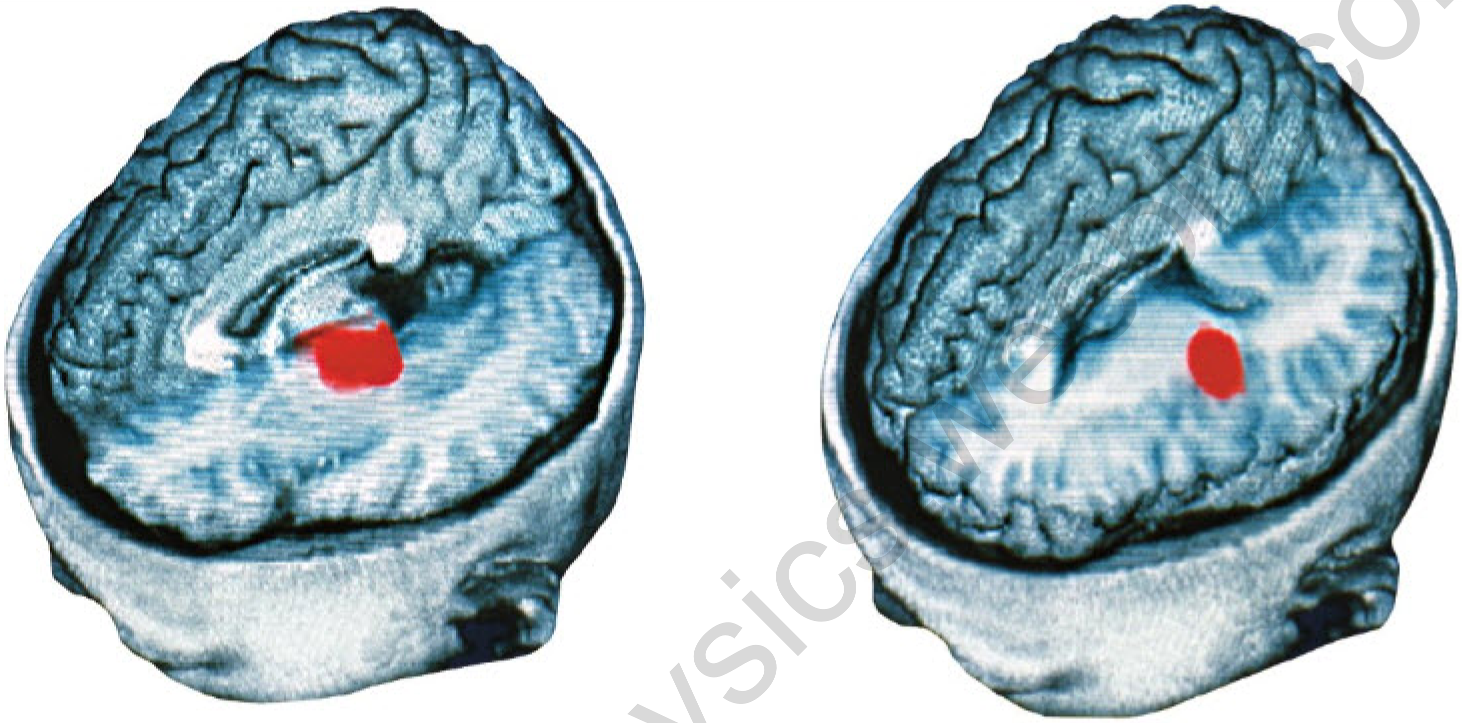
29 Relativity



If someone reads this caption to you, a specific region of your brain is activated as you process the information. On the other hand, you may try to recall the words that were read to you after a period of time has elapsed, in which case a different region of your brain is activated as you retrieve the words from your memory. How do we know? The images shown here are the result of positron emission tomography (PET) scans of the brain, in which a radioactive tracer is injected into the bloodstream. In the image on the left, a person is listening to words being read, and the hippocampus region is activated, causing a higher concentration of radioactive blood flow in that area; in the image on the right, the person tries to recall the words, and the temporoparietal region is activated. What is really amazing about these images, however, is that they show areas where matter (electrons) and antimatter (positrons emitted by the radioactive tracer) are annihilating inside the brain and sending out bursts of energy. The energy released in these annihilations is given by the familiar equation ($E = mc^2$), developed by a 26-year-old patent clerk who would soon become the most famous scientist of our time. This chapter explores some of the major contributions of Albert Einstein, including his special and general theories of relativity, and shows how they have radically altered our views of space, time, matter, and energy.

What we refer to today as "modern physics" can be thought of as having started around the beginning of the twentieth century, when two fundamentally new ways of looking at nature were introduced. One of these developments was the introduction of the quantum hypothesis by Max Planck. This will be considered in detail in the next chapter. The other revolutionary development was Albert Einstein's theory of relativity.

The one thing these new theories had in common was that they showed Newton's laws to be incomplete. As it turns out, Newton's laws apply only to objects of macroscopic size and relatively low speeds. In this chapter we explore the surprising behavior associated with speeds approaching the speed of light. We find, in fact, that clocks run slow, metersticks become shorter, and objects become more massive as their speed increases. Although this may seem like science fiction rather than science fact, experiment shows that the predictions of Einstein's theory are indeed correct.

29–1 The Postulates of Special Relativity

At a time when some scientists thought physics was almost completely understood, with only minor details to be straightened out, physics was changed forever with the introduction of the **special theory of relativity.** Published in 1905 by Albert Einstein (1879–1955), a 26-year-old patent clerk (third class) in Berne, Switzerland, relativity fundamentally altered our understanding of such basic physical concepts as time, length, mass, and energy. It may come as a surprise, then, that Einstein's theory of relativity is based on just two simply stated postulates, and that algebra is all the mathematics required to work out its main results.

The postulates of special relativity put forward by Einstein can be stated as follows:

Equivalence of Physical Laws
The laws of physics are the same in all inertial frames of reference.

Constancy of the Speed of Light
The speed of light in a vacuum, $c = 3.00 \times 10^8$ m/s, is the same in all inertial frames of reference, independent of the motion of the source or the receiver.

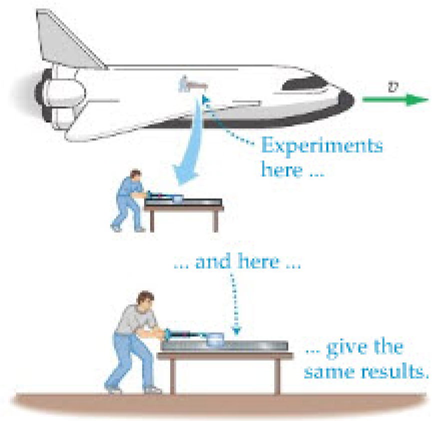
All the consequences of relativity explored in this chapter follow as a direct result of these postulates.

The first postulate is certainly reasonable. Recall from Section 5–2 that an inertial frame of reference is one in which Newton's laws of motion are obeyed. Specifically, an object with no force acting on it has zero acceleration in all inertial frames. Einstein's first postulate simply extends this notion of an inertial frame to cover *all* the known laws of physics, including those dealing with thermodynamics, electricity, magnetism, and electromagnetic waves. For example, a mechanics experiment performed on the surface of the Earth (which is approximately an inertial frame) gives the same results as when the same experiment is carried out in an airplane moving with constant velocity. In addition, the behavior of heat, magnets, and electric circuits is the same in the airplane as on the ground, as indicated in **Figure 29–1.**

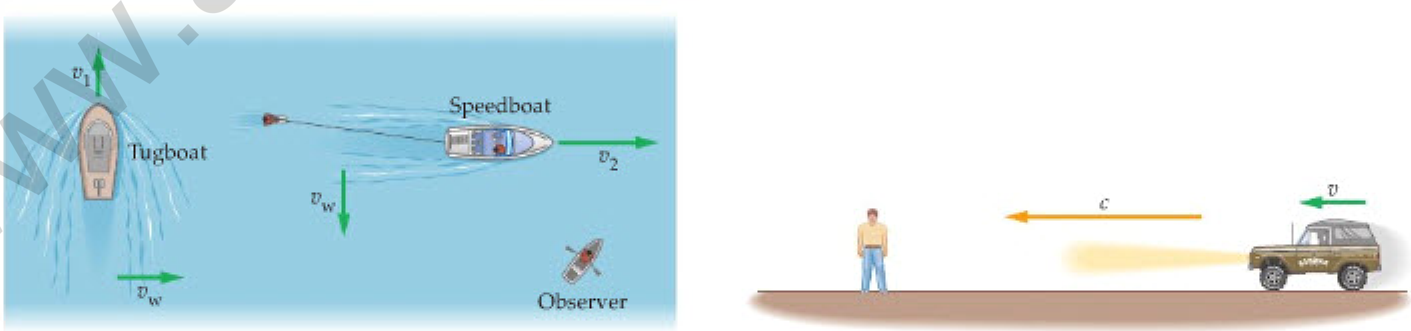
All inertial frames of reference move with constant velocity (that is, zero acceleration) relative to one another. Hence, the special theory of relativity is "special" in the sense that it restricts our considerations to frames with no acceleration. The more general case, in which accelerated motion is considered, is the subject of the *general* theory of relativity, which we discuss later in this chapter. In the case of Earth, the accelerations associated with its orbital and rotational motions are small enough to be ignored in most experiments. Thus, unless otherwise stated, we shall consider the Earth and objects moving with constant velocity relative to it to be inertial frames of reference.

The second postulate of relativity is less intuitive than the first. Specifically, it states that light travels with the same speed, c, regardless of whether the source or observer is in motion. To understand the implications of this assertion, consider for a moment the case of waves on water. In **Figure 29–2 (a)** we see an observer at

▲ Albert Einstein in his twenties.



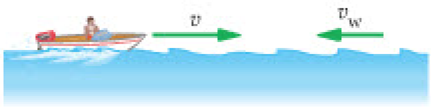
▲ **FIGURE 29–1 Inertial frames of reference**
The two observers shown are in different inertial frames of reference. According to the first postulate of relativity, physical experiments will give identical laws of nature in the two frames.



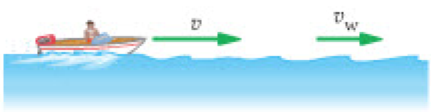
(a) Speed of water waves independent of speed of source

(b) Speed of light waves independent of speed of source

▲ **FIGURE 29–2 Wave speed versus source speed**
The speed of a wave is independent of the speed of the source that generates it. **(a)** Water waves produced by a slow-moving tugboat have the same speed as those produced by a high-powered speedboat. **(b)** The speed of a beam of light, c, is independent of the speed of its source.



(a) High relative speed



(b) Low relative speed

▲ **FIGURE 29–3 Wave speed versus observer speed**

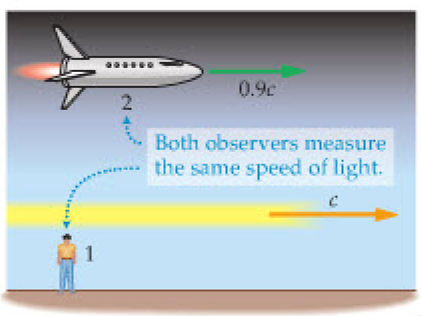
The speed of a wave depends on the speed of the observer relative to the medium through which the wave propagates. **(a)** The water waves move relative to the observer with speed $v + v_w$. **(b)** In this case, the waves move relative to the observer with speed $v - v_w$.

rest relative to the water, and two moving sources generating waves. The waves produced by both the speedboat and the tugboat travel at the characteristic speed of water waves, v_w, once they are generated. Thus the observer sees a wave speed that is independent of the speed of the source—just as postulated for light, and shown in **Figure 29–2 (b)**.

On the other hand, suppose the observer is in motion with a speed v with respect to the water. If the observer is moving to the right, and water waves are moving to the left with a speed v_w, as in **Figure 29–3 (a)**, the waves move past the observer with a speed $v + v_w$. Similarly, if the water waves are moving to the right, as in **Figure 29–3 (b)**, the observer finds them to have a speed $v - v_w$. Clearly, the fact that the observer is in motion with respect to the medium through which the waves are traveling (water in this case) means that the speed of the water waves depends on the speed of the observer.

Before Einstein's theory of relativity, it was generally accepted that a similar situation would apply to light waves. In particular, light was thought to propagate through a hypothetical medium, referred to as the *luminiferous ether*, or the ether for short, that permeates all space. Since the Earth rotates about its axis with a speed of roughly 1000 mi/h at the equator, and orbits the Sun with a speed of about 67,000 mi/h, it follows that it must move relative to the ether. If this is the case, it should be possible to detect this motion by measuring differences in the speed of light propagating in different directions—just as in the case of water waves. Extremely precise experiments were carried out to this end by the American physicists A. A. Michelson (1852–1931) and E. W. Morley (1838–1923) from 1883 to 1887. They were unable to detect *any* difference in the speed of light. More recent and accurate experiments have come to precisely the same conclusion; namely, the second postulate of relativity is an accurate description of the way light behaves.

To see how counterintuitive the second postulate can be, consider the situation illustrated in **Figure 29–4**. In this case a ray of light is propagating to the right with a speed c relative to observer 1. A second observer is moving to the right as well, with a speed of $0.9c$. Although it seems natural to think that observer 2 should see the ray of light passing with a speed of only $0.1c$, this is not the case. Observer 2, like observers in all inertial frames of reference, sees the ray go by with the speed of light, c.



▲ **FIGURE 29–4 The speed of light for different observers**

A beam of light is moving to the right with a speed c relative to observer 1. Observer 2 is moving to the right with a speed of $0.9c$. Still, from the point of view of observer 2, the beam of light is moving to the right with a speed of c, in agreement with the second postulate of relativity.

For the observations given in Figure 29–4 to be valid—that is, for both observers to measure the same speed of light—the behavior of space and time must differ from our everyday experience when speeds approach the speed of light. This is indeed the case, as we shall see in considerable detail in the next few sections. In everyday circumstances, however, the physics described by Newton's laws are perfectly adequate. In fact, Newton's laws are valid in the limit of very small speeds, whereas Einstein's theory of relativity gives correct results for all speeds from zero up to the speed of light.

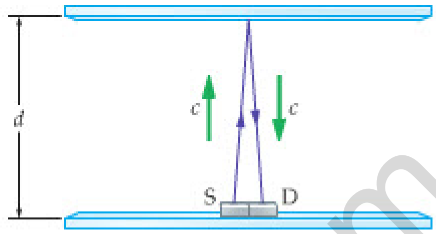
Since all inertial observers measure the same speed for light, they are all equally correct in claiming that they are at rest. For example, observer 1 in Figure 29–4 may say that he is at rest and that observer 2 is moving to the right with a speed of $0.9c$. Observer 2, however, is equally justified in saying that she is at rest and that observer 1 is in motion with a speed of $0.9c$ to the left. From the point of view of relativity, both observers are equally correct. There is no absolute rest or absolute motion, only motion relative to something else.

Finally, note that it would not make sense for observer 2 in Figure 29–4 to have a speed greater than that of light. If this were the case, it would not be possible for the light ray to pass the observer, much less to pass with the speed c. Thus we conclude that *the ultimate speed in the universe is the speed of light in a vacuum*. In the next several sections of this chapter, we shall see several more ways of arriving at precisely the same conclusion.

29–2 The Relativity of Time and Time Dilation

We generally think of time as moving forward at a constant rate, as suggested by our everyday experience. This is simply not the case, however, when dealing with speeds approaching the speed of light. If you were to observe a spaceship moving past you with a speed of $0.5c$, for example, you would notice that the clocks on the ship run slow compared with your clocks—even if they were identical in all other respects.

To calculate the difference between the rates of a moving clock and one at rest, consider the "light clock" shown in Figure 29–5. In this clock, a cycle begins when a burst of light is emitted from the light source S. The light then travels a distance d to a mirror, where it is reflected. It travels back a distance d to the detector D, and triggers the next burst of light. Each round trip of light can be thought of as one "tick" of the clock.



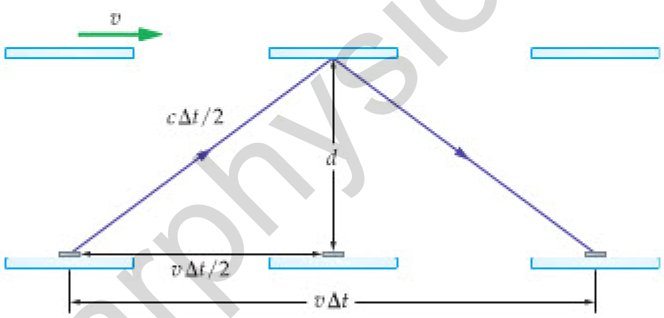
▲ **FIGURE 29–5 A stationary light clock**
Light emitted by the source S travels to a mirror a distance d away and is reflected back into the detector D. The time between emission and detection is one cycle, or one "tick," of the clock.

We begin by calculating the time interval between the ticks of this clock when it is at rest; that is, when its speed relative to the observer making the measurement is zero. Since the light covers a total distance $2d$ with a constant speed c, the time between ticks is simply

$$\Delta t_0 = \frac{2d}{c} \qquad 29\text{–}1$$

The subscript 0 indicates the clock is at rest ($v = 0$) when the measurement is made.

In contrast, consider the same light clock moving with a finite speed v, as in Figure 29–6. Notice that the light must now follow a zigzag path in order to complete a tick of the clock. Since this path is clearly longer than $2d$, and the speed of the light is still the same—according to the second postulate of relativity—the time between ticks must be greater than Δt_0. With more time elapsing between ticks, the clock runs slow. We refer to this phenomenon as **time dilation**, because the time interval for one tick has been increased—or dilated—from Δt_0 for a clock at rest relative to an observer to $\Delta t > \Delta t_0$ for a clock in motion relative to an observer.



◀ **FIGURE 29–6 A moving light clock**
A moving light clock requires a time Δt to complete one cycle. Note that the light follows a zigzag path that is longer than $2d$; hence, the time between ticks is greater for the moving clock than it is for the clock at rest.

To calculate the dilated time, Δt, notice that in the time $\Delta t/2$ the clock moves a horizontal distance $v\Delta t/2$, which is halfway to its position at the end of the tick. The distance traveled by the light in this time is $c\Delta t/2$, which is the hypotenuse of the right triangle shown in Figure 29–6. Applying the Pythagorean theorem to this triangle, we find the following relation:

$$\left(\frac{v\Delta t}{2}\right)^2 + d^2 = \left(\frac{c\Delta t}{2}\right)^2$$

Solving for the time Δt, we find

$$\Delta t = \frac{2d}{\sqrt{c^2 - v^2}} = \frac{2d}{c\sqrt{1 - v^2/c^2}}$$

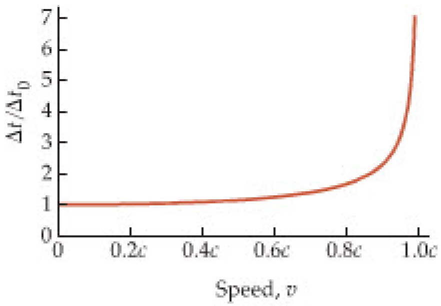
Recalling that $\Delta t_0 = 2d/c$, we can relate the two time intervals as follows:

Time Dilation

$$\Delta t = \frac{\Delta t_0}{\sqrt{1 - v^2/c^2}} \qquad 29\text{–}2$$

SI unit: s

Notice that $\Delta t = \Delta t_0$ for $v = 0$, as expected. For speeds v that are greater than zero but less than c, the denominator in Equation 29–2 is less than 1. As a result, it



▲ **FIGURE 29–7 Time dilation as a function of speed**

As the speed of a clock increases, the time required for it to advance by 1 s increases slowly at first and then rapidly near the speed of light.

follows that Δt is greater than Δt_0. Finally, in the limit that the speed v approaches the speed of light, c, we observe that the denominator of Equation 29–2 vanishes, and the time interval Δt goes to infinity. This behavior is illustrated in **Figure 29–7**, where we show the ratio $\Delta t/\Delta t_0$ as a function of the speed v. The fact that Δt goes to infinity means that it takes an infinite amount of time for one tick—in other words, as v approaches the speed of light a clock slows to the point of stopping. Clearly, then, the speed of light provides a natural upper limit to the possible speed of an object.

EXERCISE 29–1

A spaceship carrying a light clock moves with a speed of $0.500c$ relative to an observer on Earth. According to this observer, how long does it take for the spaceship's clock to advance 1.00 s?

SOLUTION

Substituting $\Delta t_0 = 1.00\text{ s}$ and $v = 0.500c$ in Equation 29–2, we obtain

$$\Delta t = \frac{\Delta t_0}{\sqrt{1 - v^2/c^2}} = \frac{1.00\text{ s}}{\sqrt{1 - (0.500c)^2/c^2}} = \frac{1.00\text{ s}}{\sqrt{1 - 0.25}} = 1.15\text{ s}$$

Even at this high speed, the relativistic effect is relatively small—only about 15%.

Equation 29–2 applies to any type of clock, not just the light clock. If this were not the case—if different clocks ran at different rates when in motion with constant velocity—the first postulate of relativity would be violated.

CONCEPTUAL CHECKPOINT 29–1 THE RATE OF TIME

A clock moving with a finite speed v is observed to run slow. If the speed of light were twice as large as it actually is, would the factor by which the clock runs slow be **(a)** increased, **(b)** decreased, or **(c)** unchanged?

REASONING AND DISCUSSION

As Figure 29–7 shows, the factor by which time is dilated increases as the speed of a clock approaches the speed of light. If the speed of light were twice as large, the speed of a moving clock would be a smaller fraction of the speed of light; hence, the time dilation factor would be less.

Extending the preceding argument, it follows that in the limit of an infinite speed of light there would be no time dilation at all. This conclusion is verified by noting that in the limit $c \to \infty$ the denominator in Equation 29–2 is equal to 1, so that Δt and Δt_0 are equal. Since the speed of light is practically infinite in everyday terms, the relativistic time dilation effect is negligible for everyday objects.

ANSWER

(b) The factor by which the clock runs slow would be decreased.

Before we proceed, it is useful to introduce some of the terms commonly employed in relativity. First, an **event** is a physical occurrence that happens at a *specified location* at a *specified time*. In three dimensions, for example, we specify an event by giving the values of the coordinates x, y, and z as well as the time t. If two events occur at the same location but at different times, the time between the events is referred to as the **proper time:**

> The *proper time* is the amount of time separating two events that occur at the *same location*.

As an example, the proper time between ticks in a light clock is the time between the emission of light (event 1) and its detection (event 2) when the clock is at rest relative to the observer. Thus, Δt_0 in Equation 29–2 is the proper time, and Δt is the corresponding time when the clock moves relative to the observer with a speed v.

PROBLEM-SOLVING NOTE

Identifying the Proper Time

The key to solving problems involving time dilation is to correctly identify the proper time, Δt_0. Simply put, the proper time is the time between events that occur at the same location.

In Exercise 29–1 we calculated the relativistic effect of time dilation for the case of a clock moving with a speed equal to half the speed of light. In everyday circumstances, however, speeds are never so large. In fact, the greatest speed a human might reasonably attain today is the speed of the space shuttle in orbit. As we saw in Chapter 12, this speed is only about 7700 m/s, or 17,000 mi/h. Although this is a rather large speed, it is still only 1/39,000th the speed of light.

To find the time dilation in a case like this we cannot simply substitute $v = 7700$ m/s into Equation 29–2, since a typical calculator does not have enough decimal places to give the correct answer. You might want to try it yourself; your calculator will probably give the incorrect result that Δt is equal to Δt_0. To find the correct answer, we must use the binomial expansion (Appendix A) to reexpress Equation 29–2 as follows:

$$\Delta t = \frac{\Delta t_0}{\sqrt{1 - v^2/c^2}} \approx \Delta t_0\left(1 + \frac{1}{2}\frac{v^2}{c^2} + \cdots\right)$$

Substituting the speed of the space shuttle and the speed of light, we find

$$\Delta t \approx \Delta t_0\left[1 + \frac{1}{2}\left(\frac{7700\text{ m/s}}{3.00 \times 10^8\text{ m/s}}\right)^2\right] = \Delta t_0(1 + 3.3 \times 10^{-10})$$

Thus, a clock on board the space shuttle runs slow by a factor of 1.00000000033. At this rate, it would take almost 100 years for the clock on the shuttle to lose 1 s compared with a clock on Earth.

Clearly, such small differences in time cannot be measured with an ordinary clock. In recent years, however, atomic clocks have been constructed that have an accuracy sufficient to put relativity to a direct test. The physicists J. C. Hafele and R. E. Keating conducted such a test in 1971 by placing one atomic clock on board a jet airplane and leaving an identical clock at rest in the laboratory. After flying the moving clock at high speed for many hours, the experimenters found it to have run slower than the clock left in the lab. The discrepancy in times agreed with the predictions of relativity. Today, if an atomic clock is transported from one location to another, the relativistic effects of time dilation must be taken into account; otherwise the clock will give a time that is behind the correct value.

Another aspect of time dilation is the fact that different observers disagree on *simultaneity*. For example, suppose observer 1 notes that two events at different locations occur at the same time. For observer 2, who is moving with a speed v relative to observer 1, these same two events are not simultaneous. Thus, relativity not only changes the rate at which time progresses for different observers, it also changes the amount of time that separates events.

Space Travel and Biological Aging

To this point, our discussion of time dilation has been applied solely to clocks. Clocks are not the only objects that show time dilation, however. In fact,

> relativistic time dilation applies equally to *all physical processes*, including chemical reactions and biological functions.

Thus an astronaut in a moving spaceship *ages more slowly* than one who remains on Earth and by precisely the same factor that a clock on the spaceship runs more slowly than one at rest. To the astronaut, however, time seems to progress as usual. The implications of time dilation for a high-speed trip to a nearby star are considered in the next Example.

EXAMPLE 29–1 BENNY AND JENNY—SEPARATED AT LAUNCH

Astronaut Benny travels to Vega, the fifth brightest star in the night sky, leaving his 35.0-year-old twin sister Jenny behind on Earth. Benny travels with a speed of 0.990c, and Vega is 25.3 light-years from Earth. **(a)** How long does the trip take from the point of view of Jenny? **(b)** How much has Benny aged when he arrives at Vega?

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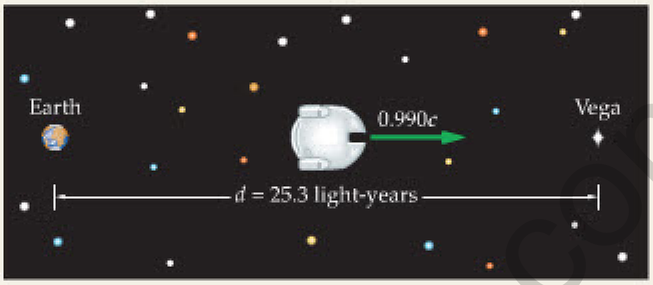
PICTURE THE PROBLEM

Our sketch shows the spacecraft as it travels in a straight line from Earth to Vega. The speed of the spacecraft relative to Earth is 99% of the speed of light, $v = 0.990c$. In addition, the distance to Vega is 25.3 light-years; that is, $d = 25.3$ ly. [*Note:* One light-year is the distance light travels with a speed c in one year. Specifically, 1 light-year = 1 ly = $c(1\text{ y})$.]



STRATEGY

The two events of interest in this problem are (1) leaving Earth and (2) arriving at Vega. For Jenny, these two events clearly occur at different locations. It follows that the time interval for her is Δt, and not the proper time, Δt_0.

For Benny, however, the two events occur at the same location—namely, just outside the spacecraft door. (In fact, from Benny's point of view the spacecraft is at rest, and the stars are in motion.) Therefore, the time interval measured by Benny is the proper time, Δt_0.

Finally, we note that Δt and Δt_0 are related by Equation 29–2; that is, $\Delta t = \Delta t_0/\sqrt{1 - v^2/c^2}$.

SOLUTION

Part (a)

1. The spacecraft covers a distance d in a time Δt with a speed $v = 0.990c$. Use $v = d/\Delta t$ to solve for the time, Δt:

$$v = \frac{d}{\Delta t}$$

$$\Delta t = \frac{d}{v} = \frac{25.3\text{ ly}}{0.990c} = \frac{25.3c(1\text{y})}{0.990c} = 25.6\text{ y}$$

Part (b)

2. Rearrange $\Delta t = \Delta t_0/\sqrt{1 - v^2/c^2}$ (Equation 29–2) to solve for the proper time, Δt_0:

$$\Delta t = \frac{\Delta t_0}{\sqrt{1 - v^2/c^2}}$$

$$\Delta t_0 = \Delta t\sqrt{1 - v^2/c^2}$$

3. Substitute $v = 0.990c$ and $\Delta t = 25.6$ y to find Δt_0:

$$\Delta t_0 = \Delta t\sqrt{1 - v^2/c^2}$$

$$= (25.6\text{ y})\sqrt{1 - \frac{(0.990c)^2}{c^2}} = 3.61\text{ y}$$

INSIGHT

Thus when Benny reaches Vega, he is only 38.6 y old, whereas Jenny, who stayed behind on Earth, is 60.6 y old.

From the point of view of Benny, the trip took 3.61 y at a speed of $0.990c$. As a result, he would say that the distance covered in traveling to Vega was only $(3.61\text{ y})(0.990c) = 3.57$ ly. We consider this result in greater detail in the next section.

PRACTICE PROBLEM

How fast must Benny travel if he wants to age only 2.00 y during his trip to Vega? [**Answer:** $v = 0.997c$]

Some related homework problems: Problem 8, Problem 14

In the following Active Example, we determine the speed of an astronaut by considering the change in her heart rate.

ACTIVE EXAMPLE 29–1 HEARTTHROB: FIND THE ASTRONAUT'S SPEED

An astronaut traveling with a speed v relative to Earth takes her pulse and finds that her heart beats once every 0.850 s. Mission control on Earth, which monitors her heart activity, observes one heartbeat every 1.40 s. What is the astronaut's speed relative to Earth?

SOLUTION *(Test your understanding by performing the calculations indicated in each step.)*

1. Identify the proper time, Δt_0, and the dilated time, Δt:

$\Delta t_0 = 0.850$ s
$\Delta t = 1.40$ s

2. Solve the time-dilation expression, $\Delta t = \Delta t_0/\sqrt{1 - v^2/c^2}$, for the speed, v: $v = c\sqrt{1 - \Delta t_0^2/\Delta t^2}$

3. Substitute numerical values: $v = 0.795c$

YOUR TURN

What speed is required for mission control to measure a time between heartbeats that is 2(1.40 s) = 2.80 s?

(Answers to **Your Turn** *problems are given in the back of the book.)*

The Decay of the Muon

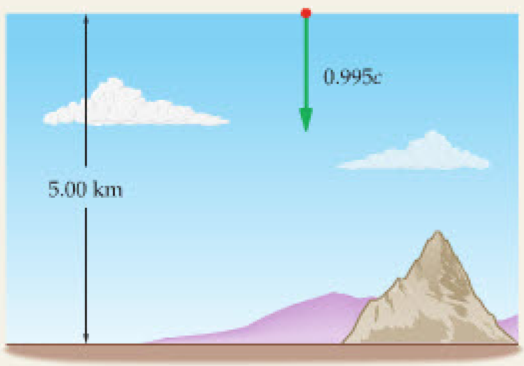
A particularly interesting example of time dilation involves subatomic particles called *muons* that are created by cosmic radiation high in Earth's atmosphere. A muon is an unstable particle; in fact, a muon at rest exists for only about 2.20×10^{-6} s, on average, before it decays. Suppose, for example, that a muon is created at an altitude of 5.00 km above the surface of the Earth. If this muon travels toward the ground with a speed of $0.995c$ for 2.20×10^{-6} s, it will cover a distance of only 657 m before decaying. Thus, without time dilation, one would conclude that the muons produced at high altitude should not reach the surface of the Earth. It is found, however, that large numbers of muons do in fact reach the ground. The reason is that they age more slowly due to their motion—just like the astronaut traveling to Vega considered in Example 29–1. The next Example examines this time dilation effect.

EXAMPLE 29–2 THE LIFE AND TIMES OF A MUON

Consider muons traveling toward Earth with a speed of $0.995c$ from their point of creation at a height of 5.00 km. **(a)** Find the average lifetime of these muons, assuming that a muon at rest has an average lifetime of 2.20×10^{-6} s. **(b)** Calculate the average distance these muons can cover before decaying.

PICTURE THE PROBLEM

Our sketch shows a muon that has just been created at an altitude of 5.00 km. The muon is heading straight for the ground with a speed $v = 0.995c$. As a result of its high speed, the muon's "internal clock" runs slow, allowing it to live longer as seen by an observer on Earth.



STRATEGY

The two events to be considered in this case are (1) the creation of a muon and (2) the decay of a muon. From the point of view of an observer on Earth, these two events occur at different locations. It follows that the corresponding time is Δt.

From the muon's point of view, it is at rest and the Earth is moving toward it at $0.995c$. Hence, for the muon, the two events occur at the same location, and the time between them is $\Delta t_0 = 2.20 \times 10^{-6}$ s. We can find the time Δt by using time dilation, $\Delta t = \Delta t_0/\sqrt{1 - v^2/c^2}$.

SOLUTION

Part (a)

1. Substitute $v = 0.995c$ and $\Delta t_0 = 2.20 \times 10^{-6}$ s into Equation 29–2:

$$\Delta t = \frac{\Delta t_0}{\sqrt{1 - v^2/c^2}} = \frac{2.20 \times 10^{-6}\ \text{s}}{\sqrt{1 - (0.995c)^2/c^2}} = 22.0 \times 10^{-6}\ \text{s}$$

Part (b)

2. Multiply $v = 0.995c$ times the time 22.0×10^{-6} s to find the average distance covered:

$$d_{av} = (0.995c)(22.0 \times 10^{-6}\ \text{s}) = 6570\ \text{m}$$

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INSIGHT

Thus relativistic time dilation allows the muons to travel about 10 times farther (6570 m instead of 657 m) than would have been expected from nonrelativistic physics. As a result, muons are detected on Earth's surface.

PRACTICE PROBLEM

Muons produced at the laboratory of the European Council for Nuclear Research (CERN) in Geneva are accelerated to high speeds. At these speeds, the muons are observed to have a lifetime 30.00 times greater than the lifetime of muons at rest. What is the speed of the muons at CERN? [**Answer:** $v = 0.9994c$]

Some related homework problems: Problem 11, Problem 13, Problem 17

29–3 The Relativity of Length and Length Contraction

Just as time is altered for an observer moving with a speed close to the speed of light, so too is distance. For example, a meterstick moving with a speed of $0.5c$ would appear noticeably shorter than a meterstick at rest. As the speed of the meterstick approaches c, its length diminishes toward zero.

To see why lengths contract, and to calculate the amount of contraction, recall the example of the twins Benny and Jenny and the trip to Vega. From Jenny's point of view on Earth, Benny's trip took 25.6 y and covered a distance of $(0.990c)(25.6\text{ y}) = 25.3$ ly. From Benny's point of view, however, the trip took only 3.61 y. Since both twins agree on their relative velocity, it follows that as far as Benny is concerned, his trip covered a distance of only $(0.990c)(3.61\text{ y}) = 3.57$ ly. Thus, from the point of view of the astronaut, Earth and Vega move by at a speed of $0.990c$, and the distance between them is not 25.3 ly, but only 3.57 ly. This is an example of length contraction, as illustrated in Figure 29–8.

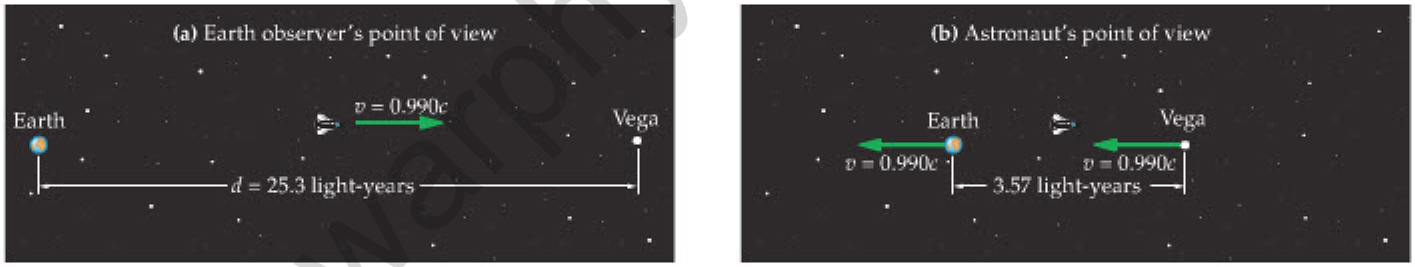


FIGURE 29–8 A relativistic trip to Vega
(a) From the Earth observer's point of view the spaceship is traveling with a speed of $0.990c$, covering a distance of 25.3 ly in a time of 25.6 y. **(b)** From the astronaut's point of view the spaceship is at rest, and Earth and Vega are moving with a speed of $0.990c$. For the astronaut the trip takes only 3.61 y and covers a contracted distance of only 3.57 ly.

In general, we would like to determine the contracted length L of an object moving with a speed v. When the object is at rest ($v = 0$), we say that its length is the **proper length,** L_0:

> The proper length is the distance between two points as measured by an observer who is at rest with respect to them.

In the Benny and Jenny Example, Jenny is at rest with respect to Earth and Vega. As a result, the distance between them as measured by Jenny is the proper length; that is, $L_0 = 25.3$ ly. The contracted length, $L = 3.57$ ly, is measured by Benny.

As for the times measured for the trip, from Jenny's point of view the two events (event 1, departing Earth; event 2, arriving at Vega) occur at different locations. As a result, she measures the *dilated time*, $\Delta t = 25.6$ y, even though she also measures the *proper length*, L_0. In contrast, Benny measures the *proper time*, $\Delta t_0 = 3.61$ y, and the *contracted length*, L. Note that one must be careful to determine

PROBLEM-SOLVING NOTE

Identifying the Proper Length

The key to solving problems involving length contraction is to correctly identify the proper length, L_0. Specifically, the proper length is the distance between two points that are at rest relative to the observer.

from the definitions given previously which observer measures the proper time and which observer measures the proper length—*it should never be assumed*, for example, that just because one observer measures the proper time that observer also measures the proper length.

We now use these observations to obtain a general expression relating L and L_0. To begin, note that both observers measure the same relative speed, v. For Jenny, the speed is $v = L_0/\Delta t$, and for Benny it is $v = L/\Delta t_0$. Setting these speeds equal we obtain

$$v = \frac{L_0}{\Delta t} = \frac{L}{\Delta t_0}$$

Solving for L in terms of L_0, we find $L = L_0(\Delta t_0/\Delta t)$. Finally, using Equation 29–2 to express Δt in terms of Δt_0 we obtain the following:

Length Contraction

$$L = L_0\sqrt{1 - \frac{v^2}{c^2}} \qquad 29\text{–}3$$

SI unit: m

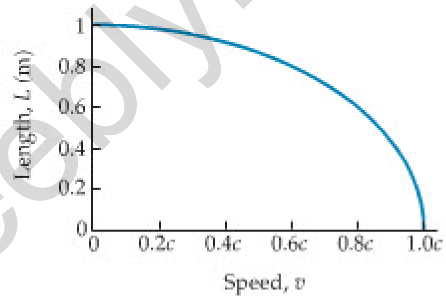
Substituting the numerical values from the Benny and Jenny Example gives us $L = (25.3\text{ ly})\sqrt{1 - (0.990c)^2/c^2} = 3.57\text{ ly}$, as expected.

Note that if $v = 0$ in Equation 29–3, we find that $L = L_0$. As v approaches the speed of light, the contracted length L approaches zero. In general, the length of a moving object is always less than its proper length. **Figure 29–9** shows L as a function of speed v for a meterstick, where we see again that the speed of light is the ultimate speed possible.

PROBLEM-SOLVING NOTE

Measuring Proper Length and Proper Time

Keep in mind that the observer who measures the proper length is *not* necessarily the same observer who measures the proper time.

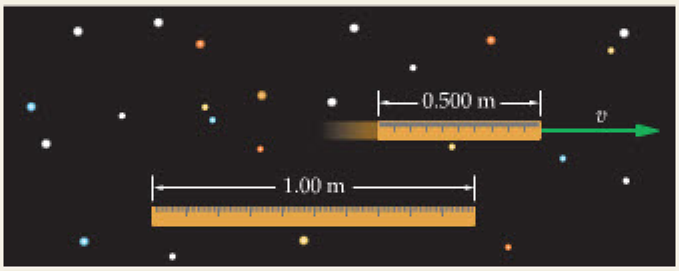


▲ **FIGURE 29–9 Length contraction**
The length of a meterstick as a function of its speed. Note that the length shrinks to zero in the limit that its speed approaches the speed of light.

EXAMPLE 29–3 HALF A METER

Find the speed for which the length of a meterstick is 0.500 m.

PICTURE THE PROBLEM
Our sketch shows a moving meterstick with a contracted length $L = 0.500$ m. The meterstick at rest has its proper length, $L_0 = 1.00$ m.



STRATEGY
We can find the desired speed by applying length contraction (Equation 29–3) to the moving meterstick. In particular, given L and L_0 we can solve $L = L_0\sqrt{1 - v^2/c^2}$ for the speed, v.

SOLUTION

1. Solve Equation 29–3 for the speed v:

$$L = L_0\sqrt{1 - \frac{v^2}{c^2}}$$

$$v = c\sqrt{1 - \frac{L^2}{L_0{}^2}}$$

2. Substitute numerical values:

$$v = c\sqrt{1 - \frac{L^2}{L_0{}^2}} = c\sqrt{1 - \frac{(0.500\text{ m})^2}{(1.00\text{ m})^2}} = 0.866c$$

INSIGHT
Note that a person traveling along with the moving meterstick would see it as having its proper length of 1.00 m. From this person's point of view, it is the *other* meterstick that is only 0.500 m long. Thus, lengths—like times—are relative, and depend on the observer making the measurement.

PRACTICE PROBLEM
Find the length of a meterstick that is moving with half the speed of light. [**Answer:** $L = 0.866$ m]

Some related homework problems: Problem 24, Problem 29

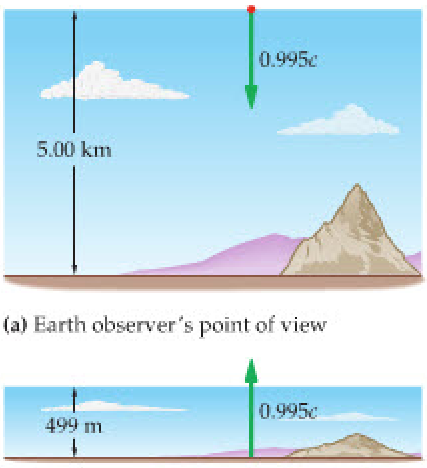


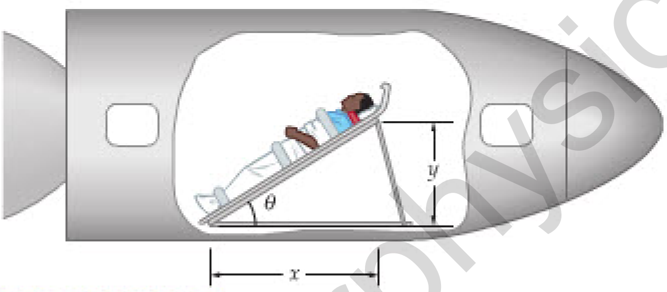
FIGURE 29–10 A muon reaches Earth's surface
(a) From Earth's point of view a muon travels downward with a speed of 0.995*c* for a distance of 5.00 km. The muon can reach Earth's surface only if it ages slowly due to its motion. **(b)** From the muon's point of view Earth is moving upward with a speed of 0.995*c*, and the distance to Earth's surface is only 499 m. From this point of view the muon reaches Earth's surface because the distance is contracted, not because the muon lives longer.

In **Figure 29–10** we illustrate the effects of length contraction for the case of a muon traveling toward Earth's surface. From the point of view of the Earth, Figure 29–10 (a), the muon travels a distance of 5.00 km at a speed of 0.995*c*. To cover this distance, the muon must live about 10 times longer than it would at rest. From the point of view of the muon, Figure 29–10 (b), the Earth is moving upward at a speed of 0.995*c*, and the distance the Earth must travel to reach the muon is only $L = (5.00\text{ km})\sqrt{1 - (0.995c)^2/c^2} = 499$ m. The Earth can easily cover this distance during the muon's resting lifetime of 2.20 μs.

One final comment: The length contraction calculated in Equation 29–3 *pertains only to lengths in the direction of relative motion*. Lengths at right angles to the relative motion are unaffected.

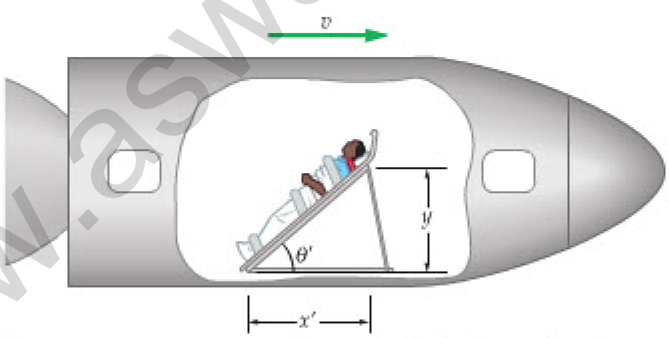
CONCEPTUAL CHECKPOINT 29–2 ANGLE OF REPOSE

An astronaut is resting on a bed inclined at an angle θ above the floor of a spaceship, as shown in the first sketch. From the point of view of an observer who sees the spaceship moving to the right with a speed approaching *c*, is the angle the bed makes with the floor **(a)** greater than, **(b)** less than, or **(c)** equal to the angle θ observed by the astronaut?



REASONING AND DISCUSSION

A person observing the spaceship moving with a speed v notices a contracted length, x', in the direction of motion, but an unchanged length, y, perpendicular to the direction of motion, as shown in the second sketch.

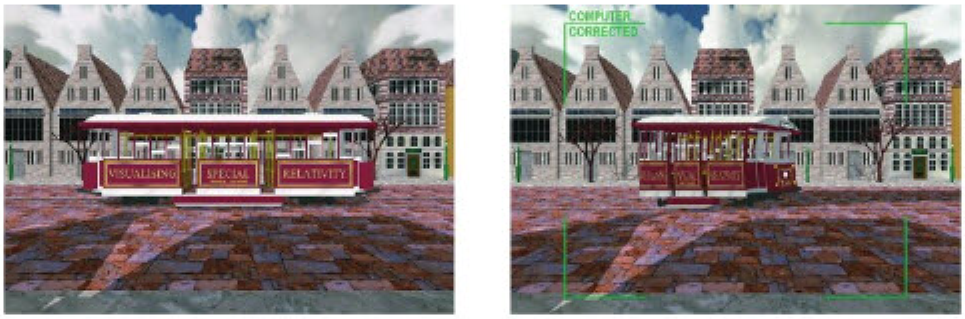


As a result of the contraction in just one direction, the bed is inclined at an angle greater than the angle θ measured by the astronaut.

ANSWER

(a) The angle will be greater than θ.

The fact that lengths in different directions contract differently has interesting effects on the way a rapidly moving object appears to the eye. Additional effects are related to the finite time it takes for light to propagate to the eye from different parts of an object. An example of the way an object would actually look if it moved past us at nearly the speed of light is shown in **Figure 29–11**.



(a) Streetcar at rest

(b) Streetcar at relativistic speed

FIGURE 29–11 Relativistic distortions

If the streetcar depicted at rest in part **(a)** moved to the left at nearly the speed of light, it would look like the computer-simulated image in part **(b)**. The streetcar would appear compressed in the direction of its motion. Because the streetcar would move significantly in the time it took for light from it to reach you, however, you would also observe other odd effects—for example, you would be able to see its back surface even from a position directly alongside it.

29–4 The Relativistic Addition of Velocities

Suppose you are piloting a spaceship in deep space, moving toward an asteroid with a speed of 25 mi/h. To signal a colleague on the asteroid, you activate a beam of light on the nose of the ship and point it in the direction of motion. Since light in a vacuum travels with the same speed c relative to all inertial observers, the speed of the light beam relative to the asteroid is simply c, *not* $c + 25$ mi/h. Clearly, then, simple addition of velocities, which seems to work just fine for everyday speeds, no longer applies.

The correct way to add velocities, valid for all speeds from zero to the speed of light, was obtained by Einstein. Consider, for example, a one-dimensional system consisting of a spaceship (1), a probe (2), and an asteroid (3), as indicated in Figure 29–12. Suppose the spaceship moves toward the asteroid with the velocity v_{13}, where the subscripts mean "the velocity of object 1 relative to object 3." Since the spaceship moves to the right in Figure 29–12 (a), its velocity is positive ($v_{13} > 0$); if it moved to the left, its velocity would be negative ($v_{13} < 0$). Now imagine that the ship sends out a probe, whose velocity relative to the ship is v_{21}, as shown in Figure 29–12 (b). The question is, how do we add velocities correctly to get the velocity of the probe relative to the asteroid, v_{23}?

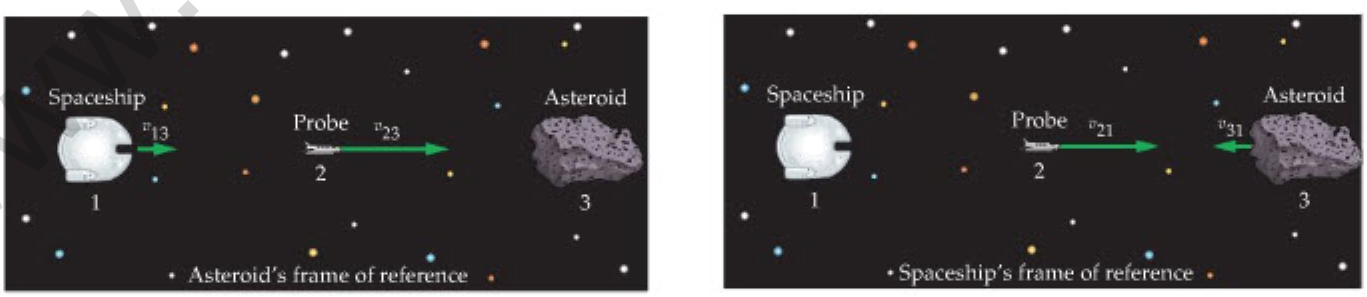
The "classical" answer is to simply add the velocities, as we did in Chapter 3; that is, $v_{23} = v_{21} + v_{13}$. Einstein showed that the correct result, taking into account relativity, contains an additional factor:

Relativistic Addition of Velocities

$$v_{23} = \frac{v_{21} + v_{13}}{1 + \dfrac{v_{21}v_{13}}{c^2}} \qquad 29\text{–}4$$

SI unit: m/s

Notice that if the speed of light were infinite, $c \to \infty$, the denominator would be $1 + v_{13}v_{21}/(\infty)^2 = 1$ and we would recover classical velocity addition. Thus, it is the *finite* speed of light that is responsible for relativistic effects.



(a) (b)

FIGURE 29–12 The velocity of a probe: Two frames of reference

A spaceship approaches an asteroid and sends a probe to investigate it. **(a)** In the asteroid's frame of reference, where the asteroid is at rest, the spaceship approaches with the velocity v_{13} and the probe approaches with the velocity v_{23}. **(b)** In the spaceship's frame of reference, the probe is launched with the velocity v_{21} and the asteroid approaches with the velocity $v_{31} = -v_{13}$. The relativistic sum of the velocities v_{13} and v_{21}, using Equation 29–4, yields the probe's velocity relative to the asteroid, v_{23}.

PROBLEM-SOLVING NOTE

Adding Velocities

To add velocities correctly, it is important to identify each velocity in terms of the object or observer relative to which it is measured, just as for velocity addition in nonrelativistic physics (Section 3–6).

To return to the example at the beginning of this section, what if the probe is actually a beam of light, with the velocity $v_{21} = c$? In this case, relativistic velocity addition gives the speed of the probe (light) relative to the asteroid as

$$v_{23} = \frac{v_{21} + v_{13}}{1 + \dfrac{v_{21}v_{13}}{c^2}} = \frac{c + v_{13}}{1 + \dfrac{c\,v_{13}}{c^2}} = \frac{c\left(1 + \dfrac{v_{13}}{c}\right)}{1 + \dfrac{v_{13}}{c}} = c$$

Thus, as expected, both the observer in the spaceship (1) and the observer on the asteroid (3) measure the same velocity for the light beam, $v_{21} = v_{23} = c$, regardless of the velocity of the ship relative to the asteroid, v_{13}.

We've seen that Equation 29–4 gives the correct result for a beam of light, but how does it work when applied to speeds much smaller than the speed of light? As an example, suppose a person on a spaceship moving at 25 mi/h throws a ball with a speed of 15 mi/h in the direction of the asteroid. According to the classical addition of velocities, the velocity of the ball relative to the asteroid is 15 mi/h + 25 mi/h = 40 mi/h. Application of Equation 29–4 gives the following result:

$$v = 39.99999999999997 \text{ mi/h}$$

Thus, in any practical measurement, the velocity of the ball relative to the asteroid will be 40 mi/h. We conclude that the classical result, $v_{23} = v_{21} + v_{13}$, although not strictly correct, is appropriate for small speeds.

EXERCISE 29–2

Suppose the spaceship described previously is approaching an asteroid with a speed of $0.750c$. If the spaceship launches a probe toward the asteroid with a speed of $0.800c$ relative to the ship, what is the speed of the probe relative to the asteroid?

SOLUTION

Substituting $v_{13} = 0.750c$ and $v_{21} = 0.800c$ in Equation 29–4 we obtain

$$v_{23} = \frac{v_{21} + v_{13}}{1 + \dfrac{v_{21}v_{13}}{c^2}} = \frac{0.800c + 0.750c}{1 + \dfrac{(0.800c)(0.750c)}{c^2}} = 0.969c$$

As expected, the speed relative to the asteroid is less than c. Note, however, that classical velocity addition gives a strikingly different prediction for the speed relative to the asteroid; namely, $v = 0.800c + 0.750c = 1.550c$. Thus, velocity addition is one more way that experiments can verify the predictions of relativity.

In the following Example and Active Example, we apply relativistic velocity addition to a variety of physical systems.

EXAMPLE 29–4 GENERATION NEXT

At starbase Faraway Point, you observe two spacecraft approaching from the same direction. The *La Forge* is approaching with a speed of $0.906c$, and the *Picard* is approaching with a speed of $0.806c$. **(a)** Find the velocity of the *La Forge* relative to the *Picard*. **(b)** Find the velocity of the *La Forge* relative to the *Picard* if the *La Forge's* direction of motion is reversed.

PICTURE THE PROBLEM

Our sketch shows the two spacecraft approaching the starbase along the same line, moving in the positive direction. The speed of the *La Forge* is $0.906c$, and the speed of the *Picard* is $0.806c$. In addition, note that we have numbered the objects in this system as follows: *Picard* (1); *La Forge* (2); Faraway Point (3). Finally, we show the *La Forge* ahead of the *Picard* in our sketch, but the final result is the same even if their positions are reversed.

INTERACTIVE FIGURE

STRATEGY

a. The key to solving a problem like this is to choose the velocities v_{13}, v_{21}, and v_{23} in a way that is consistent with our numbering scheme. In this case, we know that $v_{13} = 0.806c$ and $v_{23} = 0.906c$. The velocity we want to find, *La Forge* relative to *Picard*, is v_{21}. This can be determined by substituting the known quantities into Equation 29–4.

b. In this case the *La Forge* is traveling in the negative direction; therefore, $v_{23} = -0.906c$.

SOLUTION

Part (a)

1. Use relativistic velocity addition (Equation 29–4) and straightforward algebra to solve for v_{21}:

$$v_{23} = \frac{v_{21} + v_{13}}{1 + \dfrac{v_{21}v_{13}}{c^2}} \quad \text{or} \quad v_{21} = \frac{v_{23} - v_{13}}{1 - \dfrac{v_{23}v_{13}}{c^2}}$$

2. Substitute $v_{13} = 0.806c$ and $v_{23} = 0.906c$ to find v_{21}, the velocity of the *La Forge* relative to the *Picard*:

$$v_{21} = \frac{v_{23} - v_{13}}{1 - \dfrac{v_{23}v_{13}}{c^2}} = \frac{0.906c - 0.806c}{1 - \dfrac{(0.906c)(0.806c)}{c^2}} = 0.371c$$

Part (b)

3. Use the result from part (a), only this time with $v_{23} = -0.906c$:

$$v_{21} = \frac{v_{23} - v_{13}}{1 - \dfrac{v_{23}v_{13}}{c^2}} = \frac{(-0.906c) - 0.806c}{1 - \dfrac{(-0.906c)(0.806c)}{c^2}} = -0.989c$$

INSIGHT

A nonrelativistic calculation of the velocity of the *La Forge* relative to the *Picard* in part (a) gives 0.100*c*, considerably less than the correct value of 0.371*c* obtained from relativistic velocity addition. In part (b), a nonrelativistic calculation gives a speed of 1.712*c*, as opposed to the correct speed, 0.989*c*, which is less than *c*. Also, note that the negative value of v_{21} means the *Picard* sees the *La Forge* moving to the left.

Finally, when we set up Equation 29–4, we arbitrarily chose the *Picard*, *La Forge*, and Faraway Point to be 1, 2, and 3, respectively. It is important to note, however, that any assignment of 1, 2, and 3 is equally valid and will yield precisely the same results.

PRACTICE PROBLEM

Find the velocity of the *La Forge* relative to the *Picard* if the *Picard*'s direction of motion is reversed but the *La Forge* is still moving toward Faraway Point. [**Answer:** In this case $v_{13} = -0.806c$; hence, $v_{21} = 0.989c$. The *Picard* sees the *La Forge* moving to the right.]

Some related homework problems: Problem 40, Problem 41, Problem 42

ACTIVE EXAMPLE 29–2 FIND THE RELATIVISTIC LENGTH

As a spaceship approaches a distant planet with a speed of 0.445*c*, it launches a probe toward the planet. The proper length of the probe is 10.0 m, and its length as measured by an observer on the spaceship is 7.50 m. What is the length of the probe as measured by an observer on the planet?

SOLUTION *(Test your understanding by performing the calculations indicated in each step.)*

1. Use length contraction, Equation 29–3, to find the velocity of the probe relative to the spaceship: $v_{21} = 0.661c$
2. Use relativistic velocity addition, Equation 29–4, to find the velocity of the probe relative to the planet: $v_{13} = 0.445c$; $v_{21} = 0.661c$; $v_{23} = 0.855c$
3. Use the velocity of the probe relative to the planet to calculate its contracted length: $L = 5.19$ m

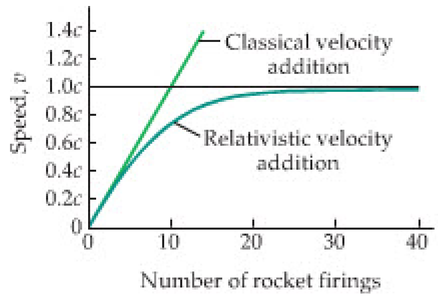
INSIGHT

Therefore, the probe has a length of 10.0 m when it is at rest relative to an observer. After it is launched from the spaceship, it has a length of 7.50 m relative to the ship, and a length of 5.19 m relative to the planet the ship is approaching.

YOUR TURN

Find the length of the probe relative to the planet if the spaceship moves *away* from the planet with a speed of 0.445*c* when it launches the probe.

(Answers to **Your Turn** *problems are given in the back of the book.)*



▲ **FIGURE 29–13 Relativistic velocity addition**

A comparison of classical velocity addition and relativistic velocity addition. Classically, a spacecraft that continues to fire its rockets attains a greater and greater speed. The correct, relativistic, result is that the spacecraft approaches the speed of light in the limit of an infinite number of rocket firings.

To get a better feeling for relativistic velocity addition, consider a spacecraft, initially at rest, that increases its speed by $0.1c$ when it fires its rockets. At first, the speed of the spacecraft increases linearly with the number of times the rockets are fired, as indicated by the line labeled "Classical velocity addition" in Figure 29–13. As the spacecraft's speed approaches the speed of light, however, further rocket firings have less and less effect, as seen in the curve labeled "Relativistic velocity addition." In the limit of infinite time—and an infinite number of rocket firings—the speed of the spacecraft approaches c, without ever attaining that value.

29–5 Relativistic Momentum

The first postulate of relativity states that the laws of physics are the same for all observers in all inertial frames of reference. Among the most fundamental of these laws are the conservation of momentum and the conservation of energy for an isolated system. In this section we consider the relativistic expression for momentum, leaving energy conservation for the next section.

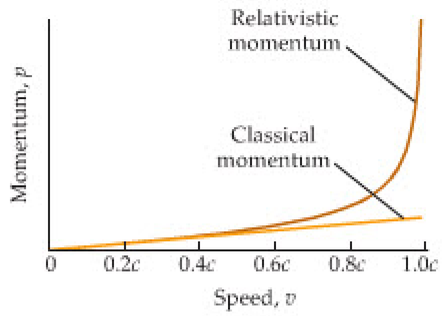
When one considers the unusual way that relativistic velocities add, as given in Equation 29–4, it comes as no surprise that the classical expression for momentum, $p = mv$, is not valid for all speeds. As an example, we saw in Chapter 9 that if a large mass with a speed v collides elastically with a small mass at rest, the small mass is given a speed $2v$. Clearly, this cannot happen if the speed of the large mass is greater than $0.5c$, since the small mass cannot have a speed greater than the speed of light. Thus the nonrelativistic relation $p = mv$ must be modified for speeds comparable to c.

A detailed analysis shows that the correct relativistic expression for the magnitude of momentum is the following:

Relativistic Momentum

$$p = \frac{mv}{\sqrt{1 - \frac{v^2}{c^2}}} \qquad 29\text{–}5$$

SI unit: kg · m/s



▲ **FIGURE 29–14 Relativistic momentum**

The magnitude of momentum as a function of speed. Classically, the momentum increases linearly with speed, as shown by the straight line. The correct relativistic momentum increases to infinity as the speed of light is approached.

As v approaches the speed of light, the relativistic momentum becomes significantly larger than the classical momentum, eventually diverging to infinity as $v \to c$, as shown in Figure 29–14. For low speeds the classical and relativistic results agree.

EXERCISE 29–3

Find **(a)** the classical and **(b)** the relativistic momentum of a 2.4-kg mass moving with a speed of $0.81c$.

SOLUTION

a. Evaluate $p = mv$:

$$p = mv = (2.4\text{ kg})(0.81)(3.00 \times 10^8\text{ m/s}) = 5.8 \times 10^8\text{ kg}\cdot\text{m/s}$$

b. Evaluate $p = mv/\sqrt{1 - v^2/c^2}$:

$$p = \frac{mv}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{(2.4\text{ kg})(0.81)(3.00 \times 10^8\text{ m/s})}{\sqrt{1 - \frac{(0.81c)^2}{c^2}}} = 9.9 \times 10^8\text{ kg}\cdot\text{m/s}$$

As expected, the relativistic momentum is larger in magnitude than the classical momentum.

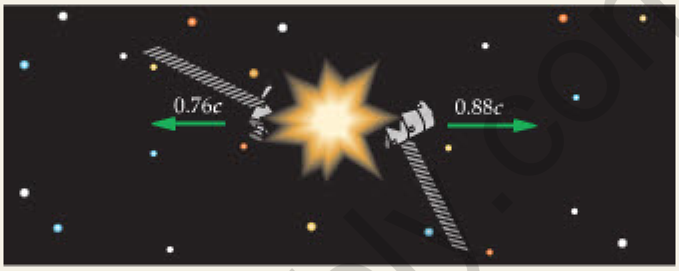
Next we consider a system in which the relativistic momentum is conserved.

EXAMPLE 29–5 THE UNKNOWN MASS

A satellite, initially at rest in deep space, explodes into two pieces. One piece has a mass of 150 kg and moves away from the explosion with a speed of 0.76*c*. The other piece moves away from the explosion in the opposite direction with a speed of 0.88*c*. Find the mass of the second piece of the satellite.

PICTURE THE PROBLEM

The two pieces of the satellite and their speeds (0.76*c* and 0.88*c*) are indicated in the sketch. Note that the two pieces move in opposite directions, which means that they move away from one another along the same line.



STRATEGY

The basic idea in this system is that because no external forces act on the satellite, its total momentum must be conserved. The initial momentum is zero; hence, the final momentum must be zero as well. This means that the pieces will move in opposite directions, as mentioned, and with momenta of equal magnitude.

Thus, we begin by calculating the magnitude of the momentum for the first piece of the satellite. Next, we set the momentum of the second piece equal to the same magnitude and solve for the mass.

SOLUTION

1. Calculate the magnitude of the momentum for the piece of the satellite with a mass $m_1 = 150$ kg and a speed $v_1 = 0.76c$:

$$p_1 = \frac{m_1 v_1}{\sqrt{1 - \frac{v_1^2}{c^2}}} = \frac{(150\ \text{kg})(0.76)(3.00 \times 10^8\ \text{m/s})}{\sqrt{1 - \frac{(0.76c)^2}{c^2}}} = 5.3 \times 10^{10}\ \text{kg} \cdot \text{m/s}$$

2. Set the momentum of the second piece of the satellite equal to the momentum of the first piece:

$$p_2 = \frac{m_2 v_2}{\sqrt{1 - \frac{v_2^2}{c^2}}} = p_1$$

3. Solve the above relation for m_2, the mass of the second piece of the satellite, and substitute the numerical values $v_2 = 0.88c$ and $p_1 = 5.3 \times 10^{10}$ kg·m/s:

$$m_2 = \left(\frac{p_1}{v_2}\right)\sqrt{1 - \frac{v_2^2}{c^2}} = \left[\frac{5.3 \times 10^{10}\ \text{kg} \cdot \text{m/s}}{(0.88)(3.00 \times 10^8\ \text{m/s})}\right]\sqrt{1 - \frac{(0.88c)^2}{c^2}} = 95\ \text{kg}$$

INSIGHT

In comparison, a classical calculation, with $p = mv$, gives the erroneous result that the mass of the second piece of the satellite is 130 kg. This overestimate of the mass is due to the omission of the factor $1/\sqrt{1 - v^2/c^2}$ in the classical momentum.

PRACTICE PROBLEM

If the mass of the second piece of the satellite had been 210 kg, what would its speed have been? [**Answer:** $v_2 = 0.64c$]

Some related homework problems: Problem 49, Problem 50

Equation 29–5 is sometimes thought of in terms of a mass that increases with speed. Suppose, for example, that an object has the mass m_0 when it is at rest; that is, its **rest mass** is m_0. When the speed of the object is v, its momentum is

$$p = \frac{m_0 v}{\sqrt{1 - \frac{v^2}{c^2}}} = \left(\frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}\right) v$$

If we now make the identification $m = m_0/\sqrt{1 - v^2/c^2}$ we can write the momentum as follows:

$$p = mv$$

Thus the classical expression for momentum can be used for all speeds if we simply interpret the mass as increasing with speed according to the expression

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}} \qquad 29\text{–}6$$

Note that m approaches infinity as $v \rightarrow c$. Hence a constant force acting on an object generates less and less acceleration, $a = F/m$, as the speed of light is approached. This gives one further way of seeing that the speed of light cannot be exceeded.

Though Equation 29–6 helps to show why an object of finite mass can never be accelerated to speeds exceeding the speed of light, it must be noted that the concept of a relativistic mass has its limitations. For example, the relativistic kinetic energy of an object is *not* obtained by simply replacing m in $\frac{1}{2}mv^2$ with the expression given in Equation 29–6, as we shall see in detail in the next section.

29–6 Relativistic Energy and $E = mc^2$

We have just seen that, from the point of view of momentum, an object's mass increases as its speed increases. Therefore, when work is done on an object, part of the work goes into increasing its speed, and part goes into increasing its mass. It follows, then, that *mass is another form of energy*. This result, like time dilation, was completely unanticipated before the introduction of the theory of relativity.

Consider, for example, an object whose mass while at rest is m_0. Einstein was able to show that when the object moves with a speed v, its **total energy**, E, is given by the following expression:

Relativistic Energy

$$E = \frac{m_0 c^2}{\sqrt{1 - \frac{v^2}{c^2}}} = mc^2 \qquad 29\text{–}7$$

SI unit: J

This is Einstein's most famous result from relativity; that is, $E = mc^2$, where m is the relativistic mass given in Equation 29–6.

Note that the total energy, E, does not vanish when the speed goes to zero, as does the classical kinetic energy. Instead, the energy of an object at rest—its **rest energy**, E_0—is

Rest Energy

$$E_0 = m_0 c^2 \qquad 29\text{–}8$$

SI unit: J

Because the speed of light is so large, it follows that the mass of an object times the *speed of light squared* is an enormous amount of energy, as illustrated in the following Exercise.

EXERCISE 29–4

Find the rest energy of a 0.12-kg apple.

SOLUTION

Substituting $m_0 = 0.12$ kg and $c = 3.00 \times 10^8$ m/s in Equation 29–8 we find

$$E_0 = m_0 c^2 = (0.12 \text{ kg})(3.00 \times 10^8 \text{ m/s})^2 = 1.1 \times 10^{16} \text{ J}$$

To put the result of Exercise 29–4 in context, let's compare it with the total yearly energy usage in the United States, which is about 10^{20} J. This means that if the rest energy of the apple could be converted entirely to usable forms of energy, it could supply the energy needs of the entire United States for about an hour.

Put another way, if the rest energy of the apple could be used to light a 100-W lightbulb, it would stay lit for about 10 million years.

This example illustrates the basic principle behind the operation of nuclear power plants, in which small decreases in mass—due to various nuclear reactions—are used to generate electrical energy. The type of reactions that power such plants are referred to as **fission reactions,** in which a large nucleus splits into smaller nuclei and neutrons, as described in Section 32–5. For example, the nucleus of a uranium-235 atom may decay into two smaller nuclei and a number of neutrons. Since the mass of the uranium nucleus is greater than the sum of the masses of the fragments of the decay, the reaction releases an enormous amount of energy. In fact, 1 lb of uranium can produce about 3×10^6 kWh of electrical energy, compared with the 1 kWh that can be produced by the combustion of 1 lb of coal.

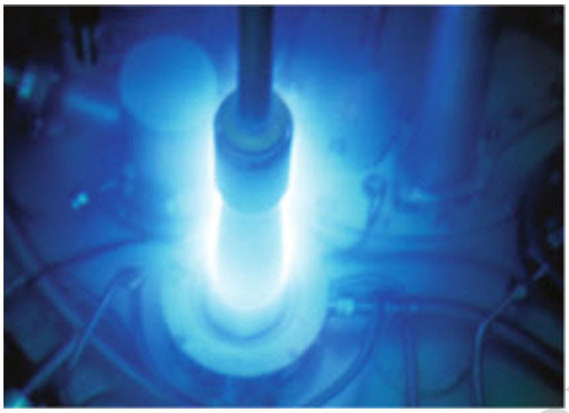
REAL-WORLD PHYSICS

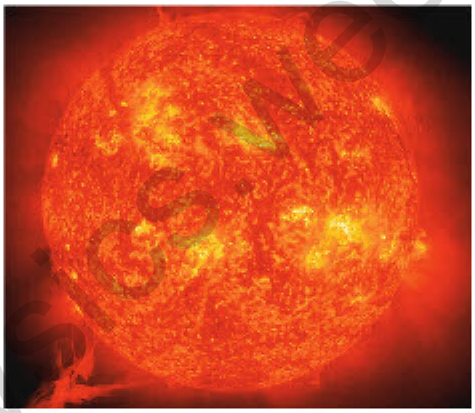
Nuclear power—converting mass to energy

The Sun is also powered by the conversion of mass to energy. In this case, however, the energy is released by **fusion reactions,** in which two very small nuclei combine to form a larger nucleus. The detailed reactions are presented in Section 32–6. In the following Example, we determine the amount of mass lost by the Sun per second.

REAL-WORLD PHYSICS

Converting mass to energy to power the Sun





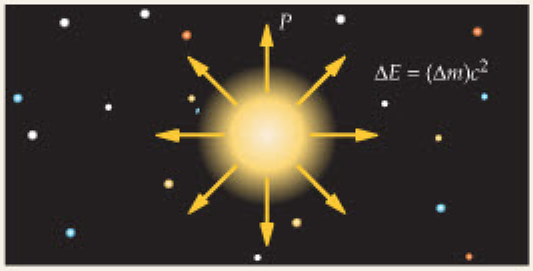
▲ A nuclear reactor (left) converts mass to energy by means of fission reactions, in which large nuclei (such as those of uranium or plutonium) are split into smaller fragments. This photo shows the blue glow referred to as Cherenkov radiation in a nuclear reactor at the Centre of Atomic Energy in Saclay, France. The Sun (right) and other stars are powered by fusion reactions, in which small nuclei (such as those of hydrogen) combine to form heavier ones (such as those of helium). Although this process is utilized in the hydrogen bomb, it has not yet been successfully harnessed as a practical source of power on Earth.

EXAMPLE 29–6 THE PRODIGAL SUN

Energy is radiated by the Sun at the rate of about 3.92×10^{26} W. Find the corresponding decrease in the Sun's mass for every second that it radiates.

PICTURE THE PROBLEM

Our sketch indicates energy radiated continuously by the Sun at the rate $P = 3.92 \times 10^{26}\text{ W} = 3.92 \times 10^{26}\text{ J/s}$. We label the energy given off in the time interval $\Delta t = 1.00$ s with ΔE. The corresponding decrease in mass is Δm.



STRATEGY

If the energy radiated by the Sun in 1.00 s is ΔE, the corresponding decrease in mass, according to the relation $E = mc^2$, is given by $\Delta m = \Delta E/c^2$.

To find ΔE, we simply recall (Chapter 7) that power is energy per time, $P = \Delta E/\Delta t$. Thus the energy radiated by the Sun in 1.00 s is $\Delta E = P\Delta t$, with $\Delta t = 1.00$ s.

SOLUTION

1. Calculate the energy radiated by the Sun in 1.00 s: $\Delta E = P\Delta t = (3.92 \times 10^{26}\text{ J/s})(1.00\text{ s}) = 3.92 \times 10^{26}\text{ J}$

2. Divide ΔE by the speed of light squared, c^2, to find the decrease in mass: $\Delta m = \dfrac{\Delta E}{c^2} = \dfrac{3.92 \times 10^{26}\text{ J}}{(3.00 \times 10^8\text{ m/s})^2} = 4.36 \times 10^9\text{ kg}$

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INSIGHT

Thus the Sun loses a rather large amount of mass each second—in fact, roughly the equivalent of 2000 space shuttles. Since the Sun has a mass of 1.99×10^{30} kg, however, the mass it loses in 1500 y is only 10^{-10} of its total mass. Even after 1.5 billion years of radiating at its present rate, the Sun will lose a mere 0.01% of its mass. Clearly, the Sun will not evaporate into space anytime soon.

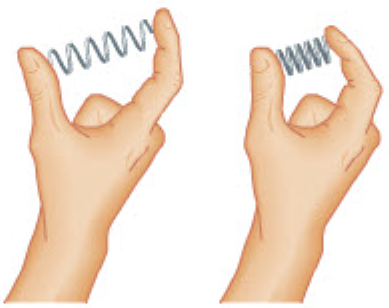
PRACTICE PROBLEM

Find the power radiated by a star whose mass decreases by the mass of the Moon (7.36×10^{22} kg) in half a million years (5.00×10^5 y). [**Answer:** $P = 4.21 \times 10^{26}$ W, slightly more than the power of the Sun.]

Some related homework problems: Problem 58, Problem 63, Problem 69

CONCEPTUAL CHECKPOINT 29–3 COMPARE THE MASS

When you compress a spring between your fingers, does its mass **(a)** increase, **(b)** decrease, or **(c)** stay the same?



REASONING AND DISCUSSION

When the spring is compressed by an amount x, its energy is increased by the amount $\Delta E = \frac{1}{2}kx^2$, as we saw in Chapter 8. Since the energy of the spring has increased, its mass increases as well, by the amount $\Delta m = \Delta E/c^2$.

ANSWER

(a) The mass of the spring increases.

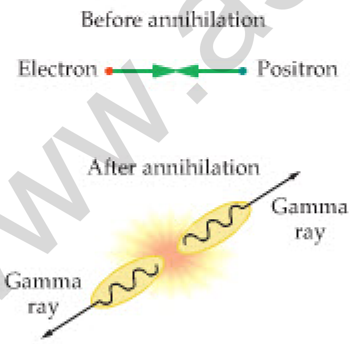
As one might expect, the increase in the mass of a compressed spring is generally too small to be measured. For example, if the energy of a spring increases by 1.00 J, its mass increases by only $\Delta m = (1.00\text{ J})/c^2 = 1.11 \times 10^{-17}$ kg.

Matter and Antimatter

A particularly interesting aspect of the equivalence of mass and energy is the existence of **antimatter.** For every elementary particle known to exist, there is a corresponding antimatter particle that has precisely the same mass but exactly the opposite charge. For example, an *electron* has a mass $m_e = 9.11 \times 10^{-31}$ kg and a charge $-e = -1.60 \times 10^{-19}$C; an *antielectron* has a mass of 9.11×10^{-31} kg and a charge equal to $+1.60 \times 10^{-19}$ C. Since an antielectron has a positive charge, it is generally referred to as a **positron.**

Antimatter is frequently created in accelerators, where particles collide at speeds approaching the speed of light. In fact, it is possible to create antiatoms in the lab made entirely of antimatter. An intriguing possibility is that the universe may actually contain entire antigalaxies of antimatter.

If this is indeed the case, one would have to be a bit careful about visiting such a galaxy, because particles of matter and antimatter have a rather interesting behavior when they meet—they **annihilate** one another. This situation is illustrated in Figure 29–15, where we show an electron and a positron coming into contact. The result is that the particles cease to exist, which satisfies charge conservation, since the net charge of the system is zero before and after the annihilation. As for energy conservation, the mass of the two particles is converted into two gamma rays, which are similar to X-rays only more energetic. Each of the gamma rays must have an energy that is at least $E = m_e c^2$. Thus, in matter-antimatter annihilation the particles vanish in a burst of radiation.



▲ **FIGURE 29–15 Electron-positron annihilation**

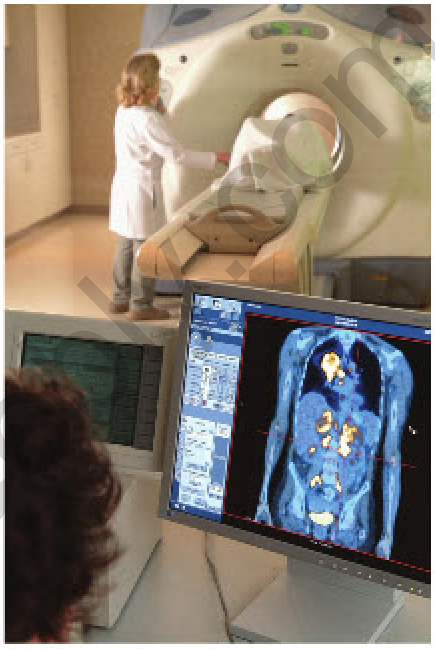
An electron and a positron annihilate when they come into contact. The result is the emission of two energetic gamma rays with no mass. The mass of the original particles has been converted into the energy of the gamma rays.

Electron-positron annihilation is the basis for the diagnostic imaging technique called positron emission tomography (PET), which is often used to examine biological processes within the brain, heart, and other organs. In a typical PET brain scan, for example, a patient is injected with glucose (the primary energy source for brain activity) that has been "tagged" with radioactive tracers. These tracers emit positrons in a nuclear reaction described in Section 32–7, and the resulting positrons, in turn, encounter electrons in the brain and undergo annihilation. The resulting gamma rays exit through the patient's skull and are monitored by the PET scanner (Figure 29–16), which converts them into false-color images showing the glucose metabolism levels within the brain. Thus, surprising as it may seem, this powerful diagnostic tool actually relies on the annihilation of matter and antimatter inside a person's brain.

The conversion between mass and energy can go the other way, as well. That is, an energetic gamma ray, which has zero mass, can be converted into a particle-antiparticle pair. For example, a gamma ray with an energy of at least $(2m_e)c^2$ can be converted into an electron and a positron; that is, the energy of the gamma ray can be converted into the rest energy of the two particles. Thus we can see that the equivalence of mass and energy, as given by the relation $E = mc^2$, has far-reaching implications.

REAL-WORLD PHYSICS: BIO

Positron emission tomography



▲ **FIGURE 29–16 A PET scanner**

In a PET scan, positrons emitted by a radioactively labeled metabolite are annihilated when they collide with electrons. The mass of the two particles is converted to energy in the form of a pair of gamma rays, which are always emitted in diametrically opposite directions. A ring of detectors surrounding the patient records the radiation and uses it to construct an image. In this way it is possible to map the location of particular metabolic activities in the body.

Relativistic Kinetic Energy

When work is done on an isolated object, accelerating it from rest to a finite speed v, its total energy increases, as given by Equation 29–7. We refer to the *increase* in the object's energy as its *kinetic energy*. Thus the total energy, E, of an object is the sum of its rest energy, m_0c^2, and its kinetic energy, K. In particular,

$$E = \frac{m_0c^2}{\sqrt{1 - \frac{v^2}{c^2}}} = m_0c^2 + K$$

Solving for the kinetic energy, we find

Relativistic Kinetic Energy

$$K = \frac{m_0c^2}{\sqrt{1 - \frac{v^2}{c^2}}} - m_0c^2 \qquad 29\text{–}9$$

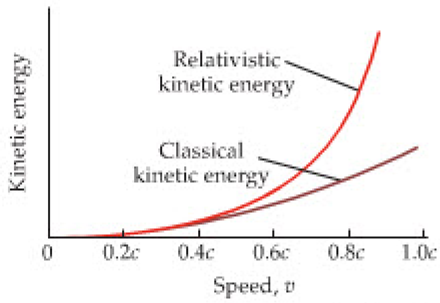
SI unit: J

As a check, note that the kinetic energy is zero when the speed is zero, as expected. A comparison between the relativistic and classical kinetic energies is presented in Figure 29–17.

Although the expression in Equation 29–9 looks nothing like the familiar classical kinetic energy, $\frac{1}{2}mv^2$, it does approach this value in the limit of small speeds. To see this, we can expand Equation 29–9 for small v using the binomial expansion given in Appendix A. The result is as follows:

$$K = \frac{m_0c^2}{\sqrt{1 - \frac{v^2}{c^2}}} - m_0c^2 = m_0c^2\left[\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}\right] - m_0c^2$$

$$= m_0c^2\left[1 + \frac{1}{2}\left(\frac{v^2}{c^2}\right) + \frac{3}{8}\left(\frac{v^2}{c^2}\right)^2 + \cdots\right] - m_0c^2$$



▲ **FIGURE 29–17 Relativistic and classical kinetic energies**

The relativistic kinetic energy (upper curve) goes to infinity as the speed of light is approached. The classical kinetic energy (lower curve) agrees with the relativistic result when the speed is small compared with the speed of light.

The second term in the square brackets is much smaller than the first term for everyday speeds. For example, if the speed v is equal to $0.00001c$ (a rather large everyday speed of roughly 6000 mi/h), the second term is only one ten-millionth of a percent of the first term. Later terms in the expansion are smaller still. Hence, for all practical purposes, the kinetic energy for low speeds is

$$K = m_0c^2\left[1 + \frac{1}{2}\left(\frac{v^2}{c^2}\right)\right] - m_0c^2 = m_0c^2 + \tfrac{1}{2}m_0v^2 - m_0c^2$$

PROBLEM-SOLVING NOTE

Rest Mass

To correctly evaluate the relativistic kinetic energy, it is necessary to use the rest mass in Equation 29–9.

Canceling the rest energy, m_0c^2, we find

$$K = \frac{1}{2}m_0v^2$$

Note that the subscript 0 in this expression simply emphasizes the fact that the mass to be used is the rest mass.

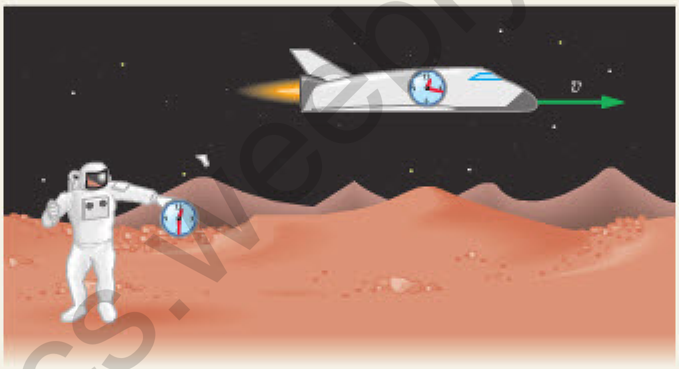
We apply the relativistic kinetic energy in the next Example.

EXAMPLE 29–7 RELATIVISTIC KINETIC ENERGY

An observer watching a high-speed spaceship pass by notices that a clock on board runs slow by a factor of 1.50. **(a)** Find the speed of the clock relative to the observer. **(b)** If the rest mass of the clock is 0.320 kg, what is its kinetic energy?

PICTURE THE PROBLEM

Our sketch shows the high-speed spaceship moving by an observer on a distant asteroid. To the observer, the clock in the ship is running slow. In fact, 1.50 s elapse on the observer's clock in the time it takes the spaceship's clock to advance only 1.00 s.



STRATEGY

a. To begin, we can find the speed of the spaceship, v, by using time dilation, as given in Equation 29–2. In particular, we set $\Delta t = \Delta t_0/\sqrt{1 - v^2/c^2} = 1.50\ \Delta t_0$ and solve for v.

b. Now that we know both the speed and the rest mass ($m_0 = 0.320$ kg) of the clock, we can find its kinetic energy by applying Equation 29–9: $K = m_0c^2/\sqrt{1 - v^2/c^2} - m_0c^2$.

SOLUTION

Part (a)

1. Use time dilation to solve for the speed, v:

$$\Delta t = \frac{\Delta t_0}{\sqrt{1 - \dfrac{v^2}{c^2}}}$$

$$v = c\sqrt{1 - \left(\frac{\Delta t_0}{\Delta t}\right)^2}$$

2. The dilated time, Δt, is greater than the proper time, Δt_0, by a factor of 1.50. Therefore, we substitute $\Delta t_0/\Delta t = 1/1.50$ in the expression for v from Step 1:

$$v = c\sqrt{1 - \left(\frac{1}{1.50}\right)^2} = 0.745c$$

Part (b)

3. Substitute $v = 0.745c$ and $m_0 = 0.320$ kg into the expression for the relativistic kinetic energy (Equation 29–9):

$$K = \frac{m_0c^2}{\sqrt{1 - \dfrac{v^2}{c^2}}} - m_0c^2 = m_0c^2\left(\frac{1}{\sqrt{1 - \dfrac{v^2}{c^2}}} - 1\right)$$

$$= m_0c^2\left(\frac{1}{\sqrt{1 - \dfrac{(0.745c)^2}{c^2}}} - 1\right) = m_0c^2(1.50 - 1)$$

$$= (0.320\text{ kg})(3.00 \times 10^8\text{ m/s})^2(1.50 - 1) = 1.44 \times 10^{16}\text{ J}$$

INSIGHT

In comparison, the classical kinetic energy at this speed is only $\frac{1}{2}m_0v^2 = \frac{1}{2}m_0(0.745c)^2 = m_0c^2(0.278) = 7.99 \times 10^{15}$ J $< m_0c^2(1.50 - 1) = 1.44 \times 10^{16}$ J. In fact, the classical kinetic energy is *always* less than the relativistic kinetic energy at any given speed, as we can see in Figure 29–17. This is because more work must be done to accelerate a particle that is becoming more massive with speed, as is the case with relativity.

PRACTICE PROBLEM

How fast must the clock be moving if its relativistic kinetic energy is to be 5.00×10^{16} J? **[Answer:** $v = 0.931c$]

Some related homework problems: Problem 55, Problem 65, Problem 66

Once again, we see that the speed of light is the ultimate speed possible for an object of finite rest mass. As is clear from Figure 29–17, the kinetic energy of an object goes to infinity as its speed approaches c. Thus to accelerate an object to the speed of light would require an infinite amount of energy. Any finite amount of work will increase the speed only to a speed less than c.

29–7 The Relativistic Universe

It is a profound understatement to say that relativity has revolutionized our understanding of the universe. If we think back over the results presented in the last several sections—time dilation, length contraction, increasing mass, mass–energy equivalence—it is clear that relativity reveals to us a universe that is far richer and more varied in its behavior than was ever imagined before. In fact, it is often said that the universe is not only stranger than we imagine, but stranger than we *can* imagine.

By way of analogy, it is almost as if we had spent our lives on a small island on the equator. We would have no knowledge of snow or deserts or mountain ranges. Although our knowledge of Earth would be valid for our small island, we would have an incomplete picture of the world. Our situation with respect to relativity is similar. Before relativity, our knowledge of the physical universe seemed complete, with only minor details to be worked out. After all, Newton's laws and other fundamental principles of physics gave correct predictions for virtually everything we experienced. What Einstein revealed with his theory of relativity, however, was that we were seeing only a small part of the whole and that the behavior we experience at low speeds cannot be extended to high speeds. We weren't missing small details—like the mid-ocean islanders, we were missing most of the picture.

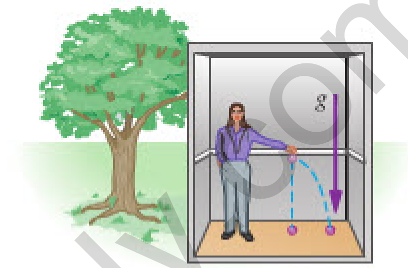
Now, it might seem that relativity plays no significant role in our daily lives, since we do not move at speeds approaching the speed of light. In many respects this observation is correct—relativity is not used to design better cars or airplanes, nor is it used to calculate the orbits needed to send astronauts to the Moon or to Mars. On the other hand, every major hospital has a particle accelerator in its basement to produce radioactive elements for various types of treatments. The accelerator brings particles to speeds very close to the speed of light; hence, relativistic effects cannot be ignored. In fact, for an accelerator to work properly, it must be constructed with relativistic effects taken into account. Similarly, GPS technology can provide accurate positional information only by taking into account relativistic effects—both those having to do with speed, as we have seen in the previous sections, as well as relativistic effects due to gravity, to be discussed in the next section. Thus, as these examples show, we now live in a world where relativity is truly an important part of our everyday lives.

29–8 General Relativity

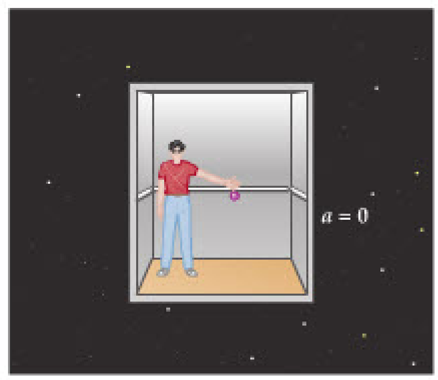
Einstein's **general theory of relativity** applies to accelerated frames of reference and to gravitation. In fact, the theory provides a link between these two types of physical processes that leads to a new interpretation of gravity.

Consider two observers, both standing within closed elevators. Observer 1 is in an elevator that is at rest on the surface of the Earth, as Figure 29–18 shows. If this observer drops or throws an object, it falls toward the floor of the elevator with an acceleration equal to the acceleration of gravity.

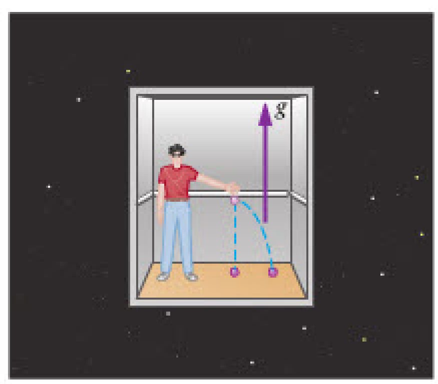
Observer 2 stands in an identical elevator located in deep space. If this elevator is at rest, or moving with a constant velocity, the observer experiences weightlessness within the elevator, as shown in Figure 29–19. If an object is released, it remains in place. Now suppose the elevator is given an upward acceleration equal to the acceleration of gravity, g, as indicated in Figure 29–20. An object that is released now remains at rest relative to the background stars while the floor of the elevator accelerates upward toward it with the acceleration g. Similarly, if observer 2 throws the ball horizontally it will follow a parabolic path to the floor, just



▲ **FIGURE 29–18 A frame of reference in a gravitational field**
An observer in an elevator at rest on Earth's surface. If the observer drops or throws a ball, it falls with a downward acceleration of g.



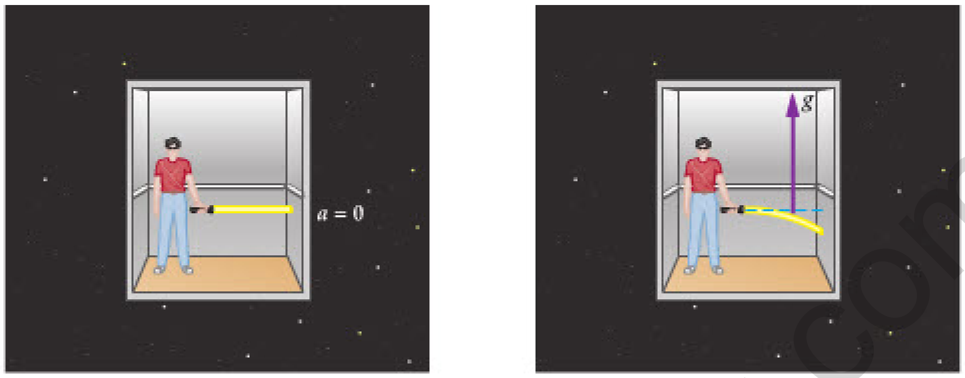
▲ **FIGURE 29–19 An inertial frame of reference with no gravitational field**
An observer in an elevator in deep space experiences weightlessness. If the observer releases an object, it remains at rest.



▲ **FIGURE 29–20 An accelerated frame of reference**
If the elevator in Figure 29–19 is given an upward acceleration of magnitude g, the observer in the elevator will note that objects that are dropped or thrown fall toward the floor of the elevator with an acceleration g, just as for the observer in Figure 29–18.

FIGURE 29–21 A light experiment in two different frames of reference

(a) In a nonaccelerating elevator, a beam of light travels on a straight line as it crosses the elevator. **(b)** In an accelerated elevator, the elevator moves upward as the light crosses the elevator; hence, the light strikes the opposite wall at a lower level. The path of the light in this case appears parabolic to the observer riding in the elevator.



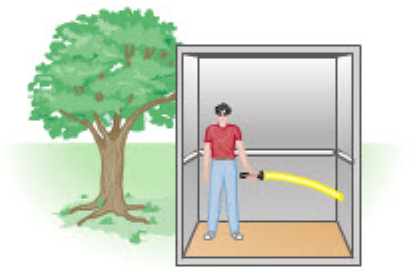


FIGURE 29–22 The principle of equivalence

By the principle of equivalence, a beam of light in a gravitational field should follow a parabolic path, just as in the accelerated elevator in Figure 29–21. The amount of bending of the light's path has been exaggerated here for clarity.

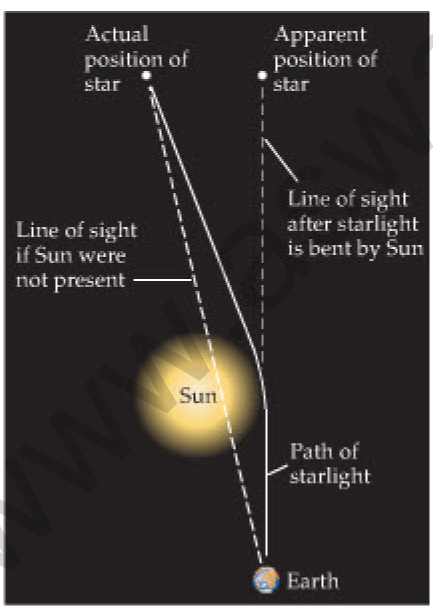


FIGURE 29–23 Gravitational bending of light

As light from a distant star passes close to the Sun, it is bent. The result is that an observer on Earth sees the star along a line of sight that is displaced away from the center of the Sun.

as for observer 1. In addition, the floor of the elevator exerts a force mg on the feet of observer 1 to give that observer (whose mass is m) an upward acceleration g.

We conclude, then, that when observer 2 conducts an experiment in his accelerating elevator, the results are the same as those obtained by observer 1 in her elevator at rest on Earth. Einstein extended these observations to a general principle, the **principle of equivalence:**

Principle of Equivalence

All physical experiments conducted in a uniform gravitational field and in an accelerated frame of reference give identical results.

Thus the two observers cannot tell, without looking outside the elevator, whether they are at rest in a gravitational field or in deep space in an accelerating elevator.

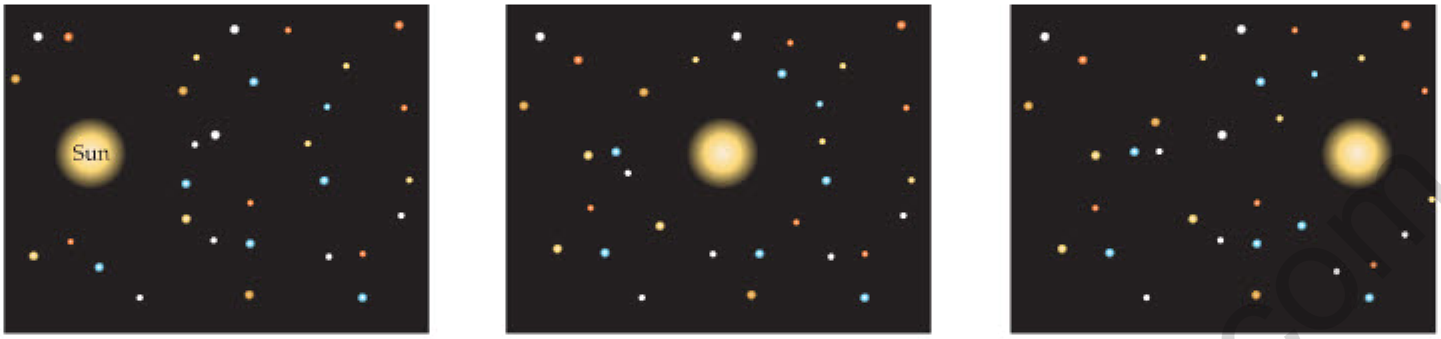
Now let's apply the principle of equivalence to a simple experiment involving light. If the observer in **Figure 29–21 (a)** shines a flashlight toward the opposite wall of the elevator (which has zero acceleration), the light strikes the wall at its initial height. If the same experiment is conducted in an accelerated elevator, as in **Figure 29–21 (b)**, the elevator is accelerating upward during the time the light travels across the elevator. Thus, by the time the light reaches the far wall, it strikes it at a lower level. In fact, the light has followed a parabolic path, just as one would expect for a ball that was projected horizontally.

Applying the principle of equivalence, Einstein concluded that a beam of light in a gravitational field must also bend downward, just as it does in an accelerated elevator; that is, *gravity bends light*. This phenomenon is illustrated in **Figure 29–22**, where the amount of bending has been exaggerated for clarity.

In order to put Einstein's prediction to the test, it is necessary to increase the amount of bending as much as possible to make it large enough to be measured. Thus, we need to use the strongest gravitational field available. In our solar system, the strongest gravitational field is provided by the Sun; hence, experiments were planned to look for the bending of light produced by the Sun.

To see what effect the Sun's gravitational field might have on light, consider the Sun and a ray of light from a distant star, as shown in **Figure 29–23**. As the light passes the Sun, it is bent, as indicated. So an observer on Earth must look in a direction that is farther from the Sun than the actual direction of the distant star—the Sun's gravitational field displaces the distant stars to apparent positions farther from the Sun. If we imagine the Sun moving in front of a background field of stars, as in **Figure 29–24**, the stars near the Sun are displaced outward. It is almost as if the Sun were a lens, slightly distorting the scene behind it.

Because the Sun is so bright, it is possible to carry out an experiment like that shown in Figures 29–23 and 29–24 only during a total eclipse of the Sun, when the Sun's light is blocked by the Moon. During the eclipse, photographs can be taken to show the positions of the background stars. Later, these photographs can be compared with photographs of the same star field taken 6 months later, when the Sun is on the other side of Earth. Comparing the photographs allows one to measure the displacement of the stars. This experiment was carried out during an expedition to

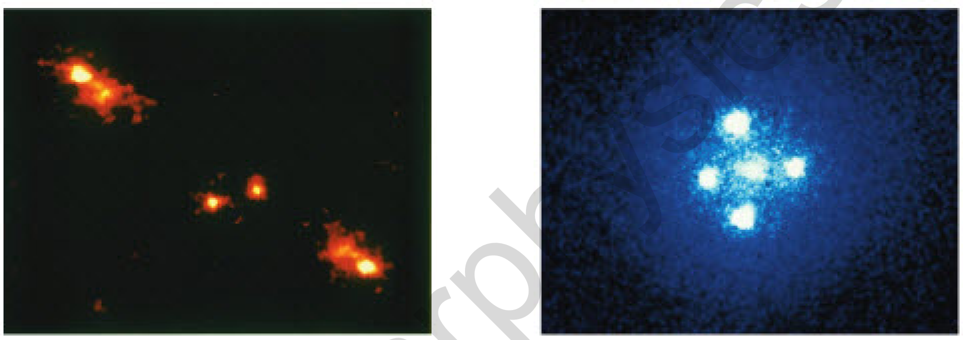


▲ **FIGURE 29–24 Bending of light near the Sun**
As the Sun moves across a starry background, the stars near it appear to be displaced outward, away from the center of the Sun. This is the effect that was used in the first experimental confirmation of general relativity.

Africa in 1919 by Sir Arthur Eddington. His results confirmed the predictions of the general theory of relativity and made Einstein a household name.

Since gravity can bend light, the more powerful the gravitational force, the more the bending, and the more dramatic the results. In Figure 12–18 we saw what can happen when a large galaxy or a cluster of galaxies, with its immense gravitational field, lies between us and a more distant galaxy. The intermediate galaxy can produce significant bending of light, resulting in multiple images of the distant galaxy that can form arcs or crosses. Some examples of such *gravitational lensing* are shown in **Figure 29–25**.

REAL-WORLD PHYSICS
Gravitational lensing



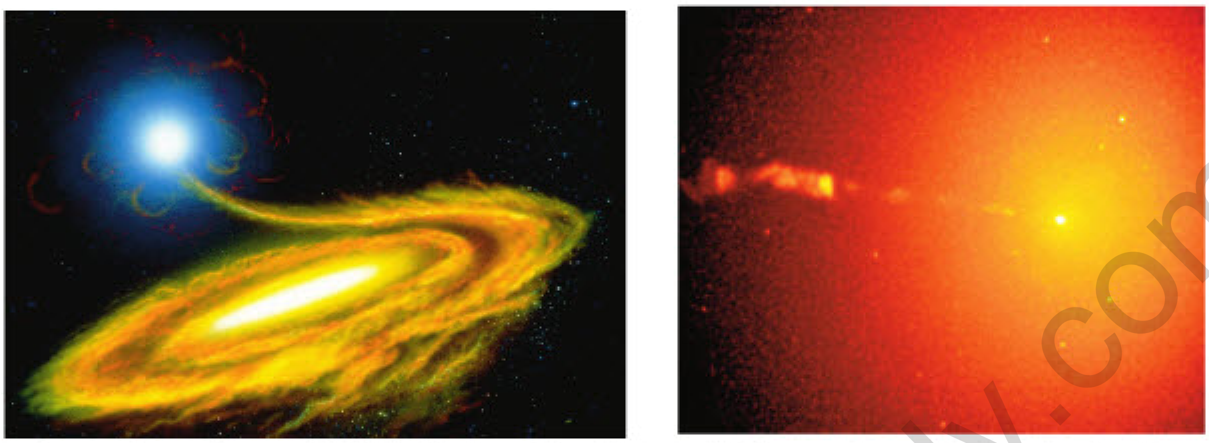
▲ **FIGURE 29–25 Gravitational lensing**
The images created by gravitational lenses take a number of forms. Sometimes the light from a distant object is stretched out into an arc or even a complete ring. In other cases, such as that of the distant quasar at left, we may see a pair of images. (The quasar appears at the upper left and lower right. The lensing galaxy is not visible in the photo—the small dots at center are unrelated objects.) In still other instances, four images may be produced, as in the famous "Einstein cross" shown at right. The lensing galaxy at the center is some 400 million light-years from us, while the quasar whose multiple images surround it is about 20 times farther away.

An intense gravitational field can also be produced when a star burns up its nuclear fuel and collapses to a very small size. In such a case, the gravitational field can become strong enough to actually trap light; that is, to bend it around to the point that it cannot escape the star. Since such a star cannot emit light of its own, it is referred to as a **black hole.** Black holes, by definition, cannot be directly observed; however, their presence can be inferred by their gravitational effects on other bodies. It is also possible to detect the intense radiation emitted by ionized matter as it falls into a black hole. (Recall from Section 25–1 that accelerated charges produce electromagnetic radiation.) By these and other means, the existence of black holes in the centers of many galaxies has been firmly established. In fact, it is now thought that black holes may be relatively common in the universe.

REAL-WORLD PHYSICS
Black holes

Recall that in Chapter 12 we calculated the escape speed for Earth. The result was

$$v_e = \sqrt{\frac{2GM_E}{R_E}}$$



Because they swallow up all light, black holes are themselves invisible. However, they can be detected indirectly in several ways. If one member of a binary star collapses to become a black hole, an accretion disk may form around it, as shown in the artist's conception at left. The accretion disk is a ring of matter wrenched from its companion, whirling around at ever greater speeds as it spirals into the black hole. The radiation emitted by this matter as it falls into the abyss is the signature of a black hole. The same mechanism, on a much vaster scale, probably accounts for the enormous radiation from active galaxies, such as M87 (right), a giant elliptical galaxy with an enormous jet of matter emanating from its nucleus. Black holes millions of times more massive than the Sun are thought to be present in the cores of such galaxies—and possibly in all galaxies.

Replacing M_E with M and R_E with R gives the escape speed for any spherical body of mass M and radius R. Now, if we set the escape speed equal to the speed of light, $v_e = c$, we find the following:

$$v_e = \sqrt{\frac{2GM}{R}} = c$$

Solving for R, we find that for an astronomical body to be a black hole, its radius must be no greater than

$$R = \frac{2GM}{c^2} \qquad 29\text{–}10$$

Although this calculation is too simplistic and not entirely correct, the final result given in Equation 29–10, known as the **Schwarzschild radius,** does indeed agree with the results of general relativity. In the following Exercise, we calculate the radius of a black hole for a specific case.

EXERCISE 29–5

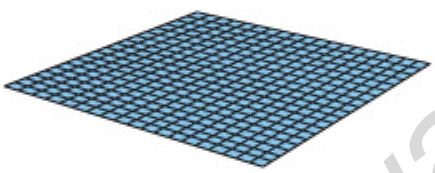
To what radius must Earth be condensed for it to become a black hole?

SOLUTION

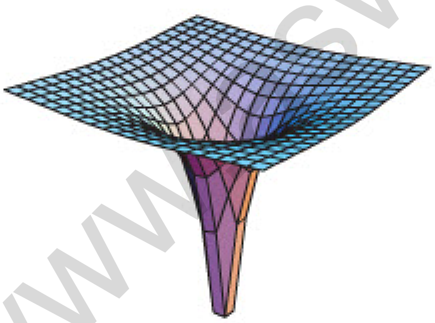
Substituting G, M_E, and c in Equation 29–10 we get

$$R = \frac{2GM_E}{c^2} = \frac{2(6.67 \times 10^{-11}\,\mathrm{N \cdot m^2/kg^2})(5.98 \times 10^{24}\,\mathrm{kg})}{(3.00 \times 10^8\,\mathrm{m/s})^2} = 8.86\ \mathrm{mm}$$

Thus Earth would have to be reduced to roughly the size of a walnut for it to become a black hole.



(a) Flat space, away from massive objects



(b) Warped space, near a massive object

FIGURE 29–26 Warped space and black holes

(a) Regions of space that are far from any large masses can be thought of as flat. In these regions, light propagates in straight lines. **(b)** Near a large concentrated mass, such as a black hole, space can be thought of as "warped." In these regions, the paths of light rays are bent.

A convenient way to visualize the effects of intense gravitational fields is to think of space as a sheet of rubber with a square array of grid lines, as shown in Figure 29–26. In the absence of mass, the sheet is flat, and the grid lines are straight, as in Figure 29–26 (a). In this case, a beam of light follows a straight-line path, parallel to a grid line. If a large mass is present, however, the sheet of rubber is deformed, as in Figure 29–26 (b), and light rays follow the curved paths of the grid lines. In cases where one large mass orbits another, as in **Figure 29–27**, the result is a series of ripples moving outward through the rubber sheet. These ripples represent **gravity waves,** one of the many intriguing predictions of general relativity.

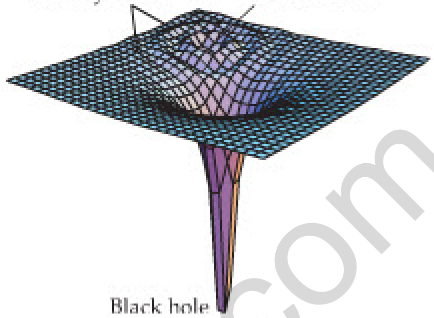
When a gravity wave passes through a given region of space, it causes a local deformation of space, as Figure 29–27 suggests. Early attempts to detect gravity waves were based on measuring the distortion a gravity wave would produce in a large metal bar. Unfortunately, the sensitivity of these devices was too low to detect the weak waves that are thought to pass through the Earth all the time. Ironically, these detectors *were* sensitive enough to detect the relatively strong gravity waves that must have accompanied the 1987 supernova explosion in the nearby Large Magellanic Cloud (a small satellite galaxy of our Milky Way). As luck would have it, however, none of the detectors were operating at the time.

The next generation of gravity wave detectors is now nearing completion. These detectors, which go by the name of Laser Interferometer Gravitational Wave Observatory, or LIGO for short, should be sensitive enough to detect several gravitational wave events per year. One type of event that LIGO will be looking for is the final death spiral of a neutron star as it plunges into a black hole. The neutron star might orbit the black hole for 150,000 years, but only in the last 15 minutes of its life does it find fame, because during those few minutes its acceleration is great enough to produce gravity waves detectable by LIGO.

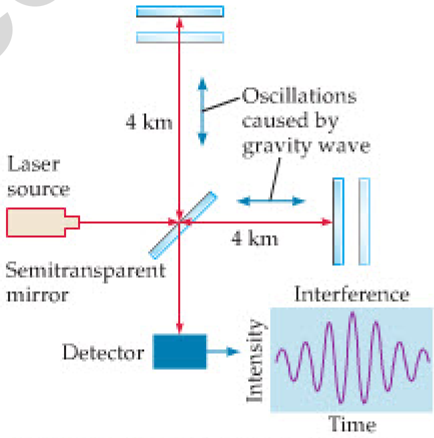
The basic operating principle of LIGO is to send a laser beam in two different directions along 4-km-long vacuum tubes. At the far end of each tube the beam is reflected back to its starting point. If the two beams travel equal distances, they will interfere constructively when reunited, just like the two rays that meet at the center of a two-slit interference pattern, as discussed in Section 28–2. If the path lengths along the two tunnels are slightly different, however, the beams will have at least partial destructive interference when they combine. The resulting difference in intensity is what LIGO will measure.

The connection with gravity waves is indicated in **Figure 29–28**. Here we see that as a gravity wave passes a LIGO facility, it will cause one tube to increase in length and the other tube to decrease in length. Hence, if the observatory can detect a small enough change in length, it will "see" the gravity wave. Just how much change in length is a gravity wave expected to produce? Incredibly, a typical strong gravity wave will change the length of a tube by less than the diameter of an atomic nucleus. Thus, the task for LIGO is to measure this small change in length in a tube that is 4 km long—no easy task, but the LIGO scientists are confident it can be done.

If gravity waves are indeed observed in the early years of the twenty-first century, as expected, it will open an entirely new window on the universe. So far, information about the universe comes to our telescopes by way of electromagnetic waves. We have certainly learned a lot about the universe in this way; in fact, whenever we observe in a different part of the electromagnetic spectrum, we find surprising new phenomena. One can only imagine what additional wonders will be observed when an entirely different spectrum—the gravity wave spectrum—is explored.



▲ **FIGURE 29–27 Gravity waves**
Gravity waves can be thought of as "ripples" in the warped space described in Figure 29–26. In the case illustrated here, a neutron star orbits a black hole. As a result of its acceleration, the neutron star emits gravity waves.



▲ **FIGURE 29–28 How LIGO detects a gravity wave**
When a gravity wave passes through the Earth, it "distorts" everything in its path. The idea behind LIGO is to detect these distortions using interference effects similar to those observed in Young's two-slit experiment (Chapter 28). In particular, LIGO sends laser light along two 4-km vacuum tubes oriented at right angles to one another. When a gravity wave passes, one arm will be lengthened and the other arm will be shortened. The resulting difference in path length changes the interference pattern, allowing us to "see" the gravity wave.

THE BIG PICTURE PUTTING PHYSICS IN CONTEXT

LOOKING BACK

The operation of a light clock, as described in Section 29–2, depends on the reflection of light from a mirror, which was studied in detail in Chapters 26 and 27.

The kinetic energy introduced originally in Chapter 7, $K = \frac{1}{2}mv^2$, is shown in this chapter to be valid only for speeds small compared to the speed of light. The generalized kinetic energy, valid for any speed, is presented in Section 29–6.

LOOKING AHEAD

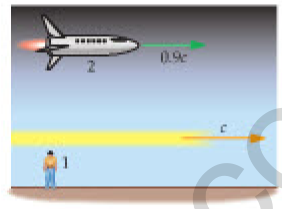
Photons, which always travel at the speed of light when propagating through a vacuum, are the ultimate relativistic particles. We shall see in Chapter 30 that even though their rest mass is zero, they still have finite momentum.

The energy released in nuclear reactions is due to a conversion of mass to energy, in accordance with $E = mc^2$ (Section 29–6). This will be discussed in detail in Chapter 32.

CHAPTER SUMMARY

29–1 THE POSTULATES OF SPECIAL RELATIVITY

Einstein's theory of relativity is based on just two postulates.



Equivalence of Physical Laws

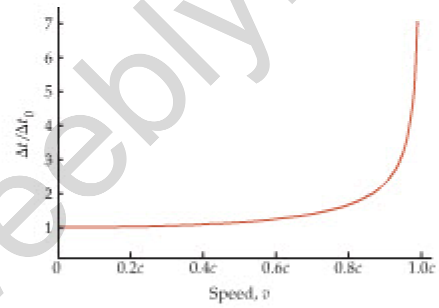
The laws of physics are the same in all inertial frames of reference.

Constancy of the Speed of Light

The speed of light in a vacuum, $c = 3.00 \times 10^8$ m/s, is the same in all inertial frames of reference, independent of the motion of the source or the receiver.

29–2 THE RELATIVITY OF TIME AND TIME DILATION

Clocks that move relative to one another keep time at different rates. In particular, a moving clock runs slower than one that is at rest relative to a given observer.



Proper Time

The proper time, Δt_0, is the amount of time separating two events that occur at the same location.

Time Dilation

If two events separated by a proper time Δt_0 occur in a frame of reference moving with a speed v relative to an observer, the dilated time measured by the observer, Δt, is given by

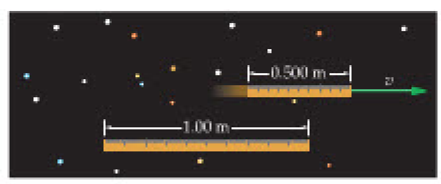
$$\Delta t = \frac{\Delta t_0}{\sqrt{1 - v^2/c^2}} \qquad 29\text{–}2$$

Space Travel and Biological Aging

Time dilation applies equally to all physical processes, including chemical reactions and biological functions.

29–3 THE RELATIVITY OF LENGTH AND LENGTH CONTRACTION

The length of an object depends on its speed relative to a given observer.



Proper Length

The proper length, L_0, is the distance between two points as measured by an observer who is at rest with respect to them.

Contracted Length

An object with a proper length L_0 moving with a speed v relative to an observer has a contracted length L given by:

$$L = L_0\sqrt{1 - \frac{v^2}{c^2}} \qquad 29\text{–}3$$

Direction of Contraction

Lengths contract only in the direction of motion.

29–4 THE RELATIVISTIC ADDITION OF VELOCITIES

Simple velocity addition, $v = v_1 + v_2$, is valid only in the limit of very small speeds.

Relativistic Addition of Velocities

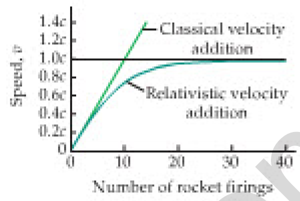
Suppose object 1 moves with a velocity v_{13} relative to object 3. If object 2 moves along the same straight line with a velocity v_{21} relative to object 1, the velocity of object 2 relative to object 3, v_{23}, is

$$v_{23} = \frac{v_{21} + v_{13}}{1 + \dfrac{v_{21}v_{13}}{c^2}} \qquad 29\text{–}4$$

Ultimate Speed

If two velocities, v_1 and v_2, are less than the speed of light, c, then their relativistic sum, v, is also less than c. Thus it is not possible to increase the speed of an object from a value less than c to a value greater than c.

29–5 RELATIVISITIC MOMENTUM

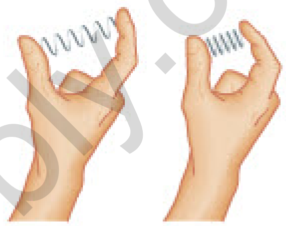


The momentum of an object of mass m and speed v is

$$p = \frac{mv}{\sqrt{1 - \frac{v^2}{c^2}}} \qquad 29\text{–}5$$

This expression is valid for all speeds between zero and the speed of light and reduces to $p = mv$ in the limit of small speeds.

29–6 RELATIVISITIC ENERGY AND $E = mc^2$



One of the most important results of relativity is that mass is another form of energy.

To put it another way, mass and energy are two different aspects of the same quantity.

Relativistic Energy

The total energy, E, of an object with rest mass m_0 and speed v is

$$E = \frac{m_0c^2}{\sqrt{1 - \frac{v^2}{c^2}}} = mc^2 \qquad 29\text{–}7$$

Rest Energy

When an object is at rest, its energy E_0 is

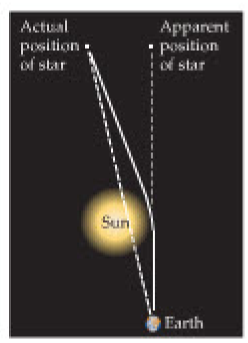
$$E_0 = m_0c^2 \qquad 29\text{–}8$$

Relativistic Kinetic Energy

The relativistic kinetic energy, K, of an object of rest mass m_0 moving with a speed v is its total energy, E, minus its rest energy, E_0. In particular,

$$K = \frac{m_0c^2}{\sqrt{1 - \frac{v^2}{c^2}}} - m_0c^2 \qquad 29\text{–}9$$

29–8 GENERAL RELATIVITY



General relativity deals with accelerated frames of reference and with gravity.

Principle of Equivalence

One of the basic principles on which general relativity is founded is the following: All physical experiments conducted in a gravitational field and in an accelerated frame of reference give identical results.

Radius of a Black Hole

For an astronomical body of mass M and radius R to be a black hole, its radius must be less than or equal to the following value, known as the Schwarzschild radius:

$$R = \frac{2GM}{c^2} \qquad 29\text{–}10$$

PROBLEM-SOLVING SUMMARY

Type of Problem	Relevant Physical Concepts	Related Examples
Find the time between two events as measured by different observers.	The time between events that occur at the same location is the proper time, Δt_0. The time measured by an observer with a relative speed v is $\Delta t = \Delta t_0/\sqrt{1 - v^2/c^2}$.	Examples 29–1, 29–2 Active Example 29–1
Find the distance between two points as measured by different observers.	The distance between points that are at rest relative to the observer is the proper length, L_0. The distance measured by an observer with a relative speed v is $L = L_0\sqrt{1 - v^2/c^2}$.	Example 29–3

Add velocities relativistically.	The correct rule for adding two velocities, v_{21} and v_{13}, to obtain the total velocity v_{23} is $v_{23} = (v_{21} + v_{13})/(1 + v_{21}v_{13}/c^2)$. In the limit of small velocities, this equation reduces to simple addition, as one would expect. For velocities comparable to the speed of light, simple addition fails, because the final velocity can never be greater than c.	Example 29–4 Active Example 29–2
Find the relativistic momentum.	The momentum of an object with a rest mass m_0 and speed v is $p = m_0 v/\sqrt{1 - v^2/c^2}$.	Example 29–5
Find the energy equivalence of a given mass.	The rest energy of an object with a rest mass m_0 is $E_0 = m_0 c^2$.	Example 29–6
Find the relativistic kinetic energy.	The kinetic energy of an object with a rest mass m_0 and speed v is $K = m_0 c^2/\sqrt{1 - v^2/c^2} - m_0 c^2$.	Example 29–7

CONCEPTUAL QUESTIONS

(Answers to odd-numbered Conceptual Questions can be found in the back of the book.)

1. Some distant galaxies are moving away from us at speeds greater than $0.5c$. What is the speed of the light received on Earth from these galaxies? Explain.
2. The speed of light in glass is less than c. Why is this not a violation of the second postulate of relativity?
3. How would velocities add if the speed of light were infinitely large? Justify your answer by considering Equation 29–4.
4. Describe some of the everyday consequences that would follow if the speed of light were 35 mi/h.
5. When we view a distant galaxy, we notice that the light coming from it has a longer wavelength (it is "red-shifted") than the corresponding light here on Earth. Is this consistent with the postulate that all observers measure the same speed of light? Explain.
6. According to the theory of relativity, the maximum speed for any particle is the speed of light. Is there a similar restriction on the maximum energy of a particle? Is there a maximum momentum? Explain.
7. Give an argument that shows that an object of finite mass cannot be accelerated from rest to a speed greater than the speed of light in a vacuum.

PROBLEMS AND CONCEPTUAL EXERCISES

Note: Answers to odd-numbered Problems and Conceptual Exercises can be found in the back of the book. **IP** *denotes an integrated problem, with both conceptual and numerical parts;* **BIO** *identifies problems of biological or medical interest;* **CE** *indicates a conceptual exercise.* **Predict/Explain** *problems ask for two responses:* **(a)** *your prediction of a physical outcome, and* **(b)** *the best explanation among three provided. On all problems, red bullets (•, ••, •••) are used to indicate the level of difficulty. (For speeds, v, that are much less than the speed of light, c, the following expansion may be used:* $1/\sqrt{1 - v^2/c^2} \approx 1 + \frac{1}{2}(v^2/c^2)$.*)*

SECTION 29–1 THE POSTULATES OF SPECIAL RELATIVITY

1. • **CE Predict/Explain** You are in a spaceship, traveling directly away from the Moon with a speed of $0.9c$. A light signal is sent in your direction from the surface of the Moon. **(a)** As the signal passes your ship, do you measure its speed to be greater than, less than, or equal to $0.1c$? **(b)** Choose the *best explanation* from among the following:
   I. The speed you measure will be greater than $0.1c$; in fact, it will be c, since all observers in inertial frames measure the same speed of light.
   II. You will measure a speed less than $0.1c$ because of time dilation, which causes clocks to run slow.
   III. When you measure the speed you will find it to be $0.1c$, which is the difference between c and $0.9c$.
2. • Albert is piloting his spaceship, heading east with a speed of $0.90c$. Albert's ship sends a light beam in the forward (eastward) direction, which travels away from his ship at a speed c. Meanwhile, Isaac is piloting his ship in the westward direction, also at $0.90c$, toward Albert's ship. With what speed does Isaac see Albert's light beam pass his ship?

SECTION 29–2 THE RELATIVITY OF TIME AND TIME DILATION

3. • **CE** A street performer tosses a ball straight up into the air (event 1) and then catches it in his mouth (event 2). For each of the following observers, state whether the time they measure between these two events is the proper time or the dilated time: **(a)** the street performer; **(b)** a stationary observer on the other side of the street; **(c)** a person sitting at home watching the performance on TV; **(d)** a person observing the performance from a moving car.
4. • **CE Predict/Explain** A clock in a moving rocket is observed to run slow. **(a)** If the rocket reverses direction, does the clock run slow, fast, or at its normal rate? **(b)** Choose the *best explanation* from among the following:
   I. The clock will run slow, just as before. The rate of the clock depends only on relative speed, not on direction of motion.
   II. When the rocket reverses direction the rate of the clock reverses too, and this makes it run fast.
   III. Reversing the direction of the rocket undoes the the time dilation effect, and so the clock will now run at its normal rate.

5. • **CE Predict/Explain** Suppose you are a traveling salesman for SSC, the Spacely Sprockets Company. You travel on a spaceship that reaches speeds near the speed of light, and you are paid by the hour. **(a)** When you return to Earth after a sales trip, would you prefer to be paid according to the clock at Spacely Sprockets universal headquarters on Earth, according to the clock on the spaceship in which you travel, or would your pay be the same in either case? **(b)** Choose the *best explanation* from among the following:
   I. You want to be paid according to the clock on Earth, because the clock on the spaceship runs slow when it approaches the speed of light.
   II. Collect your pay according to the clock on the spaceship because according to you the clock on Earth has run slow.
   III. Your pay would be the same in either case because motion is relative, and all inertial observers will agree on the amount of time that has elapsed.

6. • A neon sign in front of a café flashes on and off once every 4.1 s, as measured by the head cook. How much time elapses between flashes of the sign as measured by an astronaut in a spaceship moving toward Earth with a speed of 0.84c?

7. • A lighthouse sweeps its beam of light around in a circle once every 7.5 s. To an observer in a spaceship moving away from Earth, the beam of light completes one full circle every 15 s. What is the speed of the spaceship relative to Earth?

8. • Refer to Example 29–1. How much does Benny age if he travels to Vega with a speed of 0.9995c?

9. • As a spaceship flies past with speed v, you observe that 1.0000 s elapses on the ship's clock in the same time that 1.0000 min elapses on Earth. How fast is the ship traveling, relative to the Earth? (Express your answer as a fraction of the speed of light.)

10. • Donovan Bailey set a world record for the 100-m dash on July 27, 1996. If observers on a spaceship moving with a speed of 0.7705c relative to Earth saw Donovan Bailey's run and measured his time to be 15.44 s, find the time that was recorded on Earth.

11. • Find the average distance (in the Earth's frame of reference) covered by the muons in Example 29–2 if their speed relative to Earth is 0.750c.

12. •• **The Pi Meson** An elementary particle called a pi meson (or pion for short) has an average lifetime of 2.6×10^{-8} s when at rest. If a pion moves with a speed of 0.99c relative to Earth, find **(a)** the average lifetime of the pion as measured by an observer on Earth and **(b)** the average distance traveled by the pion as measured by the same observer. **(c)** How far would the pion have traveled relative to Earth if relativistic time dilation did not occur?

13. •• **The Σ^- Particle** The Σ^- is an exotic particle that has a lifetime (when at rest) of 0.15 ns. How fast would it have to travel in order for its lifetime, as measured by laboratory clocks, to be 0.25 ns?

14. •• **IP** **(a)** Is it possible for you to travel far enough and fast enough so that when you return from a trip, you are younger than your stay-at-home sister, who was born 5.0 y after you? **(b)** Suppose you fly on a rocket with a speed $v = 0.99c$ for 1 y, according to the ship's clocks and calendars. How much time elapses on Earth during your 1-y trip? **(c)** If you were 22 y old when you left home and your sister was 17, what are your ages when you return?

15. •• The radar antenna on a navy ship rotates with an angular speed of 0.29 rad/s. What is the angular speed of the antenna as measured by an observer moving away from the antenna with a speed of 0.82c?

16. •• An observer moving toward Earth with a speed of 0.95c notices that it takes 5.0 min for a person to fill her car with gas. Suppose, instead, that the observer had been moving away from Earth with a speed of 0.80c. How much time would the observer have measured for the car to be filled in this case?

17. •• **IP** An astronaut moving with a speed of 0.65c relative to Earth measures her heart rate to be 72 beats per minute. **(a)** When an Earth-based observer measures the astronaut's heart rate, is the result greater than, less than, or equal to 72 beats per minute? Explain. **(b)** Calculate the astronaut's heart rate as measured on Earth.

18. •• **BIO** Newly sprouted sunflowers can grow at the rate of 0.30 in. per day. One such sunflower is left on Earth, and an identical one is placed on a spacecraft that is traveling away from Earth with a speed of 0.94c. How tall is the sunflower on the spacecraft when a person on Earth says his is 2.0 in. high?

19. •• An astronaut travels to Mars with a speed of 8350 m/s. After a month (30.0 d) of travel, as measured by clocks on Earth, how much difference is there between the Earth clock and the spaceship clock? Give your answer in seconds.

20. •• As measured in Earth's frame of reference, two planets are 424,000 km apart. A spaceship flies from one planet to the other with a constant velocity, and the clocks on the ship show that the trip lasts only 1.00 s. How fast is the ship traveling?

21. •• Captain Jean-Luc is piloting the USS *Enterprise XXIII* at a constant speed $v = 0.825c$. As the *Enterprise* passes the planet Vulcan, he notices that his watch and the Vulcan clocks both read 1:00 P.M. At 3:00 P.M., according to his watch, the *Enterprise* passes the planet Endor. If the Vulcan and Endor clocks are synchronized with each other, what time do the Endor clocks read when the *Enterprise* passes by?

22. ••• **IP** A plane flies with a constant velocity of 222 m/s. The clocks on the plane show that it takes exactly 2.00 h to travel a certain distance. **(a)** According to ground-based clocks, will the flight take slightly more or slightly less than 2.00 h? **(b)** Calculate how much longer or shorter than 2.00 h this flight will last, according to clocks on the ground.

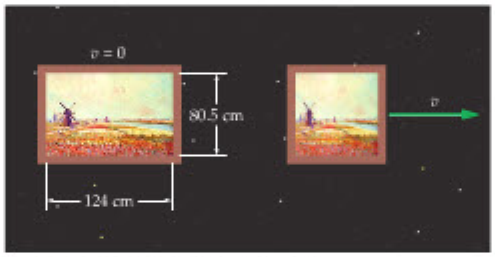
SECTION 29–3 THE RELATIVITY OF LENGTH AND LENGTH CONTRACTION

23. • **CE** If the universal speed of light in a vacuum were larger than 3.00×10^8 m/s, would the effects of length contraction be greater or less than they are now? Explain.

24. • How fast does a 250-m spaceship move relative to an observer who measures the ship's length to be 150 m?

25. • Suppose the speed of light in a vacuum were only 25.0 mi/h. Find the length of a bicycle being ridden at a speed of 20.0 mi/h as measured by an observer sitting on a park bench, given that its proper length is 1.89 m.

26. • A rectangular painting is 124 cm wide and 80.5 cm high, as indicated in Figure 29–29. At what speed, v, must the painting move parallel to its width if it is to appear to be square?



▲ **FIGURE 29–29** Problem 26

27. • The Linac portion of the Fermilab Tevatron contains a high-vacuum tube that is 64 m long, through which protons travel with an average speed $v = 0.65c$. How long is the Linac tube, as measured in the proton's frame of reference?

28. •• A cubical box is 0.75 m on a side. **(a)** What are the dimensions of the box as measured by an observer moving with a speed of $0.88c$ parallel to one of the edges of the box? **(b)** What is the volume of the box, as measured by this observer?

29. •• When parked, your car is 5.0 m long. Unfortunately, your garage is only 4.0 m long. **(a)** How fast would your car have to be moving for an observer on the ground to find your car shorter than your garage? **(b)** When you are driving at this speed, how long is your garage, as measured in the car's frame of reference?

30. •• An astronaut travels to a distant star with a speed of $0.55c$ relative to Earth. From the astronaut's point of view, the star is 7.5 ly from Earth. On the return trip, the astronaut travels with a speed of $0.89c$ relative to Earth. What is the distance covered on the return trip, as measured by the astronaut? Give your answer in light-years.

31. •• **IP** Laboratory measurements show that an electron traveled 3.50 cm in a time of 0.200 ns. **(a)** In the rest frame of the electron, did the lab travel a distance greater than or less than 3.50 cm? Explain. **(b)** What is the electron's speed? **(c)** In the electron's frame of reference, how far did the laboratory travel?

32. •• You and a friend travel through space in identical spaceships. Your friend informs you that he has made some length measurements and that his ship is 150 m long but that yours is only 120 m long. From your point of view, **(a)** how long is your friend's ship, **(b)** how long is your ship, and **(c)** what is the speed of your friend's ship relative to yours?

33. •• A ladder 5.0 m long leans against a wall inside a spaceship. From the point of view of a person on the ship, the base of the ladder is 3.0 m from the wall, and the top of the ladder is 4.0 m above the floor. The spaceship moves past the Earth with a speed of $0.90c$ in a direction parallel to the floor of the ship. Find the angle the ladder makes with the floor as seen by an observer on Earth.

34. ••• When traveling past an observer with a relative speed v, a rocket is measured to be 9.00 m long. When the rocket moves with a relative speed $2v$, its length is measured to be 5.00 m. **(a)** What is the speed v? **(b)** What is the proper length of the rocket?

35. ••• **IP** The starships *Picard* and *La Forge* are traveling in the same direction toward the Andromeda galaxy. The *Picard* moves with a speed of $0.90c$ relative to the *La Forge*. A person on the *La Forge* measures the length of the two ships and finds the same value. **(a)** If a person on the *Picard* also measures the lengths of the two ships, which of the following is observed: (i) the *Picard* is longer; (ii) the *La Forge* is longer; or (iii) both ships have the same length? Explain. **(b)** Calculate the ratio of the proper length of the *Picard* to the proper length of the *La Forge*.

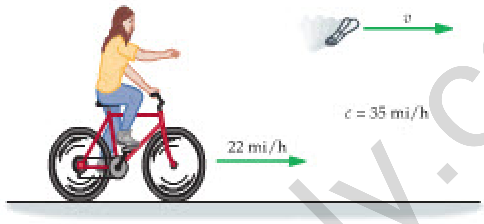
SECTION 29–4 THE RELATIVISTIC ADDITION OF VELOCITIES

36. • A spaceship moving toward Earth with a speed of $0.90c$ launches a probe in the forward direction with a speed of $0.10c$ relative to the ship. Find the speed of the probe relative to Earth.

37. • Suppose the probe in Problem 36 is launched in the opposite direction to the motion of the spaceship. Find the speed of the probe relative to Earth in this case.

38. • A spaceship moving relative to an observer with a speed of $0.70c$ shines a beam of light in the forward direction, directly toward the observer. Use Equation 29–4 to calculate the speed of the beam of light relative to the observer.

39. • Suppose the speed of light is 35 mi/h. A paper girl riding a bicycle at 22 mi/h throws a rolled-up newspaper in the forward direction, as shown in Figure 29–30. If the paper is thrown with a speed of 19 mi/h relative to the bike, what is its speed, v, with respect to the ground?



▲ **FIGURE 29–30** Problem 39

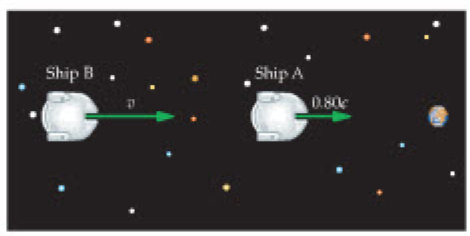
40. •• Two asteroids head straight for Earth from the same direction. Their speeds relative to Earth are $0.80c$ for asteroid 1 and $0.60c$ for asteroid 2. Find the speed of asteroid 1 relative to asteroid 2.

41. •• Two rocket ships approach Earth from opposite directions, each with a speed of $0.8c$ relative to Earth. What is the speed of one ship relative to the other?

42. •• A spaceship and an asteroid are moving in the same direction away from Earth with speeds of $0.77c$ and $0.41c$, respectively. What is the relative speed between the spaceship and the asteroid?

43. •• An electron moves to the right in a laboratory accelerator with a speed of $0.84c$. A second electron in a different accelerator moves to the left with a speed of $0.43c$ relative to the first electron. Find the speed of the second electron relative to the lab.

44. •• **IP** Two rocket ships are racing toward Earth, as shown in Figure 29–31. Ship A is in the lead, approaching the Earth at $0.80c$ and separating from ship B with a relative speed of $0.50c$. **(a)** As seen from Earth, what is the speed, v, of ship B? **(b)** If ship A increases its speed by $0.10c$ relative to the Earth, does the relative speed between ship A and ship B increase by $0.10c$, by more than $0.10c$, or by less than $0.10c$? Explain. **(c)** Find the relative speed between ships A and B for the situation described in part (b).



▲ **FIGURE 29–31** Problem 44

45. •• **IP** An inventor has proposed a device that will accelerate objects to speeds greater than c. He proposes to place the object to be accelerated on a conveyor belt whose speed is $0.80c$. Next, the entire system is to be placed on a second conveyor belt that also has a speed of $0.80c$, thus producing a final speed of $1.6c$. **(a)** Construction details aside, should you invest in this scheme? **(b)** What is the actual speed of the object relative to the ground?

SECTION 29–5 RELATIVISTIC MOMENTUM

46. • A 4.5×10^6-kg spaceship moves away from Earth with a speed of $0.75c$. What is the magnitude of the ship's **(a)** classical and **(b)** relativistic momentum?

47. • An asteroid with a mass of 8.2×10^{11} kg is observed to have a relativistic momentum of magnitude 7.74×10^{20} kg · m/s. What is the speed of the asteroid relative to the observer?

48. •• An object has a relativistic momentum that is 7.5 times greater than its classical momentum. What is its speed?

49. •• A football player with a mass of 88 kg and a speed of 2.0 m/s collides head-on with a player from the opposing team whose mass is 120 kg. The players stick together and are at rest after the collision. Find the speed of the second player, assuming the speed of light is 3.0 m/s.

50. •• In the previous problem, suppose the speed of the second player is 1.2 m/s. What is the speed of the players after the collision?

51. •• A space probe with a rest mass of 8.2×10^{7} kg and a speed of $0.50c$ smashes into an asteroid at rest and becomes embedded within it. If the speed of the probe–asteroid system is $0.26c$ after the collision, what is the rest mass of the asteroid?

52. •• At what speed does the classical momentum, $p = mv$, give an error, when compared with the relativistic momentum, of **(a)** 1.00% and **(b)** 5.00%?

53. •• A proton has 1836 times the rest mass of an electron. At what speed will an electron have the same momentum as a proton moving at $0.0100c$?

SECTION 29–6 RELATIVISTIC ENERGY AND $E = mc^2$

54. • **CE** Particles A through D have the following rest energies and total energies:

Particle	Rest Energy	Total Energy
A	$6E$	$6E$
B	$2E$	$4E$
C	$4E$	$6E$
D	$3E$	$4E$

Rank these particles in order of increasing **(a)** rest mass, **(b)** kinetic energy, and **(c)** speed. Indicate ties where appropriate.

55. • Find the work that must be done on a proton to accelerate it from rest to a speed of $0.90c$.

56. • If a neutron moves with a speed of $0.99c$, what are its **(a)** total energy, **(b)** rest energy, and **(c)** kinetic energy?

57. • A spring with a force constant of 584 N/m is compressed a distance of 39 cm. Find the resulting increase in the spring's mass.

58. • When a certain capacitor is charged, its mass increases by 8.3×10^{-16} kg. How much energy is stored in the capacitor?

59. • What minimum energy must a gamma ray have to create an electron-antielectron pair?

60. • When a proton encounters an antiproton, the two particles annihilate each other, producing two gamma rays. Assuming the particles were at rest when they annihilated, find the energy of each of the two gamma rays produced. (*Note*: The rest energies of an antiproton and a proton are identical.)

61. • A rocket with a mass of 2.7×10^{6} kg has a relativistic kinetic energy of 2.7×10^{23} J. How fast is the rocket moving?

62. •• An object has a total energy that is 5.5 times its rest energy. What is its speed?

63. •• A nuclear power plant produces an average of 1.0×10^{3} MW of power during a year of operation. Find the corresponding change in mass of reactor fuel, assuming all of the energy released by the fuel can be converted directly to electrical energy. (In a practical reactor, only a relatively small fraction of the energy can be converted to electricity.)

64. •• A helium atom has a rest mass of $m_{He} = 4.002603$ u. When disassembled into its constituent particles (2 protons, 2 neutrons, 2 electrons), the well-separated individual particles have the following masses: $m_p = 1.007276$ u, $m_n = 1.008665$ u, $m_e = 0.000549$ u. How much work is required to completely disassemble a helium atom? (*Note:* 1 u of mass has a rest energy of 931.49 MeV.)

65. •• What is the percent difference between the classical kinetic energy, $K_{cl} = \frac{1}{2}m_0v^2$, and the correct relativistic kinetic energy, $K = m_0c^2/\sqrt{1 - v^2/c^2} - m_0c^2$, at a speed of **(a)** $0.10c$ and **(b)** $0.90c$?

66. •• A proton has 1836 times the rest mass of an electron. At what speed will an electron have the same kinetic energy as a proton moving at $0.0250c$?

67. •• **IP** Consider a baseball with a rest mass of 0.145 kg. **(a)** How much work is required to increase the speed of the baseball from 25.0 m/s to 35.0 m/s? **(b)** Is the work required to increase the speed of the baseball from 200,000,025 m/s to 200,000,035 m/s greater than, less than, or the same as the amount found in part (a)? Explain. **(c)** Calculate the work required for the increase in speed indicated in part (b).

68. •• **IP** A particle has a kinetic energy equal to its rest energy. **(a)** What is the speed of this particle? **(b)** If the kinetic energy of this particle is doubled, does its speed increase by a more than, less than, or exactly a factor of 2? Explain. **(c)** Calculate the speed of a particle whose kinetic energy is twice its rest energy.

69. ••• A lump of putty with a mass of 0.240 kg and a speed of $0.980c$ collides head-on and sticks to an identical lump of putty moving with the same speed. After the collision the system is at rest. What is the mass of the system after the collision?

SECTION 29–8 GENERAL RELATIVITY

70. • Find the radius to which the Sun must be compressed for it to become a black hole.

71. •• **The Black Hole in the Center of the Milky Way** Recent measurements show that the black hole at the center of the Milky Way galaxy, which is believed to coincide with the powerful radio source Sagittarius A*, is 2.6 million times more massive than the Sun; that is, $M = 5.2 \times 10^{36}$ kg. **(a)** What is the maximum radius of this black hole? **(b)** Find the acceleration of gravity at the Schwarzschild radius of this black hole, using the expression for R given in Equation 29–10. **(c)** How does your answer to part (b) change if the mass of the black hole is doubled? Explain.

GENERAL PROBLEMS

72. • **CE** Two observers are moving relative to one another. Which of the following quantities will they always measure to have the same value: **(a)** their relative speed; **(b)** the time between two events; **(c)** the length of an object; **(d)** the speed of light in a vacuum; **(e)** the speed of a third observer?

73. • **CE** You are standing next to a runway as an airplane lands. **(a)** If you and the pilot observe a clock in the cockpit, which of you measures the proper time? **(b)** If you and the pilot observe a large clock on the control tower, which of you measures the

proper time? **(c)** Which of you measures the proper length of the airplane? **(d)** Which of you measures the proper length of the runway?

74. • **CE** Which clock runs slower relative to a clock on the North Pole: clock 1 on an airplane flying from New York to Los Angeles, or clock 2 on an airplane flying from Los Angeles to New York? Assume each plane has the same speed relative to the surface of the Earth. Explain.

75. • **CE** An apple drops from the bough of a tree to the ground. Is the mass of the apple near the top of its fall greater than, less than, or the same as its mass after it has landed? Explain.

76. • **CE Predict/Explain** Consider two apple pies that are identical in every respect, except that pie 1 is piping hot and pie 2 is at room temperature. **(a)** If identical forces are applied to the two pies, is the acceleration of pie 1 greater than, less than, or equal to the acceleration of pie 2? **(b)** Choose the *best explanation* from among the following:

I. The acceleration of pie 1 is greater because the fact that it is hot means it has the greater energy.
II. The fact that pie 1 is hot means it behaves as if it has more mass than pie 2, and therefore it has a smaller acceleration.
III. The pies have the same acceleration regardless of their temperature because they have identical rest masses.

77. • **CE** Is the mass of a warm cup of tea greater than, less than, or the same as the mass of the same cup of tea when it has cooled? Explain.

78. • **CE Predict/Explain** An uncharged capacitor is charged by moving some electrons from one plate of the capacitor to the other plate. **(a)** Is the mass of the charged capacitor greater than, less than, or the same as the mass of the uncharged capacitor? **(b)** Choose the *best explanation* from among the following:

I. The charged capacitor has more mass because it is storing energy within it, just like a compressed spring.
II. The charged capacitor has less mass because some of its mass now appears as the energy of the electric field between its plates.
III. The capacitor has the same mass whether it is charged or not because charging it only involves moving electrons from one plate to the other without changing the total number of electrons.

79. •• **Cosmic Rays** Protons in cosmic rays have been observed with kinetic energies as large as 1.0×10^{20} eV. **(a)** How fast are these protons moving? Give your answer as a fraction of the speed of light. **(b)** Show that the kinetic energy of a single one of these protons is much greater than the kinetic energy of a 15-mg ant walking with a speed of 8.8 mm/s.

80. •• An apple falls from a tree, landing on the ground 3.7 m below. How long is the apple in the air, as measured by an observer moving toward Earth with a speed of 0.89*c*?

81. •• What is the momentum of a proton with 1.50×10^3 MeV of kinetic energy? (*Note*: The rest energy of a proton is 938 MeV.)

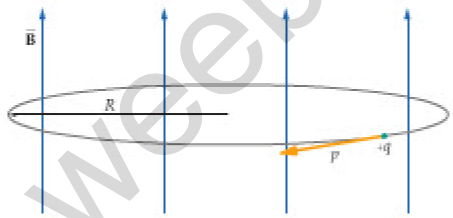
82. •• **IP** A container holding 2.00 moles of an ideal monatomic gas is heated at constant volume until the temperature of the gas increases by 112 F°. **(a)** Does the mass of the gas increase, decrease, or stay the same? Explain. **(b)** Calculate the change in mass of the gas, if any.

83. •• A ^{14}C nucleus, initially at rest, emits a beta particle. The beta particle is an electron with 156 keV of kinetic energy. **(a)** What is the speed of the beta particle? **(b)** What is the momentum of the beta particle? **(c)** What is the momentum of the nucleus after it emits the beta particle? **(d)** What is the speed of the nucleus after it emits the beta particle?

84. •• A clock at rest has a rectangular shape, with a width of 24 cm and a height of 12 cm. When this clock moves parallel to its width with a certain speed v its width and height are the same. Relative to a clock at rest, how long does it take for the moving clock to advance by 1.0 s?

85. •• A starship moving toward Earth with a speed of 0.75*c* launches a shuttle craft in the forward direction. The shuttle, which has a proper length of 12.5 m, is only 6.25 m long as viewed from Earth. What is the speed of the shuttle relative to the starship?

86. •• When a particle of charge q and momentum p enters a uniform magnetic field at right angles it follows a circular path of radius $R = p/qB$, as shown in Figure 29–32. What radius does this expression predict for a proton traveling with a speed $v = 0.99c$ through a magnetic field $B = 0.20$ T if you use (a) the nonrelativistic momentum ($p = mv$) or (b) the relativistic momentum ($p = mv/\sqrt{1 - v^2/c^2}$)?



▲ **FIGURE 29–32** Problem 86

87. •• **IP** A starship moving away from Earth with a speed of 0.75*c* launches a shuttle craft in the reverse direction, that is, toward Earth. **(a)** If the speed of the shuttle relative to the starship is 0.40*c*, and its proper length is 13 m, how long is the shuttle as measured by an observer on Earth? **(b)** If the shuttle had been launched in the forward direction instead, would its length as measured by an observer on Earth be greater than, less than, or the same as the length found in part (a)? Explain. **(c)** Calculate the length for the case described in part (b).

88. •• A 2.5-m titanium rod in a moving spacecraft is at an angle of 45° with respect to the direction of motion. The craft moves directly toward Earth at 0.98*c*. As viewed from Earth, **(a)** how long is the rod and **(b)** what angle does the rod make with the direction of motion?

89. •• Electrons are accelerated from rest through a potential difference of 276,000 V. What is the final speed predicted **(a)** classically and **(b)** relativistically?

90. •• The rest energy, m_0c^2, of a particle with a kinetic energy K and a momentum p can be determined as follows:

$$m_0c^2 = \frac{(pc)^2 - K^2}{2K}$$

Suppose a pion (a subatomic particle) is observed to have a kinetic energy $K = 35.0$ MeV and a momentum $p = 5.61 \times 10^{-20}\ \mathrm{kg^3\,m/s} = 105$ MeV/c. What is the rest energy of the pion? Give your answer in MeV.

91. •• A small star of mass m orbits a supermassive black hole of mass M. **(a)** Find the orbital speed of the star if its orbital radius is $2R$, where R is the Schwarzschild radius (Equation 29–10). **(b)** Repeat part (a) for an orbital radius equal to R.

92. •• **IP** Consider a "relativistic air track" on which two identical air carts undergo a completely inelastic collision. One cart is initially at rest; the other has an initial speed of 0.650*c*. **(a)** In classical physics, the speed of the carts after the collision would be 0.325*c*. Do you expect the final speed in this relativistic collision to be greater than or less than 0.325*c*? Explain. **(b)** Use relativistic

momentum conservation to find the speed of the carts after they collide and stick together.

93. ••• **IP** In Conceptual Checkpoint 29–2 we considered an astronaut at rest on an inclined bed inside a moving spaceship. From the point of view of observer 1, on board the ship, the astronaut has a length L_0 and is inclined at an angle θ_0 above the floor. Observer 2 sees the spaceship moving to the right with a speed v. **(a)** Show that the length of the astronaut as measured by observer 2 is

$$L = L_0\sqrt{1 - \left(\frac{v^2}{c^2}\right)\cos^2\theta_0}$$

(b) Show that the angle θ the astronaut makes with the floor of the ship, as measured by observer 2, is given by

$$\tan\theta = \frac{\tan\theta_0}{\sqrt{1 - v^2/c^2}}$$

94. ••• A pulsar is a collapsed, rotating star that sends out a narrow beam of radiation, like the light from a lighthouse. With each revolution, we see a brief, intense pulse of radiation from the pulsar. Suppose a pulsar is receding directly away from Earth with a speed of 0.800c, and the starship *Endeavor* is sent out toward the pulsar with a speed of 0.950c relative to Earth. If an observer on Earth finds that 153 pulses are emitted by the pulsar every second, at what rate does an observer on the *Endeavor* see pulses emitted?

95. ••• Show that the total energy of an object is related to its momentum by the relation $E^2 = p^2c^2 + (m_0c^2)^2$.

96. ••• Show that if $0 < v_1 < c$ and $0 < v_2 < c$ are two velocities pointing in the same direction, the relativistic sum of these velocities, v, is greater than v_1 and greater than v_2 but less than c. In particular, show that this is true even if v_1 and v_2 are greater than 0.5c.

97. ••• Show that an object with momentum p and rest mass m_0 has a speed given by

$$v = \frac{c}{\sqrt{1 + (m_0c/p)^2}}$$

98. ••• **Decay of the Σ^- Particle** When at rest, the Σ^- particle has a lifetime of 0.15 ns before it decays into a neutron and a pion. One particular Σ^- particle is observed to travel 3.0 cm in the lab before decaying. What was its speed? (*Hint:* Its speed was not $\frac{2}{3}c$.)

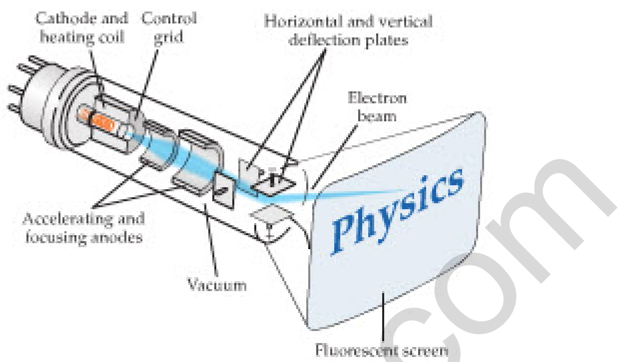
PASSAGE PROBLEMS

Relativity in a TV Set

The first televisions used cathode-ray tubes, or CRTs, to form a picture. Even today, when plasma screens, liquid-crystal displays (LCD), and digital light processing (DLP) systems are increasingly popular, the CRT is still a reliable and inexpensive choice for TVs and computer monitors.

The basic idea behind a CRT is fairly simple: use a beam of electrons to "paint" a picture on a fluorescent screen. This is illustrated in Figure 29–33. First, a heated coil at the negative terminal of the tube (the cathode) produces electrons which are accelerated toward the positive terminal (the anode) to form a beam of electrons—the so-called "cathode ray." A series of horizontal and vertical deflecting plates then direct the beam to any desired spot on a fluorescent screen to produce a glowing dot that can be seen. Moving the glowing dot rapidly around the screen, and varying its intensity with the control grid, allows one to produce a glowing image of any desired object. The first televised image—a dollar sign—was transmitted by Philo T. Farnsworth in 1927, and television inventors have been seeing dollar signs ever since.

The interior of a CRT must be a very good vacuum, typically 10^{-7} of an atmosphere or less, to ensure electrons aren't



▲ **FIGURE 29–33** A cathode-ray tube. (Problems 99, 100, 101, and 102)

scattered by air molecules on their way to the screen. Electrons in a television set are accelerated through a potential difference of 25.0 kV, which is sufficient to give them speeds comparable to the speed of light. As a result, relativity must be used to accurately determine their behavior. Thus, even in something as commonplace as a TV set, Einstein's theory of relativity proves itself to be of great practical value.

99. • Find the speed of an electron accelerated through a voltage of 25.0 kV—*ignoring* relativity. Express your answer as a fraction times the speed of light. (Speeds over about 0.1c are generally regarded as relativistic.)

A. 0.221c **B.** 0.281c

C. 0.312c **D.** 0.781c

100. • When relativistic effects are included, do you expect the speed of the electrons to be greater than, less than, or the same as the result found in the previous problem?

101. • Find the speed of the electrons in Problem 99, this time using a correct relativistic calculation. As before, express your answer as a fraction times the speed of light.

A. 0.301c **B.** 0.312c

C. 0.412c **D.** 0.953c

102. • Suppose the accelerating voltage in Problem 99 is increased by a factor of 10. What is the correct relativistic speed of an electron in this case?

A. 0.205c **B.** 0.672c

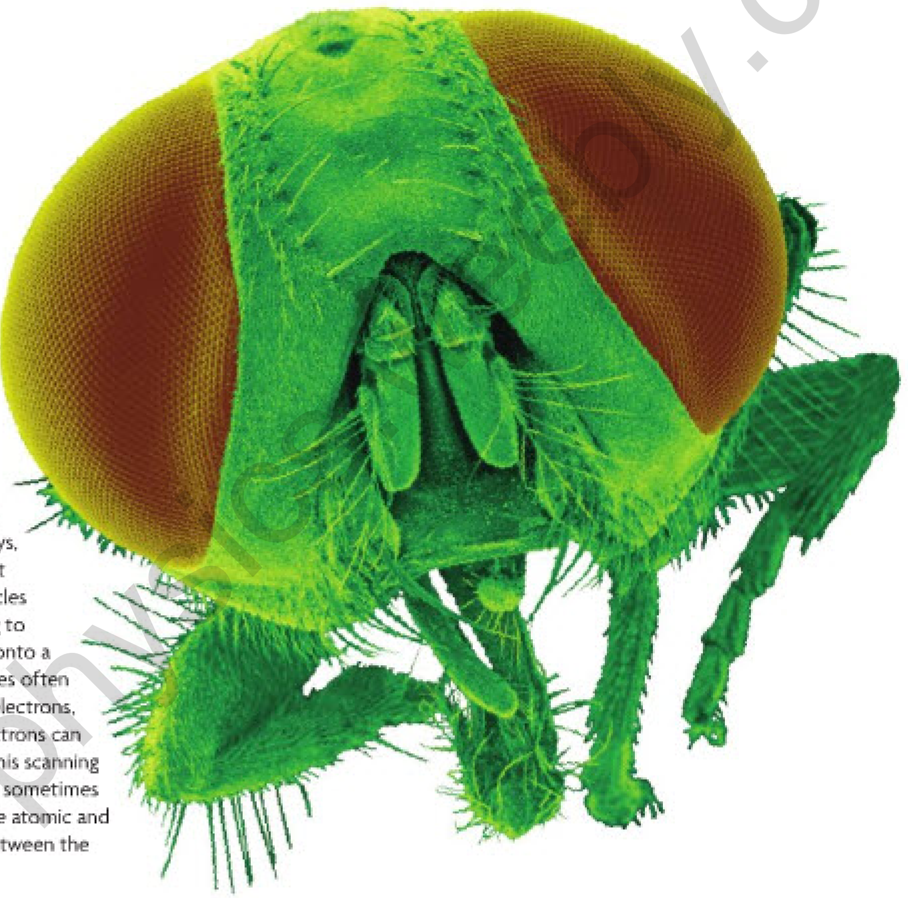
C. 0.740c **D.** 0.862c

INTERACTIVE PROBLEMS

103. •• **Referring to Example 29–4** The *Picard* approaches starbase Faraway Point with a speed of 0.806c, and the *La Forge* approaches the starbase with a speed of 0.906c. Suppose the *Picard* now launches a probe toward the starbase. **(a)** What velocity must the probe have relative to the *Picard* if it is to be at rest relative to the *La Forge*? **(b)** What velocity must the probe have relative to the *Picard* if its velocity relative to the *La Forge* is to be 0.100c? **(c)** For the situation described in part (b), what is the velocity of the probe relative to the Faraway Point starbase?

104. •• **Referring to Example 29–4** Faraway Point starbase launches a probe toward the approaching starships. The probe has a velocity relative to the *Picard* of −0.906c. The *Picard* approaches starbase Faraway Point with a speed of 0.806c, and the *La Forge* approaches the starbase with a speed of 0.906c. **(a)** What is the velocity of the probe relative to the *La Forge*? **(b)** What is the velocity of the probe relative to Faraway Point starbase?

30 Quantum Physics



Most of the images that we encounter are made with visible light. Even those that are not, such as thermograms and X-rays, employ other kinds of electromagnetic radiation. Until about 80 years ago, the idea of making a picture by means of particles rather than radiation would have seemed absurd—like trying to create a portrait by bouncing paintballs off the subject and onto a canvas. Yet by the 1920s, physicists discovered that light waves often behave like particles and, conversely, that particles, such as electrons, often behave like waves. Indeed, the wave properties of electrons can be exploited to create remarkably detailed images, such as this scanning electron micrograph of a housefly. This chapter explores the sometimes odd-seeming laws that describe the behavior of nature in the atomic and subatomic realms, and the series of revolutions in physics between the 1890s and the 1930s that uncovered them.

To understand the behavior of nature at the atomic level, it is necessary to introduce a number of new concepts to physics and to modify many others. In this chapter we consider the basic ideas of quantum physics and show that they lead to a deeper understanding of microscopic systems—in much the same way that relativity extends physics into the realm of high speeds. Relativity and quantum physics, taken together, provide the basis for what we refer to today as modern physics.

We begin this chapter by introducing the concept of *quantization*, in which a physical quantity—such as energy—varies in discrete steps rather than continuously, as in classical physics. This concept leads to the idea of the *photon*, which can be thought of as a "particle" of light. Next, we find that just as light can behave like a particle, particles—such as electrons, protons, and neutrons—can behave like waves. Finally, the wave nature of matter introduces a fundamental uncertainty to our knowledge of physical quantities and allows for such classically "forbidden" behavior as *quantum tunneling*.

30–1 Blackbody Radiation and Planck's Hypothesis of Quantized Energy

If you have ever looked through a small opening into a hot furnace, you have seen the glow of light associated with its high temperature. As unlikely as it may seem, this light played a central role in the revolution of physics that occurred in the early 1900s. It was through the study of such systems that the idea of *energy quantization*—energy taking on only discrete values—was first introduced to physics.

More precisely, physicists in the late 1800s were actively studying the electromagnetic radiation given off by a physical system known as a **blackbody.** An example of a blackbody is illustrated in Figure 30–1. Note that this blackbody has a cavity with a small opening to the outside world—much like a furnace. Light that enters the cavity through the opening is reflected multiple times from the interior walls until it is completely absorbed. It is for this reason that the system is referred to as "black," even though the material from which it is made need not be black at all.

An ideal blackbody absorbs all the light that is incident on it.

Objects that absorb much of the incident light—though not all of it—are reasonable approximations to a blackbody; objects that are highly reflective and shiny are poor representations of a blackbody.

As we saw in Section 16–6, objects that are effective at absorbing radiation are also effective at giving off radiation. Thus an ideal blackbody is also an ideal radiator. In fact, the basic experiment performed with a blackbody is the following: Heat the blackbody to a fixed temperature, T, and measure the amount of electromagnetic radiation it gives off at a given frequency, f. Repeat this measurement for a number of different frequencies, then plot the intensity of radiation versus frequency. The results of a typical blackbody experiment are shown in Figure 30–2 for a variety of different temperatures. Note that there is little radiation at low frequencies, a peak in the radiation at intermediate frequencies, and finally a fall-off to little radiation again at high frequencies.

Now, what is truly remarkable about the blackbody experiment is the following:

The distribution of energy in blackbody radiation is *independent* of the material from which the blackbody is constructed—it depends only on the temperature, T.

Therefore, a blackbody of steel and one of wood give precisely the same results when held at the same temperature. When physicists observe a phenomenon that is independent of the details of the system, it is a clear signal that they are observing something of fundamental significance. This was certainly the case with blackbody radiation.

Two aspects of the blackbody curves in Figure 30–2 are of particular importance. First, note that as the temperature is increased, the area under the curve increases. Since the total area under the curve is a measure of the total energy emitted by the blackbody, it follows that an object radiates more energy as it becomes hotter.

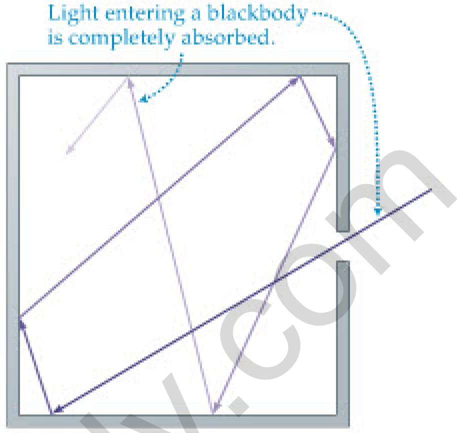
Second, note that the peak in the blackbody curve moves to higher frequency as the absolute temperature T is increased. This movement, or displacement, of the peak with temperature is described by **Wien's displacement law:**

Wien's Displacement Law

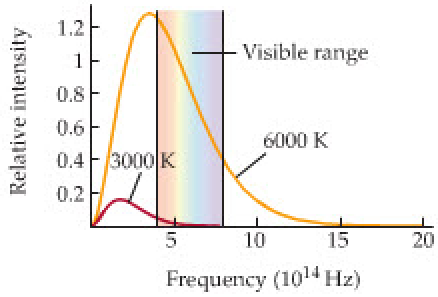
$$f_{peak} = (5.88 \times 10^{10}\ s^{-1} \cdot K^{-1})T \qquad 30\text{–}1$$

SI unit: Hz = s^{-1}

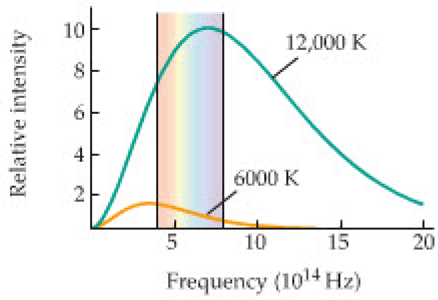
Thus, there is a direct connection between the temperature of an object and the frequency of radiation it emits most strongly.



▲ **FIGURE 30–1 An ideal blackbody**
In an ideal blackbody, incident light is completely absorbed. In the case shown here, the absorption occurs as the result of multiple reflections within a cavity. The blackbody, and the electromagnetic radiation it contains, are in thermal equilibrium at a temperature T.



(a)



(b)

▲ **FIGURE 30–2 Blackbody radiation**
Blackbody radiation as a function of frequency for various temperatures: **(a)** 3000 K and 6000 K; **(b)** 6000 K and 12,000 K. Note that as the temperature is increased, the peak in the radiation shifts toward higher frequency.

CONCEPTUAL CHECKPOINT 30–1 COMPARE TEMPERATURES

Betelgeuse is a red-giant star in the constellation Orion; Rigel is a bluish white star in the same constellation. Is the surface temperature of Betelgeuse **(a)** higher than, **(b)** lower than, or **(c)** the same as the surface temperature of Rigel?

REASONING AND DISCUSSION

Recall that red light has a lower frequency than blue light, as can be seen in Figure 25–8. It follows, from Wien's displacement law, that a red star has a lower temperature than a blue star. Therefore, Betelgeuse has the lower surface temperature.

ANSWER

(b) The surface temperature of Betelgeuse is lower than that of Rigel.

▲ All objects emit electromagnetic radiation over a range of frequencies. The frequency that is radiated most intensely depends on the object's temperature, as specified by Wien's law. The glowing bolt in this picture radiates primarily in the infrared part of the spectrum, but it is hot enough (a few thousand kelvin) so that a significant portion of its radiation falls within the red end of the visible region. The other bolts are too cool to radiate any detectable amount of visible light.

To be more specific about the conclusion given in Conceptual Checkpoint 30–1, let's consider Figure 30–2 in greater detail. At the lowest temperature shown, 3000 K, the radiation is more intense at the red end of the visible spectrum than at the blue end. An object at this temperature—like the heating coil on a stove, for example—would appear "red hot" to the eye. Even so, most of the radiation at this temperature is in the infrared, and thus is not visible to the eye at all. A blackbody at 6000 K, like the surface of the Sun, gives out strong radiation throughout the visible spectrum, though there is still more radiation at the red end than at the blue end. As a result, the light of the Sun appears somewhat yellowish. Finally, at 12,000 K a blackbody appears bluish white, and most of its radiation is in the ultraviolet. The temperature of the star Rigel is determined from the location of its radiation peak in the following Exercise.

EXERCISE 30–1

Find the surface temperature of Rigel, given that its radiation peak occurs at a frequency of 1.17×10^{15} Hz.

SOLUTION

Solving Equation 30–1 for T, we find

$$T = \frac{f_{\text{peak}}}{5.88 \times 10^{10}\ \text{s}^{-1} \cdot \text{K}^{-1}} = \frac{1.17 \times 10^{15}\ \text{Hz}}{5.88 \times 10^{10}\ \text{s}^{-1} \cdot \text{K}^{-1}} = 19{,}900\ \text{K}$$

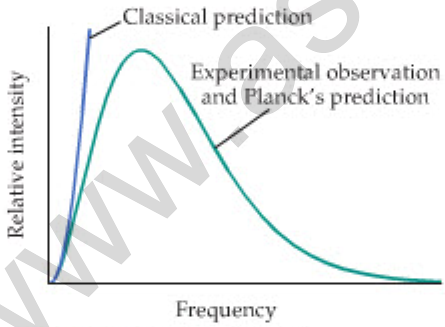
This is a little more than three times the surface temperature of the Sun. Thus blackbody radiation allows us to determine the temperature of a distant star that we may never visit.

REAL-WORLD PHYSICS

Measuring the temperature of a star

Planck's Quantum Hypothesis

Although experimental understanding of blackbody radiation was quite extensive in the late 1800s, there was a problem. Attempts to explain the blackbody curves of Figure 30–2 theoretically, using classical physics, failed—and failed miserably. To see the problem, consider the curves shown in **Figure 30–3**. The green curve is the experimental result for a blackbody at a given temperature. In contrast, the blue curve shows the prediction of classical physics. Clearly, the classical result cannot be valid, since its curve diverges to infinity at high frequency, which in turn implies that the blackbody radiates an infinite amount of energy. This unphysical divergence at high frequencies is referred to as the *ultraviolet catastrophe*.



▲ **FIGURE 30–3 The ultraviolet catastrophe**

Classical physics predicts a blackbody radiation curve that rises without limit as the frequency increases. This outcome is referred to as the ultraviolet catastrophe. By assuming energy quantization, Planck was able to derive a curve in agreement with experimental results.

The German physicist Max Planck (1858–1947) worked long and hard on this problem. Eventually, he was able to construct a mathematical formula that agreed with experiment for all frequencies. His next problem was to "derive" the equation. The only way he could do this, it turned out, was to make the following bold and unprecedented assumption: The radiation energy in a blackbody at the frequency f must be an integral multiple of a constant (h) times the frequency; that is, energy is *quantized*:

Quantized Energy

$$E_n = nhf \qquad n = 0, 1, 2, 3, \ldots \qquad 30\text{–}2$$

The constant, h, in this expression is known as **Planck's constant,** and it has the following value:

Planck's Constant, h

$$h = 6.63 \times 10^{-34}\ \mathrm{J \cdot s} \qquad 30\text{–}3$$

SI unit: J · s

This constant is recognized today as one of the fundamental constants of nature, on an equal footing with other such constants as the speed of light in a vacuum and the rest mass of an electron.

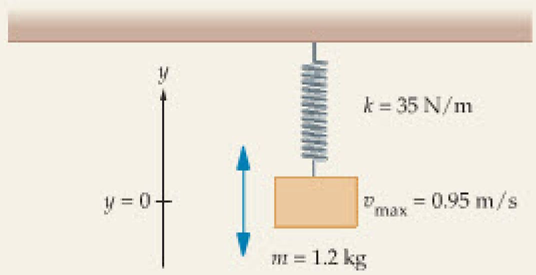
The assumption of energy quantization is quite a departure from classical physics, in which energy can take on any value at all and is related to the amplitude of a wave rather than its frequency. In Planck's calculation, the energy can have only the discrete values hf, $2hf$, $3hf$, and so on. Because of this quantization, it follows that the energy can change only in *quantum jumps* of energy no smaller than hf as the system goes from one quantum state to another. The fundamental increment, or *quantum*, of energy, hf, is incredibly small, as can be seen from the small magnitude of Planck's constant. The next Example explores the size of the quantum and the value of the *quantum number, n*, for a typical macroscopic system.

EXAMPLE 30–1 QUANTUM NUMBERS

Suppose the maximum speed of a 1.2-kg mass attached to a spring with a force constant of 35 N/m is 0.95 m/s. **(a)** Find the frequency of oscillation and total energy of this mass–spring system. **(b)** Determine the size of one quantum of energy in this system. **(c)** Assuming the energy of this system satisfies $E_n = nhf$, find the quantum number, n.

PICTURE THE PROBLEM

Our sketch shows a 1.2-kg mass oscillating on a spring with a force constant of 35 N/m. The mass has its maximum speed of $v_{max} = 0.95$ m/s when it passes through the equilibrium position. At this moment, the total energy of the system is simply the kinetic energy of the mass.



STRATEGY

a. We can find the frequency of oscillation using $\omega = \sqrt{k/m}$ (Equation 13–10) and $\omega = 2\pi f$ (Equation 13–15). The total energy is simply the kinetic energy as the mass passes through equilibrium, $E = K_{max} = \frac{1}{2}mv_{max}^2$.

b. The energy of one quantum is hf, where f is the frequency found in part (a).

c. We determine the quantum number by solving $E_n = nhf$ for n.

SOLUTION

Part (a)

1. Calculate the frequency of oscillation using $\omega = \sqrt{k/m} = 2\pi f$:

$$\omega = \sqrt{\frac{k}{m}} = 2\pi f$$

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{35\ \mathrm{N/m}}{1.2\ \mathrm{kg}}} = 0.86\ \mathrm{Hz}$$

2. Calculate the maximum kinetic energy of the mass $\left(\frac{1}{2}mv_{max}^2\right)$ to find the total, E, of the system:

$$E = \tfrac{1}{2}mv_{max}^2 = \tfrac{1}{2}(1.2\ \mathrm{kg})(0.95\ \mathrm{m/s})^2 = 0.54\ \mathrm{J}$$

Part (b)

3. The energy of one quantum is hf, where $f = 0.86$ Hz:

$$hf = (6.63 \times 10^{-34}\ \mathrm{J \cdot s})(0.86\ \mathrm{Hz}) = 5.7 \times 10^{-34}\ \mathrm{J}$$

Part (c)

4. Set $E_n = nhf$ equal to the total energy of the system and solve for n:

$$E_n = nhf$$

$$n = \frac{E_n}{hf} = \frac{0.54\ \mathrm{J}}{5.7 \times 10^{-34}\ \mathrm{J}} = 9.5 \times 10^{32}$$

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INSIGHT

The numbers found in parts (b) and (c) are incredible for their size. For example, the quantum is on the order of 10^{-34} J, as compared with the energy required to break a bond in a DNA molecule, which is on the order of 10^{-20} J. Thus the quantum for a macroscopic system is about 10^{14} times smaller than the energy needed to affect a molecule. Similarly, the number of quanta in the system, roughly 10^{33}, is comparable to the number of atoms in four Olympic-size swimming pools.

PRACTICE PROBLEM

If the quantum of energy for a 1.5-kg mass on a spring is 0.80×10^{-33} J, what is the force constant of the spring? [**Answer:** $k = 86$ N/m]

Some related homework problems: Problem 10, Problem 86

Clearly, then, the quantum numbers in typical macroscopic systems are incredibly large. As a result, a change of one in the quantum number is completely insignificant and undetectable. Similarly, the change in energy from one quantum state to the next is so small that it cannot be measured in a typical experiment; hence, for all practical purposes, the energy of a macroscopic system seems to change continuously, even though it actually changes by small increments. In contrast, in an atomic system, the energy jumps are of great importance, as we shall see in the next section.

Returning to the ultraviolet catastrophe for a moment, we can now see how Planck's hypothesis removes the unphysical divergence at high frequency predicted by classical physics. In Planck's theory, the higher the frequency f, the greater the quantum of energy, hf. Therefore, as the frequency is increased, the amount of energy required for even the smallest quantum jump increases as well. Since a blackbody has only a finite amount of energy, however, it simply cannot supply the large amount of energy required to produce an extremely high-frequency quantum jump. As a result, the amount of radiation at high frequency drops off toward zero.

Planck's theory of energy quantization leads to an adequate description of the experimental results for blackbody radiation. Still, the theory was troubling and somewhat unsatisfying to Planck and to many other physicists as well. Although the idea of energy quantization worked, at least in this case, it seemed ad hoc and more of a mathematical trick than a true representation of nature. With the work of Einstein, however, which we present in the next section, the well-founded misgivings about quantum theory began to fade away.

30–2 Photons and the Photoelectric Effect

From Max Planck's point of view, energy quantization in a blackbody was probably related to quantized vibrations of atoms in the walls of the blackbody. We are familiar, for example, with the fact that a string tied at both ends can produce standing waves at only certain discrete frequencies (Chapter 14), so perhaps atoms vibrating in a blackbody behave in a similar way, vibrating only with certain discrete energies. Certainly, Planck did not think the light in a blackbody had a quantized energy, since most physicists thought of light as being a wave, which can have any energy.

A brash young physicist named Albert Einstein, however, took the idea of quantized energy seriously and applied it to the radiation in the blackbody. Einstein proposed that light comes in bundles of energy, called **photons**, that obey Planck's hypothesis of energy quantization; that is, light of frequency f consists of photons with an energy given by the following relation:

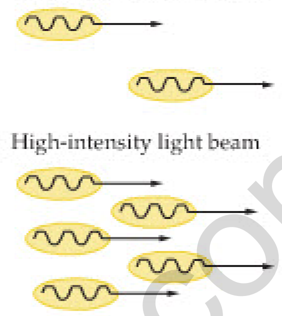
Energy of a Photon of Frequency f

$$E = hf \qquad 30\text{–}4$$

SI unit: J

Thus the energy in a beam of light of frequency f can have only the values hf, $2hf$, $3hf$, and so on. Planck's initial reaction to Einstein's suggestion was that he had gone too far with the idea of quantization. As it turns out, nothing could have been further from the truth.

In Einstein's photon model, a beam of light can be thought of as a beam of particles, each carrying the energy hf, as indicated in Figure 30–4. If the beam of light is made more intense while keeping the frequency the same, the result is that the photons in the beam are more tightly packed, so that more photons pass a given point in a given time. In this way, more photons shine on a given surface in a given time, increasing the energy delivered to the surface per time. Even so, each photon in the more intense beam has exactly the same amount of energy as those in the less intense beam. The energy of a typical photon of visible light is calculated in the next Exercise.



▲ **FIGURE 30–4 The photon model of light**

In the photon model of light, a beam of light consists of many photons, each with an energy hf. The more intense the beam, the more tightly packed the photons.

EXERCISE 30–2

Calculate the energy of a photon of yellow light with a frequency of 5.25×10^{14} Hz. Give the energy in both joules and electron-volts.

SOLUTION

Applying Equation 30–4, we find

$$E = hf = (6.63 \times 10^{-34}\ \text{J}\cdot\text{s})(5.25 \times 10^{14}\ \text{s}^{-1}) = 3.48 \times 10^{-19}\ \text{J}$$

$$= 3.48 \times 10^{-19}\ \text{J}\left(\frac{1\ \text{eV}}{1.60 \times 10^{-19}\ \text{J}}\right) = 2.18\ \text{eV}$$

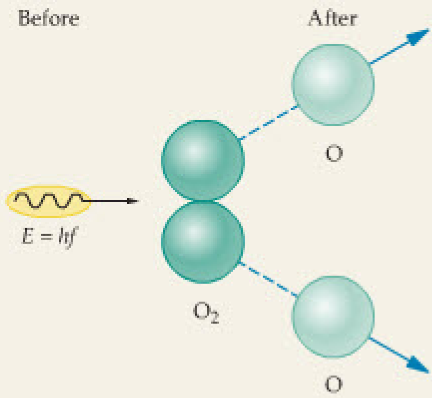
Note that the energy of a visible photon is on the order of an electron-volt (eV). This is also the typical energy scale for atomic and molecular systems, as we show in detail in the following Example.

EXAMPLE 30–2 WHEN OXYGENS SPLIT

Molecular oxygen (O_2) is a diatomic molecule. The energy required to dissociate 1 mol of O_2 to form 2 mol of atomic oxygen is 118 kcal. **(a)** Find the energy (in joules and electron-volts) required to dissociate one O_2 molecule. **(b)** Assuming the dissociation energy for one molecule is supplied by a single photon, find the frequency of the photon.

PICTURE THE PROBLEM

In our sketch we show a single photon dissociating an O_2 molecule to form two O atoms. The energy of the photon is $E = hf$.



STRATEGY

a. This part of the problem is simply a matter of converting from kcal per mole to joules per molecule. This can be accomplished by using the fact that 1 kcal = 4186 J and that Avogadro's number (Section 17–1) is 6.02×10^{23} molecules/mol. We can then convert to electron-volts using $1\ \text{eV} = 1.60 \times 10^{-19}\ \text{J}$.

b. We can find the frequency of the photon by setting hf equal to the energy E found in part (a). Since Planck's constant is given in units of J·s, we must use the energy expressed in joules from part (a).

SOLUTION

Part (a)

1. Convert 118 kcal/mol to J/molecule:

$$\left(118\frac{\text{kcal}}{\text{mol}}\right)\left(\frac{4186\ \text{J}}{1\ \text{kcal}}\right)\left(\frac{1}{6.02 \times 10^{23}\ \text{molecules/mol}}\right) = 8.21 \times 10^{-19}\ \text{J/molecule}$$

2. Convert the preceding result to eV/molecule:

$$8.21 \times 10^{-19}\frac{\text{J}}{\text{molecule}}\left(\frac{1\ \text{eV}}{1.60 \times 10^{-19}\ \text{J}}\right) = 5.13\ \text{eV/molecule}$$

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Part (b)

3. Use $E = 8.21 \times 10^{-19}\ \text{J} = hf$ to solve for the frequency, f:

$$f = \frac{E}{h} = \frac{8.21 \times 10^{-19}\ \text{J}}{6.63 \times 10^{-34}\ \text{J} \cdot \text{s}} = 1.24 \times 10^{15}\ \text{Hz}$$

INSIGHT

This frequency is in the ultraviolet. In fact, ultraviolet rays in Earth's upper atmosphere cause O_2 molecules to dissociate, freeing up atomic oxygen which can then combine with O_2 to form ozone, O_3.

PRACTICE PROBLEM

An infrared photon has a frequency of 1.00×10^{13} Hz. How much energy is carried by one mole of these photons? [**Answer:** 3990 J = 0.953 kcal]

Some related homework problems: Problem 25, Problem 26

Since photons typically have rather small amounts of energy on a macroscopic scale, it follows that enormous numbers of photons must be involved in everyday situations, as demonstrated in the following Active Example.

REAL-WORLD PHYSICS: BIO

Dark-adapted vision

ACTIVE EXAMPLE 30–1 DARK VISION: FIND THE NUMBER OF PHOTONS

Dark-adapted (scotopic) vision is possible in humans with as little as $4.00 \times 10^{-11}\ \text{W/m}^2$ of 505-nm light entering the eye. If light of this intensity and wavelength enters the eye through a pupil that is 6.00 mm in diameter, how many photons enter the eye per second?

SOLUTION *(Test your understanding by performing the calculations indicated in each step.)*

1. Calculate the area of the pupil:	$2.83 \times 10^{-5}\ \text{m}^2$
2. Multiply the intensity by the area of the pupil to find the energy entering the eye per second:	$1.13 \times 10^{-15}\ \text{J/s}$
3. Calculate the energy of a photon:	$3.94 \times 10^{-19}\ \text{J}$
4. Divide the energy of a photon into the energy per second to find the number of photons per second:	2870 photons/s

INSIGHT

Thus, even though a typical lightbulb gives off roughly 10^{18} photons per second, we need only about 10^3 photons per second to see. Our eyes are extraordinary instruments, sensitive to an incredibly wide range of intensities.

Finally, suppose an astronomer views a dim, distant galaxy with scotopic vision. The 2870 photons that enter the astronomer's eye each second are separated from one another by about 65 miles. It follows that only one photon at a time from the distant galaxy traverses the astronomer's telescope.

YOUR TURN

Suppose we consider light with a wavelength greater than 505 nm by a factor of 1.25. Will more or fewer photons be required per second with this new wavelength? By what factor will the required number of photons per second change?

(Answers to **Your Turn** *problems are given in the back of the book.)*

The Photoelectric Effect

Einstein applied his photon model of light to the **photoelectric effect**, in which a beam of light (photo-) hits the surface of a metal and ejects an electron (-electric). The effect can be measured using a device like that pictured in Figure 30–5. Note that incoming light ejects an electron—referred to as a photoelectron—from a metal plate called the emitter (E); the electron is then attracted to a collector plate (C), which is at a positive potential relative to the emitter. The result is an electric current that can be measured with an ammeter.

The minimum amount of energy necessary to eject an electron from a particular metal is referred to as the **work function**, W_0, for that metal. Work functions

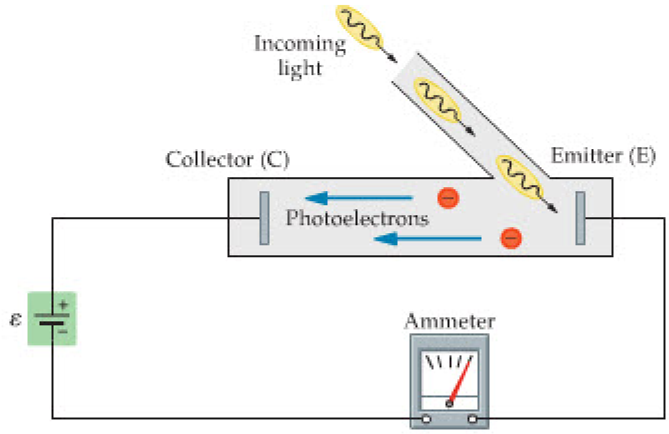


FIGURE 30–5 The photoelectric effect
The photoelectric effect can be studied with a device like that shown. Light shines on a metal plate, ejecting electrons, which are then attracted to a positively charged "collector" plate. The result is an electric current that can be measured with an ammeter.

vary from metal to metal but are typically on the order of a few electron-volts. If an electron is given an energy E by the beam of light that is greater than W_0, the excess energy goes into kinetic energy of the ejected electron. The maximum kinetic energy (K) a photoelectron can have, then, is

$$K_{max} = E - W_0 \qquad 30\text{–}5$$

Just as with blackbody radiation, the photoelectric effect exhibits behavior that is at odds with classical physics. Two of the main areas of disagreement are the following:

- Classical physics predicts that a beam of light of *any* color (frequency) can eject electrons, as long as the beam has sufficient intensity. That is, if a beam is intense enough, the energy it delivers to an electron will exceed the work function and cause it to be ejected.
- Classical physics also predicts that the maximum kinetic energy of an ejected electron should increase as the intensity of the light beam is increased. In particular, the more energy the beam delivers to the metal, the more energy that any given electron can have as it is ejected.

Although both of these predictions are reasonable—necessary, in fact, from the classical physics point of view—they simply do not agree with experiments on the photoelectric effect. In fact, experiments show the following behavior:

- To eject electrons, the incident light beam must have a frequency greater than a certain minimum value, referred to as the **cutoff frequency,** f_0. If the frequency of the light is less than f_0, it will not eject electrons, no matter how intense the beam.
- If the frequency of light is greater than the cutoff frequency, f_0, the effect of increasing the intensity is to increase the *number* of electrons that are emitted per second. The maximum kinetic energy of the electrons does not increase with the intensity of the light; the kinetic energy depends only on the frequency of the light.

As we shall see, these observations are explained quite naturally with the photon model.

First, in Einstein's model each photon has an energy determined solely by its frequency. Therefore, making a beam of a given frequency more intense simply means increasing the number of photons hitting the metal in a given time—not increasing the energy carried by a photon. An electron, then, is ejected only if an incoming photon has an energy that is at least equal to the work function: $E = hf_0 = W_0$. The *cutoff frequency* is thus defined as follows:

Cutoff Frequency, f_0

$$f_0 = \frac{W_0}{h} \qquad 30\text{–}6$$

SI unit: Hz = s^{-1}

If the frequency of the light is greater than f_0, the electron can leave the metal with a finite kinetic energy; if the frequency is less than f_0, no electrons are ejected, no matter how intense the beam. We determine a typical cutoff frequency in the next Exercise.

EXERCISE 30–3

The work function for a gold surface is 4.58 eV. Find the cutoff frequency, f_0, for a gold surface.

SOLUTION

Substitution in Equation 30–6 yields

$$f_0 = \frac{W_0}{h} = \frac{(4.58\ \text{eV})\left(\dfrac{1.60 \times 10^{-19}\ \text{J}}{1\ \text{eV}}\right)}{6.63 \times 10^{-34}\ \text{J}\cdot\text{s}} = 1.11 \times 10^{15}\ \text{Hz}$$

This frequency is in the near ultraviolet.

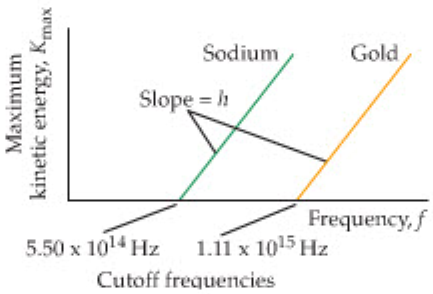


FIGURE 30–6 The kinetic energy of photoelectrons

The maximum kinetic energy of photoelectrons as a function of the frequency of light. Note that sodium and gold have different cutoff frequencies, as one might expect for different materials. On the other hand, the slope of the two lines is the same, h, as predicted by Einstein's photon model of light.

Second, the fact that a more intense beam of monochromatic light delivers more photons per time to the metal just means that more electrons are ejected per time. Since each electron receives precisely the same amount of energy, however, the maximum kinetic energy is the same regardless of the intensity. In fact, if we return to Equation 30–5 and replace the energy, E, with the energy of a photon, hf, we find

$$K_{max} = hf - W_0 \qquad 30\text{–}7$$

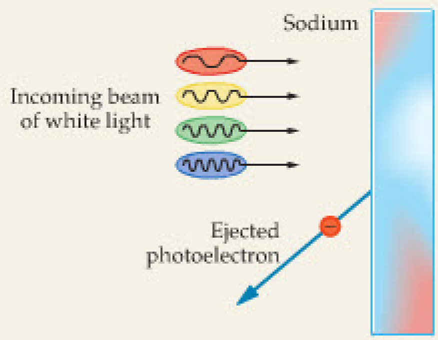
Note that K_{max} depends linearly on the frequency but is independent of the intensity. A plot of K_{max} for sodium (Na) and gold (Au) is given in **Figure 30–6**. Clearly, both lines have the same slope, h, as expected from Equation 30–7, but have different cutoff frequencies. Therefore, with the result given in Equation 30–7, Einstein was able to show that Planck's constant, h, appears in a natural way in the photoelectric effect and is not limited in applicability to the blackbody.

EXAMPLE 30–3 WHITE LIGHT ON SODIUM

A beam of white light containing frequencies between 4.00×10^{14} Hz and 7.90×10^{14} Hz is incident on a sodium surface, which has a work function of 2.28 eV. **(a)** What is the range of frequencies in this beam of light for which electrons are ejected from the sodium surface? **(b)** Find the maximum kinetic energy of the "photoelectrons" that are ejected from this surface.

PICTURE THE PROBLEM

Our sketch shows a beam of white light, represented by photons with different frequencies, incident on a sodium surface. Photoelectrons are ejected from this surface with a kinetic energy that depends on the frequency of the photon that was absorbed.



STRATEGY

a. We can find the cutoff frequency, f_0, for sodium using $f_0 = W_0/h$, with $W_0 = 2.28$ eV. Frequencies between the cutoff frequency and the maximum frequency in the beam of light, 7.90×10^{14} Hz, will eject electrons.

b. We can obtain the maximum kinetic energy for a given frequency, f, from Equation 30–7: $K_{max} = hf - W_0$. Clearly, the higher the frequency, the greater the maximum kinetic energy. It follows, then, that the greatest possible maximum kinetic energy corresponds to the highest frequency in the beam, 7.90×10^{14} Hz.

SOLUTION

Part (a)

1. Use $f_0 = W_0/h$ to calculate the cutoff frequency for sodium:

$$f_0 = \frac{W_0}{h} = \frac{(2.28\ \text{eV})\left(\dfrac{1.60 \times 10^{-19}\ \text{J}}{1\ \text{eV}}\right)}{6.63 \times 10^{-34}\ \text{J}\cdot\text{s}} = 5.50 \times 10^{14}\ \text{Hz}$$

2. The frequencies that eject electrons are those between the cutoff frequency and the highest frequency in the beam:

frequencies in this beam that eject electrons: 5.50×10^{14} Hz to 7.90×10^{14} Hz

Part (b)

3. Using $K_{max} = hf - W_0$, calculate K_{max} for the maximum frequency in the beam, $f = 7.90 \times 10^{14}$ Hz:

$$K_{max} = hf - W_0$$
$$= (6.63 \times 10^{-34}\ \text{J}\cdot\text{s})(7.90 \times 10^{14}\ \text{Hz})$$
$$- (2.28\ \text{eV})\left(\frac{1.60 \times 10^{-19}\ \text{J}}{1\ \text{eV}}\right) = 1.59 \times 10^{-19}\ \text{J}$$

INSIGHT

Note that most of the photons in a beam of white light will eject electrons from sodium, and that the maximum kinetic energy of one of these photoelectrons is about 1 eV.

PRACTICE PROBLEM

What frequency of light would be necessary to give a maximum kinetic energy of 2.00 eV to the photoelectrons from this surface? [**Answer:** 1.03×10^{15} Hz]

Some related homework problems: Problem 31, Problem 32, Problem 33

CONCEPTUAL CHECKPOINT 30–2 EJECTED ELECTRONS

Consider a photoelectric experiment such as the one illustrated in Figure 30–5. A beam of light with a frequency greater than the cutoff frequency shines on the emitter. If the frequency of this beam is increased while the intensity is held constant, does the number of electrons ejected per second from the metal surface **(a)** increase, **(b)** decrease, or **(c)** stay the same?

REASONING AND DISCUSSION

Increasing the frequency of the beam means that each photon carries more energy; however, we know that the intensity of the beam remains constant. It follows, then, that fewer photons hit the surface per time—otherwise the intensity would increase. Since fewer photons hit the surface per time, fewer electrons are ejected per time.

ANSWER

(b) The number of electrons ejected per second decreases.

Applications of the photoelectric effect are in common use all around us. For example, if you have ever dashed into an elevator as its doors were closing, you were probably saved from being crushed by the photoelectric effect. Many elevators and garage-door systems use a beam of light and a photoelectric device known as a *photocell* as a safety feature. As long as the beam of light strikes the photocell, the photoelectric effect generates enough ejected electrons to produce a detectable electric current. When the light beam is blocked—by a late arrival at the elevator, for example—the electric current produced by the photocell is interrupted and the doors are signaled to open. Similar photocells automatically turn on streetlights at dusk and measure the amount of light entering a camera.

REAL-WORLD PHYSICS

Photocells

◀ The photoelectric effect is the basic mechanism used by photovoltaic cells, which are now used to power both terrestrial devices such as pay phones (left) and the solar panels that supply electricity to the Hubble Space Telescope (right).