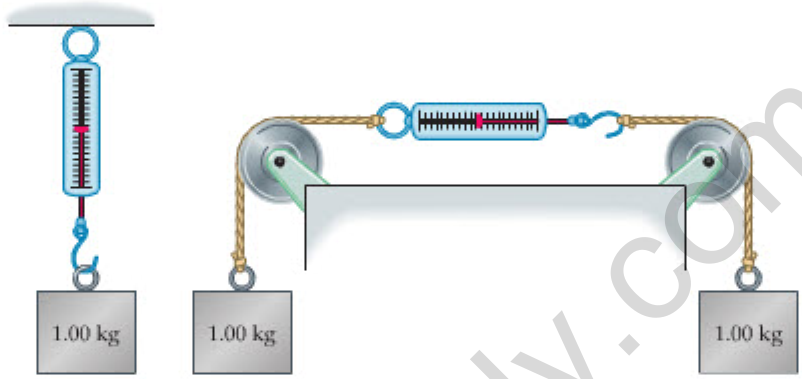
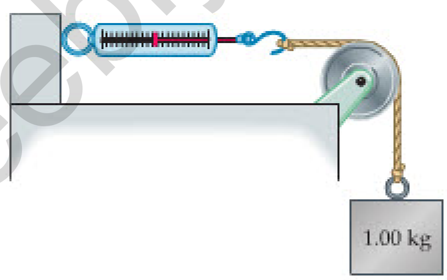
CONCEPTUAL CHECKPOINT 6–2 COMPARE THE READINGS ON THE SCALES

The scale at left reads 9.81 N. Is the reading of the scale at right **(a)** greater than 9.81 N, **(b)** equal to 9.81 N, or **(c)** less than 9.81 N?



REASONING AND DISCUSSION

Since a pulley simply changes the direction of the tension in a string without changing its magnitude, it is clear that the scale attached to the ceiling reads the same as the scale shown in the figure to the right.



There is no difference, however, between attaching the top end of the scale to something rigid and attaching it to another 1.00-kg hanging mass. In either case, the fact that the scale is at rest means that a force of 9.81 N must be exerted to the left on the top of the scale to balance the 9.81-N force exerted on the lower end of the scale. As a result, the two scales read the same.

ANSWER

(b) The reading of the scale at right is equal to 9.81 N.

Springs and Hooke's Law

Suppose you take a spring of length L, as shown in **Figure 6–8 (a)**, and attach it to a block. If you pull on the spring, causing it to stretch to a length $L + x$, the spring pulls on the block with a force of magnitude F. If you increase the length of the spring to $L + 2x$, the force exerted by the spring increases to $2F$. Similarly, if you compress the spring to a length $L - x$, the spring pushes on the block with a force of magnitude F, where F is the same force given previously. As you might expect, compression to a length $L - 2x$ results in a push of magnitude $2F$.

As a result of these experiments, we can say that a spring exerts a force that is proportional to the amount, x, by which it is stretched or compressed. Thus, if F is the magnitude of the spring force, we can say that

$$F = kx$$

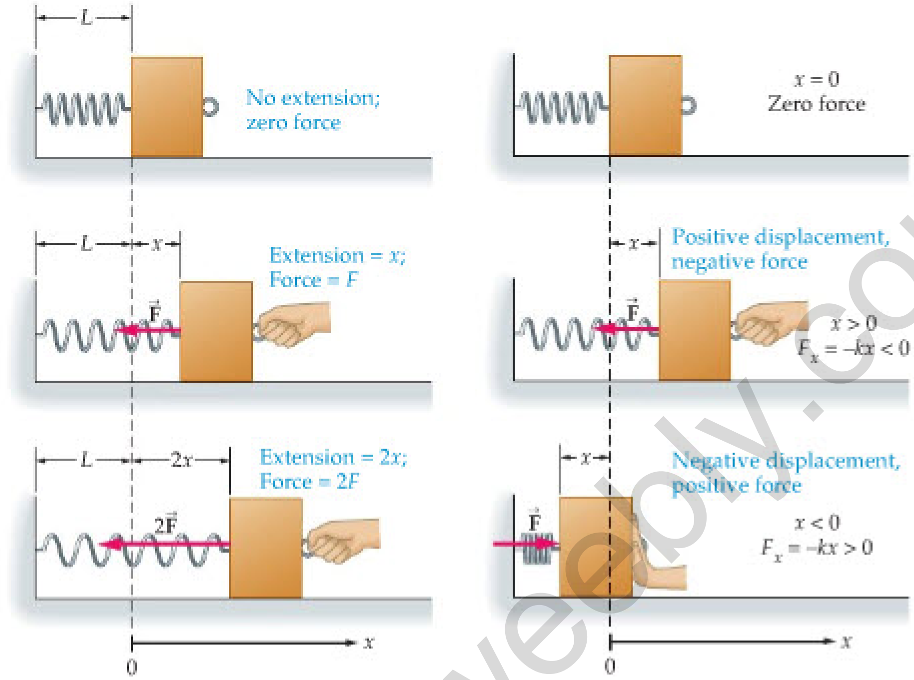
In this expression, k is a constant of proportionality, referred to as the **force constant,** or, equivalently, as the spring constant. Since F has units of newtons and x has units of meters, it follows that k has units of newtons per meter, or N/m. The larger the value of k, the stiffer the spring.

To be more precise, consider the spring shown in **Figure 6–8 (b)**. Note that we have placed the origin of the x axis at the equilibrium length of the spring—that is, at the position of the spring when no force acts on it. Now, if we stretch the spring so that the end of the spring is at a positive value of x ($x > 0$), we find that the spring exerts a force of magnitude kx in the negative x direction. Thus, the spring force (which has only an x component) can be written as

$$F_x = -kx$$

Similarly, consider compressing the spring so that its end is at a negative value of x ($x < 0$). In this case, the force exerted by the spring is of magnitude kx, and points in the positive x direction, as is shown in Figure 6–8 (b). Again, we can write the spring force as

$$F_x = -kx$$



(a) Doubling the extension doubles the force.

(b) The spring force is opposite to the displacement from equilibrium.

FIGURE 6–8 The force exerted by a spring
When dealing with a spring, it is convenient to choose the origin at the equilibrium (zero force) position. In the cases shown here, the force is strictly in the x direction, and is given by $F_x = -kx$. Note that the minus sign means that the force is opposite to the displacement; that is, the force is restoring.

To see that this is correct—that is, that F_x is positive in this case—recall that x is negative, which means that $(-x)$ is positive.

This result for the force of a spring is known as Hooke's law, after Robert Hooke (1635–1703). It is really just a good rule of thumb rather than a law of nature. Clearly, it can't work for any amount of stretching. For example, we know that if we stretch a spring far enough it will be permanently deformed, and will never return to its original length. Still, for small stretches or compressions, Hooke's law is quite accurate.

Rules of Thumb for Springs (Hooke's Law)

A spring stretched or compressed by the amount x from its equilibrium length exerts a force whose x component is given by

$$F_x = -kx \text{ (gives magnitude and direction)} \quad 6\text{–}4$$

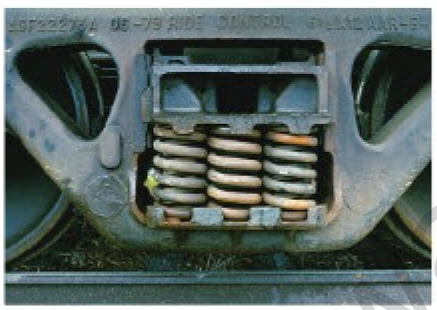
If we are interested only in the magnitude of the force associated with a given stretch or compression, we use the somewhat simpler form of Hooke's law:

$$F = kx \text{ (gives magnitude only)} \quad 6\text{–}5$$

In this text, we consider only **ideal springs**—that is, springs that are massless, and that are assumed to obey Hooke's law exactly.

Since the stretch of a spring and the force it exerts are proportional, we can now see how a spring scale operates. In particular, pulling on the two ends of a scale stretches the spring inside it by an amount proportional to the applied force. Once the scale is calibrated—by stretching the spring with a known, or reference, force—we can use it to measure other unknown forces.

Finally, it is useful to note that Hooke's law, which we've introduced in the context of ideal springs, is particularly important in physics because it applies to so much more than just springs. For example, the forces that hold atoms together are often modeled by Hooke's law—that is, as "interatomic springs"—and these are the forces that are ultimately responsible for the normal force (Chapter 5), vibrations and oscillations (Chapter 13), wave motion (Chapter 14), and even the thermal expansion of solids (Chapter 16). And this just scratches



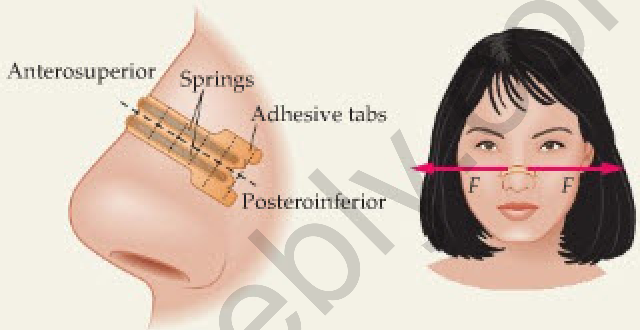
▲ Springs come in a variety of sizes and shapes. The large springs on a railroad car (top) are so stiff and heavy that you can't compress or stretch them by hand. Still, three of them are needed to smooth the ride of this car. In contrast, the delicate spiral spring inside a watch (bottom) flexes with even the slightest touch. It exerts enough force, however, to power the equally delicate mechanism of the watch.

the surface—Hooke's law comes up in one form or another in virtually every field of physics. In the following Active Example, we present a biomedical application of Hooke's law.

ACTIVE EXAMPLE 6–2 NASAL STRIPS

REAL-WORLD PHYSICS: BIO

An increasingly popular device for improving air flow through nasal passages is the nasal strip, which consists of two flat, polyester springs enclosed by an adhesive tape covering. Measurements show that a nasal strip can exert an outward force of 0.22 N on the nose, causing it to expand by 3.5 mm. **(a)** Treating the nose as an ideal spring, find its force constant in newtons per meter. **(b)** How much force would be required to expand the nose by 4.0 mm?



SOLUTION *(Test your understanding by performing the calculations indicated in each step.)*

Part (a)

1. Solve the magnitude form of Hooke's law, $F = kx$, for the force constant, k: $k = F/x$
2. Substitute numerical values for F and x: $k = 62\ \text{N/m}$

Part (b)

3. Use $F = kx$ to find the required force: $F = 0.25\ \text{N}$

INSIGHT

Even though the human nose is certainly not an ideal spring, Hooke's law is still a useful way to model its behavior when dealing with forces and the stretches they cause.

YOUR TURN

Suppose a new nasal strip comes on the market that exerts an outward force of 0.32 N. What expansion of the nose will be caused by this strip?

(Answers to **Your Turn** *problems are given in the back of the book.)*

6–3 Translational Equilibrium

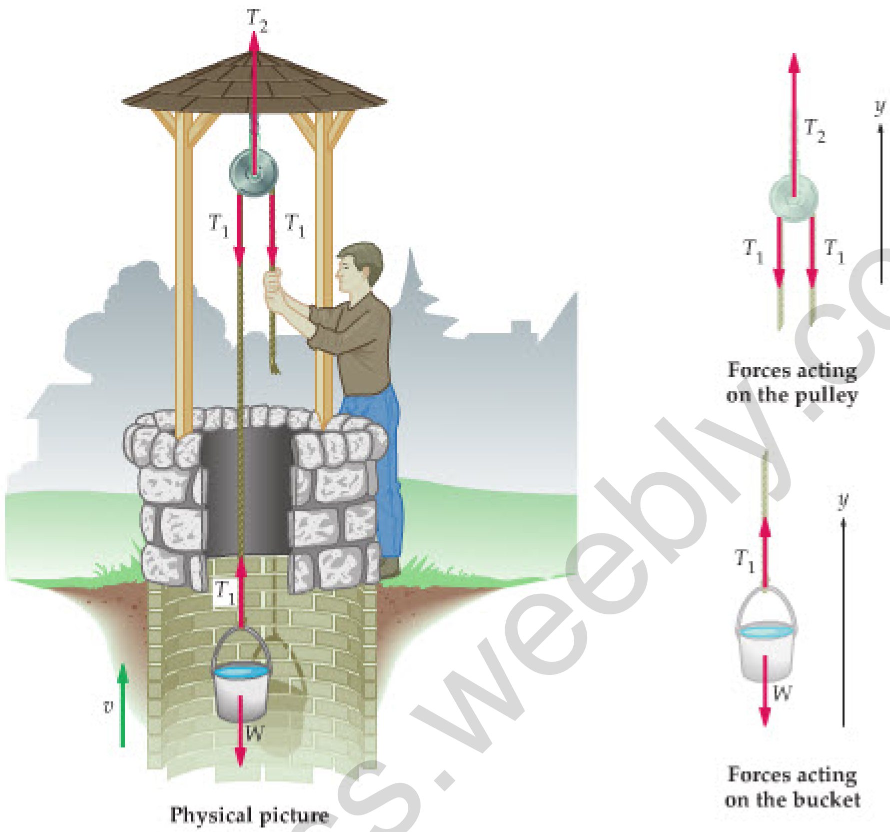
When we say that an object is in **translational equilibrium**, we mean that the net force acting on it is zero:

$$\sum \vec{F} = 0 \qquad 6\text{–}6$$

From Newton's second law, this is equivalent to saying that the object's acceleration is zero. In two-dimensional systems, translational equilibrium implies two independent conditions: $\Sigma F_x = 0$ and $\Sigma F_y = 0$. In one dimension, only one of these conditions will apply.

Later, in Chapters 10 and 11, we will study objects that have both rotational and linear motions. In such cases, rotational equilibrium will be as important as translation equilibrium. For now, however, when we say equilibrium, we simply mean translational equilibrium.

As a first example, consider the one-dimensional situation illustrated in **Figure 6–9**. Here we see a person lifting a bucket of water from a well by pulling down on a rope that passes over a pulley. If the bucket's mass is m, and it is rising with constant speed v, what is the tension T_1 in the rope attached to the bucket? In addition, what is the tension T_2 in the chain that supports the pulley?



▶ **FIGURE 6–9 Raising a bucket**
A person lifts a bucket of water from the bottom of a well with a constant speed, v. Because the speed is constant, the net force acting on the bucket must be zero.

To answer these questions, we first note that both the bucket and the pulley are in equilibrium; that is, they both have zero acceleration. As a result, the net force on each of them must be zero.

Let's start with the bucket. In Figure 6–9, we see that just two forces act on the bucket: (i) its weight $W = mg$ downward, and (ii) the tension in the rope, T_1 upward. If we take upward to be the positive direction, we can write $\Sigma F_y = 0$ for the bucket as follows:

$$T_1 - mg = 0$$

Therefore, the tension in the rope is $T_1 = mg$. Note that this is also the force the person must exert downward on the rope, as expected.

Next, we consider the pulley. In Figure 6–9, we see that three forces act on it: (i) the tension in the chain, T_2 upward, (ii) the tension in the part of the rope leading to the bucket, T_1 downward, and (iii) the tension in the part of the rope leading to the person, T_1 downward. Note that we don't include the weight of the pulley since we consider it to be ideal; that is, massless and frictionless. If we again take upward to be positive, the statement that the net force acting on the pulley is zero $(\Sigma F_y = 0)$ can be written

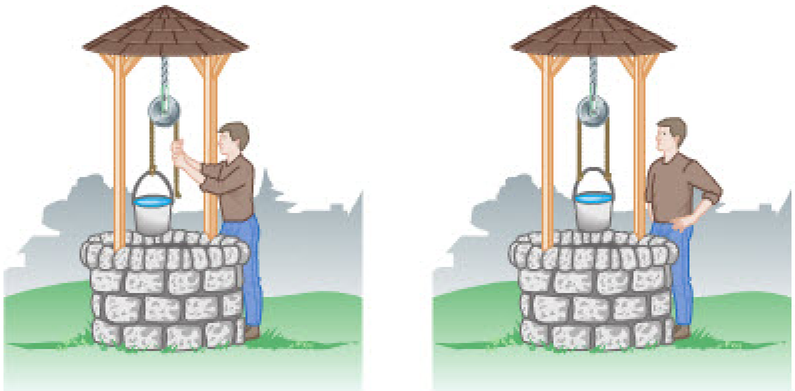
$$T_2 - T_1 - T_1 = 0$$

It follows that the tension in the chain is $T_2 = 2T_1 = 2mg$, twice the weight of the bucket of water!

In the next Conceptual Checkpoint we consider a slight variation of this situation.

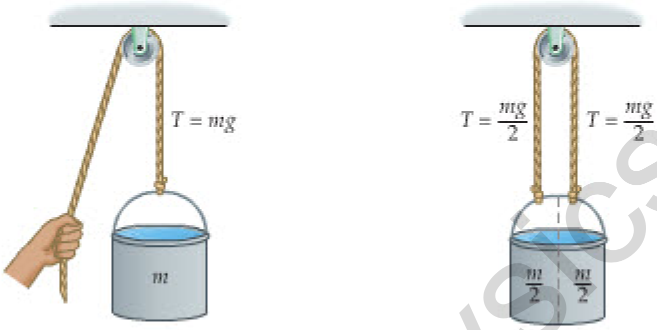
CONCEPTUAL CHECKPOINT 6–3 COMPARING TENSIONS

A person hoists a bucket of water from a well and holds the rope, keeping the bucket at rest, as at left. A short time later, the person ties the rope to the bucket so that the rope holds the bucket in place, as at right. In this case, is the tension in the rope **(a)** greater than, **(b)** less than, or **(c)** equal to the tension in the first case?



REASONING AND DISCUSSION

In the first case (left), the only upward force exerted on the bucket is the tension in the rope. Since the bucket is at rest, the tension must be equal in magnitude to the weight of the bucket. In the second case (right), the two ends of the rope exert equal upward forces on the bucket, hence the tension in the rope is only half the weight of the bucket. To see this more clearly, imagine cutting the bucket in half so that each end of the rope supports half the weight, as indicated in the accompanying diagram.



ANSWER

(b) The tension in the second case is less than in the first.

In the next two Examples, we consider two-dimensional systems in which forces act at various angles with respect to one another. Hence, our first step is to resolve the relevant vectors into their x and y components. Following that, we apply the conditions for translational equilibrium, $\Sigma F_x = 0$ and $\Sigma F_y = 0$.

EXAMPLE 6–5 SUSPENDED VEGETATION

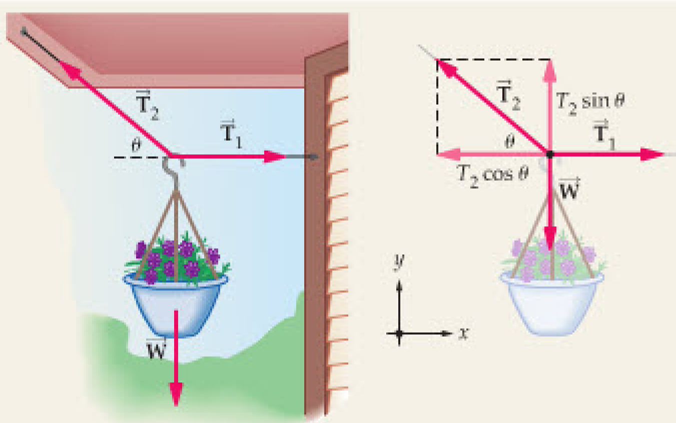
To hang a 6.20-kg pot of flowers, a gardener uses two wires—one attached horizontally to a wall, the other sloping upward at an angle of $\theta = 40.0°$ and attached to the ceiling. Find the tension in each wire.

PICTURE THE PROBLEM

We choose a typical coordinate system, with the positive x direction to the right and the positive y direction upward. With this choice, tension 1 is in the positive x direction, $\vec{\mathbf{T}}_1 = T_1\hat{\mathbf{x}}$, the weight is in the negative y direction, $\vec{\mathbf{W}} = -mg\,\hat{\mathbf{y}}$, and tension 2 has a negative x component and a positive y component, $\vec{\mathbf{T}}_2 = (-T_2 \cos\theta)\hat{\mathbf{x}} + (T_2 \sin\theta)\hat{\mathbf{y}}$.

STRATEGY

The pot is at rest, and therefore the net force acting on it is zero. As a result, we can say that (i) $\Sigma F_x = 0$ and (ii) $\Sigma F_y = 0$. These two conditions allow us to determine the magnitude of the two tensions, T_1 and T_2.



Physical picture

Free-body diagram

CONTINUED ON NEXT PAGE

CONTINUED FROM PREVIOUS PAGE

SOLUTION

1. First, resolve each of the forces acting on the pot into x and y components:

$$T_{1,x} = T_1 \qquad T_{1,y} = 0$$
$$T_{2,x} = -T_2 \cos\theta \qquad T_{2,y} = T_2 \sin\theta$$
$$W_x = 0 \qquad W_y = -mg$$

2. Now, set $\Sigma F_x = 0$. Note that this condition gives a relation between T_1 and T_2:

$$\sum F_x = T_{1,x} + T_{2,x} + W_x = T_1 + (-T_2 \cos\theta) + 0 = 0$$
$$T_1 = T_2 \cos\theta$$

3. Next, set $\Sigma F_y = 0$. This time, the resulting condition determines T_2 in terms of the weight, mg:

$$\sum F_y = T_{1,y} + T_{2,y} + W_y = 0 + T_2 \sin\theta + (-mg) = 0$$
$$T_2 \sin\theta = mg$$

4. Use the relation obtained in Step 3 to find T_2:

$$T_2 = \frac{mg}{\sin\theta} = \frac{(6.20\ \text{kg})(9.81\ \text{m/s}^2)}{\sin 40.0^\circ} = 94.6\ \text{N}$$

5. Finally, use the connection between the two tensions (obtained from $\Sigma F_x = 0$) to find T_1:

$$T_1 = T_2 \cos\theta = (94.6\ \text{N}) \cos 40.0^\circ = 72.5\ \text{N}$$

INSIGHT

Notice that even though two wires suspend the pot, they both have tensions *greater* than the pot's weight, $mg = 60.8$ N. This is an important point for architects and engineers to consider when designing structures.

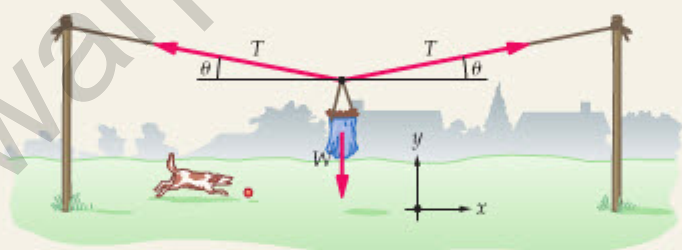
PRACTICE PROBLEM

Find T_1 and T_2 if the second wire slopes upward at the angle **(a)** $\theta = 20°$, **(b)** $\theta = 60.0°$, or **(c)** $\theta = 90.0°$. [**Answer:** **(a)** $T_1 = 167$ N, $T_2 = 178$ N **(b)** $T_1 = 35.1$ N, $T_2 = 70.2$ N **(c)** $T_1 = 0$, $T_2 = mg = 60.8$ N]

Some related homework problems: Problem 34, Problem 37

ACTIVE EXAMPLE 6–3 THE FORCES IN A LOW-TECH LAUNDRY

A 1.84-kg bag of clothespins hangs in the middle of a clothesline, causing it to sag by an angle $\theta = 3.50°$. Find the tension, T, in the clothesline.



SOLUTION *(Test your understanding by performing the calculations indicated in each step.)*

1. Find the y component for each tension: $T_y = T \sin\theta$
2. Find the y component of the weight: $W_y = -mg$
3. Set $\Sigma F_y = 0$: $T \sin\theta + T \sin\theta - mg = 0$
4. Solve for T: $T = mg/(2 \sin\theta) = 148$ N

INSIGHT

Note that we only considered the y components of force in our calculation. This is because forces in the x direction automatically balance, due to the symmetry of the system.

YOUR TURN

At what sag angle, θ, will the tension in the clothesline have a magnitude of 175 N?

(Answers to **Your Turn** *problems are given in the back of the book.)*

At 148 N, the tension in the clothesline is quite large, especially when you consider that the weight of the clothespin bag itself is only 18.1 N. The reason for such a large value is that the vertical component of the two tensions is $2T \sin \theta$, which, for $\theta = 3.50°$, is $(0.122)T$. If $(0.122)T$ is to equal the weight of the bag, it is clear that T must be roughly eight times the bag's weight.

If you and a friend were to pull on the two ends of the clothesline, in an attempt to straighten it out, you would find that no matter how hard you pulled, the line would still sag. You may be able to reduce θ to quite a small value, but as you do so the corresponding tension increases rapidly. In principle, it would take an infinite force to completely straighten the line and reduce θ to zero.

On the other hand, if θ were 90°, so that the two halves of the clothesline were vertical, the tension would be $T = mg/(2 \sin 90°) = mg/2$. In this case, each side of the line supports half the weight of the bag, as expected.

▲ Like the bag of clothespins in Active Example 6–3, this mountain climber is in static equilibrium. Since the ropes suspending the climber are nearly horizontal, the tension in them is significantly greater than the climber's weight.

6–4 Connected Objects

Interesting applications of Newton's laws arise when we consider accelerating objects that are tied together. Suppose, for example, that a force of magnitude F pulls two boxes—connected by a string—along a frictionless surface, as in **Figure 6–10**. In such a case, the string has a certain tension, T, and the two boxes have the same acceleration, a. Given the masses of the boxes and the applied force F, we would like to determine both the tension in the string and the acceleration of the boxes.

First, sketch the free-body diagram for each box. Box 1 has two horizontal forces acting on it: (i) the tension T to the left, and (ii) the force F to the right. Box 2 has only a single horizontal force, the tension T to the right. If we take the positive direction to be to the right, Newton's second law for the two boxes can be written as follows:

$$\begin{aligned} F - T &= m_1 a_1 = m_1 a \qquad \text{box 1} \\ T &= m_2 a_2 = m_2 a \qquad \text{box 2} \end{aligned} \tag{6–7}$$

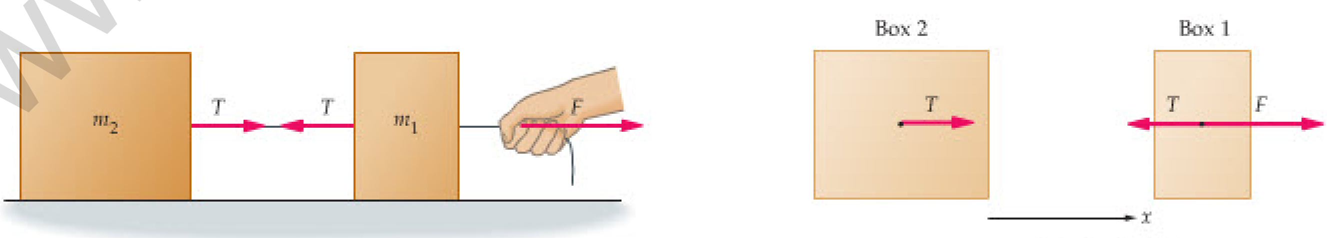
Since the boxes have the same acceleration, a, we have set $a_1 = a_2 = a$.

Next, we can eliminate the tension T by adding the two equations:

$$\begin{aligned} F - T &= m_1 a \\ T &= m_2 a \\ \hline F &= (m_1 + m_2)a \end{aligned}$$

With this result, it is straightforward to solve for the acceleration in terms of the applied force F:

$$a = \frac{F}{m_1 + m_2} \tag{6–8}$$



Physical picture

Free-body diagrams
(horizontal components only)

▲ **FIGURE 6–10 Two boxes connected by a string**
The string ensures that the two boxes have the same acceleration. This physical connection results in a mathematical connection, as shown in Equation 6–7. Note that in this case we treat each box as a separate system.

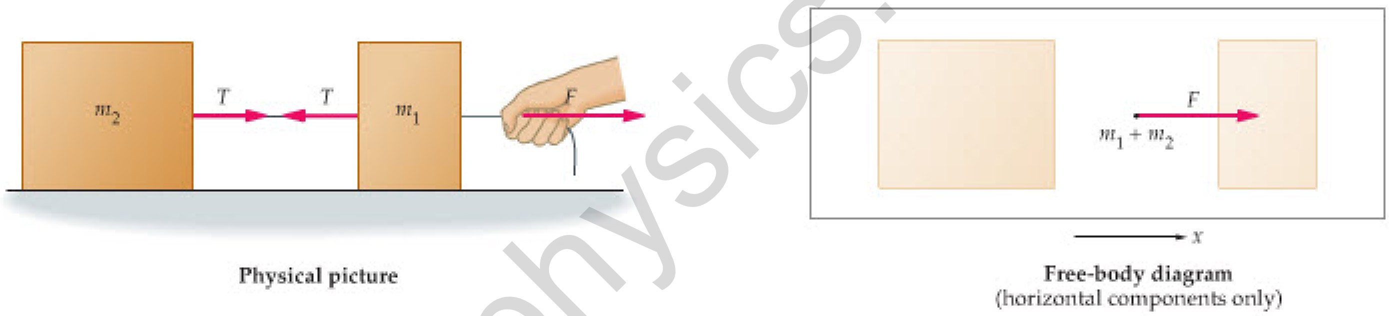
Finally, substitute this expression for a into either of the second-law equations to find the tension. The algebra is simpler if we use the equation for box 2. We find

$$T = m_2 a = \left(\frac{m_2}{m_1 + m_2}\right)F \qquad 6\text{–}9$$

It is left as an exercise to show that the equation for box 1 gives the same expression for T.

A second way to approach this problem is to treat both boxes together as a single system with a mass $m_1 + m_2$, as shown in **Figure 6–11.** The only *external* horizontal force acting on this system is the applied force F—the two tension forces are now *internal* to the system, and internal forces are not included when applying Newton's second law. As a result, the horizontal acceleration is simply $F/(m_1 + m_2)$, as given in Equation 6–8. This is certainly a quick way to find the acceleration a, but to find the tension T we must still use one of the relations given in Equations 6–7.

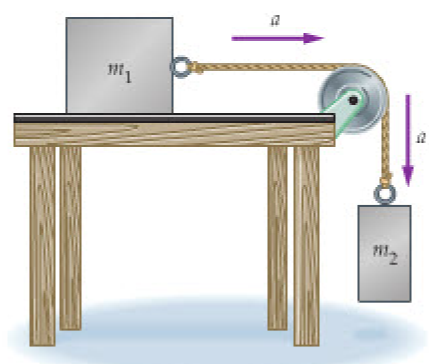
In general, we are always free to choose the "system" any way we like—we can choose any individual object, as when we considered box 1 and box 2 separately, or we can choose all the objects together. The important point is that Newton's second law is equally valid no matter what choice we make for the system, as long as we remember to include only forces *external* to *that system* in the corresponding free-body diagram.



▲ **FIGURE 6–11 Two boxes, one system**
In this case we consider the two boxes together as a single system of mass $m_1 + m_2$. The only external horizontal force acting on this system is $\vec{\mathbf{F}}$; hence the horizontal acceleration of the system is $a = F/(m_1 + m_2)$, in agreement with Equation 6–8.

CONCEPTUAL CHECKPOINT 6–4 TENSION IN THE STRING

Two masses, m_1 and m_2, are connected by a string that passes over a pulley. Mass m_1 slides without friction on a horizontal tabletop, and mass m_2 falls vertically downward. Both masses move with a constant acceleration of magnitude a. Is the tension in the string **(a)** greater than, **(b)** equal to, or **(c)** less than $m_2 g$?



REASONING AND DISCUSSION

First, note that m_2 accelerates downward, which means that the net force acting on it is downward. Only two forces act on m_2, however: the tension in the string (upward) and its weight (downward). Since the net force is downward, the tension in the string must be less than the weight, $m_2 g$.

A common misconception is that since m_2 has to pull m_1 behind it, the tension in the string must be greater than $m_2 g$. Certainly, attaching the string to m_1 has an effect on the tension. If the string were not attached, for example, its tension would be zero. Hence, m_2 pulling on m_1 increases the tension to a value greater than zero, though still less than $m_2 g$.

ANSWER

(c) The tension in the string is less than $m_2 g$.

In the next Example, we verify the qualitative conclusions given in the Conceptual Checkpoint with a detailed calculation. But first, a note about choosing a coordinate system for a problem such as this. Rather than apply the same coordinate system to both masses, it is useful to take into consideration the fact that a pulley simply changes the direction of the tension in a string. With this in mind, we choose a set of axes that "follow the motion" of the string, so that both masses accelerate in the positive x direction with accelerations of equal magnitude. Example 6–6 illustrates the use of this type of coordinate system.

PROBLEM-SOLVING NOTE

Choice of Coordinate System: Connected Objects

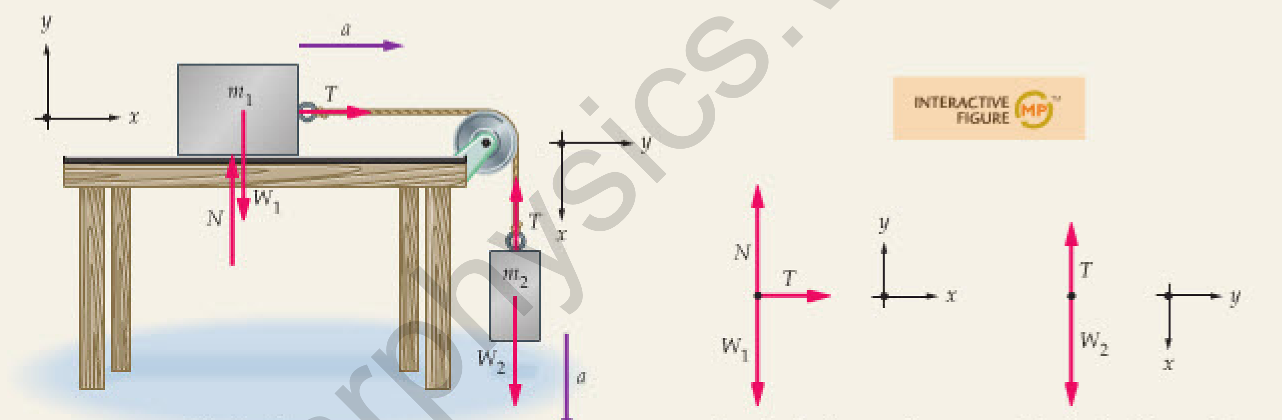
If two objects are connected by a string passing over a pulley, let the coordinate system follow the direction of the string. With this choice, both objects have accelerations of the same magnitude and in the same coordinate direction.

EXAMPLE 6–6 CONNECTED BLOCKS

A block of mass m_1 slides on a frictionless tabletop. It is connected to a string that passes over a pulley and suspends a mass m_2. Find **(a)** the acceleration of the masses and **(b)** the tension in the string.

PICTURE THE PROBLEM

Our coordinate system follows the motion of the string so that both masses move in the positive x direction. Since the masses are connected, their accelerations have the same magnitude. Thus, $a_{1,x} = a_{2,x} = a$. In addition, note that the tension, $\vec{T}$, is in the positive x direction for mass 1, but in the negative x direction for mass 2. Its magnitude, T, is the same for each mass, however. Finally, the weight of mass 2, W_2, acts in the positive x direction, whereas the weight of mass 1 is offset by the normal force, N.



Physical picture — Free-body diagram for m_1 — Free-body diagram for m_2

STRATEGY

Applying Newton's second law to the two masses yields the following relations: For mass 1, $\Sigma F_{1,x} = T = m_1 a_{1,x} = m_1 a$ and for mass 2, $\Sigma F_{2,x} = m_2 g - T = m_2 a_{2,x} = m_2 a$. These two equations can be solved for the two unknowns, a and T.

SOLUTION

Part (a)

1. First, write $\Sigma F_{1,x} = m_1 a$. Note that the only force acting on m_1 in the x direction is T:

$$\sum F_{1,x} = T = m_1 a$$

$$T = m_1 a$$

2. Next, write $\Sigma F_{2,x} = m_2 a$. In this case, two forces act in the x direction: $W_2 = m_2 g$ (positive direction) and T (negative direction):

$$\sum F_{2,x} = m_2 g - T = m_2 a$$

$$m_2 g - T = m_2 a$$

3. Sum the two relations obtained to eliminate T:

$$T = m_1 a$$
$$m_2 g - T = m_2 a$$
$$m_2 g = (m_1 + m_2) a$$

4. Solve for a:

$$a = \left(\frac{m_2}{m_1 + m_2}\right) g$$

CONTINUED ON NEXT PAGE

CONTINUED FROM PREVIOUS PAGE

Part (b)

5. Substitute a into the first relation ($T = m_1 a$) to find T: $$T = m_1 a = \left(\frac{m_1 m_2}{m_1 + m_2}\right)g$$

INSIGHT

We could just as well have determined T using $m_2 g - T = m_2 a$, though the algebra is a bit messier. Also, note that $a = 0$ if $m_2 = 0$, and that $a = g$ if $m_1 = 0$, as expected. Similarly, $T = 0$ if either m_1 or m_2 is zero. This type of check, where you connect equations with physical situations, is one of the best ways to increase your understanding of physics.

PRACTICE PROBLEM

Find the tension for the case $m_1 = 1.50$ kg and $m_2 = 0.750$ kg, and compare the tension to $m_2 g$. [**Answer:** $a = 3.27\ \text{m/s}^2$, $T = 4.91\ \text{N} < m_2 g = 7.36\ \text{N}$]

Some related homework problems: Problem 44, Problem 48

Conceptual Checkpoint 6–4 shows that the tension in the string should be less than $m_2 g$. Let's rewrite our solution for T to show that this is indeed the case. From Example 6–6 we have

$$T = \left(\frac{m_1 m_2}{m_1 + m_2}\right)g = \left(\frac{m_1}{m_1 + m_2}\right)m_2 g$$

Since the ratio $m_1/(m_1 + m_2)$ is always less than 1 (as long as m_2 is nonzero), it follows that $T < m_2 g$, as expected.

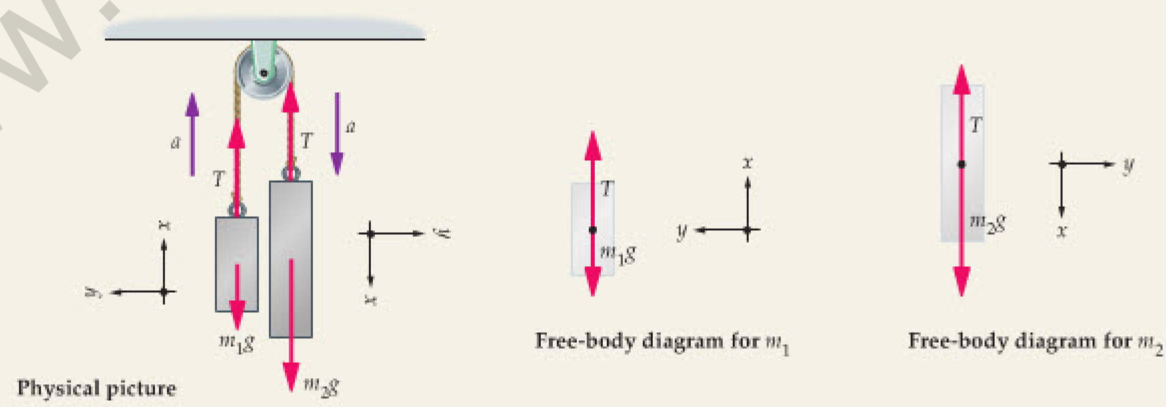
We conclude this section with a classic system that can be used to measure the acceleration of gravity. It is referred to as Atwood's machine, and it is basically two blocks of different mass connected by a string that passes over a pulley. The resulting acceleration of the blocks is related to the acceleration of gravity by a relatively simple expression, which we derive in the following Example.

EXAMPLE 6–7 ATWOOD'S MACHINE

Atwood's machine consists of two masses connected by a string that passes over a pulley, as shown below. Find the acceleration of the masses for general m_1 and m_2, and evaluate for the specific case $m_1 = 3.1$ kg, $m_2 = 4.4$ kg.

PICTURE THE PROBLEM

Our sketch shows Atwood's machine, along with our choice of coordinate directions for the two blocks. Note that both blocks accelerate in the positive x direction with accelerations of equal magnitude, a. From the free-body diagrams we can see that for mass 1 the weight is in the negative x direction and the tension is in the positive x direction. For mass 2, the tension is in the negative x direction and the weight is in the positive x direction. The tension has the same magnitude T for both masses, but their weights are different.



STRATEGY

To find the acceleration of the blocks, we follow the same strategy given in the previous Example. In particular, we start by applying Newton's second law to each block individually, using the fact that $a_{1,x} = a_{2,x} = a$. This gives two equations, both involving the tension T and the acceleration a. Eliminating T allows us to solve for the acceleration.

SOLUTION

1. Begin by writing out the expression $\Sigma F_{1,x} = m_1 a$. Note that two forces act in the x direction; T (positive direction) and $m_1 g$ (negative direction):

$$\sum F_{1,x} = T - m_1 g = m_1 a$$

2. Next, write out $\Sigma F_{2,x} = m_2 a$. The two forces acting in the x direction in this case are $m_2 g$ (positive direction) and T (negative direction):

$$\sum F_{2,x} = m_2 g - T = m_2 a$$

3. Sum the two relations obtained above to eliminate T:

$$\begin{array}{r} T - m_1 g = m_1 a \\ m_2 g - T = m_2 a \\ \hline (m_2 - m_1)g = (m_1 + m_2)a \end{array}$$

4. Solve for a:

$$a = \left(\frac{m_2 - m_1}{m_1 + m_2}\right)g$$

5. To evaluate the acceleration, substitute numerical values for the masses and for g:

$$a = \left(\frac{m_2 - m_1}{m_1 + m_2}\right)g = \left(\frac{4.4\text{ kg} - 3.1\text{ kg}}{3.1\text{ kg} + 4.4\text{ kg}}\right)(9.81\text{ m/s}^2) = 1.7\text{ m/s}^2$$

INSIGHT

Since m_2 is greater than m_1, we find that the acceleration is positive, meaning that the masses accelerate in the positive x direction. On the other hand, if m_1 were greater than m_2, we would find that a is negative, indicating that the masses accelerate in the negative x direction. Finally, if $m_1 = m_2$ we have $a = 0$, as expected.

PRACTICE PROBLEM

If m_1 is increased by a small amount, does the acceleration of the blocks increase, decrease, or stay the same? Check your answer by evaluating the acceleration for $m_1 = 3.3$ kg. [**Answer:** If m_1 is increased only slightly, the acceleration will decrease. For $m_1 = 3.3$ kg, we find $a = 1.4\text{ m/s}^2$.]

Some related homework problems: Problem 48, Problem 50

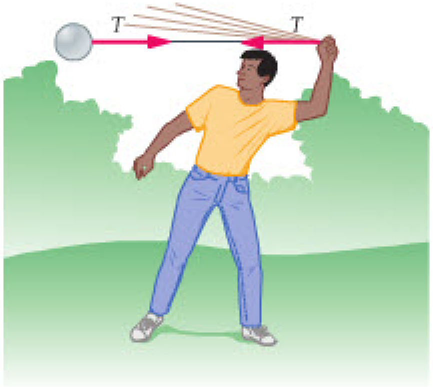
6–5 Circular Motion

According to Newton's second law, if no force acts on an object, it will move with constant speed in a constant direction. A force is required to change the speed, the direction, or both. For example, if you drive a car with constant speed on a circular track, the direction of the car's motion changes continuously. A force must act on the car to cause this change in direction. We would like to know two things about a force that causes circular motion: what is its direction, and what is its magnitude?

First, let's consider the direction of the force. Imagine swinging a ball tied to a string in a circle about your head, as shown in Figure 6–12. As you swing the ball, you feel a tension in the string pulling outward. Of course, on the other end of the string, where it attaches to the ball, the tension pulls inward, toward the center of the circle. Thus, the force the ball experiences is a force that is always directed toward the center of the circle. In summary,

> To make an object move in a circle with constant speed, a force must act on it that is directed toward the center of the circle.

Since the ball is acted on by a *force* toward the center of the circle, it follows that it must be *accelerating* toward the center of the circle. This might seem odd at



▲ **FIGURE 6–12 Swinging a ball in a circle**
The tension in the string pulls outward on the person's hand and pulls inward on the ball.

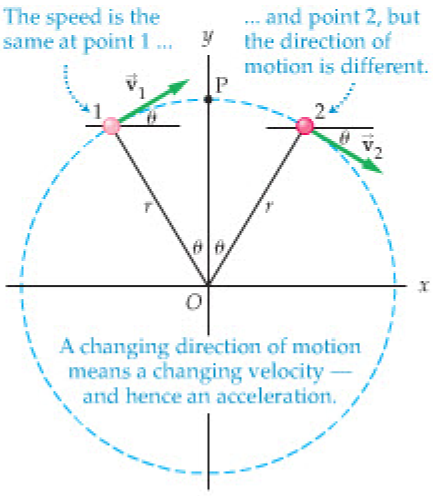


FIGURE 6–13 A particle moving with constant speed in a circular path centered on the origin

The speed of the particle is constant, but its velocity is constantly changing direction. Because the velocity changes, the particle is accelerating.

first: How can a ball that moves with constant speed have an acceleration? The answer is that acceleration is produced whenever the speed or direction of the velocity changes—and in circular motion, the direction changes continuously. The resulting center-directed acceleration is called **centripetal acceleration** (centripetal is from the Latin for "center seeking").

Let's calculate the magnitude of the centripetal acceleration, a_{cp}, for an object moving with a constant speed v in a circle of radius r. Figure 6–13 shows the circular path of an object, with the center of the circle at the origin. To calculate the acceleration at the top of the circle, at point P, we first calculate the average acceleration from point 1 to point 2:

$$\vec{a}_{av} = \frac{\Delta \vec{v}}{\Delta t} = \frac{\vec{v}_2 - \vec{v}_1}{\Delta t} \quad \text{6–10}$$

The instantaneous acceleration at P is the limit of $\vec{a}_{av}$ as points 1 and 2 move closer to P.

Referring to Figure 6–13, we see that $\vec{v}_1$ is at an angle θ above the horizontal, and $\vec{v}_2$ is at an angle θ below the horizontal. Both $\vec{v}_1$ and $\vec{v}_2$ have a magnitude v. Therefore, we can write the two velocities in vector form as follows:

$$\vec{v}_1 = (v \cos\theta)\hat{x} + (v \sin\theta)\hat{y}$$
$$\vec{v}_2 = (v \cos\theta)\hat{x} + (-v \sin\theta)\hat{y}$$

Substituting these results into $\vec{a}_{av}$ gives

$$\vec{a}_{av} = \frac{\vec{v}_2 - \vec{v}_1}{\Delta t} = \frac{-2v \sin\theta}{\Delta t}\hat{y} \quad \text{6–11}$$

Note that $\vec{a}_{av}$ points in the negative y direction—which, at point P, is toward the center of the circle.

To complete the calculation, we need Δt, the time it takes the object to go from point 1 to point 2. Since the object's speed is v, and the distance from point 1 to point 2 is $d = r(2\theta)$ where θ is measured in radians (see Appendix A, page A-2 for a discussion of radians and degrees), we find

$$\Delta t = \frac{d}{v} = \frac{2r\theta}{v} \quad \text{6–12}$$

TABLE 6.2 $\frac{\sin\theta}{\theta}$ for Values of θ Approaching Zero

θ, radians	$\frac{\sin\theta}{\theta}$
1.00	0.841
0.500	0.959
0.250	0.990
0.125	0.997
0.0625	0.999

Combining this result for Δt with the previous result for $\vec{a}_{av}$ gives

$$\vec{a}_{av} = \frac{-2v \sin\theta}{(2r\theta/v)}\hat{y} = -\frac{v^2}{r}\left(\frac{\sin\theta}{\theta}\right)\hat{y} \quad \text{6–13}$$

To find $\vec{a}$ at point P, we let points 1 and 2 approach P, which means letting θ go to zero. Table 6–2 shows that as θ goes to zero ($\theta \to 0$), the ratio $(\sin\theta)/\theta$ goes to 1:

$$\frac{\sin\theta}{\theta} \xrightarrow[\text{as } \theta \to 0]{} 1$$

Finally, then, the instantaneous acceleration at point P is

$$\vec{a} = -\frac{v^2}{r}\hat{y} = -a_{cp}\hat{y} \quad \text{6–14}$$

As mentioned, the direction of the acceleration is toward the center of the circle, and now we see that its magnitude is

$$a_{cp} = \frac{v^2}{r} \quad \text{6–15}$$

The people enjoying this carnival ride are experiencing a centripetal acceleration of roughly 10 m/s^2 directed inward, toward the axis of rotation. The force needed to produce this acceleration, which keeps the riders moving in a circular path, is provided by the horizontal component of the tension in the chains.

We can summarize these results as follows:

- When an object moves in a circle of radius r with constant speed v, its centripetal acceleration is $a_{cp} = v^2/r$.
- A force must be applied to an object to give it circular motion. For an object of mass m, the net force acting on it must have a magnitude given by

$$f_{cp} = ma_{cp} = m\frac{v^2}{r} \qquad \text{6–16}$$

and must be directed toward the center of the circle.

Note that the **centripetal force,** f_{cp}, can be produced in any number of ways. For example, f_{cp} might be the tension in a string, as in the example with the ball, or it might be due to friction between tires and the road, as when a car turns a corner. In addition, f_{cp} could be the force of gravity causing a satellite, or the Moon, to orbit the Earth. Thus, f_{cp} is a force that must be present to cause circular motion, but the specific cause of f_{cp} varies from system to system.

We now show how these results for centripetal force and centripetal acceleration can be applied in practice.

PROBLEM-SOLVING NOTE

Choice of Coordinate System: Circular Motion

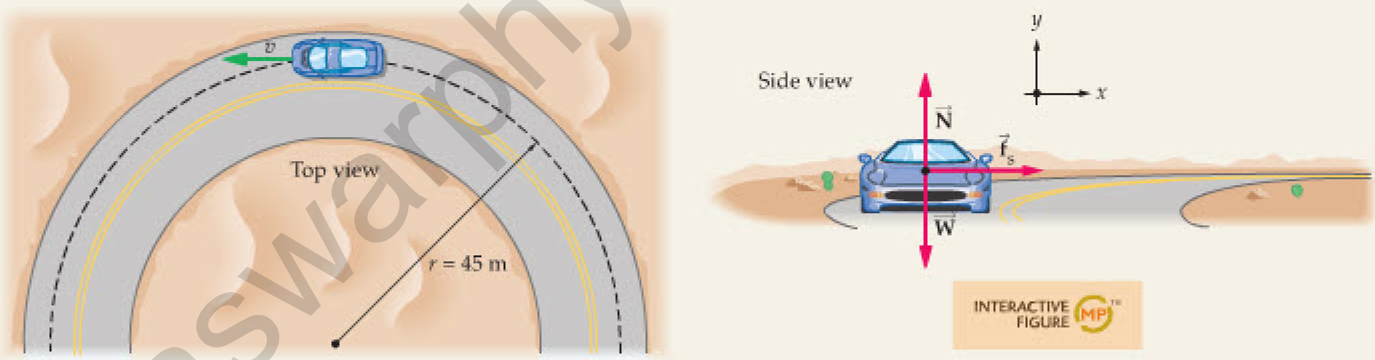
In circular motion, it is convenient to choose the coordinate system so that one axis points toward the center of the circle. Then, we know that the acceleration in that direction must be $a_{cp} = v^2/r$.

EXAMPLE 6–8 ROUNDING A CORNER

A 1200-kg car rounds a corner of radius $r = 45$ m. If the coefficient of static friction between the tires and the road is $\mu_s = 0.82$, what is the greatest speed the car can have in the corner without skidding?

PICTURE THE PROBLEM

In the first sketch we show a bird's-eye view of the car as it moves along its circular path. The next sketch shows the car moving directly toward the observer. Note that we have chosen the positive x direction to point toward the center of the circular path, and the positive y axis to point vertically upward. We also indicate the three forces acting on the car: gravity, $\vec{\mathbf{W}} = -W\hat{\mathbf{y}} = -mg\hat{\mathbf{y}}$; the normal force, $\vec{\mathbf{N}} = N\hat{\mathbf{y}}$; and the force of static friction, $\vec{\mathbf{f}}_s = \mu_s N\hat{\mathbf{x}}$.



STRATEGY

In this system, the force of static friction provides the centripetal force required for the car to move in a circular path. That is why the force of friction is at right angles to the car's direction of motion; it is directed toward the center of the circle. In addition, the friction in this case is static because the car's tires are rolling without slipping—always making static contact with the ground. Finally, if the car moves faster, more centripetal force (i.e., more friction) is required. Thus, the greatest speed for the car corresponds to the maximum static friction, $f_s = \mu_s N$. Hence, if we set $\mu_s N$ equal to the centripetal force, $ma_{cp} = mv^2/r$, we can solve for v.

SOLUTION

1. Sum the x components of force to relate the force of static friction to the centripetal acceleration of the car: $\sum F_x = f_s = ma_x$

2. Since the car moves in a circular path, with the center of the circle in the x direction, it follows that $a_x = a_{cp} = v^2/r$. Make this substitution, along with $f_s = \mu_s N$ for the force of static friction: $\mu_s N = ma_{cp} = m\dfrac{v^2}{r}$

CONTINUED ON NEXT PAGE

CONTINUED FROM PREVIOUS PAGE

3. Next, set the sum of the y components of force equal to zero, since $a_y = 0$: $\sum F_y = N - W = ma_y = 0$
4. Solve for the normal force: $N = W = mg$
5. Substitute the result $N = mg$ in Step 2 and solve for v. Notice that the mass of the car cancels:

$$\mu_s mg = m\frac{v^2}{r}$$

$$v = \sqrt{\mu_s rg}$$

6. Substitute numerical values to determine v: $v = \sqrt{(0.82)(45\ \text{m})(9.81\ \text{m/s}^2)} = 19\ \text{m/s}$

INSIGHT

Note that the maximum speed is less if the radius is smaller (tighter corner) or if μ_s is smaller (slick road). The mass of the vehicle, however, is irrelevant. For example, the maximum speed is precisely the same for a motorcycle rounding this corner as it is for a large, heavily loaded truck.

PRACTICE PROBLEM

Suppose the situation described in this Example takes place on the Moon, where the acceleration of gravity is less than it is on Earth. If a lunar rover goes around this same corner, is its maximum speed greater than, less than, or the same as the result found in Step 4? To check your answer, find the maximum speed for a lunar rover when it rounds a corner with $r = 45$ m and $\mu_s = 0.82$. (On the Moon, $g = 1.62\ \text{m/s}^2$.) [**Answer:** The maximum speed will be less. On the Moon we find $v = 7.7$ m/s.]

Some related homework problems: Problem 55, Problem 57, Problem 61

REAL-WORLD PHYSICS
Skids and banked roadways

If you try to round a corner too rapidly, you may experience a skid; that is, your car may begin to slide sideways across the road. A common bit of road wisdom is that you should turn in the direction of the skid to regain control—which, to most people, sounds counterintuitive. The advice is sound, however. Suppose, for example, that you are turning to the left and begin to skid to the right. If you turn more sharply to the left to try to correct for the skid, you simply reduce the turning radius of your car, r. The result is that the centripetal acceleration, v^2/r, becomes larger, and an even larger force would be required from the road to make the turn. The tendency to skid would therefore be increased. On the other hand, if you turn slightly to the right when you start to skid, you *increase* your turning radius and the centripetal acceleration decreases. In this case your car may stop skidding, and you can then regain control of your vehicle.

You may also have noticed that many roads are tilted, or banked, when they round a corner. The same type of banking is observed on many automobile racetracks as well. Next time you drive around a banked curve, notice that the banking tilts you in toward the center of the circular path you are following. This is by

▲ The steeply banked track at the Talladega Speedway in Alabama (left) helps to keep the rapidly moving cars from skidding off along a tangential path. Even when there is no solid roadway, however, banking can still help—airplanes bank when making turns (center) to keep from "skidding" sideways. Banking is beneficial in another way as well. Occupants of cars on a banked roadway or of a banking airplane feel no sideways force when the banking angle is just right, so turns become a safer and more comfortable experience. For this reason, some trains use hydraulic suspension systems to bank when rounding corners (right), even though the tracks themselves are level.

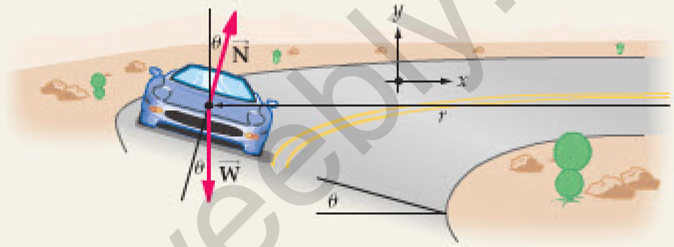
design. On a banked curve, the normal force exerted by the road contributes to the required centripetal force. If the tilt angle is just right, the normal force provides all of the centripetal force so that the car can negotiate the curve even if there is no friction between its tires and the road. The next Example determines the optimum banking angle for a given speed and given radius of turn.

EXAMPLE 6–9 BANK ON IT

If a roadway is banked at the proper angle, a car can round a corner without any assistance from friction between the tires and the road. Find the appropriate banking angle for a 900-kg car traveling at 20.5 m/s in a turn of radius 85.0 m.

PICTURE THE PROBLEM

Note that we choose the positive y axis to point vertically upward and the positive x direction to point toward the center of the circular path. Since $\vec{\mathbf{N}}$ is perpendicular to the banked roadway, it is at an angle θ to the y axis. Therefore, $\vec{\mathbf{N}} = (N\sin\theta)\hat{\mathbf{x}} + (N\cos\theta)\hat{\mathbf{y}}$ and $\vec{\mathbf{W}} = -W\hat{\mathbf{y}} = -mg\hat{\mathbf{y}}$.



STRATEGY

In order for the car to move in a circular path, there must be a force acting on it in the positive x direction. Since the weight $\vec{\mathbf{W}}$ has no x component, it follows that the normal force $\vec{\mathbf{N}}$ must supply the needed centripetal force. Thus, we find N by setting $\Sigma F_y = ma_y = 0$, since there is no motion in the y direction. Then we use N in $\Sigma F_x = ma_x = mv^2/r$ to find the angle θ.

SOLUTION

1. Start by determining N from the condition $\Sigma F_y = 0$:

$$\sum F_y = N\cos\theta - W = 0$$

$$N = \frac{W}{\cos\theta} = \frac{mg}{\cos\theta}$$

2. Next, set $\Sigma F_x = mv^2/r$:

$$\sum F_x = N\sin\theta$$

$$= ma_x = ma_{cp} = m\frac{v^2}{r}$$

3. Substitute $N = mg/\cos\theta$ (from $\Sigma F_y = 0$, Step 1) and solve for θ, using the fact that $\sin\theta/\cos\theta = \tan\theta$. Notice that, once again, the mass of the car cancels:

$$N\sin\theta = \frac{mg}{\cos\theta}\sin\theta = m\frac{v^2}{r}$$

$$\tan\theta = \frac{v^2}{gr} \quad \text{or} \quad \theta = \tan^{-1}\left(\frac{v^2}{gr}\right)$$

4. Substitute numerical values to determine θ:

$$\theta = \tan^{-1}\left[\frac{(20.5\text{ m/s})^2}{(9.81\text{ m/s}^2)(85.0\text{ m})}\right] = 26.7°$$

INSIGHT

The symbolic result in Step 3 shows that the banking angle increases with increasing speed and decreasing radius of turn, as one would expect.

From the point of view of a passenger, the experience of rounding a properly banked corner is basically the same as riding on a level road—there are no "sideways forces" to make the turn uncomfortable. There is one small difference, however—the passenger feels heavier due to the increased normal force.

PRACTICE PROBLEM

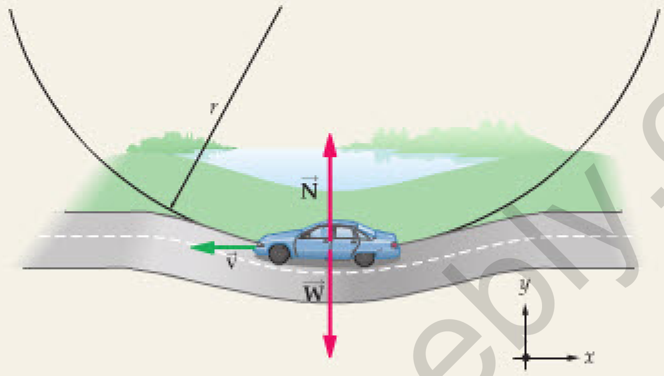
A turn of radius 65 m is banked at 30.0°. What speed should a car have in order to make the turn with no assistance from friction? [**Answer:** $v = 19$ m/s]

Some related homework problems: Problem 58, Problem 107

If you've ever driven through a dip in the road, you know that you feel momentarily heavier near the bottom of the dip, just like a passenger in Example 6–9. This change in apparent weight is due to the approximately circular motion of the car, as we show next.

ACTIVE EXAMPLE 6–4 FIND THE NORMAL FORCE

While driving along a country lane with a constant speed of 17.0 m/s, you encounter a dip in the road. The dip can be approximated as a circular arc, with a radius of 65.0 m. What is the normal force exerted by a car seat on an 80.0-kg passenger when the car is at the bottom of the dip?



SOLUTION *(Test your understanding by performing the calculations indicated in each step.)*

1. Write $\Sigma F_y = ma_y$ for the passenger: $N - mg = ma_y$
2. Replace a_y with the centripetal acceleration: $a_y = v^2/r$
3. Solve for N: $N = mg + mv^2/r$
4. Substitute numerical values: $N = 1140\ \text{N}$

INSIGHT

At the bottom of the dip the normal force is greater than the weight of the passenger, since it must also supply the centripetal force. As a result, the passenger feels heavier than usual. In this case, the 80.0-kg passenger feels as if his mass has increased by 45%, to 116 kg!

The same physics applies to a jet pilot who pulls a plane out of a high-speed dive. In that case, the magnitude of the effect can be much larger, resulting in a decrease of blood flow to the brain and eventually to loss of consciousness. Here's a case where basic physics really can be a matter of life and death.

YOUR TURN

At what speed will the magnitude of the normal force be equal to 1250 N?

(Answers to **Your Turn** *problems are given in the back of the book.)*

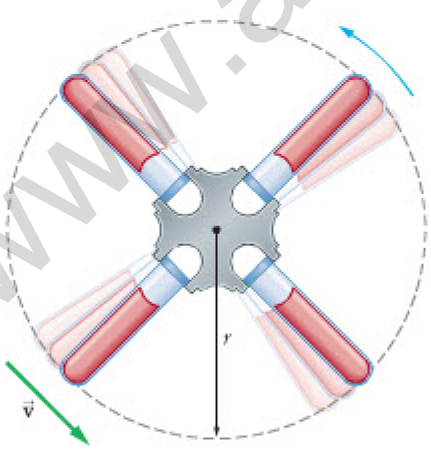
▲ A laboratory centrifuge of the kind commonly used to separate blood components.

REAL-WORLD PHYSICS: BIO

Centrifuges and ultracentrifuges

A similar calculation can be applied to a car going over the top of a bump. In that case, circular motion results in a reduced apparent weight.

Finally, we determine the acceleration produced in a **centrifuge**, a common device in biological and medical laboratories that uses large centripetal accelerations to perform such tasks as separating red and white blood cells from serum. A simplified top view of a centrifuge is shown in **Figure 6–14.**



▲ **FIGURE 6–14 Simplified top view of a centrifuge in operation**

EXERCISE 6–1

The centrifuge in Figure 6–14 rotates at a rate that gives the bottom of the test tube a linear speed of 89.3 m/s. If the bottom of the test tube is 8.50 cm from the axis of rotation, what is the centripetal acceleration experienced there?

SOLUTION

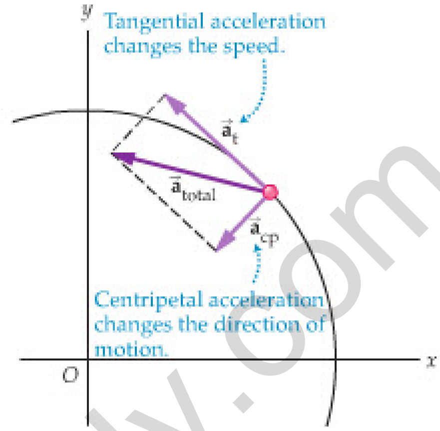
Applying the relation $a_{cp} = v^2/r$ yields

$$a_{cp} = \frac{v^2}{r} = \frac{(89.3\ \text{m/s})^2}{0.0850\ \text{m}} = 93{,}800\ \text{m/s}^2 = 9560g$$

In this expression, g is the acceleration of gravity, 9.81 m/s^2.

Thus, a centrifuge can produce centripetal accelerations that are many thousand times greater than the acceleration of gravity. In fact, devices referred to as **ultracentrifuges** can produce accelerations as great as 1 million g. Even in the relatively modest case considered in Exercise 6–1, the forces involved in a centrifuge can be quite significant. For example, if the contents of the test tube have a mass of 12.0 g, the centripetal force that must be exerted by the bottom of the tube is $(0.0120 \text{ kg})(9560\ g) = 1130$ N, or about 250 lb!

Finally, an object moving in a circular path may increase or decrease its speed. In such a case, the object has both an acceleration tangential to its path that changes its speed, $\vec{a}_t$, and a centripetal acceleration perpendicular to its path, $\vec{a}_{cp}$, that changes its direction of motion. Such a situation is illustrated in Figure 6–15. The total acceleration of the object is the vector sum of $\vec{a}_t$ and $\vec{a}_{cp}$. We will explore this case more fully in Chapter 10.



▲ **FIGURE 6–15 A particle moving in a circular path with tangential acceleration**
In this case, the particle's speed is increasing at the rate given by a_t.

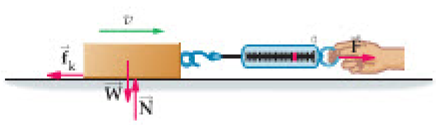
THE BIG PICTURE PUTTING PHYSICS IN CONTEXT

LOOKING BACK	LOOKING AHEAD
The equations of kinematics from Chapters 2 and 4 proved useful again in this chapter. See, in particular, Examples 6–1 and 6–2.	Our discussion of springs, and Hooke's law in particular, will be of importance when we consider oscillations in Chapter 13.
The discussion related to Figure 5–15 about angles on an inclined surface came into play when identifying the angles in Examples 6–2 and 6–9.	The basic ideas of translational equilibrium (Section 6–3) will be extended to more general objects in Chapter 11.
Our derivation of the direction and magnitude of centripetal acceleration (Section 6–5) made extensive use of our knowledge of vectors and how to resolve them into components.	Circular motion will come up again in a number of situations, but especially when we consider orbital motion in Chapter 12 and the Bohr model of the hydrogen atom in Chapter 31.

CHAPTER SUMMARY

6–1 FRICTIONAL FORCES

Frictional forces are due to the microscopic roughness of surfaces in contact. As a rule of thumb, friction is independent of the area of contact and independent of the relative speed of the surfaces.



Kinetic Friction
Friction experienced by surfaces that are in contact and moving relative to one another. The force of kinetic friction is given by

$$f_k = \mu_k N \qquad 6\text{–}1$$

In this expression, μ_k is the coefficient of kinetic friction and N is the magnitude of the normal force.

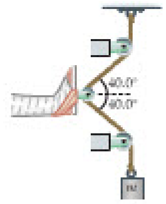
Static Friction
Friction experienced by surfaces that are in static contact. The maximum force of static friction is given by

$$f_{s,max} = \mu_s N \qquad 6\text{–}3$$

In this expression, μ_s is the coefficient of static friction and N is the magnitude of the normal force. The force of static friction can have any magnitude between zero and its maximum value.

6–2 STRINGS AND SPRINGS

Strings and springs provide a common way of exerting forces on objects. Ideal strings and springs are massless.



Tension

The force transmitted through a string. The tension is the same throughout the length of an ideal string.

Hooke's Law

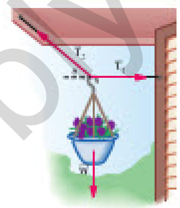
The force exerted by an ideal spring stretched by the amount x is

$$F_x = -kx \qquad 6\text{–}4$$

In words, the force exerted by a spring is proportional to the amount of stretch or compression, and is in the opposite direction.

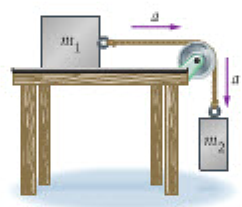
6–3 TRANSLATIONAL EQUILIBRIUM

An object is in translational equilibrium if the net force acting on it is zero. Equivalently, an object is in equilibrium if it has zero acceleration.



6–4 CONNECTED OBJECTS

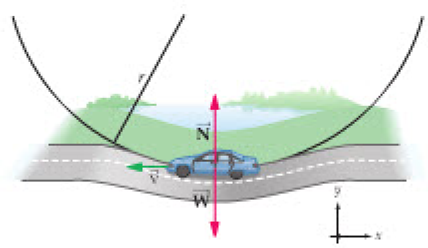
Connected objects are linked physically, and hence they are linked mathematically as well. For example, objects connected by strings have the same magnitude of acceleration.



6–5 CIRCULAR MOTION

An object moving with speed v in a circle of radius r has an acceleration of magnitude v^2/r directed toward the center of the circle. This is referred to as the centripetal acceleration, a_{cp}. If the object has a mass m, the force required for the circular motion is

$$f_{cp} = ma_{cp} = mv^2/r \qquad 6\text{–}16$$



PROBLEM-SOLVING SUMMARY

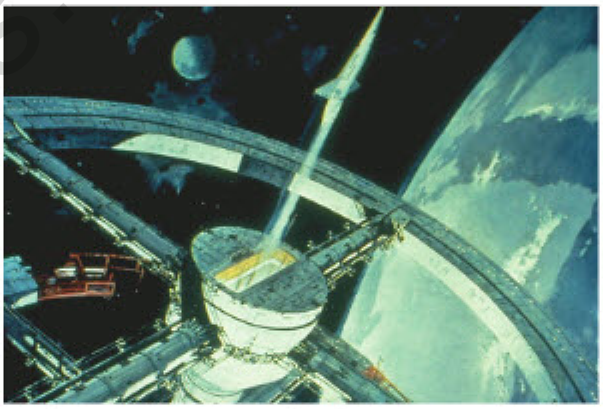
Type of Calculation	Relevant Physical Concepts	Related Examples
Find the acceleration when kinetic friction is present.	First, find the magnitude of the normal force, N. The corresponding kinetic friction has a magnitude of $f_k = \mu_k N$ and points opposite to the direction of motion. Include this force with the others when applying Newton's second law.	Examples 6–1, 6–2
Solve problems involving static friction.	Start by finding the magnitude of the normal force, N. The corresponding static friction has a magnitude between zero and $\mu_s N$. Its direction opposes motion.	Example 6–3 Active Example 6–1
Find the acceleration and the tension for masses connected by a string.	Apply Newton's second law to each mass separately. This generates two equations, which can be solved for the two unknowns, a and T.	Examples 6–6, 6–7
Solve problems involving circular motion.	Set up the coordinate system so that one axis points to the center of the circle. When applying Newton's second law to that direction, set the acceleration equal to $a_{cp} = v^2/r$.	Examples 6–8, 6–9 Active Example 6–4

CONCEPTUAL QUESTIONS

For instructor-assigned homework, go to www.masteringphysics.com

(Answers to odd-numbered Conceptual Questions can be found in the back of the book.)

1. A clothesline always sags a little, even if nothing hangs from it. Explain.
2. In the *Jurassic Park* sequel, *The Lost World*, a man tries to keep a large vehicle from going over a cliff by connecting a cable from his Jeep to the vehicle. The man then puts the Jeep in gear and spins the rear wheels. Do you expect that spinning the tires will increase the force exerted by the Jeep on the vehicle? Why or why not?
3. When a traffic accident is investigated, it is common for the length of the skid marks to be measured. How could this information be used to estimate the initial speed of the vehicle that left the skid marks?
4. In a car with rear-wheel drive, the maximum acceleration is often less than the maximum deceleration. Why?
5. A train typically requires a much greater distance to come to rest, for a given initial speed, than does a car. Why?
6. Give some everyday examples of situations in which friction is beneficial.
7. At the local farm, you buy a flat of strawberries and place them on the backseat of the car. On the way home, you begin to brake as you approach a stop sign. At first the strawberries stay put, but as you brake a bit harder, they begin to slide off the seat. Explain.
8. It is possible to spin a bucket of water in a vertical circle and have none of the water spill when the bucket is upside down. How would you explain this to members of your family?
9. Water sprays off a rapidly turning bicycle wheel. Why?
10. Can an object be in equilibrium if it is moving? Explain.
11. In a dramatic circus act, a motorcyclist drives his bike around the inside of a vertical circle. How is this possible, considering that the motorcycle is upside down at the top of the circle?
12. The gravitational attraction of the Earth is only slightly less at the altitude of an orbiting spacecraft than it is on the Earth's surface. Why is it, then, that astronauts feel weightless?
13. A popular carnival ride has passengers stand with their backs against the inside wall of a cylinder. As the cylinder begins to spin, the passengers feel as if they are being pushed against the wall. Explain.
14. Referring to Question 13, after the cylinder reaches operating speed, the floor is lowered away, leaving the passengers "stuck" to the wall. Explain.
15. Your car is stuck on an icy side street. Some students on their way to class see your predicament and help out by sitting on the trunk of your car to increase its traction. Why does this help?
16. The parking brake on a car causes the rear wheels to lock up. What would be the likely consequence of applying the parking brake in a car that is in rapid motion? (*Note:* Do *not* try this at home.)
17. **BIO** The foot of your average gecko is covered with billions of tiny hair tips—called spatulae—that are made of keratin, the protein found in human hair. A subtle shift of the electron distribution in both the spatulae and the wall to which a gecko clings produces an adhesive force by means of the van der Waals interaction between molecules. Suppose a gecko uses its spatulae to cling to a vertical windowpane. If you were to describe this situation in terms of a coefficient of static friction, μ_s, what value would you assign to μ_s? Is this a sensible way to model the gecko's feat? Explain.
18. Discuss the physics involved in the spin cycle of a washing machine. In particular, how is circular motion related to the removal of water from the clothes?
19. The gas pedal and the brake pedal are capable of causing a car to accelerate. Can the steering wheel also produce an acceleration? Explain.
20. In the movie *2001: A Space Odyssey*, a rotating space station provides "artificial gravity" for its inhabitants. How does this work?



The rotating space station from the movie *2001: A Space Odyssey* (Conceptual Question 20)

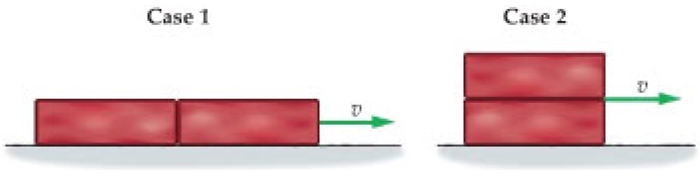
21. When rounding a corner on a bicycle or a motorcycle, the driver leans inward, toward the center of the circle. Why?
22. In *Robin Hood: Prince of Thieves*, starring Kevin Costner, Robin swings between trees on a vine that is on fire. At the lowest point of his swing, the vine burns through and Robin begins to fall. The next shot, from high up in the trees, shows Robin falling straight downward. Would you rate the physics of this scene "Good," "Bad," or "Ugly"? Explain.

PROBLEMS AND CONCEPTUAL EXERCISES

Note: Answers to odd-numbered Problems and Conceptual Exercises can be found in the back of the book. **IP** *denotes an integrated problem, with both conceptual and numerical parts;* **BIO** *identifies problems of biological or medical interest;* **CE** *indicates a conceptual exercise.* **Predict/Explain** *problems ask for two responses:* **(a)** *your prediction of a physical outcome, and* **(b)** *the best explanation among three provided. On all problems, red bullets (•, ••, •••) are used to indicate the level of difficulty.*

SECTION 6–1 FRICTIONAL FORCES

1. • **CE Predict/Explain** You push two identical bricks across a tabletop with constant speed, v, as shown in **Figure 6–16**. In case 1, you place the bricks end to end; in case 2, you stack the bricks one on top of the other. **(a)** Is the force of kinetic friction in case 1 greater than, less than, or equal to the force of kinetic friction in case 2? **(b)** Choose the *best explanation* from among the following:



▲ **FIGURE 6–16** Problem 1

I. The normal force in case 2 is larger, and hence the bricks press down more firmly against the tabletop.
II. The normal force is the same in the two cases, and friction is independent of surface area.
III. Case 1 has more surface area in contact with the tabletop, and this leads to more friction.

2. • **CE Predict/Explain** Two drivers traveling side-by-side at the same speed suddenly see a deer in the road ahead of them and begin braking. Driver 1 stops by locking up his brakes and screeching to a halt; driver 2 stops by applying her brakes just to the verge of locking, so that the wheels continue to turn until her car comes to a complete stop. **(a)** All other factors being equal, is the stopping distance of driver 1 greater than, less than, or equal to the stopping distance of driver 2? **(b)** Choose the *best explanation* from among the following:
I. Locking up the brakes gives the greatest possible braking force.
II. The same tires on the same road result in the same force of friction.
III. Locked-up brakes lead to sliding (kinetic) friction, which is less than rolling (static) friction.

3. • A baseball player slides into third base with an initial speed of 4.0 m/s. If the coefficient of kinetic friction between the player and the ground is 0.46, how far does the player slide before coming to rest?

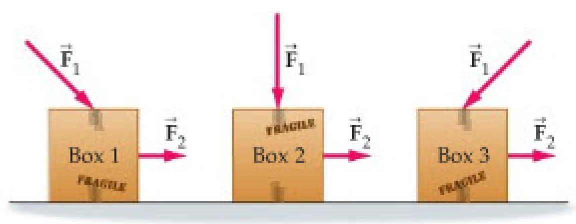
4. • A child goes down a playground slide with an acceleration of 1.26 m/s^2. Find the coefficient of kinetic friction between the child and the slide if the slide is inclined at an angle of 33.0° below the horizontal.

5. • Hopping into your Porsche, you floor it and accelerate at 12 m/s^2 without spinning the tires. Determine the minimum coefficient of static friction between the tires and the road needed to make this possible.

6. • When you push a 1.80-kg book resting on a tabletop, it takes 2.25 N to start the book sliding. Once it is sliding, however, it takes only 1.50 N to keep the book moving with constant speed. What are the coefficients of static and kinetic friction between the book and the tabletop?

7. • In Problem 6, what is the frictional force exerted on the book when you push on it with a force of 0.75 N?

8. •• **CE** The three identical boxes shown in Figure 6–17 remain at rest on a rough, horizontal surface, even though they are acted on by two different forces, $\vec{F}_1$ and $\vec{F}_2$. All of the forces labeled $\vec{F}_1$ have the same magnitude; all of the forces labeled $\vec{F}_2$ are identical to one another. Rank the boxes in order of increasing magnitude of the force static friction between them and the surface. Indicate ties where appropriate.



▲ **FIGURE 6–17** Problem 8

9. •• **IP** A tie of uniform width is laid out on a table, with a fraction of its length hanging over the edge. Initially, the tie is at rest. **(a)** If the fraction hanging from the table is increased, the tie eventually slides to the ground. Explain. **(b)** What is the coefficient of static friction between the tie and the table if the tie begins to slide when one-fourth of its length hangs over the edge?

10. •• To move a large crate across a rough floor, you push on it with a force F at an angle of 21° below the horizontal, as shown in Figure 6–18. Find the force necessary to start the crate moving, given that the mass of the crate is 32 kg and the coefficient of static friction between the crate and the floor is 0.57.

▲ **FIGURE 6–18** Problems 10, 11, and 106

11. •• In Problem 10, find the acceleration of the crate if the applied force is 330 N and the coefficient of kinetic friction is 0.45.

12. •• **IP** A 48-kg crate is placed on an inclined ramp. When the angle the ramp makes with the horizontal is increased to 26°, the crate begins to slide downward. **(a)** What is the coefficient of static friction between the crate and the ramp? **(b)** At what angle does the crate begin to slide if its mass is doubled?

13. •• **IP** A 97-kg sprinter wishes to accelerate from rest to a speed of 13 m/s in a distance of 22 m. **(a)** What coefficient of static friction is required between the sprinter's shoes and the track? **(b)** Explain the strategy used to find the answer to part (a).

14. •• **Coffee To Go** A person places a cup of coffee on the roof of her car while she dashes back into the house for a forgotten item. When she returns to the car, she hops in and takes off with the coffee cup still on the roof. **(a)** If the coefficient of static friction between the coffee cup and the roof of the car is 0.24, what is the maximum acceleration the car can have without causing the cup to slide? Ignore the effects of air resistance. **(b)** What is the smallest amount of time in which the person can accelerate the car from rest to 15 m/s and still keep the coffee cup on the roof?

15. •• **IP Force Times Distance I** At the local hockey rink, a puck with a mass of 0.12 kg is given an initial speed of v = 5.3 m/s. **(a)** If the coefficient of kinetic friction between the ice and the puck is 0.11, what distance d does the puck slide before coming to rest? **(b)** If the mass of the puck is doubled, does the frictional force F exerted on the puck increase, decrease, or stay the same? Explain. **(c)** Does the stopping distance of the puck increase, decrease, or stay the same when its mass is doubled? Explain. **(d)** For the situation considered in part (a), show that $Fd = \frac{1}{2}mv^2$.

(The significance of this result will be discussed in Chapter 7, where we will see that $\frac{1}{2}mv^2$ is the kinetic energy of an object.)

16. •• **IP** **Force Times Time** At the local hockey rink, a puck with a mass of 0.12 kg is given an initial speed of $v_0 = 6.7$ m/s. **(a)** If the coefficient of kinetic friction between the ice and the puck is 0.13, how much time t does it take for the puck to come to rest? **(b)** If the mass of the puck is doubled, does the frictional force F exerted on the puck increase, decrease, or stay the same? Explain. **(c)** Does the stopping time of the puck increase, decrease, or stay the same when its mass is doubled? Explain. **(d)** For the situation considered in part (a), show that $Ft = mv_0$. (The significance of this result will be discussed in Chapter 9, where we will see that mv is the momentum of an object.)

17. •• **Force Times Distance II** A block of mass $m = 1.95$ kg slides with an initial speed $v_i = 4.33$ m/s on a smooth, horizontal surface. The block now encounters a rough patch with a coefficient of kinetic friction given by $\mu_k = 0.260$. The rough patch extends for a distance $d = 0.125$ m, after which the surface is again frictionless. **(a)** What is the acceleration of the block when it is in the rough patch? **(b)** What is the final speed, v_f, of the block when it exits the rough patch? **(c)** Show that $-Fd = -(\mu_k mg)d = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$. (The significance of this result will be discussed in Chapter 7, where we will see that $\frac{1}{2}mv^2$ is the kinetic energy of an object.)

18. ••• **IP** The coefficient of kinetic friction between the tires of your car and the roadway is μ. **(a)** If your initial speed is v and you lock your tires during braking, how far do you skid? Give your answer in terms of v, μ, and m, the mass of your car. **(b)** If you double your speed, what happens to the stopping distance? **(c)** What is the stopping distance for a truck with twice the mass of your car, assuming the same initial speed and coefficient of kinetic friction?

SECTION 6–2 STRINGS AND SPRINGS

19. • **CE** A certain spring has a force constant k. **(a)** If this spring is cut in half, does the resulting half spring have a force constant that is greater than, less than, or equal to k? **(b)** If two of the original full-length springs are connected end to end, does the resulting double spring have a force constant that is greater than, less than, or equal to k?

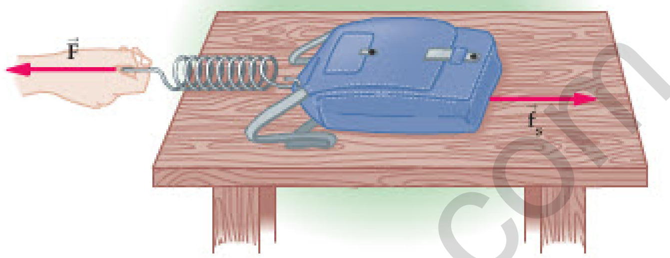
20. • Pulling up on a rope, you lift a 4.35-kg bucket of water from a well with an acceleration of 1.78 m/s^2. What is the tension in the rope?

21. • When a 9.09-kg mass is placed on top of a vertical spring, the spring compresses 4.18 cm. Find the force constant of the spring.

22. • A 110-kg box is loaded into the trunk of a car. If the height of the car's bumper decreases by 13 cm, what is the force constant of its rear suspension?

23. • A 50.0-kg person takes a nap in a backyard hammock. Both ropes supporting the hammock are at an angle of 15.0° above the horizontal. Find the tension in the ropes.

24. • **IP** A backpack full of books weighing 52.0 N rests on a table in a physics laboratory classroom. A spring with a force constant of 150 N/m is attached to the backpack and pulled horizontally, as indicated in Figure 6–19. **(a)** If the spring is pulled until it stretches 2.00 cm and the pack remains at rest, what is the force of friction exerted on the backpack by the table? **(b)** Does your answer to part (a) change if the mass of the backpack is doubled? Explain.



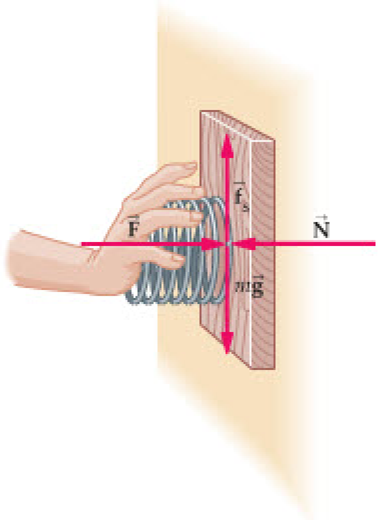
▲ **FIGURE 6–19** Problems 24 and 25

25. • If the 52.0-N backpack in Problem 24 begins to slide when the spring ($k = 150$ N/m) stretches by 2.50 cm, what is the coefficient of static friction between the backpack and the table?

26. •• **IP** The equilibrium length of a certain spring with a force constant of $k = 250$ N/m is 0.18 m. **(a)** What is the magnitude of the force that is required to hold this spring at twice its equilibrium length? **(b)** Is the magnitude of the force required to keep the spring compressed to half its equilibrium length greater than, less than, or equal to the force found in part (a)? Explain.

27. •• **IP** Illinois Jones is being pulled from a snake pit with a rope that breaks if the tension in it exceeds 755 N. **(a)** If Illinois Jones has a mass of 70.0 kg and the snake pit is 3.40 m deep, what is the minimum time that is required to pull our intrepid explorer from the pit? **(b)** Explain why the rope breaks if Jones is pulled from the pit in less time than that calculated in part (a).

28. •• **IP** A spring with a force constant of 120 N/m is used to push a 0.27-kg block of wood against a wall, as shown in Figure 6–20. **(a)** Find the minimum compression of the spring needed to keep the block from falling, given that the coefficient of static friction between the block and the wall is 0.46. **(b)** Does your answer to part (a) change if the mass of the block of wood is doubled? Explain.

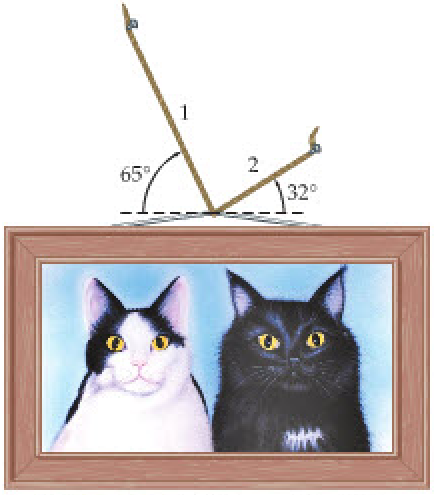


▲ **FIGURE 6–20** Problem 28

29. •• **IP** Your friend's 13.6-g graduation tassel hangs on a string from his rearview mirror. **(a)** When he accelerates from a stoplight, the tassel deflects backward toward the rear of the car. Explain. **(b)** If the tassel hangs at an angle of 6.44° relative to the vertical, what is the acceleration of the car?

30. •• In Problem 29, **(a)** find the tension in the string holding the tassel. **(b)** At what angle to the vertical will the tension in the string be twice the weight of the tassel?

31. •• **IP** A picture hangs on the wall suspended by two strings, as shown in **Figure 6–21**. The tension in string 1 is 1.7 N. **(a)** Is the tension in string 2 greater than, less than, or equal to 1.7 N? Explain. **(b)** Verify your answer to part (a) by calculating the tension in string 2. **(c)** What is the weight of the picture?



▲ **FIGURE 6–21** Problems 31 and 83

32. •• **Mechanical Advantage** The pulley system shown in **Figure 6–22** is used to lift a 52-kg crate. Note that one chain connects the upper pulley to the ceiling and a second chain connects the lower pulley to the crate. Assuming the masses of the chains, pulleys, and ropes are negligible, determine **(a)** the force $\vec{F}$ required to lift the crate with constant speed, **(b)** the tension in the upper chain, and **(c)** the tension in the lower chain.

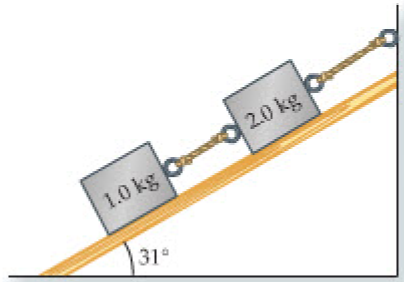
▲ **FIGURE 6–22** Problems 32 and 33

33. •• In Problem 32, determine **(a)** the force $\vec{F}$, **(b)** the tension in the upper chain, and **(c)** the tension in the lower chain, given that the crate is rising with an acceleration of 2.3 m/s^2.

SECTION 6–3 TRANSLATIONAL EQUILIBRIUM

34. • Pulling the string on a bow back with a force of 28.7 lb, an archer prepares to shoot an arrow. If the archer pulls in the center of the string, and the angle between the two halves is 138°, what is the tension in the string?

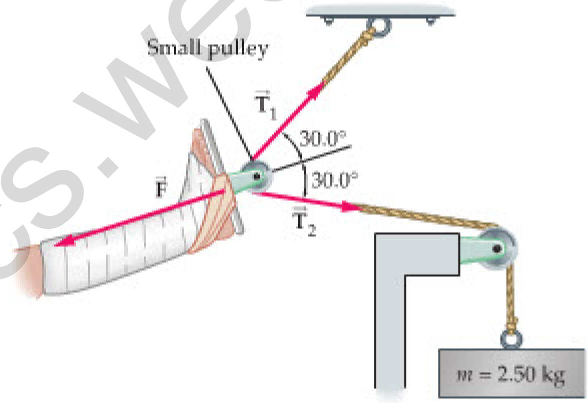
35. • In **Figure 6–23** we see two blocks connected by a string and tied to a wall. The mass of the lower block is 1.0 kg; the mass of the upper block is 2.0 kg. Given that the angle of the incline is



▲ **FIGURE 6–23** Problem 35

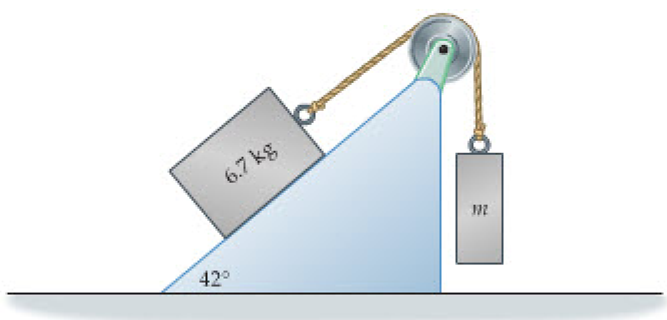
31°, find the tensions in **(a)** the string connecting the two blocks and **(b)** the string that is tied to the wall.

36. • **BIO Traction** After a skiing accident, your leg is in a cast and supported in a traction device, as shown in **Figure 6–24**. Find the magnitude of the force $\vec{F}$ exerted by the leg on the small pulley. (By Newton's third law, the small pulley exerts an equal and opposite force on the leg.) Let the mass m be 2.50 kg.



▲ **FIGURE 6–24** Problems 36 and 69

37. • Two blocks are connected by a string, as shown in **Figure 6–25**. The smooth inclined surface makes an angle of 42° with the horizontal, and the block on the incline has a mass of 6.7 kg. Find the mass of the hanging block that will cause the system to be in equilibrium. (The pulley is assumed to be ideal.)



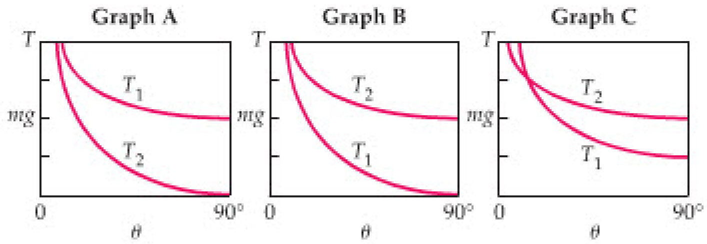
▲ **FIGURE 6–25** Problem 37

38. •• **CE Predict/Explain** **(a)** Referring to the hanging planter in Example 6–5, which of the three graphs (A, B, or C) in **Figure 6–26** shows an accurate plot of the tensions T_1 and T_2 as a function of the angle θ? **(b)** Choose the *best explanation* from among the following:

I. The two tensions must be equal at some angle between $\theta = 0$ and $\theta = 90°$.

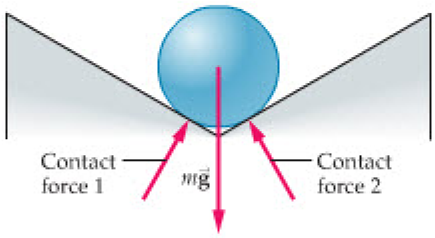
II. T_2 is greater than T_1 at all angles, and is equal to mg at $\theta = 90°$.

III. T_2 is less than T_1 at all angles, and is equal to 0 at $\theta = 90°$.



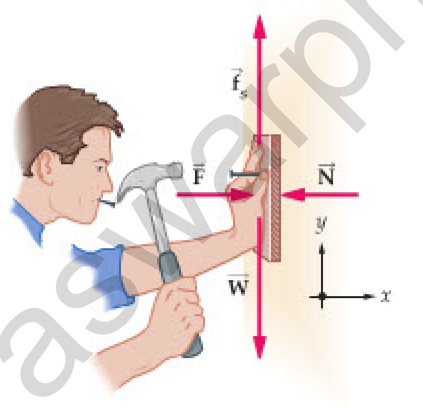
▲ **FIGURE 6–26** Problem 38

39. •• A 0.15-kg ball is placed in a shallow wedge with an opening angle of 120°, as shown in Figure 6–27. For each contact point between the wedge and the ball, determine the force exerted on the ball. Assume the system is frictionless.



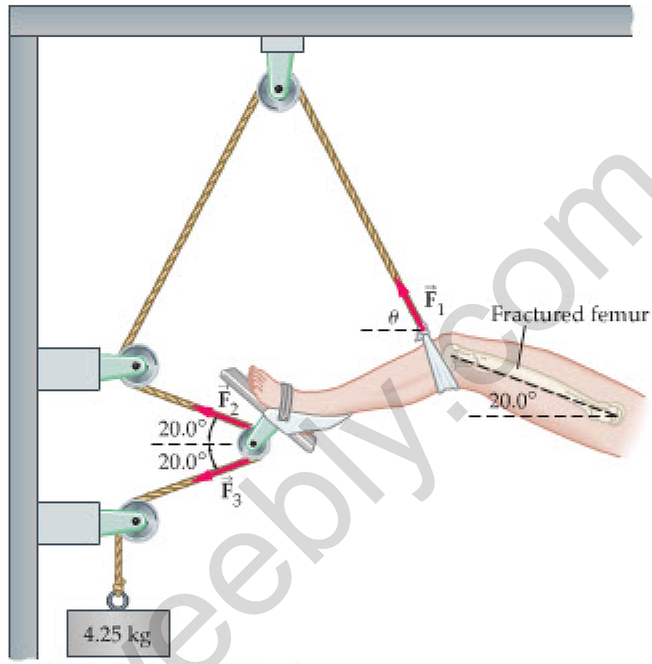
▲ **FIGURE 6–27** Problem 39

40. •• **IP** You want to nail a 1.6-kg board onto the wall of a barn. To position the board before nailing, you push it against the wall with a horizontal force $\vec{F}$ to keep it from sliding to the ground (Figure 6–28). **(a)** If the coefficient of static friction between the board and the wall is 0.79, what is the least force you can apply and still hold the board in place? **(b)** What happens to the force of static friction if you push against the wall with a force greater than that found in part (a)?



▲ **FIGURE 6–28** Problem 40

41. ••• **BIO The Russell Traction System** To immobilize a fractured femur (the thigh bone), doctors often utilize the Russell traction system illustrated in Figure 6–29. Notice that one force is applied directly to the knee, $\vec{F}_1$, while two other forces, $\vec{F}_2$ and $\vec{F}_3$, are applied to the foot. The latter two forces combine to give a force $\vec{F}_2 + \vec{F}_3$ that is transmitted through the lower leg to the knee. The result is that the knee experiences the total force $\vec{F}_{total} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3$. The goal of this traction system is to have $\vec{F}_{total}$ directly in line with the fractured femur, at an angle of 20.0° above the horizontal. Find **(a)** the angle θ required to produce this alignment of $\vec{F}_{total}$ and **(b)** the magnitude of the force, $\vec{F}_{total}$ that is applied to the femur in this case. (Assume the pulleys are ideal.)



▲ **FIGURE 6–29** Problem 41

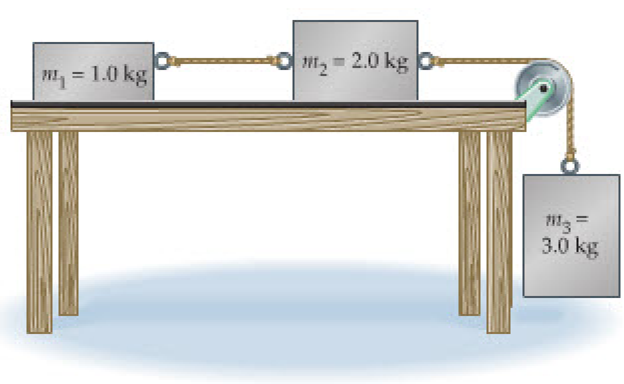
SECTION 6–4 CONNECTED OBJECTS

42. • **CE** In Example 6–6 (Connected Blocks), suppose m_1 and m_2 are both increased by a factor of 2. **(a)** Does the acceleration of the blocks increase, decrease, or stay the same? **(b)** Does the tension in the string increase, decrease, or stay the same?

43. • **CE Predict/Explain** Suppose m_1 and m_2 in Example 6–7 (Atwood's Machine) are both increased by 1 kg. Does the acceleration of the blocks increase, decrease, or stay the same? **(b)** Choose the *best explanation* from among the following:

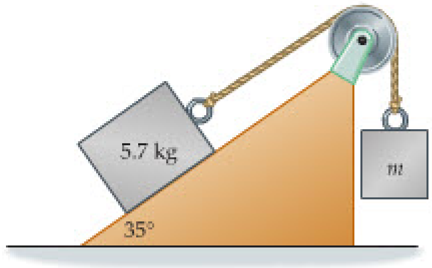
- **I.** The net force acting on the blocks is the same, but the total mass that must be accelerated is greater.
- **II.** The difference in the masses is the same, and this is what determines the net force on the system.
- **III.** The force exerted on each block is greater, leading to an increased acceleration.

44. • Find the acceleration of the masses shown in Figure 6–30, given that $m_1 = 1.0$ kg, $m_2 = 2.0$ kg, and $m_3 = 3.0$ kg. Assume the table is frictionless and the masses move freely.



▲ **FIGURE 6–30** Problems 44, 47, and 103

45. • Two blocks are connected by a string, as shown in Figure 6–31. The smooth inclined surface makes an angle of 35° with the horizontal, and the block on the incline has a mass of 5.7 kg. The mass of the hanging block is $m = 3.2$ kg. Find **(a)** the direction and **(b)** the magnitude of the hanging block's acceleration.

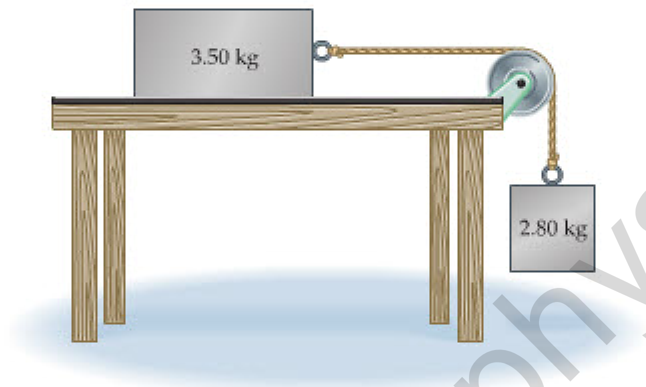


▲ **FIGURE 6–31** Problems 45 and 46

46. • Referring to Problem 45, find **(a)** the direction and **(b)** the magnitude of the hanging block's acceleration if its mass is $m = 4.2$ kg.

47. •• Referring to Figure 6–30, find the tension in the string connecting **(a)** m_1 and m_2 and **(b)** m_2 and m_3. Assume the table is frictionless and the masses move freely.

48. •• **IP** A 3.50-kg block on a smooth tabletop is attached by a string to a hanging block of mass 2.80 kg, as shown in **Figure 6–32**. The blocks are released from rest and allowed to move freely. **(a)** Is the tension in the string greater than, less than, or equal to the weight of the hanging mass? Find **(b)** the acceleration of the blocks and **(c)** the tension in the string.



▲ **FIGURE 6–32** Problem 48

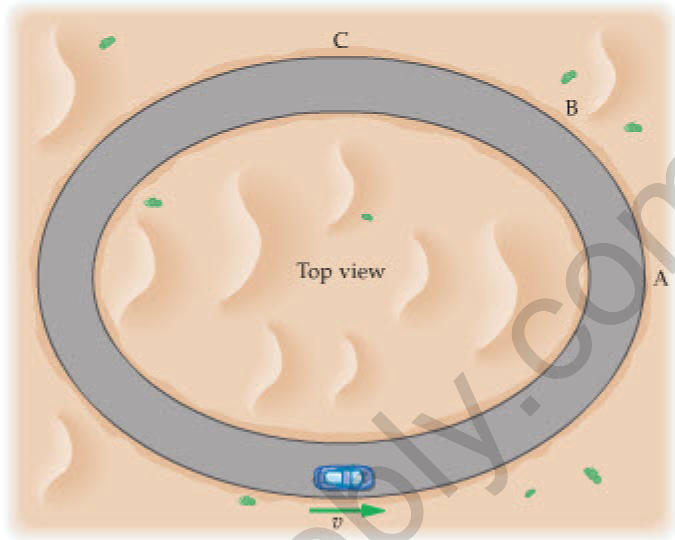
49. •• **IP** A 7.7-N force pulls horizontally on a 1.6-kg block that slides on a smooth horizontal surface. This block is connected by a horizontal string to a second block of mass $m_2 = 0.83$ kg on the same surface. **(a)** What is the acceleration of the blocks? **(b)** What is the tension in the string? **(c)** If the mass of block 1 is increased, does the tension in the string increase, decrease, or stay the same?

50. ••• **Buckets and a Pulley** Two buckets of sand hang from opposite ends of a rope that passes over an ideal pulley. One bucket is full and weighs 120 N; the other bucket is only partly filled and weighs 63 N. **(a)** Initially, you hold onto the lighter bucket to keep it from moving. What is the tension in the rope? **(b)** You release the lighter bucket and the heavier one descends. What is the tension in the rope now? **(c)** Eventually the heavier bucket lands and the two buckets come to rest. What is the tension in the rope now?

SECTION 6–5 CIRCULAR MOTION

51. • **CE** Suppose you stand on a bathroom scale and get a reading of 700 N. In principle, would the scale read more, less, or the same if the Earth did not rotate?

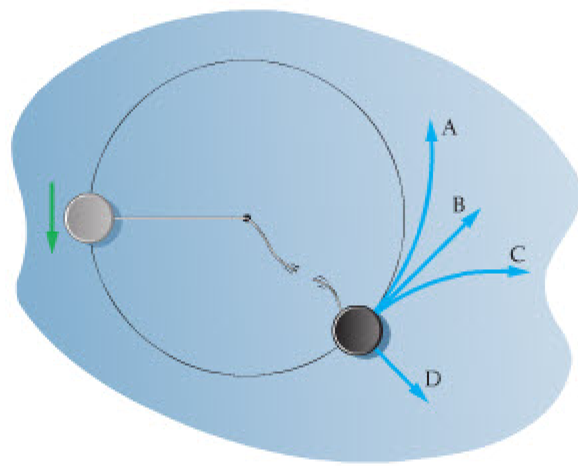
52. • **CE** A car drives with constant speed on an elliptical track, as shown in **Figure 6–33**. Rank the points A, B, and C in order of increasing likelihood that the car might skid. Indicate ties where appropriate.



▲ **FIGURE 6–33** Problem 52

53. • **CE** A car is driven with constant speed around a circular track. Answer each of the following questions with "Yes" or "No." **(a)** Is the car's velocity constant? **(b)** Is its speed constant? **(c)** Is the magnitude of its acceleration constant? **(d)** Is the direction of its acceleration constant?

54. • **CE** A puck attached to a string undergoes circular motion on an air table. If the string breaks at the point indicated in **Figure 6–34**, is the subsequent motion of the puck best described by path A, B, C, or D?



▲ **FIGURE 6–34** Problem 54

55. • When you take your 1300-kg car out for a spin, you go around a corner of radius 59 m with a speed of 16 m/s. The coefficient of static friction between the car and the road is 0.88. Assuming your car doesn't skid, what is the force exerted on it by static friction?

56. • Find the linear speed of the bottom of a test tube in a centrifuge if the centripetal acceleration there is 52,000 times the acceleration of gravity. The distance from the axis of rotation to the bottom of the test tube is 7.5 cm.

57. • **BIO A Human Centrifuge** To test the effects of high acceleration on the human body, the National Aeronautics and Space Administration (NASA) has constructed a large centrifuge at the Manned Spacecraft Center in Houston. In this device, astronauts are placed in a capsule that moves in a circular path with a radius of 15 m. If the astronauts in this centrifuge experience a centripetal acceleration 9.0 times that of gravity, what is the linear speed of the capsule?

58. • A car goes around a curve on a road that is banked at an angle of 33.5°. Even though the road is slick, the car will stay on the road without any friction between its tires and the road when its speed is 22.7 m/s. What is the radius of the curve?

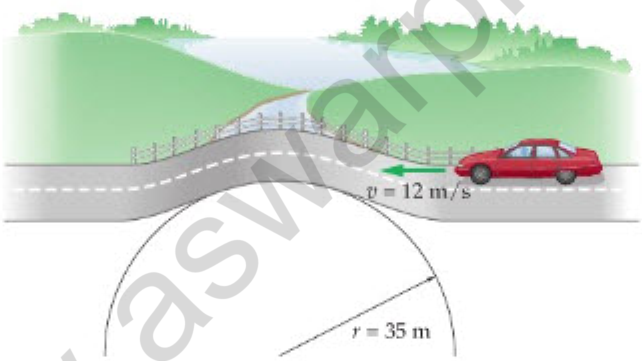
59. •• Jill of the Jungle swings on a vine 6.9 m long. What is the tension in the vine if Jill, whose mass is 63 kg, is moving at 2.4 m/s when the vine is vertical?

60. •• **IP** In Problem 59, **(a)** how does the tension in the vine change if Jill's speed is doubled? Explain. **(b)** How does the tension change if her mass is doubled instead? Explain.

61. •• **IP** **(a)** As you ride on a Ferris wheel, your apparent weight is different at the top than at the bottom. Explain. **(b)** Calculate your apparent weight at the top and bottom of a Ferris wheel, given that the radius of the wheel is 7.2 m, it completes one revolution every 28 s, and your mass is 55 kg.

A Ferris Wheel (Problems 61 and 84)

62. •• Driving in your car with a constant speed of 12 m/s, you encounter a bump in the road that has a circular cross section, as indicated in Figure 6–35. If the radius of curvature of the bump is 35 m, find the apparent weight of a 67-kg person in your car as you pass over the top of the bump.



▲ **FIGURE 6–35** Problems 62 and 63

63. •• Referring to Problem 62, at what speed must you go over the bump if people in your car are to feel "weightless"?

64. •• **IP** You swing a 4.6-kg bucket of water in a vertical circle of radius 1.3 m. **(a)** What speed must the bucket have if it is to complete the circle without spilling any water? **(b)** How does your answer depend on the mass of the bucket?

GENERAL PROBLEMS

65. • **CE** If you weigh yourself on a bathroom scale at the equator, is the reading you get greater than, less than, or equal to the reading you get if you weigh yourself at the North Pole?

66. • **CE** An object moves on a flat surface with an acceleration of constant magnitude. If the acceleration is always perpendicular to the object's direction of motion, **(a)** is the shape of the object's path circular, linear, or parabolic? **(b)** During its motion, does the object's velocity change in direction but not magnitude, change in magnitude but not direction, or change in both magnitude and direction? **(c)** Does its speed increase, decrease, or stay the same?

67. • **CE BIO** **Maneuvering a Jet** Humans lose consciousness if exposed to prolonged accelerations of more than about $7g$. This is of concern to jet fighter pilots, who may experience centripetal accelerations of this magnitude when making high-speed turns. Suppose we would like to decrease the centripetal acceleration of a jet. Rank the following changes in flight path in order of how effective they are in decreasing the centripetal acceleration, starting with the least effective: **A**, decrease the turning radius by a factor of two; **B**, decrease the speed by a factor of three; or **C**, increase the turning radius by a factor of four.

68. • **CE BIO** **Gravitropism** As plants grow, they tend to align their stems and roots along the direction of the gravitational field. This tendency, which is related to differential concentrations of plant hormones known as auxins, is referred to as *gravitropism*. As an illustration of gravitropism, experiments show that seedlings placed in pots on the rim of a rotating turntable do not grow in the vertical direction. Do you expect their stems to tilt inward—toward the axis of rotation—or outward—away from the axis of rotation?

69. • **BIO** A skateboard accident leaves your leg in a cast and supported by a traction device, as in Figure 6–24. Find the mass m that must be attached to the rope if the net force exerted by the small pulley on the foot is to have a magnitude of 37 N.

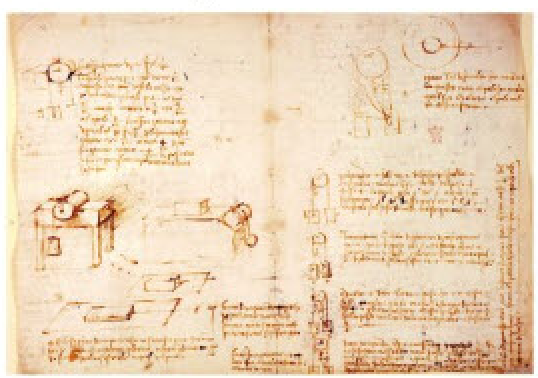
70. • Find the centripetal acceleration at the top of a test tube in a centrifuge, given that the top is 4.2 cm from the axis of rotation and that its linear speed is 77 m/s.

71. • Find the coefficient of kinetic friction between a 3.85-kg block and the horizontal surface on which it rests if an 850-N/m spring must be stretched by 6.20 cm to pull it with constant speed. Assume that the spring pulls in the horizontal direction.

72. • A child goes down a playground slide that is inclined at an angle of 26.5° below the horizontal. Find the acceleration of the child given that the coefficient of kinetic friction between the child and the slide is 0.315.

73. • When a block is placed on top of a vertical spring, the spring compresses 3.15 cm. Find the mass of the block, given that the force constant of the spring is 1750 N/m.

74. •• **The da Vinci Code** Leonardo da Vinci (1452–1519) is credited with being the first to perform quantitative experiments on friction, though his results weren't known until centuries later, due in part to the secret code (mirror writing) he used in his notebooks. Leonardo would place a block of wood on an inclined



Sketches from the notebooks of Leonardo da Vinci showing experiments he performed on friction (Problem 74) plane and measure the angle at which the block begins to slide. He reports that the coefficient of static friction was 0.25 in his experiments. At what angle did Leonardo's blocks begin to slide?

75. •• A force of 9.4 N pulls horizontally on a 1.1-kg block that slides on a rough, horizontal surface. This block is connected by a horizontal string to a second block of mass $m_2 = 1.92$ kg on the same surface. The coefficient of kinetic friction is $\mu_k = 0.24$ for both blocks. **(a)** What is the acceleration of the blocks? **(b)** What is the tension in the string?

76. •• You swing a 3.25-kg bucket of water in a vertical circle of radius 0.950 m. At the top of the circle the speed of the bucket is 3.23 m/s; at the bottom of the circle its speed is 6.91 m/s. Find the tension in the rope tied to the bucket at **(a)** the top and **(b)** the bottom of the circle.

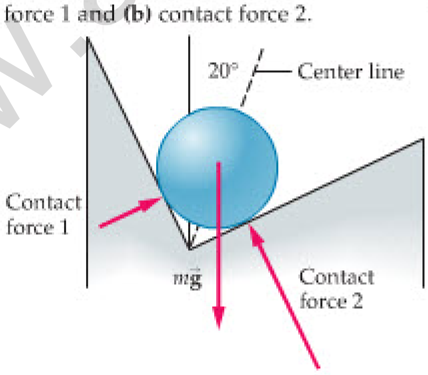
77. •• A 14-g coin slides upward on a surface that is inclined at an angle of 18° above the horizontal. The coefficient of kinetic friction between the coin and the surface is 0.23; the coefficient of static friction is 0.35. Find the magnitude and direction of the force of friction **(a)** when the coin is sliding and **(b)** after it comes to rest.

78. •• In Problem 77, the angle of the incline is increased to 25°. Find the magnitude and direction of the force of friction when the coin is **(a)** sliding upward initially and **(b)** sliding back downward later.

79. •• A physics textbook weighing 22 N rests on a table. The coefficient of static friction between the book and the table is $\mu_s = 0.60$; the coefficient of kinetic friction is $\mu_k = 0.40$. You push horizontally on the book with a force that gradually increases from 0 to 15 N, and then slowly decreases to 5.0 N, as indicated in the following table. For each value of the applied force given in the table, give the magnitude of the force of friction and state whether the book is accelerating, decelerating, at rest, or moving with constant speed.

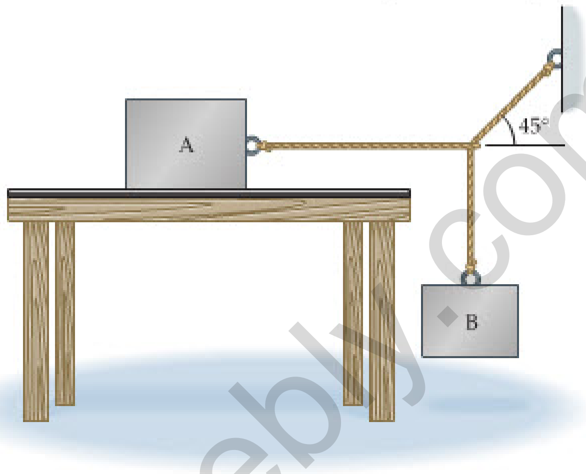
Applied force	Friction force	Motion
0		
5.0 N		
11 N		
15 N		
11 N		
8.0 N		
5.0 N		

80. •• A ball of mass m is placed in a wedge, as shown in Figure 6–36, in which the two walls meet at a right angle. Assuming the walls of the wedge are frictionless, determine the magnitude of **(a)** contact force 1 and **(b)** contact force 2.



▲ **FIGURE 6–36** Problem 80

81. •• **IP** The blocks shown in Figure 6–37 are at rest. **(a)** Find the frictional force exerted on block A given that the mass of block A is 8.82 kg, the mass of block B is 2.33 kg, and the coefficient of static friction between block A and the surface on which it rests is 0.320. **(b)** If the mass of block A is doubled, does the frictional force exerted on it increase, decrease, or stay the same? Explain.



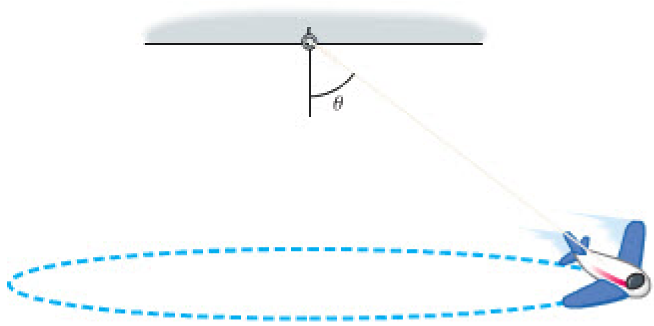
▲ **FIGURE 6–37** Problems 81 and 82

82. •• In part (a) of Problem 81, what is the maximum mass block B can have and the system still be in equilibrium?

83. •• **IP** A picture hangs on the wall suspended by two strings, as shown in Figure 6–21. The tension in string 2 is 1.7 N. **(a)** Is the tension in string 1 greater than, less than, or equal to 1.7 N? Explain. **(b)** Verify your answer to part (a) by calculating the tension in string 1. **(c)** What is the mass of the picture?

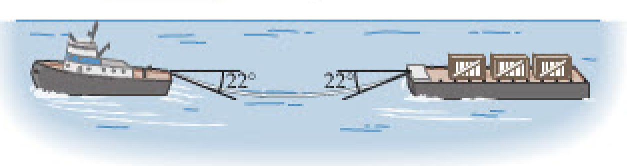
84. •• **IP** Referring to Problem 61, suppose the Ferris wheel rotates fast enough to make you feel "weightless" at the top. **(a)** How many seconds does it take to complete one revolution in this case? **(b)** How does your answer to part (a) depend on your mass? Explain. **(c)** What are the direction and magnitude of your acceleration when you are at the bottom of the wheel? Assume that its rotational speed has remained constant.

85. •• **A Conical Pendulum** A 0.075-kg toy airplane is tied to the ceiling with a string. When the airplane's motor is started, it moves with a constant speed of 1.21 m/s in a horizontal circle of radius 0.44 m, as illustrated in Figure 6–38. Find **(a)** the angle the string makes with the vertical and **(b)** the tension in the string.



▲ **FIGURE 6–38** Problem 85

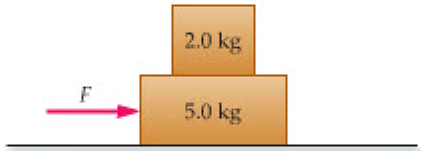
86. •• A tugboat tows a barge at constant speed with a 3500-kg cable, as shown in Figure 6–39. If the angle the cable makes with the hor-



▲ **FIGURE 6–39** Problem 86

izontal where it attaches to the barge and the tugboat is 22°, find the force the cable exerts on the barge in the forward direction.

87. •• **IP** Two blocks, stacked one on top of the other, can move without friction on the horizontal surface shown in Figure 6–40. The surface between the two blocks is rough, however, with a coefficient of static friction equal to 0.47. **(a)** If a horizontal force F is applied to the 5.0-kg bottom block, what is the maximum value F can have before the 2.0-kg top block begins to slip? **(b)** If the mass of the top block is increased, does the maximum value of F increase, decrease, or stay the same? Explain.



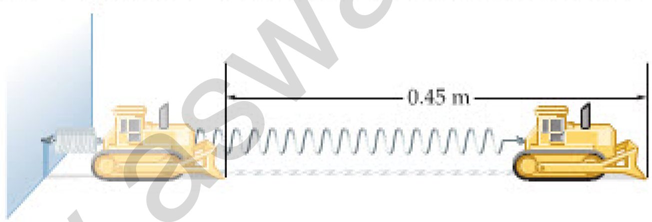
▲ **FIGURE 6–40** Problem 87

88. •• Find the coefficient of kinetic friction between a 4.7-kg block and the horizontal surface on which it rests if an 89-N/m spring must be stretched by 2.2 cm to pull the block with constant speed. Assume the spring pulls in a direction 13° above the horizontal.

89. •• **IP** In a daring rescue by helicopter, two men with a combined mass of 172 kg are lifted to safety. **(a)** If the helicopter lifts the men straight up with constant acceleration, is the tension in the rescue cable greater than, less than, or equal to the combined weight of the men? Explain. **(b)** Determine the tension in the cable if the men are lifted with a constant acceleration of 1.10 m/s^2.

90. •• At the airport, you pull a 18-kg suitcase across the floor with a strap that is at an angle of 45° above the horizontal. Find **(a)** the normal force and **(b)** the tension in the strap, given that the suitcase moves with constant speed and that the coefficient of kinetic friction between the suitcase and the floor is 0.38.

91. •• **IP** A light spring with a force constant of 13 N/m is connected to a wall and to a 1.2-kg toy bulldozer, as shown in Figure 6–41. When the electric motor in the bulldozer is turned on, it stretches the spring for a distance of 0.45 m before its tread begins to slip on the floor. **(a)** Which coefficient of friction (static or kinetic) can be determined from this information? Explain. **(b)** What is the numerical value of this coefficient of friction?

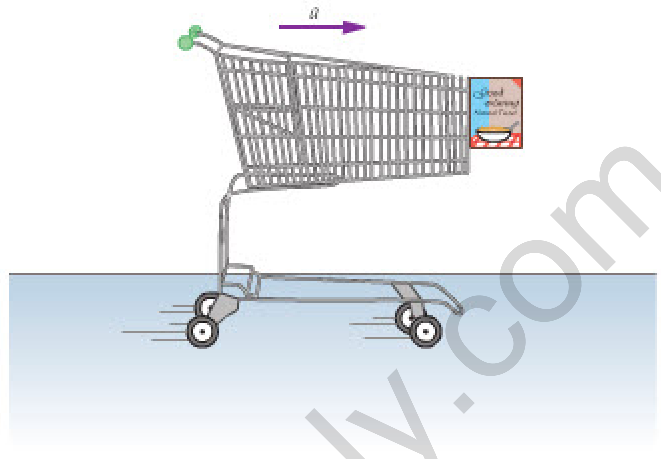


▲ **FIGURE 6–41** Problem 91

92. •• **IP** A 0.16-g spider hangs from the middle of the first thread of its future web. The thread makes an angle of 7.2° with the horizontal on both sides of the spider. **(a)** What is the tension in the thread? **(b)** If the angle made by the thread had been less than 7.2°, would its tension have been greater than, less than, or the same as in part (a)? Explain.

93. •• Find the acceleration the cart in Figure 6–42 must have in order for the cereal box at the front of the cart not to fall. Assume that the coefficient of static friction between the cart and the box is 0.38.

94. •• **IP Playing a Violin** The tension in a violin string is 2.7 N. When pushed down against the neck of the violin, the string makes an angle of 4.1° with the horizontal. **(a)** With what force must you push down on the string to bring it into contact with the neck? **(b)** If the angle were less than 4.1°, would the required force be greater than, less than, or the same as in part (a)? Explain.



▲ **FIGURE 6–42** Problem 93

95. •• **IP** A pair of fuzzy dice hangs from a string attached to your rearview mirror. As you turn a corner with a radius of 98 m and a constant speed of 27 mi/h, what angle will the dice make with the vertical? Why is it unnecessary to give the mass of the dice?

96. •• Find the tension in each of the two ropes supporting a hammock if one is at an angle of 18° above the horizontal and the other is at an angle of 35° above the horizontal. The person sleeping in the hammock (unconcerned about tensions and ropes) has a mass of 68 kg.

97. •• As your plane circles an airport, it moves in a horizontal circle of radius 2300 m with a speed of 390 km/h. If the lift of the airplane's wings is perpendicular to the wings, at what angle should the plane be banked so that it doesn't tend to slip sideways?

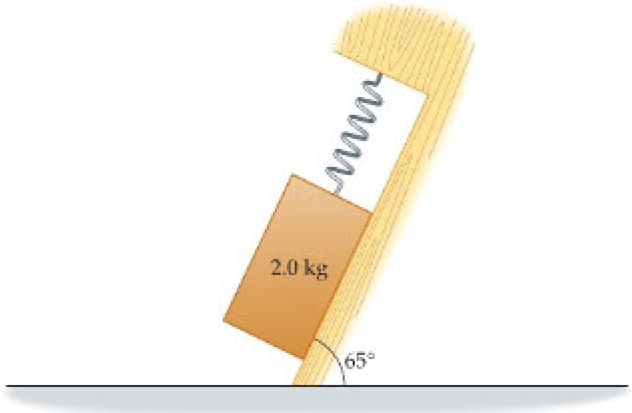
98. •• **IP** A block with a mass of 3.1 kg is placed at rest on a surface inclined at an angle of 45° above the horizontal. The coefficient of static friction between the block and the surface is 0.50, and a force of magnitude F pushes upward on the block, parallel to the inclined surface. **(a)** The block will remain at rest only if F is greater than a minimum value, F_{min}, and less than a maximum value, F_{max}. Explain the reasons for this behavior. **(b)** Calculate F_{min}. **(c)** Calculate F_{max}.

99. •• A mountain climber of mass m hangs onto a rope to keep from sliding down a smooth, ice-covered slope (Figure 6–43). Find a formula for the tension in the rope when the slope is inclined at an angle θ above the horizontal. Check your results in the limits $\theta = 0$ and $\theta = 90°$.

▲ **FIGURE 6–43** Problem 99

100. •• A child sits on a rotating merry-go-round, 2.3 m from its center. If the speed of the child is 2.2 m/s, what is the minimum coefficient of static friction between the child and the merry-go-round that will prevent the child from slipping?

101. ••• A 2.0-kg box rests on a plank that is inclined at an angle of 65° above the horizontal. The upper end of the box is attached to a spring with a force constant of 360 N/m, as shown in **Figure 6–44**. If the coefficient of static friction between the box and the plank is 0.22, what is the maximum amount the spring can be stretched and the box remain at rest?



▲ **FIGURE 6–44** Problem 101

102. ••• A wood block of mass m rests on a larger wood block of mass M that rests on a wooden table. The coefficients of static and kinetic friction between all surfaces are μ_s and μ_k, respectively. What is the minimum horizontal force, F, applied to the lower block that will cause it to slide out from under the upper block?

103. ••• Find the tension in each of the two strings shown in Figure 6–30 for general values of the masses. Your answer should be in terms of m_1, m_2, m_3, and g.

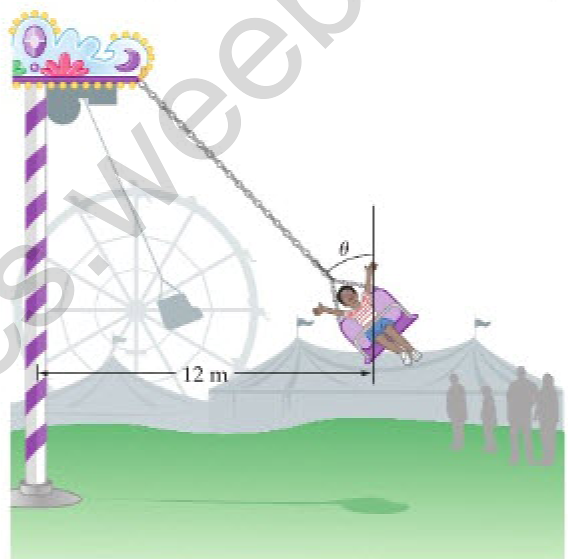
104. ••• The coefficient of static friction between a rope and the table on which it rests is μ_s. Find the fraction of the rope that can hang over the edge of the table before it begins to slip.

105. ••• A hockey puck of mass m is attached to a string that passes through a hole in the center of a table, as shown in **Figure 6–45**. The hockey puck moves in a circle of radius r. Tied to the other end of the string, and hanging vertically beneath the table, is a mass M. Assuming the tabletop is perfectly smooth, what speed must the hockey puck have if the mass M is to remain at rest?

▲ **FIGURE 6–45** Problem 105

106. ••• **The Force Needed to Move a Crate** To move a crate of mass m across a rough floor, you push down on it at an angle θ, as shown in Figure 6–18 for the special case of $\theta = 21°$. **(a)** Find the force necessary to start the crate moving as a function of θ, given that the coefficient of static friction between the crate and the floor is μ_s. **(b)** Show that it is impossible to move the crate, no matter how great the force, if the coefficient of static friction is greater than or equal to $1/\tan\theta$.

107. ••• **IP** A popular ride at amusement parks is illustrated in **Figure 6–46**. In this ride, people sit in a swing that is suspended from a rotating arm. Riders are at a distance of 12 m from the axis of rotation and move with a speed of 25 mi/h. **(a)** Find the centripetal acceleration of the riders. **(b)** Find the angle θ the supporting wires make with the vertical. **(c)** If you observe a ride like that in Figure 6–46, or as shown in the photo on page 170, you will notice that all the swings are at the same angle θ to the vertical, regardless of the weight of the rider. Explain.

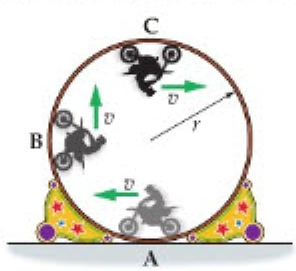


▲ **FIGURE 6–46** Problem 107

108. ••• **A Conveyor Belt** A box is placed on a conveyor belt that moves with a constant speed of 1.25 m/s. The coefficient of kinetic friction between the box and the belt is 0.780. **(a)** How much time does it take for the box to stop sliding relative to the belt? **(b)** How far does the box move in this time?

109. ••• You push a box along the floor against a constant force of friction. When you push with a horizontal force of 75 N, the acceleration of the box is 0.50 m/s^2; when you increase the force to 81 N, the acceleration is 0.75 m/s^2. Find **(a)** the mass of the box and **(b)** the coefficient of kinetic friction between the box and the floor.

110. ••• As part of a circus act, a person drives a motorcycle with constant speed v around the inside of a vertical track of radius r, as indicated in **Figure 6–47**. If the combined mass of the motorcycle and rider is m, find the normal force exerted on the motorcycle by the track at the points **(a)** A, **(b)** B, and **(c)** C.



▲ **FIGURE 6–47** Problem 110

PASSAGE PROBLEMS

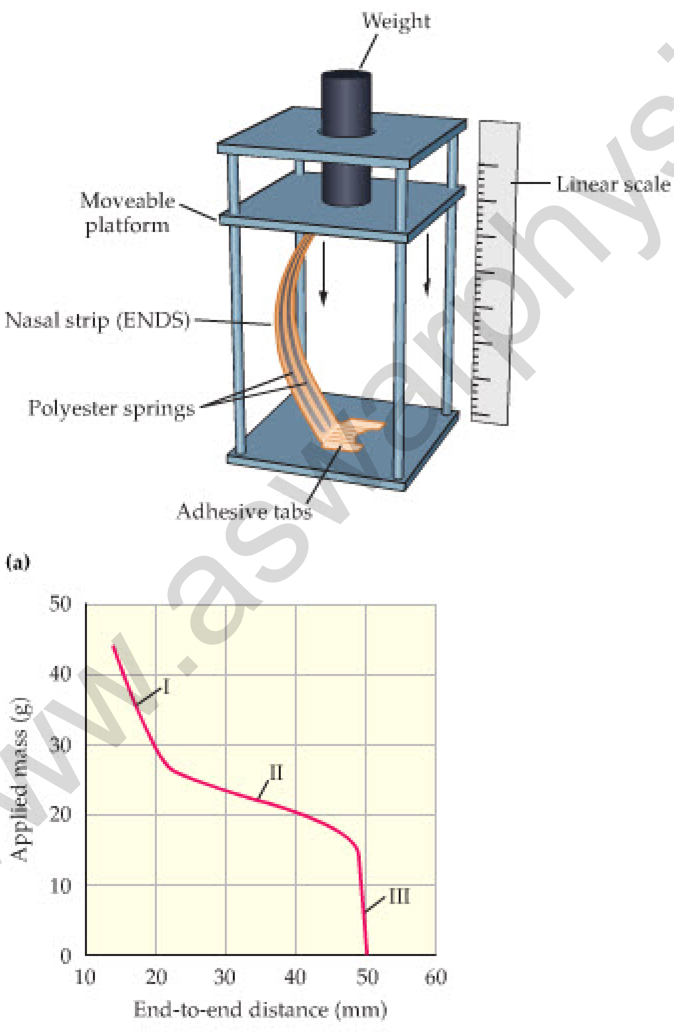
BIO Nasal Strips

People in all walks of life use nasal strips, or external nasal dilator strips (ENDS), to alleviate a number of respiratory problems. First introduced to eliminate snoring, they are now finding use in a number of other areas. For example, dentists have found that nasal strips help patients breathe better during dental procedures, making the experience considerably more pleasant for both doctor and patient. Surprisingly, horse owners have also discovered the advantage of nasal strips, and have begun to apply large "horse-sized" strips to saddle horses—as well as racing thoroughbreds—to reduce fatigue and lung stress.

One of the great advantages of ENDS is that no drugs are involved; the strips are a purely mechanical device, consisting of two flat, polyester springs enclosed by an adhesive tape covering. When applied to the nose, they exert an outward force that enlarges the nasal passages and reduces the resistance to air flow (see the illustration in Active Example 6–2). The mechanism shown in **Figure 6–48 (a)** is used to measure the behavior of these strips. For example, if a 30-g weight is placed on the moveable platform (of negligible mass), the strip is found to compress from an initial length of 50 mm to a reduced length of 19 mm, as can be seen in **Figure 6–48 (b)**.

111. • On the straight-line segment I in Figure 6–48 (b) we see that increasing the applied mass from 26 g to 44 g results in a reduction of the end-to-end distance from 21 mm to 14 mm. What is the force constant in N/m on segment I?

A. 2.6 N/m **B.** 3.8 N/m
C. 9.8 N/m **D.** 25 N/m



▲ **FIGURE 6–48** Problems 111, 112, 113, and 114

A thoroughbred racehorse with a nasal strip. Did it win by a nose?

112. • Is the force constant on segment II greater than, less than, or equal to the force constant on segment I?

113. • Which of the following is the best estimate for the force constant on segment II?

A. 0.83 N/m **B.** 1.3 N/m
C. 2.5 N/m **D.** 25 N/m

114. • Rank the straight segments I, II, and III in order of increasing "stiffness" of the nasal strip.

INTERACTIVE PROBLEMS

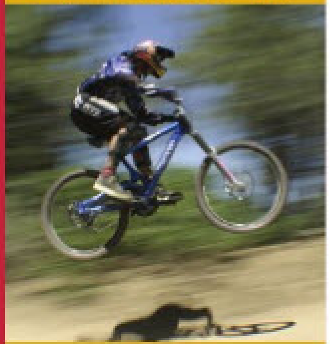
115. •• **IP Referring to Example 6–3** Suppose the coefficients of static and kinetic friction between the crate and the truck bed are 0.415 and 0.382, respectively. **(a)** Does the crate begin to slide at a tilt angle that is greater than, less than, or equal to 23.2°? **(b)** Verify your answer to part (a) by determining the angle at which the crate begins to slide. **(c)** Find the length of time it takes for the crate to slide a distance of 2.75 m when the tilt angle has the value found in part (b).

116. •• **IP Referring to Example 6–3** The crate begins to slide when the tilt angle is 17.5°. When the crate reaches the bottom of the flatbed, after sliding a distance of 2.75 m, its speed is 3.11 m/s. Find **(a)** the coefficient of static friction and **(b)** the coefficient of kinetic friction between the crate and the flatbed.

117. •• **Referring to Example 6–6** Suppose that the mass on the frictionless tabletop has the value $m_1 = 2.45$ kg. **(a)** Find the value of m_2 that gives an acceleration of 2.85 m/s^2. **(b)** What is the corresponding tension, T, in the string? **(c)** Calculate the ratio T/m_2g and show that it is less than 1, as expected.

118. •• **Referring to Example 6–8** **(a)** At what speed will the force of static friction exerted on the car by the road be equal to half the weight of the car? The mass of the car is $m = 1200$ kg, the radius of the corner is $r = 45$ m, and the coefficient of static friction between the tires and the road is $\mu_s = 0.82$. **(b)** Suppose that the mass of the car is now doubled, and that it moves with a speed that again makes the force of static friction equal to half the car's weight. Is this new speed greater than, less than, or equal to the speed in part (a)?

PHYSICS IN



PERSPECTIVE

Force, Acceleration, and Motion

Motion does not require a force—but a *change* in motion does. On these pages we explore the connections between forces, as described in Newton's laws, and the types of motion we've studied in the first six chapters.

1 Objects that experience zero net force obey Newton's first law

If the net force $\vec{F}_{\text{net}} = \Sigma\vec{F}$ acting on an object is zero, the object's motion doesn't change—the object either remains at rest or continues to move with constant velocity, as Newton's first law states.

At rest

Motion at constant velocity

This behavior is consistent with Newton's second law for a net force of zero:

If the net force acting on an object is zero ... $\Sigma\vec{F} = 0$, then $\vec{a} = \frac{\Sigma\vec{F}}{m} = 0$ ... the object has zero acceleration.

2 All objects experience forces—the question is whether the object experiences a *net* force

All objects—moving or at rest—are acted on by forces. Even in outer space, objects experience gravitational and other forces. Therefore, the *net* force on the object is the quantity that matters.

Newton's first law seems at odds with our experience: If we stop exerting a force on a moving object, the object usually stops. But that is because we must counter friction and drag forces. In the photo, the net force *on the couch* is zero even though the person exerts a steady push.

3 Moving at constant velocity is equivalent to being at rest

When you sit in a jet flying in a straight line, you feel the same as when you are sitting at home, and objects around you behave the same.

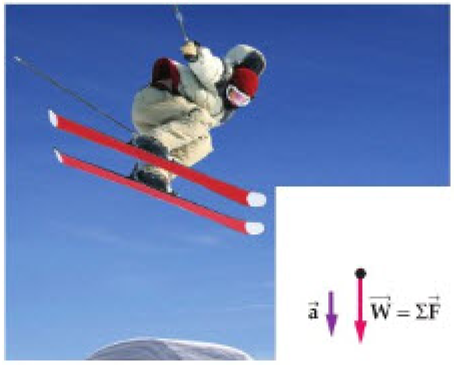
From the point of view of physics, *there is no difference* between these situations; Newton's laws hold in both. We say that both represent *inertial frames of reference*.

4 Objects that experience a nonzero net force obey Newton's second law

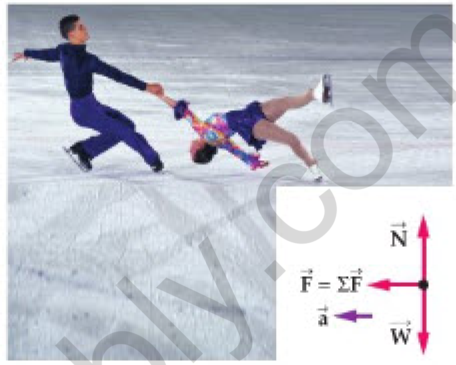
A nonzero net force accelerates an object—that is, causes its velocity to change in magnitude, direction, or both. We have studied the following three special types of accelerated motion:

Linear accelerated motion

Projectile motion



Circular motion



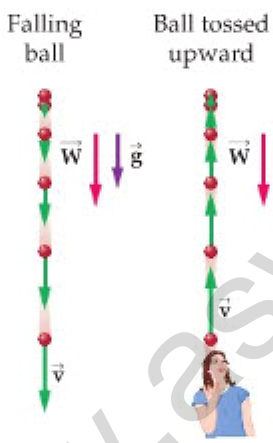
Accelerated motion obeys Newton's second law:

If a nonzero net force acts on an object ... $\Sigma\vec{F} \neq 0$, then $\vec{a} = \dfrac{\Sigma\vec{F}}{m} \neq 0$ ... the object has an acceleration in the direction of the net force that is proportional to $\Sigma\vec{F}$ and inversely proportional to m.

5 The acceleration points in the direction of the net force

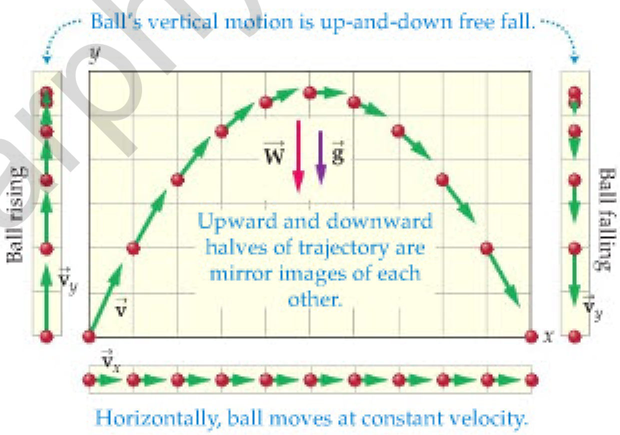
Linear accelerated motion
- Net force is parallel to motion.
- Velocity changes in magnitude but not in direction.

Special case: free fall
Constant downward acceleration $\vec{g}$

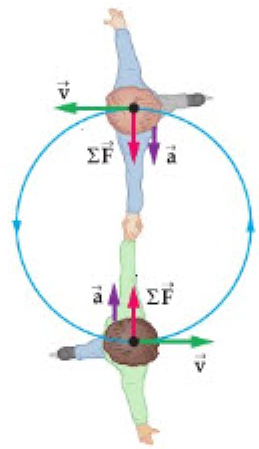


Parabolic motion
- Constant net force acts at angle to motion.
- Velocity changes in both magnitude and direction.

Special case: projectile motion
Constant downward acceleration $\vec{g}$

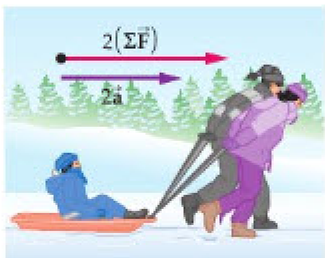


Circular motion (constant speed)
- Net force is constant in magnitude but always points toward the center of the circle. Thus, the net force is always at a right angle to the object's velocity.
- Velocity changes in direction but not in magnitude.

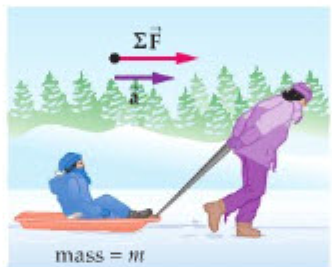


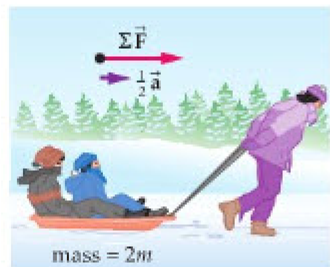
6 The acceleration magnitude is proportional to *F* and inversely proportional to *m*

Doubling the net force acting on an object doubles the object's acceleration ($\vec{a} \propto \vec{F}$).

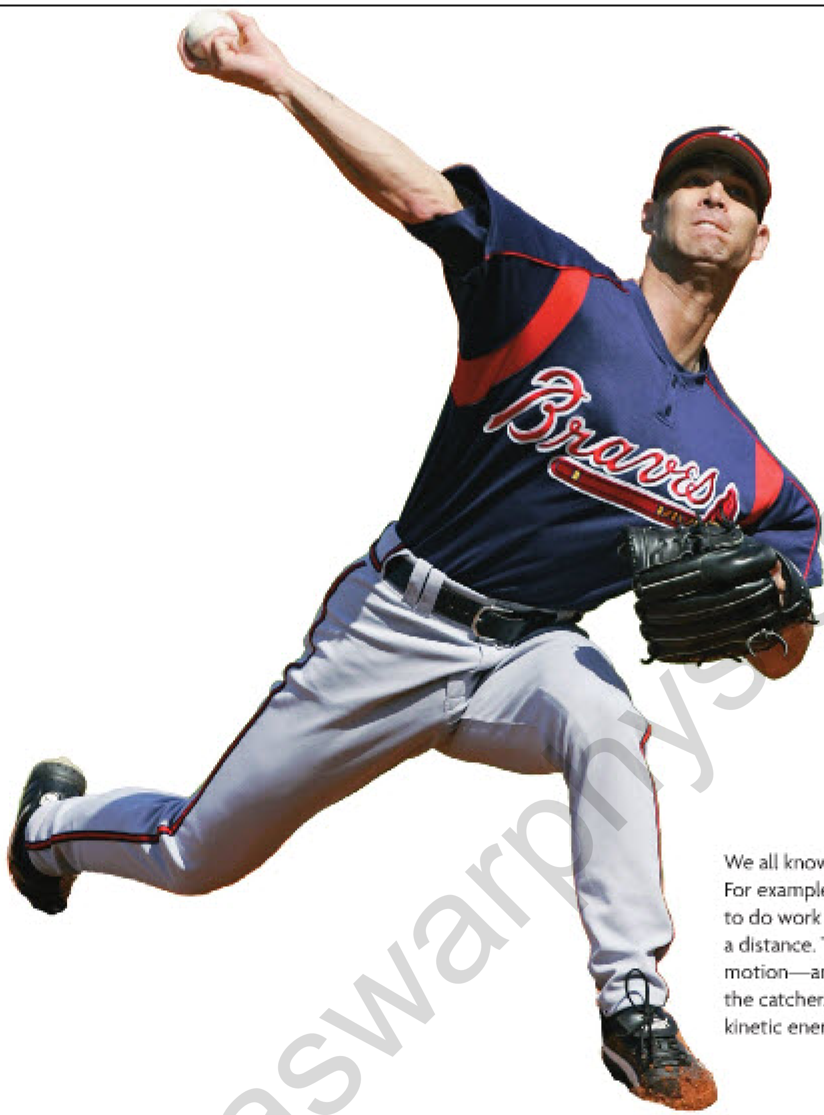


Doubling the object's mass m *halves* its acceleration ($\vec{a} \propto 1/m$).





7 Work and Kinetic Energy



We all know intuitively that motion, energy, and work are somehow related. For example, the chemical energy stored in this pitcher's muscles enables him to do work on a baseball. This means, basically, that he exerts a force on it over a distance. The work done on the ball appears as kinetic energy—the energy of motion—and when the ball is caught, its kinetic energy can in turn do work on the catcher. In this chapter we'll give precise definitions of the concepts of work, kinetic energy, and power, and explore the physical relationships among them.

The concept of force is one of the foundations of physics, as we have seen in the previous two chapters. Equally fundamental, though less obvious, is the idea that a force times the displacement through which it acts is also an important physical quantity. We refer to this quantity as the *work* done by a force.

Now, we all know what work means in everyday life: We get up in the morning and go to work, or we "work up a sweat" as we hike a mountain trail. Later in the day we eat lunch, which gives us the "energy" to continue working or to continue our hike. In this chapter we give a precise physical definition of work, and show how it is related to another important physical quantity—the energy of motion, or *kinetic energy*. When these concepts are extended in the next chapter, we are led to the rather sweeping observation that the total amount of energy in the universe remains constant at all times.

7–1 Work Done by a Constant Force

In this section we define work—in the physics sense of the word—and apply our definition to a variety of physical situations. We start with the simplest case; namely, the work done when force and displacement are in the same direction. Later in the section we generalize our definition to include cases where the force and displacement are in arbitrary directions. We conclude with a discussion of the work done on an object when it is acted on by more than one force.

Force in the Direction of Displacement

When we push a shopping cart in a store or pull a suitcase through an airport, we do work. The greater the force, the greater the work; the greater the distance, the greater the work. These simple ideas form the basis for our definition of work.

To be specific, suppose we push a box with a constant force $\vec{\mathbf{F}}$, as shown in Figure 7–1. If we move the box *in the direction of* $\vec{\mathbf{F}}$ through a displacement $\vec{\mathbf{d}}$, the **work** W we have done is Fd:

Definition of Work, *W*, When a Constant Force Is in the Direction of Displacement

$$W = Fd \quad 7\text{–}1$$

SI unit: newton-meter $(\mathrm{N \cdot m})$ = joule, J

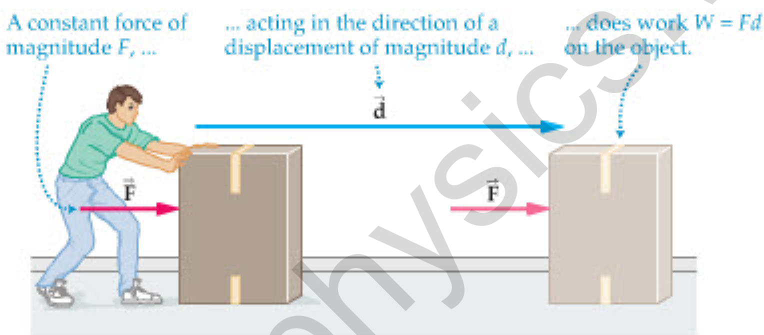


FIGURE 7–1 Work: constant force in the direction of motion

A constant force $\vec{\mathbf{F}}$ pushes a box through a displacement $\vec{\mathbf{d}}$. In this special case, where the force and displacement are in the *same* direction, the work done on the box by the force is $W = Fd$.

Note that work is the product of two magnitudes, and hence it is a scalar. In addition, notice that a small force acting over a large distance gives the same work as a large force acting over a small distance. For example, $W = (1\ \mathrm{N})(400\ \mathrm{m}) = (400\ \mathrm{N})(1\ \mathrm{m})$.

The dimensions of work are newtons (force) times meters (distance), or $\mathrm{N \cdot m}$. This combination of dimensions is called the **joule** (rhymes with "school," as commonly pronounced) in honor of James Prescott Joule (1818–1889), a dedicated physicist who is said to have conducted physics experiments even while on his honeymoon. We define a joule as follows:

Definition of the Joule, J

$$1\ \text{joule} = 1\ \mathrm{J} = 1\ \mathrm{N \cdot m} = 1(\mathrm{kg \cdot m/s^2}) \cdot \mathrm{m} = 1\ \mathrm{kg \cdot m^2/s^2} \quad 7\text{–}2$$

To get a better feeling for work and the associated units, suppose you exert a force of 82.0 N on the box in Figure 7–1 and move it in the direction of the force through a distance of 3.00 m. The work you have done is

$$W = Fd = (82.0\ \mathrm{N})(3.00\ \mathrm{m}) = 246\ \mathrm{N \cdot m} = 246\ \mathrm{J}$$

Similarly, if you do 5.00 J of work to lift a book through a vertical distance of 0.750 m, the force you exerted on the book is

$$F = \frac{W}{d} = \frac{5.00\ \mathrm{J}}{0.750\ \mathrm{m}} = \frac{5.00\ \mathrm{N \cdot m}}{0.750\ \mathrm{m}} = 6.67\ \mathrm{N}$$

TABLE 7–1 Typical Values of Work

Activity	Equivalent work (J)
Annual U.S. energy use	8×10^{19}
Mt. St. Helens eruption	10^{18}
Burning one gallon of gas	10^{8}
Human food intake/day	10^{7}
Melting an ice cube	10^{4}
Lighting a 100-W bulb for 1 minute	6000
Heartbeat	0.5
Turning page of a book	10^{-3}
Hop of a flea	10^{-7}
Breaking a bond in DNA	10^{-20}

EXERCISE 7–1

One species of Darwin's finch, *Geospiza magnirostris*, can exert a force of 205 N with its beak as it cracks open a *Tribulus* seed case. If its beak moves through a distance of 0.40 cm during this operation, how much work does the finch do to get the seed?

SOLUTION

$W = Fd = (205\text{ N})(0.0040\text{ m}) = 0.82\text{ J}$

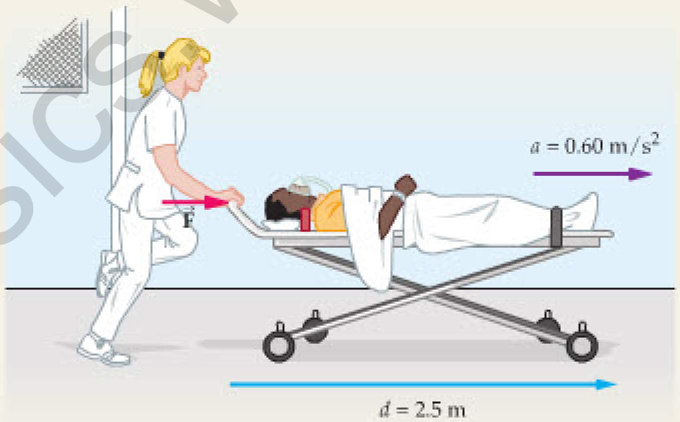
Just how much work is a joule, anyway? Well, you do one joule of work when you lift a gallon of milk through a height of about an inch, or lift an apple a meter. One joule of work lights a 100-watt lightbulb for 0.01 seconds or heats a glass of water 0.00125 degrees Celsius. Clearly, a joule is a modest amount of work in everyday terms. Additional examples of work are listed in Table 7–1.

EXAMPLE 7–1 HEADING FOR THE ER

An intern pushes a 72-kg patient on a 15-kg gurney, producing an acceleration of 0.60 m/s^2. **(a)** How much work does the intern do by pushing the patient and gurney through a distance of 2.5 m? Assume the gurney moves without friction. **(b)** How far must the intern push the gurney to do 140 J of work?

PICTURE THE PROBLEM

Our sketch shows the physical situation for this problem. Note that the force exerted by the intern is in the same direction as the displacement of the gurney; therefore, we know that $W = Fd$.



STRATEGY

We are not given the magnitude of the force F, so we cannot apply Equation 7–1 directly. However, we are given the mass and acceleration of the patient and gurney, from which we can calculate the force with $F = ma$. The work done by the intern is then $W = Fd$.

SOLUTION

Part (a)

1. First, find the force F exerted by the intern: $F = ma = (72\text{ kg} + 15\text{ kg})(0.60\text{ m/s}^2) = 52\text{ N}$

2. The work done by the intern, W, is the force times the distance: $W = Fd = (52\text{ N})(2.5\text{ m}) = 130\text{ J}$

Part (b)

3. Use $W = Fd$ to solve for the distance d: $W = Fd$ therefore $d = \frac{W}{F} = \frac{140\text{ J}}{52\text{ N}} = 2.7\text{ m}$

INSIGHT

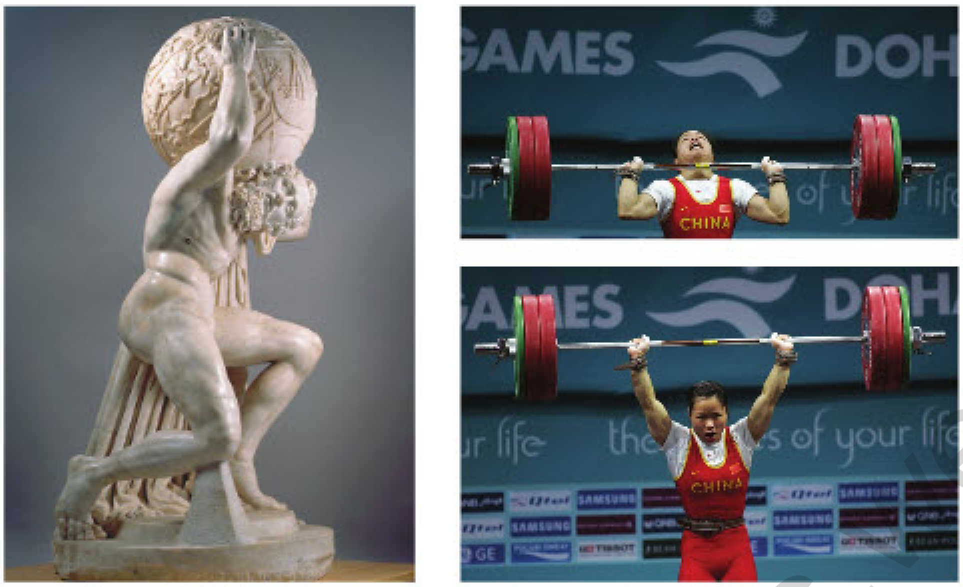
You might wonder whether the work done by the intern depends on the speed of the gurney. The answer is no. The work done on an object, $W = Fd$, doesn't depend on whether the object moves through the distance d quickly or slowly. What does depend on the speed of the gurney is the *rate* at which work is done, as we discuss in detail in Section 7–4.

PRACTICE PROBLEM

If the total mass of the gurney plus patient is halved and the acceleration is doubled, does the work done by the intern increase, decrease, or remain the same? [**Answer:** The work remains the same.]

Some related homework problems: Problem 4, Problem 5

Before moving on, let's note an interesting point about our definition of work. It's clear from Equation 7–1 that *the work W is zero if the distance d is zero*—and this is true regardless of how great the force might be. For example, if you push against a solid wall you do no work on it, even though you may become tired from your efforts. Similarly, if you stand in one place holding a 50-pound suitcase in your hand, you do no work on the suitcase. The fact that we become tired when we push against a wall or hold a heavy object is due to the repeated contraction and expansion of individual cells within our muscles. Thus, even when we are "at rest," our muscles are doing mechanical work on the microscopic level.



▲ The weightlifter at left does more work in raising 150 kilograms above her head than Atlas, who is supporting the entire world. Why?

Force at an Angle to the Displacement

In **Figure 7–2** we see a person pulling a suitcase on a level surface with a strap that makes an angle θ with the horizontal—in this case the force is at an angle to the direction of motion. How do we calculate the work now? Well, instead of force times distance, we say that work is the *component* of force in the *direction* of displacement times the magnitude of the displacement. In Figure 7–2, the component of force in the direction of the displacement is $F \cos \theta$ and the magnitude of the displacement is d. Therefore, the work is $F \cos \theta$ times d:

Definition of Work When the Angle Between a Constant Force and the Displacement Is θ

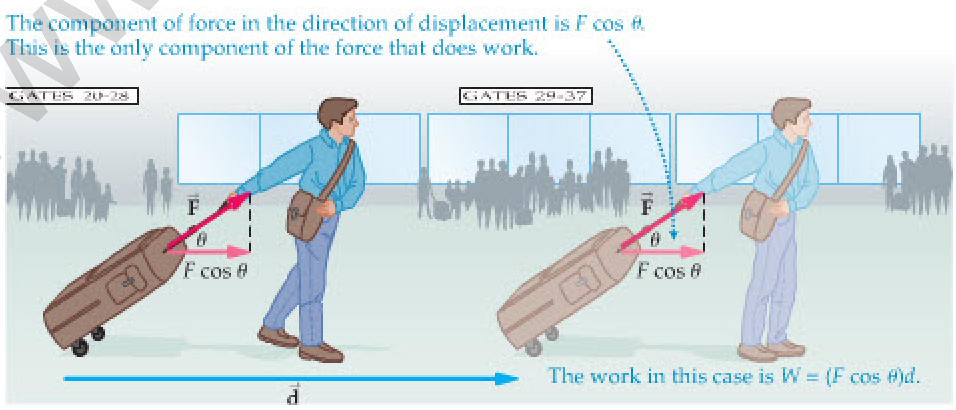
$$W = (F \cos \theta)d = Fd \cos \theta \qquad 7\text{–}3$$

SI unit: joule, J

Of course, in the case where the force is in the direction of motion, the angle θ is zero; then $W = Fd \cos 0° = Fd \cdot 1 = Fd$, in agreement with Equation 7–1.

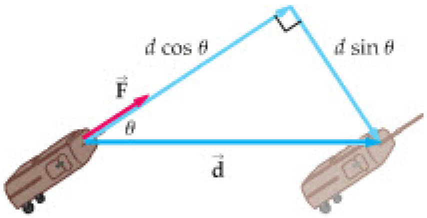
Equally interesting is a situation in which the force and the displacement are at right angles to one another. In this case $\theta = 90°$ and the work done by the force F is zero; $W = Fd \cos 90° = 0$.

This result leads naturally to an alternative way to think about the expression $W = Fd \cos \theta$. In **Figure 7–3** we show the displacement and the force for the suitcase in Figure 7–2. Notice that the displacement is equivalent to a displacement in the



▲ **FIGURE 7–2 Work: force at an angle to the direction of motion**

A person pulls a suitcase with a strap at an angle θ to the direction of motion. The component of force in the direction of motion is $F \cos \theta$, and the work done by the person is $W = (F \cos \theta)d$.



▲ FIGURE 7–3 Force at an angle to direction of motion: another look

The displacement of the suitcase in Figure 7–2 is equivalent to a displacement of magnitude $d\cos\theta$ in the direction of the force $\vec{\mathbf{F}}$, plus a displacement of magnitude $d\sin\theta$ perpendicular to the force. Only the displacement parallel to the force results in nonzero work, hence the total work done is $F(d\cos\theta)$ as expected.

direction of the force of magnitude $(d\cos\theta)$ *plus* a displacement at right angles to the force of magnitude $(d\sin\theta)$. Since the displacement at right angles to the force corresponds to zero work and the displacement in the direction of the force corresponds to a work $W = F(d\cos\theta)$, it follows that the work done in this case is $Fd\cos\theta$, as given in Equation 7–3. Thus, the work done by a force can be thought of in the following two *equivalent* ways:

(i) *Work is the component of force in the direction of the displacement times the magnitude of the displacement.*

(ii) *Work is the component of displacement in the direction of the force times the magnitude of the force.*

In either of these interpretations, the mathematical expression for work is exactly the same, $W = Fd\cos\theta$, where θ is the angle between the force vector and the displacement vector when they are placed tail-to-tail. This definition of θ is illustrated in Figure 7–3.

Finally, we can also express work as the **dot product** between the vectors $\vec{\mathbf{F}}$ and $\vec{\mathbf{d}}$; that is, $W = \vec{\mathbf{F}}\cdot\vec{\mathbf{d}} = Fd\cos\theta$. Note that the dot product, which is always a scalar, is simply the magnitude of one vector times the magnitude of the second vector times the cosine of the angle between them. We discuss the dot product in greater detail in Appendix A.

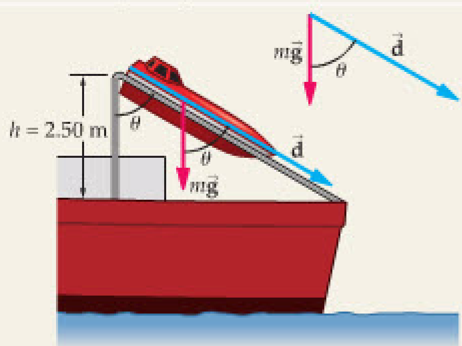
EXAMPLE 7–2 GRAVITY ESCAPE SYSTEM

REAL-WORLD PHYSICS

In a gravity escape system (GES), an enclosed lifeboat on a large ship is deployed by letting it slide down a ramp and then continuing in free fall to the water below. Suppose a 4970-kg lifeboat slides a distance of 5.00 m on a ramp, dropping through a vertical height of 2.50 m. How much work does gravity do on the boat?

PICTURE THE PROBLEM

From our sketch, we see that the force of gravity $m\vec{\mathbf{g}}$ and the displacement $\vec{\mathbf{d}}$ are at an angle θ relative to one another when placed tail-to-tail, and that θ is also the angle the ramp makes with the vertical. In addition, we note that the vertical height of the ramp is $h = 2.50$ m and the length of the ramp is $d = 5.00$ m.



STRATEGY

By definition, the work done on the lifeboat by gravity is $W = Fd\cos\theta$, where $F = mg$, $d = 5.00$ m, and θ is the angle between $m\vec{\mathbf{g}}$ and $\vec{\mathbf{d}}$. We are not given θ in the problem statement, but from the right triangle that forms the ramp we see that $\cos\theta = h/d$. Once θ is determined from the geometry of our sketch, it is straightforward to calculate W.

SOLUTION

1. First, find the component of $\vec{\mathbf{F}} = m\vec{\mathbf{g}}$ in the direction of motion:

$$F\cos\theta = (mg)\left(\frac{h}{d}\right)$$

$$= (4970\ \text{kg})(9.81\ \text{m/s}^2)\left(\frac{2.50\ \text{m}}{5.00\ \text{m}}\right) = 24{,}400\ \text{N}$$

2. Multiply by distance to find the work:

$$W = (F\cos\theta)d = (24{,}400\ \text{N})(5.00\ \text{m}) = 122{,}000\ \text{J}$$

3. Alternatively, cancel d algebraically before substituting numerical values:

$$W = Fd\cos\theta = (mg)(d)\left(\frac{h}{d}\right)$$

$$= mgh = (4970\ \text{kg})(9.81\ \text{m/s}^2)(2.50\ \text{m}) = 122{,}000\ \text{J}$$

INSIGHT

The work is simply $W = mgh$, exactly the same as if the lifeboat had fallen straight down through the height h.

Notice that working the problem symbolically, as in Step 3, results in two distinct advantages. First, it makes for a simpler expression for the work. Second, and more importantly, it shows that the distance d cancels; hence the work depends on the height h but not on d. Such a result is not apparent when we work solely with numbers, as in Steps 1 and 2.

PRACTICE PROBLEM

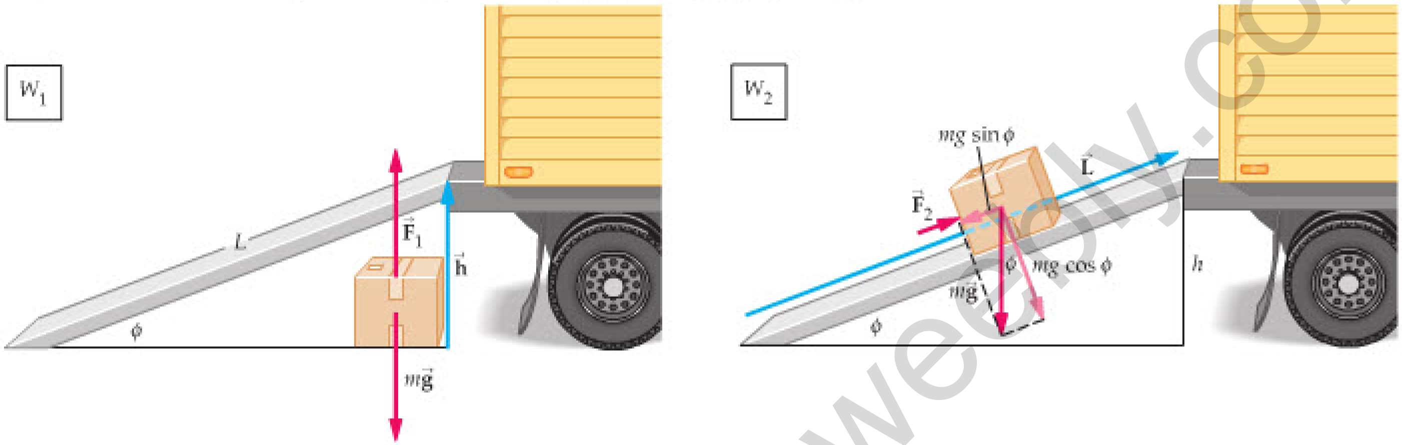
Suppose the lifeboat slides halfway to the water, gets stuck for a moment, and then starts up again and continues to the end of the ramp. What is the work done by gravity in this case? [**Answer:** The work done by gravity is exactly the same, $W = mgh$, independent of how the boat moves down the ramp.]

Some related homework problems: Problem 11, Problem 12

Next, we present a Conceptual Checkpoint that compares the work required to move an object along two different paths.

CONCEPTUAL CHECKPOINT 7–1 PATH DEPENDENCE OF WORK

You want to load a box into the back of a truck. One way is to lift it straight up through a height h, as shown, doing a work W_1. Alternatively, you can slide the box up a loading ramp a distance L, doing a work W_2. Assuming the box slides on the ramp without friction, which of the following is correct: **(a)** $W_1 < W_2$, **(b)** $W_1 = W_2$, **(c)** $W_1 > W_2$?



REASONING AND DISCUSSION

You might think that W_2 is less than W_1, since the force needed to slide the box up the ramp, F_2, is less than the force needed to lift it straight up. On the other hand, the distance up the ramp, L, is greater than the vertical distance, h, so perhaps W_2 should be greater than W_1. In fact, these two effects cancel exactly, giving $W_1 = W_2$.

To see this, we first calculate W_1. The force needed to lift the box with constant speed is $F_1 = mg$, and the height is h, therefore $W_1 = mgh$.

Next, the work to slide the box up the ramp with constant speed is $W_2 = F_2L$, where F_2 is the force required to push against the tangential component of gravity. In the figure we see that $F_2 = mg \sin \phi$. The figure also shows that $\sin \phi = h/L$; thus $W_2 = (mg \sin \phi)L = (mg)(h/L)L = mgh = W_1$.

Clearly, the ramp is a useful device—it reduces the *force* required to move the box upward from $F_1 = mg$ to $F_2 = mg(h/L)$. Even so, it doesn't decrease the amount of *work* we need to do. As we have seen, the reduced force on the ramp is offset by the increased distance.

ANSWER

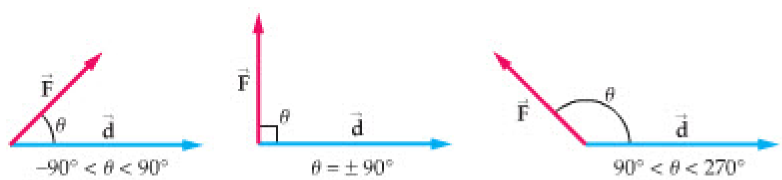
(b) $W_1 = W_2$

Negative Work and Total Work

Work depends on the angle between the force, $\vec{F}$, and the displacement (or direction of motion), $\vec{d}$. This dependence gives rise to three distinct possibilities, as shown in Figure 7–4:

(i) *Work is positive if the force has a component in the direction of motion* $(-90^\circ < \theta < 90^\circ)$.
(ii) *Work is zero if the force has no component in the direction of motion* $(\theta = \pm 90^\circ)$.
(iii) *Work is negative if the force has a component opposite to the direction of motion* $(90^\circ < \theta < 270^\circ)$.

Thus, whenever we calculate work, we must be careful about its sign and not just assume it to be positive.



(a) Positive work (b) Zero work (c) Negative work

FIGURE 7–4 Positive, negative, and zero work
Work is positive when the force is in the same general direction as the displacement and is negative if the force is generally opposite to the displacement. Zero work is done if the force is at right angles to the displacement.

PROBLEM-SOLVING NOTE

Be Careful About the Angle θ

In calculating $W = Fd\cos\theta$ be sure that the angle you use in the cosine is the angle between the force and the displacement vectors when they are placed tail to tail. Sometimes θ may be used to label a different angle in a given problem. For example, θ is often used to label the angle of a slope, in which case it may have nothing to do with the angle between the force and the displacement. To summarize: Just because an angle is labeled θ doesn't mean it's automatically the correct angle to use in the work formula.

When more than one force acts on an object, the total work is the sum of the work done by each force separately. Thus, if force $\vec{\mathbf{F}}_1$ does work W_1, force $\vec{\mathbf{F}}_2$ does work W_2, and so on, the total work is

$$W_{\text{total}} = W_1 + W_2 + W_3 + \cdots = \sum W \qquad 7\text{–}4$$

Equivalently, the total work can be calculated by first performing a vector sum of all the forces acting on an object to obtain $\vec{\mathbf{F}}_{\text{total}}$ and then using our basic definition of work:

$$W_{\text{total}} = (F_{\text{total}}\cos\theta)d = F_{\text{total}}\,d\cos\theta \qquad 7\text{–}5$$

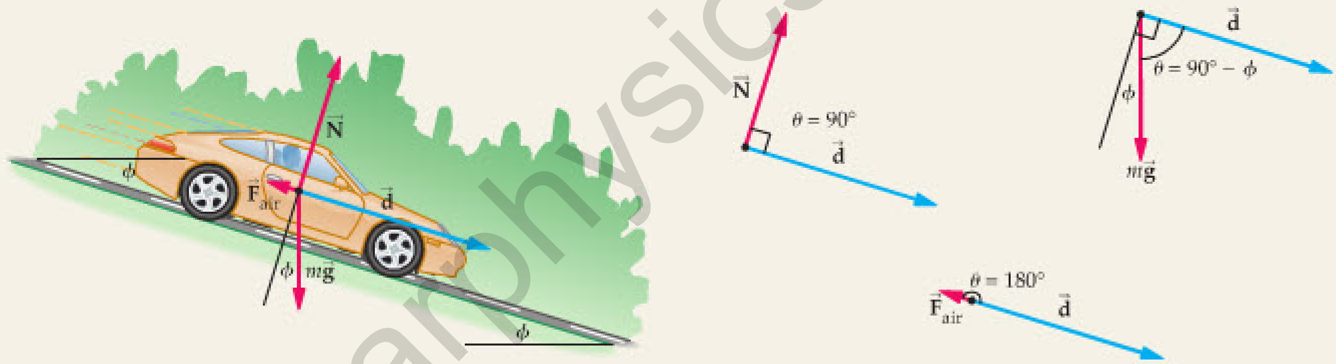
where θ is the angle between $\vec{\mathbf{F}}_{\text{total}}$ and the displacement $\vec{\mathbf{d}}$. In the next two Examples we calculate the total work in each of these ways.

EXAMPLE 7–3 A COASTING CAR I

A car of mass m coasts down a hill inclined at an angle ϕ below the horizontal. The car is acted on by three forces: (i) the normal force $\vec{\mathbf{N}}$ exerted by the road, (ii) a force due to air resistance, $\vec{\mathbf{F}}_{\text{air}}$, and (iii) the force of gravity, $m\vec{\mathbf{g}}$. Find the total work done on the car as it travels a distance d along the road.

PICTURE THE PROBLEM

Because ϕ is the angle the slope makes with the horizontal, it is also the angle between $m\vec{\mathbf{g}}$ and the downward normal direction, as was shown in Figure 5–15. It follows that the angle between $m\vec{\mathbf{g}}$ and the displacement $\vec{\mathbf{d}}$ is $\theta = 90° - \phi$. Our sketch also shows that the angle between $\vec{\mathbf{N}}$ and $\vec{\mathbf{d}}$ is $\theta = 90°$, and the angle between $\vec{\mathbf{F}}_{\text{air}}$ and $\vec{\mathbf{d}}$ is $\theta = 180°$.



STRATEGY

For each force we calculate the work using $W = Fd\cos\theta$, where θ is the angle between that particular force and the displacement $\vec{\mathbf{d}}$. The total work is the sum of the work done by each of the three forces.

SOLUTION

1. We start with the work done by the normal force, $\vec{\mathbf{N}}$. From the figure we see that $\theta = 90°$ for this force: $W_N = Nd\cos\theta = Nd\cos 90° = Nd(0) = 0$
2. For the force of air resistance, $\theta = 180°$: $W_{\text{air}} = F_{\text{air}}d\cos 180° = F_{\text{air}}d(-1) = -F_{\text{air}}d$
3. For gravity the angle θ is $\theta = 90° - \phi$, as indicated in the figure. Recall that $\cos(90° - \phi) = \sin\phi$ (see Appendix A): $W_{mg} = mgd\cos(90° - \phi) = mgd\sin\phi$
4. The total work is the sum of the individual works: $W_{\text{total}} = W_N + W_{\text{air}} + W_{mg} = 0 - F_{\text{air}}d + mgd\sin\phi$

INSIGHT

The normal force is perpendicular to the motion of the car, and thus does no work. Air resistance points in a direction that opposes the motion, so it does negative work. On the other hand, gravity has a component in the direction of motion; therefore, its work is positive. The physical significance of positive, negative, and zero work will be discussed in detail in the next section.

PRACTICE PROBLEM

Calculate the total work done on a 1550-kg car as it coasts 20.4 m down a hill with $\phi = 5.00°$. Let the force due to air resistance be 15.0 N. [**Answer:** $W_{\text{total}} = W_N + W_{\text{air}} + W_{mg} = 0 - F_{\text{air}}d + mgd\sin\phi = 0 - 306\text{ J} + 2.70 \times 10^4\text{ J} = 2.67 \times 10^4\text{ J}$]

Some related homework problems: Problem 15, Problem 81

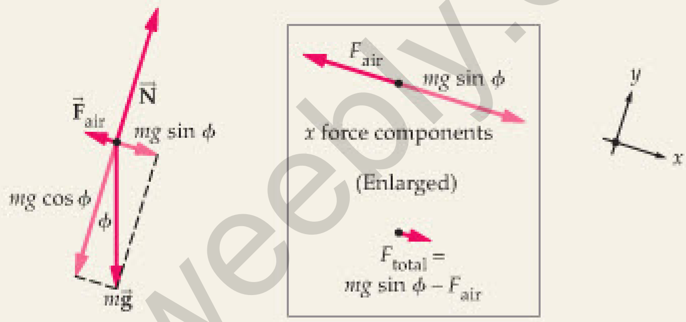
In the previous Example, we showed that the total work can be calculated by finding the work done by each force separately, and then summing the individual works. In the next Example, we take a different approach. We first sum the forces acting on the car to find F_{total}. Once the total force is determined, we calculate the total work using $W_{total} = F_{total}d\cos\theta$.

EXAMPLE 7–4 A COASTING CAR II

Consider the car described in Example 7–3. Calculate the total work done on the car using $W_{total} = F_{total}d\cos\theta$.

PICTURE THE PROBLEM

First, we choose the x axis to point down the slope, and the y axis to be at right angles to the slope. With this choice, there is no acceleration in the y direction, which means that the total force in that direction must be zero. As a result, the total force acting on the car is in the x direction. The magnitude of the total force is $mg\sin\phi - F_{air}$, as can be seen in our sketch.



STRATEGY

We begin by finding the x component of each force vector and then summing them to find the total force acting on the car. As can be seen from the figure, the total force points in the positive x direction; that is, in the same direction as the displacement. Therefore, the angle θ in $W = F_{total}d\cos\theta$ is zero.

SOLUTION

1. Referring to the figure above, we see that the *magnitude* of the total force is $mg\sin\phi$ minus F_{air}:

$$F_{total} = mg\sin\phi - F_{air}$$

2. The *direction* of $\vec{F}_{total}$ is the same as the direction of $\vec{d}$, thus $\theta = 0°$. We can now calculate W_{total}:

$$W_{total} = F_{total}d\cos\theta = (mg\sin\phi - F_{air})d\cos 0°$$
$$= mgd\sin\phi - F_{air}d$$

INSIGHT

Note that we were careful to calculate both the magnitude and the direction of the total force. The magnitude (which is always positive) gives F_{total} and the direction gives $\theta = 0°$, allowing us to use $W_{total} = F_{total}d\cos\theta$.

PRACTICE PROBLEM

Suppose the total work done on a 1620-kg car as it coasts 25.0 m down a hill with $\phi = 6.00°$ is $W_{total} = 3.75 \times 10^4$ J. Find the magnitude of the force due to air resistance. [**Answer:** $F_{air}d = -W_{total} + mgd\sin\phi = 4030$ J, thus $F_{air} = (4030\text{ J})/d = 161$ N]

Some related homework problems: Problem 15, Problem 81

The full significance of positive versus negative work is seen in the next section, where we relate the work done on an object to the change in its speed.

7–2 Kinetic Energy and the Work–Energy Theorem

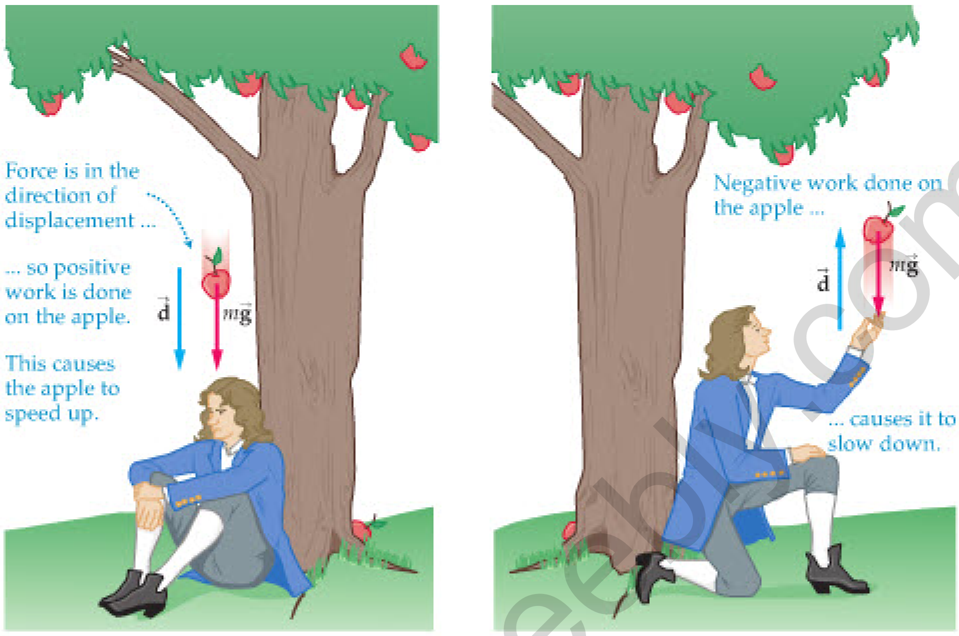
Suppose you drop an apple. As it falls, gravity does positive work on it, as indicated in Figure 7–5, and its speed increases. If you toss the apple upward, gravity does negative work, and the apple slows down. In general, whenever the total work done on an object is positive, its speed increases; when the total work is negative, its speed decreases. In this section we derive an important result, the **work–energy theorem**, which makes this connection between work and change in speed precise.

To begin, consider an apple of mass m falling through the air, and suppose that two forces act on the apple—gravity, $m\vec{g}$, and the average force of air resistance, $\vec{F}_{air}$. The total force acting on the apple, $\vec{F}_{total}$, gives the apple a constant downward acceleration of magnitude

$$a = F_{total}/m$$

Since the total force is downward and the motion is downward, the work done on the apple is positive.

FIGURE 7–5 Gravitational work
The work done by gravity on an apple that moves downward is positive. If the apple is in free fall, this positive work will result in an increase in speed. On the other hand, the work done by gravity on an apple that moves upward is negative. If the apple is in free fall, the negative work done by gravity will result in a decrease of speed.



Apple falling: $W > 0$, speed increases   Apple tossed upward: $W < 0$, speed decreases

Now, suppose the initial speed of the apple is v_i, and that after falling a distance d its speed increases to v_f. The apple falls with constant acceleration a, hence constant-acceleration kinematics (Equation 2–12) gives

$$v_f^2 = v_i^2 + 2ad$$

or, with a slight rearrangement,

$$2ad = v_f^2 - v_i^2$$

Next, substitute $a = F_{total}/m$ into this equation:

$$2\left(\frac{F_{total}}{m}\right)d = v_f^2 - v_i^2$$

Multiplying both sides by m and dividing by 2 yields

$$F_{total}d = \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2$$

where $F_{total}d$ is simply the total work done on the apple. Thus we find

$$W_{total} = \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2$$

showing that total work is directly related to change in speed, as just mentioned. Note that $W_{total} > 0$ means $v_f > v_i$, $W_{total} < 0$ means $v_f < v_i$, and $W_{total} = 0$ implies that $v_f = v_i$.

The quantity $\frac{1}{2}mv^2$ in the equation for W_{total} has a special significance in physics, as we shall see. We call it the **kinetic energy**, K:

Definition of Kinetic Energy, K

$$K = \tfrac{1}{2}mv^2 \qquad \text{7–6}$$

SI unit: $kg \cdot m^2/s^2$ = joule, J

In general, the kinetic energy of an object is the energy due to its motion. We measure kinetic energy in joules, the same units as work, and both kinetic energy and work are scalars. Unlike work, however, kinetic energy is never negative. Instead, K is always greater than or equal to zero, independent of the direction of motion or the direction of any forces.

To get a feeling for typical values of kinetic energy, consider your kinetic energy when jogging. Assuming a mass of about 62 kg and a speed of 2.5 m/s, your kinetic energy is $K = \frac{1}{2}(62\ \text{kg})(2.5\ \text{m/s})^2 = 190$ J. Additional examples of kinetic energy are given in Table 7–2.

PROBLEM-SOLVING NOTE
Work Can Be Positive, Negative, or Zero

When you calculate work, be sure to keep track of whether it is positive or negative. The distinction is important, since positive work increases speed, whereas negative work decreases speed. Zero work, of course, has no effect on speed.

TABLE 7–2 Typical Kinetic Energies

Source	Approximate kinetic energy (J)
Jet aircraft at 500 mi/h	10^9
Car at 60 mi/h	10^6
Home-run baseball	10^3
Person at walking speed	50
Housefly in flight	10^{-3}

EXERCISE 7–2

A truck moving at 15 m/s has a kinetic energy of 4.2×10^5 J. **(a)** What is the mass of the truck? **(b)** By what multiplicative factor does the kinetic energy of the truck increase if its speed is doubled?

SOLUTION

(a) $K = \frac{1}{2}mv^2$; therefore $m = 2K/v^2 = 3700$ kg. **(b)** Kinetic energy depends on the speed squared, and hence doubling the speed increases the kinetic energy by a factor of four.

In terms of kinetic energy, the work–energy theorem can be stated as follows:

Work–Energy Theorem

The total work done on an object is equal to the change in its kinetic energy:

$$W_{\text{total}} = \Delta K = \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2 \quad \text{7–7}$$

Thus, the work–energy theorem says that when a force acts on an object over a distance—doing work on it—the result is a change in the speed of the object, and hence a change in its energy of motion. Equation 7–7 is the quantitative expression of this connection.

Finally, though we have derived the work–energy theorem for a force that is constant in direction and magnitude, it is valid for any force, as can be shown using the methods of calculus. In fact, the work–energy theorem is completely general, making it one of the more important and fundamental results in physics. It is also a very handy tool for problem solving, as we shall see many times throughout this text.

PROBLEM-SOLVING NOTE

Starts from Rest Means $v_i = 0$

A problem statement that uses a phrase like "starts from rest" or "is raised from rest" is telling you that $v_i = 0$.

EXERCISE 7–3

How much work is required for a 74-kg sprinter to accelerate from rest to 2.2 m/s?

SOLUTION

Since $v_i = 0$, we have $W = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = \frac{1}{2}mv_f^2 = \frac{1}{2}(74\text{ kg})(2.2\text{ m/s})^2 = 180$ J.

We now present a variety of Examples showing how the work–energy theorem is used in practical situations.

EXAMPLE 7–5 HIT THE BOOKS

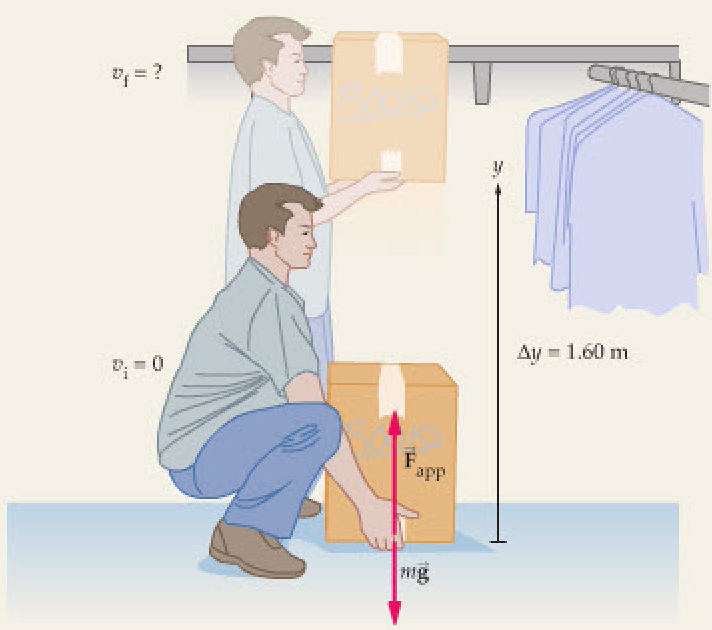
A 4.10-kg box of books is lifted vertically from rest a distance of 1.60 m with a constant, upward applied force of 52.7 N. Find **(a)** the work done by the applied force, **(b)** the work done by gravity, and **(c)** the final speed of the box.

PICTURE THE PROBLEM

Our sketch shows that the direction of motion of the box is upward. In addition, we see that the applied force, $\vec{F}_{\text{app}}$, is upward and the force of gravity, $m\vec{g}$, is downward. Finally, the box is lifted from rest ($v_i = 0$) through a distance $\Delta y = 1.60$ m.

STRATEGY

The applied force is in the direction of motion, so the work it does, W_{app}, is positive. Gravity is opposite in direction to the motion; thus its work, W_g, is negative. The total work is the sum of W_{app} and W_g, and the final speed of the box is found by applying the work–energy theorem, $W_{\text{total}} = \Delta K$.



CONTINUED ON NEXT PAGE

CONTINUED FROM PREVIOUS PAGE

SOLUTION

Part (a)

1. First we find the work done by the applied force. In this case, $\theta = 0°$ and the distance is $\Delta y = 1.60$ m:

$$W_{app} = F_{app} \cos 0° \,\Delta y = (52.7\text{ N})(1)(1.60\text{ m}) = 84.3\text{ J}$$

Part (b)

2. Next, we calculate the work done by gravity. The distance is $\Delta y = 1.60$ m, as before, but now $\theta = 180°$:

$$W_g = mg \cos 180° \,\Delta y$$
$$= (4.10\text{ kg})(9.81\text{ m/s}^2)(-1)(1.60\text{ m}) = -64.4\text{ J}$$

Part (c)

3. The total work done on the box, W_{total}, is the sum of W_{app} and W_g:

$$W_{total} = W_{app} + W_g = 84.3\text{ J} - 64.4\text{ J} = 19.9\text{ J}$$

4. To find the final speed, v_f, we apply the work–energy theorem. Recall that the box started at rest, thus $v_i = 0$:

$$W_{total} = \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2 = \tfrac{1}{2}mv_f^2$$
$$v_f = \sqrt{\frac{2W_{total}}{m}} = \sqrt{\frac{2(19.9\text{ J})}{4.10\text{ kg}}} = 3.12\text{ m/s}$$

INSIGHT

As a check on our result, we can find v_f in a completely different way. First, calculate the acceleration of the box with the result $a = (F_{app} - mg)/m = 3.04\text{ m/s}^2$. Next, use this result in the kinematic equation $v^2 = v_0^2 + 2a\Delta y$. With $v_0 = 0$ and $\Delta y = 1.60$ m, we find $v = 3.12$ m/s, in agreement with the results using the work–energy theorem.

PRACTICE PROBLEM

If the box is lifted only a quarter of the distance, is the final speed 1/8, 1/4, or 1/2 of the value found in Step 4? Calculate v_f in this case as a check on your answer. [**Answer:** Since work depends linearly on Δy, and v_f depends on the square root of the work, it follows that the final speed is $\sqrt{1/4} = \frac{1}{2}$ the value in Step 4. Letting $\Delta y = (1.60\text{ m})/4 = 0.400$ m, we find $v_f = \frac{1}{2}(3.12\text{ m/s}) = 1.56\text{ m/s}$.]

Some related homework problems: Problem 19, Problem 24, Problem 25

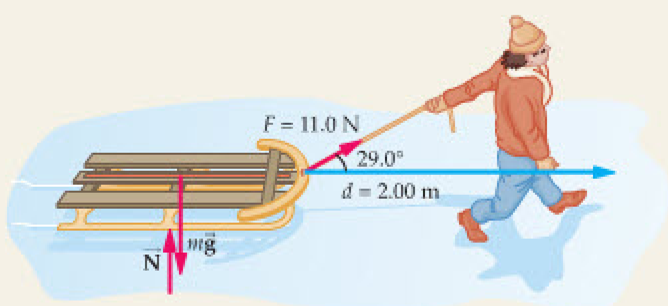
In the previous Example the initial speed was zero. This is not always the case, of course. The next Example illustrates how to use the work–energy theorem when the initial velocity is nonzero.

EXAMPLE 7–6 PULLING A SLED

A boy exerts a force of 11.0 N at 29.0° above the horizontal on a 6.40-kg sled. Find **(a)** the work done by the boy and **(b)** the final speed of the sled after it moves 2.00 m, assuming the sled starts with an initial speed of 0.500 m/s and slides horizontally without friction.

PICTURE THE PROBLEM

Our sketch shows the direction of motion and the directions of each of the forces. Note that the normal force and the force due to gravity are vertical, whereas the displacement is horizontal. The force exerted by the boy has both a vertical component, $F \sin \theta$, and a horizontal component, $F \cos \theta$.



STRATEGY

a. The forces $\vec{N}$ and $m\vec{g}$ do no work because they are at right angles to the horizontal displacement. The force exerted by the boy, however, has a horizontal component that does positive work on the sled. Therefore, the total work is simply the work done by the boy.

b. After calculating this work, we find v_f by applying the work–energy theorem with $v_i = 0.500$ m/s.

SOLUTION

Part (a)

1. The work done by the boy is $(F \cos \theta)d$, where $\theta = 29.0°$. This is also the total work done on the sled:

$$W_{boy} = (F \cos \theta)d$$
$$= (11.0\text{ N})(\cos 29.0°)(2.00\text{ m}) = 19.2\text{ J} = W_{total}$$

Part (b)

2. Use the work–energy theorem to solve for the final speed:

$$W_{total} = \Delta K = \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2$$

$$\tfrac{1}{2}mv_f^2 = W_{total} + \tfrac{1}{2}mv_i^2$$

$$v_f = \sqrt{\frac{2W_{total}}{m} + v_i^2}$$

3. Substitute numerical values to get the final answer:

$$v_f = \sqrt{\frac{2(19.2\text{ J})}{6.40\text{ kg}} + (0.500\text{ m/s})^2}$$

$$= 2.50\text{ m/s}$$

INSIGHT

If the sled had started from rest, instead of with an initial speed of 0.500 m/s, would its final speed be 2.50 m/s − 0.500 m/s = 2.00 m/s?

No. If the initial speed is zero, then $v_f = \sqrt{\frac{2W_{total}}{m}} = \sqrt{\frac{2(19.2\text{ J})}{6.40\text{ kg}}} = 2.45$ m/s. Why don't the speeds add and subtract in a straightforward way? The reason is that the work–energy theorem depends on the *square* of the speeds rather than on v_i and v_f directly.

PRACTICE PROBLEM

Suppose the sled starts with a speed of 0.500 m/s and has a final speed of 2.50 m/s after the boy pulls it through a distance of 3.00 m. What force did the boy exert on the sled? [**Answer:** $F = W_{total}/(d\cos\theta) = \Delta K/(d\cos\theta) = 7.32$ N]

Some related homework problems: Problem 28, Problem 61

The final speeds in the previous Examples could have been found using Newton's laws and the constant-acceleration kinematics of Chapter 2, as indicated in the Insight following Example 7–5. The work–energy theorem provides an alternative method of calculation that is often much easier to apply than Newton's laws. We return to this point in Chapter 8.

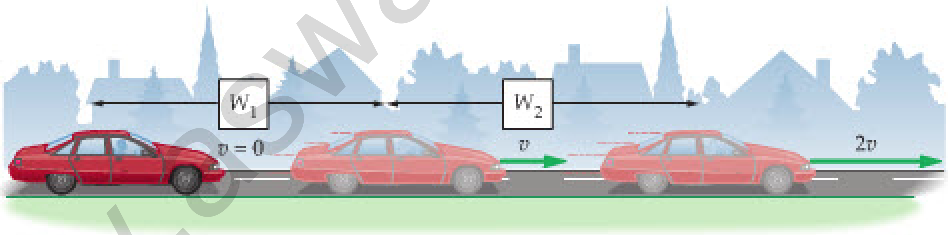
PROBLEM-SOLVING NOTE

Be Careful About Linear Reasoning

Though some relations are linear—if you *double* the mass, you *double* the kinetic energy—others are not. For example, if you *double* the speed, you *quadruple* the kinetic energy. Be careful not to jump to conclusions based on linear reasoning.

CONCEPTUAL CHECKPOINT 7–2 COMPARE THE WORK

To accelerate a certain car from rest to the speed v requires the work W_1. The work needed to accelerate the car from v to $2v$ is W_2. Which of the following is correct: **(a)** $W_2 = W_1$, **(b)** $W_2 = 2W_1$, **(c)** $W_2 = 3W_1$, **(d)** $W_2 = 4W_1$?



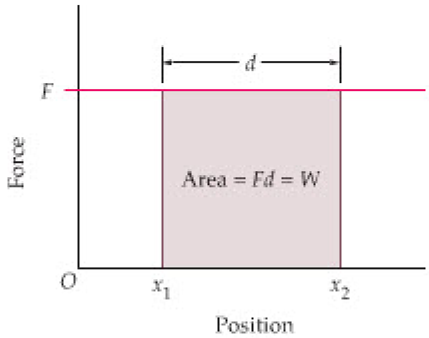
REASONING AND DISCUSSION

A common mistake is to reason that since we increase the speed by the same amount in each case, the work required is the same. It is not, and the reason is that work depends on the speed squared rather than on the speed itself.

To see how this works, first calculate W_1, the work needed to go from rest to a speed v. From the work–energy theorem, with $v_i = 0$ and $v_f = v$, we find $W_1 = \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2 = \tfrac{1}{2}mv^2$. Similarly, the work needed to go from rest, $v_i = 0$, to a speed $v_f = 2v$, is simply $\tfrac{1}{2}m(2v)^2 = 4\left(\tfrac{1}{2}mv^2\right) = 4W_1$. Therefore, the work needed to increase the speed from v to $2v$ is the difference: $W_2 = 4W_1 - W_1 = 3W_1$.

ANSWER

(c) $W_2 = 3W_1$



▲ **FIGURE 7–6 Graphical representation of the work done by a constant force**

A constant force F acting through a distance d does a work $W = Fd$. Note that Fd is also equal to the shaded area between the force line and the x axis.

7–3 Work Done by a Variable Force

Thus far we have calculated work only for constant forces, yet most forces in nature vary with position. For example, the force exerted by a spring depends on how far the spring is stretched, and the force of gravity between planets depends on their separation. In this section we show how to calculate the work for a force that varies with position.

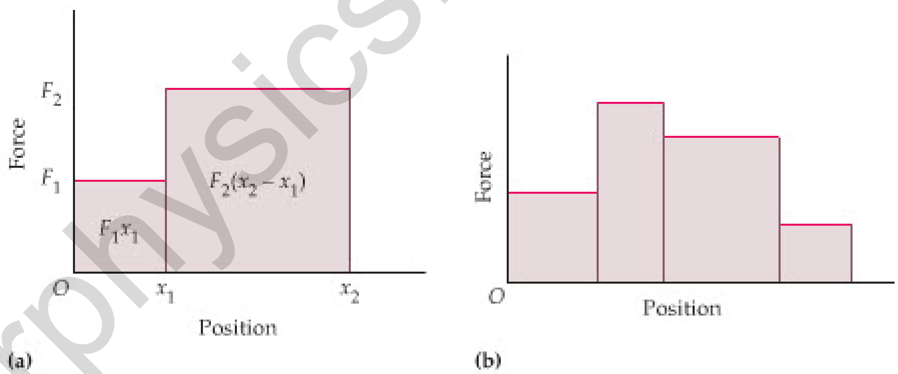
First, let's review briefly the case of a constant force, and develop a graphical interpretation of work. Figure 7–6 shows a constant force plotted versus position, x. If the force acts in the positive x direction and moves an object a distance d, from x_1 to x_2, the work it does is $W = Fd = F(x_2 - x_1)$. Referring to the figure, we see that the work is equal to the shaded area[1] between the force line and the x axis.

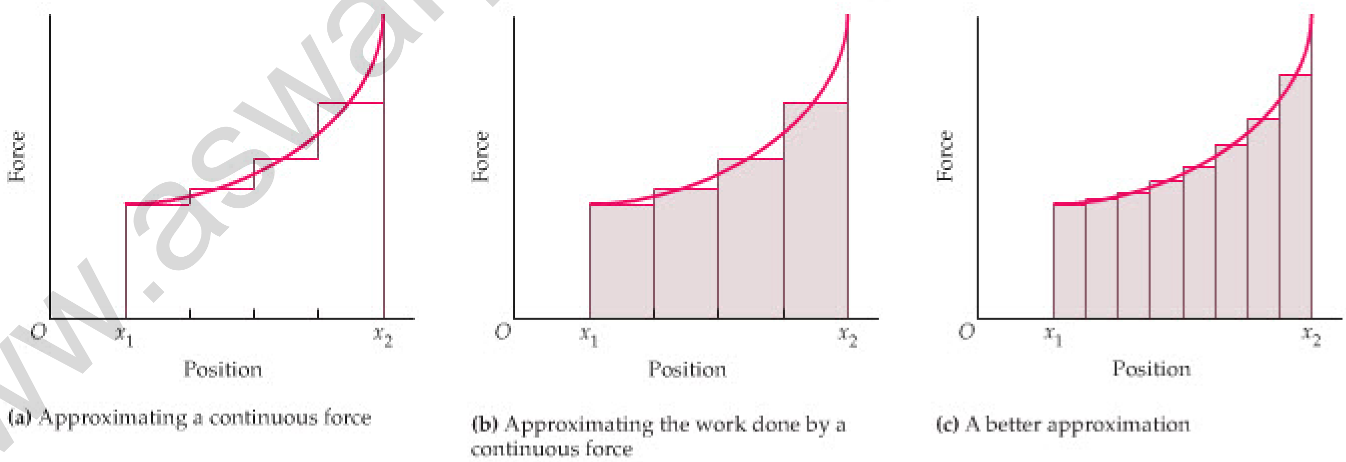
Next, consider a force that has the value F_1 from $x = 0$ to $x = x_1$ and a different value F_2 from $x = x_1$ to $x = x_2$, as in Figure 7–7 (a). The work in this case is the sum of the works done by F_1 and F_2. Therefore, $W = F_1x_1 + F_2(x_2 - x_1)$ which, again, is the area between the force lines and the x axis. Clearly, this type of calculation can be extended to a force with any number of different values, as indicated in Figure 7–7 (b).

If a force varies continuously with position, we can approximate it with a series of constant values that follow the shape of the curve, as shown in Figure 7–8 (a). It follows that the work done by the continuous force is approximately equal to the area of the corresponding rectangles, as Figure 7–8 (b) shows. The approximation can be made better by using more rectangles, as illustrated in Figure 7–8 (c). In the

▶ **FIGURE 7–7 Work done by a nonconstant force**

(a) A force with a value F_1 from 0 to x_1 and a value F_2 from x_1 to x_2 does the work $W = F_1x_1 + F_2(x_2 - x_1)$. This is simply the area of the two shaded rectangles. **(b)** If a force takes on a number of different values, the work it does is still the total area between the force lines and the x axis, just as in part (a).





▲ **FIGURE 7–8 Work done by a continuously varying force**

(a) A continuously varying force can be approximated by a series of constant values that follow the shape of the curve. **(b)** The work done by the continuous force is approximately equal to the area of the small rectangles corresponding to the constant values of force shown in part (a). **(c)** In the limit of an infinite number of vanishingly small rectangles, we see that the work done by the force is equal to the area between the force curve and the x axis.

[1]Usually, area has the dimensions of (length) × (length), or length^2. In this case, however, the vertical axis is force and the horizontal axis is distance. As a result, the dimensions of area are (force) × (distance), which in SI units is N · m = J.

limit of an infinite number of vanishingly small rectangles, the area of the rectangles becomes identical to the area under the force curve. Hence this area is the work done by the continuous force. To summarize:

The work done by a force in moving an object from x_1 to x_2 is equal to the corresponding area between the force curve and the x axis.

A case of particular interest is that of a spring. Since the force exerted by a spring is given by $F_x = -kx$ (Section 6–2), it follows that the force we must exert to hold it at the position x is $+kx$. This is illustrated in Figure 7–9, where we also show that the corresponding force curve is a straight line extending from the origin. Therefore, the work we do in stretching a spring from $x = 0$ (equilibrium) to the general position x is the shaded, triangular area shown in Figure 7–10. This area is equal to $\frac{1}{2}$(base)(height), where in this case the base is x and the height is kx. As a result, the work is $\frac{1}{2}(x)(kx) = \frac{1}{2}kx^2$. Similar reasoning shows that the work needed to compress a spring a distance x is also $\frac{1}{2}kx^2$. Therefore,

Work to Stretch or Compress a Spring a Distance *x* from Equilibrium

$$W = \frac{1}{2}kx^2 \qquad 7\text{–}8$$

SI unit: joule, J

We can get a feeling for the amount of work required to compress a typical spring in the following Exercise.

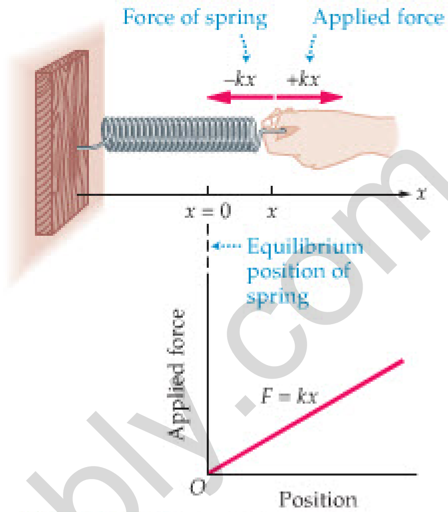


FIGURE 7–9 Stretching a spring
The force we must exert on a spring to stretch it a distance x is $+kx$. Thus, applied force versus position for a spring is a straight line of slope k.

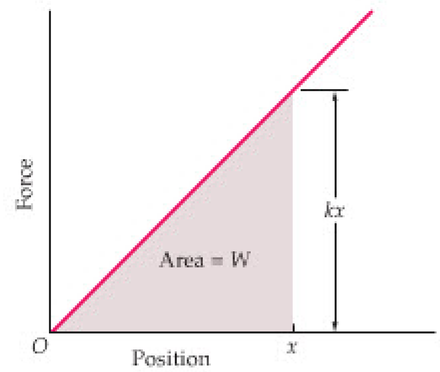


FIGURE 7–10 Work needed to stretch a spring a distance *x*
The work done is equal to the shaded area, which is a right triangle. The area of the triangle is $\frac{1}{2}(x)(kx) = \frac{1}{2}kx^2$.

EXERCISE 7–4

The spring in a pinball launcher has a force constant of 405 N/m. How much work is required to compress the spring a distance of 3.00 cm?

SOLUTION

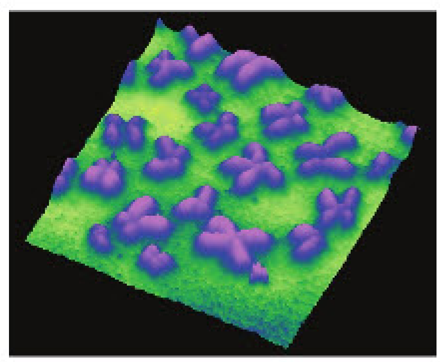
$W = \frac{1}{2}kx^2 = \frac{1}{2}(405\ \text{N/m})(0.0300\ \text{m})^2 = 0.182\ \text{J}$

Note that the work done in compressing or expanding a spring varies with the second power of x, the displacement from equilibrium. The consequences of this dependence are explored throughout the rest of this section.

Before we consider a specific example, however, recall that the results for a spring apply to more than just the classic case of a helical coil of wire. In fact, any flexible structure satisfies the relations $F_x = -kx$ and $W = \frac{1}{2}kx^2$, given the appropriate value of the force constant, k, and small enough displacements, x. Several examples were mentioned in Section 6–2.

Here we consider an example from the field of nanotechnology; namely, the cantilevers used in **atomic-force microscopy** (AFM). As we show in Example 7–7, a typical atomic-force cantilever is basically a thin silicon bar about 250 μm in length, supported at one end like a diving board, with a sharp, hanging point at the other end. When the point is pulled across the surface of a material—like an old-fashioned phonograph needle in the groove of a record—individual atoms on the surface cause the point to move up and down, deflecting the cantilever. These deflections, which can be measured by reflecting a laser beam from the top of the cantilever, are then converted into an atomic-level picture of the surface, as shown in the accompanying photograph.

A typical force constant for an AFM cantilever is on the order of 1 N/m, much smaller than the 100–500 N/m force constant of a common lab spring. The implications of this are discussed in the following Example.



Human chromosomes, as imaged by an atomic-force microscope.

EXAMPLE 7–7 FLEXING AN AFM CANTILEVER

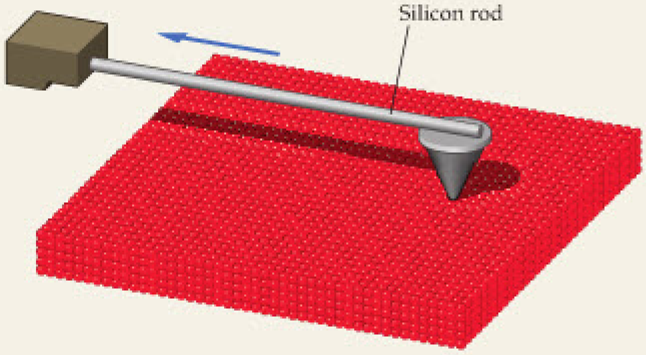
The work required to deflect a typical AFM cantilever by 0.10 nm is 1.2×10^{-20} J. **(a)** What is the force constant of the cantilever, treating it as an ideal spring? **(b)** How much work is required to increase the deflection of the cantilever from 0.10 nm to 0.20 nm?

CONTINUED ON NEXT PAGE

CONTINUED FROM PREVIOUS PAGE

PICTURE THE PROBLEM

The sketch on the left shows the cantilever and its sharp point being dragged across the surface of a material. In the sketch to the right, we show an exaggerated view of the cantilever's deflection, and indicate that it is equivalent to the stretch of an "effective" ideal spring with a force constant k.



STRATEGY

a. Given that $W = 1.2 \times 10^{-20}$ J for a deflection of $x = 0.10$ nm, we can find the effective force constant k using $W = \frac{1}{2}kx^2$.

b. To find the work required to deflect from $x = 0.10$ nm to $x = 0.20$ nm, $W_{1\to2}$, we calculate the work to deflect from $x = 0$ to $x = 0.20$ nm, $W_{0\to2}$, and then subtract the work needed to deflect from $x = 0$ to $x = 0.10$ nm, $W_{0\to1}$. (Note that we *cannot* simply assume the work to go from $x = 0.10$ nm to $x = 0.20$ nm is the same as the work to go from $x = 0$ to $x = 0.10$ nm.)

SOLUTION

Part (a)

1. Solve $W = \frac{1}{2}kx^2$ for the force constant k:

$$k = \frac{2W}{x^2} = \frac{2(1.2 \times 10^{-20}\ \text{J})}{(0.10 \times 10^{-9}\ \text{m})^2} = 2.4\ \text{N/m}$$

Part (b)

2. First, calculate the work needed to deflect the cantilever from $x = 0$ to $x = 0.20$ nm:

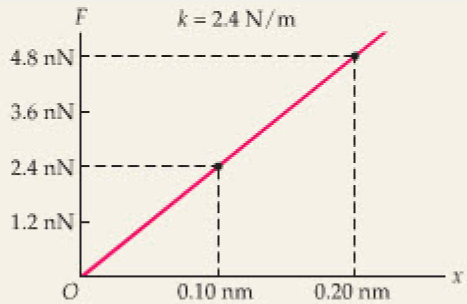
$$W_{0\to2} = \tfrac{1}{2}kx^2 = \tfrac{1}{2}(2.4\ \text{N/m})(0.2 \times 10^{-9}\ \text{m})^2 = 4.8 \times 10^{-20}\ \text{J}$$

3. Subtract from the above result the work to deflect from $x = 0$ to $x = 0.10$ nm, which the problem statement gives as 1.2×10^{-20} J:

$$W_{1\to2} = W_{0\to2} - W_{0\to1} = 4.8 \times 10^{-20}\ \text{J} - 1.2 \times 10^{-20}\ \text{J} = 3.6 \times 10^{-20}\ \text{J}$$

INSIGHT

Our results show that more energy is needed to deflect the cantilever the second 0.10 nm than to deflect it the first 0.10 nm. Why? The reason is that the force of the cantilever increases with distance; thus, the average force over the second 0.10 nm is greater than the average force over the first 0.10 nm. In fact, we can see from the adjacent figure that the average force between 0.10 nm and 0.20 nm (3.6 nN) is three times the average force between 0 and 0.10 nm (1.2 nN). It follows, then, that the work required for the second 0.10 nm is three times the work required for the first 0.10 nm.



PRACTICE PROBLEM

A second cantilever has half the force constant of the cantilever in this Example. Is the work required to deflect the second cantilever by 0.20 nm greater than, less than, or equal to the work required to deflect the cantilever in this Example by 0.10 nm? [**Answer:** Halving the force constant halves the work, but doubling the deflection quadruples the work. The net effect is that the work increases by a factor of two, to 2.4×10^{-20} J.]

Some related homework problems: Problem 32, Problem 38

An equivalent way to calculate the work for a variable force is to multiply the average force, F_{av}, by the distance, d:

$$W = F_{av}d \qquad 7\text{–}9$$

For a spring that is stretched a distance x from equilibrium the force varies linearly from 0 to kx. Thus, the average force is $F_{av} = \frac{1}{2}kx$, as indicated in Figure 7–11. Therefore, the work is

$$W = \tfrac{1}{2}kx(x) = \tfrac{1}{2}kx^2$$

As expected, our result agrees with Equation 7–8.

Finally, when you stretch or compress a spring from its equilibrium position, the work you do is always positive. The work done *by* a spring, however, may be positive or negative, depending on the situation. For example, consider a block sliding to the right with an initial speed v_0 on a smooth, horizontal surface, as shown in Figure 7–12 (a). When the block begins to compress the spring, as in Figure 7–12 (b), the spring exerts a force on the block to the left—that is, opposite to the block's direction of motion. As a result, the spring does *negative* work on the block, which causes the block's speed to decrease. Eventually the negative work done by the spring, $W = -\frac{1}{2}kx^2$, is equal in magnitude to the initial kinetic energy of the block. At this point, Figure 7–12 (c), the block comes to rest momentarily, and $W = \Delta K = K_f - K_i = 0 - K_i = -K_i = -\frac{1}{2}mv_0^2 = -\frac{1}{2}kx^2$. We apply this result in Active Example 7–1.

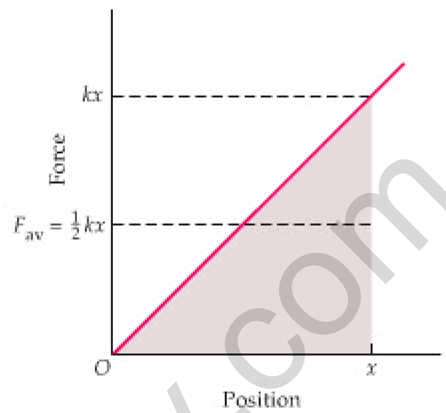
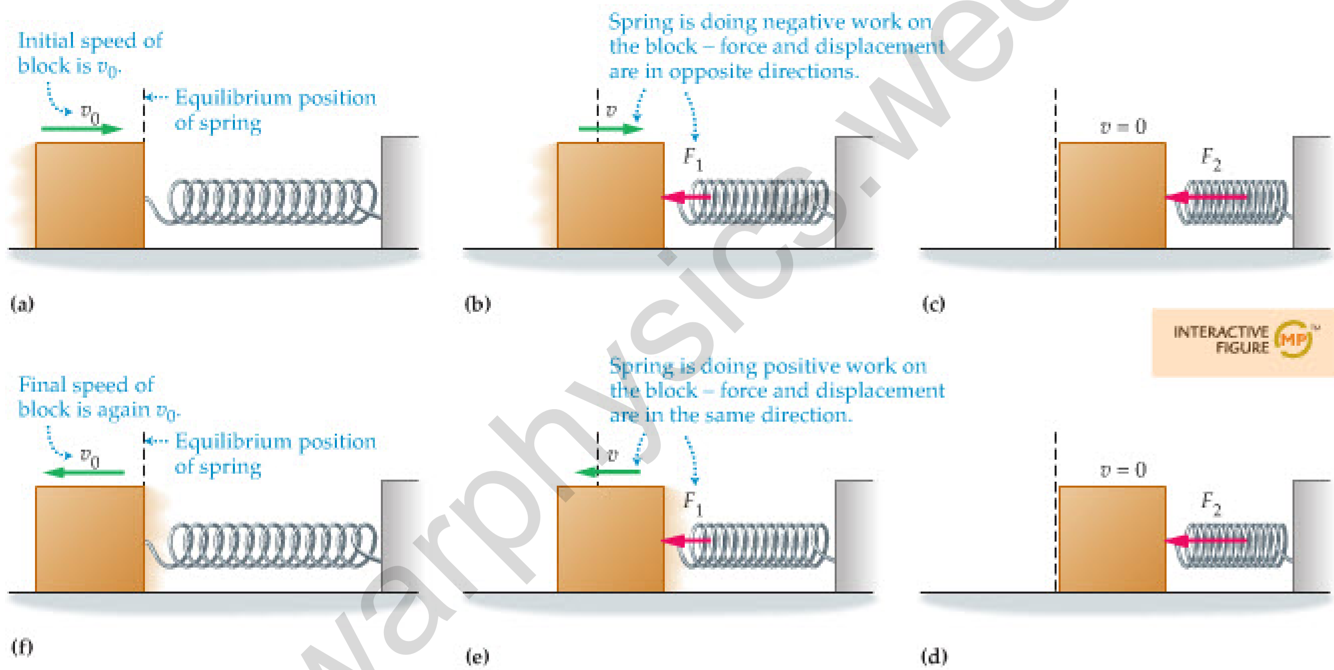


FIGURE 7–11 Work done in stretching a spring: average force

The average force of a spring from $x = 0$ to x is $F_{av} = \frac{1}{2}kx$, and the work done is $W = F_{av}d = \frac{1}{2}kx(x) = \frac{1}{2}kx^2$.



INTERACTIVE FIGURE

FIGURE 7–12 The work done by a spring can be positive or negative

(a) A block slides to the right on a frictionless surface with a speed v_0 until it encounters a spring. **(b)** The spring now exerts a force to the left—opposite to the block's motion—and hence it does negative work on the block. This causes the block's speed to decrease. **(c)** The negative work done by the spring eventually is equal in magnitude to the block's initial kinetic energy, at which point the block comes to rest momentarily. As the spring expands, **(d)** and **(e)**, it does positive work on the block and increases its speed. **(f)** When the block leaves the spring its speed is again equal to v_0.

ACTIVE EXAMPLE 7–1 A BLOCK COMPRESSES A SPRING

Suppose the block in Figure 7–12 (a) has a mass of 1.5 kg and moves with an initial speed of $v_0 = 2.2$ m/s. Find the compression of the spring, whose force constant is 475 N/m, when the block momentarily comes to rest.

SOLUTION *(Test your understanding by performing the calculations indicated in each step.)*

1. Calculate the initial and final kinetic energies of the block: $K_i = 3.6$ J, $K_f = 0$
2. Calculate the change in kinetic energy of the block: $\Delta K = -3.6$ J
3. Set the negative work done by the spring equal to the change in kinetic energy of the block: $-\frac{1}{2}kx^2 = \Delta K = -3.6$ J
4. Solve for the compression, x, and substitute numerical values: $x = 0.12$ m

CONTINUED ON NEXT PAGE