CONTINUED FROM PREVIOUS PAGE

INSIGHT

After the block comes to rest, the spring expands back to its equilibrium position, as shown in Figures 7–12 (d)–(f). During this expansion the force exerted by the spring is in the same direction as the block's motion, and hence it does *positive* work in the amount $W = \frac{1}{2}kx^2$. As a result, the block leaves the spring with the same speed it had initially.

YOUR TURN

Find the compression of the spring for the case where the mass of the block is doubled to 3.0 kg.

(Answers to **Your Turn** *problems are given in the back of the book.)*

7–4 Power

Power is a measure of how *quickly* work is done. To be precise, suppose the work W is performed in the time t. The average power delivered during this time is defined as follows:

Definition of Average Power, *P*

$$P = \frac{W}{t} \qquad 7\text{–}10$$

SI unit: J/s = watt, W

For simplicity of notation we drop the usual subscript av for an average quantity and simply understand that the power P refers to an average power unless stated otherwise.

Note that the dimensions of power are joules (work) per second (time). We define one joule per second to be a watt (W), after James Watt (1736–1819), the Scottish engineer and inventor who played a key role in the development of practical steam engines:

$$1 \text{ watt} = 1 \text{ W} = 1 \text{ J/s} \qquad 7\text{–}11$$

Of course, the watt is the unit of power used to rate the output of lightbulbs. Another common unit of power is the horsepower (hp), which is used to rate the output of car engines. It is defined as follows:

$$1 \text{ horsepower} = 1 \text{ hp} = 746 \text{ W} \qquad 7\text{–}12$$

Though it sounds like a horse should be able to produce one horsepower, in fact, a horse can generate only about 2/3 hp for sustained periods. The reason for the discrepancy is that when James Watt defined the horsepower—as a way to characterize the output of his steam engines—he purposely chose a unit that was overly generous to the horse, so that potential investors couldn't complain he was overstating the capability of his engines.

To get a feel for the magnitude of the watt and the horsepower, consider the power you might generate when walking up a flight of stairs. Suppose, for example, that an 80.0-kg person walks up a flight of stairs in 20.0 s, and that the altitude gain is 12.0 ft (3.66 m). Referring to Example 7–2 and Conceptual Checkpoint 7–1, we find that the work done by the person is $W = mgh = (80.0 \text{ kg})(9.81 \text{ m/s}^2)(3.66 \text{ m}) = 2870 \text{ J}$. To find the power, we simply divide by the time: $P = W/t = (2870 \text{ J})/(20.0 \text{ s}) = 144 \text{ W} = 0.193 \text{ hp}$. Thus, a leisurely stroll up the stairs requires about 1/5 hp or 150 W. Similarly, the power produced by a sprinter bolting out of the starting blocks is about 1 hp, and the greatest power most people can produce for sustained periods of time is roughly 1/3 to 1/2 hp. Further examples of power are given in Table 7–3.

TABLE 7–3 Typical Values of Power

Source	Approximate power (W)
Hoover Dam	1.34×10^9
Car moving at 40 mi/h	7×10^4
Home stove	1.2×10^4
Sunlight falling on one square meter	1380
Refrigerator	615
Television	200
Person walking up stairs	150
Human brain	20

REAL-WORLD PHYSICS: BIO

Human power output and flight

▲ The *Gossamer Albatross* on its record-breaking flight across the English Channel in 1979. On two occasions the aircraft actually touched the surface of the water, but the pilot was able to maintain control and complete the 22.25-mile flight.

Human-powered flight is a feat just barely within our capabilities, since the most efficient human-powered airplanes require a steady power output of about 1/3 hp. On August 23, 1977, the *Gossamer Condor*, designed by Paul MacCready and flown by Bryan Allen, became the first human-powered airplane to complete a prescribed one-mile, figure-eight course and claim the Kremer Prize of £50,000. Allen, an accomplished bicycle racer, used bicycle-like pedals to spin the propeller. Controlling the slow-moving craft while pedaling at full power was no easy task. Allen also piloted the *Gossamer Albatross*, which, in 1979, became the first (and so far the only) human-powered aircraft to fly across the English Channel. This 22.25-mile flight—from Folkestone, England, to Cap Gris-Nez, France—took 2 hours 49 minutes and required a total energy output roughly equivalent to climbing to the top of the Empire State Building 10 times.

Power output is also an important factor in the performance of a car. For example, suppose it takes a certain amount of work, W, to accelerate a car from 0 to 60 mi/h. If the average power provided by the engine is P, then according to Equation 7–10 the amount of time required to reach 60 mi/h is $t = W/P$. Clearly, the greater the power P, the less the time required to accelerate. Thus, in a loose way of speaking, we can say that the power of a car is a measure of "how fast it can go fast."

EXAMPLE 7–8 PASSING FANCY

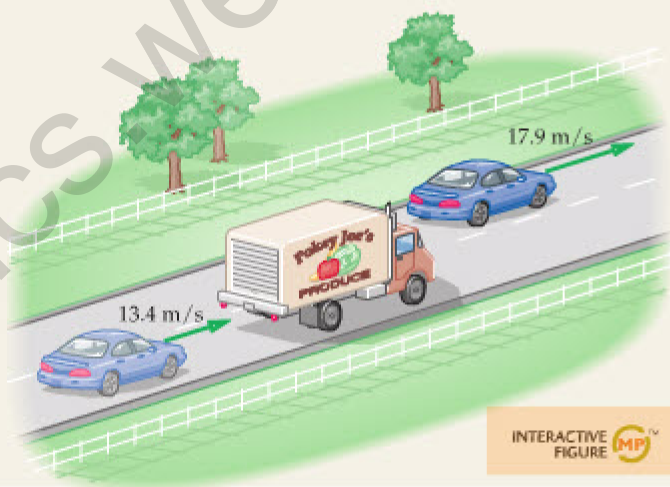
To pass a slow-moving truck, you want your fancy 1.30×10^3-kg car to accelerate from 13.4 m/s (30.0 mi/h) to 17.9 m/s (40.0 mi/h) in 3.00 s. What is the minimum power required for this pass?

PICTURE THE PROBLEM

Our sketch shows the car accelerating from an initial speed of $v_i = 13.4$ m/s to a final speed of $v_f = 17.9$ m/s. We assume the road is level, so that no work is done against gravity, and that friction and air resistance may be ignored.

STRATEGY

Power is work divided by time, and work is equal to the change in kinetic energy as the car accelerates. We can determine the change in kinetic energy from the given mass of the car and its initial and final speeds. With this information at hand, we can determine the power with the relation $P = W/t = \Delta K/t$.



SOLUTION

1. First, calculate the change in kinetic energy:

$$\Delta K = \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2 = \tfrac{1}{2}(1.30 \times 10^3\ \text{kg})(17.9\ \text{m/s})^2 - \tfrac{1}{2}(1.30 \times 10^3\ \text{kg})(13.4\ \text{m/s})^2 = 9.16 \times 10^4\ \text{J}$$

2. Divide by time to find the minimum power. (The actual power would have to be greater to overcome frictional losses.):

$$P = \frac{W}{t} = \frac{\Delta K}{t} = \frac{9.16 \times 10^4\ \text{J}}{3.00\ \text{s}} = 3.05 \times 10^4\ \text{W} = 40.9\ \text{hp}$$

INSIGHT

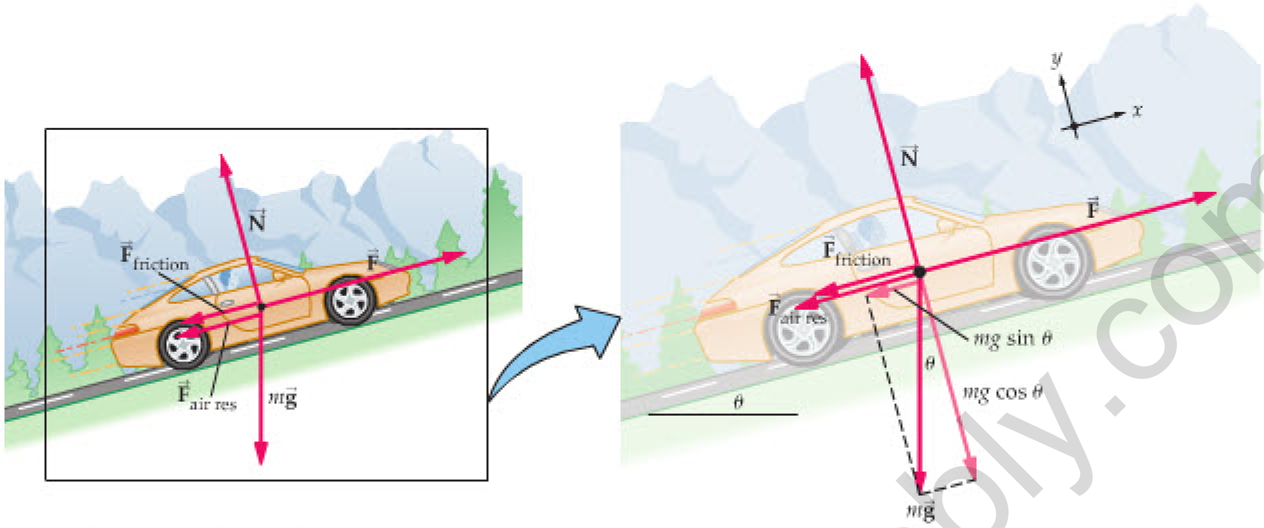
Suppose that your fancy car continues to produce the same 3.05×10^4 W of power as it accelerates from $v = 17.9$ m/s (40.0 mi/h) to $v = 22.4$ ms (50.0 mi/h). Is the time required more than, less than, or equal to 3.00 s? It will take more than 3.00 s. The reason is that ΔK is greater for a change in speed from 40.0 mi/h to 50.0 mi/h than for a change in speed from 30.0 mi/h to 40.0 mi/h, because K depends on speed squared. Since ΔK is greater, the time $t = \Delta K/P$ is also greater.

PRACTICE PROBLEM

Find the time required to accelerate from 40.0 mi/h to 50.0 mi/h with 3.05×10^4 W of power. [**Answer:** First, $\Delta K = 1.18 \times 10^5$ J. Second, $P = \Delta K/t$ can be solved for time to give $t = \Delta K/P$. Thus, $t = 3.87$ s.]

Some related homework problems: Problem 44, Problem 59

Finally, consider a system in which a car, or some other object, is moving with a constant speed v. For example, a car might be traveling uphill on a road inclined at an angle θ above the horizontal. To maintain a constant speed, the engine must exert a constant force F equal to the combined effects of friction, gravity, and air



▲ **FIGURE 7–13 Driving up a hill**
A car traveling uphill at constant speed requires a constant force, F, of magnitude $mg \sin \theta + F_{\text{air res}} + F_{\text{friction}}$, applied in the direction of motion.

resistance, as indicated in Figure 7–13. Now, as the car travels a distance d, the work done by the engine is $W = Fd$, and the power it delivers is

$$P = \frac{W}{t} = \frac{Fd}{t}$$

Since the car has a constant speed, $v = d/t$, it follows that

$$P = \frac{Fd}{t} = F\left(\frac{d}{t}\right) = Fv \qquad 7\text{–}13$$

Note that power is directly proportional to both the force and the speed. For example, suppose you push a heavy shopping cart with a force F. You produce twice as much power when you push at 2 m/s than when you push at 1 m/s, even though you are pushing no harder. It's just that the amount of work you do in a given time period is doubled.

ACTIVE EXAMPLE 7–2 FIND THE MAXIMUM SPEED

It takes a force of 1280 N to keep a 1500-kg car moving with constant speed up a slope of 5.00°. If the engine delivers 50.0 hp to the drive wheels, what is the maximum speed of the car?

SOLUTION *(Test your understanding by performing the calculations indicated in each step.)*

1. Convert the power of 50.0 hp to watts: $P = 3.73 \times 10^4$ W
2. Solve Equation 7–13 for the speed v: $v = P/F$
3. Substitute numerical values for the power and force: $v = 29.1$ m/s

INSIGHT
Thus, the maximum speed of the car on this slope is approximately 65 mi/h.

YOUR TURN
How much power is required for a maximum speed of 32.0 m/s?

*(Answers to **Your Turn** problems are given in the back of the book.)*

THE BIG PICTURE PUTTING PHYSICS IN CONTEXT

LOOKING BACK

Even though work and kinetic energy are scalar quantities, the idea of vectors, and vector components in particular (Chapter 3), was used in the definition of work in Section 7–1.

The kinematic equations of motion for constant acceleration (Chapters 2 and 4) were used in the derivation of kinetic energy in Section 7–2. In particular, we used the relation between the speed of an object and the distance through which it accelerates.

The basic concepts of force, mass, and acceleration (Chapters 5 and 6) were used throughout this chapter. One particular force, the force exerted by a spring (Chapter 6), played a key role in Section 7–3.

LOOKING AHEAD

In Chapter 8 we introduce the concept of potential energy. The combination of kinetic and potential energy is referred to as the mechanical energy, which will play a central role in our discussion of the conservation of energy.

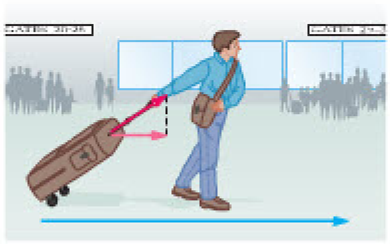
Collisions are studied in Chapter 9. As we shall see, the kinetic energy before and after a collision is an important characterizing feature. Look for the discussion of elastic versus inelastic collision in particular.

The concept of kinetic energy plays a significant role in many areas of physics. Look for it to reappear when we study rotational motion in Chapter 10, and in Section 10–5 in particular. Kinetic energy is also important when we study ideal gases in Chapter 17—in fact, Section 17–2 is titled Kinetic Theory.

CHAPTER SUMMARY

7–1 WORK DONE BY A CONSTANT FORCE

A force exerted through a distance performs mechanical work.



Force in Direction of Motion
In this, the simplest case, work is force times distance:

$$W = Fd \qquad 7\text{–}1$$

Force at an Angle θ to Motion
Work is the component of force in the direction of motion, $F \cos\theta$, times distance, d:

$$W = (F\cos\theta)d = Fd\cos\theta \qquad 7\text{–}3$$

Negative and Total Work
Work is negative if the force opposes the motion; that is, if $\theta > 90°$. If more than one force does work, the total work is the sum of the works done by each force separately:

$$W_{\text{total}} = W_1 + W_2 + W_3 + \cdots \qquad 7\text{–}4$$

Equivalently, sum the forces first to find F_{total}, then

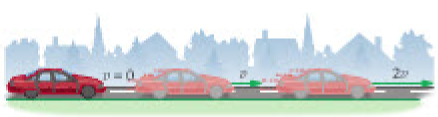
$$W_{\text{total}} = (F_{\text{total}}\cos\theta)d = F_{\text{total}}\, d\cos\theta \qquad 7\text{–}5$$

Units
The SI unit of work and energy is the joule, J:

$$1\,\text{J} = 1\,\text{N}\cdot\text{m} \qquad 7\text{–}2$$

7–2 KINETIC ENERGY AND THE WORK–ENERGY THEOREM

Total work is equal to the change in kinetic energy:



$$W_{\text{total}} = \Delta K = \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2 \qquad 7\text{–}7$$

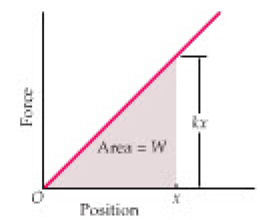
Note: To apply this theorem correctly, you must use the *total* work. Kinetic energy is one-half mass times speed squared:

$$K = \tfrac{1}{2}mv^2 \qquad 7\text{–}6$$

It follows that kinetic energy is always positive or zero.

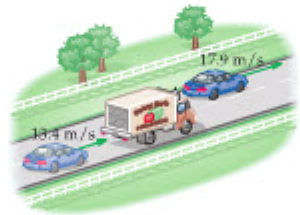
7–3 WORK DONE BY A VARIABLE FORCE

Work is equal to the area between the force curve and the displacement on the x axis. For the case of a spring force, the work to stretch or compress a distance x from equilibrium is



$$W = \tfrac{1}{2}kx^2 \qquad 7\text{–}8$$

7–4 POWER



Average power is work divided by the time required to do the work:

$$P = \frac{W}{t} \qquad 7\text{–}10$$

Equivalently, power is force times speed:

$$P = Fv \qquad 7\text{–}13$$

Units

The SI unit of power is the watt, W:

$$1\ \text{W} = 1\ \text{J/s} \qquad 7\text{–}11$$

$$746\ \text{W} = 1\ \text{hp} \qquad 7\text{–}12$$

PROBLEM-SOLVING SUMMARY

Type of Calculation	Relevant Physical Concepts	Related Examples
Find the work done by a constant force.	Work is defined as force times displacement, $W = Fd$, when F is in the direction of motion. Use $W = (F\cos\theta)d$ when there is an angle θ between the force and the direction of motion.	Examples 7–1 through 7–6
Calculate the change in speed.	The change in kinetic energy is given by the work–energy theorem, $W_{\text{total}} = \Delta K$. From this, the change in speed can be found by recalling that $K = \frac{1}{2}mv^2$. Be sure W_{total} is the total work and that it has the correct sign.	Examples 7–5, 7–6
Calculate the power.	Find the work done, then divide by time: $P = W/t$. Alternatively, find the force, then multiply by the speed: $P = Fv$.	Example 7–8 Active Example 7–2

CONCEPTUAL QUESTIONS

For instructor-assigned homework, go to www.masteringphysics.com MP™

(Answers to odd-numbered Conceptual Questions can be found in the back of the book.)

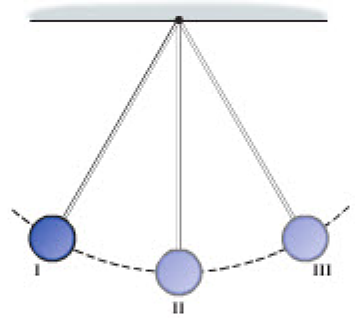
1. Is it possible to do work on an object that remains at rest?
2. A friend makes the statement, "Only the total force acting on an object can do work." Is this statement true or false? If it is true, state why; if it is false, give a counterexample.
3. A friend makes the statement, "A force that is always perpendicular to the velocity of a particle does no work on the particle." Is this statement true or false? If it is true, state why; if it is false, give a counterexample.
4. The net work done on a certain object is zero. What can you say about its speed?
5. To get out of bed in the morning, do you have to do work? Explain.
6. Give an example of a frictional force doing negative work.
7. Give an example of a frictional force doing positive work.
8. A ski boat moves with constant velocity. Is the net force acting on the boat doing work? Explain.
9. A package rests on the floor of an elevator that is rising with constant speed. The elevator exerts an upward normal force on the package, and hence does positive work on it. Why doesn't the kinetic energy of the package increase?
10. An object moves with constant velocity. Is it safe to conclude that no force acts on the object? Why, or why not?
11. Engine 1 does twice the work of engine 2. Is it correct to conclude that engine 1 produces twice as much power as engine 2? Explain.
12. Engine 1 produces twice the power of engine 2. Is it correct to conclude that engine 1 does twice as much work as engine 2? Explain.

PROBLEMS AND CONCEPTUAL EXERCISES

Note: Answers to odd-numbered Problems and Conceptual Exercises can be found in the back of the book. **IP** *denotes an integrated problem, with both conceptual and numerical parts;* **BIO** *identifies problems of biological or medical interest;* **CE** *indicates a conceptual exercise.* **Predict/Explain** *problems ask for two responses:* **(a)** *your prediction of a physical outcome, and* **(b)** *the best explanation among three provided. On all problems, red bullets (•, ••, •••) are used to indicate the level of difficulty.*

SECTION 7–1 WORK DONE BY A CONSTANT FORCE

1. • **CE** The International Space Station orbits the Earth in an approximately circular orbit at a height of $h = 375$ km above the Earth's surface. In one complete orbit, is the work done by the Earth on the space station positive, negative, or zero? Explain.
2. • **CE** A pendulum bob swings from point I to point II along the circular arc indicated in Figure 7–14. **(a)** Is the work done on the bob by gravity positive, negative, or zero? Explain. **(b)** Is the work done on the bob by the string positive, negative, or zero? Explain.
3. • **CE** A pendulum bob swings from point II to point III along the circular arc indicated in Figure 7–14. **(a)** Is the work done on



▲ **FIGURE 7–14** Problems 2 and 3

the bob by gravity positive, negative, or zero? Explain. **(b)** Is the work done on the bob by the string positive, negative, or zero? Explain.

4. • A farmhand pushes a 26-kg bale of hay 3.9 m across the floor of a barn. If she exerts a horizontal force of 88 N on the hay, how much work has she done?

5. • Children in a tree house lift a small dog in a basket 4.70 m up to their house. If it takes 201 J of work to do this, what is the combined mass of the dog and basket?

6. • Early one October, you go to a pumpkin patch to select your Halloween pumpkin. You lift the 3.2-kg pumpkin to a height of 1.2 m, then carry it 50.0 m (on level ground) to the check-out stand. **(a)** Calculate the work you do on the pumpkin as you lift it from the ground. **(b)** How much work do you do on the pumpkin as you carry it from the field?

7. • The coefficient of kinetic friction between a suitcase and the floor is 0.272. If the suitcase has a mass of 71.5 kg, how far can it be pushed across the level floor with 642 J of work?

8. •• You pick up a 3.4-kg can of paint from the ground and lift it to a height of 1.8 m. **(a)** How much work do you do on the can of paint? **(b)** You hold the can stationary for half a minute, waiting for a friend on a ladder to take it. How much work do you do during this time? **(c)** Your friend decides against the paint, so you lower it back to the ground. How much work do you do on the can as you lower it?

9. •• **IP** A tow rope, parallel to the water, pulls a water skier directly behind the boat with constant velocity for a distance of 65 m before the skier falls. The tension in the rope is 120 N. **(a)** Is the work done on the skier by the rope positive, negative, or zero? Explain. **(b)** Calculate the work done by the rope on the skier.

10. •• **IP** In the situation described in the previous problem, **(a)** is the work done on the boat by the rope positive, negative, or zero? Explain. **(b)** Calculate the work done by the rope on the boat.

11. •• A child pulls a friend in a little red wagon with constant speed. If the child pulls with a force of 16 N for 10.0 m, and the handle of the wagon is inclined at an angle of 25° above the horizontal, how much work does the child do on the wagon?

12. •• A 51-kg packing crate is pulled with constant speed across a rough floor with a rope that is at an angle of 43.5° above the horizontal. If the tension in the rope is 115 N, how much work is done on the crate to move it 8.0 m?

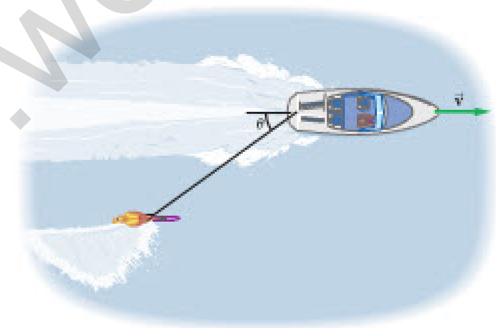
13. •• **IP** To clean a floor, a janitor pushes on a mop handle with a force of 50.0 N. **(a)** If the mop handle is at an angle of 55° above the horizontal, how much work is required to push the mop 0.50 m? **(b)** If the angle the mop handle makes with the horizontal is increased to 65°, does the work done by the janitor increase, decrease, or stay the same? Explain.

14. •• A small plane tows a glider at constant speed and altitude. If the plane does 2.00×10^5 J of work to tow the glider 145 m and the tension in the tow rope is 2560 N, what is the angle between the tow rope and the horizontal?

15. •• A young woman on a skateboard is pulled by a rope attached to a bicycle. The velocity of the skateboarder is $\vec{v} = (4.1\text{ m/s})\hat{x}$ and the force exerted on her by the rope is $\vec{F} = (17\text{ N})\hat{x} + (12\text{ N})\hat{y}$. **(a)** Find the work done on the skateboarder by the rope in 25 seconds. **(b)** Assuming the velocity of the bike is the same as that of the skateboarder, find the work the rope does on the bicycle in 25 seconds.

16. •• To keep her dog from running away while she talks to a friend, Susan pulls gently on the dog's leash with a constant force given by $\vec{F} = (2.2\text{ N})\hat{x} + (1.1\text{ N})\hat{y}$. How much work does she do on the dog if its displacement is **(a)** $\vec{d} = (0.25\text{ m})\hat{x}$, **(b)** $\vec{d} = (0.25\text{ m})\hat{y}$, or **(c)** $\vec{d} = (-0.50\text{ m})\hat{x} + (-0.25\text{ m})\hat{y}$?

17. •• Water skiers often ride to one side of the center line of a boat, as shown in Figure 7–15. In this case, the ski boat is traveling at 15 m/s and the tension in the rope is 75 N. If the boat does 3500 J of work on the skier in 50.0 m, what is the angle θ between the tow rope and the center line of the boat?



▲ **FIGURE 7–15** Problems 17 and 69

SECTION 7–2 KINETIC ENERGY AND THE WORK–ENERGY THEOREM

18. • **CE** A pitcher throws a ball at 90 mi/h and the catcher stops it in her glove. **(a)** Is the work done on the ball by the pitcher positive, negative, or zero? Explain. **(b)** Is the work done on the ball by the catcher positive, negative, or zero? Explain.

19. • How much work is needed for a 73-kg runner to accelerate from rest to 7.7 m/s?

20. • **Skylab's Reentry** When Skylab reentered the Earth's atmosphere on July 11, 1979, it broke into a myriad of pieces. One of the largest fragments was a 1770-kg lead-lined film vault, and it landed with an estimated speed of 120 m/s. What was the kinetic energy of the film vault when it landed?

21. • **IP** A 9.50-g bullet has a speed of 1.30 km/s. **(a)** What is its kinetic energy in joules? **(b)** What is the bullet's kinetic energy if its speed is halved? **(c)** If its speed is doubled?

22. •• **CE Predict/Explain** The work W_0 accelerates a car from 0 to 50 km/h. **(a)** Is the work required to accelerate the car from 50 km/h to 150 km/h equal to $2W_0$, $3W_0$, $8W_0$, or $9W_0$? **(b)** Choose the *best explanation* from among the following:
    I. The work to accelerate the car depends on the speed squared.
    II. The final speed is three times the speed that was produced by the work W_0.
    III. The increase in speed from 50 km/h to 150 km/h is twice the increase in speed from 0 to 50 km/h.

23. •• **CE** Jogger A has a mass m and a speed v, jogger B has a mass $m/2$ and a speed $3v$, jogger C has a mass $3m$ and a speed $v/2$, and jogger D has a mass $4m$ and a speed $v/2$. Rank the joggers in order of increasing kinetic energy. Indicate ties where appropriate.

24. •• **IP** A 0.14-kg pinecone falls 16 m to the ground, where it lands with a speed of 13 m/s. **(a)** With what speed would the pinecone have landed if there had been no air resistance? **(b)** Did air resistance do positive work, negative work, or zero work on the pinecone? Explain.

25. •• In the previous problem, **(a)** how much work was done on the pinecone by air resistance? **(b)** What was the average force of air resistance exerted on the pinecone?

26. •• At $t = 1.0$ s, a 0.40-kg object is falling with a speed of 6.0 m/s. At $t = 2.0$ s, it has a kinetic energy of 25 J. **(a)** What is the kinetic energy of the object at $t = 1.0$ s? **(b)** What is the speed of the object at $t = 2.0$ s? **(c)** How much work was done on the object between $t = 1.0$ s and $t = 2.0$ s?

27. •• After hitting a long fly ball that goes over the right fielder's head and lands in the outfield, the batter decides to keep going past second base and try for third base. The 62.0-kg player begins sliding 3.40 m from the base with a speed of 4.35 m/s. If the player comes to rest at third base, **(a)** how much work was done on the player by friction? **(b)** What was the coefficient of kinetic friction between the player and the ground?

28. •• **IP** A 1100-kg car coasts on a horizontal road with a speed of 19 m/s. After crossing an unpaved, sandy stretch of road 32 m long, its speed decreases to 12 m/s. **(a)** Was the net work done on the car positive, negative, or zero? Explain. **(b)** Find the magnitude of the average net force on the car in the sandy section.

29. •• **IP** **(a)** In the previous problem, the car's speed decreased by 7.0 m/s as it coasted across a sandy section of road 32 m long. If the sandy portion of the road had been only 16 m long, would the car's speed have decreased by 3.5 m/s, more than 3.5 m/s, or less than 3.5 m/s? Explain. **(b)** Calculate the change in speed in this case.

30. •• A 65-kg bicyclist rides his 8.8-kg bicycle with a speed of 14 m/s. **(a)** How much work must be done by the brakes to bring the bike and rider to a stop? **(b)** How far does the bicycle travel if it takes 4.0 s to come to rest? **(c)** What is the magnitude of the braking force?

SECTION 7–3 WORK DONE BY A VARIABLE FORCE

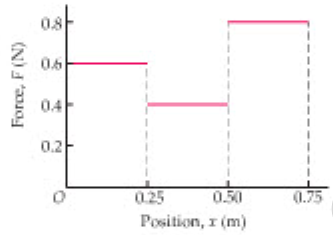
31. • **CE** A block of mass m and speed v collides with a spring, compressing it a distance Δx. What is the compression of the spring if the force constant of the spring is increased by a factor of four?

32. • A spring with a force constant of 3.5×10^4 N/m is initially at its equilibrium length. **(a)** How much work must you do to stretch the spring 0.050 m? **(b)** How much work must you do to compress it 0.050 m?

33. • A 1.2-kg block is held against a spring of force constant 1.0×10^4 N/m, compressing it a distance of 0.15 m. How fast is the block moving after it is released and the spring pushes it away?

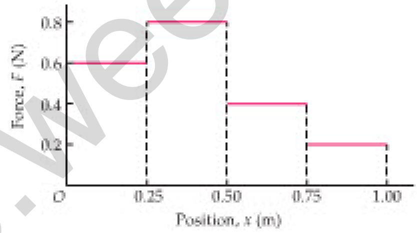
34. • Initially sliding with a speed of 2.2 m/s, a 1.8-kg block collides with a spring and compresses it 0.31 m before coming to rest. What is the force constant of the spring?

35. • The force shown in Figure 7–16 moves an object from $x = 0$ to $x = 0.75$ m. **(a)** How much work is done by the force? **(b)** How much work is done by the force if the object moves from $x = 0.15$ m to $x = 0.60$ m?



▲ **FIGURE 7–16** Problem 35

36. • An object is acted on by the force shown in Figure 7–17. What is the final position of the object if its initial position is $x = 0.40$ m and the work done on it is equal to **(a)** 0.21 J, or **(b)** −0.19 J?



▲ **FIGURE 7–17** Problems 36 and 40

37. •• **CE** A block of mass m and speed v collides with a spring, compressing it a distance Δx. What is the compression of the spring if the mass of the block is halved and its speed is doubled?

38. •• To compress spring 1 by 0.20 m takes 150 J of work. Stretching spring 2 by 0.30 m requires 210 J of work. Which spring is stiffer?

39. •• **IP** It takes 180 J of work to compress a certain spring 0.15 m. **(a)** What is the force constant of this spring? **(b)** To compress the spring an additional 0.15 m, does it take 180 J, more than 180 J, or less than 180 J? Verify your answer with a calculation.

40. •• The force shown in Figure 7–17 acts on a 1.7-kg object whose initial speed is 0.44 m/s and initial position is $x = 0.27$ m. **(a)** Find the speed of the object when it is at the location $x = 0.99$ m. **(b)** At what location would the object's speed be 0.32 m/s?

41. ••• A block is acted on by a force that varies as $(2.0 \times 10^4\ \text{N/m})x$ for $0 \le x \le 0.21$ m, and then remains constant at 4200 N for larger x. How much work does the force do on the block in moving it **(a)** from $x = 0$ to $x = 0.30$ m, or **(b)** from $x = 0.10$ m to $x = 0.40$ m?

SECTION 7–4 POWER

42. • **CE** Force F_1 does 5 J of work in 10 seconds, force F_2 does 3 J of work in 5 seconds, force F_3 does 6 J of work in 18 seconds, and force F_4 does 25 J of work in 125 seconds. Rank these forces in order of increasing power they produce. Indicate ties where appropriate.

43. • **BIO** **Climbing the Empire State Building** A new record for running the stairs of the Empire State Building was set on February 3, 2003. The 86 flights, with a total of 1576 steps, was run in 9 minutes and 33 seconds. If the height gain of each step was 0.20 m, and the mass of the runner was 70.0 kg, what was his average power output during the climb? Give your answer in both watts and horsepower.

44. • How many joules of energy are in a kilowatt-hour?

45. • Calculate the power output of a 1.4-g fly as it walks straight up a windowpane at 2.3 cm/s.

46. • An ice cube is placed in a microwave oven. Suppose the oven delivers 105 W of power to the ice cube and that it takes 32,200 J to melt it. How long does it take for the ice cube to melt?

47. • You raise a bucket of water from the bottom of a deep well. If your power output is 108 W, and the mass of the bucket and the water in it is 5.00 kg, with what speed can you raise the bucket? Ignore the weight of the rope.

48. •• In order to keep a leaking ship from sinking, it is necessary to pump 12.0 lb of water each second from below deck up a height of 2.00 m and over the side. What is the minimum horsepower motor that can be used to save the ship?

49. •• **IP** A kayaker paddles with a power output of 50.0 W to maintain a steady speed of 1.50 m/s. **(a)** Calculate the resistive force exerted by the water on the kayak. **(b)** If the kayaker doubles her power output, and the resistive force due to the water remains the same, by what factor does the kayaker's speed change?

50. •• **BIO Human-Powered Flight** Human-powered aircraft require a pilot to pedal, as in a bicycle, and produce a sustained power output of about 0.30 hp. The *Gossamer Albatross* flew across the English Channel on June 12, 1979, in 2 h 49 min. **(a)** How much energy did the pilot expend during the flight? **(b)** How many Snickers candy bars (280 Cal per bar) would the pilot have to consume to be "fueled up" for the flight? [*Note:* The nutritional calorie, 1 Cal, is equivalent to 1000 calories (1000 cal) as defined in physics. In addition, the conversion factor between calories and joules is as follows: 1 Cal = 1000 cal = 1 kcal = 4186 J.]

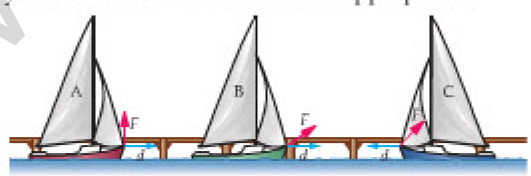
51. •• **IP** A grandfather clock is powered by the descent of a 4.35-kg weight. **(a)** If the weight descends through a distance of 0.760 m in 3.25 days, how much power does it deliver to the clock? **(b)** To increase the power delivered to the clock, should the time it takes for the mass to descend be increased or decreased? Explain.

52. •• **BIO The Power You Produce** Estimate the power you produce in running up a flight of stairs. Give your answer in horsepower.

53. ••• **IP** A certain car can accelerate from rest to the speed v in T seconds. If the power output of the car remains constant, **(a)** how long does it take for the car to accelerate from v to $2v$? **(b)** How fast is the car moving at $2T$ seconds after starting?

GENERAL PROBLEMS

54. • **CE** As the three small sailboats shown in Figure 7–18 drift next to a dock, because of wind and water currents, students pull on a line attached to the bow and exert forces of equal magnitude F. Each boat drifts through the same distance d. Rank the three boats (A, B, and C) in order of increasing work done on the boat by the force F. Indicate ties where appropriate.



▲ **FIGURE 7–18** Problem 54

55. • **CE** A youngster rides on a skateboard with a speed of 2 m/s. After a force acts on the youngster, her speed is 3 m/s. Was the work done by the force positive, negative, or zero? Explain.

56. • **CE Predict/Explain** A car is accelerated by a constant force, F. The distance required to accelerate the car from rest to the speed v is Δx. **(a)** Is the distance required to accelerate the car from the speed v to the speed $2v$ equal to Δx, $2\Delta x$, $3\Delta x$, or $4\Delta x$? **(b)** Choose the *best explanation* from among the following:

I. The final speed is twice the initial speed.
II. The increase in speed is the same in each case.
III. Work is force times distance, and work depends on the speed squared.

57. • **CE** Car 1 has four times the mass of car 2, but they both have the same kinetic energy. If the speed of car 2 is v, is the speed of car 1 equal to $v/4$, $v/2$, $2v$, or $4v$? Explain.

58. • **BIO Muscle Cells** Biological muscle cells can be thought of as nanomotors that use the chemical energy of ATP to produce mechanical work. Measurements show that the active proteins within a muscle cell (such as myosin and actin) can produce a force of about 7.5 pN and displacements of 8.0 nm. How much work is done by such proteins?

59. • When you take a bite out of an apple, you do about 19 J of work. Estimate **(a)** the force and **(b)** the power produced by your jaw muscles during the bite.

60. • A Mountain bar has a mass of 0.045 kg and a calorie rating of 210 Cal. What speed would this candy bar have if its kinetic energy were equal to its metabolic energy? [See the note following Problem 50.]

61. • A small motor runs a lift that raises a load of bricks weighing 836 N to a height of 10.7 m in 23.2 s. Assuming that the bricks are lifted with constant speed, what is the minimum power the motor must produce?

62. • You push a 67-kg box across a floor where the coefficient of kinetic friction is $\mu_k = 0.55$. The force you exert is horizontal. **(a)** How much power is needed to push the box at a speed of 0.50 m/s? **(b)** How much work do you do if you push the box for 35 s?

63. • **BIO The Beating Heart** The average power output of the human heart is 1.33 watts. **(a)** How much energy does the heart produce in a day? **(b)** Compare the energy found in part (a) with the energy required to walk up a flight of stairs. Estimate the height a person could attain on a set of stairs using nothing more than the daily energy produced by the heart.

64. • **The Atmos Clock** The Atmos clock (the so-called perpetual motion clock) gets its name from the fact that it runs off pressure variations in the atmosphere, which drive a bellows containing a mixture of gas and liquid ethyl chloride. Because the power to drive these clocks is so limited, they must be very efficient. In fact, a single 60.0-W lightbulb could power 240 million Atmos clocks simultaneously. Find the amount of energy, in joules, required to run an Atmos clock for one day.

65. •• **CE** The work W_0 is required to accelerate a car from rest to the speed v_0. How much work is required to accelerate the car **(a)** from rest to the speed $v_0/2$ and **(b)** from $v_0/2$ to v_0?

66. •• **CE** A work W_0 is required to stretch a certain spring 2 cm from its equilibrium position. **(a)** How much work is required to stretch the spring 1 cm from equilibrium? **(b)** Suppose the spring is already stretched 2 cm from equilibrium. How much additional work is required to stretch it to 3 cm from equilibrium?

67. •• After a tornado, a 0.55-g straw was found embedded 2.3 cm into the trunk of a tree. If the average force exerted on the straw by the tree was 65 N, what was the speed of the straw when it hit the tree?

68. •• You throw a glove straight upward to celebrate a victory. Its initial kinetic energy is K and it reaches a maximum height h. What is the kinetic energy of the glove when it is at the height $h/2$?

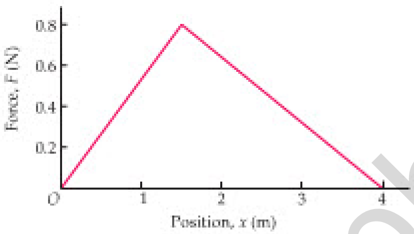
69. •• The water skier in Figure 7–15 is at an angle of 35° with respect to the center line of the boat, and is being pulled at a constant speed of 14 m/s. If the tension in the tow rope is 90.0 N, **(a)** how much work does the rope do on the skier in 10.0 s? **(b)** How much work does the resistive force of water do on the skier in the same time?

70. •• **IP** A sled with a mass of 5.80 kg is pulled along the ground through a displacement given by $\vec{\mathbf{d}} = (4.55\text{ m})\hat{\mathbf{x}}$. (Let the x axis be horizontal and the y axis be vertical.) **(a)** How much work is done on the sled when the force acting on it is $\vec{\mathbf{F}} = (2.89\text{ N})\hat{\mathbf{x}} + (0.131\text{ N})\hat{\mathbf{y}}$? **(b)** How much work is done on the sled when the force acting on it is $\vec{\mathbf{F}} = (2.89\text{ N})\hat{\mathbf{x}} + (0.231\text{ N})\hat{\mathbf{y}}$? **(c)** If the mass of the sled is increased, does the work done by the forces in parts (a) and (b) increase, decrease, or stay the same? Explain.

71. •• **IP** A 0.19-kg apple falls from a branch 3.5 m above the ground. **(a)** Does the power delivered to the apple by gravity increase, decrease, or stay the same during the time the apple falls to the ground? Explain. Find the power delivered by gravity to the apple when the apple is **(b)** 2.5 m and **(c)** 1.5 m above the ground.

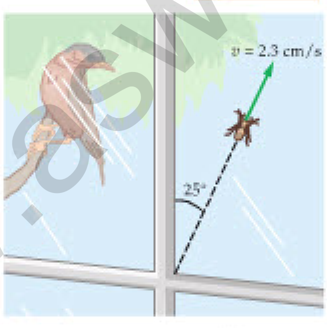
72. •• A juggling ball of mass m is thrown straight upward from an initial height h with an initial speed v_0. How much work has gravity done on the ball **(a)** when it reaches its greatest height, h_{max}, and **(b)** when it reaches ground level? **(c)** Find an expression for the kinetic energy of the ball as it lands.

73. •• The force shown in Figure 7–19 acts on an object that moves along the x axis. How much work is done by the force as the object moves from **(a)** $x = 0$ to $x = 2.0$ m, **(b)** $x = 1.0$ m to $x = 4.0$ m, and **(c)** $x = 3.5$ m to $x = 1.2$ m?



▲ **FIGURE 7–19** Problem 73

74. •• Calculate the power output of a 1.8-g spider as it walks up a windowpane at 2.3 cm/s. The spider walks on a path that is at 25° to the vertical, as illustrated in Figure 7–20.



▲ **FIGURE 7–20** Problem 74

75. •• The motor of a ski boat produces a power of 36,600 W to maintain a constant speed of 14.0 m/s. To pull a water skier at the same constant speed, the motor must produce a power of 37,800 W. What is the tension in the rope pulling the skier?

76. •• **Cookie Power** To make a batch of cookies, you mix half a bag of chocolate chips into a bowl of cookie dough, exerting a 21-N force on the stirring spoon. Assume that your force is always in the direction of motion of the spoon. **(a)** What power is needed to move the spoon at a speed of 0.23 m/s? **(b)** How much work do you do if you stir the mixture for 1.5 min?

77. •• **IP** A pitcher accelerates a 0.14-kg hardball from rest to 42.5 m/s in 0.060 s. **(a)** How much work does the pitcher do on the ball? **(b)** What is the pitcher's power output during the pitch? **(c)** Suppose the ball reaches 42.5 m/s in less than 0.060 s. Is the power produced by the pitcher in this case more than, less than, or the same as the power found in part (b)? Explain.

78. •• **Catapult Launcher** A catapult launcher on an aircraft carrier accelerates a jet from rest to 72 m/s. The work done by the catapult during the launch is 7.6×10^7 J. **(a)** What is the mass of the jet? **(b)** If the jet is in contact with the catapult for 2.0 s, what is the power output of the catapult?

79. •• **BIO Brain Power** The human brain consumes about 22 W of power under normal conditions, though more power may be required during exams. **(a)** How long can one Snickers bar (see the note following Problem 50) power the normally functioning brain? **(b)** At what rate must you lift a 3.6-kg container of milk (one gallon) if the power output of your arm is to be 22 W? **(c)** How long does it take to lift the milk container through a distance of 1.0 m at this rate?

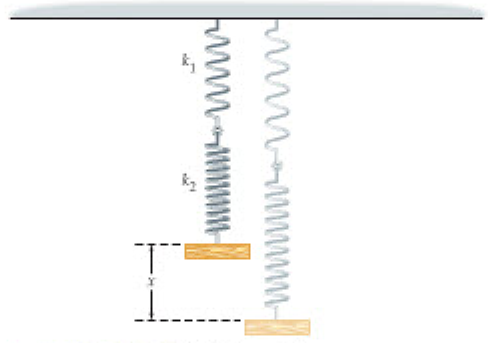
80. •• **IP** A 1300-kg car delivers a constant 49 hp to the drive wheels. We assume the car is traveling on a level road and that all frictional forces may be ignored. **(a)** What is the acceleration of this car when its speed is 14 m/s? **(b)** If the speed of the car is doubled, does its acceleration increase, decrease, or stay the same? Explain. **(c)** Calculate the car's acceleration when its speed is 28 m/s.

81. •• **Meteorite** On October 9, 1992, a 27-pound meteorite struck a car in Peekskill, NY, creating a dent about 22 cm deep. If the initial speed of the meteorite was 550 m/s, what was the average force exerted on the meteorite by the car?

An interplanetary fender bender (Problem 81)

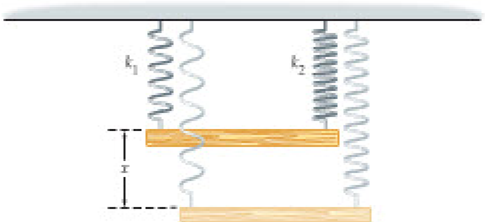
82. ••• **BIO Powering a Pigeon** A pigeon in flight experiences a force of air resistance given approximately by $F = bv^2$, where v is the flight speed and b is a constant. **(a)** What are the units of the constant b? **(b)** What is the largest possible speed of the pigeon if its maximum power output is P? **(c)** By what factor does the largest possible speed increase if the maximum power is doubled?

83. ••• **Springs in Series** Two springs, with force constants k_1 and k_2, are connected in series, as shown in Figure 7–21. How much work is required to stretch this system a distance x from the equilibrium position?



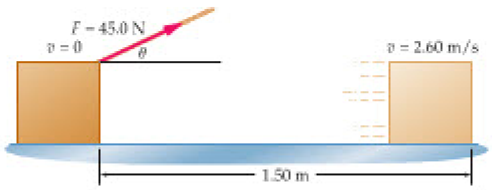
▲ **FIGURE 7–21** Problem 83

84. ••• **Springs in Parallel** Two springs, with force constants k_1 and k_2, are connected in parallel, as shown in Figure 7–22. How much work is required to stretch this system a distance x from the equilibrium position?



▲ **FIGURE 7–22** Problem 84

85. ••• A block rests on a horizontal frictionless surface. A string is attached to the block, and is pulled with a force of 45.0 N at an angle θ above the horizontal, as shown in Figure 7–23. After the block is pulled through a distance of 1.50 m, its speed is 2.60 m/s, and 50.0 J of work has been done on it. **(a)** What is the angle θ? **(b)** What is the mass of the block?



▲ **FIGURE 7–23** Problem 85

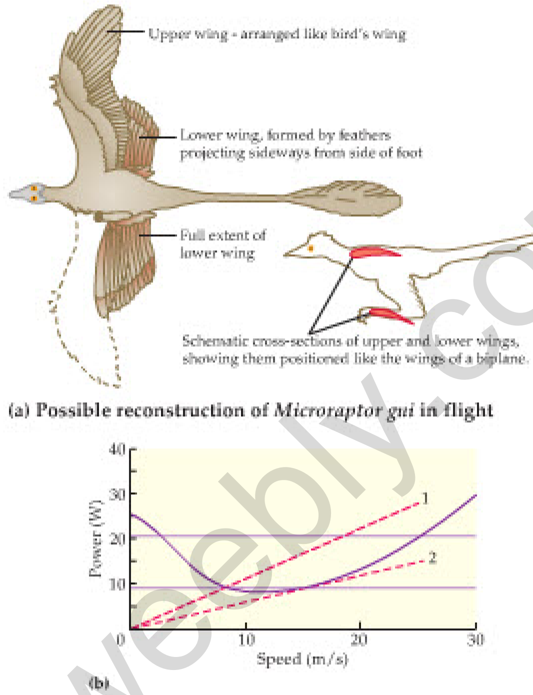
PASSAGE PROBLEMS

BIO *Microraptor gui*: The Biplane Dinosaur

The evolution of flight is a subject of intense interest in paleontology. Some subscribe to the "cursorial" (or ground-up) hypothesis, in which flight began with ground-dwelling animals running and jumping after prey. Others favor the "arboreal" (or trees-down) hypothesis, in which tree-dwelling animals, like modern-day flying squirrels, developed flight as an extension of gliding from tree to tree.

A recently discovered fossil from the Cretaceous period in China supports the arboreal hypothesis and adds a new element—it suggests that feathers on both the wings and the lower legs and feet allowed this dinosaur, *Microraptor gui*, to glide much like a biplane, as shown in Figure 7–24 (a). Researchers have produced a detailed computer simulation of *Microraptor*, and with its help have obtained the power-versus-speed plot presented in Figure 7–24 (b). This curve shows how much power is required for flight at speeds between 0 and 30 m/s. Notice that the power increases at high speeds, as expected, but is also high for low speeds, where the dinosaur is almost hovering. A minimum of 8.1 W is needed for flight at 10 m/s. The lower horizontal line shows the estimated 9.8-W power output of *Microraptor*, indicating the small range of speeds for which flight would be possible. The upper horizontal line shows the wider range of flight speeds that would be available if *Microraptor* were able to produce 20 W of power.

Also of interest are the two dashed, straight lines labeled 1 and 2. These lines represent constant ratios of power to speed; that is, a constant value for P/v. Referring to Equation 7–13, we see that $P/v = Fv/v = F$, so the lines 1 and 2 correspond to lines of constant force. Line 2 is interesting in that it has the smallest slope that still touches the power-versus-speed curve.



▲ **FIGURE 7–24** Problems 86, 87, 88, and 89

86. • Estimate the range of flight speeds for *Microraptor gui* if its power output is 9.8 W.

A. 0–7.7 m/s **B.** 7.7–15 m/s **C.** 15–30 m/s **D.** 0–15 m/s

87. • What approximate range of flight speeds would be possible if *Microraptor gui* could produce 20 W of power?

A. 0–25 m/s **B.** 25–30 m/s **C.** 2.5–25 m/s **D.** 0–2.5 m/s

88. •• How much energy would *Microraptor* have to expend to fly with a speed of 10 m/s for 1.0 minute?

A. 8.1 J **B.** 81 J **C.** 490 J **D.** 600 J

89. • Estimate the minimum force that *Microraptor* must exert to fly.

A. 0.65 N **B.** 1.3 N **C.** 1.0 N **D.** 10 N

INTERACTIVE PROBLEMS

90. •• **Referring to Figure 7–12** Suppose the block has a mass of 1.4 kg and an initial speed of 0.62 m/s. **(a)** What force constant must the spring have if the maximum compression is to be 2.4 cm? **(b)** If the spring has the force constant found in part (a), find the maximum compression if the mass of the block is doubled *and* its initial speed is halved.

91. •• **IP Referring to Figure 7–12** In the situation shown in Figure 7–12 (d), a spring with a force constant of 750 N/m is compressed by 4.1 cm. **(a)** If the speed of the block in Figure 7–12 (f) is 0.88 m/s, what is its mass? **(b)** If the mass of the block is doubled, is the final speed greater than, less than, or equal to 0.44 m/s? **(c)** Find the final speed for the case described in part (b).

92. •• **IP Referring to Example 7–8** Suppose the car has a mass of 1400 kg and delivers 48 hp to the wheels. **(a)** How long does it take for the car to increase its speed from 15 m/s to 25 m/s? **(b)** Would the time required to increase the speed from 5.0 m/s to 15 m/s be greater than, less than, or equal to the time found in part (a)? **(c)** Determine the time required to accelerate from 5.0 m/s to 15 m/s.

8 Potential Energy and Conservation of Energy

Probably everyone has seen a high jumper spring into the air, slow, hang motionless in midair for an instant, and then start to descend, picking up speed on the way. At the top of the trajectory, where has the kinetic energy gone? And how does it reappear as the jumper descends? As we answer these questions, we will find that there are other kinds of energy besides those considered in the last chapter.

One of the greatest accomplishments of physics is the concept of energy and its conservation. To realize, for example, that there is an important physical quantity that we can neither see nor touch is an impressive leap of the imagination. Even more astonishing, however, is the discovery that energy comes in a multitude of forms, and that the sum total of all these forms of energy is a constant. The universe, in short, has a certain amount of energy, and that energy simply ebbs and flows from one form to another, with the total amount remaining fixed.

In this chapter we focus on the conservation of energy, the first "conservation law" to be studied in this text. Though only a handful of conservation laws are known, they are all of central importance in physics. Not only do they give deep insight into the workings of nature, they are also practical tools in problem solving. As we shall see in this chapter, many problems that would be difficult to solve using Newton's laws can be solved with ease using the principle of energy conservation.

8–1 Conservative and Nonconservative Forces

In physics, we classify forces according to whether they are *conservative* or *nonconservative*. The key distinction is that when a **conservative force** acts, the work it does is stored in the form of energy that can be released at a later time. In this section, we sharpen this distinction and explore some examples of conservative and nonconservative forces.

Perhaps the simplest case of a conservative force is gravity. Imagine lifting a box of mass m from the floor to a height h, as in Figure 8–1. To lift the box with constant speed, the force you must exert against gravity is mg. Since the upward distance is h, the work you do on the box is $W = mgh$. If you now release the box and allow it to drop back to the floor, gravity does the same work, $W = mgh$, and in the process gives the box an equivalent amount of kinetic energy.

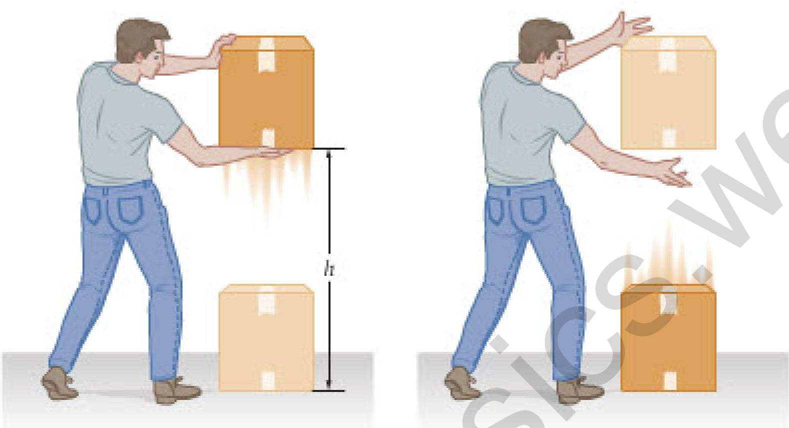


FIGURE 8–1 Work against gravity Lifting a box against gravity with constant speed takes a work mgh. When the box is released, gravity does the same work on the box as it falls. Gravity is a conservative force.

Contrast this with the force of kinetic friction, which is nonconservative. To slide a box of mass m across the floor with constant speed, as shown in Figure 8–2, you must exert a force of magnitude $\mu_k N = \mu_k mg$. After sliding the box a distance d, the work you have done is $W = \mu_k mgd$. In this case, when you release the box it simply stays put—friction does no work on it after you let go. Thus, the work done by a **nonconservative force** cannot be recovered later as kinetic energy; instead, it is converted to other forms of energy, such as a slight warming of the floor and box in our example.

The differences between conservative and nonconservative forces are even more apparent if we consider moving an object around a closed path. Consider, for example, the path shown in Figure 8–3. If we move a box of mass m along this path, the total work done by gravity is the sum of the work done on each segment of the path; that is, $W_{total} = W_{AB} + W_{BC} + W_{CD} + W_{DA}$. The work done by gravity from A to B and from C to D is zero, since the force is at right angles to the displacement on these segments. Thus $W_{AB} = W_{CD} = 0$. On the segment from B to C, gravity does negative work (displacement and force are in opposite directions),

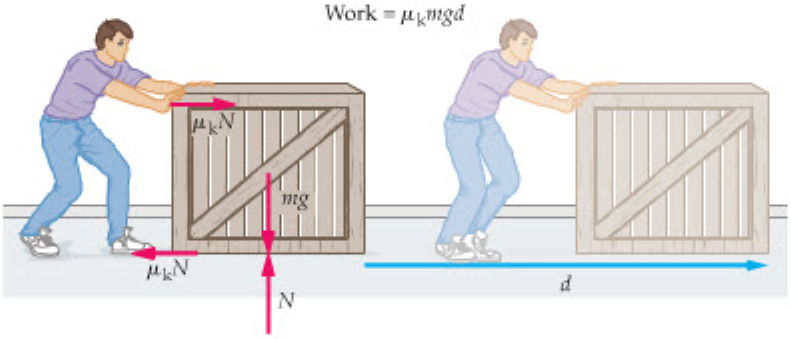


FIGURE 8–2 Work against friction Pushing a box with constant speed against friction takes a work $\mu_k mgd$. When the box is released, it quickly comes to rest and friction does no further work. Friction is a nonconservative force.

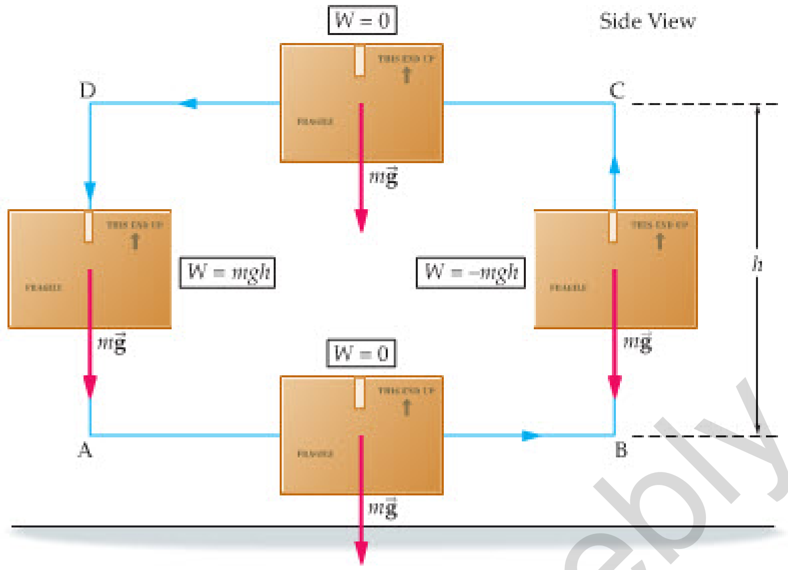


FIGURE 8–3 Work done by gravity on a closed path is zero
Gravity does no work on the two horizontal segments of the path. On the two vertical segments, the amounts of work done are equal in magnitude but opposite in sign. Therefore, the total work done by gravity on this—or any—closed path is zero.

but it does positive work from D to A (displacement and force are in the same direction). Hence, $W_{BC} = -mgh$ and $W_{DA} = mgh$. As a result, the total work done by gravity is zero:

$$W_{total} = 0 + (-mgh) + 0 + mgh = 0$$

With friction, the results are quite different. If we push the box around the closed horizontal path shown in **Figure 8–4**, the total work done by friction does not vanish. In fact, friction does the negative work $W = -f_k d = -\mu_k mgd$ on each segment. Therefore, the total work done by kinetic friction is

$$W_{total} = (-\mu_k mgd) + (-\mu_k mgd) + (-\mu_k mgd) + (-\mu_k mgd) = -4\,\mu_k mgd$$

These results lead to the following definition of a conservative force:

Conservative Force: Definition 1
A conservative force is a force that does zero total work on any closed path.

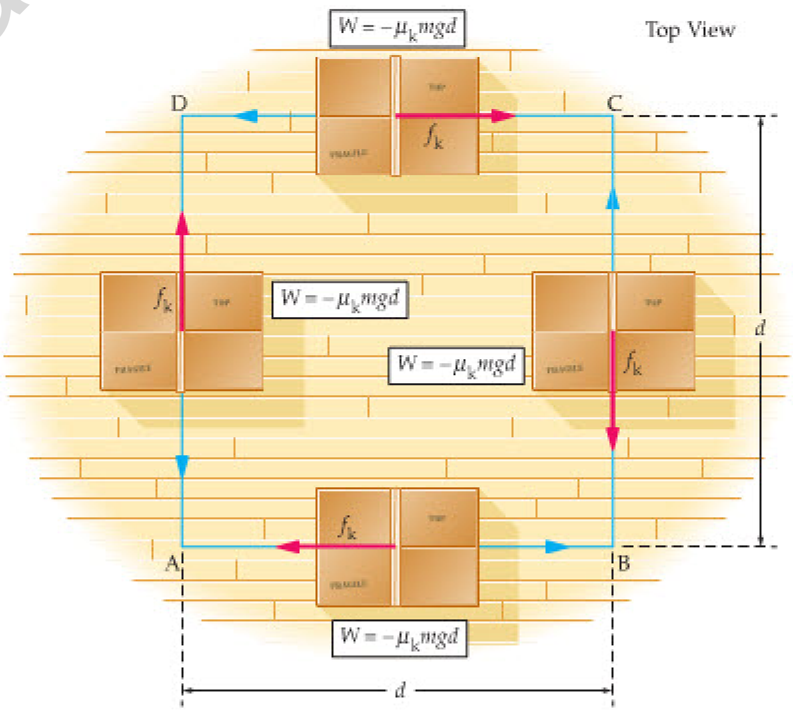
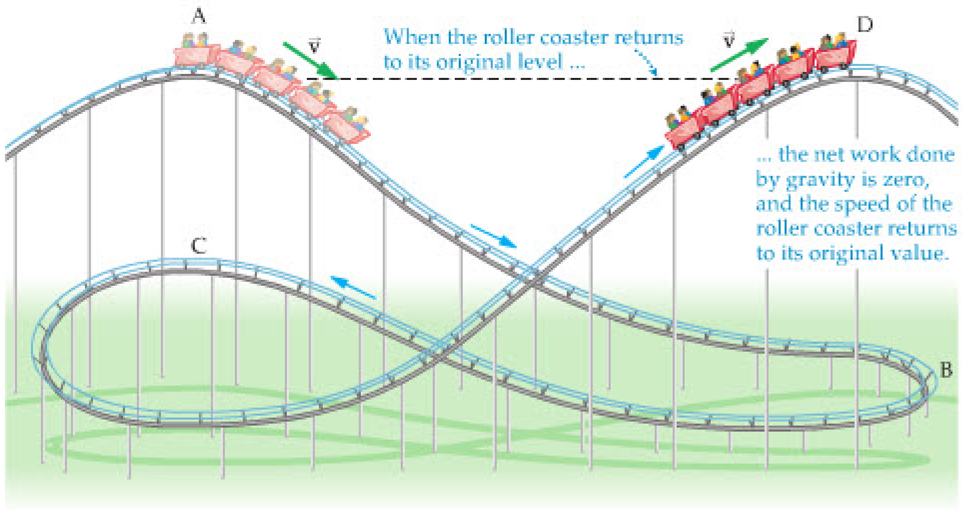


FIGURE 8–4 Work done by friction on a closed path is nonzero
The work done by friction when an object moves through a distance d is $-\mu_k mgd$. Thus, the total work done by friction on a closed path is nonzero. In this case, it is equal to $-4\,\mu_k mgd$.



▲ **FIGURE 8–5 Gravity is a conservative force**
If frictional forces can be ignored, a roller coaster car will have the same speed at points A and D, since they are at the same height. Hence, after any complete circuit of the track the speed of the car returns to its initial value. It follows that the change in kinetic energy is zero for a complete circuit, and, therefore, the work done by gravity is also zero.

A roller coaster provides a good illustration of this definition. If a car on a roller coaster has a speed v at point A in Figure 8–5, it speeds up as it drops to point B, slows down as it approaches point C, and so on. When the car returns to its original height, at point D, it will again have the speed v, as long as friction and other nonconservative forces can be neglected. Similarly, if the car completes a circuit of the track and returns to point A, it will again have the speed v. Hence, a car's kinetic energy is unchanged ($\Delta K = 0$) after *any* complete circuit of the track. From the work–energy theorem, $W_{\text{total}} = \Delta K$, it follows that the work done by gravity is zero for the closed path of the car, as expected for a conservative force.

This property of conservative forces has interesting consequences. For instance, consider the closed paths shown in Figure 8–6. On each of these paths, we know that the work done by a conservative force is zero. Thus, it follows from paths 1 and 2 that $W_{\text{total}} = W_1 + W_2 = 0$, or

$$W_2 = -W_1$$

Similarly, using paths 1 and 3 we have $W_{\text{total}} = W_1 + W_3 = 0$, or

$$W_3 = -W_1$$

As a result, we see that the work done on path 3 is the same as the work done on path 2:

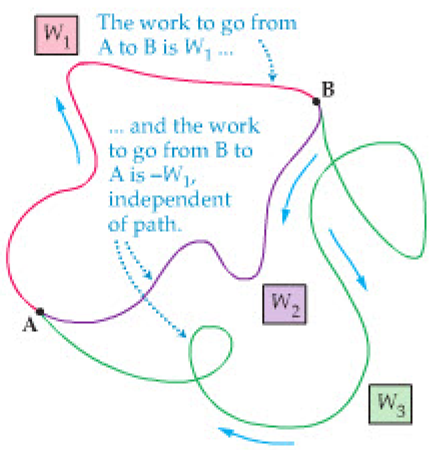
$$W_3 = W_2$$

But paths 2 and 3 are arbitrary, as long as they start at point B and end at point A. This leads to an equivalent definition of a conservative force:

Conservative Force: Definition 2
If the work done by a force in going from an arbitrary point A to an arbitrary point B is *independent of the path* from A to B, the force is conservative.

This definition is given an explicit check in Example 8–1.

Table 8–1 summarizes the different kinds of conservative and nonconservative forces we have encountered thus far in this text.



▲ **FIGURE 8–6 The work done by a conservative force is independent of path**
Considering paths 1 and 2, we see that $W_1 + W_2 = 0$, or $W_2 = -W_1$. From paths 1 and 3, however, we see that $W_1 + W_3 = 0$, or $W_3 = -W_1$. It follows, then, that $W_3 = W_2$, since they are both equal to $-W_1$; hence the work done in going from A to B is independent of the path.

TABLE 8–1 Conservative and Nonconservative Forces

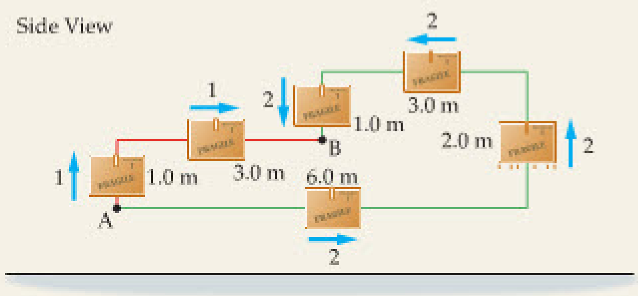
Force	Section
Conservative forces	
Gravity	5–6
Spring force	6–2
Nonconservative forces	
Friction	6–1
Tension in a rope, cable, etc.	6–2
Forces exerted by a motor	7–4
Forces exerted by muscles	5–3

EXAMPLE 8–1 DIFFERENT PATHS, DIFFERENT FORCES

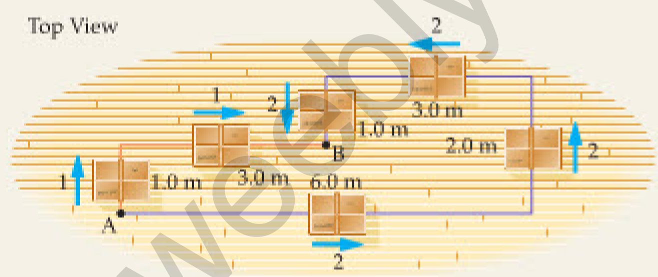
(a) A 4.57-kg box is moved with constant speed from A to B along the two paths shown at left below. Calculate the work done by gravity on each of these paths. **(b)** The same box is pushed across a floor from A to B along path 1 and path 2 at right below. If the coefficient of kinetic friction between the box and the surface is $\mu_k = 0.63$, how much work is done by friction along each path?

PICTURE THE PROBLEM

Part (a) of our sketch shows two different paths a box might be taken through in going from point A to point B. Path 1 is indicated by two red lines, indicating a vertical displacement of 1.0 m and a horizontal displacement of 3.0 m. Path 2, indicated in green, consists of two horizontal and two vertical displacements. In this case, we are interested in the work done by gravity. Part (b) shows the same basic paths—path 1 in orange and path 2 in purple—only this time on a rough floor. Here it is the force of kinetic friction that is of interest.



(a)



(b)

STRATEGY

To calculate the work for each path, we break it down into segments. Path 1 is made up of two segments, path 2 has four segments.

a. For gravity, the work is zero on horizontal segments. On vertical segments, the work done by gravity is positive when motion is downward and negative when motion is upward.

b. The work done by kinetic friction is negative on all segments of both paths.

SOLUTION

Part (a)

1. Using $W = Fd = mgy$, calculate the work done by gravity along the two segments of path 1:

$$W_1 = -(4.57\ \text{kg})(9.81\ \text{m/s}^2)(1.0\ \text{m}) + 0 = -45\ \text{J}$$

2. In the same way, calculate the work done by gravity along the four segments of path 2:

$$W_2 = 0 - (4.57\ \text{kg})(9.81\ \text{m/s}^2)(2.0\ \text{m}) + 0 + (4.57\ \text{kg})(9.81\ \text{m/s}^2)(1.0\ \text{m}) = -45\ \text{J}$$

Part (b)

3. Using $F = \mu_k N$, calculate the work done by kinetic friction along the two segments of path 1:

$$W_1 = -(0.63)(4.57\ \text{kg})(9.81\ \text{m/s}^2)(1.0\ \text{m}) - (0.63)(4.57\ \text{kg})(9.81\ \text{m/s}^2)(3.0\ \text{m}) = -110\ \text{J}$$

4. Similarly, calculate the work done by kinetic friction along the four segments of path 2:

$$W_2 = -(0.63)(4.57\ \text{kg})(9.81\ \text{m/s}^2)(6.0\ \text{m}) - (0.63)(4.57\ \text{kg})(9.81\ \text{m/s}^2)(2.0\ \text{m}) - (0.63)(4.57\ \text{kg})(9.81\ \text{m/s}^2)(3.0\ \text{m}) - (0.63)(4.57\ \text{kg})(9.81\ \text{m/s}^2)(1.0\ \text{m}) = -340\ \text{J}$$

INSIGHT

As expected, the conservative force of gravity gives the same work in going from A to B, regardless of the path. The work done by kinetic friction, however, is greater on the path of greater length.

PRACTICE PROBLEM

The work done by gravity when the box is moved from point B to a point C is 140 J. Is point C above or below point B? What is the vertical distance between points B and C? [**Answer:** Point C is 3.1 m below point B.]

Some related homework problems: Problem 2, Problem 3

8–2 Potential Energy and the Work Done by Conservative Forces

Work must be done to lift a bowling ball from the floor to a shelf. Once on the shelf, the bowling ball has zero kinetic energy, just as it did on the floor. Even so, the work done in lifting the ball has not been lost. If the ball is allowed to fall from the shelf, gravity does the same amount of work on it as you did to lift it in the first place. As a result, the work you did is "recovered" in the form of kinetic energy. Thus we say that when the ball is lifted to a new position, there is an increase in **potential energy,** U, and that this potential energy can be converted to kinetic energy when the ball falls.

▲ Because gravity is a conservative force, the work done against gravity in lifting these logs (left) can, in principle, all be recovered. If the logs are released, for example, they will acquire an amount of kinetic energy exactly equal to the work done to lift them and to the gravitational potential energy that they gained in being lifted. Friction, by contrast, is a nonconservative force. Some of the work done by this spinning grindstone (right) goes into removing material from the object being ground, while the rest is transformed into sound energy and (especially) heat. Most of this work can never be recovered as kinetic energy.

In a sense, potential energy is a storage system for energy. When we increase the separation between the ball and the ground, the work we do is stored in the form of an increased potential energy. Not only that, but the storage system is perfect, in the sense that the energy is never lost, as long as the separation remains the same. The ball can rest on the shelf for a million years, and still, when it falls, it gains the same amount of kinetic energy.

Work done against friction, however, is not "stored" as potential energy. Instead, it is dissipated into other forms of energy such as heat or sound. The same is true of other nonconservative forces. Only conservative forces have the potential-energy storage system.

Before proceeding, we should point out an interesting difference between kinetic and potential energy. Kinetic energy is given by the expression $K = \frac{1}{2}mv^2$, no matter what force might be involved. On the other hand, each different conservative force has a different expression for its potential energy. To see how this comes about, we turn now to a precise definition of potential energy.

Potential Energy, U

When a conservative force does an amount of work W_c (the subscript c stands for conservative), the corresponding potential energy U is changed according to the following definition:

Definition of Potential Energy, U

$$W_c = U_i - U_f = -(U_f - U_i) = -\Delta U \qquad 8\text{–}1$$

SI unit: joule, J

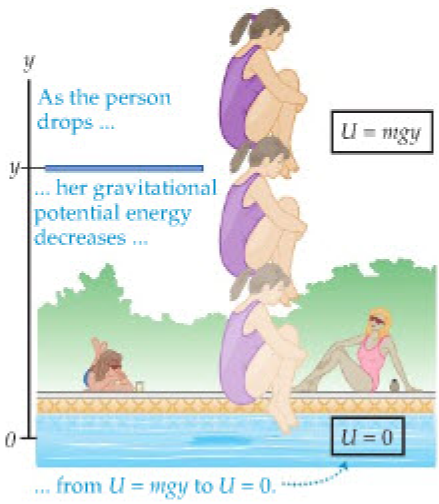
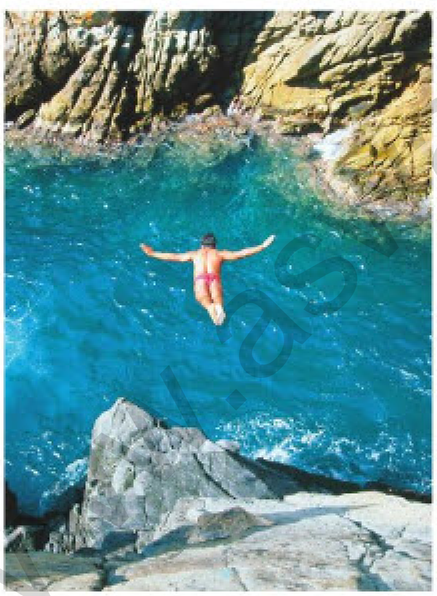


FIGURE 8–7 Gravitational potential energy
A person drops from a diving board into a swimming pool. The diving board is at the height y, and the surface of the water is at $y = 0$. We choose the gravitational potential energy to be zero at $y = 0$; hence, the potential energy is mgy at the diving board.

PROBLEM-SOLVING NOTE

Zero of Potential Energy

When working potential energy problems it is important to make a definite choice for the location where the potential energy is to be set equal to zero. Any location can be chosen, but once the choice is made, it must be used consistently.



The $U = 0$ level for the gravitational potential energy of this system can be assigned to the point where the diver starts his dive, to the water level, or to any other level. Regardless of the choice, however, his kinetic energy when he strikes the water will be exactly equal to the difference in gravitational potential energy between his launch and splashdown points.

In words, the work done by a conservative force is equal to the negative of the change in potential energy. For example, when an object falls, gravity does *positive* work on it and its potential energy *decreases*. Similarly, when an object is lifted, gravity does *negative* work and the potential energy *increases*.

Note that since work is a scalar with units of joules, the same is true of potential energy. In addition, our definition determines only the *difference* in potential energy between two points, not the actual value of the potential energy. Hence, we are free to choose the place where the potential energy is zero ($U = 0$) in much the same way we are free to choose the location of the origin in a coordinate system.

Gravity

Let's apply our definition of potential energy to the force of gravity near the Earth's surface. Suppose a person of mass m drops a distance y from a diving board into a pool, as shown in Figure 8–7. As the person drops, gravity does the work

$$W_c = mgy$$

Applying the definition given in Equation 8–1, the corresponding change in potential energy is

$$-\Delta U = U_i - U_f = W_c = mgy$$

In this expression, U_i is the potential energy when the diver is on the board, and U_f is the potential energy when the diver enters the water. Rearranging slightly, we have

$$U_i = mgy + U_f \qquad 8\text{–}2$$

Note that U_i is greater than U_f.

As mentioned above, we are free to choose $U = 0$ anywhere we like; *only the difference* in U is important. For example, if you slip and fall to the ground, you hit with the same thud whether you fall in Denver (altitude 1 mile) or in Honolulu (at sea level). It's the difference in height that matters, not the height itself. (The acceleration of gravity does vary slightly with altitude, as we shall see, but the difference is small enough to be unimportant in this case.) The only point to be careful about when choosing a location for $U = 0$ is to be consistent with the choice once it is made.

In general, we choose $U = 0$ in a convenient location. In Figure 8–7, a reasonable place for $U = 0$ is the surface of the water, where $y = 0$; that is, $U_f = 0$. Then, Equation 8–2 becomes $U_i = mgy$. If we omit the subscript on U_i, letting U stand for the potential energy at the arbitrary height y, we have

Gravitational Potential Energy (Near Earth's Surface)

$$U = mgy \qquad 8\text{–}3$$

Note that the gravitational potential energy depends only on the height, y, and is independent of horizontal position.

EXERCISE 8–1

Find the gravitational potential energy of a system consisting of a 65-kg person on a 3.0-m-high diving board. Let $U = 0$ be at water level.

SOLUTION

Substituting $m = 65$ kg and $y = 3.0$ m in Equation 8–3 yields

$$U = mgy = (65\ \text{kg})(9.81\ \text{m/s}^2)(3.0\ \text{m}) = 1900\ \text{J}$$

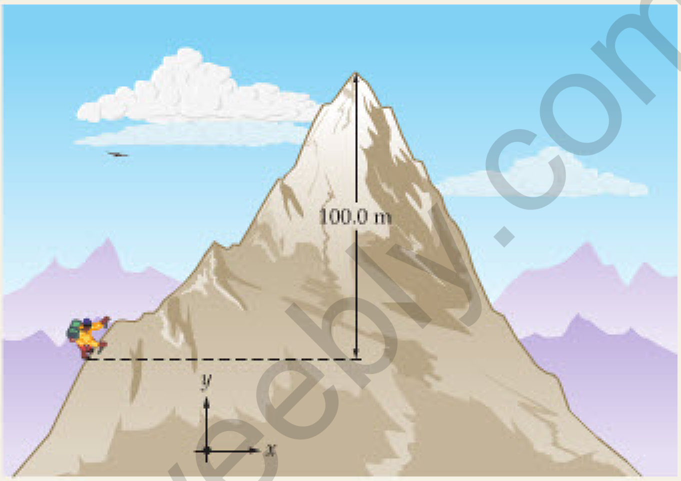
The next Example considers the change in gravitational potential energy of a mountain climber, given different choices for the location of $U = 0$.

EXAMPLE 8–2 PIKES PEAK OR BUST

An 82.0-kg mountain climber is in the final stage of the ascent of 4301-m-high Pikes Peak. What is the change in gravitational potential energy as the climber gains the last 100.0 m of altitude? Let $U = 0$ be **(a)** at sea level or **(b)** at the top of the peak.

PICTURE THE PROBLEM

Our sketch shows the mountain climber and the last 100.0 m of altitude to be climbed. We choose a typical coordinate system, with the positive y axis upward and the positive x axis to the right.



STRATEGY

The gravitational potential energy of the Earth–climber system depends only on the height y; the path followed in gaining the last 100.0 m of altitude is unimportant. The change in potential energy is $\Delta U = U_f - U_i = mgy_f - mgy_i$, where y_f is the altitude of the peak and y_i is 100.0 m less than y_f.

SOLUTION

Part (a)

1. Calculate ΔU with $y_f = 4301$ m and $y_i = 4201$ m:

$$\Delta U = mgy_f - mgy_i = (82.0\text{ kg})(9.81\text{ m/s}^2)(4301\text{ m}) - (82.0\text{ kg})(9.81\text{ m/s}^2)(4201\text{ m}) = 80{,}400\text{ J}$$

Part (b)

2. Calculate ΔU with $y_f = 0$ and $y_i = -100.0$ m:

$$\Delta U = mgy_f - mgy_i = (82.0\text{ kg})(9.81\text{ m/s}^2)(0) - (82.0\text{ kg})(9.81\text{ m/s}^2)(-100.0\text{ m}) = 80{,}400\text{ J}$$

INSIGHT

As expected, the *change* in gravitational potential energy does not depend on where we choose $U = 0$. Nor does it depend on the path taken between the initial and final points.

PRACTICE PROBLEM

Find the altitude of the climber for which the gravitational potential energy of the Earth–climber system is 1.00×10^5 J less than it is when the climber is at the summit. [**Answer:** 4180 m]

Some related homework problems: Problem 10, Problem 17

A single item of food can be converted into a surprisingly large amount of potential energy. This is shown for the case of a candy bar in Example 8–3.

EXAMPLE 8–3 CONVERTING FOOD ENERGY TO MECHANICAL ENERGY

REAL-WORLD PHYSICS: BIO

A candy bar called the Mountain Bar has a calorie content of 212 Cal = 212 kcal, which is equivalent to an energy of 8.87×10^5 J. If an 81.0-kg mountain climber eats a Mountain Bar and magically converts it all to potential energy, what gain of altitude would be possible?

PICTURE THE PROBLEM

We show the mountain climber eating the candy bar at a given level on the mountain, which we can take to be $y = 0$. The altitude gain, then, corresponds to $y = h$.

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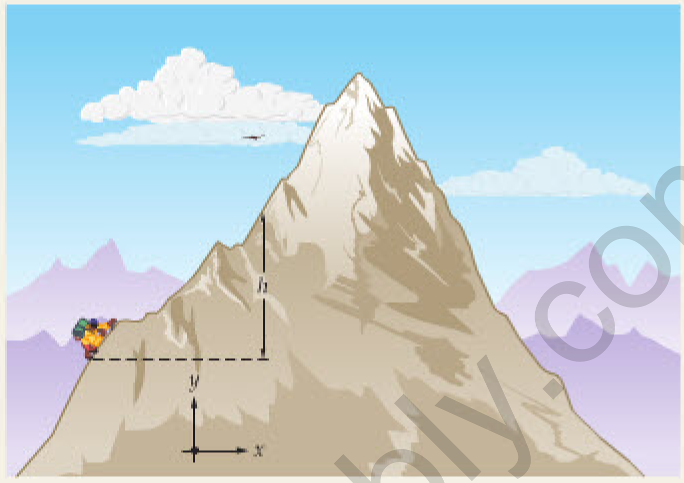
STRATEGY

The initial gravitational potential energy of the Earth–climber system is $U = 0$; the final potential energy is $U = mgh$. To find the altitude gain, set $U = mgh$ equal to the energy provided by the candy bar, 8.87×10^5 J, and solve for h.



SOLUTION

1. Solve $U = mgh$ for h:

$$U = mgh$$
$$h = \frac{U}{mg}$$

2. Substitute numerical values, with $U = 8.87 \times 10^5$ J:

$$h = \frac{U}{mg} = \frac{8.87 \times 10^5 \text{ J}}{(81.0 \text{ kg})(9.81 \text{ m/s}^2)} = 1120 \text{ m}$$

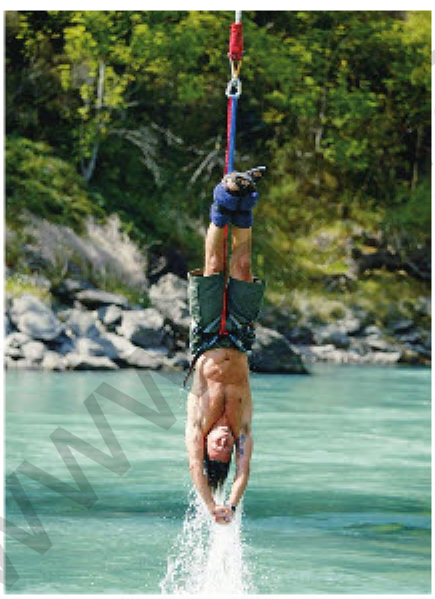
INSIGHT

This is more than two-thirds of a mile in elevation. Even if we take into account the fact that metabolic efficiency is only about 25%, the height would still be 280 m, or nearly two-tenths of a mile. It's remarkable just how much our bodies can do with so little.

PRACTICE PROBLEM

If the mass of the mountain climber is increased—by adding more items to the backpack, for example—does the possible elevation gain increase, decrease, or stay the same? Calculate the elevation gain for a climber with a mass of 91.0 kg. [**Answer:** The altitude gain will decrease. For $m = 91.0$ kg we find $h = 994$ m.]

Some related homework problems: Problem 10, Problem 17



▲ Because springs, and bungee cords, exert conservative forces, they can serve as energy storage devices. In this case, the stretched bungee cord is beginning to give up the energy it has stored, and to convert that potential energy into kinetic energy as the jumper is pulled rapidly skyward.

We have been careful *not* to say that the potential energy of the mountain climber—or any object—increases when its height increases. The reason is that the potential energy is a property of an entire system, not of its individual parts. The correct statement is that if an object is lifted, the potential energy of the Earth–object system is increased.

Springs

Consider a spring that is stretched from its equilibrium position a distance x. According to Equation 7–8, the work required to cause this stretch is $W = \frac{1}{2}kx^2$. Therefore, if the spring is released—and allowed to move from the stretched position back to the equilibrium position—it will do the same work, $\frac{1}{2}kx^2$. From our definition of potential energy, then, we see that

$$W_c = \frac{1}{2}kx^2 = U_i - U_f \qquad \text{8–4}$$

Note that in this case U_f is the potential energy when the spring is at $x = 0$ (equilibrium position), and U_i is the potential energy when the spring is stretched by the amount x.

A convenient choice for $U = 0$ is the equilibrium position of the spring. With this choice we have $U_f = 0$, and Equation 8–4 becomes $U_i = \frac{1}{2}kx^2$. Omitting the subscript i, so that U represents the potential energy of the spring for an arbitrary amount of stretch x, we have

Potential Energy of a Spring

$$U = \frac{1}{2}kx^2 \qquad \text{8–5}$$

Since U depends on x^2, which is positive even if x is negative, the potential energy of a spring is always greater than or equal to zero. Thus, a spring's potential energy increases whenever it is displaced from equilibrium.

EXERCISE 8–2

Find the potential energy of a spring with force constant $k = 680\ \mathrm{N/m}$ if it is **(a)** stretched by 5.00 cm or **(b)** compressed by 7.00 cm.

SOLUTION

Substituting $x = 0.0500\ \mathrm{m}$ and $x = -0.0700\ \mathrm{m}$ in Equation 8–5 yields

a. $U = \frac{1}{2}(680\ \mathrm{N/m})(0.0500\ \mathrm{m})^2 = 0.850\ \mathrm{J}$

b. $U = \frac{1}{2}(680\ \mathrm{N/m})(-0.0700\ \mathrm{m})^2 = 1.67\ \mathrm{J}$

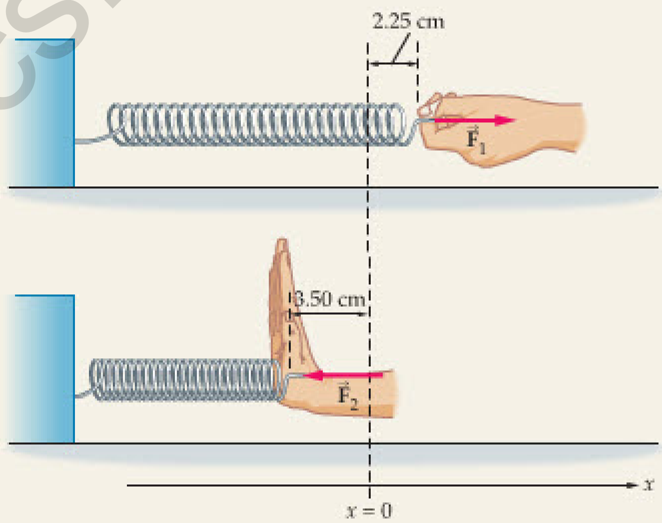
Finally, comparing Equation 8–3 to Equation 8–5, we see that the potential energies for gravity and for a spring are given by different expressions. As mentioned, each conservative force has its own potential energy.

EXAMPLE 8–4 COMPRESSED ENERGY AND THE JUMP OF A FLEA

When a force of 120.0 N is applied to a certain spring, it causes a stretch of 2.25 cm. What is the potential energy of this spring when it is **(a)** compressed by 3.50 cm or **(b)** expanded by 7.00 cm?

PICTURE THE PROBLEM

The top sketch shows the spring stretched 2.25 cm by the force $F_1 = 120.0\ \mathrm{N}$. The lower sketch shows the same spring compressed by a second force, F_2, which causes a compression of 3.50 cm. An expansion of the spring by 7.00 cm would look similar to the top sketch.



STRATEGY

From the first piece of information—a certain force causes a certain stretch—we can calculate the force constant using $F = kx$. Once we know k, we find the potential energy for either a compression or an expansion with $U = \frac{1}{2}kx^2$.

SOLUTION

1. Solve $F = kx$ for the spring constant, k:

$$F = kx$$
$$k = \frac{F}{x} = \frac{120.0\ \mathrm{N}}{0.0225\ \mathrm{m}} = 5330\ \mathrm{N/m}$$

Part (a)

2. Substitute $k = 5330\ \mathrm{N/m}$ and $x = -0.0350\ \mathrm{m}$ into the potential energy expression, $U = \frac{1}{2}kx^2$:

$$U = \tfrac{1}{2}kx^2 = \tfrac{1}{2}(5330\ \mathrm{N/m})(-0.0350\ \mathrm{m})^2 = 3.26\ \mathrm{J}$$

Part (b)

3. Substitute $k = 5330\ \mathrm{N/m}$ and $x = 0.0700\ \mathrm{m}$ into the potential energy expression, $U = \frac{1}{2}kx^2$:

$$U = \tfrac{1}{2}kx^2 = \tfrac{1}{2}(5330\ \mathrm{N/m})(0.0700\ \mathrm{m})^2 = 13.1\ \mathrm{J}$$

INSIGHT

Though this Example deals with ideal springs, the same basic physics applies to many other real-world situations. A case in point is the jump of a flea, in which a flea can propel itself up to 100 times its body length. The physics behind this feat is the slow

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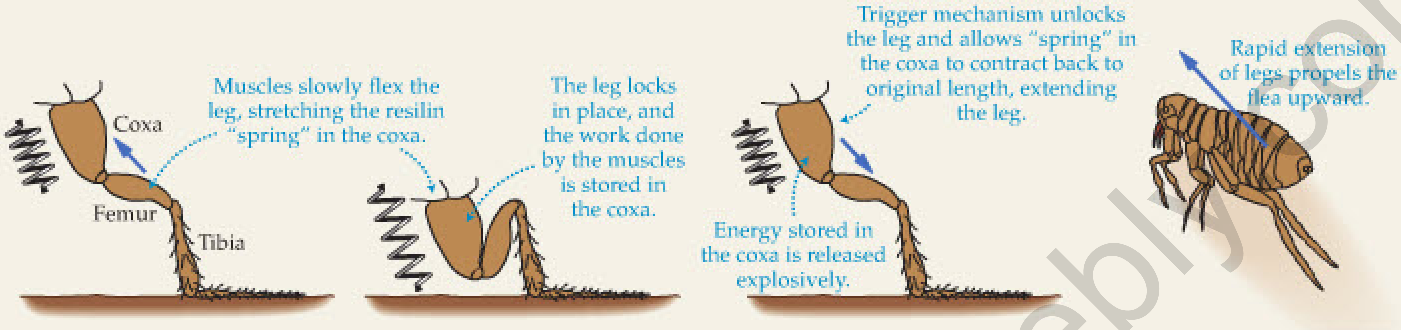
accumulation of energy in a "springy" strip of resilin in the coxa of the leg, as shown in the accompanying sketches, and the sudden release of this energy at a later time. Specifically, as the flea's muscles flex the leg, the resilin strip in the coxa is stretched, storing the work done by the muscles in the form of potential energy, $U = \frac{1}{2}kx^2$. Later, when a trigger mechanism unlocks the flexed leg, the energy stored in the resilin is released explosively—rapidly extending the leg and propelling the flea upward. See Problem 89 for a calculation using the force constant of resilin.



PRACTICE PROBLEM

What stretch is necessary for the spring in this Example to have a potential energy of 5.00 J? [**Answer:** 4.33 cm]

Some related homework problems: Problem 12, Problem 16

The jump of a flea is similar in many respects to the operation of a bow and arrow. In the latter case, the work done in slowly pulling the string back is stored in the flex of the bow. The string is held in place while aim is taken, and then released to allow the bow to return to its original shape. This propels the arrow forward with great speed—a speed many times faster than could be obtained by simply throwing the arrow with the same arm muscles that pulled back on the string. In fact, if the string returns to its original position in 1/1000th the time it took to pull the string back, the power it delivers to the arrow is magnified by a factor of 1000. Similarly, the spring-loaded jump of the flea gives it a much greater takeoff speed than if it relied solely on muscle power. An analogous process occurs in the flash unit of a camera, as we shall see in Chapter 20.

8–3 Conservation of Mechanical Energy

In this section, we show how potential energy can be used as a powerful tool in solving a variety of problems and in gaining greater insight into the workings of physical systems. To do so, we begin by defining the **mechanical energy**, E, as the sum of the potential and kinetic energies of an object:

$$E = U + K \tag{8–6}$$

The significance of mechanical energy is that it is **conserved** in systems involving only conservative forces. By conserved, we mean that its value never changes; that is, $E =$ constant. (In situations where nonconservative forces are involved, the mechanical energy can change, as when friction causes warming by converting mechanical energy to thermal energy. When *all* possible forms of energy are considered, energy is always found to be conserved.)

To show that E is conserved for conservative forces, we start with the work–energy theorem from Chapter 7:

$$W_{\text{total}} = \Delta K = K_f - K_i$$

Suppose for a moment that the system has only a single force and that the force is conservative. If this is the case, then the total work, W_{total}, is the work done by the conservative force, W_c:

$$W_{\text{total}} = W_c$$

From the definition of potential energy, we know that $W_c = -\Delta U = U_i - U_f$. Combining these results, we have

$$W_{total} = W_c$$

$$K_f - K_i = U_i - U_f$$

With a slight rearrangement we find

$$U_f + K_f = U_i + K_i$$

or

$$E_f = E_i$$

Since the initial and final points can be chosen arbitrarily, it follows that E is conserved:

$$E = \text{constant}$$

If the system has more than one conservative force, the only change to these results is to replace U with the sum of potential energies of all the forces.

To summarize:

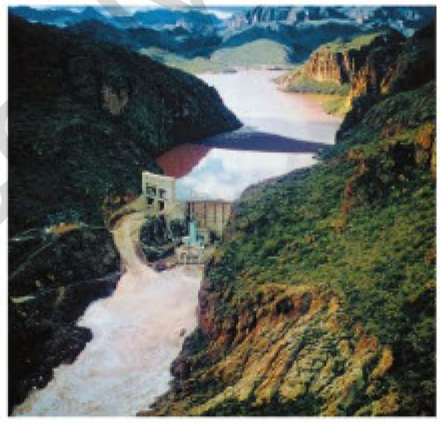
Conservation of Mechanical Energy

In systems with conservative forces only, the mechanical energy E is conserved; that is, $E = U + K = \text{constant}$.

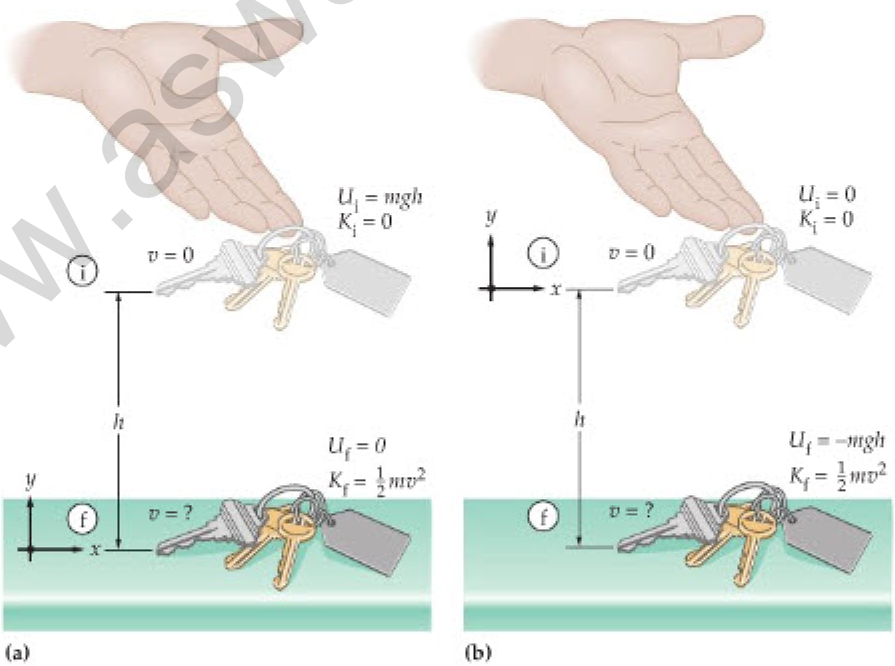
In terms of physical systems, conservation of mechanical energy means that energy can be converted between potential and kinetic forms, but that the sum remains the same. As an example, in the roller coaster shown in Figure 8–5, the gravitational potential energy decreases as the car approaches point B; as it does, the car's kinetic energy increases by the same amount. From a practical point of view, conservation of mechanical energy means that many physics problems can be solved by what amounts to simple bookkeeping.

For example, consider a key chain of mass m that is dropped to the floor from a height h, as illustrated in **Figure 8–8**. The question is, how fast are the keys moving just before they land? We know how to solve this problem using Newton's laws and kinematics, but now let's see how energy conservation can be used instead.

First, note that the only force acting on the keys is gravity—ignoring air resistance, of course—and that gravity is a conservative force. As a result, we can say



▲ A roller coaster (top) illustrates the conservation of mechanical energy. With every descent, gravitational potential energy is converted into kinetic energy; with every rise, kinetic energy is converted back into gravitational potential energy. If friction is neglected, the total mechanical energy of the car remains constant. The same principle is exploited at a pumped-storage facility, such as this one at the Mormon Flat Dam in Phoenix, Arizona (bottom). When surplus electrical power is available, it is used to pump water uphill into the reservoir. This process, in effect, stores electrical energy as gravitational potential energy. When power demand is high, the stored water is allowed to flow back downhill through the electrical generators in the dam, converting the gravitational energy to kinetic energy and the kinetic energy to electrical energy.



◀ **FIGURE 8–8 Solving a kinematics problem using conservation of energy**

(a) A set of keys falls to the floor. Ignoring frictional forces, we know that the mechanical energy at points i and f must be equal; $E_i = E_f$. Using this condition, we can find the speed of the keys just before they land. **(b)** The same physical situation as in part (a), except this time we have chosen $y = 0$ to be at the point where the keys are dropped. As before, we set $E_i = E_f$ to find the speed of the keys just before they land. The result is the same.

PROBLEM-SOLVING NOTE

Conservative Systems

A convenient approach to problems involving energy conservation is to first sketch the system, and then label the initial and final points with i and f, respectively. To apply energy conservation, write out the energy at these two points and set $E_i = E_f$.

that $E = U + K$ is constant during the entire time the keys are falling. To solve the problem, then, we pick two points on the motion of the keys, say i and f in Figure 8–8, and we set the mechanical energy equal at these points:

$$E_i = E_f \qquad 8\text{–}7$$

Writing this out in terms of potential and kinetic energies, we have

$$U_i + K_i = U_f + K_f \qquad 8\text{–}8$$

This one equation—which is nothing but bookkeeping—can be used to solve for the one unknown, the final speed.

To be specific, in Figure 8–8 (a) we choose $y = 0$ at ground level, which means that $U_i = mgh$. In addition, the fact that the keys are released from rest means that $K_i = 0$. Similarly, at point f—just before hitting the ground—the energy is all kinetic, and the potential energy is zero; that is, $U_f = 0$, $K_f = \frac{1}{2}mv^2$. Substituting these values into Equation 8–8, we find

$$mgh + 0 = 0 + \tfrac{1}{2}mv^2$$

Canceling m and solving for v yields the same result we get with kinematics:

$$v = \sqrt{2gh}$$

Suppose, instead, that we had chosen $y = 0$ to be at the release point of the keys, as in Figure 8–8 (b), so that the keys land at $y = -h$. Now, when the keys are released, we have $U_i = 0$ and $K_i = 0$, and when they land $U_f = -mgh$ and $K_f = \frac{1}{2}mv^2$. Substituting these results in $U_i + K_i = U_f + K_f$ yields

$$0 + 0 = -mgh + \tfrac{1}{2}mv^2$$

Solving for v gives the same result:

$$v = \sqrt{2gh}$$

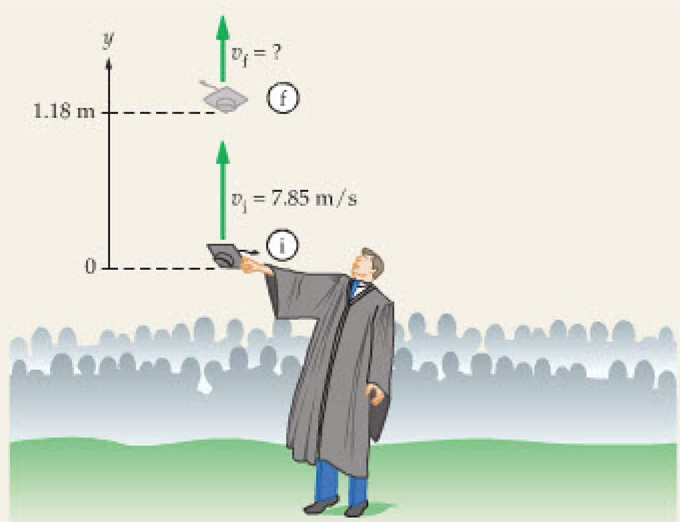
Thus, as expected, changing the zero level has no effect on the physical results.

EXAMPLE 8–5 GRADUATION FLING

At the end of a graduation ceremony, graduates fling their caps into the air. Suppose a 0.120-kg cap is thrown straight upward with an initial speed of 7.85 m/s, and that frictional forces can be ignored. **(a)** Use kinematics to find the speed of the cap when it is 1.18 m above the release point. **(b)** Show that the mechanical energy at the release point is the same as the mechanical energy 1.18 m above the release point.

PICTURE THE PROBLEM

In our sketch we choose $y = 0$ to be at the level where the cap is released with an initial speed of 7.85 m/s. In addition, note that we designate the release point as i (initial) and the point at which $y = 1.18$ m as f (final). It is the speed at point f that we wish to find.



STRATEGY

a. The cap is in free fall, which justifies the use of constant-acceleration kinematics. Since we want to relate velocity to position, we use $v_y^2 = v_{0y}^2 + 2a_y\Delta y$ (Section 2–5). In this case, $v_{0y} = 7.85$ m/s, $\Delta y = 1.18$ m, and $a_y = -g$. Substituting these values gives v_y.

b. At each point we simply calculate $E = U + K$, with $U = mgy$ and $K = \frac{1}{2}mv^2$.

SOLUTION

Part (a)

1. Use kinematics to solve for v_y: $v_y^2 = v_{0y}^2 + 2a_y\,\Delta y$
   $v_y = \pm\sqrt{v_{0y}^2 + 2a_y\,\Delta y}$

2. Substitute $v_{0y} = 7.85$ m/s, $\Delta y = 1.18$ m, and $a_y = -g$ to find v_y. Choose the plus sign, since we are interested only in the speed:
   $v_y = \sqrt{v_{0y}^2 + 2a_y\,\Delta y}$
   $= \sqrt{(7.85\text{ m/s})^2 + 2(-9.81\text{ m/s}^2)(1.18\text{ m})} = 6.20\text{ m/s}$

Part (b)

3. Calculate E_i. At this point $y_i = 0$ and $v_i = 7.85$ m/s:
   $E_i = U_i + K_i = mgy_i + \frac{1}{2}mv_i^2$
   $= 0 + \frac{1}{2}(0.120\text{ kg})(7.85\text{ m/s})^2 = 3.70\text{ J}$

4. Calculate E_f. At this point $y_f = 1.18$ m and $v_f = 6.20$ m/s:
   $E_f = U_f + K_f = mgy_f + \frac{1}{2}mv_f^2$
   $= (0.120\text{ kg})(9.81\text{ m/s}^2)(1.18\text{ m}) + \frac{1}{2}(0.120\text{ kg})(6.20\text{ m/s})^2$
   $= 1.39\text{ J} + 2.31\text{ J} = 3.70\text{ J}$

INSIGHT

As expected, E_f is equal to E_i. In the remaining Examples in this section we turn this process around; we start with $E_f = E_i$, and use this relation to find a final speed or a final height. As we shall see, this procedure of using energy conservation is a more powerful approach—it actually makes the calculations simpler.

PRACTICE PROBLEM

Use energy conservation to find the height at which the speed of the cap is 5.00 m/s. [**Answer:** 1.87 m]

Some related homework problems: Problem 30, Problem 31, and Problem 33

An interesting extension of this Example is shown in **Figure 8–9**. In this case, we are given that the speed of the cap is v_i at the height y_i, and we would like to know its speed v_f when it is at the height y_f.

To find v_f, we apply energy conservation to the points i and f:

$$U_i + K_i = U_f + K_f$$

Writing out U and K specifically for these two points yields the following:

$$mgy_i + \tfrac{1}{2}mv_i^2 = mgy_f + \tfrac{1}{2}mv_f^2$$

As before, we cancel m and solve for the unknown speed, v_f:

$$v_f = \sqrt{v_i^2 + 2g(y_i - y_f)}$$

This result is in agreement with the kinematic equation, $v_y^2 = v_{0y}^2 + 2a_y\,\Delta y$.

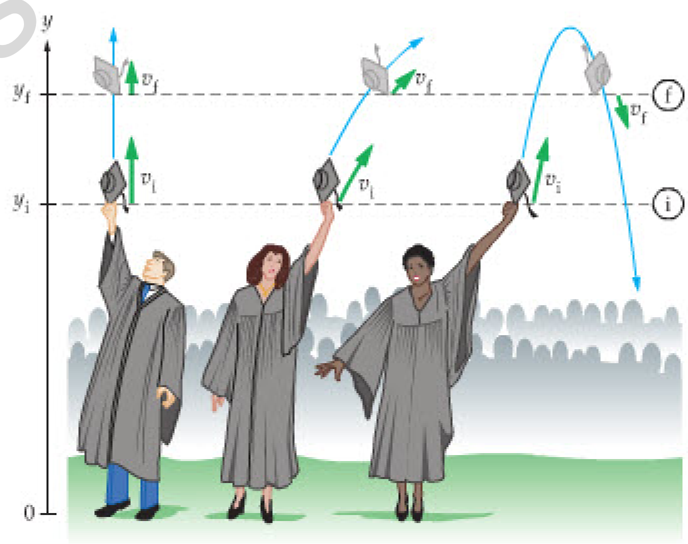


FIGURE 8–9 Speed is independent of path

If the speed of the cap is v_i at the height y_i, its speed is v_f at the height y_f, independent of the path between the two heights. This assumes, of course, that frictional forces can be neglected.

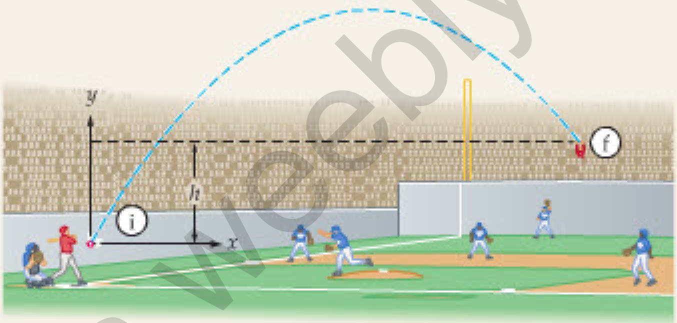
Note that v_f depends only on y_i and y_f, not on the path connecting them. This is because conservative forces such as gravity do work that is path-independent. What this means physically is that the cap has the same speed v_f at the height y_f, whether it goes straight upward or follows some other trajectory, as in Figure 8–9. All that matters is the height difference.

EXAMPLE 8–6 CATCHING A HOME RUN

In the bottom of the ninth inning, a player hits a 0.15-kg baseball over the outfield fence. The ball leaves the bat with a speed of 36 m/s, and a fan in the bleachers catches it 7.2 m above the point where it was hit. Assuming frictional forces can be ignored, find **(a)** the kinetic energy of the ball when it is caught and **(b)** its speed when caught.

PICTURE THE PROBLEM

Our sketch shows the ball's trajectory. We label the hit point i and the catch point f. At point i we choose $y_i = 0$; at point f, then, $y_f = h = 7.2$ m. In addition, we are given that $v_i = 36$ m/s; v_f is to be determined.



STRATEGY

a. Because frictional forces can be ignored, it follows that the initial mechanical energy is equal to the final mechanical energy; that is, $U_i + K_i = U_f + K_f$. Use this relation to find K_f.

b. Once K_f is determined, use $K_f = \frac{1}{2}mv_f^2$ to find v_f.

SOLUTION

Part (a)

1. Begin by writing U and K for point i:
$$U_i = 0$$
$$K_i = \tfrac{1}{2}mv_i^2 = \tfrac{1}{2}(0.15\text{ kg})(36\text{ m/s})^2 = 97\text{ J}$$

2. Next, write U and K for point f:
$$U_f = mgh = (0.15\text{ kg})(9.81\text{ m/s}^2)(7.2\text{ m}) = 11\text{ J}$$
$$K_f = \tfrac{1}{2}mv_f^2$$

3. Set the total mechanical energy at point i, $E_i = U_i + K_i$, equal to the total mechanical energy at point f, $E_f = U_f + K_f$, and solve for K_f:
$$U_i + K_i = U_f + K_f$$
$$0 + 97\text{ J} = 11\text{ J} + K_f$$
$$K_f = 97\text{ J} - 11\text{ J} = 86\text{ J}$$

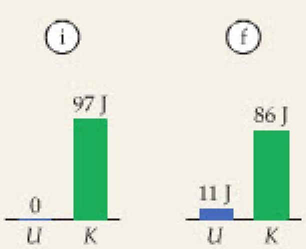
Part (b)

4. Use $K_f = \frac{1}{2}mv_f^2$ to find v_f:
$$K_f = \tfrac{1}{2}mv_f^2$$
$$v_f = \sqrt{\frac{2K_f}{m}} = \sqrt{\frac{2(86\text{ J})}{0.15\text{ kg}}} = 34\text{ m/s}$$

INSIGHT

To find the ball's speed when it was caught, we need to know the height of point f, but we don't need to know any details about the ball's trajectory. For example, it is not necessary to know the angle at which the ball leaves the bat or its maximum height.

The histograms to the right show the values of U and K at the points i and f. Notice that the energy of the system is mostly kinetic at the time the ball is caught.



PRACTICE PROBLEM

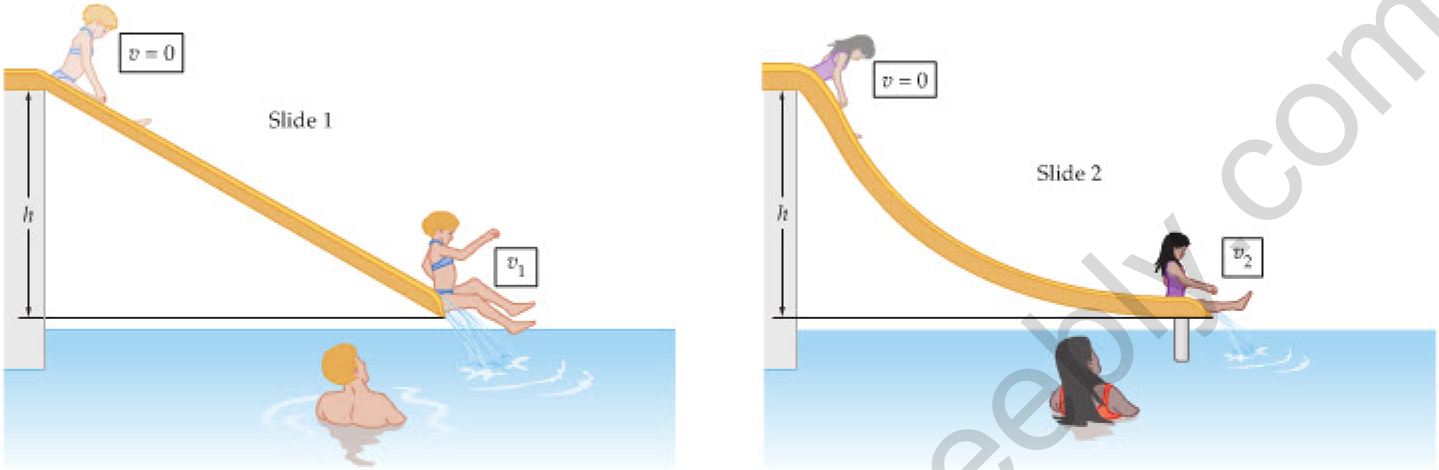
If the mass of the ball were increased, would the catch speed be greater than, less than, or the same as the value we just found? [**Answer:** The same. U and K depend on mass in the same way, hence the mass cancels.]

Some related homework problems: Problem 29, Problem 30

The connection between height difference and speeds is explored further in the following Conceptual Checkpoint and Example.

CONCEPTUAL CHECKPOINT 8–1 COMPARE THE FINAL SPEEDS

Swimmers at a water park can enter a pool using one of two frictionless slides of equal height. Slide 1 approaches the water with a uniform slope; slide 2 dips rapidly at first, then levels out. Is the speed v_2 at the bottom of slide 2 **(a)** greater than, **(b)** less than, or **(c)** the same as the speed v_1 at the bottom of slide 1?



REASONING AND DISCUSSION

In both cases, the same amount of potential energy, mgh, is converted to kinetic energy. Since the conversion of gravitational potential energy to kinetic energy is the *only* energy transaction taking place, it follows that the speed is the same for each slide.

Interestingly, although the final speeds are the same, the time required to reach the water is less for slide 2. The reason is that swimmer 2 reaches a high speed early and maintains it, whereas the speed of swimmer 1 increases slowly and steadily.

ANSWER

(c) The speeds are the same.

EXAMPLE 8–7 SKATEBOARD EXIT RAMP

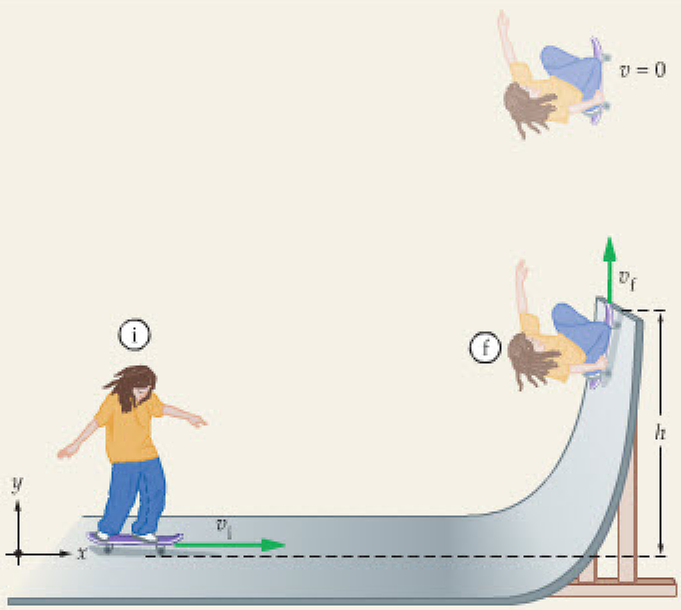
A 55-kg skateboarder enters a ramp moving horizontally with a speed of 6.5 m/s and leaves the ramp moving vertically with a speed of 4.1 m/s. Find the height of the ramp, assuming no energy loss to frictional forces.

PICTURE THE PROBLEM

We choose $y = 0$ to be the level of the bottom of the ramp, thus the gravitational potential energy is zero there. Point i indicates the skateboarder entering the ramp with a speed of 6.5 m/s; point f is the top of the ramp, where the speed is 4.1 m/s.

STRATEGY

To find h, simply set the initial energy, $E_i = U_i + K_i$, equal to the final energy, $E_f = U_f + K_f$.



SOLUTION

1. Write expressions for U_i and K_i: $U_i = mg \cdot 0 = 0$ $\quad K_i = \frac{1}{2}mv_i^2$

2. Write expressions for U_f and K_f: $U_f = mgh$ $\quad K_f = \frac{1}{2}mv_f^2$

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3. Set the total mechanical energy at point i, $E_i = U_i + K_i$, equal to the total mechanical energy at point f, $E_f = U_f + K_f$:

$$U_i + K_i = U_f + K_f$$
$$0 + \tfrac{1}{2}mv_i^2 = mgh + \tfrac{1}{2}mv_f^2$$

4. Solve for h. Note that m cancels:

$$mgh = \tfrac{1}{2}mv_i^2 - \tfrac{1}{2}mv_f^2$$
$$h = \frac{v_i^2 - v_f^2}{2g}$$

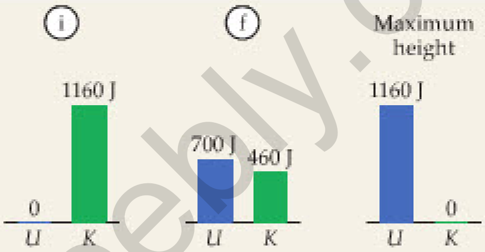
5. Substitute numerical values:

$$h = \frac{(6.5\ \text{m/s})^2 - (4.1\ \text{m/s})^2}{2(9.81\ \text{m/s}^2)} = 1.3\ \text{m}$$

INSIGHT

Note that our value for h is independent of the shape of the ramp—it is equally valid for one with the shape shown here, or one that simply inclines upward at a constant angle. In addition, the height does not depend on the person's mass, as we see in Step 4.

The histograms to the right show U and K to scale at the points i and f, as well as at the maximum height where $K = 0$.



PRACTICE PROBLEM

What is the skateboarder's maximum height above the bottom of the ramp? **[Answer:** 2.2 m**]**

Some related homework problems: Problem 29, Problem 33

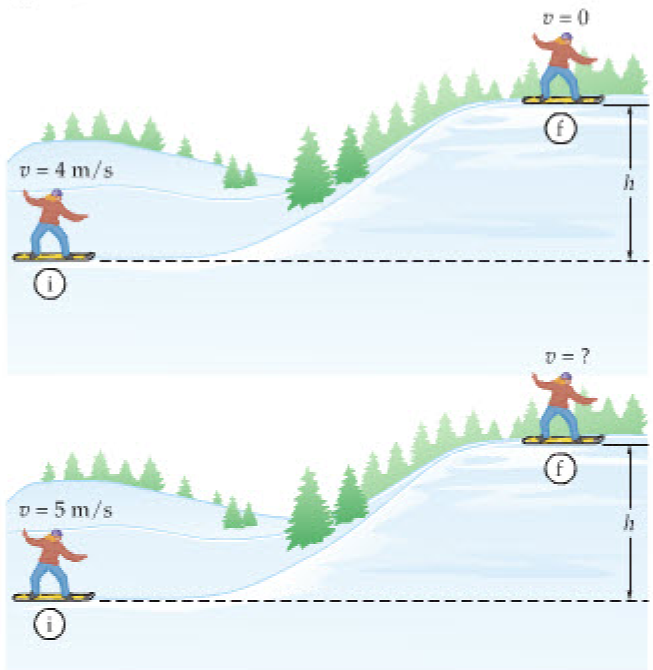
▲ Does the shape of the slide matter? (See Conceptual Checkpoint 8–1.)

It is interesting to express the equation in Step 3 from Example 8–7 in words. First, the left side of the equation is the initial kinetic energy of the skateboarder, $\tfrac{1}{2}mv_i^2$. This is the initial energy content of the system. At point f the system still has the same amount of energy, only now part of it, mgh, is in the form of gravitational potential energy. The remainder is the final kinetic energy, $\tfrac{1}{2}mv_f^2$.

Conceptual Checkpoint 8–2 considers the effect of a slight change in the initial speed of an object.

CONCEPTUAL CHECKPOINT 8–2 WHAT IS THE FINAL SPEED?

A snowboarder coasts on a smooth track that rises from one level to another. If the snowboarder's initial speed is 4 m/s, the snowboarder just makes it to the upper level and comes to rest. With a slightly greater initial speed of 5 m/s, the snowboarder is still moving to the right on the upper level. Is the snowboarder's final speed in this case **(a)** 1 m/s, **(b)** 2 m/s, or **(c)** 3 m/s?



REASONING AND DISCUSSION

A plausible-sounding answer is that since the initial speed is greater by 1 m/s in the second case, the final speed should be greater by 1 m/s as well. Therefore, the answer should be 0 + 1 m/s = 1 m/s. This is incorrect, however.

As surprising as it may seem, an increase in the initial speed from 4 m/s to 5 m/s results in an increase in the final speed from 0 to 3 m/s. This is due to the fact that kinetic energy depends on v^2 rather than v; thus, it is the difference in v^2 that counts. In this case, the initial value of v^2 increases from 16 m²/s² to 25 m²/s², for a total increase of 25 m²/s² − 16 m²/s² = 9 m²/s². The final value of v^2 must increase by the same amount, 9 m²/s² = (3 m/s)². As a result, the final speed is 3 m/s.

ANSWER

(c) The final speed of the snowboarder in the second case is 3 m/s.

Let's check the results of the previous Conceptual Checkpoint with a specific numerical example. Suppose the snowboarder has a mass of 74.0 kg. It follows that in the first case the initial kinetic energy is $K_i = \frac{1}{2}(74.0\text{ kg})(4.00\text{ m/s})^2 = 592\text{ J}$. At the top of the hill all of this kinetic energy is converted to gravitational potential energy, mgh.

In the second case, the initial speed of the snowboarder is 5.00 m/s; thus, the initial kinetic energy is $K_i = \frac{1}{2}(74.0\text{ kg})(5.00\text{ m/s})^2 = 925\text{ J}$. When the snowboarder reaches the top of the hill, 592 J of this kinetic energy is converted to gravitational potential energy, leaving the snowboarder with a final kinetic energy of 925 J − 592 J = 333 J. The corresponding speed is given by

$$\tfrac{1}{2}mv^2 = 333\text{ J}$$

$$v = \sqrt{\frac{2(333\text{ J})}{m}} = \sqrt{\frac{2(333\text{ J})}{74.0\text{ kg}}} = \sqrt{9.00\text{ m}^2/\text{s}^2} = 3.00\text{ m/s}$$

Thus, as expected, the snowboarder in the second case has a final speed of 3.00 m/s.

We conclude this section with two Examples involving springs.

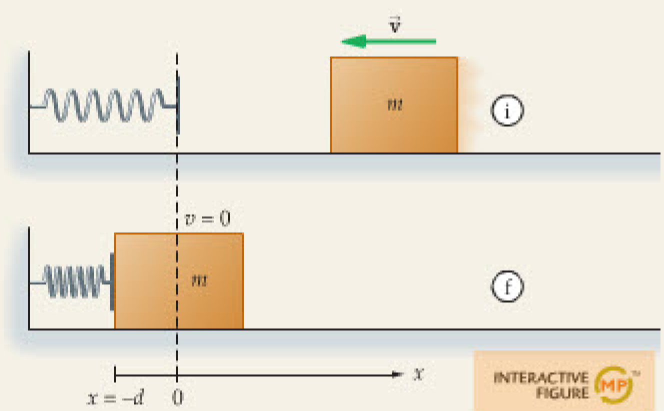
EXAMPLE 8–8 SPRING TIME

A 1.70-kg block slides on a horizontal, frictionless surface until it encounters a spring with a force constant of 955 N/m. The block comes to rest after compressing the spring a distance of 4.60 cm. Find the initial speed of the block. (Ignore air resistance and any energy lost when the block initially contacts the spring.)

PICTURE THE PROBLEM

Point i refers to times before the block makes contact with the spring, which means the block has a speed v and the end of the spring is at $x = 0$. Point f refers to the time when the block has come to rest, and the spring is compressed to $x = -d = -4.60$ cm.

We can choose the center of the block to be the $y = 0$ level. With this choice, the gravitational potential energy of the system is zero at all times.



STRATEGY

Set E_i equal to E_f to find the one unknown, v. Note that the initial energy, E_i, is the kinetic energy of the block before it reaches the spring. The final energy, E_f, is the potential energy of the compressed spring.

SOLUTION

1. Write expressions for U_i and K_i. For U, we consider only the potential energy of the spring, $U = \frac{1}{2}kx^2$: $\quad U_i = \frac{1}{2}k \cdot 0^2 = 0 \qquad K_i = \frac{1}{2}mv^2$
2. Do the same for U_f and K_f: $\quad U_f = \frac{1}{2}k(-d)^2 = \frac{1}{2}kd^2 \qquad K_f = \frac{1}{2}m \cdot 0^2 = 0$

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3. Set the initial mechanical energy, $E_i = U_i + K_i$, equal to the final mechanical energy, $E_f = U_f + K_f$, and solve for v:

$$U_i + K_i = U_f + K_f$$
$$0 + \frac{1}{2}mv^2 = \frac{1}{2}kd^2 + 0$$
$$v = d\sqrt{\frac{k}{m}}$$

4. Substitute numerical values:

$$v = d\sqrt{\frac{k}{m}} = (0.0460\text{ m})\sqrt{\frac{955\text{ N/m}}{1.70\text{ kg}}} = 1.09\text{ m/s}$$

INSIGHT

After the block comes to rest, the spring expands again, converting its potential energy back into the kinetic energy of the block. When the block leaves the spring, moving to the right, its speed is once again 1.09 m/s.

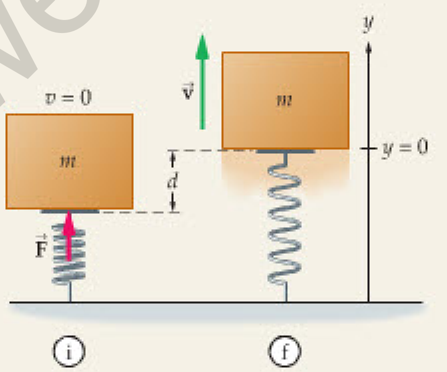
PRACTICE PROBLEM

What is the compression distance, d, if the block's initial speed is 0.500 m/s? [**Answer:** 2.11 cm]

Some related homework problems: Problem 32, Problem 34

ACTIVE EXAMPLE 8–1 FIND THE SPEED OF THE BLOCK

Suppose the spring and block in Example 8–8 are oriented vertically, as shown here. Initially, the spring is compressed 4.60 cm and the block is at rest. When the block is released, it accelerates upward. Find the speed of the block when the spring has returned to its equilibrium position.



SOLUTION *(Test your understanding by performing the calculations indicated in each step.)*

1. Write an expression for the initial mechanical energy E_i: $\quad E_i = U_i + K_i = -mgd + \frac{1}{2}kd^2 + 0$

2. Write an expression for the final mechanical energy E_f: $\quad E_f = U_f + K_f = 0 + 0 + \frac{1}{2}mv^2$

3. Set E_i equal to E_f and solve for v: $\quad -mgd + \frac{1}{2}kd^2 = \frac{1}{2}mv^2$

$$v = \sqrt{kd^2/m - 2gd}$$

4. Substitute numerical values: $\quad v = 0.535\text{ m/s}$

INSIGHT

In this system, part of the initial potential energy of the spring ($\frac{1}{2}kd^2$) goes into increasing the gravitational potential energy of the block (mgd). The remainder of the initial energy, $\frac{1}{2}kd^2 - mgd$, is converted into the block's kinetic energy.

YOUR TURN

What is the speed of the block when the spring is only halfway back to its equilibrium position?

(Answers to **Your Turn** *problems are given in the back of the book.)*

8–4 Work Done by Nonconservative Forces

Nonconservative forces change the amount of mechanical energy in a system. They might decrease the mechanical energy by converting it to thermal energy, or increase it by converting muscular work to kinetic or potential energy. In some systems, both types of processes occur at the same time.

To see the connection between the work done by a nonconservative force, W_{nc}, and the mechanical energy, E, we return once more to the work–energy theorem, which says that the *total* work is equal to the change in kinetic energy:

$$W_{total} = \Delta K$$

Suppose, for instance, that a system has one conservative and one nonconservative force. In this case, the total work is the sum of the conservative work W_c and the nonconservative work W_{nc}:

$$W_{total} = W_c + W_{nc}$$

Recalling that conservative work is related to the change in potential energy by the definition given in Equation 8–1, $W_c = -\Delta U$, we have

$$W_{total} = -\Delta U + W_{nc} = \Delta K$$

Solving this relation for the nonconservative work yields

$$W_{nc} = \Delta U + \Delta K$$

Finally, since the total mechanical energy is $E = U + K$, it follows that the *change* in mechanical energy is $\Delta E = \Delta U + \Delta K$. As a result, the nonconservative work is simply the change in mechanical energy:

$$W_{nc} = \Delta E = E_f - E_i \qquad 8\text{–}9$$

If more than one nonconservative force acts, we simply add the nonconservative work done by each such force to obtain W_{nc}.

At this point it may be useful to collect the three "working relationships" that have been introduced in the last two chapters:

$$W_{total} = \Delta K$$
$$W_c = -\Delta U \qquad 8\text{–}10$$
$$W_{nc} = \Delta E$$

Note that positive nonconservative work increases the total mechanical energy of a system, while negative nonconservative work decreases the mechanical energy—and converts it to other forms. In the next Example, for instance, part of the initial mechanical energy of a leaf is converted to heat and other forms of energy by air resistance as it falls to the ground.

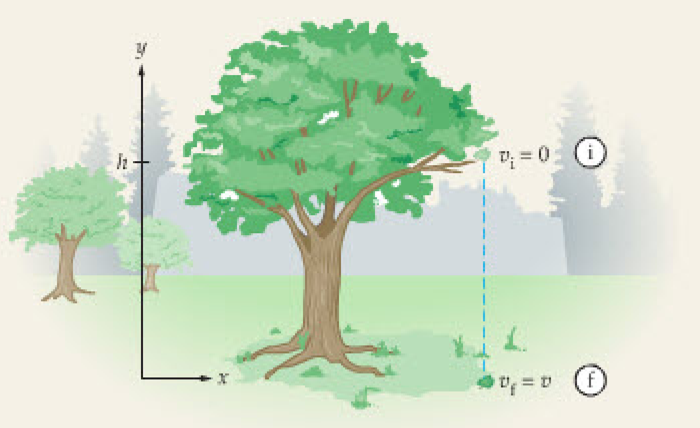
PROBLEM-SOLVING NOTE

Nonconservative Systems

Start by sketching the system and labeling the initial and final points with i and f, respectively. The initial and final mechanical energies are related to the nonconservative work by $W_{nc} = E_f - E_i$.

EXAMPLE 8–9 A LEAF FALLS IN THE FOREST: FIND THE NONCONSERVATIVE WORK

Deep in the forest, a 17.0-g leaf falls from a tree and drops straight to the ground. If its initial height was 5.30 m and its speed on landing was 1.3 m/s, how much nonconservative work was done on the leaf?



PICTURE THE PROBLEM

The leaf drops from rest at a height $y = h = 5.30$ m and lands with a speed $v = 1.3$ m/s at $y = 0$. These two points are labeled i and f, respectively.

STRATEGY

To begin, calculate the initial mechanical energy, E_i, and the final mechanical energy, E_f. Once these energies have been determined, the nonconservative work is $W_{nc} = \Delta E = E_f - E_i$.

SOLUTION

1. Evaluate U_i, K_i, and E_i:

$$U_i = mgh = (0.0170 \text{ kg})(9.81 \text{ m/s}^2)(5.30 \text{ m}) = 0.884 \text{ J}$$
$$K_i = \tfrac{1}{2}m \cdot 0^2 = 0$$
$$E_i = U_i + K_i = 0.884 \text{ J}$$

2. Next, evaluate U_f, K_f, and E_f.

$$U_f = mg \cdot 0 = 0$$
$$K_f = \tfrac{1}{2}mv^2 = \tfrac{1}{2}(0.0170 \text{ kg})(1.3 \text{ m/s})^2 = 0.014 \text{ J}$$
$$E_f = U_f + K_f = 0.014 \text{ J}$$

3. Use $W_{nc} = \Delta E$ to find the nonconservative work:

$$W_{nc} = \Delta E = E_f - E_i = 0.014 \text{ J} - 0.884 \text{ J} = -0.870 \text{ J}$$

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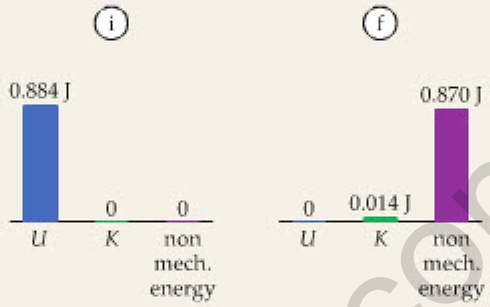
INSIGHT

Note that most of the initial mechanical energy is dissipated as the leaf falls. This is indicated in the histograms to the right. The small amount that remains (only about 1.6%) appears as the kinetic energy of the leaf just before it lands. If a cherry had fallen from the tree, it would have struck the ground with a considerably greater speed—perhaps five times the speed of the leaf. In that case, the percentage of the initial potential energy remaining as kinetic energy would have been $5^2 = 25$ times greater than the percentage retained by the leaf.



PRACTICE PROBLEM

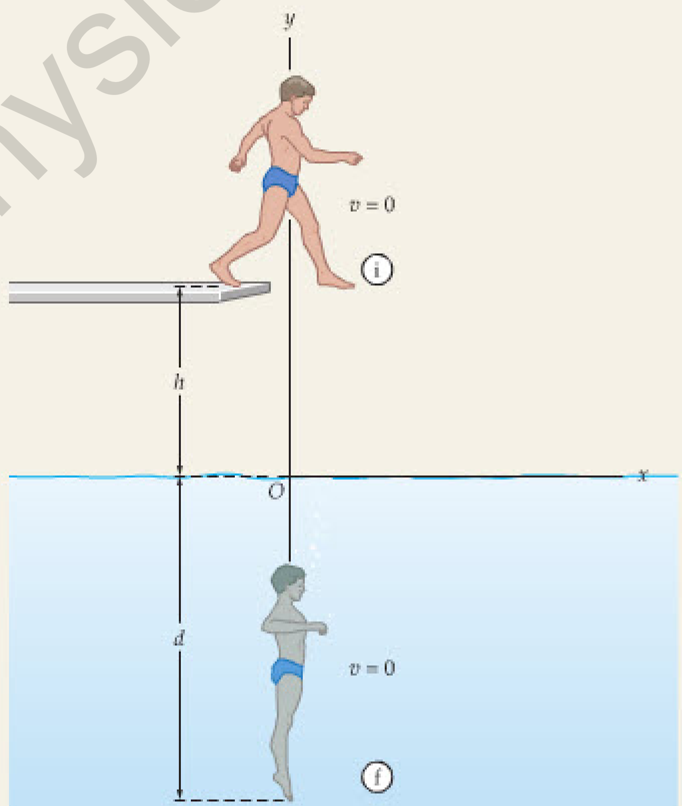
What was the average nonconservative force exerted on the leaf as it fell? [**Answer:** $W_{nc} = -Fh$, $F = -W_{nc}/h = 0.16$ N, upward]

Some related homework problems: Problem 42, Problem 43

In the following Active Example, we use a knowledge of the nonconservative work to find the depth at which a diver comes to rest.

ACTIVE EXAMPLE 8–2 FIND THE DIVER'S DEPTH

A 95.0-kg diver steps off a diving board and drops into the water 3.00 m below. At some depth d below the water's surface, the diver comes to rest. If the nonconservative work done on the diver is $W_{nc} = -5120$ J, what is the depth, d?



SOLUTION *(Test your understanding by performing the calculations indicated in each step.)*

1. Write the initial mechanical energy, E_i: $E_i = mgh + 0 = mgh$
2. Write the final mechanical energy, E_f: $E_f = mg(-d) + 0 = -mgd$

3. Set W_{nc} equal to ΔE:	$W_{nc} = \Delta E = E_f - E_i = -mgd - mgh$
4. Solve for d:	$d = -(W_{nc} + mgh)/mg$
5. Substitute numerical values:	$d = 2.49$ m

INSIGHT

Another way to write Step 3 is $E_f = E_i + W_{nc}$. In words, this equation says that the final mechanical energy is the initial mechanical energy plus the nonconservative work done on the system. In this case, $W_{nc} < 0$; hence the final mechanical energy is less than the initial mechanical energy.

YOUR TURN

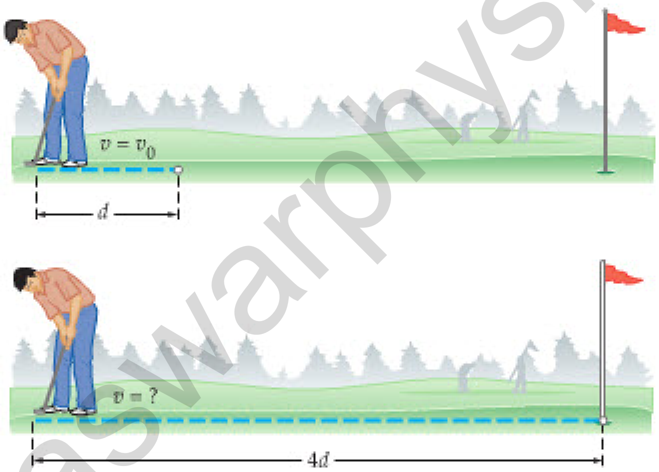
Suppose the diver descends to a depth of 3.50 m. How much nonconservative work is done in this case?

(Answers to **Your Turn** *problems are given in the back of the book.)*

We now present a Conceptual Checkpoint that further examines the relationship between nonconservative work and distance.

CONCEPTUAL CHECKPOINT 8–3 JUDGING A PUTT

A golfer badly misjudges a putt, sending the ball only one-quarter of the distance to the hole. The original putt gave the ball an initial speed of v_0. If the force of resistance due to the grass is constant, would an initial speed of **(a)** $2v_0$, **(b)** $3v_0$, or **(c)** $4v_0$ be needed to get the ball to the hole from its original position?



REASONING AND DISCUSSION

In the original putt, the ball started with a kinetic energy of $\frac{1}{2}mv_0^2$ and came to rest in the distance d. The kinetic energy was dissipated by the nonconservative force due to grass resistance, F, which does the work $W_{nc} = -Fd$. Since the change in mechanical energy is $\Delta E = 0 - \frac{1}{2}mv_0^2 = -\frac{1}{2}mv_0^2$, it follows from $W_{nc} = \Delta E$ that $Fd = \frac{1}{2}mv_0^2$. Therefore, to go four times the distance, $4d$, we need to give the ball four times as much kinetic energy. Noting that kinetic energy is proportional to v^2, we see that the initial speed need only be doubled.

ANSWER

(a) The initial speed should be doubled to $2v_0$.

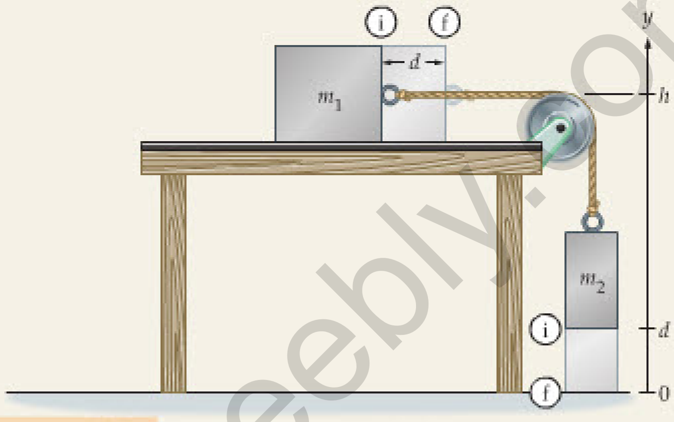
A common example of a nonconservative force is kinetic friction. In the next Example, we show how to include the effects of friction in a system that also includes kinetic energy and gravitational potential energy.

EXAMPLE 8–10 LANDING WITH A THUD

A block of mass $m_1 = 2.40$ kg is connected to a second block of mass $m_2 = 1.80$ kg, as shown here. When the blocks are released from rest, they move through a distance $d = 0.500$ m, at which point m_2 hits the floor. Given that the coefficient of kinetic friction between m_1 and the horizontal surface is $\mu_k = 0.450$, find the speed of the blocks just before m_2 lands.

PICTURE THE PROBLEM

We choose $y = 0$ to be at floor level; therefore, the gravitational potential energy of m_2 is zero when it lands. The potential energy of m_1 doesn't change during this process; it is always m_1gh. Thus, it isn't necessary to know the value of h. Note that we label the beginning and ending points with i and f, respectively.



INTERACTIVE FIGURE

STRATEGY

Since a nonconservative force (friction) is doing work in this system, we use $W_{nc} = \Delta E = E_f - E_i$. Thus, we must calculate not only the mechanical energies, E_i and E_f, but also the nonconservative work, W_{nc}. Note that E_f can be written in terms of the unknown speed of the blocks just before m_2 lands. Therefore, we can set W_{nc} equal to ΔE and solve for the final speed.

SOLUTION

1. Evaluate U_i, K_i, and E_i. Be sure to include contributions from both masses:

$$U_i = m_1gh + m_2gd$$
$$K_i = \tfrac{1}{2}m_1 \cdot 0^2 + \tfrac{1}{2}m_2 \cdot 0^2 = 0$$
$$E_i = U_i + K_i = m_1gh + m_2gd$$

2. Next, evaluate U_f, K_f, and E_f. Note that E_f depends on the unknown speed, v:

$$U_f = m_1gh + 0$$
$$K_f = \tfrac{1}{2}m_1v^2 + \tfrac{1}{2}m_2v^2$$
$$E_f = U_f + K_f = m_1gh + \tfrac{1}{2}m_1v^2 + \tfrac{1}{2}m_2v^2$$

3. Calculate the nonconservative work, W_{nc}. Recall that the force of friction is $f_k = \mu_k N = \mu_k m_1 g$, and that it points opposite to the displacement of distance d:

$$W_{nc} = -f_k d = -\mu_k m_1 g d$$

4. Set W_{nc} equal to $\Delta E = E_f - E_i$. Notice that m_1gh cancels because it occurs in both E_i and E_f.

$$W_{nc} = E_f - E_i$$
$$-\mu_k m_1 g d = \tfrac{1}{2}m_1v^2 + \tfrac{1}{2}m_2v^2 - m_2gd$$

5. Solve for v:

$$v = \sqrt{\frac{2(m_2 - \mu_k m_1)gd}{m_1 + m_2}}$$

6. Substitute numerical values:

$$v = \sqrt{\frac{2[1.80\text{ kg} - (0.450)(2.40\text{ kg})](9.81\text{ m/s}^2)(0.500\text{ m})}{1.80\text{ kg} + 2.40\text{ kg}}}$$
$$= 1.30\text{ m/s}$$

INSIGHT

Note that Step 4 can be rearranged as follows: $\tfrac{1}{2}m_1v^2 + \tfrac{1}{2}m_2v^2 = m_2gd - \mu_k m_1 gd$. Translating this to words, we can say that the final kinetic energy of the blocks is equal to the initial gravitational potential energy of m_2, minus the energy dissipated by friction.

PRACTICE PROBLEM

Find the coefficient of kinetic friction if the final speed of the blocks is 0.950 m/s. **[Answer:** $\mu_k = 0.589$]

Some related homework problems: Problem 46, Problem 51, Problem 66, Problem 106

Finally, we present an Active Example for the common situation of a system in which two different nonconservative forces do work.

ACTIVE EXAMPLE 8–3 MARATHON MAN: FIND THE HEIGHT OF THE HILL

An 80.0-kg jogger starts from rest and runs uphill into a stiff breeze. At the top of the hill the jogger has done the work $W_{nc1} = +1.80 \times 10^4$ J, air resistance has done the work $W_{nc2} = -4420$ J, and the jogger's speed is 3.50 m/s. Find the height of the hill.

SOLUTION *(Test your understanding by performing the calculations indicated in each step.)*

1. Write the initial mechanical energy, E_i:	$E_i = U_i + K_i = 0 + 0 = 0$
2. Write the final mechanical energy, E_f:	$E_f = U_f + K_f = mgh + \frac{1}{2}mv^2$
3. Set W_{nc} equal to ΔE:	$W_{nc} = \Delta E = mgh + \frac{1}{2}mv^2$
4. Use $W_{nc} = \Delta E$ to solve for h:	$h = (W_{nc} - \frac{1}{2}mv^2)/mg$
5. Calculate the total nonconservative work:	$W_{nc} = W_{nc1} + W_{nc2} = 13{,}600$ J
6. Substitute numerical values to determine h:	$h = 16.7$ m

INSIGHT
As usual when dealing with energy calculations, our final result is independent of the shape of the hill.

YOUR TURN
Suppose the jogger's mass had been 90.0 kg rather than 80.0 kg. What would be the height of the hill in this case?

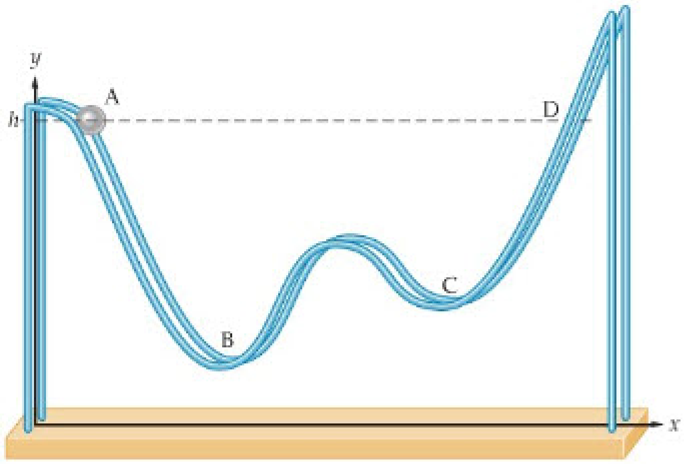
(Answers to **Your Turn** *problems are given in the back of the book.)*

▲ Highways that descend steeply are often provided with escape ramps that enable truck drivers whose brakes fail to bring their rigs to a safe stop. These ramps provide a perfect illustration of the conservation of energy. From a physics point of view, the driver's problem is to get rid of an enormous amount of kinetic energy in the safest possible way. The ramps run uphill, so some of the kinetic energy is simply converted back into gravitational potential energy (just as in a roller coaster). In addition, the ramps are typically surfaced with sand or gravel, allowing much of the initial kinetic energy to be dissipated by friction into other forms of energy, such as sound and heat.

8–5 Potential Energy Curves and Equipotentials

Figure 8–10 shows a metal ball rolling on a roller coaster–like track. Initially the ball is at rest at point A. Since the height at A is $y = h$, the ball's initial mechanical energy is $E_0 = mgh$. If friction and other nonconservative forces can be ignored, the ball's mechanical energy remains fixed at E_0 throughout its motion. Thus,

$$E = U + K = E_0$$



◀ **FIGURE 8–10 A ball rolling on a frictionless track**
The ball starts at A, where $y = h$, with zero speed. Its greatest speed occurs at B. At D, where $y = h$ again, its speed returns to zero.

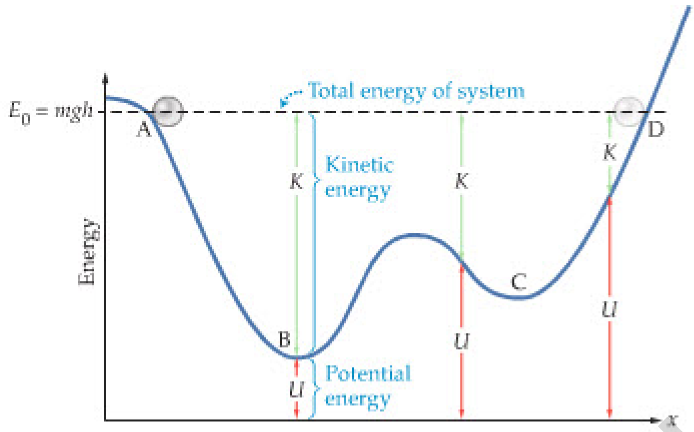


FIGURE 8–11 Gravitational potential energy versus position for the track shown in Figure 8–10
The shape of the potential energy curve is exactly the same as the shape of the track. In this case, the total mechanical energy is fixed at its initial value, $E_0 = U + K = mgh$. Because the height of the curve is U, by definition, it follows that K is the distance from the curve up to the dashed line at $E_0 = mgh$. Note that K is largest at B. In addition, K vanishes at A and D, which are turning points of the motion.

As the ball moves, its potential energy falls and rises in the same way as the track. After all, the gravitational potential energy, $U = mgy$, is directly proportional to the height of the track, y. In a sense, then, the track itself represents a graph of the corresponding potential energy.

This is shown explicitly in Figure 8–11, where we plot energy on the vertical axis and x on the horizontal axis. The potential energy U looks just like the track in Figure 8–10. In addition, we plot a horizontal line at the value E_0, indicating the constant energy of the ball. Since the potential energy plus the kinetic energy must always add up to E_0, it follows that K is the amount of energy from the potential energy curve up to the horizontal line at E_0. This is also shown in Figure 8–11.

Examining an energy plot like Figure 8–11 gives a great deal of information about the motion of an object. For example, at point B the potential energy has its lowest value, and thus the kinetic energy is greatest there. At point C the potential energy has increased, indicating a corresponding decrease in kinetic energy. As the ball continues to the right, the potential energy increases until, at point D, it is again equal to the total energy, E_0. At this point the kinetic energy is zero, and the ball comes to rest momentarily. It then "turns around" and begins to move to the left, eventually returning to point A where it again stops, changes direction, and begins a new cycle. Points A and D, then, are referred to as **turning points** of the motion.

Turning points are also seen in the motion of a mass on a spring, as indicated in Figure 8–12. Figure 8–12 (a) shows a mass pulled to the position $x = A$, and released from rest; Figure 8–12 (b) shows the potential energy of the system, $U = \frac{1}{2}kx^2$.

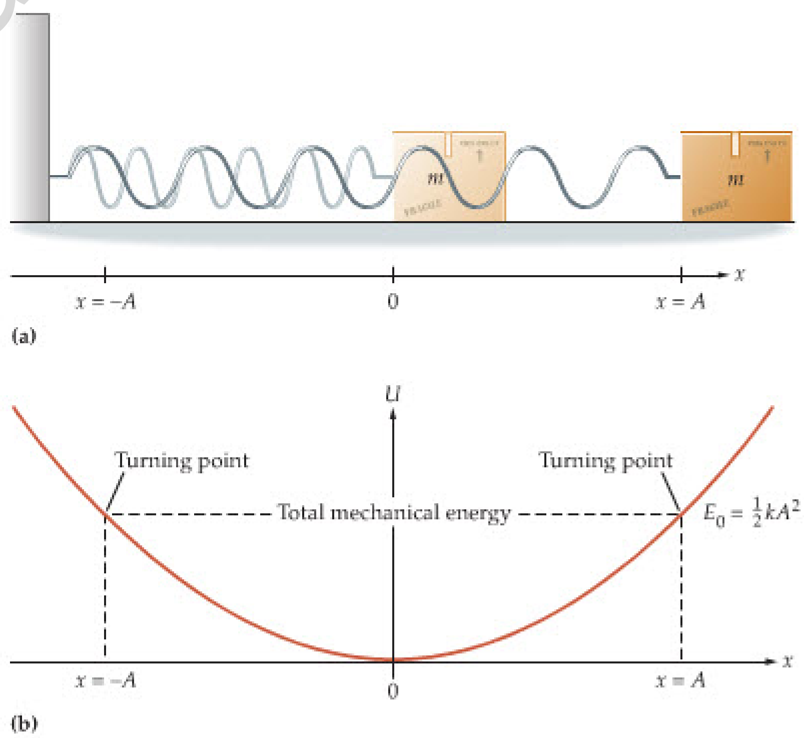


FIGURE 8–12 A mass on a spring
(a) A spring is stretched by an amount A, giving it a potential energy of $U = \frac{1}{2}kA^2$. **(b)** The potential energy curve, $U = \frac{1}{2}kx^2$, for the spring in (a). Because the mass starts at rest, its initial mechanical energy is $E_0 = \frac{1}{2}kA^2$. The mass oscillates between $x = A$ and $x = -A$.

Starting the system this way gives it an initial energy $E_0 = \frac{1}{2}kA^2$, shown by the horizontal line in Figure 8–12 (b). As the mass moves to the left, its speed increases, reaching a maximum where the potential energy is lowest, at $x = 0$. If no nonconservative forces act, the mass continues to $x = -A$, where it stops momentarily before returning to $x = A$. This type of **oscillatory motion** will be studied in detail in Chapter 13.

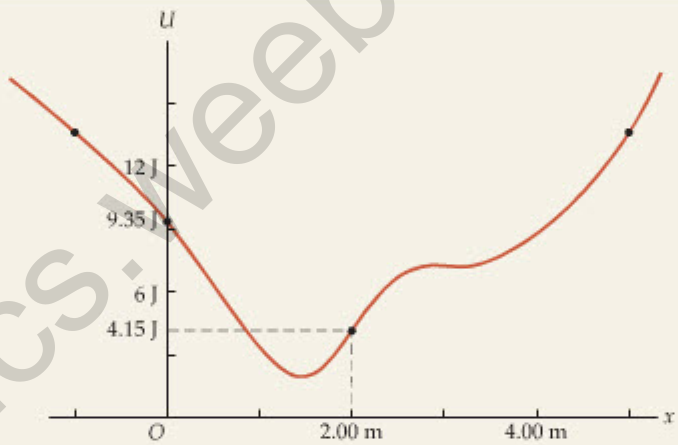
The next Example uses a potential-energy curve to find the speed of an object at a given value of x.

EXAMPLE 8–11 A POTENTIAL PROBLEM

A 1.60-kg object in a conservative system moves along the x axis, where the potential energy is as shown. A physical example would be a bead sliding on a wire with the shape of the potential energy curve. If the object's speed at $x = 0$ is 2.30 m/s, what is its speed at $x = 2.00$ m?

PICTURE THE PROBLEM

The plot shows U as a function of x. The values of U at $x = 0$ and $x = 2.00$ m are 9.35 J and 4.15 J, respectively. It follows that the object's speed at $x = 2.00$ m will be greater than its speed at $x = 0$.



STRATEGY

Since mechanical energy is conserved, we know that the total energy at $x = 0$ $(U_i + K_i)$ is equal to the total energy at $x = 2.00$ m $(U_f + K_f)$.

The problem statement gives U_i, and since we also know the speed at $x = 0$ we can use $K = \frac{1}{2}mv^2$ to calculate the corresponding kinetic energy, K_i. At $x = 2.00$ m we know the potential energy, U_f; hence we can use $U_i + K_i = U_f + K_f$ to solve for K_f. Once the final kinetic energy is known, it is possible to solve for the final speed by once again using $K = \frac{1}{2}mv^2$.

SOLUTION

1. Evaluate U_i, K_i, and E_i at $x = 0$:

$$U_i = 9.35\ \text{J}$$
$$K_i = \tfrac{1}{2}mv_i^2 = \tfrac{1}{2}(1.60\ \text{kg})(2.30\ \text{m/s})^2 = 4.23\ \text{J}$$
$$E_i = U_i + K_i = 9.35\ \text{J} + 4.23\ \text{J} = 13.58\ \text{J}$$

2. Write expressions for U_f, K_f, and E_f at $x = 2.00$ m:

$$U_f = 4.15\ \text{J}$$
$$K_f = \tfrac{1}{2}mv_f^2$$
$$E_f = U_f + K_f = 4.15\ \text{J} + \tfrac{1}{2}mv_f^2$$

3. Set E_f equal to E_i and solve for v_f:

$$4.15\ \text{J} + \tfrac{1}{2}mv_f^2 = 13.58\ \text{J}$$

Solve for v_f:

$$v_f = \sqrt{\frac{2(13.58\ \text{J} - 4.15\ \text{J})}{m}}$$

4. Substitute the numerical value of the object's mass:

$$v_f = \sqrt{\frac{2(13.58\ \text{J} - 4.15\ \text{J})}{1.60\ \text{kg}}} = 3.43\ \text{m/s}$$

INSIGHT

As we see in Step 1, the total mechanical energy of the system is 13.58 J. This means that turning points for this object occur at values of x where $U = 13.58$ J.

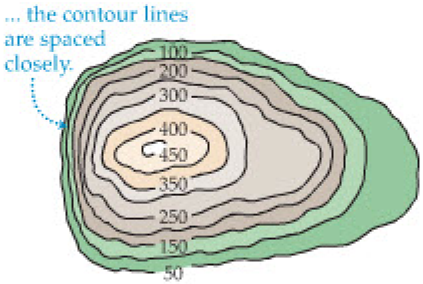
PRACTICE PROBLEM

Using the graph provided, estimate the location of the turning points for this object. [**Answer:** $x = -1.00$ m and $x = 5.00$ m]

Some related homework problems: Problem 57, Problem 58

Oscillatory motion between turning points is also observed in molecules. If the energy of oscillation is relatively small, as is usual at room temperature, the atoms in a molecule simply vibrate back and forth—like masses connected by a spring. As long as no energy is gained or lost, the molecular oscillations continue unchanged. On the other hand, if the energy of the molecule is increased by

Side view of mountain



Contour map of mountain (from above)

▲ **FIGURE 8–13** **A contour map**

A small mountain (top, in side view) is very steep on the left, more gently sloping on the right. A contour map of this mountain (bottom) shows a series of equal-altitude contour lines from 50 ft to 450 ft. Notice that the contour lines are packed close together where the terrain is steep, but are widely spaced where it is more level.

heating, or some other mechanism, the molecule will eventually dissociate—fly apart—as the atoms move to infinite separation.

In some cases, a two-dimensional plot of potential energy contours is useful. For instance, **Figure 8–13** shows a contour map of a hill. Each contour corresponds to a given altitude and, hence, to a given value of the gravitational potential energy. In general, lines corresponding to constant values of potential energy are called **equipotentials.** Since the altitude changes by equal amounts from one contour to the next, it follows that when gravitational equipotentials are packed close together, the corresponding terrain is steep. On the other hand, when the equipotentials are widely spaced, the ground is nearly flat, since a large horizontal distance is required for a given change in altitude. We shall see similar plots with similar interpretations when we study electric potential energy in Chapter 21.

THE BIG PICTURE PUTTING PHYSICS IN CONTEXT

LOOKING BACK

The concept of work, first introduced in Chapter 7 as force times distance, is used again in this chapter. For example, we use work in Section 8–1 to illustrate the difference between conservative and nonconservative forces.

Work (Chapter 7) is used again in Section 8–2 to introduce the concept of potential energy, U, and to define its change.

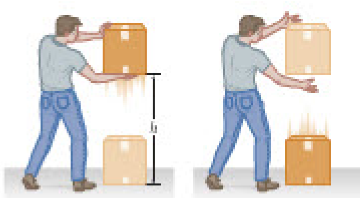
LOOKING AHEAD

Conservation of energy is one of the key elements in the study of elastic collisions. See, in particular, Section 9–6 and Example 9–7.

The wide-ranging importance of energy conservation is illustrated by its use in the following disparate topics: rotational motion (Section 10–6), gravitation (Section 12–5), oscillatory motion (Section 13–5), fluid dynamics (Section 15–7), and phase changes (Section 17–6).

CHAPTER SUMMARY

8–1 CONSERVATIVE AND NONCONSERVATIVE FORCES



Conservative forces conserve the mechanical energy of a system. Thus, in a conservative system the total mechanical energy remains constant.

Nonconservative forces convert mechanical energy into other forms of energy, or convert other forms of energy into mechanical energy.

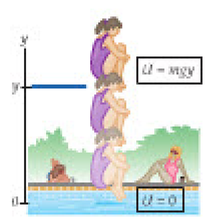
Conservative Force, Definition
A conservative force does zero total work on any closed path. In addition, the work done by a conservative force in going from point A to point B is *independent of the path* from A to B.

Examples of Conservative Forces
Gravity, spring.

Nonconservative Force, Definition
The work done by a nonconservative force on a closed path is nonzero. The work is also path-dependent.

Examples of Nonconservative Forces
Friction, air resistance, tension in ropes and cables, forces exerted by muscles and motors.

8–2 POTENTIAL ENERGY AND THE WORK DONE BY CONSERVATIVE FORCES



Potential energy, U, can "store" energy in a system. Energy in the form of potential energy can be converted to kinetic or other forms of energy.

Potential Energy, Definition
The work done by a conservative force is the negative of the change in potential energy:

$$W_c = -\Delta U = U_i - U_f \qquad 8\text{–}1$$

Zero Level
Any location can be chosen for $U = 0$. Once the choice is made, however, it must be used consistently.

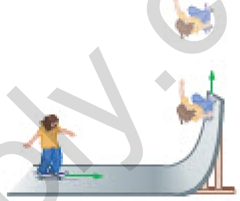
Gravity
Choosing $y = 0$ to be the zero level near Earth's surface,

$$U = mgy. \quad 8\text{–}3$$

Spring
Choosing $x = 0$ (the equilibrium position) to be the zero level,

$$U = \tfrac{1}{2}kx^2. \quad 8\text{–}5$$

8–3 CONSERVATION OF MECHANICAL ENERGY

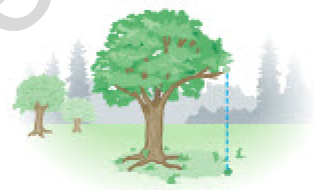


Mechanical energy, E, is conserved in systems with conservative forces only.

Mechanical Energy, Definition
Mechanical energy is the sum of the potential and kinetic energies of a system:

$$E = U + K \quad 8\text{–}6$$

8–4 WORK DONE BY NONCONSERVATIVE FORCES

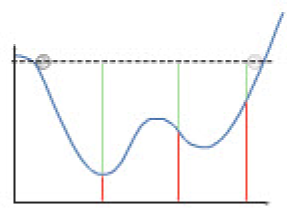


Nonconservative forces can change the mechanical energy of a system.

Change in Mechanical Energy
The work done by a nonconservative force is equal to the change in the mechanical energy of a system:

$$W_{nc} = \Delta E = E_f - E_i \quad 8\text{–}9$$

8–5 POTENTIAL ENERGY CURVES AND EQUIPOTENTIALS



A potential energy curve plots U as a function of position.

An equipotential plot shows contours corresponding to constant values of U.

Turning Points
Turning points occur where an object stops momentarily before reversing direction. At turning points the kinetic energy is zero.

Oscillatory Motion
An object moving back and forth between two turning points is said to have oscillatory motion.

PROBLEM-SOLVING SUMMARY

Type of Calculation	Relevant Physical Concepts	Related Examples
Calculate the gravitational or spring potential energy.	The potential energy for gravity is $U = mgy$; the potential energy for a spring is $U = \frac{1}{2}kx^2$.	Examples 8–2, 8–3, 8–4
Apply energy conservation in a system involving gravity.	Choose a horizontal level for $y = 0$, then use $U = mgy$.	Examples 8–6, 8–7 Active Example 8–1
Apply energy conservation in a system involving a spring.	Use $U = \frac{1}{2}kx^2$, where x measures the expansion or compression of the spring from its equilibrium position.	Example 8–8, Active Example 8–1
Find the nonconservative work done on a system.	Calculate the initial energy, E_i, and the final energy, E_f. Then use $W_{nc} = \Delta E = E_f - E_i$.	Examples 8–9, 8–10, Active Examples 8–2, 8–3

CONCEPTUAL QUESTIONS

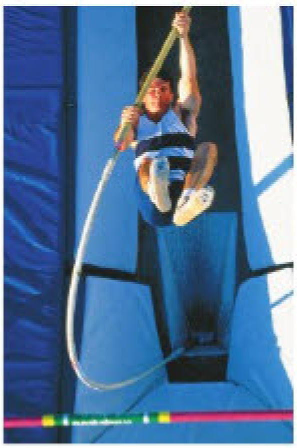
For instructor-assigned homework, go to www.masteringphysics.com MP™

(Answers to odd-numbered Conceptual Questions can be found in the back of the book.)

1. Is it possible for the kinetic energy of an object to be negative? Is it possible for the gravitational potential energy of an object to be negative? Explain.

2. An avalanche occurs when a mass of snow slides down a steep mountain slope. Discuss the energy conversions responsible for water vapor rising to form clouds, falling as snow on a mountain, and then sliding down a slope as an avalanche.

3. If the stretch of a spring is doubled, the force it exerts is also doubled. By what factor does the spring's potential energy increase?
4. When a mass is placed on top of a vertical spring, the spring compresses and the mass moves downward. Analyze this system in terms of its mechanical energy.
5. If a spring is stretched so far that it is permanently deformed, its force is no longer conservative. Why?
6. An object is thrown upward to a person on a roof. At what point is the object's kinetic energy at maximum? At what point is the potential energy of the system at maximum? At what locations do these energies have their minimum values?
7. It is a law of nature that the total energy of the universe is conserved. What do politicians mean, then, when they urge "energy conservation"?
8. Discuss the various energy conversions that occur when a person performs a pole vault. Include as many conversions as you can, and consider times before, during, and after the actual vault itself.



How many energy conversions can you identify? (Conceptual Question 8)

9. Discuss the nature of the work done by the equipment shown in this photo. What types of forces are involved?

Conservative or nonconservative? (Conceptual Question 9)

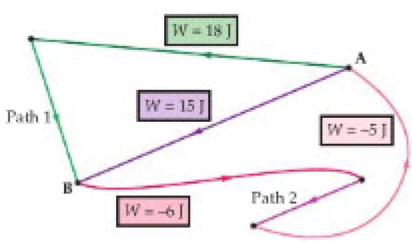
10. A toy frog consists of a suction cup and a spring. When the suction cup is pressed against a smooth surface, the frog is held down. When the suction cup lets go, the frog leaps into the air. Discuss the behavior of the frog in terms of energy conversions.
11. If the force on an object is zero, does that mean the potential energy of the system is zero? If the potential energy of a system is zero, is the force zero?
12. When a ball is thrown upward, its mechanical energy, $E = mgy + \frac{1}{2}mv^2$, is constant with time if air resistance can be ignored. How does E vary with time if air resistance cannot be ignored?
13. When a ball is thrown upward, it spends the same amount of time on the way up as on the way down—as long as air resistance can be ignored. If air resistance is taken into account, is the time on the way down the same as, greater than, or less than the time on the way up? Explain.

PROBLEMS AND CONCEPTUAL EXERCISES

Note: Answers to odd-numbered Problems and Conceptual Exercises can be found in the back of the book. **IP** *denotes an integrated problem, with both conceptual and numerical parts;* **BIO** *identifies problems of biological or medical interest;* **CE** *indicates a conceptual exercise.* **Predict/Explain** *problems ask for two responses:* ***(a)*** *your prediction of a physical outcome, and* ***(b)*** *the best explanation among three provided. On all problems, red bullets (•, ••, •••) are used to indicate the level of difficulty.*

SECTION 8–1 CONSERVATIVE AND NONCONSERVATIVE FORCES

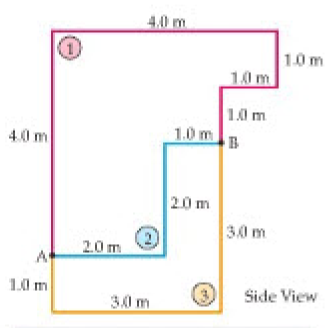
1. • **CE** The work done by a conservative force is indicated in Figure 8–14 for a variety of different paths connecting the points A and B. What is the work done by this force **(a)** on path 1 and **(b)** on path 2?



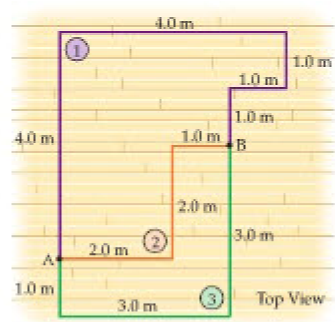
▲ **FIGURE 8–14** Problem 1

2. • Calculate the work done by gravity as a 3.2-kg object is moved from point A to point B in Figure 8–15 along paths 1, 2, and 3.

3. • Calculate the work done by friction as a 3.7-kg box is slid along a floor from point A to point B in Figure 8–16 along paths 1, 2, and 3. Assume that the coefficient of kinetic friction between the box and the floor is 0.26.

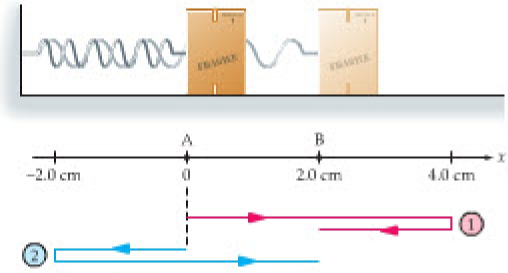


▲ **FIGURE 8–15** Problem 2



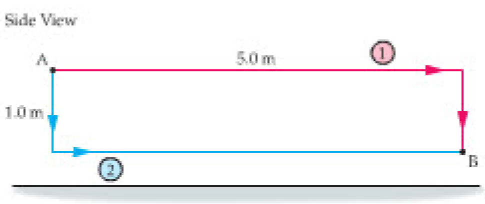
▲ **FIGURE 8–16** Problem 3

4. • **IP** A 4.1-kg block is attached to a spring with a force constant of 550 N/m, as shown in **Figure 8–17**. **(a)** Find the work done by the spring on the block as the block moves from A to B along paths 1 and 2. **(b)** How do your results depend on the mass of the block? Specifically, if you increase the mass, does the work done by the spring increase, decrease, or stay the same? (Assume the system is frictionless.)



▲ **FIGURE 8–17** Problems 4 and 6

5. • **IP** **(a)** Calculate the work done by gravity as a 5.2-kg object is moved from A to B in **Figure 8–18** along paths 1 and 2. **(b)** How do your results depend on the mass of the block? Specifically, if you increase the mass, does the work done by gravity increase, decrease, or stay the same?



▲ **FIGURE 8–18** Problem 5

6. •• In the system shown in Figure 8–17, suppose the block has a mass of 2.7 kg, the spring has a force constant of 480 N/m, and the coefficient of kinetic friction between the block and the floor is 0.16. **(a)** Find the work done on the block by the spring and by friction as the block is moved from point A to point B along path 2. **(b)** Find the work done on the block by the spring and by friction if the block is moved directly from point A to point B.

SECTION 8–2 POTENTIAL ENERGY AND THE WORK DONE BY CONSERVATIVE FORCES

7. • **CE Predict/Explain** Ball 1 is thrown to the ground with an initial downward speed; ball 2 is dropped to the ground from rest. Assuming the balls have the same mass and are released from the same height, is the change in gravitational potential energy of ball 1 greater than, less than, or equal to the change in gravitational potential energy of ball 2? **(b)** Choose the *best explanation* from among the following:
   I. Ball 1 has the greater total energy, and therefore more energy can go into gravitational potential energy.
   II. The gravitational potential energy depends only on the mass of the ball and the drop height.
   III. All of the initial energy of ball 2 is gravitational potential energy.

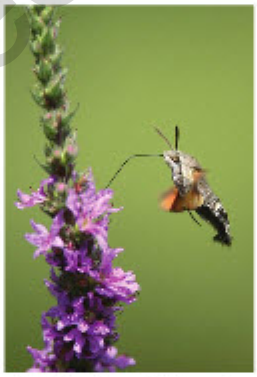
8. • **CE** A mass is attached to the bottom of a vertical spring. This causes the spring to stretch and the mass to move downward. **(a)** Does the potential energy of the spring increase, decrease, or stay the same during this process? Explain. **(b)** Does the gravitational potential energy of the Earth–mass system increase, decrease, or stay the same during this process? Explain.

9. • As an Acapulco cliff diver drops to the water from a height of 46 m, his gravitational potential energy decreases by 25,000 J. What is the diver's weight in newtons?

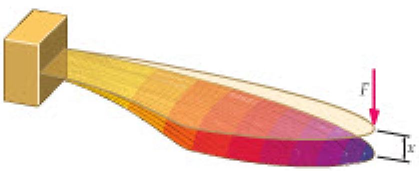
10. • Find the gravitational potential energy of an 88-kg person standing atop Mt. Everest at an altitude of 8848 m. Use sea level as the location for $y = 0$.

11. • **Jeopardy!** Contestants on the game show *Jeopardy!* depress spring-loaded buttons to "buzz in" and provide the question corresponding to the revealed answer. The force constant on these buttons is about 130 N/m. Estimate the amount of energy it takes—at a minimum—to buzz in.

12. •• **BIO The Wing of the Hawkmoth** Experiments performed on the wing of a hawkmoth (*Manduca sexta*) show that it deflects by a distance of $x = 4.8$ mm when a force of magnitude $F = 3.0$ mN is applied at the tip, as indicated in **Figure 8–19**. Treating the wing as an ideal spring, find **(a)** the force constant of the wing and **(b)** the energy stored in the wing when it is deflected. **(c)** What force must be applied to the tip of the wing to store twice the energy found in part (b)?



Hummingbird hawkmoth (*Manduca sexta*). (Problem 12)



▲ **FIGURE 8–19** Problem 12

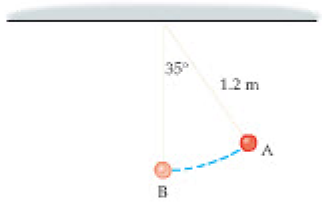
13. •• **IP** A vertical spring stores 0.962 J in spring potential energy when a 3.5-kg mass is suspended from it. **(a)** By what multiplicative factor does the spring potential energy change if the mass attached to the spring is doubled? **(b)** Verify your answer to part (a) by calculating the spring potential energy when a 7.0-kg mass is attached to the spring.

14. •• Pushing on the pump of a soap dispenser compresses a small spring. When the spring is compressed 0.50 cm, its potential energy is 0.0025 J. **(a)** What is the force constant of the spring? **(b)** What compression is required for the spring potential energy to equal 0.0084 J?

15. •• A force of 4.1 N is required to stretch a certain spring by 1.4 cm. **(a)** How far must this spring be stretched for its potential energy to be 0.020 J? **(b)** How much stretch is required for the spring potential energy to be 0.080 J?

16. •• **IP** The work required to stretch a certain spring from an elongation of 4.00 cm to an elongation of 5.00 cm is 30.5 J. **(a)** Is the work required to increase the elongation of the spring from 5.00 cm to 6.00 cm greater than, less than, or equal to 30.5 J? Explain. **(b)** Verify your answer to part (a) by calculating the required work.

17. •• A 0.33-kg pendulum bob is attached to a string 1.2 m long. What is the change in the gravitational potential energy of the system as the bob swings from point A to point B in Figure 8–20?



▲ **FIGURE 8–20** Problems 17, 34, 35, and 74

SECTION 8–3 CONSERVATION OF MECHANICAL ENERGY

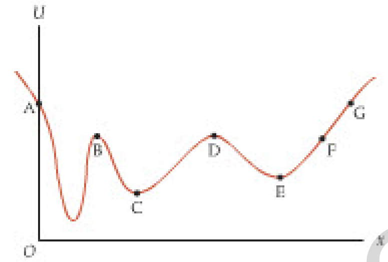
18. • **CE Predict/Explain** You throw a ball upward and let it fall to the ground. Your friend drops an identical ball straight down to the ground from the same height. Is the change in kinetic energy of your ball greater than, less than, or equal to the change in kinetic energy of your friend's ball? **(b)** Choose the *best explanation* from among the following:
   I. Your friend's ball converts all its initial energy into kinetic energy.
   II. Your ball is in the air longer, which results in a greater change in kinetic energy.
   III. The change in gravitational potential energy is the same for each ball, which means the change in kinetic energy must be the same also.

19. • **CE** Suppose the situation described in Conceptual Checkpoint 8–2 is repeated on the fictional planet Epsilon, where the acceleration due to gravity is less than it is on the Earth. **(a)** Would the height of a hill on Epsilon that causes a reduction in speed from 4 m/s to 0 be greater than, less than, or equal to the height of the corresponding hill on Earth? Explain. **(b)** Consider the hill on Epsilon discussed in part (a). If the initial speed at the bottom of the hill is 5 m/s, will the final speed at the top of the hill be greater than, less than, or equal to 3 m/s? Explain.

20. • **CE Predict/Explain** When a ball of mass m is dropped from rest from a height h, its kinetic energy just before landing is K. Now, suppose a second ball of mass $4m$ is dropped from rest from a height $h/4$. **(a)** Just before ball 2 lands, is its kinetic energy $4K$, $2K$, K, $K/2$, or $K/4$? **(b)** Choose the *best explanation* from among the following:
   I. The two balls have the same initial energy.
   II. The more massive ball will have the greater kinetic energy.
   III. The reduced drop height results in a reduced kinetic energy.

21. • **CE Predict/Explain** When a ball of mass m is dropped from rest from a height h, its speed just before landing is v. Now, suppose a second ball of mass $4m$ is dropped from rest from a height $h/4$. **(a)** Just before ball 2 lands, is its speed $4v$, $2v$, v, $v/2$, or $v/4$? **(b)** Choose the *best explanation* from among the following:
   I. The factors of 4 cancel; therefore, the landing speed is the same.
   II. The two balls land with the same kinetic energy; therefore, the ball of mass $4m$ has the speed $v/2$.
   III. Reducing the height by a factor of 4 reduces the speed by a factor of 4.

22. • **CE** For an object moving along the x axis, the potential energy of the frictionless system is shown in Figure 8–21. Suppose the object is released from rest at the point A. Rank the other points in the figure in increasing order of the object's speed. Indicate ties where appropriate.



▲ **FIGURE 8–21** Problems 22 and 23

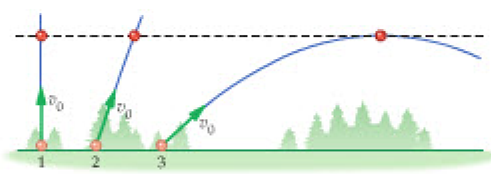
23. • **CE** Referring to Problem 22, suppose the object is released from rest at a point halfway between the points F and G. Rank the other points in the figure in increasing order of the object's speed, if the object can reach that point. Indicate ties where appropriate.

24. • At an amusement park, a swimmer uses a water slide to enter the main pool. If the swimmer starts at rest, slides without friction, and descends through a vertical height of 2.31 m, what is her speed at the bottom of the slide?

25. • In the previous problem, find the swimmer's speed at the bottom of the slide if she starts with an initial speed of 0.840 m/s.

26. • **IP** A player passes a 0.600-kg basketball downcourt for a fast break. The ball leaves the player's hands with a speed of 8.30 m/s and slows down to 7.10 m/s at its highest point. **(a)** Ignoring air resistance, how high above the release point is the ball when it is at its maximum height? **(b)** How would doubling the ball's mass affect the result in part (a)? Explain.

27. •• **CE** Three balls are thrown upward with the same initial speed v_0, but at different angles relative to the horizontal, as shown in Figure 8–22. Ignoring air resistance, indicate which of the following statements is correct: At the dashed level, **(A)** ball 3 has the lowest speed; **(B)** ball 1 has the lowest speed; **(C)** all three balls have the same speed; **(D)** the speed of the balls depends on their mass.



▲ **FIGURE 8–22** Problem 27

28. •• **IP** In a tennis match, a player wins a point by hitting the ball sharply to the ground on the opponent's side of the net. **(a)** If the ball bounces upward from the ground with a speed of 16 m/s, and is caught by a fan in the stands with a speed of 12 m/s, how high above the court is the fan? Ignore air resistance. **(b)** Explain why it is not necessary to know the mass of the tennis ball.

29. •• A 0.21-kg apple falls from a tree to the ground, 4.0 m below. Ignoring air resistance, determine the apple's kinetic energy, K, the gravitational potential energy of the system, U, and the total mechanical energy of the system, E, when the apple's height above the ground is **(a)** 4.0 m, **(b)** 3.0 m, **(c)** 2.0 m, **(d)** 1.0 m, and **(e)** 0 m. Take ground level to be $y = 0$.

30. •• **IP** A 2.9-kg block slides with a speed of 1.6 m/s on a frictionless horizontal surface until it encounters a spring. **(a)** If the block compresses the spring 4.8 cm before coming to rest, what is the force constant of the spring? **(b)** What initial speed should the block have to compress the spring by 1.2 cm?

31. •• A 0.26-kg rock is thrown vertically upward from the top of a cliff that is 32 m high. When it hits the ground at the base of the cliff, the rock has a speed of 29 m/s. Assuming that air resistance can be ignored, find **(a)** the initial speed of the rock and **(b)** the greatest height of the rock as measured from the base of the cliff.

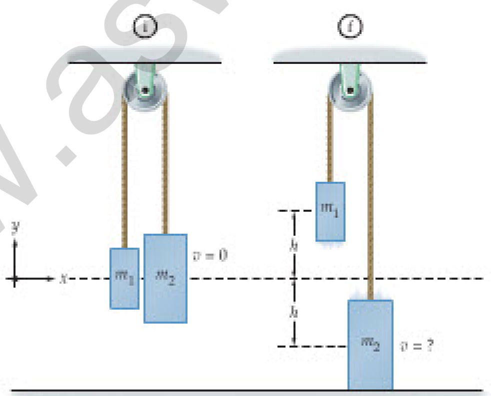
32. •• A 1.40-kg block slides with a speed of 0.950 m/s on a frictionless horizontal surface until it encounters a spring with a force constant of 734 N/m. The block comes to rest after compressing the spring 4.15 cm. Find the spring potential energy, U, the kinetic energy of the block, K, and the total mechanical energy of the system, E, for compressions of **(a)** 0 cm, **(b)** 1.00 cm, **(c)** 2.00 cm, **(d)** 3.00 cm, and **(e)** 4.00 cm.

33. •• A 5.76-kg rock is dropped and allowed to fall freely. Find the initial kinetic energy, the final kinetic energy, and the change in kinetic energy for **(a)** the first 2.00 m of fall and **(b)** the second 2.00 m of fall.

34. •• **IP** Suppose the pendulum bob in Figure 8–20 has a mass of 0.33 kg and is moving to the right at point B with a speed of 2.4 m/s. Air resistance is negligible. **(a)** What is the change in the system's gravitational potential energy when the bob reaches point A? **(b)** What is the speed of the bob at point A? **(c)** If the mass of the bob is increased, does your answer to part (a) increase, decrease, or stay the same? Explain. **(d)** If the mass of the bob is increased, does your answer to part (b) increase, decrease, or stay the same? Explain.

35. •• **IP** In the previous problem, **(a)** what is the bob's kinetic energy at point B? **(b)** At some point the bob will come to rest momentarily. Without doing an additional calculation, determine the change in the system's gravitational potential energy between point B and the point where the bob comes to rest. **(c)** Find the maximum angle the string makes with the vertical as the bob swings back and forth. Ignore air resistance.

36. ••• The two masses in the Atwood's machine shown in Figure 8–23 are initially at rest at the same height. After they are released, the large mass, m_2, falls through a height h and hits the floor, and the small mass, m_1, rises through a height h. **(a)** Find the speed of the masses just before m_2 lands, giving your answer in terms of m_1, m_2, g, and h. Assume the ropes and pulley have negligible mass and that friction can be ignored. **(b)** Evaluate your answer to part (a) for the case $h = 1.2$ m, $m_1 = 3.7$ kg, and $m_2 = 4.1$ kg.



▲ **FIGURE 8–23** Problems 36, 37, 82, and 99

37. ••• In the previous problem, suppose the masses have an initial speed of 0.20 m/s, and that m_2 is moving upward. How high does m_2 rise above its initial position before momentarily coming to rest, given that $m_1 = 3.7$ kg and $m_2 = 4.1$ kg?

SECTION 8–4 WORK DONE BY NONCONSERVATIVE FORCES

38. • **CE** You coast up a hill on your bicycle with decreasing speed. Your friend pedals up the hill with constant speed. **(a)** Ignoring friction, does the mechanical energy of the you–bike–Earth system increase, decrease, or stay the same? Explain. **(b)** Does the mechanical energy of the friend–bike–Earth system increase, decrease, or stay the same? Explain.

39. • **CE Predict/Explain** On reentry, the space shuttle's protective heat tiles become extremely hot. **(a)** Is the mechanical energy of the shuttle–Earth system when the shuttle lands greater than, less than, or the same as when it is in orbit? **(b)** Choose the *best explanation* from among the following:
   I. Dropping out of orbit increases the mechanical energy of the shuttle.
   II. Gravity is a conservative force.
   III. A portion of the mechanical energy has been converted to heat energy.

40. • Catching a wave, a 77-kg surfer starts with a speed of 1.3 m/s, drops through a height of 1.65 m, and ends with a speed of 8.2 m/s. How much nonconservative work was done on the surfer?

41. • At a playground, a 19-kg child plays on a slide that drops through a height of 2.3 m. The child starts at rest at the top of the slide. On the way down, the slide does a nonconservative work of −361 J on the child. What is the child's speed at the bottom of the slide?

42. • Starting at rest at the edge of a swimming pool, a 72.0-kg athlete swims along the surface of the water and reaches a speed of 1.20 m/s by doing the work $W_{nc1} = +161$ J. Find the nonconservative work, W_{nc2}, done by the water on the athlete.

43. • A 17,000-kg airplane lands with a speed of 82 m/s on a stationary aircraft carrier deck that is 115 m long. Find the work done by nonconservative forces in stopping the plane.

44. • **IP** The driver of a 1300-kg car moving at 17 m/s brakes quickly to 11 m/s when he spots a local garage sale. **(a)** Find the change in the car's kinetic energy. **(b)** Explain where the "missing" kinetic energy has gone.

45. •• **CE** You ride your bicycle down a hill, maintaining a constant speed the entire time. **(a)** As you ride, does the gravitational potential energy of the you–bike–Earth system increase, decrease, or stay the same? Explain. **(b)** Does the kinetic energy of you and your bike increase, decrease, or stay the same? Explain. **(c)** Does the mechanical energy of the you–bike–Earth system increase, decrease, or stay the same? Explain.

46. •• Suppose the system in Example 8–10 starts with m_2 moving downward with a speed of 1.3 m/s. What speed do the masses have just before m_2 lands?

47. •• A 42.0-kg seal at an amusement park slides from rest down a ramp into the pool below. The top of the ramp is 1.75 m higher than the surface of the water, and the ramp is inclined at an angle of 35.0° above the horizontal. If the seal reaches the water with a speed of 4.40 m/s, what are **(a)** the work done by kinetic friction and **(b)** the coefficient of kinetic friction between the seal and the ramp?

48. •• A 1.9-kg rock is released from rest at the surface of a pond 1.8 m deep. As the rock falls, a constant upward force of 4.6 N is exerted on it by water resistance. Calculate the nonconservative work, W_{nc}, done by water resistance on the rock, the gravitational potential energy of the system, U, the kinetic energy of the rock, K, and the total mechanical energy of the system, E, when the depth of the rock below the water's surface is **(a)** 0 m, **(b)** 0.50 m, and **(c)** 1.0 m. Let $y = 0$ be at the bottom of the pond.

49. •• A 1250-kg car drives up a hill that is 16.2 m high. During the drive, two nonconservative forces do work on the car: (i) the force of friction, and (ii) the force generated by the car's engine. The work done by friction is -3.11×10^5 J; the work done by the engine is $+6.44 \times 10^5$ J. Find the change in the car's kinetic energy from the bottom of the hill to the top of the hill.

50. •• **IP** An 81.0-kg in-line skater does +3420 J of nonconservative work by pushing against the ground with his skates. In addition, friction does −715 J of nonconservative work on the skater. The skater's initial and final speeds are 2.50 m/s and 1.22 m/s, respectively. **(a)** Has the skater gone uphill, downhill, or remained at the same level? Explain. **(b)** Calculate the change in height of the skater.

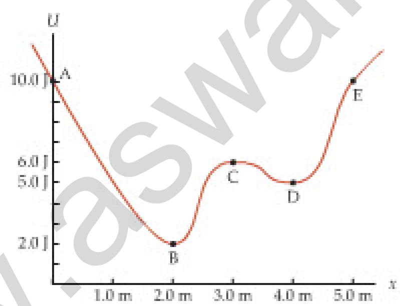
51. •• In Example 8–10, suppose the two masses start from rest and are moving with a speed of 2.05 m/s just before m_2 hits the floor. **(a)** If the coefficient of kinetic friction is $\mu_k = 0.350$, what is the distance of travel, d, for the masses? **(b)** How much conservative work was done on this system? **(c)** How much nonconservative work was done on this system? **(d)** Verify the three work relations given in Equations 8–10.

52. •• **IP** A 15,800-kg truck is moving at 12.0 m/s when it starts down a 6.00° incline in the Canadian Rockies. At the start of the descent the driver notices that the altitude is 1630 m. When she reaches an altitude of 1440 m, her speed is 29.0 m/s. Find the change in **(a)** the gravitational potential energy of the system and **(b)** the truck's kinetic energy. **(c)** Is the total mechanical energy of the system conserved? Explain.

53. ••• A 1.80-kg block slides on a rough horizontal surface. The block hits a spring with a speed of 2.00 m/s and compresses it a distance of 11.0 cm before coming to rest. If the coefficient of kinetic friction between the block and the surface is $\mu_k = 0.560$, what is the force constant of the spring?

SECTION 8–5 POTENTIAL ENERGY CURVES AND EQUIPOTENTIALS

54. • Figure 8–24 shows a potential energy curve as a function of x. In qualitative terms, describe the subsequent motion of an object that starts at rest at point A.



▲ **FIGURE 8–24** Problems 54, 55, 56, and 59

55. • An object moves along the x axis, subject to the potential energy shown in Figure 8–24. The object has a mass of 1.1 kg and starts at rest at point A. **(a)** What is the object's speed at point B? **(b)** At point C? **(c)** At point D? **(d)** What are the turning points for this object?

56. • A 1.34-kg object moves along the x axis, subject to the potential energy shown in Figure 8–24. If the object's speed at point C is 1.25 m/s, what are the approximate locations of its turning points?

57. • A 23-kg child swings back and forth on a swing suspended by 2.5-m-long ropes. Plot the gravitational potential energy of this system as a function of the angle the ropes make with the vertical, assuming the potential energy is zero when the ropes are vertical. Consider angles up to 90° on either side of the vertical.

58. •• Find the turning-point angles in the previous problem if the child has a speed of 0.89 m/s when the ropes are vertical. Indicate the turning points on a plot of the system's potential energy.

59. •• The potential energy of a particle moving along the x axis is shown in Figure 8–24. When the particle is at $x = 1.0$ m it has 3.6 J of kinetic energy. Give approximate answers to the following questions. **(a)** What is the total mechanical energy of the system? **(b)** What is the smallest value of x the particle can reach? **(c)** What is the largest value of x the particle can reach?

60. •• A block of mass $m = 0.95$ kg is connected to a spring of force constant $k = 775$ N/m on a smooth, horizontal surface. **(a)** Plot the potential energy of the spring from $x = -5.00$ cm to $x = 5.00$ cm. **(b)** Determine the turning points of the block if its speed at $x = 0$ is 1.3 m/s.

61. •• A ball of mass $m = 0.75$ kg is thrown straight upward with an initial speed of 8.9 m/s. **(a)** Plot the gravitational potential energy of the block from its launch height, $y = 0$, to the height $y = 5.0$ m. Let $U = 0$ correspond to $y = 0$. **(b)** Determine the turning point (maximum height) of this mass.

62. ••• Two blocks, each of mass m, are connected on a frictionless horizontal table by a spring of force constant k and equilibrium length L. Find the maximum and minimum separation between the two blocks in terms of their maximum speed, v_{max}, relative to the table. (The two blocks always move in opposite directions as they oscillate back and forth about a fixed position.)

GENERAL PROBLEMS

63. • **CE** You and a friend both solve a problem involving a skier going down a slope. When comparing solutions, you notice that your choice for the $y = 0$ level is different than the $y = 0$ level chosen by your friend. Will your answers agree or disagree on the following quantities: **(a)** the skier's potential energy; **(b)** the skier's change in potential energy; **(c)** the skier's kinetic energy?

64. • **CE** A particle moves under the influence of a conservative force. At point A the particle has a kinetic energy of 12 J; at point B the particle is momentarily at rest, and the potential energy of the system is 25 J; at point C the potential energy of the system is 5 J. **(a)** What is the potential energy of the system when the particle is at point A? **(b)** What is the kinetic energy of the particle at point C?

65. • **CE** A leaf falls to the ground with constant speed. Is $U_i + K_i$ for this system greater than, less than, or the same as $U_f + K_f$ for this system? Explain.

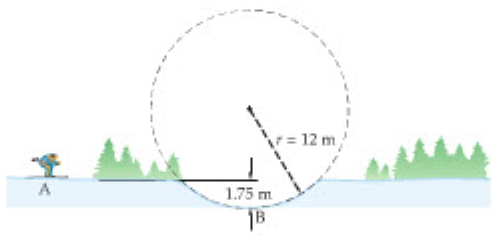
66. • **CE** Consider the two-block system shown in Example 8–10. **(a)** As block 2 descends through the distance d, does its mechanical energy increase, decrease, or stay the same? Explain. **(b)** Is the nonconservative work done on block 2 by the tension in the rope positive, negative, or zero? Explain.

67. •• **CE** Taking a leap of faith, a bungee jumper steps off a platform and falls until the cord brings her to rest. Suppose you analyze this system by choosing $y = 0$ at the platform level, and your friend chooses $y = 0$ at ground level. **(a)** Is the jumper's initial potential energy in your calculation greater than, less than, or equal to the same quantity in your friend's calculation? Explain. **(b)** Is the change in the jumper's potential energy in your calculation greater than, less than, or equal to the same quantity in your friend's calculation? Explain.

68. •• **IP** A sled slides without friction down a small, ice-covered hill. If the sled starts from rest at the top of the hill, its speed at the bottom is 7.50 m/s. **(a)** On a second run, the sled starts with a speed of 1.50 m/s at the top. When it reaches the bottom of the hill, is its speed 9.00 m/s, more than 9.00 m/s, or less than 9.00 m/s? Explain. **(b)** Find the speed of the sled at the bottom of the hill after the second run.

69. •• In the previous problem, what is the height of the hill?

70. •• A 68-kg skier encounters a dip in the snow's surface that has a circular cross section with a radius of curvature of 12 m. If the skier's speed at point A in Figure 8–25 is 8.0 m/s, what is the normal force exerted by the snow on the skier at point B? Ignore frictional forces.



▲ **FIGURE 8–25** Problem 70

71. •• **Running Shoes** The soles of a popular make of running shoe have a force constant of 2.0×10^5 N/m. Treat the soles as ideal springs for the following questions. **(a)** If a 62-kg person stands in a pair of these shoes, with her weight distributed equally on both feet, how much does she compress the soles? **(b)** How much energy is stored in the soles of her shoes when she's standing?

72. •• **Nasal Strips** The force required to flex a nasal strip and apply it to the nose is 0.25 N; the energy stored in the strip when flexed is 0.0022 J. Assume the strip to be an ideal spring for the following calculations. Find **(a)** the distance through which the strip is flexed and **(b)** the force constant of the strip.

73. •• **IP** A pendulum bob with a mass of 0.13 kg is attached to a string with a length of 0.95 m. We choose the potential energy to be zero when the string makes an angle of 90° with the vertical. **(a)** Find the potential energy of this system when the string makes an angle of 45° with the vertical. **(b)** Is the magnitude of the change in potential energy from an angle of 90° to 45° greater than, less than, or the same as the magnitude of the change from 45° to 0°? Explain. **(c)** Calculate the potential energy of the system when the string is vertical.

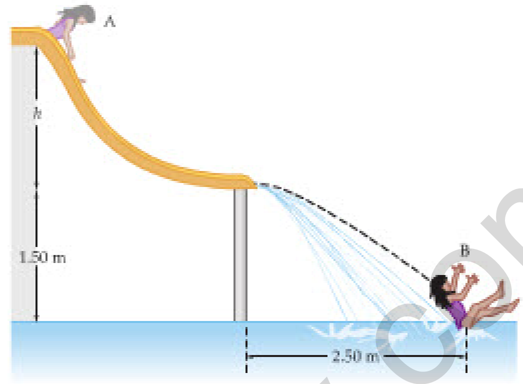
74. •• Suppose the pendulum bob in Figure 8–20 has a mass of 0.25 kg. **(a)** How much work does gravity do on the bob as it moves from point A to point B? **(b)** From point B to point A? **(c)** How much work does the string do on the bob as it moves from point A to point B? **(d)** From point B to point A?

75. •• An 1865-kg airplane starts at rest on an airport runway at sea level. **(a)** What is the change in mechanical energy of the airplane if it climbs to a cruising altitude of 2420 m and maintains a constant speed of 96.5 m/s? **(b)** What cruising speed would the plane need at this altitude if its increase in kinetic energy is to be equal to its increase in potential energy?

76. •• **IP** At the local playground a child on a swing has a speed of 2.02 m/s when the swing is at its lowest point. **(a)** To what maximum vertical height does the child rise, assuming he sits still and "coasts"? Ignore air resistance. **(b)** How do your results change if the initial speed of the child is halved?

77. •• The water slide shown in Figure 8–26 ends at a height of 1.50 m above the pool. If the person starts from rest at point A and lands in the water at point B, what is the height h of the water slide? (Assume the water slide is frictionless.)

78. •• If the height of the water slide in Figure 8–26 is $h = 3.2$ m, and the person's initial speed at point A is 0.54 m/s, what is the new horizontal distance between the base of the slide and the splashdown point of the person?

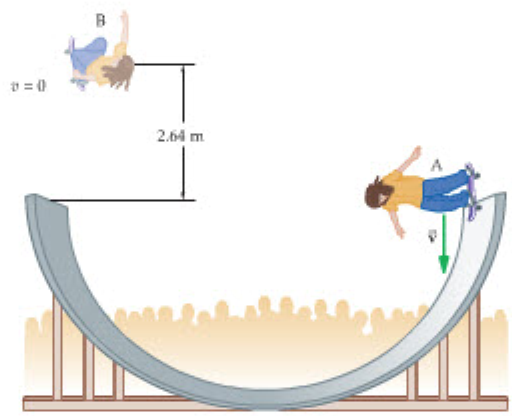


▲ **FIGURE 8–26** Problems 77 and 78

79. •• **IP** A person is to be released from rest on a swing pulled away from the vertical by an angle of 20.0°. The two frayed ropes of the swing are 2.75 m long, and will break if the tension in either of them exceeds 355 N. **(a)** What is the maximum weight the person can have and not break the ropes? **(b)** If the person is released at an angle greater than 20.0°, does the maximum weight increase, decrease, or stay the same? Explain.

80. •• **IP** A car is coasting without friction toward a hill of height h and radius of curvature r. **(a)** What initial speed, v_0, will result in the car's wheels just losing contact with the roadway as the car crests the hill? **(b)** What happens if the initial speed of the car is greater than the value found in part (a)?

81. •• A skateboarder starts at point A in Figure 8–27 and rises to a height of 2.64 m above the top of the ramp at point B. What was the skateboarder's initial speed at point A?



▲ **FIGURE 8–27** Problem 81

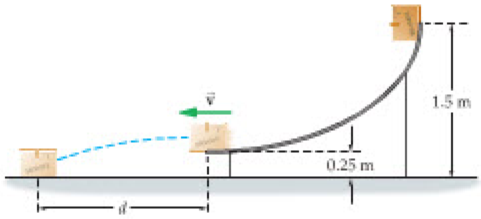
82. •• In the Atwood's machine of Problem 36, the mass m_2 remains at rest once it hits the floor, but the mass m_1 continues moving upward. How much higher does m_1 go after m_2 has landed? Give your answer for the case $h = 1.2$ m, $m_1 = 3.7$ kg, and $m_2 = 4.1$ kg.

83. •• An 8.70-kg block slides with an initial speed of 1.56 m/s up a ramp inclined at an angle of 28.4° with the horizontal. The coefficient of kinetic friction between the block and the ramp is 0.62. Use energy conservation to find the distance the block slides before coming to rest.

84. •• Repeat the previous problem for the case of an 8.70-kg block sliding down the ramp, with an initial speed of 1.56 m/s.

85. •• Jeff of the Jungle swings on a 7.6-m vine that initially makes an angle of 37° with the vertical. If Jeff starts at rest and has a mass of 78 kg, what is the tension in the vine at the lowest point of the swing?

86. •• A 1.9-kg block slides down a frictionless ramp, as shown in Figure 8–28. The top of the ramp is 1.5 m above the ground; the bottom of the ramp is 0.25 m above the ground. The block leaves the ramp moving horizontally, and lands a horizontal distance d away. Find the distance d.



▲ **FIGURE 8–28** Problems 86 and 87

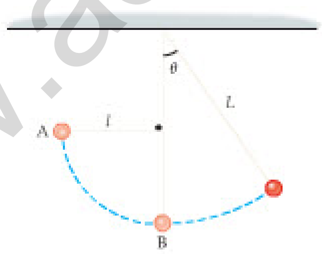
87. •• Suppose the ramp in Figure 8–28 is not frictionless. Find the distance d for the case in which friction on the ramp does -9.7 J of work on the block before it becomes airborne.

88. •• **BIO Compressing the Ground** A running track at Harvard University uses a surface with a force constant of 2.5×10^5 N/m. This surface is compressed slightly every time a runner's foot lands on it. The force exerted by the foot, according to the Saucony shoe company, has a magnitude of 2700 N for a typical runner. Treating the track's surface as an ideal spring, find **(a)** the amount of compression caused by a foot hitting the track and **(b)** the energy stored briefly in the track every time a foot lands.

89. •• **BIO A Flea's Jump** The resilin in the upper leg (coxa) of a flea has a force constant of about 26 N/m, and when the flea cocks its jumping legs, the resilin in each leg is stretched by approximately 0.10 mm. Given that the flea has a mass of 0.50 mg, and that two legs are used in a jump, estimate the maximum height a flea can attain by using the energy stored in the resilin. (Assume the resilin to be an ideal spring.)

90. ••• **IP** A trapeze artist of mass m swings on a rope of length L. Initially, the trapeze artist is at rest and the rope makes an angle θ with the vertical. **(a)** Find the tension in the rope when it is vertical. **(b)** Explain why your result for part (a) depends on L in the way it does.

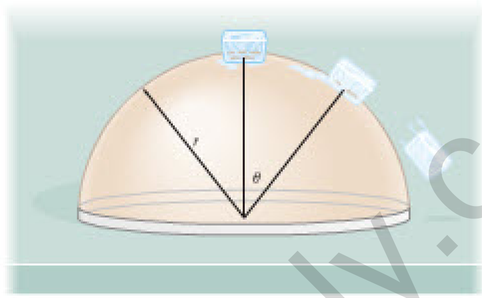
91. ••• **IP Tension at the Bottom** A ball of mass m is attached to a string of length L and released from rest at the point A in Figure 8–29. **(a)** Show that the tension in the string when the ball reaches point B is $3mg$, independent of the length l. **(b)** Give a detailed physical explanation for the fact that the tension at point B is independent of the length l.



▲ **FIGURE 8–29** Problems 91 and 92

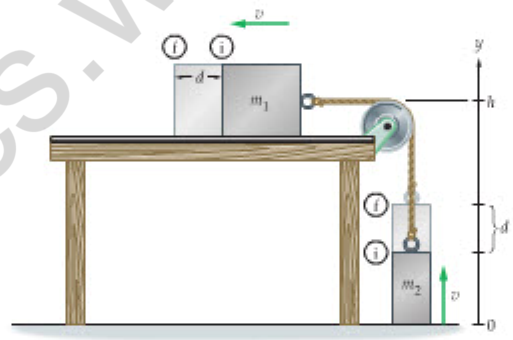
92. ••• **IP** In Figure 8–29, suppose that $L = 0.652$ m and $l = 0.325$ m. **(a)** Find the maximum angle the string makes with the vertical when the mass is released from rest at point A and swings as far to the right as it can. **(b)** At the point found in part (a), find the height of the mass above point B. Explain the physical significance of your result. **(c)** Give the angle of part (a) as a general expression in terms of L and l.

93. ••• An ice cube is placed on top of an overturned spherical bowl of radius r, as indicated in Figure 8–30. If the ice cube slides downward from rest at the top of the bowl, at what angle θ does it separate from the bowl? In other words, at what angle does the normal force between the ice cube and the bowl go to zero?



▲ **FIGURE 8–30** Problem 93

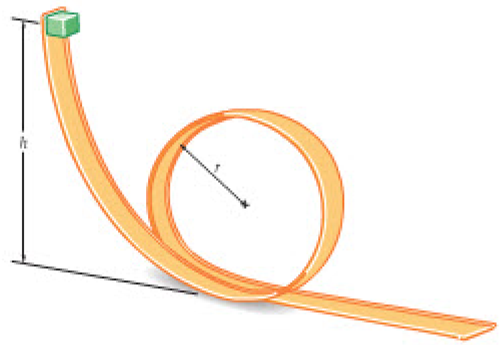
94. ••• **IP** The two blocks shown in Figure 8–31 are moving with an initial speed v. **(a)** If the system is frictionless, find the distance d the blocks travel before coming to rest. (Let $U = 0$ correspond to the initial position of block 2.) **(b)** Is the work done on block 2 by the rope positive, negative, or zero? Explain. **(c)** Calculate the work done on block 2 by the rope.



▲ **FIGURE 8–31** Problems 94 and 95

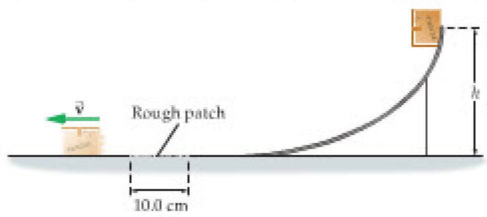
95. ••• **IP** Consider the system shown in Figure 8–31. **(a)** What initial speed v is required if the blocks $m_1 = 2.4$ kg and $m_2 = 1.1$ kg are to travel a distance $d = 6.5$ cm before coming to rest? Assume the coefficient of kinetic friction between m_1 and the tabletop is $\mu_k = 0.25$. **(b)** Is the work done on m_2 by the rope positive, negative, or zero? Explain. **(c)** Calculate the work done on m_2 by the rope.

96. ••• **IP Loop-the-Loop (a)** A block of mass m slides from rest on a frictionless loop-the-loop track, as shown in Figure 8–32. What is the minimum release height, h, required for the block to maintain contact with the track at all times? Give your answer in terms of the radius of the loop, r. **(b)** Explain why the release height obtained in part (a) is independent of the block's mass.



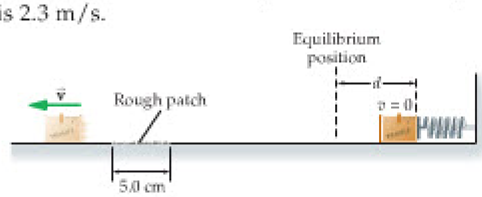
▲ **FIGURE 8–32** Problem 96

97. ••• Figure 8–33 shows a 1.75-kg block at rest on a ramp of height h. When the block is released, it slides without friction to the bottom of the ramp, and then continues across a surface that is frictionless except for a rough patch of width 10.0 cm that has a coefficient of kinetic friction $\mu_k = 0.640$. Find h such that the block's speed after crossing the rough patch is 3.50 m/s.



▲ **FIGURE 8–33** Problem 97

98. ••• In Figure 8–34 a 1.2-kg block is held at rest against a spring with a force constant $k = 730$ N/m. Initially, the spring is compressed a distance d. When the block is released, it slides across a surface that is frictionless except for a rough patch of width 5.0 cm that has a coefficient of kinetic friction $\mu_k = 0.44$. Find d such that the block's speed after crossing the rough patch is 2.3 m/s.



▲ **FIGURE 8–34** Problem 98

99. ••• **IP Using Work and Energy to Calculate Tension** Consider the Atwood's machine shown in Figure 8–23, with $h = 1.2$ m, $m_1 = 3.7$ kg, and $m_2 = 4.1$ kg. In this problem, we show how to calculate the tension in the rope using energy and work, rather than Newton's laws. **(a)** Is the change in mechanical energy for block 2 as it drops through the height h positive, negative, or zero? Explain. **(b)** Use energy conservation applied to the entire system to calculate the change in mechanical energy for block 2 as it drops through the height h. **(c)** Use your answer to part (b), and the known drop height, to find the magnitude of the tension in the rope.

PASSAGE PROBLEMS

BIO The Flight of the Dragonflies

Of all the animals you're likely to see on a summer's day, the most ancient is the dragonfly. In fact, the fossil record for dragonflies extends back over 250 million years, more than twice as long as for birds. Ancient dragonflies could be as large as a hawk, and were surely buzzing around the heads of both *T. Rex* and *Triceratops*.

Dragonflies belong to the order Odonata ("toothed jaws") and the suborder Anisoptera ("different wings"), a reference to the fact that their hindwings are wider front-to-back than their forewings. (Damselflies, in contrast, have forewings and hindwings that are the same.) Although ancient in their lineage, dragonflies are the fastest flying and most acrobatic of all insects; some of their maneuvers subject them to accelerations as great as 20g.

The properties of dragonfly wings, and how they account for such speed and mobility, have been of great interest to biologists. Figure 8–35 (a) shows an experimental setup designed to measure the force constant of Plexiglas models of wings, which are used in wind tunnel tests. A downward force is applied to the model wing at the tip (1 for hindwing, 2 for forewing) or at two-thirds the distance to the tip (3 for hindwing, 4 for forewing). As the force is varied in magnitude, the resulting deflection of the wing is measured. The results are shown in Figure 8–35 (b). Notice that significant differences are seen between the hindwings and forewings, as one might expect from their different shapes.

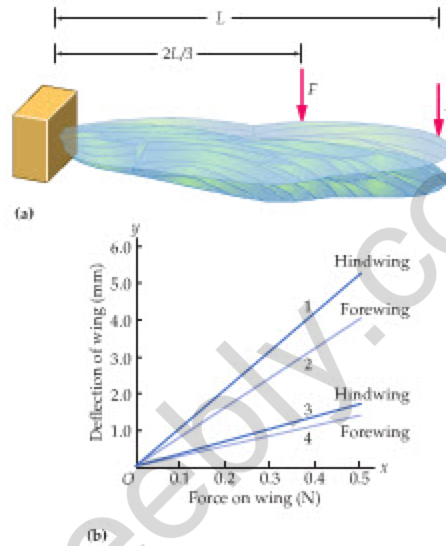


FIGURE 8–35 Problems 100, 101, 102, and 103

100. • Treating the model wing as an ideal spring, what is the force constant of the hindwing when a force is applied to its tip?

A. 94 N/m B. 130 N/m C. 290 N/m D. 330 N/m

101. • What is the force constant of the hindwing when a force is applied at two-thirds the distance from the base of the wing to the tip?

A. 94 N/m B. 130 N/m
C. 290 N/m D. 330 N/m

102. • Which of the wings is "stiffer"?

A. The hindwing. B. The forewing.
C. Depends on where the force is applied.
D. They are equally "stiff."

103. •• How much energy is stored in the forewing when a force at the tip deflects it by 3.5 mm?

A. 0.766 mJ B. 49.0 mJ C. 0.219 J D. 1.70 kJ

INTERACTIVE PROBLEMS

104. •• **IP Referring to Example 8–8** Consider a spring with a force constant of 955 N/m. **(a)** Suppose the mass of the block is 1.70 kg, but its initial speed can be varied. What initial speed is required to give a maximum spring compression of 4.00 cm? **(b)** Suppose the initial speed of the block is 1.09 m/s, but its mass can be varied. What mass is required to give a maximum spring compression of 4.00 cm?

105. •• **Referring to Example 8–8** Suppose the block is released from rest with the spring compressed 5.00 cm. The mass of the block is 1.70 kg and the force constant of the spring is 955 N/m. **(a)** What is the speed of the block when the spring expands to a compression of only 2.50 cm? **(b)** What is the speed of the block after it leaves the spring?

106. •• **Referring to Example 8–10** Suppose we would like the landing speed of block 2 to be increased to 1.50 m/s. **(a)** Should the coefficient of kinetic friction between block 1 and the tabletop be increased or decreased? **(b)** Find the required coefficient of kinetic friction for a landing speed of 1.50 m/s. Note that $m_1 = 2.40$ kg, $m_2 = 1.80$ kg, and $d = 0.500$ m.