REAL-WORLD PHYSICS
Solar energy panels

Photocells are also the basic unit in the *solar energy panels* that convert some of the energy in sunlight into electrical energy. A small version of a solar energy panel can be found on many pocket calculators. These panels are efficient enough to operate their calculators with nothing more than dim indoor lighting. Larger outdoor panels can operate billboards and safety lights in remote areas far from commercial power lines. Truly large solar panels, 240 ft in length, power the International Space Station and are so large that they make the station visible to the naked eye from Earth's surface. These applications of solar panels may only hint at the potential for solar energy in the future, however, especially when one considers that sunlight delivers about 200,000 times more energy to Earth each day than all the world's electrical energy production combined.

Finally, it is interesting to note that though Einstein is best known for his development of the theory of relativity, he was awarded the Nobel Prize in physics not for relativity but for the photoelectric effect.

30–3 The Mass and Momentum of a Photon

When we say that a photon is like a "particle" of light, we put the word *particle* in quotes because a photon is quite different from everyday particles in many important respects. For example, a typical particle can be held at rest in the hand. It can also be placed on a scale to have its mass determined. These operations are not possible with a photon.

First, photons travel with the speed of light, which means that all observers see them as having the same speed. It is not possible to stop a photon and hold it in your hand. In contrast, particles with a finite rest mass can never attain the speed of light. It follows, then, that photons must have *zero rest mass*. This condition can be seen mathematically by rewriting Equation 29–7 for the total energy, E, as follows:

$$E\sqrt{1-\frac{v^2}{c^2}} = m_0c^2 \qquad 30\text{–}8$$

Since photons travel at the speed of light, $v = c$, the left side of the equation is zero. As a result, the right side of the equation must also be zero, which can happen only if the rest mass is zero:

Rest Mass of a Photon

$$m_0 = 0 \qquad 30\text{–}9$$

Second, photons differ from everyday particles in that they have a finite momentum even though they have no mass. To see how this can be, note that the relativistic equation for momentum, Equation 29–5, can be rewritten as follows:

$$p\sqrt{1-\frac{v^2}{c^2}} = m_0v \qquad 30\text{–}10$$

Dividing Equation 30–10 by Equation 30–8 we get

$$\frac{p}{E} = \frac{v}{c^2}$$

Once again setting $v = c$ for a photon, we find that the momentum of a photon is related to its total energy as follows:

$$p = \frac{E}{c}$$

Finally, recalling that $E = hf$, and that $f = c/\lambda$, we obtain the following result:

Momentum of a Photon

$$p = \frac{hf}{c} = \frac{h}{\lambda} \qquad 30\text{–}11$$

PROBLEM-SOLVING NOTE
The Momentum of a Photon

The relation $p = mv$ is not valid for photons. In fact, photons have a finite momentum even though their rest mass is zero.

Note that a photon's momentum increases with its frequency, and thus with its energy.

As one might expect, the momentum of a typical photon of visible light is quite small, as is illustrated in the next Exercise.

EXERCISE 30–4

Calculate the momentum of a photon of yellow light with a frequency of 5.25×10^{14} Hz.

SOLUTION

Substituting $f = 5.25 \times 10^{14}$ Hz in Equation 30–11, we obtain

$$p = \frac{hf}{c} = \frac{(6.63 \times 10^{-34}\ \mathrm{J \cdot s})(5.25 \times 10^{14}\ \mathrm{Hz})}{3.00 \times 10^{8}\ \mathrm{m/s}} = 1.16 \times 10^{-27}\ \mathrm{kg \cdot m/s}$$

REAL-WORLD PHYSICS
Sailing on a beam of light

As small as the momentum of a photon is, it can still have a significant impact if the number of photons is large. For example, NASA is studying the feasibility of constructing spaceships that would be powered by huge "light sails" that transfer the momentum of photons from the Sun to the ship. A sail would reflect photons, creating a change in momentum, and hence a reaction force on the sail due to the **radiation pressure** of the photons. With a large enough sail, it may one day be possible to cruise among the stars with space-faring sailboats. In fact, the designs under consideration by NASA would result in the largest and fastest spacecraft ever constructed—10 times faster than the space shuttle.

REAL-WORLD PHYSICS
Optical tweezers

A more "down to Earth," though no less exotic, application of radiation pressure is the *optical tweezers*. Basically, an optical tweezers is a laser beam that is made more intense in the middle than near the edges. When a small, translucent object is placed in this beam, the recoil produced by photons passing through the object exerts a small force on it directed toward the center of the beam. The situation is similar to a ball suspended in the "beam" of air produced by a hair dryer. Although the force exerted by such tweezers is typically on the order of only 10^{-12} N, it is still large enough to manipulate cells, DNA, and other subcellular particles. In fact, optical tweezers can even capture, lift, and separate a single bacterium from a bacterial culture for further study.

30–4 Photon Scattering and the Compton Effect

Einstein's photon model of the photoelectric effect, published in 1905, focused on the energy of a photon, $E = hf$. The momentum of a photon, $p = hf/c = h/\lambda$, plays a key role in a different type of experiment, in which an X-ray photon undergoes a collision with an electron initially at rest. The result of this collision is that the photon is scattered, which changes its direction and energy. This type of process, referred to as the **Compton effect,** was explained in terms of the photon model of light by the American physicist Arthur Holly Compton (1892–1962) in 1923. The success of the photon model in explaining both the Compton effect and the photoelectric effect gained it widespread acceptance.

Figure 30–7 shows an X-ray photon striking an electron at rest and scattering at an angle θ with respect to the incident direction. To understand the behavior of a system like this, we treat the photon as a particle, with a certain energy and momentum, that collides with another particle (the electron) of mass m_e and initial speed zero. If we assume the electron is free to move, this collision conserves both energy and momentum.

First, to conserve energy the following relation must be satisfied:

energy of incident photon = energy of scattered photon + final kinetic energy of electron

$$hf = hf' + K \qquad \text{30–12}$$

Note that, in general, the frequency of the scattered photon, f', is less than the frequency of the incident photon, f, since part of the energy of the incident photon

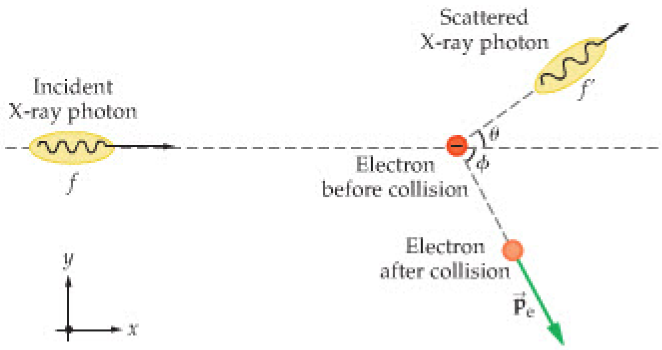


FIGURE 30–7 The Compton effect

An X-ray photon scattering from an electron at rest can be thought of as a collision between two particles. The result is a change of wavelength for the scattered photon. This is referred to as the Compton effect.

has gone into kinetic energy of the electron. Because the frequency of the scattered photon is reduced, its wavelength, $\lambda' = c/f'$, is increased.

Next, we conserve both the x and y components of momentum, using $p = h/\lambda$ as the magnitude of the incident photon's momentum, and $p' = h/\lambda'$ as the magnitude of the scattered photon's momentum. For the x component of momentum we have the following relation:

$$\frac{h}{\lambda} = \frac{h}{\lambda'}\cos\theta + p_e\cos\phi \qquad \text{30–13}$$

Note that we assume the electron has a momentum p_e at an angle ϕ to the incident direction. Conserving the y component of momentum, which initially is zero, we have

$$0 = \frac{h}{\lambda'}\sin\theta - p_e\sin\phi \qquad \text{30–14}$$

Given the initial wavelength and frequency, $\lambda = c/f$, and the scattering angle θ, we can use the three relations, Equations 30–12, 30–13, and 30–14, to solve for the three unknowns: λ', ϕ, and K.

Of particular interest is the change in wavelength produced by the scattering. We find the following result:

Compton Shift Formula

$$\Delta\lambda = \lambda' - \lambda = \frac{h}{m_e c}(1 - \cos\theta) \qquad \text{30–15}$$

SI unit: m

In this expression, the quantity $h/m_e c = 2.43 \times 10^{-12}$ m is referred to as the *Compton wavelength of an electron*. The maximum change in the photon's wavelength occurs when it scatters in the reverse direction ($\theta = 180°$), in which case the change is twice the Compton wavelength, $\Delta\lambda = 2(h/m_e c)$. On the other hand, if the scattering angle is zero ($\theta = 0$)—in which case there really is no scattering at all—the change in wavelength is zero; $\Delta\lambda = 0$.

PROBLEM-SOLVING NOTE

Scattering Angle

The angle θ in the Compton shift formula is the angle between the incident direction of an X-ray and its direction of propagation after scattering.

CONCEPTUAL CHECKPOINT 30–3 CHANGE IN WAVELENGTH

If X-rays are scattered from protons instead of from electrons, is the change in their wavelength for a given angle of scattering **(a)** increased, **(b)** decreased, or **(c)** unchanged?

REASONING AND DISCUSSION

As can be seen from Equation 30–15, the change in wavelength for a given angle is proportional to $h/m_e c$. If a proton is substituted for the electron, the change in wavelength will be proportional to $h/m_p c$ instead. Since protons have about 2000 times the mass of electrons, the change in wavelength will be reduced by a factor of about 2000.

ANSWER

(b) The change in wavelength is decreased when X-rays scatter from protons rather than from electrons.

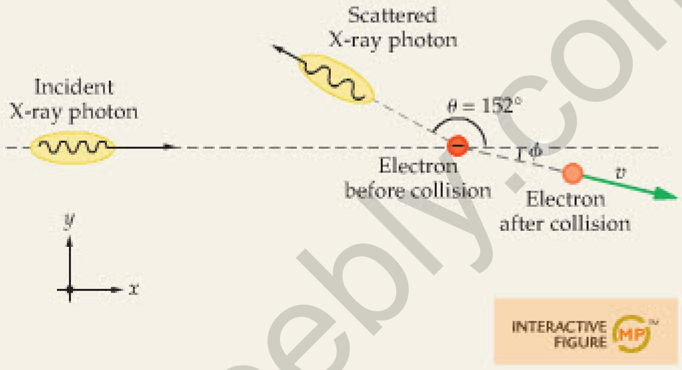
The next Example gives a detailed analysis of a photon-electron collision.

EXAMPLE 30–4 SCATTERING X-RAYS

An X-ray photon with a wavelength of 0.650 nm scatters from a free electron at rest. After scattering, the photon moves at an angle of 152° relative to the incident direction. Find **(a)** the wavelength and **(b)** the energy of the scattered photon. **(c)** Determine the kinetic energy of the recoiling electron.

PICTURE THE PROBLEM

As shown in the sketch, the incoming photon is scattered at an angle of 152° relative to its initial direction; thus, it almost heads back the way it came. After the collision, the electron moves with a speed v at an angle ϕ relative to the forward direction.



STRATEGY

a. We can find the wavelength after scattering, λ', by using the Compton shift formula, Equation 30–15. Note that we are given the initial wavelength, λ, as well as the scattering angle, θ.

b. The energy of the scattered photon is $E' = hf' = hc/\lambda'$.

c. Since energy is conserved, the kinetic energy of the electron is given by $hf = hf' + K$. We can solve this relation to yield the kinetic energy, K.

SOLUTION

Part (a)

1. Use Equation 30–15 to find the wavelength of the scattered photon:

$$\lambda' = \lambda + \frac{h}{m_e c}(1 - \cos\theta) = 0.650 \times 10^{-9}\ \text{m} + \left(\frac{6.63 \times 10^{-34}\ \text{J}\cdot\text{s}}{(9.11 \times 10^{-31}\ \text{kg})(3.00 \times 10^{8}\ \text{m/s})}\right)(1 - \cos 152^\circ)$$
$$= 6.55 \times 10^{-10}\ \text{m} = 0.655\ \text{nm}$$

Part (b)

2. The energy of the scattered photon is given by $E' = hf' = hc/\lambda'$:

$$E' = hf' = \frac{hc}{\lambda'} = \frac{(6.63 \times 10^{-34}\ \text{J}\cdot\text{s})(3.00 \times 10^{8}\ \text{m/s})}{6.55 \times 10^{-10}\ \text{m}}$$
$$= 3.04 \times 10^{-16}\ \text{J}$$

Part (c)

3. Find the initial energy of the photon using $E = hf = hc/\lambda$:

$$E = hf = \frac{hc}{\lambda} = \frac{(6.63 \times 10^{-34}\ \text{J}\cdot\text{s})(3.00 \times 10^{8}\ \text{m/s})}{6.50 \times 10^{-10}\ \text{m}}$$
$$= 3.06 \times 10^{-16}\ \text{J}$$

4. Subtract the final energy of the photon from its initial energy to find the kinetic energy of the electron:

$$K = hf - hf'$$
$$= 3.06 \times 10^{-16}\ \text{J} - 3.04 \times 10^{-16}\ \text{J}$$
$$= 2 \times 10^{-18}\ \text{J} = 10\ \text{eV}$$

INSIGHT

Notice that the energy of the X-ray photon is roughly 1900 eV = 1.9 keV both before and after scattering. Only a very small fraction of its energy is delivered to the electron. Still, the electron acquires about 10 eV of energy—enough to ionize an atom.

PRACTICE PROBLEM

At what scattering angle will the photon have a wavelength of 0.652 nm? [**Answer:** $\theta = 79.9^\circ$]

Some related homework problems: Problem 54, Problem 55

As we have seen in both the photoelectric effect and the Compton effect, a photon can behave as a particle with a well-defined energy and momentum. We also know, however, that light exhibits wavelike behavior, as when it produces interference fringes in Young's two-slit experiment (Chapter 28). That light can have such seemingly opposite attributes is one of the deepest mysteries of quantum physics and, at the same time, one of its most significant insights. In the next section we expand on this insight and extend it to the behavior of matter.

30–5 The de Broglie Hypothesis and Wave–Particle Duality

As we have seen, the Compton effect was explained in terms of photons in 1923. Another significant advance in quantum physics occurred that same year, when a French graduate student, Louis de Broglie (1892–1987), put forward a most remarkable hypothesis that would later win him the Nobel Prize in physics. His suggestion was basically the following:

> Since light, which we usually think of as a wave, can exhibit particle-like behavior, perhaps a particle of matter, like an electron, can exhibit wavelike behavior.

In particular, de Broglie proposed that the *same* relation between wavelength and momentum that Compton applied to the photon, $p = h/\lambda$, should apply to particles as well. Thus, if the momentum of a particle is p, its de Broglie wavelength is

de Broglie Wavelength

$$\lambda = \frac{h}{p} \qquad 30\text{–}16$$

SI unit: m

Note that the greater a particle's momentum, the smaller its de Broglie wavelength.

How can the idea of a wavelength for matter make sense, however, when we know that objects like baseballs and cars behave like particles, not like waves? To see how this is possible, we calculate the de Broglie wavelength of a typical macroscopic object: a 0.13-kg apple moving with a speed of 5.0 m/s. Substituting these values into $p = mv$, and using $\lambda = h/p$, we find the following wavelength: $\lambda = 1.0 \times 10^{-33}$ m. Clearly, this wavelength, which is smaller than the diameter of an atom by a factor of 10^{23}, is much too small to be observed in any macroscopic experiment. Thus, an apple could have a wavelength as given by the de Broglie relation, and we would never notice it.

In contrast, consider an electron with a kinetic energy of 10.0 eV, a typical atomic energy. Using $K = \frac{1}{2}mv^2 = p^2/2m$ to solve for the momentum p, we find that the de Broglie wavelength in this case is $\lambda = 3.88 \times 10^{-10}\ \text{m} = 3.88\ \text{Å}$. Now this wavelength, which is comparable to the size of an atom or molecule, would clearly be significant in such systems. Therefore, the de Broglie wavelength may be unobservable in macroscopic systems but all-important in atomic systems.

ACTIVE EXAMPLE 30–2 THE SPEED AND WAVELENGTH OF AN ELECTRON

How fast is an electron moving if its de Broglie wavelength is 3.50×10^{-7} m?

SOLUTION *(Test your understanding by performing the calculations indicated in each step.)*

1. Write Equation 30–16 in terms of the electron's mass and speed: $\lambda = h/mv$
2. Rearrange to solve for the speed, v: $v = h/m\lambda$
3. Substitute numerical values: $v = 2080$ m/s

INSIGHT

The relatively small value obtained for the electron's speed justifies using the nonrelativistic expression for momentum, $p = mv$.

YOUR TURN

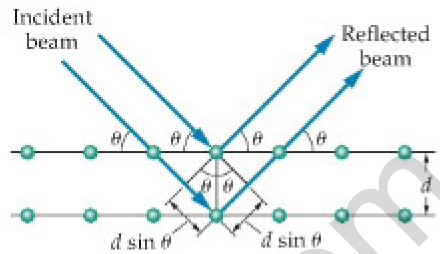
If the electron's speed is doubled, does its de Broglie wavelength increase or decrease? By what factor?

(Answers to **Your Turn** *problems are given in the back of the book.)*

In order for the de Broglie wavelength to be taken seriously, however, it must be observed experimentally. We next consider ways in which this can be done.

Diffraction of X-rays and Particles by Crystals

An especially powerful way to investigate wave properties is through the study of interference patterns. Consider, for example, directing a coherent beam of X-rays onto a crystalline substance composed of regularly spaced planes of atoms, as indicated in Figure 30–8. Notice that the reflected beam combines rays that have followed different paths, with different path lengths. If the difference in path lengths is half a wavelength, destructive interference occurs; if the difference in path lengths is one wavelength, constructive interference results, and so on. From Figure 30–8 it is clear that the difference in path lengths for rays reflecting from adjacent planes that have a spacing d is $2d \sin \theta$. Thus, constructive interference occurs when the following conditions are met:



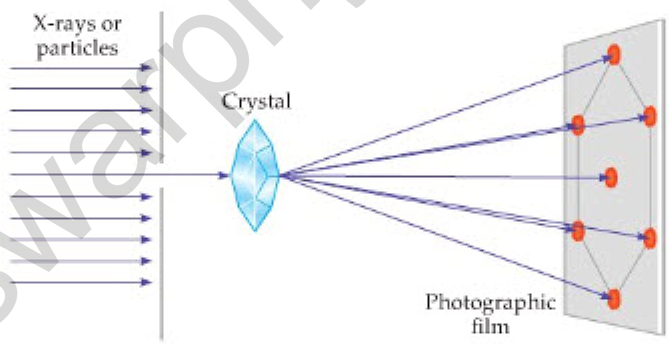
▲ **FIGURE 30–8 Scattering from a crystal**
Scattering of X-rays or particles from a crystal. Note that waves reflecting from the lower plane of atoms have a path length that is longer than the path of the upper waves by the amount $2d \sin \theta$.

Constructive Interference When Scattering from a Crystal

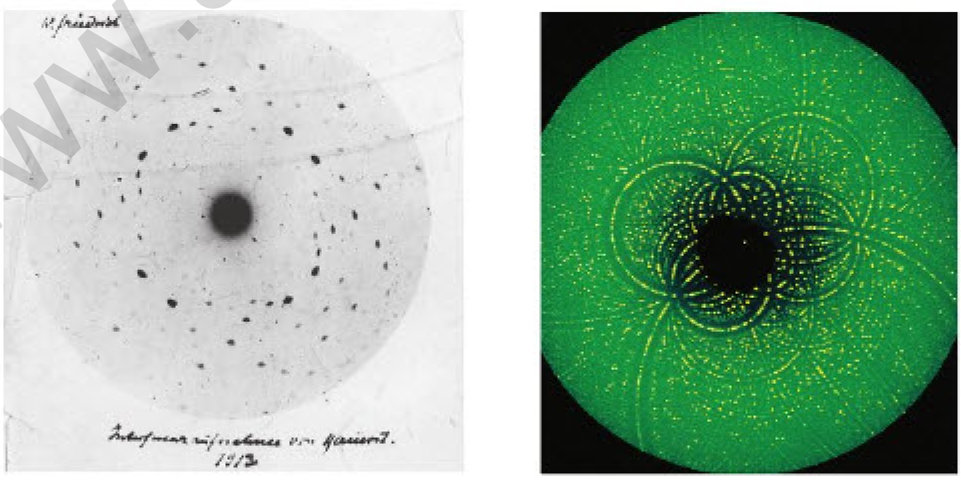
$$2d \sin \theta = m\lambda \quad m = 1, 2, 3, \ldots \qquad \text{30–17}$$

This is very similar to the condition for constructive interference in a diffraction grating, as derived in Equation 28–16. The resulting pattern of interference maxima—referred to as a **diffraction pattern**—can be projected onto photographic film, as indicated in Figure 30–9, and used to determine the geometric properties of a crystal. Examples of diffraction patterns are shown in Figure 30–10.

Now, if particles—like electrons, for example—have a wavelength comparable to atomic distances, it should be possible to produce diffraction patterns with them in much the same way as with X-rays. This is indeed the case, as was shown by the American physicists C. J. Davisson and L. H. Germer, who, in 1928, produced diffraction patterns by scattering low-energy electrons (about 54 eV) from crystals of nickel. The spacing between spots in the electron diffraction pattern allowed the researchers to determine the electron's wavelength, which verified in



◀ **FIGURE 30–9 Diffraction patterns**
Diffraction patterns can be observed by passing a beam of X-rays or particles through a crystal. The beams emerging from the crystal are at specific angles, due to constructive interference, and can be recorded on photographic film.


◀ **FIGURE 30–10 X-ray diffraction**
Because the spacing of atoms in a crystal is of the same order of magnitude as the wavelengths of X-rays, the planes of a crystal can serve as a diffraction grating for X-rays. When properly interpreted, the resulting diffraction patterns can provide remarkably detailed information about the structure of the crystal. The historic photograph at left, made by two of the pioneers of X-ray crystallography in 1912, records the diffraction pattern from a simple inorganic salt. The photograph at right, made nearly a century later, shows the much more intricate pattern produced by a large protein molecule.

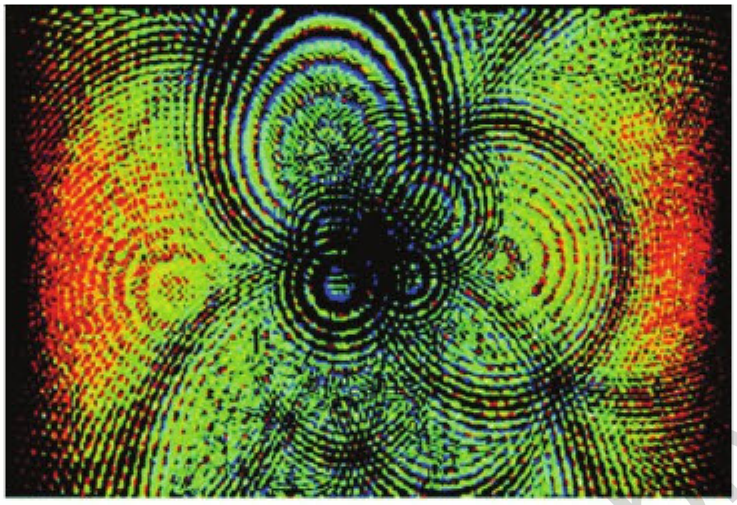


FIGURE 30–11 Neutron diffraction The neutron diffraction pattern of the protein lysozyme, a digestive enzyme.

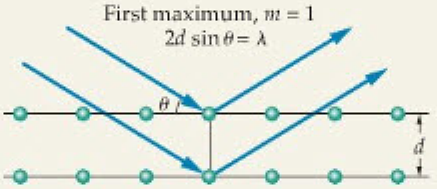
detail the de Broglie relation, $\lambda = h/p$. Similar diffraction patterns have since been observed with neutrons, hydrogen atoms, and helium atoms. An example of a neutron diffraction pattern is shown in Figure 30–11.

EXAMPLE 30–5 NEUTRON DIFFRACTION

A beam of neutrons moving with a speed of 1450 m/s is diffracted from a crystal of table salt (sodium chloride), which has an interplanar spacing of $d = 0.282$ nm. **(a)** Find the de Broglie wavelength of the neutrons. **(b)** Find the angle of the first interference maximum.

PICTURE THE PROBLEM

Our sketch shows a beam of neutrons reflecting from two adjacent atomic planes of table salt. The distance d between the planes is 0.282 nm.



STRATEGY

a. We can find the de Broglie wavelength using Equation 30–16: $\lambda = h/p$. Since the speed of the neutrons is much less than the speed of light, we can write the momentum as $p = m_n v$, where m_n is the mass of a neutron.

b. Referring to Equation 30–17, we see that the first interference maximum occurs when $m = 1$ in the relation $2d \sin\theta = m\lambda$. Thus we set $2d \sin\theta$ equal to λ and solve for θ.

SOLUTION

Part (a)

1. Calculate the de Broglie wavelength using $\lambda = h/p = h/m_n v$:

$$\lambda = \frac{h}{p}$$

$$= \frac{h}{m_n v} = \frac{6.63 \times 10^{-34}\ \text{J}\cdot\text{s}}{(1.67 \times 10^{-27}\ \text{kg})(1450\ \text{m/s})} = 0.274\ \text{nm}$$

Part (b)

2. Set $m = 1$ in $2d \sin\theta = m\lambda$:

$$2d \sin\theta = \lambda$$

3. Solve for the angle θ:

$$\theta = \sin^{-1}\left(\frac{\lambda}{2d}\right) = \sin^{-1}\left(\frac{0.274\ \text{nm}}{2(0.282\ \text{nm})}\right) = 29.1°$$

INSIGHT

Note that if we change the speed of the neutrons, we will change their wavelength. This, in turn, changes the angle of the interference maxima. This connection between the neutron speed and the interference maxima provides a detailed and precise way of verifying the de Broglie relation.

PRACTICE PROBLEM

Find the angle of the first interference maximum if the neutron speed is increased to 2250 m/s. **[Answer:** 18.2°]

Some related homework problems: Problem 62, Problem 65

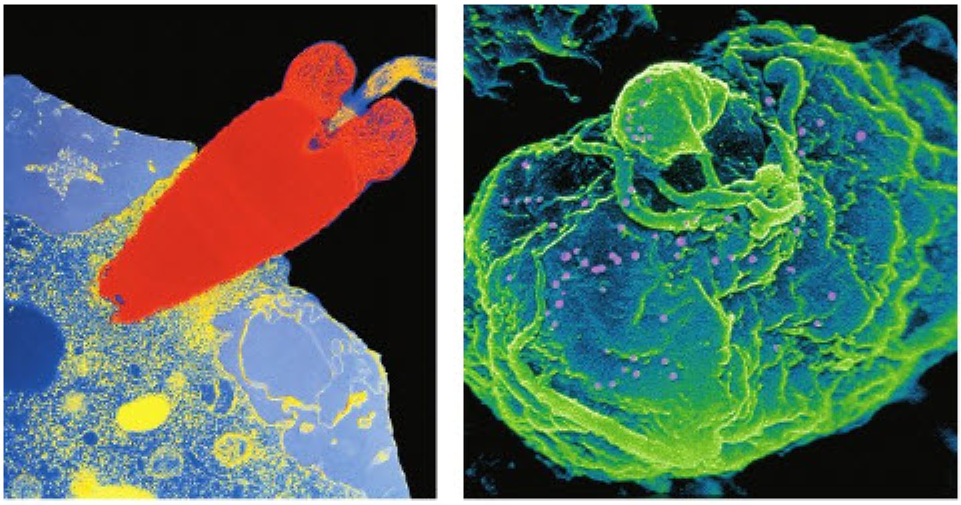


FIGURE 30–12 Electron micrography
Electron micrographs can be produced in several different ways. In a transmission electron micrograph (TEM), such as the image at left, the beam of electrons passes through the specimen, as in an ordinary light microscope. This colorized photo shows a sea urchin sperm cell penetrating the membrane of an egg. In a scanning electron micrograph (SEM), like the image at right, the electrons are reflected from the surface of the specimen (which is usually coated with a thin layer of metal atoms first) and used to produce a startlingly detailed three-dimensional image. This photo, also colorized for greater clarity, shows particles of the AIDS virus HIV (purple) emerging from an infected human cell.

In *electron microscopes*, a beam of electrons is used to form an image of an object in much the same way that a beam of light is used in a light microscope. One of the differences, however, is that the wavelength of the electrons can be much shorter than the wavelength of visible light. For example, the shortest visible wavelength is that of blue light, which is about 380 nm. In comparison, we have seen that the de Broglie wavelength of an electron with an energy of only 10.0 eV is 0.388 nm—about 1000 times smaller than the wavelength of blue light. Since the ability to resolve small objects depends on using a wavelength that is smaller than the object to be imaged, an electron microscope can see much finer detail than a light microscope, as Figure 30–12 indicates.

REAL-WORLD PHYSICS
Electron microscopes

Wave–Particle Duality

Notice that we have now come full circle in our study of waves and particles, from considering light as a wave, and then noting that it has particle-like properties, to investigating particles of matter like electrons, and finding that they have wave-like properties. This type of behavior is referred to as the **wave–particle duality.** Thus, as strange as it may seem, light is a wave, but it also comes in discrete units called photons. Electrons come in discrete units of well-defined mass and charge, but they also have wave properties.

To illustrate the wave–particle duality, we consider a two-slit experiment, as shown in Figure 30–13. If light is passed through these slits it forms interference patterns of dark and light fringes, as we saw in Chapter 28. If the intensity of the light is reduced to a very low level, it is possible to have only a single photon at a time passing through the apparatus. This photon lands on the distant screen. Eventually, as more and more photons land on the screen, the interference pattern emerges.

Similar behavior can be observed with electrons passing through a pair of slits. The results, as shown in Figure 30–14, are the direct analog of the results obtained with light. Notice that the dark portions of the interference pattern are

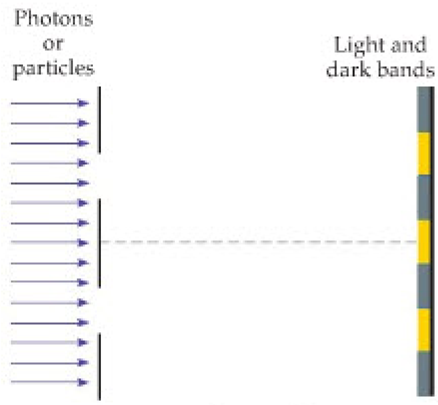


FIGURE 30–13 The two-slit experiment
When photons or particles are passed through a screen with two slits, the result is an interference pattern of light and dark bands.

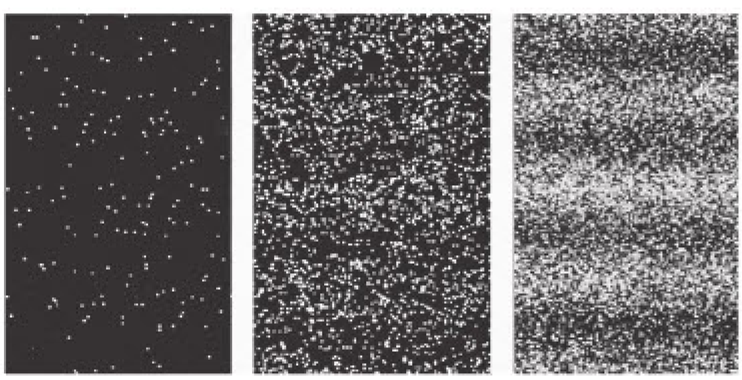


FIGURE 30–14 Creation of an interference pattern by electrons passing through two slits
At first, the electrons seem to arrive at the screen in random locations. As the number of electrons increases, however, the interference pattern becomes more evident. (Compare Figures 28–3 and 28–6.)

those places where electron waves passing through the two slits have combined to produce destructive interference—that is, at these special points an electron is essentially able to "cancel itself out" to produce a dark fringe.

Clearly, then, subatomic particles and light are quite different from objects and waves that we observe on a macroscopic level. In fact, one of the most profound and unexpected insights of quantum physics is simply that even though baseballs, apples, and people are composed of electrons, protons, and other particles, the behavior of these subatomic particles is nothing like the behavior of baseballs, apples, and people. In short, an electron is not like a BB reduced in size. An electron has properties that are different from those of any object we experience on the macroscopic level. To try to force light and electrons into categories like waves and particles is to miss the essence of their existence—they are neither one nor the other, though they have characteristics of both. Again, one is reminded of the saying "The universe is not only stranger than we imagine, it is stranger than we can imagine."

30–6 The Heisenberg Uncertainty Principle

One of the more interesting aspects of Figure 30–14 is that the dots, corresponding to observations of individual electrons, appear on the screen in random order. To be specific, each time this experiment is run, the dots appear in different locations and in different sequence—all that remains the same is the final pattern that emerges after a large number of electrons are observed. The point is that as any given electron passes through the two-slit apparatus, *it is not possible to predict exactly where that one electron will land on the screen*. We can give the probability that it will land at different locations, but the fate of each individual electron is uncertain.

This kind of "uncertainty" is inherent in quantum physics and is due to the fact that matter has wavelike properties. As a simple example, consider a beam of electrons moving in the x direction and passing through a single slit, as in **Figure 30–15**. The result of this experiment is a diffraction pattern similar to that found for light in Chapter 28 (see Figure 28–18). In particular, if the beam passes through a slit of width W, it produces a large central maximum—where the probability of detecting an electron is high—with a dark fringe on either side at an angle θ given by Equation 28–12:

$$\sin\theta = \frac{\lambda}{W} \qquad 30\text{–}18$$

If the incoming electrons have a momentum p_x, the wavelength in Equation 30–18 is given by the de Broglie relation: $\lambda = h/p_x$.

We interpret this experiment as follows: By passing the beam of electrons through the slit, we have determined their location in the y direction to within an uncertainty of roughly $\Delta y \sim W$. After passing through the slit, however, the beam spreads out to form a diffraction pattern—with some electrons acquiring a y component of momentum. Thus there is now an uncertainty in the y component of momentum, Δp_y.

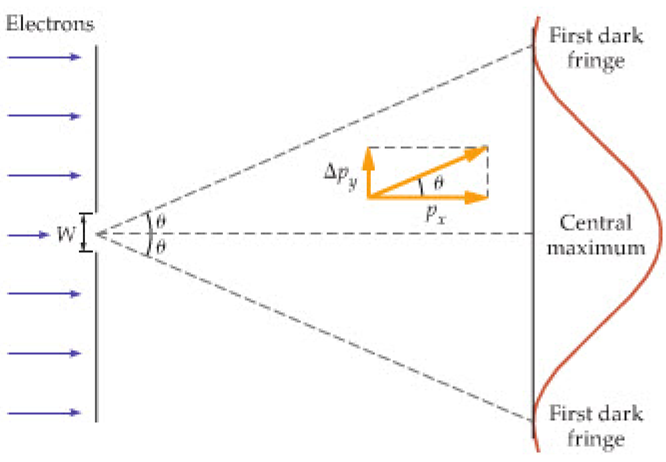


FIGURE 30–15 Diffraction pattern of electrons

Central region of the diffraction pattern formed by electrons passing through a single slit in a screen. The curve to the right is a measure of the number of electrons detected at any given location. Note that no electrons are detected at the dark fringes. This diffraction pattern is identical in form with that produced by light passing through a single slit.

There is a reciprocal relationship between Δy and Δp_y, as we show in Figure 30–16. As the width W of the slit is made smaller, to decrease Δy, the angle in Equation 30–18 increases, and the diffraction pattern spreads out, increasing Δp_y. Conversely, if the width of the slit is increased, the diffraction pattern becomes narrower; that is, increasing Δy results in a decrease in Δp_y. To summarize;

> If we know the position of a particle with greater precision, its momentum is more uncertain; if we know the momentum of a particle with greater precision, its position is more uncertain.

We can give this conclusion in mathematical form by returning to Equation 30–18 and Figure 30–15. First, assuming small angles, θ, we can use the approximation that $\sin\theta \sim \theta$ to write Equation 30–18 as $\theta \sim \lambda/W$. Similarly, from Figure 30–15 we see that $\tan\theta = \Delta p_y/p_x$. Since $\tan\theta \sim \theta$ for small angles, it follows that $\theta \sim \Delta p_y/p_x$. Equating these two expressions for θ we get

$$\theta \sim \frac{\lambda}{W} \sim \frac{\Delta p_y}{p_x}$$

Setting $p_x = h/\lambda$ and $W \sim \Delta y$, we find

$$\frac{\lambda}{\Delta y} \sim \frac{\Delta p_y}{h/\lambda}$$

Finally, canceling λ and rearranging, we obtain

$$\Delta p_y\,\Delta y \sim h$$

Thus, the product of uncertainties in position and momentum cannot be less than a certain finite amount that is approximately equal to Planck's constant.

A thorough treatment of this system yields the more precise relation first given by the German physicist Werner Heisenberg (1901–1976) in 1927, known as the **Heisenberg uncertainty principle:**

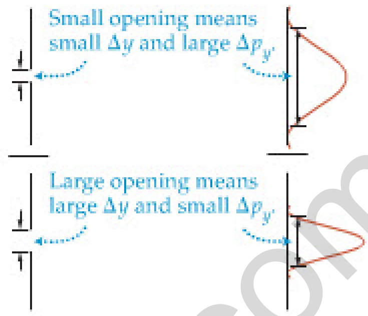
The Heisenberg Uncertainty Principle: Momentum and Position

$$\Delta p_y\,\Delta y \geq \frac{h}{2\pi} \qquad 30\text{–}19$$

In fact, Heisenberg showed that this relation is a general principle and not restricted in any way to the single-slit system considered here. There is simply an unremovable, intrinsic uncertainty in nature that is the result of the wave behavior of matter. Since the wavelength of matter, $\lambda = h/p$, depends directly on the magnitude of h, so too does the uncertainty.

One way of stating the uncertainty principle is that it is impossible to know both the position and momentum of a particle with arbitrary precision at any given time. For example, if the position is known precisely, so that Δy approaches zero, it follows that Δp_y must approach infinity, as shown in Figure 30–16. This result implies that the y component of momentum is completely uncertain. Similarly, complete knowledge of p_y ($\Delta p_y \rightarrow 0$) implies that the position y is completely uncertain.

As one might expect, the uncertainty restrictions given in Equation 30–19 have negligible impact on macroscopic systems, but are all-important in atomic and nuclear systems. This is illustrated in the following Example.



▲ **FIGURE 30–16 Uncertainty in position and momentum**

Reciprocal relationship between the uncertainty in position (Δy) and the uncertainty in momentum (Δp_y). As in Figure 30–15, the curves at the right indicate the number of electrons detected at any given location.

EXAMPLE 30–6 WHAT IS YOUR POSITION?

If the speed of an object is 35.0 m/s, with an uncertainty of 5.00%, what is the minimum uncertainty in the object's position if it is **(a)** an electron or **(b)** a volleyball (m = 0.350 kg)?

PICTURE THE PROBLEM

As shown in the sketch, the electron and the volleyball have the same speed. The fact that the electron has the smaller mass means that the values of its momentum, and of its uncertainty in momentum, are less than the corresponding values for the volleyball. As a result, the uncertainty in position will be larger for the electron.

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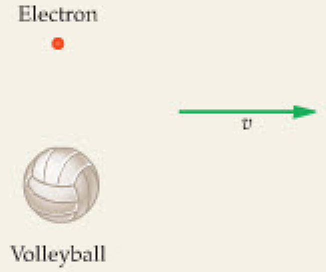
STRATEGY

The 5.00% uncertainty in speed means that the magnitude of the momentum (mass times speed) of the electron and the volleyball are also uncertain by 5.00%. The minimum uncertainty in position, then, is given by the Heisenberg uncertainty principle: $\Delta y = h/(2\pi\,\Delta p_y)$.



SOLUTION

Part (a)

1. Calculate the uncertainty in the electron's momentum:

$$\Delta p_y = 0.0500 m_e v = 0.0500(9.11 \times 10^{-31}\ \text{kg})(35.0\ \text{m/s}) = 1.59 \times 10^{-30}\ \text{kg}\cdot\text{m/s}$$

2. Use the uncertainty principle to find the minimum uncertainty in position:

$$\Delta y = \frac{h}{2\pi\,\Delta p_y} = \frac{6.63 \times 10^{-34}\ \text{J}\cdot\text{s}}{2\pi(1.59 \times 10^{-30}\ \text{kg}\cdot\text{m/s})} = 6.64 \times 10^{-5}\ \text{m}$$

Part (b)

3. Calculate the uncertainty in the volleyball's momentum:

$$\Delta p_y = 0.0500 m_{\text{volleyball}} v = 0.0500(0.350\ \text{kg})(35.0\ \text{m/s}) = 0.613\ \text{kg}\cdot\text{m/s}$$

4. As before, use the uncertainty principle to find the minimum uncertainty in position:

$$\Delta y = \frac{h}{2\pi\,\Delta p_y} = \frac{6.63 \times 10^{-34}\ \text{J}\cdot\text{s}}{2\pi(0.613\ \text{kg}\cdot\text{m/s})} = 1.72 \times 10^{-34}\ \text{m}$$

INSIGHT

To put these results in perspective, the minimum uncertainty in the position of the electron is roughly 100,000 times the size of an atom. Clearly, this is a significant uncertainty on the atomic level. On the other hand, the minimum uncertainty of the volleyball is smaller than an atom by a factor of about 10^{24}. It follows, then, that the uncertainty principle does not have measurable consequences on typical macroscopic objects.

PRACTICE PROBLEM

Repeat this problem, assuming the uncertainty in speed is reduced by a factor of 2 to 2.50%. [**Answer:** The uncertainties in position increase by a factor of 2. Electron, $\Delta y = 1.33 \times 10^{-4}$ m; volleyball, $\Delta y = 3.44 \times 10^{-34}$ m.]

Some related homework problems: Problem 70, Problem 72

The Heisenberg uncertainty principle also sets the typical energy scales in atomic and nuclear systems. For example, if an electron is known to be confined to an atom, the uncertainty in its position Δy, will be roughly 1 Å. This implies a finite uncertainty for Δp_y, which in turn implies a finite kinetic energy—even though the average value of p_y is zero.

EXAMPLE 30–7 AN ELECTRON IN A BOX

Suppose an electron is confined to a box that is about 0.50 Å on a side. If this distance is taken as the uncertainty in position of the electron, **(a)** calculate the corresponding minimum uncertainty in momentum. **(b)** Because the electron is confined to a stationary box, its average momentum is zero. The magnitude of the electron's momentum is nonzero, however. Assuming the magnitude of the electron's momentum is the same as its uncertainty in momentum, calculate the corresponding kinetic energy.

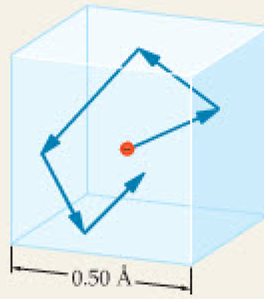
PICTURE THE PROBLEM

In our sketch we show an electron bouncing around inside a box that is 0.50 Å on a side. The rapid motion of the electron is a quantum effect, due to the uncertainty principle.

STRATEGY

a. Letting $\Delta y = 0.50$ Å, we can find the minimum uncertainty in momentum by using the Heisenberg uncertainty principle: $\Delta p_y = h/(2\pi\, \Delta y)$.

b. We can calculate the kinetic energy of the electron from its momentum by using $K = p^2/2m$. For the momentum, p, we will use the value of Δp_y obtained in part (a).



SOLUTION

Part (a)

1. Use $\Delta y = 0.50$ Å and the uncertainty principle to find Δp_y:

$$\Delta p_y = \frac{h}{2\pi\, \Delta y} = \frac{6.63 \times 10^{-34}\ \text{J}\cdot\text{s}}{2\pi(0.50 \times 10^{-10}\ \text{m})} = 2.1 \times 10^{-24}\ \text{kg}\cdot\text{m/s}$$

Part (b)

2. Set $p = \Delta p_y$, and use $K = p^2/2m$ to find the kinetic energy:

$$K = \frac{p^2}{2m} = \frac{(2.1 \times 10^{-24}\ \text{kg}\cdot\text{m/s})^2}{2(9.11 \times 10^{-31}\ \text{kg})} = 2.4 \times 10^{-18}\ \text{J} = 15\ \text{eV}$$

INSIGHT

The important point of this rough estimate of an electron's kinetic energy is that it is of the order of typical atomic energies. Thus, the fact that an electron is confined to an object the size of an atom means that its energy must be on the order of 10 eV. This is why the electron in our sketch is bouncing around inside the box rather than resting on its floor—as would be the case for a tennis ball inside a cardboard box. The uncertainty principle implies that the electron must have about 10 eV of kinetic energy when it is confined in this way; hence, it must be moving quite rapidly. We consider the case of a tennis ball in a cardboard box in the following Practice Problem.

PRACTICE PROBLEM

A tennis ball ($m = 0.06$ kg) is confined within a cardboard box 0.5 m on a side. Estimate the kinetic energy of this ball and its corresponding speed. [**Answer:** $\Delta K \sim 4 \times 10^{-67}$ J, $\Delta v \sim 4 \times 10^{-33}$ m/s. For all practical purposes, the tennis ball will appear to be at rest in the box.]

Some related homework problems: Problem 71, Problem 77, Problem 78

Finally, the uncertainty relation $\Delta y \Delta p_y \geq h/2\pi$ is just one of many forms the uncertainty principle takes. There are a number of quantities like y and p_y that satisfy the same type of relation between their uncertainties. Perhaps the most important of these uncertainty principles is the one relating the uncertainty in energy to the uncertainty in time:

The Heisenberg Uncertainty Principle: Energy and Time

$$\Delta E \Delta t \geq \frac{h}{2\pi} \qquad 30\text{–}20$$

For example, the shorter the half-life of an unstable particle, the greater the uncertainty in its energy.

CONCEPTUAL CHECKPOINT 30–4 UNCERTAINTIES

If Planck's constant were magically reduced to zero, would the uncertainties in position/momentum and energy/time be **(a)** increased or **(b)** decreased?

REASONING AND DISCUSSION

If h were zero, the product of uncertainties—$\Delta y \Delta p_y$ and $\Delta E \Delta t$—would be reduced to zero as well. The result is that position and momentum, for example, could be determined simultaneously with arbitrary accuracy; that is, with zero uncertainty. In this limit, particles would behave as predicted in classical physics, with well-defined positions and momenta at all times.

ANSWER

(b) The uncertainties would decrease to zero.

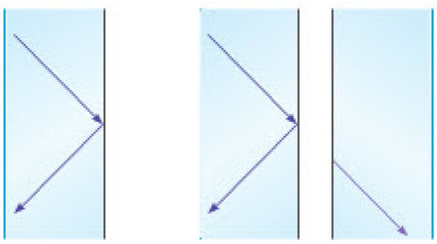
Thus, if h were zero, the classical description of particles moving along well-defined trajectories, and having no wavelike properties, would be valid. Notice, however, that Planck's constant has an extremely small magnitude, on the order of 10^{-34} in SI units. It is this small difference from zero that is responsible for the quantum behavior seen on the atomic scale.

If h were relatively large, however, the wavelike properties of matter would be apparent even on the macroscopic level. For example, consider pitching a baseball toward the catcher's glove in a universe where h is rather large. If the ball is thrown with a well-defined momentum toward the glove, its position will be uncertain, and the ball may end up anywhere. Similarly, if the catcher gives the glove a relatively small uncertainty in position, its uncertainty in momentum is large, meaning that the glove is moving rapidly about its average position. So, even if the pitcher could aim the ball to go where desired—which is not possible—there is no way to know where to aim it. If h were significantly larger than it actually is, our experience of the natural world would be very different indeed.

30–7 Quantum Tunneling

Because particles have wavelike properties, any behavior seen in waves can be found in particles as well, under the right conditions. An example is the phenomenon known as **tunneling,** in which a wave, or a quantum particle, "tunnels" through a region of space that would be forbidden to it if it were simply a small piece of matter like a classical particle.

Figure 30–17 shows a case of tunneling by light. In Figure 30–17 (a) we see a beam of light propagating within glass and undergoing total internal reflection (Chapter 26) when it encounters the glass–air interface. If light were composed of classical particles this would be the end of the story; the particles of light would simply change direction at the interface and continue propagating within the glass.



(a) Total internal reflection (b) Light tunnels across the gap

▲ **FIGURE 30–17 Optical tunneling**

(a) An incident beam of light undergoes total internal reflection from the glass–air interface. **(b)** If a second piece of glass is brought near the first piece, a weak beam of light is observed continuing in the same direction as the incident beam. We say that the light has "tunneled" across the gap.

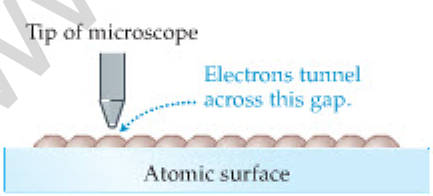
What can actually happen in such a system is more interesting, however. In Figure 30–17 (b) we show a second piece of glass brought near the first piece, but with a small vacuum gap between them. If the gap is small, but still finite, a weak beam of light is observed in the second piece of glass, propagating in the same direction as the original beam. We say that the light waves have "tunneled" across the gap. The strength of the beam of light in the second piece of glass depends very sensitively on the size of the gap through which it tunnels, decreasing exponentially as the width of the gap is increased.

Just as with light, electrons and other particles can tunnel across gaps that would be forbidden to them if they were classical particles. An electron, for instance, is not simply a point of mass; instead, it has wave properties that extend outward from it like ripples on a pond. These waves, like those of light, can "feel" the surroundings of an electron, allowing it to "tunnel" through a barrier to a region on the other side where it can propagate.

REAL-WORLD PHYSICS

Scanning tunneling microscopy

An example of electron tunneling is shown in **Figure 30–18**, where we illustrate the operation of a scanning tunneling microscope, or STM. In the lower part of the figure we see the material, or specimen, to be investigated with the microscope. The upper portion of the figure shows the key element of the microscope—a small, pointed tip of metal that can be moved up and down with piezoelectric supports. This tip is brought very close to the specimen being observed, leaving a small vacuum gap between them. Classically, electrons in the specimen are not able to move across the gap to the tip, but in reality they can tunnel to the tip and create a small electric current. The number of electrons that tunnel, and thus the magnitude of the tunneling current, depends on the width of the gap.



▲ **FIGURE 30–18 Operation of a scanning tunneling microscope**

Schematic operation of a scanning tunneling microscope.

In one version of the STM, the tunneling current between the specimen and the tip is held constant. Imagine, for example, that the tip in Figure 30–18 is scanned horizontally from left to right. Since the tunneling current is very sensitive to the size of the gap between the tip and the specimen, the tip must be moved up and down with the contours of the specimen in order to maintain a gap of constant width. The tip is moved by sending electrical voltage to the piezoelectric

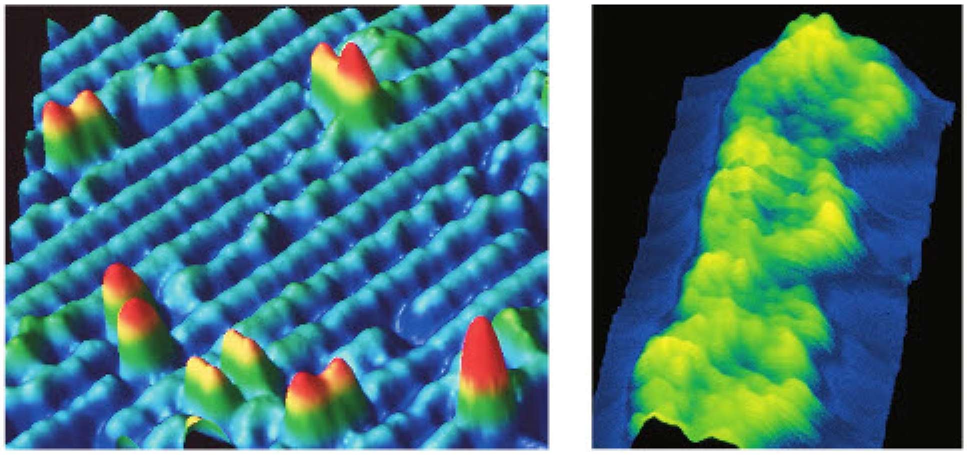


FIGURE 30–19 Scanning tunneling microscopy
STM images are particularly good at recording the "terrain" of surfaces at the atomic level. The image at left shows clusters of antimony atoms on a silicon surface. At right, a DNA molecule sits on a substrate of graphite. Three turns of the double helix are visible, magnified about 2,000,000 times.

supports; thus, the voltage going to the supports is a measure of the height of the surface being scanned and can be converted to a visual image of the surface, as in Figure 30–19. The resolution of these microscopes is on the atomic level, showing the hills and valleys created by atoms on the surface of the material being examined.

THE BIG PICTURE PUTTING PHYSICS IN CONTEXT

LOOKING BACK

The expression for total relativistic energy (Chapter 29) is used in Section 30–3 to find the momentum of a photon—even though a photon has zero rest mass. We then study collisions involving photons in Section 30–4 in the same way we studied collisions involving massive objects in Chapter 9.

Diffraction and two-slit interference (Chapter 28) play a key role in understanding neutron diffraction and electron interference in Section 30–5. In addition, diffraction forms the basis for our understanding of the uncertainty principle in Section 30–6.

LOOKING AHEAD

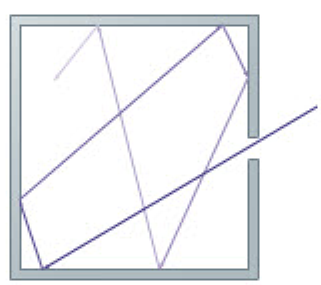
De Broglie waves, introduced in Section 30–5, appear again in Chapter 31 when we study the Bohr model of the atom. As we shall see, the allowed Bohr orbits correspond to standing waves formed from the de Broglie waves of electrons.

Photons are fascinating in that they have zero mass and yet have finite momentum and energy (Sections 30–3, 30–4 and 30–5). They also travel at the speed of light, which is not possible for any finite-mass object. We use the concept of photons again in Section 31–4 when we study atomic radiation.

CHAPTER SUMMARY

30–1 BLACKBODY RADIATION AND PLANCK'S HYPOTHESIS OF QUANTIZED ENERGY

An ideal blackbody is an object that absorbs all the light incident on it. The distribution of energy as a function of frequency within a blackbody is independent of the material from which the blackbody is made and depends only on the temperature, T.



Wien's Displacement Law
The frequency at which the radiation from a blackbody is maximum is given by the following relation:

$$f_{peak} = (5.88 \times 10^{10}\ s^{-1} \cdot K^{-1})T \qquad 30\text{–}1$$

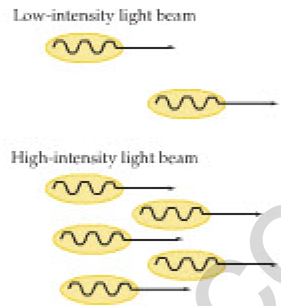
Planck's Quantum Hypothesis
Planck hypothesized that the energy in a blackbody at a frequency f must be an integer multiple of the constant $h = 6.63 \times 10^{-34}\ J \cdot s$; that is,

$$E_n = nhf \quad n = 0, 1, 2, 3, \ldots \qquad 30\text{–}2$$

The constant h is known as Planck's constant.

30–2 PHOTONS AND THE PHOTOELECTRIC EFFECT

Light is composed of particle-like photons, which carry energy in discrete amounts.



Energy of a Photon
The energy of a photon depends on its frequency. A photon with the frequency f has the energy

$$E = hf \tag{30–4}$$

Noting the relation $\lambda f = c$, we can also express the energy of a photon in terms of its wavelength, $E = hc/\lambda$.

The Photoelectric Effect
The photoelectric effect occurs when photons of light eject electrons from the surface of a metal.

Work Function
The minimum energy required to eject an electron from a particular metal is the work function, W_0.

Cutoff Frequency
To eject an electron, a photon must have an energy at least as great as W_0, and thus the minimum, or cutoff, frequency to eject an electron is

$$f_0 = \frac{W_0}{h} \tag{30–6}$$

If the frequency of the photon is greater than f_0, the ejected electron has a finite kinetic energy.

Maximum Kinetic Energy
The maximum kinetic energy of an electron ejected from a metal by a photon of frequency $f > f_0$ is

$$K_{max} = hf - W_0 \tag{30–7}$$

30–3 THE MASS AND MOMENTUM OF A PHOTON

A photon is like a "typical" particle in some respects but different in others. In particular, a photon has zero rest mass, yet it still has a nonzero momentum.

Rest Mass of a Photon
Photons, which travel through a vacuum at the speed of light, c, have zero rest mass

$$m_0 = 0 \tag{30–9}$$

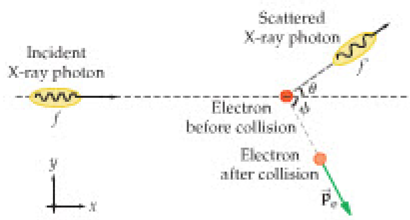
Only objects with zero rest mass can propagate at the speed of light.

Momentum of a Photon
The momentum, p, of a photon of frequency f and wavelength $\lambda = c/f$ is given by

$$p = \frac{hf}{c} = \frac{h}{\lambda} \tag{30–11}$$

30–4 PHOTON SCATTERING AND THE COMPTON EFFECT

Photons can undergo collisions with particles, in much the same way that particles can collide with other particles. In order to conserve the total energy and momentum of a system during such a collision, the frequency and wavelength of a photon will change.



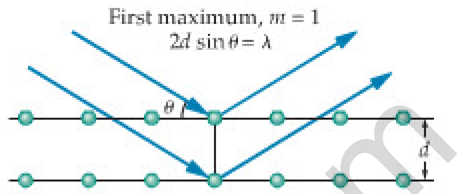
The Compton Effect
If a photon of wavelength λ undergoes a collision with an electron (mass $= m_e$) and scatters into a new direction at an angle θ from its incident direction, its new wavelength, λ', is given by the Compton shift formula:

$$\Delta\lambda = \lambda' - \lambda = \frac{h}{m_e c}(1 - \cos\theta) \tag{30–15}$$

The Compton shift formula can be applied to scattering from particles other than the electron by changing the mass in Equation 30–15.

30–5 THE DE BROGLIE HYPOTHESIS AND WAVE–PARTICLE DUALITY

The de Broglie hypothesis is basically the following: Since light displays particle-like behavior, perhaps particles display wavelike behavior. In particular, de Broglie hypothesized that particles have wavelengths.



de Broglie Wavelength

According to de Broglie, the relationship between momentum and wavelength should be the same for both light and particles. Thus, the de Broglie wavelength of a particle of momentum p is

$$\lambda = \frac{h}{p} \qquad \text{30–16}$$

Diffraction of X-Rays and Particles by Crystals

X-rays and particles can show interference effects when reflected from the atomic layers of a crystal. If the wavelength of the X-rays or particles is λ, and the spacing between atomic layers is d, the angles at which constructive interference occurs are given by

$$2d \sin\theta = m\lambda \quad m = 1, 2, 3, \ldots \qquad \text{30–17}$$

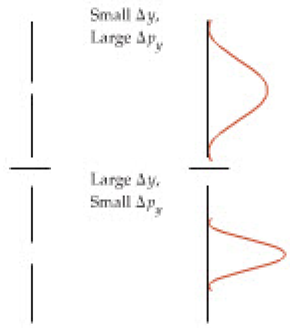
The resulting patterns produced by the constructive interference are referred to as a diffraction pattern.

Wave–Particle Duality

Light and matter display both wavelike and particle-like properties.

30–6 THE HEISENBERG UNCERTAINTY PRINCIPLE

Because particles have wavelengths and can behave as waves, their position and momentum cannot be determined simultaneously with arbitrary precision.



The Heisenberg Uncertainty Principle: Momentum and Position

In terms of momentum and position, the Heisenberg uncertainty principle states the following:

$$\Delta p_y \Delta y \geq \frac{h}{2\pi} \qquad \text{30–19}$$

Thus, as momentum is determined more precisely, the position becomes more uncertain, and vice versa.

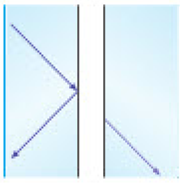
The Heisenberg Uncertainty Principle: Energy and Time

In terms of energy and time, the Heisenberg uncertainty principle states the following:

$$\Delta E \Delta t \geq \frac{h}{2\pi} \qquad \text{30–20}$$

30–7 QUANTUM TUNNELING

Particles, because of their wavelike behavior, can pass through regions of space that would be forbidden to a classical particle. This phenomenon is referred to as tunneling.



PROBLEM-SOLVING SUMMARY

Type of Problem	Relevant Physical Concepts	Related Examples
Relate the peak frequency of blackbody radiation to the temperature of the blackbody.	A blackbody at the absolute temperature T has a peak in its radiation spectrum at the frequency $f_{peak} = (5.88 \times 10^{10}\ s^{-1} \cdot K^{-1})T$. This is true regardless of the blackbody's composition.	Exercise 30–1
Find the quantum number of a macroscopic system.	If a macroscopic system has the energy E and oscillates with frequency f, the corresponding quantum number is $n = E/hf$.	Example 30–1
Relate the energy and frequency of a photon.	A photon of frequency f has an energy hf, where h is Planck's constant.	Example 30–2 Active Example 30–1

Find the maximum kinetic energy of electrons given off in the photoelectric effect.	When light of frequency f illuminates a metal surface with a work function W_0, the maximum kinetic energy of ejected electrons is $K_{max} = hf - W_0$.	Example 30–3
Relate the change in wavelength of an X-ray to the angle through which it scatters from an electron.	The change in wavelength is related to the scattering angle θ by the following relation: $$\Delta\lambda = \lambda' - \lambda = \frac{h}{m_e c}(1 - \cos\theta)$$	Example 30–4
Determine the de Broglie wavelength of a particle.	The de Broglie wavelength of a particle with momentum p is $\lambda = h/p$.	Example 30–5 Active Example 30–2
Relate the uncertainty in position to the uncertainty in momentum.	The Heisenberg uncertainty principle states that the uncertainty in momentum, Δp_y, and the uncertainty in position, Δy, are related as follows: $\Delta p_y \Delta y \geq h/2\pi$. Similarly, the uncertainties in time and energy obey $\Delta E\, \Delta t \geq h/2\pi$.	Examples 30–6, 30–7

CONCEPTUAL QUESTIONS

(Answers to odd-numbered Conceptual Questions can be found in the back of the book.)

1. Give a brief description of the "ultraviolet catastrophe."
2. How does Planck's hypothesis of energy quantization resolve the "ultraviolet catastrophe"?
3. Is there a lowest temperature below which blackbody radiation is no longer given off by an object? Explain.
4. How can an understanding of blackbody radiation allow us to determine the temperature of distant stars?
5. **Differential Fading** Many vehicles in the United States have a small American flag decal in one of their windows. If the decal has been in place for a long time, the colors will show some fading from exposure to the Sun. In fact, the red stripes are generally more faded than the blue background for the stars, as shown in the accompanying photo. Photographs and posters react in the same way, with red colors showing the most fading. Explain this effect in terms of the photon model of light.

Differential fading. (Conceptual Question 5)

6. A source of light is monochromatic. What can you say about the photons emitted by this source?
7. The relative intensity of radiation given off by a blackbody is shown in Figure 30–2. Notice that curves corresponding to different temperatures never cross one another. If two such curves did intersect, however, it would be possible to violate the second law of thermodynamics. Explain.
8. **(a)** Is it possible for a photon from a green source of light to have more energy than a photon from a blue source of light? Explain. **(b)** Is it possible for a photon from a green source of light to have more energy than a photon from a red source of light? Explain.
9. Light of a given wavelength ejects electrons from the surface of one metal but not from the surface of another metal. Give a possible explanation for this observation.
10. Why does the existence of a cutoff frequency in the photoelectric effect argue in favor of the photon model of light?
11. Why can an electron microscope resolve smaller objects than a light microscope?
12. A proton is about 2000 times more massive than an electron. Is it possible for an electron to have the same de Broglie wavelength as a proton? Explain.

PROBLEMS AND CONCEPTUAL EXERCISES

Note: Answers to odd-numbered Problems and Conceptual Exercises can be found in the back of the book. **IP** *denotes an integrated problem, with both conceptual and numerical parts;* **BIO** *identifies problems of biological or medical interest;* **CE** *indicates a conceptual exercise.* **Predict/Explain** *problems ask for two responses:* **(a)** *your prediction of a physical outcome, and* **(b)** *the best explanation among three provided. On all problems, red bullets (•, ••, •••) are used to indicate the level of difficulty.*

SECTION 30–1 BLACKBODY RADIATION AND PLANCK'S HYPOTHESIS OF QUANTIZED ENERGY

1. • **CE Predict/Explain** The blackbody spectrum of blackbody A peaks at a longer wavelength than that of blackbody B. **(a)** Is the temperature of blackbody A higher than or lower than the temperature of blackbody B? **(b)** Choose the *best explanation* from among the following:
   I. Blackbody A has the higher temperature because the higher the temperature the longer the wavelength.
   II. Blackbody B has the higher temperature because an increase in temperature means an increase in frequency, which corresponds to a decrease in wavelength.
2. • **The Surface Temperature of Betelgeuse** Betelgeuse, a red-giant star in the constellation Orion, has a peak in its radiation

at a frequency of 1.82×10^{14} Hz. What is the surface temperature of Betelgeuse?

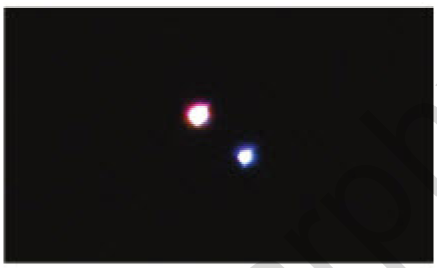
3. • What is the frequency of the most intense radiation emitted by your body? Assume a skin temperature of 95 °F. What is the wavelength of this radiation?

4. • **The Cosmic Background Radiation** Outer space is filled with a sea of photons, created in the early moments of the universe. The frequency distribution of this "cosmic background radiation" matches that of a blackbody at a temperature near 2.7 K. **(a)** What is the peak frequency of this radiation? **(b)** What is the wavelength that corresponds to the peak frequency?

5. • The Sun has a surface temperature of about 5800 K. At what frequency does the Sun emit the most radiation?

6. •• **(a)** By what factor does the peak frequency change if the Kelvin temperature of an object is doubled from 20.0 K to 40.0 K? **(b)** By what factor does the peak frequency change if the Celsius temperature of an object is doubled from 20.0 °C to 40.0 °C?

7. •• **IP A Famous Double Star** Albireo in the constellation Cygnus, which appears as a single star to the naked eye, is actually a beautiful double-star system. The brighter of the two stars is referred to as A (or Beta-01 Cygni), with a surface temperature of $T_A = 4700$ K; its companion is B (or Beta-02 Cygni), with a surface temperature of $T_B = 13{,}000$ K. **(a)** When viewed through a telescope, one star is a brilliant blue color, and the other has a warm golden color, as shown in the accompanying photo. Is the blue star A or B? Explain. **(b)** What is the ratio of the peak frequencies emitted by the two stars, (f_A/f_B)?

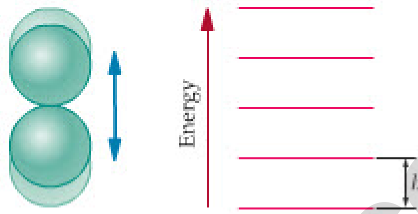


The double star Albireo in the constellation Cygnus. (Problem 7)

8. •• **IP Halogen Lightbulbs** Modern halogen lightbulbs allow their filaments to operate at a higher temperature than the filaments in standard incandescent bulbs. For comparison, the filament in a standard lightbulb operates at about 2900 K, whereas the filament in a halogen bulb may operate at 3400 K. **(a)** Which bulb has the higher peak frequency? **(b)** Calculate the ratio of peak frequencies (f_{hal}/f_{std}). **(c)** The human eye is most sensitive to a frequency around 5.5×10^{14} Hz. Which bulb produces a peak frequency closer to this value?

9. •• **IP** A typical lightbulb contains a tungsten filament that reaches a temperature of about 2850 K, roughly half the surface temperature of the Sun. **(a)** Treating the filament as a blackbody, determine the frequency for which its radiation is a maximum. **(b)** Do you expect the lightbulb to radiate more energy in the visible or in the infrared part of the spectrum? Explain.

10. •• **Exciting an Oxygen Molecule** An oxygen molecule (O_2) vibrates with an energy identical to that of a single particle of mass $m = 1.340 \times 10^{-26}$ kg attached to a spring with a force constant of $k = 1215$ N/m. The energy levels of the system are uniformly spaced, as indicated in Figure 30–20, with a separation given by hf. **(a)** What is the vibration frequency of this molecule? **(b)** How much energy must be added to the molecule to excite it from one energy level to the next higher level?



▲ **FIGURE 30–20** Problem 10

SECTION 30–2 PHOTONS AND THE PHOTOELECTRIC EFFECT

11. • **CE** A source of red light, a source of green light, and a source of blue light each produce beams of light with the same power. Rank these sources in order of increasing **(a)** wavelength of light, **(b)** frequency of light, and **(c)** number of photons emitted per second. Indicate ties where appropriate.

12. • **CE Predict/Explain** A source of red light has a higher wattage than a source of green light. **(a)** Is the energy of photons emitted by the red source greater than, less than, or equal to the energy of photons emitted by the green source? **(b)** Choose the *best explanation* from among the following:
    - I. The photons emitted by the red source have the greater energy because that source has the greater wattage.
    - II. The red-source photons have less energy than the green-source photons because they have a lower frequency. The wattage of the source doesn't matter.
    - III. Photons from the red source have a lower frequency, but that source also has a greater wattage. The two effects cancel, so the photons have equal energy.

13. • **CE Predict/Explain** A source of yellow light has a higher wattage than a source of blue light. **(a)** Is the number of photons emitted per second by the yellow source greater than, less than, or equal to the number of photons emitted per second by the blue source? **(b)** Choose the *best explanation* from among the following:
    - I. The yellow source emits more photons per second because (i) it emits more energy per second than the blue source, and (ii) its photons have less energy than those of the blue source.
    - II. The yellow source has the higher wattage, which means its photons have higher energy than the blue-source photons. Therefore, the yellow source emits fewer photons per second.
    - III. The two sources emit the same number of photons per second because the higher wattage of the yellow source compensates for the higher energy of the blue photons.

14. • **CE Predict/Explain** Light of a particular wavelength does not eject electrons from the surface of a given metal. **(a)** Should the wavelength of the light be increased or decreased in order to cause electrons to be ejected? **(b)** Choose the *best explanation* from among the following:
    - I. The photons have too little energy to eject electrons. To increase their energy, their wavelength should be increased.
    - II. The energy of a photon is proportional to its frequency; that is, inversely proportional to its wavelength. To increase the energy of the photons so they can eject electrons, one must decrease their wavelength.

15. • **CE** Light of a particular wavelength and intensity does not eject electrons from the surface of a given metal. Can electrons be ejected from the metal by increasing the intensity of the light? Explain.

16. • When a person visits the local tanning salon, they absorb photons of ultraviolet (UV) light to get the desired tan. What are the frequency and wavelength of a UV photon whose energy is 6.5×10^{-19} J?

17. • An AM radio station operating at a frequency of 880 kHz radiates 270 kW of power from its antenna. How many photons are emitted by the antenna every second?

18. • A photon with a wavelength of less than 50.4 nm can ionize a helium atom. What is the ionization potential of helium?

19. • A flashlight emits 2.5 W of light energy. Assuming a frequency of 5.2×10^{14} Hz for the light, determine the number of photons given off by the flashlight per second.

20. • Light of frequency 9.95×10^{14} Hz ejects electrons from the surface of silver. If the maximum kinetic energy of the ejected electrons is 0.180×10^{-19} J, what is the work function of silver?

21. • The work function of gold is 4.58 eV. What frequency of light must be used to eject electrons from a gold surface with a maximum kinetic energy of 6.48×10^{-19} J?

22. • **(a)** How many 350-nm (UV) photons are needed to provide a total energy of 2.5 J? **(b)** How many 750-nm (red) photons are needed to provide the same energy?

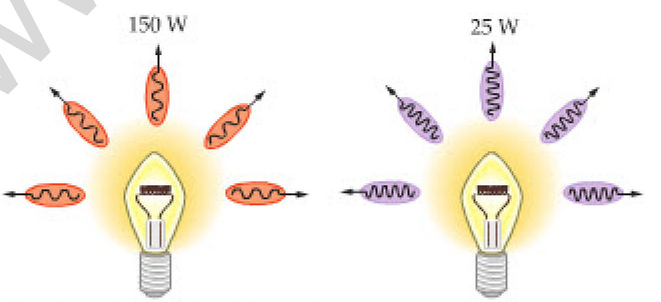
23. •• **(a)** How many photons per second are emitted by a monochromatic lightbulb ($\lambda = 650$ nm) that emits 45 W of power? **(b)** If you stand 15 m from this bulb, how many photons enter each of your eyes per second? Assume your pupil is 5.0 mm in diameter and that the bulb radiates uniformly in all directions.

24. •• **IP** Two 57.5-kW radio stations broadcast at different frequencies. Station A broadcasts at a frequency of 892 kHz, and station B broadcasts at a frequency of 1410 kHz. **(a)** Which station emits more photons per second? Explain. **(b)** Which station emits photons of higher energy?

25. •• **Dissociating the Hydrogen Molecule** The energy required to separate a hydrogen molecule into its individual atoms is 104.2 kcal per mole of H_2. **(a)** If the dissociation energy for a single H_2 molecule is provided by one photon, determine its frequency and wavelength. **(b)** In what region of the electromagnetic spectrum does the photon found in part (a) lie? (Refer to the spectrum shown in Figure 25–8.)

26. •• **(a)** How many photons are emitted per second by a He-Ne laser that emits 1.0 mW of power at a wavelength $\lambda = 632.8$ nm? **(b)** What is the frequency of the electromagnetic waves emitted by a He-Ne laser?

27. •• **IP** You have two lightbulbs of different power and color, as indicated in Figure 30–21. One is a 150-W red bulb, and the other is a 25-W blue bulb. **(a)** Which bulb emits more photons per second? **(b)** Which bulb emits photons of higher energy? **(c)** Calculate the number of photons emitted per second by each bulb. Take $\lambda_{red} = 650$ nm and $\lambda_{blue} = 460$ nm. (Most of the electromagnetic radiation given off by incandescent lightbulbs is in the infrared portion of the spectrum. For the purposes of this problem, however, assume that all of the radiated power is at the wavelengths indicated.)



▲ **FIGURE 30–21** Problem 27

28. •• The maximum wavelength an electromagnetic wave can have and still eject an electron from a copper surface is 264 nm. What is the work function of a copper surface?

29. •• **IP** Aluminum and calcium have photoelectric work functions of $W_{Al} = 4.28$ eV and $W_{Ca} = 2.87$ eV, respectively. **(a)** Which metal requires higher-frequency light to produce photoelectrons? Explain. **(b)** Calculate the minimum frequency that will produce photoelectrons from each surface.

30. •• **IP** Two beams of light with different wavelengths ($\lambda_A > \lambda_B$) are used to produce photoelectrons from a given metal surface. **(a)** Which beam produces photoelectrons with greater kinetic energy? Explain. **(b)** Find K_{max} for cesium ($W_0 = 1.9$ eV) if $\lambda_A = 620$ nm and $\lambda_B = 410$ nm.

31. •• **IP** Zinc and cadmium have photoelectric work functions given by $W_{Zn} = 4.33$ eV and $W_{Cd} = 4.22$ eV, respectively. **(a)** If both metals are illuminated by UV radiation of the same wavelength, which one gives off photoelectrons with the greater maximum kinetic energy? Explain. **(b)** Calculate the maximum kinetic energy of photoelectrons from each surface if $\lambda = 275$ nm.

32. •• White light, with frequencies ranging from 4.00×10^{14} Hz to 7.90×10^{14} Hz, is incident on a potassium surface. Given that the work function of potassium is 2.24 eV, find **(a)** the maximum kinetic energy of electrons ejected from this surface and **(b)** the range of frequencies for which no electrons are ejected.

33. •• Electromagnetic waves, with frequencies ranging from 4.00×10^{14} Hz to 9.00×10^{16} Hz, are incident on an aluminum surface. Given that the work function of aluminum is 4.28 eV, find **(a)** the maximum kinetic energy of electrons ejected from this surface and **(b)** the range of frequencies for which no electrons are ejected.

34. •• **IP** Platinum has a work function of 6.35 eV, and iron has a work function of 4.50 eV. Light of frequency 1.88×10^{15} Hz ejects electrons from both of these surfaces. **(a)** From which surface will the ejected electrons have a greater maximum kinetic energy? Explain. **(b)** Calculate the maximum kinetic energy of ejected electrons for each surface.

35. •• When light with a frequency $f_1 = 547.5$ THz illuminates a metal surface, the most energetic photoelectrons have 1.260×10^{-19} J of kinetic energy. When light with a frequency $f_2 = 738.8$ THz is used instead, the most energetic photoelectrons have 2.480×10^{-19} J of kinetic energy. Using these experimental results, determine the approximate value of Planck's constant.

36. •• **BIO Owl Vision** Owls have large, sensitive eyes for good night vision. Typically, the pupil of an owl's eye can have a diameter of 8.5 mm (as compared with a maximum diameter of about 7.0 mm for humans). In addition, an owl's eye is about 100

An apt pupil. (Problem 36)

times more sensitive to light of low intensity than a human eye, allowing owls to detect light with an intensity as small as $5.0 \times 10^{-13}\ \text{W/m}^2$. Find the minimum number of photons per second an owl can detect, assuming a frequency of 7.0×10^{14} Hz for the light.

SECTION 30–3 THE MASS AND MOMENTUM OF A PHOTON

37\. • **CE** If the momentum of a particle with finite mass is doubled, its kinetic energy increases by a factor of 4. If the momentum of a photon is doubled, by what factor does its energy increase?

38\. • The photons used in microwave ovens have a momentum of $5.1 \times 10^{-33}\ \text{kg} \cdot \text{m/s}$. **(a)** What is their wavelength? **(b)** How does the wavelength of the microwaves compare with the size of the holes in the metal screen on the door of the oven?

39\. • What speed must an electron have if its momentum is to be the same as that of an X-ray photon with a wavelength of 0.25 nm?

40\. • What is the wavelength of a photon that has the same momentum as an electron moving with a speed of 1200 m/s?

41\. • What is the frequency of a photon that has the same momentum as a neutron moving with a speed of 1500 m/s?

42\. •• A hydrogen atom, initially at rest, emits an ultraviolet photon with a wavelength of $\lambda = 122$ nm. What is the recoil speed of the atom after emitting the photon?

43\. •• A blue-green photon ($\lambda = 486$ nm) is absorbed by a free hydrogen atom, initially at rest. What is the recoil speed of the hydrogen atom after absorbing the photon?

44\. •• **IP** **(a)** Which has the greater momentum, a photon of red light or a photon of blue light? Explain. **(b)** Calculate the momentum of a photon of red light ($f = 4.0 \times 10^{14}$ Hz) and a photon of blue light ($f = 7.9 \times 10^{14}$ Hz).

45\. •• **IP** Photon A has twice the momentum of photon B. **(a)** Which photon has the greater wavelength? Explain. **(b)** If the wavelength of photon A is 333 nm, what is the wavelength of photon B?

46\. ••• A laser produces a 5.00-mW beam of light, consisting of photons with a wavelength of 632.8 nm. **(a)** How many photons are emitted by the laser each second? **(b)** The laser beam strikes a black surface and is absorbed. What is the change in the momentum of each photon that is absorbed? **(c)** What force does the laser beam exert on the black surface?

47\. ••• A laser produces a 7.50-mW beam of light, consisting of photons with a wavelength of 632.8 nm. **(a)** How many photons are emitted by the laser each second? **(b)** The laser beam strikes a mirror at normal incidence and is reflected. What is the change in momentum of each reflected photon? Give the magnitude only. **(c)** What force does the laser beam exert on the mirror?

SECTION 30–4 PHOTON SCATTERING AND THE COMPTON EFFECT

48\. • **CE** In a Compton scattering experiment, the scattered electron is observed to move in the same direction as the incident X-ray photon. What is the scattering angle of the photon? Explain.

49\. • An X-ray photon has 38.0 keV of energy before it scatters from a free electron, and 33.5 keV after it scatters. What is the kinetic energy of the recoiling electron?

50\. • In the Compton effect, an X-ray photon scatters from a free electron. Find the *change* in the photon's wavelength if it scatters at an angle of **(a)** $\theta = 30.0°$, **(b)** $\theta = 90.0°$, and **(c)** $\theta = 180.0°$ relative to the incident direction.

51\. • An X-ray scattering from a free electron is observed to change its wavelength by 3.13 pm. At what angle to the incident direction does the scattered X-ray move?

52\. •• The maximum Compton shift in wavelength occurs when a photon is scattered through 180°. What scattering angle will produce a wavelength shift of one-fourth the maximum?

53\. •• **IP** Consider two different photons that scatter through an angle of 180° from a free electron. One is a visible-light photon with $\lambda = 520$ nm, the other is an X-ray photon with $\lambda = 0.030$ nm. **(a)** Which (if either) photon experiences the greater change in wavelength as a result of the scattering? Explain. **(b)** Which photon experiences the greater percentage change in wavelength? Explain. **(c)** Calculate the percentage change in wavelength of each photon.

54\. •• An X-ray photon with a wavelength of 0.240 nm scatters from a free electron at rest. The scattered photon moves at an angle of 105° relative to its incident direction. Find **(a)** the initial momentum and **(b)** the final momentum of the photon.

55\. •• An X-ray photon scatters from a free electron at rest at an angle of 175° relative to the incident direction. **(a)** If the scattered photon has a wavelength of 0.320 nm, what is the wavelength of the incident photon? **(b)** Determine the energy of the incident and scattered photons. **(c)** Find the kinetic energy of the recoil electron.

56\. •• **IP** An X-ray photon scatters through 180° from (i) an electron or (ii) a helium atom. **(a)** In which case is the change in wavelength of the X-ray greater? Explain. **(b)** Calculate the change in wavelength for each of these two cases.

57\. ••• A photon has an energy E and wavelength λ before scattering from a free electron. After scattering through a 135° angle, the photon's wavelength has increased by 10.0%. Find the initial wavelength and energy of the photon.

58\. ••• Find the direction of propagation of the scattered electron in Problem 51, given that the incident X-ray has a wavelength of 0.525 nm and propagates in the positive x direction.

SECTION 30–5 THE DE BROGLIE HYPOTHESIS AND WAVE–PARTICLE DUALITY

59\. • **CE Predict/Explain** **(a)** As you accelerate your car away from a stoplight, does the de Broglie wavelength of the car increase, decrease, or stay the same? **(b)** Choose the *best explanation* from among the following:

I. The de Broglie wavelength will increase because the momentum of the car has increased.
II. The momentum of the car increases. It follows that the de Broglie wavelength will decrease, because it is inversely proportional to the wavelength.
III. The de Broglie wavelength of the car depends only on its mass, which doesn't change by pulling away from the stoplight. Therefore, the de Broglie wavelength stays the same.

60\. • **CE** By what factor does the de Broglie wavelength of a particle change if **(a)** its momentum is doubled or **(b)** its kinetic energy is doubled? Assume the particle is nonrelativistic.

61\. • A particle with a mass of 6.69×10^{-27} kg has a de Broglie wavelength of 7.22 pm. What is the particle's speed?

62\. • What speed must a neutron have if its de Broglie wavelength is to be equal to the interionic spacing of table salt (0.282 nm)?

63\. • A 79-kg jogger runs with a speed of 4.2 m/s. If the jogger is considered to be a particle, what is her de Broglie wavelength?

64\. • Find the kinetic energy of an electron whose de Broglie wavelength is 1.5 Å.

65. •• A beam of neutrons with a de Broglie wavelength of 0.250 nm diffracts from a crystal of table salt, which has an interionic spacing of 0.282 nm. **(a)** What is the speed of the neutrons? **(b)** What is the angle of the second interference maximum?

66. •• **IP** An electron and a proton have the same speed. **(a)** Which has the longer de Broglie wavelength? Explain. **(b)** Calculate the ratio (λ_e/λ_p).

67. •• **IP** An electron and a proton have the same de Broglie wavelength. **(a)** Which has the greater kinetic energy? Explain. **(b)** Calculate the ratio of the electron's kinetic energy to the kinetic energy of the proton.

68. •• Diffraction effects become significant when the width of an aperture is comparable to the wavelength of the waves being diffracted. **(a)** At what speed will the de Broglie wavelength of a 65-kg student be equal to the 0.76-m width of a doorway? **(b)** At this speed, how long will it take the student to travel a distance of 1.0 mm? (For comparison, the age of the universe is approximately 4×10^{17} s.)

69. ••• A particle has a mass m and an electric charge q. The particle is accelerated from rest through a potential difference V. What is the particle's de Broglie wavelength, expressed in terms of m, q, and V?

SECTION 30–6 THE HEISENBERG UNCERTAINTY PRINCIPLE

70. • A baseball (0.15 kg) and an electron both have a speed of 41 m/s. Find the uncertainty in position of each of these objects, given that the uncertainty in their speed is 5.0%.

71. • The uncertainty in position of a proton confined to the nucleus of an atom is roughly the diameter of the nucleus. If this diameter is 7.5×10^{-15} m, what is the uncertainty in the proton's momentum?

72. • The position of a 0.26-kg air-track cart is determined to within an uncertainty of 2.2 mm. What speed must the cart acquire as a result of the position measurement?

73. • The measurement of an electron's energy requires a time interval of 1.0×10^{-8} s. What is the smallest possible uncertainty in the electron's energy?

74. • A particle's energy is measured with an uncertainty of 0.0010 eV. What is the smallest possible uncertainty in our knowledge of when the particle had this energy?

75. • An excited state of a particular atom has a mean lifetime of 0.60×10^{-9} s, which we may take as the uncertainty Δt. What is the minimum uncertainty in any measurement of the energy of this state?

76. • The Σ^+ is an unstable particle, with a mean lifetime of 2.5×10^{-10} s. Its lifetime defines the uncertainty Δt for this particle. What is the minimum uncertainty in this particle's energy?

77. •• The uncertainty in an electron's position is 0.15 nm. **(a)** What is the minimum uncertainty Δp in its momentum? **(b)** What is the kinetic energy of an electron whose momentum is equal to this uncertainty ($\Delta p = p$)?

78. •• The uncertainty in a proton's position is 0.15 nm. **(a)** What is the minimum uncertainty Δp in its momentum? **(b)** What is the kinetic energy of a proton whose momentum is equal to this uncertainty ($\Delta p = p$)?

79. •• An electron has a momentum $p \approx 1.7 \times 10^{-25}$ kg·m/s. What is the minimum uncertainty in its position that will keep the relative uncertainty in its momentum ($\Delta p/p$) below 1.0%?

GENERAL PROBLEMS

80. • **CE** Suppose you perform an experiment on the photoelectric effect using light with a frequency high enough to eject electrons. If the intensity of the light is increased while the frequency is held constant, describe whether the following quantities increase, decrease, or stay the same: **(a)** The maximum kinetic energy of an ejected electron; **(b)** the minimum de Broglie wavelength of an electron; **(c)** the number of electrons ejected per second; **(d)** the electric current in the phototube.

81. • **CE** Suppose you perform an experiment on the photoelectric effect using light with a frequency high enough to eject electrons. If the frequency of the light is increased while the intensity is held constant, describe whether the following quantities increase, decrease, or stay the same: **(a)** The maximum kinetic energy of an ejected electron; **(b)** the minimum de Broglie wavelength of an electron; **(c)** the number of electrons ejected per second; **(d)** the electric current in the phototube.

82. • **CE** An electron that is accelerated from rest through a potential difference V_0 has a de Broglie wavelength λ_0. What potential difference will double the electron's wavelength? (Express your answer in terms of V_0.)

83. • **CE** A beam of particles diffracts from a crystal, producing an interference maximum at the angle θ. **(a)** If the mass of the particles is increased, with everything else remaining the same, does the angle of the interference maximum increase, decrease, or stay the same? Explain **(b)**. If the energy of the particles is increased, with everything else remaining the same, does the angle of the interference maximum increase, decrease, or stay the same? Explain.

84. • You want to construct a photocell that works with visible light. Three materials are readily available: aluminum ($W_0 = 4.28$ eV), lead ($W_0 = 4.25$ eV), and cesium ($W_0 = 2.14$ eV). Which material(s) would be suitable?

85. • **BIO Human Vision** Studies have shown that some people can detect 545-nm light with as few as 100 photons entering the eye per second. What is the power delivered by such a beam of light?

86. •• A pendulum consisting of a 0.15-kg mass attached to a 0.78-m string undergoes simple harmonic motion. **(a)** What is the frequency of oscillation for this pendulum? **(b)** Assuming the energy of this system satisfies $E_n = nhf$, find the maximum speed of the 0.15-kg mass when the quantum number is 1.0×10^{33}.

87. •• To listen to a radio station, a certain home receiver must pick up a signal of at least 1.0×10^{-10} W. **(a)** If the radio waves have a frequency of 96 MHz, how many photons must the receiver absorb per second to get the station? **(b)** How much force is exerted on the receiving antenna for the case considered in part (a)?

88. •• The latent heat for converting ice at 0 °C to water at 0 °C is 80.0 kcal/kg (Chapter 17). **(a)** How many photons of frequency 6.0×10^{14} Hz must be absorbed by a 1.0-kg block of ice at 0 °C to melt it to water at 0 °C? **(b)** How many molecules of H_2O can one photon convert from ice to water?

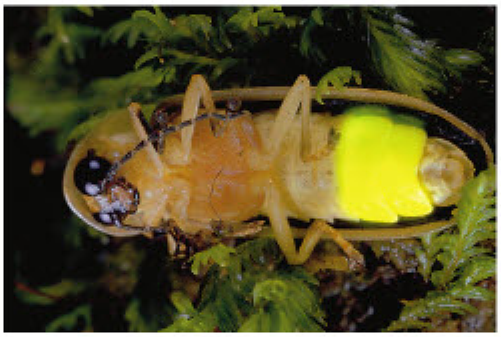
89. •• How many 550-nm photons would have to be absorbed to raise the temperature of 1.0 g of water by 1.0 C°?

90. •• A microwave oven can heat 205 mL of water from 20.0 °C to 90.0 °C in 2.00 min. If the wavelength of the microwaves is $\lambda = 12.2$ cm, how many photons were absorbed by the water? (Assume no loss of heat by the water.)

91. •• Light with a frequency of 2.11×10^{15} Hz ejects electrons from the surface of lead, which has a work function of 4.25 eV. What is the minimum de Broglie wavelength of the ejected electrons?

92. •• An electron moving with a speed of 2.7×10^6 m/s has the same momentum as a photon. Find **(a)** the de Broglie wavelength of the electron and **(b)** the wavelength of the photon.

93. •• **BIO** **The Cold Light of Fireflies** Fireflies are often said to give off "cold light." Given that the peak in a firefly's radiation occurs at about 5.4×10^{14} Hz, determine the temperature of a blackbody that would have the same peak frequency. From your result, would you say that firefly radiation is well approximated by blackbody radiation? Explain.



How cool is that? (Problem 93)

94. •• **IP** When light with a wavelength of 545 nm shines on a metal surface, electrons are ejected with speeds of 3.10×10^5 m/s or less. **(a)** Give a strategy that allows you to use the preceding information to calculate the work function and cutoff frequency for this surface. **(b)** Carry out your strategy and determine the work function and cutoff frequency.

95. •• **IP** A hydrogen atom absorbs a 486.2-nm photon. A short time later, the same atom emits a photon with a wavelength of 97.23 nm. **(a)** Has the net energy of the atom increased or decreased? Explain. **(b)** Calculate the change in energy of the hydrogen atom.

96. •• When a beam of atoms emerges from an oven at the absolute temperature T, the most probable de Broglie wavelength for a given atom is

$$\lambda_{mp} = \frac{h}{\sqrt{5mkT}}$$

In this expression, m is the mass of an atom, and k is Boltzmann's constant (Chapter 17). What is the most probable speed of a hydrogen atom emerging from an oven at 450 K?

97. •• **IP** **(a)** Does the de Broglie wavelength of a particle increase or decrease as its kinetic energy increases? Explain. **(b)** Show that the de Broglie wavelength of an electron in nanometers can be written as $\lambda = (1.23 \text{ nm})/\sqrt{K}$, where K is the kinetic energy of the electron in eV. Use classical expressions for momentum and kinetic energy.

98. ••• A jar is filled with monatomic helium gas at a temperature of 25 °C. The pressure inside the jar is one atmosphere; that is, 101 kPa. **(a)** Find the average de Broglie wavelength of the helium atoms. **(b)** Calculate the average separation between helium atoms in the jar. (*Note*: The fact that the spacing between atoms is much greater than the de Broglie wavelength means quantum effects are negligible, and the atoms can be treated as particles.)

99. ••• **The Compton Wavelength** The *Compton wavelength*, λ_C, of a particle of mass m is defined as follows: $\lambda_C = h/mc$. **(a)** Calculate the Compton wavelength of a proton. **(b)** Calculate the energy of a photon that has the same wavelength as found in part (a). **(c)** Show, in general, that a photon with a wavelength equal to the Compton wavelength of a particle has an energy that is equal to the rest energy of the particle.

100. ••• **IP** Light of frequency 8.22×10^{14} Hz ejects electrons from surface A with a maximum kinetic energy that is 2.00×10^{-19} J greater than the maximum kinetic energy of electrons ejected from surface B. **(a)** If the frequency of the light is increased, does the difference in maximum kinetic energy observed from the two surfaces increase, decrease, or stay the same? Explain. **(b)** Calculate the difference in work function for these two surfaces.

PASSAGE PROBLEMS

Millikan and the Photoelectric Effect

Robert A. Millikan (1868–1953), best known for his "oil-drop experiment" that measured the charge of an electron, also performed pioneering research on the photoelectric effect. In fact, the 1923 Nobel Prize in physics was awarded to Millikan "for his work on the elementary charge of electricity and on the photoelectric effect." Initially convinced that Einstein's theory of the photoelectric effect was wrong—because of overwhelming evidence for the wave nature of light—Millikan undertook a decade-long experimental program to study the effect. In the end, his experiments confirmed Einstein's theory in every detail and ushered in the modern view of light as having a wave–particle duality.

Millikan carried out an exhaustive set of experiments on a variety of materials. In experiments on lithium, for example, Millikan observed a maximum kinetic energy of 0.550 eV when electrons were ejected with 433.9-nm light. When light of 253.5 nm was used, he observed a maximum kinetic energy of 2.57 eV. Using results like this, Millikan was able to measure the value of Planck's constant, and to show that the value obtained from the photoelectric effect is in complete agreement with the value obtained from blackbody radiation.

101. •• What is the work function, W_0, for lithium, as determined from Millikan's results?

A. 0.0112 eV **B.** 0.951 eV
C. 1.63 eV **D.** 2.29 eV

102. • What value does Millikan obtain for Planck's constant, based on the lithium measurements? (His value is close to, but not the same as, the currently accepted value.)

A. 1.12×10^{-34} J·s **B.** 3.84×10^{-34} J·s
C. 6.14×10^{-34} J·s **D.** 6.57×10^{-34} J·s

103. • What maximum kinetic energy do you predict Millikan found when he used light with a wavelength of 365.0 nm?

A. 0.805 eV **B.** 1.08 eV
C. 2.29 eV **D.** 2.82 eV

INTERACTIVE PROBLEMS

104. •• **IP** **Referring to Example 30–4** An X-ray photon with $\lambda = 0.6500$ nm scatters from an electron, giving the electron a kinetic energy of 7.750 eV. **(a)** Is the scattering angle of the photon greater than, less than, or equal to 152°? **(b)** Find the scattering angle.

105. •• **IP** **Referring to Example 30–4** An X-ray photon with $\lambda = 0.6500$ nm scatters from an electron. The wavelength of the scattered photon is 0.6510 nm. **(a)** Is the scattering angle in this case greater than, less than, or equal to 152°? **(b)** Find the scattering angle.

31 Atomic Physics

A number of animals, including many species of coral, sponges, and even scorpions, emit beautiful and sometimes eerie colors when illuminated by ultraviolet light. We do not ordinarily expect rabbits to fall into this category. However, scientists recently inserted a jellyfish gene that directs the production of green fluorescent protein (GFP) into the fertilized egg of a rabbit. The result was Alba, the "GFP bunny"—the world's first fluorescent rabbit. This chapter explores our modern understanding of the structure of the atom, which makes it possible to explain not only the phenomenon of fluorescence but many others as well, including the properties of the chemical elements and the generation of laser light.

In today's world it is taken for granted that we, along with everything else on Earth, are made of atoms. Although it may seem surprising at first, this belief in atoms has not always been universal. As recently as the first part of the twentieth century there was still serious debate about the microscopic nature of matter. With the advent of quantum physics, however, the debate quickly faded away as atomic structure came to be understood in ever greater detail.

In this chapter we begin by developing the quantum model of the simplest of all atoms—the hydrogen atom. We then show that the basic features of hydrogen apply to more complex atoms as well. As a result, we are able to understand—in detail—the arrangement of elements in the periodic table. That quantum physics can describe the structure of an atom, and show why the various elements have their characteristic properties, is one of the greatest successes of modern science.

31–1 Early Models of the Atom

Speculations about the microscopic structure of matter have intrigued humankind for thousands of years. Ancient Greek philosophers, including Leucippus and Democritus, considered the question of what would happen if you took a small object, like a block of copper, and cut it in half, then cut it in half again, and again, for many subsequent divisions. They reasoned that eventually you would reduce the block to a single speck of copper that could not be divided further. This smallest piece of an element was called the **atom** (a + tom), which means, literally, "without division."

It was not until the late nineteenth century, however, that the question of atoms began to yield to direct scientific investigation. We now consider some of the more important early developments in atomic models that helped lead to our current understanding.

The Thomson Model: Plum Pudding

In 1897 the English physicist J. J. Thomson (1856–1940) discovered a "particle" that is smaller in size and thousands of times less massive than even the smallest atom. The **electron**, as this particle was named, was also found to have a negative electric charge—in contrast with atoms, which are electrically neutral. Thomson proposed, therefore, that atoms have an internal structure that includes both electrons and a quantity of positively charged matter. The latter would account for most of the mass of an atom, and would have a charge equal in magnitude to the charge on the electrons.

The picture of an atom that Thomson settled on is one he referred to as the "plum-pudding model." In this model, electrons are embedded in a more or less uniform distribution of positive charge—like raisins spread throughout a pudding. This model is illustrated in Figure 31–1. Although the plum-pudding model was in agreement with everything Thomson knew about atoms at the time, new experiments were soon to rule out this model and replace it with one that was more like the solar system than a pudding.

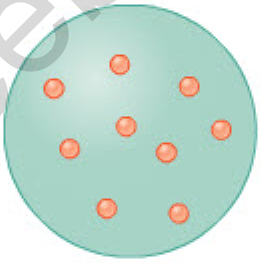


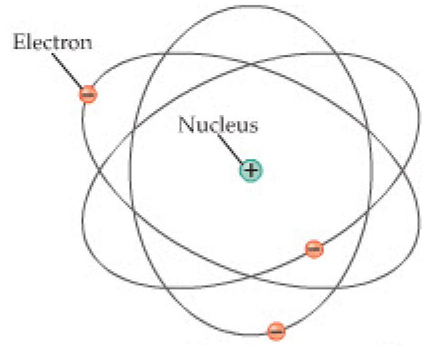
FIGURE 31–1 The plum-pudding model of an atom
The model of an atom proposed by J. J. Thomson consists of a uniform positive charge, which accounts for most of the mass of an atom, with small negatively charged electrons scattered throughout, like raisins in a pudding.

The Rutherford Model: A Miniature Solar System

Inspired by the findings and speculations of Thomson, other physicists began to investigate atomic structure. In 1909, Ernest Rutherford (1871–1937) and his coworkers Hans Geiger (1882–1945) and Ernest Marsden (1889–1970) (at that time a twenty-year-old undergraduate) decided to test Thomson's model by directing a beam of positively charged particles, known as **alpha particles**, at a thin gold foil. Since alpha particles—which were later found to be the nuclei of helium atoms—carry a positive charge, they should be deflected as they pass through the positively charged "pudding" in the gold foil. The deflection should have the following properties: (i) it should be relatively small, since the alpha particles have a substantial mass and the positive charge in the atom is spread out; and (ii) all the alpha particles should be deflected in roughly the same way, since the positive pudding fills virtually all space.

When Geiger and Marsden performed the experiment, their results were not in agreement with these predictions. In fact, most of the alpha particles passed right through the foil as if it were not there—as if the atoms in the foil were mostly empty space. Because the results were rather surprising, Rutherford suggested that the experiment be modified to look not only for alpha particles with small angles of deflection—as originally expected—but for ones with large deflections as well.

This suggestion turned out to be an inspired hunch. Not only were large-angle deflections observed, but some of the alpha particles, in fact, were found to have practically reversed their direction of motion. Rutherford was stunned. In his own words, "It was almost as incredible as if you fired a fifteen-inch shell at a piece of tissue paper and it came back and hit you."



▲ **FIGURE 31–2 The solar system model of an atom**
Ernest Rutherford proposed that an atom is like a miniature solar system, with a massive positively charged nucleus orbited by lightweight negatively charged electrons.

To account for the results of these experiments, Rutherford proposed that an atom has a structure similar to that of the solar system, as illustrated in Figure 31–2. In particular, he imagined that the lightweight, negatively charged electrons orbit a small, positively charged **nucleus** containing almost all the atom's mass. In this nuclear model of the atom, most of the atom is indeed empty space, allowing the majority of the alpha particles to pass right through. Furthermore, the positive charge of the atom is now highly concentrated in a small nucleus, rather than spread throughout the atom. This means that an alpha particle that happens to make a head-on collision with the nucleus can actually be turned around, as observed in the experiments.

To see just how small the nucleus must be in his model, Rutherford combined the experimental data with detailed theoretical calculations. His result was that the radius of a nucleus must be smaller than the diameter of the atom by a factor of about 10,000. To put this value into perspective, imagine an atom magnified in size until its nucleus is as large as the Sun. At what distance would an electron orbit in this "atomic" solar system? Using the factor given by Rutherford, we find that the orbit of the electron would have a radius essentially the same as the orbit of Pluto—inside this radius would be only empty space and the nucleus. Thus an atom must have an even larger fraction of empty space than the solar system!

Although Rutherford's nuclear model of the atom seems reasonable, it contains fatal flaws. First, an orbiting electron undergoes a centripetal acceleration toward the nucleus (Chapter 6). As we know from Section 25–1, however, any electric charge that accelerates gives off energy in the form of electromagnetic radiation. Thus, an electron continually radiating energy as it orbits is similar to a satellite losing energy to air resistance when it orbits too close to the Earth's atmosphere. Just as in the case of a satellite, an electron would spiral inward and eventually plunge into the nucleus. Since the entire process of collapse would occur in a fraction of a second (about 10^{-9} s in fact), the atoms in Rutherford's model would simply not be stable—in contrast with the observed stability of atoms in nature.

Even if we ignore the stability problem for a moment, there is another serious discrepancy between Rutherford's model and experiment. Maxwell's equations state that the frequency of radiation from an orbiting electron should be the same as the frequency of its orbit. In the case of an electron spiraling inward the frequency would increase continuously. Thus if we look at light coming from an atom, the Rutherford model indicates that we should see a continuous range of frequencies. This prediction is in striking contrast with experiments, which show that light coming from an atom has only certain discrete frequencies and wavelengths, as we discuss in the next section.

▲ Emission nebulas, like the Lagoon Nebula in Sagittarius shown here, are masses of glowing interstellar gas. The gas is excited by high-energy radiation from nearby stars and emits light at wavelengths characteristic of the atoms present, chiefly hydrogen. Much of the visible light from such nebulas is contributed by the red Balmer line of hydrogen with a wavelength of 656.3 nm, known as H-alpha.

31–2 The Spectrum of Atomic Hydrogen

A red-hot piece of metal glows with a ruddy light that represents only a small fraction of its total radiation output. As we saw in Chapter 30, the metal gives off blackbody radiation that extends in a continuous distribution over all possible frequencies. This blackbody distribution, or spectrum, of radiation is characteristic of the entire collection of atoms that make up the metal—it is not characteristic of the spectrum of light that would be given off by a single, isolated metal atom.

To see the light produced by an isolated atom, we turn our attention from a solid—where the atoms are close together and strongly interacting—to a low-pressure gas—where the atoms are far apart and have little interaction with one another. Consider, then, an experiment in which we seal a low-pressure gas in a tube. If we apply a large voltage between the ends of the tube, the gas will emit electromagnetic radiation characteristic of the individual gas atoms. When this radiation is passed through a diffraction grating (Chapter 28), it is

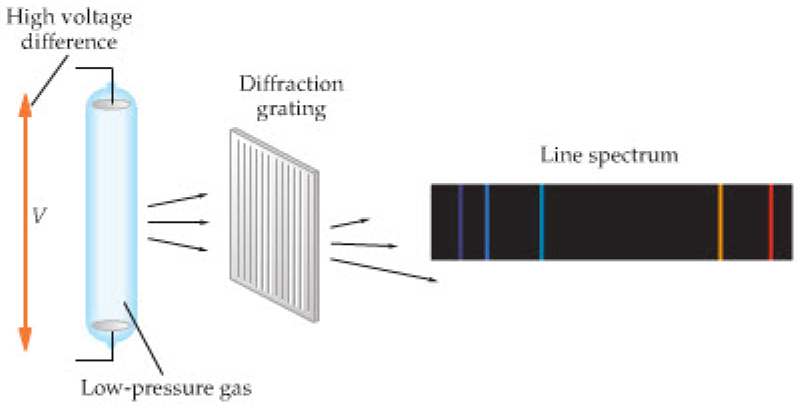


FIGURE 31–3 The line spectrum of an atom

The light given off by individual atoms, as in a low-pressure gas, consists of a series of discrete wavelengths corresponding to different colors.

separated into its various wavelengths, as indicated in Figure 31–3. The result of such an experiment is that a series of bright "lines" is observed, reminiscent of the bar codes used in supermarkets. The precise wavelength associated with each of these lines provides a sort of "fingerprint" identifying a particular type of atom, just as each product in a supermarket has its own unique bar code.

This type of spectrum, with its bright lines in different colors, is referred to as a **line spectrum.** As an example, we show the visible part of the line spectrum of atomic hydrogen in Figure 31–4 (a). Hydrogen produces additional lines in the infrared and ultraviolet parts of the electromagnetic spectrum.

The line spectrum shown in Figure 31–4 (a) is an *emission spectrum*, since it shows light that is emitted by the hydrogen atoms. Similarly, if light of all colors is passed through a tube of hydrogen gas, some wavelengths will be absorbed by the atoms, giving rise to an *absorption spectrum*, which consists of dark lines (where the atoms absorb the radiation) against an otherwise bright background. The absorption lines occur at precisely the same wavelengths as the emission lines. Figure 31–4 (b) shows the absorption spectrum of hydrogen.

The first step in developing a quantitative understanding of the hydrogen spectrum occurred in 1885, when Johann Jakob Balmer (1825–1898), a Swiss schoolteacher, used trial-and-error methods to discover the following simple formula that gives the wavelength of the visible lines in the spectrum:

$$\frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{n^2}\right) \qquad n = 3, 4, 5, \ldots \text{(Balmer series)} \qquad \text{31–1}$$

The constant, R, in this expression is known as the *Rydberg constant*. Its value is

$$R = 1.097 \times 10^7\ \text{m}^{-1}$$

Each integer value of n (3, 4, 5, . . .) in Balmer's formula corresponds to the wavelength, λ, of a different spectral line. For example, if we set $n = 5$ in Equation 31–1 we find

$$\frac{1}{\lambda} = (1.097 \times 10^7\ \text{m}^{-1})\left(\frac{1}{2^2} - \frac{1}{5^2}\right)$$

Solving for the wavelength, we have

$$\lambda = 4.341 \times 10^{-7}\ \text{m} = 434.1\ \text{nm}$$

This is the bluish line, second from the left in Figure 31–4 (a).

The collection of all lines predicted by the Balmer formula is referred to as the **Balmer series.** We consider the Balmer series in detail in the following Example.

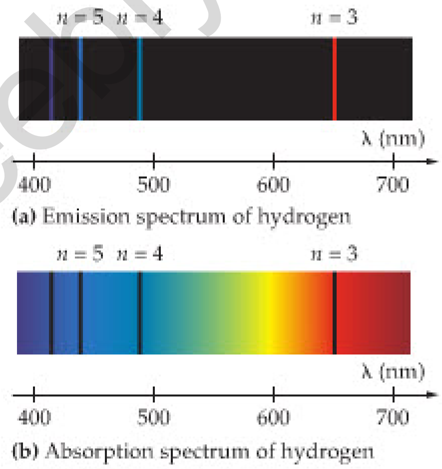


FIGURE 31–4 The line spectrum of hydrogen

The emission **(a)** and absorption **(b)** spectra of hydrogen. Note that the wavelengths absorbed by hydrogen (dark lines) are the same as those emitted by hydrogen (colored lines). The location of these lines is predicted by the Balmer series (Equation 31–1) with the appropriate values of n.

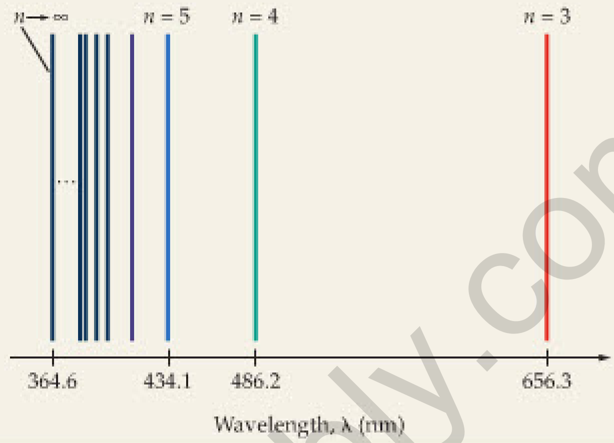
PROBLEM-SOLVING NOTE

Calculating Wavelengths for the Balmer Series

Note that the formula for the Balmer series gives the inverse of the wavelength, rather than the wavelength itself.

EXAMPLE 31–1 THE BALMER SERIES

Find **(a)** the longest and **(b)** the shortest wavelengths in the Balmer series of spectral lines.



PICTURE THE PROBLEM

In our sketch we indicate the first several lines in the Balmer series, along with their corresponding colors, using the results given in Figure 31–4 as a guide. There are an infinite number of lines in the Balmer series, as indicated by the ellipsis (three dots) to the right of the $n \to \infty$ line.

STRATEGY

By substituting the values $n = 3$, $n = 4$, and $n = 5$ in the Balmer series (Equation 31–1), we find that the wavelength decreases with increasing n. Hence, **(a)** the longest wavelength corresponds to $n = 3$, and **(b)** the shortest wavelength corresponds to $n \to \infty$.

SOLUTION

Part (a)

1. To find the longest wavelength in the Balmer series, substitute $n = 3$ in Equation 31–1:

$$\frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{3^2}\right) = (1.097 \times 10^7\ \text{m}^{-1})\left(\frac{5}{36}\right)$$

2. Invert the result in Step 1 to obtain the corresponding wavelength, λ:

$$\lambda = \frac{36}{5(1.097 \times 10^7\ \text{m}^{-1})} = 656.3\ \text{nm}$$

Part (b)

3. The shortest wavelength is found in the limit $n \to \infty$ or, equivalently, $(1/n^2) \to 0$. Make this substitution in Equation 31–1:

$$\frac{1}{\lambda} = R\left(\frac{1}{2^2} - 0\right) = (1.097 \times 10^7\ \text{m}^{-1})\left(\frac{1}{4}\right)$$

4. Invert the result in Step 3 to obtain the corresponding wavelength, λ:

$$\lambda = \frac{4}{(1.097 \times 10^7\ \text{m}^{-1})} = 364.6\ \text{nm}$$

INSIGHT

The longest wavelength corresponds to visible light with a reddish hue, whereas the shortest wavelength is well within the ultraviolet portion of the electromagnetic spectrum—it is invisible to our eyes.

PRACTICE PROBLEM

Which value of n corresponds to a wavelength of 377.1 nm in the Balmer series? [**Answer:** $n = 11$]

Some related homework problems: Problem 5, Problem 6

TABLE 31–1 Common Spectral Series of Hydrogen

n'	Series name
1	Lyman
2	Balmer
3	Paschen
4	Brackett
5	Pfund

PROBLEM-SOLVING NOTE

Correctly Applying Equation 31–2

Notice that n and n' are integers in Equation 31–2 and that the integer n must always be greater than n'.

Figure 31–5 shows that the Balmer series is not the only series of lines produced by atomic hydrogen. The series with the shortest wavelengths is the **Lyman series**—all its lines are in the ultraviolet. Similarly, the series with wavelengths just longer than those in the Balmer series is the **Paschen series.** The lines in this series are all in the infrared. The formula that gives the wavelength in all the series of hydrogen is

$$\frac{1}{\lambda} = R\left(\frac{1}{n'^2} - \frac{1}{n^2}\right) \quad n' = 1, 2, 3, \ldots$$

$$n = n' + 1, n' + 2, n' + 3, \ldots \qquad \text{31–2}$$

Referring to Equation 31–1, we see that the Balmer series corresponds to the choice $n' = 2$. Similarly, the Lyman series is given by Equation 31–2 with $n' = 1$, and the Paschen series corresponds to $n' = 3$. As we shall see later in this chapter, there is an infinite number of series in hydrogen, each corresponding to a different choice for the integer n'. The names of the most common spectral series of hydrogen are listed in Table 31–1.

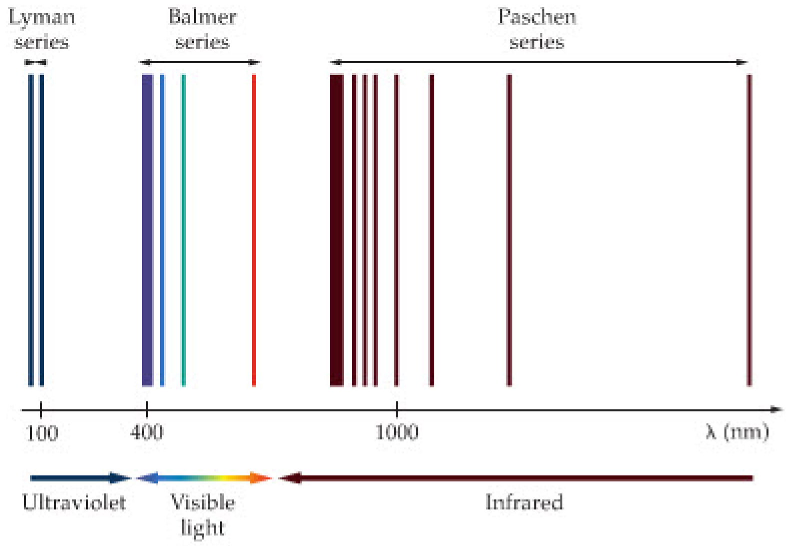


FIGURE 31–5 The Lyman, Balmer, and Paschen series of spectral lines

The first three series of spectral lines in the spectrum of hydrogen. The shortest wavelengths appear in the Lyman series. There is no upper limit to the number of series in hydrogen or to the wavelengths that can be emitted.

EXERCISE 31–1

Find **(a)** the shortest wavelength in the Lyman series and **(b)** the longest wavelength in the Paschen series.

SOLUTION

a. Substitute $n' = 1$ and $n \to \infty$ in Equation 31–2:

$$\frac{1}{\lambda} = R\left(\frac{1}{1^2} - 0\right) = (1.097 \times 10^7\ \text{m}^{-1})$$

$$\lambda = \frac{1}{(1.097 \times 10^7\ \text{m}^{-1})} = 91.16\ \text{nm}$$

b. Substitute $n' = 3$ and $n = 4$ in Equation 31–2:

$$\frac{1}{\lambda} = R\left(\frac{1}{3^2} - \frac{1}{4^2}\right) = (1.097 \times 10^7\ \text{m}^{-1})\left(\frac{7}{144}\right)$$

$$\lambda = \frac{144}{7(1.097 \times 10^7\ \text{m}^{-1})} = 1875\ \text{nm}$$

As successful as Equation 31–2 is in giving the various wavelengths of radiation produced by hydrogen, it is still just an empirical formula. It gives no insight as to *why* these particular wavelengths, and no others, are produced. The goal of atomic physicists in the early part of the twentieth century was to *derive* Equation 31–2 from basic physical principles. The first significant step in that direction is the topic of the next section.

31–3 Bohr's Model of the Hydrogen Atom

Our scientific understanding of the hydrogen atom took a giant leap forward in 1913, when Niels Bohr (1885–1962), a Danish physicist who had just earned his doctorate in physics in 1911, introduced a model that allowed him to derive Equation 31–2. Bohr's model combined elements of classical physics with the ideas of quantum physics introduced by Planck and Einstein about ten years earlier. As such, his model is a hybrid that spanned the gap between the classical physics of Newton and Maxwell and the newly emerging quantum physics.

▲ Niels Bohr, applying the principles of classical mechanics, with some members of his family.

Assumptions of the Bohr Model

Bohr's model of the hydrogen atom is based on four assumptions. Two are specific to his model and do not apply to the full quantum mechanical picture of hydrogen that will be introduced in Section 31–5. The remaining two assumptions are quite general—they apply not only to hydrogen but to all atoms.

The two specific assumptions of the Bohr model are as follows:

- The electron in a hydrogen atom moves in a circular orbit about the nucleus.

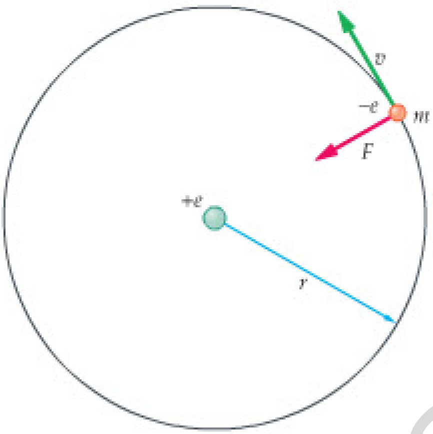
- Only certain circular orbits are allowed. In these orbits the angular momentum of the electron is equal to an integer times Planck's constant divided by 2π. That is, the angular momentum of an electron in the nth allowed orbit is $L_n = nh/2\pi$, where $n = 1, 2, 3, \ldots$.

The next two assumptions are more general:

- Electrons do not give off electromagnetic radiation when they are in an allowed orbit. Thus, the orbits are stable.
- Electromagnetic radiation is given off or absorbed only when an electron changes from one allowed orbit to another. If the energy difference between two allowed orbits is ΔE, the frequency, f, of the photon that is emitted or absorbed is given by the relation $|\Delta E| = hf$.

Notice that Bohr's model retains the classical picture of an electron orbiting a nucleus, as in Rutherford's model. It adds the stipulations, however, that only certain orbits are allowed and that no radiation is given off from these orbits. Radiation is given off *only* when an electron shifts from one orbit to another, and then the radiation is in the form of a photon that obeys Einstein's quantum relation $E = hf$. Thus, as mentioned before, the Bohr model is a hybrid that includes ideas from both classical and quantum physics. We now use this model to determine the behavior of hydrogen.

Bohr Orbits



▲ **FIGURE 31–6 A Bohr orbit**

In the Bohr model of hydrogen, electrons orbit the nucleus in circular orbits. The centripetal acceleration of the electron, v^2/r, is produced by the Coulomb force of attraction between the electron and the nucleus.

We begin by determining the radii of the allowed orbits in the Bohr model and the speed of the electrons in these orbits. There are two conditions that we must apply. First, for the electron to move in a circular orbit of radius r with a speed v, as depicted in Figure 31–6, the electrostatic force of attraction between the electron and the nucleus must be equal in magnitude to the mass of the electron times its centripetal acceleration, mv^2/r. Recalling Coulomb's law (Equation 19–5), we see that the electrostatic force between the electron (with charge $-e$) and the nucleus (with charge $+e$) has a magnitude given by ke^2/r^2. It follows that $mv^2/r = ke^2/r^2$, or, canceling one power of r,

$$mv^2 = k\frac{e^2}{r} \qquad \text{31–3}$$

Note that this relation is completely analogous to the one that was used to derive Kepler's third law (Chapter 12), except in that case the force of attraction was provided by gravity.

The second condition for an allowed orbit is that the angular momentum of the electron must be a nonzero integer n times $h/2\pi$. Since the electron moves with a speed v in a circular path of radius r, its angular momentum is $L = rmv$ (Equation 11–12). Hence, the condition for the nth allowed orbit is $L_n = r_n m v_n = nh/2\pi$, or, solving for v_n, we have

$$v_n = \frac{nh}{2\pi m r_n} \qquad n = 1, 2, 3, \ldots \qquad \text{31–4}$$

Combining these two conditions allows us to solve for the two unknowns, r_n and v_n.

For example, if we substitute v_n from Equation 31–4 into Equation 31–3, we can solve for r_n. Specifically, we find the following:

$$m\left(\frac{nh}{2\pi m r_n}\right)^2 = k\frac{e^2}{r_n}$$

Rearranging and solving for r_n, we get

$$r_n = \left(\frac{h^2}{4\pi^2 m k e^2}\right)n^2 \qquad n = 1, 2, 3, \ldots \qquad \text{31–5}$$

The quantity in parentheses is the radius for the smallest ($n = 1$) orbit. Substitution of numerical values for h, m, e, and k gives the following value for r_1:

$$r_1 = \frac{h^2}{4\pi^2 mke^2}$$

$$= \frac{(6.626 \times 10^{-34}\,\mathrm{J \cdot s})^2}{4\pi^2(9.109 \times 10^{-31}\,\mathrm{kg})(8.988 \times 10^9\,\mathrm{N \cdot m^2/C^2})(1.602 \times 10^{-19}\,\mathrm{C})^2}$$

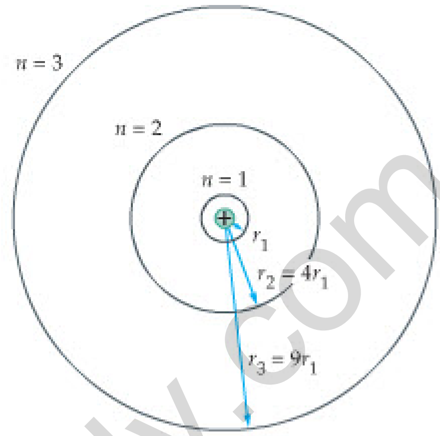
$$= 5.29 \times 10^{-11}\,\mathrm{m}$$

This radius, which is about half an angstrom and is referred to as the **Bohr radius**, is in agreement with the observed size of hydrogen atoms. Note the n^2 dependence in the radii of allowed orbits: $r_n = r_1 n^2 = (5.29 \times 10^{-11}\,\mathrm{m})n^2$. This dependence is illustrated in **Figure 31–7.**

To complete the solution, we can substitute our result for r_n (Equation 31–5) into the expression for v_n (Equation 31–4). This yields

$$v_n = \frac{nh}{2\pi m}\left(\frac{4\pi^2 mke^2}{n^2h^2}\right) = \frac{2\pi ke^2}{nh} \qquad n = 1, 2, 3, \ldots \qquad 31\text{–}6$$

Note that the speed of the electron is smaller in orbits farther from the nucleus.

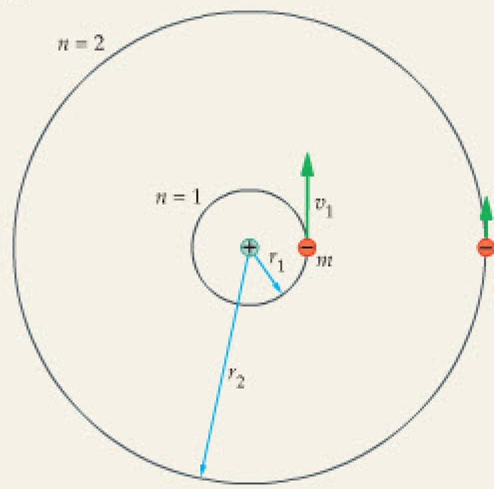


▲ **FIGURE 31–7 The first three Bohr orbits**
The first Bohr orbit has a radius $r_1 = 5.29 \times 10^{-11}$ m. The second and third Bohr orbits have radii $r_2 = 2^2 r_1 = 4r_1$, and $r_3 = 3^2 r_1 = 9r_1$, respectively. (*Note*: For clarity, the nucleus is drawn larger than its true scale relative to the size of the atom.)

EXAMPLE 31–2 FIRST AND SECOND BOHR ORBITS

Find the speed and kinetic energy of the electron in **(a)** the first Bohr orbit ($n = 1$) and **(b)** the second Bohr orbit ($n = 2$).

PICTURE THE PROBLEM
The first two orbits of the Bohr model are shown in the sketch. Note that the second orbit has a radius four times greater than the radius of the first orbit. In addition, the speed of the electron in the second orbit is half its value in the first orbit.



STRATEGY
The speed of the electron can be determined by direct substitution in $v_n = 2\pi ke^2/nh$ (Equation 31–6). Once v is determined, the kinetic energy is simply $K = \frac{1}{2}mv^2$.

Note that because the speed in the second orbit is half the speed in the first orbit, the kinetic energy is smaller by a factor of 4.

SOLUTION

Part (a)

1. Substitute $n = 1$ in Equation 31–6:

$$v_1 = \frac{2\pi ke^2}{h} = \frac{2\pi(8.99 \times 10^9\,\mathrm{N \cdot m^2/C^2})(1.60 \times 10^{-19}\,\mathrm{C})^2}{(6.63 \times 10^{-34}\,\mathrm{J \cdot s})} = 2.18 \times 10^6\,\mathrm{m/s}$$

2. The corresponding kinetic energy is $K_1 = \frac{1}{2}mv_1^{\,2}$:

$$K_1 = \tfrac{1}{2}mv_1^{\,2} = \tfrac{1}{2}(9.11 \times 10^{-31}\,\mathrm{kg})(2.18 \times 10^6\,\mathrm{m/s})^2 = 2.16 \times 10^{-18}\,\mathrm{J}$$

Part (b)

3. Divide the speed found in part (a) by 2:

$$v_2 = \tfrac{1}{2}v_1 = \tfrac{1}{2}(2.18 \times 10^6\,\mathrm{m/s}) = 1.09 \times 10^6\,\mathrm{m/s}$$

4. Similarly, divide the kinetic energy found in part (a) by 4:

$$K_2 = \tfrac{1}{2}mv_2^{\,2} = \tfrac{1}{2}m(v_1/2)^2 = \tfrac{1}{4}K_1 = \tfrac{1}{4}(2.16 \times 10^{-18}\,\mathrm{J}) = 5.40 \times 10^{-19}\,\mathrm{J}$$

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INSIGHT

The speed of the electron in the first Bohr orbit is smaller than the speed of light by a factor of about 137. In higher orbits, the electron's speed is even less. It follows that relativistic effects are small for the hydrogen atom.

In addition, note that the kinetic energy of the electron in the first Bohr orbit is approximately 13.6 eV. We shall encounter this particular energy again later in the section.

PRACTICE PROBLEM

An electron in a Bohr orbit has a kinetic energy of 8.64×10^{-20} J. Find the radius of this orbit. [**Answer:** $r = 1.32 \times 10^{-9}$ m, corresponding to $n = 5$.]

Some related homework problems: Problem 15, Problem 20, Problem 21

Bohr's model applies equally well to singly ionized helium, doubly ionized lithium, and other ions with only a single electron. In the case of singly ionized helium (one electron removed) the charge on the nucleus is $+2e$, for doubly ionized lithium (two electrons removed) it is $+3e$, and so on. In the general case, we may consider a nucleus that contains Z protons and has a charge of $+Ze$, where Z is the atomic number associated with that nucleus. Hydrogen, which has only a single proton in its nucleus, corresponds to $Z = 1$.

To be more explicit about the Z dependence, notice that the electrostatic force between an electron and a nucleus with Z protons has a magnitude of $k(e)(Ze)/r^2 = kZe^2/r^2$. Thus the results derived earlier in this section can be applied to the more general case if we simply replace e^2 with Ze^2. For example, Equation 31–3 becomes $mv^2 = kZe^2/r$. Similarly, the radius of an allowed orbit is

$$r_n = \left(\frac{h^2}{4\pi^2 mkZe^2}\right)n^2 \qquad n = 1, 2, 3, \ldots \qquad \text{31–7}$$

For example, the radius of the $n = 1$ orbit of singly ionized helium is half the radius of the $n = 1$ orbit of hydrogen.

The Energy of a Bohr Orbit

To find the energy of an electron in a Bohr orbit, we simply note that its total mechanical energy, E, is the sum of its kinetic and potential energies:

$$E = K + U = \tfrac{1}{2}mv^2 + U$$

Using the fact that $mv^2 = kZe^2/r$ for a hydrogen-like atom of atomic number Z (Equation 31–3), and the fact that the electrostatic potential energy of a charge $-e$ and a charge $+Ze$ a distance r apart is $U = -kZe^2/r$ (Equation 20–8), we find that the total mechanical energy is

$$E = \tfrac{1}{2}\left(\frac{kZe^2}{r}\right) - \frac{kZe^2}{r} = -\frac{kZe^2}{2r}$$

Finally, substituting the radius of a Bohr orbit, as given in Equation 31–7, we obtain the corresponding energy for the nth orbit:

$$E_n = -\frac{kZe^2}{2r_n} = -\left(\frac{kZe^2}{2}\right)\left(\frac{4\pi^2 mkZe^2}{h^2}\right)\frac{1}{n^2}$$

$$= -\left(\frac{2\pi^2 mk^2e^4}{h^2}\right)\frac{Z^2}{n^2} \qquad n = 1, 2, 3, \ldots \qquad \text{31–8}$$

Using the numerical values for m, k, e, and h, as we did in calculating the Bohr radius, we have

$$E_n = -(13.6\text{ eV})\frac{Z^2}{n^2} \qquad n = 1, 2, 3, \ldots \qquad \text{31–9}$$

PROBLEM-SOLVING NOTE

The Z Dependence of Hydrogen-like Ions

Note that the energy of a hydrogen-like ion depends on Z^2, the square of the atomic number.

Let's first consider the specific case of hydrogen. With $Z = 1$ we find that the energy of the orbits in hydrogen are given by the relation $E_n = -(13.6 \text{ eV})/n^2$. We plot these energies in Figure 31–8 for various values of n. This type of plot is referred to as an **energy-level diagram.** Notice that the **ground state** ($n = 1$) corresponds to the lowest possible energy of the system. The higher energy levels are referred to as **excited states.** As the integer n tends to infinity, the energy of the excited states approaches zero—the energy the electron and proton would have if they were at rest and separated by an infinite distance. Thus to **ionize** hydrogen—that is, to remove the electron from the atom—requires a minimum energy of 13.6 eV. This value, which is a specific prediction of the Bohr model, is in complete agreement with experiment.

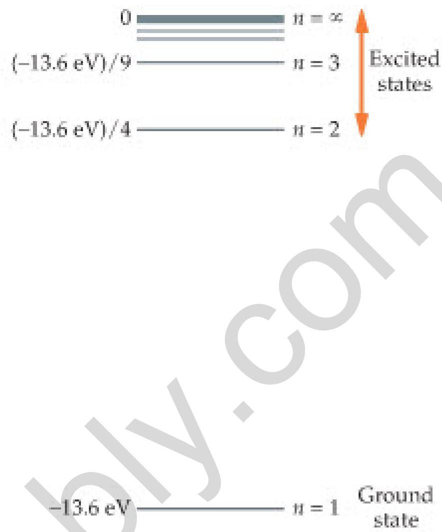


FIGURE 31–8 Energy-level diagram for the Bohr model of hydrogen

The energy of the ground state of hydrogen is -13.6 eV. Excited states of hydrogen approach zero energy. Note that the difference in energy from the ground state to the first excited state is $\frac{3}{4}(13.6 \text{ eV})$, and the energy difference from the first excited state to the zero level is only $\frac{1}{4}(13.6 \text{ eV})$.

EXERCISE 31–2

In doubly ionized lithium, a single electron orbits a lithium nucleus. Calculate the minimum energy required to remove this electron.

SOLUTION

The nucleus of lithium has a charge of $+3e$. Substitution of $Z = 3$ and $n = 1$ in Equation 31–9 yields

$$E_1 = -(13.6 \text{ eV})\left(\frac{3^2}{1^2}\right) = -122 \text{ eV}$$

Therefore, 122 eV must be added to remove the electron.

The electron in doubly ionized lithium experiences a stronger attractive force than the single electron in hydrogen. In fact, the force is greater by a factor of 3. In addition, the $n = 1$ orbit of doubly ionized lithium has one-third the radius of the $n = 1$ orbit in hydrogen. As a result, nine times as much energy is required to remove the electron from the lithium ion.

At room temperature, most hydrogen atoms are in the ground state. This is because the typical thermal energy of such atoms is too small to cause even the lowest-energy excitation from the ground state. Specifically, a typical thermal energy, $k_B T$, corresponding to room temperature ($T \sim 300$ K) is only about $\frac{1}{40}$ eV. In comparison, the energy required to excite an electron from the ground state of hydrogen to the first excited state is roughly 10 eV; that is, $\Delta E = E_2 - E_1 = (-3.40 \text{ eV}) - (-13.6 \text{ eV}) = 10.2 \text{ eV}$. Excitations to higher excited states require even more energy. As a result, typical intermolecular collisions are simply not energetic enough to produce an excited state in hydrogen.

The Spectrum of Hydrogen

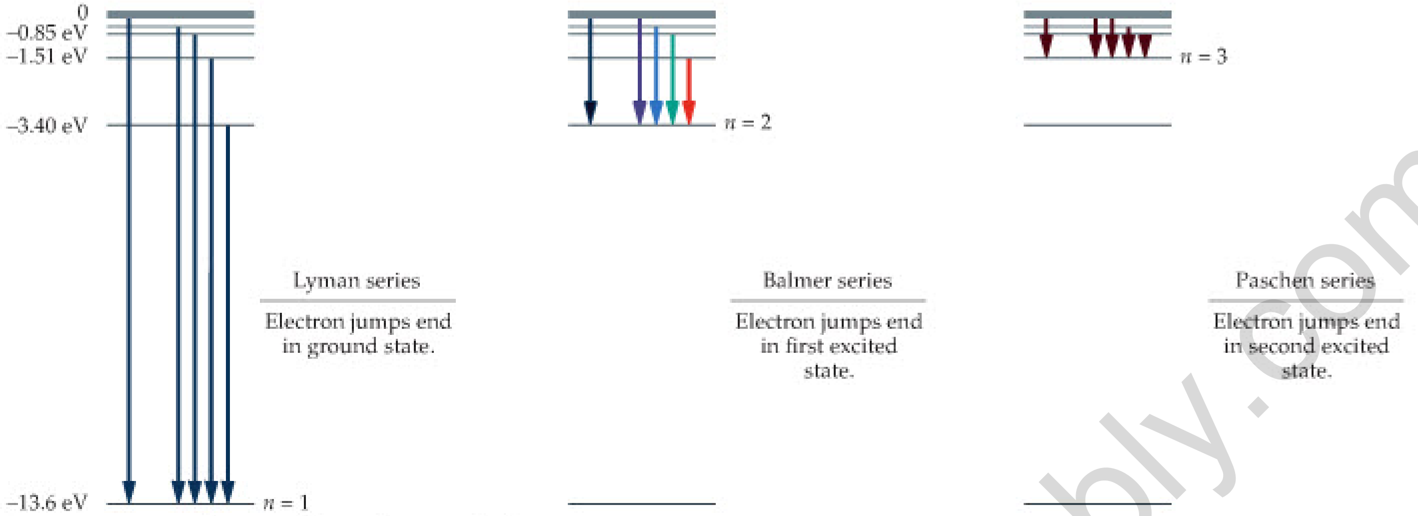
To find a formula describing the spectrum of hydrogen, we use Bohr's assumption that the frequency of emitted radiation for a change in energy equal to ΔE is given by $|\Delta E| = hf$. Since $\lambda f = c$ for electromagnetic radiation (Equation 25–4), we can rewrite this relation in terms of the wavelength as $|\Delta E| = hc/\lambda$. To find $|\Delta E|$, we recall that the energy for hydrogen is given by Equation 31–9 with $Z = 1$. Therefore, the *change* in energy as the electron moves from an excited outer orbit with $n = n_i$ to a lower orbit with $n = n_f < n_i$ has the following magnitude:

$$|\Delta E| = \left(\frac{2\pi^2 mk^2e^4}{h^2}\right)\left(\frac{1}{n_f^{\,2}} - \frac{1}{n_i^{\,2}}\right)$$

Using $|\Delta E| = hc/\lambda$ we can now solve for $1/\lambda$:

$$\frac{1}{\lambda} = \left(\frac{2\pi^2 mk^2e^4}{h^3c}\right)\left(\frac{1}{n_f^{\,2}} - \frac{1}{n_i^{\,2}}\right) \qquad 31\text{–}10$$

Comparing Equation 31–10 with Equation 31–2, we see that the expressions have precisely the same form, provided we identify n_f with n', and n_i with n. In addition, we see that the Rydberg constant in Equation 31–2, $R = 1.097 \times 10^7 \text{ m}^{-1}$, has



▲ **FIGURE 31–9 The origin of spectral series in hydrogen**

Each series of spectral lines in hydrogen is the result of electrons jumping from an excited state to a particular lower level. For the Lyman series the lower level is the ground state. The lower level for the Balmer series is the first excited state ($n = 2$), and the lower level for the Paschen series is the second excited state ($n = 3$).

been replaced by the rather unusual constant $2\pi^2mk^2e^4/h^3c$ in Equation 31–10. It is remarkable that when the known values of the fundamental constants m, k, e, h, and c are substituted into $2\pi^2mk^2e^4/h^3c$, the resulting value is precisely $1.097 \times 10^7\ \mathrm{m}^{-1}$. This completes the derivation of Equation 31–2, one of the most significant accomplishments of the Bohr model.

The origin of the line spectrum of hydrogen can be visualized in **Figure 31–9**. Notice that transitions involving an electron jumping from an excited state ($n_i = n > 1$) to the ground state ($n_f = n' = 1$) result in the Lyman series of lines in the ultraviolet. Jumps ending in the $n = 2$ level give rise to the Balmer series, and jumps ending in the $n = 3$ level give the Paschen series. The largest energy jump in each series occurs when an electron falls from $n = \infty$ to the final level. Thus each series of spectral lines has a well-defined shortest wavelength.

CONCEPTUAL CHECKPOINT 31–1 COMPARE WAVELENGTHS

The wavelength of the photon emitted when an electron in hydrogen jumps from the $n_i = 100$ state to the $n_f = 2$ state is **(a)** greater than, **(b)** less than, or **(c)** equal to the wavelength of the photon when the electron jumps from the $n_i = 2$ state to the $n_f = 1$ state.

REASONING AND DISCUSSION

We begin by noting that wavelength is inversely proportional to the energy difference between levels ($|\Delta E| = hc/\lambda$), thus the smaller the energy difference, the greater the wavelength.

Next, we note that the full range of energies in the hydrogen atom extends from −13.6 eV to 0, and that the difference in energy between $n_i = 2$ and $n_f = 1$ is three-quarters of this range. It follows that the energy difference between $n_i = 2$ and $n_f = 1$ is greater than the energy difference between *any* state with $n_i > 2$ and $n_f = 2$. In fact, the maximum energy difference ending in the state 2 is $\frac{1}{4}(-13.6\ \mathrm{eV})$, corresponding to $n_i = \infty$ and $n_f = 2$.

Since the energy difference for $n_i = 100$ and $n_f = 2$ is less than the energy difference for $n_i = 2$ and $n_f = 1$, the wavelength for $n_i = 100$ and $n_f = 2$ is the greater of the two.

ANSWER

(a) The wavelength for $n_i = 100$, $n_f = 2$ is greater than the wavelength for $n_i = 2$, $n_f = 1$.

We calculate a variety of specific wavelengths in the following Active Example.

ACTIVE EXAMPLE 31–1 FIND THE WAVELENGTHS

An electron in a hydrogen atom is in the initial state $n_i = 4$. Calculate the wavelength of the photon emitted by this electron if it jumps to the final state **(a)** $n_f = 3$, **(b)** $n_f = 2$, or **(c)** $n_f = 1$. (*Note*: To simplify the calculations, replace the constant $2\pi^2mk^2e^4/h^3c$ in Equation 31–10 with its numerical equivalent, the Rydberg constant, $R = 1.097 \times 10^7\ \text{m}^{-1}$.)

SOLUTION *(Test your understanding by performing the calculations indicated in each step.)*

Part (a)

1. Substitute $n_i = 4$ and $n_f = 3$ in Equation 31–10: $\lambda = 1875$ nm

Part (b)

2. Repeat with $n_f = 2$: $\lambda = 486.2$ nm

Part (c)

3. Finally, use $n_f = 1$: $\lambda = 97.23$ nm

INSIGHT

Other wavelengths are possible when an electron in the $n_i = 4$ state jumps to a lower state. For example, after dropping from the $n = 4$ state to the $n = 3$ state, the electron might then jump from the $n = 3$ state to the $n = 2$ state, and finally from the $n = 2$ state to the $n = 1$ state. Alternatively, the electron might first jump from the $n = 4$ state to the $n = 2$ state, and then from the $n = 2$ state to the $n = 1$ state. Thus an electron in an excited state may result in the emission of a variety of different wavelengths.

YOUR TURN

A hydrogen atom with its electron in the initial state $n_i = 5$ emits a photon with a wavelength of 434 nm. To which state did the electron jump?

(Answers to **Your Turn** *problems are given in the back of the book.)*

PROBLEM-SOLVING NOTE

The Frequency and Wavelength of a Photon

Recall that the frequency of a photon is given by $hf = |\Delta E|$, and its wavelength is given by $hc/\lambda = |\Delta E|$.

Just as an electron can *emit* a photon when it jumps to a lower level, it can also *absorb* a photon and jump to a higher level. This process occurs, however, only if the photon has the proper energy. In particular, the photon must have an energy that precisely matches the energy difference between the lower level of the electron and the higher level to which it is raised. This situation is explored in more detail in the next Active Example.

ACTIVE EXAMPLE 31–2 ABSORBING A PHOTON: WHAT IS THE FREQUENCY?

Find the frequency a photon must have if it is to raise an electron in a hydrogen atom from the $n = 3$ state to the $n = 5$ state.

SOLUTION *(Test your understanding by performing the calculations indicated in each step.)*

1. Calculate the energy of the $n = 5$ state in joules: -8.70×10^{-20} J
2. Calculate the energy of the $n = 3$ state: -2.42×10^{-19} J
3. Calculate the difference in energy between these states: 1.55×10^{-19} J
4. Set the energy of a photon equal to this energy difference: $hf = 1.55 \times 10^{-19}$ J
5. Solve for the frequency of the photon: $f = 2.34 \times 10^{14}$ Hz

INSIGHT

Of course, this frequency corresponds to one of the lines in the absorption spectrum of hydrogen. In this case, the line is in the infrared.

YOUR TURN

Suppose an electron jumps from the $n = 3$ state to the $n = 5$ state in singly ionized helium. Does the required frequency of the absorbed photon increase or decrease? By what factor?

(Answers to **Your Turn** *problems are given in the back of the book.)*

31–4 de Broglie Waves and the Bohr Model

The fact that hydrogen emits radiation only at certain well-defined wavelengths is reminiscent of the harmonics of standing waves on a string. Recall that a vibrating string tied down at both ends produces a standing wave only if an integral number of half-wavelengths fit within its length (Section 14–8). Perhaps the behavior of hydrogen can be understood in similar terms.

In 1923 de Broglie used his idea of matter waves (Section 30–5) to show that one of Bohr's assumptions could indeed be thought of as a condition for standing waves. As we saw earlier in this chapter, Bohr assumed that the angular momentum of an electron in an allowed orbit must be a nonzero integer times $h/2\pi$. Specifically,

$$rmv = n\frac{h}{2\pi} \qquad n = 1, 2, 3, \ldots$$

In Bohr's model there is no particular reason for this condition other than that it produces results in agreement with experiment.

Now, de Broglie imagined his matter waves as analogous to a wave on a string—except that in this case the "string" is not tied down at both ends. Instead, it is formed into a circle of radius r representing an electron's orbit about the nucleus, as illustrated in **Figure 31–10**. The condition for a standing wave in this case is that an integral number of wavelengths fit into the circumference of the orbit. Stated mathematically, the condition is $n\lambda = 2\pi r$, as shown in Figure 31–10 (a) for the cases $n = 7$ and $n = 8$. Other wavelengths would result in destructive interference, as Figure 31–10 (b) shows.

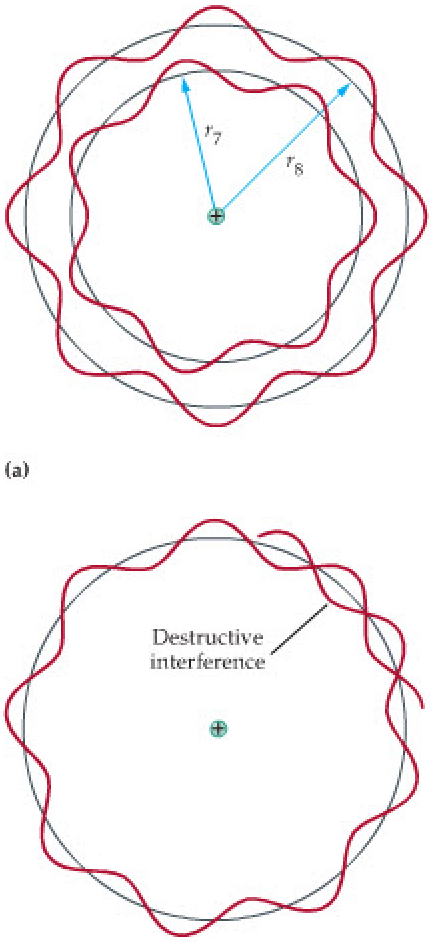
Finally, de Broglie combined the standing wave condition with his matter–wave relationship $p = h/\lambda$ (Equation 30–16). The result is as follows:

$$p = mv = \frac{h}{\lambda} = \frac{h}{(2\pi r/n)} = \frac{nh}{2\pi r} \qquad n = 1, 2, 3, \ldots$$

Multiplying both sides of this equation by r to obtain the angular momentum, we find

$$L = rmv = \frac{nh}{2\pi} \qquad n = 1, 2, 3, \ldots$$

This is precisely the Bohr orbital condition, now understood as a reflection of the wave nature of matter.



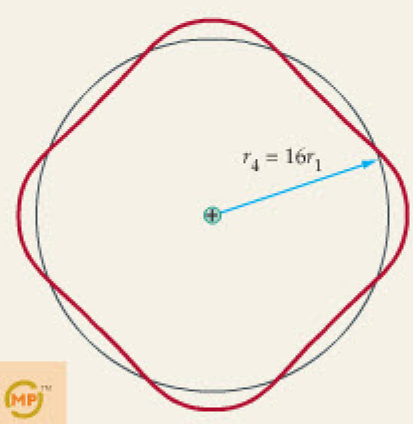
▲ **FIGURE 31–10 de Broglie wavelengths and Bohr orbits**
Bohr's condition that the angular momentum of an allowed orbit must be an integer n times $h/2\pi$ is equivalent to the condition that n de Broglie wavelengths must fit into the circumference of an orbit. **(a)** The de Broglie waves for the $n = 7$ and $n = 8$ orbits. **(b)** If an integral number of wavelengths do not fit the circumference of an orbit, the result is destructive interference.

EXAMPLE 31–3 THE WAVELENGTH OF AN ELECTRON

Find the wavelength associated with an electron in the $n = 4$ state of hydrogen.

PICTURE THE PROBLEM
Our sketch shows that four wavelengths fit around the circumference of the $n = 4$ orbit. Recall that the radius of this orbit is $4^2 = 16$ times the radius of the ground-state orbit.

STRATEGY
To find the wavelength of this matter wave we simply calculate the circumference of the $n = 4$ orbit, then divide by 4.



INTERACTIVE FIGURE

SOLUTION

1. Calculate the radius of the $n = 4$ orbit: $r_4 = 4^2 r_1 = 16(5.29 \times 10^{-11}\ \text{m}) = 8.46 \times 10^{-10}\ \text{m}$
2. Use this result to find the circumference of the orbit: $2\pi r_4 = 2\pi(8.46 \times 10^{-10}\ \text{m}) = 5.32 \times 10^{-9}\ \text{m}$
3. Divide the circumference by 4 to find the wavelength: $\lambda_4 = \frac{1}{4}(2\pi r_4) = \frac{1}{4}(5.32 \times 10^{-9}\ \text{m}) = 1.33 \times 10^{-9}\ \text{m}$

INSIGHT

An equivalent way of determining the wavelength is to use the de Broglie relation, $p = mv = h/\lambda$. Solving for the wavelength yields $\lambda_4 = h/mv_4$, and substituting v_4 from Equation 31–6 yields the same result given in Step 3.

PRACTICE PROBLEM

In which state of hydrogen does the electron have a wavelength of 2.66×10^{-9} m? [**Answer:** $n = 8$]

Some related homework problems: Problem 35, Problem 36

The striking success of de Broglie's matter waves in deriving Bohr's angular momentum condition encouraged physicists to give the idea of matter waves serious consideration. If we accept matter waves as being real, however, a large number of new questions must be addressed. For example, if particles like electrons can be described by matter waves, how do the matter waves behave? What determines the value of a matter wave at a particular location? What is the physical significance of a matter wave having a large value at one location and a small value at another location?

These questions were answered by Erwin Schrödinger (1887–1961), Max Born (1882–1970), and others. In particular, Schrödinger introduced an equation—similar in many respects to the equation that describes sound waves—to describe the behavior of matter waves. Today, this equation, known as **Schrödinger's equation,** forms the basis for quantum mechanics, which is the quantum physics version of classical mechanics. In fact, Schrödinger's equation plays the same role in quantum mechanics that Newton's laws play in classical mechanics and Maxwell's equations play in electromagnetism.

After the introduction of the Schrödinger equation, Born developed an interpretation of the matter waves that was quite different from that for mechanical waves. For example, in the case of a wave on a string, the amplitude of a wave simply represents the displacement of the string from its equilibrium position. For a matter wave, on the other hand, the amplitude is related to the *probability* of finding a particle in a particular location. Thus, matter waves do not tell us precisely where a particle is located; rather, they give the probability of finding the particle at a given place, as we shall see in detail in the next section.

▲ Although we can't see the de Broglie waves associated with electrons, the standing waves they produce are of great importance because they correspond to allowed states in atoms and molecules. One way to visualize de Broglie waves, however, is to make an analogy with mechanical standing waves. In the photos shown above, a loop of wire is oscillated vertically about the support point at the bottom of the loop. The oscillations set up waves that travel on the circumference of the wire—like de Broglie waves on a Bohr orbit in hydrogen. If the wavelength of the mechanical wave is tuned to an appropriate value—by adjusting the frequency of the oscillator—the result is a variety of different standing wave patterns, analogous to different energy levels of de Broglie waves in the Bohr model.

31–5 The Quantum Mechanical Hydrogen Atom

Although Schrödinger's equation and its solution for the hydrogen atom are beyond the scope of this text, we present here some of the main features obtained by this analysis. Other than relativistic effects, the Schrödinger equation presents our most complete understanding of the hydrogen atom and of behavior at the atomic level in general. As we shall see, many aspects of the Bohr model survive in this analysis, though there are also significant differences.

To begin, we note that whereas the Bohr model was characterized by a single quantum number, n, the quantum mechanical description of the hydrogen atom requires four quantum numbers. They are as follows:

- **The principal quantum number,** n**:** The quantum number $n = 1, 2, 3, \ldots$ plays a similar role in the quantum mechanical hydrogen atom and in Bohr's model. In Bohr's model, n is the only quantum number, and it determines the radius of an orbit, its angular momentum, and its energy. In particular, the energy in the Bohr model is $E = (-13.6\ \text{eV})/n^2$. The energy given by Schrödinger's equation is precisely the same, if we neglect small relativistic effects and small magnetic interactions within the atom.

- **The orbital angular momentum quantum number, ℓ:** In the Bohr model an electron's orbital angular momentum is determined by the quantum number n. In particular, $L_n = nh/2\pi$, where $n = 1, 2, 3, \ldots$. In the quantum mechanical solution, there is a separate quantum number, ℓ, for the orbital angular momentum. This quantum number can take on the following values for any given value of the principal quantum number, n:

$$\ell = 0, 1, 2, \ldots, (n - 1) \qquad \text{31–11}$$

The magnitude of the angular momentum for any given value of ℓ is given by the following relation:

$$L = \sqrt{\ell(\ell + 1)}\frac{h}{2\pi} \qquad \text{31–12}$$

Note that the angular momentum of the electron can have a range of values for a given n, in contrast with the Bohr model, where the angular momentum has just a single value. In particular, the electron in a hydrogen atom can have zero angular momentum; an orbiting electron in the Bohr model always has a nonzero angular momentum.

Finally, although the energy of the hydrogen atom does not depend on ℓ, the energy of more complex atoms does have an ℓ dependence, as we shall see in the next section.

- **The magnetic quantum number, m_ℓ:** If a hydrogen atom is placed in an external magnetic field, its energy is found to depend not only on n but on an additional quantum number as well. This quantum number, m_ℓ, is referred to as the magnetic quantum number. The allowed values of m_ℓ are as follows:

$$m_\ell = -\ell, -\ell + 1, -\ell + 2, \ldots, -1, 0, 1, \ldots, \ell - 2, \ell - 1, \ell \qquad \text{31–13}$$

This quantum number gives the component of orbital angular momentum vector along a specified direction, usually chosen to be the z axis. With this choice, L_z has the following values:

$$L_z = m_\ell \frac{h}{2\pi}$$

Only a single component of the orbital angular momentum can be known precisely; it is not possible to know all three components of the angular momentum simultaneously, due to the Heisenberg uncertainty principle.

- **The electron spin quantum number, m_s:** The final quantum number needed to describe the hydrogen atom is related to the angular momentum of the electron itself. Just as Earth spins on its axis at the same time that it orbits the Sun, the electron can be thought of as having both an orbital and a "spin" angular momentum. The spin quantum number for an electron takes on just two values:

$$m_s = -\tfrac{1}{2}, \tfrac{1}{2}$$

These two values correspond to the electron's spin being "up" $\left(m_s = \tfrac{1}{2}\right)$ or "down" $\left(m_s = -\tfrac{1}{2}\right)$ with respect to the z axis.

Spin angular momentum is an *intrinsic* property of an electron, like its mass and its charge—*all* electrons have exactly the *same* mass, the *same* charge, and the *same* spin angular momentum. Thus, we do not imagine the electron to be a small spinning sphere, like a microscopic planet. You can't speed up or slow down the spin of an electron. Instead, spin is simply one of the properties that defines an electron.

The energy-level structure of hydrogen in zero magnetic field is shown in Figure 31–11, along with the corresponding quantum numbers. Since the energies are the same as in the Bohr model, it follows that the spectrum will be the same as the Bohr model, and experiment.

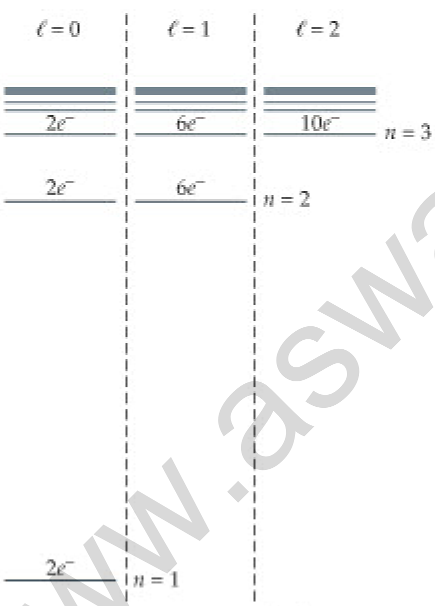


FIGURE 31–11 Energy-level structure of hydrogen
The values of the quantum mechanical energy levels for hydrogen are in complete agreement with the Bohr model. In the quantum model, however, each energy level has associated with it a specific number of "quantum states" determined by the values of all four quantum numbers, as specified in Table 31–2. In multielectron atoms, these states lead to the formation of the periodic table of elements, as we show in Section 31–6.

TABLE 31–2 States of Hydrogen for $n = 1$ and $n = 2$

***n* = 1, ℓ = 0**			**Two states**
$n = 1$	$\ell = 0$	$m_\ell = 0$	$m_s = \frac{1}{2}$
$n = 1$	$\ell = 0$	$m_\ell = 0$	$m_s = -\frac{1}{2}$
***n* = 2, ℓ = 0**			**Two states**
$n = 2$	$\ell = 0$	$m_\ell = 0$	$m_s = \frac{1}{2}$
$n = 2$	$\ell = 0$	$m_\ell = 0$	$m_s = -\frac{1}{2}$
***n* = 2, ℓ = 1**			**Six states**
$n = 2$	$\ell = 1$	$m_\ell = 1$	$m_s = \frac{1}{2}$
$n = 2$	$\ell = 1$	$m_\ell = 1$	$m_s = -\frac{1}{2}$
$n = 2$	$\ell = 1$	$m_\ell = 0$	$m_s = \frac{1}{2}$
$n = 2$	$\ell = 1$	$m_\ell = 0$	$m_s = -\frac{1}{2}$
$n = 2$	$\ell = 1$	$m_\ell = -1$	$m_s = \frac{1}{2}$
$n = 2$	$\ell = 1$	$m_\ell = -1$	$m_s = -\frac{1}{2}$

We define a **state** of hydrogen to be a specific assignment of values for each of the four quantum numbers. For example, there are two states that correspond to the lowest possible energy level of hydrogen. These are $n = 1, \ell = 0, m_\ell = 0, m_s = \frac{1}{2}$; and $n = 1, \ell = 0, m_\ell = 0, m_s = -\frac{1}{2}$. Similarly, there are two states corresponding to the $n = 2, \ell = 0$ energy level, and six states corresponding to the $n = 2, \ell = 1$ level. These states are listed in Table 31–2, and the corresponding numbers are shown in **Figure 31–11**. When we consider multielectron atoms in the next section, we shall see that the number of states associated with a given energy level determines the number of electrons (e^-) it can accommodate. Once an energy level is "filled," additional electrons must occupy higher levels. This progressive filling of energy levels ultimately leads to the periodic table of elements.

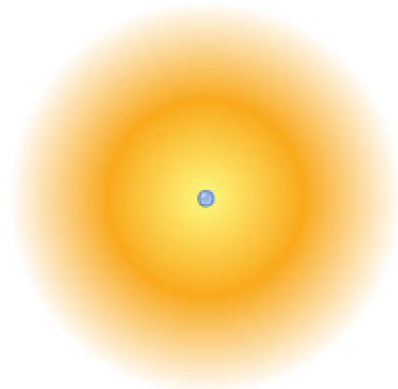


FIGURE 31–12 Probability cloud for the ground state of hydrogen

In the quantum mechanical model of hydrogen, the electron can be found at any distance from the nucleus. The probability of finding the electron at a given location is proportional to the density of the "probability cloud."

Electron Probability Clouds: Three-Dimensional Standing Waves

As mentioned in the previous section, the solution to Schrödinger's equation gives a matter wave, or **wave function**, as it is known, corresponding to a particular physical system. The wave function for hydrogen gives the probability of finding the electron at a particular location. The best way to visualize this probability distribution is in terms of a "probability cloud," as shown in **Figure 31–12**. The probability of finding the electron is greatest where the cloud is densest. In the case shown in Figure 31–12, corresponding to the ground state, $n = 1, \ell = 0, m_\ell = 0$, the probability of finding the electron is distributed with spherical symmetry. Note that the probability decreases rapidly far from the nucleus, as one would expect.

Figure 31–13 gives a different way of looking at the probability distribution for the ground state. Here we plot the probability versus distance from the nucleus. It is interesting to note that the maximum probability occurs at a distance from the nucleus equal to the Bohr radius. Thus certain aspects of the Bohr model find their way into the final solution for hydrogen. The difference, however, is that in the Bohr model the electron is always at a particular distance from the nucleus as it moves in its circular orbit. In the quantum solution to hydrogen, the electron can be found at virtually any distance from the nucleus, not just one distance.

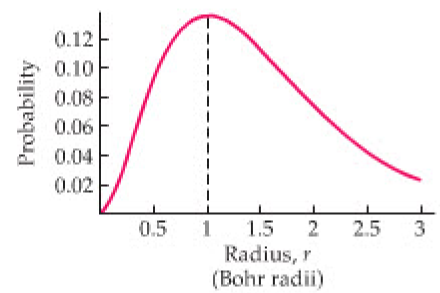


FIGURE 31–13 Probability as a function of distance

This plot shows the probability of finding an electron in the ground state of hydrogen at a given distance from the nucleus. Note that the probability is greatest at a distance equal to the Bohr radius, r_1.

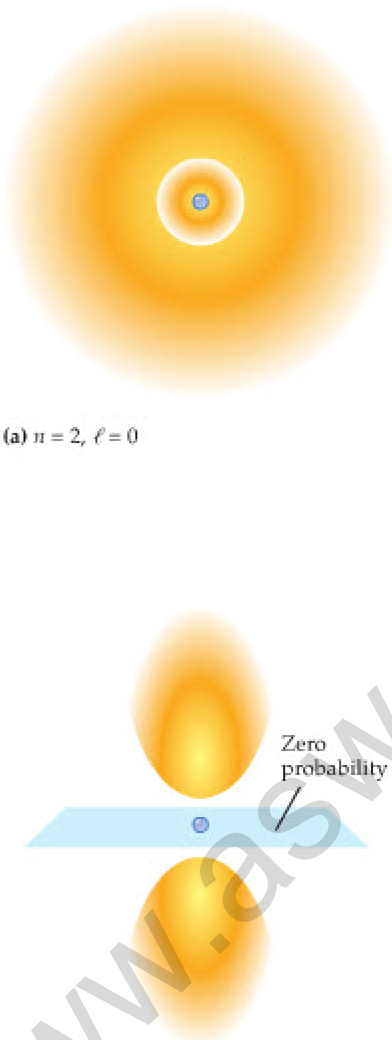
States of higher quantum number have increasingly complex probability distributions, as indicated in **Figure 31–14**. As the quantum number n is increased, for example, the basic shape of the distribution remains the same, but additional nodes appear. This is illustrated in Figure 31–14 (a) for the case $n = 2, \ell = 0$. Note that the distribution is spherically symmetric, as in the state $n = 1, \ell = 0$, but now there is a node—where the probability is zero—separating an inner and an outer portion of the distribution. As the quantum number ℓ is increased, the distributions become more complex in their shape. An example is given in Figure 31–14 (b) for the case $n = 2, \ell = 1$, and $m_\ell = 0$.

CONCEPTUAL CHECKPOINT 31–2 FINDING THE ELECTRON

The probability cloud for the $n = 2$, $\ell = 1$ state of hydrogen is shown in Figure 31–14 (b). Notice that this cloud consists of two lobes of high probability separated by a plane of zero probability. Given that an electron in this state can never be halfway between the two lobes, how is it possible that the electron is as likely to be found in the upper lobe as in the lower lobe?

REASONING AND DISCUSSION

The probability lobes of an electron are the result of a standing wave pattern, analogous to the standing waves found on a string tied down at both ends. Both the node on a string and the region of zero probability of an electron are the result of destructive interference. For example, the displacement of a string may be of equal amplitude on either side of a node—where the displacement is always zero—just as the electron probability may be high on either side of a probability node and zero in the middle. In summary, an electron does not simply move from place to place in an atom, like a small ball of charge; instead, it forms a standing wave pattern with nodes in certain locations.



(a) $n = 2$, $\ell = 0$

(b) $n = 2$, $\ell = 1$

▲ **FIGURE 31–14 Probability clouds for excited states of hydrogen**

The "probability cloud" for an electron in hydrogen increases in complexity as the quantum numbers increase. **(a)** $n = 2$, $\ell = 0$; **(b)** $n = 2$, $\ell = 1$.

31–6 Multielectron Atoms and the Periodic Table

In this section we extend our considerations of atomic physics to atoms with more than one electron. As we shall see, certain regularities arise in the properties of multielectron atoms, and these regularities are intimately related to the quantum numbers described in the previous section.

Multielectron Atoms

One of the great simplicities of the hydrogen atom is that the only electrostatic force in the atom is the attractive force between the electron and the proton. In multielectron atoms the situation is more complex. Specifically, the electrons in such atoms experience repulsive electrostatic interactions with one another, in addition to their interaction with the nucleus. Thus the simple expression for the energy of a single-electron atom given in Equation 31–9 cannot be applied to atoms with multiple electrons, since it does not include energy contributions due to the forces between electrons.

Although no simple formula analogous to Equation 31–9 exists for multielectron atoms, the energy levels of these atoms can be understood in terms of the four quantum numbers (n, ℓ, m_ℓ, m_s) used to describe hydrogen. In fact, by applying Schrödinger's equation to such atoms, we have discovered that the energy levels of multielectron atoms depend on the principal quantum number, n, and on the orbital quantum number, ℓ. For example, increasing n for a fixed value of ℓ results in an increase in energy. This relationship is illustrated in **Figure 31–15**, where we see that the energy for $n = 2$ and $\ell = 0$ is greater than the energy for $n = 1$ and $\ell = 0$. Similarly, the energy for $n = 3$ and $\ell = 1$ is greater than the energy for $n = 2$ and $\ell = 1$. It is also found that the energy increases with increasing ℓ for fixed n. Thus the energy for $n = 2$ and $\ell = 1$ is greater than the energy for $n = 2$ and $\ell = 0$.

Figure 31–15 also shows that in some cases the energy levels corresponding to different values of n can cross. For example, the energy for $n = 3$ and $\ell = 2$ is greater than the energy for $n = 4$ and $\ell = 0$. A similar crossing occurs with the $n = 4$, $\ell = 2$ state and the $n = 5$, $\ell = 0$ state. Note in all cases, however, that the energy still increases with n for fixed ℓ, and increases with ℓ for fixed n. We shall see the effect of these energy-level crossings in terms of the periodic table later in this section.

Because the energy levels of multielectron atoms depend on n and ℓ, we give specific names and designations to the various values of these quantum numbers. For example, all electrons that have the same value of n are said to be in the same **shell.** Specifically, electrons with $n = 1$ are said to be in the K shell, those with $n = 2$ are in the L shell, those with $n = 3$ are in the M shell, and so on. These designations are summarized in Table 31–3 and displayed in Figure 31–15.

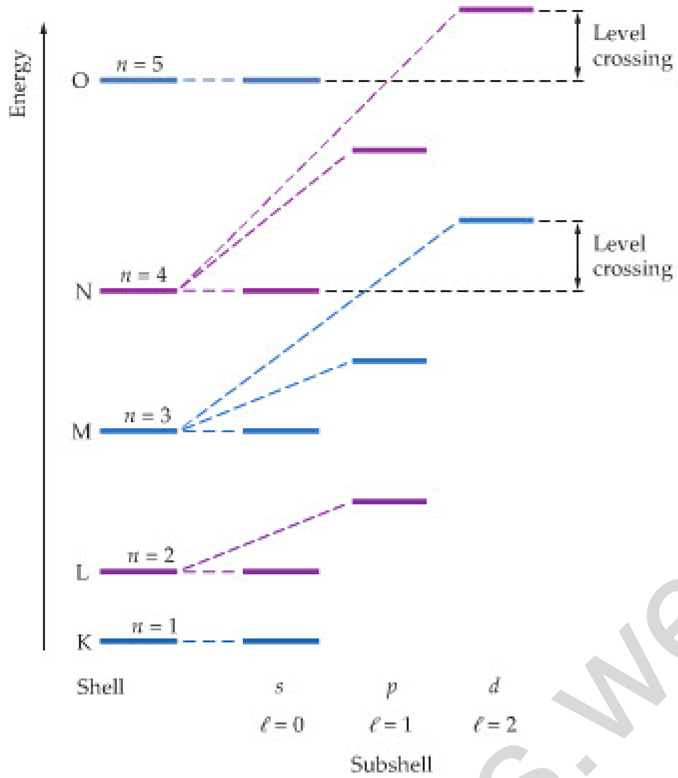


FIGURE 31–15 Energy levels in multielectron atoms

In multielectron atoms the energy increases with n for fixed ℓ and increases with ℓ for fixed n. Note the possibility of energy-level crossing. For example, the $n = 3$, $\ell = 2$ energy level is higher than the $n = 4$, $\ell = 0$ energy level.

Similarly, electrons in a given shell with the same value of ℓ are said to be in the same **subshell**, and different values of ℓ have different alphabetical designations. For example, electrons with $\ell = 0$ are said to be in the *s* subshell, those with $\ell = 1$ are in the *p* subshell, and those with $\ell = 2$ are in the *d* subshell. These names, though not particularly logical, are used for historical reasons. After the *f* subshell ($\ell = 3$), the names of subsequent subshells continue in alphabetical order, as indicated in Table 31–3.

TABLE 31–3 Shell and Subshell Designations

n	Shell
1	K
2	L
3	M
4	N
...	...
ℓ	**Subshell**
0	*s*
1	*p*
2	*d*
3	*f*
4	*g*
...	...

The Pauli Exclusion Principle

As mentioned in Section 31–3, most hydrogen atoms are in their ground state at room temperature, since typical thermal energies are not great enough to excite the electron to higher energy levels. The same is true of multielectron atoms—they too are generally found in their ground state at room temperature. The question is this: What is the ground state of a multielectron atom?

The answer to this question involves an entirely new fundamental principle of physics put forward by the Austrian physicist Wolfgang Pauli (1900–1958) in 1925. Pauli's "exclusion principle" states that no two electrons in an atom can be in the same state at the same time. That is, once an electron occupies a given state, as defined by the values of its quantum numbers, other electrons are *excluded* from occupying the same state:

The Pauli Exclusion Principle

Only one electron at a time may have a particular set of quantum numbers, n, ℓ, m_ℓ, and m_s. Once a particular state is occupied, other electrons are excluded from that state.

Because of the exclusion principle, the ground state of a multielectron atom is *not* obtained by placing all the electrons in the lowest possible energy state, as one might at first suppose. Once the lowest-energy states are occupied, additional electrons in the atom must occupy levels of higher energy. As more electrons are added to an atom, they fill up one subshell after another until all the electrons are accommodated. The situation is analogous to placing marbles

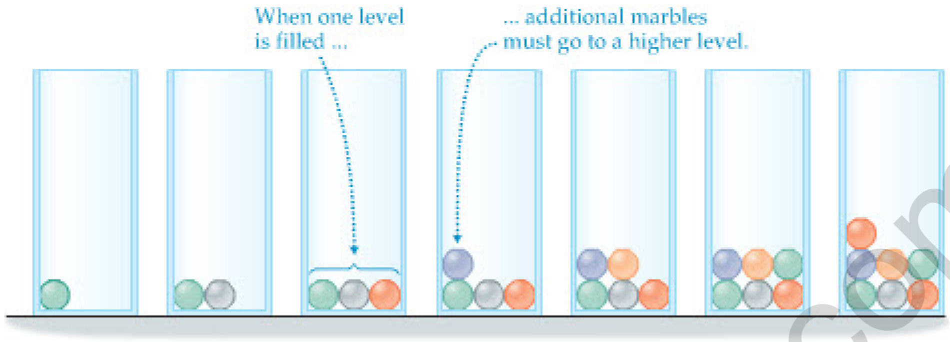


FIGURE 31–16 Filling a jar with marbles As marbles are added to a jar, they fill in one level, then another, and another. Once a given level is filled with marbles, additional marbles are excluded from that level, analogous to the filling of energy levels in multielectron atoms.

into a jar, as illustrated in **Figure 31–16**. The first few marbles occupy the lowest level of the jar, but as more marbles are added they must occupy levels of higher gravitational potential energy, simply because the lower levels are already occupied.

In the case of atoms, the lowest energy level corresponds to $n = 1$ and $\ell = 0$; that is, to the s subshell of the K shell. To completely define a state, however, we must specify all four quantum numbers: n, ℓ, m_ℓ, and m_s. First, recall that $m_\ell = 0, \pm1, \pm2, \ldots, \pm\ell$ for general ℓ. It follows that in the $\ell = 0$ state the only possible value of m_ℓ is 0. The quantum number m_s, however, can always take on two values, $m_s = +\frac{1}{2}$ and $m_s = -\frac{1}{2}$. Therefore, two electrons can occupy the $n = 1, \ell = 0$ energy level, since two different states—$n = 1, \ell = 0, m_\ell = 0, m_s -\frac{1}{2}$; and $n = 1, \ell = 0, m_\ell = 0, m_s = +\frac{1}{2}$—correspond to that level, as indicated in **Figure 31–17**.

The next higher energy level corresponds to $n = 2$ and $\ell = 0$. Again, two states correspond to this level, allowing it to hold two electrons. Lest we conclude that all levels can hold two electrons, consider the next level up: $n = 2$ and $\ell = 1$.

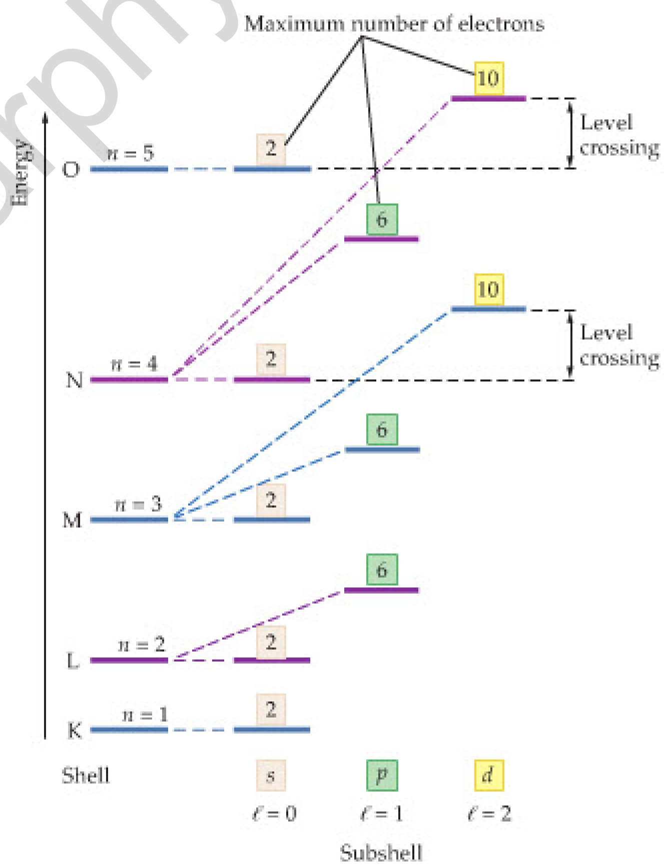


FIGURE 31–17 Maximum number of electrons in an energy level The maximum number of electrons that can occupy a given energy level in a multielectron atom is $2(2\ell + 1)$. Occupancy by more than this number of electrons would violate the Pauli exclusion principle.

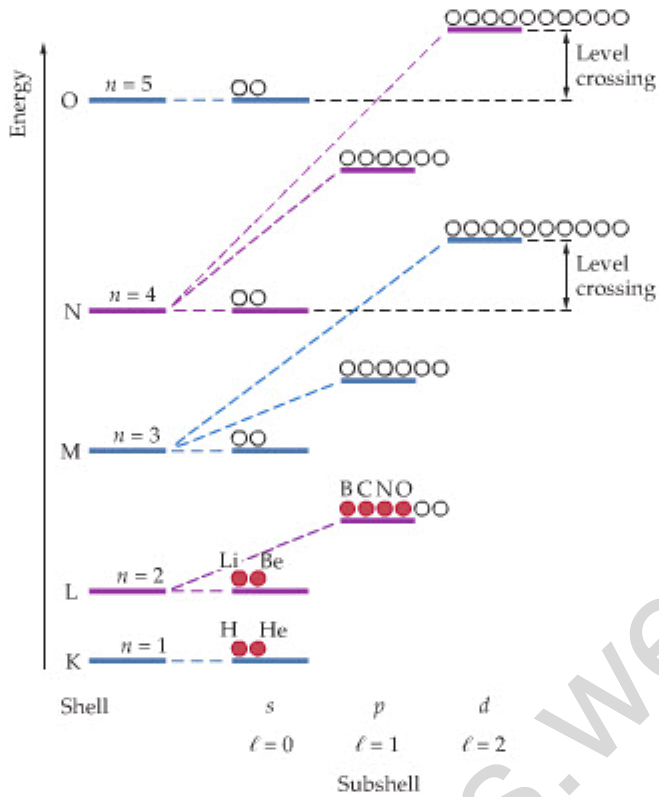


FIGURE 31–18 First eight elements of the periodic table
The elements hydrogen and helium fill the $n = 1$, $\ell = 0$ level; the elements lithium and beryllium fill the $n = 2$, $\ell = 0$ level; the elements boron, carbon, nitrogen, and oxygen fill four of the six available states in the $n = 2$, $\ell = 1$ level.

In this case, m_ℓ can take on three values: $m_\ell = 0, \pm 1$. For each of these three values, m_s can take on two values. Therefore, the $n = 2$, $\ell = 1$ energy level can accommodate 6 electrons, as shown in Figure 31–17. For general ℓ, the number of possible values of m_ℓ $(0, \pm 1, \pm 2, \ldots, \pm \ell)$ is $2\ell + 1$. When we multiply by 2 (for the number of values of m_s), we find a total number of states equal to $2(2\ell + 1)$. For example, note in Figure 31–17 that the $n = 3$, $\ell = 2$ energy level can hold $2(2 \cdot 2 + 1) = 10$ electrons.

Figure 31–18 presents the ground-state electron arrangements for the following elements: hydrogen (1 electron); helium (2 electrons); lithium (3 electrons); beryllium (4 electrons); boron (5 electrons); carbon (6 electrons); nitrogen (7 electrons); and oxygen (8 electrons). Notice that the energy levels are filled from the bottom upward, like marbles in a jar, with each level having a predetermined maximum number of electrons.

Electronic Configurations

Indicating the arrangements of electrons as in Figure 31–18 is instructive, but also somewhat cumbersome. This is especially so when we consider elements with a large number of electrons. To streamline the process, we introduce a shorthand notation that can be applied to all elements.

As an example of this notation, consider the element lithium, which has two electrons in the $n = 1$, $\ell = 0$ state and one electron in the $n = 2$, $\ell = 0$ state. This arrangement of electrons—referred to as an **electronic configuration**—will be abbreviated as follows:

$$1s^2 2s^1$$

In this expression, the $1s^2$ part indicates $n = 1$ ($1s^2$), $\ell = 0$ ($1s^2$), and an occupancy of two electrons ($1s^2$). Similarly, the $2s^1$ part indicates one electron ($2s^1$) in the $n = 2$ ($2s^1$) and $\ell = 0$ ($2s^1$) state. The general labeling is indicated in Figure 31–19.

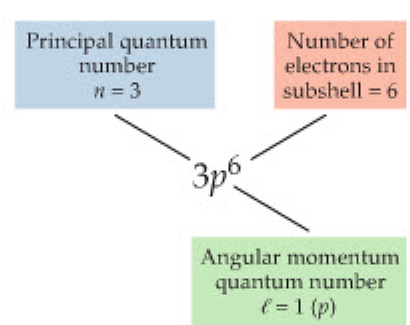


FIGURE 31–19 Designation of electronic configuration
The example shown here indicates six electrons in the $n = 3$, $\ell = 1$ (p) energy level.

EXERCISE 31–3

Use the shorthand notation just introduced to write the electronic configuration for the ground state of **(a)** nitrogen and **(b)** sodium.

SOLUTION

a. The 1s and 2s subshells of nitrogen ($Z = 7$) are filled; three electrons are in the 2p subshell:

$$1s^2\,2s^2\,2p^3$$

b. In sodium ($Z = 11$), one electron is in the 3s subshell; all the inner subshells are fully occupied:

$$1s^2\,2s^2\,2p^6\,3s^1$$

Table 31–4 presents a list of electronic configurations for the elements hydrogen (H) through potassium (K). Note that the 3*d* levels in potassium have no electrons, in agreement with the level crossing shown in Figure 31–18.

TABLE 31–4 Electronic Configurations of the Elements Hydrogen through Potassium

Atomic number	Element	Electronic configuration
1	Hydrogen (H)	$1s^1$
2	Helium (He)	$1s^2$
3	Lithium (Li)	$1s^22s^1$
4	Beryllium (Be)	$1s^22s^2$
5	Boron (B)	$1s^22s^22p^1$
6	Carbon (C)	$1s^22s^22p^2$
7	Nitrogen (N)	$1s^22s^22p^3$
8	Oxygen (O)	$1s^22s^22p^4$
9	Fluorine (F)	$1s^22s^22p^5$
10	Neon (Ne)	$1s^22s^22p^6$
11	Sodium (Na)	$1s^22s^22p^63s^1$
12	Magnesium (Mg)	$1s^22s^22p^63s^2$
13	Aluminum (Al)	$1s^22s^22p^63s^23p^1$
14	Silicon (Si)	$1s^22s^22p^63s^23p^2$
15	Phosphorus (P)	$1s^22s^22p^63s^23p^3$
16	Sulfur (S)	$1s^22s^22p^63s^23p^4$
17	Chlorine (Cl)	$1s^22s^22p^63s^23p^5$
18	Argon (Ar)	$1s^22s^22p^63s^23p^6$
19	Potassium (K)	$1s^22s^22p^63s^23p^64s^1$

The Periodic Table

Referring to the elements listed in Table 31–4, we observe a number of interesting patterns. For example, notice that the elements hydrogen, lithium, sodium, and potassium all have the same type of electronic configuration for the final (and outermost) electron in the atom. In particular, hydrogen's outermost (and only) electron is the $1s^1$ electron. In the case of lithium, we see that the outermost electron is $2s^1$, the outermost electron of sodium is $3s^1$, and for potassium it is $4s^1$. Continuing through the list of elements, we note that rubidium has a $5s^1$ outer electron, cesium has a $6s^1$ outer electron, and francium has a $7s^1$ outer electron. In each case the outermost electron is a single electron in an otherwise empty *s* subshell.

What makes this similarity in *electronic configuration* of particular interest is that these elements have similar *chemical properties* as well. In particular, the metallic members of this group of elements (lithium, sodium, potassium, rubidium, cesium, and francium) are referred to as the *alkali metals*. In each of these metals, the outermost electron is easily removed from the atom, leading to a stable, positively charged ion, as in the familiar case of the sodium ion, Na^+. Thus the regular pattern in the filling of shells leads to a regular pattern in the properties of the elements.

Grouping various elements with similar chemical properties was the motivation behind the development of the **periodic table** of elements by the Russian chemist Dmitri Mendeleev (1834–1907). The periodic table is presented in Appendix E. Notice that the elements just mentioned form Group I in the leftmost column of the table. Although Mendeleev grouped these elements strictly on the basis of their chemical properties, we can now see that the grouping also corresponds to the filling of shells, in accordance with the Pauli exclusion principle.

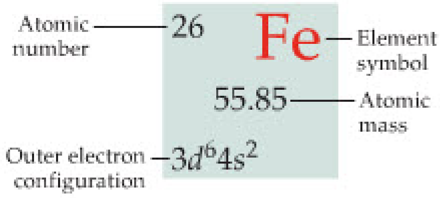
Many groups of elements appear in the periodic table. As another example, Group VII consists of the *halogens:* fluorine, chlorine, bromine, and iodine. Note that these elements also have similar configurations of their outer electrons. In this case, the outer electrons are $2p^5$, $3p^5$, and so on. Thus we see that these elements are just one electron short of filling one of the p subshells. As a result, halogens are highly reactive—they can readily acquire a single electron from another element to form a stable negative ion, as in the case of chlorine, which forms the chloride ion, Cl^-.

Group VIII consists of the *noble gases*. These elements all have completely filled subshells. Thus they do not readily gain or lose an electron. It is for this reason that the noble gases are relatively inert.

Finally, the *transition elements* represent those cases where a crossing of energy levels occurs. For example, the $4s$ subshell is filled at the element calcium ($Z = 20$). The next electron, rather than going into the $4p$ subshell, goes into the $3d$ subshell. In fact, the 10 elements from scandium ($Z = 21$) to zinc ($Z = 30$) correspond to filling of the $3d$ subshell. After this subshell is filled, additional electrons go into the $4p$ subshell in the elements gallium ($Z = 31$) to krypton ($Z = 36$).

Figure 31–20 shows the meaning of the various symbols used in the periodic table. Note that each box in the table gives the symbol for the element, its atomic mass, and its atomic number. Each box also includes the configuration of the outermost electrons in the element. In the case of iron, shown in Figure 31–20, this configuration is $3d^64s^2$.

▲ Neon (atomic number 10), one of the "noble gases," is most widely known for its role in neon tubes. Electrons in neon atoms are excited by electrical discharge through the tube. When they return to the ground state, they emit electromagnetic radiation, much of it in the red part of the visible spectrum. The other colors familiarly found in "neon signs" or "neon lights" are produced by adding other elements of the same chemical family: argon, krypton, or xenon.



▲ **FIGURE 31–20 Designation of elements in the periodic table**

An explanation of the various entries to be found for each element in the periodic table of Appendix E. The example shown here is for the element iron (Fe).

CONCEPTUAL CHECKPOINT 31–3 COMPARE THE ENERGY

The energy required to remove the outermost electron from sodium is 5.1 eV. Is the energy required to remove the outermost electron from potassium **(a)** greater than, **(b)** less than, or **(c)** equal to 5.1 eV?

REASONING AND DISCUSSION

Referring to the periodic table, we see that potassium has one electron in its outermost shell, just like sodium. In this respect the two elements are alike. The difference between the elements is that the outermost electron in potassium is in a higher energy state; that is, its electron is in an $n = 4$ state, as opposed to the $n = 3$ state in sodium. Less energy is required to remove an electron from a state of higher energy; hence, the outermost electron of potassium can be removed with less than 5.1 eV. In fact, the required energy is only 4.3 eV.

ANSWER

(b) Less energy is required to remove the outermost electron from potassium.

31–7 Atomic Radiation

We conclude this chapter with a brief investigation of various types of radiation associated with multielectron atoms. Examples range from X-rays that are energetic enough to pass through a human body, to the soft white light of a fluorescent lightbulb.

X-rays

X-rays were discovered quite by accident by the German physicist Wilhelm Roentgen (1845–1923) on November 8, 1895. Within months of their discovery they were being used in medical applications, and they have played an important role in medicine ever since. Today, the X-rays used to give diagnostic images in hospitals and dentist offices are produced by an X-ray tube similar to the one shown in Figure 31–21. The basic operating principle of this device is that an energetic beam of electrons is generated and directed at a metal target—when the electrons collide with the target, X-rays are emitted. Figure 31–22 shows a typical plot of X-ray intensity per wavelength versus the wavelength for such a device.

REAL-WORLD PHYSICS: BIO
Medical X-ray tubes

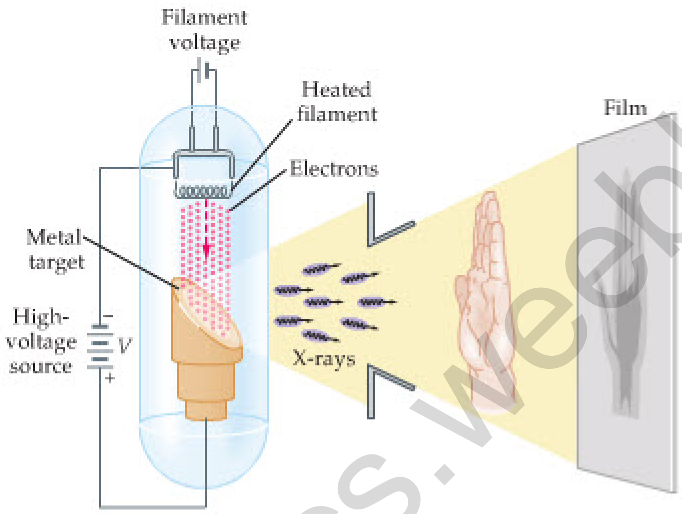


FIGURE 31–21 X-ray tube
An X-ray tube accelerates electrons in a vacuum through a potential difference and then directs the electrons onto a metal target. X-rays are produced when the electrons decelerate in the target, and as a result of the excitations they cause when they collide with the metal atoms of the target.

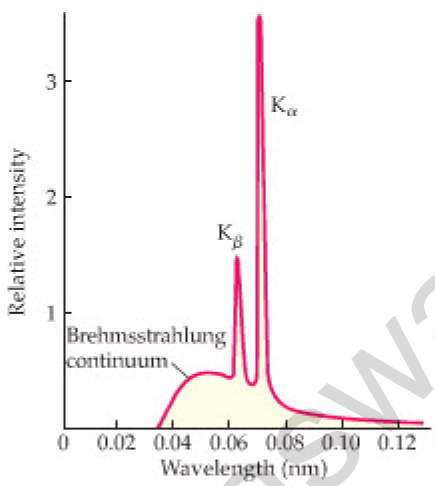


FIGURE 31–22 X-ray spectrum
The spectrum produced by an X-ray tube in which electrons are accelerated from rest through a potential difference of 35,000 V and directed against a molybdenum target.

The radiation produced by an X-ray tube is created by two completely different physical mechanisms. The first mechanism is referred to as **bremsstrahlung**, which is German for "braking radiation." What is meant by this expression is that as the energetic electrons impact the target, they undergo a rapid deceleration. This is the "braking." As we know from Chapter 25, an accelerated charge gives off electromagnetic radiation; hence, as the electrons suddenly come to rest in the target they give off high-energy radiation in the form of X-rays. These X-rays cover a wide range of wavelengths, giving rise to the continuous part of the spectrum shown in Figure 31–22.

The sharp peaks in Figure 31–22 are produced by the second physical mechanism. To understand their origin, imagine what happens if one of the electrons in the incident beam is energetic enough to knock an electron out of a target atom. In addition, suppose the ejected electron comes from the lowest energy level of the atom, that is, from the K shell. This "vacancy" is filled almost immediately when an electron from an outer shell drops to the K shell, with the energy difference given off as a photon. In an atom of large atomic number [molybdenum ($Z = 42$) is often used for a target, as is tungsten ($Z = 74$)], the photon that emerges is an X-ray. If an electron drops from the $n = 2$ level to the K shell, the sharp peak of radiation that results is called the K_α line. Similarly, if an electron drops from the $n = 3$ level to the K shell, the resulting peak is called the K_β line. These lines are shown in Figure 31–22, and the corresponding electron jumps are indicated in Figure 31–23. Because the wavelengths of these lines vary from element to element—that is, they are characteristic of a certain element—they are referred to as **characteristic X-rays.** Similar characteristic X-rays can be emitted when electrons drop in energy to fill a vacancy in the L shell, the M shell, and so on.

The energy an incoming electron must have to dislodge a K-shell electron from an atom can be estimated using results from the Bohr model. The basic idea is that the energy of a K-shell electron in an atom of atomic number Z is given approximately by Equation 31–9 [$E_n = -(13.6\text{ eV})Z^2/n^2$], with one minor modification: since there are two electrons in the K shell, each electron shields the other from the nucleus. That is, the negative charge on one electron, $-e$, partially cancels the positive charge of the nucleus, $+Ze$, giving an effective charge experienced by the second electron of $+(Z - 1)e$. Thus, replacing Z with $Z - 1$ in Equation 31–9, and