PRACTICE PROBLEM

Suppose the incident ray is in a different type of glass, with a glass–air critical angle of 40.0°. Is the index of refraction of this glass more than or less than 1.50? Verify your answer with a calculation. [**Answer:** The index of refraction is greater than 1.50. The calculated value is 1.56.]

Some related homework problems: Problem 62, Problem 63, Problem 64

Total internal reflection is frequently put to practical use. For example, many binoculars contain a set of prisms—referred to as *Porro prisms*—that use total internal reflection to "fold" a relatively long light path into the short length of the binoculars, as shown in **Figure 26–26**. This allows the user to hold with ease a relatively short device that has the same optical behavior as a set of long, unwieldy telescopes. Thus, the characteristic zigzag shape of a binocular is not a fashion statement, but a reflection of its internal optical construction.

Optical fibers are another important application of total internal reflection. These thin fibers are generally composed of a glass or plastic core with a high index of refraction surrounded by an outer coating, or cladding, with a low index of refraction. Light is introduced into the core of the fiber at one end. It then propagates along the fiber in a zigzag path, undergoing one total internal reflection after another, as indicated in **Figure 26–27**. The core is so transparent that even after light propagates through a 1-km length of fiber, the amount of absorption is roughly the same as if the light had simply passed through a glass window. In addition, the total internal reflections allow the fiber to go around corners, and even to be tied into knots, and still deliver the light to the other end.

The ability of optical fibers to convey light along curved paths has been put to good use in various fields of medicine. In particular, devices known as *endoscopes* allow physicians to examine the interior of the body by snaking a flexible tube containing optical fibers into the part of the body to be examined. For example, a type of endoscope called the bronchoscope can be inserted into the nose or throat, threaded through the bronchial tubes, and eventually placed in the lungs. There, the bronchoscope delivers light through one set of fibers and returns light to the physician through another set of fibers. In some cases, the bronchoscope can even be used to retrieve small samples from the lung for further analysis. Similarly, the colonoscope can be used to examine the colon, making it one of the most important weapons in the fight against colon cancer.

Finally, optical fibers are important components of telecommunication systems. Not only are they small, light, and flexible, but they are also immune to the type of electrical interference that can degrade information carried on copper wires. Even more important, however, is that optical fibers can carry thousands of times more information than an electric current in a wire. For example, it takes

REAL-WORLD PHYSICS

Porro prisms in binoculars

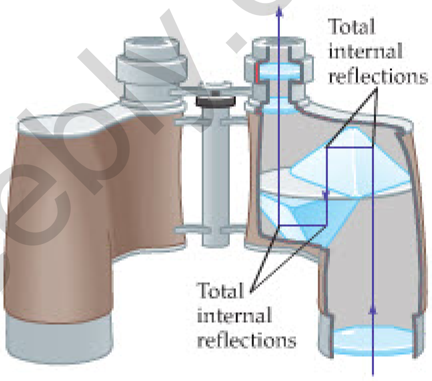


FIGURE 26–26 Porro prisms

Prisms are used to "fold" the light path within a pair of binoculars. This makes the binoculars easier to handle.

REAL-WORLD PHYSICS: BIO

Optical fibers and endoscopes

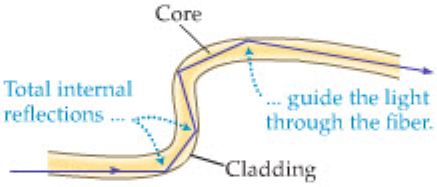
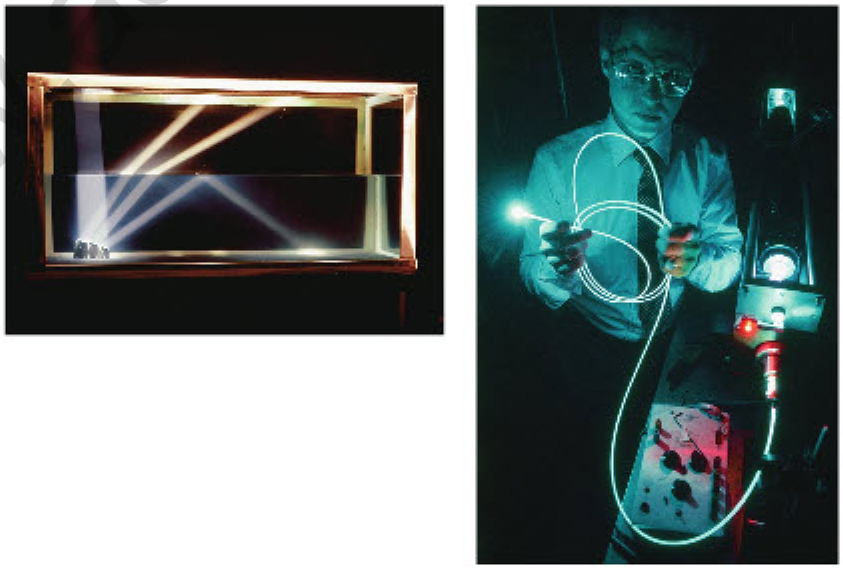


FIGURE 26–27 An optical fiber

An optical fiber channels light along its core by a series of total internal reflections between the core and the cladding.



At left, a beam of light enters a tank of water from above and is reflected by mirrors oriented at different angles. Most of the light in the first two beams passes through the water–air interface, undergoing refraction as it leaves the water. Only a small portion of the light is reflected back down into the tank. (The weak beams are hard to see, but the spots they make on the bottom of the tank are clearly visible.) The third beam, however, strikes the interface at an angle of incidence greater than the critical angle. As a result, all of the beam is reflected, as if the surface of the water were a mirror. This phenomenon of total internal reflection makes it possible to "pipe" light through tiny optical fibers such as the one shown at right.

only a single optical fiber to transmit several television programs and tens of thousands of telephone conversations, all at the same time.

Total Polarization

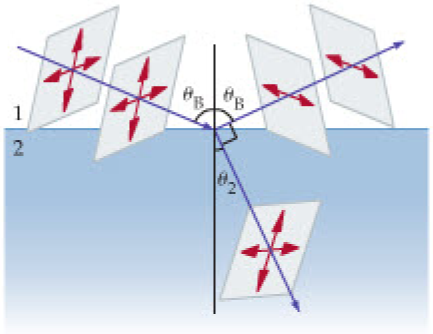


FIGURE 26–28 Brewster's angle
When light is incident at Brewster's angle, θ_B, the reflected and refracted rays are perpendicular to each other. In addition, the reflected light is completely polarized parallel to the reflecting surface.

As explained in Section 25–5, light reflected from a nonmetallic surface is generally polarized to some degree. For example, the light reflected from the surface of a lake is preferentially polarized in the horizontal direction. The polarization of reflected light is complete for one special angle of incidence, **Brewster's angle**, θ_B, defined as follows:

> Reflected light is completely polarized when the reflected and refracted beams are at right angles to one another. The direction of polarization is parallel to the reflecting surface.

This situation is illustrated in Figure 26–28. Brewster's angle is named for its discoverer, and the inventor of the kaleidoscope, the Scottish physicist Sir David Brewster (1781–1868).

To calculate Brewster's angle, we begin by applying Snell's law with an incident angle equal to θ_B:

$$n_1 \sin \theta_B = n_2 \sin \theta_2$$

Next, we note from Figure 26–28 that $\theta_B + 90° + \theta_2 = 180°$; that is, $\theta_B + \theta_2 = 90°$, or $\theta_2 = 90° - \theta_B$. Therefore, a standard trigonometric identity (Appendix A) leads to the following relation: $\sin \theta_2 = \sin(90° - \theta_B) = \cos \theta_B$. Combining this result with the preceding equation, we obtain

Brewster's Angle, θ_B

$$\tan \theta_B = \frac{n_2}{n_1} \qquad 26\text{–}13$$

We calculate Brewster's angle for a typical situation in the next Exercise.

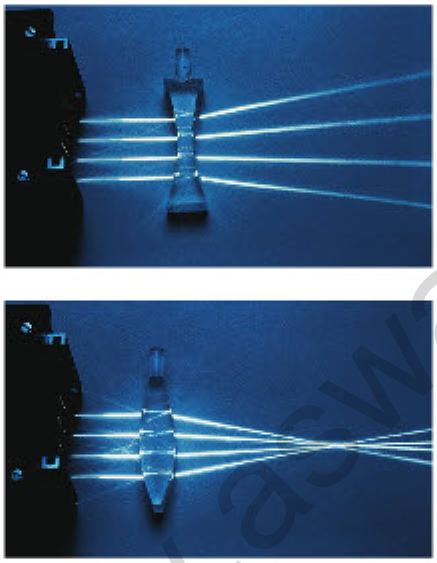
EXERCISE 26–5

Find Brewster's angle for light reflected from the top of a glass ($n = 1.50$) coffee table.

SOLUTION

Letting $n_1 = 1.00$ and $n_2 = 1.50$, we find

$$\theta_B = \tan^{-1}\left(\frac{n_2}{n_1}\right) = 56.3°$$



The paths of light rays through a concave (diverging) lens and a convex (converging) lens.

26–6 Ray Tracing for Lenses

As we have seen, a ray of light can be redirected as it passes from one medium to another. A device that takes advantage of this effect, and uses it to focus light and form images, is a **lens.** Typically, a lens is a thin piece of glass or other transparent substance that can be characterized by the effect it has on light. In particular, converging lenses take parallel rays of light and bring them together at a focus; diverging lenses cause parallel rays to spread out as if diverging from a point source. A variety of converging and diverging lenses are illustrated in Figure 26–29,

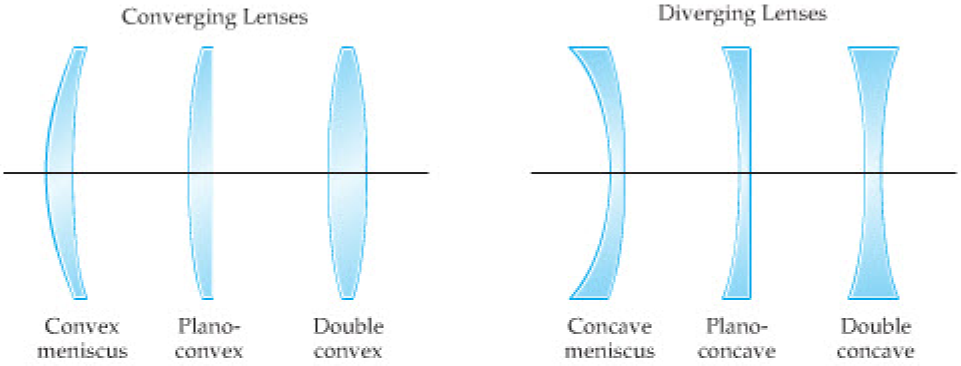
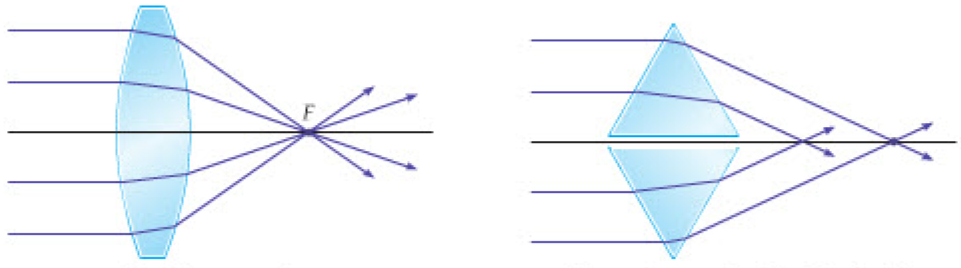


FIGURE 26–29 A variety of converging and diverging lenses
Converging and diverging lenses come in a variety of shapes. Generally speaking, converging lenses are thicker in the middle than at the edges, and diverging lenses are thinner in the middle. We will use double convex and double concave lenses when we present examples of converging and diverging lenses, respectively.



A double convex lens

Two prisms stacked "back to back"

FIGURE 26–30 A convex lens compared with a pair of prisms

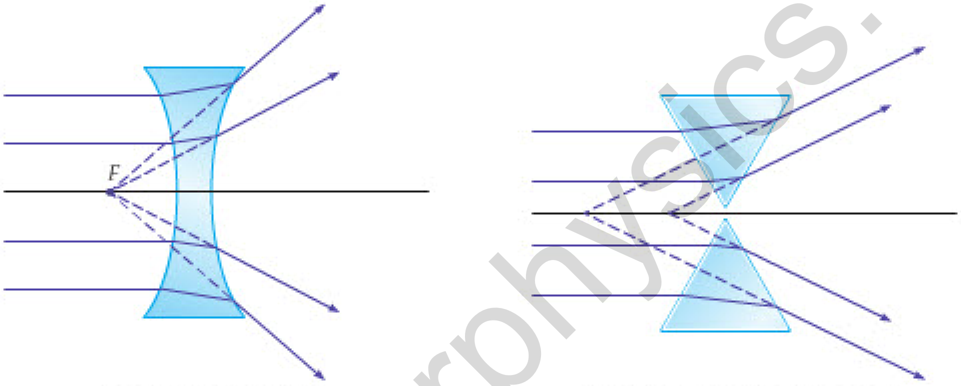
The behavior of a convex lens is similar to that of two prisms placed back to back. In both cases, light parallel to the axis is made to converge. Note that the lens, because of its curved shape, brings light to a focus at the focal point, *F*.

though we consider only the most basic types here—namely, the double concave (or simply concave) and the double convex (or simply convex).

First, consider a convex lens, as shown in Figure 26–30. To see qualitatively why such a lens is converging, note that it is similar to two prisms placed back to back. Recalling the bending of light described for a prism in Conceptual Checkpoint 26–4, we expect parallel rays of light to be brought together. In fact, lenses are shaped so that they bring parallel light to a focus at a **focal point**, *F*, along the center line, or axis, of the lens, as indicated in the figure.

Similarly, a concave lens is qualitatively the same as two prisms brought together point to point, as we see in Figure 26–31. In this case, parallel rays are bent away from the axis of the lens. When the diverging rays from a lens are extended back, they appear to originate at a focal point *F* on the axis of the lens.

Drops of dew can serve as double convex lenses, producing tiny, inverted images of objects beyond their focal points.



A double concave lens

Two prisms stacked "tip to tip"

FIGURE 26–31 A concave lens compared with a pair of prisms

A concave lens and two prisms placed point to point have similar behavior. In both cases, parallel light is made to diverge.

To determine the type of image formed by a convex or concave lens, we can use ray tracing, as we did with mirrors. The three principal rays for lenses are shown in Figures 26–32 and 26–33. Their properties are as follows:

- The *P* ray—or parallel ray—approaches the lens parallel to its axis. The *P* ray is bent so that it passes through the focal point of a convex lens or extrapolates back to the focal point on the same side of a concave lens.
- The *F* ray (focal-point ray) on a convex lens is drawn through the focal point and on to the lens, as pictured in Figure 26–32. The lens bends the ray parallel to the axis—basically the reverse of a *P* ray. For a concave lens, the *F* ray is drawn toward the focal point *on the other side* of the lens, as in Figure 26–33. Before it gets there, however, it passes through the lens and is bent parallel to the axis.
- The midpoint ray (*M* ray) goes through the middle of the lens, which is basically like a thin slab of glass. For ideal lenses, which are infinitely thin, the *M* ray continues in its original direction with negligible displacement after passing through the lens.

To illustrate ray tracing, we start with the concave lens shown in Figure 26–34. Notice that the three rays originating from the top of the object extend backward

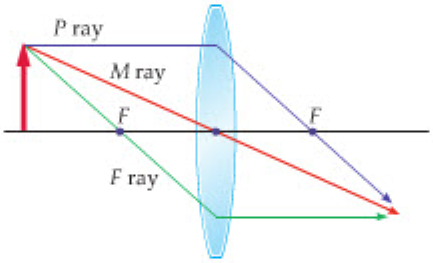
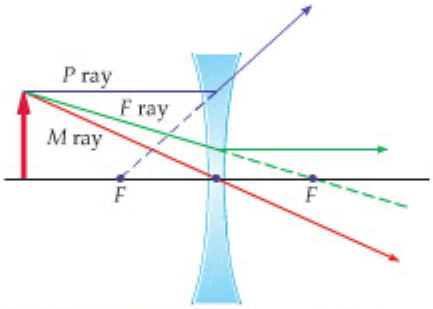
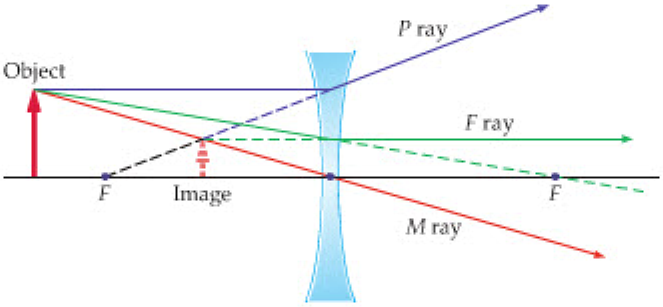


FIGURE 26–32 The three principal rays used for ray tracing with convex lenses

The image formed by a convex lens can be found by using the rays shown here. The *P* ray propagates parallel to the principal axis until it encounters the lens, where it is refracted to pass through the focal point on the far side of the lens. The *F* ray passes through the focal point on the near side of the lens, then leaves the lens parallel to the principal axis. The *M* ray passes through the middle of the lens with no deflection. Note that in each case we consider the lens to be of negligible thickness (thin-lens approximation). Therefore, the *P* and *F* rays are depicted as undergoing refraction at the center of the lens, rather than at its front and back surfaces, and the *M* ray has zero displacement. This convention is adopted in all subsequent figures.



▲ **FIGURE 26–33 The three principal rays used for ray tracing with concave lenses**

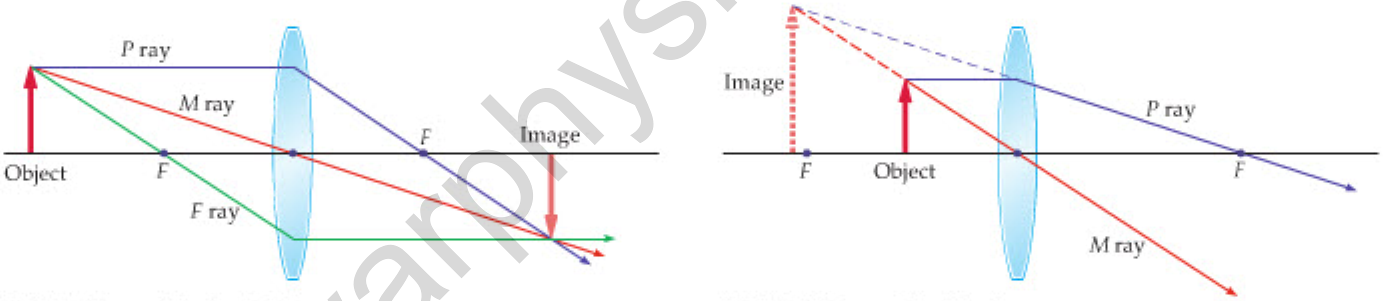


▲ **FIGURE 26–34 The image formed by a concave lens**

Ray tracing can be used to find the image produced by a concave lens. Note that the *P*, *F*, and *M* rays all extend back to the top of the virtual image, which is upright and reduced in size.

to a single point on the left side of the lens—to an observer on the right side of the lens this point is the top of the image. Our ray diagram also shows that the image is upright and reduced in size. In addition, the image is virtual, since it is on the same side of the lens as the object. These are general features of the image formed by a concave lens.

The behavior of a convex lens is more interesting in that the type of image it forms depends on the location of the object. For example, if the object is placed beyond the focal point, as in **Figure 26–35 (a)**, the image is on the opposite side of the lens and light passes through it—it is a real image. Notice as well that the image is inverted. If the object is placed between the lens and the focal point, the result, shown in **Figure 26–35 (b)**, is an image that is virtual (on the same side as the object) and upright.



(a) Object beyond focal point *F*

(b) Object between *F* and the lens

▲ **FIGURE 26–35 Ray tracing for a convex lens**

(a) The object is beyond the focal point. The image in this case is real and inverted. **(b)** The object is between the lens and the focal point. In this case the image is virtual, upright, and enlarged.

The imaging characteristics of concave and convex lenses are summarized in Table 26–3. Compare with Table 26–1, which presents the same information for mirrors.

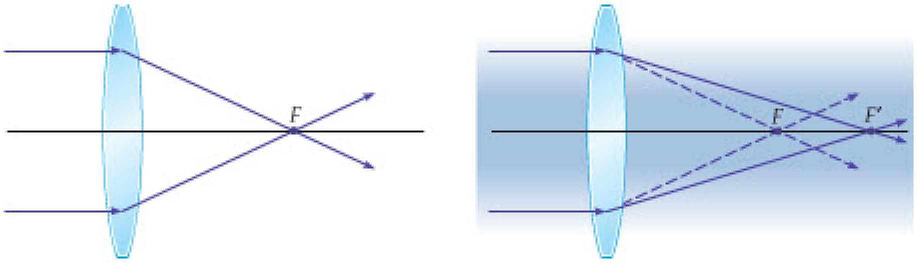
TABLE 26–3 Imaging Characteristics of Concave and Convex Lenses

CONCAVE LENS Object location	Image orientation	Image size	Image type
Arbitrary	Upright	Reduced	Virtual
CONVEX LENS **Object location**	**Image orientation**	**Image size**	**Image type**
Beyond *F*	Inverted	Reduced or enlarged	Real
Just beyond *F*	Inverted	Approaching infinity	Real
Just inside *F*	Upright	Approaching infinity	Virtual
Between lens and *F*	Upright	Enlarged	Virtual

Finally, the location of the focal point depends on the index of refraction of the lens as well as that of the surrounding medium. This effect is considered in the next Conceptual Checkpoint.

CONCEPTUAL CHECKPOINT 26–5 A LENS IN WATER

The lens shown in the diagram below (left) is generally used in air. If it is placed in water instead, does its focal length **(a)** increase, **(b)** decrease, or **(c)** stay the same?



REASONING AND DISCUSSION

In water, the difference in index of refraction between the lens and its surroundings is less than when it is in air. Therefore, recalling the discussion following Exercise 26–4, we conclude that light is bent less by the lens when it is in water, as illustrated in the second diagram (right).

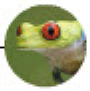
As a result, the focal length of the lens is increased.

This explains why our vision is so affected by immersing our eyes in water—the focusing ability of the eye is greatly altered by the water, as we can see from the diagram above. On the other hand, if we wear goggles, so that our eyes are still in contact with air, our vision is normal.

ANSWER

(a) The focal length increases.

REAL-WORLD PHYSICS: BIO



Underwater vision

26–7 The Thin-Lens Equation

To calculate the precise location and size of the image formed by a lens, we use an equation that is analogous to the mirror equation. This equation can be derived by referring to **Figure 26–36**, which shows the image produced by a convex lens, along with the P and M rays that locate the image.

First, note that the P ray creates two similar triangles on the right side of the lens in Figure 26–36 (a). Since the triangles are similar, it follows that

$$\frac{h_o}{f} = \frac{-h_i}{d_i - f} \qquad 26\text{–}14$$

In this expression, f is the **focal length**—that is, the distance from the lens to the focal point, F—and we use $-h_i$ on the right side of the equation, since h_i is negative for an inverted image. Next, the M ray forms another pair of similar triangles, shown in Figure 26–36 (b), from which we obtain the following:

$$\frac{h_o}{d_o} = \frac{-h_i}{d_i} \qquad 26\text{–}15$$

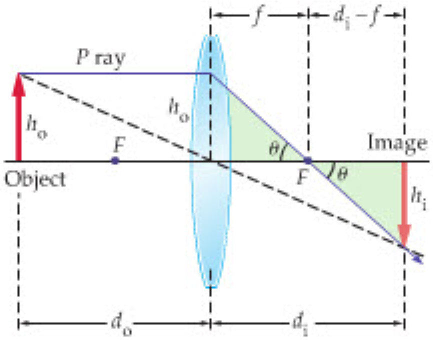
Combining these two relations, we obtain the thin-lens equation:

Thin-Lens Equation

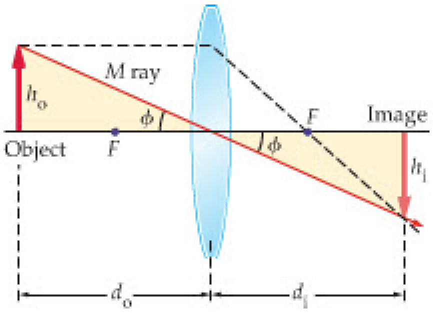
$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f} \qquad 26\text{–}16$$

Finally, the magnification, m, of the image is defined in the same way as for a mirror:

$$h_i = mh_o \qquad 26\text{–}17$$



(a) Triangles to derive Equation 26–14



(b) Triangles to derive Equation 26–15

▲ **FIGURE 26–36 Ray diagrams used to derive the thin-lens equation**

(a) The two similar triangles in this case are used to obtain Equation 26–14. **(b)** These similar triangles yield Equation 26–15.

Rearranging Equation 26–15, we find that $h_i = -(d_i/d_o)h_o$. Therefore, the magnification for a lens, just as for a mirror, is

Magnification, *m*

$$m = -\frac{d_i}{d_o} \tag{26–18}$$

As before, the sign of the magnification indicates the orientation of the image, and the magnitude gives the amount by which its size is enlarged or reduced compared with the object.

A summary of the sign conventions for lenses is as follows:

Focal Length
f is positive for converging (convex) lenses.
f is negative for diverging (concave) lenses.

Magnification
m is positive for upright images (same orientation as object).
m is negative for inverted images (opposite orientation of object).

Image Distance
d_i is positive for real images (images on the opposite side of the lens from the object).
d_i is negative for virtual images (images on the same side of the lens as the object).

Object Distance
d_o is positive for real objects (from which light diverges).
d_o is negative for virtual objects (toward which light converges).

Examples of virtual objects will be presented in Chapter 27.

We now apply the thin-lens equation and the definition of magnification to typical lens systems.

PROBLEM-SOLVING NOTE

Applying the Thin-Lens Equation

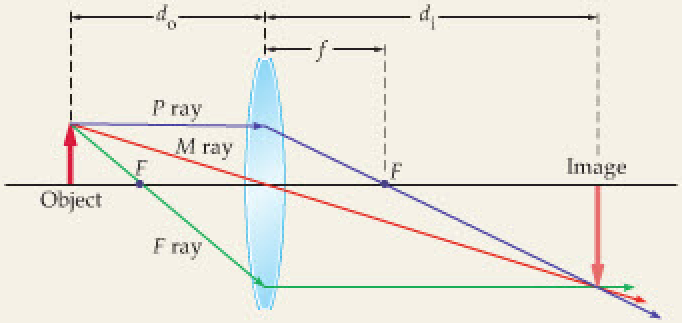
To use the thin-lens equation correctly, be careful to use the appropriate signs for all the known quantities. The final answer will also have a sign, which gives additional information about the system.

EXAMPLE 26–7 OBJECT DISTANCE AND FOCAL LENGTH

A lens produces a real image that is twice as large as the object and is located 15 cm from the lens. Find **(a)** the object distance and **(b)** the focal length of the lens.

PICTURE THE PROBLEM

Because the image is *real*, the lens must be convex, and the object must be outside the focal point, as we indicate in our sketch. [Compare with Figure 26–35 (a).] Note that the image is inverted, which means that the magnification is actually −2. In addition, the distance to the real image is given as $d_i = 15$ cm.



INTERACTIVE FIGURE

STRATEGY

To find both d_o and f requires two independent relations. One is provided by the magnification, the other by the thin-lens equation.

a. We can use the magnification, $m = -d_i/d_o$, to find the object distance, d_o. As noted before, $m = -2$ in this case.

b. We now use the values of d_i and d_o in the thin-lens equation, $1/d_o + 1/d_i = 1/f$, to find the focal length, f.

SOLUTION

Part (a)

1. Use $m = -d_i/d_o$ to find the object distance, d_o:

$$m = -\frac{d_i}{d_o} = -2 \quad \text{or} \quad d_o = -\frac{d_i}{m} = -\left(\frac{15\text{ cm}}{-2}\right) = 7.5\text{ cm}$$

Part (b)

2. Use the thin-lens equation to find $1/f$:

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{7.5\text{ cm}} + \frac{1}{15\text{ cm}} = \frac{1}{5.0\text{ cm}}$$

3. Invert $1/f$ to find the focal length, f:

$$f = \left(\frac{1}{5.0\text{ cm}}\right)^{-1} = 5.0\text{ cm}$$

INSIGHT

As expected for a convex lens, the focal length is positive. In addition, note that the object distance is greater than the focal length, in agreement with both Figure 26–35 (a) and our sketch for this Example. Finally, the magnification produced by this lens is not always -2. In fact, it depends on the precise location of the object, as we see in the following Practice Problem.

PRACTICE PROBLEM

Suppose we would like to have a magnification of -3 using the same lens. **(a)** Should the object be moved closer to the lens or farther from it? **(b)** Calculate the object and image distances for this case. [**Answer: (a)** The object should be moved closer to the lens. This moves the image farther from the lens and makes it larger. **(b)** $d_o = 6.67$ cm and $d_i = 3d_o = 20.0$ cm]

Some related homework problems: Problem 78, Problem 81, Problem 114

ACTIVE EXAMPLE 26–2 FIND THE MAGNIFICATION

An object is placed 12 cm in front of a diverging lens with a focal length of -7.9 cm. Find **(a)** the image distance and **(b)** the magnification.

SOLUTION *(Test your understanding by performing the calculations indicated in each step.)*

a. Use the thin-lens equation to find the image distance, d_i: $d_i = -4.8$ cm

b. Use $m = -d_i/d_o$ to find the magnification: $m = 0.40$

INSIGHT

Since the image distance is negative, it follows that the image is virtual and, hence, on the same side of the lens as the object, as expected for a concave (diverging) lens. In addition, the fact that the magnification is positive means that the image is upright. These numerical values correspond to the system illustrated in Figure 26–34.

YOUR TURN

If we would like a larger magnification, should the object be moved closer to the lens or farther from it? Calculate the object and image distances that give a magnification of 0.75.

(Answers to **Your Turn** *problems are given in the back of the book.)*

26–8 Dispersion and the Rainbow

As discussed earlier in this chapter, different materials, such as air, water, and glass, have different indices of refraction. There is more to the story, however. The index of refraction for a given material also depends on the frequency of the light—in general, the higher the frequency, the higher the index of refraction. This means, for example, that violet light—with its high frequency and large index of refraction—bends more when refracted by a given material than does red light. The result is that white light, with its mixture of frequencies, is spread out by refraction, so that different colors travel in different directions. This "spreading out" of light according to color is known as **dispersion.**

EXAMPLE 26–8 PRISMATICS

A flint-glass prism is made in the shape of a 30°-60°-90° triangle, as shown in the diagram. Red and violet light are incident on the prism at right angles to its vertical side. Given that the index of refraction of flint glass is 1.66 for red light and 1.70 for violet light, find the angle each ray makes with the horizontal when it emerges from the prism.

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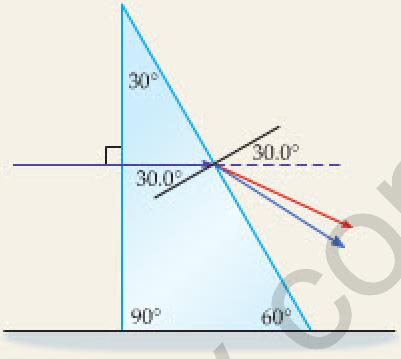


PICTURE THE PROBLEM

The prism and the red and violet rays are shown in our sketch. Note that the angle of incidence on the vertical side of the prism is 0°; hence, the angle of refraction is also 0° for both rays. On the slanted side of the prism, the rays have an angle of incidence equal to 30.0°. Their angles of refraction are different, however.

STRATEGY

To find the final angle for each ray, we apply Snell's law with the appropriate index of refraction. Note, however, that the angle of refraction is measured relative to the normal, which itself is 30.0° above the horizontal. Therefore, the angle each ray makes with the horizontal is the angle of refraction minus 30.0°.

SOLUTION

1. Apply Snell's law with $n_1 = 1.66$, $\theta_1 = 30.0°$, and $n_2 = 1.00$ to find the angle of refraction, θ_2, for red light:

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

$$\theta_2 = \sin^{-1}\left(\frac{n_1}{n_2}\sin\theta_1\right) = \sin^{-1}\left(\frac{1.66}{1.00}\sin 30.0°\right) = 56.1°$$

2. Subtract 30.0° to find the angle relative to the horizontal:

$$56.1° - 30.0° = 26.1°$$

3. Next, apply Snell's law with $n_1 = 1.70$, $\theta_1 = 30.0°$, and $n_2 = 1.00$ to find the angle of refraction, θ_2, for violet light:

$$\theta_2 = \sin^{-1}\left(\frac{n_1}{n_2}\sin\theta_1\right) = \sin^{-1}\left(\frac{1.70}{1.00}\sin 30.0°\right) = 58.2°$$

4. Subtract 30.0° to find the angle relative to the horizontal:

$$58.2° - 30.0° = 28.2°$$

INSIGHT

This is the reason for the familiar "rainbow" of colors seen with a prism. It is also the cause of a common defect of simple lenses—referred to as chromatic aberration—in which different colors focus at different points. In the next chapter, we show how two or more lenses in combination can be used to correct this problem.

PRACTICE PROBLEM

If green light emerges from the prism at an angle of 27.0° below the horizontal, what is the index of refraction for this color of light? [**Answer:** $n = 1.68$]

Some related homework problems: Problem 91, Problem 92

REAL-WORLD PHYSICS

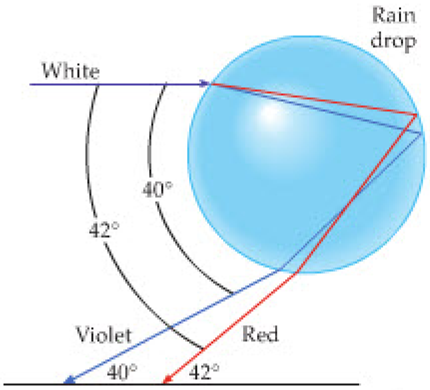
The rainbow

Perhaps the most famous and striking example of dispersion is provided by the rainbow, which is caused by the dispersion of light in droplets of rain. The physical situation is illustrated in **Figure 26–37**, which shows a single drop of rain and an incident beam of sunlight. When sunlight enters the drop, it is separated into its red and violet components by dispersion, as shown. The light then reflects from the back of the drop and finally refracts and undergoes additional dispersion as it leaves the drop.

Note that the final direction of the light is almost opposite to its incident direction, falling short by only 40° to 42°, depending on the color of the light. To be specific, violet light—which is bent the most by refraction—changes its direction of propagation by 320°, so that it is 40° away from moving in the direction of the Sun. Red light, which is not bent as much as violet light, changes its direction by only 318° and hence moves in a direction that is 42° away from the Sun.

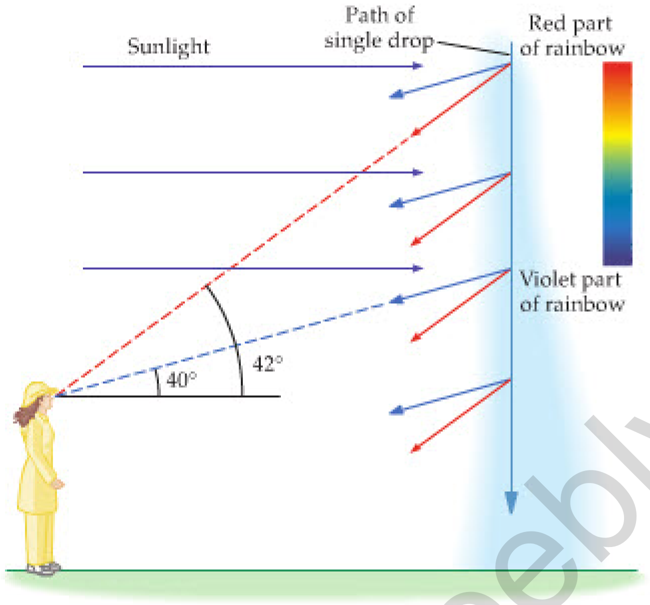
To see how the rainbow is formed in the sky, imagine standing with your back to the setting Sun, looking toward an area where rain is falling. Consider a single drop as it falls toward the ground. This drop, like all the other drops in the area, is sending out light of all colors in different directions. When the drop is at an angle of 42° above the horizontal, we see the red light coming from it, as indicated in **Figure 26–38**. As the drop continues to fall, its angle above the horizontal decreases. Eventually it reaches a height where its angle with the horizontal is 40°, at which point the violet light from the drop reaches our eye. In between, the drop has sent all the other colors of the rainbow to us for our enjoyment.

▲ A rainbow over Isaac Newton's childhood home in the manor house of Woolsthorpe, near Grantham, Lincolnshire, England. (Note the apple tree near the right side of the house.)



▲ **FIGURE 26–37 Dispersion in a raindrop**

White light entering a raindrop is spread out by dispersion into its various color components—like red and violet. (The angles shown in this figure are exaggerated for clarity.) This is the basic mechanism responsible for the formation of a rainbow. Rays of light that reflect twice inside the raindrop before exiting produce the "secondary" rainbow, which is above the normal, or "primary," rainbow. The sequence of colors in a secondary rainbow is reversed from that in the primary rainbow. A faint secondary bow can be seen in the photograph on the previous page.



▲ **FIGURE 26–38 How rainbows are produced**

As a single drop of rain falls toward the ground, it sends all the colors of the rainbow to an observer. Note that the top of the rainbow is red; the bottom is violet. (The angles in this figure have been exaggerated for clarity.)

THE BIG PICTURE PUTTING PHYSICS IN CONTEXT

LOOKING BACK

The concepts of light rays and wave fronts relate directly to the discussion of light propagation in Chapter 25.

We make extensive use of trigonometric functions in Section 26–5, where the refraction of light is presented. Note especially the calculations in Example 26–5. Trigonometric functions were first introduced in Chapter 3 when we discussed vector components.

LOOKING AHEAD

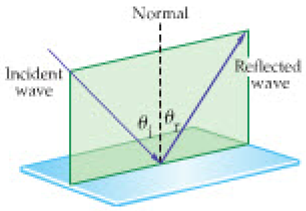
The material on dispersion and the rainbow (Section 26–8) relates directly to chromatic aberration, one of the lens aberrations discussed in Section 27–6.

The index of refraction plays an important role in Chapter 28, when we study interference in thin films. As we shall see, the wavelength of light depends on the index of refraction of the medium in which it propagates. The wavelength, in turn, is needed to determine whether the light experiences constructive or destructive interference.

CHAPTER SUMMARY

26–1 THE REFLECTION OF LIGHT

The direction of light can be changed by reflecting it from a shiny surface.



Wave Fronts

A wave front is a surface on which the phase of a wave is constant.

Rays

The direction of wave propagation is indicated by rays, which are always at right angles to wave fronts.

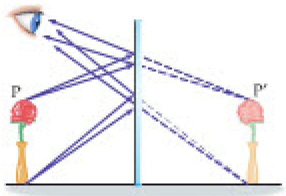
Law of Reflection

The law of reflection states that the angle of reflection, θ_r, is equal to the angle of incidence, θ_i; that is, $\theta_r = \theta_i$.

Specular/Diffuse Reflection

A smooth surface reflects light in a single direction (specular reflection). A rough surface reflects light in many directions (diffuse reflection).

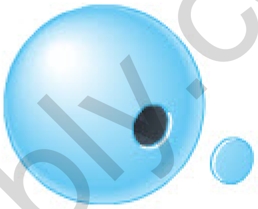
26–2 FORMING IMAGES WITH A PLANE MIRROR



The image formed by a plane mirror has the following characteristics:

- The image is upright, but appears reversed right to left.
- The image appears to be the same distance behind the mirror that the object is in front of the mirror.
- The image is the same size as the object.

26–3 SPHERICAL MIRRORS



A spherical mirror has a spherical reflecting surface. A convex spherical mirror has a reflecting surface that bulges outward. A concave spherical mirror has a hollowed reflecting surface.

Focal Point and Focal Length for a Convex Mirror
A convex mirror reflects rays that are parallel to its principal axis so that they diverge, as if they had originated from a focal point, F, behind the mirror. The focal length of a convex mirror is

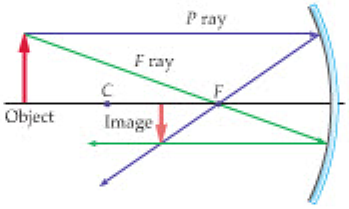
$$f = -\tfrac{1}{2}R \qquad 26\text{–}2$$

Focal Point and Focal Length for a Concave Mirror
A concave mirror reflects rays that are parallel to its principal axis so that they pass through a point known as the focal point, F. The distance from the surface of the mirror to the focal point is the focal length, f. The focal length for a mirror with a radius of curvature R is

$$f = \tfrac{1}{2}R \qquad 26\text{–}3$$

Spherical Aberration and Paraxial Rays
Paraxial rays are rays that are close to the principal axis of a mirror. Rays that are farther from the axis produce a blurred effect known as spherical aberration.

26–4 RAY TRACING AND THE MIRROR EQUATION



The location, size, and orientation of an image produced by a mirror can be found qualitatively using a ray diagram or quantitatively using a relation known as the mirror equation.

Ray Tracing
Ray tracing involves drawing two or three of the rays that have particularly simple behavior. These rays originate at a point on the object and intersect at the corresponding point on the image.

Real/Virtual Images
An image is said to be real if light passes through the apparent position of the image itself; it is virtual if light does not pass through the image.

Mirror Equation
The mirror equation relates the object distance, d_o, image distance, d_i, and focal length, f:

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f} \qquad 26\text{–}6$$

The focal length is positive for a concave mirror, negative for a convex mirror. Similarly, the image distance is positive for an image in front of the mirror and negative for an image behind the mirror.

Magnification
The magnification of an image is

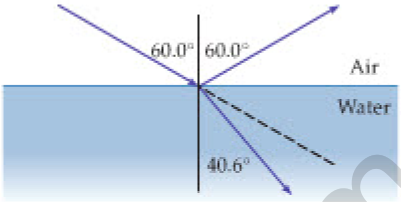
$$m = -\frac{d_i}{d_o} \qquad 26\text{–}8$$

26–5 THE REFRACTION OF LIGHT

Refraction is the change in direction of light due to a change in its speed.

Index of Refraction
The index of refraction, n, quantifies how much a medium slows the speed of light. In particular, the speed of light in a medium is

$$v = \frac{c}{n} \qquad 26\text{–}10$$



Snell's Law
Snell's law relates the index of refraction and angle of incidence in one medium (n_1, θ_1) to the index of refraction and angle of refraction in another medium (n_2, θ_2):

$$n_1 \sin\theta_1 = n_2 \sin\theta_2 \qquad 26\text{–}11$$

Qualitative Properties of Refraction
Refracted light is bent closer to the normal in a medium where its speed is reduced and away from the normal in a medium where its speed is increased.

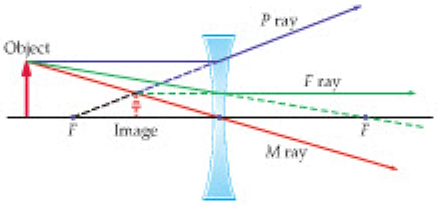
Total Internal Reflection
When light in a medium in which its speed is relatively low encounters a medium in which its speed is greater, the light will be totally reflected back into its original medium if its angle of incidence exceeds the critical angle, θ_c, given by

$$\sin\theta_c = \frac{n_2}{n_1} \qquad 26\text{–}12$$

Total Polarization
Reflected light is totally polarized parallel to the surface when the reflected and refracted rays are at right angles. This condition occurs at Brewster's angle, θ_B, given by

$$\tan\theta_B = \frac{n_2}{n_1} \qquad 26\text{–}13$$

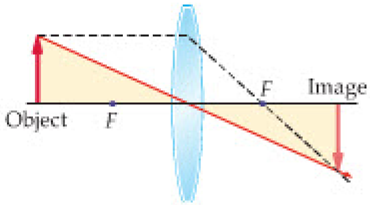
26–6 RAY TRACING FOR LENSES



As with mirrors, ray tracing is a convenient way to determine the qualitative features of an image formed by a lens.

Lens
A lens is an object that uses refraction to bend light and form images.

26–7 THE THIN-LENS EQUATION



The precise location of an image formed by a lens can be obtained using the thin-lens equation, which has the same form as the mirror equation.

Thin-Lens Equation
The thin-lens equation relates the object distance, d_o, the image distance, d_i, and the focal length, f, for a lens:

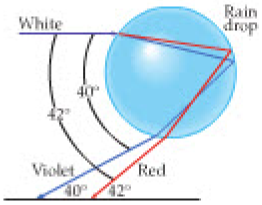
$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f} \qquad 26\text{–}16$$

Magnification
The magnification, m, of an image formed by a lens is given by the same expression used for the images produced by mirrors:

$$m = -\frac{d_i}{d_o} \qquad 26\text{–}18$$

Sign Conventions
The focal length is positive for a converging lens, negative for a diverging lens. The magnification is positive for an upright image, negative for an inverted image. The image distance is positive when the image is on the opposite side of a lens from the object, negative when it is on the same side of the lens.

26–8 DISPERSION AND THE RAINBOW



The index of refraction depends on frequency—generally, the higher the frequency, the higher the index of refraction. This difference in index of refraction causes light of different colors to be refracted in different directions (dispersion). Rainbows are caused by dispersion within raindrops.

PROBLEM-SOLVING SUMMARY

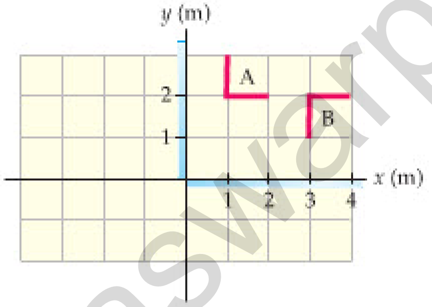
Type of Problem	Relevant Physical Concepts	Related Examples
Find the approximate location and size of the image produced by a spherical mirror.	Begin by drawing the three principal rays, P, F, and C, from the top of the object. The intersection of these rays gives the location of the top of the image.	Example 26–3
Determine precise values for the location and size of the image produced by a spherical mirror.	The mirror equation, $1/d_o + 1/d_i = 1/f$, relates the object distance, d_o, the image distance, d_i, and the focal length, f. The magnification of the image is given by $m = -(d_i/d_o)$.	Example 26–4 Active Example 26–1
Relate the angles of incidence and refraction.	The angles of incidence and refraction as a beam of light passes from one medium to another are related by Snell's law: $n_1 \sin\theta_1 = n_2 \sin\theta_2$.	Example 26–5
Find the critical angle for total internal reflection.	A light beam in a medium with an index of refraction n_1 may undergo total internal reflection when it encounters a second medium with an index of refraction $n_2 < n_1$. The critical angle at which total internal reflection begins is given by the relation $\sin\theta_c = n_2/n_1$.	Example 26–6
Determine precise values for the location and size of the image produced by a convex or concave lens.	The thin-lens equation, $1/d_o + 1/d_i = 1/f$, relates the object distance, d_o, the image distance, d_i, and the focal length, f. The magnification of the image is given by $m = -(d_i/d_o)$.	Example 26–7 Active Example 26–2

CONCEPTUAL QUESTIONS

For instructor-assigned homework, go to www.masteringphysics.com

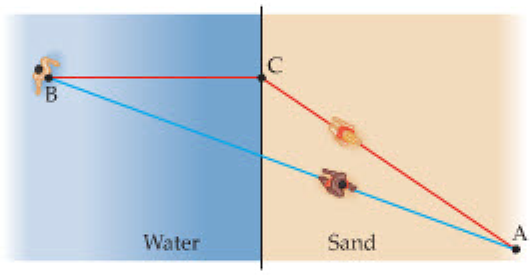
(Answers to odd-numbered Conceptual Questions can be found in the back of the book.)

1. Two plane mirrors meet at right angles at the origin, as indicated in Figure 26–39. Suppose an L-shaped object has the position and orientation labeled A. Draw the location and orientation of *all* the images of object A formed by the two mirrors.



▲ **FIGURE 26–39** Conceptual Questions 1 and 2

2. Two plane mirrors meet at right angles at the origin, as indicated in Figure 26–39. Suppose an L-shaped object has the position and orientation labeled B. Draw the location and orientation of *all* the images of object B formed by the two mirrors.
3. What is the radius of curvature of a plane mirror? What is its focal length? Explain.
4. Dish receivers for satellite TV always use the concave side of the dish, never the convex side. Explain.
5. Suppose you would like to start a fire by focusing sunlight onto a piece of paper. In Conceptual Checkpoint 26–2 we saw that a concave mirror would be better than a convex mirror for this purpose. At what distance from the mirror should the paper be held for best results?
6. When light propagates from one medium to another, does it always bend toward the normal? Explain.
7. A swimmer at point B in Figure 26–40 needs help. Two lifeguards depart simultaneously from their tower at point A, but they follow different paths. Although both lifeguards run with equal speed on the sand and swim with equal speed in the water, the lifeguard who follows the longer path, ACB, arrives at point B before the lifeguard who follows the shorter, straight-line path from A to B. Explain.



▲ **FIGURE 26–40** Conceptual Question 7

8. When you observe a mirage on a hot day, what are you actually seeing when you gaze at the "pool of water" in the distance?
9. Explain the difference between a virtual and a real image.
10. Sitting on a deserted beach one evening, you watch as the last bit of the Sun approaches the horizon. Just before the Sun disappears from sight, is the top of the Sun actually above or below the horizon? That is, if Earth's atmosphere could be instantly removed just before the Sun disappeared, would the Sun still be visible, or would it be below the horizon? Explain.
11. A large, empty coffee mug sits on a table. From your vantage point the bottom of the mug is not visible. When the mug is filled with water, however, you *can* see the bottom of the mug. Explain.
12. **The Disappearing Eyedropper** The accompanying photograph shows eyedroppers partially immersed in oil (left) and water (right). Explain why the dropper is invisible in the oil.

What happened to the dropper? (Conceptual Questions 12 and 13)

13. **The Invisible Man** In the H. G. Wells novel *The Invisible Man*, a person becomes invisible by altering his index of refraction to match that of air. This is the idea behind the disappearing eyedropper in Conceptual Question 12. If the invisible man could actually do this, would he be able to see? Explain.

14. **What's the Secret?** The top of **Figure 26–41** shows the words SECRET CODE written in different colors. If you place a cylindrical rod of glass or plastic just above the words, you find that SECRET appears inverted, but CODE does not. Explain.

▲ **FIGURE 26–41** Conceptual Question 14

PROBLEMS AND CONCEPTUAL EXERCISES

Note: Answers to odd-numbered Problems and Conceptual Exercises can be found in the back of the book. **IP** *denotes an integrated problem, with both conceptual and numerical parts;* **BIO** *identifies problems of biological or medical interest;* **CE** *indicates a conceptual exercise.* **Predict/Explain** *problems ask for two responses:* **(a)** *your prediction of a physical outcome, and* **(b)** *the best explanation among three provided. On all problems, red bullets (•, ••, •••) are used to indicate the level of difficulty.*

(The outside medium is assumed to be air, with an index of refraction of 1.00, unless specifically stated otherwise.)

SECTION 26–1 THE REFLECTION OF LIGHT

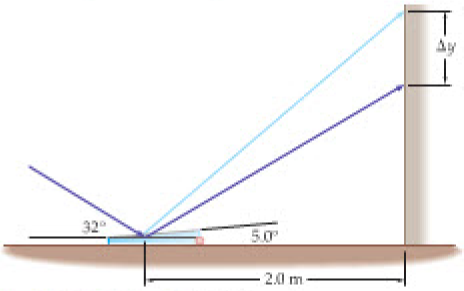
1. • A laser beam is reflected by a plane mirror. It is observed that the angle between the incident and reflected beams is 28°. If the mirror is now rotated so that the angle of incidence increases by 5.0°, what is the new angle between the incident and reflected beams?

2. •• The reflecting surfaces of two mirrors form a vertex with an angle of 120°. If a ray of light strikes mirror 1 with an angle of incidence of 55°, find the angle of reflection of the ray when it leaves mirror 2.

3. •• A ray of light reflects from a plane mirror with an angle of incidence of 37°. If the mirror is rotated by an angle θ, through what angle is the reflected ray rotated?

4. •• **IP** A small vertical mirror hangs on the wall, 1.40 m above the floor. Sunlight strikes the mirror, and the reflected beam forms a spot on the floor 2.50 m from the wall. Later in the day, you notice that the spot has moved to a point 3.75 m from the wall. **(a)** Were your two observations made in the morning or in the afternoon? Explain. **(b)** What was the change in the Sun's angle of elevation between your two observations?

5. •• Sunlight enters a room at an angle of 32° above the horizontal and reflects from a small mirror lying flat on the floor. The reflected light forms a spot on a wall that is 2.0 m behind the mirror, as shown in **Figure 26–42**. If you now place a pencil under the edge of the mirror nearer the wall, tilting it upward by 5.0°, how much higher on the wall (Δy) is the spot?

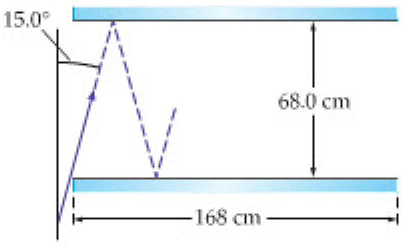


▲ **FIGURE 26–42** Problem 5

6. •• You stand 1.50 m in front of a wall and gaze downward at a small vertical mirror mounted on it. In this mirror you can see the reflection of your shoes. If your eyes are 1.85 m above your feet, through what angle should the mirror be tilted for you to see your eyes reflected in the mirror? (The location of the mirror remains the same, only its angle to the vertical is changed.)

7. •• **IP** Standing 2.3 m in front of a small vertical mirror, you see the reflection of your belt buckle, which is 0.72 m below your eyes. **(a)** What is the vertical location of the mirror relative to the level of your eyes? **(b)** What angle do your eyes make with the horizontal when you look at the buckle? **(c)** If you now move backward until you are 6.0 m from the mirror, will you still see the buckle, or will you see a point on your body that is above or below the buckle? Explain.

8. •• How many times does the light beam shown in **Figure 26–43** reflect from **(a)** the top and **(b)** the bottom mirror?



▲ **FIGURE 26–43** Problems 8 and 102

SECTION 26–2 FORMING IMAGES WITH A PLANE MIRROR

9. • **CE** If you view a clock in a mirror, do the hands rotate clockwise or counterclockwise?

Which way do the hands go? (Problem 9)

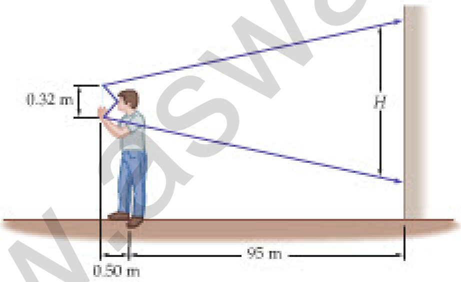
10. • A 12.5-foot-long, nearsighted python is stretched out perpendicular to a plane mirror, admiring its reflected image. If the greatest distance to which the snake can see clearly is 26.0 ft, how close must its head be to the mirror for it to see a clear image of its tail?

11. •• **(a)** How rapidly does the distance between you and your mirror image decrease if you walk directly toward a mirror with a speed of 2.6 m/s? **(b)** Repeat part (a) for the case in which you walk toward a mirror but at an angle of 38° to its normal.

12. •• You are 1.9 m tall and stand 3.2 m from a plane mirror that extends vertically upward from the floor. On the floor 1.5 m in front of the mirror is a small table, 0.80 m high. What is the minimum height the mirror must have for you to be able to see the top of the table in the mirror?

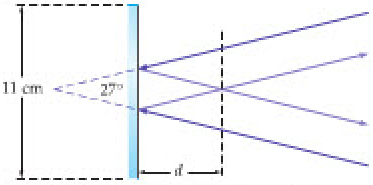
13. •• The rear window in a car is approximately a rectangle, 1.3 m wide and 0.30 m high. The inside rearview mirror is 0.50 m from the driver's eyes, and 1.50 m from the rear window. What are the minimum dimensions for the rearview mirror if the driver is to be able to see the entire width and height of the rear window in the mirror without moving her head?

14. •• **IP** You hold a small plane mirror 0.50 m in front of your eyes, as shown in **Figure 26–44** (not to scale). The mirror is 0.32 cm high, and in it you see the image of a tall building behind you. **(a)** If the building is 95 m behind you, what vertical height of the building, H, can be seen in the mirror at any one time? **(b)** If you move the mirror closer to your eyes, does your answer to part (a) increase, decrease, or stay the same? Explain.



▲ **FIGURE 26–44** Problems 14 and 113

15. •• Two rays of light converge toward each other, as shown in **Figure 26–45**, forming an angle of 27°. Before they intersect, however, they are reflected from a circular plane mirror with a diameter of 11 cm. If the mirror can be moved horizontally to the left or right, what is the greatest possible distance d from the mirror to the point where the reflected rays meet?



▲ **FIGURE 26–45** Problem 15

16. •• For a corner reflector to be effective, its surfaces must be precisely perpendicular. Suppose the surfaces of a corner reflector left on the Moon's surface by the Apollo astronauts formed a 90.001° angle with each other. If a laser beam is bounced back to Earth from this reflector, how far (in kilometers) from its starting point will the reflected beam strike Earth? For simplicity, assume the beam reflects from only two sides of the reflector, and that it strikes the first surface at precisely 45°.

SECTION 26–3 SPHERICAL MIRRORS

17. • **CE** Astronomers often use large mirrors in their telescopes to gather as much light as possible from faint distant objects. Should the mirror in their telescopes be concave or convex? Explain.

18. • A section of a sphere has a radius of curvature of 0.86 m. If this section is painted with a reflective coating on both sides, what is the focal length of **(a)** the convex side and **(b)** the concave side?

19. • A mirrored-glass gazing globe in a garden is 31.9 cm in diameter. What is the focal length of the globe?

20. • Sunlight reflects from a concave piece of broken glass, converging to a point 15 cm from the glass. What is the radius of curvature of the glass?

SECTION 26–4 RAY TRACING AND THE MIRROR EQUATION

21. • **CE** You hold a shiny tablespoon at arm's length and look at the back side of the spoon. **(a)** Is the image you see of yourself upright or inverted? **(b)** Is the image enlarged or reduced? **(c)** Is the image real or virtual?

22. • **CE** You hold a shiny tablespoon at arm's length and look at the front side of the spoon. **(a)** Is the image you see of yourself upright or inverted? **(b)** Is the image enlarged or reduced? **(c)** Is the image real or virtual?

23. • **CE** An object is placed in front of a convex mirror whose radius of curvature is R. What is the greatest distance behind the mirror that the image can be formed?

24. • **CE** An object is placed to the left of a concave mirror, beyond its focal point. In which direction will the image move when the object is moved farther to the left?

25. • **CE** An object is placed to the left of a convex mirror. In which direction will the image move when the object is moved farther to the left?

26. • A small object is located 30.0 cm in front of a concave mirror with a radius of curvature of 40.0 cm. Where will the image be formed?

27. • Use ray diagrams to show whether the image formed by a convex mirror increases or decreases in size as an object is brought closer to the mirror's surface.

28. • An object with a height of 46 cm is placed 2.4 m in front of a concave mirror with a focal length of 0.50 m. **(a)** Determine the approximate location and size of the image using a ray diagram. **(b)** Is the image upright or inverted?

29. • Find the location and magnification of the image produced by the mirror in Problem 28 using the mirror and magnification equations.

30. • An object with a height of 46 cm is placed 2.4 m in front of a convex mirror with a focal length of −0.50 m. **(a)** Determine the approximate location and size of the image using a ray diagram. **(b)** Is the image upright or inverted?

31. • Find the location and magnification of the image produced by the mirror in Problem 30 using the mirror and magnification equations.

32. •• During a daytime football game you notice that a player's reflective helmet forms an image of the Sun 4.8 cm behind the surface of the helmet. What is the radius of curvature of the helmet, assuming it to be roughly spherical?

33. •• **IP** A magician wishes to create the illusion of a 2.74-m-tall elephant. He plans to do this by forming a virtual image of a 50.0-cm-tall model elephant with the help of a spherical mirror. **(a)** Should the mirror be concave or convex? **(b)** If the model must be placed 3.00 m from the mirror, what radius of curvature is needed? **(c)** How far from the mirror will the image be formed?

34. •• A person 1.7 m tall stands 0.66 m from a reflecting globe in a garden. **(a)** If the diameter of the globe is 18 cm, where is the image of the person, relative to the surface of the globe? **(b)** How large is the person's image?

35. •• Shaving/makeup mirrors typically have one flat and one concave (magnifying) surface. You find that you can project a magnified image of a lightbulb onto the wall of your bathroom if you hold the mirror 1.8 m from the bulb and 3.5 m from the wall. **(a)** What is the magnification of the image? **(b)** Is the image erect or inverted? **(c)** What is the focal length of the mirror?

36. •• **The Hale Telescope** The 200-inch-diameter concave mirror of the Hale telescope on Mount Palomar has a focal length of 16.9 m. An astronomer stands 20.0 m in front of this mirror. **(a)** Where is her image located? Is it in front of or behind the mirror? **(b)** Is her image real or virtual? How do you know? **(c)** What is the magnification of her image?

37. •• A concave mirror produces a virtual image that is three times as tall as the object. **(a)** If the object is 28 cm in front of the mirror, what is the image distance? **(b)** What is the focal length of this mirror?

38. •• A concave mirror produces a real image that is three times as large as the object. **(a)** If the object is 22 cm in front of the mirror, what is the image distance? **(b)** What is the focal length of this mirror?

39. •• The virtual image produced by a convex mirror is one-quarter the size of the object. **(a)** If the object is 36 cm in front of the mirror, what is the image distance? **(b)** What is the focal length of this mirror?

40. •• **IP** A 5.7-ft-tall shopper in a department store is 17 ft from a convex security mirror. The shopper notices that his image in the mirror appears to be only 6.4 in. tall. **(a)** Is the shopper's image upright or inverted? Explain. **(b)** What is the mirror's radius of curvature?

41. •• You view a nearby tree in a concave mirror. The inverted image of the tree is 3.8 cm high and is located 7.0 cm in front of the mirror. If the tree is 23 m from the mirror, what is its height?

42. ••• A shaving/makeup mirror produces an erect image that is magnified by a factor of 2.2 when your face is 25 cm from the mirror. What is the mirror's radius of curvature?

43. ••• A concave mirror with a focal length of 36 cm produces an image whose distance from the mirror is one-third the object distance. Find the object and image distances.

SECTION 26–5 THE REFRACTION OF LIGHT

44. • **CE Predict/Explain** When a ray of light enters a glass lens surrounded by air, it slows down. **(a)** As it leaves the glass, does its speed increase, decrease, or stay the same? **(b)** Choose the *best explanation* from among the following:
   I. Its speed increases because the ray is now propagating in a medium with a smaller index of refraction.
   II. The speed decreases because the speed of light decreases whenever light moves from one medium to another.
   III. The speed will stay the same because the speed of light is a universal constant.

45. • **CE Samurai Fishing** A humorous scene in Akira Kurosawa's classic film *The Seven Samurai* shows the young samurai Kikuchiyo wading into a small stream and plucking a fish from it for his dinner. **(a)** As Kikuchiyo looks through the water to the fish, does he see it in the general direction of point 1 or point 2 in Figure 26–46? **(b)** If the fish looks up at Kikuchiyo, does it see Kikuchiyo's head in the general direction of point 3 or point 4?

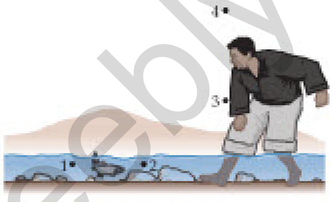


FIGURE 26–46 Problem 45

46. • **CE** When color A and color B are sent through a prism, color A is bent more than color B. Which color travels more rapidly in the prism? Explain.

47. • **CE Day Versus Night** **(a)** Imagine for a moment that the Earth has no atmosphere. Over the period of a year, is the number of daylight hours at your home greater than, less than, or equal to the number of nighttime hours? **(b)** Repeat part (a), only this time take into account the Earth's atmosphere.

48. • **CE Predict/Explain** A kitchen has twin side-by-side sinks. One sink is filled with water, the other is empty. **(a)** Does the sink with water appear to be deeper, shallower, or the same depth as the empty sink? **(b)** Choose the *best explanation* from among the following:
   I. The sink with water appears deeper because you have to look through the water to see the bottom.
   II. Water bends the light, making an object under the water appear to be closer to the surface. Thus the water-filled sink appears shallower.
   III. The sinks are identical, and therefore have the same depth. This doesn't change by putting water in one of them.

49. • **CE** A light beam undergoes total internal reflection at the interface between medium A, in which it propagates, and medium B, on the other side of the interface. Which medium has the greater index of refraction? Explain.

50. • Light travels a distance of 0.960 m in 4.00 ns in a given substance. What is the index of refraction of this substance?

51. • Find the ratio of the speed of light in water to the speed of light in diamond.

52. • **Ptolemy's Optics** One of the many works published by the Greek astronomer Ptolemy (A.D. ca. 100–170) was *Optics*. In this book Ptolemy reports the results of refraction experiments he conducted by observing light passing from air into water. His results are as follows: angle of incidence = 10.0°, angle of refraction = 8.00°; angle of incidence = 20.0°, angle of refraction = 15.5°. Find the percentage error in the calculated index of refraction of water for each of Ptolemy's measurements.

53. • Light enters a container of benzene at an angle of 43° to the normal; the refracted beam makes an angle of 27° with the normal. Calculate the index of refraction of benzene.

54. • The angle of refraction of a ray of light traveling into an ice cube from air is 38°. Find the angle of incidence.

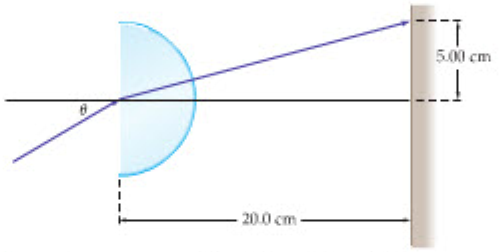
55. • **IP (a)** Referring to Problem 54, suppose the ice melts, but the angle of refraction remains the same. Is the corresponding angle of incidence greater than, less than, or the same as it was for ice? Explain. **(b)** Calculate the angle of incidence for part (a).

56. •• A submerged scuba diver looks up toward the calm surface of a freshwater lake and notes that the Sun appears to be 35° from the vertical. The diver's friend is standing on the shore of the lake. At what angle above the horizon does the friend see the sun?

57. •• A pond with a total depth (ice + water) of 3.25 m is covered by a transparent layer of ice, with a thickness of 0.38 m. Find the time required for light to travel vertically from the surface of the ice to the bottom of the pond.

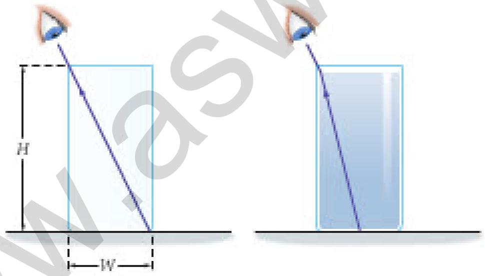
58. •• Light is refracted as it travels from a point A in medium 1 to a point B in medium 2. If the index of refraction is 1.33 in medium 1 and 1.51 in medium 2, how long does it take light to go from A to B, assuming it travels 331 cm in medium 1 and 151 cm in medium 2?

59. •• You have a semicircular disk of glass with an index of refraction of $n = 1.52$. Find the incident angle θ for which the beam of light in **Figure 26–47** will hit the indicated point on the screen.



▲ **FIGURE 26–47** Problems 59 and 66

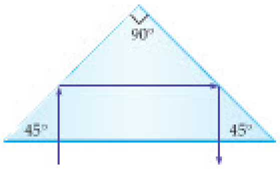
60. •• The observer in **Figure 26–48** is positioned so that the far edge of the bottom of the empty glass (not to scale) is just visible. When the glass is filled to the top with water, the center of the bottom of the glass is just visible to the observer. Find the height, H, of the glass, given that its width is $W = 6.2$ cm.



▲ **FIGURE 26–48** Problem 60

61. •• A coin is lying at the bottom of a pool of water that is 6.5 feet deep. Viewed from directly above the coin, how far below the surface of the water does the coin appear to be? (The coin is assumed to be small in diameter; therefore, we can use the small-angle approximations $\sin\theta \cong \tan\theta \cong \theta$.)

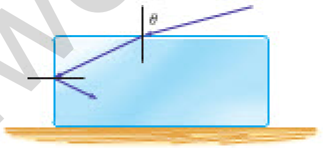
62. •• A ray of light enters the long side of a 45°-90°-45° prism and undergoes two total internal reflections, as indicated in **Figure 26–49**. The result is a reversal in the ray's direction of propagation. Find the minimum value of the prism's index of refraction, n, for these internal reflections to be total.



▲ **FIGURE 26–49** Problems 62 and 63

63. •• When the prism in Problem 62 is immersed in a fluid with an index of refraction of 1.21, the internal reflections shown in Figure 26–49 are still total. The reflections are no longer total, however, when the prism is immersed in a fluid with $n = 1.43$. Use this information to set upper and lower limits on the possible values of the prism's index of refraction.

64. •• **IP** A glass paperweight with an index of refraction n rests on a desk, as shown in **Figure 26–50**. An incident ray of light enters the horizontal top surface of the paperweight at an angle $\theta = 77°$ to the vertical. **(a)** Find the minimum value of n for which the internal reflection on the vertical surface of the paperweight is total. **(b)** If θ is decreased, is the minimum value of n increased or decreased? Explain.



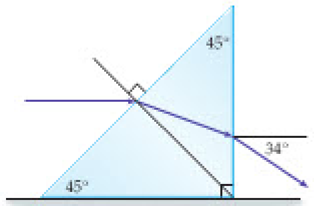
▲ **FIGURE 26–50** Problems 64 and 65

65. •• **IP** Suppose the glass paperweight in Figure 26–50 has an index of refraction $n = 1.38$. **(a)** Find the value of θ for which the reflection on the vertical surface of the paperweight exactly satisfies the condition for total internal reflection. **(b)** If θ is increased, is the reflection at the vertical surface still total? Explain.

66. •• **IP** Consider the physical system shown in Figure 26–47 and described in Problem 59. **(a)** If the index of refraction of the glass is increased, will the desired value of θ increase or decrease? Explain. **(b)** Find the value of θ for the case of flint glass ($n = 1.66$).

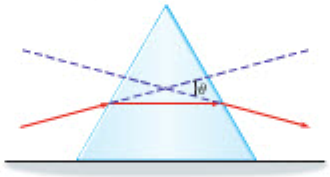
67. •• While studying physics at the library late one night, you notice the image of the desk lamp reflected from the varnished tabletop. When you turn your Polaroid sunglasses sideways, the reflected image disappears. If this occurs when the angle between the incident and reflected rays is 110°, what is the index of refraction of the varnish?

68. ••• A horizontal beam of light enters a 45°-90°-45° prism at the center of its long side, as shown in **Figure 26–51**. The emerging ray moves in a direction that is 34° below the horizontal. What is the index of refraction of this prism?



▲ **FIGURE 26–51** Problems 68 and 123

69. ••• A laser beam enters one of the sloping faces of the equilateral glass prism ($n = 1.42$) in **Figure 26–52** and refracts through the prism. Within the prism the light travels horizontally. What is the angle θ between the direction of the incident ray and the direction of the outgoing ray?



▲ **FIGURE 26–52** Problems 69 and 92

SECTION 26–6 RAY TRACING FOR LENSES

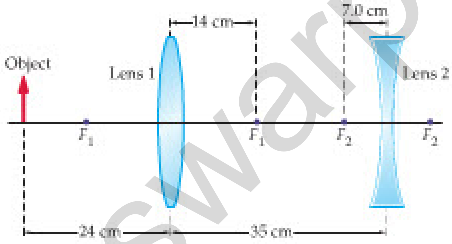
70. • **(a)** Use a ray diagram to determine the approximate location of the image produced by a concave lens when the object is at a distance $\frac{1}{2}|f|$ from the lens. **(b)** Is the image upright or inverted? **(c)** Is the image real or virtual? Explain.

71. • **(a)** Use a ray diagram to determine the approximate location of the image produced by a concave lens when the object is at a distance $2|f|$ from the lens. **(b)** Is the image upright or inverted? **(c)** Is the image real or virtual? Explain.

72. • An object is a distance $f/2$ from a convex lens. **(a)** Use a ray diagram to find the approximate location of the image. **(b)** Is the image upright or inverted? **(c)** Is the image real or virtual? Explain.

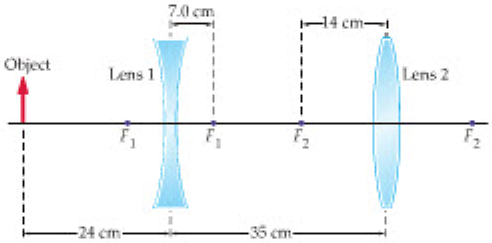
73. • An object is a distance $2f$ from a convex lens. **(a)** Use a ray diagram to find the approximate location of the image. **(b)** Is the image upright or inverted? **(c)** Is the image real or virtual? Explain.

74. •• Two lenses that are 35 cm apart are used to form an image, as shown in Figure 26–53. Lens 1 is converging and has a focal length $f_1 = 14$ cm; lens 2 is diverging and has a focal length $f_2 = -7.0$ cm. The object is placed 24 cm to the left of lens 1. **(a)** Use a ray diagram to find the approximate location of the image. **(b)** Is the image upright or inverted? **(c)** Is the image real or virtual? Explain.



▲ **FIGURE 26–53** Problems 74 and 83

75. •• Two lenses that are 35 cm apart are used to form an image, as shown in Figure 26–54. Lens 1 is diverging and has a focal length $f_1 = -7.0$ cm; lens 2 is converging and has a focal length $f_2 = 14$ cm. The object is placed 24 cm to the left of lens 1. **(a)** Use a ray diagram to find the approximate location of the image. **(b)** Is the image upright or inverted? **(c)** Is the image real or virtual? Explain.



▲ **FIGURE 26–54** Problems 75 and 84

SECTION 26–7 THE THIN-LENS EQUATION

76. • A convex lens is held over a piece of paper outdoors on a sunny day. When the paper is held 26 cm below the lens, the sunlight is focused on the paper and the paper ignites. What is the focal length of the lens?

77. • A concave lens has a focal length of −32 cm. Find the image distance and magnification that result when an object is placed 29 cm in front of the lens.

78. • When an object is located 46 cm to the left of a lens, the image is formed 17 cm to the right of the lens. What is the focal length of the lens?

79. • An object with a height of 2.54 cm is placed 36.3 mm to the left of a lens with a focal length of 35.0 mm. **(a)** Where is the image located? **(b)** What is the height of the image?

80. •• A lens for a 35-mm camera has a focal length given by $f = 55$ mm. **(a)** How close to the film should the lens be placed to form a sharp image of an object that is 5.0 m away? **(b)** What is the magnification of the image on the film?

81. •• **IP** An object is located to the left of a convex lens whose focal length is 34 cm. The magnification produced by the lens is $m = 3.0$. **(a)** To increase the magnification to 4.0, should the object be moved closer to the lens or farther away? Explain. **(b)** Calculate the distance through which the object should be moved.

82. •• **IP** You have two lenses at your disposal, one with a focal length $f_1 = +40.0$ cm, the other with a focal length $f_2 = -40.0$ cm. **(a)** Which of these two lenses would you use to project an image of a lightbulb onto a wall that is far away? **(b)** If you want to produce an image of the bulb that is enlarged by a factor of 2.00, how far from the wall should the lens be placed?

83. •• **(a)** Determine the distance from lens 1 to the final image for the system shown in Figure 26–53. **(b)** What is the magnification of this image?

84. •• **(a)** Determine the distance from lens 1 to the final image for the system shown in Figure 26–54. **(b)** What is the magnification of this image?

85. •• **IP** An object is located to the left of a concave lens whose focal length is −34 cm. The magnification produced by the lens is $m = \frac{1}{3}$. **(a)** To decrease the magnification to $m = \frac{1}{4}$, should the object be moved closer to the lens or farther away? **(b)** Calculate the distance through which the object should be moved.

86. •• **IP BIO** Albert is nearsighted, and without his eyeglasses he can focus only on objects less than 2.2 m away. **(a)** Are Albert's eyeglasses concave or convex? Explain. **(b)** To correct Albert's nearsightedness, his eyeglasses must produce a virtual, upright image at a distance of 2.2 m when viewing an infinitely distant object. What is the focal length of Albert's eyeglasses?

87. •• A small insect viewed through a convex lens is 1.4 cm from the lens and appears twice its actual size. What is the focal length of the lens?

88. ••• **IP** A friend tells you that when he takes off his eyeglasses and holds them 23 cm above a printed page the image of the print is erect but reduced to 0.67 of its actual size. **(a)** Is the image real or virtual? How do you know? **(b)** What is the focal length of your friend's glasses? **(c)** Are the lenses in the glasses concave or convex? Explain.

89. ••• **IP** A friend tells you that when she takes off her eyeglasses and holds them 23 cm above a printed page the image of the print is erect but enlarged to 1.5 times its actual size. **(a)** Is the image real or virtual? How do you know? **(b)** What is the focal length of your friend's glasses? **(c)** Are the lenses in the glasses concave or convex? Explain.

SECTION 26–8 DISPERSION AND THE RAINBOW

90\. • **CE Predict/Explain** You take a picture of a rainbow with an infrared camera, and your friend takes a picture at the same time with visible light. **(a)** Is the height of the rainbow in the infrared picture greater than, less than, or the same as the height of the rainbow in the visible-light picture? **(b)** Choose the *best explanation* from among the following:

I. The height will be greater because the top of a rainbow is red, and so infrared light would be even higher.
II. The height will be less because infrared light is below the visible spectrum.
III. A rainbow is the same whether seen in visible light or infrared; therefore the height is the same.

91\. •• The index of refraction for red light in a certain liquid is 1.320; the index of refraction for violet light in the same liquid is 1.332. Find the dispersion ($\theta_v - \theta_r$) for red and violet light when both are incident on the flat surface of the liquid at an angle of 45.00° to the normal.

92\. •• A horizontal incident beam consisting of white light passes through an equilateral prism, like the one shown in Figure 26–52. What is the dispersion ($\theta_v - \theta_r$) of the outgoing beam if the prism's index of refraction is $n_v = 1.505$ for violet light and $n_r = 1.421$ for red light?

93\. •• The focal length of a lens is inversely proportional to the quantity ($n - 1$), where n is the index of refraction of the lens material. The value of n, however, depends on the wavelength of the light that passes through the lens. For example, one type of flint glass has an index of refraction of $n_r = 1.572$ for red light and $n_v = 1.605$ in violet light. Now, suppose a white object is placed 24.00 cm in front of a lens made from this type of glass. If the red light reflected from this object produces a sharp image 55.00 cm from the lens, where will the violet image be found?

GENERAL PROBLEMS

94\. • **CE Jurassic Park** A *T. rex* chases the heroes of Steven Spielberg's *Jurassic Park* as they desperately try to escape in their Jeep. The *T. rex* is closing in fast, as they can see in the outside rearview mirror. Near the bottom of the mirror they also see the following helpful message: OBJECTS IN THE MIRROR ARE CLOSER THAN THEY APPEAR. Is this mirror concave or convex? Explain.

95\. • **CE** The receiver for a dish antenna is placed in front of the concave surface of the dish. If the radius of curvature of the dish is R, how far in front of the dish should the receiver be placed? Explain.

96\. • **CE Predict/Explain** If a lens is immersed in water, its focal length changes, as discussed in Conceptual Checkpoint 26–5. **(a)** If a spherical mirror is immersed in water, does its focal length increase, decrease, or stay the same? **(b)** Choose the *best explanation* from among the following:

I. The focal length will increase because the water will cause more bending of light.
II. Water will refract the light. This, combined with the reflection due to the mirror, will result in a decreased focal length.
III. The focal length stays the same because it depends on the fact that the angle of incidence is equal to the angle of reflection for a mirror. This is unaffected by the presence of the water.

97\. • **CE Predict/Explain** A glass slab surrounded by air causes a sideways displacement in a beam of light. **(a)** If the slab is now placed in water, does the displacement it causes increase, decrease, or stay the same? **(b)** Choose the *best explanation* from among the following:

I. The displacement of the beam increases because of the increased refraction due to the water.
II. The displacement of the beam is decreased because with water surrounding the slab there is a smaller difference in index of refraction between the slab and its surroundings.
III. The displacement stays the same because it is determined only by the properties of the slab; in particular, the material it is made of and its thickness.

98\. • **CE** Referring to Conceptual Question 12, suppose the same type of glass used in an eyedropper is made into a convex lens with a focal length f. If this lens is immersed in the oil of the bottle on the left in the photo, will its focal length be 0, $f/2$, $2f$, or ∞? (*Hint:* See Conceptual Checkpoint 26–5.)

99\. • **CE** Two identical containers are filled with different transparent liquids. The container with liquid A appears to have a greater depth than the container with liquid B. Which liquid has the greater index of refraction? Explain.

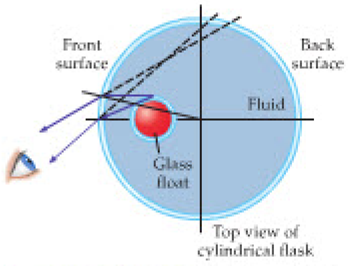
100\. •• **CE** Is the image you see in a three-dimensional corner reflector upright or inverted?

101\. •• **CE Inverse Lenses** Suppose we mold a hollow piece of plastic into the shape of a double concave lens. The "lens" is watertight, and its interior is filled with air. We now place this lens in water and shine a beam of light on it. **(a)** Does the lens converge or diverge the beam of light? Explain. **(b)** If our hollow lens is double convex instead, does it converge or diverge a beam of light when immersed in water? Explain.

102\. •• **IP** Suppose the separation between the two mirrors in Figure 26–43 is increased by moving the top mirror upward. **(a)** Will this affect the number of reflections made by the beam of light? If so, how? **(b)** What is the total number of reflections made by the beam of light when the separation between the mirrors is 145 cm?

103\. •• Standing 2.0 m in front of a small vertical mirror you see the reflection of your belt buckle, which is 0.70 m below your eyes. If you remain 2.0 m from the mirror but climb onto a stool, how high must the stool be to allow you to see your knees in the mirror? Assume that your knees are 1.2 m below your eyes.

104\. •• **IP Apparent Size of Floats in a Termometro Lentos** The Galileo thermometer, or Termometro Lentos (slow thermometer in Italian), consists of a vertical, cylindrical flask containing a fluid and several glass floats of different color. The floats all have the same dimensions, but they appear to differ in size depending on their location within the cylinder. **(a)** Does a float near the front surface of the cylinder (the surface closest to you) appear to be larger or smaller than a float near the back surface? **(b)** Figure 26–55 shows a ray diagram for a float near the front surface of the cylinder. Draw a ray diagram for a float at the center of the cylinder, and show that the change in apparent size agrees with your answer to part (a).

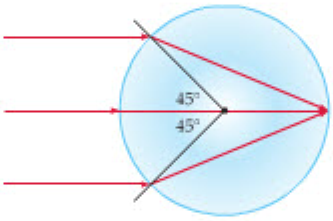


▲ **FIGURE 26–55** Problem 104

105\. •• **(a)** Find the two locations where an object can be placed in front of a concave mirror with a radius of curvature of 39 cm such that its image is twice its size. **(b)** In each of these cases, state whether the image is real or virtual, upright or inverted.

106\. •• A convex mirror with a focal length of −85 cm is used to give a truck driver a view behind the vehicle. **(a)** If a person who is 1.7 m tall stands 2.2 m from the mirror, where is the person's image located? **(b)** Is the image upright or inverted? **(c)** What is the size of the image?

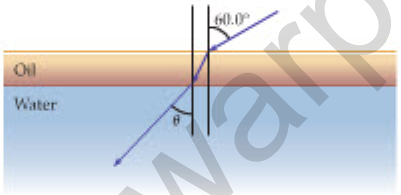
107\. •• **IP** The three laser beams shown in **Figure 26–56** meet at a point at the back of a solid, transparent sphere. **(a)** What is the index of refraction of the sphere? **(b)** Is there a finite index of refraction that will make the three beams come to a focus at the center of the sphere? If your answer is yes, give the required index of refraction; if your answer is no, explain why not.



▲ **FIGURE 26–56** Problem 107

108\. •• The speed of light in substance A is x times greater than the speed of light in substance B. Find the ratio n_A/n_B in terms of x.

109\. •• **IP** A film of oil, with an index of refraction of 1.48 and a thickness of 1.50 cm, floats on a pool of water, as shown in **Figure 26–57**. A beam of light is incident on the oil at an angle of 60.0° to the vertical. **(a)** Find the angle θ the light beam makes with the vertical as it travels through the water. **(b)** How does your answer to part (a) depend on the thickness of the oil film? Explain.

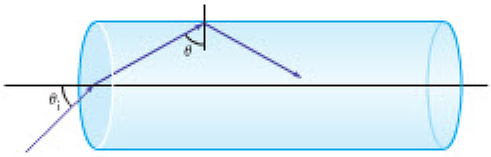


▲ **FIGURE 26–57** Problems 109, 110, and 111

110\. •• **IP** Consider the physical system shown in Figure 26–57. For this problem we assume that the angle of incidence at the air–oil interface can be varied from 0° to 90°. **(a)** What is the maximum possible value for θ, the angle of refraction in the water? **(b)** If an oil with a larger index of refraction is used, does your answer to part (a) increase or decrease? Explain.

111\. •• **IP** Consider the physical system shown in Figure 26–57, only this time let the direction of the light rays be reversed. **(a)** Find the angle of incidence θ at the water–oil interface such that the condition for total internal reflection at the oil–air surface is exactly satisfied. **(b)** If θ is decreased, is the reflection at the oil–air interface still total? Explain.

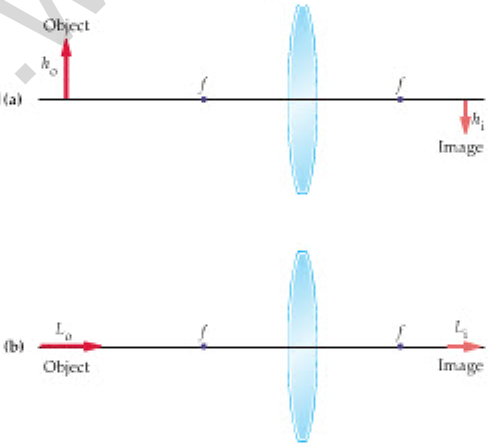
112\. •• **Figure 26–58** shows a ray of light entering one end of an optical fiber at an angle of incidence $\theta_i = 50.0°$. The index of refraction of the fiber is 1.62. **(a)** Find the angle θ the ray makes with the normal when it reaches the curved surface of the fiber. **(b)** Show that the internal reflection from the curved surface is total.



▲ **FIGURE 26–58** Problems 112 and 120

113\. •• Suppose the person's eyes in Figure 26–44 are 1.6 m above the ground and that the small plane mirror can be moved up or down. **(a)** Find the height of the bottom of the mirror such that the lowest point the person can see on the building is 19.6 m above the ground. **(b)** With the mirror held at the height found in part (a), what is the highest point on the building the person can see?

114\. •• An arrow 2.00 cm long is located 75.0 cm from a lens that has a focal length $f = 30.0$ cm. **(a)** If the arrow is perpendicular to the principal axis of the lens, as in **Figure 26–59 (a)**, what is its lateral magnification, defined as h_i/h_o? **(b)** Suppose, instead, that the arrow lies *along* the principal axis, extending from 74.0 cm to 76.0 cm from the lens, as indicated in **Figure 26–59 (b)**. What is the longitudinal magnification of the arrow, defined as L_i/L_o? (*Hint*: Use the thin-lens equation to locate the image of each end of the arrow.)



▲ **FIGURE 26–59** Problems 114 and 115

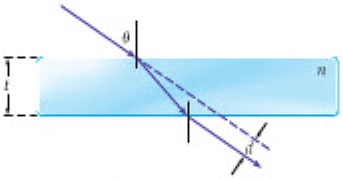
115\. •• Repeat Problem 114, this time for a diverging lens with a focal length $f = -30.0$ cm.

116\. ••• A convex lens with $f_1 = 20.0$ cm is mounted 40.0 cm to the left of a concave lens. When an object is placed 30.0 cm to the left of the convex lens, a real image is formed 60.0 cm to the right of the concave lens. What is the focal length f_2 of the concave lens?

117\. ••• Two thin lenses, with focal lengths f_1 and f_2, are placed in contact. What is the effective focal length of the double lens?

118\. ••• When an object is placed a distance d_o in front of a curved mirror, the resulting image has a magnification m. Find an expression for the focal length of the mirror, f, in terms of d_o and m.

119\. ••• **A Slab of Glass** Give a symbolic expression for the sideways displacement d of a light ray passing through the slab of glass shown in **Figure 26–60**. The thickness of the glass is t, its index of refraction is n, and the angle of incidence is θ.



▲ **FIGURE 26–60** Problem 119

120. **•••** Referring to Figure 26–58, show that the internal reflection from the curved surface of the fiber is always total for any incident angle θ_i, provided the index of refraction of the fiber exceeds $\sqrt{2}$.

121. **•••** **Least Time** A beam of light propagates from point A in medium 1 to point B in medium 2, as shown in **Figure 26–61.** The index of refraction is different in these two media; therefore, the light follows a refracted path that obeys Snell's law. **(a)** Calculate the time required for light to travel from A to B along the refracted path. **(b)** Compare the time found in part (a) with the time it takes for light to travel from A to B along a straight-line path. (Note that the time on the straight-line path is longer than the time on the refracted path. In general, the shortest time between two points in different media is along the path given by Snell's law.)

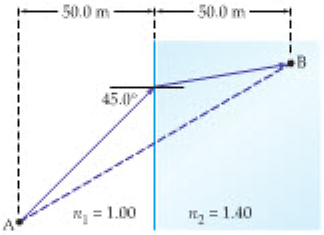


FIGURE 26–61 Problem 121

122. **•••** The ray of light shown in **Figure 26–62** passes from medium 1 to medium 2 to medium 3. The index of refraction in medium 1 is n_1, in medium 2 it is $n_2 > n_1$, and in medium 3 it is $n_3 > n_2$. Show that medium 2 can be ignored when calculating the angle of refraction in medium 3; that is, show that $n_1 \sin\theta_1 = n_3 \sin\theta_3$.

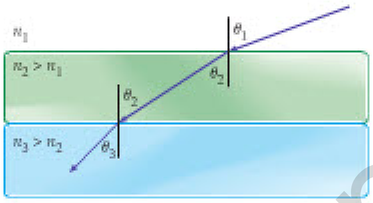


FIGURE 26–62 Problem 122

123. **•••** **IP** A beam of light enters the sloping side of a 45°-90°-45° glass prism with an index of refraction $n = 1.66$. The situation is similar to that shown in Figure 26–51, except that the angle of incidence of the incoming beam can be varied. **(a)** Find the angle of incidence for which the reflection on the vertical side of the prism exactly satisfies the condition for total internal reflection. **(b)** If the angle of incidence is increased, is the reflection at the vertical surface still total? Explain. **(c)** What is the minimum value of n such that a horizontal beam like that in Figure 26–51 undergoes total internal reflection at the vertical side of the prism?

PASSAGE PROBLEMS

The Focal Length of a Lens

A number of factors play a role in determining the focal length of a lens. First and foremost is the shape of the lens. As a general rule, a lens that is thicker in the middle will converge light, a lens that is thinner in the middle will diverge light.

Another important factor is the index of refraction of the lens material, n_{lens}. For example, imagine comparing two lenses with identical shapes but made of different materials. The lens with the larger index of refraction bends light more, bringing it to a focus in a shorter distance. As a result, a larger index of refraction implies a smaller focal length. In fact, the focal length of a lens surrounded by air ($n = 1$) is given by the **lens maker's formula:**

$$\frac{1}{f_{\text{in air}}} = (n_{\text{lens}} - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

In this expression R_1 and R_2 are the radii of curvature of the front and back surfaces of the lens, respectively. For given values of R_1 and R_2—that is, for a given shape—the focal length of the lens becomes smaller as the index of refraction increases.

A lens is not always surrounded by air, however. More generally, the fluid in which the lens is immersed may have an index of refraction given by n_{fluid}. In this case, the focal length is given by

$$\frac{1}{f_{\text{in fluid}}} = \left(\frac{n_{\text{lens}} - n_{\text{fluid}}}{n_{\text{fluid}}}\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

It follows, then, that the focal lengths of a lens surrounded by air or by a general fluid are related by

$$f_{\text{in fluid}} = \left[\frac{(n_{\text{lens}} - 1)n_{\text{fluid}}}{n_{\text{lens}} - n_{\text{fluid}}}\right] f_{\text{in air}}$$

This relation shows that the surrounding fluid can change the magnitude of the focal length, or even cause it to become infinite. The fluid can also change the sign of the focal length, which determines whether the lens is diverging or converging.

124. **•** A converging lens with a focal length in air of $f = +5.25$ cm is made from ice. What is the focal length of this lens if it is immersed in benzene? (Refer to Table 26–2.)

A. −20.7 cm　**B.** −18.1 cm

C. −12.8 cm　**D.** −11.2 cm

125. **•** A diverging lens with $f = -12.5$ cm is made from ice. What is the focal length of this lens if it is immersed in ethyl alcohol? (Refer to Table 26–2.)

A. 102 cm　**B.** 105 cm

C. 118 cm　**D.** 122 cm

126. **•** Calculate the focal length of a lens in water, given that the index of refraction of the lens is $n_{\text{lens}} = 1.52$ and its focal length in air is 25.0 cm. (Refer to Table 26–2.)

A. 57.8 cm　**B.** 66.0 cm

C. 91.0 cm　**D.** 104 cm

127. **•** Suppose a lens is made from fused quartz (glass), and that its focal length in air is −7.75 cm. What is the focal length of this lens if it is immersed in benzene? (Refer to Table 26–2.)

A. −130 cm　**B.** 134 cm

C. 141 cm　**D.** −145 cm

INTERACTIVE PROBLEMS

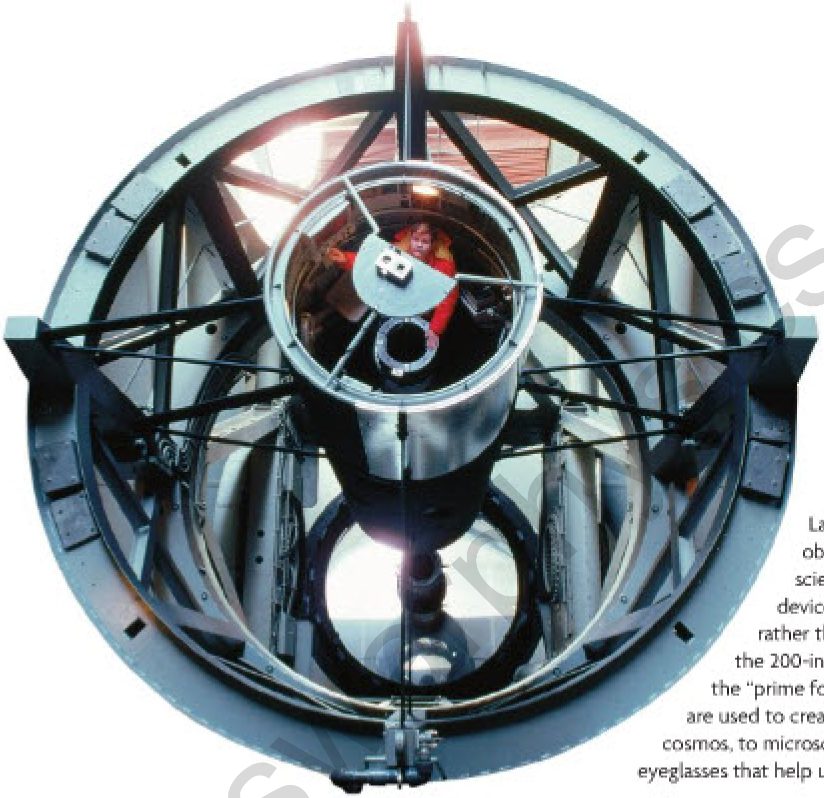
128. **••** **Referring to Example 26–3** Suppose the radius of curvature of the mirror is 5.0 cm. **(a)** Find the object distance that gives an upright image with a magnification of 1.5. **(b)** Find the object distance that gives an inverted image with a magnification of −1.5.

129. **••** **IP** **Referring to Example 26–3** An object is 4.5 cm in front of the mirror. **(a)** What radius of curvature must the mirror have if the image is to be 2.2 cm in front of the mirror? **(b)** What is the magnification of the image? **(c)** If the object is moved closer to the mirror, does the magnification of the image increase in magnitude, decrease in magnitude, or stay the same?

130. **••** **Referring to Example 26–7** **(a)** What object distance is required to give an image with a magnification of +2.0? Assume that the focal length of the lens is +5.0 cm. **(b)** What is the location of the image in this case?

131. **••** **IP** **Referring to Example 26–7** Suppose the convex lens is replaced with a concave lens with a focal length of −5.0 cm. **(a)** Where must the object be placed to form an image with a magnification of 0.50? **(b)** What is the location of the image in this case? **(c)** If we now move the object closer to the lens, does the magnification of the image increase, decrease, or stay the same?

27 Optical Instruments



Large modern astronomical telescopes are seldom used for visual observations. More often, the light they capture is directed to a scientific instrument, such as a camera, spectrograph, or semiconducting device. When a human observer is involved, it's often within the telescope rather than behind it. The astronomer in this photo is actually perched above the 200-inch mirror of the great Hale reflecting telescope on Mount Palomar, at the "prime focus" of the telescope. This chapter explores how mirrors and lenses are used to create a variety of optical devices, from telescopes that let us scan the cosmos, to microscopes that give us access to the world of the very small—and even to eyeglasses that help us read a newspaper.

Human vision can be aided in a number of ways by the practical application of optics. For example, a pair of glasses or contact lenses can correct a person's faulty eyesight to produce normal vision. Optics can also extend the abilities of human sight by magnifying very small objects to a visible size or by making distant objects seem near at hand. In this chapter, after discussing the optics of a normal eye and the closely related behavior of a camera, we consider a variety of optical instruments designed to either correct or extend our abilities to see the natural world.

27–1 The Human Eye and the Camera 948

27–2 Lenses in Combination and Corrective Optics 951

27–3 The Magnifying Glass 957

27–4 The Compound Microscope 961

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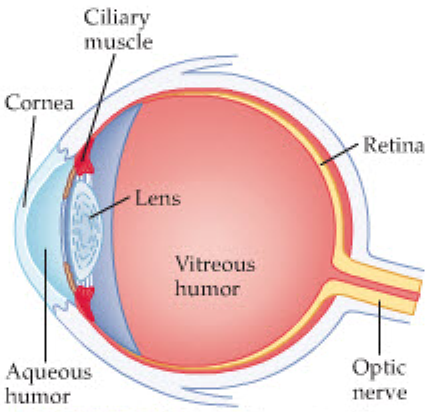


▲ **FIGURE 27–1 Basic elements of the human eye**

Light enters the eye through the cornea and the lens. It is focused onto the retina by the ciliary muscles, which change the shape of the lens.

27–1 The Human Eye and the Camera

The Human Eye

The eye is a marvelously sensitive and versatile optical instrument, allowing us to observe objects as distant as the stars or as close as the book in our hands. Perhaps more amazing is the fact that the eye can accomplish all this even though its basic structure is that of a spherical bag of water 2.5 cm in diameter. The slight differences that set the eye apart from a bag of water make all the difference in terms of optical performance, however.

The fundamental elements of an eye are illustrated in Figure 27–1. Basically, light enters the eye through the transparent outer coating of the eye, the *cornea*, and then passes through the *aqueous humor*, the adjustable *lens*, and the jellylike *vitreous humor* before reaching the light-sensitive *retina* at the back of the eye, as illustrated in Figure 27–2. The retina is covered with millions of small structures known as *rods* and *cones*, which, when stimulated by light, send electrical impulses along the *optic nerve* to the brain. How the nerve impulses are processed, so that we interpret the upside-down image on the retina as a right-side-up object, is another story altogether; here we concentrate on the optical properties of the eye.

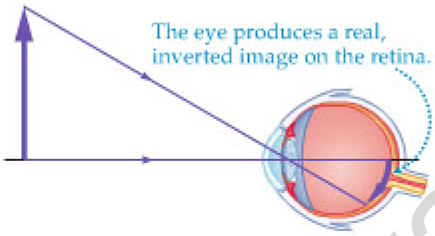
To begin, we note that most of the refraction needed to produce an image occurs at the cornea, as light first enters the eye. The reason is that the difference in index of refraction is greater at the air–cornea interface than at any interface within the eye. Specifically, the index of refraction of air is $n = 1.00$, whereas that of the cornea is about $n = 1.38$, just slightly greater than the index of refraction of water ($n = 1.33$). When light passes from the cornea to the aqueous humor, the index of refraction changes from $n = 1.38$ to $n = 1.33$. Next, light encounters the lens ($n = 1.40$) and then the vitreous humor ($n = 1.34$) before arriving at the retina.

In the end, the lens accounts for only about a quarter of the total refraction produced by the eye—but it is a crucial contribution nonetheless. By altering the shape of the lens with the *ciliary muscles*, we are able to change the precise amount of refraction the lens produces, which, in turn, changes its focal length. Specifically, when we view a distant object, our ciliary muscles are *relaxed*, as shown in Figure 27–3 (a), allowing the lens to be relatively flat. As a result, it causes little refraction and its focal length is at its greatest. When we view a nearby object, the lens must shorten its focal length and cause more refraction, as shown in Figure 27–3 (b). Thus, the ciliary muscles *tense* to give the lens a greater curvature. The process of changing the shape of the lens, and hence adjusting its focal length, is referred to as **accommodation.** Producing the proper accommodation is no easy feat for a newborn but is automatic for an adult.

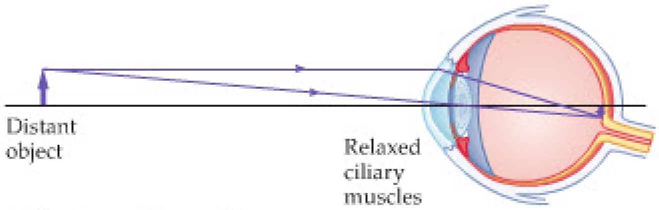
The fact that the ciliary muscles must be tensed to focus on nearby objects means that our eyes can "tire" from muscular strain. That is why it is beneficial to pause occasionally from reading and to look off into the distance. Viewing distant

REAL-WORLD PHYSICS: BIO

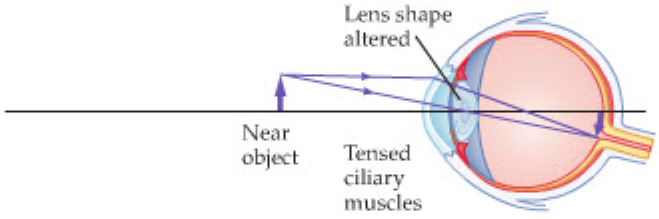
Optical properties of the human eye



▲ **FIGURE 27–2 Image production in the eye**



(a) Viewing a distant object



(b) Viewing a near object

▶ **FIGURE 27–3 Accommodation in the human eye**

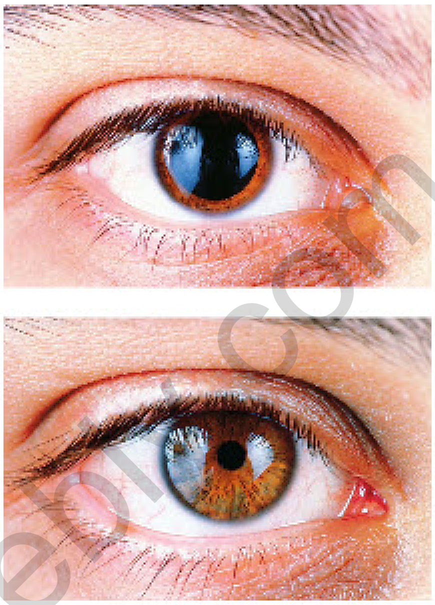
(a) When the eye is viewing a distant object, the ciliary muscles are relaxed and the focal length of the lens is at its greatest. **(b)** When the eye is focusing on a near object, the ciliary muscles are tensed, changing the shape and reducing the focal length of the lens.

objects allows the ciliary muscles to relax, thus reducing the strain on our eyes to a minimum.

The lens can be distorted only so much, however; hence, there is a limit to how close the eye can focus. The shortest distance at which a sharp focus can be obtained is the **near point**—anything closer will appear fuzzy no matter how hard we try to focus on it. For young people the near-point distance, N, is typically about 25 cm, but it increases with age. Persons 40 years of age may experience a near point that is 40 cm from the eye, and in later years the near point may move to 500 cm or more. The extension of the near point to greater distances with age is referred to as *presbyopia* (or "short arm" syndrome) and is due to the lens becoming less flexible. Thus, as one ages it is not uncommon to have to move a piece of paper away from the eyes in order to focus, and eventually reading glasses may be necessary.

At the other end of the scale, the **far point** is the greatest distance an object can be from the eye and still be in focus. Since we can focus on the Moon and stars, it is clear that the normal far point is essentially infinity.

Finally, the amount of light that reaches the retina is controlled by a colored diaphragm called the *iris*. As the iris expands or contracts it adjusts the size of the *pupil*, the opening through which light enters the eye. In bright light the pupil closes down to about 1 mm in diameter. On the darkest nights, the dark-adapted pupil can open up to a diameter of about 7.0 mm.



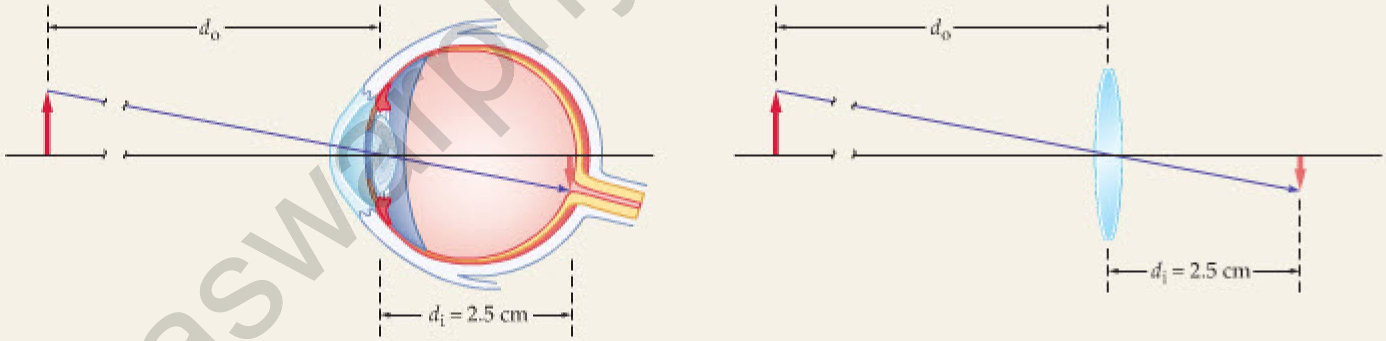
▲ The pigmented iris of the human eye responds automatically to changing levels of illumination, dilating the pupil in dim light and contracting it in bright light.

EXAMPLE 27–1 JOURNEY TO NEAR POINT

The near-point distance of a given eye is $N = 25$ cm. Treating the eye as if it were a single thin lens a distance 2.5 cm from the retina, find the focal length of the lens when it is focused on an object **(a)** at the near point and **(b)** at infinity. (Typical values for the effective lens–retina distance range from 1.7 cm to 2.5 cm.)

PICTURE THE PROBLEM

The eye, and the simplified thin-lens equivalent, are shown in the sketch. Note that the horizontal axis is broken in order to bring the object and eye into the same sketch. The object and image distances are indicated as well.



STRATEGY

The focal length can be found using the thin-lens equation, $1/d_o + 1/d_i = 1/f$. The image distance is $d_i = 2.5$ cm for both (a) and (b). For part (a) the object distance is $d_o = 25$ cm; for part (b) the object distance is $d_o = \infty$.

SOLUTION

Part (a)

1. Substitute $d_o = 25$ cm and $d_i = 2.5$ cm into the thin-lens equation and solve for f:

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{25\text{ cm}} + \frac{1}{2.5\text{ cm}} = 0.44\text{ cm}^{-1}$$

$$f = \frac{1}{0.44\text{ cm}^{-1}} = 2.3\text{ cm}$$

Part (b)

2. Substitute $d_o = \infty$ and $d_i = 2.5$ cm into the thin-lens equation and solve for f:

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{\infty} + \frac{1}{2.5\text{ cm}} = \frac{1}{2.5\text{ cm}}$$

$$f = 2.5\text{ cm}$$

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INSIGHT

Note that the effective focal length of the eye changes by only about 2 mm in changing focus from the near point to infinity. Thus, if the shape of the eye is changed even slightly—so that the distance from the lens to the retina is increased or decreased by a couple millimeters—the eye will no longer be able to function properly. We return to this point in the next section when we discuss near- and farsightedness.

PRACTICE PROBLEM

At what object distance is the effective focal length of the eye equal to 2.4 cm? [**Answer:** $d_o = 60$ cm]

Some related homework problems: Problem 4, Problem 5

Perhaps you have noticed that when you are looking at a light-colored background, or a clear sky, one or more "spots" may float across your field of vision. Many people have these "floaters." In fact, some people are occasionally fooled by one of these spots into thinking a fly is buzzing about their head. For this reason, these floaters are called *muscae volitantes*, which means, literally, "flying flies." Muscae volitantes are caused by cells, cell fragments, or other small impurities suspended either in the vitreous humor of the eye or in the lens itself. These are usually harmless but can be symptoms of a detached retina.

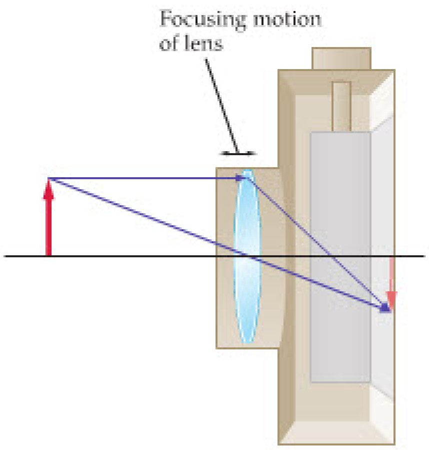


FIGURE 27–4 Basic elements of a camera

A camera forms a real, inverted image on photographic film or an electronic sensor, and is focused by moving the lens back and forth. Unlike the adjustable shape of the human eye, the shape of the camera lens does not change.

The Camera

A simple camera, such as the one illustrated in **Figure 27–4**, operates in much the same way as the eye. In particular, the lens of the camera forms a real, inverted image on a light-sensitive material—which in this case is either photographic film or, more commonly these days, the charge-coupled device (CCD) in a digital camera. The focusing mechanism is different, however. To focus a digital camera at different distances the lens is moved either toward or away from the CCD. Thus, the eye focuses by changing the shape of a stationary lens; the camera focuses by moving a lens of fixed shape.

ACTIVE EXAMPLE 27–1 FIND THE DISPLACEMENT OF THE LENS

A simple camera uses a thin lens with a focal length of 50.0 mm. How far, and in what direction, must the lens be moved to change the focus of the camera from a person 20.0 m away to a person only 3.00 m away?

SOLUTION *(Test your understanding by performing the calculations indicated in each step.)*

1. Calculate the image distance for an object distance of 20.0 m: $d_{i1} = 5.01$ cm
2. Calculate the image distance for an object distance of 3.00 m: $d_{i2} = 5.08$ cm
3. Find the difference in image distance: 0.07 cm = 0.7 mm

INSIGHT

Since the image distance is greater for the person at 3.00 m, it follows that the lens must be moved *away* from the film by 0.7 mm to change the focus the desired amount. Note how little displacement is required.

YOUR TURN

Suppose the lens is moved an additional 0.7 mm away from the film. At what distance is the camera focused now?

(Answers to **Your Turn** *problems are given in the back of the book.)*

REAL-WORLD PHYSICS

Speed and aperture settings on a camera

The aperture of a camera is analogous to the pupil of an eye, and like the pupil, its size can be adjusted—the greater the size, the more light that is available to make an image. Photographers often characterize the size of the aperture with a dimensionless quantity called the ***f*-number,** which is defined as follows:

$$f\text{-number} = \frac{\text{focal length}}{\text{diameter of aperture}} = \frac{f}{D} \tag{27–1}$$

Notice that the larger the diameter of the aperture, the smaller the *f*-number; in fact, $D = f/(f\text{-number})$.

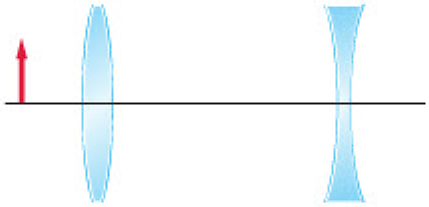
Professional-grade cameras have aperture settings indicated by a sequence of *f*-numbers such as the following:

$$2, \quad 2.8, \quad 4, \quad 5.6, \quad 8, \quad 11, \quad 16$$

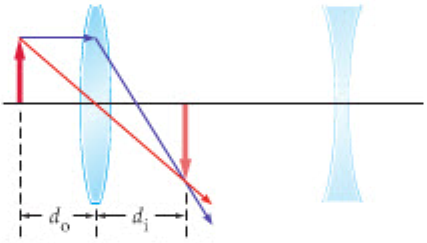
For example, a camera lens with a focal length of 50.0 mm and an aperture setting of 4 has an aperture diameter of $D = (50.0\text{ mm})/4 = 12.5\text{ mm}$. This is often referred to by photographers as an $f/4$ setting for the aperture. Similarly, turning the aperture ring to the $f/2$ setting opens the aperture to a diameter of $(50.0\text{ mm})/2 = 25.0\text{ mm}$. Since the aperture is a circular opening, the area through which light enters the camera, $A = \pi D^2/4$, varies as the square of the aperture diameter D and thus inversely as the square of the *f*-number.

The amount of light that falls on the photographic film or CCD in a camera is also controlled by the shutter speed. A shutter speed of 1/500, for example, means that the shutter is open for only 1/500 of a second—very effective for "freezing" the motion of a high-speed object. Changing the shutter speed to 1/250 doubles the time the shutter is open and, hence, doubles the light received by the film. Typical camera shutter speeds are 1/1000, 1/500, 1/250, 1/125, and so on. (On professional-grade cameras, these speeds are indicated on a dial as 1000, 500, 250, 125 to save space.)

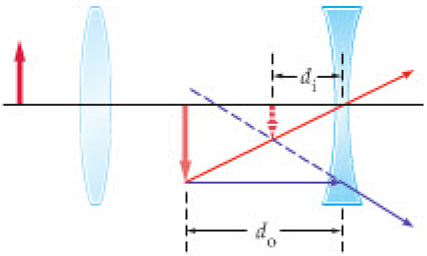
Suppose, for example, that a photograph receives the proper amount of light when the shutter speed is 1/500 and the aperture *f*-number is 4. If the photographer decides to take a second shot with a shutter speed of 1/250, what *f*-number is required to maintain the correct exposure? Since the slower shutter speed doubles the light entering the camera, the area of the aperture must be halved to compensate. To halve the area, the diameter must be *reduced* by a factor of $\sqrt{2} \approx 1.4$, which means the *f*-number must be *increased* by a factor of 1.4. Hence, the photographer must change the aperture setting to $4 \times (1.4) = 5.6$. Note that each of the *f*-numbers listed earlier is larger than the previous one by a factor of roughly 1.4, leading to a factor of 2 difference in the light received by the film.



(a) Original object



(b) Image from first lens



(c) Image from second lens

▲ **FIGURE 27–5 A two-lens system**

In this system, a convex and a concave lens are separated by 50.0 cm. **(a)** An object is placed 20.0 cm to the left of the convex lens, whose focal length is 10.0 cm. **(b)** The image formed by the convex lens is 20.0 cm to its right. This image is the object for the concave lens. **(c)** The object for this lens is 30.0 cm to its left. Because the focal length of this lens is −12.5 cm, it forms an image 8.82 cm to its left. This is the final image of the system.

27–2 Lenses in Combination and Corrective Optics

Whereas a normal eye can provide sharp images for objects over a wide range of distances, some eyes have difficulty focusing on distant objects, and others are unable to focus as close as the normal near point. Problems such as these can be corrected with glasses or contact lenses used in combination with the eye's lens. In general, a combination of lenses can have beneficial properties not possible with a single lens, as we shall see throughout the rest of this chapter. First, in this section, we discuss how to analyze a system with more than one lens; we then apply these results to correcting near- and farsightedness.

The basic operating principle for a system consisting of more than one lens is the following:

> The *image* produced by one lens acts as the *object* for the next lens. This is true regardless of whether the image produced by the first lens is real or virtual, or whether it is in front of or behind the second lens.

This principle finds many applications, since most optical instruments—like cameras, microscopes, and telescopes—use a number of lenses to produce the desired results.

As an example of a lens system, consider the two lenses shown in Figure 27–5 (a). An object is 20.0 cm to the left of the convex lens, which is 50.0 cm from a concave lens to its right. Given that the focal lengths of the convex and concave lenses are

10.0 cm and −12.5 cm, respectively, we would like to find the location and orientation of the image produced by the two lenses acting together.

The first step is illustrated in **Figure 27–5 (b)**, where we determine the image formed by the convex lens using a ray diagram. The precise location of "image 1" is given by the thin-lens equation:

$$\frac{1}{d_o}+\frac{1}{d_i}=\frac{1}{f} \quad \text{or} \quad \frac{1}{20.0\text{ cm}}+\frac{1}{d_i}=\frac{1}{10.0\text{ cm}}$$

$$d_i = 20.0\text{ cm}$$

Note that image 1 is inverted and 20.0 cm to the right of the convex lens.

The next step is to note that image 1 is 50.0 cm − 20.0 cm = 30.0 cm to the left of the concave lens. Considering image 1 to be the object for the concave lens, we obtain the ray diagram shown in **Figure 27–5 (c)**. The thin-lens equation, with $d_o = 30.0$ cm and $f = -12.5$ cm, yields the following image distance:

$$\frac{1}{d_o}+\frac{1}{d_i}=\frac{1}{f} \quad \text{or} \quad \frac{1}{30.0\text{ cm}}+\frac{1}{d_i}=\frac{1}{-12.5\text{ cm}}$$

$$d_i = -8.82\text{ cm}$$

Thus the final image of the two-lens system is 8.82 cm to the left of the concave lens.

PROBLEM-SOLVING NOTE

Finding the Image for a Multilens System

To find the final image of a multilens system, consider the lenses one at a time. In particular, find the image for the first lens, then use that image as the object for the next lens, and so on.

The orientation and size of the final image can be found as follows:

> The total magnification produced by a lens system is equal to the *product* of the magnifications produced by each lens individually.

Using Equation 26–18, $m = -d_i/d_o$, we find that the first lens produces magnification $m_1 = -(20.0\text{ cm})/(20.0\text{ cm}) = -1$. Similarly, the second lens causes a magnification of $m_2 = -(-8.82\text{ cm})/(30.0\text{ cm}) = 0.294$. The total magnification of the system, then, is $m = m_1 m_2 = -0.294$, showing that the final image is inverted and reduced in size by a factor of 0.294 compared with the original object. This value is in agreement with the results shown in the ray diagrams of Figure 27–5.

The following Active Example considers the effect of reversing the order of the two lenses.

ACTIVE EXAMPLE 27–2 FIND THE FINAL IMAGE

Find the location and orientation of the final image produced by the system shown in Figure 27–5 if the positions of the two lenses are reversed.

SOLUTION *(Test your understanding by performing the calculations indicated in each step.)*

1. Use the thin-lens equation to find the image distance for the concave lens, with $d_{o1} = 20.0$ cm: $d_{i1} = -7.69$ cm
2. Find the object distance for the convex lens: $d_{o2} = 57.7$ cm
3. Use this object distance and the thin-lens equation to find the location of the final image: $d_{i2} = 12.1$ cm

INSIGHT

The final image is 12.1 cm to the right of the convex lens. The magnification produced by the first lens is $m_1 = -d_{i1}/d_{o1} = 0.385$, and that produced by the second lens is $m_2 = -d_{i2}/d_{o2} = -0.210$. Hence, the total magnification is $m = m_1 m_2 = -0.0809$, showing that the final image is reduced in size by a factor of 0.0809 and inverted. Notice how different the results are when we simply reverse the order of the lenses—just like looking through the wrong end of a pair of binoculars.

YOUR TURN

Consider again the system shown in Figure 27–5. If we replace the concave lens with a concave mirror whose focal length is $f = 12.5$ cm, what are the location and magnification of the final image?

(Answers to **Your Turn** *problems are given in the back of the book.)*

We now show how a two-lens system (the eye plus an external lens) can correct abnormal vision.

Nearsightedness

When a person with normal vision relaxes the ciliary muscles of the eye, an object at infinity is in focus. In a nearsighted (myopic) person, however, a totally relaxed eye focuses only out to a finite distance from the eye—the far point. Thus a person with this condition is said to be nearsighted because objects near the eye can be focused, whereas objects beyond the far point are fuzzy.

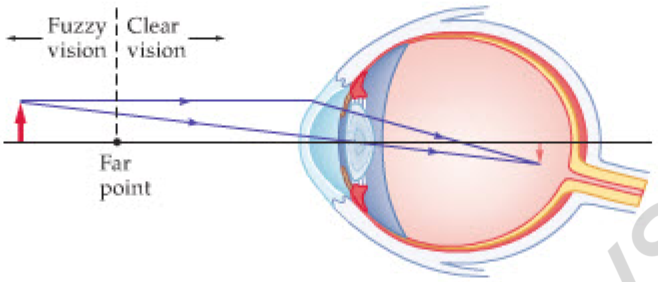
The problem in this situation is that the eye converges the light coming into it in too short a distance—in other words, the focal length of the eye is less than the distance from the lens to the retina. This condition is illustrated in **Figure 27–6**, where we see that an object at infinity forms an image in front of the retina, because of the elongation of the eye. The effect need not be large; as we saw in the previous section, an elongation of only a millimeter or two is enough to cause a problem.

To correct this condition, we need to "undo" some of the excess convergence produced by the eye, so that images again fall on the retina. This can be done by placing a *diverging* lens in front of the eye. Specifically, consider an object at infinity—which would ordinarily appear blurry to a nearsighted person. If a concave lens with the proper focal length produces a virtual image of this object at the nearsighted person's far point, as in **Figure 27–7**, the person's relaxed eye can now focus on the object. We consider this situation in the next Example.

PROBLEM-SOLVING NOTE

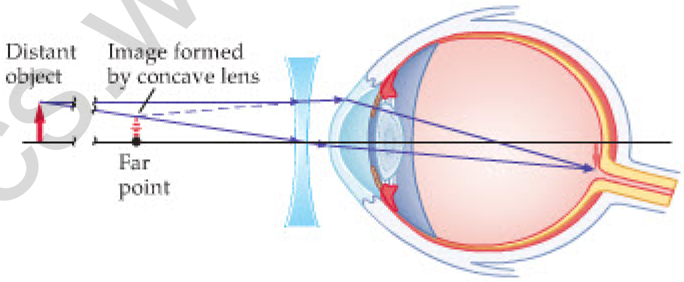
Correcting Nearsightedness

To correct for nearsightedness, one must use a lens that produces an image at the person's far-point distance when the object is at infinity. Note that the far point is closer to the lens in a pair of glasses than it is to the eye, since the glasses are a finite distance in front of the eyes.



▲ **FIGURE 27–6 Eye shape and nearsightedness**

An eye that is elongated can cause nearsightedness. In this case, an object at infinity comes to a focus in front of the retina.



▲ **FIGURE 27–7 Correcting nearsightedness**

A diverging lens in front of the eye can correct for nearsightedness. The concave lens focuses light from an object beyond the far point to produce an image that is at the far point. The eye can now focus on the image of the object.

EXAMPLE 27–2 EXTENDED VISION

REAL-WORLD PHYSICS: BIO

A nearsighted person has a far point that is 323 cm from her eye. If the lens in a pair of glasses is 2.00 cm in front of this person's eye, what focal length must it have to allow her to focus on distant objects?

PICTURE THE PROBLEM

In our sketch, we show an object at infinity and the corresponding image produced by the concave lens. As is usual with a concave lens, the image is upright, reduced, and on the same side of the lens as the object. In addition, the image is placed at the person's far point, where the eye can focus on it.

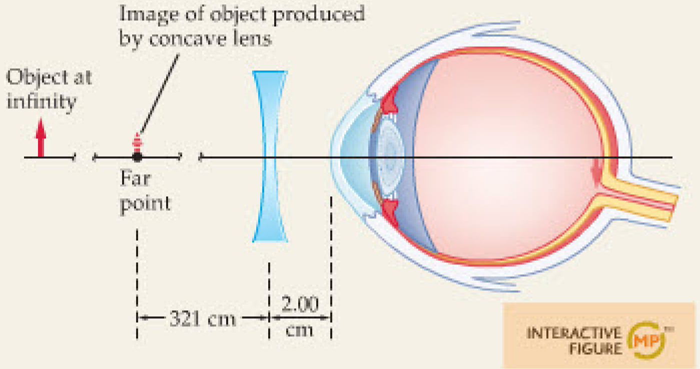
STRATEGY

We can find the focal length of the lens using the thin-lens equation, $1/d_o + 1/d_i = 1/f$.

In this case, $d_o = \infty$, since the object is infinitely far away.

As for the image, it must be 323 cm from the eye, which is 323 cm − 2.00 cm = 321 cm in front of the lens. Thus, the image distance is $d_i = -321$ cm, where the minus sign is required because the image is on the same side of the lens as the object.

With these values for d_o and d_i, it is straightforward to find the focal length, f.



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SOLUTION

1. Substitute $d_o = \infty$ and $d_i = -321$ cm in the thin-lens equation: $\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f} = \frac{1}{\infty} + \frac{1}{-321\text{ cm}}$
2. Solve for the focal length, f: $f = -321$ cm

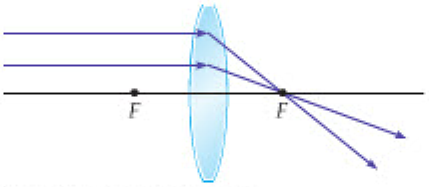
INSIGHT

With a lens of this focal length, the person can focus on distant objects with a relaxed eye. In addition, note that the focal length is equal to the image distance, $f = d_i$. This is always the case when the object distance is infinite.

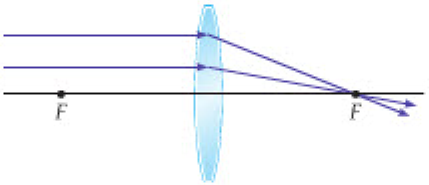
PRACTICE PROBLEM

If these glasses are used to view an object 525 cm from the eye, how far from the eye is the image produced by the concave lens? [**Answer:** The image is 201 cm in front of the eye.]

Some related homework problems: Problem 31, Problem 33, Problem 34



(a) Large refractive power



(b) Small refractive power

FIGURE 27–8 Refractive power

Light is bent (refracted) more by a lens with a short focal length than one with a long focal length. Therefore, lens **(a)** has a greater refractive power than lens **(b)**.

The ability of a lens to refract light—its **refractive power**—is related to its focal length. For example, the shorter the focal length, the more strongly a lens refracts light, as indicated in **Figure 27–8**. Thus refractive power depends inversely on the focal length. By definition, then, we say that the refractive power of a lens is $1/f$, where f is measured in meters:

Refractive Power

$$\text{refractive power} = \frac{1}{f} \qquad 27\text{–}2$$

SI unit: diopter $= \text{m}^{-1}$

Lenses are typically characterized by optometrists in terms of diopters rather than in terms of focal length.

As an example of the meaning of diopters, a lens with a refractive power of 10.0 diopters has a focal length of $1/(10.0\text{ m}^{-1}) = 10.0$ cm (a converging lens), and a lens with a refractive power of -10.0 diopters has a focal length of -10.0 cm (a diverging lens). In Example 27–2, the lens required to correct nearsightedness had a refractive power of $1/(-3.21\text{ m}) = -0.312$ diopter.

ACTIVE EXAMPLE 27–3 FIND THE REFRACTIVE POWER

A person has a far point that is 5.50 m from his eyes. If this person is to wear glasses that are 2.00 cm from his eyes, what refractive power, in diopters, must his lenses have?

SOLUTION *(Test your understanding by performing the calculations indicated in each step.)*

1. Identify the object distance: $d_o = \infty$
2. Identify the image distance: $d_i = -548$ cm
3. Use the thin-lens equation to calculate the focal length of the lenses: $f = -548$ cm
4. Convert f to meters and invert to find the refractive power: refractive power $= -0.182$ diopter

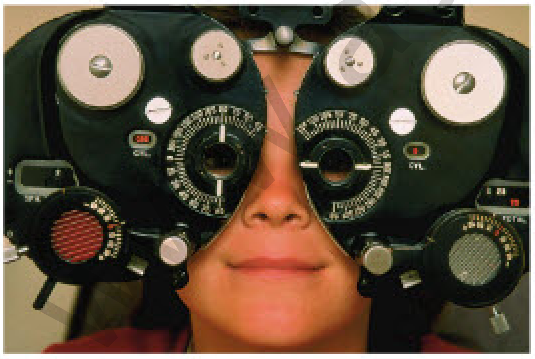
INSIGHT

The fact that this person's far point is farther away (closer to infinity) than the far point in Example 27–2 means that the lenses do not have to be as "strong" to correct the vision. As a result, the magnitude of the refractive power in this case (0.182 diopter) is less than the corresponding magnitude (0.312 diopter) in Example 27–2.

YOUR TURN

A person wears glasses with a refractive power of -0.500 diopter. If the glasses are 2.00 cm in front of the eyes, what is the distance from the eyes to the far point?

(Answers to **Your Turn** *problems are given in the back of the book.)*



In order to prescribe the right corrective lenses, it is necessary to measure the refractive properties of the patient's eyes. This device, known as a *phoropter*, allows the optometrist to see how lenses with various optical characteristics affect the patient's vision.

Today, in addition to glasses and contact lenses, a number of medical procedures are available to correct nearsightedness. Some of these procedures involve using laser beams to reshape the cornea of the eye. These techniques are discussed in Chapter 31, where we consider lasers in detail. Here we present two alternative procedures that change the shape of a cornea by mechanical means.

REAL-WORLD PHYSICS: BIO

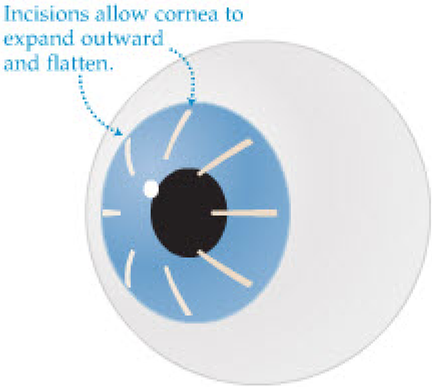
Intracorneal rings and Intacts

Perhaps the simplest such technique is the implantation of either an *intracorneal ring* or an *Intact* in the cornea of an eye. An Intact, in particular, consists of two small, clear crescents made of the same material used in contact lenses. These crescents are slipped into "tunnels" that are cut into the cornea. When the crescents are in place they tend to stretch the cornea outward and flatten its surface. This flattening increases the focal length of the eye and corrects for nearsightedness by −1.0 to −3.0 diopters. If necessary, the Intacts can be removed and replaced with others that cause more or less flattening of the cornea. Intracorneal rings are similar, except that they consist of a single ring rather than the two crescents used in Intacts.

REAL-WORLD PHYSICS: BIO

Radial keratotomy

A more complex method involves making radial incisions in the cornea with a highly precise diamond blade that cuts to a specified depth. This method is referred to as *radial keratotomy* or RK (**Figure 27–9**). The cuts allow the peripheral parts of the cornea to bulge outward, which, in turn, causes the central portion to flatten. As with an Intact, the flattening of the cornea corrects for nearsightedness.



▲ **FIGURE 27–9 Radial keratotomy**

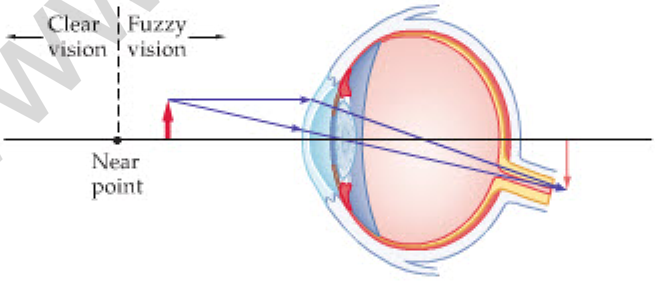
In radial keratotomy, a series of radial incisions is made around the periphery of the cornea. This allows the cornea to expand outward, resulting in a flattening of the cornea's central region. The reduced curvature of the cornea increases the focal length of the eye, allowing it to focus on distant objects.

Farsightedness

A person who is farsighted (hyperopic) can see clearly beyond a certain distance—the near point—but cannot focus on closer objects. Basically, the vision of a farsighted person differs from that of a person with normal vision by having a near point that is much farther from the eye than the usual 25 cm. As a result, a farsighted person is typically unable to read clearly, since a book would have to be held at such a great distance to come into focus.

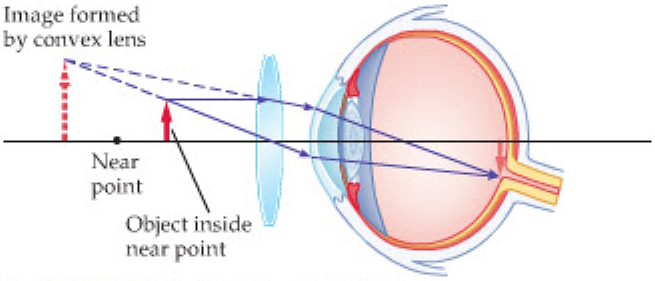
Farsightedness can be caused by an eyeball that is shorter than normal, as illustrated in **Figure 27–10**, or by a lens that becomes sufficiently stiff with age that it can no longer take on the shape required to focus on nearby objects. In such cases, rays from an object inside the near point are brought to a focus behind the retina. Thus the focal length of the farsighted eye is too large—stated another way, the farsighted eye does not converge the incoming light strongly enough to focus on the retina.

This problem can be corrected by "preconverging" the light—that is, by using a converging lens in front of the eye to add to its insufficient convergence. For example, suppose an object is inside a person's near point, as in **Figure 27–11**. If a converging lens placed in front of the eye can produce an image that is far away—that is, beyond the near point—the farsighted person can view the object with ease. Such a system is considered in the next Example.



▲ **FIGURE 27–10 Eye shape and farsightedness**

An eye that is shorter than normal can cause farsightedness. Note that an object inside the near point comes to a focus behind the retina.



▲ **FIGURE 27–11 Correcting farsightedness**

A converging lens in front of the eye can correct for farsightedness. The convex lens focuses light from an object inside the near point to produce an image that is beyond the near point. The eye can now focus on the image of the object.

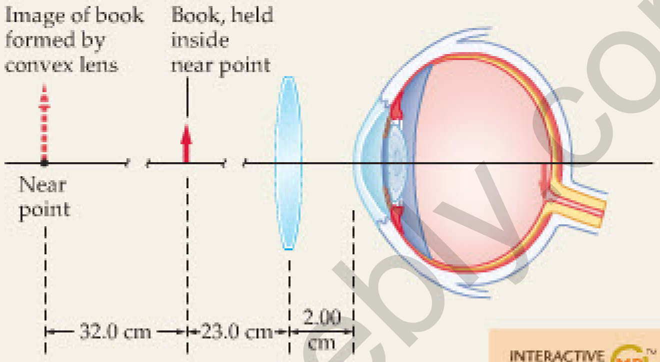
EXAMPLE 27–3 HIS VISION IS A FAR SIGHT BETTER

REAL-WORLD PHYSICS: BIO

A farsighted person wears glasses that enable him to read a book held at a distance of 25.0 cm from his eyes, even though his near-point distance is 57.0 cm. If his glasses are at a distance of 2.00 cm from his eyes, find the focal length and refractive power required of his lenses to place the image of the book at the near point.

PICTURE THE PROBLEM

The physical situation is shown in our sketch. Notice that we use a convex lens, and that the image produced by the lens is upright and farther from the eye than the book. These features are in agreement with the qualitative characteristics shown in Figure 27–11. In our case, however, the image of the book is exactly at the near point.



INTERACTIVE FIGURE

STRATEGY

The focal length of the lens can be found using the thin-lens equation. First, the object distance is $d_o = 23.0$ cm, taking into account the 2.00 cm between the glasses and the eye. Similarly, the desired image distance is $d_i = -55.0$ cm. Note that the minus sign is required on the image distance, since the image is on the same side of the lens as the object.

SOLUTION

1. Use the thin-lens equation to find the focal length, f:

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{23.0\text{ cm}} + \frac{1}{-55.0\text{ cm}} = 0.0253\text{ cm}^{-1}$$

$$f = \frac{1}{0.0253\text{ cm}^{-1}} = 39.5\text{ cm}$$

2. The refractive power is $1/f$, with f measured in meters:

$$\text{refractive power} = \frac{1}{f} = \frac{1}{0.395\text{ m}} = 2.53\text{ diopters}$$

INSIGHT

Notice that the book is between the lens and its focal point. As a result, the image is upright [see Figure 26–35 (b)], as desired for reading glasses. In addition, the image is virtual, as was also the case for the concave lens in Example 27–2. Thus, even though a "virtual image" can sound like one that isn't real or of practical importance, you are viewing a virtual image every time you look through a pair of glasses.

PRACTICE PROBLEM

A second person has a near-point distance that is greater than 57.0 cm. Is the refractive power of the lenses required for this person greater than or less than 2.53 diopters? To check your answer, calculate the refractive strength for a near-point distance of 67.0 cm. [**Answer:** The refractive power must be greater than 2.53 diopters. We find 2.81 diopters for a near-point distance of 67.0 cm.]

Some related homework problems: Problem 30, Problem 32

CONCEPTUAL CHECKPOINT 27–1 EYEGLASSES TO START A FIRE

Bill and Ted are on an excellent camping trip when they decide to start a fire by focusing sunlight with a pair of eyeglasses. If Bill is nearsighted and Ted is farsighted, should they use **(a)** Bill's glasses or **(b)** Ted's glasses?

REASONING AND DISCUSSION

To focus the parallel rays of light from the Sun to a point requires a converging lens. As we have seen, nearsightedness is corrected with a diverging lens; farsightedness is corrected with a converging lens. Ted's eyeglasses are converging; therefore, they should be the ones used to start a fire.

ANSWER

(b) Ted's eyeglasses would be the more excellent choice.

To consider the effect of a pair of contact lenses, we simply take into account that contacts are placed directly against the eye. This means that the eye–object distance is the same as the lens–object distance. We make use of this fact in the following Active Example.

ACTIVE EXAMPLE 27–4 CONTACT: FIND THE FOCAL LENGTH OF CONTACT LENSES

Find the focal length of a pair of contact lenses that will allow a person with a near-point distance of 145 cm to read a newspaper held 25.1 cm from the eyes.

SOLUTION *(Test your understanding by performing the calculations indicated in each step.)*

1. Identify the object distance: $d_o = 25.1$ cm
2. Identify the image distance: $d_i = -145$ cm
3. Use the thin-lens equation to calculate the focal length: $f = 30.4$ cm

INSIGHT

Note that the image distance is negative, since the image is on the same side of the lens as the object.

YOUR TURN

Suppose a second person has a near-point distance of 205 cm. Is the focal length of the second person's contacts greater than or less than the focal length of the first person's contacts? Find the focal length of the second person's contacts.

(Answers to **Your Turn** *problems are given in the back of the book.)*

PROBLEM-SOLVING NOTE

Correcting Farsightedness

To correct for farsightedness, one must use a lens that produces an image at the person's near-point distance when the object is closer than the near point. Note that the distance from the object to the lens in a pair of glasses is less than the distance from the object to the eye.

PROBLEM-SOLVING NOTE

Finding the Focal Length of Contact Lenses

Contact lenses are so named because they are in direct contact with the eye. Therefore, the object distance for a contact lens is the same as the distance from the eye to the object. Similarly, the near point is the same distance from both the contact lens and the eye.

REAL-WORLD PHYSICS: BIO

Keratometers

An important step in assuring that contact lenses fit a patient's eye properly is to measure the radius of curvature of the cornea. This is accomplished with a device known as a *keratometer*. To begin the procedure, a brightly lit object is brought near the eye. The light from the object reflects from the front surface of the cornea, just as light reflects from a convex, spherical mirror. In the next step of the procedure, the keratometer measures the magnification of the mirror image produced by the cornea. Finally, a straightforward application of the mirror equation determines the radius of curvature. A specific example of a keratometer in use is given in Problem 90.

Another eye condition that requires corrective optics is **astigmatism.** In most cases, astigmatism is due to an irregular curvature of the cornea, with a greater curvature in one direction than in another. For example, the curvature in the vertical plane may be greater than the curvature in the horizontal plane. As a result, if the eye is focused for light coming into it from one direction, it will be out of focus for light arriving from a different direction. Almost everyone has some degree of astigmatism, usually quite mild, but in serious cases it can cause distorted or blurry vision at all distances.

27–3 The Magnifying Glass

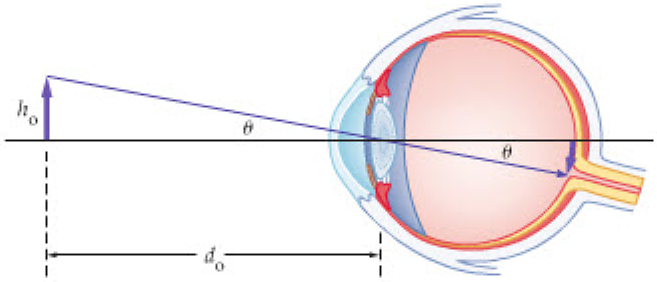
A **magnifying glass** is nothing more than a simple convex lens. Working together with the eye as part of a two-lens system, a *magnifier* can make objects appear to be many times larger than their actual size. As we shall see, the magnifying glass works by moving the near point closer to the eye—much as a converging lens corrects farsightedness. Basically, the magnifier allows an object to be viewed from a reduced distance, and this is what makes it appear larger.

To be more specific about the apparent size of an object, we consider the angle it subtends on the retina—after all, the more area an image takes up on the retina, the larger it will seem to us. For example, suppose an object of height h_o is a distance d_o from the eye, as in Figure 27–12 (a). The image formed by this object subtends an angle θ on the retina of the eye, as shown in the figure. If the angle is small, as is often the case, it can be approximated by the tangent of the angle, since $\tan\theta \approx \theta$ for small angles. Referring to the figure, we see that the angle θ is approximately

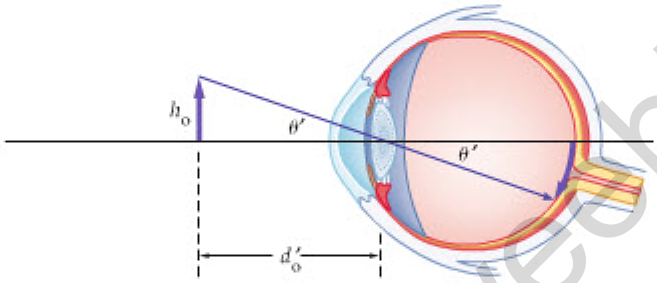
$$\theta \approx \frac{h_o}{d_o}$$

FIGURE 27–12 Angular size and distance

The angular size of an object depends on its distance from the eye, even though its height, h_o, remains the same.



(a) Small angular size



(b) Same object, larger angular size

If the object is now moved closer to the eye, to a distance d'_o, the angle subtended by the image is larger. Using the small-angle approximation again, and referring to **Figure 27–12 (b)**, we find that the new angle is

$$\theta' \approx \frac{h_o}{d'_o} > \theta$$

Thus, moving an object closer to the eye increases its apparent size, since its image covers a larger portion of the retina. There is a limit, however, to how close an object can come to the unaided eye and still be in focus—the near point. This is where a magnifier comes into play.

Suppose, then, that you would like to see as much detail as possible on a small object—a feather, perhaps, or a flower petal. With the unaided eye you can bring the object to the near point, a distance N from the eye, as in **Figure 27–13 (a)**. If the height of the object is h_o, the angular size of the object on the retina is approximately

$$\theta = \frac{h_o}{N}$$

Now, consider placing a convex lens of focal length $f < N$ just in front of the eye, as shown in **Figure 27–13 (b)**. If the object is brought to the focal point of this lens, its image will be infinitely far from the eye, where it can be viewed in focus with ease. As we can see from the figure, the angular size of the image is approximately

$$\theta' = \frac{h_o}{f}$$

Note that θ' is greater than θ—since f is less than N—and hence the object appears larger. The factor by which the object is enlarged, referred to as the **angular magnification,** M, is defined as follows:

Angular Magnification, *M*

$$M = \frac{\theta'}{\theta} \qquad 27\text{–}3$$

SI unit: dimensionless

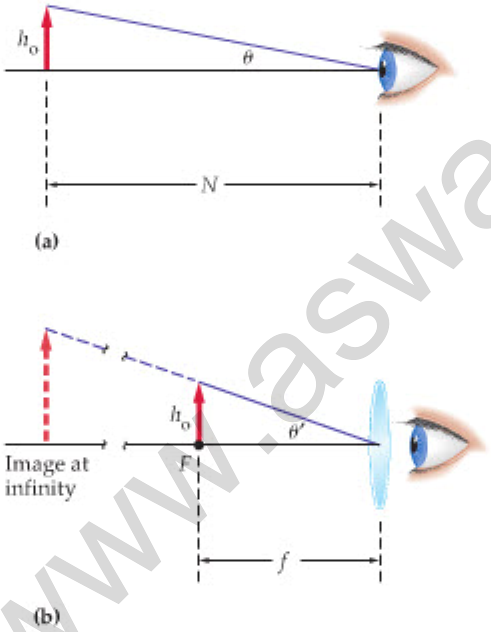


FIGURE 27–13 How a simple magnifier works

(a) An object viewed with the unaided eye at the near point of the eye subtends an angle $\theta \approx h_o/N$. **(b)** With a magnifier, the object can be viewed from the distance f, which is less than N. As a result, the angular size of the object is $\theta' \approx h_o/f$, which is greater than θ.

Using the angles θ and θ' obtained previously, we find

$$M = \frac{h_o/f}{h_o/N} = \frac{N}{f} \qquad 27\text{–}4$$

A typical situation is considered in the following Exercise.

EXERCISE 27–1

A person with a near-point distance of 30 cm examines a stamp with a magnifying glass. If the magnifying glass produces an angular magnification of 6, what is its focal length?

SOLUTION

Solving $M = N/f$ for the focal length we find

$$f = \frac{N}{M} = \frac{30\text{ cm}}{6} = 5\text{ cm}$$

Note that an object examined with a magnifier appears larger precisely because it is closer to the eye. In fact, the angular size of the object in Figure 27–13 (b) is h_o/f regardless of whether the magnifier is present or not. If the magnifier is not present, however, the object is out of focus, and then its increased angular size is of no practical value. The magnifier is beneficial in that it brings the object into focus at this close distance.

In addition, since the image produced by the magnifier is at infinity, the rays entering the eye are parallel. This means that a person can view the image with a completely relaxed eye. If the same object is viewed with the unaided eye at the near point, the ciliary muscles are fully tensed, causing eye strain. Thus, not only does the magnifier enlarge the object, it also makes it more comfortable to view.

Now, of course, it is also possible to produce an enlarged image with a convex lens without holding the lens close to the eye. In fact, if the lens is held at some distance from the eye, and just under a focal length from the object to be viewed, we see an enlarged image. There are two distinct disadvantages to holding a magnifier far from the eye, however. First, it is difficult to hold the lens motionless at a distance, as opposed to bracing it against the face. Second, when the lens is held at a distance, only a small portion of the object can be viewed at any given time. When the lens is held close up, however, the entire object can be viewed at once.

As we shall see in the next two sections, the magnifier plays an important role in both the microscope and the telescope.

▲ A magnifying glass produces an enlarged image when held near an object. The image is upright and virtual.

CONCEPTUAL CHECKPOINT 27–2 USING A MAGNIFIER

Person 1, with a near-point distance of 25 cm, and person 2, with a near-point distance of 50 cm, both use the same magnifying glass. Does **(a)** person 1 or **(b)** person 2 benefit more from using the magnifier?

REASONING AND DISCUSSION

The person with the greater near-point distance cannot see an object as closely with the unaided eye as can the person with the smaller near-point distance. With the magnifier, however, both people can view an object from the same close-in distance. Hence, the person with the larger near-point distance benefits more from the magnifier.

ANSWER

(b) Person 2, with the 50-cm near-point distance, benefits more.

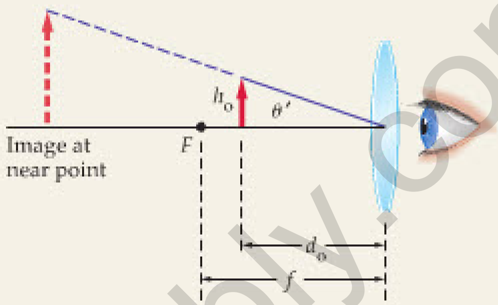
It is possible to obtain a magnification that is slightly greater than the value N/f given in Equation 27–4. After all, this magnification is for an image formed at infinity; if the image were closer to the eye, it would appear larger. The closest the image can be—and still be in focus—is the near point. In the next Example we compare the magnifications that result when the image is at infinity and at the near point.

EXAMPLE 27–4 COMPARING MAGNIFICATIONS

A biologist with a near-point distance of $N = 26$ cm examines an insect wing through a magnifying glass whose focal length is 4.3 cm. Find the angular magnification when the image produced by the magnifier is **(a)** at infinity and **(b)** at the near point.

PICTURE THE PROBLEM

Our sketch shows the situation in which the image formed by the magnifying glass is at the near point. To produce an image at this point, the object must be closer to the magnifier than its focal point. Therefore, the object appears larger in this case than it does in the case where the object is at the focal length and the image is at infinity.



STRATEGY

a. The magnification when the image is at infinity is $M = N/f$, as given in Equation 27–4.

b. The angular size of the object in this case is $\theta' \approx h_o/d_o$, where d_o is the object distance that places the image at the near point. Thus the magnification is $M = \theta'/\theta = (h_o/d_o)/(h_o/N) = N/d_o$. Note that this result differs from the magnification in part (a) only in that f has been replaced with d_o. All that remains is to calculate d_o using the thin-lens equation.

SOLUTION

Part (a)

1. Substitute $N = 26$ cm and $f = 4.3$ cm into $M = N/f$:

$$M = \frac{N}{f} = \frac{26\text{ cm}}{4.3\text{ cm}} = 6.0$$

Part (b)

2. Use the thin-lens equation to find d_o, given $f = 4.3$ cm and $d_i = -26$ cm:

$$\frac{1}{d_o} = \frac{1}{f} - \frac{1}{d_i} = \frac{1}{4.3\text{ cm}} - \frac{1}{-26\text{ cm}} = 0.27\text{ cm}^{-1}$$

$$d_o = \frac{1}{0.27\text{ cm}^{-1}} = 3.7\text{ cm}$$

3. Calculate the magnification using $M = N/d_o$:

$$M = \frac{N}{d_o} = \frac{26\text{ cm}}{3.7\text{ cm}} = 7.0$$

INSIGHT

We see that moving the object closer to the lens, which brings the image in from infinity to the near point, results in an increase in magnification of 1.0—from 6.0 to 7.0.

PRACTICE PROBLEM

What is the magnification if the object is placed 4.0 cm in front of the magnifying glass? [**Answer:** $M = N/d_o = 6.5$]

Some related homework problems: Problem 50, Problem 52

Thus we see from Example 27–4 that the magnification of a magnifying glass with the object at a distance d_o is $M = N/d_o$, where N is the near-point distance of the observer. In the special case of an image at infinity, the object distance is the focal length, and $M = N/f$. It can be shown (see Problem 106) that the greatest magnification—which occurs when the image is at the near point—is $M = 1 + N/f$. The magnification results are summarized here:

$$M = \frac{N}{f} \quad \text{(image at infinity)}$$

$$M = 1 + \frac{N}{f} \quad \text{(image at near point)} \qquad 27\text{–}5$$

Note that Example 27–4 is simply a special case of these general expressions.

The early microscopes produced by Antonie van Leeuwenhoek (1632–1723) were, in fact, simply powerful magnifying glasses using a single lens mounted in a hole in a flat metal plate. The object to be examined with the microscope was placed on the head of a small, movable pin placed just below the lens. The observer would then hold the top surface of the lens close to the eye for the most stable, wide-angle view. Leeuwenhoek's instruments were capable of magnifying objects by as much as 275 times, enough that he could make the first detailed microscopic descriptions of single-celled animals, red blood cells, plant cells, and much more. Microscopes in common use today have more than one lens, as we describe in the next section.

27–4 The Compound Microscope

Although a magnifying glass is a useful device, higher magnifications and improved optical quality can be obtained with a **microscope.** The simplest microscope consists of two converging lenses fixed at either end of a tube. Such an instrument, illustrated in Figure 27–14, is sometimes referred to as a *compound microscope*.

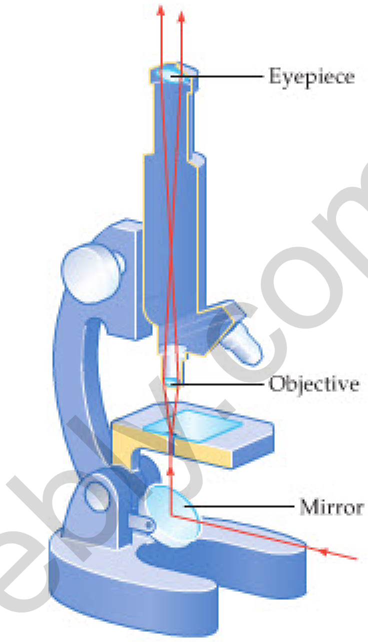


FIGURE 27–14 Basic elements of a compound microscope

A compound microscope consists of two lenses—an objective and an eyepiece—fixed at either end of a movable tube.

The basic optical elements of a microscope are the **objective** and the **eyepiece.** The objective is a converging lens with a relatively short focal length that is placed near the object to be viewed. It forms a real, inverted, and enlarged image, as shown in Figure 27–15. The precise location of the image is adjusted when the microscope is focused by moving the objective up or down. This image serves as the object for the second lens in the microscope—the eyepiece. In fact, the eyepiece is simply a magnifier that views the image of the objective, giving it an additional enlargement.

The final magnification of the microscope, then, is simply the product of the magnification of the objective and the magnification of the eyepiece. For example, a microscope might have a 10× eyepiece (meaning it magnifies 10 times) and a 50× objective. When these two lenses are used together, the magnification of the microscope is 500×.

In a typical situation, the object to be examined is placed only a small distance beyond the focal point of the objective, which means that $d_o \approx f_{\text{objective}}$. The magnification produced by the objective is given by Equation 26–18:

$$m_{\text{objective}} = -\frac{d_i}{d_o} \approx -\frac{d_i}{f_{\text{objective}}}$$

Next, the image formed by the objective is essentially at the focal point of the eyepiece. This means that the eyepiece forms a virtual image at infinity that the observer can view with a relaxed eye. The angular magnification of the eyepiece is given by Equation 27–4:

$$M_{\text{eyepiece}} = \frac{N}{f_{\text{eyepiece}}}$$

Multiplying these magnifications, we find the total magnification of the microscope:

$$M_{\text{total}} = m_{\text{objective}}M_{\text{eyepiece}} = \left(-\frac{d_i}{f_{\text{objective}}}\right)\left(\frac{N}{f_{\text{eyepiece}}}\right)$$
$$= -\frac{d_iN}{f_{\text{objective}}f_{\text{eyepiece}}} \tag{27–6}$$

The minus sign indicates that the image is inverted.

Typical numerical values are considered in the following Example.

PROBLEM-SOLVING NOTE

Lens Placement in a Microscope

In a working microscope, the distance between the lenses is greater than the sum of their focal lengths.

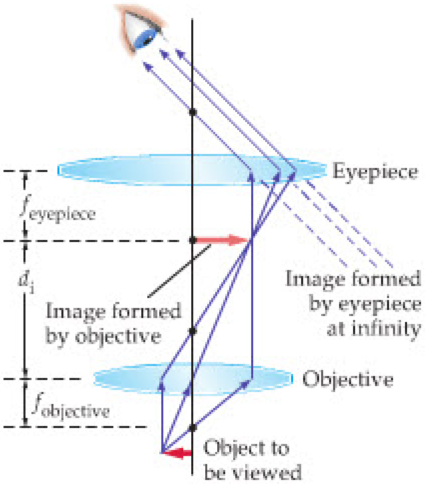


FIGURE 27–15 The operation of a compound microscope

In a compound microscope the object is placed just outside the focal point of the objective. The resulting enlarged image is then enlarged further by the eyepiece, which is basically a magnifying glass.

EXAMPLE 27–5 A MICROSCOPIC VIEW

In biology class, a student with a near-point distance of $N = 25$ cm uses a microscope to view an amoeba. If the objective has a focal length of 1.0 cm, the eyepiece has a focal length of 2.5 cm, and the amoeba is 1.1 cm from the objective, what is the magnification produced by the microscope?

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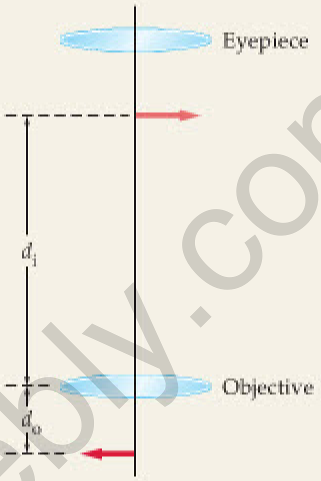
PICTURE THE PROBLEM

The lenses of the microscope are shown in the sketch. Note that the distance from the objective to the object is d_o and the distance from the objective to its image is d_i.



STRATEGY

The magnification of the microscope is given by Equation 27–6. The only unknown in this expression is the image distance, d_i. We can find this by using the thin-lens equation.

SOLUTION

1. Use the thin-lens equation to find the image distance, d_i:

$$\frac{1}{d_i} = \frac{1}{f} - \frac{1}{d_o} = \frac{1}{1.0\text{ cm}} - \frac{1}{1.1\text{ cm}} = 0.091\text{ cm}^{-1}$$

$$d_i = \frac{1}{0.091\text{ cm}^{-1}} = 11\text{ cm}$$

2. Use Equation 27–6 to find the magnification of the microscope:

$$M_{\text{total}} = -\frac{d_i N}{f_{\text{objective}} f_{\text{eyepiece}}} = -\frac{(11\text{ cm})(25\text{ cm})}{(1.0\text{ cm})(2.5\text{ cm})} = -110$$

INSIGHT

Thus the amoeba appears 110 times larger and is inverted. If the amoeba is to be viewed with a relaxed eye, the image formed by the objective should be at the focal point of the eyepiece, which will then form an image at infinity. Therefore, the length of the tube containing the objective and eyepiece is 11 cm + 2.5 cm = 13.5 cm in this case.

PRACTICE PROBLEM

If the focal length of the eyepiece is increased, does the magnitude of the magnification increase or decrease? Check your response by calculating the magnification when the focal length of the eyepiece is 3.5 cm. [**Answer:** The magnification is reduced in magnitude. Its new value is −79.]

Some related homework problems: Problem 58, Problem 61

27–5 Telescopes

A telescope is similar in many respects to a microscope—both use two converging lenses to give a magnified image of an object with small angular size. In the case of a microscope the object itself is small and close at hand; in the case of the telescope the object may be as large as a galaxy, but its angular size can be very small because of its great distance. The major difference between these instruments is that the telescope must deal with an object that is essentially infinitely far away.

For this reason, it is clear that the light entering the objective of a telescope from a distant object is focused at the focal point of the objective, as shown in Figure 27–16. If the image formed by the objective has a height h_i, the angular size of the object is approximately

$$\theta = -\frac{h_i}{f_{\text{objective}}}$$

Note that the minus sign is included because h_i is negative for the inverted image formed by the objective.

As in the microscope, the image of the objective is the object for the eyepiece, which is basically a magnifier. Thus, if the image of the objective is placed

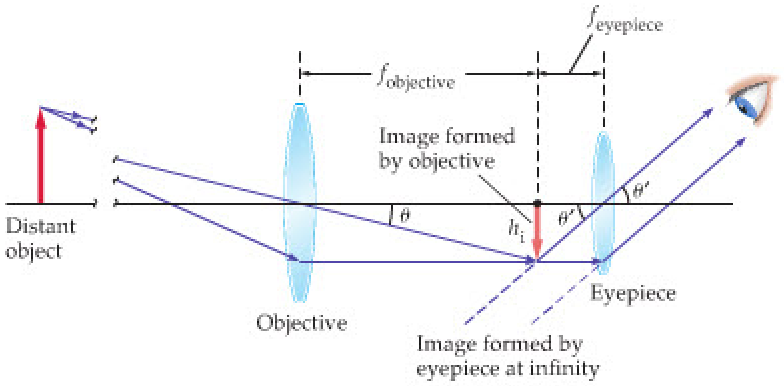


FIGURE 27–16 Basic elements of a telescope

A telescope focuses light from a distant object at its focal point. The image of the objective is placed at the focal point of the eyepiece to produce an enlarged image that can be viewed with a relaxed eye.

at the focal point of the eyepiece, it will form an image that is at infinity, as indicated in Figure 27–16. In this configuration, the observer can view the final image of the telescope with a completely relaxed eye. The angular size of the image formed by the eyepiece, θ', is shown in Figure 27–16. Clearly, this angle is approximately

$$\theta' = -\frac{h_i}{f_{\text{eyepiece}}}$$

To find the total angular magnification of the telescope, we take the ratio of θ' to θ:

$$M_{\text{total}} = \frac{\theta'}{\theta} = \frac{f_{\text{objective}}}{f_{\text{eyepiece}}} \tag{27–7}$$

Thus, for example, a telescope with an objective whose focal length is 1500 mm and an eyepiece whose focal length is 10.0 mm produces an angular magnification of 150.

PROBLEM-SOLVING NOTE

Lens Placement in a Telescope

In a working telescope, the distance between the lenses is approximately equal to the sum of their focal lengths. In addition, the focal length of the eyepiece is significantly less than that of the objective.

CONCEPTUAL CHECKPOINT 27–3 COMPARING TELESCOPES

Two telescopes have identical eyepieces, but telescope A is twice as long as telescope B. Is the magnification of telescope A **(a)** greater than the magnification of telescope B, **(b)** less than the magnification of telescope B, or **(c)** is there no way to tell?

REASONING AND DISCUSSION

Note in Figure 27–16 that the total length of the telescope is $f_{\text{objective}}$ plus f_{eyepiece}. Thus, because the scopes have identical eyepieces, it follows that telescope A must have a greater objective focal length than telescope B. We know, then, from $M_{\text{total}} = f_{\text{objective}}/f_{\text{eyepiece}}$, that the magnification of telescope A is greater than the magnification of telescope B.

ANSWER

(a) Telescope A has a greater magnification than telescope B.

Telescopes using two or more lenses, as in Figure 27–16, are referred to as *refractors*. In fact, the first telescopes constructed for astronomical purposes, made by Galileo starting in 1609, were refractors. Galileo's telescopes differed from the telescopes in common use today, however, in that they used a diverging lens for the eyepiece. We consider this type of telescope in Problems 75, 86, and 94. By the end of 1609, Galileo had produced a telescope whose angular magnification was 20. This was more than enough to enable him to see—for the first time in human history—mountains on the Moon, stars in the Milky Way, the phases of Venus, and moons orbiting Jupiter. As a result of his telescopic observations, Galileo became a firm believer in the Copernican model of the solar system.

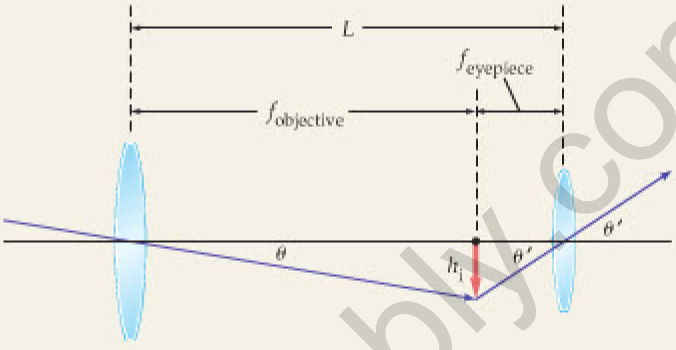
In the next Example we consider a standard refractor with two converging lenses. In particular, we show how the length of such a telescope is related to the focal lengths of its objective and eyepiece.

EXAMPLE 27–6 CONSIDERING THE TELESCOPE AT LENGTH

A telescope has a magnification of 40.0 and a length of 1230 mm. What are the focal lengths of the objective and eyepiece?

PICTURE THE PROBLEM

Our sketch shows that the overall length of the telescope, L, is equal to the sum of the focal lengths of the objective and the eyepiece. In addition, note that the objective forms an inverted image of the distant object, which the eyepiece enlarges. The final image seen by the observer, then, is inverted.



STRATEGY

We can determine the two unknowns, $f_{\text{objective}}$ and f_{eyepiece}, from the two independent bits of information given in the problem statement:

The magnification, $M_{\text{total}} = f_{\text{objective}}/f_{\text{eyepiece}}$, is equal to 40.0.

The total length, $L = f_{\text{objective}} + f_{\text{eyepiece}}$, is equal to 1230 mm.

Combining this information gives us the two focal lengths.

SOLUTION

1. Use the magnification equation to write $f_{\text{objective}}$ in terms of f_{eyepiece}:

$$M_{\text{total}} = \frac{f_{\text{objective}}}{f_{\text{eyepiece}}} = 40.0$$

$$f_{\text{objective}} = 40.0 f_{\text{eyepiece}}$$

2. Substitute $f_{\text{objective}} = 40.0 f_{\text{eyepiece}}$ into the expression for the total length of the telescope:

$$L = f_{\text{objective}} + f_{\text{eyepiece}}$$

$$= 40.0 f_{\text{eyepiece}} + f_{\text{eyepiece}} = 41.0 f_{\text{eyepiece}} = 1230\text{ mm}$$

3. Divide 1230 mm by 41.0 to find the focal length of the eyepiece:

$$f_{\text{eyepiece}} = \frac{1230\text{ mm}}{41.0} = 30.0\text{ mm}$$

4. Multiply f_{eyepiece} by 40.0 to find the focal length of the objective:

$$f_{\text{objective}} = 40.0 f_{\text{eyepiece}} = 40.0(30.0\text{ mm}) = 1200\text{ mm}$$

INSIGHT

The focal lengths found in this example are typical of those used in many popular amateur telescopes. A telescope with a magnification of 40.0 can easily show the moons of Jupiter and the rings of Saturn.

PRACTICE PROBLEM

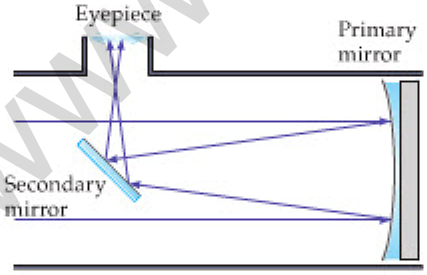
A telescope 1820 mm long has an objective with a focal length of 1780 mm. Find the focal length of the eyepiece and magnification of the telescope. [**Answer:** $f_{\text{eyepiece}} = 40.0$ mm, $M_{\text{total}} = 44.5$]

Some related homework problems: Problem 68, Problem 69, Problem 78

Because telescopes are typically used to view objects that are very dim, it is desirable to have an objective with as large a diameter, or aperture, as possible. If the diameter is doubled, for example, the light gathered by a telescope increases by the same factor as the area of its objective: $2^2 = 4$. In addition, a larger aperture results in a higher-resolution image, as we shall see in the next chapter.

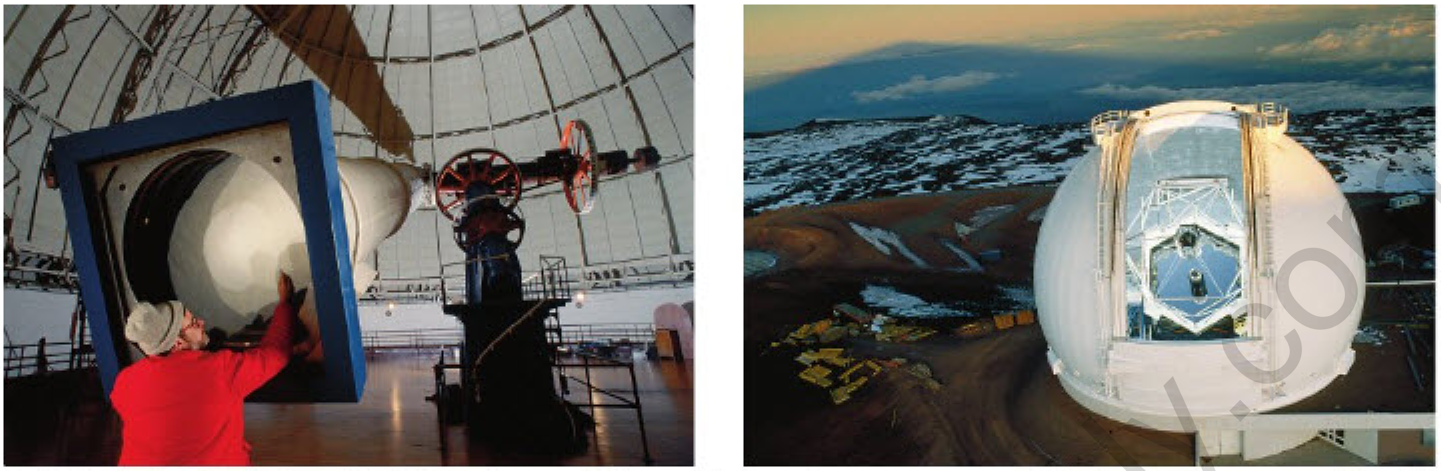
For a refractor, a large aperture means a very large and heavy piece of glass. In fact, the world's largest refractor, the Yerkes refractor at Williams Bay, Wisconsin, has an objective that is only 1 m across. If refractors were made much larger, the objective lens would sag and distort under its own weight.

The telescopes with the largest apertures are reflectors, one example of which is illustrated in Figure 27–17. Invented by Isaac Newton in 1671, and referred to as a Newtonian reflector, this type of telescope uses a mirror in place of an objective lens. As with the refractor, the mirror forms an image which the eyepiece then magnifies. Since a mirror can be much thinner and lighter than a lens, and can be supported all over its back surface instead of just around the edges as with a lens, it has many advantages for a large scope. In fact, the largest telescopes in the world are the twin Keck reflecting telescopes atop Hawaii's Mauna Kea. These telescopes have hexagonal objective mirrors that are 10 m across, which means



▲ **FIGURE 27–17 A Newtonian reflecting telescope**

The large primary mirror collects incoming light and reflects it off a small secondary mirror into the eyepiece.



▲ At left, the 1-m objective lens of the Yerkes refractor—the largest refracting telescope ever constructed—being dusted to remove spiders. If lenses were made much larger than this, they would sag and distort under their own weight. At right, one of the twin 10-m Keck reflecting telescopes atop Mauna Kea on the Big Island of Hawaii, where the air is cold, thin, and dry—ideal conditions for astronomical observation. Clearly, the use of interchangeable objectives, commonly found in microscopes, is not feasible for large telescopes. Instead, magnification is varied by using eyepieces with different focal lengths. It is rare for a person to actually look through one of these telescopes, however—instead, they are usually fitted with sophisticated cameras, spectrographs, or other instruments.

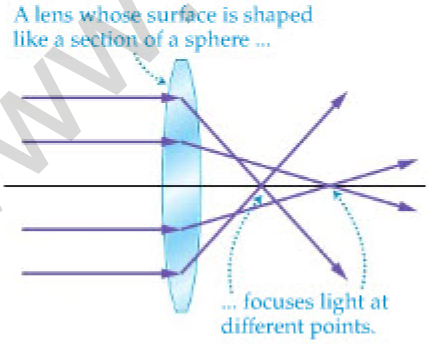
that each gathers 100 times as much light as the Yerkes refractor. When used together, the Keck telescopes constitute the world's largest pair of binoculars!

27–6 Lens Aberrations

An ideal lens brings all parallel rays of light that strike it together at a single focal point. Real lenses, however, never quite live up to the ideal. Instead, a real lens blurs the focal point into a small but finite region of space. This, in turn, blurs the image it forms. The deviation of a lens from ideal behavior is referred to as an **aberration.**

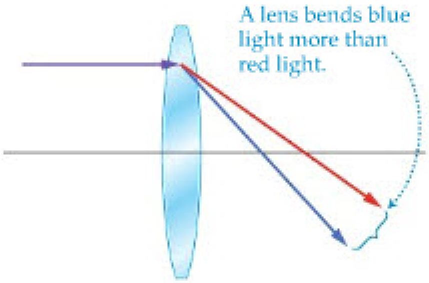
One example is **spherical aberration,** which is related to the shape of a lens. Figure 27–18 shows parallel rays of light passing through a lens with spherical aberration; note that the rays do not meet at a single focal point. This is completely analogous to the spherical aberration present in mirrors with a spherical cross section, as shown in Figure 26–13. To prevent spherical aberration, a lens must be ground and polished to a very precise nonspherical shape.

Another common aberration in lenses is **chromatic aberration,** which is due to the unavoidable dispersion present in any refracting material. Just as a prism splits white light into a spectrum of colors, each color bent by a different amount, so too does a lens bend light of different colors by different amounts as shown in Figure 27–19. The result is that white light passing through a lens does not focus to



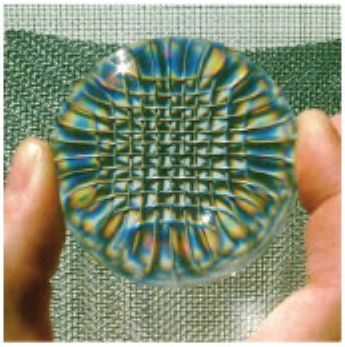
▲ **FIGURE 27–18 Spherical aberration**

In a lens with spherical aberration, light striking the lens at different locations comes together at different focal points.



▲ **FIGURE 27–19 Chromatic aberration in a converging lens**

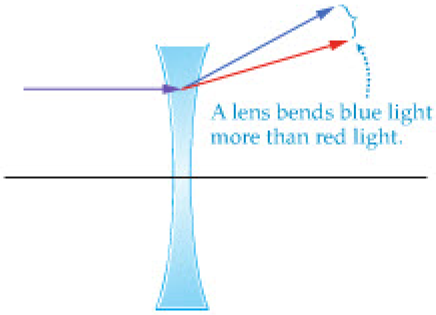
Chromatic aberration is caused by the fact that blue light bends more than red light as it passes through a lens. As a result, different colors have different focal points.



▲ Chromatic aberration typical of a simple lens.

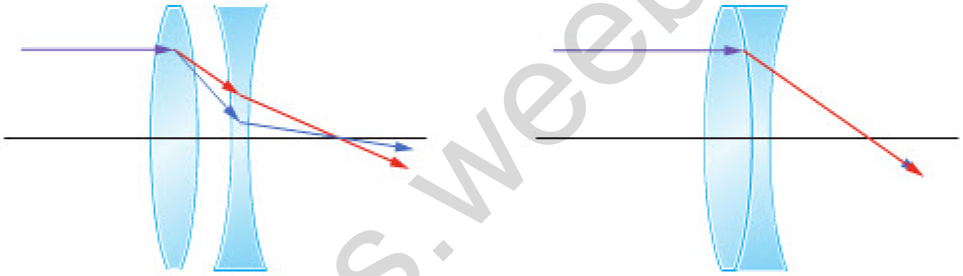
a single point. This is why you sometimes see a fringe of color around an image seen through a simple lens.

Chromatic aberration can be corrected by combining two or more lenses to form a compound lens. For example, suppose a convex lens made from glass A bends red and blue light as shown in Figure 27–19. Note that the blue light is bent more, as expected. A second lens, concave this time, is made of glass B. This lens also bends blue light more than red light, but since it is a diverging lens, it bends both the red and blue light in the opposite direction compared with the convex lens, as we see in **Figure 27–20**. If the amount of bending of blue light compared with red light is different for glasses A and B, it is possible to construct a convex and a concave lens in such a way that the opposite directions in which they bend red and blue light can be made to cancel while still causing the light to come to a focus. This combination is illustrated in **Figure 27–21**. Such a lens is said to be **achromatic.** Three lenses connected similarly can produce even better correction for chromatic aberration. Lenses of this type are referred to as **apochromatic.**



▲ **FIGURE 27–20 Chromatic aberration in a diverging lens**

A diverging lens also bends blue light more than red light, though the rays are bent in the opposite direction compared with the bending by a converging lens.


Two lenses canceling chromatic aberration

An achromatic doublet

▶ **FIGURE 27–21 Correcting for chromatic aberration**

An achromatic doublet is a combination of a converging and a diverging lens made of different types of glass. In the converging lens the blue light is bent toward the axis of the lens more than the red light. In the diverging lens the opposite is the case. The net result is that the red and blue light pass through the same focal point.

Many lenses in optical instruments are not single lenses at all but, instead, are compound achromatic lenses. For example, the "lens" in a 35-mm camera may actually contain five or more individual lenses—some of them converging, some diverging, some made of one type of glass, some made of another. A zoom lens for a camera not only must correct for chromatic aberration, but must also be able to change its focal length. Thus, it is not uncommon for such a lens to contain as many as twelve individual lenses.

THE BIG PICTURE PUTTING PHYSICS IN CONTEXT

LOOKING BACK	LOOKING AHEAD
Ray diagrams from Chapter 26 are used throughout this chapter to determine the location, orientation, and size of an image. We also extend ray diagrams to cases involving more than a single lens.	Light has been treated as straight-line rays in both Chapters 26 and 27. In Chapter 28, however, we consider the wave properties of light, and show that rays—while useful—do not tell the whole story.
Dispersion, which was discussed in Chapter 26 in relation to rainbows, appears again in this chapter when we consider chromatic aberration in Section 27–6.	Ray diagrams showing light reflected from a mirror are used in relativity (Chapter 29). There we analyze a "light clock," and show that a moving clock runs at a slower rate than a clock at rest.

CHAPTER SUMMARY

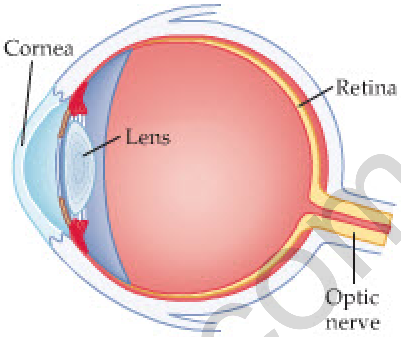
27–1 THE HUMAN EYE AND THE CAMERA

The human eye forms a real, but inverted, image on the retina; a camera forms a real, but inverted, image on light-sensitive material.

Focusing the Eye

The eye is focused by the ciliary muscles, which change the shape of the lens. This process is referred to as accommodation.

Focusing a Camera
A camera is focused by moving the lens closer to or farther away from the light-sensitive material. The shape of the lens is unchanged.



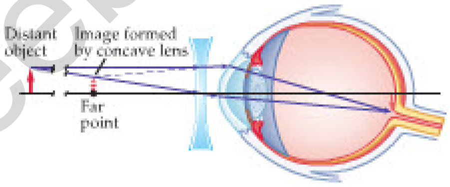
Near Point
The near point is the closest distance to which the eye can focus. A typical value for the near-point distance, N, is 25 cm.

Far Point
The far point is the greatest distance at which the eye can focus. In a normal eye the far point is infinity.

***f*-Number**
The f-number of a lens relates the diameter of the aperture, D, to the focal length, f, as follows:

$$f\text{-number} = \frac{\text{focal length}}{\text{diameter of aperture}} = \frac{f}{D} \qquad 27\text{–}1$$

27–2 LENSES IN COMBINATION AND CORRECTIVE OPTICS



The basic idea used in analyzing systems with more than one lens is the following: The *image* produced by one lens acts as the *object* for the next lens.

In addition, the total magnification of a system of lenses is given by the product of the magnifications produced by each lens individually.

Nearsightedness
Nearsightedness is a condition in which clear vision is restricted to a region relatively close to the eye—in other words, the far point is not at infinity but instead is a finite distance from the eye. This condition can be corrected with diverging lenses placed in front of the eyes.

Farsightedness
A person who is farsighted can see clearly only at relatively large distances from the eye—that is, the person's near point is much farther from the eye than the more typical values of 25 to 40 cm. Farsightedness can be corrected by placing converging lenses in front of the eyes.

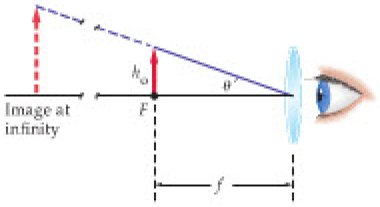
Refractive Power and Diopters
The refractive power of a lens refers to its ability to bend light and is measured in diopters. It is defined as follows:

$$\text{refractive power} = \frac{1}{f} \qquad 27\text{–}2$$

In this expression, the focal length, f, must be measured in meters.

The greater the magnitude of the refractive power, the more strongly the lens bends light. A positive refractive power indicates a converging lens; a negative refractive power indicates a diverging lens.

27–3 THE MAGNIFYING GLASS



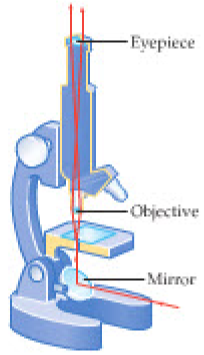
A magnifying glass is simply a converging lens. It works by allowing an object to be viewed at a distance less than the near-point distance. Since the distance is reduced, the angular size is increased.

Magnification
A person with a near-point distance N using a magnifying glass with a focal length f experiences the following magnifications, M:

$$M = \frac{N}{f} \qquad \text{(image at infinity)}$$

$$M = 1 + \frac{N}{f} \qquad \text{(image at near point)} \qquad 27\text{–}5$$

27–4 THE COMPOUND MICROSCOPE



A compound microscope uses two lenses in combination—an objective and an eyepiece—to produce a magnified image.

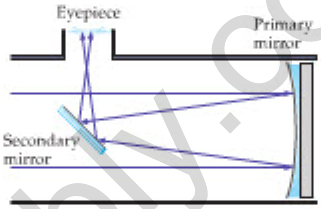
The object to be viewed is placed just outside the focal length of the objective. The image formed by the objective is then viewed by the eyepiece, giving additional magnification.

Magnification
The magnification produced by a compound microscope is

$$M_{\text{total}} = -\frac{d_i N}{f_{\text{objective}} f_{\text{eyepiece}}} \qquad 27\text{–}6$$

where N is the near-point distance and d_i is the image distance for the objective. The magnifications quoted for microscopes assume a near-point distance of 25 cm.

27–5 TELESCOPES



A telescope provides magnified views of distant objects using two lenses. The objective lens focuses the incoming light at its focal point; the eyepiece magnifies the image formed by the objective.

Magnification
The total magnification produced by a telescope is

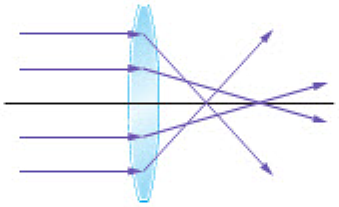
$$M_{\text{total}} = \frac{f_{\text{objective}}}{f_{\text{eyepiece}}} \qquad 27\text{–}7$$

Length of a Telescope
The length, L, of a telescope is the sum of the focal lengths of its two lenses:

$$L = f_{\text{objective}} + f_{\text{eyepiece}}$$

Reflecting Telescopes
A reflecting telescope uses a mirror in place of an objective lens. The largest telescopes are reflectors, as are many amateur telescopes.

27–6 LENS ABERRATIONS



Any deviation of a lens from ideal behavior is referred to as an aberration.

Spherical Aberration
In spherical aberration, parallel rays of light passing through a lens fail to go through a single focal point. Spherical aberration is related to the shape of a lens.

Chromatic Aberration
Chromatic aberration results from dispersion within a refracting material. It causes different colors to focus at different points. Achromatic lenses correct for chromatic aberration by combining two lenses with different refractive properties.

PROBLEM-SOLVING SUMMARY

Type of Problem	Relevant Physical Concepts	Related Examples
Find the effective focal length of an eye when focused on a given object.	Use the thin-lens equation with an image distance of 1.7 cm to 2.5 cm to take into account the distance from the lens to the retina.	Example 27–1
Determine the optical behavior of a multilens system.	Apply the thin-lens equation to each lens one at a time. Let the image of one lens be the object for the next lens.	Active Example 27–2
Find the focal length of corrective glasses.	In the case of a nearsighted person, make sure the image is no farther away from the eye than the far point. For a farsighted person, make sure the image is no closer to the eye than the near point.	Examples 27–2, 27–3 Active Examples 27–3, 27–4

CONCEPTUAL QUESTIONS

For instructor-assigned homework, go to www.masteringphysics.com

(Answers to odd-numbered Conceptual Questions can be found in the back of the book.)

1. Why is it restful to your eyes to gaze off into the distance?
2. If a lens is cut in half through a plane perpendicular to its surface, does it show only half an image?
3. If your near-point distance is N, how close can you stand to a mirror and still be able to focus on your image?
4. When you open your eyes underwater, everything looks blurry. Can this be thought of as an extreme case of nearsightedness or farsightedness? Explain.
5. Would you benefit more from a magnifying glass if your near-point distance is 25 cm or if it is 15 cm? Explain.
6. When you use a simple magnifying glass, does it matter whether you hold the object to be examined closer to the lens than its focal length or farther away? Explain.
7. Is the final image produced by a telescope real or virtual? Explain.
8. Does chromatic aberration occur in mirrors? Explain.

PROBLEMS AND CONCEPTUAL EXERCISES

Note: Answers to odd-numbered Problems and Conceptual Exercises can be found in the back of the book. **IP** *denotes an integrated problem, with both conceptual and numerical parts;* **BIO** *identifies problems of biological or medical interest;* **CE** *indicates a conceptual exercise.* **Predict/Explain** *problems ask for two responses:* **(a)** *your prediction of a physical outcome, and* **(b)** *the best explanation among three provided. On all problems, red bullets (•, ••, •••) are used to indicate the level of difficulty.*

(The outside medium is assumed to be air, with an index of refraction of 1.00, unless specifically stated otherwise.)

SECTION 27–1 THE HUMAN EYE AND THE CAMERA

1. • **CE Predict/Explain BIO Octopus Eyes** To focus its eyes, an octopus does not change the shape of its lens, as is the case in humans. Instead, an octopus moves its rigid lens back and forth, as in a camera. This changes the distance from the lens to the retina and brings an object into focus. **(a)** If an object moves closer to an octopus, must the octopus move its lens closer to or farther from its retina to keep the object in focus? **(b)** Choose the *best explanation* from among the following:
   - I. The lens must move closer to the retina—that is, farther away from the object—to compensate for the object moving closer to the eye.
   - II. When the object moves closer to the eye, the image produced by the lens will be farther behind the lens; therefore, the lens must move farther from the retina.

2. • Your friend is 1.9 m tall. **(a)** When she stands 3.2 m from you, what is the height of her image formed on the retina of your eye? (Consider the eye to consist of a thin lens 2.5 cm from the retina.) **(b)** What is the height of her image when she is 4.2 m from you?

3. • Which forms the larger image on the retina of your eye: a 43-ft tree seen from a distance of 210 ft, or a 12-in. flower viewed from a distance of 2.0 ft?

4. • Approximating the eye as a single thin lens 2.60 cm from the retina, find the eye's near-point distance if the smallest focal length the eye can produce is 2.20 cm.

5. • Referring to Problem 4, what is the focal length of the eye when it is focused on an object at a distance of **(a)** 285 cm and **(b)** 28.5 cm?

6. • Four camera lenses have the following focal lengths and *f*-numbers:

Lens	Focal length (mm)	*f*-number
A	150	$f/1.2$
B	150	$f/5.6$
C	35	$f/1.2$
D	35	$f/5.6$

Rank these lenses in order of increasing aperture diameter. Indicate ties where appropriate.

7. • **BIO** The focal length of the human eye is approximately 1.7 cm. **(a)** What is the *f*-number for the human eye in bright light, when the pupil diameter is 2.0 mm? **(b)** What is the *f*-number in dim light, when the pupil diameter has expanded to 7.0 mm?

8. • **IP** A camera with a 55-mm-focal-length lens has aperture settings of 2.8, 4, 8, 11, and 16. **(a)** Which setting has the largest aperture diameter? **(b)** Calculate the five possible aperture diameters for this camera.

9. • The actual frame size of "35-mm" film is 24 mm × 36 mm. You want to take a photograph of your friend, who is 1.9 m tall. Your camera has a 55-mm-focal-length lens. How far from the camera should your friend stand in order to produce a 36-mm-tall image on the film?

10. • To completely fill a frame of "35-mm" film, the image produced by a camera must be 36 mm high. If a camera has a focal length of 150 mm, how far away must a 2.0-m-tall person stand to produce an image that fills the frame?

11. •• You are taking a photograph of a poster on the wall of your dorm room, so you can't back away any farther than 3.0 m to take the shot. The poster is 0.80 m wide and 1.2 m tall, and you want the image to fit in the 24-mm × 36-mm frame of the film in your camera. What is the longest focal length lens that will work?

12. •• A photograph is properly exposed when the aperture is set to $f/8$ and the shutter speed is 125. Find the approximate shutter speed needed to give the same exposure if the aperture is changed to $f/2.4$.

13. •• You are taking pictures of the beach at sunset. Just before the Sun sets, a shutter speed of $f/11$ produces a properly exposed picture. Shortly after the Sun sets, however, your light meter indicates that the scene is only one-quarter as bright as before. **(a)** If you don't change the aperture, what approximate shutter speed is needed for your second shot? **(b)** If, instead, you keep the shutter speed at 1/100 s, what approximate *f*-stop will be needed for the second shot?

14. •• **IP** You are taking a photograph of a horse race. A shutter speed of 125 at $f/5.6$ produces a properly exposed image, but the running horses give a blurred image. Your camera has *f*-stops of 2, 2.8, 4, 5.6, 8, 11, and 16. **(a)** To use the shortest possible exposure time (i.e., highest shutter speed), which *f*-stop should you use? **(b)** What is the shortest exposure time you can use and *still* get a properly exposed image?

15. •• **The Hale Telescope** The 200-in. (5.08-m) diameter mirror of the Hale telescope on Mount Palomar has a focal length $f = 16.9$ m. **(a)** When the detector is placed at the focal point of the mirror (the "prime focus"), what is the *f*-ratio for this telescope? **(b)** The coudé focus arrangement uses additional mirrors to bend the light path and increase the effective focal length to 155.4 m. What is the *f*-ratio of the telescope when the coudé focus is being used? (*Coudé* is French for "elbow," since the light path is "bent like an elbow." This arrangement is useful when the light needs to be focused onto a distant instrument.)

SECTION 27–2 LENSES IN COMBINATION AND CORRECTIVE OPTICS

16. • **CE Predict/Explain** Two professors are stranded on a deserted island. Both wear glasses, though one is nearsighted and the other is farsighted. **(a)** Which person's glasses should be used to focus the rays of the Sun and start a fire? **(b)** Choose the *best explanation* from among the following:
    - I. A nearsighted person can focus close, so that person's glasses should be used to focus the sunlight on a piece of moss at a distance of a couple inches.
    - II. A farsighted person can't focus close, so the glasses to correct that person's vision are converging. A converging lens is what you need to concentrate the rays of the Sun.

17. • **CE** A clerk at the local grocery store wears glasses that make her eyes look larger than they actually are. Is the clerk nearsighted or farsighted? Explain.