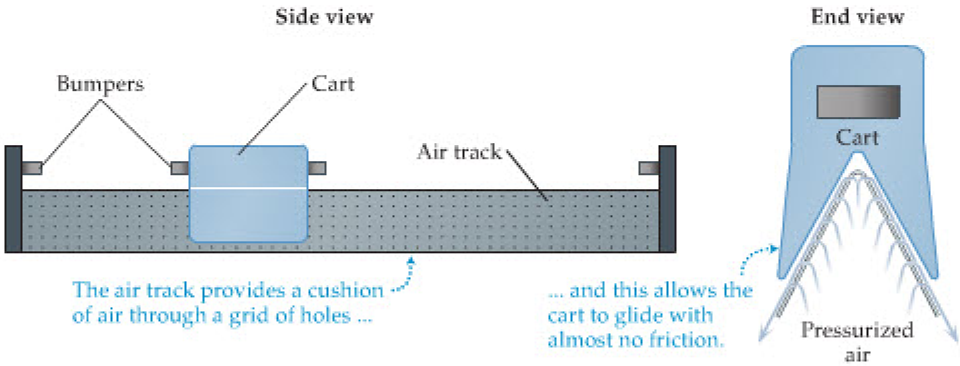


FIGURE 5–1 The air track
An air track provides a cushion of air on which a cart can ride with virtually no friction.

exerts a force on the cart, causing it to change its direction of motion. After bouncing off the bumper, the cart again moves with constant velocity. If the track could be extended to infinite length, and could be made perfectly frictionless, the cart would simply keep moving with constant velocity forever.

Newton's first law of motion summarizes these observations in the following statements:

Newton's First Law

An object at rest remains at rest as long as no net force acts on it.

An object moving with constant velocity continues to move with the same speed and in the same direction as long as no net force acts on it.

Notice the recurring phrase, "no net force," in these statements. It is important to realize that this can mean one of two things: (i) no force acts on the object; or (ii) forces act on the object, but they sum to zero. We shall see examples of the second possibility later in this chapter and again in the next chapter.

Newton's first law, which was first enunciated by Galileo, is also known as the **law of inertia,** which is appropriate since the literal meaning of the word *inertia* is "laziness." Speaking loosely, we can say that matter is "lazy," in that it won't change its motion unless forced to do so. For example, if an object is at rest, it won't start moving on its own. If an object is already moving with constant velocity, it won't alter its speed *or* direction, unless a force causes the change. We call this property of matter its inertia.

According to Newton's first law, being at rest and moving with constant velocity are actually equivalent. To see this, imagine two observers: one is in a train moving with constant velocity; the second is standing next to the tracks, at rest on the ground. The observer in the train places an ice cube on a dinner tray. From that person's point of view—that is, in that person's **frame of reference**—the ice cube has no net force acting on it and it is at rest on the tray. It obeys the first law. In the frame of reference of the observer on the ground, the ice cube has no net force on it and it moves with constant velocity. This also agrees with the first law. Thus Newton's first law holds for both observers: They both see an ice cube with zero net force moving with constant velocity—it's just that for the first observer the constant velocity happens to be zero.

In this example, we say that each observer is in an **inertial frame of reference;** that is, a frame of reference in which the law of inertia holds. In general, if one frame is an inertial frame of reference, then any frame of reference that moves with constant velocity relative to that frame is also an inertial frame of reference. Thus, if an object moves with constant velocity in one inertial frame, it is always possible to find another inertial frame in which the object is at rest. It is in this sense that there really isn't any difference between being at rest and moving with constant velocity. It's all relative—relative to the frame of reference the object is viewed from.

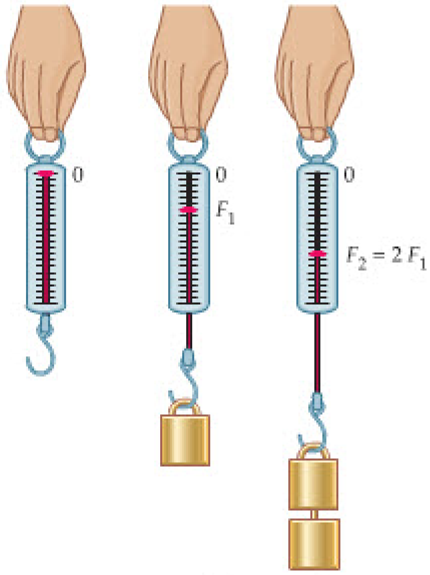
This gives us a more compact statement of the first law:

If the net force on an object is zero, its velocity is constant.

An air track provides a nearly frictionless environment for experiments involving linear motion.

As an example of a frame of reference that is not inertial, imagine that the train carrying the first observer suddenly comes to a halt. From the point of view of that observer, there is still no net force on the ice cube. However, because of the rapid braking, the ice cube flies off the tray. In fact, the ice cube simply continues to move forward with the same constant velocity while the *train* comes to rest. To the observer on the train, it appears that the ice cube has accelerated forward, even though no force acts on it, which is in violation of Newton's first law.

In general, any frame that accelerates relative to an inertial frame is a noninertial frame. The surface of the Earth accelerates slightly, due to its rotational and orbital motions, but since the acceleration is so small, it may be considered an excellent approximation to an inertial frame of reference. Unless specifically stated otherwise, we will always consider the surface of the Earth to be an inertial frame.



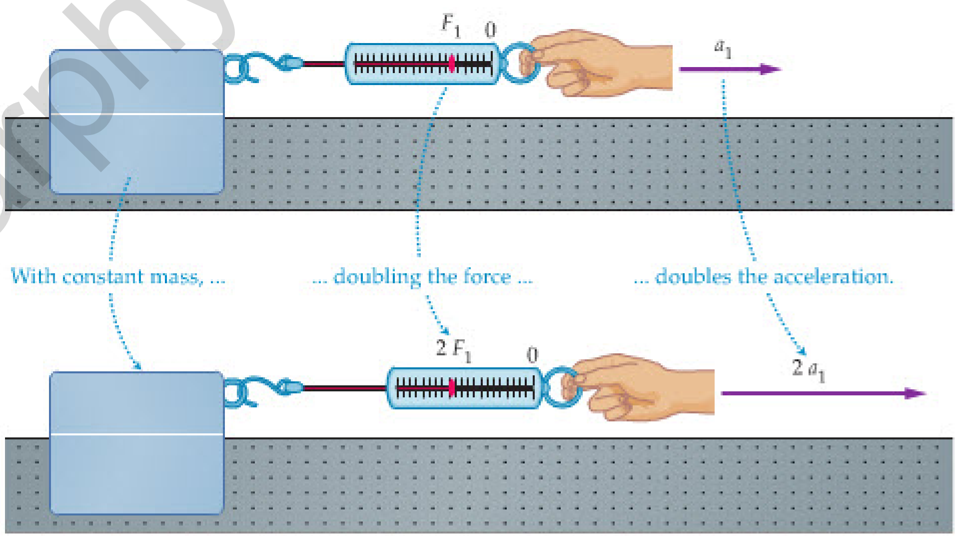
▲ **FIGURE 5–2 Calibrating a "force meter"** With two weights, the force exerted by the scale is twice the force exerted when only a single weight is attached.

5–3 Newton's Second Law of Motion

To hold an object in your hand, you have to exert an upward force to oppose, or "balance," the force of gravity. If you suddenly remove your hand so that the only force acting on the object is gravity, it accelerates downward, as discussed in Chapter 2. This is one example of Newton's second law, which states, basically, that unbalanced forces cause accelerations.

To explore this in more detail, consider a spring scale of the type used to weigh fish. The scale gives a reading of the force, F, exerted by the spring contained within it. If we hang one weight from the scale, it gives a reading that we will call F_1. If two identical weights are attached, the scale reads $F_2 = 2F_1$, as indicated in **Figure 5–2.** With these two forces marked on the scale, we are ready to perform some force experiments.

First, attach the scale to an air-track cart, as in **Figure 5–3.** If we pull with a force F_1, we observe that the cart accelerates at the rate a_1. If we now pull with a force $F_2 = 2F_1$, the acceleration we observe is $a_2 = 2a_1$. Thus, the acceleration is proportional to the force—the greater the force, the greater the acceleration.



▶ **FIGURE 5–3 Acceleration is proportional to force** The spring calibrated in Figure 5–2 is used to accelerate a mass on a "frictionless" air track. If the force is doubled, the acceleration is also doubled.

Second, instead of doubling the force, let's double the mass of the cart by connecting two together, as in **Figure 5–4.** In this case, if we pull with a force F_1 we find an acceleration equal to $\frac{1}{2}a_1$. Thus, the acceleration is inversely proportional to mass—the greater the mass, the less the acceleration.

Combining these results, we find that in this simple case—with just one force in just one direction—the acceleration is given by

$$a = \frac{F}{m}$$

Rearranging the equation yields the form of Newton's law that is perhaps best known, $F = ma$.

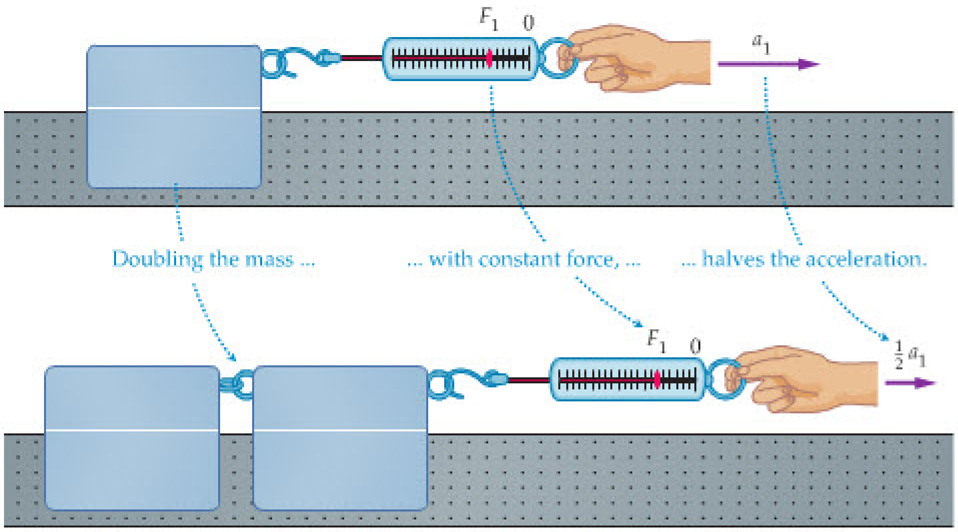


FIGURE 5–4 Acceleration is inversely proportional to mass
If the mass of an object is doubled but the force remains the same, the acceleration is halved.

In general, there may be several forces acting on a given mass, and these forces may be in different directions. Thus, we replace F with the sum of the force vectors acting on a mass:

$$\text{sum of force vectors} = \vec{\mathbf{F}}_{\text{net}} = \sum \vec{\mathbf{F}}$$

The notation, $\Sigma\vec{\mathbf{F}}$, which uses the Greek letter sigma (Σ), is read "sum $\vec{\mathbf{F}}$." Recalling that acceleration is also a vector, we arrive at the formal statement of Newton's second law of motion:

Newton's Second Law

$$\vec{\mathbf{a}} = \frac{\sum \vec{\mathbf{F}}}{m} \quad \text{or} \quad \sum \vec{\mathbf{F}} = m\vec{\mathbf{a}} \qquad 5\text{–}1$$

In words:

If an object of mass m is acted on by a net force $\Sigma\vec{\mathbf{F}}$, it will experience an acceleration $\vec{\mathbf{a}}$ that is equal to the net force divided by the mass. Because the net force is a vector, the acceleration is also a vector. In fact, the direction of an object's acceleration is the *same* as the direction of the net force acting on it.

One should note that Newton's laws cannot be derived from anything more basic. In fact, this is what we mean by a law of nature. The validity of Newton's laws, and all other laws of nature, comes directly from comparisons with experiment.

In terms of vector components, an equivalent statement of the second law is:

$$\sum F_x = ma_x \qquad \sum F_y = ma_y \qquad \sum F_z = ma_z \qquad 5\text{–}2$$

Note that Newton's second law holds independently for each coordinate direction. This component form of the second law is particularly useful when solving problems.

Let's pause for a moment to consider an important special case of the second law. Suppose an object has zero net force acting upon it. This may be because no forces act on it at all, or because it is acted on by forces whose vector sum is zero. In either case, we can state this mathematically as:

$$\sum \vec{\mathbf{F}} = 0$$

Now, according to Newton's second law, we conclude that the acceleration of this object must be zero:

$$\vec{\mathbf{a}} = \frac{\sum \vec{\mathbf{F}}}{m} = \frac{0}{m} = 0$$

But if an object's acceleration is zero, its velocity must be constant. In other words, if the net force on an object is zero, the object moves with constant velocity. This is

▲ Even though the tugboat exerts a large force on this ship, the ship's acceleration is small. This is because the acceleration of an object is inversely proportional to its mass, and the mass of the ship is enormous. The force exerted on the unfortunate hockey player is much smaller. The resulting acceleration is much larger, however, due to the relatively small mass of the player compared to that of the ship.

Newton's first law. Thus we see that Newton's first and second laws are consistent with one another.

Forces are measured in units called, appropriately enough, the **newton (N).** In particular, one newton is defined as the force required to give one kilogram of mass an acceleration of 1 m/s^2. Thus,

$$1\text{ N} = (1\text{ kg})(1\text{ m/s}^2) = 1\text{ kg}\cdot\text{m/s}^2 \quad 5\text{–}3$$

In everyday terms, a newton is roughly a quarter of a pound. Note that a force in newtons divided by a mass in kilograms has the units of acceleration:

$$\frac{1\text{ N}}{1\text{ kg}} = \frac{1\text{ kg}\cdot\text{m/s}^2}{1\text{ kg}} = 1\text{ m/s}^2 \quad 5\text{–}4$$

Other common units for force are presented in Table 5–2. Typical forces and their magnitudes in newtons are listed in Table 5–3.

TABLE 5–2 Units of Mass, Acceleration, and Force

System of units	Mass	Acceleration	Force
SI	kilogram (kg)	m/s^2	newton (N)
cgs	gram (g)	cm/s^2	dyne (dyn)
British	slug	ft/s^2	pound (lb)

(*Note:* 1 N = 10^5 dyne = 0.225 lb.)

TABLE 5–3 Typical Forces in Newtons (N)

Main engines of space shuttle	31,000,000
Pulling force of locomotive	250,000
Thrust of jet engine	75,000
Force to accelerate a car	7000
Weight of adult human	700
Weight of an apple	1
Weight of a rose	0.1
Weight of an ant	0.001

EXERCISE 5–1

The net force acting on a Jaguar XK8 has a magnitude of 6800 N. If the car's acceleration is 3.8 m/s^2, what is its mass?

SOLUTION

Since the net force and the acceleration are always in the same direction, we can replace the vectors in Equation 5–1 with magnitudes. Solving $\Sigma F = ma$ for the mass yields

$$m = \frac{\sum F}{a} = \frac{6800\text{ N}}{3.8\text{ m/s}^2} = 1800\text{ kg}$$

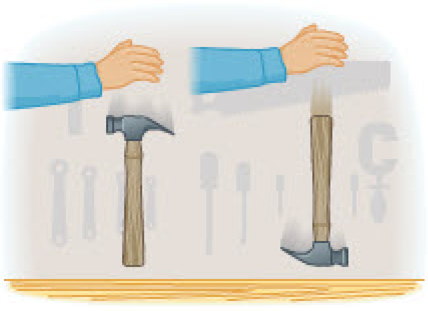
The following Conceptual Checkpoint presents a situation in which both Newton's first and second laws play an important role.

CONCEPTUAL CHECKPOINT 5–1 TIGHTENING A HAMMER

The metal head of a hammer is loose. To tighten it, you drop the hammer down onto a table. Should you **(a)** drop the hammer with the handle end down, **(b)** drop the hammer with the head end down, or **(c)** do you get the same result either way?

REASONING AND DISCUSSION

It might seem that since the same hammer hits against the same table in either case, there shouldn't be a difference. Actually, there is.



In case (a) the handle of the hammer comes to rest when it hits the table, but the head continues downward until a force acts on it to bring it to rest. The force that acts on it is supplied by the handle, which results in the head being wedged more tightly onto the handle. Since the metal head is heavy, the force wedging it onto the handle is great. In case (b) the head of the hammer comes to rest, but the handle continues to move until a force brings it to rest. The handle is lighter than the head, however; thus the force acting on it is less, resulting in less tightening.

ANSWER

(a) Drop the hammer with the handle end down.

A similar effect occurs when you walk—with each step you take you tamp your head down onto your spine, as when dropping a hammer handle end down.

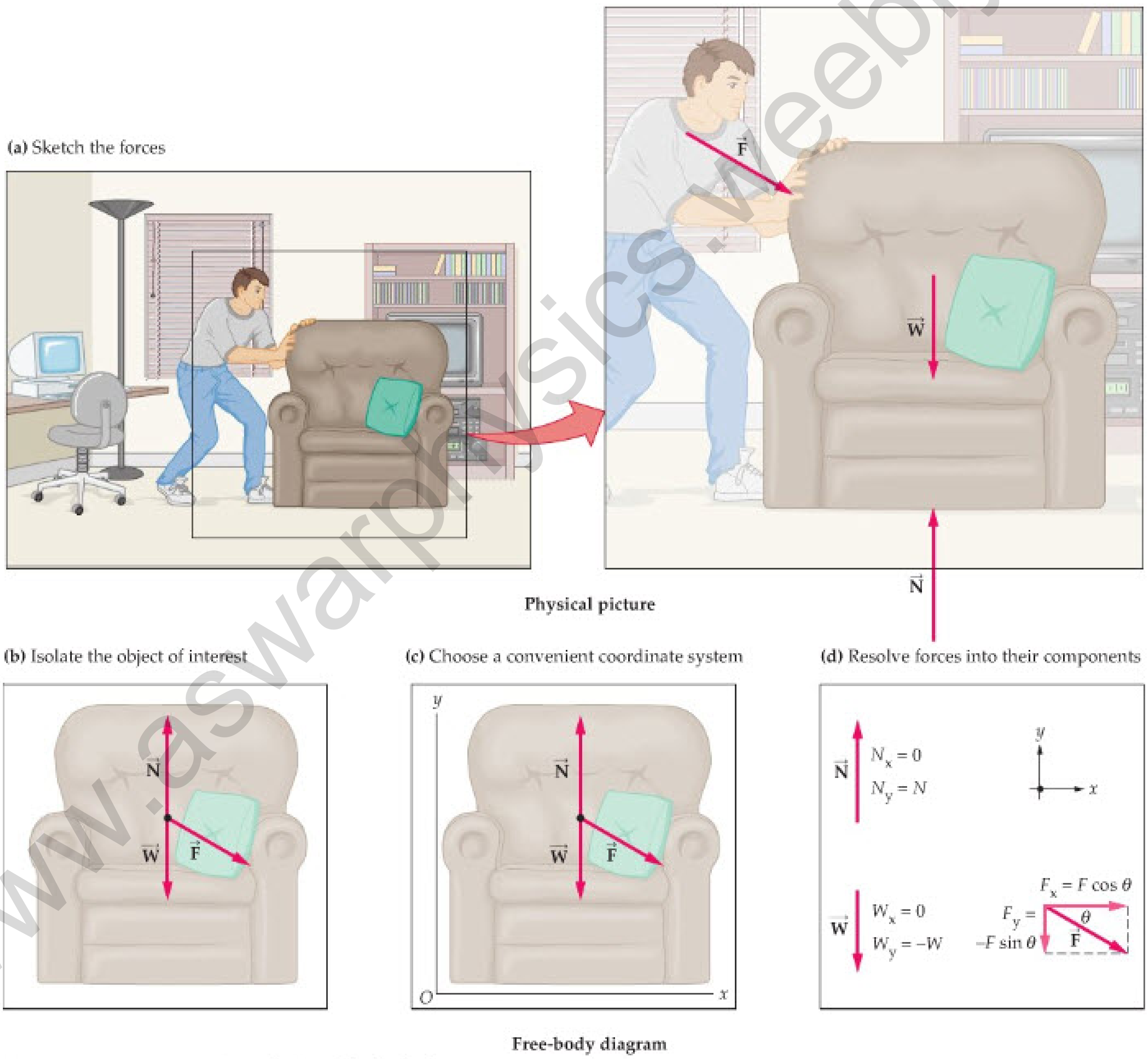
This causes you to grow shorter during the day! Try it. Measure your height first thing in the morning, then again before going to bed. If you're like many people, you'll find that you have shrunk by an inch or so during the day.

REAL-WORLD PHYSICS: BIO
How walking affects your height

Free-Body Diagrams

When solving problems involving forces and Newton's laws, it is essential to begin by making a sketch that indicates *each and every external force* acting on a given object. This type of sketch is referred to as a **free-body diagram.** If we are concerned only with nonrotational motion, as is the case in this and the next chapter, we treat the object of interest as a point particle and apply each of the forces acting on the object to that point, as Figure 5–5 shows. Once the forces are drawn, we choose a coordinate system and resolve each force into components. At this point, Newton's second law can be applied to each coordinate direction separately.

PROBLEM-SOLVING NOTE
External Forces
External forces acting on an object fall into two main classes: (i) Forces at the point of contact with another object, and (ii) forces exerted by an external agent, such as gravity.



▲ **FIGURE 5–5 Constructing and using a free-body diagram**
The four basic steps in constructing and using a free-body diagram are illustrated in these sketches. **(a)** Sketch all of the external forces acting on an object of interest. Note that only forces acting *on* the object are shown; none of the forces exerted *by* the object are included. **(b)** Isolate the object and treat it as a point particle. **(c)** Choose a convenient coordinate system. This will often mean aligning a coordinate axis to coincide with the direction of one or more forces in the system. **(d)** Resolve each of the forces into components using the coordinate system of part (c).

For example, in Figure 5–5 there are three external forces acting on the chair. One is the force $\vec{F}$ exerted by the person. In addition, gravity exerts a downward force, $\vec{W}$, which is simply the weight of the chair. Finally, the floor exerts an upward force on the chair that prevents it from falling toward the center of the Earth. This force is referred to as the *normal force*, $\vec{N}$, because it is perpendicular (that is, normal) to the surface of the floor. We will consider the weight and the normal force in greater detail in Sections 5–6 and 5–7, respectively.

We can summarize the steps involved in constructing a free-body diagram as follows:

Sketch the Forces
Identify and sketch all of the external forces acting on an object. Sketching the forces roughly to scale will help in estimating the direction and magnitude of the net force.

Isolate the Object of Interest
Replace the object with a point particle of the same mass. Apply each of the forces acting on the object to that point.

Choose a Convenient Coordinate System
Any coordinate system will work; however, if the object moves in a known direction, it is often convenient to pick that direction for one of the coordinate axes. Otherwise, it is reasonable to choose a coordinate system that aligns with one or more of the forces acting on the object.

Resolve the Forces into Components
Determine the components of each force in the free-body diagram.

Apply Newton's Second Law to Each Coordinate Direction
Analyze motion in each coordinate direction using the component form of Newton's second law, as given in Equation 5–2.

These basic steps are illustrated in Figure 5–5. Note that the figures in this chapter use the labels "Physical picture" to indicate a sketch of the physical situation and "Free-body diagram" to indicate a free-body sketch.

We start by applying this procedure to a simple one-dimensional example, saving two-dimensional systems for Section 5–5. Suppose, for instance, that you hold a book at rest in your hand. What is the magnitude of the upward force that your hand must exert to keep the book at rest? From everyday experience, we expect that the upward force must be equal in magnitude to the weight of the book, but let's see how this result can be obtained directly from Newton's second law.

We begin with a sketch of the physical situation, as shown in **Figure 5–6 (a)**. The corresponding free-body diagram, in **Figure 5–6 (b)**, shows just the book, represented by a point, and the forces acting on it. Note that two forces act on the book: (i) the downward force of gravity, $\vec{W}$, and (ii) the upward force, $\vec{F}$, exerted by your hand. Only the forces acting *on* the book are included in the free-body diagram.

Now that the free-body diagram is drawn, we indicate a coordinate system so that the forces can be resolved into components. In this case all the forces are vertical. Thus we draw a y axis in the vertical direction in Figure 5–6 (b). Note that we have chosen upward to be the positive direction. With this choice, the y components of the forces are $F_y = F$ and $W_y = -W$. It follows that

$$\sum F_y = F - W$$

Using the y component of the second law ($\Sigma F_y = ma_y$) we find

$$F - W = ma_y$$

Since the book remains at rest, its acceleration is zero. Thus, $a_y = 0$, which gives

$$F - W = ma_y = 0 \qquad \text{or} \qquad F = W$$

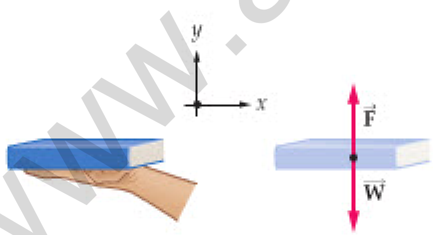
as expected.

Next, we consider a situation where the net force acting on an object is nonzero, meaning that its acceleration is also nonzero.

PROBLEM-SOLVING NOTE

Picture the Problem

In problems involving Newton's laws, it is important to begin with a free-body diagram and to identify all the external forces that act on an object. Once these forces are identified and resolved into their components, Newton's laws can be applied in a straightforward way. It is crucial, however, that only external forces acting on the object be included, and that none of the external forces be omitted.



(a) Physical picture (b) Free-body diagram

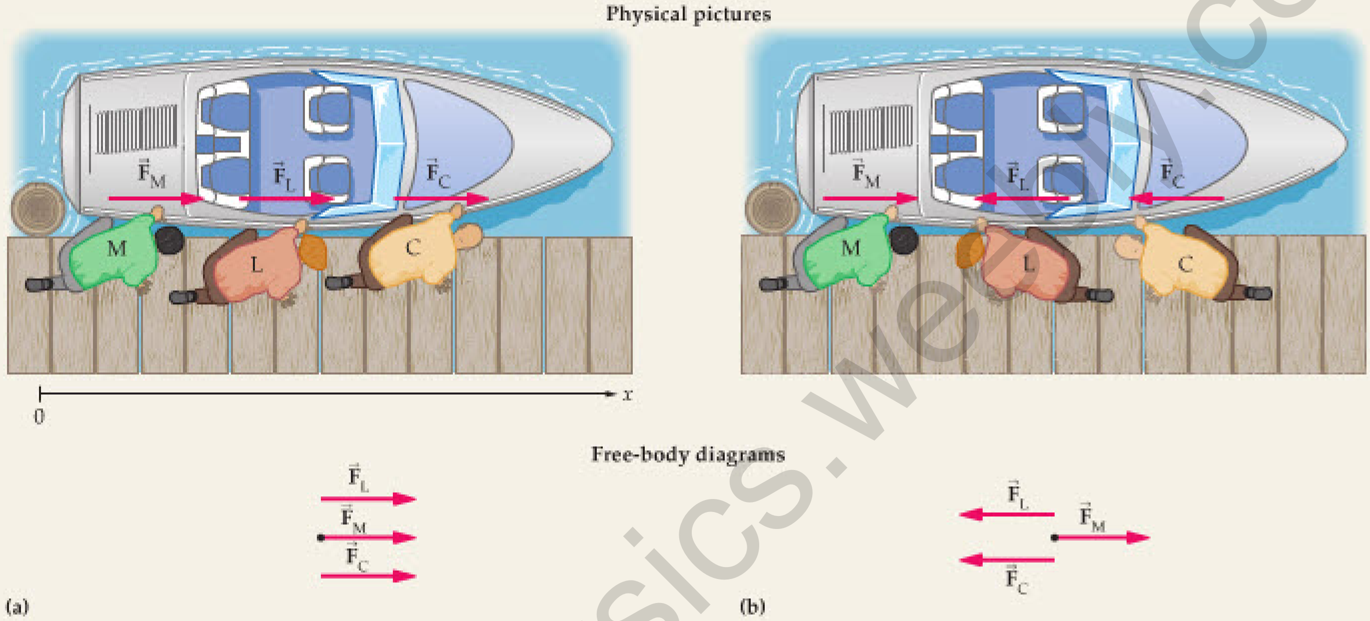
▲ **FIGURE 5–6 A book supported in a person's hand**
(a) The physical situation. **(b)** The free-body diagram for the book, showing the two external forces acting on it. We also indicate our choice for a coordinate system.

EXAMPLE 5–1 THREE FORCES

Moe, Larry, and Curly push on a 752-kg boat that floats next to a dock. They each exert an 80.5-N force parallel to the dock. **(a)** What is the acceleration of the boat if they all push in the same direction? Give both direction and magnitude. **(b)** What are the magnitude and direction of the boat's acceleration if Larry and Curly push in the opposite direction to Moe's push?

PICTURE THE PROBLEM

In our sketch we indicate the three relevant forces acting on the boat: $\vec{F}_M$, $\vec{F}_L$, and $\vec{F}_C$. Note that we have chosen the positive x direction to the right, in the direction that all three push for part (a). Therefore, all three forces have a positive x component in part (a). In part (b), however, the forces exerted by Larry and Curly have negative x components.



STRATEGY

Since we know the mass of the boat and the forces acting on it, we can find the acceleration using $\sum F_x = ma_x$. Even though this problem is one-dimensional, it is important to think of it in terms of vector components. For example, when we sum the x components of the forces, we are careful to use the appropriate signs—just as we always do when dealing with vectors.

SOLUTION

Part (a)

1. Write out the x component for each of the three forces: $F_{M,x} = F_{L,x} = F_{C,x} = 80.5\text{ N}$

2. Sum the x components of force and set equal to ma_x: $\sum F_x = F_{M,x} + F_{L,x} + F_{C,x} = 241.5\text{ N} = ma_x$

3. Divide by the mass to find a_x. Since a_x is positive, the acceleration is to the right, as expected: $a_x = \frac{\sum F_x}{m} = \frac{241.5\text{ N}}{752\text{ kg}} = 0.321\text{ m/s}^2$

Part (b)

4. Again, start by writing the x component for each force: $F_{M,x} = 80.5\text{ N}$; $F_{L,x} = F_{C,x} = -80.5\text{ N}$

5. Sum the x components of force and set equal to ma_x:

$$\sum F_x = F_{M,x} + F_{L,x} + F_{C,x} = 80.5\text{ N} - 80.5\text{ N} - 80.5\text{ N} = -80.5\text{ N} = ma_x$$

6. Solve for a_x. In this case a_x is negative, indicating an acceleration to the left: $a_x = \frac{\sum F_x}{m} = \frac{-80.5\text{ N}}{752\text{ kg}} = -0.107\text{ m/s}^2$

INSIGHT

The results of this Example are in agreement with everyday experience: three forces in the same direction cause more acceleration than three forces in opposing directions. The method of using vector components and being careful about their signs gives the expected results in a simple situation like this, and also works in more complicated situations where everyday experience may be of little help.

PRACTICE PROBLEM

If Moe, Larry, and Curly all push to the right with 85.0-N forces, and the boat accelerates at 0.530 m/s^2, what is its mass? **[Answer:** 481 kg]

Some related homework problems: Problem 2, Problem 4

In some problems, we are given information that allows us to calculate an object's acceleration using the kinematic equations of Chapters 2 and 4. Once the acceleration is known, the second law can be used to find the net force that caused the acceleration.

REAL-WORLD PHYSICS
Astronaut jet packs

For example, suppose that an astronaut uses a jet pack to push a satellite toward the space shuttle. These jet packs, which are known to NASA as Manned Maneuvering Units, or MMUs, are basically small "one-person rockets" strapped to the back of an astronaut's spacesuit. An MMU contains pressurized nitrogen gas that can be released through varying combinations of 24 nozzles spaced around the unit, producing a force of about 10 pounds. The MMUs contain enough propellant for a six-hour EVA (extra-vehicular activity).

We show the physical situation in **Figure 5–7 (a)**, where an astronaut pushes on a 655-kg satellite. The corresponding free-body diagram for the satellite is shown in **Figure 5–7 (b)**. Note that we have chosen the x axis to point in the direction of the push. Now, if the satellite starts at rest and moves 0.675 m after 5.00 seconds of pushing, what is the force, F, exerted on it by the astronaut?

(a) Physical picture (b) Free-body diagram

FIGURE 5–7 An astronaut using a jet pack to push a satellite
(a) The physical situation. **(b)** The free-body diagram for the satellite. Only one force acts on the satellite, and it is in the positive x direction.

Clearly, we would like to use Newton's second law (basically, $\vec{\mathbf{F}} = m\vec{\mathbf{a}}$) to find the force, but we know only the mass of the satellite, not its acceleration. We can find the acceleration, however, by assuming constant acceleration (after all, the force is constant) and using the kinematic equation relating position to time: $x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2$. We can choose the initial position of the satellite to be $x_0 = 0$, and we are given that it starts at rest, thus $v_{0x} = 0$. Hence,

$$x = \frac{1}{2}a_x t^2$$

Since we know the distance covered in a given time, we can solve for the acceleration:

$$a_x = \frac{2x}{t^2} = \frac{2(0.675\ \text{m})}{(5.00\ \text{s})^2} = 0.0540\ \text{m/s}^2$$

Now that kinematics has provided the acceleration, we use the x component of the second law to find the force. Only one force acts on the satellite, and its x component is F; thus,

$$\sum F_x = F = ma_x$$

$$F = ma_x = (655\ \text{kg})(0.0540\ \text{m/s}^2) = 35.4\ \text{N}$$

This force corresponds to a push of about 8 lb.

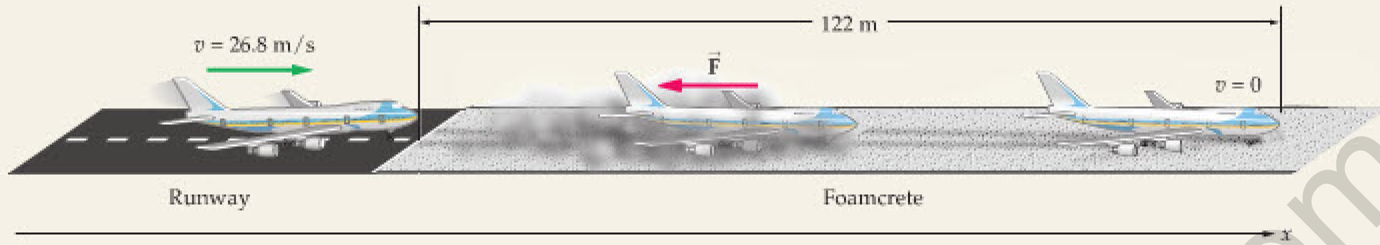
Another problem in which we use kinematics to find the acceleration is presented in the following Active Example.

▲ A technician inspects the landing gear of an airliner in a test of Foamcrete, a solid paving material that is just soft enough to collapse under the weight of an airliner. A plane that has run off the runway will slow safely to a stop as its wheels plow through the crumbling Foamcrete.

ACTIVE EXAMPLE 5–1 THE FORCE EXERTED BY FOAMCRETE

REAL-WORLD PHYSICS

Foamcrete is a substance designed to stop an airplane that has run off the end of a runway, without causing injury to passengers. It is solid enough to support a car, but crumbles under the weight of a large airplane. By crumbling, it slows the plane to a safe stop. For example, suppose a 747 jetliner with a mass of 1.75×10^5 kg and an initial speed of 26.8 m/s is slowed to a stop in 122 m. What is the magnitude of the average retarding force $\vec{\mathbf{F}}$ exerted by the Foamcrete on the plane?



SOLUTION *(Test your understanding by performing the calculations indicated in each step.)*

1. Use $v^2 = v_0^2 + 2a_x\,\Delta x$ to find the plane's average acceleration: $a_x = -2.94\ \text{m/s}^2$
2. Sum the forces in the x direction. Let F represent the magnitude of the force $\vec{F}$: $\sum F_x = -F$
3. Set the sum of forces equal to mass times acceleration: $-F = ma_x$
4. Solve for the magnitude of the average force, F: $F = -ma_x = 5.15 \times 10^5\ \text{N}$

INSIGHT
Though the plane moves in the positive direction, its acceleration, and the net force exerted on it, are in the negative direction. As a result, the plane's speed decreases with time.

YOUR TURN
Find the plane's stopping distance if the magnitude of the average force exerted by the Foamcrete is doubled.

(Answers to **Your Turn** *problems are given in the back of the book.)*

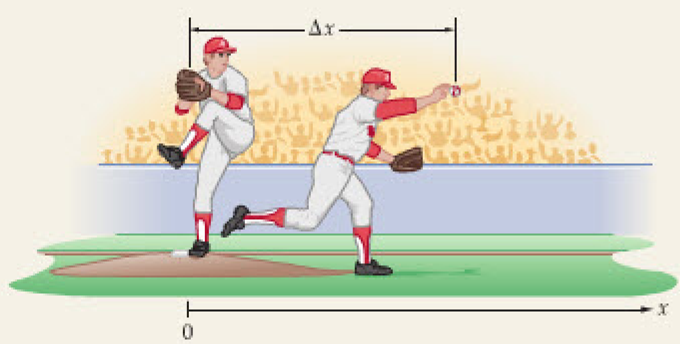
Note again the care we take with the signs. The plane's acceleration is negative, hence the net force acting on it, $\vec{F}$, is in the negative x direction. On the other hand, the magnitude of the force, F, is positive, as is always the case for magnitudes.

Finally, we end this section with an estimation problem.

EXAMPLE 5–2 PITCH MAN: ESTIMATE THE FORCE ON THE BALL

A pitcher throws a 0.15-kg baseball, accelerating it from rest to a speed of about 90 mi/h. Estimate the force exerted by the pitcher on the ball.

PICTURE THE PROBLEM
We choose the x axis to point in the direction of the pitch. Also indicated in the sketch is the distance over which the pitcher accelerates the ball, Δx. Since we are interested only in the pitch, and not in the subsequent motion of the ball, we ignore the effects of gravity.



STRATEGY
We know the mass, so we can find the force with $F_x = ma_x$ if we can estimate the acceleration. To find the acceleration, we start with the fact that $v_0 = 0$ and $v \approx 90$ mi/h. In addition, we can see from the sketch that a reasonable estimate for Δx is about 2.0 m. Combining these results with the kinematic equation $v^2 = v_0^2 + 2a_x\Delta x$ yields the acceleration, which we then use to find the force.

SOLUTION

1. Starting with the fact that 60 mi/h = 1 mi/min, perform a rough back-of-the-envelope conversion of 90 mi/h to meters per second: $v \approx 90\ \text{mi/h} = \dfrac{1.5\ \text{mi}}{\text{min}} \approx \dfrac{2400\ \text{m}}{60\ \text{s}} = 40\ \text{m/s}$
2. Solve $v^2 = v_0^2 + 2a_x\,\Delta x$ for the acceleration, a_x. Use the estimates $\Delta x \approx 2.0$ m and $v \approx 40$ m/s: $a_x = \dfrac{v^2 - v_0^2}{2\,\Delta x} \approx \dfrac{(40\ \text{m/s})^2 - 0}{2(2.0\ \text{m})} = 400\ \text{m/s}^2$
3. Find the corresponding force with $F_x = ma_x$: $F_x = ma_x \approx (0.15\ \text{kg})(400\ \text{m/s}^2) = 60\ \text{N} \approx 10\ \text{lb}$

INSIGHT
On the one hand, this is a sizable force, especially when you consider that the ball itself weighs only about 1/3 lb. Thus, the pitcher exerts a force on the ball that is about 30 times greater than the force exerted by Earth's gravity. It follows that ignoring gravity during the pitch is a reasonable approximation.

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On the other hand, you might say that 10 lb isn't that much force for a person to exert. That's true, but this force is being exerted with an average speed of about 20 m/s, which means that the pitcher is actually generating about 1.5 horsepower—a sizable power output for a person. We will cover power in detail in Chapter 7, and relate it to human capabilities.

PRACTICE PROBLEM

What is the approximate speed of the pitch if the force exerted by the pitcher is $\frac{1}{2}(60\text{ N}) = 30\text{ N}$? **[Answer:** 30 m/s or 60 mi/h]

Some related homework problems: Problem 5, Problem 8

Another way to find the acceleration is to estimate the amount of time it takes to make the pitch. However, since the pitch is delivered so quickly—about 1/10 s—estimating the time would be more difficult than estimating the distance Δx.

5–4 Newton's Third Law of Motion

Nature never produces just one force at a time; *forces always come in pairs*. In addition, the forces in a pair, which always act on *different objects*, are equal in magnitude and opposite in direction. This is Newton's third law of motion.

Newton's Third Law

For every force that acts on an object, there is a reaction force acting on a different object that is equal in magnitude and opposite in direction.

In a somewhat more specific form:

If object 1 exerts a force $\vec{F}$ on object 2, then object 2 exerts a force $-\vec{F}$ on object 1.

This law, more commonly known by its abbreviated form, "for every action there is an equal and opposite reaction," completes Newton's laws of motion.

Figure 5–8 illustrates some action-reaction pairs. Notice that there is always a reaction force, whether the action force pushes on something hard to move, like a refrigerator, or on something that moves with no friction, like an air-track cart. In some cases, the reaction force tends to be overlooked, as when the Earth exerts a *downward* gravitational force on the space shuttle, and the shuttle exerts an equal and opposite *upward* gravitational force on the Earth. Still, the reaction force always exists.

Another important aspect of the third law is that the action-reaction forces always act on *different* objects. This, again, is illustrated in Figure 5–8. Thus, in drawing a free-body diagram, only one of the action-reaction pair of forces would be drawn for a given object. The other force in the pair would appear in the free-body diagram of a different object. As a result, *the two forces do not cancel*.

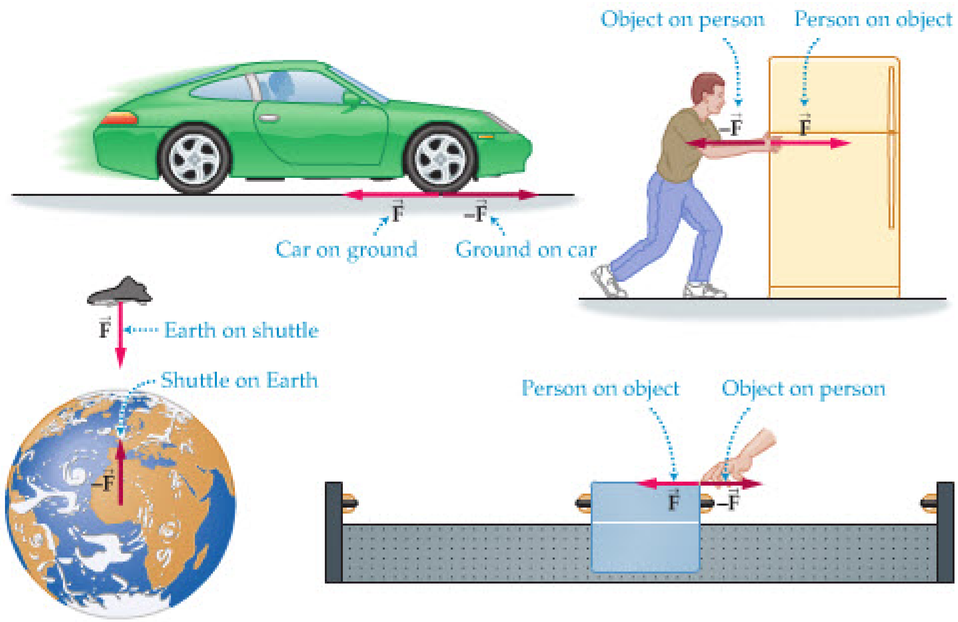


FIGURE 5–8 Examples of action-reaction force pairs

For example, consider a car accelerating from rest, as in Figure 5–8. As the car's engine turns the wheels, the tires exert a force on the road. By the third law, the road exerts an equal and opposite force on the car's tires. It is this second force—which acts on the car through its tires—that propels the car forward. The force exerted by the tires on the road does not accelerate the car.

Since the action-reaction forces act on different objects, they generally produce different accelerations. This is the case in the next Example.

EXAMPLE 5–3 TIPPY CANOE

Two groups of canoeists meet in the middle of a lake. After a brief visit, a person in canoe 1 pushes on canoe 2 with a force of 46 N to separate the canoes. If the mass of canoe 1 and its occupants is $m_1 = 150$ kg, and the mass of canoe 2 and its occupants is $m_2 = 250$ kg, **(a)** find the acceleration the push gives to each canoe. **(b)** What is the separation of the canoes after 1.2 s of pushing?

PICTURE THE PROBLEM

We have chosen the positive x direction to point from canoe 1 to canoe 2. With this choice, the force exerted on canoe 2 is $\vec{\mathbf{F}}_2 = (+46\text{ N})\hat{\mathbf{x}}$. By Newton's third law, the force exerted on the person in canoe 1, and thus on canoe 1 itself if the person is firmly seated, is $\vec{\mathbf{F}}_1 = (-46\text{ N})\hat{\mathbf{x}}$. For convenience, we have placed the origin at the point where the canoes touch.

Physical picture   Free-body diagrams

STRATEGY

From Newton's third law, the force on canoe 1 is equal in magnitude to the force on canoe 2—the masses of the canoes are different, however, and therefore their accelerations are different as well. **(a)** We can find the acceleration of each canoe by solving $\sum F_x = ma_x$ for a_x. **(b)** The kinematic equation relating position to time, $x = x_0 + v_{0x}t + \frac{1}{2}a_xt^2$, can then be used to find the displacement of each canoe.

SOLUTION

Part (a)

1. Use Newton's second law to find the acceleration of canoe 2:

$$a_{2,x} = \frac{\sum F_{2,x}}{m_2} = \frac{46\text{ N}}{250\text{ kg}} = 0.18\text{ m/s}^2$$

2. Do the same calculation for canoe 1. Note that the acceleration of canoe 1 is in the negative direction:

$$a_{1,x} = \frac{\sum F_{1,x}}{m_1} = \frac{-46\text{ N}}{150\text{ kg}} = -0.31\text{ m/s}^2$$

Part (b)

3. Use $x = x_0 + v_{0x}t + \frac{1}{2}a_xt^2$ to find the position of canoe 2 at $t = 1.2$ s. From the problem statement, we know the canoes start at the origin ($x_0 = 0$) and at rest ($v_{0x} = 0$):

$$x_2 = \tfrac{1}{2}a_{2,x}t^2 = \tfrac{1}{2}(0.18\text{ m/s}^2)(1.2\text{ s})^2 = 0.13\text{ m}$$

4. Repeat the calculation for canoe 1:

$$x_1 = \tfrac{1}{2}a_{1,x}t^2 = \tfrac{1}{2}(-0.31\text{ m/s}^2)(1.2\text{ s})^2 = -0.22\text{ m}$$

5. Subtract the two positions to find the separation of the canoes:

$$x_2 - x_1 = 0.13\text{ m} - (-0.22\text{ m}) = 0.35\text{ m}$$

INSIGHT

The same magnitude of force acts on each canoe; hence the lighter one has the greater acceleration and the greater displacement. If the heavier canoe were replaced by a large ship of great mass, both vessels would still accelerate as a result of the push. However, the acceleration of the large ship would be so small as to be practically imperceptible. In this case, it would appear as if only the canoe moved, whereas, in fact, both vessels move.

PRACTICE PROBLEM

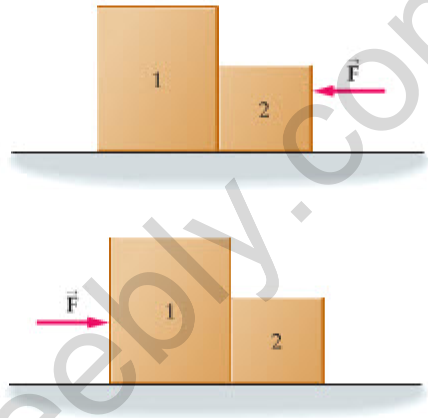
If the mass of canoe 2 is increased, does its acceleration increase, decrease, or stay the same? Check your answer by calculating the acceleration for the case where canoe 2 is replaced by a 25,000-kg ship. [**Answer:** The acceleration will decrease. In this case, $a = 0.0018\text{ m/s}^2$.]

Some related homework problems: Problem 18, Problem 19

When objects are touching one another, the action-reaction forces are often referred to as **contact forces.** The behavior of contact forces is explored in the following Conceptual Checkpoint.

CONCEPTUAL CHECKPOINT 5–2 CONTACT FORCES

Two boxes—one large and heavy, the other small and light—rest on a smooth, level floor. You push with a force $\vec{F}$ on either the small box or the large box. Is the contact force between the two boxes **(a)** the same in either case, **(b)** larger when you push on the large box, or **(c)** larger when you push on the small box?



REASONING AND DISCUSSION

Since the same force pushes on the boxes, you might think the force of contact is the same in both cases. It is not. What we can conclude, however, is that the boxes have the same acceleration in either case—the same net force acts on the same total mass, so the same acceleration, a, results.

To find the contact force between the boxes, we focus our attention on each box individually, and note that *Newton's second law must be satisfied for each of the boxes, just as it is for the entire two-box system.* For example, when the external force is applied to the small box, the only force acting on the *large* box (mass m_1) is the contact force; hence, the contact force must have a magnitude equal to m_1a. In the second case, the only force acting on the *small* box (mass m_2) is the contact force, and so the magnitude of the contact force is m_2a. Since m_1 is greater than m_2, it follows that the force of contact is larger when you push on the small box, m_1a, than when you push on the large box, m_2a.

To summarize, the contact force is larger when *it* must push the larger box.

ANSWER

(c) The contact force is larger when you push on the small box.

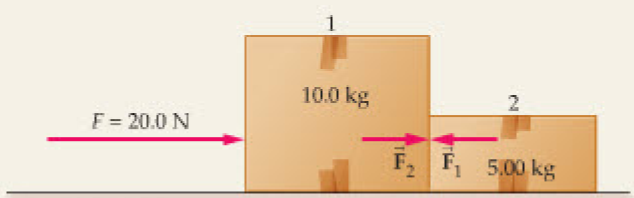
In the next Example, we calculate a numerical value for the contact force in a system similar to that described in Conceptual Checkpoint 5–2. We also show explicitly that Newton's third law is required for a full analysis of this system.

EXAMPLE 5–4 WHEN PUSH COMES TO SHOVE

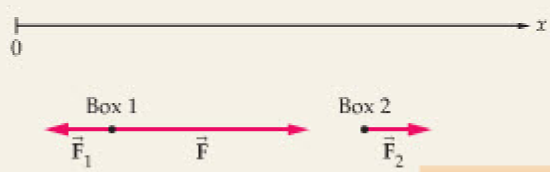
A box of mass $m_1 = 10.0$ kg rests on a smooth, horizontal floor next to a box of mass $m_2 = 5.00$ kg. If you push on box 1 with a horizontal force of magnitude $F = 20.0$ N, **(a)** what is the acceleration of the boxes? **(b)** What is the force of contact between the boxes?

PICTURE THE PROBLEM

We choose the x axis to be horizontal and pointing to the right. Thus, $\vec{F} = (20.0\text{ N})\hat{x}$. The contact forces are labeled as follows: $\vec{F}_1$ is the contact force exerted on box 1; $\vec{F}_2$ is the contact force exerted on box 2. By Newton's third law, the contact forces have the same magnitude, f, but point in opposite directions. With our coordinate system, we have $\vec{F}_1 = -f\hat{x}$ and $\vec{F}_2 = f\hat{x}$.



Physical picture



Free-body diagrams

INTERACTIVE FIGURE

STRATEGY

a. Since the two boxes are in contact, they have the same acceleration. We find this acceleration with Newton's second law; that is, we divide the net horizontal force by the total mass of the two boxes.

b. Now let's consider the system consisting solely of box 2. The mass in this case is 5.00 kg, and the only horizontal force acting on the system is $\vec{F}_2$. Thus, we can find f, the magnitude of $\vec{F}_2$, by requiring that box 2 have the acceleration found in part (a).

SOLUTION

Part (a)

1. Find the net horizontal force acting on the two boxes. Note that $\vec{F}_1$ and $\vec{F}_2$ are equal in magnitude but opposite in direction. Hence, they sum to zero; $\vec{F}_1 + \vec{F}_2 = 0$:

$$\sum_{\substack{\text{both}\\\text{boxes}}} F_x = F = 20.0\text{ N}$$

2. Divide the net force by the total mass, $m_1 + m_2$, to find the acceleration of the boxes:

$$a_x = \frac{\sum F_x}{m_1 + m_2} = \frac{20.0\text{ N}}{(10.0\text{ kg} + 5.00\text{ kg})} = \frac{20.0\text{ N}}{15.0\text{ kg}} = 1.33\text{ m/s}^2$$

Part (b)

3. Find the net horizontal force acting on box 2, and set it equal to the mass of box 2 times its acceleration:

$$\sum_{\text{box2}} F_x = F_{2,x} = f = m_2 a_x$$

4. Determine the magnitude of the contact force, f, by substituting numerical values for m_2 and a_x:

$$f = m_2 a_x = (5.00\text{ kg})(1.33\text{ m/s}^2) = 6.67\text{ N}$$

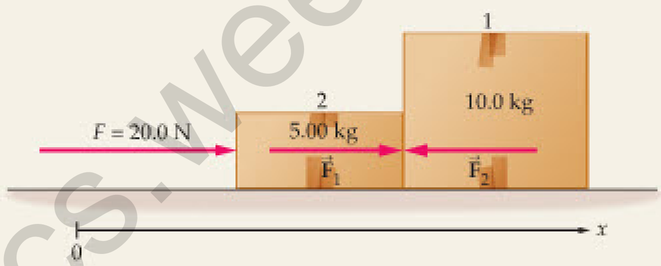
INSIGHT

Since the net horizontal force acting on box 1 is $F - f = 20.0\text{ N} - 6.67\text{ N} = 13.3\text{ N}$, it follows that its acceleration is $(13.3\text{ N})/(10.0\text{ kg}) = 1.33\text{ m/s}^2$. Thus, as expected, box 1 and box 2 have precisely the same acceleration.

If box 2 were not present, the 20.0-N force acting on box 1 would give it an acceleration of 2.00 m/s^2. As it is, the contact force between the boxes slows box 1 so that its acceleration is less than 2.00 m/s^2, and accelerates box 2 so that its acceleration is greater than zero. The precise value of the contact force is simply the value that gives both boxes the same acceleration.

PRACTICE PROBLEM

Suppose the relative positions of the boxes are reversed, so that F pushes on the small box, as shown here. Calculate the contact force for this case, and show that the force is greater than 6.67 N, as expected from Conceptual Checkpoint 5–2. [**Answer:** The contact force in this case is 13.3 N, double its previous value. This follows because the box being pushed has twice the mass of the box that was pushed originally.]



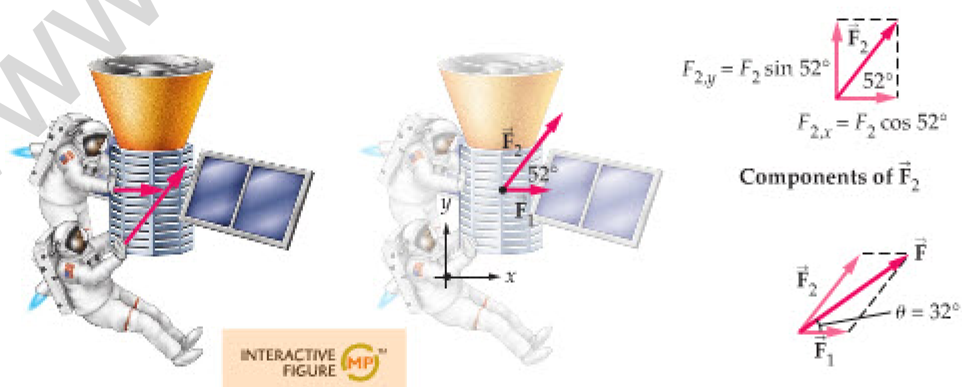
Some related homework problems: Problem 20, Problem 21

5–5 The Vector Nature of Forces: Forces in Two Dimensions

When we presented Newton's second law in Section 5–3, we said that an object's acceleration is equal to the net force acting on it divided by its mass. For example, if only a single force acts on an object, its acceleration is found to be in the same direction as the force. If more than one force acts on an object, experiments show that its acceleration is in the direction of the vector sum of the forces. Thus forces are indeed vectors, and they exhibit all the vector properties discussed in Chapter 3.

The mass of an object, on the other hand, is simply a positive number with no associated direction. It represents the amount of matter in an object.

As an example of the vector nature of forces, suppose two astronauts are using jet packs to push a 940-kg satellite toward the space shuttle, as shown in **Figure 5–9**. With the coordinate system indicated in the figure, astronaut 1 pushes in the positive x direction and astronaut 2 pushes in a direction 52° above the x axis.



(a) Physical picture (b) Free-body diagram Total force

FIGURE 5–9 Two astronauts pushing a satellite with forces that differ in magnitude and direction
The acceleration of the satellite can be found by calculating a_x and a_y separately, then combining these components to find a and θ.

If astronaut 1 pushes with a force of magnitude $F_1 = 26$ N and astronaut 2 pushes with a force of magnitude $F_2 = 41$ N, what are the magnitude and direction of the satellite's acceleration?

The easiest way to solve a problem like this is to treat each coordinate direction independently of the other, just as we did many times when studying two-dimensional kinematics in Chapter 4. Thus, we first resolve each force into its x and y components. Referring to Figure 5–9, we see that for the x direction

$$F_{1,x} = F_1$$
$$F_{2,x} = F_2 \cos 52°$$

For the y direction

$$F_{1,y} = 0$$
$$F_{2,y} = F_2 \sin 52°$$

Next, we find the acceleration in the x direction by using the x component of Newton's second law:

$$\sum F_x = ma_x$$

Applied to this system, we have

$$\sum F_x = F_{1,x} + F_{2,x} = F_1 + F_2 \cos 52° = 26\text{ N} + (41\text{ N}) \cos 52° = 51\text{ N} = ma_x$$

Solving for the acceleration yields

$$a_x = \frac{\sum F_x}{m} = \frac{51\text{ N}}{940\text{ kg}} = 0.054\text{ m/s}^2$$

Similarly, in the y direction we start with

$$\sum F_y = ma_y$$

This gives

$$\sum F_y = F_{1,y} + F_{2,y} = 0 + F_2 \sin 52° = (41\text{ N}) \sin 52° = 32\text{ N} = ma_y$$

As a result, the y component of acceleration is:

$$a_y = \frac{\sum F_y}{m} = \frac{32\text{ N}}{940\text{ kg}} = 0.034\text{ m/s}^2$$

Thus, the satellite accelerates in both the x and the y directions. Its total acceleration has a magnitude of

$$a = \sqrt{a_x^2 + a_y^2} = \sqrt{(0.054\text{ m/s}^2)^2 + (0.034\text{ m/s}^2)^2} = 0.064\text{ m/s}^2$$

From Figure 5–9 we expect the total acceleration to be in a direction above the x axis but at an angle less than 52°. Straightforward calculation yields

$$\theta = \tan^{-1}\left(\frac{a_y}{a_x}\right) = \tan^{-1}\left(\frac{0.034\text{ m/s}^2}{0.054\text{ m/s}^2}\right) = \tan^{-1}(0.63) = 32°$$

This is the same direction as the total force in Figure 5–9, as expected.

The following Example and Active Example give further practice with resolving force vectors and using Newton's second law in component form.

PROBLEM-SOLVING NOTE

Component-by-Component Application of Newton's Laws

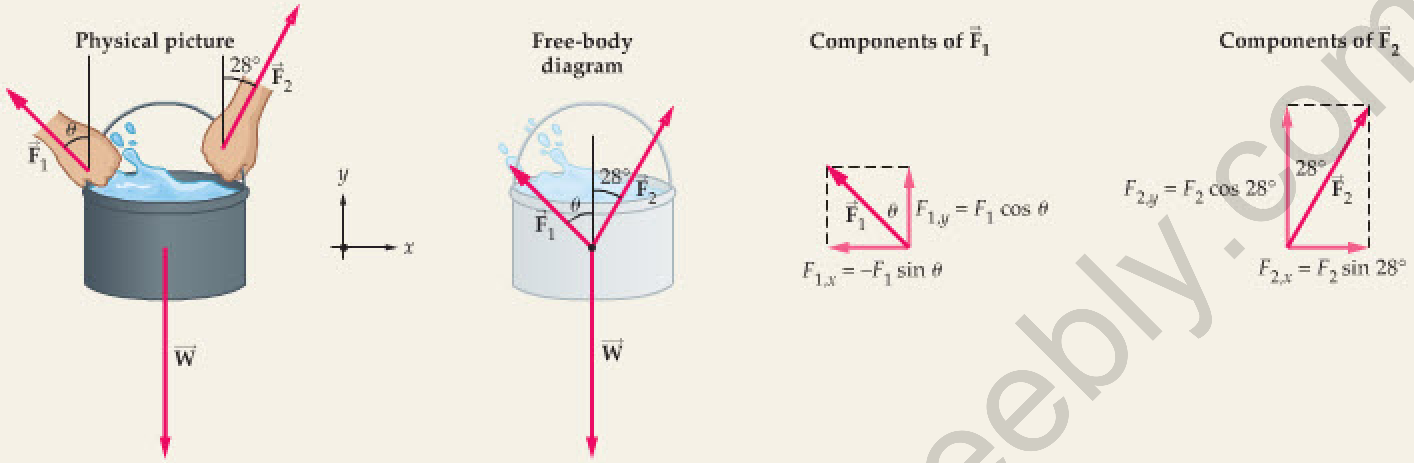
Newton's laws can be applied to each coordinate direction independently of the others. Therefore, when drawing a free-body diagram, be sure to include a coordinate system. Once the forces are resolved into their x and y components, the second law can be solved for each component separately. Working in a component-by-component fashion is the systematic way of using Newton's laws.

EXAMPLE 5–5 JACK AND JILL

Jack and Jill lift upward on a 1.30-kg pail of water, with Jack exerting a force $\vec{\mathbf{F}}_1$ of magnitude 7.0 N and Jill exerting a force $\vec{\mathbf{F}}_2$ of magnitude 11 N. Jill's force is exerted at an angle of 28° with the vertical, as shown below. **(a)** At what angle θ with respect to the vertical should Jack exert his force if the pail is to accelerate straight upward? **(b)** Determine the acceleration of the pail of water, given that its weight, $\vec{\mathbf{W}}$, has a magnitude of 12.8 N. (The simple connection between an object's mass and weight is presented in the next section.)

PICTURE THE PROBLEM

Our physical picture and free-body diagram show the pail and the three forces acting on it, as well as the angles relative to the vertical. In the panels at the right, we show the x and y components of the forces $\vec{\mathbf{F}}_1$ and $\vec{\mathbf{F}}_2$. Notice, in particular, that $F_{1,x} = -F_1 \sin\theta$ and $F_{1,y} = F_1 \cos\theta$. Similarly, $F_{2,x} = F_2 \sin 28°$ and $F_{2,y} = F_2 \cos 28°$.



STRATEGY

a. We want the acceleration to be purely vertical. This means that the x component of acceleration must be zero, $a_x = 0$. For a_x to be zero it is necessary that the sum of forces in the x direction be zero, $\Sigma F_x = 0$. Since the x component of $\vec{\mathbf{F}}_1$ depends on the angle θ, the equation $\Sigma F_x = 0$ can be used to find θ.

b. Once the appropriate angle is found, we can use it to find the y component of $\vec{\mathbf{F}}_1$. Add this result to the y component of $\vec{\mathbf{F}}_2$. We're not done yet, though—to find the total y component of the force, $\Sigma \vec{\mathbf{F}}_y$, we must also add the weight of the pail, which points in the negative y direction. Finally, divide the total force by the mass of the pail, $m = 1.30$ kg, to obtain its acceleration, $a_y = (\Sigma F_y)/m$.

SOLUTION

Part (a)

1. Begin by writing out the x component of each force. Note that $\vec{\mathbf{W}}$ has no x component and that the x component of $\vec{\mathbf{F}}_1$ points in the negative x direction:

$$F_{1,x} = -F_1 \sin\theta \qquad F_{2,x} = F_2 \sin 28° \qquad W_x = 0$$

2. Sum the x components of force and set equal to zero. Note that θ is the only unknown in this equation:

$$\sum F_x = -F_1 \sin\theta + F_2 \sin 28° + 0 = ma_x = 0 \text{ or}$$
$$F_1 \sin\theta = F_2 \sin 28°$$

3. Solve for $\sin\theta$ and then for θ:

$$\sin\theta = \frac{F_2 \sin 28°}{F_1} = \frac{(11\text{ N}) \sin 28°}{7.0\text{ N}} = 0.74$$
$$\theta = \sin^{-1}(0.74) = 48°$$

Part (b)

4. First, determine the y component of each force. Note that $\vec{\mathbf{W}}$ points in the negative y direction and that the y components of both $\vec{\mathbf{F}}_1$ and $\vec{\mathbf{F}}_2$ are positive:

$$F_{1,y} = F_1 \cos\theta = (7.0\text{ N}) \cos 48° = 4.7\text{ N}$$
$$F_{2,y} = F_2 \cos 28° = (11\text{ N}) \cos 28° = 9.7\text{ N}$$
$$W_y = -W = -12.8\text{ N}$$

5. Sum the y components of force and divide by the mass m to obtain the acceleration of the pail of water:

$$\sum F_y = F_1 \cos\theta + F_2 \cos 28° - W$$
$$= 4.7\text{ N} + 9.7\text{ N} - 12.8\text{ N} = 1.6\text{ N}$$
$$a_y = (\sum F_y)/m = (1.6\text{ N})/(1.3\text{ kg}) = 1.2\text{ m/s}^2$$

INSIGHT

Note that only the y components of $\vec{\mathbf{F}}_1$ and $\vec{\mathbf{F}}_2$ contribute to the vertical acceleration of the pail. The x components of the applied forces influence only the horizontal motion—they have no effect at all on the vertical acceleration of the pail. In this case the horizontal components of the applied forces cancel, and hence the pail moves straight upward with an acceleration of 1.2 m/s².

Finally, in the next section we shall see that the weight W of an object of mass m is $W = mg$. In this case, $W = (1.3\text{ kg})(9.81\text{ m/s}^2) = 12.8\text{ N}$.

PRACTICE PROBLEM

At what angle must Jack exert his force for the pail to accelerate straight upward if **(a)** $\vec{\mathbf{F}}_2$ is at an angle of 19° with the vertical or **(b)** $\vec{\mathbf{F}}_2$ is at an angle of 35° with the vertical? [**Answer: (a)** 31°, **(b)** 64°]

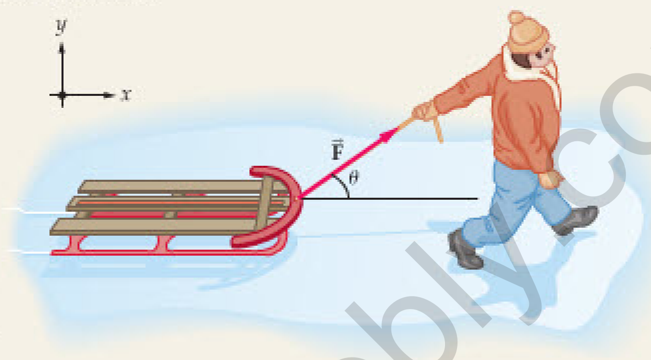
Some related homework problems: Problem 28, Problem 33

ACTIVE EXAMPLE 5–2 FIND THE SPEED OF THE SLED

A 4.60-kg sled is pulled across a smooth ice surface. The force acting on the sled is of magnitude 6.20 N and points in a direction 35.0° above the horizontal. If the sled starts at rest, how fast is it going after being pulled for 1.15 s?

SOLUTION *(Test your understanding by performing the calculations indicated in each step.)*

1. Find the x component of $\vec{\mathbf{F}}$: $F_x = 5.08\text{ N}$
2. Apply Newton's second law to the x direction: $\sum F_x = F_x = ma_x$
3. Solve for the x component of acceleration: $a_x = 1.10\text{ m/s}^2$
4. Use $v_x = v_{0x} + a_x t$ to find the speed of the sled: $v_x = 1.27\text{ m/s}$



INSIGHT

Note that the y component of $\vec{\mathbf{F}}$ has no effect on the acceleration of the sled.

YOUR TURN

Suppose the angle of the force above the horizontal is decreased, and the sled is again pulled from rest for 1.15 s. **(a)** Is the final speed of the sled greater than, less than, or the same as before? Explain. **(b)** Find the final speed of the sled for the case $\theta = 25.0°$.

(Answers to **Your Turn** *problems are given in the back of the book.)*

5–6 Weight

When you step onto a scale to weigh yourself, the scale gives a measurement of the pull of Earth's gravity. This is your weight, W. Similarly, the weight of any object on the Earth's surface is simply the gravitational force exerted on it by the Earth.

- The weight, W, of an object on the Earth's surface is the gravitational force exerted on it by the Earth.

As we know from everyday experience, the greater the mass of an object, the greater its weight. For example, if you put a brick on a scale and weigh it, you might get a reading of 9.0 N. If you put a second, identical brick on the scale—which doubles the mass—you will find a weight of 2(9.0 N) = 18 N. Clearly, there must be a simple connection between weight, W, and mass, m.

To see exactly what this connection is, consider taking one of the bricks just mentioned and letting it drop in free fall. As indicated in Figure 5–10, the only force acting on the brick is its weight, W, which is downward. If we choose upward to be the positive direction, we have

$$\sum F_y = -W$$

In addition, we know from Chapter 2 that the brick moves downward with an acceleration of $g = 9.81\text{ m/s}^2$ regardless of its mass. Thus,

$$a_y = -g$$

Using these results in Newton's second law

$$\sum F_y = ma_y$$

we find

$$-W = -mg$$

Therefore, the weight of an object of mass m is $W = mg$:

Definition: Weight, *W*

$$W = mg \qquad 5\text{–}5$$

SI unit: newton, N

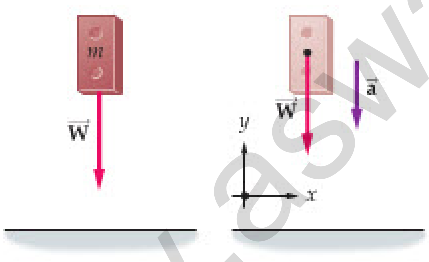


FIGURE 5–10 Weight and mass

A brick of mass m has only one force acting on it in free fall—its weight, $\vec{\mathbf{W}}$. The resulting acceleration has a magnitude $a = g$; hence $W = mg$.

Note that there is a clear distinction between weight and mass. Weight is a gravitational force, measured in newtons; mass is a measure of the inertia of an object, and it is given in kilograms. For example, if you were to travel to the Moon, your mass would not change—you would have the same amount of matter in you, regardless of your location. On the other hand, the gravitational force on the Moon's surface is less than the gravitational force on the Earth's surface. As a result, you would weigh less on the Moon than on the Earth, even though your mass is the same.

To be specific, on Earth an 81.0-kg person has a weight given by

$$W_{\text{Earth}} = mg_{\text{Earth}} = (81.0\ \text{kg})(9.81\ \text{m/s}^2) = 795\ \text{N}$$

In contrast, the same person on the Moon, where the acceleration of gravity is $1.62\ \text{m/s}^2$, weighs only

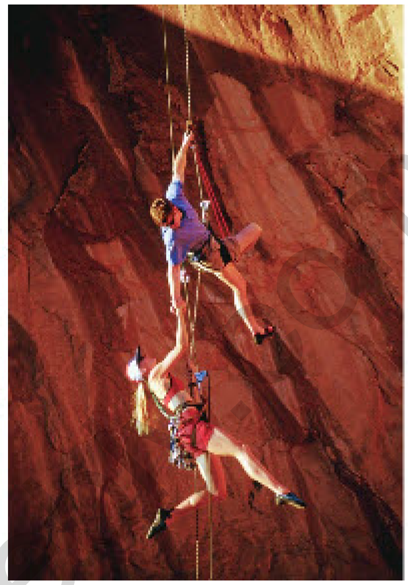
$$W_{\text{Moon}} = mg_{\text{Moon}} = (81.0\ \text{kg})(1.62\ \text{m/s}^2) = 131\ \text{N}$$

This is roughly one-sixth the weight on Earth. If, sometime in the future, there is a Lunar Olympics, the Moon's low gravity would be a boon for pole-vaulters, gymnasts, and others.

Finally, since weight is a force—which is a vector quantity—it has both a magnitude and a direction. Its magnitude, of course, is mg, and its direction is simply the direction of gravitational acceleration. Thus, if $\vec{\mathbf{g}}$ denotes a vector of magnitude g, pointing in the direction of free-fall acceleration, the weight of an object can be written in vector form as follows:

$$\vec{\mathbf{W}} = m\vec{\mathbf{g}}$$

We use the weight vector and its magnitude, mg, in the next Example.



▲ At the moment this picture was taken, the acceleration of both climbers was zero because the net force acting on them was zero. In particular, the upward forces exerted on the lower climber by the other climber and the ropes exactly cancel the downward force that gravity exerts on her.

EXAMPLE 5–6 WHERE'S THE FIRE?

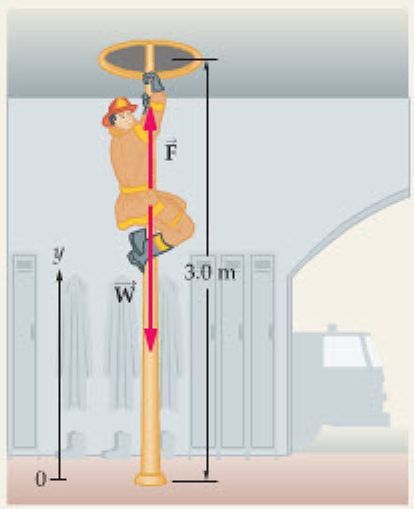
The fire alarm goes off, and a 97-kg fireman slides 3.0 m down a pole to the ground floor. Suppose the fireman starts from rest, slides with constant acceleration, and reaches the ground floor in 1.2 s. What was the upward force $\vec{\mathbf{F}}$ exerted by the pole on the fireman?

PICTURE THE PROBLEM

Our sketch shows the fireman sliding down the pole and the two forces acting on him: the upward force exerted by the pole, $\vec{\mathbf{F}}$, and the downward force of gravity, $\vec{\mathbf{W}}$. We choose the positive y direction to be upward, therefore $\vec{\mathbf{F}} = F\hat{\mathbf{y}}$ and $\vec{\mathbf{W}} = (-mg)\hat{\mathbf{y}}$. In addition, we choose $y = 0$ to be at ground level.

STRATEGY

The basic idea in approaching this problem is to apply Newton's second law to the y direction: $\sum F_y = ma_y$. The acceleration is not given directly, but we can find it using the kinematic equation $y = y_0 + v_{0y}t + \frac{1}{2}a_yt^2$. Substituting the result for a_y into Newton's second law, along with $W_y = -W = -mg$, allows us to solve for the unknown force, $\vec{\mathbf{F}}$.



Physical picture

Free-body diagram

SOLUTION

1. Solve $y = y_0 + v_{0y}t + \frac{1}{2}a_yt^2$ for a_y, using the fact that $v_{0y} = 0$:

$$y = y_0 + v_{0y}t + \tfrac{1}{2}a_yt^2 = y_0 + \tfrac{1}{2}a_yt^2$$

$$a_y = \frac{2(y - y_0)}{t^2}$$

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2. Substitute $y = 0$, $y_0 = 3.0$ m, and $t = 1.2$ s to find the acceleration: $$a_y = \frac{2(0 - 3.0\text{ m})}{(1.2\text{ s})^2} = -4.2\text{ m/s}^2$$

3. Sum the forces in the y direction: $$\sum F_y = F - mg$$

4. Set the sum of the forces equal to mass times acceleration: $$F - mg = ma_y$$

5. Solve for F, the y component of the force exerted by the pole. Use the result for F to write the force vector $\vec{\mathbf{F}}$: $$F = mg + ma_y = m(g + a_y)$$ $$= (97\text{ kg})(9.81\text{ m/s}^2 - 4.2\text{ m/s}^2) = 540\text{ N}$$ $$\vec{\mathbf{F}} = (540\text{ N})\,\hat{\mathbf{y}}$$

INSIGHT

How is it that the pole exerts a force on the fireman? Well, by wrapping his arms and legs around the pole as he slides, the fireman exerts a downward force on the pole. By Newton's third law, the pole exerts an upward force of equal magnitude on the fireman. These forces are due to friction, which we shall study in detail in Chapter 6.

PRACTICE PROBLEM

What is the fireman's acceleration if the force exerted on him by the pole is 650 N? **[Answer:** $a_y = -3.1\text{ m/s}^2$**]**

Some related homework problems: Problem 36, Problem 40

Apparent Weight

We have all had the experience of riding in an elevator and feeling either heavy or light, depending on its motion. For example, when an elevator moving downward comes to rest by accelerating upward, we feel heavier. On the other hand, we feel lighter when an elevator moving upward comes to rest by accelerating downward. In short, the motion of an elevator can give rise to an **apparent weight** that differs from our true weight. Why?

The reason is that our sensation of weight in this case is due to the force exerted on our feet by the floor of the elevator. If this force is greater than our weight, mg, we feel heavy; if it is less than mg, we feel light.

As an example, imagine you are in an elevator that is moving with an upward acceleration a, as indicated in Figure 5–11. Two forces act on you: (i) your weight, W, acting downward; and (ii) the upward normal force exerted on your feet by the floor of the elevator. Let's call the second force W_a, since it represents your apparent weight—that is, W_a is the force that pushes upward on your feet and gives you the sensation of your "weight" pushing down on the floor. We can find W_a by applying Newton's second law to the vertical direction.

To be specific, the sum of the forces acting on you is

$$\sum F_y = W_a - W$$

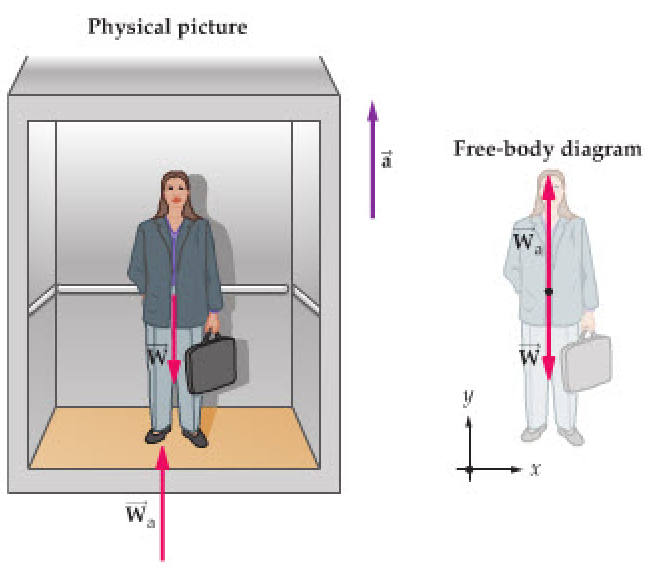


FIGURE 5–11 Apparent weight

A person rides in an elevator that is accelerating upward. Because the acceleration is upward, the net force must also be upward. As a result, the force exerted on the person by the floor of the elevator, $\vec{\mathbf{W}}_a$, must be greater than the person's weight, $\vec{\mathbf{W}}$. This means that the person feels heavier than normal.

By Newton's second law, this sum must equal ma_y. Since $a_y = a$, we find

$$W_a - W = ma$$

Solving for the apparent weight, W_a, yields

$$W_a = W + ma = mg + ma = m(g + a) \qquad \text{5–6}$$

Note that W_a is greater than your weight, mg, and hence you feel heavier. In fact, your apparent weight is precisely what it would be if you were suddenly "transported" to a planet where the acceleration of gravity is $g + a$ instead of g.

On the other hand, if the elevator accelerates downward, so that $a_y = -a$, your apparent weight is found by simply replacing a with $-a$ in Equation 5–6:

$$W_a = W - ma = mg - ma = m(g - a) \qquad \text{5–7}$$

In this case you feel lighter than usual.

We explore these results in the next Example, in which we consider weighing a fish on a scale. The reading on the scale is equal to the upward force it exerts on an object. Thus, the upward force exerted by the scale is the apparent weight, W_a.

EXAMPLE 5–7 HOW MUCH DOES THE SALMON WEIGH?

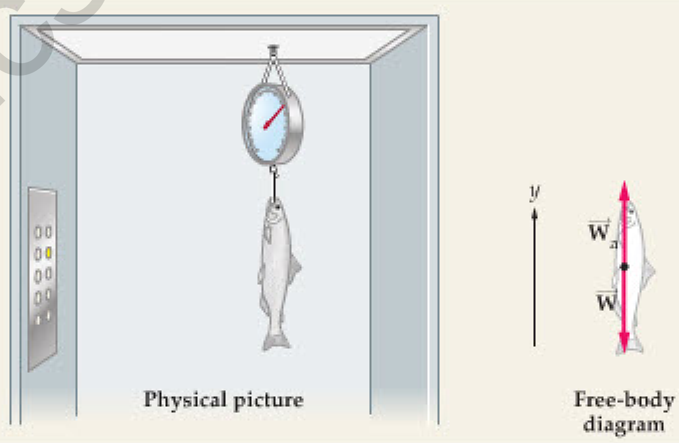
As part of an attempt to combine physics and biology in the same class, an instructor asks students to weigh a 5.0-kg salmon by hanging it from a fish scale attached to the ceiling of an elevator. What is the apparent weight of the salmon, $\vec{\mathbf{W}}_a$, if the elevator **(a)** is at rest, **(b)** moves with an upward acceleration of 2.5 m/s^2, or **(c)** moves with a downward acceleration of 3.2 m/s^2?

PICTURE THE PROBLEM

The free-body diagram for the salmon shows the weight of the salmon, $\vec{\mathbf{W}}$, and the force exerted by the scale, $\vec{\mathbf{W}}_a$. Note that upward is the positive direction. Therefore, the y component of $\vec{\mathbf{W}}$ is $-W = -mg$ and the y component of $\vec{\mathbf{W}}_a$ is W_a.

STRATEGY

We know the weight, $W = mg$, and the acceleration, a. To find the apparent weight, W_a, we use $\sum F_y = ma_y$. **(a)** Set $a_y = 0$. **(b)** Set $a_y = 2.5\ \text{m/s}^2$. **(c)** Set $a_y = -3.2\ \text{m/s}^2$.



SOLUTION

Part (a)

1. Sum the y component of the forces and set equal to mass times the y component of acceleration, with $a_y = 0$: $\sum F_y = W_a - W = ma_y = 0$
2. Solve for W_a, then write the vector $\vec{\mathbf{W}}$: $W_a = W = mg = (5.0\ \text{kg})(9.81\ \text{m/s}^2) = 49\ \text{N}$; $\vec{\mathbf{W}}_a = (49\ \text{N})\hat{\mathbf{y}}$

Part (b)

3. Again, sum the forces and set equal to mass times acceleration, this time with $a_y = a = 2.5\ \text{m/s}^2$: $\sum F_y = W_a - W = ma_y = ma$
4. Solve for W_a, then write the vector $\vec{\mathbf{W}}_a$: $W_a = W + ma = mg + ma = 49\ \text{N} + (5.0\ \text{kg})(2.5\ \text{m/s}^2) = 62\ \text{N}$; $\vec{\mathbf{W}}_a = (62\ \text{N})\hat{\mathbf{y}}$

Part (c)

5. Finally, sum the forces and set equal to mass times acceleration, with $a_y = -a = -3.2\ \text{m/s}^2$: $\sum F_y = W_a - W = ma_y = -ma$

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6. Solve for W_a, then write the vector $\overrightarrow{\mathbf{W}}_a$:

$$W_a = W - ma$$
$$= mg - ma = 49\ \text{N} - (5.0\ \text{kg})(3.2\ \text{m/s}^2) = 33\ \text{N}$$
$$\overrightarrow{\mathbf{W}}_a = (33\ \text{N})\hat{\mathbf{y}}$$

INSIGHT

When the salmon is at rest, or moving with constant velocity, its acceleration is zero and the apparent weight is equal to the actual weight, mg. In part (b) the apparent weight is greater than the actual weight because the scale must exert an upward force capable not only of supporting the salmon, but of accelerating it upward as well. In part (c) the apparent weight is less than the actual weight. In this case the net force acting on the salmon is downward, and hence its acceleration is downward.

PRACTICE PROBLEM

What is the elevator's acceleration if the scale gives a reading of **(a)** 55 N or **(b)** 45 N? [**Answer: (a)** $a_y = 1.2\ \text{m/s}^2$, **(b)** $a_y = -0.80\ \text{m/s}^2$]

Some related homework problems: Problem 38, Problem 39

REAL-WORLD PHYSICS: BIO

Simulating weightlessness

▲ Astronaut candidates pose for a floating class picture during weightlessness training aboard the "vomit comet."

Let's return for a moment to Equation 5–7:

$$W_a = m(g - a)$$

This result indicates that a person feels lighter than normal when riding in an elevator with a downward acceleration a. In particular, if the elevator's downward acceleration is g—that is, if the elevator is in free fall—it follows that $W_a = m(g - g) = 0$. Thus, a person feels "weightless" (zero apparent weight) in a freely falling elevator!

NASA uses this effect when training astronauts. Trainees are sent aloft in a KC-135 airplane affectionately known as the "vomit comet" (since many trainees experience nausea along with the weightlessness). To generate an experience of weightlessness, the plane flies on a parabolic path—the same path followed by a projectile in free fall. Each round of weightlessness lasts about half a minute, after which the plane pulls up to regain altitude and start the cycle again. On a typical flight, trainees experience about 40 cycles of weightlessness. Many scenes in the movie *Apollo 13* were shot in 30-second takes aboard the vomit comet.

This idea of free-fall weightlessness applies to more than just the vomit comet. In fact, astronauts in orbit experience weightlessness for the same reason—they and their craft are actually in free fall. As we shall see in detail in Chapter 12 (Gravity), orbital motion is just a special case of free fall.

CONCEPTUAL CHECKPOINT 5–3 ELEVATOR RIDE

If you ride in an elevator moving upward with constant speed, is your apparent weight **(a)** the same as, **(b)** greater than, or **(c)** less than mg?

REASONING AND DISCUSSION

If the elevator is moving in a straight line with constant speed, its acceleration is zero. Now, if the acceleration is zero, the net force must also be zero. Hence, the upward force exerted by the floor of the elevator, W_a, must equal the downward force of gravity, mg. As a result, your apparent weight is equal to mg.

Note that this conclusion agrees with Equations 5–6 and 5–7, with $a = 0$.

ANSWER

(a) Your apparent weight is the same as mg.

5–7 Normal Forces

As you get ready for lunch, you take a can of soup from the cupboard and place it on the kitchen counter. The can is now at rest, which means that its acceleration is zero, so the net force acting on it is also zero. Thus, you know that the

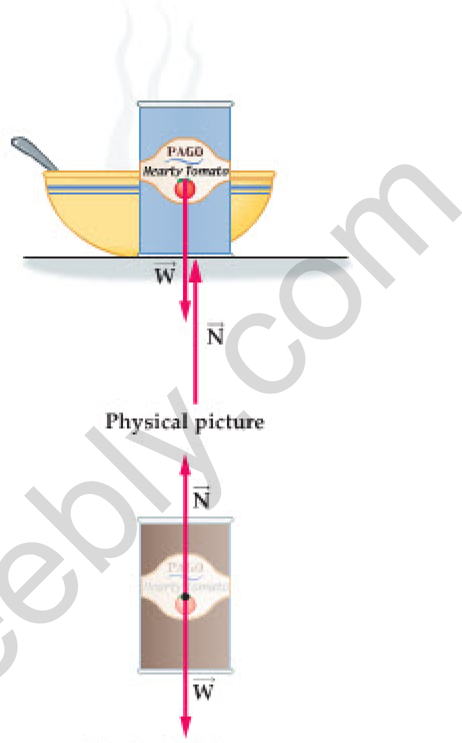


FIGURE 5–12 The normal force is perpendicular to a surface
A can of soup rests on a kitchen counter, which exerts a normal (perpendicular) force, $\vec{\mathbf{N}}$, to support it. In the special case shown here, the normal force is equal in magnitude to the weight, $W = mg$, and opposite in direction.

downward force of gravity is being opposed by an upward force exerted by the counter, as shown in Figure 5–12. As we have mentioned before, this force is referred to as the **normal force, $\vec{\mathbf{N}}$**. The reason the force is called normal is that it is *perpendicular to the surface*, and in mathematical terms, *normal* simply means perpendicular.

The origin of the normal force is the interaction between atoms in a solid that act to maintain its shape. When the can of soup is placed on the countertop, for example, it causes an imperceptibly small compression of the surface of the counter. This is similar to compressing a spring, and just like a spring, the countertop exerts a force to oppose the compression. Therefore, the greater the weight placed on the countertop, the greater the normal force it exerts to oppose being compressed.

In the example of the soup can and the countertop, the magnitude of the normal force is equal to the weight of the can. This is a special case, however. In general, the normal force may be greater than or less than the weight of an object.

To see how this can come about, consider pulling a 12.0-kg suitcase across a smooth floor by exerting a force, $\vec{\mathbf{F}}$, at an angle θ above the horizontal. The weight of the suitcase is $mg = (12.0\text{ kg})(9.81\text{ m/s}^2) = 118\text{ N}$. The normal force will have a magnitude less than this, however, because the force $\vec{\mathbf{F}}$ has an upward component that supports part of the suitcase's weight. To be specific, suppose that $\vec{\mathbf{F}}$ has a magnitude of 45.0 N and that $\theta = 20.0°$. What is the normal force exerted by the floor on the suitcase?

The situation is illustrated in Figure 5–13, where we show the three forces acting on the suitcase: (i) the weight of the suitcase, $\vec{\mathbf{W}}$, (ii) the force $\vec{\mathbf{F}}$, and (iii) the normal force, $\vec{\mathbf{N}}$. We also indicate a typical coordinate system in the figure, with the x axis horizontal and the y axis vertical. Now, the key to solving a problem like this is to realize that since the suitcase does not move in the y direction, its y component of acceleration is zero; that is, $a_y = 0$. It follows, from Newton's second law, that the sum of the y components of force must also equal zero; that is, $\sum F_y = ma_y = 0$. Using this condition, we can solve for the one force that is unknown, $\vec{\mathbf{N}}$.

To find $\vec{\mathbf{N}}$, then, we start by writing out the y component of each force. For the weight we have $W_y = -mg = -118\text{ N}$; for the applied force, $\vec{\mathbf{F}}$, the y component is $F_y = F\sin 20.0° = (45.0\text{ N})\sin 20.0° = 15.4\text{ N}$; finally, the y component of the normal force is $N_y = N$. Setting the sum of the y components of force equal to zero yields

$$\sum F_y = W_y + F_y + N_y = -mg + F\sin 20.0° + N = 0$$

Solving for N gives

$$N = mg - F\sin 20.0° = 118\text{ N} - 15.4\text{ N} = 103\text{ N}$$

In vector form,

$$\vec{\mathbf{N}} = N_y\hat{\mathbf{y}} = (103\text{ N})\hat{\mathbf{y}}$$

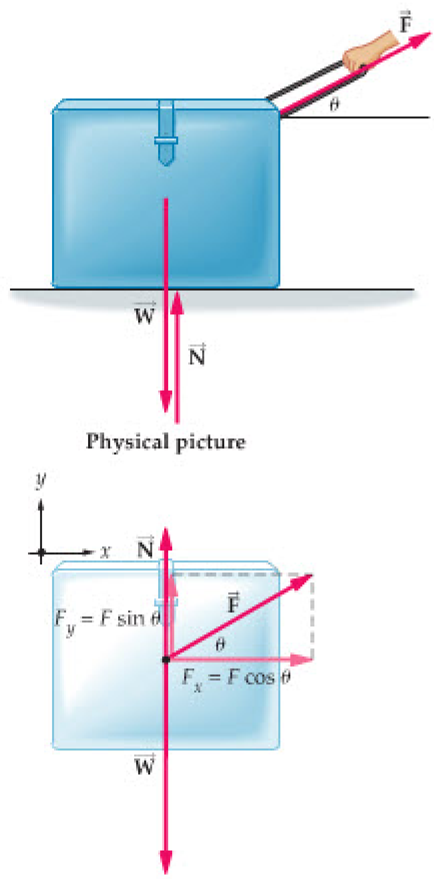


FIGURE 5–13 The normal force may differ from the weight
A suitcase is pulled across the floor by an applied force of magnitude F, directed at an angle θ above the horizontal. As a result of the upward component of $\vec{\mathbf{F}}$, the normal force $\vec{\mathbf{N}}$ will have a magnitude less than the weight of the suitcase.

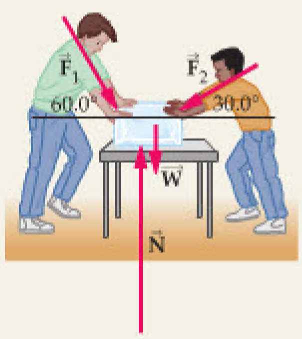
Thus, as mentioned, the normal force has a magnitude less than $mg = 118\text{ N}$ because the y component of $\vec{\mathbf{F}}$, $F_y = F \sin 20.0°$, supports part of the weight. In the following Example, however, the applied forces cause the normal force to be greater than the weight.

EXAMPLE 5–8 ICE BLOCK

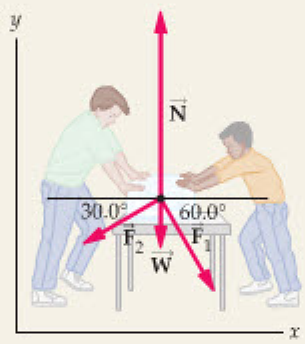
A 6.0-kg block of ice is acted on by two forces, $\vec{\mathbf{F}}_1$ and $\vec{\mathbf{F}}_2$, as shown in the diagram. If the magnitudes of the forces are $F_1 = 13\text{ N}$ and $F_2 = 11\text{ N}$, find **(a)** the acceleration of the ice and **(b)** the normal force exerted on it by the table.

PICTURE THE PROBLEM

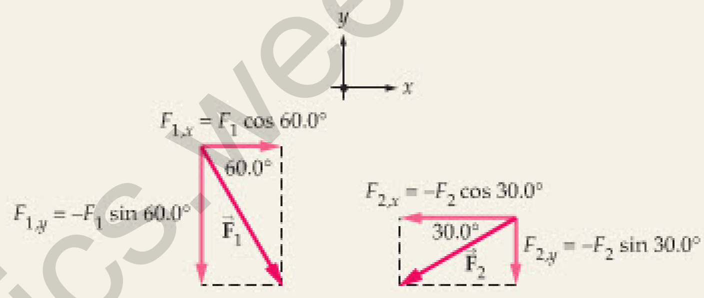
The sketch shows our choice of coordinate system, as well as all the forces acting on the block of ice. Note that $\vec{\mathbf{F}}_1$ has a positive x component and a negative y component; $\vec{\mathbf{F}}_2$ has negative x and y components. The weight and the normal force have only y components, therefore $W_x = 0$, $W_y = -W = -mg$, $N_x = 0$, and $N_y = N$.



Physical picture



Free-body diagram



Components of $\vec{\mathbf{F}}_1$ (enlarged) Components of $\vec{\mathbf{F}}_2$ (enlarged)

STRATEGY

The basic idea in this problem is to apply Newton's second law to the x and y directions separately. **(a)** The block can accelerate only in the horizontal direction; thus we find the acceleration by solving $\Sigma F_x = ma_x$ for a_x. **(b)** There is no motion in the y direction, and therefore the acceleration in the y direction is zero. Hence, we can find the normal force $\vec{\mathbf{N}}$ by setting $\Sigma F_y = ma_y = 0$.

SOLUTION

Part (a)

1. Write out the x component of each force:
$$F_{1,x} = F_1 \cos 60.0° = (13\text{ N}) \cos 60.0° = 6.5\text{ N}$$
$$F_{2,x} = -F_2 \cos 30.0° = -(11\text{ N}) \cos 30.0° = -9.5\text{ N}$$
$$N_x = 0 \qquad W_x = 0$$

2. Sum the x components of force:
$$\sum F_x = F_{1,x} + F_{2,x} + N_x + W_x$$
$$= 6.5\text{ N} - 9.5\text{ N} + 0 + 0 = -3.0\text{ N}$$

3. Divide by the mass to obtain the acceleration:
$$a_x = \frac{\sum F_x}{m} = \frac{-3.0\text{ N}}{6.0\text{ kg}} = -0.50\text{ m/s}^2$$
$$\vec{\mathbf{a}} = (-0.50\text{ m/s}^2)\hat{\mathbf{x}}$$

Part (b)

4. Write out the y component of each force:
$$F_{1,y} = -F_1 \sin 60° = -(13\text{ N}) \sin 60.0° = -11\text{ N}$$
The only force we don't know is the normal. We represent its magnitude by N:
$$F_{2,y} = -F_2 \sin 30° = -(11\text{ N}) \sin 30.0° = -5.5\text{ N}$$
$$N_y = N \qquad W_y = -W = -mg$$

5. Sum the y components of force:
$$\sum F_y = F_{1,y} + F_{2,y} + N_y + W_y$$
$$= -11\text{ N} - 5.5\text{ N} + N - mg$$

6. Set this sum equal to 0 since the acceleration in the y direction is zero, and solve for N:	$-11\text{ N} - 5.5\text{ N} + N - mg = 0$ $N = 11\text{ N} + 5.5\text{ N} + mg$ $= 11\text{ N} + 5.5\text{ N} + (6.0\text{ kg})(9.81\text{ m/s}^2) = 75\text{ N}$
7. Finally, we write the normal force in vector form:	$\vec{\mathbf{N}} = (75\text{ N})\hat{\mathbf{y}}$

INSIGHT

The block accelerates to the left, even though the force acting to the right, $\vec{\mathbf{F}}_1$, has a greater magnitude than the force acting to the left, $\vec{\mathbf{F}}_2$. This is because $\vec{\mathbf{F}}_2$ has the greater x component. Also, note that the normal force is greater in magnitude than the weight, $mg = 59\text{ N}$.

In general, the normal force exerted by a surface is just as large as is necessary to prevent motion of an object into the surface. If the required force is larger than the material can provide, the surface will break.

PRACTICE PROBLEM

At what angle must $\vec{\mathbf{F}}_2$ be applied if the block of ice is to have zero acceleration? **[Answer:** $a_x = 0$ implies $F_1 \cos 60.0° = F_2 \cos\theta$. Thus, $\theta = 54°$.]

Some related homework problems: Problem 44, Problem 50

To this point, we have considered surfaces that are horizontal, in which case the normal force is vertical. When a surface is inclined, the normal force is still at right angles to the surface, even though it is no longer vertical. This is illustrated in Figure 5–14. (If friction is present, a surface may also exert a force that is parallel to its surface. This will be considered in detail in Chapter 6.)

When choosing a coordinate system for an inclined surface, it is generally best to have the x and y axes of the system parallel and perpendicular to the surface, respectively, as in Figure 5–15. One can imagine the coordinate system to be "bolted down" to the surface, so that when the surface is tilted the coordinate system tilts along with it.

With this choice of coordinate system, there is no motion in the y direction, even on the inclined surface, and the normal force points in the positive y direction. Thus, the condition that determines the normal force is still $\Sigma F_y = ma_y = 0$, as before. In addition, if the object slides on the surface, its motion is purely in the x direction.

Finally, if the surface is inclined by an angle θ, note that the weight—which is still vertically downward—is at the same angle θ with respect to the negative

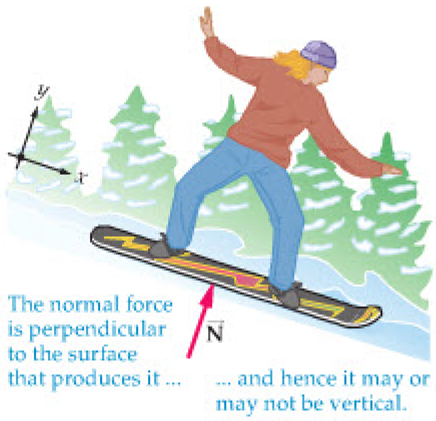


FIGURE 5–14 An object on an inclined surface
The normal force $\vec{\mathbf{N}}$ is always at right angles to the surface; hence, it is not always in the vertical direction.

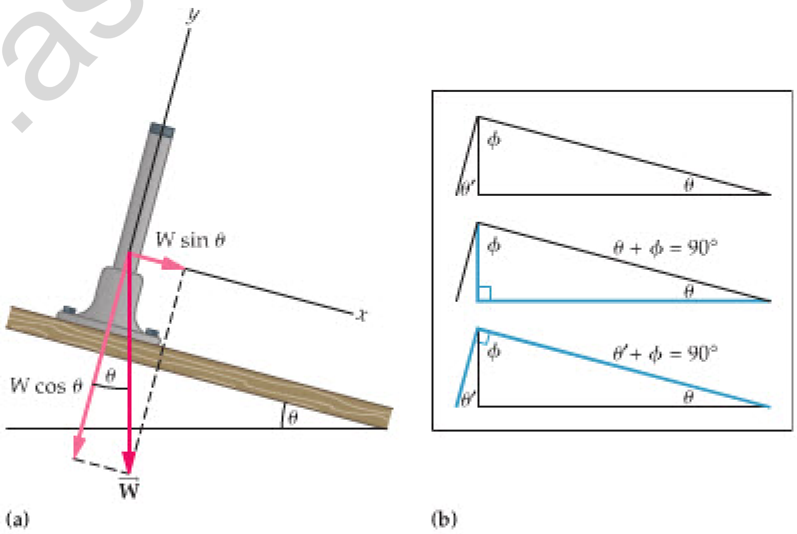


FIGURE 5–15 Components of the weight on an inclined surface
Whenever a surface is tilted by an angle θ, the weight $\vec{\mathbf{W}}$ makes the same angle θ with respect to the negative y axis. This is proven in part (b), where we show that $\theta + \phi = 90°$, and that $\theta' + \phi = 90°$. From these results it follows that $\theta' = \theta$. The component of the weight perpendicular to the surface is $W_y = -W\cos\theta$; the component parallel to the surface is $W_x = W\sin\theta$.

y axis, as shown in Figure 5–15. As a result, the x and y components of the weight are

$$W_x = W \sin\theta = mg \sin\theta \qquad 5\text{–}8$$

and

$$W_y = -W \cos\theta = -mg \cos\theta \qquad 5\text{–}9$$

Let's quickly check some special cases of these results. First, if $\theta = 0$ the surface is horizontal, and we find $W_x = 0$, $W_y = -mg$, as expected. Second, if $\theta = 90°$ the surface is vertical; therefore, the weight is parallel to the surface, pointing in the positive x direction. In this case, $W_x = mg$ and $W_y = 0$.

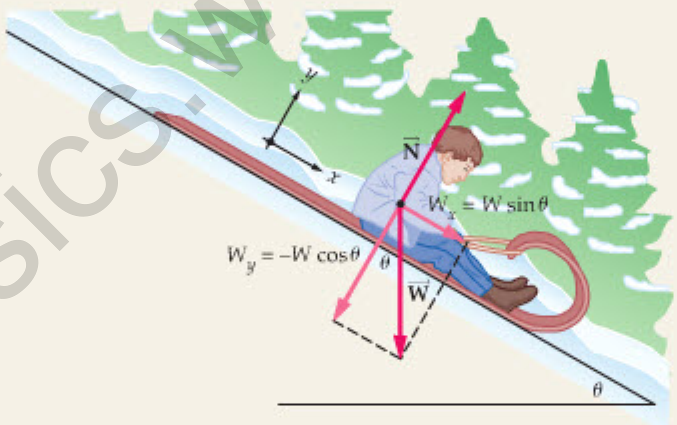
The next Example shows how to use the weight components to find the acceleration of an object on an inclined surface.

EXAMPLE 5–9 TOBOGGAN TO THE BOTTOM

A child of mass m rides on a toboggan down a slick, ice-covered hill inclined at an angle θ with respect to the horizontal. **(a)** What is the acceleration of the child? **(b)** What is the normal force exerted on the child by the toboggan?

PICTURE THE PROBLEM

We choose the x axis to be parallel to the slope, with the positive direction pointing downhill. Similarly, we choose the y axis to be perpendicular to the slope, pointing up and to the right. With these choices, the x component of $\vec{\mathbf{W}}$ is positive, $W_x = W \sin\theta$, and its y component is negative, $W_y = -W\cos\theta$. Finally, the x component of the normal force is zero, $N_x = 0$, and its y component is positive, $N_y = N$.



STRATEGY

Note that only two forces act on the child: (i) the weight, $\vec{\mathbf{W}}$, and (ii) the normal force, $\vec{\mathbf{N}}$. **(a)** We find the child's acceleration by solving $\sum F_x = ma_x$ for a_x. **(b)** Because there is no motion in the y direction, the y component of acceleration is zero. Therefore, we can find the normal force by setting $\sum F_y = ma_y = 0$.

SOLUTION

Part (a)

1. Write out the x components of the forces acting on the child: $N_x = 0 \quad W_x = W\sin\theta = mg\sin\theta$
2. Sum the x components of the forces and set equal to ma_x: $\sum F_x = N_x + W_x = mg\sin\theta = ma_x$
3. Divide by the mass m to find the acceleration in the x direction: $a_x = \dfrac{\sum F_x}{m} = \dfrac{mg\sin\theta}{m} = g\sin\theta$

Part (b)

4. Write out the y components of the forces acting on the child: $N_y = N \quad W_y = -W\cos\theta = -mg\cos\theta$
5. Sum the y components of the forces and set the sum equal to zero, since $a_y = 0$: $\sum F_y = N_y + W_y = N - mg\cos\theta = ma_y = 0$
6. Solve for the magnitude of the normal force, N: $N - mg\cos\theta = 0 \quad \text{or} \quad N = mg\cos\theta$
7. Write the normal force in vector form: $\vec{\mathbf{N}} = (mg\cos\theta)\hat{\mathbf{y}}$

INSIGHT

Note that for θ between 0 and 90° the acceleration of the child is *less* than the acceleration of gravity. This is because only a *component* of the weight is causing the acceleration.

Let's check some special cases of our general result, $a_x = g\sin\theta$. First, let $\theta = 0$. In this case, we find zero acceleration; $a_x = g\sin 0 = 0$. This makes sense because with $\theta = 0$ the hill is actually level, and we don't expect an acceleration. Second, let $\theta = 90°$. In this case, the hill is vertical, and the toboggan should drop straight down in free fall. This also agrees with our general result; $a_x = g\sin 90° = g$.

PRACTICE PROBLEM

What is the child's acceleration if its mass is doubled to $2m$? [**Answer:** The acceleration is still $a_x = g \sin\theta$. As in free fall, the acceleration produced by gravity is independent of mass.]

Some related homework problems: Problem 45, Problem 49

THE BIG PICTURE PUTTING PHYSICS IN CONTEXT

LOOKING BACK

The fact that a constant force produces a constant acceleration gives special significance to the discussion of constant acceleration in Chapters 2 and 4.

All forces are vectors, and therefore the ability to use and manipulate vectors confidently is essential to a full and complete understanding of forces. Again, we see the importance of the vector material presented in Chapter 3.

As with two-dimensional kinematics in Chapter 4, where motion in the x and y directions were seen to be independent, the x and y components of force are independent as well. In particular, acceleration in the x direction depends only on the x component of force, and acceleration in the y direction depends only on the y component of force.

LOOKING AHEAD

Forces are a central theme throughout physics. In particular, we shall see in Chapters 7 and 8 that a force acting on an object over a distance changes its energy.

Another important application of forces is in the study of collisions. Central to this topic is the concept of momentum, a physical quantity that is changed when a force acts on an object over a period of time.

In this chapter we introduced the force law for gravity near the Earth's surface, $F = mg$. The more general law of gravity, valid at any location, is introduced in Chapter 12. Similarly, the force laws for electricity and magnetism are presented in Chapters 19 and 22, respectively.

CHAPTER SUMMARY

5–1 FORCE AND MASS

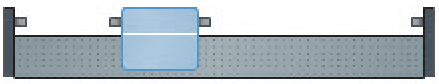
Force

A push or a pull.

Mass

A measure of the difficulty in accelerating an object. Equivalently, a measure of the quantity of matter in an object.

5–2 NEWTON'S FIRST LAW OF MOTION



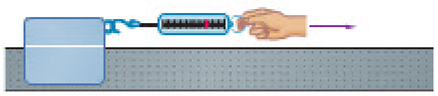
First Law (Law of Inertia)

If the net force on an object is zero, its velocity is constant.

Inertial Frame of Reference

Frame of reference in which the first law holds. All inertial frames of reference move with constant velocity relative to one another.

5–3 NEWTON'S SECOND LAW OF MOTION



Second Law

An object of mass m has an acceleration $\vec{a}$ given by the net force $\Sigma\vec{F}$ divided by m. That is

$$\vec{a} = \sum \vec{F}/m \qquad 5\text{–}1$$

Component Form

$$a_x = \sum F_x/m \qquad a_y = \sum F_y/m \qquad a_z = \sum F_z/m \qquad 5\text{–}2$$

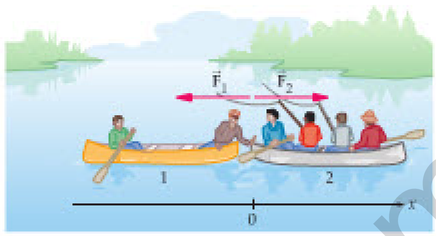
SI Unit: Newton (N)

$$1\ \text{N} = 1\ \text{kg}\cdot\text{m/s}^2 \qquad 5\text{–}3$$

Free-Body Diagram

A sketch showing all external forces acting on an object.

5–4 NEWTON'S THIRD LAW OF MOTION

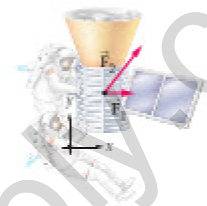


Physical picture

Third Law
For every force that acts on an object, there is a reaction force acting on a different object that is equal in magnitude and opposite in direction.

Contact Forces
Action-reaction pair of forces produced by physical contact of two objects.

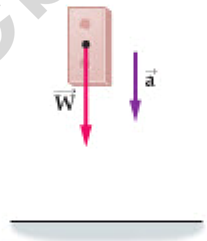
5–5 THE VECTOR NATURE OF FORCES: FORCES IN TWO DIMENSIONS



Forces are vectors.

Newton's second law can be applied to each component of force separately and independently.

5–6 WEIGHT



Gravitational force exerted *by the Earth on an object*.

On the surface of the Earth the weight, W, of an object of mass m has the magnitude

$$W = mg \qquad 5\text{–}5$$

Apparent Weight
Force felt from contact with the floor or a scale in an accelerating system. For example, the sensation of feeling heavier or lighter in an accelerating elevator.

5–7 NORMAL FORCES

Force exerted by a surface that is *perpendicular* to the surface.

The normal force is equal to the weight of an object only in special cases. In general, the normal force is greater than or less than the object's weight.

PROBLEM-SOLVING SUMMARY

Type of Calculation	Relevant Physical Concepts	Related Examples
Find the acceleration of an object.	Solve Newton's second law for each component of the acceleration; that is, $a_x = \Sigma F_x/m$ and $a_y = \Sigma F_y/m$.	Examples 5–1, 5–3, 5–4, 5–5, 5–8, 5–9 Active Examples 5–1, 5–2
Solve problems involving action-reaction forces.	Apply Newton's third law, being careful to note that the action-reaction forces act on different objects.	Examples 5–3, 5–4
Find the normal force exerted on an object.	Since there is no acceleration in the normal direction, set the sum of the normal components of force equal to zero.	Examples 5–8, 5–9

CONCEPTUAL QUESTIONS

For instructor-assigned homework, go to www.masteringphysics.com

(Answers to odd-numbered Conceptual Questions can be found in the back of the book.)

1. Driving down the road, you hit the brakes suddenly. As a result, your body moves toward the front of the car. Explain, using Newton's laws.
2. You've probably seen pictures of someone pulling a tablecloth out from under glasses, plates, and silverware set out for a formal dinner. Perhaps you've even tried it yourself. Using Newton's laws of motion, explain how this stunt works.
3. As you read this, you are most likely sitting quietly in a chair. Can you conclude, therefore, that you are at rest? Explain.

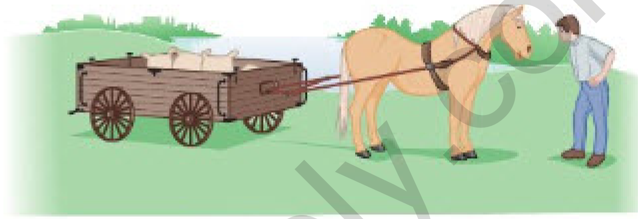
4. When a dog gets wet, it shakes its body from head to tail to shed the water. Explain, in terms of Newton's first law, why this works.

A dog uses the principle of inertia to shake water from its coat. (Conceptual Question 4)

5. A young girl slides down a rope. As she slides faster and faster she tightens her grip, increasing the force exerted on her by the rope. What happens when this force is equal in magnitude to her weight? Explain.
6. A drag-racing car accelerates forward because of the force exerted on it by the road. Why, then, does it need an engine? Explain.
7. A block of mass m hangs from a string attached to a ceiling, as shown in Figure 5–16. An identical string hangs down from the bottom of the block. Which string breaks if **(a)** the lower string is pulled with a slowly increasing force or **(b)** the lower string is jerked rapidly downward? Explain.

▲ **FIGURE 5–16** Conceptual Question 7

8. An astronaut on a space walk discovers that his jet pack no longer works, leaving him stranded 50 m from the spacecraft. If the jet pack is removable, explain how the astronaut can still use it to return to the ship.
9. Two untethered astronauts on a space walk decide to take a break and play catch with a baseball. Describe what happens as the game of catch progresses.
10. What are the action-reaction forces when a baseball bat hits a fast ball? What is the effect of each force?
11. In Figure 5–17 Wilbur asks Mr. Ed, the talking horse, to pull a cart. Mr. Ed replies that he would like to, but the laws of nature just won't allow it. According to Newton's third law, he says, if he pulls on the wagon it pulls back on him with an equal force. Clearly, then, the net force is zero and the wagon will stay put. How should Wilbur answer the clever horse?



▲ **FIGURE 5–17** Conceptual Question 11

12. A whole brick has more mass than half a brick, thus the whole brick is harder to accelerate. Why doesn't a whole brick fall more slowly than half a brick? Explain.
13. The force exerted by gravity on a whole brick is greater than the force exerted by gravity on half a brick. Why, then, doesn't a whole brick fall faster than half a brick? Explain.
14. Is it possible for an object at rest to have only a single force acting on it? If your answer is yes, provide an example. If your answer is no, explain why not.
15. Is it possible for an object to be in motion and yet have zero net force acting on it? Explain.
16. A bird cage, with a parrot inside, hangs from a scale. The parrot decides to hop to a higher perch. What can you say about the reading on the scale **(a)** when the parrot jumps, **(b)** when the parrot is in the air, and **(c)** when the parrot lands on the second perch? Assume that the scale responds rapidly so that it gives an accurate reading at all times.
17. Suppose you jump from the cliffs of Acapulco and perform a perfect swan dive. As you fall, you exert an upward force on the Earth equal in magnitude to the downward force the Earth exerts on you. Why, then, does it seem that you are the one doing all the accelerating? Since the forces are the same, why aren't the accelerations?
18. A friend tells you that since his car is at rest, there are no forces acting on it. How would you reply?
19. Since all objects are "weightless" in orbit, how is it possible for an orbiting astronaut to tell if one object has more mass than another object? Explain.
20. To clean a rug, you can hang it from a clothesline and beat it with a tennis racket. Use Newton's laws to explain why beating the rug should have a cleansing effect.
21. If you step off a high board and drop to the water below, you plunge into the water without injury. On the other hand, if you were to drop the same distance onto solid ground, you might break a leg. Use Newton's laws to explain the difference.
22. A moving object is acted on by a net force. Give an example of a situation in which the object moves **(a)** in the same direction as the net force, **(b)** at right angles to the net force, or **(c)** in the opposite direction of the net force.
23. Is it possible for an object to be moving in one direction while the net force acting on it is in another direction? If your answer is yes, provide an example. If your answer is no, explain why not.
24. Since a bucket of water is "weightless" in space, would it hurt to kick the bucket? Explain.

25. In the movie *The Rocketeer*, a teenager discovers a jet-powered backpack in an old barn. The backpack allows him to fly at incredible speeds. In one scene, however, he uses the backpack to rapidly accelerate an old pickup truck that is being chased by "bad guys." He does this by bracing his arms against the cab of the pickup and firing the backpack, giving the truck the acceleration of a drag racer. Is the physics of this scene "Good," "Bad," or "Ugly?" Explain.

26. List three common objects that have a weight of approximately 1 N.

PROBLEMS AND CONCEPTUAL EXERCISES

Note: Answers to odd-numbered Problems and Conceptual Exercises can be found in the back of the book. **IP** *denotes an integrated problem, with both conceptual and numerical parts;* **BIO** *identifies problems of biological or medical interest;* **CE** *indicates a conceptual exercise.* **Predict/Explain** *problems ask for two responses:* **(a)** *your prediction of a physical outcome, and* **(b)** *the best explanation among three provided. On all problems, red bullets (•, ••, •••) are used to indicate the level of difficulty.*

SECTION 5–3 NEWTON'S SECOND LAW OF MOTION

1. • **CE** An object of mass m is initially at rest. After a force of magnitude F acts on it for a time T, the object has a speed v. Suppose the mass of the object is doubled, and the magnitude of the force acting on it is quadrupled. In terms of T, how long does it take for the object to accelerate from rest to a speed v now?

2. • On a planet far, far away, an astronaut picks up a rock. The rock has a mass of 5.00 kg, and on this particular planet its weight is 40.0 N. If the astronaut exerts an upward force of 46.2 N on the rock, what is its acceleration?

3. • In a grocery store, you push a 12.3-kg shopping cart with a force of 10.1 N. If the cart starts at rest, how far does it move in 2.50 s?

4. • You are pulling your little sister on her sled across an icy (frictionless) surface. When you exert a constant horizontal force of 120 N, the sled has an acceleration of 2.5 m/s^2. If the sled has a mass of 7.4 kg, what is the mass of your little sister?

5. • A 0.53-kg billiard ball initially at rest is given a speed of 12 m/s during a time interval of 4.0 ms. What average force acted on the ball during this time?

6. • A 92-kg water skier floating in a lake is pulled from rest to a speed of 12 m/s in a distance of 25 m. What is the net force exerted on the skier, assuming his acceleration is constant?

7. •• **CE Predict/Explain** You drop two balls of equal diameter from the same height at the same time. Ball 1 is made of metal and has a greater mass than ball 2, which is made of wood. The upward force due to air resistance is the same for both balls. **(a)** Is the drop time of ball 1 greater than, less than, or equal to the drop time of ball 2? **(b)** Choose the *best explanation* from among the following:

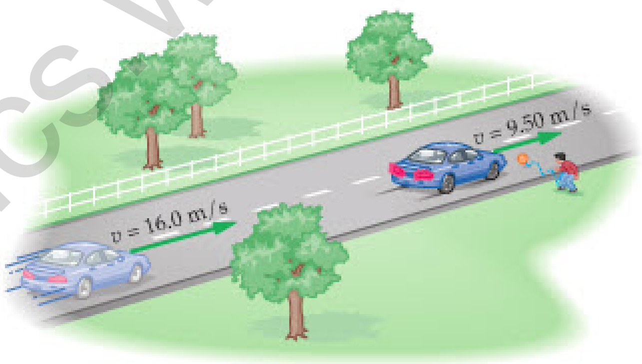
I. The acceleration of gravity is the same for all objects, regardless of mass.
II. The more massive ball is harder to accelerate.
III. Air resistance has less effect on the more massive ball.

8. •• **IP** A 42.0-kg parachutist is moving straight downward with a speed of 3.85 m/s. **(a)** If the parachutist comes to rest with constant acceleration over a distance of 0.750 m, what force does the ground exert on her? **(b)** If the parachutist comes to rest over a shorter distance, is the force exerted by the ground greater than, less than, or the same as in part (a)? Explain.

9. •• **IP** In baseball, a pitcher can accelerate a 0.15-kg ball from rest to 98 mi/h in a distance of 1.7 m. **(a)** What is the average force exerted on the ball during the pitch? **(b)** If the mass of the ball is increased, is the force required of the pitcher increased, decreased, or unchanged? Explain.

10. •• A major-league catcher gloves a 92-mi/h pitch and brings it to rest in 0.15 m. If the force exerted by the catcher is 803 N, what is the mass of the ball?

11. •• Driving home from school one day, you spot a ball rolling out into the street (Figure 5–18). You brake for 1.20 s, slowing your 950-kg car from 16.0 m/s to 9.50 m/s. **(a)** What was the average force exerted on your car during braking? **(b)** How far did you travel while braking?



▲ **FIGURE 5–18** Problem 11

12. •• **Stopping a 747** A 747 jetliner lands and begins to slow to a stop as it moves along the runway. If its mass is 3.50×10^5 kg, its speed is 27.0 m/s, and the net braking force is 4.30×10^5 N, **(a)** what is its speed 7.50 s later? **(b)** How far has it traveled in this time?

13. •• **IP** A drag racer crosses the finish line doing 202 mi/h and promptly deploys her drag chute (the small parachute used for braking). **(a)** What force must the drag chute exert on the 891-kg car to slow it to 45.0 mi/h in a distance of 185 m? **(b)** Describe the strategy you used to solve part (a).

SECTION 5–4 NEWTON'S THIRD LAW OF MOTION

14. • **CE Predict/Explain** A small car collides with a large truck. **(a)** Is the magnitude of the force experienced by the car greater than, less than, or equal to the magnitude of the force experienced by the truck? **(b)** Choose the *best explanation* from among the following:

I. Action-reaction forces always have equal magnitude.
II. The truck has more mass, and hence the force exerted on it is greater.
III. The massive truck exerts a greater force on the lightweight car.

15\. • **CE Predict/Explain** A small car collides with a large truck. **(a)** Is the acceleration experienced by the car greater than, less than, or equal to the acceleration experienced by the truck? **(b)** Choose the *best explanation* from among the following:

I. The truck exerts a larger force on the car, giving it the greater acceleration.

II. Both vehicles experience the same magnitude of force, therefore the lightweight car experiences the greater acceleration.

III. The greater force exerted on the truck gives it the greater acceleration.

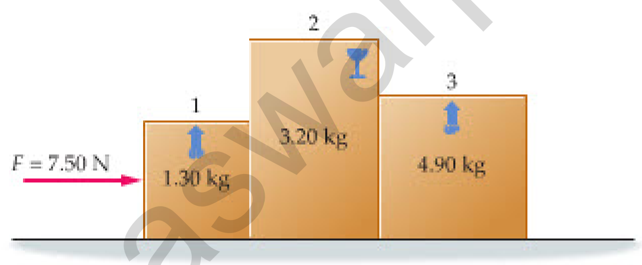
16\. • You hold a brick at rest in your hand. **(a)** How many forces act on the brick? **(b)** Identify these forces. **(c)** Are these forces equal in magnitude and opposite in direction? **(d)** Are these forces an action-reaction pair? Explain.

17\. • Referring to Problem 16, you are now accelerating the brick upward. **(a)** How many forces act on the brick in this case? **(b)** Identify these forces. **(c)** Are these forces equal in magnitude and opposite in direction? **(d)** Are these forces an action-reaction pair? Explain.

18\. •• On vacation, your 1400-kg car pulls a 560-kg trailer away from a stoplight with an acceleration of 1.85 m/s^2. **(a)** What is the net force exerted on the trailer? **(b)** What force does the trailer exert on the car? **(c)** What is the net force acting on the car?

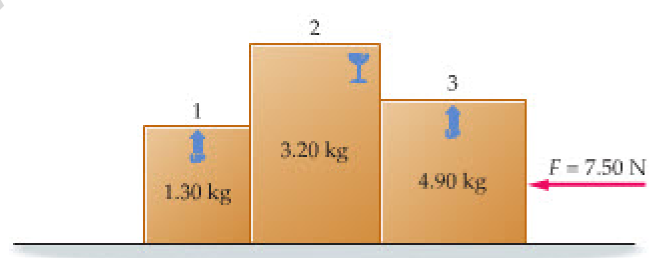
19\. •• **IP** A 71-kg parent and a 19-kg child meet at the center of an ice rink. They place their hands together and push. **(a)** Is the force experienced by the child more than, less than, or the same as the force experienced by the parent? **(b)** Is the acceleration of the child more than, less than, or the same as the acceleration of the parent? Explain. **(c)** If the acceleration of the child is 2.6 m/s^2 in magnitude, what is the magnitude of the parent's acceleration?

20\. •• A force of magnitude 7.50 N pushes three boxes with masses $m_1 = 1.30$ kg, $m_2 = 3.20$ kg, and $m_3 = 4.90$ kg, as shown in Figure 5–19. Find the magnitude of the contact force **(a)** between boxes 1 and 2, and **(b)** between boxes 2 and 3.



▲ **FIGURE 5–19** Problem 20

21\. •• A force of magnitude 7.50 N pushes three boxes with masses $m_1 = 1.30$ kg, $m_2 = 3.20$ kg, and $m_3 = 4.90$ kg, as shown in Figure 5–20. Find the magnitude of the contact force **(a)** between boxes 1 and 2, and **(b)** between boxes 2 and 3.



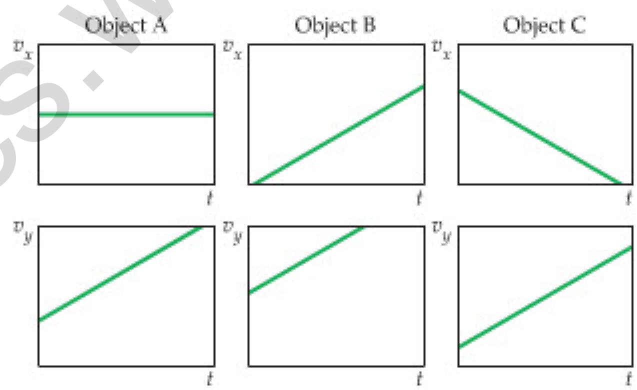
▲ **FIGURE 5–20** Problem 21

22\. •• **IP** Two boxes sit side-by-side on a smooth horizontal surface. The lighter box has a mass of 5.2 kg; the heavier box has a mass of 7.4 kg. **(a)** Find the contact force between these boxes when a horizontal force of 5.0 N is applied to the light box. **(b)** If the 5.0-N force is applied to the heavy box instead, is the contact force between the boxes the same as, greater than, or less than the contact force in part (a)? Explain. **(c)** Verify your answer to part (b) by calculating the contact force in this case.

SECTION 5–5 THE VECTOR NATURE OF FORCES

23\. • **CE** A skateboarder on a ramp is accelerated by a nonzero net force. For each of the following statements, state whether it is always true, never true, or sometimes true. **(a)** The skateboarder is moving in the direction of the net force. **(b)** The acceleration of the skateboarder is at right angles to the net force. **(c)** The acceleration of the skateboarder is in the same direction as the net force. **(d)** The skateboarder is instantaneously at rest.

24\. • **CE** Three objects, A, B, and C, have x and y components of velocity that vary with time as shown in Figure 5–21. What is the direction of the net force acting on **(a)** object A, **(b)** object B, and **(c)** object C, as measured from the positive x axis? (All of the nonzero slopes have the same magnitude.)

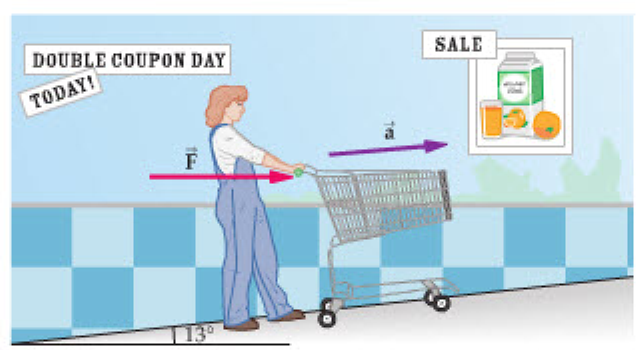


▲ **FIGURE 5–21** Problem 24

25\. • A farm tractor tows a 3700-kg trailer up an 18° incline with a steady speed of 3.2 m/s. What force does the tractor exert on the trailer? (Ignore friction.)

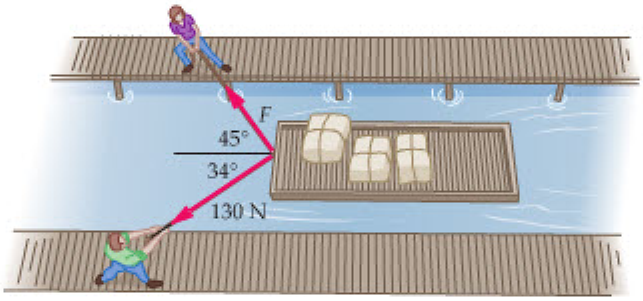
26\. • A surfer "hangs ten," and accelerates down the sloping face of a wave. If the surfer's acceleration is 3.25 m/s^2 and friction can be ignored, what is the angle at which the face of the wave is inclined above the horizontal?

27\. • A shopper pushes a 7.5-kg shopping cart up a 13° incline, as shown in Figure 5–22. Find the magnitude of the horizontal force, $\vec{F}$, needed to give the cart an acceleration of 1.41 m/s^2.



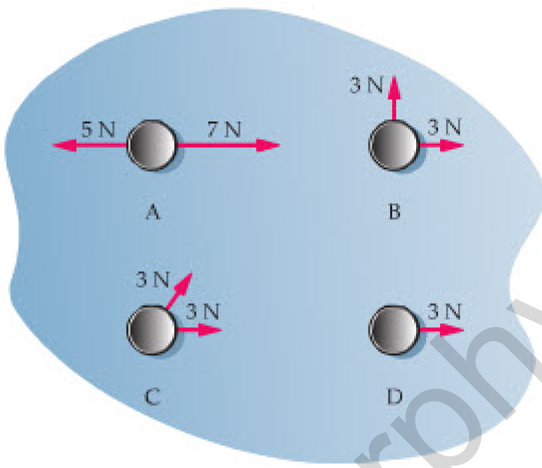
▲ **FIGURE 5–22** Problem 27

28. • Two crewmen pull a raft through a lock, as shown in Figure 5–23. One crewman pulls with a force of 130 N at an angle of 34° relative to the forward direction of the raft. The second crewman, on the opposite side of the lock, pulls at an angle of 45°. With what force should the second crewman pull so that the net force of the two crewmen is in the forward direction?



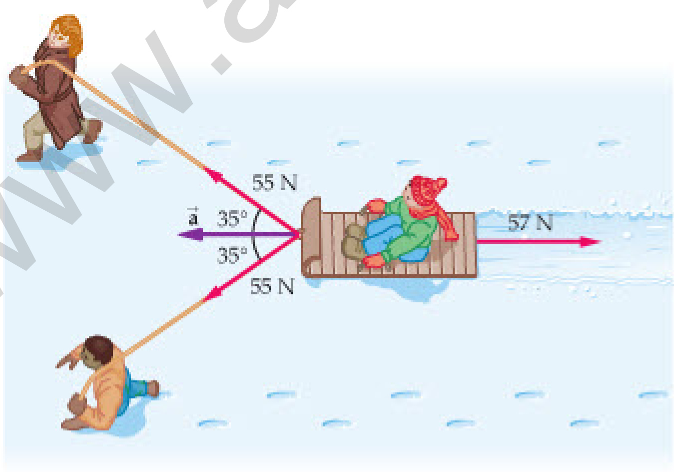
▲ **FIGURE 5–23** Problem 28

29. •• **CE** A hockey puck is acted on by one or more forces, as shown in Figure 5–24. Rank the four cases, A, B, C, and D, in order of the magnitude of the puck's acceleration, starting with the smallest. Indicate ties where appropriate.



▲ **FIGURE 5–24** Problem 29

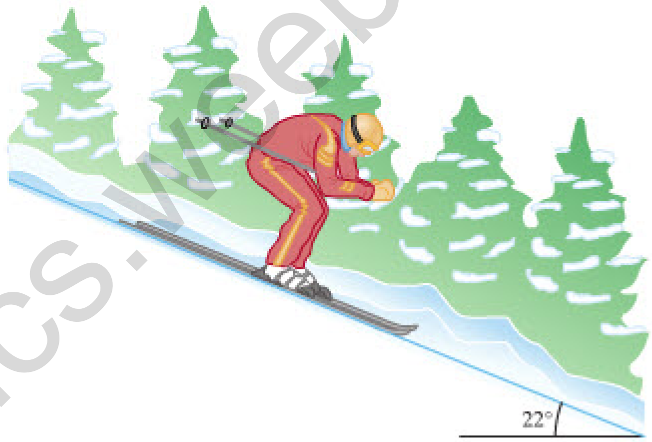
30. •• To give a 19-kg child a ride, two teenagers pull on a 3.7-kg sled with ropes, as indicated in Figure 5–25. Both teenagers pull with a force of 55 N at an angle of 35° relative to the forward direction, which is the direction of motion. In addition, the snow exerts a retarding force on the sled that points opposite to the direction of motion, and has a magnitude of 57 N. Find the acceleration of the sled and child.



▲ **FIGURE 5–25** Problem 30

31. •• **IP** Before practicing his routine on the rings, a 67-kg gymnast stands motionless, with one hand grasping each ring and his feet touching the ground. Both arms slope upward at an angle of 24° above the horizontal. **(a)** If the force exerted by the rings on each arm has a magnitude of 290 N, and is directed along the length of the arm, what is the magnitude of the force exerted by the floor on his feet? **(b)** If the angle his arms make with the horizontal is greater that 24°, and everything else remains the same, is the force exerted by the floor on his feet greater than, less than, or the same as the value found in part (a)? Explain.

32. •• **IP** A 65-kg skier speeds down a trail, as shown in Figure 5–26. The surface is smooth and inclined at an angle of 22° with the horizontal. **(a)** Find the direction and magnitude of the net force acting on the skier. **(b)** Does the net force exerted on the skier increase, decrease, or stay the same as the slope becomes steeper? Explain.



▲ **FIGURE 5–26** Problems 32 and 45

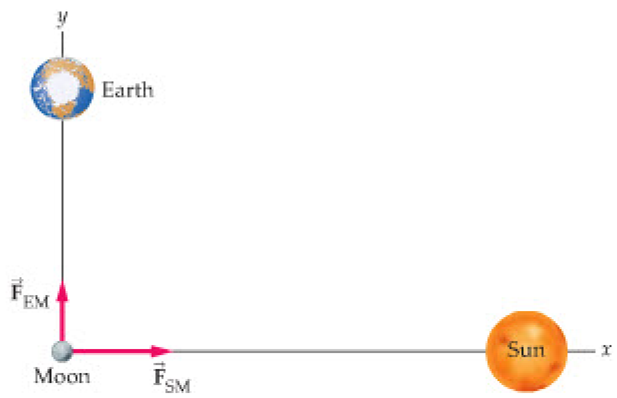
33. •• An object acted on by three forces moves with constant velocity. One force acting on the object is in the positive x direction and has a magnitude of 6.5 N; a second force has a magnitude of 4.4 N and points in the negative y direction. Find the direction and magnitude of the third force acting on the object.

34. •• A train is traveling up a 3.73° incline at a speed of 3.25 m/s when the last car breaks free and begins to coast without friction. **(a)** How long does it take for the last car to come to rest momentarily? **(b)** How far did the last car travel before momentarily coming to rest?

35. •• **The Force Exerted on the Moon** Figure 5–27 shows the Earth, Moon, and Sun (not to scale) in their relative positions at the time when the Moon is in its third-quarter phase. Though few people realize it, the force exerted on the Moon by the Sun is actually greater than the force exerted on the Moon by the Earth. In fact, the force exerted on the Moon by the Sun has a magnitude of $F_{SM} = 4.34 \times 10^{20}$ N, whereas the force exerted by the Earth has a magnitude of only $F_{EM} = 1.98 \times 10^{20}$ N. These forces are indicated to scale in Figure 5–28. Find **(a)** the direction and **(b)** the magnitude of the net force acting on the Moon. **(c)** Given that the mass of the Moon is $M_M = 7.35 \times 10^{22}$ kg, find the magnitude of its acceleration at the time of the third-quarter phase.

SECTION 5–6 WEIGHT

36. • You pull upward on a stuffed suitcase with a force of 105 N, and it accelerates upward at 0.705 m/s^2. What are **(a)** the mass and **(b)** the weight of the suitcase?



▲ **FIGURE 5–27** Problem 35

37. • **BIO Brain Growth** A newborn baby's brain grows rapidly. In fact, it has been found to increase in mass by about 1.6 mg per minute. **(a)** How much does the brain's weight increase in one day? **(b)** How long does it take for the brain's weight to increase by 0.15 N?

38. • Suppose a rocket launches with an acceleration of 30.5 m/s^2. What is the apparent weight of an 92-kg astronaut aboard this rocket?

39. • At the bow of a ship on a stormy sea, a crewman conducts an experiment by standing on a bathroom scale. In calm waters, the scale reads 182 lb. During the storm, the crewman finds a maximum reading of 225 lb and a minimum reading of 138 lb. Find **(a)** the maximum upward acceleration and **(b)** the maximum downward acceleration experienced by the crewman.

40. •• **IP** As part of a physics experiment, you stand on a bathroom scale in an elevator. Though your normal weight is 610 N, the scale at the moment reads 730 N. **(a)** Is the acceleration of the elevator upward, downward, or zero? Explain. **(b)** Calculate the magnitude of the elevator's acceleration. **(c)** What, if anything, can you say about the velocity of the elevator? Explain.

41. •• When you weigh yourself on good old *terra firma* (solid ground), your weight is 142 lb. In an elevator your apparent weight is 121 lb. What are the direction and magnitude of the elevator's acceleration?

42. •• **IP BIO Flight of the Samara** A 1.21-g samara—the winged fruit of a maple tree—falls toward the ground with a constant speed of 1.1 m/s (Figure 5–28). **(a)** What is the force of air resistance exerted on the samara? **(b)** If the constant speed of descent is greater than 1.1 m/s, is the force of air resistance greater than, less than, or the same as in part (a)? Explain.

▲ **FIGURE 5–28** Problem 42

43. ••• When you lift a bowling ball with a force of 82 N, the ball accelerates upward with an acceleration a. If you lift with a force of 92 N, the ball's acceleration is $2a$. Find **(a)** the weight of the bowling ball, and **(b)** the acceleration a.

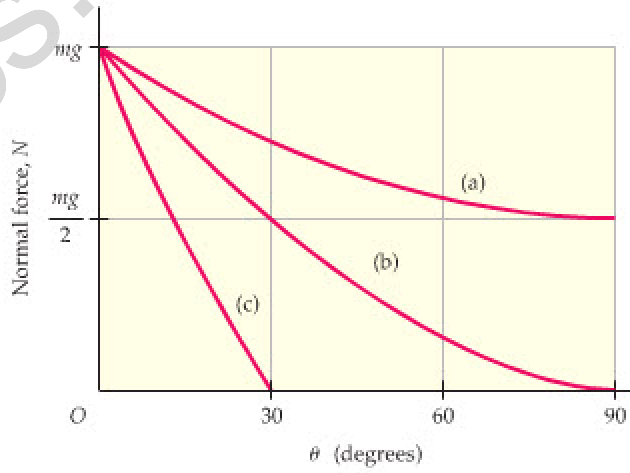
SECTION 5–7 NORMAL FORCES

44. • A 23-kg suitcase is being pulled with constant speed by a handle that is at an angle of 25° above the horizontal. If the normal force exerted on the suitcase is 180 N, what is the force F applied to the handle?

45. • **(a)** Draw a free-body diagram for the skier in Problem 32. **(b)** Determine the normal force acting on the skier.

46. • A 9.3-kg child sits in a 3.7-kg high chair. **(a)** Draw a free-body diagram for the child, and find the normal force exerted by the chair on the child. **(b)** Draw a free-body diagram for the chair, and find the normal force exerted by the floor on the chair.

47. •• Figure 5–29 shows the normal force as a function of the angle θ for the suitcase shown in Figure 5–13. Determine the magnitude of the force $\vec{F}$ for each of the three curves shown in Figure 5–29. Give your answer in terms of the weight of the suitcase, mg.



▲ **FIGURE 5–29** Problem 47

48. •• A 5.0-kg bag of potatoes sits on the bottom of a stationary shopping cart. **(a)** Sketch a free-body diagram for the bag of potatoes. **(b)** Now suppose the cart moves with a constant velocity. How does this affect your free-body diagram? Explain.

49. •• **IP** **(a)** Find the normal force exerted on a 2.9-kg book resting on a surface inclined at 36° above the horizontal. **(b)** If the angle of the incline is reduced, do you expect the normal force to increase, decrease, or stay the same? Explain.

50. •• **IP** A gardener mows a lawn with an old-fashioned push mower. The handle of the mower makes an angle of 35° with the surface of the lawn. **(a)** If a 219-N force is applied along the handle of the 19-kg mower, what is the normal force exerted by the lawn on the mower? **(b)** If the angle between the surface of the lawn and the handle of the mower is increased, does the normal force exerted by the lawn increase, decrease, or stay the same? Explain.

51. ••• An ant walks slowly away from the top of a bowling ball, as shown in Figure 5–30. If the ant starts to slip when the normal

force on its feet drops below one-half its weight, at what angle θ does slipping begin?

▲ **FIGURE 5–30** Problem 51

GENERAL PROBLEMS

52. • **CE Predict/Explain** Riding in an elevator moving upward with constant speed, you begin a game of darts. **(a)** Do you have to aim your darts higher than, lower than, or the same as when you play darts on solid ground? **(b)** Choose the *best explanation* from among the following:
   I. The elevator rises during the time it takes for the dart to travel to the dartboard.
   II. The elevator moves with constant velocity. Therefore, Newton's laws apply within the elevator in the same way as on the ground.
   III. You have to aim lower to compensate for the upward speed of the elevator.

53. • **CE Predict/Explain** Riding in an elevator moving with a constant upward acceleration, you begin a game of darts. **(a)** Do you have to aim your darts higher than, lower than, or the same as when you play darts on solid ground? **(b)** Choose the *best explanation* from among the following:
   I. The elevator accelerates upward, giving its passengers a greater "effective" acceleration of gravity.
   II. You have to aim lower to compensate for the upward acceleration of the elevator.
   III. Since the elevator moves with a constant acceleration, Newton's laws apply within the elevator the same as on the ground.

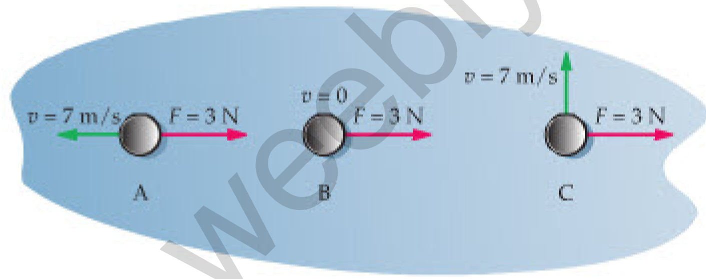
54. • **CE** Give the direction of the net force acting on each of the following objects. If the net force is zero, state "zero." **(a)** A car accelerating northward from a stoplight. **(b)** A car traveling southward and slowing down. **(c)** A car traveling westward with constant speed. **(d)** A skydiver parachuting downward with constant speed. **(e)** A baseball during its flight from pitcher to catcher (ignoring air resistance).

55. • **CE Predict/Explain** You jump out of an airplane and open your parachute after an extended period of free fall. **(a)** To decelerate your fall, must the force exerted on you by the parachute be greater than, less than, or equal to your weight? **(b)** Choose the *best explanation* from among the following:
   I. Parachutes can only exert forces that are less than the weight of the skydiver.
   II. The parachute exerts a force exactly equal to the skydiver's weight.
   III. To decelerate after free fall, the net force acting on a skydiver must be upward.

56. • In a tennis serve, a 0.070-kg ball can be accelerated from rest to 36 m/s over a distance of 0.75 m. Find the magnitude of the average force exerted by the racket on the ball during the serve.

57. • A 51.5-kg swimmer with an initial speed of 1.25 m/s decides to coast until she comes to rest. If she slows with constant acceleration and stops after coasting 2.20 m, what was the force exerted on her by the water?

58. •• **CP** Each of the three identical hockey pucks shown in Figure 5–31 is acted on by a 3-N force. Puck A moves with a speed of 7 m/s in a direction opposite to the force; puck B is instantaneously at rest; puck C moves with a speed of 7 m/s at right angles to the force. Rank the three pucks in order of the magnitude of their acceleration, starting with the smallest. Indicate ties with an equal sign.



▲ **FIGURE 5–31** Problem 58

59. •• **IP The VASIMR Rocket** NASA plans to use a new type of rocket, a Variable Specific Impulse Magnetoplasma Rocket (VASIMR), on future missions. A VASIMR can produce 1200 N of thrust (force) when in operation. If a VASIMR has a mass of 2.2×10^5 kg, **(a)** what acceleration will it experience? Assume that the only force acting on the rocket is its own thrust, and that the mass of the rocket is constant. **(b)** Over what distance must the rocket accelerate from rest to achieve a speed of 9500 m/s? **(c)** When the rocket has covered one-quarter the acceleration distance found in part (b), is its average speed 1/2, 1/3, or 1/4 its average speed during the final three-quarters of the acceleration distance? Explain.

60. •• An object of mass $m = 5.95$ kg has an acceleration $\vec{a} = (1.17\text{ m/s}^2)\hat{x} + (-0.664\text{ m/s}^2)\hat{y}$. Three forces act on this object: $\vec{F}_1$, $\vec{F}_2$, and $\vec{F}_3$. Given that $\vec{F}_1 = (3.22\text{ N})\hat{x}$ and $\vec{F}_2 = (-1.55\text{ N})\hat{x} + (2.05\text{ N})\hat{y}$, find $\vec{F}_3$.

61. •• At the local grocery store, you push a 14.5-kg shopping cart. You stop for a moment to add a bag of dog food to your cart. With a force of 12.0 N, you now accelerate the cart from rest through a distance of 2.29 m in 3.00 s. What was the mass of the dog food?

62. •• **IP BIO The Force of Running** Biomechanical research has shown that when a 67-kg person is running, the force exerted on each foot as it strikes the ground can be as great as 2300 N. **(a)** What is the ratio of the force exerted on the foot by the ground to the person's body weight? **(b)** If the only forces acting on the person are (i) the force exerted by the ground and (ii) the person's weight, what are the magnitude and direction of the person's acceleration? **(c)** If the acceleration found in part (b) acts for 10.0 ms, what is the resulting change in the vertical component of the person's velocity?

63. •• **IP BIO Grasshopper Liftoff** To become airborne, a 2.0-g grasshopper requires a takeoff speed of 2.7 m/s. It acquires this speed by extending its hind legs through a distance of 3.7 cm. **(a)** What is the average acceleration of the grasshopper during takeoff? **(b)** Find the magnitude of the average net force exerted

on the grasshopper by its hind legs during takeoff. **(c)** If the mass of the grasshopper increases, does the takeoff acceleration increase, decrease, or stay the same? **(d)** If the mass of the grasshopper increases, does the required takeoff force increase, decrease, or stay the same? Explain.

64. •• **Takeoff from an Aircraft Carrier** On an aircraft carrier, a jet can be catapulted from 0 to 155 mi/h in 2.00 s. If the average force exerted by the catapult is 9.35×10^5 N, what is the mass of the jet?

A jet takes off from the flight deck of an aircraft carrier. (Problem 64)

65. •• **IP** An archer shoots a 0.024-kg arrow at a target with a speed of 54 m/s. When it hits the target, it penetrates to a depth of 0.083 m. **(a)** What was the average force exerted by the target on the arrow? **(b)** If the mass of the arrow is doubled, and the force exerted by the target on the arrow remains the same, by what multiplicative factor does the penetration depth change? Explain.

66. •• An apple of mass $m = 0.13$ kg falls out of a tree from a height $h = 3.2$ m. **(a)** What is the magnitude of the force of gravity, mg, acting on the apple? **(b)** What is the apple's speed, v, just before it lands? **(c)** Show that the force of gravity times the height, mgh, is equal to $\frac{1}{2}mv^2$. (We shall investigate the significance of this result in Chapter 8.) Be sure to show that the dimensions are in agreement as well as the numerical values.

67. •• An apple of mass $m = 0.22$ kg falls from a tree and hits the ground with a speed of $v = 14$ m/s. **(a)** What is the magnitude of the force of gravity, mg, acting on the apple? **(b)** What is the time, t, required for the apple to reach the ground? **(c)** Show that the force of gravity times the time, mgt, is equal to mv. (We shall investigate the significance of this result in Chapter 9.) Be sure to show that the dimensions are in agreement as well as the numerical values.

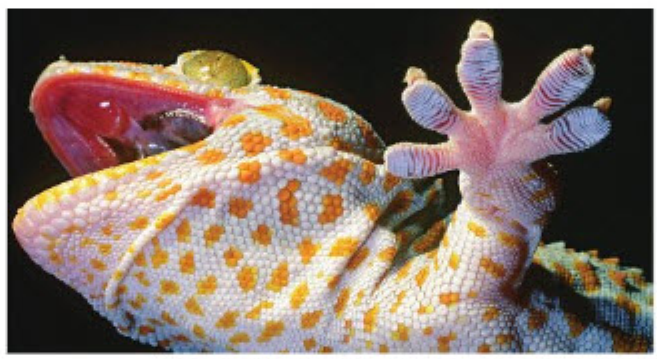
68. •• **BIO The Fall of *T. rex*** Paleontologists estimate that if a *Tyrannosaurus rex* were to trip and fall, it would have experienced a force of approximately 260,000 N acting on its torso when it hit the ground. Assuming the torso has a mass of 3800 kg, **(a)** find the magnitude of the torso's upward acceleration as it comes to rest. (For comparison, humans lose consciousness with an acceleration of about $7g$.) **(b)** Assuming the torso is in free fall for a distance of 1.46 m as it falls to the ground, how much time is required for the torso to come to rest once it contacts the ground?

69. •• **Deep Space I** The NASA spacecraft *Deep Space I* was shut down on December 18, 2001, following a three-year journey to the asteroid Braille and the comet Borrelly. This spacecraft used a solar-powered ion engine to produce 0.064 ounces of thrust (force) by stripping electrons from neon atoms and accelerating the resulting ions to 70,000 mi/h. The thrust was only as much as the weight of a couple sheets of paper, but the engine operated continuously for 16,000 hours. As a result, the speed of the spacecraft increased by 7900 mi/h. What was the mass of *Deep Space I*? (Assume that the mass of the neon gas is negligible.)

70. •• Your groceries are in a bag with paper handles. The handles will tear off if a force greater than 51.5 N is applied to them. What is the greatest mass of groceries that can be lifted safely with this bag, given that the bag is raised **(a)** with constant speed, or **(b)** with an acceleration of 1.25 m/s^2?

71. •• **IP** While waiting at the airport for your flight to leave, you observe some of the jets as they take off. With your watch you find that it takes about 35 seconds for a plane to go from rest to takeoff speed. In addition, you estimate that the distance required is about 1.5 km. **(a)** If the mass of a jet is 1.70×10^5 kg, what force is needed for takeoff? **(b)** Describe the strategy you used to solve part (a).

72. •• **BIO Gecko Feet** Researchers have found that a gecko's foot is covered with hundreds of thousands of small hairs (*setae*) that allow it to walk up walls and even across ceilings. A single foot pad, which has an area of 1.0 cm^2, can attach to a wall or ceiling with a force of 11 N. **(a)** How many 250-g geckos could be suspended from the ceiling by a single foot pad? **(b)** Estimate the force per square centimeter that your body exerts on the soles of your shoes, and compare with the 11 N/cm^2 of the sticky gecko foot.



A Tokay gecko (*Gekko gecko*) shows off its famous feet. (Problem 72)

73. •• Two boxes are at rest on a smooth, horizontal surface. The boxes are in contact with one another. If box 1 is pushed with a force of magnitude $F = 12.00$ N, the contact force between the boxes is 8.50 N; if, instead, box 2 is pushed with the force F, the contact force is 12.00 N − 8.50 N = 3.50 N. In either case, the boxes move together with an acceleration of 1.70 m/s^2. What is the mass of **(a)** box 1 and **(b)** box 2?

74. ••• **IP** Responding to an alarm, a 102-kg fireman slides down a pole to the ground floor, 3.3 m below. The fireman starts at rest and lands with a speed of 4.2 m/s. **(a)** Find the average force exerted on the fireman by the pole. **(b)** If the landing speed is half that in part (a), is the average force exerted on the fireman by the pole doubled? Explain. **(c)** Find the average force exerted on the fireman by the pole when the landing speed is 2.1 m/s.

75. ••• For a birthday gift, you and some friends take a hot-air balloon ride. One friend is late, so the balloon floats a couple of feet off the ground as you wait. Before this person arrives, the combined weight of the basket and people is 1220 kg, and the balloon is neutrally buoyant. When the late arrival climbs up into the basket, the balloon begins to accelerate downward at $0.56\ m/s^2$. What was the mass of the last person to climb aboard?

76. ••• A baseball of mass m and initial speed v strikes a catcher's mitt. If the mitt moves a distance Δx as it brings the ball to rest, what is the average force it exerts on the ball?

77. ••• When two people push in the same direction on an object of mass m they cause an acceleration of magnitude a_1. When the same people push in opposite directions, the acceleration of the object has a magnitude a_2. Determine the magnitude of the force exerted by each of the two people in terms of m, a_1, and a_2.

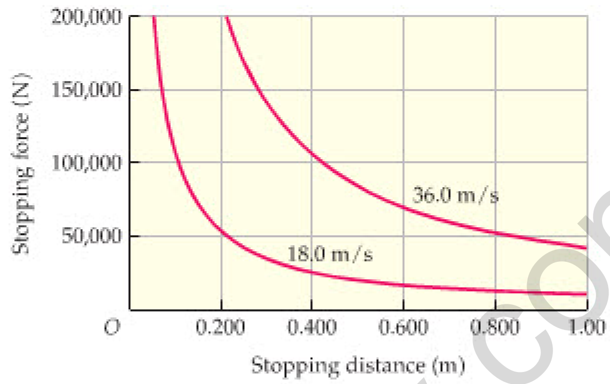
78. ••• An air-track cart of mass $m_1 = 0.14$ kg is moving with a speed $v_0 = 1.3$ m/s to the right when it collides with a cart of mass $m_2 = 0.25$ kg that is at rest. Each cart has a wad of putty on its bumper, and hence they stick together as a result of their collision. Suppose the average contact force between the carts is $F = 1.5$ N during the collision. **(a)** What is the acceleration of cart 1? Give direction and magnitude. **(b)** What is the acceleration of cart 2? Give direction and magnitude. **(c)** How long does it take for both carts to have the same speed? (Once the carts have the same speed the collision is over and the contact force vanishes.) **(d)** What is the final speed of the carts, v_f? **(e)** Show that m_1v_0 is equal to $(m_1 + m_2)v_f$. (We shall investigate the significance of this result in Chapter 9.)

PASSAGE PROBLEMS

BIO Increasing Safety in a Collision

Safety experts say that an automobile accident is really a succession of three separate collisions. These can be described as follows: (1) the automobile collides with an obstacle and comes to rest; (2) people within the car continue to move forward until they collide with the interior of the car, or are brought to rest by a restraint system like a seatbelt or an air bag; (3) organs within the occupants' bodies continue to move forward until they collide with the body wall and are brought to rest. Not much can be done about the third collision, but the effects of the first two can be mitigated by increasing the distance over which the car and its occupants are brought to rest.

For example, the severity of the first collision is reduced by building collapsible "crumple zones" into the body of a car, and by placing compressible collision barriers near dangerous obstacles like bridge supports. The second collision is addressed primarily through the use of seatbelts and air bags. These devices reduce the force that acts on an occupant to survivable levels by increasing the distance over which he or she comes to rest. This is illustrated in Figure 5–32, where we see the force exerted on a 65.0-kg driver who slows from an initial speed of 18.0 m/s (lower curve) or 36.0 m/s (upper curve) to rest in a distance ranging from 5.00 cm to 1.00 m.



▲ **FIGURE 5–32** Problems 79, 80, 81, and 82

79. • The combination of "crumple zones" and air bags/seatbelts might increase the distance over which a person stops in a collision to as great as 1.00 m. What is the magnitude of the force exerted on a 65.0-kg driver who decelerates from 18.0 m/s to 0.00 m/s over a distance of 1.00 m?

A. 162 N **B.** 585 N
C. 1.05×10^4 N **D.** 2.11×10^4 N

80. • A driver who does not wear a seatbelt continues to move forward with a speed of 18.0 m/s (due to inertia) until something solid like the steering wheel is encountered. The driver now comes to rest in a much shorter distance—perhaps only a few centimeters. Find the magnitude of the net force acting on a 65.0-kg driver who is decelerated from 18.0 m/s to rest in 5.00 cm.

A. 3240 N **B.** 1.17×10^4 N
C. 2.11×10^5 N **D.** 4.21×10^5 N

81. • Suppose the initial speed of the driver is doubled to 36.0 m/s. If the driver still has a mass of 65.0 kg, and comes to rest in 1.00 m, what is the magnitude of the force exerted on the driver during this collision?

A. 648 N **B.** 1170 N
C. 2.11×10^4 N **D.** 4.21×10^4 N

82. • If both the speed and stopping distance of a driver are doubled, by what factor does the force exerted on the driver change?

A. 0.5 **B.** 1
C. 2 **D.** 4

INTERACTIVE PROBLEMS

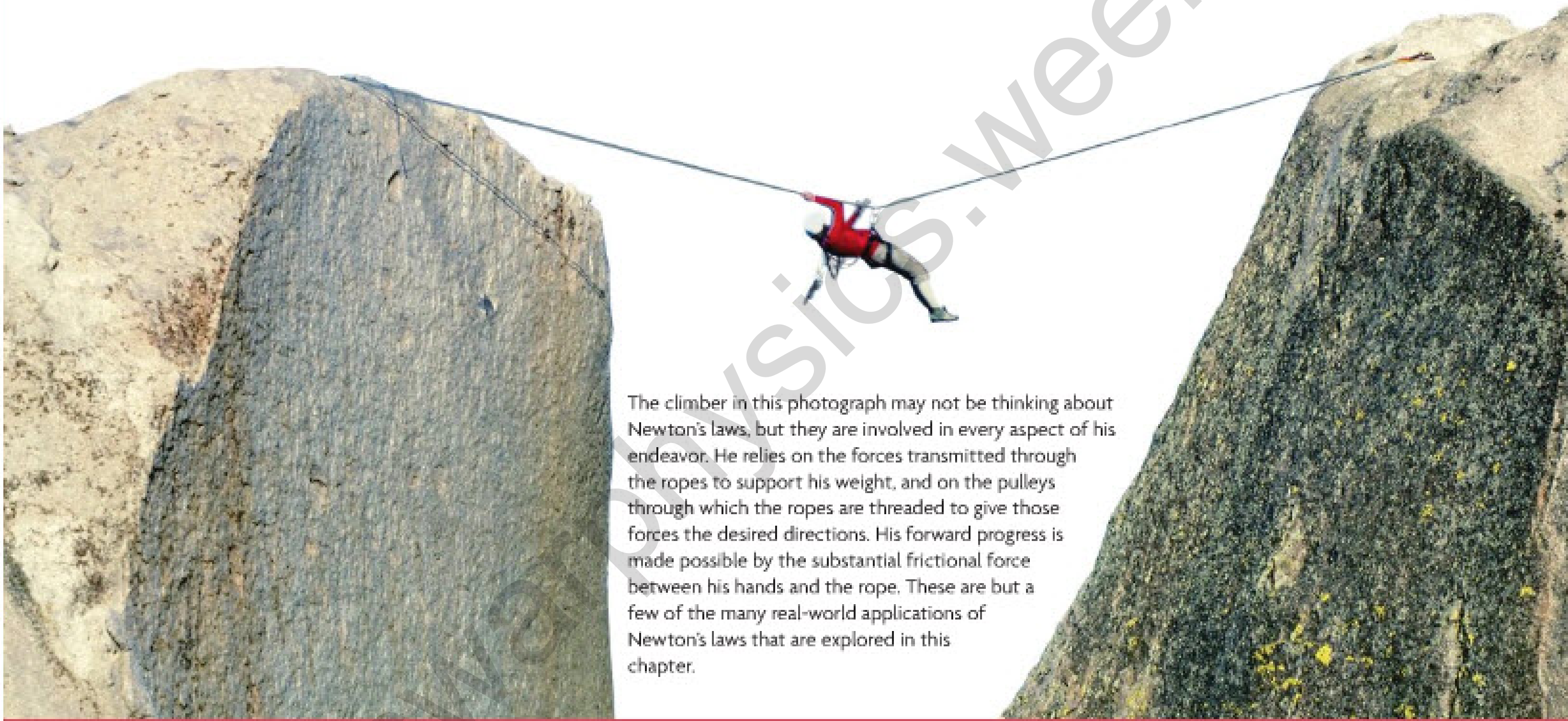
83. •• **IP Referring to Example 5–4** Suppose that we would like the contact force between the boxes to have a magnitude of 5.00 N, and that the only thing in the system we are allowed to change is the mass of box 2—the mass of box 1 is 10.0 kg and the applied force is 20.0 N. **(a)** Should the mass of box 2 be increased or decreased? Explain. **(b)** Find the mass of box 2 that results in a contact force of magnitude 5.00 N. **(c)** What is the acceleration of the boxes in this case?

84. •• **Referring to Example 5–4** Suppose the force of 20.0 N pushes on two boxes of unknown mass. We know, however, that the acceleration of the boxes is $1.20\ m/s^2$ and the contact force has a magnitude of 4.45 N. Find the mass of **(a)** box 1 and **(b)** box 2.

85. •• **IP Referring to Figure 5–9** Suppose the magnitude of $\vec{F}_2$ is increased from 41 N to 55 N, and that everything else in the system remains the same. **(a)** Do you expect the direction of the satellite's acceleration to be greater than, less than, or equal to 32°? Explain. Find **(b)** the direction and **(c)** the magnitude of the satellite's acceleration in this case.

86. •• **IP Referring to Figure 5–9** Suppose we would like the acceleration of the satellite to be at an angle of 25°, and that the only quantity we can change in the system is the magnitude of $\vec{F}_1$. **(a)** Should the magnitude of $\vec{F}_1$ be increased or decreased? Explain. **(b)** What is the magnitude of the satellite's acceleration in this case?

6 Applications of Newton's Laws



The climber in this photograph may not be thinking about Newton's laws, but they are involved in every aspect of his endeavor. He relies on the forces transmitted through the ropes to support his weight, and on the pulleys through which the ropes are threaded to give those forces the desired directions. His forward progress is made possible by the substantial frictional force between his hands and the rope. These are but a few of the many real-world applications of Newton's laws that are explored in this chapter.

Newton's laws of motion can be applied to an immense variety of systems, a sampling of which was discussed in Chapter 5. In this chapter we extend our discussion of Newton's laws by introducing new types of forces and by considering new classes of systems.

For example, we begin by considering the forces due to friction between two surfaces. As we shall see, the force of friction is different depending on whether the surfaces are in static contact, or are moving relative to one another—an important consideration in antilock braking systems. And though friction may seem like something that should be eliminated, we show that it is actually essential to life as we know it.

Next, we investigate the forces exerted by strings and springs, and show how these forces can safely suspend a mountain climber over a chasm, or cushion the ride of a locomotive. Finally, we consider the key role that force plays in making circular motion possible.

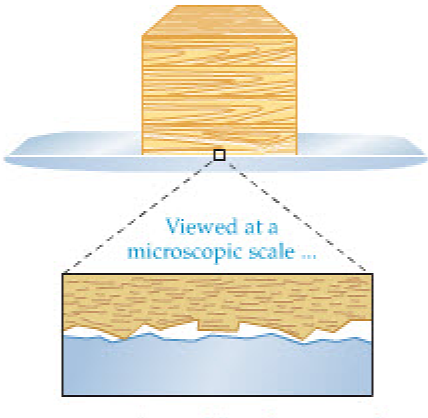


FIGURE 6–1 The origin of friction
Even "smooth" surfaces have irregularities when viewed at the microscopic level. This type of roughness contributes to friction.

6–1 Frictional Forces

In Chapter 5 we always assumed that surfaces were smooth and that objects could slide without resistance to their motion. No surface is perfectly smooth, however. When viewed on the atomic level, even the "smoothest" surface is actually rough and jagged, as indicated in **Figure 6–1**. To slide one such surface across another requires a force large enough to overcome the resistance of microscopic hills and valleys bumping together. This is the origin of the force we call **friction.**

We often think of friction as something that should be reduced, or even eliminated if possible. For example, roughly 20% of the gasoline you buy does nothing but overcome friction within your car's engine. Clearly, reducing that friction would be most desirable.

On the other hand, friction can be helpful—even indispensable—in other situations. Suppose, for example, that you are standing still and then decide to begin walking forward. The force that accelerates you is the force of friction between your shoes and the ground. We simply couldn't walk or run without friction—it's hard enough when friction is merely reduced, as on an icy sidewalk. Similarly, starting or stopping a car, or even turning a corner, all require friction. Friction is an important and common feature of everyday life.

Since friction is caused by the random, microscopic irregularities of a surface, and since it is greatly affected by other factors such as the presence of lubricants, there is no simple "law of nature" for friction. There are, however, some very useful rules of thumb that give us rather accurate, approximate results for calculating frictional forces. In what follows, we describe these rules of thumb for the two types of friction most commonly used in this text—kinetic friction and static friction.

Kinetic Friction

As its name implies, kinetic friction is the friction encountered when surfaces slide against one another with a finite relative speed. The force generated by this friction, which will be designated with the symbol f_k, acts to oppose the sliding motion at the point of contact between the surfaces.

A series of simple experiments illustrates the main characteristics of kinetic friction. First, imagine attaching a spring scale to a rough object, like a brick, and pulling it across a table, as shown in **Figure 6–2**. If the brick moves with constant velocity, Newton's second law tells us that the net force on the brick must be zero. Hence, the force read on the scale, F, has the same magnitude as the force of kinetic friction, f_k. Now, if we repeat the experiment, but this time put a second brick on top of the first, we find that the force needed to pull the brick with constant velocity is doubled, to $2F$.

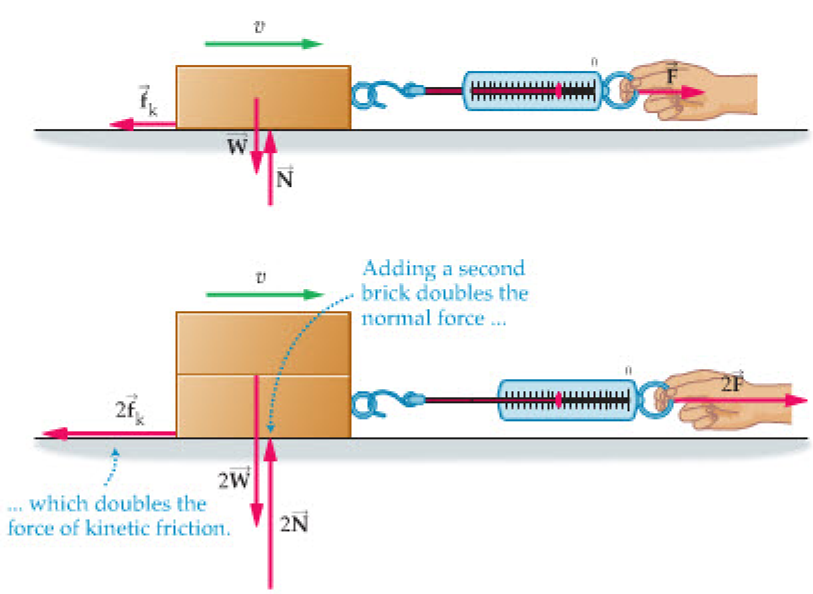


FIGURE 6–2 The force of kinetic friction depends on the normal force
In the top part of the figure, a force F is required to pull the brick with constant speed v. Thus the force of kinetic friction is $f_k = F$. In the bottom part of the figure, the normal force has been doubled, and so has the force of kinetic friction, to $f_k = 2F$.

From this experiment we see that when we double the normal force—by stacking up two bricks, for example—the force of kinetic friction is also doubled. In general, the force of kinetic friction is found to be proportional to the magnitude of the normal force, N. Stated mathematically, this observation can be written as follows:

$$f_k = \mu_k N \qquad 6\text{–}1$$

The constant of proportionality, μ_k (pronounced "mew sub k"), is referred to as the **coefficient of kinetic friction.** In Figure 6–2 the normal force is equal to the weight of the bricks, but this is a special case. The normal force is greater than the weight if someone pushes down on the bricks, and this would cause more friction, or less than the weight if the bricks are placed on an incline. The former case is considered in several homework problems, and the latter case is considered in Examples 6–2 and 6–3.

Since f_k and N are both forces, and hence have the same units, we see that μ_k is a dimensionless number. The coefficient of kinetic friction is always positive, and typical values range between 0 and 1, as indicated in Table 6–1. The interpretation of μ_k is simple: If $\mu_k = 0.1$, for example, the force of kinetic friction is one-tenth of the normal force. Simply put, the greater μ_k the greater the friction; the smaller μ_k the smaller the friction.

TABLE 6–1 Typical Coefficients of Friction

Materials	Kinetic, μ_k	Static, μ_s
Rubber on concrete (dry)	0.80	1–4
Steel on steel	0.57	0.74
Glass on glass	0.40	0.94
Wood on leather	0.40	0.50
Copper on steel	0.36	0.53
Rubber on concrete (wet)	0.25	0.30
Steel on ice	0.06	0.10
Waxed ski on snow	0.05	0.10
Teflon on Teflon	0.04	0.04
Synovial joints in humans	0.003	0.01

As we know from everyday experience, the force of kinetic friction tends to oppose motion, as shown in Figure 6–2. Thus, $f_k = \mu_k N$ is not a vector equation, because N is perpendicular to the direction of motion. When doing calculations with the force of kinetic friction, we use $f_k = \mu_k N$ to find its magnitude, and we draw its direction so that it is opposite to the direction of motion.

There are two more friction experiments of particular interest. First, suppose that when we pull a brick, we initially pull it at the speed v, then later at the speed $2v$. What forces do we measure? It turns out that the force of kinetic friction is approximately the same in each case—it certainly does not double when we double the speed. Second, let's try standing the brick on end, so that it has a smaller area in contact with the table. If this smaller area is half the previous area, is the force halved? No, the force remains essentially the same, regardless of the area of contact.

We summarize these observations with the following three rules of thumb for kinetic friction:

Rules of Thumb for Kinetic Friction

The force of kinetic friction between two surfaces is:

1. Proportional to the magnitude of the normal force, N, between the surfaces:

$$f_k = \mu_k N$$

2. Independent of the relative speed of the surfaces.
3. Independent of the area of contact between the surfaces.

▲ Friction plays an important role in almost everything we do. Sometimes it is desirable to reduce friction; in other cases we want as much friction as possible. For example, it is more fun to ride on a water slide (upper) if the friction is low. Similarly, an engine operates more efficiently when it is oiled. When running, however, we need friction to help us speed up, slow down, and make turns. The sole of this running shoe (lower), like a car tire, is designed to maximize friction.

Again, these rules are useful and fairly accurate, though they are still only approximate. For simplicity, when we do calculations involving kinetic friction in this text, we will use these rules as if they were exact.

Before we show how to use f_k in calculations, we should make a comment regarding rule 3. This rule often seems rather surprising and counterintuitive. How is it that a larger area of contact doesn't produce a larger force? One way to think about this is to consider that when the area of contact is large, the normal force is spread out over a large area, giving a small force per area, F/A. As a result, the microscopic hills and valleys are not pressed too deeply against one another. On the other hand, if the area is small, the normal force is concentrated in a small region, which presses the surfaces together more firmly, due to the large force per area. The net effect is roughly the same in either case.

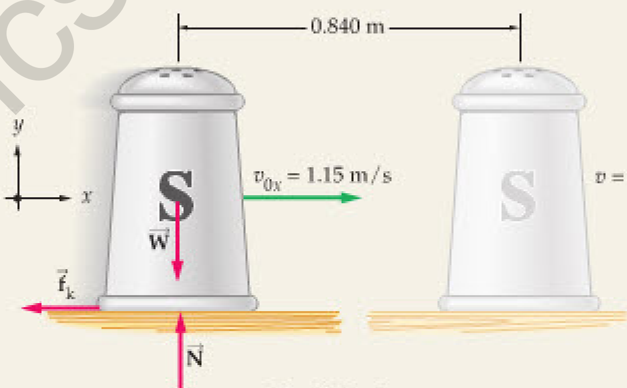
Now, let's consider a commonly encountered situation in which kinetic friction plays a decisive role.

EXAMPLE 6–1 PASS THE SALT—PLEASE

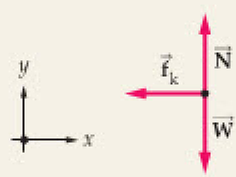
Someone at the other end of the table asks you to pass the salt. Feeling quite dashing, you slide the 50.0-g salt shaker in their direction, giving it an initial speed of 1.15 m/s. **(a)** If the shaker comes to rest with constant acceleration in 0.840 m, what is the coefficient of kinetic friction between the shaker and the table? **(b)** How much time is required for the shaker to come to rest if you slide it with an initial speed of 1.32 m/s?

PICTURE THE PROBLEM

We choose the positive x direction to be the direction of motion, and the positive y direction to be upward. Two forces act in the y direction; the shaker's weight, $\vec{\mathbf{W}} = -W\hat{\mathbf{y}} = -mg\hat{\mathbf{y}}$, and the normal force, $\vec{\mathbf{N}} = N\hat{\mathbf{y}}$. Only one force acts in the x direction: the force of kinetic friction, $\vec{\mathbf{f}}_k = -\mu_k N\hat{\mathbf{x}}$. Note that the shaker moves through a distance of 0.840 m with an initial speed $v_{0x} = 1.15$ m/s.



Physical picture



Free-body diagram

STRATEGY

a. Since the frictional force has a magnitude of $f_k = \mu_k N$, it follows that $\mu_k = f_k/N$. Therefore, we need to find the magnitudes of the frictional force, f_k, and the normal force, N. To find f_k we set $\sum F_x = ma_x$, and find a_x with the kinematic equation $v_x^2 = v_{0x}^2 + 2a_x\Delta x$. To find N we set $a_y = 0$ (since there is no motion in the y direction) and solve for N using $\sum F_y = ma_y = 0$.

b. The coefficient of kinetic friction is independent of the sliding speed, and hence the acceleration of the shaker is also independent of the speed. As a result, we can use the acceleration from part (a) in the relation $v_x = v_{0x} + a_x t$ to find the sliding time.

SOLUTION

Part (a)

1. Set $\sum F_x = ma_x$ to find f_k in terms of a_x:

$$\sum F_x = -f_k = ma_x \quad \text{or} \quad f_k = -ma_x$$

2. Determine a_x by using the kinematic equation relating velocity to position, $v_x^2 = v_{0x}^2 + 2a_x\Delta x$:

$$v_x^2 = v_{0x}^2 + 2a_x\Delta x$$

$$a_x = \frac{v_x^2 - v_{0x}^2}{2\Delta x} = \frac{0 - (1.15\text{ m/s})^2}{2(0.840\text{ m})} = -0.787\text{ m/s}^2$$

3. Set $\sum F_y = ma_y = 0$ to find the normal force, N:

$$\sum F_y = N + (-W) = ma_y = 0 \quad \text{or} \quad N = W = mg$$

4. Substitute $N = mg$ and $f_k = -ma_x$ (with $a_x = -0.787\text{ m/s}^2$) into $\mu_k = f_k/N$ to find μ_k:

$$\mu_k = \frac{f_k}{N} = \frac{-ma_x}{mg} = \frac{-a_x}{g} = \frac{-(-0.787\text{ m/s}^2)}{9.81\text{ m/s}^2} = 0.0802$$

Part (b)

5. Use $a_x = -0.787\ \mathrm{m/s^2}$, $v_{0x} = 1.32\ \mathrm{m/s}$, and $v_x = 0$ in $v_x = v_{0x} + a_x t$ to solve for the time, t:

$$v_x = v_{0x} + a_x t \quad \text{or}$$

$$t = \frac{v_x - v_{0x}}{a_x} = \frac{0 - (1.32\ \mathrm{m/s})}{-0.787\ \mathrm{m/s^2}} = 1.68\ \mathrm{s}$$

INSIGHT

Note that m canceled in Step 4, so our result for the coefficient of friction is independent of the shaker's mass. For example, if we were to slide a shaker with twice the mass, but with the same initial speed, it would slide the same distance. It is unlikely this independence would have been apparent if we had worked the problem numerically rather than symbolically. Part (b) shows that the same comments apply to the sliding time—it too is independent of the shaker's mass.

PRACTICE PROBLEM

Given the same initial speed and a coefficient of kinetic friction equal to 0.120, what are **(a)** the acceleration of the shaker, and **(b)** the distance it slides? [**Answer: (a)** $a_x = -1.18\ \mathrm{m/s^2}$, **(b)** 0.560 m]

Some related homework problems: Problem 3, Problem 18

In the next Example we consider a system that is inclined at an angle θ relative to the horizontal. As a result, the normal force responsible for the kinetic friction is less than the weight of the object. To be very clear about how we handle the force vectors in such a case, we begin by resolving each vector into its x and y components.

PROBLEM-SOLVING NOTE

Choice of Coordinate System: Incline

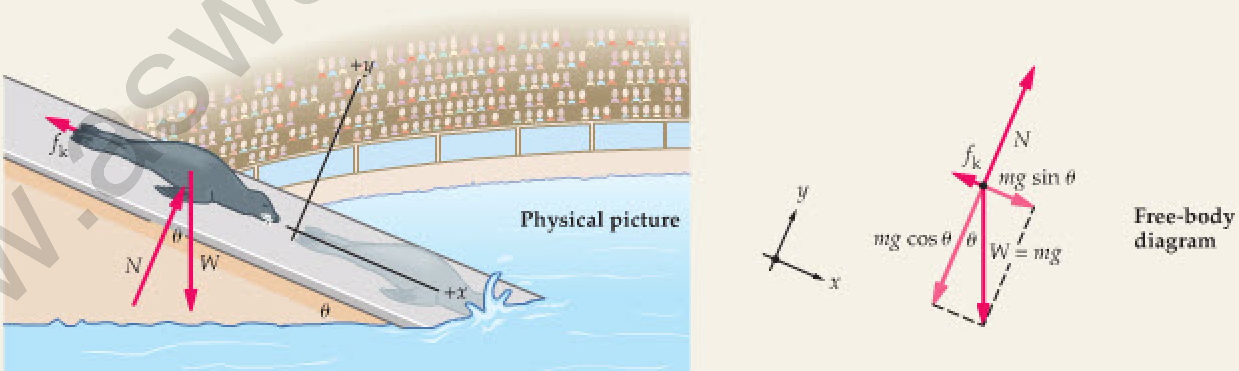
On an incline, align one axis (x) parallel to the surface, and the other axis (y) perpendicular to the surface. That way the motion is in the x direction. Since no motion occurs in the y direction, we know that $a_y = 0$.

EXAMPLE 6–2 MAKING A BIG SPLASH

A trained sea lion slides from rest with constant acceleration down a 3.0-m-long ramp into a pool of water. If the ramp is inclined at an angle of 23° above the horizontal and the coefficient of kinetic friction between the sea lion and the ramp is 0.26, how long does it take for the sea lion to make a splash in the pool?

PICTURE THE PROBLEM

As is usual with inclined surfaces, we choose one axis to be parallel to the surface and the other to be perpendicular to it. In our sketch, the sea lion accelerates in the positive x direction ($a_x > 0$), having started from rest, $v_{0x} = 0$. We are free to choose the initial position of the sea lion to be $x_0 = 0$. There is no motion in the y direction, and therefore $a_y = 0$. Finally, we note from the free-body diagram that $\vec{\mathbf{N}} = N\hat{\mathbf{y}}$, $\vec{\mathbf{f}}_k = -\mu_k N\hat{\mathbf{x}}$, and $\vec{\mathbf{W}} = (mg\sin\theta)\hat{\mathbf{x}} + (-mg\cos\theta)\hat{\mathbf{y}}$.



STRATEGY

We can use the kinematic equation relating position to time, $x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2$, to find the time of the sea lion's slide. It will be necessary, however, to first determine the acceleration of the sea lion in the x direction, a_x.

To find a_x we apply Newton's second law to the sea lion. First, we can find N by setting $\Sigma F_y = ma_y$ equal to zero (since $a_y = 0$). It is important to start by finding N because we need it to find the force of kinetic friction, $f_k = \mu_k N$. Using f_k in the sum of forces in the x direction, $\Sigma F_x = ma_x$, allows us to solve for a_x and, finally, for the time.

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SOLUTION

1. We begin by resolving each of the three force vectors into x and y components:

$$N_x = 0 \qquad N_y = N$$
$$f_{k,x} = -f_k = -\mu_k N \qquad f_{k,y} = 0$$
$$W_x = mg\sin\theta \qquad W_y = -mg\cos\theta$$

2. Set $\Sigma F_y = ma_y = 0$ to find N:

$$\sum F_y = N - mg\cos\theta = ma_y = 0$$

We see that N is less than the weight, mg:

$$N = mg\cos\theta$$

3. Next, set $\Sigma F_x = ma_x$:

$$\sum F_x = mg\sin\theta - \mu_k$$

Note that the mass cancels in this equation:

$$= mg\sin\theta - \mu_k mg\cos\theta = ma_x$$

4. Solve for the acceleration in the x direction, a_x:

$$a_x = g(\sin\theta - \mu_k\cos\theta)$$
$$= (9.81\ \text{m/s}^2)[\sin 23° - (0.26)\cos 23°]$$
$$= 1.5\ \text{m/s}^2$$

5. Use $x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2$ to find the time when the sea lion reaches the bottom. We choose $x_0 = 0$, and we are given that $v_{0x} = 0$, hence we set $x = \frac{1}{2}a_x t^2 = 3.0$ m and solve for t:

$$x = \tfrac{1}{2}a_x t^2$$
$$t = \sqrt{\frac{2x}{a_x}} = \sqrt{\frac{2(3.0\ \text{m})}{1.5\ \text{m/s}^2}} = 2.0\ \text{s}$$

INSIGHT

Note that we don't need the sea lion's mass to find the time. On the other hand, if we wanted the magnitude of the force of kinetic friction, $f_k = \mu_k N = \mu_k mg\cos\theta$, the mass would be needed.

It is useful to compare the sliding salt shaker in Example 6–1 with the sliding sea lion in this Example. In the case of the salt shaker, friction is the only force acting along the direction of motion (opposite to the direction of motion, in fact), and it brings the object to rest. Because of the slope on which the sea lion slides, however, it experiences both a component of its weight in the forward direction and the friction force opposite to the motion. Since the component of the weight is the larger of the two forces, the sea lion accelerates down the slope—friction only acts to slow its progress.

PRACTICE PROBLEM

How long would it take the sea lion to reach the water if there were no friction in this system? [**Answer:** 1.3 s]

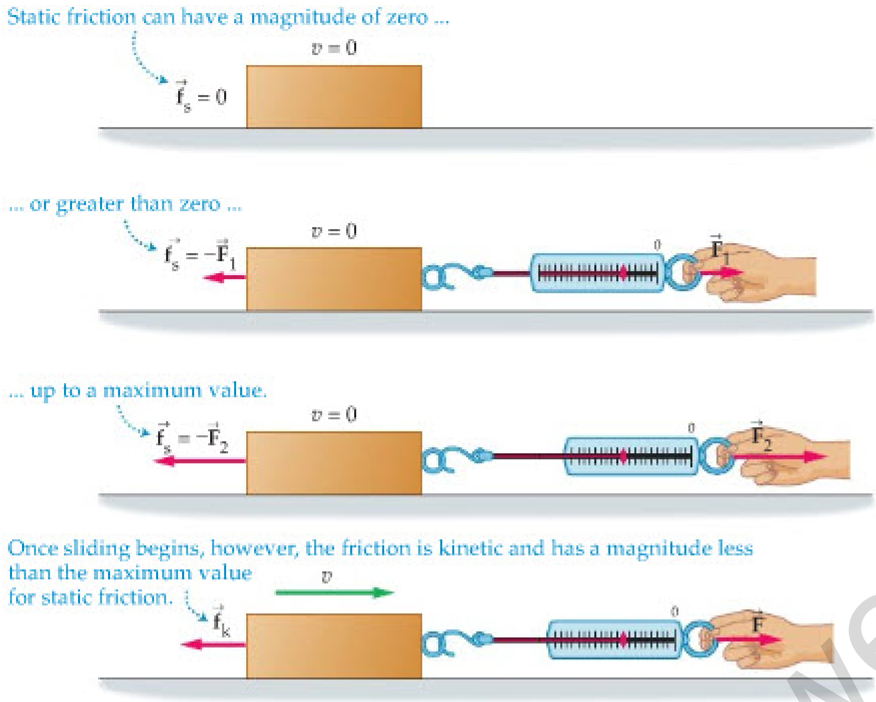
Some related homework problems: Problem 11, Problem 72

Static Friction

Static friction tends to keep two surfaces from moving relative to one another. It, like kinetic friction, is due to the microscopic irregularities of surfaces that are in contact. In fact, static friction is typically stronger than kinetic friction because when surfaces are in static contact, their microscopic hills and valleys can nestle down deeply into one another, thus forming a strong connection between the surfaces that may even include molecular bonding. In kinetic friction, the surfaces bounce along relative to one another and don't become as firmly enmeshed.

As we did with kinetic friction, let's use the results of some simple experiments to determine the rules of thumb for static friction. We start with a brick at rest on a table, with no horizontal force pulling on it, as in **Figure 6–3.** Of course, in this case the force of static friction is zero; no force is needed to keep the brick from sliding.

Next, attach a spring scale to the brick and pull with a small force of magnitude F_1, a force small enough that the brick doesn't move. Since the brick is still at rest, it follows that the force of static friction, f_s, is equal in magnitude to the applied force; that is, $f_s = F_1$. Now, increase the applied force to a new value, F_2, which is still small enough that the brick stays at rest. In this case, the force of static friction has also increased so that $f_s = F_2$. If we continue increasing the applied force, we eventually reach a value beyond which the brick starts to move and kinetic friction takes over, as shown in the figure. Thus, there is an upper limit to the force that can be exerted by static friction, and we call this upper limit $f_{s,max}$.



◀ **FIGURE 6–3 The maximum limit of static friction**

As the force applied to an object increases, so does the force of static friction—up to a certain point. Beyond this maximum value, static friction can no longer hold the object, and it begins to slide. Now kinetic friction takes over.

To summarize, the force of static friction, f_s, can have any value between zero and $f_{s,max}$. This can be written mathematically as follows:

$$0 \le f_s \le f_{s,max} \qquad 6\text{–}2$$

Imagine repeating the experiment, only now with a second brick on top of the first. This doubles the normal force and it also doubles the maximum force of static friction. Thus, the maximum force is proportional to the magnitude of the normal force, or

$$f_{s,max} = \mu_s N \qquad 6\text{–}3$$

The constant of proportionality is called μ_s (pronounced "mew sub s"), the **coefficient of static friction.** Note that μ_s, like μ_k, is dimensionless. Typical values are given in Table 6–1. In most cases, μ_s is greater than μ_k, indicating that the force of static friction is greater than the force of kinetic friction, as mentioned. In fact, it is not uncommon for μ_s to be greater than 1, as in the case of rubber in contact with dry concrete.

Finally, two additional comments regarding the nature of static friction: (i) Experiments show that static friction, like kinetic friction, is independent of the area of contact. (ii) The force of static friction is not in the direction of the normal force, thus $f_{s,max} = \mu_s N$ is not a vector relation. The direction of f_s is parallel to the surface of contact, and opposite to the direction the object would move if there were no friction.

These observations are summarized in the following rules of thumb:

Rules of Thumb for Static Friction

The force of static friction between two surfaces has the following properties:

1. It takes on any value between zero and the maximum possible force of static friction, $f_{s,max} = \mu_s N$:

$$0 \le f_s \le \mu_s N$$

2. It is independent of the area of contact between the surfaces.
3. It is parallel to the surface of contact, and in the direction that opposes relative motion.

▲ The coefficient of static friction between two surfaces depends on many factors, including whether the surfaces are dry or wet. On the desert floor of Death Valley, California, occasional rains can reduce the friction between rocks and the sandy ground to such an extent that strong winds can move the rocks over considerable distances. This results in linear "rock trails," which record the direction of the winds at different times.

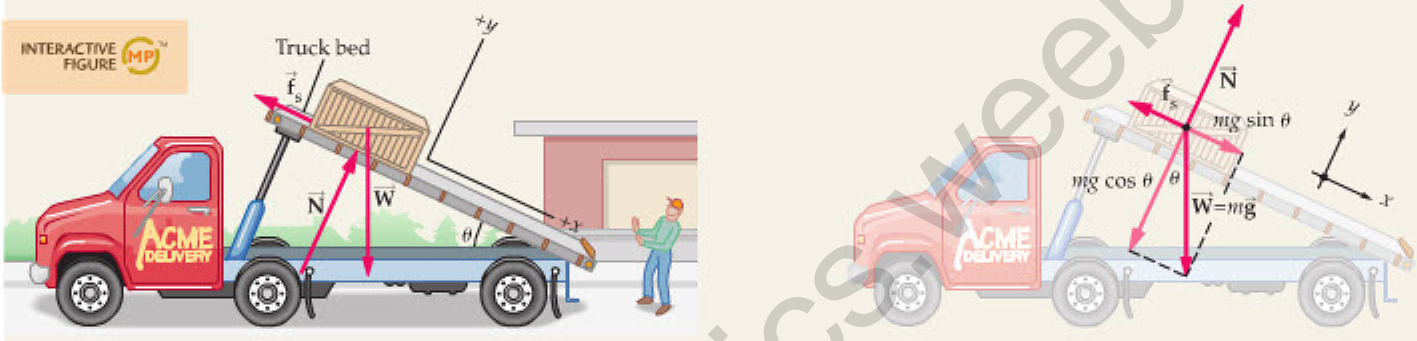
Next, we consider a practical method of determining the coefficient of static friction. As with the last Example, we begin by resolving all relevant force vectors into their x and y components.

EXAMPLE 6–3 SLIGHTLY TILTED

A flatbed truck slowly tilts its bed upward to dispose of a 95.0-kg crate. For small angles of tilt the crate stays put, but when the tilt angle exceeds 23.2°, the crate begins to slide. What is the coefficient of static friction between the bed of the truck and the crate?

PICTURE THE PROBLEM

We align our coordinate system with the incline, and choose the positive x direction to point down the slope. Note that three forces act on the crate: the normal force, $\vec{\mathbf{N}} = N\hat{\mathbf{y}}$, the force of static friction, $\vec{\mathbf{f}}_s = -\mu_s N\hat{\mathbf{x}}$, and the weight, $\vec{\mathbf{W}} = (mg \sin\theta)\hat{\mathbf{x}} + (-mg\cos\theta)\hat{\mathbf{y}}$.



Physical picture **Free-body diagram**

STRATEGY

When the crate is on the verge of slipping, but has not yet slipped, its acceleration is zero in both the x and y directions. In addition, "verge of slipping" means that the magnitude of the static friction is at its maximum value, $f_s = f_{s,max} = \mu_s N$. Thus, we set $\Sigma F_y = ma_y = 0$ to find N, then use $\Sigma F_x = ma_x = 0$ to find μ_s.

SOLUTION

1. Resolve the three force vectors acting on the crate into x and y components:

$$N_x = 0 \qquad N_y = N$$
$$f_{s,x} = -f_{s,max} = -\mu_s N \qquad f_{s,y} = 0$$
$$W_x = mg\sin\theta \qquad W_y = -mg\cos\theta$$

2. Set $\Sigma F_y = ma_y = 0$, since $a_y = 0$.

$$\sum F_y = N_y + f_{s,y} + W_y = N + 0 - mg\cos\theta = ma_y = 0$$

Solve for the normal force, N:

$$N = mg\cos\theta$$

3. Set $\Sigma F_x = ma_x = 0$, since the crate is at rest, and use the result for N obtained in Step 2:

$$\sum F_x = N_x + f_{s,x} + W_x = ma_x = 0$$
$$= 0 - \mu_s N + mg\sin\theta$$
$$= 0 - \mu_s mg\cos\theta + mg\sin\theta$$

4. Solve the expression for the coefficient of static friction, μ_s:

$$\mu_s mg\cos\theta = mg\sin\theta$$
$$\mu_s = \frac{mg\sin\theta}{mg\cos\theta} = \tan\theta = \tan 23.2° = 0.429$$

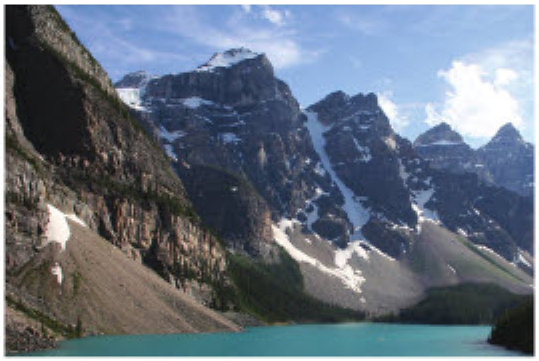
INSIGHT

In general, if an object is on the verge of slipping when the surface on which it rests is tilted at an angle θ_c, the coefficient of static friction between the object and the surface is $\mu_s = \tan\theta_c$. Note that this result is independent of the mass of the object. In particular, the critical angle for this crate is precisely the same whether it is filled with feathers or lead bricks.

PRACTICE PROBLEM

Find the magnitude of the force of static friction acting on the crate. [**Answer:** $f_{s,max} = \mu_s N = 367$ N]

Some related homework problems: Problem 12, Problem 82

▲ The angle that the sloping sides of a sand pile (left) make with the horizontal is determined by the coefficient of static friction between grains of sand, in much the same way that static friction determines the angle at which the crate in Example 6–3 begins to slide. The same basic mechanism determines the angle of the cone-shaped mass of rock debris at the base of a cliff, known as a talus slope (right).

Recall that static friction can have magnitudes less than its maximum possible value. This point is emphasized in the following Active Example.

ACTIVE EXAMPLE 6–1 THE FORCE OF STATIC FRICTION

In the previous Example, what is the magnitude of the force of static friction acting on the crate when the truck bed is tilted at an angle of 20.0°?

SOLUTION *(Test your understanding by performing the calculations indicated in each step.)*

1. Sum the x components of force acting on the crate: $\sum F_x = 0 - f_s + mg\sin\theta$
2. Set this sum equal to zero (since $a_x = 0$) and solve for the magnitude of the static friction force, f_s: $f_s = mg\sin\theta$
3. Substitute numerical values, including $\theta = 20.0°$: $f_s = 319\text{ N}$

INSIGHT

Notice that the force of static friction in this case has a magnitude (319 N) that is less than the value of 367 N found in the Practice Problem of Example 6–3, even though the coefficient of static friction is precisely the same.

YOUR TURN

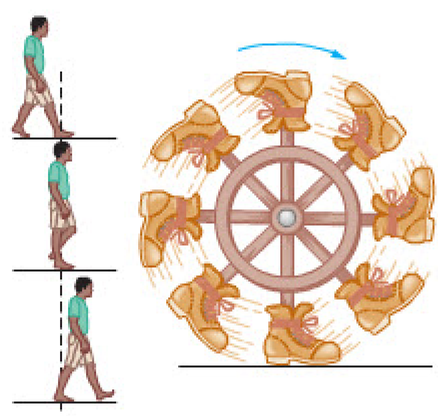
At what tilt angle will the force of static friction have a magnitude of 225 N?

(Answers to **Your Turn** *problems are given in the back of the book.)*

Finally, friction often enters into problems dealing with vehicles with rolling wheels. In Conceptual Checkpoint 6–1, we consider which type of friction is appropriate in such cases.

CONCEPTUAL CHECKPOINT 6–1 FRICTION FOR ROLLING TIRES

A car drives with its tires rolling freely. Is the friction between the tires and the road **(a)** kinetic or **(b)** static?



REASONING AND DISCUSSION

A reasonable-sounding answer is that because the car is moving, the friction between its tires and the road must be kinetic friction—but this is not the case.

Actually, the friction is static because the bottom of the tire is in static contact with the road. To understand this, watch your feet as you walk. Even though you are moving, each foot is in static contact with the ground once you step down on it. Your foot doesn't move again until you lift it up and move it forward for the next step. A tire can be thought of as a succession of feet arranged in a circle, each of which is momentarily in static contact with the ground.

ANSWER

(b) The friction between the tires and the road is static friction.

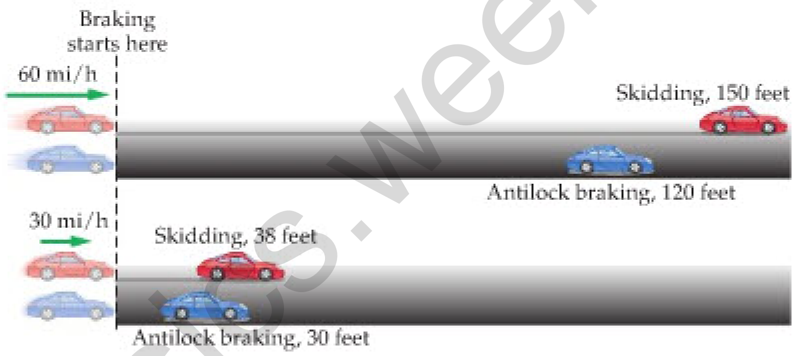
REAL-WORLD PHYSICS
Antilock braking systems

To summarize, if a car skids, the friction acting on it is kinetic; if its wheels are rolling, the friction is static. Since static friction is generally greater than kinetic friction, it follows that a car can be stopped in less distance if its wheels are rolling (static friction) than if its wheels are locked up (kinetic friction). This is the idea behind the antilock braking systems (ABS) that are available on many cars. When the brakes are applied in a car with ABS, an electronic rotation sensor at each wheel detects whether the wheel is about to start skidding. To prevent skidding, a small computer automatically begins to modulate the hydraulic pressure in the brake lines in short bursts, causing the brakes to release and then reapply in rapid succession. This allows the wheels to continue rotating, even in an emergency stop, and for static friction to determine the stopping distance. Figure 6–4 shows a comparison of braking distances for cars with and without ABS. An added benefit of ABS is that a driver is better able to steer and control a braking car if its wheels are rotating.

(a) Front wheels locked; rear wheels free to turn

(b) Rear wheels locked; front wheels free to turn

▲ **Static Versus Kinetic Friction** Each of the two photos above shows five images of a toy car as it slides down an inclined surface. **(a)** In this photo the front wheels are locked, and skid on the surface, but the rear wheels roll without slipping. This means the front wheels experience kinetic friction and the rear wheels experience static friction. Because the force of kinetic friction is usually less than the force of static friction, the front wheels go down the incline first, pulling the rear wheels behind. **(b)** The situation is reversed in this photo, and the rear wheels are the ones that skid and experience a smaller frictional force. As a result, the rear wheels slide down the incline more quickly than the front wheels, causing the car to spin around. This change in behavior, which could be dangerous in a real-life situation, illustrates the significant differences between static and kinetic friction.



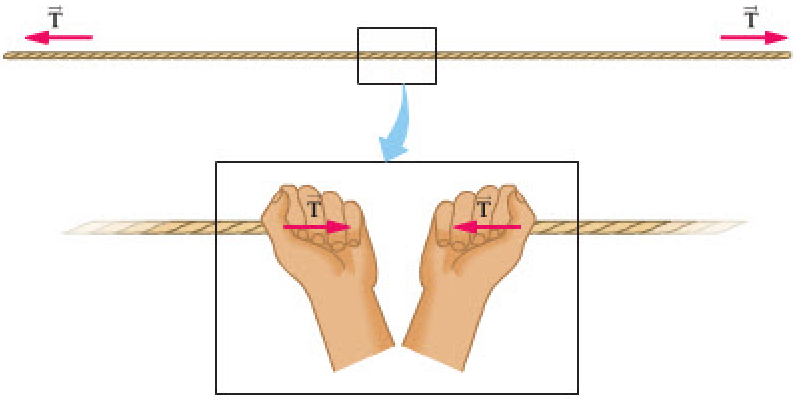
▲ **FIGURE 6–4 Stopping distance with and without ABS**
Antilock braking systems (ABS) allow a car to stop with static friction rather than kinetic friction—even in a case where a person slams on the brakes. As a result, the braking distance is reduced, due to the fact that μ_s is typically greater than μ_k. Professional drivers can beat the performance of ABS by carefully adjusting the force they apply to the brake pedal during a stop, but ABS provides essentially the same performance—within a few percent—for a person who simply pushes the brake pedal to the floor and holds it there.

6–2 Strings and Springs

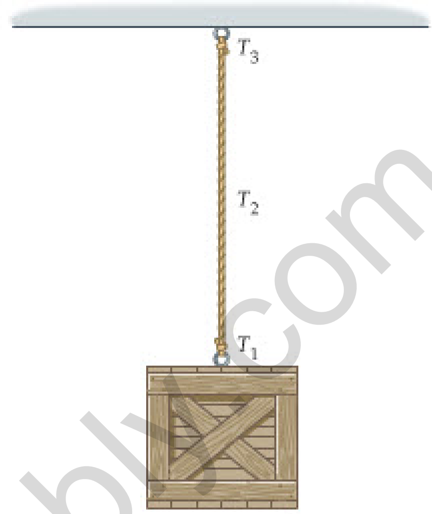
A common way to exert a force on an object is to pull on it with a string, a rope, a cable, or a wire. Similarly, you can push or pull on an object if you attach it to a spring. In this section we discuss the basic features of strings and springs and how they transmit forces.

Strings and Tension

Imagine picking up a light string and holding it with one end in each hand. If you pull to the right with your right hand with a force T and to the left with your left hand with a force T, the string becomes taut. In such a case, we say that there is a **tension** T in the string. To be more specific, if your friend were to cut the string at some point, the tension T is the force pulling the ends apart, as illustrated in Figure 6–5—that is, T is the force your friend would have to exert with each hand to hold the cut ends together. Note that at any given point, the tension pulls equally to the right and to the left.



▲ **FIGURE 6–5 Tension in a string**
A string, pulled from either end, has a tension, T. If the string were to be cut at any point, the force required to hold the ends together is T.



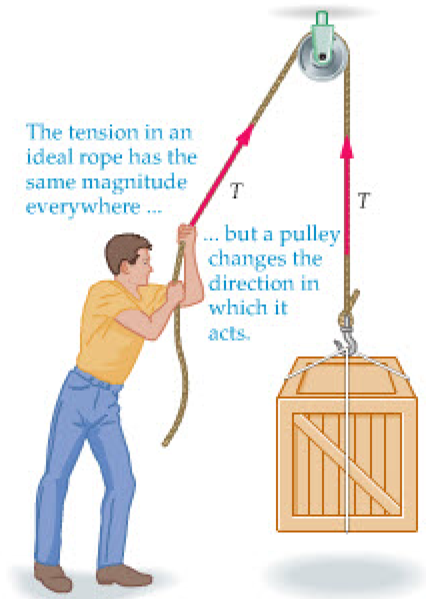
▲ **FIGURE 6–6 Tension in a heavy rope**
Because of the weight of the rope, the tension is noticeably different at points 1, 2, and 3. As the rope becomes lighter, however, the difference in tension decreases. In the limit of a rope of zero mass, the tension is the same throughout the rope.

As an example, consider a rope that is attached to the ceiling at one end, and to a box with a weight of 105 N at the other end, as shown in Figure 6–6. In addition, suppose the rope is uniform, and that it has a total weight of 2.00 N. What is the tension in the rope (i) where it attaches to the box, (ii) at its midpoint, and (iii) where it attaches to the ceiling?

First, the rope holds the box at rest; thus, the tension where the rope attaches to the box is simply the weight of the box, $T_1 = 105$ N. At the midpoint of the rope, the tension supports the weight of the box, plus the weight of half the rope. Thus, $T_2 = 105\text{ N} + \frac{1}{2}(2.00\text{ N}) = 106$ N. Similarly, at the ceiling the tension supports the box plus all of the rope, giving a tension of $T_3 = 107$ N. Note that the tension pulls down on the ceiling but pulls up on the box.

From this discussion, we can see that the tension in the rope changes slightly from top to bottom because of the mass of the rope. If the rope had less mass, the difference in tension between its two ends would also be less. In particular, if the rope's mass were to be vanishingly small, the difference in tension would vanish as well. In this text, we will assume that all ropes, strings, wires, and so on are practically massless—unless specifically stated otherwise—and, hence, that the tension is the same throughout their length.

Pulleys are often used to redirect a force exerted by a string, as indicated in Figure 6–7. In the ideal case, a pulley has no mass and no friction in its bearings. Thus, *an ideal pulley simply changes the direction of the tension in a string, without changing its magnitude*. If a system contains more than one pulley, however, it is possible to arrange them in such a way as to "magnify a force," even if each pulley itself merely redirects the tension in a string. The traction device considered in the next Example shows one way this can be accomplished in a system that uses three ideal pulleys.



▲ **FIGURE 6–7 A pulley changes the direction of a tension**

EXAMPLE 6–4 A BAD BREAK: SETTING A BROKEN LEG WITH TRACTION

A traction device employing three pulleys is applied to a broken leg, as shown in the sketch. The middle pulley is attached to the sole of the foot, and a mass m supplies the tension in the ropes. Find the value of the mass m if the force exerted on the sole of the foot by the middle pulley is to be 165 N.

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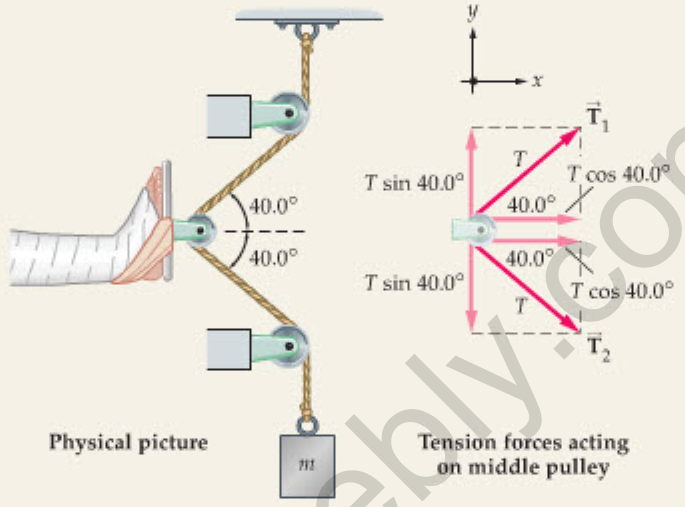
PICTURE THE PROBLEM

Our sketch shows the physical picture as well as the tension forces acting on the middle pulley. Notice that on the upper portion of the rope the tension is $\vec{\mathbf{T}}_1 = (T \cos 40.0°)\hat{\mathbf{x}} + (T \sin 40.0°)\hat{\mathbf{y}}$; on the lower portion it is $\vec{\mathbf{T}}_2 = (T \cos 40.0°)\hat{\mathbf{x}} + (-T \sin 40.0°)\hat{\mathbf{y}}$.



STRATEGY

We begin by noting that the rope supports the hanging mass m. As a result, the tension in the rope, T, must be equal in magnitude to the weight of the mass: $T = mg$.

Next, the pulleys simply change the direction of the tension without changing its magnitude. Therefore, the net force exerted on the sole of the foot is the sum of the tension T at 40.0° above the horizontal plus the tension T at 40.0° below the horizontal. We will calculate the net force component by component.

Once we calculate the net force acting on the foot, we set it equal to 165 N and solve for the tension T. Finally, we find the mass using the relation $T = mg$.

SOLUTION

1. First, consider the tension that acts upward and to the right on the middle pulley. Resolve this tension into x and y components:

$$T_{1,x} = T \cos 40.0° \qquad T_{1,y} = T \sin 40.0°$$

2. Next, consider the tension that acts downward and to the right on the middle pulley. Resolve this tension into x and y components. Note the minus sign in the y component:

$$T_{2,x} = T \cos 40.0° \qquad T_{2,y} = -T \sin 40.0°$$

3. Sum the x and y components of force acting on the middle pulley. We see that the net force acts only in the x direction, as one might expect from symmetry:

$$\sum F_x = T \cos 40.0° + T \cos 40.0° = 2T \cos 40.0°$$

$$\sum F_y = T \sin 40.0° - T \sin 40.0° = 0$$

4. Step 3 shows that the net force acting on the middle pulley is $2T \cos 40.0°$. Set this force equal to 165 N and solve for T:

$$2T \cos 40.0° = 165 \text{ N}$$

$$T = \frac{165 \text{ N}}{2 \cos 40.0°} = 108 \text{ N}$$

5. Solve for the mass, m, using $T = mg$:

$$T = mg$$

$$m = \frac{T}{g} = \frac{108 \text{ N}}{9.81 \text{ m/s}^2} = 11.0 \text{ kg}$$

INSIGHT

As pointed out earlier, this pulley arrangement "magnifies the force" in the sense that a 108-N weight attached to the rope produces a 165-N force exerted on the foot by the middle pulley. Note that the tension in the rope always has the same value—$T = 108$ N—as expected with ideal pulleys, but because of the arrangement of the pulleys the force applied to the foot by the rope is $2T \cos 40.0° > T$.

In addition, notice that the force exerted on the foot by the middle pulley produces an opposing force in the leg that acts in the direction of the head (a cephalad force), as desired to set a broken leg and keep it straight as it heals.

PRACTICE PROBLEM

(a) Would the required mass m increase or decrease if the angles in this device were changed from 40.0° to 30.0°? **(b)** Find the mass m for an angle of 30.0°. [**Answer:** **(a)** The required mass m would decrease. **(b)** 9.71 kg]

Some related homework problems: Problem 23, Problem 26, Problem 36