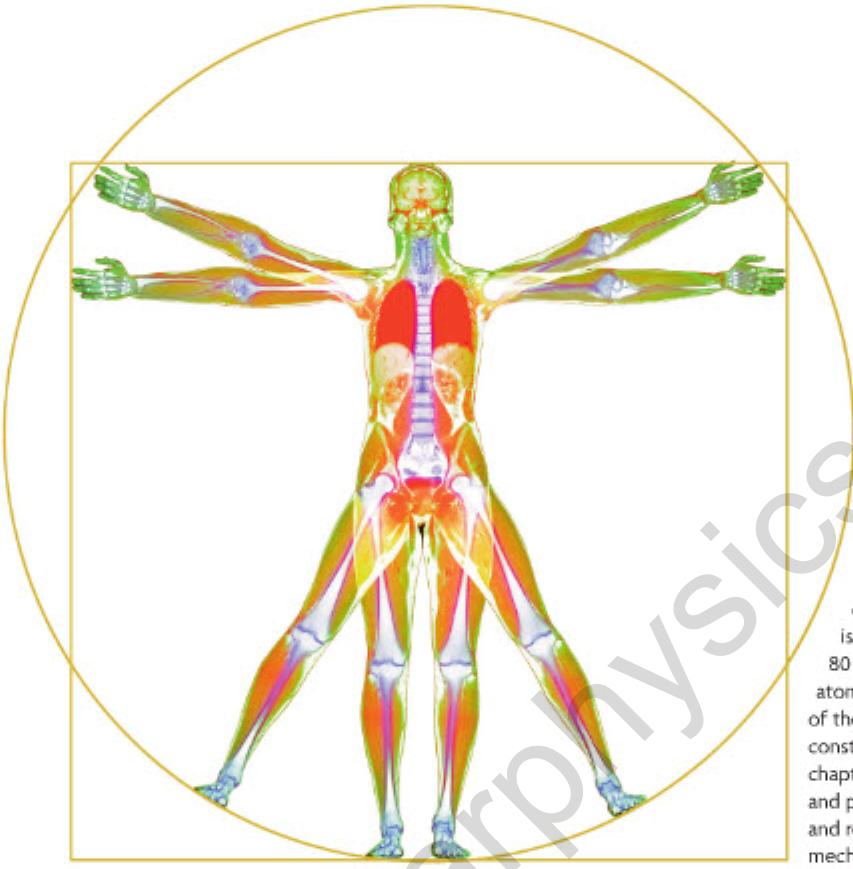
24 Alternating-Current Circuits



Magnetic resonance imaging (MRI), which produced the modern version of Leonardo da Vinci's *Vitruvian Man* shown here, would not be possible without alternating-current (ac) electric circuits. In fact, the key component of all MRI machines is an ac circuit that oscillates with a frequency in the range of 15 to 80 MHz. At these frequencies, the circuit resonates with hydrogen atoms within a person's body. By detecting how much each portion of the body responds to the resonance effect, a computer can construct detailed images of the body's internal structure. In this chapter, we explore many aspects of ac circuits, including resonance, and point out the close connections between inductors, capacitors, and resistors in electric circuits, and masses, springs, and friction in mechanical systems.

We conclude our study of electric circuits with a consideration of the type of circuit most common in everyday usage—a circuit with an alternating current. In an alternating-current (ac) circuit, the polarity of the voltage and the direction of the current undergo periodic reversals. Typically, the time dependence of the voltage and current is sinusoidal, with a frequency of 60 cycles per second being the standard in the United States.

As we shall see, ac circuits require that we generalize the notion of resistance to include the time-dependent effects associated with capacitors and inductors. In particular, the voltage and current in an ac circuit may not vary with time in exactly the same way—they are often "out of step" with each other. These effects and others are found in ac circuits but not in direct-current circuits.

Finally, we also consider the fact that some electric circuits have natural frequencies of oscillation, much like a pendulum or a mass on a spring. One of the results of having a natural frequency is that resonance effects are to be expected. Indeed, electric circuits do show resonance—in fact, this is how you are able to tune your radio or television to the desired station. Clearly, then, ac circuits have a rich variety of behaviors that make them not only very useful but also most interesting from a physics point of view.

24–1 Alternating Voltages and Currents

When you plug a lamp into a wall socket, the voltage and current supplied to the lightbulb vary sinusoidally with a steady frequency, completing 60 cycles each second. Because the current periodically reverses direction, we say that the wall socket provides an **alternating current.** A simplified alternating-current (ac) circuit diagram for the lamp is shown in Figure 24–1. In this circuit, we indicate the bulb by its equivalent resistance, R, and the wall socket by an **ac generator,** represented as a circle enclosing one cycle of a sine wave.

The voltage delivered by an ac generator, which is plotted in Figure 24–2 (a), can be represented mathematically as follows:

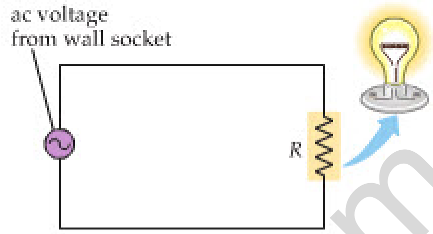
$$V = V_{max} \sin \omega t \quad \text{24–1}$$

In this expression, V_{max} is the largest value attained by the voltage during a cycle, and the angular frequency is $\omega = 2\pi f$, where $f = 60$ Hz. (Note the similarity between the sinusoidal time dependence in an ac circuit and the time dependence given in Section 13–2 for simple harmonic motion.) From Ohm's law, $I = V/R$, it follows that the current in the lightbulb is

$$I = \frac{V}{R} = \left(\frac{V_{max}}{R}\right) \sin \omega t = I_{max} \sin \omega t \quad \text{24–2}$$

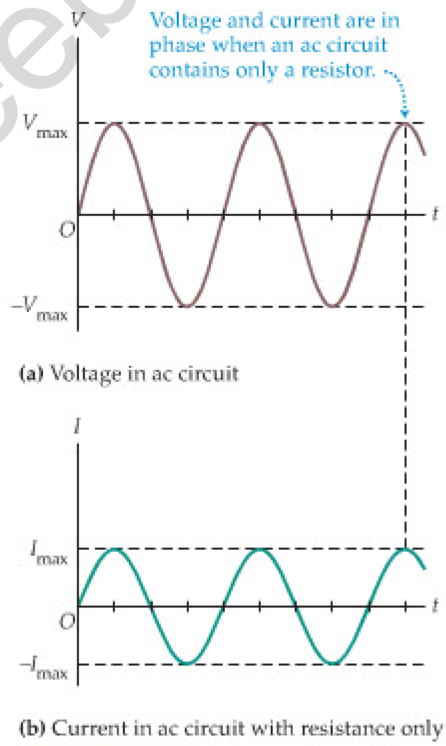
This result is plotted in Figure 24–2 (b).

Notice that the voltage and current plots have the same time variation. In particular, the voltage reaches its maximum value at precisely the same time as the current. We express this relationship between the voltage and current in a resistor by saying that they are *in phase* with one another. As we shall see later in this chapter, other circuit elements, like capacitors and inductors, have different phase relationships between the current and voltage. For these elements, the current and voltage reach maximum values at different times.



▲ **FIGURE 24–1 An ac generator connected to a lamp**

Simplified alternating-current circuit diagram for a lamp plugged into a wall socket. The lightbulb is replaced in the circuit with its equivalent resistance, R.



▲ **FIGURE 24–2 Voltage and current for an ac resistor circuit**

(a) An ac voltage described by $V = V_{max} \sin \omega t$. **(b)** The alternating current, $I = I_{max} \sin \omega t = (V_{max}/R) \sin \omega t$, corresponding to the ac voltage in (a). Note that the voltage of the generator and the current in the resistor are in phase with each other; that is, their maxima and minima occur at precisely the same times.

Phasors

A convenient way to represent an alternating voltage and the corresponding current is with counterclockwise rotating vectors referred to as **phasors.** Phasors allow us to take advantage of the connection between uniform circular motion and linear, sinusoidal motion—just as we did when we studied oscillations in Chapter 13. In that case, we related the motion of a peg on a rotating turntable to the oscillating motion of a mass on a spring by projecting the position of the peg onto a screen. We use a similar projection with phasors.

For example, Figure 24–3 shows a vector of magnitude V_{max} rotating about the origin with an angular speed ω. This rotating vector is the voltage phasor. If the voltage phasor makes an angle $\theta = \omega t$ with the x axis, it follows that its y component is $V_{max} \sin \omega t$; that is, if we project the voltage phasor onto the y axis, the projection gives the instantaneous value of the voltage, $V = V_{max} \sin \omega t$, in agreement with Equation 24–1. In general,

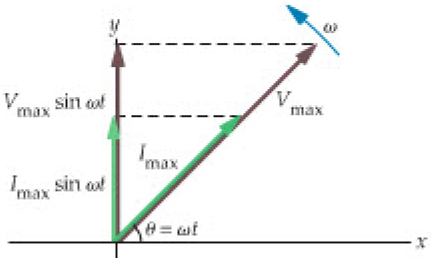
> the instantaneous value of a quantity represented by a phasor is the projection of the phasor onto the y axis.

Figure 24–3 also shows the current phasor, represented by a rotating vector of magnitude $I_{max} = V_{max}/R$. The current phasor points in the same direction as the voltage phasor; hence, the instantaneous current is $I = I_{max} \sin \omega t$, as in Equation 24–2.

The fact that the voltage and current phasors always point in the same direction for a resistor is an equivalent way of saying that the voltage and current are in phase:

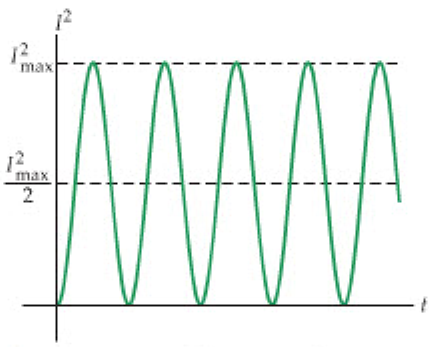
> The voltage phasor for a resistor points in the same direction as the current phasor.

In circuits that also contain capacitors or inductors, the current and voltage phasors will usually point in different directions.



▲ **FIGURE 24–3 Phasor diagram for an ac resistor circuit**

Since the current and voltage are in phase in a resistor, the corresponding phasors point in the same direction at all times. Both phasors rotate about the origin with an angular speed ω, and the vertical component of each is the instantaneous value of that quantity.



▲ **FIGURE 24–4 The square of a sinusoidally varying current**

Note that I^2 varies symmetrically about the value $\frac{1}{2}I^2_{max}$. The average of I^2 over time, then, is $\frac{1}{2}I^2_{max}$.

Root Mean Square (rms) Values

Notice that both the voltage and the current in Figure 24–2 have average values that are zero. Thus, V_{av} and I_{av} give very little information about the actual behavior of V and I. A more useful type of average, or mean, is the **root mean square**, or **rms** for short.

To see the significance of a *root mean square*, consider the current as a function of time, as given in Equation 24–2. First, we *square* the current to obtain

$$I^2 = I^2_{max} \sin^2 \omega t$$

Clearly, I^2 is always positive; hence, its average value will not vanish. Next, we calculate the *mean* value of I^2. This can be done by inspecting Figure 24–4, where we plot I^2 as a function of time. As we see, I^2 varies *symmetrically* between 0 and I^2_{max}; that is, it spends equal amounts of time above and below the value $\frac{1}{2}I^2_{max}$. Hence, the mean value of I^2 is half of I^2_{max}:

$$(I^2)_{av} = \tfrac{1}{2}I^2_{max}$$

Finally, we take the square *root* of this average so that our final result is a current rather than a current squared. This calculation yields the rms value of the current:

$$I_{rms} = \sqrt{(I^2)_{av}} = \frac{1}{\sqrt{2}}I_{max} \qquad 24\text{–}3$$

In general, any quantity x that varies with time as $x = x_{max} \sin \omega t$ or $x = x_{max} \cos \omega t$ obeys the same relationships among its average, maximum, and rms values:

RMS Value of a Quantity with Sinusoidal Time Dependence

$$(x^2)_{av} = \tfrac{1}{2}x^2_{max}$$

$$x_{rms} = \frac{1}{\sqrt{2}}x_{max} \qquad 24\text{–}4$$

As an example, the rms value of the voltage in an ac circuit is

$$V_{rms} = \frac{1}{\sqrt{2}}V_{max} \qquad 24\text{–}5$$

This result is applied to standard household voltages in the following Exercise.

EXERCISE 24–1

Typical household circuits operate with an rms voltage of 120 V. What is the maximum, or peak, value of the voltage in these circuits?

SOLUTION

Solving Equation 24–5 for the maximum voltage, V_{max}, we find

$$V_{max} = \sqrt{2}V_{rms} = \sqrt{2}(120\ \text{V}) = 170\ \text{V}$$

Whenever we refer to an ac generator in this chapter, we shall assume that the time variation is sinusoidal, so that the relations given in Equation 24–4 hold. If a different form of time variation is to be considered, it will be specified explicitly. Because the rms and maximum values of a sinusoidally varying quantity are proportional, it follows that *any* relation between rms values, like $I_{rms} = V_{rms}/R$, is equally valid as a relation between maximum values, $I_{max} = V_{max}/R$.

We now show how rms values are related to the average power consumed by a circuit. Referring again to the lamp circuit shown in Figure 24–1, we see that the instantaneous power dissipated in the resistor is $P = I^2R$. Using the time dependence $I = I_{max} \sin \omega t$, we find

$$P = I^2R = I^2_{max}R \sin^2 \omega t$$

Note that P is always positive, as one might expect; after all, a current always dissipates energy as it passes through a resistor, regardless of its direction. To find the average power dissipated in the resistor, we note again that the average of $\sin^2 \omega t$ is $\frac{1}{2}$, thus

$$P_{av} = I_{max}^2 R(\sin^2 \omega t)_{av} = \tfrac{1}{2} I_{max}^2 R$$

In terms of the rms current, $I_{rms} = I_{max}/\sqrt{2}$, it follows that

$$P_{av} = I_{rms}^2 R$$

Therefore, we arrive at the following conclusion:

> $P = I^2 R$ gives the instantaneous power consumption in both ac and dc circuits. To find the *average* power in an ac circuit, we simply replace I with I_{rms}.

Similar conclusions apply to many other dc and instantaneous formulas as well. For example, the power can also be written as $P = V^2/R$. Using the time-dependent voltage of an ac circuit, we have

$$P = \frac{V^2}{R} = \left(\frac{V_{max}^2}{R}\right)\sin^2 \omega t$$

Clearly, the average power is

$$P_{av} = \left(\frac{V_{max}^2}{R}\right)\left(\frac{1}{2}\right) = \frac{V_{rms}^2}{R} \qquad \text{24–6}$$

As before, we convert the dc power, $P = V^2/R$, to an average ac power by using the rms value of the voltage.

The close similarity between dc expressions and the corresponding ac expressions with rms values is one of the advantages of working with rms values. Another is that electrical meters, such as ammeters and voltmeters, generally give readings of rms values, rather than peak values, when used in ac circuits. In the remainder of this chapter, we shall make extensive use of rms quantities.

PROBLEM-SOLVING NOTE

Maximum Versus RMS Values

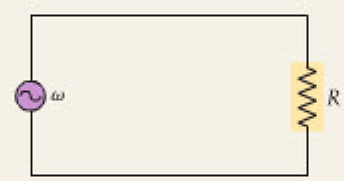
When reading a problem statement, be sure to determine whether a given voltage or current is a maximum value or an rms value. If an rms current is given, for example, it follows that the maximum current is $I_{max} = \sqrt{2} I_{rms}$.

EXAMPLE 24–1 A RESISTOR CIRCUIT

An ac generator with a maximum voltage of 24.0 V and a frequency of 60.0 Hz is connected to a resistor with a resistance $R = 265\ \Omega$. Find **(a)** the rms voltage and **(b)** the rms current in the circuit. In addition, determine **(c)** the average and **(d)** the maximum power dissipated in the resistor.

PICTURE THE PROBLEM

The circuit in this case consists of a 60.0-Hz ac generator connected directly to a 265-Ω resistor. The maximum voltage of the generator is 24.0 V.



STRATEGY

a. The rms voltage is simply $V_{rms} = V_{max}/\sqrt{2}$.

b. Ohm's law gives the rms current, $I_{rms} = V_{rms}/R$.

c. The average power can be found using $P_{av} = I_{rms}^2 R$ or $P_{av} = V_{rms}^2/R$.

d. The maximum power, $P_{max} = V_{max}^2/R$, is twice the average power, from Equation 24–6.

SOLUTION

Part (a)

1. Use $V_{rms} = V_{max}/\sqrt{2}$ to find the rms voltage: $V_{rms} = \frac{V_{max}}{\sqrt{2}} = \frac{24.0\ \text{V}}{\sqrt{2}} = 17.0\ \text{V}$

Part (b)

2. Divide the rms voltage by the resistance, R, to find the rms current: $I_{rms} = \frac{V_{rms}}{R} = \frac{17.0\ \text{V}}{265\ \Omega} = 0.0642\ \text{A}$

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Part (c)

3. Use $I_{rms}^2 R$ to find the average power: $P_{av} = I_{rms}^2 R = (0.0642\text{ A})^2(265\ \Omega) = 1.09\text{ W}$

4. Use V_{rms}^2/R to find the average power: $P_{av} = \dfrac{V_{rms}^2}{R} = \dfrac{(17.0\text{ V})^2}{265\ \Omega} = 1.09\text{ W}$

Part (d)

5. Use $P_{max} = V_{max}^2/R$ to find the maximum power: $P_{max} = \dfrac{V_{max}^2}{R} = 2\left(\dfrac{V_{rms}^2}{R}\right) = 2(1.09\text{ W}) = 2.18\text{ W}$

INSIGHT

Note that the instantaneous power in the resistor, $P = (V_{max}^2/R)\sin^2 \omega t$, oscillates symmetrically between 0 and twice the average power. This is another example of the type of averaging illustrated in Figure 24–4.

PRACTICE PROBLEM

Suppose we would like the average power dissipated in the resistor to be 5.00 W. Should the resistance be increased or decreased, assuming the same ac generator? Find the required value of R. [**Answer:** The resistance should be decreased. The required resistance is $R = 57.8\ \Omega$.]

Some related homework problems: Problem 3, Problem 4

CONCEPTUAL CHECKPOINT 24–1 COMPARE AVERAGE POWER

If the frequency of the ac generator in Example 24–1 is increased, does the average power dissipated in the resistor **(a)** increase, **(b)** decrease, or **(c)** stay the same?

REASONING AND DISCUSSION

None of the results in Example 24–1 depend on the frequency of the generator. For example, the relation $V_{rms} = V_{max}/\sqrt{2}$ depends only on the fact that the voltage varies sinusoidally with time and not at all on the frequency of the oscillations. The same frequency independence applies to the rms current and the average power.

These results are due to the fact that resistance is independent of frequency. In contrast, we shall see later in this chapter that the behavior of capacitors and inductors does indeed depend on frequency.

ANSWER

(c) The average power remains the same.

Safety Features in Household Electric Circuits

In today's technological world, with electrical devices as common as a horse and buggy in an earlier era, we sometimes forget that household electric circuits pose potential dangers to homes and their occupants. For example, if several electrical devices are plugged into a single outlet, the current in the wires connected to that outlet may become quite large. The corresponding power dissipation in the wires ($P = I^2R$) can turn them red hot and lead to a fire.

To protect against this type of danger, household circuits use fuses and circuit breakers. In the case of a fuse, the current in a circuit must flow through a thin metal strip enclosed within the fuse. If the current exceeds a predetermined amount (typically 15 A) the metal strip becomes so hot that it melts and breaks the circuit. Thus, when a fuse "burns out," it is an indication that too many devices are operating on that circuit.

Circuit breakers provide similar protection with a switch that incorporates a bimetallic strip (Figure 16–5). When the bimetallic strip is cool, it closes the switch, allowing current to flow. When the strip is heated by a large current, however, it bends enough to open the switch and stop the current. Unlike the fuse, which cannot be used after it burns out, the circuit breaker can be reset when the bimetallic strip cools and returns to its original shape.

REAL-WORLD PHYSICS: BIO
Electric shock hazard

Household circuits also pose a threat to the occupants of a home, and at much lower current levels than the 15 A it takes to trigger a fuse or a circuit breaker. For example, it takes a current of only about 0.001 A to give a person a mild tingling sensation. Currents in the range of 0.01 A to 0.02 A produce muscle spasms that make it difficult to let go of the wire delivering the current, or may even result in respiratory arrest. When currents reach 0.1 A to 0.2 A, the heartbeat is interrupted by an uncoordinated twitching referred to as ventricular fibrillation. As a result, currents in this range can prove to be fatal in a matter of seconds.

REAL-WORLD PHYSICS
Polarized plugs and grounded plugs

Several strategies are employed to reduce the danger of electrical shock. The first line of defense is the *polarized plug*, in which one prong of the plug is wider than the other prong. The corresponding wall socket will accept the plug in only one orientation, with the wide prong in the wide receptacle. The narrow receptacle of the outlet is wired to the high-potential side of the circuit; the wide receptacle is connected to the low-potential side, which is essentially at "ground" potential. A polarized plug provides protection by ensuring that the case of an electrical appliance, which manufacturers design to be connected to the wide prong, is at low potential. Furthermore, when an electrical device with a polarized plug is turned off, the high potential extends only from the wall outlet to the switch, leaving the rest of the device at zero potential.

The next line of defense against accidental shock is the three-prong grounded plug. In this plug, the rounded third prong is connected directly to ground when plugged into a three-prong receptacle. In addition, the third prong is wired to the case of an electrical appliance. If something goes wrong within the appliance, and a high-potential wire comes into contact with the case, the resulting current flows through the third prong, rather than through the body of a person who happens to touch the case.

REAL-WORLD PHYSICS
Ground fault circuit interrupter

An even greater level of protection is provided by a device known as a *ground fault circuit interrupter* (GFCI). The basic operating principle of an interrupter is illustrated in **Figure 24–5**. Note that the wires carrying an ac current to the protected appliance pass through a small iron ring. When the appliance operates normally, the two wires carry equal currents in opposite directions—in one wire the current goes to the appliance, in the other the same current returns from the appliance. Each of these wires produces a magnetic field (Equation 22–9), but because their currents are in opposite directions the magnetic fields are in opposite directions as well. As a result, the magnetic fields of the two wires cancel. If a malfunction occurs in the appliance—say, a wire frays and contacts the case of the appliance—current that would ordinarily return through the power cord may pass through the user's body instead and into the ground. In such a situation, the wire carrying current to the appliance now produces a net magnetic field within the iron ring that varies with the frequency of the ac generator. The changing magnetic field in the ring induces a current in the sensing coil wrapped around the ring, and the induced current triggers a circuit breaker in the interrupter. This cuts the flow of current to the appliance within a millisecond, protecting the user. In newer homes, interrupters are built directly into the wall sockets. The same protection can be obtained, however, by plugging a ground fault interrupter into an unprotected wall socket and then plugging an appliance into the interrupter.

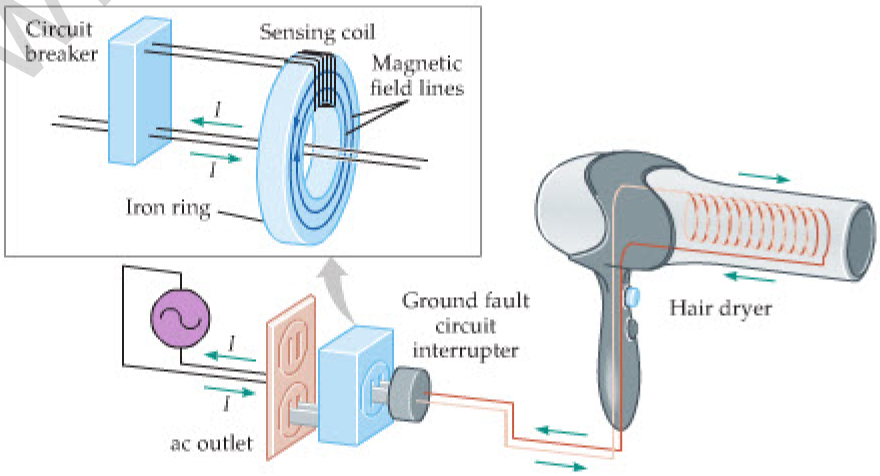
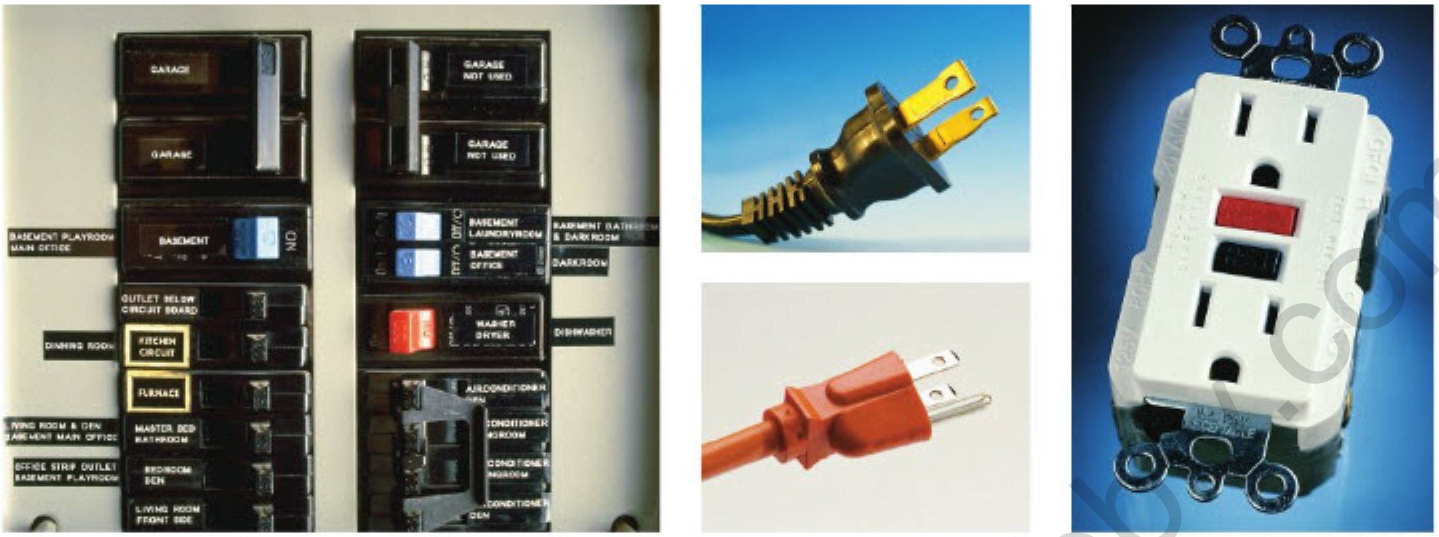


FIGURE 24–5 The ground fault circuit interrupter

A short circuit in an appliance will ordinarily blow the fuse or trip the circuit breaker in the electrical line supplying power to the circuit. However, although fuses and circuit breakers help prevent electrical fires, these devices are not very good at preventing electric shock. This is because they are activated by the heat produced when the current becomes abnormally high—for example, when a "hot" wire touches a conductor that is in contact with the ground. If you happen to be that conductor, the fuse or breaker may act too slowly to prevent serious injury or even death. A ground fault circuit interrupter, by contrast, can cut off the current in a shorted circuit in less than a millisecond, before any harm can be done.



▲ Electricity is everywhere in the modern house, and electricity can be dangerous. Many common safety devices help us to minimize the risks associated with this very convenient form of energy. Whereas older homes typically have fuse boxes, most new houses are protected by circuit breaker panels (left). Both function in a similar way—if too much current is being drawn (perhaps as the result of a short circuit), the heat produced "blows" the fuse (by melting a metal strip) or "trips" the circuit breaker (by bending a bimetallic strip). Either way, the circuit is interrupted and current is cut off before the wires can become hot enough to start a fire.

The danger of electric shock can be reduced by the use of polarized plugs (center, top) or three-prong, grounded plugs (center, bottom) on appliances. Each of these provides a low-resistance path to ground that can be wired to the case of an appliance. In the event that a "hot" wire touches the case, most of the current will flow through the grounded wire rather than through the user. Even more protection is afforded by a ground fault circuit interrupter, or GFCI (right). This device, which is much faster and more sensitive than an ordinary circuit breaker, utilizes magnetic induction to interrupt the current in a circuit that has developed a short.

24–2 Capacitors in ac Circuits

In an ac circuit containing only a generator and a capacitor, the relationship between current and voltage is different in many important respects from the behavior seen with a resistor. In this section we explore these differences and show that the average power consumed by a capacitor is zero.

Capacitive Reactance

▲ **FIGURE 24–6 An ac generator connected to a capacitor**

The rms current in an ac capacitor circuit is $I_{rms} = V_{rms}/X_C$, where the capacitive reactance is $X_C = 1/\omega C$. Therefore, the rms current in this circuit is proportional to the frequency of the generator.

Consider a simple circuit consisting of an ac generator and a capacitor, as in Figure 24–6. The generator supplies an rms voltage, V_{rms}, to the capacitor. We would like to answer the following question: How is the rms current in this circuit related to the capacitance of the capacitor, C, and the frequency of the generator, ω?

To answer this question requires the methods of calculus, but the final result is quite straightforward. In fact, the rms current is simply

$$I_{rms} = \omega C V_{rms} \qquad 24\text{–}7$$

In analogy with the expression $I_{rms} = V_{rms}/R$ for a resistor, we will find it convenient to rewrite this result in the following form:

$$I_{rms} = \frac{V_{rms}}{X_C} \qquad 24\text{–}8$$

In this expression, X_C is referred to as the **capacitive reactance;** it plays the same role for a capacitor that resistance does for a resistor. In particular, to find the rms current in a resistor we divide V_{rms} by R; to find the rms current that flows into one side of a capacitor and out the other side, we divide V_{rms} by X_C. Comparing Equations 24–7 and 24–8, we see that the capacitive reactance can be written as follows:

Capacitive Reactance, X_C

$$X_C = \frac{1}{\omega C} \qquad 24\text{–}9$$

SI unit: ohm, Ω

It is straightforward to show that the unit of $X_C = 1/\omega C$ is the ohm, the same unit as for resistance.

EXERCISE 24–2

Find the capacitive reactance of a 22-μF capacitor in a 60.0-Hz circuit.

SOLUTION

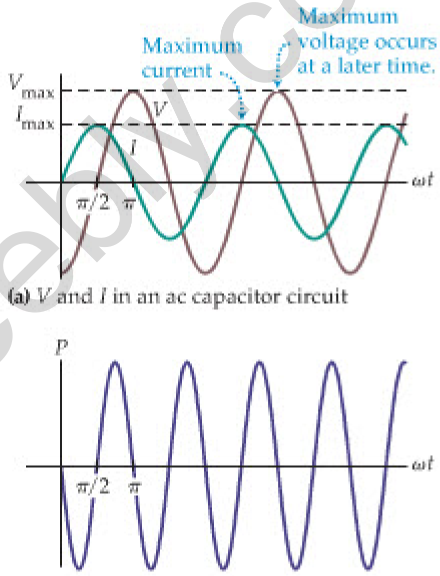
Using Equation 24–9, and recalling that $\omega = 2\pi f$, we find

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi(60.0\ \mathrm{s^{-1}})(22 \times 10^{-6}\ \mathrm{F})} = 120\ \Omega$$

Unlike resistance, the capacitive reactance of a capacitor depends on the frequency of the ac generator. For example, at low frequencies the capacitive reactance becomes very large, and hence the rms current, $I_{rms} = V_{rms}/X_C$, is very small. This is to be expected. In the limit of low frequency, the current in the circuit becomes constant; that is, the ac generator becomes a dc battery. In this case we know that the capacitor becomes fully charged, and the current ceases to flow.

In the limit of large frequency, the capacitive reactance is small and the current is large. The reason is that at high frequency the current changes direction so rapidly that there is never enough time to fully charge the capacitor. As a result, the charge on the capacitor is never very large, and therefore it offers essentially no resistance to the flow of charge.

The behavior of a capacitor as a function of frequency is considered in the next Exercise.



(a) V and I in an ac capacitor circuit

(b) Power in an ac capacitor circuit

▲ **FIGURE 24–7 Time variation of voltage, current, and power for a capacitor in an ac circuit**

(a) Note that the time dependences are different for the voltage and the current. In particular, the voltage reaches its maximum value $\pi/2$ rad, or 90°, *after* the current. Thus we say that the voltage *lags* the current by 90°. **(b)** The power consumed by a capacitor in an ac circuit, $P = IV$, has an average value of zero.

EXERCISE 24–3

Suppose the capacitance in Figure 24–6 is 4.5 μF and the rms voltage of the generator is 120 V. Find the rms current in the circuit when the frequency of the generator is **(a)** 60.0 Hz and **(b)** 6.00 Hz.

SOLUTION

Applying Equations 24–8 and 24–9, we find for part (a)

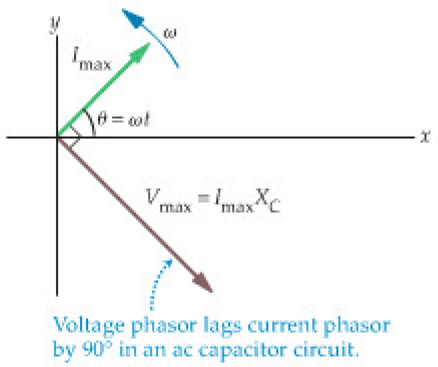
$$I_{rms} = \frac{V_{rms}}{X_C} = \omega C V_{rms} = (2\pi)(60.0\ \mathrm{s^{-1}})(4.5 \times 10^{-6}\ \mathrm{F})(120\ \mathrm{V}) = 0.20\ \mathrm{A}$$

Similarly, for part (b)

$$I_{rms} = \frac{V_{rms}}{X_C} = \omega C V_{rms} = (2\pi)(6.00\ \mathrm{s^{-1}})(4.5 \times 10^{-6}\ \mathrm{F})(120\ \mathrm{V}) = 0.020\ \mathrm{A}$$

When the frequency is reduced, the reactance becomes larger and the current decreases, as expected.

Notice that the rms current in a capacitor is proportional to its capacitance, C, for all frequencies. A capacitor with a large capacitance can store and release large amounts of charge, which results in a large current. Finally, recall that rms expressions like $I_{rms} = V_{rms}/X_C$ (Equation 24–8) are equally valid in terms of maximum quantities; that is, $I_{max} = V_{max}/X_C$.



▲ **FIGURE 24–8 Current and voltage phasors for a capacitor**

Both the current and voltage phasors rotate counterclockwise about the origin. The voltage phasor lags the current phasor by 90°. This means that the voltage points in a direction that is 90° clockwise from the direction of the current.

Phasor Diagrams: Capacitor Circuits

Figure 24–7 (a) shows the time dependence of the voltage and current in an ac capacitor circuit. Notice that the voltage and current are not in phase. For example, the current has its maximum value at the time $\omega t = \pi/2 = 90°$, whereas the voltage does not reach its maximum value until a later time, when $\omega t = \pi = 180°$. In general,

> the voltage across a capacitor *lags* the current by 90°.

This 90° difference between current and voltage can probably best be seen in a phasor diagram. For example, in **Figure 24–8** we show both the current and the

PROBLEM-SOLVING NOTE

Drawing a Capacitor Phasor

When drawing a voltage phasor for a capacitor, always draw it at right angles to the current phasor. In addition, be sure the capacitor phasor is 90° *clockwise* from the current phasor.

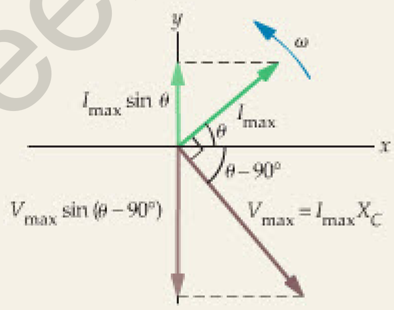
voltage phasors for a capacitor. The current phasor, with a magnitude I_{max}, is shown at the angle $\theta = \omega t$; it follows that the instantaneous current is $I = I_{max} \sin \omega t$. (As a matter of consistency, all phasor diagrams in this chapter will show the current phasor at the angle $\theta = \omega t$.) The voltage phasor, with a magnitude $V_{max} = I_{max} X_C$, is at right angles to the current phasor, pointing in the direction $\theta = \omega t - 90°$. The instantaneous value of the voltage is $V = V_{max} \sin(\omega t - 90°)$. Because the phasors rotate counterclockwise, we see that the voltage phasor lags the current phasor.

EXAMPLE 24–2 INSTANTANEOUS VOLTAGE

Consider a capacitor circuit, as in Figure 24–6, where the capacitance is $C = 50.0\ \mu F$, the maximum current is 2.10 A, and the frequency of the generator is 60.0 Hz. At a given time, the current in the capacitor is 0.500 A and increasing. What is the voltage across the capacitor at this time?

PICTURE THE PROBLEM

The appropriate phasor diagram for this system is shown in our sketch. To see how this diagram was drawn, we first recall that the current has a positive value and is increasing. The fact that it is positive means it must point in a direction θ between 0 and 180°. The additional fact that it is increasing means θ must be between 0 and 90°; if it were between 90° and 180°, it would be decreasing with time. Finally, the voltage phasor lags the current phasor by 90° in a capacitor.



STRATEGY

There are several pieces that we must put together to complete this solution. First, we know the maximum current, I_{max}, and the instantaneous current, $I = I_{max} \sin \omega t = I_{max} \sin \theta$. We can solve this relation for the angle θ.

Second, the maximum voltage is given by $V_{max} = I_{max} X_C = I_{max}/\omega C$.

Finally, the instantaneous voltage is $V = V_{max} \sin(\theta - 90°)$, as indicated in the sketch.

SOLUTION

1. Use $I = I_{max} \sin \theta$ to find the angle θ of the current phasor above the x axis:
$$\theta = \sin^{-1}\left(\frac{I}{I_{max}}\right) = \sin^{-1}\left(\frac{0.500\ \text{A}}{2.10\ \text{A}}\right) = 13.8°$$

2. Find the maximum voltage in the circuit using $V_{max} = I_{max}/\omega C$:
$$V_{max} = \frac{I_{max}}{\omega C} = \frac{2.10\ \text{A}}{2\pi(60.0\ \text{s}^{-1})(50.0 \times 10^{-6}\ \text{F})} = 111\ \text{V}$$

3. Calculate the instantaneous voltage across the capacitor with $V = V_{max} \sin(\theta - 90°)$:
$$V = V_{max} \sin(\theta - 90°)$$
$$= (111\ \text{V}) \sin(13.8° - 90°) = -108\ \text{V}$$

INSIGHT

We can see from the phasor diagram that the current is positive and increasing in magnitude, whereas the voltage is negative and decreasing in magnitude. It follows that both the current and the voltage are becoming more positive with time.

PRACTICE PROBLEM

What is the voltage across the capacitor when the current in the circuit is 2.10 A? Is the voltage increasing or decreasing at this time? [**Answer:** $V = 0$, increasing]

Some related homework problems: Problem 12, Problem 13

To gain a qualitative understanding of the phase relation between the voltage and current in a capacitor, note that at the time when $\omega t = \pi/2$ the voltage across the capacitor in Figure 24–7 (a) is zero. Since the capacitor offers no resistance to the flow of current at this time, the current in the circuit is now a maximum. As the current continues to flow, charge builds up on the capacitor, and its voltage increases. This causes the current to decrease. At the time when $\omega t = \pi$ the capacitor voltage reaches a maximum and the current vanishes. As the current begins to flow in the opposite direction, charge flows out of the capacitor and its voltage decreases. When the voltage goes to zero, the current is once again a maximum, though this time in the opposite direction. It follows, then, that the variations of current and voltage are 90° out of phase—that is, when one is a maximum or a minimum, the other is zero—just like position and velocity in simple harmonic motion.

Power

As a final observation on the behavior of a capacitor in an ac circuit, we consider the power it consumes. Recall that the instantaneous power for any circuit can be written as $P = IV$. In **Figure 24–7 (b)** we plot the power $P = IV$ corresponding to the current and voltage shown in Figure 24–7 (a). The result is a power that changes sign with time.

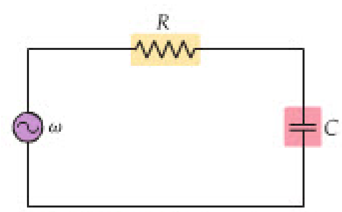
In particular, note that the power is negative when the current and voltage have opposite signs, as between $\omega t = 0$ and $\omega t = \pi/2$, but is positive when they have the same sign, as between $\omega t = \pi/2$ and $\omega t = \pi$. This means that between $\omega t = 0$ and $\omega t = \pi/2$ the capacitor draws energy from the generator, but between $\omega t = \pi/2$ and $\omega t = \pi$ it delivers energy back to the generator. As a result, the average power as a function of time is zero, as can be seen by the symmetry about zero power in Figure 24–7 (b). Thus, *a capacitor in an ac circuit consumes zero net energy*.

24–3 *RC* Circuits

We now consider a resistor and a capacitor in series in the same ac circuit. This leads to a useful generalization of resistance known as the impedance.

Impedance

The circuit shown in **Figure 24–9** consists of an ac generator, a resistor, R, and a capacitor, C, connected in series. It is assumed that the values of R and C are known, as well as the maximum voltage, V_{max}, and angular frequency, ω, of the generator. In terms of these quantities we would like to determine the maximum current in the circuit and the maximum voltages across both the resistor and the capacitor.

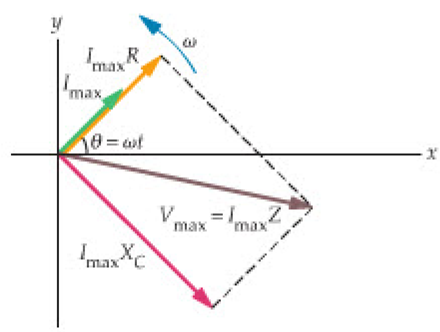


▲ **FIGURE 24–9 An alternating-current *RC* circuit**

An ac *RC* circuit consists of a generator, a resistor, and a capacitor connected in series.

To begin, we note that the magnitudes of the voltages are readily determined. For instance, the magnitude of the maximum voltage across the resistor is $V_{max,R} = I_{max}R$, and for the capacitor it is $V_{max,C} = I_{max}X_C = I_{max}/\omega C$. The total voltage in this circuit is *not* the sum of these two voltages, however, because they are not in phase—they do not attain their maximum values at the same time. To take these phase differences into account we turn to a phasor diagram.

Figure 24–10 shows the phasor diagram for a simple *RC* circuit. To construct this diagram, we start by drawing the current phasor with a length I_{max} at the angle $\theta = \omega t$, as indicated in the figure. Next, we draw the voltage phasor associated with the resistor. This has a magnitude of $I_{max}R$ and points in the same direction as the current phasor. Finally, we draw the voltage phasor for the capacitor. This phasor has a magnitude of $I_{max}X_C$ and points in a direction that is rotated 90° clockwise from the current phasor.



▲ **FIGURE 24–10 Phasor diagram for an *RC* circuit**

The maximum current in an *RC* circuit is $I_{max} = V_{max}/Z$, where V_{max} is the maximum voltage of the ac generator and $Z = \sqrt{R^2 + X_C{}^2}$ is the impedance of the circuit. The maximum voltage across the resistor is $V_{max,R} = I_{max}R$ and the maximum voltage across the capacitor is $V_{max,C} = I_{max}X_C$.

Now, to obtain the total voltage in the circuit in terms of the individual voltages, we perform a vector sum of the resistor-voltage phasor and the capacitor-voltage phasor, as indicated in Figure 24–10. The total voltage, then, is the hypotenuse of the right triangle formed by these two phasors. Its magnitude, using the Pythagorean theorem, is simply

$$V_{max} = \sqrt{V_{max,R}^2 + V_{max,C}^2} \qquad 24\text{–}10$$

Substituting the preceding expressions for the voltages across R and C, we find

$$V_{max} = \sqrt{(I_{max}R)^2 + (I_{max}X_C)^2} = I_{max}\sqrt{R^2 + X_C{}^2}$$

The last quantity in the preceding expression is given a special name; it is called the **impedance**, Z:

Impedance in an *RC* Circuit

$$Z = \sqrt{R^2 + X_C{}^2} = \sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2} \qquad 24\text{–}11$$

SI unit: ohm, Ω

Clearly, the impedance has units of ohms, the same as those of resistance and reactance.

EXERCISE 24–4

A given *RC* circuit has $R = 135\ \Omega$, $C = 28.0\ \mu F$, and $f = 60.0$ Hz. What is the impedance of the circuit?

SOLUTION

Applying Equation 24–11 with $\omega = 2\pi f$, we obtain

$$Z = \sqrt{R^2 + X_C^2} = \sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}$$

$$= \sqrt{(135\ \Omega)^2 + \left(\frac{1}{2\pi(60.0\ s^{-1})(28.0 \times 10^{-6}\ F)}\right)^2} = 165\ \Omega$$

We are now in a position to calculate the maximum current in an *RC* circuit. First, given *R*, *C*, and $\omega = 2\pi f$, we can determine the value of *Z* using Equation 24–11. Next, we solve for the maximum current using the relation

$$V_{max} = I_{max}\sqrt{R^2 + X_C^2} = I_{max}Z$$

which yields

$$I_{max} = \frac{V_{max}}{Z}$$

In a circuit containing only a resistor, the maximum current is $I_{max} = V_{max}/R$; thus, we see that the impedance *Z* is indeed a generalization of resistance that can be applied to more complex circuits.

To check some limits of *Z*, note that in the capacitor circuit discussed in the previous section (Figure 24–6), the resistance is zero. Hence, in that case the impedance is

$$Z = \sqrt{0 + X_C^2} = X_C$$

The maximum voltage across the capacitor, then, is $V_{max} = I_{max}/Z = V_{max}/X_C$, as expected. Similarly, in a resistor circuit with no capacitor, as in Figure 24–1, the capacitive reactance is zero; hence,

$$Z = \sqrt{R^2 + 0} = R$$

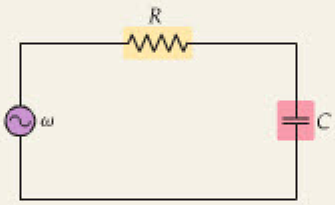
Thus, *Z* includes X_C and *R* as special cases.

EXAMPLE 24–3 FIND THE FREQUENCY

An ac generator with an rms voltage of 110 V is connected in series with a 35-Ω resistor and a 11-μF capacitor. The rms current in the circuit is 1.2 A. What are **(a)** the impedance and **(b)** the capacitive reactance of this circuit? **(c)** What is the frequency, *f*, of the generator?

PICTURE THE PROBLEM

The appropriate circuit is shown in our sketch. We are given the rms voltage of the generator, $V_{rms} = 110$ V; the resistance, $R = 35\ \Omega$; and the capacitance, $C = 11\ \mu F$. The only remaining variable that affects the current is the frequency of the generator, *f*.



STRATEGY

a. The impedance, *Z*, can be solved directly from $V_{rms} = I_{rms}Z$.
b. Once the impedance is known, we can use $Z = \sqrt{R^2 + X_C^2}$ to find the capacitive reactance, X_C.
c. Now that the reactance is known, we can use the relation $X_C = 1/\omega C = 1/2\pi fC$ to solve for the frequency.

SOLUTION

Part (a)

1. Use $V_{rms} = I_{rms}Z$ to find the impedance, *Z*:

$$Z = \frac{V_{rms}}{I_{rms}} = \frac{110\ V}{1.2\ A} = 92\ \Omega$$

Part (b)

2. Solve for X_C in terms of Z and R:

$$Z = \sqrt{R^2 + X_C^2}$$

$$X_C = \sqrt{Z^2 - R^2} = \sqrt{(92\ \Omega)^2 - (35\ \Omega)^2} = 85\ \Omega$$

Part (c)

3. Use the value of X_C from part (b) to find the frequency, f:

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$$

$$f = \frac{1}{2\pi X_C C} = \frac{1}{2\pi(85\ \Omega)(11 \times 10^{-6}\ \text{F})} = 170\ \text{Hz}$$

INSIGHT

Note that the rms voltage across the resistor is $V_{\text{rms},R} = I_{\text{rms}} R = 42$ V, and the rms voltage across the capacitor is $V_{\text{rms},C} = I_{\text{rms}} X_C = 102$ V. As expected, these voltages *do not* add up to the generator voltage of 110 V—in fact, they add up to a considerably larger voltage. The point is, however, that the sum of 42 V and 102 V is physically meaningless because the voltages are 90° out of phase, and hence do not occur at the same time. If the voltages are combined as in Equation 24–10, which takes into account the 90° phase difference, we find $V_{\text{rms}} = \sqrt{(V_{\text{rms},R}^2 + V_{\text{rms},C}^2)} = \sqrt{(42\ \text{V})^2 + (102\ \text{V})^2} = 110$ V, as expected.

PRACTICE PROBLEM

(a) If the frequency in this circuit is increased, do you expect the rms current to increase, decrease, or stay the same? **(b)** What is the rms current if the frequency is increased to 250 Hz? [**Answer: (a)** The rms current increases with frequency because the capacitive reactance decreases as the frequency is increased. **(b)** At 250 Hz the current is 1.6 A.]

Some related homework problems: Problem 20, Problem 21

ACTIVE EXAMPLE 24–1 FIND THE RESISTANCE

An ac generator with an rms voltage of 110 V and a frequency of 60.0 Hz is connected in series with a 270-μF capacitor and a resistor of resistance R. What value must R have if the rms current in this circuit is to be 1.7 A?

SOLUTION *(Test your understanding by performing the calculations indicated in each step.)*

1. Solve for the resistance: $R = \sqrt{Z^2 - X_C^2}$
2. Find the impedance of the circuit: $Z = 65\ \Omega$
3. Find the capacitive reactance: $X_C = 9.8\ \Omega$
4. Substitute numerical values: $R = 64\ \Omega$

INSIGHT

At this frequency the capacitor has relatively little effect in the circuit, as can be seen by comparing X_C, R, and Z.

YOUR TURN

At what frequency is the capacitive reactance equal to 64 Ω? What is the current in the circuit at this frequency, assuming all other variables remain the same?

(Answers to **Your Turn** *problems are given in the back of the book.)*

We can also gain considerable insight about an ac circuit with qualitative reasoning—that is, without going through detailed calculations like those just given. This is illustrated in the following Conceptual Checkpoint.

CONCEPTUAL CHECKPOINT 24–2 COMPARE BRIGHTNESS: CAPACITOR CIRCUIT

Shown in the sketch are two circuits with identical ac generators and lightbulbs. Circuit 2 differs from circuit 1 by the addition of a capacitor in series with the light. Does the lightbulb in circuit 2 shine **(a)** more brightly, **(b)** less brightly, or **(c)** with the same intensity as that in circuit 1?

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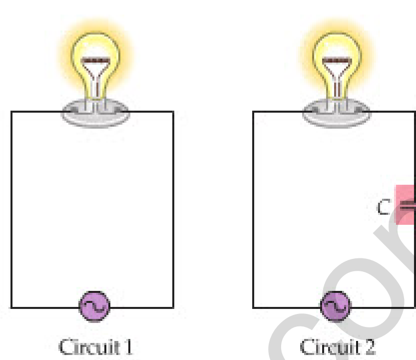
REASONING AND DISCUSSION

Since circuit 2 has both a resistance (in the lightbulb) and a capacitive reactance, its impedance, $Z = \sqrt{R^2 + X_C^2}$, is greater than that of circuit 1, which has only the resistance R. Therefore, the current in circuit 2, $I_{rms} = V_{rms}/Z$, is less than the current in circuit 1. As a result, the average power dissipated in the lightbulb, $P_{av} = I_{rms}^2 R$, is less in circuit 2, so its bulb shines less brightly.



Note that the bulb dims, even though no power is consumed by the capacitor.

ANSWER

(b) The light bulb in circuit 2 shines less brightly.

Phase Angle and Power Factor

We have seen how to calculate the current in an RC circuit and how to find the voltages across each element. Next we consider the phase relation between the total voltage in the circuit and the current. As we shall see, there is a direct connection between this phase relation and the power consumed by a circuit.

To do this, consider the phasor diagram shown in Figure 24–11 (a). The **phase angle**, ϕ, between the voltage and the current can be read off the diagram as indicated in Figure 24–11 (b). Clearly, the cosine of ϕ is given by the following:

$$\cos\phi = \frac{I_{max}R}{I_{max}Z} = \frac{R}{Z} \qquad 24\text{–}12$$

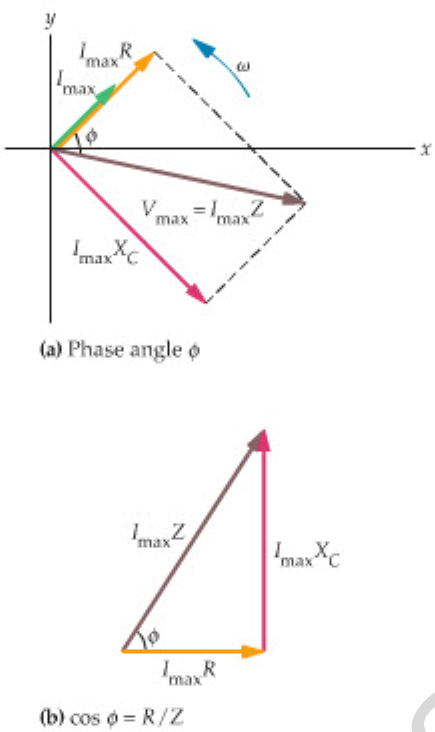


FIGURE 24–11 Phase angle for an *RC* circuit

(a) The phase angle, ϕ, is the angle between the voltage phasor, $V_{max} = I_{max}Z$, and the current phasor, I_{max}. Because I_{max} and $I_{max}R$ are in the same direction, we can say that ϕ is the angle between $I_{max}Z$ and $I_{max}R$. **(b)** From this triangle we can see that $\cos\phi = R/Z$, a result that is valid for any circuit.

As we shall see, both the magnitude of the voltage and its phase angle relative to the current play important roles in the behavior of a circuit.

Consider, for example, the power consumed by the circuit shown in Figure 24–9. One way to obtain this result is to recall that no power is consumed by the capacitor at all, as was shown in the previous section. Thus the total power of the circuit is simply the power dissipated by the resistor. This power can be written as

$$P_{av} = I_{rms}^2 R$$

An equivalent expression for the power is obtained by replacing one power of I_{rms} with the expression $I_{rms} = V_{rms}/Z$. This replacement yields

$$P_{av} = I_{rms}^2 R = I_{rms}I_{rms}R = I_{rms}\left(\frac{V_{rms}}{Z}\right)R$$

Finally, recalling that $R/Z = \cos\phi$, we can write the power as follows:

$$P_{av} = I_{rms}V_{rms}\cos\phi \qquad 24\text{–}13$$

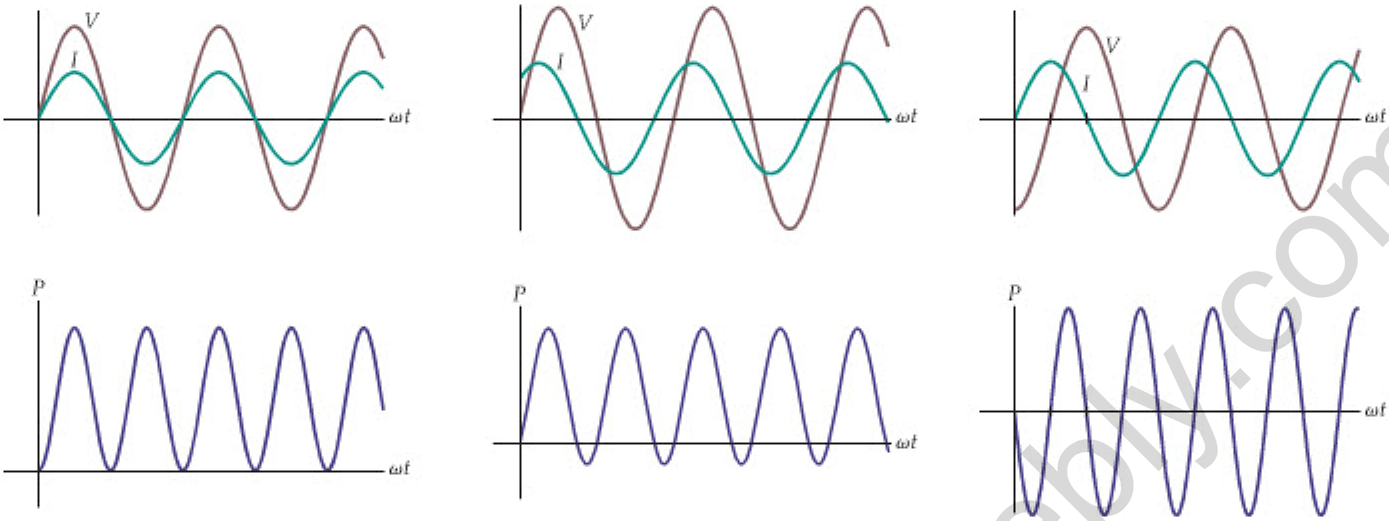
Thus, a knowledge of the current and voltage in a circuit, along with the value of $\cos\phi$, gives the power. The multiplicative factor $\cos\phi$ is referred to as the **power factor.**

Writing the power in terms of ϕ allows one to get a feel for the power in a circuit simply by inspecting the phasor diagram. For example, in the case of a circuit with only a capacitor, the phasor diagram (Figure 24–8) shows that the angle between the current and voltage has a magnitude of 90°, so the power factor is zero; $\cos\phi = \cos 90° = 0$. Thus no power is consumed in this circuit, as expected. In contrast, in a purely resistive circuit the phasor diagram shows that $\phi = 0$, as in Figure 24–3, giving a power factor of 1. In this case, the power is simply $P_{av} = I_{rms}V_{rms}$. Therefore, the angle between the current and the voltage in a phasor diagram gives an indication of the power being used by the circuit. We explore this feature in greater detail in the next Example and in Figure 24–12.

PROBLEM-SOLVING NOTE

The Angle in the Power Factor

The angle in the power factor, $\cos\phi$, is the angle between the voltage phasor and the current phasor. Be careful not to identify ϕ with the angle between the voltage phasor and the x or y axis.



(a) V and I in phase ($\phi = 0$) (b) V lags I by 45° ($\phi = 45°$) (c) V lags I by 90° ($\phi = 90°$)

▲ **FIGURE 24–12 Voltage, current, and power for various phase angles, ϕ**

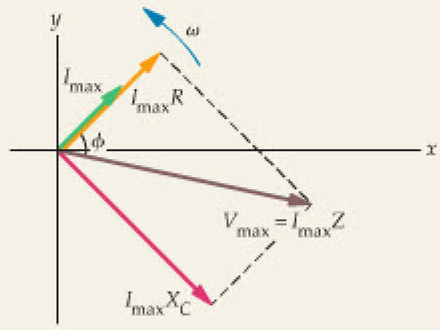
(a) $\phi = 0$. In this case the power is always positive. **(b)** $\phi = 45°$. The power oscillates between positive and negative values, with an average that is positive. **(c)** $\phi = 90°$. The power oscillates symmetrically about its average value of zero.

EXAMPLE 24–4 POWER AND RESISTANCE

A certain *RC* circuit has an ac generator with an rms voltage of 240 V. The rms current in the circuit is 2.5 A, and it leads the voltage by 56°. Find **(a)** the value of the resistance, *R*, and **(b)** the average power consumed by the circuit.

PICTURE THE PROBLEM

The phasor diagram appropriate to this circuit is shown in the sketch. Note that the voltage lags the current by the angle $\phi = 56°$.



STRATEGY

a. To find the resistance in the circuit, recall that $\cos\phi = R/Z$; thus, $R = Z\cos\phi$. We can find the impedance, Z, from $V_{rms} = I_{rms}Z$, since we know both V_{rms} and I_{rms}.

b. The average power consumed by the circuit is simply $P_{av} = I_{rms}V_{rms}\cos\phi$.

SOLUTION

Part (a)

1. Use $V_{rms} = I_{rms}Z$ to find the impedance of the circuit: $Z = \dfrac{V_{rms}}{I_{rms}} = \dfrac{240\text{ V}}{2.5\text{ A}} = 96\ \Omega$

2. Now use $\cos\phi = R/Z$ to find the resistance: $R = Z\cos\phi = (96\ \Omega)\cos 56° = 54\ \Omega$

Part (b)

3. Calculate the average power with the expression $P_{av} = I_{rms}V_{rms}\cos\phi$: $P_{av} = I_{rms}V_{rms}\cos\phi = (2.5\text{ A})(240\text{ V})\cos 56° = 340\text{ W}$

INSIGHT

Of course, the average power consumed by this circuit is simply the average power dissipated in the resistor: $P_{av} = I_{rms}^2R = (2.5\text{ A})^2(54\ \Omega) = 340\text{ W}$. Note that care must be taken if the average power dissipated in the resistor is calculated using $P_{av} = V_{rms}^2/R$. The potential pitfall is that one might use 240 V for the rms voltage; but this is the rms voltage of the generator, *not* the rms voltage across the resistor. This problem didn't arise with $P_{av} = I_{rms}^2R$ because the same current flows through both the generator and the resistor.

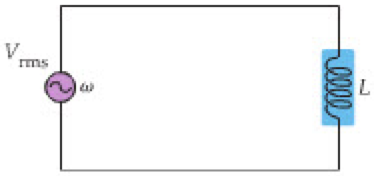
PRACTICE PROBLEM

What is the capacitive reactance in this circuit? **[Answer:** $X_C = 79\ \Omega$**]**

Some related homework problems: Problem 22, Problem 23, Problem 24

24–4 Inductors in ac Circuits

We turn now to the case of ac circuits containing inductors. As we shall see, the behavior of inductors is, in many respects, just the opposite of that of capacitors.



▲ **FIGURE 24–13 An ac generator connected to an inductor**

The rms current in an ac inductor circuit is $I_{rms} = V_{rms}/X_L$, where the inductive reactance is $X_L = \omega L$.

Inductive Reactance

The voltage across a capacitor is given by $V = IX_C$, where $X_C = 1/\omega C$ is the capacitive reactance. Similarly, the voltage across an inductor connected to an ac generator, as in Figure 24–13, can be written as $V = IX_L$, which defines the **inductive reactance**, X_L. The precise expression for X_L, in terms of frequency and inductance, can be derived using the methods of calculus. The result of such a calculation is the following:

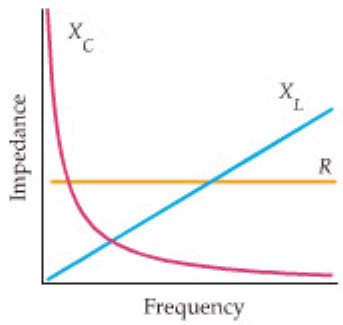
Inductive Reactance, X_L

$$X_L = \omega L \qquad 24\text{–}14$$

SI unit: ohm, Ω

A plot of X_L versus frequency is given in Figure 24–14, where it is compared with X_C and R. The rms current in an ac inductor circuit is $I_{rms} = V_{rms}/X_L = V_{rms}/\omega L$.

Note that X_L increases with frequency, in contrast with the behavior of X_C. This is easily understood when one recalls that the voltage across an inductor has a magnitude given by $\mathcal{E} = L\,\Delta I/\Delta t$. Thus the higher the frequency, the more rapidly the current changes with time, and hence the greater the voltage across the inductor.



▲ **FIGURE 24–14 Frequency variation of the inductive reactance, X_L, the capacitive reactance, X_C, and the resistance, R**

The resistance is independent of frequency. In contrast, the capacitive reactance becomes large with decreasing frequency, and the inductive reactance becomes large with increasing frequency.

EXERCISE 24–5

A 21-mH inductor is connected to an ac generator with an rms voltage of 24 V and a frequency of 60.0 Hz. What is the rms current in the inductor?

SOLUTION

The rms current is $I_{rms} = V_{rms}/X_L = V_{rms}/\omega L$. Substituting numerical values, we find

$$I_{rms} = \frac{V_{rms}}{\omega L} = \frac{24\ \text{V}}{2\pi(60.0\ \text{s}^{-1})(21 \times 10^{-3}\ \text{H})} = 3.0\ \text{A}$$

Phasor Diagrams: Inductor Circuits

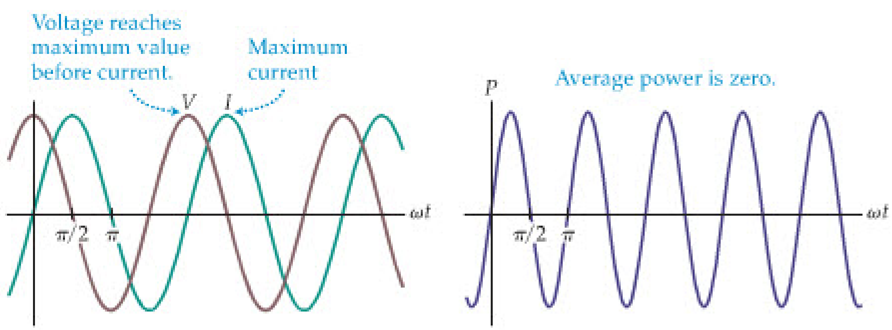
Now that we can calculate the magnitude of the voltage across an inductor, we turn to the question of the phase of the inductor's voltage relative to the current in the circuit. With both phase and magnitude determined, we can construct the appropriate phasor diagram for an inductor circuit.

Suppose the current in an inductor circuit is as shown in Figure 24–15 (a). At time zero the current is zero, but increasing at its maximum rate. Since the voltage across an inductor depends on the rate of change of current, it follows that the inductor's voltage is a maximum at $t = 0$. When the current reaches a maximum value, at the time $\omega t = \pi/2$, its rate of change becomes zero; hence, the voltage across the inductor falls to zero at that point, as is also indicated in Figure 24–15 (a).

PROBLEM-SOLVING NOTE

Drawing an Inductor Phasor

When drawing a voltage phasor for an inductor, always draw it at right angles to the current phasor. In addition, be sure the inductor phasor is 90° *counterclockwise* from the current phasor.



(a) V and I in an ac inductor circuit

(b) Power in an ac inductor circuit

▶ **FIGURE 24–15 Voltage, current, and power in an ac inductor circuit**

(a) Voltage V and current I in an inductor. Note that V reaches a maximum $\pi/2$ rad = 90° *before* the current. Thus, the voltage *leads* the current by 90°. **(b)** The power consumed by an inductor. Note that the average power is zero.

Thus we see that the current and the inductor's voltage are a quarter of a cycle (90°) out of phase. More specifically, since the voltage reaches its maximum *before* the current reaches its maximum, we say that the voltage *leads* the current:

The voltage across an inductor *leads* the current by 90°.

Note that this, again, is just the opposite of the behavior in a capacitor.

The phase relation between current and voltage in an inductor is shown with a phasor diagram in Figure 24–16. Here we see the current, I_{max}, and the corresponding inductor voltage, $V_{max} = I_{max}X_L$. Note that the voltage is rotated counterclockwise (that is, ahead) of the current by 90°.

Because of the 90° angle between the current and voltage, the power factor for an inductor is zero; $\cos 90° = 0$. Thus, an ideal inductor—like an ideal capacitor—consumes zero average power. That is,

$$P_{av} = I_{rms}V_{rms}\cos\phi = I_{rms}V_{rms}\cos 90° = 0$$

The instantaneous power in an inductor alternates in sign, as shown in Figure 24–15 (b). Thus energy enters the inductor at one time, only to be given up at a later time, for a net gain on average of zero energy.

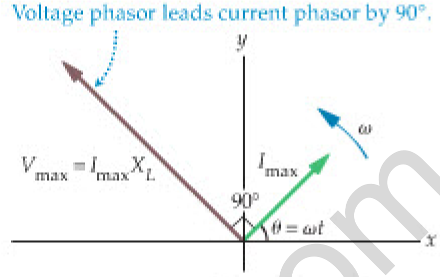


FIGURE 24–16 Phasor diagram for an ac inductor circuit

The maximum value of the inductor's voltage is $I_{max}X_L$, and its angle is 90° ahead (counterclockwise) of the current.

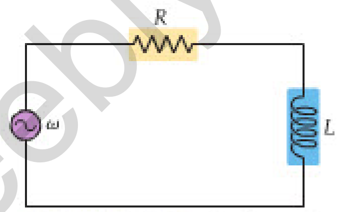


FIGURE 24–17 An alternating-current *RL* circuit

An ac *RL* circuit consists of a generator, a resistor, and an inductor connected in series.

RL Circuits

Next, we consider an ac circuit containing both a resistor and an inductor, as shown in Figure 24–17. The corresponding phasor diagram is drawn in Figure 24–18, where we see the resistor voltage in phase with the current, and the inductor voltage 90° ahead. The total voltage, of course, is given by the vector sum of these two phasors. Therefore, the magnitude of the total voltage is

$$V_{max} = \sqrt{(I_{max}R)^2 + (I_{max}X_L)^2} = I_{max}\sqrt{R^2 + X_L^2} = I_{max}Z$$

This expression defines the impedance, just as with an *RC* circuit. In this case, the impedance is

Impedance in an *RL* Circuit

$$Z = \sqrt{R^2 + X_L^2} = \sqrt{R^2 + (\omega L)^2} \qquad \text{24–15}$$

SI unit: ohm, Ω

Note that the impedance for an *RL* circuit has the same form as for an *RC* circuit, except that X_L replaces X_C. Similarly, the power factor for an *RL* circuit can be written as follows:

$$\cos\phi = \frac{R}{Z} = \frac{R}{\sqrt{R^2 + (\omega L)^2}}$$

We consider an *RL* circuit in the following Example.

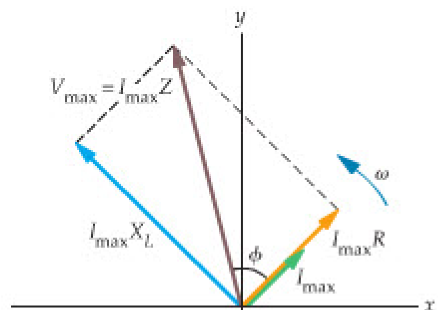


FIGURE 24–18 Phasor diagram for an *RL* circuit

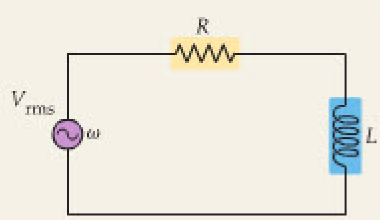
The maximum current in an *RL* circuit is $I_{max} = V_{max}/Z$, where V_{max} is the maximum voltage of the ac generator and $Z = \sqrt{R^2 + X_L^2}$ is the impedance of the circuit. The angle between the maximum voltage phasor and the current phasor is ϕ. Note that the voltage leads the current.

EXAMPLE 24–5 *RL* CIRCUIT

A 0.380-H inductor and a 225-Ω resistor are connected in series to an ac generator with an rms voltage of 30.0 V and a frequency of 60.0 Hz. Find the rms values of **(a)** the current in the circuit, **(b)** the voltage across the resistor, and **(c)** the voltage across the inductor.

PICTURE THE PROBLEM

Our sketch shows a 60.0-Hz generator connected in series with a 225-Ω resistor and a 0.380-H inductor. Because of the series connection, the same current flows through each of the circuit elements.



STRATEGY

a. The rms current in the circuit is $I_{rms} = V_{rms}/Z$, where the impedance is $Z = \sqrt{R^2 + (\omega L)^2}$.

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b. The rms voltage across the resistor is $V_{rms,R} = I_{rms}R$.

c. The rms voltage across the inductor is $V_{rms,L} = I_{rms}X_L = I_{rms}\,\omega L$.

SOLUTION

Part (a)

1. Calculate the impedance of the circuit:

$$Z = \sqrt{R^2 + X_L^2} = \sqrt{R^2 + (\omega L)^2}$$
$$= \sqrt{(225\ \Omega)^2 + [2\pi(60.0\ s^{-1})(0.380\ H)]^2} = 267\ \Omega$$

2. Use Z to find the rms current:

$$I_{rms} = \frac{V_{rms}}{Z} = \frac{30.0\ V}{267\ \Omega} = 0.112\ A$$

Part (b)

3. Multiply I_{rms} by the resistance, R, to find the rms voltage across the resistor:

$$V_{rms,R} = I_{rms}R = (0.112\ A)(225\ \Omega) = 25.2\ V$$

Part (c)

4. Multiply I_{rms} by the inductive reactance, X_L, to find the rms voltage across the inductor:

$$V_{rms,L} = I_{rms}X_L = I_{rms}\omega L$$
$$= (0.112\ A)2\pi(60.0\ s^{-1})(0.380\ H) = 16.0\ V$$

INSIGHT

As with the *RC* circuit, the individual voltages *do not* add up to the generator voltage. However, the generator rms voltage *is* equal to $\sqrt{V_{rms,R}^2 + V_{rms,L}^2}$.

PRACTICE PROBLEM

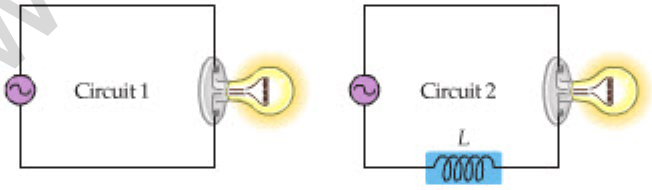
If the frequency in this circuit is increased, do you expect the rms current to increase, decrease, or stay the same? What is the current if the frequency is increased to 125 Hz? [**Answer:** The current will decrease with frequency. At 125 Hz the current is 0.0803 A.]

Some related homework problems: Problem 32, Problem 33

The preceding Practice Problem shows that the current in an *RL* circuit decreases with increasing frequency. Conversely, as we saw in Example 24–3, an *RC* circuit has the opposite behavior. These results are summarized and extended in Figure 24–19, where we show the current in *RL* and *RC* circuits as a function of frequency. It is assumed in this plot that both circuits have the same resistance, *R*. The horizontal line at the top of the plot indicates the current that would flow if the circuits contained *only* the resistor.

CONCEPTUAL CHECKPOINT 24–3 COMPARE BRIGHTNESS: INDUCTOR CIRCUIT

Shown in the sketch below are two circuits with identical ac generators and lightbulbs. Circuit 2 differs from circuit 1 by the addition of an inductor in series with the light. Does the lightbulb in circuit 2 shine **(a)** more brightly, **(b)** less brightly, or **(c)** with the same intensity as that in circuit 1?

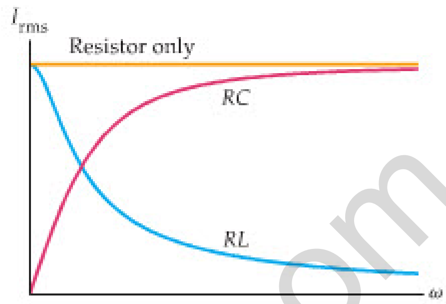


REASONING AND DISCUSSION

Since circuit 2 has both a resistance and an inductive reactance, its impedance, $Z = \sqrt{R^2 + X_L{}^2}$, is greater than that of circuit 1. Therefore, the current in circuit 2, $I_{rms} = V_{rms}/Z$, is less than the current in circuit 1, and hence the average power dissipated in lightbulb 2 is less.

ANSWER

(b) The lightbulb in circuit 2 shines less brightly.



▶ **FIGURE 24–19 RMS currents in *RL* and *RC* circuits as a function of frequency**

A circuit with only a resistor has the same rms current regardless of frequency. In an *RC* circuit the current is low where the capacitive reactance is high (low frequency), and in the *RL* circuit the current is low where the inductive reactance is high (high frequency).

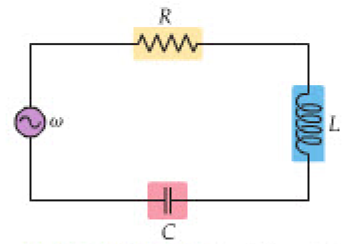
The basic principle illustrated in Conceptual Checkpoint 24–3 is used commercially in light dimmers. When you rotate the knob on a light dimmer in one direction or the other, adjusting the light's intensity to the desired level, what you are actually doing is moving an iron rod into or out of the coils of an inductor. This changes both the inductance of the inductor and the intensity of the light. For example, if the inductance is increased—by moving the iron rod deeper within the inductor's coil—the current in the circuit is decreased and the light dims. The advantage of dimming a light in this way is that no energy is dissipated by the inductor. In contrast, if one were to dim a light by placing a resistor in the circuit, the reduction in light would occur at the expense of wasted energy in the form of heat in the resistor. This needless dissipation of energy is avoided with the inductive light dimmer.

REAL-WORLD PHYSICS

Light dimmers

24–5 *RLC* Circuits

After having considered *RC* and *RL* circuits separately, we now consider circuits with all three elements, *R*, *L*, and *C*. In particular, if *R*, *L*, and *C* are connected in series, as in Figure 24–20, the resulting circuit is referred to as an ***RLC* circuit.** We now consider the behavior of such a circuit, using our previous results as a guide.



▲ **FIGURE 24–20 An alternating-current *RLC* circuit**

An ac *RLC* circuit consists of a generator, a resistor, an inductor, and a capacitor connected in series. The voltage may lead or lag the current, depending on the frequency of the generator and the values of *L* and *C*.

Phasor Diagram

As one might expect, a useful way to analyze the behavior of an *RLC* circuit is with the assistance of a phasor diagram. Thus, the phasor diagram for the circuit in Figure 24–20 is shown in Figure 24–21. Note that in addition to the current phasor, we show three separate voltage phasors corresponding to the resistor, inductor, and capacitor. The phasor diagram also shows that the voltage of the resistor is in phase with the current, the voltage of the inductor is 90° ahead of the current, and the voltage of the capacitor is 90° behind the current.

To find the total voltage in the system we must, as usual, perform a vector sum of the three voltage phasors. This process can be simplified if we first sum the inductor and capacitor phasors, which point in opposite directions along the same line. In the case shown in Figure 24–21 we assume for specificity that X_L is greater than X_C, so that the sum of these two voltage phasors is $I_{max}X_L - I_{max}X_C$. Combining this result with the phasor for the resistor voltage, and applying the Pythagorean theorem, we obtain the total maximum voltage:

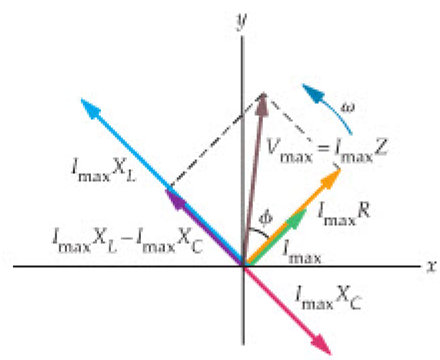
$$V_{max} = \sqrt{(I_{max}R)^2 + (I_{max}X_L - I_{max}X_C)^2} = I_{max}\sqrt{R^2 + (X_L - X_C)^2} = I_{max}Z$$

The impedance of the circuit is thus defined as follows:

Impedance of an *RLC* Circuit

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} \qquad 24\text{–}16$$

SI unit: ohm, Ω



▲ **FIGURE 24–21 Phasor diagram for a typical *RLC* circuit**

In the case shown here, we assume that X_L is greater than X_C. All the results are the same for the opposite case, X_C greater than X_L, except that the phase angle ϕ changes sign, as can be seen in Equation 24–17.

Note that this result for the impedance contains the expressions given for the *RC* and *RL* circuits as special cases. For example, in the *RC* circuit the inductance is zero; hence, $X_L = 0$, and the impedance becomes

$$Z = \sqrt{R^2 + X_C{}^2}$$

This relation is identical with the result given for the *RC* circuit in Equation 24–11.

In addition to the magnitude of the total voltage, we are also interested in the phase angle ϕ between it and the current. From the phasor diagram it is clear that this angle is given by

$$\tan\phi = \frac{I_{max}(X_L - X_C)}{I_{max}R} = \frac{X_L - X_C}{R} \qquad 24\text{–}17$$

In particular, if X_L is greater than X_C, as in Figure 24–21, then ϕ is positive and the voltage leads the current. On the other hand, if X_C is greater than X_L it follows that ϕ is negative, so the voltage lags the current. In the special case $X_C = X_L$ the phase angle is zero, and the current and voltage are in phase. There is special significance to this last case, as we shall see in greater detail in the next section.

Finally, the power factor, $\cos\phi$, can also be obtained from the phasor diagram. Referring again to Figure 24–21, we see that

$$\cos\phi = \frac{R}{Z}$$

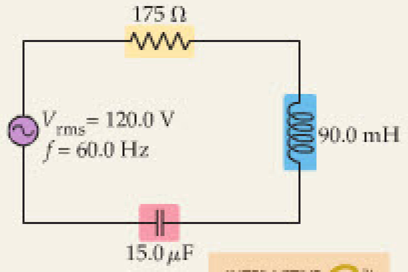
Note that this is precisely the result given earlier for RC and RL circuits.

EXAMPLE 24–6 DRAW YOUR PHASORS!

An ac generator with a frequency of 60.0 Hz and an rms voltage of 120.0 V is connected in series with a 175-Ω resistor, a 90.0-mH inductor, and a 15.0-μF capacitor. Draw the appropriate phasor diagram for this system and calculate the phase angle, ϕ.

PICTURE THE PROBLEM

Our sketch shows the RLC circuit described in the problem statement. In particular, the 60.0-Hz generator is connected in series with a 175-Ω resistor, a 90.0-mH inductor, and a 15.0-μF capacitor.



STRATEGY

To draw an appropriate phasor diagram we must first determine the values of X_C and X_L. These values determine immediately whether the voltage leads the current or lags it.

The precise value of the phase angle between the current and voltage is given by $\tan\phi = (X_L - X_C)/R$.

SOLUTION

1. Calculate the capacitive and inductive reactances:

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi(60.0\ \text{s}^{-1})(15.0 \times 10^{-6}\ \text{F})} = 177\ \Omega$$

$$X_L = \omega L = 2\pi(60.0\ \text{s}^{-1})(90.0 \times 10^{-3}\ \text{H}) = 33.9\ \Omega$$

2. Use $\tan\phi = (X_L - X_C)/R$ to find the phase angle, ϕ:

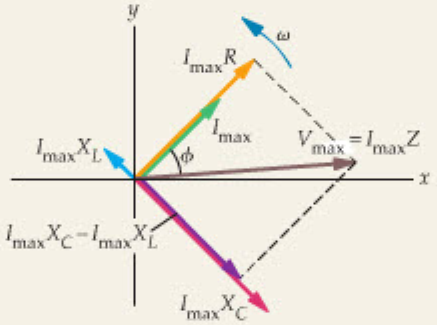
$$\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right)$$

$$= \tan^{-1}\left(\frac{33.9\ \Omega - 177\ \Omega}{175\ \Omega}\right) = -39.3°$$

INSIGHT

We can now draw the phasor diagram for this circuit. First, the fact that X_C is greater than X_L means that the voltage of the circuit lags the current and that the phase angle is negative. In fact, we know that the phase angle has a magnitude of 39.3°. It follows that the phasor diagram for this circuit is as shown in the diagram.

Note that the lengths of the phasors $I_{max}R$, $I_{max}X_C$, and $I_{max}X_L$ in this diagram are drawn in proportion to the values of R, X_C, and X_L, respectively.



PRACTICE PROBLEM

Find the impedance and rms current for this circuit. [**Answer:** Z = 226 Ω, $I_{rms} = V_{rms}/Z$ = 0.531 A]

INTERACTIVE FIGURE

Some related homework problems: Problem 52, Problem 53

The phase angle can also be obtained from the power factor: $\cos\phi = R/Z = (175\ \Omega)/(226\ \Omega) = 0.774$. Notice, however, that $\cos^{-1}(0.774) = \pm 39.3°$. Thus, because of the symmetry of cosine, the power factor determines only the magnitude of ϕ, not its sign. The sign can be obtained from the phasor diagram, of course, but using $\phi = \tan^{-1}(X_L - X_C)/R$ yields both the magnitude and sign of ϕ.

ACTIVE EXAMPLE 24–2 FIND THE INDUCTOR VOLTAGE

The circuit elements shown in the sketch are connected to an ac generator at points A and B. If the rms voltage of the generator is 41 V and its frequency is 75 Hz, what is the rms voltage across the inductor?

SOLUTION *(Test your understanding by performing the calculations indicated in each step.)*

1. Calculate the capacitive reactance: $X_C = 61\ \Omega$
2. Calculate the inductive reactance: $X_L = 71\ \Omega$
3. Use Equation 24–16 to determine the impedance of the circuit: $Z = 43\ \Omega$
4. Divide V_{rms} by Z to find the rms current: $I_{rms} = 0.95$ A
5. Multiply I_{rms} by X_L to find the rms voltage of the inductor: $V_{rms,L} = 67$ V

INSIGHT

Note that the voltage across individual components in a circuit can be larger than the applied voltage. In this case the applied voltage is 41 V, whereas the voltage across the inductor is 67 V.

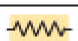
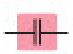
YOUR TURN

Find the rms voltage across **(a)** the resistor and **(b)** the capacitor.

(Answers to **Your Turn** *problems are given in the back of the book.)*

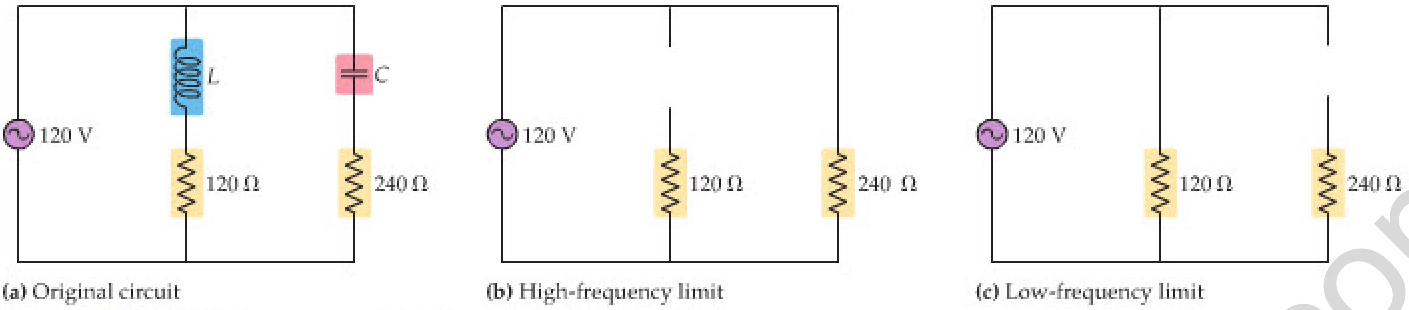
Table 24–1 summarizes the various results for ac circuit elements (R, L, and C) and their combinations (RC, RL, RLC).

TABLE 24–1 Properties of AC Circuit Elements and Their Combinations

Circuit element	Impedance, Z	Average power, P_{av}	Phase angle, ϕ	Phasor diagram
	$Z = R$	$P_{av} = I_{rms}^2 R = V_{rms}^2/R$	$\phi = 0°$	Figure 24–3
	$Z = X_C = \dfrac{1}{\omega C}$	$P_{av} = 0$	$\phi = -90°$	Figure 24–8
	$Z = X_L = \omega L$	$P_{av} = 0$	$\phi = +90°$	Figure 24–16
	$Z = \sqrt{R^2 + X_C^2} = \sqrt{R^2 + \left(\dfrac{1}{\omega C}\right)^2}$	$P_{av} = I_{rms}V_{rms}\cos\phi$	$-90° < \phi < 0°$	Figure 24–10
	$Z = \sqrt{R^2 + X_L^2} = \sqrt{R^2 + (\omega L)^2}$	$P_{av} = I_{rms}V_{rms}\cos\phi$	$0° < \phi < 90°$	Figure 24–18
	$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{R^2 + \left(\omega L - \dfrac{1}{\omega C}\right)^2}$	$P_{av} = I_{rms}V_{rms}\cos\phi$	$-90° < \phi < 0°$ $(X_C > X_L)$ $0° < \phi < 90°$ $(X_C < X_L)$	Figure 24–21

Large and Small Frequencies

In the limit of very large or small frequencies, the behavior of capacitors and inductors is quite simple, allowing us to investigate more complex circuits



▲ **FIGURE 24–22 High-frequency and low-frequency limits of an ac circuit**

(a) A complex circuit containing resistance, inductance, and capacitance. Although this is not a simple *RLC* circuit, we can still obtain useful results about the circuit in the limits of high and low frequencies. **(b)** The high-frequency limit, in which the inductor is essentially an open circuit, and the capacitor behaves like an ideal wire. **(c)** The low-frequency limit. In this case, the inductor is like an ideal wire, and the capacitor acts like an open circuit.

containing *R*, *L*, and *C*. For example, in the limit of large frequency the reactance of an inductor becomes very large, whereas that of a capacitor becomes vanishingly small. This means that a capacitor acts like a segment of ideal wire, with no resistance, and an inductor behaves like a very large resistor with practically no current—essentially, an open circuit.

By applying these observations to the circuit in **Figure 24–22 (a)**, for example, we can predict the current supplied by the generator at high frequencies. First, we replace the capacitor with an ideal wire and the inductor with an open circuit. This results in the simplified circuit shown in **Figure 24–22 (b)**. Clearly, the current in this circuit is 0.50 A; hence, we expect the current in the original circuit to approach this value as the frequency is increased.

In the opposite extreme of very small frequency we obtain the behavior that would be expected if the ac generator were replaced with a battery. Specifically, the reactance of an inductor vanishes as the frequency goes to zero, whereas that of a capacitor becomes extremely large. Thus, the roles of the inductor and capacitor are now reversed; it is the inductor that acts like an ideal wire, and the capacitor that behaves like an open circuit. For small frequencies, then, the circuit in Figure 24–22 (a) will behave the same as the circuit shown in **Figure 24–22 (c)**. The current in this circuit, and in the original circuit at low frequency, is 1.0 A.

EXAMPLE 24–7 FIND *R*

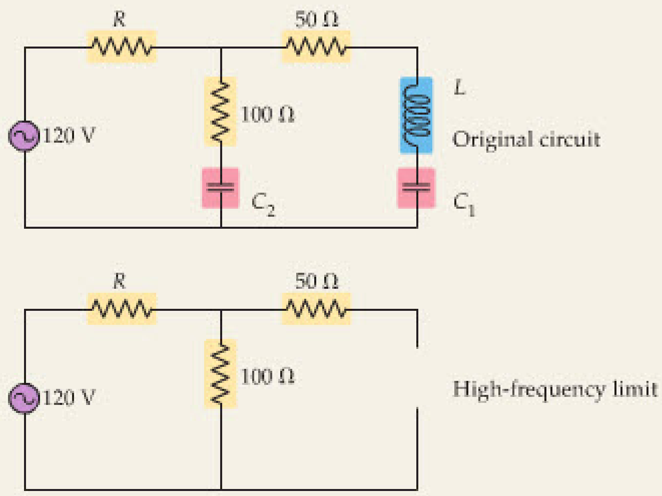
The circuit shown in the sketch is connected to an ac generator with an rms voltage of 120 V. What value must *R* have if the rms current in this circuit is to approach 1.0 A at high frequency?

PICTURE THE PROBLEM

The top diagram shows the original circuit with its various elements. The high-frequency behavior of this circuit is indicated in the bottom diagram, where the inductor has been replaced with an open circuit, and the capacitors have been replaced with ideal wires.

STRATEGY

The high-frequency circuit has only one remaining path through which current can flow. On this path the resistors with resistance *R* and 100 Ω are in series. Therefore, the total resistance of the circuit is $R_{total} = R + 100\ \Omega$. Finally, the rms current is $I_{rms} = V_{rms}/R_{total}$. Setting I_{rms} equal to 1.0 A gives us the value of *R*.



SOLUTION

1. Calculate the total resistance of the high-frequency circuit: $R_{\text{total}} = R + 100\ \Omega$

2. Write an expression for the rms current in the circuit: $I_{\text{rms}} = \dfrac{V_{\text{rms}}}{R_{\text{total}}} = \dfrac{V_{\text{rms}}}{R + 100\ \Omega}$

3. Solve for the resistance R: $R = \dfrac{V_{\text{rms}}}{I_{\text{rms}}} - 100\ \Omega = \dfrac{120\ \text{V}}{1.0\ \text{A}} - 100\ \Omega = 20\ \Omega$

INSIGHT

Note that no values are given for the capacitances and the inductance. At high enough frequencies the precise values of these quantities are unimportant.

PRACTICE PROBLEM

What is the rms current in this circuit in the limit of small frequency? [**Answer:** The current approaches zero.]

Some related homework problems: Problem 50, Problem 51

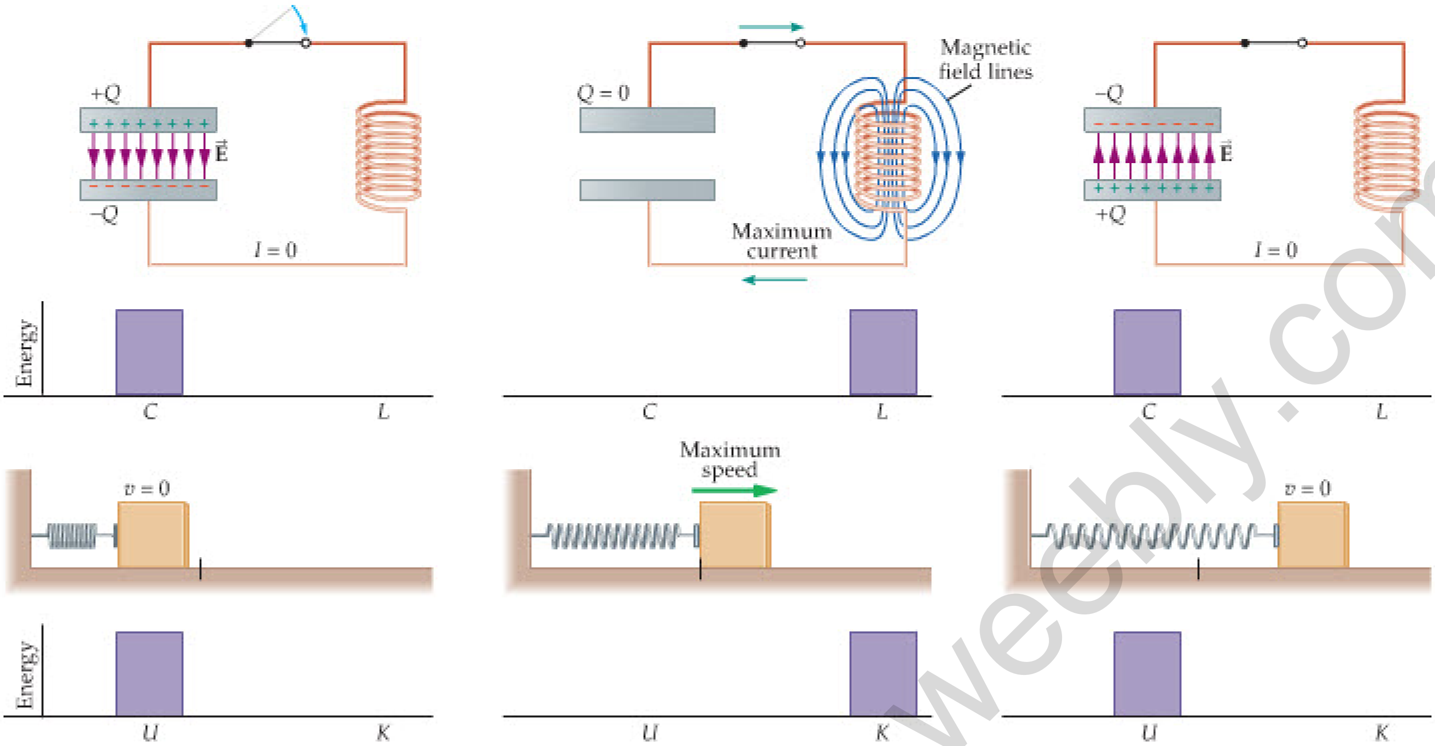
24–6 Resonance in Electric Circuits

Many physical systems have natural frequencies of oscillation. For example, we saw in Chapter 13 that a child on a swing oscillates about the vertical with a definite, characteristic frequency determined by the length of the swing and the acceleration of gravity. Similarly, an object attached to a spring oscillates about its equilibrium position with a frequency determined by the "stiffness" of the spring and the mass of the object. Certain electric circuits have analogous behavior—their electric currents oscillate with certain characteristic frequencies. In this section we consider some examples of "oscillating" electric circuits.

LC Circuits

Perhaps the simplest circuit that displays an oscillating electric current is an ***LC* circuit** with no generator; that is, one that consists of nothing more than an inductor and a capacitor. Suppose, for example, that at $t = 0$ a charged capacitor has just been connected to an inductor and that there is no current in the circuit, as shown in **Figure 24–23 (a)**. Since the capacitor is charged, it has a voltage, $V = Q/C$, which causes a current to begin flowing through the inductor, as in **Figure 24–23 (b)**. Soon all the charge drains from the capacitor and its voltage drops to zero, but the current continues to flow because an inductor resists changes in its current. In fact, the current continues flowing until the capacitor becomes charged enough in the *opposite* direction to stop the current, as in **Figure 24–23 (c)**. At this point, the current begins to flow back the way it came, and the same sequence of events occurs again, leading to a *series of oscillations* in the current.

In the ideal case, the oscillations can continue forever, since neither an inductor nor a capacitor dissipates energy. The situation is completely analogous to a mass oscillating on a spring with no friction, as we indicate in Figure 24–23. At $t = 0$ the capacitor has a charge of magnitude Q on its plates, which means it stores the energy $U_C = Q^2/2C$. This situation is analogous to a spring being compressed a distance x and storing the potential energy $U = \frac{1}{2}kx^2$. At a later time the charge on the capacitor is zero, so it no longer stores any energy. The energy is not lost, however. Instead, it is now in the inductor, which carries a current I and stores the energy $U_L = \frac{1}{2}LI^2 = U_C$. In the mass–spring system, this corresponds to the mass being at the equilibrium position of the spring. At this time all the system's energy is the kinetic energy of the mass, $K = \frac{1}{2}mv^2 = U$, and none is stored in the spring. As the current continues to flow, it charges the capacitor with the opposite polarity until it reaches the magnitude Q and stores the same energy, U_C, as at $t = 0$. In the mass–spring system, this corresponds to the spring being stretched by the same distance x that it was originally compressed, which again stores all the initial energy as potential energy.



(a) Maximum charge **(b)** Maximum current **(c)** Maximum (opposite) charge

▲ **FIGURE 24–23 Oscillations in an *LC* circuit with no generator**

The current oscillations in an LC circuit are analogous to the oscillations of a mass on a spring. **(a)** At the moment the switch is closed, all the energy in the circuit is stored in the electric field of the charged capacitor. This is analogous to a mass at rest against a compressed spring, where all the energy of the system is stored in the spring. **(b)** A quarter of the way through a cycle, the capacitor is uncharged and the current in the inductor is a maximum. At this time, all the energy in the circuit is stored in the magnetic field of the inductor. In the mass–spring analog, the spring is at equilibrium and the mass has its maximum speed. All the system's energy is now in the form of kinetic energy. **(c)** After half a cycle, the capacitor is fully charged in the opposite direction and holds all the system's energy. This corresponds to a fully extended spring (with all the system's energy) and the mass at rest.

Thus, we see that a close analogy exists between a capacitor and a spring, and between an inductor and a mass. In addition, the charge on the capacitor is analogous to the displacement of the spring, and the current in the inductor is analogous to the speed of the mass. Thus, for example, the energy stored in the inductor, $\frac{1}{2}LI^2$, corresponds precisely to the kinetic energy of the mass, $\frac{1}{2}mv^2$. Comparing the potential energy of a spring, $\frac{1}{2}kx^2$, and the energy stored in a capacitor, $Q^2/2C$, we see that the stiffness of a spring is analogous to $1/C$. This makes sense, because a capacitor with a large capacitance, C, can store large quantities of charge with ease, just as a spring with a small force constant (if C is large, then $k = 1/C$ is small) can be stretched quite easily.

In the mass–spring system the natural angular frequency of oscillation is determined by the characteristics of the system. In particular, recall from Section 13–8 that

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \quad \text{or} \quad \omega_0 = 2\pi f_0 = \sqrt{\frac{k}{m}}$$

The natural frequency of the LC circuit can be determined by noting that the rms voltage across the capacitor C in Figure 24–23 must be the same as the rms voltage across the inductor L. This condition can be written as follows:

$$V_{\text{rms},C} = V_{\text{rms},L}$$

$$I_{\text{rms}}X_C = I_{\text{rms}}X_L$$

$$I_{\text{rms}}\left(\frac{1}{\omega_0 C}\right) = I_{\text{rms}}(\omega_0 L)$$

Solving for ω_0, we find

Natural Frequency of an *LC* Circuit

$$\omega_0 = \frac{1}{\sqrt{LC}} = 2\pi f \qquad \text{24–18}$$

SI unit: s^{-1}

Note again the analogy between this result and that for a mass on a spring: if we make the following replacements, $m \rightarrow L$ and $k \rightarrow 1/C$, in the mass–spring result we find the expected LC result:

$$\omega_0 = \sqrt{\frac{k}{m}} = \sqrt{\frac{1}{LC}}$$

We summarize the mass–spring/LC circuit analogies in Table 24–2.

TABLE 24–2 Analogies Between a Mass on a Spring and an *LC* Circuit

Mass–spring system		*LC* circuit	
Position	x	Charge	q
Velocity	$v = \Delta x/\Delta t$	Current	$I = \Delta q/\Delta t$
Mass	m	Inductance	L
Force constant	k	Inverse capacitance	$1/C$
Natural Frequency	$\omega_0 = \sqrt{k/m}$	Natural frequency	$\omega_0 = \sqrt{1/LC}$

EXERCISE 24–6

It is desired to tune the natural frequency of an LC circuit to match the 88.5-MHz broadcast signal of an FM radio station. If a 1.50-μH inductor is to be used in the circuit, what capacitance is required?

SOLUTION

Solving $\omega_0 = 1/\sqrt{LC}$ for the capacitance, we find

$$C = \frac{1}{\omega_0^2 L} = \frac{1}{[2\pi(88.5 \times 10^6 \text{ s}^{-1})]^2(1.50 \times 10^{-6} \text{ H})} = 2.16 \times 10^{-12} \text{ F}$$

Resonance

Whenever a physical system has a natural frequency, we can expect to find resonance when it is driven near that frequency. In a mass–spring system, for example, if we move the top end of the spring up and down with a frequency near the natural frequency, the displacement of the mass can become quite large. Similarly, if we push a person on a swing at the right frequency, the amplitude of motion will increase. These are examples of resonance in mechanical systems.

To drive an electric circuit, we can connect it to an ac generator. As we adjust the frequency of the generator, the current in the circuit will be a maximum at the natural frequency of the circuit. For example, consider the circuit shown in Figure 24–20. Here an ac generator drives a circuit containing an inductor, a capacitor, and a resistor. As we have already seen, the inductor and capacitor together establish a natural frequency $\omega_0 = 1/\sqrt{LC}$, and the resistor provides for energy dissipation.

The phasor diagram for this circuit is like the one shown in Figure 24–21. Recall that the maximum current in this circuit is

$$I_{max} = \frac{V_{max}}{Z}$$

Thus, the smaller the impedance, the larger the current. Hence, to obtain the largest possible current, we must have the smallest Z. Recall that the impedance is given by

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

Writing Z in terms of frequency, we have

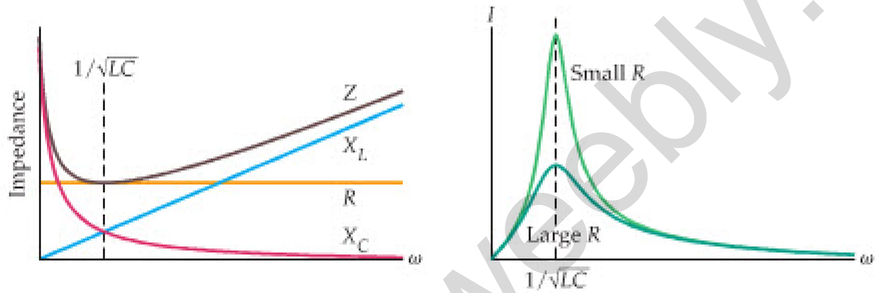
$$Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} \qquad 24\text{–}19$$

PROBLEM-SOLVING NOTE

A Circuit at Resonance

The resonance frequency of an *RLC* circuit depends only on the inductance, L, and the capacitance, C; that is, $\omega_0 = 1/\sqrt{LC}$. On the other hand, the impedance at resonance depends only on the resistance, R; that is, $Z = R$.

This expression for Z is plotted in Figure 24–24 (a). Notice that the smallest value of the impedance is $Z = R$, and that this value is attained precisely at the frequency where $X_L = X_C$. We can see this mathematically by setting $X_L = X_C$ in the expression $Z = \sqrt{R^2 + (X_L - X_C)^2}$, which yields $Z = \sqrt{R^2 + 0} = R$, as expected. The frequency at which $X_L = X_C$ is the frequency for which $\omega L = 1/\omega C$. This frequency is $\omega = 1/\sqrt{LC} = \omega_0$, the *natural frequency* found in Equation 24–18 for *LC* circuits.



(a) Impedance is a minimum ($Z = R$) at resonance

(b) Resonance curves for the current

FIGURE 24–24 Resonance in an *RLC* circuit

(a) The impedance, Z, of an *RLC* circuit varies as a function of frequency. The minimum value of Z—which corresponds to the largest current—occurs at the resonance frequency $\omega_0 = 1/\sqrt{LC}$, where $X_C = X_L$. At this frequency $Z = R$. **(b)** Typical resonance peaks for an *RLC* circuit. The location of the peak is independent of resistance, but the resonance effect becomes smaller and more spread out with increasing R.

Figure 24–24 (b) shows typical plots of the current in an *RLC* circuit. Note that the current peaks at the resonance frequency. Note also that increasing the resistance, although it reduces the maximum current in the circuit, does not change this frequency. It does, however, make the resonance peak flatter and broader. As a result, the resonance effect occurs over a wide range of frequencies and gives only a small increase in current. If the resistance is small, however, the peak is high and sharp. In this case, the resonance effect yields a large current that is restricted to a very small range of frequencies.

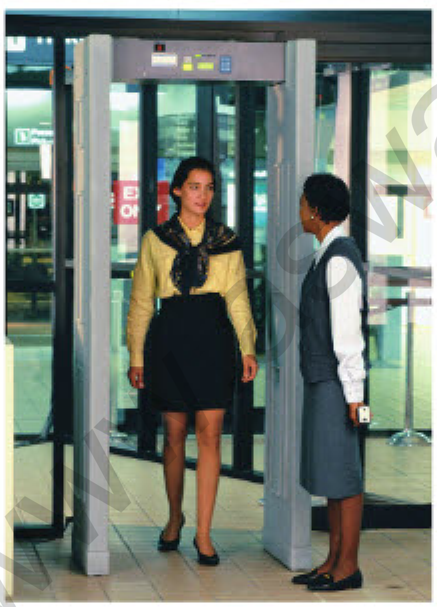
REAL-WORLD PHYSICS

Tuning a radio or television

Radio and television tuners use low-resistance *RLC* circuits so they can pick up one station at a frequency f_1 without also picking up a second station at a nearby frequency f_2. In a typical radio tuner, for example, turning the knob of the tuner rotates one set of capacitor plates between a second set of plates, effectively changing both the plate separation and the plate area. This changes the capacitance in the circuit and the frequency at which it resonates. If the resonance peak is high and sharp—as occurs with low resistance—only the station broadcasting at the resonance frequency will be picked up and amplified. Other stations at nearby frequencies will produce such small currents in the tuning circuit that they will be undetectable.

REAL-WORLD PHYSICS

Metal detectors



Metal detectors also use resonance in *RLC* circuits, although in this case it is the inductance that changes rather than the capacitance. For example, when you walk through a metal detector at an airport, you are actually walking through a large coil of wire—an inductor. Metal objects on your person cause the inductance of the coil to increase slightly. This increase in inductance results in a small decrease in the resonance frequency of the *RLC* circuit to which the coil is connected. If the resonance peak is sharp and high, even a slight change in frequency results in a large change in current. It is this change in current that sets off the detector, indicating the presence of metal.

CONCEPTUAL CHECKPOINT 24–4 PHASE OF THE VOLTAGE

An *RLC* circuit is driven at its resonance frequency. Is its voltage **(a)** ahead of, **(b)** behind, or **(c)** in phase with the current?

REASONING AND DISCUSSION

At resonance the capacitive and inductive reactances are equal, which means that the voltage across the capacitor is equal in magnitude and opposite in direction to the voltage across the inductor. As a result, the net voltage in the system is simply the voltage across the resistor, which is in phase with the current.

ANSWER

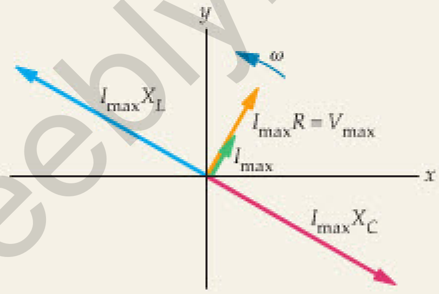
(c) The voltage and current are in phase.

EXAMPLE 24–8 A CIRCUIT IN RESONANCE

An ac generator with an rms voltage of 25 V is connected in series to a 10.0-Ω resistor, a 53-mH inductor, and a 65-μF capacitor. Find **(a)** the resonance frequency of the circuit and **(b)** the rms current at resonance. In addition, sketch the phasor diagram at resonance.

PICTURE THE PROBLEM

As mentioned in the previous Conceptual Checkpoint, at resonance the magnitude of the voltage across the inductor is equal to the magnitude of the voltage across the capacitor. Since the phasors corresponding to these voltages point in opposite directions, however, they cancel. This leads to a net voltage phasor that is simply $I_{max}R$ in phase with the current. Finally, note that the lengths of the phasors $I_{max}R$, $I_{max}X_C$, and $I_{max}X_L$ are drawn in proportion to the values of R, X_C, and X_L, respectively.



STRATEGY

a. We find the resonance frequency by substituting numerical values into $\omega_0 = 1/\sqrt{LC} = 2\pi f_0$.

b. At resonance the impedance of the circuit is simply the resistance; that is, $Z = R$. Thus, the rms current in the circuit is $I_{rms} = V_{rms}/Z = V_{rms}/R$.

SOLUTION

Part (a)

1. Calculate the resonance frequency:

$$\omega_0 = \frac{1}{\sqrt{LC}} = 2\pi f_0$$

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

$$= \frac{1}{2\pi\sqrt{(53 \times 10^{-3}\,\text{H})(65 \times 10^{-6}\,\text{F})}} = 86\ \text{Hz}$$

Part (b)

2. Determine the impedance at resonance:

$$Z = R = 10.0\ \Omega$$

3. Divide the rms voltage by the impedance to find the rms current:

$$I_{rms} = \frac{V_{rms}}{Z} = \frac{25\ \text{V}}{10.0\ \Omega} = 2.5\ \text{A}$$

INSIGHT

If the frequency of this generator is increased above resonance, the inductive reactance, X_L, will be larger than the capacitive reactance, X_C, and hence the voltage will lead the current. If the frequency is lowered below resonance the voltage will lag the current.

PRACTICE PROBLEM

What is the magnitude of the rms voltage across the capacitor? [**Answer:** 71 V. Note again that the voltage across a given circuit element can be much larger than the applied voltage.]

Some related homework problems: Problem 67, Problem 69

ACTIVE EXAMPLE 24–3 FIND THE OFF-RESONANCE CURRENT

Referring to the system in the previous Example, find the rms current when the generator operates at a frequency that is twice the resonance frequency.

SOLUTION *(Test your understanding by performing the calculations indicated in each step.)*

1. Calculate the inductive reactance at the frequency $f = 2(86\ \text{Hz})$: $X_L = 57\ \Omega$

2. Calculate the capacitive reactance at the frequency $f = 2(86\ \text{Hz})$: $X_C = 14\ \Omega$

CONTINUED ON NEXT PAGE

CONTINUED FROM PREVIOUS PAGE

3. Use these results plus the resistance R to find the impedance, Z: $Z = 44\ \Omega$

4. Divide the rms voltage by the impedance to find the rms current: $I_{rms} = 0.57\ A$

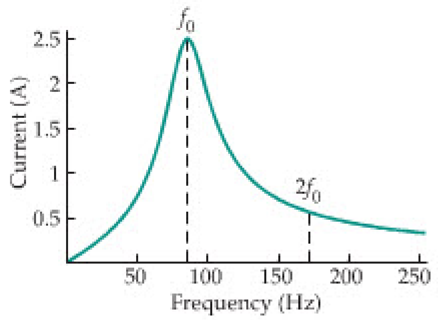
INSIGHT

As expected, the inductive reactance is greater than the capacitive reactance at this frequency. In addition, note that the rms current has been reduced by roughly a factor of 5 compared with its value in Example 24–8.

YOUR TURN

At what two frequencies is the current in this circuit equal to 1.5 A?

(Answers to **Your Turn** *problems are given in the back of the book.)*



▲ **FIGURE 24–25 RMS current in an *RLC* circuit**

This plot shows the rms current versus frequency for the circuit considered in Example 24–8 and Active Example 24–3. The currents determined in these Examples occur at the frequencies indicated by the vertical dashed lines.

The rms current as a function of frequency for the system considered in the previous Example and Active Example is shown in **Figure 24–25**. Notice that the two currents just calculated, at $f_0 = 86$ Hz and $f = 2f_0 = 2(86\ \text{Hz}) = 172$ Hz, are indicated on the graph.

THE BIG PICTURE PUTTING PHYSICS IN CONTEXT

LOOKING BACK

The concepts of current, resistance, capacitance, and inductance (Chapters 21 and 23) are used throughout this chapter. We also make extensive use of electrical power (Chapter 21).

Alternating-current circuits have many similarities with oscillating mechanical systems. In particular, this chapter presents detailed connections between the behavior of a mass on a spring (Chapter 13) and the behavior of an *RLC* circuit.

LOOKING AHEAD

An ac generator appears again in Chapter 25, where we show how the alternating current in an electric circuit can produce electromagnetic waves.

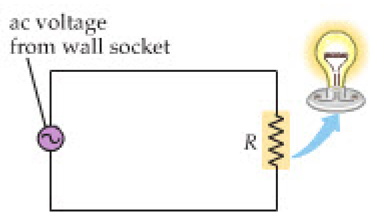
Electromagnetic waves can be detected with an LC circuit whose frequency is matched to the frequency of the electromagnetic wave. This is discussed in Section 25–1.

CHAPTER SUMMARY

24–1 ALTERNATING VOLTAGES AND CURRENTS

An ac generator produces a voltage that varies with time as

$$V = V_{max} \sin \omega t \qquad 24\text{–}1$$



Phasors

A phasor is a rotating vector representing a voltage or a current in an ac circuit. Phasors rotate counterclockwise about the origin in the x-y plane with an angular speed ω. The y component of a phasor gives the instantaneous value of that quantity.

rms Values

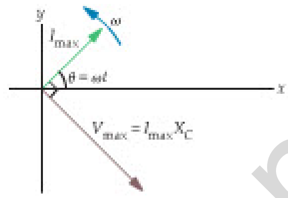
The rms, or root mean square, of a quantity x is the square root of the average value of x^2. For any quantity x that varies with time as a sine or a cosine, the rms value is

$$x_{rms} = \frac{1}{\sqrt{2}} x_{max} \qquad 24\text{–}4$$

Standard dc formulas like $P = I^2R$ can be converted to ac average formulas by using rms values. For example, $P_{av} = I^2_{rms}R$.

24–2 CAPACITORS IN AC CIRCUITS

A capacitor in an ac circuit has a current that depends on the frequency and is out of phase with the voltage.



Capacitive Reactance
The rms current in a capacitor is

$$I_{rms} = \frac{V_{rms}}{X_C} \qquad 24\text{–}8$$

where X_C is the capacitive reactance. It is defined as follows:

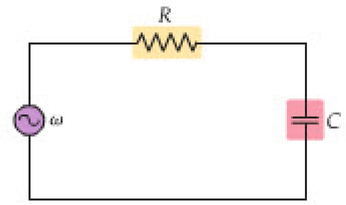
$$X_C = \frac{1}{\omega C} \qquad 24\text{–}9$$

Phase Relation Between Voltage and Current in a Capacitor
The voltage across a capacitor lags the current by 90°.

Phasor Diagram for Capacitors
In a phasor diagram, the voltage of a capacitor is drawn at an angle that is 90° clockwise from the direction of the current phasor.

24–3 *RC* CIRCUITS

To analyze an *RC* circuit, the 90° phase difference between the resistor and capacitor voltages must be taken into account.



Impedance
The impedance, *Z*, of an *RC* circuit is analogous to the resistance in a simple resistor circuit and takes into account both the resistance, *R*, and the capacitive reactance, X_C:

$$Z = \sqrt{R^2 + X_C^2} = \sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2} \qquad 24\text{–}11$$

Voltage and Current
The rms voltage and current in an *RC* circuit are related by

$$V_{rms} = I_{rms}\sqrt{R^2 + X_C^2} = I_{rms}Z$$

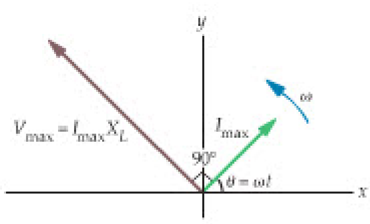
Power Factor
If the phase angle between the current and voltage in an *RC* circuit is ϕ, the average power consumed by the circuit is

$$P_{av} = I_{rms}V_{rms}\cos\phi \qquad 24\text{–}13$$

where $\cos\phi$ is referred to as the power factor.

24–4 INDUCTORS IN AC CIRCUITS

Inductors in ac circuits have frequency-dependent currents, and voltages that are out of phase with the current.



Inductive Reactance
The inductive reactance is

$$X_L = \omega L \qquad 24\text{–}14$$

The rms current in an inductor is

$$I_{rms} = \frac{V_{rms}}{X_L}$$

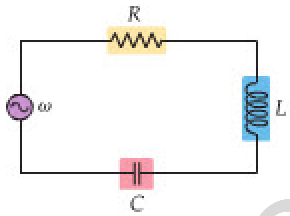
Phasor Diagram for Inductors
The voltage across an inductor leads the current by 90°. Thus, in a phasor diagram, the voltage phasor for an inductor is rotated 90° counterclockwise from the current phasor.

***RL* Circuits**
The impedance of an *RL* circuit is

$$Z = \sqrt{R^2 + X_L^2} = \sqrt{R^2 + (\omega L)^2} \qquad 24\text{–}15$$

24–5 *RLC* CIRCUITS

An *RLC* circuit consists of a resistor, *R*, an inductor, *L*, and a capacitor, *C*, connected in series to an ac generator.



Impedance

The impedance of an *RLC* circuit is

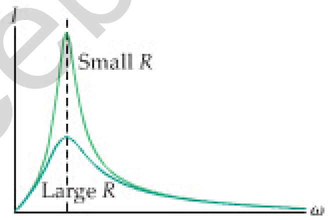
$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} \qquad 24\text{–}16$$

Large and Small Frequencies

In the limit of large frequencies, inductors behave like open circuits and capacitors are like ideal wires. For very small frequencies, the behaviors are reversed; inductors are like ideal wires, and capacitors act like open circuits.

24–6 RESONANCE IN ELECTRIC CIRCUITS

Electric circuits can have natural frequencies of oscillation, just like a pendulum or a mass on a spring.



***LC* Circuits**

Circuits containing only an inductor and a capacitor have a natural frequency given by

$$\omega_0 = \frac{1}{\sqrt{LC}} = 2\pi f_0 \qquad 24\text{–}18$$

Resonance

An *RLC* circuit connected to an ac generator has maximum current at the frequency $\omega = 1/\sqrt{LC}$. This effect is referred to as resonance.

PROBLEM-SOLVING SUMMARY

Type of Problem	Relevant Physical Concepts	Related Examples
Find the average power dissipated in a resistor.	To find the average power dissipated in a resistor, simply replace I with I_{rms} in $P = I^2R$, or replace V with V_{rms} in $P = V^2/R$.	Example 24–1
Calculate the rms voltage across a capacitor in an ac circuit.	In analogy with $V = IR$ for a resistor, the rms voltage across a capacitor is $V_{rms} = I_{rms}X_C$, where $X_C = 1/\omega C$.	Example 24–2
Find the impedance and rms voltage or rms current in an *RC* circuit.	The impedance in an *RC* circuit is $Z = \sqrt{R^2 + X_C^2} = \sqrt{R^2 + (1/\omega C)^2}$. The voltage and current in an *RC* circuit are related by the relation $V_{rms} = I_{rms}Z$.	Example 24–3 Active Example 24–1
Find the average power consumed by an ac circuit.	In an ac circuit, the average power consumed is $P_{av} = I_{rms}V_{rms}\cos\phi$, where the power factor ($\cos\phi$) is defined as follows: $\cos\phi = R/Z$.	Example 24–4
Find the impedance and rms voltage or rms current in an *RL* circuit.	The impedance in an *RL* circuit is $Z = \sqrt{R^2 + X_L^2} = \sqrt{R^2 + (\omega L)^2}$. The voltage and current in an *RL* circuit are related by $V_{rms} = I_{rms}Z$.	Example 24–5
Find the impedance and rms voltage or rms current in an *RLC* circuit.	The impedance in an *RLC* circuit is $Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$. The voltage and current in an *RLC* circuit are related by the relation $V_{rms} = I_{rms}Z$.	Example 24–6 Active Example 24–2
Determine the resonance frequency and impedance of an *RLC* circuit.	In a circuit with a capacitance C and an inductance L, the resonance frequency is $\omega_0 = 1/\sqrt{LC}$. At resonance, the impedance of an *RLC* circuit is simply equal to its resistance; that is, $Z = R$.	Example 24–8 Active Example 24–3

CONCEPTUAL QUESTIONS

For instructor-assigned homework, go to www.masteringphysics.com MP

(Answers to odd-numbered Conceptual Questions can be found in the back of the book.)

1. How can the rms voltage of an ac circuit be nonzero when its average value is zero? Explain.
2. Why is the current in an ac circuit not always in phase with its voltage?
3. Does an *LC* circuit consume any power? Explain.
4. An *LC* circuit is driven at a frequency higher than its resonance frequency. What can be said about the phase angle, ϕ, for this circuit?
5. An *LC* circuit is driven at a frequency lower than its resonance frequency. What can be said about the phase angle, ϕ, for this circuit?
6. In Conceptual Checkpoint 24–3 we considered an ac circuit consisting of a lightbulb in series with an inductor. The effect of the inductor was to cause the bulb to shine less brightly. Would the same be true in a direct-current (dc) circuit? Explain.
7. How do the resistance, capacitive reactance, and inductive reactance change when the frequency in a circuit is increased?
8. In the analogy between an *RLC* circuit and a mass on a spring, what is the analog of the current in the circuit? Explain.
9. In the analogy between an *RLC* circuit and a mass on a spring, the mass is analogous to the inductance, and the spring constant is analogous to the inverse of the capacitance. Explain.
10. Two *RLC* circuits have different values of *L* and *C*. Is it possible for these two circuits to have the same resonance frequency? Explain.
11. Can an *RLC* circuit have the same impedance at two different frequencies? Explain.

PROBLEMS AND CONCEPTUAL EXERCISES

Note: Answers to odd-numbered Problems and Conceptual Exercises can be found in the back of the book. **IP** *denotes an integrated problem, with both conceptual and numerical parts;* **BIO** *identifies problems of biological or medical interest;* **CE** *indicates a conceptual exercise.* **Predict/Explain** *problems ask for two responses:* **(a)** *your prediction of a physical outcome, and* **(b)** *the best explanation among three provided. On all problems, red bullets (•, ••, •••) are used to indicate the level of difficulty.*

SECTION 24–1 ALTERNATING VOLTAGES AND CURRENTS

1. • An ac generator produces a peak voltage of 55 V. What is the rms voltage of this generator?

2. • **European Electricity** In many European homes the rms voltage available from a wall socket is 240 V. What is the maximum voltage in this case?

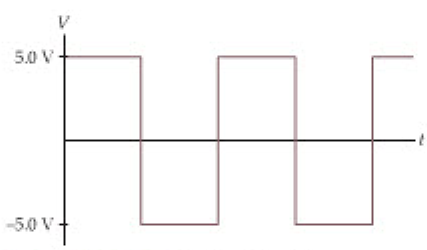
3. • An rms voltage of 120 V produces a maximum current of 2.1 A in a certain resistor. Find the resistance of this resistor.

4. • The rms current in an ac circuit with a resistance of 150 Ω is 0.85 A. What are **(a)** the average and **(b)** the maximum power consumed by this circuit?

5. • A 3.33-kΩ resistor is connected to a generator with a maximum voltage of 141 V. Find **(a)** the average and **(b)** the maximum power delivered to this circuit.

6. •• A "75-watt" lightbulb uses an average power of 75 W when connected to an rms voltage of 120 V. **(a)** What is the resistance of the lightbulb? **(b)** What is the maximum current in the bulb? **(c)** What is the maximum power used by the bulb at any given instant of time?

7. ••• **Square-Wave Voltage I** The relationship $V_{rms} = V_{max}/\sqrt{2}$ is valid only for voltages that vary sinusoidally. Find the relationship between V_{rms} and V_{max} for the "square-wave" voltage shown in Figure 24–26.



▲ **FIGURE 24–26** Problem 7

SECTION 24–2 CAPACITORS IN AC CIRCUITS

8. • The reactance of a capacitor is 65 Ω at a frequency of 57 Hz. What is its capacitance?

9. • The capacitive reactance of a capacitor at 60.0 Hz is 105 Ω. At what frequency is its capacitive reactance 72.5 Ω?

10. • A 105-μF capacitor is connected to an ac generator with an rms voltage of 20.0 V and a frequency of 100.0 Hz. What is the rms current in this circuit?

11. • The rms voltage across a 0.010-μF capacitor is 1.8 V at a frequency of 52 Hz. What are **(a)** the rms and **(b)** the maximum current through the capacitor?

12. •• An ac generator with a frequency of 30.0 Hz and an rms voltage of 12.0 V is connected to a 45.5-μF capacitor. **(a)** What is the maximum current in this circuit? **(b)** What is the current in the circuit when the voltage across the capacitor is 5.25 V and increasing? **(c)** What is the current in the circuit when the voltage across the capacitor is 5.25 V and decreasing?

13. •• The maximum current in a 22-μF capacitor connected to an ac generator with a frequency of 120 Hz is 0.15 A. **(a)** What is the maximum voltage of the generator? **(b)** What is the voltage across the capacitor when the current in the circuit is 0.10 A and increasing? **(c)** What is the voltage across the capacitor when the current in the circuit is 0.10 A and decreasing?

14. •• **IP** An rms voltage of 20.5 V with a frequency of 1.00 kHz is applied to a 0.395-μF capacitor. **(a)** What is the rms current in this circuit? **(b)** By what factor does the current change if the frequency of the voltage is doubled? **(c)** Calculate the current for a frequency of 2.00 kHz.

15. •• A circuit consists of a 1.00-kHz generator and a capacitor. When the rms voltage of the generator is 0.500 V, the rms current in the circuit is 0.430 mA. **(a)** What is the reactance of the capacitor at 1.00 kHz? **(b)** What is the capacitance of the capacitor? **(c)** If the rms voltage is maintained at 0.500 V, what is the rms current at 2.00 kHz? At 10.0 kHz?

16. •• **IP** A capacitor has an rms current of 21 mA at a frequency of 60.0 Hz when the rms voltage across it is 14 V. **(a)** What is the capacitance of this capacitor? **(b)** If the frequency is increased, will the current in the capacitor increase, decrease, or stay the same? Explain. **(c)** Find the rms current in this capacitor at a frequency of 410 Hz.

17. •• A 0.22-μF capacitor is connected to an ac generator with an rms voltage of 12 V. For what range of frequencies will the rms current in the circuit be less than 1.0 mA?

18. •• At what frequency will a generator with an rms voltage of 504 V produce an rms current of 7.50 mA in a 0.0150-μF capacitor?

SECTION 24–3 *RC* CIRCUITS

19. • Find the impedance of a 60.0-Hz circuit with a 45.5-Ω resistor connected in series with a 95.0-μF capacitor.

20. • An ac generator with a frequency of 105 Hz and an rms voltage of 22.5 V is connected in series with a 10.0-kΩ resistor and a 0.250-μF capacitor. What is the rms current in this circuit?

21. • The rms current in an *RC* circuit is 0.72 A. The capacitor in this circuit has a capacitance of 13 μF and the ac generator has a frequency of 150 Hz and an rms voltage of 95 V. What is the resistance in this circuit?

22. •• A 65.0-Hz generator with an rms voltage of 135 V is connected in series to a 3.35-kΩ resistor and a 1.50-μF capacitor. Find **(a)** the rms current in the circuit and **(b)** the phase angle, ϕ, between the current and the voltage.

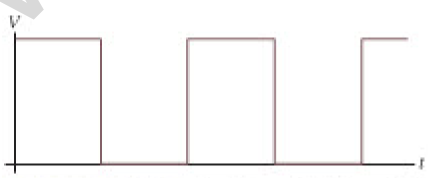
23. •• **(a)** At what frequency must the circuit in Problem 22 be operated for the current to lead the voltage by 23.0°? **(b)** Using the frequency found in part (a), find the average power consumed by this circuit.

24. •• **(a)** Sketch the phasor diagram for an ac circuit with a 105-Ω resistor in series with a 32.2-μF capacitor. The frequency of the generator is 60.0 Hz. **(b)** If the rms voltage of the generator is 120 V, what is the average power consumed by the circuit?

25. •• Find the power factor for an *RC* circuit connected to a 70.0-Hz generator with an rms voltage of 155 V. The values of *R* and *C* in this circuit are 105 Ω and 82.4 μF, respectively.

26. •• **IP** **(a)** Determine the power factor for an *RC* circuit with $R = 4.0$ kΩ and $C = 0.35$ μF that is connected to an ac generator with an rms voltage of 24 V and a frequency of 150 Hz. **(b)** Will the power factor for this circuit increase, decrease, or stay the same if the frequency of the generator is increased? Explain.

27. ••• **Square-Wave Voltage II** The "square-wave" voltage shown in Figure 24–27 is applied to an *RC* circuit. Sketch the shape of the instantaneous voltage across the capacitor, assuming the time constant of the circuit is equal to the period of the applied voltage.



▲ **FIGURE 24–27** Problems 27, 42, and 98

SECTION 24–4 INDUCTORS IN AC CIRCUITS

28. • **CE Predict/Explain** When a long copper wire of finite resistance is connected to an ac generator, as shown in Figure 24–28 (a), a certain amount of current flows through the wire. The wire is now wound into a coil of many loops and reconnected to the generator, as indicated in Figure 24–28 (b). **(a)** Is the current supplied to the coil greater than, less than, or the same as the current supplied to the uncoiled wire? **(b)** Choose the *best explanation* from among the following:

   I. More current flows in the circuit because the coiled wire is an inductor, and inductors tend to keep the current flowing in an ac circuit.

   II. The current supplied to the circuit is the same because the wire is the same. Simply wrapping the wire in a coil changes nothing.

   III. Less current is supplied to the circuit because the coiled wire acts as an inductor, which increases the impedance of the circuit.

▲ **FIGURE 24–28** Problem 28

29. • An inductor has a reactance of 56.5 Ω at 75.0 Hz. What is its reactance at 60.0 Hz?

30. • What is the rms current in a 77.5-mH inductor when it is connected to a 60.0-Hz generator with an rms voltage of 115 V?

31. • What rms voltage is required to produce an rms current of 2.1 A in a 66-mH inductor at a frequency of 25 Hz?

32. •• A 525-Ω resistor and a 295-mH inductor are connected in series with an ac generator with an rms voltage of 20.0 V and a frequency of 60.0 Hz. What is the rms current in this circuit?

33. •• The rms current in an *RL* circuit is 0.26 A when it is connected to an ac generator with a frequency of 60.0 Hz and an rms voltage of 25 V. **(a)** Given that the inductor has an inductance of 145 mH, what is the resistance of the resistor? **(b)** Find the rms voltage across the resistor. **(c)** Find the rms voltage across the inductor. **(d)** Use your results from parts (b) and (c) to show that $\sqrt{V_{\mathrm{rms},R}^2 + V_{\mathrm{rms},L}^2}$ is equal to 25 V.

34. •• An ac generator with a frequency of 1.34 kHz and an rms voltage of 24.2 V is connected in series with a 2.00-kΩ resistor and a 315-mH inductor. **(a)** What is the power factor for this circuit? **(b)** What is the average power consumed by this circuit?

35. •• **IP** An rms voltage of 22.2 V with a frequency of 1.00 kHz is applied to a 0.290-mH inductor. **(a)** What is the rms current in this circuit? **(b)** By what factor does the current change if the frequency of the voltage is doubled? **(c)** Calculate the current for a frequency of 2.00 kHz.

36. •• A 0.22-μH inductor is connected to an ac generator with an rms voltage of 12 V. For what range of frequencies will the rms current in the circuit be less than 1.0 mA?

37. •• The phase angle in a certain *RL* circuit is 76° at a frequency of 60.0 Hz. If $R = 2.7$ Ω for this circuit, what is the value of the inductance, *L*?

38. •• An *RL* circuit consists of a resistor $R = 68$ Ω, an inductor, $L = 31$ mH, and an ac generator with an rms voltage of 120 V. **(a)** At what frequency will the rms current in this circuit be 1.5 A? For this frequency, what are **(b)** the rms voltage across the resistor, $V_{\mathrm{rms},R}$, and **(c)** the rms voltage across the inductor, $V_{\mathrm{rms},L}$? **(d)** Show that $V_{\mathrm{rms},R} + V_{\mathrm{rms},L} > 120$ V, but that $\sqrt{V_{\mathrm{rms},R}^2 + V_{\mathrm{rms},L}^2} = 120$ V.

39. •• **(a)** Sketch the phasor diagram for an ac circuit with a 105-Ω resistor in series with a 22.5-mH inductor. The frequency of the generator is 60.0 Hz. **(b)** If the rms voltage of the generator is 120 V, what is the average power consumed by the circuit?

40. •• **IP** In Problem 37, does the phase angle increase, decrease, or stay the same when the frequency is increased? Verify your answer by calculating the phase angle at 70.0 Hz.

41. •• A large air conditioner has a resistance of 7.0 Ω and an inductive reactance of 15 Ω. If the air conditioner is powered by a 60.0-Hz generator with an rms voltage of 240 V, find **(a)** the impedance of the air conditioner, **(b)** its rms current, and **(c)** the average power consumed by the air conditioner.

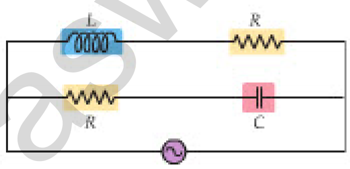
42. ••• **Square-Wave Voltage III** The "square-wave" voltage shown in Figure 24–27 is applied to an *RL* circuit. Sketch the shape of the instantaneous voltage across the inductor, assuming the time constant of the circuit is much less than the period of the applied voltage.

SECTION 24–5 *RLC* CIRCUITS

43. • **CE** An inductor and a capacitor are to be connected to a generator. Will the generator supply more current at *high* frequency if the inductor and capacitor are connected in series or in parallel? Explain.

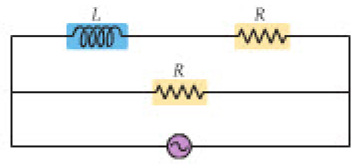
44. • **CE** An inductor and a capacitor are to be connected to a generator. Will the generator supply more current at *low* frequency if the inductor and capacitor are connected in series or in parallel? Explain.

45. • **CE Predict/Explain (a)** When the ac generator in Figure 24–29 operates at high frequency, is the rms current in the circuit greater than, less than, or the same as when the generator operates at low frequency? **(b)** Choose the *best explanation* from among the following:
   I. The current is the same because at high frequency the inductor is like an open circuit, and at low frequency the capacitor is like an open circuit. In either case the resistance of the circuit is *R*.
   II. Less current flows at high frequency because in that limit the inductor acts like an open circuit, allowing no current to flow.
   III. More current flows at high frequency because in that limit the capacitor acts like an ideal wire of zero resistance.



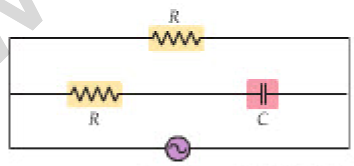
▲ **FIGURE 24–29** Problems 45 and 75

46. • **CE Predict/Explain (a)** When the ac generator in Figure 24–30 operates at high frequency, is the rms current in the circuit greater than, less than, or the same as when the generator operates at low frequency? **(b)** Choose the *best explanation* from among the following:
   I. The current at high frequency is greater because the higher the frequency the more charge that flows through a circuit.
   II. Less current flows at high frequency because in that limit the inductor is like an open circuit and current has only one path to flow through.
   III. The inductor has zero resistance, and therefore the resistance of the circuit is the same at all frequencies. As a result the current is the same at all frequencies.



▲ **FIGURE 24–30** Problems 46, 50, and 76

47. • **CE Predict/Explain (a)** When the ac generator in Figure 24–31 operates at high frequency, is the rms current in the circuit greater than, less than, or the same as when the generator operates at low frequency? **(b)** Choose the *best explanation* from among the following:
   I. The capacitor has no resistance, and therefore the resistance of the circuit is the same at all frequencies. As a result the current is the same at all frequencies.
   II. Less current flows at high frequency because in that limit the capacitor is like an open circuit and current has only one path to flow through.
   III. More current flows at high frequency because in that limit the capacitor is like a short circuit and current has two parallel paths to flow through.



▲ **FIGURE 24–31** Problems 47, 51, 75, and 76

48. • Find the rms voltage across each element in an *RLC* circuit with $R = 9.9\ \text{k}\Omega$, $C = 0.15\ \mu\text{F}$, and $L = 25$ mH. The generator supplies an rms voltage of 115 V at a frequency of 60.0 Hz.

49. • What is the impedance of a 1.50-kΩ resistor, a 105-mH inductor, and a 12.8-μF capacitor connected in series with a 60.0-Hz ac generator?

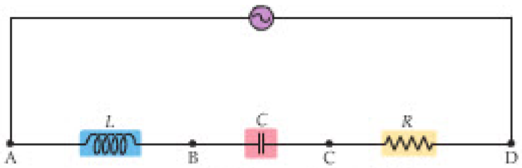
50. • Consider the circuit shown in Figure 24–30. The ac generator in this circuit has an rms voltage of 65 V. Given that $R = 15\ \Omega$ and $L = 0.22$ mH, find the rms current in this circuit in the limit of **(a)** high frequency and **(b)** low frequency.

51. • Consider the circuit shown in Figure 24–31. The ac generator in this circuit has an rms voltage of 75 V. Given that $R = 15\ \Omega$ and $C = 41\ \mu\text{F}$, find the rms current in this circuit in the limit of **(a)** high frequency and **(b)** low frequency.

52. •• What is the phase angle in an *RLC* circuit with $R = 9.9\ \text{k}\Omega$, $C = 1.5\ \mu\text{F}$, and $L = 250$ mH? The generator supplies an rms voltage of 115 V at a frequency of 60.0 Hz.

53. •• **IP** An *RLC* circuit has a resistance of 105 Ω, an inductance of 85.0 mH, and a capacitance of 13.2 μF. **(a)** What is the power factor for this circuit when it is connected to a 125-Hz ac generator? **(b)** Will the power factor increase, decrease, or stay the same if the resistance is increased? Explain. **(c)** Calculate the power factor for a resistance of 525 Ω.

54. •• An ac voltmeter, which displays the rms voltage between the two points touched by its leads, is used to measure voltages in the circuit shown in Figure 24–32. In this circuit, the ac generator has an rms voltage of 6.00 V and a frequency of 30.0 kHz. The inductance in the circuit is 0.300 mH, the capacitance is 0.100 μF, and the resistance is 2.50 Ω. What is the reading on a voltmeter when it is connected to points **(a)** A and B, **(b)** B and C, **(c)** A and C, and **(d)** A and D?



▲ **FIGURE 24–32** Problems 54 and 55

55. •• **IP** Consider the ac circuit shown in Figure 24–32, where we assume that the values of R, L, and C are the same as in the previous problem, and that the rms voltage of the generator is still 6.00 V. The frequency of the generator, however, is doubled to 60.0 kHz. Calculate the rms voltage across **(a)** the resistor, R, **(b)** the inductor, L, and **(c)** the capacitor, C. **(d)** Do you expect the sum of the rms voltages in parts (a), (b), and (c) to be greater than, less than, or equal to 6.00 V? Explain.

56. •• **(a)** Sketch the phasor diagram for an ac circuit with a 105-Ω resistor in series with a 22.5-mH inductor and a 32.2-μF capacitor. The frequency of the generator is 60.0 Hz. **(b)** If the rms voltage of the generator is 120 V, what is the average power consumed by the circuit?

57. •• A generator connected to an RLC circuit has an rms voltage of 120 V and an rms current of 34 mA. If the resistance in the circuit is 3.3 kΩ and the capacitive reactance is 6.6 kΩ, what is the inductive reactance of the circuit?

58. ••• **Manufacturing Plant Power** A manufacturing plant uses 2.22 kW of electric power provided by a 60.0-Hz ac generator with an rms voltage of 485 V. The plant uses this power to run a number of high-inductance electric motors. The plant's total resistance is $R = 25.0\ \Omega$ and its inductive reactance is $X_L = 45.0\ \Omega$. **(a)** What is the total impedance of the plant? **(b)** What is the plant's power factor? **(c)** What is the rms current used by the plant? **(d)** What capacitance, connected in series with the power line, will increase the plant's power factor to unity? **(e)** If the power factor is unity, how much current is needed to provide the 2.22 kW of power needed by the plant? Compare your answer with the current found in part (c). (Because power-line losses are proportional to the square of the current, a utility company will charge an industrial user with a low power factor a higher rate per kWh than a company with a power factor close to unity.)

SECTION 24–6 RESONANCE IN ELECTRIC CIRCUITS

59. • **CE** A capacitor and an inductor connected in series have a period of oscillation given by T. At the time $t = 0$ the capacitor has its maximum charge. In terms of T, what is the first time after $t = 0$ that **(a)** the current in the circuit has its maximum value and **(b)** the energy stored in the electric field is a maximum?

60. • **CE Predict/Explain** In an RLC circuit a second capacitor is added in *series* to the capacitor already present. **(a)** Does the resonance frequency increase, decrease, or stay the same? **(b)** Choose the *best explanation* from among the following:
    I. The resonance frequency stays the same because it depends only on the resistance in the circuit.
    II. Adding a capacitor in series increases the equivalent capacitance, and this decreases the resonance frequency.
    III. Adding a capacitor in series decreases the equivalent capacitance, and this increases the resonance frequency.

61. • **CE** In an RLC circuit a second capacitor is added in *parallel* to the capacitor already present. Does the resonance frequency increase, decrease, or stay the same? Explain.

62. • An RLC circuit has a resonance frequency of 2.4 kHz. If the capacitance is 47 μF, what is the inductance?

63. • At resonance, the rms current in an RLC circuit is 2.8 A. If the rms voltage of the generator is 120 V, what is the resistance, R?

64. •• **CE** The resistance in an RLC circuit is doubled. **(a)** Does the resonance frequency increase, decrease, or stay the same? Explain. **(b)** Does the maximum current in the circuit increase, decrease, or stay the same? Explain.

65. •• **CE** The voltage in a sinusoidally driven RLC circuit leads the current. **(a)** If we want to bring this circuit into resonance by changing the frequency of the generator, should the frequency be increased or decreased? Explain. **(b)** If we want to bring this circuit into resonance by changing the inductance instead, should the inductance be increased or decreased? Explain.

66. •• A 115-Ω resistor, a 67.6-mH inductor, and a 189-μF capacitor are connected in series to an ac generator. **(a)** At what frequency will the current in the circuit be a maximum? **(b)** At what frequency will the impedance of the circuit be a minimum?

67. •• **IP** An ac generator of variable frequency is connected to an RLC circuit with $R = 12\ \Omega$, $L = 0.15$ mH, and $C = 0.20$ mF. At a frequency of 1.0 kHz, the rms current in the circuit is larger than desired. Should the frequency of the generator be increased or decreased to reduce the current? Explain.

68. •• **(a)** Find the frequency at which a 33-μF capacitor has the same reactance as a 33-mH inductor. **(b)** What is the resonance frequency of an LC circuit made with this inductor and capacitor?

69. •• Consider an RLC circuit with $R = 105\ \Omega$, $L = 518$ mH, and $C = 0.200$ μF. **(a)** At what frequency is this circuit in resonance? **(b)** Find the impedance of this circuit if the frequency has the value found in part (a), but the capacitance is increased to 0.220 μF. **(c)** What is the power factor for the situation described in part (b)?

70. •• **IP** An RLC circuit has a resonance frequency of 155 Hz. **(a)** If both L and C are doubled, does the resonance frequency increase, decrease, or stay the same? Explain. **(b)** Find the resonance frequency when L and C are doubled.

71. •• An RLC circuit has a capacitance of 0.29 μF. **(a)** What inductance will produce a resonance frequency of 95 MHz? **(b)** It is desired that the impedance at resonance be one-fifth the impedance at 11 kHz. What value of R should be used to obtain this result?

GENERAL PROBLEMS

72. • **CE BIO Persistence of Vision** Although an incandescent lightbulb appears to shine with constant intensity, this is an artifact of the eye's persistence of vision. In fact, the intensity of a bulb's light rises and falls with time due to the alternating current used in household circuits. If you could perceive these oscillations, would you see the light attain maximum brightness 60 or 120 times per second? Explain.

73. • **CE** An inductor in an LC circuit has a maximum current of 2.4 A and a maximum energy of 36 mJ. When the current in the inductor is 1.2 A, what is the energy stored in the capacitor?

74. • **CE** An RLC circuit is driven at its resonance frequency. Is its impedance greater than, less than, or equal to R? Explain.

75. • **CE Predict/Explain** Suppose the circuits shown in Figures 24–29 and 24–31 are connected to identical batteries, rather than to ac generators. **(a)** Assuming the value of R is the same in the two circuits, is the current in Figure 24–29 greater than, less

than, or the same as the current in Figure 24–31? **(b)** Choose the *best explanation* from among the following:

I. The circuits have the same current because the capacitor acts like an open circuit and the inductor acts like a short circuit.

II. The current in Figure 24–29 is larger because it has more circuit elements, each of which can carry current.

III. The current in Figure 24–31 is larger because it has fewer circuit elements, meaning less resistance to current flow.

76. • **CE** Suppose the circuits shown in Figures 24–30 and 24–31 are connected to identical batteries, rather than to ac generators. Assuming the value of R is the same in the two circuits, is the current in Figure 24–30 greater than, less than, or the same as the current in Figure 24–31? Explain.

77. • **CE Predict/Explain** Consider a circuit consisting of a lightbulb and a capacitor, as shown in circuit 2 of Conceptual Checkpoint 24–2. **(a)** If the frequency of the generator is increased, does the intensity of the lightbulb increase, decrease, or stay the same? **(b)** Choose the *best explanation* from among the following:

I. As the frequency increases it becomes harder to force current through the capacitor, and therefore the intensity of the lightbulb decreases.

II. The intensity of the lightbulb increases because as the frequency becomes higher the capacitor acts more like a short circuit, allowing more current to flow.

III. The intensity of the lightbulb is independent of frequency because the circuit contains a capacitor but not an inductor.

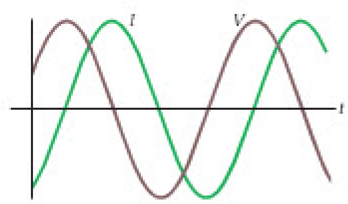
78. • **CE** Consider a circuit consisting of a lightbulb and an inductor, as shown in Conceptual Checkpoint 24–3. If the frequency of the generator is increased, does the intensity of the lightbulb increase, decrease, or stay the same? Explain.

79. • A 4.40-μF and an 8.80-μF capacitor are connected in *parallel* to a 60.0-Hz generator operating with an rms voltage of 115 V. What is the rms current supplied by the generator?

80. • A 4.40-μF and an 8.80-μF capacitor are connected in *series* to a 60.0-Hz generator operating with an rms voltage of 115 V. What is the rms current supplied by the generator?

81. • A 10.0-μF capacitor and a 30.0-μF capacitor are connected in parallel to an ac generator with a frequency of 60.0 Hz. What is the capacitive reactance of this pair of capacitors?

82. •• **CE** A generator drives an RLC circuit with the voltage V shown in Figure 24–33. The corresponding current I is also shown in the figure. **(a)** Is the inductive reactance of this circuit greater than, less than, or equal to its capacitive reactance? Explain. **(b)** Is the frequency of this generator greater than, less than, or equal to the resonance frequency of the circuit? Explain.



▲ **FIGURE 24–33** Problem 82

83. •• **IP** Consider the RLC circuit shown in Example 24–6, and the corresponding phasor diagram given in the Insight. **(a)** On the basis of the phasor diagram, can you conclude that the resonance frequency of this circuit is greater than, less than, or equal to 60.0 Hz? Explain. **(b)** Calculate the resonance frequency for this circuit. **(c)** The impedance of this circuit at 60.0 Hz is 226 Ω. What is the impedance at resonance?

84. •• **IP** When a certain resistor is connected to an ac generator with a maximum voltage of 15 V, the average power dissipated in the resistor is 22 W. **(a)** What is the resistance of the resistor? **(b)** What is the rms current in the circuit? **(c)** We know that $P_{av} = I_{rms}^2 R$, and hence it seems that reducing the resistance should reduce the average power. On the other hand, we also know that $P_{av} = V_{rms}^2/R$, which suggests that reducing R increases P_{av}. Which conclusion is correct? Explain.

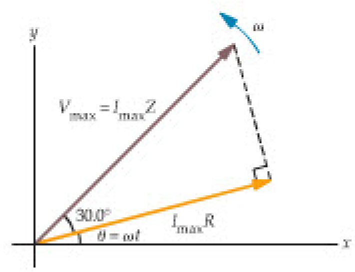
85. •• A 9.5-Hz generator is connected to a capacitor. If the current in the generator has its maximum value at $t = 0$, what is the earliest possible time that the voltage across the capacitor is a maximum?

86. •• The voltage across an inductor reaches its maximum value 25 ms before the current supplied by the generator reaches *its* maximum value. What is the lowest possible frequency at which the generator operates?

87. •• Find the average power consumed by an RC circuit connected to a 60.0-Hz generator with an rms voltage of 122 V. The values of R and C in this circuit are 3.30 kΩ and 2.75 μF, respectively.

88. •• A 1.15-kΩ resistor and a 505-mH inductor are connected in series to a 1250-Hz generator with an rms voltage of 14.2 V. **(a)** What is the rms current in the circuit? **(b)** What capacitance must be inserted in series with the resistor and inductor to reduce the rms current to half the value found in part (a)?

89. •• **IP *RLC* Phasor** The phasor diagram for an RLC circuit is shown in Figure 24–34. **(a)** If the resistance in this circuit is 525 Ω, what is the impedance? **(b)** If the frequency in this circuit is increased, will the impedance increase, decrease, or stay the same? Explain.



▲ **FIGURE 24–34** Problems 89, 90, and 91

90. •• **IP** Figure 24–34 shows the phasor diagram for an RLC circuit in which the impedance is 337 Ω. **(a)** What is the resistance, R, in this circuit? **(b)** Is this circuit driven at a frequency that is greater than, less than, or equal to the resonance frequency of the circuit? Explain.

91. •• **IP** An RLC circuit has a resistance $R = 25$ Ω and an inductance $L = 160$ mH, and is connected to an ac generator with a frequency of 55 Hz. The phasor diagram for this circuit is shown in Figure 24–34. Find **(a)** the impedance, Z, and **(b)** the capacitance, C, for this circuit. **(c)** If the value of C is decreased, will the impedance of the circuit increase, decrease, or stay the same? Explain.

92. •• **IP Black-Box Experiment** You are given a sealed box with two electrical terminals. The box contains a 5.00-Ω resistor in series with either an inductor or a capacitor. When you attach an ac generator with an rms voltage of 0.750 V to the terminals of the box, you find that the current increases with increasing frequency. **(a)** Does the box contain an inductor or a capacitor?

Explain. **(b)** When the frequency of the generator is 25.0 kHz, the rms current is 87.2 mA. What is the capacitance or inductance of the unknown component in the box?

93. •• **IP** A circuit is constructed by connecting a 1.00-kΩ resistor, a 252-μF capacitor, and a 515-mH inductor in series. **(a)** What is the highest frequency at which the impedance of this circuit is equal to 2.00 kΩ? **(b)** To reduce the impedance of this circuit, should the frequency be increased or decreased from its value in part (a)? Explain.

94. •• An *RLC* circuit with $R = 25.0\ \Omega$, $L = 325$ mH, and $C = 45.2\ \mu$F is connected to an ac generator with an rms voltage of 24 V. Determine the average power delivered to this circuit when the frequency of the generator is **(a)** equal to the resonance frequency, **(b)** twice the resonance frequency, and **(c)** half the resonance frequency.

95. •• **A Light-Dimmer Circuit** The intensity of a lightbulb with a resistance of 120 Ω is controlled by connecting it in series with an inductor whose inductance can be varied from $L = 0$ to $L = L_{max}$. This "light dimmer" circuit is connected to an ac generator with a frequency of 60.0 Hz and an rms voltage of 110 V. **(a)** What is the average power dissipated in the lightbulb when $L = 0$? **(b)** The inductor is now adjusted so that $L = L_{max}$. In this case, the average power dissipated in the lightbulb is one-fourth the value found in part (a). What is the value of L_{max}?

96. ••• An electric motor with a resistance of 15 Ω and an inductance of 53 mH is connected to a 60.0-Hz ac generator. **(a)** What is the power factor for this circuit? **(b)** In order to increase the power factor of this circuit to 0.80, a capacitor is connected in series with the motor and inductor. Find the required value of the capacitance.

97. ••• **IP Tuning a Radio** A radio tuning circuit contains an *RLC* circuit with $R = 5.0\ \Omega$ and $L = 2.8\ \mu$H. **(a)** What capacitance is needed to produce a resonance frequency of 85 MHz? **(b)** If the capacitance is increased above the value found in part (a), will the impedance increase, decrease, or stay the same? Explain. **(c)** Find the impedance of the circuit at resonance. **(d)** Find the impedance of the circuit when the capacitance is 1% higher than the value found in part (a).

98. ••• If the maximum voltage in the square wave shown in Figure 24–27 is V_{max}, what are **(a)** the average voltage, V_{av}, and **(b)** the rms voltage, V_{rms}?

99. ••• An ac generator supplies an rms voltage of 5.00 V to an *RC* circuit. At a frequency of 20.0 kHz the rms current in the circuit is 45.0 mA; at a frequency of 25.0 kHz the rms current is 50.0 mA. What are the values of *R* and *C* in this circuit?

100. ••• An ac generator supplies an rms voltage of 5.00 V to an *RL* circuit. At a frequency of 20.0 kHz the rms current in the circuit is 45.0 mA; at a frequency of 25.0 kHz the rms current is 40.0 mA. What are the values of *R* and *L* in this circuit?

101. ••• An *RC* circuit consists of a resistor $R = 32\ \Omega$, a capacitor $C = 25\ \mu$F, and an ac generator with an rms voltage of 120 V. **(a)** At what frequency will the rms current in this circuit be 2.9 A? For this frequency, what are **(b)** the rms voltage across the resistor, $V_{rms,R}$, and **(c)** the rms voltage across the capacitor, $V_{rms,C}$? **(d)** Show that $V_{rms,R} + V_{rms,C} > 120$ V, but that $\sqrt{V_{rms,R}^2 + V_{rms,C}^2} = 120$ V.

PASSAGE PROBLEMS

Playing a Theremin

As mentioned in Chapter 20, a theremin is a musical instrument you play without touching. You may not have heard of a theremin before, but you have certainly heard one being played—in fact, theremins are responsible for the eerie background music on many science fiction movies, such as *The Day the Earth Stood Still*. A theremin-like instrument also plays a prominent part in the Beach Boys hit song "Good Vibrations."

So how *do* you play a theremin if you don't actually touch it? Well, a theremin has two antennas that extend from the main body of the instrument, a horizontal loop antenna to control the volume, and a vertical linear antenna to control the pitch. Varying the distance between your hand and the vertical antenna, for example, adjusts the capacitance of the *RLC* circuit to which that antenna is attached. Changing the capacitance changes the resonant frequency and that, in turn, changes the frequency of the note played by the instrument.

To produce notes spanning a full five octaves, the theremin employs a clever mechanism referred to as the heterodyne principle. Rather than produce the audio frequencies directly, the theremin uses two radio frequency oscillators, one at a fixed frequency, the other with a frequency controlled by the thereminist. The beat frequency between these two radio frequencies, which is in the audio frequency range, is what you actually hear. This mechanism is essentially the same as that used in FM (frequency-modulated) radio.

102. • Suppose a theremin uses an oscillator with a fixed frequency of 90.1 MHz and an *RLC* circuit with $R = 1.5\ \Omega$, $L = 2.08\ \mu H$, and $C = 1.50$ pF. What is the beat frequency of these two oscillators? (Audio frequencies range from about 20 Hz to 20,000 Hz.)

A. 3740 Hz B. 5100 Hz
C. 4760 Hz D. 9000 Hz

103. • If the thereminist moves one of her fingers and increases the capacitance of the system slightly, does the beat frequency increase, decrease, or stay the same?

104. • Find the new beat frequency if the thereminist increases the capacitance by 0.100% over its value in Problem 102. All other quantities stay the same.

A. 761 Hz B. 41,300 Hz
C. 41,900 Hz D. 86,300 Hz

105. • What is the rms current in the theremin's *RLC* circuit (Problem 102) if it is attached to an ac generator with an rms voltage of 25.0 V and a frequency of 90.0 MHz?

A. 2.14 mA B. 3.46 mA
C. 8.06 A D. 16.7 A

INTERACTIVE PROBLEMS

106. •• **IP Referring to Example 24–6** Suppose we would like to change the phase angle for this circuit to $\phi = -25.0°$, and that we would like to accomplish this by changing the resistor to a value other than 175 Ω. The inductor is still 90.0 mH, the capacitor is 15.0 μF, the rms voltage is 120.0 V, and the ac frequency is 60.0 Hz. **(a)** Should the resistance be increased or decreased? Explain. **(b)** Find the resistance that gives the desired phase angle. **(c)** What is the rms current in the circuit with the resistance found in part (b)?

107. •• **IP Referring to Example 24–6** You plan to change the frequency of the generator in this circuit to produce a phase angle of smaller magnitude. The resistor is still 175 Ω, the inductor is 90.0 mH, the capacitor is 15.0 μF, and the rms voltage is 120.0 V. **(a)** Should you increase or decrease the frequency? Explain. **(b)** Find the frequency that gives a phase angle of −22.5°. **(c)** What is the rms current in the circuit at the frequency found in part (b)?

25 Electromagnetic Waves

Most people think they know exactly what the world looks like—all you have to do, after all, is open your eyes and look. There's more to it than that, however. We all know, for example, that the simple yellow flower shown here (left) is a daisy. But is that really what a daisy looks like? To a bee—with its ability to see ultraviolet light invisible to us—the very same daisy is a flower with three concentric circles forming a "bull's-eye" centered on the nectar (right). Such "nectar guides" are a common feature of flowers as viewed by bees, but are generally invisible to humans. This chapter explores the nature and properties of electromagnetic radiation—the kind we know as visible light, and many other kinds as well.

Electricity and magnetism can seem very different in many ways, but they are actually intimately related—in fact, electric and magnetic fields can be considered as different aspects of the same thing, like the two sides of a coin. For example, we have seen that an electric current produces a magnetic field, and a changing magnetic field produces an electric field. Because of fundamental connections like these, we refer to the phenomena of electricity and magnetism together as **electromagnetism.**

In this chapter we consider one of the most significant manifestations of electromagnetism; namely, that electric and magnetic fields can work together to create traveling waves called **electromagnetic waves.** As we shall see, these waves are responsible for everything from radio and TV signals, to the visible light we see all around us, to the X-rays that reveal our internal structure, and much more. In addition, the prediction, discovery, and technological development of electromagnetic waves are a fascinating success story in the history of science.

25–1 The Production of Electromagnetic Waves

Electromagnetic waves were predicted, and their properties were studied theoretically, decades before they were first produced with electric circuits in the lab. The prediction came from Scottish physicist James Clerk Maxwell (1831–1879), who, in 1864, hypothesized that since a changing magnetic field produces an electric field (Faraday's law) a changing electric field should similarly produce a magnetic field. In effect, Maxwell suggested a sort of "symmetry" between electric and magnetic fields.

Maxwell followed up on his suggestion by working out its mathematical consequences. Among these was that electric and magnetic fields, acting together, could produce an *electromagnetic wave* that travels with the speed of light. As a result, he proposed that visible light—which had previously been thought of as a completely separate phenomenon from electricity and magnetism—was, in fact, an electromagnetic wave. His theory also implied that electromagnetic waves would not be limited to visible light and that it should be possible to produce them with oscillating electric circuits similar to those studied in the previous chapter.

The first production and observation of electromagnetic waves in the lab were carried out by the German physicist Heinrich Hertz (1857–1894) in 1887. Hertz used what was basically an *LC* circuit to generate an alternating current and found that energy could be transferred from this circuit to a similar circuit several meters away. He was able to show, in addition, that the energy transfer exhibited such standard wave phenomena as reflection, refraction, interference, diffraction, and polarization. There could be no doubt that waves were produced by the first circuit and that they propagated across the room to the second circuit. Even more significantly, he was able to show that the speed of the waves was roughly the speed of light, as predicted by Maxwell.

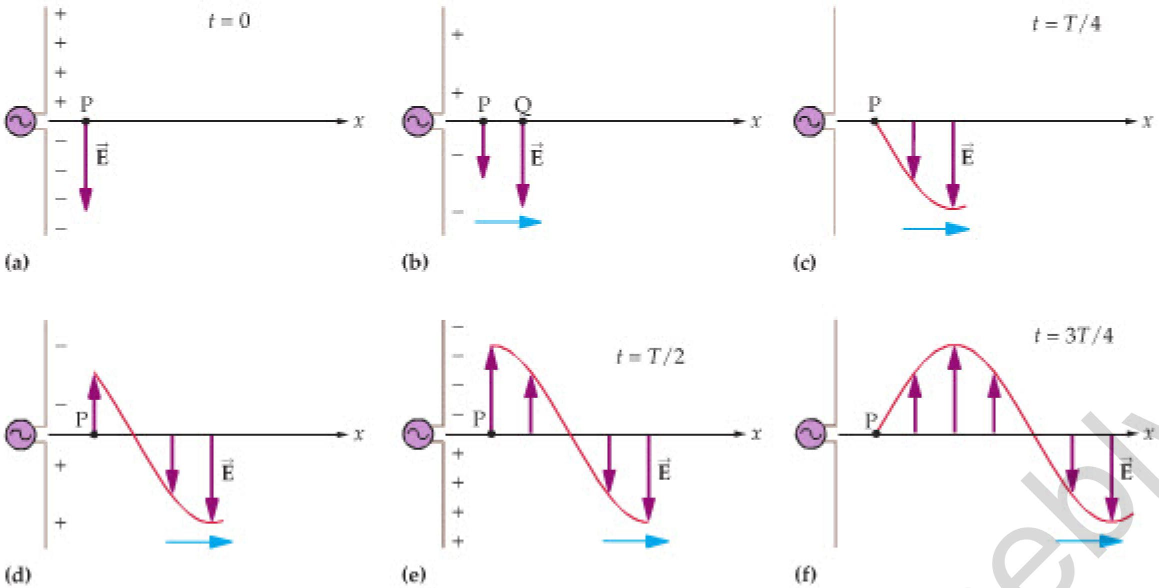
It took only a few years for Hertz's experimental apparatus to be refined and improved to the point where it could be used in practical applications. The first to do so was Guglielmo Marconi (1874–1937), who immediately recognized the implications of the electromagnetic-wave experiments—namely, that waves could be used for communications, eliminating the wires necessary for telegraphy. He patented his first system in 1896 and gained worldwide attention when, in 1901, he received a radio signal in St. John's, Newfoundland, that had been sent from Cornwall, England. When Maxwell died, electromagnetic waves were still just a theory; twenty years later, they were revolutionizing communications.

▲ Electromagnetic waves are produced by (and detected as) oscillating electric currents in a wire or similar conducting element. The actual antenna is often much smaller than is commonly imagined—the bowl-shaped structures that we tend to think of as antennas, such as these microwave relay dishes, serve to focus the transmitted beam in a particular direction or concentrate the received signal on the actual detector.

To gain an understanding of electromagnetic waves, consider the simple electric circuit shown in **Figure 25–1**. Here we show an ac generator of period T connected to the center of an antenna, which is basically a long, straight wire with a break in the middle. Suppose at time $t = 0$ the generator gives the upper segment of the antenna a maximum positive charge and the lower segment a maximum negative charge, as shown in Figure 25–1 (a). A positive test charge placed on the x axis at point P will experience a downward force; hence, the electric field there is downward. A short time later, when the charge on the antenna is reduced in magnitude, the electric field at P also has a smaller magnitude. We show this result in Figure 25–1 (b).

More importantly, Figure 25–1 (b) also shows that the electric field produced at time $t = 0$ has not vanished, nor has it simply been replaced with the new, reduced-magnitude field. Instead, the original field has *moved farther away from the antenna*, to point Q. The reason that the reduction in charge on the antenna is felt at point P *before* it is felt at point Q is simply that it takes a finite time for this change in charge to be felt at a distance. This is analogous to the fact that a person near a lightning strike hears the thunder before a person who is half a mile away, or that a wave pulse on a string takes a finite time to move from one end of the string to the other.

After the generator has completed one-quarter of a cycle, at time $t = \frac{1}{4}T$, the antenna is uncharged and the field vanishes, as in Figure 25–1 (c). Still later the charges on the antenna segments change sign, giving rise to an electric field that points upward, as we see in Figures 25–1 (d) and (e). The field vanishes again after



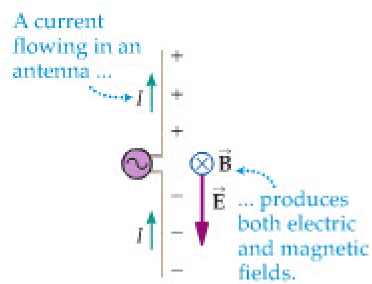
▲ **FIGURE 25–1 Producing an electromagnetic wave**

A traveling electromagnetic wave produced by an ac generator attached to an antenna. **(a)** At $t = 0$ the electric field at point P is downward. **(b)** A short time later, the electric field at P is still downward, but now with a reduced magnitude. Note that the field created at $t = 0$ has moved to point Q. **(c)** After one-quarter of a cycle, at $t = \frac{1}{4}T$, the electric field at P vanishes. **(d)** The charge on the antenna has reversed polarity now, and the electric field at P points upward. **(e)** When the oscillator has completed half a cycle, $t = \frac{1}{2}T$, the field at point P is upward and of maximum magnitude. **(f)** At $t = \frac{3}{4}T$ the field at P vanishes again. The fields produced at earlier times continue to move away from the antenna.

three-quarters of a cycle, at $t = \frac{3}{4}T$, as shown in Figure 25–1 (f). Immediately after this time, the electric field begins to point downward once more. The net result is a wavelike electric field moving steadily away from the antenna. To summarize:

> The electric field produced by an antenna connected to an ac generator propagates away from the antenna, analogous to a wave on a string moving away from your hand as you wiggle it up and down.

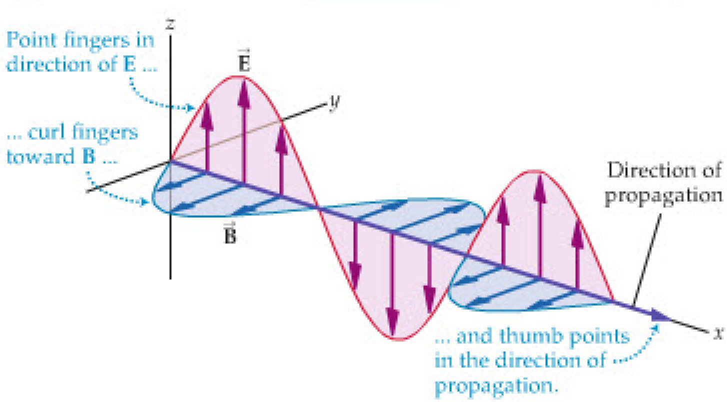
This is really only half of the electromagnetic wave, however; the other half is a similar wave in the magnetic field. To see this, consider **Figure 25–2**, where we show the current in the antenna flowing upward at a time when the upper segment is positive. Pointing the thumb of the right hand in the direction of the current, and curling the fingers around the wire, as specified in the magnetic field RHR, we see that $\vec{B}$ points into the page at the same time that $\vec{E}$ points downward. It follows, then, that $\vec{E}$ and $\vec{B}$ are at right angles to each other. A more detailed analysis shows that $\vec{E}$ and $\vec{B}$ are perpendicular to each other at all times, and that they are also in phase; that is, when the magnitude of $\vec{E}$ is at its maximum, so is the magnitude of $\vec{B}$.



▲ **FIGURE 25–2 Field directions in an electromagnetic wave**

At a time when the electric field produced by the antenna points downward, the magnetic field points into the page. In general, the electric and magnetic fields in an electromagnetic wave are always at right angles to each other.

Combining the preceding results, we can represent the electric and magnetic fields in an electromagnetic wave as shown in **Figure 25–3.** Notice that not only are



◀ **FIGURE 25–3 The right-hand rule applied to an electromagnetic wave**

An electromagnetic wave propagating in the positive x direction. Note that $\vec{E}$ and $\vec{B}$ are perpendicular to each other and in phase. The direction of propagation is given by the thumb of the right hand, after pointing the fingers in the direction of $\vec{E}$ and curling them toward $\vec{B}$.

$\vec{E}$ and $\vec{B}$ perpendicular to each other, they are also perpendicular to the direction of propagation; hence, electromagnetic waves are **transverse** waves. (See Section 14–1 for a comparison of various types of waves.) Finally, the direction of propagation is given by another right-hand rule:

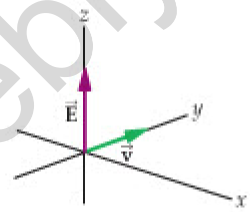
Direction of Propagation for Electromagnetic Waves

Point the fingers of your right hand in the direction of $\vec{E}$, curl your fingers toward $\vec{B}$, and your thumb will point in the direction of propagation.

This rule is consistent with the direction of propagation shown in Figure 25–3.

CONCEPTUAL CHECKPOINT 25–1 DIRECTION OF MAGNETIC FIELD

An electromagnetic wave propagates in the positive y direction, as shown in the sketch. If the electric field at the origin is in the positive z direction, is the magnetic field at the origin in **(a)** the positive x direction, **(b)** the negative x direction, or **(c)** the negative y direction?



REASONING AND DISCUSSION

Pointing the fingers of the right hand in the positive z direction (the direction of $\vec{E}$), we see that in order for the thumb to point in the direction of propagation (the positive y direction) the fingers must be curled toward the positive x direction. Therefore, $\vec{B}$ points in the positive x direction.

ANSWER

(a) $\vec{B}$ is in the positive x direction.

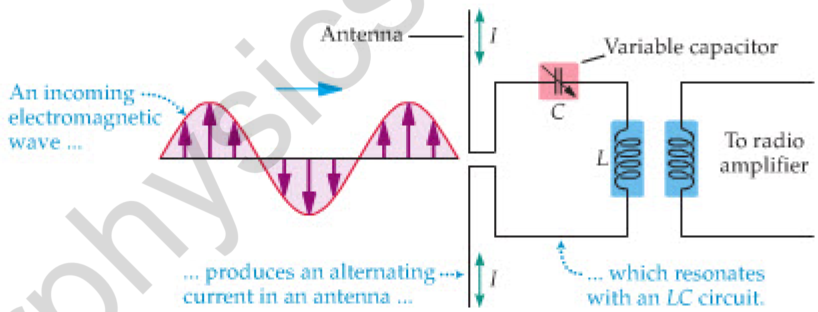


FIGURE 25–4 Receiving radio waves

Basic elements of a tuning circuit used to receive radio waves. First, an incoming wave sets up an alternating current in the antenna. Next, the resonance frequency of the LC circuit is adjusted to match the frequency of the radio wave, resulting in a relatively large current in the circuit. This current is then fed into an amplifier to further increase the signal.

REAL-WORLD PHYSICS

Radio and television communications

Electromagnetic waves can be detected in much the same way they are generated. Suppose, for instance, that an electromagnetic wave moves to the right, as in Figure 25–4. As the wave continues to move, its electric field exerts a force on electrons in the antenna that is alternately up and down, resulting in an alternating current. Thus the electromagnetic field makes the antenna behave much like an ac generator. If the antenna is connected to an LC circuit, as indicated in the figure, the resulting current can be relatively large if the resonant frequency of the circuit matches the frequency of the wave. This is the basic principle behind radio and television tuners. In fact, when you turn the tuning knob on a radio, you are actually changing the capacitance or the inductance in an LC circuit and, therefore, changing the resonance frequency.

Finally, though we have discussed the production of electromagnetic waves by means of an electric circuit and an antenna, this is certainly not the only way such waves can be generated. In fact, any time an electric charge is accelerated, it will radiate:

Accelerated charges radiate electromagnetic waves.

This condition applies no matter what the cause of the acceleration. In addition, the intensity of radiated electromagnetic waves depends on the orientation of the acceleration relative to the viewer. For example, viewing the antenna perpendicular to its length, so that the charges accelerate at right angles to the line of sight, results in maximum intensity, as illustrated in Figure 25–5. Conversely, viewing the antenna straight down from above, in the same direction as the acceleration, results in zero intensity.

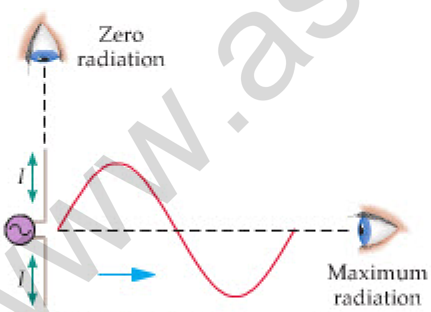


FIGURE 25–5 Electromagnetic waves and the line of sight

Electromagnetic radiation is greatest when charges accelerate at right angles to the line of sight. Zero radiation is observed when the charges accelerate along the line of sight. These observations apply to electromagnetic waves of all frequencies.

25–2 The Propagation of Electromagnetic Waves

A sound wave or a wave on a string requires a medium through which it can propagate. For example, when the air is pumped out of a jar containing a ringing bell, its sound can no longer be heard. In contrast, we can still *see* that the bell is ringing. Thus, light can propagate through a vacuum, as can all other types of electromagnetic waves, such as radio waves and microwaves. In fact, electromagnetic waves travel through a vacuum with the maximum speed that *any* form of energy can have, as we discuss in detail in Chapter 29.

The Speed of Light

All electromagnetic waves travel through a vacuum with precisely the same speed, c. Since light is the form of electromagnetic wave most familiar to us, we refer to c as the *speed of light in a vacuum*. The approximate value of this speed is as follows:

Speed of Light in a Vacuum

$$c = 3.00 \times 10^8 \text{ m/s} \qquad 25\text{–}1$$

This is a large speed, corresponding to about 186,000 mi/s. Put another way, a beam of light could travel around the world about seven times in a single second. In air the speed of light is slightly less than it is in a vacuum, and in denser materials, such as glass or water, the speed of light is reduced to about two-thirds of its vacuum value.

EXERCISE 25–1

The distance between Earth and the Sun is 1.50×10^{11} m. How long does it take for light to cover this distance?

SOLUTION

Recalling that speed is distance divided by time, it follows that the time t to cover a distance d is $t = d/v$. Using $v = c$, we find

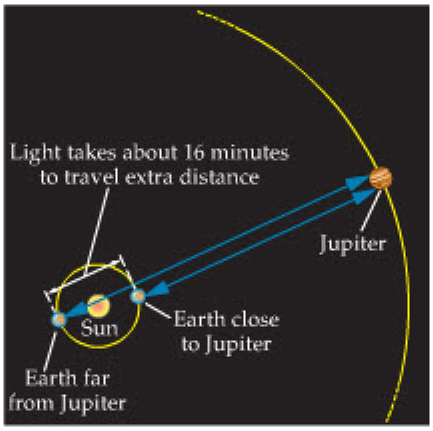
$$t = \frac{d}{c} = \frac{1.50 \times 10^{11} \text{ m}}{3.00 \times 10^8 \text{ m/s}} = 500 \text{ s}$$

Noting that 500 s is $8\frac{1}{3}$ min, we say that Earth is about 8 light-minutes from the Sun.

Because the speed of light is so large, its value is somewhat difficult to determine. The first scientific attempt to measure the speed of light was made by Galileo (1564–1642), who used two lanterns for the experiment. Galileo opened the shutters of one lantern, and an assistant a considerable distance away was instructed to open the shutter on the second lantern as soon as he observed the light from Galileo's lantern. Galileo then attempted to measure the time that elapsed before he saw the light from his assistant's lantern. Since there was no perceptible time lag, beyond the normal human reaction time, Galileo could conclude only that the speed of light must be very great indeed.

The first to give a finite, numerical value to the speed of light was the Danish astronomer Ole Romer (1644–1710), though he did not set out to measure the speed of light at all. Romer was measuring the times at which the moons of Jupiter disappeared behind the planet, and he noticed that these eclipses occurred earlier when Earth was closer to Jupiter and later when Earth was farther away from Jupiter. This difference is illustrated in Figure 25–6. From the results of Exercise 25–1, we know that light requires about 16 minutes to travel from one side of Earth's orbit to the other, and this is roughly the discrepancy in eclipse times observed by Romer. In 1676 he announced a value for the speed of light of 2.25×10^8 m/s.

The first laboratory measurement of the speed of light was performed by the French scientist Armand Fizeau (1819–1896). The basic elements of his experiment, shown in Figure 25–7, are a mirror and a rotating, notched wheel. Light passing through one notch travels to a mirror a considerable distance away, is reflected



▲ FIGURE 25–6 Using Jupiter to determine the speed of light

When the Earth is at its greatest distance from Jupiter, light takes about 16 minutes longer to travel between them. This time lag allowed Ole Romer to estimate the speed of light.

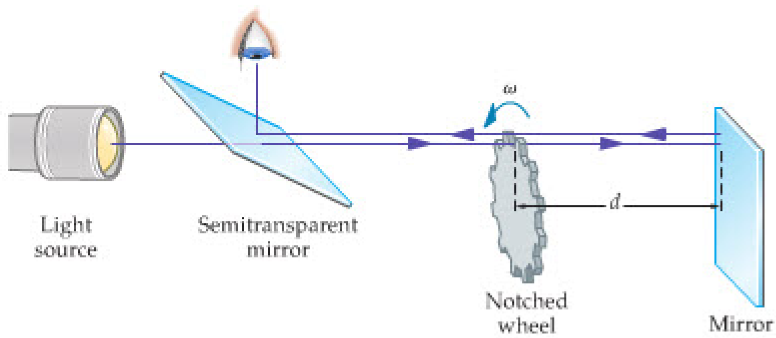


FIGURE 25–7 Fizeau's experiment to measure the speed of light
If the time required for light to travel to the far mirror and back is equal to the time it takes the wheel to rotate from one notch to the next, light will pass through the wheel and on to the observer.

back, and then, if the rotational speed of the wheel is adjusted properly, passes through the *next notch* in the wheel. By measuring the rotational speed of the wheel and the distance from the wheel to the mirror, Fizeau was able to obtain a value of 3.13×10^8 m/s for the speed of light.

Today, experiments to measure the speed of light have been refined to such a degree that we now use it to *define* the meter, as was mentioned in Chapter 1. Thus, by definition, the speed of light in a vacuum is

$$c = 299\,792\,458 \text{ m/s}$$

For most routine calculations, however, the value $c = 3.00 \times 10^8$ m/s is adequate.

Maxwell's theoretical description of electromagnetic waves allowed him to obtain a simple expression for c in terms of previously known physical quantities. In particular, he found that c could be written as follows:

$$c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}} \qquad 25\text{–}2$$

Recall that $\varepsilon_0 = 8.85 \times 10^{-12}\ \text{C}^2/(\text{N}\cdot\text{m}^2)$ occurs in the expression for the electric field due to a point charge; in fact, ε_0 determines the strength of the electric field. The constant $\mu_0 = 4\pi \times 10^{-7}\ \text{T}\cdot\text{m/A}$ plays an equivalent role for the magnetic field. Thus, Maxwell was able to show that these two constants, which were determined by electrostatic and magnetostatic measurements, also combine to yield the speed of light—again demonstrating the symmetrical role that electric and magnetic fields play in electromagnetic waves. Substituting the values for ε_0 and μ_0 we find

$$c = \frac{1}{\sqrt{(8.85 \times 10^{-12}\ \text{C}^2/(\text{N}\cdot\text{m}^2))(4\pi \times 10^{-7}\ \text{T}\cdot\text{m/A})}} = 3.00 \times 10^8 \text{ m/s}$$

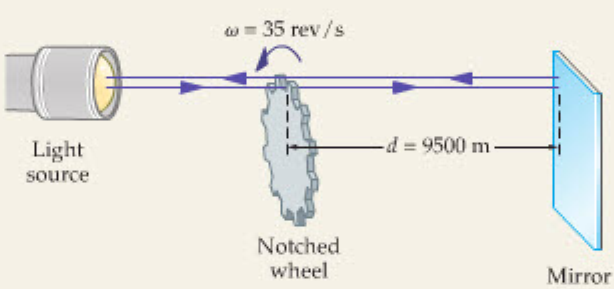
Clearly, Maxwell's theoretical expression agrees with experiment.

EXAMPLE 25–1 FIZEAU'S RESULTS

Consider a Fizeau experiment in which the wheel has 450 notches and rotates with a speed of 35 rev/s. Light passing through one notch travels to the mirror and back just in time to pass through the next notch. If the distance from the wheel to the mirror is 9500 m, what is the speed of light obtained by this measurement?

PICTURE THE PROBLEM

Our sketch shows an experimental setup similar to Fizeau's. The notched wheel is 9500 m from a mirror and spins with an angular speed of 35 rev/s. We show a few notches in our sketch, which represent the 450 notches on the actual wheel.



STRATEGY

The speed of light is the distance traveled, $2d$, divided by the time required, Δt. To find the time, we note that the wheel rotates from one notch to the next during this time; that is, it rotates through an angle $\Delta\theta = (1/450)$ rev. Knowing the rotational speed, ω, of the wheel, we can find the time using the relation $\Delta\theta = \omega\,\Delta t$ (Section 10–1).

SOLUTION

1. Find the time required for the wheel to rotate from one notch to the next: $\Delta t = \dfrac{\Delta\theta}{\omega} = \dfrac{(1/450)\text{ rev}}{35\text{ rev/s}} = 6.3 \times 10^{-5}\text{ s}$

2. Divide the time into the distance to find the speed of light: $c = \dfrac{2d}{\Delta t} = \dfrac{2(9500\text{ m})}{6.3 \times 10^{-5}\text{ s}} = 3.0 \times 10^{8}\text{ m/s}$

INSIGHT

Note that even with a rather large distance for the round trip, the travel time of the light is small, only 0.063 millisecond. This illustrates the great difficulty experimentalists faced in attempting to make an accurate measurement of c.

PRACTICE PROBLEM

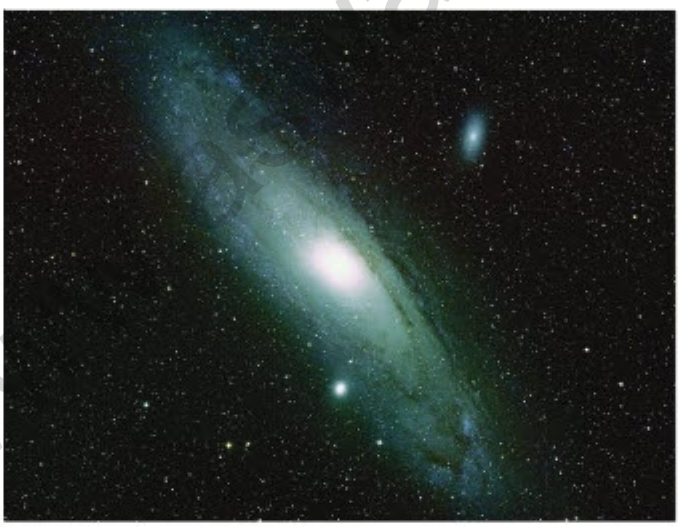
If the wheel has 430 notches, what rotational speed is required for the return beam of light to pass through the next notch? [**Answer:** $\omega = 37$ rev/s]

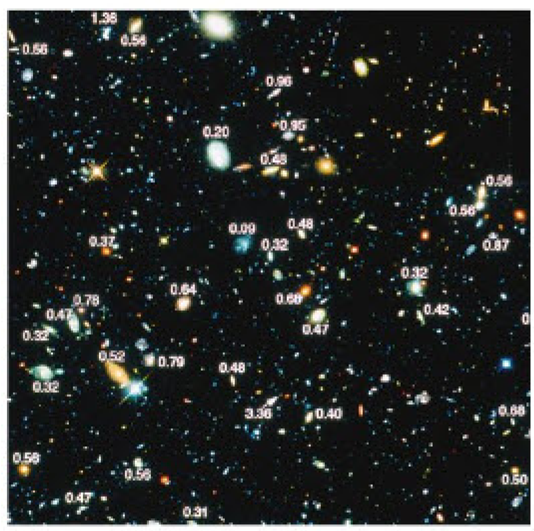
Some related homework problems: Problem 16, Problem 17

Although the speed of light is enormous by earthly standards, it is useful to look at it from an astronomical perspective. Imagine, for example, that you could shrink the solar system to fit onto a football field, with the Sun at one end zone and Pluto at the other. On this scale, Earth would be a grain of sand located at the 2.5-yard line from the Sun, and light would take 8 min to cover that distance. To travel to Pluto, at the other end of the field, light would require about 5.5 hr. Thus, on this scale, the speed of light is like the crawl of a small caterpillar. When one recalls that the solar system is but a speck on the outskirts of the Milky Way galaxy, and that the nearest major galaxy to our own—the Andromeda galaxy—is about 2.2 million light-years away, the speed of light doesn't appear so great after all.

The Doppler Effect

In Section 14–5 we discussed the Doppler effect for sound waves—the familiar increase or decrease in frequency as a source of sound approaches or recedes. A similar Doppler effect applies to electromagnetic waves. There are two fundamental differences, however. First, sound waves require a medium through which to travel, whereas light can propagate across a vacuum. Second, the speed of sound can be different for different observers. For example, an observer approaching a source of sound measures an increased speed of sound, whereas an observer detecting sound from a moving source measures the usual speed of sound. For





▲ Even traveling at 300 million meters per second, light from the Andromeda galaxy (left) takes over 2 million years to reach us. Yet Andromeda is one of our nearest cosmic neighbors. A sense of the true vastness of the universe is provided by the image known as the Hubble Deep Field (right). This long-exposure photograph, taken from orbit by the Hubble Space Telescope, shows over 1600 galaxies when examined closely. Most of them exhibit a Doppler red shift—that is, their light is shifted to lower frequencies by the Doppler effect, indicating that they are receding from Earth as the universe expands. The red shifts marked on the photo correspond to distances ranging from about 1.3 billion light-years to over 13 billion light-years (nearly 10^{23} miles).

this reason, the Doppler effect with sound is different for a moving observer than it is for a moving source (see Figure 14–18 for a direct comparison). In contrast, the speed of electromagnetic waves is *independent* of the motion of the source and observer, as we shall see in Chapter 29. Therefore, there is just one Doppler effect for electromagnetic waves, and it depends only on the *relative speed* between the observer and source.

For source speeds u that are small compared with the speed of light, the observed frequency f' from a source with frequency f is

$$f' = f\left(1 \pm \frac{u}{c}\right) \qquad 25\text{–}3$$

PROBLEM-SOLVING NOTE

Evaluating the Doppler Shift

Since everyday objects generally move with speeds much less than the speed of light, the Doppler-shifted frequency differs little from the original frequency. To see the Doppler effect more clearly, it is often useful to calculate the difference in frequency, $f' - f$, rather than the Doppler-shifted frequency itself.

Note that u in this expression is a speed and hence is always positive. The appropriate sign in front of the term u/c is chosen for a given situation—the plus sign applies to a source that is approaching the observer, the minus sign to a receding source. In addition, u is a *relative* speed between the source and the observer, both of which may be moving. For example, if an observer is moving in the positive x direction with a speed of 5 m/s, and a source ahead of the observer is moving in the positive x direction with a speed of 12 m/s, the relative speed is $u = 12\text{ m/s} - 5\text{ m/s} = 7\text{ m/s}$. Since the distance between the observer and source is increasing with time in this case, we would choose the minus sign in Equation 25–3.

EXERCISE 25–2

An FM radio station broadcasts at a frequency of 88.5 MHz. If you drive your car toward the station at 32.0 m/s, what change in frequency do you observe?

SOLUTION

We can find the change in frequency, $f' - f$, using Equation 25–3:

$$f' - f = f\frac{u}{c} = (88.5 \times 10^6\text{ Hz})\frac{32.0\text{ m/s}}{3.00 \times 10^8\text{ m/s}} = 9.44\text{ Hz}$$

Thus, the frequency changes by only 9.44 Hz = 0.00000944 MHz.

REAL-WORLD PHYSICS

Doppler radar

Common applications of the Doppler effect include the radar units used to measure the speed of automobiles, and the Doppler radar that is used to monitor the weather. In Doppler radar, electromagnetic waves are sent out into the atmosphere and are reflected back to the receiver. The change in frequency of the reflected beam relative to the outgoing beam provides a way of measuring the speed of the clouds and precipitation that reflected the beam. Thus, Doppler radar gives more information than just where a rainstorm is located; it also tells how it is moving. Measurements of this type are particularly important for airports, where information regarding areas of possible wind shear can be crucial for safety.

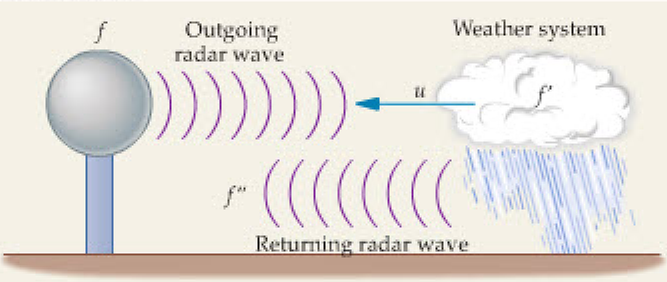
EXAMPLE 25–2 NEXRAD

REAL-WORLD PHYSICS

The Doppler weather radar used by the National Weather Service is referred to as Nexrad, which stands for next-generation radar. Nexrad commonly operates at a frequency of 2.7 GHz. If a Nexrad wave reflects from an approaching weather system moving with a speed of 28 m/s, find the difference in frequency between the outgoing and returning waves.

PICTURE THE PROBLEM

Our sketch shows the outgoing radar wave, the incoming weather system, and the returning radar wave. The speed of the weather system relative to the radar station is $u = 28$ m/s, and the frequency of the outgoing wave is $f = 2.7$ GHz.



STRATEGY

Two Doppler effects are involved in this system. First, the outgoing wave is seen to have a frequency $f' = f(1 + u/c)$ by the weather system, since it is moving toward the source. The waves reflected by the weather system, then, have the frequency f'. Since the weather system acts like a moving source of radar with frequency f', an observer at the radar facility detects a frequency $f'' = f'(1 + u/c)$. Thus, given u and f, we can calculate the difference, $f'' - f$.

SOLUTION

1. Use $f' = f(1 + u/c)$ to calculate the difference in frequency, $f' - f$:

$$f' - f = fu/c = \frac{(2.7 \times 10^9\ \text{Hz})(28\ \text{m/s})}{3.00 \times 10^8\ \text{m/s}} = 250\ \text{Hz}$$

2. Now, use $f'' = f'(1 + u/c)$ to find the difference between f'' and f':

$$f'' - f' = f'u/c$$

3. Use the results of Step 1 to replace f' with $f + 250$ Hz:

$$f'' - (f + 250\ \text{Hz}) = (f + 250\ \text{Hz})u/c$$

4. Solve for the frequency difference, $f'' - f$:

$$f'' - f = (f + 250\ \text{Hz})u/c + 250\ \text{Hz}$$
$$= \frac{(2.7 \times 10^9\ \text{Hz} + 250\ \text{Hz})(28\ \text{m/s})}{3.00 \times 10^8\ \text{m/s}} + 250\ \text{Hz}$$
$$= 500\ \text{Hz}$$

INSIGHT

Notice that we focus on the *difference* in frequency between the very large numbers $f = 2{,}700{,}000{,}000$ Hz and $f'' = 2{,}700{,}000{,}500$ Hz. Clearly, it is more convenient to simply write $f'' - f = 500$ Hz.

In addition, note that the two Doppler shifts in this problem are analogous to the two Doppler shifts we found for the case of a train approaching a tunnel in Example 14–6.

PRACTICE PROBLEM

Find the difference in frequency if the weather system is receding with a speed of 21 m/s. [**Answer:** $f'' - f = -380$ Hz]

Some related homework problems: Problem 23, Problem 25

25–3 The Electromagnetic Spectrum

When white light passes through a prism it spreads out into a rainbow of colors, with red on one end and violet on the other. All these various colors of light are electromagnetic waves, of course; they differ only in their frequency and, hence, their wavelength. The relationship between frequency and wavelength for any wave with a speed v is simply $v = f\lambda$, as was shown in Section 14–11. Because all electromagnetic waves in a vacuum have the same speed, c, it follows that f and λ are related as follows:

$$c = f\lambda \qquad 25\text{–}4$$

Thus, as the frequency of an electromagnetic wave increases, its wavelength decreases.

In the following Example we calculate the frequency of red and violet light, given the corresponding wavelengths. Notice that the wavelengths are given in units of nanometers (nm), where $1\ \text{nm} = 10^{-9}$ m. Occasionally the wavelength of light is given in terms of a non-SI unit referred to as the *angstrom* (Å), defined as follows: $1\text{Å} = 10^{-10}$ m.

EXAMPLE 25–3 ROSES ARE RED, VIOLETS ARE VIOLET

Find the frequency of red light, with a wavelength of 700.0 nm, and violet light, with a wavelength of 400.0 nm.

PICTURE THE PROBLEM

The visible electromagnetic spectrum, along with representative wavelengths, is shown in our diagram. In addition to the wavelengths of 700.0 nm for red light and 400.0 nm for violet light, we include 600.0 nm for yellowish orange light and 500.0 nm for greenish blue light.

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STRATEGY

We obtain the frequency by rearranging $c = f\lambda$ to yield $f = c/\lambda$.

SOLUTION

1. Substitute $\lambda = 700.0$ nm for red light: $$f = \frac{c}{\lambda} = \frac{3.00 \times 10^8 \text{ m/s}}{700.0 \times 10^{-9} \text{ m}} = 4.29 \times 10^{14} \text{ Hz}$$

2. Substitute $\lambda = 400.0$ nm for violet light: $$f = \frac{c}{\lambda} = \frac{3.00 \times 10^8 \text{ m/s}}{400.0 \times 10^{-9} \text{ m}} = 7.50 \times 10^{14} \text{ Hz}$$

INSIGHT

The frequency of visible light is extremely large. In fact, even for the relatively low frequency of red light, it takes only 2.33×10^{-15} s to complete one cycle. The *range* of visible frequencies is relatively small, however, when compared with other portions of the electromagnetic spectrum.

PRACTICE PROBLEM

What is the wavelength of light with a frequency of 5.25×10^{14} Hz? [**Answer:** 571 nm]

Some related homework problems: Problem 27, Problem 28

▲ Since the development of the first radiotelescopes in the 1950s, the radio portion of the electromagnetic spectrum has provided astronomers with a valuable new window on the universe. These antennas are part of the Very Large Array (VLA), located in San Augustin, New Mexico.

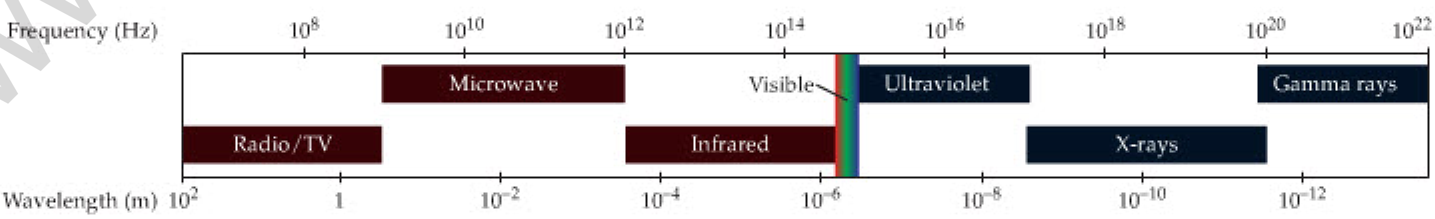
In principle, the frequency of an electromagnetic wave can have any positive value, and this full range of frequencies is known as the **electromagnetic spectrum.** Certain bands of the spectrum are given special names, as indicated in Figure 25–8. For example, we have just seen that visible light occupies a relatively narrow band of frequencies from 4.29×10^{14} Hz to 7.50×10^{14} Hz. In what follows, we discuss the various regions of the electromagnetic spectrum of most relevance to humans and our technology, in order of increasing frequency.

Radio Waves

($f \sim 10^6$ Hz to 10^9 Hz, $\lambda \sim$ 300 m to 0.3 m) The lowest-frequency electromagnetic waves of practical importance are *radio* and *television waves* in the frequency range of roughly 10^6 Hz to 10^9 Hz. Waves in this frequency range are produced in a variety of ways. For example, molecules and accelerated electrons in space give off radio waves, which radio astronomers detect with large dish receivers. Radio waves are also produced as a piece of adhesive tape is slowly peeled from a surface, as you can confirm by holding a transistor radio near the tape and listening for pops and snaps coming from the speaker. Most commonly, the radio waves we pick up with our radios and televisions are produced by alternating currents in metal antennas.

Microwaves

($f \sim 10^9$ Hz to 10^{12} Hz, $\lambda \sim$ 300 mm to 0.3 mm) Electromagnetic radiation with frequencies from 10^9 Hz to about 10^{12} Hz are referred to as *microwaves*. Waves in this frequency range are used to carry long-distance telephone conversations, as well as to cook our food. Microwaves, with wavelengths of about 1 mm to 30 cm,



▲ **FIGURE 25–8 The electromagnetic spectrum**

Note that the visible portion of the spectrum is relatively narrow. The boundaries between various bands of the spectrum are not sharp but, instead, are somewhat arbitrary.