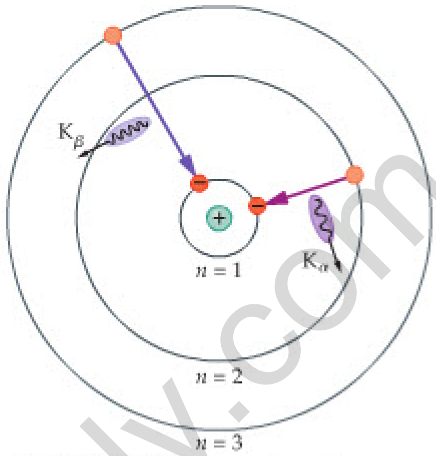
setting $n = 1$, we obtain a reasonable estimate for the energy of a K-shell electron:

$$E_K = -(13.6\text{ eV})\frac{(Z-1)^2}{1^2} \qquad 31\text{–}14$$

We apply this result in the following Active Example.



▲ **FIGURE 31–23 Production of characteristic X-rays**

When an electron strikes a metal atom in the target of an X-ray tube, it may knock one of the two K-shell ($n = 1$) electrons out of the atom. The resulting vacancy in the K shell will be filled by an electron dropping from a higher shell. If an electron drops from the $n = 2$ shell to the $n = 1$ shell, we say that the resulting photon is a K_α X-ray. Similarly, if an electron in the $n = 3$ shell drops to the $n = 1$ shell, the result is a K_β X-ray. Clearly, the K_β X-ray has the greater energy and the shorter wavelength, as we see in Figure 31–22.

ACTIVE EXAMPLE 31–3 FIND THE VOLTAGE OF AN X-RAY TUBE

Estimate the minimum energy an incoming electron must have to knock a K-shell electron out of a tungsten atom ($Z = 74$).

SOLUTION *(Test your understanding by performing the calculations indicated in each step.)*

1. Calculate $(Z-1)^2$:	5329
2. Multiply −13.6 eV by $(Z-1)^2$:	−72,500 eV

INSIGHT

An electron at rest an infinite distance from a tungsten atom has an energy of zero. Therefore, a minimum energy of 72,500 eV must be supplied to the electron to remove it from the K shell of tungsten.

Recall that an electron gains an energy of 1 eV when it accelerates through a potential difference of 1 V. Thus a potential difference of at least 72,500 V = 72.5 kV is required to produce characteristic X-rays with a tungsten target. Typical X-ray tubes, like those used in dental offices, operate in the 100-kV range.

YOUR TURN

Suppose an X-ray tube has a voltage of only 35 kV. What is the largest value of Z for which this tube can knock out a K-shell electron?

(Answers to **Your Turn** *problems are given in the back of the book.)*

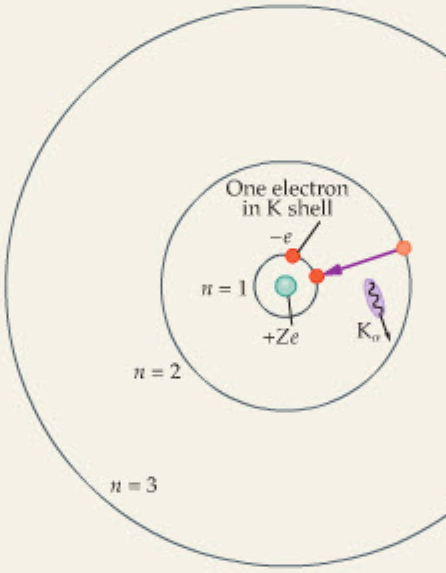
Equation 31–14 can also be used to obtain an estimate of the K_α wavelength for a given element. We show how this can be done for the case of molybdenum in the next Example.

EXAMPLE 31–4 K_α FOR MOLYBDENUM

Estimate the K_α wavelength for molybdenum ($Z = 42$).

PICTURE THE PROBLEM

Our sketch indicates the electron jump that is responsible for the K_α X-ray; that is, from $n = 2$ to $n = 1$. Note that the net charge from the K shell outward is $+(Z-1)e$.



STRATEGY

To find the wavelength of a K_α X-ray, we start with the relationship between the change in energy of an electron and the wavelength of the corresponding photon, $|\Delta E| = hf = hc/\lambda$. Once we have calculated ΔE, we can find the wavelength using $\lambda = hc/|\Delta E|$.

To find ΔE, we first calculate the energy of an electron in the K shell of molybdenum, using $E_K = -(13.6\text{ eV})(Z-1)^2/1^2$ with $Z = 42$.

Next, we calculate the energy of an electron in the L shell ($n = 2$) of molybdenum, since it is an L-shell electron that fills the vacancy in the K shell and emits a K_α X-ray. Note that an L-shell electron sees a nucleus with an effective charge of $+(Z-1)e$. This follows because there is one electron in the K shell, and this electron partially screens the nucleus. As a result, the energy of an L-shell electron is given by the following expression: $E_L = -(13.6\text{ eV})(Z-1)^2/2^2$.

Finally, with these two energies determined, the change in energy is simply $\Delta E = E_K - E_L$.

CONTINUED ON NEXT PAGE

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SOLUTION

1. Calculate the energy of a K-shell electron, E_K:

$$E_K = -(13.6\text{ eV})\frac{(Z-1)^2}{1^2} = -(13.6\text{ eV})\frac{(42-1)^2}{1^2} = -22{,}900\text{ eV}$$

2. Calculate the energy of an L-shell electron, E_L:

$$E_L = -(13.6\text{ eV})\frac{(Z-1)^2}{2^2} = -(13.6\text{ eV})\frac{(42-1)^2}{2^2} = -5720\text{ eV}$$

3. Determine the change in energy, ΔE, of an electron that jumps from the L shell to the K shell:

$$\Delta E = E_K - E_L = -22{,}900\text{ eV} - (-5720\text{ eV}) = -17{,}200\text{ eV}$$

4. Calculate the wavelength corresponding to ΔE:

$$\lambda = \frac{hc}{|\Delta E|} = \frac{(6.63\times10^{-34}\text{ J}\cdot\text{s})(3.00\times10^{8}\text{ m/s})}{(17{,}200\text{ eV})(1.60\times10^{-19}\text{ J/eV})} = 7.23\times10^{-11}\text{ m} = 0.0723\text{ nm}$$

INSIGHT

Comparing our result with Figure 31–22, which shows the X-ray spectrum for molybdenum, we see that our approximate wavelength of 0.0723 nm is in good agreement with experiment.

PRACTICE PROBLEM

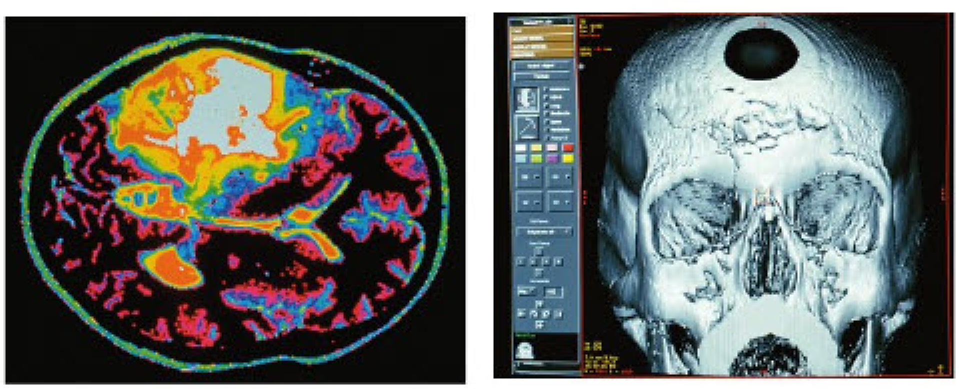
Which element has a K_α peak at a wavelength of approximately 0.155 nm? [**Answer:** Copper, $Z = 29$]

Some related homework problems: Problem 61, Problem 62, Problem 81

As mentioned earlier in this section, X-rays were put to medical use as soon as people found out about their properties and how to produce them. In fact, Roentgen himself, whose first article about X-rays was published in late December 1895, produced an X-ray image of his wife's left hand, clearly showing the bones in her fingers and her wedding ring. Less than two months later, in February 1896, American physicians were starting to test them on patients. One of the earliest patients was a young boy named Eddie McCarthy, who had his broken forearm X-rayed. A New Yorker by the name of Tolson Cunningham had a bullet removed from his leg after its position was determined by a 45-minute X-ray exposure.

REAL-WORLD PHYSICS: BIO
Computerized axial tomography

Standard X-rays can be difficult to interpret, however, since they cast shadows of all the body materials they pass through onto a single sheet of film. It is somewhat like placing several transparencies on top of one another and trying to decipher their individual contents. With the advent of high-speed computers, a new type of X-ray image is now possible. In a **computerized axial tomography scan** (CAT scan), thin beams of X-rays are directed through the body from a variety of directions. The intensity of the transmitted beam is detected for each direction, and the results are sent to a computer for processing. The result is an image that shows the physician a "cross-sectional slice" through the body. In this way, each part of the body can be viewed individually and with clarity. If a series of such slices are stacked together in the computer, they can give a three-dimensional view of the body's interior.



▶ The false-color CAT scan at left represents a horizontal section through the brain, revealing a large benign tumor (the white and orange area at top). A series of CAT scans can also be combined to create remarkable three-dimensional images such as the one at right.

Lasers

The production of light by humans advanced significantly when flames were replaced by the lightbulb. An even greater advancement occurred in 1960, however, when the first laser was developed. Lasers produce light that is intense, highly collimated (uni-directional), and pure in its color. Because of these properties, lasers are used in a multitude of technological applications, ranging from supermarket scanners to CDs, from laser pointers to eye surgery. In fact, lasers are now almost as common in everyday life as the lightbulb.

To understand just what a laser is and what makes it so special, we start with its name. The word **laser** is an acronym for **l**ight **a**mplification by the **s**timulated **e**mission of **r**adiation. As we shall see, the properties of stimulated emission lead directly to the amplification of light.

To begin, consider two energy levels in an atom, and suppose an electron occupies the higher of the two levels. If the electron is left alone, it will eventually drop to the lower level in a time that is typically about 10^{-8} s, giving off a photon in the process referred to as **spontaneous emission.** The photon given off in this process can propagate in any direction.

In contrast, suppose the electron in the excited state just described is not left alone. For example, a photon with an energy equal to the energy difference between the two energy levels might pass near the electron, which *enhances* the probability that the electron will drop to the lower level. That is, the incident photon can *stimulate* the emission of a second photon by the electron. The photon given off in this process of **stimulated emission** has the same energy as the incident photon, the same phase, and propagates in the same direction, as indicated in Figure 31–24. This accounts for the fact that laser light is highly focused and of a single color.

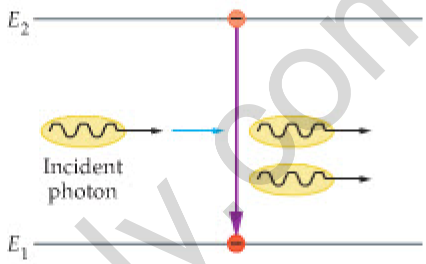


FIGURE 31–24 Stimulated emission

A photon with an energy equal to the difference $E_2 - E_1$ can enhance the probability that an electron in the state E_2 will drop to the state E_1 and emit a photon. When a photon stimulates the emission of a second photon in this way, the new photon has the same frequency, direction, and phase as the incident photon. If each of the two photons resulting from this process in turn produces two photons, the total number of photons can increase exponentially, resulting in an intense beam of light.

As for the amplification of light, notice that a single photon entering an excited atom can cause two identical photons to exit the atom. If each of these two photons encounters another excited atom and undergoes the same process, the number of photons increases to four. Continuing in this manner, the photons undergo a sort of "chain reaction" that doubles the number of photons with each generation. It is this property of stimulated emission that results in **light amplification.**

In order for the light amplification process to work, it is necessary that photons continue to encounter atoms with electrons in excited states. Under ordinary conditions this will not be the case, since most electrons are in the lowest possible energy levels. For laser action to occur, atoms must first be prepared in an excited state. Then, before the electrons have a chance to drop to a lower level by way of spontaneous emission, the process of stimulated emission can proceed. This requires what is known as a **population inversion,** in which more electrons are found in the excited state than in a lower state. In addition, the excited state must be one that lasts for a relatively long time so that photons will continue to encounter excited atoms. A long-lived excited state is referred to as a **metastable state.** So to produce a laser, one needs a metastable excited state with a population inversion.

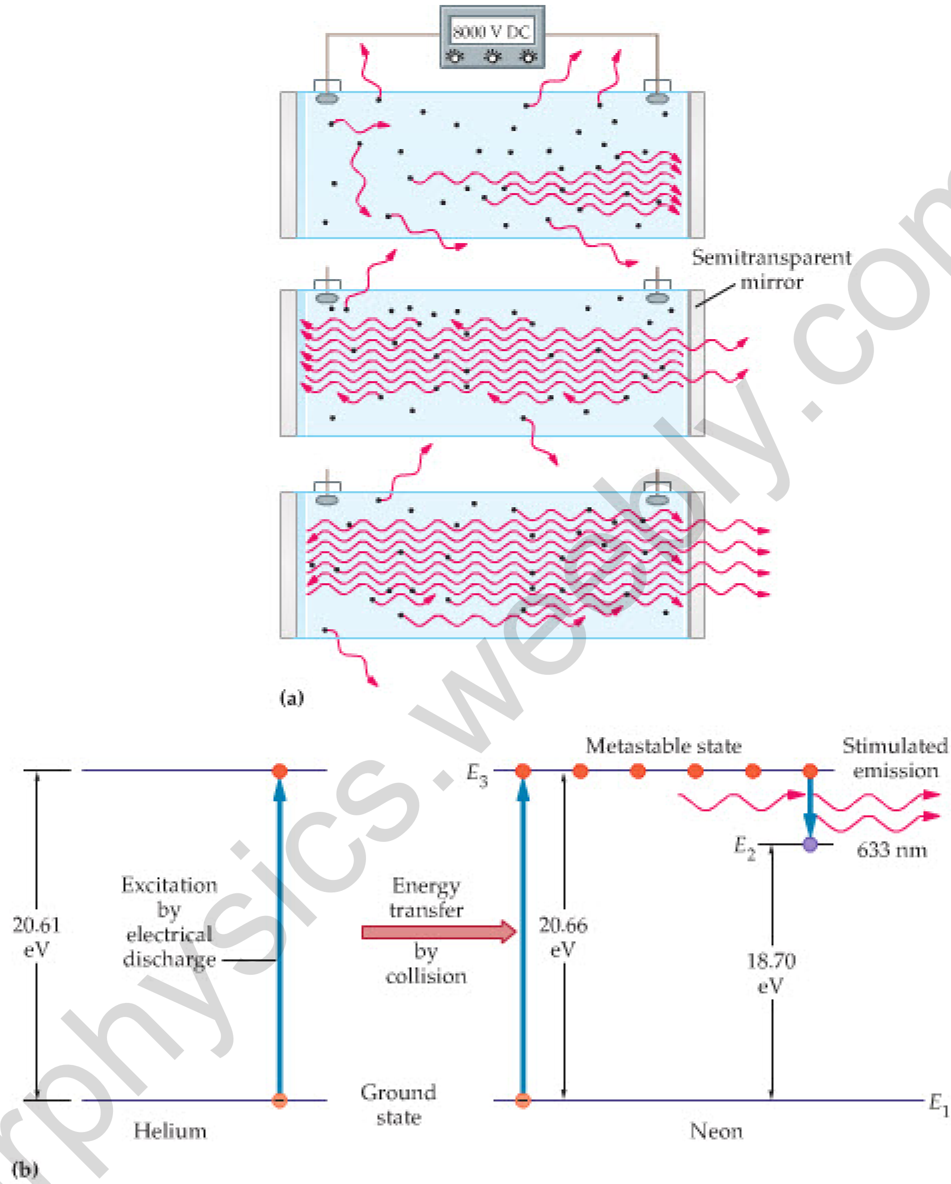
REAL-WORLD PHYSICS

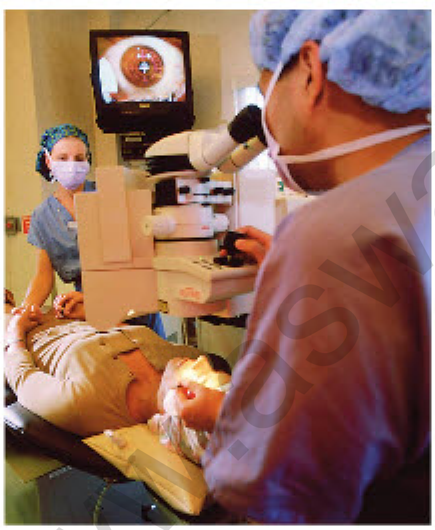
Helium-neon laser

A specific example of a laser is the **helium-neon laser,** shown schematically in Figure 31–25 (a), in which the neon atoms produce the laser light. The appropriate energy-level diagrams for neon are shown in Figure 31–25 (b). The excited state E_3 is metastable—electrons promoted to that level stay in the level for a relatively long time. Electrons are excited to this level by an electrical power supply connected to the tube containing the helium-neon mixture. The power supply causes electrons to move through the tube, colliding with neon atoms and exciting them. Similarly, excited helium atoms colliding with the neon atoms also cause excitations to the E_3 level. Since this level is metastable, the excitation processes can cause a population inversion, setting the stage for laser action. Stimulated emission then occurs, allowing the electrons to drop to the lower level, E_2. The electrons subsequently proceed through various intermediate steps to the ground state. It is the emission of light between the levels E_3 and E_2 that results in laser light. Since the difference in energy between these levels is 1.96 eV, the light coming out of the laser is red, with a wavelength of 633 nm.

▶ FIGURE 31–25 The helium-neon laser

(a) A schematic representation of the basic features of a helium-neon laser. **(b)** The relevant energy levels in helium and neon that result in the laser action.





▲ A doctor uses a microscope and video monitor to ensure precise cutting of the corneal flap in the LASIK procedure. A UV laser will then remove some of the material under the flap, flattening the cornea and correcting the patient's nearsightedness.

REAL-WORLD PHYSICS: BIO

Laser eye surgery

To enhance the output of a laser, the light is reflected back and forth between mirrors. In fact, this reflection produces a resonant condition in the light, much like standing waves in an organ pipe. A schematic of a laser in operation is shown in **Figure 31–25 (a)**.

A recent medical application of lasers involves several types of *laser eye surgery*. In these techniques, a laser that emits high-energy photons in the UV range (typically at wavelengths of 193 nm) is used to reshape the cornea and correct nearsightedness. For example, in LASIK (**la**ser in **si**tu **k**eratomileusis) eye surgery the procedure begins with a small mechanical shaver known as a microkeratome cutting a flap in the cornea, leaving a portion of the cornea uncut to serve as a hinge. After the mechanical cut is made, the corneal flap is folded back, exposing the middle portion of the cornea as shown in **Figure 31–26 (a)**. Next, an *excimer laser* sends pulses of UV light onto the cornea, each pulse vaporizing a small layer of corneal material (0.1 to 0.5 μm in thickness) with no heating. This process continues until the cornea is flattened just enough to correct the nearsightedness, after which the corneal flap is put back into place.

Photorefractive keratectomy (PRK) is similar to LASIK eye surgery, except that material is removed directly from the surface of the cornea, without the use of a corneal flap, as shown in **Figure 31–26 (b)**. To correct nearsightedness, the laser beam is directed onto the central portion of the cornea (left), resulting in a flattening of the cornea. To correct farsightedness, it is necessary to increase the

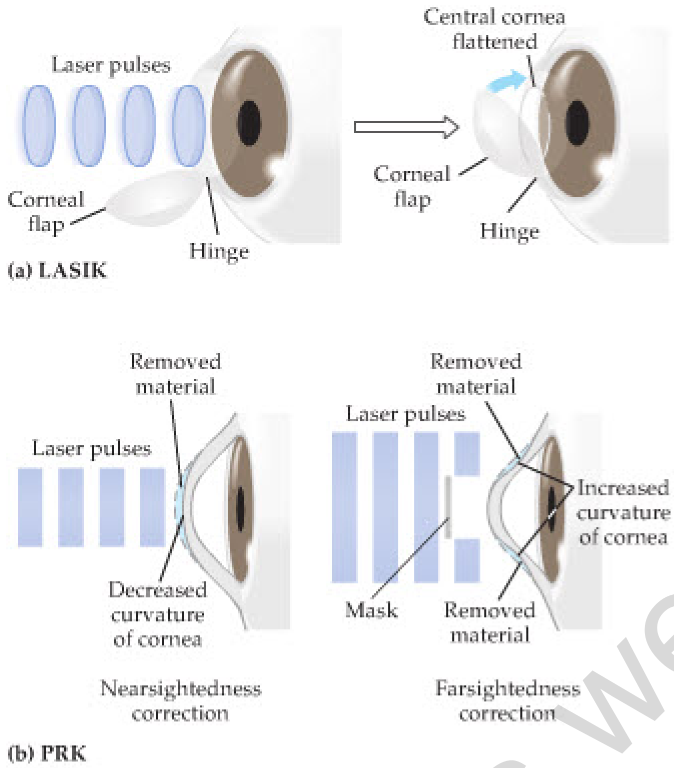


FIGURE 31–26 Laser vision correction

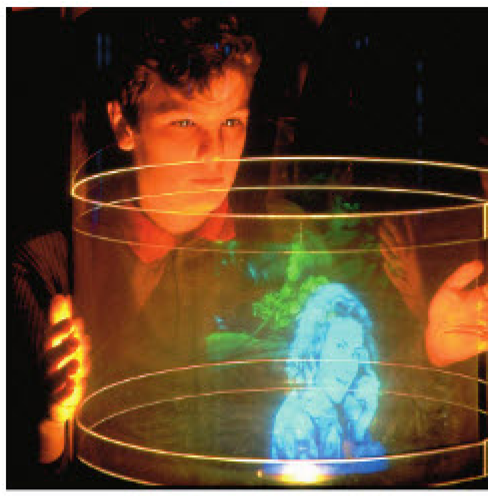
(a) In LASIK eye surgery, a flap of the cornea is cut and folded back. Next, an ultraviolet excimer laser is used to vaporize some of the underlying corneal material. When the flap is replaced, the cornea is flatter than it was, correcting the patient's nearsightedness. **(b)** In the PRK procedure, the laser removes material directly from the corneal surface. If the cornea is too curved, producing nearsightedness, the laser beam is directed at its center and the cornea is flattened. If the cornea is too flat, producing farsightedness, the central region is masked and material is removed from the periphery, increasing the curvature.

curvature of the cornea. This is accomplished by masking the central portion of the cornea so that the laser removes only peripheral portions of the cornea (right). In both cases, it is necessary to keep the beam focused at the desired location on the eye. This is difficult, because the eye routinely moves by small amounts roughly every 15 ms. In the most sophisticated application of PRK, these eye movements can be tracked and the aiming of the laser beam corrected accordingly.

REAL-WORLD PHYSICS: BIO
Photodynamic therapy

Another medical application of lasers is known as *photodynamic therapy*, or PDT. In this type of therapy, light-sensitive chemicals (such as porphyrins) are injected into the bloodstream and are taken up by cells throughout the body. These chemicals are found to remain in cancerous cells for greater periods of time than in normal cells. Thus, after an appropriate time interval, the light-sensitive chemicals are preferentially concentrated in cancerous cells. If a laser beam with the precise wavelength absorbed by the light-sensitive chemicals illuminates the cancer cells, the resulting chemical reactions kill the cancer cells without damaging the adjacent normal cells. The laser beam can be directed to the desired location using a flexible fiber-optic cable in conjunction with a bronchoscope to treat lung cancer or with an endoscope to treat esophageal cancer.

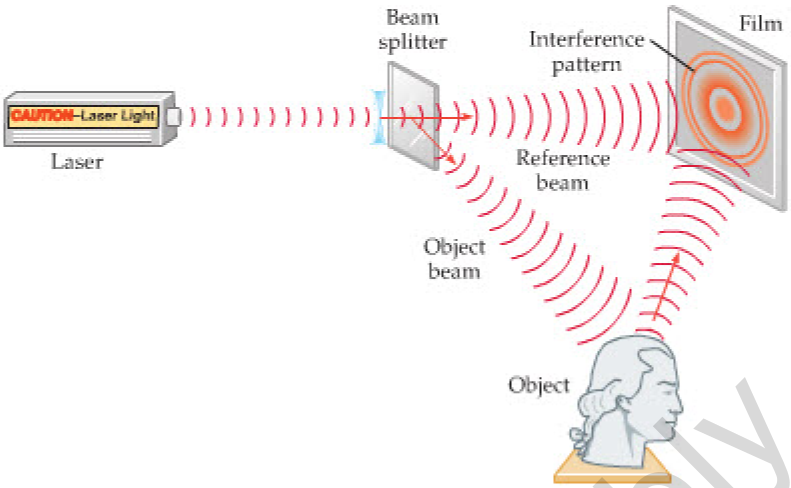
PDT is also used to treat certain types of age-related macular degeneration (AMD). For most people, macular degeneration results when abnormal blood vessels behind the retina leak fluid and blood into the central region of the retina, or macula, causing it to degenerate. In this case, light-sensitive chemicals are preferentially taken up by the abnormal blood vessels; hence, a laser beam of the proper wavelength can destroy these blood vessels without damaging the normal structures in the retina.



▲ A hologram creates a three-dimensional image in empty space. The image can be viewed from different angles to reveal different parts of the original subject.

Finally, lasers can also be used to take three-dimensional photographs known as **holograms.** A typical setup for taking a hologram is shown in Figure 31–27. Notice that the hologram is produced with no focusing lenses, in contrast with normal photography. The basic procedure in holography begins with the splitting of a laser beam into two separate beams. One beam, the reference beam, is directed onto the photographic film. The second beam, called the object beam, is directed onto the object to be recorded in the hologram. The object beam reflects from various parts of the object and then combines with the reference beam on the film. Because the laser light is coherent, and because the object and reference beams travel

REAL-WORLD PHYSICS
Holography

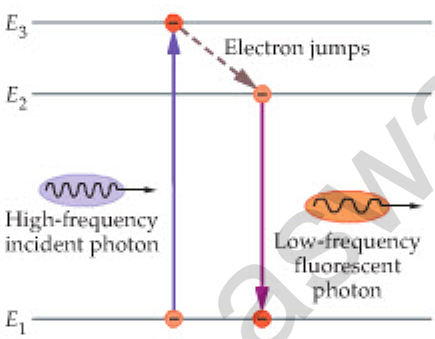


▶ **FIGURE 31–27 Holography**
To create a hologram, laser light is split into two beams. One, the reference beam, is directed onto the photographic film. The other is reflected from the surface of an object onto the film, where it combines with the reference beam to create an interference pattern. When this pattern is illuminated with laser light of the same wavelength, a three-dimensional image of the original object is produced.

different distances, the combined light results in an interference pattern. In fact, if you look at a hologram in normal light it simply looks like a confusing mass of swirls and lines. When a laser beam illuminates the hologram, however, the interference pattern causes the laser light to propagate away from the hologram in exactly the same way as the light that originally produced the interference pattern. Thus, a person viewing the hologram sees precisely the same patterns of light that would have been observed when the hologram was recorded.

Holograms give a true three-dimensional image. When you view a hologram, you can move your vantage point to see different parts of the scene that is recorded. In particular, viewing the hologram from one angle may obscure an object in the background, but by moving your head, you can look around the foreground objects to get a clear view of the obscured object. In addition, you have to adjust your focus as you shift your gaze from foreground to background objects, just as in the real world. Finally, if you cut a hologram into pieces, each piece still shows the entire scene! This is analogous to your ability to see everything in your front yard through a small window just as you can through a large window—both show the entire scene.

The holograms on your credit cards are referred to as *rainbow holograms*, because they are designed to be viewed with white light containing all the colors of the rainbow. Although these holograms give an impression of three-dimensionality, they do not compare in quality to a hologram viewed with a laser.



▲ **FIGURE 31–28 The mechanism of fluorescence**
In fluorescence, a high-frequency photon raises an electron to an excited state. When the electron drops back to the ground state, it may do so by way of various intermediate states. The jumps between intermediate states produce photons of lower frequency, which are observed as the phenomenon of fluorescence.

Fluorescence and Phosphorescence

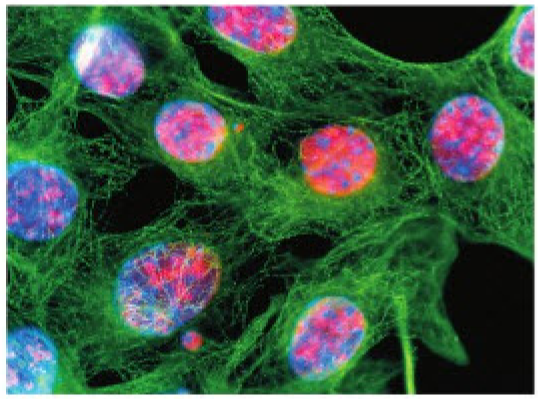
In the Insight to Active Example 31–1 it was noted that an electron in an excited state can emit photons of various energies as it falls to the ground state. This type of behavior is at the heart of fluorescence and phosphorescence.

Consider the energy levels shown in **Figure 31–28**. If an atom with these energy levels absorbs a photon of energy $E_3 - E_1$, it can excite an electron from state E_1 to state E_3. In some atoms, the most likely way for the electron to return to the ground state is by first jumping to level E_2 and then jumping to level E_1. The photons emitted in these jumps have less energy than the photon that caused the excitation in the first place. Hence, in a system like this, an atom is illuminated with a photon of one frequency, and it subsequently emits photons of lower energy and lower frequency. The emission of light of lower frequency after illumination by a higher frequency is referred to as **fluorescence.** In essence, fluorescence can be thought of as a conversion process, in which photons of high frequency are converted to photons of lower frequency.

REAL-WORLD PHYSICS
Fluorescent lightbulbs

Perhaps the most common example of fluorescence is the *fluorescent lightbulb*. This device uses fluorescence to convert high-frequency ultraviolet light to lower-frequency visible light. In particular, the tube of a fluorescent light contains mercury vapor. When electricity is applied to one of these tubes, a filament is heated, producing electrons. These electrons are accelerated by an applied voltage. The

▲ The fingerprints on this mug become clearly visible when treated with fluorescent dye and illuminated by ultraviolet light.



▲ In the technique known as immunofluorescence, an antibody molecule is linked to a fluorescent dye molecule, making it possible to visualize cellular structures and components that are otherwise largely invisible. This photograph utilized antibodies that bind to a protein found in intermediate filaments. Such filaments (seen here as lacy fibers resembling a spiderweb) are a component of the cytoskeleton, the cellular scaffolding that enables cells to maintain or change their shape and move materials about.

electrons strike mercury atoms in the tube, exciting them, and they give off ultraviolet light as they decay to their ground state. This process is not particularly useful in itself, since the ultraviolet light is invisible to us. However, the inside of the tube is coated with a phosphor that absorbs the ultraviolet light and then emits a lower-frequency light that is visible. Thus, several different physical processes must take place before a fluorescent lightbulb produces visible light.

Fluorescence finds many other less familiar applications as well. In forensics, the analysis of a crime scene is enhanced by the fact that human bones and teeth are fluorescent. Thus, illuminating a crime scene with ultraviolet light can make items of interest stand out for easy identification. In addition, the use of a fluorescent dye can make fingerprints visible with great clarity.

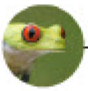
REAL-WORLD PHYSICS: BIO
Applications of fluorescence in forensics

Many creatures produce fluorescence in their bodies, as well. For example, several types of coral glow brightly when illuminated with ultraviolet light. It is also well known that scorpions are strongly fluorescent, giving off a distinctive green light. In fact, it is often possible to discern a greenish cast when viewing a scorpion in sunlight. At night in the desert, scorpions stand out with a bright green glow when a person illuminates the area with a portable ultraviolet light. This aids researchers who would like to find certain scorpions for study, and campers who are just as interested in avoiding scorpions altogether.

REAL-WORLD PHYSICS: BIO
Detecting scorpions at night

The green fluorescence produced by the jellyfish *Aequorea victoria* finds many uses in biological experiments. The gene that produces the green fluorescent protein (GFP) can serve as a marker to identify whether an organism has incorporated a new segment of DNA into its genome. For example, bacterial colonies that incorporate the GFP gene can be screened by eye simply by viewing the colony

◄ A variety of creatures, including the scorpion at left, are naturally fluorescent when illuminated with ultraviolet light. So too are many minerals, such as the one at right.

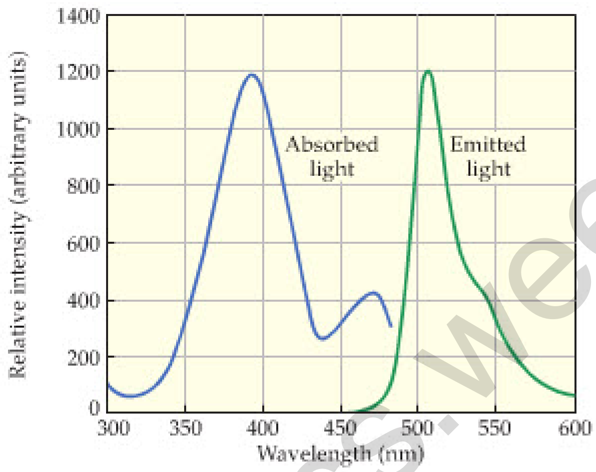


REAL-WORLD PHYSICS: BIO

The GFP bunny

under an ultraviolet light. Recently, GFP has been inserted into the genome of a white rabbit, giving rise to the "GFP bunny." The bunny appears normal in white light, but when viewed under light with a wavelength of 392 nm, it glows with a bright green light at 509 nm (see p. 1078). The fluorescence spectrum of GFP is shown in **Figure 31–29**.

Phosphorescence is similar to fluorescence, except that phosphorescent materials continue to give off a secondary glow long after the initial illumination that excited the atoms. In fact, phosphorescence may persist for periods of time ranging from a few seconds to several hours, as on the hands of a watch that glow in the dark.



▶ **FIGURE 31–29 The fluorescence spectrum of GFP**
The green fluorescent protein (GFP) strongly absorbs light with a wavelength of about 400 nm (violet). It reemits green light with a wavelength of 509 nm.

THE BIG PICTURE PUTTING PHYSICS IN CONTEXT

LOOKING BACK

We've come a long way in our study of physics, from its earliest beginnings with Galileo and Newton to the quantum revolution of the twentieth century with Einstein, Planck, Bohr, Schrödinger, Heisenberg, and others. As we look back on this journey, we see that even the most exotic predictions and discoveries of modern physics are grounded in the fundamentals that have been presented throughout this book. For example:

Bohr orbits (Section 31–3) for the hydrogen atom are calculated in the same way that gravitational orbits of planets and satellites were calculated in Chapter 12.

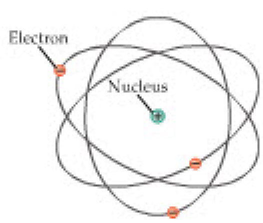
The energy of a Bohr orbit (Section 31–3) is determined using the electric potential energy introduced in Chapter 20.

Photons emitted from a hydrogen atom in the Bohr model obey the relation $E = hf$, which was used to understand blackbody radiation and the photoelectric effect in Chapter 30.

And finally, we showed in Section 31–4 that Bohr orbits can be understood in terms of standing waves, just like the standing waves on a string studied in Chapter 14, only this time using the de Broglie waves introduced in Chapter 30.

CHAPTER SUMMARY

31–1 EARLY MODELS OF THE ATOM



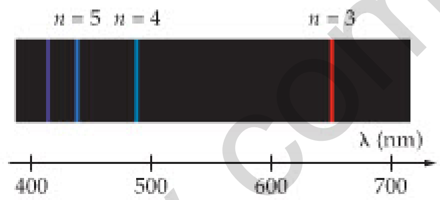
Atoms are the smallest unit of a given element. If an atom is broken down into smaller pieces, it loses the properties that characterized the element.

The Thomson Model: Plum Pudding
In Thomson's model, an atom is imagined to be like a positively charged pudding with negatively charged electrons scattered throughout.

The Rutherford Model: A Miniature Solar System
Rutherford discovered that an atom is somewhat like an atomic-scale solar system: mostly empty space, with most of its mass concentrated in the nucleus. The electrons were thought to orbit the nucleus.

31–2 THE SPECTRUM OF ATOMIC HYDROGEN

Excited atoms of hydrogen in a low-pressure gas give off light of specific wavelengths. This is referred to as the spectrum of hydrogen.



Line Spectra
The spectrum of hydrogen is a series of bright lines of well-defined wavelengths.

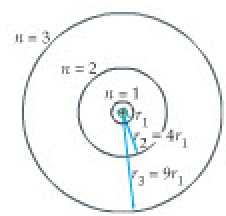
Series
Hydrogen's line spectrum is formed by a series of lines that are grouped together. The wavelengths of these series are given by the expression

$$\frac{1}{\lambda} = R\left(\frac{1}{n'^2} - \frac{1}{n^2}\right) \qquad n' = 1, 2, 3, \ldots \qquad n = n' + 1, n' + 2, n' + 3, \ldots \qquad 31\text{–}2$$

Each line in a given series corresponds to a different value of n. The different series correspond to different values of n'. For example, $n' = 1$ is the Lyman series, $n' = 2$ is the Balmer series, and $n' = 3$ is the Paschen series.

31–3 BOHR'S MODEL OF THE HYDROGEN ATOM

Bohr's model of hydrogen is basically a solar-system model, with the electron orbiting the nucleus. In Bohr's model, however, only certain orbits are allowed.



Assumptions of the Bohr Model
The Bohr model assumes the following: (i) Electrons move in circular orbits about the nucleus; (ii) allowed orbits must have an angular momentum equal to $L_n = nh/2\pi$, where $n = 1, 2, 3, \ldots$; (iii) electrons in allowed orbits do not give off electromagnetic radiation; and (iv) radiation is emitted only when electrons jump from one orbit to another.

Bohr Orbits
The radii of allowed orbits in the Bohr model are given by

$$r_n = \left(\frac{h^2}{4\pi^2 mkZe^2}\right)n^2 = (5.29 \times 10^{-11}\,\text{m})n^2 \qquad n = 1, 2, 3\ldots \qquad 31\text{–}7$$

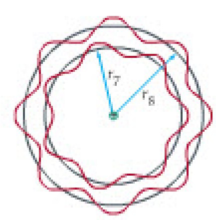
The Energy of a Bohr Orbit
The energy of an allowed Bohr orbit is

$$E_n = -(13.6\ \text{eV})\frac{Z^2}{n^2} \qquad n = 1, 2, 3, \ldots \qquad 31\text{–}9$$

These expressions correspond to hydrogen when $Z = 1$.

31–4 de BROGLIE WAVES AND THE BOHR MODEL

De Broglie was able to show that the allowed orbits of the Bohr model correspond to standing matter waves of the electrons. In particular, an allowed orbit in Bohr's model has a circumference equal to an integer times the wavelength of the electron in that orbit.

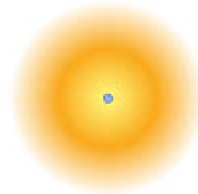


31–5 THE QUANTUM MECHANICAL HYDROGEN ATOM

The correct description of the hydrogen atom is derived from Schrödinger's equation. It agrees in many key ways with the Bohr model but has significant differences as well.

Quantum Numbers
The quantum mechanical hydrogen atom is described by four quantum numbers. They are as follows: (i) the principal quantum number, n, which is analogous to n in the Bohr model; (ii) the orbital angular momentum quantum number, ℓ, which takes on the values $\ell = 0, 1, 2, \ldots, (n - 1)$; the orbital angular momentum has a magnitude given by $L = \sqrt{\ell(\ell + 1)}(h/2\pi)$; (iii) the magnetic quantum number, m_ℓ, for which the allowed values are $m_\ell = -\ell, -\ell + 1, -\ell + 2, \ldots,$

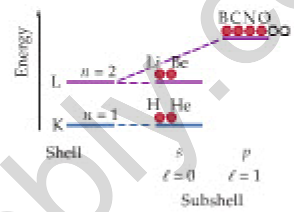
$-1, 0, 1, \ldots, \ell - 2, \ell - 1, \ell$; the z component of the orbital angular momentum is $L_z = m_\ell(h/2\pi)$; and (iv) the electron spin quantum number, m_s, which can have the values $m_s = -\frac{1}{2}, \frac{1}{2}$.



Electron Probability Clouds: Three-Dimensional Standing Waves
In the quantum mechanical hydrogen atom, the electron does not orbit at a precise distance from the nucleus. Instead, the electron distribution is represented by a probability cloud, where the densest regions of the cloud correspond to regions of highest probability.

31–6 MULTIELECTRON ATOMS AND THE PERIODIC TABLE

As electrons are added to atoms, the properties of the atoms change in a regular and predictable way.



Multielectron Atoms
Energy levels in a multielectron atom depend on n and ℓ. The energy increases with increasing n for fixed ℓ, and with increasing ℓ for fixed n.

Shells and Subshells
Electrons with the same value of n are said to be in the same shell. Electrons in a given shell with the same value of ℓ are said to be in the same subshell.

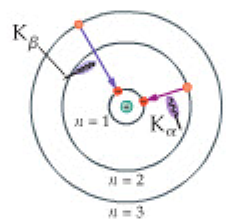
The Pauli Exclusion Principle
The Pauli exclusion principle states that only a single electron may have a particular set of quantum numbers. This means that it is not possible for all the electrons in a multielectron atom to occupy the lowest energy level.

Electronic Configurations
The arrangement of electrons is indicated by the electronic configuration. For example, the configuration $1s^2$ indicates that 2 electrons ($1s^2$) are in the $n = 1$ ($1s^2$), $\ell = 0$ ($1s^2$) state.

The Periodic Table
As electrons fill subshells of progressively higher energy, they produce the elements of the periodic table. Atoms with the same configuration of outermost electrons generally have similar chemical properties.

31–7 ATOMIC RADIATION

Atoms can give off radiation ranging from X-rays to visible light to infrared rays.



X-rays
X-rays characteristic of a particular element are given off when an electron in an inner shell is knocked out of the atom, and an electron from an outer shell drops down to take its place.

Lasers
A laser is a device that produces light amplification by the stimulated emission of radiation.

Fluorescence and Phosphorescence
When an electron in an atom is excited to a high energy level, it may return to the ground state through a series of lower-energy jumps. These jumps give off radiation of longer wavelength than the radiation that caused the original excitation.

PROBLEM-SOLVING SUMMARY

Type of Problem	Relevant Physical Concepts	Related Examples
Find the wavelength of a spectral line in hydrogen.	All the spectral lines of hydrogen are described by Equation 31–2. Note that n must be greater than n', and that $n' = 1$ gives the Lyman series, $n' = 2$ gives the Balmer series, $n' = 3$ gives the Paschen series, and so on.	Example 31–1 Exercise 31–1
Find the radius and energy of a Bohr orbit in hydrogen. Also determine the speed of the electron.	The radius of a Bohr orbit varies with n^2 as follows: $r_n = r_1 n^2 = (5.29 \times 10^{-11}\text{ m})n^2$. The speed of the electron varies as $1/n$, as shown in Equation 31–6. The energy associated with an orbit is the kinetic energy of the electron plus the potential energy of the system, $U = -ke^2/r$.	Example 31–2 Active Example 31–2

Determine the wavelength (or frequency) of a photon that is emitted or absorbed when an electron jumps from one Bohr orbit to another.	When an electron jumps from an initial state n_i to a final state n_f, the wavelength of the associated photon is given by $\frac{1}{\lambda} = \left(\frac{2\pi^2 mk^2 e^4}{h^3 c}\right)\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right) = (1.097 \times 10^7\ \text{m}^{-1})\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$ If $n_i > n_f$, the photon is emitted; if $n_i < n_f$, the photon is absorbed. The photon's wavelength and frequency are related by the expression $\lambda f = c$.	Active Examples 31–1, 31–2
Calculate the wavelength of an electron in a Bohr orbit.	An integral number of wavelengths fit around the circumference of a Bohr orbit, starting with one wavelength for the ground state.	Example 31–3
Estimate the energy of a K-shell electron and the wavelength of a K_α X-ray for an element with atomic number Z.	The nuclear charge of $+Ze$ is partially screened by a K-shell electron, giving an effective nuclear charge of $+(Z-1)e$. Therefore, we use $Z - 1$ in Equation 31–9, which results in Equation 31–14.	Example 31–4 Active Example 31–3

CONCEPTUAL QUESTIONS

(Answers to odd-numbered Conceptual Questions can be found in the back of the book.)

1. Give a reason why the Thomson plum-pudding model does not agree with experimental observations.
2. Give a reason why the Rutherford solar-system model does not agree with experimental observations.
3. Cite one example of how the Bohr model disagrees with the quantum mechanical model of the hydrogen atom.
4. What observation led Rutherford to propose that atoms have a small nucleus containing most of the atom's mass?
5. Do you expect the light given off by **(a)** a neon sign or **(b)** an incandescent lightbulb to be continuous in distribution or in the form of a line spectrum? Explain.
6. In principle, how many spectral lines are there in any given series of hydrogen? Explain.
7. Is there an upper limit to the radius of an allowed Bohr orbit? Explain.
8. **(a)** Is there an upper limit to the wavelength of lines in the spectrum of hydrogen? Explain. **(b)** Is there a lower limit? Explain.
9. The principal quantum number, n, can increase without limit in the hydrogen atom. Does this mean that the energy of the hydrogen atom also can increase without limit? Explain.
10. For each of the following configurations of outermost electrons, state whether the configuration is allowed by the rules of quantum mechanics. If the configuration is not allowed, give the rule or rules that are violated. **(a)** $2d^1$, **(b)** $1p^7$, **(c)** $3p^5$, **(d)** $4g^6$.
11. **(a)** In the quantum mechanical model of the hydrogen atom, there is one value of n for which the angular momentum of the electron must be zero. What is this value of n? **(b)** Can the angular momentum of the electron be zero in states with other values of n? Explain.
12. Would you expect characteristic X-rays to be emitted by **(a)** helium atoms or **(b)** lithium atoms in their ground state? Explain.
13. The elements fluorine, chlorine, and bromine are found to exhibit similar chemical properties. Explain.

PROBLEMS AND CONCEPTUAL EXERCISES

Note: Answers to odd-numbered Problems and Conceptual Exercises can be found in the back of the book. **IP** *denotes an integrated problem, with both conceptual and numerical parts;* **BIO** *identifies problems of biological or medical interest;* **CE** *indicates a conceptual exercise.* **Predict/Explain** *problems ask for two responses:* **(a)** *your prediction of a physical outcome, and* **(b)** *the best explanation among three provided. On all problems, red bullets (•, ••, •••) are used to indicate the level of difficulty.*

SECTION 31–1 EARLY MODELS OF THE ATOM

1. • The electron in a hydrogen atom is typically found at a distance of about 5.3×10^{-11} m from the nucleus, which has a diameter of about 1.0×10^{-15} m. If you assume the hydrogen atom to be a sphere of radius 5.3×10^{-11} m, what fraction of its volume is occupied by the nucleus?
2. • Referring to Problem 1, suppose the nucleus of the hydrogen atom were enlarged to the size of a baseball (diameter = 7.3 cm). At what typical distance from the center of the baseball would you expect to find the electron?
3. •• Copper atoms have 29 protons in their nuclei. If the copper nucleus is a sphere with a diameter of 4.8×10^{-15} m, find the work required to bring an alpha particle (charge = $+2e$) from rest at infinity to the "surface" of the nucleus.
4. •• In Rutherford's scattering experiments, alpha particles (charge = $+2e$) were fired at a gold foil. Consider an alpha particle with an initial kinetic energy K heading directly for the nucleus of a gold atom (charge = $+79e$). The alpha particle will come to rest when all its initial kinetic energy has been converted to electrical potential energy. Find the distance of closest approach between the alpha particle and the gold nucleus for the case $K = 3.0$ MeV.

SECTION 31–2 THE SPECTRUM OF ATOMIC HYDROGEN

5. • Find the wavelength of the Balmer series spectral line corresponding to $n = 15$.

6. • What is the smallest value of n for which the wavelength of a Balmer series line is less than 400 nm?

7. • Find the wavelength of the three longest-wavelength lines of the Lyman series.

8. • Find the wavelength of the three longest-wavelength lines of the Paschen series.

9. • Find **(a)** the longest wavelength in the Lyman series and **(b)** the shortest wavelength in the Paschen series.

10. •• In Table 31–1 we see that the Paschen series corresponds to $n' = 3$ in Equation 31–2, and that the Brackett series corresponds to $n' = 4$. **(a)** Show that the ranges of wavelengths of these two series overlap. **(b)** Is there a similar overlap between the Balmer series and the Paschen series? Verify your answer.

SECTION 31–3 BOHR'S MODEL OF THE HYDROGEN ATOM

11. • **CE Predict/Explain (a)** If the mass of the electron were magically doubled, would the ionization energy of hydrogen increase, decrease, or stay the same? **(b)** Choose the *best explanation* from among the following:
    - **I.** The ionization energy would increase because the increased mass would mean the electron would orbit closer to the nucleus and would require more energy to move to infinity.
    - **II.** The ionization energy would decrease because a more massive electron is harder to hold in orbit, and therefore it is easier to remove the electron and leave the hydrogen ionized.
    - **III.** The ionization energy would be unchanged because, just like in gravitational orbits, the orbit of the electron is independent of its mass. As a result, there is no change in the energy required to move it to infinity.

12. • **CE** Consider the Bohr model as applied to the following three atoms: (A) neutral hydrogen in the state $n = 2$; (B) singly ionized helium in the state $n = 1$; (C) doubly ionized lithium in the state $n = 3$. Rank these three atoms in order of increasing Bohr radius. Indicate ties where appropriate.

13. • **CE** Consider the Bohr model as applied to the following three atoms: (A) neutral hydrogen in the state $n = 3$; (B) singly ionized helium in the state $n = 2$; (C) doubly ionized lithium in the state $n = 1$. Rank these three atoms in order of increasing energy. Indicate ties where appropriate.

14. • **CE** An electron in the $n = 1$ Bohr orbit has the kinetic energy K_1. In terms of K_1, what is the kinetic energy of an electron in the $n = 2$ Bohr orbit?

15. • Find the ratio v/c for an electron in the first excited state ($n = 2$) of hydrogen.

16. • Find the magnitude of the force exerted on an electron in the ground-state orbit of the Bohr model.

17. • How much energy is required to ionize hydrogen when it is in the $n = 4$ state?

18. • Find the energy of the photon required to excite a hydrogen atom from the $n = 2$ state to the $n = 5$ state.

19. •• **CE** In the Bohr model, the potential energy of a hydrogen atom in the nth orbit has a value we will call U_n. What is the potential energy of a hydrogen atom when the electron is in the $(n + 1)$th Bohr orbit? Give your answer in terms of U_n and n.

20. •• A hydrogen atom is in its second excited state, $n = 3$. Using the Bohr model of hydrogen, find **(a)** the linear momentum and **(b)** the angular momentum of the electron in this atom.

21. •• Referring to Problem 20, find **(a)** the kinetic energy of the electron, **(b)** the potential energy of the atom, and **(c)** the total energy of the atom. Give your results in eV.

22. •• Initially, an electron is in the $n = 3$ state of hydrogen. If this electron acquires an additional 1.23 eV of energy, what is the value of n in the final state of the electron?

23. •• Identify the initial and final states if an electron in hydrogen emits a photon with a wavelength of 656 nm.

24. •• **IP** An electron in hydrogen absorbs a photon and jumps to a higher orbit. **(a)** Find the energy the photon must have if the initial state is $n = 3$ and the final state is $n = 5$. **(b)** If the initial state was $n = 5$ and the final state $n = 7$, would the energy of the photon be greater than, less than, or the same as that found in part (a)? Explain. **(c)** Calculate the photon energy for part (b).

25. •• **IP** Consider the following four transitions in a hydrogen atom:

    (i) $n_i = 2, n_f = 6$ (ii) $n_i = 2, n_f = 8$

    (iii) $n_i = 7, n_f = 8$ (iv) $n_i = 6, n_f = 2$

    Find **(a)** the longest- and **(b)** the shortest-wavelength photon that can be emitted or absorbed by these transitions. Give the value of the wavelength in each case. **(c)** For which of these transitions does the atom lose energy? Explain.

26. •• **IP Muonium** Muonium is a hydrogen-like atom in which the electron is replaced with a muon, a fundamental particle with a charge of $-e$ and a mass equal to $207m_e$. (The muon is sometimes referred to loosely as a "heavy electron.") **(a)** What is the Bohr radius of muonium? **(b)** Will the wavelengths in the Balmer series of muonium be greater than, less than, or the same as the wavelengths in the Balmer series of hydrogen? Explain. **(c)** Calculate the longest wavelength of the Balmer series in muonium.

27. •• **IP (a)** Find the radius of the $n = 4$ Bohr orbit of a doubly ionized lithium atom (Li^{2+}, $Z = 3$). **(b)** Is the energy required to raise an electron from the $n = 4$ state to the $n = 5$ state in Li^{2+} greater than, less than, or equal to the energy required to raise an electron in hydrogen from the $n = 4$ state to the $n = 5$ state? Explain. **(c)** Verify your answer to part (b) by calculating the relevant energies.

28. •• Applying the Bohr model to a triply ionized beryllium atom (Be^{3+}, $Z = 4$), find **(a)** the shortest wavelength of the Lyman series for Be^{3+} and **(b)** the ionization energy required to remove the final electron in Be^{3+}.

29. •• **(a)** Calculate the time required for an electron in the $n = 2$ state of hydrogen to complete one orbit about the nucleus. **(b)** The typical "lifetime" of an electron in the $n = 2$ state is roughly 10^{-8} s—after this time the electron is likely to have dropped back to the $n = 1$ state. Estimate the number of orbits an electron completes in the $n = 2$ state before dropping to the ground state.

30. •• **IP** The kinetic energy of an electron in a particular Bohr orbit of hydrogen is 1.35×10^{-19} J. **(a)** Which Bohr orbit does the electron occupy? **(b)** Suppose the electron moves away from the nucleus to the next higher Bohr orbit. Does the kinetic energy of the electron increase, decrease, or stay the same? Explain. **(c)** Calculate the kinetic energy of the electron in the orbit referred to in part (b).

31. •• **IP** The potential energy of a hydrogen atom in a particular Bohr orbit is -1.20×10^{-19} J. **(a)** Which Bohr orbit does the electron occupy in this atom? **(b)** Suppose the electron moves away from the nucleus to the next higher Bohr orbit. Does the potential energy of the atom increase, decrease, or stay the

same? Explain. **(c)** Calculate the potential energy of the atom for the orbit referred to in part (b).

32. ••• Consider a head-on collision between two hydrogen atoms, both initially in their ground state and moving with the same speed. Find the minimum speed necessary to leave both atoms in their $n = 2$ state after the collision.

33. ••• A hydrogen atom is in the initial state $n_i = n$, where $n > 1$. **(a)** Find the frequency of the photon that is emitted when the electron jumps to state $n_f = n - 1$. **(b)** Find the frequency of the electron's orbital motion in the state n. **(c)** Compare your results for parts (a) and (b) in the limit of large n.

SECTION 31–4 DE BROGLIE WAVES AND THE BOHR MODEL

34. • **CE Predict/Explain** **(a)** Is the de Broglie wavelength of an electron in the $n = 2$ Bohr orbit of hydrogen greater than, less than, or equal to the de Broglie wavelength in the $n = 1$ Bohr orbit? **(b)** Choose the *best explanation* from among the following:

I. The de Broglie wavelength in the nth state is $2\pi r/n$, where r is proportional to n^2. Therefore, the wavelength increases with increasing n, and is greater for $n = 2$ than for $n = 1$.

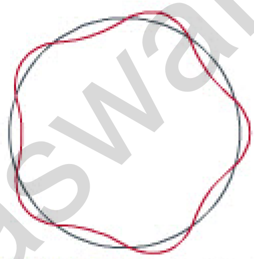
II. The de Broglie wavelength of an electron in the nth state is such that n wavelengths fit around the circumference of the orbit. Therefore, $\lambda = 2\pi r/n$ and the wavelength for $n = 2$ is less than for $n = 1$.

III. The de Broglie wavelength depends on the mass of the electron, and that is the same regardless of which state of the hydrogen atom the electron occupies.

35. • Find the de Broglie wavelength of an electron in the ground state of the hydrogen atom.

36. •• Find an expression for the de Broglie wavelength of an electron in the nth state of the hydrogen atom.

37. •• What is the radius of the hydrogen-atom Bohr orbit shown in **Figure 31–30**?



▲ **FIGURE 31–30** Problem 37

38. •• **(a)** Find the kinetic energy (in eV) of an electron whose de Broglie wavelength is equal to 0.5 Å, a typical atomic size. **(b)** Repeat part (a) for an electron with a wavelength equal to 10^{-15} m, a typical nuclear size.

SECTION 31–5 THE QUANTUM MECHANICAL HYDROGEN ATOM

39. • What are the allowed values of ℓ when the principal quantum number is $n = 5$?

40. • How many different values of m_ℓ are possible when the principal quantum number is $n = 4$?

41. • Give the value of the quantum number ℓ, if one exists, for a hydrogen atom whose orbital angular momentum has a magnitude of **(a)** $\sqrt{6}(h/2\pi)$, **(b)** $\sqrt{15}(h/2\pi)$, **(c)** $\sqrt{30}(h/2\pi)$, or **(d)** $\sqrt{36}(h/2\pi)$.

42. •• **IP** Hydrogen atom number 1 is known to be in the 4f state. **(a)** What is the energy of this atom? **(b)** What is the magnitude of this atom's orbital angular momentum? **(c)** Hydrogen atom number 2 is in the 5d state. Is this atom's energy greater than, less than, or the same as that of atom 1? Explain. **(d)** Is the magnitude of the orbital angular momentum of atom 1 greater than, less than, or the same as that of atom 2? Explain.

43. •• **IP** A hydrogen atom has an orbital angular momentum with a magnitude of $10\sqrt{57}(h/2\pi)$. **(a)** Determine the value of the quantum number ℓ for this atom. **(b)** What is the minimum possible value of this atom's principal quantum number, n? Explain. **(c)** If $10\sqrt{57}(h/2\pi)$ is the *maximum* orbital angular momentum this atom can have, what is its energy?

44. •• **IP** The electron in a hydrogen atom with an energy of −0.544 eV is in a subshell with 18 states. **(a)** What is the principal quantum number, n, for this atom? **(b)** What is the maximum possible orbital angular momentum this atom can have? **(c)** Is the number of states in the subshell with the next lowest value of ℓ equal to 16, 14, or 12? Explain.

45. •• **IP** Consider two different states of a hydrogen atom. In state I the maximum value of the magnetic quantum number is $m_\ell = 3$; in state II the corresponding maximum value is $m_\ell = 2$. Let L_{I} and L_{II} represent the magnitudes of the orbital angular momentum of an electron in states I and II, respectively. **(a)** Is L_{I} greater than, less than, or equal to L_{II}? Explain. **(b)** Calculate the ratio $L_{\text{I}}/L_{\text{II}}$.

SECTION 31–6 MULTIELECTRON ATOMS AND THE PERIODIC TABLE

46. • **CE** How many electrons can occupy **(a)** the 2p subshell and **(b)** the 3p subshell?

47. • **CE** **(a)** How many electrons can occupy the 3d subshell? **(b)** How many electrons can occupy the $n = 2$ shell?

48. • **CE** The electronic configuration of a given atom is $1s^2 2s^2 2p^6 3s^2 3p^1$. How many electrons are in this atom?

49. • Give the electronic configuration for the ground state of carbon.

50. • List the values of the four quantum numbers (n, ℓ, m_ℓ, m_s) for each of the electrons in the ground state of neon.

51. • Give the electronic configuration for the ground state of nitrogen.

52. • Give a list of all possible sets of the four quantum numbers (n, ℓ, m_ℓ, m_s) for electrons in the 3s subshell.

53. • Give a list of all possible sets of the four quantum numbers (n, ℓ, m_ℓ, m_s) for electrons in the 3p subshell.

54. •• List the values of the four quantum numbers (n, ℓ, m_ℓ, m_s) for each of the electrons in the ground state of magnesium.

55. •• The configuration of the outer electrons in Ni is $3d^8 4s^2$. Write out the complete electronic configuration for Ni.

56. •• Determine the number of different sets of quantum numbers possible for each of the following shells: **(a)** $n = 2$, **(b)** $n = 3$, **(c)** $n = 4$.

57. ••• Generalize the results of Problem 56 and show that the number of different sets of quantum numbers for the nth shell is $2n^2$.

58. •• Suppose that the 5d subshell is filled in a certain atom. Write out the 10 sets of four quantum numbers (n, ℓ, m_ℓ, m_s) for the electrons in this subshell.

SECTION 31–7 ATOMIC RADIATION

59. • **CE Predict/Explain (a)** In an X-ray tube, do you expect the wavelength of the characteristic X-rays to increase, decrease, or stay the same if the energy of the electrons striking the target is increased? **(b)** Choose the *best explanation* from among the following:
    I. Increasing the energy of the incoming electrons will increase the wavelength of the emitted X-rays.
    II. When the energy of the incoming electrons is increased, the energy of the X-rays is also increased; this, in turn, decreases the wavelength.
    III. The wavelength of characteristic X-rays depends only on the material used in the metal target, and does not change if the energy of incoming electrons is increased.

60. • **CE** Is the wavelength of the radiation that excites a fluorescent material greater than, less than, or equal to the wavelength of the radiation the material emits? Explain.

61. • Using the Bohr model, estimate the wavelength of the K_α X-ray in nickel ($Z = 28$).

62. • Using the Bohr model, estimate the energy of a K_α X-ray emitted by lead ($Z = 82$).

63. •• The K-shell ionization energy of iron is 8500 eV, and its L-shell ionization energy is 2125 eV. What is the wavelength of K_α X-rays emitted by iron?

64. •• An electron drops from the L shell to the K shell and gives off an X-ray with a wavelength of 0.0205 nm. What is the atomic number of this atom?

65. •• Consider an X-ray tube that uses platinum ($Z = 78$) as its target. **(a)** Use the Bohr model to estimate the minimum kinetic energy electrons must have in order for K_α X-rays to just appear in the X-ray spectrum of the tube. **(b)** Assuming the electrons are accelerated from rest through a voltage V, estimate the minimum voltage necessary to produce the K_α X-rays.

66. •• **BIO Photorefractive Keratectomy** A person's vision may be improved significantly by having the cornea reshaped with a laser beam, in a procedure known as photorefractive keratectomy. The excimer laser used in these treatments produces ultraviolet light with a wavelength of 193 nm. **(a)** What is the difference in energy between the two levels that participate in stimulated emission in the excimer laser? **(b)** How many photons from this laser are required to deliver an energy of 1.58×10^{-13} J to the cornea?

GENERAL PROBLEMS

67. • **CE** Consider the following three transitions in a hydrogen atom: (A) $n_i = 5, n_f = 2$; (B) $n_i = 7, n_f = 2$; (C) $n_i = 7, n_f = 6$. Rank the transitions in order of increasing **(a)** wavelength and **(b)** frequency of the emitted photon. Indicate ties where appropriate.

68. • **CE** Suppose an electron is in the ground state of hydrogen. **(a)** What is the highest-energy photon this system can absorb without dissociating the electron from the proton? Explain. **(b)** What is the lowest-energy photon this system can absorb? Explain.

69. • **CE** The electronic configuration of a particular carbon atom is $1s^2 2s^2 2p^1 3s^1$. Is this atom in its ground state or in an excited state? Explain.

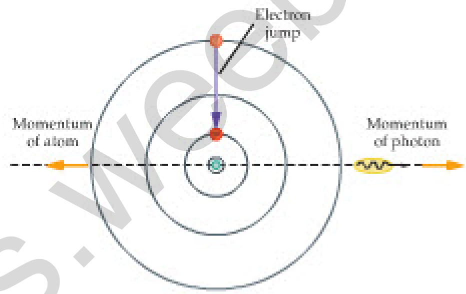
70. • **CE** The electronic configuration of a particular potassium atom is $1s^2 2s^2 2p^6 3s^2 3p^6 3d^1$. Is this atom in its ground state or in an excited state? Explain.

71. • **CE** Do you expect the ionization energy of sodium (Na) to be greater than, less than, or equal to the ionization energy of lithium (Li)? Explain.

72. • Find the minimum frequency a photon must have if it is to ionize the ground state of the hydrogen atom.

73. •• It was pointed out in Section 31–3 that intermolecular collisions at room temperature do not have enough energy to cause an excitation in hydrogen from the $n = 1$ state to the $n = 2$ state. Given that the average kinetic energy of a hydrogen atom in a high-temperature gas is $\frac{3}{2}kT$ (where k is Boltzmann's constant), find the minimum temperature required for atoms to have enough thermal energy to excite electrons from the ground state to the $n = 2$ state.

74. •• The electron in a hydrogen atom makes a transition from the $n = 4$ state to the $n = 2$ state, as indicated in Figure 31–31. **(a)** Determine the linear momentum of the photon emitted as a result of this transition. **(b)** Using your result to part (a), find the recoil speed of the hydrogen atom, assuming it was at rest before the photon was emitted.



▲ **FIGURE 31–31** Problems 74 and 75

75. •• **IP** Referring to Problem 74, find **(a)** the energy of the emitted photon and **(b)** the kinetic energy of the hydrogen atom after the photon is emitted. **(c)** Do you expect the sum of the energies in parts (a) and (b) to be greater than, less than, or the same as the difference in energy between the $n = 4$ and $n = 2$ states of hydrogen? Explain.

76. •• **BIO Laser Eye Surgery** In laser eye surgery, the laser emits a 1.45-ns pulse focused on a spot that is 34.0 μm in diameter. **(a)** If the energy contained in the pulse is 2.75 mJ, what is the power per square meter (the irradiance) associated with this beam? **(b)** Suppose a molecule with a diameter of 0.650 nm is irradiated by the laser beam. How much energy does the molecule receive in one pulse from the laser? (The energy obtained in part (b) is more than enough to dissociate a molecule.)

77. •• Consider an electron in the ground-state orbit of the Bohr model of hydrogen. **(a)** Find the time required for the electron to complete one orbit about the nucleus. **(b)** Calculate the current (in amperes) corresponding to the electron's motion.

78. •• A particular Bohr orbit in a hydrogen atom has a total energy of −0.85 eV. What are **(a)** the kinetic energy of the electron in this orbit and **(b)** the electric potential energy of the system?

79. •• The element helium is named for the Sun because that is where it was first observed. **(a)** What is the shortest wavelength that one would expect to observe from a singly ionized helium atom in the atmosphere of the Sun? **(b)** Suppose light with a wavelength of 388.9 nm is observed from singly ionized helium. What are the initial and final values of the quantum number n corresponding to this wavelength?

80. •• An ionized atom has only a single electron. The $n = 6$ Bohr orbit of this electron has a radius of 2.72×10^{-10} m. Find **(a)** the atomic number Z of this atom and **(b)** the total energy E of its $n = 3$ Bohr orbit.

81. •• Find the approximate wavelength of K_β X-rays emitted by molybdenum ($Z = 42$), and compare your result with Figure

31–22. (*Hint*: An electron in the M shell is shielded from the nucleus by the single electron in the K shell, plus all the electrons in the L shell.)

82. •• Referring to the hint given in Problem 81, estimate the wavelength of L_α X-rays in molybdenum.

83. •• **IP The Pickering Series** In 1896, the American astronomer Edward C. Pickering (1846–1919) discovered an unusual series of spectral lines in light from the hot star Zeta Puppis. After some time, it was determined that these lines are produced by singly ionized helium. In fact, the "Pickering series" is produced when electrons drop from higher levels to the $n = 4$ level of He^+. Spectral lines in the Pickering series have wavelengths given by

$$\frac{1}{\lambda} = C\left(\frac{1}{16} - \frac{1}{n^2}\right)$$

In this expression, $n = 5, 6, 7, \ldots$. **(a)** Do you expect the constant C to be greater than, less than, or equal to the Rydberg constant R? Explain. **(b)** Find the numerical value of C. **(c)** Pickering lines with $n = 6, 8, 10, \ldots$ correspond to Balmer lines in hydrogen with $n = 3, 4, 5, \ldots$. Verify this assertion for the $n = 6$ Pickering line.

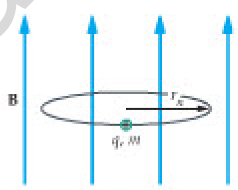
84. •• **IP Rydberg Atoms** There is no limit to the size a hydrogen atom can attain, provided it is free from disruptive outside influences. In fact, radio astronomers have detected radiation from large, so-called "Rydberg atoms" in the diffuse hydrogen gas of interstellar space. **(a)** Find the smallest value of n such that the Bohr radius of a single hydrogen atom is greater than 8.0 microns, the size of a typical single-celled organism. **(b)** Find the wavelength of radiation this atom emits when its electron drops from level n to level $n - 1$. **(c)** If the electron drops one more level, from $n - 1$ to $n - 2$, is the emitted wavelength greater than or less than the value found in part **(b)**? Explain.

85. ••• Consider a particle of mass m, charge q, and constant speed v moving perpendicular to a uniform magnetic field of magnitude B, as shown in Figure 31–32. The particle follows a circular path. Suppose the angular momentum of the particle about the center of its circular motion is quantized in the following way: $mvr = n\hbar$, where $n = 1, 2, 3, \ldots$, and $\hbar = h/2\pi$.

a. Show that the radii of its allowed orbits have the following values:

$$r_n = \sqrt{\frac{n\hbar}{qB}}$$

b. Find the speed of the particle in each allowed orbit.



▲ **FIGURE 31–32** Problem 85

86. ••• Consider a particle of mass m confined in a one-dimensional box of length L. In addition, suppose the matter wave associated with this particle is analogous to a wave on a string of length L that is fixed at both ends. Using the de Broglie relationship, show that **(a)** the quantized values of the linear momentum of the particle are

$$p_n = \frac{nh}{2L} \qquad n = 1, 2, 3, \ldots$$

and **(b)** the allowed energies of the particle are

$$E_n = n^2\left(\frac{h^2}{8mL^2}\right) \qquad n = 1, 2, 3, \ldots$$

87. ••• Show that the time required for an electron in the nth Bohr orbit of hydrogen to circle the nucleus once is given by

$$T = T_1 n^3 \qquad n = 1, 2, 3, \ldots$$

where $T_1 = h^3/4\pi^2 mk^2e^4$.

PASSAGE PROBLEMS

BIO Welding a Detached Retina

As a person ages, a normal part of the process is a shrinkage of the vitreous gel—the gelatinous substance that fills the interior of the eye. When this happens, the usual result is that the gel pulls away cleanly from the retina, with little or no adverse effect on the person's vision. This is referred to as posterior vitreous detachment. In some cases, however, the vitreous membrane that surrounds the vitreous gel pulls on the retina as the gel contracts, eventually creating a hole or a tear in the retina itself. At this point, fluid can seep through the hole in the retina and separate it from the underlying supporting cells—the retinal pigment epithelium. This process, known as rhegmatogenous retinal detachment, causes a blind spot in the person's vision. If not treated immediately, a retinal detachment can lead to permanent vision loss.

One way to treat a detached retina is to "weld" it back in place using a laser beam. This type of operation is performed with an argon laser because the blue-green light it produces passes through the vitreous gel with little absorption or damage, but is strongly absorbed by the red pigments in the retina and the retinal epithelium. An argon laser produces light consisting primarily of two wavelengths, 488.0 nm (blue-green) and 514.5 nm (green), and has a power output ranging between 1 W and 20 W.

88. •• Suppose an argon laser emits 1.49×10^{19} photons per second, half with a wavelength of 488.0 nm and half with a wavelength of 514.5 nm. What is the power output of this laser in watts?

A. 1.49 W B. 5.76 W
C. 5.92 W D. 6.07 W

89. •• A different type of laser also emits 1.49×10^{19} photons per second. If all of its photons have a wavelength of 414.0 nm, is its power output greater than, less than, or equal to the power output of the argon laser in Problem 88?

90. •• What is the power output of the laser in Problem 89?

A. 1.23 W B. 2.39 W
C. 4.80 W D. 7.16 W

91. •• What is the energy difference (in eV) between the states of an argon atom that are responsible for a photon with a wavelength of 514.5 nm?

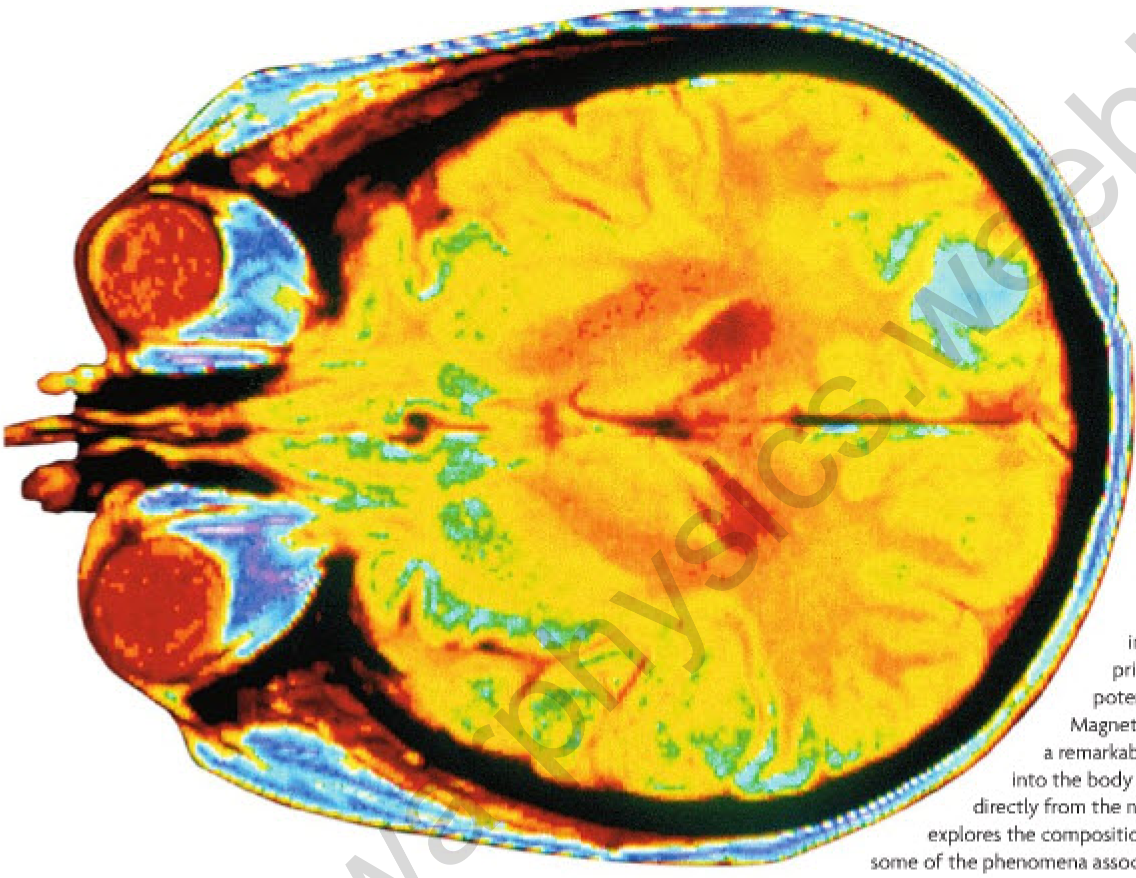
A. 2.13 eV B. 2.42 eV
C. 3.87 eV D. 6.40 eV

INTERACTIVE PROBLEMS

92. •• **IP Referring to Example 31–3** Suppose the electron is in a state whose standing wave consisting of two wavelengths. **(a)** Is the wavelength of this standing wave greater than or less than 1.33×10^{-9} m? **(b)** Find the wavelength of this standing wave.

93. •• **Referring to Example 31–3** **(a)** Which state has a de Broglie wavelength of 3.99×10^{-9} m? **(b)** What is the Bohr radius of this state?

32 Nuclear Physics and Nuclear Radiation



Until about a century ago, the only way for physicians to explore the inside of the human body was to cut it open. The discovery of X-rays gave us our first noninvasive imaging technique, but at a price—high-energy X-rays are potentially harmful to living tissues. Magnetic resonance imaging (MRI) opens a remarkable (and very safe) new window into the body by utilizing signals that come directly from the nuclei of its atoms. This chapter explores the composition of the atomic nucleus and some of the phenomena associated with it.

In the previous chapter our focus was almost entirely on the electrons in an atom. We studied their orbits and energies, their jumps from orbit to orbit, and the photons they emitted or absorbed. The nucleus played little role in these considerations. It was treated as a point object at the center of the atom, providing the electrostatic force necessary to hold the atom together.

The nucleus is much more than a point, however. Most nuclei contain a number of strongly interacting particles packed closely together in a more or less spherical assembly. The energies associated with changes inside a nucleus are orders of magnitude greater than those involved in chemical reactions, which involve only the electrons. It is for this reason that the Sun, which is powered by nuclear reactions, can burn for many billions of years—if the Sun were powered by chemical reactions it would have burned out after giving off light for only a few million years. This chapter considers the physics at play in the nucleus, discusses the nuclear reactions that occur in the Sun and in nuclear power plants, and describes a number of biomedical applications related to the nucleus.

32–1 The Constituents and Structure of Nuclei

The simplest nucleus is that of the hydrogen atom. This nucleus consists of a single **proton,** whose mass is about 1836 times greater than the mass of an electron and whose electric charge is $+e$. All other nuclei contain neutrons in addition to protons. The **neutron** is an electrically neutral particle (its electric charge is zero) with a mass just slightly greater than that of the proton. No other particles are found in nuclei. Collectively, protons and neutrons are referred to as **nucleons.**

Nuclei are characterized by the number and type of nucleons they contain. First, the **atomic number,** Z, is defined as the number of protons in a nucleus. In an electrically neutral atom, the number of electrons will also be equal to Z. Next, the number of neutrons in a nucleus is designated by the **neutron number,** N. Finally, the total number of nucleons in a nucleus is the **mass number,** A. These definitions are summarized in Table 32–1. Clearly, the mass number is the sum of the atomic number and the neutron number:

$$A = Z + N \qquad 32\text{–}1$$

TABLE 32–1 Numbers That Characterize a Nucleus

Z	Atomic number = number of protons in nucleus
N	Neutron number = number of neutrons in nucleus
A	Mass number = number of nucleons in nucleus

A special notation is used to indicate the composition of a nucleus. Consider, for example, an unstable but very useful form of carbon known as carbon-14. The nucleus of carbon-14 is written as follows:

$$^{14}_{6}\text{C}$$

In this expression, C represents the chemical element carbon. The number 6 is the atomic number of carbon, $Z = 6$, and the number 14 is the mass number of this nucleus, $A = 14$. This means that carbon-14 has 14 nucleons in its nucleus. The neutron number can be found by solving Equation 32–1 for N: $N = A - Z = 14 - 6 = 8$. Thus the nucleus of carbon-14 consists of 6 protons and 8 neutrons. The most common form of carbon is carbon-12, whose nucleus is designated as follows: $^{12}_{6}\text{C}$. This nucleus has 6 protons and 6 neutrons.

In general, the nucleus of an arbitrary element, X, with atomic number Z and mass number A, is represented as

$$^{A}_{Z}X$$

Note that once a given element is specified, the value of Z is known. As a result, the subscript Z is sometimes omitted.

EXERCISE 32–1

a. Give the symbol for a nucleus of aluminum that contains 14 neutrons.

b. Tritium is a type of "heavy hydrogen." The nucleus of tritium can be written as $^{3}_{1}\text{H}$. What is the number of protons and neutrons in a tritium nucleus?

SOLUTION

a. Looking up aluminum in the periodic table in Appendix E, we find that $Z = 13$. In addition, we are given that $N = 14$. Therefore, $A = Z + N = 27$, and hence the symbol for this nucleus is $^{27}_{13}\text{Al}$.

b. We obtain the number of protons from the subscript; therefore, $Z = 1$. The number of neutrons, from Equation 32–1, is $N = A - Z$, where A is the superscript. Therefore, the number of neutrons is $N = 3 - 1 = 2$.

All nuclei of a given element have the same number of protons, Z. They may have different numbers of neutrons, N, however. Nuclei with the same value of Z but different values of N are referred to as **isotopes.** For example, $^{12}_{6}\text{C}$ and $^{13}_{6}\text{C}$ are two isotopes of carbon, with $^{12}_{6}\text{C}$ being the most common one, constituting about 98.89% of naturally occurring carbon. About 1.11% of natural carbon is $^{13}_{6}\text{C}$. Values for the percentage abundance of various isotopes can be found in Appendix F.

Also given in Appendix F are the atomic masses of many common isotopes. These masses are given in terms of the **atomic mass unit,** u, defined so that the mass of one atom of $^{12}_{6}\text{C}$ is exactly 12 u. The value of u is as follows:

Definition of Atomic Mass Unit, u

$$1 \text{ u} = 1.660540 \times 10^{-27} \text{ kg} \qquad 32\text{–}2$$

SI unit: kg

Protons have a mass just slightly greater than 1 u, and the neutron is slightly more massive than the proton. The precise masses of the proton and neutron are given in Table 32–2, along with the mass of the electron.

TABLE 32–2 Mass and Charge of Particles in the Atom

Particle	Mass (kg)	Mass (MeV/c^2)	Mass (u)	Charge (C)
Proton	1.672623×10^{-27}	938.28	1.007276	$+1.6022 \times 10^{-19}$
Neutron	1.674929×10^{-27}	939.57	1.008665	0
Electron	9.109390×10^{-31}	0.511	0.0005485799	-1.6022×10^{-19}

When we consider nuclear reactions later in this chapter, an important consideration will be the energy equivalent of a given mass, as given by Einstein's famous relation, $E = mc^2$ (Equation 30–7). The energy equivalent of one atomic mass unit is

$$E = mc^2 = (1 \text{ u})c^2$$

$$= (1.660540 \times 10^{-27} \text{ kg})(2.998 \times 10^8 \text{ m/s})^2\left(\frac{1 \text{ eV}}{1.6022 \times 10^{-19} \text{ J}}\right)$$

$$= 931.5 \text{ MeV}$$

where 1 MeV $= 10^6$ eV. When we consider that the ionization energy of hydrogen is only 13.6 eV, it is clear that the energy equivalent of nucleons is enormous compared with typical atomic energies. In general, energies involving the nucleus are on the order of MeV, and energies associated with the electrons in an atom are on the order of eV. Finally, because mass and energy can be converted from one form to the other, it is common to express the atomic mass unit in terms of energy as follows:

$$1 \text{ u} = 931.5 \text{ MeV}/c^2 \qquad 32\text{–}3$$

The masses of the proton, neutron, and electron are also given in units of MeV/c^2 in Table 32–2.

Nuclear Size and Density

To obtain an estimate for the size of a nucleus, Rutherford did a simple calculation using energy conservation. He considered the case of a particle of charge $+q$ and mass m approaching a nucleus of charge $+Ze$ with a speed v. He further assumed that the approach was head-on. At some distance, d, from the center of the nucleus, the incoming particle comes to rest instantaneously, before turning around. It follows that the radius of the nucleus is less than d. In Example 32–1 we obtain a symbolic expression for the distance d.

EXAMPLE 32–1 SETTING A LIMIT ON THE RADIUS OF A NUCLEUS

A particle of mass m, charge $+q$, and speed v heads directly toward a distant, stationary nucleus of charge $+Ze$. Find the distance of closest approach between the incoming particle and the center of the nucleus.

PICTURE THE PROBLEM

The incoming particle moves on a line that passes through the center of the nucleus. Far from the nucleus, the particle's speed is v. The particle turns around (comes to rest instantaneously) a distance d from the center of the nucleus.

STRATEGY

We can find the distance d by applying energy conservation. In particular, the initial energy of the system is the kinetic energy of the particle, $\frac{1}{2}mv^2$, assuming the particle approaches the nucleus from infinity. The final energy is the electric potential energy, $U = kq_1q_2/r = k(Ze)q/d$. Setting these energies equal to each other allows us to solve for d.

SOLUTION

1. Write an expression for the initial energy of the system: $E_i = \frac{1}{2}mv^2$
2. Write an expression for the final energy of the system: $E_f = \frac{k(Ze)q}{d}$
3. Set the final energy equal to the initial energy and solve for d: $\frac{1}{2}mv^2 = \frac{k(Ze)q}{d}$

$$d = \frac{kZeq}{\left(\frac{1}{2}mv^2\right)}$$

INSIGHT

Notice that the distance of closest approach is inversely proportional to the initial kinetic energy of the incoming particle and directly proportional to the charge on the nucleus.

PRACTICE PROBLEM

Find the distance at which the speed of the incoming particle is equal to $\frac{1}{2}v$. [**Answer:** $\frac{kZeq}{\frac{3}{4}\left(\frac{1}{2}mv^2\right)}$]

Some related homework problems: Problem 6, Problem 7

Using the result obtained in Example 32–1, Rutherford found that for an alpha particle approaching a gold nucleus in one of his experiments, the distance of closest approach was 3.2×10^{-14} m. A similar calculation for alpha particles fired at silver atoms gives a closest approach distance of 2.0×10^{-14} m. This suggests that the size of the nucleus varies from element to element; in particular, the nucleus of silver is smaller than that of gold. In fact, more careful measurements since Rutherford's time have established that the average radius of a nucleus of mass number A is given approximately by the following expression:

$$r = (1.2 \times 10^{-15}\ \text{m})A^{1/3} \qquad 32\text{–}4$$

Notice that the length scale of the nucleus is on the order of 10^{-15} m, as opposed to the length scale of an atom, which is on the order of 10^{-10} m. Recall that 10^{-15} m is referred to as a *femtometer* (fm). To honor the pioneering work of Enrico Fermi (1901–1954) in the field of nuclear physics, the femtometer is often referred to as the **fermi:**

Definition of the Fermi, fm

1 fermi = 1 fm = 10^{-15} m

SI unit: m

We now use Equation 32–4 to find the radius of a particular nucleus.

EXERCISE 32–2

Find the radius of a $^{14}_{6}C$ nucleus.

SOLUTION

Substitute $A = 14$ in Equation 32–4:

$$r = (1.2 \times 10^{-15}\ \text{m})(14)^{1/3} = 2.9 \times 10^{-15}\ \text{m} = 2.9\ \text{fm}$$

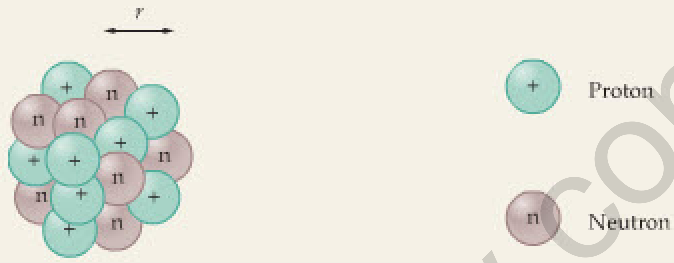
The fact that the radius of a nucleus depends on $A^{1/3}$ has interesting consequences for the density of the nucleus, and these are explored in the next Example.

EXAMPLE 32–2 NUCLEAR DENSITY

Using the expression $r = (1.2 \times 10^{-15}\ \text{m})A^{1/3}$, calculate the density of a nucleus with mass number A.

PICTURE THE PROBLEM

Our sketch shows a collection of neutrons and protons in a densely packed nucleus of radius r.



STRATEGY

To find the density of a nucleus, we must divide its mass, M, by its volume, V. Ignoring the small difference in mass between a neutron and a proton, we can express the mass of a nucleus as $M = Am$, where $m = 1.67 \times 10^{-27}$ kg. The volume of a nucleus is simply the volume of a sphere of radius r: $V = 4\pi r^3/3$.

SOLUTION

1. Write an expression for the mass of a nucleus: $M = Am = A(1.67 \times 10^{-27}\ \text{kg})$
2. Write an expression for the volume of a nucleus:
$$V = \tfrac{4}{3}\pi r^3 = \tfrac{4}{3}\pi[(1.2 \times 10^{-15}\ \text{m})A^{1/3}]^3 = \tfrac{4}{3}\pi(1.7 \times 10^{-45}\ \text{m}^3)A$$
3. Divide the mass by the volume to find the density:
$$\rho = \frac{M}{V} = \frac{Am}{\tfrac{4}{3}\pi(1.7 \times 10^{-45}\ \text{m}^3)A} = \frac{(1.67 \times 10^{-27}\ \text{kg})}{\tfrac{4}{3}\pi(1.7 \times 10^{-45}\ \text{m}^3)} = 2.3 \times 10^{17}\ \text{kg/m}^3$$

INSIGHT

Note that the density of a nucleus is found to be *independent* of the mass number, A. This means that a nucleus can be thought of as a collection of closely packed nucleons, much like a group of marbles in a bag. The neutrons in a nucleus serve to separate the protons, thereby reducing their mutual electrostatic repulsion.

The density of a nucleus is incredibly large. For example, a single teaspoon of nuclear matter would weigh about a trillion tons.

PRACTICE PROBLEM

Find the surface area of a nucleus in terms of the mass number, A, assuming it to be a sphere. [**Answer:** area = $(1.8 \times 10^{-29}\ \text{m}^2)A^{2/3}$]

Some related homework problems: Problem 8, Problem 9

Nuclear Stability

We know that like charges repel one another, and that the force of repulsion increases rapidly with decreasing distance. It follows that protons in a nucleus, with a separation of only about a fermi, must exert relatively large forces on one another. Applying Coulomb's law (Equation 19–5), we find the following force for two protons (charge $+e$) separated by a distance of 10^{-15} m:

$$F = \frac{ke^2}{r^2} = 230\ \text{N}$$

The acceleration such a force would give to a proton is $a = F/m = (230\ \text{N})/(1.67 \times 10^{-27}\ \text{kg}) = 1.4 \times 10^{29}\ \text{m/s}^2$, which is about 10^{28} times greater than the acceleration of gravity! Thus, if protons in the nucleus experienced only the electrostatic force, the nucleus would fly apart in an instant. It follows that large attractive forces must also act within the nucleus.

The attractive force that holds a nucleus together is called the **strong nuclear force.** This force has the following properties:

- The strong force is short range, acting only to distances of a couple fermis.
- The strong force is attractive and acts with nearly equal strength between protons and protons, protons and neutrons, and neutrons and neutrons.

In addition, the strong nuclear force does not act on electrons. As a result, it has no effect on the chemical properties of an atom.

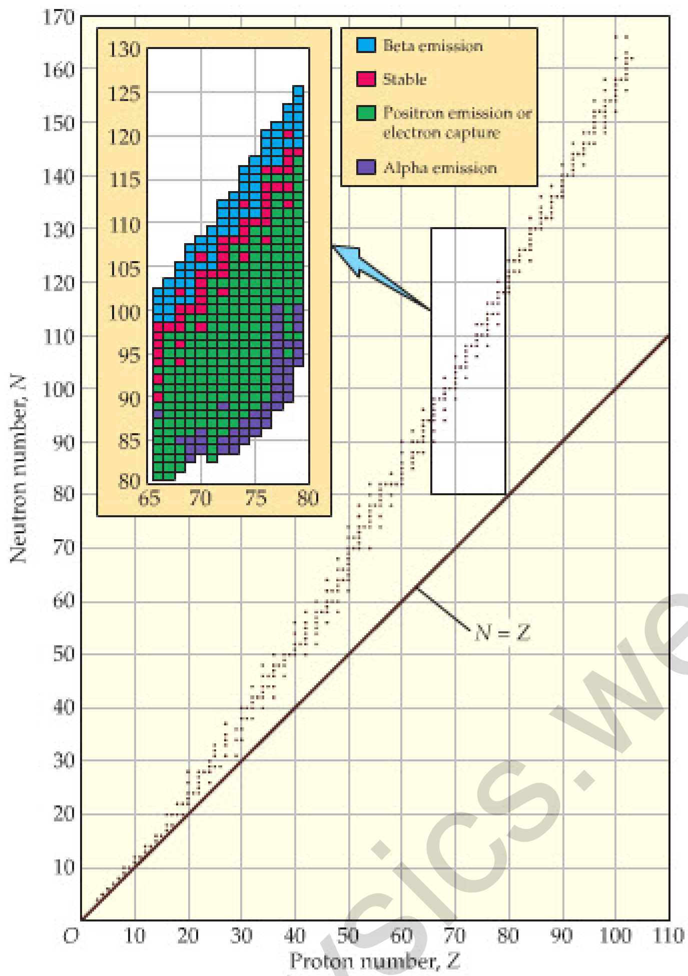


FIGURE 32–1 ***N* and *Z* for stable and unstable nuclei**

Stable nuclei with proton numbers less than 104 are indicated by small red dots. Notice that large nuclei have significantly more neutrons, *N*, than protons, *Z*. The inset shows unstable nuclei and their decay modes for proton numbers between 65 and 80.

It is the competition between the repulsive electrostatic forces and the attractive strong nuclear forces that determines whether a given nucleus is stable. **Figure 32–1** shows the neutron number, *N*, and atomic number (proton number), *Z*, for nuclei that are stable. Note that nuclei of relatively small atomic number are most stable when the numbers of protons and neutrons in the nucleus are approximately equal, $N = Z$. For example, ${}^{12}_{6}\text{C}$ and ${}^{13}_{6}\text{C}$ are both stable. As the atomic number increases, however, we see that the points corresponding to stable nuclei deviate from the line $N = Z$. In fact, we see that large stable nuclei tend to contain significantly more neutrons than protons, as in the case of ${}^{185}_{75}\text{Re}$. Since all nucleons experience the strong nuclear force, but only the protons experience the electrostatic force, the neutrons effectively "dilute" the nuclear charge density, reducing the effect of the repulsive forces that otherwise would make the nucleus disintegrate.

As the number of protons in a nucleus increases, however, a point is reached at which the strong nuclear forces are no longer able to compensate for the repulsive forces between protons. In fact, the largest number of protons in a stable nucleus is $Z = 83$, corresponding to the element bismuth. Nuclei with more than 83 protons are simply not stable, as can be seen by noting that all elements with $Z > 83$ in Appendix F decay in a finite time—that is, they have a finite half-life. (We shall discuss the half-life of unstable nuclei in detail in Section 32–3.) The nuclei of many well-known elements, such as radon and uranium, disintegrate—decay—in a finite time. We turn now to a discussion of the various ways in which an unstable nucleus decays.

32–2 Radioactivity

An unstable nucleus does not last forever—sooner or later it changes its composition by emitting a particle of one type or another. Alternatively, a nucleus in an excited state may rearrange its nucleons into a lower-energy state and emit a high-energy photon. We refer to such processes as the **decay** of a nucleus, and the various emissions that result are known collectively as **radioactivity.**

When a nucleus undergoes radioactive decay, the mass of the system decreases. That is, the mass of the initial nucleus before decay is greater than the mass of the resulting nucleus plus the mass of the emitted particle. The difference in mass, $\Delta m < 0$, appears as a release of energy, according to the relation $E = |\Delta m|c^2$. The mass difference for any given decay can be determined by referring to Appendix F. Note that the atomic masses listed in Appendix F are the masses of neutral atoms; that is, the values given in the table include the mass of the electrons in an atom. This factor must be considered whenever the mass difference of a reaction is calculated, as we shall see later in this section.

Three types of particles with mass are given off during the various processes of radioactive decay. They are as follows:

- Alpha (α) particles, which are the nuclei of ${}^4_2\text{He}$. Note that an alpha particle consists of two protons and two neutrons. When a nucleus decays by giving off alpha particles, we say that it emits α rays.
- Electrons, also referred to as beta (β) particles. The electrons given off by a nucleus are called β rays, or β^- rays to be more precise. (The minus sign is a reminder that the charge of an electron is $-e$.)
- **Positrons,** which have the same mass as an electron but a charge of $+e$. If a nucleus gives off positrons, we say that it emits β^+ rays. (A positron, which is short for "positive electron," is the **antiparticle** of the ordinary electron. Positrons will be considered in greater detail in Section 32–7.)

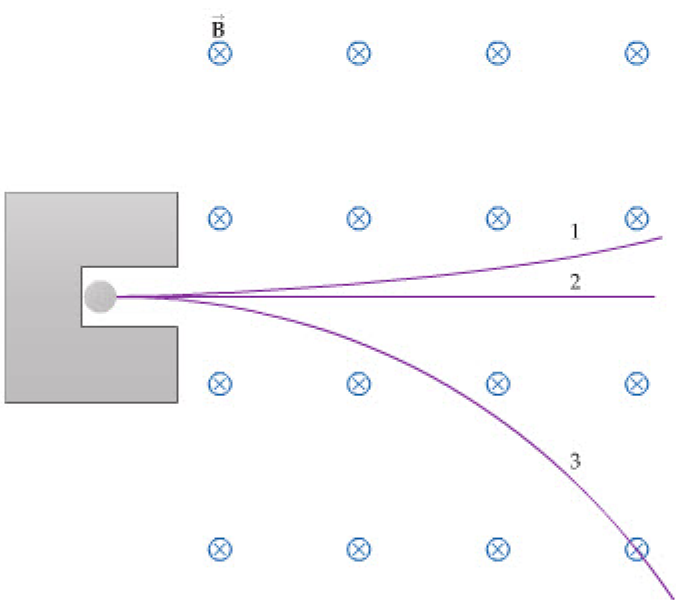
Finally, radioactivity may take the form of a photon rather than a particle with nonzero mass:

- A nucleus in an excited state may emit a high-energy photon, a gamma (γ) ray, and drop to a lower-energy state.

The following Conceptual Checkpoint examines the behavior of radioactivity in a magnetic field.

CONCEPTUAL CHECKPOINT 32–1 IDENTIFY THE RADIATION

A sample of radioactive material is placed at the bottom of a small hole drilled into a piece of lead. The sample emits α rays, β^- rays, and γ rays into a region of constant magnetic field. It is observed that the radiation follows three distinct paths, 1, 2, and 3, as shown in the sketch. Identify each path with the corresponding type of radiation: **(a)** path 1, α rays; path 2, β^- rays; path 3, γ rays; **(b)** path 1, β^- rays; path 2, γ rays; path 3, α rays; or **(c)** path 1, α rays; path 2, γ rays; path 3, β^- rays.



REASONING AND DISCUSSION

First, because γ rays are uncharged, they are not deflected by the magnetic field. It follows that path 2 corresponds to γ rays.

Next, the right-hand rule for the magnetic force (Section 22–2) indicates that positively charged particles will be deflected upward, and negatively charged particles will be deflected downward. As a result, path 1 corresponds to α rays, and path 3 corresponds to β^- rays.

ANSWER

(c) Path 1, α rays; path 2, γ rays; path 3, β^- rays

Radioactivity was discovered by the French physicist Antoine Henri Becquerel (1852–1908) in 1896 when he observed that uranium was able to expose photographic emulsion, even when the emulsion was covered. Thus radioactivity has the ability to penetrate various materials. In fact, the various types of radioactivity were initially named according to their ability to penetrate, starting with α rays, which are the least penetrating. Typical penetrating abilities for α, β, and γ rays are as follows:

- α rays can barely penetrate a sheet of paper.
- β rays (both β^- and β^+) can penetrate a few millimeters of aluminum.
- γ rays can penetrate several centimeters of lead.

We turn now to a detailed examination of each of these types of decay.

PROBLEM-SOLVING NOTE

The Effects of Alpha Decay

In alpha decay, the total number of protons is the same before and after the reaction. The same is true of the total number of neutrons. On the other hand, the mass number of the daughter nucleus is 4 less than the mass number of the parent nucleus. Similarly, the atomic number of the daughter nucleus is 2 less than the atomic number of the parent nucleus.

Alpha Decay

When a nucleus decays by giving off an α particle (^4_2He), it loses two protons and two neutrons. As a result, its atomic number, Z, decreases by 2, and its mass number decreases by 4. Symbolically, we can write this process as follows:

$$^A_Z X \longrightarrow {}^{A-4}_{Z-2}Y + {}^4_2\text{He}$$

where X is referred to as the **parent nucleus**, and Y is the **daughter nucleus.** Notice that the sum of the atomic numbers on the right side of this process is equal to the atomic number on the left side; similar remarks apply to the mass numbers.

The next Example considers the alpha decay of uranium-238. We first use conservation of atomic number and mass number to determine the identity of the daughter nucleus. Next, we use the mass difference to calculate the amount of energy released by the decay.

PROBLEM-SOLVING NOTE

Atomic Masses Include Electrons

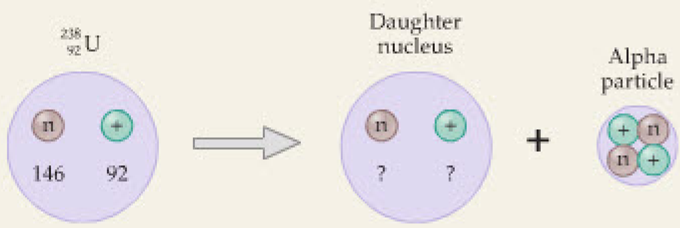
When calculating the mass difference in a nuclear reaction, be sure to note that the atomic masses in Appendix F include the electrons that would be present in a neutral atom. The only item in Appendix F to which this does *not* apply is the neutron.

EXAMPLE 32–3 URANIUM DECAY

Determine **(a)** the daughter nucleus and **(b)** the energy released when $^{238}_{92}\text{U}$ undergoes alpha decay.

PICTURE THE PROBLEM

Our sketch shows the specified decay of $^{238}_{92}\text{U}$ into a daughter nucleus plus an α particle. Note that the number of neutrons and protons is indicated for $^{238}_{92}\text{U}$. The α particle consists of two neutrons and two protons.



STRATEGY

a. We can identify the daughter nucleus by requiring that the total number of neutrons and protons be the same before and after the decay.

b. To find the energy, we first calculate the mass before and after the decay. The magnitude of the difference in mass, $|\Delta m|$, times the speed of light squared, c^2, gives the amount of energy released.

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SOLUTION

Part (a)

1. Determine the number of neutrons and protons in the daughter nucleus. Add these numbers together to obtain the mass number of the daughter nucleus:

$$N = 146 - 2 = 144$$
$$Z = 92 - 2 = 90$$
$$A = N + Z = 144 + 90 = 234$$

2. Referring to Appendix F, we see that the daughter nucleus is thorium-234:

$${}^{234}_{90}\text{Th}$$

Part (b)

3. Use Appendix F to find the initial mass of the system; that is, the mass of a ${}^{238}_{92}\text{U}$ atom:

$$m_i = 238.050786 \text{ u}$$

4. Use Appendix F to find the final mass of the system; that is, the mass of a ${}^{234}_{90}\text{Th}$ atom plus the mass of a ${}^{4}_{2}\text{He}$ atom:

$$m_f = 234.043596 \text{ u} + 4.002603 \text{ u} = 238.046199 \text{ u}$$

5. Calculate the mass difference and the corresponding energy release (recall that $1 \text{ u} = 931.5 \text{ MeV}/c^2$):

$$\Delta m = m_f - m_i = 238.046199 \text{ u} - 238.050786 \text{ u} = -0.004587 \text{ u}$$

$$E = |\Delta m|c^2 = (0.004587 \text{ u})\left(\frac{931.5 \text{ MeV}/c^2}{1 \text{ u}}\right)c^2 = 4.273 \text{ MeV}$$

INSIGHT

Each of the masses used in this decay includes the electrons of the corresponding neutral atom. Since the number of electrons initially (92) is the same as the number of electrons in a thorium atom (90) plus the number of electrons in a helium atom (2), the electrons make no contribution to the total mass difference, Δm.

PRACTICE PROBLEM

Find the daughter nucleus and energy released when ${}^{226}_{88}\text{Ra}$ undergoes alpha decay. [**Answer:** ${}^{222}_{86}\text{Rn}$, 4.871 MeV]

Some related homework problems: Problem 20, Problem 21

A considerable amount of energy is released in the alpha decay of uranium-238. As indicated in **Figure 32–2**, this energy appears as kinetic energy of the daughter nucleus and the α particle as they move off in opposite directions. The following Conceptual Checkpoint compares the kinetic energy of the daughter nucleus with that of the α particle.

CONCEPTUAL CHECKPOINT 32–2 COMPARE KINETIC ENERGIES

When a stationary ${}^{238}_{92}\text{U}$ nucleus decays into a ${}^{234}_{90}\text{Th}$ nucleus and an α particle, is the kinetic energy of the α particle **(a)** greater than, **(b)** less than, or **(c)** the same as the kinetic energy of the ${}^{234}_{90}\text{Th}$ nucleus?

REASONING AND DISCUSSION

Because no external forces are involved in the decay process, it follows that the momentum of the system is conserved. Letting subscript 1 refer to the ${}^{234}_{90}\text{Th}$ nucleus, and subscript 2 to the α particle, the condition for momentum conservation is $m_1v_1 = m_2v_2$. Solving for the speed of the α particle, we have $v_2 = (m_1/m_2)v_1$; that is, the α particle has the greater speed, since m_1 is greater than m_2.

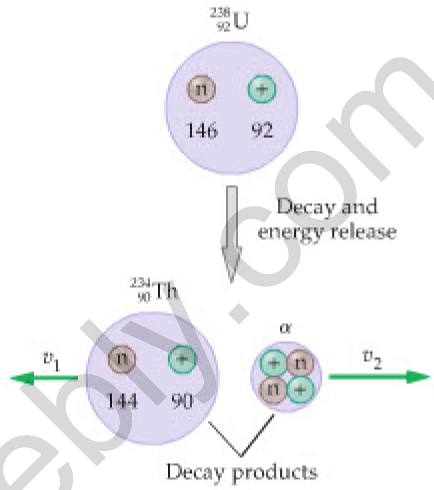
The fact that the α particle has the greater speed does not, in itself, ensure that its kinetic energy is the greater of the two. After all, the α particle also has the smaller mass. To compare the kinetic energies, note that the kinetic energy of the ${}^{234}_{90}\text{Th}$ nucleus is $\frac{1}{2}m_1{v_1}^2$. The kinetic energy of the α particle is $\frac{1}{2}m_2{v_2}^2 = \frac{1}{2}m_2[(m_1/m_2)v_1]^2 = \frac{1}{2}m_1{v_1}^2(m_1/m_2) > \frac{1}{2}m_1{v_1}^2$. Therefore, the α particle carries away the majority of the kinetic energy released in the decay. In this particular case, the α particle has a kinetic energy that is $m_1/m_2 = 234/4 = 58.5$ times greater than the kinetic energy of the ${}^{234}_{90}\text{Th}$ nucleus.

ANSWER

(a) The α particle has the greater kinetic energy.

Although you may not be aware of it, many homes are protected from the hazards of fire by a small device—a smoke detector—that uses the alpha decay of a man-made radioactive isotope, $^{241}_{95}Am$. In this type of smoke detector, a minute quantity of $^{241}_{95}Am$ is placed between two metal plates connected to a battery or other source of emf. The α particles emitted by the radioactive source ionize the air, allowing a measurable electric current to flow between the plates. As long as this current flows, the smoke detector remains silent. When smoke enters the detector, however, the ionized air molecules tend to stick to the smoke particles and become neutralized. This reduces the current and triggers the alarm. These "ionization" smoke detectors are more sensitive than the "photoelectric" detectors that rely on the thickness of smoke to dim a beam of light.

REAL-WORLD PHYSICS
Smoke detector



▲ FIGURE 32–2 Alpha decay of uranium-238

When $^{238}_{92}U$ decays into $^{234}_{90}Th$ and an alpha particle, the mass of the system decreases. The "lost" mass is actually converted into energy; it appears as the kinetic energy of the $^{234}_{90}Th$ nucleus and the alpha particle.

Beta Decay

The basic process that occurs in beta decay is the conversion of a neutron to a proton and an electron:

$$^1_0n \longrightarrow {}^1_1p + e^-$$

Thus when a nucleus decays by giving off an electron, its mass number is unchanged (since protons and neutrons count equally in determining A), but its atomic number increases by 1. This process can be represented as follows:

$$^A_ZX \longrightarrow {}_{Z+1}^{\;\;A}Y + e^-$$

Similarly, if a nucleus undergoes a different type of decay in which it gives off a positron, the process can be written

$$^A_ZX \longrightarrow {}_{Z-1}^{\;\;A}Y + e^+$$

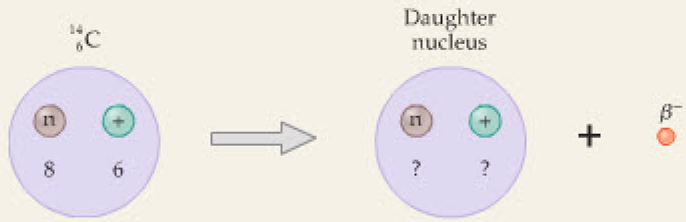
In the next Example we determine the energy that is released as carbon-14 undergoes beta decay.

EXAMPLE 32–4 BETA DECAY OF CARBON-14

Find **(a)** the daughter nucleus and **(b)** the energy released when $^{14}_{6}C$ undergoes β^- decay.

PICTURE THE PROBLEM

In our sketch we show $^{14}_{6}C$ giving off a β^- particle and converting into a daughter nucleus. The number of neutrons and protons in the $^{14}_{6}C$ nucleus is indicated. Note that the β^- particle is not a nucleon.



STRATEGY

a. We can identify the daughter nucleus by requiring that the total number of nucleons be the same before and after the decay. The number of neutrons will be decreased by 1, and the number of protons will be increased by 1.

b. To find the energy, we begin by calculating the mass before and after the decay. The magnitude of the difference in mass, $|\Delta m|$, times the speed of light squared, c^2, gives the amount of energy released.

SOLUTION

Part (a)

1. Determine the number of neutrons and protons in the daughter nucleus. Add these numbers together to obtain the mass number of the daughter nucleus:
   $N = 8 - 1 = 7$
   $Z = 6 + 1 = 7$
   $A = N + Z = 7 + 7 = 14$

2. Referring to Appendix F, we see that the daughter nucleus is nitrogen-14:
   $^{14}_{7}N$

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Part (b)

3. Use Appendix F to find the initial mass of the system; that is, the mass of a $^{14}_{6}C$ atom:

$$m_i = 14.003242\ u$$

4. Use Appendix F to find the final mass of the system, which is simply the mass of a $^{14}_{7}N$ atom (the mass of the β^- particle is included in the mass of $^{14}_{7}N$, as we point out in the Insight):

$$m_f = 14.003074\ u$$

5. Calculate the mass difference and the corresponding energy release (recall that $1\ u = 931.5\ MeV/c^2$):

$$\Delta m = m_f - m_i = 14.003074\ u - 14.003242\ u = -0.000168\ u$$

$$E = |\Delta m| c^2 = (0.000168\ u)\left(\frac{931.5\ MeV/c^2}{1\ u}\right)c^2 = 0.156\ MeV$$

INSIGHT

With regard to the masses used in this calculation, note that the mass of $^{14}_{6}C$ includes the mass of its 6 electrons. Similarly, the mass of $^{14}_{7}N$ includes the mass of 7 electrons in the neutral $^{14}_{7}N$ atom. However, when the $^{14}_{6}C$ nucleus converts to a $^{14}_{7}N$ nucleus, the number of electrons orbiting the nucleus is still 6. In effect, the newly created $^{14}_{7}N$ atom is missing one electron; that is, the mass of the $^{14}_{7}N$ atom includes the mass of one too many electrons. Therefore, it is not necessary to add the mass of an electron (representing the β^- particle) to the final mass of the system, because this extra electron mass is already included in the mass of the $^{14}_{7}N$ atom.

PRACTICE PROBLEM

Find the daughter nucleus and the energy released when $^{234}_{90}Th$ undergoes β^- decay. [**Answer:** $^{234}_{91}Pa$, 0.274 MeV]

Some related homework problems: Problem 22, Problem 24

PROBLEM-SOLVING NOTE

The Effects of Beta Decay

In β^- decay, the number of neutrons decreases by 1, and the number of protons increases by 1. The mass number is unchanged.

Referring to Conceptual Checkpoint 32–2, we would expect the kinetic energy of an electron emitted during beta decay to account for most of the energy released by the decay process. In fact, energy conservation allows us to predict the precise amount of kinetic energy the electron should have. It turns out, however, that when the kinetic energy of emitted electrons is measured, a range of values is obtained, as indicated in Figure 32–3. Specifically, we find that all electrons given off in beta decay have energies that are less than would be predicted by energy conservation. On closer examination it is found that beta decay seems to violate conservation of linear and angular momentum as well! For these reasons, beta decay was an interesting and intriguing puzzle for physicists.

The resolution of this puzzle was given by Pauli in 1930, when he proposed that the "missing" energy and momentum were actually carried off by a particle that was not observed in the experiments. For this particle to have been unobserved, it must have zero charge and little or no mass. Fermi dubbed Pauli's hypothetical particle the **neutrino**, meaning, literally, "little neutral one." We now know that neutrinos do in fact exist and that they account exactly for the missing energy and momentum. They interact so weakly with matter, however, that it wasn't until 1950 that they were observed experimentally. Recent experiments on neutrinos given off by the Sun provide the best evidence yet that the mass of a neutrino is in fact finite—though extremely small. In fact, the best estimate of the neutrino mass at this time is that it is less than about $7\ eV/c^2$. For comparison, the mass of the electron is $511{,}000\ eV/c^2$.

To give an indication of just how weakly neutrinos interact with matter, only one in every 200 million neutrinos that pass through the Earth interacts with it in any way. As far as the neutrinos are concerned, it is almost as if the Earth did not exist. Right now, in fact, billions of neutrinos are passing through your body every second without the slightest effect.

We can now write the correct expression for the decay of a neutron. Indicating the electron neutrino with the symbol ν_e, we have the following:

$$^{1}_{0}n \longrightarrow {}^{1}_{1}p + e^- + \overline{\nu}_e \qquad 32\text{–}5$$

The bar over the neutrino symbol indicates that the neutrino given off in β^- decay is actually an **antineutrino**, the antiparticle counterpart of the neutrino (just as the

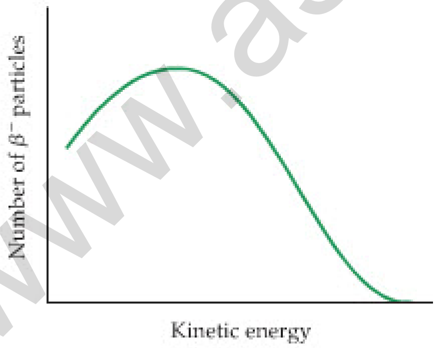


FIGURE 32–3 Energy of electrons emitted in β^- decay

When electrons are emitted during β^- decay, they come off with a range of energies. This indicates that another particle (the neutrino) must also be taking away some of the energy.

positron is the antiparticle of the electron). The neutrino itself is given off in β^+ decay.

Gamma Decay

An atom in an excited state can emit a photon when one of its electrons drops to a lower-energy level. Similarly, a nucleus in an excited state can emit a photon as it decays to a state of lower energy. Since nuclear energies are so much greater than typical atomic energies, the photons given off by a nucleus are highly energetic. In fact, these photons have energies that place them well beyond X-rays in the electromagnetic spectrum. We refer to such high-energy photons as **gamma (γ) rays.**

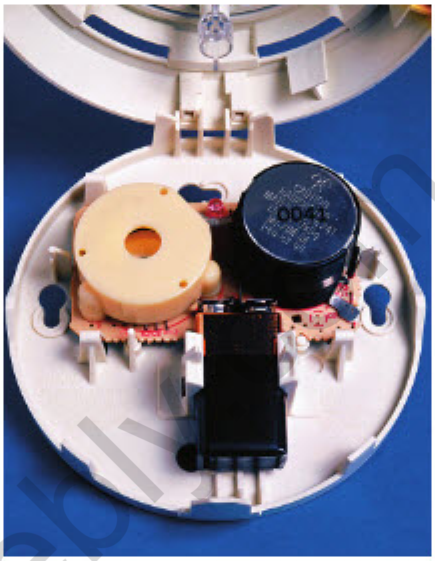
As an example of a situation in which a γ ray can be given off, consider the following beta decay:

$$^{14}_{6}\text{C} \longrightarrow {}^{14}_{7}\text{N}^* + e^- + \bar{\nu}_e$$

The asterisk on the nitrogen symbol indicates that the nitrogen nucleus has been left in an excited state as a result of the beta decay. Subsequently, the nitrogen nucleus may decay to its ground state with the emission of a γ ray:

$$^{14}_{7}\text{N}^* \longrightarrow {}^{14}_{7}\text{N} + \gamma$$

Notice that neither the atomic number nor the mass number is changed by the emission of a γ ray.



▲ Smoke detectors like this one make use of a synthetic radioactive isotope, americium-241. The alpha particles emitted when this isotope decays ionize air molecules, making them able to conduct a small current. Smoke particles neutralize the ions, interrupting the current and setting off an alarm.

ACTIVE EXAMPLE 32–1 GAMMA-RAY EMISSION: FIND THE CHANGE IN MASS

A $^{226}_{88}$Ra nucleus in an excited state emits a γ ray with a wavelength of 6.67×10^{-12} m. Find the decrease in mass of the $^{226}_{88}$Ra nucleus as a result of this process.

SOLUTION *(Test your understanding by performing the calculations indicated in each step.)*

1. Find the frequency of the γ ray: $f = c/\lambda = 4.50 \times 10^{19}$ Hz
2. Calculate the energy of the γ ray photon: $E = hf = 0.186$ MeV
3. Determine the mass difference corresponding to the energy of the photon: $|\Delta m| = E/c^2 = 0.000200$ u

INSIGHT

As a result of emitting this γ ray, the mass of the $^{226}_{88}$Ra nucleus decreases by an amount that is about one-third the mass of the electron.

YOUR TURN

If the wavelength of the emitted gamma ray is doubled, by what factor does the mass difference change?

*(Answers to **Your Turn** problems are given in the back of the book.)*

Radioactive Decay Series

Consider an unstable nucleus that decays and produces a daughter nucleus. If the daughter nucleus is also unstable, it will eventually decay and produce its own daughter nucleus, which may in turn be unstable. In such cases, an original parent nucleus can produce a series of related nuclei referred to as a **radioactive decay series.** An example of a radioactive decay series is shown in Figure 32–4. In this case, the parent nucleus is $^{235}_{92}$U, and the final nucleus of the series is $^{207}_{82}$Pb, which is stable.

Notice that several of the intermediate nuclei in this series can decay in two different ways—either by alpha decay or by beta decay. Thus there are various "paths" a $^{235}_{92}$U nucleus can follow as it transforms into a $^{207}_{82}$Pb nucleus. In addition, the intermediate nuclei in this series decay fairly rapidly, at least on a geological time scale. For example, any actinium-227 that was present when the Earth formed would have decayed away long ago. The fact that actinium-227 is still

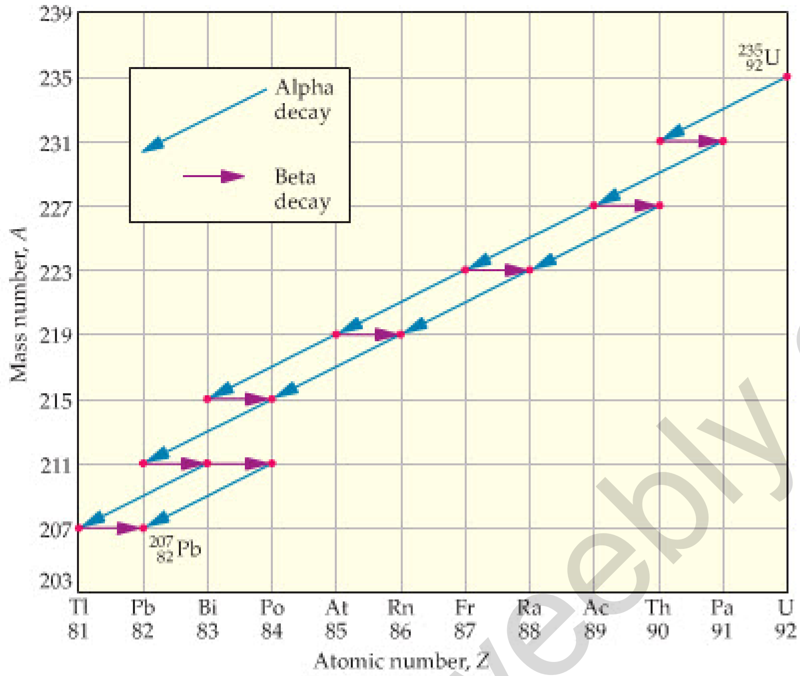


FIGURE 32–4 Radioactive decay series of $^{235}_{92}U$
When $^{235}_{92}U$ decays, it passes through a number of intermediate nuclei before reaching the stable end of the series, $^{207}_{82}Pb$. Note that some intermediary nuclei can decay in only one way, whereas others have two decay possibilities.

found on the Earth today in natural uranium deposits is due to its continual production in this and other decay series.

Activity

The rate at which nuclear decay occurs—that is, the number of decays per second—is referred to as the **activity.** A highly active material has many nuclear decays occurring every second. For example, a typical sample of radium (usually a fraction of a gram) might have 10^5 to 10^{10} decays per second.

The unit we use to measure activity is the curie, named in honor of Pierre (1859–1906) and Marie (1867–1934) Curie, pioneers in the study of radioactivity. The **curie (Ci)** is defined as follows:

$$1 \text{ curie} = 1 \text{ Ci} = 3.7 \times 10^{10} \text{ decays/s} \qquad 32\text{–}6$$

The reason for this choice is that 1 Ci is roughly the activity of 1 g of radium. In SI units, we measure activity in terms of the **becquerel** (Bq):

$$1 \text{ becquerel} = 1 \text{ Bq} = 1 \text{ decay/s} \qquad 32\text{–}7$$

The units of activity most often encountered in practical applications are the millicurie ($1 \text{ mCi} = 10^{-3} \text{ Ci}$) and the microcurie ($1\ \mu\text{Ci} = 10^{-6} \text{ Ci}$).

EXERCISE 32–3

A sample of radium has an activity of 15 μCi. How many decays per second occur in this sample?

SOLUTION

Using the definition given in Equation 32–6, we find

$$15\ \mu\text{Ci} = 15 \times 10^{-6} \text{ Ci}$$

$$= (15 \times 10^{-6} \text{ Ci})\left(\frac{3.7 \times 10^{10} \text{ decays/s}}{1 \text{ Ci}}\right) = 5.6 \times 10^5 \text{ decays/s}$$

32–3 Half-Life and Radioactive Dating

The phenomenon of radioactive decay, though fundamentally random in its behavior, has certain properties that make it useful as a type of "nuclear clock." In fact, it has been discovered that radioactive decay can be used to date numerous items of interest from the recent—and not so recent—past. In this section we

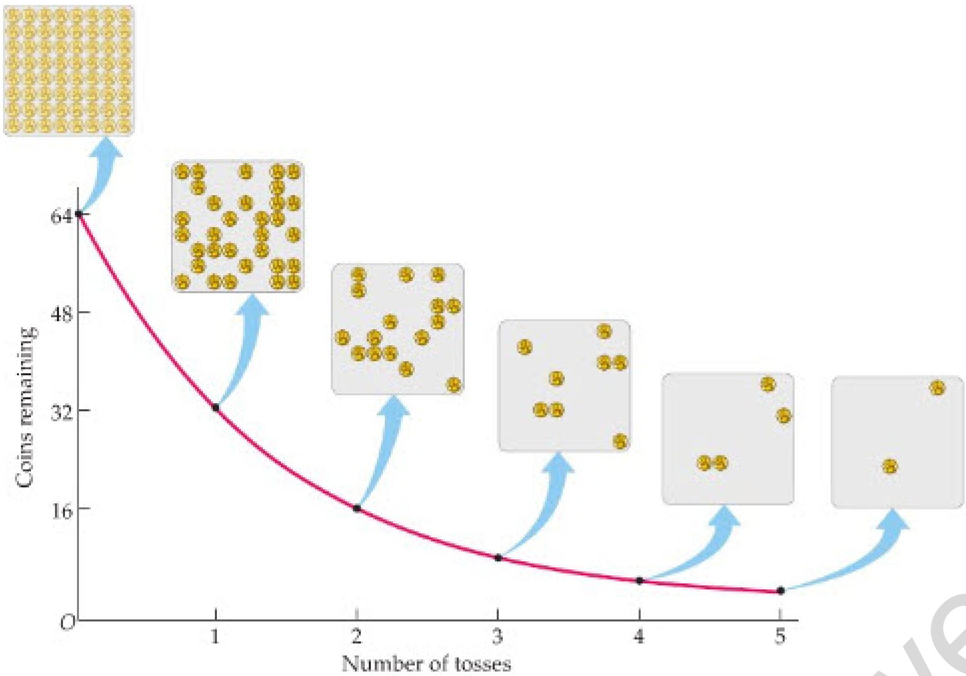


FIGURE 32–5 Tossing coins as an analogy for nuclear decay

The points on this graph show the number of coins remaining (on average) if one starts with 64 coins and removes half with each round of tosses. The curve is a plot of the mathematical function $64e^{-(0.693)t}$, where $t = 1$ means one round of tosses, $t = 2$ means two rounds, and so on.

consider the behavior of radioactive decay as a function of time, introduce the concept of a half-life, and show explicitly how these concepts can be applied to dating.

To begin, consider an analogy in which coins represent nuclei, and the side that comes up when a given coin is tossed determines whether the corresponding nucleus decays. Suppose, for example, that we toss a group of 64 coins and remove any coin that comes up tails. We expect that—on average—32 coins will be removed after the first round of tosses. Which coins will be removed—and the precise number that will be removed—cannot be known, because the flip of a coin, like the decay of a nucleus, is a random process. When we toss the remaining 32 coins, we expect an average of 16 more to be removed, and so on, with each round of tosses decreasing the number of coins by a factor of 2. The results after the first few rounds are shown by the points in **Figure 32–5**.

Also shown in Figure 32–5 is a smooth curve representing the following mathematical function:

$$N = (64)e^{-(\ln 2)t} = (64)e^{-(0.693)t} \qquad 32\text{–}8$$

where N represents the number of coins, and the time variable, t, represents the number of rounds of tosses. For example, if we set $t = 1$ in Equation 32–8, we find $N = 32$, and if we set $t = 2$, we find $N = 16$. This type of "exponential dependence" is a general feature whenever the number of some quantity increases or decreases by a constant factor with each constant interval of time. Examples include the balance in a bank account with compounding interest and the population of the Earth as a function of time.

When nuclei decay, their behavior is much like that of the coins in our analogy. Which nucleus will decay in a given interval of time, and the precise number that will decay, are controlled by a random process that causes the decay—on average—of a given fraction of the original number of nuclei. Thus the number of nuclei, N, remaining at time t is given by an expression analogous to Equation 32–8:

$$N = N_0 e^{-\lambda t} \qquad 32\text{–}9$$

where N_0 is the number of nuclei present at time $t = 0$, and the constant λ is referred to as the **decay constant**. In the analogy of the coins, $N_0 = 64$ and $\lambda = 0.693\ s^{-1}$. Note that the larger the value of the decay constant, the more rapidly the number of nuclei decreases with time. **Figure 32–6** shows the dependence on λ graphically.

PROBLEM-SOLVING NOTE

Consistent Units and the Decay Constant

When calculating the number of nuclei present at a given time, be sure to use consistent units for the time, t, and the decay constant, λ. For example, if you express λ in units of y^{-1}, measure the time in units of y.

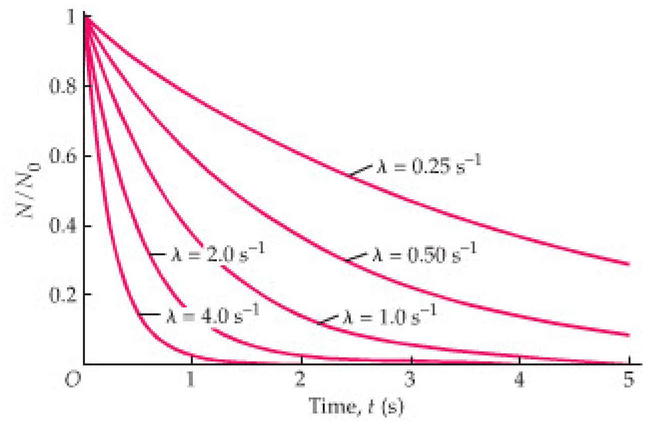


FIGURE 32–6 Dependence on the decay constant

The larger the decay constant, λ, the more rapidly the population of a group of nuclei decreases. In this plot, the value of λ doubles as we move downward from one curve to the next.

ACTIVE EXAMPLE 32–2 FIND THE RADON LEVEL

Radon can pose a health risk when high levels become trapped in the basement of a house. Suppose 4.75×10^7 radon atoms are in a basement at a time when it is sealed to prevent any additional radon from entering. Given that the decay constant of radon is $0.181\ \mathrm{d}^{-1}$, how many radon atoms remain in the basement after **(a)** 7 d and **(b)** 14 d?

SOLUTION *(Test your understanding by performing the calculations indicated in each step.)*

Part (a)

1. Evaluate Equation 32–9 with $t = 7$ d: $N = 1.34 \times 10^7$

Part (b)

1. Evaluate Equation 32–9 with $t = 14$ d: $N = 3.77 \times 10^6$

INSIGHT

Notice that after 7 d the number of radon atoms has decreased by a factor of about 3.55. After 14 d, the number has decreased by a factor of about $3.55^2 = 12.6$. It follows that every 7 d the number of radon atoms decreases by another factor of 3.55.

YOUR TURN

What period of time is required for the number of radon atoms to decrease by a factor of 2.00?

(Answers to ***Your Turn*** *problems are given in the back of the book.)*

Radon-222 ($^{222}_{86}\mathrm{Rn}$) is an isotope produced in a radioactive decay series that includes uranium-238 and radium-226. Since uranium is naturally present in certain kinds of rocks and the soils derived from them, radon, which is a gas, can accumulate in basements and similar enclosed underground spaces that lack adequate ventilation. Radon-222 is itself radioactive, undergoing alpha decay with a half-life of about 4 days, and although its concentration is generally small, it may produce radiation levels great enough to be a health hazard if exposure is prolonged. Homeowners in many parts of the country use test kits to monitor the radiation levels produced by radon.

A useful way to characterize the rate at which a given type of nucleus decays is in terms of its half-life, which is defined as follows:

> The **half-life** of a given type of radioactive nucleus is the time required for the number of such nuclei to decrease by a factor of 2; that is, for the number to decrease from N_0 to $\frac{1}{2}N_0$, from $\frac{1}{2}N_0$ to $\frac{1}{4}N_0$, and so on.

We can solve for this time, call it $T_{1/2}$, by setting $N = \frac{1}{2}N_0$ in Equation 32–9:

$$\frac{1}{2}N_0 = N_0 e^{-\lambda T_{1/2}}$$

Canceling N_0 and taking the natural logarithm of both sides of the equation, we find

$$T_{1/2} = \frac{\ln 2}{\lambda} = \frac{0.693}{\lambda} \qquad \text{32–10}$$

Notice that a large decay constant corresponds to a short half-life, in agreement with the plots shown in Figure 32–6.

CONCEPTUAL CHECKPOINT 32–3 HOW MANY NUCLEI?

A system consists of N_0 radioactive nuclei at time $t = 0$. The number of nuclei remaining after *half* a half-life (that is, at time $t = \frac{1}{2}T_{1/2}$) is **(a)** $\frac{1}{4}N_0$, **(b)** $\frac{3}{4}N_0$, or **(c)** $\frac{1}{\sqrt{2}}N_0$?

REASONING AND DISCUSSION

Referring to the Insight following Active Example 32–2, we note that if the number of nuclei decreases by a factor f in the time $\frac{1}{2}T_{1/2}$, it will decrease by the factor f^2 in the time $2\left(\frac{1}{2}T_{1/2}\right) = T_{1/2}$. We know, however, that the number of nuclei remaining at the time $T_{1/2}$ is $\frac{1}{2}N_0$. It follows that $f^2 = \frac{1}{2}$, or that $f = \frac{1}{\sqrt{2}}$. Therefore, the number of nuclei remaining at the time $\frac{1}{2}T_{1/2}$ is $\frac{1}{\sqrt{2}}N_0$.

ANSWER

(c) At half a half-life, the number of nuclei remaining is $\frac{1}{\sqrt{2}}N_0$.

Now, the property that makes radioactivity so useful as a clock is that its **decay rate, *R*,** or **activity,** depends on time in a straightforward way. To see this, think back to the analogy of the coins. The number of coins that decay (are removed) on the first round of tosses is $32 = \frac{1}{2}(64)$, the number that decay on the second round is $16 = \frac{1}{2}(32)$, and so on. That is, the number that decay in any given interval of time is proportional to the number present at the beginning of the interval.

The same type of analysis applies to nuclei. Therefore, the number of nuclei that decay, ΔN, in a given time interval, Δt, is proportional to the number, N, that are present at time t:

$$R = \left|\frac{\Delta N}{\Delta t}\right| = \lambda N \qquad 32\text{–}11$$

There are two points to note regarding this equation. First, observe that the number of nuclei is decreasing, $\Delta N < 0$. It is for this reason that we take the absolute value of the quantity $\Delta N/\Delta t$. Second, notice that the proportionality constant is simply λ, the decay constant. That λ is the correct constant of proportionality can be shown using calculus.

Combining Equation 32–11 with Equation 32–9, we obtain the time dependence of the activity, R:

$$R = \lambda N_0 e^{-\lambda t} = R_0 e^{-\lambda t} \qquad 32\text{–}12$$

Note that $R_0 = \lambda N_0$ is the initial value of the activity. We apply this relation in the following Active Example.

ACTIVE EXAMPLE 32–3 FIND THE ACTIVITY OF RADON

Referring to Active Example 32–2, calculate how many radon atoms disintegrate per second **(a)** initially and **(b)** after 7 d.

SOLUTION *(Test your understanding by performing the calculations indicated in each step.)*

Part (a)

1. Calculate the activity for $N_0 = 4.75 \times 10^7$. Be sure to convert the decay constant to the unit s^{-1}: $R = \lambda N_0 = 99.5$ decays/s

Part (b)

2. Repeat the calculation for $N = 1.34 \times 10^7$: $R = \lambda N = 28.1$ decays/s

INSIGHT

We see that the initial activity (number of decays per time) is 99.5 Bq. In terms of the curie, the initial activity is 0.00269 μCi.

YOUR TURN

How long does it take after the basement is sealed for the activity of the radon to decrease to 10.0 Bq?

(Answers to **Your Turn** *problems are given in the back of the book.)*

Referring to Equation 32–12, we see that the basic idea of radioactive dating is simply this: If we know the initial activity of a sample, R_0, and we also know the sample's activity now, R, we can find the corresponding time, t, as follows:

$$\frac{R}{R_0} = e^{-\lambda t}$$

$$t = -\frac{1}{\lambda}\ln\frac{R}{R_0} = \frac{1}{\lambda}\ln\frac{R_0}{R} \qquad 32\text{–}13$$

The current activity, R, can be measured in the lab, but how can we know the initial activity, R_0? We address the question next for the specific case of carbon-14.

Carbon-14 Dating

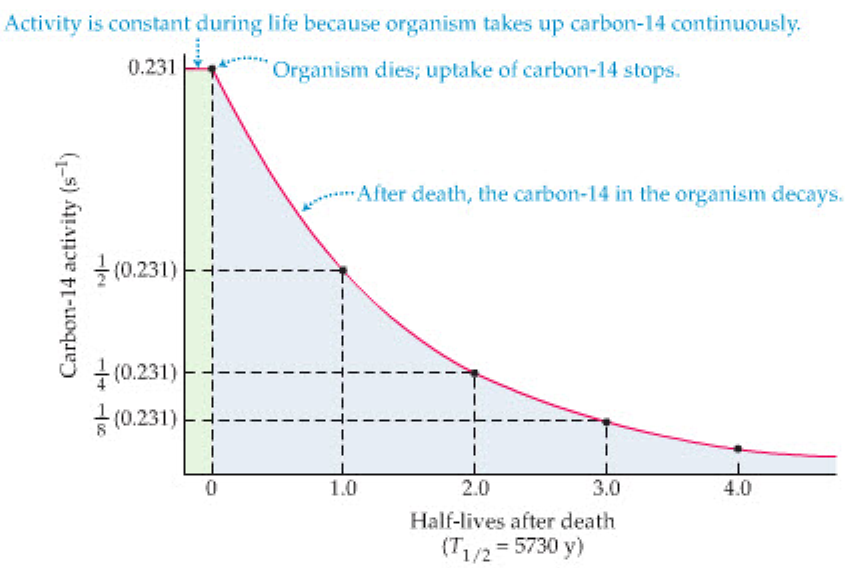
To determine the initial activity of carbon-14 requires a basic knowledge of the role it plays in Earth's biosphere. First, we note that carbon-14 is unstable, with a half-life of 5730 y. It follows that the carbon-14 initially present in any *closed* system will decay away to practically nothing in a time of several half-lives, yet the ratio of carbon-14 to carbon-12 in Earth's atmosphere remains approximately constant at the value 1.20×10^{-12}. Evidently, Earth's atmosphere is not a closed system, at least as far as carbon-14 is concerned.

This is indeed the case. Cosmic rays, which are high-energy particles from outer space, are continuously entering Earth's upper atmosphere and initiating nuclear reactions in nitrogen-14 (a stable isotope). These reactions result in a steady production of carbon-14. Thus, the steady level of carbon-14 in the atmosphere is a result of the balance between the *production rate* due to cosmic rays and the *decay rate* due to the properties of the carbon-14 nucleus.

▲ The Iceman, found in the Italian Alps in 1991. His age was established by means of radiocarbon dating.

We note that living organisms have the same ratio of carbon-14 to carbon-12 as the atmosphere, since they continuously exchange carbon with their surroundings. When an organism dies, however, the exchange of carbon ceases and the carbon-14 in the organism (wood, bone, shell, etc.) begins to decay. This process is illustrated in Figure 32–7, where we see that the carbon-14 activity of an organism is constant until it dies, at which point it decreases exponentially with a half-life of 5730 y.

All that remains to implement carbon-14 dating is to determine the initial activity, R_0. For convenience, we calculate R_0 for a 1-g sample of carbon. As we know from Chapter 31, 12 g of carbon-12 consists of Avogadro's number of atoms, 6.02×10^{23}. Therefore, 1 g consists of $(6.02 \times 10^{23})/12 = 5.02 \times 10^{22}$ carbon-12 atoms. Multiplying this number of atoms by the ratio of carbon-14 to carbon-12, 1.20×10^{-12}, shows that the number of carbon-14 atoms in the 1-g sample is



▶ **FIGURE 32–7 Activity of carbon-14**
While an organism is living and exchanging carbon with the atmosphere, its carbon-14 activity remains constant. When the organism dies, the carbon-14 activity decays exponentially with a half-life of 5730 years.

6.02×10^{10}. Next, we need the decay constant, which we obtain by rearranging Equation 32–10:

$$\lambda = \frac{\ln 2}{T_{1/2}} = \frac{0.693}{T_{1/2}}$$

Using $T_{1/2} = 5730\text{ y} = 1.81 \times 10^{11}\text{ s}$, we find $\lambda = 3.83 \times 10^{-12}\text{ s}^{-1}$. Finally, the initial activity of a 1-g sample of carbon is

$$R_0 = \lambda N_0 = (3.83 \times 10^{-12}\text{ s}^{-1})(6.02 \times 10^{10}) = 0.231\text{ Bq} \qquad 32\text{–}14$$

This is the initial activity used in Figure 32–7. It follows that 5730 y after an organism dies, its carbon-14 activity per gram of carbon will have decreased to about $\frac{1}{2}(0.231\text{ Bq}) = 0.116\text{ Bq}$.

The next Example applies this basic idea to a real-world case of some interest—the Iceman of the Alps.

REAL-WORLD PHYSICS
Dating the Iceman

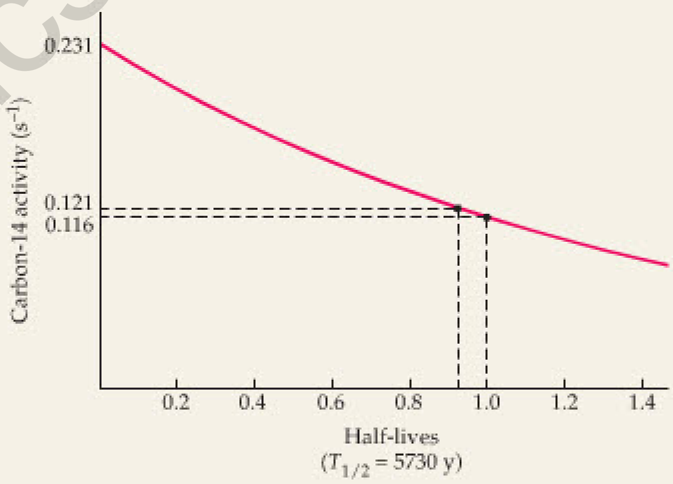
EXAMPLE 32–5 AGE OF THE ICEMAN: YOU DON'T LOOK A DAY OVER 5000

Early in the afternoon of September 19, 1991, a German couple hiking in the Italian Alps noticed something brown sticking out of the ice 8 to 10 m ahead of them. At first they took the object to be a doll or some rubbish. As they got closer, however, it became apparent that the object they had discovered was the body of a person trapped in the ice, with only the top part of the body exposed. Subsequent investigation revealed the remarkably well-preserved body to be that of a Stone Age man who had died in the mountains and become entombed in the ice. When the carbon-14 dating method was applied to the remains of the Iceman and some of the materials he had carried with him, it was found that the carbon-14 activity was about 0.121 Bq per gram of carbon. Using this information, date the remains of the Iceman.

PICTURE THE PROBLEM
Our sketch shows the decay of the carbon-14 activity of a gram of carbon as a function of time. The initial activity of carbon-14 in such a sample is 0.231 Bq.

STRATEGY
We can obtain the age of the remains directly from Equation 32–13: $t = (1/\lambda)\ln(R_0/R)$. In this case, $R_0 = 0.231$ Bq, and $R = 0.121$ Bq. Since an answer in years would be most useful, we express the decay constant as $\lambda = 0.693/T_{1/2}$, with $T_{1/2} = 5730$ y.

Note that the observed activity of 0.121 Bq is slightly greater than $\frac{1}{2}(0.231\text{ Bq}) = 0.116$ Bq; hence, we expect the age of the remains to be slightly less than the half-life of 5730 y.



SOLUTION

1. Determine the value of the decay constant, λ, in units of y^{-1}:

$$\lambda = \frac{0.693}{5730\text{ y}} = 1.21 \times 10^{-4}\text{ y}^{-1}$$

2. Substitute λ, R_0, and R into Equation 32–13:

$$t = \frac{1}{\lambda}\ln\left(\frac{R_0}{R}\right) = \frac{1}{(1.21 \times 10^{-4}\text{ y}^{-1})}\ln\left(\frac{0.231\text{ Bq}}{0.121\text{ Bq}}\right) = 5340\text{ y}$$

INSIGHT
We conclude that the Iceman, who has been dubbed Ötzi, died in the mountains during the Stone Age, some 5340 y ago. Detailed examination of Ötzi's body and possessions indicates he was probably an itinerant sheepherder and/or hunter. He met his end in a violent fashion, however. Recent CT scans confirm that Ötzi was killed by an arrow that entered through his shoulder blade and lodged less than an inch from his left lung.

PRACTICE PROBLEM
If the remains of another Iceman of the same age are found in the year 2991, what will be the carbon-14 activity of 1 g of carbon? [**Answer:** 0.107 Bq]

Some related homework problems: Problem 35, Problem 36

As useful as carbon-14 dating is, it is limited to time spans of only a few half-lives, say, 10,000 to 15,000 y. Beyond that range, the current activity will be so small that accurate measurements will be difficult. To measure dates on different time scales, different radioactive isotopes must be used. Other frequently used isotopes and their half-lives are $^{210}_{82}$Pb (22.3 y), $^{40}_{19}$K (1.28×10^9 y), and $^{238}_{92}$U (4.468×10^9 y).

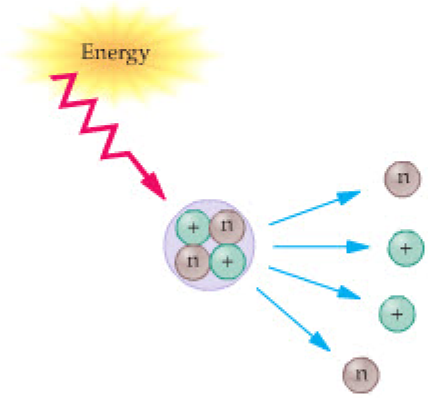


FIGURE 32–8 The concept of binding energy
The minimum energy that must be supplied to a stable nucleus to break it into its constituent nucleons is referred to as the binding energy. Because the stable nucleus is the lower-energy state, it has less mass than the sum of the masses of its individual constituents.

32–4 Nuclear Binding Energy

An α particle consists of two protons and two neutrons. Does it follow that the mass of an α particle is twice the mass of a proton plus twice the mass of a neutron? One would certainly think so, but in fact this is not the case. Alpha particles, and all other stable nuclei containing more than one nucleon, have a mass that is *less* than the mass of the individual nucleons added together.

As strange as this result may seem, it is just one more manifestation of Einstein's theory of relativity. In particular, the reduction in mass of a nucleus, compared with the mass of its constituents, corresponds to a reduction in its energy, according to the relation $E = mc^2$. This reduction in energy is referred to as the **binding energy** of the nucleus. To separate a nucleus into its individual nucleons requires an energy at least as great as the binding energy; therefore, the binding energy indicates how firmly a given nucleus is held together. We illustrate this concept in **Figure 32–8**.

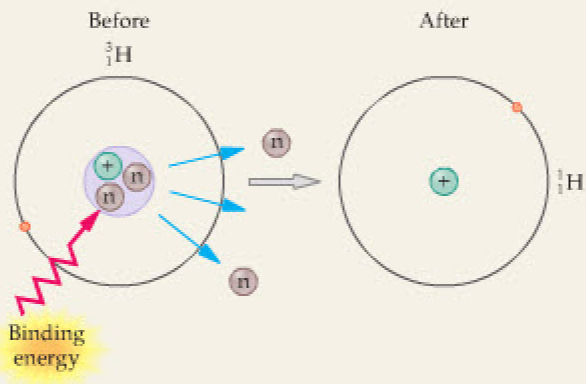
The next Example uses $E = (\Delta m)c^2$ and the atomic masses given in Appendix F to calculate the binding energy of tritium.

EXAMPLE 32–6 THE BINDING ENERGY OF TRITIUM

Using the information given in Appendix F, calculate the binding energy of tritium, $^3_1$H.

PICTURE THE PROBLEM
In our sketch we show energy being added to the nucleus of tritium, which consists of one proton and two neutrons. The result is one hydrogen atom and two neutrons.

STRATEGY
To find the binding energy, we simply calculate Δm and multiply by c^2. The only point to be careful about is that, with the exception of the neutron, all the masses given in Appendix F are for neutral atoms. This means that the mass of tritium includes the mass of one electron. The same is true of the hydrogen atom, however. It follows that the electron mass will cancel when we calculate Δm.



SOLUTION

1. Let the initial mass be the mass of tritium: $m_i = 3.016049$ u
2. The final mass is the mass of hydrogen plus two neutrons: $m_f = 1.007825 \text{ u} + 2(1.008665 \text{ u}) = 3.025155$ u
3. Calculate Δm: $\Delta m = m_f - m_i = 3.025155 \text{ u} - 3.016049 \text{ u} = 0.009106$ u
4. Find the corresponding binding energy (recall that $1 \text{ u} = 931.5 \text{ MeV}/c^2$): $E = (\Delta m)c^2 = (0.009106 \text{ u})\left(\frac{931.5 \text{ MeV}/c^2}{1 \text{ u}}\right)c^2 = 8.482$ MeV

INSIGHT
We see that it takes about 8.5 MeV to separate the nucleons of tritium. Compare this with the fact that only 13.6 eV is required to remove the electron from tritium. As we have noted before, nuclear processes typically involve energies in the MeV range, whereas atomic processes require energies in the eV range.

PRACTICE PROBLEM
Calculate the binding energy of an α particle. [**Answer:** 28.296 MeV]

Some related homework problems: Problem 41, Problem 42

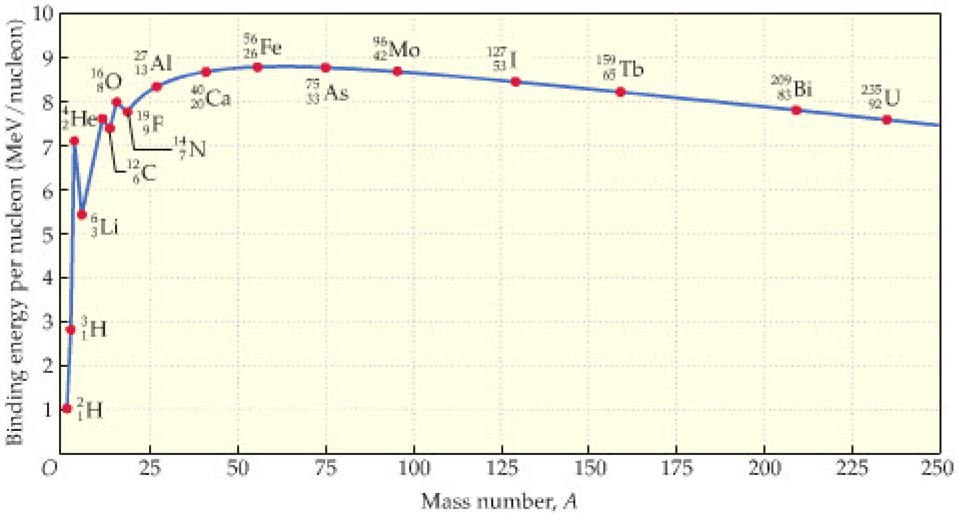


FIGURE 32–9 The curve of binding energy
This graph shows the binding energy per nucleon for a variety of nuclei. Notice that the binding energy per nucleon for tritium is approximately 2.8 MeV, in agreement with Example 32–6.

In addition to knowing the binding energy of a given nucleus, it is also of interest to know the binding energy per nucleon. In the case of tritium, there are three nucleons; hence, the binding energy per nucleon is $\frac{1}{3}(8.482\text{ MeV}) = 2.827\text{ MeV}$. **Figure 32–9** presents the binding energy per nucleon for various stable nuclei as a function of the mass number, A. Notice that the binding-energy curve rises rapidly to a maximum near $A = 60$ and then decreases slowly to a value near 7.4 MeV per nucleon for larger A.

It follows that nuclei with A in the range $A = 50$ to $A = 75$ are the most stable nuclei in nature. In addition, the fact that the binding energy per nucleon changes very little for large A means that the strong nuclear force "saturates," in the sense that adding more nucleons does not add to the binding energy per nucleon. This phenomenon can be understood as a consequence of the short range of the nuclear force. Because of this short range, each nucleon interacts with only a few nucleons that are close neighbors to it in the nucleus. Nucleons on the other side of the nucleus are too far away to interact. Thus, from the point of view of a given nucleon, the attractive energy it feels due to other nucleons in the nucleus is essentially the same whether the nucleus has 150 nucleons or 200 nucleons—only the nearby nucleons interact.

The fact that the binding-energy curve has a maximum has important implications, as will be shown in the next two sections. For example, energy can be released when large nuclei split into smaller nuclei (fission), or when small nuclei combine to form a larger nucleus (fusion).

PROBLEM-SOLVING NOTE

Interpreting the Mass Difference

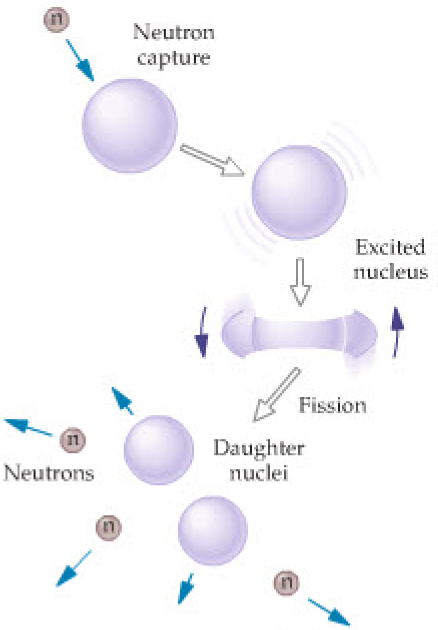
If the mass difference due to a reaction is negative, $\Delta m = m_f - m_i < 0$, it follows that energy is *released* by the reaction. If Δm is positive, energy must be *supplied* to the system to make the reaction occur.

32–5 Nuclear Fission

A new type of physical phenomenon was discovered in 1939 when Otto Hahn and Fritz Strassman found that, under certain conditions, a uranium nucleus can split apart into two smaller nuclei. This process is called **nuclear fission.** As we shall see, nuclear fission releases an amount of energy that is many orders of magnitude greater than the energy released in chemical reactions. This fact has had a profound impact on the course of human events over the last several decades.

To obtain a rough estimate of just how much energy is released as a result of a fission reaction, consider the binding-energy curve in Figure 32–9. For a large nucleus like uranium-235, $^{235}_{92}$U, we see that the binding energy per nucleon is approximately 7.5 MeV. If this nucleus splits into two nuclei with roughly half the mass number ($A \sim 235/2 \sim 115$), we see that the binding energy per nucleon increases to roughly 8.3 MeV. If all 235 nucleons in the original uranium nucleus release $(8.3\text{ MeV} - 7.5\text{ MeV}) = 0.8\text{ MeV}$ of energy, the total energy release is

$$(235\text{ nucleons})(0.8\text{ MeV/nucleon}) = 200\text{ MeV}$$



▲ **FIGURE 32–10 Nuclear fission**
When a large nucleus captures a neutron, it may become excited and ultimately split into two smaller nuclei. This is the process of fission.

Note that this is the energy released by a *single nucleus* undergoing fission. In contrast, the combustion of a *single molecule* of gasoline releases an energy of only about 2 eV. Thus the nuclear reaction gives off about one hundred million times more energy than the chemical reaction!

The first step in a typical fission reaction occurs when a slow neutron is absorbed by a uranium-235 nucleus. This step increases the mass number of the nucleus by one and leaves it in an excited state:

$$^{1}_{0}\text{n} + ^{235}_{92}\text{U} \longrightarrow ^{236}_{92}\text{U}^*$$

The excited nucleus oscillates wildly and becomes highly distorted, as depicted in Figure 32–10. In many respects, the nucleus behaves like a spinning drop of water. Like a drop of water, the nucleus can distort only so much before it breaks apart into smaller pieces; that is, before it undergoes fission.

There are about 90 different ways in which the uranium-235 nucleus can undergo fission. Typically, 2 or 3 neutrons (2.47 on average) are released during the fission process, in addition to the two smaller nuclei that are formed. The reason that neutrons are released can be seen by examining Figure 32–1, where we show N and Z for various nuclei. A large nucleus, like uranium-235, contains a higher percentage of neutrons than a smaller nucleus—that is, the larger nuclei deviate more from the $N = Z$ line. Thus, if $^{235}_{92}$U were to simply break in two—keeping the same percentage of neutrons and protons in each piece—the smaller nuclei would have too many neutrons to be stable. As a result, neutrons are typically given off in a fission reaction.

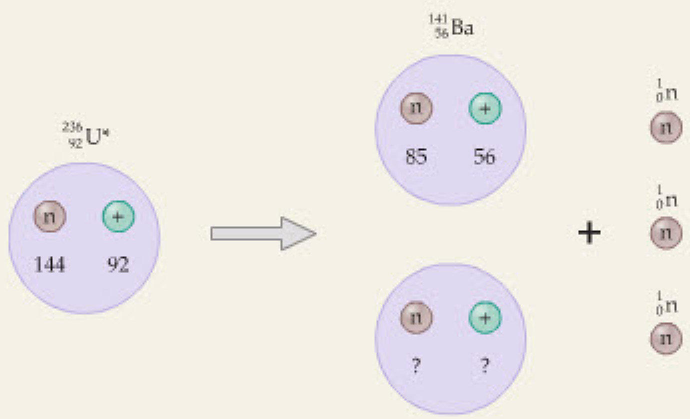
One of the possible fission reactions for $^{235}_{92}$U is considered in the following Example.

EXAMPLE 32–7 A FISSION REACTION OF URANIUM-235

When uranium-235 captures a neutron, it may undergo the following fission reaction: $^{1}_{0}\text{n} + ^{235}_{92}\text{U} \longrightarrow ^{236}_{92}\text{U}^* \longrightarrow ^{141}_{56}\text{Ba} + ? + 3^{1}_{0}\text{n}$. **(a)** Complete the reaction and **(b)** determine the energy it releases.

PICTURE THE PROBLEM

The indicated fission reaction is shown in our sketch, starting with the $^{236}_{92}$U* nucleus. The known numbers of protons and neutrons are indicated for both $^{236}_{92}$U* and $^{141}_{56}$Ba. The corresponding numbers for the unidentified nucleus are to be determined.



STRATEGY

a. The total number of protons must be the same before and after the reaction, as must the number of neutrons. By conserving protons and neutrons, we can determine the missing numbers on the unidentified nucleus.

b. As in other reactions, we calculate the difference in mass, $|\Delta m|$ and multiply by c^2. The same number of electrons appears on both sides of the reaction; hence, the electron mass will cancel when we calculate $|\Delta m|$.

SOLUTION

Part (a)

1. Determine the number of protons, Z, neutrons, N, and nucleons, A, in the unidentified nucleus. When calculating the number of neutrons, be sure to include the three individual neutrons given off in the reaction:
$Z = 92 - 56 = 36$
$N = 144 - 85 - 3 = 56$
$A = Z + N = 36 + 56 = 92$

2. Use Z and A to specify the unidentified nucleus: $^{92}_{36}$Kr

Part (b)

3. Use Appendix F to find the initial mass of the system; that is, the mass of a $^{235}_{92}$U atom plus the mass of a neutron: $m_i = 235.043925\text{ u} + 1.008665\text{ u} = 236.052590\text{ u}$

4. Use Appendix F to find the final mass of the system, including both nuclei and the three neutrons:

$$m_f = 140.914406 \text{ u} + 91.926111 \text{ u} + 3(1.008665 \text{ u}) = 235.866512 \text{ u}$$

5. Calculate the mass difference and the corresponding energy release (recall that $1 \text{ u} = 931.5 \text{ MeV}/c^2$):

$$\Delta m = m_f - m_i = 235.866512 \text{ u} - 236.052590 \text{ u} = -0.186078 \text{ u}$$

$$E = |\Delta m|c^2 = (0.186078 \text{ u})\left(\frac{931.5 \text{ MeV}/c^2}{1 \text{ u}}\right)c^2 = 173.3 \text{ MeV}$$

INSIGHT

Thus, as suggested by our crude calculation earlier in this section, the energy given off by a typical fission reaction is on the order of 200 MeV.

PRACTICE PROBLEM

Complete the following reaction, and determine the energy it releases: ${}^1_0\text{n} + {}^{235}_{92}\text{U} \longrightarrow {}^{236}_{92}\text{U}^* \longrightarrow {}^{140}_{54}\text{Xe} + ? + 2{}^1_0\text{n}$. **[Answer:** ${}^{94}_{38}\text{Sr}$, 184.7 MeV]

Some related homework problems: Problem 48, Problem 49

Chain Reactions

The fact that fission reactions of ${}^{235}_{92}\text{U}$ give off more than one neutron on average has significant implications. To see why, recall that the fission of ${}^{235}_{92}\text{U}$ is initiated by the absorption of a neutron in the first place. So the neutrons given off by one ${}^{235}_{92}\text{U}$ fission reaction may cause additional fission reactions in other ${}^{235}_{92}\text{U}$ nuclei. A reaction that proceeds from one nucleus to another in this fashion is referred to as a **chain reaction.**

Suppose, for the sake of discussion, that a fission reaction gives off two neutrons and that both neutrons induce additional fissions in other nuclei. These nuclei in turn give off two neutrons. Starting with one nucleus to begin the chain reaction, we have two nuclei in the second generation of the chain, four nuclei in the third generation, and so on, as indicated in Figure 32–11. After only 100 generations, the number of nuclei undergoing fission is 1.3×10^{30}. If each of these reactions gives off 200 MeV of energy, the total energy release after just 100 generations is 4.1×10^{19} J. To put this into everyday terms, this is enough energy to supply the needs of the entire United States for half a year. Clearly, a rapidly developing **runaway** chain reaction, like the one just described, would result in the explosive release of an enormous amount of energy.

A great deal of effort has gone into controlling the chain reaction of ${}^{235}_{92}\text{U}$. If the release of energy can be kept to a manageable and usable level—avoiding

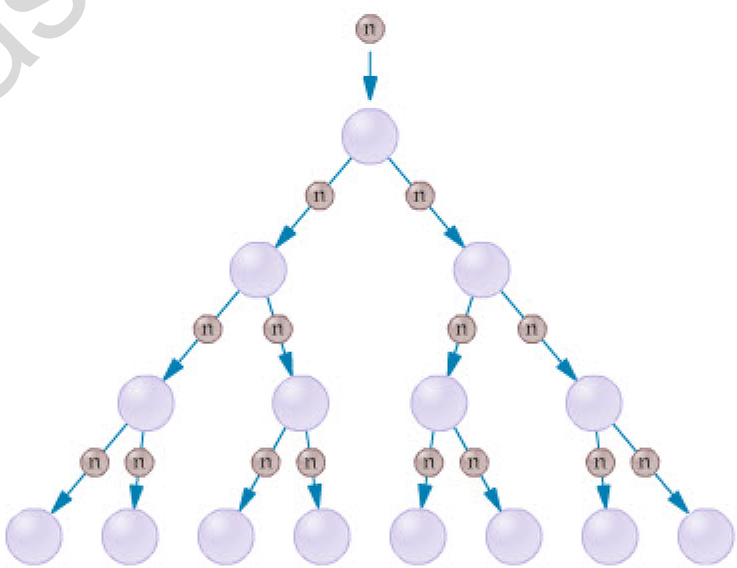
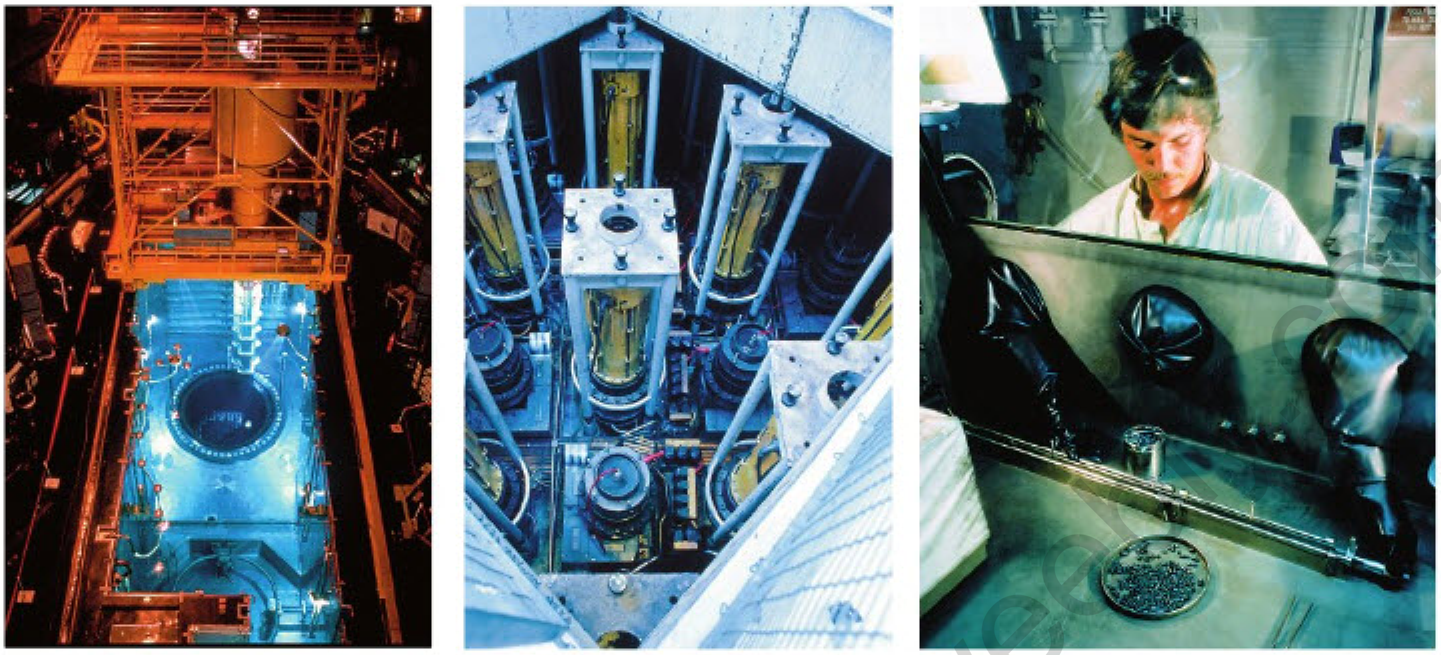


FIGURE 32–11 A chain reaction

A chain reaction occurs when a neutron emitted by one fission reaction induces a fission reaction in another nucleus. In a runaway chain reaction, the number of neutrons given off by one reaction that cause an additional reaction is greater than one. In the case shown here, two neutrons from one reaction induce additional reactions. In a controlled chain reaction, like those used in nuclear power plants, only one neutron, on average, induces additional reactions.



▲ (Left) The core of a nuclear reactor at a power plant. The core sits in a pool of water, which provides cooling while absorbing stray radiation. The blue glow suffusing the pool is radiation from electrons traveling through the water at relativistic velocities. Above the core is a crane used to replace the fuel rods when their radioactive material becomes depleted. (Center) A closer view of a reactor core. The structures projecting above the core house the mechanism that lowers and raises the control rods, thereby regulating the rate of fission in the reactor. The tubes that are nearly flush with the top of the core house the fuel rods. (Right) A technician loads pellets of fissionable material into fuel rods.

explosions or meltdowns—it follows that a powerful source of energy is at our disposal.

REAL-WORLD PHYSICS
Nuclear reactors

The first controlled nuclear chain reaction was achieved by Fermi in 1942, using a racquetball court at the University of Chicago for his improvised laboratory. His reactor consisted of blocks of uranium (the fuel) stacked together with blocks of graphite (the moderator) to form a large "pile." In such a **nuclear reactor,** the **moderator** slows the neutrons given off during fission, making it more likely that they will be captured by other uranium nuclei and cause additional fissions. The reaction rate is adjusted with **control rods** made of a material (such as cadmium) that is very efficient at absorbing neutrons. With the control rods fully inserted into the pile, any reaction that begins quickly dies out because neutrons are absorbed rather than allowed to cause additional fissions. As the control rods are pulled partway out of the pile, more neutrons become available to induce reactions. When, on average, one neutron given off by any fission reaction produces an additional reaction, the reactor is said to be **critical.** If the control rods are pulled out even farther from the pile, the number of neutrons causing fission reactions in the next generation becomes greater than one, and a runaway reaction begins to occur. Nuclear reactors that produce power for practical applications are operated near their critical condition by continuous adjustment of the placement of the control rods.

Fermi's original reactor, which was designed to test the basic scientific principles underlying fission reactions, produced a power of only about 0.5 W when operating near its critical condition. This is the power necessary to operate a flashlight bulb. In comparison, modern-day nuclear power reactors typically produce 1000 MW of power, enough to power an entire city.

32–6 Nuclear Fusion

When two light nuclei combine to form a more massive nucleus, the reaction is referred to as **nuclear fusion.** The binding-energy curve in Figure 32–9 shows that when light nuclei undergo fusion, the resulting nucleus will have a greater binding energy per nucleon than the original nuclei. This means that *the larger nucleus formed by fusion has less mass than the sum of the masses of the original light nuclei.* The mass difference appears as an energy ($E = mc^2$) given off by the reaction.

It is not easy to initiate a fusion reaction. The reason is that to combine two small nuclei, say two protons, into a single nucleus requires that the initial nuclei have a kinetic energy great enough to overcome the Coulomb repulsion the protons exert on each other. The temperature required to give protons the needed kinetic energy is about 10^7 K. When the temperature is high enough to initiate fusion, we say that the resulting process is a **thermonuclear fusion reaction.**

One place where such high temperatures may be encountered is at the core of a star. In fact, all stars generate energy by the process of thermonuclear reactions. Most stars (including the Sun) fuse hydrogen to produce helium. At this very moment, the Sun is converting roughly 600,000,000 tons of hydrogen into helium every second. Some stars also fuse helium or other heavier elements.

In its early stages a star begins as an enormous cloud of gas and dust. As the cloud begins to fall inward under the influence of its own gravity, it converts gravitational potential energy into kinetic energy. This means that the gas heats up. When the temperature becomes high enough to begin the fusion process, the resulting release of energy tends to stabilize the star, preventing its further collapse. A star like the Sun can burn its hydrogen for about 10 billion years, producing a remarkably stable output of energy so important to life on Earth. When its hydrogen fuel is depleted, the Sun will enter a new phase of its life, becoming a red giant and converting its fusion process to one that fuses helium. When the Sun enters its red giant phase, it will expand greatly in size, eventually engulfing the Earth and vaporizing it.

REAL-WORLD PHYSICS

Powering the Sun: the proton-proton cycle

The Sun is powered by the **proton-proton cycle** of fusion reactions, as first described by Hans Bethe (1906–2005). This cycle consists of three steps. The first two steps are as follows:

$$ {}^{1}_{1}\text{H} + {}^{1}_{1}\text{H} \longrightarrow {}^{2}_{1}\text{H} + \text{e}^{+} + \nu_{\text{e}} $$

$$ {}^{1}_{1}\text{H} + {}^{2}_{1}\text{H} \longrightarrow {}^{3}_{2}\text{He} + \gamma $$

A number of reactions are possible for the third step, but the dominant one is

$$ {}^{3}_{2}\text{He} + {}^{3}_{2}\text{He} \longrightarrow {}^{4}_{2}\text{He} + {}^{1}_{1}\text{H} + {}^{1}_{1}\text{H} $$

The overall energy production during this cycle is about 27 MeV.

In the next Active Example we show how the mass difference in a deuterium-deuterium reaction can be used to calculate the amount of energy released.

ACTIVE EXAMPLE 32–4 THE DEUTERIUM-DEUTERIUM REACTION

Find the energy released in the deuterium-deuterium reaction, ${}^{2}_{1}\text{H} + {}^{2}_{1}\text{H} \longrightarrow {}^{3}_{1}\text{H} + {}^{1}_{1}\text{H}$.

SOLUTION *(Test your understanding by performing the calculations indicated in each step.)*

1. Calculate the initial mass: $m_i = 4.028204$ u
2. Calculate the final mass: $m_f = 4.023874$ u
3. Find the difference in mass: $\Delta m = -0.004330$ u
4. Convert the mass difference to an energy release: $E = |\Delta m|c^2 = 4.033$ MeV

INSIGHT

The tritium produced in this reaction has a half-life of 12.33 y. Deuterium is stable.

YOUR TURN

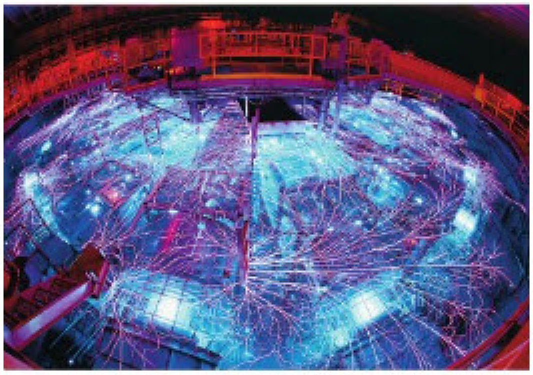
Find the energy released in the reaction ${}^{3}_{2}\text{He} + {}^{3}_{2}\text{He} \longrightarrow {}^{4}_{2}\text{He} + {}^{1}_{1}\text{H} + {}^{1}_{1}\text{H}$.

(Answers to **Your Turn** *problems are given in the back of the book.)*

Just as nuclear fission reactions can be controlled, allowing for the generation of usable power, it would be desirable to control fusion as well. In fact, there are many potential advantages of fusion over fission. For example, fusion reactions yield more energy from a given mass of fuel than fission reactions. In addition, one type of fuel for a fission reactor, ^2_1H, is readily obtained from seawater. To date, however, more energy is required to produce sustained fusion reactions in the lab than is released by the reactions. Still, researchers are close to the break-even point, and many are confident that in time controlled fusion reactions that give off usable amounts of energy will be possible.

REAL-WORLD PHYSICS
Man-made fusion

Most attempts at controlled nuclear fusion employ one of two basic methods. In both methods the basic idea is to overcome the repulsive electrical forces that keep nuclei apart by giving them sufficiently large kinetic energies, which can be accomplished by heating the fuel to temperatures on the order of 10^7 K.



▲ Nuclear fusion, which powers the stars, may eventually turn out to be the clean, inexpensive, and renewable energy source that our society needs. But fusion requires enormous temperatures and pressures, and so far the problems involved in creating a practical fusion technology have not been overcome. Several different approaches have been explored in the effort to produce sustained nuclear fusion. One of them, embodied in the Z machine at Sandia National Laboratories in New Mexico, shown here, involves the compression of a plasma by an intense burst of X-rays. So far, temperatures of nearly 2 million degrees have been attained, but only for very brief periods of time.

At temperatures like these, all atoms are completely ionized, forming a gas of electrons and nuclei known as a *plasma*. To initiate fusion in the plasma, one must maintain the plasma long enough for collisions to occur between the appropriate nuclei. This can be accomplished using a technique known as **magnetic confinement,** in which powerful magnetic fields confine a plasma within a "magnetic bottle." The trapped plasma is kept away from the walls of the container—preventing their melting—and is heated until fusion begins.

Another approach to reaching the high temperatures and pressures required for fusion is by way of **inertial confinement.** In this technique, one begins by dropping a small solid pellet of fuel into a vacuum chamber. When the pellet reaches the center of the chamber it is bombarded from all sides by high-power laser beams. These beams heat and vaporize the surface of the pellet in an almost instantaneous event. The heating that causes fusion is still to come, however. As the vaporized exterior of the pellet expands rapidly outward, it exerts an inward "thrust" that causes the remainder of the pellet to implode. This implosion is so violent that the pellet's temperature and pressure rise to levels sufficient to ignite the desired fusion reactions.

32–7 Practical Applications of Nuclear Physics

Nuclear physics, though it involves objects none of us will ever touch or see, nevertheless has significant impact on our everyday life. In this section we consider a number of ways in which nuclear physics affects us all, either directly or indirectly.

Biological Effects of Radiation

Although nuclear radiation can be beneficial when used to image our bodies, or to treat a variety of diseases, it can also be harmful to living tissues. For example, the high-energy photons, electrons, and α particles given off by radioactive decay can ionize a neutral atom by literally "knocking" one or more of its electrons out of the atom. In fact, since typical nuclear decays give off energies in the MeV range, and typical ionization energies are in the 10 eV range, it follows that a single α, β, or γ particle can ionize thousands of atoms or molecules.

REAL-WORLD PHYSICS: BIO
Radiation and cells

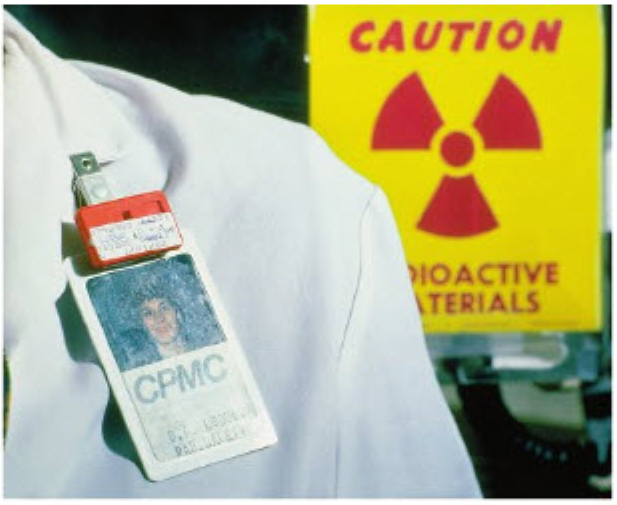
Such ionization can be harmful to a living cell by altering the structure of some of its molecules. The result can be a cell that no longer functions or behaves normally, or even a cell that will soon die. This effect is the basis for radiation treatments of cancer, which seek to concentrate high doses of radiation on a cancerous tumor in order to kill the malignant cells. But just what is a "high dose," and how do we quantify radiation dosage?

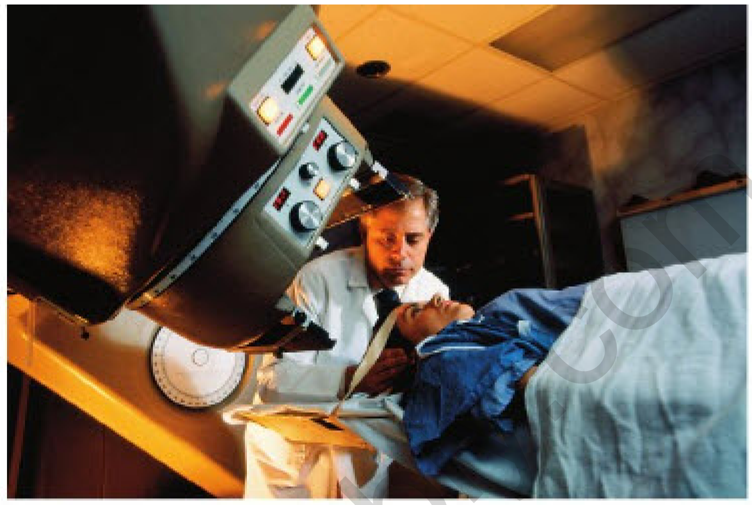
The first radiation unit to be defined, called the **roentgen** (R), is directly related to the amount of ionization caused by X-rays or γ rays. Suppose such radiation is sent through a mass, m, of dry air at standard temperature and pressure (STP). If the radiation creates ions in the air with a total charge of q, we say that the dose of radiation delivered to the air is proportional to q/m. Specifically, we say that the dosage of X-rays or γ rays is 1 R if an ionization charge of 2.58×10^{-4} C is produced in 1 kg of dry air.

Definition of the Roentgen, R

$$1\ \text{R} = 2.58 \times 10^{-4}\ \text{C/kg} \qquad \text{(X-rays or } \gamma \text{ rays in dry air at STP)} \qquad 32\text{–}15$$

SI unit: C/kg





▲ Because radiation can kill or damage living cells, people who work with radioactive materials or other sources of ionizing radiation (such as X-ray tubes) must be careful to monitor their cumulative exposure. One way to do this is with a simple film badge (left). When the photographic film is developed, the degree of darkening records the amount of radiation it has received. Ironically, the fact that radiation can be lethal to cells also makes it a valuable therapeutic tool in the fight against cancer. At right, radiation from cobalt-60 is being used to attack a malignant tumor. Today, many hospitals have their own particle accelerators to manufacture short-lived radioactive isotopes for the treatment of different kinds of cancer or to use as tracers.

The roentgen is one key measure of radiation dosage, but there are others. The focus of the **rad,** an acronym for **r**adiation **a**bsorbed **d**ose, is the amount of energy that is absorbed by the irradiated material, regardless of the type of radiation that delivers the energy. For example, if a 1-kg sample of any material absorbs 0.01 J of energy, we say that it has received a dose of 1 rad:

Definition of the Rad

$$1 \text{ rad} = 0.01 \text{ J/kg} \qquad \text{(any type of radiation)} \qquad 32\text{–}16$$

SI unit: J/kg

Because the rad depends only on the amount of energy absorbed, and not on the type of radiation, more information is needed if we are to have an indication of the biological effect a certain dosage will produce. For example, a 1-rad dose of X-rays is far less likely to cause a cataract in the cornea of the eye than a 1-rad dose of neutrons. To take into account such differences, we introduce a quantity called the **r**elative **b**iological **e**ffectiveness, or **RBE.** The standard used for comparison is the biological effect produced by a 1-rad dose of 200-keV X-rays. Thus, the RBE is defined as follows:

Definition of Relative Biological Effectiveness, RBE

$$\text{RBE} = \frac{\text{the dose of 200-keV X-rays necessary to produce a given biological effect}}{\text{the dose of a particular type of radiation necessary to produce the same biological effect}} \qquad 32\text{–}17$$

SI unit: dimensionless

Representative RBE values are given in Table 32–3. Note that the larger the RBE for a certain type of radiation, the greater the biological damage caused by a given dosage of that radiation.

TABLE 32–3 Relative Biological Effectiveness, RBE, for Different Types of Radiation

Type of radiation	RBE
Heavy ions	20
α rays	10–20
Protons	10
Fast neutrons	10
Slow neutrons	4–5
β rays	1.0–1.7
γ rays	1
200-kev X-Rays	1

Combining the dosage in rad and the RBE value for a given radiation yields a new unit of radiation referred to as the **biologically equivalent dose.** This unit is measured in **rem,** which stands for **r**oentgen **e**quivalent in **m**an. To be specific, dosage in rem is defined as follows:

Definition of Roentgen Equivalent in Man, rem

$$\text{dose in rem} = \text{dose in rad} \times \text{RBE} \qquad 32\text{–}18$$

SI unit: J/kg

TABLE 32–4 Typical Radiation Dosages

Source of radiation	Radiation dose (rem/y)
Inhaled radon	~0.200
Medical/dental examinations	0.040
Cosmic rays	0.028
Natural radioactivity in the Earth and atmosphere	0.028
Nuclear medicine	0.015

Defined in this way, 1 rem of radiation produces the same amount of biological damage, no matter which type of radiation is involved. In addition, the larger the dosage in rem, the greater the biological damage. Referring to Table 32–3, we see that 1 rad of 200-keV X-rays produces a radiation dose of 1 rem, whereas 1 rad of protons produces a dose of 10 rem. The dosage of radiation we receive per year from cosmic rays is about 28 mrem = 0.028 rem. Other typical values of radiation dosages are presented in Table 32–4. (For comparison, a dose of 50 to 100 rem damages blood-forming tissues, whereas a dose of 500 rem usually results in death.)

EXERCISE 32–4

A biological sample receives a dose of 456 rad from neutrons with an RBE of 6.20. **(a)** Find the dosage in rem. **(b)** If this same dosage in rem is to be delivered by α rays with an RBE of 13.0, what dosage in rad is required?

SOLUTION

a. The dose in rem is the product of the dose in rad and the RBE:

$$(456\text{ rad})(6.20) = 2830\text{ ram}$$

b. The dose in rad is the dose in rem divided by the RBE:

$$\frac{2830\text{ rem}}{13.0} = 218\text{ rad}$$

Radioactive Tracers

REAL-WORLD PHYSICS: BIO
Radioactive tracers

The phenomenon of radioactivity has found applications of many types in medicine and biology. One of these applications employs a radioactive isotope as a sort of "identification tag" or "tracer" to determine the location and quantity of a substance of interest.

For example, an artificially produced radioactive isotope of iodine, ${}^{131}_{53}\text{I}$, is used to determine the condition of a person's thyroid gland. A healthy thyroid plays an important role in the distribution of iodine—a necessary nutrient—throughout the body. To see if the thyroid is functioning as it should, a patient drinks a small quantity of radioactive sodium iodide, which incorporates the isotope ${}^{131}_{53}\text{I}$. A couple of hours later, the radiation given off by the patient's thyroid gland is measured, giving a direct indication of the amount of sodium iodide the gland has processed.

Because radioactive tracers differ from their nonradioactive counterparts only in the composition of their nuclei, they undergo the same chemical and metabolic reactions. This makes them useful in diagnosing and treating a variety of different conditions. For example, chromium-51 is used to label red blood cells and to quantify gastrointestinal protein loss, copper-64 is used to study genetic diseases affecting copper metabolism, and yttrium-90 is used for liver cancer therapy.

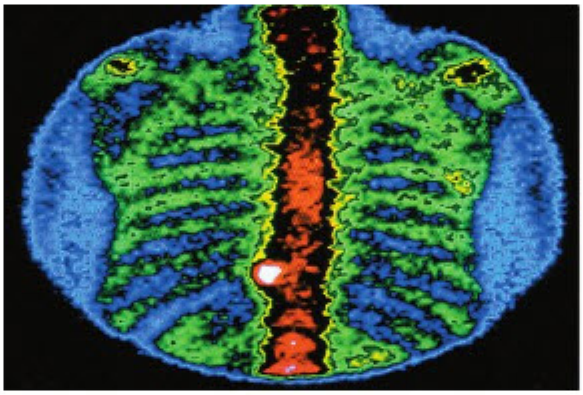
PET Scans

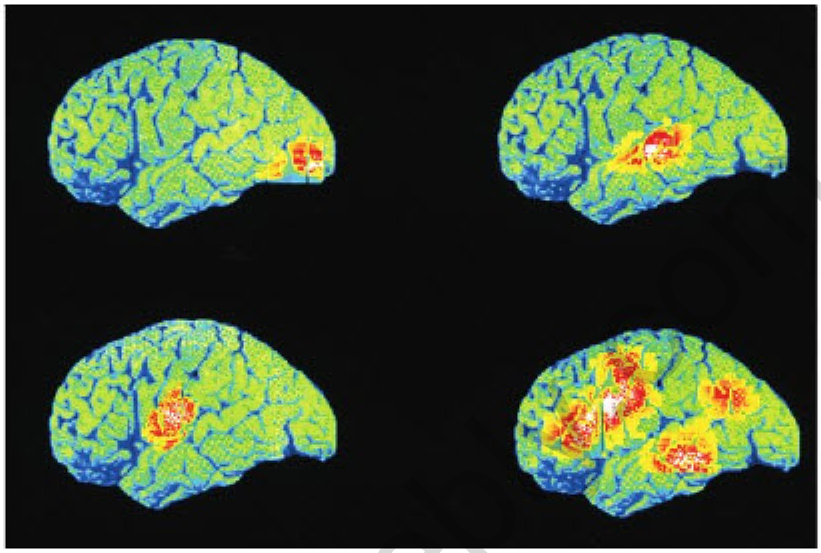
REAL-WORLD PHYSICS: BIO
Positron-emission tomography

Perhaps the most amazing medical use of radioactive decay is in the diagnostic technique known as **positron emission tomography** (PET). Like the images made using radioactive tracers, PET scans are produced with radiation that *emerges from within* the body, as opposed to radiation that is generated externally and is then passed through the body. Remarkably, the radiation in a PET scan is produced by the *annihilation of matter and antimatter* within the patient's body! It sounds like science fiction, but it is, in fact, a valuable medical tool.

To produce a PET scan, a patient is given a radiopharmaceutical, like fluorodeoxyglucose (FDG), which contains an atom that decays by giving off a positron (the antiparticle to the electron). In the case of FDG, a fluorine-18 atom is attached to a molecule of glucose, the basic energy fuel of cells. The fluorine-18 atom undergoes the following decay with a half-life of 110 minutes:

$${}^{18}_{9}\text{F} \longrightarrow {}^{18}_{8}\text{O} + e^{+} + \nu_e$$

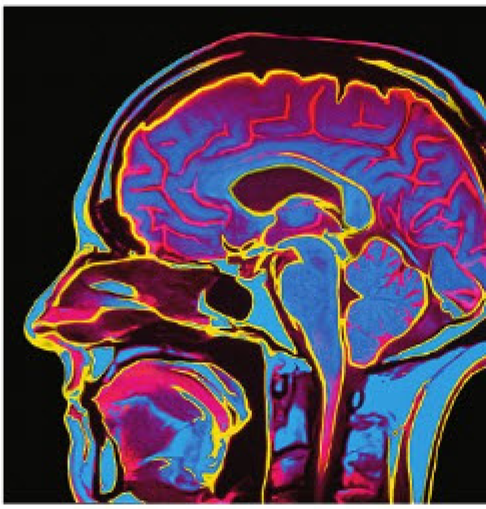




▲ Whereas X-ray images are created by passing a beam of radiation through the body, other imaging techniques make use of radiation produced within the body. One method involves the use of radioactive isotopes that tend to become concentrated in particular tissues or structures, such as the thyroid gland or the bones. Radiation from that component can then be measured, or even used to create an image, as in the bone scan at left, where areas of abnormally high radiation (white) indicate the presence of malignancy. A sophisticated variant of this approach is used to produce PET scans (see text), which are most commonly employed to visualize areas of the body where cellular energy production is most intense. The series of images at right records brain activity associated with seeing words (upper left), hearing words (upper right), speaking words (lower left), and thinking about words while uttering them (lower right). (The image of the brain, superimposed for reference, is not part of the PET scan.)

Almost immediately after the positron is emitted in this decay, it encounters an electron, and the two particles annihilate each other in a burst of energy. In fact, the annihilation process generates two powerful γ rays moving in opposite directions. As these γ rays emerge from the body, specialized detectors observe them and determine their point of origin. The resulting computerized image shows the areas in the body where glucose metabolism is most intense.

PET scans are particularly useful in examining the brain. For example, a PET scan using FDG can show which regions of the brain are most active when a person is performing a specific mental task, like counting, speaking, or translating a foreign language. The scans can also show abnormality in brain function, as when one side of a brain becomes more active than the other during an epileptic seizure. Finally, PET scans can be used to locate tumors in the brain and other parts of the body, and to monitor the progress of their treatment.



▲ Magnetic resonance imaging (MRI) is a safe, noninvasive technique for visualizing internal body structures. No ionizing radiation is involved, so the risk of tissue damage is minimal. MRI images generally show soft tissue with greater clarity than do ordinary X-rays.

Magnetic Resonance Imaging

A diagnostic technique that gives images similar to those obtained with computerized tomography (Section 31–7) is **magnetic resonance imaging**, or MRI. The basic physical mechanism used in MRI is the interaction of a nucleus with an externally applied magnetic field.

Consider the simplest nucleus—a single proton. Protons have a magnetic moment, like a small bar magnet, and when they are placed in a constant magnetic field they precess about the field with a frequency proportional to the field strength. In addition, protons have a spin of one-half, like electrons; hence, their spin can be either "up" or "down" relative to the direction of the constant magnetic field. When an oscillating magnetic field is applied perpendicular to the constant magnetic field, it can cause protons to "flip" from one spin state to the other if the frequency of oscillation is in resonance with the proton's precessional frequency. These spin flips result in the absorption or release of energy in the radio portion of the electromagnetic spectrum, which can be detected electronically.

By varying the strength of the constant magnetic field as a function of position, spin flips can be detected in various parts of the body being examined. Using a computer to combine signals from various positions allows for the generation of detailed cross-sectional images, as in CAT scans.

REAL-WORLD PHYSICS: BIO
Magnetic resonance imaging (MRI)

One of the advantages of MRI, however, is that the photons associated with the magnetic fields used in the imaging process are very low in energy. In fact, typical energies are in the range of 10^{-7} eV, much less than typical ionization energies; hence, MRI causes very little cellular damage. In contrast, photons used in CAT scans have energies that can range from 10^4 to 10^6 eV, more than enough to produce cellular damage.

32–8 Elementary Particles

Scientists have long sought to identify the fundamental building blocks of all matter, the **elementary particles.** At one point, it was thought that atoms were elementary particles—one for each element. As we saw in the previous chapter, however, this idea was put to rest when atoms were discovered to be made up of electrons, protons, and neutrons. Of these three particles, only the electron is presently considered to be elementary—protons and neutrons are now known to be composed of still smaller particles. In addition, approximately 300 new particles were discovered in the last half of the twentieth century, most of which are unstable and have lifetimes of only 10^{-6} to 10^{-23} s.

Although a complete accounting of the current theories of elementary particles is beyond the scope of this text, we shall outline the basic insights that have been derived from these theories and from related experiments. We begin by describing the four fundamental forces of nature, since particles can be categorized according to which of these forces they experience.

The Fundamental Forces of Nature

Although nature presents us with a myriad of different physical phenomena—from tornadoes and volcanoes to sunspots and comets to galaxies and black holes—all are the result of just *four* fundamental forces. This is one example of the simplicity that physicists see in nature. These forces, in order of diminishing strength, are the strong nuclear force, the electromagnetic force, the weak nuclear force, and gravity. If we assign a strength of 1 to the strong nuclear force, for purposes of comparison, the strength of the electromagnetic force is 10^{-2}, the strength of the weak nuclear force is 10^{-6}, and the strength of the gravitational force is an incredibly tiny 10^{-43}. These results are summarized in Table 32–5.

TABLE 32–5 The Fundamental Forces

Force	Relative Strength	Range
Strong nuclear	1	≈ 1 fm
Electromagnetic	10^{-2}	Infinite ($\propto 1/r^2$)
Weak nuclear	10^{-6}	$\approx 10^{-3}$ fm
Gravitational	10^{-43}	Infinite ($\propto 1/r^2$)

All objects of finite mass experience gravitational forces. This is one reason why gravity is such an important force in the universe, even though it is spectacularly weak. Similarly, objects with a finite charge experience electromagnetic forces. As for the weak and strong nuclear forces, some particles experience only the weak force, whereas others experience both the weak and the strong force. We turn now to a discussion of particles that fall into these latter two categories.

Leptons

Particles that are acted on by the weak nuclear force but not by the strong nuclear force are referred to as **leptons.** There are only six leptons known to exist, all of which are listed in Table 32–6. The most familiar of these are the electron and its corresponding neutrino—both of which are stable. No internal structure has ever been detected in any of these particles. As a result, all six leptons have the status of true elementary particles.

TABLE 32–6 Leptons

Particle	Particle Symbol	Antiparticle Symbol	Rest Energy (MeV)	Lifetime (s)
Electron	e^- or β^-	e^+ or β^+	0.511	Stable
Muon	μ^-	μ^+	105.7	2.2×10^{-6}
Tau	τ^-	τ^+	1784	10^{-13}
Electron neutrino	ν_e	$\overline{\nu}_e$	≈0	Stable
Muon neutrino	ν_μ	$\overline{\nu}_\mu$	≈0	Stable
Tau neutrino	ν_τ	$\overline{\nu}_\tau$	≈0	Stable

The weak nuclear force is responsible for most radioactive decay processes, such as beta decay. It is also a force of extremely short range. In fact, the weak force can be felt only by particles that are separated by roughly one-thousandth the diameter of a nucleus. Beyond that range, the weak force has practically no effect at all.

Hadrons

Hadrons are particles that experience both the weak and the strong nuclear force. They are also acted on by gravity, since all hadrons have finite mass. The two most familiar hadrons, of course, are the proton and the neutron. A partial list of the hundreds of hadrons known to exist is given in Table 32–7. Notice that the proton is the only stable hadron (though some theories suggest that even it may decay with the incredibly long half-life of 10^{35} y).

TABLE 32–7 Hadrons

	Particle	Particle Symbol	Antiparticle Symbol	Rest Energy (MeV)	Lifetime (s)
MESONS					
	Pion	π^+	π^-	139.6	2.6×10^{-8}
		π^0	π^0	135.0	0.8×10^{-16}
	Kaon	K^+	K^-	493.7	1.2×10^{-8}
		$K_S{}^0$	$\overline{K}_S{}^0$	497.7	0.9×10^{-10}
		$K_L{}^0$	$\overline{K}_L{}^0$	497.7	5.2×10^{-8}
	Eta	η^0	η^0	548.8	$<10^{-18}$
BARYONS					
	Proton	p	$\overline{p}$	938.3	Stable
	Neutron	n	$\overline{n}$	939.6	900
	Sigma	Σ^+	$\overline{\Sigma}^-$	1189	0.8×10^{-10}
		Σ^0	$\overline{\Sigma}^0$	1192	6×10^{-20}
		Σ^-	$\overline{\Sigma}^+$	1197	1.6×10^{-10}
	Omega	Ω^-	Ω^+	1672	0.8×10^{-10}

The strong nuclear force is the only force powerful enough to hold a nucleus together. It is a short-range force, extending only to distances comparable to the diameter of a nucleus, but within that range it is strong enough to counteract the intense electromagnetic repulsion between positively charged protons. Outside the nucleus, however, the strong nuclear force is of negligible strength.

In striking contrast with leptons, none of the hadrons are elementary particles. In fact, all hadrons are composed of either two or three smaller particles called **quarks.** Hadrons formed from two quarks are referred to as **mesons;** those formed from three quarks are **baryons.** The properties of quarks will be considered next.

Quarks

To account for the internal structure observed in hadrons, Murray Gell-Mann (1929–) and George Zweig (1937–) independently proposed in 1963 that all