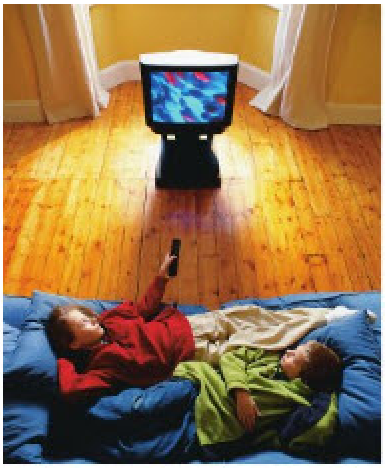


◀ We use infrared rays all the time, even though they are invisible to us. If you change the channel with a remote control, your signal is sent by an infrared ray; if you move your hand in front of a no-touch water faucet, an infrared ray detects the motion. In contrast, snakes called pit vipers can actually "see" infrared rays with the "pit" organs located just in front of their eyes. What must a remote control look like to one of these creatures?

are the highest-frequency electromagnetic waves that can be produced by electronic circuitry.

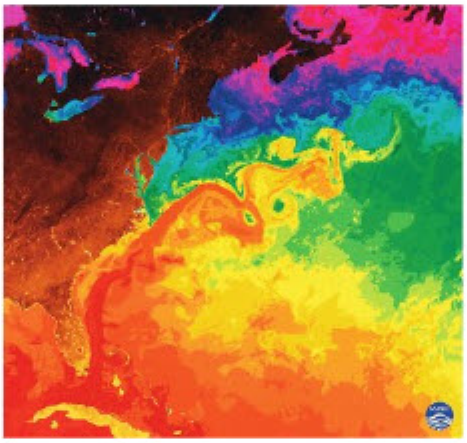
Infrared Waves

($f \sim 10^{12}$ Hz to 4.3×10^{14} Hz, $\lambda \sim 0.3$ mm to 700 nm) Electromagnetic waves with frequencies just below that of red light—roughly 10^{12} Hz to 4.3×10^{14} Hz—are known as *infrared* rays. These waves can be felt as heat on our skin but cannot be seen with our eyes. Many creatures, including various types of pit vipers, have specialized infrared receptors that allow them to "see" the infrared rays given off by a warm-blooded prey animal, even in total darkness. Infrared rays are often generated by the rotations and vibrations of molecules. In turn, when infrared rays are absorbed by an object, its molecules rotate and vibrate more vigorously, resulting in an increase in the object's temperature. Finally, many remote controls—for items ranging from TVs to DVD players to gas fireplaces—operate on a beam of infrared light with a wavelength of about 1000 nm. This infrared light is so close to the visible spectrum and so low in intensity that it cannot be felt as heat.

REAL-WORLD PHYSICS: BIO
Infrared receptors in pit vipers

Visible Light

($f \sim 4.3 \times 10^{14}$ Hz to 7.5×10^{14} Hz, $\lambda \sim 700$ nm to 400 nm) The portion of the electromagnetic spectrum most familiar to us is the spectrum of visible light, represented by the full range of colors seen in a rainbow. Each of the different colors, as perceived by our eyes and nervous system, is nothing more than an electromagnetic wave with a different frequency. Waves in this frequency range (4.3×10^{14} to 7.5×10^{14} Hz) are produced primarily by electrons changing their positions within an atom, as we discuss in detail in Chapter 31.

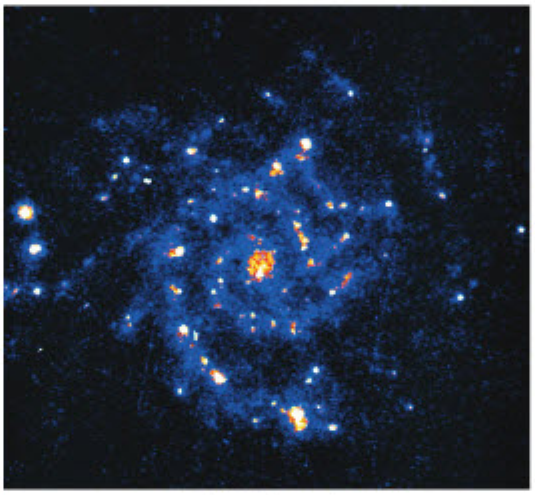


◀ Photographs made with infrared radiation are often called thermograms, since most infrared wavelengths can be felt as heat by the human skin. Thermograms provide a useful remote sensing technique for measuring temperature. In the photo on the left, the areas of the cat's head that are warmest (pink) and coolest (blue) can be clearly identified. In the photo on the right, an infrared satellite image of the Atlantic Ocean off the coast of North America, warmer colors are used to indicate higher sea surface temperatures. The swirling red streak running from lower left toward the upper right is the Gulf Stream.

REAL WORLD-PHYSICS: BIO
Biological effects of ultraviolet light

Ultraviolet Light

($f \sim 7.5 \times 10^{14}$ Hz to 10^{17} Hz, $\lambda \sim 400$ nm to 3 nm) When electromagnetic waves have frequencies just above that of violet light—from about 7.5×10^{14} Hz to 10^{17} Hz—they are called *ultraviolet* or *UV rays*. Although these rays are invisible, they often make their presence known by causing suntans with moderate exposure. More prolonged or intense exposure to UV rays can have harmful consequences, including an increased probability of developing a skin cancer. Fortunately, most of the UV radiation that reaches Earth from the Sun is absorbed in the upper atmosphere by ozone (O_3) and other molecules. A significant reduction in the ozone concentration in the stratosphere could result in an unwelcome increase of UV radiation on Earth's surface.



▲ Most ultraviolet radiation cannot penetrate Earth's atmosphere, but ultraviolet astronomical photographs, such as the one shown here, have been taken by cameras in orbit. The bright spots in this image of a spiral galaxy are areas of intense star formation, populated by hot young stars that radiate heavily in the ultraviolet.

X-Rays

($f \sim 10^{17}$ Hz to 10^{20} Hz, $\lambda \sim 3$ nm to 0.003 nm) As the frequency of electromagnetic waves is raised even higher, into the range between about 10^{17} Hz to 10^{20} Hz, we reach the part of the spectrum known as *X-rays*. Typically, the X-rays used in medicine are generated by the rapid deceleration of high-speed electrons projected against a metal target, as we show in Section 31–17. These energetic rays, which are only weakly absorbed by the skin and soft tissues, pass through our bodies rather freely, except when they encounter bones, teeth, or other relatively dense material. This property makes X-rays most valuable for medical diagnosis, research, and treatment. Still, X-rays can cause damage to human tissue, and it is desirable to reduce unnecessary exposure to these rays as much as possible.

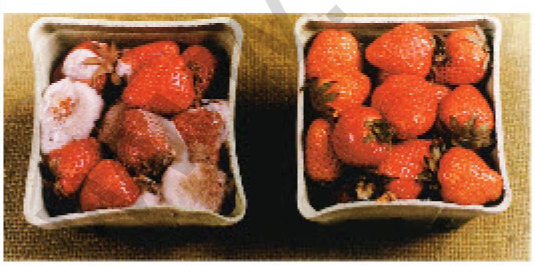
Gamma Rays

($f \sim 10^{20}$ Hz and higher, $\lambda \sim 0.003$ nm and smaller) Finally, electromagnetic waves with frequencies above about 10^{20} Hz are generally referred to as *gamma* (γ) *rays*. These rays, which are even more energetic than X-rays, are often produced as neutrons and protons rearrange themselves within a nucleus, or when a particle collides with its antiparticle, and the two annihilate each other. These processes are discussed in detail in Chapter 32. Gamma rays are also highly penetrating and destructive to living cells. It is for this reason that they are used to kill cancer cells and, more recently, microorganisms in food. Irradiated food, however, is a concept that has yet to become popular with the general public, even though NASA has irradiated astronauts' food since the 1960s. If you happen to see irradiated food in the grocery store, you will know that it has been exposed to γ rays from cobalt-60 for 20 to 30 minutes.

REAL-WORLD PHYSICS: BIO
Irradiated food

Notice that the visible part of the electromagnetic spectrum, so important to life on Earth, is actually the smallest of the frequency bands we have named. This accounts for the fact that a rainbow produces only a narrow band of color in the sky—if the visible band were wider, the rainbow would be wider as well. It should be remembered, however, that there is nothing particularly special about the visible band; in fact, it is even species dependent. For example, some bees and butterflies can see ultraviolet light, and, as mentioned previously, certain snakes can form images from infrared radiation.

One of the main factors in determining the visible range of frequencies is Earth's atmosphere. For example, if one examines the transparency of the atmosphere as a function of frequency, it is found that there is a relatively narrow range of frequencies for which the atmosphere is highly transparent. As eyes evolved in living systems on Earth, they could have evolved to be sensitive to various different frequency ranges. It so happens, however, that the range of frequencies that most animal eyes can detect matches nicely with the range of frequencies that the atmosphere allows to reach Earth's surface. This is a nice example of natural adaptation.



▲ The use of radiation to preserve food can be quite effective, but the technique is still controversial. Both boxes of strawberries shown here were stored for about 2 weeks at refrigerator temperature. Before storage, the box at right was irradiated to kill microorganisms and mold spores.

25–4 Energy and Momentum in Electromagnetic Waves

All waves transmit energy, and electromagnetic waves are no exception. When you walk outside on a sunny day, for example, you feel warmth where the sunlight strikes your skin. The energy creating this warm sensation originated in the Sun, and it has just completed a 93-million-mile trip when it reaches your body. In fact, the energy necessary for most of the life on Earth is transported here across the vacuum of space by electromagnetic waves traveling at the speed of light.

That electromagnetic waves carry energy with them is no surprise when you recall that they are composed of electric and magnetic fields, each of which has an associated energy density. For example, in Section 20–6 we showed that the energy density of an electric field of magnitude E is

$$u_E = \tfrac{1}{2}\varepsilon_0 E^2$$

Similarly, we showed in Section 23–9 that the energy density of a magnetic field of magnitude B is

$$u_B = \frac{1}{2\mu_0}B^2$$

It follows that the total energy density, u, of an electromagnetic wave is simply

$$u = u_E + u_B = \tfrac{1}{2}\varepsilon_0 E^2 + \frac{1}{2\mu_0}B^2 \qquad 25\text{–}5$$

As expected, both E and B contribute to the total energy carried by a wave. Not only that, but it can be shown that the electric and magnetic energy densities in an electromagnetic wave are, in fact, equal to each other—again demonstrating the symmetrical role played by the electric and magnetic fields. Thus, the total energy density of an electromagnetic field can be written in the following equivalent forms:

$$u = \tfrac{1}{2}\varepsilon_0 E^2 + \frac{1}{2\mu_0}B^2 = \varepsilon_0 E^2 = \frac{1}{\mu_0}B^2 \qquad 25\text{–}6$$

Since E and B vary sinusoidally with time, as indicated in Figure 25–3, it follows that their average values are zero—just like the current or voltage in an ac circuit. Therefore, to find the average energy density of an electromagnetic wave, we must use the rms values of E and B:

$$u_{av} = \tfrac{1}{2}\varepsilon_0 E_{rms}^2 + \frac{1}{2\mu_0}B_{rms}^2 = \varepsilon_0 E_{rms}^2 = \frac{1}{\mu_0}B_{rms}^2 \qquad 25\text{–}7$$

Recall that the rms value of a sinusoidally varying quantity x is related to its maximum value as $x_{rms} = x_{max}/\sqrt{2}$. Thus, for the electric and magnetic fields in an electromagnetic wave we have

$$E_{rms} = \frac{E_{max}}{\sqrt{2}}$$

$$B_{rms} = \frac{B_{max}}{\sqrt{2}} \qquad 25\text{–}8$$

The fact that the electric and magnetic energy densities are equal in an electromagnetic wave has a further interesting consequence. Setting u_E equal to u_B we obtain

$$\tfrac{1}{2}\varepsilon_0 E^2 = \frac{1}{2\mu_0}B^2$$

Rearranging slightly, and taking the square root, we find

$$E = \frac{1}{\sqrt{\varepsilon_0\mu_0}}B$$

Finally, from the fact that the speed of light is given by the relation $c = 1/\sqrt{\varepsilon_0 \mu_0}$, it follows that

$$E = cB \qquad 25\text{–}9$$

Thus, not only does an electromagnetic wave have both an electric and a magnetic field, the fields must also have the specific ratio $E/B = c$.

EXERCISE 25–3

At a given instant of time the electric field in a beam of sunlight has a magnitude of 510 N/C. What is the magnitude of the magnetic field at this instant?

SOLUTION

Using $E = cB$, or $B = E/c$, we find

$$B = \frac{E}{c} = \frac{510 \text{ N/C}}{3.00 \times 10^8 \text{ m/s}} = 1.7 \times 10^{-6} \text{ T}$$

Note that the units are consistent, since $1 \text{ T} = 1 \text{ N}/(\text{C} \cdot \text{m/s})$.

The amount of energy a wave delivers to a unit area in a unit time is referred to as its **intensity**, I. (Equivalently, since power is energy per time, the intensity of a wave is the power per unit area.) Imagine, for example, an electromagnetic wave of area A moving in the positive x direction, as in Figure 25–9. In the time Δt the wave moves through a distance $c\Delta t$; hence, all the energy in the volume $\Delta V = A(c\Delta t)$ is deposited on the area A in this time. Because energy is equal to the energy density times the volume, it follows that the energy in the volume ΔV is $\Delta U = u\Delta V$. Therefore, the intensity of the wave (energy per area per time) is

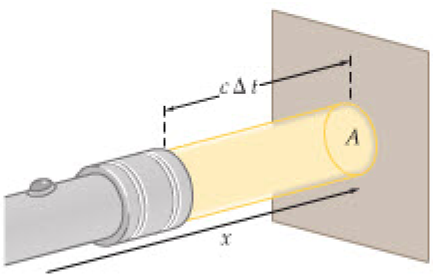


FIGURE 25–9 The energy in a beam of light

A beam of light of cross-sectional area A shines on a surface. All the light energy contained in the volume $\Delta V = A(c\Delta t)$ strikes the surface in the time Δt.

$$I = \frac{\Delta U}{A\Delta t} = \frac{u(Ac\Delta t)}{A\Delta t} = uc$$

Averaged over time, the intensity is $I_{av} = u_{av}c$. In terms of the electric and magnetic fields, we have

$$I = uc = \tfrac{1}{2}c\varepsilon_0 E^2 + \frac{1}{2\mu_0}cB^2 = c\varepsilon_0 E^2 = \frac{c}{\mu_0}B^2 \qquad 25\text{–}10$$

As before, to calculate an average intensity, we must replace E and B with their rms values.

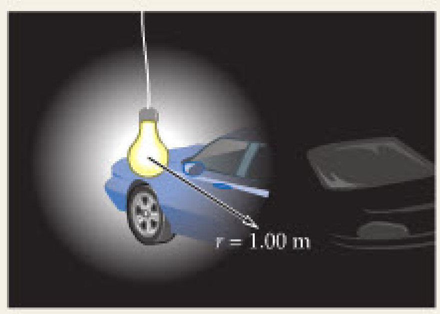
Notice that the intensity is proportional to the square of the fields. This is analogous to the case of simple harmonic motion, where the energy of oscillation is proportional to the square of the amplitude, as we found in Section 14–15.

EXAMPLE 25–4 LIGHTBULB FIELDS

A garage is illuminated by a single incandescent lightbulb dangling from a wire. If the bulb radiates light uniformly in all directions, and consumes an average electrical power of 50.0 W, what are **(a)** the average intensity of the light and **(b)** the rms values of E and B at a distance of 1.00 m from the bulb? (Assume that 5.00% of the electrical power consumed by the bulb is converted to light.)

PICTURE THE PROBLEM

The physical situation is pictured in the sketch. We assume that all the power radiated by the bulb passes uniformly through the area of a sphere of radius $r = 1.00$ m centered on the bulb.



STRATEGY

a. Recall that intensity is power per unit area; therefore, $I_{av} = P_{av}/A$. In this case the area is $A = 4\pi r^2$, the surface area of a sphere of radius r. The average power of the light is 5.00% of 50.0 W.

b. Once we know the intensity, I_{av}, we obtain the fields using Equation 25–10. Since the intensity is an average value, the corresponding fields are rms values.

SOLUTION

Part (a)

1. Calculate the average intensity at the surface of the sphere of radius $r = 1.00$ m:

$$I_{av} = \frac{P_{av}}{A} = \frac{(50.0\ \text{W})(0.0500)}{4\pi(1.00\ \text{m})^2} = 0.199\ \text{W/m}^2$$

Part (b)

2. Use $I_{av} = c\varepsilon_0 E_{rms}^2$ to find E_{rms}:

$$I_{av} = c\varepsilon_0 E_{rms}^2$$

$$E_{rms} = \sqrt{\frac{I_{av}}{c\varepsilon_0}} = \sqrt{\frac{0.199\ \text{W/m}^2}{(3.00 \times 10^8\ \text{m/s})(8.85 \times 10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)}}$$

$$= 8.66\ \text{N/C}$$

3. Use $I_{av} = cB_{rms}^2/\mu_0$ to find B_{rms}:

$$I_{av} = cB_{rms}^2/\mu_0$$

$$B_{rms} = \sqrt{\frac{\mu_0 I_{av}}{c}} = \sqrt{\frac{(4\pi \times 10^{-7}\ \text{N}\cdot\text{s}^2/\text{C}^2)(0.199\ \text{W/m}^2)}{(3.00 \times 10^8\ \text{m/s})}}$$

$$= 2.89 \times 10^{-8}\ \text{T}$$

4. As a check, use the relation $E = cB$ and the results from Steps 2 and 3 to calculate the speed of light, c:

$$c = \frac{E_{rms}}{B_{rms}} = \frac{8.66\ \text{N/C}}{2.89 \times 10^{-8}\ \text{T}} = 3.00 \times 10^8\ \text{m/s}$$

INSIGHT

Notice that the electric field in the light from the bulb has a magnitude (8.66 N/C = 8.66 V/m) that is relatively easy to measure. The magnetic field, however, is so small that a direct measurement would be difficult. This is common in electromagnetic waves because $B = E/c$, and c is such a large number.

PRACTICE PROBLEM

If the distance from the bulb is doubled, does E_{rms} increase, decrease, or stay the same? Check your answer by calculating E_{rms} at $r = 2.00$ m. [**Answer:** E_{rms} decreases with increasing distance from the bulb. For $r = 2.00$ m, we find $E_{rms} = \frac{1}{2}(8.66\ \text{N/C}) = 4.33\ \text{N/C}$.]

Some related homework problems: Problem 51, Problem 62

ACTIVE EXAMPLE 25–1 FIND E AND B IN A BEAM OF LIGHT

A small laser emits a cylindrical beam of light 1.00 mm in diameter with an average power of 5.00 mW. Find the maximum values of E and B in this beam.

SOLUTION *(Test your understanding by performing the calculations indicated in each step.)*

1. Divide the average power of the beam by its area to find the intensity: $I_{av} = 6370\ \text{W/m}^2$
2. Use $I_{av} = c\varepsilon_0 E_{rms}^2$ to find the rms electric field: $E_{rms} = 1550\ \text{N/C}$
3. Multiply E_{rms} by $\sqrt{2}$ to find E_{max}: $E_{max} = 2190\ \text{N/C}$
4. Divide E_{max} by c to find B_{max}: $B_{max} = 7.30 \times 10^{-6}\ \text{T}$

INSIGHT

To find the cross-sectional area of the cylindrical beam in Step 1 we used $A = \pi d^2/4$.

YOUR TURN

Suppose the beam spreads out to twice its initial diameter. By what factor do E_{max} and B_{max} change?

(Answers to **Your Turn** *problems are given in the back of the book.)*

Finally, an electromagnetic wave also carries momentum. In fact, it can be shown that if a total energy U is absorbed by a given area, the momentum, p, that it receives is

$$p = \frac{U}{c} \qquad 25\text{–}11$$

For an electromagnetic wave absorbed by an area A, the total energy received in the time Δt is $U = u_{av}Ac\Delta t$; hence, the momentum, Δp, received in this time is

$$\Delta p = \frac{u_{av}Ac\Delta t}{c} = \frac{I_{av}A\Delta t}{c}$$

Since the average force is $F_{av} = \Delta p/\Delta t = u_{av}A = I_{av}A/c$, it follows that the average pressure (force per area) is simply

$$\text{pressure}_{av} = \frac{I_{av}}{c} \qquad \text{25–12}$$

The pressure exerted by light is commonly referred to as **radiation pressure.**

CONCEPTUAL CHECKPOINT 25–2 MOMENTUM TRANSFER

When an electromagnetic wave carrying an energy U is absorbed by an object, the momentum the object receives is $p = U/c$. If, instead, the object reflects the wave, is the momentum the object receives **(a)** more than, **(b)** less than, or **(c)** the same as when it absorbs the wave?

REASONING AND DISCUSSION

When the object reflects the wave, it must supply not only enough momentum to stop the wave, $p = U/c$, but an equivalent amount of momentum to send the wave back in the opposite direction. Thus, the momentum the object receives is $p = 2U/c$. This situation is completely analogous to the momentum transfer of a ball that either sticks to a wall or bounces back the way it came.

ANSWER

(a) The object receives twice as much momentum.

Radiation pressure is a very real effect, though in everyday situations it is too small to be noticed. In principle, turning on a flashlight should give the user a "kick," like the recoil from firing a gun. In practice, the effect is much too small to be felt. In the next Exercise, we calculate the radiation pressure due to sunlight to get a feel for the magnitudes involved.

EXERCISE 25–4

On a sunny day, the average intensity of sunlight on Earth's surface is about $1.00 \times 10^3\ \text{W/m}^2$. Find **(a)** the average radiation pressure due to the sunlight and **(b)** the average force exerted by the light on a 1.00-m × 2.50-m beach towel. For part (b), assume that the towel absorbs all the light that falls on it.

SOLUTION

a. Divide the average intensity by the speed of light to obtain the average pressure:

$$\text{pressure}_{av} = \frac{I_{av}}{c} = \frac{1.00 \times 10^3\ \text{W/m}^2}{3.00 \times 10^8\ \text{m/s}} = 3.33 \times 10^{-6}\ \text{N/m}^2$$

b. Multiply pressure times area to find the force exerted on the towel:

$$F_{av} = \text{pressure}_{av}A = (3.33 \times 10^{-6}\ \text{N/m}^2)(1.00\ \text{m} \times 2.50\ \text{m}) = 8.33 \times 10^{-6}\ \text{N}$$

Since a newton is about a quarter of a pound, it follows that the force exerted on the beach towel by sunlight is incredibly small.

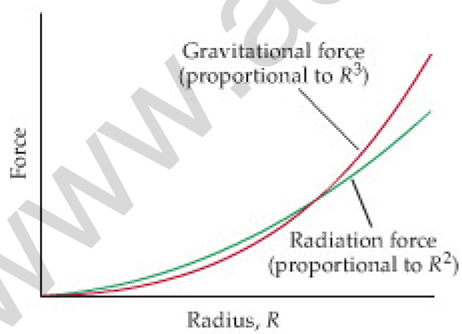


FIGURE 25–10 Radiation versus gravitational forces

The forces exerted on a small particle of radius R by gravity and radiation pressure. For large R the gravitational force dominates; for small R the radiation force is larger.

Even though the radiation force on a beach towel is negligible, the effects of radiation pressure can be significant on particles that are very small. Consider a small object of radius R drifting through space somewhere in our solar system. The gravitational attraction it feels from the Sun depends on its mass. Since mass is proportional to volume, and volume is proportional to the cube of the particle's radius, the gravitational force on the particle varies as R^3. On the other hand, the radiation pressure is exerted over the area of the object, and the area is proportional to R^2. As R becomes smaller, the radiation pressure, with its R^2 dependence, decreases less rapidly than the gravitational force, with its R^3 dependence, as illustrated in Figure 25–10. Thus, for small enough particles, the radiation pressure from sunlight is actually more important than the gravitational force. It is for this reason that dust particles given off by a comet are "blown" away by sunlight, giving the comet a long tail that streams away from the sun. Some imaginative thinkers have envisaged a "sailing" ship designed to travel through the cosmos using this light-pressure "wind."

Finally, it has been discovered that the energy carried by light comes in discrete units, as if there were "particles" of energy in a light beam. So, in many ways, light behaves just like any other wave, showing the effects of diffraction and interference, whereas in other ways it acts like a particle. Therefore, light, and all electromagnetic waves, have a "dual" nature, exhibiting both wave and particle properties. As we shall see when we consider quantum physics in Chapter 30, this *wave–particle duality* plays a fundamental role in our understanding of modern physics.

▲ Like many comets, comet Hale-Bopp, which passed relatively close to Earth in 1997, developed two tails as it approached the Sun. The straighter, blue tail in this photograph is gas boiling off the comet's head as volatile material is vaporized by the Sun's heat. The curved, whiter tail consists mostly of dust particles, blown away by the pressure of sunlight.

25–5 Polarization

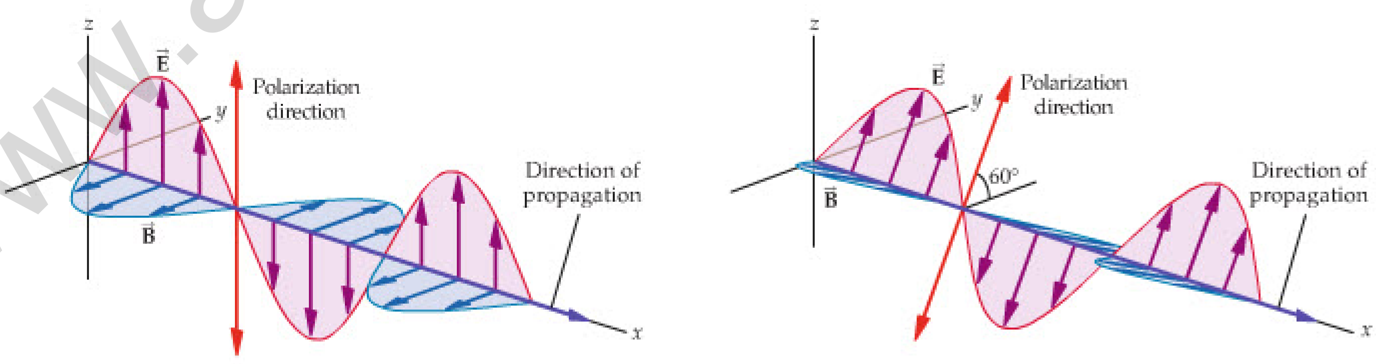
When looking into the blue sky of a crystal-clear day, we see light that is fairly uniform—as long as we refrain from looking too close to the Sun. However, for some animals, like the common honeybee, the light in the sky is far from uniform. The reason is that honeybees are sensitive to the **polarization** of light, an ability that aids in their navigation from hive to flower and back.

To understand what is meant by the polarization of light, or any other electromagnetic wave, consider the electromagnetic waves pictured in **Figure 25–11**. Each of these waves has an electric field that points along a single line. For example, the electric field in Figure 25–11 (a) points in either the positive or negative z direction. We say, then, that this wave is **linearly polarized** in the z direction. A wave of this sort might be produced by a straight-wire antenna oriented along the z axis. Similarly, the direction of polarization for the wave in Figure 25–11 (b) is in the y-z plane at an angle of 60° relative to the y direction. In general, *the polarization of an electromagnetic wave refers to the direction of its electric field.*

A convenient way to represent the polarization of a beam of light is shown in **Figure 25–12**. In part (a) of this figure, we indicate light that is polarized in the vertical direction. Not all light is polarized, however. In part (b) of Figure 25–12 we show light that is a combination of many waves with polarizations in different, random directions. Such light is said to be **unpolarized.** A common incandescent lightbulb produces unpolarized light because each atom in the heated filament sends out light of random polarization. Similarly, the light from the Sun is unpolarized.

Passing Light Through Polarizers

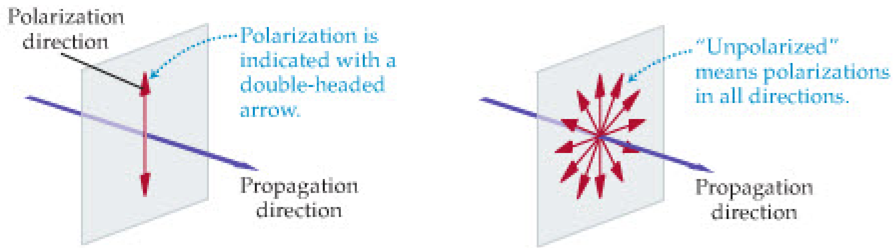
A beam of unpolarized light can be polarized in a number of ways, including by passing it through a **polarizer.** To be specific, a polarizer is a material that is



(a) Polarization in z direction **(b)** Polarization 60° to y direction

▲ **FIGURE 25–11 Polarization of electromagnetic waves**

The polarization of an electromagnetic wave is the direction along which its electric field vector, $\vec{E}$, points. The cases shown illustrate **(a)** polarization in the z direction and **(b)** polarization in the y-z plane, at an angle of 60° with respect to the y axis.



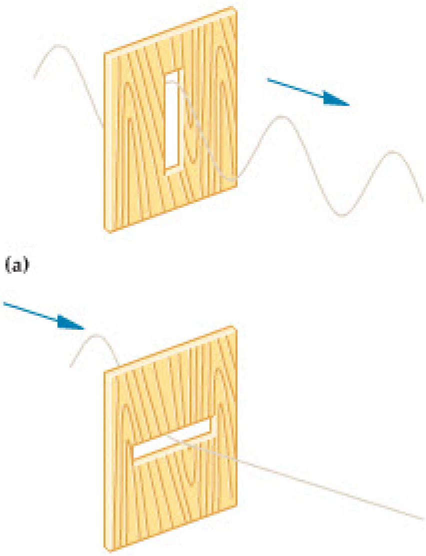
(a) Vertically polarized light **(b)** Unpolarized light

▶ **FIGURE 25–12 Polarized versus unpolarized light**

A beam of light that is **(a)** polarized in the vertical direction and **(b)** unpolarized.

composed of long, thin, electrically conductive molecules oriented in a specific direction. When a beam of light strikes a polarizer, it is readily absorbed if its electric field is parallel to the molecules; light whose electric field is perpendicular to the molecules passes through the material with little absorption. As a result, the light that passes through a polarizer is preferentially polarized along a specific direction. Common examples of a polarizer are the well-known Polaroid sheets used to make Polaroid sunglasses.

A simple mechanical analog of a polarizer is shown in **Figure 25–13**. Here we see a wave that displaces a string in the vertical direction as it propagates toward a slit cut into a block of wood. If the slit is oriented vertically, as in Figure 25–13 (a), the wave passes through unhindered. Conversely, when the slit is oriented horizontally it stops the wave, as indicated in Figure 25–13 (b). A polarizer performs a similar function on a beam of light.



(a) (b)

▲ **FIGURE 25–13 A mechanical analog of a polarizer**

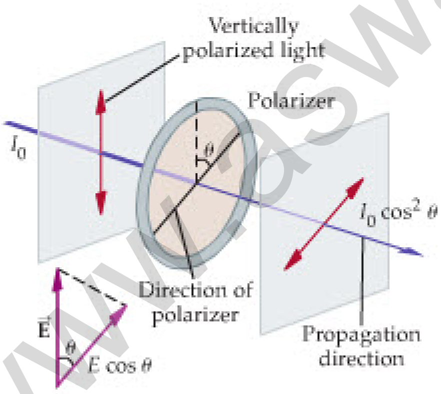
(a) The polarization of the wave is in the same direction as the polarizer; hence, the wave passes through unaffected. **(b)** The polarization of the wave is at right angles to the direction of the polarizer. In this case the wave is absorbed.

We now consider what happens when a beam of light polarized in one direction encounters a polarizer oriented in a different direction. The situation is illustrated in **Figure 25–14**, where we see light with a vertical polarization and intensity I_0 passing through a polarizer with its preferred direction—its **transmission axis**—at an angle θ to the vertical. As shown in the figure, the component of $\vec{E}$ along the transmission axis is $E\cos\theta$. Recalling that the intensity of light is proportional to the electric field squared, we see that the intensity, I, of the transmitted beam is reduced by the factor $\cos^2\theta$. Therefore,

Law of Malus

$$I = I_0 \cos^2 \theta \qquad 25\text{–}13$$

This result is known as the **law of Malus**, after the French engineer Etienne-Louis Malus (1775–1812). Notice that the intensity is unchanged if $\theta = 0$, and is zero if $\theta = 90°$.



▲ **FIGURE 25–14 Transmission of polarized light through a polarizer**

A polarized beam of light, with intensity I_0, encounters a polarizer oriented at an angle θ relative to the polarization direction. The intensity of light transmitted through the polarizer is $I = I_0 \cos^2 \theta$. After passing through the polarizer, the light is polarized in the same direction as the polarizer.

EXERCISE 25–5

Vertically polarized light with an intensity of 515 W/m^2 passes through a polarizer oriented at an angle θ to the vertical. Find the transmitted intensity of the light for **(a)** $\theta = 10.0°$, **(b)** $\theta = 45.0°$, and **(c)** $\theta = 90.0°$.

SOLUTION

Applying $I = I_0 \cos^2 \theta$ we obtain

a. $I = (515\ \text{W/m}^2)\cos^2 10.0° = 499\ \text{W/m}^2$

b. $I = (515\ \text{W/m}^2)\cos^2 45.0° = 258\ \text{W/m}^2$

c. $I = (515\ \text{W/m}^2)\cos^2 90.0° = 0$

Just as important as the change in intensity is what happens to the *polarization* of the transmitted light:

> The transmitted beam of light is no longer polarized in its original direction; it is now polarized in the direction of the polarizer.

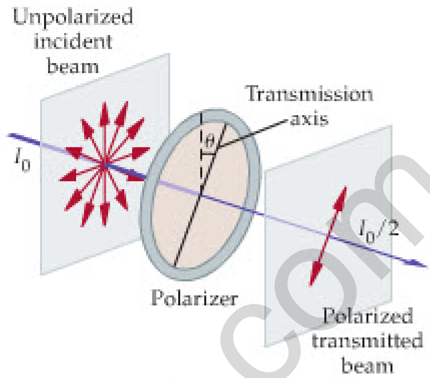
This effect also is illustrated in Figure 25–14. Thus, a polarizer changes both the intensity *and* the polarization of a beam of light.

Figure 25–15 shows an unpolarized beam of light with an intensity I_0 encountering a polarizer. In this case, there is no single angle θ; instead, to obtain the transmitted intensity, we must average $\cos^2\theta$ over all angles. This has already been done in Section 24–1, where we considered rms values in ac circuits. As was shown there, the average of $\cos^2\theta$ is one-half; thus, the transmitted intensity for an unpolarized beam with an intensity of I_0 is

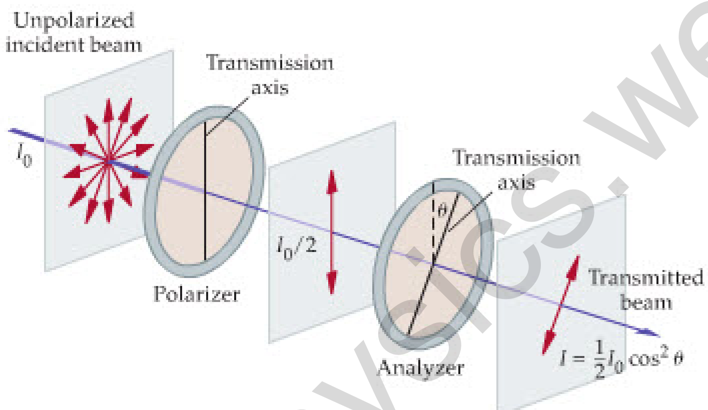
Transmitted Intensity for an Unpolarized Beam

$$I = \tfrac{1}{2}I_0 \qquad 25\text{–}14$$

A common type of polarization experiment is shown in **Figure 25–16**. An unpolarized beam is first passed through a polarizer to give the light a specified polarization. The light next passes through a second polarizer, referred to as the **analyzer**, whose transmission axis is at an angle θ relative to the polarizer. The orientation of the analyzer can be adjusted to give a beam of light of variable intensity and polarization. We consider a situation of this type in the next Example.



▲ **FIGURE 25–15 Transmission of unpolarized light through a polarizer**
When an unpolarized beam of intensity I_0 passes through a polarizer, the transmitted beam has an intensity of $\frac{1}{2}I_0$ and is polarized in the direction of the polarizer.



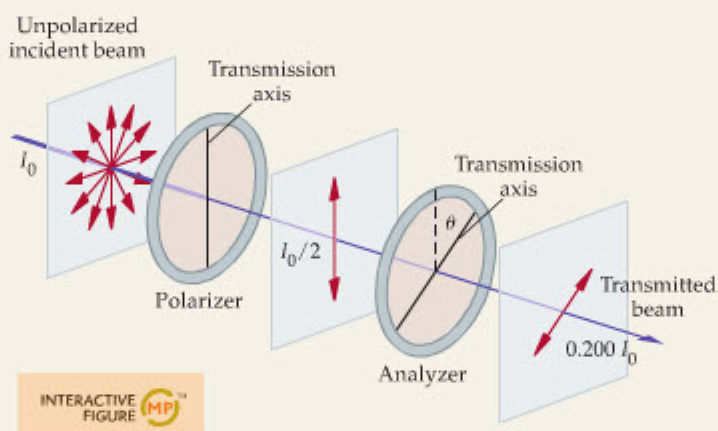
◀ **FIGURE 25–16 A polarizer and an analyzer**
An unpolarized beam of intensity I_0 is polarized in the vertical direction by a polarizer with a vertical transmission axis. Next, it passes through another polarizer, the analyzer, whose transmission axis is at an angle θ relative to the transmission axis of the polarizer. The final intensity of the beam is $I = \frac{1}{2}I_0\cos^2\theta$.

EXAMPLE 25–5 ANALYZE THIS

In the polarization experiment shown in our sketch, the final intensity of the beam is $0.200\,I_0$. What is the angle θ between the transmission axes of the analyzer and polarizer?

PICTURE THE PROBLEM

The experimental setup is shown in the sketch. As indicated, the intensity of the unpolarized incident beam, I_0, is reduced to $\frac{1}{2}I_0$ after passing through the first polarizer. The second polarizer reduces the intensity further, to $0.200\,I_0$.



INTERACTIVE FIGURE

STRATEGY

It is clear from the sketch that the analyzer reduces the intensity of the light by a factor of 1/2.50; that is, the intensity is reduced from $I_0/2$ to $(1/2.50)(I_0/2) = I_0/5.00 = 0.200\,I_0$. Thus we must find the angle θ that gives this reduction. Recalling that an analyzer reduces intensity according to the relation $I = I_0\cos^2\theta$, we set $\cos^2\theta = 1/2.50$ and solve for θ.

SOLUTION

1. Set $\cos^2\theta$ equal to 1/2.50 and solve for $\cos\theta$:

$$\cos^2\theta = \frac{1}{2.50}$$

$$\cos\theta = \frac{1}{\sqrt{2.50}}$$

2. Solve for the angle θ:

$$\theta = \cos^{-1}\left(\frac{1}{\sqrt{2.50}}\right) = 50.8°$$

CONTINUED ON NEXT PAGE

CONTINUED FROM PREVIOUS PAGE

INSIGHT

Since the analyzer absorbs part of the light as the beam passes through, it also absorbs energy. Therefore, in principle, the analyzer would experience a slight heating. As always, energy must be conserved.

PRACTICE PROBLEM

If the angle θ is increased slightly, does the final intensity increase, decrease, or stay the same? Check your answer by finding the final intensity for 60.0°. [**Answer:** The final intensity decreases. For $\theta = 60.0°$, the final intensity is $I = 0.125\ I_0$.]

Some related homework problems: Problem 74, Problem 76, Problem 80

PROBLEM-SOLVING NOTE

Transmission Through Polarizing Filters

The intensity of light transmitted through a pair of polarizing filters depends only on the relative angle θ between the filters; it is independent of whether the filters are rotated clockwise or counterclockwise relative to each other.

ACTIVE EXAMPLE 25–2 FIND THE TRANSMITTED INTENSITY

Calculate the transmitted intensity for the following two cases: **(a)** A vertically polarized beam of intensity I_0 passes through a polarizer with its transmission axis at 60° to the vertical. **(b)** A vertically polarized beam of intensity I_0 passes through two polarizers, the first with its transmission axis at 30° to the vertical, and the second with its transmission axis rotated an additional 30° to the vertical. (*Note:* In both cases, the final beam is polarized at 60° to the vertical.)

SOLUTION *(Test your understanding by performing the calculations indicated in each step.)*

Part (a)

1. Calculate the final intensity using $I = I_0 \cos^2\theta$, with $\theta = 60°$: $I = \frac{1}{4} I_0$

Part (b)

2. Calculate the intermediate intensity using $I = I_0 \cos^2\theta$, with $\theta = 30°$: $I = \frac{3}{4} I_0$
3. Calculate the final intensity using $I = I_0 \cos^2\theta$, with $\theta = 30°$ again: $I = \frac{9}{16} I_0$

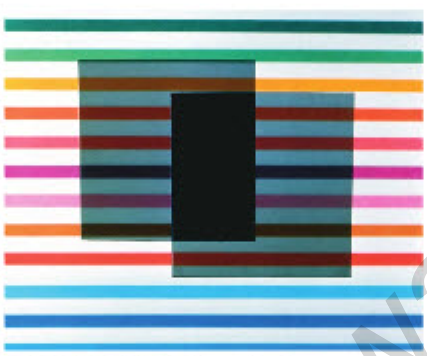
INSIGHT

Even though the direction of polarization is rotated by a total of 60° in each case, the final intensity is more than twice as great when two polarizers are used instead of just one. In general, the more polarizers that are used—and, hence, the more smoothly the direction of polarization changes—the greater the final intensity.

YOUR TURN

Calculate the transmitted intensity when three polarizers are used, the first at 20.0° to the vertical, the second at 40.0° to the vertical, and the third at 60.0° to the vertical. Compare your result with $\frac{9}{16} I_0 = 0.563\ I_0$ found with two polarizers.

(Answers to **Your Turn** *problems are given in the back of the book.)*



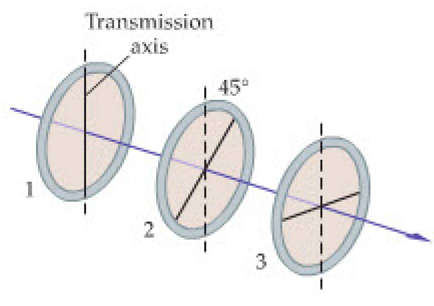
▲ **FIGURE 25–17 Overlapping polarizing sheets**

When unpolarized light strikes a single layer of polarizing material, half of the light is transmitted. This is true regardless of how the transmission axis is oriented. But no light at all can pass through a pair of polarizing filters with axes at right angles (crossed polarizers).

Polarizers with transmission axes at right angles to one another are referred to as "crossed polarizers." The transmission through a pair of crossed polarizers is zero, since $\theta = 90°$ in Malus's law. Crossed polarizers are illustrated in Figure 25–17 and are referred to in the following Conceptual Checkpoint.

CONCEPTUAL CHECKPOINT 25–3 LIGHT TRANSMISSION

Consider a set of three polarizers. Polarizer 1 has a vertical transmission axis, and polarizer 3 has a horizontal transmission axis. Taken together, polarizers 1 and 3 are a pair of crossed polarizers. Polarizer 2, with a transmission angle at 45° to the vertical, is placed between polarizers 1 and 3, as shown in the sketch. A beam of unpolarized light shines on polarizer 1 from the left. Is the transmission of light through the three polarizers **(a)** zero or **(b)** nonzero?



REASONING AND DISCUSSION

Since polarizers 1 and 3 are still crossed, it might seem that no light can be transmitted. When one recalls, however, that a polarizer causes a beam to have a polarization in the direction of its transmission axis, it becomes clear that transmission is indeed possible.

To be specific, some of the light that passes through polarizer 1 will also pass through polarizer 2, since the angle between their transmission axes is less than 90°. After passing through polarizer 2, the light is polarized at 45° to the vertical. As a result, some of this light can pass through polarizer 3 because, again, the angle between the polarization direction and the transmission axis is less than 90°.

In fact, the intensity of the incident light is reduced by a factor of 2 when it passes through polarizer 1, by a factor of 2 when it passes through polarizer 2 (since $\cos^2 45° = \frac{1}{2}$), and again by a factor of 2 when it passes through polarizer 3. The final intensity, then, is one-eighth the incident intensity.

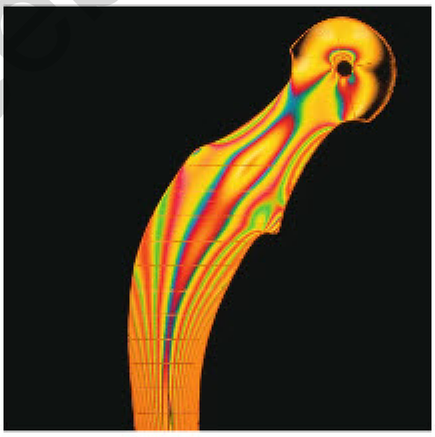
ANSWER

(b) The transmission is nonzero.

There are many practical uses for crossed polarizers. For example, engineers often construct a plastic replica of a building, bridge, or similar structure to study the relative amounts of stress in its various parts in a technique known as *photoelastic stress analysis*. Dentists use the same technique to study stresses in teeth, and doctors use stress analysis when they design prosthetic joints. In this application, the plastic replica plays the role of polarizer 2 in Conceptual Checkpoint 25–3. In particular, in those regions of the structure where the stress is high, the plastic acts to rotate the plane of polarization and—like polarizer 2 in the Conceptual Checkpoint—allows light to pass through the system. By examining such models with crossed polarizers, engineers can gain valuable insight into the safety of the structures they plan to build, dentists can determine where a tooth is likely to break, and doctors can see where an artificial hip joint needs to be strengthened.

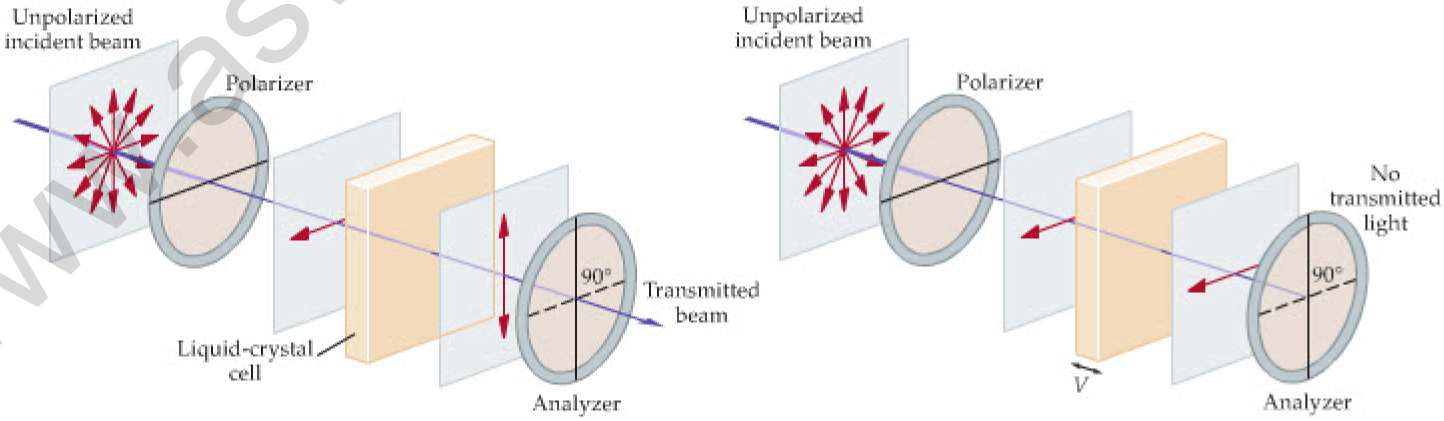
REAL-WORLD PHYSICS

Photoelastic stress analysis



▲ In photoelastic stress analysis, a plastic model of an object being studied is placed between crossed polarizers. In this case, the object is a prosthetic hip joint. If the polarization of the light is unchanged by the plastic, it will not pass through the second polarizer. In areas where the plastic is stressed, however, it rotates the plane of polarization, allowing some of the light to pass through.

Another use of crossed polarizers is in the operation of a liquid-crystal display (LCD). There are basically three essential elements to each active area of an LCD—two crossed polarizers and a thin cell that holds a fluid composed of long, thin molecules known as a **liquid crystal.** The liquid crystal is selected for its ability to rotate the direction of polarization, and the thickness of the liquid-crystal cell is adjusted to give a rotation of 90°. Thus, in its "off" state, the liquid crystal rotates the direction of polarization and light passes through the crossed polarizers, as illustrated in Figure 25–18 (a). In this state, the LCD is transparent—it allows ambient light to enter the display, reflect off the back, and exit the display, giving it the characteristic light background. When a voltage is applied to the liquid crystal, it no longer rotates the direction of polarization, and light is no longer transmitted through that area of the display. Thus, in the "on" state, shown in Figure 25–18 (b), an area of the LCD appears black, which is how the black characters are



(a) Off (transmitted light gives bright background) **(b) On** (dark characters formed where no light is transmitted)

▲ **FIGURE 25–18 Basic operation of a liquid-crystal display (LCD)**

(a) In the "off" state, the liquid crystal in the cell rotates the polarization of the light by 90°, allowing light to pass through the display. This produces the light background of the display. **(b)** In the "on" state, a voltage V is applied to the liquid-crystal cell, which means that the polarization of the light is no longer rotated. As a result, no light is transmitted and this element of the display appears black. This is how the dark characters are created on the display.

REAL-WORLD PHYSICS
Liquid crystal displays (LCDs)

formed on the light background. Since very little energy is required to give the voltage necessary to turn a liquid-crystal cell "on," the LCD is very energy efficient. In addition, the LCD uses light already present in the environment; it does not need to produce its own light, as do some displays.

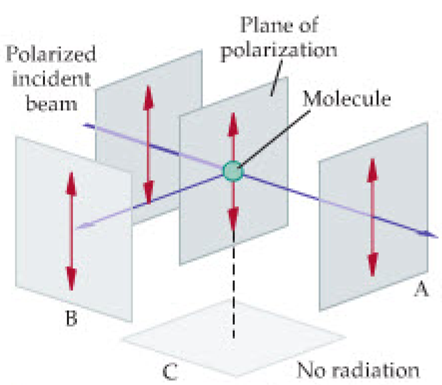
Finally, there are many organic compounds that are capable of rotating the polarization direction of a beam of light. Such compounds, which include sugar, turpentine, and tartaric acid, are said to be **optically active.** When a solution containing optically active molecules is placed between crossed polarizers, the amount of light that passes through the system gives a direct measure of the concentration of the active molecules in the solution.

Polarization by Scattering and Reflection

When unpolarized light is scattered from atoms or molecules, as in the atmosphere, or reflected by a solid or liquid surface, the light can acquire some degree of polarization. The basic reason for this is illustrated in Figure 25–19. In this case, we consider a vertically polarized beam of light that encounters a molecule. The light causes electrons in the molecule to oscillate in the vertical direction, that is, in the direction of the light's electric field. As we know, an accelerating charge radiates—thus, the molecule radiates as if it were a small vertical antenna, and this radiation is what we observe as scattered light. We also know, however, that an antenna gives off maximum radiation in the direction perpendicular to its length, and no radiation at all along its axis. Therefore, an observer at point A or B in Figure 25–19 sees maximum scattered radiation, whereas an observer at point C sees no radiation at all.

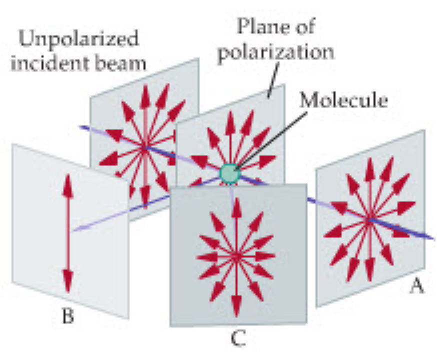
If we apply this same idea to an initially unpolarized beam of light scattering from a molecule, we find the situation pictured in Figure 25–20. In this case, electrons in the molecule oscillate in all directions within the plane of polarization; hence, an observer in the forward direction, at point A, sees scattered light of all polarizations—that is, unpolarized light. An observer at point B, however, sees no radiation from electrons oscillating horizontally, only from those oscillating vertically. Hence this observer sees vertically polarized light. An observer at an intermediate angle sees an intermediate amount of polarization.

This mechanism produces polarization in the light coming from the sky. In particular, maximum polarization is observed in a direction at right angles to the Sun, as can be seen in Figure 25–21. Thus, to a creature that is sensitive to the polarization of light, like a bee or certain species of birds, the light from the sky varies



▲ **FIGURE 25–19 Vertically polarized light scattering from a molecule**

The molecule radiates scattered light, much like a small antenna. At points A and B strong scattered rays are observed; at point C no radiation is seen.



▲ **FIGURE 25–20 Unpolarized light scattering from a molecule**

In the forward direction, A, the scattered light is unpolarized. At right angles to the initial beam of light, B, the scattered light is polarized. Along other directions the light is only partially polarized.

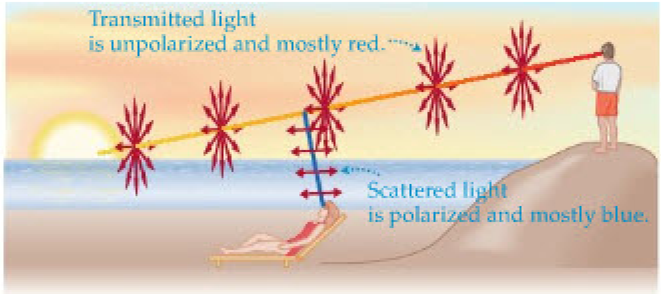


FIGURE 25–21 The effects of scattering on sunlight

The scattering of sunlight by the atmosphere produces polarized light for an observer looking at right angles to the direction of the Sun. This observer also sees more blue light than red. An observer looking toward the Sun sees unpolarized light that contains more red light than blue.

with the direction relative to the Sun. Experiments show that such creatures can use polarized light as an aid in navigation.

REAL-WORLD PHYSICS: BIO

Navigating using polarized light from the sky

Another point of interest follows from the general observation that the amount of light scattered from a molecule is greatest when the wavelength of the light is comparable to the size of the molecule. The molecules in the atmosphere are generally much smaller than the wavelength of visible light, but blue light, with its relatively short wavelength, scatters more effectively than red light, with

As it is scattered by air molecules and dust particles in the atmosphere, sunlight becomes polarized. The shorter wavelengths at the blue end of the spectrum are scattered most effectively, creating our familiar blue skies. Sunsets are red (top) because much of the blue light has been scattered as it passes through the atmosphere on the way to our eyes. The polarization of light from the sky can be demonstrated with a pair of photographs like those at left—one taken without a polarizing filter (left center), the other with a polarizer (left bottom). A smooth surface such as a lake (right) can also act as a polarizing filter. That is why the sky is darker and the clouds more easily seen in the reflected image than in the direct view.

REAL-WORLD PHYSICS
Why the sky is blue

its longer wavelength. Similarly, microscopic particles of dust in the upper atmosphere scatter the short-wavelength blue light most effectively. This basic mechanism is the answer to the age-old question, Why is the sky blue? Similarly, a sunset appears red because you are viewing the Sun through the atmosphere, and most of the Sun's blue light has been scattered off in other directions. In fact, the blue light that is missing from your sunset—giving it a red color—is the blue light of someone else's blue sky, as indicated in Figure 25–21.

REAL-WORLD PHYSICS
How Polaroid sunglasses cut reflected glare

Polarization also occurs when light reflects from a smooth surface, like the top of a table or the surface of a calm lake. A typical reflection situation, with unpolarized light from the Sun reflecting from the surface of a lake, is shown in Figure 25–22. When the light encounters molecules in the water, their electrons oscillate in the plane of polarization. For an observer at point A, however, only oscillations at right angles to the line of sight give rise to radiation. As a result, the reflected light from the lake is polarized horizontally. Polaroid sunglasses take advantage of this effect by using sheets of Polaroid material with a vertical transmission axis. With this orientation, the horizontally polarized reflected light—the glare—is not transmitted.

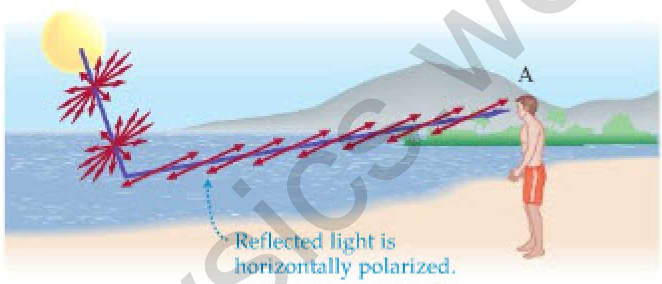


FIGURE 25–22 Polarization by reflection
Because the observer sees no radiation from electrons moving along the line of sight, the radiation that is observed is polarized horizontally. Polaroid sunglasses with a vertical transmission axis reduce this kind of reflected glare.

This presents a potential problem for the makers of digital watches and electronic devices with LCD displays. Suppose the person using such a device is wearing Polaroid sunglasses—not an unusual circumstance. As we saw in Figure 25–18, the light emerging from an LCD display is linearly polarized. If the polarization direction is vertical, the light will pass through the sunglasses and the display can be read as usual. On the other hand, if the polarization direction of the display is horizontal, it will appear completely black through a pair of Polaroid sunglasses—no light will pass through at all. The same effect can be seen by observing an LCD display through a pair of Polaroid sunglasses and slowly rotating the glasses or the display through 90°. Try this experiment sometime with a computer display or a digital watch, and you will see the result shown in the accompanying photographs. Clearly, it would not be wise to make an LCD display with a horizontal polarization direction—a person wearing Polaroid sunglasses would think the display was broken.

To eliminate the glare from reflected light, which is horizontally polarized, the lenses of Polaroid sunglasses have a vertical transmission axis. LCD displays are usually constructed so that the polarized light that comes from them is also vertically polarized. Thus a person wearing Polaroid glasses can view an LCD display (left). If the pair of glasses or the LCD device is rotated 90°, the display becomes invisible (right).

THE BIG PICTURE PUTTING PHYSICS IN CONTEXT

LOOKING BACK

Electric and magnetic fields (from Chapters 19 and 22, respectively) are used throughout this chapter. In fact, we see in Sections 25–1 and 25–2 that $\vec{E}$ and $\vec{B}$ fields are intimately linked, and that together they can produce light waves, radio waves, infrared waves, and other forms of electromagnetic radiation.

The Doppler effect, which we studied for the case of sound in Chapter 14, is applied to light in Section 25–2. We find that the situation is simpler for light than it was for sound in that there is only a single Doppler effect for light, as opposed to two distinct cases with sound.

In Section 25–4 we study the energy, momentum, and intensity associated with electromagnetic waves. These are the same concepts introduced in Chapter 8 (energy), Chapter 9 (momentum), and Chapter 14 (intensity), respectively.

LOOKING AHEAD

The straight-line propagation of light presented in Sections 25–1 and 25–2 is the basis for the geometrical optics to be presented in Chapters 26 and 27.

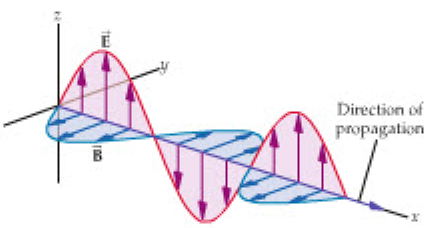
The speed of light, $c = 3.00 \times 10^8$ m/s, applies to all forms of electromagnetic radiation propagating through a vacuum. When electromagnetic radiation propagates through a medium like glass, however, its speed is reduced. This leads to a change in direction of propagation referred to as refraction, as we shall see in Section 26–5.

Light is a fascinating phenomenon, with characteristics of both waves and particles. In this chapter we focused on the wavelike properties of light, and we will expand on this topic in Chapter 28. The particle-like aspects of light will be presented in Chapter 30, when we study quantum physics.

CHAPTER SUMMARY

25–1 THE PRODUCTION OF ELECTROMAGNETIC WAVES

Electromagnetic waves are traveling waves of oscillating electric and magnetic fields.



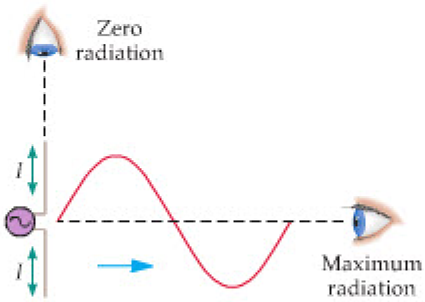
$\vec{E}$ and $\vec{B}$
The electric and magnetic fields in an electromagnetic wave are perpendicular to each other and to the direction of propagation. They are also in phase.

Direction of Propagation
To find the direction of propagation of an electromagnetic wave, point the fingers of your right hand in the direction of $\vec{E}$, then curl them toward $\vec{B}$. Your thumb will be pointing in the direction of propagation.

Generation of Electromagnetic Waves
Any accelerated charge will radiate energy in the form of electromagnetic waves.

25–2 THE PROPAGATION OF ELECTROMAGNETIC WAVES

Electromagnetic waves can travel through a vacuum, and all electromagnetic waves in a vacuum have precisely the same speed, $c = 3.00 \times 10^8$ m/s. This is referred to as the *speed of light in a vacuum*.

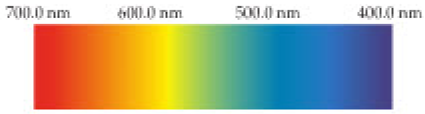


Doppler Effect
Electromagnetic radiation experiences a Doppler effect that is analogous to that observed in sound waves. For relative speeds, u, that are small compared with the speed of light, c, the Doppler effect shifts the frequency of an electromagnetic wave as follows:

$$f' = f\left(1 \pm \frac{u}{c}\right) \qquad 25\text{–}3$$

where the plus sign is used when source and receiver are approaching, the minus sign when they are receding.

25–3 THE ELECTROMAGNETIC SPECTRUM



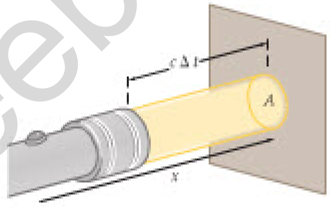
Electromagnetic waves can have any frequency from zero to infinity. The entire range of waves with different frequencies is referred to as the electromagnetic spectrum. Several "bands" of frequency are given special names.

Frequency Bands in the Electromagnetic Spectrum
Radio/TV, 10^6 Hz to 10^9 Hz; microwaves, 10^9 Hz to 10^{12} Hz; infrared, 10^{12} Hz to 10^{14} Hz; visible, 4.29×10^{14} Hz to 7.50×10^{14} Hz; ultraviolet, 10^{15} Hz to 10^{17} Hz; X-rays, 10^{17} Hz to 10^{20} Hz; γ rays, above 10^{20} Hz.

Frequency–Wavelength Relationship
The frequency, f, and wavelength, λ, of all electromagnetic waves in a vacuum are related as follows:

$$c = f\lambda \qquad 25\text{–}4$$

25–4 ENERGY AND MOMENTUM IN ELECTROMAGNETIC WAVES



Electromagnetic waves carry both energy and momentum, shared equally between the electric and magnetic fields.

Energy Density
The energy density, u, of an electromagnetic wave can be written in several equivalent forms:

$$u = \tfrac{1}{2}\varepsilon_0 E^2 + \frac{1}{2\mu_0}B^2 = \varepsilon_0 E^2 = \frac{1}{\mu_0}B^2 \qquad 25\text{–}6$$

Ratio of Electric and Magnetic Fields
In an electromagnetic field, the magnitudes E and B are related as follows:

$$E = cB \qquad 25\text{–}9$$

Intensity
The intensity, I, of an electromagnetic field is the power per area. It can be expressed in the following forms:

$$I = uc = \tfrac{1}{2}c\varepsilon_0 E^2 + \frac{1}{2\mu_0}cB^2 = c\varepsilon_0 E^2 = \frac{c}{\mu_0}B^2 \qquad 25\text{–}10$$

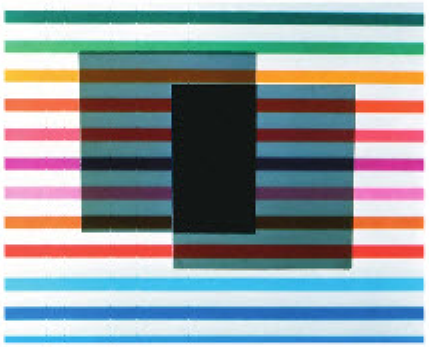
Momentum
An electromagnetic wave that delivers an energy U to an object transfers the momentum

$$p = \frac{U}{c} \qquad 25\text{–}11$$

Radiation Pressure
If an electromagnetic wave shines on an object with an average intensity I_{av}, the average pressure exerted on the object by the radiation is

$$\text{pressure}_{av} = \frac{I_{av}}{c} \qquad 25\text{–}12$$

25–5 POLARIZATION



The polarization of a beam of light is the direction along which its electric field points. An unpolarized beam has components with polarization in random directions.

Polarizer
A polarizer transmits only light whose electric field has a component in the direction of the polarizer's transmission axis.

Law of Malus
If light with intensity I_0 encounters a polarizer with a transmission axis at a direction θ relative to its polarization, the transmitted intensity, I, is

$$I = I_0 \cos^2\theta \qquad 25\text{–}13$$

Unpolarized Light Passing Through a Polarizer
When an unpolarized beam of light of intensity I_0 passes through a polarizer, the transmitted intensity, I, is reduced by half:

$$I = \tfrac{1}{2}I_0 \qquad 25\text{–}14$$

Polarization by Scattering
Light scattered by the atmosphere is polarized when viewed at right angles to the Sun.

Polarization by Reflection
When light reflects from a horizontal surface, like a tabletop or the surface of a lake, it is partially polarized in the horizontal direction.

PROBLEM-SOLVING SUMMARY

Type of Problem	Relevant Physical Concepts	Related Examples
Find the difference in frequency produced by the Doppler effect.	If an object moves with speed u and emits electromagnetic waves of speed c and frequency f, the Doppler-shifted frequency is $f' = f(1 \pm u/c)$. In this expression, the plus sign corresponds to a source approaching the observer; the minus sign corresponds to the source receding from the observer.	Example 25–2
Relate the frequency, f, wavelength, λ, and speed of an electromagnetic wave.	Electromagnetic waves in a vacuum always travel at the speed of light, c, regardless of their frequency. If either the frequency or wavelength of a wave is known, the other quantity can be obtained from the general relation $c = \lambda f$.	Example 25–3
Relate the intensity of an electromagnetic wave to its electric and magnetic fields.	An electromagnetic wave always consists of both electric and magnetic fields. The intensity of a wave, I (energy per area per time), can be related to either field as follows: $I = c\varepsilon_0 E^2 = cB^2/\mu_0$.	Example 25–4 Active Example 25–1
Find the average radiation pressure exerted by an electromagnetic wave.	When electromagnetic waves fall on an object, they exert a pressure known as the radiation pressure. The average radiation pressure, P_{av}, is directly proportional to the average intensity of the wave; that is, $P_{av} = I_{av}/c$.	Exercise 25–4
Find the transmitted intensity of unpolarized light passing through a polarizing filter.	When unpolarized light of intensity I_0 passes through a polarizing filter, its intensity is cut in half; that is, $I = \frac{1}{2}I_0$.	Example 25–5
Find the transmitted intensity of polarized light passing through a polarizing filter.	When polarized light of intensity I_0 passes through a polarizing filter, its transmitted intensity is $I = I_0 \cos^2\theta$, where θ is the angle between the polarization direction of the light and the polarization direction of the filter.	Example 25–5 Active Example 25–2

CONCEPTUAL QUESTIONS

For instructor-assigned homework, go to www.masteringphysics.com

(Answers to odd-numbered Conceptual Questions can be found in the back of the book.)

1. Explain why the "invisible man" would be unable to see.
2. The magnitude of the Doppler effect tells how rapidly a weather system is moving. What determines whether the system is approaching or receding?
3. Explain why radiation pressure is more significant on a grain of dust in interplanetary space when the grain is very small.
4. While wearing your Polaroid sunglasses at the beach, you notice that they reduce the glare from the water better when you are sitting upright than when you are lying on your side. Explain.
5. You want to check the time while wearing your Polaroid sunglasses. If you hold your forearm horizontally, you can read the time easily. If you hold your forearm vertically, however, so that you are looking at your watch sideways, you notice that the display is black. Explain.
6. **BIO Polarization and the Ground Spider** The ground spider *Drassodes cupreus*, like many spiders, has several pairs of eyes. It has been discovered that one of these pairs of eyes acts as a set of polarization filters, with one eye's polarization direction oriented at 90° to the other eye's polarization direction. In addition, experiments show that the spider uses these eyes to aid in navigating to and from its burrow. Explain how such eyes might aid navigation.
7. The electromagnetic waves we pick up on our radios are typically polarized. In contrast, the indoor light we see every day is typically unpolarized. Explain.
8. You are given a sheet of Polaroid material. Describe how to determine the direction of its transmission axis if none is indicated on the sheet.
9. Can sound waves be polarized? Explain.
10. At a garage sale you find a pair of "Polaroid" sunglasses priced to sell. You are not sure, however, if the glasses are truly Polaroid, or if they simply have tinted lenses. How can you tell which is the case? Explain.
11. **3-D Movies** Modern-day 3-D movies are produced by projecting two different images onto the screen, with polarization directions that are at 90° relative to one another. Viewers must wear headsets with polarizing filters to experience the 3-D effect. Explain how this works.

PROBLEMS AND CONCEPTUAL EXERCISES

Note: Answers to odd-numbered Problems and Conceptual Exercises can be found in the back of the book. **IP** *denotes an integrated problem, with both conceptual and numerical parts;* **BIO** *identifies problems of biological or medical interest;* **CE** *indicates a conceptual exercise.* **Predict/Explain** *problems ask for two responses:* **(a)** *your prediction of a physical outcome, and* **(b)** *the best explanation among three provided. On all problems, red bullets (•, ••, •••) are used to indicate the level of difficulty.*

SECTION 25–1 THE PRODUCTION OF ELECTROMAGNETIC WAVES

1. • **CE** If the electric field in an electromagnetic wave is increasing in magnitude at a particular time, is the magnitude of the magnetic field at the same time increasing or decreasing? Explain.

2. • The electric field of an electromagnetic wave points in the positive y direction. At the same time, the magnetic field of this wave points in the positive z direction. In what direction is the wave traveling?

3. • An electric charge on the x axis oscillates sinusoidally about the origin. A distant observer is located at a point on the $+y$ axis. **(a)** In what direction will the electric field oscillate at the observer's location? **(b)** In what direction will the magnetic field oscillate at the observer's location? **(c)** In what direction will the electromagnetic wave propagate at the observer's location?

4. • An electric charge on the z axis oscillates sinusoidally about the origin. A distant observer is located at a point on the $+y$ axis. **(a)** In what direction will the electric field oscillate at the observer's location? **(b)** In what direction will the magnetic field oscillate at the observer's location? **(c)** In what direction will the electromagnetic wave propagate at the observer's location?

5. •• Give the direction (N, S, E, W, up, or down) of the missing quantity for each of the four electromagnetic waves listed in Table 25–1.

TABLE 25–1 Problem 5

Direction of propagation	Direction of electric field	Direction of magnetic field
N	W	**(a)**
N	**(b)**	W
up	S	**(c)**
(d)	down	S

6. •• Give the direction ($\pm x$, $\pm y$, $\pm z$) of the missing quantity for each of the four electromagnetic waves listed in Table 25–2.

TABLE 25–2 Problem 6

Direction of propagation	Direction of electric field	Direction of magnetic field
$+x$	$+y$	**(a)**
$+x$	**(b)**	$+y$
$-y$	$+z$	**(c)**
(d)	$+z$	$+y$

7. •• **IP** At a particular instant of time, a light beam traveling in the positive z direction has an electric field given by $\vec{E} = (6.22\text{ N/C})\hat{x} + (2.87\text{ N/C})\hat{y}$. The magnetic field in the beam has a magnitude of 2.28×10^{-8} T at the same time. **(a)** Does the magnetic field at this time have a z component that is positive, negative, or zero? Explain. **(b)** Write $\vec{B}$ in terms of unit vectors.

8. •• **IP** A light beam traveling in the negative z direction has a magnetic field $\vec{B} = (3.02 \times 10^{-9}\text{ T})\hat{x} + (-5.28 \times 10^{-9}\text{ T})\hat{y}$ at a given instant of time. The electric field in the beam has a magnitude of 1.82 N/C at the same time. **(a)** Does the electric field at this time have a z component that is positive, negative, or zero? Explain. **(b)** Write $\vec{E}$ in terms of unit vectors.

SECTION 25–2 THE PROPAGATION OF ELECTROMAGNETIC WAVES

9. • **CE** Three electromagnetic waves have electric and magnetic fields pointing in the directions shown in Figure 25–23. For each of the three cases, state whether the wave propagates in the $+x$, $-x$, $+y$, $-y$, $+z$, or $-z$ direction.

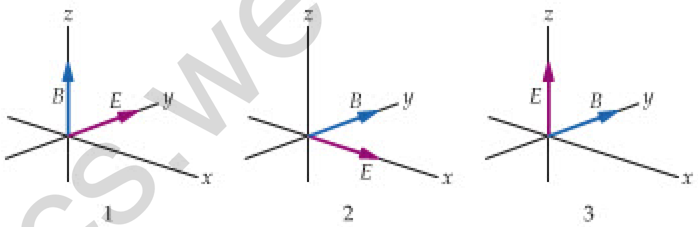


FIGURE 25–23 Problem 9

10. • The light-year (ly) is a unit of distance commonly used in astronomy. It is defined as the distance traveled by light in a vacuum in one year. **(a)** Express 1 ly in km. **(b)** Express the speed of light, c, in units of ly per year. **(c)** Express the speed of light in feet per nanosecond.

11. • Alpha Centauri, the closest star to the sun, is 4.3 ly away. How far is this in meters?

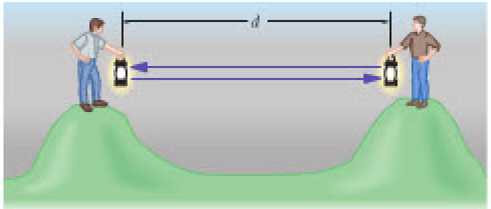
12. • **Mars Rover** When the Mars rover was deployed on the surface of Mars in July 1997, radio signals took about 12 min to travel from Earth to the rover. How far was Mars from Earth at that time?

13. •• A distant star is traveling directly away from Earth with a speed of 37,500 km/s. By what factor are the wavelengths in this star's spectrum changed?

14. •• **IP** A distant star is traveling directly toward Earth with a speed of 37,500 km/s. **(a)** When the wavelengths in this star's spectrum are measured on Earth, are they greater or less than the wavelengths we would find if the star were at rest relative to us? Explain. **(b)** By what fraction are the wavelengths in this star's spectrum shifted?

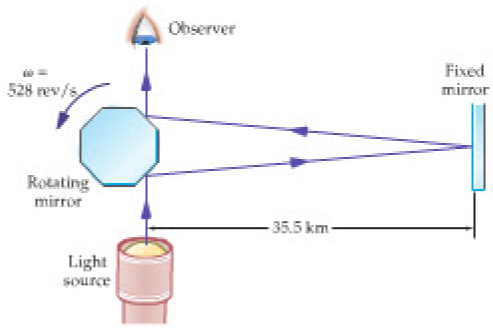
15. •• **IP** The frequency of light reaching Earth from a particular galaxy is 12% lower than the frequency the light had when it was emitted. **(a)** Is this galaxy moving toward or away from Earth? Explain. **(b)** What is the speed of this galaxy relative to the Earth? Give your answer as a fraction of the speed of light.

16. •• **Measuring the Speed of Light** Galileo attempted to measure the speed of light by measuring the time elapsed between his opening a lantern and his seeing the light return from his assistant's lantern. The experiment is illustrated in Figure 25–24. What distance, d, must separate Galileo and his assistant in order for the human reaction time, $\Delta t = 0.2$ s, to introduce no more than a 15% error in the speed of light?



▲ **FIGURE 25–24** Problem 16

17. •• **Measuring the Speed of Light: Michelson** In 1926, Albert Michelson measured the speed of light with a technique similar to that used by Fizeau. Michelson used an eight-sided mirror rotating at 528 rev/s in place of the toothed wheel, as illustrated in Figure 25–25. The distance from the rotating mirror to a distant reflector was 35.5 km. If the light completed the 71.0-km round trip in the time it took the mirror to complete one-eighth of a revolution, what is the speed of light?



▲ **FIGURE 25–25** Problems 17 and 92

18. •• **Communicating with the *Voyager* Spacecraft** When the *Voyager I* and *Voyager II* spacecraft were exploring the outer planets, NASA flight controllers had to plan the crafts' moves well in advance. How many seconds elapse between the time a command is sent from Earth and the time the command is received by *Voyager* at Neptune? Assume the distance from Earth to Neptune is 4.5×10^{12} m.

19. •• A father and his daughter are interested in the same baseball game. The father sits next to his radio at home and listens to the game; his daughter attends the game and sits in the outfield bleachers. In the bottom of the ninth inning a home run is hit. If the father's radio is 132 km from the radio station, and the daughter is 115 m from home plate, who hears the home run first? (Assume that there is no time delay between the baseball being hit and its sound being broadcast by the radio station. In addition, let the speed of sound in the stadium be 343 m/s.)

20. •• **IP** **(a)** How fast would a motorist have to be traveling for a yellow ($\lambda = 590$ nm) traffic light to appear green ($\lambda = 550$ nm) because of the Doppler shift? **(b)** Should the motorist be traveling toward or away from the traffic light to see this effect? Explain.

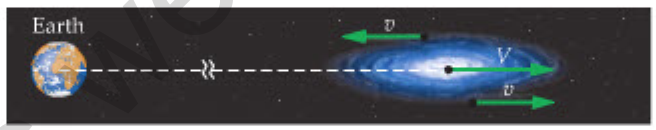
21. •• Most of the galaxies in the universe are observed to be moving away from Earth. Suppose a particular galaxy emits orange light with a frequency of 5.000×10^{14} Hz. If the galaxy is receding from Earth with a speed of 3325 km/s, what is the frequency of the light when it reaches Earth?

22. •• Two starships, the *Enterprise* and the *Constitution*, are approaching each other head-on from a great distance. The separation between them is decreasing at a rate of 782.5 km/s. The *Enterprise* sends a laser signal toward the *Constitution*. If the *Constitution* observes a wavelength $\lambda = 670.3$ nm, what wavelength was emitted by the *Enterprise*?

23. •• Baseball scouts often use a radar gun to measure the speed of a pitch. One particular model of radar gun emits a microwave signal at a frequency of 10.525 GHz. What will be the increase in frequency if these waves are reflected from a 90.0-mi/h fastball headed straight toward the gun? (*Note*: 1 mi/h = 0.447 m/s)

24. •• A state highway patrol car radar unit uses a frequency of 8.00×10^9 Hz. What frequency difference will the unit detect from a car receding at a speed of 44.5 m/s from a stationary patrol car?

25. •• Consider a spiral galaxy that is moving directly away from Earth with a speed $V = 3.600 \times 10^5$ m/s at its center, as shown in Figure 25–26. The galaxy is also rotating about its center, so that points in its spiral arms are moving with a speed $v = 6.400 \times 10^5$ m/s relative to the center. If light with a frequency of 8.230×10^{14} Hz is emitted in both arms of the galaxy, what frequency is detected by astronomers observing the arm that is moving **(a)** toward and **(b)** away from Earth? (Measurements of this type are used to map out the speed of various regions in distant, rotating galaxies.)



▲ **FIGURE 25–26** Problems 25 and 93

26. ••• **IP** A highway patrolman sends a 24.150-GHz radar beam toward a speeding car. The reflected wave is lower in frequency by 4.04 kHz. **(a)** Is the car moving toward or away from the radar gun? Explain. **(b)** What is the speed of the car? [*Hint*: For small values of x, the following approximation may be used: $(1 + x)^2 \approx 1 + 2x$.]

SECTION 25–3 THE ELECTROMAGNETIC SPECTRUM

27. • **BIO Dental X-rays** X-rays produced in the dentist's office typically have a wavelength of 0.30 nm. What is the frequency of these rays?

28. • Find the frequency of blue light with a wavelength of 460 nm.

29. • Yellow light has a wavelength $\lambda = 590$ nm. How many of these waves would span the 1.0-mm thickness of a dime?

30. • How many red wavelengths ($\lambda = 705$ nm) tall are you?

31. • A cell phone transmits at a frequency of 1.75×10^8 Hz. What is the wavelength of the electromagnetic wave used by this phone?

32. • **BIO Human Radiation** Under normal conditions, humans radiate electromagnetic waves with a wavelength of about 9.0 microns. **(a)** What is the frequency of these waves? **(b)** To what portion of the electromagnetic spectrum do these waves belong?

33. • **BIO UV Radiation** Ultraviolet light is typically divided into three categories. UV-A, with wavelengths between 400 nm and 320 nm, has been linked with malignant melanomas. UV-B radiation, which is the primary cause of sunburn and other skin cancers, has wavelengths between 320 nm and 280 nm. Finally, the region known as UV-C extends to wavelengths of 100 nm. **(a)** Find the range of frequencies for UV-B radiation. **(b)** In which of these three categories does radiation with a frequency of 7.9×10^{14} Hz belong?

34. • **Communicating with a Submarine** Normal radiofrequency waves cannot penetrate more than a few meters below the surface of the ocean. One method of communicating with submerged submarines uses very low frequency (VLF) radio waves. What is the wavelength (in air) of a 10.0-kHz VLF radio wave?

35. •• **IP** When an electromagnetic wave travels from one medium to another with a different speed of propagation, the frequency of the wave remains the same. Its wavelength, however, changes. **(a)** If the wave speed decreases, does the wavelength increase or decrease? Explain. **(b)** Consider a case where the wave speed decreases from c to $\frac{3}{4}c$. By what factor does the wavelength change?

36. •• **IP** **(a)** Which color of light has the higher frequency, red or violet? **(b)** Calculate the frequency of blue light with a wavelength of 470 nm, and red light with a wavelength of 680 nm.

37. •• ULF (ultra low frequency) electromagnetic waves, produced in the depths of outer space, have been observed with wavelengths in excess of 29 million kilometers. What is the period of such a wave?

38. •• A television is tuned to a station broadcasting at a frequency of 6.60×10^7 Hz. For best reception, the rabbit-ear antenna used by the TV should be adjusted to have a tip-to-tip length equal to half a wavelength of the broadcast signal. Find the optimum length of the antenna.

39. •• An AM radio station's antenna is constructed to be $\lambda/4$ tall, where λ is the wavelength of the radio waves. How tall should the antenna be for a station broadcasting at a frequency of 810 kHz?

40. •• As you drive by an AM radio station, you notice a sign saying that its antenna is 112 m high. If this height represents one quarter-wavelength of its signal, what is the frequency of the station?

41. •• Find the difference in wavelength ($\lambda_1 - \lambda_2$) for each of the following pairs of radio waves: **(a)** $f_1 = 50$ kHz and $f_2 = 52$ kHz, **(b)** $f_1 = 500$ kHz and $f_2 = 502$ kHz.

42. •• Find the difference in frequency ($f_1 - f_2$) for each of the following pairs of radio waves: **(a)** $\lambda_1 = 300.0$ m and $\lambda_2 = 300.5$ m, **(b)** $\lambda_1 = 30.0$ m and $\lambda_2 = 30.5$ m.

SECTION 25–4 ENERGY AND MOMENTUM IN ELECTROMAGNETIC WAVES

43. • **CE** If the rms value of the electric field in an electromagnetic wave is doubled, **(a)** by what factor does the rms value of the magnetic field change? **(b)** By what factor does the average intensity of the wave change?

44. • **CE** The radiation pressure exerted by beam of light 1 is half the radiation pressure of beam of light 2. If the rms electric field of beam 1 has the value E_0, what is the rms electric field in beam 2?

45. • The maximum magnitude of the electric field in an electromagnetic wave is 0.0400 V/m. What is the maximum magnitude of the magnetic field in this wave?

46. • What is the rms value of the electric field in a sinusoidal electromagnetic wave that has a maximum electric field of 88 V/m?

47. •• The magnetic field in an electromagnetic wave has a peak value given by $B = 3.7\ \mu$T. For this wave, find (a) the peak electric field strength, (b) the peak intensity, and (c) the average intensity.

48. •• What is the maximum value of the electric field in an electromagnetic wave whose maximum intensity is 5.00 W/m^2?

49. •• What is the maximum value of the electric field in an electromagnetic wave whose average intensity is 5.00 W/m^2?

50. •• **IP** Electromagnetic wave 1 has a maximum electric field of $E_0 = 52$ V/m, and electromagnetic wave 2 has a maximum magnetic field of $B_0 = 1.5\ \mu$T. (a) Which wave has the greater intensity? (b) Calculate the intensity of each wave.

51. •• A 65-kW radio station broadcasts its signal uniformly in all directions. (a) What is the average intensity of its signal at a distance of 250 m from the antenna? (b) What is the average intensity of its signal at a distance of 2500 m from the antenna?

52. •• At what distance will a 45-W lightbulb have the same apparent brightness as a 120-W bulb viewed from a distance of 25 m? (Assume that both bulbs convert electrical power to light with the same efficiency, and radiate light uniformly in all directions.)

53. •• What is the ratio of the sunlight intensity reaching Pluto compared with the sunlight intensity reaching Earth? (On average, Pluto is 39 times as far from the Sun as is Earth.)

54. •• **IP** In the following, assume that lightbulbs radiate uniformly in all directions and that 5.0% of their power is converted to light. (a) Find the average intensity of light at a point 2.0 m from a 120-W red lightbulb ($\lambda = 710$ nm). (b) Is the average intensity 2.0 m from a 120-W blue lightbulb ($\lambda = 480$ nm) greater than, less than, or the same as the intensity found in part (a)? Explain. (c) Calculate the average intensity for part (b).

55. •• A 5.0-mW laser produces a narrow beam of light. How much energy is contained in a 1.0-m length of its beam?

56. •• What length of a 5.0-mW laser's beam will contain 9.5 mJ of energy?

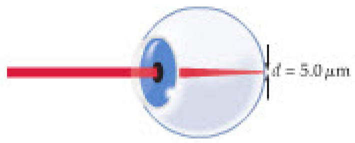
57. •• **Sunlight Intensity** After filtering through the atmosphere, the Sun's radiation illuminates Earth's surface with an average intensity of 1.0 kW/m^2. Assuming this radiation strikes the 15-m $\times$ 45-m black, flat roof of a building at normal incidence, calculate the average force the radiation exerts on the roof.

58. •• **IP** (a) Find the electric and magnetic field amplitudes in an electromagnetic wave that has an average energy density of 1.0 J/m^3. (b) By what factor must the field amplitudes be increased if the average energy density is to be doubled to 2.0 J/m^3?

59. •• **Lasers for Fusion** Two of the most powerful lasers in the world are used in nuclear fusion experiments. The NOVA laser produces 40.0 kJ of energy in a pulse that lasts 2.50 ns, and the NIF laser (under construction) will produce a 10.0-ns pulse with 3.00 MJ of energy. (a) Which laser produces more energy in each pulse? (b) Which laser produces the greater average power during each pulse? (c) If the beam diameters are the same, which laser produces the greater average intensity?

60. •• **BIO** You are standing 2.5 m from a 150-W lightbulb. **(a)** If the pupil of your eye is a circle 5.0 mm in diameter, how much energy enters your eye per second? (Assume that 5.0% of the lightbulb's power is converted to light.) **(b)** Repeat part (a) for the case of a 1.0-mm-diameter laser beam with a power of 0.50 mW.

61. •• **BIO** **Laser Safety** A 0.75-mW laser emits a narrow beam of light that enters the eye, as shown in Figure 25–27. **(a)** How much energy is absorbed by the eye in 0.2 s? **(b)** The eye focuses this beam to a tiny spot on the retina, perhaps 5.0 μm in diameter. What is the average intensity of light (in W/cm^2) at this spot? **(c)** Damage to the retina can occur if the average intensity of light exceeds 1.0×10^{-2} W/cm^2. By what factor has the intensity of this laser beam exceeded the safe value?



▲ **FIGURE 25–27** Problems 61 and 94

62. •• Find the rms electric and magnetic fields at a point 2.50 m from a lightbulb that radiates 75.0 W of light uniformly in all directions.

63. •• A 0.50-mW laser produces a beam of light with a diameter of 1.5 mm. **(a)** What is the average intensity of this beam? **(b)** At what distance does a 150-W lightbulb have the same average intensity as that found for the laser beam in part (a)? (Assume that 5.0% of the bulb's power is converted to light.)

64. •• A laser emits a cylindrical beam of light 2.4 mm in diameter. If the average power of the laser is 2.8 mW, what is the rms value of the electric field in the laser beam?

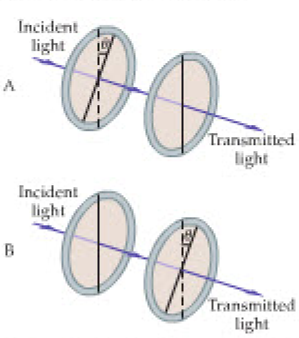
65. •• **(a)** If the laser in Problem 64 shines its light on a perfectly absorbing surface, how much energy does the surface receive in 12 s? **(b)** What is the radiation pressure exerted by the beam?

66. ••• **BIO Laser Surgery** Each pulse produced by an argon–fluoride excimer laser used in PRK and LASIK ophthalmic surgery lasts only 10.0 ns but delivers an energy of 2.50 mJ. **(a)** What is the power produced during each pulse? **(b)** If the beam has a diameter of 0.850 mm, what is the average intensity of the beam during each pulse? **(c)** If the laser emits 55 pulses per second, what is the average power it generates?

67. ••• A pulsed laser produces brief bursts of light. One such laser emits pulses that carry 0.350 J of energy but last only 225 fs. **(a)** What is the average power during one of these pulses? **(b)** Assuming the energy is emitted in a cylindrical beam of light 2.00 mm in diameter, calculate the average intensity of this laser beam. **(c)** What is the rms electric field in this wave?

SECTION 25–5 POLARIZATION

68. • **CE Predict/Explain** Consider the two polarization experiments shown in Figure 25–28. **(a)** If the incident light is unpolarized, is the transmitted intensity in case A greater than, less than, or the same as the transmitted intensity in case B? **(b)** Choose the *best explanation* from among the following:
   I. The transmitted intensity is the same in either case; the first polarizer lets through one-half the incident intensity, and the second polarizer is at an angle θ relative to the first.
   II. Case A has a smaller transmitted intensity than case B because the first polarizer is at an angle θ relative to the incident beam.
   III. Case B has a smaller transmitted intensity than case A because the direction of polarization is rotated by an angle θ in the clockwise direction in case B.

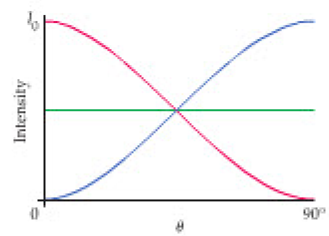


▲ **FIGURE 25–28** Problems 68, 69, and 70

69. • **CE Predict/Explain** Consider the two polarization experiments shown in Figure 25–28. **(a)** If the incident light is polarized in the horizontal direction, is the transmitted intensity in case A greater than, less than, or the same as the transmitted intensity in case B? **(b)** Choose the *best explanation* from among the following:
   I. The two cases have the same transmitted intensity because the angle between the polarizers is θ in each case.
   II. The transmitted intensity is greater in case B because all of the initial beam gets through the first polarizer.
   III. The transmitted intensity in case B is smaller than in case A; in fact, the transmitted intensity in case B is zero because the first polarizer is oriented vertically.

70. • **CE** Suppose linearly polarized light is incident on the polarization experiments shown in Figure 25–28. In what direction, relative to the vertical, must the incident light be polarized if the transmitted intensity is to be the same in both experiments? Explain.

71. • **CE** An incident beam of light with an intensity I_0 passes through a polarizing filter whose transmission axis is at an angle θ to the vertical. As the angle is changed from $\theta = 0$ to $\theta = 90°$, the intensity as a function of angle is given by one of the curves in Figure 25–29. Give the color of the curve corresponding to an incident beam that is **(a)** unpolarized, **(b)** vertically polarized, and **(c)** horizontally polarized.



▲ **FIGURE 25–29** Problem 71

72. • Vertically polarized light with an intensity of 0.55 W/m^2 passes through a polarizer whose transmission axis is at an angle of 65.0° with the vertical. What is the intensity of the transmitted light?

73. • A person riding in a boat observes that the sunlight reflected by the water is polarized parallel to the surface of the water. The person is wearing polarized sunglasses with the polarization axis vertical. If the wearer leans at an angle of 21.5° to the vertical, what fraction of the reflected light intensity will pass through the sunglasses?

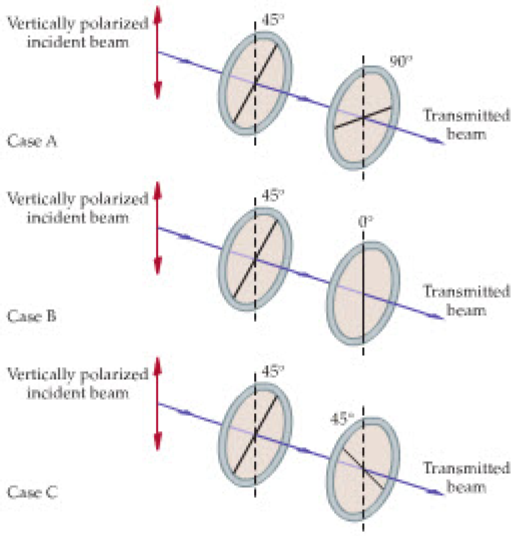
74. •• Unpolarized light passes through two polarizers whose transmission axes are at an angle of 30.0° with respect to each other. What fraction of the incident intensity is transmitted through the polarizers?

75. •• In Problem 74, what should be the angle between the transmission axes of the polarizers if it is desired that one-tenth of the incident intensity be transmitted?

76. •• Unpolarized light with intensity I_0 falls on a polarizing filter whose transmission axis is vertical. The axis of a second polarizing filter makes an angle of θ with the vertical. Plot a graph that shows the intensity of light transmitted by the second filter (expressed as a fraction of I_0) as a function of θ. Your graph should cover the range $\theta = 0°$ to $\theta = 360°$.

77. •• **IP** A beam of vertically polarized light encounters two polarizing filters, as shown in Figure 25–30. **(a)** Rank the three

cases, A, B, and C, in order of increasing transmitted intensity. Indicate ties where appropriate. **(b)** Calculate the transmitted intensity for each of the cases in Figure 25–30, assuming that the incident intensity is 37.0 W/m^2. Verify that your numerical results agree with the rankings in part (a).

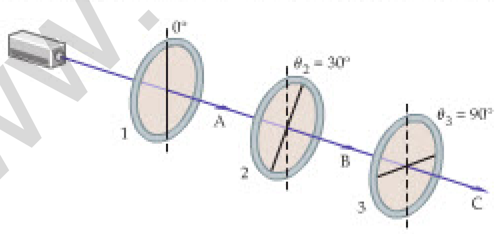


▲ **FIGURE 25–30** Problems 77 and 78

78\. •• **IP** Repeat Problem 77, this time assuming that the polarizers to the left in Figure 25–30 are at an angle of 22.5° to the vertical rather than 45°. The incident intensity is again 37.0 W/m^2.

79\. •• **IP BIO Optical Activity** Optically active molecules have the property of rotating the direction of polarization of linearly polarized light. Many biologically important molecules have this property, some causing a counterclockwise rotation (negative rotation angle), others causing a clockwise rotation (positive rotation angle). For example, a 5.00 gram per 100 mL solution of *l*-leucine causes a rotation of −0.550°; the same concentration of *d*-glutamic acid causes a rotation of 0.620°. **(a)** If placed between crossed polarizers, which of these solutions transmits the greater intensity? Explain. **(b)** Find the transmitted intensity for each of these solutions when placed between crossed polarizers. The incident beam is unpolarized and has an intensity of 12.5 W/m^2.

80\. •• A helium–neon laser emits a beam of unpolarized light that passes through three Polaroid filters, as shown in Figure 25–31. The intensity of the laser beam is I_0. **(a)** What is the intensity of the beam at point A? **(b)** What is the intensity of the beam at point B? **(c)** What is the intensity of the beam at point C? **(d)** If filter 2 is removed, what is the intensity of the beam at point C?



▲ **FIGURE 25–31** Problems 80, 81, 100, and 101

81\. ••• Referring to Figure 25–31, suppose that filter 3 is at a general angle θ with the vertical, rather than the angle 90°. **(a)** Find an expression for the transmitted intensity as a function of θ. **(b)** Plot your result from part (a), and determine the maximum transmitted intensity. **(c)** At what angle θ does maximum transmission occur?

GENERAL PROBLEMS

82\. • **CE** Suppose the magnitude of the electric field in an electromagnetic wave is doubled. **(a)** By what factor does the magnitude of the magnetic field change? **(b)** By what factor does the maximum intensity of the wave change?

83\. • **CE** If "sailors" of the future use radiation pressure to propel their ships, should the surfaces of their sails be absorbing or reflecting? Explain.

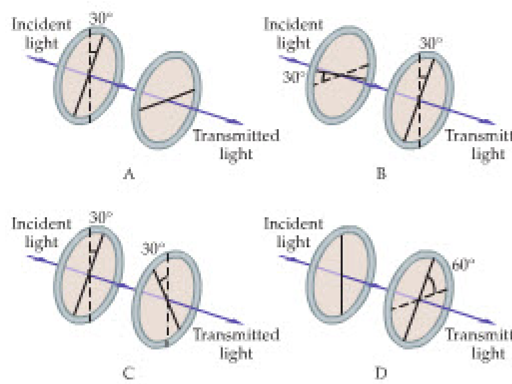
84\. • Sunlight at the surface of Earth has an average intensity of about 1.00×10^3 W/m^2. Find the rms values of the electric and magnetic fields in the sunlight.

85\. • **BIO** A typical medical X-ray has a frequency of 1.50×10^{19} Hz. What is the wavelength of such an X-ray?

86\. • How many hydrogen atoms, 0.10 nm in diameter, must be placed end to end to fit into one wavelength of 410-nm violet light?

87\. • **BIO Radiofrequency Ablation** In radiofrequency (RF) ablation, a small needle is inserted into a cancerous tumor. When radiofrequency oscillating currents are sent into the needle, ions in the neighboring tissue respond by vibrating rapidly, causing local heating to temperatures as high as 100°C. This kills the cancerous cells and, because of the small size of the needle, relatively few of the surrounding healthy cells. A typical RF ablation treatment uses a frequency of 750 kHz. What is the wavelength that such radio waves would have in a vacuum?

88\. •• **CE** Figure 25–32 shows four polarization experiments in which unpolarized incident light passes through two polarizing filters with different orientations. Rank the four cases in order of increasing amount of transmitted light. Indicate ties where appropriate.



▲ **FIGURE 25–32** Problem 88

89\. •• **IP** **(a)** What minimum intensity must a laser beam have if it is to levitate a tiny black (100% absorbing) sphere of radius $r = 0.5$ μm and mass $= 1.6 \times 10^{-15}$ kg? Comment on the feasibility of such levitation. **(b)** If the radius of the sphere is doubled but its mass remains the same, will the minimum intensity be greater than, less than, or equal to the value found in part (a)? Explain. **(c)** Find the minimum intensity for the situation described in part (b).

90\. •• **The *Apollo 11* Reflector** One of the experiments placed on the Moon's surface by *Apollo 11* astronauts was a reflector that is used to measure the Earth–Moon distance with high accuracy. A laser beam on Earth is bounced off the reflector, and its round-trip travel time is recorded. If the travel time can be measured to within an accuracy of 0.030 ns, what is the uncertainty in the Earth–Moon distance?

91. •• The H_β line of the hydrogen atom's spectrum has a normal wavelength $\lambda_\beta = 486$ nm. This same line is observed in the spectrum of a distant quasar, but lengthened by 20.0 nm. What is the speed of the quasar relative to Earth, assuming it is moving along our line of sight?

92. •• **IP** Suppose the distance to the fixed mirror in Figure 25–25 is decreased to 20.5 km. **(a)** Should the angular speed of the rotating mirror be increased or decreased to ensure that the experiment works as described in Problem 17? **(b)** Find the required angular speed, assuming the speed of light is 3.00×10^8 m/s.

93. •• **IP** Suppose the speed of the galaxy in Problem 25 is increased by a factor of 10; that is, $V = 3.600 \times 10^6$ m/s. The speed of the arms, v, and the frequency of the light remain the same. **(a)** Does the arm near the top of Figure 25–26 show a red shift (toward lower frequency) or a blue shift (toward higher frequency)? Does the lower arm show a red or a blue shift? Explain. What frequency is detected by astronomers observing **(b)** the upper arm and **(c)** the lower arm?

94. •• **IP BIO** Consider the physical situation illustrated in Figure 25–27. **(a)** Is E_{rms} in the incident laser beam greater than, less than, or the same as E_{rms} where the beam hits the retina? Explain. **(b)** If the intensity of the beam at the retina is equal to the damage threshold, 1.0×10^{-2} W/cm^2, what is the value of E_{rms} at that location? **(c)** If the diameter of the spot on the retina is reduced by a factor of 2, by what factor does the intensity increase? By what factor does E_{rms} increase?

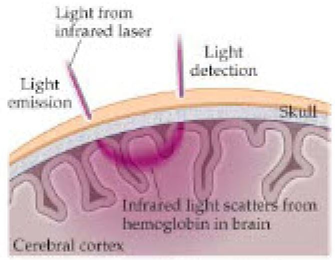
95. •• **BIO Polaroid Vision in a Spider** Experiments show that the ground spider *Drassodes cupreus* uses one of its several pairs of eyes as a polarization detector. In fact, the two eyes in this pair have polarization directions that are at right angles to one another. Suppose linearly polarized light with an intensity of 825 W/m^2 shines from the sky onto the spider, and that the intensity transmitted by one of the polarizing eyes is 232 W/m^2. **(a)** For this eye, what is the angle between the polarization direction of the eye and the polarization direction of the incident light? **(b)** What is the intensity transmitted by the other polarizing eye?

96. •• A state highway patrol car radar unit uses a frequency of 9.00×10^9 Hz. What frequency difference will the unit detect from a car approaching a parked patrol car with a speed of 35.0 m/s?

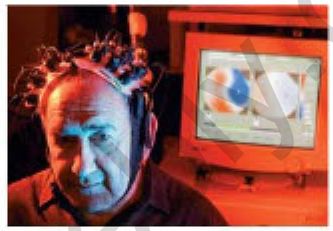
97. •• What is the ratio of the sunlight intensity reaching Mercury compared with the sunlight intensity reaching Earth? (On average, Mercury's distance from the Sun is 0.39 that of Earth's.)

98. •• What area is needed for a solar collector to absorb 45.0 kW of power from the Sun's radiation if the collector is 75.0% efficient? (At the surface of Earth, sunlight has an average intensity of 1.00×10^3 W/m^2.)

99. •• **BIO Near-Infrared Brain Scans** Light in the near-infrared (close to visible red) can penetrate surprisingly far through human tissue, a fact that is being used to "illuminate" the interior of the brain in a noninvasive technique known as near-infrared spectroscopy (NIRS). In this procedure, illustrated in Figure 25–33, an optical fiber carrying a beam of infrared laser light with a power of 1.5 mW and a cross-sectional diameter of 1.2 mm is placed against the skull. Some of the light enters the brain, where it scatters from hemoglobin in the blood. The scattered light is picked up by a detector and analyzed by a computer. **(a)** According to the **Beer–Lambert law,** the intensity of light, I, decreases with penetration distance, d, as $I = I_0 e^{-\mu d}$, where I_0 is the initial intensity of the beam and $\mu = 4.7$ cm^{-1} for a typical case. Find the intensity of the laser beam after it penetrates through 3.0 cm of tissue. **(b)** Find the electric field of the initial light beam.



▲ **FIGURE 25–33** The basic elements of a near-infrared brain scan. (Problem 99)



A patient undergoing a near-infrared brain scan. (Problem 99)

100. •• Three polarizers are arranged as shown in Figure 25–31. If the incident beam of light is unpolarized and has an intensity of 1.60 W/m^2, find the transmitted intensity **(a)** when $\theta_2 = 25.0°$ and $\theta_3 = 50.0°$, and **(b)** when $\theta_2 = 50.0°$ and $\theta_3 = 25.0°$.

101. •• Repeat Problem 100, this time assuming an incident beam that is vertically polarized. The intensity of the incident beam is 1.60 W/m^2.

102. •• A lightbulb emits light uniformly in all directions. If the rms electric field of this light is 16.0 N/C at a distance of 1.35 m from the bulb, what is the average total power radiated by the bulb?

103. •• **IP** A beam of light is a mixture of unpolarized light with intensity I_u and linearly polarized light with intensity I_p. The polarization direction for the polarized light is vertical. When this mixed beam of light is passed through a polarizer that is vertical, the transmitted intensity is 16.8 W/m^2; when the polarizer is at an angle of 55.0° with the vertical, the transmitted intensity is 8.68 W/m^2. **(a)** Is I_u greater than, less than, or equal to I_p? Explain. **(b)** Calculate I_u and I_p.

104. ••• **BIO** As mentioned in Problem 95, one pair of eyes in a particular species of ground spider has polarization directions that are at right angles to one another. Suppose that linearly polarized light is incident on such a spider. **(a)** Prove that the transmitted intensity of one eye plus the transmitted intensity from the other eye is equal to the incident intensity. **(b)** If the transmitted intensities for the two eyes are 163 W/m^2 and 662 W/m^2, through what angle must the spider rotate to make the transmitted intensities equal to one another?

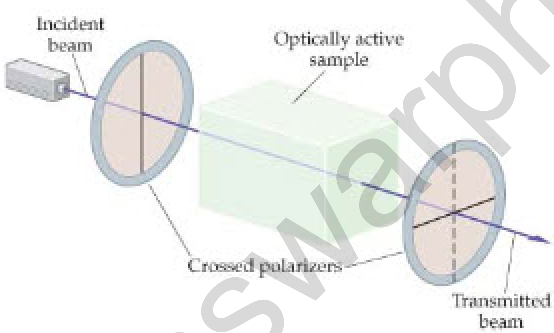
105. ••• A typical home may require a total of 2.00×10^3 kWh of energy per month. Suppose you would like to obtain this energy from sunlight, which has an average daylight intensity of 1.00×10^3 W/m^2. Assuming that sunlight is available 8.0 h per day, 25 d per month (accounting for cloudy days), and that you have a way to store energy from your collector when the Sun isn't shining, determine the smallest collector size that will provide the needed energy, given a conversion efficiency of 25%.

106. ••• At the top of Earth's atmosphere, sunlight has an average intensity of 1360 W/m^2. If the average distance from Earth to the Sun is 1.50×10^{11} m, at what rate does the Sun radiate energy?

107. ••• **IP** A typical laser used in introductory physics laboratories produces a continuous beam of light about 1.0 mm in diameter. The average power of such a laser is 0.75 mW. What are **(a)** the average intensity, **(b)** the peak intensity, and **(c)** the average energy density of this beam? **(d)** If the beam is reflected from a mirror, what is the maximum force the laser beam can exert on it? **(e)** Describe the orientation of the laser beam relative to the mirror for the case of maximum force.

108. ••• Four polarizers are set up so that the transmission axis of each successive polarizer is rotated clockwise by an angle θ relative to the previous polarizer. Find the angle θ for which unpolarized light is transmitted through these four polarizers with its intensity reduced by a factor of 25.

109. ••• **BIO Optical Activity of Sugar** The sugar concentration in a solution (e.g., in a urine specimen) can be measured conveniently by using the *optical activity* of sugar and other asymmetric molecules. In general, an optically active molecule, like sugar, will rotate the plane of polarization through an angle that is proportional to the thickness of the sample and to the concentration of the molecule. To measure the concentration of a given solution, a sample of known thickness is placed between two polarizing filters that are at right angles to each other, as shown in Figure 25–34. The intensity of light transmitted through the two filters can be compared with a calibration chart to determine the concentration. **(a)** What percentage of the incident (unpolarized) light will pass through the first filter? **(b)** If no sample is present, what percentage of the initial light will pass through the second filter? **(c)** When a particular sample is placed between the two filters, the intensity of light emerging from the second filter is 40.0% of the incident intensity. Through what angle did the sample rotate the plane of polarization? **(d)** A second sample has half the sugar concentration of the first sample. Find the intensity of light emerging from the second filter in this case.



▲ **FIGURE 25–34** Problem 109

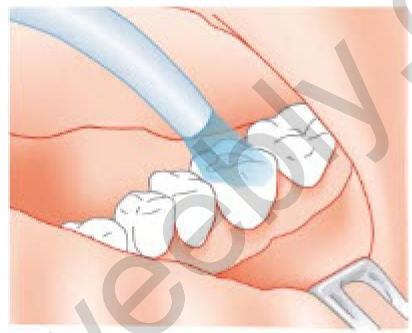
PASSAGE PROBLEMS

Visible-Light Curing in Dentistry

An essential part of modern dentistry is visible-light curing (VLC), a procedure that hardens the restorative materials used in fillings, veneers, and other applications. These "curing lights" work by activating molecules known as photoinitiators within the restorative materials. The photoinitiators, in turn, start a process of polymerization that causes monomers to link together to form a tough, solid polymer network. Thus, with VLC a dentist can apply and shape soft restorative materials as desired, shine a bright light on the result as shown in Figure 25–35, and in 20 seconds have a completely hardened—or cured—final product.

The most common photoinitiator is camphoroquinone (CPQ). To cure CPQ in the least time, one should illuminate it with light having a wavelength of approximately 465 nm. Many VLC units use a halogen light, but there are some drawbacks to this approach. First, the filament in a halogen light is heated to a temperature of about 3000 K, which can cause heat degradation of components in the curing unit itself. Second, less than 1% of the energy given off by a halogen bulb is visible light, so a halogen bulb must have a high power rating to produce the desired light intensity.

More recently, VLC units have begun to use LEDs as their light source. These lights stay cool, emit their energy output as visible light at the desired wavelength, and provide light with an intensity as high as 1000 mW/cm^2, which is about 10 times the intensity of sunlight on the surface of the Earth.



▲ **FIGURE 25–35** An intense beam of light cures, or hardens, the restorative material used to fill a cavity. (Problems 110, 111, 112, and 113)

110. • What is the color of the light that is most effective at activating the photoinitiator CPQ?

A. red **B.** yellow
C. green **D.** blue

111. • What is the frequency of the light that is most effective at activating a CPQ molecule?

A. 140 Hz **B.** 1.00×10^{14} Hz
C. 6.45×10^{14} Hz **D.** 1.55×10^{15} Hz

112. • Suppose a VLC unit uses an LED that produces light with an average intensity of 975 mW/cm^2. What is the rms value of the electric field in this beam of light?

A. 606 N/C **B.** 1920 N/C
C. 3.67×10^6 N/C **D.** 3.32×10^7 N/C

113. • How much radiation pressure does the beam of light in Problem 112 exert on a tooth, assuming the tooth absorbs all the light?

A. 3.25×10^{-5} N/m^2 **B.** 5.70×10^{-3} N/m^2
C. 3.67×10^6 N/m^2 **D.** 1.10×10^{15} N/m^2

INTERACTIVE PROBLEMS

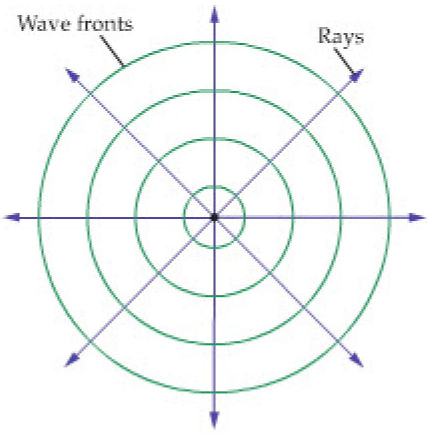
114. •• **IP Referring to Example 25–5** Suppose the incident beam of light is linearly polarized in the same direction θ as the transmission axis of the analyzer. The transmission axis of the polarizer remains vertical. **(a)** What value must θ have if the transmitted intensity is to be 0.200 I_0? **(b)** If θ is increased from the value found in part (a), does the transmitted intensity increase, decrease, or stay the same? Explain.

115. •• **Referring to Example 25–5** Suppose the incident beam of light is linearly polarized in the vertical direction. In addition, the transmission axis of the analyzer is at an angle of 80.0° to the vertical. What angle should the transmission axis of the polarizer make with the vertical if the transmitted intensity is to be a maximum?

26 Geometrical Optics

We all learn early in life that light travels in straight lines—but we also learn, when we first look into a mirror or through a glass of water, that these lines can take some odd bends and turns. The mirrors shown here, for example, present a distorted view of their surroundings, simply because light striking their surface is reflected onto new directions. As we shall see in this chapter, optical devices that change the direction of light—such as mirrors and lenses—follow simple geometrical laws that account for their ability to form images. By analyzing the direction of light rays before and after they encounter a mirror or lens, we can determine the precise location, size, and orientation of the image it produces.

When you look into a mirror, or through a pair of binoculars, you see images of various objects in your surroundings. These images are formed by mirrors or lenses that redirect light coming from the objects. In the case of mirrors, light is *reflected* onto a new path. In lenses the speed of light is reduced, resulting in a change of direction referred to as *refraction*. By changing the direction in which light moves, both mirrors and lenses can be used to create images of various sizes and orientations.



▲ **FIGURE 26–1 Wave fronts and rays**
In this case, the wave fronts indicate the crests of water waves. The rays indicate the local direction of motion.

26–1 The Reflection of Light

Perhaps the simplest way to change the direction of light is by reflection from a shiny surface. To understand this process in detail, it is convenient to describe light in terms of "wave fronts" and "rays." As we shall see later in this chapter, these concepts are equally useful in understanding the behavior of lenses.

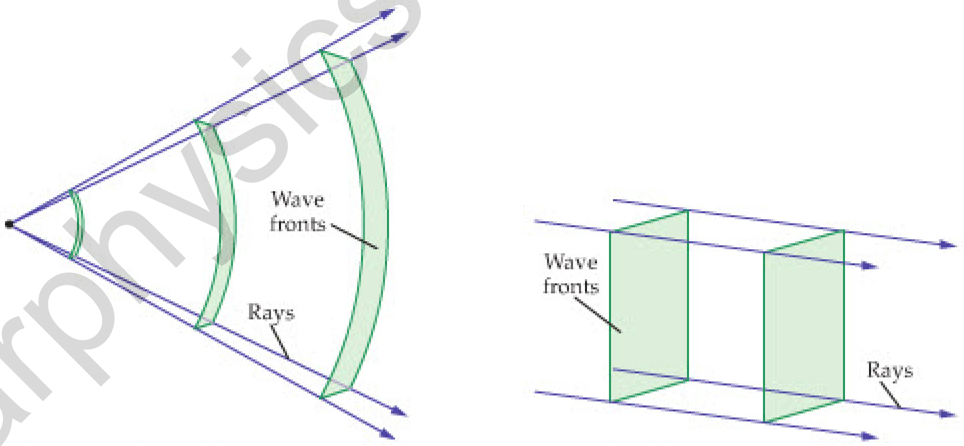
Wave Fronts and Rays

Consider the waves created by a rock dropped into a still pool of water. As we know, these waves form concentric outward-moving circles. A simplified representation of this system is given in Figure 26–1, where the circles indicate the crests of the waves. We refer to these circles as **wave fronts.** In addition, the radial motion of the waves is indicated by the outward-pointing arrows, referred to as **rays.** Notice that the rays are always at right angles to the wave fronts.

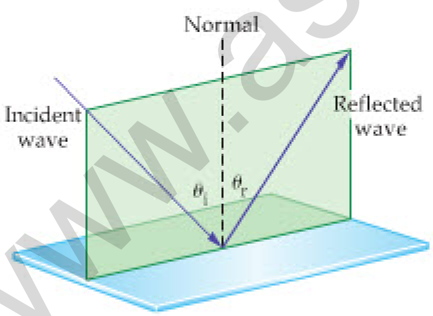
A similar situation applies to electromagnetic waves radiated by a small source, as illustrated in Figure 26–2 (a). In this case, however, the waves move outward in three dimensions, giving rise to spherical wave fronts. As expected, **spherical waves** such as these have rays that point radially outward.

In Figure 26–2 (b) we show that as one moves farther from the source of spherical waves the wave fronts become increasingly flat and the rays more nearly parallel. In the limit of increasing distance, the wave fronts approach perfectly flat planes. Such **plane waves,** with their planar wave fronts and parallel rays, are useful idealizations for investigating the properties of mirrors and lenses.

▶ **FIGURE 26–2 Spherical and planar wave fronts**
(a) As spherical waves move farther from their source, the wave fronts become increasingly flat. **(b)** In the limit of large distances the wave fronts are planes, and the corresponding rays are parallel to one another.



Finally, we usually simplify our representation of light beams even further by omitting the wave fronts and plotting only one, or a few, rays. For example, in Figure 26–3 both the incident plane wave and the reflected plane wave are shown as single rays pointing in the direction of propagation. The direction of these rays is considered next.



▲ **FIGURE 26–3 Reflection from a smooth surface**
In this simplified representation, the incident and reflected waves are indicated by single rays pointing in the direction of propagation. Notice that the angle of reflection, θ_r, is equal to the angle of incidence, θ_i. In addition, the incident ray, reflected ray, and the normal all lie in the same plane.

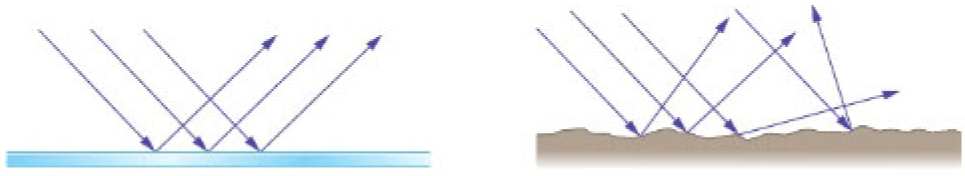
The Law of Reflection

To characterize the behavior of light as it reflects from a mirror or other shiny object, we begin by drawing the normal, which is simply a line perpendicular to the reflecting surface at the point of incidence. Relative to the normal, the incident ray strikes the surface at the angle θ_i, the *angle of incidence*, as shown in Figure 26–3. Similarly, the *angle of reflection*, θ_r, is the angle the reflected ray makes with the normal. The relationship between these two angles is very simple—they are equal:

Law of Reflection

$$\theta_r = \theta_i \qquad 26\text{–}1$$

Note, in addition, that the incident ray, the normal, and the reflected ray all lie in the same plane, as is also clear from Figure 26–3.



(a) Specular reflection (b) Diffuse reflection

FIGURE 26–4 Reflection from smooth and rough surfaces
(a) A smooth surface gives specular reflection, in which all the reflected light propagates in a single direction. **(b)** A rough surface gives rise to reflected waves propagating over a range of directions. This process is referred to as diffuse reflection.

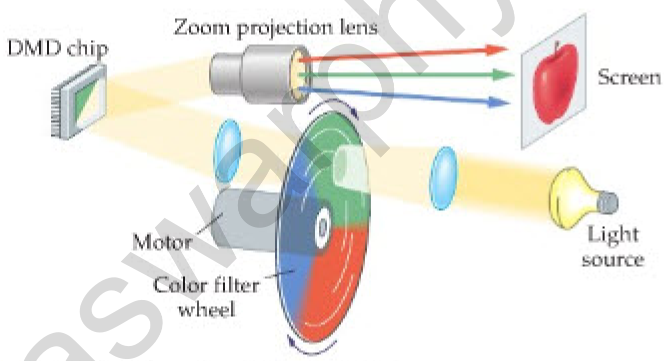
The reflection of light from a smooth shiny surface, as in Figure 26–4 (a), is referred to as **specular reflection.** Notice that all the reflected light moves in the same direction. In contrast, if a surface is rough, as in Figure 26–4 (b), the reflected light is sent out in a variety of directions, giving rise to **diffuse reflection.** For example, when the surface of a road is wet, the water creates a smooth surface, and headlights reflecting from the road undergo specular reflection. As a result, the reflected light goes in a single direction, giving rise to an intense glare. When the same road is dry its surface is microscopically rough; hence, light is reflected in many directions and glare is not observed. The law of reflection is obeyed in either case, of course; it is the surface that is different, not the underlying physics.

A novel variation on specular versus diffuse reflection occurs in a new type of electronic chip known as a Digital Micromirror Device (DMD). These small devices consist of as many as 1.3 million microscopic plane mirrors, each of which, though smaller than the diameter of a human hair, can be oriented independently in response to electrical signals. The reflection from each micromirror is specular, and if all 1.3 million mirrors are oriented in the same direction, the DMD acts like a small plane mirror. Conversely, if the mirrors are oriented randomly, the reflection from the DMD is diffuse. When a DMD is used to project a movie, as will soon be the case in certain theaters, each micromirror will play the role of a single pixel in the projected image. In such a system, the light directed onto the DMD cycles rapidly from red to green to blue, and each of the mirrors reflects only the appropriate colors for that pixel onto the screen (Figure 26–5). The result is a projected image of great brilliance and vividness that eliminates the need for film.

▲ When rays of light are reflected from a smooth, flat surface, both the incident and reflected rays make identical angles with the normal. Consequently, parallel rays are reflected in the same direction, as shown in Figure 26–4 (a). The result is specular reflection, which produces a "mirror image" of the source of the rays.

REAL-WORLD PHYSICS

Micromirror devices and digital movie projection

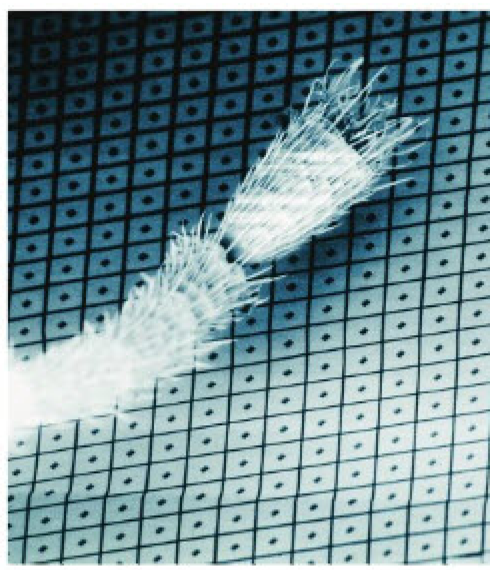


▲ **FIGURE 26–5 A Digital Micromirror Projection System**
A digital projection system based on micromirrors reflects incoming light onto a distant screen. Each micromirror produces one pixel of the final image. The color and intensity of a given pixel are determined by the amount of red, green, and blue light the corresponding micromirror reflects to the screen.

26–2 Forming Images with a Plane Mirror

We are all familiar with looking at ourselves in a mirror. If the mirror is perfectly flat—that is, a **plane mirror**—we see an upright image of ourselves as far behind the mirror as we are in front. In addition, the image is reversed right to left; for example, if we raise our right hand, the mirror image raises its left hand. Lettering read in a mirror is reversed right to left as well. This is the reason ambulances and other emergency vehicles use mirror-image writing on the front of their vehicles—when viewed in the rear-view mirror of a car, the writing looks normal. In this section we use the law of reflection to derive these and other results.

Before we delve into the details of mirror images, however, let's pause for a moment to consider the process by which physical objects produce images in our



▲ An ant's leg provides a sense of scale in this photo of an array of Digital Micromirror Device (DMD) mirrors. Each mirror has an area of 16 μm^2 and can pivot 10° in either direction about a diagonal axis.

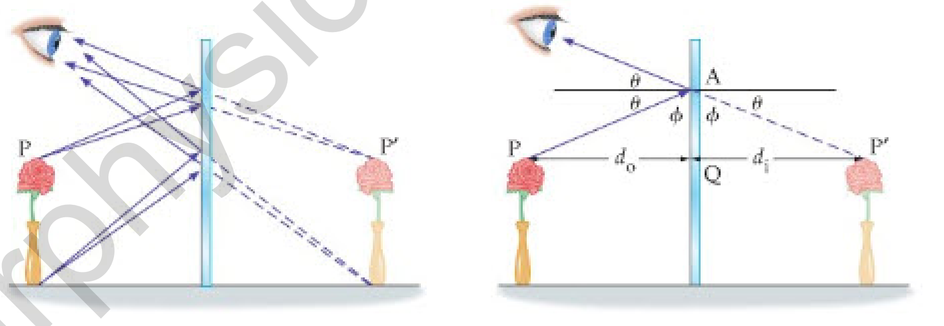
▲ Leonardo da Vinci (1452–1519), the quintessential Renaissance man, tried to keep the contents of his notebooks private by setting down his observations in "mirror writing."

eyes. First, any object that is near you at this moment is bathed in light coming at it from all directions. As this object reflects the incoming light back into the room, every point on it acts like an omnidirectional source of light. When we view the object, the light coming from any given point enters our eyes and is focused to a point on our retina. This is the case *for every point* that we can see on the object—each point on the object is detected by a different point on the retina. This results in a one-to-one mapping between the physical object and the image on the retina.

The formation of a mirror image occurs in a similar manner, except that the light from an object reflects from a mirror before it enters our eyes. This is illustrated in **Figure 26–6 (a)**, where we show an object—a small flower—placed in front of a plane mirror. Rays of light leaving the top of the flower at point P are shown reflecting from the mirror and entering the eye of an observer. To the observer, it appears that the rays are emanating from point P′ *behind the mirror*.

We can show that the image is the same distance behind the mirror as the object is in front. Consider **Figure 26–6 (b)**, where we indicate the distance of the object from the mirror by d_o, and the distance from the image to the mirror by d_i. One ray from the top of the flower is shown reflecting from the mirror and entering the observer's eye. We also show the extension of the reflected ray back to the image. By the law of reflection, if the angle of incidence at point A is θ, the angle of reflection is also θ. Therefore, the straight line from the observer to the image cuts across the normal line with an angle θ on either side, as shown. Clearly, then, the angles indicated by ϕ must also be equal to one another. Combining these results, we see that triangle PAQ shares a side and two adjacent angles with triangle P′AQ; hence, the two triangles are equal. It follows that the distance d_o is equal to the distance d_i, as expected.

▶ **FIGURE 26–6 Locating a mirror image** **(a)** Rays of light from point P at the top of the flower appear to originate from point P′ behind the mirror. **(b)** Construction showing that the distance from the object to the mirror is the same as the distance from the image to the mirror.



(a) Image formed by a plane mirror **(b)** Object distance (d_o) equals image distance (d_i)

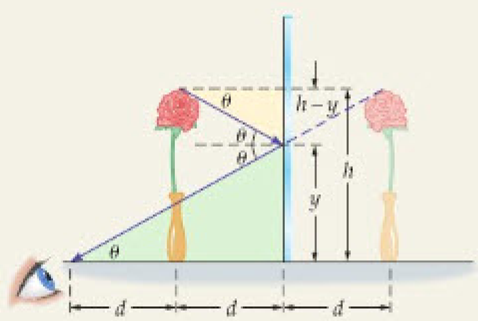
In the following Example, we once again apply the law of reflection to the flower and its mirror image.

EXAMPLE 26–1 REFLECTING ON A FLOWER

An observer is at table level, a distance d to the left of a flower of height h. The flower itself is a distance d to the left of a mirror, as shown in the sketch. Note that a ray of light propagating from the top of the flower to the observer's eye reflects from the mirror at a height y above the table. Find y in terms of the height of the flower, h.

PICTURE THE PROBLEM

The physical situation is shown in the diagram, along with a ray from the top of the flower to the eye of the observer. The point where the ray hits the mirror is a height y above the table. The flower is a distance d to the left of the mirror, and its image is a distance d to the right of the mirror. Finally, the observer's eye is a distance $2d$ to the left of the mirror.



STRATEGY

To find y, we must use the fact that the angle of reflection is equal to the angle of incidence. In the diagram we indicate these two angles by the symbol θ. It follows, then, that $\tan\theta$ obtained from the small yellow triangle must be equal to $\tan\theta$ obtained from the larger green triangle. Setting these expressions for $\tan\theta$ equal to one another yields a relation that can be solved for y.

SOLUTION

1. Write an expression for tan θ using the yellow triangle: $\tan\theta = \dfrac{h-y}{d}$

2. Now, write a similar expression for tan θ using the green triangle: $\tan\theta = \dfrac{y}{2d}$

3. Set the two expressions for tan θ equal to one another: $\dfrac{h-y}{d} = \dfrac{y}{2d}$

4. Rearrange the preceding equation and solve for y: $h - y = \dfrac{y}{2}$

$$y = \frac{2}{3}h$$

INSIGHT

Note that the observer will see the entire image of the flower in a section of mirror that is only two-thirds the height of the flower.

PRACTICE PROBLEM

If the observer moves farther from the base of the flower, does the point of reflection move upward, downward, or stay at the same location? As a check on your answer, calculate the point of reflection for the case where the distance between the observer and the base of the flower is $2d$. [**Answer:** The point of reflection moves upward. In this case, we find $y = \frac{3}{4}h$.]

Some related homework problems: Problem 5, Problem 7

To summarize, the basic features of reflection by a plane mirror are the following:

Properties of Mirror Images Produced by Plane Mirrors

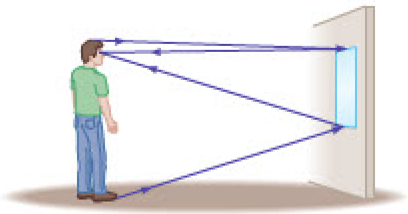
- A mirror image is upright, but appears reversed right to left.
- A mirror image appears to be the same distance behind the mirror that the object is in front of the mirror.
- A mirror image is the same size as the object.

CONCEPTUAL CHECKPOINT 26–1 HEIGHT OF MIRROR

To save expenses, you would like to buy the shortest mirror that will allow you to see your entire body. Should the mirror be **(a)** half your height, **(b)** two-thirds your height, or **(c)** equal to your height?

REASONING AND DISCUSSION

First, to see your feet, the mirror must extend from your eyes downward to a point halfway between your eyes and feet, as shown in the sketch.



Similarly, the mirror must extend upward from your eyes half the distance to the top of your head. Altogether, then, the mirror must have a height equal to half the distance from your eyes to your feet plus half the distance from your eyes to the top of your head—that is, half your total height.

ANSWER

(a) The mirror needs to be only half your height.

An application of mirror images that has been used in military aircraft for many years, and is now beginning to appear in commercial automobiles, is the *heads-up display*. In the case of a car, a small illuminated display screen is recessed in the dashboard, out of direct sight of the driver. The screen shows important information, like the speed of the car, in mirror image. The driver sees the information not by looking directly at the screen—which is hidden from view—but by looking at its reflection in the windshield. Thus, while still looking straight ahead (heads up), the driver can see both the road and the reading of the speedometer.

A similar device is used in theaters to provide subtitles for the hearing impaired. In this case, a transparent plastic screen is mounted on the arm of a person's chair. This screen is adjusted in such a way that the person can look through it to see the movie, and at the same time see the reflection of a screen in the back of the theater that gives the subtitles in mirror-image form.

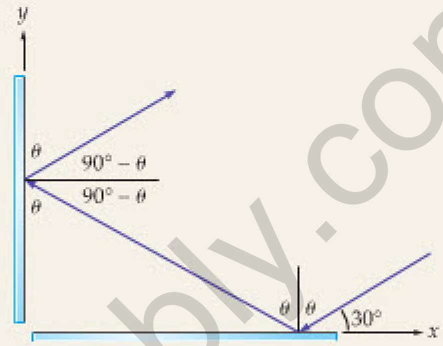
▲ In this heads-up display in an airplane cockpit, important flight information is reflected on a transparent screen in the windshield, enabling the pilot to view the data without diverting attention from the scene ahead.

EXAMPLE 26–2 TWO-DIMENSIONAL CORNER REFLECTOR

Two mirrors are placed at right angles, as shown in the diagram. An incident ray of light makes an angle of 30° with the x axis and reflects from the lower mirror. Find the angle the outgoing ray makes with the y axis after it reflects once from each mirror.

PICTURE THE PROBLEM

The physical system is shown in the diagram. Notice that the normal for the first reflection is vertical, whereas it is horizontal for the second reflection. It follows that the angle of incidence for the first reflection is $\theta = 90° - 30° = 60°$. Similarly, the angle of incidence for the second reflection is $90° - \theta = 30°$.



SOLUTION

We apply the law of reflection to each of the two reflections. For the first reflection, the angle of incidence is $\theta = 60°$; hence, the reflected ray also makes a 60° angle with the vertical.

At the second reflection, the normal is horizontal; hence, the angle of incidence is $90° - \theta = 30°$. After the second reflection, the outgoing ray is 30° above the horizontal and hence it makes an angle $\theta = 60°$ with respect to the y axis, as shown.

INSIGHT

Note that the outgoing ray travels parallel to the incoming ray, but in exactly the opposite direction. This is true regardless of the value of the initial angle θ.

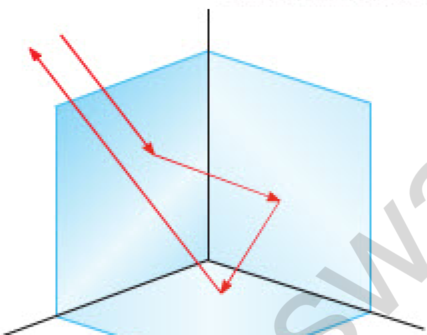
PRACTICE PROBLEM

If the incoming ray hits the horizontal mirror farther to the right, is the angle the outgoing ray makes with the vertical greater than θ, equal to θ, or less than θ? [**Answer:** The angle of the outgoing ray is still equal to θ. The distance between the incoming and outgoing rays would be increased, however.]

Some related homework problems: Problem 8, Problem 16

REAL-WORLD PHYSICS

Corner reflectors and the Earth–Moon distance



▲ FIGURE 26–7 A corner reflector

A three-dimensional corner reflector constructed from three plane mirrors at right angles to one another. A ray entering the reflector is sent back in the direction from which it came.

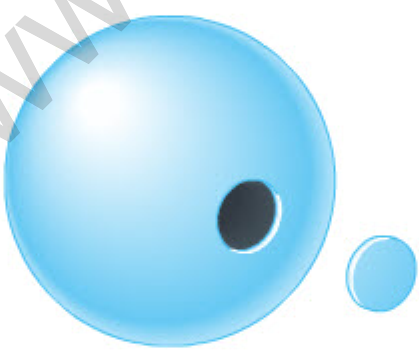
If three plane mirrors are joined at right angles, as in Figure 26–7, the result is a **corner reflector.** A corner reflector behaves in three dimensions the same as two mirrors at right angles in two dimensions; namely, a ray incident on the corner reflector is sent back in the same direction from which it came. This type of behavior has led to many useful applications for corner reflectors.

One of the more interesting applications involves the only Apollo experiments still returning data from the Moon. On *Apollos 11, 14,* and *15* the astronauts placed retroreflector arrays consisting of 100 corner reflectors on the lunar surface. Scientists at observatories on Earth can send a laser beam to the appropriate location on the Moon, where the retroreflector sends it back to its source. By measuring the round-trip travel time of the light, it is possible to determine the Earth–Moon distance to an accuracy of about 3 cm out of a total distance of roughly 385,000 km! Over the years, these measurements have revealed that the Moon is moving away from Earth at the rate of 3.8 cm per year.

26–3 Spherical Mirrors

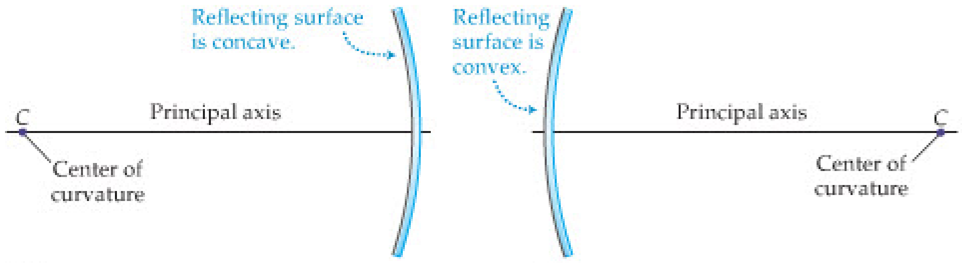
We now consider another type of mirror that is encountered frequently in everyday life—the spherical mirror. These mirrors get their name from the fact that they have the same shape as a section of a sphere, as is shown in Figure 26–8, where a portion of a spherical shell of radius R is cut away from the rest of the shell. If the outside of this spherical section is a reflecting surface, the result is a **convex** mirror; if the inside surface is reflecting, we have a **concave** mirror.

Convex and concave spherical mirrors are illustrated in Figure 26–9, where we also indicate the **center of curvature,** C, and the **principal axis.** The center of



◀ FIGURE 26–8 Spherical mirrors

A spherical mirror has the same shape as a section of a sphere. If the outside surface of this section is reflecting, the mirror is convex. If the inside surface reflects, the mirror is concave.



(a) Concave mirror (b) Convex mirror

FIGURE 26–9 Concave and convex mirrors
(a) A concave mirror and its center of curvature, C, which is on the same side as the reflecting surface. **(b)** A convex mirror and its center of curvature, C. In this case C is on the opposite side of the mirror from its reflecting surface.

curvature is the center of the sphere with radius R of which the mirror is a section, and the principal axis is a straight line drawn through the center of curvature and the midpoint of the mirror. Note that the principal axis intersects the mirror at right angles.

To investigate the behavior of spherical mirrors, suppose that a beam of light is directed toward either a convex or a concave mirror along its principal axis. For example, several parallel rays of light are approaching a convex mirror in Figure 26–10. After reflecting from the mirror, the rays diverge as if they originated from a single point behind the mirror called the **focal point**, F. (This is strictly true only for light rays close to the principal axis, as we point out in greater detail later in this section. All of our results for spherical mirrors assume rays close to the axis.)

Now, let's find the **focal length;** that is, the distance from the surface of the mirror to the focal point. This can be done with the aid of Figure 26–11, which shows a single ray reflecting from the mirror. The first thing to notice about this diagram is that a straight line drawn through the center of curvature always intersects the mirror at right angles; hence, the line through C is the normal to the surface at the point of incidence, A. Since the incoming ray is parallel to the principal axis, it follows that the angle of incidence, θ, is equal to the angle FCA. Next, the law of reflection states that the angle of reflection must equal θ, which means that the angle CAF is also θ. We see, then, that CAF is an isosceles triangle, with the sides CF and FA of equal length. Finally, for small angles θ, the length CF is approximately equal to half the length $CA = R$; that is, $CF \sim \frac{1}{2}R$. Therefore, to this same approximation, the distance FB is also $\frac{1}{2}R$.

Thus, when considering a convex mirror of radius R, we will always use the following result for the focal length:

Focal Length for a Convex Mirror of Radius *R*

$$f = -\tfrac{1}{2}R \qquad 26\text{–}2$$

SI unit: m

The minus sign in this expression is used to indicate that the focal point lies behind the mirror. This is part of a general sign convention for mirrors that will be discussed in detail in the next section.

The situation is similar for a concave mirror. First, rays parallel to the principal axis are reflected by the mirror and brought together at a focal point, F, as shown in Figure 26–12. The same type of analysis used for the convex mirror can be applied to the concave mirror as well, with the result that the focal point is a distance $\frac{1}{2}R$ in front of the mirror. Thus, for a concave mirror,

Focal Length for a Concave Mirror of Radius *R*

$$f = \tfrac{1}{2}R \qquad 26\text{–}3$$

SI unit: m

This time f is positive, since the focal point is in front of the mirror. Note that in this case the rays of light actually pass through the focal point, in contrast to the behavior of the convex mirror.

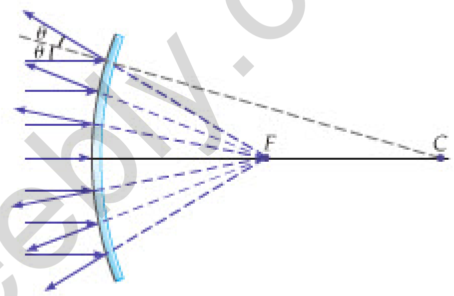


FIGURE 26–10 Parallel rays on a convex mirror
When parallel rays of light reflect from a convex mirror, they diverge as if originating from a focal point halfway between the surface of the mirror and its center of curvature.

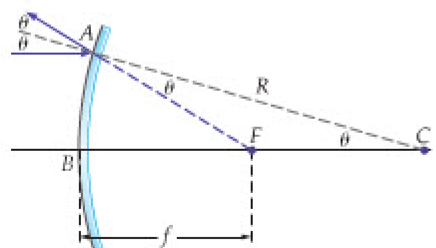


FIGURE 26–11 Ray diagram for a convex mirror
Ray diagram used to locate the focal point, F.

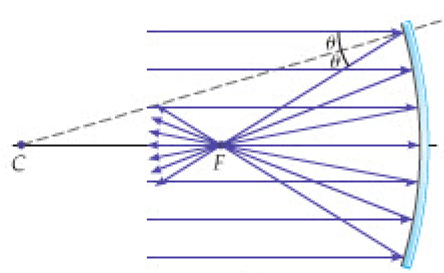


FIGURE 26–12 Parallel rays on a concave mirror
Parallel rays reflecting from a concave mirror pass through the focal point F, which is halfway between the surface of the mirror and the center of curvature.

CONCEPTUAL CHECKPOINT 26–2 STARTING A FIRE

Suppose you would like to use the Sun to start a fire in the wilderness. Which type of mirror, concave or convex, would work best?

REASONING AND DISCUSSION

First, note that rays from the Sun are essentially parallel, since it is at such a great distance. Therefore, if a convex mirror is used, the situation is like that shown in Figure 26–10, in which the rays are spread out after reflection. Conversely, a concave mirror will bring the rays together at a point, as in Figure 26–12. Clearly, by focusing the sunlight at one point, the concave mirror will stand a better chance of starting a fire.

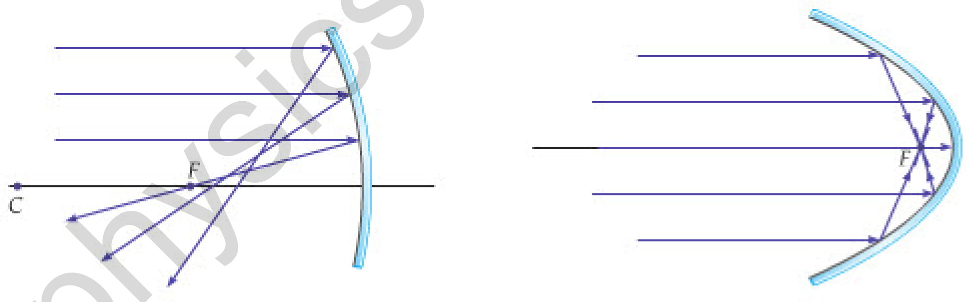
ANSWER

The concave mirror is the one to use.

REAL-WORLD PHYSICS

Parabolic mirrors

A final point regards the approximation made earlier in this section; namely, that the angle θ is small. This is equivalent to saying that the distance between the principal axis of the mirror and the incoming rays is much less than the radius of curvature of the mirror, R. When rays are displaced from the axis by distances comparable to the radius R, as in Figure 26–13 (a), the result is that they do not all pass through the focal point—the farther a ray is from the axis, the more it misses the focal point. In a case like this, the mirror will produce a blurred image, an effect known as **spherical aberration.** This effect can be reduced to undetectable levels by restricting the incoming rays to only those near the axis. These axis-hugging rays are referred to as **paraxial rays.**



(a) A spherical mirror blurs the focus

(b) A parabolic mirror has a single focal point

FIGURE 26–13 Spherical aberration and the parabolic mirror

(a) Rays far from the axis of a spherical mirror are not reflected through its focal point. The result is a blurred image referred to as spherical aberration. **(b)** A mirror with a parabolic cross section brings all parallel rays to a focus at one point, regardless of how far they are from the axis of the mirror.

Another way to eliminate spherical aberration is to construct a mirror with a *parabolic* cross section, as shown in Figure 26–13 (b). One of the key properties of a parabola is that rays parallel to its axis are reflected through the same point F regardless of their distance from the axis. Thus a parabolic mirror produces a sharp image from all the rays coming into it. For this reason, astronomical mirrors, like that of the Hale telescope on Mount Palomar, California, are polished to a parabolic shape to give the greatest possible light-gathering ability and the sharpest possible images.

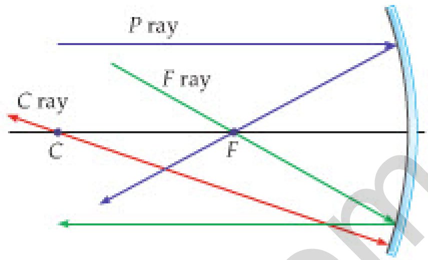
The same principle works in reverse as well. For example, if a source of light is placed at the focal point of a parabolic mirror, as at point F in Figure 26–13 (b), the mirror will redirect the light into an intense, unidirectional beam that can be aimed in a precise direction. Applications of this effect include flashlights, car headlights, and the giant arc lights that sweep across the sky to announce a grand opening.

▲ The mirror of the Hubble Space Telescope (HST) was to have a perfectly parabolic shape. Unfortunately, owing to a mistake in the grinding of the mirror, the images produced by the HST were marred by spherical aberration. This necessitated a repair mission to the orbiting telescope, during which astronauts installed corrective optics to compensate for the defect.

26–4 Ray Tracing and the Mirror Equation

The images formed by a spherical mirror can be more varied than those produced by a plane mirror. In a plane mirror the image is always upright, the same size as the object, and the same distance from the mirror as the object. In the case of spherical mirrors, the image can be either upright or inverted, larger or smaller than the object, and closer or farther from the mirror than the object.

To find the orientation, size, and location of an image in a spherical mirror, we use two techniques. The first, referred to as **ray tracing,** gives the orientation of the image as well as qualitative information on its location and size. If drawn carefully to scale, a ray diagram can also give quantitative results. The second method, using a relation referred to as the **mirror equation,** provides precise quantitative information without the need for accurate scale drawings. Both methods are presented in this section.



▲ **FIGURE 26–14 Principal rays used in ray tracing for a concave mirror**
The parallel ray (*P* ray) reflects through the focal point. The focal ray (*F* ray) reflects parallel to the axis, and the center-of-curvature ray (*C* ray) reflects back along its incoming path.

Ray Tracing

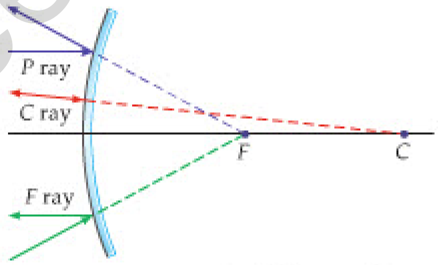
The basic idea behind ray tracing is to follow the path of representative rays of light as they reflect from a mirror and form an image. This was done for the plane mirror in Section 26–2. Ray tracing for spherical mirrors is a straightforward extension of the same basic techniques.

There are three rays with simple behavior that are used most often in ray tracing with spherical mirrors. These rays are illustrated in **Figure 26–14** for a concave mirror, and in **Figure 26–15** for a convex mirror. We start with the parallel ray (*P* ray), which, as its name implies, is parallel to the principal axis of the mirror. As we know from the previous section, a parallel ray is reflected through the focal point of a concave mirror, as shown by the purple ray in Figure 26–14. Similarly, a *P* ray reflects from a convex mirror along a line that *extends back* through the focal point, as with the purple ray in Figure 26–15.

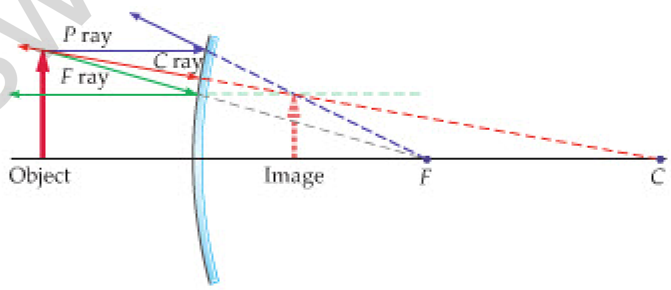
Next, a ray that passes through the focal point of a concave mirror is reflected parallel to the axis, as indicated by the green ray in Figure 26–14. Thus, in a sense, a focal-point ray (*F* ray) is the reverse of a *P* ray. In general, any ray is equally valid in either the forward or reverse direction. The corresponding *F* ray for a convex mirror is shown in green in Figure 26–15.

Finally, note that any straight line extending from the center of curvature intersects the mirror at right angles. Thus, a ray moving along such a path is reflected back along the same path. Center-of-curvature rays (*C* rays) are illustrated in red in Figures 26–14 and 26–15.

To see how these rays can be used to obtain an image, consider the convex mirror shown in **Figure 26–16.** In front of the mirror is an object, represented symbolically by the red arrow. Also indicated in the figure are the three rays described above. Note that these rays diverge from the mirror as if they had originated from the tip of the orange arrow behind the mirror. This arrow is the image of the object; in fact, since no light passes through the image, we call it a **virtual image.** As we can see from the diagram, the virtual image is upright, smaller than the object, and located between the mirror and the focal point *F*.



▲ **FIGURE 26–15 Principal rays used in ray tracing for a convex mirror**
The parallel ray (*P* ray) reflects along a direction that extends back to the focal point. Similarly, a focal ray (*F* ray) moves toward the focal point until it reflects from the mirror, after which it moves parallel to the axis. A center-of-curvature ray (*C* ray) is directed toward the center of curvature. It reflects from the mirror back along its incoming path.



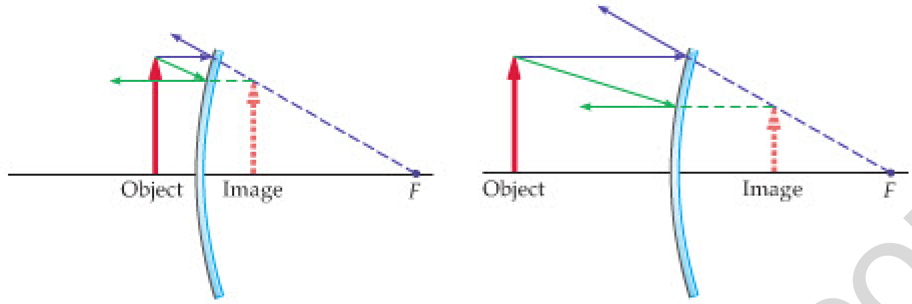
◀ **FIGURE 26–16 Image formation with a convex mirror**
Ray diagram showing an image formed by a convex mirror. The three outgoing rays (*P*, *F*, and *C*) extend back to a single point at the top of the image.

Even though we drew three rays in Figure 26–16, any two would have given the intersection point at the tip of the virtual image. This is commonly the case with ray diagrams. When possible, it is useful to draw all three rays as a check on your results.

Finally, in the limit that the object is very close to a convex mirror, the mirror is essentially flat and behaves like a plane mirror. Thus the virtual image will be about the same distance behind the mirror that the image is in front, and about the same size as the object. Conversely, if the object is far from the mirror, the image is very small and practically at the focal point. These limits are illustrated in **Figure 26–17.**

FIGURE 26–17 Image size and location in a convex mirror

(a) When an object is close to a convex mirror, the image is practically the same size and distance from the mirror. **(b)** In the limit that the object is very far from the mirror, the image is small and close to the focal point.



(a) An object close to a convex mirror

(b) An object far from a convex mirror

PROBLEM-SOLVING NOTE

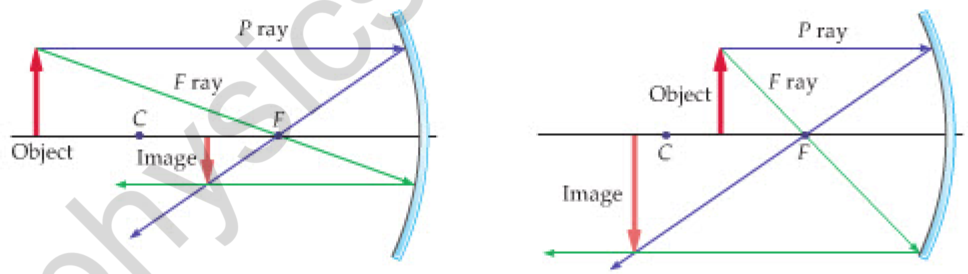
Using Rays to Locate the Image of a Spherical Mirror

To find the top of an image, draw the three rays, *P*, *F*, and *C*, from the top of the object. The rays will either intersect at the top of the image (real image) or extend backward to the top of the image (virtual image). When drawing the rays, remember that a *P* ray is *p*arallel to the axis of the mirror, the *C* ray goes either through or toward the mirror's center of curvature, and the *F* ray goes either through or toward the *f*ocal point of the mirror.

Next we consider the image formed by a concave mirror. In particular, we examine the following three cases: (i) The object is farther from the mirror than the center of curvature; (ii) the object is between the center of curvature and the focal point; and (iii) the object is between the mirror and the focal point. The *F* and *P* rays for case (i) are drawn in **Figure 26–18 (a)**, showing that the image is inverted, closer to the mirror, and smaller than the object. In addition, because light rays pass through the image, we call it a **real image.** The corresponding ray diagram for case (ii) is shown in **Figure 26–18 (b)**. Note that the image is again real and inverted, but it is now farther from the mirror and larger than the object—as the object moves closer to the focal point, an observer will see a large inverted image of the object far in front of the mirror. The *C* ray was not useful in these cases and therefore was omitted from the diagrams.

FIGURE 26–18 Image formation with a concave mirror

Ray diagrams showing the image formed by a concave mirror when the object is **(a)** beyond the center of curvature and **(b)** between the center of curvature and the focal point.



(a) An object beyond *C*

(b) An object between *C* and *F*

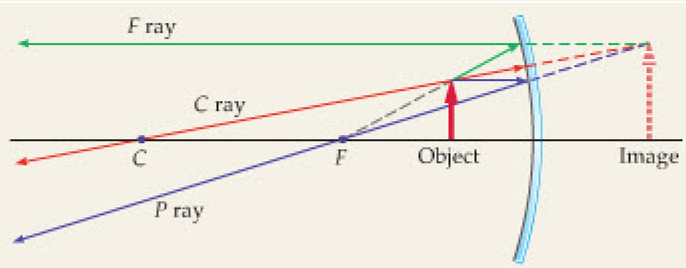
The final case, in which the object is between the mirror and the focal point, is considered in the next Example.

EXAMPLE 26–3 IMAGE FORMATION

Use a ray diagram to find the location, size, and orientation of the image formed by a concave mirror when the object is between the mirror and the focal point.

PICTURE THE PROBLEM

The physical system is shown in the diagram, along with the three principal rays. Notice that after reflection these three rays diverge from one another, just as if they had originated at a point behind the mirror.



INTERACTIVE FIGURE

SOLUTION

The three outgoing rays (*P*, *F*, and *C*) extend back to form an image behind the mirror that is upright (same orientation as the object) and enlarged. We now discuss these three rays one at a time:

***P* ray:** The *P* ray is the most straightforward of the three. It starts parallel to the axis, then reflects through the focal point.

***F* ray:** The *F* ray does not go through the focal point, as is usually the case. Instead, it starts on a line that *extends back* to the focal point, then reflects parallel to the axis.

C ray: The C ray starts at the top of the object, contacts the mirror at right angles, then reflects back along its initial path and through the center of curvature.

INSIGHT

Makeup mirrors are concave mirrors with fairly large focal lengths. The person applying makeup is between the mirror and its focal point, as is the object in this Example, and therefore sees an upright and enlarged image, as desired.

PRACTICE PROBLEM

If the object in the diagram is moved closer to the mirror, does the image increase or decrease in size? [**Answer:** As the object moves closer, the mirror behaves more like a plane mirror. Thus, the image becomes smaller, so that it is closer in size to the object.]

Some related homework problems: Problem 27, Problem 30

CONCEPTUAL CHECKPOINT 26–3 REARVIEW MIRRORS

The passenger-side rearview mirrors in newer cars often have warning labels that read, OBJECTS IN MIRROR ARE CLOSER THAN THEY APPEAR. Are these rearview mirrors concave or convex?

REASONING AND DISCUSSION

Objects in the mirror are closer than they appear because the mirror produces an image that is reduced in size, which makes the object look as if it is farther away. In addition, we know that the rearview mirror always gives an upright image, no matter how close or far away the object. The mirror that always produces upright and reduced images is the convex mirror.

ANSWER

The mirrors are convex.

The imaging characteristics of convex and concave mirrors are summarized in Table 26–1.

TABLE 26–1 Imaging Characteristics of Convex and Concave Spherical Mirrors

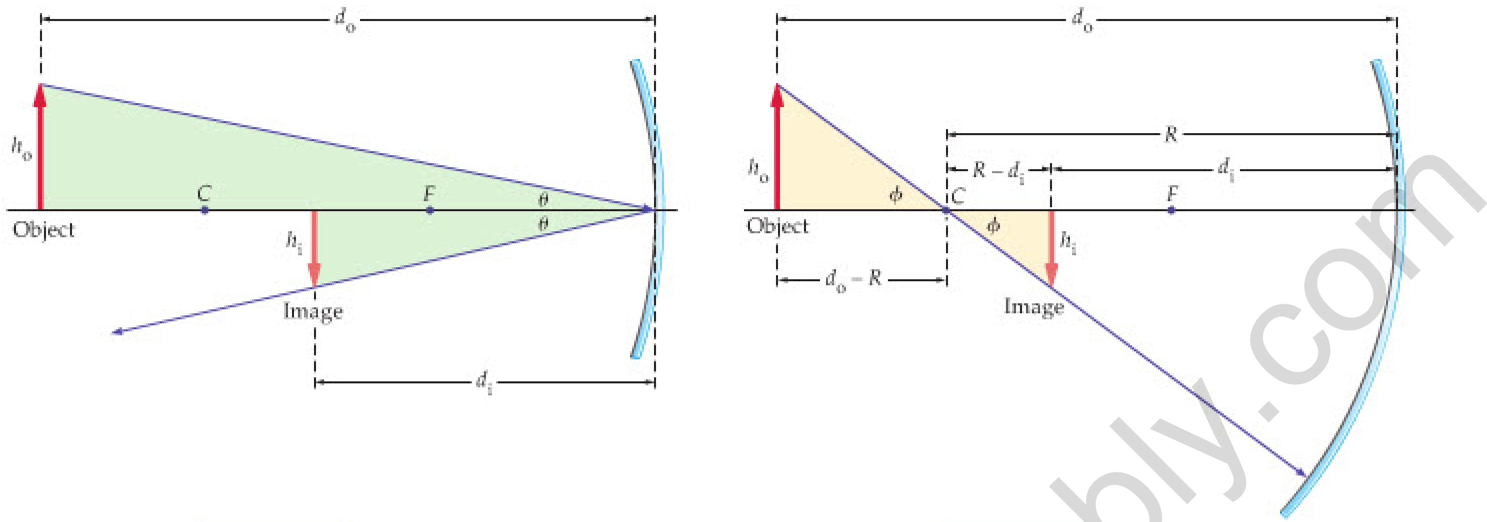
CONVEX MIRROR Object location	Image orientation	Image size	Image type
Arbitrary	Upright	Reduced	Virtual
CONCAVE MIRROR **Object location**	**Image orientation**	**Image size**	**Image type**
Beyond C	Inverted	Reduced	Real
C	Inverted	Same as object	Real
Between F and C	Inverted	Enlarged	Real
Just beyond F	Inverted	Approaching infinity	Real
Just inside F	Upright	Approaching infinity	Virtual
Between mirror and F	Upright	Enlarged	Virtual

We now show how to obtain results about the images formed by mirrors in a quantitative manner.

The Mirror Equation

The mirror equation is a precise mathematical relationship between the object distance and the image distance for a given mirror. To obtain this relation, we use the ray diagrams shown in **Figure 26–19**. Note that the distance from the mirror to the object is d_o, the distance from the mirror to the image is d_i, and the distance from the mirror to the center of curvature is R. In addition, the height of the object is h_o, and the height of the image is h_i. Since the image is inverted, its height is negative; thus $-h_i$ is positive.

The ray in Figure 26–19 (a) hits the mirror at its midpoint, where the principal axis is the normal to the mirror. As a result, the ray reflects at an angle θ below



(a) Triangles to derive Equation 26–4

(b) Triangles to derive Equation 26–5

▲ **FIGURE 26–19 Ray diagrams used to derive the mirror equation**
(a) The two similar triangles in this case are used to obtain Equation 26–4. **(b)** These similar triangles yield Equation 26–5.

the principal axis that is equal to its incident angle θ above the axis. Therefore, the two green triangles in this diagram are similar, from which it follows that $h_o/d_o = (-h_i)/d_i$, or

$$\frac{h_o}{-h_i} = \frac{d_o}{d_i} \qquad 26\text{–}4$$

Next, Figure 26–19 (b) shows the C ray for this mirror. From the figure it is clear that the two yellow triangles in this diagram are also similar, since they are both right triangles and share the common angle ϕ. Thus, $h_o/(d_o - R) = (-h_i)/(R - d_i)$, or

$$\frac{h_o}{-h_i} = \frac{d_o - R}{R - d_i} \qquad 26\text{–}5$$

Setting these two expressions for $h_o/(-h_i)$ equal gives us

$$\frac{d_o}{d_i} = \frac{d_o - R}{R - d_i} \quad \text{or} \quad 1 = \frac{1 - \dfrac{R}{d_o}}{\dfrac{R}{d_i} - 1}$$

Rearranging, we find $R/d_o + R/d_i = 2$. Finally, dividing by R and recalling that $f = \frac{1}{2}R$ for a concave mirror, we get the **mirror equation:**

The Mirror Equation

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f} \qquad 26\text{–}6$$

We apply the mirror equation to a simple system in the following Exercise.

EXERCISE 26–1

The concave side of a spoon has a focal length of 5.00 cm. Find the image distance for this "mirror" when the object distance is **(a)** 25.0 cm, **(b)** 9.00 cm, and **(c)** 2.00 cm. These three cases correspond to Figure 26–18 (a), Figure 26–18 (b), and Example 26–3, respectively.

SOLUTION

Solving Equation 26–6 for d_i, we obtain the following results:

a. $d_i = \dfrac{d_o f}{d_o - f} = \dfrac{(25.0\text{ cm})(5.00\text{ cm})}{25.0\text{ cm} - 5.00\text{ cm}} = 6.25\text{ cm}$

b. $d_i = \dfrac{d_o f}{d_o - f} = \dfrac{(9.00\text{ cm})(5.00\text{ cm})}{9.00\text{ cm} - 5.00\text{ cm}} = 11.3\text{ cm}$

c. $d_i = \dfrac{d_o f}{d_o - f} = \dfrac{(2.00\text{ cm})(5.00\text{ cm})}{2.00\text{ cm} - 5.00\text{ cm}} = -3.33\text{ cm}$

Note that the image distance in Exercise 26–1 is negative when the object is closer to the mirror than the focal point [$d_o < f$, as in part (c)]. We know from Example 26–3 that this is also the case where the image is *behind* the mirror. Thus, the *sign* of the image distance indicates the *side* of the mirror on which the image is located. As long as we are discussing signs, it should be noted that the mirror equation applies equally well to a *convex* mirror, as long as we recall that the focal length in this case is *negative*. The following Exercise calculates the image distance for the convex mirror shown in Figure 26–16.

EXERCISE 26–2

The convex mirror in Figure 26–16 has a 20.0-cm radius of curvature. Find the image distance for this mirror when the object distance is 6.33 cm, as it is in Figure 26–16.

SOLUTION

Recalling that $f = -\frac{1}{2}R$ for a convex mirror, we find $f = -10.0$ cm and

$$d_i = \frac{d_o f}{d_o - f} = \frac{(6.33\text{ cm})(-10.0\text{ cm})}{6.33\text{ cm} - (-10.0\text{ cm})} = -3.88\text{ cm}$$

This result agrees with the image shown in Figure 26–16.

Next, we consider the height of an image, which is given by the relation in Equation 26–4. Solving this equation for h_i we find

$$h_i = -\left(\frac{d_i}{d_o}\right)h_o \qquad 26\text{–}7$$

The ratio of the height of the image to the height of the object is defined as the **magnification,** m; that is, $m = h_i/h_o$. From Equation 26–7, we see that

Magnification, *m*

$$m = \frac{h_i}{h_o} = -\frac{d_i}{d_o} \qquad 26\text{–}8$$

The sign of the magnification gives the orientation of the image. For example, if both d_o and d_i are positive, as in Figure 26–18, the magnification is negative and the image is inverted. Conversely, if the image is behind the mirror, so that d_i is negative, the magnification is positive, and the image is upright. An example of this case is shown in Example 26–3 and Figure 26–16. Finally, the magnitude of the magnification gives the factor by which the size of the image is increased or decreased compared with the object. In the special case of an image with the same size and orientation as the object, as in a plane mirror, the magnification is one.

The sign conventions for mirrors are summarized below:

Focal Length
f is positive for concave mirrors.
f is negative for convex mirrors.

Magnification
m is positive for upright images.
m is negative for inverted images.

Image Distance
d_i is positive for images in front of a mirror (real images).
d_i is negative for images behind a mirror (virtual images).

PROBLEM-SOLVING NOTE

Applying the Mirror Equation

To use the mirror equation correctly, be careful to use the appropriate signs for all the known quantities. The final answer will also have a sign, which gives additional information about the system.

Object Distance

d_o is positive for objects in front of a mirror (real objects).

d_o is negative for objects behind a mirror (virtual objects).

The case of a negative object distance—that is, a virtual object—can occur when the image from one mirror serves as the object for another mirror or lens. For example, if mirror 1 produces an image that is behind mirror 2, we say that the image of mirror 1 is a virtual object for mirror 2. Such situations will be considered in the next chapter.

We now apply the mirror and magnification equations to specific Examples.

EXAMPLE 26–4 CHECKING IT TWICE

After leaving some presents under the tree, Santa notices his image in a shiny, spherical Christmas ornament. The ornament is 8.50 cm in diameter and 1.10 m away from Santa. Curious to know the location and size of his image, Santa consults a book on physics. Knowing that Santa likes to "check it twice," what results should he obtain, assuming his height is 1.75 m?

PICTURE THE PROBLEM

The physical situation is illustrated in the sketch, along with the image Santa sees in the ornament. Because the spherical ornament is a convex mirror, the image it forms is upright and reduced.

STRATEGY

The ornament is a convex mirror of radius $R = \frac{1}{2}(8.50\ \text{cm}) = 4.25\ \text{cm}$. We can find the location of the image, then, by using the mirror equation with the focal length given by $f = -\frac{1}{2}R$.

Once we have determined the location of the image, we can find its size using $h_i = -(d_i/d_o)h_o = mh_o$, where $h_o = 1.75\ \text{m}$.

SOLUTION

1. Calculate the focal length for the ornament:

$$f = -\tfrac{1}{2}R = -\tfrac{1}{2}(4.25\ \text{cm}) = -0.0213\ \text{m}$$

2. Use the mirror equation to find the image distance, d_i:

$$\frac{1}{d_i} = \frac{1}{f} - \frac{1}{d_o} = \frac{1}{(-0.0213\ \text{m})} - \frac{1}{1.10\ \text{m}} = -47.9\ \text{m}^{-1}$$

$$d_i = \frac{1}{(-47.9\ \text{m}^{-1})} = -0.0209\ \text{m}$$

3. Determine the magnification of the image using $m = -(d_i/d_o)$:

$$m = -\left(\frac{d_i}{d_o}\right) = -\left(\frac{-0.0209\ \text{m}}{1.10\ \text{m}}\right) = 0.0190$$

4. Find the image height with $h_i = mh_o$:

$$h_i = mh_o = (0.0190)(1.75\ \text{m}) = 0.0333\ \text{m}$$

INSIGHT

Thus, Santa's image is 2.09 cm behind the surface of the ornament—about halfway between the surface and the center—and 3.33 cm high. Note that his image fits on the surface of the ornament with room to spare, and therefore Santa can see the reflection of his entire body.

PRACTICE PROBLEM

How far from the ornament must Santa stand if his image is to be 1.75 cm behind its surface? [**Answer:** 9.81 cm]

Some related homework problems: Problem 31, Problem 34, Problem 39

ACTIVE EXAMPLE 26–1 SAY AHHH: FIND THE MAGNIFICATION OF THE TOOTH

A dentist uses a small mirror attached to a thin rod to examine one of your teeth. When the tooth is 1.20 cm in front of the mirror, the image it forms is 9.25 cm behind the mirror. Find **(a)** the focal length of the mirror and **(b)** the magnification of the image.

SOLUTION *(Test your understanding by performing the calculations indicated in each step.)*

1. Use the information given to identify the object and image distances: $d_o = 1.20\ \text{cm}, d_i = -9.25\ \text{cm}$

Part (a)

2. Use the mirror equation to calculate the focal length: $f = 1.38\text{ cm}$

Part (b)

3. Use $m = -(d_i/d_o)$ to find the magnification: $m = 7.71$

INSIGHT

Because the focal length is positive, and the magnification is greater than one, we conclude that the dentist's mirror is concave. Note that it will make the tooth look 7.71 times larger than life. A convex mirror, in contrast, always produces a magnification less than one.

YOUR TURN

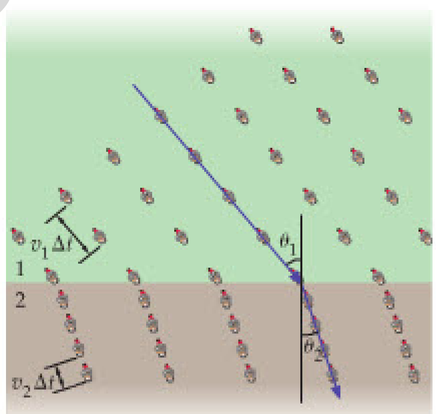
If the mirror is moved closer to the tooth, will the magnification of the tooth increase or decrease? Find the magnification if the distance from the tooth to the mirror is reduced to 1.00 cm.

(Answers to **Your Turn** *problems are given in the back of the book.)*

Finally, recall that a spherical mirror behaves like a plane mirror when the object is close to the mirror. But just what exactly do we mean by *close*? Well, in this case, *close* means that the object distance should be small in comparison with the radius of curvature. For example, if the radius of curvature were to go to infinity, the mirror would behave like a plane mirror for all object distances. Simply put, a sphere with $R \to \infty$ has a surface that is essentially as flat as a plane. Letting $f = \frac{1}{2}R$ go to infinity in the mirror equation yields

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f} \to 0$$

Therefore, in this limit $d_i = -d_o$, as expected for a plane mirror.



▲ FIGURE 26–20 An analogy for refraction

As a marching band moves from an area where the ground is solid to one where it is soft and muddy, the direction of motion changes.

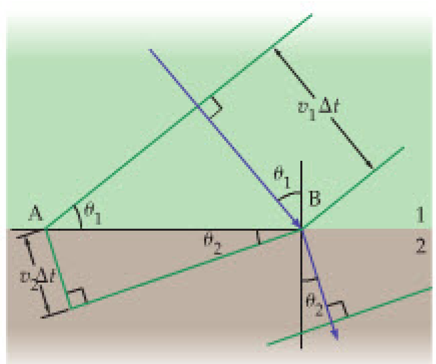
26–5 The Refraction of Light

When a wave propagates from a medium in which its speed is v_1 to another in which its speed is $v_2 \neq v_1$, it will, in general, change its direction of motion. This phenomenon is called **refraction.** To understand the cause of refraction, consider the behavior of a marching band as it moves from a solid section of ground to an area where the ground is muddy, as indicated in Figure 26–20. In this analogy, the rows of the marching band correspond to the wave fronts of a traveling wave, and the solid and muddy sections of the ground represent media in which the wave speed is different.

If the speed of the marchers on solid ground is v_1, then after a time Δt they have advanced a distance $v_1 \Delta t$. On the other hand, if the marchers in the mud move at the reduced speed v_2, they advance only a distance $v_2 \Delta t$ in the same time. This causes a bend in the line of marchers, as can be seen in Figure 26–20, and therefore a change in the direction of motion.

Figure 26–21 shows a simplified version of Figure 26–20, with three "wave fronts" taking the place of the marchers. Also shown is a ray drawn at right angles to the wave fronts, and a normal to the interface between the two types of ground. The angle of incidence is θ_1, and from the figure we can see that this is also the angle between the incoming wave front and the interface. The outgoing wave front and its ray are characterized by a different angle, θ_2. From the geometry of the figure, we see that the green and brown triangles share a common side, AB. Therefore,

$$\sin\theta_1 = \frac{v_1 \Delta t}{\text{AB}} \quad \text{and} \quad \sin\theta_2 = \frac{v_2 \Delta t}{\text{AB}}$$

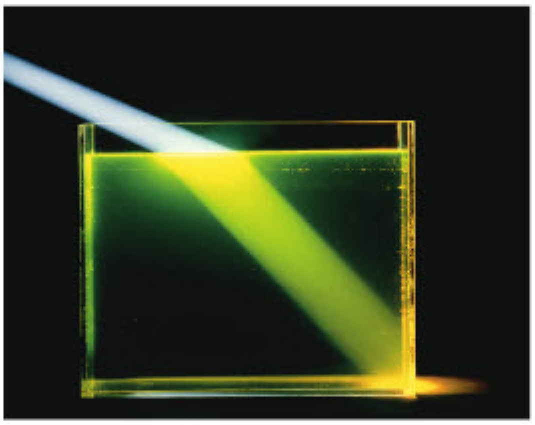


▲ FIGURE 26–21 The basic mechanism of refraction

Refraction is the bending of wave fronts and a change in direction of propagation due to a change in speed.

TABLE 26–2 Index of Refraction for Common Substances

Substance	Index of refraction, n
Solids	
Diamond	2.42
Flint glass	1.66
Crown glass	1.52
Fused quartz (glass)	1.46
Ice	1.31
Liquids	
Benzene	1.50
Ethyl alcohol	1.36
Water	1.33
Gases	
Carbon dioxide	1.00045
Air	1.000293



▲ The water in this tank contains a fluorescent dye, making it easier to see the refraction of the beam as it passes from the air into the water.

Eliminating the common factor $\Delta t/AB$ yields

$$\frac{\sin\theta_1}{v_1} = \frac{\sin\theta_2}{v_2} \qquad 26\text{–}9$$

Thus, we see that the direction of propagation is directly related to the speed of propagation.

The speed of light, in turn, depends on the medium through which it travels. For example, we know that in a vacuum the speed of light is $c = 3.00 \times 10^8$ m/s. When light propagates through water, however, its speed is reduced by a factor of 1.33. In general, the speed of light in a given medium, v, is determined by the medium's **index of refraction**, n, defined as follows:

Definition of the Index of Refraction, n

$$v = \frac{c}{n} \qquad 26\text{–}10$$

Representative values of the index of refraction for a variety of media are given in Table 26–2.

EXERCISE 26–3

How long does it take for light to travel 2.50 m in water?

SOLUTION

The speed of light in water is c/n, where $n = 1.33$. Therefore, the time required to cover 2.50 m is

$$t = \frac{d}{v} = \frac{d}{(c/n)} = \frac{2.50\text{ m}}{\left(\dfrac{3.00 \times 10^8\text{ m/s}}{1.33}\right)} = 1.11 \times 10^{-8}\text{ s}$$

Returning to the direction of propagation, let's suppose light has the speed $v_1 = c/n_1$ in one medium and $v_2 = c/n_2$ in a second medium. The direction of propagation in these two media is related by Equation 26–9:

$$\frac{\sin\theta_1}{(c/n_1)} = \frac{\sin\theta_2}{(c/n_2)}$$

Elimination of the common factor c yields the following relation, known as Snell's law:

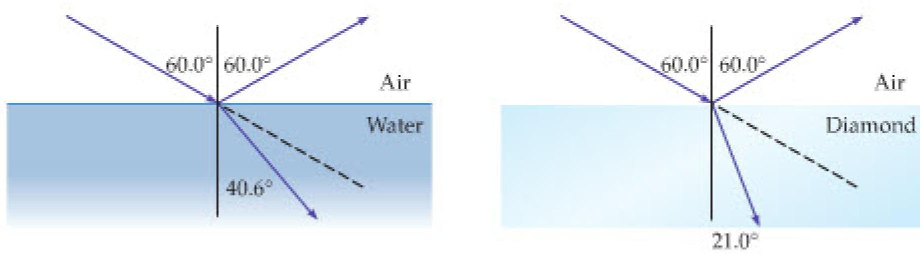
Snell's Law

$$n_1 \sin\theta_1 = n_2 \sin\theta_2 \qquad 26\text{–}11$$

A typical application of Snell's law is given in the following Exercise.

EXERCISE 26–4

A beam of light in air enters **(a)** water ($n = 1.33$) or **(b)** diamond ($n = 2.42$) at an angle of 60.0° relative to the normal. Find the angle of refraction for each case.



SOLUTION

Since the beam starts in air, we refer to Table 26–2 and set $n_1 = 1.000293$, or simply $n_1 = 1.00$ to three significant figures.

a. With $n_2 = 1.33$ we find

$$\theta_2 = \sin^{-1}\left(\frac{1.00}{n_2}\sin\theta_1\right) = \sin^{-1}\left(\frac{1.00}{1.33}\sin 60.0^\circ\right) = 40.6^\circ$$

b. Setting $n_2 = 2.42$, we get

$$\theta_2 = \sin^{-1}\left(\frac{1.00}{n_2}\sin\theta_1\right) = \sin^{-1}\left(\frac{1.00}{2.42}\sin 60.0^\circ\right) = 21.0^\circ$$

From the preceding Exercise we can see that the greater the difference in index of refraction between two different materials, the greater the difference in direction of propagation. In addition, light is bent closer to the normal in the medium where its speed is less. Of course, the opposite is true when light passes into a medium in which its speed is greater, as can be seen by reversing the incident and refracted rays. The qualitative features of refraction are as follows:

- When a ray of light enters a medium where the index of refraction is increased, and hence the speed of the light is *decreased*, the ray is bent *toward* the normal.
- When a ray of light enters a medium where the index of refraction is decreased, and hence the speed of the light is *increased*, the ray is bent *away* from the normal.
- There is no change in direction of propagation if there is no change in index of refraction. The greater the change in index of refraction, the greater the change in propagation direction.
- If a ray of light goes from one medium to another along the normal, it is undeflected, regardless of the index of refraction.

The last property listed follows directly from Snell's law: If θ_1 is zero, then $0 = n_2 \sin\theta_2$, which means that $\theta_2 = 0$. Refraction is explored further in the following Example.

PROBLEM-SOLVING NOTE

Applying Snell's Law

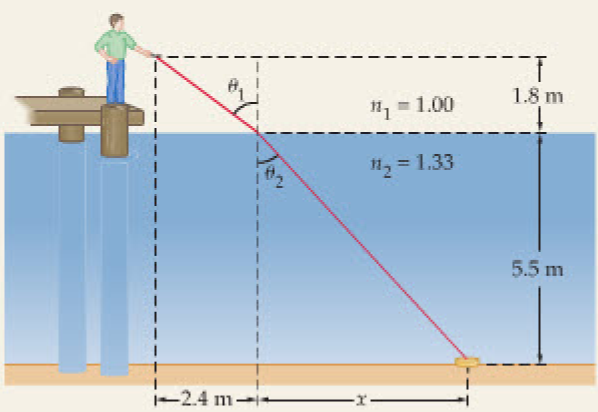
To apply Snell's law correctly, recall that the two angles in $n_1 \sin\theta_1 = n_2 \sin\theta_2$ are always measured relative to the normal at the interface. Also, note that each angle is associated with its corresponding index of refraction. For example, the angle θ_1 is the angle to the normal in the substance with an index of refraction equal to n_1.

EXAMPLE 26–5 SITTING ON A DOCK OF THE BAY

One night, while on vacation in the Caribbean, you walk to the end of a dock and, for no particular reason, shine your laser pointer into the water. When you shine the beam of light on the water a horizontal distance of 2.4 m from the dock, you see a glint of light from a shiny object on the sandy bottom—perhaps a gold doubloon. If the pointer is 1.8 m above the surface of the water, and the water is 5.5 m deep, what is the horizontal distance from the end of the dock to the shiny object?

PICTURE THE PROBLEM

The person standing at the end of the dock and the shiny object on the bottom are shown in the sketch. Note also that all the known distances are indicated, along with the angle of incidence, θ_1, and the angle of refraction, θ_2. Finally, the appropriate indices of refraction from Table 26–2 are given as well.



STRATEGY

We can use Snell's law and basic trigonometry to find the horizontal distance to the shiny object.

First, the information given in the problem determines the angle of incidence, θ_1. In particular, we can see from the sketch that $\tan\theta_1 = (2.4\text{ m})/(1.8\text{ m})$.

Second, Snell's law, $(n_1 \sin\theta_1 = n_2 \sin\theta_2)$, gives the angle of refraction, θ_2.

Finally, the sketch shows that the horizontal distance to the shiny object is $2.4\text{ m} + x$. We can find the distance x from θ_2, since $\tan\theta_2 = x/(5.5\text{ m})$.

CONTINUED ON NEXT PAGE

CONTINUED FROM PREVIOUS PAGE

SOLUTION

1. Find the angle of incidence from the information given in the problem: $\theta_1 = \tan^{-1}\left(\frac{2.4\text{ m}}{1.8\text{ m}}\right) = 53°$

2. Use Snell's law to calculate the angle of refraction: $\theta_2 = \sin^{-1}\left(\frac{n_1}{n_2}\sin\theta_1\right) = \sin^{-1}\left[\left(\frac{1.00}{1.33}\right)\sin 53°\right] = 37°$

3. Calculate x using $\tan\theta_2 = x/(5.5\text{ m})$: $\tan\theta_2 = \frac{x}{5.5\text{ m}}$

   $x = (5.5\text{ m})\tan\theta_2 = (5.5\text{ m})\tan 37° = 4.1\text{ m}$

4. Add 2.4 m to x to find the total horizontal distance to the shiny object: $\text{distance} = 2.4\text{ m} + x = 2.4\text{ m} + 4.1\text{ m} = 6.5\text{ m}$

INSIGHT

Notice that if the water were to be removed, the incident beam of light would continue along its original path, characterized by the angle θ_1, and overshoot the hoped-for doubloon by a significant distance. Similarly, if you were to simply stand at the end of the dock and look out into the water at a glint of gold on the bottom, you would be looking in a direction (again characterized by θ_1) that is too high—the gold would be below your line of sight and closer to you than you think.

PRACTICE PROBLEM

If the index of refraction for water were 1.35 instead of 1.33, would the doubloon be farther from the dock, nearer the dock, or the same distance calculated above? Check your answer by calculating the distance with $n_2 = 1.35$. [**Answer:** The doubloon would be closer to the dock. The horizontal distance in this case would be 6.4 m.]

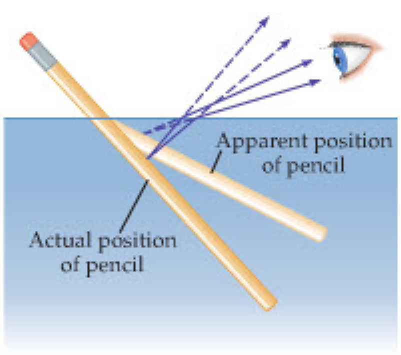
Some related homework problems: Problem 52, Problem 56, Problem 60

REAL-WORLD PHYSICS
Apparent depth

Refraction is responsible for a number of common "optical illusions." For example, we all know that a pencil placed in a glass of water appears to be bent, though it is still perfectly straight. The cause of this illusion is shown in Figure 26–22, where we see that rays leaving the water bend away from the normal and make the pencil appear to be above its actual position. This is an example of what is known as *apparent depth*, in which an object appears to be closer to the water's surface than it really is. The relation between the true depth and apparent depth is considered in Problem 61.

REAL-WORLD PHYSICS
Mirages

Similarly, refraction can cause a mirage, which makes hot, dry ground in the distance appear to be covered with water. Basically, hot air near the surface is less dense—and hence has a smaller index of refraction—than the cooler air higher



▲ **FIGURE 26–22 Refraction and the "bent" pencil**

Refraction causes a pencil to appear bent when placed in water. Note that rays leaving the water are bent away from the normal and hence extend back to a point that is higher than the actual position of the pencil.

▲ One of the most common mirages, often seen in hot weather, makes a stretch of road look like the surface of a lake. The blue color that so resembles water to our eyes is actually an image of the sky, refracted by the hot, low-density air above the road.

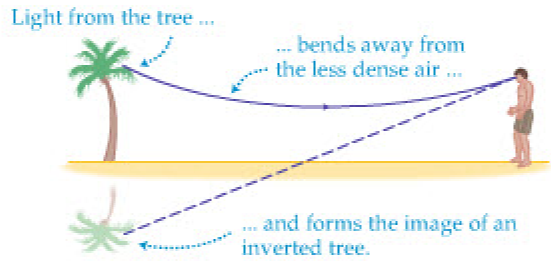


FIGURE 26–23 A mirage
A mirage is produced when light bends upward due to the low index of refraction of heated air near the ground.

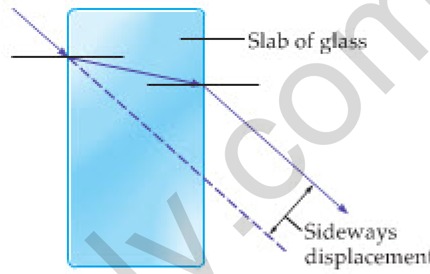


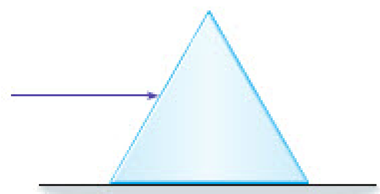
FIGURE 26–24 Light propagating through a glass slab
When a ray of light passes through a glass slab, it first refracts toward the normal, then away from the normal. The net result is that the ray continues in its original direction but is displaced sideways by a finite distance.

up. Thus, as light propagates toward the ground, it bends away from the normal until, eventually, it travels upward and enters the eye of an observer, as indicated in Figure 26–23. What appears to be a reflecting pool of water in the distance, then, is actually an image of the sky.

Finally, if light passes through a refracting slab, like the sheet of glass in Figure 26–24, it undergoes two refractions—one at each surface of the slab. The first refraction bends the light rays closer to the normal, and the second refraction bends the rays away from the normal. As can be seen in the figure, the two changes in direction cancel, so that the final direction of the light is the same as its original direction. The light has been displaced slightly, however, by an amount proportional to the thickness of the slab. The displacement distance is calculated in Problem 119.

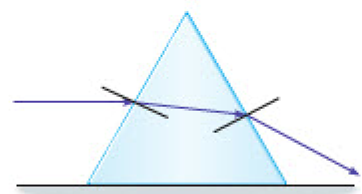
CONCEPTUAL CHECKPOINT 26–4 REFRACTION IN A PRISM

A horizontal ray of light encounters a prism, as shown in the first diagram. After passing through the prism, is the ray **(a)** deflected upward, **(b)** still horizontal, or **(c)** deflected downward?



REASONING AND DISCUSSION

When the ray enters the prism, it is bent toward the normal, which deflects it *downward*, as shown in the second diagram. When it leaves through the opposite side of the prism, it is bent away from the normal. Because the sides of a prism are angled in opposite directions, however, bending away from the normal in the second refraction also causes a *downward* deflection.



The net result, then, is a downward deflection of the ray.

ANSWER

(c) The ray deflects downward.

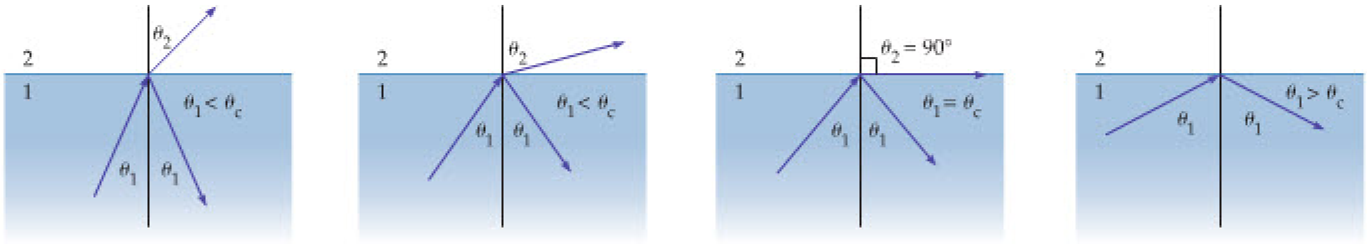
We will use the results of this Conceptual Checkpoint in the next section when we investigate the behavior of lenses. For now, we turn to two additional phenomena associated with refraction.

Total Internal Reflection

Figure 26–25 (a) shows a ray of light in water encountering a water–air interface. In such a case, it is observed that part of the light is reflected back into the water at the interface—as from the surface of a mirror—while the rest emerges into the air along a direction that is bent away from the normal according to Snell's law. If the angle of incidence is increased, as in Figure 26–25 (b), the angle of refraction increases as well. At some critical angle of incidence, θ_c, the refracted beam no longer enters the air but instead is directed parallel to the water–air interface (Figure 26–25 (c)). In this case, the angle of refraction is 90°. For angles of incidence greater than the critical angle, as in Figure 26–25 (d), it is observed that all the light is reflected back into the water. This phenomenon is referred to as **total internal reflection.**

We can find the critical angle for total internal reflection by setting $\theta_2 = 90°$ and applying Snell's law:

$$n_1 \sin \theta_c = n_2 \sin 90° = n_2$$



(a) Small angle of incidence **(b)** Larger angle of incidence **(c)** Refracted beam parallel to interface **(d)** Total internal reflection

▲ **FIGURE 26–25 Total internal reflection**

Total internal reflection can occur when light propagates from a region with a high index of refraction to one with a lower index of refraction. Part **(a)** shows an incident ray in medium 1 encountering the interface with medium 2, which has a lower index of refraction. A portion of the ray is transmitted to region 2 at the angle θ_2, given by Snell's law, and a portion of the ray is reflected back into medium 1 at the angle of incidence, θ_1. The sum of the intensities of the refracted and reflected rays equals the intensity of the incident ray. In part **(b)** the angle of incidence has been increased, and the refracted beam makes a smaller angle with the interface. When the angle of incidence equals the critical angle, θ_c, as in part **(c)**, the refracted ray propagates parallel to the interface. For incident angles greater than θ_c there is no refracted ray at all, as shown in part **(d)**, and all of the incident intensity goes into the reflected ray.

Therefore, the critical angle is given by the following relation:

Critical Angle for Total Internal Reflection, θ_c

$$\sin \theta_c = \frac{n_2}{n_1} \qquad 26\text{–}12$$

Since $\sin \theta$ is always less than or equal to 1, the index of refraction, n_1, must be larger than the index of refraction n_2 if the Equation 26–12 is to give a physical solution. Thus, total internal reflection can occur only when light in one medium encounters an interface with another medium in which the speed of light is greater. For example, light moving from water to air can undergo total internal reflection, as shown in Figure 26–25, but light moving from air to water cannot.

PROBLEM-SOLVING NOTE

Total Internal Reflection

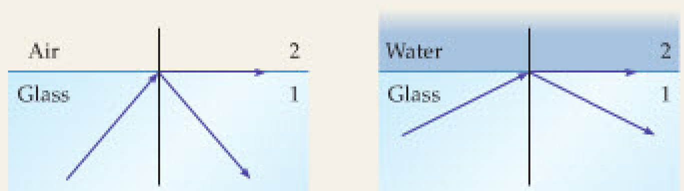
Remember that total internal reflection can occur only on the side of an interface between two different substances that has the greater index of refraction.

EXAMPLE 26–6 LIGHT TOTALLY REFLECTED

Find the critical angle for light traveling from glass ($n = 1.50$) to **(a)** air ($n = 1.00$) and **(b)** water ($n = 1.33$).

PICTURE THE PROBLEM

Our sketch shows the two cases considered in the problem. Note that in each case the incident medium is glass; therefore $n_1 = 1.50$. For part (a) it follows that $n_2 = 1.00$, and for part (b) $n_2 = 1.33$.



STRATEGY

The critical angle, θ_c, is defined in Equation 26–12. It is straightforward to obtain θ_c by simply substituting the appropriate indices of refraction for each case in the relation $\theta_c = \sin^{-1}(n_2/n_1)$.

SOLUTION

Part (a)

1. Solve Equation 26–12 for θ_c, using $n_1 = 1.50$ and $n_2 = 1.00$:

$$\theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right) = \sin^{-1}\left(\frac{1.00}{1.50}\right) = 41.8°$$

Part (b)

2. Solve Equation 26–12 for θ_c, using $n_1 = 1.50$ and $n_2 = 1.33$:

$$\theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right) = \sin^{-1}\left(\frac{1.33}{1.50}\right) = 62.5°$$

INSIGHT

Note that in the case of water the two indices of refraction are closer in value; hence, light escapes from glass to water over a wider range of incident angles (0° to 62.5°) than from glass to air (0° to 41.8°). In general, if the indices of refraction of two media are close in value, only light rays with large angles of incidence will undergo total internal reflection.