18. • **CE** The umpire at a baseball game wears glasses that make his eyes look smaller than they actually are. Is the umpire nearsighted or farsighted? Explain.

19. • Construct a ray diagram for Active Example 27–2.

20. • The cornea of a normal human eye has an optical power of +43.0 diopters. What is its focal length?

21. • A myopic student is shaving without his glasses. If his eyes have a far point of 1.6 m, what is the greatest distance he can stand from the mirror and still see his image clearly?

22. • An eyeglass prescription calls for a lens with an optical power of +2.7 diopters. What is the focal length of this lens?

23. •• Two thin lenses, with $f_1 = +25.0$ cm and $f_2 = -42.5$ cm, are placed in contact. What is the focal length of this combination?

24. •• Two thin lenses have refractive powers of +4.00 diopters and −2.35 diopters. What is the refractive power of the two if they are placed in contact? (Note that these are the same two lenses described in the previous problem.)

25. •• Two concave lenses, each with $f = -12$ cm, are separated by 6.0 cm. An object is placed 24 cm in front of one of the lenses. Find **(a)** the location and **(b)** the magnification of the final image produced by this lens combination.

26. •• **IP BIO** The focal length of a relaxed human eye is approximately 1.7 cm. When we focus our eyes on a close-up object, we can change the refractive power of the eye by about 16 diopters. **(a)** Does the refractive power of our eyes increase or decrease by 16 diopters when we focus closely? Explain. **(b)** Calculate the focal length of the eye when we focus closely.

27. •• **IP BIO Diopter Change in Diving Cormorants** Double-crested cormorants (*Phalacrocorax auritus*) are extraordinary birds—they can focus on objects in the air, just like we can, but they can also focus underwater as they pursue their prey. To do so, they have one of the largest *accommodation ranges* in nature—that is, they can change the focal length of their eyes by amounts that are greater than is possible in other animals. When a cormorant plunges into the ocean to catch a fish, it can change the refractive power of its eyes by about 45 diopters, as compared to only 16 diopters of change possible in the human eye. **(a)** Should this change of 45 diopters be an increase or a decrease? Explain. **(b)** If the focal length of the cormorant's eyes is 4.2 mm before it enters the water, what is the focal length after the refractive power changes by 45 diopters?

28. •• A converging lens of focal length 8.000 cm is 20.0 cm to the left of a diverging lens of focal length −6.00 cm. A coin is placed 12.0 cm to the left of the converging lens. Find **(a)** the location and **(b)** the magnification of the coin's final image.

29. •• Repeat Problem 28, this time with the coin placed 18.0 cm to the right of the diverging lens.

30. •• Find the focal length of contact lenses that would allow a farsighted person with a near-point distance of 176 cm to read a book at a distance of 10.1 cm.

31. •• Find the focal length of contact lenses that would allow a nearsighted person with a 135-cm far point to focus on the stars at night.

32. •• What focal length should a pair of contact lenses have if they are to correct the vision of a person with a near point of 56 cm?

33. •• A nearsighted person wears contacts with a focal length of −8.5 cm. If this person's far-point distance with her contacts is 8.5 m, what is her uncorrected far-point distance?

34. •• Without his glasses, Isaac can see objects clearly only if they are less than 4.5 m from his eyes. What focal length glasses worn 2.1 cm from his eyes will allow Isaac to see distant objects clearly?

35. •• A person whose near-point distance is 49 cm wears a pair of glasses that are 2.0 cm from her eyes. With the aid of these glasses, she can now focus on objects 25 cm away from her eyes. Find the focal length and refractive power of her glasses.

36. •• A pair of eyeglasses is designed to allow a person with a far-point distance of 2.50 m to read a road sign at a distance of 25.0 m. Find the focal length required of these glasses if they are to be worn **(a)** 2.00 cm or **(b)** 1.00 cm from the eyes.

37. •• **IP** Your favorite aunt can read a newspaper only if it is within 15.0 cm of her eyes. **(a)** Is your aunt nearsighted or farsighted? Explain. **(b)** Should your aunt wear glasses that are converging or diverging to improve her vision? Explain. **(c)** How many diopters of refractive power must her glasses have if they are worn 2.00 cm from the eyes and allow her to read a newspaper at a distance of 25.0 cm?

38. •• **IP** The relaxed eyes of a patient have a refractive power of 48.5 diopters. **(a)** Is this patient nearsighted or farsighted? Explain. **(b)** If this patient is nearsighted, find the far point. If this person is farsighted, find the near point. (For the purposes of this problem, treat the eye as a single-lens system, with the retina 2.40 cm from the lens.)

39. •• **IP** You are comfortably reading a book at a distance of 24 cm. **(a)** What is the refractive power of your eyes? **(b)** Does the refractive power of your eyes increase or decrease when you move the book farther away? Explain. (For the purposes of this problem, treat the eye as a single-lens system, with the retina 2.40 cm from the lens.)

40. •• Without glasses, your Uncle Albert can see things clearly only if they are between 25 cm and 170 cm from his eyes. **(a)** What power eyeglass lens will correct your uncle's myopia? Assume the lenses will sit 2.0 cm from his eyes. **(b)** What is your uncle's near point when wearing these glasses?

41. •• A 2.05-cm-tall object is placed 30.0 cm to the left of a converging lens with a focal length $f_1 = 20.5$ cm. A diverging lens, with a focal length $f_2 = -42.5$ cm, is placed 30.0 cm to the right of the first lens. How tall is the final image of the object?

42. •• A simple camera telephoto lens consists of two lenses. The objective lens has a focal length $f_1 = +39.0$ cm. Precisely 36.0 cm behind this lens is a concave lens with a focal length $f_2 = -10.0$ cm. The object to be photographed is 4.00 m in front of the objective lens. **(a)** How far behind the concave lens should the film be placed? **(b)** What is the linear magnification of this lens combination?

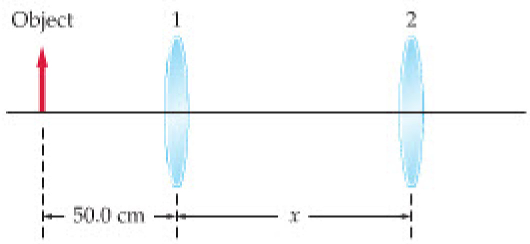
43. •• **IP** With unaided vision, a librarian can focus only on objects that lie at distances between 5.0 m and 0.50 m. **(a)** Which type of lens (converging or diverging) is needed to correct his nearsightedness? Explain. **(b)** Which type of lens will correct his farsightedness? Explain. **(c)** Find the refractive power needed for each part of the bifocal eyeglass lenses that will give the librarian normal visual acuity from 25 cm out to infinity. (Assume the lenses rest 2.0 cm from his eyes.)

44. •• **IP** With unaided vision, a physician can focus only on objects that lie at distances between 5.0 m and 0.50 m. **(a)** Which type of lens (converging or diverging) is needed to correct her nearsightedness? Explain. **(b)** Which type of lens will correct her farsightedness? Explain. **(c)** Find the refractive power needed for each part of the bifocal contact lenses that will give the physician normal visual acuity from 25 cm out to infinity.

45. •• A person's prescription for her new bifocal glasses calls for a refractive power of −0.445 diopter in the distance-vision part, and a power of +1.85 diopters in the close-vision part. What are the near and far points of this person's uncorrected vision? Assume the glasses are 2.00 cm from the person's eyes, and that the person's near-point distance is 25.0 cm when wearing the glasses.

46. •• A person's prescription for his new bifocal eyeglasses calls for a refractive power of −0.0625 diopter in the distance-vision part and a power of +1.05 diopters in the close-vision part. Assuming the glasses rest 2.00 cm from his eyes and that the corrected near-point distance is 25.0 cm, determine the near and far points of this person's uncorrected vision.

47. ••• Two lenses, with $f_1 = +20.0$ cm and $f_2 = +30.0$ cm, are placed on the x axis, as shown in Figure 27–22. An object is fixed 50.0 cm to the left of lens 1, and lens 2 is a variable distance x to the right of lens 1. Find the lateral magnification and location of the final image relative to lens 2 for the following cases: **(a)** $x = 115$ cm; **(b)** $x = 30.0$ cm; **(c)** $x = 0$. **(d)** Show that your result for part (c) agrees with the relation for the effective focal length of two lenses in contact, $1/f_{\text{eff}} = 1/f_1 + 1/f_2$.



▲ **FIGURE 27–22** Problems 47 and 102

48. ••• A converging lens with a focal length of 4.0 cm is to the left of a second identical lens. When a feather is placed 12 cm to the left of the first lens, the final image is the same size and orientation as the feather itself. What is the separation between the lenses?

SECTION 27–3 THE MAGNIFYING GLASS

49. • The Moon is 3476 km in diameter and orbits the Earth at an average distance of 384,400 km. **(a)** What is the angular size of the Moon as seen from Earth? **(b)** A penny is 19 mm in diameter. How far from your eye should the penny be held to produce the same angular diameter as the Moon?

50. •• A magnifying glass is a single convex lens with a focal length of $f = +14.0$ cm. **(a)** What is the angular magnification when this lens forms a (virtual) image at $-\infty$? How far from the object should the lens be held? **(b)** What is the angular magnification when this lens forms a (virtual) image at the person's near point (assumed to be 25 cm)? How far from the object should the lens be held in this case?

51. •• **IP** A student has two lenses, one of focal length $f_1 = 5.0$ cm and the other with focal length $f_2 = 13$ cm. **(a)** When used as a simple magnifier, which of these lenses can produce the greater magnification? Explain. **(b)** Find the maximum magnification produced by each of these lenses.

52. •• A beetle 4.73 mm long is examined with a simple magnifier of focal length $f = 10.1$ cm. If the observer's eye is relaxed while using the magnifier, and has a near-point distance of 25.0 cm, what is the apparent length of the beetle?

53. •• To engrave wishes of good luck on a watch, an engraver uses a magnifier whose focal length is 8.65 cm. If the image formed by the magnifier is at the engraver's near point of 25.6 cm, find **(a)** the distance between the watch and the magnifier and **(b)** the angular magnification of the engraving. Assume the magnifying glass is directly in front of the engraver's eyes.

54. •• A jeweler examines a diamond with a magnifying glass. If the near-point distance of the jeweler is 20.8 cm, and the focal length of the magnifying glass is 7.50 cm, find the angular magnification when the diamond is held at the focal point of the magnifier. Assume the magnifying glass is directly in front of the jeweler's eyes.

55. •• In Problem 54, find the angular magnification when the diamond is held 5.59 cm from the magnifying glass.

56. ••• A person with a near-point distance of 25 cm finds that a magnifying glass gives an angular magnification that is 1.5 times larger when the image of the magnifier is at the near point than when the image is at infinity. What is the focal length of the magnifying glass?

SECTION 27–4 THE COMPOUND MICROSCOPE

57. • **CE** You have two lenses: lens 1 with a focal length of 0.45 cm and lens 2 with a focal length of 1.9 cm. If you construct a microscope with these lenses, which one should you use as the objective? Explain.

58. • A compound microscope has an objective lens with a focal length of 2.2 cm and an eyepiece with a focal length of 5.4 cm. If the image produced by the objective is 12 cm from the objective, what magnification does this microscope produce?

59. • **BIO** A typical red blood cell subtends an angle of only 1.9×10^{-5} rad when viewed at a person's near-point distance of 25 cm. Suppose a red blood cell is examined with a compound microscope in which the objective and eyepiece are separated by a distance of 12.0 cm. Given that the focal length of the eyepiece is 2.7 cm, and the focal length of the objective is 0.49 cm, find the magnitude of the angle subtended by the red blood cell when viewed through this microscope.

60. •• The medium-power objective lens in a laboratory microscope has a focal length $f_{\text{objective}} = 4.00$ mm. **(a)** If this lens produces a lateral magnification of −40.0, what is its "working distance"; that is, what is the distance from the object to the objective lens? **(b)** What is the focal length of an eyepiece lens that will provide an overall magnification of 125?

61. •• A compound microscope has the objective and eyepiece mounted in a tube that is 18.0 cm long. The focal length of the eyepiece is 2.62 cm, and the near-point distance of the person using the microscope is 25.0 cm. If the person can view the image produced by the microscope with a completely relaxed eye, and the magnification is −4525, what is the focal length of the objective?

62. •• In Problem 61, what is the distance between the objective lens and the object to be examined?

63. •• The barrel of a compound microscope is 15 cm in length. The specimen will be mounted 1.0 cm from the objective, and the eyepiece has a 5.0-cm focal length. Determine the focal length of the objective lens.

64. •• A compound microscope uses a 75.0-mm lens as the objective and a 2.0-cm lens as the eyepiece. The specimen will be mounted 122 mm from the objective. Determine **(a)** the barrel length and **(b)** the total magnification produced by the microscope.

65. ••• The "tube length" of a microscope is defined to be the difference between the (objective) image distance and objective focal length: $L = d_i - f_{\text{objective}}$. Many microscopes are standardized to a tube length of $L = 160$ mm. Consider such a microscope whose objective lens has a focal length $f_{\text{objective}} = 7.50$ mm.

(a) How far from the object should this lens be placed? (b) What focal length eyepiece would give an overall magnification of −55? (c) What focal length eyepiece would give an overall magnification of −110?

SECTION 27–5 TELESCOPES

66. • **CE** Two telescopes of different length produce the same angular magnification. Is the focal length of the long telescope's eyepiece greater than or less than the focal length of the short telescope's eyepiece? Explain.

67. • **CE** To construct a telescope, you are given a lens with a focal length of 32 mm and a lens with a focal length of 1600 mm. (a) On the basis of focal length alone, which lens should be the objective and which the eyepiece? Explain. (b) What magnification would this telescope produce?

68. • A grade school student plans to build a 35-power telescope as a science fair project. She starts with a magnifying glass with a focal length of 5.0 cm as the eyepiece. What focal length is needed for her objective lens?

69. • A 55-power refracting telescope has an eyepiece with a focal length of 5.0 cm. How long is the telescope?

70. • An amateur astronomer wants to build a small refracting telescope. The only lenses available to him have focal lengths of 5.00 cm, 10.0 cm, 20.0 cm, and 30.0 cm. (a) What is the greatest magnification that can be obtained using two of these lenses? (b) How long is the telescope with the greatest magnification?

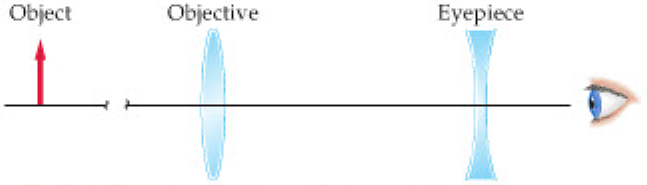
71. • A pirate sights a distant ship with a spyglass that gives an angular magnification of 22. If the focal length of the eyepiece is 11 mm, what is the focal length of the objective?

72. •• A telescope has lenses with focal lengths $f_1 = +30.0$ cm and $f_2 = +5.0$ cm. (a) What distance between the two lenses will allow the telescope to focus on an infinitely distant object and produce an infinitely distant image? (b) What distance between the lenses will allow the telescope to focus on an object that is 5.0 m away and to produce an infinitely distant image?

73. •• Jason has a 25-power telescope whose objective lens has a focal length of 120 cm. To make his sister appear smaller than normal, he turns the telescope around and looks through the objective lens. What is the angular magnification of his sister when viewed through the "wrong" end of the telescope?

74. •• **Roughing It with Science** A professor shipwrecked on Hooligan's Island decides to build a telescope from his eyeglasses and some coconut shells. Fortunately, the professor's eyes require different prescriptions, with the left lens having a power of +5.0 diopters and the right lens having a power of +2.0 diopters. (a) Which lens should he use as the objective? (b) What is the angular magnification of the professor's telescope?

75. •• **Galileo's Telescope** Galileo's first telescope used a convex objective lens with a focal length $f = 1.7$ m and a concave eyepiece, as shown in Figure 27–23. When this telescope is focused on an infinitely distant object, and produces an infinitely distant image, its angular magnification is +3.0. (a) What is the focal length of the eyepiece? (b) How far apart are the two lenses?



▲ **FIGURE 27–23** Problems 75, 86, and 94

76. •• The Moon has an angular size of 0.50° when viewed with unaided vision from Earth. Suppose the Moon is viewed through a telescope with an objective whose focal length is 53 cm and an eyepiece whose focal length is 25 mm. What is the angular size of the Moon as seen through this telescope?

77. •• In Problem 76, an eyepiece is selected to give the Moon an angular size of 15°. What is the focal length of this eyepiece?

78. •• A telescope is 275 mm long and has an objective lens with a focal length of 257 mm. (a) What is the focal length of the eyepiece? (b) What is the magnification of this telescope?

GENERAL PROBLEMS

79. • **CE Predict/Explain BIO Intracorneal Ring** An intracorneal ring is a small plastic device implanted in a person's cornea to change its curvature. By changing the shape of the cornea, the intracorneal ring can correct a person's vision. (a) If a person is nearsighted, should the ring increase or decrease the cornea's curvature? (b) Choose the *best explanation* from among the following:
   I. The intracorneal ring should increase the curvature of the cornea so that it bends light more. This will allow it to focus on light coming from far away.
   II. The intracorneal ring should decrease the curvature of the cornea so it's flatter and bends light less. This will allow parallel rays from far away to be focused.

80. • **CE BIO** The lens in a normal human eye, with aqueous humor on one side and vitreous humor on the other side, has a refractive power of 15 diopters. Suppose a lens is removed from an eye and surrounded by air. In this case, is its refractive power greater than, less than, or equal to 15 diopters? Explain.

81. • **CE** An optical system consists of two lenses, one with a focal length of 0.50 cm and the other with a focal length of 2.3 cm. If the separation between the lenses is 12 cm, is the instrument a microscope or a telescope? Explain.

82. • **CE** An optical system consists of two lenses, one with a focal length of 50 cm and the other with a focal length of 2.5 cm. If the separation between the lenses is 52.5 cm, is the instrument a microscope or a telescope? Explain.

83. • **CE Predict/Explain BIO Treating Cataracts** When the lens in a person's eye becomes clouded by a cataract, the lens can be removed with a process called phacoemulsification and replaced with a man-made intraocular lens. The intraocular lens restores clear vision, but its focal length cannot be changed to allow the user to focus at different distances. In most cases, the intraocular lens is adjusted for viewing of distant objects, and corrective glasses are worn when viewing nearby objects. (a) Should the refractive power of the corrective glasses be positive or negative? (b) Choose the *best explanation* from among the following:
   I. The refractive power should be positive—converging—because the intraocular lens will make the person farsighted.
   II. A negative refractive power is required to bring the focal point of the intraocular lens in from infinity to a finite value.

84. •• **IP** The greatest refractive power a patient's eyes can produce is 44.1 diopters. (a) Is this patient nearsighted or farsighted? Explain. (b) If this patient is nearsighted, find the far point. If this person is farsighted, find the near point. (For the purposes of this problem, treat the eye as a single-lens system, with the retina 2.40 cm from the lens.)

85. •• **IP** You are observing a rare species of bird in a distant tree with your unaided eyes. (a) What is the refractive power of your eyes? (b) Does the refractive power of your eyes increase

or decrease when you shift your view to the guidebook in your hands? Explain. (For the purposes of this problem, treat the eye as a single-lens system, with the retina 2.40 cm from the lens.)

86. •• Galileo's original telescope (Figure 27–23) used a convex objective and a concave eyepiece. Use a ray diagram to show that this telescope produces an upright image when a distant object is being viewed. Assume that the eyepiece is to the right of the object and that the right-hand focal point of the eyepiece is just to the left of the objective's right-hand focal point. In addition, assume that the focal length of the eyepiece has a magnitude that is about one-quarter the focal length of the objective.

87. •• **IP** For each of the following cases, use a ray diagram to show that the angular sizes of the image and the object are identical if both angles are measured *from the center of the lens*. **(a)** A convex lens with the object outside the focal length. **(b)** A convex lens with the object inside the focal length. **(c)** A concave lens with the object outside the focal length. **(d)** Given that the angular size does not change, how does a simple magnifier work? Explain.

88. •• **IP** You have two lenses, with focal lengths $f_1 = +2.60$ cm and $f_2 = +20.4$ cm. **(a)** How would you arrange these lenses to form a magnified image of the Moon? **(b)** What is the maximum angular magnification these lenses could produce? **(c)** How would you arrange the same two lenses to form a magnified image of an insect? **(d)** If you use the magnifier of part (c) to view an insect, what is the angular magnification when the insect is held 2.90 cm from the objective lens?

89. •• **BIO** The eye is actually a multiple-lens system, but we can approximate it with a single-lens system for most of our purposes. When the eye is focused on a distant object, the optical power of the equivalent single lens is +41.4 diopters. **(a)** What is the effective focal length of the eye? **(b)** How far in front of the retina is this "equivalent lens" located?

90. •• **BIO Fitting Contact Lenses with a Keratometer** When a patient is being fitted with contact lenses, the curvature of the patient's cornea is measured with an instrument known as a keratometer. A lighted object is held near the eye, and the keratometer measures the magnification of the image formed by reflection from the front of the cornea. If an object is held 10.0 cm in front of a patient's eye, and the reflected image is magnified by a factor of 0.035, what is the radius of curvature of the patient's cornea?

91. •• **Pricey Stamp** A rare 1918 "Jenny" stamp, depicting a misprinted, upside-down Curtiss JN-4 "Jenny" airplane, sold at auction for $525,000. A collector uses a simple magnifying glass to examine the "Jenny," obtaining a linear magnification of 2.5 when the stamp is held 2.76 cm from the lens. What is the focal length of the magnifying glass?

92. •• **IP** A person needs glasses with a refractive power of −1.35 diopters to be able to focus on distant objects. **(a)** Is this person nearsighted or farsighted? Explain. **(b)** What is this person's (unaided) far point?

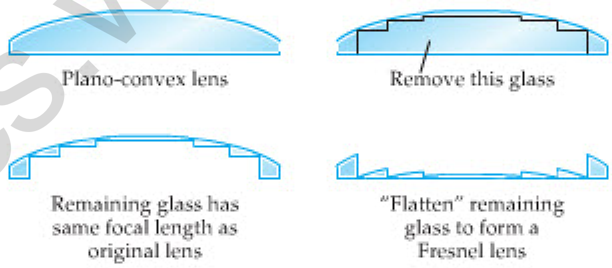
93. •• **IP BIO A Big Eye** The largest eye ever to exist on Earth belonged to an extinct species of ichthyosaur, *Temnodontosaurus platyodon*. This creature had an eye that was 26.4 cm in diameter. It is estimated that this ichthyosaur also had a relatively large pupil, giving it an effective aperture setting of about $f/1.1$. **(a)** Assuming its pupil was one-third the diameter of the eye, what was the approximate focal length of the ichthyosaur's eye? **(b)** When the ichthyosaur narrowed its pupil in bright light, did its f-number increase or decrease? Explain.

94. •• Consider a Galilean telescope, as illustrated in Figure 27–23, constructed from two lenses with focal lengths of 75.6 cm and −18.0 mm. **(a)** What is the distance between these lenses if an infinitely distant object is to produce an infinitely distant image? **(b)** What is the angular magnification when the lenses are separated by the distance calculated in part (a)?

95. •• A converging lens forms a virtual object 12 cm to the right of a second lens that has a refracting power of 3.75 diopter. **(a)** Where is the image? **(b)** Is the image real or virtual? Explain.

96. •• A farsighted person uses glasses with a refractive power of 3.6 diopters. The glasses are worn 2.5 cm from his eyes. What is this person's near point when not wearing glasses?

97. ••• **Landing on an Aircraft Carrier** The Long-Range Lineup System (LRLS) used to ensure safe landings on aircraft carriers consists of a series of Fresnel lenses of different colors. Each lens focuses light in a different, specific direction, and hence which light a pilot sees on approach determines whether the plane is above, below, or on the proper landing path. The basic idea behind a Fresnel lens, which has the same optical properties as an ordinary lens, is shown in Figure 27–24, along with a photo of the LRLS. Suppose an object (a lightbulb in this case) is 17.1 cm behind a Fresnel lens, and that the corresponding image is a distance $d_i = d$ in front of the lens. If the object is moved to a distance of 12.0 cm behind the lens, the image distance doubles to $d_i = 2d$. In the LRLS, it is desired to have the image of the lightbulb at infinity. What object distance will give this result for this particular lens?



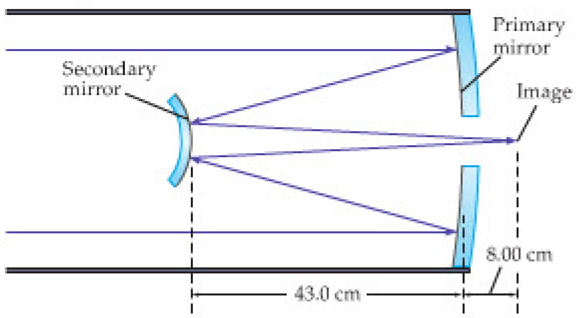
(a)

(b)

▲ **FIGURE 27–24 Fresnel lenses and the Long-Range Lineup System**
(a) A lens causes light to refract at its surface; therefore, the interior glass can be removed without changing its optical properties. This produces a Fresnel lens, which is much lighter than the original lens. **(b)** If an airplane is on the correct approach path, the pilot will see an amber light, called the "meatball," in line with the row of blue lights.

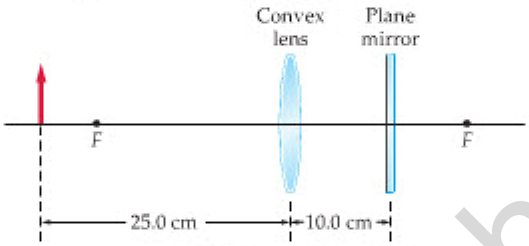
98. ••• When using a telescope to photograph a faint astronomical object, you need to maximize the amount of light energy that falls on each square millimeter of the image on the film. For a given telescope and object, the total light that falls on the film is proportional to the length of the exposure, so a long exposure will reveal fainter objects than a short exposure. Show that for a given length of exposure, the brightness of the image is inversely proportional to the *square* of the f-number of the telescope system.

99. ••• A Cassegrain astronomical telescope uses two mirrors to form the image. The larger (concave) objective mirror has a focal length $f_1 = +50.0$ cm. A small convex secondary mirror is mounted 43.0 cm in front of the primary. As shown in Figure 27–25, light is reflected from the secondary through a hole in the center of the primary, thereby forming a real image 8.00 cm behind the primary mirror. What is the radius of curvature of the secondary mirror?



▲ **FIGURE 27–25** Problem 99

100. ••• **IP** A convex lens ($f = 20.0$ cm) is placed 10.0 cm in front of a plane mirror. A matchstick is placed 25.0 cm in front of the lens, as shown in Figure 27–26. **(a)** If you look through the lens toward the mirror, where will you see the image of the matchstick? **(b)** Is the image real or virtual? Explain. **(c)** What is the magnification of the image? **(d)** Is the image upright or inverted?

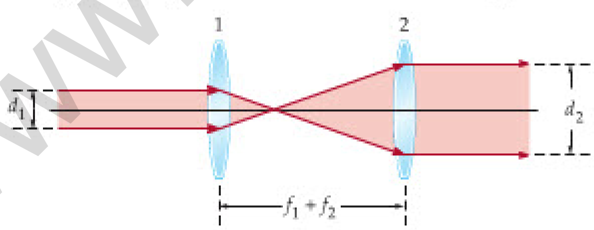


▲ **FIGURE 27–26** Problems 100 and 101

101. ••• Repeat Problem 100 for the case where the converging lens is replaced with a diverging lens with $f = -20.0$ cm. Everything else in the problem remains the same.

102. ••• Repeat Problem 47 for the case where lens 1 is replaced with a diverging lens with $f_1 = -20.0$ cm. Everything else in the problem remains the same.

103. ••• The diameter of a collimated laser beam can be expanded or reduced by using two converging lenses, with focal lengths f_1 and f_2, mounted a distance $f_1 + f_2$ from each other, as shown in Figure 27–27. What is the ratio of the two beam diameters, (d_1/d_2), expressed in terms of the focal lengths?



▲ **FIGURE 27–27** Problem 103

104. ••• Consider three lenses with focal lengths of 25.0 cm, −15.0 cm, and 11.0 cm positioned on the x axis at $x = 0$, $x = 0.400$ m, and $x = 0.500$ m, respectively. An object is at $x = -122$ cm. Find **(a)** the location and **(b)** the orientation and magnification of the final image produced by this lens system.

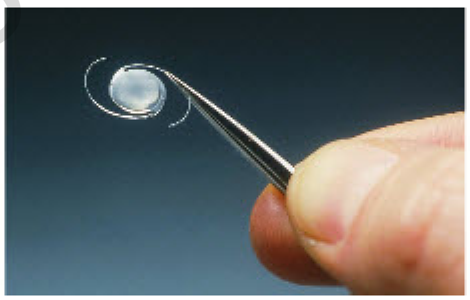
105. ••• Because a concave lens cannot form a real image of a real object, it is difficult to measure its focal length precisely. One method uses a second, convex, lens to produce a virtual object for the concave lens. Under the proper conditions, the concave lens will form a real image of the virtual object! A student conducting a laboratory project on concave lenses makes the following observations: When a lamp is placed 42.0 cm to the left of a particular convex lens, a real (inverted) image is formed 37.5 cm to the right of the lens. The lamp and convex lens are kept in place while a concave lens is mounted 15.0 cm to the right of the convex lens. A real image of the lamp is now formed 35.0 cm to the right of the concave lens. What is the focal length of each lens?

106. ••• A person with a near-point distance N uses a magnifying glass with a focal length f. Show that the greatest magnification that can be achieved with this magnifier is $M = 1 + N/f$.

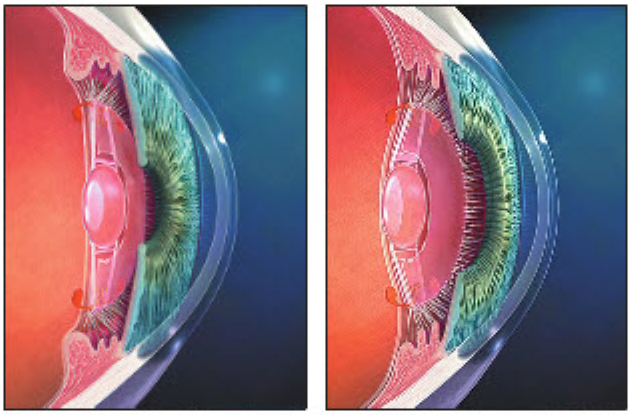
PASSAGE PROBLEMS

BIO Cataracts and Intraocular Lenses

A cataract is an opacity or "cloudiness" that develops in the lens of an eye. The result can be serious degradation of vision, or even blindness. In fact, cataracts are the leading cause of blindness worldwide, and in the United States 60% of the population between the ages of 65 and 74 have cataracts to some extent. Cataracts can be caused by prolonged exposure to electromagnetic radiation of almost any form, including microwaves,



(a) A spring-loaded, nonadaptive intraocular lens



Forward position (close focus) Rear position (focus at infinity)

(b) An adaptive intraocular lens in action

▲ **FIGURE 27–28 Intraocular Lenses**

In cases of severe cataract, an intraocular lens (IOL) can be implanted in place of the eye's natural lens. An adaptive IOL makes use of the same muscles that change he shape of the natural lens, although the lens moves forward and back rather than changing shape.

ultraviolet rays, and infrared rays. For example, cataracts are unusually common among airline pilots, who encounter intense UV exposure at high altitude, and glassblowers, who are exposed to infrared radiation for long periods of time.

Cataracts are generally treated by removing the affected lens with a technique referred to as phacoemulsification. After the natural lens is removed, it is replaced with a man-made, intraocular lens, or IOL. In many cases, the IOL is rigid; neither its focal length nor location can be changed. These lenses are designed to allow the eye to see clearly at infinity, but corrective glasses or contacts must be worn for close vision. More recently, adaptive IOLs have been developed that flex when the focusing muscles of the eye contract, thus allowing a degree of accommodation. This is illustrated in Figure 27–28, where we see the IOL move forward to focus on a close object. Notice that the focal length of the adaptive IOL is fixed, just as with a normal IOL, but the eye muscles can change its location—the same as in a camera when it focuses.

107. • A patient receives a rigid IOL whose focus cannot be changed—it is designed to provide clear vision of objects at infinity. The patient will use corrective contacts to allow for close vision. Should the refractive power of the corrective contacts be positive or negative?

108. • Referring to the previous problem, find the refractive power of contacts that will allow the patient to focus on a book at a distance of 23.0 cm.

A. 0.0435 diopter **B.** 0.230 diopter

C. 4.35 diopters **D.** 8.70 diopters

109. • Suppose a flexible, adaptive IOL has a focal length of 3.00 cm. How far forward must the IOL move to change the focus of the eye from an object at infinity to an object at a distance of 50.0 cm?

A. 1.9 mm **B.** 2.8 mm

C. 3.1 mm **D.** 3.2 mm

INTERACTIVE PROBLEMS

110. •• **IP Referring to Example 27–2** Suppose a person's eyeglasses have a focal length of −301 cm, are 2.00 cm in front of the eyes, and allow the person to focus on distant objects. **(a)** Is this person's far point greater than or less than 323 cm, which is the far point for glasses the same distance from the eyes and with a focal length of −321 cm? Explain. **(b)** Find the far point for this person.

111. •• **IP Referring to Example 27–2** In Example 27–2, a person has a far-point distance of 323 cm. If this person wears glasses 2.00 cm in front of the eyes with a focal length of −321 cm, distant objects can be brought into focus. Suppose a second person's far point is 353 cm. **(a)** Is the magnitude of the focal length of the eyeglasses that allow this person to focus on distant objects greater than or less than 321 cm? Assume the glasses are 2.00 cm in front of the eyes. **(b)** Find the required focal length for the second person's eyeglasses.

112. •• **IP Referring to Example 27–3** Suppose a person's eyeglasses have a refractive power of 2.75 diopters and that they allow the person to focus on an object that is just 25.0 cm from the eye. The glasses are 2.00 cm in front of the eyes. **(a)** Is this person's near point greater than or less than 57.0 cm, which is the near-point distance when the glasses have a refractive power of 2.53 diopters? Explain. **(b)** Find the near point for this person.

113. •• **IP Referring to Example 27–3** Suppose a person's near-point distance is 67.0 cm. **(a)** Is the refractive power of the eyeglasses that allow this person to focus on an object just 25.0 cm from the eye greater than or less than 2.53 diopters, which is the refractive power when the near-point distance is 57.0 cm? The glasses are worn 2.00 cm in front of the eyes. **(b)** Find the required refractive power for this person's eyeglasses.

28 Physical Optics: Interference and Diffraction

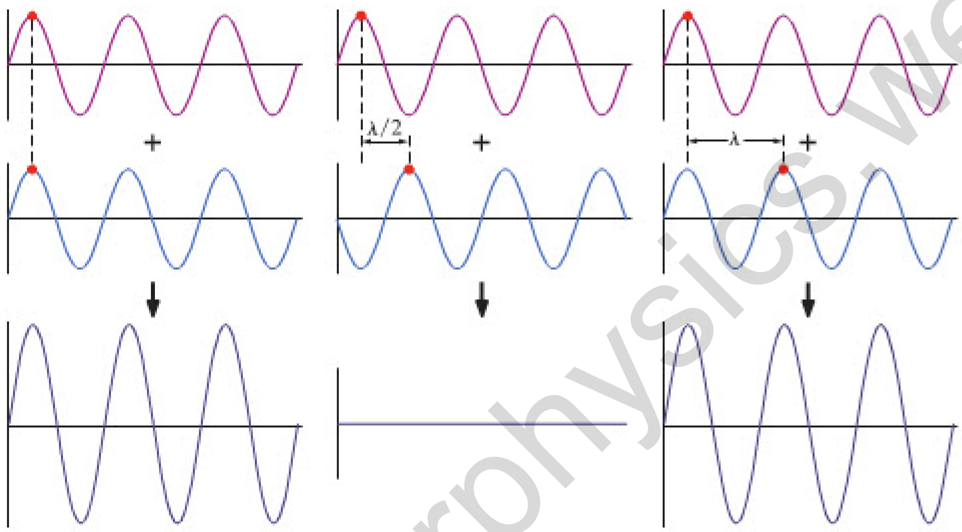
Most of the colors that we see every day can be understood in terms of absorption and reflection—an old-fashioned fire engine absorbs green light and reflects red light, a grassy lawn absorbs red light and reflects green light, and so on. Yet the distinctive iridescent hues of this blue morpho butterfly are created by different physical mechanisms. These mechanisms, known as interference and diffraction, cannot be understood solely in terms of geometrical optics. Instead, they have their origin in the wave nature of light, the subject of this chapter.

In our discussion of optics to this point we have treated light in terms of "rays" that propagate in straight lines. This description is valid in a wide variety of circumstances, as we have seen, though it ignores the fact that light is an electromagnetic *wave*. There are situations, however, in which the ray model fails and the wave nature of light is of central importance. For example, any time we deal with objects or sources that have characteristic dimensions comparable to the wavelength of light, we shall find new effects not predicted by ray optics. In this chapter we show that the wave properties of light are responsible for such phenomena as the operation of a CD, the appearance of images on a television screen, and the brilliant iridescent colors of a butterfly's wing.

28–1 Superposition and Interference

One of the fundamental aspects of wave behavior is **superposition,** in which the net displacement caused by a combination of waves is the algebraic sum of the displacements caused by each wave individually. If waves add to cause a larger displacement, we say that they interfere **constructively;** if the net displacement is reduced, the interference is **destructive.** Examples of wave superposition on a string are given in Figures 14–19 and 14–20. Because light also exhibits wave behavior, with propagating electric and magnetic fields, it too can show interference effects and a resulting increase or decrease in brightness.

Interference is noticeable, however, only if certain conditions are met. In the case of light, for example, the light should be **monochromatic;** that is, it should have a single color and hence a single frequency. In addition, if two or more sources of light are to show interference, they must maintain a constant phase relationship with one another—that is, they must be **coherent.** Sources whose relative phases vary randomly with time show no discernible interference patterns and are said to be **incoherent.** Incoherent light sources include incandescent and fluorescent lights. In contrast, lasers emit light that is both monochromatic and coherent.



(a) In phase

(b) Half wavelength out of phase

(c) Full wavelength out of phase

FIGURE 28–1 Constructive and destructive interference

(a) Waves that are in phase add to give a larger amplitude. This is constructive interference. **(b)** Waves that are half a wavelength out of phase interfere destructively. If the individual waves have equal amplitudes, as here, their sum will have zero amplitude. **(c)** When waves are one wavelength out of phase, the result is again constructive interference, exactly the same as when the waves are in phase.

The basic principle that determines whether waves will interfere constructively or destructively is their phase relative to one another. For example, if two waves have zero phase difference—that is, the waves are in phase—they add constructively, and the net result is an increased amplitude, as **Figure 28–1 (a)** indicates. If waves of equal amplitude are 180° out of phase, however, the net result is zero amplitude and destructive interference, as indicated in **Figure 28–1 (b)**. Notice that a *180° difference in phase* corresponds to waves being out of step by *half a wavelength*. Similarly, if the phase difference between two waves is 360°—which corresponds to one full wavelength—the interference is again constructive, as in **Figure 28–1 (c)**.

Finally, Figure 28–1 shows the blue wave shifted to the right, meaning it is *ahead* of the red wave by half a wavelength in part (b), and *ahead* by a full wavelength in part (c). The same results are obtained, however, if the blue wave is *behind* the red wave. Thus, for example, a phase difference of −180° produces destructive interference, the same as in Figure 28–1 (b), and a phase difference of −360° produces constructive interference, just as in Figure 28–1 (c).

Let's apply the preceding observations to a system consisting of two radio antennas radiating electromagnetic waves of frequency f and wavelength λ, as in **Figure 28–2.** If the antennas are connected to the same transmitter, they emit waves that are in phase and coherent. When waves from each antenna reach point P_0, they have traveled the same distance—that is, the same number of wavelengths—and hence they are still in phase. As a result, point P_0 experiences constructive

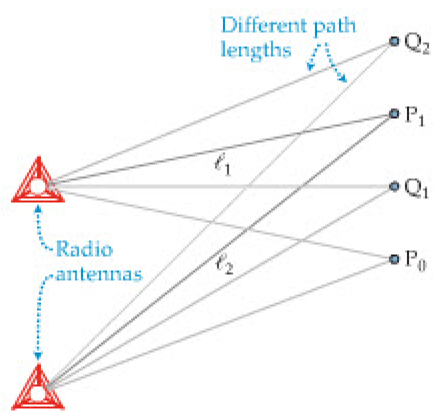
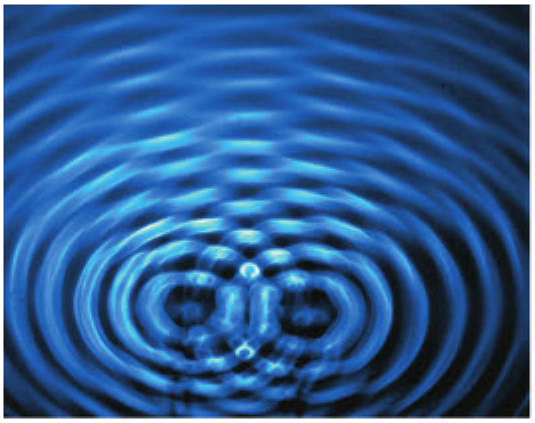


FIGURE 28–2 Two radio antennas transmitting the same signal

At point P_0, midway between the antennas, the waves travel the same distance, and hence they interfere constructively. At point P_1 the distance ℓ_2 is greater than the distance ℓ_1 by one wavelength; thus, P_1 is also a point of constructive interference. At Q_1 the distance ℓ_2 is greater than the distance ℓ_1 by half a wavelength, and the waves interfere destructively at that point.



▲ Two sets of water waves, radiating outward in circular patterns from point sources, create an interference pattern where they overlap.

interference and the radio signal is strong. This location corresponds to the situation shown in Figure 28–1 (a).

To reach point P_1, the waves from the two antennas must travel different distances, ℓ_1 and ℓ_2, as indicated in Figure 28–2. If the difference in these distances is one wavelength, $\ell_2 - \ell_1 = \lambda$, the waves are 360° out of phase, and point P_1 also has constructive interference. This location corresponds to Figure 28–1 (c). Similarly, if the difference in path length to a point P_m is an integer $m = 0, 1, 2, \ldots$ times the wavelength, $\ell_2 - \ell_1 = m\lambda$, that point will also be a location of constructive interference.

Finally, the difference in path length to point Q_1 is half a wavelength, $\ell_2 - \ell_1 = \lambda/2$, and hence the waves cancel there, just as in Figure 28–1 (b). Destructive interference also occurs at Q_2, where the difference in path length is one and a half wavelengths, $\ell_2 - \ell_1 = 3\lambda/2$. In general, for $m = 1, 2, 3, \ldots$, destructive interference occurs at points Q_m where the difference in path lengths is $\ell_2 - \ell_1 = \left(m - \frac{1}{2}\right)\lambda$.

To summarize, *constructive interference* satisfies the conditions

$$\ell_2 - \ell_1 = m\lambda \qquad m = 0, 1, 2, \ldots \qquad \textit{(constructive interference)}$$

Similarly, *destructive interference* satisfies the conditions

$$\ell_2 - \ell_1 = \left(m - \tfrac{1}{2}\right)\lambda \qquad m = 1, 2, 3, \ldots \qquad \textit{(destructive interference)}$$

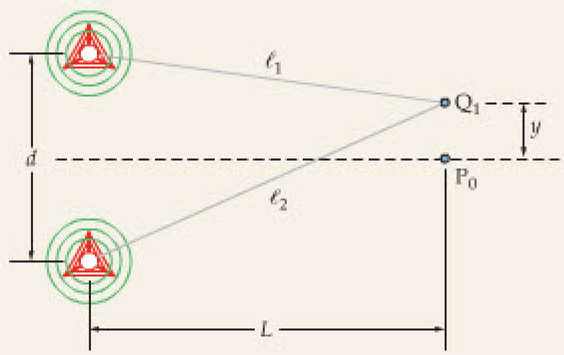
These conditions are applied to a specific physical system in the following Example.

EXAMPLE 28–1 TWO MAY NOT BE BETTER THAN ONE

Two friends tune their radios to the same frequency and pick up a signal transmitted simultaneously by a pair of antennas. The friend who is equidistant from the antennas, at P_0, receives a strong signal. The friend at point Q_1 receives a very weak signal. Find the wavelength of the radio waves if $d = 7.50$ km, $L = 14.0$ km, and $y = 1.88$ km. Assume that Q_1 is the first point of minimum signal as one moves away from P_0 in the y direction.

PICTURE THE PROBLEM

Our sketch shows the radio antennas and the two locations mentioned in the problem statement. Notice that the radio antennas are separated by a distance $d = 7.50$ km in the y direction, and that the points P_0 and Q_1 have a y-direction separation of $y = 1.88$ km. The distance to P_0 and Q_1 in the x direction is $L = 14.0$ km.



STRATEGY

Since point Q_1 is the first *minimum* in the y direction from the *maximum* at point P_0, we know that the path difference, $\ell_2 - \ell_1$, is half a wavelength. Thus we can determine λ by calculating the lengths ℓ_2 and ℓ_1 and setting their difference equal to $\lambda/2$.

SOLUTION

1. Calculate the path length ℓ_1:

$$\ell_1 = \sqrt{L^2 + \left(\frac{d}{2} - y\right)^2}$$

$$= \sqrt{(14.0\text{ km})^2 + \left(\frac{7.50\text{ km}}{2} - 1.88\text{ km}\right)^2} = 14.1\text{ km}$$

2. Calculate the path length ℓ_2:

$$\ell_2 = \sqrt{L^2 + \left(\frac{d}{2} + y\right)^2}$$

$$= \sqrt{(14.0\text{ km})^2 + \left(\frac{7.50\text{ km}}{2} + 1.88\text{ km}\right)^2} = 15.1\text{ km}$$

3. Set $\ell_2 - \ell_1$ equal to $\lambda/2$ and solve for the wavelength:

$$\ell_2 - \ell_1 = \tfrac{1}{2}\lambda$$

$$\lambda = 2(\ell_2 - \ell_1) = 2(15.1\text{ km} - 14.1\text{ km}) = 2.0\text{ km}$$

INSIGHT

We see that radio waves are rather large. In fact, the distance from one crest to the next (the wavelength) for these waves is about 1.2 miles. Recalling that radio waves travel at the speed of light, the corresponding frequency is $f = c/\lambda = 150$ kHz.

PRACTICE PROBLEM

Suppose the wavelength broadcast by these antennas is changed, and that the y distance between P_0 and Q_1 (first minimum) increases as a result. Is the new wavelength greater than or less than 2.0 km? Find the wavelength for the case $y = 2.91$ km. [**Answer:** Greater than 2.0 km; $\lambda = 3.0$ km]

Some related homework problems: Problem 1, Problem 3, Problem 4

28–2 Young's Two-Slit Experiment

We now consider a classic physics experiment that not only demonstrates the wave nature of light, but also allows one to determine the wavelength of a beam of light, just as we did for the radio waves in Example 28–1. The experiment was first performed in 1801 by the English physician and physicist Thomas Young (1773–1829), whose medical background contributed to his studies of vision, and whose love of languages made him a key figure in deciphering the Rosetta Stone. The experiment, in simplest form, consists of a beam of monochromatic light that passes through a small slit in a screen and then illuminates two slits, S_1 and S_2, in a second screen. After the light passes through the two slits, it shines on a distant screen, as **Figure 28–3** shows, where an interference pattern of bright and dark "fringes" is observed.

In this "two-slit experiment" the slit in the first screen serves only to produce a small source of light that prevents the interference pattern on the distant screen from becoming smeared out. The key elements in the experiment are the two slits in the second screen. Since they are equidistant from the single slit, as shown in Figure 28–3, the light passing through them has the same phase. Thus the two slits act as monochromatic, coherent sources of light—analogous to the two radio antennas in Example 28–1—as needed to produce interference.

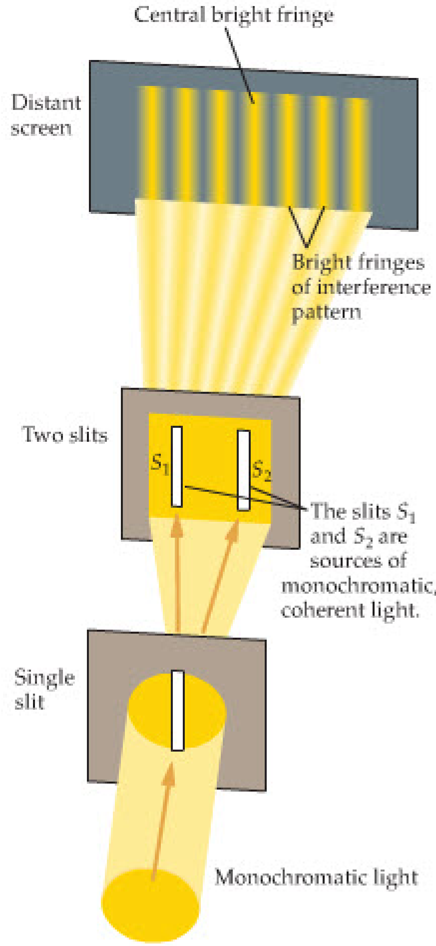
If light were composed of small particles, or "corpuscles" as Newton referred to them, they would simply pass straight through the two slits and illuminate the distant screen directly behind each slit. If light is a wave, on the other hand, each slit acts as the source of new waves, analogous to water waves passing through two small openings. This is referred to as **Huygens's principle** and is illustrated in **Figure 28–4**. Notice that light radiates away from the slits in all forward directions—not just in the direction of the incoming light. The result is that light is spread out over a large area on the distant screen; it is not localized in small regions directly behind the slits. Thus, an experiment like this can readily distinguish between the two models of light.

Of key importance is the fact that the illumination of the distant screen is not only spread out, but also consists of alternating bright and dark fringes, as indicated in Figure 28–3. These fringes are the direct result of constructive and destructive interference. For example, the central bright fringe is midway between the two slits; hence, the path lengths for light from the slits are equal. Because light coming from the slits is in phase, it follows that the light is also in phase at the midpoint, giving rise to constructive interference—just like at point P_0 in Figure 28–2

The next bright fringe occurs when the difference in path length from the two slits is equal to one wavelength of light, as with point P_1 in Figure 28–2. In most experimental situations the distance to the screen is much greater than the separation d of the slits; hence, the light travels to a point on the screen along approximately parallel paths, as indicated in **Figure 28–5**. Therefore, the path difference, $\Delta\ell$, is

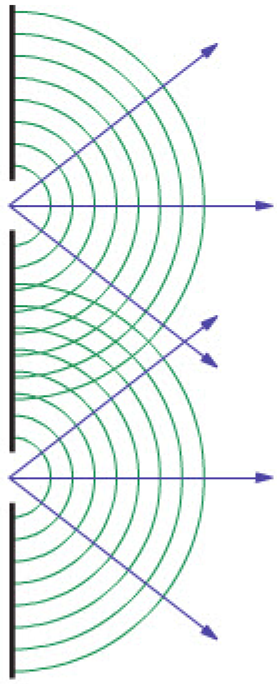
$$\Delta\ell = d\sin\theta$$

As a result, the bright fringe closest to the midpoint occurs at the angle θ given by the condition $d\sin\theta = \lambda$, or $\sin\theta = \lambda/d$. In general, a bright fringe occurs whenever

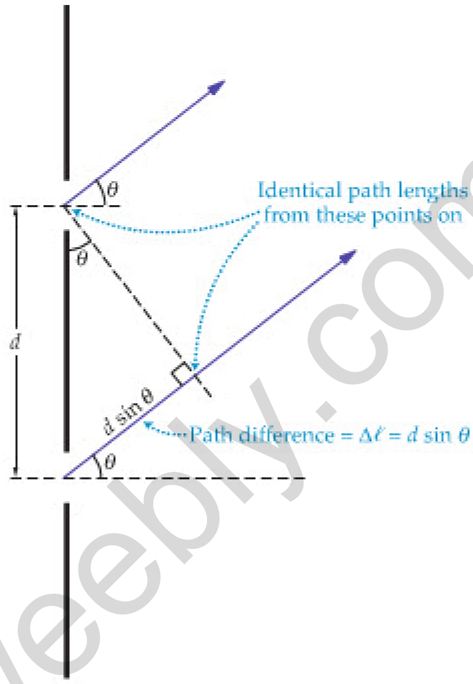


▲ **FIGURE 28–3 Young's two-slit experiment**

The first screen produces a small source of light that illuminates the two slits, S_1 and S_2. After passing through these slits, the light spreads out into an interference pattern of alternating bright and dark fringes on a distant screen.



▲ **FIGURE 28–4 Huygens's principle**
According to Huygens's principle, each of the two slits in Young's experiment acts as a source of light waves propagating outward in all forward directions. It follows that light from the two sources can overlap, resulting in an interference pattern.



▲ **FIGURE 28–5 Path difference in the two-slit experiment**
Light propagating from two slits to a distant screen along parallel paths; note that the paths make an angle θ relative to the normal to the slits. The difference in path length is $\Delta \ell = d \sin \theta$, where d is the slit separation.

the path difference, $\Delta \ell = d \sin \theta$, is equal to $m\lambda$, where $m = 0, \pm 1, \pm 2, \ldots$. Therefore, we find that bright fringes satisfy the following conditions:

Conditions for Bright Fringes (Constructive Interference) in a Two-Slit Experiment

$$d \sin \theta = m\lambda \qquad m = 0, \pm 1, \pm 2, \ldots \qquad \text{28–1}$$

Note that the $m = 0$ fringe occurs at $\theta = 0$, which corresponds to the central fringe. In addition, positive values of m indicate fringes above the central bright fringe; negative values indicate fringes below the central bright fringe.

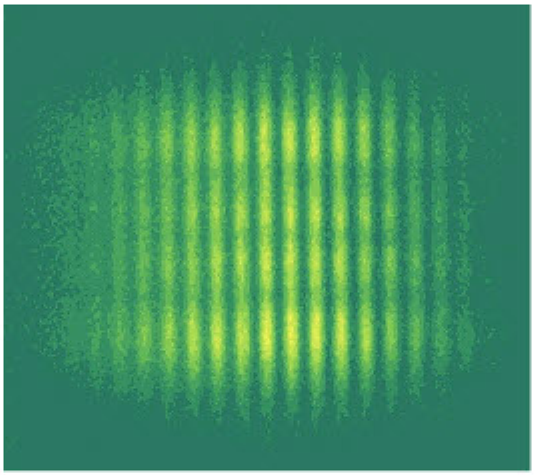
Between the bright fringes we find dark fringes, where destructive interference occurs. The condition for a dark fringe is that the difference in path lengths be $\pm\lambda/2, \pm 3\lambda/2, \pm 5\lambda/2, \ldots$. Notice that the dark fringes are analogous to the points $Q_1, Q_2, \ldots$ in Figure 28–2. It follows that the angles corresponding to dark fringes are given by the following conditions:

Conditions for Dark Fringes (Destructive Interference) in a Two-Slit Experiment

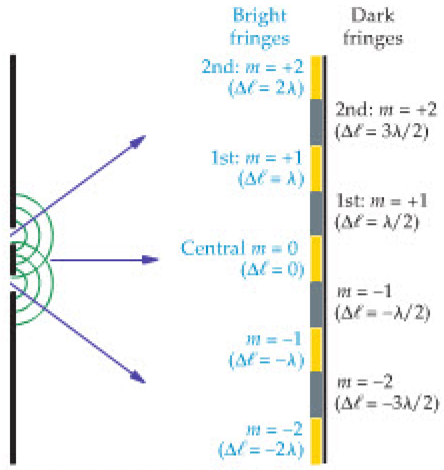
$$d \sin \theta = \left(m - \tfrac{1}{2}\right)\lambda \qquad m = 1, 2, 3, \ldots \qquad \text{(above central bright fringe)}$$
$$d \sin \theta = \left(m + \tfrac{1}{2}\right)\lambda \qquad m = -1, -2, -3, \ldots \qquad \text{(below central bright fringe)} \qquad \text{28–2}$$

Clearly, $m = +1$ corresponds to a path difference of $\Delta \ell = \lambda/2$, the value $m = +2$ corresponds to a path difference of $\Delta \ell = 3\lambda/2$, and so on. These dark fringes are above the central bright fringe. Similarly, $m = -1$ corresponds to a path difference of $-\lambda/2$, and $m = -2$ corresponds to a path difference of $-3\lambda/2$. These dark fringes are below the central bright fringe. **Figure 28–6** shows the numbering systems for both bright and dark fringes. (All problems in this text refer to fringes above the central bright fringe, so only the first set of conditions in Equation 28–2 will be needed.)

Since $\sin \theta$ is less than or equal to 1, it follows from Equation 28–1 that d must be greater than or equal to λ to show the $m = \pm 1$ bright fringes. In a typical



▲ An interference pattern created by monochromatic laser light passing through two slits.



▲ **FIGURE 28–6 The two-slit pattern**
Numbering systems for bright and dark fringes.

situation d may be 100 times larger than λ, which means that the angle to the first dark or bright fringe will be roughly half a degree. If d is too much larger than λ, however, the angle between successive minima and maxima is so small that they tend to merge together, making the interference pattern difficult to discern.

EXERCISE 28–1

Red light ($\lambda = 752$ nm) passes through a pair of slits with a separation of 6.20×10^{-5} m. Find the angles corresponding to **(a)** the first bright fringe and **(b)** the second dark fringe above the central bright fringe.

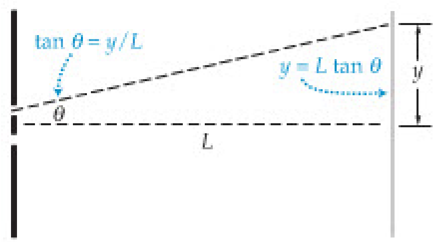
SOLUTION

a. Referring to Figure 28–6 we find that $m = +1$ corresponds to the first bright fringe above the central bright fringe; hence,

$$\theta = \sin^{-1}\left(m\frac{\lambda}{d}\right) = \sin^{-1}\left[(1)\frac{7.52 \times 10^{-7}\text{ m}}{6.20 \times 10^{-5}\text{ m}}\right] = 0.695°$$

b. In this case $m = +2$, therefore

$$\theta = \sin^{-1}\left[\left(m - \tfrac{1}{2}\right)\frac{\lambda}{d}\right] = \sin^{-1}\left[\left(2 - \tfrac{1}{2}\right)\frac{7.52 \times 10^{-7}\text{ m}}{6.20 \times 10^{-5}\text{ m}}\right] = 1.04°$$



▲ **FIGURE 28–7 Linear distance in an interference pattern**
If light propagates at an angle θ relative to the normal to the slits, it is displaced a linear distance $y = L\tan\theta$ on the distant screen.

A convenient way to characterize the location of interference fringes is in terms of their linear distance from the central fringe, as indicated in **Figure 28–7**. If the distance to the screen is L—and L is much greater than the slit separation d—it follows that the linear distance y is given by the following expression:

Linear Distance from Central Fringe

$$y = L\tan\theta \qquad 28\text{–}3$$

In the next Example we show how a measurement of the linear distance between fringes can determine the wavelength of light.

PROBLEM-SOLVING NOTE

Angular Versus Linear Position of Fringes

The angle at which a bright or dark fringe occurs is determined by the wavelength of the light and the separation of the slits. The linear position of a fringe on a screen is determined by the distance from the slits to the screen.

EXAMPLE 28–2 BLUE LIGHT SPECIAL

Two slits with a separation of 8.5×10^{-5} m create an interference pattern on a screen 2.3 m away. **(a)** If the tenth bright fringe above the central fringe is a linear distance of 12 cm above the central fringe, what is the wavelength of light used in the experiment? **(b)** What is the linear distance from the central bright fringe to the tenth dark fringe above it?

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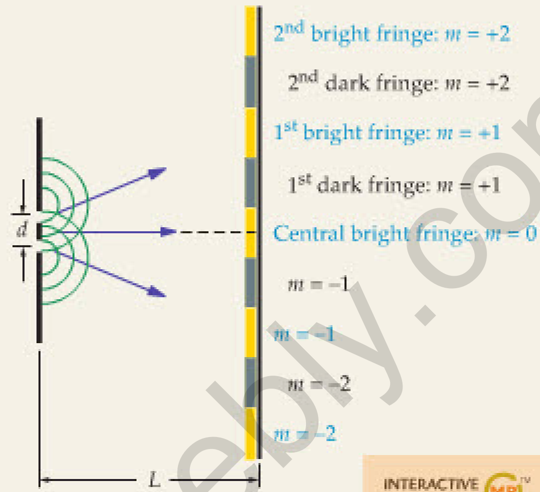
PICTURE THE PROBLEM

Our sketch shows that the first bright fringe above the central fringe corresponds to $m = +1$ in Equation 28–1, the second bright fringe corresponds to $m = +2$, and so on. Therefore, $m = +10$ in Equation 28–1 gives the position of the tenth bright fringe. Similarly, the first dark fringe corresponds to $m = +1$, and so the tenth dark fringe is given by $m = +10$ in the top equation of Equation 28–2.

Finally, we note that the separation of the slits is $d = 8.5 \times 10^{-5}$ m, the distance to the screen is $L = 2.3$ m, and the vertical distance to the $m = +10$ bright fringe is $y = 12$ cm $= 0.12$ m.



INTERACTIVE FIGURE

STRATEGY

a. To find the wavelength, we first determine the angle to the tenth fringe using $y = L \tan\theta$ (Equation 28–3). Once we know θ, we use the condition for bright fringes to determine the wavelength. That is, we set $m = +10$ in Equation 28–1 ($d \sin\theta = m\lambda$) and solve for λ.

b. We use $d \sin\theta = (m - \frac{1}{2})\lambda$ (Equation 28–2) with $m = +10$ and λ from part (a) to determine the angle θ. Next, we substitute θ in $y = L \tan\theta$ to find the linear distance.

SOLUTION

Part (a)

1. Calculate the angle to the tenth bright fringe:

$$y = L \tan\theta$$

$$\theta = \tan^{-1}\left(\frac{y}{L}\right) = \tan^{-1}\left(\frac{0.12\text{ m}}{2.3\text{ m}}\right) = 3.0°$$

2. Use $\sin\theta = m\lambda/d$ to find the wavelength:

$$\lambda = \frac{d}{m}\sin\theta = \left(\frac{8.5 \times 10^{-5}\text{ m}}{10}\right)\sin(3.0°)$$

$$= 4.4 \times 10^{-7}\text{ m} = 440\text{ nm}$$

Part (b)

3. Find the angle corresponding to the tenth dark fringe:

$$d \sin\theta = \left(m - \tfrac{1}{2}\right)\lambda$$

$$\theta = \sin^{-1}\left[\left(m - \tfrac{1}{2}\right)\lambda/d\right]$$

$$= \sin^{-1}\left[\left(10 - \tfrac{1}{2}\right)(4.4 \times 10^{-7}\text{ m})/(8.5 \times 10^{-5}\text{ m})\right] = 2.8°$$

4. Use $y = L \tan\theta$ to find the linear distance:

$$y = L \tan\theta = (2.3\text{ m})\tan(2.8°) = 0.11\text{ m}$$

INSIGHT

Note that we have expressed the wavelength of the light in nanometers, a common unit of measure for light waves. Referring to the electromagnetic spectrum shown in Example 25–3, we see that light with a wavelength of 440 nm is dark blue.

PRACTICE PROBLEM

(a) If the wavelength of light used in this experiment is increased, does the linear distance to the tenth bright fringe above the central fringe increase, decrease, or stay the same? **(b)** Check your reasoning by calculating the linear distance to the tenth bright fringe for a wavelength of 550 nm. [**Answer: (a)** Increase; **(b)** $y = 0.15\text{ m} > 0.12\text{ m}$]

Some related homework problems: Problem 17, Problem 23, Problem 25

Finally, we consider the effect of changing the medium through which the light propagates in a two-slit experiment.

CONCEPTUAL CHECKPOINT 28–1 FRINGE SPACING

A two-slit experiment is performed in the air. Later, the same apparatus is immersed in water and the experiment is repeated. When the apparatus is in water, are the interference fringes **(a)** more closely spaced, **(b)** more widely spaced, or **(c)** spaced the same as when the apparatus was in air?

REASONING AND DISCUSSION

The angles corresponding to bright fringes are related to the wavelength by the equation $d \sin \theta = m\lambda$. From this relation it is clear that if λ is increased, the angle θ (and hence the spacing between fringes) also increases; if λ is decreased, the angle θ decreases. Thus the behavior of the two-slit experiment in water depends on how the wavelength of light changes in water.

Recall that when light goes from air ($n = 1.00$) to a medium in which the index of refraction is $n > 1$, the speed of propagation decreases by the factor n:

$$v = \frac{c}{n}$$

The frequency of light, f, is unchanged throughout as it goes from one medium to another. Therefore, the fact that the speed $v = \lambda f$ decreases by a factor n means that the wavelength λ decreases by the same factor. Hence, if the wavelength of light is λ when $n = 1$, its wavelength in a medium with an index of refraction $n > 1$ is

$$\lambda_n = \frac{\lambda}{n} \qquad 28\text{–}4$$

As a result, the wavelength of light is less in water than in air, and therefore, the interference fringes are more closely spaced when the experiment is performed in water.

ANSWER

(a) The fringes are more closely spaced.

The relation $\lambda_n = \lambda/n$ plays an important role in the interference observed in thin films, as we shall see in the next section.

28–3 Interference in Reflected Waves

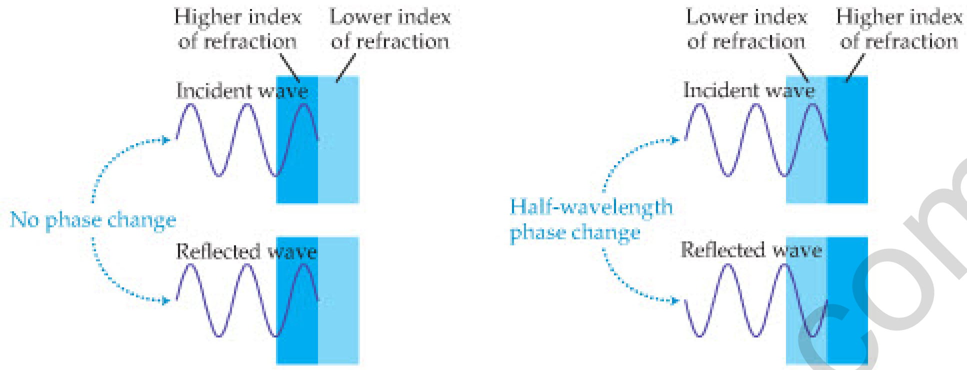
Waves that reflect from objects at different locations can interfere with one another, just like the light from two different slits. In fact, interference due to reflected waves is observed in many everyday circumstances, as we show next. Before we can understand the physics behind this type of interference, however, we must note that reflected waves change their phase in two completely different ways. First, the phase changes in proportion to the distance the waves travel—just as with light in the two-slit experiment. For example, the phase of a wave that travels half a wavelength changes by 180°, and the phase of a wave that travels one wavelength changes by 360°. Second, the phase of a reflected wave can change as a result of the reflection process itself. We begin by considering the latter type of phase change.

Phase Changes Due to Reflection

Phase changes due to reflection have been discussed before in connection with waves on a string in Chapter 14. In particular, we observed at that time that a wave on a string reflects differently depending on whether the end of the string is tied to a solid support, as in Figure 14–7, or is free to move up and down, as in Figure 14–8. Specifically, the wave with a loose end is reflected back exactly as it approached the end; that is, there is no phase change. Conversely, a wave on a string that is tied down is inverted when reflected. This is equivalent to changing the phase of the wave by 180°, or half a wavelength.

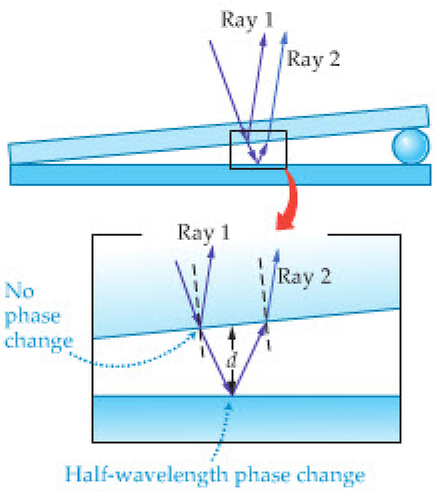
Since light is a wave, it undergoes an analogous phase change on reflection. As indicated in **Figure 28–8 (a)**, a light wave that encounters a region with a lower index of refraction is reflected with no phase change, like a wave on a string whose end is free to move. In contrast, when light encounters a region with a larger index of refraction, as in **Figure 28–8 (b)**, it is reflected with a phase change of half a wavelength, like a wave on a string whose end is fixed. This half-wavelength phase change also applies to light reflected from a solid surface, such as a mirror.

▶ **FIGURE 28–8 Phase change with reflection**
(a) An electromagnetic wave reflects with no phase change when it encounters a medium with a lower index of refraction. **(b)** An electromagnetic wave reflects with a half-wavelength (180°) phase change when it encounters a medium with a larger index of refraction.



(a) Reflection from lower index of refraction

(b) Reflection from higher index of refraction



▲ **FIGURE 28–9 An air wedge**
In an air wedge, interference occurs between the light reflected from the bottom surface of the top plate of glass (ray 1) and light reflected from the top surface of the bottom plate of glass (ray 2). (The two rays are shown widely separated for clarity; in reality, they should be almost on top of one another.)

PROBLEM-SOLVING NOTE
Interference of Reflected Light
When determining whether reflected light interferes constructively or destructively, it is essential to take into account the phase changes that can occur under reflection.

To summarize:

> There is no phase change when light reflects from a region with a lower index of refraction.
>
> There is a half-wavelength phase change when light reflects from a region with a higher index of refraction, or from a solid surface.

We now apply these observations to the case of an air wedge.

Air Wedge

An interesting example of reflection interference is provided by two plates of glass that touch at one end and have a small separation at the other, as shown in Figure 28–9. Note that the air between the plates occupies a thin, wedge-shaped region; hence, this type of arrangement is referred to as an **air wedge.**

The predominant interference effect in this system is between light reflected from the bottom surface of the top glass plate and light reflected from the upper surface of the lower plate, since these surfaces are physically so close together. Consider, for example, the two rays illustrated in Figure 28–9. Ray 1 reflects at the glass-to-air interface; it experiences no phase change. Ray 2 travels a distance d through the air ($n = 1.00$), reflects from the air-to-glass interface, then travels essentially the same distance d in the opposite direction before rejoining ray 1 (the rays in Figure 28–9 are shown widely separated for clarity). Since the reflection from air to glass results in a 180° phase change—the same phase change as if the wave had traveled half a wavelength—the *effective* path length of ray 2 is

$$\Delta \ell_{\text{eff}} = d + \tfrac{1}{2}\lambda + d = \tfrac{1}{2}\lambda + 2d$$

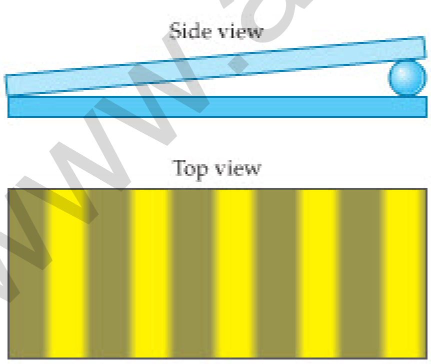
If the effective path length is an integer number of wavelengths, $\lambda/2 + 2d = m\lambda$, rays 1 and 2 will interfere constructively. Dividing by the wavelength, we have the following condition for **constructive interference:**

$$\frac{1}{2} + \frac{2d}{\lambda} = m \qquad m = 1, 2, 3, \ldots \qquad \text{28–5}$$

Similarly, if the effective path length of ray 2 is an odd half integer there will be **destructive interference:**

$$\frac{1}{2} + \frac{2d}{\lambda} = m + \frac{1}{2} \qquad m = 0, 1, 2, \ldots \qquad \text{28–6}$$

Since the distance between the plates, d, increases linearly with the distance from the point where the glass plates touch, it follows that the dark and bright interference fringes are evenly spaced, as shown in Figure 28–10.



▲ **FIGURE 28–10 Interference fringes in an air wedge**
The interference fringes in an air wedge are regularly spaced, as shown in the top view of the wedge.

CONCEPTUAL CHECKPOINT 28–2 DARK OR BRIGHT FRINGE?

Is the point where the glass plates touch in an air wedge **(a)** a dark fringe or **(b)** a bright fringe?

REASONING AND DISCUSSION

At the point where the glass plates touch, the separation d is zero. Setting d equal to zero in Equation 28–5 gives $\frac{1}{2} = m$, which can never be satisfied with an integer value of m. As a result, we conclude that the point of contact is not a bright fringe.

Considering Equation 28–6 in the limit of $d = 0$ yields $\frac{1}{2} = m + \frac{1}{2}$, which is satisfied by $m = 0$. Therefore, the point of contact of the glass plates is the first dark fringe in the system.

We can understand the origin of the dark fringe at the point of contact by recalling that ray 1 undergoes no phase change on reflection, whereas ray 2 experiences a 180° phase change due to reflection. It follows, then, that when the path difference, $2d$, approaches zero, rays 1 and 2 will cancel with destructive interference.

ANSWER

(a) A dark fringe occurs where the two glass plates touch.

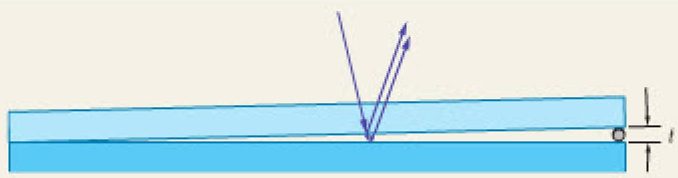
The next Example uses the number of fringes observed in an air wedge to calculate the thickness of a human hair.

EXAMPLE 28–3 SPLITTING HAIRS

An air wedge is formed by placing a human hair between two glass plates on one end, and allowing them to touch on the other end. When this wedge is illuminated with red light ($\lambda = 771$ nm), it is observed to have 179 bright fringes. How thick is the hair?

PICTURE THE PROBLEM

The air wedge used in this experiment is shown in the sketch. Note that the separation of the plates on the end with the hair is equal to the thickness of the hair, t. It is at this point that the 179th bright fringe is observed.



STRATEGY

Recall that the condition for bright fringes is $\frac{1}{2} + 2d/\lambda = m$, and that $m = 1$ corresponds to the first bright fringe, $m = 2$ corresponds to the second bright fringe, and so on. Clearly, then, the 179th bright fringe is given by $m = 179$. Substituting this value for m in $\frac{1}{2} + 2d/\lambda = m$, and setting the plate separation, d, equal to the thickness of the hair, t, allows us to solve for t.

SOLUTION

1. Solve the bright-fringe condition, $\frac{1}{2} + 2d/\lambda = m$, for the plate separation, d:

$$\tfrac{1}{2} + \frac{2d}{\lambda} = m$$

$$d = \frac{\lambda}{2}\left(m - \tfrac{1}{2}\right)$$

2. Set $m = 179$, and solve for the hair thickness, $t = d$:

$$t = \frac{\lambda}{2}\left(m - \tfrac{1}{2}\right)$$

$$= \frac{(771 \times 10^{-9}\ \text{m})}{2}\left(179 - \tfrac{1}{2}\right)$$

$$= 6.88 \times 10^{-5}\ \text{m} = 68.8\ \mu\text{m}$$

INSIGHT

Thus, a human hair has a diameter of roughly 70 micrometers. Note that to measure the thickness of a hair we have used a "ruler" with units that are comparable to the distance to be measured. In this case, the hair has a thickness about 100 times larger than the wavelength of the light that we used.

PRACTICE PROBLEM

If a thicker hair is used in this experiment, will the number of bright fringes increase, decrease, or stay the same? How many bright fringes will be observed if the hair has a thickness of 80.0 μm? [**Answer:** Increase; 208 fringes]

Some related homework problems: Problem 37, Problem 42

REAL-WORLD PHYSICS
Newton's rings

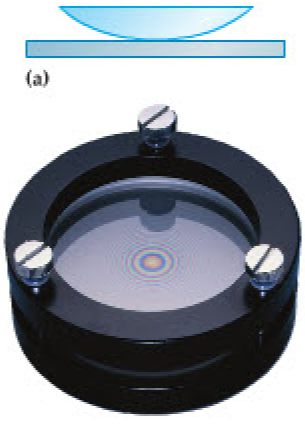
Clearly, the location of interference fringes is very sensitive to even extremely small changes in plate separation. This can be illustrated dramatically by simply pressing down lightly on the upper plate with a finger. Even though the finger causes no visible change in the glass plate, the fringes are observed to move. By measuring the displacement of the fringes it is possible to calculate the tiny deflection, or bend, the finger caused in the plate. Devices using this type of mechanism are frequently used to show small displacements that would be completely invisible to the naked eye.

Newton's Rings

A system similar to an air wedge, but with a slightly different geometry, is shown in Figure 28–11 (a). In this case, the upper glass plate is replaced by a curved piece of glass with a spherical cross section. Still, the mechanism producing interference is the same as before. The result is a series of circular interference fringes, as shown in Figure 28–11 (b), referred to as **Newton's rings.**

Notice that the fringes become more closely spaced as one moves farther from the center of the pattern. The reason is that the curved surface of the upper piece of glass moves away from the lower plate at a progressively faster rate. As a result, the horizontal distance required to go from one fringe to the next becomes less as one moves away from the center.

Newton's rings can be used to test the shape of a lens. Imperfections in the ring pattern indicate slight distortions in the lens. As in the case of an air wedge, even a very small change in shape can cause a significant displacement of the interference fringes.



▲ **FIGURE 28–11 A system for generating Newton's rings**
(a) A variation on the air wedge is produced by placing a piece of glass with a spherical cross section on top of a plate of glass. **(b)** A top view of the system shown in part (a). The circular interference fringes are referred to as Newton's rings.

Thin Films

Perhaps the most commonly observed examples of interference are provided by thin films, such as those found in soap bubbles and oil slicks. We are all familiar, for example, with the swirling patterns of color that are seen on the surface of a bubble. These colors are the result of the constructive and destructive interference that can occur when white light reflects from a thin film. In particular, some colors undergo destructive interference and are *eliminated* from the incident light, while others colors are *enhanced* by constructive interference.

To determine the conditions for constructive and destructive interference in a thin film, consider Figure 28–12. Here we show a thin film of thickness t and index of refraction $n > 1$, with air ($n = 1.00$) on either side. To analyze this system, we proceed in much the same way as we did for the air wedge earlier in this section. Specifically, we focus on the phase change of rays 1 and 2, taking into account phase changes due to both reflection and path-length difference.

First, note that ray 1 reflects from the air-to-film interface; hence, its phase changes by half a wavelength. The effective path length for ray 1, then, is

$$\ell_{\text{eff},1} = \tfrac{1}{2}\lambda$$

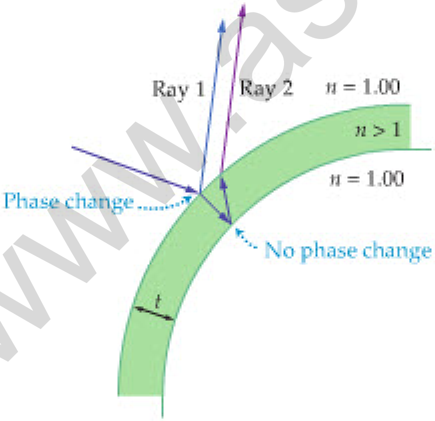
Dividing by the wavelength, λ, gives the phase change of ray 1 in terms of the wavelength:

$$\frac{\ell_{\text{eff},1}}{\lambda} = \frac{1}{2} \qquad 28\text{–}7$$

Recall that if two rays of light differ in phase by half a wavelength, the result is destructive interference.

Next, ray 2 reflects from the film-to-air interface; hence, it experiences no phase change due to reflection. It does, however, have a phase change as a result of traveling an extra distance $2t$ through the film. Thus the effective path length for ray 2 is

$$\ell_{\text{eff},2} = 2t$$



▲ **FIGURE 28–12 Interference in thin films**
The phase of ray 1 changes by half a wavelength due to reflection; the phase of ray 2 changes by $2t/\lambda_n$, where λ_n is the wavelength of light within the thin film.

To put this path length in terms of the wavelength, we must recall that if the wavelength of light in a vacuum is λ_{vacuum}, its wavelength in a medium with an index of refraction n is

$$\lambda_n = \frac{\lambda_{vacuum}}{n}$$

Dividing the path length by λ_n gives us the phase change of ray 2 in terms of the wavelength within the film:

$$\frac{\ell_{eff,2}}{\lambda_n} = \frac{2t}{\lambda_n} = \frac{2nt}{\lambda_{vacuum}} \qquad 28\text{–}8$$

Finally, we can calculate the difference in phase changes for rays 1 and 2 using the preceding results:

$$\text{difference in phase changes} = \frac{2nt}{\lambda_{vacuum}} - \frac{1}{2} \qquad 28\text{–}9$$

Note that in the limit of zero film thickness, $t = 0$, the phase difference is $-\frac{1}{2}$. This corresponds to destructive interference, since moving a wave back half a wavelength is equivalent to moving it ahead half a wavelength. In general, then, destructive interference occurs if any of the following conditions are satisfied:

$$\frac{2nt}{\lambda_{vacuum}} - \frac{1}{2} = -\frac{1}{2}, \frac{1}{2}, \frac{3}{2}, \ldots$$

Adding $\frac{1}{2}$ to each side yields our final result for **destructive interference:**

$$\frac{2nt}{\lambda_{vacuum}} = m \qquad m = 0, 1, 2, \ldots \qquad 28\text{–}10$$

Similarly, if the phase difference between the rays is equal to an integer, m, the result is **constructive interference:**

$$\frac{2nt}{\lambda_{vacuum}} - \frac{1}{2} = m \qquad m = 0, 1, 2, \ldots \qquad 28\text{–}11$$

These conditions are applied in the next Example.

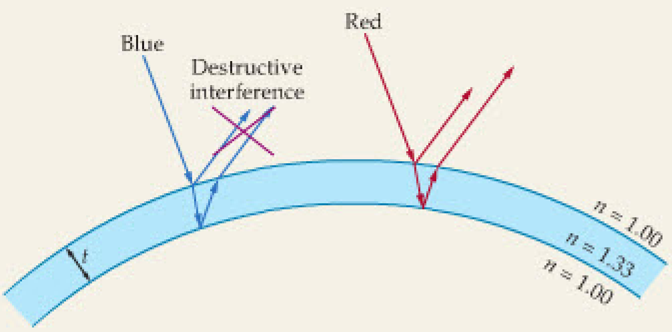
▲ The swirling colors typical of soap bubbles are created by interference, both destructive and constructive, which eliminates certain wavelengths from the reflected light while enhancing others. Which colors are removed or enhanced at a given point depends on the precise thickness of the film in that region.

EXAMPLE 28–4 RED LIGHT SPECIAL

A beam consisting of red light (λ_{vacuum} = 662 nm) and blue light (λ_{vacuum} = 465 nm) is directed at right angles onto a thin soap film. If the film has an index of refraction $n = 1.33$ and is suspended in air ($n = 1.00$), find the smallest nonzero thickness for which it appears red in reflected light.

PICTURE THE PROBLEM

Our sketch shows a soap film of thickness t and index of refraction $n = 1.33$ suspended in air ($n = 1.00$). We consider each of the colors in the incoming beam of light separately, with one blue ray and one red ray. If the film is to look red in reflected light, it follows that the reflected blue light must cancel due to destructive interference, as indicated.



STRATEGY

As mentioned above, the desired thickness of the film is such that blue light satisfies one of the conditions for destructive interference given in Equation 28–10. These conditions are $2nt/\lambda_{vacuum} = m$, where m is equal to 0, 1, 2, and so on. Because the case $m = 0$ corresponds to zero thickness, $t = 0$, it follows that the smallest nonzero thickness is given by $m = 1$.

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SOLUTION

1. Solve $2nt/\lambda_{vacuum} = m$ for the thickness t:

$$\frac{2nt}{\lambda_{vacuum}} = m \quad \text{or} \quad t = \frac{m\lambda_{vacuum}}{2n}$$

2. Calculate the thickness using $m = 1$:

$$t = \frac{m\lambda_{vacuum}}{2n} = \frac{(1)(465 \text{ nm})}{2(1.33)} = 175 \text{ nm}$$

INSIGHT

Although blue light is canceled at this thickness, red light is not. In fact, repeating the preceding calculation shows that red light is not canceled until the thickness of the film is 249 nm.

PRACTICE PROBLEM

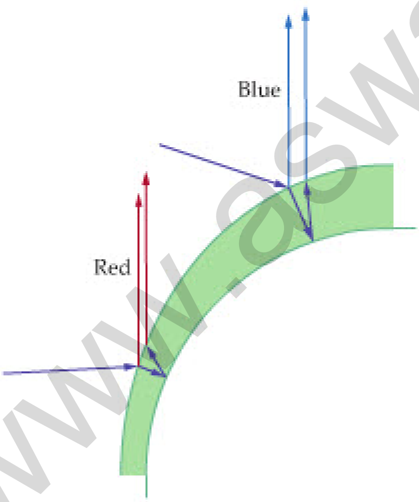
What is the smallest thickness of film that gives *constructive* interference for the blue light in this system? [**Answer:** $t = 87.4$ nm]

Some related homework problems: Problem 32, Problem 38

The connection between the thickness of a film and the color it shows in reflected light is illustrated in **Figure 28–13**. Notice that in thicker regions of the film, the longer wavelengths of light interfere destructively, resulting in reflected light that is bluish. In the limit of zero thickness the condition for destructive interference, $2nt/\lambda_{vacuum} = m$, is satisfied for all wavelengths with m set equal to zero. Hence, an extremely thin film appears dark in reflected light—it is essentially one large dark fringe.

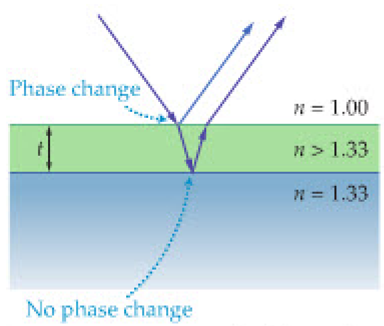
Another example of a thin-film interference is provided by a film floating on a liquid or coating a solid surface. For example, **Figure 28–14** shows a thin film floating on water. If the index of refraction of the film is greater than that of water, the situation in terms of interference is essentially the same as for a thin film suspended in air. In particular, the ray reflected from the top surface of the film undergoes a phase change of half a wavelength; the ray reflected from the bottom of the film has no phase change due to reflection. Thus the interference conditions given in Equation 28–10 and 28–11 still apply.

On the other hand, suppose a thin film has an index of refraction that is greater than 1.00 but less than the index of refraction of the material on which it is supported, as in **Figure 28–15**. In a case like this, there is a reflection phase change at both the upper and the lower surfaces of the film. The condition for destructive interference for this type of system is discussed in the following Active Example.



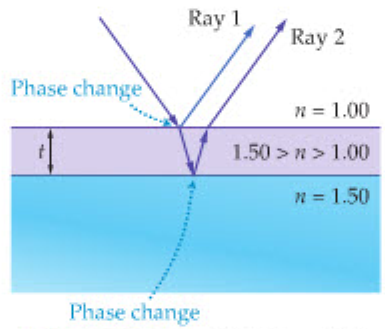
▲ FIGURE 28–13 Thickness and color in a thin film

Thicker portions of thin film appear blue, since the long-wavelength red light experiences destructive interference. Thinner regions appear red because the short-wavelength blue light interferes destructively.



▲ FIGURE 28–14 A thin film with one phase change

If the index of refraction of the film is greater than that of the water, the situation in terms of phase changes is essentially the same as for a thin film suspended in air.



▲ FIGURE 28–15 A thin film with two phase changes

A thin film is applied to a material with a relatively large index of refraction. If the index of refraction of the film is less than that of the material that supports it, there will be a phase change for reflections from both surfaces of the film. Films of this type are often used in nonreflective coatings.

ACTIVE EXAMPLE 28–1 NONREFLECTIVE COATING: FIND THE THICKNESS

Camera lenses ($n = 1.52$) are often coated with a thin film of magnesium fluoride ($n = 1.38$). These "nonreflective coatings" use destructive interference to reduce unwanted reflections. Find the condition for destructive interference in this case, and calculate the minimum thickness required to give destructive interference for light in the middle of the visible spectrum (yellow-green light, $\lambda_{vacuum} = 565$ nm).

SOLUTION *(Test your understanding by performing the calculations indicated in each step.)*

1. Give the phase change (in units of the wavelength) for ray 1 in Figure 28–15: $\frac{1}{2}$
2. Give the phase change (in units of the wavelength) for ray 2 in Figure 28–15: $\frac{1}{2} + 2t/\lambda_n$
3. Set the difference in phase changes equal to one-half: $2t/\lambda_n = \frac{1}{2}$
4. Solve for the thickness: $t = \lambda_n/4$
5. Substitute numerical values: $t = 102$ nm

INSIGHT

Because the thickness of the film should be a quarter of a wavelength, these nonreflective films are often referred to as quarter-wave coatings.

YOUR TURN

Suppose a different coating material with a larger index of refraction is used on this lens. Is the desired minimum thickness greater than or less than it was with magnesium fluoride? Calculate the new minimum thickness, assuming the index of refraction for the coating material is $n = 1.45$.

(Answers to **Your Turn** *problems are given in the back of the book.)*

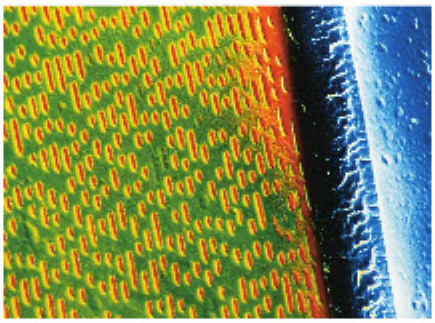
▲ The lenses of binoculars and cameras often have a blue, purple, or amber tint, the product of their antireflection coating. The coating is a thin film that reduces reflection from the lens surfaces by destructive interference.

REAL-WORLD PHYSICS
Nonreflective coating

Interference in CDs

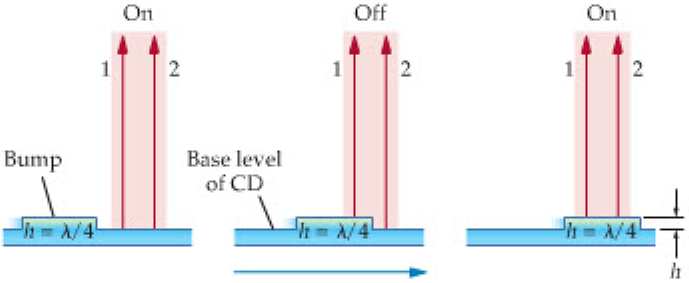
Destructive interference plays a crucial role in the operation of a CD. The basic idea behind these devices is that information is encoded in the form of a series of "bumps" on an otherwise smooth reflecting surface. A laser beam directed onto the surface is reflected back to a detector, and as the intensity of the reflected beam varies due to the bumps, the information on the CD is decoded—similar to using dots and dashes to send information in Morse code.

Imagine the laser beam illuminating a small area on the surface of a CD. As a bump moves into the beam, as in Figure 28–16, there are two components to the reflected beam—one from the top of the bump, the other from the bottom. If these two beams are out of phase by half a wavelength, there will be destructive interference and the detector will receive a weak signal. When the beam is reflected solely from the top of the bump, the detector again receives a strong signal, since there is no longer an interference effect. As the bump moves out of the beam we again have the condition for destructive interference, and the detector signal again falls. Thus the bumps give the detector a series of "on" and "off" signals that can be converted to sound, pictures, or other types of information.



▲ This microscopic view of the surface of a CD reveals the pattern of tiny bumps that encodes the information. From the back, these areas take the form of indentations—hence they are commonly known as "pits," even though to the laser beam scanning the CD they appear as raised areas.

REAL-WORLD PHYSICS
Reading the information on a CD



◀ **FIGURE 28–16 Reading information on a CD**

As a "bump" on a CD moves through the laser beam, the detector receives a weak "off" signal when the bump enters and leaves the beam. When the beam reflects entirely from the top of a bump, or from the base level of the CD, the detector receives a strong "on" signal.

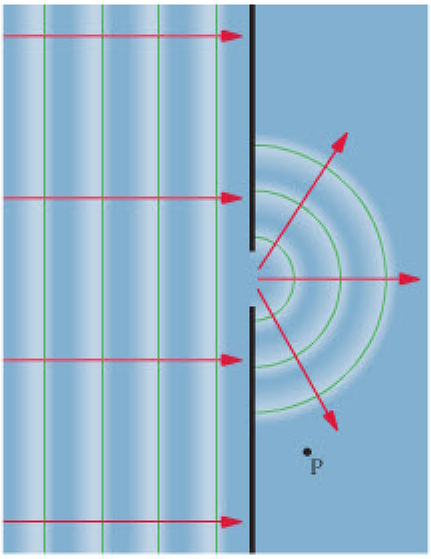
Let's find the necessary height h of a bump if the red light of a ruby laser, with a wavelength of 694 nm, is to have destructive interference. Figure 28–16 shows the situation, in which ray 1 reflects from the top of a bump, and ray 2 reflects from the base level of the CD. The path difference between the rays is $2h$; thus, to have destructive interference, $2h$ must be equal to half a wavelength, $2h = \lambda/2$, or

$$h = \frac{\lambda}{4}$$

In the case of a ruby laser, the required height of a bump is about 174 nm.

28–4 Diffraction

If light is indeed a wave, it must exhibit behavior similar to that displayed by the water waves in Figure 28–17. Note that the waves are initially traveling directly to the right. After passing through the gap in the barrier, however, they spread out and travel in all possible forward directions, in accordance with Huygens's principle. Thus, an observer at point P detects waves even though she is not on a direct line with the initial waves and the gap. In general, waves always bend, or **diffract**, when they pass by a barrier or through an opening.

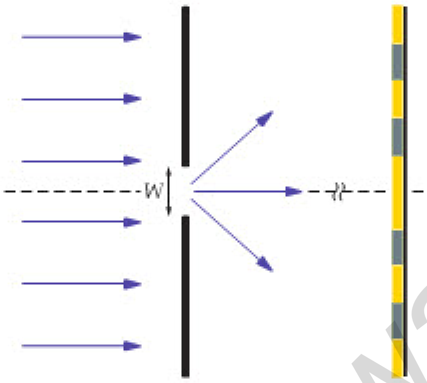


▲ **FIGURE 28–17 Diffraction of water waves**

As water waves pass through an opening, they diffract, or change direction. Thus an observer at point P detects waves even though this point is not on a line with the original direction of the waves and the opening. All waves exhibit similar diffraction behavior.

A familiar example of diffraction is the observation that you can hear a person talking even when that person is out of sight around a corner. The sound waves from the person bend around the corner, just like the water waves in Figure 28–17. It might seem, then, that light cannot be a wave, since it does not bend around a corner along with the sound. There is a significant difference between sound and light waves, however; namely, their wavelengths differ by many orders of magnitude—from a meter or so for sound to about 10^{-7} m for light. As we shall see, the angle through which a wave bends is greater the larger the wavelength of the wave; hence, diffraction effects in light are typically small compared with those in sound waves and water waves.

To investigate the diffraction of light we start by considering the behavior of a beam of light as it passes through a single, narrow slit in a screen.



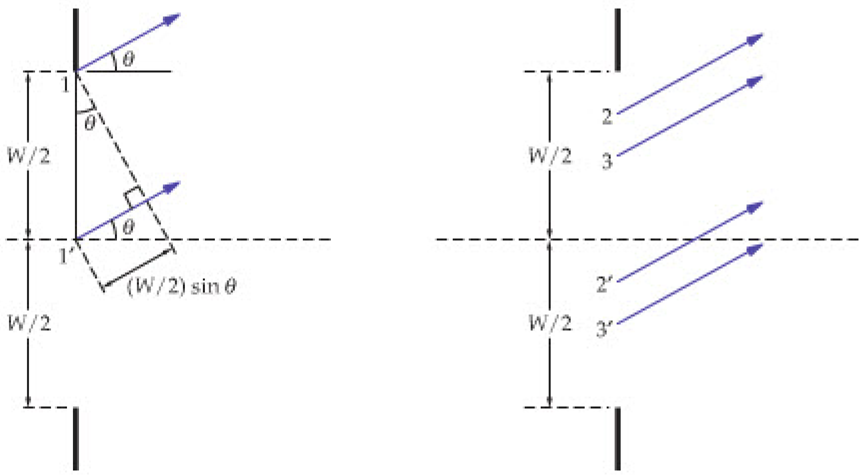
▲ **FIGURE 28–18 Single-slit diffraction**

When light of wavelength λ passes through a slit of width W, a "diffraction pattern" of bright and dark fringes is formed.

Single-Slit Diffraction

Consider monochromatic light of wavelength λ passing through a narrow slit of width W, as shown in Figure 28–18. After passing through the slit the light shines on a distant screen, which we assume to be much farther from the slit than the width W. According to Huygens's principle, each point within the slit can be considered as a source of new waves that radiate toward the screen. The interference of these sources with one another generates a diffraction pattern.

We can understand the origin of a single-slit diffraction pattern by referring to Figure 28–19, where we show light propagating to the screen from various points in



(a) Path difference for rays 1 and 1′

(b) Same path difference for all other pairs of rays

▶ **FIGURE 28–19 Locating the first dark fringe in single-slit diffraction**

The location of the first dark fringe in single-slit diffraction can be determined by considering pairs of waves radiating from the top and bottom half of a slit. **(a)** A wave pair originating at points 1 and 1′ has a path difference of $(W/2)\sin\theta$. These waves interfere destructively if the path difference is equal to half a wavelength. **(b)** The rest of the light coming from the slit can be considered to consist of additional wave pairs, like 2 and 2′, 3 and 3′, and so on. Each wave pair has the same path difference.

the slit. For example, consider waves from points 1 and 1′ propagating to the screen at an angle θ relative to the initial direction of the light. Since the screen is distant, the waves from 1 and 1′ travel on approximately parallel paths to reach the screen. From the figure, then, it is clear that the path difference for these waves is $(W/2)\sin\theta$. Similarly, the same path difference applies to the "wave pairs" from points 2 and 2′, the wave pairs from points 3 and 3′, and so on through all points in the slit.

In the forward direction, $\theta = 0°$, the path difference is zero; $(W/2)\sin 0° = 0$. As a result, all wave pairs interfere constructively, giving maximum intensity at the center of the diffraction pattern. However, if θ is increased until the path difference is half a wavelength, $(W/2)\sin\theta = \lambda/2$, each wave pair experiences destructive interference. Thus, the *first minimum*, or dark fringe, in the diffraction pattern occurs at the angle given by

$$W\sin\theta = \lambda$$

To find the second dark fringe, imagine dividing the slit into four regions, as illustrated in Figure 28–20. Within the upper two regions, we perform the same wave-pair construction described above; the same is done with the lower two regions. In this case, the path difference between wave pairs 1 and 1′ is $(W/4)\sin\theta$. As before, destructive interference first occurs when the path difference is $\lambda/2$. Solving for $\sin\theta$ in this case gives us the condition for the *second dark fringe*:

$$W\sin\theta = 2\lambda$$

The next dark fringe can be found by dividing the slit into six regions, with a path difference between wave pairs of $(W/6)\sin\theta$. In this case, the condition for destructive interference is $W\sin\theta = 3\lambda$. In general, then, dark fringes satisfy the following conditions:

Conditions for Dark Fringes in Single-Slit Interference

$$W\sin\theta = m\lambda \qquad m = \pm 1, \pm 2, \pm 3, \ldots \qquad \text{28–12}$$

Note that by including both positive and negative values for m, we have taken into account the symmetry of the diffraction pattern about its midpoint.

EXERCISE 28–2

Monochromatic light passes through a slit of width 1.2×10^{-5} m. If the first dark fringe of the resulting diffraction pattern is at angle $\theta = 3.25°$, what is the wavelength of the light?

SOLUTION

Solving Equation 28–12 for the wavelength gives us

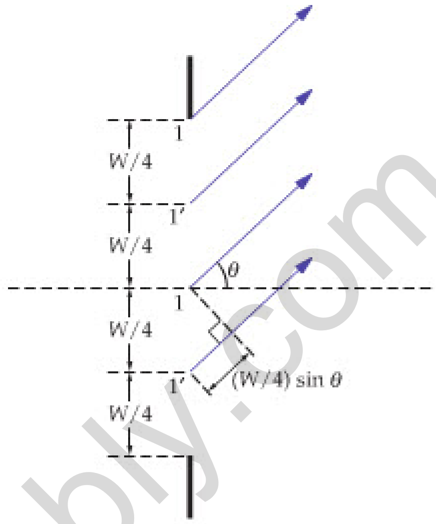
$$\lambda = \frac{W\sin\theta}{m} = \frac{(1.2 \times 10^{-5}\ \text{m})\sin(3.25°)}{1} = 680\ \text{nm}$$

Notice that we use $m = 1$, since this is the *first* dark fringe.

The bright fringes in a diffraction pattern consist of the central fringe plus additional bright fringes approximately halfway between successive dark fringes. Note, in addition, that the central fringe is about twice as wide as the other bright fringes. In the small-angle approximation ($\sin\theta \sim \theta$), the central fringe extends from $\theta = \lambda/W$ to $\theta = -\lambda/W$, and hence its width is given by the following:

$$\text{approximate angular width of central fringe} = 2\frac{\lambda}{W} \qquad \text{28–13}$$

Thus we see that the wavelength λ plays a crucial role in diffraction patterns and that λ/W gives the characteristic angle of "bending" produced by diffraction. In the following Conceptual Checkpoint we consider the role played by the width of the slit.



▲ **FIGURE 28–20 Locating the second dark fringe in single-slit diffraction**
To find the second dark fringe in a single-slit diffraction pattern, we divide the slit into four regions and consider wave pairs that originate from points separated by a distance $W/4$. Destructive interference occurs when the path difference, $(W/4)\sin\theta$, is half a wavelength.

▲ Upon close inspection, the shadow of a sharp edge is seen to consist of numerous fringes produced by diffraction.

CONCEPTUAL CHECKPOINT 28–3 WIDTH OF CENTRAL BRIGHT FRINGE

If the width of the slit through which light passes is reduced, does the central bright fringe **(a)** become wider, **(b)** become narrower, or **(c)** remain the same size?

REASONING AND DISCUSSION

It might seem that making the slit narrower will cause the diffraction pattern to be narrower as well. Recall, however, that the diffraction pattern is produced by waves propagating from all parts of the slit. If the slit is wide, the incoming wave passes through with little deflection. If it is small, on the other hand, it acts like a point source, and light is radiated over a broad range of angles. Therefore, the smaller slit produces a wider central fringe.

This result is also confirmed by considering Equation 28–13, where we see that a smaller value of W results in a wider central fringe.

ANSWER

(a) The central bright fringe is wider.

As mentioned earlier, waves diffract whenever they encounter some sort of barrier or opening. It follows, then, that the shadow cast by an object is really not as sharp as it may seem. On closer examination, as shown in the photo on the previous page, the shadow of an object such as a pair of scissors actually consists of a tightly spaced series of diffraction fringes. Thus, shadows are not the sharp boundaries implied by geometrical optics but, instead, are smeared out on a small length scale. Under ordinary conditions the diffraction pattern in a shadow is not readily visible. However, a similar diffraction pattern can be observed by simply holding two fingers close together before your eyes—try it.

EXAMPLE 28–5 EXPLORING THE DARK SIDE

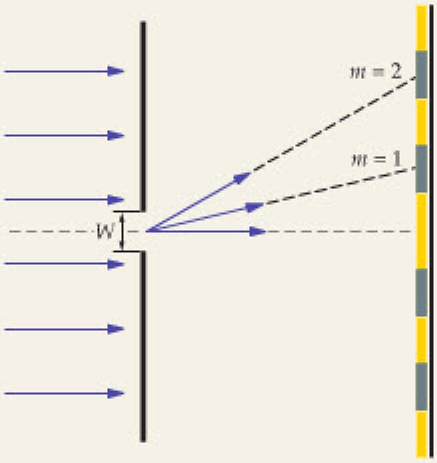
Light with a wavelength of 511 nm forms a diffraction pattern after passing through a single slit of width 2.20×10^{-6} m. Find the angle associated with **(a)** the first and **(b)** the second dark fringe above the central bright fringe.

PICTURE THE PROBLEM

In our sketch we identify the first and second dark fringes above the central bright fringe. Note that the first dark fringe corresponds to $m = 1$, and the second corresponds to $m = 2$. Finally, the width of the slit is $W = 2.20 \times 10^{-6}$ m.

STRATEGY

We can find the desired angles by using the condition for dark fringes, $W \sin\theta = m\lambda$ (Equation 28–12). As mentioned above, we use $m = 1$ for part (a), and $m = 2$ for part (b). The values of λ and W are given in the problem statement.



INTERACTIVE FIGURE

SOLUTION

Part (a)

1. Solve for θ using $m = 1$:

$$\theta = \sin^{-1}\left(\frac{m\lambda}{W}\right) = \sin^{-1}\left[\frac{(1)(511 \times 10^{-9}\text{ m})}{2.20 \times 10^{-6}\text{ m}}\right] = 13.4°$$

Part (b)

1. Solve for θ using $m = 2$:

$$\theta = \sin^{-1}\left(\frac{m\lambda}{W}\right) = \sin^{-1}\left[\frac{(2)(511 \times 10^{-9}\text{ m})}{2.20 \times 10^{-6}\text{ m}}\right] = 27.7°$$

INSIGHT

Notice that the angle to the second dark fringe is *not* simply twice the angle to the first dark fringe. This is because the angle θ depends on the sine function, which is not linear. If you look at a plot of the sine function, as in Figure 13–15, you will see that $\sin\theta$ is slightly less than θ for the angles considered here. Therefore, to double the value of $\sin\theta$ you must increase θ by slightly more than a factor of 2.

PRACTICE PROBLEM

Suppose the wavelength of light in this experiment is changed to give the first dark fringe at an angle greater than 13.4°. Is the required wavelength greater than or less than 511 nm? Check your answer by calculating the wavelength required to give the first dark fringe at $\theta = 15.0°$. [**Answer:** The wavelength must be larger; $\lambda = 569$ nm.]

Some related homework problems: Problem 44, Problem 45, Problem 50

ACTIVE EXAMPLE 28–2 FIND THE LINEAR DISTANCE

In a single-slit experiment, light passes through the slit and forms a diffraction pattern on a screen 2.31 m away. If the wavelength of light is 632 nm, and the width of the slit is 4.20×10^{-5} m, find the linear distance on the screen from the center of the diffraction pattern to the first dark fringe.

SOLUTION *(Test your understanding by performing the calculations indicated in each step.)*

1. Find the angle to the first dark fringe: $\theta = 0.862°$
2. Use $y = L \tan\theta$ to find the linear distance: $y = 3.48$ cm

INSIGHT

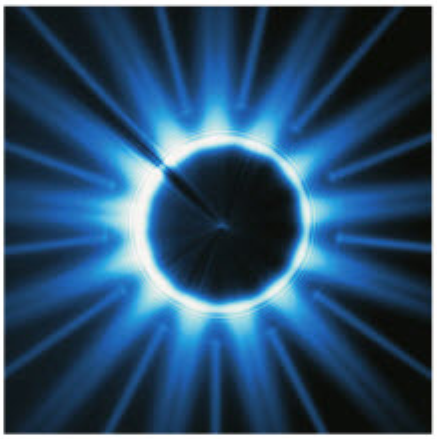
Note that the linear distance is found using Equation 28–3, just as for the two-slit experiment.

YOUR TURN

Find the linear distance from the center of the pattern to the second dark fringe.

(Answers to **Your Turn** *problems are given in the back of the book.)*

The photo to the right shows a particularly interesting diffraction phenomenon: the shadow produced by a penny. As expected, the edges of the shadow show diffraction fringes, but of even greater interest is the bright point of light seen in the center of the shadow. This is referred to as "Poisson's bright spot," after the French scientist Siméon D. Poisson (1781–1840) who predicted its existence. It should be noted that Poisson did not believe that light is a wave. In fact, he used his prediction of a bright spot to show that the wave model of light was absurd and must be rejected—after all, how could a bright spot occur in the darkest part of a shadow? When experiments soon after his prediction showed conclusively that the bright spot does exist, however, the wave model of light gained almost universal acceptance. We know today that light has both wave and particle properties, a fact referred to as the *wave–particle duality*. This will be discussed in detail in Chapter 30.

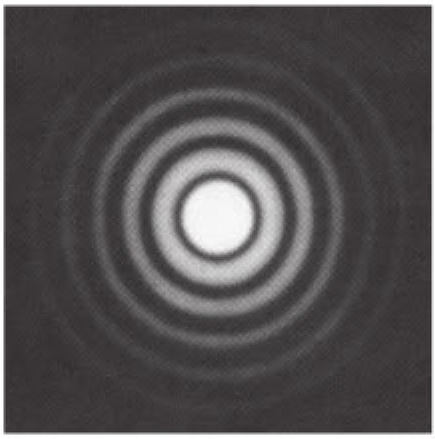


▲ The paradoxical bright dot at the center of a circular shadow, known as Poisson's bright spot, is a convincing proof of the wave nature of light.

28–5 Resolution

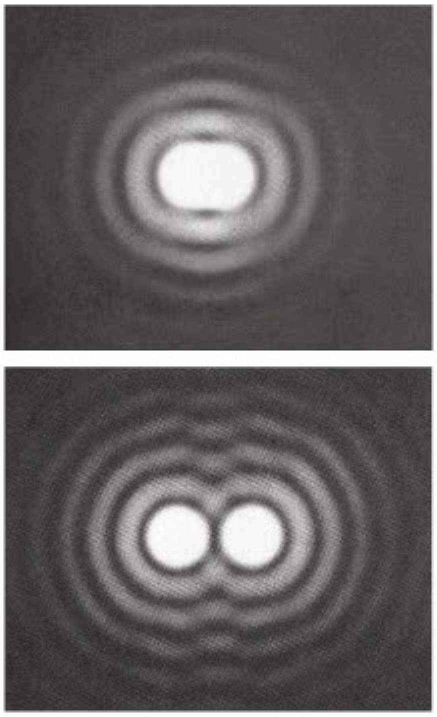
Diffraction affects, and ultimately limits, the way we see the world. There is a difference, for example, in how sharply we can see a scene depending on the size of our pupil—in general, the larger the pupil, the sharper the vision. For example, an eagle, which has pupils that are even larger than ours, can see a small creature on the ground with greater acuity than is possible for a person. Similarly, a camera or a telescope with a large aperture can "see" with greater detail than the human eye. The sharpness of vision—in particular, the ability to visually separate closely spaced objects—is referred to as **resolution.**

The way in which diffraction affects resolution can be seen by considering the diffraction pattern created by a circular aperture—such as the pupil of an eye. Just as with the diffraction pattern of a slit, a circular aperture creates a pattern of alternating bright and dark regions. The difference, as one might expect, is that the diffraction pattern of a circular opening is circular in shape, as we see in Figure 28–21.



▲ **FIGURE 28–21 Diffraction from a circular opening**

Light passing through a circular aperture creates a circular diffraction pattern of alternating bright and dark regions.



▲ **FIGURE 28–22 Resolving two point sources: Rayleigh's criterion**

If the angular separation between two sources is not great enough (top), their diffraction patterns overlap to the point where they appear as a single elongated source. With greater angular separation (bottom), the individual sources can be distinguished as separate.

In a slit pattern, the first dark fringe occurs at the angle given by $\sin\theta = \lambda/W$, where W is the width of the slit. A circular aperture of diameter D produces a central bright spot and a dark fringe at an angle θ from the center line given by the following expression:

First Dark Fringe for the Diffraction Pattern of a Circular Opening

$$\sin\theta = 1.22\frac{\lambda}{D} \qquad 28\text{–}14$$

So we see that the change in geometry from a slit of width W to a circular opening of diameter D is reflected in the replacement of $(1)\lambda/W$ with $(1.22)\lambda/D$.

The physical significance of this result is that even if you focus perfectly on a point source of light, it will form a circular image of finite size on the retina. This blurs the image, replacing a point with a small circle. Thus, the diffraction of light through the pupil limits the resolution of your eye. In addition, it should be noted that the wavelength in Equation 28–14 refers to the wavelength in the medium in which the diffraction pattern is observed. For example, the wavelength that should be used in considering the eye is $\lambda_n = \lambda/n$, where n is the eye's average index of refraction (approximately $n = 1.36$).

Diffraction-induced smearing also makes it difficult to visually separate objects that are close to one another. In particular, if two closely spaced sources of light are smeared by diffraction, the circles they produce may overlap, making it difficult to tell if there are two sources or only one. The condition that is used to determine whether two sources can be visually separated is called **Rayleigh's criterion:**

> If the first dark fringe of one circular diffraction pattern passes through the center of a second diffraction pattern, the two sources responsible for the patterns will appear to be a single source.

This condition is illustrated in Figure 28–22.

To put Rayleigh's criterion in quantitative terms, note that for small angles (as is usually the case) the location of the first dark fringe is given by $\theta = 1.22\lambda/D$. Therefore, *two objects can be seen as separate only if their angular separation is greater than the following minimum:*

Rayleigh's Criterion

$$\theta_{\min} = 1.22\frac{\lambda}{D} \qquad 28\text{–}15$$

Note that the larger the diameter D of the aperture, the smaller the angular separation that can be resolved and, hence, the greater the resolution.

EXERCISE 28–3

Find $\theta_{\min}$ for yellow light (551 nm) and an aperture diameter of 5.00 mm.

SOLUTION

Substituting into Equation 28–15 we find

$$\theta_{\min} = 1.22\frac{\lambda}{D} = 1.22\left(\frac{551 \times 10^{-9}\ \text{m}}{5.00 \times 10^{-3}\ \text{m}}\right) = 0.000134\ \text{rad} = 0.00770°$$

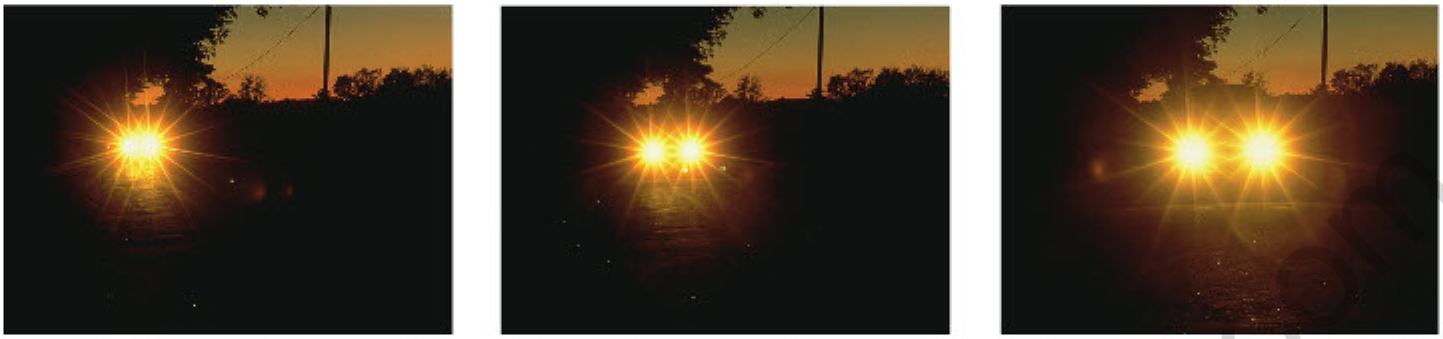
Because our pupils have diameters of about 5.00 mm, the small value of $\theta_{\min}$ indicates that human vision has the potential for relatively high resolution.

PROBLEM-SOLVING NOTE

Angular Resolution and the Index of Refraction

When applying the condition $\theta_{\min} = 1.22\lambda/D$, it is important to remember that λ refers to the wavelength in the region between the aperture and the screen (retina, film, etc.) on which the diffraction pattern is observed. If the index of refraction in this region is n, the wavelength is reduced from λ to λ/n. (See Conceptual Checkpoint 28–1.)

An example of diffraction-limited resolution is illustrated in the accompanying photos. In the first photo we see a brilliant light in the distance that may be the single headlight of an approaching motorcycle or the unresolved image of two headlights on a car. If we are seeing the headlights of a car, the angular separation between them will increase as the car approaches. When the angular separation



▲ Resolving the headlights of an approaching car. If the headlights were true point sources and the atmosphere perfectly transparent (or absent), the individual headlights could be distinguished at a much greater distance, as Example 28–6 shows.

exceeds $1.22\lambda/D$, as in the second photo, we are able to distinguish the two headlights as separate sources of light. As the car continues to approach, its individual headlights become increasingly distinct, as shown in the third photo.

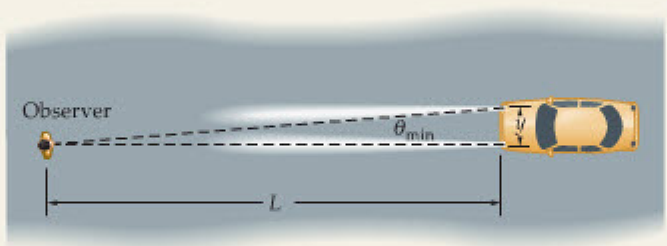
The distance at which the two headlights can be resolved increases as the size of the aperture increases. This dependence is considered in detail in the following Example.

EXAMPLE 28–6 MOTORCYCLE OR CAR?

The linear distance separating the headlights of a car is 1.1 m. Assuming light of 460 nm, a pupil diameter of 5.0 mm, and an average index of refraction for the eye of 1.36, find the maximum distance at which the headlights can be distinguished as two separate sources of light.

PICTURE THE PROBLEM

Our sketch shows the car a distance L from the observer, with the headlights separated by a linear distance $y = 1.1$ m. The condition for the headlights to be resolved is that their angular separation be at least θ_{min}.



STRATEGY

To find the maximum distance, we must first determine the minimum angular separation, which is given by $\theta_{min} = 1.22\lambda_n/D$. In this expression, $\lambda_n = \lambda/1.36$.

Once we know θ_{min}, we can find the distance L using the trigonometric relation $\tan\theta_{min} = y/L$, which follows directly from our sketch.

SOLUTION

1. Find the minimum angular separation for the headlights to be resolved:

$$\theta_{min} = 1.22\frac{\lambda/n}{D}$$

$$= 1.22\left[\frac{(460 \times 10^{-9}\ \text{m})/1.36}{5.0 \times 10^{-3}\ \text{m}}\right] = 8.3 \times 10^{-5}\ \text{rad}$$

2. Solve the relation $\tan\theta_{min} = y/L$ for the distance L:

$$L = \frac{y}{\tan\theta_{min}}$$

3. Substitute numerical values to find L:

$$L = \frac{y}{\tan\theta_{min}} = \frac{1.1\ \text{m}}{\tan(8.3 \times 10^{-5}\ \text{rad})} = 13{,}000\ \text{m}$$

INSIGHT

Thus, the car must be about 8 mi away before the headlights appear to merge. This, of course, is the ideal case. In the real world, the finite size of the headlights and the blurring effects of the atmosphere greatly reduce the maximum distance.

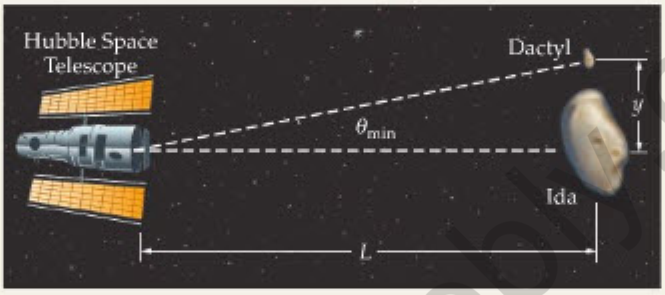
PRACTICE PROBLEM

If the pupil diameter for an eagle is 6.2 mm, from what distance can it resolve the car's headlights under ideal conditions? [**Answer:** About 9.9 mi]

Some related homework problems: Problem 62, Problem 82

ACTIVE EXAMPLE 28–3 RESOLVING IDA AND DACTYL

The asteroid Ida is orbited by its own small "moon" called Dactyl. If the separation between these two asteroids is 2.5 km, what is the maximum distance at which the Hubble Space Telescope (aperture diameter = 2.4 m) can still resolve them with 550-nm light?



SOLUTION *(Test your understanding by performing the calculations indicated in each step.)*

1. Calculate the minimum angular separation for the asteroids to be resolved: $\theta_{min} = 2.8 \times 10^{-7}$ rad
2. Express L in terms of y and θ_{min}: $L = y/\tan(\theta_{min})$
3. Substitute numerical values to find L: $L = 8.9 \times 10^{9}$ m $= 5.5 \times 10^{6}$ mi

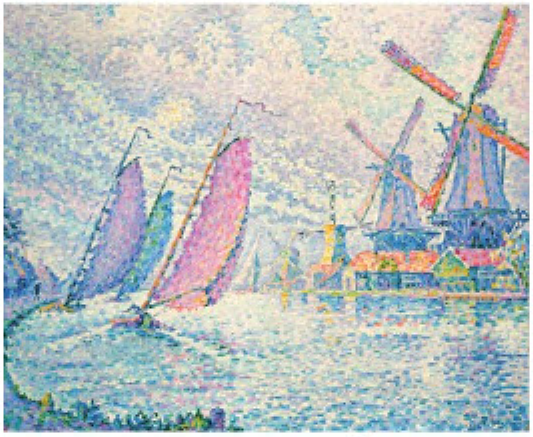
INSIGHT

If the asteroids are farther away than this distance, the Hubble Space Telescope will not be able to image them as two separate objects. In fact, the asteroids are close enough to be viewed separately, as we see in the photo on page 390.

YOUR TURN

If the aperture diameter of the telescope is increased, does the maximum resolution distance increase or decrease? Calculate the maximum resolution distance for an aperture diameter of 3.0 m.

(Answers to **Your Turn** *problems are given in the back of the book.)*



▲ **FIGURE 28–23 Pointillism**
Paul Signac's *The Mills at Owerschie* (1905), an example of pointillism.

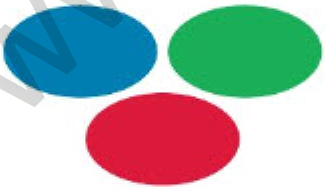
REAL-WORLD PHYSICS: BIO
Pointillism and painting

We conclude by considering two interesting examples of diffraction-limited resolution. First, in the artistic style known as *pointillism*, the artist applies paint to a canvas in the form of small dots of color. When viewed from a distance the individual dots cannot be resolved and the painting appears to be painted with continuous colors. An example is given in Figure 28–23. If this painting is viewed from a distance of more than a few meters, the color dots blend into one another.

REAL-WORLD PHYSICS: BIO
Color television image

The second example is the formation of a picture on the screen of a television. Although a television can show all the colors of the rainbow, it in fact produces only three colors—red, green, and blue, the so-called additive primaries. These three colors are grouped together tightly to form the "pixels" on the screen, as Figure 28–24 shows. From a distance the three individual color spots can no longer be distinguished, and the eye sees the net effect of the three colors combined. Since any color can be created with the proper amounts of the three primary colors—red, green, and blue—the television screen can reproduce any picture.

To see this effect in action, try the following: look for a region on a television screen or a computer monitor where the picture is yellow. Since yellow is created by mixing red and green light equally, you will see on close examination (perhaps with the aid of a magnifying glass) that pixels in the yellow region of the screen have both the red and green dots illuminated, but the blue dots are dark. As you slowly move away from the screen, note that the red and green dots merge, leaving the brain with the sensation of a yellow light, even though there are no yellow color dots on the screen.

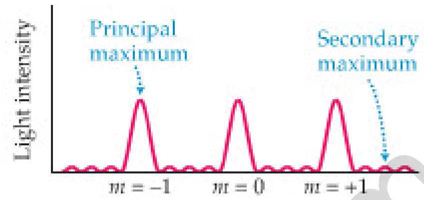


▲ **FIGURE 28–24 Pixels on a television screen**
A typical pixel on the screen of a color television consists of three closely spaced color spots: one red, one blue, and one green. These are the only colors the television actually produces.

28–6 Diffraction Gratings

As we saw earlier in this chapter, a screen with one or two slits can produce striking patterns of interference fringes. It is natural to wonder what interference effects may be produced if the number of slits is increased significantly. In general, we refer to a system with a large number of slits as a **diffraction grating.** There are many ways of producing a grating; for example, one might use a diamond stylus to cut thousands of slits in the aluminum coating on a sheet of glass. Alternatively, one might photoreduce an image of parallel lines onto a strip of film. In some cases it is possible to produce gratings with as many as 40,000 slits—or "lines," as they are often called—per centimeter.

The interference pattern formed by a diffraction grating consists of a series of sharp, widely spaced bright fringes—called *principal maxima*—separated by relatively dark regions with a number of weak secondary maxima, as indicated in **Figure 28–25** for the case of five slits. In the limit of a large number of slits, the principal maxima become more sharply peaked, and the secondary maxima become insignificant. As one might expect, the angle at which a principal maximum occurs depends on the wavelength of light that passes through the grating. In this way, a grating acts much like a prism—sending the various colors of white light off in different directions. A grating, however, can spread the light out over a wider range of angles than a prism.



▲ **FIGURE 28–25 Diffraction pattern for five slits**

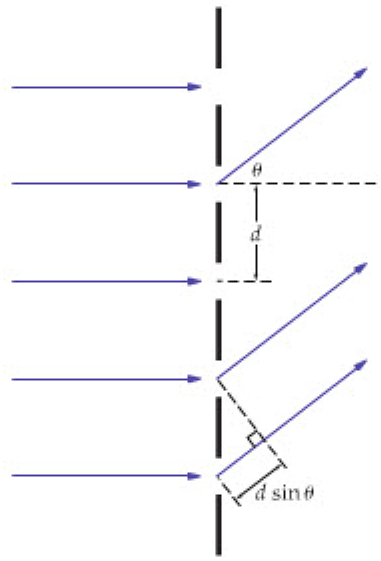
The interference pattern produced by a diffraction grating with five slits. The large "principal" maxima are sharper than the maxima in the two-slit apparatus. The small "secondary" maxima are negligible compared with the principal maxima.

To determine the angles at which principal maxima are found, consider a grating with a large number of slits, each separated from the next by a distance d, as shown in **Figure 28–26.** A beam of light with wavelength λ is incident on the grating from the left and is diffracted onto a distant screen. At an angle θ to the incident direction the path difference between successive slits is $d \sin\theta$, as Figure 28–26 shows. Therefore, constructive interference, and a principal maximum, occurs when the path difference is an integral number of wavelengths, λ:

Constructive Interference in a Diffraction Grating

$$d \sin\theta = m\lambda \qquad m = 0, \pm 1, \pm 2, \ldots \qquad 28\text{–}16$$

Notice that the angle θ becomes larger as d is made smaller. In particular, if a grating has more lines per centimeter (smaller d), light passing through it will be spread out over a larger range of angles.



▲ **FIGURE 28–26 Path-length difference in a diffraction grating**

A simple diffraction grating consists of a number of slits with a spacing d. The difference in path length for rays from neighboring slits is $d \sin\theta$.

EXERCISE 28–4

Find the slit spacing necessary for 450-nm light to have a first-order ($m = 1$) principal maximum at 15°.

SOLUTION

Solving Equation 28–16 for d we obtain

$$d = \frac{m\lambda}{\sin\theta} = \frac{(1)(450 \times 10^{-9}\ \text{m})}{\sin 15^\circ} = 1.7 \times 10^{-6}\ \text{m}$$

A grating is often characterized in terms of its number of lines per unit length, N. For example, a particular grating might have 2250 lines per centimeter. The corresponding slit separation, d, is simply the inverse of the number of lines per length. In this case the slit separation is $d = 1/N = 1/(2250\ \text{cm}^{-1}) = 4.44 \times 10^{-4}\ \text{cm} = 4.44 \times 10^{-6}\ \text{m}$.

EXAMPLE 28–7 A SECOND-ORDER MAXIMUM

When 546-nm light passes through a particular diffraction grating, a second-order principal maximum is observed at an angle of 16.0°. How many lines per centimeter does this grating have?

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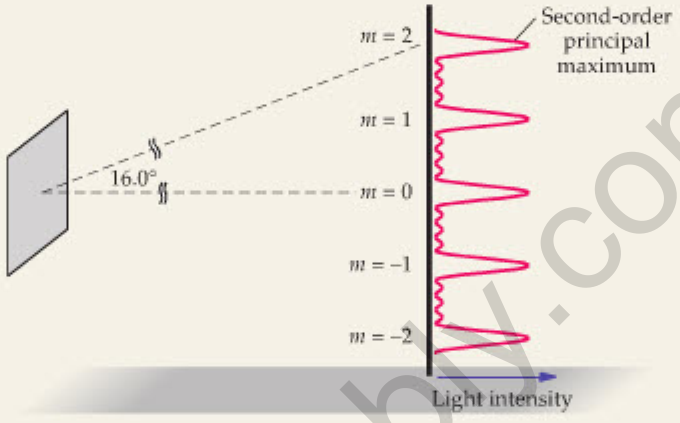
PICTURE THE PROBLEM

Our sketch shows the first few principal maxima on either side of the center of the diffraction pattern. The second-order maximum ($m = 2$) is the second maximum above the central peak, and it occurs at an angle of 16.0°.



STRATEGY

First, we can use $\sin\theta = m\lambda/d$ to find the necessary spacing d, given that m, λ, and θ are specified in the problem statement. Next, the number of lines per centimeter, N, is simply the inverse of the spacing d; that is, $N = 1/d$.

SOLUTION

1. Calculate the distance d between slits: $$d = \frac{m\lambda}{\sin\theta} = \frac{(2)(546 \times 10^{-9}\ \text{m})}{\sin 16.0°} = 3.96 \times 10^{-6}\ \text{m}$$

2. Take the inverse of d to find the number of lines per meter: $$N = \frac{1}{d} = \frac{1}{3.96 \times 10^{-6}\ \text{m}} = 2.53 \times 10^{5}\ \text{m}^{-1}$$

3. Convert to lines per centimeter: $$N = 2.53 \times 10^{5}\ \text{m}^{-1}\left(\frac{1\ \text{m}}{100\ \text{cm}}\right) = 2530\ \text{cm}^{-1}$$

INSIGHT

Thus, this diffraction grating must have 2530 lines per centimeter. Though this is a lot of lines to pack into a distance of one centimeter, it is common to find this many lines or more in a typical diffraction grating.

PRACTICE PROBLEM

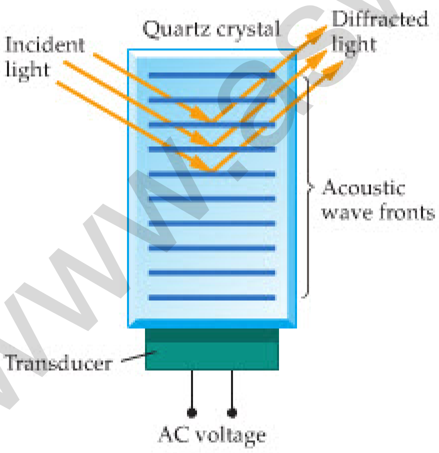
If the grating is ruled with more lines per centimeter, does the angle to the second-order maximum increase, decrease, or stay the same? Check your answer by calculating the angle for a grating with 3530 lines per centimeter. [**Answer:** Increase; $\theta = 22.7°$]

Some related homework problems: Problem 63, Problem 64, Problem 74

REAL-WORLD PHYSICS

Acousto-optic modulation

Of the many ways to produce a diffraction grating, one of the more novel is by *acousto-optic modulation* (AOM). In this technique, light is diffracted not by a series of slits but by a series of high-density wave fronts produced by a sound wave propagating through a solid or a liquid. For example, in the AOM device shown in Figure 28–27, sound waves propagate through a quartz crystal, producing a series of closely spaced, parallel wave fronts. An incoming beam of light diffracts from these wave fronts, giving rise to an intense outgoing beam. If the sound is turned off, however, the incoming light passes through the crystal without being deflected. Thus, simply turning the sound on or off causes the diffracted beam to be switched on or off, whereas changing the frequency of the sound can change the angle of the diffracted beam. Many laser printers use AOMs to control the laser beam responsible for "drawing" the desired image on a light-sensitive surface.



▲ **FIGURE 28–27 An acousto-optical modulator**

Acousto-optical modulators use sound waves, and the density variations they produce, to diffract light. The angle at which the light diffracts can be controlled by the frequency of the sound.

X-ray Diffraction

There is another type of diffraction grating that is not made by clever applications of technology but occurs naturally—the crystal. The key characteristic of a crystal is that it has a regular, repeating structure. In particular, crystals generally consist of regularly spaced planes of atoms or ions; these planes, just like the wave fronts in an AOM, can diffract an incoming beam of electromagnetic radiation.

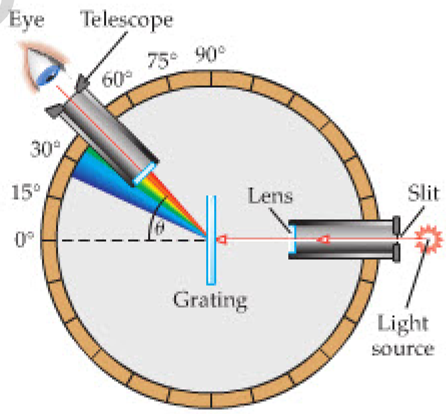
For a diffraction grating to be effective, however, the wavelength of the radiation, λ, must be comparable to the spacing, d, in Equation 28–16. In a typical crystal, the spacing between atomic planes is roughly an angstrom; that is, $d \sim 10^{-10}\ \text{m} \sim 0.1\ \text{nm}$. Notice that this distance is much less than the wavelength of visible light, which is roughly 400 to 700 nm. Therefore, visible light will not produce useful diffraction effects from a crystal. However, if we consider the full electromagnetic spectrum, as presented in Figure 25–8, we see that wavelengths of 0.1 nm fall within the X-ray portion of the spectrum. Indeed, X-rays produce vivid diffraction patterns when sent through crystals (Figure 28–28).

Today, *X-ray diffraction* is a valuable scientific tool. First, the angle at which principal maxima occur in an X-ray diffraction pattern can determine the precise distance between various planes of atoms in a particular crystal. Second, the symmetry of the pattern determines the type of crystal structure. More sophisticated analysis of X-ray diffraction patterns—in particular, the angles and intensities of diffraction maxima—can be used to help determine the structures of even large organic molecules. In fact, it was in part through examination of X-ray diffraction patterns that J. D. Watson and F. H. C. Crick were able to deduce the double-helix structure of DNA in 1953.

▲ **FIGURE 28–28 X-ray diffraction**
X-ray diffraction pattern produced by DNA. A photo like this one helped Watson and Crick deduce the double-helix structure of the DNA molecule in 1953.

Grating Spectroscopes

As noted earlier in this section, a grating can produce a wide separation in the various colors contained in a beam of light. This phenomenon is used as a means of measuring the corresponding wavelengths with an instrument known as a *grating spectroscope*. As we see in Figure 28–29, light entering a grating spectroscope is diffracted as it passes through a grating. Next, the angle of diffraction of a given color is determined by a small telescope mounted on a rotating base. Finally, application of Equation 28–16 allows one to determine the wavelength of the light to great precision. Devices of this type have played a key role in elucidating the expanding nature of the universe, as we show in the following Active Example.



▲ **FIGURE 28–29 A grating spectroscope**
Because gratings can spread light out over a wider angle than prisms, they are used in most modern spectroscopes.

ACTIVE EXAMPLE 28–4 FIND THE WAVELENGTHS

A grating spectroscope with a line separation of 4.600×10^{-6} m is used to analyze light from the distant quasar (quasi-stellar object) designated 3C 273. With this instrument it is determined that light from the quasar exhibits a Doppler "red shift" (Section 25–2), indicating motion away from Earth. For example, some of the light given off by hydrogen atoms in the lab has a principal maximum at an angle of 6.067°. When this same hydrogen light is analyzed from the quasar, it is found to have a principal maximum at an angle of 7.030°. Find the wavelength in the lab, and the red-shifted wavelength from the quasar.

SOLUTION *(Test your understanding by performing the calculations indicated in each step.)*

1. Use Equation 28–16 to solve for λ: $\lambda = d \sin \theta$
2. Substitute $\theta = 6.067°$: $\lambda = 486.2$ nm
3. Substitute $\theta = 7.030°$: $\lambda = 563.0$ nm

INSIGHT
Thus we find that the wavelength from the quasar is 15.8% longer than the wavelength in the lab. If we combine this information with the Doppler effect for light (Section 25–2), we can determine the speed at which the quasar is receding from Earth.

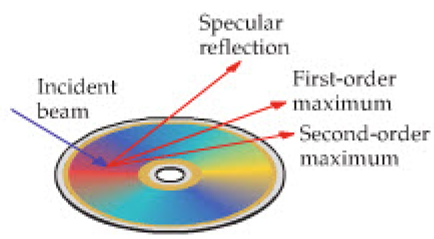
YOUR TURN
Find the recession speed of the quasar.

(Answers to **Your Turn** *problems are given in the back of the book.)*

REAL-WORLD PHYSICS
Measuring the red shift of a quasar

Reflection Gratings

Yet another way to produce a diffraction grating is to inscribe lines on a reflecting surface, with the regions between the lines acting as coherent sources of light. An everyday example of this type of *reflection grating* is a CD. As we saw earlier in this chapter, the information on a CD is encoded in the form of a series of bumps, and these bumps spiral around the CD, creating a tightly spaced set of lines. When a beam of monochromatic light is incident on a CD, as in Figure 28–30, a series of

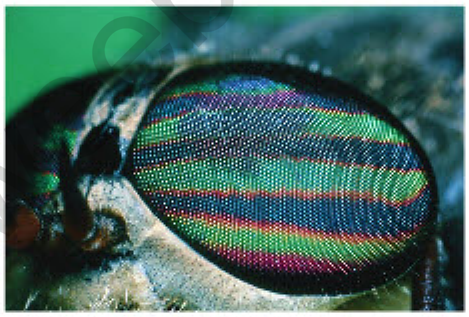


▲ **FIGURE 28–30 Reflection from a CD**
When a laser beam shines on a CD, a number of reflected beams are observed. The most intense of these is the specular beam, whose angle of reflection is equal to its angle of incidence—the same as if the CD were a plane mirror. Additional reflected beams are observed at angles corresponding to the principal maxima of the grating.

reflected beams is created. One beam reflects at an angle equal to the incident angle—this is referred to as the *specular beam*, since it is the beam that would be expected from a smooth plane mirror. In addition, a number of other reflected beams are observed, each corresponding to a different principal maximum of the grating.

REAL-WORLD PHYSICS: BIO
Iridescence in nature

If white light is reflected from a CD, different colors in the incident light are reflected at different angles. This is why light reflected from a CD shows the colors of the rainbow. A similar effect occurs in light reflected from feathers, or from the wing of a butterfly like the blue morpho shown on page 976. For example, a microscopic examination of a butterfly wing shows that it consists of a multitude of plates, much like shingles on a roof. On each of these plates is a series of closely spaced ridges. These ridges act like the grooves on a CD, producing reflected light of different color in different directions. This type of coloration is referred to as **iridescence.** The next time you are able to examine an iridescent object, notice how the color changes as you change the angle from which it is viewed.



▶ Diffraction can occur when light falls on any surface having grooves with a spacing comparable to the wavelength of the light. Many common surfaces can act as diffraction gratings, including artificially created ones, such as the CDs at left, and natural ones, such as the fly's eye at right. If the incident light is white, comprising a range of different wavelengths, diffraction gives rise to a rainbow effect.

THE BIG PICTURE PUTTING PHYSICS IN CONTEXT

LOOKING BACK

Superposition and interference in light waves (Section 28–1) are completely analogous to superposition and interference in sound waves, as discussed in Chapter 14.

Phase changes due to the reflection of light from an interface (Section 28–3) is just like the phase changes seen when a wave on a string reflects from an end that is either tied down or free to move. See in particular Section 14–2.

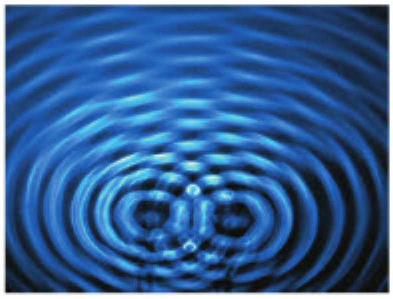
LOOKING AHEAD

One of the key experiments related to relativity was the measurement of the speed of light in different directions by Michelson and Morley (Chapter 29). Their experiment was based on observing interference fringes, just like those seen in Young's two-slit experiment (Section 28–2).

The Heisenberg uncertainty principle that is so fundamental to quantum physics can be understood as analogous to the diffraction of light, as we shall see in Section 30–6.

CHAPTER SUMMARY

28–1 SUPERPOSITION AND INTERFERENCE



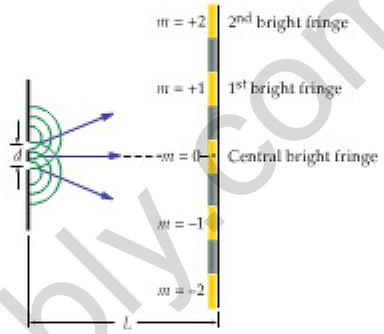
The simple addition of two or more waves to give a resultant wave is referred to as superposition. When waves are superposed, the result may be a wave of greater amplitude (constructive interference) or of reduced amplitude (destructive interference).

Monochromatic Light
Monochromatic light consists of waves with a single frequency and, hence, a single color.

Coherent/Incoherent Light
Light waves that maintain a constant phase relationship with one another are referred to as coherent. Light waves in which the relative phases vary randomly with time are said to be incoherent.

28–2 YOUNG'S TWO-SLIT EXPERIMENT

Interference effects in light are shown clearly in Young's two-slit experiment, in which light passing through two slits forms bright and dark interference "fringes."



Conditions for Bright Fringes
Bright fringes in a two-slit experiment occur at angles θ given by the following relation:

$$d \sin \theta = m\lambda \qquad m = 0, \pm 1, \pm 2, \ldots \qquad 28\text{–}1$$

In this expression, λ is the wavelength of the light and d is the separation of the slits. The various values of the integer m correspond to different bright fringes.

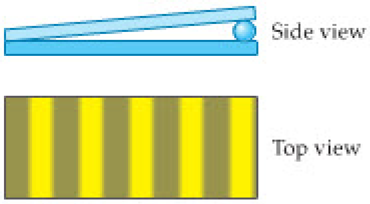
Conditions for Dark Fringes
The locations of dark fringes in a two-slit experiment are given by the following:

$$d \sin \theta = \left(m - \tfrac{1}{2}\right)\lambda \quad m = 1, 2, 3, \ldots \quad \text{(above central bright fringe)}$$
$$d \sin \theta = \left(m + \tfrac{1}{2}\right)\lambda \quad m = -1, -2, -3, \ldots \quad \text{(below central bright fringe)} \qquad 28\text{–}2$$

Linear Distance
If the screen on which the interference pattern is projected in a two-slit experiment is a distance L from the slits, the linear distance to a given bright or dark fringe is $y = L \tan \theta$.

28–3 INTERFERENCE IN REFLECTED WAVES

Light waves reflected from different locations can interfere, just like light from the slits in a two-slit experiment.



Phase Changes Due to Reflection
No phase change occurs when light is reflected from a region with a lower index of refraction, whereas a 180° (half-wavelength) phase change occurs when light reflects from a region with a higher index of refraction, or from a solid surface.

Air Wedge
Two plates of glass that touch on one end and have a small separation on the other end form an air wedge. When light of wavelength λ shines on an air wedge, bright fringes occur when the separation between the plates, d, is such that

$$\frac{1}{2} + \frac{2d}{\lambda} = m \qquad m = 1, 2, 3, \ldots \qquad 28\text{–}5$$

Similarly, dark fringes occur when the following conditions are satisfied:

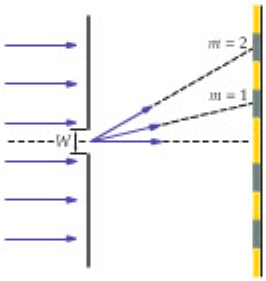
$$\frac{1}{2} + \frac{2d}{\lambda} = m + \frac{1}{2} \qquad m = 0, 1, 2, \ldots \qquad 28\text{–}6$$

Newton's Rings
When a piece of glass with a spherical cross section is placed on a flat sheet of glass, the resulting interference fringes form a set of concentric circles known as Newton's rings.

Thin Films
Thin films, like those in a soap bubble, can produce colors in reflected light by eliminating other colors with destructive interference.

28–4 DIFFRACTION

When a wave encounters an obstacle, or passes through an opening, it changes direction. This phenomenon is referred to as diffraction.



Single-Slit Diffraction
When monochromatic light of wavelength λ passes through a single slit of width W, it forms a diffraction pattern of alternating bright and dark fringes.

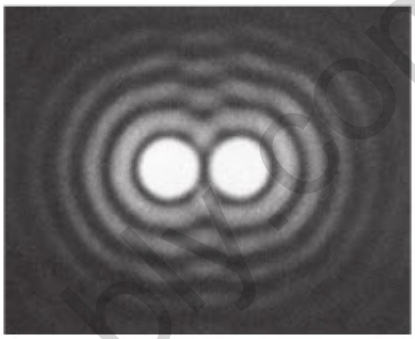
Condition for Dark Fringes
The condition that determines the location of dark fringes in single-slit diffraction is

$$W \sin \theta = m\lambda \qquad m = \pm 1, \pm 2, \pm 3, \ldots \qquad 28\text{–}12$$

Bright Fringes
Bright fringes are located approximately halfway between successive dark fringes. In addition, the central bright fringe is approximately twice as wide as the other bright fringes.

28–5 RESOLUTION

Resolution refers to the ability of a visual system, like the eye or a camera, to distinguish closely spaced objects.



First Dark Fringe
A circular aperture of diameter D produces a circular diffraction pattern in which the first dark fringe occurs at the angle θ given by the following condition:

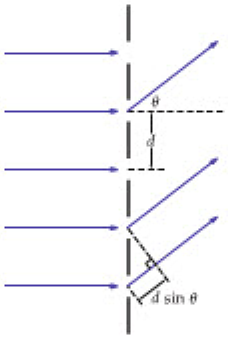
$$\sin\theta = 1.22\frac{\lambda}{D} \quad \text{28–14}$$

Rayleigh's Criterion: Qualitative Statement
Rayleigh's criterion states that two objects become blurred together when the first dark fringe of one object's diffraction pattern passes through the center of the other object's diffraction pattern.

Rayleigh's Criterion: Quantitative Statement
In quantitative terms, Rayleigh's criterion states that if the angular separation between two objects is less than a certain minimum, $\theta_{min} = 1.22\lambda/D$, they will appear to be a single object.

28–6 DIFFRACTION GRATINGS

A diffraction grating is a large number of slits through which a beam of light can pass.



Principal Maxima
The principal maxima produced by a diffraction grating occur at the angles given by the following conditions:

$$d\sin\theta = m\lambda \qquad m = 0, \pm1, \pm2, \ldots \quad \text{28–16}$$

In this expression, d is the distance between successive slits and λ is the wavelength of light.

Number of Lines per Centimeter
Diffraction gratings are often characterized by the number of lines, or slits, they have per centimeter. If the number of lines per centimeter is N, the spacing between slits is $d = 1/N$, where d is measured in centimeters.

Reflection Gratings
Diffraction gratings can also be constructed from a reflecting surface with a large number of reflecting lines, like a CD or a butterfly wing.

Iridescence
When white light shines on a reflecting grating, different colors in the light are reflected at different angles. The color effects produced in this way are referred to as iridescence.

PROBLEM-SOLVING SUMMARY

Type of Problem	Relevant Physical Concepts	Related Examples
Determine the angular and linear positions of fringes in a two-slit experiment.	The angular positions are determined by constructive and destructive interference, with the results given in Equations 28–1 and 28–2. The linear distance is given by simple geometry, as in Equation 28–3.	Example 28–2
Determine whether reflection results in constructive or destructive interference.	When reflection is involved, the condition for constructive versus destructive interference involves both the difference in path length and the phase change that may result from reflection.	Examples 28–3, 28–4 Active Example 28–1
Find the angle corresponding to dark fringes in a diffraction pattern.	A single slit produces a dark fringe when a ray from the top of the slit follows a path that is an integer number of half wavelengths longer than the path followed by a ray starting at the center of the slit. The result is the condition given in Equation 28–12.	Example 28–5 Active Example 28–2

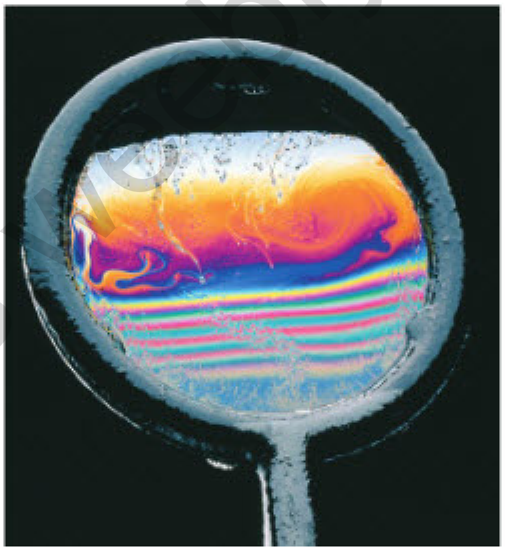
Determine whether two nearby objects can be resolved.	To be resolved, nearby objects must have an angular separation that is at least $1.22\,\lambda/D$, where D is the diameter of the aperture and λ is the wavelength of the light.	Example 28–6 Active Example 28–3
Find the location of the principal maxima produced by a diffraction grating.	The principal maxima in a diffraction grating pattern are at the same angular positions as the bright fringes in a two-slit experiment, as given in Equation 28–1.	Example 28–7 Active Example 28–4

CONCEPTUAL QUESTIONS

For instructor-assigned homework, go to www.masteringphysics.com MP™

(Answers to odd-numbered Conceptual Questions can be found in the back of the book.)

1. When two light waves interfere destructively, what happens to their energy?
2. What happens to the two-slit interference pattern if the separation between the slits is less than the wavelength of light?
3. If a radio station broadcasts its signal through two different antennas simultaneously, does this guarantee that the signal you receive will be stronger than from a single antenna? Explain.
4. How would you expect the interference pattern of a two-slit experiment to change if white light is used instead of monochromatic light?
5. Suppose a sheet of glass is placed in front of one of the slits in a two-slit experiment. If the thickness of the glass is such that the light reaching the two slits is 180° out of phase, how does this affect the interference pattern?
6. Describe the changes that would be observed in the two-slit interference pattern if the entire experiment were to be submerged in water.
7. Explain why the central spot in Newton's rings is dark.
8. Two identical sheets of glass are coated with films of different materials but equal thickness. The colors seen in reflected light from the two films are different. Give a reason that can account for this observation.
9. Spy cameras use lenses with very large apertures. Why are large apertures advantageous in such applications?
10. A cat's eye has a pupil that is elongated in the vertical direction. How does the resolution of a cat's eye differ in the horizontal and vertical directions?
11. Which portion of the soap film in the accompanying photograph is thinnest? Explain.



Conceptual Question 11

12. The color of an iridescent object, like a butterfly wing or a feather, appears to be different when viewed from different directions. The color of a painted surface appears the same from all viewing angles. Explain the difference.

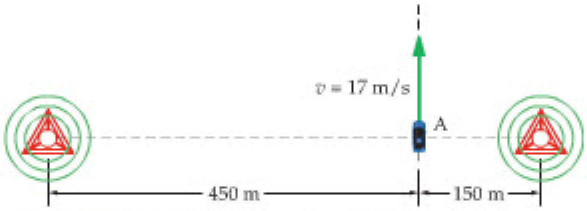
PROBLEMS AND CONCEPTUAL EXERCISES

Note: Answers to odd-numbered Problems and Conceptual Exercises can be found in the back of the book. **IP** *denotes an integrated problem, with both conceptual and numerical parts;* **BIO** *identifies problems of biological or medical interest;* **CE** *indicates a conceptual exercise.* **Predict/Explain** *problems ask for two responses:* **(a)** *your prediction of a physical outcome, and* **(b)** *the best explanation among three provided. On all problems, red bullets (•, ••, •••) are used to indicate the level of difficulty.*

(In all problems involving sound waves, take the speed of sound to be 343 m/s.)

SECTION 28–1 SUPERPOSITION AND INTERFERENCE

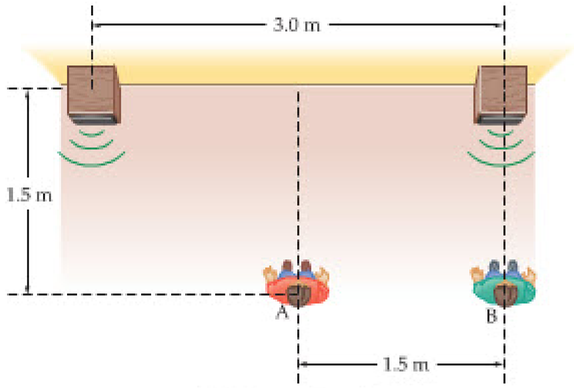
1. • Two sources emit waves that are coherent, in phase, and have wavelengths of 26.0 m. Do the waves interfere constructively or destructively at an observation point 78.0 m from one source and 143 m from the other source?
2. • Repeat Problem 1 for observation points that are **(a)** 91.0 m and 221 m and **(b)** 44.0 m and 135 m from the two sources.
3. •• Two sources emit waves that are in phase with each other. What is the longest wavelength that will give constructive interference at an observation point 161 m from one source and 295 m from the other source?
4. •• A person driving at 17 m/s crosses the line connecting two radio transmitters at right angles, as shown in Figure 28–31. The transmitters emit identical signals in phase with each other, which the driver receives on the car radio. When the car is at



▲ **FIGURE 28–31** Problems 4 and 11

point A, the radio picks up a maximum net signal. **(a)** What is the longest possible wavelength of the radio waves? **(b)** How long after the car passes point A does the radio experience a minimum in the net signal? Assume that the wavelength has the value found in part (a).

5. •• Two students in a dorm room listen to a pure tone produced by two loudspeakers that are in phase. Students A and B in Figure 28–32 hear a maximum sound. What is the lowest possible frequency of the loudspeakers?



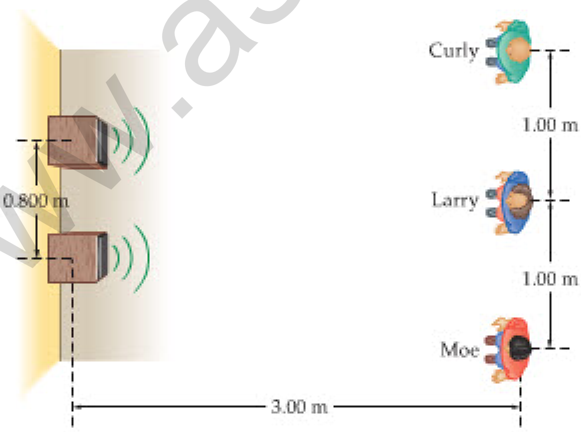
▲ **FIGURE 28–32** Problems 5 and 6

6. •• If the loudspeakers in Problem 5 are 180° out of phase, determine whether a 185-Hz tone heard at location B is a maximum or a minimum.

7. •• A microphone is located on the line connecting two speakers that are 0.845 m apart and oscillating in phase. The microphone is 2.55 m from the midpoint of the two speakers. What are the lowest two frequencies that produce an interference maximum at the microphone's location?

8. •• A microphone is located on the line connecting two speakers that are 0.845 m apart and oscillating 180° out of phase. The microphone is 2.25 m from the midpoint of the two speakers. What are the lowest two frequencies that produce an interference maximum at the microphone's location?

9. •• Moe, Larry, and Curly stand in a line with a spacing of 1.00 m. Larry is 3.00 m in front of a pair of stereo speakers 0.800 m apart, as shown in Figure 28–33. The speakers produce a single-frequency tone, vibrating in phase with each other. What are the two lowest frequencies that allow Larry to hear a loud tone while Moe and Curly hear very little?



▲ **FIGURE 28–33** Problems 9 and 10

10. •• **IP** In Figure 28–33 the two speakers emit sound that is 180° out of phase and of a single frequency, f. **(a)** Does Larry hear a sound intensity that is a maximum or a minimum? Does your answer depend on the frequency of the sound? Explain. **(b)** Find the lowest two frequencies that produce a maximum sound intensity at the positions of Moe and Curly.

11. •• **IP** Suppose the car radio in Problem 4 picks up a minimum net signal at point A. **(a)** What is the largest possible value for the wavelength of the radio waves? **(b)** If the radio transmitters use a wavelength that is half the value found in part (a), will the car radio pick up a net signal at point A that is a maximum or a minimum? Explain. **(c)** What is the second largest wavelength that will result in a minimum signal at point A?

SECTION 28–2 YOUNG'S TWO-SLIT EXPERIMENT

12. • **CE** Consider a two-slit interference pattern, with monochromatic light of wavelength λ. What is the path difference $\Delta\ell$ for **(a)** the fourth bright fringe and **(b)** the third dark fringe above the central bright fringe? Give your answers in terms of the wavelength of the light.

13. • **CE** **(a)** Does the path-length difference $\Delta\ell$ increase or decrease as you move from one bright fringe of a two-slit experiment to the next bright fringe farther out? **(b)** What is $\Delta\ell$ in terms of the wavelength λ of the light?

14. • **CE Predict/Explain** A two-slit experiment with red light produces a set of bright fringes. **(a)** Will the spacing between the fringes increase, decrease, or stay the same if the color of the light is changed to blue? **(b)** Choose the *best explanation* from among the following:
   I. The spacing between the fringes will increase because blue light has a greater frequency than red light.
   II. The fringe spacing decreases because blue light has a shorter wavelength than red light.
   III. Only the wave property of light is important in producing the fringes, not the color of the light. Therefore the spacing stays the same.

15. • **CE** A two-slit experiment with blue light produces a set of bright fringes. Will the spacing between the fringes increase, decrease, or stay the same if **(a)** the separation of the slits is decreased, or **(b)** the experiment is immersed in water?

16. • Laser light with a wavelength $\lambda = 670$ nm illuminates a pair of slits at normal incidence. What slit separation will produce first-order maxima at angles of $\pm 35°$ from the incident direction?

17. • Monochromatic light passes through two slits separated by a distance of 0.0334 mm. If the angle to the third maximum above the central fringe is 3.21°, what is the wavelength of the light?

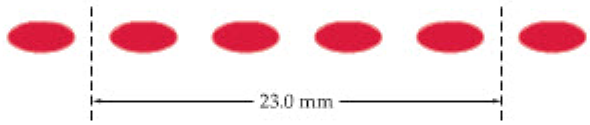
18. • In Young's two-slit experiment, the first dark fringe above the central bright fringe occurs at an angle of 0.31°. What is the ratio of the slit separation, d, to the wavelength of the light, λ?

19. •• **IP** A two-slit experiment with slits separated by 48.0×10^{-5} m produces a second-order maximum at an angle of 0.0990°. **(a)** Find the wavelength of the light used in this experiment. **(b)** If the slit separation is increased but the second-order maximum stays at the same angle, does the wavelength increase, decrease, or stay the same? Explain. **(c)** Calculate the wavelength for a slit separation of 68.0×10^{-5} m.

20. •• A two-slit pattern is viewed on a screen 1.00 m from the slits. If the two third-order minima are 22.0 cm apart, what is the width (in cm) of the central bright fringe?

21. •• Light from a He–Ne laser ($\lambda = 632.8$ nm) strikes a pair of slits at normal incidence, forming a double-slit interference pattern on a screen located 1.40 m from the slits. Figure 28–34

shows the interference pattern observed on the screen. What is the slit separation?



▲ **FIGURE 28–34** Problems 21 and 24

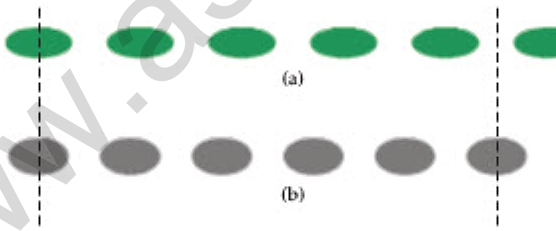
22. •• Light with a wavelength of 546 nm passes through two slits and forms an interference pattern on a screen 8.75 m away. If the linear distance on the screen from the central fringe to the first bright fringe above it is 5.36 cm, what is the separation of the slits?

23. •• A set of parallel slits for optical interference can be made by holding two razor blades together (carefully!) and scratching a pair of lines on a glass microscope slide that has been painted black. When monochromatic light strikes these slits at normal incidence, an interference pattern is formed on a distant screen. The thickness of each razor blade used to make the slits is 0.230 mm, and the screen is 2.50 m from the slits. If the center-to-center separation of the fringes is 7.15 mm, what is the wavelength of the light?

24. •• **IP** Suppose the interference pattern shown in Figure 28–34 is produced by monochromatic light passing through two slits, with a separation of 135 μm, and onto a screen 1.20 m away. **(a)** What is the wavelength of the light? **(b)** If the frequency of this light is increased, will the bright spots of the pattern move closer together or farther apart? Explain.

25. •• A physics instructor wants to produce a double-slit interference pattern large enough for her class to see. For the size of the room, she decides that the distance between successive bright fringes on the screen should be at least 2.50 cm. If the slits have a separation $d = 0.0220$ mm, what is the minimum distance from the slits to the screen when 632.8-nm light from a He–Ne laser is used?

26. •• **IP** When green light ($\lambda = 505$ nm) passes through a pair of double slits, the interference pattern shown in **Figure 28–35 (a)** is observed. When light of a different color passes through the same pair of slits, the pattern shown in **Figure 28–35 (b)** is observed. **(a)** Is the wavelength of the second color longer or shorter than 505 nm? Explain. **(b)** Find the wavelength of the second color. (Assume that the angles involved are small enough to set $\sin\theta \approx \tan\theta$.)

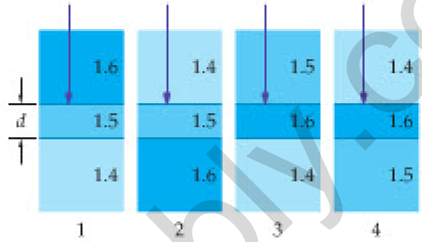


▲ **FIGURE 28–35** Problems 26 and 27

27. •• **IP** The interference pattern shown in Figure 28–35 (a) is produced by green light with a wavelength of $\lambda = 505$ nm passing through two slits with a separation of 127 μm. After passing through the slits, the light forms a pattern of bright and dark spots on a screen located 1.25 m from the slits. **(a)** What is the distance between the two vertical, dashed lines in Figure 28–35 (a)? **(b)** If it is desired to produce a more tightly packed interference pattern, like the one shown in Figure 28–35 (b), should the frequency of the light be increased or decreased? Explain.

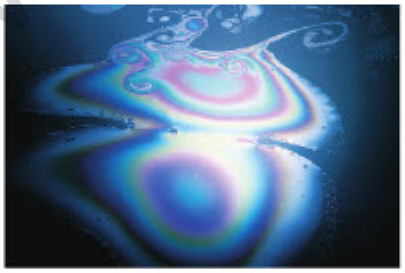
SECTION 28–3 INTERFERENCE IN REFLECTED WAVES

28. • **CE** Figure 28–36 shows four different cases where light of wavelength λ reflects from both the top and the bottom of a thin film of thickness d. The indices of refraction of the film and the media above and below it are indicated in the figure. For which of the cases will the two reflected rays undergo constructive interference if **(a)** $d = \lambda/4$ or **(b)** $d = \lambda/2$?



▲ **FIGURE 28–36** Problem 28

29. • **CE** The oil film floating on water in the accompanying photo appears dark near the edges, where it is thinnest. Is the index of refraction of the oil greater than or less than that of the water? Explain.



Light reflected from a film of oil. (Problem 29)

30. • A soap bubble with walls 401 nm thick floats in air. If this bubble is illuminated perpendicularly with sunlight, what wavelength (and color) will be absent in the reflected light? Assume that the index of refraction of the soap film is 1.33. (Refer to Example 25–3 for the connection between wavelength and color.)

31. • A soap film ($n = 1.33$) is 825 nm thick. White light strikes the film at normal incidence. What visible wavelengths will be constructively reflected if the film is surrounded by air on both sides? (Refer to Example 25–3 for the range of visible wavelengths.)

32. • White light is incident on a soap film ($n = 1.30$) in air. The reflected light looks bluish because the red light ($\lambda = 670$ nm) is absent in the reflection. What is the minimum thickness of the soap film?

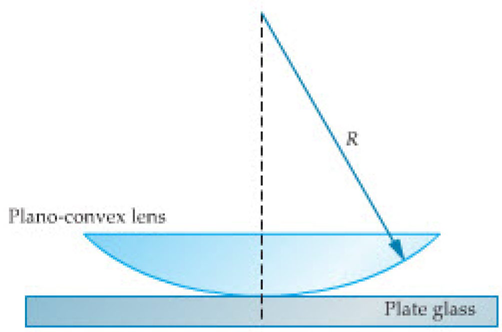
33. • A 742-nm-thick soap film ($n_{film} = 1.33$) rests on a glass plate ($n_{glass} = 1.52$). White light strikes the film at normal incidence. What visible wavelengths will be constructively reflected from the film? (Refer to Example 25–3 for the range of visible wavelengths.)

34. • An oil film ($n = 1.38$) floats on a water puddle. You notice that green light ($\lambda = 521$ nm) is absent in the reflection. What is the minimum thickness of the oil film?

35. •• A radio broadcast antenna is 36.00 km from your house. Suppose an airplane is flying 2.230 km above the line connecting the broadcast antenna and your radio, and that waves reflected from the airplane travel 88.00 wavelengths farther than waves that travel directly from the antenna to your house. **(a)** Do you observe constructive or destructive interference

between the direct and reflected waves? (*Hint:* Does a phase change occur when the waves are reflected?) **(b)** The situation just described occurs when the plane is above a point on the ground that is two-thirds of the way from the antenna to your house. What is the wavelength of the radio waves?

36. •• **IP** **Newton's Rings** Monochromatic light with $\lambda = 648$ nm shines down on a plano-convex lens lying on a piece of plate glass, as shown in Figure 28–37. When viewed from above, one sees a set of concentric dark and bright fringes, referred to as Newton's rings (See Figure 28–11 for a photo of Newton's rings.). **(a)** If the radius of the twelfth dark ring from the center is measured to be 1.56 cm, what is the radius of curvature, R, of the lens? **(b)** If light with a longer wavelength is used with this system, will the radius of the twelfth dark ring be greater than or less than 1.56 cm? Explain.



▲ **FIGURE 28–37** Problems 36 and 90

37. •• Light is incident from above on two plates of glass, separated on both ends by small wires of diameter $d = 0.600\ \mu$m. Considering only interference between light reflected from the bottom surface of the upper plate and light reflected from the upper surface of the lower plate, state whether the following wavelengths give constructive or destructive interference: **(a)** $\lambda = 600.0$ nm; **(b)** $\lambda = 800.0$ nm; **(c)** $\lambda = 343.0$ nm.

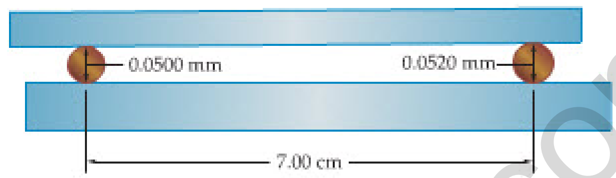
38. •• **(a)** What is the minimum soap-film thickness ($n = 1.33$) in air that will produce constructive interference in reflection for red ($\lambda = 652$ nm) light? **(b)** Which visible wavelengths will destructively interfere when reflected from this film? (Refer to Example 25–3 for the range of visible wavelengths.)

39. •• **IP** A thin layer of magnesium fluoride ($n = 1.38$) is used to coat a flint-glass lens ($n = 1.61$). **(a)** What thickness should the magnesium fluoride film have if the reflection of 565-nm light is to be suppressed? Assume that the light is incident at right angles to the film. **(b)** If it is desired to suppress the reflection of light with a higher frequency, should the coating of magnesium fluoride be made thinner or thicker? Explain.

40. ••• White light is incident normally on a thin soap film ($n = 1.33$) suspended in air. **(a)** What are the two minimum thicknesses that will constructively reflect yellow ($\lambda = 590$ nm) light? **(b)** What are the two minimum thicknesses that will *destructively* reflect yellow ($\lambda = 590$ nm) light?

41. ••• A thin coating ($t = 340.0$ nm, $n = 1.480$) is placed on a glass lens. Which visible ($400\text{ nm} < \lambda < 700\text{ nm}$) wavelength(s) will be absent in the reflected beam if **(a)** the glass has an index of refraction $n = 1.350$, and **(b)** the glass has an index of refraction $n = 1.675$?

42. ••• Two glass plates are separated by fine wires with diameters $d_1 = 0.0500$ mm and $d_2 = 0.0520$ mm, as indicated in Figure 28–38. The wires are parallel and separated by a distance of 7.00 cm. If monochromatic light with $\lambda = 589$ nm is incident from above, what is the distance (in cm) between adjacent dark bands in the reflected light? (Consider interference only between light reflected from the bottom surface of the upper plate and light reflected from the upper surface of the lower plate.)



▲ **FIGURE 28–38** Problem 42

SECTION 28–4 DIFFRACTION

43. • **CE** A single-slit diffraction pattern is formed on a distant screen. Assuming the angles involved are small, by what factor will the width of the central bright spot on the screen change if **(a)** the wavelength is doubled, **(b)** the slit width is doubled, or **(c)** the distance from the slit to the screen is doubled?

44. • What width single slit will produce first-order diffraction minima at angles of $\pm 23°$ from the central maximum with 690-nm light?

45. • Diffraction also occurs with sound waves. Consider 1300-Hz sound waves diffracted by a door that is 84 cm wide. What is the angle between the two first-order diffraction minima?

46. • Green light ($\lambda = 546$ nm) strikes a single slit at normal incidence. What width slit will produce a central maximum that is 2.50 cm wide on a screen 1.60 m from the slit?

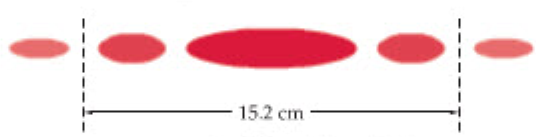
47. • Light with a wavelength of 676 nm passes through a slit 7.64 μm wide and falls on a screen 1.85 m away. Find the linear distance on the screen from the central bright fringe to the first bright fringe above it.

48. • Repeat Problem 47, only this time find the distance on the screen from the central bright fringe to the third dark fringe above it.

49. •• **IP** A single slit is illuminated with 610-nm light, and the resulting diffraction pattern is viewed on a screen 2.3 m away. **(a)** If the linear distance between the first and second dark fringes of the pattern is 12 cm, what is the width of the slit? **(b)** If the slit is made wider, will the distance between the first and second dark fringes increase or decrease? Explain.

50. •• How many dark fringes will be produced on either side of the central maximum if green light ($\lambda = 553$ nm) is incident on a slit that is 8.00 μm wide?

51. •• **IP** The diffraction pattern shown in Figure 28–39 is produced by passing He–Ne laser light ($\lambda = 632.8$ nm) through a single slit and viewing the pattern on a screen 1.50 m behind the slit. **(a)** What is the width of the slit? **(b)** If monochromatic yellow light with a wavelength of 591 nm is used with this slit instead, will the distance indicated in Figure 28–39 be greater than or less than 15.2 cm? Explain.



▲ **FIGURE 28–39** Problems 51 and 91

52. •• A screen is placed 1.00 m behind a single slit. The central maximum in the resulting diffraction pattern on the screen is 1.60 cm wide—that is, the two first-order diffraction minima are separated by 1.60 cm. What is the distance between the two second-order minima?

SECTION 28–5 RESOLUTION

53. • **CE Predict/Explain** **(a)** In principle, do your eyes have greater resolution on a dark cloudy day or on a bright sunny day? **(b)** Choose the *best explanation* from among the following:
   I. Your eyes have greater resolution on a cloudy day because your pupils are open wider to allow more light to enter the eye.
   II. Your eyes have greater resolution on a sunny day because the bright light causes your pupil to narrow down to a smaller opening.

54. • **CE** Is resolution greater with blue light or red light, all other factors being equal? Explain.

55. • Two point sources of light are separated by 5.5 cm. As viewed through a 12-μm-diameter pinhole, what is the maximum distance from which they can be resolved **(a)** if red light ($\lambda = 690$ nm) is used, or **(b)** if violet light ($\lambda = 420$ nm) is used?

56. • A spy camera is said to be able to read the numbers on a car's license plate. If the numbers on the plate are 5.0 cm apart, and the spy satellite is at an altitude of 160 km, what must be the diameter of the camera's aperture? (Assume light with a wavelength of 550 nm.)

57. • **Splitting Binary Stars** As seen from Earth, the red dwarfs Krüger 60A and Krüger 60B form a binary star system with an angular separation of 2.5 arc seconds. What is the smallest diameter telescope that could theoretically resolve these stars using 550-nm light? (*Note:* 1 arc sec = 1/3600°)

58. • Find the minimum aperture diameter of a camera that can resolve detail on the ground the size of a person (2.0 m) from an SR-71 Blackbird airplane flying at an altitude of 27 km. (Assume light with a wavelength of 450 nm.)

59. • **The Resolution of Hubble** The Hubble Space Telescope (HST) orbits Earth at an altitude of 613 km. It has an objective mirror that is 2.4 m in diameter. If the HST were to look down on Earth's surface (rather than up at the stars), what is the minimum separation of two objects that could be resolved using 550-nm light? [*Note:* The HST is used only for astronomical work, but a (classified) number of similar telescopes are in orbit for spy purposes.]

60. •• A lens that is "optically perfect" is still limited by diffraction effects. Suppose a lens has a diameter of 120 mm and a focal length of 640 mm. **(a)** Find the angular width (that is, the angle from the bottom to the top) of the central maximum in the diffraction pattern formed by this lens when illuminated with 540-nm light. **(b)** What is the linear width (diameter) of the central maximum at the focal distance of the lens?

61. •• The resolution of a telescope is ultimately limited by the diameter of its objective lens or mirror. A typical amateur astronomer's telescope may have a 6.0-in.-diameter mirror. **(a)** What is the minimum angular separation (in arc seconds) of two stars that can be resolved with a 6.0-in. scope? (Take λ to be at the center of the visible spectrum, about 550 nm, and see Problem 57 for the definition of an arc second.) **(b)** What is the minimum distance (in km) between two points on the Moon's surface that can be resolved by a 6.0-in. scope? (*Note:* The average distance from Earth to the Moon is 384,400 km.)

62. •• Early cameras were little more than a box with a pinhole on the side opposite the film. **(a)** What angular resolution would you expect from a pinhole with a 0.50-mm diameter? **(b)** What is the greatest distance from the camera at which two point objects 15 cm apart can be resolved? (Assume light with a wavelength of 520 nm.)

SECTION 28–6 DIFFRACTION GRATINGS

63. • A grating has 787 lines per centimeter. Find the angles of the first three principal maxima above the central fringe when this grating is illuminated with 655-nm light.

64. • Suppose you want to produce a diffraction pattern with X-rays whose wavelength is 0.030 nm. If you use a diffraction grating, what separation between lines is needed to generate a pattern with the first maximum at an angle of 14°? (For comparison, a typical atom is a few tenths of a nanometer in diameter.)

65. • A diffraction grating has 2200 lines/cm. What is the angle between the first-order maxima for red light ($\lambda = 680$ nm) and blue light ($\lambda = 410$ nm)?

66. • A diffraction grating with 345 lines/mm is 1.00 m in front of a screen. What is the wavelength of light whose first-order maxima will be 16.4 cm from the central maximum on the screen?

67. • The yellow light from a helium discharge tube has a wavelength of 587.5 nm. When this light illuminates a certain diffraction grating it produces a first-order principal maximum at an angle of 1.250°. Calculate the number of lines per centimeter on the grating.

68. •• **IP** The second-order maximum produced by a diffraction grating with 560 lines per centimeter is at an angle of 3.1°. **(a)** What is the wavelength of the light that illuminates the grating? **(b)** If a grating with a larger number of lines per centimeter is used with this light, is the angle of the second-order maximum greater than or less than 3.1°? Explain.

69. •• White light strikes a grating with 7600 lines/cm at normal incidence. How many complete visible spectra will be formed on either side of the central maximum? (Refer to Example 25–3 for the range of visible wavelengths.)

70. •• White light strikes a diffraction grating (890 lines/mm) at normal incidence. What is the highest-order visible maximum that is formed? (Refer to Example 25–3 for the range of visible wavelengths.)

71. •• White light strikes a diffraction grating (760 lines/mm) at normal incidence. What is the longest wavelength that forms a second-order maximum?

72. •• A light source emits two distinct wavelengths [$\lambda_1 = 430$ nm (violet); $\lambda_2 = 630$ nm (orange)]. The light strikes a diffraction grating with 450 lines/mm at normal incidence. Identify the colors of the first eight interference maxima on either side of the central maximum.

73. •• A laser emits two wavelengths ($\lambda_1 = 420$ nm; $\lambda_2 = 630$ nm). When these two wavelengths strike a grating with 450 lines/mm, they produce maxima (in different orders) that coincide. **(a)** What is the order (m) of each of the two overlapping lines? **(b)** At what angle does this overlap occur?

74. •• **IP** When blue light with a wavelength of 465 nm illuminates a diffraction grating, it produces a first-order principal maximum but no second-order maximum. **(a)** Explain the absence of higher-order principal maxima. **(b)** What is the maximum spacing between lines on this grating?

75. •• Monochromatic light strikes a diffraction grating at normal incidence before illuminating a screen 2.10 m away. If the first-order maxima are separated by 1.53 m on the screen, what is the distance between the two second-order maxima?

76. ••• A diffraction grating with a slit separation d is illuminated by a beam of monochromatic light of wavelength λ. The diffracted beam is observed at an angle ϕ relative to the incident direction. If the plane of the grating bisects the angle between the incident and diffracted beams, show that the mth maximum will be observed at an angle that satisfies the relation $m\lambda = 2d\sin(\phi/2)$, with $m = 0, \pm1, \pm2, \ldots$.

GENERAL PROBLEMS

77. • **CE** Monochromatic light with a wavelength λ passes through a single slit of width W and forms a diffraction pattern of alternating bright and dark fringes. **(a)** If the width of the slit is decreased, do the dark fringes move outward or inward? Explain. **(b)** What width is necessary for the first dark fringe to move outward to infinity? Give your answer in terms of λ.

78. • **CE Predict/Explain** **(a)** If a thin liquid film floating on water has an index of refraction less than that of water, will the film appear bright or dark in reflected light as its thickness goes to zero? **(b)** Choose the *best explanation* from among the following:
   I. The film will appear bright because as the thickness of the film goes to zero the phase difference for reflected rays goes to zero.
   II. The film will appear dark because there is a phase change at both interfaces, and this will cause destructive interference of the reflected rays.

79. • **CE** If the index of refraction of an eye could be magically reduced, would the eye's resolution increase or decrease? Explain.

80. • **CE** In order to increase the resolution of a camera, should its f-number be increased or decreased? Explain.

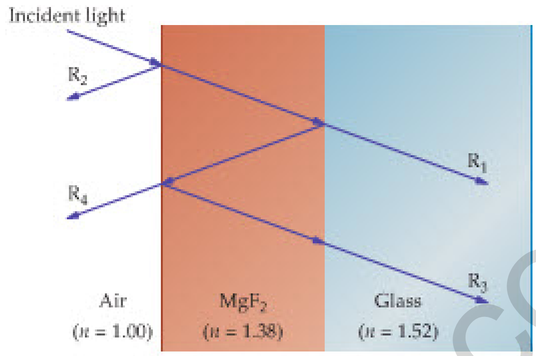
81. • Diffraction effects often involve small angles, and we usually make the approximation $\sin\theta \cong \tan\theta$. To see how accurate this approximation is, complete the following table.

θ (deg)	θ (rad)	$\sin\theta$	$\tan\theta$	$\sin\theta/\tan\theta$
0.0100°				
1.00°				
5.00°				
10.0°				
20.0°				
30.0°				
40.0°				

82. •• When reading the printout from a laser printer, you are actually looking at an array of tiny dots. If the pupil of your eye is 4.3 mm in diameter when reading a page held 28 cm from your eye, what is the minimum separation of adjacent dots that can be resolved? (Assume light with a wavelength of 540 nm, and use 1.36 as the index of refraction for the interior of the eye.)

83. •• The headlights of a pickup truck are 1.32 m apart. What is the greatest distance at which these headlights can be resolved as separate points of light on a photograph taken with a camera whose aperture has a diameter of 12.5 mm? (Take $\lambda = 555$ nm.)

84. •• **Antireflection Coating** A glass lens ($n_{glass} = 1.52$) has an antireflection coating of MgF_2 ($n = 1.38$). **(a)** For 517-nm light, what minimum thickness of MgF_2 will cause the reflected rays R_2 and R_4 in Figure 28–40 to interfere destructively, assuming normal incidence? **(b)** Interference will also occur between the forward-moving rays R_1 and R_3 in Figure 28–40. What minimum thickness of MgF_2 will cause these two rays to interfere constructively?



▲ **FIGURE 28–40** Problem 84

85. •• **IP** White light reflected at normal incidence from a soap bubble ($n = 1.33$) in air produces an interference maximum at $\lambda = 575$ nm but no interference minima in the visible spectrum. **(a)** Explain the absence of interference minima in the visible. **(b)** What are the possible thicknesses of the soap film? (Refer to Example 25–3 for the range of visible wavelengths.)

86. •• A thin film of oil ($n = 1.30$) floats on water ($n = 1.33$). When sunlight is incident at right angles to this film, the only colors that are enhanced by reflection are blue (458 nm) and red (687 nm). Estimate the thickness of the oil film.

87. •• The yellow light of sodium, with wavelengths of 588.99 nm and 589.59 nm, is normally incident on a grating with 494 lines/cm. Find the linear distance between the first-order maxima for these two wavelengths on a screen 2.55 m from the grating.

88. •• **IP** A thin soap film ($n = 1.33$) suspended in air has a uniform thickness. When white light strikes the film at normal incidence, violet light ($\lambda_V = 420$ nm) is constructively reflected. **(a)** If we would like green light ($\lambda_G = 560$ nm) to be constructively reflected, instead, should the film's thickness be increased or decreased? **(b)** Find the new thickness of the film. (Assume the film has the minimum thickness that can produce these reflections.)

89. •• **IP** A thin film of oil ($n = 1.40$) floats on water ($n = 1.33$). When sunlight is incident at right angles to this film, the only colors that are absent from the reflected light are blue (458 nm) and red (687 nm). Estimate the thickness of the oil film.

90. •• **IP** Sodium light, with a wavelength of $\lambda = 589$ nm, shines downward onto the system shown in Figure 28–37. When viewed from above, you see a series of concentric circles known as Newton's rings. **(a)** Do you expect a bright or a dark spot at the center of the pattern? Explain. **(b)** If the radius of curvature of the plano-convex lens is $R = 26.1$ m, what is the radius of the tenth-largest dark ring? (Only rings of nonzero radius will be counted as "rings.")

91. •• **IP** Figure 28–39 shows a single-slit diffraction pattern formed by light passing through a slit of width $W = 11.2\ \mu$m and illuminating a screen 0.855 m behind the slit. **(a)** What is the wavelength of the light? **(b)** If the width of the slit is decreased, will the distance indicated in Figure 28–39 be greater than or less than 15.2 cm? Explain.

92. •• **BIO Entoptic Halos** Images produced by structures within the eye (like lens fibers or cell fragments) are referred to as entoptic images. These images can sometimes take the form of "halos" around a bright light seen against a dark background. The halo in such a case is actually the bright outer rings of a circular diffraction pattern, like Figure 28–21, with the central bright spot not visible because it overlaps the direct image of the light. Find the diameter of the eye structure that causes a

circular diffraction pattern with the first dark ring at an angle of 3.7° when viewed with monochromatic light of wavelength 630 nm. (Typical eye structures of this type have diameters on the order of 10 μm. Also, the index of refraction of the vitreous humor is 1.336.)

93. ••• White light is incident on a soap film ($n = 1.33$, thickness = 800.0 nm) suspended in air. If the incident light makes a 45° angle with the normal to the film, what visible wavelength(s) will be constructively reflected? (Refer to Example 25–3 for the range of visible wavelengths.)

94. ••• **IP** A system like that shown in Figure 28–26 consists of N slits, each transmitting light of intensity I_0. The light from each slit has the same phase and the same wavelength. The net intensity I observed at an angle θ due to all N slits is

$$I = I_0\left[\frac{\sin(N\phi/2)}{\sin(\phi/2)}\right]^2$$

In this expression, $\phi = (2\pi d/\lambda)\sin\theta$, where λ is the wavelength of the light. **(a)** Show that the intensity in the limit $\theta \to 0$ is $I = N^2 I_0$. This is the maximum intensity of the interference pattern. **(b)** Show that the first points of zero intensity on either side of $\theta = 0$ occur at $\phi = 2\pi/N$ and $\phi = -2\pi/N$. **(c)** Does the central maximum ($\theta = 0$) of this pattern become narrower or broader as the number of slits is increased? Explain.

95. ••• Two plates of glass are separated on both ends by small wires of diameter d. Derive an expression for the condition for constructive interference when light of wavelength λ is incident normally on the plates. Consider only interference between waves reflected from the bottom of the top plate and the top of the bottom plate.

96. ••• A curved piece of glass with a radius of curvature R rests on a flat plate of glass. Light of wavelength λ is incident normally on this system. Considering only interference between waves reflected from the curved (lower) surface of glass and the top surface of the plate, show that the radius of the nth dark ring is

$$r_n = \sqrt{n\lambda R - n^2\lambda^2/4}$$

97. ••• **BIO The Resolution of the Eye** The resolution of the eye is ultimately limited by the pupil diameter. What is the smallest diameter spot the eye can produce on the retina if the pupil diameter is 4.25 mm? Assume light with a wavelength of $\lambda = 550$ nm. (*Note*: The distance from the pupil to the retina is 25.4 mm. In addition, the space between the pupil and the retina is filled with a fluid whose index of refraction is $n = 1.36$.)

PASSAGE PROBLEMS

Resolving Lines on an HDTV

The American Television Systems Committee (ATSC) sets the standards for high-definition television (HDTV). One of the approved HDTV formats is 1080p, which means 1080 horizontal lines scanned progressively (p)—that is, one line after another in sequence from top to bottom. Another standard is 1080i, which stands for 1080 lines interlaced (i). In this system it takes two scans of the screen to show a complete picture: the first scan shows the "even" horizontal lines, the second scan shows the "odd" horizontal lines. Interlacing was the norm for television displays until the 1970s, and is still used in most standard-definition TVs today. Progressive scanning became more popular with the advent of computer monitors, and is used today in LCD, DLP, and plasma HDTVs.

In addition, the ATSC sets the standard for the shape of displays. For example, it defines a "wide screen" to be one with a 16:9 ratio; that is, the width of the display is greater than the height by the factor 16/9. This ratio is just a little larger than the golden ratio, $\phi = (1 + \sqrt{5})/2 = 1.618\ldots$, which is generally believed to be especially pleasing to the eye. Whatever the shape or definition of a TV, the ATSC specifies that it project 30 frames per second on a progressive display, or 60 fields per second on an interlace display, where each field is half the horizontal lines.

For the following problems, assume that 1080 horizontal lines are displayed on a television with a screen that is 15.7 inches high (32-inch diagonal), and that the light coming from the screen has a wavelength of 645 nm. Also, assume that the pupil of your eye has a diameter of 5.50 mm, and that the index of refraction of the interior of the eye is 1.36.

98. • What is the minimum angle your eye can resolve, according to the Rayleigh criterion and the above assumptions?

A. 0.862×10^{-4} rad **B.** 1.05×10^{-4} rad
C. 1.43×10^{-4} rad **D.** 1.95×10^{-4} rad

99. • What is the linear separation between horizontal lines on the screen?

A. 0.0235 mm **B.** 0.145 mm
C. 0.369 mm **D.** 0.926 mm

100. • What is the angular separation of the horizontal lines as viewed from a distance of 12.0 feet?

A. 1.01×10^{-4} rad **B.** 2.53×10^{-4} rad
C. 2.56×10^{-4} rad **D.** 12.1×10^{-4} rad

101. • According to the Rayleigh criterion, what is the closest you can be to the TV screen before resolving the individual horizontal lines? (In practice you can be considerably closer than this distance before resolving the lines.)

A. 3.51 ft **B.** 4.53 ft
C. 11.5 ft **D.** 14.0 ft

INTERACTIVE PROBLEMS

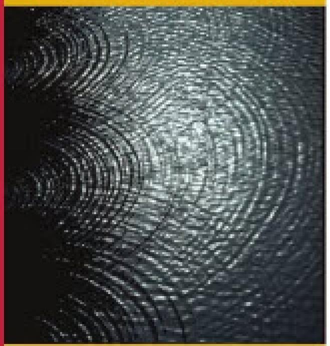
102. •• **IP Referring to Example 28–2** Suppose we change the slit separation to a value other than 8.5×10^{-5} m, with the result that the linear distance to the tenth bright fringe above the central bright fringe increases from 12 cm to 18 cm. The screen is still 2.3 m from the slits, and the wavelength of the light is 440 nm. **(a)** Did we increase or decrease the slit separation? Explain. **(b)** Find the new slit separation.

103. •• **IP Referring to Example 28–2** The wavelength of the light is changed to a value other than 440 nm, with the result that the linear distance to the *seventh* bright fringe above the central bright fringe is 12 cm. The screen is still 2.3 m from the slits, and the slit separation is 8.5×10^{-5} m. **(a)** Is the new wavelength longer or shorter than 440 nm? Explain. **(b)** Find the new wavelength.

104. •• **IP Referring to Example 28–5** The light used in this experiment has a wavelength of 511 nm. **(a)** If the width of the slit is decreased, will the angle to the first dark fringe above the central bright fringe increase or decrease? Explain. **(b)** Find the angle to the first dark fringe if the reduced slit width is 1.50×10^{-6} m.

105. •• **IP Referring to Example 28–5** The width of the slit in this experiment is 2.20×10^{-6} m. **(a)** If the frequency of the light is decreased, will the angle to the first dark fringe above the central bright fringe increase or decrease? Explain. **(b)** Find the angle to the first dark fringe if the reduced frequency is 5.22×10^{14} Hz.

PHYSICS IN



PERSPECTIVE

Waves and Particles: A Theme of Modern Physics

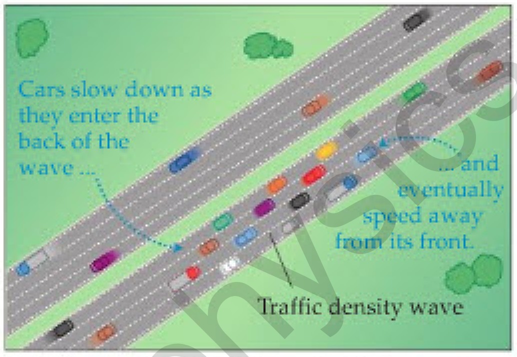
We usually think of waves and matter as distinct. In these pages we review what waves are and how we recognize them. Then we show that waves and matter are not so distinct after all.

❶ What is a wave?

A wave is a traveling disturbance. Some waves propagate through a medium, such as water or air, but some—such as light—can travel in a vacuum. Waves often carry energy, although the particles within a wave usually just oscillate back and forth as the wave passes by. The speed of a wave is determined by the properties of the medium through which the wave travels. As the following examples show, a "wave" can take many forms.

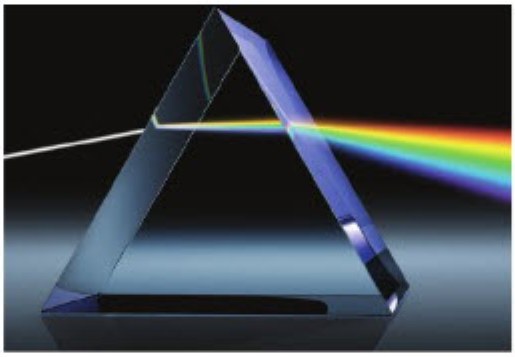
Kelvin-Helmholtz waves The ripple-shaped clouds in this photo mark a type of wave that forms in the shear zone between atmospheric layers moving at different velocities.

Kelvin-Helmholtz waves are also common in Jupiter's atmosphere, where intense wind bands shear against each other. The white waves to the left of the Great Red Spot are each larger than our moon.

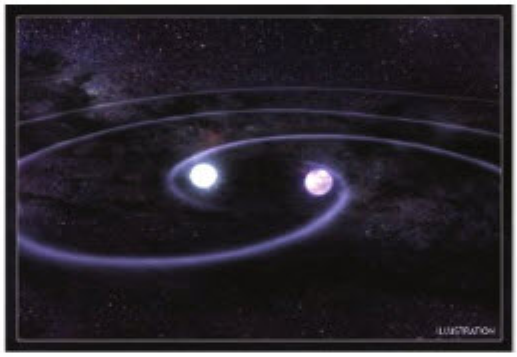


Density waves in traffic Clumpy traffic represents a type of *density wave*. At certain traffic densities, transient disturbances can set up long-lasting, self-sustaining density waves, with cars slowing as they approach the rear of the wave and eventually speeding away from its front. The wave itself may propagate in the direction opposite to the motion of the cars.

Galactic density waves The arms of a spiral galaxy are also density waves—zones in which the stars and gas that orbit the galactic center are unusually tightly packed. The arm propagates because its extra density strengthens its gravitational pull, speeding up the matter approaching its leading edge and slowing the matter leaving its trailing edge. The arms are bright because their high density ignites star formation.



Electromagnetic waves As we learned in Chapter 25, light and other electromagnetic waves represent oscillating, mutually generating electric and magnetic fields. Electromagnetic waves can propagate in a vacuum—they do not require a material medium.

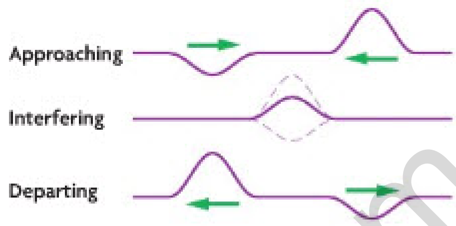


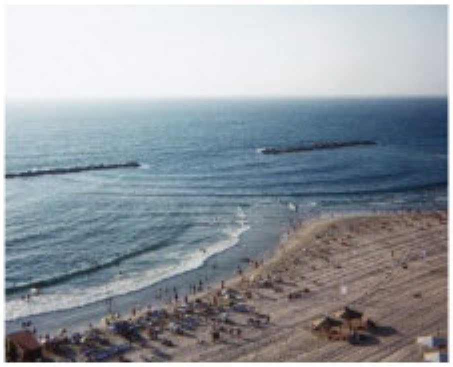
Gravitational waves Einstein's theory of general relativity implies that moving masses can produce waves in the fabric of spacetime. This artist's impression shows the pattern of gravitational waves (actually invisible) that would be produced by a pair of white dwarf stars orbiting each other.

2 What properties do all waves share?

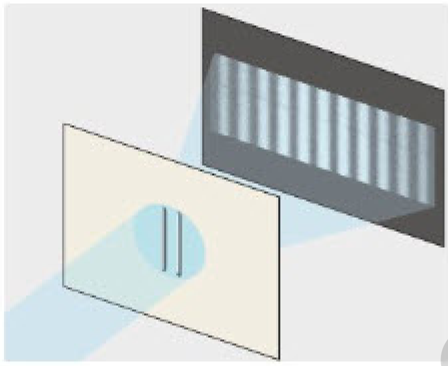
- All waves exhibit *superposition* and its consequence, *interference*. When two or more waves of the same type coincide in space, their effects are additive, as diagrammed at right for two waves moving in opposite directions on a string.
- All waves also exhibit *diffraction*: they bend around obstacles. Diffraction is a consequence of the way in which waves propagate and interfere.

If a phenomenon exhibits interference and diffraction, you know it is a wave.





Interference and diffraction: Water waves Water waves passing through slits in a breakwater diffract and interfere. The sand has formed a point at a node in the interference pattern.



Interference and diffraction: Light This is the diffraction pattern formed when a wave (light in this case) passes through a double slit.

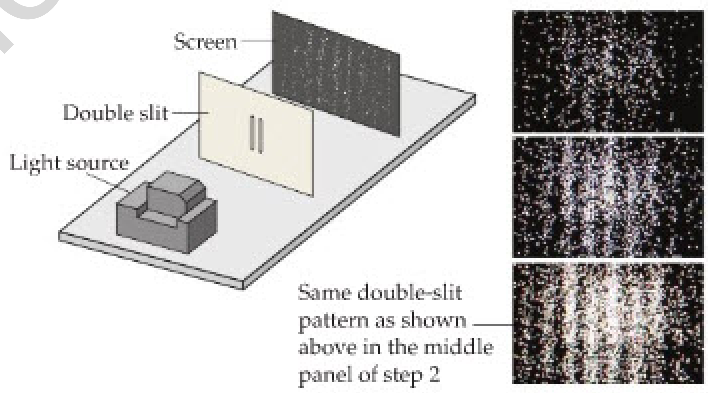
Standing wave in a vibrating cup of coffee A standing wave will form whenever identical waves travel in opposite directions—as when a wave reflects back on itself.

3 Light consists of waves—or does it?

Light exhibits interference and diffraction, so clearly it's a wave.

However, if you shine light of very low intensity through a double slit onto a CCD chip, what you'll see at first is not a faint diffraction pattern, but instead a random pattern of sharp dots, as if tiny "light bullets" are striking the CCD. Over time, though, these dots accumulate to form the diffraction pattern that is diagnostic of a wave phenomenon!

As we'll explore in Chapter 30, light is both wavelike and particle-like—our everyday experience tells us that these phenomena are mutually exclusive, but in reality they are not.

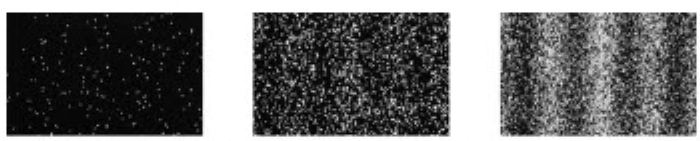


4 Matter is made of particles—or is it?

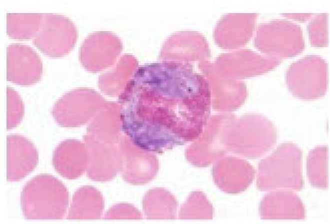
Now shoot electrons through a double slit. Initially you see the random dots you expect if electrons act like little bullets—but over time the dots accumulate to form a classic diffraction pattern, meaning that the electrons must be waves! This pattern forms even if you fire the electrons through the double slit one at a time—meaning that *each individual electron* "knows" about both slits and interferes with itself.

Thus, electrons, like light, have both wave and particle properties. As we'll learn in Chapter 30, this finding applies to all particles—therefore, surprising as it may seem, solid objects also have wavelike properties.

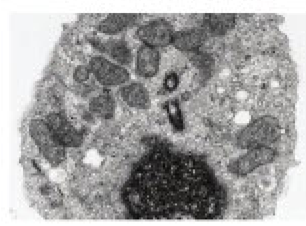
The wavelengths of electrons and other particles are much shorter than those of light. That is why an electron microscope can see smaller details than a light microscope can. (Recall from Section 28-5 that resolution is limited by wavelength.)



Two-slit diffraction pattern formed by electrons. This pattern appears even if the electrons pass through the device one by one.



Light micrograph of blood cells



Electron micrograph showing structure in a single cell