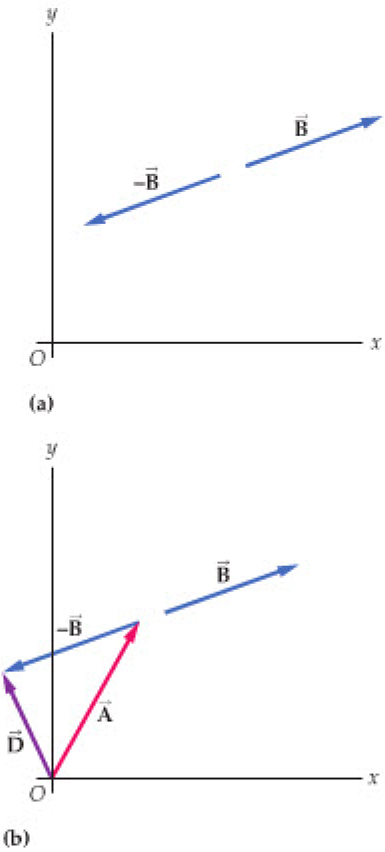


▲ **FIGURE 3–14 Vector subtraction**
(a) The vector $\vec{B}$ and its negative $-\vec{B}$. (b) A vector construction for $\vec{D} = \vec{A} - \vec{B}$.

For example, the vectors $\vec{B}$ and $-\vec{B}$ are indicated in Figure 3–14 (a). Thus, to subtract $\vec{B}$ from $\vec{A}$, simply reverse the direction of $\vec{B}$ and add it to $\vec{A}$, as indicated in Figure 3–14 (b).

In terms of components, you subtract vectors by simply subtracting the components. For example, if

$$\vec{D} = \vec{A} - \vec{B}$$

then

$$D_x = A_x - B_x$$

and

$$D_y = A_y - B_y$$

Once the components of $\vec{D}$ are found, its magnitude and direction angle can be calculated as usual.

EXERCISE 3–5

a. For the vectors given in Figure 3–12, find the components of $\vec{D} = \vec{A} - \vec{B}$.

b. Find D and θ and compare with the vector $\vec{D}$ shown in Figure 3–14 (b).

SOLUTION

a. $D_x = -1.26$ m, $D_y = 2.96$ m

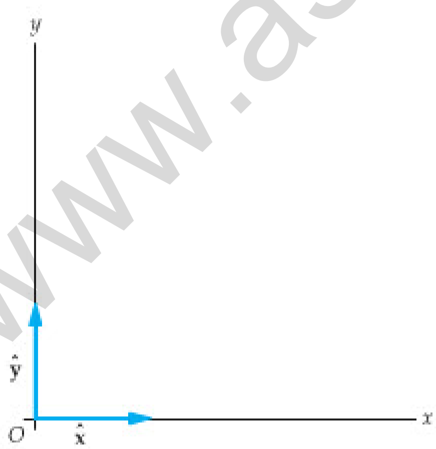
b. $D = 3.22$ m, $\theta = -66.9° + 180° = 113°$. In Figure 3–14 (b) we see that $\vec{D}$ is shorter than $\vec{B}$, which has a magnitude of 4.00 m, and its direction angle is somewhat greater than 90°, in agreement with our numerical results.

3–4 Unit Vectors

Unit vectors provide a convenient way of expressing an arbitrary vector in terms of its components, as we shall see. But first, let's define what we mean by a unit vector. In particular, the unit vectors $\hat{x}$ and $\hat{y}$ are defined to be dimensionless vectors of unit magnitude pointing in the positive x and y directions, respectively:

- The x unit vector, $\hat{x}$, is a dimensionless vector of unit length pointing in the positive x direction.
- The y unit vector, $\hat{y}$, is a dimensionless vector of unit length pointing in the positive y direction.

Figure 3–15 shows $\hat{x}$ and $\hat{y}$ on a two-dimensional coordinate system. Since unit vectors have no physical dimensions—like mass, length, or time—they are used to specify direction only.



▲ **FIGURE 3–15 Unit vectors**
The unit vectors $\hat{x}$ and $\hat{y}$ point in the positive x and y directions, respectively.

Multiplying Unit Vectors by Scalars

To see the utility of unit vectors, consider the effect of multiplying a vector by a scalar. For example, multiplying a vector by 3 increases its magnitude by a factor of 3, but does not change its direction, as shown in Figure 3–16. Multiplying by −3 increases the magnitude by a factor of 3 *and* reverses the direction of the vector. This is also shown in Figure 3–16. In the case of unit vectors—which have a magnitude of 1 and are dimensionless—multiplication by a scalar results in a vector with the same magnitude and dimensions as the scalar.

For example, if a vector $\vec{A}$ has the scalar components $A_x = 5$ m and $A_y = 3$ m, we can write it as

$$\vec{A} = (5\text{ m})\hat{x} + (3\text{ m})\hat{y}$$

www.aswarphysics.weebly.com

We refer to the quantities $(5\text{ m})\hat{\mathbf{x}}$ and $(3\text{ m})\hat{\mathbf{y}}$ as the x and y **vector components** of the vector $\vec{\mathbf{A}}$. In general, an arbitrary two-dimensional vector $\vec{\mathbf{A}}$ can always be written as the sum of a vector component in the x direction and a vector component in the y direction:

$$\vec{\mathbf{A}} = A_x\hat{\mathbf{x}} + A_y\hat{\mathbf{y}}$$

This is illustrated in Figure 3–17 (a). An equivalent way of representing the vector components of a vector is illustrated in Figure 3–17 (b). In this case we see that the vector components are the *projection* of a vector onto the x and y axes. The sign of the vector components is positive if they point in the positive x or y direction, and negative if they point in the negative x or y direction. This is how vector components will generally be shown in later chapters.

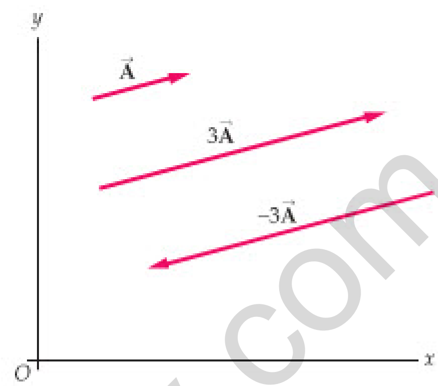
Finally, note that vector addition and subtraction are straightforward with unit vector notation:

$$\vec{\mathbf{C}} = \vec{\mathbf{A}} + \vec{\mathbf{B}} = (A_x + B_x)\hat{\mathbf{x}} + (A_y + B_y)\hat{\mathbf{y}}$$

and

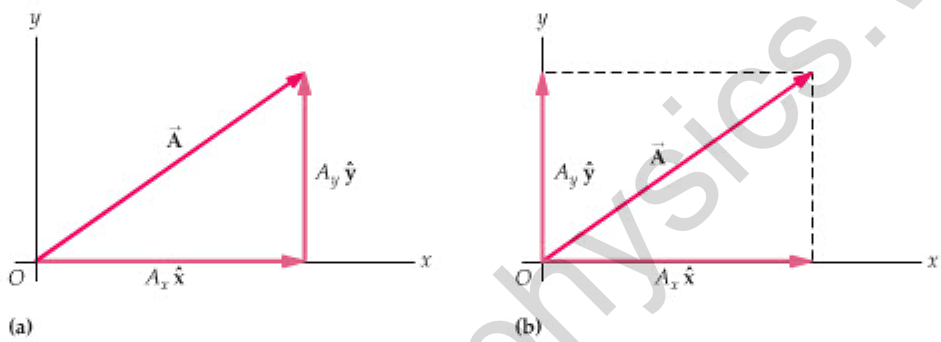
$$\vec{\mathbf{D}} = \vec{\mathbf{A}} - \vec{\mathbf{B}} = (A_x - B_x)\hat{\mathbf{x}} + (A_y - B_y)\hat{\mathbf{y}}$$

Clearly, unit vectors provide a useful way to keep track of the x and y components of a vector.



▲ **FIGURE 3–16 Multiplying a vector by a scalar**

Multiplying a vector by a positive scalar different from 1 will change the length of the vector but leave its direction the same. If the vector is multiplied by a negative scalar its direction is reversed.



◀ **FIGURE 3–17 Vector components**

(a) A vector $\vec{\mathbf{A}}$ can be written in terms of unit vectors as $\vec{\mathbf{A}} = A_x\hat{\mathbf{x}} + A_y\hat{\mathbf{y}}$. **(b)** Vector components can be thought of as the projection of the vector onto the x and y axes. This method of representing vector components will be used frequently in subsequent chapters.

3–5 Position, Displacement, Velocity, and Acceleration Vectors

In Chapter 2 we discussed four different one-dimensional vectors: position, displacement, velocity, and acceleration. Each of these quantities had a direction associated with it, indicated by its sign; positive meant in the positive direction, negative meant in the negative direction. Now we consider these vectors again, this time in two dimensions, where the possibilities for direction are not so limited.

Position Vectors

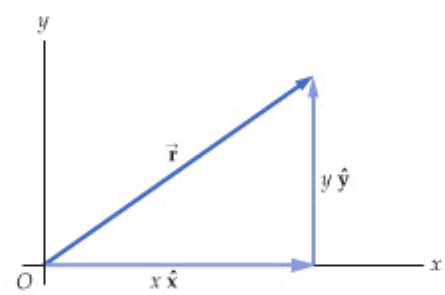
To begin, imagine a two-dimensional coordinate system, as in Figure 3–18. Position is indicated by a vector from the origin to the location in question. We refer to the position vector as $\vec{\mathbf{r}}$; its units are meters, m.

Definition: Position Vector, $\vec{\mathbf{r}}$

position vector $= \vec{\mathbf{r}}$ 3–1

SI unit: meter, m

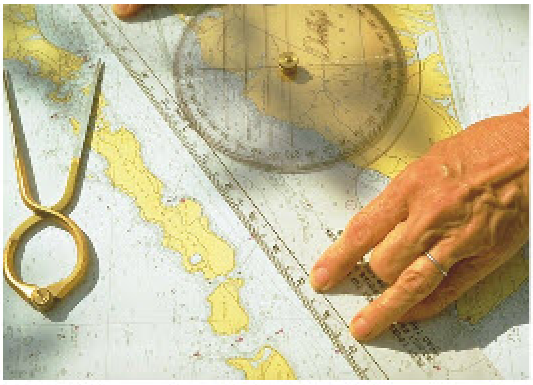
In terms of unit vectors, the position vector is simply $\vec{\mathbf{r}} = x\hat{\mathbf{x}} + y\hat{\mathbf{y}}$.



▲ **FIGURE 3–18 Position vector**

The position vector $\vec{\mathbf{r}}$ points from the origin to the current location of an object. The x and y vector components of $\vec{\mathbf{r}}$ are $x\hat{\mathbf{x}}$ and $y\hat{\mathbf{y}}$, respectively.

www.aswarphysics.weebly.com



▲ A map can be used to determine the direction and magnitude of the displacement vector from your initial position to your destination.

Displacement Vectors

Now, suppose that initially you are at the location indicated by the position vector $\vec{\mathbf{r}}_i$, and that later you are at the final position represented by the position vector $\vec{\mathbf{r}}_f$. Your displacement vector, $\Delta\vec{\mathbf{r}}$, is the change in position:

Definition: Displacement Vector, $\Delta\vec{\mathbf{r}}$

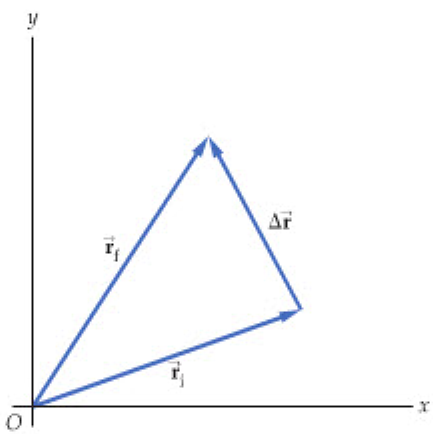
$$\Delta\vec{\mathbf{r}} = \vec{\mathbf{r}}_f - \vec{\mathbf{r}}_i \qquad 3\text{–}2$$

SI unit: meter, m

Rearranging this definition slightly, we see that

$$\vec{\mathbf{r}}_f = \vec{\mathbf{r}}_i + \Delta\vec{\mathbf{r}}$$

That is, the final position is equal to the initial position plus the change in position. This is illustrated in **Figure 3–19**, where we see that $\Delta\vec{\mathbf{r}}$ extends from the head of $\vec{\mathbf{r}}_i$ to the head of $\vec{\mathbf{r}}_f$.



▲ **FIGURE 3–19 Displacement vector**

The displacement vector $\Delta\vec{\mathbf{r}}$ is the change in position. It points from the head of the initial position vector $\vec{\mathbf{r}}_i$ to the head of the final position vector $\vec{\mathbf{r}}_f$. Thus $\vec{\mathbf{r}}_f = \vec{\mathbf{r}}_i + \Delta\vec{\mathbf{r}}$ or $\Delta\vec{\mathbf{r}} = \vec{\mathbf{r}}_f - \vec{\mathbf{r}}_i$.

Velocity Vectors

Next, the average velocity vector is defined as the displacement vector $\Delta\vec{\mathbf{r}}$ divided by the elapsed time Δt.

Definition: Average Velocity Vector, $\vec{\mathbf{v}}_{av}$

$$\vec{\mathbf{v}}_{av} = \frac{\Delta\vec{\mathbf{r}}}{\Delta t} \qquad 3\text{–}3$$

SI unit: meter per second, m/s

Since $\Delta\vec{\mathbf{r}}$ is a vector, it follows that $\vec{\mathbf{v}}_{av}$ is also a vector; it is the vector $\Delta\vec{\mathbf{r}}$ times the scalar $(1/\Delta t)$. Thus $\vec{\mathbf{v}}_{av}$ is parallel to $\Delta\vec{\mathbf{r}}$ and has the units m/s.

EXERCISE 3–6

A dragonfly is observed initially at the position $\vec{\mathbf{r}}_i = (2.00\text{ m})\hat{\mathbf{x}} + (3.50\text{ m})\hat{\mathbf{y}}$. Three seconds later it is at the position $\vec{\mathbf{r}}_f = (-3.00\text{ m})\hat{\mathbf{x}} + (5.50\text{ m})\hat{\mathbf{y}}$. What was the dragonfly's average velocity during this time?

SOLUTION

$$\vec{\mathbf{v}}_{av} = (\vec{\mathbf{r}}_f - \vec{\mathbf{r}}_i)/\Delta t = [(-5.00\text{ m})\hat{\mathbf{x}} + (2.00\text{ m})\hat{\mathbf{y}}]/(3.00\text{ s})$$
$$= (-1.67\text{ m/s})\hat{\mathbf{x}} + (0.667\text{ m/s})\hat{\mathbf{y}}$$

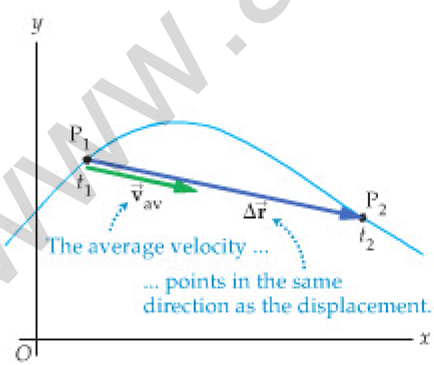
To help visualize $\vec{\mathbf{v}}_{av}$, imagine a particle moving in two dimensions along the blue path shown in **Figure 3–20**. If the particle is at point P_1 at time t_1, and at P_2 at time t_2, its displacement is indicated by the vector $\Delta\vec{\mathbf{r}}$. The average velocity is parallel to $\Delta\vec{\mathbf{r}}$, as indicated in Figure 3–20. It makes sense physically that $\vec{\mathbf{v}}_{av}$ is parallel to $\Delta\vec{\mathbf{r}}$; after all, on *average* you have moved in the direction of $\Delta\vec{\mathbf{r}}$ during the time from t_1 to t_2. To put it another way, a particle that starts at P_1 at the time t_1 and moves with the velocity $\vec{\mathbf{v}}_{av}$ until the time t_2 will arrive in precisely the same location as the particle that follows the blue path.

By considering smaller and smaller time intervals, as in **Figure 3–21**, it is possible to calculate the instantaneous velocity vector:

Definition: Instantaneous Velocity Vector, $\vec{\mathbf{v}}$

$$\vec{\mathbf{v}} = \lim_{\Delta t \to 0} \frac{\Delta\vec{\mathbf{r}}}{\Delta t} \qquad 3\text{–}4$$

SI unit: meter per second, m/s

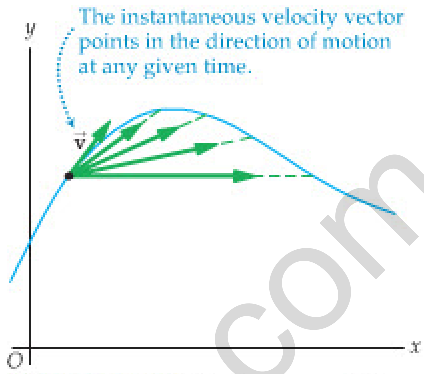


▲ **FIGURE 3–20 Average velocity vector**

The average velocity, $\vec{\mathbf{v}}_{av}$, points in the same direction as the displacement, $\Delta\vec{\mathbf{r}}$, for any given interval of time.

www.aswarphysics.weebly.com

As can be seen in Figure 3–21, the instantaneous velocity at a given time is tangential to the path of the particle at that time. In addition, the magnitude of the velocity vector is the speed of the particle. Thus, the instantaneous velocity vector tells you both how fast a particle is moving and in what direction.



▲ **FIGURE 3–21 Instantaneous velocity vector**

The instantaneous velocity vector $\vec{\mathbf{v}}$ is obtained by calculating the average velocity vector over smaller and smaller time intervals. In the limit of vanishingly small time intervals, the average velocity approaches the instantaneous velocity, which points in the direction of motion.

EXERCISE 3–7

Find the speed and direction of motion for a rainbow trout whose velocity is $\vec{\mathbf{v}} = (3.7\text{ m/s})\hat{\mathbf{x}} + (-1.3\text{ m/s})\hat{\mathbf{y}}$.

SOLUTION

speed $= v = \sqrt{(3.7\text{ m/s})^2 + (-1.3\text{ m/s})^2} = 3.9\text{ m/s}$, $\theta = \tan^{-1}\left(\dfrac{-1.3\text{ m/s}}{3.7\text{ m/s}}\right) = -19°$,

that is, 19° below the x axis.

Acceleration Vectors

Finally, the average acceleration vector over an interval of time, Δt, is defined as the change in the velocity vector, $\Delta\vec{\mathbf{v}}$, divided by the scalar Δt.

Definition: Average Acceleration Vector, $\vec{\mathbf{a}}_{av}$

$$\vec{\mathbf{a}}_{av} = \frac{\Delta\vec{\mathbf{v}}}{\Delta t} \qquad 3\text{–}5$$

SI unit: meter per second per second, m/s^2

An example is given in **Figure 3–22**, where we show the initial and final velocity vectors corresponding to two different times. Since the change in velocity is defined as

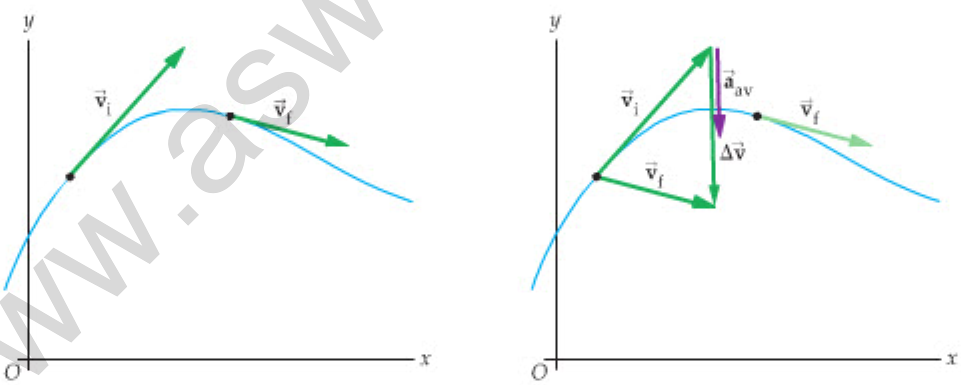
$$\Delta\vec{\mathbf{v}} = \vec{\mathbf{v}}_f - \vec{\mathbf{v}}_i$$

it follows that

$$\vec{\mathbf{v}}_f = \vec{\mathbf{v}}_i + \Delta\vec{\mathbf{v}}$$

as indicated in Figure 3–22. Thus, $\Delta\vec{\mathbf{v}}$ is the vector extending from the head of $\vec{\mathbf{v}}_i$ to the head of $\vec{\mathbf{v}}_f$, just as $\Delta\vec{\mathbf{r}}$ extends from the head of $\vec{\mathbf{r}}_i$ to the head of $\vec{\mathbf{r}}_f$ in Figure 3–19. The direction of $\vec{\mathbf{a}}_{av}$ is the direction of $\Delta\vec{\mathbf{v}}$, as shown in **Figure 3–22(b)**.

Can an object accelerate if its speed is constant? Absolutely—if its direction changes. Consider a car driving with a constant speed on a circular track, as



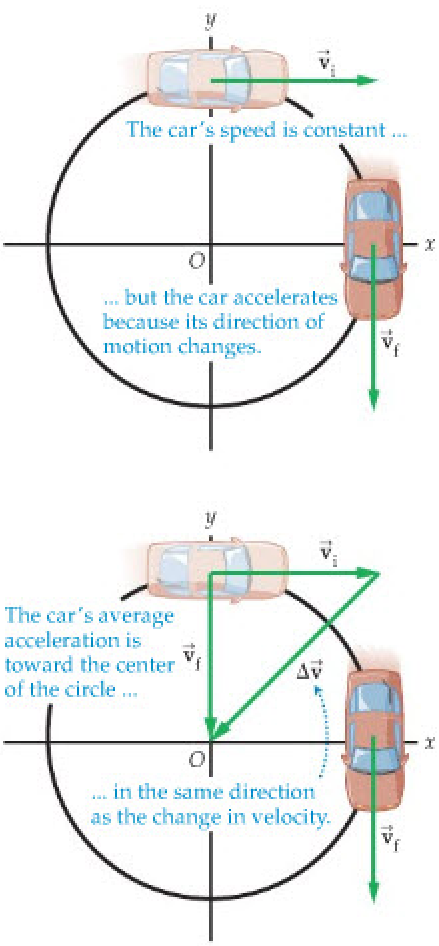
(a) The instantaneous velocity at two different times

(b) The average acceleration points in the same direction as the change in velocity

▲ **FIGURE 3–22 Average acceleration vector**

(a) As a particle moves along the blue path its velocity changes in magnitude and direction. At the time t_i the velocity is $\vec{\mathbf{v}}_i$; at the time t_f the velocity is $\vec{\mathbf{v}}_f$. **(b)** The average acceleration vector $\vec{\mathbf{a}}_{av} = \Delta\vec{\mathbf{v}}/\Delta t$ points in the direction of the change in velocity vector $\Delta\vec{\mathbf{v}}$. We obtain $\Delta\vec{\mathbf{v}}$ by moving $\vec{\mathbf{v}}_f$ so that its tail coincides with the tail of $\vec{\mathbf{v}}_i$, and then drawing the arrow that connects the head of $\vec{\mathbf{v}}_i$ to the head of $\vec{\mathbf{v}}_f$. Note that $\vec{\mathbf{a}}_{av}$ need not point in the direction of motion, and in general it doesn't.

www.aswarphysics.weebly.com



▲ **FIGURE 3–23 Average acceleration for a car traveling in a circle with constant speed**
Although the speed of this car never changes, it is still accelerating—due to the change in its direction of motion. For the time interval depicted, the car's average acceleration is in the direction of $\Delta\vec{v}$, which is toward the center of the circle. (As we shall see in Chapter 6, the car's acceleration is always toward the center of the circle.)

shown in Figure 3–23. Suppose that the initial velocity of the car is $\vec{v}_i = (12\ \text{m/s})\hat{x}$, and that 10.0 s later its final velocity is $\vec{v}_f = (-12\ \text{m/s})\hat{y}$. Note that the speed is 12 m/s in each case, but the velocity is different because the *direction* has changed. Calculating the average acceleration, we find a nonzero acceleration:

$$\vec{a}_{av} = \frac{\Delta\vec{v}}{\Delta t} = \frac{\vec{v}_f - \vec{v}_i}{10.0\ \text{s}}$$
$$= \frac{(-12\ \text{m/s})\hat{y} - (12\ \text{m/s})\hat{x}}{10.0\ \text{s}} = (-1.2\ \text{m/s}^2)\hat{x} + (-1.2\ \text{m/s}^2)\hat{y}$$

Thus, a change in direction is just as important as a change in speed in producing an acceleration. We shall study circular motion in detail in Chapter 6.

Finally, by going to infinitesimally small time intervals, $\Delta t \to 0$, we can define the instantaneous acceleration:

Definition: Instantaneous Acceleration Vector, $\vec{a}$

$$\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta\vec{v}}{\Delta t} \qquad 3\text{–}6$$

SI unit: meter per second per second, m/s^2

ACTIVE EXAMPLE 3–2 FIND THE AVERAGE ACCELERATION

A car is traveling northwest at 9.00 m/s. Eight seconds later it has rounded a corner and is now heading north at 15.0 m/s. What are the magnitude and direction of its average acceleration during those 8.00 seconds?

Let the positive x direction be east, and the positive y direction be north.

SOLUTION *(Test your understanding by performing the calculations indicated in each step.)*

1. Write out $\vec{v}_i$: $\vec{v}_i = (-6.36\ \text{m/s})\hat{x} + (6.36\ \text{m/s})\hat{y}$
2. Write out $\vec{v}_f$: $\vec{v}_f = (15.0\ \text{m/s})\hat{y}$
3. Calculate $\Delta\vec{v}$: $\Delta\vec{v} = (6.36\ \text{m/s})\hat{x} + (8.64\ \text{m/s})\hat{y}$
4. Find $\vec{a}_{av}$: $\vec{a}_{av} = (0.795\ \text{m/s}^2)\hat{x} + (1.08\ \text{m/s}^2)\hat{y}$
5. Determine a_{av} and θ: $a_{av} = 1.34\ \text{m/s}^2$, $\theta = 53.6°$ north of east

YOUR TURN

Find the magnitude and direction of the average acceleration if the same change in velocity occurs in 4.00 s rather than 8.00 s.

(Answers to **Your Turn** *problems are given in the back of the book.)*

▶ The velocities of these cyclists change in both magnitude and direction as they slow to negotiate a series of sharp curves and then speed up again. Both kinds of velocity change involve an acceleration.

www.aswarphysics.weebly.com

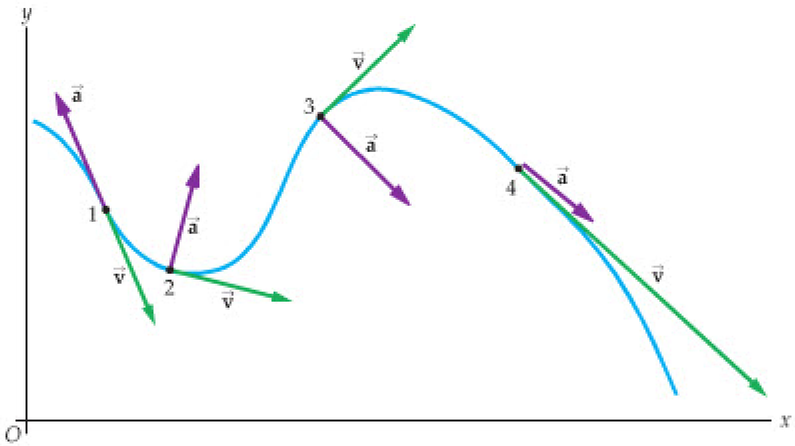


FIGURE 3–24 Velocity and acceleration vectors for a particle moving along a winding path

The acceleration of a particle need not point in the direction of motion. At point (1) the particle is slowing down, at (2) it is turning to the left, at (3) it is turning to the right, and, finally, at point (4) it is speeding up.

Note carefully the following critical distinctions between the velocity vector and the acceleration vector:

- The velocity vector, $\vec{\mathbf{v}}$, is always in the direction of a particle's motion.
- The acceleration vector, $\vec{\mathbf{a}}$, can point in directions other than the direction of motion, and in general it does.

An example of a particle's motion, showing the velocity and acceleration vectors at various times, is presented in **Figure 3–24**.

Note that in all cases the velocity is tangential to the motion, though the acceleration points in various directions. When the acceleration is perpendicular to the velocity of an object, as at points (2) and (3) in Figure 3–24, its speed remains constant while its direction of motion changes. At points (1) and (4) in Figure 3–24 the acceleration is antiparallel (opposite) or parallel to the velocity of the object, respectively. In such cases, the direction of motion remains the same while the speed changes. Throughout the next chapter we shall see further examples of motion in which the velocity and acceleration are in different directions.

3–6 Relative Motion

A good example of the use of vectors is in the description of relative motion. Suppose, for example, that you are standing on the ground as a train goes by at 15.0 m/s, as shown in **Figure 3–25**. Inside the train, a free-riding passenger is walking in the forward direction at 1.2 m/s relative to the train. How fast is the passenger moving relative to you? Clearly, the answer is $1.2\ \mathrm{m/s} + 15.0\ \mathrm{m/s} = 16.2\ \mathrm{m/s}$. What if the passenger had been walking with the same speed, but toward the back of the train? In this case, you would see the passenger going by with a speed of $-1.2\ \mathrm{m/s} + 15.0\ \mathrm{m/s} = 13.8\ \mathrm{m/s}$.

Let's generalize these results. Call the velocity of the *t*rain relative to the *g*round $\vec{\mathbf{v}}_{tg}$, the velocity of the *p*assenger relative to the *t*rain $\vec{\mathbf{v}}_{pt}$, and the velocity of the *p*assenger relative to the *g*round $\vec{\mathbf{v}}_{pg}$. As we saw in the previous paragraph, the velocity of the *p*assenger relative to the *g*round is

$$\vec{\mathbf{v}}_{pg} = \vec{\mathbf{v}}_{pt} + \vec{\mathbf{v}}_{tg} \qquad 3\text{–}7$$

This vector addition is illustrated in **Figure 3–26** for the two cases we discussed.

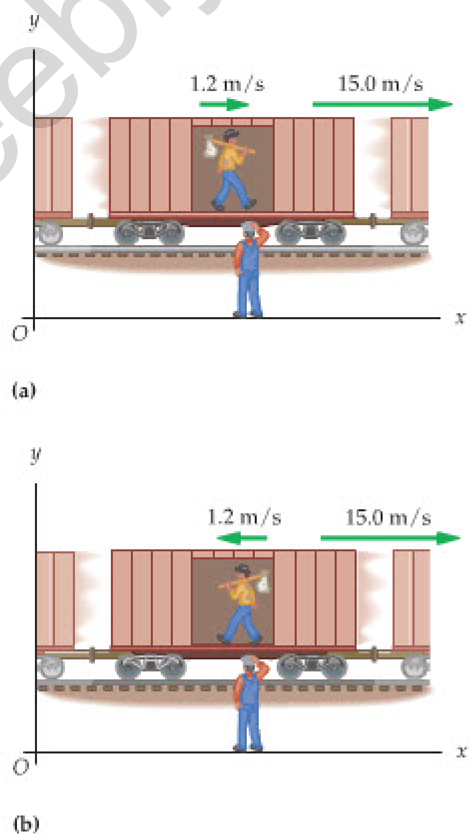


FIGURE 3–25 Relative velocity of a passenger on a train with respect to a person on the ground

(a) The passenger walks toward the front of the train. **(b)** The passenger walks toward the rear of the train.

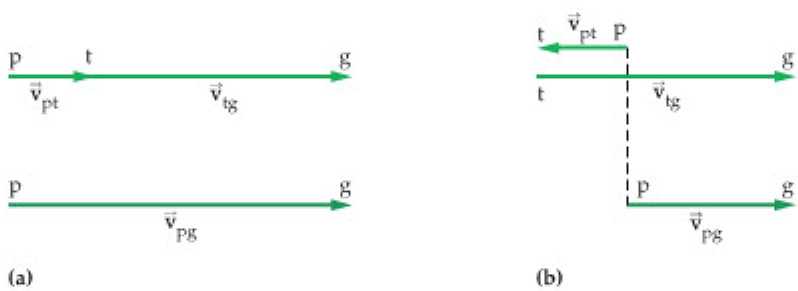
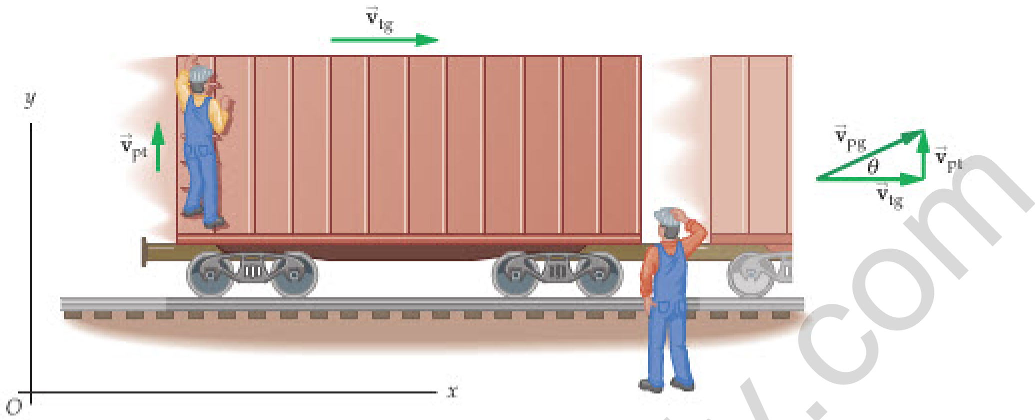


FIGURE 3–26 Adding velocity vectors

Vector addition to find the velocity of the passenger with respect to the ground for **(a)** Figure 3–25 (a) and **(b)** Figure 3–25 (b).

www.aswarphysics.weebly.com



▲ **FIGURE 3–27 Relative velocity in two dimensions**

A person climbs up a ladder on a moving train with velocity $\vec{\mathbf{v}}_{pt}$ relative to the train. If the train moves relative to the ground with a velocity $\vec{\mathbf{v}}_{tg}$, the velocity of the person on the train relative to the ground is $\vec{\mathbf{v}}_{pg} = \vec{\mathbf{v}}_{pt} + \vec{\mathbf{v}}_{tg}$.

Though this example dealt with one-dimensional motion, Equation 3–7 is valid for velocity vectors pointing in arbitrary directions. For example, instead of walking on the car's floor, the passenger might be climbing a ladder to the roof of the car, as in **Figure 3–27**. In this case $\vec{\mathbf{v}}_{pt}$ is vertical, $\vec{\mathbf{v}}_{tg}$ is horizontal, and $\vec{\mathbf{v}}_{pg}$ is simply the vector sum $\vec{\mathbf{v}}_{pt} + \vec{\mathbf{v}}_{tg}$.

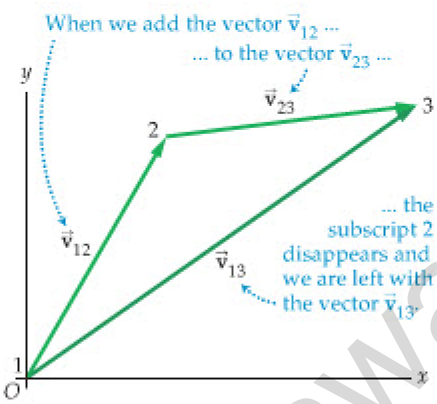
EXERCISE 3–8

Suppose the passenger in Figure 3–27 is climbing a vertical ladder with a speed of 0.20 m/s, and the train is slowly coasting forward at 0.70 m/s. Find the speed and direction of the passenger relative to the ground.

SOLUTION

$$\vec{\mathbf{v}}_{pg} = (0.70\ \text{m/s})\hat{\mathbf{x}} + (0.20\ \text{m/s})\hat{\mathbf{y}};\ \text{thus}$$

$$v_{pg} = \sqrt{(0.70\ \text{m/s})^2 + (0.20\ \text{m/s})^2} = 0.73\ \text{m/s},\ \theta = \tan^{-1}(0.20/0.70) = 16°$$



▲ **FIGURE 3–28 Vector addition used to determine relative velocity**

Note that the subscripts in Equation 3–7 follow a definite pattern. On the left-hand side of the equation we have the subscripts pg. On the right-hand side we have two sets of subscripts, pt and tg; note that a pair of t's has been inserted between the p and the g. This pattern always holds for any relative motion problem, though the subscripts will be different when referring to different objects. Thus, we can say quite generally that

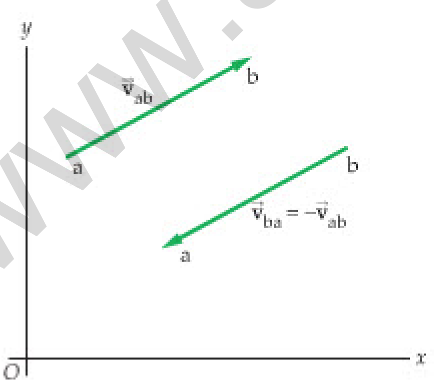
$$\vec{\mathbf{v}}_{13} = \vec{\mathbf{v}}_{12} + \vec{\mathbf{v}}_{23} \qquad 3\text{–}8$$

where, in the train example, we can identify 1 as the *p*assenger, 2 as the *t*rain, and 3 as the *g*round.

The vector addition in Equation 3–8 is shown in **Figure 3–28**. For convenience in seeing how the subscripts are ordered in the equation, we have labeled the tail of each vector with its first subscript and the head of each vector with its second subscript.

One final note about velocities and their subscripts: Reversing the subscripts reverses the velocity. This is indicated in **Figure 3–29**, where we see that

$$\vec{\mathbf{v}}_{ba} = -\vec{\mathbf{v}}_{ab}$$



▲ **FIGURE 3–29 Reversing the subscripts of a velocity reverses the corresponding velocity vector**

Physically, what we are saying is that if you are riding in a car due *north* at 20 m/s relative to the ground, then the ground, relative to you, is moving due *south* at 20 m/s.

Let's apply these results to a two-dimensional example.

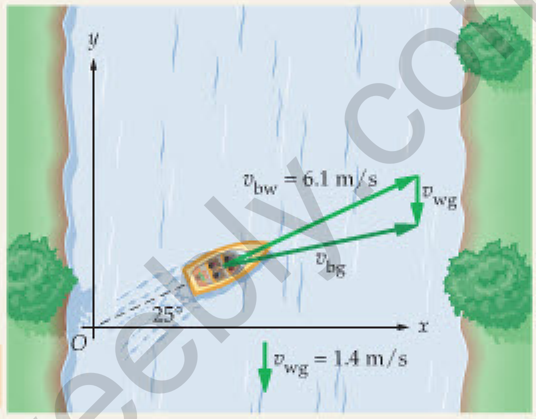
www.aswarphysics.weebly.com

EXAMPLE 3–2 CROSSING A RIVER

REAL-WORLD PHYSICS You are riding in a boat whose speed relative to the water is 6.1 m/s. The boat points at an angle of 25° upstream on a river flowing at 1.4 m/s. **(a)** What is your velocity relative to the ground? **(b)** Suppose the speed of the boat relative to the water remains the same, but the direction in which it points is changed. What angle is required for the boat to go straight across the river?

PICTURE THE PROBLEM

We choose the x axis to be perpendicular to the river, and the y axis to point upstream. With these choices the velocity of the boat relative to the water is 25° above the x axis. In addition, the velocity of the water relative to the ground has a magnitude of 1.4 m/s and points in the negative y direction.



STRATEGY

If the water were still, the boat would move in the direction in which it is pointed. With the water flowing downstream, as shown, the boat will move in a direction closer to the x axis. **(a)** To find the velocity of the boat we use $\vec{v}_{13} = \vec{v}_{12} + \vec{v}_{23}$ with 1 referring to the boat (b), 2 referring to the water (w), and 3 referring to the ground (g). **(b)** To go "straight across the river" means that the velocity of the boat relative to the ground should be in the x direction. Thus, we choose the angle θ that cancels the y component of velocity.

INTERACTIVE FIGURE

SOLUTION

Part (a)

1. Rewrite $\vec{v}_{13} = \vec{v}_{12} + \vec{v}_{23}$ with $1 \rightarrow$ b, $2 \rightarrow$ w, and $3 \rightarrow$ g: $\vec{v}_{bg} = \vec{v}_{bw} + \vec{v}_{wg}$

2. From our sketch we see that the water flows at 1.4 m/s in the negative y direction relative to the ground: $\vec{v}_{wg} = (-1.4\ \mathrm{m/s})\hat{\mathbf{y}}$

3. The velocity of the boat relative to the water is given in the problem statement: $\vec{v}_{bw} = (6.1\ \mathrm{m/s})\cos 25°\,\hat{\mathbf{x}} + (6.1\ \mathrm{m/s})\sin 25°\,\hat{\mathbf{y}} = (5.5\ \mathrm{m/s})\hat{\mathbf{x}} + (2.6\ \mathrm{m/s})\hat{\mathbf{y}}$

4. Carry out the vector sum in Step 1 to find $\vec{v}_{bg}$: $\vec{v}_{bg} = (5.5\ \mathrm{m/s})\hat{\mathbf{x}} + (2.6\ \mathrm{m/s} - 1.4\ \mathrm{m/s})\hat{\mathbf{y}} = (5.5\ \mathrm{m/s})\hat{\mathbf{x}} + (1.2\ \mathrm{m/s})\hat{\mathbf{y}}$

Part (b)

5. To cancel the y component of $\vec{v}_{bg}$, we choose the angle θ that gives 1.4 m/s for the y component of $\vec{v}_{bw}$: $(6.1\ \mathrm{m/s})\sin\theta = 1.4\ \mathrm{m/s}$

6. Solve for θ. With this angle, we see that the y component of $\vec{v}_{bg}$ in Step 4 will be zero: $\theta = \sin^{-1}(1.4/6.1) = 13°$

INSIGHT

(a) Note that the speed of the boat relative to the ground is $\sqrt{(5.5\ \mathrm{m/s})^2 + (1.2\ \mathrm{m/s})^2} = 5.6\ \mathrm{m/s}$, and the direction angle is $\theta = \tan^{-1}(1.2/5.5) = 12°$ upstream. **(b)** The speed of the boat in this case is equal to the x component of its velocity, since the y component is zero. Therefore, its speed is $(6.1\ \mathrm{m/s})\cos 13° = 5.9\ \mathrm{m/s}$.

PRACTICE PROBLEM

Find the speed and direction of the boat relative to the ground if the river flows at 4.5 m/s. [**Answer:** $v_{bg} = 5.8\ \mathrm{m/s}$, $\theta = -19°$. In this case, a person on the ground sees the boat going slowly downstream, even though the boat itself points upstream.]

Some related homework problems: Problem 50, Problem 53, Problem 55

Suppose the problem had been to find the velocity of the boat relative to the water so that it goes straight across the river at 5.0 m/s. That is, we want to find $\vec{v}_{bw}$ such that $\vec{v}_{bg} = (5.0\ \mathrm{m/s})\hat{\mathbf{x}}$. One approach is to simply solve $\vec{v}_{bg} = \vec{v}_{bw} + \vec{v}_{wg}$ for $\vec{v}_{bw}$, which gives

$$\vec{v}_{bw} = \vec{v}_{bg} - \vec{v}_{wg} \qquad 3\text{–}9$$

Another approach is to go back to our general relation, $\vec{v}_{13} = \vec{v}_{12} + \vec{v}_{23}$ and choose 1 to be the *boat*, 2 to be the *ground*, and 3 to be the *water*. With these substitutions we find

$$\vec{v}_{bw} = \vec{v}_{bg} + \vec{v}_{gw}$$

www.aswarphysics.weebly.com

This is the same as Equation 3–9, since $\vec{\mathbf{v}}_{gw} = -\vec{\mathbf{v}}_{wg}$. In either case, the desired velocity of the boat relative to the water is

$$\vec{\mathbf{v}}_{bw} = (5.0\ \text{m/s})\hat{\mathbf{x}} + (1.4\ \text{m/s})\hat{\mathbf{y}}$$

which corresponds to a speed of 5.2 m/s and a direction angle of 16° upstream.

THE BIG PICTURE PUTTING PHYSICS IN CONTEXT

LOOKING BACK

In Chapter 2 we indicated direction with + and − signs, since only two directions were possible. With the results from this chapter we can now deal with quantities that point in any direction at all.

The vector quantities we have considered so far are position, displacement, velocity, and acceleration. These quantities are important throughout our study of mechanics.

LOOKING AHEAD

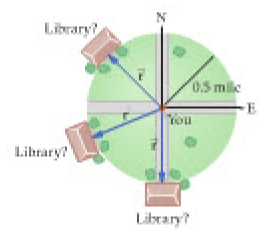
In Chapter 4 we will consider kinematics in two dimensions. As we shall see, the vectors developed in this chapter will play a key role in that study. In particular, vectors will allow us to analyze two-dimensional motion as a combination of two completely independent one-dimensional motions.

In Chapter 5 we will introduce one of the most important concepts in all of physics—force. It is a vector quantity. Other important vector quantities to be introduced in later chapters include linear momentum (Chapter 9), angular momentum (Chapter 11), electric field (Chapter 19), and magnetic field (Chapter 22).

www.aswarphysics.weebly.com

CHAPTER SUMMARY

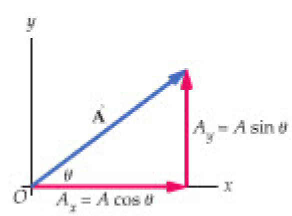
3–1 SCALARS VERSUS VECTORS



Scalar
A number with appropriate units. Examples of scalar quantities include time and length.

Vector
A quantity with both a magnitude and a direction. Examples include displacement, velocity, and acceleration.

3–2 THE COMPONENTS OF A VECTOR



x Component of Vector $\vec{\mathbf{A}}$
$A_x = A\cos\theta$, where θ is measured relative to the x axis.

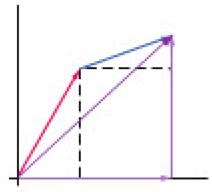
y Component of Vector $\vec{\mathbf{A}}$
$A_y = A\sin\theta$, where θ is measured relative to the x axis.

Sign of the Components
A_x is positive if $\vec{\mathbf{A}}$ points in the positive x direction, and negative if it points in the negative x direction. Similar remarks apply to A_y.

Magnitude of Vector $\vec{\mathbf{A}}$
The magnitude of $\vec{\mathbf{A}}$ is $A = \sqrt{A_x^2 + A_y^2}$.

Direction Angle of Vector $\vec{\mathbf{A}}$
The direction angle of $\vec{\mathbf{A}}$ is $\theta = \tan^{-1}(A_y/A_x)$, where θ is measured relative to the x axis.

3–3 ADDING AND SUBTRACTING VECTORS



Graphical Method

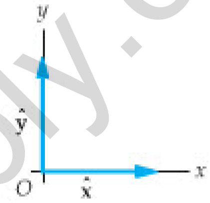
To add $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$, place them so that the tail of $\vec{\mathbf{B}}$ is at the head of $\vec{\mathbf{A}}$. The sum $\vec{\mathbf{C}} = \vec{\mathbf{A}} + \vec{\mathbf{B}}$ is the arrow from the tail of $\vec{\mathbf{A}}$ to the head of $\vec{\mathbf{B}}$. See Figure 3–8.

To find $\vec{\mathbf{A}} - \vec{\mathbf{B}}$, place $\vec{\mathbf{A}}$ and $-\vec{\mathbf{B}}$ head-to-tail and draw an arrow from the tail of $\vec{\mathbf{A}}$ to the head of $-\vec{\mathbf{B}}$. See Figure 3–14.

Component Method

If $\vec{\mathbf{C}} = \vec{\mathbf{A}} + \vec{\mathbf{B}}$, then $C_x = A_x + B_x$ and $C_y = A_y + B_y$. If $\vec{\mathbf{C}} = \vec{\mathbf{A}} - \vec{\mathbf{B}}$, then $C_x = A_x - B_x$ and $C_y = A_y - B_y$.

3–4 UNIT VECTORS



***x* Unit Vector**

Written $\hat{\mathbf{x}}$, the x unit vector is a dimensionless vector of unit length in the positive x direction.

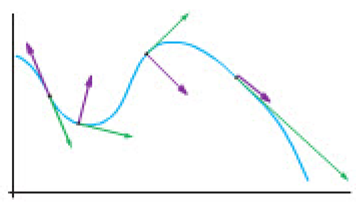
***y* Unit Vector**

Written $\hat{\mathbf{y}}$, the y unit vector is a dimensionless vector of unit length in the positive y direction.

Vector Addition

$\vec{\mathbf{A}} + \vec{\mathbf{B}} = (A_x + B_x)\hat{\mathbf{x}} + (A_y + B_y)\hat{\mathbf{y}}$

3–5 POSITION, DISPLACEMENT, VELOCITY, AND ACCELERATION VECTORS



Position Vector

The position vector $\vec{\mathbf{r}}$ points from the origin to a particle's location.

Displacement Vector

The displacement vector $\Delta\vec{\mathbf{r}}$ is the change in position; $\Delta\vec{\mathbf{r}} = \vec{\mathbf{r}}_f - \vec{\mathbf{r}}_i$.

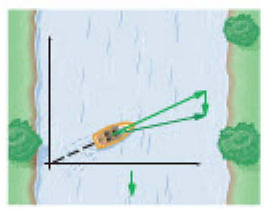
Velocity Vector

The velocity vector $\vec{\mathbf{v}}$ points in the direction of motion and has a magnitude equal to the speed.

Acceleration Vector

The acceleration vector $\vec{\mathbf{a}}$ indicates how quickly and in what direction the velocity is changing. It need not point in the direction of motion.

3–6 RELATIVE MOTION



Velocity of Object 1 Relative to Object 3

$\vec{\mathbf{v}}_{13} = \vec{\mathbf{v}}_{12} + \vec{\mathbf{v}}_{23}$, where object 2 can be anything.

Reversing the Subscripts on a Velocity

$\vec{\mathbf{v}}_{12} = -\vec{\mathbf{v}}_{21}$.

PROBLEM-SOLVING SUMMARY

Type of Problem	Relevant Physical Concepts	Related Examples
Add or subtract vectors.	Resolve the vectors into x and y components, then add or subtract the components.	Active Example 3–1 Exercise 3–5
Calculate the average velocity.	Divide the displacement, $\Delta\vec{\mathbf{r}}$, by the elapsed time, Δt.	Exercise 3–6
Calculate the average acceleration.	Divide the change in velocity, $\Delta\vec{\mathbf{v}}$, by the elapsed time, Δt.	Active Example 3–2
Find the relative velocity of object 1 with respect to object 3.	Use $\vec{\mathbf{v}}_{13} = \vec{\mathbf{v}}_{12} + \vec{\mathbf{v}}_{23}$ with the appropriate choices for 1, 2, and 3.	Example 3–2 Exercise 3–8

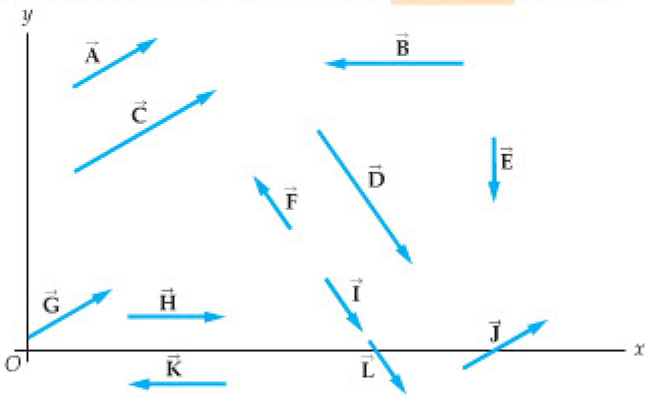
www.aswarphysics.weebly.com

CONCEPTUAL QUESTIONS

For instructor-assigned homework, go to www.masteringphysics.com MP

(Answers to odd-numbered Conceptual Questions can be found in the back of the book.)

1. For the following quantities, indicate which is a scalar and which is a vector: **(a)** the time it takes for you to run the 100-yard dash; **(b)** your displacement after running the 100-yard dash; **(c)** your average velocity while running; **(d)** your average speed while running.
2. Which, if any, of the vectors shown in **Figure 3–30** are equal?



▲ **FIGURE 3–30** Conceptual Question 2

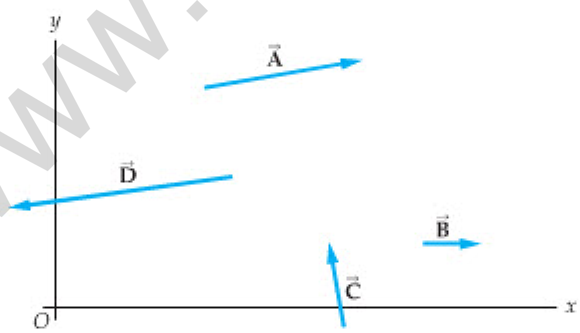
3. Given that $\vec{A} + \vec{B} = 0$, **(a)** how does the magnitude of $\vec{B}$ compare with the magnitude of $\vec{A}$? **(b)** How does the direction of $\vec{B}$ compare with the direction of $\vec{A}$?
4. Can a component of a vector be greater than the vector's magnitude?
5. Suppose that $\vec{A}$ and $\vec{B}$ have nonzero magnitude. Is it possible for $\vec{A} + \vec{B}$ to be zero?
6. Can a vector with zero magnitude have one or more components that are nonzero? Explain.
7. Given that $\vec{A} + \vec{B} = \vec{C}$, and that $A^2 + B^2 = C^2$, how are $\vec{A}$ and $\vec{B}$ oriented relative to one another?
8. Given that $\vec{A} + \vec{B} = \vec{C}$, and that $A + B = C$, how are $\vec{A}$ and $\vec{B}$ oriented relative to one another?
9. Given that $\vec{A} + \vec{B} = \vec{C}$, and that $A - B = C$, how are $\vec{A}$ and $\vec{B}$ oriented relative to one another?
10. Vector $\vec{A}$ has x and y components of equal magnitude. What can you say about the possible directions of $\vec{A}$?
11. The components of a vector $\vec{A}$ satisfy the relation $A_x = -A_y \neq 0$. What are the possible directions of $\vec{A}$?
12. Use a sketch to show that two vectors of unequal magnitude cannot add to zero, but that three vectors of unequal magnitude can.
13. Rain is falling vertically downward and you are running for shelter. To keep driest, should you hold your umbrella vertically, tilted forward, or tilted backward? Explain.
14. When sailing, the wind feels stronger when you sail upwind ("beating") than when you are sailing downwind ("running"). Explain.

PROBLEMS AND CONCEPTUAL EXERCISES

Note: Answers to odd-numbered Problems and Conceptual Exercises can be found in the back of the book. **IP** *denotes an integrated problem, with both conceptual and numerical parts;* **BIO** *identifies problems of biological or medical interest;* **CE** *indicates a conceptual exercise.* **Predict/Explain** *problems ask for two responses:* **(a)** *your prediction of a physical outcome, and* **(b)** *the best explanation among three provided. On all problems, red bullets (•, ••, •••) are used to indicate the level of difficulty.*

SECTION 3–2 THE COMPONENTS OF A VECTOR

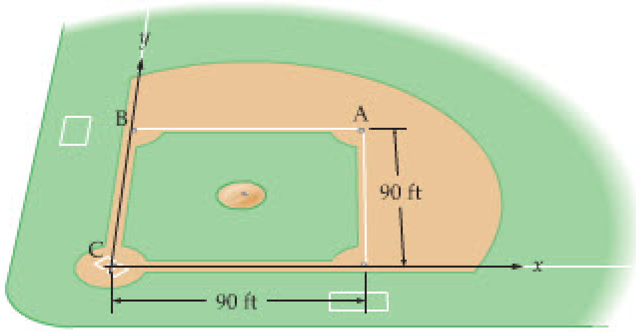
1. • **CE** Suppose that each component of a certain vector is doubled. **(a)** By what multiplicative factor does the magnitude of the vector change? **(b)** By what multiplicative factor does the direction angle of the vector change?
2. • **CE** Rank the vectors in **Figure 3–31** in order of increasing magnitude.



▲ **FIGURE 3–31** Problems 2, 3, and 4

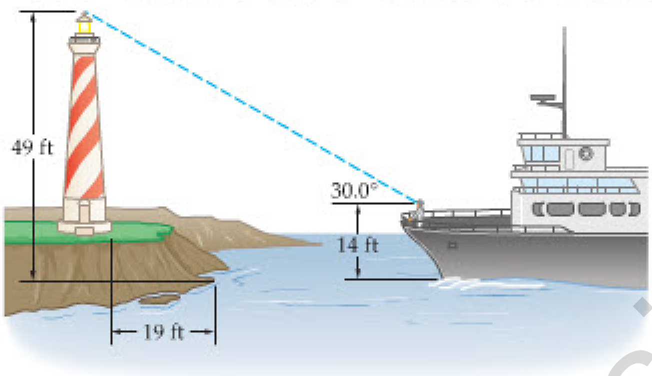
3. • **CE** Rank the vectors in Figure 3–31 in order of increasing value of their x component.
4. • **CE** Rank the vectors in Figure 3–31 in order of increasing value of their y component.
5. • The press box at a baseball park is 32.0 ft above the ground. A reporter in the press box looks at an angle of 15.0° below the horizontal to see second base. What is the horizontal distance from the press box to second base?
6. • You are driving up a long, inclined road. After 1.2 miles you notice that signs along the roadside indicate that your elevation has increased by 530 ft. **(a)** What is the angle of the road above the horizontal? **(b)** How far do you have to drive to gain an additional 150 ft of elevation?
7. • **A One-Percent Grade** A road that rises 1 ft for every 100 ft traveled horizontally is said to have a 1% grade. Portions of the Lewiston grade, near Lewiston, Idaho, have a 6% grade. At what angle is this road inclined above the horizontal?
8. • Find the x and y components of a position vector $\vec{r}$ of magnitude $r = 75$ m, if its angle relative to the x axis is **(a)** 35.0° and **(b)** 65.0°.
9. • A baseball "diamond" **(Figure 3–32)** is a square with sides 90 ft in length. If the positive x axis points from home plate to first base, and the positive y axis points from home plate to third base, find the displacement vector of a base runner who has just hit **(a)** a double, **(b)** a triple, or **(c)** a home run.

www.aswarphysics.weebly.com



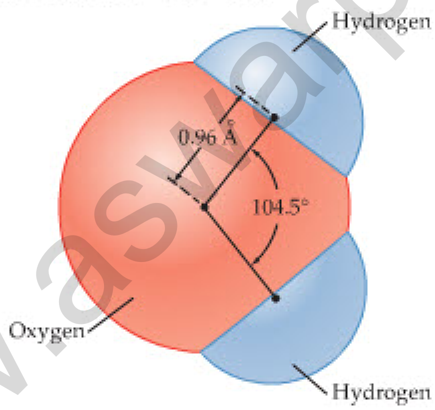
▲ **FIGURE 3–32** Problem 9

10. •• A lighthouse that rises 49 ft above the surface of the water sits on a rocky cliff that extends 19 ft from its base, as shown in **Figure 3–33**. A sailor on the deck of a ship sights the top of the lighthouse at an angle of 30.0° above the horizontal. If the sailor's eye level is 14 ft above the water, how far is the ship from the rocks?



▲ **FIGURE 3–33** Problem 10

11. •• H_2O A water molecule is shown schematically in **Figure 3–34**. The distance from the center of the oxygen atom to the center of a hydrogen atom is 0.96 Å, and the angle between the hydrogen atoms is 104.5°. Find the center-to-center distance between the hydrogen atoms. ($1 \text{ Å} = 10^{-10}$ m.)



▲ **FIGURE 3–34** Problem 11

12. •• **IP** The x and y components of a vector $\vec{\mathbf{r}}$ are $r_x = 14$ m and $r_y = -9.5$ m, respectively. Find **(a)** the direction and **(b)** the magnitude of the vector $\vec{\mathbf{r}}$. **(c)** If both r_x and r_y are doubled, how do your answers to parts (a) and (b) change?

13. •• **IP The Longitude Problem** In 1755, John Harrison (1693–1776) completed his fourth precision chronometer, the H4, which eventually won the celebrated Longitude Prize. (For the human drama behind the Longitude Prize, see *Longitude*, by Dava Sobel.) When the minute hand of the H4 indicated 10 minutes past the hour, it extended 3.0 cm in the horizontal direction. **(a)** How long was the H4's minute hand? **(b)** At 10 minutes past the hour, was the extension of the minute hand in the vertical direction more than, less than, or equal to 3.0 cm? Explain. **(c)** Calculate the vertical extension of the minute hand at 10 minutes past the hour.

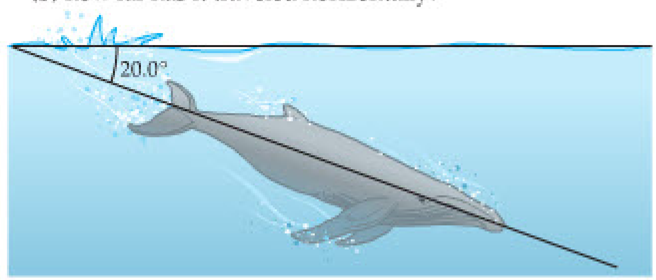
Not just a watch! The Harrison H4. (Problem 13)

14. •• You drive a car 680 ft to the east, then 340 ft to the north. **(a)** What is the magnitude of your displacement? **(b)** Using a sketch, estimate the direction of your displacement. **(c)** Verify your estimate in part (b) with a numerical calculation of the direction.

15. •• Vector $\vec{\mathbf{A}}$ has a magnitude of 50 units and points in the positive x direction. A second vector, $\vec{\mathbf{B}}$, has a magnitude of 120 units and points at an angle of 70° below the x axis. Which vector has **(a)** the greater x component, and **(b)** the greater y component?

16. •• A treasure map directs you to start at a palm tree and walk due north for 15.0 m. You are then to turn 90° and walk 22.0 m; then turn 90° again and walk 5.00 m. Give the distance from the palm tree, and the direction relative to north, for each of the four possible locations of the treasure.

17. •• A whale comes to the surface to breathe and then dives at an angle of 20.0° below the horizontal **(Figure 3–35)**. If the whale continues in a straight line for 150 m, **(a)** how deep is it, and **(b)** how far has it traveled horizontally?



▲ **FIGURE 3–35** Problem 17

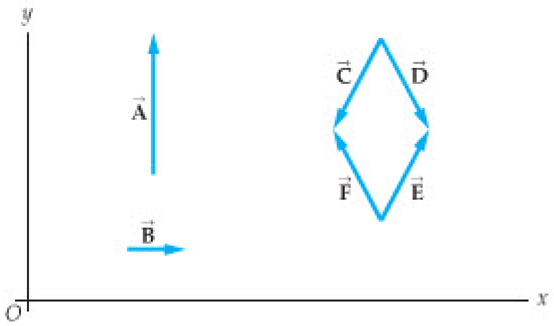
SECTION 3–3 ADDING AND SUBTRACTING VECTORS

18. • **CE** Consider the vectors $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ shown in **Figure 3–36**. Which of the other four vectors in the figure ($\vec{\mathbf{C}}$, $\vec{\mathbf{D}}$, $\vec{\mathbf{E}}$, and $\vec{\mathbf{F}}$) best represents the *direction* of **(a)** $\vec{\mathbf{A}} + \vec{\mathbf{B}}$, **(b)** $\vec{\mathbf{A}} - \vec{\mathbf{B}}$, and **(c)** $\vec{\mathbf{B}} - \vec{\mathbf{A}}$,?

19. • **CE** Refer to **Figure 3–36** for the following questions: **(a)** Is the magnitude of $\vec{\mathbf{A}} + \vec{\mathbf{D}}$ greater than, less than, or equal to the magnitude of $\vec{\mathbf{A}} + \vec{\mathbf{E}}$? **(b)** Is the magnitude of $\vec{\mathbf{A}} + \vec{\mathbf{E}}$ greater than, less than, or equal to the magnitude of $\vec{\mathbf{A}} + \vec{\mathbf{F}}$?

20. • A vector $\vec{\mathbf{A}}$ has a magnitude of 40.0 m and points in a direction 20.0° below the positive x axis. A second vector, $\vec{\mathbf{B}}$, has a magnitude of 75.0 m and points in a direction 50.0° above the positive x axis. **(a)** Sketch the vectors $\vec{\mathbf{A}}$, $\vec{\mathbf{B}}$, and $\vec{\mathbf{C}} = \vec{\mathbf{A}} + \vec{\mathbf{B}}$. **(b)** Using the component method of vector addition, find the magnitude and direction of the vector $\vec{\mathbf{C}}$.

www.aswarphysics.weebly.com



▲ **FIGURE 3–36** Problems 18 and 19

21. • An air traffic controller observes two airplanes approaching the airport. The displacement from the control tower to plane 1 is given by the vector $\vec{A}$, which has a magnitude of 220 km and points in a direction 32° north of west. The displacement from the control tower to plane 2 is given by the vector $\vec{B}$, which has a magnitude of 140 km and points 65° east of north. **(a)** Sketch the vectors $\vec{A}$, $-\vec{B}$, and $\vec{D} = \vec{A} - \vec{B}$. Notice that $\vec{D}$ is the displacement from plane 2 to plane 1. **(b)** Find the magnitude and direction of the vector $\vec{D}$.

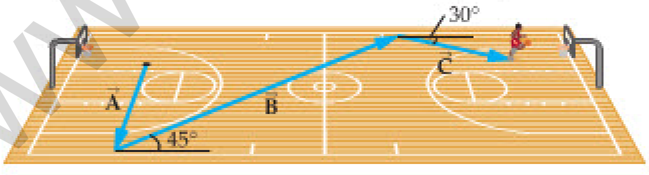
22. • The initial velocity of a car, $\vec{v}_i$, is 45 km/h in the positive x direction. The final velocity of the car, $\vec{v}_f$, is 66 km/h in a direction that points 75° above the positive x axis. **(a)** Sketch the vectors $-\vec{v}_i$, $\vec{v}_f$, and $\Delta\vec{v} = \vec{A}_f - \vec{A}_i$. **(b)** Find the magnitude and direction of the change in velocity, $\Delta\vec{v}$.

23. •• Vector $\vec{A}$ points in the positive x direction and has a magnitude of 75 m. The vector $\vec{C} = \vec{A} + \vec{B}$ points in the positive y direction and has a magnitude of 95 m. **(a)** Sketch $\vec{A}$, $\vec{B}$, and $\vec{C}$. **(b)** Estimate the magnitude and direction of the vector $\vec{B}$. **(c)** Verify your estimate in part (b) with a numerical calculation.

24. •• Vector $\vec{A}$ points in the negative x direction and has a magnitude of 22 units. The vector $\vec{B}$ points in the positive y direction. **(a)** Find the magnitude of $\vec{B}$ if $\vec{A} + \vec{B}$ has a magnitude of 37 units. **(b)** Sketch $\vec{A}$ and $\vec{B}$.

25. •• Vector $\vec{A}$ points in the negative y direction and has a magnitude of 5 units. Vector $\vec{B}$ has twice the magnitude and points in the positive x direction. Find the direction and magnitude of **(a)** $\vec{A} + \vec{B}$, **(b)** $\vec{A} - \vec{B}$, and **(c)** $\vec{B} - \vec{A}$.

26. •• A basketball player runs down the court, following the path indicated by the vectors $\vec{A}$, $\vec{B}$, and $\vec{C}$ in Figure 3–37. The magnitudes of these three vectors are $A = 10.0$ m, $B = 20.0$ m, and $C = 7.0$ m. Find the magnitude and direction of the net displacement of the player using **(a)** the graphical method and **(b)** the component method of vector addition. Compare your results.



▲ **FIGURE 3–37** Problem 26

SECTION 3–4 UNIT VECTORS

27. • A particle undergoes a displacement $\Delta\vec{r}$ of magnitude 54 m in a direction 42° below the x axis. Express $\Delta\vec{r}$ in terms of the unit vectors $\hat{x}$ and $\hat{y}$.

28. • A vector has a magnitude of 3.50 m and points in a direction that is 145° counterclockwise from the x axis. Find the x and y components of this vector.

29. • A vector $\vec{A}$ has a length of 6.1 m and points in the negative x direction. Find **(a)** the x component and **(b)** the magnitude of the vector $-3.7\,\vec{A}$.

30. • The vector $-5.2\,\vec{A}$ has a magnitude of 34 m and points in the positive x direction. Find **(a)** the x component and **(b)** the magnitude of the vector $\vec{A}$.

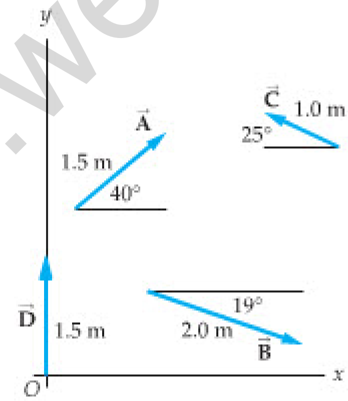
31. • Find the direction and magnitude of the vectors.
(a) $\vec{A} = (5.0\text{ m})\hat{x} + (-2.0\text{ m})\hat{y}$,
(b) $\vec{B} = (-2.0\text{ m})\hat{x} + (5.0\text{ m})\hat{y}$, and **(c)** $\vec{A} + \vec{B}$.

32. • Find the direction and magnitude of the vectors.
(a) $\vec{A} = (25\text{ m})\hat{x} + (-12\text{ m})\hat{y}$,
(b) $\vec{B} = (2.0\text{ m})\hat{x} + (15\text{ m})\hat{y}$, and **(c)** $\vec{A} + \vec{B}$.

33. • For the vectors given in Problem 32, express **(a)** $\vec{A} - \vec{B}$ and **(b)** $\vec{B} - \vec{A}$ in unit vector notation.

34. • Express each of the vectors in Figure 3–38 in unit vector notation.

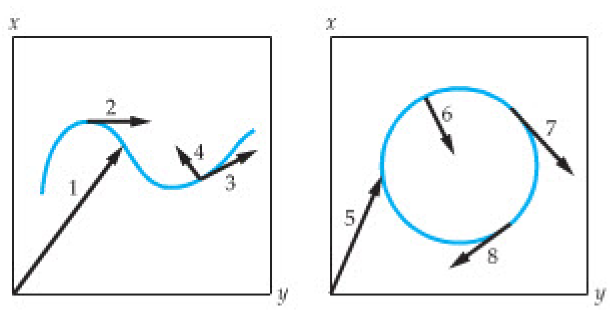
35. •• Referring to the vectors in Figure 3–38, express the sum $\vec{A} + \vec{B} + \vec{C}$ in unit vector notation.



▲ **FIGURE 3–38** Problems 34 and 35

SECTION 3–5 POSITION, DISPLACEMENT, VELOCITY, AND ACCELERATION VECTORS

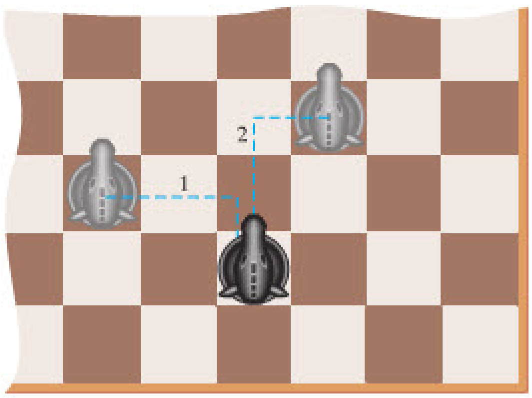
36. • **CE** The blue curves shown in Figure 3–39 display the constant-speed motion of two different particles in the x-y plane. For each of the eight vectors in Figure 3–39, state whether it is **(a)** a position vector, **(b)** a velocity vector, or **(c)** an acceleration vector for the particles.



▲ **FIGURE 3–39** Problem 36

37. • **IP Moving the Knight** Two of the allowed chess moves for a knight are shown in Figure 3–40. **(a)** Is the magnitude of displacement 1 greater than, less than, or equal to the magnitude of displacement 2? Explain. **(b)** Find the magnitude and direction of the knight's displacement for each of the two moves. Assume that the checkerboard squares are 3.5 cm on a side.

www.aswarphysics.weebly.com



▲ **FIGURE 3–40** Problem 37

38. • **IP** In its daily prowl of the neighborhood, a cat makes a displacement of 120 m due north, followed by a 72-m displacement due west. **(a)** Find the magnitude and direction of the displacement required for the cat to return home. **(b)** If, instead, the cat had first prowled 72 m west and then 120 m north, how would this affect the displacement needed to bring it home? Explain.

39. • If the cat in Problem 38 takes 45 minutes to complete the 120-m displacement and 17 minutes to complete the 72-m displacement, what are the magnitude and direction of its average velocity during this 62-minute period of time?

40. • What are the direction and magnitude of your total displacement if you have traveled due west with a speed of 27 m/s for 125 s, then due south at 14 m/s for 66 s?

41. •• You drive a car 1500 ft to the east, then 2500 ft to the north. If the trip took 3.0 minutes, what were the direction and magnitude of your average velocity?

42. •• **IP** A jogger runs with a speed of 3.25 m/s in a direction 30.0° above the x axis. **(a)** Find the x and y components of the jogger's velocity. **(b)** How will the velocity components found in part (a) change if the jogger's speed is halved?

43. •• You throw a ball upward with an initial speed of 4.5 m/s. When it returns to your hand 0.92 s later, it has the same speed in the downward direction (assuming air resistance can be ignored). What was the average acceleration vector of the ball?

44. •• A skateboarder rolls from rest down an inclined ramp that is 15.0 m long and inclined above the horizontal at an angle of $\theta = 20.0°$. When she reaches the bottom of the ramp 3.00 s later her speed is 10.0 m/s. Show that the average acceleration of the skateboarder is $g \sin\theta$, where $g = 9.81\ \mathrm{m/s^2}$.

45. •• Consider a skateboarder who starts from rest at the top of a ramp that is inclined at an angle of 17.5° to the horizontal. Assuming that the skateboarder's acceleration is $g \sin 17.5°$, find his speed when he reaches the bottom of the ramp in 3.25 s.

46. ••• **IP The Position of the Moon** Relative to the center of the Earth, the position of the Moon can be approximated by

$$\vec{r} = (3.84 \times 10^8\ \mathrm{m})\{\cos[(2.46 \times 10^{-6}\ \mathrm{radians/s})t]\hat{x} + \sin[(2.46 \times 10^{-6}\ \mathrm{radians/s})t]\hat{y}\}$$

where t is measured in seconds. **(a)** Find the magnitude and direction of the Moon's average velocity between $t = 0$ and $t = 7.38$ days. (This time is one-quarter of the 29.5 days it takes the Moon to complete one orbit.) **(b)** Is the instantaneous speed of the Moon greater than, less than, or the same as the average speed found in part (a)? Explain.

47. ••• **The Velocity of the Moon** The velocity of the Moon relative to the center of the Earth can be approximated by

$$\vec{v} = (945\ \mathrm{m/s})\{-\sin[(2.46 \times 10^{-6}\ \mathrm{radians/s})t]\hat{x} + \cos[(2.46 \times 10^{-6}\ \mathrm{radians/s})t]\hat{y}\}$$

where t is measured in seconds. To approximate the instantaneous acceleration of the Moon at $t = 0$, calculate the magnitude and direction of the average acceleration between the times **(a)** $t = 0$ and $t = 0.100$ days and **(b)** $t = 0$ and $t = 0.0100$ days. (The time required for the Moon to complete one orbit is 29.5 days.)

SECTION 3–6 RELATIVE MOTION

48. • **CE** The accompanying photo shows a KC-10A Extender using a boom to refuel an aircraft in flight. If the velocity of the KC-10A is 125 m/s due east relative to the ground, what is the velocity of the aircraft being refueled relative to **(a)** the ground, and **(b)** the KC-10A?

Air-to-air refueling. (Problem 48)

49. • As an airplane taxies on the runway with a speed of 16.5 m/s, a flight attendant walks toward the tail of the plane with a speed of 1.22 m/s. What is the flight attendant's speed relative to the ground?

50. • Referring to part (a) of Example 3–2, find the time it takes for the boat to reach the opposite shore if the river is 35 m wide.

51. •• As you hurry to catch your flight at the local airport, you encounter a moving walkway that is 85 m long and has a speed of 2.2 m/s relative to the ground. If it takes you 68 s to cover 85 m when walking on the ground, how long will it take you to cover the same distance on the walkway? Assume that you walk with the same speed on the walkway as you do on the ground.

52. •• In Problem 51, how long would it take you to cover the 85-m length of the walkway if, once you get on the walkway, you immediately turn around and start walking in the opposite direction with a speed of 1.3 m/s relative to the walkway?

53. •• **IP** The pilot of an airplane wishes to fly due north, but there is a 65-km/h wind blowing toward the east. **(a)** In what direction should the pilot head her plane if its speed relative to the air is 340 km/h? **(b)** Draw a vector diagram that illustrates your result in part (a). **(c)** If the pilot decreases the air speed of the plane, but still wants to head due north, should the angle found in part (a) be increased or decreased?

54. •• A passenger walks from one side of a ferry to the other as it approaches a dock. If the passenger's velocity is 1.50 m/s due north relative to the ferry, and 4.50 m/s at an angle of 30.0° west of north relative to the water, what are the direction and magnitude of the ferry's velocity relative to the water?

www.aswarphysics.weebly.com

55. •• You are riding on a Jet Ski at an angle of 35° upstream on a river flowing with a speed of 2.8 m/s. If your velocity relative to the ground is 9.5 m/s at an angle of 20.0° upstream, what is the speed of the Jet Ski relative to the water? (*Note*: Angles are measured relative to the x axis shown in Example 3–2.)

56. •• **IP** In Problem 55, suppose the Jet Ski is moving at a speed of 12 m/s relative to the water. **(a)** At what angle must you point the Jet Ski if your velocity relative to the ground is to be perpendicular to the shore of the river? **(b)** If you increase the speed of the Jet Ski relative to the water, does the angle in part (a) increase, decrease, or stay the same? Explain. (*Note:* Angles are measured relative to the x axis shown in Example 3–2.)

57. ••• **IP** Two people take identical Jet Skis across a river, traveling at the same speed relative to the water. Jet Ski A heads directly across the river and is carried downstream by the current before reaching the opposite shore. Jet Ski B travels in a direction that is 35° upstream and arrives at the opposite shore directly across from the starting point. **(a)** Which Jet Ski reaches the opposite shore in the least amount of time? **(b)** Confirm your answer to part (a) by finding the ratio of the time it takes for the two Jet Skis to cross the river. (*Note:* Angles are measured relative to the x axis shown in Example 3–2.)

GENERAL PROBLEMS

58. • **CE Predict/Explain** Consider the vectors $\vec{\mathbf{A}} = (1.2\text{ m})\hat{\mathbf{x}}$ and $\vec{\mathbf{B}} = (-3.4\text{ m})\hat{\mathbf{x}}$. **(a)** Is the magnitude of vector $\vec{\mathbf{A}}$ greater than, less than, or equal to the magnitude of vector $\vec{\mathbf{B}}$? **(b)** Choose the *best explanation* from among the following:
    I. The number 3.4 is greater than the number 1.2.
    II. The component of $\vec{\mathbf{B}}$ is negative.
    III. The vector $\vec{\mathbf{A}}$ points in the positive x direction.

59. • **CE Predict/Explain** Two vectors are defined as follows: $\vec{\mathbf{A}} = (-2.2\text{ m})\hat{\mathbf{x}}$ and $\vec{\mathbf{B}} = (1.4\text{ m})\hat{\mathbf{y}}$. **(a)** Is the magnitude of $1.4\,\vec{\mathbf{A}}$ greater than, less than, or equal to the magnitude of $2.2\,\vec{\mathbf{B}}$? **(b)** Choose the *best explanation* from among the following:
    I. The vector $\vec{\mathbf{A}}$ has a negative component.
    II. A number and its negative have the same magnitude.
    III. The vectors $1.4\,\vec{\mathbf{A}}$ and $2.2\,\vec{\mathbf{B}}$ point in opposite directions.

60. • You slide a box up a loading ramp that is 10.0 ft long. At the top of the ramp the box has risen a height of 3.00 ft. What is the angle of the ramp above the horizontal?

61. • Find the direction and magnitude of the vector $2\vec{\mathbf{A}} + \vec{\mathbf{B}}$, where $\vec{\mathbf{A}} = (12.1\text{ m})\hat{\mathbf{x}}$ and $\vec{\mathbf{B}} = (-32.2\text{ m})\hat{\mathbf{y}}$.

62. •• **CE** The components of a vector $\vec{\mathbf{A}}$ satisfy $A_x < 0$ and $A_y < 0$. Is the direction angle of $\vec{\mathbf{A}}$ between 0° and 90°, between 90° and 180°, between 180° and 270°, or between 270° and 360°?

63. •• **CE** The components of a vector $\vec{\mathbf{B}}$ satisfy $B_x > 0$ and $B_y < 0$. Is the direction angle of $\vec{\mathbf{B}}$ between 0° and 90°, between 90° and 180°, between 180° and 270°, or between 270° and 360°?

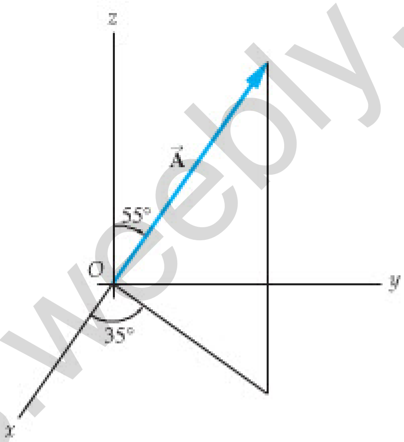
64. •• It is given that $\vec{\mathbf{A}} - \vec{\mathbf{B}} = (-51.4\text{ m})\hat{\mathbf{x}}$, $\vec{\mathbf{C}} = (62.2\text{ m})\hat{\mathbf{x}}$, and $\vec{\mathbf{A}} + \vec{\mathbf{B}} + \vec{\mathbf{C}} = (13.8\text{ m})\hat{\mathbf{x}}$. Find the vectors $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$.

65. •• **IP** Two students perform an experiment with a train and a ball. Michelle rides on a flatcar pulled at 8.35 m/s by a train on a straight, horizontal track; Gary stands at rest on the ground near the tracks. When Michelle throws the ball with an initial angle of 65.0° above the horizontal, from her point of view, Gary sees the ball rise straight up and back down above a fixed point on the ground. **(a)** Did Michelle throw the ball toward the front of the train or toward the rear of the train? Explain. **(b)** What was the initial speed of Michelle's throw? **(c)** What was the initial speed of the ball as seen by Gary?

66. •• An off-roader explores the open desert in her Hummer. First she drives 25° west of north with a speed of 6.5 km/h for 15 minutes, then due east with a speed of 12 km/h for 7.5 minutes. She completes the final leg of her trip in 22 minutes. What are the direction and speed of travel on the final leg? (Assume her speed is constant on each leg, and that she returns to her starting point at the end of the final leg.)

67. •• Find the x, y, and z components of the vector $\vec{\mathbf{A}}$ shown in Figure 3–41, given that $A = 65$ m.

68. •• A football is thrown horizontally with an initial velocity of $(16.6\text{ m/s})\hat{\mathbf{x}}$. Ignoring air resistance, the average acceleration of the football over any period of time is $(-9.81\text{ m/s}^2)\hat{\mathbf{y}}$. **(a)** Find the velocity vector of the ball 1.75 s after it is thrown. **(b)** Find the magnitude and direction of the velocity at this time.

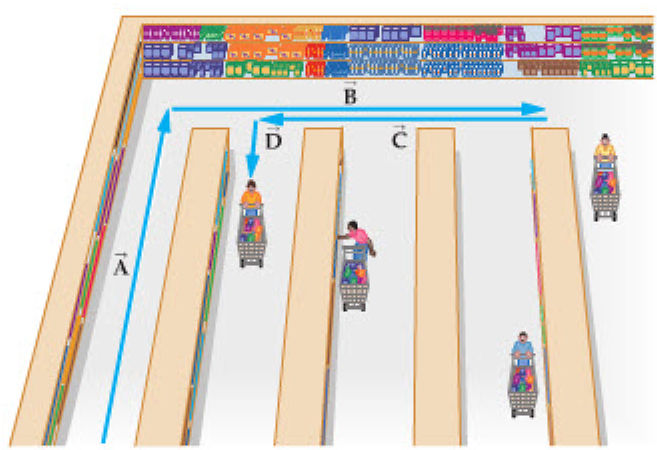


▲ **FIGURE 3–41** Problem 67

69. •• As a function of time, the velocity of the football described in Problem 68 can be written as $\vec{\mathbf{v}} = (16.6\text{ m/s})\hat{\mathbf{x}} - [(9.81\text{ m/s}^2)t]\hat{\mathbf{y}}$. Calculate the average acceleration vector of the football for the time periods **(a)** $t = 0$ to $t = 1.00$ s, **(b)** $t = 0$ to $t = 2.50$ s, and **(c)** $t = 0$ to $t = 5.00$ s. (If the acceleration of an object is constant, its average acceleration is the same for all time periods.)

70. •• Two airplanes taxi as they approach the terminal. Plane 1 taxies with a speed of 12 m/s due north. Plane 2 taxies with a speed of 7.5 m/s in a direction 20° north of west. **(a)** What are the direction and magnitude of the velocity of plane 1 relative to plane 2? **(b)** What are the direction and magnitude of the velocity of plane 2 relative to plane 1?

71. •• A shopper at the supermarket follows the path indicated by vectors $\vec{\mathbf{A}}$, $\vec{\mathbf{B}}$, $\vec{\mathbf{C}}$, and $\vec{\mathbf{D}}$ in Figure 3–42. Given that the



▲ **FIGURE 3–42** Problem 71

www.aswarphysics.weebly.com

vectors have the magnitudes $A = 51$ ft, $B = 45$ ft, $C = 35$ ft, and $D = 13$ ft, find the total displacement of the shopper using **(a)** the graphical method and **(b)** the component method of vector addition. Give the direction of the displacement relative to the direction of vector $\vec{A}$.

72. •• Initially, a particle is moving at 4.10 m/s at an angle of 33.5° above the horizontal. Two seconds later, its velocity is 6.05 m/s at an angle of 59.0° below the horizontal. What was the particle's average acceleration during these 2.00 seconds?

73. •• A passenger on a stopped bus notices that rain is falling vertically just outside the window. When the bus moves with constant velocity, the passenger observes that the falling raindrops are now making an angle of 15° with respect to the vertical. **(a)** What is the ratio of the speed of the raindrops to the speed of the bus? **(b)** Find the speed of the raindrops, given that the bus is moving with a speed of 18 m/s.

74. •• **A Big Clock** The clock that rings the bell known as Big Ben has an hour hand that is 9.0 feet long and a minute hand that is 14 feet long, where the distance is measured from the center of the clock to the tip of each hand. What is the tip-to-tip distance between these two hands when the clock reads 12 minutes after four o'clock?

75. •• **IP** Suppose we orient the x axis of a two-dimensional coordinate system along the beach at Waikiki. Waves approaching the beach have a velocity relative to the shore given by $\vec{v}_{ws} = (1.3 \text{ m/s})\hat{y}$. Surfers move more rapidly than the waves, but at an angle to the beach. The angle is chosen so that the surfers approach the shore with the same speed as the waves. **(a)** If a surfer has a speed of 7.2 m/s relative to the water, what is her direction of motion relative to the positive x axis? **(b)** What is the surfer's velocity relative to the wave? **(c)** If the surfer's speed is increased, will the angle in part (a) increase or decrease? Explain.

76. ••• **IP** Referring to Example 3–2, **(a)** what heading must the boat have if it is to land directly across the river from its starting point? **(b)** How much time is required for this trip if the river is 25.0 m wide? **(c)** Suppose the speed of the boat is increased, but it is still desired to land directly across from the starting point. Should the boat's heading be more upstream, more downstream, or the same as in part (a)? Explain.

77. ••• Vector $\vec{A}$ points in the negative x direction. Vector $\vec{B}$ points at an angle of 30.0° above the positive x axis. Vector $\vec{C}$ has a magnitude of 15 m and points in a direction 40.0° below the positive x axis. Given that $\vec{A} + \vec{B} + \vec{C} = 0$, find the magnitudes of $\vec{A}$ and $\vec{B}$.

78. ••• As two boats approach the marina, the velocity of boat 1 relative to boat 2 is 2.15 m/s in a direction 47.0° east of north. If boat 1 has a velocity that is 0.775 m/s due north, what is the velocity (magnitude and direction) of boat 2?

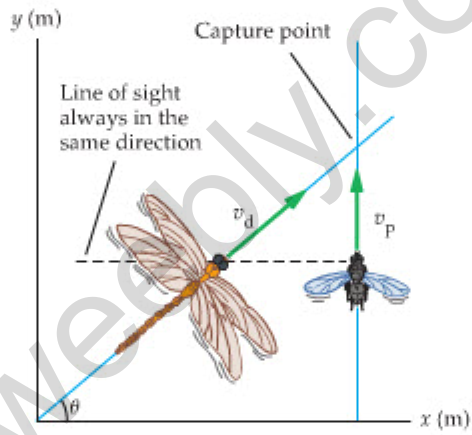
PASSAGE PROBLEMS

BIO Motion Camouflage in Dragonflies

Dragonflies, whose ancestors were once the size of hawks, have prowled the skies in search of small flying insects for over 250 million years. Faster and more maneuverable than any other insect, they even fold their front two legs in flight and tuck them behind their head to be as streamlined as possible. They also employ an intriguing stalking strategy known as "motion camouflage" to approach their prey almost undetected.

The basic idea of motion camouflage is for the dragonfly to move in such a way that the line of sight from the prey to the dragonfly is always in the same direction. Moving in this way, the dragonfly appears almost motionless to its prey, as if it were an object at infinity. Eventually the prey notices the dragonfly has grown in size and is therefore closer, but by that time it's too late for it to evade capture.

A typical capture scenario is shown in Figure 3–43, where the prey moves in the positive y direction with the constant speed $v_p = 0.750$ m/s, and the dragonfly moves at an angle $\theta = 48.5°$ to the x axis with the constant speed v_d. If the dragonfly chooses its speed correctly, the line of sight from the prey to the dragonfly will always be in the same direction—parallel to the x axis in this case.



▲ **FIGURE 3–43** Problems 79, 80, 81, and 82

79. • What speed must the dragonfly have if the line of sight, which is parallel to the x axis initially, is to remain parallel to the x axis?

A. 0.562 m/s　　B. 0.664 m/s
C. 1.00 m/s　　D. 1.13 m/s

80. • Suppose the dragonfly now approaches its prey along a path with $\theta > 48.5°$, but it still keeps the line of sight parallel to the x axis. Is the speed of the dragonfly in this new case greater than, less than, or equal to its speed in Problem 79?

81. • What is the correct "motion camouflage" speed of approach for a dragonfly pursuing its prey at the angle $\theta = 68.5°$?

A. 0.295 m/s　　B. 0.698 m/s
C. 0.806 m/s　　D. 2.05 m/s

82. •• If the dragonfly approaches its prey with a speed of 0.950 m/s, what angle θ is required to maintain a constant line of sight parallel to the x axis?

A. 37.9°　　B. 38.3°
C. 51.7°　　D. 52.1°

INTERACTIVE PROBLEMS

83. •• **IP Referring to Example 3–2** Suppose the speed of the boat relative to the water is 7.0 m/s. **(a)** At what angle to the x axis must the boat be headed if it is to land directly across the river from its starting position? **(b)** If the speed of the boat relative to the water is increased, will the angle needed to go directly across the river increase, decrease, or stay the same? Explain.

84. ••• **Referring to Example 3–2** Suppose the boat has a speed of 6.7 m/s relative to the water, and that the dock on the opposite shore of the river is at the location $x = 55$ m and $y = 28$ m relative to the starting point of the boat. **(a)** At what angle relative to the x axis must the boat be pointed in order to reach the other dock? **(b)** With the angle found in part (a), what is the speed of the boat relative to the ground?

www.aswarphysics.weebly.com

4 Two-Dimensional Kinematics

When you hear the word "projectile," you probably think of an artillery shell or perhaps a home run into the upper deck. But as we'll see in this chapter, the term applies to any object moving under the influence of gravity alone. For example, each of these juggling balls undergoes projectile motion as it moves from one hand to the other. In this chapter we will explore the physical laws that govern such motion, and will learn—among other things—that these balls follow a parabolic path.

We now extend our study of kinematics to motion in two dimensions. This allows us to consider a much wider range of physical phenomena observed in everyday life. Of particular interest is **projectile motion,** the motion of objects that are initially launched—or "projected"—and that then continue moving under the influence of gravity alone. Examples of projectile motion include balls thrown from one person to another, water spraying from a hose, salmon leaping over rapids, and divers jumping from the cliffs of Acapulco.

The main idea of this chapter is quite simple: Horizontal and vertical motions are independent. That's it. For example, a ball thrown horizontally with a speed *v* continues to move with the same speed *v* in the horizontal direction, even as it falls with an increasing speed in the vertical direction. Similarly, the time of fall is the same whether a ball is dropped from rest straight down, or thrown horizontally. Simply put, each motion continues as if the other motion were not present.

This chapter develops and applies the idea of independence of motion to many common physical systems.

4–1	Motion in Two Dimensions	83
4–2	Projectile Motion: Basic Equations	86
4–3	Zero Launch Angle	88
4–4	General Launch Angle	92
4–5	Projectile Motion: Key Characteristics	96

www.aswarphysics.weebly.com

4–1 Motion in Two Dimensions

In this section we develop equations of motion to describe objects moving in two dimensions. First, we consider motion with constant velocity, determining x and y as functions of time. Next, we investigate motion with constant acceleration. We show that the one-dimensional kinematic equations of Chapter 2 can be extended in a straightforward way to apply to two dimensions.

Constant Velocity

To begin, consider the simple situation shown in Figure 4–1. A turtle starts at the origin at $t = 0$ and moves with a constant speed $v_0 = 0.26\ \mathrm{m/s}$ in a direction 25° above the x axis. How far has the turtle moved in the x and y directions after 5.0 seconds?

First, note that the turtle moves in a straight line a distance

$$d = v_0 t = (0.26\ \mathrm{m/s})(5.0\ \mathrm{s}) = 1.3\ \mathrm{m}$$

as indicated in Figure 4–1(a). From the definitions of sine and cosine given in the previous chapter, we see that

$$x = d\cos 25^\circ = 1.2\ \mathrm{m}$$
$$y = d\sin 25^\circ = 0.55\ \mathrm{m}$$

An alternative way to approach this problem is to treat the x and y motions separately. First, we determine the speed of the turtle in each direction. Referring to Figure 4–1(b), we see that the x component of velocity is

$$v_{0x} = v_0\cos 25^\circ = 0.24\ \mathrm{m/s}$$

and the y component is

$$v_{0y} = v_0\sin 25^\circ = 0.11\ \mathrm{m/s}$$

Next, we find the distance traveled by the turtle in the x and y directions by multiplying the speed in each direction by the time:

$$x = v_{0x}t = (0.24\ \mathrm{m/s})(5.0\ \mathrm{s}) = 1.2\ \mathrm{m}$$

and

$$y = v_{0y}t = (0.11\ \mathrm{m/s})(5.0\ \mathrm{s}) = 0.55\ \mathrm{m}$$

This is in agreement with our previous results. To summarize, we can think of the turtle's actual motion as a combination of separate x and y motions.

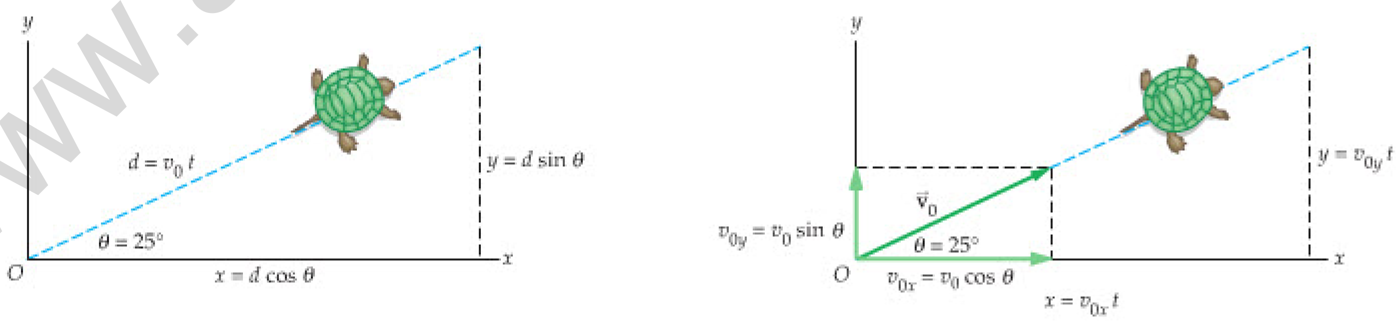
In general, the turtle might start at a position $x = x_0$ and $y = y_0$ at time $t = 0$. In this case, we have

$$x = x_0 + v_{0x}t \qquad \text{4–1}$$

and

$$y = y_0 + v_{0y}t \qquad \text{4–2}$$

as the x and y equations of motion.



(a) (b)

▲ **FIGURE 4–1 Constant velocity**

A turtle walks from the origin with a speed of $v_0 = 0.26\ \mathrm{m/s}$. **(a)** In a time t the turtle moves through a straight-line distance of $d = v_0 t$; thus the x and y displacements are $x = d\cos\theta$, $y = d\sin\theta$. **(b)** Equivalently, the turtle's x and y components of velocity are $v_{0x} = v_0\cos\theta$ and $v_{0y} = v_0\sin\theta$; hence $x = v_{0x}t$ and $y = v_{0y}t$.

www.aswarphysics.weebly.com

Compare these equations with Equation 2–11, $x = x_0 + v_0 t + \frac{1}{2}at^2$, which gives position as a function of time in one dimension. When acceleration is zero, as it is for the turtle, Equation 2–11 reduces to $x = x_0 + v_0 t$. Replacing v_0 with the x component of the velocity, v_{0x}, yields Equation 4–1. Similarly, replacing each x in Equation 4–1 with y converts it to Equation 4–2, the y equation of motion.

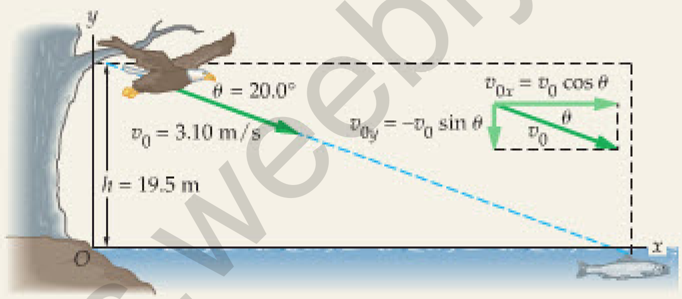
A situation illustrating the use of Equations 4–1 and 4–2 is given in Example 4–1.

EXAMPLE 4–1 THE EAGLE DESCENDS

An eagle perched on a tree limb 19.5 m above the water spots a fish swimming near the surface. The eagle pushes off from the branch and descends toward the water. By adjusting its body in flight, the eagle maintains a constant speed of 3.10 m/s at an angle of 20.0° below the horizontal. **(a)** How long does it take for the eagle to reach the water? **(b)** How far has the eagle traveled in the horizontal direction when it reaches the water?

PICTURE THE PROBLEM

We set up our coordinate system so that the eagle starts at $x_0 = 0$ and $y_0 = h = 19.5$ m. The water level is $y = 0$. As indicated in our sketch, $v_{0x} = v_0 \cos\theta$ and $v_{0y} = -v_0 \sin\theta$, where $v_0 = 3.10$ m/s and $\theta = 20.0°$. Notice that both components of the eagle's velocity are constant, and therefore the equations of motion given in Equations 4–1 and 4–2 apply.



STRATEGY

As usual in such problems, it is best to treat the eagle's flight as a combination of separate x and y motions. Since we are given the constant speed of the eagle, and the angle at which it descends, we can find the x and y components of its velocity. We then use the y equation of motion, $y = y_0 + v_{0y}t$, to find the time t when the eagle reaches the water. Finally, we use this value of t in the x equation of motion, $x = x_0 + v_{0x}t$, to find the horizontal distance the bird travels.

SOLUTION

Part (a)

1. Begin by determining v_{0x} and v_{0y}:

$$v_{0x} = v_0 \cos\theta = (3.10 \text{ m/s}) \cos 20.0° = 2.91 \text{ m/s}$$
$$v_{0y} = -v_0 \sin\theta = -(3.10 \text{ m/s}) \sin 20.0° = -1.06 \text{ m/s}$$

2. Now, set $y = 0$ in $y = y_0 + v_{0y}t$ and solve for t:

$$y = y_0 + v_{0y}t = h + v_{0y}t = 0$$
$$t = -\frac{h}{v_{0y}} = -\frac{19.5 \text{ m}}{(-1.06 \text{ m/s})} = 18.4 \text{ s}$$

Part (b)

3. Substitute $t = 18.4$ s into $x = x_0 + v_{0x}t$ to find x:

$$x = x_0 + v_{0x}t = 0 + (2.91 \text{ m/s})(18.4 \text{ s}) = 53.5 \text{ m}$$

INSIGHT

Notice how the two minus signs in Step 2 combine to give a positive time. One minus sign comes from setting $y = 0$, the other from the fact that v_{0y} is negative. No matter where we choose the origin, or what direction we choose to be positive, the time will always have the same value.

As mentioned in the problem statement, the eagle cannot travel in a straight line by simply dropping from the tree limb—it has to adjust its wings and tail to produce enough lift to balance the force of gravity. Airplanes do the same thing when they adjust their flight surfaces to make a smooth landing.

PRACTICE PROBLEM

What is the location of the eagle 2.00 s after it takes flight? [**Answer:** $x = 5.82$ m, $y = 17.4$ m]

Some related homework problems: Problem 2, Problem 3

Constant Acceleration

To study motion with constant acceleration in two dimensions we repeat what was done in one dimension in Chapter 2, but with separate equations for both x and y. For example, to obtain x as a function of time we start with $x = x_0 + v_0 t + \frac{1}{2}at^2$ (Equation 2–11), and replace both v_0 and a with the corresponding x components, v_{0x} and a_x. This gives

$$x = x_0 + v_{0x}t + \tfrac{1}{2}a_x t^2 \qquad \text{4–3(a)}$$

www.aswarphysics.weebly.com

To obtain y as a function of time, we write y in place of x in Equation 4–3(a):

$$y = y_0 + v_{0y}t + \tfrac{1}{2}a_y t^2 \qquad \text{4–3(b)}$$

These are the position-versus-time equations of motion for two dimensions. (In three dimensions we introduce a third coordinate direction and label it z. We would then simply replace x with z in Equation 4–3(a) to obtain z as a function of time.)

The same approach gives velocity as a function of time. Start with Equation 2–7, $v = v_0 + at$, and write it in terms of x and y components. This yields

$$v_x = v_{0x} + a_x t \qquad \text{4–4(a)}$$

$$v_y = v_{0y} + a_y t \qquad \text{4–4(b)}$$

Note that we simply repeat everything we did for one dimension, only now with separate equations for the x and y components.

Finally, we can write $v^2 = v_0{}^2 + 2a\Delta x$ in terms of components as well:

$$v_x{}^2 = v_{0x}{}^2 + 2a_x\Delta x \qquad \text{4–5(a)}$$

$$v_y{}^2 = v_{0y}{}^2 + 2a_y\Delta y \qquad \text{4–5(b)}$$

The following table summarizes our results:

Table 4–1 Constant-Acceleration Equations of Motion

Position as a function of time	Velocity as a function of time	Velocity as a function of position
$x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2$	$v_x = v_{0x} + a_x t$	$v_x{}^2 = v_{0x}{}^2 + 2a_x\Delta x$
$y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2$	$v_y = v_{0y} + a_y t$	$v_y{}^2 = v_{0y}{}^2 + 2a_y\Delta y$

These are the fundamental equations that will be used to obtain *all* of the results presented throughout the rest of this chapter. Though it may appear sometimes that we are writing new sets of equations for different special cases, the equations aren't new—what we are actually doing is simply writing these equations again, but with specific values substituted for the constants that appear in them.

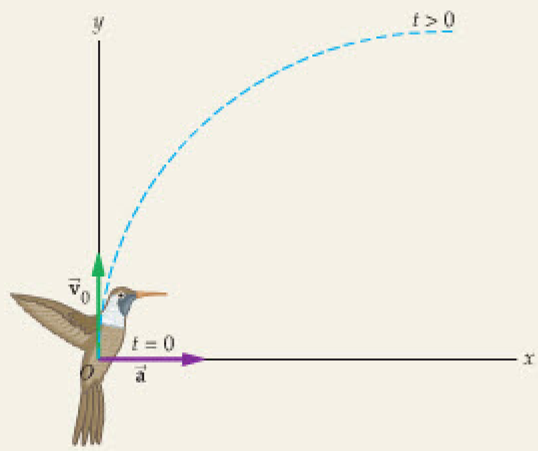
www.aswarphysics.weebly.com

EXAMPLE 4–2 A HUMMER ACCELERATES

A hummingbird is flying in such a way that it is initially moving vertically with a speed of 4.6 m/s and accelerating horizontally at 11 m/s^2. Assuming the bird's acceleration remains constant for the time interval of interest, find **(a)** the horizontal and vertical distances through which it moves in 0.55 s and **(b)** its x and y velocity components at $t = 0.55$ s.

PICTURE THE PROBLEM

In our sketch we have placed the origin of a two-dimensional coordinate system at the location of the hummingbird at the initial time, $t = 0$. In addition, we have chosen the initial direction of motion to be in the positive y direction, and the direction of acceleration to be in the positive x direction. As a result, it follows that $x_0 = y_0 = 0$, $v_{0x} = 0$, $v_{0y} = 4.6$ m/s, $a_x = 11$ m/s^2, and $a_y = 0$. As the hummingbird moves upward, its x component of velocity increases, resulting in a curved path, as shown.



STRATEGY

(a) Since we want to relate position and time, we find the horizontal position of the hummingbird using $x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2$, and the vertical position using $y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2$. **(b)** The velocity components as a function of time can be found using $v_x = v_{0x} + a_x t$ and $v_y = v_{0y} + a_y t$.

CONTINUED ON NEXT PAGE

CONTINUED FROM PREVIOUS PAGE

SOLUTION

Part (a)

1. Use $x = x_0 + v_{0x}t + \frac{1}{2}a_xt^2$ to find x at $t = 0.55$ s: $x = x_0 + v_{0x}t + \frac{1}{2}a_xt^2 = 0 + 0 + \frac{1}{2}(11\ \text{m/s}^2)(0.55\ \text{s})^2 = 1.7\ \text{m}$
2. Use $y = y_0 + v_{0y}t + \frac{1}{2}a_yt^2$ to find y at $t = 0.55$ s: $y = y_0 + v_{0y}t + \frac{1}{2}a_yt^2 = 0 + (4.6\ \text{m/s})(0.55\ \text{s}) + 0 = 2.5\ \text{m}$

Part (b)

3. Use $v_x = v_{0x} + a_xt$ to find v_x at $t = 0.55$ s: $v_x = v_{0x} + a_xt = 0 + (11\ \text{m/s}^2)(0.55\ \text{s}) = 6.1\ \text{m/s}$
4. Use $v_y = v_{0y} + a_yt$ to find v_y at $t = 0.55$ s: $v_y = v_{0y} + a_yt = 4.6\ \text{m/s} + (0)(0.55\ \text{s}) = 4.6\ \text{m/s}$

INSIGHT

In 0.55 s the hummingbird moves 1.7 m horizontally and 2.5 m vertically. The horizontal position of the bird will eventually increase more rapidly with time than the vertical position, due to the t^2 dependence of x as compared with the t dependence of y. This results in a curved, parabolic path for the hummingbird, as shown in our sketch. The bird's velocity at 0.55 s is $v = \sqrt{v_x^2 + v_y^2} = \sqrt{(6.1\ \text{m/s})^2 + (4.6\ \text{m/s})^2} = 7.6\ \text{m/s}$ at an angle of $\theta = \tan^{-1}(v_y/v_x) = \tan^{-1}[(4.6\ \text{m/s})/(6.1\ \text{m/s})] = 37°$ above the x axis. It's clear the angle of flight must be less than 45° at this time, since the x component of velocity is greater than the y component.

PRACTICE PROBLEM

How much time is required for the hummingbird to move 2.0 m horizontally from its initial position? [**Answer:** $t = 0.60$ s]

Some related homework problems: Problem 4, Problem 5, Problem 62

4–2 Projectile Motion: Basic Equations

We now apply the independence of horizontal and vertical motions to projectiles. Just what do we mean by a projectile? Well, a **projectile** is an object that is thrown, kicked, batted, or otherwise launched into motion and then allowed to follow a path determined solely by the influence of gravity. As you might expect, this covers a wide variety of physical systems.

In studying projectile motion we make the following assumptions:

- air resistance is ignored
- the acceleration due to gravity is constant, downward, and has a magnitude equal to $g = 9.81\ \text{m/s}^2$
- the Earth's rotation is ignored

Air resistance can be significant when a projectile moves with relatively high speed or if it encounters a strong wind. In many everyday situations, however, like tossing a ball to a friend or dropping a book, air resistance is relatively insignificant. As for the acceleration due to gravity, $g = 9.81\ \text{m/s}^2$, this value varies slightly from place to place on the Earth's surface and decreases with increasing altitude. In addition, the rotation of the Earth can be significant when considering projectiles that cover great distances. Little error is made in ignoring the variation of g or the rotation of the Earth, however, in the examples of projectile motion considered in this chapter.

Let's incorporate these assumptions into the equations of motion given in the previous section. Suppose, as in Figure 4–2, that the x axis is horizontal and the y axis is vertical, with the positive direction upward. Since downward is the negative direction, it follows that

$$a_y = -9.81\ \text{m/s}^2 = -g$$

Gravity causes no acceleration in the x direction. Thus, the x component of acceleration is zero:

$$a_x = 0$$

With these acceleration components substituted into the fundamental constant-acceleration equations of motion (Table 4–1) we find:

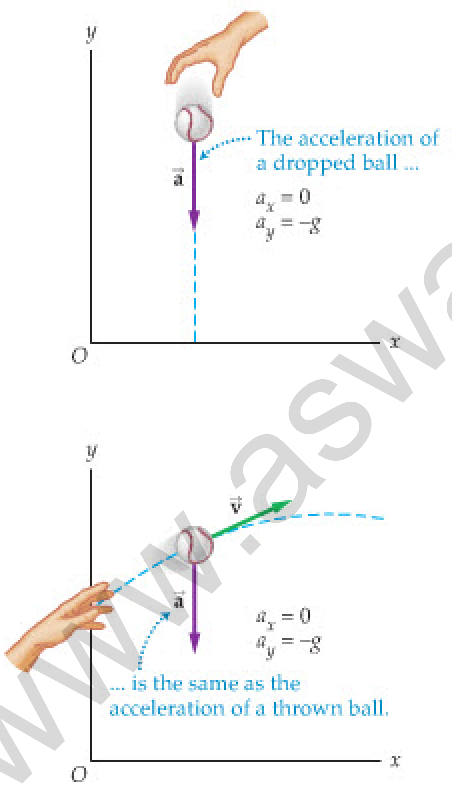
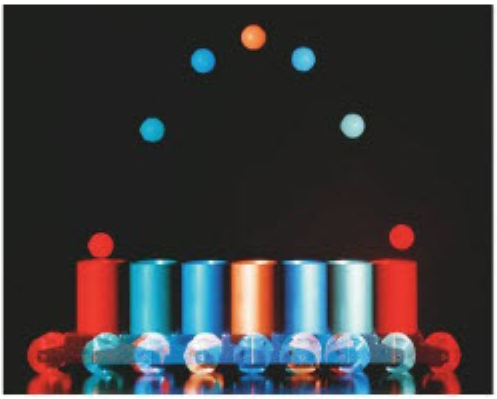


FIGURE 4–2 Acceleration in free fall
All objects in free fall have acceleration components $a_x = 0$ and $a_y = -g$ when the coordinate system is chosen as shown here. This is true regardless of whether the object is dropped, thrown, kicked, or otherwise set into motion.

www.aswarphysics.weebly.com



▲ In the multiple-exposure photo at left, a ball is projected upward from a moving cart. The ball retains its initial horizontal velocity; as a result, it follows a parabolic path and remains directly above the cart at all times. When the ball lands, it falls back into the cart, just as it would if the cart had been at rest. (In this sequence, the exposures were made at equal time intervals with light of different colors, making it easier to follow the relative motion of the ball and the cart.) In the photo at right, the pilot ejection seat of a jet fighter is being ground-tested. Here too the horizontal and vertical motions are independent; thus, the test dummy is still almost directly above the cockpit from which it was ejected. Note, however, that air resistance is beginning to reduce the dummy's horizontal velocity. Eventually, it will fall far behind the speeding plane.

Projectile Motion ($a_x = 0, a_y = -g$)

$$x = x_0 + v_{0x}t \qquad v_x = v_{0x} \qquad {v_x}^2 = {v_{0x}}^2$$
$$y = y_0 + v_{0y}t - \tfrac{1}{2}gt^2 \qquad v_y = v_{0y} - gt \qquad {v_y}^2 = {v_{0y}}^2 - 2g\Delta y \qquad \text{4–6}$$

PROBLEM-SOLVING NOTE

Acceleration of a Projectile

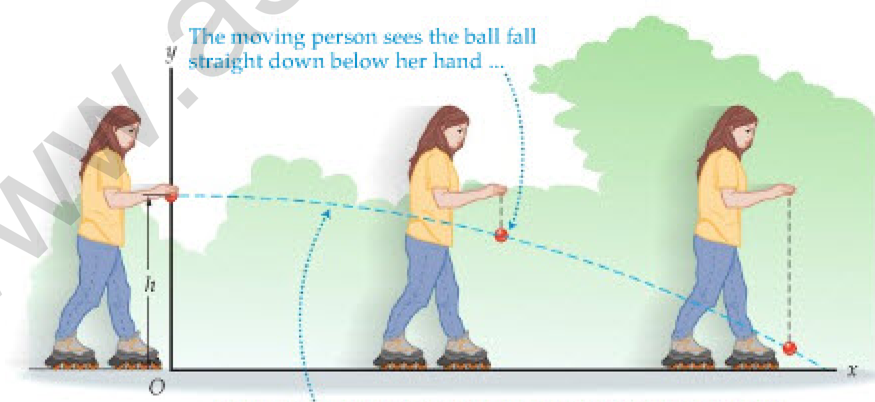
When the x axis is chosen to be horizontal and the y axis points vertically upward, it follows that the acceleration of an ideal projectile is $a_x = 0$ and $a_y = -g$.

Note that in these expressions the positive y direction is upward and the quantity g is positive. All of our studies of *projectile motion* will use Equations 4–6 as our fundamental equations—again, special cases will simply correspond to substituting specific values for the constants.

A simple demonstration illustrates the independence of horizontal and vertical motions in projectile motion. First, while standing still, drop a rubber ball to the floor and catch it on the rebound. Note that the ball goes straight down, lands near your feet, and returns almost to the level of your hand in about a second.

Next, walk—or roller skate—with constant speed before dropping the ball, then observe its motion carefully. To you, its motion looks the same as before: It goes straight down, lands near your feet, bounces straight back up, and returns in about one second. This is illustrated in Figure 4–3. The fact that you were moving in the horizontal direction the whole time had no effect on the ball's vertical motion—the motions were independent.

To an observer who sees you walking by, the ball follows a curved path, as shown. The precise shape of this curved path is determined in the next section.

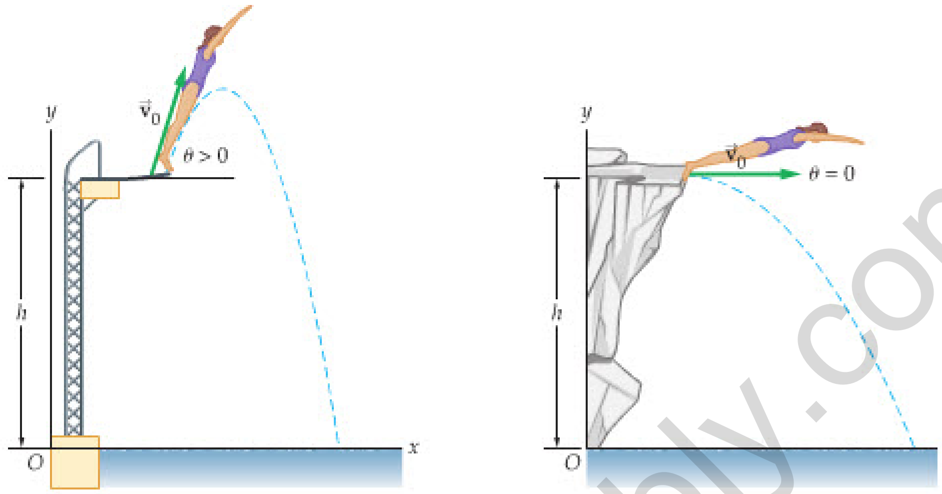


▲ **FIGURE 4–3 Independence of vertical and horizontal motions**

When you drop a ball while walking, running, or skating with constant velocity, it appears to you to drop straight down from the point where you released it. To a person at rest, the ball follows a curved path that combines horizontal and vertical motions.

▲ This rollerblader may not be thinking about independence of motion, but the ball she released illustrates the concept perfectly; it continues to move horizontally with constant speed—even though she's no longer touching it—at the same time that it accelerates vertically downward.

www.aswarphysics.weebly.com



(a) (b)

▲ **FIGURE 4–4 Launch angle of a projectile**

(a) A projectile launched at an angle above the horizontal, $\theta > 0$. A launch below the horizontal would correspond to $\theta < 0$. **(b)** A projectile launched horizontally, $\theta = 0$. In this section we consider $\theta = 0$. The next section deals with $\theta \neq 0$.

4–3 Zero Launch Angle

A special case of some interest is a projectile launched horizontally, so that the angle between the initial velocity and the horizontal is $\theta = 0$. We devote this section to a brief look at this type of motion.

Equations of Motion

Suppose you are walking with a speed v_0 when you release a ball from a height h, as discussed in the previous section. If we choose ground level to be $y = 0$ and the release point to be directly above the origin, the initial position of the ball is given by

$$x_0 = 0$$

and

$$y_0 = h$$

This is illustrated in Figure 4–3.

The initial velocity is horizontal, corresponding to $\theta = 0$ in **Figure 4–4**. As a result, the x component of the initial velocity is simply the initial speed:

$$v_{0x} = v_0 \cos 0° = v_0$$

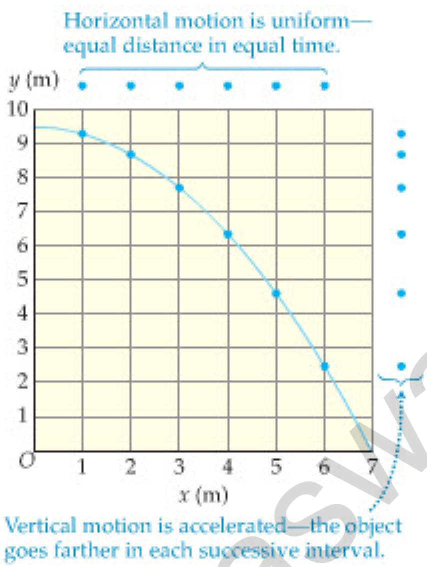
and the y component of the initial velocity is zero:

$$v_{0y} = v_0 \sin 0° = 0$$

Substituting these specific values into our fundamental equations for projectile motion (Equations 4–6) gives the following simplified results for zero launch angle ($\theta = 0$):

$$x = v_0 t \qquad v_x = v_0 = \text{constant} \qquad v_x^2 = v_0^2 = \text{constant}$$
$$y = h - \tfrac{1}{2}gt^2 \qquad v_y = -gt \qquad v_y^2 = -2g\Delta y \tag{4–7}$$

Note that the x component of velocity remains the same for all time and that the y component steadily decreases with time. As a result, x increases linearly with time, and y decreases with a t^2 dependence. Snapshots of this motion at equal time intervals are shown in **Figure 4–5**.



▲ **FIGURE 4–5 Trajectory of a projectile launched horizontally**

In this plot, the projectile was launched from a height of 9.5 m with an initial speed of 5.0 m/s. The positions shown in the plot correspond to the times $t = 0.20$ s, 0.40 s, 0.60 s, ... Note the uniform motion in the x direction, and the accelerated motion in the y direction.

PROBLEM-SOLVING NOTE

Identify Initial Conditions

The launch point of a projectile determines x_0 and y_0. The initial velocity of a projectile determines v_{0x} and v_{0y}.

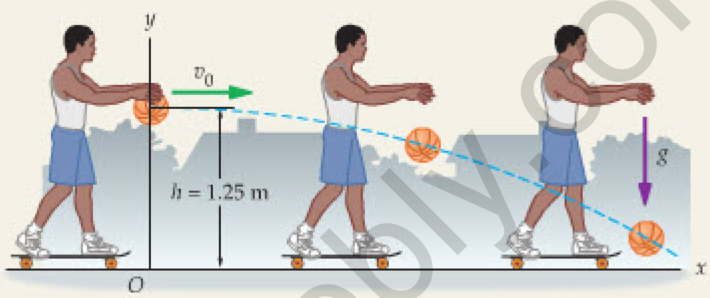
www.aswarphysics.weebly.com

EXAMPLE 4–3 DROPPING A BALL

A person skateboarding with a constant speed of 1.30 m/s releases a ball from a height of 1.25 m above the ground. Given that $x_0 = 0$ and $y_0 = h = 1.25$ m, find x and y for **(a)** $t = 0.250$ s and **(b)** $t = 0.500$ s. **(c)** Find the velocity, speed, and direction of motion of the ball at $t = 0.500$ s.

PICTURE THE PROBLEM

The ball starts at $x_0 = 0$ and $y_0 = h = 1.25$ m. Its initial velocity is horizontal, therefore $v_{0x} = v_0 = 1.30$ m/s and $v_{0y} = 0$. In addition, it accelerates with the acceleration due to gravity in the negative y direction, $a_y = -g$, and moves with constant speed in the x direction, $a_x = 0$.



STRATEGY

The x and y positions are given by $x = v_0 t$ and $y = h - \frac{1}{2}gt^2$, respectively. We simply substitute time into these expressions. Similarly, the velocity components are $v_x = v_0$ and $v_y = -gt$.

SOLUTION

Part (a)

1. Substitute $t = 0.250$ s into the x and y equations of motion:

$$x = v_0 t = (1.30 \text{ m/s})(0.250 \text{ s}) = 0.325 \text{ m}$$
$$y = h - \tfrac{1}{2}gt^2$$
$$= 1.25 \text{ m} - \tfrac{1}{2}(9.81 \text{ m/s}^2)(0.250 \text{ s})^2 = 0.943 \text{ m}$$

Part (b)

2. Substitute $t = 0.500$ s into the x and y equations of motion: Note that the ball is only about an inch above the ground at this time:

$$x = v_0 t = (1.30 \text{ m/s})(0.500 \text{ s}) = 0.650 \text{ m}$$
$$y = h - \tfrac{1}{2}gt^2$$
$$= 1.25 \text{ m} - \tfrac{1}{2}(9.81 \text{ m/s}^2)(0.500 \text{ s})^2 = 0.0238 \text{ m}$$

Part (c)

3. First, calculate the x and y components of the velocity at $t = 0.500$ s using $v_x = v_0$ and $v_y = -gt$:

$$v_x = v_0 = 1.30 \text{ m/s}$$
$$v_y = -gt = -(9.81 \text{ m/s}^2)(0.500 \text{ s}) = -4.91 \text{ m/s}$$

4. Use these components to determine $\vec{\mathbf{v}}$, v, and θ:

$$\vec{\mathbf{v}} = (1.30 \text{ m/s})\hat{\mathbf{x}} + (-4.91 \text{ m/s})\hat{\mathbf{y}}$$
$$v = \sqrt{v_x^2 + v_y^2}$$
$$= \sqrt{(1.30 \text{ m/s})^2 + (-4.91 \text{ m/s})^2} = 5.08 \text{ m/s}$$
$$\theta = \tan^{-1}\frac{v_y}{v_x} = \tan^{-1}\frac{(-4.91 \text{ m/s})}{1.30 \text{ m/s}} = -75.2°$$

INSIGHT

Note that the x position of the ball does not depend on the acceleration of gravity, g, and that its y position does not depend on the initial horizontal speed of the ball, v_0. For example, if the person is running when he drops the ball, the ball is moving faster in the horizontal direction, and it keeps up with the person when it is dropped. Its vertical motion doesn't change at all, however; it drops to the ground in exactly the same time and bounces back to the same height as before.

PRACTICE PROBLEM

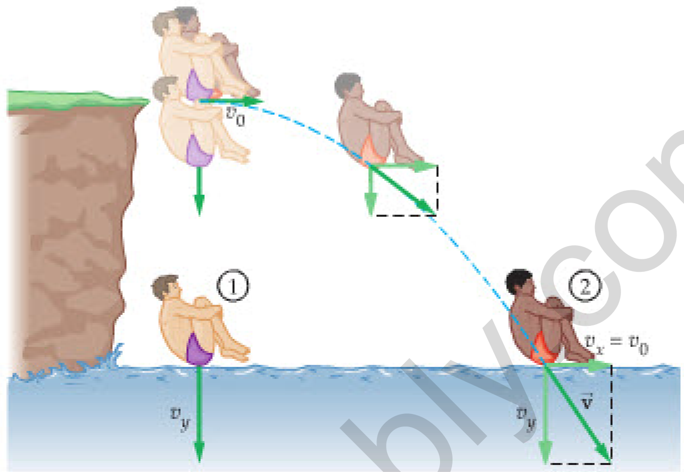
How long does it take for the ball to land? [**Answer:** Referring to the results of part (b), it is clear that the time of landing is slightly greater than 0.500 s. Setting $y = 0$ gives a precise answer; $t = \sqrt{2h/g} = 0.505$ s.]

Some related homework problems: Problem 15, Problem 16, Problem 20

www.aswarphysics.weebly.com

CONCEPTUAL CHECKPOINT 4–1 COMPARE SPLASHDOWN SPEEDS

Two youngsters dive off an overhang into a lake. Diver 1 drops straight down, diver 2 runs off the cliff with an initial horizontal speed v_0. Is the splashdown speed of diver 2 **(a)** greater than, **(b)** less than, or **(c)** equal to the splashdown speed of diver 1?



REASONING AND DISCUSSION

Note that neither diver has an initial y component of velocity, and that they both fall with the same vertical acceleration—the acceleration due to gravity. Therefore, the two divers fall for the same amount of time, and their y components of velocity are the same at splashdown. Since diver 2 also has a nonzero x component of velocity, unlike diver 1, the speed of diver 2 is greater.

ANSWER

(a) The speed of diver 2 is greater than that of diver 1.

REAL-WORLD PHYSICS

The parabolic trajectory of projectiles

▲ Lava bombs (top) and fountain jets (bottom) trace out parabolic paths, as is typical in projectile motion. The trajectories are only slightly altered by air resistance.

Parabolic Path

Just what is the shape of the curved path followed by a projectile launched horizontally? This can be found by combining $x = v_0 t$ and $y = h - \frac{1}{2}gt^2$, which allows us to express y in terms of x. First, solve for time using the x equation. This gives

$$t = x/v_0$$

Next, substitute this result into the y equation to eliminate t:

$$y = h - \tfrac{1}{2}g\left(\frac{x}{v_0}\right)^2 = h - \left(\frac{g}{2v_0{}^2}\right)x^2 \qquad 4\text{–}8$$

Note that y has the form

$$y = a + bx^2$$

where $a = h$ = constant and $b = -g/2v_0{}^2$ = constant. This is the equation of a parabola that curves downward, a characteristic shape in projectile motion.

Landing Site

Where does a projectile land if it is launched horizontally with a speed v_0 from a height h?

The most direct way to answer this question is to set $y = 0$ in Equation 4–8, since $y = 0$ corresponds to ground level. This gives

$$0 = h - \left(\frac{g}{2v_0{}^2}\right)x^2$$

Solving for x yields the landing site:

$$x = v_0\sqrt{\frac{2h}{g}} \qquad 4\text{–}9$$

Note that we have chosen the positive sign for the square root since the projectile was launched in the positive x direction, and hence lands at a positive value of x.

A useful alternative approach is to find the time of landing with the kinematic relations given in Equation 4–7, and then substitute this time into $x = v_0 t$. This approach is illustrated in the next Example.

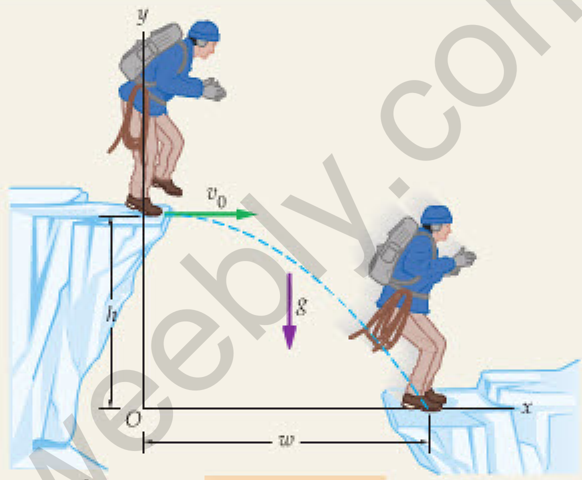
www.swarphysics.weebly.com

EXAMPLE 4–4 JUMPING A CREVASSE

A mountain climber encounters a crevasse in an ice field. The opposite side of the crevasse is 2.75 m lower, and is separated horizontally by a distance of 4.10 m. To cross the crevasse, the climber gets a running start and jumps in the horizontal direction. **(a)** What is the minimum speed needed by the climber to safely cross the crevasse? If, instead, the climber's speed is 6.00 m/s, **(b)** where does the climber land, and **(c)** what is the climber's speed on landing?

PICTURE THE PROBLEM

The mountain climber jumps from $x_0 = 0$ and $y_0 = h = 2.75$ m. The landing site for part (a) is $x = w = 4.10$ m and $y = 0$. Note that the y position of the climber *decreases* by h, and therefore $\Delta y = -h = -2.75$ m. As for the initial velocity, we are given that $v_{0x} = v_0$ and $v_{0y} = 0$. Finally, with our choice of coordinates it follows that $a_x = 0$ and $a_y = -g$.



STRATEGY

We can model the climber as a projectile, and apply our equations for projectile motion with a horizontal launch.

a. From Equations 4–7 we have that $x = v_0 t$ and $y = h - \frac{1}{2}gt^2$. Setting $y = 0$ determines the time of landing. Using this time in the x equation gives the horizontal landing position in terms of the initial speed.

b. We can now use the relation from part (a) to find x in terms of $v_0 = 6.00$ m/s.

c. We already know v_x, since it remains constant, and we can calculate v_y using $v_y^2 = -2g\Delta y$ (Equations 4–7). With the velocity components known, we can use the Pythagorean theorem to find the speed.

SOLUTION

Part (a)

1. Set $y = h - \frac{1}{2}gt^2$ equal to zero (landing condition) and solve for the corresponding time t:

$$y = h - \tfrac{1}{2}gt^2 = 0$$
$$t = \sqrt{\frac{2h}{g}}$$

2. Substitute this expression for t into the x equation of motion, $x = v_0 t$, and solve for the speed, v_0:

$$x = v_0 t = v_0\sqrt{\frac{2h}{g}} \quad \text{or} \quad v_0 = x\sqrt{\frac{g}{2h}}$$

3. Substitute numerical values in this expression:

$$v_0 = x\sqrt{\frac{g}{2h}} = (4.10\ \text{m})\sqrt{\frac{9.81\ \text{m/s}^2}{2(2.75\ \text{m})}} = 5.48\ \text{m/s}$$

Part (b)

4. Substitute $v_0 = 6.00$ m/s into the expression for x obtained in Step 2, $x = v_0\sqrt{2h/g}$:

$$x = v_0\sqrt{\frac{2h}{g}} = (6.00\ \text{m/s})\sqrt{\frac{2(2.75\ \text{m})}{9.81\ \text{m/s}^2}} = 4.49\ \text{m}$$

Part (c)

5. Use the fact that the x component of velocity does not change to determine v_x, and use $v_y^2 = -2g\Delta y$ to determine v_y. For v_y, note that we choose the minus sign for the square root because the climber is moving downward:

$$v_x = v_0 = 6.00\ \text{m/s}$$
$$v_y = \pm\sqrt{-2g\Delta y}$$
$$= -\sqrt{-2(9.81\ \text{m/s}^2)(-2.75\ \text{m})} = -7.35\ \text{m/s}$$

6. Use the Pythagorean theorem to determine the speed:

$$v = \sqrt{v_x^2 + v_y^2}$$
$$= \sqrt{(6.00\ \text{m/s})^2 + (-7.35\ \text{m/s})^2} = 9.49\ \text{m/s}$$

INSIGHT

The minimum speed needed to safely cross the crevasse is 5.48 m/s. If the initial horizontal speed is 6.00 m/s, the climber will land 4.49 m − 4.10 m = 0.39 m beyond the edge of the crevasse with a speed of 9.49 m/s.

PRACTICE PROBLEM

(a) When the climber's speed is the minimum needed to cross the crevasse, $v_0 = 5.48$ m/s, how long is the climber in the air? **(b)** How long is the climber in the air when $v_0 = 6.00$ m/s? [**Answer: (a)** $t = x/v_0 = (4.10\ \text{m})/(5.48\ \text{m/s}) = 0.748$ s. **(b)** $t = x/v_0 = (4.49\ \text{m})/(6.00\ \text{m/s}) = 0.748$ s. The times are the same! The answer to both parts is simply the time needed to fall through a height h; $t = \sqrt{2h/g} = 0.748$ s.]

Some related homework problems: Problem 11, Problem 12, Problem 17

www.aswarphysics.weebly.com

PROBLEM-SOLVING NOTE

Use Independence of Motion

Projectile problems can be solved by breaking the problem into its x and y components, and then solving for the motion of each component separately.

CONCEPTUAL CHECKPOINT 4–2 MINIMUM SPEED

If the height h is increased in the previous example but the width w remains the same, does the minimum speed needed to cross the crevasse **(a)** increase, **(b)** decrease, or **(c)** stay the same?

REASONING AND DISCUSSION

If the height is greater, the time of fall is also greater. Since the climber is in the air for a greater time, the horizontal distance covered for a given initial speed is also greater. Thus, if the width of the crevasse is the same, a lower initial speed allows for a safe crossing.

ANSWER

(b) The minimum speed decreases.

4–4 General Launch Angle

We now consider the more general case of a projectile launched at an arbitrary angle with respect to the horizontal. This means we can no longer use the simplifications associated with zero launch angle. As always, we return to our basic equations for projectile motion (Equations 4–6), and this time we simply let θ be nonzero.

Figure 4–6 (a) shows a projectile launched with an initial speed v_0 at an angle θ above the horizontal. Since the projectile starts at the origin, the initial x and y positions are zero:

$$x_0 = y_0 = 0$$

The components of the initial velocity are determined as indicated in Figure 4–6 (b):

$$v_{0x} = v_0 \cos\theta$$

and

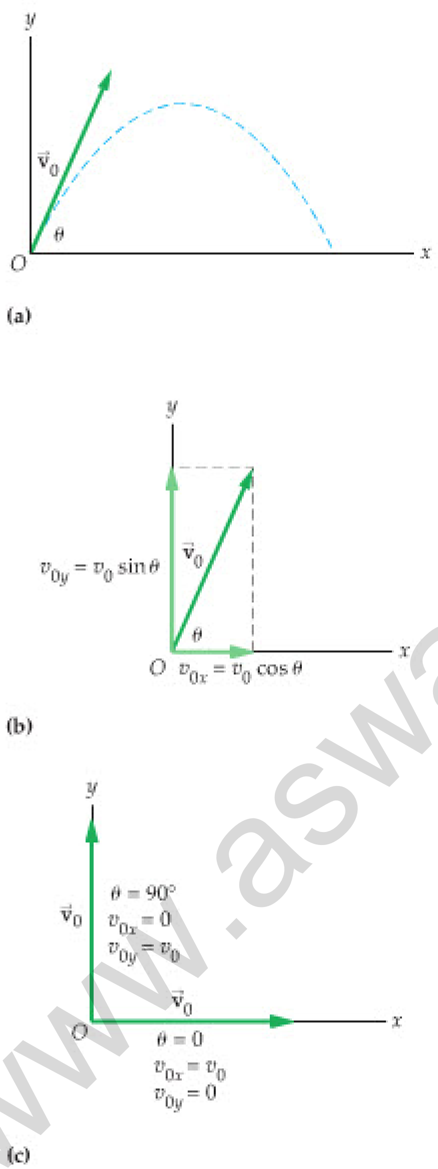
$$v_{0y} = v_0 \sin\theta$$

As a quick check, note that if $\theta = 0$, then $v_{0x} = v_0$ and $v_{0y} = 0$. Similarly, if $\theta = 90°$ we find $v_{0x} = 0$ and $v_{0y} = v_0$. These checks are depicted in Figure 4–6 (c).

Substituting these results into the basic equations for projectile motion yields the following results for a general launch angle:

$$x = (v_0\cos\theta)t \qquad v_x = v_0\cos\theta \qquad v_x^2 = v_0^2\cos^2\theta$$
$$y = (v_0\sin\theta)t - \tfrac{1}{2}gt^2 \qquad v_y = v_0\sin\theta - gt \qquad v_y^2 = v_0^2\sin^2\theta - 2g\Delta y \qquad \text{4–10}$$

Note that these equations, which are valid for any launch angle, reduce to the simpler Equations 4–7 when we set $\theta = 0$ and $y_0 = h$. In the next two Exercises, we use Equations 4–10 to calculate a projectile's position and velocity for three equally spaced times.



▲ **FIGURE 4–6 Projectile with an arbitrary launch angle**

(a) A projectile launched from the origin at an angle θ above the horizontal. **(b)** The x and y components of the initial velocity. **(c)** Velocity components in the limits $\theta = 0$ and $\theta = 90°$.

EXERCISE 4–1

A projectile is launched from the origin with an initial speed of 20.0 m/s at an angle of 35.0° above the horizontal. Find the x and y positions of the projectile at times **(a)** $t = 0.500$ s, **(b)** $t = 1.00$ s, and **(c)** $t = 1.50$ s.

SOLUTION

a. $x = 8.19$ m, $y = 4.51$ m,

b. $x = 16.4$ m, $y = 6.57$ m,

c. $x = 24.6$ m, $y = 6.17$ m. Note that x increases steadily; y increases, then decreases.

www.aswarphysics.weebly.com

EXERCISE 4–2

Referring to Exercise 4–1, find the velocity of the projectile at times **(a)** $t = 0.500$ s, **(b)** $t = 1.00$ s, and **(c)** $t = 1.50$ s.

SOLUTION

a. $\vec{v} = (16.4 \text{ m/s})\hat{x} + (6.57 \text{ m/s})\hat{y}$,

b. $\vec{v} = (16.4 \text{ m/s})\hat{x} + (1.66 \text{ m/s})\hat{y}$,

c. $\vec{v} = (16.4 \text{ m/s})\hat{x} + (-3.24 \text{ m/s})\hat{y}$.

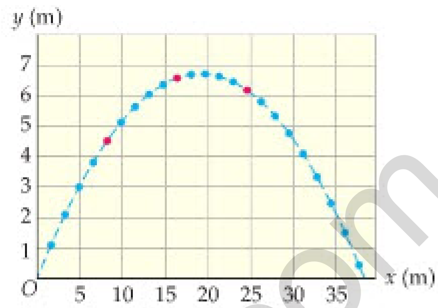
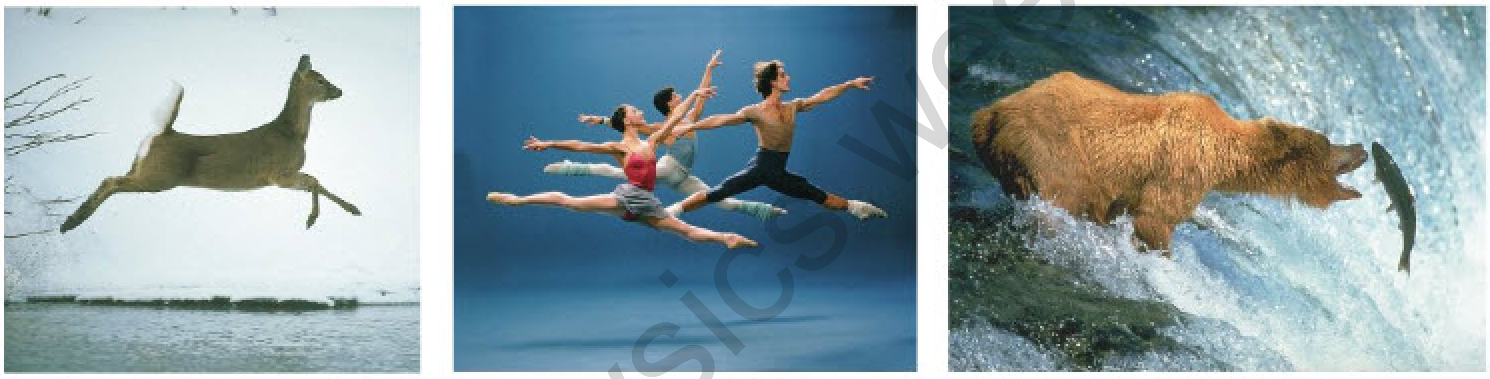


FIGURE 4–7 Snapshots of a trajectory This plot shows a projectile launched from the origin with an initial speed of 20.0 m/s at an angle of 35.0° above the horizontal. The positions shown in the plot correspond to the times $t = 0.1$ s, 0.2 s, 0.3 s, ... Red dots mark the positions considered in Exercises 4–1 and 4–2.

Figure 4–7 shows the projectile referred to in the previous Exercises for a series of times spaced by 0.10 s. Note that the points in Figure 4–7 are not evenly spaced in terms of position, even though they are evenly spaced in time. In fact, the points bunch closer together at the top of the trajectory, showing that a comparatively large fraction of the flight time is spent near the highest point. This is why it seems that a basketball player soaring toward a slam dunk, or a ballerina performing a grand jeté, is "hanging" in air.



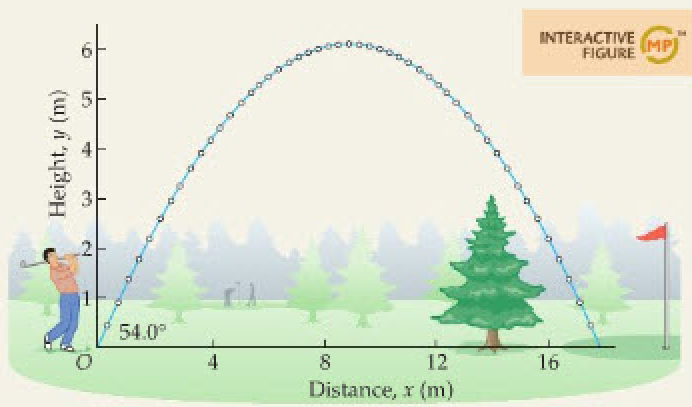
"Hanging" in air near the peak of a jump requires no special knack—in fact, it's an unavoidable consequence of the laws of physics. This phenomenon, which makes big leapers (such as deer and dancers) look particularly graceful, can also make life more dangerous for salmon fighting their way upstream to spawn.

EXAMPLE 4–5 A ROUGH SHOT

Chipping from the rough, a golfer sends the ball over a 3.00-m-high tree that is 14.0 m away. The ball lands at the same level from which it was struck after traveling a horizontal distance of 17.8 m—on the green, of course. **(a)** If the ball left the club 54.0° above the horizontal and landed on the green 2.24 s later, what was its initial speed? **(b)** How high was the ball when it passed over the tree?

PICTURE THE PROBLEM

Our sketch shows the ball taking flight from the origin, $x_0 = y_0 = 0$, with a launch angle of 54.0°, and arcing over the tree. The individual points along the parabolic trajectory correspond to equal time intervals.



STRATEGY

a. Since the projectile moves with constant speed in the x direction, the x component of velocity is simply horizontal distance divided by time. Knowing v_x and θ, we can find v_0 from $v_x = v_0 \cos\theta$.

b. We can use $x = (v_0 \cos\theta)t$ to find the time when the ball is at $x = 14.0$ m. Substituting this time into $y = (v_0 \sin\theta)t - \frac{1}{2}gt^2$ gives the height.

SOLUTION

Part (a)

1. Divide the horizontal distance, d, by the time of flight, t, to obtain v_x:

$$v_x = \frac{d}{t} = \frac{17.8 \text{ m}}{2.24 \text{ s}} = 7.95 \text{ m/s}$$

CONTINUED ON NEXT PAGE

www.aswarphysics.weebly.com

CONTINUED FROM PREVIOUS PAGE

2. Use $v_x = v_0 \cos\theta$ to find v_0, the initial speed: $v_x = v_0 \cos\theta$ or $v_0 = \frac{v_x}{\cos\theta} = \frac{7.95 \text{ m/s}}{\cos 54.0°} = 13.5 \text{ m/s}$

Part (b)

3. Use $x = (v_0 \cos\theta)t$ to find the time when $x = 14.0$ m. Recall that $x_0 = 0$: $x = (v_0 \cos\theta)t$ or $t = \frac{x}{v_0 \cos\theta} = \frac{14.0 \text{ m}}{7.95 \text{ m/s}} = 1.76 \text{ s}$

4. Evaluate $y = (v_0 \sin\theta)t - \frac{1}{2}gt^2$ at the time found in Step 3. Recall that $y_0 = 0$:

$$y = (v_0 \sin\theta)t - \tfrac{1}{2}gt^2$$
$$= [(13.5 \text{ m/s}) \sin 54.0°](1.76 \text{ s}) - \tfrac{1}{2}(9.81 \text{ m/s}^2)(1.76 \text{ s})^2$$
$$= 4.03 \text{ m}$$

INSIGHT

The ball clears the top of the tree by 1.03 m and lands on the green 0.48 s later. When it lands, its speed (in the absence of air resistance) is again 13.5 m/s—the same as when it was launched. This result will be verified in the next section.

PRACTICE PROBLEM

What are the speed and direction of the ball when it passes over the tree? [**Answer:** To find the ball's speed and direction, note that $v_x = 7.95$ m/s and $v_y = v_0 \sin\theta - gt = -6.34$ m/s. It follows that $v = \sqrt{v_x^2 + v_y^2} = 10.2$ m/s and $\theta = \tan^{-1}(v_y/v_x) = -38.6°$.]

Some related homework problems: Problem 31, Problem 39

ACTIVE EXAMPLE 4–1 AN ELEVATED GREEN

A golfer hits a ball from the origin with an initial speed of 30.0 m/s at an angle of 50.0° above the horizontal. The ball lands on a green that is 5.00 m above the level where the ball was struck.

a. How long is the ball in the air?

b. How far has the ball traveled in the horizontal direction when it lands?

c. What are the speed and direction of motion of the ball just before it lands?

SOLUTION *(Test your understanding by performing the calculations indicated in each step.)*

Part (a)

1. Let $y = (v_0 \sin\theta)t - \frac{1}{2}gt^2 = 5.00$ m and solve for t: $t = 0.229$ s, 4.46 s

2. When $t = 0.229$ s, the ball is moving upward; when $t = 4.46$ s, the ball is on the way down. Choose the later time: $t = 4.46$ s

Part (b)

3. Substitute $t = 4.46$ s into $x = (v_0 \cos\theta)t$: $x = 86.0$ m

Part (c)

4. Use $v_x = v_0 \cos\theta$ to calculate v_x: $v_x = 19.3$ m/s

5. Substitute $t = 4.46$ s into $v_y = v_0 \sin\theta - gt$ to find v_y: $v_y = -20.8$ m/s

6. Calculate v and θ: $v = 28.4$ m/s, $\theta = -47.1°$

YOUR TURN

How long is the ball in the air if the green is 5.00 m *below* the level where the ball was struck?

(Answers to **Your Turn** *problems are given in the back of the book.)*

The next Example presents a classic situation in which two projectiles collide. One projectile is launched from the origin, and thus its equations of motion are given by Equations 4–10. The second projectile is simply dropped from a height, which is a special case of the equations of motion in Equations 4–7 with $v_0 = 0$.

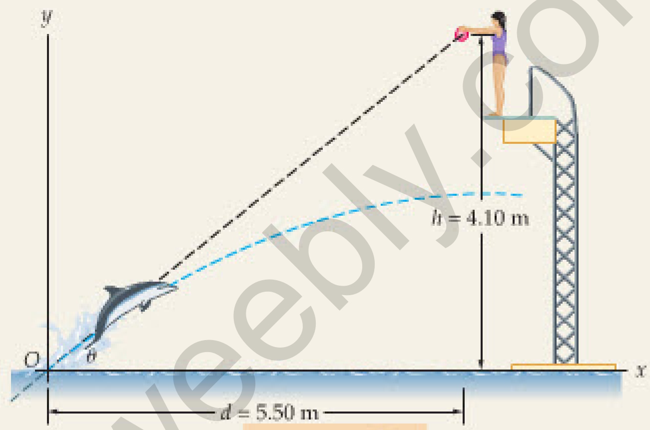
www.aswarphysics.weebly.com

EXAMPLE 4–6 A LEAP OF FAITH

A trained dolphin leaps from the water with an initial speed of 12.0 m/s. It jumps directly toward a ball held by the trainer a horizontal distance of 5.50 m away and a vertical distance of 4.10 m above the water. In the absence of gravity the dolphin would move in a straight line to the ball and catch it, but because of gravity the dolphin follows a parabolic path well below the ball's initial position, as shown in the sketch. If the trainer releases the ball the instant the dolphin leaves the water, show that the dolphin and the falling ball meet.

PICTURE THE PROBLEM

In our sketch we have the dolphin leaping from the water at the origin $x_0 = y_0 = 0$ with an angle above the horizontal given by $\theta = \tan^{-1}(h/d)$. The initial position of the ball is $x_0 = d = 5.50$ m and $y_0 = h = 4.10$ m, and its initial velocity is zero. The ball drops straight down with the acceleration of gravity, $a_y = -g$.



STRATEGY

We want to show that when the dolphin is at $x = d$, its height above the water is the same as the height of the ball above the water. To do this we first find the time when the dolphin is at $x = d$, then calculate y for the dolphin at this time. Next, we calculate y of the ball at the same time and then check to see if they are equal.

Since the ball drops from rest from a height h, its y equation of motion is $y = h - \frac{1}{2}gt^2$, as in Equations 4–7 in Section 4–3.

SOLUTION

1. Calculate the angle at which the dolphin leaves the water:

$$\theta = \tan^{-1}\left(\frac{h}{d}\right) = \tan^{-1}\left(\frac{4.10\text{ m}}{5.50\text{ m}}\right) = 36.7°$$

2. Use this angle and the initial speed to find the time t when the x position of the dolphin, x_d, is equal to 5.50 m. The x equation of motion is $x_d = (v_0 \cos\theta)t$:

$$x_d = (v_0\cos\theta)t = [(12.0\text{ m/s})\cos 36.7°]t = (9.62\text{ m/s})t = 5.50\text{ m}$$

$$t = \frac{5.50\text{ m}}{9.62\text{ m/s}} = 0.572\text{ s}$$

3. Evaluate the y position of the dolphin, y_d, at $t = 0.572$ s. The y equation of motion is $y_d = (v_0 \sin\theta)t - \frac{1}{2}gt^2$:

$$y_d = (v_0\sin\theta)t - \tfrac{1}{2}gt^2$$
$$= [(12.0\text{ m/s})\sin 36.7°](0.572\text{ s}) - \tfrac{1}{2}(9.81\text{ m/s}^2)(0.572\text{ s})^2$$
$$= 4.10\text{ m} - 1.60\text{ m} = 2.50\text{ m}$$

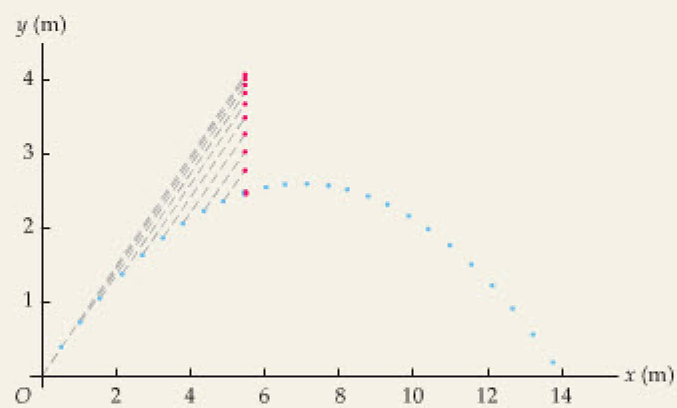
4. Finally, evaluate the y position of the ball, y_b, at $t = 0.572$ s. The ball's equation of motion is $y_b = h - \frac{1}{2}gt^2$:

$$y_b = h - \tfrac{1}{2}gt^2 = 4.10\text{ m} - \tfrac{1}{2}(9.81\text{ m/s}^2)(0.572\text{ s})^2$$
$$= 4.10\text{ m} - 1.60\text{ m} = 2.50\text{ m}$$

INSIGHT

In the absence of gravity, both the dolphin and the ball would be at $x = 5.50$ m and $y = 4.10$ m at $t = 0.572$ s. Because of gravity, however, the dolphin and the ball fall below their zero-gravity positions—and by the same amount, 1.60 m. In fact, from the point of view of the dolphin, the ball is always at the same angle of 36.7° above the horizontal until it is caught.

This is shown in the accompanying plot, where the red dots show the position of the ball at ten equally spaced times, and the blue dots show the position of the dolphin at the corresponding times. In addition, the dashed lines from the dolphin to the ball all make the same angle with the horizontal, 36.7°.

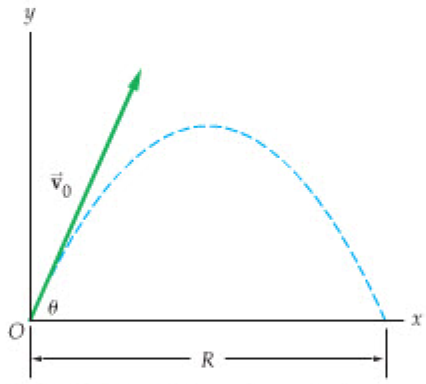


PRACTICE PROBLEM

At what height does the dolphin catch the ball if it leaves the water with an initial speed of 8.00 m/s? [**Answer:** $y_d = y_b = 0.493$ m. If the dolphin's initial speed is less than 7.50 m/s, it reenters the water before catching the ball.]

Some related homework problems: Problem 31, Problem 40

www.aswarphysics.weebly.com



▲ **FIGURE 4–8 Range of a projectile**
The range R of a projectile is the horizontal distance it travels between its takeoff and landing positions.

4–5 Projectile Motion: Key Characteristics

We conclude this chapter with a brief look at some additional characteristics of projectile motion that are both interesting and useful. In all cases our results follow as a direct consequence of the fundamental kinematic equations (Equations 4–10) describing projectile motion.

Range

The **range**, R, of a projectile is the horizontal distance it travels before landing. We consider the case shown in Figure 4–8, where the initial and final elevations are the same ($y = 0$). One way to obtain the range, then, is as follows: (i) Find the time when the projectile lands by setting $y = 0$ in the expression $y = (v_0 \sin\theta)t - \frac{1}{2}gt^2$; (ii) Substitute the time found in (i) into the x equation of motion.

Carrying out the first part of the calculation yields the following:

$$(v_0 \sin\theta)t - \tfrac{1}{2}gt^2 = 0 \quad \text{or} \quad (v_0 \sin\theta)t = \tfrac{1}{2}gt^2$$

Clearly, $t = 0$ is a solution to this equation—corresponding to the initial condition—but the solution we seek is a time that is greater than zero. We can find the desired time by dividing both sides of the equation by t. This gives

$$(v_0 \sin\theta) = \tfrac{1}{2}gt \quad \text{or} \quad t = \left(\frac{2v_0}{g}\right)\sin\theta \qquad \text{4–11}$$

This is the time when the projectile lands—also known as the time of flight.

Now, substitute this time into $x = (v_0 \cos\theta)t$ to find the value of x when the projectile lands:

$$x = (v_0 \cos\theta)t = (v_0 \cos\theta)\left(\frac{2v_0}{g}\right)\sin\theta = \left(\frac{2{v_0}^2}{g}\right)\sin\theta\cos\theta$$

This value of x is the range, R, thus

$$R = \left(\frac{2{v_0}^2}{g}\right)\sin\theta\cos\theta$$

Using the trigonometric identity $\sin 2\theta = 2\sin\theta\cos\theta$, as given in Appendix A, we can write this more compactly as follows:

$$R = \left(\frac{{v_0}^2}{g}\right)\sin 2\theta \quad \textit{(same initial and final elevation)} \qquad \text{4–12}$$

PROBLEM-SOLVING NOTE

Use the Same Math Regardless of the Initial Conditions

Once an object is launched, its trajectory follows the kinematic equations of motion, regardless of specific differences in the initial conditions. Thus, our equations of motion can be used to derive any desired characteristic of projectile motion, including range, symmetry, and maximum height.

ACTIVE EXAMPLE 4–2 FIND THE INITIAL SPEED

A football game begins with a kickoff in which the ball travels a horizontal distance of 45 yd and lands on the ground. If the ball was kicked at an angle of 40.0° above the horizontal, what was its initial speed?

SOLUTION *(Test your understanding by performing the calculations indicated in each step.)*

1. Solve Equation 4–12 for the initial speed v_0: $v_0 = \sqrt{gR/\sin 2\theta}$
2. Convert the range to meters: $R = 41$ m
3. Substitute numerical values: $v_0 = 20$ m/s

INSIGHT

Note that we choose the positive square root in Step 1 because we are interested only in the *speed* of the ball, which is always positive.

YOUR TURN

Suppose the initial speed of the ball is increased by 10%, to 22 m/s. By what percentage does the range increase?

(Answers to **Your Turn** *problems are given in the back of the book.)*

www.aswarphysics.weebly.com

Note that R depends inversely on the acceleration of gravity, g—thus the smaller g, the larger the range. For example, a projectile launched on the Moon, where the acceleration of gravity is only about 1/6 that on Earth, travels about six times as far as it would on Earth. It was for this reason that astronaut Alan Shepard simply couldn't resist the temptation of bringing a golf club and ball with him on the third lunar landing mission in 1971. He ambled out onto the Fra Mauro Highlands and became the first person to hit a tee shot on the Moon. His distance was undoubtedly respectable—unfortunately, his ball landed in a sand trap.

REAL-WORLD PHYSICS
Golf on the Moon

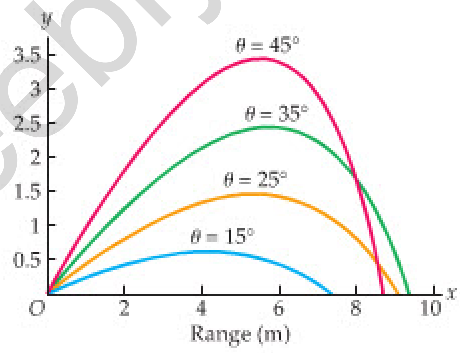
Now, what launch angle gives the greatest range? From Equation 4–12 we see that R varies with angle as $\sin 2\theta$; thus R is largest when $\sin 2\theta$ is largest—that is, when $\sin 2\theta = 1$. Since $\sin 90^\circ = 1$, it follows that $\theta = 45^\circ$ gives the maximum range. Thus

$$R_{\text{max}} = \frac{v_0^2}{g} \qquad \text{4–13}$$

As expected, the range (Equation 4–12) and maximum range (Equation 4–13) depend strongly on the initial speed of the projectile—they are both proportional to v_0^2.

Note that these results are specifically for the case where a projectile lands at the same level from which it was launched. If a projectile lands at a higher level, for example, the launch angle that gives maximum range is greater than 45°, and if it lands at a lower level, the angle for maximum range is less than 45°.

Finally, the range given here applies only to the ideal case of no air resistance. In cases where air resistance is significant, as in the flight of a rapidly moving golf ball, for example, the overall range of the ball is reduced. In addition, the maximum range occurs for a launch angle less than 45° (Figure 4–9). The reason is that with a smaller launch angle the golf ball is in the air for less time, giving air resistance less time to affect its flight.



▲ **FIGURE 4–9 Projectiles with air resistance**
Projectiles with the same initial speed but different launch angles showing the effects of air resistance. Notice that the maximum range occurs for a launch angle less than 45°, and that the projectiles return to the ground at a steeper angle than the launch angle.

Symmetry in Projectile Motion

There are many striking symmetries in projectile motion, beginning with the graceful symmetry of the parabola itself. As a first example, recall that earlier in this section, in Equation 4–11, we found the time when a projectile lands:

$$t = \left(\frac{2v_0}{g}\right)\sin\theta$$

Now, by symmetry, the time it takes a projectile to reach its highest point (in the absence of air resistance) should be just half this time. After all, the projectile moves in the x direction with constant speed, and the highest point—by symmetry—occurs at $x = \frac{1}{2}R$.

This all seems reasonable, but is there another way to check? Well, at the highest point the projectile is moving horizontally, thus its y component of velocity is zero. Let's find the time when $v_y = 0$ and compare with the time to land:

$$v_y = v_{0y} - gt = v_0 \sin\theta - gt = 0$$
$$t = \left(\frac{v_0}{g}\right)\sin\theta \qquad \text{4–14}$$

As expected from symmetry, the time at the highest point is one-half the time at landing.

▲ To be successful, a juggler must master the behavior of projectile motion. Physicist Richard Feynman shows that just knowing the appropriate equations is not enough; one must also practice. In this sense, learning to juggle is similar to learning to solve physics problems.

There is another interesting symmetry concerning speed. Recall that when a projectile is launched, its y component of velocity is $v_y = v_0 \sin\theta$. When the projectile lands, at time $t = (2v_0/g)\sin\theta$, its y component of velocity is

$$v_y = v_0\sin\theta - gt = v_0\sin\theta - g\left(\frac{2v_0}{g}\right)\sin\theta = -v_0\sin\theta$$

www.aswarphysics.weebly.com

This is exactly the opposite of the y component of the velocity when it was launched. Since the x component of velocity is always the same, it follows that when the projectile lands, its speed, $v = \sqrt{v_x^2 + v_y^2}$, is the same as when it was launched—as one might expect from symmetry.

The velocities are different, however, since the direction of motion is different at launch and landing. Even so, there is still a symmetry—the initial velocity is *above* the horizontal by the angle θ; the landing velocity is *below* the horizontal by the same angle θ.

So far, these results have referred to launching and landing, which both occur at $y = 0$. The same symmetry extends to any level, though. That is, at a given height the speed of a projectile is the same on the way up as on the way down. In addition, the angle of the velocity above the horizontal on the way up is the same as the angle below the horizontal on the way down. This is illustrated in **Figure 4–10** and in the next Conceptual Checkpoint.

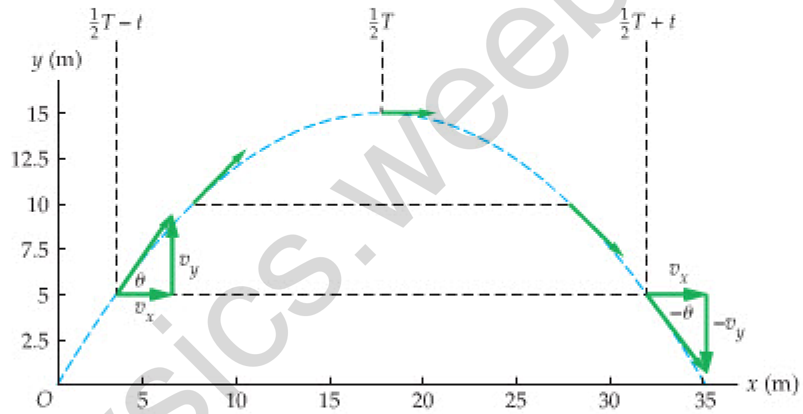
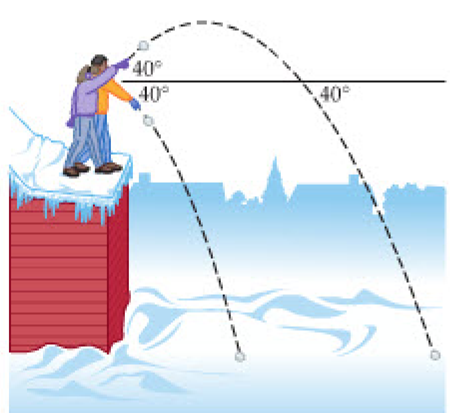


FIGURE 4–10 Velocity vectors for a projectile launched at the origin

At a given height the speed (length of velocity vector) is the same on the way up as on the way down. The direction of motion on the way up is above the horizontal by the same amount that it is below the horizontal on the way down. In this case, the total time of flight is T, and the greatest height is reached at the time $T/2$. Notice that the speed is the same at the time $(T/2) - t$ as it is at the time $(T/2) + t$.

CONCEPTUAL CHECKPOINT 4–3 COMPARE LANDING SPEEDS

You and a friend stand on a snow-covered roof. You both throw snowballs with the same initial speed, but in different directions. You throw your snowball downward, at 40° *below* the horizontal; your friend throws her snowball upward, at 40° *above* the horizontal. When the snowballs land on the ground, is the speed of your snowball **(a)** greater than, **(b)** less than, or **(c)** the same as the speed of your friend's snowball?



REASONING AND DISCUSSION

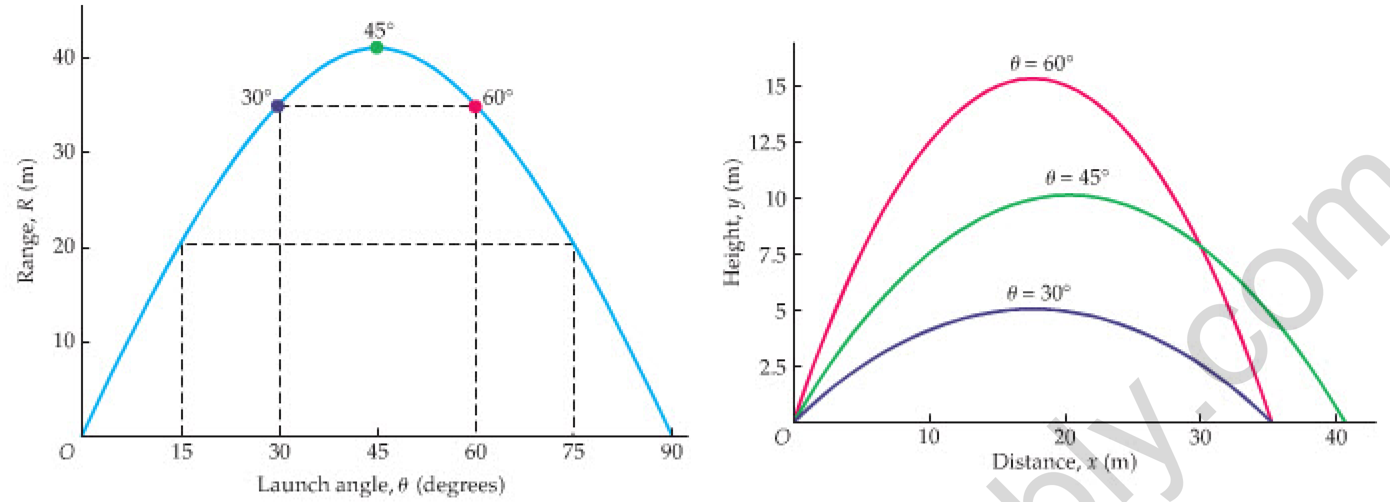
One consequence of symmetry in projectile motion is that when your friend's snowball returns to the level of the throw, its speed will be the same as the initial speed. In addition, it will be moving downward, at 40° below the horizontal. From that point on its motion is the same as that of your snowball; thus it lands with the same speed.

What if you throw your snowball horizontally? Or suppose you throw it straight down? In either case, the final speed is unchanged! In fact, for a given initial speed, the speed on landing simply doesn't depend on the direction in which you throw the ball. This is shown in Homework Problems 35 and 76. We return to this point in Chapter 8 when we discuss potential energy and energy conservation.

ANSWER

(c) The snowballs have the same speed.

www.aswarphysics.weebly.com



(a) Launch angles that are greater or less than 45° by the same amount give the same range.

(b) Projectiles with $\theta = 30°$ and $\theta = 60°$ follow different paths but have the same range.

▲ **FIGURE 4–11 Range and launch angle in the absence of air resistance**

(a) A plot of range versus launch angle for a projectile launched with an initial speed of 20 m/s. Note that the maximum range occurs at $\theta = 45°$. Launch angles equally greater than or less than 45°, such as 30° and 60°, give the same range. **(b)** Trajectories of projectiles with initial speeds of 20 m/s and launch angles of 60°, 45°, and 30°. The projectiles with launch angles of 30° and 60° land at the same location.

As our final example of symmetry, consider the range R. A plot of R versus launch angle θ is shown in **Figure 4–11 (a)** for $v_0 = 20$ m/s. Note that in the absence of air resistance, R is greatest at $\theta = 45°$, as pointed out previously. In addition, we can see from the figure that the range for angles equally above or below 45° is the same. For example, if air resistance is negligible, the range for $\theta = 30°$ is the same as the range for $\theta = 60°$, as we can see in both parts (a) and (b) of Figure 4–11.

Symmetries such as these are just some of the many reasons why physicists find physics to be "beautiful" and "aesthetically pleasing." Discovering such patterns and symmetries in nature is really what physics is all about. A physicist does not consider the beauty of projectile motion to be diminished by analyzing it in detail. Just the opposite—detailed analysis reveals deeper, more subtle, and sometimes unexpected levels of beauty.

▲ An archerfish would have trouble procuring its lunch without an instinctive grasp of projectile motion.

Maximum Height

Let's follow up on an observation made earlier in this section, namely, that a projectile is at maximum height when its y component of velocity is zero. In fact, we will use this observation to determine the maximum height of an arbitrary projectile. This can be accomplished with the following two-step calculation: (i) Find the time when $v_y = 0$; (ii) Substitute this time into the y-versus-t equation of motion, $y = (v_0 \sin \theta)t - \frac{1}{2}gt^2$. This calculation is carried out in the next Example.

EXAMPLE 4–7 WHAT A SHOT!

The archerfish hunts by dislodging an unsuspecting insect from its resting place with a stream of water expelled from the fish's mouth. Suppose the archerfish squirts water with an initial speed of 2.30 m/s at an angle of 19.5° above the horizontal. When the stream of water reaches a beetle on a leaf at height h above the water's surface, it is moving horizontally.

a. How much time does the beetle have to react?

b. What is the height h of the beetle?

c. What is the horizontal distance d between the fish and the beetle when the water is launched?

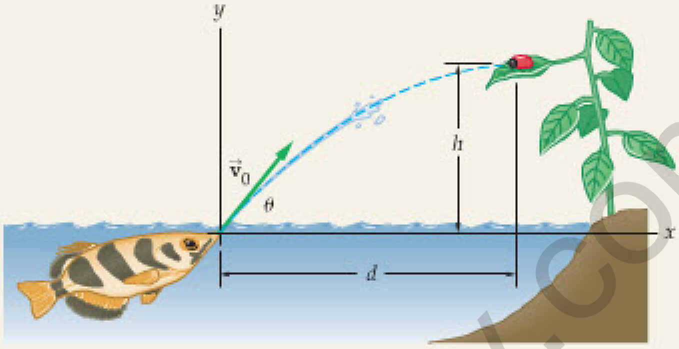
CONTINUED ON NEXT PAGE

www.aswarphysics.weebly.com

CONTINUED FROM PREVIOUS PAGE

PICTURE THE PROBLEM

Our sketch shows the fish squirting water from the origin, $x_0 = y_0 = 0$, and the beetle at $x = d, y = h$. The stream of water starts off with a speed $v_0 = 2.30$ m/s at an angle $\theta = 19.5°$ above the horizontal. Note that the water is moving horizontally when it reaches the beetle.



STRATEGY

a. Because the stream of water is moving horizontally when it reaches the beetle, it is at the top of its parabolic trajectory, as can be seen in Figure 4–10. This means that its y component of velocity is zero. Therefore, we can set $v_y = 0$ in $v_y = v_0 \sin\theta - gt$ and solve for the time t.

b. To find the maximum height of the stream of water, and of the beetle, we substitute the time found in part (a) into $y = (v_0 \sin\theta)t - \frac{1}{2}gt^2$.

c. Similarly, we can find the horizontal distance d by substituting the time from part (a) into $x = (v_0 \cos\theta)t$.

SOLUTION

Part (a)

1. Set $v_y = v_0 \sin\theta - gt$ equal to zero and solve for the corresponding time t:

$$v_y = v_{0y} - gt = v_0 \sin\theta - gt = 0$$
$$t = \frac{v_0 \sin\theta}{g}$$

2. Substitute numerical values to determine the reaction time:

$$t = \frac{v_0 \sin\theta}{g} = \frac{(2.30\ \text{m/s})\sin 19.5°}{9.81\ \text{m/s}^2} = 0.0783\ \text{s}$$

Part (b)

3. To calculate the height, we substitute $t = (v_0 \sin\theta)/g$ into $y = (v_0 \sin\theta)t - \frac{1}{2}gt^2$:

$$y = (v_0 \sin\theta)\left(\frac{v_0 \sin\theta}{g}\right) - \frac{1}{2}g\left(\frac{v_0 \sin\theta}{g}\right)^2 = \frac{(v_0 \sin\theta)^2}{2g}$$

4. Substitute numerical values to find the height h:

$$h = \frac{(v_0 \sin\theta)^2}{2g} = \frac{[(2.30\ \text{m/s})\sin 19.5°]^2}{2(9.81\ \text{m/s}^2)} = 0.0300\ \text{m}$$

Part (c)

5. We can find the horizontal distance d using x as a function of time, $x = (v_0 \cos\theta)t$:

$$x = (v_0 \cos\theta)t$$
$$d = [(2.30\ \text{m/s})\cos 19.5°](0.0783\ \text{s}) = 0.170\ \text{m}$$

INSIGHT

To hit the beetle, the fish aims 19.5° above the horizontal. For comparison, note that the straight-line angle to the beetle is $\tan^{-1}(0.0300/0.170) = 10.0°$. Therefore, the fish cannot aim directly at its prey if it wants a meal.

Finally, note that by working symbolically in Step 3 we have derived a general result for the maximum height of a projectile. In particular, we find $y_{max} = (v_0 \sin\theta)^2/2g$, a result that is valid for any launch speed and angle. As a check of our result, note that if we launch a projectile straight upward ($\theta = 90°$), the maximum height is $y_{max} = v_0{}^2/2g$. Comparing with the one-dimensional kinematics of Chapter 2, if an object is thrown straight upward with an initial speed v_0, and the object accelerates downward with the acceleration of gravity, $a = -g$, it comes to rest ($v = 0$) after covering a vertical distance Δy given by $0 = v_0{}^2 + 2(-g)\Delta y$. Solving for the distance yields $\Delta y = v_0{}^2/2g = y_{max}$. This is an example of the internal consistency that characterizes all of physics.

PRACTICE PROBLEM

How far does the stream of water go if it happens to miss the beetle? [**Answer:** By symmetry, the distance d is half the range. Thus the stream of water travels a distance $R = 2d = 0.340$ m.]

Some related homework problems: Problem 81, Problem 82

www.aswarphysics.weebly.com

THE BIG PICTURE PUTTING PHYSICS IN CONTEXT

LOOKING BACK

This chapter provides a number of opportunities to use the vector methods developed in Chapter 3. In Section 4–4, for example, we resolve a velocity vector into its x and y components, and then use the components in Equations 4–10.

The equations of one-dimensional kinematics derived in Chapter 2 are used again in this chapter, even though we are now studying kinematics in two dimensions. For example, the equations in Table 4–1 are the same as those used in Chapter 2, only now applied individually to the x and y directions.

LOOKING AHEAD

The basic idea behind projectile motion will be used again in Chapter 12, when we consider orbital motion. See, in particular, the illustration presented in Section 12–1.

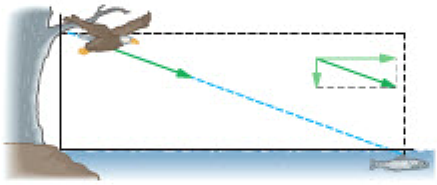
Two-dimensional kinematics comes up again when we study the motion of charged particles (like electrons) in electric fields. To see the connection, compare Figures 19–41 and 22–10 (a) with the person jumping a crevasse in Example 4–4. The same basic principles apply.

www.aswarphysics.weebly.com

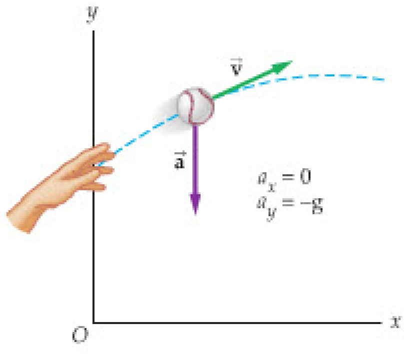
CHAPTER SUMMARY

4–1 MOTION IN TWO DIMENSIONS

Independence of Motion
Components of motion in the x and y directions can be treated independently of one another. Thus, two-dimensional motion with constant acceleration is described by the same kinematic equations derived in Chapter 2, only now written in terms of x and y components.



4–2 PROJECTILE MOTION: BASIC EQUATIONS



Projectile motion refers to the path of an object after it is thrown, kicked, batted, or otherwise launched into the air. For the ideal case, we assume no air resistance and a constant downward acceleration of magnitude g.

Acceleration Components
In projectile motion, with the x axis horizontal and the y axis upward, the components of the acceleration of gravity are

$$a_x = 0$$
$$a_y = -g$$

***x* and *y* as Functions of Time**
The x and y equations of motion are

$$x = x_0 + v_{0x}t \qquad \text{4–6}$$
$$y = y_0 + v_{0y}t - \tfrac{1}{2}gt^2$$

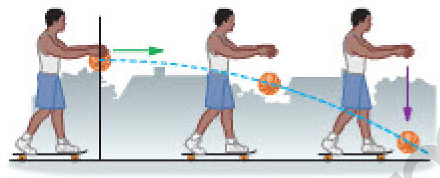
v_x and v_y as Functions of Time
The velocity components vary with time as follows:

$$v_x = v_{0x} \qquad \text{4–6}$$
$$v_y = v_{0y} - gt$$

v_x and v_y as Functions of Displacement
v_x and v_y vary with displacement as

$$v_x^2 = v_{0x}^2 \qquad \text{4–6}$$
$$v_y^2 = v_{0y}^2 - 2g\Delta y$$

4–3 ZERO LAUNCH ANGLE



Equations of Motion

A projectile launched horizontally from $x_0 = 0$, $y_0 = h$ with an initial speed v_0 has the following equations of motion:

$$x = v_0 t \qquad v_x = v_0 \qquad {v_x}^2 = {v_0}^2$$
$$y = h - \tfrac{1}{2}gt^2 \qquad v_y = -gt \qquad {v_y}^2 = -2g\Delta y \qquad \text{4–7}$$

Parabolic Path

The path followed by a projectile launched horizontally with an initial speed v_0 is described by

$$y = h - \left(\frac{g}{2{v_0}^2}\right)x^2 \qquad \text{4–8}$$

This path is a parabola.

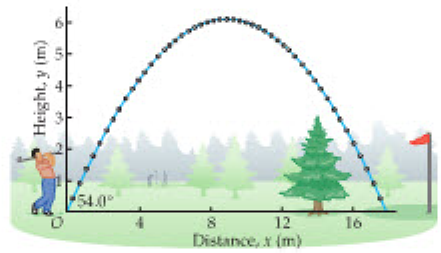
Landing Site

The landing site of a projectile launched horizontally is

$$x = v_0\sqrt{\frac{2h}{g}} \qquad \text{4–9}$$

In this expression, v_0 is the initial speed and h is the initial height. Note that this result is simply the speed in the x direction multiplied by the time of fall.

4–4 GENERAL LAUNCH ANGLE

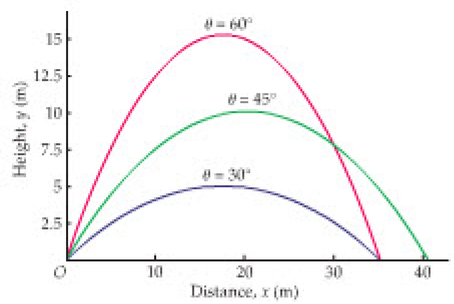


Launch from the Origin

The equations of motion for a launch from the origin with an initial speed v_0 at an angle of θ with respect to the horizontal are

$$x = (v_0 \cos\theta)t \qquad v_x = v_0\cos\theta \qquad {v_x}^2 = {v_0}^2\cos^2\theta$$
$$y = (v_0\sin\theta)t - \tfrac{1}{2}gt^2 \qquad v_y = v_0\sin\theta - gt \qquad {v_y}^2 = {v_0}^2\sin^2\theta - 2g\Delta y \qquad \text{4–10}$$

4–5 PROJECTILE MOTION: KEY CHARACTERISTICS



Range

The range of a projectile launched from the origin with an initial speed v_0 and a launch angle θ is

$$R = \left(\frac{{v_0}^2}{g}\right)\sin 2\theta \qquad \text{4–12}$$

This expression applies only to projectiles that land at the same level from which they were launched.

Symmetry

Projectile motion exhibits many symmetries. For example, the speed of a projectile depends only on its height and not on whether it is moving upward or downward.

Maximum Height

The maximum height of a projectile above its launch site is

$$y_{max} = \frac{{v_0}^2\sin^2\theta}{2g}$$

In this equation, v_0 is the initial speed and θ is the launch angle.

www.aswarphysics.weebly.com

PROBLEM-SOLVING SUMMARY

Type of Problem	Relevant Physical Concepts	Related Examples
Study two-dimensional motion with constant acceleration.	Motion in the x direction is independent of motion in the y direction. This is the basis for the equations of motion given in Table 4–1. Note that these equations are the same as the kinematic equations of Chapter 2, only written in terms of x and y components.	Examples 4–1, 4–2
Find the location and velocity of a projectile launched horizontally.	When a projectile is launched horizontally with a speed v_0 its initial velocity components are $v_{0x} = v_0$ and $v_{0y} = 0$. Make these substitutions in the equations of projectile motion given in Equations 4–6.	Examples 4–3, 4–4 Conceptual Checkpoints 4–1, 4–2
Find the location and velocity of a projectile launched with an arbitrary launch angle.	If a projectile is launched at an angle θ, its initial velocity components are $v_{0x} = v_0 \cos\theta$ and $v_{0y} = v_0 \sin\theta$. Make these substitutions in the equations of projectile motion given in Equations 4–6.	Examples 4–5, 4–6, 4–7 Active Examples 4–1, 4–2

CONCEPTUAL QUESTIONS

For instructor-assigned homework, go to www.masteringphysics.com

www.aswarphysics.weebly.com

(Answers to odd-numbered Conceptual Questions can be found in the back of the book.)

1. What is the acceleration of a projectile when it reaches its highest point? What is its acceleration just before and just after reaching this point?
2. A projectile is launched with an initial speed of v_0 at an angle θ above the horizontal. It lands at the same level from which it was launched. What was its average velocity between launch and landing? Explain.
3. A projectile is launched from level ground. When it lands, its direction of motion has rotated clockwise through 60°. What was the launch angle? Explain.
4. In a game of baseball, a player hits a high fly ball to the outfield. **(a)** Is there a point during the flight of the ball where its velocity is parallel to its acceleration? **(b)** Is there a point where the ball's velocity is perpendicular to its acceleration? Explain in each case.
5. A projectile is launched with an initial velocity of $\vec{v} = (4\text{ m/s})\hat{x} + (3\text{ m/s})\hat{y}$. What is the velocity of the projectile when it reaches its highest point? Explain.
6. A projectile is launched from a level surface with an initial velocity of $\vec{v} = (2\text{ m/s})\hat{x} + (4\text{ m/s})\hat{y}$. What is the velocity of the projectile just before it lands? Explain.
7. Do projectiles for which air resistance is nonnegligible, such as a bullet fired from a rifle, have maximum range when the launch angle is greater than, less than, or equal to 45°? Explain.
8. Two projectiles are launched from the same point at the same angle above the horizontal. Projectile 1 reaches a maximum height twice that of projectile 2. What is the ratio of the initial speed of projectile 1 to the initial speed of projectile 2? Explain.
9. A child rides on a pony walking with constant velocity. The boy leans over to one side and a scoop of ice cream falls from his ice cream cone. Describe the path of the scoop of ice cream as seen by **(a)** the child and **(b)** his parents standing on the ground nearby.
10. Driving down the highway, you find yourself behind a heavily loaded tomato truck. You follow close behind the truck, keeping the same speed. Suddenly a tomato falls from the back of the truck. Will the tomato hit your car or land on the road, assuming you continue moving with the same speed and direction? Explain.
11. A projectile is launched from the origin of a coordinate system where the positive x axis points horizontally to the right and the positive y axis points vertically upward. What was the projectile's launch angle with respect to the x axis if, at its highest point, its direction of motion has rotated **(a)** clockwise through 50° or **(b)** counterclockwise through 30°? Explain.

PROBLEMS AND CONCEPTUAL EXERCISES

Note: Answers to odd-numbered Problems and Conceptual Exercises can be found in the back of the book. **IP** *denotes an integrated problem, with both conceptual and numerical parts;* **BIO** *identifies problems of biological or medical interest;* **CE** *indicates a conceptual exercise.* **Predict/Explain** *problems ask for two responses:* **(a)** *your prediction of a physical outcome, and* **(b)** *the best explanation among three provided. On all problems, red bullets (•, ••, •••) are used to indicate the level of difficulty.*

(Air resistance should be ignored in the problems for this chapter, unless specifically stated otherwise.)

SECTION 4–1 MOTION IN TWO DIMENSIONS

1. • **CE Predict/Explain** As you walk briskly down the street, you toss a small ball into the air. **(a)** If you want the ball to land in your hand when it comes back down, should you toss the ball straight upward, in a forward direction, or in a backward direction, relative to your body? **(b)** Choose the *best explanation* from among the following:
   - I. If the ball is thrown straight up you will leave it behind.
   - II. You have to throw the ball in the direction you are walking.
   - III. The ball moves in the forward direction with your walking speed at all times.

2. • A sailboat runs before the wind with a constant speed of 4.2 m/s in a direction 32° north of west. How far **(a)** west and **(b)** north has the sailboat traveled in 25 min?

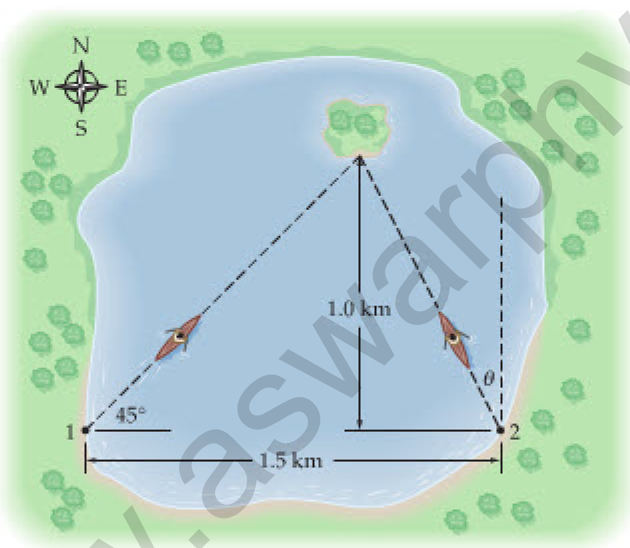
3. • As you walk to class with a constant speed of 1.75 m/s, you are moving in a direction that is 18.0° north of east. How much time does it take to change your displacement by **(a)** 20.0 m east or **(b)** 30.0 m north?

4. • Starting from rest, a car accelerates at 2.0 m/s^2 up a hill that is inclined 5.5° above the horizontal. How far **(a)** horizontally and **(b)** vertically has the car traveled in 12 s?

5. •• **IP** A particle passes through the origin with a velocity of $(6.2\ m/s)\hat{y}$. If the particle's acceleration is $(-4.4\ m/s^2)\hat{x}$, **(a)** what are its x and y positions after 5.0 s? **(b)** What are v_x and v_y at this time? **(c)** Does the speed of this particle increase with time, decrease with time, or increase and then decrease? Explain.

6. •• An electron in a cathode-ray tube is traveling horizontally at 2.10×10^9 cm/s when deflection plates give it an upward acceleration of 5.30×10^{17} cm/s^2. **(a)** How long does it take for the electron to cover a horizontal distance of 6.20 cm? **(b)** What is its vertical displacement during this time?

7. •• Two canoeists start paddling at the same time and head toward a small island in a lake, as shown in **Figure 4–12**. Canoeist 1 paddles with a speed of 1.35 m/s at an angle of 45° north of east. Canoeist 2 starts on the opposite shore of the lake, a distance of 1.5 km due east of canoeist 1. **(a)** In what direction relative to north must canoeist 2 paddle to reach the island? **(b)** What speed must canoeist 2 have if the two canoes are to arrive at the island at the same time?



▲ **FIGURE 4–12** Problem 7

SECTION 4–3 ZERO LAUNCH ANGLE

8. • **CE Predict/Explain** Two divers run horizontally off the edge of a low cliff. Diver 2 runs with twice the speed of diver 1. **(a)** When the divers hit the water, is the horizontal distance covered by diver 2 twice as much, four times as much, or equal to the horizontal distance covered by diver 1? **(b)** Choose the *best explanation* from among the following:

I. The drop time is the same for both divers.
II. Drop distance depends on t^2.
III. All divers in free fall cover the same distance.

9. • **CE Predict/Explain** Two youngsters dive off an overhang into a lake. Diver 1 drops straight down, and diver 2 runs off the cliff with an initial horizontal speed v_0. **(a)** Is the splashdown speed of diver 2 greater than, less than, or equal to the splashdown speed of diver 1? **(b)** Choose the *best explanation* from among the following:

I. Both divers are in free fall, and hence they will have the same splashdown speed.
II. The divers have the same vertical speed at splashdown, but diver 2 has the greater horizontal speed.
III. The diver who drops straight down gains more speed than the one who moves horizontally.

10. • An archer shoots an arrow horizontally at a target 15 m away. The arrow is aimed directly at the center of the target, but it hits 52 cm lower. What was the initial speed of the arrow?

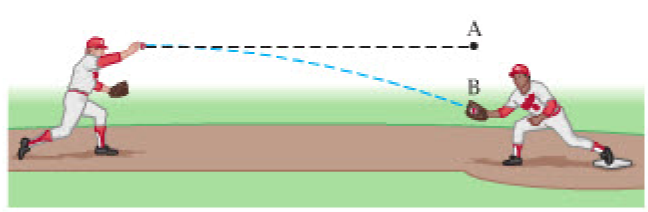
11. • **Victoria Falls** The great, gray-green, greasy Zambezi River flows over Victoria Falls in south central Africa. The falls are approximately 108 m high. If the river is flowing horizontally at 3.60 m/s just before going over the falls, what is the speed of the water when it hits the bottom? Assume the water is in free fall as it drops.

12. • A diver runs horizontally off the end of a diving board with an initial speed of 1.85 m/s. If the diving board is 3.00 m above the water, what is the diver's speed just before she enters the water?

13. • An astronaut on the planet Zircon tosses a rock horizontally with a speed of 6.95 m/s. The rock falls through a vertical distance of 1.40 m and lands a horizontal distance of 8.75 m from the astronaut. What is the acceleration of gravity on Zircon?

14. •• **IP Pitcher's Mounds** Pitcher's mounds are raised to compensate for the vertical drop of the ball as it travels a horizontal distance of 18 m to the catcher. **(a)** If a pitch is thrown horizontally with an initial speed of 32 m/s, how far does it drop by the time it reaches the catcher? **(b)** If the speed of the pitch is increased, does the drop distance increase, decrease, or stay the same? Explain. **(c)** If this baseball game were to be played on the Moon, would the drop distance increase, decrease, or stay the same? Explain.

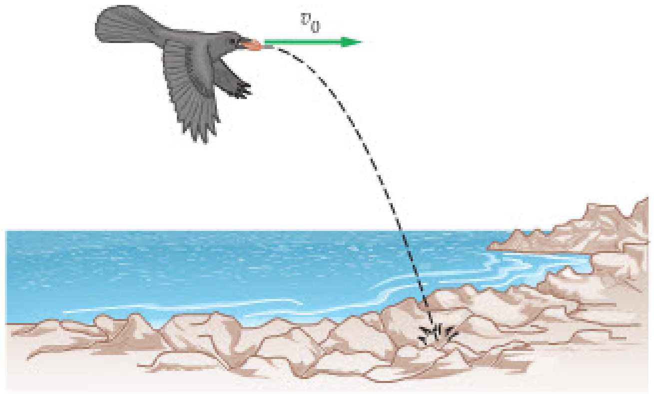
15. •• Playing shortstop, you pick up a ground ball and throw it to second base. The ball is thrown horizontally, with a speed of 22 m/s, directly toward point A **(Figure 4–13)**. When the ball reaches the second baseman 0.45 s later, it is caught at point B. **(a)** How far were you from the second baseman? **(b)** What is the distance of vertical drop, AB?



▲ **FIGURE 4–13** Problem 15

16. •• **IP** A crow is flying horizontally with a constant speed of 2.70 m/s when it releases a clam from its beak **(Figure 4–14)**. The clam lands on the rocky beach 2.10 s later. Just before the clam lands, what is **(a)** its horizontal component of velocity, and **(b)** its vertical component of velocity? **(c)** How would your answers to parts (a) and (b) change if the speed of the crow were increased? Explain.

www.aswarphysics.weebly.com

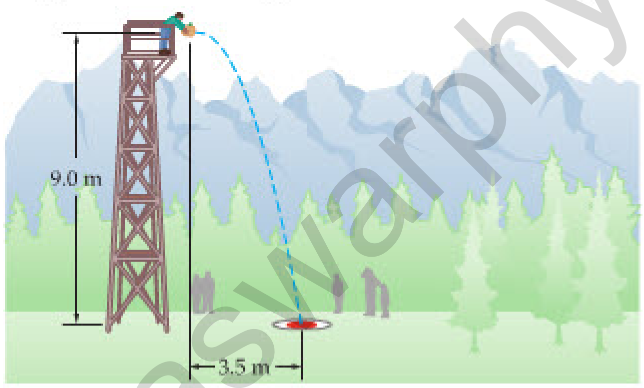


▲ **FIGURE 4–14** Problem 16

17. •• A mountain climber jumps a 2.8-m-wide crevasse by leaping horizontally with a speed of 7.8 m/s. **(a)** If the climber's direction of motion on landing is −45°, what is the height difference between the two sides of the crevasse? **(b)** Where does the climber land?

18. •• **IP** A white-crowned sparrow flying horizontally with a speed of 1.80 m/s folds its wings and begins to drop in free fall. **(a)** How far does the sparrow fall after traveling a horizontal distance of 0.500 m? **(b)** If the sparrow's initial speed is increased, does the distance of fall increase, decrease, or stay the same?

19. •• **Pumpkin Toss** In Denver, children bring their old jack-o-lanterns to the top of a tower and compete for accuracy in hitting a target on the ground (Figure 4–15). Suppose that the tower is 9.0 m high and that the bull's-eye is a horizontal distance of 3.5 m from the launch point. If the pumpkin is thrown horizontally, what is the launch speed needed to hit the bull's-eye?

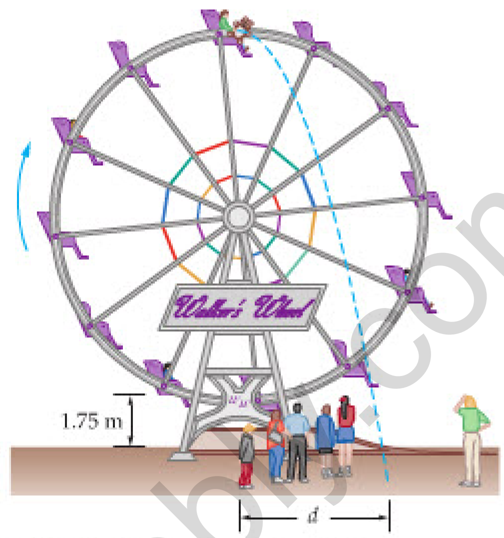


▲ **FIGURE 4–15** Problems 19 and 20

20. •• If, in the previous problem, a jack-o-lantern is given an initial horizontal speed of 3.3 m/s, what are the direction and magnitude of its velocity **(a)** 0.75 s later, and **(b)** just before it lands?

21. •• Fairgoers ride a Ferris wheel with a radius of 5.00 m (Figure 4–16). The wheel completes one revolution every 32.0 s. **(a)** What is the average speed of a rider on this Ferris wheel? **(b)** If a rider accidentally drops a stuffed animal at the top of the wheel, where does it land relative to the base of the ride? (Note: The bottom of the wheel is 1.75 m above the ground.)

22. •• **IP** A swimmer runs horizontally off a diving board with a speed of 3.32 m/s and hits the water a horizontal distance of 1.78 m from the end of the board. **(a)** How high above the water was the diving board? **(b)** If the swimmer runs off the board

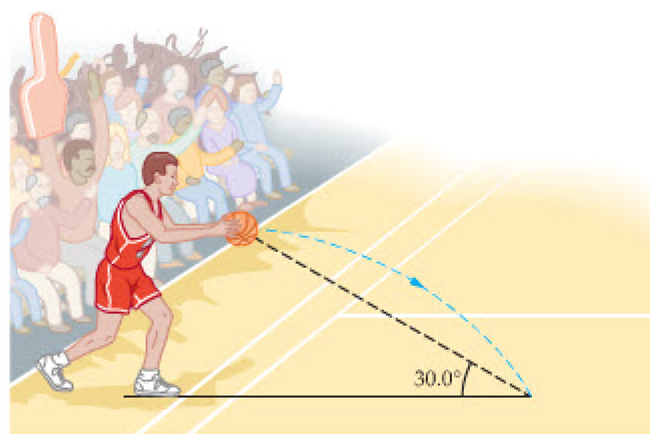


▲ **FIGURE 4–16** Problems 21 and 42

with a reduced speed, does it take more, less, or the same time to reach the water?

23. •• **Baseball and the Washington Monument** On August 25, 1894, Chicago catcher William Schriver caught a baseball thrown from the top of the Washington Monument (555 ft, 898 steps). **(a)** If the ball was thrown horizontally with a speed of 5.00 m/s, where did it land? **(b)** What were the ball's speed and direction of motion when caught?

24. ••• A basketball is thrown horizontally with an initial speed of 4.20 m/s (Figure 4–17). A straight line drawn from the release point to the landing point makes an angle of 30.0° with the horizontal. What was the release height?



▲ **FIGURE 4–17** Problem 24

25. ••• **IP** A ball rolls off a table and falls 0.75 m to the floor, landing with a speed of 4.0 m/s. **(a)** What is the acceleration of the ball just before it strikes the ground? **(b)** What was the initial speed of the ball? **(c)** What initial speed must the ball have if it is to land with a speed of 5.0 m/s?

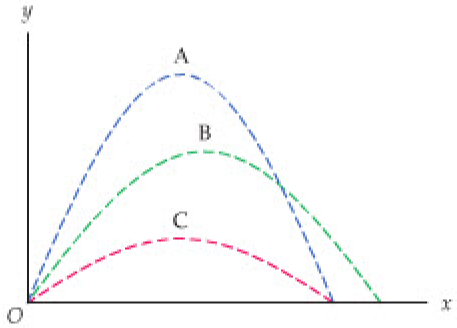
SECTION 4–4 GENERAL LAUNCH ANGLE

26. • **CE** A certain projectile is launched with an initial speed v_0. At its highest point its speed is $\frac{1}{2}v_0$. What was the launch angle of the projectile?

A. 30° **B.** 45° **C.** 60° **D.** 75°

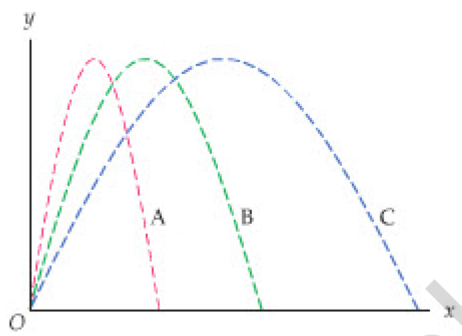
www.aswarphysics.weebly.com

27. • **CE** Three projectiles (A, B, and C) are launched with the same initial speed but with different launch angles, as shown in Figure 4–18. Rank the projectiles in order of increasing **(a)** horizontal component of initial velocity and **(b)** time of flight. Indicate ties where appropriate.



▲ **FIGURE 4–18** Problem 27

28. • **CE** Three projectiles (A, B, and C) are launched with different initial speeds so that they reach the same maximum height, as shown in Figure 4–19. Rank the projectiles in order of increasing **(a)** initial speed and **(b)** time of flight. Indicate ties where appropriate.



▲ **FIGURE 4–19** Problem 28

29. • A second baseman tosses the ball to the first baseman, who catches it at the same level from which it was thrown. The throw is made with an initial speed of 18.0 m/s at an angle of 37.5° above the horizontal. **(a)** What is the horizontal component of the ball's velocity just before it is caught? **(b)** How long is the ball in the air?

30. • Referring to the previous problem, what are the y component of the ball's velocity and its direction of motion just before it is caught?

31. • A cork shoots out of a champagne bottle at an angle of 35.0° above the horizontal. If the cork travels a horizontal distance of 1.30 m in 1.25 s, what was its initial speed?

32. • A soccer ball is kicked with a speed of 9.85 m/s at an angle of 35.0° above the horizontal. If the ball lands at the same level from which it was kicked, how long was it in the air?

33. •• In a game of basketball, a forward makes a bounce pass to the center. The ball is thrown with an initial speed of 4.3 m/s at an angle of 15° below the horizontal. It is released 0.80 m above the floor. What horizontal distance does the ball cover before bouncing?

34. •• Repeat the previous problem for a bounce pass in which the ball is thrown 15° *above* the horizontal.

35. •• **IP** Snowballs are thrown with a speed of 13 m/s from a roof 7.0 m above the ground. Snowball A is thrown straight downward; snowball B is thrown in a direction 25° above the horizontal. **(a)** Is the landing speed of snowball A greater than, less than, or the same as the landing speed of snowball B? Explain. **(b)** Verify your answer to part (a) by calculating the landing speed of both snowballs.

36. •• In the previous problem, find the direction of motion of the two snowballs just before they land.

37. •• A golfer gives a ball a maximum initial speed of 34.4 m/s. **(a)** What is the longest possible hole-in-one for this golfer? Neglect any distance the ball might roll on the green and assume that the tee and the green are at the same level. **(b)** What is the minimum speed of the ball during this hole-in-one shot?

38. •• What is the highest tree the ball in the previous problem could clear on its way to the longest possible hole-in-one?

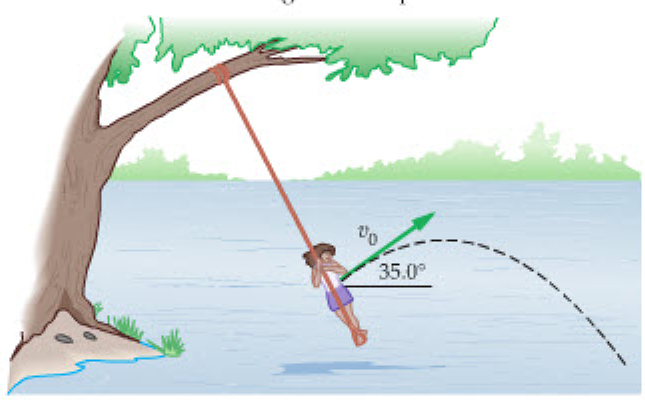
39. •• The "hang time" of a punt is measured to be 4.50 s. If the ball was kicked at an angle of 63.0° above the horizontal and was caught at the same level from which it was kicked, what was its initial speed?

40. •• In a friendly game of handball, you hit the ball essentially at ground level and send it toward the wall with a speed of 18 m/s at an angle of 32° above the horizontal. **(a)** How long does it take for the ball to reach the wall if it is 3.8 m away? **(b)** How high is the ball when it hits the wall?

41. •• **IP** In the previous problem, **(a)** what are the magnitude and direction of the ball's velocity when it strikes the wall? **(b)** Has the ball reached the highest point of its trajectory at this time? Explain.

42. •• A passenger on the Ferris wheel described in Problem 21 drops his keys when he is on the way up and at the 10 o'clock position. Where do the keys land relative to the base of the ride?

43. •• On a hot summer day, a young girl swings on a rope above the local swimming hole (Figure 4–20). When she lets go of the rope her initial velocity is 2.25 m/s at an angle of 35.0° above the horizontal. If she is in flight for 0.616 s, how high above the water was she when she let go of the rope?



▲ **FIGURE 4–20** Problem 43

44. •• A certain projectile is launched with an initial speed v_0. At its highest point its speed is $v_0/4$. What was the launch angle?

SECTION 4–5 PROJECTILE MOTION: KEY CHARACTERISTICS

45. • **Punkin Chunkin** In Sussex County, Delaware, a post-Halloween tradition is "Punkin Chunkin," in which contestants build cannons, catapults, trebuchets, and other devices to launch pumpkins and compete for the greatest distance. Though hard to believe, pumpkins have been projected a distance of 4086 feet in this contest. What is the minimum initial speed needed for such a shot?

www.aswarphysics.weebly.com

46. • A dolphin jumps with an initial velocity of 12.0 m/s at an angle of 40.0° above the horizontal. The dolphin passes through the center of a hoop before returning to the water. If the dolphin is moving horizontally when it goes through the hoop, how high above the water is the center of the hoop?

47. • A player passes a basketball to another player who catches it at the same level from which it was thrown. The initial speed of the ball is 7.1 m/s, and it travels a distance of 4.6 m. What were **(a)** the initial direction of the ball and **(b)** its time of flight?

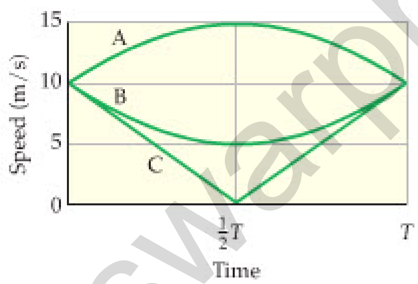
48. • A golf ball is struck with a five iron on level ground. It lands 92.2 m away 4.30 s later. What were **(a)** the direction and **(b)** the magnitude of the initial velocity?

49. • **A Record Toss** Babe Didrikson holds the world record for the longest baseball throw (296 ft) by a woman. For the following questions, assume that the ball was thrown at an angle of 45.0° above the horizontal, that it traveled a horizontal distance of 296 ft, and that it was caught at the same level from which it was thrown. **(a)** What was the ball's initial speed? **(b)** How long was the ball in the air?

50. • In the photograph to the left on page 87, suppose the cart that launches the ball is 11 cm high. Estimate **(a)** the launch speed of the ball and **(b)** the time interval between successive stroboscopic exposures.

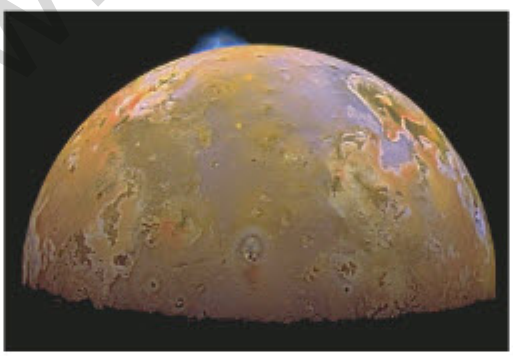
51. •• **CE Predict/Explain** You throw a ball into the air with an initial speed of 10 m/s at an angle of 60° above the horizontal. The ball returns to the level from which it was thrown in the time T. **(a)** Referring to Figure 4–21, which of the plots (A, B, or C) best represents the speed of the ball as a function of time? **(b)** Choose the *best explanation* from among the following:

I. Gravity causes the ball's speed to increase during its flight.
II. The ball has zero speed at its highest point.
III. The ball's speed decreases during its flight, but it doesn't go to zero.



▲ **FIGURE 4–21** Problem 51

52. •• **IP Volcanoes on Io** Astronomers have discovered several volcanoes on Io, a moon of Jupiter. One of them, named Loki, ejects lava to a maximum height of 2.00×10^5 m. **(a)** What is the initial speed of the lava? (The acceleration of gravity on Io is $1.80\ m/s^2$.) **(b)** If this volcano were on Earth, would the maximum height of the ejected lava be greater than, less than, or the same as on Io? Explain.



A volcano on Io, the innermost moon of Jupiter, displays the characteristic features of projectile motion. (Problem 52)

53. •• **IP** A soccer ball is kicked with an initial speed of 10.2 m/s in a direction 25.0° above the horizontal. Find the magnitude and direction of its velocity **(a)** 0.250 s and **(b)** 0.500 s after being kicked. **(c)** Is the ball at its greatest height before or after 0.500 s? Explain.

54. •• A second soccer ball is kicked with the same initial speed as in Problem 53. After 0.750 s it is at its highest point. What was its initial direction of motion?

55. •• **IP** A golfer tees off on level ground, giving the ball an initial speed of 46.5 m/s and an initial direction of 37.5° above the horizontal. **(a)** How far from the golfer does the ball land? **(b)** The next golfer in the group hits a ball with the same initial speed but at an angle above the horizontal that is greater than 45.0°. If the second ball travels the same horizontal distance as the first ball, what was its initial direction of motion? Explain.

56. •• **IP** One of the most popular events at Highland games is the hay toss, where competitors use a pitchfork to throw a bale of hay over a raised bar. Suppose the initial velocity of a bale of hay is $\vec{v} = (1.12\ m/s)\hat{x} + (8.85\ m/s)\hat{y}$. **(a)** After what minimum time is its speed equal to 5.00 m/s? **(b)** How long after the hay is tossed is it moving in a direction that is 45.0° below the horizontal? **(c)** If the bale of hay is tossed with the same initial speed, only this time straight upward, will its time in the air increase, decrease, or stay the same? Explain.

GENERAL PROBLEMS

57. • **CE** Child 1 throws a snowball horizontally from the top of a roof; child 2 throws a snowball straight down. Once in flight, is the acceleration of snowball 2 greater than, less than, or equal to the acceleration of snowball 1?

58. • **CE** The penguin to the left in the accompanying photo is about to land on an ice floe. Just before it lands, is its speed greater than, less than, or equal to its speed when it left the water?

This penguin behaves much like a projectile from the time it leaves the water until it touches down on the ice. (Problem 58)

59. • **CE Predict/Explain** A person flips a coin into the air and it lands on the ground a few feet away. **(a)** If the person were to perform an identical coin flip on an elevator rising with constant speed, would the coin's time of flight be greater than, less than, or equal to its time of flight when the person was at rest? **(b)** Choose the *best explanation* from among the following:

I. The floor of the elevator is moving upward, and hence it catches up with the coin in mid flight.

www.aswarphysics.weebly.com

II. The coin has the same upward speed as the elevator when it is tossed, and the elevator's speed doesn't change during the coin's flight.
III. The coin starts off with a greater upward speed because of the elevator, and hence it reaches a greater height.

60. • **CE Predict/Explain** Suppose the elevator in the previous problem is rising with a constant upward acceleration, rather than constant velocity. **(a)** In this case, would the coin's time of flight be greater than, less than, or equal to its time of flight when the person was at rest? **(b)** Choose the *best explanation* from among the following:
I. The coin has the same acceleration once it is tossed, whether the elevator accelerates or not.
II. The elevator's upward speed increases during the coin's flight, and hence it catches up with the coin at a greater height than before.
III. The coin's downward acceleration is less than before because the elevator's upward acceleration partially cancels it.

61. • A train moving with constant velocity travels 170 m north in 12 s and an undetermined distance to the west. The speed of the train is 32 m/s. **(a)** Find the direction of the train's motion relative to north. **(b)** How far west has the train traveled in this time?

62. • Referring to Example 4–2, find **(a)** the x component and **(b)** the y component of the hummingbird's velocity at the time $t = 0.72$ s. **(c)** What is the bird's direction of travel at this time, relative to the positive x axis?

63. • A racket ball is struck in such a way that it leaves the racket with a speed of 4.87 m/s in the horizontal direction. When the ball hits the court, it is a horizontal distance of 1.95 m from the racket. Find the height of the racket ball when it left the racket.

64. •• **IP** A hot-air balloon rises from the ground with a velocity of $(2.00\ \text{m/s})\hat{\mathbf{y}}$. A champagne bottle is opened to celebrate takeoff, expelling the cork horizontally with a velocity of $(5.00\ \text{m/s})\hat{\mathbf{x}}$ relative to the balloon. When opened, the bottle is 6.00 m above the ground. **(a)** What is the initial velocity of the cork, as seen by an observer on the ground? Give your answer in terms of the x and y unit vectors. **(b)** What are the speed of the cork and its initial direction of motion as seen by the same observer? **(c)** Determine the maximum height above the ground attained by the cork. **(d)** How long does the cork remain in the air?

65. •• Repeat the previous problem, this time assuming that the balloon is *descending* with a speed of 2.00 m/s.

66. •• **IP** A soccer ball is kicked from the ground with an initial speed of 14.0 m/s. After 0.275 s its speed is 12.9 m/s. **(a)** Give a strategy that will allow you to calculate the ball's initial direction of motion. **(b)** Use your strategy to find the initial direction.

67. •• A particle leaves the origin with an initial velocity $\vec{\mathbf{v}} = (2.40\ \text{m/s})\hat{\mathbf{x}}$, and moves with constant acceleration $\vec{\mathbf{a}} = (-1.90\ \text{m/s}^2)\hat{\mathbf{x}} + (3.20\ \text{m/s}^2)\hat{\mathbf{y}}$. **(a)** How far does the particle move in the x direction before turning around? **(b)** What is the particle's velocity at this time? **(c)** Plot the particle's position at $t = 0.500$ s, 1.00 s, 1.50 s, and 2.00 s. Use these results to sketch position versus time for the particle.

68. •• When the dried-up seed pod of a scotch broom plant bursts open, it shoots out a seed with an initial velocity of 2.62 m/s at an angle of 60.5° above the horizontal. If the seed pod is 0.455 m above the ground, **(a)** how long does it take for the seed to land? **(b)** What horizontal distance does it cover during its flight?

69. •• Referring to Problem 68, a second seed shoots out from the pod with the same speed but with a direction of motion 30.0° below the horizontal. **(a)** How long does it take for the second seed to land? **(b)** What horizontal distance does it cover during its flight?

70. •• A shot-putter throws the shot with an initial speed of 12.2 m/s from a height of 5.15 ft above the ground. What is the range of the shot if the launch angle is **(a)** 20.0°, **(b)** 30.0°, or **(c)** 40.0°?

71. •• **Pararescue Jumpers** Coast Guard pararescue jumpers are trained to leap from helicopters into the sea to save boaters in distress. The rescuers like to step off their helicopter when it is "ten and ten", which means that it is *ten* feet above the water and moving forward horizontally at *ten* knots. What are **(a)** the speed and **(b)** the direction of motion as a pararescuer enters the water following a ten and ten jump?

72. •• A ball thrown straight upward returns to its original level in 2.75 s. A second ball is thrown at an angle of 40.0° above the horizontal. What is the initial speed of the second ball if it also returns to its original level in 2.75 s?

73. •• **IP** To decide who pays for lunch, a passenger on a moving train tosses a coin straight upward with an initial speed of 4.38 m/s and catches it again when it returns to its initial level. From the point of view of the passenger, then, the coin's initial velocity is $(4.38\ \text{m/s})\hat{\mathbf{y}}$. The train's velocity relative to the ground is $(12.1\ \text{m/s})\hat{\mathbf{x}}$. **(a)** What is the minimum speed of the coin relative to the ground during its flight? At what point in the coin's flight does this minimum speed occur? Explain. **(b)** Find the initial speed and direction of the coin as seen by an observer on the ground. **(c)** Use the expression for y_{max} derived in Example 4–7 to calculate the maximum height of the coin, as seen by an observer on the ground. **(d)** Calculate the maximum height of the coin from the point of view of the passenger, who sees only one-dimensional motion.

74. •• **IP** A cannon is placed at the bottom of a cliff 61.5 m high. If the cannon is fired straight upward, the cannonball just reaches the top of the cliff. **(a)** What is the initial speed of the cannonball? **(b)** Suppose a second cannon is placed at the top of the cliff. This cannon is fired horizontally, giving its cannonballs the same initial speed found in part (a). Show that the range of this cannon is the same as the maximum range of the cannon at the base of the cliff. (Assume the ground at the base of the cliff is level, though the result is valid even if the ground is not level.)

75. •• **Shot Put Record** The men's world record for the shot put, 23.12 m, was set by Randy Barnes of the United States on May 20, 1990. If the shot was launched from 6.00 ft above the ground at an initial angle of 42.0°, what was its initial speed?

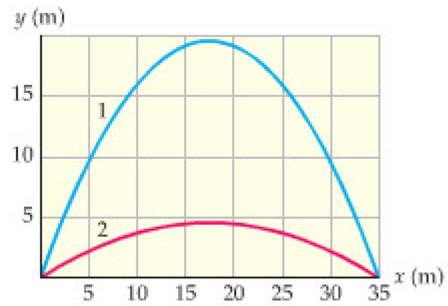
76. •• Referring to Conceptual Checkpoint 4–3, suppose the two snowballs are thrown from an elevation of 15 m with an initial speed of 12 m/s. What is the speed of each ball when it is 5.0 m above the ground?

77. •• **IP** A hockey puck just clears the 2.00-m-high boards on its way out of the rink. The base of the boards is 20.2 m from the point where the puck is launched. **(a)** Given the launch angle of the puck, θ, outline a strategy that you can use to find its initial speed, v_0. **(b)** Use your strategy to find v_0 for $\theta = 15.0°$.

78. •• Referring to Active Example 4–2, suppose the ball is punted from an initial height of 0.750 m. What is the initial speed of the ball in this case?

79. •• **A "Lob" Pass Versus a "Bullet"** A quarterback can throw a receiver a high, lazy "lob" pass or a low, quick "bullet" pass. These passes are indicated by curves 1 and 2, respectively, in **Figure 4–22**. **(a)** The lob pass is thrown with an initial speed of 21.5 m/s and its time of flight is 3.97 s. What is its launch angle?

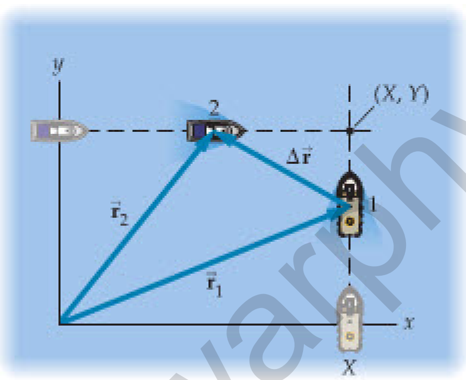
www.aswarphysics.weebly.com



▲ **FIGURE 4–22** Problem 79

(b) The bullet pass is thrown with a launch angle of 25.0°. What is the initial speed of this pass? **(c)** What is the time of flight of the bullet pass?

80. ••• **Collision Course** A useful rule of thumb in boating is that if the heading from your boat to a second boat remains constant, the two boats are on a collision course. Consider the two boats shown in Figure 4–23. At time $t = 0$, boat 1 is at the location $(X, 0)$ and moving in the positive y direction; boat 2 is at $(0, Y)$ and moving in the positive x direction. The speed of boat 1 is v_1. **(a)** What speed must boat 2 have if the boats are to collide at the point (X, Y)? **(b)** Assuming boat 2 has the speed found in part (a), calculate the displacement from boat 1 to boat 2, $\Delta\vec{r} = \vec{r}_2 - \vec{r}_1$. **(c)** Use your results from part (b) to show that $(\Delta r)_y/(\Delta r)_x = -Y/X$, independent of time. This shows that $\Delta\vec{r} = \vec{r}_2 - \vec{r}_1$ maintains a constant direction until the collision, as specified in the rule of thumb.



▲ **FIGURE 4–23** Problem 80

81. ••• As discussed in Example 4–7, the archerfish hunts by dislodging an unsuspecting insect from its resting place with a stream of water expelled from the fish's mouth. Suppose the archerfish squirts water with a speed of 2.15 m/s at an angle of 52.0° above the horizontal, and aims for a beetle on a leaf 3.00 cm above the water's surface. **(a)** At what horizontal distance from the beetle should the archerfish fire if it is to hit its target in the least time? **(b)** How much time will the beetle have to react?

82. ••• **(a)** What is the greatest horizontal distance from which the archerfish can hit the beetle, assuming the same squirt speed and direction as in Problem 81? **(b)** How much time does the beetle have to react in this case?

83. ••• Find the launch angle for which the range and maximum height of a projectile are the same.

84. ••• A mountain climber jumps a crevasse of width W by leaping horizontally with speed v_0. **(a)** If the height difference between the two sides of the crevasse is h, what is the minimum value of v_0 for the climber to land safely on the other side? **(b)** In this case, what is the climber's direction of motion on landing?

85. ••• Prove that the landing speed of a projectile is independent of launch angle for a given height of launch.

86. •• **Maximum Height and Range** Prove that the maximum height of a projectile, H, divided by the range of the projectile, R, satisfies the relation $H/R = \frac{1}{4}\tan\theta$.

87. •• **Landing on a Different Level** A projectile fired from $y = 0$ with initial speed v_0 and initial angle θ lands on a different level, $y = h$. Show that the time of flight of the projectile is

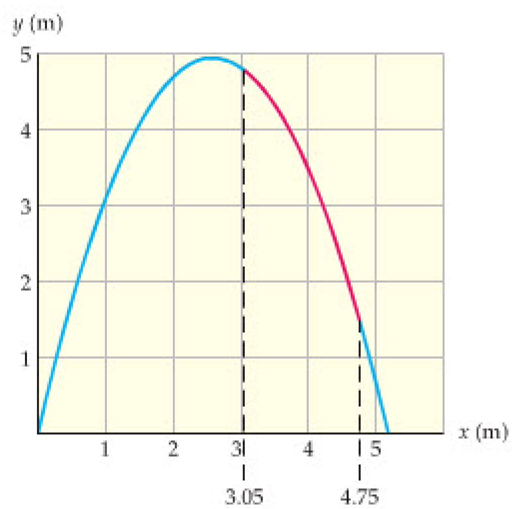
$$T = \tfrac{1}{2}T_0\left(1 + \sqrt{1 - \frac{h}{H}}\right)$$

where T_0 is the time of flight for $h = 0$ and H is the maximum height of the projectile.

88. ••• A mountain climber jumps a crevasse by leaping horizontally with speed v_0. If the climber's direction of motion on landing is θ below the horizontal, what is the height difference h between the two sides of the crevasse?

89. •••**IP** Referring to Problem 73, suppose the initial velocity of the coin tossed by the passenger is $\vec{v} = (-2.25\ \text{m/s})\hat{x} + (4.38\ \text{m/s})\hat{y}$. The train's velocity relative to the ground is still $(12.1\ \text{m/s})\hat{x}$. **(a)** What is the minimum speed of the coin relative to the ground during its flight? At what point in the coin's flight does this minimum speed occur? Explain. **(b)** Find the initial speed and direction of the coin as seen by an observer on the ground. **(c)** Use the expression for y_{max} derived in Example 4–7 to calculate the maximum height of the coin, as seen by an observer on the ground. **(d)** Repeat part (c) from the point of view of the passenger. Verify that both observers calculate the same maximum height.

90. ••• **Projectiles: Coming or Going?** Most projectiles continually move farther from the origin during their flight, but this is not the case if the launch angle is greater than $\cos^{-1}\left(\frac{1}{3}\right) = 70.5°$. For example, the projectile shown in Figure 4–24 has a launch angle of 75.0° and an initial speed of 10.1 m/s. During the portion of its motion shown in red, it is moving closer to the origin—it is moving away on the blue portions. Calculate the distance from the origin to the projectile **(a)** at the start of the red portion, **(b)** at the end of the red portion, and **(c)** just before the projectile lands. Notice that the distance for part (b) is the smallest of the three.



▲ **FIGURE 4–24** Problem 90

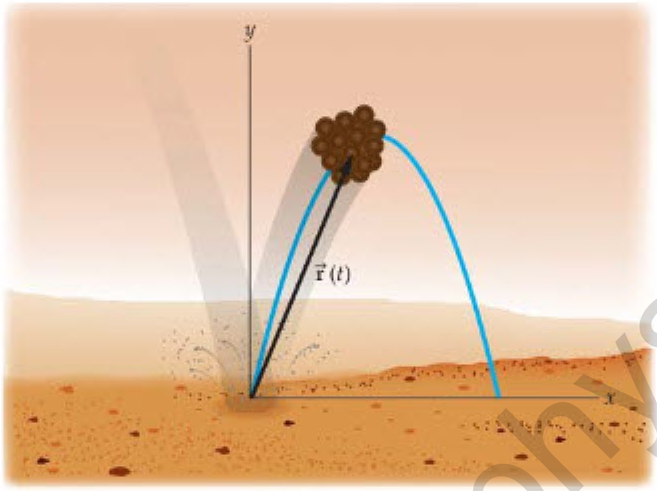
www.aswarphysics.weebly.com

PASSAGE PROBLEMS

Landing Rovers on Mars

When the twin Mars exploration rovers, *Spirit* and *Opportunity*, set down on the surface of the red planet in January of 2004, their method of landing was both unique and elaborate. After initial braking with retro rockets, the rovers began their long descent through the thin Martian atmosphere on a parachute until they reached an altitude of about 16.7 m. At that point a system of four air bags with six lobes each were inflated, additional retro rocket blasts brought the craft to a virtual standstill, and the rovers detached from their parachutes. After a period of free fall to the surface, with an acceleration of 3.72 m/s^2, the rovers bounced about a dozen times before coming to rest. They then deflated their air bags, righted themselves, and began to explore the surface.

Figure 4–25 shows a rover with its surrounding cushion of air bags making its first contact with the Martian surface. After a typical first bounce the upward velocity of a rover would be 9.92 m/s at an angle of 75.0° above the horizontal. Assume this is the case for the problems that follow.



▲ **FIGURE 4–25** Problems 91, 92, 93, and 94

91. • What is the maximum height of a rover between its first and second bounces?

A. 2.58 m **B.** 4.68 m
C. 12.3 m **D.** 148 m

92. • How much time elapses between the first and second bounces?

A. 1.38 s **B.** 2.58 s
C. 5.15 s **D.** 5.33 s

93. • How far does a rover travel in the horizontal direction between its first and second bounces?

A. 13.2 m **B.** 49.4 m
C. 51.1 m **D.** 98.7 m

94. •• What is the average velocity of a rover between its first and second bounces?

A. 0
B. 2.57 m/s in the *x* direction
C. 9.92 m/s at 75.0° above the *x* axis
D. 9.58 m/s in the *y* direction

INTERACTIVE PROBLEMS

95. •• **Referring to Example 4–5 (a)** At what launch angle *greater* than 54.0° does the golf ball just barely miss the top of the tree in front of the green? Assume the ball has an initial speed of 13.5 m/s, and that the tree is 3.00 m high and is a horizontal distance of 14.0 m from the launch point. **(b)** Where does the ball land in the case described in part (a)? **(c)** At what launch angle *less* than 54.0° does the golf ball just barely miss the top of the tree in front of the green? **(d)** Where does the ball land in the case described in part (c)?

96. •• **Referring to Example 4–5** Suppose that the golf ball is launched with a speed of 15.0 m/s at an angle of 57.5° above the horizontal, and that it lands on a green 3.50 m above the level where it was struck. **(a)** What horizontal distance does the ball cover during its flight? **(b)** What increase in initial speed would be needed to increase the horizontal distance in part (a) by 7.50 m? Assume everything else remains the same.

97. •• **Referring to Example 4–6** Suppose the ball is dropped at the horizontal distance of 5.50 m, but from a new height of 5.00 m. The dolphin jumps with the same speed of 12.0 m/s. **(a)** What launch angle must the dolphin have if it is to catch the ball? **(b)** At what height does the dolphin catch the ball in this case? **(c)** What is the minimum initial speed the dolphin must have to catch the ball before it hits the water?

98. •• **IP Referring to Example 4–6** Suppose we change the dolphin's launch angle to 45.0°, but everything else remains the same. Thus, the horizontal distance to the ball is 5.50 m, the drop height is 4.10 m, and the dolphin's launch speed is 12.0 m/s. **(a)** What is the vertical distance between the dolphin and the ball when the dolphin reaches the horizontal position of the ball? We refer to this as the "miss distance." **(b)** If the dolphin's launch speed is reduced, will the miss distance increase, decrease, or stay the same? **(c)** Find the miss distance for a launch speed of 10.0 m/s.

www.aswarphysics.weebly.com

5 Newton's Laws of Motion

Bobsledders know that a force is required to accelerate an object. In fact, the greater the force, the greater the acceleration. What they may not realize, however, is that forces always come in pairs that are equal in magnitude but opposite in direction. For example, when these athletes push on the bobsled, it pushes back on them with equal strength. All of these observations follow directly from Newton's three laws of motion, the subject of this chapter.

We are all subject to Newton's laws of motion, whether we know it or not. You can't move your body, drive a car, or toss a ball in a way that violates his rules. In short, our very existence is constrained and regulated by these three fundamental statements concerning matter and its motion.

Yet Newton's laws are surprisingly simple, especially when you consider that they apply equally well to galaxies, planets, comets, and yes, even apples falling from trees. In this chapter we present the three laws of Newton, and we show how they can be applied to everyday situations. Using them, we go beyond a simple description of motion, as in kinematics, to a study of the *causes* of motion, referred to as **dynamics.**

With the advent of Newtonian dynamics in 1687, science finally became quantitative and predictive. Edmund Halley, inspired by Newton's laws, used them to predict the return of the comet that today bears his name. In all of recorded history, no one had ever before predicted the appearance of a comet; in fact, they were generally regarded as supernatural apparitions. Though Halley didn't live to see his comet's return, his correct prediction illustrated the power of Newton's laws in a most dramatic and memorable way.

Today, we still recognize Newton's laws as the indispensable foundation for all of physics. It would be nice to say that these laws are the complete story when it comes to analyzing motion, but that is not the case. In the early part of the last century, physicists discovered that Newton's laws must be modified for objects moving at speeds near that of light and for objects comparable in size to atoms. In the world of everyday experience, however, Newton's laws still reign supreme.

5–1	Force and Mass	112
5–2	Newton's First Law of Motion	112
5–3	Newton's Second Law of Motion	114
5–4	Newton's Third Law of Motion	122
5–5	The Vector Nature of Forces: Forces in Two Dimensions	125
5–6	Weight	128
5–7	Normal Forces	132

www.aswarphysics.weebly.com

5–1 Force and Mass

A **force,** simply put, is a push or a pull. When you push on a box to slide it across the floor, for example, or pull on the handle of a wagon to give a child a ride, you are exerting a force. Similarly, when you hold this book in your hand, you exert an upward force to oppose the downward pull of gravity. If you set the book on a table, the table exerts the same upward force you exerted a moment before. Forces are truly all around us.

Now, when you push or pull on something, there are two quantities that characterize the force you are exerting. The first is the strength or **magnitude** of your force; the second is the **direction** in which you are pushing or pulling. Because a force is determined by both a magnitude and a direction, it is a vector. We consider the vector properties of forces in more detail in Section 5–5.

In general, an object has several forces acting on it at any given time. In the previous example, a book at rest on a table experiences a downward force due to gravity and an upward force due to the table. If you push the book across the table, it also experiences a horizontal force due to your push. The total, or net, force exerted on the book is the vector sum of the individual forces acting on it.

After the net force acting on an object, the second key ingredient in Newton's laws is the **mass** of an object, which is a measure of how difficult it is to change its velocity—to start an object moving if it is at rest, to bring it to rest if it is moving, or to change its direction of motion. For example, if you throw a baseball or catch one thrown to you, the force required is not too great. But if you want to start a car moving or to stop one that is coming at you, the force involved is much greater. It follows that the mass of a car is greater than the mass of a baseball.

In agreement with everyday usage, mass can also be thought of as a measure of the quantity of matter in an object. Thus, it is clear that the mass of an automobile, for example, is much greater than the mass of a baseball, but much less than the mass of Earth. We measure mass in units of kilograms (kg), where one kilogram is defined as the mass of a standard cylinder of platinum-iridium, as discussed in Chapter 1. A list of typical masses is given in Table 5–1.

TABLE 5–1 Typical Masses in Kilograms (kg)

Earth	5.97×10^{24}
Space shuttle	2,000,000
Blue whale (largest animal on Earth)	178,000
Whale shark (largest fish)	18,000
Elephant (largest land animal)	5400
Automobile	1200
Human (adult)	70
Gallon of milk	3.6
Quart of milk	0.9
Baseball	0.145
Honeybee	0.00015
Bacterium	10^{-15}

These properties of force and mass are developed in detail in the next three sections.

5–2 Newton's First Law of Motion

If you've ever stood in line at an airport, pushing your bags forward a few feet at a time, you know that as soon as you stop pushing the bags, they stop moving. Observations such as this often lead to the erroneous conclusion that a force is required for an object to move. In fact, according to Newton's first law of motion, a force is required only to *change* an object's motion.

What is missing in this analysis is the force of friction between the bags and the floor. When you stop pushing the bags, it is not true that they stop moving because they no longer have a force acting on them. On the contrary, there is a rather large *frictional force* between the bags and the floor. It is this force that causes the bags to come to rest.

To see how motion is affected by reducing friction, imagine that you slide on dirt into second base during a baseball game. You won't slide very far before stopping. On the other hand, if you slide with the same initial speed on a sheet of ice—where the friction is much less than on a ball field—you slide considerably farther. If you could reduce the friction more, you would slide even farther.

In the classroom, air tracks allow us to observe motion with practically no friction. An example of such a device is shown in **Figure 5–1**. Note that air is blown through small holes in the track, creating a cushion of air for a small "cart" to ride on. A cart placed at rest on a level track remains at rest—unless you push on it to get it started.

Once set in motion, the cart glides along with constant velocity—constant speed in a straight line—until it hits a bumper at the end of the track. The bumper

www.aswarphysics.weebly.com