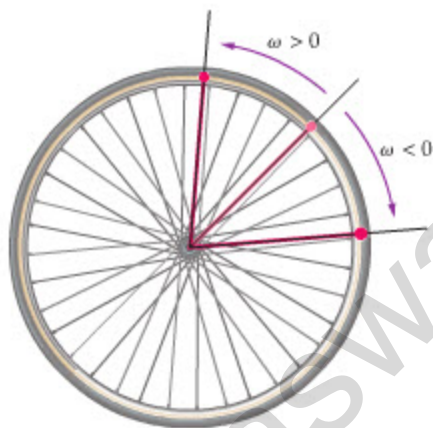




▲ Star trails provide a clear illustration of the relationship between angle, arc, and radius in circular motion. The stars, of course, do not actually move like this, but because of the Earth's rotation they appear to follow circular paths across the sky each night, with Polaris, the North Star, very near the axis of rotation. This photo was made by opening the camera shutter for an extended period of time. Notice that each star moves through the same angle in the course of the exposure. However, the farther a star is from the axis of rotation, the longer the arc it traces out in a given period of time. (Can you estimate the length of the exposure?)



▲ FIGURE 10-4 Angular speed and velocity

Counterclockwise rotation is defined to correspond to a positive angular velocity, ω . Similarly, clockwise rotation corresponds to a negative angular velocity. The magnitude of the angular velocity is referred to as the angular speed.

In addition to the average angular velocity, we can define an **instantaneous angular velocity** as the limit of ω_{av} as the time interval, Δt , approaches zero. The instantaneous angular velocity, then, is

$$\omega = \lim_{\Delta t \rightarrow 0} \frac{\Delta\theta}{\Delta t}$$

10-4

SI unit: $\text{rad/s} = \text{s}^{-1}$

Generally, we shall refer to the instantaneous angular velocity simply as the angular velocity.

Note that we call ω the angular velocity, not the angular speed. The reason is that ω can be positive or negative, depending on the sense of rotation. For example, if the red paint spot rotates in the counterclockwise sense, the angular position, θ , increases. As a result, $\Delta\theta$ is positive and therefore, so is ω . Similarly, clockwise rotation corresponds to a negative $\Delta\theta$ and hence a negative ω .

Sign Convention for Angular Velocity

By convention:

- $\omega > 0$ counterclockwise rotation
- $\omega < 0$ clockwise rotation

The sign convention for angular velocity is illustrated in **Figure 10-4**. In analogy with linear motion, the sign of ω indicates the *direction* of the angular velocity *vector*, as we shall see in detail in **Chapter 11**. Similarly, the magnitude of the angular velocity is the **angular speed**, just as in the one-dimensional case.

In **Exercise 10-1** we utilize the definitions and conversion factors presented so far in this section.

EXERCISE 10-1

(a) An old phonograph record rotates clockwise at $33\frac{1}{3}$ rpm (revolutions per minute). What is its angular velocity in rad/s ? (b) If a CD rotates at 22.0 rad/s , what is its angular speed in rpm?

SOLUTION

- a. Convert from rpm to rad/s , and note that clockwise rotation corresponds to a negative angular velocity:

$$\omega = -33\frac{1}{3} \text{ rpm} = \left(-33\frac{1}{3} \frac{\text{rev}}{\text{min}}\right) \left(\frac{2\pi \text{ rad}}{1 \text{ rev}}\right) \left(\frac{1 \text{ min}}{60 \text{ s}}\right) = -3.49 \text{ rad/s}$$

- b. Converting angular speed from rad/s to rpm gives

$$\omega = \left(22.0 \frac{\text{rad}}{\text{s}}\right) \left(\frac{1 \text{ rev}}{2\pi \text{ rad}}\right) \left(\frac{60 \text{ s}}{1 \text{ min}}\right) = 210 \frac{\text{rev}}{\text{min}} = 210 \text{ rpm}$$

Note that the same symbol, ω , is used for both angular velocity and angular speed in **Exercise 10-1**. Which quantity is meant in a given situation will be clear from the context in which it is used.

As a simple application of angular velocity, consider the following question: An object rotates with a constant angular velocity, ω . How much time, T , is required for it to complete one full revolution?

To solve this problem, note that since ω is constant, the instantaneous angular velocity is equal to the average angular velocity. That is,

$$\omega = \omega_{av} = \frac{\Delta\theta}{\Delta t}$$

In one revolution, we know that $\Delta\theta = 2\pi$ and $\Delta t = T$. Therefore,

$$\omega = \frac{\Delta\theta}{\Delta t} = \frac{2\pi}{T}$$

Finally, solving for T we find

$$T = \frac{2\pi}{\omega}$$

The time to complete one revolution, T , is referred to as the **period**.

Definition of Period, T

$$T = \frac{2\pi}{\omega}$$

10-5

SI unit: second, s

EXERCISE 10-2

Find the period of a record that is rotating at 45 rpm.

SOLUTION

To apply $T = 2\pi/\omega$ we must first express ω in terms of rad/s:

$$45 \text{ rpm} = \left(45 \frac{\text{rev}}{\text{min}}\right) \left(\frac{2\pi \text{ rad}}{1 \text{ rev}}\right) \left(\frac{1 \text{ min}}{60 \text{ s}}\right) = 4.7 \text{ rad/s}$$

Now we can calculate the period:

$$T = \frac{2\pi}{\omega} = \frac{2\pi \text{ rad}}{4.7 \text{ rad/s}} = 1.3 \text{ s}$$

Angular Acceleration, α

If the angular velocity of the rotating bicycle wheel increases or decreases with time, we say that the wheel experiences an **angular acceleration**, α . The average angular acceleration is the change in angular velocity, $\Delta\omega$, in a given interval of time, Δt :

Definition of Average Angular Acceleration, α_{av}

$$\alpha_{av} = \frac{\Delta\omega}{\Delta t}$$

10-6

SI unit: radian per second per second (rad/s^2) = s^{-2}

Note that the SI units of α are rad/s^2 , which, since rad is dimensionless, is simply s^{-2} .

As expected, the instantaneous angular acceleration is the limit of α_{av} as the time interval, Δt , approaches zero:

Definition of Instantaneous Angular Acceleration, α

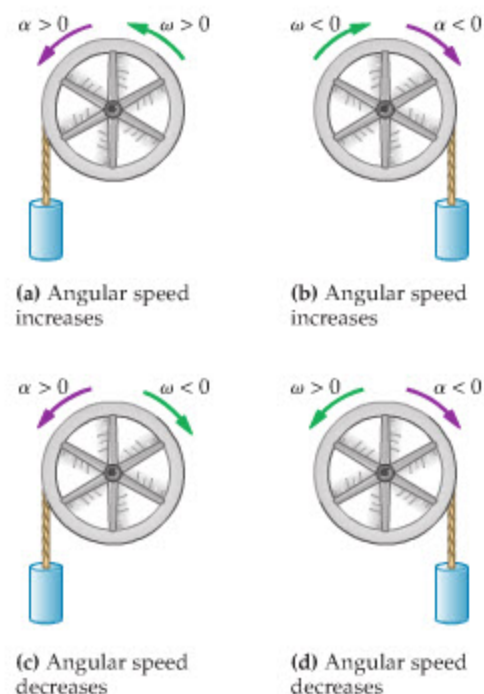
$$\alpha = \lim_{\Delta t \rightarrow 0} \frac{\Delta\omega}{\Delta t}$$

10-7

SI unit: $\text{rad/s}^2 = \text{s}^{-2}$

When referring to the instantaneous angular acceleration, we will usually just say angular acceleration.

The sign of the angular acceleration is determined by whether the change in angular velocity is positive or negative. For example, if ω is becoming more positive, so that ω_f is greater than ω_i , it follows that α is positive. Similarly, if ω is becoming more negative, so that ω_f is less than ω_i , it follows that α is negative. Therefore, if ω and α have the same sign, the speed of rotation is increasing. If ω and α have opposite signs, the speed of rotation is decreasing. This is illustrated in **Figure 10-5**.



▲ FIGURE 10-5 Angular acceleration and angular speed

When angular velocity and acceleration have the same sign, as in (a) and (b), the angular speed increases. When angular velocity and angular acceleration have opposite signs, as in (c) and (d), the angular speed decreases.

EXERCISE 10-3

As the wind dies, a windmill that was rotating at 2.1 rad/s begins to slow down with a constant angular acceleration of 0.45 rad/s². How long does it take for the windmill to come to a complete stop?

SOLUTION

If we choose the initial angular velocity to be positive, the angular acceleration is negative, corresponding to a deceleration. Hence, Equation 10-6 gives

$$\Delta t = \frac{\Delta\omega}{\alpha_{av}} = \frac{\omega_f - \omega_i}{\alpha} = \frac{0 - 2.1 \text{ rad/s}}{-0.45 \text{ rad/s}^2} = 4.7 \text{ s}$$

10-2 Rotational Kinematics

Just as the kinematics of Chapter 2 described linear motion, rotational kinematics describes rotational motion. In this section, as in Chapter 2, we concentrate on the important special case of constant acceleration.

As an example of a system with constant angular acceleration, consider the pulley shown in Figure 10-6. Wrapped around the circumference of the pulley is a string, with a mass attached to its free end. When the mass is released, the pulley begins to rotate—slowly at first, but then faster and faster. As we shall see in Chapter 11, the pulley is accelerating with constant angular acceleration.

Since α is constant, it follows that the average and instantaneous angular accelerations are equal. Hence,

$$\alpha = \alpha_{av} = \frac{\Delta\omega}{\Delta t}$$

Suppose the pulley starts with the initial angular velocity ω_0 at time $t = 0$, and that at the later time t its angular velocity is ω . Substituting these values into the preceding expression for α yields

$$\alpha = \frac{\Delta\omega}{\Delta t} = \frac{\omega - \omega_0}{t - 0} = \frac{\omega - \omega_0}{t}$$

Rearranging, we see that the angular velocity, ω , varies with time as follows:

$$\omega = \omega_0 + \alpha t \quad 10-8$$

EXERCISE 10-4

If the angular velocity of the pulley in Figure 10-6 is -8.4 rad/s at a given time, and its angular acceleration is -2.8 rad/s², what is the angular velocity of the pulley 1.5 s later?

SOLUTION

The angular velocity, ω , is found by applying Equation 10-8:

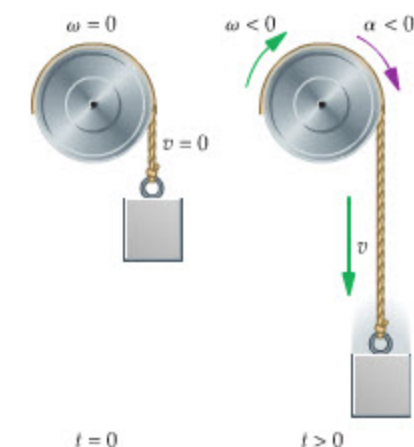
$$\omega = \omega_0 + \alpha t = -8.4 \text{ rad/s} + (-2.8 \text{ rad/s}^2)(1.5 \text{ s}) = -12.6 \text{ rad/s}$$

Note that the angular speed has increased, as expected, since ω and α have the same sign.

Note the close analogy between Equation 10-8 for angular velocity and the corresponding relation for linear velocity, Equation 2-7:

$$v = v_0 + at$$

Clearly, the equation for angular velocity can be obtained from our previous equation for linear velocity by replacing v with ω and replacing a with α . This type of analogy between linear and angular quantities can be most useful both in deriving angular equations—by starting with linear equations and using analogies—and in obtaining a better physical understanding of angular systems. Several linear-to-angular analogs are listed in the adjacent table.



▲ FIGURE 10-6 A pulley with constant angular acceleration

A mass is attached to a string wrapped around a pulley. As the mass falls, it causes the pulley to increase its angular speed with a constant angular acceleration.

Linear Quantity	Angular Quantity
x	θ
v	ω
a	α

Using these analogies, we can rewrite all the kinematic equations in Chapter 2 in angular form. The following table gives both the linear kinematic equations and their angular counterparts.

Linear Equation ($a = \text{constant}$)		Angular Equation ($\alpha = \text{constant}$)	
$v = v_0 + at$	2-7	$\omega = \omega_0 + \alpha t$	10-8
$x = x_0 + \frac{1}{2}(v_0 + v)t$	2-10	$\theta = \theta_0 + \frac{1}{2}(\omega_0 + \omega)t$	10-9
$x = x_0 + v_0t + \frac{1}{2}at^2$	2-11	$\theta = \theta_0 + \omega_0t + \frac{1}{2}\alpha t^2$	10-10
$v^2 = v_0^2 + 2a(x - x_0)$	2-12	$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$	10-11

In solving kinematic problems involving rotation, we apply these angular equations in the same way that the linear equations were applied in Chapter 2. In a sense, then, this material is a review—since the mathematics is essentially the same. The only difference comes in the physical interpretation of the results. We will emphasize the rotational interpretations throughout the chapter.

PROBLEM-SOLVING NOTE

Rotational Kinematics

Using analogies between linear and angular quantities often helps when solving problems involving rotational kinematics.



EXAMPLE 10-1 THROWN FOR A CURVE

To throw a curve ball, a pitcher gives the ball an initial angular speed of 36.0 rad/s . When the catcher gloves the ball 0.595 s later, its angular speed has decreased (due to air resistance) to 34.2 rad/s . (a) What is the ball's angular acceleration, assuming it to be constant? (b) How many revolutions does the ball make before being caught?

PICTURE THE PROBLEM

We choose the ball's initial direction of rotation to be positive. As a result, the angular acceleration will be negative. We can also identify the initial angular velocity to be $\omega_0 = 36.0 \text{ rad/s}$, and the final angular velocity to be $\omega = 34.2 \text{ rad/s}$.

STRATEGY

The problem states that the angular acceleration of the ball is constant. It follows that Equations 10-8 to 10-11 apply to its rotation.

- To relate angular velocity to time, we use $\omega = \omega_0 + \alpha t$. This can be solved for α .
- To relate angle to time we use $\theta = \theta_0 + \omega_0t + \frac{1}{2}\alpha t^2$. The angular displacement of the ball is $\theta - \theta_0$.



SOLUTION

Part (a)

- Solve $\omega = \omega_0 + \alpha t$ for the angular acceleration, α :

$$\begin{aligned}\omega &= \omega_0 + \alpha t \\ \alpha &= \frac{\omega - \omega_0}{t} \\ \alpha &= \frac{34.2 \text{ rad/s} - 36.0 \text{ rad/s}}{0.595 \text{ s}} = -3.03 \text{ rad/s}^2\end{aligned}$$

Part (b)

- Use $\theta = \theta_0 + \omega_0t + \frac{1}{2}\alpha t^2$ to calculate the angular displacement of the ball:

$$\begin{aligned}\theta - \theta_0 &= \omega_0t + \frac{1}{2}\alpha t^2 \\ &= (36.0 \text{ rad/s})(0.595 \text{ s}) + \frac{1}{2}(-3.03 \text{ rad/s}^2)(0.595 \text{ s})^2 \\ &= 20.9 \text{ rad}\end{aligned}$$

- Convert the angular displacement to revolutions:

$$\theta - \theta_0 = 20.9 \text{ rad} = 20.9 \text{ rad} \left(\frac{1 \text{ rev}}{2\pi \text{ rad}} \right) = 3.33 \text{ rev}$$

CONTINUED ON NEXT PAGE

CONTINUED FROM PREVIOUS PAGE

INSIGHT

The ball rotates through three-and-one-third revolutions during its time in flight.

An alternative method of solution is to use the kinematic relation given in Equation 10–9. This procedure yields $\theta - \theta_0 = \frac{1}{2}(\omega_0 + \omega)t = 20.9 \text{ rad}$, in agreement with our previous result.

PRACTICE PROBLEM

(a) What is the angular velocity of the ball 0.500 s after it is thrown? (b) What is the ball's angular velocity after it completes its first full revolution? [Answer: (a) Use $\omega = \omega_0 + \alpha t$ to find $\omega = 34.5 \text{ rad/s}$. (b) Use $\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$ to find $\omega = 35.5 \text{ rad/s}$.]

Some related homework problems: Problem 19, Problem 22

EXAMPLE 10–2 WHEEL OF MISFORTUNE

On a certain game show, contestants spin a wheel when it is their turn. One contestant gives the wheel an initial angular speed of 3.40 rad/s . It then rotates through one-and-one-quarter revolutions and comes to rest on the BANKRUPT space. (a) Find the angular acceleration of the wheel, assuming it to be constant. (b) How long does it take for the wheel to come to rest?

PICTURE THE PROBLEM

We choose the initial angular velocity to be positive, $\omega_0 = +3.40 \text{ rad/s}$, and indicate it with a counterclockwise rotation in our sketch. Since the wheel slows to a stop, the angular acceleration must be negative; that is, in the clockwise direction. After a rotation of 1.25 rev the wheel will read BANKRUPT.

STRATEGY

As in Example 10–1, we can use the kinematic equations for constant angular acceleration, Equations 10–8 to 10–11.

- To begin, we are given the initial angular velocity, $\omega_0 = +3.40 \text{ rad/s}$, the final angular velocity, $\omega = 0$ (the wheel comes to rest), and the angular displacement, $\theta - \theta_0 = 1.25 \text{ rev}$. We can find the angular acceleration using $\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$.
- Knowing the angular velocity and acceleration, we can find the time with $\omega = \omega_0 + \alpha t$.

**SOLUTION****Part (a)**

- Solve $\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$ for the angular acceleration, α :

$$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$$

$$\alpha = \frac{\omega^2 - \omega_0^2}{2(\theta - \theta_0)}$$

- Convert $\theta - \theta_0 = 1.25 \text{ rev}$ to radians:

$$\theta - \theta_0 = 1.25 \text{ rev} = 1.25 \text{ rev} \left(\frac{2\pi \text{ rad}}{1 \text{ rev}} \right) = 7.85 \text{ rad}$$

- Substitute numerical values to find α :

$$\alpha = \frac{\omega^2 - \omega_0^2}{2(\theta - \theta_0)} = \frac{0 - (3.40 \text{ rad/s})^2}{2(7.85 \text{ rad})} = -0.736 \text{ rad/s}^2$$

Part (b)

- Solve $\omega = \omega_0 + \alpha t$ for the time, t :

$$\omega = \omega_0 + \alpha t$$

$$t = \frac{\omega - \omega_0}{\alpha}$$

- Substitute numerical values to find t :

$$t = \frac{\omega - \omega_0}{\alpha} = \frac{0 - 3.40 \text{ rad/s}}{-0.736 \text{ rad/s}^2} = 4.62 \text{ s}$$

INSIGHT

Note that it was not necessary to define a reference line; that is, a direction for $\theta = 0$. All we need to know is the angular displacement, $\theta - \theta_0$, not the individual angles θ and θ_0 . Finally, notice that we can also solve Equation 10-9 for the time in part (b), which yields $t = 2(\theta - \theta_0)/(\omega_0 + \omega) = 4.62$ s, as expected.

PRACTICE PROBLEM

What is the angular speed of the wheel after one complete revolution? [Answer: $\omega = 1.52$ rad/s]

Some related homework problems: Problem 18, Problem 20

Finally, we consider a pulley that is rotating in such a way that initially it is lifting a mass with speed v . Gravity acting on the mass causes it and the pulley to slow and momentarily come to rest.

ACTIVE EXAMPLE 10-1 FIND THE TIME TO REST

A pulley rotating in the counterclockwise direction is attached to a mass suspended from a string. The mass causes the pulley's angular velocity to decrease with a constant angular acceleration $\alpha = -2.10$ rad/s². (a) If the pulley's initial angular velocity is $\omega_0 = 5.40$ rad/s, how long does it take for the pulley to come to rest? (b) Through what angle does the pulley turn during this time?

SOLUTION (Test your understanding by performing the calculations indicated in each step.)

- | | |
|--|---|
| 1. (a) Relate angular velocity to time: | $\omega = \omega_0 + \alpha t$ |
| 2. Solve for the time, t : | $t = (\omega - \omega_0)/\alpha$ |
| 3. Substitute numerical values: | $t = 2.57$ s |
| 4. (b) Use $\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$ to solve for $\theta - \theta_0$: | $\theta - \theta_0 = \omega_0 t + \frac{1}{2}\alpha t^2 = 6.94$ rad |
| 5. Alternatively, use $\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$: | $\theta - \theta_0 = (\omega^2 - \omega_0^2)/2\alpha = 6.94$ rad |

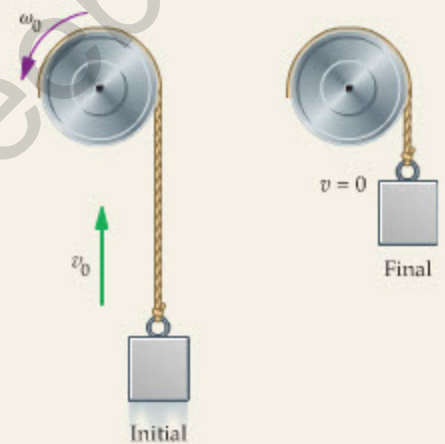
INSIGHT

After the pulley comes to rest, it immediately begins to rotate in the clockwise direction as the mass falls. The pulley's angular acceleration is constant—it has the same value before the pulley stops, when it stops, and after it begins rotating in the opposite direction. This is analogous to a projectile thrown straight upward, where the linear velocity starts out positive, goes to zero, then changes sign, all while the linear acceleration remains constant in the negative direction.

YOUR TURN

Find the angular displacement of the pulley at the time when its angular velocity is half its initial value.

(Answers to **Your Turn** problems are given in the back of the book.)



10-3 Connections Between Linear and Rotational Quantities

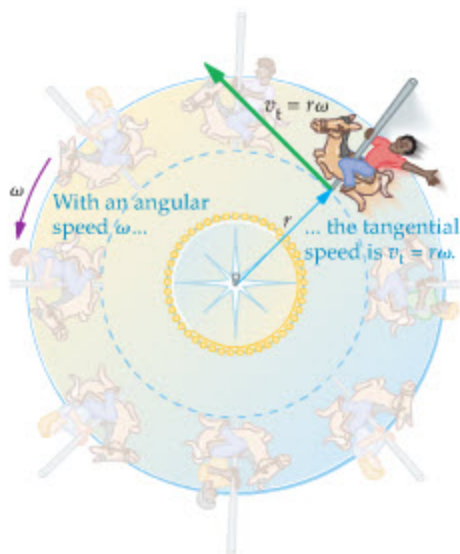
At a local county fair a child rides on a merry-go-round. The ride completes one circuit every $T = 7.50$ s. Therefore, the angular velocity of the child, from Equation 10-5, is

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{7.50 \text{ s}} = 0.838 \text{ rad/s}$$

The path followed by the child is circular, with the center of the circle at the axis of rotation of the merry-go-round. In addition, at any instant of time the child is moving in a direction that is *tangential* to the circular path, as Figure 10-7 shows. What is the tangential speed, v_t , of the child? In other words, what is the speed of the wind in the child's face?

We can find the child's tangential speed by dividing the circumference of the circular path, $2\pi r$, by the time required to complete one circuit, T . Thus,

$$v_t = \frac{2\pi r}{T} = r\left(\frac{2\pi}{T}\right)$$



▲ FIGURE 10-7 Angular and linear speed
Overhead view of a child riding on a merry-go-round. The child's path is a circle centered on the merry-go-round's axis of rotation. At any given time the child is moving tangential to the circular path with a speed $v_t = r\omega$.



REAL-WORLD PHYSICS
The operation of a CD

Because $2\pi/T$ is simply ω , we can express the tangential speed as follows:

Tangential Speed of a Rotating Object

$$v_t = r\omega$$

10-12

SI unit: m/s

Note that ω must be given in rad/s for this relation to be valid.

In the case of the merry-go-round, if the radius of the child's circular path is $r = 4.25$ m, the tangential speed is $v_t = r\omega = (4.25 \text{ m})(0.838 \text{ rad/s}) = 3.56 \text{ m/s}$. When it is clear that we are referring to the tangential speed, we will often drop the subscript t , and simply write $v = r\omega$.

An interesting application of the relation between linear and angular speeds is provided in the operation of a compact disk (CD). As you know, a CD is played by shining a laser beam onto the disk, and then converting the pattern of reflected light into a pattern of sound waves. For proper operation, however, the linear speed of the disk where the laser beam shines on it must be maintained at the constant value of 1.25 m/s. As the CD is played, the laser beam scans the disk in a spiral track from near the center outward to the rim. In order to maintain the required linear speed, the angular speed of the disk must decrease as the beam scans outward. The required angular speeds are determined in the following Exercise.

EXERCISE 10-5

Find the angular speed a CD must have to give a linear speed of 1.25 m/s when the laser beam shines on the disk (a) 2.50 cm and (b) 6.00 cm from its center.

SOLUTION

a. Using $v = 1.25$ m/s and $r = 0.0250$ m in Equation 10-12, we find

$$\omega = \frac{v}{r} = \frac{1.25 \text{ m/s}}{0.0250 \text{ m}} = 50.0 \text{ rad/s} = 477 \text{ rpm}$$

b. Similarly, with $r = 0.0600$ m we find

$$\omega = \frac{v}{r} = \frac{1.25 \text{ m/s}}{0.0600 \text{ m}} = 20.8 \text{ rad/s} = 199 \text{ rpm}$$

Thus, a CD slows from about 500 rpm to roughly 200 rpm as it plays.

How do the angular and tangential speeds of an object vary from one point to another? We explore this question in the following Conceptual Checkpoint.

CONCEPTUAL CHECKPOINT 10-1 COMPARE THE SPEEDS

Two children ride on a merry-go-round, with child 1 at a greater distance from the axis of rotation than child 2. Is the angular speed of child 1 (a) greater than, (b) less than, or (c) the same as the angular speed of child 2?

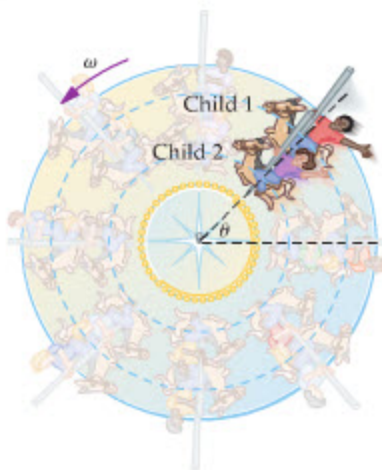
REASONING AND DISCUSSION

At any given time, the angle θ for child 1 is the same as the angle for child 2, as shown. Therefore, when the angle for child 1 has gone through 2π , for example, so has the angle for child 2. As a result, they have the same angular speed. In fact, *each and every point on the merry-go-round has exactly the same angular speed.*

The tangential speeds are different, however. Child 1 has the greater tangential speed since he travels around a larger circle in the same time that child 2 travels around a smaller circle. This is in agreement with the relation $v = r\omega$, since the radius to child 1 is greater than the radius to child 2. That is, $v_1 = r_1\omega > v_2 = r_2\omega$.

ANSWER

(c) The angular speeds are the same.





◀ In the photo at left, two identical plastic letter “E”s have been placed on a rotating turntable at different distances from the axis of rotation. The stretching and blurring of the image of the outermost letter clearly show that it is moving faster than the letter closer to the axis. Similarly, the boy near the rim of this playground merry-go-round is moving faster than the girl near the hub.

Because the children on the merry-go-round move in a circular path, they experience a centripetal acceleration, a_{cp} (Section 6-5). The centripetal acceleration is always directed toward the axis of rotation and has a magnitude given by

$$a_{cp} = \frac{v^2}{r}$$

Note that the speed v in this expression is the tangential speed, $v = v_t = r\omega$, and therefore the centripetal acceleration in terms of ω is

$$a_{cp} = \frac{(r\omega)^2}{r}$$

Canceling one power of r , we have

Centripetal Acceleration of a Rotating Object

$$a_{cp} = r\omega^2$$

10-13

SI unit: m/s^2

If the radius of a child’s circular path on the merry-go-round is 4.25 m, and the angular speed of the ride is 0.838 rad/s, the centripetal acceleration of the child is $a_{cp} = r\omega^2 = (4.25 \text{ m})(0.838 \text{ rad/s})^2 = 2.98 \text{ m/s}^2$.

Though the centripetal acceleration of a merry-go-round is typically only a fraction of the acceleration of gravity, rotating devices referred to as **centrifuges** can produce centripetal accelerations many times greater than gravity. For example, the world’s most powerful research centrifuge, operated by the U.S. Army Corps of Engineers, can subject 2.2-ton payloads to accelerations as high as 350g (350 times greater than the acceleration of gravity). This centrifuge is used to study earthquake engineering and dam erosion. The Air Force uses centrifuges to subject prospective jet pilots to the accelerations they will experience during rapid flight maneuvers, and in the future NASA may even use a human-powered centrifuge for gravity studies aboard the International Space Station.

REAL-WORLD PHYSICS

The centrifuge



▲ The large centrifuge shown at left, at the Gagarin Cosmonaut Training Center, is used to train Russian cosmonauts for space missions. This device, which rotates at 36 rpm, can produce a centripetal acceleration of over 290 m/s^2 , 30 times the acceleration of gravity. The device at right is a microhematocrit centrifuge, used to separate blood cells from plasma. The volume of red blood cells in a given quantity of whole blood is a major factor in determining the oxygen-carrying capacity of the blood, an important clinical indicator.

The centrifuges most commonly encountered in everyday life are those found in virtually every medical laboratory in the world. These devices, which can produce centripetal accelerations in excess of 13,000g, are used to separate blood cells from blood plasma. They do this by speeding up the natural tendency of cells to settle out of plasma from days to minutes. The ratio of the packed cell volume to the total blood volume gives the *hematocrit value*, which is a useful clinical indicator of blood quality. In the next Example we consider the operation of a *microhematocrit centrifuge*, which measures the hematocrit value of a small (micro) sample of blood.


REAL-WORLD PHYSICS: BIO
The microhematocrit centrifuge
EXAMPLE 10-3 THE MICROHEMATOCRIT

REAL-WORLD PHYSICS: BIO

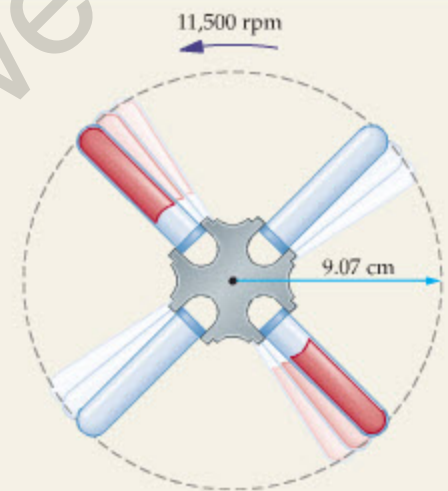
In a microhematocrit centrifuge, small samples of blood are placed in heparinized capillary tubes (heparin is an anticoagulant). The tubes are rotated at 11,500 rpm, with the bottoms of the tubes 9.07 cm from the axis of rotation. (a) Find the linear speed of the bottom of the tubes. (b) What is the centripetal acceleration at the bottom of the tubes?

PICTURE THE PROBLEM

Our sketch shows a top view of the centrifuge, with the capillary tubes rotating at 11,500 rpm. Notice that the bottoms of the tubes move in a circular path of radius 9.07 cm.

STRATEGY

- Linear and angular speeds are related by $v = r\omega$. Once we convert the angular speed to rad/s we can use this relation to determine v .
- The centripetal acceleration is $a_{cp} = r\omega^2$. Using ω from part (a) yields the desired result.


SOLUTION
Part (a)

- Convert the angular speed, ω , to radians per second:

$$\begin{aligned}\omega &= (11,500 \text{ rev/min}) \left(\frac{2\pi \text{ rad}}{1 \text{ rev}} \right) \left(\frac{1 \text{ min}}{60 \text{ s}} \right) \\ &= 1.20 \times 10^3 \text{ rad/s}\end{aligned}$$

- Use $v = r\omega$ to calculate the linear speed:

$$v = r\omega = (0.0907 \text{ m})(1.20 \times 10^3 \text{ rad/s}) = 109 \text{ m/s}$$

Part (b)

- Calculate the centripetal acceleration using $a_{cp} = r\omega^2$:

$$a_{cp} = r\omega^2 = (0.0907 \text{ m})(1.20 \times 10^3 \text{ rad/s})^2 = 131,000 \text{ m/s}^2$$

- As a check, calculate the centripetal acceleration using $a_{cp} = v^2/r$:

$$a_{cp} = \frac{v^2}{r} = \frac{(109 \text{ m/s})^2}{0.0907 \text{ m}} = 131,000 \text{ m/s}^2$$

INSIGHT

Note that every point on a tube has the same angular speed. As a result, points near the top of a tube have smaller linear speeds and centripetal accelerations than do points near the bottom of a tube. In this case, the bottoms of the tubes experience a centripetal acceleration about 13,400 times greater than the acceleration of gravity on the surface of the Earth; that is, $a_{cp} = 131,000 \text{ m/s}^2 = 13,400g$.

PRACTICE PROBLEM

What angular speed must this centrifuge have if the centripetal acceleration at the bottom of the tubes is to be $98,100 \text{ m/s}^2$ ($\approx 10,000g$)? [Answer: $\omega = \sqrt{a_{cp}/r} = 1040 \text{ rad/s} = 9930 \text{ rpm}$]

Some related homework problems: Problem 34, Problem 37

If the angular speed of the merry-go-round in Conceptual Checkpoint 10-1 changes, the tangential speed of the children changes as well. It follows, then, that the children will experience a tangential acceleration, a_t . We can determine a_t by considering the relation $v_t = r\omega$. If ω changes by the amount $\Delta\omega$, with r remaining constant, the corresponding change in tangential speed is

$$\Delta v_t = r\Delta\omega$$

If this change in ω occurs in the time Δt , the tangential acceleration is

$$a_t = \frac{\Delta v_t}{\Delta t} = r \frac{\Delta\omega}{\Delta t}$$

Since $\Delta\omega/\Delta t$ is the angular acceleration, α , we find that

Tangential Acceleration of a Rotating Object

$$a_t = r\alpha$$

10-14

SI unit: m/s^2

As with the tangential speed, we will often drop the subscript t in a_t when no confusion will arise.

In general, the children on the merry-go-round may experience both tangential and centripetal accelerations at the same time. Recall that a_t is due to a changing tangential speed, and that a_{cp} is caused by a changing direction of motion, even if the tangential speed remains constant. To summarize:

Tangential Versus Centripetal Acceleration

$a_t = r\alpha$ due to changing angular speed

$a_{cp} = r\omega^2$ due to changing direction of motion

As the names suggest, the tangential acceleration is always tangential to an object's path; the centripetal acceleration is always perpendicular to its path.

In cases in which both the centripetal and tangential accelerations are present, the total acceleration is the vector sum of the two, as indicated in **Figure 10-8**. Note that \vec{a}_t and \vec{a}_{cp} are at right angles to one another, and hence the magnitude of the total acceleration is given by the Pythagorean theorem:

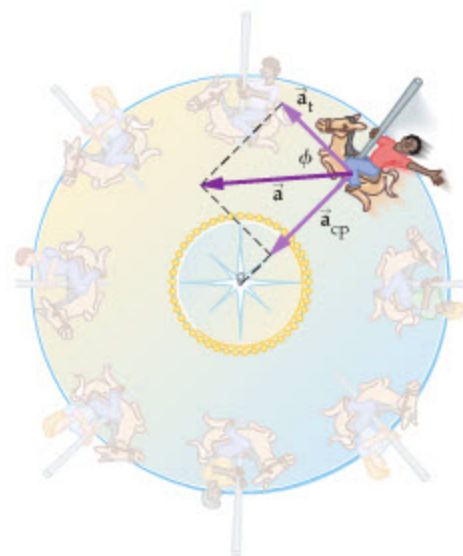
$$a = \sqrt{a_t^2 + a_{cp}^2}$$

The direction of the total acceleration, measured relative to the tangential direction, is

$$\phi = \tan^{-1}\left(\frac{a_{cp}}{a_t}\right)$$

This angle is shown in **Figure 10-8**.

In the next Active Example, we consider an object that is rotating with a constant angular acceleration, α . In this case, the tangential acceleration, $a_t = r\alpha$, is constant in magnitude. On the other hand, the centripetal acceleration, $a_{cp} = r\omega^2$, changes with time since the angular speed changes.



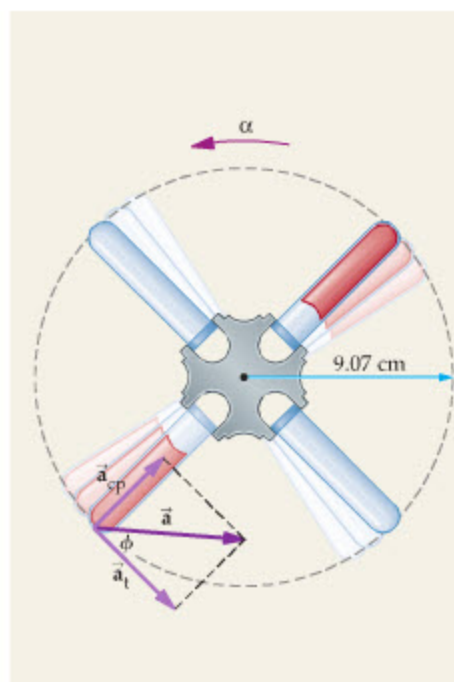
▲ FIGURE 10-8 Centripetal and tangential acceleration

If the angular speed of the merry-go-round is increased, the child will experience two accelerations: (i) a tangential acceleration, \vec{a}_t , and (ii) a centripetal acceleration, \vec{a}_{cp} . The child's total acceleration, \vec{a} , is the vector sum of \vec{a}_t and \vec{a}_{cp} .

ACTIVE EXAMPLE 10-2 FIND THE ACCELERATION

Suppose the centrifuge in **Example 10-3** is starting up with a constant angular acceleration of 95.0 rad/s^2 . (a) What are the magnitudes of the centripetal, tangential, and total accelerations of the bottom of a tube when the angular speed is 8.00 rad/s ? (b) What angle does the total acceleration make with the direction of motion?

CONTINUED ON NEXT PAGE



▶ **FIGURE 10-9** Rolling without slipping

A wheel of radius r rolling without slipping. During one complete revolution, the center of the wheel moves forward through a distance $2\pi r$.

CONTINUED FROM PREVIOUS PAGE

SOLUTION (Test your understanding by performing the calculations indicated in each step.)

Part (a)

1. Calculate the centripetal acceleration: $a_{cp} = r\omega^2 = 5.80 \text{ m/s}^2$
2. Calculate the tangential acceleration: $a_t = r\alpha = 8.62 \text{ m/s}^2$
3. Find the magnitude of the total acceleration: $a = \sqrt{a_{cp}^2 + a_t^2} = 10.4 \text{ m/s}^2$

Part (b)

4. Find the angle ϕ for the total acceleration: $\phi = \tan^{-1}(a_{cp}/a_t) = 33.9^\circ$

INSIGHT

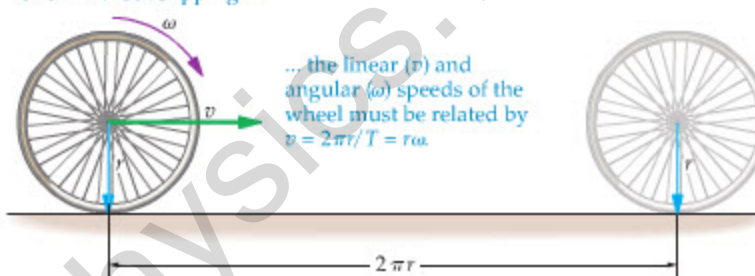
Note that all points on a tube have the same angular speed. In addition, all points have the same angular acceleration. In contrast, different points have different centripetal and tangential accelerations, due to their dependence on the distance r from the axis of rotation.

YOUR TURN

Find the magnitude and direction of the total acceleration of a point halfway between the top and bottom of a tube.

(Answers to **Your Turn** problems are given in the back of the book.)

To roll without slipping ...



... the linear (v) and angular (ω) speeds of the wheel must be related by $v = 2\pi r/T = r\omega$.

10-4 Rolling Motion

We began this chapter with a bicycle wheel rotating about its axle. In that case, the axle was at rest and every point on the wheel, such as the spot of red paint, moved in a circular path about the axle. We would like to consider a different situation now. Suppose the bicycle wheel is rolling freely, as indicated in **Figure 10-9**, with no slipping between the tire and the ground. The wheel still rotates about the axle, but the axle itself is moving in a straight line. As a result, the motion of the wheel is a combination of both rotational motion and linear (or **translational**) motion.

To see the connection between the wheel's rotational and translational motions, we show one full rotation of the wheel in **Figure 10-9**. During this rotation, the axle translates forward through a distance equal to the circumference of the wheel, $2\pi r$. Because the time required for one rotation is the period, T , the translational speed of the axle is

$$v = \frac{2\pi r}{T}$$

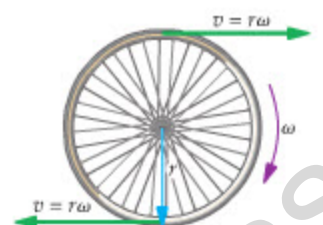
Recalling that $\omega = 2\pi/T$, we find

$$v = r\omega = v_t$$

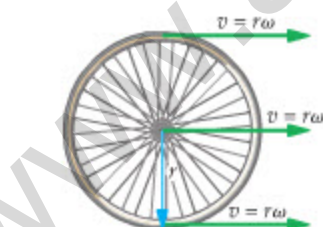
10-15

Hence, the translational speed of the axle is equal to the tangential speed of a point on the rim of a wheel spinning with angular speed ω .

A rolling object, then, combines rotational motion with angular speed ω , and translational motion with linear speed $v = r\omega$, where r is the radius of the object. Let's consider these two motions one at a time. First, in **Figure 10-10 (a)** we show pure rotational motion with angular speed ω . In this case, the axle is at rest, and points at the top and bottom of the wheel have tangential velocities that are equal in magnitude, $v = r\omega$, but point in opposite directions.



(a) Pure rotational motion



(b) Pure translational motion

▶ **FIGURE 10-10** Rotational and translational motions of a wheel

(a) In pure rotational motion, the velocities at the top and bottom of the wheel are in opposite directions. (b) In pure translational motion, each point on the wheel moves with the same speed in the same direction.

Next, we consider translational motion with speed $v = r\omega$. This is illustrated in **Figure 10-10 (b)**, where we see that each point on the wheel moves in the same direction with the same speed. If this were the only motion the wheel had, it would be skidding across the ground, instead of rolling without slipping.

Finally, we combine these two motions by simply adding the velocity vectors in **Figures 10-10 (a)** and **(b)**. The result is shown in **Figure 10-11**. At the top of the wheel the two velocity vectors are in the same direction, so they sum to give a speed of $2r\omega$. At the axle, the velocity vectors sum to give a speed $r\omega$. Finally, at the bottom of the wheel, the velocity vectors from rotation and translation have equal magnitude, but are in opposite directions. As a result, these velocities cancel, giving a speed of zero where the wheel is in contact with the ground.

The fact that the bottom of the wheel is instantaneously at rest, so that it is in static contact with the ground, is precisely what is meant by “rolling without slipping.” Thus, a wheel that rolls without slipping is just like the situation when you are walking—even though your body as a whole moves forward, the soles of your shoes are momentarily at rest every time you place them on the ground. This point was discussed in detail in Conceptual Checkpoint 6-1.

EXERCISE 10-6

A car with tires of radius 32 cm drives on the highway at 55 mph. **(a)** What is the angular speed of the tires? **(b)** What is the linear speed of the tops of the tires?

SOLUTION

- a. Using Equation 10-15 we find

$$\omega = \frac{v}{r} = \frac{(55 \text{ mph}) \left(\frac{0.447 \text{ m/s}}{1 \text{ mph}} \right)}{0.32 \text{ m}} = 77 \text{ rad/s}$$

This is about 12 revolutions per second.

- b. The tops of the tires have a speed of $2v = 110 \text{ mph}$.

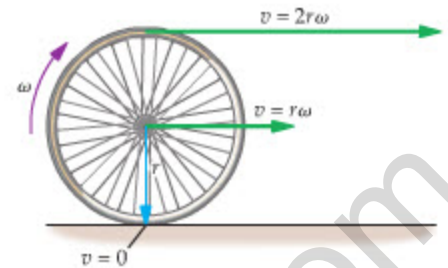
10-5 Rotational Kinetic Energy and the Moment of Inertia

An object in motion has kinetic energy, whether that motion is translational, rotational, or a combination of the two. In translational motion, for example, the kinetic energy of a mass m moving with a speed v is $K = \frac{1}{2}mv^2$. We cannot use this expression for a rotating object, however, because the speed v of each particle within a rotating object varies with its distance r from the axis of rotation, as we have seen in **Equation 10-12**. Thus, there is no unique value of v for an entire rotating object. On the other hand, there is a unique value of ω , the angular speed, that applies to all particles in the object.

To see how the kinetic energy of a rotating object depends on its angular speed, we start with a particularly simple system consisting of a rod of length r and negligible mass rotating about one end with an angular speed ω . Attached to the other end of the rod is a point mass m , as **Figure 10-12** shows. To find the kinetic energy of the mass, recall that its linear speed is $v = r\omega$ (**Equation 10-12**). Therefore, the translational kinetic energy of the mass m is

$$K = \frac{1}{2}mv^2 = \frac{1}{2}m(r\omega)^2 = \frac{1}{2}(mr^2)\omega^2 \quad 10-16$$

Notice that the kinetic energy of the mass depends not only on the angular speed squared (analogous to the way the translational kinetic energy depends on the linear speed squared), but also on the radius squared—that is, the kinetic energy depends on the *distribution* of mass in the rotating object. To be specific, mass near the axis of rotation contributes little to the kinetic energy since its speed ($v = r\omega$) is small. On the other hand, the farther a mass is from the axis of rotation, the greater its speed v for a given angular velocity, and thus the greater its kinetic energy.

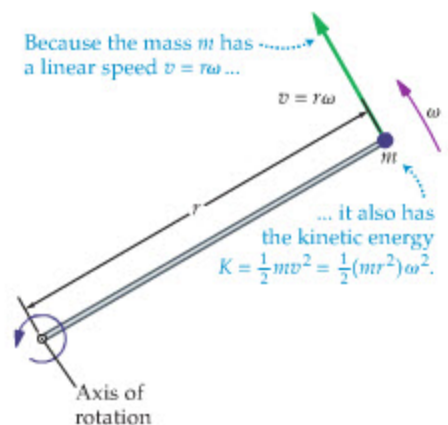


▲ FIGURE 10-11 Velocities in rolling motion

In a wheel that rolls without slipping, the point in contact with the ground is instantaneously at rest. The center of the wheel moves forward with the speed $v = r\omega$, and the top of the wheel moves forward with twice that speed, $v = 2r\omega$.



▲ This photograph of a rolling wheel gives a visual indication of the speed of its various parts. The bottom of the wheel is at rest at any instant, so the image there is sharp. The top of the wheel has the greatest speed, and the image there shows the most blurring. (Compare **Figure 10-11**.)



▲ FIGURE 10-12 Kinetic energy of a rotating object

As this rod rotates about the axis of rotation with an angular speed ω , the mass has a speed of $v = r\omega$. It follows that the kinetic energy of the mass is $K = \frac{1}{2}mv^2 = \frac{1}{2}(mr^2)\omega^2$.

You have probably noticed that the kinetic energy in Equation 10-16 is similar in form to the translational kinetic energy. Instead of $\frac{1}{2}(m)v^2$, we now have $\frac{1}{2}(mr^2)\omega^2$. Clearly, then, the quantity mr^2 plays the role of the mass for the rotating object. This “rotational mass” is given a special name in physics: the **moment of inertia**, I . Thus, in general, the kinetic energy of an object rotating with an angular speed ω can be written as:

Rotational Kinetic Energy

$$K = \frac{1}{2}I\omega^2$$

10-17

SI unit: J

The greater the moment of inertia—which some books call the rotational inertia—the greater an object’s rotational kinetic energy. As we have just seen, in the special case of a point mass m a distance r from the axis of rotation, the moment of inertia is simply $I = mr^2$.

We now show how to find the moment of inertia for an object of arbitrary, fixed shape, as in Figure 10-13. Suppose, for example, that this object rotates about the axis indicated in the figure with an angular speed ω . To calculate the kinetic energy of the object, we first imagine dividing it into a collection of small mass elements, m_i . We then calculate the kinetic energy of each element and sum over all elements. This extends to a large number of mass elements what we did for the single mass m .

Following this plan, the total kinetic energy of an arbitrary rotating object is

$$K = \sum \left(\frac{1}{2}m_i v_i^2 \right)$$

In this expression, m_i is the mass of one of the small mass elements and v_i is its speed. If m_i is at the radius r_i from the axis of rotation, as indicated in Figure 10-13, its speed is $v_i = r_i\omega$. Note that it is not necessary to write a separate angular speed, ω_i , for each element, because *all* mass elements of the object have exactly the same angular speed, ω . Therefore,

$$K = \sum \left(\frac{1}{2}m_i r_i^2 \omega^2 \right) = \frac{1}{2} \left(\sum m_i r_i^2 \right) \omega^2$$

Now, in analogy with our results for the single mass, we can define the moment of inertia, I , as follows:

Definition of Moment of Inertia, I

$$I = \sum m_i r_i^2$$

10-18

SI unit: $\text{kg} \cdot \text{m}^2$

The precise value of I for a given object depends on its distribution of mass. A simple example of this dependence is given in the following Exercise.

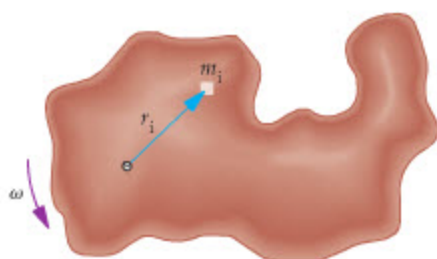
EXERCISE 10-7

Use the general definition of the moment of inertia, as given in Equation 10-18, to find the moment of inertia for the dumbbell-shaped object shown in Figure 10-14. Note that the axis of rotation goes through the center of the object and points out of the page. In addition, assume that the masses may be treated as point masses.

SOLUTION

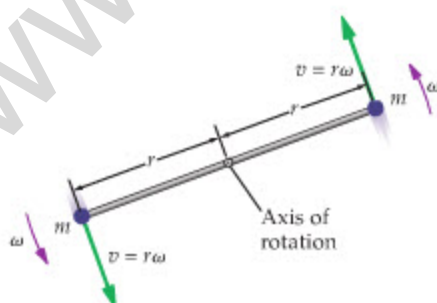
Referring to Figure 10-14, we see that $m_1 = m_2 = m$ and $r_1 = r_2 = r$. Therefore, the moment of inertia is

$$I = \sum m_i r_i^2 = m_1 r_1^2 + m_2 r_2^2 = mr^2 + mr^2 = 2mr^2$$



▲ FIGURE 10-13 Kinetic energy of a rotating object of arbitrary shape

To calculate the kinetic energy of an object of arbitrary shape as it rotates about an axis with angular speed ω , imagine dividing it into small mass elements, m_i . The total kinetic energy of the object is the sum of the kinetic energies of all the mass elements.



▲ FIGURE 10-14 A dumbbell-shaped object rotating about its center

The connection between rotational kinetic energy and the moment of inertia is explored in more detail in the following Example.

EXAMPLE 10-4 NOSE TO THE GRINDSTONE

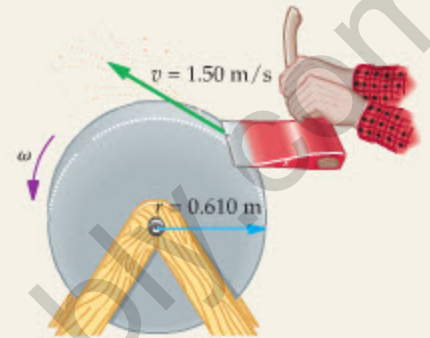
A grindstone with a radius of 0.610 m is being used to sharpen an ax. (a) If the linear speed of the stone relative to the ax is 1.50 m/s, and the stone's rotational kinetic energy is 13.0 J, what is its moment of inertia? (b) If the linear speed is doubled to 3.00 m/s, what is the corresponding kinetic energy of the grindstone?

PICTURE THE PROBLEM

Our sketch shows the grindstone spinning with an angular speed ω , which is not given in the problem statement. We do know, however, that the linear speed of the grindstone at its rim is $v = 1.50$ m/s and that its radius is $r = 0.610$ m. At this rate of rotation, the stone has a kinetic energy of 13.0 J.

STRATEGY

- Recall that rotational kinetic energy and moment of inertia are related by $K = \frac{1}{2}I\omega^2$; thus $I = 2K/\omega^2$. We are not given ω , but we can find it from the connection between linear and angular speed, $v = r\omega$. Thus, we begin by finding ω . We then use ω , along with the kinetic energy K , to find I .
- Find the new angular speed with $\omega = v/r$. Use I from part (a), along with $K = \frac{1}{2}I\omega^2$, to find the new kinetic energy.

**SOLUTION****Part (a)**

- Find the angular speed of the grindstone:

$$\omega = \frac{v}{r} = \frac{1.50 \text{ m/s}}{0.610 \text{ m}} = 2.46 \text{ rad/s}$$

- Solve for the moment of inertia in terms of kinetic energy:

$$K = \frac{1}{2}I\omega^2 \quad \text{or} \quad I = \frac{2K}{\omega^2}$$

- Substitute numerical values for K and ω :

$$I = \frac{2K}{\omega^2} = \frac{2(13.0 \text{ J})}{(2.46 \text{ rad/s})^2} = 4.30 \text{ J} \cdot \text{s}^2 = 4.30 \text{ kg} \cdot \text{m}^2$$

Part (b)

- Find the angular speed of the grindstone corresponding to $v = 3.00$ m/s:

$$\omega = \frac{v}{r} = \frac{3.00 \text{ m/s}}{0.610 \text{ m}} = 4.92 \text{ rad/s}$$

- Determine the kinetic energy, K , using the moment of inertia, I , from part (a):

$$K = \frac{1}{2}I\omega^2 = \frac{1}{2}(4.30 \text{ kg} \cdot \text{m}^2)(4.92 \text{ rad/s})^2 = 52.0 \text{ J}$$

INSIGHT

(a) We found I by relating it to the rotational kinetic energy of the grindstone. Later in this section we show how to calculate the moment of inertia of a disk directly, given its radius and mass. (b) Doubling the linear speed, v , results in a doubling of the angular speed, ω . The kinetic energy K depends on ω^2 ; therefore doubling ω increases K by a factor of 4, from 13.0 J to $4(13.0 \text{ J}) = 52.0 \text{ J}$.

PRACTICE PROBLEM

When the ax is pressed firmly against the grindstone for sharpening, the angular speed of the grindstone decreases. If the rotational kinetic energy of the grindstone is cut in half to 6.50 J, what is its angular speed? [Answer: The moment of inertia is unchanged; it depends only on the size, shape, and mass of the grindstone. Hence, $\omega = \sqrt{2K/I} = 1.74$ rad/s, which is smaller than the original $\omega = 2.46$ rad/s by a factor of $\sqrt{2}$.]

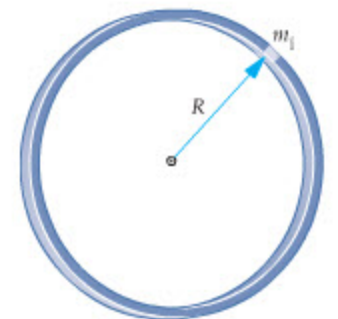
Some related homework problems: Problem 56, Problem 57

We return now to the dependence of the moment of inertia on the particular shape, or mass distribution, of an object. Suppose, for example, that a mass M is formed into the shape of a hoop of radius R . In addition, consider the case where the axis of rotation is perpendicular to the plane of the hoop and passes through its center, as shown in Figure 10-15. This is similar to a bicycle wheel rotating about its axle, if one ignores the spokes. In terms of small mass elements, we can write the moment of inertia as

$$I = \sum m_i r_i^2$$

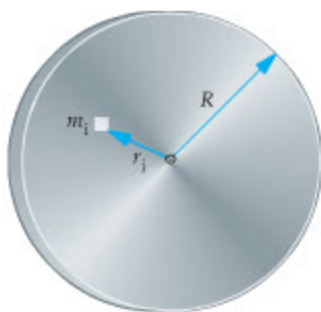
Each mass element of the hoop, however, is at the same radius R from the axis of rotation; that is, $r_i = R$. Hence, the moment of inertia in this case is

$$I = \sum m_i r_i^2 = \sum m_i R^2 = (\sum m_i) R^2$$



▲ FIGURE 10-15 The moment of inertia of a hoop

Consider a hoop of mass M and radius R . Each small mass element is at the same distance, R , from the center of the hoop. The moment of inertia in this case is $I = MR^2$.



▲ **FIGURE 10-16** The moment of inertia of a disk

Consider a disk of mass M and radius R . Mass elements for the disk are at distances from the center ranging from 0 to R . The moment of inertia in this case is $I = \frac{1}{2}MR^2$.

Clearly, the sum of all the elementary masses is simply the total mass of the hoop, $\Sigma m_i = M$. Therefore, the moment of inertia of a hoop of mass M and radius R is

$$I = MR^2 \text{ (hoop)}$$

In contrast, if the same mass, M , is formed into a uniform *disk* of the same radius, R , the moment of inertia is different. To see this, note that it is no longer true that $r_i = R$ for all mass elements. In fact, most of the mass elements are closer to the axis of rotation than was the case for the hoop, as indicated in **Figure 10-16**. Thus, since the r_i are generally less than R , the moment of inertia will be smaller for the disk than for the hoop. A detailed calculation, summing over all mass elements, yields the following result:

$$I = \frac{1}{2}MR^2 \text{ (disk)}$$

As expected, I is less for the disk than for the hoop.

EXERCISE 10-8

If the grindstone in **Example 10-4** is a uniform disk, what is its mass?

SOLUTION

Applying the preceding equation yields

$$M = \frac{2I}{R^2} = \frac{2(4.30 \text{ kg} \cdot \text{m}^2)}{(0.610 \text{ m})^2} = 23.1 \text{ kg}$$

Thus, the grindstone has a weight of roughly 51 lb.

Table 10-1 collects moments of inertia for a variety of objects. Note that in all cases the moment of inertia is of the form $I = (\text{constant})MR^2$. It is only the constant in front of MR^2 that changes from one object to another.

Note also that objects of the same general shape but with different mass distributions—such as solid and hollow spheres—have different moments of inertia. In particular, a hollow sphere has a larger I than a solid sphere of the same mass, for the same reason that a hoop's moment of inertia is greater than a disk's—more of its mass is at a greater distance from the axis of rotation. Thus, I is a measure of both the shape *and* the mass distribution of an object.

TABLE 10-1 Moments of Inertia for Uniform, Rigid Objects of Various Shapes and Total Mass M

<p>Hoop or cylindrical shell $I = MR^2$</p>	<p>Disk or solid cylinder $I = \frac{1}{2}MR^2$</p>	<p>Disk or solid cylinder (axis at rim) $I = \frac{3}{2}MR^2$</p>	<p>Long thin rod (axis through midpoint) $I = \frac{1}{12}ML^2$</p>	<p>Long thin rod (axis at one end) $I = \frac{1}{3}ML^2$</p>
<p>Hollow sphere $I = \frac{2}{3}MR^2$</p>	<p>Solid sphere $I = \frac{2}{5}MR^2$</p>	<p>Solid sphere (axis at rim) $I = \frac{7}{5}MR^2$</p>	<p>Solid plate (axis through center, in plane of plate) $I = \frac{1}{12}ML^2$</p>	<p>Solid plate (axis perpendicular to plane of plate) $I = \frac{1}{12}M(L^2 + W^2)$</p>

Consider, for example, the moment of inertia of the Earth. If the Earth were a uniform sphere of mass M_E and radius R_E , its moment of inertia would be $\frac{2}{5}M_ER_E^2 = 0.4M_ER_E^2$. In fact, the Earth's moment of inertia is only $0.331M_ER_E^2$, considerably less than for a uniform sphere. This is due to the fact that the Earth is not homogeneous, but instead has a dense inner core surrounded by a less dense outer core and an even less dense mantle. The resulting concentration of mass near its axis of rotation gives the Earth a much smaller moment of inertia than it would have if its mass were uniformly distributed.

On the other hand, if the polar ice caps were to melt and release their water into the oceans, the Earth's moment of inertia would increase. This is because mass that had been near the axis of rotation (in the polar ice) would now be distributed more or less uniformly around the Earth (in the oceans). With more of the Earth's mass at greater distances from the axis of rotation, the moment of inertia would increase. If such an event were to occur, not only would the moment of inertia increase, but the length of the day would increase as well. We shall discuss the reasons for this in the next chapter.

The moment of inertia of an object also depends on the location and orientation of the axis of rotation. If the axis of rotation is moved, all of the r_i change, leading to a different result for I . This is investigated for the dumbbell system in the following Conceptual Checkpoint.



▲ The distribution of mass in the Earth is not uniform. Dense materials, like iron and nickel, have concentrated near the center, while less dense materials, like silicon and aluminum, have risen to the surface. This concentration of mass near the axis of rotation lowers the Earth's moment of inertia.

REAL-WORLD PHYSICS

Moment of inertia of the Earth



CONCEPTUAL CHECKPOINT 10-2 COMPARE THE MOMENTS OF INERTIA

If the dumbbell-shaped object in Figure 10-14 is rotated about one end, is its moment of inertia (a) more than, (b) less than, or (c) the same as the moment of inertia about its center? As before, assume that the masses can be treated as point masses.

REASONING AND DISCUSSION

As we saw in Exercise 10-7, the moment of inertia about the center of the dumbbell is $I = 2mR^2$. When the axis is at one end, that mass is at the radius $r = 0$, and the other mass is at $r = 2R$. Therefore, the moment of inertia is

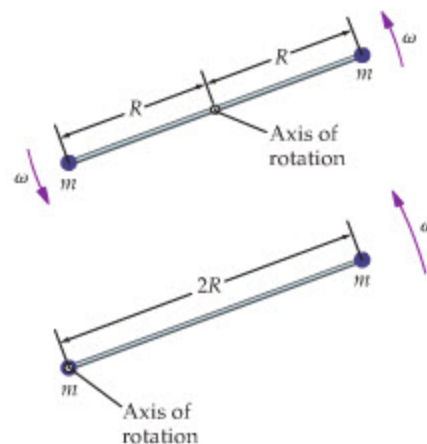
$$I = \sum m_i r_i^2 = m \cdot 0 + m(2R)^2 = 4mR^2$$

Thus, the moment of inertia doubles when the axis of rotation is moved from the center to one end.

The reason I increases is that the moment of inertia depends on the radius squared. Hence, even small increases in r can cause significant increases in I . By moving the axis to one end, the radius to the other mass is increased to its greatest possible value. As a result, I increases.

ANSWER

(a) The moment of inertia is greater about one end than about the center.



Finally, we summarize in the accompanying table the similarities between the translational kinetic energy, $K = \frac{1}{2}mv^2$, and the rotational kinetic energy, $K = \frac{1}{2}I\omega^2$. As expected, we see that the linear speed, v , has been replaced with the angular speed, ω . In addition, note that the mass m has been replaced with the moment of inertia I .

As suggested by these analogies, the moment of inertia I plays the same role in rotational motion that mass plays in translational motion. For example, the larger I the more resistant an object is to any change in its angular velocity—an object with a large I is difficult to start rotating, and once it is rotating, it is difficult to stop. We shall see further applications of this analogy in the next chapter when we consider angular momentum.

Linear Quantity	Angular Quantity
v	ω
m	I
$\frac{1}{2}mv^2$	$\frac{1}{2}I\omega^2$

10-6 Conservation of Energy

In this section, we consider the mechanical energy of objects that roll without slipping, and show how to apply energy conservation to such systems. In addition, we consider objects that rotate as a string or rope unwinds: for example, a pulley

with a string wrapped around its circumference, or a yo-yo with a string wrapped around its axle. As long as the unwinding process and the rolling motion occur without slipping, the two situations are basically the same—at least as far as energy considerations are concerned.

To apply energy conservation to rolling objects, we first need to determine the kinetic energy of rolling motion. In Section 10-4 we saw that rolling motion is a combination of rotation and translation. It follows, then, that the kinetic energy of a rolling object is simply the sum of its translational kinetic energy, $\frac{1}{2}mv^2$, and its rotational kinetic energy, $\frac{1}{2}I\omega^2$:



PROBLEM-SOLVING NOTE

Energy Conservation with Rotational Motion

When applying energy conservation to a system with rotational motion, be sure to include the rotational kinetic energy, $\frac{1}{2}I\omega^2$.

Kinetic Energy of Rolling Motion

$$K = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

10-19

Note that I in this expression is the moment of inertia about the center of the rolling object.

We can simplify the expression for the kinetic energy of a rolling object by using the fact that linear and angular speeds are related. In fact, recall that $v = r\omega$ (Equation 10-12), which can be rewritten as $\omega = v/r$. Substituting this into our expression for the rolling kinetic energy yields

Kinetic Energy of Rolling Motion: Alternative Form

$$K = \frac{1}{2}mv^2 + \frac{1}{2}I\left(\frac{v}{r}\right)^2 = \frac{1}{2}mv^2\left(1 + \frac{I}{mr^2}\right)$$

10-20

Since $I = (\text{constant})mr^2$, the last term in Equation 10-20 is a constant that depends on the shape and mass distribution of the rolling object.

A special case of some interest is the point particle. In this case, by definition, all of the mass is at a single point. Therefore, $r = 0$, and hence $I = 0$. Substituting $I = 0$ in either Equation 10-19 or Equation 10-20 yields $K = \frac{1}{2}mv^2$, as expected.

Next, we apply Equations 10-19 and 10-20 to a disk that rolls with no slipping.

EXAMPLE 10-5 LIKE A ROLLING DISK

A 1.20-kg disk with a radius of 10.0 cm rolls without slipping. If the linear speed of the disk is 1.41 m/s, find (a) the translational kinetic energy, (b) the rotational kinetic energy, and (c) the total kinetic energy of the disk.

PICTURE THE PROBLEM

Because the disk rolls without slipping, the angular speed and the linear speed are related by $v = r\omega$. Note that the linear speed is $v = 1.41$ m/s and the radius is $r = 10.0$ cm. Finally, we are given that the mass of the disk is 1.20 kg.

STRATEGY

We calculate each contribution to the kinetic energy separately. The linear kinetic energy, of course, is simply $\frac{1}{2}mv^2$. For the rotational kinetic energy, $\frac{1}{2}I\omega^2$, we must use the fact that the moment of inertia for a disk is $I = \frac{1}{2}mr^2$. Finally, since the disk rolls without slipping, its angular speed is $\omega = v/r$.

SOLUTION

Part (a)

1. Calculate the translational kinetic energy, $\frac{1}{2}mv^2$:

$$\frac{1}{2}mv^2 = \frac{1}{2}(1.20 \text{ kg})(1.41 \text{ m/s})^2 = 1.19 \text{ J}$$

Part (b)

2. Calculate the rotational kinetic energy symbolically, using $I = \frac{1}{2}mr^2$ and $\omega = v/r$:

$$\frac{1}{2}I\omega^2 = \frac{1}{2}\left(\frac{1}{2}mr^2\right)\left(\frac{v}{r}\right)^2 = \frac{1}{2}\left(\frac{1}{2}mv^2\right)$$

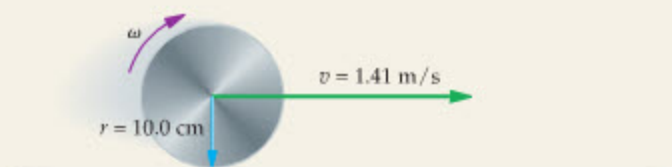
3. Substitute the numerical value for $\frac{1}{2}mv^2$ (the translational kinetic energy) obtained in Step 1:

$$\frac{1}{2}I\omega^2 = \frac{1}{2}(1.19 \text{ J}) = 0.595 \text{ J}$$

Part (c)

4. Sum the kinetic energies obtained in parts (a) and (b):

$$K = 1.19 \text{ J} + 0.595 \text{ J} = 1.79 \text{ J}$$



5. Note that the same result is obtained using Equation 10-20:

$$\begin{aligned} K &= \frac{1}{2}mv^2 \left(1 + \frac{I}{mr^2} \right) = \frac{1}{2}mv^2 \left(1 + \frac{1}{2} \right) \\ &= \frac{3}{2} \left(\frac{1}{2}mv^2 \right) = \frac{3}{2}(1.19 \text{ J}) = 1.79 \text{ J} \end{aligned}$$

INSIGHT

The symbolic result in Step 2 shows that the rotational kinetic energy of a uniform disk rolling without slipping is precisely one-half the disk's translational kinetic energy. Thus, 2/3 of the disk's kinetic energy is translational, 1/3 rotational. This result is independent of the disk's radius, as we can see by the cancellation of the radius r in Step 2.

To understand this cancellation, note that a larger disk has a larger moment of inertia, since it has mass farther from the axis of rotation. On the other hand, the larger disk also has a smaller angular speed, since the angular speed is inversely proportional to the radius: $\omega = v/r$. These two effects cancel, giving the same rotational kinetic energy for uniform disks of any radius—provided their linear speed is the same.

PRACTICE PROBLEM

Repeat this problem for the case of a rolling, hollow sphere. [Answer: (a) 1.19 J, (b) 0.793 J, (c) 1.98 J]

Some related homework problems: Problem 58, Problem 62

CONCEPTUAL CHECKPOINT 10-3 COMPARE KINETIC ENERGIES

A solid sphere and a hollow sphere of the same mass and radius roll without slipping at the same speed. Is the kinetic energy of the solid sphere (a) more than, (b) less than, or (c) the same as the kinetic energy of the hollow sphere?

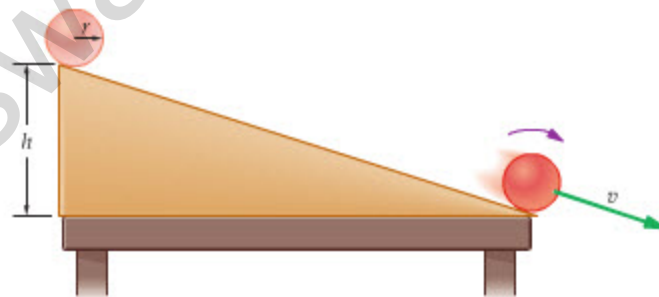
REASONING AND DISCUSSION

Both spheres have the same translational kinetic energy since they have the same mass and speed. The rotational kinetic energy, however, is proportional to the moment of inertia. Since the hollow sphere has the greater moment of inertia, it has the greater kinetic energy.

ANSWER

(b) The solid sphere has less kinetic energy than the hollow sphere.

Now that we can calculate the kinetic energy of rolling motion, we show how to apply it to energy conservation. For example, consider an object of mass m , radius r , and moment of inertia I at the top of a ramp, as shown in Figure 10-17. The object is released from rest and allowed to roll to the bottom, a vertical height h below the starting point. What is the object's speed on reaching the bottom?



◀ **FIGURE 10-17** An object rolls down an incline

An object starts at rest at the top of an inclined plane and rolls without slipping to the bottom. The speed of the object at the bottom depends on its moment of inertia—a larger moment of inertia results in a lower speed.

The simplest way to solve this problem is to use energy conservation. To do so, we set the initial mechanical energy at the top (i) equal to the final mechanical energy at the bottom (f). That is,

$$K_i + U_i = K_f + U_f$$

Since we are dealing with rolling motion, the kinetic energy is

$$K = \frac{1}{2}mv^2 \left(1 + \frac{I}{mr^2} \right)$$

The potential energy is simply that due to the uniform gravitational field. Therefore,

$$U = mgy$$

With $y = h$ at the top of the ramp and the object starting at rest, we have

$$K_i + U_i = 0 + mgh = mgh$$

Similarly, with $y = 0$ at the bottom of the ramp and the object rolling with a speed v , we find

$$K_f + U_f = \frac{1}{2}mv^2\left(1 + \frac{I}{mr^2}\right) + 0 = \frac{1}{2}mv^2\left(1 + \frac{I}{mr^2}\right)$$

Setting the initial and final energies equal yields

$$mgh = \frac{1}{2}mv^2\left(1 + \frac{I}{mr^2}\right)$$

Solving for v , we find

$$v = \sqrt{\frac{2gh}{1 + \frac{I}{mr^2}}}$$

Let's quickly check one special case: namely, $I = 0$. With this substitution, we find

$$v = \sqrt{2gh}$$

This is the speed an object would have after falling straight down with no rotation through a distance h . Thus, setting $I = 0$ means there is no rotational kinetic energy, and hence the result is the same as for a point particle. As I becomes larger, the speed at the bottom of the ramp is smaller.

CONCEPTUAL CHECKPOINT 10-4 WHICH OBJECT WINS THE RACE?

A disk and a hoop of the same mass and radius are released at the same time at the top of an inclined plane. Does the disk reach the bottom of the plane **(a)** before, **(b)** after, or **(c)** at the same time as the hoop?

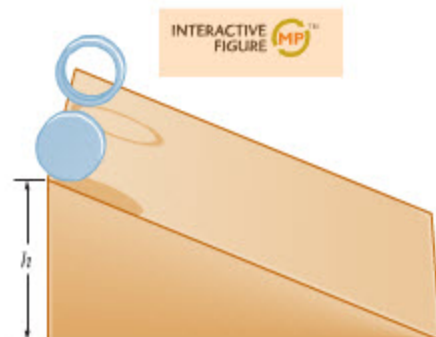
REASONING AND DISCUSSION

As we have just seen, the larger the moment of inertia, I , the smaller the speed, v . Hence the object with the larger moment of inertia (the hoop in this case) loses the race to the bottom, because its speed is less than the speed of the disk at any given height.

Another way to think about this is to recall that both objects have the same mechanical energy to begin with, namely, mgh . For the hoop, more of this initial potential energy goes into rotational kinetic energy, since the hoop has the larger moment of inertia; therefore, less energy is left for translational motion. As a result, the hoop moves more slowly and loses the race.

ANSWER

(a) The disk wins the race by reaching the bottom before the hoop.



In the next Conceptual Checkpoint, we consider the effects of a surface that changes from nonslip to frictionless.

CONCEPTUAL CHECKPOINT 10-5 COMPARE HEIGHTS

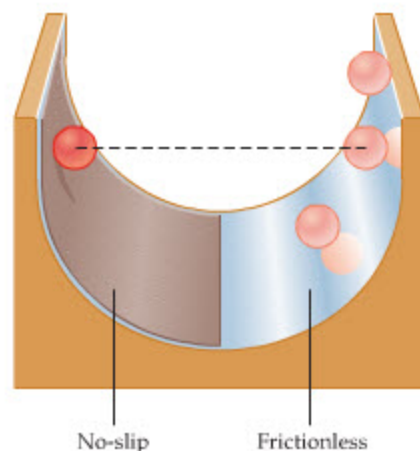
A ball is released from rest on a no-slip surface, as shown. After reaching its lowest point, the ball begins to rise again, this time on a frictionless surface. When the ball reaches its maximum height on the frictionless surface, is it **(a)** at a greater height, **(b)** at a lesser height, or **(c)** at the same height as when it was released?

REASONING AND DISCUSSION

As the ball descends on the no-slip surface, it begins to rotate, increasing its angular speed until it reaches the lowest point of the surface. When it begins to rise again, there is no friction to slow the rotational motion; thus, the ball continues to rotate with the same angular speed it had at its lowest point. Therefore, some of the ball's initial gravitational potential energy remains in the form of rotational kinetic energy. As a result, less energy is available to be converted back into gravitational potential energy, and the height is less.

ANSWER

(b) The height on the frictionless side is less.



We can also apply energy conservation to the case of a pulley, or similar object, with a string that winds or unwinds without slipping. In such cases, the relation $v = r\omega$ is valid and we can follow the same methods applied to an object that rolls without slipping.

EXAMPLE 10-6 SPINNING WHEEL

A block of mass m is attached to a string that is wrapped around the circumference of a wheel of radius R and moment of inertia I . The wheel rotates freely about its axis and the string wraps around its circumference without slipping. Initially the wheel rotates with an angular speed ω , causing the block to rise with a linear speed v . To what height does the block rise before coming to rest? Give a symbolic answer.

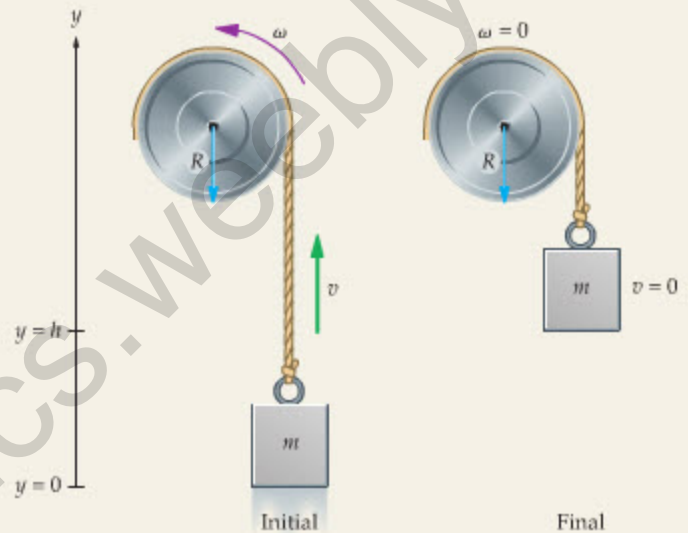
PICTURE THE PROBLEM

Note in our sketch that we choose the origin of the y axis to be at the initial height of the block. The positive y direction, as usual, is chosen to be upward. When the block comes to rest, then, it is at the height $y = h > 0$, where h is to be determined from the initial speed of the block and the properties of the wheel.

STRATEGY

The problem statement gives two key pieces of information. First, the string wraps onto the disk without slipping; therefore, $v = R\omega$. Second, the wheel rotates freely, which means that the mechanical energy of the system is conserved. Thus, at the height h the initial kinetic energy of the system has been converted to gravitational potential energy. This condition can be used to find h .

Before we continue, note that the mechanical energy of the system includes the following contributions: (i) linear kinetic energy for the block, (ii) rotational kinetic energy for the wheel, and (iii) gravitational potential energy for the block. We do not include the gravitational potential energy of the wheel because its height does not change.



SOLUTION

- Write an expression for the initial mechanical energy of the system, E_i , including all three contributions mentioned in the Strategy:
- Write an expression for the final mechanical energy of the system, E_f :
- Set the initial and final mechanical energies equal to one another, $E_i = E_f$:
- Solve for the height, h :

$$E_i = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 + mgy$$

$$= \frac{1}{2}mv^2 + \frac{1}{2}I\left(\frac{v}{R}\right)^2 + 0$$

$$E_f = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 + mgy$$

$$= 0 + 0 + mgh$$

$$E_i = \frac{1}{2}mv^2 + \frac{1}{2}I\left(\frac{v}{R}\right)^2 = \frac{1}{2}mv^2\left(1 + \frac{I}{mR^2}\right)$$

$$= E_f = mgh$$

$$h = \left(\frac{v^2}{2g}\right)\left(1 + \frac{I}{mR^2}\right)$$

INSIGHT

If the block were moving upward with speed v on its own—not attached to anything—it would rise to the height $h = v^2/2g$. We recover this result if $I = 0$, since in that case it is as if the wheel were not there. If the wheel is there, and I is nonzero, the block rises to a height that is *greater* than $v^2/2g$. The reason is that the wheel has kinetic energy, in addition to the kinetic energy of the block, and the sum of these kinetic energies must be converted to gravitational potential energy before the block and the wheel stop moving.

PRACTICE PROBLEM

Suppose the wheel is a disk with a mass equal to the mass m of the block. Find an expression for the height h in this case. [Answer: The moment of inertia of the wheel is $I = \frac{1}{2}mR^2$. Therefore, $h = (3/2)(v^2/2g)$.]

Some related homework problems: Problem 66, Problem 70, Problem 73

The situation with a yo-yo is similar, as we see in the next Active Example.

ACTIVE EXAMPLE 10-3 FIND THE YO-YO'S SPEED

Yo-yo man releases a yo-yo from rest and allows it to drop, as he keeps the top end of the string stationary. The mass of the yo-yo is 0.056 kg, its moment of inertia is $2.9 \times 10^{-5} \text{ kg} \cdot \text{m}^2$, and the radius, r , of the axle the string wraps around is 0.0064 m. What is the linear speed, v , of the yo-yo after it has dropped through a height $h = 0.50 \text{ m}$?

SOLUTION (Test your understanding by performing the calculations indicated in each step.)

- | | |
|--|-------------------------------------|
| 1. Write the initial energy of the system: | $E_i = mgh$ |
| 2. Write the final energy of the system: | $E_f = \frac{1}{2}mv^2(1 + I/mr^2)$ |
| 3. Set $E_f = E_i$ and solve for v : | $v = \sqrt{2gh/(1 + I/mr^2)}$ |
| 4. Substitute numerical values: | $v = 0.85 \text{ m/s}$ |

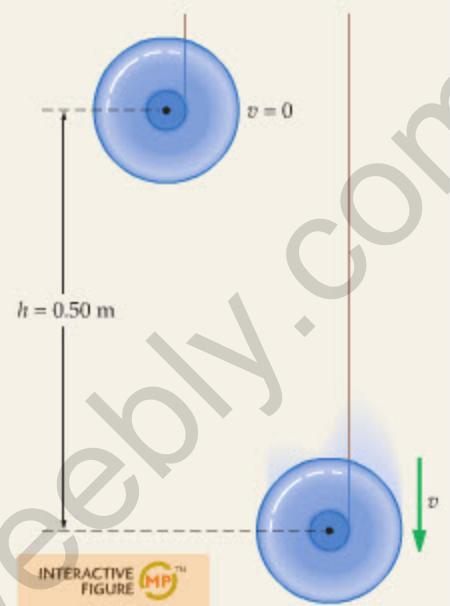
INSIGHT

The linear speed of the yo-yo is $v = r\omega$, where r is the radius of the axle from which the string unwraps without slipping. Therefore, the r in the term I/mr^2 is the radius of the axle. The outer radius of the yo-yo affects its moment of inertia, but since I is given to us in the problem statement, the outer radius is not pertinent.

YOUR TURN

If the yo-yo's moment of inertia is increased, does its final speed increase, decrease, or stay the same? Calculate the final speed for the case $I = 3.9 \times 10^{-5} \text{ kg} \cdot \text{m}^2$.

(Answers to **Your Turn** problems are given in the back of the book.)

**THE BIG PICTURE** PUTTING PHYSICS IN CONTEXT**LOOKING BACK**

Our definitions of position, velocity, and acceleration from Chapter 2 are generalized in Section 10-1 to apply to rotational motion. We then use the kinematics of Chapters 2 and 4 in Section 10-2 to relate these quantities. The basic equations of motion are the same; only the names have been changed.

The kinetic energy, first defined in Chapter 7, plays a key role in defining the moment of inertia in Section 10-5.

Conservation of energy (Chapter 8) is just as important in rotational motion as it is in linear motion. We apply it to rotational motion in Section 10-6.

LOOKING AHEAD

In Chapter 11 we relate force to angular acceleration, in much the same way that force and acceleration are related in linear motion. This results in the concept of torque in Section 11-1.

Just as linear speed is related to linear momentum (Chapter 9), angular speed is related to angular momentum. This is discussed in detail in Section 11-6.

Though a bit surprising at first, rotational motion is directly related to the motion of a pendulum swinging back and forth, and to the motion of a mass oscillating up and down on a spring. These connections are established in Section 13-3.

CHAPTER SUMMARY**10-1 ANGULAR POSITION, VELOCITY, AND ACCELERATION**

To describe rotational motion, rotational analogues of position, velocity, and acceleration are defined.

Angular Position

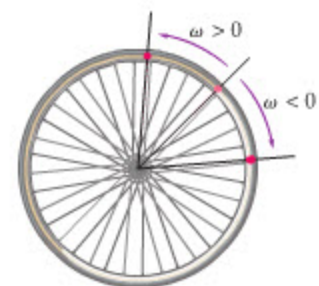
Angular position, θ , is the angle measured from an arbitrary reference line:

$$\theta \text{ (in radians)} = \text{arc length}/\text{radius} = s/r \quad 10-2$$

Angular Velocity

Angular velocity, ω , is the rate of change of angular position. The average angular velocity is

$$\omega_{av} = \frac{\Delta\theta}{\Delta t} \quad 10-3$$



The instantaneous angular velocity is the limit of ω_{av} as Δt approaches zero:

$$\omega = \lim_{\Delta t \rightarrow 0} \frac{\Delta\theta}{\Delta t} \quad 10-4$$

Angular Acceleration

Angular acceleration, α , is the rate of change of angular velocity. The average angular acceleration is

$$\alpha_{av} = \frac{\Delta\omega}{\Delta t} \quad 10-6$$

The instantaneous angular acceleration is the limit of α_{av} as Δt approaches zero:

$$\alpha = \lim_{\Delta t \rightarrow 0} \frac{\Delta\omega}{\Delta t} \quad 10-7$$

Period of Rotation

The period, T , is the time required to complete one full rotation. If the angular velocity is constant, T is related to ω as follows:

$$T = \frac{2\pi}{\omega} \quad 10-5$$

Sign Convention

Counterclockwise rotations are positive; clockwise rotations are negative.

10-2 ROTATIONAL KINEMATICS

Rotational kinematics is the description of angular motion, in the same way that linear kinematics describes linear motion. In both cases, we assume constant acceleration.

Linear–Angular Analogues

Rotational kinematics is related to linear kinematics by the following linear–angular analogies:

Linear Quantity	Angular Quantity
x	θ
v	ω
a	α

Kinematic Equations (Constant Acceleration)

The equations of rotational kinematics are the same as the equations of linear kinematics, with the substitutions indicated by the linear–angular analogies:

Linear Equation	Angular Equation
$v = v_0 + at$ 2-7	$\omega = \omega_0 + \alpha t$ 10-8
$x = x_0 + \frac{1}{2}(v_0 + v)t$ 2-10	$\theta = \theta_0 + \frac{1}{2}(\omega_0 + \omega)t$ 10-9
$x = x_0 + v_0t + \frac{1}{2}at^2$ 2-11	$\theta = \theta_0 + \omega_0t + \frac{1}{2}\alpha t^2$ 10-10
$v^2 = v_0^2 + 2a(x - x_0)$ 2-12	$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$ 10-11

10-3 CONNECTIONS BETWEEN LINEAR AND ROTATIONAL QUANTITIES

A point on a rotating object follows a circular path. At any instant of time, the point is moving in a direction tangential to the circle, with a linear speed and acceleration. The linear speed and acceleration are related to the angular speed and acceleration.

Tangential Speed

The tangential speed, v_t , of a point on a rotating object is

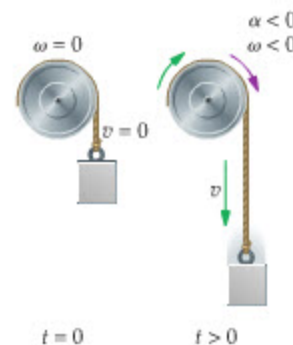
$$v_t = r\omega \quad 10-12$$

Centripetal Acceleration

The centripetal acceleration, a_{cp} , of a point on a rotating object is

$$a_{cp} = r\omega^2 \quad 10-13$$

Centripetal acceleration is due to a change in direction of motion.



Tangential Acceleration

The tangential acceleration, a_t , of a point on a rotating object is

$$a_t = r\alpha \quad 10-14$$

Tangential acceleration is due to a change in speed.

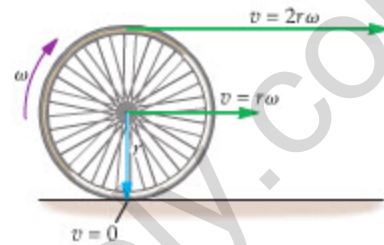
Total Acceleration

The total acceleration of a rotating object is the vector sum of its tangential and centripetal accelerations.

10-4 ROLLING MOTION

Rolling motion is a combination of translational and rotational motions. An object of radius r , rolling without slipping, translates with linear speed v and rotates with angular speed

$$\omega = v/r \quad 10-15$$

**10-5 ROTATIONAL KINETIC ENERGY AND THE MOMENT OF INERTIA**

Rotating objects have kinetic energy, just as objects in linear motion have kinetic energy.

Rotational Kinetic Energy

The kinetic energy of a rotating object is

$$K = \frac{1}{2}I\omega^2 \quad 10-17$$

The quantity I is the moment of inertia.

Moment of Inertia, Discrete Masses

The moment of inertia, I , of a collection of masses, m_i , at distances r_i from the axis of rotation is

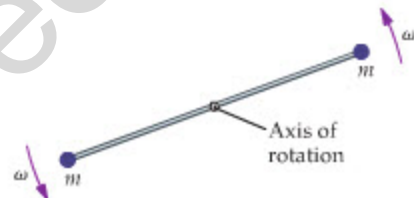
$$I = \sum m_i r_i^2 \quad 10-18$$

Moment of Inertia, Continuous Distribution of Mass

In a continuous object, the moment of inertia is calculated by dividing the object into a collection of small mass elements and summing $m_i r_i^2$ for each element. Results for a variety of continuous objects are collected in [Table 10-1 on p. 314](#).

Linear–Angular Analogue

The moment of inertia is the rotational analogue to mass in linear systems. In particular, an object with a large moment of inertia is hard to start rotating and hard to stop rotating.

**10-6 CONSERVATION OF ENERGY**

Energy conservation can be applied to a variety of rotational systems in the same way that it is applied to translational systems.

Kinetic Energy of Rolling Motion

The kinetic energy of an object that rolls without slipping is

$$K = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 \quad 10-19$$

Since rolling without slipping implies that $\omega = v/r$, the kinetic energy can be written as follows:

$$K = \frac{1}{2}mv^2 + \frac{1}{2}I\left(\frac{v}{r}\right)^2 = \frac{1}{2}mv^2\left(1 + \frac{I}{mr^2}\right) \quad 10-20$$

Energy Conservation


Conservation of mechanical energy is a statement that the initial kinetic plus potential energy is equal to the final kinetic plus potential energy: $K_i + U_i = K_f + U_f$. By taking into account both rotational and translational kinetic energy, energy conservation can be applied in the same way as was done for linear systems.



PROBLEM-SOLVING SUMMARY

Type of Problem	Relevant Physical Concepts	Related Examples
Apply rotational kinematics with constant angular acceleration.	Rotational kinematics is completely analogous to the linear kinematics studied in Chapter 2. Angular problems are solved in the same way as the corresponding linear problems.	Example 10-1, Example 10-2 Active Example 10-1
Relate linear and angular motion.	Linear speed and angular speed are related by $v = r\omega$. Similarly, linear and angular accelerations are related by $a = r\alpha$. The centripetal acceleration of an object in circular motion is $a_{cp} = r\omega^2$.	Example 10-3 Active Example 10-2
Find the rotational kinetic energy of an object.	Rotational kinetic energy is given by $K = \frac{1}{2}I\omega^2$. The moment of inertia, I , plays the same role in rotational motion as the mass in linear motion.	Example 10-4, Example 10-5
Apply energy conservation to a rotational system.	To use energy conservation in a system with rotational motion, it is necessary to include the kinetic energy of rotation as one of the forms of energy.	Example 10-6 Active Example 10-3

CONCEPTUAL QUESTIONS

For instructor-assigned homework, go to www.masteringphysics.com 

(Answers to odd-numbered Conceptual Questions can be found in the back of the book.)

- A rigid object rotates about a fixed axis. Do all points on the object have the same angular speed? Do all points on the object have the same linear speed? Explain.
- Can you drive your car in such a way that your tangential acceleration is zero while at the same time your centripetal acceleration is nonzero? Give an example if your answer is yes, state why not if your answer is no.
- Can you drive your car in such a way that your tangential acceleration is nonzero while at the same time your centripetal acceleration is zero? Give an example if your answer is yes, state why not if your answer is no.
- The fact that the Earth rotates gives people in New York a linear speed of about 750 mi/h. Where should you stand on the Earth to have the smallest possible linear speed?
- At the local carnival you and a friend decide to take a ride on the Ferris wheel. As the wheel rotates with a constant angular speed, your friend poses the following questions: (a) Is my linear velocity constant? (b) Is my linear speed constant? (c) Is the magnitude of my centripetal acceleration constant? (d) Is the direction of my centripetal acceleration constant? What is your answer to each of these questions?
- Why should changing the axis of rotation of an object change its moment of inertia, given that its shape and mass remain the same?
- Give a common, everyday example for each of the following: (a) An object that has zero rotational kinetic energy but nonzero translational kinetic energy. (b) An object that has zero translational kinetic energy but nonzero rotational kinetic energy. (c) An object that has nonzero rotational and translational kinetic energies.
- Two spheres have identical radii and masses. How might you tell which of these spheres is hollow and which is solid?
- At the grocery store you pick up a can of beef broth and a can of chunky beef stew. The cans are identical in diameter and weight. Rolling both of them down the aisle with the same initial speed, you notice that the can of chunky stew rolls much farther than the can of broth. Why?
- Suppose we change the race shown in Conceptual Checkpoint 10-4 so that a hoop of radius R and mass M races a hoop of radius R and mass $2M$. (a) Does the hoop with mass M finish before, after, or at the same time as the hoop with mass $2M$? Explain. (b) How would your answer to part (a) change if the hoops had different radii? Explain.

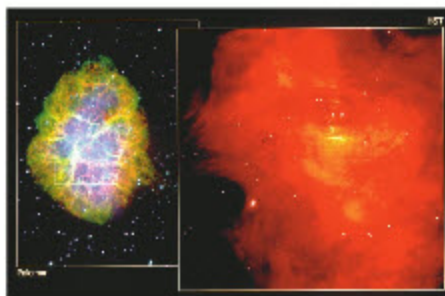
PROBLEMS AND CONCEPTUAL EXERCISES

Note: Answers to odd-numbered Problems and Conceptual Exercises can be found in the back of the book. **IP** denotes an integrated problem, with both conceptual and numerical parts; **BIO** identifies problems of biological or medical interest; **CE** indicates a conceptual exercise. **Predict/Explain** problems ask for two responses: (a) your prediction of a physical outcome, and (b) the best explanation among three provided. On all problems, red bullets (\bullet , $\bullet\bullet$, $\bullet\bullet\bullet$) are used to indicate the level of difficulty.

SECTION 10-1 ANGULAR POSITION, VELOCITY, AND ACCELERATION

- The following angles are given in degrees. Convert them to radians: 30° , 45° , 90° , 180° .
- The following angles are given in radians. Convert them to degrees: $\pi/6$, 0.70π , 1.5π , 5π .
- Find the angular speed of (a) the minute hand and (b) the hour hand of the famous clock in London, England, that rings the bell known as Big Ben.
- Express the angular velocity of the second hand on a clock in the following units: (a) rev/hr, (b) deg/min, and (c) rad/s.
- Rank the following in order of increasing angular speed: an automobile tire rotating at 2.00×10^3 deg/s, an electric drill rotating at 400.0 rev/min, and an airplane propeller rotating at 40.0 rad/s.
- A spot of paint on a bicycle tire moves in a circular path of radius 0.33 m. When the spot has traveled a linear distance of 1.95 m, through what angle has the tire rotated? Give your answer in radians.

7. • What is the angular speed (in rev/min) of the Earth as it orbits about the Sun?
8. • Find the angular speed of the Earth as it spins about its axis. Give your result in rad/s.
9. • **The Crab Nebula** One of the most studied objects in the night sky is the Crab nebula, the remains of a supernova explosion observed by the Chinese in 1054. In 1968 it was discovered that a pulsar—a rapidly rotating neutron star that emits a pulse of radio waves with each revolution—lies near the center of the Crab nebula. The period of this pulsar is 33 ms. What is the angular speed (in rad/s) of the Crab nebula pulsar?



The photo at left is a true-color visible light image of the Crab nebula. In the false-color breakout, the pulsar can be seen as the left member of the pair of stars just above the center of the frame. (Problems 9 and 106)

10. •• **IP** A 3.5-inch floppy disk in a computer rotates with a period of 2.00×10^{-1} s. What are (a) the angular speed of the disk and (b) the linear speed of a point on the rim of the disk? (c) Does a point near the center of the disk have an angular speed that is greater than, less than, or the same as the angular speed found in part (a)? Explain. (Note: A 3.5-inch floppy disk is 3.5 inches in diameter.)
11. •• The angle an airplane propeller makes with the horizontal as a function of time is given by $\theta = (125 \text{ rad/s})t + (42.5 \text{ rad/s}^2)t^2$. (a) Estimate the instantaneous angular velocity at $t = 0.00$ s by calculating the average angular velocity from $t = 0.00$ s to $t = 0.010$ s. (b) Estimate the instantaneous angular velocity at $t = 1.000$ s by calculating the average angular velocity from $t = 1.000$ s to $t = 1.010$ s. (c) Estimate the instantaneous angular velocity at $t = 2.000$ s by calculating the average angular velocity from $t = 2.000$ s to $t = 2.010$ s. (d) Based on your results from parts (a), (b), and (c), is the angular acceleration of the propeller positive, negative, or zero? Explain. (e) Calculate the average angular acceleration from $t = 0.00$ s to $t = 1.00$ s and from $t = 1.00$ s to $t = 2.00$ s.

SECTION 10-2 ROTATIONAL KINEMATICS

12. • **CE** An object at rest begins to rotate with a constant angular acceleration. If this object rotates through an angle θ in the time t , through what angle did it rotate in the time $t/2$?
13. • **CE** An object at rest begins to rotate with a constant angular acceleration. If the angular speed of the object is ω after the time t , what was its angular speed at the time $t/2$?
14. • In **Active Example 10-1**, how long does it take before the angular velocity of the pulley is equal to -5.0 rad/s?
15. • In **Example 10-2**, through what angle has the wheel turned when its angular speed is 2.45 rad/s?
16. • The angular speed of a propeller on a boat increases with constant acceleration from 12 rad/s to 26 rad/s in 2.5 revolutions. What is the acceleration of the propeller?

17. • The angular speed of a propeller on a boat increases with constant acceleration from 11 rad/s to 28 rad/s in 2.4 seconds. Through what angle did the propeller turn during this time?
18. •• After fixing a flat tire on a bicycle you give the wheel a spin. (a) If its initial angular speed was 6.35 rad/s and it rotated 14.2 revolutions before coming to rest, what was its average angular acceleration? (b) For what length of time did the wheel rotate?
19. •• **IP** A ceiling fan is rotating at 0.96 rev/s. When turned off, it slows uniformly to a stop in 2.4 min. (a) How many revolutions does the fan make in this time? (b) Using the result from part (a), find the number of revolutions the fan must make for its speed to decrease from 0.96 rev/s to 0.48 rev/s.
20. •• A discus thrower starts from rest and begins to rotate with a constant angular acceleration of 2.2 rad/s². (a) How many revolutions does it take for the discus thrower's angular speed to reach 6.3 rad/s? (b) How much time does this take?
21. •• **Half Time** At 3:00 the hour hand and the minute hand of a clock point in directions that are 90.0° apart. What is the first time after 3:00 that the angle between the two hands has decreased by half to 45.0° ?



When the little hand is on the 3 and the big hand is on the 12 . . . (Problem 21)

22. •• **BIO** A centrifuge is a common laboratory instrument that separates components of differing densities in solution. This is accomplished by spinning a sample around in a circle with a large angular speed. Suppose that after a centrifuge in a medical laboratory is turned off, it continues to rotate with a constant angular deceleration for 10.2 s before coming to rest. (a) If its initial angular speed was 3850 rpm, what is the magnitude of its angular deceleration? (b) How many revolutions did the centrifuge complete after being turned off?
23. •• **The Slowing Earth** The Earth's rate of rotation is constantly decreasing, causing the day to increase in duration. In the year 2006 the Earth took about 0.840 s longer to complete 365 revolutions than it did in the year 1906. What was the average angular acceleration of the Earth during this time? Give your answer in rad/s².
24. •• **IP** A compact disk (CD) speeds up uniformly from rest to 310 rpm in 3.3 s. (a) Describe a strategy that allows you to calculate the number of revolutions the CD makes in this time. (b) Use your strategy to find the number of revolutions.
25. •• When a carpenter shuts off his circular saw, the 10.0 -inch-diameter blade slows from 4440 rpm to 0.00 rpm in 2.50 s. (a) What is the angular acceleration of the blade? (b) What is the distance traveled by a point on the rim of the blade during the deceleration? (c) What is the magnitude of the net displacement of a point on the rim of the blade during the deceleration?
26. •• **The World's Fastest Turbine** The drill used by most dentists today is powered by a small air turbine that can operate at angular speeds of $350,000$ rpm. These drills, along with ultrasonic dental drills, are the fastest turbines in the world—far exceeding the angular speeds of jet engines. Suppose a drill starts at rest and comes up to operating speed in 2.1 s. (a) Find the angular accel-

ation produced by the drill, assuming it to be constant. (b) How many revolutions does the drill bit make as it comes up to speed?

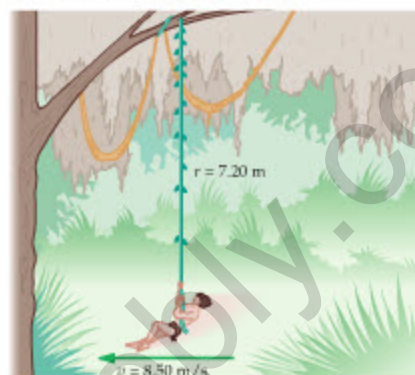


An air-turbine dentist drill—faster than a jet engine. (Problem 26)

SECTION 10-3 CONNECTIONS BETWEEN LINEAR AND ROTATIONAL QUANTITIES

27. • **CE Predict/Explain** Two children, Jason and Betsy, ride on the same merry-go-round. Jason is a distance R from the axis of rotation; Betsy is a distance $2R$ from the axis. Is the rotational period of Jason greater than, less than, or equal to the rotational period of Betsy? (b) Choose the *best explanation* from among the following:
- The period is greater for Jason because he moves more slowly than Betsy.
 - The period is greater for Betsy since she must go around a circle with a larger circumference.
 - It takes the same amount of time for the merry-go-round to complete a revolution for all points on the merry-go-round.
28. • **CE** Referring to the previous problem, what are (a) the ratio of Jason's angular speed to Betsy's angular speed, (b) the ratio of Jason's linear speed to Betsy's linear speed, and (c) the ratio of Jason's centripetal acceleration to Betsy's centripetal acceleration?
29. • **CE Predict/Explain A Tall Building** The world's tallest building is the Taipei 101 Tower in Taiwan, which rises to a height of 508 m (1667 ft). (a) When standing on the top floor of the building, is your angular speed due to the Earth's rotation greater than, less than, or equal to your angular speed when you stand on the ground floor? (b) Choose the *best explanation* from among the following:
- The angular speed is the same at all distances from the axis of rotation.
 - At the top of the building you are farther from the axis of rotation and hence you have a greater angular speed.
 - You are spinning faster when you are closer to the axis of rotation.
30. • The hour hand on a certain clock is 8.2 cm long. Find the tangential speed of the tip of this hand.
31. • Two children ride on the merry-go-round shown in Conceptual Checkpoint 10-1. Child 1 is 2.0 m from the axis of rotation, and child 2 is 1.5 m from the axis. If the merry-go-round completes one revolution every 4.5 s, find (a) the angular speed and (b) the linear speed of each child.
32. • The outer edge of a rotating Frisbee with a diameter of 29 cm has a linear speed of 3.7 m/s. What is the angular speed of the Frisbee?
33. • A carousel at the local carnival rotates once every 45 seconds. (a) What is the linear speed of an outer horse on the carousel, which is 2.75 m from the axis of rotation? (b) What is the linear speed of an inner horse that is 1.75 m from the axis of rotation?

34. •• **IP** Jeff of the Jungle swings on a vine that is 7.20 m long (Figure 10-18). At the bottom of the swing, just before hitting the tree, Jeff's linear speed is 8.50 m/s. (a) Find Jeff's angular speed at this time. (b) What centripetal acceleration does Jeff experience at the bottom of his swing? (c) What exerts the force that is responsible for Jeff's centripetal acceleration?



▲ FIGURE 10-18 Problems 34 and 35

35. •• Suppose, in Problem 34, that at some point in his swing Jeff of the Jungle has an angular speed of 0.850 rad/s and an angular acceleration of 0.620 rad/s^2 . Find the magnitude of his centripetal, tangential, and total accelerations, and the angle his total acceleration makes with respect to the tangential direction of motion.
36. •• A compact disk, which has a diameter of 12.0 cm, speeds up uniformly from 0.00 to 4.00 rev/s in 3.00 s. What is the tangential acceleration of a point on the outer rim of the disk at the moment when its angular speed is (a) 2.00 rev/s and (b) 3.00 rev/s?
37. •• **IP** When a compact disk with a 12.0-cm diameter is rotating at 5.05 rad/s, what are (a) the linear speed and (b) the centripetal acceleration of a point on its outer rim? (c) Consider a point on the CD that is halfway between its center and its outer rim. Without repeating all of the calculations required for parts (a) and (b), determine the linear speed and the centripetal acceleration of this point.
38. •• **IP** As Tony the fisherman reels in a "big one," he turns the spool on his fishing reel at the rate of 3.0 complete revolutions every second (Figure 10-19). (a) If the radius of the reel is 3.7 cm, what is the linear speed of the fishing line as it is reeled in? (b) How would your answer to part (a) change if the radius of the reel were doubled?



▲ FIGURE 10-19 Problem 38

39. •• A Ferris wheel with a radius of 9.5 m rotates at a constant rate, completing one revolution every 36 s. Find the direction and magnitude of a passenger's acceleration when (a) at the top and (b) at the bottom of the wheel.

40. •• Suppose the Ferris wheel in the previous problem begins to decelerate at the rate of 0.22 rad/s^2 when the passenger is at the top of the wheel. Find the direction and magnitude of the passenger's acceleration at that time.
41. •• **IP** A person swings a 0.52-kg tether ball tied to a 4.5-m rope in an approximately horizontal circle. (a) If the maximum tension the rope can withstand before breaking is 11 N , what is the maximum angular speed of the ball? (b) If the rope is shortened, does the maximum angular speed found in part (a) increase, decrease, or stay the same? Explain.
42. •• To polish a filling, a dentist attaches a sanding disk with a radius of 3.20 mm to the drill. (a) When the drill is operated at $2.15 \times 10^4 \text{ rad/s}$, what is the tangential speed of the rim of the disk? (b) What period of rotation must the disk have if the tangential speed of its rim is to be 275 m/s ?
43. •• In the previous problem, suppose the disk has an angular acceleration of 232 rad/s^2 when its angular speed is 640 rad/s . Find both the tangential and centripetal accelerations of a point on the rim of the disk.
44. •• **The Bohr Atom** The Bohr model of the hydrogen atom pictures the electron as a tiny particle moving in a circular orbit about a stationary proton. In the lowest-energy orbit the distance from the proton to the electron is $5.29 \times 10^{-11} \text{ m}$, and the linear speed of the electron is $2.18 \times 10^6 \text{ m/s}$. (a) What is the angular speed of the electron? (b) How many orbits about the proton does it make each second? (c) What is the electron's centripetal acceleration?
45. ••• A wheel of radius R starts from rest and accelerates with a constant angular acceleration α about a fixed axis. At what time t will the centripetal and tangential accelerations of a point on the rim have the same magnitude?

SECTION 10-4 ROLLING MOTION

46. • **CE** As you drive down the highway, the top of your tires are moving with a speed v . What is the reading on your speedometer?
47. •• The tires on a car have a radius of 31 cm . What is the angular speed of these tires when the car is driven at 15 m/s ?
48. • A child pedals a tricycle, giving the driving wheel an angular speed of 0.373 rev/s (Figure 10-20). If the radius of the wheel is 0.260 m , what is the child's linear speed?



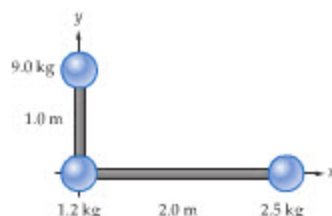
▲ FIGURE 10-20 Problem 48

49. • A soccer ball, which has a circumference of 70.0 cm , rolls 14.0 yards in 3.35 s . What was the average angular speed of the ball during this time?
50. •• As you drive down the road at 17 m/s , you press on the gas pedal and speed up with a uniform acceleration of 1.12 m/s^2 for 0.65 s . If the tires on your car have a radius of 33 cm , what is their angular displacement during this period of acceleration?

51. •• **IP** A bicycle coasts downhill and accelerates from rest to a linear speed of 8.90 m/s in 12.2 s . (a) If the bicycle's tires have a radius of 36.0 cm , what is their angular acceleration? (b) If the radius of the tires had been smaller, would their angular acceleration be greater than or less than the result found in part (a)?

SECTION 10-5 ROTATIONAL KINETIC ENERGY AND THE MOMENT OF INERTIA

52. • **CE Predict/Explain** The minute and hour hands of a clock have a common axis of rotation and equal mass. The minute hand is long, thin, and uniform; the hour hand is short, thick, and uniform. (a) Is the moment of inertia of the minute hand greater than, less than, or equal to the moment of inertia of the hour hand? (b) Choose the *best explanation* from among the following:
- The hands have equal mass, and hence equal moments of inertia.
 - Having mass farther from the axis of rotation results in a greater moment of inertia.
 - The more compact hour hand concentrates its mass and has the greater moment of inertia.
53. • **CE Predict/Explain** Tons of dust and small particles rain down onto the Earth from space every day. As a result, does the Earth's moment of inertia increase, decrease, or stay the same? (b) Choose the *best explanation* from among the following:
- The dust adds mass to the Earth and increases its radius slightly.
 - As the dust moves closer to the axis of rotation, the moment of inertia decreases.
 - The moment of inertia is a conserved quantity and cannot change.
54. • **CE • Predict/Explain** Suppose a bicycle wheel is rotated about an axis through its rim and parallel to its axle. (a) Is its moment of inertia about this axis greater than, less than, or equal to its moment of inertia about its axle? (b) Choose the *best explanation* from among the following:
- The moment of inertia is greatest when an object is rotated about its center.
 - The mass and shape of the wheel remain the same.
 - Mass is farther from the axis when the wheel is rotated about the rim.
55. • The moment of inertia of a 0.98-kg bicycle wheel rotating about its center is $0.13 \text{ kg} \cdot \text{m}^2$. What is the radius of this wheel, assuming the weight of the spokes can be ignored?
56. • What is the kinetic energy of the grindstone in Example 10-4 if it completes one revolution every 4.20 s ?
57. • An electric fan spinning with an angular speed of 13 rad/s has a kinetic energy of 4.6 J . What is the moment of inertia of the fan?
58. • Repeat Example 10-5 for the case of a rolling hoop of the same mass and radius.
59. •• **CE** The L-shaped object in Figure 10-21 can be rotated in one of the following three ways: case 1, rotation about the x axis; case 2, rotation about the y axis; and case 3, rotation about the



▲ FIGURE 10-21 Problem 59

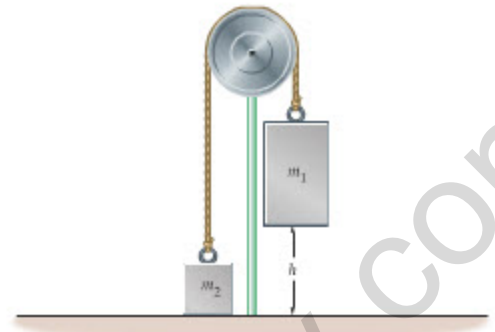
z axis (which passes through the origin perpendicular to the plane of the figure). Rank these three cases in order of increasing moment of inertia. Indicate ties where appropriate.

60. •• **IP** A 12-g CD with a radius of 6.0 cm rotates with an angular speed of 34 rad/s. (a) What is its kinetic energy? (b) What angular speed must the CD have if its kinetic energy is to be doubled?
61. •• When a pitcher throws a curve ball, the ball is given a fairly rapid spin. If a 0.15-kg baseball with a radius of 3.7 cm is thrown with a linear speed of 48 m/s and an angular speed of 42 rad/s, how much of its kinetic energy is translational and how much is rotational? Assume the ball is a uniform, solid sphere.
62. •• **IP** A basketball rolls along the floor with a constant linear speed v . (a) Find the fraction of its total kinetic energy that is in the form of rotational kinetic energy about the center of the ball. (b) If the linear speed of the ball is doubled to $2v$, does your answer to part (a) increase, decrease, or stay the same? Explain.
63. •• Find the rate at which the rotational kinetic energy of the Earth is decreasing. The Earth has a moment of inertia of $0.331 M_E R_E^2$, where $R_E = 6.38 \times 10^6$ m and $M_E = 5.97 \times 10^{24}$ kg, and its rotational period increases by 2.3 ms with each passing century. Give your answer in watts.
64. •• A lawn mower has a flat, rod-shaped steel blade that rotates about its center. The mass of the blade is 0.65 kg and its length is 0.55 m. (a) What is the rotational energy of the blade at its operating angular speed of 3500 rpm? (b) If all of the rotational kinetic energy of the blade could be converted to gravitational potential energy, to what height would the blade rise?

SECTION 10-6 CONSERVATION OF ENERGY

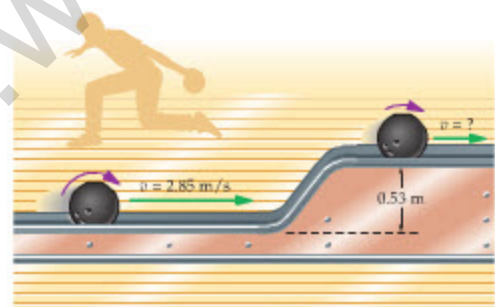
65. • **CE** Consider the physical situation shown in Conceptual Checkpoint 10-5. Suppose this time a ball is released from rest on the frictionless surface. When the ball comes to rest on the no-slip surface, is its height greater than, less than, or equal to the height from which it was released?
66. • Suppose the block in Example 10-6 has a mass of 2.1 kg and an initial upward speed of 0.33 m/s. Find the moment of inertia of the wheel if its radius is 8.0 cm and the block rises to a height of 7.4 cm before momentarily coming to rest.
67. • Through what height must the yo-yo in Active Example 10-3 fall for its linear speed to be 0.65 m/s?
68. •• **CE** Suppose we change the race shown in Conceptual Checkpoint 10-4 to a race between three different disks. Let disk 1 have a mass M and a radius R , disk 2 have a mass M and a radius $2R$, and disk 3 have a mass $2M$ and a radius R . Rank the three disks in the order in which they finish the race. Indicate ties where appropriate.
69. •• Calculate the speeds of (a) the disk and (b) the hoop at the bottom of the inclined plane in Conceptual Checkpoint 10-4 if the height of the incline is 0.82 m.
70. •• **IP Atwood's Machine** The two masses ($m_1 = 5.0$ kg and $m_2 = 3.0$ kg) in the Atwood's machine shown in Figure 10-22 are released from rest, with m_1 at a height of 0.75 m above the floor. When m_1 hits the ground its speed is 1.8 m/s. Assuming that the pulley is a uniform disk with a radius of 12 cm, (a) outline a strategy that allows you to find the mass of the pulley. (b) Implement the strategy given in part (a) and determine the pulley's mass.
71. •• In Conceptual Checkpoint 10-5, assume the ball is a solid sphere of radius 2.9 cm and mass 0.14 kg. If the ball is released from rest at a height of 0.78 m above the bottom of the track on the no-slip side, (a) what is its angular speed when it is on the

frictionless side of the track? (b) How high does the ball rise on the frictionless side?



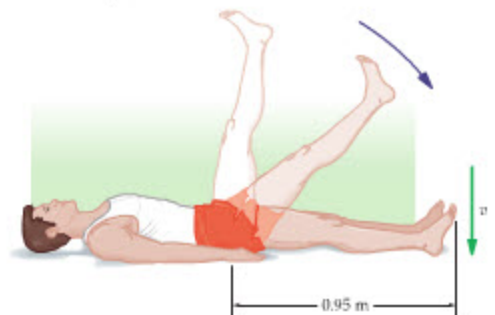
▲ FIGURE 10-22 Problem 70

72. •• **IP** After you pick up a spare, your bowling ball rolls without slipping back toward the ball rack with a linear speed of 2.85 m/s (Figure 10-23). To reach the rack, the ball rolls up a ramp that rises through a vertical distance of 0.53 m. (a) What is the linear speed of the ball when it reaches the top of the ramp? (b) If the radius of the ball were increased, would the speed found in part (a) increase, decrease, or stay the same? Explain.



▲ FIGURE 10-23 Problem 72

73. •• **IP** A 1.3-kg block is tied to a string that is wrapped around the rim of a pulley of radius 7.2 cm. The block is released from rest. (a) Assuming the pulley is a uniform disk with a mass of 0.31 kg, find the speed of the block after it has fallen through a height of 0.50 m. (b) If a small lead weight is attached near the rim of the pulley and this experiment is repeated, will the speed of the block increase, decrease, or stay the same? Explain.
74. •• After doing some exercises on the floor, you are lying on your back with one leg pointing straight up. If you allow your leg to fall freely until it hits the floor (Figure 10-24), what is the tangential speed of your foot just before it lands? Assume the leg can be treated as a uniform rod 0.95 m long that pivots freely about the hip.



▲ FIGURE 10-24 Problem 74

75. ••• A 2.0-kg solid cylinder (radius = 0.10 m, length = 0.50 m) is released from rest at the top of a ramp and allowed to roll without slipping. The ramp is 0.75 m high and 5.0 m long. When the cylinder reaches the bottom of the ramp, what are

(a) its total kinetic energy, (b) its rotational kinetic energy, and (c) its translational kinetic energy?

76. ••• A 2.5-kg solid sphere (radius = 0.10 m) is released from rest at the top of a ramp and allowed to roll without slipping. The ramp is 0.75 m high and 5.6 m long. When the sphere reaches the bottom of the ramp, what are (a) its total kinetic energy, (b) its rotational kinetic energy, and (c) its translational kinetic energy?

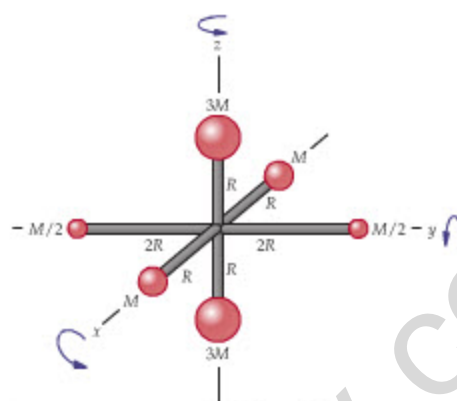
GENERAL PROBLEMS

77. • CE When you stand on the observation deck of the Empire State Building in New York, is your linear speed due to the Earth's rotation greater than, less than, or the same as when you were waiting for the elevators on the ground floor?
78. • CE **Hard-Boiled Versus Raw Eggs** One way to tell whether an egg is raw or hard boiled—without cracking it open—is to place it on a kitchen counter and give it a spin. If you do this to two eggs, one raw the other hard boiled, you will find that one spins considerably longer than the other. Is the raw egg the one that spins a long time, or the one that stops spinning in a short time?
79. • CE When the Hoover Dam was completed and the reservoir behind it filled with water, did the moment of inertia of the Earth increase, decrease, or stay the same?
80. • **Weightless on the Equator** In Quito, Ecuador, near the equator, you weigh about half a pound less than in Barrow, Alaska, near the pole. Find the rotational period of the Earth that would make you feel weightless at the equator. (With this rotational period, your centripetal acceleration would be equal to the acceleration due to gravity, g .)
81. • A diver completes $2\frac{1}{2}$ somersaults during a 2.3-s dive. What was the diver's average angular speed during the dive?
82. • What linear speed must a 0.065-kg hula hoop have if its total kinetic energy is to be 0.12 J? Assume the hoop rolls on the ground without slipping.
83. • **BIO Losing Consciousness** A pilot performing a horizontal turn will lose consciousness if she experiences a centripetal acceleration greater than 7.00 times the acceleration of gravity. What is the minimum radius turn she can make without losing consciousness if her plane is flying with a constant speed of 245 m/s?
84. •• CE Place two quarters on a table with their rims touching, as shown in Figure 10-25. While holding one quarter fixed, roll the other one—without slipping—around the circumference of the fixed quarter until it has completed one round trip. How many revolutions has the rolling quarter made about its center?



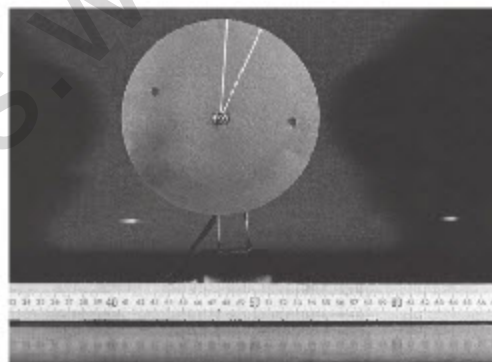
▲ FIGURE 10-25 Problem 84

85. • CE The object shown in Figure 10-26 can be rotated in three different ways: case 1, rotation about the x axis; case 2, rotation about the y axis; and case 3, rotation about the z axis. Rank these three cases in order of increasing moment of inertia. Indicate ties where appropriate.



▲ FIGURE 10-26 Problem 85

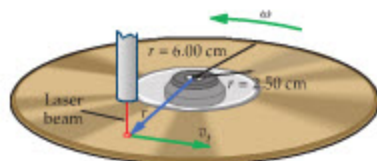
86. •• The accompanying double-exposure photograph illustrates a method for determining the speed of a BB. The circular disk in the upper part of the photo rotates with a constant angular speed of 50.4 revolutions per second. A single white radial line drawn on the disk is seen in two locations in the double exposure. Below the disk are two bright images of a BB taken during the two exposures. Use the information given here and in the photo to estimate the speed of the BB.



Speeding BB and spinning wheel. (Problems 86 and 87)

87. •• Referring to the previous problem, (a) estimate the linear speed of a point on the rim of the rotating disk. (b) By comparing the arc length between the two white lines to the distance covered by the BB, estimate the speed of the BB. (c) What radius must the disk have for the linear speed of a point on its rim to be the same as the speed of the BB? (d) Suppose a 1.0-g lump of putty is stuck to the rim of the disk. What centripetal force is required to hold the putty in place?
88. •• **IP When the Hands Align** A mathematically inclined friend e-mails you the following instructions: "Meet me in the cafeteria the first time after 2:00 P.M. today that the hands of a clock point in the same direction." (a) Is the desired meeting time before, after, or equal to 2:10 P.M.? Explain. (b) Is the desired meeting time before, after, or equal to 2:15 P.M.? Explain. (c) When should you meet your friend?
89. •• **IP** A diver runs horizontally off the end of a diving tower 3.0 m above the surface of the water with an initial speed of 2.6 m/s. During her fall she rotates with an average angular speed of 2.2 rad/s. (a) How many revolutions has she made when she hits the water? (b) How does your answer to part (a) depend on the diver's initial speed? Explain.
90. •• **IP** A potter's wheel of radius 6.8 cm rotates with a period of 0.52 s. What are (a) the linear speed and (b) the centripetal acceleration of a small lump of clay on the rim of the wheel? (c) How do your answers to parts (a) and (b) change if the period of rotation is doubled?

91. •• **IP** **Playing a CD** The record in an old-fashioned record player always rotates at the same angular speed. With CDs, the situation is different. For a CD to play properly, the point on the CD where the laser beam shines must have a linear speed $v_t = 1.25$ m/s, as indicated in **Figure 10-27**. (a) As the CD plays from the center outward, does its angular speed increase, decrease, or stay the same? Explain. (b) Find the angular speed of a CD when the laser beam is 2.50 cm from its center. (c) Repeat part (b) for the laser beam 6.00 cm from the center. (d) If the CD plays for 66.5 min, and the laser beam moves from 2.50 cm to 6.00 cm during this time, what is the CD's average angular acceleration?



▲ **FIGURE 10-27** Problem 91

92. •• **BIO** **Roller Pigeons** Pigeons are bred to display a number of interesting characteristics. One breed of pigeon, the "roller," is remarkable for the fact that it does a number of backward somersaults as it drops straight down toward the ground. Suppose a roller pigeon drops from rest and free falls downward for a distance of 14 m. If the pigeon somersaults at the rate of 12 rad/s, how many revolutions has it completed by the end of its fall?
93. •• As a marble with a diameter of 1.6 cm rolls down an incline, its center moves with a linear acceleration of 3.3 m/s². (a) What is the angular acceleration of the marble? (b) What is the angular speed of the marble after it rolls for 1.5 s from rest?
94. •• A rubber ball with a radius of 3.2 cm rolls along the horizontal surface of a table with a constant linear speed v . When the ball rolls off the edge of the table, it falls 0.66 m to the floor below. If the ball completes 0.37 revolution during its fall, what was its linear speed, v ?
95. •• A college campus features a large fountain surrounded by a circular pool. Two students start at the northernmost point of the pool and walk slowly around it in opposite directions. (a) If the angular speed of the student walking in the clockwise direction (as viewed from above) is 0.045 rad/s and the angular speed of the other student is 0.023 rad/s, how long does it take before they meet? (b) At what angle, measured clockwise from due north, do the students meet? (c) If the difference in linear speed between the students is 0.23 m/s, what is the radius of the fountain?
96. •• **IP** A yo-yo moves downward until it reaches the end of its string, where it "sleeps." As it sleeps—that is, spins in place—its angular speed decreases from 35 rad/s to 25 rad/s. During this time it completes 120 revolutions. (a) How long did it take for the yo-yo to slow from 35 rad/s to 25 rad/s? (b) How long does it take for the yo-yo to slow from 25 rad/s to 15 rad/s? Assume a constant angular acceleration as the yo-yo sleeps.
97. •• **IP** (a) An automobile with tires of radius 32 cm accelerates from 0 to 45 mph in 9.1 s. Find the angular acceleration of the tires. (b) How does your answer to part (a) change if the radius of the tires is halved?
98. •• **IP** In Problems 75 and 76 we considered a cylinder and a solid sphere, respectively, rolling down a ramp. (a) Which object do you expect to have the greater speed at the bottom of the ramp? (b) Verify your answer to part (a) by calculating the speed of the cylinder and of the sphere when they reach the bottom of the ramp.

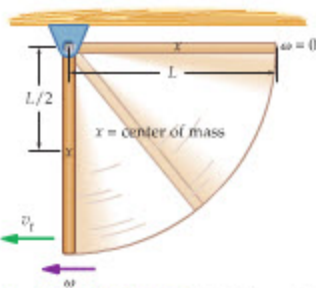
99. •• A centrifuge (Problem 22) with an angular speed of 6050 rpm produces a maximum centripetal acceleration equal to 6840g (that is, 6840 times the acceleration of gravity). (a) What is the diameter of this centrifuge? (b) What force must the bottom of the sample holder exert on a 15.0-g sample under these conditions?
100. •• **A Yo-Yo with a Brain** Yomega ("The yo-yo with a brain") is constructed with a clever clutch mechanism in its axle that allows it to rotate freely and "sleep" when its angular speed is greater than a certain critical value. When the yo-yo's angular speed falls below this value, the clutch engages, causing the yo-yo to climb the string to the user's hand. If the moment of inertia of the yo-yo is 7.4×10^{-5} kg·m², its mass is 0.11 kg, and the string is 1.0 m long, what is the smallest angular speed that will allow the yo-yo to return to the user's hand?



A brain or just a clutch?
(Problem 100)

101. •• The rotor in a centrifuge has an initial angular speed of 430 rad/s. After 8.2 s of constant angular acceleration, its angular speed has increased to 550 rad/s. During this time, what were (a) the angular acceleration of the rotor and (b) the angle through which it turned?
102. •• **BIO** A honey bee has two pairs of wings that can beat 250 times a second. Estimate (a) the maximum angular speed of the wings and (b) the maximum linear speed of a wing tip.
103. •• The Sun, with Earth in tow, orbits about the center of the Milky Way galaxy at a speed of 137 miles per second, completing one revolution every 240 million years. (a) Find the angular speed of the Sun relative to the center of the Milky Way. (b) Find the distance from the Sun to the center of the Milky Way.
104. •• A person walks into a room and switches on the ceiling fan. The fan accelerates with constant angular acceleration for 15 s until it reaches its operating angular speed of 1.9 rotations/s—after that its speed remains constant as long as the switch is "on." The person stays in the room for a short time; then, 5.5 minutes after turning the fan on, she switches it off again and leaves the room. The fan now decelerates with constant angular acceleration, taking 2.4 minutes to come to rest. What is the total number of revolutions made by the fan, from the time it was turned on until the time it stopped?
105. •• **BIO** **Preventing Bone Loss in Space** When astronauts return from prolonged space flights, they often suffer from bone loss, resulting in brittle bones that may take weeks for their bodies to rebuild. One solution may be to expose astronauts to periods of substantial "g forces" in a centrifuge carried aboard their spaceship. To test this approach, NASA conducted a study in which four people spent 22 hours each in a compartment attached to the end of a 28-foot arm that rotated with an angular speed of 10.0 rpm. (a) What centripetal acceleration did these volunteers experience? Express your answer in terms of g. (b) What was their linear speed?

106. ••• **Angular Acceleration of the Crab Nebula** The pulsar in the Crab nebula (Problem 9) was created by a supernova explosion that was observed on Earth in A.D. 1054. Its current period of rotation (33.0 ms) is observed to be increasing by 1.26×10^{-5} seconds per year. (a) What is the angular acceleration of the pulsar in rad/s^2 ? (b) Assuming the angular acceleration of the pulsar to be constant, how many years will it take for the pulsar to slow to a stop? (c) Under the same assumption, what was the period of the pulsar when it was created?
107. ••• A thin, uniform rod of length L and mass M is pivoted about one end, as shown in Figure 10–28. The rod is released from rest in a horizontal position, and allowed to swing downward without friction or air resistance. When the rod is vertical, what are (a) its angular speed ω and (b) the tangential speed v_t of its free end?

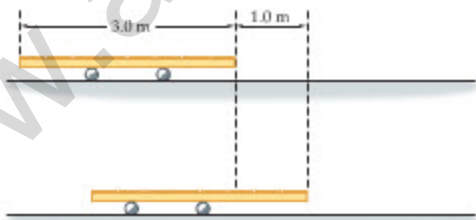


▲ FIGURE 10–28 Problem 107

108. ••• **Center of Percussion** In the previous problem, suppose a small metal ball of mass $m = 2M$ is attached to the rod a distance d from the pivot. The rod and ball are released from rest in the horizontal position. (a) Show that when the rod reaches the vertical position, the speed of its tip is

$$v_t = \sqrt{3gL} \sqrt{\frac{1 + 4(d/L)}{1 + 6(d/L)^2}}$$

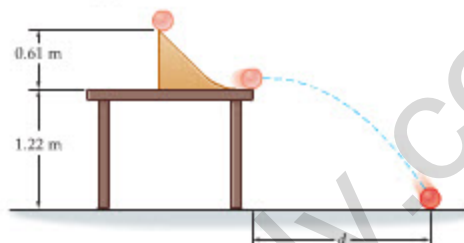
- (b) At what finite value of d/L is the speed of the rod the same as it is for $d = 0$? (This value of d/L is the **center of percussion**, or “sweet spot,” of the rod.)
109. ••• A wooden plank rests on two soup cans laid on their sides. Each can has a diameter of 6.5 cm, and the plank is 3.0 m long. Initially, one can is placed 1.0 m inward from either end of the plank, as Figure 10–29 shows. The plank is now pulled 1.0 m to the right, and the cans roll without slipping. (a) How far does the center of each can move? (b) How many rotations does each can make?



▲ FIGURE 10–29 Problem 109

110. ••• A person rides on a 12-m-diameter Ferris wheel that rotates at the constant rate of 8.1 rpm. Calculate the magnitude and direction of the force that the seat exerts on a 65-kg person when he is (a) at the top of the wheel, (b) at the bottom of the wheel, and (c) halfway up the wheel.
111. ••• **IP** A solid sphere with a diameter of 0.17 m is released from rest; it then rolls without slipping down a ramp, dropping

through a vertical height of 0.61 m. The ball leaves the bottom of the ramp, which is 1.22 m above the floor, moving horizontally (Figure 10–30). (a) Through what horizontal distance d does the ball move before landing? (b) How many revolutions does the ball make during its fall? (c) If the ramp were to be made frictionless, would the distance d increase, decrease, or stay the same? Explain.



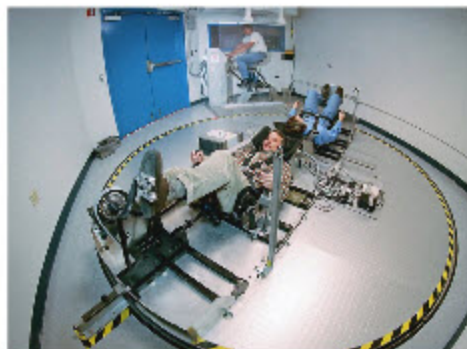
▲ FIGURE 10–30 Problem 111

PASSAGE PROBLEMS

BIO Human-Powered Centrifuge

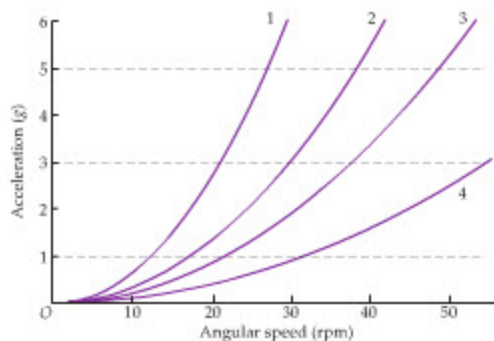
Space travel is fraught with hazards, not the least of which are the many side effects of prolonged weightlessness, including weakened muscles, bone loss, decreased coordination, and unsteady balance. If you are fortunate enough to go on a trip to Mars, which could take more than a year each way, you might be a bit “weak in the knees” by the time you arrive. This could lead to problems when you try to take your first “small step” on the surface.

To counteract these effects, NASA is looking into ways to provide astronauts with “portable gravity” on long space flights. One method under consideration is the human-powered centrifuge, which not only subjects the astronauts to artificial gravity, but also gives them aerobic exercise. The device is basically a rotating, circular platform on which two astronauts lie supine along a diameter, head-to-head at the center, with their feet at opposite rims, as shown in the accompanying photo. The radius of the platform in this test model is 6.25 ft. As one astronaut pedals to rotate the platform, the astronaut facing the other direction can exercise in the artificial gravity. Alternatively, a third astronaut on a stationary bicycle can provide the rotation for the other two.



Human-powered centrifuge, designed to give astronauts exercise and artificial gravity during long space flights.

Figure 10–31 shows the centripetal acceleration (in g s) produced by a rotating platform at four different radii. Notice that the acceleration increases as the square of the angular speed. Also indicated in Figure 10–31 are acceleration levels corresponding to 1, 3, and 5 g s. It is thought that enhanced gravitational effects may be desirable since the astronauts will experience the artificial gravity for only relatively brief periods of time during the flight.



▲ FIGURE 10-31 Problems 112, 113, 114, and 115

112. • Rank the four curves shown in Figure 10-31 in order of increasing radius. Indicate ties where appropriate.
113. • What angular speed (in rpm) must the platform in this test model have to give a centripetal acceleration of 5.00 *g*s at the rim?
- A. 5.07 rpm B. 26.1 rpm
C. 36.2 rpm D. 48.5 rpm
114. • Which of the curves shown in Figure 10-31 corresponds to the test model?
- A. 1 B. 2
C. 3 D. 4
115. •• Estimate the radius corresponding to curve 4 in Figure 10-31.
- A. 0.03 ft B. 0.3 ft
C. 3 ft D. 6 ft

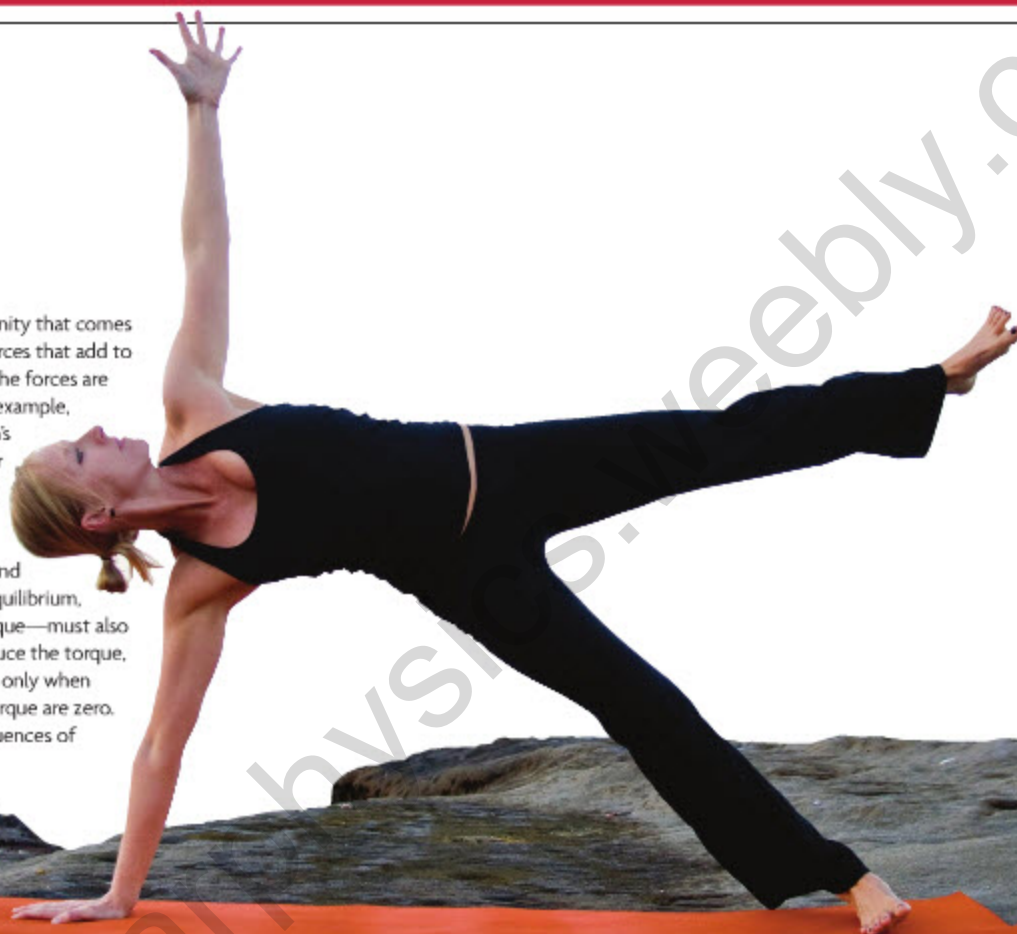
INTERACTIVE PROBLEMS

116. •• Referring to Conceptual Checkpoint 10-4 Suppose we race a disk and a hollow spherical shell, like a basketball. The spherical shell has a mass M and a radius R ; the disk has a mass $2M$ and a radius $2R$. (a) Which object wins the race? If the two objects are released at rest, and the height of the ramp is $h = 0.75$ m, find the speed of (b) the disk and (c) the spherical shell when they reach the bottom of the ramp.
117. •• Referring to Conceptual Checkpoint 10-4 Consider a race between the following three objects: object 1, a disk; object 2, a solid sphere; and object 3, a hollow spherical shell. All objects have the same mass and radius. (a) Rank the three objects in the order in which they finish the race. Indicate a tie where appropriate. (b) Rank the objects in order of increasing kinetic energy at the bottom of the ramp. Indicate a tie where appropriate.
118. •• Referring to Active Example 10-3 (a) Suppose the radius of the axle the string wraps around is increased. Does the speed of the yo-yo after falling through a given height increase, decrease, or stay the same? (b) Find the speed of the yo-yo after falling from rest through a height $h = 0.50$ m if the radius of the axle is 0.0075 m. Everything else in Active Example 10-3 remains the same.
119. •• Referring to Active Example 10-3 Suppose we use a new yo-yo that has the same mass as the original yo-yo and an axle of the same radius. The new yo-yo has a different mass distribution—most of its mass is concentrated near the rim. (a) Is the moment of inertia of the new yo-yo greater than, less than, or the same as that of the original yo-yo? (b) Find the moment of inertia of the new yo-yo if its speed after dropping from rest through a height $h = 0.50$ m is $v = 0.64$ m/s.

11

Rotational Dynamics and Static Equilibrium

Equilibrium, and the sense of serenity that comes with it, involves more than just forces that add to zero—it also depends on where the forces are applied. To keep from falling, for example, the forces exerted on this woman's hand and foot must add up to her total weight. But the total weight must be shared between her hand and foot in just the right way, or else her body will rotate and the pose will be lost. To ensure equilibrium, a new physical quantity—the torque—must also be zero. In this chapter we introduce the torque, and show that equilibrium occurs only when both the net force and the net torque are zero. We will also consider the consequences of nonzero torque.



In the previous chapter we learned how to describe uniformly accelerated rotational motion, but we did not discuss how a given angular acceleration is caused by a given force. The connection between forces and angular acceleration is the focus of this chapter.

We begin by defining a quantity that is the rotational equivalent of force. This quantity is called the *torque*. Although torque may not be as familiar a term as force, your muscles are exerting torques on your body at this very moment. In fact, every time you raise an arm, extend a finger, or stretch a leg, you exert torques to carry out these motions. Thus,

our ability to move from place to place, or to hold our body still, is intimately related to our ability to exert precisely controlled torques on our limbs.

We also introduce the notion of *angular momentum* in this chapter and show that it is related to torque in essentially the same way that linear momentum is related to force. As a result, it follows that angular momentum is conserved when the net external torque acting on a system is zero. Thus, conservation of angular momentum joins conservation of energy and conservation of linear momentum as one of the fundamental principles on which all physics is based.

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11-1 Torque

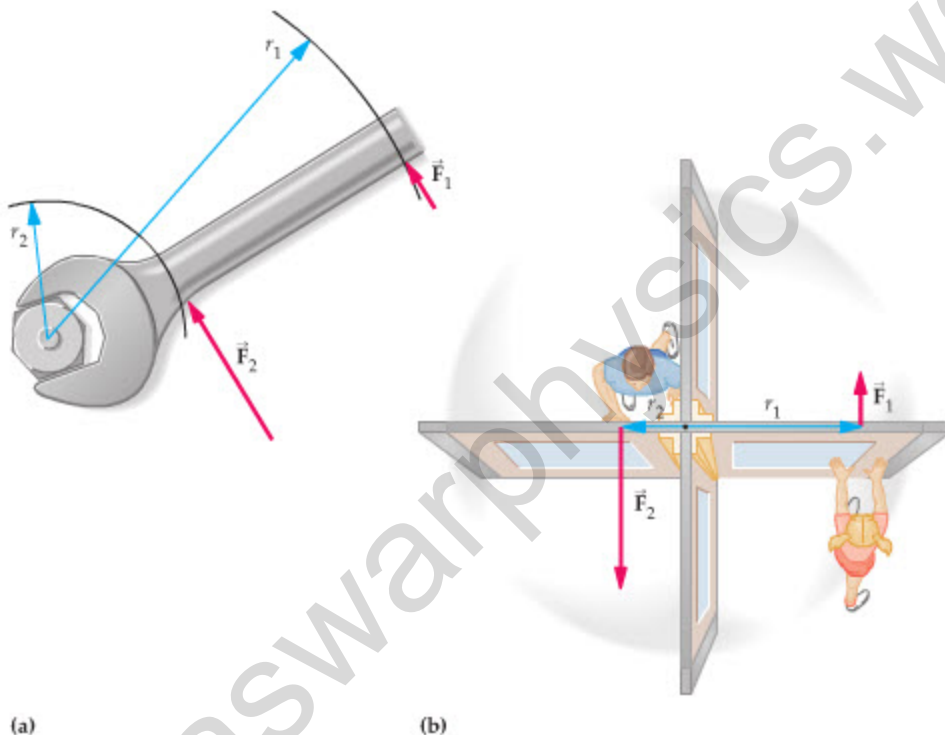
Suppose you want to loosen a nut by rotating it counterclockwise with a wrench. If you have ever used a wrench in this way, you probably know that the nut is more likely to turn if you apply your force as far from the nut as possible, as indicated in **Figure 11-1 (a)**. Applying a force near the nut would not be very effective—you could still get the nut to turn, but it would require considerably more effort! Similarly, it is much easier to open a revolving door if you push far from the axis of rotation, as indicated in **Figure 11-1 (b)**. Clearly, then, the tendency for a force to cause a rotation increases with the distance, r , from the axis of rotation to the force. As a result, it is useful to define a quantity called the **torque**, τ , that takes into account both the magnitude of the force, F , and the distance from the axis of rotation, r :

Definition of Torque, τ , for a Tangential Force

$$\tau = rF$$

SI unit: $\text{N} \cdot \text{m}$

Note that the torque increases with both the force and the distance.



▲ The long handle of this wrench enables the user to produce a large torque without having to exert a very great force.

FIGURE 11-1 Applying a torque

(a) When a wrench is used to loosen a nut, less force is required if it is applied far from the nut. (b) Similarly, less force is required to open a revolving door if it is applied far from the axis of rotation.

Equation 11-1 is valid only when the applied force is *tangential* to a circle of radius r centered on the axis of rotation, as indicated in **Figure 11-1**. The more general case is considered later in this section. First, we use **Equation 11-1** to determine how much force is needed to open a swinging door, depending on where we apply the force.

EXERCISE 11-1

To open the door in **Figure 11-1 (b)** a tangential force F is applied at a distance r from the axis of rotation. If the minimum torque required to open the door is $3.1 \text{ N} \cdot \text{m}$, what force must be applied if r is (a) 0.94 m or (b) 0.35 m ?

SOLUTION

(a) Setting $\tau = r_1 F_1 = 3.1 \text{ N} \cdot \text{m}$, we find that the required force is

$$F_1 = \frac{\tau}{r_1} = \frac{3.1 \text{ N} \cdot \text{m}}{0.94 \text{ m}} = 3.3 \text{ N}$$

PROBLEM-SOLVING NOTE

The Units of Torque

Note that the units of torque are $\text{N} \cdot \text{m}$, the same as the units of work. Though their units are the same, torque, τ , and work, W , represent different physical quantities and should not be confused with one another.

(b) Repeat the calculation, this time with $r_2 = 0.35$ m:

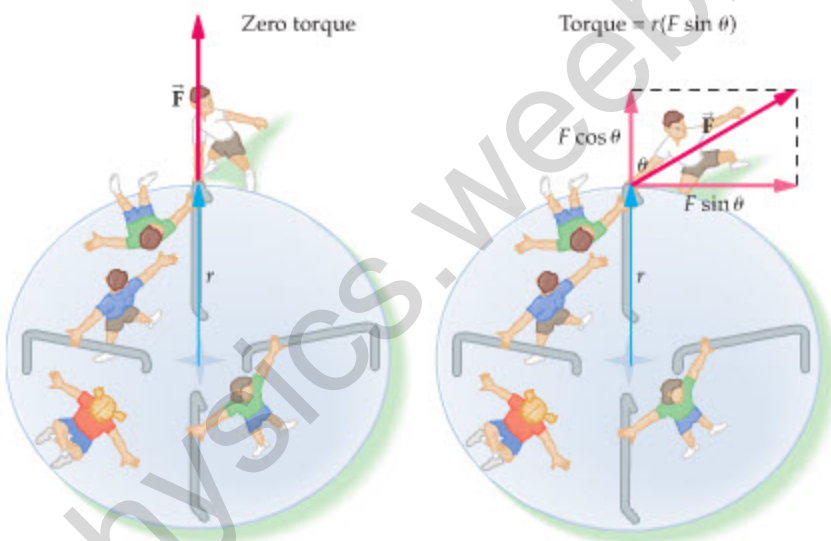
$$F_2 = \frac{\tau}{r_2} = \frac{3.1 \text{ N} \cdot \text{m}}{0.35 \text{ m}} = 8.9 \text{ N}$$

As expected, the required force is greater when it is applied closer to the hinges.

To this point we have considered tangential forces only. What happens if you exert a force in a direction that is not tangential? Suppose, for example, that you pull on a playground merry-go-round in a direction that is radial—that is, along a line that extends through the axis of rotation—as in **Figure 11–2 (a)**. In this case, your force has no tendency to cause a rotation. Instead, the axle of the merry-go-round simply exerts an equal and opposite force, and the merry-go-round remains at rest. Similarly, if you were to push or pull in a radial direction on a swinging door it would not rotate. We conclude that a *radial force produces zero torque*.

► **FIGURE 11–2** Only the tangential component of a force causes a torque

(a) A radial force causes no rotation. In this case, the force \vec{F} is opposed by an equal and opposite force exerted by the axle of the merry-go-round. The merry-go-round does not rotate. (b) A force applied at an angle θ with respect to the radial direction. The radial component of this force, $F \cos \theta$, causes no rotation; the tangential component, $F \sin \theta$, can cause a rotation.



(a) A radial force produces zero torque

(b) Only the tangential component of force causes a torque

On the other hand, what if your force is at an angle θ relative to a radial line, as shown in **Figure 11–2 (b)**? To analyze this case, we first resolve the force vector \vec{F} into radial and tangential components. Referring to the figure, we see that the radial component has a magnitude of $F \cos \theta$, and the tangential component has a magnitude of $F \sin \theta$. Because it is the tangential component alone that causes rotation, we define the torque to have a magnitude of $r(F \sin \theta)$. That is,

General Definition of Torque, τ

$$\tau = r(F \sin \theta)$$

SI units: $\text{N} \cdot \text{m}$

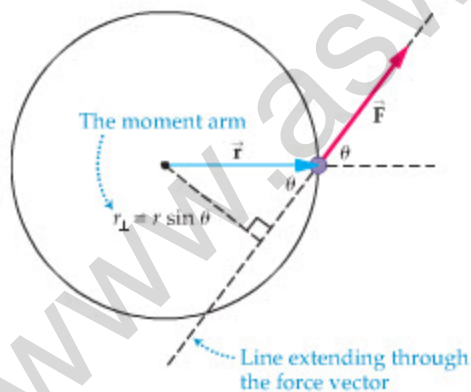
11–2

(More generally, the torque can be defined as the **cross product** between the vectors \vec{r} and \vec{F} ; that is, $\vec{\tau} = \vec{r} \times \vec{F}$. The cross product is discussed in detail in Appendix A.)

As a quick check, note that a radial force corresponds to $\theta = 0$. In this case, $\tau = r(F \sin 0) = 0$, as expected. If the force is tangential, however, it follows that $\theta = \pi/2$. This gives $\tau = r(F \sin \pi/2) = rF$, in agreement with **Equation 11–1**.

An equivalent way to define the torque is in terms of the **moment arm**, r_{\perp} . The idea here is to extend a line through the force vector, as in **Figure 11–3**, and then draw a second line from the axis of rotation perpendicular to the line of the force. The perpendicular distance from the axis of rotation to the line of the force is defined to be r_{\perp} . From the figure, we see that

$$r_{\perp} = r \sin \theta$$



► **FIGURE 11–3** The moment arm

To find the moment arm, r_{\perp} , for a given force, first extend a line through the force vector. This line is sometimes referred to as the “line of action.” Next, drop a perpendicular line from the axis of rotation to the line of the force. The perpendicular distance is $r_{\perp} = r \sin \theta$.

In addition, we note that a simple rearrangement of the torque expression in Equation 11-2 yields

$$\tau = r(F \sin \theta) = (r \sin \theta)F$$

Thus, the torque can be written as the moment arm times the force:

$$\tau = r_{\perp} F \quad 11-3$$

Just as a force applied to an object gives rise to a linear acceleration, a torque applied to an object gives rise to an angular acceleration. For example, if a torque acts on an object at rest, the object will begin to rotate; if a torque acts on a rotating object, the object's angular velocity will change. In fact, the greater the torque applied to an object, the greater its angular acceleration, as we shall see in the next section. For this reason, the sign of the torque is determined by the same convention used in Section 10-1 for angular acceleration:

Sign Convention for Torque

By convention, if a torque τ acts alone, then

- $\tau > 0$ if the torque causes a counterclockwise angular acceleration
- $\tau < 0$ if the torque causes a clockwise angular acceleration

In a system with more than one torque, the sign of each torque is determined by the type of angular acceleration *it alone* would produce. The net torque acting on the system, then, is the sum of each individual torque, taking into account the proper sign. This is illustrated in the following Example.



▲ The net torque on the wheel of this ship is the sum of the torques exerted by the two helmsmen. At the moment pictured, they are both exerting negative torques on the wheel, causing it to rotate in the clockwise direction. This will turn the boat to its left—or, in nautical terms, to port.

PROBLEM-SOLVING NOTE

The Sign of Torques

The sign of a torque is determined by the direction of rotation it would cause if it were the only torque acting in the system.

EXAMPLE 11-1 TORQUES TO THE LEFT AND TORQUES TO THE RIGHT

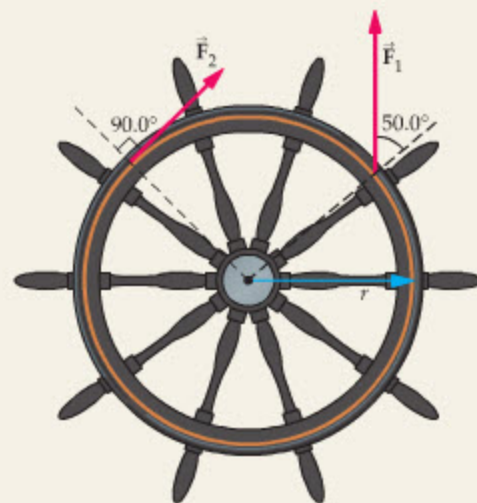
Two helmsmen, in disagreement about which way to turn a ship, exert the forces shown below on a ship's wheel. The wheel has a radius of 0.74 m, and the two forces have the magnitudes $F_1 = 72$ N and $F_2 = 58$ N. Find (a) the torque caused by \vec{F}_1 and (b) the torque caused by \vec{F}_2 . (c) In which direction does the wheel turn as a result of these two forces?

PICTURE THE PROBLEM

Our sketch shows that both forces are applied at the distance $r = 0.74$ m from the axis of rotation. However, $F_1 = 72$ N is at an angle of 50.0° relative to the radial direction, whereas $F_2 = 58$ N is tangential, which means that its angle relative to the radial direction is 90.0° .

STRATEGY

For each force, we find the magnitude of the corresponding torque, using $\tau = rF \sin \theta$. As for the signs of the torques, we must consider the angular acceleration each force alone would cause. \vec{F}_1 acting alone would cause the wheel to accelerate counterclockwise, hence its torque is positive. \vec{F}_2 would accelerate the wheel clockwise if it acted alone, hence its torque is negative. If the sum of the two torques is positive, the wheel accelerates counterclockwise; if the sum of the two torques is negative, the wheel accelerates clockwise.



SOLUTION

Part (a)

- Use Equation 11-2 to calculate the torque due to \vec{F}_1 . Recall that this torque is positive:

$$\tau_1 = rF_1 \sin 50.0^\circ = (0.74 \text{ m})(72 \text{ N}) \sin 50.0^\circ = 41 \text{ N} \cdot \text{m}$$

Part (b)

- Similarly, calculate the torque due to \vec{F}_2 . Recall that this torque is negative:

$$\tau_2 = -rF_2 \sin 90.0^\circ = -(0.74 \text{ m})(58 \text{ N}) = -43 \text{ N} \cdot \text{m}$$

Part (c)

- Sum the torques from parts (a) and (b) to find the net torque:

$$\tau_{\text{net}} = \tau_1 + \tau_2 = 41 \text{ N} \cdot \text{m} - 43 \text{ N} \cdot \text{m} = -2 \text{ N} \cdot \text{m}$$

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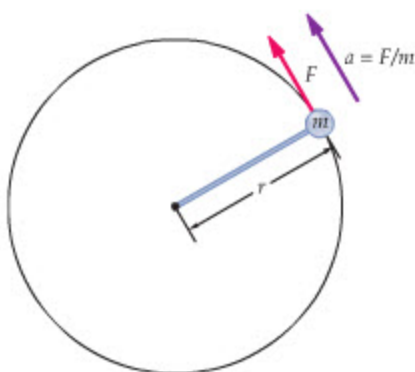
INSIGHT

Because the net torque is negative, the wheel accelerates clockwise. Thus, even though \vec{F}_2 is the smaller force, it has the greater effect in determining the wheel's direction of acceleration. This is because \vec{F}_2 is applied tangentially, whereas \vec{F}_1 is applied in a direction that is partially radial.

PRACTICE PROBLEM

What magnitude of \vec{F}_2 would yield zero net torque on the wheel? [Answer: $F_2 = 55 \text{ N}$]

Some related homework problems: Problem 1, Problem 3



▲ FIGURE 11-4 Torque and angular acceleration

A tangential force F applied to a mass m gives it a linear acceleration of magnitude $a = F/m$. The corresponding angular acceleration is $\alpha = \tau/I$, where $\tau = rF$ and $I = mr^2$.

11-2 Torque and Angular Acceleration

In the previous section we indicated that a torque causes a change in the rotational motion of an object. To be more precise, a single torque, τ , acting on an object causes the object to have an angular acceleration, α . In this section we develop the specific relationship between τ and α .

Consider, for example, a small object of mass m connected to an axis of rotation by a light rod of length r , as in **Figure 11-4**. If a tangential force of magnitude F is applied to the mass, it will move with an acceleration given by Newton's second law:

$$a = \frac{F}{m}$$

From **Equation 10-14**, we know that the linear and angular accelerations are related by

$$\alpha = \frac{a}{r}$$

Combining these results yields the following expression for the angular acceleration:

$$\alpha = \frac{a}{r} = \frac{F}{mr}$$

Finally, multiplying both numerator and denominator by r gives

$$\alpha = \left(\frac{r}{r}\right) \frac{F}{mr} = \frac{rF}{mr^2}$$

Now this last result is rather interesting, since the numerator and denominator have simple interpretations. First, the numerator is the torque, $\tau = rF$, for the case of a tangential force (**Equation 11-1**). Second, the denominator is the moment of inertia of a single mass m rotating at a radius r ; that is, $I = mr^2$. Therefore, we find that

$$\alpha = \frac{rF}{mr^2} = \frac{\tau}{I}$$

or, rewriting slightly,

$$\tau = I\alpha$$

Thus, once we calculate the torque, as described in the previous section, we can find the angular acceleration of a system using $\tau = I\alpha$. Notice that the angular acceleration is directly proportional to the torque, and inversely proportional to the moment of inertia—that is, a large moment of inertia means a small angular acceleration.

Now, the relationship $\tau = I\alpha$ was derived for the special case of a tangential force and a single mass rotating at a radius r . However, the result is completely general. For example, in a system with more than one torque, the relation $\tau = I\alpha$

is replaced with $\tau_{\text{net}} = \Sigma\tau = I\alpha$, where τ_{net} is the net torque acting on the system. This gives us the *rotational* version of Newton's second law:

Newton's Second Law for Rotational Motion

$$\Sigma\tau = I\alpha$$

11-4

If only a single torque acts on a system, we will simply write $\tau = I\alpha$.

EXERCISE 11-2

A light rope wrapped around a disk-shaped pulley is pulled tangentially with a force of 0.53 N. Find the angular acceleration of the pulley, given that its mass is 1.3 kg and its radius is 0.11 m.

SOLUTION

The torque applied to the disk is

$$\tau = rF = (0.11 \text{ m})(0.53 \text{ N}) = 5.8 \times 10^{-2} \text{ N}\cdot\text{m}$$

Since the pulley is a disk, its moment of inertia is given by

$$I = \frac{1}{2}mr^2 = \frac{1}{2}(1.3 \text{ kg})(0.11 \text{ m})^2 = 7.9 \times 10^{-3} \text{ kg}\cdot\text{m}^2$$

Thus, the angular acceleration of the pulley is

$$\alpha = \frac{\tau}{I} = \frac{5.8 \times 10^{-2} \text{ N}\cdot\text{m}}{7.9 \times 10^{-3} \text{ kg}\cdot\text{m}^2} = 7.3 \text{ rad/s}^2$$

It is easy to remember the rotational version of Newton's second law, $\Sigma\tau = I\alpha$, by using analogies between rotational and linear quantities. We have already seen that I is the analogue of m , and that α is the analogue of a . Similarly, τ , which causes an angular acceleration, is the analogue of F , which causes a linear acceleration. To summarize:

Linear Quantity	Angular Quantity
m	I
a	α
F	τ

Thus, just as $\Sigma F = ma$ describes linear motion, $\Sigma\tau = I\alpha$ describes rotational motion.

EXAMPLE 11-2 A FISH TAKES THE LINE

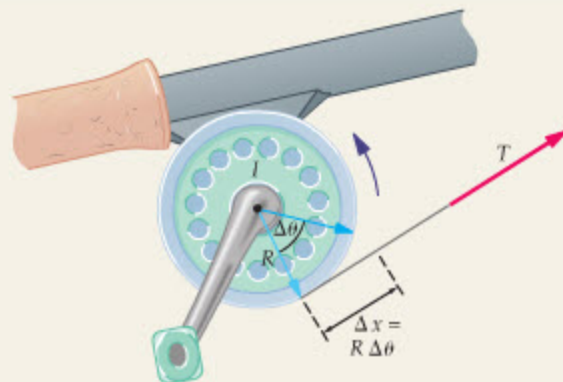
A fisherman is dozing when a fish takes the line and pulls it with a tension T . The spool of the fishing reel is at rest initially and rotates without friction (since the fisherman left the drag off) as the fish pulls for a time t . If the radius of the spool is R , and its moment of inertia is I , find (a) the angular displacement of the spool, (b) the length of line pulled from the spool, and (c) the final angular speed of the spool.

PICTURE THE PROBLEM

Our sketch shows the fishing line being pulled tangentially from the spool with a tension T . Because the radius of the spool is R , the torque produced by the line is $\tau = RT$. Also note that as the spool rotates through an angle $\Delta\theta$, the line moves through a linear distance $\Delta x = R\Delta\theta$. Finally, the spool starts at rest, hence $\omega_0 = 0$.

STRATEGY

This is basically an angular kinematics problem, as in Chapter 10, but in this case we must first calculate the angular acceleration using $\alpha = \tau/I$. Once α is known, we can find the angular displacement, $\Delta\theta$, using $\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$. Similarly, we can find the angular speed of the spool, ω , using $\omega = \omega_0 + \alpha t$.



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SOLUTION

- Calculate the torque acting on the spool. Note that $\theta = 90^\circ$, since the pull is tangential. The radius is $r = R$, and the force applied to the reel is the tension in the line, T :
- Using the result just obtained for the torque, find the angular acceleration of the reel:

$$\tau = rF \sin \theta = RT \sin 90^\circ = RT$$

$$\alpha = \frac{\tau}{I} = \frac{RT}{I}$$

Part (a)

- Calculate the angular displacement $\Delta\theta = \theta - \theta_0$:

$$\Delta\theta = \theta - \theta_0 = \omega_0 t + \frac{1}{2}\alpha t^2 = \frac{1}{2}\alpha t^2 = \left(\frac{RT}{2I}\right)t^2$$

Part (b)

- Calculate the length of line pulled from the spool with $\Delta x = R \Delta\theta$:

$$\Delta x = R \Delta\theta = \left(\frac{R^2 T}{2I}\right)t^2$$

Part (c)

- Use $\omega = \omega_0 + \alpha t$ to find the final angular speed:

$$\omega = \omega_0 + \alpha t = \left(\frac{RT}{I}\right)t$$

INSIGHT

Note that the final angular speed can also be obtained from the kinematic equation relating angular speed and angular distance; $\omega^2 = \omega_0^2 + 2\alpha \Delta\theta = 0 + 2(RT/I)(RT/2I)t^2 = (RT/I)^2 t^2$.

This calculation also applies to other situations in which a “line” is pulled from a “reel.” Examples include telephone line or sewing thread pulled from a spool.

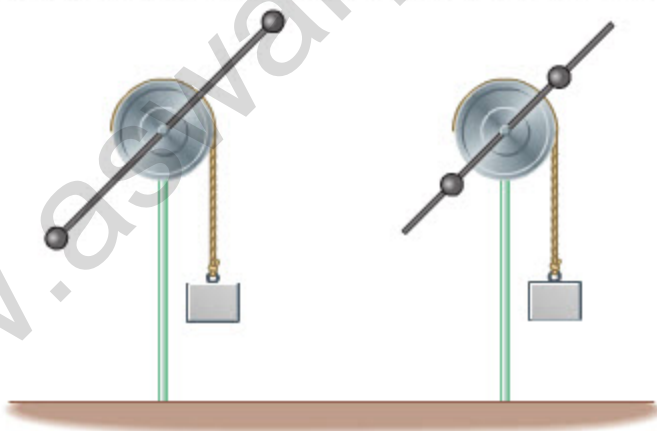
PRACTICE PROBLEM

How fast is the line moving at time t ? [Answer: $v = R\omega = (R^2 T/I)t$]

Some related homework problems: Problem 10, Problem 19

CONCEPTUAL CHECKPOINT 11-1 WHICH BLOCK LANDS FIRST?

The rotating systems shown below differ only in that the two spherical movable masses are positioned either far from the axis of rotation (left), or near the axis of rotation (right). If the hanging blocks are released simultaneously from rest, is it observed that (a) the block at left lands first, (b) the block at right lands first, or (c) both blocks land at the same time?

**REASONING AND DISCUSSION**

The net external torque, supplied by the hanging blocks, is the same for each of these systems. However, the moment of inertia of the system at right is less than that of the system at left because the movable masses are closer to the axis of rotation. Since the angular acceleration is inversely proportional to the moment of inertia ($\alpha = \tau_{\text{net}}/I$), the system at right has the greater angular acceleration, and it wins the race.

ANSWER

(b) The block at right lands first.

EXAMPLE 11-3 DROP IT

A person holds his outstretched arm at rest in a horizontal position. The mass of the arm is m and its length is 0.740 m . When the person releases his arm, allowing it to drop freely, it begins to rotate about the shoulder joint. Find (a) the initial angular acceleration of the arm, and (b) the initial linear acceleration of the man's hand. (*Hint:* In calculating the torque, assume the mass of the arm is concentrated at its midpoint. In calculating the angular acceleration, use the moment of inertia of a uniform rod of length L about one end; $I = \frac{1}{3}mL^2$.)

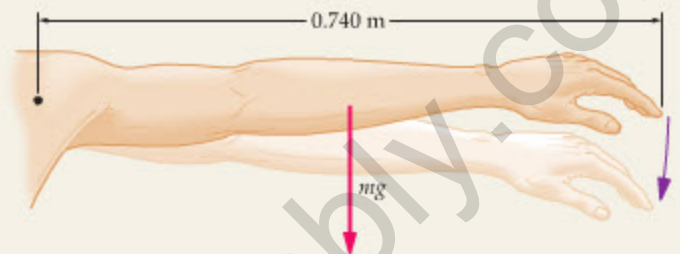
PICTURE THE PROBLEM

The arm is initially horizontal and at rest. When released, it rotates downward about the shoulder joint. The force of gravity, mg , acts at a distance of $(0.740\text{ m})/2 = 0.370\text{ m}$ from the shoulder.

STRATEGY

The angular acceleration, α , can be found using $\tau = I\alpha$. In this case, the initial torque is $\tau = mg(L/2)$, where $L = 0.740\text{ m}$, and the moment of inertia is $I = \frac{1}{3}mL^2$.

Once the initial angular acceleration is found, the corresponding linear acceleration is obtained from $a = r\alpha$.

**SOLUTION****Part (a)**

- Use $\tau = I\alpha$ to find the angular acceleration, α :
- Write expressions for the initial torque, τ , and the moment of inertia, I :
- Substitute τ and I into the expression for the angular acceleration. Note that the mass of the arm cancels:
- Substitute numerical values:

$$\alpha = \frac{\tau}{I}$$

$$\tau = mg\frac{L}{2}$$

$$I = \frac{1}{3}mL^2$$

$$\alpha = \frac{\tau}{I} = \frac{mgL/2}{mL^2/3} = \frac{3g}{2L}$$

$$\alpha = \frac{3g}{2L} = \frac{3(9.81\text{ m/s}^2)}{2(0.740\text{ m})} = 19.9\text{ rad/s}^2$$

Part (b)

- Use $a = r\alpha$ to calculate the linear acceleration at the man's hand, a distance $r = L$ from the shoulder:

$$a = L\alpha = L\left(\frac{3g}{2L}\right) = \frac{3}{2}g = 14.7\text{ m/s}^2$$

INSIGHT

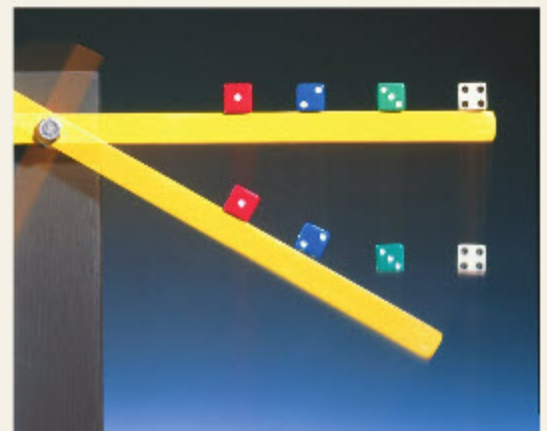
Note that the linear acceleration of the hand is 1.50 times greater than the acceleration of gravity, regardless of the mass of the arm. This can be demonstrated with the following simple experiment: Hold your arm straight out with a pen resting on your hand. Now, relax your deltoid muscles, and let your arm rotate freely downward about your shoulder joint. Notice that as your arm falls downward, your hand moves more rapidly than the pen, which appears to "lift off" your hand. The pen drops with the acceleration of gravity, which is clearly less than the acceleration of the hand. This effect can be seen in the adjacent photo.

PRACTICE PROBLEM

At what distance from the shoulder is the initial linear acceleration of the arm equal to the acceleration of gravity?

[Answer: Set $a = r\alpha$ equal to g . This gives $r = 2L/3 = 0.493\text{ m}$.]

Some related homework problems: Problem 13, Problem 15



▲ As a rod of length L rotates freely about one end, points farther from the axle than $2L/3$ have an acceleration greater than g (see the Practice Problem for Example 11-3). Thus, the rod falls out from under the last two dice.

11-3 Zero Torque and Static Equilibrium

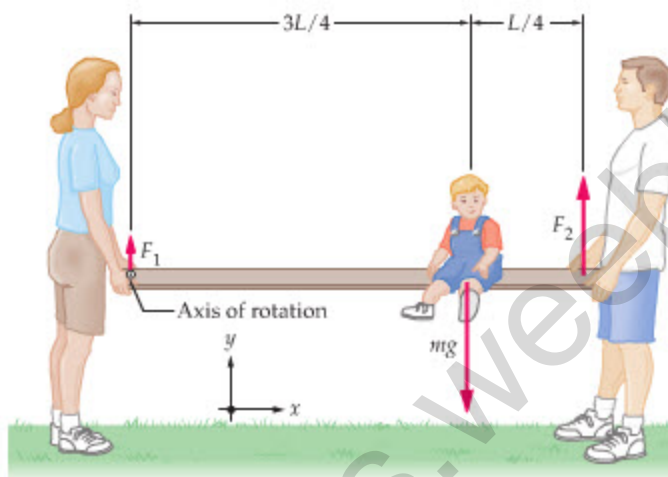
The parents of a young boy are supporting him on a long, lightweight plank, as illustrated in **Figure 11-5**. If the mass of the child is m , the upward forces exerted by the parents must sum to mg ; that is,

$$F_1 + F_2 = mg$$

This condition ensures that the net force acting on the plank is zero. It *does not*, however, guarantee that the plank remains at rest.

FIGURE 11-5 Forces required for static equilibrium

Two parents support a child on a lightweight plank of length L . For the calculation described in the text, we choose the axis of rotation to be the left end of the plank.



To see why, imagine for a moment that the parent on the right lets go of the plank and that the parent on the left increases her force until it is equal to the weight of the child. In this case, $F_1 = mg$ and $F_2 = 0$, which clearly satisfies the force equation we have just written. Since the right end of the plank is no longer supported, however, it drops toward the ground while the left end rises. In other words, the plank rotates in a clockwise sense. For the plank to remain completely at rest, with no translation or rotation, we must impose the following *two* conditions: First, the net force acting on the plank must be zero, so that there is no translational acceleration. Second, the net torque acting on the plank must also be zero, so that there is no rotational acceleration. If both of these conditions are met, an extended object, like the plank, will remain at rest if it starts at rest. To summarize:

Conditions for Static Equilibrium

For an extended object to be in static equilibrium, the following two conditions must be met:

- (i) The net force acting on the object must be zero,

$$\sum F_x = 0, \quad \sum F_y = 0 \quad 11-5$$

- (ii) The net torque acting on the object must be zero,

$$\sum \tau = 0 \quad 11-6$$

Note that these two conditions are independent of one another; that is, satisfying one does *not* guarantee that the other is satisfied.

Let's apply these conditions to the plank that supports the child. First, we consider the forces acting on the plank, with upward chosen as the positive direction, as in **Figure 11-5**. Setting the net force equal to zero yields

$$F_1 + F_2 - mg = 0$$

Clearly, this agrees with the force equation we wrote down earlier.

Next, we apply the torque condition. To do so, we must first choose an axis of rotation. For example, we might take the left end of the plank to be the axis, as in **Figure 11-5**. With this choice, we see that the force F_1 exerts zero torque, since it

acts directly through the axis of rotation. On the other hand, F_2 acts at the far end of the plank, a distance L from the axis. In addition, F_2 would cause a counter-clockwise (positive) rotation if it acted alone, as we can see in Figure 11-5. Therefore, the torque due to F_2 is

$$\tau_2 = F_2 L$$

Finally, the weight of the child, mg , acts at a distance of $3L/4$ from the axis, and would cause a clockwise (negative) rotation if it acted alone. Hence, its torque is negative:

$$\tau_{mg} = -mg\left(\frac{3L}{4}\right)$$

Setting the net torque equal to zero, then, yields the following condition:

$$F_2 L - mg\left(\frac{3L}{4}\right) = 0$$

This torque condition, along with the force condition in $F_1 + F_2 - mg = 0$, can be used to determine the two unknowns, F_1 and F_2 . For example, we can begin by canceling L in the torque equation to find F_2 :

$$F_2 = \frac{3}{4}mg$$

Substituting this result into the force condition gives

$$F_1 + \frac{3}{4}mg - mg = 0$$

Therefore, F_1 is

$$F_1 = \frac{1}{4}mg$$

These two forces support the plank, and keep it from rotating. As one might expect, the force nearest the child is greatest.

Our choice of the left end of the plank as the axis of rotation was completely arbitrary. In fact, if an object is in static equilibrium, the net torque acting on it is zero, regardless of the location of the axis of rotation. Hence, we are free to choose an axis of rotation that is most convenient for a given problem. In general, it is useful to pick the axis to be at the location of one of the unknown forces. This eliminates that force from the torque condition, and simplifies the remaining algebra. We consider an alternative choice for the axis of rotation in the following Active Example.

PROBLEM-SOLVING NOTE

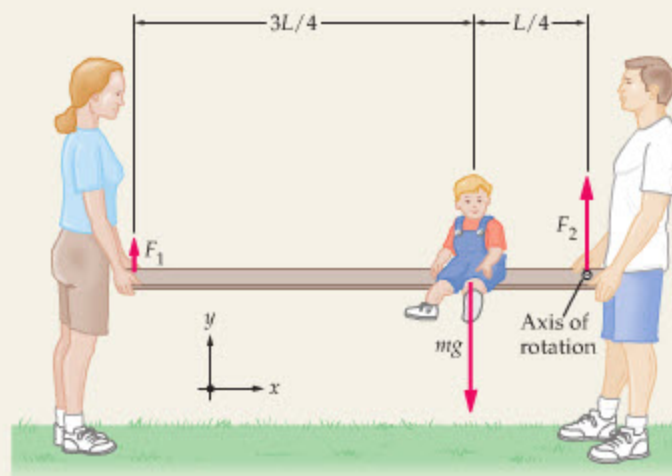
Axis of Rotation



Any point in a system may be used as the axis of rotation when calculating torque. It is generally best, however, to choose an axis that gives zero torque for at least one of the unknown forces in the system. Such a choice simplifies the algebra needed to solve for the forces.

ACTIVE EXAMPLE 11-1 FIND THE FORCES: AXIS ON THE RIGHT

A child of mass m is supported on a light plank by his parents, who exert the forces F_1 and F_2 as indicated. Find the forces required to keep the plank in static equilibrium. Use the right end of the plank as the axis of rotation.



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SOLUTION (Test your understanding by performing the calculations indicated in each step.)

1. Set the net force acting on the plank equal to zero:

$$F_1 + F_2 - mg = 0$$

2. Set the net torque acting on the plank equal to zero:

$$-F_1(L) + mg\left(\frac{1}{4}L\right) = 0$$

3. Note that the torque condition involves only one of the two unknowns, F_1 . Use this condition to solve for F_1 :

$$F_1 = \frac{1}{4}mg$$

4. Substitute F_1 into the force condition to solve for F_2 :

$$F_2 = mg - \frac{1}{4}mg = \frac{3}{4}mg$$

INSIGHT

As expected, the results are identical to those obtained previously. Note that in this case the torque produced by the child would cause a counterclockwise rotation, hence it is positive. Thus, the magnitude and sign of the torque produced by a given force depend on the location chosen for the axis of rotation.

YOUR TURN

Suppose the child moves to a new position, with the result that the force exerted by the father is reduced to $0.60mg$. Did the child move to the left or to the right? How far did the child move?

(Answers to **Your Turn** problems are given in the back of the book.)

A third choice for the axis of rotation is considered in Problem 24. As expected, all three choices give the same results.

In the next Example, we show that the forces supporting a person or other object sometimes act in different directions. To emphasize the direction of the forces, we solve the Example in terms of the components of the relevant forces.

EXAMPLE 11-4 TAKING THE PLUNGE

A 5.00-m-long diving board of negligible mass is supported by two pillars. One pillar is at the left end of the diving board, as shown below; the other is 1.50 m away. Find the forces exerted by the pillars when a 90.0-kg diver stands at the far end of the board.

PICTURE THE PROBLEM

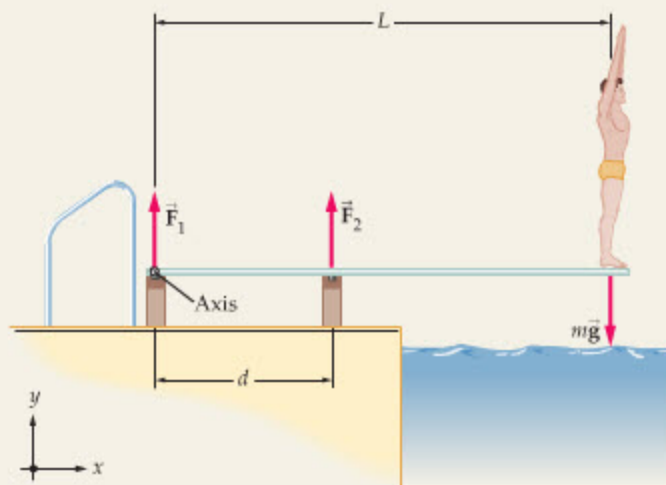
We choose upward to be the positive direction for the forces. When calculating torques, we use the left end of the diving board as the axis of rotation. Note that \vec{F}_2 would cause a counterclockwise rotation if it acted alone, so its torque is positive. On the other hand, $m\vec{g}$ would cause a clockwise rotation, so its torque is negative. Finally, \vec{F}_2 acts at a distance d from the axis of rotation, and $m\vec{g}$ acts at a distance L .

STRATEGY

As usual in static equilibrium problems, we use the conditions of (i) zero net force and (ii) zero net torque to determine the unknown forces, \vec{F}_1 and \vec{F}_2 . In this system all forces act in the positive or negative y direction; thus we need only set the net y component of force equal to zero.

SOLUTION

1. Set the net y component of force acting on the diving board equal to zero:
2. Calculate the torque due to each force, using the left end of the board as the axis of rotation. Note that each force is at right angles to the radius and that \vec{F}_1 goes directly through the axis of rotation:
3. Set the net torque acting on the diving board equal to zero:
4. Solve the torque equation for the force $F_{2,y}$:



$$\sum F_y = F_{1,y} + F_{2,y} - mg = 0$$

$$\tau_1 = F_{1,y}(0) = 0$$

$$\tau_2 = F_{2,y}(d)$$

$$\tau_3 = -mg(L)$$

$$\sum \tau = F_{1,y}(0) + F_{2,y}(d) - mg(L) = 0$$

$$F_{2,y} = mg(L/d)$$

$$= (90.0 \text{ kg})(9.81 \text{ m/s}^2)(5.00 \text{ m}/1.50 \text{ m}) = 2940 \text{ N}$$

5. Use the force equation to determine $F_{1,y}$:

$$\begin{aligned} F_{1,y} &= mg - F_{2,y} \\ &= (90.0 \text{ kg})(9.81 \text{ m/s}^2) - 2940 \text{ N} = -2060 \text{ N} \end{aligned}$$

INSIGHT

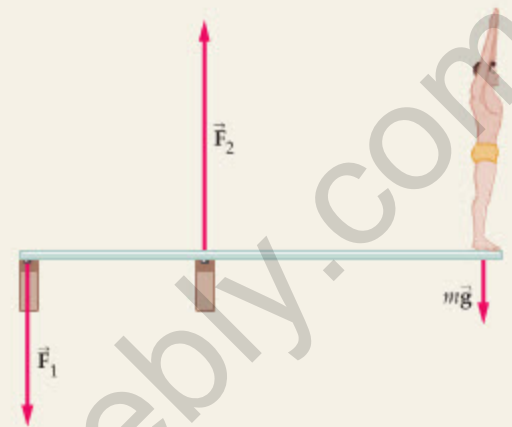
The first point to notice about our solution is that $F_{1,y}$ is negative, which means that \vec{F}_1 is actually directed *downward*, as shown to the right. To see why, imagine for a moment that the board is no longer connected to the first pillar. In this case, the board would rotate clockwise about the second pillar, and the left end of the board would move upward. Thus, a downward force is required on the left end of the board to hold it in place.

The second point is that both pillars exert forces with magnitudes that are considerably larger than the diver's weight, $mg = 883 \text{ N}$. In particular, the first pillar must pull downward with a force of $2.33mg$, while the second pillar pushes upward with a force of $2.33mg + mg = 3.33mg$. This is not unusual. In fact, it is common for the forces in a structure, such as a bridge, a building, or the human body, to be much greater than the weight it supports.

PRACTICE PROBLEM

Find the forces exerted by the pillars when the diver is 1.00 m from the right end. [Answer: $F_{1,y} = -1470 \text{ N}$, $F_{2,y} = 2350 \text{ N}$]

Some related homework problems: Problem 26, Problem 32

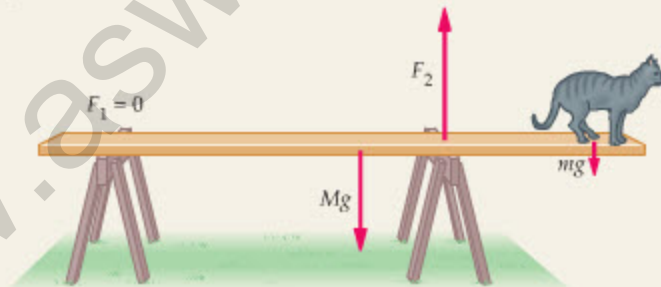


To this point we have ignored the mass of the plank holding the child and the diving board holding the swimmer, since they were described as lightweight. If we want to consider the torque exerted by an extended object of finite mass, however, we can simply treat it as if all its mass were concentrated at its center of mass, as was done in similar situations in Section 9-7. We consider such a system in the next Active Example.

ACTIVE EXAMPLE 11-2

WALKING THE PLANK: FIND THE MASS

A cat walks along a uniform plank that is 4.00 m long and has a mass of 7.00 kg. The plank is supported by two sawhorses, one 0.440 m from the left end of the board and the other 1.50 m from its right end. When the cat reaches the right end, the plank just begins to tip. What is the mass of the cat?



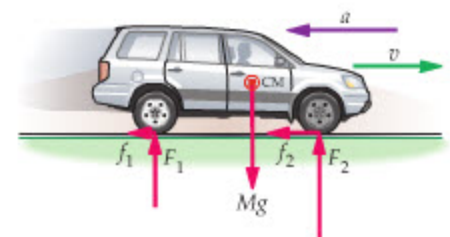
SOLUTION (Test your understanding by performing the calculations indicated in each step.)

- Since the board is just beginning to tip, there is no weight on the left sawhorse: $F_1 = 0$
- Calculate the torque about the right sawhorse: $Mg(0.500 \text{ m}) - mg(1.50 \text{ m}) = 0$
- Solve the torque equation for the mass of the cat, m : $m = 0.333M = 2.33 \text{ kg}$

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REAL-WORLD PHYSICS

Applying the brakes



▲ As the brakes are applied on this SUV, rotational equilibrium demands that the normal forces exerted on the front tires be greater than the normal forces exerted on the rear tires—which is why braking cars are “nose down” during a rapid stop. For this reason, many cars use disk brakes for the front wheels and less powerful drum brakes for the rear wheels. As the disk brakes wear, they tend to coat the front wheels with dust from the brake pads, which give the front wheels a characteristic “dirty” look.

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INSIGHT

Note that we did not include a torque for the left sawhorse, since F_1 is zero. As an exercise, you might try repeating the calculation with the axis of rotation at the left sawhorse, or at the center of mass of the plank.

YOUR TURN

Write both the zero force and zero torque conditions for the case where the axis of rotation is at the left sawhorse.

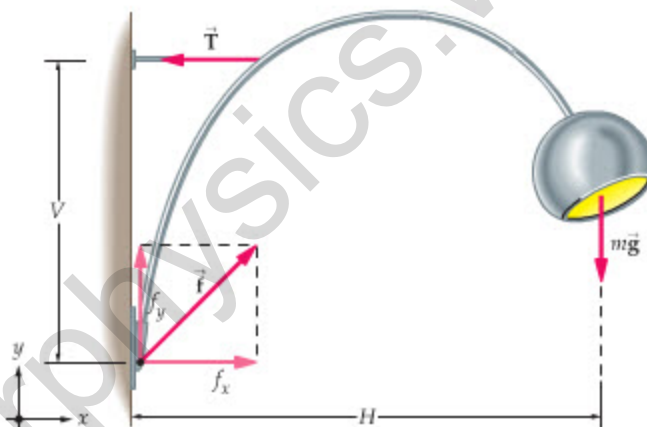
(Answers to **Your Turn** problems are given in the back of the book.)

Forces with Both Vertical and Horizontal Components

Note that all of the previous examples have dealt with forces that point either directly upward or directly downward. We now consider a more general situation, where forces may have both vertical and horizontal components. For example, consider the wall-mounted lamp (sconce) shown in **Figure 11-6**. The sconce consists of a light curved rod that is bolted to the wall at its lower end. Suspended from the upper end of the rod, a horizontal distance H from the wall, is the lamp of mass m . The rod is also connected to the wall by a horizontal wire a vertical distance V above the bottom of the rod.

► **FIGURE 11-6** A lamp in static equilibrium

A wall-mounted lamp of mass m is suspended from a light curved rod. The bottom of the rod is bolted to the wall. The rod is also connected to the wall by a horizontal wire a vertical distance V above the bottom of the rod.



Now, suppose we are designing this sconce to be placed in the lobby of a building on campus. To ensure its structural stability, we would like to know the tension T the wire must exert and the vertical and horizontal components of the force \vec{f} that must be exerted by the bolt on the rod. This information will be important in deciding on the type of wire and bolt to be used in the construction.

To find these forces, we apply the same conditions as before: the net force and the net torque must be zero. In this case, however, forces may have both horizontal and vertical components. Thus, the condition of zero net force is really two separate conditions: (i) zero net force in the horizontal direction; and (ii) zero net force in the vertical direction. These two conditions plus (iii) zero net torque, allow for a full solution of the problem.

We begin with the torque condition. A convenient choice for the axis of rotation is the bottom end of the rod, since this eliminates one of the unknown forces (\vec{f}). With this choice we can readily calculate the torques acting on the rod by using the moment arm expression for the torque, $\tau = r_{\perp} F$ (Equation 11-3). We find

$$\sum \tau = T(V) - mg(H) = 0$$

This relation can be solved immediately for the tension, giving

$$T = mg(H/V)$$

Note that the tension is increased if the wire is connected closer to the bottom of the rod; that is, if V is reduced.

Next, we apply the force conditions. First, we sum the y components of all the forces and set the sum equal to zero:

$$\sum F_y = f_y - mg = 0$$

Thus, the vertical component of the force exerted by the bolt simply supports the weight of the lamp:

$$f_y = mg$$

Finally, we sum the x components of the forces and set that sum equal to zero:

$$\sum F_x = f_x - T = 0$$

Clearly, the x component of the force exerted by the bolt is of the same magnitude as the tension, but it points in the opposite direction:

$$f_x = T = mg(H/V)$$

The bolt, then, pushes upward on the rod to support the lamp, and at the same time it pushes to the right to keep the rod from rotating.

For example, suppose the lamp in Figure 11-6 has a mass of 2.00 kg, and that $V = 12.0$ cm and $H = 15.0$ cm. In this case, we find the following forces:

$$T = mg(H/V) = (2.00 \text{ kg})(9.81 \text{ m/s}^2)(15.0 \text{ cm})/(12.0 \text{ cm}) = 24.5 \text{ N}$$

$$f_x = T = 24.5 \text{ N}$$

$$f_y = mg = (2.00 \text{ kg})(9.81 \text{ m/s}^2) = 19.6 \text{ N}$$

Note that f_x and T are greater than the weight, mg , of the lamp. Just as we found with the diving board in Example 11-4, the forces required of structural elements can be greater than the weight of the object to be supported—an important consideration when designing a structure like a bridge, an airplane, or a scone. The same effect occurs in the human body. We find in Problem 25, for example, that the force exerted by the biceps to support a baseball in the hand is several times larger than the baseball's weight. Similar conclusions apply to muscles throughout the body.

In Example 11-5 we consider another system in which forces have both vertical and horizontal components.



▲ The chains that support this sign maintain it in a state of translational and rotational equilibrium. The forces in the chains are most easily analyzed by resolving them into vertical and horizontal components and applying the conditions for equilibrium. In particular, the net vertical force, the net horizontal force, and the net torque must all be zero.

REAL-WORLD PHYSICS

Forces required for structural stability



EXAMPLE 11-5 ARM IN A SLING

A hiker who has broken his forearm rigs a temporary sling using a cord stretching from his shoulder to his hand. The cord holds the forearm level and makes an angle of 40.0° with the horizontal where it attaches to the hand. Considering the forearm and hand to be uniform, with a total mass of 1.30 kg and a length of 0.300 m, find (a) the tension in the cord and (b) the horizontal and vertical components of the force, \vec{f} , exerted by the humerus (the bone of the upper arm) on the radius and ulna (the bones of the forearm).

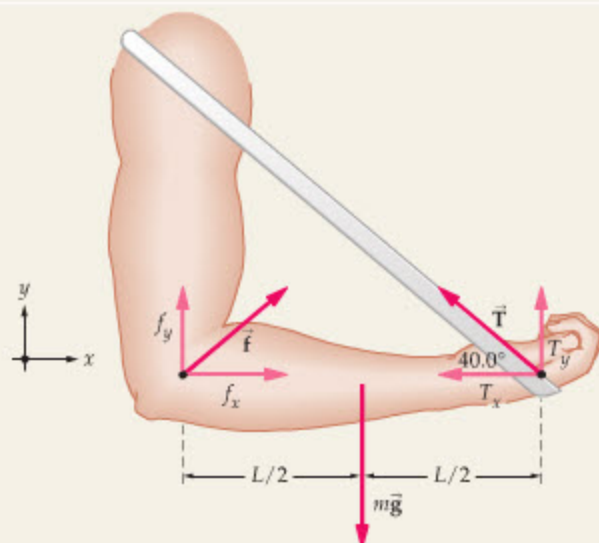
PICTURE THE PROBLEM

In our sketch, we use the typical conventions for the positive x and y directions. In addition, since the forearm and hand are assumed to be a uniform object, we indicate the weight mg as acting at its center. The length of the forearm and hand is $L = 0.300$ m. Finally, two other forces act on the forearm: (i) the tension in the cord, \vec{T} , at an angle of 40.0° above the negative x axis, and (ii) the force \vec{f} exerted at the elbow joint.

STRATEGY

In this system there are three unknowns: T , f_x , and f_y . These unknowns can be determined using the following three conditions: (i) net torque equals zero; (ii) net x component of force equals zero; and (iii) net y component of force equals zero.

We start with the torque condition, using the elbow joint as the axis of rotation. As we shall see, this choice of axis eliminates f , and gives a direct solution for the tension T . Next, we use T and the two force conditions to determine f_x and f_y .



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SOLUTION**Part (a)**

1. Calculate the torque about the elbow joint. Note that f causes zero torque, mg causes a negative torque, and the vertical component of T causes a positive torque. The horizontal component of T produces no torque, since it is on a line with the axis.
2. Solve the torque condition for the tension, T :

$$\sum \tau = (T \sin 40.0^\circ)L - mg(L/2) = 0$$

$$T = \frac{mg}{2 \sin 40.0^\circ} = \frac{(1.30 \text{ kg})(9.81 \text{ m/s}^2)}{2 \sin 40.0^\circ} = 9.92 \text{ N}$$

Part (b)

3. Set the sum of the x components of force equal to zero, and solve for f_x :
4. Set the sum of the y components of force equal to zero, and solve for f_y :

$$\sum F_x = f_x - T \cos 40.0^\circ = 0$$

$$f_x = T \cos 40.0^\circ = (9.92 \text{ N}) \cos 40.0^\circ = 7.60 \text{ N}$$

$$\sum F_y = f_y - mg + T \sin 40.0^\circ = 0$$

$$f_y = mg - T \sin 40.0^\circ$$

$$= (1.30 \text{ kg})(9.81 \text{ m/s}^2) - (9.92 \text{ N}) \sin 40.0^\circ = 6.38 \text{ N}$$

INSIGHT

It is not necessary to determine T_x and T_y separately, since we know the direction of the cord. In particular, it is clear from our sketch that the components of \vec{T} are $T_x = -T \cos 40.0^\circ = -7.60 \text{ N}$ and $T_y = T \sin 40.0^\circ = 6.38 \text{ N}$.

Did you notice that \vec{f} is at an angle of 40.0° with respect to the positive x axis, the same angle that \vec{T} makes with the negative x axis? The reason for this symmetry, of course, is that mg acts at the center of the forearm. If mg were to act closer to the elbow, for example, \vec{f} would make a larger angle with the horizontal, as we see in the following Practice Problem.

PRACTICE PROBLEM

Suppose the forearm and hand are nonuniform, and that the center of mass is located at a distance of $L/4$ from the elbow joint. What are T , f_x , and f_y in this case? [Answer: $T = 4.96 \text{ N}$, $f_x = 3.80 \text{ N}$, $f_y = 9.56 \text{ N}$. In this case, \vec{f} makes an angle of 68.3° with the horizontal.]

Some related homework problems: Problem 33, Problem 94

ACTIVE EXAMPLE 11-3 DON'T WALK UNDER THE LADDER: FIND THE FORCES

An 85-kg person stands on a lightweight ladder, as shown. The floor is rough; hence, it exerts both a normal force, f_1 , and a frictional force, f_2 , on the ladder. The wall, on the other hand, is frictionless; it exerts only a normal force, f_3 . Using the dimensions given in the figure, find the magnitudes of f_1 , f_2 , and f_3 .

SOLUTION (Test your understanding by performing the calculations indicated in each step.)

1. Set the net torque acting on the ladder equal to zero. Use the bottom of the ladder as the axis:

$$f_3(a) - mg(b) = 0$$
2. Solve for f_3 :

$$f_3 = mg(b/a) = 150 \text{ N}$$
3. Sum the x components of force and set equal to zero:

$$f_2 - f_3 = 0$$
4. Solve for f_2 :

$$f_2 = f_3 = 150 \text{ N}$$
5. Sum the y components of force and set equal to zero:

$$f_1 - mg = 0$$
6. Solve for f_1 :

$$f_1 = mg = 830 \text{ N}$$

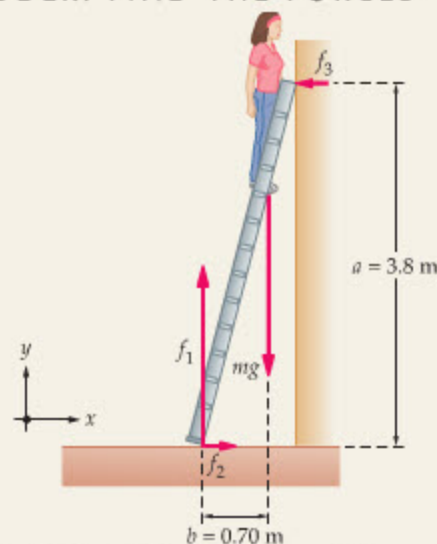
INSIGHT

If the floor is quite smooth, the ladder might slip—it depends on whether the coefficient of static friction is great enough to provide the needed force $f_2 = 150 \text{ N}$. In this case, the normal force exerted by the floor is $N = f_1 = 830 \text{ N}$. Therefore, if the coefficient of static friction is greater than 0.18 [since $0.18(830 \text{ N}) = 150 \text{ N}$], the ladder will stay put. Ladders often have rubberized pads on the bottom in order to increase the static friction, and hence increase the safety of the ladder.

YOUR TURN

Write both the zero force and zero torque conditions for the case where the axis of rotation is at the top of the ladder.

(Answers to Your Turn problems are given in the back of the book.)



11-4 Center of Mass and Balance

Suppose you decide to construct a mobile. To begin, you tie a thread to a light rod, as in **Figure 11-7**. Note that the rod extends a distance x_1 to the left of the thread and a distance x_2 to the right. At the left end of the rod you attach an object of mass m_1 . What mass, m_2 , should be attached to the right end if the rod is to be balanced?

From the discussions in the previous sections, it is clear that if the rod is to be in static equilibrium (balanced), the net torque acting on it must be zero. Taking the point where the thread is tied to the rod as the axis of rotation, this zero-torque condition can be written as:

$$m_1 g(x_1) - m_2 g(x_2) = 0$$

Canceling g and rearranging, we find

$$m_1 x_1 = m_2 x_2$$

This gives the following result for m_2 :

$$m_2 = m_1(x_1/x_2)$$

For example, if $x_2 = 2x_1$, it follows that m_2 should be one-half of m_1 .

Let's now consider a slightly different question: Where is the center of mass of m_1 and m_2 ? Choosing the origin of the x axis to be at the location of the thread, as indicated in **Figure 11-7**, we can use the definition of the center of mass, **Equation 9-13**, to find x_{cm} :

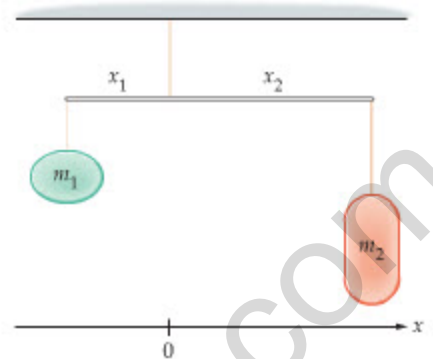
$$x_{cm} = \frac{m_1(-x_1) + m_2(x_2)}{m_1 + m_2} = -\left(\frac{m_1 x_1 - m_2 x_2}{m_1 + m_2}\right)$$

Referring to the zero-torque condition in **Equation 11-7**, we see that $m_1 x_1 - m_2 x_2 = 0$; hence the center of mass is at the origin:

$$x_{cm} = 0$$

This is precisely where the string is attached. We conclude, then, that the rod balances when the center of mass is directly below the point from which the rod is suspended. This is a general result.

Let's apply this result to the case of the mobile shown in the next Example.



▲ FIGURE 11-7 Zero torque and balance One section of a mobile. The rod is balanced when the net torque acting on it is zero. This is equivalent to having the center of mass directly under the suspension point.

EXAMPLE 11-6 A WELL-BALANCED MEAL

As a grade-school project, students construct a mobile representing some of the major food groups. Their completed artwork is shown below. Find the masses m_1 , m_2 , and m_3 that are required for a perfectly balanced mobile. Assume the strings and the horizontal rods have negligible mass.

PICTURE THE PROBLEM

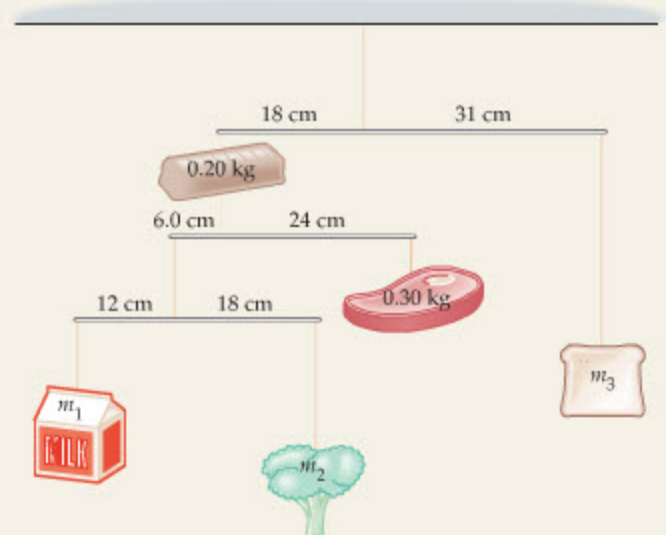
The dimensions of the horizontal rods, and the values of the given masses, are indicated in our sketch. Note that each rod is balanced at its suspension point.

STRATEGY

We can find all three unknown masses by repeatedly applying the condition for balance, $m_1 x_1 = m_2 x_2$.

First, we apply the balance condition to m_1 and m_2 , with the distances $x_1 = 12$ cm and $x_2 = 18$ cm. This gives a relation between m_1 and m_2 .

To get a second relation between m_1 and m_2 , we apply the balance condition again at the next higher level of the mobile. That is, the mass ($m_1 + m_2$) at the distance 6.0 cm must balance the mass 0.30 kg at the distance 24 cm. These two conditions determine m_1 and m_2 .



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