.02 eV cause fission in ^{235}U. For ^{238}U, we use fast neutrons to make a fuel (developed in Breeder reactor).

2. Confusion between fermions and bosons.

⇒ Fermions are spin half particles and follow Pauli's exclusion principle. They follow Fermi Dirac statistics.

Bosons are integral spin particles. They follow Bose–Einstein statistics.

3. Confusion between half life and average life.

⇒ Half life $t_{\frac{1}{2}} = \frac{0.693}{\lambda}$ (the time in which particles reduce to half the amount)

$$t_{av} = \frac{1}{\lambda}$$

4. Confusion between Curie and Rutherford and radiation dose.

⇒ 1 Ci = 3.7×10^{10} dps, 1 R = 10^6 dps } Practical units of activity

1 Bq = 1dps ⟶ SI unit of activity

1 Gy = 1 J/kg, 1 rad = 0.01 J/kg = 0.01 Gy } Radiation dose

1 Sv = (RBE) × absorbed dose (Gy), 1 rem = 0.01 Sv } Relative biological effectiveness

For a normal man permitted radiation dose is 50 μ Sv per annum.

5. Not recalling the formula between time/activity.

⇒ $N = N_o e^{-\lambda t}$ After n half times $N = \frac{N_o}{2^n}$

6. Non clarity on concepts of radioactive equilibrium.

⇒ When rate of decay of daughter is equal to rate of decay of parent, radioactive equilibrium occurs, i.e., $\lambda_1 N_1 = \lambda_2 N_2$.

7. Assuming that all β-particles emitted from a source have constant energy.

⇒ Their energies are different. This effect is explained by neutrino hypothesis of two particle emission.

8. Considering that nuclear force is a central force.

⇒ Nuclear force is a short range attractive force. It is independent of charge. It depends upon spin and distance.

9. Considering neutrinos as particles having mass.

⇒ Neutrinos are massless, chargeless particles. They have spin $\frac{\hbar}{2}$, i.e., they are Fermions. The linear momentum and spin vectors are mutually opposite for antineutrino as illustrated in Fig. 35.10.

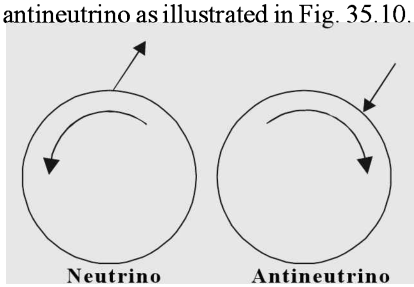


Fig. 35.10

10. Non clarity on the process of absorption of γ-rays.

⇒ The γ-radiation is absorbed by ionization, photo electric effect, compton scattering and by pair production. The absorption coefficient $\alpha \propto \lambda^3$ and $\alpha \propto Z^4$. $I = I_o e^{-\alpha x}$ is the absorption law.

11. Considering that nuclear density depends upon mass number.

⇒ Nuclear density is independent of mass number as

$$\frac{A}{\frac{4}{3}\pi R^3} = \frac{A}{\frac{4}{3}\pi R_0^3 A} = \frac{3}{4\pi R_0^3}.$$

12. Considering that a neutron cannot be a Fermion as it has no charge.

⇒ Chargeless particles like neutron and neutrino are Fermions as they are half spin particles.

13. Considering neutron can not have magnetic dipole moment as it is chargeless.

⇒ Neutron has magnetic moment. It is attributed to the fact that neutrons are composed of quarks.

14. Assuming that electrons are also present inside the nucleus as during β-emission electrons are emitted.

⇒ Electrons cannot live inside the nucleus otherwise their speed will exceed speed of light. It is a neutron which decays to proton and electron through a complicated process that electrons are emitted during β-decay.

15. Considering that a radioactive element can emit any particle electron, proton, neutron or alpha.

⇒ Naturally occuring radioactive elements emit α, β and γ. Elements with $Z > 83$ are all radioactive.

Artificially prepared radioactive samples can decay even by γ-emission or neutron or proton emission.

16. Assuming neutron as the most stable particle as it has no charge.

⇒ The neutron can not exist independently. Its life is hardly 7 minute. When it is outside the nucleus.

17. During positron emission taking into account mass of one electron as in β^--emission.

⇒ In positron emission $Q = [M_{Reactant} - M_{Product} - 2\, m_e]\, c^2$

SOLVED PROBLEMS

1. In the reaction $^{2}_{1}H + ^{3}_{1}H \rightarrow ^{4}_{2}He + ^{1}_{0}n$, if the binding energies of $^{2}_{1}H$, $^{3}_{1}H$ and $^{4}_{2}He$ are respectively a, b and c (in MeV) then the energy (in MeV) released is

(a) $a + b + c$ (b) $a + b - c$

(c) $c - a - b$ (d) $c + a - b$

(CBSE PMT 2005)

Solution (b) $Q = (M_R - M_P)\, C^2 = a + b - c$

2. Fission of nuclei is possible because the binding energy per nucleon in them

(a) increases with mass number of low mass.

(b) decreases with mass number at low mass number.

(c) increases with mass number at high mass number.

(d) decreases with mass number at high mass number.

(CBSE PMT 2005)

Solution (d)

3. A star converts all its He in Oxygen. Find the amount of energy released per nucleus of oxygen. He = 4.0026 amu O = 15.9994 amu

(a) 7.26 MeV (b) 7 MeV

(c) 10.24 MeV (d) 5.12 MeV

(IIT Screening 2005)

Solution (c) $E = \Delta mc^2$

$= [4 \times 4.0026 - 15.9994]\, 931.5 = 10.24$ MeV.

4. The intensity of gamma radiation from a given source is I. On passing through 36 mm of lead it is reduced to $\frac{I}{8}$. The thickness of lead which reduces the intensity to $\frac{I}{2}$ is

(a) 6 mm (b) 9 mm

(c) 18 mm (d) 12 mm

(AIEEE 2005)

Solution (d) $\frac{\frac{I}{2}}{\frac{I}{8}} = \frac{e^{-\mu x}}{e^{-\mu 36}}$ or $\frac{8}{2} = \frac{e^{36\mu}}{e^{\mu x}}$ or $\frac{2^3}{2} = \frac{(e^{\mu x})^3}{e^{\mu x}}$

$(e^{\mu x})^3 = e^{36\mu}$ $\therefore x = 12$

5. Starting with a sample of pure ^{66}Cu, $\frac{7}{8}$ of it decays into Zn in 15 minutes the corresponding half life is

(a) 10 min (b) 15 min

(c) 5 min (d) 7 ½ min

(AIEEE 2005)

Solution (c) Sample left $= \frac{1}{8} = \frac{1}{2^n}$

n = 3 i.e. 3 half lines have passed

$3\, t_{1/2}$ = 15 or $t_{1/2}$ = 5 min.

6. The radius of $^{27}_{13}$Al is 3.6 Fermi. Find the radius of $^{125}_{52}$Te nucleus.

(a) 6 Fermi (b) 8 Fermi

(c) 4 Fermi (d) 5 Fermi

(AIEEE 2005)

Solution (a) $\frac{R_1}{R_2} = \frac{A_1^{1/3}}{A_2^{1/3}}$

$R_2 = \left(\frac{125}{27}\right)^{1/3} \times 3.6 = 6$ Fermi.

7. A. It is not possible to use ^{35}Cl as the fuel for fusion energy

R. The binding energy ^{35}Cl is too small

(AIIMS 2005)

(a) Both A and R are correct and R is correct explanation of A.

(b) A and R are correct but R is not correct explanation of A.

(c) A is true but R is false.

(d) both A & R are false.

Solution (c)

8. $^{222}_{86}A \rightarrow\, ^{210}_{84}B$ in this reaction how many α and β emissions have occurred

(a) $6\alpha, 3\beta$ (b) $3\alpha, 4\beta$

(c) $4\alpha, 3\beta$ (d) $3\alpha, 6\beta$

(BHU 2005)

Solution (b) Since the mass number has decreased by 12 $\therefore$ 3α emissions have occurred. The charge number will decrease by 6 with 3α emission. 4β emission will make charge 84 units.

9. The phenomenon of radioactivity

(a) is an exothermic change which increases or decreases with temperature.

(b) increases on applied pressure.

(c) is a nuclear process does not depend upon external factors.

(d) none of these.

Solution (c)

10. Mean life of a radioactive sample is 100 s. Find its half life in minutes.

(a) 0.693 (b) 1

(c) 10^{-4} (d) 1.155

(CET Karnataka 2005)

Solution (d) $t_{av} = \frac{1}{\lambda}$ 100 s

$t_{1/2} = \frac{.693}{\lambda} = 69.3\, s$

11. Consider two nuclei of same radioactive nuclide. One of the nuclei was created in a supernova explosion 5 billion years ago. The other was created in a nuclear reactor 5 minutes ago. The probability of decay during the next time is

(a) different of each nuclei.

(b) nuclei created in explosion decays first.

(c) nuclei created in the reactor decays first.

(d) independent of time of creation.

Solution (d) It depends only on the number of nuclei present at that time.

12. Protons are placed in a magnetic field in the Z-direction (magnitude = 2.3 T). The energy difference between a state with Z component of proton spin angular momentum parallel to the field and antiparallel to the field is.

(a) 4.05×10^{7} eV (b) 4.05×10^{-7} eV

(c) 2.025×10^{7} eV (d) 2.025×10^{-7} eV

Solution (b) $U_1 = |S_Z| B$

$$= -2.7928 \times (2.3\, T) \times 3.152 \times 10^{-8} \left(\frac{eV}{T}\right)$$

$= -2.025 \times 10^{-7}$ eV (When B and $|S_Z|$ are parallel.

$U_2 = +2.025 \times 10^{-7}$ eV when B and $|S_Z|$ are antiparallel.

$\therefore \Delta U = U_2 - U_1 = 4.05 \times 10^{-7}$ eV

13. The hyperfine lines in the spectrum is related to

(a) Zeeman effect. (b) Stark effect.

(c) Lande's splitting. (d) nuclear magnetic spin.

Solution (d)

14. Find the binding energy of ${}^{62}_{28}Ni$. Given $m_H = 1.008\, u$, $m_n = 1.0087\, u$, ${}^{62}_{28}m = 61.9238\, u$

(a) 545.3 MeV (b) 595.3 MeV

(c) 645.3 MeV (d) 695.3 MeV

Solution (a) $E_B = (28\, m_H + 34\, m_n - 61.9238)\, 931.5$

$= 545.3$ MeV

15. ${}^{57}_{27}\text{Co}$ will emit _________ radiation.

(a) β^- (b) β^+

(c) α (d) electron capture

Solution (d) ${}^{57}_{27}\text{Co}$ decays to ${}^{57}_{26}\text{Fe}$. The mass ${}^{57}_{26}\text{Fe}$ is $0.000897\, \upsilon$ less than ${}^{57}_{27}\text{Co}$ makes it suitable for electron capture.

16. ^{57}Co decays by electron capture. Its half life is 272 days. Find the activity left after a year if present activity is 2 μ Ci.

(a) 0.788 μCi (b) 0.431 μCi

(c) 0.39 μCi (d) none of these

Solution (a) $\lambda = \dfrac{0.693}{t_{1/2}} = 2.95 \times 10^{-8} s^{-1}$

$$N_o = \frac{-dN/dt}{\lambda} = \frac{7.4 \times 10^4}{2.95 \times 10^{-8}} = 2.51 \times 10^{12} \text{ nuclei.}$$

$$N_{(t)} = N_o e^{-\lambda t} = 2.51 \times 10^{12}\, e^{-2.95 \times 10^{-8} \times 3.156 \times 10^{7}}$$

$= 0.394\, (2.51 \times 10^{12})$.

Activity = $\lambda\, N(t) = .394\, (2.5 \times 10^{12}) \times 2.95 \times 10^{-8} = 0.788\, \mu$Ci

Alternative method $\dfrac{dN(t)}{dt} = \dfrac{dN(o)}{dt} e^{-\lambda t}$

$= (2\, \mu\text{Ci})\, (e^{-2.95 \times 10^{-8} \times 3.156 \times 10^{7}})$

$= 2\, (.394) = 0.788\, \mu$Ci.

17. During a diagnostic X-ray examination a 1.2 kg portion of the broken leg receives an equivalent dose of 0.4 m Sv. Find the absorbed dose in m Gy and number of X-ray photons received if energy of X-ray is 50 KeV.

(a) 0.4 $m\,Gy$, 3×10^{15} (b) 0.32 $m\,Gy$, 3×10^{10}

(c) 0.4 $m\,Gy$, 6×10^{10} (d) 0.32 $m\,Gy$, 3×10^{15}

Solution (c) X-ray RBE = 1 $\therefore$ absorbed dose = $\dfrac{0.4 mSv}{1 Sv/Gy}$

$D = 0.4\, m\,Gy$.

Total energy absorbed = $0.4 \times 10^{-3} \times 1.2$

$= 4.8 \times 10^{-4}$ J $= 3 \times 10^{15}$ eV

Number of photons of X-ray absorbed

$$= \frac{3 \times 10^{15}}{50 \times 10^{3}} = 6 \times 10^{10}.$$

18. When ${}^{7}_{3}\text{Li}$ ($M_{Li} = 7.016004\, u$) is bombarded by a proton two α-particles result $(MHe = 4.002603 u)$. Find the reaction energy.

(a) 13.35 MeV (b) 14.85 MeV

(c) 16.08 MeV (d) 17.35 MeV

Solution (d)

$Q = [7.016004 + 1.007825 - 2\,(4.002603)] \times 931.5$

$= .018623 \times 931.5 = 17.35$ MeV

19. What mass of ${}^{235}_{92}U$ has to undergo fission each day to provide 3000 MsW of power each day.

(a) 3.2 g (b) 320 g

(c) 3.2 kg (d) 32 kg

Solution (c) 1 fission or 235 u gives 200 MeV.

Mass of uranium = $\dfrac{m \times 200 \times 1.6 \times 10^{-13}}{235 \times 1.66 \times 10^{-27}}$

$= 10^6 \times 3000 \times 86400$ or $m = 3.2$ kg

20. A bone fragment found in a cave contains 0.21 times as much $^{14}_{6}C$ as an equal amount of carbon in air when the organism containing bone died. Find the approximate age of fragment $t_{1/2}$ of $^{14}C = 5730$ years.

(a) $1.15 \times 10^4 y$ (b) $1.3 \times 10^4 y$
(c) $1.24 \times 10^4 y$ (d) $1.4 \times 10^4 y$

Solution (b) $\lambda = \dfrac{0.693}{t_{1/2}} = \dfrac{0.693}{5730} = 1.209 \times 10^{-4} y^{-1}$

$\dfrac{N}{N_o} = e^{-\lambda t} = 0.21$ or $t = \dfrac{2.303 \log \dfrac{1}{0.21}}{\lambda}$

$t = \dfrac{2.303(.6794)}{1.209 \times 10^{-4}} = \dfrac{1.564 \times 10^4}{1.2} = 1.3 \times 10^4 y.$

21. Two Nucleons are at a separation of 1 fermi. Proton have a charge 1.6×10^{-19} C, the nuclear force between them is F_1 both are neutrons, F_2 if both are protons, F_3 if one neutron and one proton then

(a) $F_1 = F_2 > F_3$ (b) $F_1 = F_2 = F_3$
(c) $F_1 < F_2 < F_3$ (d) $F_1 > F_2 > F_3$

Solution (b) Nuclear force is independent of charge.

22. In which of the following decays atomic number increases.

(a) α (b) β^+
(c) β^- (d) γ

Solution (c) $^{A}_{Z}X \rightarrow {}^{A}_{Z+1}Y + {}^{0}_{-1}e + \bar{\nu}$

23. As the mass number A varies which of the quantity related to nucleus does not change

(a) mass. (b) volume.
(c) binding energy. (d) density.

Solution (d)

24. For the nuclie with mass number > 100

(a) binding energy of the nucleus decreases on an average as A increases.
(b) binding energy of the nucleus increases on an average as A increases.
(c) The two nuclei fuse to form a bigger nuclide releasing energy.
(d) The nucleus essentially breaks up into two nuclides of equal mass releasing energy.

Solution (a) See Binding Energy Curve.

25. The half life of ^{226}Ra is 1602 year. Calculate the activity of 0.1 g of $Ra\,Cl_2$ assuming all the Ra atoms are ^{226}Ra and mass of Cl atom is 35.5 g/mol.

(a) 1.8×10^8 dps (b) 2.8×10^8 dps
(c) 1.8×10^9 dps (d) 2.8×10^9 dps

Solution (d) Number of ^{226}Ra atoms present

$= \dfrac{.1}{297} \times 6.02 \times 10^{23}.$

Activity $= \lambda N = \dfrac{.693}{.602 \times 3.156 \times 10^7} \times \dfrac{1}{297}\, 6.02 \times 10^{22}$

$= 2.8 \times 10^9$ dps

TYPICAL PROBLEMS

26. A radioactive sample decays in two modes. In one mode its half life is t_1 and in the other mode its half life is t_2. Find the overall half life

(a) $t_1 + t_2$ (b) $\dfrac{t_1 + t_2}{2}$
(c) $\dfrac{t_1 t_2}{t_1 + t_2}$ (d) $\dfrac{t_1 t_2}{t_1 - t_2}$

Solution (c) $\lambda_{eff} = \lambda_1 + \lambda_2$ or $\dfrac{1}{t_{eff}} = \dfrac{1}{t_1} + \dfrac{1}{t_2}.$

27. Consider $^{25}_{13}Al \rightarrow {}^{25}_{12}Mg + {}^{0}_{+1}e + \nu$

$m_{AL} = 24.990423\,u; m_{Mg}$

$= 24.485839\,u$. Find the Q value of reaction.

(a) 3.3 MeV (b) 1.3 MeV
(c) 2.3 MeV (d) 5.3 MeV

Solution (a) $Q = [24.990432u - 24.985839u - 2m_e]\,c^2$

$= .004593\,(931.5) - 1.102 = 3.3$ MeV

28. A vessel of 125 cc contains $^{3}_{1}H$ $(t_{1/2} = 12.3y)$ at 500 kPa and 300 K. Find the activity of the gas.

(a) .754 Ci (b) 7.24 Ci
(c) 72.4 Ci (d) 724 Ci

Solution (d) $PV = n\,RT$ or number of moles $n = \dfrac{PV}{RT}$

$$n = \frac{500\times10^3\times125\times10^{-6}}{8.31\times300} = 25\times10^{-3}$$

$$\text{Activity } A = \lambda N = \frac{0.693\times25\times10^{-3}\times6.02\times10^{23}}{12.3\times3.156\times10^7\times3.7\times10^{10}} = 724$$

29. Consider fusion of *He* plasma. At what temperature fusion at a distance 2 fm is possible?

(a) $2.23\times10^9 K$ (b) $22.3\times10^9 K$
(c) $2.23\times10^8 K$ (d) none of these

Solution (b) $\frac{3}{2}kT = \frac{(2e)^2}{4\pi\varepsilon_o r}$

$$\Rightarrow T = \frac{8\times(1.6\times10^{-19})^2\times9\times10^9}{3\times1.38\times10^{-23}\times2\times10^{-15}}$$

$$= 2.23\times10^{10} K$$

30. Calculate the energy which can be obtained from 1 kg of H_2O through the fusion reaction ${}^2_1H + {}^2_1H \rightarrow {}^3_1H + p$. Assume 1.5×10^{-2} % of the water contains deuterium. The whole deuterium is consumed in the fusion reaction.

(a) 2820 J (b) 2820×10^4 J
(c) 2820×10^6 J (d) 2820×10^8 J

Solution (c) $D_2O = 1\times1.5\times10^{-4}\text{ kg} = 0.15\text{ g}$

$$\text{Number of } {}^2_1H = \frac{0.15\times6.023\times10^{23}}{20} = 4.5\times10^{21}$$

$Q = (2\times2.014102 - 3.016049 - 1.008)\,931.5\times4.5\times10^{21}\times1.6\times10^{-13}$ J

$= 0.004155\times931.5\times7.2\times10^8 = 2820$ MJ

31. A human body excretes certain material by a law similar to radioactivity. The body excretes half the amount injected in 24 h. Find the time in which activity falls to 3 μCi. If a person is injected technitium ($t_{1/2} = 6h$) and its activity just after the injection is 6 μCi.

(a) 4.8 *h* (b) 6 *h*
(c) 6.3 *h* (d) none of these

Solution (a) $\frac{1}{t} = \frac{1}{t_1} + \frac{1}{t_2}$ or

$$t = \frac{t_1t_2}{t_1+t_2} = \frac{24\times6}{24+6} = 4.8\ h$$

QUESTIONS FOR PRACTICE

1. As the mass number *A* increases, the binding energy per nucleon in a nucleus

(a) increases. (b) decreases.
(c) remains the same.
(d) varies in a way that depends on the actual value of *A*.

2. Which of the following is a wrong description of binding energy of a nucleus ?

(a) It is the energy required to break a nucleus into its constituent nucleons.
(b) It is the energy made available when free nucleons combine to form a nucleus.
(c) It is the sum of the rest mass energies of its nucleons minus the rest mass energy of the nucleus.
(d) It is the sum of the kinetic energy of all the nucleons in the nucleus.

3. In one average-life,

(a) half the active nuclei decay.
(b) less than half the active nuclei decay.
(c) more than half the active nuclei decay.
(d) all the nuclei decay.

4. In a radioactive decay, neither the atomic number nor the mass number changes. Which of the following particles is emitted in the decay ?

(a) Proton. (b) Neutron.
(c) Electron. (d) Photon.

5. During a negative beta decay,

(a) an atomic electron is ejected.
(b) an electron which is already present within the nucleus is ejected.
(c) a neutron in the nucleus decays emitting an electron.
(d) a proton in the nucleus decays emitting an electron.

6. A freshly prepared radioactive source of half-life 2 *h* emits radiation of intensity which is 64 times the permissible safe level. The minimum time after which it would be possible to work safely with this source is

(a) 6 *h*. (b) 12 *h*.
(c) 24 *h*. (d) 128 *h*.

7. The decay constant of a radioactive sample is λ. The half-life and the average-life of the sample are respectively

(a) $\frac{1}{\lambda}$ and $\left(\frac{\log_e^2}{\lambda}\right)$ (b) $\left(\frac{\log_e^2}{\lambda}\right)$ and $\frac{1}{\lambda}$

(c) $\lambda(\log_e^2)$ and $\frac{1}{\lambda}$ (d) $\frac{\lambda}{\log_e^2}$ and $\frac{1}{\lambda}$.

8. Consider a sample of a pure beta-active material.

(a) All the beta particles emitted have the same energy.

(b) The beta particles originally exist inside the nucleus and are ejected at the time of beta decay.

(c) The antineutrino emitted in a beta decay has zero mass and hence, zero momentum.

(d) The active nucleus changes to one of its isobars after the beta decay.

9. Free ^{238}U nuclei kept in a train emit alpha particles. When the train is stationary and a uranium nucleus decays, a passenger measures that the seperation between the alpha particle and the recoiling nucleus becomes x, time t after the decay. If a decay takes place when the train is moving at a uniform speed υ, the distance between the alpha particle and the recoiling nucleus at a time t after the decay, as measured by the passenger will be

(a) $x+\upsilon t$ (b) $x-\upsilon t$

(c) x

(d) depends on the directions of the train.

10. During a nuclear fission reaction

(a) a heavy nucleus breaks into two fragments by itself.

(b) a light nucleus bombared by thermal neutrons breaks up.

(c) a heavy nucleus bombared by thermal neutrons breaks up.

(d) two light nuclei combine to give a heavier nucleus and possibly other products.

11. A free neutron decays to a proton but a free proton does not decay to a neutron. This is because

(a) neutron is a composite particle made of a proton and an electron whereas proton is a fundamental particle.

(b) neutron is an uncharged particle whereas proton is a charged particle.

(c) neutron has larger rest mass than the proton.

(d) weak forces can operate in a neutron but not in a proton.

12. As the mass number A increases, which of the following quantities related to a nucleus do not change?

(a) Mass. (b) Volume.

(c) Density. (d) Binding energy.

13. In which of the following decays the element reduce does not change ?

(a) α-decay (b) β^+-decay

(c) β^--decay (d) γ-decay.

14. Two lithium nuclei in a lithium vapour at room temperature do not combine to form a carbon nucleus because

(a) a lithium nucleus is more tightly bound than a carbon nucleus.

(b) carbon nucleus is unstable particle.

(c) it is not energetically favourable.

(d) coulomb repulsion does not allow the nuclei to come very close.

15. Magnetic field does not cause deflection in

(a) α-rays. (b) beta-plus rays.

(c) beta-minus rays. (d) gamma rays.

16. An α-particle is bombarded on ^{14}N. As a result, a ^{17}O nucleus is formed and a particle is emitted. This particle is a

(a) neutron. (b) proton.

(c) electron. (d) positron.

17. Ten grams of ^{57}Co kept in an open container beta-decays with a half-life of 270 days. The weight of the material inside the container after 540 days will be very nearly

(a) 10 g. (b) 5 g.

(c) 2.5 g. (d) 1.25 g.

18. Which of the following are electromagnetic waves ?

(a) α-rays. (b) beta-plus rays.

(c) beta-minus rays. (d) gamma rays.

19. Before 1900 the activity per mass of atmospheric carbon due to the presence of $14C$ averaged about 0.255 Bq per gram of C $14C$.?

(a) 1 ^{14}C in every 10^{14} ^{12}C.

(b) 3 ^{14}C in every 10^{12} ^{12}C.

(c) 4 ^{14}C in every 10^{10} ^{12}C.

(d) 2 ^{14}C in every 10^{11} ^{12}C.

20. The fusion of two nuclider will require a temp of the order of

(a) 10^6K (b) 10^7K

(c) 10^8K (d) 10^9K

21. If F_{NN}, F_{NP}, F_{PP} denotes net force between neutron and neutron, neutron and proton, proton and proton then

(a) $F_{NN} = F_{NP} = F_{PP}$ (b) $F_{NN} = F_{NP} > F_{PP}$

(c) $F_{NN} = F_{NP} < F_{PP}$ (d) $F_{NN} > F_{NP} > F_{PP}$

22. $^{221}_{87}Ra$ under goes radioactive decay with $t_{1/2} = 4$ days. What is the probability that a neucleus under goes a decay in two half lives (8 days)? IIT 2006

(a) 1 (b) 1/2

(c) 3/4 (d) 1/4

23. A γ -ray of energy 1900 MeV is absorbed by

(a) electron - positron pair

(b) photo electric effect

(c) proton-antiproton pair

(d) producing heat in the substance

24. ${}_1^2H + {}_4^9Be \rightarrow X + {}_2^4He$. Identify X.

(a) ${}_3^7Li$ (b) ${}_3^6Li$

(c) ${}_4^7Be$ (d) $2\,{}_2^3He$

25. Two identical nuclei A and B of the same radioactive element undergo β decay. A emits a β-particle and changes to A'. B emits a β-particle and then a γ-ray photon immediately afterwards, and changes to B'.

(a) A' and B' have the same atomic number and mass number.

(b) A' and B' have the same atomic number but different mass numbers.

(c) A' and B' have different atomic number but the same mass number.

(d) A' and B' are isotopes.

26. A and B are isotopes. B and C are isobars. All three are radioactive.

(a) A, B and C must belong to the same element.

(b) A, B and C may belong to the same element.

(c) It is possible that A will change to B through a radio-active-decay process.

(d) It is possible that B will change to C through a radio-active-decay process.

27. The decay constant of a radioactive sample is λ. Its half-life is $T_{1/2}$ and mean life is T.

(a) $T_{1/2} = \frac{1}{\lambda},\ T = \frac{\text{In } 2}{\lambda}$ (b) $T_{1/2} = \frac{\text{In } 2}{\lambda},\ T = \frac{1}{\lambda}$

(c) $T_{1/2} = \lambda \text{ In } 2,\ T = \frac{1}{\lambda}$ (d) $T_{1/2} = \frac{\lambda}{\text{In } 2},\ T = \frac{\text{In } 2}{\lambda}$.

28. The count rate from 100 cm^3 of a radioactive liquid is c. Some of this liquid is now discarded. The count rate of the remaining liquid is found to be c/10 after three half-lives. The volume of the remaining liquid, in cm^3, is

(a) 20 (b) 40

(c) 60 (d) 80

29. The value of A in the following reaction is

${}_4Be^9 + {}_2He^4 = {}_6C^A + {}_0n^1$

(a) 14 (b) 10

(c) 12 (d) 16

30. In the fusion process there are

(a) isotopes of hydrogen. (b) isotopes of helium.

(c) isotopes of carbon. (d) isotopes.

31. For fast chain reaction, the size of U^{235} block, as compared to its critical size, must be

(a) greater. (b) smaller.

(c) same. (d) anything.

32. One of the characteristics of nuclear reactions is that in their decayed or fused parts

(a) total mass number keeps on changing.

(b) total charge number keeps on changing.

(c) total charge number remains constant.

(d) all of the above.

33. Out of atom bomb and hydrogen bomb of same capacity which one is more harmful?

(a) Atom bomb.

(b) Hydrogen bomb.

(c) Sometimes atom bomb sometimes hydrogen bomb.

(d) Nothing can be said.

34. When beryllium is bombarded by á-particles, then we obtain

(a) electron. (b) proton.

(c) positron. (d) neutron.

35. The fissionable material used in a breeder reactor is—

(a) ${}_{92}U^{235}$ (b) ${}_{94}Pu^{239}$

(c) ${}_{90}Th^{234}$ (d) ${}_6C^{12}$

36. Hydrogen bomb is based on

(a) controlled chain reaction.

(b) uncontrolled chain reaction.

(c) nuclear fusion. (d) nuclear fission.

37. Atomic reactor is based on

(a) controlled chain reaction.

(b) uncontrolled chain reaction.

(c) nuclear fission. (d) nuclear fusion.

38. The ratio of the volume of atom to volume of nucleus is

(a) 10^5 (b) 10

(c) 10^{15} (d) 10^{10}

39. The fission of U^{238} is possible by

(a) only fast neutrons.

(b) only slow neutrons.

(c) fast as well as slow neutrons.

(d) fast protons.

40. The first atomic reactor was made by

(a) Fermi. (b) Bohr.

(c) Taylor. (d) Rutherford.

41. When the number of nucleons in the nucleus increased then the binding energy per nucleon

(a) decreases continuously with A.

(b) increases continuously with A.

(c) remains constant with A.

(d) first increases with A and then decreases.

42. The necessary condition for nuclear fusion is

(a) high temperature and high pressure.

(b) low temperature and low pressure.

(c) high temperature and low pressure.

(d) low temperature and high pressure.

43. In stable nuclei the number of protons (Z) and number of neutrons (N) are related as

(a) $N > Z$. (b) $N < Z$.

(c) $N = Z$. (d) $N = Z = 0$.

44. Which of the following is the best nuclear fuel ?

(a) U^{236} (b) Pu^{239}

(c) Np^{239} (d) Th^{236}

45. The critical mass of fissionable material is

(a) 75 Kg (b) 1 Kg

(c) 20 Kg (d) 10 Kg

46. The energy of neutrons obtained during fission is approximately

(a) 2KeV (b) 4GeV

(c) 1MeV (d) Zero

47. The energy emitted per second by sun is approximately

(a) 3.8×10^{26} Joule. (b) 3.8×10^{14} Joule.

(c) 3.8×10^{12} Joule. (d) 3.8×10^{-26} Joule.

48. ${}_{92}U^{235} + {}_{0}n_{1} = {}_{54}Xe^{139} + \ldots.Y + {}_{20}n^{1}$ Here Y is

(a) ${}_{38}Sr^{95}$ (b) ${}_{38}Si^{95}$

(c) ${}_{38}Ge^{95}$ (d) ${}_{38}Ni^{95}$

49. The energy released in the explosion of atom bomb is mainly due to

(a) nuclear fission. (b) nuclear fusion.

(c) chemical reaction.

(d) radioactive disintegration.

50. The difference between the atom U^{235} and U^{238} is that

(a) U^{238} contains 3 neutrons more.

(b) U^{238} contains 3 neutrons and 3 electrons more.

(c) U^{238} contains 3 protons more.

(d) U^{238} contains 3 proton and 3 electrons more.

51. The mass number (A) of a nucleus as compared to its charge number (Z) is

(a) always greater. (b) always less.

(c) some times equal.

(d) sometimes equal and sometimes more.

52. The fissionable materials used in the bomb dropped on the city Nagasaki of Japan in 1945 was

(a) Pu. (b) U.

(c) Th. (d) Np.

53. The energy equivalent to 1kg of matter is (in Joule)

(a) 10^{17}. (b) 10^{20}.

(c) 10^{11}. (d) 10^{14}.

54. 1 a.m.u. is equivalent to

(a) 931MeV. (b) 139MeV.

(c) 93MeV. (d) 39MeV.

55. The binding energy of a nucleus is equivalent to

(a) the mass of nucleus. (b) the mass of proton.

(c) the mass of neutron.

(d) the mass defect of nucleus.

56. The atomic weight of uranium isotope, which is easily fissionable, is

(a) 235. (b) 238.

(c) 234. (d) 236.

57. The critical mass of U^{235} can be reduced by

(a) putting a neutron reflector around it.

(b) heating it.

(c) mixing impurity in it.

(d) putting neutron absorber around it.

58. The fusion process is possible at high temperatures because at high temperatures

(a) the nucleus disintegrates.

(b) molecules disintegrate.

(c) atoms become ionised.

(d) the nuclei get sufficient energy so as to overcome the Coulomb repulsive force.

59. For making atom bomb, what else is needed except U^{235} ?

(a) Neutron. (b) Proton.

(c) Electron. (c) Meson.

60. The mass defect for helium nucleus is 0.0304 a.m.u. The binding energy per nucleon of helium nucleus is

(a) 28.3 MeV (b) 7.075 MeV

(c) 9.31 MeV (d) 200 MeV

61. The most suitable material for moderator in a nuclear reactor is

(a) D_2O (b) Cd
(c) B (d) ${}_{92}U^{235}$

62. Which of the following reactions is impossible?
(a) ${}_2He^4 + {}_4Be^9 = {}_0n^1 + {}_6c^{12}$
(b) ${}_2He^4 + {}_7N^{14} = {}_1H^1 + {}_8O^{17}$
(c) $4({}_1H^1) = {}_2He^4 + 2({}_{-1}e^0)$
(d) ${}_3Li^7 + {}_1H^1 = {}_4Be^8$

63. In nuclear reactor which of the following quantities is conserved ?
(a) Only energy. (b) Only mass.
(c) Only momentum.
(d) Mass, energy and momentum.

64. For maintaining sustained chain reaction, the following is required.
(a) Protons. (b) Electrons.
(c) Neutrons. (d) Positrons.

65. Two deutrons fuse to form a helium nucleus and energy is released, because the mass of helium nucleus is
(a) equal to that of two deutrons.
(b) less than that of two deutrons.
(c) more than that of two deutrons.
(d) all of the above.

66. The energy of thermal neutrons is nearly
(a) 0.25eV. (b) 0.025eV.
(c) 200MeV. (d) 0.025 Joule.

67. The correct relation between the packing fraction P and mass number A is
(a) $P = \frac{M-A}{A}$ (b) $P = \frac{M+A}{A}$
(c) $P = \frac{A}{M-A}$ (d) $P = \frac{A}{M+A}$

68. The curve between binding energy per nucleon (E) and mass number A is

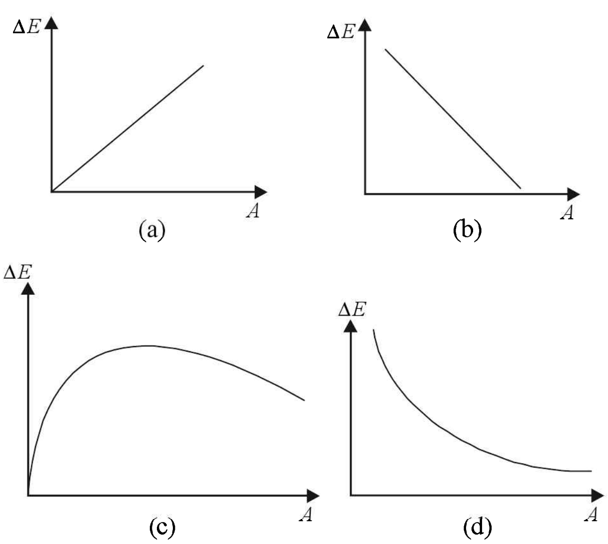


Fig. 35.11

69. Sun maintains its shining because of
(a) the fission of helium.
(b) chemical reaction.
(c) fusion of hydrogen nuclei.
(d) burning of carbon.

70. Critical mass is minimum mass necessary for
(a) chain reaction (b) fusion
(c) hydrogen bomb (d) all of the above

71. The ratio r of the rate of production of neutrons from uranium nucleus to the rate of leakage of neutrons for sustained chain reaction is
(a) $r > 1$ (b) $1 > r$
(d) $1 > r^2$ (c) $r^2 = 1$

72. The temperature necessary for fusion reactions is
(a) $3 \times 10^3 K$ (b) $3 \times 10^6 K$
(c) $3 \times 10^2 K$ (d) $3 \times 10^4 K$

73. Which of the following is a neutron absorber ?
(a) Pb (b) Ag
(c) Cd (d) Cu

74. A neutron decays to
(a) one p, one ν one β^+.
(b) one β^+, one β^- and ν.
(c) one p, one β^- and one $\bar{u}$.
(d) all of the above.

75. The nucleus ${}_{48}Cd^{115}$, after the emission two successive negative β-particles, changes to
(a) ${}_{49}Sn^{114}$. (b) ${}_{50}Sn^{113}$.
(c) ${}_{46}Pa^{115}$. (d) ${}_{50}Sn^{115}$.

76. The particle which is deflected by an electric field is
(a) α-particle. (b) neutron.
(c) x rays. (d) γ rays.

77. The maximum frequency is of
(a) blue light. (b) γ-rays.
(c) ultraviolet rays. (d) infrared rays.

78. The half life of a radioactive substance, as compared to its mean life, is
(a) 30% (b) 60%
(c) 70% (d) 100%

79. The nucleus ${}_{92}X^{234}$ emits 3α-particle and then one β-particle. The end product will be
(a) ${}_{84}Y^{222}$ (b) ${}_{87}Y^{228}$
(c) ${}_{84}Y^{228}$ (d) ${}_{87}Y^{222}$

80. The half life of radium is 1600 years. The fraction of radium atoms that remain undecayed after 4800 years will be

(a) $\frac{1}{4}$ (b) $\frac{1}{16}$

(c) $\frac{1}{8}$ (d) $\frac{1}{8}$

81. The half life of radioactive substance is 6 years. The time taken by 12 gms of this substance to decay completely will be

(a) ∞ (b) 48 years

(c) 18 years (d) 72 years

82. The ionising power is

(a) same in all the three.

(b) maximum in α- particles.

(c) maximum in β-particles.

(d) maximum in γ-rays.

83. The uranium nucleus $_{92}U^{238}$ emits an α-particle and resulting nucleus emits one β-particle. The atomic number and mass number of the final nucleus will respectively be

(a) 91,234 (b) 90,234

(c) 91,238 (d) 92,234

84. If the decay constant of radium is 4.28×10^{-4} per year, then its half life will approximately be

(a) 1240 years. (b) 1620 years.

(c) 2000 years. (d) 2260 years.

85. The particle emitted in the nuclear reaction

$_ZX^A = _{Z+1}Y^A +$ .... will be

(a) α particle (b) β^- particle

(c) β^+ particle (d) Photon

86. The fraction of atoms of radioactive element that decays in 6 days is 7/8. The fraction that decays in 10 days will be

(a) $\frac{77}{80}$ (b) $\frac{71}{80}$

(c) $\frac{31}{32}$ (d) $\frac{15}{16}$

87. The two elements, with same number of electrons but different mass number, are known as

(a) isotopes. (b) isomers.

(c) isotones. (d) isobars.

88. The decay constant of a radioactive sample is λ. The values of its half life and mean life will respectively be

(a) $\frac{1}{\lambda}, \log_e 2$. (b) $\frac{\log_e 2}{\lambda}, \frac{1}{\lambda}$.

(c) $\frac{1}{\lambda}, \frac{1}{\lambda^2}$. (d) $\lambda \log_e 2, \frac{1}{\lambda}$.

89. According to Geiger–Nuttal law the curve between $\log \lambda$ and $\log R$ will be

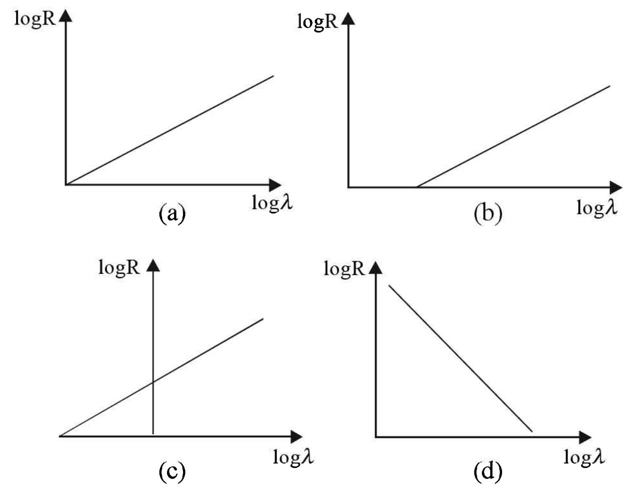


Fig. 35.12

90. The S.I. unit of radioactivity is

(a) Becquerl. (b) Curie.

(c) Rutherford. (d) Roentgen.

91. The activity of a radioactivity substances is

(a) $\frac{\lambda dN}{dt}$ (b) $\frac{dN}{dt}$

(c) $\frac{N d\lambda}{dt}$ (d) $\frac{1}{\lambda}\frac{dN}{dt}$

92. The nucleus obtained after α-emission from the nucleus $_yA^x$ is

(a) $_{y-2}B^{x-2}$ (b) $_{y+2}B^{x+4}$

(c) $_yB^x$ (d) $_{y-2}B^{x-4}$

93. An α-particle and a proton enter a perpendicular uniform magnetic field. The period of revolution of α-particles as compared to that of proton is

(a) 2 (b) 3

(c) 4 (d) 1

94. The particles emitted by a radioactive element are

(a) neutral.

(b) emitted from the nucleus.

(c) elelctromagnetic radiations.

(d) electrons revolving round the nucleus.

95. In majority of radioactive elements, the ratio of the number of neutrons to that of protons

(a) decreases. (b) increases.

(c) remains constant.

(d) sometimes decreases sometimes increases.

96. The mass of α-particle is

(a) $4\,M_p$ (b) $4M_n$

(c) $2\,M_p + 2\,M_n$ (d) $2\,M_c + 2\,M_p$

97. If $_5B^{11}$ converts into $_6C^{11}$, then the particle emitted in this process will be

(a) electron. (b) proton.

(c) neutron. (d) positron.

98. Carbon dating is

(a) a process to determine the age of archaeological samples.

(b) a medicine for treatment of cancer.

(c) a process to determine the age of a meteorite.

(d) a process of treatment of diseases in animals.

99. If the half life of a radioactive material is 100 days, then its half life after 10 days will become

(a) 50 days. (b) 200 days.

(c) 400 days. (d) 100 days.

100. If 10% of a radioactive substance decays in every 5 years, then the percentage of the substance that will have decayed in 20 years will be

(a) 40%. (b) 50%.

(c) 65.6%. (d) 34.4%.

101. If the half lives of a radioactive element for α and β decay are 4 years and 12 years respectively, then the percentage of the element that remains after 12 years will be

(a) 6.25%. (b) 12.5%.

(c) 25%. (d) 50%.

102. The masses of two radioactive substances are same and their half lives are 1 year and 2 years respectively. The ratio of their activities after six years will be

(a) 1 : 4 (b) 1 : 2

(c) 2 : 1 (d) 4 : 1

103. An atom with mass number A_1 converts into another atom with mass number A_2 after radioactive decay. The correct statement is

(a) A_2 can never be equal to A_1.

(b) the value of A_1 will be less than A_2.

(c) the value of A_2 will not be more than that of A_1.

(d) the value of A_1 will not be more than that of A_2.

104. If there are N nuclear particles in a nucleus of radius R, then the number of nuclear particles in radius 3R will be

(a) N (b) $2N$

(c) $27N$ (d) $\frac{21}{3}N$

105. The intensity of γ-rays from a source (I_0) reduces to $\frac{I_0}{8}$ after passing through 48 mm thick sheet of lead. The thickness of the sheet for obtaining intensity equal to $\frac{I_0}{2}$ will be

(a) 48 mm (b) 24 mm

(c) 16 mm (d) 8 mm

106. The specific activity of Cobalt-57, whose half life is 270 days, will be

(a) 8478 Curie/gm. (b) 847.8 Curie/gm.

(c) 84.78 Curie/gm. (d) 1 Curie/gm.

107. The mass of radon-222 corresponding to 1 Curie will be, if its half is 3.8 days

(a) 6.46×10^{-8} gm. (b) 64.60×10^{-7} gm.

(c) 6.46×10^{-10} gm. (d) 64.60×10^{-10} gm.

108. The particle X in the following nuclear reaction is

$_7N^{13} = {_6C^{13}} + {_1e^0} + X$

(a) P (b) v

(c) e^- (d) P

109. The particle X in the following nuclear reaction is

$_5B^{10} + {_2He^4} = {_7N^{13}} + X$

(a) P (b) α

(c) e (d) n

110. The particle X in the following nuclear reaction is

$_3Li + {_1H^1} = {_2He^4} + X$

(a) α (b) n

(c) e (d) p

111. The fraction of a radioactive material which remains active after time t is 9/16. The fraction which remains active after time t/2 will be

(a) $\frac{4}{5}$ (b) $\frac{7}{8}$

(c) $\frac{3}{5}$ (d) $\frac{3}{4}$

112. The half life of a radioactive substance is 34.65 minute If 10^{22} atoms are active at any inslant of time, then its activity will be

(a) 1×10^{18} disintegrations/sec.

(b) 2×10^{18} disintegrations/sec.

(c) 3.33×10^{18} disintegrations/sec.

(d) 4×10^{18} disintegrations/sec.

113. The half life of Sr is 28 years. The fraction of the specimen that remains undecayed after 40 years will be

(a) 21%. (b) 37%.

(c) 45%. (d) 63%.

114. The weight based ratio of U^{238} and Pb^{226} in a sample of rock is 4 : 3. If the half life of U^{238} is 4.5×10^9 years then the age of rock is

(a) 9.0×10^9 years. (b) 6.3×10^9 years.

(c) 4.5×10^9 years. (d) 3.78×10^9 years.

115. Which of the following reactions is correct ?

(a) There are 78 neutrons in $_{78}Pt^{192}$

(b) $_{84}Po^{214} \longrightarrow {}_{82}Pb^{210} + \beta$

(c) $_{92}U^{238} \longrightarrow {}_{90}Th^{234} + {}_{2}He^{4}$

(d) $_{90}Th^{234} \longrightarrow {}_{91}Pa^{234} + {}_{2}He^{4}$

116. The initial number of atoms of a radioactive element with half life 100 days, is 9.6×10^{20}. The number of atoms remaining undecayed after 500 days, will be

(a) 9.6×10^{20} (b) 3.84×10^{20}
(c) 3×10^{20} (d) 19.2×10^{20}

117. Initially 480 α-particles per minute are being emitted by a radioactive substance. This number reduces to 240 after 2 hours, The number of α-particles being emitted per minute after next 4 hour will be

(a) 0 (b) 60
(c) 80 (d) 120

118. The fraction of Cs^{137} atoms decaying in one year is 1/27 Its mean life will be

(a) 27 years. (b) 18.2 years.
(c) 13.5 years. (d) 0.037 years.

119. The mass of U^{234}, with half life 2.5×10^5 years, corresponding to its activity of 1 Curie will be

(a) 163.62 gm. (b) 1.438×10^{-11} gm.
(c) 3.7×10^{-10} gm. (d) 3.7×10^{10} gm.

120. If a radioactive substance decays for time interval equal to its mean life, then its fraction that remains undecayed will be

(a) 0.6322. (b) 0.3678.
(c) 0.50. (d) 0.75.

121. The half life of radioactive nuclei is 3 minute. What fraction of 1 gm of this element will remain after 9 minute?

(a) $\frac{1}{2}$ (b) $\frac{1}{4}$
(c) $\frac{1}{8}$ (d) $\frac{1}{16}$

122. The half lives of radioactive elements X and Y are 3 minute and 27 minute respectively. If the activities of both are same, then the ratio of number of atoms of X and Y will be

(a) 1 : 9 (b) 1 : 10
(c) 1 : 1 (d) 9 : 1

123. If the half life of a radioactive substance is T, then the fraction of its initial mass that remains after time $T/2$ will be

(a) $\frac{\sqrt{2}-1}{\sqrt{2}}$ (b) $\frac{3}{4}$
(c) $\frac{1}{2}$ (d) $\frac{1}{\sqrt{2}}$

124. In the radioactive decay process of uranium the initial nuclide is $_{92}U^{238}$ and the final nuclide is $_{82}Pb^{206}$. When uranium nucleus decays to lead, then the number of α- and β-particles emitted will respectively be

(a) 8, 6 (b) 8, 4
(c) 6, 8 (d) 4, 8

125. The rate of decay of radioactive element at a given instant of time is 10^3 disintegration/second. If the half life of this element is I second, then the rate of decay after 3 second will be

(a) 12 per sec. (b) 50 per sec.
(c) 500 per sec. (d) 125 per sec.

126. The $\log_e N$-$\log_e t$ curve for radioactive material is

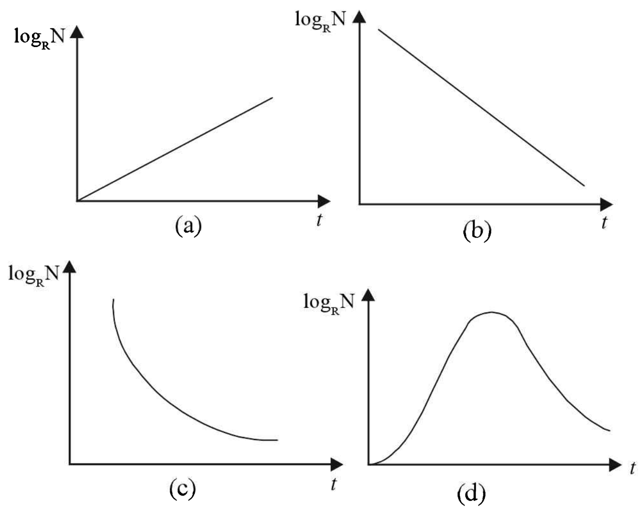


Fig. 35.13

127. An archaeologist analyses a wooden sample of prehistorical structure and finds that the ratio of C^{14} to ordinary carbon in it is only on fourth of that in the calls of living plants. If the half life of C^{14} is 5700 years, then the age of the wood will be

(a) 22,800 years. (b) 5700 years.
(c) 2450 years. (d) 11,400 years.

128. As a result of radioactive disintegration the nucleus $_{92}U^{238}$ converts into $_{91}Pa^{234}$. The particles emitted during the process are

(a) 2 β particle, 2p.
(b) 1 α particle, 1 β particle.
(c) 2 β particle, 1n (neutron).
(d) 1p, 2n.

129. A nucleus, with mass number m and atomic number n, emits one α-particle and one β-particle. The mass number and atomic number of the resulting nucleus will respectively be

(a) $(m-2), n$. (b) $(m-4), (n-1)$.
(c) $(m-4), (n-2)$. (d) $(m+4), (n-1)$.

130. The half life of C^{14} is 5730 years. What fraction of C^{14} will remain unchanged after 5 half lives ?

(a) $\frac{1}{16}$ (b) $\frac{1}{8}$

(c) $\frac{1}{64}$ (d) $\frac{1}{32}$

131. The half life of radioactive radon is 3.8 days. The time in which 1/20 fraction of radon remains undecayed, is

(a) 16.5 days. (b) 76 days.

(c) 3.8 days. (d) 33 days.

132. A given sample contains 16 gms of radioactive material whose half life is two days. The quantity of radioactive material that remains in that sample after 32 days will be

(a) $\frac{1}{2}$ g (b) $\frac{1}{8}$ g

(c) $\frac{1}{4}$ g (d) less then 1 mg

133. The rate of decay of a radioactive element at any instant is 10 disintegrations per second. If the half life of the element is 1 second then the rate of decay after one second will be

(a) 500 per sec. (b) 1000 per sec.

(c) 250 per sec. (d) 2000 per sec.

134. The particles emitted by a radioactive substance are deflected in a magnetic field. The particle may be

(a) neutrons. (b) electrons.

(c) protons. (d) hydrogen atoms.

135. Which of the following radiations can penetrate 20 cm thickness of steel ?

(a) β-particals. (b) α-particles.

(c) γ-particles. (d) ultraviolet rays.

136. The particles not emitted by a radioactive substance are

(a) γ-rays. (b) electrons.

(c) protons. (d) helium nuclei.

137. The isotope used in carbon dating is

(a) C^{14} (b) C^{12}

(c) C^{13} (d) N^{14}

138. The phenomenon explained by tunnel effect is

(a) β-decay. (b) α-decay.

(c) γ-decay. (d) All of above.

Answers to Questions for Practice

1.	(d)	**2.**	(d)	**3.**	(c)	**4.**	(d)	**5.**	(c)	**6.**	(b)	**7.**	(b)
8.	(d)	**9.**	(c)	**10.**	(c)	**11.**	(c)	**12.**	(c)	**13.**	(d)	**14.**	(d)
15.	(d)	**16.**	(b)	**17.**	(a)	**18.**	(d)	**19.**	(b)	**20.**	(d)	**21.**	(c)
22.	(c)	**23.**	(c)	**24.**	(a)	**25.**	(a)	**26.**	(d)	**27.**	(b)	**28.**	(d)
29.	(c)	**30.**	(a)	**31.**	(a)	**32.**	(c)	**33.**	(b)	**34.**	(b)	**35.**	(b)
36.	(c)	**37.**	(a)	**38.**	(c)	**39.**	(a)	**40.**	(a)	**41.**	(d)	**42.**	(a)
43.	(c)	**44.**	(b)	**45.**	(d)	**46.**	(c)	**47.**	(a)	**48.**	(a)	**49.**	(a)
50.	(a)	**51.**	(d)	**52.**	(a)	**53.**	(a)	**54.**	(a)	**55.**	(d)	**56.**	(a)
57.	(a)	**58.**	(d)	**59.**	(a)	**60.**	(b)	**61.**	(a)	**62.**	(c)	**63.**	(d)
64.	(c)	**65.**	(b)	**66.**	(b)	**67.**	(a)	**68.**	(c)	**69.**	(c)	**70.**	(a)
71.	(a)	**72.**	(b)	**73.**	(c)	**74.**	(c)	**75.**	(d)	**76.**	(a)	**77.**	(b)
78.	(c)	**79.**	(d)	**80.**	(c)	**81.**	(a)	**82.**	(b)	**83.**	(a)	**84.**	(b)
85.	(b)	**86.**	(c)	**87.**	(a)	**88.**	(a)	**89.**	(b)	**90.**	(a)	**91.**	(b)
92.	(d)	**93.**	(a)	**94.**	(b)	**95.**	(a)	**96.**	(c)	**97.**	(a)	**98.**	(a)
99.	(d)	**100.**	(d)	**101.**	(a)	**102.**	(a)	**103.**	(c)	**104.**	(c)	**105.**	(c)
106.	(a)	**107.**	(b)	**108.**	(b)	**109.**	(d)	**110.**	(a)	**111.**	(d)	**112.**	(c)
113.	(b)	**114.**	(d)	**115.**	(c)	**116.**	(c)	**117.**	(b)	**118.**	(a)	**119.**	(a)
120.	(b)	**121.**	(c)	**122.**	(a)	**123.**	(d)	**124.**	(a)	**125.**	(d)	**126.**	(b)
127.	(d)	**128.**	(b)	**129.**	(b)	**130.**	(d)	**131.**	(a)	**132.**	(d)	**133.**	(a)
134.	(b)	**135.**	(c)	**136.**	(c)	**137.**	(a)	**138.**	(b)				

Explanations

22. (c) required probability = $1 - e^{-dt} = 1 - e^{-\lambda(2T)} = 1$ (2T)

$$1 - e^{-\frac{\log e^2}{\pi}(2T)} = 1 - \frac{1}{4} = \frac{3}{4}.$$

28. (d) Initial count rate (*CR*) for 1 cm^3 of liquid = $\frac{C}{100}$ 2

after 3 half-lives, *CR* for 1 cm^3 of liquid = $\frac{1}{8} \times \frac{C}{100}$

Let the volume of the remaining liquid = $V\,cm^3$.

$\therefore$ *CR* of this liquid = $V \times \frac{C}{800} = \frac{C}{10}$

or $V = 80$.

36

Semiconductors

BRIEF REVIEW

We can have four types of conductors:

(a) super conductors (b) good conductors

(c) semiconductors (d) bad conductors or insulators

Superconductivity was first discovered by K. Onnes in 1911.

Super conductors have zero resistance at low temperatures below a certain maximum called critical temperature. They are perfect diamagnets (Meissner effect). If temperature is greater than a critical temperature T_C, they become normal conductors. They become normal conductor even if a magnetic field greater than critical magnetic field is applied. BCS (Bardeen Cooper Schereffer) theory, according to this theory current is carried by electron pair called cooper pair instead of individual electrons. The highest temperature at which superconductor is known is 160 K. Note that cooper pair is a boson. Though, yet new theory has not arrived but there is little evidence that at high temperature electron pair could exist.

Good conductors are metals. Their resistivity increases with rise in temperature according to

$$\rho(T) = \rho_{(o)}[1 + \alpha T]$$

Semiconductors have a unique property that their conductivity increases with rise in temperature. Fig. 36.1 illustrates how resistivity falls with rise in temperature. This phenomenon can be explained on the basis of band theory. The energy bands which are completely filled at 0 K are called valence bands. The bands with higher energy are called conduction bands. We will refer to valence band as the top most filled band and conduction band as lowest conduction band. E_v is the topmost energy of valence band and E_c is the bottom most energy of conduction band, the $E_g = E_c - E_v$ represents forbidden energy gap.

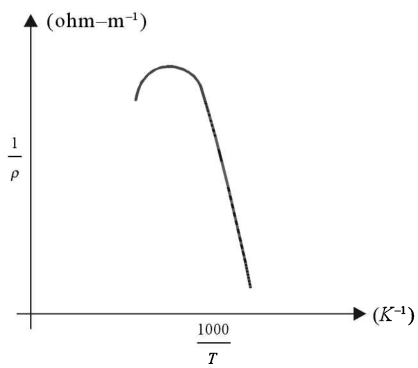


Fig. 36.1 Variation of resistivity vs temperature in a semiconductor

In metals $E_g \to 0$, that is, valence band and conduction band overlap so that a large number of electrons lie in the conduction band.

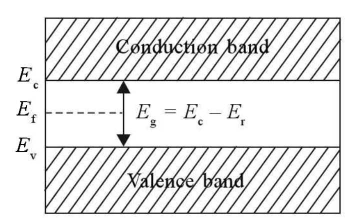


Fig. 36.2 Energy band diagram of a semiconductor

In semiconductors $E_g \sim 1$ eV. At room temperature it is about 1–2 % filled. At 0 K semiconductor is a perfect insulator.

Fermi level is an imaginary level which lies between valence band and conduction band such that the probability of finding an **electron is 50 % or $\frac{1}{2}$.**

At 0 *K* Fermi level is the highest filled level. Fermi level is used as reference level.

E_g (for Ge) = 0.71 eV and E_g (for Si) = 1.12 eV

In insulator, $E_g \sim 6$ eV. For example, E_g (for diamond) = 6.3 eV

Semiconductors are of two types (a) **intrinsic** (b) **extrinsic** or **doped** semiconductors.

In intrinsic semiconductor no impurity, from 13th or 15th group of the periodic table has been added. So that density of holes in valence band is equal to density of electrons in conduction band, that is, $n_i = h_i$ where n is electron density and h is hole density (subscript '*i*' stands for intrinsic).

Extrinsic semiconductor is of two types *p*-type and *n*-type. In *p*-type majority carriers are holes. Thus, $h_p > n_p$. Impurities from 13th group (*B, Al, Ga, In*) is added to make a semiconductor *p*-type. Such a type of impurity is called acceptor impurity. Acceptor level E_A lies very close to valence band (VB).

In *n*-type semiconductor majority carriers are electrons. It is made by doping donor impurity, i.e., impurity from 15th group of periodic table like P, As, Bi, Sb. Thus, $n_n > h_n$. Donor level lies very close to conduction band (CB).

In thermal equilibrium condition $n_e h_e = n_i^2$ (subscript *e* denotes extrinsic). Fig. 36.3 (a) and 36.3 (c) show energy band diagram for *p*- and *n*-type semiconductor. In heavily doped *p*- or *n*-type acceptor or donor impurity level lies inside *VB* and *CB* respectively. Fig. 36.3 (b) shows heavily doped *p*-type.

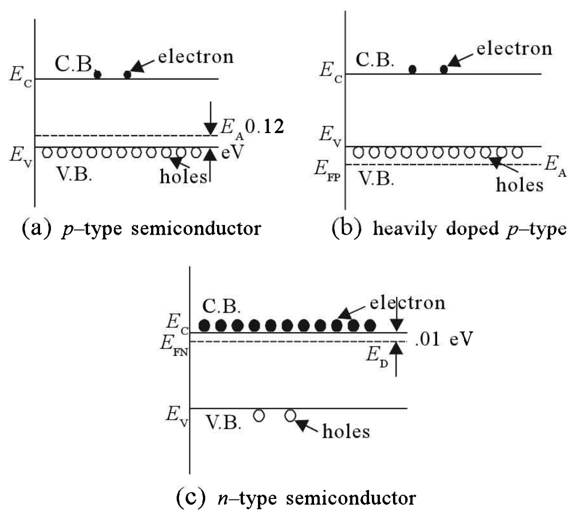


Fig. 36.3 Energy band diagram of extrinsic semiconductors.

Conductivity in semi conductors is due to holes and electrons both. Moreover, total conduction is due to diffusion and drift currents. In an intrinsic semiconductor, conductivity $\sigma = ne\,\mu_n + he\,\mu_h$

where μ_n and μ_h are mobility of electrons and holes respectively. For *n*-type semiconductor.

Assuming $n \cong N_D$

$\sigma = ne\,\mu_n = N_D e \mu_n$

For *p*-type semiconductor

Assuming $h \cong N_A$

$\sigma = he\,\mu_n = N_A e \mu_h$

***pn* junction** When *p*- and *n*-type semiconductors of same material, either both of Si or both of *Ge* are fused together, or, *n*-type is grown on *p*-type, then such a device is called *pn* junction or semiconductor diode.

Potential barrier and Depletion layer Due to charge density gradient, electrons from *n*-type move towards *p*-type close to the junction and are accepted by acceptor impurity atoms present there. Similarly, holes from *p*-type ionize impurity atoms present close to the junction forming a **fictitious battery** V_B or **potential barrier** V_B. Potential barrier and depletion layer are illustrated in Fig. 36.4 (a) and 36.4 (b).

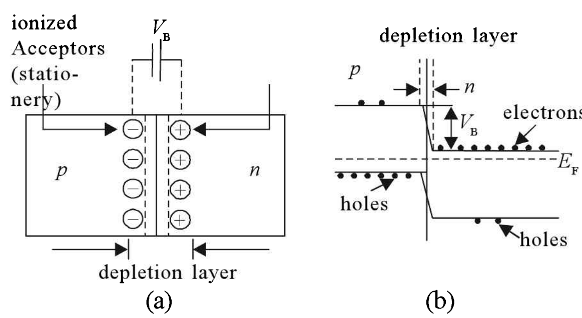


Fig. 36.4 (a) pn junction (b) Energy band diagram of a p-n junction

Depletion layer is the layer close to the junction devoid of carriers due to migration of charge carriers (electrons and holes) and acceptance by acceptor and donor impurity atoms.

Forward biasing If positive terminal of an external battery is connected to *p*-type and its negative terminal to *n*-type of the *pn* junction then such a biasing is called forward biasing. Forward biasing reduces potential barrier and hence, depletion layer width decreases. The current is due to majority carriers. See Fig. 36.5 (a) and Fig. 36.6. Current is quite large when applied, with forward voltage > V_B or V_r.

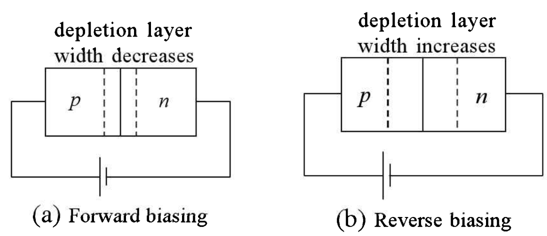


Fig. 36.5 Biasing of p-n junction

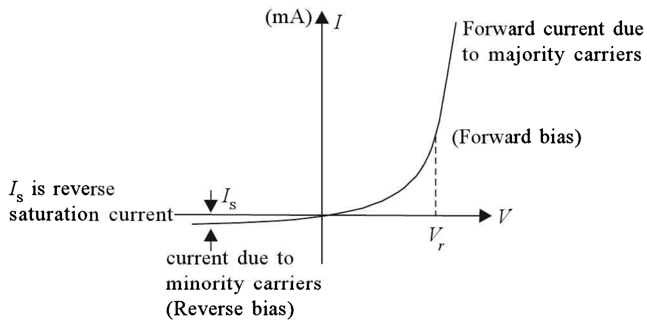


Fig. 36.6 Characteristics of pn junction

Reverse biasing If positive terminal of external battery is connected to n-type and negative terminal to p-type as shown in Fig. 36.5 (b), then such a biasing is called **reverse biasing** or **back biasing**. It increases the barrier potential and hence, depletion layer width. Extremely low current due to minority carrier flows. The current is nearly constant and is termed as reverse saturation current as shown in Fig. 36.6. Fig. 36.7 shows circuit symbol for an ideal diode. It is clear from characteristic curve shown in Fig. 36.6 that pn junction acts very closely like a valve.

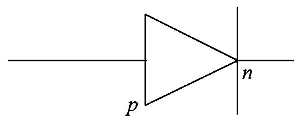


Fig. 36.7 Circuit symbol of an ideal pn junction

Equation of current in a pn junction $I = I_S\left[e^{V/V_T} - 1\right]$ where I_S is reverse saturation current, V is applied potential, V_T = thermal voltage $V_T = \dfrac{kT}{e} = 0.026$ V at 300 K where k is Boltzmann's constant, T is temperature (Kelvin) and e is charge on an electron.

Dynamic or Incremental Resistance

$r = \dfrac{\Delta V}{\Delta I} = \dfrac{dV}{dI}$ is called dynamic resistance. It may be determined from the characteristic curve as shown in Fig. 36.8. Its value is different at different points. From the diode equation $\dfrac{dI}{dV} = \dfrac{I_S}{V_T}e^{V/V_T} = \dfrac{I}{V_T}$

or $r_f = \dfrac{dV}{dI} = \dfrac{V_T}{I} = \dfrac{26mV}{1mA} = 26\ \Omega$

In forward bias $I = 1$ mA $r_f = 26\ \Omega$ (low).

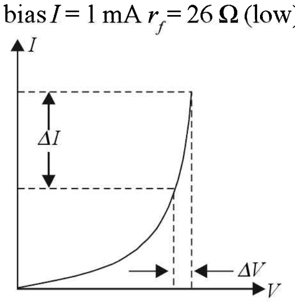


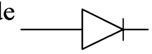
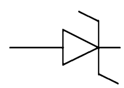
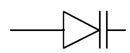
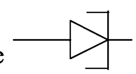
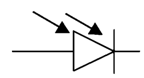
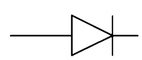
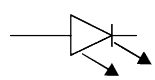
Fig. 36.8 Dynamic resistance determination

In reverse bias case $r_r = \dfrac{V_T}{I}$ as $I \to 0$, $r_r \to \infty$. In an actual diode $r_r \geq 10^4\ \Omega$.

Ideal diode Ideal diode is like a voltage controlled switch. When forward biased it acts like an ON switch (zero resistance) and when reverse biased it acts like an OFF switch (infinite resistance).

Table 36.1 lists some of important types of diodes and their applications along with their circuit symbols.

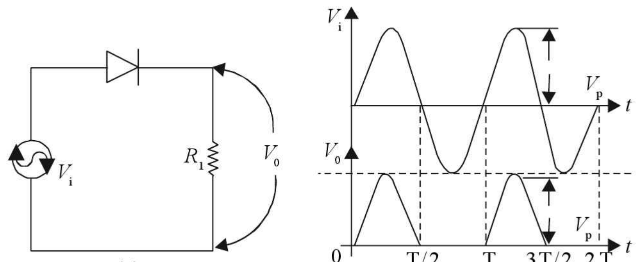
Table 36.1 Types of p-n junctions

Type of diode	Circuit symbol	Applications
1. General purpose diode		Demodulator, voltage multiplier clipping, clamping, rectifier, peak detector, waveshaping.
2. Avalanche or Zener or breakdown diode		Load regulator, reference voltage formation.
3. Varactor or Varicap		Frequency modulation (FM) voltage to frequency converter.
4. Tunnel diode or Esaki diode		Oscillator, Astable/ monostable multivibrator
5. Photo diode		Burglar alarm, fire alarm, remote sensing, automatic switching of light, nuclear detector, communication system
6. Switching diode		Logic gates
7. Light emitting diode (LED)		Indicator, remote control, optical fiber communication, system alpha-numeric (7-segment, 14-segment displays) devices.

Rectifier It converts AC to unidirectional pulsating output. In other words it converts AC to DC.

Rectifiers are of two types

(a) Half Wave Rectifier (b) Full Wave Rectifier



(a) Half wave rectifier

(b) Its input/output characteristics

Fig. 36.9

Half-wave rectifier Fig. 36.9 (a) illustrates half wave rectifier and Fig. 36.9 (b) shows the input/output characteristics.

In a half-wave rectifier

$$V_o = V_i = V_P \sin \omega t \quad 0 < t < \frac{T}{2}$$

$$V_o = 0 \qquad \frac{T}{2} < t < T$$

$$V_{out}(dc) = \frac{V_P}{\pi}$$

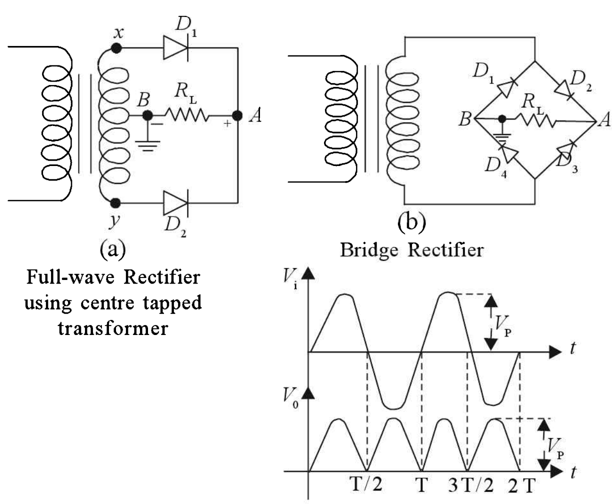
$$V_{out}(rms) = \frac{V_P}{2}.$$

$$\text{Ripple factor } \gamma = \frac{V_{AC}}{V_{DC}} = \sqrt{\left(\frac{V_{rms}}{V_{DC}}\right)^2 - 1} = 1.21$$

$$\text{Rectification efficiency } \eta = \frac{P_{DC}}{P_{rms}} \times 100 = 40.6\%$$

Frequency of output signal = frequency of input signal

Full-wave rectifier gives output in both the half cycles. Circuit and input/output characteristic are shown in Fig. 36.10 (a), 36.10 (b) and 36.10 (c) respectively.



(a) Full-wave Rectifier using centre tapped transformer

(b) Bridge Rectifier

(c) Input/output characteristics

Fig. 36.10

Note that a full-wave rectifier can be made using a centre tapped transformer or a bridge rectifier.

For a full-wave recitifier

$$V_o = V_i = V_P \sin \omega t \qquad 0 < t < \frac{T}{2}$$

$$V_o = -V_P \sin \omega t \qquad \frac{T}{2} < t < T$$

$$V_{out}(DC) = \frac{2V_P}{\pi} \;;$$

$$V_{out}(rms) = \frac{V_P}{\sqrt{2}}$$

$$\text{Ripple factor } \gamma = \frac{V_{AC}}{V_{DC}} = 0.48$$

$$\text{Rectification efficiency } \eta = \frac{P_{DC}}{P_{rms}} \times 100 = 81.2\,\%$$

Frequency of output signal = 2 × Frequency of input signal. Fig. 36.10 (c) or the characteristics mentioned suggest we shall prefer a full wave rectifier.

Bridge Rectifier is preferred as the diodes used shall have peak inverse voltage (PIV) half that of the value needed in full wave rectifier, made using centre tapped transformer.

Negative Resistance See Fig. 36.11. In the region AB. $I \propto \frac{1}{V}$. This region is termed as negative resistance region. The devices which show negative resistance are

(a) tunnel diode.

(b) tetrode (vacuum tube).

(c) thyristors.

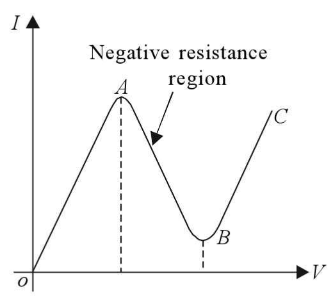


Fig. 36.11 Negative resistance illustration

Photo diodes are operated in **reverse bias.** If frequency of incident radiation is such that $hf \geq E_g$ then conductivity will increase as electrons from valence band jump to conduction band creating conduction electrons (in CB) and holes (in V.B.).

Thus, if $hf \geq E_g$ or $\frac{hc}{\lambda} \geq E_g$ or $\lambda \leq \frac{hc}{E_g}$ conductivity will increase.

LED (Light emitting diode) If the diode is forward biased and band gap E_g is such that $\lambda = \frac{hc}{E_g}$ lies in visible region for a transition of electron from conduction band to valence band then light will be emitted. The band gap E_g of *Ge* or *Si* does not warrant visible light emission. Therefore, GaAs, GaAlAs. GaInP, GaAlP, InP etc. are used to make LEDs which emit light in visible region. By varying % contents band gap E_g in such compounds can be varied.

Drawback of diode Note from Table 36.1 pn junction or diode cannot be used as an amplifier.

Transistor is made of words TRANSfer + resISTOR (TRANS from transfer and ISTOR from resistor). Thus, transistor is a device which gives transfer of resistor without changing the current at input or output, i.e., same current flows through input and output while resistance at the two places are different. This device is designed to make amplifier. Obviously if $R_{out} > R_{input}$ then $P = I^2 R$ gives us clue that output power is more than input power. This is the principle of amplifier.

Transistor is basically of three types (a) UJT (uni junction transistor), (b) BJT (bipolar junction transistor), (c) FET (Field effect transistor). BJT is of two types *npn* and *pnp*. FET is of three types JFET (Junction field effect transistor), MOSFET (Metal oxide semiconductor field effect transistor), IGFET (Insulated gate field effect transistor).

In a BJT, emitter is heavily doped, base should be extremely thin. Fig. 36.12 (a) shows *pnp*-transistor and its circuit symbol, and Fig. 36.12 (b) shows *npn*-transistor and its circuit symbol.

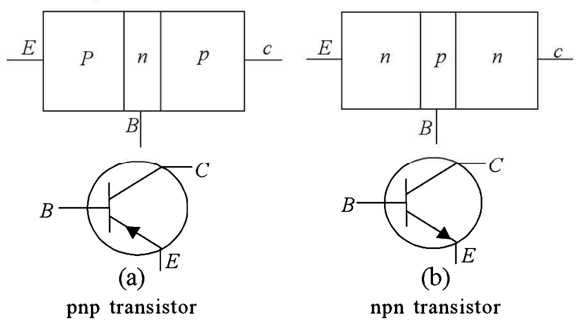


Fig. 36.12 Bipolar junction transistor

A transistor can be considered as a junction. Then,

$$I_E = I_B + I_C$$

$$I_C = \beta I_B + (\beta + 1) I_{CBO}$$

$I_C = \alpha I_E + I_{CBO}$. I_{CBO} is collector base Junction current when emitter is open.

The term I_{CBO} is temperature dependent. I_{CBO} doubles for every 8-10°C rise in temperature.

α is current gain for common base amplifier and $\alpha < 1$. β is current gain for common emitter amplifier and $\beta > 1$. Each transistor circuit requires temperature compensation. Therefore, self bias arrangement is used which automatically, adjusts/bias to compensate the effect of temperature.

$$\beta = \frac{I_C}{I_B} \ (>1); \qquad \alpha = \frac{I_C}{I_E} \ (<1);$$

$$\beta = \frac{\alpha}{1-\alpha}; \qquad \alpha = \frac{\beta}{1+\beta}.$$

A transistor can operate in three regions. In saturation region, transistor acts like an ON switch (dynamic resistance is 8Ω). In cut off region, transistor behaves as an OFF switch (resistance ≥ 10^4 Ω). In active region transistor acts as an amplifier. See Fig. 36.13 to understand these regions. Note that in active region characteristics are equidistant and parallel for equal change in input, i.e., output is directly proportional to input. Hence, active region is also called linear region.

Cut off region is achieved when both collector base junction and emitter base junction are reverse biased. Saturation region is achieved when both collector base (CB) junction and emitter base (EB) junction are forward biased.

In active region emitter base junction is forward biased and collector base junction is reverse biased. Cut off and saturation regions are used in logic gates.

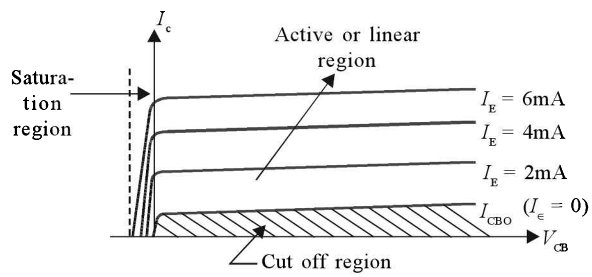


Fig. 36.13 Output characteristics of common base

Transconductance or transfer conductance or mutual conductance $g_m = \frac{\partial I_C}{\partial V_{BE}}$.

Amplifier is a circuit which gives power gain. Table 36.2 shows characteristic of common base (CB) Amplifier, common emitter (CE) and common collector (CC) Amplifiers.

One can note from Table 36.2 that

(i) **CB Amplifier** is a **voltage amplifier** as it amplifies only voltage.

(ii) **CC Amplifier** is a **current amplifier** as it amplifes only current.

(iii) **CE Amplifier** amplifies both current and voltage. It gives a **phase shift of 180°** between input and output signal. This is the reason, it is used to make a **NOT** gate or **inverter**. Do not get the notion that CE amplifier is the best as it gives A_i and A_v both > 1.

Common collector amplifier is also known as a **buffer amplifier** or an **emitter follower.**

In another classification amplifiers may be of four types: class A, class B, class AB and class C.

Class A amplifier It amplifies complete signal (0–360°) using a single transistor. It is used when the signal is small,

Table 36.2 Characteristics of Amplifiers

Property	Common base Amplifier	Common emitter Amplifier	Common collector Amplifier
Input impedance (Z_i)	Low	medium high	medium high
Output impedance (Z_o)	high	high	low
Current gain (A_i)	$A_i = \alpha < 1$	$A_i = \beta > 1$	$A_i = (\beta + 1) > 1$
Voltage gain (A_v)	$A_v = A_i \dfrac{R_L}{r_e} > 1$ $= \alpha \dfrac{R_L}{r_e} > 1$	$A_v = A_i \dfrac{R_L}{r_b} > 1$ $= \beta \dfrac{R_L}{r_b} > 1$	$A_v = A_i \dfrac{R_L}{r_b} < 1.$ $= (\beta + 1) \dfrac{R_L}{r_b} < 1$
Power gain ($A_p = A_v . A_i$)	$A_p = \alpha^2 \dfrac{R_L}{r_e} > 1$	$A_p = \beta^2 \dfrac{R_L}{r_b} > 1$	$A_p = (\beta + 1)^2 \dfrac{R_L}{r_b} > 1.$
Phase shift between input and output signal	nil	180° or π-rad	nil

Key words → Low ~ 25-30 Ω, medium high ~ 200 Ω, high ≥ 10^4 Ω r_e = dynamic resistance of emitter, r_b = dynamic resistance of base.

i.e., at the input or first stage or preamplifier stage. The transistor is biased in the active region in the midway as illustrated in Fig. 36.14. Q-point or operating point shows the bias point.

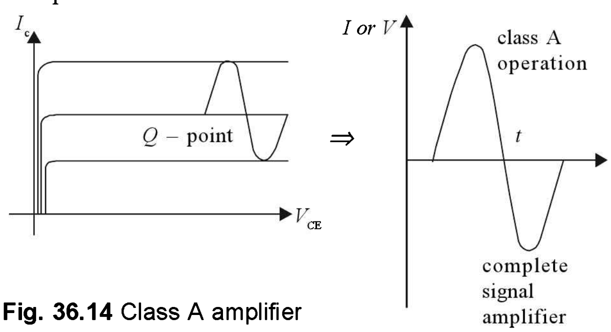


Fig. 36.14 Class A amplifier

Class B amplifier As shown in Fig. 36.15 Q-point in class B amplifiers is in cut off region. Therefore, they amplify only half the signal 0–180° or 180–360°.

Therefore, two transistors are required to amplify complete signal. At the output stage of amplifier system signal becomes large and is amplified using class B push-pull amplifier (two transistors one amplifier 0-180° and the other 180°–360°).

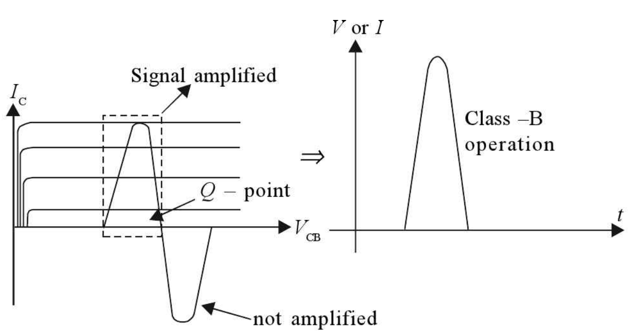


Fig. 36.15 Class B amplifier

Class AB Amplifier It amplifies > 180° but less than 360° using a single transistor as illustrated in Fig. 36.16(a). Q-point is slightly above cut off region. So a part of the back half signal is also amplified.

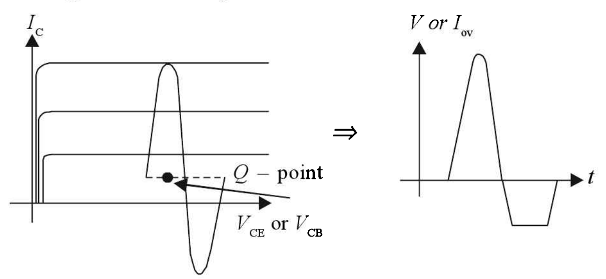


Fig. 36.16(a) Class AB amplifier

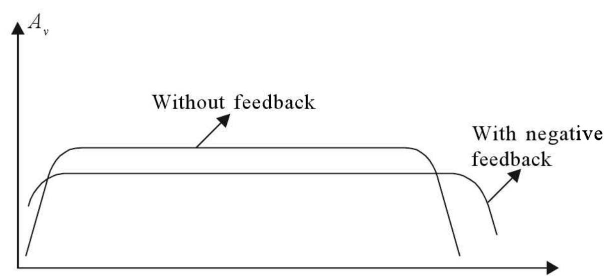


Fig. 36.16(b)

Class C amplifier They amplify only pulses when the signal is large. Class C amplifiers are used in transmitters. The figure shows amplifier frequency response curve. Note band-width increases with negative feedback.

Oscillator LC tank circuit is a basic oscillator. Due to dissipative element (internal resistance of the coil), the oscillations are damped. Hence, positive feedback or negative resistance is required to achieve sustained oscillations.

Barkhausen criterion should be satisfied to achieve sustained oscillation

$\beta A_v \geq 1$ where β is feedback factor and A_v is voltage gain. The criterion lists two points:

(a) there should be positive feedback.

(b) feedback factor $\beta \geq \dfrac{1}{A_v}$.

Therefore, a frequency selective feedback network is employed so that at a particular frequency called the frequency of oscillation $\beta A_v \geq 1$. This frequency is frequency of oscillation. Oscillators may be of two types (a) Audio-frequency oscillators (AFO) (b) Radio frequency oscillator (RFO).

To design an AF oscillicator, one requires an RC circuit. RC phase shift oscillator and Wein's bridge oscillator are popular AF oscillators. Now-a-days, normally operational amplifier (op-amp) is used to design amplifiers and oscillators. AF oscillators have frequency ≤ 20 kHz.

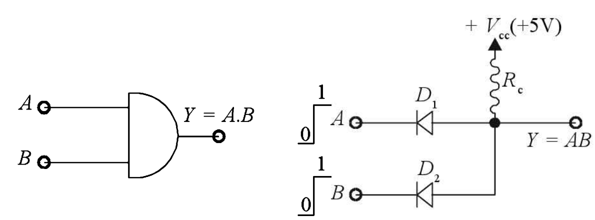
Radio frequency oscillators are LC oscillators. Hartley, Colpitt's, Clapp's, Crystal oscillators are popular RF oscillators. RF oscillators operate at high frequency > 100 kHz. They are used to generate carrier wave and as a local oscillator in a radio receiver.

In another categorization, oscillators may be of two types a) sinusoidal or sine/cosine wave generator b) relaxation oscillators. Relaxation oscillators generate any wave other than sine or cosine, that is, square, rectangular, triangular, sawtooth etc.

Logic gates Logic is of two types (a) positive logic (b) negative logic.

In positive logic high state +5V is assigned a '1' and low state (0V) a value '0' as illustrate in Fig. 36.17. In **negative logic** high state is assigned a '0' while a low state is assigned a '1'.

AND gate A positive logic AND gate assumes high state if and only if all the inputs are high. Circuit symbol of two input AND gate is shown in Fig. 36.17 (a) and circuit implementation using switching diodes in Fig. 36.17 (b). Operation symbol is '.' as it behaves like multiplication.



(a) circuit symbol of AND gate

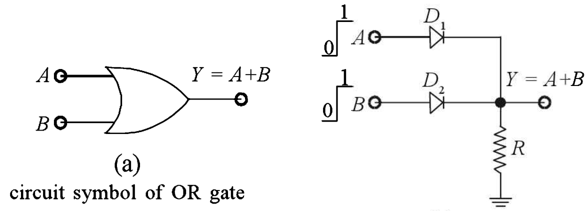
(b) circuit implementation of AND gate

Fig. 36.17

Table 36.3 Truth Table of AND gate

A	B	$Y = A.B$
0	0	0
0	1	0
1	0	0
1	1	1

OR gate A positive logic OR gate assumes a high state if any of the input is high. Fig. 36.18 (a) shows circuit symbol and Fig. 36.18 (b) shows the circuit implementation of OR using switching diodes. Operation symbols of OR is '+'.



(a) circuit symbol of OR gate

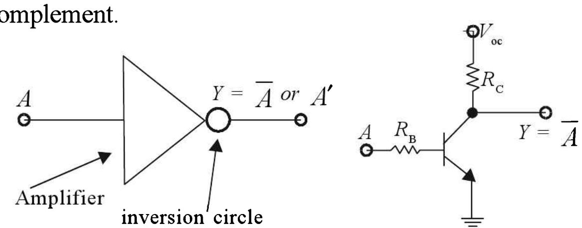
(b) circuit implement ation of OR gate

Fig. 36.18

Table 36.4 Truth Table of OR gate

A	B	$Y = A + B$
0	0	0
0	1	1
1	0	1
1	1	1

NOT gate or Inverter A NOT gate inverts the input ,i.e., *a* '0' input appears as *a* '1' or vice versa Fig. 36.19 (a) shows circuit symbol and Fig. 36.19 (b) circuit implementation using a switching transistor. Operator symbol a bar or a complement.



(a) circuit symbol of NOT

(b) circuit implementation

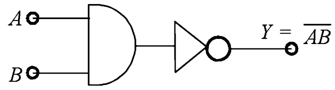
Fig. 36.19

Table 36.5 Truth Table of NOT gate

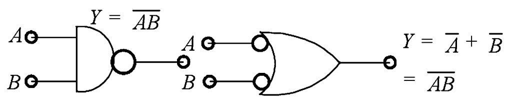
A	$Y = \overline{A}$
0	1
1	0

Note that NOT gate is a unary gate. All other gates are binary gates.

NAND (Negated AND) Output of AND is negated or inverted. Fig. 36.20 shows NAND gate implementation and its circuit symbols.



(a)
NAND = AND + NOT or NOT of AND



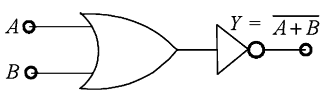
(b)
circuit symbol of NAND

Fig. 36.20

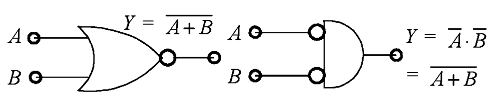
Table 36.6 Truth Table of NAND

A	B	$Y = \overline{AB}$
0	0	1
0	1	1
1	0	1
1	1	0

NOR (Negated OR) Output of OR gate is negated or inverted. Fig. 36.21 shows NOR gate implementation and its circuit symbols.



(a)
NOR = OR + NOT or NOT of OR



(b)
circuit symbol of NOR

Fig. 36.21

Table 36.7 Truth Table of NOR

A	B	$Y = \overline{A+B}$
0	0	1
0	1	0
1	0	0
1	1	0

Universal gate A gate which possesses following properties is called a universal gate

(a) Any gate/logic can be developed using a single gate or combination of similar gates.

(b) They follow associative or commutative laws.

(c) They can be manufactured economically.

NAND and NOR qualify these properties and hence, are termed as universal gates.

De-Morgan Laws

(i) $\overline{A+B} = \overline{A}\;\overline{B}$ (ii) $\overline{AB} = \overline{A} + \overline{B}$.

Duality principle When positive logic is changed to negative logic or vice versa AND changes to OR; OR changes to AND; NAND changes to NOR and NOR changes to NAND. Note a change of 0 with 1, and, 1 with 0 in a truth table will reveal this result.

Phantom OR or wired OR: Fig 36.22 shows Phantom OR or wired OR.

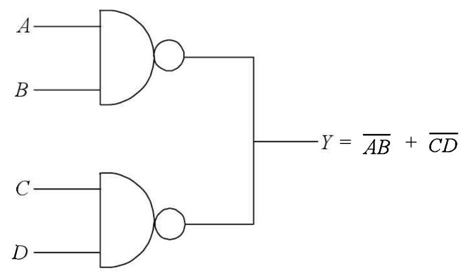
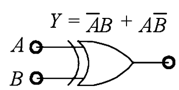
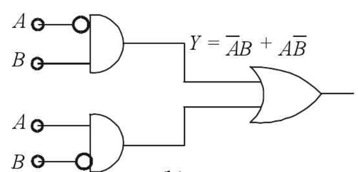


Fig. 36.22 Wired OR

XOR (Exclusive OR): Fig. 36.23 shows XOR gate. The output of XOR acts as sum bit of half Adder.



(a)
circuit symbol of XOR



(b)
implemeutation of XOR

Fig. 36.23

Conversion of gates: Fig. 36.24 shows NOT from NAND. Fig. 36.25 shows NOT from NOR.

NOT from NAND

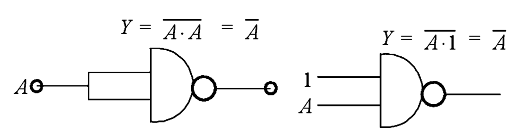


Fig. 36.24 Short both the inputs of NAND to get NOT

NOT from NOR

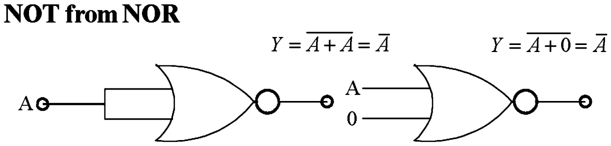


Fig. 36.25 Short both the inputs of NAND to get NOT

AND from NAND

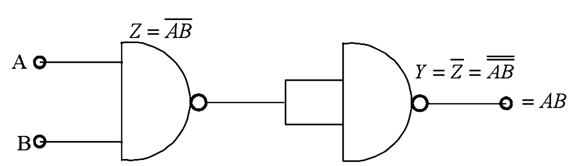


Fig. 36.26

AND from NOR

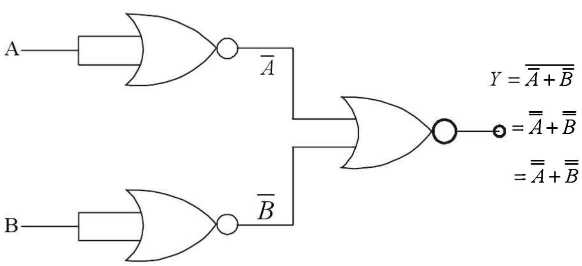


Fig. 36.27

OR from NAND

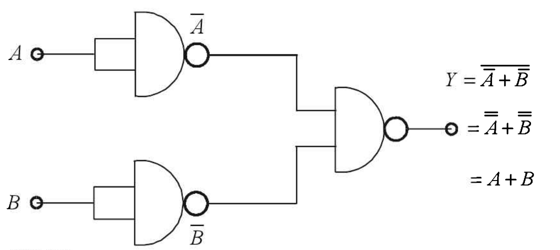


Fig. 36.28

OR from NOR

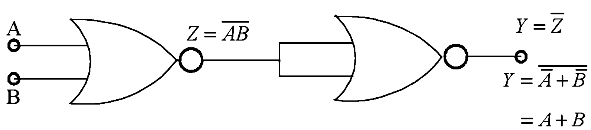


Fig. 36.29

Half Adder It is used to add two bits Augend and Addend. Sum bit S can be implemented using XOR and carry bit can be implemented using AND. Fig. 36.30 (a) shows circuit symbol of Half Adder and Fig. 36.30 (b) shows its gate implementation.

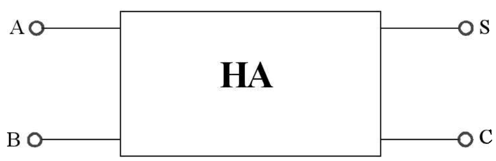


Fig. 36.30 (a) Half Adder

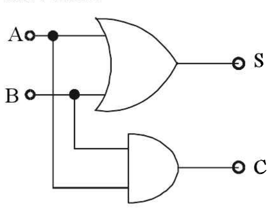


Fig. 36.30 (b) Gate implementation of half adder

Table 36.8 Truth Table of Half Adder

Augend A	Addend B	Sum S	Carry C
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

Full Adder It is used to add 3 bits—Augend (A_n), Addend (B_n) and carry from previous state (C_{n-1}). Sum (S_n) and carry to next state (C_n) are outputs. Fig. 36.31 (a) shows circuit symbol and Fig. 36.30 (b) shows gate implementation of Full Adder.

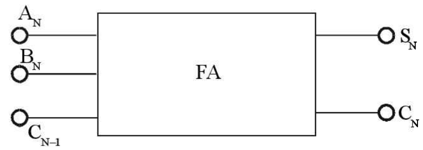


Fig. 36.31 (a)

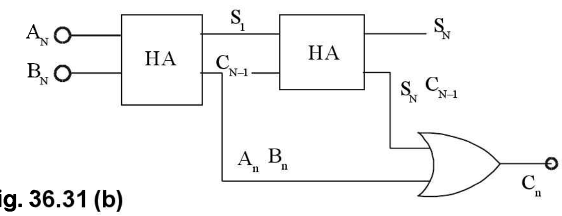


Fig. 36.31 (b)

Table 36.9 Truth Table of Full Adder

Augend A_n	Addend B_n	Carry from previous state C_{n-1}	Sum S_n	Carry C_n
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	1	0
1	1	1	1	1

From the table

$$S_n = \overline{A}_n\, \overline{B}_n\, C_{n-1} + \overline{A}_n\, B_n\, \overline{C}_{n-1} + A_n\, \overline{B}_n\, \overline{C}_{n-1} + A_n B_n C_{n-1}$$

$$= \left(\overline{A}_n\overline{B}_n + A_nB_n\right) C_{n-1} + \left(\overline{A}_nB_n + A_n\overline{B}_n\right) C_{n-1}$$

$$= A_n \oplus B_n \oplus C_{n-1}$$

$$C_n = \overline{A}_n\, B_n C_{n-1} + A_n\, \overline{B}_n\, C_{n-1} + A_n B_n\, \overline{C}_{n-1} + A_n B_n C_n$$

$$= S_1\, C_{n-1} + A_n B_n$$

Integrated Circuit (IC) If all the circuit elements like transistor, diode, capacitor resistor are built on a single chip and inter connected to form a complete circuit, then such a circuit is called an integrated circuit or an IC. ICs are in general of two types

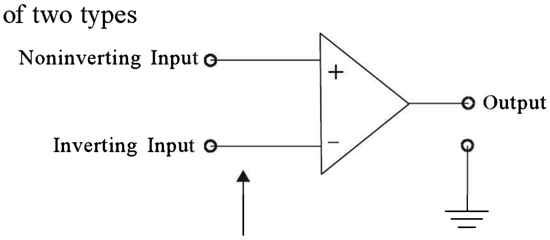


Fig. 36.32 Linear IC or op-amplifier

(a) Analog or linear or op-amplifier

and (b) digital or binary or logic families.

Analog ICs contain operational amplifiers (op-amp). These are direct coupled differential amplifiers. Fig. 36.32 shows circuit symbol of op-amp. Note, it has an inverting input terminal (–) and a non inverting input terminal (+). The difference between inverting and non inverting inputs is amplified. Gain of such amplifiers is very high ($10^5 - 10^8$). Therefore, an external feedback system is used to curtail gain. This amplifier can be used for any mathematical operation like addition, subtraction, multiplication, log, antilog, differentiation and integration.

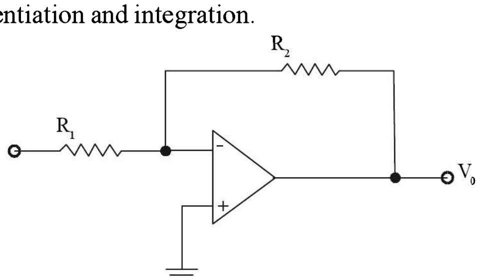


Fig. 36.33 Inverting amplifier or scale changer

In Fig. 36.34 configuration

$\frac{V_o}{V_i} = -\frac{R_2}{R_1}$. The Ratio $\frac{R_2}{R_1}$ decides the voltage gain under feedback condition.

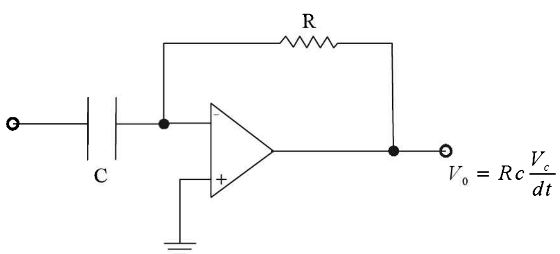


Fig. 36.34 (a) Differentiation circuit

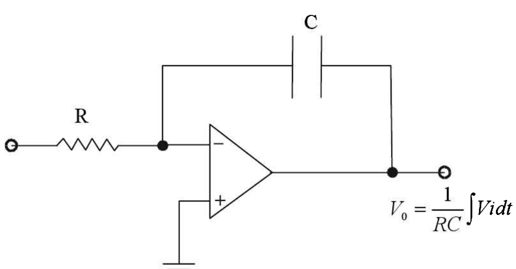


Fig. 36.34 (b) Integrating circuit

Fig. 36.34 (a) and Fig. 36.34 (b) show differentiation and integration implementation using op-amp.

Now-a-days, CMOS technology, biCMOS, GaAs and opto electronic ICs are being made. biCMOS, GaAs and opto-electronics ICs are very high speed devices.

• Short Cuts and Points to Note

1. Energy bands are formed due to degenerated energy levels in a crystal structure or bulk of the material.
2. The property of the semiconductor that resistivity falls with rise in temperature can be explained using band theory.
3. A semiconductor is a perfect insulator at 0 *K*.
4. In a semiconductor, conduction is due to drift as well as diffusion currents.
5. The mobility of electrons is 2–3 times higher than that of holes. Therefore, *npn* devices or n-channel MOSFET are faster than *pnp* devices or p-channel MOSFET.
6. In intrinsic or pure semiconductors $n_i = h_i$.
7. In extrinsic semiconductor, if n_e is the electron density and h_e is the hole density the material will be *n*-type if $n_e > n_i$ or $n_e > h_e$. The material will be p-type if $h_e > n_e$ or $h_e > n_i$. Moreover, in thermal equilibrium $n_e h_e = n_i^2$.
8. In metals, valence band and conduction band overlap therefore, $E_g = 0$. In semiconductors $E_g \sim 1$ eV and insulators have $E_g \sim 6$ eV.
9. In *n*-type semiconductor, conductivity
   $\sigma_n = ne\,\mu_n \cong N_D\,e\,\mu_n$.
   In *p*-type semiconductor conductivity
   $\sigma_p = he\,\mu_h \cong N_A\,e\,\mu_h$.
   In intr]insic semiconductor conductivity
   $\sigma_{\text{intrinsic}} = n_i\,e\,\mu_n + h_i\,e\,\mu_n$.
10. A *pn* junction or a diode may be assumed ideal diode. It may be assumed to act like an ON switch when forward biased and like an OFF switch when reverse biased. That is, diode shows full conduction ($r = 0$) when forward biased and no conduction ($r = \infty$) when reverse biased.
11. On forward biasing the diode resistance $r \to 0$, depletion layer width decreases. Current is mostly diffusion. (Actual current is diffusion current + drift current).

    On Reverse biasing the diode $r \to \infty$, depletion layer width increases. Current is drift current only.
12. In photo diodes conduction will increase (They are operated reverse biased) if wavelength of incident radiation $\lambda \le \frac{hc}{E_g}$.
13. In an LED light will be emitted f a wavelength $\lambda = \frac{hc}{E_g}$. Since, *Ge* and *Si* will emit IR they can be used in remote sensing, Robots etc. LEDs emitting light in visible region are made from Ga As, Ga In P, Ga In As, Ga Al As etc, i.e., it is an alloy of 13_{th} and 15_{th} group element forming a semiconductor where E_g depends on concentration of their constituents.

14. 60 Carbon atoms forming a football like structure behave as a semiconductor.

15. Diode cannot be used as an amplifier since it is a two terminal device.

16. Transistor amplifies by converting power of dc source into AC (of the signal applied). It uses the principle $P = I^2R$. If $R_{in} << R_{out}$ and current at input and output remains unchanged then power gain is obtained.

17. Current gain $A_i = \alpha < 1$

Voltage gain $A_v = \alpha \dfrac{R_L}{r_e} > 1$

Power gain $A_p = \alpha^2 \dfrac{R_L}{r_e} > 1$

No phase shift between input and output.

} In common base (CB) amplifier $\alpha = h_{FB}$

18. Current gain $A_i = \beta > 1$

Voltage gain $A_v = \beta \dfrac{R_L}{r_b} > 1$

Power gain $A_p = \beta^2 \dfrac{R_L}{r_b} > 1$

Phase shift = 180° or π rad

} In common emitter (CE) amplifier $\beta = h_{FE}$

19. Current gain $A_1 = \beta + 1$; Power gain $A_p = (\beta + 1)^2 \dfrac{R_l}{r_b} > 1$

Voltage gain $A_v = (\beta + 1)\dfrac{R_l}{r_b} < 1$;

Phase shift nil

} In common collector (CC) amplifier

Common collector amplifier is also called Power amplifier, Buffer amplifier, Current booster or Emitter follower.

20. $I_E = I_C + I_B$; $\alpha = \dfrac{I_C}{I_E}$; $\beta = \dfrac{I_C}{I_B}$, $\beta = \dfrac{\alpha}{1-\alpha}$ and $\alpha = \dfrac{\beta}{1+\beta}$.

21. Oscillator can be designed with any of the two techniques. Either use negative resistance device or positive feedback. When in the frequency selective network, Barkhausen criterion is satisfied then oscillations are generated. ($\beta A_v \geq 1$). Fig. 36.35 shows block diagram of an oscillator. *LC* oscillators are high frequency of *RF* oscillators while *RC* oscillators are low frequency or Audio frequency oscillators.

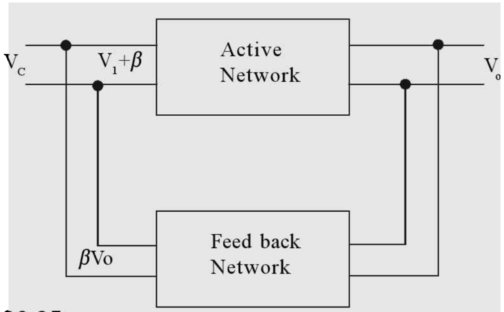


Fig. 36.35

22. Use the relations in order to simplify logical expressions.

(i) $A + A = A$ its dual $A \cdot A = A$

(ii) $A + \bar{A} = 1$ its dual $A \cdot \bar{A} = 0$

(iii) $A + 1 = A$ its dual $A \cdot 1 = A$

(iv) $A + AB = A$ its dual $A\,(A + B) = A$

(v) $A + \bar{A}B = A + B$ its dual $A\,(\bar{A} + B) = AB$

(vi) $\overline{A+B} = \bar{A}\bar{B}$; $\overline{AB} = \bar{A} + \bar{B}$.

(vii) $\bar{\bar{A}} = A$; $1 + 1 = 1 + 0 = 1$

$1.0 = 0$; $1.1 = 1$

23. NAND and NOR gates are universal gates. NOT gate is unipolar. All other gates are bipolar.

24. Devices like tunnel diode, thyristor, tetrode have negative resistance.

25. Transistors may operate in 3 regions. (a) Cut off region is obtained if both emitter base junction and collector base junction are reverse biased. It acts like an OFF switch. (b) Active or linear region is achieved when EB junction is forward biased ($V_{EB} >$ 0.6 V) and collector base (CB) junction is reverse biased. Amplifiers can be made only in this region. (c) Saturation region is achieved when both EB and CB junction are forward biased. In this region transistor acts like an ON switch.

26. Logic circuit are of two types combinational and sequential. Sequential circuits possess memory. In combinational circuit output depends only on the present inputs. In sequential circuits output depends not only on present input but on past inputs in chronological order.

27. Integrated circuits are of two types. a) Analog b) digital or logic family. Analog ICs has op-amps used in amplifiers, oscillators, D/A converter, timer circuits, power supplies–regulated and SMPS, function enerator filters, modulation and demodulation, phase locked loops etc.

Digital ICs contain simple logic gates, Adders, multiplexers, CPU, ASIC (Application specific integrated circuits). RAMs, ROMs, combinational locks, registers, counters, decoders, code converters etc. Digital circuits in general can be divided into SSI, MSI, LSI, VLSI and ULSI.

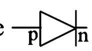
28. Op-amps are high gain dc coupled differential amplifiers and can be used even in mathematical operation like addition, difference, multiplication, log, antilog, differentiation, integration, scale changer etc.

29. If n amplifiers are connected in tandem having individual gains $A_1, A_2, \ldots A_n$. Then, the overall gain is $A_{net} = A_1 A_2 \ldots A_n$.

30. High speed ICs are made with GaAs, BICMOS. Optical fibers are even added to increase the speed.

• Caution

1. Forgetting the valve action of *pn* junction diode.

⇒ In *pn* junction current flow from p to n side 

, i.e.; arrow mark side.

2. Assuming that output of rectifier is dc, hence, its frequency is zero.

⇒ Frequency of output of half wave rectifier is same as that of input signal and frequency of output of full wave rectifier is twice that of input signal.

3. Assuming that rectification efficiency of a half wave rectifier is 50% and that of a full wave rectifier is 100% as in half wave rectifier half of the signal and in full wave rectifier complete signal is obtained.

⇒ Rectification efficiency of half wave rectifier is 40.6% and that of a full wave rectifier is 81.2%.

4. Assuming amplification means increasing the amplitude of current or of voltage.

⇒ Amplitude should be increased along with increase in power.

5. Not able to recall current gain in CE amplifier and CB amplifiers.

⇒ In CB amplifier current gain $A_i < 1$. $A_i = \alpha = h_{FB} = \frac{I_C}{I_E}$.

In CE amplifier current gain $A_i > 1$. $A_i = \beta = h_{FE} = \frac{I_C}{I_B}$.

6. Considering transistor cannot be used as Rectifier.

⇒ If only collector base or emitter base junction is considered then rectifier can be designed.

7. Assuming that Kirchhoff's laws cannot be applied in electronic circuits.

⇒ Kirchhoff's laws can be applied in circuits containing transistors or *pn* junction. A transistor can be considered a junction, therefore, $I_E = I_C + I_B$.

8. Not remembering the formulae for voltage gain and power gain.

⇒ Voltage gain $A_v = \alpha \frac{R_L}{r_e} > 1$ in CB amplifier.

Power gain $A_p = A_v . A_i - \alpha^2 \frac{R_L}{r_e} > 1$ in CB amplifier.

Voltage gain $A_v = \beta \frac{R_L}{r_b} > 1$ in CE amplifier.

Power gain $A_p = A_v . A_i = \beta^2 \frac{R_L}{r_b}$ in CE amplifier.

Note A_i, A_v or A_p is a ratio, therefore, they are dimensionless.

9. Assuming that the oscillator can be developed only with L and C.

⇒ Low frequency or audio frequency oscillators are made with R and C. Remember that basic requirement to make an oscillator is to fulfil the Barkhausen criterion ($\beta A_v \geq 1$).

10. Assuming in binary/logic circuits 1 + 1 = 2.

⇒ 1 + 1 is OR operation ∴ 1 + 1 = 1

and in binary number addition 1 + 1 = 10 (2 written in binary numbers).

11. Assuming in a circuit $V_i = 10$ mV, gain $A_v = 10^6$ then output must be 10^4 V.

⇒ Output in no case can exceed the dc biasing voltage applied.

12. Assuming amplifiers do not have internal source of distortion of signal.

⇒ Temperature dependence of minority carriers, causes thermal runaway. Moreover, the characteristics do not remain parallel and equidistant for large signal variation. Self bias system are to be used to prevent thermal runaway and large signals are amplified using Push-pull class B amplifiers.

13. Assuming mobility of hole and electron are equal.

⇒ Mobility of electron is 2-3 times larger than that of holes.

SOLVED PROBLEMS

1. Choose the only false statement from the following

(a) In conductors valence and conduction band may overlap.

(b) Substances with energy gap of the order of 10 eV are insulators.

(c) The resistivity of semiconductor increases with rise in temperature.

(d) The conductivity of semiconductor increases with rise in temperature.

[CBSE PMT 2005]

Solution (c) is false.

2. Zener diode is used for

(a) Amplification. (b) Rectification.

(c) Stabilization.

(d) Producing oscillations in oscillator.

[CBSE PMT 2005]

Solution (c)

3. Application of forward bias to a *pn* junction.

(a) widens the depletion zone.

(b) increases the potential difference across the depletion zone.

(c) increases the number of donors in the *n* side.

(d) increases the electric field in the depletion zone.

[CBSE PMT 2005]

Solution (c)

4. Carbon, Silicon and Germanium atoms have 4 Valence electrons each. Their valence and conduction bands are separated by energy band gaps represented by $(E_g)_C$, $(E_g)_{Si}$, $(E_g)_{Ge}$ respectively. Which one of the following relationship is true in their case?

(a) $(E_g)_C > (E_g)_{Si}$ (b) $(E_g)_C < (E_g)_{Si}$

(c) $(E_g)_C = (E_g)_{Si}$ (d) $(E_g)_C < (E_g)_{Ge}$

[CBSE PMT 2005]

Solution (a) The allotrope of C (diamond) is insulator.

$\therefore (E_g)_C > (E_g)_{Si}$

5. The electrical conductivity of semiconductor increases when electromagnetic radiation of wavelength shorter than 2480 nm is incident on it. The band gap in eV for semiconductor is

(a) 1.1 eV (b) 2.5 eV

(c) 0.5 eV (d) 0.7 eV

[AIEEE 2005]

Solution (c) $E_g(\text{eV}) = \dfrac{1240}{\lambda(nm)} = \dfrac{1240}{2480} = \dfrac{1}{2} = 0.5\text{ eV}$

6. In a common base amplifier, the phase difference between the input signal voltage and output signal voltage is

(a) $\pi/4$ (b) π

(c) 0 (d) $\dfrac{\pi}{2}$

[AIEEE 2005]

Solution (c) In CE amplifier only a phase shift of π exists.

7. In a full wave rectifier circuit operating from 50 Hz mains frequency, the fundamental frequency in the ripple would be

(a) 50 Hz (b) 100 Hz

(c) 25 Hz (d) 70.7 Hz

[AIEEE 2005]

Solution (b) $f_{(out)} = 2 f_{input}$

8. Which of the following gates is a universal gate

(a) OR (b) AND

(c) NOT (d) NAND

[AIIMS 2005]

Solution (d)

9. Consider an *npn* transistor amplifier in CE configuration. The current gain in the transistor is 100. If the collector changes by 1 mA. What will be the change in emitter current?

(a) 1.1 mA (b) 1.01 mA

(c) 0.01 mA (d) 10 mA

[AIIMS 2005]

Solution (b) $\beta = 100$ and $\alpha = \dfrac{\beta}{1+\beta} = \dfrac{100}{101}$;

Given $\dfrac{\Delta I_C}{\Delta I_E} = \dfrac{100}{101} = \dfrac{1mA}{\Delta I_E}$.

$\therefore \Delta I_E = 1.01$ mA.

10. In a semiconducting material the mobilities of electron and hole are μ_e and μ_h respectively. Which of the following is true?

(a) $\mu_e > \mu_h$ (b) $\mu_e = \mu_h$

(c) $\mu_h > \mu_h$ (d) $\mu_e > 0$; $\mu_h > 0$

Solution (a) mobility of electrons is 2 to 3 times larger than that of hole.

11. The voltage gain of the amplifier shown in Fig 36.36 is

(a) 10 (b) 100

(c) 9.9 (d) 1000

Solution (b) $A_v = \frac{R_f}{R_i} = \frac{100k\Omega}{1k\Omega} = 100.$

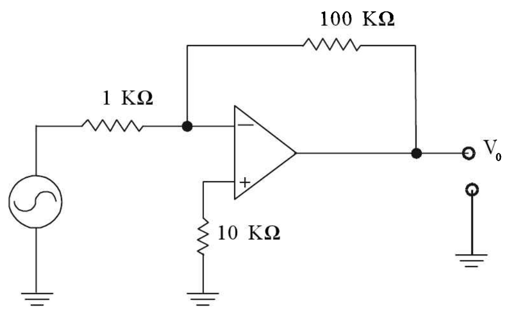


Fig. 36.36

12. *A*. The logic gate NOT can be built using diode.

R. The output and input voltage of the diode have 180° phase shift.

(a) *A* and *R* are true and *R* is correct explanation of *A*.

(b) *A* and *R* are true but *R* is not correct explanation of *A*.

(c) *A* is true but *R* is false.

(d) both *A* and *R* are false.

[AIIMS 2005]

Solution (d) NOT gate is made using CE transistor configuration.

13. *A*. The number of electrons in a *p*-type semiconductor is less than number of electrons in a pure silicon semiconductor at room temperature. *R*. It is due to law of mass action.

(a) *A* and *R* are true and *R* is correct explanation of *A*.

(b) *A* and *R* are true but *R* is not correct explanation of *A*.

(c) *A* is true but *R* is false.

(d) *A* and *R* both are false.

[AIIMS 2005]

Solution (a) According to law of massaction $h_e n_e = n_i^2$

14. *A*. In a CE transistor amplifier the input current is much less than output current. *R*. The common emitter transistor amplifier has very high input impedance.

(a) *A* and *R* are true and *R* is correct explanation of *A*.

(b) *A* and *R* are true but *R* is not correct explanation of *A*.

(c) *A* is true but *R* is false.

(d) *A* and *R* both are false.

[AIIMS 2005]

Solution (c) See table 36.2. The input impedance is moderately high.

15. The amplification factor of a triode valve is 15. If the grif voltage is changed by 0.3 V. The change in plate voltage in order to keep the current constant (in volt) is

(a) 0.02 V (b) 0.002 V

(c) 4.5 V (d) 5.0 V

[BHU 2005]

Solution (c) $\mu = \frac{\Delta V_p}{\Delta V_g} \Rightarrow 15 = \frac{\Delta V_p}{0.3}$ or $\Delta V_p = 4.5$ V

16. In a full wave rectifier, input ac current has a frequency ν, the output frequency of the current is

(a) 2ν (b) $\nu/2$

(c) ν (d) none

[BHU 2005]

Solution (a)

17. The forward voltage of the diode is increased, the width of depletion layer

(a) increases. (b) decreases.

(c) fluctuates. (d) no change.

[CET Karnataka 2005]

Solution (b)

18. Identify the property which is not characteristic for a semiconductor.

(a) At a very low temperature it behaves as an insulator.

(b) At higher temperatures two types of charge carriers will cause conductivity.

(c) The charge carriers are electrons and holes in the Valence band at higher temperature.

(d) The semiconductor is electrically neutral.

Solution (c) Electrons exist in conduction band and holes exist in Valence band.

19. The type of transition in Fig 36.37 takes place in

(a) Si (b) Ge

(c) C (d) Ga As

Solution (d) This is an example of direct band gap used in LED and Lasers.

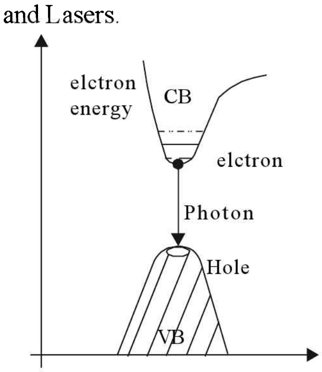


Fig. 36.37

20. If $V = V_m \sin \omega t$ is input then find the voltage at P.

(a) V_m (b) $\frac{3}{2} V_m$

(c) $\frac{V_m}{2}$ (d) $2V_m$

Solution (d) This is the circuit of voltage doubler.

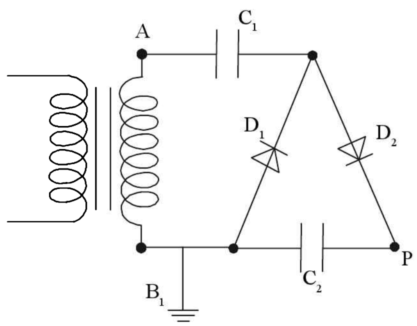


Fig. 36.38

21. The current through diode D_1 is

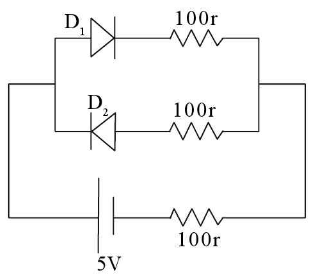


Fig. 36.39

(a) 2.5 mA (b) 25 mA

(c) zero (d) 33 mA

Solution (b) $\frac{5}{200} = 25\,\text{mA}$.

22. The capacitance shown by reverse biased *pn* junction is termed as

(a) fixed capacitance. (b) drift capacitance.

(c) diffusion capacitance. (d) plate capacitance.

Solution (c)

23. A silicon diode has knee or cut-in voltage equal to ...... Volts.

(a) 0.2 V (b) 0.3 V

(c) 0.6 V (d) 0.8 V

(e) none

Solution (c)

24. Tunnel diode is another name for

(a) power diode. (b) varactor diode.

(c) Photo diode. (d) Esaki diode.

(e) zener diode.

Solution (d)

25. To make a Full wave rectifier, the *ac* input be applied across

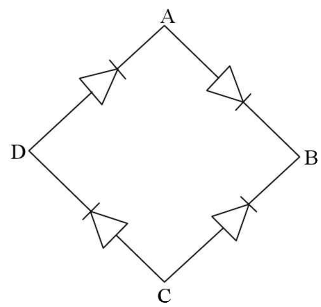


Fig. 36.40

(a) BD (b) BC

(c) AD (d) AC

(e) AB

Solution (d) and output is taken across BD.

26. Ebers Moll model describes the working of a

(a) pn junction diode. (b) BJT.

(c) MOSFET.

(d) UJT (unijunction transistor).

Solution (b)

27. A transistor has $h_{FE} = 95$, find h_{FB}.

(a) 1.9 (b) 0.20

(c) 0.94 (d) 0.99

Solution (d) $h_{FB} = \alpha$; $h_{FE} = \beta$

$$\alpha = \frac{\beta}{\beta + 1} = \frac{95}{96}.$$

28. If temperature rises by 10°C which of the following current doubles.

(a) I_C (b) I_B

(c) I_{CBO} (d) I_E

(e) none

Solution (c)

29. An amplifier has low output impedance and high input impedance. It is a ________________.

(a) CB amplifier. (b) CE amplifier.

(c) CC amplifier. (d) Pushpull class B amplifier.

Solution (c)

30. The circuit shown in Fig 36.41 used in nuclear physics as

(a) a coincidence circuit.

(b) an anti coincidence circuit.

(c) delayed coincidence circuit.

(d) none.

Solution (b)

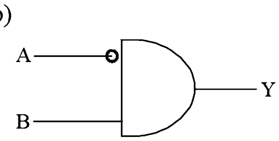


Fig. 36.41

31. In which of the following circuits capacitor is discharged if it was charged initially.

Solution (a)

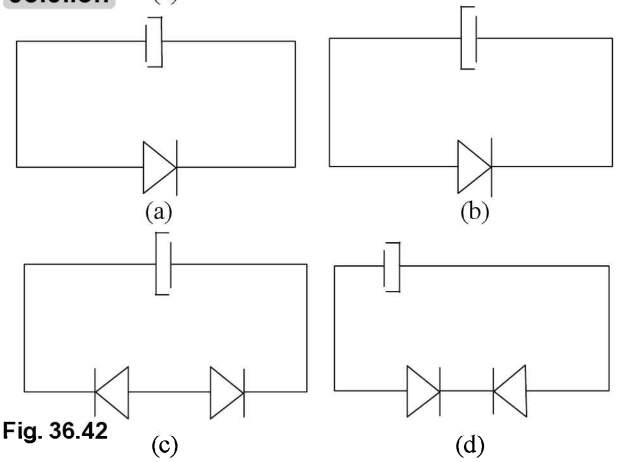


Fig. 36.42

32. From the truth table find the gate it represents

A	B	Y
0	0	1
0	1	0
1	0	0
1	1	1

(a) XOR (b) Associated NAND
(c) XNOR (d) NOR

Solution (c) $Y = \overline{A}\ \overline{B} + AB$ or $Y = \overline{\overline{A}B + A\overline{B}}$

33. A circuit used as local oscillator in Radio receivers is

(a) op-amp. (b) AF oscillator.
(c) RF oscillator. (d) phase locked loop.

Solution (c)

34. For an input sinusoidal wave train in Fig 36.43, the output wave form is..........

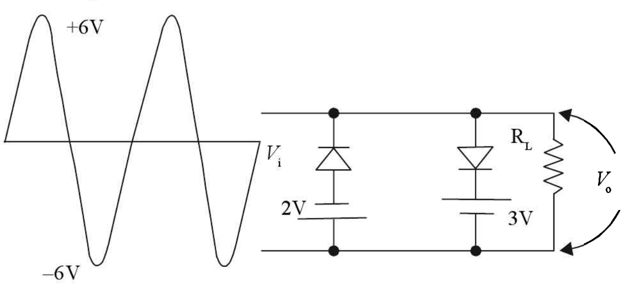


Fig. 36.43

Solution (d) Shunt clipping circuit operation

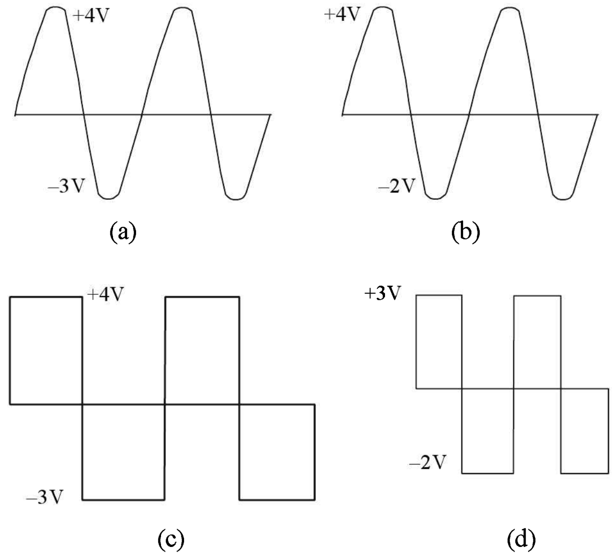


Fig. 36.44

35. The conductivity of a pure semiconductor is roughly ∝ $T^{3/2}e^{-\Delta E/2kT}$ where ΔE is band gap. The band gap for Ge is 0.74 eV at 4 K and 0.67 eV at 300 K. By what factors does the conductivity of pure Ge increases as the temperature rises from 4 K to 300 K.

(a) 10^{40} (b) 10^{48}
(c) 10^{201} (d) 10^{461}

Solution (d) $$\frac{\sigma_{300K}}{\sigma_{4K}} = \left(\frac{300}{4}\right)^{3/2}\left(\frac{e^{-\frac{.67}{2k\times300}}}{e^{-\frac{.74}{2k\times4}}}\right)$$

$$= (75)^{3/2}\left(\frac{e^{\frac{.74}{8\times1.38\times10^{-23}}}}{e^{\frac{.67}{600\times1.38\times10^{-23}}}}\right) \cong 10^{461}$$

36. Find binary equivalent of $(75.50)_{10}$

(a) 1001011.11 (b) 1001011.10
(c) 10011.11 (d) 1100101.01

Solution (a) 1001011.11

TYPICAL PROBLEMS

37. Find the resistance between A and B in fig 36.45(a).

(a) 7.5 Ω (b) 30 Ω
(c) 15 Ω (d) 3.75 Ω

Solution (c) The equivalent circuit in Fig 36.45(b) is

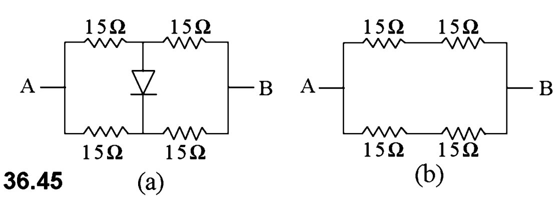


Fig. 36.45

38. The drift current in the diode of the circuit shown in Fig 36.46 is 20 μA. Find the potential drop across the diode.

(a) 5.02 V (b) 4.98 V
(c) 5.0 V (d) 0 V

Solution (b) $V_{\text{diode}} = 5 - IR$
$= 5 - 20 \times 10^{-6} \times 100 = 4.98$ V

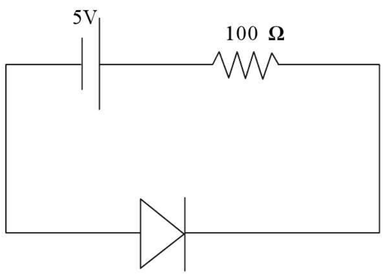


Fig. 36.46

39. Evaluate $X = A\,\overline{B}\overline{C} + \overline{B}\,\overline{C}\,\overline{A} + C\,\overline{A}\,\overline{B}$

(a) $\overline{A}\,\overline{B} + \overline{A}\,\overline{C}$ (b) $\overline{A}\,\overline{B} + \overline{B}\,\overline{C}$
(c) $\overline{A}\,\overline{C} + AC$ (d) $\overline{A}\,\overline{B} + AB$

Solution (b) $X = \overline{B}\,\overline{C}\,(A + \overline{A}) + \overline{A}\,\overline{B}\,(\overline{C} + C)$
$= \overline{B}\,\overline{C} + \overline{A}\,\overline{B}$.

40. Evaluate $P = AB + ABC$

(a) AB (b) C
(c) ABC (d) BC 15Ω 15Ω

Solution (a) $AB\,(1 + C) = AB$

41. Evaluate $Y_1 = \overline{AB} + AB$ and $Y_2 = A + \overline{A}B$ 15Ω 15Ω

(a) $0, A + B$ (b) $1, A + B$
(c) $\overline{A}, B$ (d) $\overline{A}B, AB$.

Solution (b) $Y_1 = \overline{AB} + AB$,
Put $AB = X$ then $Y_1 = \overline{X} + X = 1$.
$Y_2 = A\,(1 + B) + \overline{A}\,B = A + (A + \overline{A})B = A + B$

42. In a *pn* junction depletion region width is 400 nm when an electric field 6 × 10⁵ V/m exists in it. (a) Find the height of potential barrier. (b) Find the minimum KE of electron which can diffuse from *n*- to *p*-side.

Solution $V_b = E \times W_{\text{barrier}} = 6 \times 10^5 \times 400 \times 10^{-9} = 0.24$ V
and KE of electron = 0.24 eV.

43. The band gap in ZnO is 3.2 eV. If an electron from Conduction Band combines with a hole in Valence band then find the maximum wavelength of photon emitted.

(a) 387.5 nm (b) 297.5 nm
(c) 437.5 nm (d) 367.5 nm

Solution (a) $\lambda\,(\text{nm}) = \dfrac{1240}{3.2} = 387.5$ nm

44. In which region transistor with $h_{FE} = 80$ operates. Also find R_B.

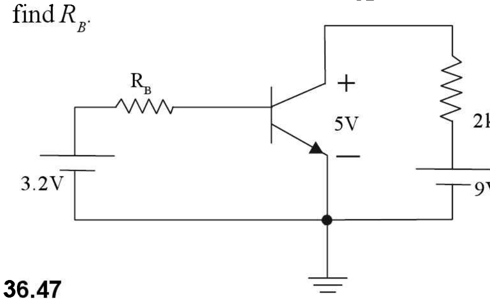


Fig. 36.47

Solution Since $V_{CB} < V_{CC}$ and not equal to zero. Therefore, transistor operates in active region.

Apply KVL in output port $I_C = \dfrac{9-5}{2 \times 10^3} = 2$ mA;

$$I_B = \frac{2 \times 10^3}{80} = 2.5 \times 10^{-5}\ \text{A}.$$

Apply KVL in input port

$R_B = \dfrac{3-0.7}{2.5 \times 10^{-5}} = 92$ kΩ. (0.7 V is V_{BE} in Active region).

45. Coincidence circuit used in nuclear physics is equivalent to

(a) OR gate. (b) AND gate.
(c) NAND gate. (d) XOR gate.

Solution (b)

46. Find V_o in the given circuit.

Solution $V_o = 0.3\left[\dfrac{10\ \text{k}\Omega}{1\ \text{k}\Omega}\right] + 1\left[\dfrac{10\ \text{k}\Omega}{10\ \text{k}\Omega}\right] + 1.2\left[\dfrac{10}{5}\right]$
$= 3 + 1 + 2.4 = 6.4$ V

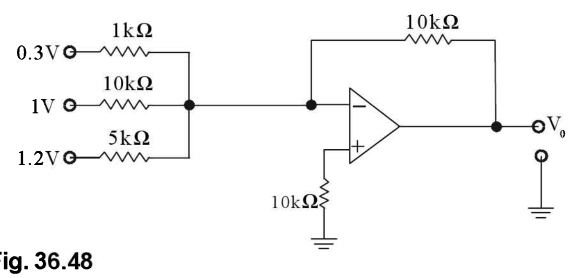


Fig. 36.48

47. Name the circuit shown and find frequency of oscillation

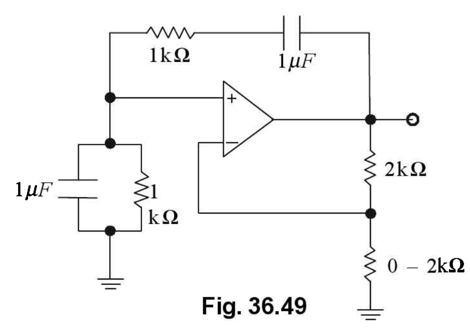


Fig. 36.49

(a) RC phase shift oscillator, 320 Hz.
(b) Wein's bridge oscillators 160 Hz.
(c) Hartley oscillator, 320 Hz.
(d) Clapp's oscillator, 160 Hz.

Solution (b) $f_o = \dfrac{1}{2\pi RC} = \dfrac{1}{2\pi \times 10^3 \times 10^{-6}} \simeq 160\text{ Hz}$

48. A free running multivibrator generates

(a) square wave. (b) sine wave.
(c) triangular wave. (d) Sawtooth wave.

Solution (a)

49. Schmitt trigger converts

(a) square wave to sine wave.
(b) square wave to triangular wave.
(c) sine wave to square wave.
(d) sine wave to triangular wave.

Solution (c)

QUESTIONS FOR PRACTICE

1. A transistor has a base current of 1mA and emitter current 100mA. The collector current will be

(a) 100mA. (b) 1 mA.
(c) 99 mA. (d) none of these.

2. In the above problem, the current transfer ratio will be

(a) 0.90. (b) 0.99.
(c) 1.1. (d) none of these.

3. In Q. 1, the current amplification factor (β) will be

(a) 89 (b) 95
(c) 99 (d) 101

4. A transistor has $a = 0.95$. If the emitter current is 10mA, then collector current will be

(a) 9.5 mA. (b) 10mA.
(c) 0.95 mA. (d) none of these.

5. In the above problem, the base current will be

(a) 0.1mA. (b) 0.2 mA.
(c) 0.3 mA. (d) 0.5 mA.

6. In Q. 4 the current amplification factor will be

(a) 11 (b) 19
(c) 35 (d) 79

7. In a transistor amplifier, $\beta = 62$, $R_L = 5000\Omega$ and internal resistance of the transistor is 500Ω. The voltage amplification of the amplifier will be

(a) 500. (b) 620.
(c) 780. (d) 950.

8. In the above problem, the power amplification will be

(a) 25580. (b) 33760.
(c) 38440. (d) none of these.

9. The change in the collector current, in a transistor of AC current gain 150, for 100μA change in its base current will be

(a) 0.15 A. (b) 0.015 A.
(c) 1.5 A. (d) none of these.

10. NOR gate is a combination of

(a) OR gate and NOT gate.
(b) OR gate and AND gate.
(c) OR gate and OR gate.
(d) none of these.

11. NAND gate is the combination of

(a) AND gate and NOT gate.
(b) AND gate and OR gate.
(c) NOT gate and OR gate.
(d) NOT gate and NOT gate.

12. The given truth table is for

A	*X*
0	1
1	0

(a) OR gate. (b) AND gate.
(c) NOT gate. (d) none of these.

13. The only function of a NOT gate is to

(a) stop a signal.
(b) recomplement a signal.
(c) invert an input signal.
(d) act as a universal gate.

14. The output of a two input OR gate is 0 only when its

(a) either input is one. (b) both inputs are one.
(c) either input is zero. (d) both inputs are zero.

15. An AND gate

(a) implements logic addition.
(b) is equivalent to a series switching circuit.
(c) is equivalent to a parallel switching circuit.
(d) is a universal gate.

16. Digital circuits can be made by respective use of

(a) AND gates. (b) OR gales.
(c) NOT gates. (d) NAND gates.

17. One way in which the operation of an NPN transistor differs from that of a PNP transistor is that

(a) the emitter junction is reverse biased in NPN.
(b) the emitter junction injects minority carriers into the base region of the PNP.
(c) the emitter injects holes into the base of the PNP and electrons into the base region of NPN
(d) the emitter injects holes into the base of NPN.

18. NPN transistors are preferred to PNP transistors because they have

(a) low cost.
(b) low dissipation energy.
(c) capable of handling large power.
(d) electrons have high mobility than holes and hence high mobility of energy.

19. If the base and collector of a transistor are in forward bias, then it can not be used as

(a) a switch. (b) an amplifier.
(c) an oscillator. (d) all the above.

20. A transistor having $\alpha = 0.99$ is used in a common-base amplifier. If the load resistance is 4.5 $k\Omega$ and the dynamic resistance of the emitter junction is 50Ω, the voltage gain of the amplifier will be

(a) 79.1. (b) 89.1.
(c) 99.1. (d) none of these.

21. In the above problem, the power gain will be

(a) 88.2 (b) 98.2
(c) 78.2 (d) none of these

22. The binary equivalent of 25 is

(a) 111001 (b) 11001
(c) 10001 (d) 10011

23. The decimal equivalent of 1111 is

(a) 25 (b) 35
(c) 15 (d) 5

24. To get AND gate from NAND gate, we need

(a) two NAND gates.
(b) two NOT gates obtained from NAND gates.
(c) one NAND gate and one NOT gate obtained from NAND gate.
(d) 3 NAND gates and one NOT gate obtained from NAND gate.

25. If the frequency of input alternating potential is n, then the ripple frequency of output potential of full wave rectifier will be

(a) $2n$ (b) n
(c) $\frac{n}{2}$ (c) $\frac{n}{4}$

26. In NPN transistor the arrow head on emitter represents that the conventional current flows from

(a) base to emitter. (b) emitter to base.
(c) emitter to collector. (d) base to collector.

27. The π section filter circuit in a full wave rectifier is

28. The expression for the efficiency of full wave rectifier is

(a) $\eta = \dfrac{81.2}{1+\dfrac{r_p}{R_L}}\%$ (b) $\eta = \dfrac{40.6}{1+\dfrac{r_p}{R_L}}\%$

(c) $\eta = \dfrac{20.3}{1+\dfrac{r_p}{R_L}}\%$ (d) $\eta = \dfrac{100}{1+\dfrac{r_p}{R_L}}\%$

29. The value of ripple factor for half wave rectifier is

(a) 121%. (b) 40.6%.
(c) 81.2%. (d) 48.2%.

30. How many diodes are used in a bridge rectifier?

(a) 1 (b) 2
(c) 3 (d) 4

31. The T.V. signals transmitted from moon are received on earth whereas signals transmitted from Jodhpur can not be received at places farther than 100 Km because

(a) there is no atmosphere as compared to that on earth.
(b) there is no atmosphere as compared to that on moon.
(c) t.v. signals propagate in straight lines and not along the curvature of earth.
(d) strong gravitational field acts on T.V. signals.

32. When two semiconductors of *P*-type and *N*-type are brought in contact with each other, the *P-N* junction formed behaves like

(a) an oscillator. (b) a condenser.
(c) an amplifier. (d) a conductor.

33. On increasing the reverse voltage in a *P-N* junction diode the value of reverse current will

(a) gradually increase. (b) suddenly increase.
(c) remain constant. (d) gradually decrease.

34. On increasing current in a semiconductor diode, the contact potential will

(a) increase. (b) decrease.
(c) remain constant. (d) become zero.

35. In forward bias the depletion layer behaves like

(a) an insulator (b) a conductor
(c) a semiconductor (d) all of the above

36. The contact potential across the junction plane in a junction diode is
(a) zero.
(b) positive at *P* and negative at *N*.
(c) infinity.
(d) negative at *P* and positive at *N*.

37. The output potential in a full wave rectifier is
(a) alternating. (b) fully direct.
(c) fluctuating direct. (d) all of the above.

38. Transistor is presumed to the more suitable for amplifier then a triode, because
(a) its output impedance is high.
(b) no heat is required in it.
(c) it can tolerate high power.
(d) it can tolerate high temperature variations.

39. On reverse biasing the *P-N* junction, its potential barrier becomes
(a) narrow. (b) broad.
(c) zero. (d) constant.

40. In a junction transistor the emitter, base and collector are respectively analogous to the following in a triode.
(a) cathode, grid and plate.
(b) plate, cathode and grid.
(c) grid, cathode and plate.
(d) plate, grid and cathode.

41. In depletion layer there are
(a) only holes. (b) only electrons.
(c) both holes and electrons.
(d) neither electrons nor holes.

42. The order of magnitude of current in the reverse bias connection of a junction diode is
(a) *mA* (b) μA
(c) *A* (d) *KA*

43. In an NPN transistor the values of base current and collector current are 100μÅ and 9 *mA* respectively, the emitter current will be
(a) 9.1 *mA*. (b) 18.2 *mA*.
(c) 9.1 μA. (d) 18.2 μA.

44. In binary system, 11000101 represents the following number on decimal system
(a) 4 (b) 401
(c) 197 (d) 204

45. Which gate is represented by the symbolic diagram given below
(a) AND gate. (b) NAND gate.
(c) OR gate. (d) NOR gate.

46. In common emitter amplifier circuit, phase reversal takes place when output is taken between
(a) base and collector. (b) collector and emitter.
(c) both of the above. (d) none of these.

47. A transistor amplifies a weak current signal because collector current is
(a) β times i_b. (b) β times i_c.
(c) β times I_B. (d) β times I_C.

48. In common collector circuit, output resistance is
(a) very high. (b) moderate.
(c) low. (d) very low.

49. In common collector circuit, voltage gain is
(a) less than one. (b) more than one.
(c) one. (d) none of these.

50. For a transistor, the current amplification factor is 0.8. The transistor is connected in common emitter configuration. The change in collector current when the base current changes by 6 mA is
(a) 6mA. (b) 4.8 mA.
(c) 24 mA. (d) 8mA.

51. The alternating current gain of a transistor in common base arrangement is 0.98. Find the change in collector current corresponding to a change of 5.0 mA in emitter current. The change is base current will be
(a) 10^{-1}A. (b) 10^{-2}A.
(c) 10^{-3}A. (d) 10^{-4}A.

52. In a transistor circuit, when the base current is increased by 50 μA, keeping collector voltage fixed at 2 volt, the collector current increases by 1.0 mA. The current amplification factor of the transistor will be
(a) 10. (b) 20.
(c) 30. (d) 40.

53. The value of a transistor is 0.9. What would be the change in the collector current corresponding to a change of 4 mA in the base current in a common emitter arrangement?
(a) 36 mA (b) 72 mA
(c) 18 mA (d) none of these

54. For a transistor, in common emitter arrangement, the alternating current gain β is given by
(a) $\beta = \left(\dfrac{\Delta I_C}{\Delta I_b}\right) V_c$ = constant.

(b) $\beta = \left(\frac{\Delta I_b}{\Delta I_c}\right) V_c$ = constant.

(c) $\beta = \left(\frac{\Delta I_c}{\Delta I_e}\right) V_c$ = constant.

(d) $\beta = \left(\frac{\Delta I_e}{\Delta I_c}\right) V_c$ = constant.

55. For a transistor, in common base arrangement, the alternating current gain is given by

(a) $\alpha = \left(\frac{\Delta I_C}{\Delta I_b}\right) V_c$ = constant

(b) $\alpha = \left(\frac{\Delta I_b}{\Delta I_C}\right) V_c$ = constant.

(c) $\alpha = \left(\frac{\Delta I_C}{\Delta I_e}\right) V_c$ = constant.

(d) $\alpha = \left(\frac{\Delta I_e}{\Delta I_C}\right) V_c$ = constant.

56. In a transistor, the emitter circuit resistance is 100Ω and the collector resistance is $100k\,\Omega$. The power gain, if the emitter and collector currents are presumed to be equal, will be

(a) 10^1 (b) 10^2

(c) 10^3 (d) 10^4

57. In a common base circuit at $V_c = 3V$, a change in emitter current from 12.0 mA to 18.5 mA produces a change in collector current from 11.8mA to 17.4 mA. The current gain will be

(a) 0.9521 (b) 0.8615

(c) 0.7351 (d) none of these

58. In a common base circuit, if the collector base voltage is changed by 0.6V, collector current changes by 0.02mA.
The output resistance will be

(a) $10^4\,\Omega$ (b) $2 \times 10^4\,\Omega$

(c) $3 \times 10^4\,\Omega$ (d) $4 \times 10^4\,\Omega$

59. For a common base amplifier, the values of resistance gain and voltage gain are 3000 and 2800 respectively. The current gain will be

(a) 0.93 (b) 0.83

(c) 0.73 (d) 0.63

60. In the above problem, the power gain of the amplifier will be

(a) 1351.2 (b) 1575.3

(c) 2173.6 (d) 2613.3

61. In a common base circuit $\alpha = 0.96$. If base current is 60μA, then emitter current will be

(a) 0.5 mA (b) 1.5 mA

(c) 0.5 A (d) 1.5 A

62. In the above problem, the collector current will be

(a) 0.44 A (b) 1.44 A

(c) 0.44 mA (d) 1.44 mA

63. The current gain for a common emitter amplifier is 54. If the emitter current is 6.8 mA, then base current will be

(a) 0.486 mA (b) 0.239 mA

(c) 0.124 mA (d) none of these

64. In the above problem, the collector current will be

(a) 6.676 mA. (b) 5.382 mA.

(c) 4.987 mA. (d) none of these.

65. In a given transistor, the emitter current is changed by 2.1mA. This results in a change of 2 mA in the collector current and a change of 1.05 V in the emitter-base voltage.
The input resistance is

(a) 5000 Ω (b) 3000 Ω

(c) 1000 Ω (d) 500 Ω

66. In the above problem, the AC gain of the transistor (α_{AC}) will be

(a) 0.90 (b) 0.95

(c) 0.99 (d) none of these

67. In Q. 65, if the transistor is used in common emitter configuration, then the value of β_{AC} will be

(a) 9 (b) 19

(c) 29 (d) 39

68. Which property of solid gives them a sharp melting point?

(a) Greater viscosity. (b) Higher rate of cooling.

(c) High melting point.

(d) Bond strength remains constant

69. Which of the following properties can be different along different directions in a crystalline solids?

(a) Electrical conductivity. (b) Refractive index.

(c) Mechanical strength. (d) All of the above.

70. The unit cell of the shape of match box is called

(a) cubic. (b) tetragonal.

(c) orthorhombic. (d) rhomboheral.

71. Semiconductor devices are

(a) temperature dependent. (b) voltage dependent.

(c) current dependent. (d) none of these.

72. The forbidden energy gap in an insulator is of the order of

(a) 1 Mev. (b) 0.1 Mev.

(c) 4 Mev. (d) 5 Mev.

73. In an N-type semiconductor, donor valence band is

(a) above the conduction band of the host crystal.

(b) close to the valence band of the host crystal.

(c) close to the conduction band of the host crystal.

(d) below to the valence band of the host crystal.

74. The forbidden energy gaps in Ge and Si are 0.7 eV and 1.1 eV respectively. It implies that

(a) both Ge and Si are perfect conductors at very low temperature but very good insulator at room temperature.

(b) both Si and Ge are perfect insulators at all temperatures.

(c) both Si and Ge are good insulators at low temperatures but start conduction at room temperature with Si a somewhat better conductor than Ge.

(d) same as (3) but with Ge showing better conductivity at room temperature.

75. A p-type semiconductor has acceptor level 57 mev above the valence band. The maximum wavelength of light required to create a hole is

(a) 57 Å (b) 57×10^{-3} Å

(c) 217100 Å (d) 11.61×10^{-11}Å

76. A silicon specimen is made into a *p*-type semiconductor by doping, on an average one indium atom per 5×10^7 silicon atoms. If the number density of atom in the silicon specimen is 5×10^{28} atoms/m^3, then the number of acceptor atoms in silicon per cm^3 will be

(a) 2.5×10^{30} (b) 1.0×10^{13}

(c) 1.0×10^{15} (d) 2.5×10^{36}

Answers to Questions for Practice

1.	(c)	**2.**	(b)	**3.**	(c)	**4.**	(a)	**5.**	(d)	**6.**	(b)	**7.**	(b)
8.	(c)	**9.**	(b)	**10.**	(a)	**11.**	(a)	**12.**	(c)	**13.**	(c)	**14.**	(d)
15.	(b)	**16.**	(d)	**17.**	(c)	**18.**	(d)	**19.**	(b)	**20.**	(b)	**21.**	(a)
22.	(b)	**23.**	(c)	**24.**	(c)	**25.**	(a)	**26.**	(a)	**27.**	(a)	**28.**	(a)
29.	(a)	**30.**	(d)	**31.**	(c)	**32.**	(b)	**33.**	(b)	**34.**	(b)	**35.**	(b)
36.	(d)	**37.**	(c)	**38.**	(b)	**39.**	(b)	**40.**	(a)	**41.**	(a)	**42.**	(a)
43.	(b)	**44.**	(c)	**45.**	(d)	**46.**	(b)	**47.**	(a)	**48.**	(a)	**49.**	(b)
50.	(c)	**51.**	(d)	**52.**	(b)	**53.**	(a)	**54.**	(a)	**55.**	(c)	**56.**	(c)
57.	(b)	**58.**	(c)	**59.**	(a)	**60.**	(d)	**61.**	(b)	**62.**	(d)	**63.**	(c)
64.	(a)	**65.**	(d)	**66.**	(b)	**67.**	(b)	**68.**	(d)	**69.**	(d)	**70.**	(c)
71.	(a)	**72.**	(d)	**73.**	(c)	**74.**	(d)	**75.**	(c)	**76.**	(c)		

37

Special Theory of Relativity

BRIEF REVIEW

Ether theory was discarded by Michelson–Morley experiment. Thus, it was established that no medium is required for light or *em* waves to propagate in the space.

Strictly speaking the Earth is not an inertial frame of reference. A body at rest on the earth's surface is accelerated due to earth's rotation about its axis and revolution about the sun.

Einstein's postulates

1. The laws of physics remain invariant in all inertial frames of reference.
2. The speed of light in vacuum remains same in all inertial frames of reference. Moreover, it is independent of motion of the source. The speed of light in free space is c.

Einstein's 2nd Postulate implies It is impossible for an inertial observer to travel at c, the speed of light in vacuum.

Gallilean Transformation If s and s' be two frames of reference with s' moving with a velocity v along positive x-direction with respect to s then co-ordinates of an event (x, y, z, t) in s frame are related to the co-ordinates (x', y', z', t') of the same event in s' frame as

$$x' = x - vt, y = y', z = z', t = t'$$

These transformations violate both the postulates of special theory of relativity.

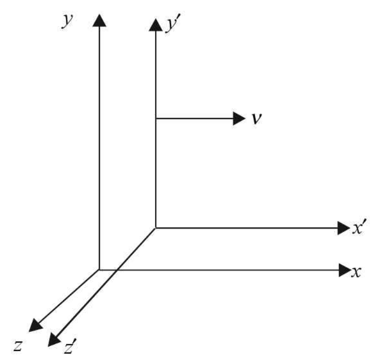


Fig. 37.1

Relativity of Simultaneity Whether or not two events at different x-axis locations are simultaneous depends on the state of motion of the observer. That is, time interval between two events may be different in different frames of reference.

Proper time is the time interval between two events that occur at the same point.

Lorentz Transformation

$$x' = \frac{x - vt}{\sqrt{1 - \frac{v^2}{c^2}}}; \qquad x = \frac{x' - vt'}{\sqrt{1 - \frac{v^2}{c^2}}};$$

$$y' = y, z' = z$$

$$t' = \frac{t - \frac{v}{c^2}x}{\sqrt{1 - \frac{v^2}{c^2}}}; \qquad t = \frac{t' + \frac{v}{c^2}x'}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Length contraction $L = L_o\sqrt{1-\frac{v^2}{c^2}}$ where L_o is original length.

Mass variation $m^* = \frac{m_o}{\sqrt{1-\frac{v^2}{c^2}}}$ where m_o is rest mass.

Time dilation $t = \frac{\tau}{\sqrt{1-\frac{v^2}{c^2}}}$, i.e., a moving clock appears to run slow. The time interval t appears to be lengthened or dilated.

Relative Velocity $V_{rel} = \frac{u-v}{1-\frac{uv}{c^2}}$

$$KE = mc^2 - m_oc^2 = m_oc^2\left[\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}-1\right]$$

Relativistic momentum $p = mv = \frac{m_ov}{\sqrt{1-\frac{v^2}{c^2}}}$

Relativistic force $F = \frac{dp}{dt} = \frac{ma}{\left(1-\frac{v^2}{c^2}\right)^{3/2}}$ ($\vec{F}$ and $\vec{v}$ act along same line).

Relation between momentum and total energy

$E^2 = m_o^2c^4 + p^2c^2$

Doppler effect $f = \sqrt{\frac{c-v}{c+v}}\,f_o$ when the source is receding.

$$v = \frac{\left(\frac{f}{f_o}\right)^2 - 1}{\left(\frac{f}{f_o}\right)^2 + 1}c$$

or $f = \sqrt{\frac{c+v}{c-v}}\,f_o$ when the source is approaching.

If v is extremely small, then, $\frac{\Delta f}{f} = \frac{v}{c}$.

Cerenkov radiation travels with speed > c. Tachyons are hypothetical particles assumed to travel with a speed > c.

• Short Cuts and Points to Note

1. **Proper frame:** It is common practice to speak of the frame in which the object is at rest as proper frame of the object. The length of the object in this frame is called proper length. The time as measured by the clock attached to this frame is called proper time. Proper time interval may be regarded as the time interval between two events occuring at the same point in s' frame or the time interval as measured by a single clock situated at the same point in s' frame.

   Lorentz transformation supports the postulates of special theory of relativity. According to this transformation, length contraction occurs if the body tries to move with a speed comparable to speed of light.

2. $L = L_o\sqrt{1-\frac{v^2}{c^2}}$; $L \to 0$ as $v \to c$

3. Time dilation $t = \frac{\tau}{\sqrt{1-\frac{v^2}{c^2}}}$

   if $v \to c, t \to \infty$ and $z \to 0$

4. At high speeds mass varies as $m^* = \frac{m_o}{\sqrt{1-\frac{v^2}{c^2}}}$

   when $v \to c, m^* \to \infty$

5. Law of conservation of momentum is valid in special theory of relativity.

6. Relativistic momentum $p = mv = \frac{m_ov}{\sqrt{1-\frac{v^2}{c^2}}}$.

7. Force $F = \frac{d}{dt}(p) = \frac{d}{dt}\frac{m_ov}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{m_ov}{\left(1-\frac{v^2}{c^2}\right)^{3/2}}$

   $\vec{F}$ and $\vec{v}$ act and along same direction.

   $a = \frac{F}{m}\left(1-\frac{v^2}{c^2}\right)^{3/2}$ Note that as speed increases a decreases and if $v \to c, a \to 0$

8. $KE = mc^2 - m_oc^2$

$$= m_o c^2 \left[\frac{1}{\sqrt{1-\frac{v^2}{c^2}}} - 1\right]$$

9. **Relative Velocity** $v_{rel} = \dfrac{u-v}{1-\dfrac{vu}{c^2}}$

10. Relativistic relation between total energy E and momentum p

$E^2 = m_o^2 c^4 + p^2 c^2$

11. Relativistic relation between KE and momentum

$$KE = m_o c^2 \left[1 + \frac{p^2}{m_o^2 c^2} - 1\right]$$

(xii) Doppler effect $f = f_o \sqrt{\dfrac{c-v}{c+v}}$ when source is receding

and $f = f_o \sqrt{\dfrac{c+v}{c-v}}$ when the source is approaching.

- **Caution**

1. Considering that relativistic mechanics or special theory of relativity can be applied when a particle/ body is moving with c (speed of light).

⇒ Whenever velocity is high ($\geq 10^7\ ms^{-1}$) relativistic conditions hold true.

2. Considering that energy possessed by a body cannot exceed $m_o c^2$.

⇒ Particle can have energy $> m_o c^2$. But velocity will be less than c. At very large velocities ($v \to c$) mass increases as $m^* = \dfrac{m_o}{\sqrt{1-\dfrac{v^2}{c^2}}}$.

3. Considering when two photons are moving in opposite direction or two electrons are moving in opposite direction with 0.9 c, then relative velocity is $2c$ and $1.8c$ respectively.

⇒ Apply relativistic mechanics rule. Remember v can never exceed c according to Einstein theory of relativity. Some exceptions have been found Cerenkov radiations and some other travel with a speed around 1.1 c or 1.2 c. However, not much work has been done on these radiations.

4. Considering 1 MeV electron has speed $\sqrt{2}\,c$.

⇒ As $v \to c$, $m \to m^* = \dfrac{m_o}{\sqrt{1-\dfrac{v^2}{c^2}}}$ and $v < c$. Therefore apply the relation $m^* c^2 = m_o c^2 + KE$

Then, to find v use $m^* = \dfrac{m_o}{\sqrt{1-\dfrac{v^2}{c^2}}}$.

SOLVED PROBLEMS

1. An electron is moving opposite to an electric field of magnitude $E = 5 \times 10^5$ N/C. Find the magnitude of momentum and acceleration when speed is 0.99 c.

(a) 1.9×10^{-19} kg ms^{-1}, 7.3×10^{15} ms^{-2}.

(b) 1.9×10^{-19} kg ms^{-1}, $2.5 \times 10^{+14}$ ms^{-2}.

(c) 2.7×10^{-24} kg ms^{-1}, 2.5×10^{15} ms^{-2}.

(d) none of these.

Solution (b) $p = \dfrac{m_o v}{\sqrt{1-\dfrac{v^2}{c^2}}} = \dfrac{9.1\times10^{-31} \times .99 \times 3\times10^8}{\sqrt{1-.98}}$

$= (7.07)(9.1 \times 10^{-23})\, 2.97$

$= 1.9 \times 10^{-19}$ kg ms^{-1}

$$a = \frac{F}{m}\left(\sqrt{1-\frac{v^2}{c^2}}\right)^3 = \frac{qE}{m}(1-.98)^3$$

$$= \frac{1.6\times10^{-19}\times5\times10^5}{9.1\times10^{-31}} \times \sqrt{8\times10^{-6}}$$

$$= \frac{8\times10^{-14}}{9.1\times10^{-31}} \times 2\sqrt{2} \times 10^{-3} = 2.5 \times 10^{14}\ \text{ms}^{-2}$$

2. The blue light observed from a jet has a frequency 6.66 $\times 10^{14}$ Hz. In the reference frame of the jet its frequency is 5.55×10^{13} Hz. Find the speed of jet.

(a) 0.9 c (b) 0.932 c

(c) 0.945 c (d) 0.986 c

Solution (d) $v = \dfrac{\left(\dfrac{f}{f_o}\right)^2 - 1}{\left(\dfrac{f}{f_o}\right)^2 + 1}\, c = \dfrac{12^2-1}{12^2+1}\, c = 0.986\, c$

3. Spaceship moving with a speed 0.9 c with reference to earth fires a robot space probe in the direction of its motion. With a speed 0.7 c relative to the spaceship. Find the velocity of robot with respect to earth.

(a) 0.961 c (b) 0.938 c
(c) 0.964 c (d) 0.982 c

Solution (d) $v = \dfrac{0.7c + 0.9c}{1 + \dfrac{0.9 \times 0.7c^2}{c^2}} = 0.982\,c$

4. The observers on earth are 56.4 m apart. How far apart a crew on spaceship moving with 0.99 c will measure them to be ?

(a) 5.64 m (b) 7.96 m
(c) 16.2 m (d) 54.2 m

Solution (b) $l = l_o \sqrt{1 - \dfrac{v^2}{c^2}}$

$= (56.4)\sqrt{1 - (.99)^2}$

$= 56.4\sqrt{.02} = 7.96\,\text{m}$

5. A student found $\dfrac{e}{m}$ using thomson method and found $\dfrac{2}{3}$ of the actual value. She claimed to commit no mistake. The possible reason of such a result is

(a) she did experiment wrong.
(b) she used quarks instead of electrons.
(c) She was telling a lie.
(d) The electron was moving with a speed 0.743 c.

Solution (d) $\dfrac{3}{2} m_o = \dfrac{m_o}{\sqrt{1 - \dfrac{v^2}{c^2}}}$

or $1 - \dfrac{v^2}{c^2} = \dfrac{4}{9}$ or $v = \dfrac{\sqrt{5}c}{3}$

6. Two protons are moving in opposite direction with 0.8 c then, their relative velocity is

(a) 0.978 c. (b) 0.95 c.
(c) 1.28 c. (d) 1.6 c.

Solution (a) $v = \dfrac{.8c + .8c}{1 + (.8)(.8)} = \dfrac{1.6c}{1.64} = 0.978\,c$

7. Find the velocity of electron accelerated by 5 MV.

(a) 0.981 c (b) 0.99 c
(c) 0.996 c (d) 0.972 c

Solution (c) $E = \dfrac{mc^2}{\sqrt{1 - \dfrac{v^2}{c^2}}} = m_o c^2 + \text{eV}$

or $m^* c^2 = m_o c^2 + \text{eV} = (.51 + 5.0)\,\text{MeV}$

$m^* = \dfrac{m_o}{\sqrt{1 - \dfrac{v^2}{c^2}}}$ or $\dfrac{v^2}{c^2} = \dfrac{120}{121}$

or $v = 0.996\,c$

TYPICAL PROBLEMS

8. An alien spacecraft is flying overhead of a great distance as you stand in your backyard. You see its search light blink for 0.19 s. The officer in the spacecraft measures the search light is ON for 12 ms. Which is the proper time? At what speed the spacecraft is flying.

Solution The proper time is 0.19 s

using $t = \dfrac{\tau}{\sqrt{1 - \dfrac{v^2}{c^2}}}$ or $\dfrac{\tau}{t} = \dfrac{12 \times 10^{-13}}{.19}$

$= \sqrt{1 - \dfrac{v^2}{c^2}} = \dfrac{12}{190} = \dfrac{6}{95}$.

$v^2 = \left(\dfrac{1.36}{9025}\right) c^2$ or $v = 0.998\,c$

9. By what minimum amount does the mass of 4 kg of ice increase when the ice melts at 0°C to form water at 0°C.

(a) 1.48×10^{-11} kg (b) 1.48×10^{-12} kg
(c) 1.48×10^{-10} kg (d) 1.48×10^{-13} kg

Solution (a) $\Delta m_o c^2 = E = mL$

$\Delta m_o = \dfrac{4 \times 10^3 \times 80 \times 4.2}{(3 \times 10^8)^2} = \dfrac{32 \times 4.2 \times 10^{-12}}{9}$

$= 1.48 \times 10^{-11}$ kg

10. Two particles created in a high energy accelerator move in opposite directions. The speed of one particle as measured in the lab is $0.65\,c$ and their relative speed is $0.95\,c$. Find the speed of the other particle.

(a) $0.3\,c$ (b) $0.652\,c$

(c) $0.784\,c$ (d) $0.842\,c$

Solution (c) $0.95\,c = \dfrac{0.65c+u}{1+\dfrac{0.65cu}{c^2}}$ or $0.95\,c + 0.95\,(.65\,u)$

$= 0.65\,c + u\; 0.30\,c = u\,(1-0.6175)$

or $u = \dfrac{0.30c}{0.3825} = 0.784\,c$

11. The average life time of pion (π^-) is $2.6 \times 10^{-8}\,s$ as measured in the proper frame and $4.2 \times 10^{-7}\,s$ in the laboratory frame. Find the speed of π^- and distance covered by π^- during its average life time.

(a) $0.95\,c$, 124 m (b) $0.996\,c$, 126 m

(c) $0.89\,c$, 108 m (d) $0.91\,c$, 114 m

Solution (b) $t = \dfrac{\tau}{\sqrt{1-\dfrac{v^2}{c^2}}}$

or $1 - \dfrac{v^2}{c^2} = \left(\dfrac{\tau}{t}\right)^2$

or $v = c\sqrt{1-\left(\dfrac{\tau}{t}\right)^2} = c\sqrt{1-\left(\dfrac{2.6\times10^{-8}}{4.2\times10^{-7}}\right)^2}$

$= 0.996\,c$

$x = 0.996 \times 3 \times 10^8 \times 4.2 \times 10^{-7} = 126$ m

12. If KE Vs speed graph is plotted then which of the following is correct.

(a) 1 (b) 2

(c) 3 (d) 4

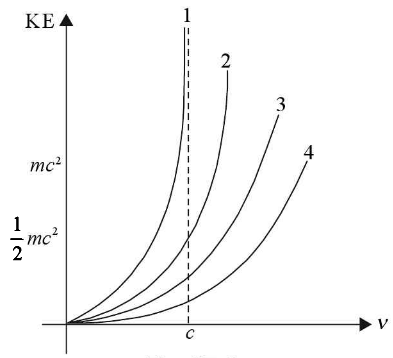


Fig. 37.2

Solution (a) KE can exceed mc^2 but velocity cannot exceed c.

13. Find the magnitude of the force to be given to a 0.145 kg base ball if it is moving with $0.9\,c$. Acceleration to be given is 1 ms^{-2}.

(a) $0.145\,N$ (b) $2.4\sqrt{3}\,N$

(c) $12.72\,N$ (d) 1.776 N

Solution (d) $F = \dfrac{ma}{\left(1-\dfrac{v^2}{c^2}\right)^{3/2}} = \dfrac{.145\times1}{(1-.81)^{3/2}} = 1.776$ N

14. How much work be done to increase the speed of a particle of mass m to increase from $0.9\,c$ to $0.99\,c$?

Solution $K = \dfrac{mc^2}{\sqrt{1-\dfrac{v^2}{c^2}}} - mc^2,$

$$\Delta K = mc^2\left[\frac{1}{\sqrt{1-(.99)^2}} - \frac{1}{\sqrt{1-(.9)^2}}\right]$$

$$= mc^2\left[\frac{1}{\sqrt{.02}} - \frac{1}{\sqrt{.19}}\right] = mc^2\,[7.07-2.32]$$

$= 4.75\,mc^2$

QUESTIONS FOR PRACTICE

1. A spaceship in space will have

(a) clocks running slower than a stationary clock by a factor $\sqrt{1-\dfrac{v^2}{c^2}}$

(b) its length shrunk in the direction of relative motion by a factor of $\sqrt{1-\dfrac{v^2}{c^2}}$

(c) its mass increased by a factor of $\dfrac{1}{\sqrt{1-\dfrac{v^2}{c^2}}}$

(d) all the above

2. Einstein's mass energy relation ($E = mc^2$) shows that

(a) mass disappears to reappear as energy.

(b) energy disappears to reappear as energy.

(c) mass and energy are two different forms of the same entity.

(d) all the statements are correct.

3. A rod of length L_0 moving with a velocity of 0.8c in a direction inclined at 60° to its own length. The apparent length along the direction of motion will be

(a) $0.1\,L_0$ (c) $0.2\,L_0$

(c) $0.3\,L_0$ (d) $0.4\,L_0$

4. In the above problem, the apparent length perpendicular to the direction of motion will be

(a) $\frac{3}{2}L_0$ (b) $\frac{\sqrt{3}}{2}L_0$

(c) $\frac{5}{2}L_0$ (d) $\frac{\sqrt{5}}{2}L_0$

5. In Q. 3, the apparent length of the moving rod will be

(a) $0.916\,L_0$ (b) L_0

(c) $1.021\,L_0$ (d) none of these

6. In Q. 3, the percentage contraction in the rod will be

(a) 0.84% (b) 8.4%

(c) 84% (d) none of these

7. A beam of particles of half life 2.8×10^{-6}s travels in the laboratory with $v = 0.96$c. The apparent lifetime of the particles will be

(a) 1×10^{-6}s (b) 1×10^{-5}s

(c) 1×10^{-4}s (d) 1×10^{-3}s

8. In the above problem, the distance travelled by the beam before the flux fails to half its initial value, will be

(a) 2.88×10^{3}m (b) 2.88×10^{2} m

(c) 2.88×10^{1}m (d) 2.88×10^{0} m

9. If l_0^3 is the rest volume of a cube, then the volume viewed from a reference frame moving with uniform velocity v parallel to an edge of the cube, will be

(a) $l_0^3\left(1-\frac{v^2}{c^2}\right)^{3/2}$ (b) $l_0^3\left(1-\frac{v^2}{c^2}\right)$

(c) $l_0^3\sqrt{1-\frac{v^2}{c^2}}$ (d) none of these

10. At what speed should a clock be moved so that it may appear to lose 1 minute in each hour ?

(a) 1.9×10^{7}m/s (b) 2.7×10^{7}m/s

(c) 5.4×10 m/s (d) none of these

11. A spaceship, moving away from earth with speed 0.9c, fires a missile in the same direction as its motion, with a speed 0.9c relative to spaceship. What will be its speed relative to earth ?

(a) 0.745c (b) 0.859c

(c) 0.994 c (d) c

12. The energy equivalent to amu in MeV will be

(a) 0.931 MeV (b) 9.31 MeV

(c) 93.1 MeV (d) 931 MeV

13. A body of specific heat 0.2 kilocal/ $kg^{-1}\,°C^{-1}$ is heated through 100°C, the percentage increase in its mass will be

(a) 9.3×10^{-8}% (b) 9.3×10^{-9}%

(c) 9.3×10^{-10}% (d) 9.3×10^{-11}%

14. The velocity of π-mesons, whose observed mean life is 2.5×10^{-7} s and proper life is

(a) 0.795 c (b) 0.895 c

(c) 0.995 c (d) none of these

15. A certain particle called μ-mesons has a life time 2×10^{-6} sec. If it is travelling with a speed of 2.994×10^{8}m then its mean life time will be

(a) 31.7×10^{-6}s (b) 3.17×10^{-6}s

(c) 0.317×10^{-6}s (d) none of these

16. In the above problem, how far does the particle go during one mean life ?

(a) 6770 m (b) 9510 m

(c) 1850 m (d) none of these

17. In Q. 15, what distance would be travelled by the particle without relativistic effects ?

(a) 598.8 m (b) 5988 m

(c) 59.88 m (d) 5.988 m

18. A charged particle shows an acceleration of 4.2×10^{10} m/s^2 under an electric field at low speed. The acceleration of the particle under the same field when its speed becomes 2.88×10^{8} m/s will be

(a) 4.2×10^{10} m/s^2 (b) 1.176×10^{10}m/s^2

(c) 2.88×10^{8} m/s^2 (d) none of these

19. The mass of an electron having kinetic energy 1.5 MeV will be

(a) 9.1×10^{-31} kg. (b) 3.58×10^{-31}kg.

(c) 35.8×10^{-31} kg. (d) none of these.

20. In the above problem, the velocity of electron will be

(a) 3×10^{8}m/s. (b) 2.9×10^{8} m/s.

(c) 2.8×10^{8} m/s. (d) 2.7×10^{8} m/s.

21. An electron is moving with a speed 0.99c. Its relativistic mass will be

(a) 9.1×10^{-31} kg. (b) 64.5×10^{-31} kg.

(c) 98.5×10^{-31} kg. (d) none of these.

22. In the above problem, the total energy of electron will be

(a) 5.8×10^{-11} Joule. (b) 5.8×10^{-12} Joule.
(c) 5.8×10^{-13} Joule. (d) none of these.

23. In Q. 21, the ratio of Newtonian kinetic energy to the relativistic energy will be
(a) 0.05 (b) 0.06
(c) 0.07 (d) 0.08

24. At what velocity the kinetic energy of a particle is equal to the rest mass of energy?
(a) $\sqrt{5}/2$ (b) $\sqrt{3}/2c$
(c) $c/2$ (d) None of these

25. The e/m measured by an experimenter is 1/3rd of the usual value. The electron is moving with a speed
(a) 0.842 c (b) 0.724 c
(c) 0.866 c (d) 0.943 c

26. At what speed the mass of a body will be almost doubled?
(a) 0.77 c (b) 0.87 c
(c) 0.97 c (d) None of these

27. Which of the following is not assumed to be absolute in newtonian mechanics?
(a) Mass (b) State of rest or motion
(c) Space (d) Time

28. An electron is moving with speed 0.99 c. The total energy of electron will be
(a) 5.8×10^{-13} J (b) 5.8×10^{-12} J
(c) 5.8×10^{-11} J (d) none of these

29. A rod of length L_0 moving with a velocity of 0.8 c in a direction inclined at 60° to its own length. The apparent length along the direction of motion will be
(a) $0.4 L_0$ (b) $0.3 L_0$
(c) $0.2 L_0$ (d) $0.1 L_0$

30. One kilogran of mass is completely converted in to heat energy. The heat produced in kilocalories will be
(a) 2.1×10^{16} (b) 2.1×10^{13}
(c) 2.1×10^{10} (d) none of these

31. A beam of particles of half-life 2.8×10^{-6} s travels in the laboratory with $v = 0.96\,c$. The apparent life time of the particles will be
(a) 1×10^{-3} s (b) 1×10^{-4} s
(c) 1×10^{-5} s (d) 1×10^{-6} s

32. The energy equivalent to amu in MeV will be
(a) 0.931 MeV (b) 931 MeV
(c) 9.31 MeV (d) 93.1 MeV

33. The velocity of an electron having kinetic energy of 1.5 MeV will be
(a) 2.7×10^8 ms^{-1} (b) 2.8×10^8 ms^{-1}
(c) 2.9×10^8 ms^{-1} (d) 3×10^8 ms^{-1}

34. The mass of an electron having kinetic energy of 1.5 MeV will be
(a) 3.58×10^{-31} kg (b) 9.1×10^{-31} kg
(c) 35.8×10^{-31} kg (d) none of these

35. A certain particle called μ-meson has a life time 2×10^{-6} s. If it is travelling with a speed of 2.994×10^8 ms^{-1}, then its mean life time will be
(a) 0.317×10^{-6} s (b) 3.17×10^{-6} s
(c) 31.7×10^{-6} s (d) none of these

36. An electron is being chased by a photon. The speed of the electron is 0.9 c. Their relative velocity is
(a) c. (b) 0.9 c.
(c) 0.1 c. (d) none of these.

37. An electron is moving with speed 0.99 c. The ratio of newtonium kinetic energy to the relativistic energy will be
(a) 0.08. (b) 0.07.
(c) 0.06. (d) 0.04.

38. A charged particle shows an acceleration of 4.2×10^{10} ms^{-2} under an electric field at low speed. The acceleration of the particle under the same field when its speed becomes 2.88×10^8 ms^{-1} will be
(a) 2.88×10^8 ms^{-2}. (b) 1.176×10^{10} ms^{-2}.
(c) 4.2×10^{10} ms^{-2}. (d) none of these.

39. A charged particle of π meson whose observed life 2.5×10^{-7} s and proper life is 3.5×10^{-7} s is
(a) 0.7 c. (b) 0.841 c.
(c) 0.795 c. (d) 1.95 c.

40. Two photons recede from each other. Their relative velocity will be
(a) $c/2$. (b) c.
(c) 2 c. (d) zero.

41. A body of specific heat 0.2 kcalkg^{-1} °C^{-1} is heated through 100°C. The percentage increase in its mass will be
(a) 9.3×10^{-10}%. (b) 9.3×10^{-11}%.
(c) 9.3×10^{-8}%. (d) 9.3×10^{-9}%.

42. An electron is moving with a speed 0.99 c. Its relativistic mass will be
(a) 98.5×10^{-31} kg. (b) 64.5×10^{-31} kg.
(c) 9.1×10^{-31} kg. (d) none of these.

43. Two photons approach each other. Their relative velocity will be
(a) slightly less than c. (b) $c/2$.
(c) c. (d) $c - v$.

44. A beam of light moves towards right with speed c. If earth also moves towards right with speed v, then the speed of beam of light relative to earth will be?

(a) $\sqrt{c^2+v^2}$ (b) c

(c) $c+v$ (d) $c-v$

45. Whose experimental work proved that the velocity of light is a universal and natural constant?

(a) Lorentz. (b) Einstein.

(c) Maxwell. (d) Michelson.

46. On the annihilation of a particle and its antiparticle the energy released is E. The mass of each particle will be

(a) E/c. (b) $E/2c$.

(c) E/c^2. (d) $\frac{E}{2c^2}$.

47. Special theory of relativity deals with the events in the frames of reference which move with constant

(a) acceleration. (b) momentum.

(c) space interval. (d) time interval.

48. A proton has a charge q when at rest. When it acquires a velocity $c/2$, its charge becomes

(a) $q\sqrt{1-(1/2)^2}$. (b) q.

(c) $\frac{q}{\sqrt{1-(1/2)^2}}s$. (d) infinity.

49. Einstein proposed the general theory of relativity in

(a) 1916. (b) 1905.

(c) 1904. (d) 1900.

50. At what speed should a clock be moved so that it may appear to lose 1 minute in each hour?

(a) $5.4 \times 10^7 ms^{-1}$. (b) $2.7 \times 10^7 ms^{-1}$.

(c) $1.9 \times 10^7 ms^{-1}$. (d) none of these.

Answers to Questions for Practice

1.	(d)	**2.**	(d)	**3.**	(c)	**4.**	(b)	**5.**	(a)	**6.**	(b)	**7.**	(b)
8.	(a)	**9.**	(c)	**10.**	(c)	**11.**	(c)	**12.**	(d)	**13.**	(d)	**14.**	(c)
15.	(a)	**16.**	(b)	**17.**	(a)	**18.**	(b)	**19.**	(c)	**20.**	(b)	**21.**	(b)
22.	(c)	**23.**	(d)	**24.**	(b)	**25.**	(d)	**26.**	(b)	**27.**	(b)	**28.**	(a)
29.	(b)	**30.**	(b)	**31.**	(c)	**32.**	(b)	**33.**	(c)	**34.**	(c)	**35.**	(c)
36.	(a)	**37.**	(a)	**38.**	(b)	**39.**	(a)	**40.**	(b)	**41.**	(b)	**42.**	(b)
43.	(c)	**44.**	(b)	**45.**	(d)	**46.**	(d)	**47.**	(b)	**48.**	(b)	**49.**	(a)
50.	(a)												

Explanations

28. (a) $m^* = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}} = 7.1\, m_0; E = 7.1\, m_0c^2.$

29. (b) $L = \left[L_0\sqrt{1-\frac{v^2}{c^2}}\right]\cos 60° = 0.3\, L_0$

30. (b) $H = \frac{m_0c^2}{4200} = \frac{1\times9\times10^{16}}{4.2\times10^3} = 2.1 \times 10^3$ kcal.

35. (c) $t = \frac{\tau}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{20\times10^{-6}}{\sqrt{1-(0.998)^2}} = 31.7 \times 10^{-6}$ s

38. (b) $m^* = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{m_0}{\sqrt{1-(0.96)^2}} = \frac{m_0}{0.28}$

$$a_{new} = 0.28(4.2 \times 10^{10})$$
$$= 1.176 \times 10^8 ms^{-2}.$$

39. (a) $\frac{2.5}{3.5} = \sqrt{\frac{1-v^2}{c^2}}$

or $1-\frac{25}{49} = \frac{v^2}{c^2}$

or $v = \frac{\sqrt{24}}{7}c = 0.7\,c.$

38

Principles of Communications

BRIEF REVIEW

Communication The transfer of information from one point to another may be termed as communication. In order to convey the information over a long distance, a communication system is required. Within a communication system the information transfer is achieved usually by superimposing or modulating the information on to an electromagnetic wave (carrier). The modulated carrier is then transmitted to the required destination where it is received and original information is retrieved by means of demodulation or detection. Sophisticated techniques have been developed for this process, using electromagnetic carrier waves operating at radio frequencies as well as microwave and millimeter wave frequencies or even infrared rays in optical communication.

Transducer It is a device which converts a physical quantity varying with time into electrical signal or vice versa. It is of two types: input transducer and output transducer. **Input transducer** converts a **physical quantity varying with time into electrical signal.** For example, microphone converts sound into electrical signal, thermocouple or thermistor convert temperature into electrical signal. An **output transducer** converts **electrical signal** back to a **physical quantity** varying with time. For example, a speaker converts electrical signal back to sound. A heater converts electrical signal back to thermal variations.

Passive transducers require a source of emf to operate while **Active transducers** do not require any electrical energy to work. For example microphone and thermistor are passive transducers while solar call is an active transducer.

Channel is used to refer to the frequency range allocated to a particular transmission, for example, a TV channel.

Noise is the introduction of unwanted signal or some distortion in the process of transmission and reception. The signal gets deteriorated. Since the noise will be received along with the signal and if noise is several times the signal, it may mask the signal making it unineligible.

Modulation is of two types: **analog** and **digital**. In analog modulation some characteristic of high frequency sine wave (called carrier) is varied in accordance with the instantaneous value of modulating signal. If **amplitude of carrier** wave is varied in accordance with the **instantaeous value** of the **modulating signal** then **amplitude modulation** (AM) results. If the frequency of the carrier is varied in accordance with the instantaneous value of the **modulating signal** then **frequency modulation** (FM) results. If phase of the carrier wave is **varied in accordance to the instantaneous value** of the **modulating signal** then **phase modulation** (PM) results. FM and PM may be categorized as angle modulation.

Modulation is required for long distance communication. If we transmit sound wave directly say at 20 kHz (largest frequency of sound) then the length of antenna required = $= \frac{\lambda}{2} = 7.5$ km $\left(\lambda = \frac{c}{f} = \frac{3\times10^8}{20\times10^3} = 15km\right)$ which is impractical in present day technology. The other reason in favour of modulation is that if all the radio stations transmit at 20 kHz (or same frequency) their signal will mix up and nothing audible will be heard.

Amplitude Modulation (AM) Let $e_c = E_c \sin \omega_c t$ be the carrier wave and $e_m = E_m \sin \omega_m t$ be the modulating signal. Then modulated signal e is given by

$$e = (E_c + E_m \sin \omega_m t) \sin \omega_c t.$$

$$e = E_c\left(1+\frac{E_m}{E_c}\sin\omega_m t\right)\sin\omega_c t$$

$$= E_c\,(1+m_a \sin\omega_m t)\sin\omega_c t \text{ where } m_a = \frac{E_m}{E_c}$$

is modulation index. It is normally expressed in % and should be less than 100%.

From Fig. 38.1 $m_a = \dfrac{E_{\max}-E_c}{E_c} = \dfrac{E_c - E_{\min}}{E_c} = \dfrac{E_{\max}-E_{\min}}{2E_c}$

$$= \frac{E_{\max}-E_{\min}}{E_{\max}+E_{\min}}.$$

$$e = E_c\,(1+m_a \sin\omega_m t)\sin\omega_c t$$

$$= E_c \sin\omega_c t + \frac{m_a}{2}E_c\cos(\omega_c+\omega_m)\,t - \frac{m_a}{2}\cos(\omega_c-\omega_m)\,t.$$

shows that spectrum of AM will consist of carrier wave, lower slide band (LSB) or component of $(\omega_c - \omega_m)$ and upper side band (USB) or component of $(\omega_c + \omega_m)$ as illustrated in Fig. 38.2.

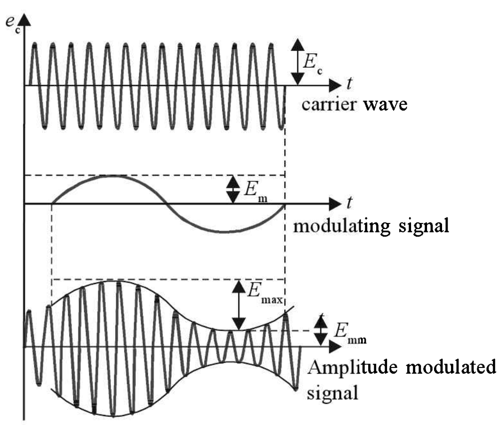


Fig. 38.1 Amplitude Modulation illustration

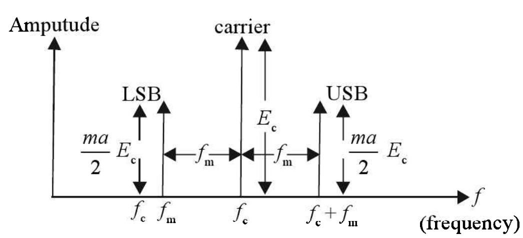


Fig. 38.2 Spectrum of Amplitude Modulation

$$P_{\text{Total}} = P_{\text{Carrier}} + P_{\text{LSB}} + P_{\text{USB}}$$

$$= P_{\text{Carrier}}\left[1+\frac{m_a^2}{2}\right]$$

$$\text{or } \frac{P_{\text{Total}}}{P_{\text{Carrier}}} = 1+\frac{m_a^2}{2}$$

$$\frac{I_{\text{total}}}{I_{\text{carrier}}} = \sqrt{1+\frac{m_a^2}{2}}$$

If several modulating signals are present then

$$m_{\text{total}} = \sqrt{m_1^2+m_2^2+m_3^2+\ldots.}$$

Moreover, total modulation index should not exceed unity. Collector and base modulation are two methods of modulation. Collector modulation is preferred as it results in better linearity and more power output.

Types of AM

A3 → double side band full carrier.

A3A→ single side band reduced carrier.

A3H → single side band full carrier.

A3J → single side band suppressed carrier called SSB (Single Side band) transmission.

A3B → two independent side bands with suppressed carrier.

A5C → vestigial side band transmission (used for video transmission in TV)

Frequency Modulation Mathematically, the instantaneous frequency of the frequency modulated signal is given by $f = f_c\,(1 + k\,E_m \cos\omega_m t)$ where f_c is unmodulated or average carrier frequency, k is a conversion factor which converts voltage to frequency. Since $\cos\omega_m t$ will lie between ±1, therefore, f lies between $f_c\,(1 \pm k\,E_m)$ maximum frequency deviation $\delta = k f_c E_m$

We may also write $\omega = \omega_c\,(1 + k\,E_m \cos\omega_m t)$ and angle θ is given by

$$\theta = \int \omega\,dt = \omega_c t + \frac{\omega_c k E_m \sin\omega_m t}{\omega_m}$$

$$= \omega_c t + \frac{f_c K\,E_m \sin\omega_m t}{f_m}$$

$$= \omega_c t + \frac{\delta}{f_m}\sin\omega_m t$$

The instantaneous voltage of *FM* signal 'thus' becomes

$$e = A\sin\left[\omega_c t + \frac{\delta}{f_m}\sin\omega_m t\right]$$

$$= A\sin\left[\omega_c t + m_f \sin\omega_m t\right]$$

Where m_f is frequency modulation index and

$$m_f = \frac{\delta}{f_m} = \frac{k\,E_m f_c}{f_m}.$$

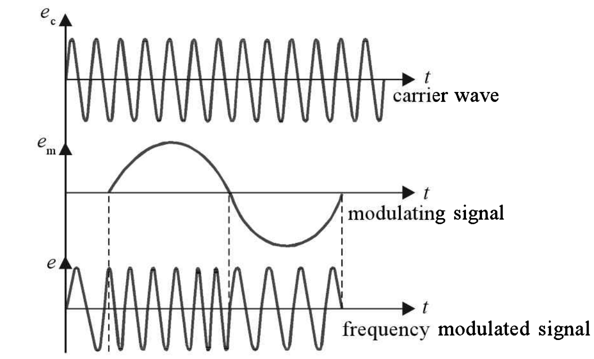


Fig. 38.3 FM illustration

Figure 38.3 shows *FM* signal illustration. To describe *FM* spectrum, Bessels function are required. It has been found that for $m_f = 2.4$, 5.5, 8.6 and 11.8 and so on carrier component completely vanishes. These values are called eigen values. These help in finding bandwidth and measuring deviation δ.

To a good approximation for $m_f > 6$, bandwidth $\Delta = 2\,(f_m + \delta)$. Otherwise, look into the table for a given m_f. Find the highest *J* coefficient for which modulation index has values < 0.01 then $\Delta = 2 \times f_m \times$ highest needed side band.

Phase Modulation If the phase in the equation $e_c = E_c \sin(\omega_c t + \phi)$ is varied in accordance with instantaneous value of modulating signal, the resulting wave is phase modulated. The expression for a *PM* wave is $e = A \sin(\omega_c t + \phi \sin \omega_m t)$ where ϕ is maximum value of phase shift. We can write $m_p = \phi_m$ as modulation index.

The difference in *FM* and *PM* is that the modulation index in *FM* is proportional to amplitude of modulating signal and in *FM* it is inversely proportional to modulating frequency. Thus, if frequency of modulating signal is changed, modulation index in *PM* will remain constant while in *FM* it will vary.

If *FM* is received on *PM* receivers bass frequencies will have more deviation of phase than a *PM* transmitters would have given them, i.e., signal will be unduly bass boosted. If *PM* is received on an *FM* receiver it will lack bass and may be connected by applying a bass booster.

Note ***AM*** is a **long distance transmission** as it operates on frequencies which are reflected by ionosphere. Moreover, its circuits are simpler. However, *AM* is **noise prone**.

FM is **short distance transmission**. Repeaters are required for long distance *FM* communication. Its circuits are complex. However, **it is noise immune** because it is detected from frequency deviation and not from amplitude variation (where noise resides).

Ground or Surface Waves These waves are vertically polarised and progress along the surface of the earth. Vertical polarization prevents short circuiting of electric component. A wave induces current in the ground over which it passes and thus, loses some energy by absorption. However, this is made up by energy diffracted downwards from a wavefront and act like a leakage capacitor as illustrated in Fig. 38.4 (a) and (b).

Attenuation also occurs due to diffraction as angle of tilt of successive wavefronts increases as shown in Fig. 38.4 (c).

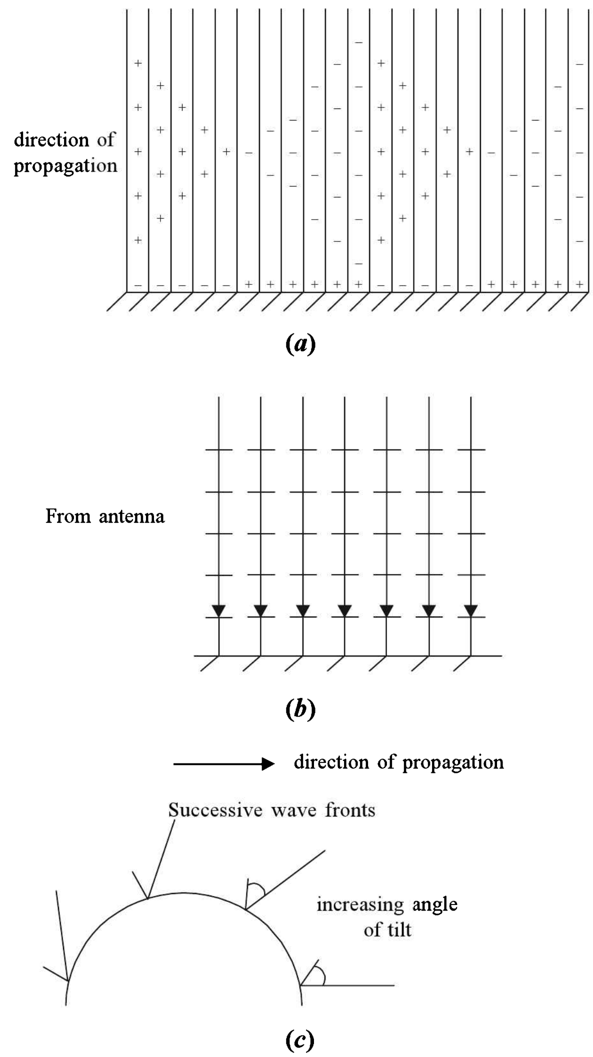


Fig. 38.4 Ground wave propagation

Electric field at a distance *d* from antenna due to ground waves is given by

$$E = \frac{120\pi h_t I}{\lambda d}$$ and the signal received by the receiving antenna of height h_r in volts is given by

$$\text{V (volts)} = \frac{120\pi h_t h_r I}{\lambda d}$$ where $120\,\pi = 377\ \Omega$ is characteristic impedance; *I* is antenna current, h_t = effective height of transmitting antenna and λ is wavelength.

VLF Propagation When propagation is over a good conductor like sea water at frequencies below 100 kHz attenuation is small. Ship communication uses frequency 10 Hz – 110 kHz. The VLF antennas are inefficient, high powered and use tallest mast.

Sky wave propagation – the ionosphere Ionosphere is the upper portion of the atmosphere, which continually absorbs large quantities of radiant energy from the sun, thus, becoming heated and ionized. Temperature, density, composition and type of radiation received stratify the ionosphere. The most important ionizing agent are *uv*, α, β, γ radiations from the sun as well as cosmic rays and meteors. The overall result as shown in Fig. 38.5 is a range of four main layers. D, E, F_1 and F_2 in ascending order. The last two combine at night to form a single layer.

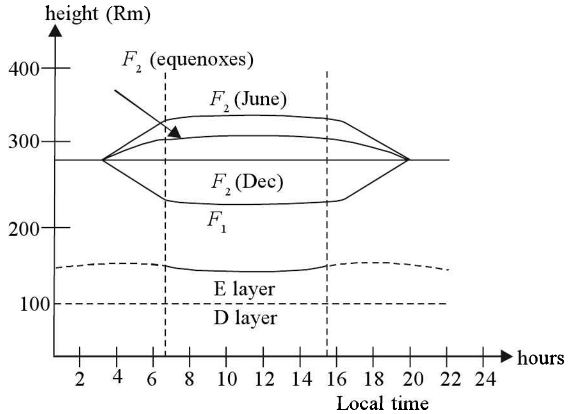


Fig. 38.5 Illustration of ionosphere layers

The D layer is the lowest existing at an average height of 70 km with an average thickness of 10 km. The degree of its ionization depends upon the altitude of the sun. It disappears at night. It reflects LF and VLF rays and absorbs MF and HF waves to a certain extent.

The E layer is a thin layer of very large density. Like D layer it also disappears at night. If reflects MF and surface waves and some HF waves in day time.

The F_1 layer exists at a height of 180–200 km and combines with F_2 at night. F_1 layer absorbs HF waves. F_2 layer is most important reflecting layer for HF. Its height is 250-400 km with an average thickness of 200 km and average height 300 km.

Reflection Mechanism As the ionization density increases the refractive index of the layer decreases. The incident ray is gradually bent and suffers total internal reflection as illustrated in Fig. 38.6. Figure 38.7 illustrates skip distance and effect of ionosphere on rays of varying angle of incidence. Large angle rays are bent while short angle of incidence rays escape.

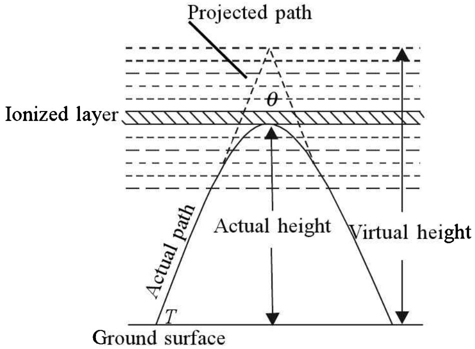


Fig. 38.6 Actual and virtual heights of an ionized layer

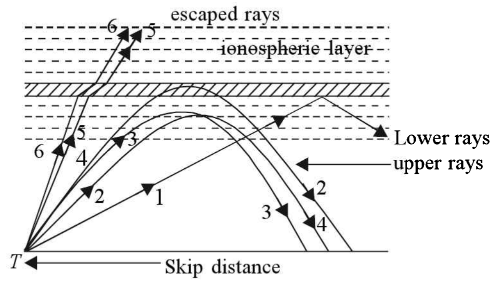


Fig. 38.7 Effect of ionosphere on rays of varying incidence

Skip distance is the shortest distance from a transmitter T, measured along the surface of the earth at which a sky wave of fixed frequency ($> f_c$) will be returned to the earth. Note that it is frequency specific and depends upon angle of incidence.

Critical frequency (f_c) For a given layer, it is the highest frequency that will be returned to earth by having been beamed straight up at it $f_c = 9\sqrt{N}$.

Maximum usable frequency (*muf*) Maximum limiting frequency is the frequency of the ray incident at some specific angle of incidence which will return to the earth from ionosphere.

$muf = f_c \sec\theta$ also known as secent law.

Space waves travel in straight lines. They depend upon line of sight. Their propagation is limited by the curvature of the earth. They are not reflected from ionosphere.

Radio horizon for space waves = $\frac{3}{4}$ optical horizon. The emperical formula gives the radio horizon. $d_t\,(k) = 4\sqrt{h_t}$. Where d_t is distance from transmitting antenna and h_t is height of transmitting antenna. This formula applies to receiving antenna also. Thus, $d = d_t + d_r = 4\sqrt{h_t} + 4\sqrt{h_r}$. For, example, if $h_t = 225$ m then radio horizon $= 4\sqrt{h_t} = 60$ km. Figure 38.8 illustrates radio horizon for space waves. Links longer than 100 km are hardly used in commercial communication that is, repeaters are then required.

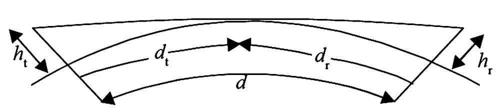


Fig. 38.8 Height of transmitting and receiving antenna

Super refraction or duct The microwaves due to decreasing refractive index just 30 m above the ground bend complete bending takes place as illustrated in Fig. 38.9. Microwaves are, thus, continuously refracted from the duct and reflected from the ground can travel 1000 km.

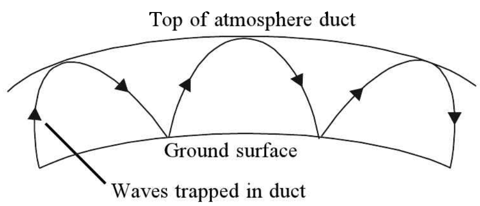


Fig. 38.9 Wave propagation and duct illustration

Tropospheric scattering As shown in Fig. 38.10, two directional antennas are pointed so that their beams intersect midway between them above the horizon. If they are UHF transmitting and receiving antenna then sufficient radio energy reaches to receiving antenna. Best results are seen if frequencies are 900 MHz, 2000 MHz and 5000 MHz. This tropospheric scattering occurs within 15 km above the ground.

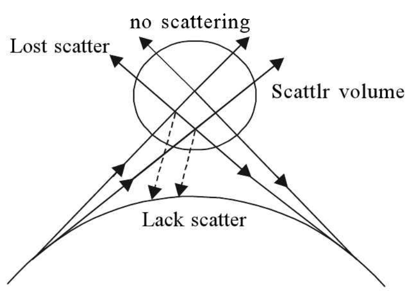


Fig. 38.10 Illustration of scattering and propagation of VHF

Satellite and Probe tracking The requirement of tracking and communicating with satellites in close orbits involve the use of fast rotating circularly polarized antennas together with fairly low noise and medium power transmitter and receivers.

Pulse modulation Pulse modulation may be subdivided into two categories – **analog** and **digital.** In analog forms the sample may be infinitely varying while in digital form a code is sent which indicates the sample amplitude to the nearest predetermined level.

Pulse amplitude modulation (PAM) and **pulse time modulation** (PTM) are both **analog** while **pulse code modulation** (PCM) and **delta modulation** are **digital** form of pulse modulations.

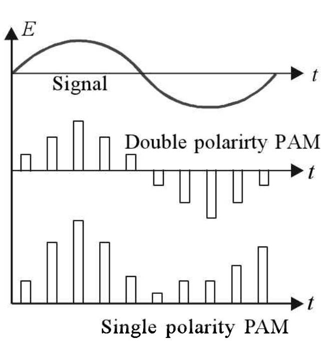


Fig. 38.11

Pulse Amplitude Modulation (PAM) is the simplest form of Pulse modulation. As illustrated in Fig. 38.11, it is of two types; single polarity PAM and double polarity PAM. In double polarity PAM the pulse height is made proportional to the amplitude of the signal at the instant of the sampling. The pulse train is then transmitted by wire, cable or modulated using carrier before transmitting through antenna. In single polarity PAM the height of the pulse is made larger than the maximum negative value of the signal so that it remains positive even when the input is negative. PAM is hardly used but whenever used, pulses are made to frequency modulate the carrier and termed as PAM FM.

Pulse Time Modulation (PTM) In PTM the signal is sampled as before but pulses indicating instantaneous sample amplitude have constant amplitude. However, one of their timing characteristic is varied and is made proportional to the amplitude of the signal at that instant. The variable characteristic may be width, position or frequency. Thus, PTM is of three types. **Pulse width modulation (PWM)** or **PDM (Pulse duration modulation) Pulse position modulation (PPM)** and **Pulse frequency modulation**. The last one, that is, pulse frequency modulation has no practical significance.

Sampling Theorem If the sampling rate in any pulse modulation system exceeds twice the maximum signal frequency only then the original signal frequency can be reconstructed in the receiver with negligibly small distortion.

PWM or PDM As illustrated in Fig. 38.12, the pulse width is made proportional to the instantaneous value of the signal at the instant of sampling. For example, zero amplitude at any instant gives 1 μs as the average width of the pulse. If the recurrence rate of the pulse is 8 kHz, the time between the occurrence of next pulse is $\frac{10^6}{8000} = 125\ \mu s$. It is quite sufficient to accomodate the varying widths and also to permit time division multiplexing. However, power varies at every duration.

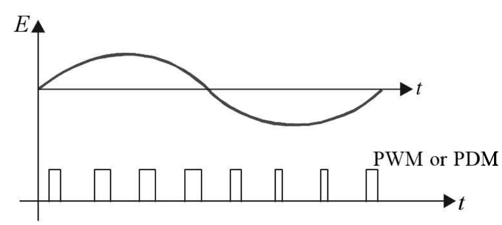


Fig. 38.12 Pulse width modulation

PPM PPM is generated from PWM as illustrated in Fig. 38.13. Each pulse has a leading and a trailing edge. Repitition rate of leading edge is fixed but not that of the trailing edge. Their positions depend on pulse width (which in turn depends on the amplitude of the signal at that instant). Thus, trailing edge of the PWM pulses are infact pulse position modulated. If the PWM pulse train is differentiated, the leading and trailing edges separate. If the position of trailing edge of unmodulated wave is considered as origin then the trailing edge of pulse width modulated (PWM) wave will arrive later or earlier than 0 displacement (origin) points. They will, therefore, have a time displacement to the instantaneous value of given signal. The leading edge of the pulses are removed using a diode clipper or rectifier and remaining pulses are pulse position modulated. The main advantage of PPM is it requires lesser power transmitter.

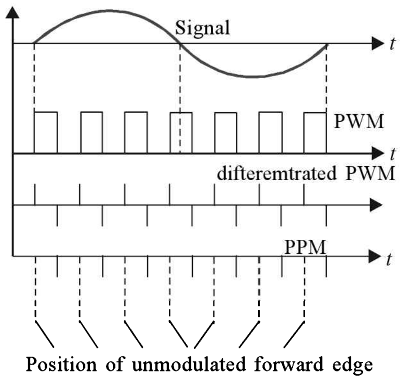


Fig. 38.13 PPM illustration

Pulse Coded Modulation (PCM) It is a digital process. Number of bits required = $\log_2 N$ or $2^n = N$ where N are equiprobable events. For example, to transmit 26 english letters, you require number of bits $2^5 = 32$ and $2^4 = 16$, i.e., 5 bits will be required. If at least 8 bits are used then almost all letters, numerals etc can be covered. PCM also uses sampling technique but to generate codes. Fig. 38.14 explains how a signal can be quantized or coded. For example, consider the signal lying between ± 8V when the signal voltage is less than –6V code is 000. When the voltage –4V < V < –2V code is 010. When 2V < V < 4V the code is 101 and so on. Since there is quantization error, to minimize the quantization error number of bits should be large. If n-bit system is used then channel capacity $C = 2\ \delta f\, n$ where δf is band width of the channel. Total number of information sent in time t is

$H = 2\ \delta f n t$

PCM requires complex encoding and quantizing circuit. Some extra bits are added for listing and error detection. It also requires larger bandwidth as compared to analog system. Its main advantages is that it can be made computer compatible using ASCII codes.

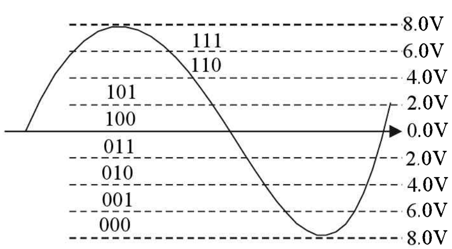


Fig. 38.14 PCM illustration

Delta Modulation The simplest form of delta modulation is differential PCM. Just one bit per sample is sent to indicate whether the signal is smaller or longer than the previous case. It uses a simple coding and decoding procedures, and quantizing process is also quite simple. In delta modulation the transmission rate must be close to 100 k bits s^{-1}, to give the same performance as for a telephone channel as PCM gives with 64 k bit s^{-1}. (8000 samples $s^{-1} \times 8$ bit per sample).

Detection Systems The simplest detector uses a peak detector (rectifier + capacitor filter) to detect AM wave. The diode will rectify and give either positive or negative half cycle and capacitor filter gives only peak value. Thereby detecting information from the carrier. The simple circuit and output is shown in Fig. 38.15. However, the information is slightly distorted as we cannot retrieve exact input because capacitor charges or discharges nearly linearly for short intervals. But to a large extent it is replica of input. Normally, we use negative part of the input for detection (in the figure positive part is shown), as it helps in achieving AGC (Automatic gain control).

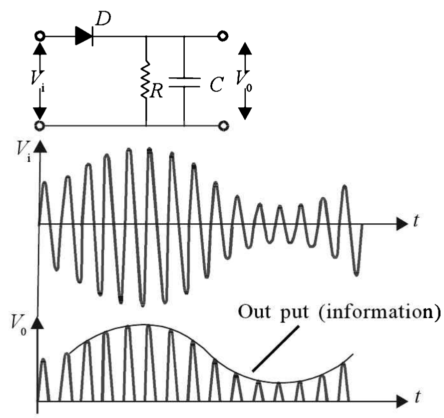


Fig. 38.15 AM detection

FM can be generated using balanced modulator or varactor diode.

FM detection The simplest FM detection is achieved using an LC circuit operated at OFF resonant frequency as shown in Fig. 38.16. See how simple it is to convert frequency variation into voltage variation using LC oscillator. The frequency variation is converted to current variation and hence, voltage variation (if taken across a resistor).

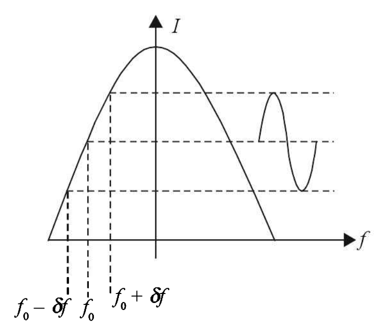


Fig. 38.16 FM detection illustration

ASK (Amplitude Shift Keying) Binary ASK is also called OOK (ON OFF keying). As is clear from Fig. 38.17.

When '1' is transmitted wave is present and when a '0' is transmitted wave is absent Fig. 38.17 (a). shows transmission of binary 10 110.

FSK (Frequency Shift Keying) During transmission of a '1', frequency increases (say doubles as shown in Fig. 38.17 (b). It remains unchanged during transmission of a '0'. It is another way of frequency modulation.

PSK (Phase Shift Keying) Phase shift of 180° or change of half wavelength is observed during a transition $0 \to 1$ or $1 \to 0$ as illustrated in Fig. 38.17 (c).

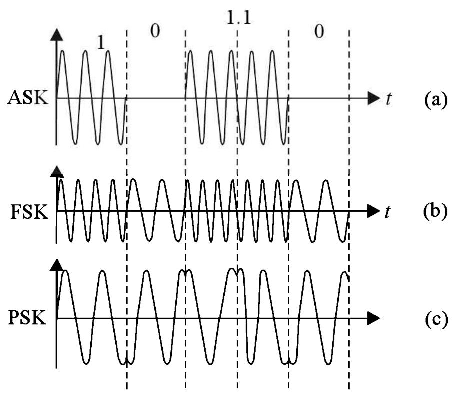


Fig. 38.17 ASK, FSK and PSK illustration

PSK may be used in Telex or Telegraphy. The carrier may be phase shifted by +90° for a mark, and by –90° for a space. In the four phase systems possible phase shifts are +135°, +45°, –45°, –135° so that two bit of information can be indicated instead of one as in the other systems.

Transmission Lines Transmission line is a mode of carrying the information. The equivalent circuit of a transmission line is shown in Fig. 38.18 (a) and (b). The common transmission lines are twisted pair, twin parallel line and coaxial cable.

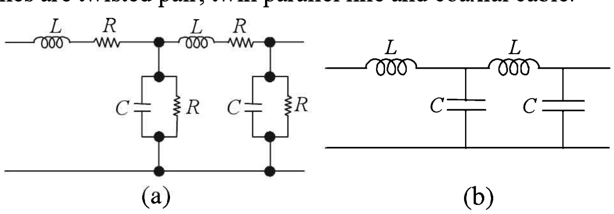


Fig. 38.18 Equivalent circuit of a transmission line

Fig. 38.19 (a), (b) and (c) shows respectively twisted pair, parallel twin line and coaxial cable structures.

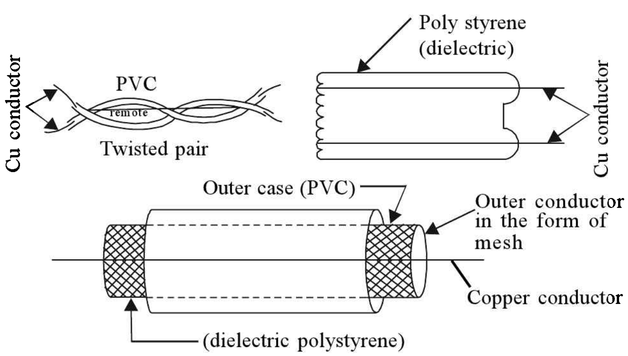


Fig. 38.19 Cables

Characteristic Impedance (Z_o) from Fig. 38.18 (a) series impedance per unit length $Z = R + jL\omega$ (Ω/m)

Shunt impedance per unit length $Y = G + jC\omega$ (s/m)

Characteristic Impedance $Z_o = \sqrt{\dfrac{Z}{Y}} = \sqrt{\dfrac{R + jL\omega}{G + jC\omega}}$ Ω. It is to be used in telephony.

At radio frequencies the equivalent circuit is as shown in Fig. 38.18 (b) and $Z_o = \sqrt{\dfrac{jL\omega}{jC\omega}} = \sqrt{\dfrac{L}{C}}$. Note that characteristic impedance is resistive at radio frequencies.

Using cable geometry as shown in Fig. 38.20.

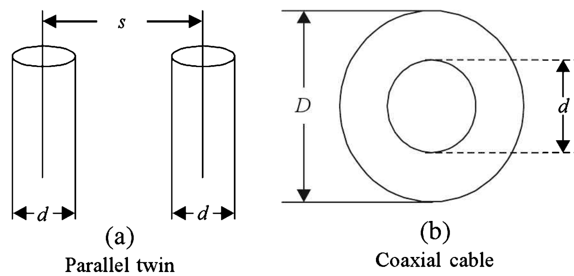


Fig. 38.20

For a parallel twin wire $Z_o = 276 \log \dfrac{2s}{d}$ Ω.

For a coaxial cable $Z_o = \dfrac{138}{\sqrt{k}} \log \dfrac{D}{d}$ Ω. Where k is dielectric constant of the insulation. The usual range of characteristic impedance for a balanced line is 150 to 600 Ω and 40 to 150 Ω for coaxial cables.

Losses in transmissions lines There are three ways of energy impressed at the transmitting end of a transmission line to dissipate before reaching the load: (a) radiation (b) conductor heating c) dielectric heating.

Standing Wave Ratio (SWR) The ratio of maximum current to minimum current along a transmission line is called SWR.

$$\text{SWR} = \frac{I_{max}}{I_{min}} = \frac{V_{max}}{V_{min}}$$

$$= \frac{Z_o}{R_L} \text{ or } \frac{R_L}{Z_o} \text{ (whichever is larger).}$$

For long distance communication satellite links and microwave links (space wave communication) are used. Optical fiber is also used for long distance communication. Repeaters are required after a suitable distance in all systems of transmission lines barring satellite communication. Table 38.1 lists the band width and repeater distance for each of the transmission line. Wavelength of interest in optical fiber range are 0.8 μm to 1.7 μm which lie in *IR* region.

Transmission line	Band width	Repeater required after a distance
Twisted pair	2-3 MHz	1-2 km
Twin wire	30-60 MHz	3-4 km
Coaxial cable	upto 18 G Hz	5-10 km
Optical fiber	enormous	10-100 km

Table 38.1

Acceptance angle A ray which is incident at the core/cladding interface at angle $\geq \theta_c$ will be transmitted. The ray enters the fiber core at an angle θ_a to the fiber axis and is refracted at the air core interface before transmission to the core/cladding interface at critical angle θ_c. Thus, θ_a is the maximum angle to the axis of the fiber at which light may enter the fiber in order to be propagated. This angle θ_a is called acceptance angle,

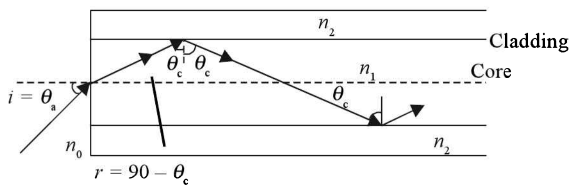


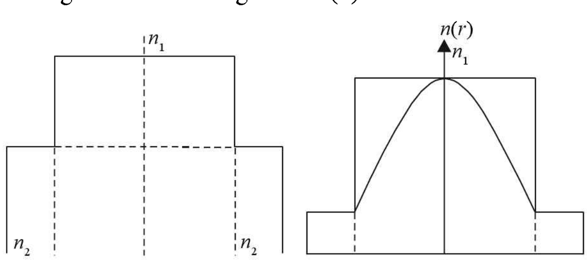
Fig. 38.21 Propagation of ray in optical fibre

$$\sin \theta_a = \frac{1}{n_0} \sqrt{n_1^2 - n_2^2}$$

$$\text{or } \theta_a = \sin^{-1}\left[\frac{\sqrt{n_1^2 - n_2^2}}{n_0}\right]$$

Types of Optical Fibers

In general, fibers may be divided into two types: index fibers and graded index fibers. Graded index do not have constant refractive index in the core but a decreasing core index $n(r)$ with radial distance r. The maximum value n_1 is at the axis and n_2 (minimum value) beyond the core radius 'a' in the cladding as shown in Fig. 38.22 (b).



(a) Stepindex fiber (b) graded index fiber

Fig. 38.22

$$n(r) = n_1\left[1 - 2\Delta\left(\frac{r}{a}\right)^{\alpha}\right]^{1/2} \quad r < a.$$

$$= n_2 \quad r \geq a \text{ for}$$

where Δ is relative refractive index.

In step index fiber the core has fixed refractive index n_1 and cladding a fixed index n_2.

In another way we can divide the optical fibers as single mode and multimode fibers.

Single mode fibers have small core diameter and large cladding diameter while multimode fibers have large core diameter as illustrated in Fig. 38.23 (a) and (b) respectively.

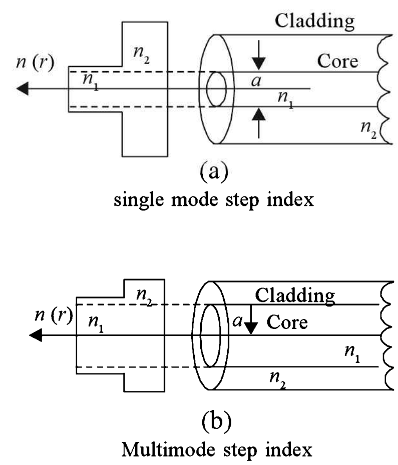


Fig. 38.23

Losses in fibers Factors which cause losses in fibers are intrinsic absorption, linear scattering, Rayleigh scattering, Mie scattering, simulated brillouin zone scattering, Raman scattering, interference of modes, dispersion, diffraction etc.

$$\text{Attenuation } (dB) = 10 \log \frac{P_1}{P_0}.$$

If α_{dB} is attenuation (in dB) per unit length then

$$\alpha_{dB}\, l = 10 \log \frac{P_1}{P_0}.$$

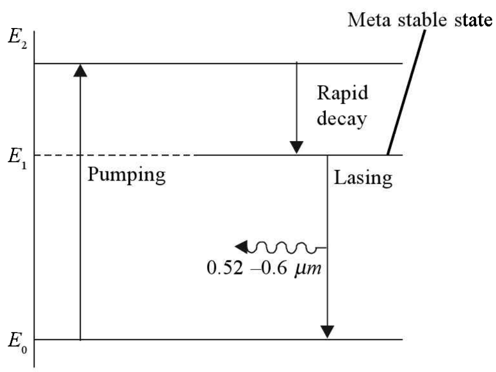
LASER (Light Amplification by Stimulated Emission of Radiation) Light in general can be emitted in two ways (a) spontaneous emission (b) stimulated emission. In spontaneous emission electrons (or atoms) de-excite in a purely random manner. For example, in an incandascent bulb.

In stimulated emission a photon having energy equal to energy difference $(E_2 - E_1)$ interacts with the atom in the higher energy state E_2 causing it to return to the lower energy state E_1 resulting in creation of second photon. Both the photons happen to be in phase and have the same polarization and hence, amplitudes are added up or energy increased which is called light amplification. In a normal case, according to Einstein's condition

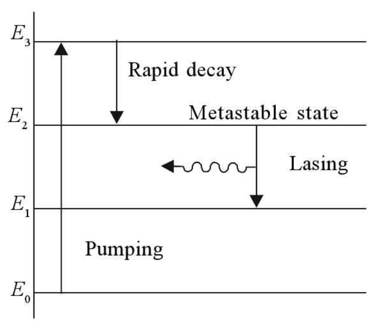
$$\frac{\text{stimulated emission rate}}{\text{spontaneous emission rate}} = \frac{1}{\exp\frac{hf}{kT} - 1}.$$

That is, spontaneous emission is much more dominant mechanism in thermal equilibrium. Therefore, to form laser suitable conditions are to be created so that stimulated emission dominates. Notable among these are **population inversion** and **resonating cavity.**

Population inversion Under normal condition (thermal equilibrium) lower energy levels contain more electrons (as they are filled) than higher energy levels (as they are empty). It is necessary to create non equilibrium condition such that number of electrons in higher levels are more. That is termed as population inversion and is achieved by means of pumping. Optical pumping and electrical pumping are two popular methods of excitations. Fig. 38.24 (a) illustrates 3 level lasing system as in Ruby laser and Fig. 38.24 (b) illustrates a 4-level transition. The disadvantage of a level system is that transition E_0 to E_2 should be very fast, i.e., pumping rate should be very large. In 4-level system E_2 is metastable state and transition E_2 to E_1 gives lasing action.



(a)



(b)

Fig. 38.24 Lasing action illustration

Resonating Cavity

Febry-perot resonator is required to achieve stimulating radiation or optical feedback and hence, lasing oscillations are generated. The resonator consists of two mirrors M_1 and M_2 at the ends as shown in Fig. 38.25. M_1 is perfectly reflecting while M_2 is partially reflecting and semitransparent. The optical output which is laser is obtained from M_2 some of the rays after reflection from M_2 and M_1 act as stimulating radiations.

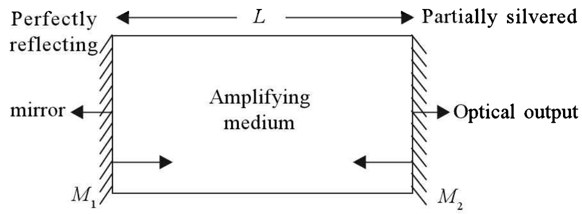


Fig. 38.25 Febry perot cavity

Types of LASERS LASERS are of three types: solid, liquid and gas lasers. Ruby laser, glass laser, semiconductor lasers are examples of solid lasers. High powered solid lasers are pulsed. Some organic liquids are used to form liquid laser. He-Ne and CO_2 lasers are examples of gas lasers. CO_2 laser is used in surgery.

Lasers used in communication system are semiconductor lasers, Nd: YAG (a glass laser), glass fiber laser. Diode laser and Nd: YAG laser is used in cosmetic surgery and hair removal triple treatment etc.

Total laser efficiency (η_T) =

$$\frac{\text{Total number of output photons}}{\text{Total number of injected electrons}}$$

External power efficiency $\eta_{EP} = \eta_T \left[\frac{E_g}{V}\right]$.

High power lasers are used to cut thick/hard metal sheet/bars or may be used in welding. Lasers are used in bloodless operations and to see the inner view of the body by piercing a small hole. LASIK is used for eye operation sight correction and so on. Laser printers are also available. Laser is used to read and store data in the CD (compact disc).

Semiconductor lasers are formed with *Ga As, Al Ga As, Al Ga P, P Al Ga P* etc. coupled cavity or distributed feedback lasers are used in communication systems.

LEDs are also used in fiber optics. Edge emitted, truncated edge emitting *InGaAs P* V-grooved edge emitting and mesa structure edge emitting LEDs are used. All are made with *Ga As* or III-V semiconductor. Superluminescent *AlGaAs* LEDs are also in use.

Reliability Reliability is defined in two ways, electrical reliability and optical reliability.

Electrical Reliability R_E (*dB*)

$$= 10\log_{10}\frac{\text{electric power output (at detector)}}{\text{electric power input (at source)}}$$

$$= 10\log\frac{I_{out}^2}{I_{in}^2} = 20\log\frac{I_{out}}{I_{in}}$$

Optical raliability R_o (*dB*) = $10\log_{10}$

$$\frac{\text{optical power output}}{\text{optical power input}} = 10\log\frac{I_{out}}{I_{in}}.$$

The $-3dB$ point occurs when $\frac{I_{out}}{I_{in}} = \frac{1}{2}$.

Photo Detectors

Avalanche photo diodes, p-i-n photo diodes, photo transistors and photo-conductine detectors are used as detectors in optical fiber communication.

Modem (Modulator-demodulator) Modem is required in two-way communication system. If digital system is used Modem converts analog signal to digital form before it is transmitted. It receives a digital signal and converts it into analog form. Modems are 'therefore' attached with telephone lines so that two-way communcation is feasible. Fax machine is also a form of modem. FAX (Facsimile) is used to transmit data written on a paper or text written on a paper.

Remote sensing Remote sensing utilizes energy of electromagnetic radiations to detect and quantity information about an object that is not in contact with the sensing device. IRS-1A and IRS-IB are Indian remote sensing satellites. These are placed in sun synchronous orbits around the earth. The important applications of remote sensing satellites are a) spying work b) ground survey, water survey, forest survey, survey of fishing zones in sea/oceans, underground locations of coal, oil, minerals, ores etc. c) preparation of wasteland maps. d) draught estimation e) estimation of crop yield f) detection of crop deseases etc.

• Short Cuts and Points to Note

1. Modulation is required for long distance communication. Modulation is of two types, analog and digital. Analog modulation is subdivided to wave modulation and pulse modulation. Wave modulation may be divided to amplitude modulation (AM) and angle modulation. Angle modulation is of two types: frequency modulation (FM) and phase modulation (PM). Frequency modulation is noise immune but comparatively short distance communication (upto 50-60 km). Beyond that either repeaters are required or satellite is to be used. Amplitude modulation is noise prone. AM frequency range for medium wave band (Radio) 550 kHz to 1650 kHz and short wave band 3 MHz to 10 MHz.

FM Radio has frequency band 80 MHz to 120 MHz.

2. Modulation index for AM wave

$$m_a = \frac{E_m}{E_c} = \frac{E_{max} - E_c}{E_c} = \frac{E_c - E_{min}}{E_c}$$

$$= \frac{E_{max} - E_{min}}{E_{max} + E_{min}} = \frac{E_{max} - E_{min}}{2E_c}.$$

If several modulating signals are present then m_{net} = $\sqrt{m_1^2 + m_2^2 + m_3^2 + ...}$ $m_{net} \le 1$.

3. Total Power in AM Wave

$$P_{Total} = P_{Carrier}\left[1 + \frac{m_a^2}{2}\right];\ \frac{I_{Total}}{I_{Carrier}} = \sqrt{1 + \frac{m_a^2}{2}}$$

4. Maximum deviation in FM $\delta = kf_c E_m$,

$$\theta = \omega_c t + \frac{\delta}{f_m} \sin \omega_m t.$$

Frequency modulation index $\frac{\delta}{f_m} = \frac{kf_c E_m}{f_m}$.

Bessels functions are required to find analytical behaviour, band width etc.

5. Band width in FM = $2f_m \times$ highest needed side band if $m_f \le 6$.

Band width = $2\,(f_m + 8)$ if $m_f > 6$.

6. Ground waves propagate along the surface of the earth. These are vertically polarised to prevent the short circuiting of electrical component. Electric field at a distance d is given by $E = \frac{120\pi h_t I}{\lambda d}$ and signal received by an antenna of height h_r is given by V (volts) = $\frac{120\pi h_t h_r I}{\lambda d}$.

7. Sky waves and space waves are reflected by ionosphere if frequency signal is < 10 MHz. The critical $f_c = 9\sqrt{N}$ where N is ionization density and maximum useable frequency (muf) = $f_c \sec \theta$.

Attentuation factor $\alpha = k\sqrt{\frac{f_c}{f}}$

Refractive index = $\sqrt{k} = \sqrt{\frac{1+81N}{f^2}}$.

8. Radio horizon = $\frac{4}{3}$ optical horizon. The emperical formula is

$$d = d_t + d_r \Rightarrow a\,(km) = 4\left(\sqrt{h_t} + \sqrt{h_r}\right).$$

9. Duct or super refraction of microwaves occurs due to temperature inversion zones present in the space. For UHF rays troposcatter or forward scatter propagation occurs. Duct is effective if frequency range 900 MHz, 2000 MHz and 5000 MHz for UHF.

10. In satellite communication uplink and down link frequencies differ. Downlink frequency being slightly higher. See Fig. 38.26.

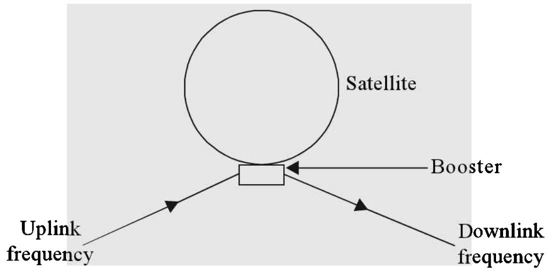


Fig. 38.26

11. Pulse modulation is of two types – analog and digital. PAM and PTM are examples of analog modulation while PCM is an example of digital modulation. PTM may be of two types PWM (or PDM) and PPM. Delta modulation which is a differential PCM is another form of digital modulation.
12. Receivers may be of two types tuned radio frequency (TRF) receivers and super heterodyne receivers. Super heterodyene receivers use local oscillators and intermediate frequency amplifiers before the signal is detected. In this way, reception becomes free of signal frequency but depends only on intermediate frequency.
13. Intermediate frequency (IF) for AM radio is 455 kHz and 10.7 MHz for FM and IF for TV operating at VHF may be set between 26 to 46 MHz and for UHF range it is 36 to 46 MHz. Those operating at microwave have IF between 60 to 70 MHz.
14. A rectifier with peak detection (capacitor filter) is used in the AM wave detection. FM detection is achieved using an LC circuit tuned at OFF resonante frequency.
15. ASK (Amplitude Shift Keying) is also called ON OFF keying (OOK). ASK, FSK and PSK are used to transmit binary data using analog means.
16. Transmission lines have equivalent circuit LCR (at low frequencies) and LC (at high frequencies). Twisted pair, twin-line and co-axial cable, optical fiber, microwave links, satellite link etc. are used as transmission lines. The last two are termed as space wave communication. Twisted pairs twin line and co-axial cable work in electrical regime. Optical fibers operate in IR region.
17. Standing wave ratio (SWR) = $\frac{I_{max}}{I_{min}} = \frac{V_{max}}{V_{min}}$ or SWR $= \frac{Z_o}{R_L}$ or SWR $= \frac{R_L}{Z_o}$ whichever is greater. Bandwidth between half power points in an antenna $\phi_{(0)} = \frac{70\lambda}{D}$ where D is mouth diameter. and bandwidth between nulls $\phi_0 = 2\phi$
18. Acceptance angle θ_a in an optical fiber is the maximum angle of incidence for which incident wave can be propagated through the fiber (by means of total internal Reflection)

$$\theta_a = \sin^{-1}\left[\frac{1}{n_0}\sqrt{n_1^2 - n_2^2}\right].$$

If α_{dB} is attenuation per unit length then $\alpha_{dB} = 10 \log \frac{P_2}{P_1}$ or $I = I_o e^{-\alpha l}$.

Total attenuation in $dB = \alpha_{dB}\, l$.

19. Lasing materials must possess metastable state. Metastable state has an average life time ~ ms (10^{-3} s). In lasing materials the difference of energies just above metastable state and metastable state or metastable state and just lower state must lie in the range of interest.
20. Population inversion corresponds to increasing the density of electrons in the excited state.
21. Febry Perot Cavity or resonator is required to generate feedback oscillation or stimulating radiations.
22. APDs (Avalanche photo diodes) are best suited for detection in fiber optic communication.
23. External power efficiency $\eta_{EP} = \eta_T \left[\frac{E_g}{r}\right]$ where η_T is total efficiency

$$\eta_T = \frac{\text{Total number of output photons}}{\text{Total number of injected electrons}}.$$

24. Electrical Reliability $R_E(dB) = 10 \log_{10} \frac{I_{out}^2}{I_{in}^2} = 20 \log \frac{I_{out}}{I_{in}}$ and – 3dB point occurs when $\frac{I_{out}}{I_{in}} = \frac{1}{\sqrt{2}}$.

Optical Reliability $R_o(dB) = 10 \log_{10} \frac{I_{out}}{I_{in}}$ and – 3dB point occurs when $\frac{I_{out}}{I_{in}} = \frac{1}{2}$.

25. Modem (Modulator-demodulator) are used for two way communications. In digital systems, modems convert analog to digital or vice versa. Telephone and Fax are simple modems.
26. Transducers are of two types self excited and those which require excitation source. Self excited do not require battery while others require battery.
27. A communication satelliate covers 42% area of the whole globe. Therefore at least 3 satellites are

required to cover the whole globe. Maximum number of geostationary (or communication satellites) which can be operative at a time = 180 at a spacing of 2° each.

28. Note the frequency band and their applications listed in Table 38.2.

Table 38.2

	Frequency band	Application
1.	VLF < 100 kHz	Navigation communication
2.	Medium frequency 300 kHZ – 3 MHz	Medium wave, local and distant radio (AM)
3.	High frequency 3 MHz – 30 MHz	Short wave radio (AM), amateur Radio and communication Radio.
4.	Very high frequency (VHF) (30 MHz – 300 MHz)	FM radio, Police communication, meteorology.
5.	U H F 300 MHz – 3 GHz	TV (bands 4 and 5), aircraft landing system.
6.	Microwaves > 3 GHz	Radar, communcation satellites, mobile telephones and TV links.

29. Lasers are monochromatic, coherent and collimated.

30. Optical fibers are single mode step index, multimode step index, single mode graded index and multimode graded index type. Single mode type have smaller cross-section of the core and multimode fibers have larger core cross-section. Parabolic index variation is most preferred in present day technology.

31. MASER is microwave amplification by stimulated emission of radiation. It is used as a microwave amplifier or oscillator. The principle of MASER is identical to that of LASER. Only frequency range is $\le 10^{11}$ Hz in masers.

32. Light intensity is modulated in optical fiber communication as a simplest means of modulation. However, all methods like FM, ASK, FSK, PSK, digital modulation can be employed even in optical communication system.

33. If h is the height of antenna then the range to which it can serve is $r = \sqrt{2Rh}$ where R is the radius of the earth.

- **Caution**

1. Considering all pulse modulations as a form of digital modulation.

⇒ Only PCM, delta and differential PCM are digital. PPM, PDM or PWM, PAM are analog modulation techniques.

2. Assuming there is no difference between phase and frequency modulation.

⇒ In FM, $m_f \propto \frac{1}{f_m}$ and in PM modulating index does not vary with modulating frequency. FM is more noise immune.

3. Assuming AM with both the sidebands is advantageous.

⇒ AM with single side band suppressed carrier is better as its power is whole intelligence.

4. Considering that with several waves, modulation indices are added algebraically.

⇒ $m_{tot} = \sqrt{m_1^2 + m_2^2 + m_3^2 + ...}$.

5. Considering all antennas to be isotropic.

⇒ Mostly antennas are directional. For example, antennas like dish, butterfly, yag and so on are directional. With an isotropic antenna range covered is $\sqrt{2\text{Re}\,h}$.

6. Considering that ionosphere reflects all the waves.

⇒ Ionosphere reflects waves of VHF order. UHF and microwaves are transparent to ionosphere. The critical frequency $f_c = 9\sqrt{N}$ where N is ion density.

7. Assuming any number of geostationary satellites can be installed in space.

⇒ Minimum 3 geostationary/communication satellites are required to cover the whole globe. Maximum number of satellites (communication) which can be launched is 180 at a separation of 2°.

8. Considering mobile phones operate at HF.

⇒ Mobile phones operate at microwaves.

9. Considering modems are used in one way communication like radio or T.V.

⇒ Modems are used in two-way communication. If the communication system is digital (for example, with computers) then Analog to digital converter (ADC) or digital to analog converter (DAC) act like modems.

10. Assuming transducers only convert time varying physical quantities into electrical signal.

⇒ They also convert electrical signal back to physical quantities. Such transducers are called output transducers. The former type of transducers are called input transducers.

11. Considering that light (laser) in fiber optics can be only intensity or amplitude modulated.

⇒ All possible modulation techniques may also be employed in optical communication including AM, FM, PM, PAM, PCM and so on. Even ASK, FSK and PSK can be used.

12. Assuming microwaves can be generated using transistors or vacuum tubes only.

⇒ Gunn diode, IMPATT diode, PIN diodes, Tunnel diodes can be used to generate microwaves.

13. Not understanding the meaning of Q-spoiling in lasers.

⇒ Decreasing the pulse duration without altering the average power.

14. Not understanding the meaning of duty cycle of a pulse train.

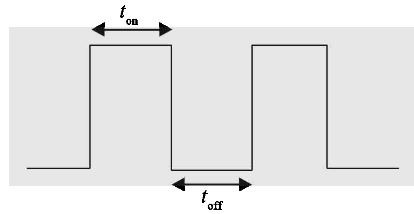


Fig. 38.27 Duty cycle illustration

⇒ $\dfrac{t_{ON}}{t_{ON}+t_{OFF}}$ = duty cycle (See Fig. 38.27).

15. Not understanding the directivity of an antenna.

⇒ Maximum directive gain is called directivity. For a lossless antenna directivity = power gain A_p and for a antenna with paraboloid reflectors $A_p = 6\left(\dfrac{D}{\lambda}\right)^2$.

SOLVED PROBLEMS

1. For sky wave propagation of a 10 MHz signal, what should be the minimum electron density in the ionosphere?

(a) ~ 1.2×10^{12} m^{-3} (b) ~ 10^6 m^{-3}

(c) ~ 10^4 m^{-3} (d) ~ 10^{22} m^{-3}

(AIIMS 2005)

Solution (a) $f_c = 9\sqrt{N}$ or $N = \dfrac{f_c^2}{81}$

$= \sim 1.2 \times 10^{12}$ m^{-3}

2. *A*. Diode lasers are used as optical sources in optical communication

R. diode lasers consume less energy.

(a) Both *A* and *R* are correct and *R* is correct explanation of *A*.

(b) Both *A* and *R* are correct but *R* is not correct explanation of *A*.

(c) *A* is correct but *R* is false.

(d) both *A* and *R* are false.

(AIIMS 2005)

Solution (b) diode lasers are small in size and consume less energy.

3. *A*. Television signals are received through sky wave propagation

R. The ionosphere reflects electromagnetic waves of frequencies greater than a certain critical frequency.

(a) Both *A* and *R* are correct and *R* is correct explanation of *A*.

(b) *A* and R are correct but *R* is not correct explanation of *A*.

(c) *A* is correct but *R* is false.

(d) both *A* & *R* are false.

(AIIMS 2005)

Solution (d)

4. Q-spoiling in a laser stands for

(a) increasing pulse duration time without decreasing average power.

(b) decreasing pulse duration while keeping average power constant.

(c) decreasing pulse duration and average power of the laser.

(d) increasing pulse duration and decreasing average power.

Solution (b)

5. The beamwidth between nulls of a 2 m paraboloid is ________ if 6 GHz reflector is used.

(a) 1.75° (b) 2.25°

(c) 3.0° (d) 3.5°

Solution (d) $\lambda = \dfrac{3\times10^8}{6\times10^9} = 0.05$ m and $\phi_o = 2 \times \dfrac{70\lambda}{D}$

$= \dfrac{140\times0.05}{2} = 3.5°$

6. Calculate the gain of a paraboloid antenna of a 2 m diameter reflector used at 6 GHz.

(a) 9600 (b) 5600
(c) 7600 (d) none of these

Solution (a) $A_P = 6\left(\frac{D}{\lambda}\right)^2 = 6\left(\frac{200}{5}\right)^2 = 9600\,\lambda$

$\because\ 1 = \frac{3\times10^8}{6\times10^9} = 5$ cm.

7. A communication receiver is also capable to receive
(a) E-mails. (b) Morse-code.
(c) SMS. (d) MMS.

Solution (b)

8. If f_s is signal frequency and f_i is intermediate frequency, then frequency of local oscillator is
(a) $f_s - f_i$ (b) $\frac{f_s + f_i}{2}$
(c) $\sqrt{f_s f_i}$ (d) $f_s + f_i$

Solution (d) $f_o = f_s + f_i$

9. The image frequency is ________ if f_s is signal frequency and f_i is intermediate frequency.
(a) $f_s - f_i$ (b) $\sqrt{f_s f_i}$
(c) $f_s + 2f_i$ (d) $\frac{f_s + f_i}{2}$.

Solution (c) $f_{\text{image}} = f_s + 2f_i$

10. An 80 MHz carrier is modulated by 400 Hz sine wave. The carrier voltage is 5V and the frequency deviation is 20 kHz. Find modulation index.
(a) 25 (b) 50
(c) 400 (d) 5
(e) 20

Solution (b) $m_f = \frac{\delta}{f_m} = \frac{20000}{400} = 50.$

11. A 100 MHz carrier is frequency modulated by 500 Hz sine wave. The voltage of modulating signal is 3.6 V and deviation is 24 kHz. If the amplitude of modulating signal drops to 2.4 V. What will be the deviation?
(a) 16 kHz (b) 24 kHz
(c) 36 kHz (d) 18 kHz

Solution (a) $\delta_2 = \delta_1 \times \frac{E_{m2}}{E_{m1}} = 24 \times \frac{2.4}{3.6} = 16$ kHz.

12. If $e_c = 24 \sin 10^6 \pi t$ and $E_m = 12 \sin 500\,\pi t$ are carrier and modulating signal then the modulation index is
(a) 50%. (b) 60%.
(c) 40%. (d) 46%.

Solution (a) $m_a = \frac{E_m}{E_c} = \frac{12}{24} \times 100 = 50\%$

13. A 600 W carrier is modulated to a depth of 75% by a 400 Hz sine wave. Find the total antenna power.
(a) 769 W (b) 796 W
(c) 679 W (d) 637.5 W

Solution (a) $P_{\text{tot}} = P_{\text{carrier}}\left(1 + \frac{m_a^2}{2}\right)$

$= 600\left(1 + \frac{(.75)^2}{2}\right) = 768.75$ W.

14. A transmitter radiates 12 kW power at 80% modulation index. The power in carrier is nearly
(a) 8.21 kW. (b) 6.66 kW.
(c) 7.4 kW. (d) 9.09 kW.

Solution (d) $P_{\text{carrier}} = \frac{P_{\text{Total}}}{1 + m_a^2/2} = \frac{12}{1.32} = 9.09$ kW.

15. When only carrier is transmitted antenna current observed is 8A. When it is modulated with 500 Hz sine wave antenna current becomes 9.6A. Find % modulation.
(a) 80% (b) 20%
(c) 94.26% (d) 83.76%

Solution (c) $m_a = \sqrt{2\left(\frac{I_{\text{Total}}}{I_{\text{Carrier}}}\right)^2 - 1} = \sqrt{\left[(1.20)^2 - 1\right]2}$

$= .9426 = 94.26\%.$

16. If minimum voltage in an AM wave was found to be 2V and maximum voltage 10V. Find % modulation index.
(a) 80% (b) 66.67%
(c) 64.25% (d) 76.25%

Solution (b) $m_a = \frac{E_{\max} - E_{\min}}{E_{\max} + E_{\min}} = \frac{10-2}{10+2} = \frac{2}{3} = 66.67\%$

17. The antenna current is 8A when only carrier is transmitted and 9.6A when AM wave is transmitted. If carrier power is 10 kW, find modulating power.
(a) 14.4 kW (b) 1.2 kW
(c) 2.0 kW (d) 4.4 kW

Solution (d) $\frac{P_{Total}}{P_{Carrier}} = \left(\frac{I_{total}}{I_{carrier}}\right)^2$

$\therefore P_{tot} = 10\left(\frac{9.6}{8}\right)^2 = 14.4\,\text{kW}$

Modulating power $P_{mod} = 14.4 - 10 = 4.4$ kW.

18. Find the acceptance angle, assume core and cladding having refractive index 1.52 and 1.48 respectively.

(a) 20° 30′ (b) 23° 16′
(c) 33° 16′ (d) 18° 30′

Solution (a) $\sin\theta_a = \frac{1}{n_0}\sqrt{n_1^2 - n_2^2} = \frac{1}{1}\sqrt{1.52^2 - 1.48^2}$

$\sin\theta_a = \sqrt{(3)(.04)} = 0.35$ or $\theta_a = 20°\,30'$.

19. Mean optical power launched into an 8 km fiber is 120 μW and mean output power is 3 μW. Find the attenuation per km.

(a) 2 *dB* (km)$^{-1}$ (b) 2.5 *dB* (km)$^{-1}$
(c) 3.1 *dB* (km)$^{-1}$ (d) 4 *dB* (km)$^{-1}$.

Solution (a) $\frac{\alpha_{dB}}{l} = \frac{10\log\frac{P_{out}}{P_{input}}}{l}$

$= \frac{10\log\frac{120}{3}}{8} = \frac{10(1.6021)}{8}$

$= 2.025\, dB\,(\text{km})^{-1}$

20. If the band width of a semiconductor is 0.8 eV. Find the wavelength emitted.

(a) 155 nm (b) 1550 nm
(c) 15.5 nm (d) none of these

Solution (b) $\lambda = \frac{1240}{0.8} = 1550$ nm

21. On a day critical frequency is 10 MHz and on another day it is 11 MHz. Find the ratio of electron densities.

(a) 1.1 (b) 0.9
(c) 1.21 (d) none of these

Solution (c) $\frac{N_2}{N_1} = \left(\frac{f_2}{f_1}\right)^2 = 1.21$

22. 1% of 10^{12} Hz of a setellite link was used for telephoney. Find the number of channels (or subscribers) if each channel is of 8 kHz.

(a) 2.5×10^7 (b) 1.25×10^7
(c) 2.5×10^8 (d) 1.25×10^8

Solution (b) 1% of 10^{12} Hz $= 10^{10}$ Hz

no. of channels $= \frac{10^{10}}{8\times10^3} = 1.25\times10^6$

23. Which of the following is not an analog modulation?

(a) PAM (b) FM
(c) δ -modulation (d) PPM

Solution (c)

24. A5C (Vestigial side band transmission) is used in _______ application.

(a) FM radio (b) AM radio
(c) Cellphone (d) Television

Solution (d)

25. To send E-mail we require

(a) modulator. (b) detector.
(c) modem. (d) FAX.

Solution (c) Modem

TYPICAL PROBLEMS

26. When 80% AM is transmitted total power is 10 kW. If carrier and one side band are suppressed. What will be the power saved ?

(a) 8.28 kW (b) 8.08 kW
(c) 7.88 kW (d) 8.78 kW

Solution (d) $P_{tot} = P_{carrier}\left[1 + \frac{m_a^2}{2}\right]$

$P_{carrier} = \frac{10}{1+\frac{0.64}{2}} = 7.58\,\text{kW}$

Power saved $= P_{carrier}\left[1 + \frac{m_a^2}{4}\right]$

$= 7.58\,[1 + .16] = 8.78\,\text{kW}$

27. The minimum number of bits required to transmit correct selection of an event out of 39 equiprobable events is

(a) 5 (b) 4

(c) 6 (d) 3

Solution (c) $2^N = 39$ $\therefore$ N = 6

28. The output current of a 60% modulated AM generator is 1.5A. To what value will the current rise if the generator is additionally modulated by another audio wave of modulation index 0.7? Also find new modulation index.

Solution $I = I_C\sqrt{1+\frac{m_1^2}{2}}$

$$I_C = \frac{1}{\sqrt{1+\frac{m_1^2}{2}}} = \frac{1.5}{\sqrt{1+\frac{(.6)^2}{2}}} = \frac{1.5}{\sqrt{1.18}}$$

$$I_{new} = I_C\sqrt{1+\frac{m_1^2}{2}+\frac{m_2^2}{2}}$$

$$= \frac{1.5}{\sqrt{1.18}}\sqrt{1+0.18+\frac{0.49}{2}} = 1.64\text{ A}$$

$$m_a = \sqrt{m_1^2+m_2^2} = \sqrt{(0.6)^2+(0.7)^2}$$

$$= \sqrt{.36+.49} = \sqrt{0.85} = 0.92 \text{ or } 92\%.$$

29. Find the bandwidth required for an FM signal, if modulating frequency is 2 kHz and maximum deviation is 10 kHz.

Solution $m_f = \frac{\delta}{f_m} = \frac{10}{2} = 5$

For $m_f = 5$. Highest J coefficient is $J8$.

Thus, 8th pair sidebands is the farthest from the carrier to be included here. Thus, bandwidth $\Delta = 2 \times 8 \times 2 =$ 32 kHz.

30. The equation of an angle modulated voltage is $e = 10 \sin(10^8 t + 3 \sin 10^4 t)$. Which form of angle modulation is this? Calculate carrier frequency and deviation.

Solution FM or PM. $f_c = \frac{10^8}{2\pi} = 4.77$ MHz.

Deviation $\delta = m f_m = 3\left(\frac{10^4}{2\pi}\right) = 4.77$ MHz.

31. Channel capacity per bit for a channel bandwidth δf and number of coding levels N according to Hartley Law in the absence of noise is

(a) $2\delta f N$. (b) $2\delta f \log_e N$.

(c) $2\delta f \log_{10} N$. (d) $2\delta f \log_2 N$.

Solution (d) Channel capacity = $2\delta f \log_2 N$ (noise free and of a noisy channel the channel capacity is

$$C = \delta f \log_2\left(1+\frac{S}{N}\right).$$

32. The centre frequency of an LC circuit is 50 MHz Transconductance by a fixed capacitor of 50 pF. Linearly with gate voltage between the limits 0 – 9 mA The FET is used in a capacitive reactance modulator with $X_{Cdg} = 8 R_{gs}$. Find the total frequency variation.

Solution $C_{min} = 0; C_{max} = \frac{I_m}{2\pi fn} = \frac{9\times10^{-3}}{2\pi\times10^7\times5\times8}$

$$= \frac{9\times10^{-11}}{8\pi} = 3.58\times10^{-12}\text{F}$$

$$\frac{f_{max}}{f_{min}} = \frac{\frac{1}{2\pi\sqrt{LC}}}{\frac{1}{2\pi\sqrt{L(C+C_{max})}}}$$

$$= \sqrt{\frac{C+C_{max}}{C}} = \sqrt{1+\frac{C_g}{C}}$$

$$= \sqrt{1+\frac{3.58}{50}} = 1.0352$$

$$\frac{f_{max}}{f_{min}} = \frac{f+\delta}{f-\delta} = 1.0352$$

$$f+\delta = (f-\delta)\times 1.0352$$

$$2.0352\delta = 0.0352f$$

$$\delta = \frac{.0352f}{2.0352} = \frac{50\times10^6(.0352)}{2.0352}$$

Total frequency deviation = $2\delta = 2 \times 0.865 = 1.73$ MHz.

33. An optical fiber has numerical aperture 0.4. Find the acceptance angle for maridional rays and skew rays which change direction by 100° at each reflection.

Solution θ_a (meridional) = $\sin^{-1}$(NA) = $\sin^{-1}$ (0.4)

= 23°36' $2r = 100°$; $r = 50°$

$$\theta_{as} \text{ (skew rays)} = \sin^{-1}\left[\frac{NA}{\cos r}\right]$$

$$= \sin^{-1}\left[\frac{0.4}{\cos 50}\right] = 38°30'.$$

34. A ruby laser crystal is 4 cm long (refractive index = 1.78). The peak emission wavelength is 0.55 μm. Determine the number of longitudinal modes and their frequency separation.

Solution Number of modes

$$m = \frac{2nL}{\lambda} = \frac{2\times1.78\times4\times10^{-2}}{0.55\times10^{-6}} = 2.6\times10^5$$

Frequency separation

$$\delta f = \frac{C}{2nL} = \frac{3\times10^8}{2\times1.78\times4\times10^{-2}} = 2.1 \text{ GHz.}$$

35. Find the information carrying capacity of 8 kHz band width line having signal to noise ratio at the input of the receiver = 28 *dB*.

Solution $10 \log \frac{S}{N} = 28$ or

$$\frac{S}{N} = \text{antilog } 2.8 = 631$$

$$C = 8000 \log_2 (1+631)$$

$$= 8000\,(9.304) = 74432 \text{ bit/s.}$$

36. The total power efficiency of an injection laser using *GaAs* active region is 20%. The voltage applied is 3.0 V and band gap is 1.5 eV. Find the external power efficiency.

(a) 10 % (b) 40 %

(c) 20 % (d) 15.3 %

Solution (a) $\eta_{ext} = \eta_{tot}\left[\frac{E_g}{V_{app}}\right] = 20\left[\frac{1.5}{3.0}\right] = 10\,\%$

37. A ruby laser crystal 4.4 cm long (ref. index 1.8) gives peak emission wavelength 0.55 μm. Find the number of longitudinal modes and their frequency separation.

Solution number of modes $= \frac{2nL}{\lambda}$

$$= \frac{2\times1.8\times4.4\times10^{-2}}{0.55\times10^{-6}} = 2.88\times10^5$$

Frequency separation $\delta f = \frac{C}{2nL}$

$$= \frac{3\times10^8}{2\times1.8\times4.4\times10^{-2}} = 1.92 \text{ GHz.}$$

38. Characteristic impedance for a coaxial cable is

(a) $\frac{276}{\sqrt{k}} \log \frac{D}{d}$ (b) $\frac{138}{\sqrt{k}} \log \frac{D}{d}$

(c) $\frac{328}{\sqrt{k}} \log \frac{D}{d}$ (d) none of these

Solution (b)

QUESTIONS FOR PRACTICE

1. The main disadvantage of PCM is

(a) its incompatibility with PDM.

(b) the complex circuitry required.

(c) the large bandwidth required.

(d) the high error rate during quantisation.

2. An on-line, real time data transmission system is most likely to require a circuit that is

(a) simplex. (b) semi duplex.

(c) duplex. (d) time-shared.

3. Compounding is used

(a) to protect small signals in PCM from quantising distortion.

(b) in PCM transmitters to allow amplitude limiting in the receiver.

(c) in PCM receivers, to overcome impulse noise.

(d) to overcome quantising noise in PCM.

4. Which of the following is not a PTM?

(a) PDM (b) PWM

(c) PPM (d) PAM

5. Which of the following system is analog modulation system?

(a) PCM (b) differential PCM

(c) delta (d) PWM

6. The height of a TV antenna is 200 m. The population density is 4000 per km^2. Find the population benefitted.

(a) 3.2×10^5 (b) 3.2×10^6

(c) 3.2×10^7 (d) 3.2×10^8

7. If the pulse repetition frequency is 300 Hz, the maximum unambiguous range will be

(a) 500 km. (b) 500 nmi.

(c) 5000 m. (d) 500 radar mile.

8. Out of the following, which has the maximum band width?

(a) twin line pair. (b) twisted pair.

(c) coaxial cable. (d) none of these.

9. When microwave signals propagate along the curvature of the earth, this effect is called

(a) ducting. (b) tropospheric scatter.
(c) ionospheric reflection. (d) Faraday effect.

10. Which of the following frequencies cannot be used for reliable communication beyond horizon without use of repeaters?
(a) 15 MHz (b) 900 MHz
(c) 5 GHz (d) 25 kHz

11. Critical frequency for E_2 layer is
(a) 1 – 10 MHz (b) 3 – 30 MHz
(c) 10 – 30 MHz (d) 5 – 12 MHz

12. During night, the ionosphere consists of
(a) D and E layers. (b) E and F layers.
(c) D, E and F layers. (d) only F layer.

13. The main component of atmosphere responsible for absorption of electromagnetic waves is
(a) N_2. (b) water vapours.
(c) N_2 and water vapour. (d) O_2 and water vapour.

14. Characteristic impedance of free space is given as
(a) $\sqrt{\frac{\mu_0}{\varepsilon_0}}$ (b) $\sqrt{\mu_0\varepsilon_0}$
(c) $\sqrt{\frac{\varepsilon_0}{\mu_0}}$ (d) $\frac{\mu_0}{\varepsilon_0}$

15. In a short circuited line, the first current node lies at a distance of
(a) $\lambda/4$ from the short circuit point.
(b) $\lambda/2$ from the short circuit point.
(c) λ from the short circuit point.
(d) $3\lambda/4$ from the short circuit point.

16. In an AM broadcasting system, modulation index is very unlikely to cross
(a) 100%. (b) 70%.
(c) 80%. (d) 50%.

17. A receiver has poor IF selectivity and, therefore, it will also have
(a) poor sensitivity. (b) double spotting.
(c) the variable selectivity. (d) poor padder capacitor.

18. The ratio of maximum to minimum frequency for a medium frequency broadcasting receiver is
(a) 2 : 1. (b) 2.4 : 1.
(c) 2.8 : 1. (d) 3.4 : 1.

19. A superheterodyne receiver with an IF = 455 kHz is tuned to 1250 kHz. The image frequency is
(a) 795 kHz. (b) 1705 kHz.
(c) 1910 kHz. (d) 2160 kHz.

20. Superheterodyne receiver replaced tuned frequency receiver because the latter suffered from
(a) inadequate selectivity at high frequencies.
(b) gain variation over frequency.
(c) instability.
(d) insufficient gain and sensitivity.

21. The saving in power in case of SSB suppressed carrier over DSB full carrier with 100% modulation is
(a) 50%. (b) 33.33%.
(c) 66.67%. (d) 83.3%.

22. In PM modulation index is
(a) directly proportional to modulating amplitude.
(b) inversely proportional to modulating frequency.
(c) directly proportional to modulating frequency.
(d) inversely proportional to modulating amplitude.

23. An FM signal with a modulation index m_f is passed through a frequency tripler. The wave in output of the tripler will have a modulation index of
(a) $3m_f$. (b) m_f.
(c) $m_f/3$. (d) $9m_f$.

24. An FM signal with a deviation δ is passed through a mixer and has its frequency reduced 5 fold. The deviation in the output of the mixer is
(a) δ. (b) $\delta/5$.
(c) 2.5δ. (d) 5δ.

25. Vestigial sideband transmission is used in
(a) radio transmission (AM).
(b) TV transmission (AM).
(c) radio transmission (digital).
(d) TV transmission (digital).

26. The total power in AM transmission is
(a) $P_{\text{carrier}}\left(1+m_a^2\right)$ (b) $P_{\text{carrier}}\left(1+\frac{m_a^2}{2}\right)$
(c) $P_{\text{carrier}}\left(1+\frac{m_a^2}{4}\right)$ (d) $P_{\text{carrier}}\left(1+2m_a^2\right)$

27. A carrier is modulated simultaneously by 3 sine waves of modulation indices 0.3, 0.4 and 0.45 respectively, the net modulation index is
(a) 1.15. (b) 0.67.
(c) 0.57. (d) none of these.

28. The band width required by an AM signal is equal to
(a) f_C. (b) f_m.
(c) $2f_m$. (d) $\frac{f_m}{2}$.

29. The number of AM broadcast stations that can be accommodated in a 300 kHz band width for the highest modulating frequency 15 kHz will be

(a) 10. (b) 12.
(c) 15. (d) 8.

30. The Shannon-Hartley theorem states that

(a) redundancy is essential.
(b) only binary codes may be used.
(c) the maximum rate of information transmission depends on the depth of modulation.
(d) the maximum rate of information transmission depends upon the channel bandwidth.

31. If the probability of occurrence of message is $p = \frac{1}{8}$ then the amount of information conveyed is

(a) 4 bits. (b) 8 bits.
(c) 3 bits. (d) 1 bit.

32. Super solar cycle repeats after every

(a) 11 years. (b) 50 years.
(c) 100 years. (d) 1 year.

33. The frequency which will pass through a D-region having refractive index 0.5 and electron density 400 cm^{-2} is

(a) 400.2 kHz. (b) 303.23 kHz.
(c) 207.3 kHz. (d) none of these.

34. In an FM system a 7 kHz signal modulates 108 MHz carrier so that frequency deviation is 50 kHz. The carrier swing is

(a) 7.143. (b) 8.
(c) 0.71. (d) 350.

35. The radio signal used to modulate $60 \sin(2\pi \times 10^6 t)$ is $15 \sin 300\pi t$. The depth of modulation is

(a) 50%. (b) 40%.
(c) 25%. (d) 15%.

36. The amplitude modulated current is given by $i = 125 [1 + 0.6 \sin 2900t] \sin 10^6 t$. The depth of modulation is

(a) 60%. (b) 6%.
(c) 36%. (d) none of these.

37. In AM the complete information can be transmitted using

(a) carrier and both the sidebands.
(b) carrier and one sideband.
(c) only carrier.
(d) only one sideband .

38. The waves used in telecommunication are

(a) IR. (b) UV.
(c) microwave. (d) cosmic rays.

39. If n_1 and n_2 are refractive index of core and cladding then

(a) $n_1 = n_2$. (b) $n_1 > n_2$.
(c) $n_1 < n_2$. (d) no confirmed relation.

40. The bandwidth in PDM is determined from

(a) minimum pulse width. (b) maximum pulse width.
(c) position of pulses. (d) none of these above.

41. Modems are

(a) A/D converters. (b) D/A converters.
(c) both (a) and (b). (d) none of these.

42. The digital transmission is mostly achieved via

(a) telephone links. (b) satellite links.
(c) computer links. (d) none of these.

43. The optimum frequency for a cable is

(a) 2.5 GHz. (b) 3 GHz.
(c) 10 GHz. (d) 300 GHz.

44. Quantisation is done in which index increases the band width in case of

(a) PCM. (b) PAM.
(c) PPM. (d) PWM.
(e) all of these.

45. The increase in modulation index increases the band width in case of

(a) FM. (b) AM.
(c) vestigial sideband transmission.
(d) none of these.

46. The channel band width allotted to FM radio is

(a) 100 kHz. (b) 150 kHz.
(c) 200 kHz. (d) 300 kHz.

47. For efficient transmission and reception the height of antenna should at least be

(a) $\lambda/2$. (b) λ.
(c) $\lambda/3$. (d) $\lambda/4$.

48. Which of the following is false? Modulation is used to

(a) separate different transmission.
(b) reduce the bandwidth.
(c) allow the use of practicable antenna.
(d) ensure transmission to long distances.

49. In a communication system, noise is most likely to affect the signal

(a) at the transmitter.
(b) in the channel or in the transmission line.
(c) in the information source.
(d) at the receiver.

Answers to Questions for Practice

1.	(c)	**2.**	(c)	**3.**	(a)	**4.**	(d)	**5.**	(d)	**6.**	(c)	**7.**	(a)
8.	(c)	**9.**	(a)	**10.**	(c)	**11.**	(d)	**12.**	(b)	**13.**	(d)	**14.**	(a)
15.	(a)	**16.**	(d)	**17.**	(a)	**18.**	(c)	**19.**	(d)	**20.**	(d)	**21.**	(d)
22.	(a)	**23.**	(a)	**24.**	(a)	**25.**	(b)	**26.**	(b)	**27.**	(b)	**28.**	(c)
29.	(a)	**30.**	(d)	**31.**	(b)	**32.**	(c)	**33.**	(c)	**34.**	(a)	**35.**	(c)
36.	(a)	**37.**	(d)	**38.**	(c)	**39.**	(b)	**40.**	(a)	**41.**	(c)	**42.**	(b)
43.	(d)	**44.**	(a)	**45.**	(a)	**46.**	(c)	**47.**	(d)	**48.**	(b)	**49.**	(b)

Explanations

6.(c) $r = \sqrt{2RH} = \sqrt{2\times 6400(0.2)}$

Population benefitted $P = \pi r^2$(Population density)

$= 2 \times 6400 \times 0.2 \times 3.14 \times 4 \times 10^3$

$= 3.2 \times 10^7$.

7.(a) $d = \dfrac{3\times 10^8}{2\times 300} = 500$ km.

20.(d) $f_{\text{image}} = f_0 + f_i = f_s + 2f_i = 1250 + 2(455) = 2160$ kHz.

27.(b) $m_{\text{net}} = \sqrt{m_1^2 + m_2^2 + m_3^2} = \sqrt{0.9 + 0.16 + 0.2025} = 0.67$.

29.(a)Number of channels

$= \dfrac{BW}{2\times \text{highest modulating frequency}}$

$= \dfrac{300\times 10^3}{2\times 15\times 10^3} = 10$.

32.(c)$\mu = 0.5$ $N = 400$ electrons/cm^2 $\mu = \sqrt{1 - \dfrac{81N}{f_C^2}}$

or $0.5 = \sqrt{1 - \dfrac{81\times 400}{f_C^2}}$ or $f_C = 207.33$ kHz.

33.(c)Carrier swing $= \dfrac{\text{frequency deviation}}{\text{modulating frequency}}$

$= \dfrac{50}{7} = 7.143$.

34.(a) $m_a = \dfrac{E_m}{E_C} = \dfrac{15}{60} \times 100 = 25\%$.

35.(c) $m_a = 0.6$ or 60%.

SELF TEST 1

1. The $(V-I)$ curve of an ideal diode is

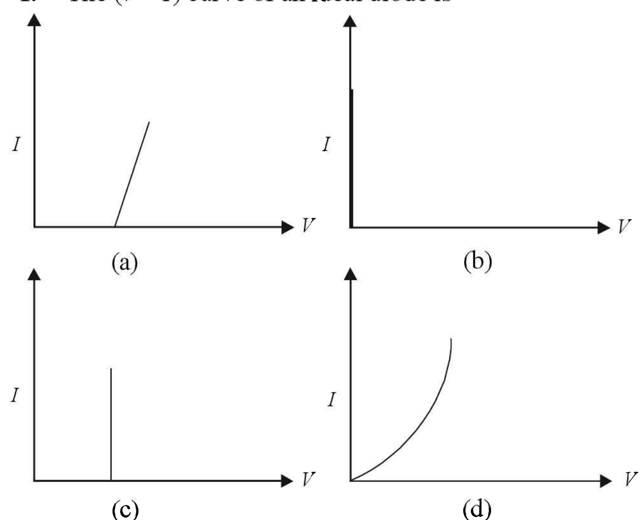


Fig. 1

2. A transistor has

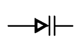
(a) resistance (b) dynamic resistance

(c) conductance (d) none of these

3. A real *p-n* junction diode behaves as

(a) a switch (b) a capacitor

(c) an R - C circuit (d) none of these

4. The circuit symbol  represents

(a) varactor diode (b) capacitor

(c) tunnel diode (d) Amplifier

5. In Fig. 2 which portion of the curve represents negative resistance?

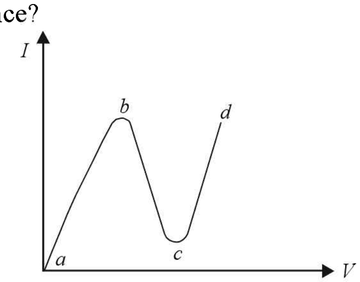


Fig. 2

(a) *ab* (b) *bc*

(c) *cd* (d) *abc*

6. Which of the following is not a negative resistance device?

(a) Tunnel diode (b) Tetrode

(c) Thyristor (d) Triode

7. Bridge rectifier will give the frequency of output____ if the frequency of input singal is f

(a) zero (b) f

(c) $2f$ (d) none

8. Ripple factor of a fullwave rectifier is

(a) 1.0 (b) 1.21

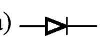
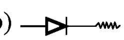
(c) 0.48 (d) 0.72

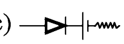
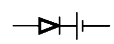
9. The efficiency of a fullwave rectifier is

(a) 40.8 % (b) 92.8 %

(c) 81.6 % (d) 73.4 %

10. Which of the following represents a *p-n* junction more closely?

(a)  (b) 

(c)  (d) 

11. A semiconductor diode cannot be used as

(a) a rectifier (b) a clipper

(c) a voltage multiplier (d) an amplifier

12. A *p-n* junction gives the following data:

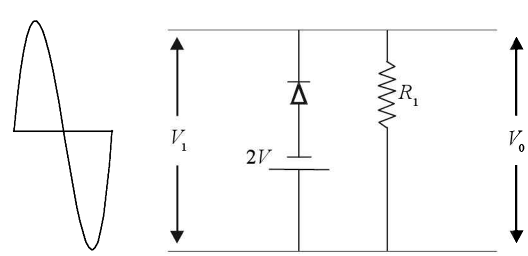
V (volts)	0.2	0.4	0.5	0.6	0.65
$I\,(mA)$	0	0.1	0.2	1.4	3.00

Then the resistance of the diode is

(a) 20Ω (b) 31Ω

(c) 25Ω (d) none of these

13. The shape of output in the given circuit.



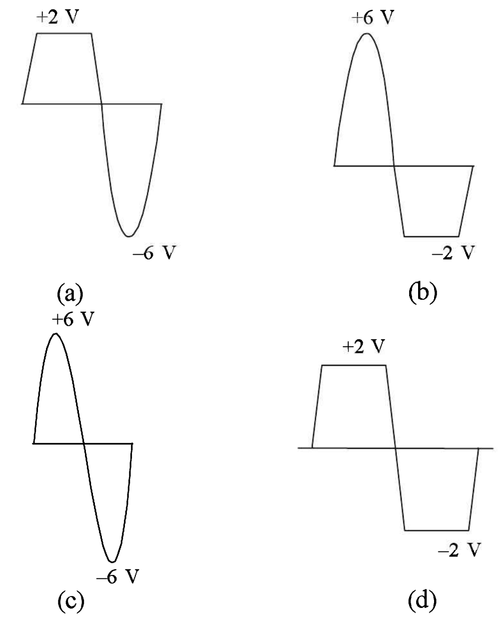


Fig. 3

14. Clipping circuit cannot be made with

(a) transducer (b) transistor

(c) diode (d) triode

15. Clamping restores

(a) ac value to a signal (b) dc value of a signal

(c) both ac and dc values (d) none of these

16. Dead zone is found in a

(a) series clipper (b) clamper

(c) *UJT* (d) transducer

17. Fig. 4 shows two $p-n$ junction diodes connected back to back. In which model it is equivalent to a transistor

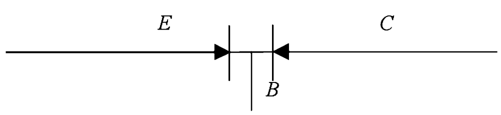


Fig. 4

(a) Ebers Moll (b) Shockley

(c) Bell (d) Schottky

18. In present day technology we prefer

(a) *p-MOS ICs* (b) *n-MOS ICs*

(c) *C MOS ICs* (d) *H-MOS ICs*

19. An amplifier gives

(a) current gain (b) voltage gain

(c) both (a) and (b) (d) power gain

20. A fullwave rectifier is made using a transformer whose rating is 12 – 0 – 12 V. If load resistance used is 500 Ω, then load current is

(a) 22 mA (b) 15 mA

(c) 18 mA (d) 34 mA

21. A transistor has $h_{FE} = 50$ and $I_B = 20\ \mu A$. Then I_E will be

(a) 1 mA (b) 1.02 mA

(c) 1.04 mA (d) 0.98 mA

22. A transistor shall have emitter

(a) heavily doped (b) moderately doped

(c) lightly doped (d) none of these

23. A transistor with $h_{FE} = 60$ is used to make a *CE* amplifier. Its voltage gain will be ____ if $r_i = 1k\,\Omega$ and $R_L = 5\ k\Omega$

(a) 60 (b) 280

(c) 300 (d) 150

24. The output Y in Fig. 5 is

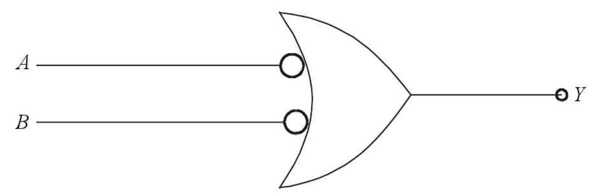


Fig. 5

(a) $\bar{A}\ \bar{B}$ (b) $\bar{A}$ +B

(c) $\overline{AB}$ (d) A + $\bar{B}$

25. An *XOR* can be used as

(a) half adder (b) full adder

(c) multiplexer (d) none of these

26. In an *IC* we cannot make

(a) *L* (b) *R*

(c) *C* (d) transistor

27. In Fig. 6, the transistor used has h_{FE} = 50. Determine base resistance R_B

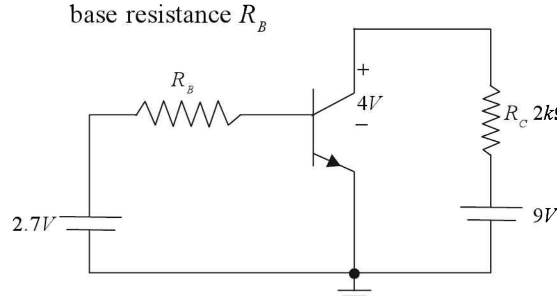


Fig. 6

(a) 30 $k\Omega$ (b) 38 $k\Omega$

(c) 40 $k\Omega$ (d) 50 $k\Omega$

28. A transistor has $h_{FB} = 0.98$. Determine h_{FE} and if $I_B = 40\ \mu A$, find I_c

(a) 49,2 mA (b) 50, 1.96 mA

(c) 49, 1.96 mA (d) 50,2 mA

29. The transistor used in Fig. 7 has h_{FB} = 0.99. Find collector to base voltage.

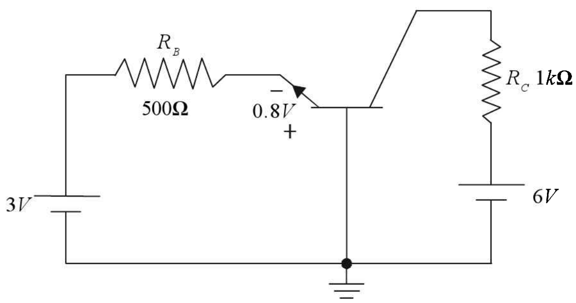


Fig. 7

(a) 0V (b) 0.2 V
(c) 6 V (d) 4 V

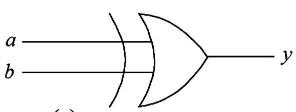
30. Two *RC* coupled amplifiers are connected in tandem with gains 15 and 10, respectively. The overall gain is
(a) 12.5 (b) 75
(c) 100 (d) 150

31. *RC* coupling is used to
(a) prevent distortion (b) prevent noise
(c) both (a) and (b) (d) none of these

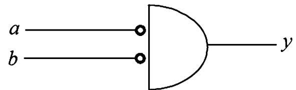
32. An amplifier is made using a transistor of $h_{FE} = 60$, $R_L = 10\,k\Omega$ $r_i = 200\,\Omega$. If the transistor is used in *CB* configuration, then voltage gain is approximately
(a) 49 (b) 60
(c) 61 (d) 30

33. A class-*B* amplifier is made by biasing the transistor in
(a) active region (b) saturation region
(c) cut-off region (d) none of these

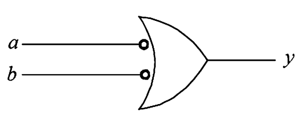
34. Which of the following represents an *NOR* gate?



(a)



(b)



(c) (d) none of these

Fig. 8

35. If n_e is electron density in an extrinsic semiconductor. If h_i is intrinsic hole density, then for $n_e < h_i$, the semiconductor is
(a) *n*-type (b) *p*-type
(c) compensated
(d) insufficient data to reply

36. In an extrinsic semiconductor donor impurity atoms 5×10^{19} /cc are added. Determine the type of semiconductor and electron concentration in it.
(a) *p*-type, 2.5×10^{19} /cc (b) *n*-type, 5×10^{19} /cc
(c) *n*-type, 2.5×10^{19} /cc (d) *p*-type, 5×10^{19} /cc

37. If $Y = AB + \bar{A}B$ then to implement *Y* how many gates are required?
(a) 1 (b) 2
(c) 3 (d) none

38. If $X = A + \bar{A}B$ then number of gates required in it are
(a) 1 (b) 2
(c) 3 (d) none

39. The operator symbol of *NOR* gate is
(a) $\uparrow$ (b) $\odot$
(c) $\downarrow$ (d) $\oplus$

40. The universal gate is used
(a) to perform any logic
(b) to perform combinational logic
(c) to perform threshold logic
(d) to perform high level logic

41. *Ga As* is used to make
(a) homo *p-n* junction (b) hetero *p-n* junction
(c) abrupt *p-n* junction (d) none of these

42. Al is doped in
(a) *Si* (b) *Ga As*
(c) both (a) and (b) (d) none of these

43. To make a solar cell which of the following semiconductor should be used?
(a) *Si* (b) *Ga As*
(c) *Ge* (d) none of these

44. The characteristics shown in Fig. 9 is given by

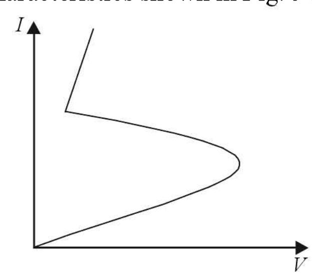


Fig. 9

(a) *UJT* (b) *SCR*
(c) tunnel diode (d) none of these

45. Base current in a bipolar junction transistor consists of
(a) fraction of emitter current
(b) fraction of collector current
(c) both (a) and (b)
(d) none of these

46. If temperature of the room in which transistor is placed is increased by 10° C then I_{CBO}
(a) remains same (b) gets halved
(c) gets doubled (d) none of these

47. To protect thermal runaway we use

(a) double battery biasing (b) self biasing

(c) single battery biasing (d) none of these

48. A full adder is used to add

(a) 2 binary bits (b) 3 binary bits

(c) 4 binary bits (d) none of these

49. The condition to make an oscillator is

(a) $\beta A_v < 1$ (b) $\beta A_v > 1$

(c) $\beta A_v \geq 1$ (d) none of these

50. pnp transistor is

(a) *UJT* (b) *BJT*

(c) JFET (d) *p*-MOSEFT

Answers

1.	(b)	**2.**	(b)	**3.**	(c)	**4.**	(a)	**5.**	(b)	**6.**	(d)	**7.**	(c)
8.	(c)	**9.**	(c)	**10.**	(c)	**11.**	(d)	**12.**	(b)	**13.**	(b)	**14.**	(a)
15.	(b)	**16.**	(a)	**17.**	(a)	**18.**	(c)	**19.**	(d)	**20.**	(a)	**21.**	(b)
22.	(a)	**23.**	(c)	**24.**	(c)	**25.**	(a)	**26.**	(a)	**27.**	(c)	**28.**	(c)
29.	(b)	**30.**	(d)	**31.**	(b)	**32.**	(a)	**33.**	(c)	**34.**	(b)	**35.**	(b)
36.	(b)	**37.**	(d)	**38.**	(a)	**39.**	(a)	**40.**	(a)	**41.**	(b)	**42.**	(b)
43.	(b)	**44.**	(b)	**45.**	(c)	**46.**	(c)	**47.**	(b)	**48.**	(b)	**49.**	(c)
50.	(b)												

Explanations

1(b)

2(b) because device is non-ohmic.

3(c) because *p-n* junction behaves as a capacitor as well as resistance.

4(a) It is the symbol of varactor diode.

5(b) In bc region the current is inversely proportional to voltage which is inverse of ohm's law. Such devices are called negative resistance devices.

6(d)

7(c)

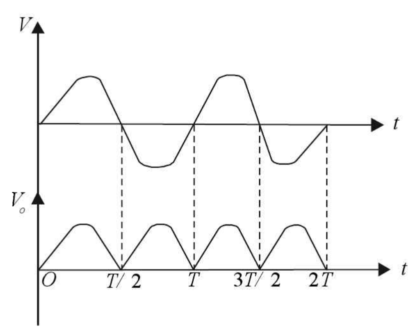


Fig. 10

We find from the fig. showing input and output of a bridge rectifier that output repeats after $T/2$. Hence its frequency is double the input frequency.

8(c)

9(c)

10(c)

11(d)

12(b) $$r = \frac{dV}{dI} = \frac{\Delta V}{\Delta I} = \frac{0.65-0.6}{(3.0-1.4)\times 10^{-3}} = 31\Omega$$

13(b)

14(a)

15(b)

16(a)

17(a)

18(c)

19(d)

20(a) $$I_L = \frac{2V_P}{\pi R_L} = \frac{2\times 12\sqrt{2}}{3.143\times 500} \cong \frac{34}{3.14\times 500}$$
$$\cong 22\ mA$$

21(b) $$I_E = (\beta+1)I_B$$
$$= 51\times 20\times 10^{-6} = 1.02\times 10^{-3} \text{ or } 1.02\ mA$$

22(a)

23(c) $$A_v = \beta\frac{R_L}{r_i} = 60\times\frac{5}{1} = 300$$

24(c) It is equivalent to NAND gate.

25(a)

26(a)

27(c) As the transistor operates in active region, therefore,
$$V_{BE} = 0.7\,V$$
$$I_c = \frac{V_{CC}-V_{CE}}{R_C} = \frac{9-4}{2\times 10^3} = 2.5\ mA;$$
$$I_B = \frac{I_C}{\beta} = \frac{2.5\times 10^{-3}}{50} = 50\ \mu A$$

$$R_B = \frac{V_{BB} - V_{BE}}{I_B} = \frac{2.7 - 0.7}{50 \times 10^{-6}} = 40\,k\Omega$$

28(c) $h_{FE} = \frac{h_{FB}}{1 - h_{FB}} = \frac{0.98}{0.02} = 49$

$I_C = h_{FE}\, I_B = 49\,(40 \times 10^{-6}) = 1.96\,mA$

29(b) As the transistor operates in saturation region, therefore

$V_{CE} = 0.2\,V$

30(d) $A = A_1 \times A_2 = 15 \times 10 = 150$

31(b)

32(a) $\alpha = \frac{60}{61}$ and $A_v = \alpha \frac{R_L}{r_i} = \frac{60}{61} \times \frac{10000}{200} \cong 49$

33(c)

34(b)

35(b)

36(b) Since the impurity B donor type, therefore semiconductor is n-type

$n_c \approx N_D$, therefore

$n_e = 5 \times 10^{19}/cc$

37(d) $X = AB + \overline{A}B = (A + \overline{A})B = 1\,B$

$= B$ (therefore no gate is required)

38(a) $X = A + \overline{A}\,B = A.1 + \overline{A}\,B$

$= A\,(1 + B) + \overline{A}\,B$

$= A + (A + \overline{A})\,B$

$= A + 1.B = A + B$

Therefore 1 or gate is required

39(c),

40(a)

41(b)

42(b)

43(b)

44(b)

45(c)

46(c) with every 8 – 10° C rise in temperature I_{CBO} gets doubled.

47(b)

48(b) These are augend, addend and carry from previous stage.

49(c)

50(b)

SELF TEST 2

1. The momentum of e which is accelerated with 100 v is

(a) 5.4×10^{-24} kg ms^{-1} (b) 2.7×10^{-24} kg ms^{-1}

(c) 3.8×10^{-24} kg ms^{-1} (d) 4.9×10^{-24} kg ms^{-1}

2. Which of the following is most unstable?

(a) Proton (b) Positron

(c) Neutrino (d) Neutron

3. Hard X-ray has

(a) higher wavelength (b) higher frequency

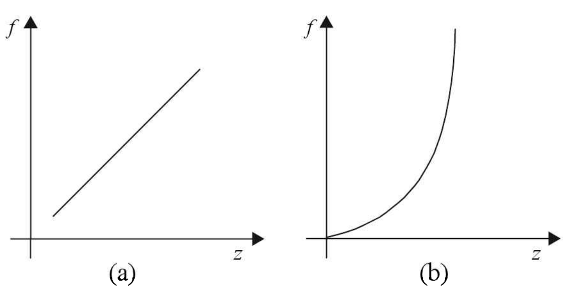
(c) more intensity (d) more penetrating power

4. The cut off wavelength in a Coolidge tube is 40 pm. The operating voltage is

(a) 21 kV (b) 27 kV

(c) 31 kV (d) 40 kV

5. In X-rays frequency f Vs atomic number z curve is



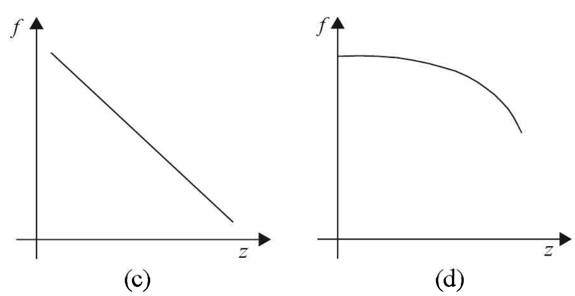


Fig. 1

6. If the potential difference is increased between cathode and anode in a Coolidge tube then which of the statement is incorrect?

(a) Electrons cause secondary emission

(b) Harder x-rays are generated

(c) Intensity of x-ray increases

(d) Wavelength of photons decrease

7. When white light is passed through a sample of hydrogen gas at room temp, dark lines are observed in

(a) Lyman series (b) Balmer series

(c) Brackett series (d) all (a), (b), (c)

8. How many wavelength are emitted by atomic hydrogen in visible region?

(a) 2 (b) 3

(c) 4 (d) 5 (e) 7

9. In a laser tube all the photons
(a) have same energy (b) have same wavelength
(c) move with same speed (d) move in same direction

10. The maximum angular speed of an electron in a stationary orbit of hydrogen is
(a) 4.1×10^{16} rad s^{-1} (b) 2.8×10^{16} rad s^{-1}
(c) 1.4×10^{16} rad s^{-1} (d) 3.14×10^{16} rad s^{-1}

11. The ratio of magnetic dipole moment to angular momentum in a hydrogen like atom is
(a) $\frac{e}{m}$ (b) $\frac{2e}{m}$
(c) $\frac{e}{2m}$ (d) n

12. The average life of an electron in n = 2 in a hydrogen atom is 10^{-8}s. The number of revolutions made by electron before it jumps to n = 1 is
(a) $8.8_2 \times 10^8$ (b) $8.8_2 \times 10^7$
(c) $8.8_2 \times 10^{10}$ (d) $8.8_2 \times 10^6$

13. Find the work functions of a metal so that H_∞ line of Balmer series can emit photo electrons
(a) 1.89 eV (b) 1.93 eV
(c) 2.01 eV (d) 2.12 eV

14. A positive ion having just one electron ejects it if a photon of wavelength ≤ 22.8 Å is incident. Identify reion.
(a) He^+ (b) Li^{++}
(c) Be^{+3} (d) B^{+4}

15. Find the temperature at which KE a gas molecube is equal to BE (Binding energy) of hydrogen
(a) $10^5 K$ (b) $1.05 \times 10^5 K$
(c) $1.15 \times 10^5 K$ (d) $2.25 \times 10^5 K$

16. Whenever a photon is emitted in Balmer series, it is followed by a photon in Lyman series of wavelength.
(a) 102 *nm* (b) 110 *nm*
(c) 116 *nm* (d) 121 *nm*

17. Radiation from a hydrogen discharge tube are incident on a cesium photocathode (ϕ= 1.91). The maximum KE of electrons emitted is
(a) 12.69 eV (b) 8.29 eV
(c) 1.49 eV (d) none of these

18. A neutron of 12.5 eV collides with a hydrogen atom at rest. Assuming mass of neutron is equal to mass of it. The KE of neutron after the collision is
(a) 6.25 eV (b) zero eV
(c) 12.5 eV (d) 8 eV

19. A beam of light having wavelength distributed uniformly between 450 nm to 550nm passes through a sample of hydrogen gas. Which wavelength will have the least intensity in the transmitted beam.
(a) 450 nm (b) 470 nm
(c) 481 nm (d) 487 nm

20. A hydrogen in ground state absorbs a radiation of 50 nm. If the whole energy is taken up by the electron with what energy the electron will be ejected.
(a) 6.8 eV (b) 9.3 eV
(c) 11.2 eV (d) 13.6 eV

21. The magnetic field in an *em* wave is given by $B = 200\mu T \sin 4 \times 10^{-15} (t - \frac{x}{c})$. The average energy density is
(a) 1.6×10^{-2} Jm^{-3} (b) 1.2×10^{-2} Jm^{-3}
(c) 3.2×10^{-2} Jm^{-3} (d) none of these

22. The sunlight reaching the earth has maximum electric field 810 Vm^{-1}. The maximum magnetic field in this light is
(a) 2.7×10^{-6} T (b) 2.7×10^{-7} T
(c) 2.7×10^{-5} T (d) none of these

23. Area of the plates of a parallel plate capacitor is A, separation between the plates is *d*. It is joined to a battery of emf ε and internal resistance R at t = 0. Consider a plane surface of area parallel to the plates and situated symmetrically between them. The displacement through this surface as a function of time is
(a) $\frac{\varepsilon}{R} e - \frac{td}{RA\varepsilon_o}$ (b) $\frac{2\varepsilon}{R} e - \frac{td}{RA\varepsilon_o}$
(c) $\frac{\varepsilon}{2R} e - \frac{td}{RA\varepsilon_o}$ (d) $\frac{\varepsilon}{2R} e - \frac{td}{RA\varepsilon_o}$

24. A nonohmic device follows the rule $i = KV^{3/2}$. The current is 10 mA when V = 100V find the current when V = 150V
(a) 18.3 mA (b) 21 mA
(c) 15 mA (d) none of these

25. A monotromatic source of light operating at 200 W emits 4×10^{20} photons s^{-1}. The wavelength is
(a) 500 nm (b) 437 nm
(c) 400 nm (d) 320 nm

26. Find the incorrect statement.
(a) A photon can be completely absorbed by a free electron
(b) Electron can be expressed as a wave packet
(c) Electron will acquire some KE
(d) None of these

27. The mximum energy of the photo electrons ejected when light of wavelength 350 nm is incident on a material of work function 1.9 eV
(a) 2.64 eV (b) 2.46 eV
(c) non of these (d) 1.98 eV

28. When frequency of incident light is doubled then
(a) KE is doubled (b) KE is more than doubled
(c) KE is less than doubled (d) none of these

29. The linear momentum of a photo electrons emitted when light of wavelength 400 nm is incident on a metal having work function 2.5 eV

(a) 3.4×10^{-25} kg ms^{-1} (b) 4.2×10^{-25} kg ms^{-1}
(c) 2.4×10^{-25} kg ms^{-1} (d) none

30. 1.1 V is the negative potential needed to stop electrons when a metal is exposed with 400 nm. The threshold wavelength is
(a) 390 nm (b) 620 nm
(c) 1128 nm (d) 572 nm

31. If the electric field of a light incident on a metal plate is $E = E_0 \sin(1.57 \times 10^7 m^{-1})(x - ct)$ work function of metal surface is 1.9 V. Find the stopping potential.
(a) 1.2 V (b) 2.1 V
(c) 3.1 V (d) none

32. A 100 W light bulb is placed at the centre of a spherical surface of radius 20 cm. If 60% of energy supplied is converted into light and that the surface of chamber is perfectly absorbing then pressure exerted on the surface of the chamber is
(a) 4×10^{-6}Pa (b) 4×10^{-7}Pa
(c) 4×10^{-8}Pa (d) 4×10^{-5}Pa

33. The half life of a radioactive dose is 3h. The present activity level is 64 times higher than the permissible limit. It is to be administered to a patient. After what time it be administered.
(a) 9 h (b) 15 h
(c) 18 h (d) 21 h

34. Stopping potential is shown against frequency. Find the work function

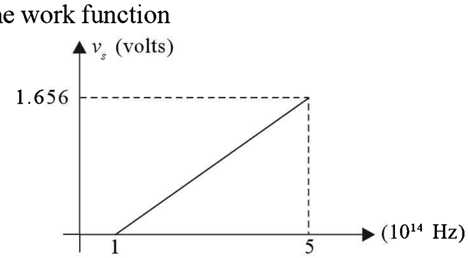


Fig. 2

(a) 0.414 eV (b) 0.828 eV
(c) 0.446 eV (d) 0.513 eV

35. The ratio of density of nuctens to density of atom is
(a) 10^{-12} (b) 10^{-15}
(c) 10^{12} (d) 10^{15}

36. Complete the reaction

$$^{26}_{13}Al \rightarrow ^{25}_{12}Mg +$$

(a) $\beta^+ + \bar{\nu}$ (b) $\beta^+ + \bar{\nu}$
(c) $\beta^+ + \bar{\nu}$ + X–ray (d) $\beta^+ + \bar{\nu}$ + X–ray

37. The count rate of a sample falls from $4 \times 10^6 s^{-1}$ to $1 \times 10^6 s^{-1}$ in 20 hrs. The count rate after 100 hrs will be
(a) $10^4 s^{-1}$ (b) $3.4 \times 10 s^{-1}$
(c) $3.2 \times 10^{-3} s^{-1}$ (d) $3.9 \times 10^{-3} s^{-1}$

38. The Atomic mass of 197.79 Au is 196.96 u. The Binding energy per nucleor
(a) 8.02 MeV (b) 7.78 MeV
(c) 7.95 MeV (d) 6.95 MeV

39. Masses of $^{11}_{6}C$ and $^{11}_{5}B$ are respectively 11.0114 u and 11.0093u. The maximum energy a positron can have is
(a) 0.93 4 MeV (b) 0.866 MeV
(c) 0.634 MeV (d) 1.956 MeV

40. What will be the selling rate of a one month old $^{32}_{15}P$ (t½ = 14.3 days) if it was orginally purchased at Rs 800/= ? Assume no profit no loss.
(a) Rs 200 (b) Rs 193
(c) Rs 187 (d) none of these

41. Half life of a radio isotope is 10h. The total number of disintegration in the tenth hour measured from a time when the activity was 1 Ci is
(a) 1.91×10^{10} (b) 1.92×10^{11}
(c) 6.92×10^{12} (d) 6.92×10^{13}

42. The graph between count rate R and time t is

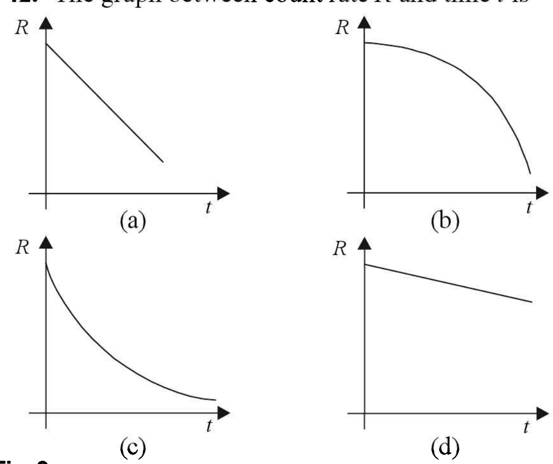


Fig. 3

43. In an experiment 1 mole of radioactive substance ($t_{½}$ = 14.3 days) in solution form was injected into the roots of a plant. The plant was allowed 70 h to settle down and then activity was measured in its fruit. If the activity measured is 1μ Ci. What % of activity was transmitted from the root to the fruit in steady state.
(a) 1.26×10^{-13} (b) 1.0×10^{-14}
(c) 0.8×10^{-13} (d) 0.6×10^{-14}

44. The energy released per fission reaction of $^{235}_{92}U$ is
(a) 200 MeV (b) 100 MeV
(c) 600 MeV (d) 880 MeV

45. The number of neutrons emitted per fission reaction on an average is
(a) 2 (b) 3
(c) 2.48 (d) 2.59

46. Thermal electrons possess energy
(a) 1.2 eV (b) .12 eV
(c) .012 eV (d) none of these

47. In the fission reaction of $^{235}_{92}U$ the energy carried by neutron is
(a) < 1 MeV (b) 2 MeV
(c) 5 MeV (d) 30 MeV

48. Enriched uranium contains % 235 U
(a) 0.7% (b) 3%
(c) 1% (d) 40%

49. β-particles are
(a) electrons of any origin
(b) electrons of nuclear origin
(c) electrons of atomic origin
(d) electrons present in the nucleus

50. Two γ photons are moving in opposite directions their relative velocity is
(a) c (b) $2c$
(c) $\frac{c}{2}$ (d) $\frac{10c}{9}$
(e) $\frac{11c}{8}$

51. A sample emits α and β radiations in a successive chain. The half life of α us t_1 and of β is t_2. The combined half life is
(a) $t_1 - t_2$ (b) $\frac{t_1 t_2}{2}$
(c) $\frac{t_1 t_2}{t_1 + t_2}$ (d) $\frac{t_1 t_2}{t_1 - t_2}$
(d) none of these

52. If $t_{½}$ is half life of a sample then its average life is _____
(a) $0.693\, t_{½}$ (b) $\frac{t_{\frac{1}{2}}}{0.693}$
(c) ∞ (d) $6.93\, t_{½}$

53. F_{PP}, F_{PN}, F_{NN} is the net force between proton and proton, proton and neutron and neutron and neutron then
(a) $F_{PP} > F_{PN} > F_{NN}$ (b) $F_{PN} = F_{NN} > F_{PP}$
(c) $F_{PN} = F_{PP} = F_{NN}$ (d) $F_{NN} > F_{PN} > F_{PP}$
(e) none of these

54. The mass of carbon $^{12}_{6}C$ is
(a) exact 12 u (b) > 12 u
(c) < 12 u (d) depends on the origin of carbon

55. During β^--emission
(a) an electron of atomic origin is emitted
(b) electrons already present in nucleus is emitted
(c) a neutron is converted to a proton, an electron and a neutrino
(d) all β-particles have equal energy

56. Which of the following is most deviated when an electric field is applied
(a) α (b) β^-
(c) β^+ (d) γ
(e) neutrino

57. The mass number of a nucleus is
(a) always greater than atomic mass number
(b) always less than atomic mass number
(c) equal to its atomic mass number
(d) sometimes more than and sometimes equal its atomic mass number

58. In hotter stars of temperature $\simeq 10^8$K use
(a) H $\rightarrow$ He cycle (b) proton – carbon cycle
(c) He – C cycle (d) C $\rightarrow$ N cycle

59. The model which explains fission reaction is
(a) charged bubble model (b) liquid drop model
(c) proton decay model (d) Gammow's model

60. In breeder reactor
(a) less efficient fuel than ^{235}U is produced
(b) more efficient fuel than ^{235}U fuel is produced
(c) a fuel equally efficient to ^{235}U is produced
(d) no nuclear fuel is produced

61. α-emission is explained using
(a) liquid drop model (b) tunneling
(c) charged bubble model (d) shell model

Answers

1.	(a)	**2.**	(d)	**3.**	(b,d)	**4.**	(c)	**5.**	(b)	**6.**	(a)	**7.**	(a)
8.	(d)	**9.**	(c)	**10.**	(a)	**11.**	(c)	**12.**	(d)	**13.**	(a)	**14.**	(a)
15.	(b)	**16.**	(d)	**17.**	(a)	**18.**	(b)	**19.**	(d)	**20.**	(c)	**21.**	(a)
22.	(a)	**23.**	(b)	**24.**	(a)	**25.**	(b)	**26.**	(a)	**27.**	(a)	**28.**	(b)
29.	(b)	**30.**	(b)	**31.**	(a)	**32.**	(b)	**33.**	(c)	**34.**	(a)	**35.**	(d)
36.	(d)	**37.**	(d)	**38.**	(c)	**39.**	(a)	**40.**	(c)	**41.**	(d)	**42.**	(c)
43.	(a)	**44.**	(a)	**45.**	(c)	**46.**	(c)	**47.**	(c)	**48.**	(b)	**49.**	(b)
50.	(a)	**51.**	(c)	**52.**	(b)	**53.**	(b)	**54.**	(a)	**55.**	(c)	**56.**	(a)
57.	(d)	**58.**	(b)	**59.**	(b)	**60.**	(b)	**61.**	(b)				

Explanations

1(a) $\text{KE} = \frac{1}{2}mv^2 = 100v$

$p = \sqrt{2km} = \sqrt{2 \times 100 \times 1.6 \times 10^{-19} \times 9 \times 10^{-31}}$

$= 5.4 \times 10^{-24}\,\text{kg. ms}^{-1}$

2(d)

3(b,d)

4(c) $\lambda = \frac{1240}{E(ev)}$ or $V = \frac{1240\times10^{-19}}{40\times10^{-12}} = 31\text{kv}$

5(b)

6(a)

7(a)

8(d)

9(c)

10(a) $mrw^2 = \frac{e^2}{4\pi 60 r^2}$ or $w = \sqrt{\frac{e^2}{4\pi 60 mr^3}}$

11(c) $\frac{M}{L} = \frac{iA}{mvr} = \frac{\frac{ev}{2\pi r}\pi r^2}{mver} = \frac{e}{zm}$

12(d) $0 = $ wt no. of revolutions $= \frac{\theta}{2\pi} = \frac{wt}{2\pi} =$

$$= \sqrt{\frac{\frac{e^2}{4\pi tomr^3}t}{2\pi}}$$

$$= \sqrt{\frac{1.6\times10^{-19}\times1.6\times10^{-19}\times9\times10^{9}}{9\times10^{-31}\times(2.12\times10^{-18})^3}}\times\frac{10^{-8}}{2\pi}$$

$= 8.82\times10^{6}$

13(a) $E_3 - E_2 = 3.4 - 1.51 = 1.89\,\text{eV}$

14(a) $E(ev) = \frac{1240}{22.8} = 54\text{-}43\text{ eV}$ $\frac{13.6\times z^2}{n^2} = 54.4$ or $z = 2$ taking $n = 1$

15(b) $\frac{3}{2}kT = 13.6$ or $T = \frac{13.6\times2}{3\times8.62\times10^{-5}}$

$= 1.05\times10^{5}\text{K}$

16(d) $13.6 - 3.4 = 10.2$ $\lambda = \frac{1240}{10.2} = 122\text{nm}$

17(a) $13.6 - 1.91 = 12.69\text{eV}$

18(b)

19(d) $13.6\left[\frac{1}{4} - \frac{1}{16}\right] = 2.55$ $\lambda = \frac{1240}{2.55} = 487\text{nm}$

20(c) $\lambda = \frac{1240}{50} = 24.8\text{ ev}$ $KE = 24.8 - 13.6 = 11.2\,\text{eV}$

21(a) $\frac{B_0^2}{2\mu_o} = \frac{200\times10^{-6}\times200\times10^{-6}}{2\times4\pi\times10^{-7}} = 1.6\times10^{-2}\,\text{Jm}^{-3}$

22(a) $B = \frac{E}{C} = \frac{810}{3\times10^8} = 2.7\times10^{-6}\text{T}$

23(b) $q = q_o(1 - e^{-t/RC})$

$$\frac{lq}{dt} = \frac{q_o}{Rc}e^{-t/Rc}$$

$$i = \frac{E}{2R}e^{-\frac{td}{R\varepsilon_0}}$$

24(a) $\frac{i_1}{l_2} = \left(\frac{V_1}{V_2}\right)^{\frac{3}{2}}$ or $i_2 = 10\times(1.5)^{\frac{3}{2}} = 18.3\text{ mA}$

25(b) The energy of each photon $= \frac{200}{4\times10^{20}} = 5\times10^{-19}\text{ J}$

$= \frac{5}{1.6}eV$ $\lambda = \frac{1240\times1.6}{5}$

26(a)

27(a) $E(eV) = \frac{1240}{350} = 3.54\text{eV}$

$KE = 3.54 - 1.9 = 2.64\,\text{eV}$

28(b) $KE = hf - \phi$

$2KE = h(2f) - 2\phi$ or $h(2f) - \phi = 2KE + \phi$

29(b) $KE_{max} = hf - \phi = 3.1 - 2.5 = 0.6\,\text{eV}$

$p = \sqrt{2(KE)m}$

$\sqrt{2\times0.6\times1.6\times10^{-19}\times9\times10^{-31}} = 4.2\times10^{-25}\text{kgms}^{-1}$

30(b) $\phi = 3.1 - 1.1 = 2\text{eV}$ $\lambda_{th} = \frac{1240}{2} = 620\,\text{nm}$

31(a) $\lambda = \frac{2\pi}{K} = \frac{2\pi\times10^{-7}}{57} = 4\times10^{-7}$

$E(ev) = \frac{1240}{400} = 3.1\text{ev} = 3.1 - 1.9 = 1.2\text{V}$

32(b) $F = \frac{P}{C}$ Pressure $P = \frac{F}{A} = \frac{P}{CA} = \frac{60}{3\times10^8\times4\pi(.2)^2}$

$= 4\times10^{-7}\,\text{Pa}$

33(c) $2^6 = 64$ $\therefore$ after 6 half lives or 18 h.

34(a) $\phi = hf = \frac{6.64\times10^{-34}\times10^{14}}{1.6\times10^{-19}} = 0.414\,eV$

35(d) $\frac{A\frac{4}{3}\pi R^3_{nucleus}}{A\frac{4}{3}\pi R^3_{atom}}$

36(d)

37(d) $t_{½} = \frac{20}{2} = 10\text{h}$. A after 100 hr means after 10 half lives

the count rate will be $\frac{4\times10^6}{2^{10}} = 3.9\times10^{-3}\,\text{s}^{-1}$

38(c) $\frac{\text{Binding energy BE}}{\text{no. of nucleons}} = \frac{\Delta mc^2}{197}$

$= \dfrac{(118m_n + 79M_H - 196.96u)c^2}{197}$

$= \dfrac{[118(1.008665) + 79(1.007825) - 196.96]931.5}{197}$

=7.95 MeV nucleon

39(a) $\Delta mc^2 = \{11.004 - 11.0093\}4c^2 - 2\,m_ec^2$

$= 1.95\text{MeV} - 1.022 = 0.934\text{MeV}$

40(c) $N = N_o e^{-\lambda t} = N_o e^{-30(.693)/14.3}$

or $\dfrac{N}{N_o} = e^{-30(.693)/14.3} = 0.2335$

Cost = Rs 800 (.2335) = Rs 187/=

41(d) Avg. dps in 10th hour

$= \dfrac{\text{dps at 9th hour + dps at } 10^{th} \text{ hour}}{2}$

$= 3.7 \times 10^{10} (e^{\frac{9(.693)}{10}} + .5) = 3.7 \times 10^{10} \times .52$

Total disintegration in 10th hour $= 3.7 \times 10^{10} \times .52 \times 3600$

$= 6.92 \times 10^{13}$

42(c) $N = N_o e^{-\lambda t}$ graph is to be plotted

43(a) The steady rate is obtained after a long time, ie, after 7 – 8 half lives.

$\therefore N = N_o e = N_o (.0067)$

$N = 6.023 \times 10^{23} \times .0067 = 40 \times 10^{20}$

At that time acitivity in fruit will be $3.7 \times 10^{-4} (e^5)$

$= 3.7 \times 1.5 \times 10^6$

% activity in fruit $\dfrac{3.7 \times 1.5 \times 10^6 \times 100}{40 \times 10^{20}}$

$= 1.26 \times 10^{-13}$ %

44(a)

45(c)

46(c)

47(c)

48(b)

49(b)

50(a)

51(c) $\dfrac{1}{t} = \dfrac{1}{t_1} + \dfrac{1}{t_2}$

52(b) $tav = \dfrac{1}{\lambda} = \dfrac{t_{\frac{1}{2}}}{.693}$

53(b)

54(a)

55(c)

56(a)

57(d)

58(b)

59(b)

60(b)

SELF TEST 3

1. Given that the mass of neutron or proton is 1840 times the mass of electron. What is the ratio of specific charge of electron to that of α-particle?

(a) $\dfrac{1}{1840}$ (b) $\dfrac{1}{3680}$

(c) 1840 (d) 3680

2. In an electron microscope if the potential is increased from 20 kV to 80 kV the resolving power R of the microscope will become

(a) R (b) $2R$

(c) $4R$ (d) $\dfrac{R}{2}$

3. The cyclotron was deviced by

(a) E. Lawrence (b) Lorentz

(c) Oersted (d) Maxwell

4. A beam of electrons, accelerated through 50 kV strikes against a target. Which of the following x-ray frequency will be missing?

(a) 4×10^8 Hz (b) 6×10^8 Hz

(c) 12×10^{18} Hz (d) 14×10^{18} Hz

5. The wavelength of K_α line for an element of atomic number 57 is λ. What is wavelength of K_α line for an element of atomic number 29?

(a) λ (b) 2λ

(c) 4λ (d) 8λ

6. A photocell is illuminated by a small bright source placed 1m away. When the same source of light is placed 2m away, which of the following will be true about the electrons emitted by photo cathode?

(a) Each electron carries one-quarter of the previous energy

(b) Each electron carries one-quarter of the previous momenta

(c) electrons are half as numerous

(d) electrons are one-quarter as numerous

7. The dynamic mass of photon is given as

(a) $\frac{h\nu}{c}$ (b) $\frac{h\lambda}{c}$

(c) $\frac{h}{c\lambda}$ (d) $\frac{h}{c\nu}$

8. Light of two different frequencies whose photons have energies 1eV and 2.5 eV respectively are incident on a metal of work function 0.5 eV. The ratio of maximum speeds of emitted electron is

(a) 1 : 5 (b) 1 : 4

(c) 1 : 2 (d) 1 : 1

9. Maximum Kinetic energy (E_k) of a photoelectron varies with the frequency f of incident radiation as:

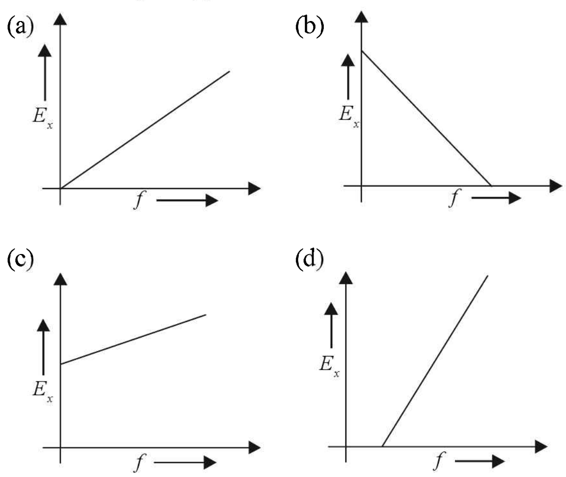


Fig. 1

10. The de Broglie wavelength of a neutron is λ at 27°C. What will be wavelength at 927°C?

(a) $\frac{\lambda}{2}$ (b) $\frac{\lambda}{3}$

(c) $\frac{\lambda}{4}$ (d) $\frac{\lambda}{9}$

11. The number of α-particles scattered from a target at 15° is 4×10^6. What is number of particle scattered at angle 30° under similar conditions?

(a) $\frac{1}{4}\times10^6$ (b) 2×10^6

(c) 8×10^6 (d) 16×10^6

12. Consider the following statements about Rutherford's Model of atom:

(1) Greater the impact parameter greater will be deflection of α-particle

(2) Cotangent of half the angle of deflection is proportional to impact parameter

(3) Electrons are bounded to nucleus by coulomb force

(4) Electron makes a shell like structure around nucleus

Select the correct answer using codes given below:

(a) 1 and 4 are correct (b) 2 and 3 are correct

(c) 1 and 3 are correct (d) 3 and 4 are correct

13. The ratio of energy of the orbital electron in first orbit to that in second orbit is

(a) 2 (b) 4

(c) 8 (d) 16

14. The orbital electron of the hydorgen atom jumps from the ground state to a higher energy state & its velocity is reduced to one-third of its initial value. If the radius of the orbit in ground state is r, then what is radius of the new orbit?

(a) $2\,r$ (b) $3\,r$

(c) $4\,r$ (d) $9\,r$

15. In an atom, the two electrons revolve around the nucleus. What is the ratio of their time periods in first excited state and the ground state?

(a) 2 (b) 4

(c) 8 (d) 16

16. Suppose the mass of electron decreases by 25%. How will it affect Rydberg's constant?

(a) Remain unchanged (b) Becomes one fourth

(c) Reduced to 75% of its initial value

(d) It is doubled

17. Which of the following transitions in the hydrogen atom gives an absorption line of higher frequency?

(a) $n = 1$ to $n = 3$ (b) $n = 3$ to $n = 1$

(c) $n = 1$ to $n = 2$ (d) $n = 2$ to $n = 1$

18. The wavelength of the first line of Lyman series is 121.6 nm for hydrogen atom. What is the wavelength of the second member of Balmer series?

(a) 30.4 nm (b) 60.8 nm

(c) 243.2 nm (d) 486.4 nm

19. In the following figure, showing energy level of certain atom, when the system moves from $2E$ level to E, a photon of wavelength λ is emitted. The wavelength of photon produced during its transition from level $\frac{4E}{3}$ to E is:

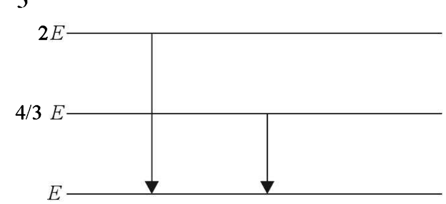


Fig. 2

(a) $\frac{\lambda}{3}$ (b) $\frac{3\lambda}{4}$

(c) $\frac{4\lambda}{3}$ (d) 3λ

20. Energy levels A, B, C of a certain atom corresponds to increasing values of energy, i.e. $E_A < E_B < E_C$. If $\lambda_1, \lambda_2, \lambda_3$ are the wavelength of radiation corresponding to the transition C to B, B to A and C to A respectively, which of the following statement is correct?

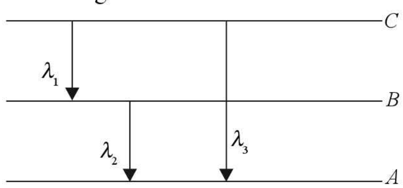


Fig. 3

(a) $\lambda_3 = \lambda_1 + \lambda_2$ (b) $\lambda_3 = \frac{\lambda_1\lambda_2}{\lambda_1 + \lambda_2}$

(c) $\lambda_3^2 = \lambda_1^2 + \lambda_2^2$ (d) $\lambda_1 + \lambda_2 + \lambda_3 = 0$

21. If the size of atom is enlarged such that the nucleus has a size of small seed (1 mm diameter) then the electron orbits will have the size of

(a) a big seed (1 cm diameter)

(b) a cricket ball (10 cm diameter)

(c) a cricket playground (100 m diameter)

(d) the earth

22. Who is credited, with discovery of proton?

(a) Chadwick (b) Rutherford

(c) Milikan (d) Thomson

23. The binding energy per nucleon is almost constant for many nuclei. To what characteristic of nuclear forces does it point?

(a) Saturative nature (b) Short range

(c) Attractive nature (d) Exchange force

24. If the speed of light were 50% of the present value, by what percentage the energy released will be decreased?

(a) 100% (b) 75%

(c) 50% (d) 25%

25. The phenomenon of pair production is

(a) the production of an electron and a positron from γ-rays

(b) ejection of an electron from a metal surface when exposed to ultraviolet light

(c) ejection of electron from nucleus

(d) ionisation of neutral atom

26. A gamma ray photon creates an electron positron pair. If the rest mass energy of electron is 0.5 MeV and the total kinetic energy of electron positron pair is 0.78 MeV, the energy of gamma ray photon must be

(a) 0.78 MeV (b) 1.78 MeV

(c) 1.28 MeV (d) 0.28 MeV

27. From the following equations select the possible nuclear fusion reaction

(a) $^{13}_{6}C + ^{1}_{1}H \rightarrow ^{14}_{6}C + 4.3\,\text{MeV}$

(b) $^{12}_{6}C + ^{1}_{1}H \rightarrow ^{13}_{7}C + 2\,\text{MeV}$

(c) $^{14}_{7}C + ^{1}_{1}H \rightarrow ^{16}_{8}O + 7.3\,\text{MeV}$

(d) $^{235}_{92}C + ^{1}_{0}n \rightarrow ^{140}_{54}Xe + ^{94}_{38}Sr + 2\left(^{1}_{0}n\right) + \gamma + 200\,\text{MeV}$

28. After two hour $\frac{1}{16}$th of initial amount of a certain radioactive isotope remains undecayed. The half life of the isotope is

(a) 15 min (b) 30 min

(c) 45 min (d) 60 min

29. What determines the half life of a radioactive sample?

(a) Temperature (b) Pressure

(c) Nature of substance (d) All of these

30. A solid that is not transparent to visible light and its electrical conductivity increases with rise in temperature is formed by

(a) hydrogen binding (b) covalent binding

(c) metallic binding (d) ionic binding

31. If 10% of a radioactive substance decays in 5 days then amount of substance left after 25 days will be nearly

(a) 40% (b) 50%

(c) 60% (d) 70%

32. A piece of copper and germanium are cooled from room temperature to 100K. Then their resistivities

(a) both decrease (b) both increase

(c) that of copper decreases and germanium increases

(d) that of copper increases and germanium decreases

33. What is the ratio of atomic radius 'r' and lattice parameter "a" of bcc system

(a) 2 (b) $\frac{1}{2}$

(c) $\frac{1}{2\sqrt{2}}$ (d) $\sqrt{\frac{3}{4}}$

34. The possible no. of Bravais lattices in space is

(a) 5 (b) 7

(c) 13 (d) 14

35. Which of the following doesnot indicate the nature of settling down of the constituent particles in the crystal structure?

(a) Square close packing (b) Hexagonal close packing

(c) Cubic close packing (d) Face close packing

36. Below are list of the impurities that may be added to obtain p-type crystal.

(I) Antimony, Bismuth (II) Boron, Aluminium

(III) Indium, Galium (IV) Nitrogen, Phosphorous

Which of them can be used in actual practice

(a) I & II (b) II & III

(c) III & IV (d) IV & I

37. When a large no. of atoms get packed together to form solids, what happens to the energy of electrons in the outermost shell of the atoms?

(a) It is same as that of a free atom

(b) It is different from that of free atom but is same for all atom

(c) There may be as many energy level as the number of electrons in the crystal

(d) None of these

38. The Depletion layer in the p-n junction region is caused by

(a) drift of holes

(b) diffusion of charge carrier

(c) migration of impurity ions

(d) drift of electron

39. On increasing the reverse bias to a large value in p-n junction diode current

(a) increases slowly (b) remains fixed

(c) suddenly increases (d) decreases slowly

40. In the diagram the input is across terminal A and C, and the output is across B and D. Then the output is

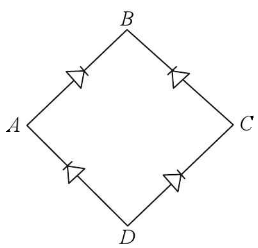


Fig. 4

(a) zero (b) same as the input

(c) full wave rectified (d) half-wave rectified

41. When npn transistor is used as an amplifier then

(a) electron moves from base to collector

(b) hole move from emitter to base

(c) electron moves from collector to base

(d) hole moves from base to emitter

42. The correct relationship between the two current gain α and β in a transistor is

(a) $\beta = \dfrac{\alpha}{1+\alpha}$ (b) $\alpha = \dfrac{\beta}{1-\beta}$

(c) $\alpha = \dfrac{\beta}{1+\beta}$ (d) $\alpha = \dfrac{1+\beta}{\beta}$

43. What is voltage gain in common emitter amplifier, where input resistance is 3Ω and load resistance 24Ω, $\beta = 0.6$?

(a) increases slowly (b) remains fixed

(c) suddenly increases (d) decreases slowly

44. Which of the following gates will have an output 1

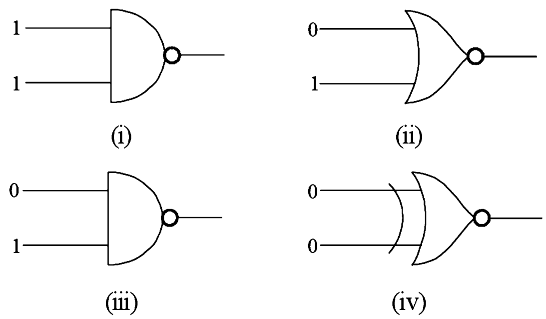


Fig. 5

(a) (iv) (b) (i)

(c) (ii) (d) (iii)

45. The truth table given below is for ______ gate

A	B	C
0	0	0
0	1	1
1	0	1
1	1	0

(a) XOR (b) OR

(c) AND (d) NAND

46. At what velocity the kinetic energy of a particle is equal to the rest mass energy?

(a) $\dfrac{c}{2}$ (b) $\sqrt{3}\dfrac{c}{2}$

(c) $\sqrt{5}\dfrac{c}{2}$ (d) none

47. Einstein was awarded Noble Prize for

(a) photoelectric effect

(b) special theory of relativity

(c) general theory of relativity

(d) cosmological prediction

48. The figure represent the observed intensity of X-ray emitted by an X-ray tube, as a function of wavelength. The sharp peak A and B denotes

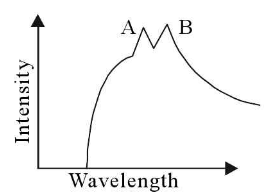


Fig. 6

(a) band spectrum

(b) continuous spectrum

(c) characteristic radiation

(d) white radiation

49. When a beam of accelerated electron hits a target, a continuous X-ray spectrum is emitted from the target. Which one of the following wavelength is absent in the X-ray spectrum if the X-ray tube is operating at 40,000 volts

(a) 1.5 A° (b) 0.5 A° (c) 0.25 A° (d) 1.0 A°

50. Black hole is

(a) hole in ozone layer in stratosphere
(b) related to reflection of light
(c) related to schwarzchild radius
(d) hole in stars.

Answers

1.	(d)	**2.**	(b)	**3.**	(a)	**4.**	(d)	**5.**	(c)	**6.**	(d)	**7.**	(c)
8.	(c)	**9.**	(d)	**10.**	(a)	**11.**	(a)	**12.**	(b)	**13.**	(b)	**14.**	(d)
15.	(c)	**16.**	(c)	**17.**	(a)	**18.**	(d)	**19.**	(d)	**20.**	(b)	**21.**	(c)
22.	(b)	**23.**	(c)	**24.**	(b)	**25.**	(a)	**26.**	(b)	**27.**	(b)	**28.**	(b)
29.	(c)	**30.**	(b)	**31.**	(c)	**32.**	(c)	**33.**	(d)	**34.**	(d)	**35.**	(d)
36.	(b)	**37.**	(c)	**38.**	(b)	**39.**	(c)	**40.**	(c)	**41.**	(d)	**42.**	(c)
43.	(b)	**44.**	(d)	**45.**	(d)	**46.**	(b)	**47.**	(a)	**48.**	(c)	**49.**	(c)
50.	(c)												

Explanations

1(d) 3680; $M_\alpha = 4M_P$

$Q_\alpha = 2Q_P$

$\alpha = \alpha$ particles

p ≡ proton

e ≡ electron

2(b) $2R$;

Resolving Power (RP) $\propto \frac{1}{\lambda}$

Also, $\frac{1}{\lambda} \propto$ momentum $\propto \sqrt{\text{Voltage}}$

Voltage increases 4 fold, causing RP to increase twice.

3(c) E. Lawrence

4(d) 14×10^{18} Hz; $f_{max} = \frac{eV}{h} = 12.12 \times 10^{18}$ Hz. Frequency above this value will be absent.

5(c) 4λ, $\lambda \propto \frac{1}{(z-1)^2}$

6(d) The intensity of light varies inversely as square of distance

7(c) $mc^2 = h\upsilon$, Hence $m = \frac{hv}{c^2} = \frac{h}{c\lambda}$

8(c) 1 : 2

$\frac{1}{2}mv_1^2 = 1 - 0.5 = 0.5\,\text{eV}$

$\frac{1}{2}mv_2^2 = 2.5 - 0.5 = 2.0\,\text{eV}$

Where $\frac{v_1}{v_2} = \sqrt{\frac{0.5}{2.0}} = 1:2$

9(d) $hv = hv_0 + E_K$ the graph between E_K and f is a straight line shown.

10(a) $\frac{\lambda}{2}$

$\lambda = h/\sqrt{2\text{m} \times \text{K.E.}} = h/\sqrt{2\text{m} \times \frac{3}{2}\text{kT}} = h/\sqrt{3\text{mkT}}$

ie, $\lambda \propto \frac{1}{\sqrt{T}}$

11(a) $N\phi = \frac{1}{\left(\sin\frac{\phi}{2}\right)^4}$

Here $\frac{N_{30}}{N_{15}} = \frac{\sin^4 15^0}{\sin^4 30^0}$, Also, $\sin^2 15 = (1 - \cos 30)/2$

12(b) 2 and 3 are correct

13(b) 4; $E_n \propto \frac{1}{n^2}$

14(d) $9r$, $v \propto \frac{1}{n}$ & $r \propto n^2$

15(c) $T \propto r^{\frac{3}{2}} \Rightarrow T \propto n^3$

16(c) $R \propto m$; new mass of electron is now 0.75

17(a) $E = hv = R\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$

For $n = 1$ to $n = 3$

$\frac{1}{(1)^2} - \frac{1}{(3)^2} = \frac{8}{9}$ (higher)

18(d) Lyman: $\frac{1}{\lambda}=R\left[\frac{1}{1}-\frac{1}{n_0^2}\right]$

$$\frac{1}{\lambda_1}=R\left[1-\frac{1}{4}\right]=\frac{3R}{4}$$

Balmer: $\frac{1}{\lambda}=R\left[\frac{1}{2^2}-\frac{1}{n^2}\right]=R\left[\frac{1}{4}-\frac{1}{4^2}\right]=\frac{3}{16}R$

$$\lambda_2=\frac{3R}{4}\times\frac{16}{3R}M=4M=486.4$$

19(d) $\frac{hc}{\lambda}=2E-E=E$

Also, $\frac{hc}{\lambda}=\frac{4}{3}E-E=\frac{E}{3}$ Thus $\lambda'=3\lambda$

20(b) $\lambda_3=\frac{\lambda_1\lambda_2}{\lambda_1+\lambda_2}$;

Note $\frac{hc}{\lambda_3}=\frac{hc}{\lambda}+\frac{hc}{\lambda_2}$

21(c)a cricket playground; size of atomic orbit $= 10^5$ (size of nucleus).

22(b)Rutherford; It was discovered through the disintegration of nitrogen nucleus by bombarding it with α-particle.

23(c)Attractive force

24(b)75% if speed of light is $\frac{c}{2}$ then $E'=\frac{mc^2}{4}$

25(a) The production of an electron and positron from γ-ray

26(b) 1.78 MeV; Energy of photon = mass energy of electron and positron + their K.E.

27(b)

28(b) 30 min; $A=A_o\left(\frac{1}{2}\right)^n$, here $n=4$, Hence

half life $=\frac{2}{4}$hr $=30$ minute

29(c) nature of substance

30(b) covalent binding

31(c) 60%, Amount left $=\left(\frac{9}{10}\right)^5=0.59=59\%$

32(c) decrease in copper & increase in germanium

33(d) $\sqrt{\frac{3}{4}}$

34(d) 14

35(d) face close packing

36(b) for p-type crystal impurity should be trivalent

37(c) There may be as many energy levels as the number of electrons

38(b) diffusion of charge carrier

39(c) suddenly increases giving breakdown

40(c) full wave rectified

41(d) because EB junction is forward biased.

42(c) $\alpha=\frac{\beta}{1+\beta}$

43(b) 4.8; Voltage gain = current gain × Resistance gain

$$=\beta\times\frac{R_o}{R_i}=0.6\times\frac{24}{3}=4.8$$

44(d) It is NAND gate

45(a) XOR gate table

46(b) $\sqrt{3}\frac{c}{2}$; $m^*c^2=m_oc^2$ i.e. $m^*c^2-m_oc^2=m_oc^2$, i.e. $m^*=2m_o$

and $m^*=\frac{m_o}{\sqrt{1-\frac{v^2}{c^2}}}$ Þ $v=\frac{\sqrt{3}}{2}c$

47(a) Photoelectric effect

48(c) Characteristic radiation

49(c) $l_{min}=\frac{12400}{V}=0.31A°$

50(c)

QUESTIONS FROM COMPETITIVE EXAMINATIONS

AIEEE 2002

1. If 13.6 eV energy is required to ionize the hydrogen atom, then the energy required to remove an electron from $n = 2$ is:

(a) 10.2 eV (b) 0 eV
(c) 3.4 eV (d) 6.8 eV

2. At absolute zero, Si acts as:

(a) non-metal (b) metal
(c) insulator (d) none of these

3. Which of the following are not electromagnetic waves?

(a) Cosmic-rays (b) γ-rays
(c) β-rays (d) X-rays

4. If N_0 is the original mass of the substance of half-life period $t_{\frac{1}{2}} = 5$ years, then the amount of substance left after 15 years is:

(a) $\frac{N_0}{8}$ (b) $\frac{N_0}{16}$
(c) $\frac{N_0}{2}$ (d) $\frac{N_0}{4}$

5. The energy band gap is maximum in:

(a) metals (b) superconductors
(c) insulators (d) semiconductors

6. The part of a transistor which is most heavily doped to produce large number of majority carriers is:

(a) emitter (b) base
(c) collector
(d) can be any of the above three

7. At a specific instant emission of radio active compound is deflected in a magnetic field. The compound can emit:

(a) electrons (b) protons
(c) He^{2+} (d) neutrons

8. Sodium and copper have work functions 2.3 eV and 4.5 eV respectively. Then the ratio of the wavelengths is nearest to:

(a) 1 : 2 (b) 4 : 1
(c) 2: 1 (d) 1 : 4

9. Formation of covalent bonds in compounds exhibits:

(a) wave nature of electron
(b) particle nature of electron
(c) both wave and particle nature of electron
(d) none of these

Answers

1.	(c)	**2.**	(c)	**3.**	(a, c)	**4.**	(a)	**5.**	(c)	**6.**	(a)	**7.**	(a, c)
8.	(c)	**9.**	(a)										

Explanations

1. (c) Energy required to remove an electron from nth orbit is

$$E_n = -\frac{13.6}{n^2}$$

Here, $n = 2$

Therefore, $E_2 = -\frac{13.6}{2^2} = -3.4\ V$

2. (c)

3. (a,c)

4. (a) N_0 is the initial amount of substance and N is the amount left after decay.

Thus, $N = N_0 \left(\frac{1}{2}\right)^n$

$$n = \frac{15}{5} = 3 = \frac{N_0}{8}$$

5. (c) E_g is max in insulators

6. (a)

7. (a,c) Electrons, protons, and helium atoms are deflected in magnetic field. A natural radioactive compound can emit electrons and He^{2+}

8. (c) $\phi = \frac{hc}{\lambda} = \frac{\phi_{Cu}}{\phi_{Na}} = \frac{\lambda_{Na}}{\lambda_{Cu}} = \frac{4.5}{2.3}$

= 2 (nearly)

9. (a) Formation of covalent bonds exhibits the wave nature of particles in compounds.

AIEEE 2003

1. Consider telecommunication through optical fibres. Which of the following statements is not true?
(a) Optical fibres can be of graded refractive index
(b) Optical fibres are subjected to electromagnetic interference from outside.
(c) Optical fibres have extremely low transmission loss
(d) Optical fibres may have homogeneous core with a suitable cladding

2. Which of the following radiations has the least wavelength?
(a) γ-rays (b) β-rays
(c) α-rays (d) X-rays

3. When U^{238} nucleus originally at rest, decays by emitting an alpha particle having a speed u, the recoil speed of the residual nucleus is:
(a) $\frac{4u}{238}$ (b) $-\frac{4u}{234}$
(c) $\frac{u}{234}$ (d) $\frac{4u}{234}$

4. A radioactive sample at any instant has its disintegration rate 5000 disintegrations per minute. After 5 minutes, the rate is 1250 disintegrations per minute. Then, the decay constant (per minute)
(a) 0.4 ln 2 (b) 0.2 ln 2
(c) 0.1 ln 2 (d) 0.8 ln 2

5. A nucleus with Z = 92 emits the following in a sequence: $\alpha, \alpha, -\beta, \alpha, \alpha, \alpha, \alpha,; -\beta, -\beta, \alpha\ \beta^+, \alpha$. The Z of the resulting nucleus is:
(a) 76 (b) 78
(c) 82 (d) 74

6. Two identical, photocathodes receive light of frequencies f_1 and f_2. If the velocities of the photoelectrons (of mass m) coming out are respectively v_1 and v_2, then:
(a) $v_1^2 - v_2^2 = \frac{2h}{m}(f_1 - f_2)$
(b) $v_1^2 + v_2^2 = \left[\frac{2h}{m}(f_1 + f_2)\right]^{\frac{1}{2}}$
(c) $v_1^2 + v_2^2 = \frac{2h}{m}(f_1 + f_2)$
(d) $v_1^2 - v_2^2 = \left[\frac{2h}{m}(f_1 - f_2)\right]^{\frac{1}{2}}$

7. Which of the following cannot be emitted by radioactive substances during their decay?
(a) Protons (b) Neutrons
(c) Helium nuclei (d) Electrons

8. In the nuclear fusion reaction,
$${}_1^2H + {}_1^3H \longrightarrow {}_2^4H + n$$
given that repulsive potential energy between the two nuclei is ~ $7.7 \times 10^{-14}\ J$, the temperature at which the gases must be heated to initiate the reaction is nearly [Boltzmann's constant $k = 1.38 \times 10^{-23}\ J/K$]:
(a) $10^7 K$ (b) $10^5 K$
(c) $10^3 K$ (d) $10^9 K$

9. Which of the following atoms has the lowest ionization potential?
(a) ${}_7^{14}N$ (b) ${}_{55}^{133}Cs$
(c) ${}_{18}^{40}Ar$ (d) ${}_8^{16}O$

10. The wavelengths involved in the spectrum of deuterium (${}_1^2D$) are slightly different from that of hydrogen spectrum, because:
(a) size of the two nuclei are different
(b) nuclear forces are different in the two cases
(c) masses of the two nuclei are different
(d) attraction between the electron and the nucleus is different in the two cases

11. In the middle of the depletion layer of reverse biased p-n junction, the:
(a) electric field is zero
(b) potential is maximum
(c) electric field is maximum
(d) potential is zero

12. If the binding energy of the electron in a hydrogen atom is 13.6 eV, the energy required to remove the electron from the first excited state of Li^{2+} is:
(a) 30.6 eV (b) 13.6 eV
(c) 3.4 eV (d) 122.4 eV

Answers

1.	(b)	**2.**	(c)	**3.**	(b)	**4.**	(a)	**5.**	(b)	**6.**	(a)	**7.**	(a)
8.	(d)	**9.**	(b)	**10.**	(c)	**11.**	(a)	**12.**	(a)				

Explanations

1. (b)
2. (c)
3. (b) By conservation of linear momentum
$0 = 234v + 4u$

$$v = -\frac{4u}{234}$$

4. (a) $\frac{N_0\lambda}{N_0\lambda e^{-5\lambda}} = \frac{5000}{1250} = 4$

$e^{5\lambda} = 4$
$5\lambda = 2 \log_e 2$
$\lambda = 0.4\, ln\, 2$

5. (b) Since, 8 α-particles and 2β-particles are emitted so, new atomic number

$Z' = Z - 8 \times 2 + 2 \times 1$
$= 92 - 16 + 2 = 78$

6. (a) $hf = hf_0 + \frac{1}{2}mv^2$

Hence, $v_1^2 = \frac{2hf_1}{m} - \frac{2hf_0}{m}$

$$v_1^2 - v_2^2 = \frac{2h}{m}[f_1 - f_2]$$

7. (a) Protons cannot be emitted by radioactive substances during decay.

8. (d) $\frac{3}{2}kT = 7.7 \times 10^{-14}$ J

$$T = \frac{2 \times 7.7 \times 10^{-4}}{3 \times 1.38 \times 10^{-23}}$$
$$= 3.7 \times 10^9 \text{ K}$$

9. (b) the last element of group 1 has least ionization energy.
10. (c)Masses of two nuclei are different.
11. (a)

12. (a) $E = -Z^2 \frac{13.6}{n^2}$ eV

For first excited state $n = 2$

$$E_2 = -3^2 \times \frac{13.6}{4}$$

Ionisation energy for first excited state of Li^{2+} is 30.6 eV.

AIEEE 2004

1. The thermistors are usually made of:
(a) metals with low temperature coefficient of resistivity
(b) metals with high temperature coefficient of resistivity
(c) metal oxides with high temperature coefficient of resistivity
(d) semiconducting materials having low temperature coefficient of resistivity

2. According to Einstein's photoelectric equation, the plot of the kinetic energy of the emitted photoelectrons from a metal *Vs* the frequency of the incident radiation gives a straight line whose slope:
(a) depends on the nature of the metal used
(b) depends on the intensity of the radiation
(c) depends both on the intensity of the radiation and the metal used
(d) is the same for all metals and independent of the intensity of the radiation

3. The work function of a substance is 4.0 eV. The longest wavelength of light that can cause photoelectron emission from this substance is approximately:
(a) 540 nm (b) 400 nm
(c) 310 nm (d) 220 nm

4. A nucleus disintegrates into two nuclear parts which have their velocities in the ratio 2 : 1. The ratio of their nuclear sizes will be:
(a) $2^{\frac{1}{3}} : 1$ (b) $1 : 3^{\frac{1}{2}}$
(c) $3^{\frac{1}{2}} : 1$ (d) $1 : 2^{\frac{1}{3}}$

5. The binding energy per nucleon of deuteron (^2_1H) and helium nucleus (^4_2He) is 1.1 MeV and 7 MeV respectively. If two deuteron nuclei react to from a single helium nucleus, then the energy released is:
(a) 13.9 MeV (b) 26.9 MeV
(c) 23.6 MeV (d) 19.2 MeV

6. An α-particle of energy 5 MeV is scattered through 180° by a fixed uranium nucleus. The distance of the closest approach is of the order of:
(a) 1 Å (b) 10^{-10} cm
(c) 10^{-12} cm (d) 10^{-15} cm

7. When *npn* transistor is used as an amplifier:
(a) electrons move from base to collector
(b) holes move from emitter to base
(c) electrons move from collector to base
(d) holes moves from base to emitter

8. For a transistor amplifier in common emitter configuration for load impedance of $1k\Omega$ ($h_{fe} = 50$ and $h_{oe} = 25\ \mu A/V$), the current gain is:

(a) – 5.2 (b) – 15.7
(c) – 24.8 (d) – 48.78

9. A piece of copper and another of germanium are cooled from room temperature to 77*K*, the resistance of:

(a) each of them increases
(b) each of them decreases
(c) copper decreases and germanium increases
(d) copper increases and germanium decreases

10. The manifestation of band structure in solids is due to:

(a) Heisenberg's uncertainty principle
(b) Pauli's exclusion principle
(c) Bohr's correspondence principle
(d) Boltzmann's law

11. When *p-n* junction diode is forward biased, then:

(a) the deplection region is reduced and barrier height is increased
(b) the depletion region is widened and barrier height is reduced
(c) both the depletion region and barrier height are reduced
(d) both the depletion region and barrier height are increased

Answers

1.	(c)	**2.**	(d)	**3.**	(c)	**4.**	(d)	**5.**	(c)	**6.**	(c)	**7.**	(d)
8.	(d)	**9.**	(c)	**10.**	(b)	**11.**	(c)						

Explanations

1. (c)

2. (d) slope = $\frac{kE_{max}}{f} = h$ = const.

3. (c) $\lambda_{max} = \frac{1240}{4} = 310$ nm

4. (d) Law of conservation of momentum gives

$m_1v_1 = m_2v_2$

or $\frac{m_1}{m_2} = \frac{v_2}{v_1}$

but $m \propto r^3$

$\therefore \frac{m_1}{m_2} = \frac{r_1^3}{r_2^3}$

$= \frac{v_2}{v_1} \Rightarrow \frac{r_1}{r_2} = \left(\frac{1}{2}\right)^{1/3}$

5. (d) $_1H^2 + {}_1H^2 \longrightarrow {}_2He^4$ + energy

Total BE of deutron = 2 (1.1) = 2.2 MeV

energy released = 28 – 2 (2.2) = 23.6 MeV

6. (c) distance of closest approach

$$r = \frac{(2e)(ze)}{4\pi\varepsilon_0 (KE)}$$

or $r = \frac{9\times10^9\times2\times92\times(1.6\times10^{-19})^2}{5\times10^6\times1.6\times10^{-19}}$

$\therefore r = 5.3 \times 10^{-14}$ m $\approx 10^{-12}$ cm

7. (d) When forward bias is applied to EB junction of an *npn*-transistor, then it works as an amplifier. In forward biased *npn*-transistor, electrons move from emitter to base and holes move from base to emitter.

8. (d) For a transistor amplifier in common emitter configuration, current gain

$$A_i = \frac{h_{fe}}{1+h_{oe}R_L}$$

where h_{fe} and h_{oe} are hybrid parameters of a transistor.

$$\therefore \quad A_i = -\frac{50}{1+25\times10^{-6}\times1\times10^3} = -48.78$$

9. (c) resistance of metals $\propto \Delta T$ and

resistance of semiconductors $\propto \frac{1}{\Delta T}$

Therefore, it is clear that resistance of conductor decreases with decrease in temperature. In case of semiconductor, resistance increases with decrease in tempearature

10.(b)

11.(c)

AIEEE 2005

1. The intensity of gamma radiation from a given source is *I*. On passing through 36 mm of lead, it is reduced to $\frac{I}{8}$. The thickness of lead, which will reduce the intensity to $\frac{I}{2}$ will be:

(a) 6 mm (b) 9 mm
(c) 18 mm (d) 12 mm

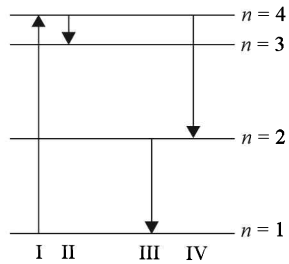
2. The electrical conductivity of a semiconductor increases when electromagnetic radiation of wavelength shorter than 2480 *nm*, is incident on it. The band gap in (eV) for the semiconductor is:
(a) 1.1 eV (b) 2.5 eV
(c) 0.5 eV (d) 0.7 eV

3. A photocell is illuminated by a small bright source placed 1 m away. When the same source of light is placed $\frac{1}{2}$ m away, the number of electrons emitted by photocathode would:
(a) decrease by a factor of 4
(b) increase by a factor of 4
(c) decrease by a factor of 2
(d) increase by a factor of 2

4. Starting with a sample of pure ^{66}Cu, $\frac{7}{8}$ of it decays into *Zn* in 15 minutes. The corresponding half-life is:
(a) 10 minute (b) 15 minute
(c) 5 minute (d) $7\frac{1}{2}$ minute

5. If radius of the $^{27}_{13}$Al nucleus is estimated to be 3.6 Fermi, then the radius of $^{125}_{52}$Te nucleus be nearly:
(a) 6 Fermi (b) 8 Fermi
(c) 4 Fermi (d) 5 Fermi

6. The diagram shows the energy levels for an electron in a certain atom. Which transition shown represents the emission of a photon with the most energy?



(a) III (b) IV
(c) I (d) II

7. If the kinetic energy of a free electron doubles, its de-Broglie wavelength changes by the factor:
(a) $\frac{1}{2}$ (b) 2
(c) $\frac{1}{\sqrt{2}}$ (d) $\sqrt{2}$

8. In a common base amplifier, the phase difference between the input signal voltage and output voltage is:
(a) $\frac{\pi}{4}$ (b) π
(c) zero (d) $\frac{\pi}{2}$

9. In a full wave rectifier circuit operating, from 50 Hz mains frequency, the fundamental frequency in the ripple would be:
(a) 50 Hz (b) 25 Hz
(c) 100 Hz (d) 70.7 Hz

10. A nuclear transformation is denoted by $X(n, \alpha) \rightarrow {}^7_3Li$. Which of the following is the nucleus of element X?
(a) $^{12}_{6}C$ (b) $^{10}_{5}B$
(c) $^{9}_{6}B$ (d) $^{11}_{4}Be$

Answers

1.	(d)	**2.**	(c)	**3.**	(b)	**4.**	(c)	**5.**	(a)	**6.**	(a)	**7.**	(c)
8.	(c)	**9.**	(c)	**10.**	(b)								

Explanations

1. (d)
$$I' = Ie^{-\mu x}$$
or
$$-\mu x = \log\frac{I'}{I}$$
$$-\mu.36 = \log\frac{I}{81} \quad ...(1)$$
$$-\mu x' = \log\frac{I}{21} \quad ...(2)$$
From eq. (1) and (2),
$$\frac{36}{x'} = \frac{3\log\left(\frac{1}{2}\right)}{\log\frac{1}{2}}$$
$\therefore\ x' = 12$ mm

2. (c) $E = \frac{1240}{2480} = \frac{1}{2} = 0.5$ eV

3. (b) as $I \propto \frac{1}{r^2}$

4. (c) Amount left = $\frac{1}{8} = \left(\frac{1}{2}\right)^3$

$\therefore \quad 3t_{1/2} = 15$

$t_{1/2} = 5$ min

5. (a) $R \propto R_0 (A)^{1/3}$

$$\frac{R_{Al}}{R_{Te}} = \frac{(A_{Al})^{\frac{1}{3}}}{(A_{Te})^{\frac{1}{3}}}$$

$$= \frac{(27)^{\frac{1}{3}}}{(125)^{\frac{1}{3}}} = \frac{3}{5}$$

$\therefore \; R_{Te} = \frac{5}{3} \times 3.6 = 6$ Fermi

6. (a) $E = Rhc\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$

$$E_{(4 \to 3)} = Rhc\left[\frac{1}{3^2} - \frac{1}{4^2}\right]$$

$$= Rhc\left[\frac{7}{9 \times 16}\right] = 0.05\ Rhc$$

$$E_{(4 \to 2)} = Rhc\left[\frac{1}{2^2} - \frac{1}{4^2}\right]$$

$$= Rhc\left[\frac{3}{16}\right] = 0.2\ Rhc$$

$$E_{(2 \to 1)} = Rhc\left[\frac{1}{(1)^2} - \frac{1}{(2)^2}\right]$$

$$= Rhc = \left[\frac{3}{4}\right] = 0.75\ Rhc$$

$$E_{(1 \to 3)} = Rhc\left[\frac{1}{(3)^2} - \frac{1}{(1)^2}\right]$$

$$= \frac{-8}{9} Rhc = -0.9\ Rhc$$

$\therefore$ Thus, III transition gives most energy.

7. (c) $\lambda = \frac{h}{\sqrt{2m(KE)}} = \frac{h}{p}$

$$\therefore \; \frac{\lambda_2}{\lambda_1} = \frac{\sqrt{KE_1}}{\sqrt{KE_2}}$$

$$= \frac{\sqrt{KE_1}}{\sqrt{2KE_1}}$$

or $\frac{\lambda_2}{\lambda_1} = \frac{1}{\sqrt{2}}$

8. (c)

9. (c) $f_{out} = 2f_{in} = 2 \times 50 = 100$ Hz

10. (b) ${}_{z}^{A}X + {}_{0}^{1}n \rightarrow {}_{3}^{7}Li + {}_{2}^{4}He$

It implies that

A + 1 = 7 + 4

$\Rightarrow$ A = 10

and Z + 0 = 3 + 2

$\Rightarrow$ Z = 5

Thus, it is Boron ${}_{5}^{10}B$.

AIEEE 2006

1. In a common-base mode of a transistor, the collector current is 5.488 *mA* for an emitter current of 5.60 *mA*. The value of the base current amplification factor (β) will be:

(a) 49 (b) 50
(c) 51 (d) 48

2. The threshold frequency for a metallic surface corresponds to an energy of 6.2 eV and the stopping potential for a radiation incident on this surface is 5V. The incident radiation lies in:

(a) ultraviolet region (b) infrared region
(c) visible region (d) *X*-ray region

3. An alpha nucleus of energy $\frac{1}{2}mv^2$ bombards a heavy nuclear target of charge *Ze*. Then the distance of closest approach for the alpha nucleus will be proportional to:

(a) v^2 (b) $\frac{1}{m}$
(c) $\frac{1}{v^4}$ (d) $\frac{1}{Ze}$

4. The time taken by a photoelectron to come out after the photon strikes is approximately:

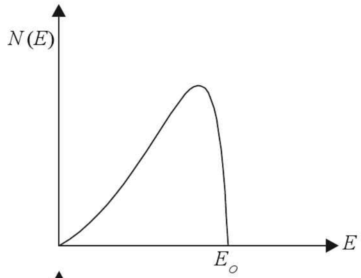
(a) $10^{-4} s$ (b) $10^{-10}\ s$
(c) $10^{-16} s$ (d) $10^{-1} s$

5. When ${}^{3}Li^{7}$ nuclei are bombarded by protons, and the resultant nuclei are ${}_{4}Be^{8}$, the emitted particles will be:

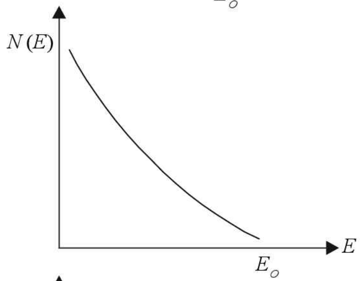
(a) alpha particles (b) beta particles
(c) gamma photons (d) neutrons

6. The energy spectrum of β-particles [number $N(E)$ as a function of β-energy E] emitted from a radioactive source is:

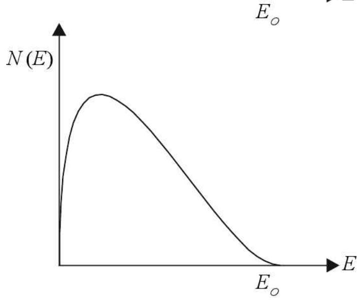
(a)



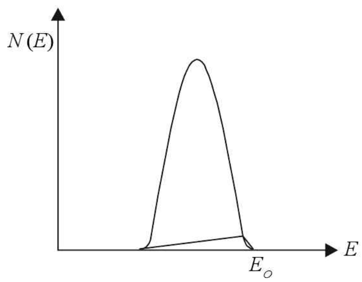
(b)



(c)



(d)



7. A solid which is not transparent to visible light and whose conductivity increases with temperature is formed by:

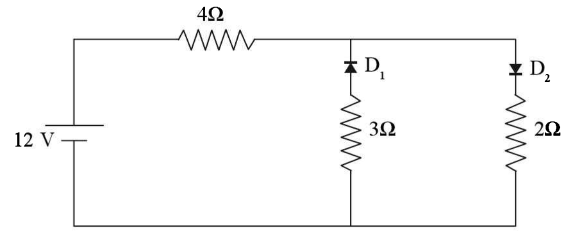
(a) ionic binding
(b) covalent binding
(c) van der Waal's binding
(d) metallic binding

8. If the ratio of the concentration of electrons to that of holes in a semiconductor is $\frac{7}{5}$ and the ratio of currents is $\frac{7}{4}$, then what is the ratio of their drift velocities?

(a) $\frac{5}{8}$ (b) $\frac{4}{5}$

(c) $\frac{5}{4}$ (d) $\frac{4}{7}$

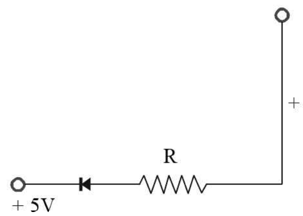
9. The circuit has two oppositely connected ideal diodes in parallel. What is the current flowing in the circuit?



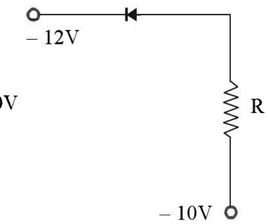
(a) 1.71 *A* (b) 2.00 *A*
(c) 2.31 *A* (d) 1.33 *A*

10. In the following, which one of the diodes is reverse biased?

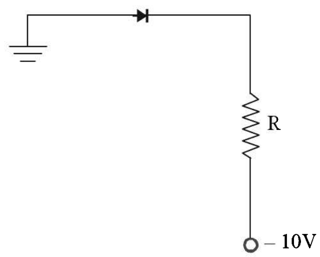
(a)



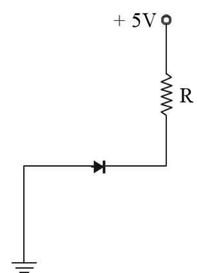
(b)



(c)

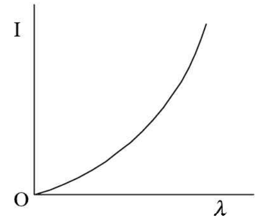


(d)

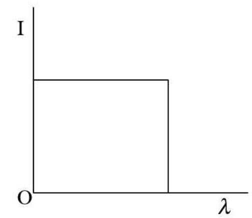


11. The anode voltage of a photocell is kept fixed. The wavelength λ of the light falling on the cathode is gradually changed. The plate current *I* of the photocell varies as follows:

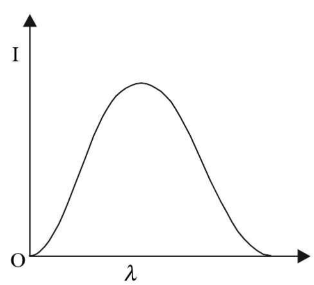
(a)



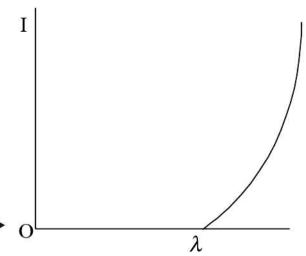
(b)



(c)



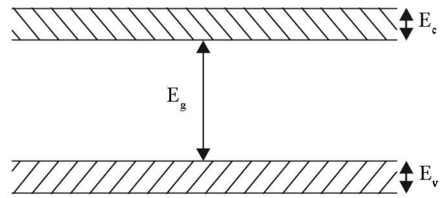
(d)



12. If the binding energy per nucleon in ${}^7_3Li$ and ${}^4_2He$ nuclei are 5.60 MeV and 7.06 MeV respectively, then in the reaction:
$p + {}^7_3Li \rightarrow 2\,{}^4_2He$ energy of proton must be:
(a) 28.24 MeV (b) 17.28 MeV
(c) 1.46 MeV (d) 39.2 MeV

13. The 'rad' is the correct unit used to report the measurement of:
(a) the ability of a beam of gamma ray photons to produce ions in a target
(b) the energy delivered by radiation to a target
(c) the biological effect of radiation
(d) the rate of decay of a radioactive source

14. If the lattice constant of this semiconductor is decreased, then which of the following is correct?



(a) All E_c, E_g, Ev increase
(b) E_c and E_v increase, but E_g increases
(c) E_c and E_v decrease, but E_g increases
(d) All E_c, E_g, E_v decrease

Answers

1.	(a)	**2.**	(a)	**3.**	(b)	**4.**	(b)	**5.**	(c)	**6.**	(c)	**7.**	(b)
8.	(c)	**9.**	(b)	**10.**	(d)	**11.**	(b)	**12.**	(b)	**13.**	(c)	**14.**	(c)

Explanations

1. (a) $\beta = \frac{I_C}{I_B}$ and $I_E = I_C + I_B$

$\therefore\ \beta = \frac{I_C}{I_E - I_C} = \frac{5.488}{5.60 - 5.488} = 49$

2. (a) $hv = hv_0 = 6.2\text{eV}$, $eV_S = 5\text{eV}$
From Einstein's photoelectric equation
$hv = hv_0 + eV_S = 6.2 + 5 = 11.2eV$

$\lambda = \frac{1240}{11.2}$

$= 110.89$ nm i.e. *uv* light

3. (b) $r = \frac{(Ze)(2e)}{4\pi\varepsilon_0 (KE)}$

4. (b) The photoelectric effect is an instantaneous phenomenon (experimentally proved). It takes approx. time of the order of $10^{-10}\ s$.

5. (c) The nuclear reaction can be represented as

${}_3Li^7 + {}_1H^1 \longrightarrow {}_4Be^8 + {}_ZX^A$

Applying conservation of atomic number (change)
$3 + 1 = 4 + Z \Rightarrow Z = 0$
Applying conservation of atomic mass
$7 + 1 = 8 + A \Rightarrow A = 0$
Thus, the emitted particles are γ-photons (${}_0X^0$).

6. (c) Energy spectrum of emitted β-particles from a radioactive source is drawn as shown in (c).

7. (b)

8. (c) $I = n_e A v_d$

$\therefore\ \frac{I_e}{I_h} = \frac{n_e \times (v_d)_e}{n_h \times (v_d)_h}$

Here, $\frac{n_e}{n_h} = \frac{7}{5}$, $\frac{I_e}{I_h} = \frac{7}{4}$

$\therefore\ \frac{7}{4} = \frac{7}{5} \times \frac{(v_d)_e}{(v_d)_h}$

$\Rightarrow \frac{(v_d)_e}{(v_d)_h} = \frac{5}{7} \times \frac{7}{4}$

$= \frac{5}{4}$

9. (b) In the given circuit Diode D_1 is reverse biased while D_2 is forward biased, so the circuit can be redrawn as:
[$\therefore$ For ideal diodes, reverse biased means open and forward biased means short]
Apply *KVL* to get current flowing through the circuit
$-12 + 4i + 2i = 0$

$\Rightarrow i = \frac{12}{6} = 2A$

10. (d) For reverse biasing of an ideal diode, the potential of *n*-side should be higher than potential of *p*-side. Only option (*d*) is satisfying the criterion for reverse biasing.

12. (b) Energy of proton + 7 × 5.60 = 2 × [4 × 7.06]
$\therefore$ Energy of proton = 17.28 MeV

13. (c) 'rad' is used to measure biological effect of radiation.

14. (c) If lattice constant of semiconductor is decreased, then E_c and E_v decrease but Eg increases.

AIIMS 2005

1. For skywave propagation of a 10 MHz signal, what should be the minimum electron density in ionosphere?

(a) ~ 1.2×10^{12} m^{-3} (b) ~ 10^6 m^{-3}
(c) ~ 10^{14} m^{-3} (d) ~ 10^{22} m^{-3}

2. Which of the following logic gates is a universal gate?

(a) OR (b) NOT
(c) AND (d) NAND.

3. Consider an n-p-n transistor amplifier in common-emitter configuration. The current gain of the transistor is 100. If the collector current changes by 1 *mA*, what will be the change in emitter current?

(a) 11 *mA* (b) 1.01 *mA*
(c) 0.01 *mA* (d) 10 *mA*.

4. The ground state energy of hydrogen atom is – 13.6 eV. What is the potential energy of the electron in this state?

(a) 0 eV (b) – 27.2 eV
(c) 1 eV (d) 2 eV.

5. Solid targets of different elements are bombarded by highly energetic electron beams. The frequency (f) of the characteristic X-rays emitted from different targets varies with atomic number Z as

(a) $f \propto \sqrt{Z}$ (b) $f \propto Z^2$
(c) $f \propto Z$ (d) $f \propto Z^{\frac{3}{2}}$

6. In a semiconducting material the mobilities of electrons and holes are μ_e and μ_h respectively. Which of the following is true?

(a) $\mu_e > \mu_h$ (b) $\mu_e < \mu_h$
(c) $\mu_e = \mu_h$ (d) $\mu_e < 0$; $\mu_h > 0$.

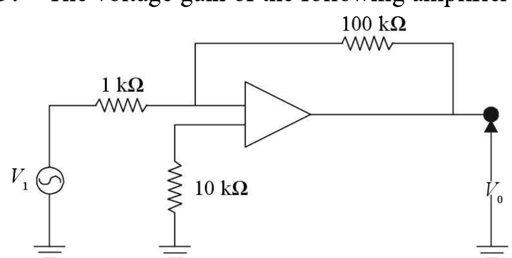
7. A radioactive material has a half-life of 10 days. What fraction of the material would remain after 30 days?

(a) 0.5 (b) 0.25
(c) 0.125 (d) 0.33.

8. According to Hubble's law, the redshift (Z) of a receding galaxy and its distance r from earth are related as

(a) $Z \propto r$ (b) $Z \propto \frac{1}{r}$
(c) $Z \propto \frac{1}{r^2}$ (d) $Z \propto r^{\frac{3}{2}}$.

9. The voltage gain of the following amplifier is



(a) 10 (b) 100
(c) 1000 (d) 9.9

Answers

1.	(a)	**2.**	(d)	**3.**	(b)	**4.**	(b)	**5.**	(b)	**6.**	(a)	**7.**	(c)
8.	(a)	**9.**	(b)										

Explanations

1. (a) $f_c = 9\,(N_{max})^{\frac{1}{2}}$

where N is the density of electron/m^3.

$$\frac{10\times10^6}{9} = (N_{max})^{\frac{1}{2}}$$

$$N_{max} = \left(\frac{10\times10^6}{9}\right)^2 \simeq 1.2 \times 10^{12}\ \text{m}^{-3}.$$

2. (d) NAND and NOR gate are called universal gate.

3. (b) Current gain = $\frac{\Delta I_C}{\Delta I_B}$

$\therefore\ \Delta I_B = \frac{1mA}{100} = 10^{-2}\,mA.$

As $\Delta I_E = \Delta I_B + \Delta I_C = (0.01 + 1)\,mA = 1.01\,mA.$

4. (b) PE = 2 binding energy = – 27.2 eV

5. (b) According to Moseley's law

$(Z - \sigma) \propto \sqrt{f}$ or, $f \propto (Z - \sigma)^2$.

6. (a) $\mu_e > \mu_h$. It is 2 to 3 times larger.

7. (c) $t_{\frac{1}{2}}$ = 10 days, $3t_{\frac{1}{2}}$ = 30 days

$$\frac{N_1}{N_0} = \left(\frac{1}{2}\right)^3$$

8. (a) Hubble's law is a statement of a direct correlation between the distance (r) to a galaxy and its recessional velocity as determined by the red shift (Z). It is stated as $V_Z = H_0 r$,

where V_Z = recessional velocity

H_0 = Hubble constant and r = distance.

9. (b) This is an example of operational amplifier in this voltage gain.

$$A = \frac{V_0}{V_i} = \frac{R_f}{R_i}$$

$$= \frac{100k\Omega}{1k\Omega} = 100.$$

BHU 2005

1. Who discovered X-rays?

(a) Roentgen (b) Marie Curie

(c) Rutherford (d) All.

2. The light rays having photons of energy 1.8 eV are falling on a metal surface having a work function 1.2 eV. What is the stopping potential to be applied to stop the emitting electrons?

(a) 3 eV (b) 1.2 eV

(c) 0.6 eV (d) 1.4 eV.

3. ${}_{86}A^{222} \rightarrow {}_{84}B^{210}$. In this reaction how many α and β particles are emitted?

(a) $6\alpha, 3\beta$ (b) $3\alpha, 4\beta$

(c) $4\alpha, 3\beta$ (d) $3\alpha, 6\beta$.

4. The phenomenon of radioactivtity is

(a) exothermic change which increases or decreases with temperature

(b) increases on applied pressure.

(c) nuclear process does not depend on external factors

(d) none of the above.

5. The amplification factor of a triode valve is 15. If the grid voltage is changed by 0.3 volt the change in plate voltage in order to keep the current constant (in volt) is

(a) 0.02 (b) 0.002

(c) 4.5 (d) 5.0.

6. Which of the following have highest specific charge?

(a) Positron (b) Proton

(c) He (d) None of these.

7. In a full wave rectifiers, input a.c. current has a frequency 'v'. The out put frequency of current is

(a) $\frac{v}{2}$ (b) v

(c) $2v$ (d) none.

8. Radius of first Bohr orbit is r. What is the radius of 2nd Bohr orbit?

(a) $8r$ (b) $2r$

(c) $4r$ (d) $2\sqrt{2}\,r$

Answers

1. (a)	**2.** (c)	**3.** (b)	**4.** (c)	**5.** (c)	**6.** (b)	**7.** (c)
8. (c)						

Explanations

1. (a) X-rays were discovered by Prof. Roentgen in 1895.

2. (c) $h\upsilon - \phi = eV_S$

= 1.8 eV – 1.2 eV = 0.6 eV. $\therefore$ V_S = 0.6 V

3. (b) ${}_{86}A^{222} \rightarrow {}_{84}B^{210}$

Decrease in mass number = 222 – 210 = 12

$\therefore$ α-particle ejected = $\frac{12}{4}$ = 3.

Decrease in atomic number = 86 – 84 = 2.

The actual decrease, decrease should have been

= 3 × 2 = 6

Increase due to β-emission = 6 – 2 = 4

Therefore, 4 β^- has also been emitted.

4. (c)

5. (c) Amplification factor (μ)

$$= \frac{\text{change in plate potential}}{\text{change in grid potential}}$$

$15 = \frac{\Delta V_P}{0.3} \Rightarrow \Delta V_P = 4.5$ V.

6. (b) $\frac{e}{m_e} = \frac{1.6\times10^{-19}}{9.1\times10^{-31}} = 1.75 \times 10^{11} Ckg^{-1}$

$\frac{e}{m_p} = \frac{1.6\times10^{-19}}{1.67\times10^{-27}} = 9..58 \times 10^{7} Ckg^{-1}$

$\frac{e}{m_\alpha} = \frac{2\times1.6\times10^{-19}}{6.6\times10^{-27}}$

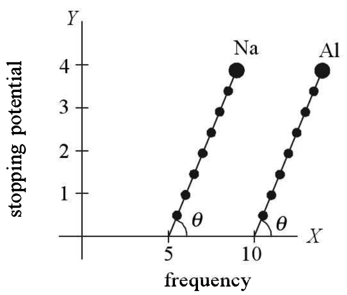
$= 4.84 \times 10^{7}\ Ckg^{-1}$

7. (c) In full wave rectifier $v_{out} = 2v_{input}$

8. (c) $r_n = n^2 r$

KARNATAKA 2005

1. From the figure describing photoelectric effect we may infer correctly that



(a) Na and Al both have the same threshod frequency
(b) maximum kinetic energy for both the metals depend linearly on the frequency
(c) the stopping potentials are different for Na and Al for the same change in frequency
(d) Al is a better photo sensitive material than Na.

2. The electron in a hydrogen atom makes a transition from $n = n_1$ to $n = n_2$ state. The time period of the electron in the initial state (n_1) is eight times that in the final state (n_2). The possible values of n_1 and n_2 are

(a) $n_1 = 8, n_2 = 1$ (b) $n_1 = 4, n_2 = 2$
(c) $n_1 = 2, n_2 = 4$ (d) $n_1 = 1, n_2 = 8$

3. If the forward voltage in a diode is increased, the width of the depletion region

(a) increases (b) decreases
(c) fluctuates (d) no change

4. Two nucleons are at a separation of one Fermi. Protons have a charge of + 1.6 × 10⁻¹⁹ C. The net nuclear force between them is F_1, if both are neutrons, F_2 and F_3 if one is proton and the other is neutron. Then

(a) $F_1 = F_2 < F_3$ (b) $F_1 = F_2 = F_3$
(c) $F_1 < F_2 < F_3$ (d) $F_1 > F_2 > F_3$

5. The energy that should be added to an electron to reduce its de Brogile wavelength from one nm to 0.5 *nm* is

(a) four times the intital energy
(b) equal to the initial energy
(c) twice the initial energy
(d) thrice the initial energy.

6. Mean life of a radioactive sample is 100 seconds. Then its half-life (in minutes) is

(a) 0.693 (b) 1
(c) 10^{-4} (d) 1.155

7. Consider two nuclei of the same rdioactive nuclide. One of the nuclei was created in supernova explosion 5 billion years ago. The other was created in a nuclear reactor 5 minutes ago. The probability of decay during the next time is

(a) different for each nuclei
(b) nuclei created in explosion decays first
(c) nuclei created in the reactor decays first
(d) independent of the time of creation

8. Bohr's atom model assumes

(a) the nucleus is of infinite mass and is at rest
(b) electrons in a quantized orbit will not radiate energy
(c) mass of electron remains constant
(d) all the above conditions

9. Identify the property of which is not characteristic for a semiconductor?

(a) at a very low temperatures, it behaves like an insulator
(b) at higher temperature two types of charge carriers will cause conductivity
(c) the charge carriers are electrons and holes in the valence band at higher temperatures
(d) the semiconductor is electrically neutral.

10. Identify the wrong statement in the following. Coulomb's law correctly describes the electric force that

(a) binds the electrons of an atom to its nucleus
(b) binds the protons and neutrons in the nucleus of an atom
(c) binds atoms together to form molecules
(d) binds atoms and molecules to form solids.

Answers

1.	(b)	**2.**	(b)	**3.**	(b)	**4.**	(b)	**5.**	(c)	**6.**	(d)	**7.**	(d)
8.	(b)	**9.**	(c)	**10.**	(b)								

Explanations

1. (b)

2. (b) $T^2 \propto r^{3/2}$

3. (b)

4. (b) nuclear force is equal

5. (c) $\lambda = \frac{hc}{E}$

6. (d) $\frac{t_1}{2} = \frac{0.693}{\lambda} = 0.693 \times 100 = 1.155$ minute

7. (d)

8. (b)

9. (c)

10. (b)

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7 Model Test Papers

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Model Test Paper 3
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MODEL TEST PAPER 1

1. A wire is bent in the shape shown in Fig. 1 and carries a current I, the magnetic field at the centre O is

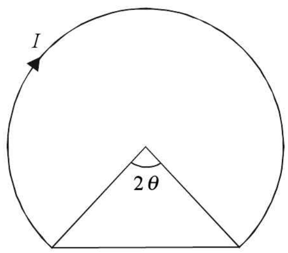


Fig. 1

(a) $\frac{\mu_0 i}{2\pi r}(\theta - \tan\theta)$ (b) $\frac{\mu_0 i}{2\pi r}(2\pi - 2\theta)$

(c) $\frac{\mu_0 i}{2\pi r}(\pi - \theta + \tan\theta)$ (d) $\frac{\mu_0 i}{2\pi r}(\pi - \theta + \frac{\tan\theta}{\pi})$

2. A transformer has turn ratio 2. Load current is 20 A. Efficiency of the transformer is 80% and output power is 2400 W. The primary voltage is

(a) 240 V (b) 300 V

(c) 120 V (d) 60 V

3. Find the resistance across AB in the given figure. Assume each resistance = R

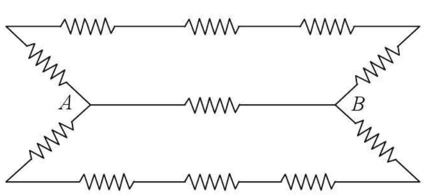


Fig. 2

(a) $\frac{5}{7}R$ (b) $\frac{2}{5}R$

(c) $\frac{3}{7}R$ (d) none of these

4. Which of the following is not a fermion?

(a) electron (b) neutron

(c) coper pair (d) proton

5. A capacitor is half filled by a dielectric of stength k_0 as shown in Fig. 3. If area of plates is A and separation is d, then net capacitance is

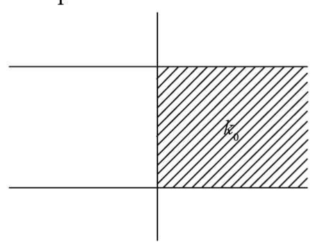


Fig. 3

(a) $\frac{A\varepsilon_0}{2d}[1 + k_0]$ (b) $\frac{A\varepsilon_0}{d}[1 + k_0]$

(c) $\frac{A\varepsilon_0 k_0}{2d}$ (d) none of these

6. If each capacitor is C, then capacitance between A and B is

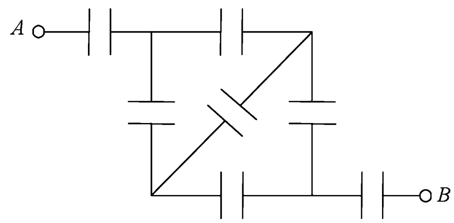


Fig. 4

(a) C (b) $2C$

(c) $\frac{C}{3}$ (d) $\frac{C}{2}$

7. If a square cage of side 10 cm made of wire carrying current 2A is placed 10 cm away from a long wire carrying current 2A, then the force acting on the wire cage is

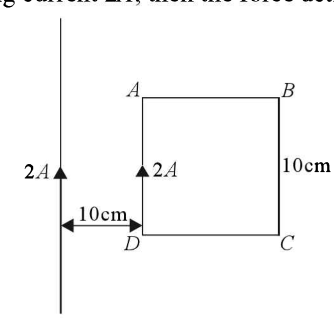


Fig. 5

(a) attractive (b) repulsive
(c) no force (d) only torque acts

8. When switch S is closed, time taken for the capacitor to charge to 9.5 V is

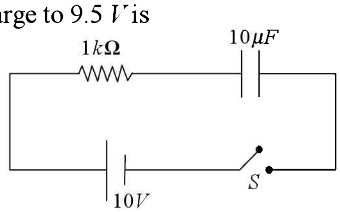


Fig. 6

(a) 3 ms (b) 30 ms
(c) 300 ms (d) none of these

9. A spring of spring constant k and length l is cut into 4 equal parts. Then spring constant of each spring is

(a) $\frac{K}{4}$ (b) $\frac{K}{2}$
(c) $2K$ (d) $4K$

10. The minimum distance between object and its real image in case of a lens of focal length f is

(a) $2f$ (b) $\frac{3}{2}f$
(c) $4f$ (d) $3f$

11. Mobility of a charged particle is defined as
(a) velocity of the particle per unit mass
(b) velocity per unit electric field
(c) velocity per unit volt
(d) velocity

12. Two lenses of Power $4D$ and -1.5 D are combined together and kept in a medium of refractive index greater than that of the lens. The combination acts as

(a) convex lens (b) concave lens
(c) concave mirror (d) none of these

13. The current I in the circuit is

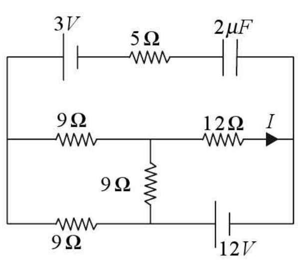


Fig. 7

(a) $\frac{2}{3}A$ (b) $\frac{4}{7}A$
(c) $\frac{1}{3}A$ (d) none of these

14. Resistivity decreases with increase in ____in metals.

(a) temperature (b) pressure
(c) volume (d) none of these

15. Emission spectrum of He will be ____spectrum.

(a) line (b) band
(c) line or band (d) dark bands

16. If both the inputs of a two input NAND gate are shorted then circuit will behave as ____ gate.

(a) AND (b) NOR
(c) XOR (d) NOT

17. A projectile is thrown at an angle θ with the horizontal with a velocity V. Then Range is R. With what angle with respect to the vertical it should be thrown with same velocity so that it acquires the same range?

(a) θ (b) $\frac{3\theta}{2}$
(c) $\frac{\theta}{2}$ (d) 2θ

18. The wavelength of H_β line of Balmer series is nearly

(a) 625 nm (b) 582 nm
(c) 517 nm (d) 487 nm

19. XYZ is a triangle formed with 6 Ω each. W is mid point of the resistance connected between Y and Z. The resistance between A and B is

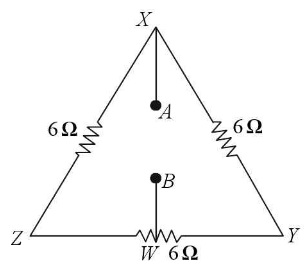


Fig. 8

(a) 4.5 Ω (b) 6 Ω
(c) 7.5 Ω (d) 9 Ω

20. V_{AB} in the adjoining circuit is

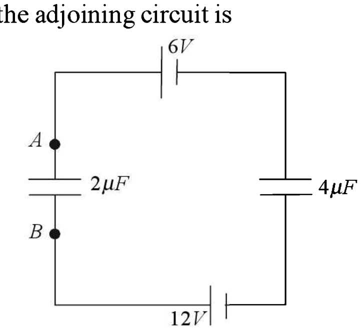


Fig. 9

(a) 2 V (b) 4 V
(c) −2 V (d) −4 V

21. Carbon resistances are used in $T.V.$ receivers because
(a) resistance does not change with temperature
(b) resistance does not change with frequency
(c) resistance does not change with applied voltage
(d) resistance does not change with current

22. Two charges $4\mu C$ each are fixed at $(0, a, c)$ and $(0, -a, 0)$. A charge $-2\mu C$ is brought from $(-b, 0, 0)$ to origin. The nature of equilibrium at $(\frac{-b}{2}, 0, 0)$ is

(a) stable (b) unstable
(c) neutral
(d) sometimes stable and sometimes unstable

23. A particle is moving in a potential region given by $V = (xy + yz + zx)$ the electric field acting is

(a) $-A\,[(y+z)\,\hat{i} + (x+z)\,\hat{j} + (x+y)\,\hat{k}\,]$
(b) $A\left[x\hat{i} + y\hat{j} + z\hat{k}\right]$
(c) $A\,[(y+z)\hat{i} + (x+z)\hat{j} + (x+y)\hat{k}\,]$
(d) $-A\,[y\hat{i} + z\hat{j} + x\hat{k}]$

24. The energy of photon of wavelength 600 nm in eV is nearly

(a) 0.206 eV (b) 1.06 eV
(c) 2.06 eV (d) 20.6 eV

25. A long wire carries a current I. Find the magnetic field at point P if it is bent as shown in Fig. 10.

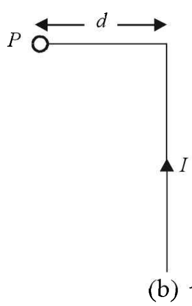


Fig. 10

(a) $\frac{\mu_0 I}{4\pi\varepsilon_0 d}$ (b) $\frac{\mu_0 I}{2\pi d}$
(c) $\frac{\mu_0 I}{4d}$ (d) $\frac{\mu_0 I}{4\pi d}$

26. In a house AC voltage measured by ac voltmeter is 200 V. If the line frequency is 50 Hz then equation of voltage is

(a) $V = 200 \sin 100\,\pi t$ (b) $V = 288 \sin 100\,\pi t$
(c) $V = 142 \sin 100\,\pi t$ (d) none of these

27. In an LC circuit $L = 0.2\ mH$ and $C = 10\ \mu F$, the fundamental frequency of the circuit is nearly

(a) 353 Hz (b) 451 Hz
(c) 2 kHz (d) none of these

28. The focal length of the lens (plano-convex) silvered from curved side if its focal length without silvering is f will be

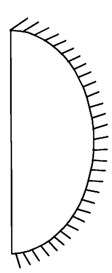


Fig. 11

(a) $(\mu - 1)f$ (b) $\frac{(\mu - 1)f}{\mu}$
(c) $\frac{(\mu - 1)f}{2\mu}$ (d) $\frac{(2\mu)f}{\mu - 1}$

29. Tuning fork of frequency 400 Hz is vibrated with 802 Hz tuning fork. Number of beats heard are

(a) none (b) 402
(c) 20 (d) 2

30. A spring of spring constant k is fixed to a wall on one end and connected to a block of mass m and charge q on the other end and placed on a smooth horizontal plane. If electric field E is switched ON then maximum displacement which can be produced is

(a) $\frac{qE}{k^2}$ (b) $\frac{q^2E}{k^2}$
(c) $\frac{qE^2}{k}$ (d) $\frac{qE}{k}$

31. A Coolidge tube is operating at 20 kV/ 5 mA. The threshold wavelength of x-ray is nearly

(a) 1 A° (b) 0.62 A°
(c) 0.51 A (d) cannot be predicted

32. 8 cells each of emf 1.5 V and internal resistance 0.75 Ω were to be connected in series. Two of them were connected wrongly. The maximum current through a load of 6Ω will be

(a) 1 A (b) 0.75 A
(c) 0.5 A (d) cannot be predicted

33. A non conducting solid sphere of radius R is charged volumetrically by Q, then electric field intensity at a distance x $(0 < x < R)$

(a) zero (b) $\frac{Q}{4\pi\varepsilon_0 x^2}$
(c) $\frac{Qx}{4\pi\varepsilon_0 R^3}$ (d) $\frac{Q}{4\pi\varepsilon_0 (R - x)^2}$

34. The densities of wood and benzene at 0° C are 880 kg/m^3 and 900 kg/m^3 respectively. The coefficient of volume expansion are 1.2×10^{-3} /°C and 1.5×10^{-3} /°C for wood and benzene respectively. The temperature at which piece of wood will just sink in the benzene is

(a) 43.3° C (b) 59.1° C
(c) 73.3°C (d) 83.3°C

35. If the rms speed of oxygen is 345 ms^{-1}, then rms speed of hydrogen at the same temperature will be

(a) 345 ms^{-1} (b) 690 ms^{-1}
(c) 1035 ms^{-1} (d) 1380 ms^{-1}

36. The correct PV diagram for a given V–T diagram will be

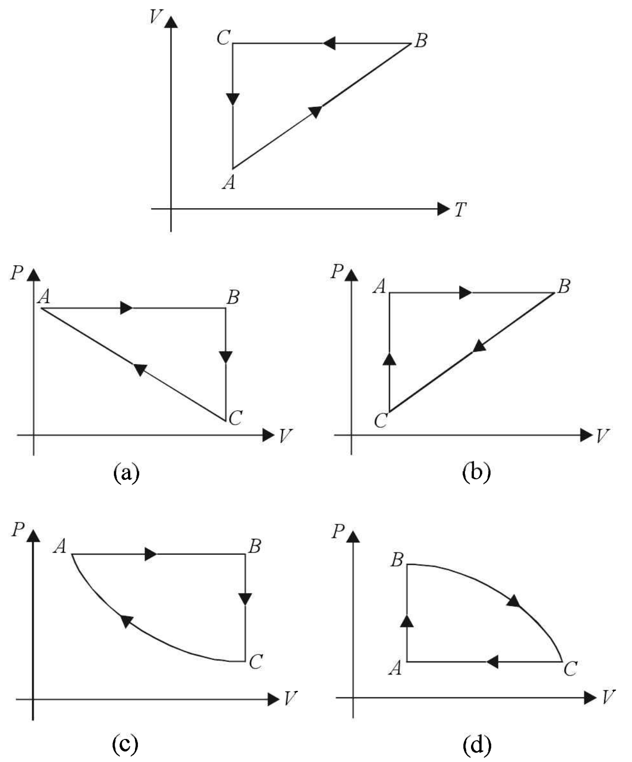


Fig. 12

37. The figure shows a graph between pressure and density for an ideal gas at two different temperatures, then

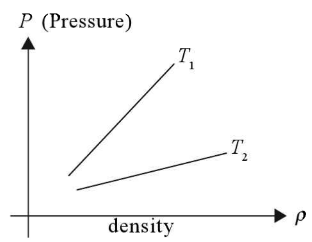


Fig. 13

(a) $T_1 = T_2$ (b) $T_1 > T_2$

(c) $T_1 < T_2$ (d) any of (a), (b), or (c)

38. The quantity $\frac{PV}{kT}$ represents ----- of the gas.

(a) mass (b) number of moles

(c) K.E. (d) number of molecules

39. The specific heat capacity of a body

(a) is finite but not zero (b) may be zero

(c) is infinite (d) $-\infty < Cv < \infty$

40. The heat absorbed in the system is

(a) 31.4 J (b) 3.14 J

(c) 314 J (d) none of these

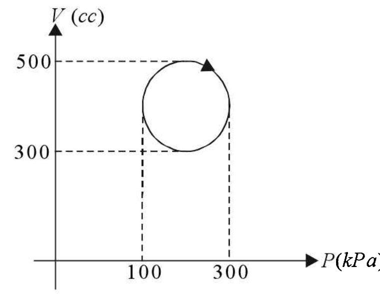


Fig. 14

41. Mutual inductance of a triode valve is

(a) $\frac{dV_P}{dI_g}$ (b) $\frac{dI_P}{dV_g}$

(c) $\frac{dV_P}{dI_P}$ (d) $\frac{dI_g}{dV_P}$

42. In an adiabatic process on a gas with γ = 1.4, the pressure is increased by 0.5 %. The volume decreases by about

(a) 0.5 % (b) 0.7 %

(c) 1 % (d) 0.36 %

43. Thermal potential is equivalent to

(a) thermal energy (b) temperature

(c) thermal coefficient (d) $\frac{KA}{l}$

44. The temperature of a body falls from 40° C to 36° C in 5 minutes. The temperature in the next 5 minutes will be ____ if the temperature of surroundings is 16° C.

(a) $\frac{98}{3}$ °C (b) 32° C

(c) 34° C (d) none of these

45. One end of a metal rod is kept in a furnace. In steady state, the temperature of the rod

(a) increases (b) decreases

(c) remains constant (d) is non-uniform

46. Potential energy of a dipole placed in a uniform electric field is

(a) $\vec{p} \times \vec{E}$ (b) $\vec{p} \cdot \vec{E}$

(c) ΔpE (d) $p\Delta E$

47. A string passes over two pulleys. The masses 5 kg each are connected at two ends. The tension in the string is nearly

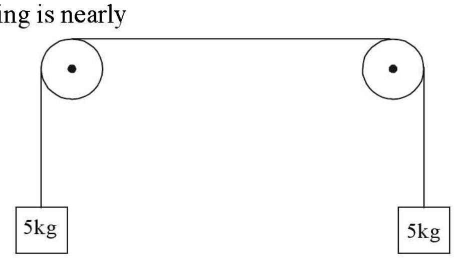


Fig. 15

(a) 50 N (b) 100 N
(c) 25 N (d) 75 N

48. The rain is falling vertically with a speed 5 ms^{-1}. A man is moving due North with a speed 8 ms^{-1}. The angle at which he should keep his umbrella to protect himself from rains is
(a) $\tan^{-1}(1.6)$ north of vertical
(b) $\tan^{-1}(0.625)$ north of vertical
(c) $\tan^{-1}(1.6)$ along north
(d) $\tan^{-1}(0.625)$ along south.

49. Inertial force is applied when the frame of reference is
(a) inertial (b) non-inertial
(c) both (a) and (b)
(d) Insufficient data to reply

50. In the steady state, a spring of spring constant K gives an extension y when a load M is applied. The maximum extension produced with the same load is
(a) y (b) $2y$
(c) $3y$ (d) cannot be predicted.

51. The radius of gyration of a hollow sphere about its tangent is
(a) $\sqrt{\frac{2}{3}}R$ (b) $\sqrt{\frac{2}{5}}R$
(c) $\sqrt{\frac{5}{3}}R$ (d) $\sqrt{\frac{7}{5}}R$

52. An open pipe of length l is closed and filled half with water. The fundamental frequency
(a) increases (b) decreases
(c) remains unchanged (d) none of these

53. The amplitude of vibration of any particle in a standing wave, produced along a stretched string depends upon
(a) frequency of incident wave
(b) location of particles
(c) time
(d) time period of reflected wave

54. A tuning capacitor has 11 plates and maximum capacitance between two successive plates is 5 μF. The maximum capacitance of the tuning capacitor is
(a) 55 μF (b) 50 μF
(c) 45 μF (d) 40 μF

55. Binary equivalent of $(25.75)_{10}$ is
(a) 11001.10 (b) 11001.11
(c) 10101.11 (d) 10110.11

56. A band pass filter is equivalent to ____ circuit.
(a) RC (b) RL
(c) LC (d) any of (a), (b) or (c)

57. Resistivity ρ of a semiconductor is related to temperature T as
(a) $\rho \propto T$ (b) $\rho \propto T^2$
(c) $\rho \propto T^{-2}$ (d) $\rho \propto T^{-1}$

58. Phase difference of π-radian is introduced between input and output voltage in
(a) common base amplifier
(b) common emitter amplifier
(c) common collector amplifier
(d) All of (a), (b) and (c)

59. Bremsstrahlung radiation is
(a) mechanical wave (b) e-m wave
(c) particle-wave (d) shockwave

60. Had the speed of light been $\frac{c}{2}$ instead of c, the Rydberg's constant would be
(a) $\frac{R}{2}$ (b) $\frac{R}{4}$
(c) R (d) $2R$

61. If A voltmeter (ac) reads 240 V, then it is
(a) mean voltage (b) rms voltage
(c) peak voltage (d) peak to peak voltage

62. The dimensional formula of $\varepsilon_0 E^2$ is
(a) ML^2T^{-2} (b) MLT^{-2}
(c) $ML^{-1}T^{-2}$ (d) ML^0T^{-2}

63. Two particles are moving in concentric horizontal circles of radii r_1 and r_2 respectively such that they always face each other, then the ratio of their angular velocities is (Given $r_1 > r_2$)
(a) $\frac{r_1}{r_2}$ (b) $\frac{r_1 - r_2}{r_2}$
(c) $\frac{r_1}{r_1 - r_2}$ (d) 1

64. If a plane is looping the loop such that its velocity at the lowest point is $\sqrt{6gr}$, then its velocity at the highest point is
(a) $\sqrt{gr}$ (b) $\sqrt{2gr}$
(c) $\sqrt{3gr}$ (d) $\sqrt{\frac{3}{2}gr}$

65. The value of C_p in a diatomic gas is
(a) $\frac{7}{2}R$ (b) $\frac{5}{2}R$
(c) $2R$ (d) $\frac{9}{2}R$

66. The reduced mass of a system of two particles of mass M and $2M$ is

(a) $\frac{2M}{3}$ (b) $\frac{M}{3}$

(c) $\frac{M}{2}$ (d) $\frac{3M}{2}$

67. In the circuit shown V_{AB} = ____ volts

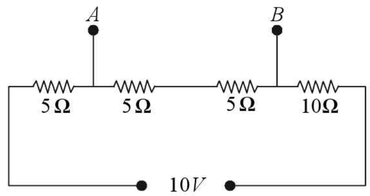


Fig. 16

(a) 4 V (b) 6 V

(c) 2 V (d) 3.5 V

68. If the polarizing angle for a medium is 60°, then its critical angle will be

(a) $\theta = \sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$ (b) $\theta = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$

(c) $\theta = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$ (d) none of these

69. The moment of inertia of a disc of Mass M, radius R parallel to diameter of the disc is

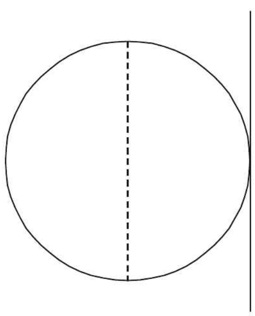


Fig. 17

(a) $\frac{MR^2}{2}$ (b) $\frac{3MR^2}{2}$

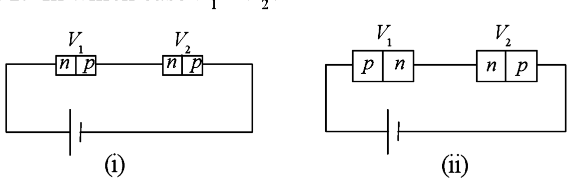
(c) $\frac{5MR^2}{4}$ (d) $\frac{5MR^2}{2}$

70. A particle falls from a height h such that it travels 6 m in its last 0.2s then height h is equal to ____ m (Take g = 10 ms^{-2})

(a) 42 m (b) 48.05 m

(c) 38.95 m (d) none of these

71. In which case $V_1 = V_2$?



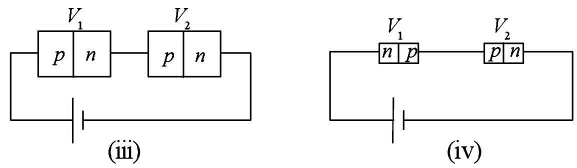


Fig. 18

(a) (i) and (ii) (b) (ii) and (iv)

(c) (iii) and (iv) (d) (i) and (iii)

72. If $\alpha = 0.98$ for a transistor then β = ____

(a) 98 (b) 49

(c) 9.8 (d) 4.9

73. Mosley's law is related with

(a) frequency of x-rays

(b) frequency of x-rays and atomic number

(c) frequency of x-rays and mass number

(d) speed of x-rays and atomic number

74. If maximum electric field is 100 N/m^{-2}, then magnitude of maximum magnetic field will be

(a) $3.33 \times 10^{-9} T$ (b) $33.3 \times 10^{-9} T$

(c) $33.3 \times 10^{-8} T$ (d) $3.33 \times 10^{-8} T$

75. A rod of length l is fixed at one end and is rotated at an angular speed ω. If a magnetic field B is applied perpendicular to the plane of the rod then emf induced is

(a) $Bl\,\omega$ (b) $Bl^2\omega$

(c) $\frac{Bl^2\omega}{2}$ (d) $Bl\,\omega^2$

76. Sound waves cannot ____

(a) be polarised (b) be diffracted

(c) be dispersed (d) produce interference

77. The curve between seebeck emf and temperature is

(a) parabolic (b) hyperbolic

(c) elliptical (d) logrithmic

78. Ultrasonic waves can be produced in

(a) open pipe (b) drum

(c) piano (d) quartz

79. If a particle moves along a path described by $x = at + bt^2$ then it has

(a) constant acceleration (b) constant velocity

(c) variable acceleration (d) none of these

80. Doppler effect in sound cannot be produced if

(a) source is moving with a speed > speed of sound

(b) source is moving with a speed ≥ speed of light

(c) observer is moving with a speed less than speed of sound

(d) source is moving with a speed less than speed of sound

81. In a sonometer experiment a wire of mass per unit length μ is replaced by $\frac{\mu}{2}$ then fundamental frequency increases by

(a) $\sqrt{2}$ times (b) 2 times
(c) 2^{-1} (d) $2^{-1/2}$ times

82. Two particles are moving in opposite directions in a circle of radius *r* with same speed, then their resultant is

(a) linear motion (b) circular motion
(c) SHM (d) projectile

83. The momentum at the highest point in case of a projectile fired with a velocity *v* making an angle θ with vertical is (Given mass is *m*).

(a) zero (b) mv cosθ
(c) $mv\sin\theta$ (d) none of these

84. The gravitational field inside a sphere of radius *R* and mass *M* at a point distant *x* from the surface will be

(a) $\frac{GM}{(R-x)^2}$ (b) $\frac{GM(R-x)}{R^3}$
(c) zero (d) $\frac{GMx}{R^3}$

85. The maximum number of geostationery satellites which can be put into orbit, working all together is

(a) 3 (b) 30
(c) 90 (d) 180

86. According to Bohr's theory the radius of 2*nd* orbit of He^+ is

(a) 0.53$A°$ (b) 1.06$A°$
(c) 2.12$A°$ (d) none of these

87. The electric field due to a short dipole of moment *p* at any point at a distance *x* from the centre of the dipole is

(a) $\frac{2p\cos\theta}{x^3}$ (b) $\frac{p\sqrt{1+3\cos^2\theta}}{4\pi\varepsilon_0 x^3}$
(c) $\frac{p\sqrt{3\cos^2\theta-1}}{4\pi\varepsilon_0 x^3}$ (d) $\frac{p\sqrt{3\cos^2\theta+1}}{4\pi\varepsilon_0 x^2}$

88. The self inductance of coil is 16 μH when a core is added the self inductance becomes 96 μH. The susceptibility of the core is

(a) 5 (b) 6
(c) 112 (d) 80

89. In tan A position, the arms of the magnetometer are parallel to

(a) East-West (b) North-South
(c) magnetic needle (d) none of these

90. Two photo cathodes *A* and *B* have work function 1.0eV and 2.5eV respectively, if a radiation of 4eV is incident on them then ratio of maximum velocities of photoelectron in two is

(a) $\sqrt{2}$ (b) 2
(c) 2^{-1} (d) $\frac{1}{\sqrt{2}}$

91. Coherent sources to the accurate term are made

(a) by division of wave front
(b) by division of amplitude
(c) both (*a*) and (*b*)
(d) none of these

92. The ratio of maximum to minimum intensity when one of the slits is covered with a paper of transparency of light energy $\frac{4}{9}$ is

(a) $\frac{2}{3}$ (b) $\frac{81}{16}$
(c) 25 (d) $\frac{25}{9}$

93. A car is accelerated suddenly at $1ms^{-2}$ while it is moving on a circular track of radius 100 m with a speed 10 ms^{-1}. The net acceleration is

(a) $1ms^{-2}$ (b) $\sqrt{2}$ ms^{-2}
(c) 2 ms^{-2} (d) cannot be predicted

94. Pitch corresponds to

(a) linear distance moved in one rotation
(b) circular distance moved in one rotation
(c) least count
(d) none of these

95. A charged particle enters a magentic field at 60° then its path is

(a) circular (b) elliptical
(c) helical (d) hyperbolic

96. Two wheels of radius ratio 1:3 are connected by a common belt. The smaller is rotated at a rate 2 rad /s^2. The angular velocity of bigger wheel after 10 *s* is

(a) 20 rad/*s* (b) 10 rad/s
(c) $\frac{20}{3}$ rad/*s* (d) none of these

97. Bragg's law is related with

(a) polarization (b) diffraction
(c) interference (d) photoelectric effect

98. Current density in the largest section of 4 × 3 × 2 cm^3 parallelo piped if it carries a current 2*A* is

(a) 3.32×10^3 A/m^2 (b) $2.5 \times 10^3 Am^{-2}$
(c) $1.66 \times 10^3 Am^{-2}$ (d) none of these

99. Wires *A* and *B* made of Cu and Fe are connected in parallel to a voltage source V_0. If their diameters and lengths are the same then in which case heat energy developed in the same time will be more?

(a) in wire *A* (b) in wire *B*
(c) equal in both (d) none of these

100. F_{PP}, F_{PN} and F_{NN} are net forces between proton and proton, proton and neutron, and neutron and neutron respectively, then

(a) $F_{PP} = F_{Pn} = F_{NN}$ (b) $F_{PP} > F_{PN} > F_{NN}$
(c) $F_{NN} = F_{PN} > F_{PP}$ (d) $F_{PP} = F_{PN} > F_{NN}$

Answers

1.	(c)	**2.**	(b)	**3.**	(a)	**4.**	(c)	**5.**	(a)	**6.**	(c)	**7.**	(a)
8.	(b)	**9.**	(d)	**10.**	(c)	**11.**	(b)	**12.**	(b)	**13.**	(a)	**14.**	(b)
15.	(a)	**16.**	(d)	**17.**	(a)	**18.**	(d)	**19.**	(a)	**20.**	(d)	**21.**	(b)
22.	(a)	**23.**	(a)	**24.**	(c)	**25.**	(d)	**26.**	(b)	**27.**	(a)	**28.**	(c)
29.	(d)	**30.**	(d)	**31.**	(d)	**32.**	(c)	**33.**	(c)	**34.**	(d)	**35.**	(d)
36.	(c)	**37.**	(b)	**38.**	(d)	**39.**	(d)	**40.**	(a)	**41.**	(b)	**42.**	(d)
43.	(b)	**44.**	(a)	**45.**	(d)	**46.**	(b)	**47.**	(a)	**48.**	(a)	**49.**	(b)
50.	(b)	**51.**	(c)	**52.**	(c)	**53.**	(b)	**54.**	(b)	**55.**	(b)	**56.**	(c)
57.	(d)	**58.**	(b)	**59.**	(b)	**60.**	(d)	**61.**	(b)	**62.**	(c)	**63.**	(d)
64.	(b)	**65.**	(a)	**66.**	(a)	**67.**	(a)	**68.**	(a)	**69.**	(c)	**70.**	(b)
71.	(d)	**72.**	(b)	**73.**	(b)	**74.**	(c)	**75.**	(c)	**76.**	(a)	**77.**	(a)
78.	(d)	**79.**	(a)	**80.**	(a)	**81.**	(a)	**82.**	(c)	**83.**	(c)	**84.**	(b)
85.	(d)	**86.**	(b)	**87.**	(b)	**88.**	(a)	**89.**	(a)	**90.**	(a)	**91.**	(c)
92.	(c)	**93.**	(b)	**94.**	(a)	**95.**	(c)	**96.**	(c)	**97.**	(b)	**98.**	(c)
99.	(a)	**100.**	(c)										

Explanations

1(c) Find the magnetic field due to curved part and straight part of the wire and add them.

$$B=\frac{\mu_0 I}{2r}\left(\frac{2\pi-2\theta}{2\pi}\right)+\frac{\mu_0 I}{4\pi r\cos\theta}[2\sin\theta]$$

2(b) Input Power $= 2400\times\frac{5}{4} = 3000\,W\, I_p = \frac{20}{2} = 10A$

$$V_P = \frac{3000}{10} = 300\,V$$

3(a) $R_{AB} = \frac{(2.5\times R)}{3.5R} = \frac{5}{7}R$

4(c)

5(a)

6(c)

7(a)

8(b) $t = 3\tau = 3RC$

9(d) Spring constant $k \propto \frac{1}{\ell}$

10(c)

11(b)

12(b)

13(a) $R_{net} = \frac{18\times 9}{18+9}+12 = 18\,\Omega\,; I = \frac{12}{18} = \frac{2}{3}A$

14(b) $\rho \propto \frac{1}{p}$

15(a)

16(d)

17(a)

18(d) $\lambda = \frac{1240}{2.55} = 487\,nm$

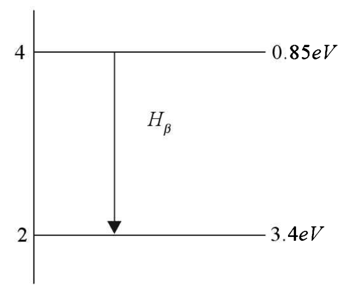


Fig. 19

19(a) Req = 9119 = 4.5 Ω

20(d) $V_{BA} = 6\times\frac{4}{4+2} = 4V;\, V_{AB} = -V_{BA}$

21(b)

22(a)

23(a) $E = -\left[\frac{\hat{i}\partial}{\partial x}+\frac{\hat{j}\partial}{\partial y}+\hat{k}\frac{\partial}{\partial z}\right]V$

24(c)

25(d)

26(b) $200\sqrt{2}\sin 100\pi t$

27(a) $f = \frac{1}{2\pi\sqrt{LVC}} = \frac{1}{2\pi\sqrt{2\times 10^{-9}}} = \frac{10^3}{2\sqrt{2}}$

28(c) $\frac{1}{f'} = \frac{2}{f}+\frac{1}{f_M} = 2\frac{(\mu-1)}{R}+\frac{2}{R} = \frac{2\mu}{R}$

$$\frac{f'}{f} = \frac{(\mu-1)}{2\mu}$$

29(d) $802-2(400) = 2$

30(d) $kx = qE$ or $x = \frac{qE}{k}$

31(b) $\lambda = \frac{1240\times 10^{-9}}{20\times 10^{+3}} = 0.62A°$

32(c) $I = \frac{8(1.5)-2(2\times 1.5)}{6+8(.75)} = 0.5A$

33(c)

34(d) $\frac{880}{V(1+1.2\times10^{-3}T)} = \frac{900}{V(1+1.5\times10^{-3}T)}$

or $240 \times 10^{-3} T = 20$ or $T = 83.3°$ C

35(d) $V_H = 4V_0$

36(c)

37(c)

38(d)

39(d)

40(a)

41(b)

42(d) PV^{γ} = const $\frac{\Delta P}{P} = \frac{\gamma\Delta V}{V}$ or $\frac{\Delta r}{V} = \frac{.5}{1.4}$

43(b)

44(a) $\frac{40-16}{36-16} = \frac{36-16}{T-16}$ or $T-16 = \frac{400}{24} = 16.67°$C ;

$T = 32.67°$ C

45(d)

46(b)

47(a) $T = 5g = 50N$

48(a) $\tan\beta = \frac{8}{5}$ or $\beta = \tan^{-1}(1.6)$ North of vertical

49(b)

50(b) $y = \frac{F}{k}$; $\frac{1}{2}ky^2_{max} = F.y$ or $y_{max} = \frac{2F}{k} = 2y$

51(c) $Mk^2 = \frac{2}{3}MR^2 + MR^2$ or $k = \sqrt{\frac{5}{3}}R$

52(c) $f_{open} = \frac{v}{2\ell}$; $f_{closed} = \frac{v}{4(\ell/2)} = f_{open}$

53(b)

54(b)

55(b)

56(c)

57(d)

58(b)

59(b) Continuous X-ray is called Bremsstrahlung radiation

60(d)

61(b)

62(c) $\varepsilon_0 E^2$ represents energy density

63(d)

64(b) $v^2 - u^2 = 2(g)h$, $v^2_{HP} = 6gr = 4gr = 2gr$

65(a) $Cp = Cv + R = \frac{5}{2}R + R = \frac{7}{2}R$

66(a) Use $\frac{1}{\mu} = \frac{1}{M_1} + \frac{1}{M_2}$

67(a) $V_{AB} = \frac{10}{25}\times10 = 4V$

68(a) $\mu = \tan 60 = \sqrt{3} = \frac{1}{\sin C}$ or $C = \sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$

69(c) $I = \frac{MR^2}{4} + MR^2 = \frac{5}{4}MR^2$

70(b)

71(d)

72(b) $\beta = \frac{\alpha}{1-\alpha}$

73(b) $\sqrt{f} \propto (z-a)$

74(c) $\frac{E_o}{c} = B_o$

75(c)

76(a)

77(a) emf $\varepsilon = \alpha T + \beta T^2$

78(d)

79(a)

80(a)

81(a)

82(c)

83(c)

84(b) Distance from centre is $R-x$. Then $\frac{F}{G} = \frac{GM(R-x)}{R^3}$

85(d) At a slot of 2° each

86(b) Use $r_n = \frac{n^2}{z}r_B$

87(b)

88(a) $\chi = \mu_r - 1 = 5$; $\mu_r = \frac{96}{16} = 6$

89 (a)

90(a) $\frac{v_1}{v_2} = \frac{\sqrt{h\upsilon - h\upsilon_{01}}}{\sqrt{h\upsilon - h\upsilon_{02}}} = \sqrt{\frac{4-1.0}{4-2.5}} = \sqrt{2}$

91(c)

92(c) $\frac{I_{max}}{I_{min}} = \frac{(1+2/3)^2}{(1-2/3)^2} = 25$

93(b) $a = \sqrt{a_r^2 + a_t^2} = \sqrt{1^2+1^2} = \sqrt{2}ms^{-2}$

94(a)

95(c)

96(c) $v_1 = v_2$ or $r_1\omega_1 = r_2\omega_2$ or $\omega_2 = \frac{r_1}{r_2}\omega_1$

97 (b)

98(c) $J = \frac{I}{a^2} = \frac{2}{4\times3\times10^{-4}} = \frac{2\times10^4}{12}$

$= 1.66 \times 10^3$ A/m²

99(a)

100(c)

MODEL TEST PAPER 2

1. The resistance between A and B is

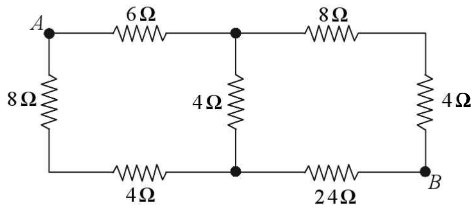


Fig. 1

(a) 12 Ω (b) 11 Ω

(c) 10 Ω (d) none of these

2. For what value of X the reading in ammeter will be zero

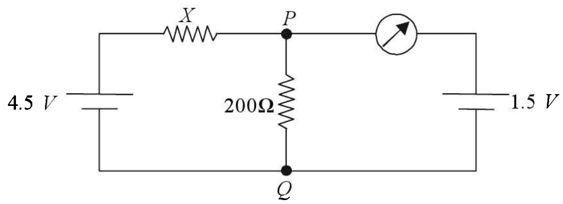


Fig. 2

(a) 400 Ω (b) 350 Ω

(c) 200 Ω (d) none of these

3. A 10 V voltmeter is to be made from a 100 μA de-Arsenval moment and internal resistance 100 Ω. The resistance to be connected in series is nearly

(a) $10^3\,\Omega$ (b) $10^5\,\Omega$

(c) $10^4\,\Omega$ (d) $9 \times 10^4\,\Omega$

4. The dimensional formula for potential difference is

(a) $[ML^2T^{-2}A^{-1}]$ (b) $MLT^{-3}A^{-1}$

(c) $MLT^{-3}A$ (d) $ML^2T^{-3}A^{-1}$

5. A ball of mass m and charge q is suspended with a silk thread of length l when an electric field is switched on then the silk thread makes an angle of 37° with the vertical then electric field is

(a) $\dfrac{mg}{q}$ (b) $\dfrac{3\,mg}{4\,q}$

(c) $\dfrac{2\,mg}{3q}$ (d) $\dfrac{mg}{3q}$

6. Two carnot engines work between temperatures 927°C and θ, θ and 27°C such that their effciencies are equal, then θ will be

(a) 500 °C (b) 327 °C

(c) 600 °C (d) none of these

7. Lapse rate is relaled to

(a) radiation (b) conduction

(c) convection (d) all of (a), (b) and (c)

8. The coulomb forces become ineffective if the distance is less than

(a) 1 mm (b) 1 μm

(c) 1 nm (d) 1 fm

9. 2 g of hydrogen is heated by 10° C at constant pressure. The amount of heat required is

(a) 207.5 J (b) 290.5 J

(c) 415 J (d) 581 J

10. C_V for an ideal gas in an isothermal process is

(a) zero (b) finite but not zero

(c) infinite (d) none of these

11. Force between the plates of a parallel plate capacitor having charge Q, area of plates A, separation between plates d, is

(a) $\dfrac{Q^2}{A}$ (b) $\dfrac{Q^2}{A\varepsilon_0}$

(c) $\dfrac{Q^2}{2A\varepsilon_0}$ (d) $\dfrac{2Q^2}{A\varepsilon_0}$

12. If r_1 and r_2 are radii of two concentric hollow spheres ($r_1 < r_2$) and a dielectric of constant k is inserted bteween the two layers then capacity of the system is

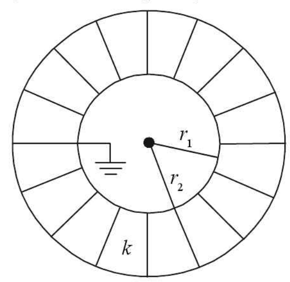


Fig. 3

(a) $\dfrac{4\pi\varepsilon_0 r_1 r_2 k}{r_2 - r_1}$ (b) $\dfrac{4\pi\varepsilon_0 r_1 r_2}{r_2 - r_1}$

(c) $4\pi\varepsilon_0 r_2$ (d) $\dfrac{4\pi\varepsilon_0 k r_1 r_2}{r_2 - r_1} + 4\pi\varepsilon_0 r_2$

13. The uniform electric field is obtained due to

(a) long wire (b) cylinder

(c) sphere (d) long plate

14. Electric field inside a nonconducting sphere is proportional to

(a) R (b) R^{-2}

(c) R^{-1} (d) R^{-3}

15. Two charged particles are 1cm apart. The minimum possible force between them is

(a) 2.3×10^{-24} (b) 2.3×10^{-23}

(c) 2.3×10^{-25} (d) cannot be predicted

16. The amount of work done in assembling 3 charged particles on the vertices of an equilateral triangle as shown in Fig. 4 is

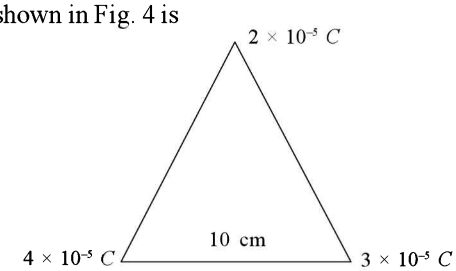


Fig. 4

(a) 234 J (b) 23.4 J
(c) 2.34 J (d) none of these

17. Electric potential energy of a uniformly charged sphere is ....electric potential energy of a uniformly charged thin spherical shell.

(a) greater than (b) equal to
(c) less than (d) cannot be predicted

18. A charge Q is placed at the centre of an imaginary hemispherical surface. The flux through the surface of hemisphere is

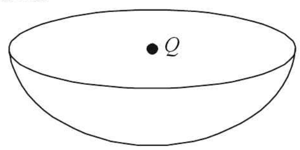


Fig. 5

(a) $\frac{Q}{\varepsilon_0}$ (b) $\frac{Q}{2\varepsilon_0}$
(c) $\frac{Q}{3\varepsilon_0}$ (d) $\frac{3Q}{4\varepsilon_0}$

19. The capacitance of a cylindrical capacitor of radius r_1 and r_2 ($r_2 > r_1$) and length l is

(a) $2\pi\varepsilon_0 l$ (b) $2\pi\varepsilon_0 (r_2 - r_1)$
(c) $\frac{2\pi\varepsilon_0 l}{\log_e \frac{r_2}{r_1}}$ (d) $2\pi\varepsilon_0 l \log_e \frac{r_2}{r_1}$

20. The charge on C_1 in the circuit shown is

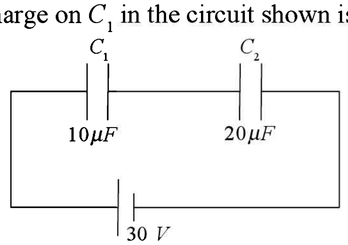


Fig. 6

(a) 200 μC (b) 300 μC
(c) 400 μC (d) 100 μC

21. Tuner capacitor is........capacitor.

(a) a fixed (b) a variable
(c) an electrolytic (d) none of these

22. The ratio of surface charge density of two spheres in the given Fig. 7 is

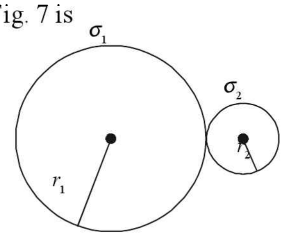


Fig. 7

(a) $\frac{\sigma_1}{\sigma_2} = \frac{r_1}{r_2}$ (b) $\frac{\sigma_1}{\sigma_2} = \frac{r_2}{r_1}$
(c) $\frac{\sigma_1}{\sigma_2} = \frac{r_2 - r_2}{r_2 + r_1}$ (d) none of these

23. Fig. 8 shows a parallel plate capacitor having square plates of edge a and separation 'd'. The gap is filled with a dielectric of constaut $k = k_o + \alpha x$ varying from left to right. The capacity of the capacitance is

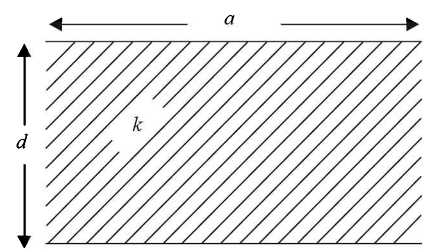


Fig. 8

(a) $\frac{\varepsilon_0 a^2}{d} [k_0 + \alpha a]$ (b) $\frac{\varepsilon_0 a^2}{2d} [k_0 + \alpha a]$
(c) $\frac{\varepsilon_0 a^2}{d} [k_0 + \alpha \frac{a^2}{2}]$ (d) $\frac{\varepsilon_0 a^2}{d} \left[k_o + \frac{\alpha a}{2}\right]$

24. J = σE describes

(a) Gauss law (b) Faraday's law
(c) Ohm's law (d) Kirchhoff's law

25. Open circuit voltage can be determined

(a) experimentally (b) theoretically only
(c) both (a) and (b) (d) cannot be determined.

26. The ammeter shown in fig. 9 consists of a 480Ω coil ammeter to a 20Ω shunt. The reading of ammeter is

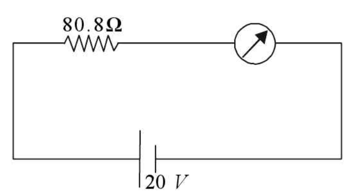


Fig. 9

(a) 0.2 A (b) 0.1 A
(c) 0.125 A (d) none of these

27. A 10Ω wire is bent in the form of a semicircle with diameter. The resistance across the diameter is nearly

(a) 2.4 Ω (b) 3.3 Ω
(c) 10 Ω (d) 4.28 Ω

28. A 100W bulb and a 25 W bulb both rated at 220 V are connected in series with a 220 V source ..... watt bulb glows brighter

(a) 100 W (b) 25 W

(c) both will glow equally bright

(d) none of these

29. In the Thomson effect constant σ has dimensions of

(a) $MLT^{-2}K^{-1}$ (b) $MLT^{-3}A^{-1}$

(c) $MLT^{-2}A^{-1}K^{-1}$ (d) $MLT^{-3}A^{-1}$

30. A wire is bent into an equilateral triangle of side l. It carries a current i. If a magnetic field B is applied perpendicular to the plane of wire cage, then net force acting on the triangle is

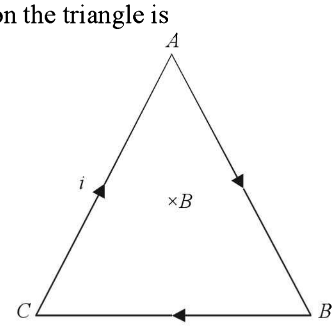


Fig. 10

(a) zero (b) ilB

(c) 3 ilB (d) $\sqrt{2}\, ilB$

31. A proton and an α – particle of equal KE enter in a magnetic field. The ratio of the radii described by them is

(a) $\frac{r_p}{r_\alpha} = \frac{1}{2}$ (b) $\frac{r_p}{r_\alpha} = 1$

(c) $\frac{r_p}{r_\alpha} = \frac{1}{\sqrt{2}}$ (d) $\frac{r_p}{r_\alpha} = \frac{1}{4}$

32. A long wire carrying current i is bent into a semicircle of radius d on one end as shown. The magnetic field at the centre C is

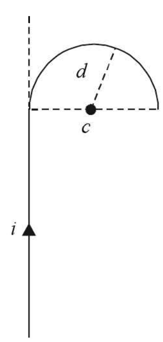


Fig. 11

(a) $\frac{\mu i}{4\pi d}[1+\pi]$ (b) $\frac{\mu_o i}{4\pi d}$

(c) $\frac{\mu_o i}{4\pi d}[1+2\pi]$ (d) none of these

33. Current in the load $R_L = 10\,\Omega$ in the given circuit is

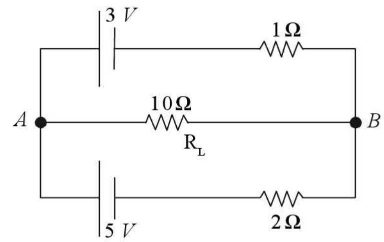


Fig. 12

(a) $\frac{8}{13}A$ (b) $\frac{11}{32}A$

(c) $\frac{7}{29}A$ (d) none of these

34. If two capacitors 10μ F and 5μ F are charged to 5 V and 9 V respectively then their common potential is

(a) 6.3 V (b) 7V

(c) 9V (d) 5 V

35. A line voltage has 5 A rating. If 100 W bulbs are connected in parallel, then the maximum number of bulbs to be connected in parallel are

(a) 5 (b) 7

(c) 9 (d) 11

36. Thermistors are most alike

(a) metals (b) insulators

(c) superconductors (d) semiconductors

37. A capacitor discharges to 95% in ...... time in a series RC circuit.

(a) RC (b) 2RC

(c) 3RC (d) 5RC

38. The resistance between a and b in the given circuit is

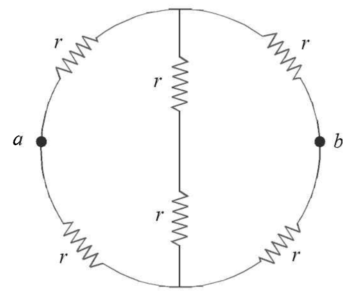


Fig. 13

(a) r (b) 2 r

(c) $\frac{3}{2}r$ (d) none of these

39. If the length of wire AB is 40 cm then point X so that galvanometer gives zero deflection is

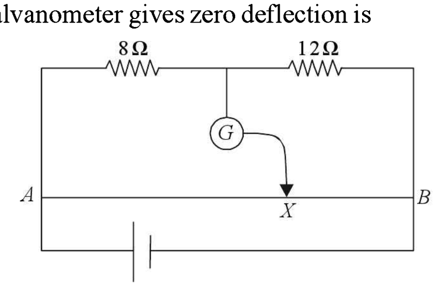


Fig. 14

(a) 16 cm from A (b) 16 cm from B
(c) 24cm from A (d) none of these

40. The charge on capacitor C_1 at any instant t is when the switch S is made on.

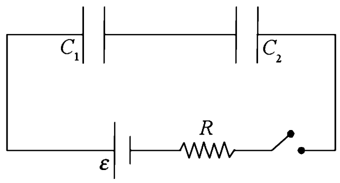


Fig. 15

(a) $\frac{C_1C_2}{C_1+C_2}\varepsilon$ (b) $\frac{C_1C_2}{C_1+C_2}\left[1-e^{\frac{-t}{R\left(\frac{C_1C_2}{C_1+C_2}\right)}}\right]\varepsilon$

(c) $(C_1+C_2)\left[1-e^{-t}\frac{RC_1C_2}{C_1+C_2}\right]\varepsilon$ (d) none of these

41. The current required to deposit 0.972 g Cr in 3 hours if ECE of Cr is 0.00018 g/C is
(a) 0.25 A (b) 0.5 A
(c) 1.5 A (d) 30 A

42. An inductance stores energy in ......form.
(a) electric (b) magnetic
(c) mechanical (d) potential

43. The peak value of an ac is 5A and its frequency is 60Hz. After what time the current reaches its peak value starting from zero.
(a) $\frac{1}{60}s$ (b) $\frac{1}{120}s$
(c) $\frac{1}{180}s$ (d) $\frac{1}{240}s$

44. Turn ratio in a transformer corresponds to
(a) $\frac{\text{number of turns in primary}}{\text{number of turns in secondary}}$
(b) $\frac{\text{number of turns in secondary}}{\text{number of turns the primary}}$
(c) either (a) or (b)
(d) none of these

45. If self inductances of two coils are 25μH and 36μH, then maximum value of mutual inductance between them is
(a) 25 μH (b) 30 μH
(c) 36 μH (d) 28 μH

46. When object and screen are fixed and lens is varied to form the image. The size of the two images at two different positions of lens are16cm and 4cm high. The size of object is
(a) 8cm (b) 12cm
(c) 64cm (b) 4cm

47. When plate potental varies from 40 to 45V in a triode value, the plate current varies from 10 to 12.5 mA. The dynamic plate resistance is
(a) 2Ω (b) 20Ω
(b) 200Ω (d) 2000Ω

48. Stopping potential decreases if ...... of incident beam on a photocathode decreases.
(a) intensity (b) wavelength
(c) amplitude (d) frequency

49. The momentum of a photon of energy $h\upsilon$ is
(a) zero (b) $2\frac{h\upsilon}{c}$
(c) $h\frac{\upsilon}{c}$ (d) $\frac{h\upsilon}{2c}$

50. A UV lamp emits 280nm wavelength and is incident on a material having work function 2.5eV. The stopping potential is
(a) 4.4 V (b) 2.5 V
(c) 2.8 V (d) 1.9 V

51. The radius of 3rd excited state of hydrogen atom is
(a) 8.48 nm (b) 8.48 A°
(c) 8.48 μm (d) 8.48 pm

52. Simulated emission is a term used in
(a) photoelectric effect (b) incandescent lamp
(c) vacuum tube (d) laser

53. To study crystal structure by Lau's method, we employ
(a) γ -rays (b) light rays
(c) uv rays (d) X -rays

54. The higher the satellite.........is the time period of rotation.
(a) more (b) less
(c) does not depend upon height
(d) none of these

55. The weight of a 1kg body in a geostationary satellite is
(a) zero (b) 10 N
(c) >10N (d) <10N

56. The logic gate used in addition of two bits is
(a) OR (b) AND
(c) NOR (d) XOR

57. An ideal diode will have resistance in forward bias
(a) zero (b) 20 Ω
(c) 200 Ω (d) 20 kΩ

58. In breeder reactor .....fuel is produced.
(a) U^{236} (b) U^{239}
(c) Pu^{239} (d) Th^{232}

59. The frequency of output of a full wave rectifier is.... if frequency of input signal is 50Hz.
(a) 50 Hz (b) 25 Hz
(c) 100 Hz (d) 175 Hz

60. A point mass slides from a height of $6r$ on an inclined plane which ends into a loop of radius r. The velocity at the top of the loop is

(a) $\sqrt{6rg}$ (b) $\sqrt{rg}$

(c) $\sqrt{5rg}$ (d) $\sqrt{8rg}$

61. A pendulum has amplitude of vibration 10 cm and time period $2s$, then the velociy at 2cm from extreme position is

(a) 8cm s^{-1} (b) 6cm s^{-1}

(c) 8π cm s^{-1} (d) 6π cm s^{-1}

62. If $x = 6t^2 + 4t$ is the equation of motion of a particle where x is in m and t is in seconds then initial velocity of the particle is

(a) 4 ms^{-1} (b) 6 ms^{-1}

(c) 12 ms^{-1} (d) zero

63. The velocity of P with respect to Q from the diagram is

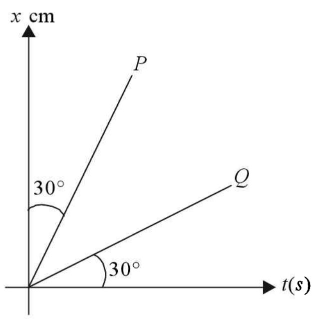


Fig. 16

(a) $\frac{V_Q}{3}$ (b) $\frac{V_Q}{\sqrt{3}}$

(c) $2V_Q$ (d) $3V_Q$

64. If a helicopter on flood relief mission drops a food packet from a height of 1km while it is moving horizontally with a speed 100 ms^{-1}, then food packet falls at a horizontal distance

(a) $10\sqrt{2}$ m (b) $100\sqrt{2}$ m

(c) $\sqrt{2}$ km (d) $1000\sqrt{2}$ km

65. The maximum value of time period of a pendulum on the earth is

(a) infinity (b) 82.4 hour

(c) 82.4 min (d) 82.4s

66. A particle of mass 0.2mg is charged to 500 electrons. The electric field required to keep it in equilibrium is

(a) 2.5×10^{12}N/C (b) 5×10^{12}N/C

(c) 5×10^{15}N/C (d) 2.5×10^{15} N/C

67. An equiconvex lens of focal length f is dipped in water. Its focal length in water is

(a) $2f$ (b) $2.5f$

(c) $3f$ (d) $4f$

68. An equiconvex lens has focal length f and radius R. If it is made of a material of refractive index 1.5 then

(a) $f = \frac{R}{2}$ (b) $f = R$

(c) $f = \frac{3}{2}$ (d) $f = \frac{2}{3}R$

69. A prism abc shown in Fig. 17 deviates a parallel beam of light by

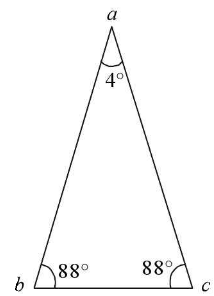


Fig. 17

(a) 2 rad (b) 4 rad

(c) 44° (d) 2°

70. A wave of amplitude 3cm is superimposed with another wave of amplitude 2cm, then the ratio of maximum to minimum intensity is

(a) 5 (b) 25

(c) $\sqrt{5}$ (d) none of these

71. If temperature on a day is 34°C the velocity of sound wave on that day will be nearly

(a) 332 ms^{-1} (b) 353ms^{-1}

(c) 370 ms^{-1} (d) cannot be predicted

72. 20g of ice at -5°C is mixed with 40g of water of 40°C. The temperature of the mixture will be

(a) – 1°C (b) zero

(c) + 1°C (d) + 18°C

73. The moment of inertia of an annular disc of radii r_1 and r_2 and mass m is

(a) $m\left(r_1^2 + r_1^2\right)$ (b) $\frac{m}{2}\left(r_1^2 + r_1^2\right)$

(c) $m\left(r_1^2 - r_1^2\right)$ (d) $\frac{m}{2}\left(r_1^2 - r_1^2\right)$

74. A train when 1 km away from a station blows a whistle of 400 Hz. If it is approaching with a velocity 20 ms^{-1} then frequency as heard by the man is nearly

(a) 456.4 Hz (b) 426.4 Hz

(c) 446.3 Hz (d) none of these

75. If the train moving with a speed 20ms^{-1} due east turns north in 3 s then its acceleration is

(a) zero (b) $20\sqrt{2}$ ms^{-1}

(c) $\frac{20}{3}$ ms^{-2} (d) $\frac{20}{3}\sqrt{2}$ ms^{-2}

76. A boat is 10 m long. A man of 60 kg takes 2 seconds to travel from one end to another. Find the distance moved by the boat. Given mass of the boat =300 kg

(a) 10 m (b) $\frac{10}{3}$ m

(c) $\frac{5}{3}$ m (d) zero

77. A bubble of radius 2 cm if taken 70 m deep in water will have a radius

(a) 1 cm (b) 2 cm

(c) 1 mm (d) 2 mm

78. A concave mirror of focal length f is taken. Few drops of water are sprinkled on it. The object needle and its image will coincide at

(a) $2f$ (b) $\frac{3}{2}f$

(c) f (d) $\frac{2}{3}f$

79. If a ball falls from a height 80 m and 20% of its energy is lost on impact with the floor, it will rise to a height

(a) 60 m (b) 64 m

(c) 68 m (d) 72 m

80. Find the acceleration of the blocks in the figure shown. Given ($\beta > \alpha$).

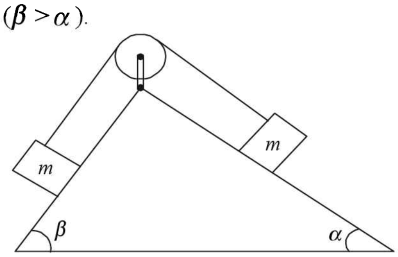


Fig. 18

(a) $\frac{g}{2}$ (b) $\frac{g}{2}\sin\beta$

(c) $\frac{g}{2}\sin\alpha$ (d) $\frac{g}{2}(\sin\beta - \sin\alpha)$

81. A particle of mass 2 g is placed at the edge of the blade of a 48 inch fan. If fan rotates at 600 rpm, then force experienced by the particle is

(a) 4.8 N (b) 19.2 N

(c) 3.6 N (d) 9.6 N

82. If $x = at^2$ and $y = bt^2$ are the equations of motion of a particle then its velocity will be

(a) $2t\sqrt{a^2+b^2}$ (b) $2t(a+b)$

(c) $2t^2(a+b)$ (d) none of these

83. A block of mass m is placed on an inclinced plane of inclination θ. The maximum and mimimum forces respectively required to just move the block are

(a) mg sinθ, 0 (b) mg sinθ, mg cosθ

(c) mg, mg sinθ (d) mg, mg cos θ

84. If $y = 10\sin(10^3\pi t - 0.5x)$ where x is in meters and t in seconds, then find the frequency of tuning fork which is in unison with the wave

(a) 10^3 Hz (b) 500 Hz

(c) 250 Hz (d) 125 Hz

85. If the tuning forks of frequency 400Hz, 401Hz and 402Hz are set into vibrations, then frequency of beats produced is

(a) 2 Hz (b) 1 Hz

(c) 3 Hz (d) no beats are produced

86. A candle is lit if the intensity at 0.5m away is I_o. The intensity 2 m away will be

(a) $\frac{I_o}{4}$ (b) $\frac{I_o}{8}$

(c) $\frac{I_o}{16}$ (d) $\frac{I_o}{2}$

87. The maximum audible syllable frequency is

(a) 20 Hz (b) 10 Hz

(c) 5 Hz (d) 20 khz

88. Three stars have magnitudes –4, –2 and +2. The brightest star has magnitude

(a) 2 (b) – 2

(c) – 4 (d) none of these

89. The apparent volumetric expansion coefficient of glycerine is 15×10^{-4}/K and if linear expansion cofficient of glass is 6×10^{-5} K^{-1} then real expansion coefficient of expansion of glycerine is

(a) 24×10^{-4}/K^{-1} (b) 22.1×10^{-4}/K^{-1}

(c) 19.8×10^{-4}/K^{-1} (d) 16.8×10^{-4}/K^{-1}

90. If 10th fringe of 700 nm coincides with 14th fringe of an unknown wavelength in Young's double slit experiment, then unknown wavelength is

(a) 500 nm (b) 550 nm

(c) 400 nm (d) 470 nm

91. If a block of mass 2 kg moves according to equation $x = \frac{t^3}{2}$r then work done by the particle in 5 seconds is

(a) 3125 J (b) 1562.5 J

(c) 520 J (d) none of these

92. A radioactive element has half life 2.5 hours. Its amount left after 7.5 hours is

(a) $\frac{1}{4}$ (b) $\frac{1}{6}$

(c) $\frac{1}{8}$ (d) none of these

93. A pendulum of length l suspended from the ceiling of a car. If the car is accelerated at rate a ms^{-2} then time period of oscillation is

(a) $T = 2\pi\sqrt{\dfrac{l}{g}}$ (b) $T = 2\pi\sqrt{\dfrac{l}{(g^2+a^2)^{\frac{1}{2}}}}$

(b) $T = 2\pi\sqrt{\dfrac{l}{(g+a)}}$ (d) none of these

94. Three particles are placed at the vertices of an equilateral triangle of length l and they move towards each other with a velocity V. Then time in which they meet is

(a) $\dfrac{2l}{3v}$ (b) $\dfrac{l}{v}$

(c) $\dfrac{3l}{2v}$ (d) none of these

95. A toy train consists of 3 identical compartments A, B and C. It is being pulled by a constant force F. The ratio of the tensions in the string AB to BC is

(a) 3 : 1 (b) 2 : 1

(c) 1 : 1 (d) 1 : 2

96. A particle drops from a height 45 m. The distance travelled by it in the last second is (take g = 10m5^2).

(a) 10 m (a) 4.5 m

(a) 25 m (d) none of these

97. An athlete runs with a velocity 10 ms^{-1}. The longest jump he can have is

(a) 10 m (b) 20 m

(c) 15 m (d) none of these

98. A lift is moving up with a velocity 10 ms^{-1}. A man of 60 kg is on this lift. The apparent weight on the lift will be

(a) 600 N (b) 900 N

(c) 750 N (d) none of these

99. Centripetal force and its reaction act on

(a) same body (b) different bodies

(c) either (a) or (b) (d) none of these

100. A mercury drop has radius 1cm—surface tension T = .036 N/m. The excess pressure is

(a) 3.6 Pa (b) 7.2 Pa

(c) 10.8 Pa (d) none of these

Answers

1.	(a)	**2.**	(a)	**3.**	(b)	**4.**	(d)	**5.**	(b)	**6.**	(b)	**7.**	(c)
8.	(d)	**9.**	(b)	**10.**	(c)	**11.**	(c)	**12.**	(d)	**13.**	(d)	**14.**	(d)
15.	(a)	**16.**	(a)	**17.**	(b)	**18.**	(b)	**19.**	(c)	**20.**	(a)	**21.**	(b)
22.	(b)	**23.**	(d)	**24.**	(c)	**25.**	(c)	**26.**	(a)	**27.**	(a)	**28.**	(b)
29.	(d)	**30.**	(a)	**31.**	(b)	**32.**	(a)	**33.**	(b)	**34.**	(a)	**35.**	(d)
36.	(d)	**37.**	(c)	**38.**	(a)	**39.**	(a)	**40.**	(b)	**41.**	(b)	**42.**	(b)
43.	(d)	**44.**	(a)	**45.**	(b)	**46.**	(a)	**47.**	(d)	**48.**	(d)	**49.**	(c)
50.	(d)	**51.**	(b)	**52.**	(d)	**53.**	(d)	**54.**	(a)	**55.**	(a)	**56.**	(d)
57.	(a)	**58.**	(c)	**59.**	(c)	**60.**	(d)	**61.**	(d)	**62.**	(a)	**63.**	(c)
64.	(c)	**65.**	(c)	**66.**	(a)	**67.**	(d)	**68.**	(b)	**69.**	(d)	**70.**	(b)
71.	(b)	**72.**	(b)	**73.**	(b)	**74.**	(a)	**75.**	(d)	**76.**	(c)	**77.**	(a)
78.	(b)	**79.**	(b)	**80.**	(d)	**81.**	(a)	**82.**	(a)	**83.**	(a)	**84.**	(b)
85.	(b)	**86.**	(c)	**87.**	(d)	**88.**	(c)	**89.**	(d)	**90.**	(a)	**91.**	(b)
92.	(c)	**93.**	(b)	**94.**	(a)	**95.**	(b)	**96.**	(c)	**97.**	(a)	**98.**	(a)
99.	(b)	**100.**	(b)										

Explanations

1(a) Use Wheatstone bridge approach

2(a) If potential drop across $PQ = 1.5\ V$

Then $I = 0$ or $1.5 = \dfrac{4.5\times200}{x+200}$

3(b) $10 = 100\times10^{-6}(R+100)$

or $R = 10^5\Omega$

4(d)

5(b) $qE = T\sin 37$ and $mg = T\cos 37$

$\tan 37 = \dfrac{qE}{mg}$ or $E = \dfrac{3mg}{4q}$

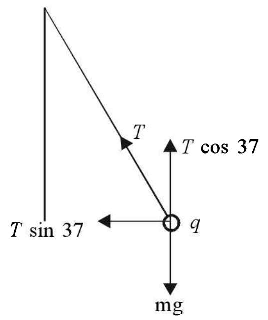


Fig. 19

6(b) $\frac{1200}{\theta} = \frac{\theta}{300}$ or $\theta = 600$K or θ = 327°C

7(c)

8(d)

9(b) $\Delta Q = nc_p\Delta\theta = 1 \times \frac{7}{2} \times 8.3 \times 10 = \frac{581}{2}$ J = 290.5 J

10(c)

11(c) $F = \frac{Q\sigma}{2\in_o} = \frac{Q^{2.}}{2A\in_o}$

12(d)

13(d)

14(d)

15(a) Take electronic charges i.e. $F = \frac{(1.6\times10^{-19})^2\times9\times10^9}{(10^{-2})^2}$

16(a) Take *PE*.

17(b)

18(b)

19(c)

20(a)

21(b)

22(b)

23(d) Consider a strip of width dx at a distance *x* from the left

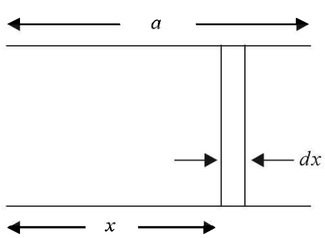


Fig. 20

$$dC = \frac{(k_o + \alpha x)\varepsilon_o a.dx}{d}$$

$$C = \frac{\in_o a}{d}\int_0^a (k_o + \alpha x)dx = \frac{\varepsilon_o a^2}{d}[k_o + \frac{\alpha a}{2}]$$

24(c)

25(c)

26(a)

27(a) Let *r* be resistance/radius

$\pi r + 2r = 10$

$$r = \frac{10}{\pi + 2}$$

$$r_{eq} = \frac{2r + \pi r}{10}$$

$$= \frac{2\pi}{10}\left(\frac{10}{\pi+2}\right)^2 = 2.42\Omega$$

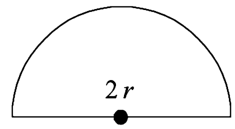


Fig. 21

28(b) The bulb of lower rating glows brighter

29(d)

30(a) Because of triangle law.

31(b) $\frac{r_p}{r_\alpha} = \frac{\sqrt{(2KE)M_p}/q_p}{\sqrt{(2KE)M_\alpha}/q_\alpha} = \frac{1}{1}$

32(a) $B = \frac{\mu_o I}{4\pi d} + \frac{\mu_o I}{4d} = \frac{\mu_o I}{4\pi d}(1+\pi)$

33(b) $i = \frac{(3\times2+5\times1)/(1+2)}{10+\frac{1\times2}{1+2}} = \frac{11}{32}A$

34(a) $V_{com} = \frac{C_1V_1 + C_2V_2}{C_1 + C_2} = \frac{10\times5+5\times9}{10+5} = 6.3\,V$

35(d) $n = \frac{220\times5}{100} = 11$

36(d)

37(c)

38(a) It is a wheatstone bridge with equal resistors.

39(a) $\frac{8}{x} = \frac{12}{40-x}$

40(b) Use C_1 and C_2 in series

$$Q = \frac{\varepsilon C_1C_2}{C_1+C_2}(1 - e^{-t/\tau})$$

41(b) Use $m = z\,it$

42(b)

43(d)

44(a)

45(b) $M = \sqrt{L_1L_2}$

46(a) $O = \sqrt{I_1I_2} = \sqrt{16\times4} = 8$ cm

47(d) $r_p = \frac{\Delta V_p}{\Delta I_p} = \frac{45-40}{(12.5-10)\times 10^{-3}} = 2000\,\Omega$

48(d)

49(c) $p = E/c = h\upsilon/c$

50(d)

51(b) $r_n = n^2 r_B = 16(.53) = 8.48$A°

52(d)

53(d)

54(a) $T^2 \propto R^3$

55(a)

56(d)

57(a)

58(c)

59(c) $f_{\text{output}} = 2 f_{\text{input}}$

60(d) $\frac{Mv^2}{2} = Mg\,(4r)$ or $v = \sqrt{8rg}$

61(d) $v = \frac{2\pi}{T}\sqrt{x_0^2 - x^2} = \frac{2\pi}{2}\sqrt{10^2 - 8^2} = 6\pi$ cms^{-1}

62(a) $\text{v} = \frac{dx}{dt}$ $t = 0 = 4$ ms^{-1}

63(c) $V_{p/V2} = \frac{\tan 60}{\tan 30} = 3$; $V_{PQ} = V_P - V_Q = 2V_Q$

64(c) $t = \sqrt{\frac{2h}{g}} = 10\sqrt{2}$ sec; $x = 100 \times 10\sqrt{2} = \sqrt{2}\,km$

65(c)

66(a) use $E = \frac{mg}{q}$

67(d)

68(b) $\frac{1}{f} = (\mu - 1)\left[\frac{1}{R} + \frac{1}{R}\right] = 0.5\left(\frac{2}{R}\right) = \frac{1}{R}$

69(d) $(\mu - 1)\,\alpha = S$ or $S = 0.5\,(4°) = 2°$

70(b) $\frac{I_{max}}{I_{min}} = \frac{(3+2)^2}{(3-2)^2} = 25$

71(b) $v\,(T) = 332\sqrt{\frac{307}{273}} = 353$ms^{-1}

72(b) Since even complete ice will not melt.

73(b)

74(a) $f_{ap} = 400 \times \frac{332}{332-20}$

75(d)

76(c) $60 \times 10 = 360\,x$ or $x = \frac{5}{3}$ m

77(a) use $P_1V_1 = P_2V_2$

78(b) $2\,f' = \frac{2f}{4/3} = 3f/2$

79(b) $mg\,h' = 0.8\,mgh$ or $h' = .8\,(80) = 64$m

80(d) $a = \frac{(m_1 \sin\theta_1 - m_2 \sin\theta_2)g}{m_1 + m_2} = \frac{g}{2}(\sin\beta - \sin\alpha)$

81(a) Radius of fan = 24 inch = 60 cm

$F = mrw^2$

82(a) $v_x = \frac{dx}{dt} = 2\text{at}$; $v_y = \frac{dy}{dt} = 2\text{bt}$; $v = 2\text{t}\sqrt{a^2 + b^2}$

83(a)

84(b) $f = \frac{\omega}{2\pi} = \frac{10^3\pi}{2\pi} = 500$ Hz

85(b)

86(c)

87(d)

88(c)

89(d)

90(a)

91(b) $W = \Delta KE = \frac{1}{2}\times 2\left(\frac{9t^4}{4}\right)$

92(c)

93(b)

94(a) $t = \frac{l}{v + v\cos 60} = \frac{2l}{3v}$

95(b)

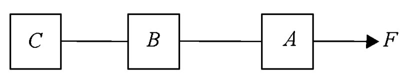


Fig. 21

$a = F/3m$

$T_{AB} = 2ma = \frac{2F}{3}$ and $T_{BC} = ma = \frac{F}{3}$

96(c) $\frac{1}{2}gt^2 = 45$ or $t = 3s$ and $s_{nth} = 5\,(2 \times 3 - 1) = 25$

97(a) $R_{\max} = \frac{V^2}{g}$

98(a) As there is no acceleration of lift.

99(b)

100(b) $\frac{2T}{r} = \frac{2\times .036}{10^{-2}} = 7.2\,Pa$

MODEL TEST PAPER 3

1. The dimensional formula for (μ) permeability of the medium is

(a) $MLT^{-2}A^{-2}$ (b) $ML^{\circ}T^{-2}A^{-1}$

(c) $MLT^{-2}A^{-1}$ (d) $M^{\circ}LT^{-2}A^{-2}$

2. 8g of oxygen is heated by 20°C at constant volume. Then heat energy required is

(a) 10 cal (b) 25 cal

(c) 15 cal (d) 20 cal

3. If the temperature at sea level is 18°C then temperature at a height of 2 km from the sea-level is

(a) 6° C (b) 10° C

(c) 12° C (d) 18° C

4. In a constant volume gas thermometer, the pressure gets doubled when the bulb is immersed in a hot bath from that at NTP. Then temperature of the bath is

(a) 273.14 K (b) 273.14° C

(c) 546.28° C (d) none of these

5. The electric field at the center of the ring of radius r having charge Q is

(a) $\frac{Q}{4\pi\varepsilon_0 r^2}$ (b) zero

(c) $\frac{Q}{4\pi\varepsilon_0 r^3}$ (d) $\frac{Q}{4\pi\varepsilon_0 r^{3/2}}$

6. The device with which uniform electric field is obtained is

(a) a long wire (b) a long plate

(c) a sphere (d) a ring

7. If each capacitor is C then effective capacitance of the circuit is

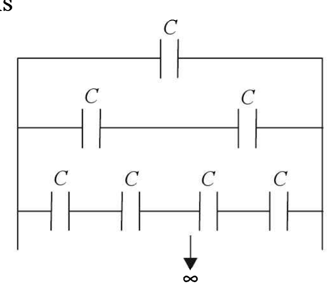


Fig. 1

(a) $\frac{C}{2}$ (b) C

(c) $\frac{3C}{2}$ (d) 2 C

8. 729 small drops each of radius 0.1cm and charge 1nc each coalesce to form a big drop. Then potential on larger drop is

(a) 8100 V (b) 8100 V

(c) 7.29×10^5 V (d) none of these

9. Two identical positive charges (q) each are situated at (0, a) and (0, – a). A charge $-q$ at rest is released from the point (2 a, 0). The charge $-q$ will

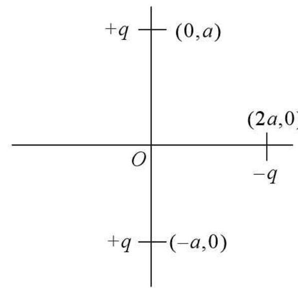


Fig. 2

(a) go to infinite distance

(b) execute SHM

(c) move towards origin and becomes stationary

(d) execute periodic motion but not SHM

10. The intensity of electric field is perpendicular to the initial velocity of proton, then path of proton in this electric field is

(a) circular (b) parabolic

(c) elliptical (d) linear

11. If point XY are short circuited, then resistance across AB is

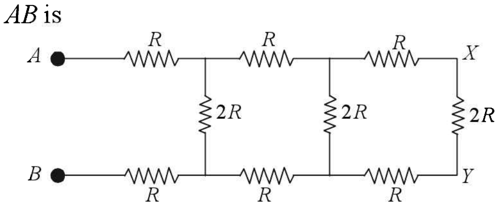


Fig. 3

(a) $\frac{15}{2}R$ (b) $\frac{16}{5}R$

(c) $\frac{3}{2}R$ (d) $\frac{6}{5}R$

12. The potential across AB in the figure shown is

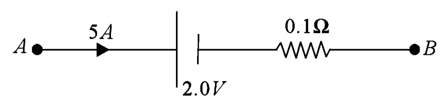


Fig. 4

(a) 2.0 V (b) 2.5 V

(c) 1.5 V (d) none of these

13. In the adjoining circuit, ammeter and voltmeter readings are 2 A and 120 V respectively. If R = 75 Ω then resistance of voltmeter is

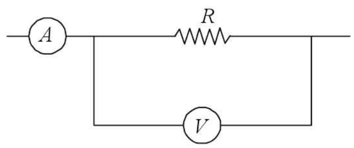


Fig. 5

(a) 300 Ω (b) 200 Ω

(c) 100 Ω (d) 75 Ω

14. The current in the resistance, in branch BD is

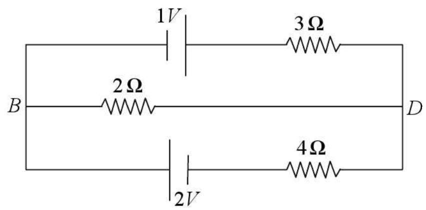


Fig. 6

(a) $-\frac{1}{13}A$ (b) $\frac{2}{13}A$

(c) $-\frac{3}{13}A$ (d) $-\frac{4}{13}A$

15. The following wires are made of the same material. The wire whose resistance is maximum is

(a) 2 mm diameter and × 60 m length

(b) 1 mm diameter and × 40 m length

(c) 2 mm diameter and × 40 m length

(d) 1 mm diameter and × 60 m length

16. The voltmeter shown in the figure reads 18V across 50 Ω resistance. Then resistance of the voltmeter is

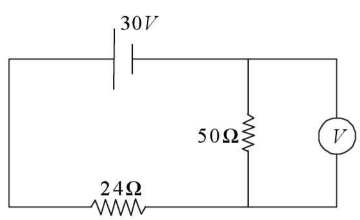


Fig. 7

(a) 50 Ω (b) 90 Ω

(c) 130 Ω (d) none of these

17. Two capacitors 4 μF and 10 μF were connected together after being charged to 10 V and 12 V, respectively. A dielectric of constant 2.5 is added in between plates of 4 μF capacitor before connecting. The charge after connection on this capacitor is

(a) 40 μC (b) 80 μC

c) 60 μC (d) 75 μC

18. The capacitance between A and B is ___μF if each capacitor is 2μF.

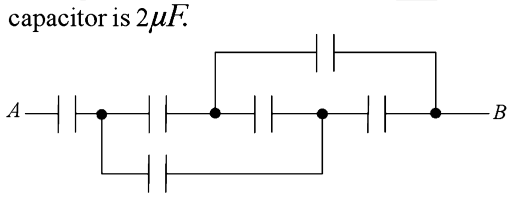


Fig. 8

(a) 2 μF (b) 1 μF

(c) 4 μF (d) $\frac{1}{2}\mu F$

19. The internal resistance of an ideal voltage source is

(a) infinity (b) zero

(c) finite but not zero (d) may have any value

20. The attractive nuclear force between two protons is given by $F = \frac{Ce^{-kr}}{r^2}$ then dimensions of C and k are respectively

(a) MLT^{-1}, L (b) ML^3T^{-2}, L^{-1}

(c) ML^3T^{-2}, L (d) none of these

21. The speed of electron in ground state of hydrogen atom is

(a) 1.4×10^6 ms^{-1} (b) 2.18×10^6 ms^{-1}

(c) 4.2×10^6 ms^{-1} (d) none of these

22. 12 J of work is done against an electric field to take a charge 6 mC from A to B then $V_B - V_A$ is

(a) 200 V (b) 2000 V

(c) 150 V (d) 1500 V

23. Two charged particles each having charge 20μc are brought from infinity to a separation of 10 cm. The increase in electric potential energy is

(a) 3.6 J (b) 36 J

(c) 1.8 J (d) 18 J

24. The electric field at x = 6 cm in the figure shown is

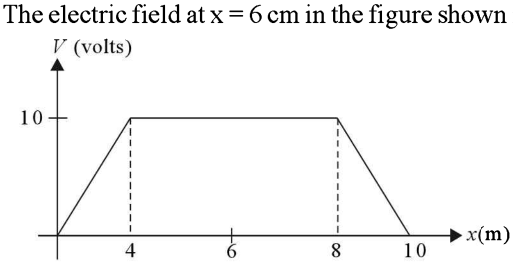


Fig. 9

(a) 1.66 V/m (b) 2.5 V/m

(c) zero (d) none of these

25. At neutral temperature thermoelectric power in a thermo couple is

(a) zero (b) finite but not zero

(c) infinite

(d) depends upon temperature difference between two junctions.

26. If 1A current is passed for half an hour through a silver voltmeter then silver deposited is nearly

(a) 2 g (b) 1g

(c) 0.2 g (d) 0.1g

27. A 1 MeV proton enters a magnetic field of 2 mT. The cyclotron frequency is

(a) $10^5 Hz$ (b) $10^4 Hz$

(c) $\frac{10^4}{\pi} Hz$ (d) $\frac{10^5}{\pi} Hz$

28. *A* wire is bent as shown and carries a current *i*. If a magnetic field *B* is switched on perpendicular to the plane of wire, then force experienced by the wire is

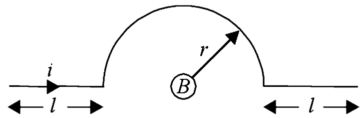


Fig. 10

(a) $2\,ilB + \pi riB$ (b) $2\,ilB + 2\,irB$
(c) $2\,ilB + irB$ (d) none of these

29. *A* charged particle having charge 10^{-6}C enters a magnetic field making an angle 60° with a velocity 10^3 m/s. If its mass is $2\,\mu g$ then pitch is ____ if $B = 2\,T$

(a) $2\,\pi$ meter (b) π meters
(c) $\frac{2}{3}\,\pi$ meters (d) $\frac{\pi}{2}$ meters

30. When an electric field E is applied, a charged particle having charge q goes undeviated in a magnetic field B. If electric field is perpendicular to magnetic field and charged particle moves perpendicular to both E and B the velocity of the particle is

(a) $\frac{E}{B}$ (b) $\frac{2E}{B}$
(c) $\frac{E^2}{B}$ (d) $\frac{E}{2B^2}$

31. A particle moves in a circle of diameter 1cm under the action of magnetic field 0.4T. An electric field $200\,Vm^{-1}$makes the path straight. The charge/mass ratio of the particle is

(a) 12.5×10^4 C/kg (b) 2.5×10^5 C/kg
(c) 5×10^5 C/kg (d) none of these

32. A circular coil has radius 2 cm and 500 turns. It carries current of 1*A*. Its axis makes an angle of 30° with uniform magnetic field $B = 0.4$T. Find the torque acting on the coil.

(a) 13 Nm^2 (b) 1.3 Nm^2
(c) 0.13 Nm^2 (d) none of these

33. Fig. 11 shows a loop having two circular arcs joined by radial lines. The magnetic field at the centre of the loop is

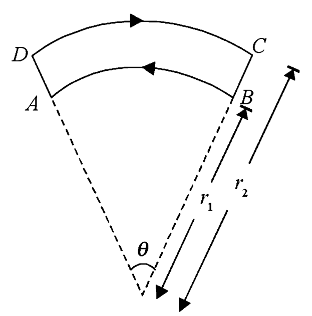


Fig. 11

(a) $\frac{\mu_0 i\theta}{2\pi}(r_2 - r_1)$ (b) $\frac{\mu_0 i\theta}{2\pi}\left[\frac{1}{r_1} - \frac{1}{r_2}\right]$
(c) $\frac{\mu_0 l}{2\pi}\cos\theta\left[\frac{1}{r_1} - \frac{1}{r_2}\right]$ (d) none of these

34. A magnet is 10 cm long and its pole strength is 120C GS units. The magnetic field strength at distance 20 cm, on its axis from it, is

(a) $3.4 \times 10^{-2}\,T$ (b) $3.4 \times 10^{-3}\,T$
(c) $3.4 \times 10^{-4}\,T$ (d) $3.4 \times 10^{-5}\,T$

35. The apparent angle of dip at a place is 30°. The axis of the dip circle points at that place 60° with the horizontal magnetic field of earth, the true value of dip is

(a) $\tan^{-1}\sqrt{\frac{3}{2}}$ (b) $\tan^{-1}\frac{1}{3\sqrt{2}}$
(c) $\tan^{-1}\frac{1}{\sqrt{6}}$ (d) $\tan^{-1}\frac{1}{2\sqrt{3}}$

36. Galvanometer constant in a moving coil Galvanometer is

(a) $\frac{k}{nAB}$ (b) $\frac{ABn}{k}$
(c) $\frac{nA}{kB}$ (d) $\frac{nk}{AB}$

37. A magnet is suspended by a vertical string attached to its mid point. If horizontal components of the earth's field is 25 μT and its vertical component 40 μT, find the angle made by it in equilibrium position.

(a) $\tan^{-1}(1.5)$ (b) $\tan^{-1}(1.6)$
(c) $\tan^{-1}(.625)$ (d) none of these

38. Band pass filter is ____ circuit.

(a) *RL* series (b) *RC* series circuit
(c) Series *LCR* (d) Parallel LC

39. If a magnet of magnetic moment *M* is cut into two equal pieces along the length, then its dipole moment becomes

(a) $\frac{M}{2}$ (b) $\frac{M}{4}$
(c) $\frac{M}{8}$ (d) M

40. The magnitude of earth's field is $3.4 \times 10^{-5}\,T$ at the magnetic equator of the earth. The magnitude at the geomagnetic poles will be

(a) $3.4 \times 10^{-5}\,T$ (b) $5.1 \times 10^{-5}\,T$
(c) $6.8 \times 10^{-5}\,T$ (d) none of these

41. The percentage increase in the magnetic field *B* when space within the current carrying toroid is filled with aluminium. The susceptibility of *Al* is 2.1×10^5.

(a) 2.1% (b) 2.1×10^{-1}%
(c) 2.1×10^{-2}% (d) 2.1×10^{-3}%

42. The maximum value of permeability of μ –metal is 0.126 T–m/A. The maximum value of susceptibility is

(a) 10^5 (b) 10^4

(c) 1.26×10^3 (d) 1.26

43. When current changes from + 5 A to – 5 A in 0.2s an average emf 0.2 V is generated. The self inductance of the coil is

(a) 2 mH (b) 4 mH

(c) 6 mH (d) none of these

44. A conducting circular loop is placed in uniform magnetic field $B = 0.02$T with its plane perpendicular to the field. If the radius starts shrinking at 1mm/s, then induced emf at the instant when radius is 2 cm

(a) 2.5 V (b) 2.5 mV

(c) 2.5 μV (d) 2.5 nV

45. A coil is joined to battery of 6V and current is 12A. This coil is connected to a capacitor and a 6V ac source in series. If the current and voltage are in phase then rms value of current is

(a) 12A (b) $6\sqrt{2}A$

(c) $12\sqrt{2}A$ (d) none of these

46. The wavelength of an electromagnetic wave is 5 mm. If E_0 is 30 V/m and is in y-direction while the wave is moving in x-direction, then equation for magnetic field is

(a) $B = 10^{-7} \sin \frac{2\pi \times 10^3}{5} (ct - x)$ along y-direction

(b) $B = 10^{-7} \sin \frac{2\pi \times 10^3}{5} (ct - x)$ along z-direction

(c) $B = 10^{-7} \sin \frac{2\pi \times 10^3}{5} (ct - x)$ along x-direction

(d) none of these

47. A triode valve has amplification factor 21 and dynamic plate resistance 10 $k\Omega$. The gain when connected with a load of 20 $k\Omega$ is

(a) 21 (b) 17

(c) 14 (d) none of these

48. The wavelength of 10 keV neutron is

(a) 2.86×10^{-10} m (b) 2.86×10^{-11} m

(c) 2.86×10^{-13} m (d) 2.86×10^{-14} m

49. A TV transmitter has height 200 m and if the population density is 1000 /km^2, then the population benefitted is

(a) 3×10^5 (b) 3×10^6

(c) 4×10^5 (d) 8×10^6

50. If intensity of incident radiation is increased in a photocell by 10 % then photo current will increase by

(a) 10 % (b) less than 10 %

(c) more than 10 % (d) 5 %

51. The excitation energy of a hydrogen like ion in its first excited state is 40.8 eV. The energy needed to remove the electron from the ion is

(a) 40.8 eV (b) 54.4 eV

(c) 68.0 eV (d) none of these

52. The theory which can be applied to study atomic structure of any atom is

(a) Rutherford theroy (b) Bohr's therory

(c) Maxwell's theory (d) Schrodinger wave theroy

53. Metastable state means a state having life time ___sec.

(a) 10^{-9} s (b) 10^{-8} s

(c) 10^{-6} s (d) 10^{-3} s

54. The wavelength of He - Ne laser is

(a) 700 nm (b) 632.8 nm

(c) 761.4 nm (d) 657.3 nm

55. Light from the Balmer series of hydrogen is able to eject photo electrons from a metal. What could be maximum work function of the metal?

(a) 1.59 eV (b) 0.86 eV

(c) 3.4 eV (d) cannot be predicted

56. Brehmsstrahlung radiations are generated

(a) due to deceleration of charged particles

(b) when an electron jumps from a higher energy state to a lower energy state

(c) when an electron is captured by nucleus

(d) none of these

57. To regulate output in a supply, we use

(a) capacitor (b) zener diode

(c) photo diode (d) LED

58. To measure temperature $\sim$ 5000K ____ is used

(a) thermistor

(b) platinum resistance thermometer

(c) constant volume gas thermometer

(d) pyrometer

59. A uniform string of mass m and length L is lying on a smooth horizontal plane. It is being pulled with a force F applied at one of the ends. The tension in the string at a distance y from it is

(a) $\frac{L}{2y}F$ (b) $\frac{L}{y}F$

(c) $\left(1 - \frac{y}{L}\right)F$ (d) $F\frac{y}{L}$

60. The excess pressure in a soap bubble of radius 2 cm will be ____. Given surface tension 0.34 N/m.

(a) 34 N/m^2 (b) 51 N/m^2

(c) 68 N/m^2 (d) none of these

61. The first resonance length for a closed pipe is 50 cm. then second resonance position will be

(a) 50 cm (b) 100 cm
(c) 150 cm (d) 250 cm

62. Angular momentum and moment of inertia are respectively

(a) scalar and scalar (b) scalar and vector
(c) vector and tensor (d) vector and vector

63. An astronomical telescope is made using two lenses 45 cm apart. The magnifying power is 8. The focal length of the lenses are

(a) 36 cm, 9 cm (b) 48 cm, 6 cm
(c) 20 cm, 5 cm (d) 40 cm, 5 cm

64. A man is moving due east and wind appears to blow from north. The actual direction of air blow is towards

(a) north-west (b) north-east
(c) south-east (d) south-west

65. A bullet of mass 10 g moving with a velocity 500 ms^{-1} strikes a block of mass 2 kg suspended from a 5 m string. The centre of gravity of the block rises to height 0.1m. The velocity of the bullet after emerging out of the block will be

(a) 217.2 ms^{-1} (b) 128.2 ms^{-1}
(c) 172.1 ms^{-1} (d) 151.4 ms^{-1}

66. The velocity of ripples on water surface depends upon the wave length λ density of water ρ and acceleration due to gravity g. Which of the following relations is correct?

(a) $v^2 \propto g\lambda$ (b) $v^2 \propto \frac{g\lambda}{\rho}$
(c) $v^2 \propto g\rho$ (d) $v^2 \propto \frac{\lambda}{g\rho}$

67. A particle travels according to the equation $x = 2t^3 + 3t^2 - 4t - 9$. The velocity when acceleration is zero is

(a) 3.5 ms^{-1} (b) 4.5 ms^{-1}
(c) 5.5 ms^{-1} (d) 6.5 ms^{-1}

68. The pulley shown has a moment of inertia I about its axis and mass m. The time period of vertical oscillation of its centre of mass, if the spring has spring constant p and spring does not slip over the pulley is

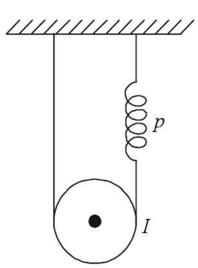


Fig. 12

(a) $T = 2\pi\sqrt{\frac{m}{p}}$ (b) $T = 2\pi\sqrt{\frac{\frac{I}{r^2}+m}{4p}}$
(c) $T = 2\pi\sqrt{\frac{\frac{I}{r^2}+m}{2p}}$ (d) $T = 2\pi\sqrt{\frac{\frac{I}{r^2}+m}{p}}$

69. The luminous flux of a 10 watt light source of wavelength 600 nm will be ___ if the relative luminosity for 600 nm is 0.6

(a) 6850 lm (b) 4110 lm
(c) 5550 lm (d) 9870 lm

70. If x_1, x_2 are the distances of the object and image from the first and second focal points of the lens in air then focal length of the lens is

(a) $x_2 - x_1$ (b) $2x_1 - x_2$
(c) $x_2 + x_1$ (d) $\sqrt{x_1 x_2}$

71. The coefficient of friction of the tyres of a car on a greasy road is 0.2. The largest speed the car can have while travelling round the corner of radius 20 m without skidding is

(a) 6.28 ms^{-1} (b) 3.14 ms^{-1}
(c) 9.12 ms^{-1} (d) none of these

72. The velocity of the particle from adjoining graph after 3 seconds is

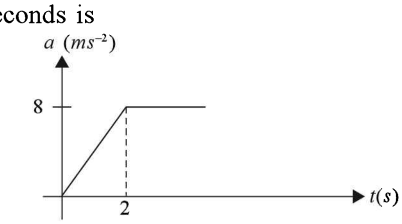


Fig. 13

(a) 8 ms^{-1} (b) 16 ms^{-1}
(c) 24 ms^{-1} (d) none of these

73. A balloon is moving up with a speed 12 ms^{-1} at a height of 80 m and has an acceleration 2 ms^{-2}. It drops a packet when at a height of 80 m. The time taken by the packet to reach ground is nearly

(a) 3.745 s (b) 5.88 s
(c) 9.32 s (d) 12.1 s

74. Two tuning forks A and B have frequencies 140 Hz and 160 Hz. Number of beats heard are

(a) 20 (b) 10
(c) 2 (d) none

75. The third over tone of 250Hz is

(a) 750 Hz (b) 500 Hz
(c) 1500 Hz (d) 1000 Hz

76. A hockey puck weighing 400 g slides on ice for 20 m before coming to rest. If its intial speed was 10 ms^{-1} then coefficient of dynamic friction is

(a) 0.1 (b) 0.2
(c) 0.25 (d) 0.15

77. Moment of inertia of a cone of mass M and radius R and height h about an axis xox' as shown in the figure is

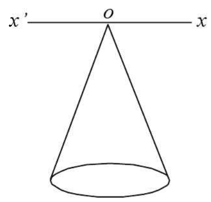


Fig. 14

(a) $\frac{3}{10}M(R^2+h^2)$ (b) $\frac{3}{5}M\left(\frac{R^2}{4}+\frac{h^2}{2}\right)$

(c) $\frac{3}{10}M\left(\frac{R^2}{2}+2h^2\right)$ (d) none of these

78. The gravitational intensity at a point distant x from the surface inside the spherical shell of radius R is

(a) zero (b) $\frac{GM(R-x)^2}{R^3}$

(c) $\frac{GM(R-x)}{R^3}$ (d) none of these

79. A spring gun projects a ball at an angle of 45° above the horizontal range is 10 m. The height attained by the ball is

(a) 2.5 m (b) 5 m

(c) 1.25 m (d) 2 m

80. A spherical ball rolls on a table without slipping. The ratio of rotational to total energy is

(a) $\frac{2}{5}$ (b) $\frac{2}{7}$

(c) $\frac{3}{5}$ (d) $\frac{3}{7}$

81. A cracker at rest explodes to a large number of parts. Then centre of mass

(a) describes a straight line (b) describes a parabola

(c) remains unchanged (d) cannot be predicted

82. A rod of length L rotates in a horizontal plane with an angular speed ω. A ball of mass m is suspended by a string of same length L. If it makes an angle θ with the vertical, then angular speed of rotation is

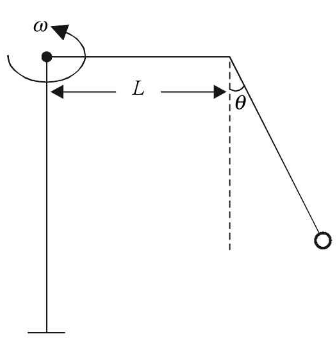


Fig. 15

(a) $\frac{g\tan\theta}{L(1+\sin\theta)}$ (b) $\left[\frac{g\tan\theta}{L(1+\sin\theta)}\right]^{1/2}$

(c) $\frac{g\sin\theta}{L(1+\cos\theta)}$ (d) $\frac{g\cos\theta}{L(1+\sin\theta)}$

83. When a rectangular glass slab is placed on a white written paper, the writing disappears from the edges. The minimum refractive index of the slab is

(a) $\sqrt{2}$ (b) 1.5

(c) $\sqrt{3}$ (d) 1.6

84. A block A is placed on a table and is tied with a string passing over the pulley. The other end of the string is connected with block B. If mass of A is 15 kg and that of B is 5 kg and coefficient of friction is 0.2 between block and the table. Then minimum value of mass of block C to be placed on A so that the system remains in the equilbrium is

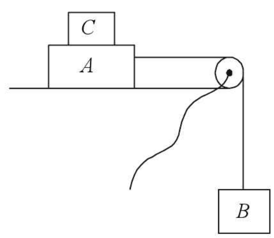


Fig. 16

(a) 5 kg (b) 10 kg

(c) 15 kg (d) none of these

85. The mass of a 2 kgf weight on the surface of moon is

(a) 2 kg f (b) 2 kg

(c) 1 kg f (d) $\frac{1}{3}$ kg f

86. The energy of thermal neutron is

(a) 0.1 eV (b) 0.6 eV

(c) 2.1 eV (d) 3.4 eV

87. The velocity of sound at 40°C will be ____ if velocity at 20°C is 350 ms^{-1}.

(a) 340 ms^{-1} (b) 351.5 ms^{-1}

(c) 363 ms^{-1} (d) 374.5 ms^{-1}

88. The dimensions of coefficient of rolling friction is

(a) $[M^\circ L^\circ T]$ (b) $[M^\circ L T^\circ]$

(c) $[M^1 L^\circ T^\circ]$ (d) $[M^\circ L^\circ T^\circ]$

89. A particle of mass m slides down an inclined plane of inclination θ. Lift is moving up with an acceleration a. If the length of the base of inclined plane is L, then time taken to reach at the bottom is

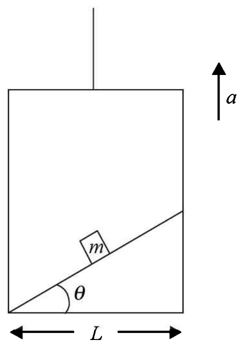


Fig. 17

(a) $\sqrt{\dfrac{L}{(g+a)\sin\theta}}$ (b) $\sqrt{\dfrac{2L}{(g+a)\sin\theta}}$

(c) $\sqrt{\dfrac{2L}{(g+a)\sin\theta\cos\theta}}$ (d) $\sqrt{\dfrac{L}{(g+a)\sin\theta\cos\theta}}$

90. Wave length of two notes are $\frac{90}{175}$ and $\frac{90}{173}$ m in air. They produce 4 beats each with a third note of fixed frequency. The velocity of sound in air is

(a) 340 ms^{-1} (b) 350 ms^{-1}

(c) 332 ms^{-1} (d) 360 ms^{-1}

91. The average momentum of a molecule of an ideal gas depends upon

(a) volume (b) number of moles

(c) temperature (d) none of these

92. *A* particle of mass m projected with a velocity *V* making an angle θ with the horizontal bursts at the maximum height into two particles of equal masses then path followed by them is

(a) one retraces the path travelled and other continues to move in the same path as it would have otherwise moved

(b) though both move in parabolic paths but different from the original path

(c) they travel in a straight line, one vertically up and the other vertically down.

(d) none of these

93. In perfectly inelastic collision and in partially inelastic collision particles after collision

(a) move with different velocites in both cases

(b) move with same velocity and different velocities respectively

(c) come to rest and move with same velocity respectively

(d) come to rest and move with different velocities respectively

94. Emission of nuclear radiations is related with

(a) elastic collision

(b) perfectly inelastic collision

(c) inelastic collision

(d) none of these

95. An accelerated particle

(a) may have zero velocity

(b) always has a finite velocity but not zero

(c) may have infinite velocity

(d) none of these

96. The coefficient of static friction μ_s is

(a) < 1 (b) ≤ 1

(c) $-1 < \mu_s < 1$ (d) $0 < \mu_s < 1$

97. A particle falls from a height *H*, when it falls down by a distance *x*, it meets an inclined plane and its velocity becomes horizontal then *x* in terms of *H* will be ____so that it takes maximum time to reach ground.

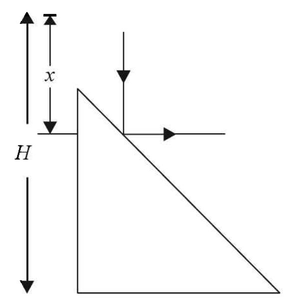


Fig. 18

(a) $x = \frac{H}{2}$ (b) $x = \frac{H}{3}$

(c) $x = \frac{H}{4}$ (d) $x = H$

98. A block *A* of mass 2 kg moving with 5 km/h strikes elastically with another block *B* of same mass moving with 3 km/h then after collision velocity of block *B* will be

(a) 3 km/ *h* (b) 5 km/*h*

(c) 2 km/*h* (d) none of these

99. The ratio of radii of the planets P_1 and P_2 is K_1. The ratio of acceleration due to gravity on them is K_2, then ratio of their escape velocities is

(a) $K_1 K_2$ (b) $\frac{K_1}{K_2}$

(c) $\sqrt{\frac{K_1}{K_2}}$ (d) $\sqrt{K_1 K_2}$

100. *A* wire of length *l*, area of cross-section *A*, Young's modulus *Y* and linear coefficient of expansion α is heated through t° C . The work that can be performed by the rod when heated is

(a) $(YA\,\alpha t)(l\,\alpha\,t)$ (b) $\frac{1}{2} YA\,\alpha^2 t^2 l$

(c) $\frac{YA\,\alpha^2 t^2 l}{4}$ (d) $2\,YA\alpha^2 t^2 l$

Answers

1.	(a)	**2.**	(b)	**3.**	(a)	**4.**	(b)	**5.**	(b)	**6.**	(b)	**7.**	(d)
8.	(c)	**9.**	(d)	**10.**	(b)	**11.**	(b)	**12.**	(b)	**13.**	(a)	**14.**	(a)
15.	(d)	**16.**	(c)	**17.**	(b)	**18.**	(b)	**19.**	(b)	**20.**	(c)	**21.**	(b)
22.	(b)	**23.**	(b)	**24.**	(c)	**25.**	(a)	**26.**	(a)	**27.**	(d)	**28.**	(b)
29.	(b)	**30.**	(a)	**31.**	(b)	**32.**	(c)	**33.**	(b)	**34.**	(d)	**35.**	(d)
36.	(a)	**37.**	(b)	**38.**	(c)	**39.**	(a)	**40.**	(c)	**41.**	(d)	**42.**	(a)
43.	(b)	**44.**	(c)	**45.**	(a)	**46.**	(b)	**47.**	(c)	**48.**	(c)	**49.**	(d)
50.	(b)	**51.**	(b)	**52.**	(d)	**53.**	(d)	**54.**	(b)	**55.**	(c)	**56.**	(a)
57.	(b)	**58.**	(d)	**59.**	(c)	**60.**	(c)	**61.**	(c)	**62.**	(c)	**63.**	(d)
64.	(b)	**65.**	(a)	**66.**	(a)	**67.**	(c)	**68.**	(b)	**69.**	(b)	**70.**	(d)
71.	(a)	**72.**	(b)	**73.**	(b)	**74.**	(d)	**75.**	(d)	**76.**	(c)	**77.**	(c)
78.	(a)	**79.**	(a)	**80.**	(b)	**81.**	(c)	**82.**	(b)	**83.**	(a)	**84.**	(b)
85.	(b)	**86.**	(a)	**87.**	(c)	**88.**	(b)	**89.**	(c)	**90.**	(d)	**91.**	(c)
92.	(b)	**93.**	(b)	**94.**	(c)	**95.**	(a)	**96.**	(d)	**97.**	(a)	**98.**	(b)
99.	(d)	**100.**	(b)										

Explanations

1(a)

2(b) $\Delta Q = nCv\Delta T = \frac{1}{4}\times\frac{5}{2}R \times 20 = 25 \text{ cal}$

3(a) Use lapse rate $T = 18 - 2 \times 6 = 6°\text{C}$

4(b) $\Delta T = \frac{\Delta P}{P}\times 273.14 = 273.14$

$T = 273.14 + 273.14 = 546.28\text{K}$

5(b)

6(b)

7(d) $C_{\text{eff.}} = \frac{C}{1-1/2} = 2C$

8(c) $V_{\text{big}} = n^{2/3}V_{\text{small}} = 81 \times \frac{10^{-9}\times 9\times 10^{9}}{10^{-3}} = 7.29 \times 10^{5}V$

9(d)

10(b)

11(b)

12(b) $V_{AB} = 2 + 5\,(0.1) = 2.5V$

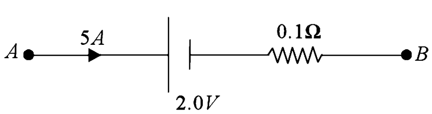


Fig. 19

13(a) $\left(\frac{75x}{75+x}\right)2 = 120$ or $x = 300\,\Omega$

14(a) $\varepsilon_{eq} = \frac{1\times 4 - 3\times 2}{4+3} = -\frac{2}{7}V$

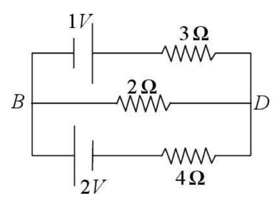


Fig. 20

$r = \frac{3\times 4}{3+4} = \frac{12}{7}$,

$I_{BD} = \frac{\frac{2}{7}}{2+\frac{12}{7}} = \frac{1}{13}A$

15(d) $R \propto l$ and $R \propto \frac{1}{r^2}$

16(c) $18 = \frac{30\times x}{24+x}$

or $x = 36$; $36 = \frac{R.50}{50+R}$ or $R \sim 130\,\Omega$

17(b) $V = \frac{4\times 10 + 10\times 12}{10+10} = 8V$; $Q = CV = 8 \times 10 = 80\,\mu$ C

18(b) A—||—||—B $= \frac{c}{2} = \frac{2}{2} = 1\,\mu F$

19(b)

20(c)

21(b) $v = \frac{c}{137} = \frac{3\times 10^{8}}{137} = 2.18 \times 10^{6} ms^{-1}$

22(b) $W = q\Delta V$ or $\Delta V = \frac{12}{6\times10^{-3}} = 2\times10^3 V$

23(b) $\Delta PE = \frac{q_1 q_2}{4\pi \in_o r}$

$= \frac{20\times20\times10^{-12}\times9\times10^9}{0.1} = 36J$

24(c) $E = \frac{-dv}{dx} = 0 \because V$ is constant at $x = 6$ m.

25(a)

26(a) Current through the circuit = 1A

charge passed in 1/2 hour $= 1\times\frac{1}{2}\times 60\times 60. = 1800C$

$m = \frac{1800\times107.9g}{96500} = 2$ g

27(d) $f_c = \frac{qB}{2\pi m} = \frac{1.6\times10^{-19}\times2\times10^{-3}}{2\pi\times1.6\times10^{-27}}$

$= \frac{10^5}{\pi} Hz$

28(b) $F = 2\,irB + 2ilB = 2\,i(l + r)B$

29(b) Use $V\cos\theta \times T$ = pitch

30(a) $qvB = qE$ or $v = E/B$

31(b) $\frac{rqB}{m} = v = \frac{E}{B}$

or $\frac{q}{m} = \frac{E}{B^2 r}$

$= \frac{400\times10^2\times10^2}{16}$

$= 2.5\times10^5$ C/kg

32(c) $\tau = ni\,A\times B = 500\times(\pi\times4\times10^{-4})\times(.4)\sin 30$

$= 4\pi\times10^{-2} = 0.13\,N-m^2$

33(b)

34(d) ICGS unit = 0.1 Am.

$B = \frac{2\mu_o md}{4\pi(d^2-l^2)^2}$

35(d) $\frac{V}{H} = \tan\delta$ and $\frac{V}{H\cos\theta} = \tan\delta'$

$\tan\delta = \tan\delta'\cos\theta = \tan 30\cos 60 = \frac{1}{\sqrt{3}}\times\frac{1}{2}$

$= \frac{1}{2\sqrt{3}}$

36(a) $niAB = k\theta$ or $\frac{i}{\theta} = \frac{k}{nAB}$

37(b) $\tan\delta = \frac{B_V}{B_H}$

38(c)

39(a) $M' = \frac{m}{2}(\ell) = \frac{M}{2}$

40(c) $B = B_{eq}\sqrt{3\sin^2\lambda+1} = 3.4\times10^{-5}\times2 = 6.8\times10^{-5}T$

41(d)

42(a) $\mu_r = \frac{\mu}{\mu_0}$; $\chi = \mu_r - 1$

43(b) $L = \frac{V}{dI/dt} = \frac{\frac{0.2}{10}}{0.2} = 4$ mH

44(c) $\frac{d\phi}{dt} = 2\pi rB\frac{dr}{dt}$

$= 2\pi\times2\times10^{-2}\times.02\times10^{-3}$

$= 2.5\,\mu V$

45(a) $\because X_L = X_C$

$\therefore Z = r$ and I remains same

46(b)

47(c) $A = \frac{\mu R_L}{R_L + r_P} = \frac{21\times20}{30} = 14$

48(c) $\lambda = \frac{h}{p}$ and $p = \sqrt{2(KE)m}$ or $\lambda = \frac{0.286\times10^{-10}}{\sqrt{10^4}}$

49(d) $(\pi r^2)\times10^3$, $r = \sqrt{200\times6.4\times10^6}$

$r = 10^4\sqrt{12.8}$

Population benefitted $= 12.8\times\pi\times\frac{10^{11}}{10^6}$

$= 4\times10^6$ people

50(b)

51(b) $E = Rhc\,z^2\left(\frac{1}{1^2}-\frac{1}{2^2}\right)$

or $E = 13.6\times z^2\times\frac{3}{4} = 40.8 \quad\Rightarrow z = 2$

$= BE = (-13.6)\frac{z^2}{1^2}$

$= -54.4\,eV$

52(d)

53(d)

54(b)

55(c) $E = 13.6\left(\frac{1}{2^2}-\frac{1}{\infty}\right)$

$= 3.4eV = \phi$

56(a)

57(b)

58(d)

59(c) $a = \frac{F}{m}$

tension $T = \frac{m}{L}(L-y)\,a = F(1-\frac{y}{L})$

60(c) $\Delta p = \frac{4T}{r} = \frac{4\times .34}{2\times 10^{-2}}$

$= 68\ \text{N/m}^2$

61(c) $l_2 = 3l_1$

62(c)

63(d)

64(b) $V_{wm} = V_w - V_m$

65(a) $mu + 0 = mv_2 + MV_1$

$\frac{1}{2}MV_1^2 = Mgh,\ or\ V_1 = \sqrt{2}\ \text{ms}^{-1}$

$.01\times 500 = 2\sqrt{2} + .01V_2$

$5 - 2.828 = .01V_2$ or $V_2 = 217.2\ \text{ms}^{-1}$

66(a)

67(c) $\frac{dx}{dt} = 6t^2 - 6t - 4$

$= 6\left(\frac{1}{4}\right) - 6\left(\frac{1}{2}\right) - 4 = \frac{11}{2}\ \text{ms}^{-1}$

$\frac{d^2x}{dt^2} = 12t - 6 = 0$ or $t = 0.5s$

68(b)

69(b) $10\times 685\times 0.6 = 4110\ lm$

70(d)

71(a) use $V = \sqrt{\mu rg}$

$= \sqrt{.2\times 20\times 9.8} = 6.28\ \text{ms}^{-1}$

72(b) Area under curve

73(b) $-80 = 12t - 5t^2$ or $t^2 - 2.4t - 16 = 0$

74(d)

75(d)

76(c) Use $\frac{1}{2}mv^2 = \mu mgs$

or $\mu = \frac{10\times 10}{2\times 10\times 20} = 0.25$

77(c)

78(a)

79(a) $R = \frac{u^2}{g}$

$\Rightarrow u = 10\text{ms}^{-1};\ h = \frac{u^2\sin^2 45}{2g}$

80(b)

81(c)

82(b) $T\cos\theta = mg;\ T\sin\theta = mL(1+\sin\theta)\omega^2$

83(a)

84(b)

85(b)

86(a)

87(c) $V = V_0\sqrt{\frac{273+t}{273}};\ \frac{350}{V_2}\sqrt{\frac{293}{313}}$

or $V_2 = 363\ \text{ms}^{-1}$

88(b)

89(c) $\frac{1}{2}(g+a)\sin\theta\ t^2 = \frac{L}{\cos\theta}$

90(d) $8 = \frac{v}{\lambda_1} - \frac{v}{\lambda_2};\ \frac{175v}{90} - \frac{173v}{90} = 8$

or $2v = 720\ \text{ms}^{-1}$ or $v = 360\ \text{ms}^{-1}$

91(c)

92(b)

93(b)

94(c)

95(a)

96(d)

97(a) For x and $H - x$ are separately

maximum $x = H - x$ or $x = \frac{H}{2}$

98(b)

99(d) $\frac{Ve_1}{Ve_2} = \sqrt{\frac{2R_1g_1}{2R_2g_2}} = \sqrt{K_1K_2}$

100(b) $W = \frac{1}{2}F\Delta l$

MODEL TEST PAPER 4

1. If percentage error in measurement of length of the pendulum is 2% and in measurement of time is 1%, then % error in measurement of g' is

(a) zero (b) 2%

(c) 3% (d) 4%

2. If $p = A + \frac{B}{V}$ then dimensions of B are

(a) MLT^{-2} (b) ML^2T^{-2}

(c) ML^5T^{-2} (d) ML^2T^{-3}

3. $x = ut + at^2$ is a consequence of

(a) Newton's Ist law (b) Newton's IInd law

(c) Newton's IIIrd law (d) none of these

4. If velocities of particles A and B are depicted in the figure then V_{AB}

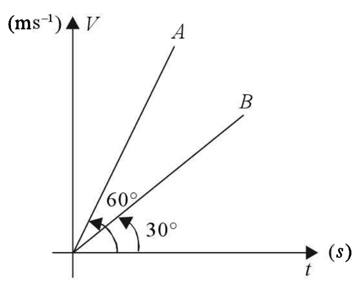


Fig. 1

(a) is zero (b) continuously decreases

(c) is constant (d) continuously increases

5. A particle of mass 3 kg is moving under the action of a central force whose PE is given by $U(r) = 10\, r^3$. For what energy and angular momentum the orbit will be a radius of 10 m.

(a) 2.5×10^4 J, 3000 kg m^{2}s^{-1}

(b) 0.5×10^4 J, 3000 kg m^{2}s^{-1}

(c) 1.5×10^4 J, 3000 kg m^{2}s^{-1}

(d) none of these

6. A car is moving in a circular track of radius 10 m with a speed of 10 ms^{-1} A plumb bob is suspended from the ceiling of a car by a light rigid rod of length 1m. The angle made by the rod with the track is

(a) 90° (b) 60°

(c) 30° (d) 45°

7. Two projectiles of mass, m each are fired in opposite direction, with same speed V from the same point with angles α and β with respect to horizontal then the change in momentum at any point is

(a) $2\text{ mv} \cos(\alpha - \beta)$ (b) $2\text{ mv} \cos\left(\frac{\alpha+\beta}{2}\right)$

(c) $3\text{ mv} \sin\left(\frac{\alpha-\beta}{2}\right)$ (d) $2\text{ mv} \cos\left(\frac{\alpha-\beta}{2}\right)$

8. A constant force (in magnitude) acts on a body always perpendicular to its motion. The motion is in a plane. It follows that

(a) velocity is constant (b) acceleration is constant

(c) KE is constant

(d) it moves in a circular path.

9. A straight narrow tunnel is bored in a sphere of radius a from surface to centre of the sphere. Density of the sphere is ρ. The amount of work done to bring a small mass m from centre to the surface of the sphere is

(a) $\frac{2}{3}\pi mG\rho a^2$ (b) $\frac{1}{3}\pi mG\rho a^2$

(c) $\frac{4}{3}\pi mG\rho a^2$ (d) none of these

10. The speed with which earth shall rotate, so that the apparent weight of the body at equator is zero, is

(a) 1.241×10^{-3} rad/s (b) 1.421×10^{-3} rad/s

(c) 1.124×10^{-3} rad/s (d) none of these

11. The amount of work done to take a mass m to a height $\frac{R}{2}$ is

(a) $mg\frac{R}{2}$ (b) $mg\frac{R}{3}$

(c) $mg\frac{R}{4}$ (d) $mg\frac{3}{4}R$

12. Poisson ratio lies theoretically between

(a) 0 and +0.25 (b) 0 and +0.5

(c) −1 to +0.5 (d) none of these

13. The velocity of efflux of kerosene oil from a tank in which pressure is 2 atm. Density of kerosene is 0.8 g/cc

(a) 5.3 ms^{-1} (b) 10.6 ms^{-1}

(c) 15.8 ms^{-1} (d) none of these

14. Two β - particles are moving in opposite directions by 0.8c each. Then their relative velocity is

(a) 1.6 c (b) 0.8 c

(c) c (d) 0.975 c

15. Two rods A and B of diffierent materials having Young's modulus Y_A and Y_B and thermal coefficient of linear expansion α_A and α_B respectively are joined end to end and the composite rod is clamped then stress developed when heated to t°c is

(a) $(\alpha_A + \alpha_B) Y_A Y_B t$
(b) $\frac{(\alpha_A + \alpha_b) Y_A Y_B t}{(Y_A + Y_B)}$
(c) $\frac{(\alpha_A - \alpha_b) Y_A Y_B t}{(Y_A - Y_B)}$
(d) $t(\alpha_A - \alpha_B) Y_A Y_B$

16. Change in entropy in an adiabatic process is
(a) zero
(b) finite but not zero
(c) infinite
(d) negative

17. At low temperatures in solids $C_V \propto T^3$. This law is called
(a) Debye law
(b) Dulong and Petit's law
(c) Rayleigh Jean's law
(d) Einstein's law

18. The rms speed of oxygen at 27° C is
(a) 212 ms^{-1}
(b) 344 ms^{-1}
(c) 484 ms^{-1}
(d) 624 ms^{-1}

19. The change in entropy when 10g of water is heated from 0° C to 40° C
(a) 5.43 cal k^{-1}
(b) 2.83 cal k^{-1}
(c) 1.37 cal k^{-1}
(d) 10.58 cal k^{-1}

20. In a Kundt's tube distance between two successive heaps of lycopodium powder is 30 cm. The frequency of the tuning fork used is ____ if the velocity of sound wave is 340 ms^{-1}.
(a) 626.6Hz
(b) 506.3 Hz
(c) 926.1Hz
(d) 566.6 Hz

21. A wave $y = 10 \sin 5\pi\ (340\,t - x)$ is travelling in the air as medium where t is in seconds and x in metres. Then intensity of the wave is nearly ____ (Given density of air .00032 g/cm³)
(a) 175 k W/m²
(b) 150 k W/m²
(c) 172 W/m²
(d) 150 W/m²

22. If the velocity of sound in steel is 5200 ms^{-1}, then angle of incidence in air for which total internal reflection will occur from steel is
(a) 3° 39′
(b) 6° 11′
(c) 1° 19′
(d) 4° 23′

23. A stylus is attached with a tuning fork which touches gently to a falling smooth plate. The distance between 8 waves consecutively noted is 10 cm and 36 cm respectively. Then frequency of the tuning fork is
(a) 31.2 Hz
(b) 36.7 Hz
(c) 42.4 Hz
(d) 49.2 Hz

24. A disc has 25 slits, a bright point is marked on the tuning fork vibrating with a frequency f. The bright point, appears stationary when frequency of revolution is 14 rev/s. Then f is
(a) 150 Hz
(b) 250 Hz
(c) 350 Hz
(d) 450 Hz

25. A complex periodic wave can be analysed using
(a) Laplace theorem
(b) Fourier theorem
(c) Doppler's theorem
(d) Regnault theorem

26. A spring oscillator and a pendulum oscillator used in two watches keep correct time on earth. These two are taken on the moon. What happens on the moon?
(a) only spring clock keeps correct time
(b) only pendulum clock keeps correct time
(c) both clocks keeps correct time
(d) neither of the clocks keeps correct time

27. A vibrating tuning fork tied to the end of a string 2 m long is whirled round in a circle. It makes 2 revolutions per second. The ratio of highest to the lowest notes heard by an observer situated in the plane of the tuning fork is ____. Velocity of sound is 340 ms^{-1}
(a) 1.23
(b) 1.31
(c) 1.47
(d) 1.59

28. A pipe is running full of water. At a certain point A, it tapers from 60 cm to 20 cm diameter at B. The pressure difference between A and B is 100 cm of water column. The rate of flow of water through pipe is
(a) 0.14 m³/s
(b) 0.6 m³/s
(c) 1.4 m³/s
(d) 6 m³/s

29. Radius of gryration of a system whose moment of inertia about an axis is $\frac{7}{5} MR^2$, will be
(a) $\frac{7}{5} R$
(b) $\frac{5}{7} R$
(c) $\sqrt{\frac{7}{5}} R$
(d) $\sqrt{\frac{5}{7}} R$

30. Work done per unit volume in a strain when a stretching force is applied is
(a) $F \times Sl$
(b) $\frac{1}{2} F \times Sl$
(c) stress × strain
(d) $\frac{1}{2}$ stress × strain

31. Slug is a unit of ____
(a) mass
(b) momentum
(c) angular momemtum
(d) magnetic field

32. If image needle and object needle are fixed at two points and lens when at a position L_1 gives image of object needle at the image needle with magnification m_1 and lens is displaced by a distance d to a point L_2, the image of object needle is again formed at image needle but with a magnification m_2, then focal length of the lens is
(a) $\frac{m_1 - m_2}{d}$
(b) $\frac{m_1 + m_2}{d}$
(c) $\frac{d}{m_1 + m_2}$
(d) $\frac{d}{m_1 - m_2}$

33. An achromat combination acts as a converging lens and has focal length 30 cm and f if ratio of their dispersive powers is $\frac{2}{3}$ then f = ____cm

(a) – 20 cm (b) 45 cm
(c) 20 cm (d) – 45 cm

34. According to the principle of photometers, illuminating power of the two sources are

(a) $\frac{L_1}{L_2} = \frac{r_2^2}{r_1^2}$ (b) $\frac{L_1}{L_2} = \frac{r_1^2}{r_2^2}$

(c) $\frac{L_1}{L_2} = \frac{r_1^{-2}}{r_2^{-2}}$ (d) $\frac{L_1}{L_2} = \frac{r_2^{+3}}{r_1^3}$

35. If white light is used in young's double slit experiment then central fringe is ____ and bordered by ___

(a) white, red (b) white, violet
(c) red, blue (d) violet, red

36. There are 15,000 lines per inch in a grating and sodium wave length is used, then maximum orders of spectrum which can be seen is

(a) 2 (b) 3
(c) 4 (d) 5

37. Limit of resolution in a microscope is _____ and resolving power is maximum when ____ colour light is used

(a) $\frac{0.16\lambda}{\mu \sin i}$, red (b) $\frac{0.61\lambda}{\mu \sin i}$, violet

(c) $\frac{0.61\lambda}{\mu}$, red (d) $\frac{1.22\lambda}{\mu \sin i}$, violet

38. Nicol prism is made using

(a) glass (b) quartz
(c) tourmaline (d) iceland spar

39. A convex mirror of focal length 12 cm is introduced between lens and image needle such that image is formed at the object itself. The object is placed 15 cm in front of a 10 cm biconvex lens. The position of convex mirror from the lens is

(a) 6 cm (b) 5 cm
(c) 7.5 cm (d) 2 cm

40. In medical science, x-rays are used for detection of fractures in bones. These x-rays are

(a) soft x-rays (b) hard x-rays
(c) characteristic x-rays (d) none of these

41. In a material k_α, k_β, L_α and L_β lines are being emitted if $\lambda_{k\alpha}$, $\lambda_{k\beta}$, $\lambda_{L\alpha}$ and $\lambda_{L\beta}$ are the wavelengths of the lines emitted from it then

(a) $\lambda_{k\alpha} = \lambda_{k\beta} - \lambda_{L\beta} - \lambda_{L\alpha}$ (b) $\frac{1}{\lambda_{k\alpha}} = \frac{1}{\lambda_{k\beta}} + \frac{1}{\lambda_{L\alpha}}$

(c) $\frac{1}{\lambda_{k\beta}} = \frac{1}{\lambda_{k\alpha}} + \frac{1}{\lambda_{L\alpha}}$ (d) $\frac{1}{\lambda_{k\beta}} = \frac{1}{\lambda_{k\alpha}} + \frac{1}{\lambda_{L\beta}}$

42. The figure shows energy-band diagram of a ____

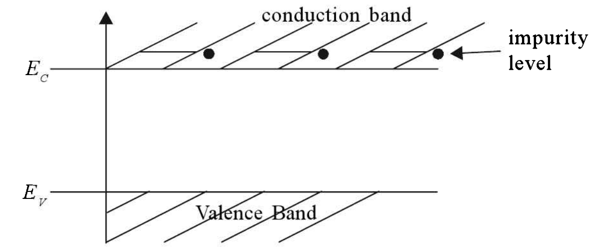


Fig. 2

(a) p-type semiconductor
(b) n-type semiconductor
(c) heavily doped p-type semiconductor
(d) heavily doped n-type semiconductor

43. In an intrinsic semiconductor, Fermi-level lies

(a) in valence band
(b) in conduction band
(c) exactly in the middle of valence band and conduction band
(d) somewhere in between V. B. and C. B. but not exactly in the middle of them

44. The acceptor density in a p - type semiconductor of resistivity 0.2Ω m is _____ . Given mobility of holes is 1.2×10^4 cm²/V–S

(a) 3.6×10^{11}/m³ (b) 2.6×10^{11}/m³
(c) 1.2×10^{11}/m³ (d) none of these

45. A jet plane is travelling west at a 1800 kmh^{-1}. The earth's magnetic field at the location is 5.0×10^{-4} T and - angle of dip is 30°, then potential difference developed across the ends of the wings 25 m long is

(a) 31.25 V (b) 3.125 V
(c) 312.5 V (d) none of these

46. An emf of 50 mV is developed in a coil when crrrent in the neighbouring coil varies from 10 A to 5 A in 0.1s. The mutual inductance of the coil is

(a) IH (b) 10^{-1}H
(c) 10^{-2}H (d) 10^{-3}H

47. The equivalent inductance in the circuit is

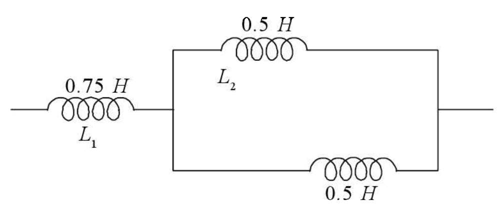


Fig. 3

(a) 1 H (b) 0.25 H
(c) 0.5 H (d) none of these

48. The electrical bulb rated 500 W at 100 V is used in a circuit having 200V supply. The value of the resistance that must be connected in series so that bulb delivers 500 W is

(a) 100 Ω (b) 60 Ω
(c) 40 Ω (d) 20 Ω

49. Two copper spheres one solid and other hollow of the same radius are charged to the same potential, then

(a) solid sphere has more charge than on hollow
(b) hollow sphere has more charge than solid
(c) both have equal charge
(d) none of these

50. If x is electric susceptibility, α is atomic polarisability and N number density of atoms then for $\alpha N << 1$

(a) $x = \frac{\alpha}{N}$ (b) $x = \alpha N$
(c) $x = \frac{N}{\alpha}$ (d) $x = (\alpha N)^{-1}$

51. In a parallel plate capacitor, the capacitance increases from 5 μF to 40 μF on introducing a dielectric medium. The susceptibility of the medium is

(a) 8 (b) 7
(c) 6 (d) none of these

52. An 80 μF capacitor is charged by a 50 V battery. The capacitor is disconnected from the battery and then connected across an uncharged 320 μF capacitor. The charge on this capacitor will be

(a) 8×10^{-3} (b) 3.2×10^{-3}
(c) 4×10^{-3} (d) none of these

53. For C = μF the net capacitance between A and B will be 1 μF.

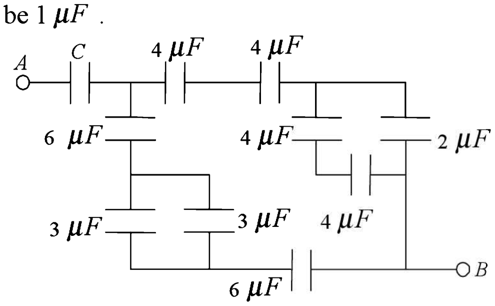


Fig. 4

(a) 1 μF (b) $\frac{10}{7}$ μF
(c) $\frac{7}{10}$ μF (d) $\frac{3}{2}$ μF

54. If two capacitors C_1 and C_2 are connected in series, their effective value is 3 μF but when connected in parallel their effective value is 16 μF. Their magnitudes are

(a) 8 μF, 8 μF (b) 10 μF, 6 μF
(c) 12 μF, 4 μF (d) 9 μF, 7 μF

55. The unit of temperature coefficient of resistivity is

(a) ohm – m K^{-1} (b) K^{-1}
(c) ohm K^{-1} (d) ohm $m^{-1} K^{-1}$

56. The $I - V$ graphs for a given material are shown in the figure at two different temperatures T_1 and T_2. Then

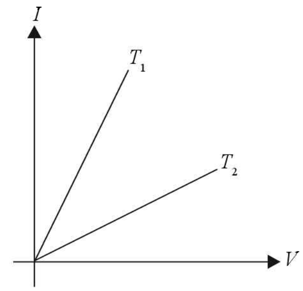


Fig. 5

(a) $T_1 = T_2$ (b) $T_1 > T_2$
(c) $T_1 < T_2$ (d) $T_1 \leq T_2$

57. The current I in the circuit is

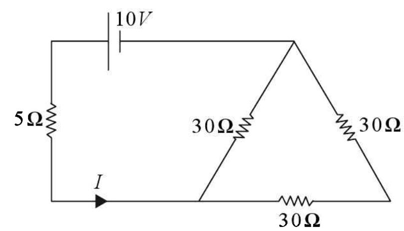


Fig. 6

(a) $\frac{2}{5} A$ (b) $\frac{2}{7} A$
(c) $\frac{2}{13} A$ (d) $\frac{1}{9} A$

58. Currents I_1 and I_2 in the circuit

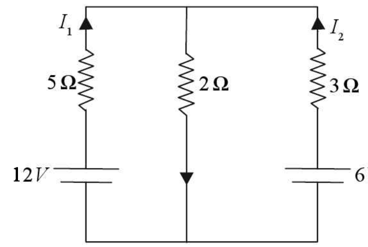


Fig. 7

(a) $\frac{48}{31} A, \frac{18}{31} A$ (b) $\frac{66}{31} A, \frac{48}{31} A$
(c) $\frac{18}{31} A, \frac{66}{31} A$ (d) none of these

59. Three equal resistors, each R ohm, are connected as shown in figure. The battery has emf 2 V and internal resistance 0.1Ω. For what value of R the heat generated is maximum?

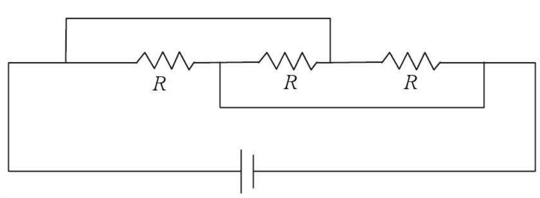


Fig. 8

(a) 0.1Ω (b) 0.2 Ω

(c) 0.3 Ω (d) none of these

60. If heat produced in 5 Ω resistance is 10 cal s^{-1} in the circuit shown, then heat produced in 4 Ω resistance is

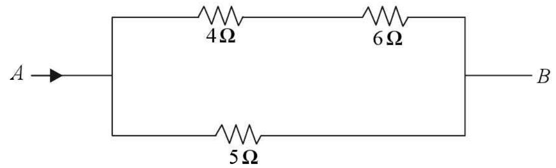


Fig. 9

(a) 2 cal s^{-1} (b) 4 cal s^{-1}

(c) 5 cal s^{-1} (d) 1 cal s^{-1}

61. A long wire shown in the figure carries 6 A current. The magnitude of magnetic field at the centre is ____ if radius of the bent part is 2 cm.

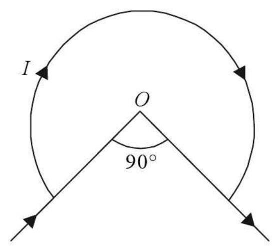


Fig. 10

(a) $1.41 \times 10^{-4} T$ (b) $1.14 \times 10^{-4} T$

(c) $14.1 \times 10^{-4} T$ (d) $9 \times 10^{-3} T$

62. The cyclotron oscillator frequency is 10^7 Hz. The magnetic field for accelerating protons is ____ if the radius of its dees is 60 cm

(a) 6.28 T (b) 3.14 T

(c) 1.28 T (d) 0.63 T

63. Permeability of Bi is 0.9983. To which class of magnetic materials does, it belongs

(a) paramagnetic (b) ferromagnetic

(c) diamagnetic (d) ferrimagnetic

64. If horizontal component of earth's magnetic field at a place is 3.6×10^{-4} T, then angle of dip at that place is _____. Given earth's magnetic field at that place is $6 \times 10^{-4} T$

(a) 37° (b) 53°

(c) 47° (d) cannot be predicted

65. Magnetic hysterisis corresponds to

(a) same is elastic hysterisis

(b) lagging of magnetic induction behind the magnetizing field

(c) leading of magnetic induction over magnetizing field

(d) susceptibility

66. The order of magnetic moment of an atom is

(a) 10^{-11} Am^2 (b) 10^{-15} Am^2

(c) 10^{-19} Am^2 (d) 10^{-23} Am^2

67. The magnetic flux of a coil is given by $\varphi = (5t^2 + 10t - 5)\, m$ weber pointing perpendicular downwards the plane of paper. The emf induced in the loop at $t = 5s$ is

(a) 6 V (b) 0.6 V

(c) 0.06 V (d) 0.006 V

68. A passive device when connected in parallel blocks dc, it is

(a) L (b) C

(c) R (d) pnp transistor

69. The impedance of load should be ____ if source impedance is $20 - 16j$ for maximum power to be transmitted

(a) 20 – 16j (b) 20 + 16j

(c) 10 + 8j (d) 10 – 8j

70. An LC circuit contains $L = 20\ mH$, $C = 50\ \mu F$ with initial charge on capacitor 10 mC. If the circuit is closed at $t = 0$ then the early instant when the energy stored is

(a) 0.78 ms (b) 1.58 ms

(c) 3.16 ms (d) 6.32 ms

71. A charged particle oscillates about its mean position 10^8 times in 1s then frequency of em waves produced by it is

(a) 2.5×10^7 Hz (b) 5×10^7 Hz

(c) 7.5×10^7 Hz (d) none of these

72. Poynting vector is _____ and its unit is ____.

(a) $\vec{B} \times \vec{H}$ Wb/m^2 (b) (b) $\vec{E} \times \vec{H}$, $Wb\,m^{-2}$

(c) $\vec{E} \times \vec{B}$ $Wb\,m^{-2}$ (d) none of these

73. In a capacitor if I_d and I_c denote displacement and conduction current respectively during its charging then

(a) $I_d = I_c$ (b) $I_d > I_c$

(c) $I_d < I_c$

(d) no relation can be established between I_d and I_c

74. The order of height of ozone layer in ionosphere from the surface of the earth is

(a) 12 km (b) 30 km

(c) 50 km (d) 400 km

75. If a red glass is heated in dark room, it appears

(a) blue (b) white

(c) red (d) green

76. The spectrum produced by Hg lamp is

(a) continuous (b) band

(c) line (d) absorption

77. The star is receding with ___ velocity if its frequency decreases by 0.2 %.

(a) 1.5×10^5 ms^{-1} (b) 3×10^5 ms^{-1}

(c) 4.5×10^5 ms^{-1} (d) 6×10^5 ms^{-1}

78. Two bullets are fired simultaneously, horizontally with 40 ms^{-1} and 120 ms^{-1} from the same place. The bullet which hits the ground first is

(a) the one moving with 40 ms^{-1}

(b) the one moving with 120 ms^{-1}

(c) both hit the ground simultaneously

(d) depends on their masses

79. Six particles situated at the vertices of a regular hexagon of side l move at a constant speed V. Each particle maintains a direction towards the particle at the next corner. The time the particles will take to meet each other is:

(a) $\frac{3a}{2V}$ (b) $\frac{2a}{3V}$

(c) $\frac{2a}{V}$ (d) $\frac{2a}{\sqrt{3V}}$

80. A pendulum of length l has bob of mass m. It is oscillating in a vertical plane. The point where the tension in the string is mg $\cos\theta$ is

(a) mean position

(b) extreme position

(c) somewhere between mean and extreme position

(d) never achieved

81. A ball falls on an inclined plane of inclination θ from a height h above the point of impact and makes a prefectly elastic collision. Where will it hit the plane again?

(a) 8 $h\cos\theta$ (b) 4 $h\cos\theta$

(c) 4 $h\sin\theta$ (d) 8 $h\sin\theta$

82. The gravitational field due to a mass distribution is given by E = $\frac{K}{x^3}$ in x-directron. The gravitational potential at x is

(a) $\frac{-K}{x^2}$ (b) $\frac{-K}{2x^2}$

(c) $\frac{K}{2x^2}$ (d) $\frac{K}{2x}$

83. A satellite is orbiting the earth close to its surface. A particle is to be projected from the satellite to just escape from the earth. The escape speed with respect to the earth is v_e. Its speed to projection

(a) will be less than V_e (b) greater than V_e

(c) equal to V_e

(d) will depend upon the direction of projection

84. In the equation of damped oscillation

$$\frac{md^2x}{dt^2} + kx + b\frac{dx}{dt} = 0$$

The damping factor is

(a) k (b) b

(c) $\frac{k}{2m}$ (d) $\frac{b}{2m}$

85. A uniform rope of length 12 m and mass 6 kg hangs vertically from a rigid support. A block of mass 2 kg is attached to the free end of the rope. A transverse wave of wavelength 0.06 m is produced at the lower end of the rope. The wavelength of the wave at the top of the rope is

(a) 0.06 m (b) 0.09 m

(c) 0.12 m (d) 0.15 m

86. Two amplifiers having gain A_1 and A_2 are connected in tandem, the net gain is

(a) $A_1 + A_2$ (b) $\frac{A_1 + A_2}{2}$

(c) $A_1 A_2$ (d) $\sqrt{A_1 A_2}$

87. The activity of a radioactive material falls to $\frac{1}{16}$ th of its initial value in 30 hours. The half - life of the material is

(a) 7.5 years (b) 2.5 years

(c) 10 years (d) 6 years

88. ${}^{x}_{y}A \rightarrow {}^{x}_{y+1}B + {}_{-1}e^0 +$ ____ complete the reaction.

(a) none (b) ν

(c) $\bar{\nu}$ (d) μ

89. The life time of quasi-stable state is of the order of

(a) $10^3 s$ (b) 1s

(c) $10^{-3} s$ (d) 2.5 A°

90. The angle of reflection for first order monochromatic x-rays from a crystal whose atomic spacing is 2.5 A° is 15°. The wavelength of the x-rays is

(a) 1.294 A° (b) 0.647 A°

(c) 1.9 A° (d) 2.5 A°

91. The α - particle beam of KE 4 MeV is incident on gold foil. The distance of nearest approach is of the order of

(a) 10^{-12}m (b) 10^{-14}m

(c) 10^{-16}m (d) 10^{-10}m

92. The radius of second excited state of L_i+ + is

(a) 4.77 A° (b) 2.12 A°

(c) 1.59 A° (d) 1.06 A°

93. When an electron makes a transition from third to first orbit in an atom, the wavelength emitted is λ. When the electron makes a transition from third to second orbit and second orbit to first orbit, the wavelengths emitted are λ_1 and λ_2 respectively then

(a) $\lambda = \lambda_1 + \lambda_2$ (b) $\frac{1}{\lambda} = \frac{1}{\lambda_1} + \frac{1}{\lambda_2}$

(c) $\lambda = \lambda_1 - \lambda_2$ (d) $\lambda_2 = \lambda - \lambda_1$

94. The binding energy of second excited state of L_i+ + according to Bohr's model is

(a) – 108 eV (b) – 40.8 eV

(c) – 13.6 eV (d) none of these

95. If the electron is present in the fifth excited state in hydrogen atom, then maximum number of lines which can be emitted is

(a) 15 (b) 10

(c) 24 (d) 12

96. Model of nucleus is called

(a) shell model (b) Bohr's model

(c) Rutherford model (d) Dirac's model

97. We know that Bohr's model is not applicable to He^+. Then which theory is applied to it

(a) perturbation theory

(b) Schrodinger wave equation

(c) Debye model

(d) Einstein's model

98. The tip of an iron needle behaves as a magnet because

(a) iron is ferromagnetic

(b) tip of the needle contains a single domain

(c) needle gets magnetized during manufacturing

(d) none of these

99. Resistance of a thermistor ____ with rise in temperature

(a) increases (b) decreases

(c) both (a) and (b) (d) remains constant

100. An *LSI* has ____ components.

(a) 10^2 (b) 10^3

(c) 10^4 (d) 10^5

Answers

1.	(d)	**2.**	(b)	**3.**	(d)	**4.**	(d)	**5.**	(a)	**6.**	(d)	**7.**	(b)
8.	(d)	**9.**	(a)	**10.**	(a)	**11.**	(b)	**12.**	(c)	**13.**	(c)	**14.**	(d)
15.	(b)	**16.**	(a)	**17.**	(a)	**18.**	(c)	**19.**	(c)	**20.**	(d)	**21.**	(a)
22.	(a)	**23.**	(d)	**24.**	(c)	**25.**	(b)	**26.**	(a)	**27.**	(d)	**28.**	(a)
29.	(c)	**30.**	(d)	**31.**	(a)	**32.**	(d)	**33.**	(d)	**34.**	(b)	**35.**	(b)
36.	(b)	**37.**	(b)	**38.**	(d)	**39.**	(a)	**40.**	(a)	**41.**	(c)	**42.**	(d)
43.	(c)	**44.**	(b)	**45.**	(b)	**46.**	(d)	**47.**	(a)	**48.**	(d)	**49.**	(c)
50.	(b)	**51.**	(b)	**52.**	(b)	**53.**	(b)	**54.**	(c)	**55.**	(b)	**56.**	(c)
57.	(a)	**58.**	(a)	**59.**	(c)	**60.**	(a)	**61.**	(a)	**62.**	(d)	**63.**	(c)
64.	(b)	**65.**	(b)	**66.**	(d)	**67.**	(c)	**68.**	(b)	**69.**	(b)	**70.**	(c)
71.	(d)	**72.**	(b)	**73.**	(a)	**74.**	(c)	**75.**	(d)	**76.**	(c)	**77.**	(d)
78.	(c)	**79.**	(c)	**80.**	(b)	**81.**	(d)	**82.**	(c)	**83.**	(d)	**84.**	(d)
85.	(c)	**86.**	(c)	**87.**	(a)	**88.**	(c)	**89.**	(c)	**90.**	(a)	**91.**	(b)
92.	(c)	**93.**	(b)	**94.**	(c)	**95.**	(a)	**96.**	(a)	**97.**	(a)	**98.**	(b)
99.	(b)	**100.**	(c)										

Explanations

1(d) $\frac{\Delta g}{g}\times 100 = \frac{\Delta L}{L}+\frac{2\Delta T}{T} = 2 + 1\,(2) = 4\%$

2(b) $B = PV' = ML^2T^{-2}$

3(d) $\because$

4(d) $a_A > a_B$

5(a) $F = \frac{du}{dr} = 30\,r^2 = \frac{mv^2}{r}$

$\Rightarrow v = 100\,\text{ms}^{-1}$

$E = KF + PE = \frac{1}{2}\times 3\,(100)^2 + 10\,(10)^3$

$= 2.5\times 10^4\,\text{J}$

$L = mvr = 3\times 100\times 10 = 3000\ \text{kg m}^2/\text{s}$

6(d) $\tan\theta = \frac{V'^2}{Rg} = \frac{10^2}{10\times 10} = 1$ or $\theta = 45°$

7(b) $\Delta Px = mv\,(\cos\alpha + \cos\beta)$

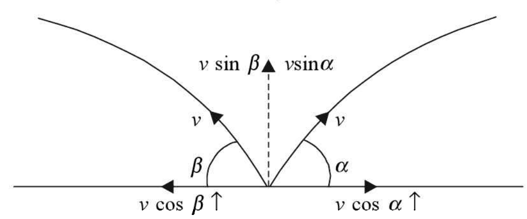


Fig. 11

$\Delta Py = mv\,(\sin\alpha - \sin\beta)$

$\Delta P = \sqrt{\Delta Px^2 + \Delta P^2 y} = mv\sqrt{2+2\cos(\alpha+\beta)}$

$= 2\,mv\cos\frac{(\alpha+\beta)}{2}$

8(d) $\because$ acceleration and velocity are perpendicular.

9(a) $W = m\Delta V = m\left(\frac{GM}{2a}\right)$

$= \frac{mG}{2a}\left(\frac{4}{3}\pi a^3 \rho\right) = \frac{2G}{3} m\pi\rho a^2$

10(a) 17 times the speed of the earth.

11(b) $W = \Delta PE\, GMm \left[\frac{1}{2} - \frac{2}{3R}\right] = GMm\left[\frac{1}{R} - \frac{2}{3R}\right]$

$= \frac{GMm}{3R} = \text{mg}\ \frac{R}{3}$

12(c)

13(c) $v = \sqrt{\frac{p}{\rho}} = \sqrt{\frac{2\times 10^5}{800}} = 15.8\ \text{ms}^{-1}$

14(d) $V_{rel} = \frac{\frac{U_1 + U_2}{1 + U_1 U_2}}{C^2} = \frac{1.6c}{1.64} = 0.975\ c$

15(b)

16(a)

17(a)

18(c) $c = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3\times 8.32\times 300}{32\times 10^{-3}}} = 484\ \text{ms}^{-1}$

19(c) $\Delta s = mc\log_e \frac{T_2}{T_1} = 10\times 2.303\ log\ \frac{313}{273} = 1.37\ \text{cal k}^{-1}$

20(d) $f = \frac{V}{\lambda} = \frac{340}{0.6} = 566.6\ \text{Hz}$

21(a) $I = 2\pi^2 \rho y_0^2 f^2 v$

$= 2\times 10\times 0.32\times (.1)^2\ (850)^2\ (340)$

$= 175\ \text{kWm}^{-2}$

22(a) $\mu = \frac{\sin i}{\sin r} = \frac{\sin i}{\sin 90} = \frac{332}{5200} = .0638$ or $i = 3°\ 39'$

23(d) $f = m\sqrt{\frac{g}{d_2 - d_1}}$

$= 8\sqrt{\frac{9.8}{26\times 10^{-2}}} \sim 49.25\ \text{Hz}$

24(c) $F = mp = 25\times 14$

25(b)

26(a) As time period is independent of g

27(d) $\frac{n_1}{n_2} = \frac{340 + 2\times 2\pi\times 2}{340 - 2\times 2\pi\times 2} = 1.59$

28(a) $v = \sqrt{\frac{2p}{\rho}};\ \frac{dv}{dt} = av$

$= \pi\,(.1)^2\sqrt{2\times 10}$

$= .14\ \text{m}^3\text{s}^{-1}$

29(c) $Mk^2 = I =$ or $k = \sqrt{\frac{7}{5}}R$

30(d)

31(a)

32(d)

33(d) $\frac{f_1}{f_2} = -\frac{\omega_1}{\omega_2}$ or $f_2 = 30\left(\frac{-3}{2}\right) = 45$ cm

34(b) $\frac{L_1}{r_1^2} = \frac{L_2}{r_2^2}$

35(b)

36(b) $n = \frac{a+b}{\lambda}$

37(b)

38(d)

39(a)

40(a)

41(c)

42(d)

43(c)

44(b) $\rho = \frac{1}{\sigma} = \frac{1}{h\mu_h e}$

or $h \cong N_A = \frac{1}{\rho\mu_h e} = \frac{1}{0.2\times 1.2\times 10^8\times 1.6\times 10^{-19}}$

$= 2.6\times 10^{11}$

45(b) $V = (B\sin 30)\, l\, v$

46(d) $M = \frac{V}{di/dt} = \frac{50\times 10^{-3}\times .1}{5} = 1\,mH$

47(a)

48(d) $R_B = \frac{V^2}{P} = 20\ \Omega;\ I(R_B + R) = 200\ V$

or $R = 20\ \Omega$; current thorugh Bulb $= \frac{100}{20} = 5\,A$

49(c) $\because V = \frac{q}{4\pi\varepsilon_0 r}$

50(b)

51(b) $x = k - 1$

52(b) $Q = CV = 10^{-6}\times 80\times 50 = 4\times 10^{-3}C$

new potential $V' = \frac{Q}{c_1 + c_2}\ \frac{4\times 10^{-3}}{400\times 10^{-6}} = 10\text{V}$

charge on second capacitor $Q = C_2\, V' = 3.2\times 10^{-3}\, C$

53(b) $\frac{4}{3} + 2 = \frac{10}{3};\ \frac{\frac{10}{3}.C}{\frac{10}{3} + C} = 1$

$C = \frac{10}{7}\ \mu F$

54(c)

55(b) $\alpha = \frac{d\rho}{\rho dT}$

56(c)

57(a)

58(a) Apply loop law

59(c) For P to be maximum $r = R_L$

R_L net $= \frac{R}{3} = 0.1\,\Omega$ or $R = 0.3\,\Omega$

60(a) $\frac{I^2 5}{\left(\frac{I}{2}\right)^2 4} = \frac{10}{x}$ or $x = 2$ cal s^{-1}

61(a) $B = \frac{\mu_0 I}{2r}\left(\frac{3}{4}\right) = \frac{4\pi\times10^{-7}\times6}{2\times2\times10^{-2}}\times\frac{3}{4} = 1.41\times10^{-4}\,T$

62(d) $T = \frac{2\pi m}{qB}$ or $f = \frac{qB}{2\pi m}$

Thus $B = \frac{2\pi fm}{e} = \frac{6.28\times10^7\times1.6\times10^{-27}}{1.6\times10^{-19}}$

$= 0.628\,T$

63(c) Permeability is less than 1.

64(b) $\cos\delta = \frac{B_H}{B} = \frac{3.6\times10^{-4}}{6\times10^{-4}} = 0.6$ or $\delta = 53°$

65(b)

66(d)

67(c) $\frac{d\phi}{dt} = (10t + 10)\times10^{-3} = (10\times5 + 10)10^{-3}$

$= 0.06V$

68(b)

69(b) $Z_L = Z_S^* = 20 + 16j$

70(b) The instant at which charge on capacitor is zero *i.e.*

$Q = Q_o\cos\omega t$ or $t = \frac{T}{4} = \frac{\pi\sqrt{LC}}{2} = 1.58$ ms

71(d) As $f = 10^8$ Hz

72(b)

73(a)

74(c)

75(d) It will appear of complementary colour, i.e., cyan.

76(c)

77(d) $\frac{v}{c} = \frac{0.2}{100}$ or $v = 6\times10^5$ ms^{-1}

78(c)

79(c) $t = \frac{a}{v - v\cos 60} = \frac{2a}{v}$

80(b)

81(d)

82(c) $V = -\int_\infty^x E.\,dx = \frac{K}{2x^2}$

83(d)

84(d)

85(c) $V = f\lambda$

or $\sqrt{\frac{T}{\mu}} = f\lambda$

or $\frac{\sqrt{T}}{\lambda} = f\sqrt{\mu}$

Thus $\frac{\sqrt{T_1}}{\lambda_1} = \frac{\sqrt{T_2}}{\lambda_2}$ or $\frac{\sqrt{2}}{.06} = \frac{\sqrt{8}}{\lambda}$

or $\lambda = 0.12\,m$

86(c)

87(a) $16 = 2^4$ or $4t_{1/2} = 30$h or $t_{1/2} = 7.5$h

88(c)

89(c)

90(a) Use $2d\sin\theta = n\lambda$

91(b) $r = \frac{2(ze)e}{4\pi\varepsilon_0(KE)}$

92(c) $r_n = \frac{n^2}{z} r_B = \frac{9\times.53}{3} = 1.59$ A°

93(b) $\frac{hc}{\lambda} = \frac{hc}{\lambda_1} + \frac{hc}{\lambda 2}$

94(c) $E_n = \frac{z^2E_1}{n^2} = \frac{9}{9}\times(-13.6)\,eV = -13.6\,eV$

95(a) No. of lines $= \frac{n(n-1)}{2} = \frac{6\times5}{2} = 15$

96(a)

97(a)

98(b)

99(b)

100(c)

MODEL TEST PAPER 5

1. The dimensions of solar constant are

(a) $[MT^{-3}]$ (b) $[ML^2T^{-3}]$

(c) $[MT^{-3}k^{-4}]$ (d) $ML^{-1}T^{-3}K^{-4}]$

2. If F_r and F_θ are radial and angular forces and r is perpendicular distance then torque τ is given by (F is total force)

(a) $\tau = F_r \times r$ (b) $\tau = F_\theta \times r$

(c) $F_\theta \times r + F_r \times r$ (d) none of these

3. Maximum particle velocity is equal to

(a) wave velocity × maximum strain

(b) $\dfrac{\text{wave velocity}}{\text{maximum strain}}$

(c) wave velocity

(d) maximum wave velocity × strain

4. The overtones produced in a closed pipe are ___ multiples of fundamental frequency f_o

(a) odd (b) even

(c) all integral (d) none of these

5. A thermometer reads 80° C when mercury level is 8.4 cm, it reads 40° C when mercury level is 5.6 cm. The temperature when mercury level is 4.8 cm is

(a) 31.2° C (b) 33.3° C

(c) 30.1° C (d) 28.6° C

6. When one moves up by 2 km, the temperature falls by

(a) 6° C (b) 8° C

(c) 10° C (d) 12° C

7. Specific heat of solids according to Dulong and Petit's law is ___ cal g^{-1} °C^{-1}

(a) 2 (b) 4

(c) 6 (d) 8

8. The work done in the given indicator diagram is

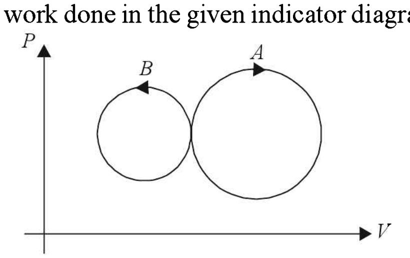


Fig. 1

(a) negative (b) positive

(c) cannot be predicted (d) positive or negative

9. Thermal resistance is

(a) inversely proportional to length of conductor

(b) directly proportional to length of the conductor

(c) depends only on the nature of material

(d) none of these

10. $\lambda_m T = b$ (constant) is called

(a) Rayleigh Jean's law (b) Dulong Petit's law

(c) Wein's displacement law

(d) none of these

11. The moment of inertia of a rectangular lamina of length l, width b and having mass M, through its centre of mass and perpendicular to the lamina is

(a) $M\left(\dfrac{l^2+b^2}{2}\right)$ (b) $M\left(\dfrac{l^2+b^2}{3}\right)$

(c) $M\left(\dfrac{l^2+b^2}{6}\right)$ (d) $M\left(\dfrac{l^2+b^2}{12}\right)$

12. The unit of magnetic flux is

(a) Tesla (b) $A - m$

(c) Gauss (d) Maxwell

13. The significant figures in 0.08760 are

(a) 3 (b) 5

(c) 4 (d) 6

14. Specific resistance of a material was measured using

$$\sigma = \frac{\pi r^2 R}{l}$$

If $R = 54 \pm 2\Omega, l = 136.0 \pm 0.1\text{cm}, r = 0.26 \pm 0.02$ cm then percentage error in specific resistance is

(a) 9.2 % (b) 12.4 %

(c) 15.8 % (d) 19.2 %

15. The displacement of the $(a - t)$ graph shown will be

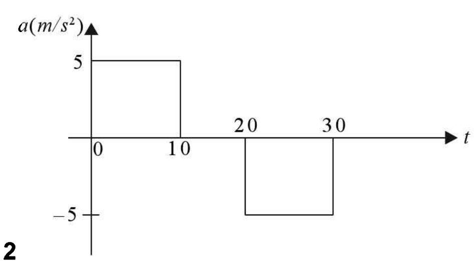


Fig. 2

(a) 250 m (b) 500 m

(c) 750 m (d) 1000 m

16. The velocity at $t = 0$ is

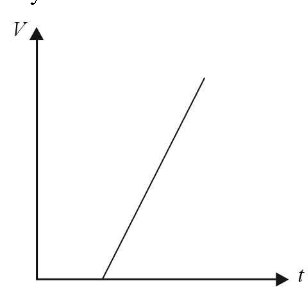


Fig. 3

(a) negative (b) positive
(c) zero (d) cannot be predicted

17. Two vectors are $5\,i + 3\,j$ and $6\,i - j$. Their resultant has magnitude

(a) $5\sqrt{5}$ (b) 5
(c) 10 (d) 125

18. If maximum height achieved is h when a particle is thrown up then maximum range will be

(a) h (b) $2\,h$
(c) $3\,h$ (d) $\frac{3}{2}\,h$

19. The range on an inclined plane of inclination α will be maximum for a projectile projected with a velocity u making an angle θ with the horizontal when

(a) $\theta = 2\alpha$ (b) $\alpha = 2\theta - \frac{\pi}{2}$
(c) $\alpha = \frac{\pi}{2} - \theta$ (d) $\alpha = \pi - 2\,\theta$

20. A vehicle of mass m having distance between inner wheel a, having centre of gravity at a height h from the ground taking a turn of radius r will have maximum velocity

(a) $V_{max} = \sqrt{r\mu g}$ (b) $V_{max} = \sqrt{\frac{rga}{\tan\theta h}}$
(c) $V_{max} = \sqrt{\frac{rag}{2h}}$ (d) $V_{max} = \sqrt{\frac{rag}{h}}$

21. A cubical room is made of mirrors. An ant is moving along the diagonal of the floor in a straight line such that the velocity of the image in two adjacent wall mirrors is 20 cm/s. Then velocity of the image of the ant in the ceiling mirror is

(a) 20 cm/s (b) 10 cm/s
(c) $10\sqrt{2}$ cm/s (d) $20\sqrt{2}$ cm/s

22. A projectile is fired at an angle θ with velocity $3i + 4j$ on return to the ground, its velocity will be

(a) $-3i - 4j$ (b) $-3i + 4j$
(c) $3i - 4j$ (d) $3\,i + 4\,j$

23. If $y = x - \frac{x^2}{2}$ is the trajectory of a particle, then the time of flight is

(a) $\frac{2}{\sqrt{g}}$ (b) $\frac{3}{\sqrt{g}}$
(c) $\frac{2\sqrt{2}}{\sqrt{g}}$ (d) $\frac{4}{\sqrt{g}}$

24. A given object takes n times as much time to slide down a 45° rough inclined plane as it takes to slide down a perfectly inclined plane. The coefficient of friction between the object and inclined plane is

(a) $\left(1 - \frac{1}{n^2}\right)$ (b) $\left(\frac{1}{1-n^2}\right)$
(c) $\sqrt{1 - \frac{1}{n^2}}$ (d) $\sqrt{\frac{1}{1-n^2}}$

25. A chain of mass m and length l placed on a frictionless table such that its $\frac{l}{n}$ part is over hanging. The amount of work done in pulling it up is

(a) $mg\,\frac{l}{n}$ (b) $mg\,\frac{l}{n^2}$
(c) $mg\,\frac{l}{2n^2}$ (d) $mg\,\frac{l}{2n}$

26. The coefficient of restitution for a perfectly elastic collision is

(a) -1 (b) 0
(c) 1 (d) ∞

27. A particle of mass 4 m at rest explodes into 3 fragments. Two fragments of mass m each move at speed V each perpendicular to one another. Then energy of the fragment of mass $2\,m$ will be

(a) mV^2 (b) $\frac{3}{2}\,mV^2$
(c) $2\,mV^2$ (d) $\frac{mv^2}{2}$

28. A sphere, a disc and a ring of the same radius slide down an inclined plane of angle θ, then ____ touches the ground at the last.

(a) sphere (b) ring
(c) disc
(d) all of them touch the ground at a time

29. One meter rod is kept horizontal and is hinged at one end and is allowed to fall down. The velocity of the other end at bottom is

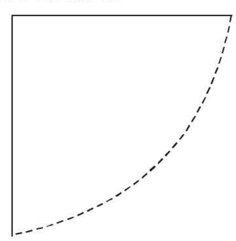


Fig. 4

(a) $\sqrt{3g}$ (b) $\sqrt{2g}$
(c) $\sqrt{g}$ (d) none of these

30. If the distance between earth and sun becomes one fourth of the present distance, then duration of the year will be

(a) $\frac{1}{2}$ year (b) $\frac{1}{4}$ year
(c) $\frac{1}{6}$ year (d) $\frac{1}{8}$ year

31. If a capillary tube is of insufficient length, then if it is dipped in a liquid

(a) liquid will fall down

(b) extra liquid will stick on the walls of the capillary tube

(c) extra liquid will collect as a drop on the top of capillary tube

(d) none of these

32. If two wires of equal length and equal area of cross-section made of steel and copper wire having young modulus Y_s and Y_c respectively are stretched by equal weights then their strains have ratio

(a) $\frac{Y_s}{Y_c}$ (b) $\frac{Y_s}{Y_c}$

(c) $\frac{2Y_s}{Y_c}$ (d) none of these

33. A slab consists of two parellel layers of two different materials of same thickness having thermal conductivities K_1 and K_2, then find combined conductivity.

(a) $k_1 + k_2$ (b) $\frac{k_1+k_2}{2}$

(c) $\frac{k_1+k_2}{k_1k_2}$ (d) $\frac{2k_1k_2}{k_1+k_2}$

34. For a sharp-edged circular hole the area of vena contracta is _________ of the hole area.

(a) 10 % (b) 32 %

(c) 51 % (d) 62 %

35. Kater's pendulum is a ____ pendulum.

(a) simple (b) compound

(c) torsional (d) none of these

36. Two particles are moving in opposite but uniform circular motions of radius r and of angular velocity 'ω' each then their resultant is

(a) circular motion (b) motion in a straight line

(c) *SHM* of amplitude r (d) *SHM* of amplitude $2r$

37. The radius of Si nucleus is

(a) 3.3×10^{-15} (b) 4.3×10^{-15}

(c) 5.3×10^{-15} (d) $2.3 \times 10^{-15} m$

38. The fission process is based on ____ theory.

(a) liquid drop model (b) binding force

(c) Yukawa theroy (d) Becquerel theory

39. A radioactive source and a capacitor in *RC* discharging circuit have undecayed nuclei to charge ratio constant. If $R = 2\ k\ \Omega$ and $C = 10\ \mu F$, then half life of the radioactive sample is nearly

(a) 12 *ms* (b) 39 *ms*

(c) 71 *ms* (d) 139 *ms*

40. If H_α - line of Balmer series is 567.5 nm, then H_β line of paschen series will be nearly

(a) 798 *nm* (b) 912 *nm*

(c) 1054 *nm* (d) 1128 *nm*

41. Which of the following devices shows a negative resistance?

(a) tunnel diode (b) zener diode

(c) varactor diode (d) photo diode

42. Dark current is a term associated with

(a) current in a dark room (b) photo cell

(c) transformer (d) vacuum tube

43. A capacitor is designed in the from of a staircase as shown in Fig. 5. If the base area is A then capacity of the system is

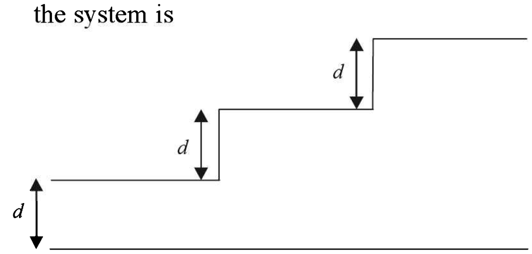


Fig. 5

(a) $\frac{A\varepsilon_o}{d}$ (b) $\frac{3A\varepsilon_o}{2d}$

(c) $\frac{5}{9}\frac{A\varepsilon_o}{d}$ (d) $\frac{11}{18}\frac{A\varepsilon_o}{d}$

44. A particle of mass m, charge q is thrown at a speed u against an electric field E. The distance travelled by it before coming momentarily to rest is

(a) $\frac{mu^2}{2qE}$ (b) $\frac{mu}{2qE}$

(c) $\frac{mu^2}{qE}$ (d) $\frac{2mu^2}{qE}$

45. A positive charge q is placed in front of a conducting solid cube. Electric field at the centre of the cube due to charges appearing on its surface is

(a) zero (b) $\frac{q}{8\pi\varepsilon_o d^2}$

(c) $\frac{q}{4\pi\varepsilon_o d^2}$ (d) $\frac{q}{24\pi\varepsilon_o d^2}$

46. A particle having a charge 2×10^{-6}C and a mass 100 g is placed at the bottom of a smooth inclined plane of inclination 30°. Where should another paticle B of same mass and charge be placed on the incline so that it may remain in equilibrium?

(a) 15 cm from bottom (b) 21 cm from bottom

(c) 27 cm from bottom (d) 33 cm from bottom

47. The potential difference is ____ when a charge 0.01C moves from A to B when work done is 12 J.

(a) 1200 V (b) 120 V

(c) 0.12 V (d) 1.2 V

48. A charge Q is placed at a distance $l/2$ above the centre of a horizontal square surface of edge l. The electric flux passing through it is

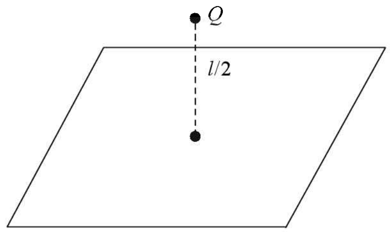


Fig. 6

(a) $\frac{Q}{\varepsilon_o}$ (b) $\frac{Q}{2\varepsilon_o}$

(c) $\frac{Q}{4\varepsilon_o}$ (d) $\frac{Q}{6\varepsilon_o}$

49. Two large conducting plates are placed parallel to each other at a separation of 2 cm. An electron starting from rest near one of the plates reaches the other plate in 2 µs. The surface charge density on the inner surfaces is

(a) 0.505×10^{-12} C/m² (b) 5.05×10^{-12} C/m²

(c) 5.05×10^{-10} C/m² d) none of these

50. The charge on 25 µF capacitor is

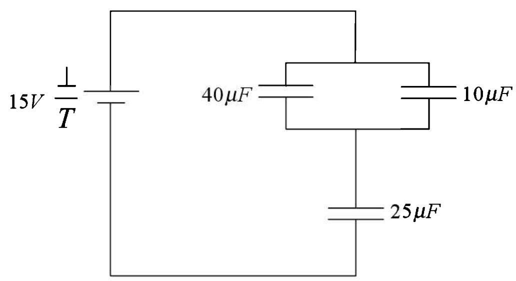


Fig. 7

(a) 100 µC (b) 250 µC

(c) 375 µC (d) none of these

51. The capacitance between A and B of the infinite series is

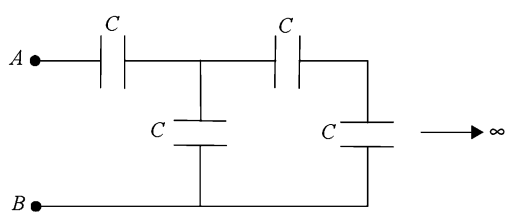


Fig. 8

(a) $\frac{1+\sqrt{5}}{2}C$ (b) $\frac{\sqrt{5}-1}{2}C$

(c) $2+\sqrt{5}\,C$ (d) $1+\sqrt{5}\,C$

52. A parallel plate capacitor is connected to a battery of emf Vo. Battery is removed after the capacitor is charged and a dielectric of strength k is introduced then potential difference between the two plates is

(a) V_0 (b) $k\,V_0$

(c) $\frac{V_0}{k}$ (d) none of these

53. A parallel plate capacitor with plate area 100 cm² and separation between the plates 1cm is connected across a battery of emf 24 V. The force of attraction between the plates is

(a) $2 \times 10^{-7} N$ (b) $2.5 \times 10^{-7} N$

(c) $3 \times 10^{-7} N$ (d) $3.5 \times 10^{-7} N$

54. The colour code of 1Ω resistance is

(a) Black Brown Black (b) Brown Black Black

(c) Black Black Black (d) Brown Black Gold

55. If $I\ (mA) = 10\ V^2$ for a device, then resistance of the device when $V = 40\ V$ is

(a) 800 Ω (b) 20 Ω

(c) 1.25 Ω (d) none of these

56. The current through resistance R in the circuit shown is

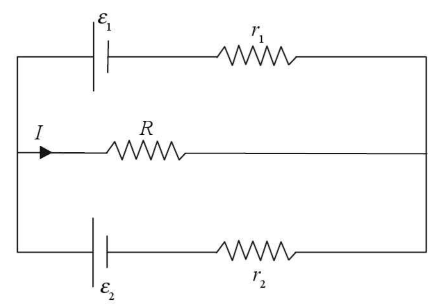


Fig. 9

(a) $\frac{\varepsilon_1 r_2 + \varepsilon_2 r_1}{(r_1 + r_2)R}$ (b) $\frac{\varepsilon_1 r_2 + \varepsilon_2 r_1}{r_1 r_2 + (r_1 + r_2)R}$

(c) $\frac{\varepsilon_1 r_1 + \varepsilon_2 r_2}{(r_1 + r_2)R}$ (d) $\frac{\varepsilon_1 r_1 + \varepsilon_2 r_2}{r_1 r_2 + (r_1 + r_2)R}$

57. If each resistance is r then resistance across BC is

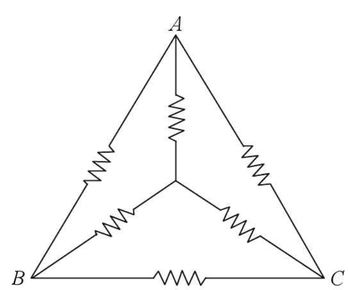


Fig. 10

(a) r (b) $\frac{r}{2}$

(c) $\frac{r}{3}$ (d) $\frac{r}{4}$

58. The currents I_1, I_2 and I_3 in the circuit are

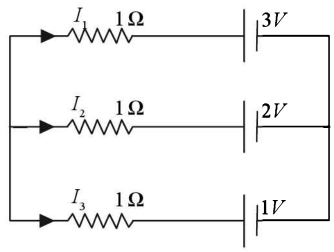


Fig. 11

(a) $-1A, 0, -1A$ (b) $1A, 0, -1A$

(c) $-1A, 0, 1A$ (d) $0, 1A, -1A$

59. The potential difference $V_a - V_b$ in the given circuit is

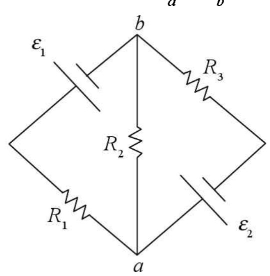


Fig. 12

(a) $\dfrac{(\varepsilon_1+\varepsilon_2)R_3}{R_1+R_2+R_3}$ (b) $\dfrac{(\varepsilon_1-\varepsilon_2)R_3}{R_1+R_2+R_3}$

(c) $\dfrac{(\varepsilon_1 R_2+\varepsilon_2 R_1)R_2}{R_1R_2+R_2R_3+R_3R_1}$ (d) none of these

60. The charge on $1\mu F$ capacitor in the circuit is

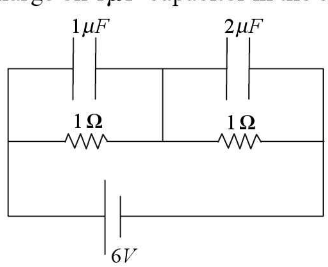


Fig. 13

(a) 1 μC (b) 2 μC

(c) 3 μC (d) zero

61. Emf generated in copper/Iron thermocouple is 0.8 V. When same temperature difference was maintained emf generated between Iron/constanton was 1.3 V. Then emf between copper/ constantan thermocouple for the same temperature difference is

(a) –0.5 V (b) +0.5 V

(c) –2.1 V (d) 2.1 V

62. A lead accumulator when fully discharged (in order to be recharged) shall have emf

(a) 0.2 V (b) 1.2 V

(c) 0.8 V (d) 1.8 V

63. A charge 2 μC moves with a speed $2 \times 10^{+6}$ m/s along + x direction. A magnetic field B $(0.2\ \hat{j} + 0.4\ \hat{k})$ T exists in space. The magnetic force is

(a) $0.8\ \vec{k}$ (b) $1.6\ \vec{k}$

(c) $0.8k+j$ (d) $0.8k-1.6j$

64. Maximum torque acting on a loop having 100 turns of radius 2 cm, carrying a current 1 A when placed in a magnetic field of 2 m T is

(a) $4\pi \times 10^{-3}$ Nm (b) $8\pi \times 10^{-3}$ Nm

(c) $8\pi \times 10^{-5}$ Nm (d) $4\pi \times 10^{-5}$ Nm

65. The magnetic field at O in the given loop of radius 10 cm when a current 2 A passes through it is

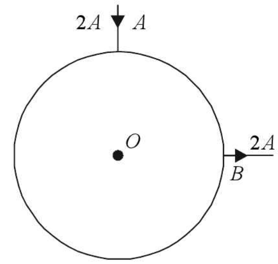


Fig. 14

(a) zero (b) $4\pi \times 10^{-6}$ T

(c) $2\pi \times 10^{-6}$ T (d) $\pi \times 10^{-6}$ T

66. The magnetic field existing in the region is given by $\hat{B} = B_0\left(1-\dfrac{x}{a}\right)\hat{k}$. A square loop of edge a, carrying current i, is placed with its edges parallel to $x-y$ axes. Magnitude of net magnetic force is

(a) zero (b) $i B_0 a$

(c) $2 i B_0 a$ (d) $\sqrt{2}\ i B_0 a$

67. A particle having mass m and charge q is released from the origin in a region in which magnetic field and electric field are given by $\vec{B} = B_0\ \hat{j}$ and $\vec{E} = E_0\ \hat{k}$. The speed of the particle as a function of its z - coordinate

(a) $\sqrt{\dfrac{qE_0 z}{2m}}$ (b) $\sqrt{\dfrac{qE_0 z}{m}}$

(c) $\sqrt{\dfrac{2qE_0 z}{m}}$ (d) zero

68. In the figure shown $\oint B.\,dl = \mu_0 i$ where i = ___

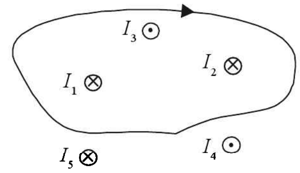


Fig. 15

(a) $i = I_1 + I_2 - I_3$ (b) $I_1 + I_2 + I_5 - I_3 - I_4$

(c) $I_1 + I_2 + I_3 - I_4 - I_5$ (d) $I_1 + I_2 + I_3$

69. Two long wires carry current 2 A each. They are parallel and are separated by 50 cm. The magnetic field at point P (mid point of separation between two wires) is

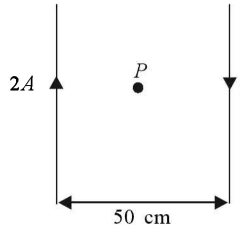


Fig. 16

(a) $16 \times 10^{-7} T$ (b) $32 \times 10^{-7} T$
(c) $64 \times 10^{-7} T$ (d) $8 \times 10^{-7} T$

70. The magnetic field at the centre of a circular ring of radius 20 cm is

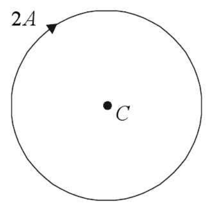


Fig. 17

(a) $4\pi \times 10^{-7} T$ (b) $10\pi \times 10^{-1} T$
(c) $20\pi \times 10^{-7} T$ (d) $40\pi \times 10^{-7} T$

71. Two short magnets are connected as shown. The magnetic moment at a point P distant d from the centre is

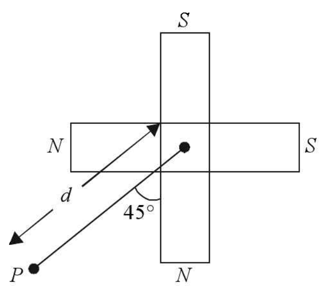


Fig. 18

(a) $\frac{\mu_0 M}{4\pi d^3}$ (b) $\frac{\mu_0 \sqrt{2} M}{4\pi d^3}$
(c) $\frac{\mu_0 2\sqrt{2} M}{4\pi d^3}$ (d) $\frac{\mu_0 2M}{4\pi d^3}$

72. If A wire of length l is stretched to double its length, then resistivity of the wire will

(a) become double (b) become 4 times
(c) become half (d) remain unchanged

73. Between two successive collisions a free electron in the absence of applied electric field moves

(a) in a straight line
(b) in a curved path
(c) in any manner, i.e, curved or straight line
(d) none of these

74. A photo cell has work function 3.2 eV, then photo, electric emission is possible by

(a) 100 W incandescent bulb
(b) 40 W flourescent lamp
(c) 400 W sodium lamp
(d) 10 W uv source

75. A beam of white light is incident on a plane surface absorbing 70% of light and reflecting rest. If the incident beam carries 10 W power then force exerted by it is

(a) $2 \times 10^{-8} N$ (b) $2.3 \times 10^{-8} N$
(c) $4.3 \times 10^{-8} N$ (d) $0.3 \times 10^{-8} N$

76. When light radiation is incident on a substance then

(a) photon can be completely absorbed by a free electron
(b) photon cannot be completely absorbed by a free electron
(c) photon is also released along with free electron
(d) none of these

77. The momentum of a photon of wavelength 600 nm is

(a) 3×10^{-34} kg ms^{-1} (b) 2.2×10^{-26} kg ms^{-1}
(c) 1.1×10^{-27} kg ms^{-1} (d) 3.1×10^{-28} kg ms^{-1}

78. A solenoid of inductance 50 mH and resistance 10Ω is connected to a battery of 6 V. The time elapsed before the current acquires half the steady state value is

(a) 1.5 ms (b) 2.5 ms
(c) 3.5 ms (d) none of these

79. Two conducting circular loops of radius R_1 and R_2 are placed in the same plane with their centres coinciding. If $R_2 >> R_1$ then mutual inductance between them is ____ (Assume current is established in outer coil).

(a) $\frac{\mu_0 \pi R_1^2}{2R_2}$ (b) $\frac{\mu_0 \pi R_2^2}{2R_1}$
(c) $\frac{\mu_0 \pi (R_2^2 + R_1^2)}{2R_1 + R_2}$ (d) $\frac{\mu_0 \pi (R_2^2 - R_1^2)}{2R_2}$

80. In an RL circuit $R = 1\Omega$, $L = 4H$ and a battery of emf 6 V is switched ON and $t = 0$. The power dissipated at $t = 4\,s$ is

(a) 14 J (b) 7 J
(c) 1.4 J (d) none of these

81. The power consumed in a series LCR circuit is ___ at resonance.

(a) minimum (b) maximum
(c) equal to that at off resonance frequencies
(d) zero

82. Bandwidth of a series LCR circuit is

(a) $R^2C^2 + 4LC$ (b) $2\sqrt{R^2C^2 + 4LC}$
(c) $\frac{\sqrt{R^2C^2 + 4LC}}{LC}$ (d) none of these

83. A radio operates on 6 V dc. A transformer of 18 turns in secondary is used to convert ac mains to 6 VAC which is further rectified to give 6 V dc. The number of turns in the primary are

(a) 330 (b) 660

(c) 600 (d) none of thes

84. The electric current in a circuit is given by $i = i_0 \frac{t}{\tau}$ then $r\,ms$ value in the period $0 < t < \tau$

(a) $\frac{i_0}{2}$ (b) $\frac{i_0}{\sqrt{2}}$

(c) $\frac{i_0}{3}$ (d) $\frac{i_0}{\sqrt{3}}$

85. The height of an antenna which can serve an area 314 km^2 is nearly

(a) 8 m (b) 80 m

(c) 18 m (d) 180 m

86. Two needles are fixed 100 cm apart. At a position of lens lying in between , the image of one needle is formed over the other. If lens is displaced by 10 cm again the image of one needle is formed over the other, then focal length of the lens is

(a) 26.5 cm (b) 24. 75 cm

(c) 21.75 cm (d) none of these

87. A lens of power 4 D is immersed from one side in water then the focal length of the system is_____ (μ = 1.5)

(a) 50 cm (b) 100 cm

(c) 150 cm (d) none of these

88. To resolve sodium light the number of lines required in a plane transmission grating in first order spectrum is

(a) 1000 (b) 1500

(c) 500 (d) 4000

89. Accroding to Moseley's law, wavelength and atomic number of characteristic x-ray of the element are related as

(a) $\sqrt{\lambda} \propto z$ (b) $\lambda \propto z$

(c) $z \propto \lambda^{-1}$ (d) $z \propto \lambda^{-1/2}$

90. $B + \overline{B}$ C can be implemented using

(a) OR gate (b) AND gate

(c) NOT gate

(d) all (*a*) , (*b*) and (*c*) are required

91. (137.25) in binary number is

(a) 10001001.01 (b) 10010001.01

(c) 10001010.01 (d) none of these

92. A transistor with h_{FE} = 49 is used in a commonbase amplifier. The maximum value of current gain in a CB amplifier will be

(a) 98 (b) 49

(c) 0.49 (d) 0.98

93. Specific activity of a radio active sample is

(a) activity/mass (b) Ci

(c) Bq (d) λN

94. In a breeder reactor fuel used is

(a) ${}^{239}_{92}U$ (b) ${}^{236}_{92}U$

(c) ${}^{239}_{92}Pu$ (d) ${}^{239}_{93}Np$

95. In interference experiment with double slits, one of the slit is twice wider of the other. Then ratio of intensity of bright to dark fringe is

(a) 2 (b) 4

(c) 3 (d) 9

96. In case of a plane mirror (thick) the brightest image is

(a) first (b) second

(c) third (d) fourth

97. The mirror which gives only positive magnification is

(a) plane (b) convex

(c) concave (d) (a) and (b) both

98. To measure temperature accurate up to 10^{-3} °C we shall use_______ thermometer.

(a) pyrometer (b) mercury

(c) thermocouple (d) thermistor

99. The reflected and refracted rays are at right angles to each other and angle of refraction is 37°, then refractive index of the medium is

(a) $\frac{3}{2}$ (b) $\frac{4}{3}$

(c) $\frac{1}{\sin 37}$ (d) $\frac{1}{\cos 37}$

100. The pressure of air in the bulb of a constant volume gas thermometer is 73 cm of Hg at 0°C, 100.3 cm at 100° C and 77.8 cm of Hg at room temperature, then room temperature is

(a) 17°C (b) 27°C

(c) 7°C (d) none of these

Answers

1.	(a)	**2.**	(b)	**3.**	(a)	**4.**	(a)	**5.**	(d)	**6.**	(d)	**7.**	(c)
8.	(b)	**9.**	(b,c)	**10.**	(c)	**11.**	(d)	**12.**	(d)	**13.**	(c)	**14.**	(b)
15.	(d)	**16.**	(a)	**17.**	(a)	**18.**	(b)	**19.**	(b)	**20.**	(c)	**21.**	(d)
22.	(c)	**23.**	(a)	**24.**	(a)	**25.**	(c)	**26.**	(c)	**27.**	(d)	**28.**	(d)
29.	(a)	**30.**	(d)	**31.**	(c)	**32.**	(d)	**33.**	(a)	**34.**	(d)	**35.**	(b)
36.	(d)	**37.**	(a)	**38.**	(a)	**39.**	(d)	**40.**	(d)	**41.**	(a)	**42.**	(b)
43.	(d)	**44.**	(a)	**45.**	(c)	**46.**	(c)	**47.**	(a)	**48.**	(a)	**49.**	(a)
50.	(b)	**51.**	(b)	**52.**	(c)	**53.**	(b)	**54.**	(d)	**55.**	(c)	**56.**	(b)
57.	(c)	**58.**	(c)	**59.**	(c)	**60.**	(c)	**61.**	(d)	**62.**	(d)	**63.**	(d)
64.	(c)	**65.**	(a)	**66.**	(b)	**67.**	(c)	**68.**	(a)	**69.**	(b)	**70.**	(c)
71.	(b)	**72.**	(d)	**73.**	(b)	**74.**	(d)	**75.**	(c)	**76.**	(b)	**77.**	(c)
78.	(c)	**79.**	(a)	**80.**	(a)	**81.**	(b)	**82.**	(c)	**83.**	(b)	**84.**	(d)
85.	(a)	**86.**	(b)	**87.**	(a)	**88.**	(a)	**89.**	(d)	**90.**	(a)	**91.**	(a)
92.	(d)	**93.**	(a)	**94.**	(a)	**95.**	(d)	**96.**	(b)	**97.**	(a, b)	**98.**	(d)
99.	(b)	**100.**	(a)										

Explanations

1(a)

2(b)

3(a)

4(a)

5(d) $\frac{80-40}{80-T} = \frac{8.4-5.6}{8.4-4.8}$

or $\frac{40}{80-T} = \frac{2.8}{3.6}$

or $2.8T = 80 \times 2.8 - 40 \times 3.6$

$= 40\,(5.6-3.6)\,T\,\frac{80}{2.8}$

$= \frac{20}{.7} = 28.57\ °C$

6(d)

7(c)

8(b)

9(b,c) $R_{\text{Thermal}} = \frac{l}{KA}$

10(c)

11(d)

12(d) C G S unit is Maxwell and SI unit is weber.

13(c)

14(b)

15(d)

16(a)

17(a) $\vec{x} = 5\hat{i} + 3\hat{j} + 6\hat{i} - \hat{j}$; $|x| = \sqrt{11^2+2^2} = 5\sqrt{5}$

18(b)

19(b)

20(c) $\frac{mv^2}{r}\ h = \text{mg}\ \frac{a}{2}$

21(d) $v \cos 45 = 20$ or $v = 20\sqrt{2}cms^{-1}$

22(c)

23(a)

24(a)

25(c) $\omega = \frac{mg}{n}\left(\frac{l}{2n}\right) = \frac{mgl}{2n^2}$

26(c)

27(d)

28(b)

29(a) $Mg\,\frac{l}{2} = \frac{1}{2} I\,\omega^2$

or $\frac{mg}{2} = \frac{1}{2} m \frac{l^2}{3} \omega^2$

$\Rightarrow\ l\,\omega = v = \sqrt{3g}$

30(d) $T^2 \propto R^3$

31(c)

32(b)

33(a)

34(d)

35(b)

36(d)

37(a) use $R = R_0 A^{1/3}$

38(a)

39(d) $\frac{N}{Q} = \frac{N_0 e^{-\lambda t}}{Q_0 e^{-t/RC}}$

$\Rightarrow \quad \lambda = \frac{1}{RC}$

or $\quad t_{1/2} = \frac{0.693}{\lambda}$

40(d) $\frac{\lambda_l}{\lambda} = \frac{\left[\frac{1}{2^2} - \frac{1}{3^2}\right]}{\left[\frac{1}{3^2} - \frac{1}{5^2}\right]} = \frac{\frac{5}{36}}{\frac{16}{9\times 25}} = \frac{5\times 9\times 25}{36\times 16} \frac{125}{64}$

$\lambda = \lambda_1 \frac{125}{64} = 567.5 \times \frac{125}{64} \cong 1128\, nm$

41(a)

42(b)

43(d) Three capacitors are in parallel

$C = C_1 + C_2 + C_3$

$= \frac{A\varepsilon_0}{3d} + \frac{A\varepsilon_0}{6d} + \frac{A\varepsilon_0}{9d} = \frac{11A\varepsilon_0}{18d}$

44(a) $v^2 - u^2 = 2\,as$ or $s = \frac{u^2}{2a}$

$= \frac{u^2 m}{2qE}$

45(c)

46(c) $mg\sin\theta = \frac{q^2}{4\pi\varepsilon_0 x^2}$ or $x^2 = \frac{2\times 4\times 10^{-12}\times 9\times 10^9}{9.8\times .1}$

or $\quad x = 27$cm

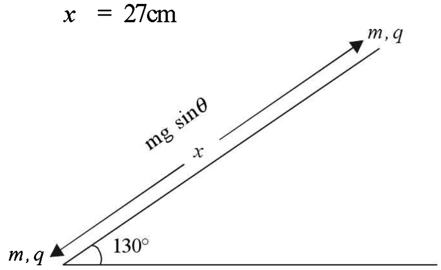


Fig. 19

47(a)

48(d)

49(a) $x = \frac{1}{2} a t^2 \left(\frac{eE}{m}\right) t^2 = \left(\frac{e\sigma}{m}\right) t^2$

50(b)

51(b)

52(c) $v = \frac{Q}{kC} = \frac{CV_o}{kC} = \frac{V_o}{k}$

53(b) $F = \frac{1}{2}\frac{CV^2}{d} = \frac{\varepsilon_0 A V^2}{2d^2}$

54(d)

55(c) $I = 10V^2$

or $\quad dI = 20V dv$

or $\quad r = \frac{dV}{dI} = \frac{10^3}{800} = 1.25\Omega$

56(b) $E_{eq} = \frac{\varepsilon_1 r_2 + \varepsilon_2 r_4}{r_1 + r_2}, I = \frac{\varepsilon er}{\frac{r_1 r_2}{r_1 + r_2} + R}$

57(a) $r_{eq} = r/2$

58(c)

59(c) $V_{ab} = \frac{\varepsilon_{eq} R_2}{\frac{R_1 R_3}{R_1 + R_3} + R_2}$ and $\varepsilon_{eq} = \frac{\varepsilon_1 R_3 + \varepsilon_2 R_1}{R_1 + R_3}$

60(c)

61(d) $\varepsilon_{\text{Copper constantan}} = \varepsilon_{\text{Copper Iron}} + \varepsilon_{\text{Iron constantan}}$

62(d)

63(d) $F = 2\times 10^{-6}(2\times 10^6 \hat{i}) \times (0.2\hat{j} + 0.4\hat{k})$

64(c) $\tau = nI(AB)$

65(a)

66(b)

67(c) $V^2 = 2\,as$

or $= \sqrt{2as}$

$= \sqrt{2\frac{qE_0}{m} z}$

68(a)

69(b) $B = \frac{2\mu_0 i}{2\pi d} = \frac{2\times 4\pi\times 10^{-7}\times 2}{2\pi\times 0.25}$

70(c) $B = \frac{\mu_0 i}{2r}$

71(a) use $B = \frac{\mu_0 M}{4\pi d^3}\sqrt{3\cos^2\theta + 1}$

72(d)

73(b)

74(d)

75(c) $F = \frac{7}{c} + \frac{2(3)}{c} = \frac{13}{3\times 10^8} N$

76(b)

77(c) $p = \frac{h}{\lambda}$

78(c) $\frac{1}{2} = (1 - e^{\frac{-t}{\tau}})$

79(a) $B = \frac{\mu_0 i}{2R_2}$

$$\phi = \frac{\mu_0 i}{2R_2} \pi R_1^2$$

$$M = \frac{\phi}{i} = \frac{\mu\pi R_1^2}{2R_2}$$

80(a) $\tau = \frac{L}{R} = 4\,s$

$$i = \frac{\varepsilon}{R}(1 - e^{-1})$$
$$= 6 \times (.633)$$
$$= 3.8A;$$
$$P = i^2R = 3.8^2 \times 1 = 14 \text{ J}$$

81(b)

82(c)

83(b) $\frac{N}{18} = \frac{220}{6}$ or $N = 660$

84(d) $i_{rms}^2 = \frac{1}{\tau}\int_0^\tau i^2 dt = \frac{i_0^2}{3}$

85(a) Use $(2\,Rh)\,\pi = 314$ or $h = \frac{100}{12800}$ km

86(b) Use $f = \frac{D^2 - d^2}{4d} = \frac{100^2 - 10^2}{400} = 24.75$ cm

87(a) $f^l = 2f$

88(a) $R.P = \frac{\lambda}{d\lambda} = nN$

for $n = 1$

$$N = \frac{5893}{6} = 983$$

89(d)

90(a) $B + \overline{B}C = B + C$

91(a)

92(d) $\alpha = \frac{\beta}{1+\beta} = \frac{49}{50} = .98$

93(a)

94(a)

95(d)

96(b)

97(d)

98(d)

99(b) $\mu = \tan 53 = 4/3$

100(a) $\frac{T - 0}{100} = \frac{77.8 - 73}{100.3 - 73}$

MODEL TEST PAPER 6

1. If $x = a^n b^m c^p, y = \dfrac{a^n b^m}{c^p}, z = \dfrac{a^n}{b^m c^p}$
   (a) The error in x = error in y > error in z
   (b) The error in z > error in y > error in x
   (c) Error in $x = y <$ error in z.
   (d) error $m\,x$ = error in y = Error in z

2. Mark the wrong statement/statements.
   (a) for the quantities energy or kinetic energy of a body, the dimensions are the same
   (b) if the dimensions are the same, they denote the same physical quanity
   (c) the dimensions of two different quantity may be same
   (d) The dimensions of a physical quantity in two system of units must be same

3. If the intial velocity is zero and the acceleration of a body is 3 t, the distance travelled in 5 seconds is given by
   (a) 187.5 m (b) 62.5 m
   (c) 125 m (d) none.

4. When a body is falling down freely from a height, the relation between distance and time is given by
   (a) straight line with increasing time
   (b) it has the shape of a circle
   (c) it is a parabola with decreasing curve and then remains constant
   (d) it is a parabola with increasing curve and then remains constant.

5. A projectile is fired with respect to horizontal ground at an angle of 45° with an initial velocity of $40\sqrt{2}$ m/s. Select the wrong statement.
   (a) the horizontal distance travelled by the projectile in 1s is half of that travelled in 2s and the horizontal distance travelled in 6s is half of that travelled 12s.
   (b) the horizontal distance travelled in 6s is less than the distance travelled in 4s
   (c) the horizontal distance travelled in 8s = the horizontal distance travelled in 16s
   (d) none

6. When a body is falling freely from a height, its maximum potential energy is equal to its maximum kinetic energy. When a satellite is turning round the earth in an orbit of radius r, the magnitude of
   (a) the potential energy of the satellite is equal to the kinetic energy of the satellite
   (b) the magnitude of potential energy is twice the kinetic energy
   (c) the kinetic energy is double the potential energy
   (d) The potential energy = binding energy

7. A man is travelling horizontally at 3 m/s towards east and the raindrops are falling vertically at 4 m/s. At what angle should he hold the umbrella?
   (a) at an angle θ to the vertical in the north-west direction where $\theta = \sin^{-1}\dfrac{3}{5}$
   (b) vertically
   (c) at $\theta = \tan^{-1}\dfrac{3}{4}$ east of vertical
   (d) at an angle $\theta = \sin^{-1}\dfrac{4}{5}$ to the vertical

8. Vectors $\vec{A}$ and $\vec{B}$ are given by $\vec{A} = (2\hat{i} - 3\hat{j} + 2\hat{k})$ and $\vec{B} = (4\hat{i} - 6\hat{j} + 4\hat{k})$, the angle between $\vec{A}$ and $\vec{B}$ is given by
   (a) 90° (b) 45°
   (c) 0° (d) none

9. When a particle of mass m is making a vertical rotation with an angular velocity ω, at the maximum height, if the tension is T, then
   (a) $T = mg + mr\omega^2$
   (b) $T = mg - mr\omega^2$
   (c) $T = 0$
   (d) $T = mr\omega^2 - mg$

10. If two spherical shells A and B of masses 2 kg and 5 kg and radii 0.1m and 0.3 m, respectively, roll down the inclined plane starting from rest.

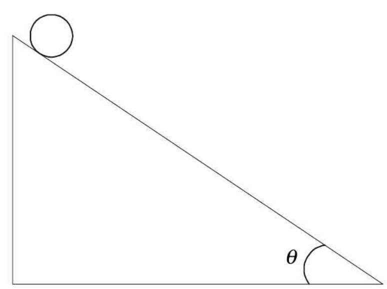


Fig. 1

   (a) the heavier mass will roll down the inclined plane faster and has an acceleration $g\sin\theta$
   (b) the lighter one will roll faster with an acceleration $g\sin\theta$
   (c) both will reach the ground with the same velocity and their acceleration will be less than $g\sin\theta$.
   (d) both will reach at the same time and their acceleration will be less than $g\sin\theta$

11. Moment of inertia about $EF, I_2 =$

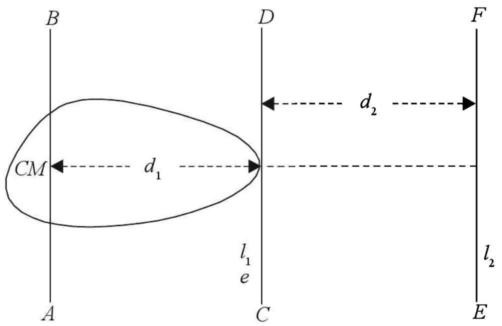


Fig. 2

(a) $I_1 + Md_2^2$ (b) I_1

(c) $I_2 = I_1 + Md_2^2$ (d) $I_2 = I_1 + Md_2^2 + 2Md_1d_2$

12. The work done by a body of mass m moving with uniform acceleration a towards the centre, in rotating through π degrees is

(a) $ma\,\pi r$ (b) zero

(c) $ma.\,2r$ (d) none.

13. A block of mass m is projected up an inclined plane with a velocity v. If there is friction between the block and the inclined plane, the minimum velocity v so that block reaches the top is

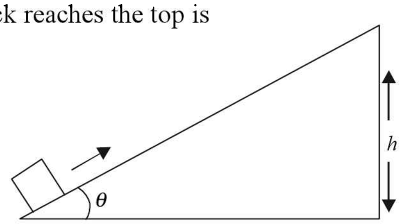


Fig. 3

(a) $\sqrt{2g\sin\theta h}$ (b) $\sqrt{2\mu_k gh\cot\theta}$

(c) $\sqrt{2\mu_k gh\cot\theta + 2gh}$ (d) $\sqrt{2gh - 2\mu_k gh\cot\theta}$

14. A and B are soap bubbles formed by filling air. If the radius of A is smaller than B, if these two bubbles are now connected to each other.

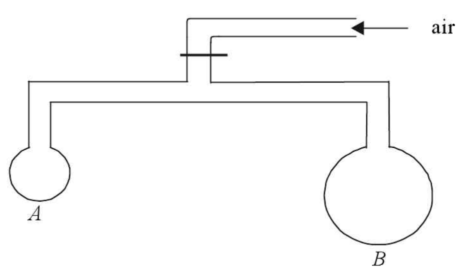


Fig. 4

(a) air flows from B to A

(b) air flows from A to B

(c) there is steady condition. No air will flow fromv A to B or B to A

(d) none.

15. The position of the hole for getting the maximum range of efflux

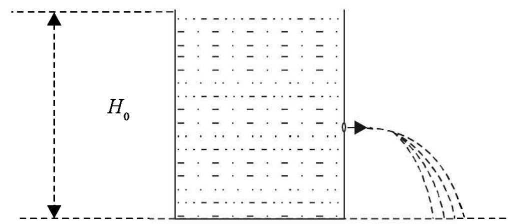


Fig. 5

(a) should be little above the bottom

(b) at $H_0/2$ (c) at the top

(d) the range attained will only depend on the total quantity of water in the tank and not the position of the hole.

16. If a manometer is made of two narrow tubes A and B of radii r_1 and r_2 ($r_1 > r_2$) and T is the surface tension of a liquid of density 10^3 kg/m³. The liquid level in A will be

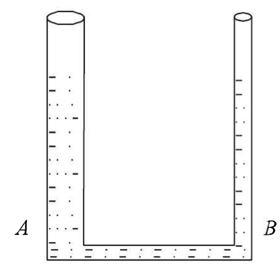


Fig. 6

(a) equal to B (b) lower than B

(c) higher than B (d) cannot say

17. AD, BC are adiabatics. The ratio of volumes

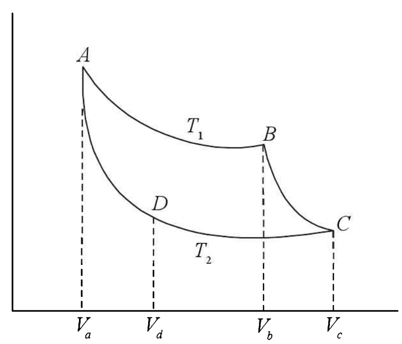


Fig. 7

(a) $\frac{V_a}{V_d} > \frac{V_b}{V_c}$ (b) $\frac{V_a}{V_d} = \frac{V_b}{V_c}$

(c) $\frac{V_a}{V_d} < \frac{V_b}{V_d}$ (d) none

18. The electric field due to a semicircular ring of charges at the centre is $\vec{E} = \frac{\lambda \times 2}{4\pi\varepsilon_0\pi}$. Therefore, the electric field at the centre of a circular ring is

(a) $\frac{4\lambda}{4\pi\varepsilon_0 r}$ (b) $\frac{2\lambda}{4\pi\varepsilon_0 r}$

(c) zero (d) none.

19. The magnetic field at the centre of a semicircular wire of radius 'a' carrying a current i is $\frac{\mu_o i}{4a}$. The magnetic field at the centre of the coil is

(a) $\frac{\mu_0 i}{2a}$ (b) zero

(c) $\frac{\mu_0 i}{4\pi}\frac{1}{r^2}$ (d) none

20. Two metal spheres A and B of radius 5 cm and 20 cm are kept at a large distance and connected by a long wire. If the charges on A and B are + 5μC and + 10μC resp., charge will flow

(a) from A to B (b) from B to A

(c) from higher electric field to lower electric field

(d) charge will not transfer

21. The total capacitance is

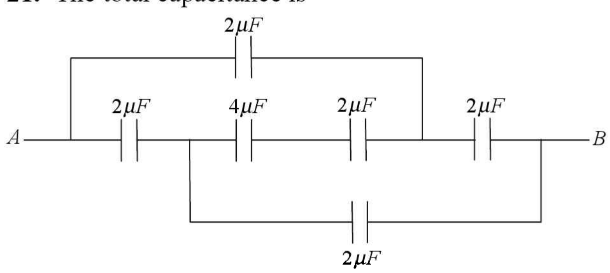


Fig. 8

(a) 4μF (b) 2 μF

(c) 1μF (d) 0.5μF

22. All resistances have equal values, 1Ω each. The current in the circuit is

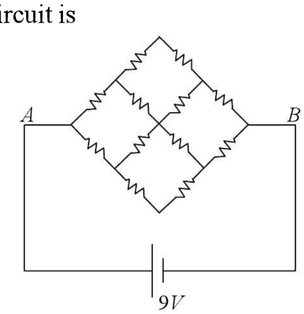


Fig. 9

(a) 6A (b) 9A

(c) 4A I (d) 3A

23. The difference between the electrical field lines due to a charge and those due to a magnet are

(a) the magnetic field lines start from a north pole and end in south poles. The lines are closed

(b) the magnetic field lines start from a south pole and end in north pole

(c) magnets are always dipoles, electric charges can exist as isolated charges

(d) electric dipoles exist but magnets are monopoles

24. The ratio of the magnetic moment to the angular momentum of an electron orbiting in the hydrogen atom according to classical physics, is given by

(a) $\frac{e}{m}$ (b) $\frac{e}{2m}$

(c) $\frac{e}{mc}$ (d) none

25. Bohr's assumption that the angular momentum of the electron in H atom is $n\hbar$ is similar to

(a) quantisation of energy

(b) quantisation of the de Broglie wave similar to waves on a sonometer wire or open tube

(c) similar to waves in a closed tube

(d) quantisation of compton wave length

26. Torque $\vec{\tau} = \vec{r}\times\vec{F}$, then which of the following is incorrect?

(a) $\vec{\tau}\perp\vec{r}$ (b) $\vec{\tau}\perp\vec{a_t}$

(c) $\vec{r}\perp\vec{a_t}$ (d) $\vec{a_t} \parallel \vec{F}$

27. Two rods each of length l and mass m form L shape. The moment of inertia about an axis passing through the point of joining and perpendicular to the plane of L-section

(a) $2ml^2$ (b) $\frac{ml^2}{6}$

(c) $\frac{2ml^2}{3}$ (d) $\frac{ml^2}{3}$

28. If the temperature of a semiconductor is increased from room temperature, then its resistance

(a) increases (b) decreases

(c) first decreases then increases (d) none

29. The relation between λ_1 λ_2 and λ_3 is

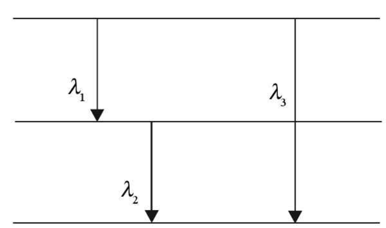


Fig. 10

(a) $\lambda_3 = \lambda_1 + \lambda_2$ (b) $\frac{1}{\lambda_1} + \frac{1}{\lambda_2} = \frac{1}{\lambda_3}$

(c) $\lambda_3 = \lambda_2 - \lambda_1$ (d) $\lambda_3 = \frac{\lambda_1\lambda_2}{\lambda_1 - \lambda_2}$

30. In the given circuit, which of the following statement is incorrect?

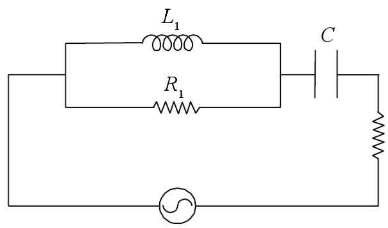


Fig. 11

(a) the phase difference between the currents in L_1 and R_1, is $\pi/2$

(b) the phase difference between the potential difference across C and R_2 is $\pi/2$

(c) the phase difference between current through C and R_2 is $\pi/2$

(d) the phase shift between V_L and $R_1$is zero

31. If two conductors of infinite length carry the same current in the direction shown. The magnetic field at P due to A and B is

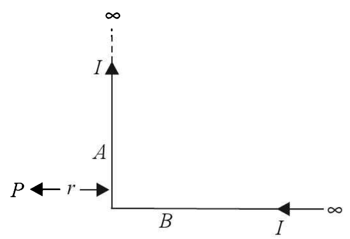


Fig. 12

(a) zero (b) $\frac{\mu_o i}{2\pi r}$

(c) $\frac{\mu_o i}{4\pi r}$ (d) $\frac{\mu_o i}{\pi r}$

32. If the kinetic energy of photons produced from a metal by irradiating the metal with 4000 A° radiation was 1.6 eV, the kinetic energy of photons produced by 6000 A° will be

(a) 2.4 eV (b) 1.6 eV

(c) 1.0 eV (d) 0.6 eV

33. The ground state energy of the electron in the hydrogen atom is – 13.6 eV. The ionization energy of H atom is

(a) –13.6 eV (b) 13.6 eV

(c) depends on the number of the orbits

(d) $\frac{-13.6eV}{2}$

34. What is the maximum wavelength that can be detected by a semiconductor photo detector if the band gap of the semiconductor Eg = 0.75 eV?

(a) 165.3 A° (b) 1653 A°

(c) 16530 A° (d) 165.3 nm

35. In an X-ray tube (cu – target), if the excitation energy of the K level is 9.5 Kev, is it possible to have $K\alpha$ X-ray if $V_{app} = 8kV$

(a) Yes, λ = 1.55 A° (b) No, λ bremsstrahlung = 1.55 A°

(c) both Kα and bremsstrahlung radiations are obtained

(d) neither Kα nor bremsstrahlung radiation is obtained

36. Which of the following substances has highest value of dielectric constant?

(a) water (b) glass

(c) strontium titanate (d) ebonite

37. An electric dipole is placed in a homogeneous electric field with its dipole moment vector antiparallel to the electric field direction.

(a) the dipole is in stable equilibrium

(b) the dipole is in unstasble equilibrium

(c) the dipole is not in equilibrium

(d) the dipole is in neutral equilibrium

38. Choose the correct statement.

(a) electric lines of forces never make closed paths

(b) electric lines of forces always make closed paths

(c) electric lines of forces are always parallel

(d) electric lines of forces may make closed path

39. Dielectric constant of a substance

(a) increases with increase in temperature

(b) decreases with increase in temperature

(c) is not affected with change in temperature

(d) may increase or decrease with increase in temperature

40. Which of the following statements is correct (E is the electric field intensity)?

(a) $\oint E.\,dl$ is always zero

(b) $\oint E.\,dl$ is may be zero

(c) $\oint E.\,dl$ is always non-zero

(d) $\oint E.\,dl$ may not be zero

41. A parallel plate air filled capacitor is charged with a supply of V volts. Supply is disconnected and then the separation between the plates is doubled. What is the ratio of electric field intensity between the plates initially and finally?

(a) 1 : 1 (b) 1 : 2

(c) 2 : 1 (d) 4 : 1

42. A charged parallel plate capacitor has charge 'q' and electric field intensity 'E' between the plates. The force of attraction between the plates is

(a) zero (b) qE

(c) $\frac{1}{2}qE$ (d) 2 qE

43. Choose the correct statement.

(a) $\oint E.\,ds$ is always zero

(b) $\oint B.\,ds$ is always zero

(c) $\oint B.\,ds$ may be zero

(d) $\oint E.\,ds$ is always non-zero

44. Two conducting concentric spherical shells of radii a and b ($a < b$) are charged to potentials V_a and V_b respectively. They are connected by a conducting wire.

(a) the total charge of inner will flow to the outer shell for all values of V_a and V_b

(b the charge of inner shell will flow to the outer shell only when $V_a > V_b$

(c) the charge of inner shell will flow to the outer shell only when V_a is negative and V_b is positive

(d) the charge may flow from the outer to inner shell.

45. Force on a charge in an electric field

(a) is always conservative

(b) is always non-conservative

(c) may be conservative

(d) is always zero

46. A rocket is going vertically upwards with a net acceleration of $2g$. A stone is dropped from the rocket. The acceleration of the stone just after it is dropped is

(a) g upwards (b) g downwards

(c) $2g$ upwards (d) zero

47. A body of mass 1 kg is projected at an angle of 60° with the horizontal with a velocity of 100 ms^{-1}. What is the total change in momentum of the body after 2 seconds of its projection?

(a) 20 Newton second (b) 10 Newton second

(c) 70 Newton second (d) none is correct

48. Choose the correct statement.

(a) force acting on a body is always along its momentum

(b) force acting on a body may be along its momentum.

(c) force acting on a body is always perpendicular to its momentum

(d) force acting on a body is always anti-parallel to its momentum

49. Two bodies of masses 1kg and 2 kg are placed at coordinates (0,0) and (4,3). Find the coordinates of its centre of mass.

(a) (2, 3/2) (b) (8/3, 2)

(c) (4/3, 1) (d) none is correct

50. A body of mass 'm' is moved in a closed path. The net work done on a body

(a) is always zero (b) is always non-zero

(c) may be zero

(d) is zero only if the force is perpendicular to motion.

51. A body is subjected to a decreasing acceleration

(a) velocity of body will remain constant

(b) velocity of body will increase

(c) velocity of body will decrease

(d) the body will change its direction of motion after some time.

52. A spring of force constant 'K' is cut into two parts whose lengths are in the ratio of 2 : 1. In what ratio the forces should be applied on them so as to compress them by the same displacement?

(a) 1 : 2 (b) 2 : 1

(c) $\sqrt{2}$: 1 (d) 1 : 4

53. The bob of a simple pendulum is hollow. When it is filled half with the liquid A its time period is T_1 and when it is completely filled with the liquid B period is T_2. The densities of A and B are in the ratio of 2 :1. Then

(a) $T_1 > T_2$ (b) $T_2 > T_1$

(c) $T_1 = T_2$ (d) insufficient data

54. A 20 μF capacitor is connected to a 6 volt battery through a key. On closing the key, the energy in the condenser is W_1 joules and the energy taken from the battery is W_2 joules. Then,

(a) $W_1 = 720 \times 10^{-6}$; $W_2 = 0$.

(b) $W_1 = W_2 = 360 \times 10^{-6}$

(c) $W_1 = 360 \times 10^{-6}$; $W_2 = 540 \times 10^{-6}$

(d) $W_1 = 360 \times 10^{-6}$; $W_2 = 720 \times 10^{-6}$

55. A proton and an α-particle both are accelerated under the same potential difference and then projected perpendicular to a magnetic field. What is the ratio of radii of their orbits in the magnetic field?

(a) 1 : $\sqrt{2}$ (b) 1 : 2

(c) $\sqrt{2}$: 1 (d) 2 : 1

56. A current carrying conductor has.

(a) only electric field

(b) only magnetic field

(c) both electric and magnetic fields

(d) neither electric nor magnetic field

57. A soild cylindrical conductor has current I flowing through it, then

(a) the magnetic field is maximum at its axis

(b) the magnetic field is zero as its surface

(c) the magnetic field is non-zero at all points lying between its axis and surface

(d) the magnetic field is zero at points within the conductor

58. A current carrying metallic conductor is heated

(a) drift velocity of electrons increases

(b) drift velocity of electrons decreases

(c) speed of electrons decreases

(c) relaxation time increases

59. A heater filament is connected to a source of e.m.f. When the filament starts glowing, it is merged in a tank of ice. The power loss in the filament

(a) is not affected (b) increases

(c) decreases (d) becomes zero

60. A simple pendulum of length l has a bob of mass m and charge q. It oscillates with a time period T. Now if a charge q is placed at the point of suspension, the time period of the pendulum

(a) will become more than T

(b) will become less than T

(c) will not be affected

(d) will become $T\sqrt{2}$

61. The thermoelectric e.m.f. generated in a thermocouple is given by $E = 500\,T - T^2$ where T is in centigrade if the cold junction is at 50°C what is the temperature of inversion?

(a) 400° C (b) 500° C

(b) 250° (d) none is correct

62. 100 volts *dc* when applied across a resistance dissipates "*P*" watts. Calculate the peak value of that *ac* which dissipates half of this power when applied across the same resistance

(a) 100 volt (b) 50 volt

(c) $\frac{100}{\sqrt{2}}$ volt (d) 100 $\sqrt{2}$ volt

63. A charged particle is projected perpendicular to a time varying magnetic field.

(a) the energy of charged particle will change due to magnetic field

(b) the energy of charged particle will change due to electric field

(c) the particle will move in a circular path

(d) the energy of charged particle will not change

64. To a coil when 100 V DC is applied current is found to be 1 A. To the same coil when 200 V, 50 Hz AC is applied current is again found to be 1 A. What is the power factor of the coil?

(a) 1 (b) $\frac{1}{2}$

(c) $\frac{1}{\sqrt{2}}$ (d) $\frac{1}{\sqrt{3}}$

65. If the rate of rotation of the coil in a generator is increased two times

(a) Emf induced is doubled

(b) frequency of induced emf is doubled

(c) frequency doubles but emf will be half

(d) none of the above is correct.

Answers

1.	(d)	**2.**	(b)	**3.**	(b)	**4.**	(d)	**5.**	(c)	**6.**	(b)	**7.**	(c)
8.	(c)	**9.**	(d)	**10.**	(c,d)	**11.**	(d)	**12.**	(b)	**13.**	(c)	**14.**	(b)
15.	(b)	**16.**	(b)	**17.**	(b)	**18.**	(c)	**19.**	(a)	**20.**	(a)	**21.**	(a)
22.	(a)	**23.**	(c)	**24.**	(b)	**25.**	(b)	**26.**	(c)	**27.**	(c)	**28.**	(b)
29.	(b)	**30.**	(c)	**31.**	(c)	**32.**	(d)	**33.**	(b)	**34.**	(c)	**35.**	(b)
36.	(c)	**37.**	(b)	**38.**	(d)	**39.**	(b)	**40.**	(d)	**41.**	(a)	**42.**	(c)
43.	(b)	**44.**	(a)	**45.**	(c)	**46.**	(b)	**47.**	(a)	**48.**	(b)	**49.**	(b)
50.	(c)	**51.**	(b)	**52.**	(a)	**53.**	(a)	**54.**	(d)	**55.**	(a)	**56.**	(b)
57.	(c)	**58.**	(b)	**59.**	(b)	**60.**	(c)	**61.**	(a)	**62.**	(a)	**63.**	(b)
64.	(b)	**65.**	(a, b)										

Explanations

1(d) We write maximum possible error

$$\therefore \quad \frac{dx}{x} = \frac{dy}{y} = \frac{dz}{z} = \frac{nda}{a} + \frac{mdb}{b} + \frac{pdc}{c}$$

2 (b)Torque and work have the same dimensions but they are two different physical quantities.

3(b) $a = \frac{dv}{dt}$ or $dv = adt$

$$\therefore \quad v = \int 3t\,dt = \frac{3t^2}{2}$$

$$\because \quad v = \frac{ds}{dt} \text{ or } ds = vdt$$

$$\text{or} \quad S = \int_0^5 \frac{3}{2}t^2\,dt = \frac{t^3}{2}\Big|_0^5 = \frac{125}{2} = 62.5\text{ m}$$

4(d) $u = 0;\ s = \frac{1}{2}gt^2$

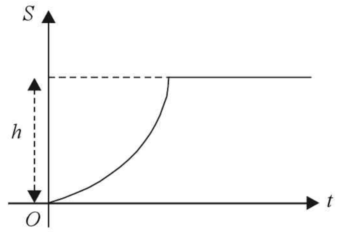


Fig. 13

The curve is a parabola with increasing distance. After reaching the ground it becomes stationary.

5(b)

6(b) In a bound system $|PE| = 2\,\text{KE}$

7(c) $\vec{V}_{rm} = -4\hat{j} - 3\hat{\tau}$

$\tan\theta = \frac{3}{4}$ or $\theta = \tan^{-1}\frac{3}{4}$ east of vertical

8(c) Since $\vec{B} = 2\vec{A}$ $\therefore$ $\vec{A}$ and $\vec{B}$ are parallel

Hence $\theta = 0°$

9(d) $T + mg = m\omega^2 r$

$\therefore \quad T = m\omega^2 r - mg$

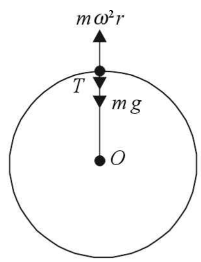


Fig. 14

10(c,d) $a = \dfrac{g\sin\theta}{1 + \dfrac{I}{MR^2}} = \dfrac{g\sin\theta}{1 + \dfrac{2}{3}}$

$= \frac{3}{5} g\sin\theta$

Since $V^2 = 2al$ $\therefore$ velocity is independent of mass and radius

11(d) $I_1 = I_{com} + M\,d_1^2$

$I_2 = I_{com} + M\,(d_1 + d_2)^2$

$I_2 = I_1 + M d_2^2 + 2 M d_1 d_2$

12(b) No work is done in uniform circular motion.

13(c) $\frac{h}{l} = \sin\theta$

or $l = \dfrac{h}{\sin\theta}$

$a = (g\sin\theta + \mu_k g\cos\theta)$

and $v = \sqrt{2al}$

$v = \sqrt{2gh + 2\mu_k gh\cot\theta}$

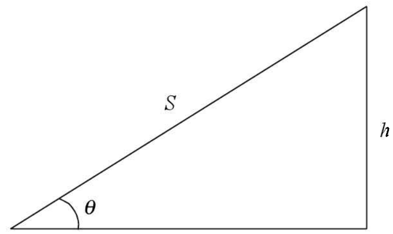


Fig. 15

14(b) $\Delta P = \frac{4T}{R}$ Since $R_A < R_B$, $P_A > P_B$

and air flows from higher pressure to lower pressure.

15(b) The range is maximum when $h = \frac{H_0}{2}$.

16(b) $\because h_1 r_1 = h_2 r_2$

Since $r_1 > r_2$, therefore $h_2 > h_1$

17(b) $TV^{\gamma-1}$ = constant for adiabatic processes

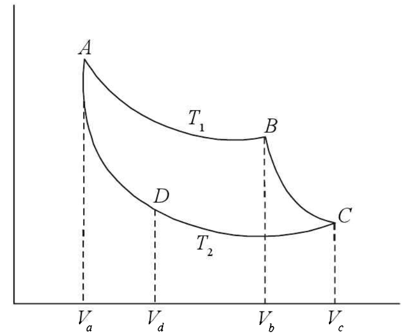


Fig. 16

$\therefore \quad \dfrac{T_1 V_a^{\gamma-1}}{T_2 V_d^{\gamma-1}} = \dfrac{T_1 V_b^{\gamma-1}}{T_2 V_c^{\gamma-1}}$

$\Rightarrow \quad \dfrac{V_a}{V_d} = \dfrac{V_b}{V_c}$

18(c) The electric fields are in the same plane, in opposite direction. They cancel each other.

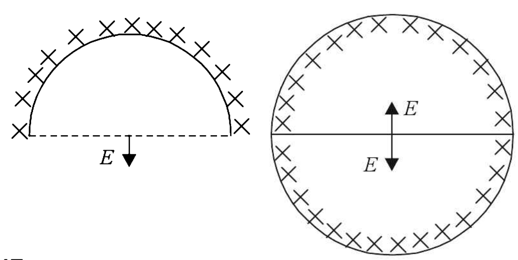


Fig. 17

19(a) $B = \dfrac{\mu_0 i}{2a}$

The magnetic field is perpendicular to the plane containing the current element and radius. Therefore, they add up.

20(a) Charges flow from a higher to lower potential, therefore, charge flow A to B till their potentials become equal.

21(b) It is a Wheatstone bridge.

22(a)

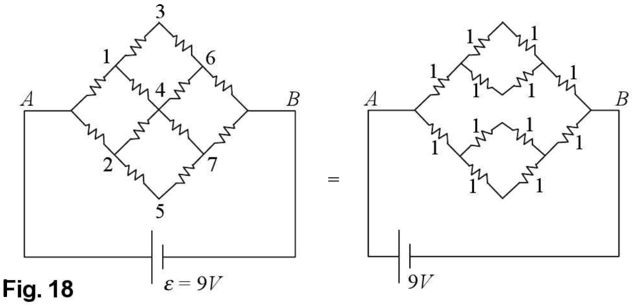


Fig. 18

$$r_{AB} = \frac{3}{2}r$$

$$I = \frac{9}{\frac{3}{2}} = 6A$$

23 (a,c)

24(b)The angular momentum of the electron $L = mr^2\omega$

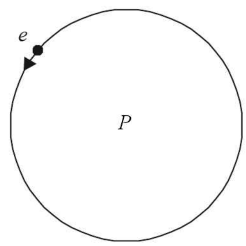


Fig. 19

The magnetic moment of the orbiting electron

$$\vec{\mu} = iA = e.f.\ \pi r^2 = e.\ \pi r^2 \frac{\omega}{2\pi}$$

$$\frac{\vec{\mu}}{L} = e\pi r^2 \frac{\omega}{2\pi} \times \frac{1}{mr^2.\omega} = \frac{e}{2m}$$

26(c)

27(c)

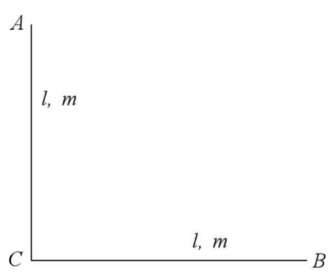


Fig. 20

$$I = \frac{ml^2}{3} \text{ for one rod}$$

$$I_{total} = \frac{2ml^2}{3}$$

29(b) $\dfrac{hc}{\lambda_1} + \dfrac{hc}{\lambda_2} = \dfrac{hc}{\lambda_3}$ or $\dfrac{1}{\lambda_3} = \dfrac{1}{\lambda_1} + \dfrac{1}{\lambda_2}$

31(c) $B_{total} = B_B + B_A$

$$= o + \frac{\mu_o i}{4\pi r}[\sin 90 + \sin 0] = \frac{\mu_o i}{4\pi r}$$

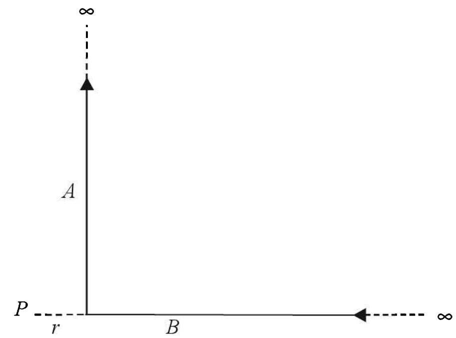


Fig. 21

32(d)$\therefore$ $K.E.$ of the photo electrons = 2.1 – 1.5 = 0.6 eV.

Case (i) $E(\text{eV}) = \dfrac{1240}{400} = 3.1\ eV$

$KE_{max} = 1.6 = 3.1 - \phi$ or $\phi = 1.5\ eV$

case(ii) $E = \dfrac{1240}{600} = 2.66\ eV$ $KE = 2.06 - 1.5 = 0.56\ eV$

34(c) $\lambda = \dfrac{12400}{0.75} = 16530\ \text{A}°.$

35(b) One cannot get K X-rays if the applied voltage is less than the excitation energy of the level *i.e.* 9.5 *kev.*. However, even at lower applied potentials, one can get some other series and a continuous spectrum or bremsstrahlung X-rays. The short wavelength limit is given by

$$\frac{hc}{v} = \frac{12400 eVA°}{8000} = 1.55\ \text{A}°$$

36(c)

37(b)

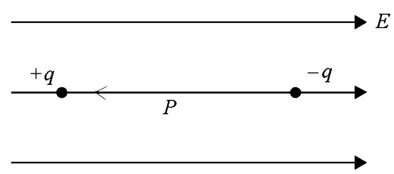


Fig. 22

If it is rotated a little from the equilibrium position, the dipole will not regain its position.

38(d)Electric lines of forces due to a charge do not make closed paths whereas the electric lines of forces created due to changing magnetic field make closed paths.

39(b)

40(d) $\oint E.\,dl = 0$ for purely electrostatic field $\oint E.\,dl = \dfrac{d\phi_B}{dt}$ for the electric fields created due to changing magnetic field.

41(a)Since supply has been disconnected charge and hence charge density on the plates is constant.

$$E = \frac{\sigma}{\varepsilon_0}$$

42(c)

43(b) $\oint B.ds = 0$ because magnetic monopole does not exist.

44(a)

45(c)Force on a charge in the electric field of a charge is conservative whereas force on a charge in the electric field created due to changing magnetic field is non-conservative.

46(b)

47(a) $\Delta p = F\Delta t$
$= mg \times \Delta t = 1 \times 10 \times 2 = 20\,NS.$

48(b) Force on a body is along the direction of change in momentum.

49(b) $\bar{x} = \dfrac{m_1x_1 + m_2x_2}{m_1 + m_2} = \dfrac{1\times0\times+2\times4}{3} = \dfrac{8}{3}$

similarly. $\bar{y} = \dfrac{1\times0+2\times3}{3} = \dfrac{6}{3} = 2$

50(c) $\oint dw = 0$ if force is conservative

$\oint dw \neq 0$ if force is non-conservative

51(b) Velocity of body will increase at decreasing rate.

52(a) The spring constants of the parts will be in the ratio of 1:2.

53(a) In the first case the centre of mass will be below the centre of sphere and hence the effective length is more than when it is completely filled.

54(d) Energy stored in capacitor is

$E = \dfrac{1}{2}CV^2$ Work done by the battery is twice the energy stored by the capacitor.

55(a) $R = \dfrac{mv}{Bq}$ also $q.V = \dfrac{1}{2}mv^2$

$m^2v^2 = 2qVm$ or $R = \dfrac{\sqrt{2qVm}}{Bq}$ so $R \propto \sqrt{\dfrac{m}{q}}$.

or $\dfrac{R_P}{R_\alpha} = \sqrt{\dfrac{\frac{1}{1}}{\frac{4}{2}}} = \dfrac{1}{\sqrt{2}}$

56(b) The net charge of a current carrying conductor is zero so the electric field is also zero.

57(c)

58(b) Due to increase in temperature the speed of electrons increases so relaxation time decreases.
as $v_d \propto \tau$, therefore v_d decreases

59(b)Due to decrease in temperature, resistance will decrease.

$P = \dfrac{V^2}{R}$

60(c)Placing charge "q" at the the point of suspension will only increase the tension in the string.

61(a) $\dfrac{dE}{dT} = 500 - 2T = 0$ at Neutral Temperature

So $T = 250°C$

$T_i - T_n = T_n - T_c$

So $T_i = 450 - 50 = 400°C$

62(a) $\left(\dfrac{V_0}{\sqrt{2}}\right)^2 \dfrac{1}{R} = \dfrac{1}{2}\left[\dfrac{(100)^2}{R}\right]$ $V_0 = 100$ volt

63(b) Changing magnetic field produces electric field.

64(b) $R = \dfrac{100}{1} = 100\Omega$

$|z| = \dfrac{200}{1} = 200\Omega$

Power factor $\cos\phi = \dfrac{R}{Z} = \dfrac{1}{2}$

65 (a, b)

MODEL TEST PAPER 7

1. The percentage error in measurement of mass and speed are 2% and 3% respectively. The maximum error in the estimation of kinetic energy obtained by measuring mass and speed is.

   (a) 11% (c) 8%

   (c) 5% (d) 1%

2. With what speed must a ball be thrown down for it to bounce 10 m higher than its original level? Neglect any loss of energy in striking the ground

   (a) data is incomplete (b) 14 ms^{-1}

   (c) 20 ms^{-1} (d) 5 ms^{-1}

3. Three vectors $\vec{A}$, $\vec{B}$, $\vec{C}$ satisfy the relation $\vec{A} \cdot \vec{B} = 0$ and $\vec{A} \cdot \vec{C} = 0$. The vector $\vec{A}$ is parallel to:

   (a) $\vec{B}$ (c) $\vec{C}$

   (c) $\vec{B} \cdot \vec{C}$ (d) $\vec{B} \times \vec{C}$

4. Two skaters A and B of masses 40 and 60 kg respectively stand facing each other 5 m apart. They then pull on a light rope stretched between them. How far each has moved when they meet.?

   (a) both move 2.5 m

   (b) A moves 2 m and B moves 3 m

   (c) A moves 3 m and B moves 2m

   (d) A moves 1.5 m and B moves 3.5 m

5. A lift is moving downward with an acceleration = g. A body of mass M kept on the floor is pulled horizontally. If the coefficient of friction is μ then the frictional resistance offered by the body is

   (a) Mg (b) μMg

   (c) 2μMg (d) 0

6. The free end of a thread wound on a bobbin of inner radius r and outer radius R is passed round a nail A. The thread is pulled at a constant speed v. The velocity of centre of bobbin at the instant when the thread forms angle α with vertical. (Assume that the bobbin rolls over the horizontal surface without slipping) is .

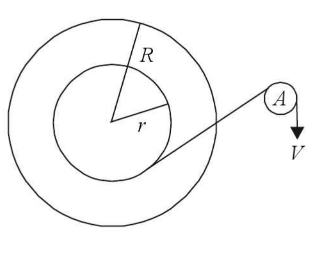


Fig. 1

   (a) vR (b) $vR/R \sin\alpha$

   (c) $vR/R \sin \alpha - r$ (d) $\dfrac{rv}{R\sin\alpha - r}$

7. The slope of kinetic energy versus displacement curve of a particle in motion is

   (a) equal to the acceleration of the particle

   (b) inversely proportional to acceleration

   (c) directly proportional to a acceleration

   (d) none of these

8. if $\vec{r} = (2\hat{i} + 3\hat{j})$ m and $\vec{p} = (3\hat{i} - \hat{j})$ kg ms^{-1} then value of angular momentum is

   (a) $11\hat{k}$ (b) $11\hat{j}$

   (c) $-11\hat{j}$ (d) $-11\hat{k}$

9. If the radius of the earth were to shrink 1% its mass remaining same, the acceleration due to gravity on earth's surface would

   (a) decrease by 2% (b) decrease by 1%

   (c) increase by 1% (d) increase by 2%

10. Kepler's second law states that the straight line joining the planet to the sun sweeps out equal areas in equal times. The statement is equivalent to saying that

    (a) total acceleration is zero

    (b) transverse acceleration is 0

    (c) tangential acceleration is 0

    (d) radial acceleration is 0

11. Thermodynamic variables are

    (a) mass and temperature of system

    (b) P, V, T

    (c) P, V, T and S (entropy)

    (d) U, F, G, H

12. Water can be made to boil at 0°C. If the pressure of surrounding is

    (a) 760 mm of Hg (b) 76 mm of Hg

    (c) 40 *mm* of Hg (d) 4 mm of Hg

13. The ratio of densities of the bodies is 3 : 4 and specific heat in ratio 4 : 3. The ratio of their thermal capacity for unit volume is

    (a) 9 : 16 (b) 16 : 9

    (c) 2 : 1 (d) 1 : 1

14. Four curves A, B, C and D are meant for a given amount of gas. The curve which represents adiabatic changes are

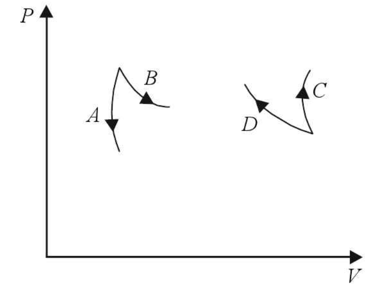


Fig. 2

(a) A and B (b) C and D

(c) A and C (d) B and D

15. A carnot's engine takes 300 calories of heat at 500 K and rejects 150 cal to sink. The temperature of sink is

(a) 1000 K (b) 750 K

(c) 250 K (d) 125 K

16. If E is the total energy emitted by a body at a temperature T K and E (max) is the maximum energy emitted by it at the same temperature then:

(a) $E \propto T^4$ $E_{max} \propto T^5$

(b) $E \propto T^4$ $E_{max} \propto T^{-5}$

(c) $E \propto T^{-4}$ $E_{max} \propto T^4$

(d) $E \propto T^{-4}$ $E_{max} \propto T^4$

17. Two forks of frequencies 250 Hz and 256 Hz produce beats. If a maxima is produced just now after how much time the minima is produced at the same place

(a) 1/8 sec (b) 1/24 sec

(c) 1/6 sec (d) 1/12 sec

18. Two waves of intensities I and $4I$ produce interference. The intensity of constructive interference in

(a) $5I$ (b) $7I$

(c) $9I$ (d) I

19. For 1°F rise in temperature the velocity of sound in air increases by nearly

(a) 0.61 ms^{-1} (b) 0.34 ms^{-1}

(c) 1.19 ms^{-1} (d) $(0.61)^2$ ms^{-1}

20. A wire under tension vibrates with frequency of 450 per sec. What would be the fundamental frequency if the wire were half as long, twice as thick under one fourth tension?

(a) 225 Hz (b) 190 Hz

(c) 247 Hz (d) 174 Hz

21. Doppler shift in frequency doesn't depend on

(a) frequency of wave (b) velocity of source

(c) velocity of observer

(d) distance from the source to listner

22. Two charges are 40 μC and –20 μC are some distance apart. Now they are touched and kept at the same distance. The ratio of initial to final force between them is

(a) 8 : 1 (b) 4 : 1

(c) 1 : 8 (d) 1: 4

23. A cyclinder of radius R and length L is placed in a uniform electric field E acting parallel to the axis of the cylinder. The total flux from the surface of cylinder is

(a) $2\pi R^2 E$ (b) $\pi R^2 / E$

(c) $(2\pi R^2 + 2\pi RL)$ (d) 0

24. With a rise in temperature, the dielectric constant K of a liquid

(a) increases (b) decreases

(c) remains unchanged (d) change irractically

25. Can a sphere of radius 1m have a charge of 1C?

(a) yes (b) no

(c) depend on metal

(d) depend on nature of charge

26. Three uncharged capacitors C_1, C_2 and C_3 are connected as shown in the figure. The potential of A, B and D are ϕ_1, ϕ_2, ϕ_3. Find the potential at the junction O.

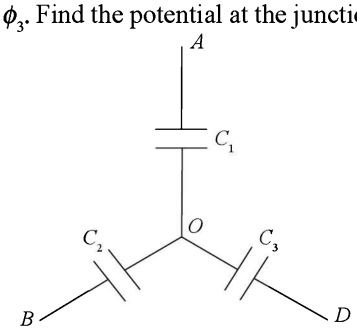


Fig. 3

(a) $\phi_1 + \phi_2 + \phi_3$

(b) $(\phi c_1 + \phi_2 c_2 + \phi_3 c_3) / (c_1 + c_2 + c_3)$

(c) $(\phi_1 - \phi_2) + (\phi_2 - \phi_3) + (\phi_3 - \phi_1)$

(d) $(\phi_1 - \phi_2)(\phi_2 - \phi_3)(\phi_3 - \phi_1)$

27. The $P.D.$ between A and B is 23 volts. The $P.D.$ across 3 μF capacitor is

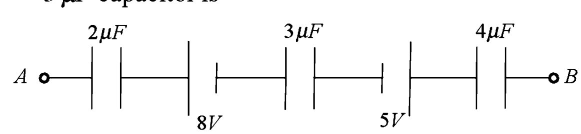


Fig. 4

(a) 7 (b) 8

(c) 23 (d) 4

28. The current I and voltage V graphs for a given metallic wire at two different temperatures T_1 and T_2 are shown in Fig 5. It concludes

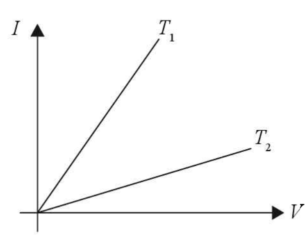


Fig. 5

(a) $T_1 > T_2$ (b) $T_1 = T_2$
(c) $T_1 < T_2$ (d) $T_1 = 2T_2$

29. In the network shown in the figure the effective resistance between A and B is... if each resistance is 1ohm.

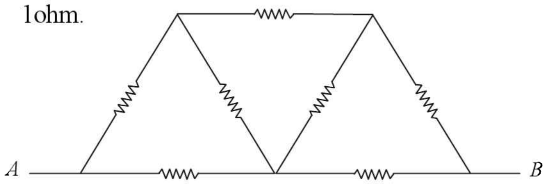


Fig. 6

(a) 4/3 (b) 3/2
(c) 7 (d) 8/7

30. Masses of three wires of same metals are in the ratio 1 : 2 : 3 and their length are in ratio of 3 : 2 : 1. Electrical resistance of these wires will be in ratio:

(a) 1 : 1 : 1 (b) 1 : 2 : 3
(c) 9 : 4 : 1 (d) 27 : 6 : 1

31. A battery of internal resistance r supplies maximum power to a load of resistance R if

(a) $R = r$ (b) $R = r/2$
(c) $R = 3r$ (d) $R = 2r$

32. Neutral temperature of a thermocouple is

(a) constant
(b) decreases with decrease in temperature of cold junction
(c) increases with decrease in temperature of cold junction
(d) increases with increase in temperature of cold junction

33. In a copper voltameter experiment current is decreased to one-fourth of initial value The time duration for which current is passed is doubled. Amount of copper deposited will be

(a) same as before (b) half of previous value
(c) 4 times of previous value
(d) one-sixth of previous value

34. Which of the following graphs shows the variation of magnetic induction B with distance r for a long current carrying wire?

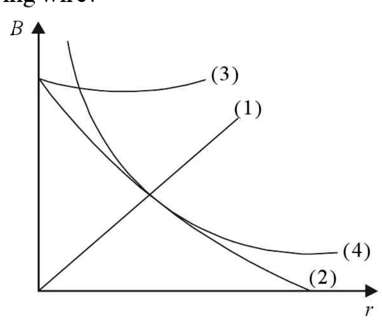


Fig. 7

(a) 1 (b) 2
(c) 3 (d) 4

35. A proton, a deutron and an α - particle with same kinetic energy enter a region of magnetic field to perpendicular magnetic field. The ratio of radius of their circular path is

(a) $1 : \sqrt{2} : 1$ (b) $1 : \sqrt{2} : \sqrt{2}$
(c) $\sqrt{2} : 1 : 1$ (d) $\sqrt{2} : \sqrt{2} : 1$

36. Magnetic induction at O will be

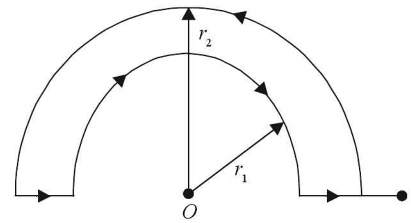


Fig. 8

(a) $\frac{\mu_0 i}{2(r_1 + r_2)}$ (b) $\frac{\mu_0 i}{2r_1} + \frac{\mu_0 i}{2r_2}$
(c) $\frac{\mu_0 i}{4r_1} - \frac{\mu_0 i}{4r_2}$ (d) $\frac{\mu_0 i}{4r_1} + \frac{\mu_0 i}{4r_2}$

37. In a wire of irregular cross-section a steady current flows. If J_a, J_b, J_c, μ_a, μ_b, μ_c are respectively current density and drift velocity at point A, B and C then

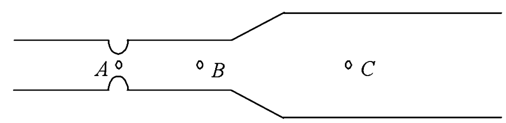


Fig. 9

(a) $J_a = J_b = J_c$ (b) $\mu_a = \mu_b = \mu_c$
(c) $J_a > J_b > J_c$ (d) $\mu_a > \mu_b > \mu_c$

38. A coil of resistance R and inductance L is connected to a battery of emf E volt. The final current in coil is

(a) E/R (b) E/L
(c) $\sqrt{E/R^2 + L^2}$ (d) $\sqrt{EL/R^2 + L^2}$

39. A coil and a bulb are connected in series with 12 volt *dc.* source. On inserting a soft iron core in coil the intensity will

(a) remain the same (b) increase
(c) decrease
(d) will change in a periodic manner

40. What is changed during reflection of electromagnetic wave?

(a) frequency (b) wavelength
(c) speed (d) amplitude

41. Graph for energy and wavelength of photon is

(a) straight line (b) hyperbola
(c) parabola (d) ellipse

42. The focal length of a concave mirror is f and the distance from object to principal focus is x. Then magnification is

(a) $f + x / f$ (b) f / x
(c) $\sqrt{f / x}$ (d) f^2 / x^2

43. Which of the following graph correctly represents magnification versus distance of object for a concave mirror?

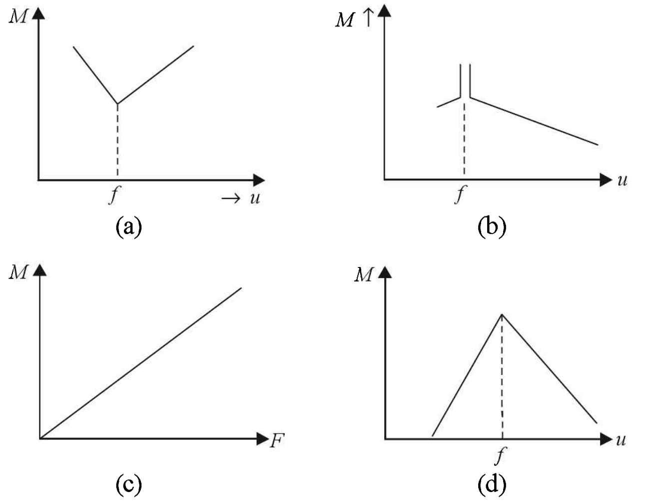


Fig. 10

44. If there were no atmosphere, the length of day would

(a) decrease (b) increase
(c) remain same (d) depend on weather

45. The Cauchy relation is

(a) $\mu = B + A\lambda$ (b) $\mu = A + B/\lambda^2$
(c) $\mu = AB/\lambda$ (d) $\mu = A^2\lambda + B$

46. The angle of a prism is A. If the angle of minimum deviation is $(180 - 2A)$ then retractive index of material of prism is

(a) $\sin A/2$ (b) $\cos A/2$
(c) $\tan A/2$ (d) $\cot A/2$

47. When the object is at distance μ_1 and μ_2 a real and a virtual image are formed respectively and are of same size. The focal length of lens is

(a) $(u_1 + u_2)/2$ (b) $\sqrt{u_1 u_2/2}$
(c) $\sqrt{u_1 u_2}$ (d) $\frac{(u_1 + u_2)^2}{4}$

48. If a bulb of 100 watt is taken away from photo cell, then photo-current I and distance between source and cell (d) are related as

(a) $I \propto d$ (b) $I \propto d^2$
(c) $I \propto 1/d$ (d) $I \propto 1/d^2$

49. For transistor

(a) $I_C > I_E$ (b) $I_C < I_E$
(c) $I_C = I_E$ (d) none

50. The atomic packing factor for bcc cell is

(a) π (b) $\pi/6$
(c) $\frac{\sqrt{3}\pi}{8}$ (d) $\frac{\sqrt{2}}{6}\pi$

Answers

1.	(b)	**2.**	(b)	**3.**	(d)	**4.**	(c)	**5.**	(d)	**6.**	(c)	**7.**	(c)
8.	(d)	**9.**	(d)	**10.**	(b)	**11.**	(c)	**12.**	(d)	**13.**	(d)	**14.**	(c)
15.	(a)	**16.**	(a)	**17.**	(d)	**18.**	(c)	**19.**	(b)	**20.**	(a)	**21.**	(d)
22.	(a)	**23.**	(d)	**24.**	(b)	**25.**	(b)	**26.**	(b)	**27.**	(b)	**28.**	(c)
29.	(d)	**30.**	(d)	**31.**	(a)	**32.**	(a)	**33.**	(b)	**34.**	(d)	**35.**	(a)
36.	(c)	**37.**	(c)	**38.**	(a)	**39.**	(c)	**40.**	(d)	**41.**	(b)	**42.**	(b)
43.	(b)	**44.**	(a)	**45.**	(b)	**46.**	(d)	**47.**	(a)	**48.**	(d)	**49.**	(b)
50.	(c)												

Explanations

1(b) $\frac{\Delta KE}{KE} = \frac{\Delta m}{m} + \frac{2\Delta v}{v} = 2 + 2(3) = 8\%$

2(b) If there is no energy loss, after rebound when it gains its level it will have the same speed in upward direction to attain a height 10 m

$$v = \sqrt{2gh} = \sqrt{2 \times 10 \times 9.8}$$
$$= \sqrt{196} = 14 \text{ ms}^{-1}$$

3(d) From given data it is clear that A $\perp C$ and also A $\perp B$. Thus, it has same direction as $\vec{B} \times \vec{C}$.

4(c) As there is no external force skaters will meet at their COM. Thus problem reduces to determination of COM. of from each

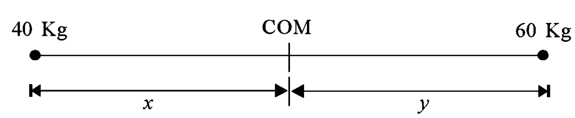


Fig. 12

$$x = \frac{60 \times 5}{40 + 60} = 3 \text{ and similarly } y = 2$$

5(d) As in this case normal reactional force is 0. Thus frictional resistance $= \mu N = 0$.

6(c) $V_A = V_{A/cm} + V_{cm}$

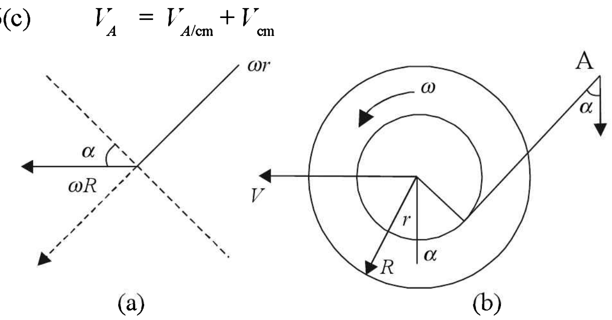


Fig. 13

The resultant velocity at A (refer. to fig 13 (a)) and geometry $\omega r - \omega R \sin\alpha = V$

$$\Rightarrow \quad \omega = \frac{v}{r - R\sin\alpha}$$

$$\therefore \quad V_{cm} = \frac{vR}{r - R\sin\alpha} \text{ or } V_A = -V_{cm} = \frac{vR}{R\sin\alpha - r}$$

7(c) $KE = 1/2\, mv^2$

$$\frac{d(KE)}{dx} = mv\frac{dv}{dx} = ma$$

⇒ Slope of KE versus x curve is proportional to acceleration

8(d)

$$\vec{r} \times \vec{p} = (2\hat{i} + 3\hat{j}) \times (3\hat{i} - \hat{j}) = -11\hat{k}$$

9(d)

$$g = \frac{GM}{R^2}$$

or $dg/g = (-2)\, dR/R = (-2)(-1) = 2\%$

10(b)

11(c)

12(d)

13(d) Ratio of thermal capacity

$$\frac{s_1}{s_2} \times \frac{\rho_1}{\rho_2} = 4/3 \times 3/4 = 1:1$$

14(c)

15(a) For carnot cycle $Q_1/T_1 = Q_2/T_2$

$$T_2 = \frac{Q_2 T_1}{Q_1} = \frac{150 \times 500}{300} = 250K$$

16(a)

17(d) Frequency of beat in 2 $(n_1 - n_2) = 2 \times 6 = 12$

∴ time of another maxima = $\frac{1}{12}$ sec

18(c) $I = I_1 + I_2 + 2\sqrt{I_1 I_2}$

$$= I\left(1 + 4 + 2\sqrt{1 \times 4}\right) = 1 + 4 + 4 = 91$$

19(b) $I°F = \frac{9}{5}\, °C$

∴ Velocity due to 1°C rise

$= 9/5 \times .61 = 0.34\ ms^{-1}$

20(a) $f = \frac{1}{2l}\sqrt{\frac{T}{\mu}}$

$$\therefore \quad \frac{f_1}{f_2} = \frac{l_1}{l_2}\sqrt{\frac{T_1 A_2}{T_2 A_1}}$$

or $\frac{f}{450}$ = or $\frac{2}{1}\sqrt{\frac{1 \times 1}{4 \times 4}}$ or f = 225 Hz

21(d)

22(a) Out of 40 μC 20 μC will neutralize –20 μC charge. The new charge on two will be 10μC each.

$$\frac{F_1}{F_2} = \frac{(q_1 q_2)_1}{(q_1 q_2)_2} = \frac{(2 \times 4)}{1 \times 1} = \frac{8}{1}$$

23(d)

24(b)

25(b) NO. Because electric breakdown occurs. The air has delectric breakdown value $3 \times 10^6\ V/m$.

26(b) Net charge at junction should be O.

$$\therefore \quad (\phi_1 - \phi_0)C_1 + (\phi_2 - \phi_0)C_2 + (\phi_3 - \phi_0)C_3 = 0$$

$$\text{or} \quad \phi_0 = \frac{\phi_1 C_2 + \phi_2 C_2 + \phi_3 C_3}{C_1 + C_2 + C_3}$$

27(b) $\frac{q}{2} - 8 + \frac{q}{3} + 5 + \frac{q}{4} = 23$

$$\Rightarrow \quad q\left\{\frac{1}{2} + \frac{1}{3} + \frac{1}{4}\right\} = 26$$

$$\Rightarrow \quad q\left(\frac{6 + 4 + 3}{12}\right) = 26$$

$$\Rightarrow \quad q = \frac{26 \times 12}{13} = 24\ \mu\text{ C}$$

∴ Pot. drop across 3/4F cap. $\phi = \frac{q}{3}\ \frac{24}{3} \cong 8\ \mu C$

28(c)

29(d) The reduced circuit is

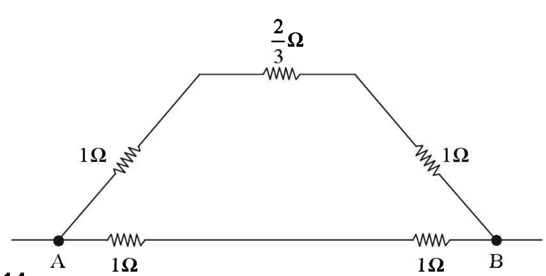


Fig. 14

$$R_{eq} = \frac{\frac{8}{3}x2}{8/3 + 2} = \frac{8}{7}\Omega$$

30(d) $R = \rho \frac{l}{A}$ $R_1 : R_2 : R_3 = \frac{l_1}{A_1} : \frac{l_2}{A_2} : \frac{l_3}{A_3}$

$= 27 : 6 : 1$

31(a)

32(a)

33(b)

34(d)

35(a) $\frac{mv^2}{r} = qvB$

$\Rightarrow \frac{1}{2} mv^2 = \frac{1}{2} qv B.r$

$\Rightarrow \frac{1}{2} mv^2 = \frac{1}{2} qB\, vr$

for Const. KE. vr = const. ....(i)

Also, $V - \sqrt{\frac{KE.2}{n}} \Rightarrow v = \frac{K}{\sqrt{m}}$ ...(ii)

from (i) and (ii)

$\frac{r}{Vm}$ is const

$\therefore$ $r_1 : r_2 : r_3 = \frac{1}{\sqrt{m_1}} : \frac{1}{\sqrt{m_2}} : \frac{1}{\sqrt{m_3}}$

$= 1 : \sqrt{2} : 1$

36(c)

37(c)

38(a)

39(c) $A_s\ L' = \mu_r L$ will increase and hence $|z| = \sqrt{R^2 + (L'\omega)^2}$ will increase, I will decrease.

40(d)

41(b)

42 (b) $u = (f + x)$ $\therefore \frac{1}{v} = \frac{1}{f} - \frac{1}{u} = \frac{1}{f+x} - \frac{1}{f}$

$= \frac{x}{f(f+x)}$

$\therefore$ $m = v/u = f/x$

43(b)

44(a)

45(b)

46(d) For minimum angle of deviation

$$\mu = \frac{\sin\left(\frac{A+Dm}{2}\right)}{\sin\frac{A}{2}} = \frac{\sin\left(\frac{A+180-2A}{2}\right)}{\sin A/2}$$

$$= \cot\frac{A}{2}$$

47(a) From Newton's law $f = \frac{u_1 + u_2}{2}$

48(d) As bulb is taken away thus intensity of light falling over it will be inversely porportional to the square of distance from point, thus the result.

49(b)

50(c)

MODEL TEST PAPER 8

1. The time period of oscillation for a circular disc if it is made to oscillate about a horizontal axis passing through its rim is

(a) $T = 2\pi\sqrt{\frac{R}{g}}$ (b) $T = 2\pi\sqrt{\frac{2R}{g}}$

(c) $T = 2\pi\sqrt{\frac{3R}{g}}$ (d) $T = 2\pi\sqrt{\frac{3R}{2g}}$

2. A compound pendulum is suspended from itc $C.G.$ then its time period of oscillation is ____ where K is radius of gyration

(a) $T = 2\pi\sqrt{\frac{2K}{g}}$ (b) $T = 2\pi\sqrt{\frac{K}{g}}$

(c) zero (d) infinity

3. Two uniform discs roll down two inclined planes of length l and 2 l as shown in the figure from the same height h. The speed at the points A and B are related as

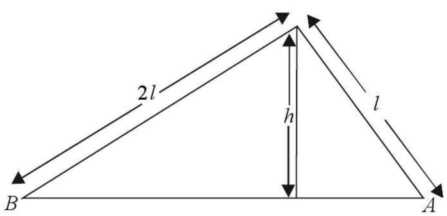


Fig. 1

(a) $V_A = V_B$ (b) $V_A = 2\,V_B$

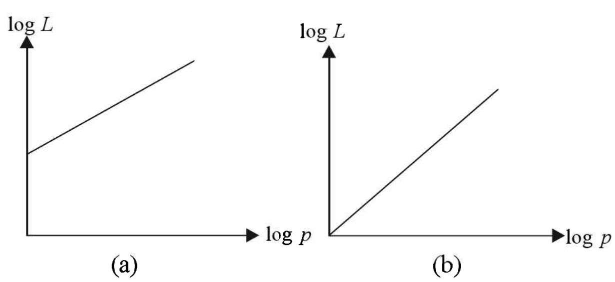
(c) $V_B = \frac{V_A}{4}$ (d) $V_B = \frac{3}{4}\,V_A$

4. The moments of inertia of a spherical shell and a solid sphere of same mass about their diameters are same. The ratio of their radius is

(a) 3 : 2 (b) 3 : 5

(c) $\sqrt{3} : \sqrt{5}$ (d) $\sqrt{3} : \sqrt{2}$

5. Select the correct curve between log L and log p where L and p denote angular momentum and linear momentum respectively



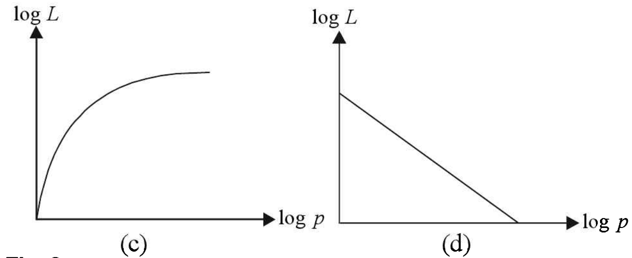


Fig. 2

6. A force varies with time as shown in the figure. The average force will be

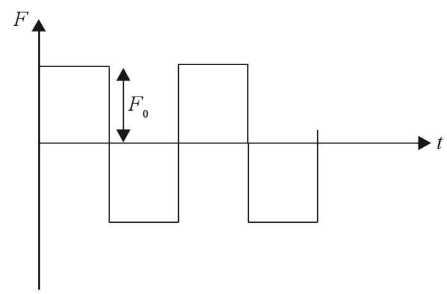


Fig. 3

(a) F_0 (b) $\frac{F_0}{\sqrt{2}}$

(c) $\frac{F_0}{2}$ (d) zero

7. Two forces $(-2i + 3j + k)$ and $(i + 2j - 4k)$ are acting at a common point. The angle between the forces is

(a) 90° (b) 60°

(c) 30° (d) none

8. A particle moves along a curve then

(a) velocity of the particle varies but acceleration remains constant in magnitude

(b) velocity of the particle is constant but acceleration varies

(c) velocity and acceleration both remain constant in magnitude

(d) velocity and acceleration both vary

9. If u represents potential energy then the condition $\frac{d^2u}{dx^2} = 0$ represents

(a) unstable equilibrium (b) stable equilibrium

(c) neutral equilibrium (d) none of these

10. The correct relation between kinetic energy K and angular momentum L of a particle in Rutherford scattering problem is

(a) $K = \dfrac{L}{2mr}$ (b) $\dfrac{L^2}{2mr}$

(c) $\dfrac{L^2}{2mr^2}$ (d) none

11. Coefficient of rolling friction has dimensions

(a) $[M°L°T°]$ (b) $[M°LT°]$

(c) $[MLT°]$ (d) none

12. If an ac voltage is given by $V = at$, $0 < t < T$ where a is a known finite constant, then rms value will be

(a) $\dfrac{a}{\sqrt{2}}$ (b) $\dfrac{aT}{\sqrt{2}}$

(c) $\dfrac{aT}{\sqrt{3}}$ (d) none

13. If in the figure 4 m = 5 kg and M = 11kg. Pulley and string are massless. The coefficient of friction between all surfaces in contact is 0.4. Then force required to move M with constant velocity is

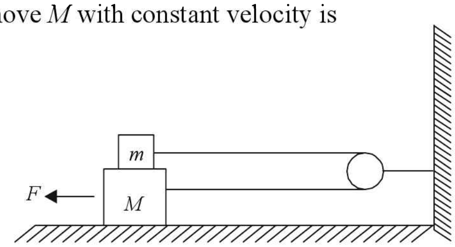


Fig. 4

(a) 5 N (b) 20 N

(c) 45 N (d) none

14. Recoil velocity of a nucleus after it has emitted a photon is

(a) $\dfrac{h}{\lambda}$ (b) $\dfrac{h}{\lambda M}$

(c) $\sqrt{\dfrac{hv}{M}}$ (d) none

where h is Planck's constant; λ wavelength; v frequency and M mass of the nucleus.

15. A radioactive nucleus ^{238}U decays to ^{234}Th by means of α-emission. If the velocity of α-particle is 1.4×10^7 m/s and its KE 4.1 MeV then KE of remaining nucleus is

(a) 0.07 MeV (b) 0.17 MeV

(c) 0.27 MeV (d) none

16. A block of mass m is moving on a frictionless track as shown in Fig. 5 with a constant velocity V. If k is the spring constant then maximum compression will be

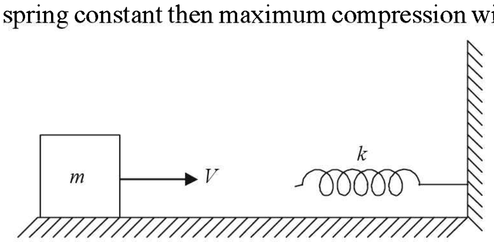


Fig. 5

(a) $\sqrt{\dfrac{mv}{k}}$ (b) $\dfrac{mv^2}{k}$

(c) $\sqrt{\dfrac{mv^2}{k}}$ (d) $\sqrt{\dfrac{v^2}{mk}}$

17. A particle moves in a potential region $U = R(x^2 + y^2 + z^2)$. The force acting on the particle is

(a) $R(x\hat{i} + y\hat{j} + z\hat{k})$ (b) $-R(x\hat{i} + y\hat{j} + z\hat{k})$

(c) $-2R(x\hat{i} + y\hat{j} + z\hat{k})$ (d) $\dfrac{R}{2}(x\hat{i} + y\hat{j} + z\hat{k})$

18. A three-phase transformer gives an output 440 V when measured by an ac voltmeter. This voltage is

(a) average value (b) peak value

(c) peak to peak value (d) rms value

19. Find the peak to peak voltage if an ac ammeter reads 1.2 A

(a) 1.7 A (b) 3.4 A

(c) 2.4 A (d) none

20. A particle of mass m slips from point A in Fig. 6. Find the force it exerts on track at Q.

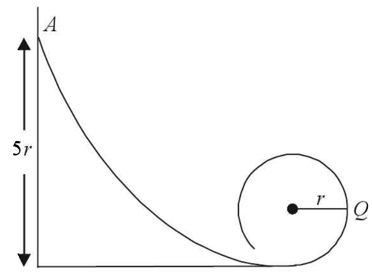


Fig. 6

(a) 8 mg (b) 3 mg

(c) 5 mg (d) none

21. A satellite will have circular orbit if eccentricity e ___ and total energy ($KE + PE$) is...

(a) e < 1, zero (b) e > 1, positive

(c) e = 0, zero (d) e = 0, negative

22. The parking orbit of a communication satellite is ... high from the surface of the earth.

(a) 3600 km (b) 36000 km

(c) 42000 km (d) 28000 km

23. What should be the velocity given to a particle if it has to reach a height equal to radius of the earth?

(a) 6 km/s (b) 7 km/s

(c) 8 km/s (d) none

24. The ratio of wavelengths corresponding to maximum intensity of emission of radiation of two bodies is 1 : 195. The ratio of their temperatures is:

(a) 1 : 195 (b) $1 : (195)^2$

(c) 195 : 1 (d) $(195)^2 : 1$

25. Newton's law of cooling is applicable to
(a) induced convectional losses
(b) natural convectional losses
(c) covectional losses
(d) in all (a), (b) or (c)

26. A black body emits radiations at the rate 5.67 watt/cm^2. Its temp is.
(a) $10^4 K$ (b) $10^3 K$
(c) $10^2 K$ (d) $10 K$

27. 40% of the radiations incident on a body are absorbed and 30% are transmitted then its coefficient of reflection is
(a) 0.2 (b) 0.3
(c) 0.4 (d) 0.7

28. A cup of tea cools from 80°C in 2 minutes when the temperature of the surrounding is 30°C. Time taken to cool from 60°C to 50°C is
(a) 1 min 12 s (b) 1 min 24 s
(c) 1 min 36 s (d) 1min 48 s

29. A pendulum is hung from the ceiling of a car accelerating at a rate 'a' in horizontal direction, then time period of oscillation is given by
(a) $2\pi\sqrt{\frac{l}{g-a}}$ (b) $2\pi\sqrt{\frac{l}{g+a}}$
(c) $2\pi\sqrt{\frac{l}{\sqrt{g^2+a^2}}}$ (d) $2\pi\sqrt{\frac{l}{\sqrt{g^2+a^2}}}$

30. A small ball of radius r rolls down a hemispherical bowl of radius R as shown in Fig. 7, then time period of oscillation of the ball is

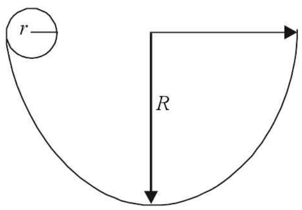


Fig. 7

(a) $T = 2\pi\sqrt{\frac{R}{g}}$ (b) $T = 2\pi\sqrt{\left(\frac{R+r}{g}\right)\frac{5}{7}}$
(c) $T = 2\pi\sqrt{\frac{(R-r)7}{5g}}$ (d) $T = 2\pi\sqrt{\frac{7r}{5g}}$

31. 9 tuning forks are arranged in series in increasing order of frequency. When they are struck, at a time 6 beats are produced. If the last tuning fork has a frequency 3 times the frequency of the first then find the frequency of first.
(a) 24 Hz (b) 68 Hz
(c) 121 Hz (d) 177 Hz

32. Decibel is unit of
(a) loudness (b) pitch
(c) quality (d) sound

33. Which of the following curves best represents the relation between velocity of sound and pressure?

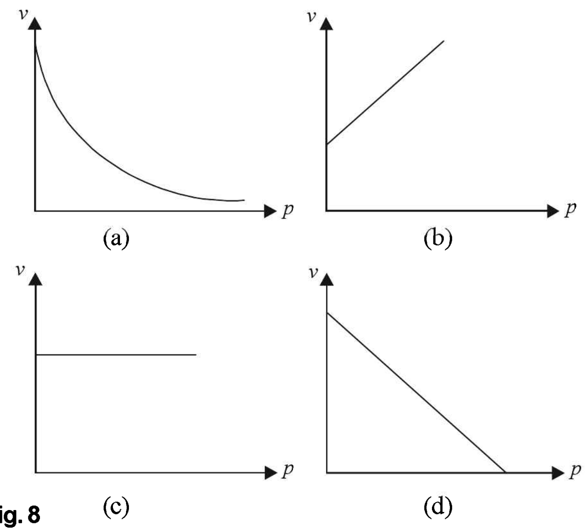


Fig. 8

34. Pitch of which sound will be maximum
(a) music (b) noise
(c) harsh sound (d) sharp sound

35. 7th overtone is equal to .... harmonic of a sound wave.
(a) 7th (b) 8th
(c) 6 th (d) none

36. A tuning fork is vibrating with second overtone. If its fundamental frequency is 256 Hz, find the number of nodes in the tuning fork.
(a) 2 (b) 4
(c) 6 (d) 5

37. The velocity of sound in air is 340 m/s. The fundamental frequency of an open pipe of length 50 cm will be
(a) 50 s^{-1} (b) 130 s^{-1}
(c) 340 s^{-1} (d) 680 s^{-1}

38. Transverse waves can be produced in a medium having
(a) bulk modulus only (b) Young's modulus
(c) modulus of rigidity (d) all of the above

39. Two tuning forks vibrating at 512Hz and 516Hz respectively. The time interval between two consecutive beats is
(a) 25 ms (b) 0.25 s
(c) 0.5 s (d) none

40. Which of the following represents a stationary wave?
(a) $y = 2y_o \sin kx \cos \omega t$ (b) $y = y_o \cos \omega t$
(c) $y = y_o \sin k(x - vt)$ (d) $y = y_o \sin \frac{2\pi}{\lambda}(vt - x)$

41. Velocity of sound in water is 1346 m/s and density of water is 1g/cc then bulk modulus of water is

(a) $1.8 \times 10^9\,Pa$ (b) $1.8 \times 10^8\,Pa$

(c) $1.8 \times 10^6\,Pa$ (d) $1.8 \times 10^6\,Pa$

42. The velocity of a plane is 2.3 Mach then it is flying with

(a) infrasonic speed (b) sonic speed

(c) supersonic speed (d) ultrasonic speed

43. Equations of waves produced by two sound sources are $y_1 = 4 \sin 400\,\pi\,t$, $y_2 = 3 \sin 404\,\pi\,t$ then a person hears... beat/s and .... intensity ratio $\left(\frac{I_{max}}{I_{max}}\right)$

(a) 4, 7 : 1 (b) 2, 7 : 1

(c) 4, 49 : 1 (d) 2, 49 : 1

44. Two capacitors of capacity 4 μF and 5 μF have charges $10\mu C$ and 20 μC respectively. What will be the charge on each after they are combined together?

(a) 10 μC, 20 μC (b) $\frac{20}{3}\ \mu C$, $\frac{70}{3}\ \mu C$

(c) $\frac{40}{3}\mu C$, $\frac{50}{3}$ C (d) 15 μC, 15 μC

45. The inner sphere of a spherical capacitor is earthed as shown in Fig. 9, then capacity of such a capacitor is

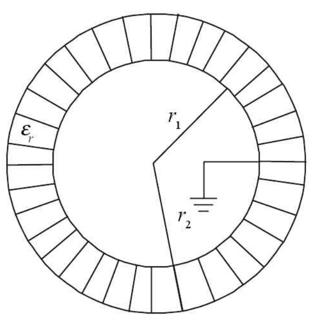


Fig. 9

(a) $4\pi\varepsilon_0\varepsilon_r \frac{r_1 r_2}{r_2 - r_1}$ (b) $4\pi\varepsilon_0\varepsilon_r r_2$

(c) $4\pi\varepsilon_0\varepsilon_r (r_1 + r_2)$ (d) $4\pi\varepsilon_0\left[\varepsilon_r \frac{r_1 r_2}{r_2 - r_1} + r_2\right]$

46. If n steps of capacitors each of capacity C as shown in Fig. 10 are arranged, then equivalent capacity is given by

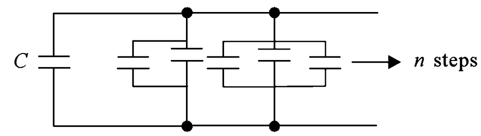


Fig. 10

(a) n^2C (b) $(n^2 + n)\,C$

(c) $\frac{(n^2 + n)C}{2}$ (d) $(n^2 - n)\,\frac{C}{2}$

47. Four capacitors of equal capacity are connected as shown in Fig. 11 If $C_{PQ} = 1\ \mu F$, then each capacitor is

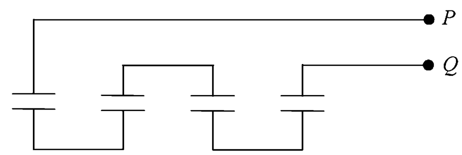


Fig. 11

(a) 1 μF (b) 2 μF

(c) 3 μF (d) 4 μF

48. 27 small drops coalesce to form a big drop. If each small drop has a potential V, then potential of big drop is

(a) 3 V (b) 9 V

(c) 27 V (d) none

49. A wire of 3Ω is bent into an equilateral Δ. Find the resistance across one of the sides.

(a) $\frac{2}{3}$ Ω (b) $\frac{3}{2}$ Ω

(c) 1Ω (d) none

50. The potential difference across $x - y$ in Fig. 12

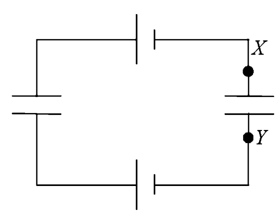


Fig. 12

(a) $\frac{(E_1 - E_2)C_1}{C_1 + C_2}$ (b) $\frac{(E_1 - E_2)C_2}{C_1 + C_2}$

(c) zero (d) none

Answers

1.	(d)	**2.**	(d)	**3.**	(a)	**4.**	(c)	**5.**	(a)	**6.**	(d)	**7.**	(a)
8.	(d)	**9.**	(c)	**10.**	(c)	**11.**	(b)	**12.**	(c)	**13.**	(d)	**14.**	(b)
15.	(a)	**16.**	(c)	**17.**	(c)	**18.**	(d)	**19.**	(b)	**20.**	(a)	**21.**	(d)
22.	(b)	**23.**	(c)	**24.**	(c)	**25.**	(a)	**26.**	(b)	**27.**	(b)	**28.**	(c)
29.	(d)	**30.**	(c)	**31.**	(a)	**32.**	(a)	**33.**	(c)	**34.**	(d)	**35.**	(b)
36.	(c)	**37.**	(c)	**38.**	(b,c)	**39.**	(b)	**40.**	(a)	**41.**	(a)	**42.**	(c)
43.	(d)	**44.**	(c)	**45.**	(d)	**46.**	(c)	**47.**	(d)	**48.**	(b)	**49.**	(a)
50.	(a)												

Explanations

1(d) $T = 2\pi\sqrt{\dfrac{l+\dfrac{R^2}{l}}{g}}$

$= 2\pi\sqrt{\dfrac{R+\dfrac{R^2}{2R}}{g}} = 2\pi\sqrt{\dfrac{3R}{2g}}$

2(d) $T = 2\pi\sqrt{\dfrac{I}{mgl}}$. Since if point of suspension is centre of gravity then $l = 0$, hence $T \to \infty$

3(a) According to law of conservation of energy

$Mgh = \frac{1}{2}MV_A^2 = \frac{1}{2}MV_B^2$

$\therefore V_A = V_B$

4(c) $I_{d\,\text{shell}} = I_{d\,\text{solid sphere}}$ or $\frac{2}{3}MR^2_{\text{shell}} = 2/5\ MR^2_{\text{sphere}}$

$\Rightarrow \dfrac{R_{\text{shell}}}{R_{\text{sphere}}} = \sqrt{\dfrac{3}{5}}$

5(a) $L = Pr$ taking $\log L = \log p + \log r$. Compare it with $y = mx + c$. It is a straight line not passing through origin.

6(d)

7(a) Because dot product becomes zero.

8(d)

9(c)

10(c) $L = mvr$

or $v = \dfrac{L}{mr}$, $K = \dfrac{1}{2}mv^2 = \dfrac{L^2}{2mr^2}$

11(b)

12(c) $V^2_{rms} = \dfrac{1}{T}\int_0^T V^2\,dt = \dfrac{a^2T^2}{3}$ and

$V_{rms} = \dfrac{aT}{\sqrt{3}}$

13(d) $T = \mu mg$ ... (1)

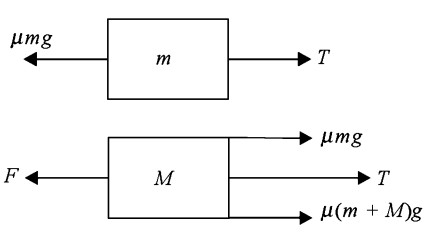


Fig. 13

$F = \mu mg + T + \mu(m+M)g$ ... (2)

$= 3\mu mg + \mu Mg$ [from (1) and (2)]

$= 3\times 0.4\times 5\times 10 + 0.4\times 11\times 10$

$= 60 + 44 = 104\,N$

14 (b) $V_{\text{recoil}} = \dfrac{p}{M} = \dfrac{E}{cM} = \dfrac{hv}{cM} = \dfrac{h}{\lambda M}$

15(a) $E_k = \dfrac{p^2}{2M}$ or $E_k \propto \dfrac{1}{m}$

$\therefore \dfrac{E_{Kth}}{K_{K\alpha}} = \dfrac{m_\alpha}{m_{th}}$ or $E_{kth} = \dfrac{4\times 4.1}{234} = 0.07\,MeV$

16(c) $\frac{1}{2}mv^2 = \frac{1}{2}kx^2$

$\Rightarrow x = \sqrt{\dfrac{mv^2}{k}}$

17(c) $\because F = \left[\dfrac{\partial U}{\partial x}i + \dfrac{\partial U}{\partial y}j + \dfrac{\partial U}{\partial z}k\right]$

18(d)

19(b) $2\times 1.2\times\sqrt{2} = 2.4\times 1.414 = 3.4\,A$

20(a) $\frac{1}{2}mv^2 = mgh$

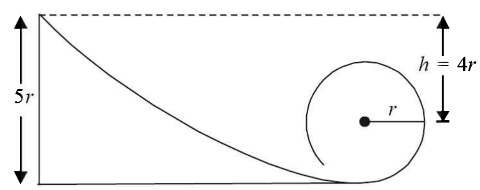


Fig. 14

$Mv^2 = 8\,mgr$, $F = \dfrac{mv^2}{r} = 8\,mg$

21(d)

22(b)

23 (c) $V = \sqrt{Rg} = 8\text{ km/s}$

24 (c) use $\lambda_m T$ = constant, then $\dfrac{\lambda_1}{\lambda_2} = \dfrac{T_2}{T_1}$

25 (a)

26(b) $E = \sigma T^4$, $T^4 = \dfrac{E}{\sigma}$

$T^4 = \dfrac{5.67\times 10^4}{5.67\times 10^{-8}} = 10^{12}$

or $T = 10^3 K$

27(b) $1 = a + t + r$

or $1 = 0.4 + 0.3 + r$

or $r = 0.3$

28(c) $$\frac{\frac{80-60}{2}}{\frac{60-50}{t}} = \frac{K\left[\frac{80+60}{2}-30\right]}{K\left[\frac{60+50}{2}-30\right]}$$

$$= \frac{10t}{10} = \frac{40}{25}$$

$\Rightarrow \quad t = 1.6 \text{ min or } 1 \text{ min } 36\,s$

29(d) $\because$ g and a are perpendicular to each other. Therefore, net acceleration is $\sqrt{a^2+g^2}$.

30(c) $$T = 2\pi\sqrt{\frac{I}{mgl}}$$

$$= 2\pi\sqrt{\frac{7}{5}\frac{(R-r)}{g}}$$

31(a) $$n_1 = \frac{(N-1)}{(r-1)} = \frac{8\times 6}{2} = 24\,\text{Hz}$$

where N = number of tuning forks arranged in series

b = number of beats

r = multiple of first tuning fork frequency the last one is

32(a)

33(c) As velocity of sound does not depend upon pressure.

34(d)

35(b)

36(c) No. of nodes = $2n$ where n is number of harmonic.

37(c) $$f = \frac{v}{\lambda} = \frac{v}{2l} = \frac{340}{1} = 340\ \text{s}^{-1}$$

38(b) and (c)

39(b) No. of beats /s = 4 time between two consecutive beats $= \frac{1}{4} = 0.25s$

40(a)

41(a) $$V = \sqrt{\frac{K}{\rho}}$$

or $K = V^2\rho = 1346^2 \times 1000$

$(\because \rho = 1\text{g/cc} = 10^3\,\text{kg}/m^3) = 1.8\times 10^9\,Pa$

42(c)

43(d) $2\pi f_1 = 400\,\pi$

or $f_1 = 200\,\text{Hz}$

$2\pi f_2 = 404\,\pi$

or $f_2 = 202\,\text{Hz}$

$\therefore$ no of beats $= 2s^{-1}$

$$\frac{I_{\max}}{I_{\min}} = \frac{(4+3)^2}{(4-3)^2} = \frac{49}{1}$$

44(c) $$V = \frac{Q_1+Q_2}{C_1+C_2}$$

$$= \frac{30}{9} = \frac{10}{3}V$$

$$Q_1' = C_1V = 4\times\frac{10}{3} = \frac{40}{3}\,\mu C$$

$$Q_2' = C_2V$$

$$= 5\times\frac{10}{3} = \frac{50}{3}\,\mu C$$

45(d)

46(c) Capacitors are in a AP

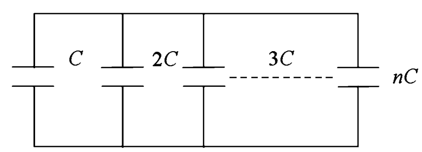


Fig. 15

$$\therefore \quad C_{eq} = \frac{n(n+1)}{2}C$$

47(d) Since $\frac{C}{4} = 1\,\mu F$

(in series $C_{eq} = \frac{C}{n}$)

$\therefore \quad C = 4\,\mu F$

48(b) $V_{big} = (n)^{2/3}V_{small}$

$= (27)^{2/3}V = 9V$

49(a) $\because$ 2Ω and 1Ω appear to be in parallel as shown

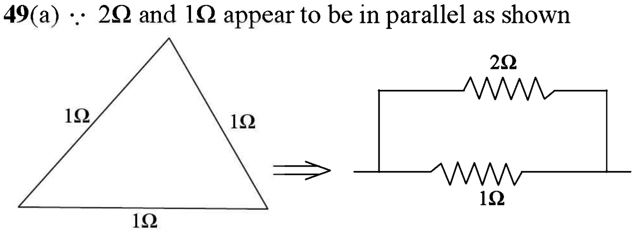


Fig. 16

50(a) Since E_1 and E_2 are in negative series

$\therefore$ the equivalent value of emf is $E_1 - E_2$

$$V_2 = \frac{(E_1-E_2)C_1}{C_1+C_2}$$

MODEL TEST PAPER 9

1. A Boson is a particle having

(a) zero spin (b) spin = $\pm\frac{1}{2}$

(c) any of (a) or (b) (d) none of these

2. A fermion is a particle

(a) which follows Maxwell's statistics

(b) which follows Pauli's exclusion principle

(c) having integral spin

(d) none of these

3. Which of the following is not a boson?

(a) proton (b) cooper-pair

(c) photon (d) phonon

4. If F_e and F_n are electrostatic and nuclear forces acting at a distance 100 cm between two protons then

(a) $F_e < F_n$ (b) $F_e = F_n$

(c) $F_e > F_n$

(d) F_e may be nearly equal to F_n

5. *AC* current is

(a) *a* scalar (b) *a* vector

(c) *a* phasor (d) none of these

6. What is output voltage V_o in Fig 1.

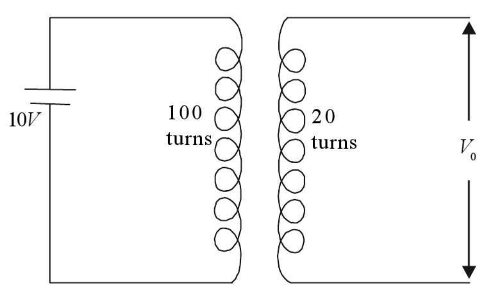


Fig. 1

(a) 2 volts (b) 5 volts

(c) 10 volts (d) zero

7. The power of an ac motor is expressed in

(a) watts (b) volt ampere

(c) any of (a) or (b) (d) none of these

8. Epoch is ____

(a) phase difference

(b) frequency deviation

(c) initial phase angle of a phasor or of a wave

(d) phase modulation

9. *SHM* is ____ motion.

(a) isochronous (b) isotropic

(c) isoamplitude (d) none of these

10. A fly wheel will possess

(a) larger angular acceleration

(b) larger angular velocity

(c) larger angular momentum

(d) larger torque

11. Coriolis force equals

(a) inertial force $2\, m\, \omega\, v$ (b) centripetal force $mr\,\omega^2$

(c) centrifugal force mv^2/r (d) none of these

12. Conservative force means

(a) the total energy of a particle acted upon by such a force is zero

(b) the work done against such a force in closed path is zero

(c) the total energy of a particle acted upon by such a force is conserved

(d) both (b) and (c)

13. The wheels of automobiles are made hollow to have

(a) larger moment of inertia

(b) larger acceleration

(c) larger efficiency

(d) larger angular velocity

14. The condition for a mechanical transverse wave to traverse through a medium is

(a) it shall have young's modulus

(b) it shall have bulk modulus

(c) it shall possess modulus of rigidity

(d) it shall have both bulk modulus and modulus of rigidity

15. Two sources of frequency 405 and 200 Hz are sounded together then

(a) no beats will be heard

(b) 5 beats will be heard

(c) 205 beats will be heard

(d) none of these

16. Molar heat capacity in an isothermal process is

(a) finite but not zero (b) zero

(c) infinity (d) R

17. Molar heat capacity in an adiabatic process is

(a) R (b) zero
(c) finite but not zero (d) infinity

18. Two bodies P and Q having equal surface areas are held at temperatures 20°C and 40°C. Thermal radiation ratio of $P : Q$ in a given time is

(a) 1 : 1.13 (b) 1 : 2
(c) 1 : 16 (d) 1 : 4

19. 1 ohm resistance is colour coded. The colour bands in sequence will be

(a) Black, Brown, Black (b) Black, Black, Black
(c) Brown, Black, Gold (d) Brown, Black, Silver

20. A conductor has resistivity 2 ohm – m. Its length is doubled and diameter halved. The new specific resistance is

(a) 16 ohm – m (b) 8 ohm – m
(c) 4 ohm – m (d) 2 ohm – m

21. A galvanometer of current sensitivity 10 μA/division has 20 divisions. It is to be converted into a voltmeter of 5 V. The resistance to be connected in series is____. The internal resistance of galvanometer is 200 Ω

(a) 20 $k\Omega$ (b) 24.8 $k\Omega$
(c) 2.48 $k\Omega$ (d) 2 $k\Omega$

22. A capacitor of 20 μF is being charged with a series resistance 1 $k\Omega$ and 10 V supply. The time in which it charges to 9.5 V is

(a) 20 ms (b) 60 ms
(c) 40 ms (d) 50 ms

23. The flow is streamlined if Reynold's number is

(a) equal to 2000 (b) less than 2000
(c) greater than 2000 (d) equal to 3000

24. A viscometer when filled with glycerine (density 1.26 g/cc and viscosity 8.5 poise) takes 45s to get empty. The viscosity of castor oil will be .... if it is emptied in 69 s (density 0.87 g/cc)

(a) 10.121 Poise (b) 9.102 Poise
(c) 7.95 Poise (d) 8.54 Poise

25. Nutation represents

(a) precession in the top (b) wobbling in the top
(c) spinning of top
(d) translation motion of top

26. A racing car moving with constant acceleration, covers two successive kilometers in 30 s and 20 s respectively. Find the acceleration of the car and the initial speed.

(a) $\frac{2}{3}\ ms^{-2}, \frac{70}{3}\ ms^{-1}$ (b) $\frac{3}{2}\ ms^{-2}, \frac{70}{3}\ ms^{-1}$
(c) $\frac{2}{3}\ ms^{-2}, \frac{35}{3}\ ms^{-1}$ (d) $\frac{2}{3}\ ms^{-2}, \frac{140}{3}\ ms^{-1}$

27. A ball is dropped from a balloon which is 290 m above the ground and ascending at 14 m/s the maximum height to which it can reach is

(a) 200 m (b) 300 m
(c) 75 m (d) 150 m

28. In the previous question the position and velocity of the ball 5 sec after being dropped is

(a) 237.5 m, $\frac{70}{3}$ m/s (b) 237.5 m, 35 m/s
(c) 300 m, 35 m/s (d) 300 m, $\frac{70}{3}$ m/s

29. A cricketer hits a ball with a velocity 25 m/s at 60° above the horizontal. Find how far above the ground it passes over a fielder 50 m from the bat.

(a) 4.9 m (b) 16 m
(c) 9.8 m (d) 8.2 m

30. An aeroplane moving horizontally at a speed of 200 m/s and at a height of 8 km is to drop a bomb on a target. At what horizontal distance from the target should the bomb be released?

(a) 1960 m (b) 980 m
(c) 8080 m (d) 4900 m

31. A stone is projected from the ground with a velocity of 25 m/s. Two seconds later, it just clears a wall 5 m high. Find the angle of projection of the stone.

(a) 90° (b) 45°
(c) 60° (d) 30°

32. A stone is projected from the ground with a velocity of 25 m/s. Two second later, it just clears a wall 5 m high. Find the greatest height reached.

(a) 7.8 m (b) 9.8 m
(c) 19.6 m (d) 4.9 m

33. A lorry travelling on the level ground at 40 km/h can be stopped by its brakes in a distance 16 m. Find the speed from which it can be brought to rest in the same distance when descending a hill whose angle of slope is $\sin^{-1}\left(\frac{1}{15}\right)$

(a) 5.07 m/s (b) 10.14 m/s
(c) 15.07 m/s (d) 20.28 m/s

34. The force required just to move a body up an inclined plane is double the force required just to prevent the body sliding down it. Then coefficient of friction is μ. Find the angle of inclination of the plane.

(a) $\tan^{-1}(2\mu)$ (b) $\tan^{-1}\mu$
(c) $\tan^{-1}\left(\frac{1}{\mu}\right)$ (d) $\tan^{-1}(3\mu)$

35. A motor car has a distance of 1.1 m between wheels, its centre of gravity is 62 cm above the ground and the coefficient of friction between the wheels and the road is 0.8 when the car travels round a curve at speed, it is more likely to skid or to overturn? What is the maximum possible speed if the centre of gravity describes a circle of radius 16 m? The road surface is horizontal.

(a) 5.42 m/s (b) 21.6 m/s
(c) 10.84 m/s (d) 9.8 m/s

36. A horse pulls a wagon of 5000 kg from rest against a constant resistance of 90 N. The pull exerted initially is 600 N and it decreases uniformly with the distance covered to 400 N at a distance of 15 m from the start. Find the velocity of the wagon at this point.

(a) 3.14 m/s (b) 21.6 m/s
(c) 0.76 m/s (d) 1.57 m/s

37. Find the radius of the orbit of a satellite of the earth which orbits in an equatorial plane with a period of revolution of 24 hours. The mass of the earth = 6.0×10^{24} kg . Assume the value of G.

(a) 4.2×10^8m (b) 4.2×10^7m
(c) 8.4×107 m (d) 4.2×10^9m

38. Three identical bodies of mass m each are located at the vertices of an equilateral triangle of side L. At what speed must they move if they all revolve under the influence of one another's gravity in a circular orbit circumscribing the triangle while still presenting the equilateral triangle.

(a) mG/L (b) $m^2 G/L$
(c) $(mG/L)^{1\backslash 2}$ (d) $(L/mG)^{1\backslash 2}$

39. An artificial satellite is revolving round the earth in a circular orbit at a height of 600 km from the surface of the earth. Find the speed and the time period of revolution of the satellite. (Radius of the earth = 6400 km)

(a) 7.86 km/*s*, 96 min (b) 7.86 km/s, 48 min
(c) 786 km/*s*, 96 min (d) 7.86 m/s, 96 sec.

40. A machine gun fires 50 gm bullets at a speed of 1km/s. The gunner holding the machine gun in his hands can exert an average force of 180 N against the gun. Determine the maximum number of bullets he can fire per minute.

(a) 16 (b) 512
(c) 216 (d) 108

41. A body of 2.0 kg mass makes an elastic collision with another body at rest and afterwards continues to move in the original direction but with one-fourth of its original speed. What is the mass of the struck body?

(a) $\frac{7}{5}$ kg (b) $\frac{5}{6}$ kg
(c) $\frac{3}{5}$ kg (d) $\frac{6}{5}$ kg

42. A 100 km/h wind blows normally against one wall of a house having an area of 50 m^2. Calculate the force exerted on the wall if the air moves parallel to the wall after striking it and has a density 1.134 kg/m^3

(a) 260.4 kN (b) 393.8 kN
(c) 120.2 kN (d) 468.2 kN

43. A piece of sugar weighting 40 g is coated with 5.76 g of wax of specific gravity 0.96. If the coated sugar weighs 14.76 g in water, find the specific gravity of sugar.

(a) 0.9 gm/cm^3 (b) 1.6 gm/cm^3
(c) 0.8 gm/cm^3 (d) 1.8 gm/cm^2

44. A piece of an alloy of mass 96 gm is composed of two metals whose specific gravities are 11.4 and 7.4. If the weight of the alloy is 86 g in water, find the mass of each metal in the alloy.

(a) 78 gm 18 cm (b) 62.7 gm, 33.3 gm
(c) 68.3, 27.8 (d) 55.1 gm, 40.9 gm

45. A weighing scale is adjusted to zero. Particles fall from a height of 4.9 m before colliding with the balance pan on the scale, the collisions are elastic i.e, the particles rebound upward with the same speed. If each particle has a mass of 5 gm and collisions occur at the rate of 50 particles per second, what is the scale reading in gm?

(a) 450 g (b) 560 g
(c) 500 g (d) 482 g

46. A cubical lump of ice (specific gravity 0.92) has embedded in it a piece of iron (specific gravity 7.76) of mass 1gm. The lump of ice floats in water and gradually melts, but retains its cubical form. Find the edge of the cube when it sinks.

(a) 11.25 mm (b) 44.5 m
(c) 2.225 cm (d) 22.25 mm

47. A rectangular tank 2 m deep, 3 m broad and 4 m long is filled with water. Calculate the magnitude and the point of action of the resultant thrust due to the water on one of the sides.

(a) $\frac{4}{3}$ m (b) $\frac{1}{3}$ m
(c) $\frac{5}{3}$ m (d) $\frac{2}{3}$ m

48. The angle between two equal forces so that their resultant is one-third of any of the forces is

(a) 120° 30′ (b) 150°
(c) 140° 56′ (d) 75° 30′

49. A simple pendulum is set up in a lift which has a downward acceleration greater than the acceleration due to gravity. The pendulum executes oscillations with period.

(a) $2\pi\sqrt{\dfrac{L}{a-g}}$ (b) $2\pi\sqrt{\dfrac{L}{a+g}}$

(c) $\dfrac{1}{2\pi}\sqrt{\dfrac{L}{a-g}}$ (d) $2\pi\sqrt{\dfrac{L}{g-a}}$

50. A body of 2 kg mass makes a perfectly inelastic collision with another body at rest and afterwards, continues to move in the original direction but with one-fourth of its original speed. The mass of the struck body is

(a) 8 kg (b) 8 kg

(c) 2 kg (d) 6 kg

Answers

1.	(a)	**2.**	(b)	**3.**	(a)	**4.**	(c)	**5.**	(c)	**6.**	(d)	**7.**	(b)
8.	(c)	**9.**	(a)	**10.**	(c)	**11.**	(a)	**12.**	(d)	**13.**	(a)	**14.**	(d)
15.	(b)	**16.**	(c)	**17.**	(b)	**18.**	(a)	**19.**	(c)	**20.**	(d)	**21.**	(b)
22.	(b)	**23.**	(b)	**24.**	(b)	**25.**	(b)	**26.**	(a)	**27.**	(b)	**28.**	(b)
29.	(d)	**30.**	(c)	**31.**	(d)	**32.**	(a)	**33.**	(b)	**34.**	(d)	**35.**	(c)
36.	(d)	**37.**	(b)	**38.**	(c)	**39.**	(a)	**40.**	(c)	**41.**	(d)	**42.**	(b)
43.	(b)	**44.**	(b)	**45.**	(c)	**46.**	(c)	**47.**	(a)	**48.**	(c)	**49.**	(a)
50.	(d)												

Explanations

1(a) Bosons are particles which have zero or integral spin and follow Bose-Einstein statistics.

2(b) Fermions are particles which have spin half odd integral multiple.They follow Fermi-Dirac statistics. They also follow Pauli's exclusion principle i.e. in a subshell two fermions with opposite spins can be accommodated.

3(a) Proton is a fermion. All others are bosons.

4(c) Nuclear force has a range of the order of 10 femtometer (10^{-14} m). At a distance much larger than 10 fm Nuclear force is much less than electrostatic force.

5(c) *AC* current is a phasor as it possesses a magnitude and phase angle e.g. $I = I_0 \sin(\omega t + \phi)$ will have amplitude $= I_0$ and phase $= \phi$

6(d) Transformer acts on the principle of mutual induction. Therefore, it works when ac is applied, if dc is inputted, output is zero.

7(b) We know ac power is $P = V.I\cos\phi$ where V and I are rms values of voltage and current, respectively, and $\cos\phi$ is power factor. Because in motors $\cos\phi$ will depend upon local conditions. Therefore, power consumed will be different at different places for this reason exact wattage cannot be represented.

8(c) Consider $y = a\sin(\omega t + \phi)$ ϕ is the initial phase angle which is called epoch or angle of repose.

9(a) In SHM time period of vibrations is constant. Therefore, it is called isochronous (equal-time) motion.

10(c)A flywheel possesses larger angular momentum. Because of this even a small jerk will allow the flyweel to complete the rotation.

11(a)Consider a turntable rotating with a uniform speed ω about a fixed axis. Assume a small cart moves outward on a radial track mounted on the turn table. Let V be the velocity of the cart. As the time elapses cart possesses, besides radial acceleration $a_r = \omega^2 r$, an inertial acceleration $a_r = w^2$ a tangential acceleration $a_t = Wv_r$. The inertial force $m_{a_t} = 2$ mwv is called coriolis force. It plays an important part in the motions of earth's atmosphere.

12(d)

13(a)Because hollow surfaces have larger moment of inertia. Moment of inertia in rotational motion acts as mass in linear motion.

14(d)Transverse waves require both bulk modulus and modulus of rigidity since air does not possess modulus of rigidity, therefore, transverse waves cannot traverse through it.

15(b) Beats are also produced by a combination of tones.

$405 - 200 = 205$ Hz

200 Hz and 205 Hz tones give 5 beats per second.

16(c)Molar heat capacity is given by

$$C = \frac{\Delta Q}{n\Delta T} \text{ In an isothermal process}$$

$\because \Delta T = 0, \therefore C =$

17(b)In case of adiabatic process $\Delta Q = 0$. Therefore, with the agruments given in solution of 16th question, we conclude $C_{\text{adiabatic}} = 0$

18(a)Radiation energy is directly proportional to T^4

$$\frac{E_P}{E_Q} = \frac{(273+20)^4}{(273+40)^4} = \left(\frac{293}{313}\right)^4 = \frac{1}{1.13}$$

19(c)First colour is never black. The scheme is

$ab \times 10^{-c}$

$10 \times 10^{-1} = 1$

$1 \Rightarrow$ Brown

$0 \Rightarrow$ Back

$10^{-1} \Rightarrow$ Gold

20(d) Resistivity or specific resistance is an inherent property of the conductor and hence remains constant for a given conductor.

21(b) Series Resistance R_s is given by

$$R_s = \frac{V}{I_g} - R_g$$

$$\Rightarrow \frac{5}{200\times10^{-6}} - 200$$

$$= 24800$$

or $= 24.8\ k\Omega$

22(b)A capacitor takes $3\tau = 3\ RC$ time to charge to 95% value of maximum 9.5 V = 95% of 10 V $\therefore t = 3\ RC = 60$ ms.

23(b) If Reynold's number is less than 2000 or velocity of fluid is less than 20c m/s then the fluid has streamlined motion or steady motion. If Reynold's number is > 3000. Then flow becomes turbulant.

24(b) $\frac{\eta_1}{\eta_2} = \frac{\rho_1 t_1}{\rho_2 t_2}$ where n = viscosity, ρ = density and t time taken by the fluid.

25(b) A spinning top has 3 types of motion (i) spinning (ii) precession (iii) wobbling or nutation

26(a) $1000 = 30\,u + \frac{1}{2}(30)^2 a = 30\,u + 450\,a$

$2000 = 50u + \frac{1}{2}(50)^2 a = 50u + 1250\,a$

or $100 = 3\,u + 45\,a$

$200 = 5\,u + 125\,a$

Solving for u and a

$a = \frac{2}{3}\ m/s^{-2}$ and $u = \frac{70}{3}\ m/s^{-1}$

27(b) Take upward direction as positive

$u = 14\ m/s,\ a = -9.8\ m/s^2,\ V = 0,\ S = ?$

using $v^2 - u^2 = 2\ as$

$0 - 14^2 = -2 \times 9.8 \times s$

$$s = \frac{14^2}{2\times9.8} = 10\,m$$

Maximum height reached = 290 + 10 = 300

28(b) $u = 14\ m/s,\ a = 9.8\ m/s^2,\ t = 5s,\ = s\ ?,\ v = ?$

$$s = ut + \frac{1}{2}at^2$$

$$= 14 \times 5 - \frac{1}{2} \times 9.8 \times 5^2 = -52.5\ \text{m}$$

Height above the ground = 290 – 52.5 = 237.5 m

$v = u + \text{at} = 14 - 9.8 \times 5 = -35$ m/s

velocity downward = 35 m/s

29(d) $$y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$$

$$= 50\sqrt{3} - \frac{9.8(50)^2}{2\times(25)^2\left(\frac{1}{2}\right)^2}$$

$= 8.2$ m

30(c) If t is the time taken for the bomb to drop on the target, using

$$h = \frac{1}{2}gt^2 \Rightarrow 8.0 \times 10^3 = \frac{1}{2} \times 9.8 \times t^2$$

$$t = \sqrt{\frac{2\times8\times10^3}{9.8}} = 40.4\,s$$

The horizontal distance covered by the bomb during this time = 200 × 40.4 = 8080 m

31(d) $$h = u_y t - \frac{1}{2}gt^2$$

$5 = 25\sin\alpha(2) - 52^2$

$\sin\alpha = \frac{1}{2}$ or $\alpha = 30°$

with the horizontal

32. Greatest height reached

$$h = \frac{u^2\sin^2\theta}{2g}$$

$$= \frac{25\times25\times1}{2\times10\times4}$$

$= 7.8\,m$

33(b) The speed of the lorry = 40 km/h = $\frac{40\times5}{18} = \frac{100}{9}$ m/s

Distance in which it is stopped = 16 m.

$$o = \left(\frac{100}{9}\right)^2 + 2a \times 16$$

$$a = -\frac{100^2}{9^2\times32} = -3.86\ \text{m/s}^2$$

force of brakes = Ma = 3.86 M Newton, where M is the mass of the lorry.

net retardation

$= a_{\text{brakes}} - g\sin\alpha$

$= 3.86 - 9.8 \times \frac{1}{15}$

$= 3.21\ \text{ms}^{-2}$

Let u_1 be the velocity required.

$0 = u_1^2 - 2 \times 3.21 \times 16$

$u_1 = \sqrt{2 \times 3.21 \times 16} = 10.14$ m/s

34(d) If F_1 is the force required to move a body up the inclined plane,

$F_{up} = W\sin\theta + \mu W\cos\theta$

In the second case,

$F_{down} = W\sin\theta - \mu W\cos\theta$

It is given that $F_1 = 2F_2$

$\therefore \quad W\sin\theta + \mu W\cos\theta = 2(W\sin\theta - \mu W\cos\theta)$

$\sin\theta + \mu\cos\theta = 2\sin\theta - 2\mu\cos\theta$

$\sin\theta = 3\mu\cos\theta$

$\tan\theta = 3\mu$:

$\theta = \tan^{-1}(3\mu)$

35(c) Here $\mu = 0.8$, $a = \dfrac{1.1}{2} = 0.55\ m$

$h = 0.62\ m.$

Substituting numerical values,

$V_s' = \sqrt{0.8gr} = 0.894\sqrt{gr}\ m/s$

$V_t' = \sqrt{\dfrac{0.55gr}{0.62}} = 0.94\sqrt{gr}\ m/s$

It is seen that V_s' is less that V_t'.

If the velocity increases gradually, V_s' will be attained first and the car will skid.

Hence skidding will occur before overturning

$V_s' = \sqrt{0.8 \times 9.8 \times 15} = 10.84\ m/s$

36(d) If a graph is drawn between force and displacement, the area under the graph gives the work done and therefore the kinetic energy.

Initial pull = 600 N

Effective force = 600 – 90 = 510 N

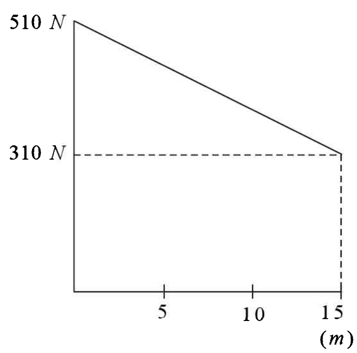


Fig. 2

Effective force after covering 15 m.

$= 400 - 90 = 310\ N$

Area under the graph $= \dfrac{1}{2}mv^2$

$= \dfrac{510 + 310}{2} \times 15 = 6150\ J$

$= \dfrac{1}{2} \times 5000 \times v^2 = 6150$

$v^2 = \dfrac{2 \times 6150}{5000}$ or $v = \sqrt{\dfrac{2 \times 615}{500}} = 1.57\ m/s.$

37(b) $\dfrac{GM_E M_s}{r^2} = M_s\,\omega^2 r$, where r is the radius of the orbit

$\omega^2 = \dfrac{GM_E}{r^3}$ .... (1)

The period of revolution $T = 24 \times 60 \times 60$ second

$\omega^2 = \dfrac{4\pi^2}{24^2 \times 60^2 \times 60^2} = \dfrac{GM_E}{r^3} = \dfrac{6.7 \times 10^{-11} \times 6 \times 10^{24}}{r^3}$

$r^3 = \dfrac{6.7 \times 6 \times 10^{-11} \times 10^{24} \times 24^2 \times 60^2 \times 60^2}{4\pi^2}$

$= 76.03 \times 10^{21}$

$r = (76)^{1/3} \times 10^7 = 4.2 \times 10^7\ m$

38(c) Consider the mass m at c. Let F be the force with which the mass is attracted by the masses at B and A.

The resultant of these two forces

$= 2F = \cos 30° = 2F.\dfrac{\sqrt{3}}{2} = F\sqrt{3}$

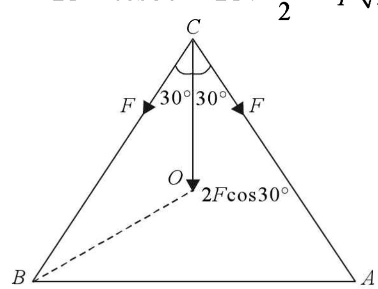


Fig. 3

The radius of the circle $= \propto = \dfrac{L}{\sqrt{3}}$

The centripetal force $= F\sqrt{3} = \dfrac{mv^2}{r} = \dfrac{mv^2\sqrt{3}}{L}$

Therefore $F = \dfrac{mv^3}{L}$, $F = \dfrac{-Gm^2}{L^2}$

force (1) and (2) $mv^2 = \dfrac{Gm^2}{L} = v = \sqrt{\dfrac{mG}{L}}$

39(a) $g_0 = \dfrac{GM}{R_E^2}$ where $g_0 = 9.8\ m/s^2$, $g = \dfrac{GM}{(R_E + 600)^2}$

$\dfrac{g_0}{g} = \left[\dfrac{R_E + 600}{R_E}\right]^2 = \left(\dfrac{7000}{6400}\right)^2 = \left[\dfrac{35}{32}\right]^2$

$g = \left(\dfrac{32}{35}\right)^2 \times 9.8$

$v_0 = \sqrt{g(R_E + 600)} = \sqrt{\left(\dfrac{32}{35}\right)^2 \times 9.8 \times 7 \times 10^6}$

$= \dfrac{32}{35}\sqrt{9.8 \times 7 \times 10^6}$

$$T = \frac{2\pi(R_E + 600)}{V_0}$$

$$= \frac{2\pi \times 7000 \times 10^3 \times 35}{32\sqrt{9.8 \times 7 \times 10^6}} \text{ seconds}$$

40(c) Let n be number of bullets that can be fired per sec

Force $= nmu = n \times 50 \times 10^{-3} \times 1000 = 180\,N$

$$n = \frac{180 \times 10^3}{50 \times 1000} = 3.6$$

Number of bullets that can be fired per minute

$= 60 \times 3.6 = 216$

41(d) Let m be the mass of the struck body. Initial momentum $= 2u$. Let v be the velocity of struck body after collision

$$2u = 2 \times \frac{u}{4} + mv \quad \text{... (1)}$$

From Newton's experimental law, for an elastic collision, the velocity of approach = the velocity of separation

$$u - 0 = v - \frac{u}{4} \quad \text{... (2)}$$

From (1), $u\left(2 - \frac{1}{2}\right) = mv,\ \frac{3}{2}u = mv$

From (2), $\frac{5}{4}u = v,\ m = \frac{3}{2} \times \frac{4}{5} = \frac{6}{5}$ kg

42(b) $F = \rho A v^2 = 1.134 \times 50 \times \left(\frac{250}{9}\right)^2$

$= 393.8\,KN$

43(b) Mass of sugar = 40 *gm*.

Mass of wax = 5.76 *gm*.

Mass of sugar and wax in air = 45.76 *gm.*

Mass of sugar and wax in water = 14.76 *gm.*

Apparent loss of weight = 31.00 *gm*.

Volume of sugar and wax = 31.00 cm^3

Volume of wax alone $= \frac{5.76}{0.96} = 6\ cm^3$

Volume of sugar $= 31 - 6 = 25\ cm^3$

Density of sugar $= \frac{40}{25} = 1.6\ gm/cm^3$

44(b) Let mass of the metal of specific gravity 11.4 be M

The volume of the metal $= \frac{M}{11.4}\ cm^3$

The volume of the other metal $= \frac{96 - M}{7.4}\ cm^3$

Total volume of the alloy $= \frac{M}{11.4} + \frac{96 - M}{7.4}\ cm^3$

$$= \frac{M}{11.4} + \frac{96 - M}{7.4} = 10$$

or $\quad 7.4M + 11.4 \times 96 - 11.4M = 10 \times 11.4 \times 7.4$

$M = 62.7\,g$

Mass of the other metal $= 96 - 62.7 = 33.3\,g$

45(c) The particle falls from a height of 4.9 m.

$v^2 = 2gh,$

$= v = \sqrt{2gh}$

The velocity of the particle as it hits the pan

$= \sqrt{2 \times 9.8 \times 4.9} = 9.8\ m/s$

The momentum of the particle before collision

$= 5 \times 10^{-3} \times 9.8\,Ns$

As the particle rebounds with the same speed the momentum after collision

$= -5 \times 10^{-3} \times 9.8\,NS$

Change in momentum of each particle

$= 2 \times 5 \times 10^{-3} \times 9.8\,Ns$

Rate of change of momentum

$= 50 \times 2 \times 5 \times 10^{-3} \times 9.8\,N$

Reading of the balance $= \frac{50 \times 2 \times 5 \times 10^{-3} \times 9.8}{9.8}$

$= 500\,g$

46(c) and (d) Let a be the edge of the cube when it sinks, so that the volume of the cube $= a^3$

Upward thrust due to displaced water $= a^3$ *gm. wt*

Mass of iron = 1gm

Volume of ice $= \frac{(a^3 - 1)}{0.92}$ cm^3

Volume of iron $= \frac{1}{7.76}$ cm^3

$$= \frac{a^3 - 1}{0.92} + \frac{1}{7.76} = a^3$$

$$0.08\,a^3 = 1 - \frac{0.92}{7.76}$$

$$a^3 = \frac{6.84}{7.76 \times 0.08} = 11.02$$

$$a = 2.225\text{ cm} = 22.25\text{ mm}$$

47(a) $\quad P = \rho gh = 10^3 \times 9.8 \times 1 = 9.8 \times 10^3\ \text{Nm}^{-2}$

Thrust $= 9.8 \times 10^3 \times 4 \times 2 = 76.4 \times 10^3$ N

The point of action is at the centre of pressure

$= \frac{2}{3} \times \text{depth} = \frac{2}{3} \times 2 = \frac{4}{3}$ m below the surface

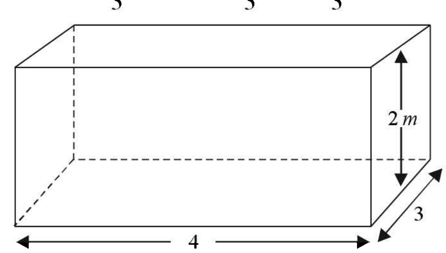


Fig. 3

48(c) $2F\cos\frac{\theta}{2} = \frac{F}{3}$ or $\cos\frac{\theta}{2} = \frac{1}{6}$

$$\therefore \quad \frac{\theta}{2} = \cos^{-1}\left(\frac{1}{6}\right)$$

$$= 70° \; 28'$$

$$\Rightarrow \quad \theta = 140° \, 56'$$

49(a) The pendulum bob will appear to float up tied to the string from below. If T is tension in string.

$T + mg = ma$

$$\Rightarrow \quad T = m(a-g)$$

$\therefore$ Time period $= 2\pi\sqrt{\frac{L}{a-g}}$

50(d) Let the mass of the struck body be m kg.

$$2.0 \times v = (2+m)\frac{v}{4}$$

$$\therefore \quad 2 = \frac{2+m}{4}$$

$$m + 2 = 8$$

$$\Rightarrow \quad m = 6\,\text{kg}$$

MODEL TEST PAPER 10

1. Calculate the energy stored in capacitor in the circuit shown below.

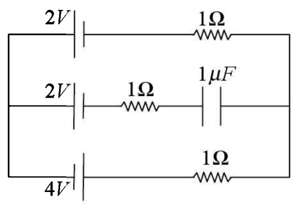


Fig. 1

(a) 3 μJ (b) 4.5μJ

(c) 2μJ (d) none

2. In the circuit shown below what is the energy associated to the inductor. Also find out power loss in the resistance R.

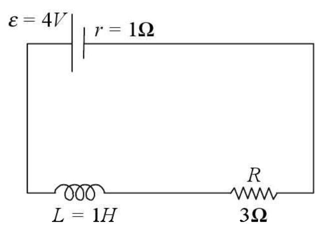


Fig. 2

(a) 0 and 3 W (b) 1 J and $\frac{48}{5}$ W

(c) 0.5J and 3 W (d) 1J and 3W

3. Two similar heater filaments A and B are connected in series and operated across a battery. When the filaments are glowing at their maximum, filament A is immersed in a tank of ice.

(a) power loss in A increases

(b) power loss in B increases

(c) power loss in B drcreases

(d) power loss in the entire circuit remains same

4. 100 volts DC when applied across a resistance R dissipates P watts. What must be the peak value of AC applied which dissipates half of this power?

(a) 50 V (b) 50 $\sqrt{2}$ V

(c) 100 V (d) 100 $\sqrt{2}$ V

5. Across a series LCR circuit a source of alternating e.m.f is applied. Potential difference across each component is found to be 100 volt. When capacitor is short circuited then what is the potential across resistor?

(a) 100 V (b) 100 $\sqrt{2}$ V

(c) 200 V (d) 50$\sqrt{2}$ V

6. A rod of length 'l' is revolving clockwise in a magnetic field $\vec{B}$ perpendicular to the plane of revolution with a constant angular velocity ω, about an axis passing through O and parallel of $\vec{B}$. N is the middle point of the rod. What is the ratio of induced e.m.f. between O and N to induced e.m.f. between N and M?

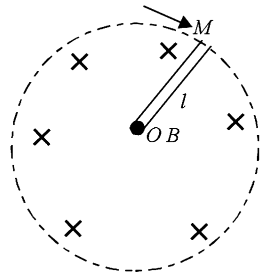


Fig. 3

(a) 1 : 2 (b) 1 : 3

(c) 1 :1 (d) none is correct

7. An x-ray tube is operating at 50 KV. The minimum wavelength of x-ray produced is

(a) .1 $\mathring{A}$ (b) 0.25 $\mathring{A}$

(c) 2.5 $\mathring{A}$ (d) 10 Å

8. A H atom in the ground state absorbs 10.2 eV in 10^{-8} sec. The torque that acts on the electron is

(a) 1.05×10^{-26} $N\text{–}m$ (b) 2.1×10^{-26} $N\text{–}m$

(c) 1.05×10^{-34} $N\text{–}m$ (d) None.

9. Assuming the orbit of electrons to be circular in 'H' atom. The ratio of areas of first and second orbit is

(a) 1 : 4 (b) 1 : 9

(c) 1 : 16 (d) None.

10. Which of the following range corresponds to visible light?

(a) 1 to 2 eV (b) 2 to 3 eV

(c) 3 to 4 eV (d) 4 to 5 eV

11. Induced electric field lines

(a) always make closed paths

(b) do not make closed paths

(c) may make closed paths

(d) are always straight

12. A capacitor of 1μF charged with a supply of 4 volt. The resistance of the charging circuit is 1Ω. What is the heat dissipated in the charging circuit in charging process?

(a) 16 J (b) 16 μJ

(c) 8μJ (d) 4 J

13. The force of attraction between the plates of a parallel plate capacitor is (q – charge on each plate E – electric field intensity between the plates

(a) $q.E$ (b) $2\,q\,E$

(c) $\frac{1}{2}\,qE$ (d) None.

14. Choose the correct statement.

(a) force on a charge in an electric field is conservative

(b) force on a charge in an electric field is non conservative

(c) force on a charge in an induced electric field may be conservative

(d) force on a charge in an induced electric field is always zero

15. A solid cylindrical long conductor of radius 'R' is carrying a steady current I.

(a) the magnetic field at all interior points of conductor is zero

(b) the magnetic field at all interior points of a conductor is constant

(c) the magnetic field only at axis is zero

(d) the magnetic field at the surface of conductor is zero

16. Unit of 'mobility' of charge carrier is

(a) ms^{-1} (b) $m^2v^{-1}s^{-1}$

(c) $mv^{-1}s^{-1}$ (d) None.

17. Find the condition so that the points A and B are at same potential in the following circuit.

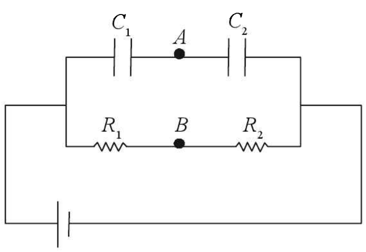


Fig. 4

(a) $\frac{R_1}{R_2} = \frac{C_2}{C_1}$ (b) $\frac{R_1}{R_2} = \frac{C_1}{C_2}$

(c) $\frac{R_1}{R_2^2} = \frac{C_1}{C_2^2}$ (d) none

18. A beam of protons moving at a speed of $10^6\ ms^{-1}$, is carrying a current of $1A$. How many protons are in 1m length of the beam.

(a) 6.25×10^{18} (b) 6.25×10^{12}

(c) 6.25×10^{15} (d) 6.25×10^{24}

19. A pn junction diode is used as a half wave rectifier. Input is fed for both half cycles of ac but output is obtained only for the first half cycle.

(a) the energy of the second half cycle is wasted as heat in the rectifier circuit

(b) the energy of second half cycle is used to increase depletion zone

(b) the energy of second half cycle is transferred to the next half cycle

(d) there is no input energy fed for the second half cycle and so no output is there

20. The truth table shown below correspond to which gate.

A	***B***	***Y***
0	0	1
0	1	1
1	0	1
1	1	0

(a) NAND (b) NOR

(c) OR (d) AND

21. An air filled parallel plate capacitor having plate area "A" and separation "d" between them is introduced with two slabs of equal volume, one metal and other of dielectric constant $K = 2$ as shown. What is the equivalent capacitance?

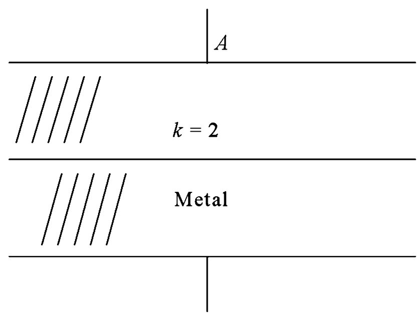


Fig. 5

(a) $\frac{2\varepsilon_0 A}{d}$ (b) $\frac{4\varepsilon_0 A}{d}$

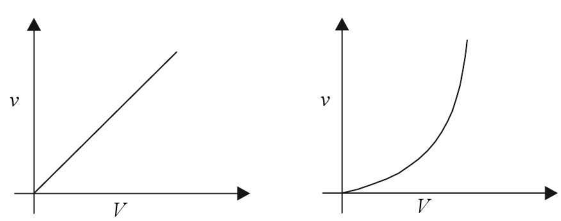
(c) infinite (d) zero

22. An electron is revolving round the nucleus in a circular orbit of radius R with velocity v. The magnetic dipole moment of the system is

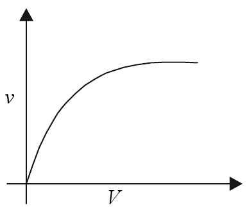
(a) $\frac{evR}{2}$ (b) $\frac{\mu_0 ev}{4\pi R^3}$

(c) zero (d) none

23. A charged particle of charge 'q' and mass m is accelerated under 'V' volts. Which is the correct graph plotted between its velocity gained and potential of acceleration?



(a) (b)



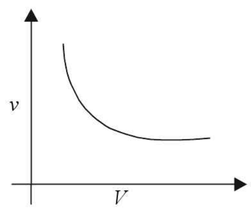


Fig. 6

24. A long wire is carrying a steady current I. An observer sitting in a platform moves down with a velocity equal to drift velocity of electron. The magetic field observed by the man is at distance R.

(a) $\frac{\mu_0 I}{2\pi R}$ (b) $\frac{2\mu_0 I}{2\pi R}$

(c) $\frac{3\mu_0 I}{2\pi R}$ (d) zero

25. A magnetic configuration has net non-zero magnetic moment.

(a) the magnetic configuration has N–S poles

(b) the magnetic configuration does not have N–S poles

(c) the magnetic configuration may have N– S poles

(d) the magnetic configuration is due to permanent magnet

26. Magnetic field at the center of hydrogen atom when it is in the ground state is

(a) 12.4 Tesla (b) 10^3 Tesla

(c) 1Tesla (d) none of these

27. Magnetic field at the centre of H– atom when it is in the first excited state is

(a) 12.4 T (b) $10^3 T$

(c) 1T (d) none of these

28. The threshold wave length for photoelectric effect to the surface of certain material is 5860 $\AA$. The photoelectric emission will take place when this material is illuminated with

(a) 100 watt IR Lamp (b) 1 Watt UV Lamp

(c) 60 Watt Red Lamp (d) 200 Watt Red Lamp

29. A photon of energy 12.75 eV is incident on H–atom, it is absorbed. The excited state to which H–atom goes

(a) first excited state (b) second excited state

(c) third excited state (d) none of these

30. The maximum number of spectral lines emitted in the above question

(a) 4 (b) 5

(c) 6 (d) 8

31. The spectral lines emitted in the above question belong to

(a) Balmer series (b) Lyman

(c) Paschen (d) all of these

32. A radioactive sample was kept in a room accidentally. The activity of the room increases 32 times the normal activity of the room. If half-life time of the sample is 15 days. The days in which the room can be safely occupied

(a) 60 days (b) 75 days

(c) 100 days (d) 50 days

33. If half life time of a sample is 10 years. The days within which radioactivity will be over

(a) 100 yrs (b) 10 yrs

(c) 50 yrs (d) infinite

34. Half life time of Polonium is 138 days. The last nucleus of this radioactive sample will decay in

(a) 138 days (b) 2 × 138 days

(c) 3 × 138 days (d) unpredictable

35. The electromagnetic waves which cannot be obtained due to the process of bremsstrahlung are

(a) X-rays (b) γ - rays

(c) IR (d) heat radiations

36. The most energetic particles are

(a) α -rays (b) β - rays

(c) γ -rays (d) cosmic rays

37. Two bulbs 25 W, 220 V and 100 W, 220 V connected in series. So that the power of first bulb is 4 W. Now the source voltage is

(a) 440 V (b) 220 V

(c) 110 V (d) 55 V

38. A resistance wire of 81Ω is uniformly stretched till its resistance becomes 256 Ω. If the original thickness is 8 mm. Now the thickness is

(a) 2 mm (b) 4 mm

(c) 6 mm (d) 1mm

39. Range of an ammeter of resistance X is to be increased by n times. The shunt resistance = ?

(a) $\frac{x}{n}$ (b) nx

(c) $\frac{x}{n-1}$ (d) $n - 1\,(x)$

40. Volt meter reading

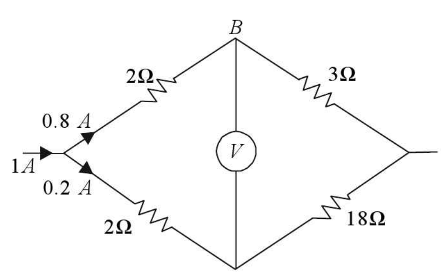


Fig. 7

(a) 1.6 V (b) 1.2 V
(c) 0.8 V (d) 0.4 V

41. $V = 12\sqrt{2}\,\sin(2000\,t)$ If $I = 2$A then resonant frequency f_o is = ?

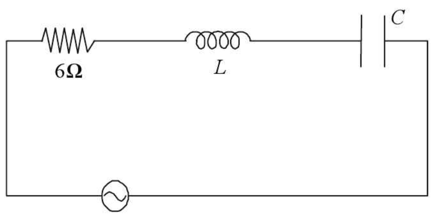


Fig. 8

(a) 2000 Hz (b) 4000 Hz
(c) $\dfrac{1000}{\pi}$ Hz (d) $\dfrac{2000}{\pi}$ Hz

42. A steel wire of length 'l' has a magnetic moment M. It is then bent into a semi-circular arc. The new magnetic moment is

(a) M (b) $2M/\pi$
(c) $\pi M/l$ (d) Ml/π

43. The period of oscillation of a magnet in a vibration magnetometer is 2 sec. The period of oscillation of a magnet whose magnetic moment is four times that of the first magnet is.

(a) 1sec (b) 5 sec
(c) 8 sec (d) 0.5 sec

44. Two identical thin bar magnets each of length 'l' and pole strength m are placed at right angles to each other with the north pole of one touching the south pole of the other. The resulting magnetic moment of the magnet is

(a) ml (b) $\sqrt{2}\,ml$
(c) m^2l^2 (d) $ml\sqrt{2}$

45. Isogonic lines on a magnetic map will have

(a) zero angle of dip
(b) zero angle of declination
(c) the same angle of dip
(d) the same angle of declination

46. The B-H curve (a) and (b) in fig. 9, are associated with

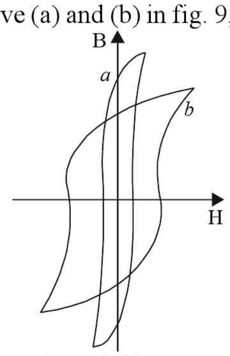


Fig. 9

(a) a diamagnetic and (b) paramagnetic respectively
(b) a paramagnetic and ferromagnetic respectively
(c) soft iron and steel respectively
(d) steel and soft iron respectively

Answers

1. (b)	**2.** (c)	**3.** (b)	**4.** (c)	**5.** (d)	**6.** (b)	**7.** (b)
8. (a)	**9.** (c)	**10.** (b)	**11.** (a)	**12.** (c)	**13.** (c)	**14.** (a)
15. (c)	**16.** (b)	**17.** (a)	**18.** (b)	**19.** (a)	**20.** (a)	**21.** (b)
22. (a)	**23.** (c)	**24.** (a)	**25.** (a)	**26.** (a)	**27.** (d)	**28.** (b)
29. (c)	**30.** (c)	**31.** (d)	**32.** (b)	**33.** (d)	**34.** (d)	**35.** (b)
36. (c)	**37.** (c)	**38.** (c)	**39.** (a)	**40.** (b)	**41.** (c)	**42.** (b)
43. (a)	**44.** (b)	**45.** (d)	**46.** (c)			

Explanations

1(b) Current will not flow through capacitors in the circuit. Current in the circuit excluding capacitor = 3 A. In the loop $ABCD$ $2 - 3 + V_c - 2 = 0$ or $V_c = 3\ V$

Energy stored $E = \dfrac{1}{2}CV_c^2 = \dfrac{1}{2} \times 10^{-6} \times 3^2 = q\mu J$

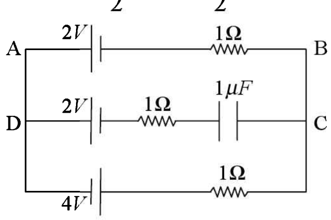


Fig. 10

2(b) $I = \dfrac{4}{3+1} = 1A$

$E_L = \dfrac{1}{2}LI^2 = \dfrac{1}{2} \times 1 \times 1^2 = 0.5\,J$

$P_R = I^2R = 1^2 \times 3 = 3W$

3(b) As the filament A is merged in ice tank its temperature reduces and so the resistance also reduces. Current will increase in the circuit.

4(c) $\left(\dfrac{E_0}{\sqrt{2}}\right)^2 \times \dfrac{1}{R} = \dfrac{1}{2}\left(\dfrac{(100)^2}{R}\right)$

$E_0 = 100$ volt

5(d) Net potential

$$V = \sqrt{(100)^2 + (100-100)^2} = 100 \text{ Volts}$$

when capacitor is short circuited

then $V = \sqrt{V_R^2 + V_L^2}$ as V_L & V_R are equal

$$\text{so } V_L = \frac{100}{\sqrt{2}} = 50\sqrt{2}\,V$$

6(b) $E_{OM} = \frac{1}{2} B\omega l^2$

$$E_{ON} = \frac{1}{2} B\omega l \left(\frac{l}{2}\right)^2 = \frac{1}{8} B\omega l$$

$$E_{NM} = E_{OM} - E_{ON}$$

$$= \frac{1}{2} B\omega l^2 = -\frac{1}{8} B\omega l^2 = \frac{3}{8} B\omega l^2$$

7(b) $\lambda_{min}(nm) = \frac{1240}{V} = \frac{1240}{50\times10^3} = 248\times10^{-13}$

$= 0.248$ Å

8(a) By absorbing 10.2 eV electron will go from first to second orbit.

$$\tau = \frac{dL}{dt} = \frac{\frac{h}{2\pi}}{10^{-8}} = \frac{6.6\times10^{-34}}{2\times3.14\times10^{-8}}$$

$= 1.05\times10^{-26}\,Nm$

9(c) $\frac{R_1}{R_2} = \frac{1}{4} = R \propto n^2$ and $\frac{A_1}{A_2} = \left(\frac{R_1}{R_2}\right)^2 = \frac{1}{16}$

10(b)

11(a)

12(c) The heat dissipated with charging circuit is equal to energy stored by the capacitor

$$\frac{1}{2}CV^2 = \frac{1}{2}(1\mu F)(4)^2 = 8\mu J.$$

13(c) $q\frac{\sigma}{2\varepsilon_o} = \frac{q^2}{2A\varepsilon_o} = \frac{qE}{2}$

14(a)

15(c)

16(b)

17(a) $V \propto \frac{1}{C}$ whereas $V \propto R$ (in series)

18(b) Number of protons passing a given point /sec. $= 6.25\times10^{18}$

Number of protons/m length of the beam

$$= \frac{6.25\times10^{18}}{10^6} = 6.25\times10^{12}$$

19(a)

20(a)

21(b) $\frac{\varepsilon_0 A}{\frac{d/2}{2} + \frac{d/2}{\infty}} = \frac{4\varepsilon_0 A}{d}$

22(a) $M = IA' = \frac{ev}{2\pi R} \times \pi R^2 = \frac{eVR}{2}$

23(c) As $v \propto \sqrt{V}$ or $v^2 \propto V$

24(a)

25(a) For a magnetic configuration to have N-S poles, it must have net non -zero magnetic moment.

26(a) $B = \frac{\mu_0 I}{2r}$

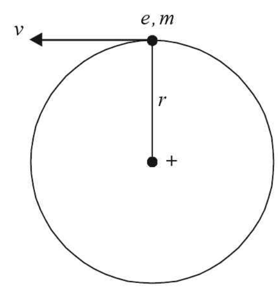


Fig. 11

$$I = \frac{e}{T} = \frac{\frac{e}{2\pi r}}{v} = \frac{ev}{2\pi r},\ B = \frac{\mu_0 ev}{2r(2\pi r)}$$

In ground state of H atom $v = \frac{c}{137}$

$$B = \frac{\mu_0 ec}{4\pi r^2\times137},\ r = 0.53\,\text{Å}$$

$$= \frac{10^{-7}\times1.6\times10^{-19}\times3\times10^8}{(0.53\times10^{-10})^2\times137}$$

$$\frac{1.6\times3\times10^2}{(0.53\times0.53)\times137} = \frac{4.8}{38.48}\times10^2$$

or $B = 12.4$ tesla

27(d) In Ist excited state $r = 0.53\times4$ Å

$$v = \frac{c}{2\times137}$$

$$\therefore \quad B = \frac{\mu_0 I}{2r} = \frac{\mu_0 ec}{4\pi(0.53\times4\times10^{-10})^2\times2\times137}$$

$$= \frac{12.4}{32},\ B = 0.38\,T$$

28(b) Condition of photoelectric effect is $\lambda \le \lambda_0$

Now $\lambda_0 = 5860 A°$

$\lambda uv < 5860 A°$ Only $U.V$ satisfies the above condition

29(c) $12.75 = 13.6\left[1 - \frac{1}{n^2}\right]$ or $n^2 = 16$, i.e., $n = +4$

∴ 3rd excited state.

30(c) $\frac{n(n-1)}{2} = \frac{4(3)}{2} = 6$

31(d)

32(b) $32 = 2^N$

$N = 5$

$\therefore$ after $5t_{1/2}$ = 75 days the room is suitable for occupancy

33(d)

34(d)

35(b)

36(d)

37(c) $R = \dfrac{220^2}{100} = 484\Omega$

$$4 = \left(\frac{V}{484+1936}\right)^2 \times 1936$$

or $V = 110V$

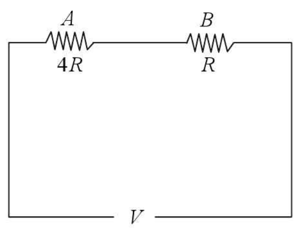


Fig. 12

38(c) $R = \rho \dfrac{l}{a}$ R'

$$= \rho \frac{l'}{a} = \rho \frac{al}{a'^2}\left(al = a'l' = \frac{al}{a'}\right)$$

$$\therefore \frac{a^2}{a'^2} = \frac{R'}{R} = \frac{256}{81}$$

$$r = \frac{3}{4}\sqrt{r} = \frac{3}{4} \times 8\,\text{mm} = 6\,\text{mm}$$

39(a) $nI\,(S) = I\,(x)$

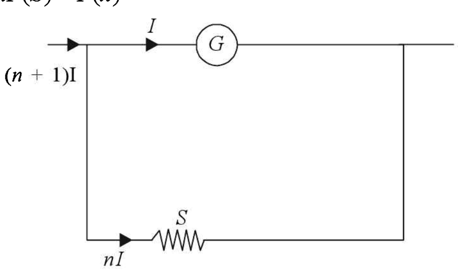


Fig. 13

$$S = \frac{x}{n}$$

40(b)

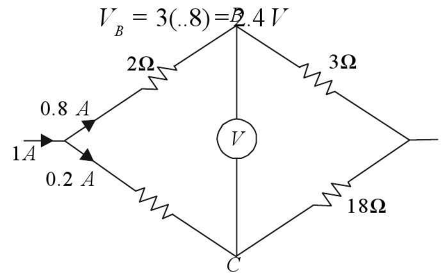


Fig. 14

$$V_C = 0.2\,(18) = 3.6\,V$$

$$V_{BC} = -1.2\,V$$

41(c) $I = \dfrac{V}{R}$ only at resonance in LCR circuit

$\omega_0 = 2000$

frequency $f = \dfrac{\omega_0}{2\pi} = \dfrac{2000}{2\pi} = \dfrac{1000}{\pi}$ Hz

42(b)

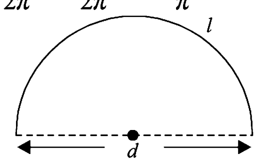


Fig. 15

$$\frac{\pi d}{2} = l$$

$d = \dfrac{2l}{\pi}$ New magnetic moment $= m \times d$

$$= m \times \frac{2l}{\pi} = \left(\frac{2M}{\pi}\right)$$

43(a) From relation $2\pi\sqrt{\dfrac{I}{MB_H}}$

or $\dfrac{T_1}{T_2} = \dfrac{\sqrt{M_2}}{M_1}$

or, $\dfrac{2}{T_2} = \dfrac{\sqrt{4}}{1}$

or $T_2 = 1$sec.

44(b) The resultant magnetic moment of a magnet is

$M^2 = M_1^2 + M_2^2 + 2M_1M_2\cos\theta$

Here $M_1 = ml$

$M_2 = ml$

$\theta = 90°$

$\therefore$ $M^2 = m^2l^2 + m^2l^2$

$\therefore$ $M = \sqrt{2}\,ml$

45(d) The same angle of declination

46(c) Retentivity of soft iron is more than that of steel.

APPENDIX 1

Useful Mathematical Relations

ALGEBRA

$a^{-x} = \frac{1}{a^x}$ $\qquad a^{(x+y)} = a^x a^y$ $\qquad a^{(x-y)} = \frac{a^x}{a^y}$

Logarithms:

If $\log a = x$, then $a = 10x$. $\qquad \log a + \log b = \log(ab)$ $\qquad \log a - \log b = \log(a/b)$ $\qquad \log(an) = n \log a$

If $\ln a = x$, then $a = e^x$. $\qquad \ln a + \ln b = \ln(ab)$ $\qquad \ln a - \ln b = \ln(a/b)$ $\qquad \ln(a^n) = n \ln a$

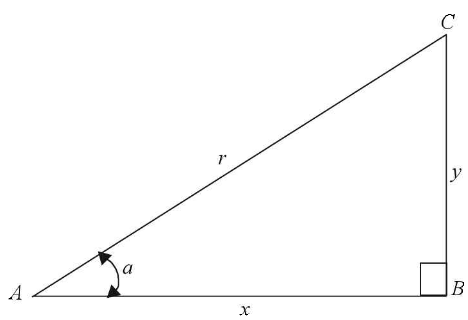
Quadratic formula: If $ax^2 + bx + c = 0$, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

BINOMIAL THEOREM

$$(a+b)^n = a^n + na^{n-1}b + \frac{n(n-1)a^{n-2}b^2}{2!} + \frac{n(n-1)(n-2)a^{n-3}b^3}{3!} + \ldots\ldots$$

TRIGONOMETRY

In the right triangle ABC, $x^2 + y^2 = r^2$.



Definitions of the trigonometric functions: $\sin a = y/r$ $\qquad \cos a = x/r$ $\qquad \tan a = y/x$

Identities:

$\sin^2 a + \cos^2 a = 1$ $\qquad \tan a = \frac{\sin a}{\cos a}$

$\sin 2a = 2 \sin a \cos a$ $\qquad \cos 2a = \cos^2 a - \sin^2 a = 2\cos^2 a - 1 = 1 - 2\sin^2 a$

$\sin \frac{1}{2} a = \sqrt{\frac{1 - \cos a}{2}}$ $\qquad \cos \frac{1}{2} a = \sqrt{\frac{1 + \cos a}{2}}$

$\sin(-a) = -\sin a$ $\qquad \sin(a \pm b) = \sin a \cos b \pm \cos a \sin b$

$\cos(-a) = \cos a$ $\qquad \cos(a \pm b) = \cos a \cos b \mp \sin a \sin b$

$\sin(a \pm \pi/2) = \pm \cos a$ $\qquad \sin a + \sin b = 2 \sin \frac{1}{2}(a+b) \cos \frac{1}{2}(a-b)$

$\cos(a \pm \pi/2) = \mp \sin a$ $\qquad \cos a + \cos b = 2 \cos \frac{1}{2}(a+b) \cos \frac{1}{2}(a-b)$

GEOMETRY

Circumference of circle of radius r: $C = 2\pi r$

Area of circle of radius r: $A = \pi r^2$

Volume of sphere of radius r: $V = 4\pi r^3/3$

Surface area of sphere of radius r: $A = 4\pi r^2$

Volume of cylinder of radius r and height h: $V = \pi r^2 h$

CALCULUS

Derivatives:

$$\frac{d}{dx}x^n = nx^{n-1}$$

$$\frac{d}{dx}\sin ax = a\cos ax$$

$$\frac{d}{dx}\cos ax = -a\sin ax$$

$$\frac{d}{dx}e^{ax} = ae^{ax}$$

$$\frac{d}{dx}\text{In } ax = \frac{1}{x}$$

Integrals:

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} \quad (n \neq -1)$$

$$\int \frac{dx}{x} = \text{In } x$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}$$

$$\int \frac{dx}{\sqrt{a^2 - x^2}} = arc\sin\frac{x}{a}$$

$$\int \frac{dx}{\sqrt{x^2 + a^2}} = \text{In}\left(x + \sqrt{x^2 + a^2}\right)$$

$$\int \frac{dx}{x^2 + a^2} = \frac{1}{a}arc\tan\frac{x}{a}$$

$$\int \frac{dx}{\left(x^2 + a^2\right)^{3/2}} = \frac{1}{a^2}\frac{1}{\sqrt{x^2 + a^2}}$$

$$\int \frac{x\,dx}{\left(x^2 + a^2\right)^{3/2}} = -\frac{1}{\sqrt{x^2 + a^2}}$$

Power series (convergent for range of x shown):

$$(1 + x)^n = 1 + nx + \frac{n(n-1)x^2}{2!} + \frac{n(n-1)(n-2)x^3}{3!} + \ldots\ldots (|x| < 1)$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \ldots\ldots \quad (\text{all } x)$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \ldots\ldots \quad (\text{all } x)$$

$$\tan x = x + \frac{x^3}{3} + \frac{2x^2}{15} + \frac{17x^7}{315} + \ldots\ldots \left(|x| < \frac{\pi}{2}\right)$$

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots\ldots (\text{all } x)$$

$$\text{In}(1 + x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \ldots\ldots (|x| < 1)$$

APPENDIX 2

The Greek Alphabet

Name	Capital	Lower case
Aplha	A	α
Beta	B	β
Gamma	Γ	γ
Delta	Δ	δ
Epsilon	E	ε
Zeta	Z	ζ
Eta	H	η
Theta	Θ	θ
Iota	I	ι
Kappa	K	κ
Lambda	Λ	λ
Mu	M	μ
Nu	N	ϖ
Xi	Ξ	ξ
Omicron	O	o
Pi	Π	π
Rho	Π	π
Sigma	Σ	σ
Tau	T	τ
Upsilon	Y	υ
Phi	Φ	ϕ
Chi	X	χ
Psi	Ψ	ψ
Omega	Ω	ω

APPENDIX 3

Periodic Table of Elements

Group / Period	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
1	1 **H** 1.008																	2 **He** 4.003
2	3 **Li** 6.941	4 **Be** 9.012											5 **B** 10.811	6 **C** 12.011	7 **N** 14.007	8 **O** 15.999	9 **F** 18.998	10 **Ne** 20.180
3	11 **Na** 22.990	12 **Mg** 24.305											13 **Al** 26.982	14 **Si** 28.086	15 **P** 30.974	16 **S** 32.065	17 **Cl** 35.453	18 **Ar** 39.948
4	19 **K** 39.098	20 **Ca** 40.078	21 **Sc** 44.956	22 **Ti** 47.867	23 **V** 50.942	24 **Cr** 51.996	25 **Mn** 54.938	26 **Fe** 55.845	27 **Co** 58.933	28 **Ni** 58.693	29 **Cu** 63.546	30 **Zn** 65.409	31 **Ga** 69.723	32 **Ge** 72.64	33 **As** 74.922	34 **Se** 78.96	35 **Br** 79.904	36 **Kr** 83.80
5	37 **Rb** 85.468	38 **Sr** 87.62	39 **Y** 88.906	40 **Zr** 91.224	41 **Nb** 92.906	42 **Mo** 95.94	43 **Tc** (98)	44 **Ru** 101.07	45 **Rh** 102.905	46 **Pd** 106.42	47 **Ag** 107.868	48 **Cd** 112.411	49 **In** 114.818	50 **Sn** 118.710	51 **Sb** 121.760	52 **Te** 127.60	53 **I** 126.904	54 **Xe** 131.293
6	55 **Cs** 132.905	56 **Ba** 137.327	71 **Lu** 174.967	72 **Hf** 178.49	73 **Ta** 180.948	74 **W** 183.84	75 **Re** 186.207	76 **Os** 109.23	77 **Ir** 192.27	78 **Pt** 195.078	79 **Au** 196.967	80 **Hg** 200.59	81 **Tl** 204.383	82 **Pb** 207.2	83 **Bi** 208.980	84 **Po** (209)	85 **At** (210)	86 **Rn** (222)
7	87 **Fr** (223)	88 **Ra** (226)	103 **Lr** (262)	104 **Rf** (261)	105 **Db** (262)	106 **Sg** (266)	107 **Bh** (264)	108 **Hs** (269)	109 **Mt** (268)	110 **Ds** (271)	111 **Uuu** (272)	112 **Uub** (285)	113 **Uut**	114 **Uuq** (285)	115 **Uup**	116 **Uuh**	117 **Uus**	118 **Uno**

Lanthanoids	57 **La** 138.905	58 **Ce** 104.116	59 **Or** 140.908	60 **Bd** 144.24	61 **Pm** (145)	62 **Sm** 150.36	63 **Eu** 151.964	64 **Gd** 157.25	65 **Tb** 158.925	66 **Dy** 162.500	67 **Ho** 164.930	68 **Er** 167.259	69 **Tm** 168.934	70 **Yb** 173.04
Actinoids	89 **Ac** (227)	90 **Th** (232)	91 **Pa** (231)	92 **U** (238)	93 **Np** (237)	94 **Pu** (244)	95 **Am** (243)	96 **Cm** (247)	97 **Bk** (247)	98 **Cf** (251)	99 **Es** (252)	100 **Fm** (257)	101 **Md** (258)	102 **No** (259

For each element the average atomic mass of the mixture of isotopes occuring in nature is shown. For elements having no stable isotope, the approximate atomic mass of the longest-lived isotope is shown in parentheses. For elements that have been predicted but not yet detected, no atomic mass is given. All atomic masses are expressed in atomic mass unit (1 u = 1.66053873(13) × 10^{-27} kg), equivalent to grams per mole (g/mol).

APPENDIX 4

Unit Conversion Factors

LENGTH

1 m = 100 cm = 1000 mm = 10^6 μm = 10^9 nm
1 km = 1000 m = 0.6214 mi
1 m = 3.281 ft = 39.37 in.
1 cm = 0.3937 in.
1 in. = 2.540 cm
1 ft = 30.48 cm
1yd = 91.44 cm
1 mi = 5280 ft = 1.609 km
1 Å = 10^{-10} m = 10^{-8} cm = 10^{-1} nm
1 nautical mile = 6080 ft
1 light year = 9.461 × 10^{15} m

AREA

1 cm^2 = 0.155in.^{2}
1 m^2 = 10^4 cm^2 = 10.76 ft^2
1 in.^{2} = 6.452 cm^2
1 ft = 144 in.^{2} = 0.0929 m^2

VOLUME

1 liter = 1000 cm^3 = 10^{-3} m^3 = 0.03531 ft^3 = 61.02 in.^{3}
1 ft^3 = 0.02832 m^3 = 28.32 liters = 7.477 gallons
1 gallon = 3.788 liters

TIME

1 min = 60 s
1 h = 3600 s
1 d = 86,400 s
1 y = 365.24 d = 3.156 × 10^7 s

ANGLE

1 rad = 57.30° = 180°/π
1° = 0.01745 rad = π/180 rad
1 revolution = 360° = 2πrad
1 rev/min (rpm) = 0.1047 rad/s

SPEED

1 m/s = 3.281ft /s
1 ft /s = 0.3048 m/s
1 mi /min = 60 mi /h = 88 ft /s
1 km /h = 0.2778 m /s = 0.6214 mi /h
1 mi /h = 1.466 ft /s = 0.4470 m /s = 1.609 km /h
1 furlong /fortnight = 1.662 × 10^{-4} m /s

ACCELERATION

1 m/s^2 = 100 cm/s^2 = 3.281 ft /s^2
1 cm/s^2 = 0.01 m/s^2 = 0.03281 ft/s^2
1 ft/s^2 = 0.3048 m/s^2 = 30.48 cm/s^2
1 mi /h . s = 1.467 ft/s^2

MASS

1 kg = 10^3 g = 0.0685 slug
1 g = 6.85 × 10^{-27} kg
1 slug = 11.59 kg
1 u = 1.661 × 10^{-27} kg
1 kg has a weight of 2.205 lb when g = 9.80 m/s^2

FORCE

1 N = 10^5 dyn = 0.2248 lb
1 lb = 4.448 N = 4.448 × 10^5 dyn

PRESSURE

1 Pa = 1 N/m^2 = 1.450 × 10^{-4} lb/in.^{2} = 0.209 lb/ft^2
1 bar = 10^5 Pa
1 lb/in.^{2} = 6895 Pa
1 lb/ft2 = 47.88 Pa
1 atm = 1.013 × 10^5 Pa = 1.013 bar
= 14.7 lb/in.^{2} = 2117 lb/ft^2
1 mm Hg = 1 torr = 133.3 Pa

ENERGY

1 J = 10^{-7} erg = 0.239 cal
1 cal = 4.186 J (based on 15° calorie)
1 ft .lb = 1.356 J
1 Btu = 1055 J = 252 cal = 778 ft . lb
1 eV = 1.602 × 10^{-19} J
1 k Wh = 3.600 × 10^6 J

MASS–ENERGY EQUIVALENCE

1 kg — 8.988 × 10^{16} J
1 u — 931.5 MeV
1 eV — 1.074 × 10^{-9} u

POWER

1 W = 1 J/s
1 hp = 746 W = 550 ft . lb/s
1 Btu/h = 0.293 W

APPENDIX 5

Numerical Constants

FUNDAMENTAL PHYSICAL CONSTANTS*

Name	*Symbol*	*Value*
Speed of light	c	2.99792458×10^8 m/s
Magnitude of charge of electron	e	$1.602176462(63) \times 10^{-19}$ C
Gravitational constant	G	$6.673(10) \times 10^{-11}$ N.m^2/kg^2
Planck's constant	h	$6.62606876(52) \times 10^{-34}$ J.s
Boltzmann constant	k	$1.3806503(24) \times 10^{-23}$ J/K
Avogadro's number	N_A	$6.02214199(47) \times 10^{23}$ molecules/mol
Gas constant	R	8.314472(15) J/mol . K
Mass of electron	m_e	$9.10938188(72) \times 10^{-31}$ kg
Mass of proton	m_p	$1.67262158(13) \times 10^{-27}$ kg
Mass of neutron	m_n	$1.67492716(13) \times 10^{-27}$ kg
Premeability of free space	μ_0	$4\pi \times 10^{-7}$ Wb/A . m
Permittivity of free space	$\varepsilon_0 = 1/\mu_0 c^2$	$8.854187817 \ldots \times 10^{-12}$ C^2/N . m^2
	$1/4\pi\varepsilon_0$	$8.987551787 \ldots \times 10^9$ N.m^2/C^2

OTHER USEFUL CONSTANTS*

Mechanical equivalent of heat		4.186 J/cal (15° calorie)
Standard atmospheric pressure	1 atm	1.01325×10^5 Pa
Absolute zero	0 K	−273.15°C
Electron volt	1 eV	$1.602176462(63) \times 10^{-19}$ J
Atomic mass unit	1 u	$1.66053873(13) \times 10^{-27}$ kg
Electron rest energy	$m_e c^2$	0.510998902(21) MeV
Volume of ideal gas (0°C and 1 atm)		22.413996(39) liter/mol
Acceleration due to gravity (standard)	g	9.80665 m/s^2

*Source: National Institute of Standards and Technology (**http://physics.nist.gov/cuu**). Numbers in parentheses show the uncertainty in the final digits of the main number; for example, the number 1.6454(21) means 1.6454 $\pm$ 0.0021. Values shown without uncertainties are exact.

APPENDIX 6

Exponential and Hyperbolic Functions

x	e^x	e^{-x}	θ° ($gd\ x$)	cosh x (secθ)	sinh x (tanθ)	tanh x (sineθ)	logcosh x	log sinh x
.1	1.1052	.9048	5.720	1.0050	.1002	.0997	.0022	$\bar{1}$.0007
.2	1.2214	.8187	11.384	1.0201	.2013	.1974	.0086	$\bar{1}$.3039
.3	1.3499	.7408	16.937	1.0453	.3045	.2913	.0193	$\bar{1}$.4836
.4	1.4918	.6703	22.331	1.0811	.4108	.3799	.0339	$\bar{1}$.6136
.5	1.6487	.6065	27.524	1.1276	.5211	.4621	.0522	$\bar{1}$.7169
.6	1.8221	.5488	32.483	1.1855	.6367	.5370	.0739	$\bar{1}$.8093
.7	2.0138	.4966	37.183	1.2552	.7586	.6044	.0987	$\bar{1}$.8800
.8	2.2255	.4493	41.608	1.3374	.8881	.6640	.1263	$\bar{1}$.9485
.9	2.4596	.4066	45.750	1.4331	1.0265	.7163	.1563	0.0114
1.0	2.7183	.3679	49.605	1.5431	1.1752	.7616	.1884	0.0701
1.1	3.0042	.3329	53.178	1.6685	1.3356	.8005	.2223	0.1257
1.2	3.3201	.3012	56.476	1.8107	1.5095	.8337	.2578	0.1788
1.3	3.6693	.2725	59.511	1.9709	1.6984	.8617	.2947	0.2300
1.4	4.0552	.2466	62.295	2.1509	1.9043	.8854	.3326	0.2797
1.5	4.4817	.2231	64.843	2.3524	2.1293	.9051	.3715	0.3282
1.6	4.9530	.2019	67.171	2.5775	2.3756	.9217	.4112	0.3758
1.7	5.4739	.1827	69.294	2.8283	2.6456	.9354	.4515	0.4225
1.8	6.0496	.1653	71.228	3.1075	2.9422	.9468	.4924	0.4687
1.9	6.6859	.1496	72.987	3.4177	3.2682	.9562	.5337	0.5143
2.0	7.3891	.1353	74.584	3.7622	3.6269	.9640	.5754	0.5595
2.1	8.1662	.1225	76.037	4.1443	4.0219	.9705	.6175	.6044
2.2	9.0250	.1108	77.354	4.5679	4.4571	.9757	.6597	.6491
2.3	9.9742	.1003	78.549	5.0372	4.9370	.9801	.7022	.6935
2.4	11.023	.0907	79.633	5.5569	5.4662	.9837	.7448	.7377
2.5	12.183	.0821	80.615	6.1323	6.0502	.9866	.7876	.7818
2.6	13.464	.0743	81.513	6.7690	6.6947	.9890	.8305	0.8257
2.7	14.880	.0672	82.310	7.4735	7.4063	.9910	.8735	0.8696
2.8	16.445	.0608	83.040	8.2527	8.1919	.9926	.9166	0.9134
2.9	18.174	.0550	83.707	9.1146	9.0596	.9940	.9597	0.9571
3.0	20.086	.0498	84.301	10.068	10.018	.9951	1.0029	1.0008
3.1	22.198	.0450	84.841	11.121	11.076	.9959	1.0462	1.0444
3.2	24.533	.0408	85.336	12.287	12.246	.9967	1.0894	1.0880
3.3	27.113	.0369	85.775	13.575	13.538	.9973	1.1327	1.1316
3.4	29.964	.0334	86.177	14.999	14:965	.9978	1.1761	1.1751
3.5	33.115	.0302	86.541	16.573	16.543	.9982	1.2194	1.2186
3.6	36.598	.0273	86.870	18.313	18.285	.9985	1.2628	1.2621
3.7	40.447	.0247	87.168	20.236	20.211	.9988	1.3061	1.3056
3.8	44.701	.0224	87.437	22.362	22.339	.9990	1.3495	1.3491
3.9	49.402	.0202	87.681	24.711	24.691	.9992	1.3929	1.3925
4.0	54.598	.0183	87.903	27.308	27.290	.9993	1.4363	1.4360
4.1	60.340	.0166	88.104	30.178	30.162	.9995	1.4797	1.4795
4.2	66.686	.0150	88.281	33.351	33.336	.9996	1.5231	1.5229
4.3	73.700	.0136	88.447	36.857	36.843	.9996	1.5665	.5664
4.4	81.451	.0123	88.591	40.732	40.719	.9997	1.6099	.6098
4.5	90.017	.0111	88.728	45.014	45.003	.9997	1.6533	.6532
4.6	99.484	.0101	88.849	49.747	49.737	.9998	1.6968	.6967
4.7	109.95	.0091	88.957	54.978	54.969	.9998	1.7402	.7401
4.8	121.51	.0082	89.055	60.759	60.751	.9999	1.7836	.7836
4.9	134.29	.0074	89.146	67.149	67.141	.9999	1.8270	.8270
5.0	148.41	.0067	89.227	74.210	74.203	.9999	1.8705	1.8704

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PAPER 2007

1. A circular disc of radius R is removed from a bigger circular disc of radius 2R such that the circumferences of the discs coincide. The centre of mass of the new disc is $\frac{\alpha}{R}$ from the centre of the bigger disc. The value of α is

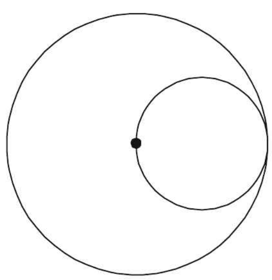
(a) $\frac{1}{4}$ (b) $\frac{1}{3}$

(c) $\frac{1}{2}$ (d) $\frac{1}{6}$

Solution **(b)** $\sigma = \frac{M}{\pi(2R)^2}$

$$m = \sigma \pi R^2 = \frac{M}{4}$$

$$\bar{x} = \frac{M(o) - \frac{M}{4}(R)}{M - \frac{M}{4}} = \frac{1}{3R}$$



2. A round uniform body of radius R, mass M and moment of inertia 'I' rolls down (without slipping) an inclined plane making an angle θ with the horizontal. Then its acceleration is

(a) $\frac{g\sin\theta}{1 - MR^2/I}$ (b) $\frac{g\sin\theta}{1 + I/MR^2}$

(c) $\frac{g\sin\theta}{1 + MR^2/I}$ (d) $\frac{g\sin\theta}{1 - I/MR^2}$

Solution **(b)** $a = \frac{g\sin\theta}{1 + \frac{I}{MR^2}}$

3. Angular momentum of the particle rotating with a central force is constant due to

(a) Constant torque
(b) Constant force
(c) Constant linear momentum
(d) Zero torque

Solution **(d)** When $\tau = o = \frac{dL}{dt}$

L = const.

4. A 2 kg block slides on a horizontal floor with a speed of 4 m/s. It strikes a uncompressed spring, and compresses it till the bolock is motionless. The kinetic friction force is 15 N and spring constant is 10,000 N/m. The spring compresses by

(a) 8.5 cm (b) 5.5 cm
(c) 2.5 cm (d) 11.0 cm

Solution **(b)** $\frac{1}{2} mv^2 = F_f x + \frac{1}{2} kx^2$

$\frac{1}{2} \times 2\,(4)^2 = 15x + \frac{10000}{2} x^2$

or x = 5.5 cm

5. A particle is projected at 60° to the horizontal with a kinetic energy K. The kinetic energy at the highest point is

(a) K/2 (b) K
(c) Zero (d) K/4

Solution **(d)** $KE = \frac{1}{2} mv^2 (\cos 60)^2 = \frac{K}{4}$

6. In a Young's double slit experiment the intensity at a point where the path difference is $\frac{\lambda}{6}$ (λ being the wavelength of the light used) is I. If I_0 denotes the maximum intensity, $\frac{I}{I_0}$ is equal to

(a) $\frac{3}{4}$ (b) $\frac{1}{\sqrt{2}}$

(c) $\frac{\sqrt{3}}{2}$ (d) $\frac{1}{2}$

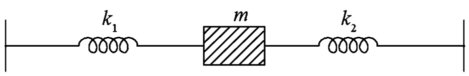
Solution **(a)** $\phi = \frac{2\pi}{\lambda} \frac{\lambda}{6} = \frac{\pi}{3}$

$I = I = I' + I' + 2I' \cos 60 = 3I'$

$$I_0 = 4I'$$

$$\frac{I}{I_0} = \frac{3}{4}$$

7. Two springs, of force constants k_1 and k_2, are connected to a mass m as shown. The frequency of oscillation of the mass is f. If both k_1 and k_2 are made four times their original values, the frequency of oscillation becomes



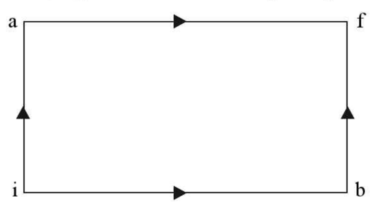
(a) $2f$ (b) $f/2$

(c) $f/4$ (d) $4f$

Solution (a) $f = \frac{1}{2\pi}\sqrt{\frac{(k_1+k_2)}{m}}$

$$f' = \frac{1}{2\pi}\sqrt{\frac{4(k_1+k_2)}{m}} \text{ or } f' = 2f$$

8. When a system is taken from state i to state f along the path iaf, it is found that $Q = 50$ cal and $W = 20$ cal. Along the path ibf, $Q = 36$ cal. W along the path ibf is



(a) 14 cal (b) 6 cal

(c) 16 cal (d) 66 cal

Solution (b) $dQ = dU + dW$

$50 = dU + 20$ or $dU = 30$ cal

case (ii) $dQ = dU + dW$

$36 = 30 + dW$ or $dW = 6$ cal

9. A particle of mass m executes simple harmonic motion with amplitude 'a' and frequency 'v'. The average kinetic energy during its motion from the position of equilibrium to the end is

(a) $2\pi^2 m a^2 v^2$ (b) $\pi^2 m a^2 v^2$

(c) $\frac{1}{4} m a^2 v^2$ (d) $4\pi^2 m a^2 v^2$

Solution (b) $\langle KE \rangle = \frac{1}{2} mw^2 a^2 \int_o^{T/4} \cos^2 wt dt$

$$= \frac{1}{2}\frac{mw^2a^2}{T/4}\int_o^{T/4}\left(\frac{1-\sin 2wt}{2}\right)dt$$

$$= \frac{1}{2}\frac{mw^2a^2}{T/4} \times \frac{1}{2} \times \frac{T}{4} = \frac{mw^2a^2}{4} = m\pi^2 v^2 a^2$$

10. The displacement of an object attached to a spring and executing simple harmonic motion is given by $x = 2 \times 10^{-2} \cos \pi t$ metres. The time at which the maximum speed first occurs is

(a) 0.25 s (b) 0.5 s

(c) 0.75 s (d) 0.125 s

Solution (b) $v = \frac{dx}{dt} = 2 \times 10^{-2} \pi \sin \pi t$

for v to be maximum, $\sin \pi t = 1$ or $\pi t = \frac{\pi}{2}$

$$t = \frac{1}{2} \text{ sec}$$

11. In an a.c. circuit the voltage applied is $E = E_0 \sin \omega t$. The resulting current in the circuit is $I = I_0 \sin (\omega t - \frac{\pi}{2})$. The power consumption in the circuit is given by

(a) $P = \sqrt{2}\, E_o I_o$ (b) $P = \frac{E_o I_o}{\sqrt{2}}$

(c) $P = $ zero (d) $P = \frac{E_o I_o}{2}$

Solution (c) $P = \frac{E_o}{\sqrt{2}} \frac{I_o}{\sqrt{2}} \cos \frac{\pi}{2} = o$

12. An electric charge 10^{-3} μC is placed at the origin (0, 0) of $X - Y$ co-ordinate system. Two points A and B are situated at $(\sqrt{2}, \sqrt{2})$ and (2, 0) respectively. The potential difference between the points A and B will be

(a) 4.5 volt (b) 9 volt

(c) zero (d) 2 volt

Solution (c) $\Delta V = \frac{q}{4\pi\varepsilon_o}\left[\frac{1}{r_f} - \frac{1}{r_i}\right] = \frac{q}{4\pi\varepsilon_o}\left[\frac{1}{2} - \frac{1}{2}\right] = 0$

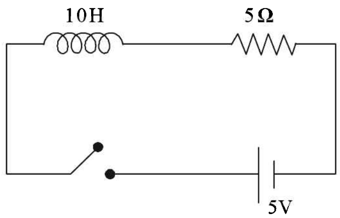
13. A battery is used to charge a parallel plate capacitor till the potential difference between the plates becomes equal to the electromotive force of the battery. The ratio of the energy stored in the capacitor and the work done by the battery will be

(a) $\frac{1}{2}$ (b) 1

(c) 2 (d) $\frac{1}{4}$

Solution (a) $\frac{E_{cap}}{E_{battery}} = \frac{(QV)/2}{QV} = \frac{1}{2}$

14. An ideal coil of 10H is connected in series with a resistance of 5Ω and a battery of 5V. 2 seconds after the connection is made, the current flowing in amperes in the circuit is



(a) $(1-e^{-1})$ (b) $(1-e)$

(c) e (d) e^{-1}

Solution (a) $\tau = \frac{L}{R} = 2$

$$I = I_0(1-e^{-t/\tau})$$
$$= 1(1-e^{-2/2}) = (1-e^{-1})$$

15. A long straight wire of radius 'a' carries a steady current i. The current is uniformly distributed across its cross section. The ratio of the magnetic field at $\frac{a}{2}$ and $2a$ is

(a) $\frac{1}{2}$ (b) $\frac{1}{4}$

(c) 4 (d) 1

Solution (b) $\oint B.dl = \mu_o I_{in}$

$$B_{in} = \frac{\mu_o Ix}{2\pi r^2}$$

$$B_{out} = \frac{\mu_o I}{2\pi x}$$

$$\frac{B_{in}}{B_{out}} = \frac{a/2}{2a} = \frac{1}{4}$$

16. A current I flows along the length of an infinitely long, straight, thin walled pipe. Then

(a) the magnetic field at all points inside the pipe is the same, but not zero

(b) the magnetic field is zero only on the axis of the pipe

(c) the magnetic field is different at different points inside the pipe

(d) the magnetic field at any point inside the pipe is zero.

Solution (d) Magnetic field inside the thin-walled pipe is zero every where.

17. If M_O is the mass of an oxygen isotope ${}_8O^{17}$, M_P and M_N are the masses of a proton and a neutron respectively, the nuclear binding energy of the isotope is

(a) $(M_O - 17\,M_N)\,C^2$

(b) $(M_O - 8\,M_P)\,C^2$

(c) $(M_O - 8M_P - 9M_N)\,C^2$

(d) $M_O\,C^2$

Solution (c)

18. In gamma ray emission from a nucleus

(a) only the proton number changes

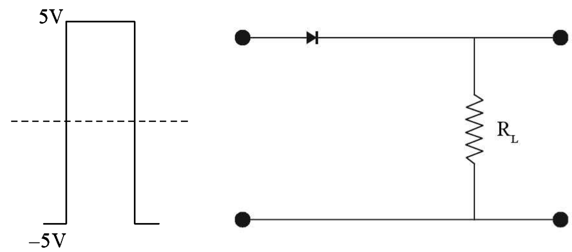
(b) both the neutron number and the proton number change

(c) there is no change in the proton number and the neutron number

(d) only the neutron number changes

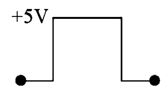
Solution (c)

19. If in a p-n junction diode, a square input signal of 10 V is applied as shown

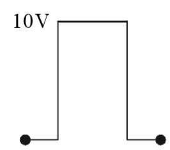


Then the output signal across R_L will be

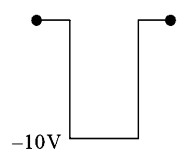
(a)



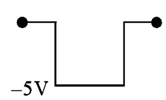
(b)



(c)



(d)



Solution (a) because it is a half-wave rectifier.

20. Photon of frequency v has a momentum associated with it. If c is the velocity of light, the momentum is

(a) hv/c (b) v/c

(c) hvc (d) hv/c^2

Solution (a) $P = \frac{h}{\lambda} = \frac{hv}{c}$

21. The velocity of a particle is $v = v_0 + gt + ft^2$. If its position is $x = 0$ at $t = 0$, then its displacement after unit time ($t = 1$) is

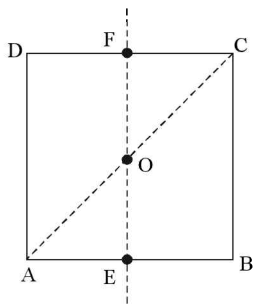
(a) $v_0 + g/2 + f$ (b) $v_0 + 2g + 3f$
(c) $v_0 + g/2 + f/3$ (d) $v_0 + g + f$

Solution (c) $x = \int v dt = v_0 t + \frac{gt^2}{2} + \frac{ft^3}{3}$.

Put $t = 1$

$$x = v_0 + \frac{g}{2} + \frac{f}{3}$$

22. For the given uniform square lamina ABCD, whose centre is O,



(a) $I_{AC} = \sqrt{2}\, I_{EF}$ (b) $\sqrt{2}\, I_{AC} = I_{EF}$
(c) $I_{AD} = 3I_{EF}$ (d) $I_{AC} = I_{EF}$

Solution (d) $I_{AC} = I_{EF} = \frac{I_o}{2}$

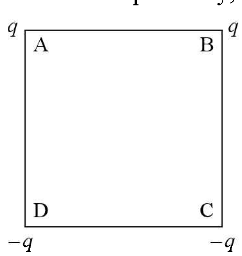
23. A point mass oscillates along the x-axis according to the law $x = x_0 \cos(\omega t - \pi/4)$. If the acceleration of the particle is written as $a = A \cos(\omega t + \delta)$, then

(a) $A = x_0 \omega^2$, $\delta = 3\pi/4$
(b) $A = x_0$, $\delta = -\pi/4$
(c) $A = x_0 \omega^2$, $\delta = \pi/4$
(d) $A = x_0 \omega^2$, $\delta = -\pi/4$

Solution (a) $a = -x_o w^2 \cos\left(wt - \frac{\pi}{4}\right)$

$$a = x_o w^2 \cos\left(wt + \frac{3\pi}{4}\right)$$

24. Charges are placed on the vertices of a square as shown. Let $\vec{E}$ be the electric field and V the potential at the centre. If the charges on A and B are interchanged with those on D and C respectively, then



(a) $\vec{E}$ changes, V remains unchanged
(b) $\vec{E}$ remains unchanged, V changes
(c) Both $\vec{E}$ and V change
(d) $\vec{E}$ and V remain unchanged

Solution (a) as E is vector and V is scalar

25. The half-life period of a radio-active element X is same as the mean life time of another radio-active element Y. Initially they have the same number of atoms. Then

(a) X and Y decay at same rate always
(b) X will decay faster than Y
(c) Y will decay faster than X
(d) X and Y have same decay rate initially

Solution (c) $t_{1/2}(1) = \frac{.693}{\lambda} = \frac{1}{\lambda_2}$ i.e. $\lambda_2 > \lambda_1$

26. A Carnot engine, having an efficiency of $\eta = 1/10$ as heat engine, is used as a refrigerator. If the work done on the system is 10 J, the amount of energy absorbed from the reservoir at lower temperature is

(a) 100 J (b) 99 J
(c) 90 J (d) 1 J

Solution (c) $\frac{1}{10} = \frac{Q_1 - Q_2}{Q_1} = \frac{10}{Q_1}$

$$Q_1 = 100\text{ J}$$
$$9Q_1 = 10Q_2$$
or $Q_2 = 90$ J

27. Carbon, silicon and germanium have four valence electrons each. At room temperature which one of the following statements is most appropriate?

(a) The number of free electrons for conduction is significant only in Si and Ge but small in C.
(b) The number of free conduction electrons is significant in C but small in Si and Ge.
(c) The number of free conduction electrons is negligibly small in all the three.
(d) The number of free electrons for conduction is significant in all the three.

Solution (a) $\because$ C (diamond) has Eg = 6.1 eV

28. A charged particle with charge q enters a region of constant, uniform and mutually orthogonal fields $\vec{E}$ and $\vec{B}$ with a velocity $\vec{v}$ perpendicular to both $\vec{E}$ and $\vec{B}$, and comes out without any change in magnitude or direction of $\vec{V}$. Then

(a) $\vec{v} = \vec{B} \times \vec{E} / E^2$
(b) $\vec{v} = \vec{E} \times \vec{B} / B^2$

(c) $\vec{v} = \vec{B} \times \vec{E} / B^2$

(d) $\vec{v} = \vec{E} \times \vec{B} / E^2$

Solution **(b)** $qvB = Eq$

$$v = \frac{E}{B}$$

Since $\vec{v}$ is perpendicular to both $\vec{E}$ and $\vec{B}$

$$\therefore \quad \vec{v} = \frac{\vec{E}\times\vec{B}}{B^2}$$

29. The potential at a point x (measured in μm) due to some charges situated on the x-axis is given by

V $(x) = 20/(x^2 - 4)$ Volts

The electric field E at $x = 4$ μm is given by

(a) 10/9 Volt/μm and in the +ve x direction

(b) 5/3 Volt/μm and in the –ve x direction

(c) 5/3 Volt/μm and in the +ve x direction

(d) 10/9 Volt/μm and in the –ve x direction

Solution **(a)** $E = \dfrac{-dv}{dx}$

$$= -\frac{20(-1)(2x)}{(x^2-4)^2} = \frac{10}{9}$$

30. Which of the following transitions in hydrogen atoms emit photons of highest frequency?

(a) $n = 1$ to $n = 2$ (b) $n = 2$ to $n = 6$

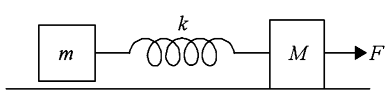
(c) $n = 6$ to $n = 2$ (d) $n = 2$ to $n = 1$

Solution **(d)** $\Delta E = 13.6\left[\dfrac{1}{1^2} - \dfrac{1}{2^2}\right]$ is largest.

31. A block of mass 'm' is connected to another block of mass 'M' by a spring constant 'k'. The blocks are kept on a smooth horizontal plane. Initially the blocks are at rest and the spring is unstretched. Then a constant force 'F' starts acting on the block of mass 'M' to pull it. Find the force on the block of mass 'm'.

(a) $\dfrac{MF}{(m+M)}$ (b) $\dfrac{mF}{M}$

(c) $\dfrac{(M+m)F}{m}$ (d) $\dfrac{mF}{(m+M)}$



Solution **(d)** $a = \dfrac{F}{m+M}$

$$F_2 = ma = \frac{mF}{m+M}$$

32. Two lenses of power –15D and +5D are in contact with each other. The focal length of the combination is

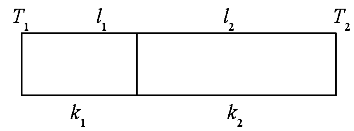
(a) + 10 cm (b) – 20 cm

(c) – 10 cm (d) + 20 cm

Solution **(c)** $P = -15 + 5 = -10$D

$$f = \frac{100}{P} = -10 \text{ cm}$$

33. One end of a thermally insulated rod is kept at a temperature T_1 and the other at T_2. The rod is composed of two sections of lengths l_1 and l_2 and thermal conductivities k_1 and k_2 respectively. The temperature at the interface of the two sections is



(a) $(k_1 l_1 T_1 + k_2 l_2 T_2) / (k_1 l_1 + k_2 l_2)$

(b) $(k_2 l_2 T_1 + k_1 l_1 T_2) / (k_1 l_1 + k_2 l_2)$

(c) $(k_2 l_1 T_1 + k_1 l_2 T_2) / (k_2 l_1 + k_1 l_2)$

(d) $(k_1 l_2 T_1 + k_2 l_1 T_2) / (k_1 l_2 + k_2 l_1)$

Solution **(d)** $I_1 = I_2$

$$\frac{k_1A}{l_1}(T_1 - T) = \frac{k_2A}{l_2}(T - T_2)$$

$$(k_1 l_2 + k_2 l_1)\, T = (k_1 l_2 T_1 + k_2 l_1 T_2)$$

or $T = \dfrac{k_1l_2T_1 + k_2l_1T_2}{k_1l_2 + k_2l_1}$

34. A sound absorber attenuates the sound level by 20 dB. The intensity decreases by a factor of

(a) 100 (b) 1000

(c) 10000 (d) 10

Solution **(a)** $-20 = 10 \log \dfrac{I}{I_o}$

$$I = \frac{I_o}{100}$$

35. If C_p and C_v denote the specific heats of nitrogen per unit mass at constant pressure and constant volume respectively, then

(a) $C_p - C_v = 28R$ (b) $C_p - C_v = R/28$

(c) $C_p - C_v = R/14$ (d) $C_p - C_v = R$

Solution **(b)** $C_p - C_v = \dfrac{R}{M} = \dfrac{R}{28}$

36. A charged particle moves through a magnetic field perpendicular to its direction. Then

(a) kinetic energy changes but the momentum is constant

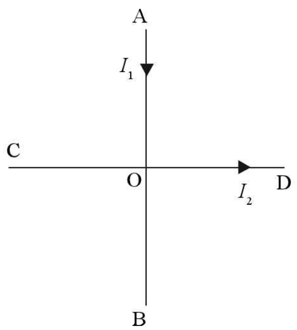
(b) the momentum changes but the kinetic energy is constant

(c) both momentum and kinetic energies of the particle are not constant

(d) both momentum and kinetic energies of the particle are contsant

Solution **(b)** As direction varies momentum varies but KE remains constant.

37. Two identical conducting wires AOB and COD are placed at right angles to each other. The wire AOB carries an electric current I_1 and COD carries a current I_2. The magnetic field on a point lying at a distance 'd' from O, in a direction perpendicular to the plane of the wires AOB and COD, will be given by



(a) $\frac{\mu_0}{2\pi d}(I_1^2+I_2^2)$ (b) $\frac{\mu_0}{2\pi}\left(\frac{I_1+I_2}{d}\right)^{1/2}$

(c) $\frac{\mu_0}{2\pi d}(I_1^2+I_2^2)^{1/2}$ (d) $\frac{\mu_0}{2\pi d}(I_1+I_2)$

Solution **(c)** $\vec{B} = \frac{\mu_0 I_1}{2\pi d}\hat{i} - \frac{\mu_0 I_2}{2\pi d}\hat{j}$

$$|\vec{B}| = \frac{\mu_0}{2\pi d}\sqrt{I_1^2+I_2^2}$$

38. The resistance of a wire is 5 ohm at 50°C and 6 ohm at 100°C. The resistance of the wire at 0°C will be

(a) 3 ohm (b) 2 ohm

(c) 1 ohm (d) 4 ohm

Solution **(d)** $6 = R_o(1+\alpha\,100)$

or $\frac{6-R_o}{5-R_o} = \frac{R_o\alpha 100}{R_o\alpha 50} = \frac{2}{1}$

$R_o = 4\Omega$

39. A parallel plate condenser with a dielectric of dielectric constant K between the plates has a capacity C and is charged to a potential V volts. The dielectric slab is slowly removed from between the plates and then reinserted. The net work done by the system in this process is

(a) zero (b) ½ $(K-1)CV^2$

(c) $CV^2(K-1)/K$ (d) $(K-1)CV^2$

Solution **(a)** $W = \Delta E = \frac{1}{2}CV^2 - \frac{1}{2}CV^2 = 0$

40. If g_E and g_m are the accelerations due to gravity on the surfaces of the earth and the moon respectively and if Millikan's oil drop experiment could be performed on the two surfaces, one will find the ratio $\frac{\text{electronic charge on the moon}}{\text{electronic charge on the earth}}$ to be

(a) g_M/g_E (b) 1

(c) 0 (d) g_E/g_M

Solution **(b)**