2–1 Position, Distance, and Displacement

The first step in describing the motion of a particle is to set up a **coordinate system** that defines its position. An example of a coordinate system in one dimension is shown in Figure 2–1. This is simply an x axis, with an origin (where $x = 0$) and an arrow indicating the positive direction—the direction in which x increases. In setting up a coordinate system, we are free to choose the origin and the positive direction as we like, but once we make a choice we must be consistent with it throughout any calculations that follow.

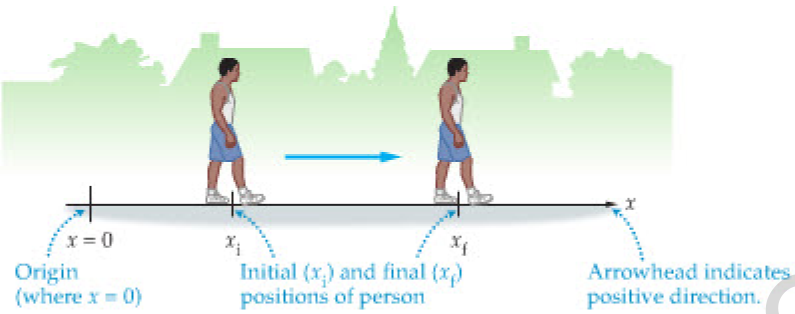


FIGURE 2–1 A one-dimensional coordinate system
You are free to choose the origin and positive direction as you like, but once your choice is made, stick with it.

The particle in Figure 2–1 is a person who has moved to the right from an initial position, x_i, to a final position, x_f. Because the positive direction is to the right, it follows that x_f is greater than x_i; that is, $x_f > x_i$.

Now that we've seen how to set up a coordinate system, let's use one to investigate the situation sketched in Figure 2–2. Suppose that you leave your house, drive to the grocery store, and then return home. The **distance** you've covered in your trip is 8.6 mi. In general, distance is defined as follows:

Definition: Distance

distance = total length of travel

SI unit: meter, m

Using SI units, the distance in this case is

$$8.6 \text{ mi} = (8.6 \text{ mi})\left(\frac{1609 \text{ m}}{1 \text{ mi}}\right) = 1.4 \times 10^4 \text{ m}$$

FIGURE 2–2 One-dimensional coordinates
The locations of your house, your friend's house, and the grocery store in terms of a one-dimensional coordinate system.

In a car, the distance traveled is indicated by the odometer. Note that distance is always positive and, because it has no direction associated with it, it is a scalar, as discussed in Chapter 1.

Another useful way to characterize a particle's motion is in terms of the **displacement**, Δx, which is simply the change in position.

Definition: Displacement, Δx

displacement = change in position = final position − initial position

$$\text{displacement} = \Delta x = x_f - x_i \qquad \text{2–1}$$

SI unit: meter, m

Notice that we use the delta notation, Δx, as a convenient shorthand for the quantity $x_f - x_i$. (See Appendix A for a complete discussion of delta notation.) Also, note that Δx can be positive (if the final position is to the right of the initial position, $x_f > x_i$), negative (if the final position is to the left of the initial position, $x_f < x_i$), or zero (if the final and initial positions are the same, $x_f = x_i$). In fact, the displacement is a one-dimensional vector, as defined in Chapter 1, and its direction (right or left) is given by its sign (positive or negative, respectively).

The SI units of displacement are meters—the same as for distance—but displacement and distance are really quite different. For example, in the round trip from your house to the grocery store and back the distance traveled is 8.6 mi, whereas the displacement is zero because $x_f = 2.1 \text{ mi} = x_i$. Suppose, instead, that you go from your house to the grocery store and then to your friend's house. On this trip the distance is 10.7 mi, but the displacement is

$$\Delta x = x_f - x_i = (0) - (2.1 \text{ mi}) = -2.1 \text{ mi}$$

As mentioned in the previous paragraph, the minus sign means your displacement is in the negative direction, that is, to the left.

ACTIVE EXAMPLE 2–1 FIND THE DISTANCE AND DISPLACEMENT

Calculate **(a)** the distance and **(b)** the displacement for a trip from your friend's house to the grocery store and then to your house.

SOLUTION *(Test your understanding by performing the calculations indicated in each step.)*

Part (a)

1. Add the distances for the various parts of the total trip: $2.1 \text{ mi} + 4.3 \text{ mi} + 4.3 \text{ mi} = 10.7 \text{ mi}$

Part (b)

2. Determine the initial position for the trip, using Figure 2–2: $x_i = 0$
3. Determine the final position for the trip, using Figure 2–2: $x_f = 2.1 \text{ mi}$
4. Subtract x_i from x_f to find the displacement: $\Delta x = 2.1 \text{ mi}$

YOUR TURN

Suppose we choose the origin in Figure 2–2 to be at your house, rather than at your friend's house. In this case, find **(a)** the distance and **(b)** the displacement for the trip from your friend's house to the grocery store and then to your house.

(Answers to **Your Turn** *problems are given in the back of the book.)*

2–2 Average Speed and Velocity

The next step in describing motion is to consider how rapidly an object moves. For example, how long does it take for a Randy Johnson fastball to reach home plate? How far does an orbiting space shuttle travel in one hour? How fast do your eyelids move when you blink? These are examples of some of the most basic questions regarding motion, and in this section we learn how to answer them.

The simplest way to characterize the rate of motion is with the **average speed:**

$$\text{average speed} = \frac{\text{distance}}{\text{elapsed time}} \qquad 2\text{–}2$$

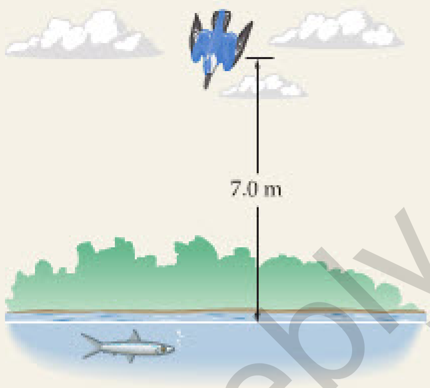
The dimensions of average speed are distance per time or, in SI units, meters per second, m/s. Both distance and elapsed time are positive; thus average speed is always positive.

EXAMPLE 2–1 THE KINGFISHER TAKES A PLUNGE

A kingfisher is a bird that catches fish by plunging into water from a height of several meters. If a kingfisher dives from a height of 7.0 m with an average speed of 4.00 m/s, how long does it take for it to reach the water?

PICTURE THE PROBLEM

As shown in the sketch, the kingfisher moves in a straight line through a vertical distance of 7.0 m. The average speed of the bird is 4.00 m/s.



STRATEGY

By rearranging Equation 2–2 we can solve for the elapsed time.

SOLUTION

1. Rearrange Equation 2–2 to solve for elapsed time:

$$\text{elapsed time} = \frac{\text{distance}}{\text{average speed}}$$

2. Substitute numerical values to find the time:

$$\text{elapsed time} = \frac{7.0\ \cancel{\text{m}}}{4.00\ \cancel{\text{m}}/\text{s}} = \frac{7.0}{4.00}\,\text{s} = 1.8\ \text{s}$$

INSIGHT

Note that Equation 2–2 is not just a formula for calculating the average speed. It relates speed, time, and distance. Given any two of these quantities, Equation 2–2 can be used to find the third.

PRACTICE PROBLEM

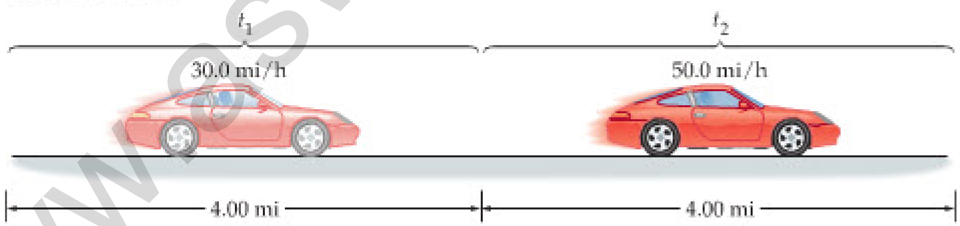
A kingfisher dives with an average speed of 4.6 m/s for 1.4 s. What was the height of the dive? [**Answer:** distance = (average speed) (elapsed time) = (4.6 m/s) (1.4 s) = 6.4 m]

Some related homework problems: Problem 13, Problem 15

Next, we calculate the average speed for a trip consisting of two parts of equal length, each traveled with a different speed.

CONCEPTUAL CHECKPOINT 2–1 AVERAGE SPEED

You drive 4.00 mi at 30.0 mi/h and then another 4.00 mi at 50.0 mi/h. Is your average speed for the 8.00-mi trip **(a)** greater than 40.0 mi/h, **(b)** equal to 40.0 mi/h, or **(c)** less than 40.0 mi/h?



REASONING AND DISCUSSION

At first glance it might seem that the average speed is definitely 40.0 mi/h. On further reflection, however, it is clear that it takes more time to travel 4.00 mi at 30.0 mi/h than it does to travel 4.00 mi at 50.0 mi/h. Therefore, you will be traveling at the lower speed for a greater period of time, and hence your average speed will be *less* than 40.0 mi/h—that is, closer to 30.0 mi/h than to 50.0 mi/h.

ANSWER

(c) The average speed is less than 40.0 mi/h.

To confirm the conclusion of the Conceptual Checkpoint, we simply apply the definition of average speed to find its value for this trip. We already know that the distance traveled is 8.00 mi; what we need now is the elapsed time. On the first 4.00 mi the time is

$$t_1 = \frac{4.00 \text{ mi}}{30.0 \text{ mi/h}} = (4.00/30.0) \text{ h}$$

The time required to cover the second 4.00 mi is

$$t_2 = \frac{4.00 \text{ mi}}{50.0 \text{ mi/h}} = (4.00/50.0) \text{ h}$$

Therefore, the elapsed time for the entire trip is

$$t_1 + t_2 = (4.00/30.0) \text{ h} + (4.00/50.0) \text{ h} = 0.213 \text{ h}$$

This gives the following average speed:

$$\text{average speed} = \frac{8.00 \text{ mi}}{0.213 \text{ h}} = 37.6 \text{ mi/h} < 40.0 \text{ mi/h}$$

Note that a "guess" will never give a detailed result like 37.6 mi/h; a systematic, step-by-step calculation is required.

In many situations, there is a quantity that is even more useful than the average speed. It is the **average velocity**, v_{av}, and it is defined as displacement per time:

Definition: Average velocity, v_{av}

$$\text{average velocity} = \frac{\text{displacement}}{\text{elapsed time}}$$

$$v_{av} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i} \qquad \text{2–3}$$

SI unit: meter per second, m/s

Not only does the average velocity tell us, on average, how fast something is moving, it also tells us the *direction* the object is moving. For example, if an object moves in the positive direction, then $x_f > x_i$, and $v_{av} > 0$. On the other hand, if an object moves in the negative direction, it follows that $x_f < x_i$, and $v_{av} < 0$. As with displacement, the average velocity is a one-dimensional vector, and its direction is given by its sign. Average velocity gives more information than average speed; hence it is used more frequently in physics.

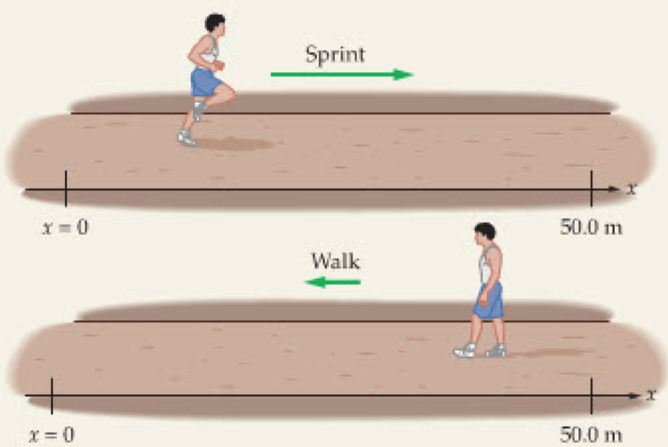
In the next Example, pay close attention to the positive and negative signs.

EXAMPLE 2–2 SPRINT TRAINING

An athlete sprints 50.0 m in 8.00 s, stops, and then walks slowly back to the starting line in 40.0 s. If the "sprint direction" is taken to be positive, what are **(a)** the average sprint velocity, **(b)** the average walking velocity, and **(c)** the average velocity for the complete round trip?

PICTURE THE PROBLEM

In our sketch we set up a coordinate system with the sprint going in the positive x direction, as described in the problem. For convenience, we choose the origin to be at the starting line. The finish line, then, is at $x = 50.0$ m.



STRATEGY

In each part of the problem we are asked for the average velocity and we are given information for times and distances. All that is needed, then, is to determine $\Delta x = x_f - x_i$ and $\Delta t = t_f - t_i$ in each case and apply Equation 2–3.

SOLUTION

Part (a)

1. Apply Equation 2–3 to the sprint, with $x_f = 50.0\text{ m}$, $x_i = 0$, $t_f = 8.00\text{ s}$, and $t_i = 0$:

$$v_{av} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i} = \frac{50.0\text{ m} - 0}{8.00\text{ s} - 0} = \frac{50.0}{8.00}\text{ m/s} = 6.25\text{ m/s}$$

Part (b)

2. Apply Equation 2–3 to the walk. In this case, $x_f = 0$, $x_i = 50.0\text{ m}$, $t_f = 48.0\text{ s}$, and $t_i = 8.00\text{ s}$:

$$v_{av} = \frac{x_f - x_i}{t_f - t_i} = \frac{0 - 50.0\text{ m}}{48.0\text{ s} - 8.00\text{ s}} = -\frac{50.0}{40.0}\text{ m/s} = -1.25\text{ m/s}$$

Part (c)

3. For the round trip, $x_f = x_i = 0$; thus $\Delta x = 0$:

$$v_{av} = \frac{\Delta x}{\Delta t} = \frac{0}{48.0\text{ s}} = 0$$

INSIGHT

Note that the sign of the velocities in parts (a) and (b) indicates the direction of motion; positive for motion to the right, negative for motion to the left. Also, notice that the average *speed* for the entire 100.0-m trip ($100.0\text{ m}/48.0\text{ s} = 2.08\text{ m/s}$) is nonzero, even though the average velocity vanishes.

PRACTICE PROBLEM

If the average velocity during the walk is -1.50 m/s, how long does it take the athlete to walk back to the starting line? [**Answer:** $\Delta t = \Delta x/v_{av} = (-50.0\text{ m})/(-1.50\text{ m/s}) = 33.3\text{ s}$]

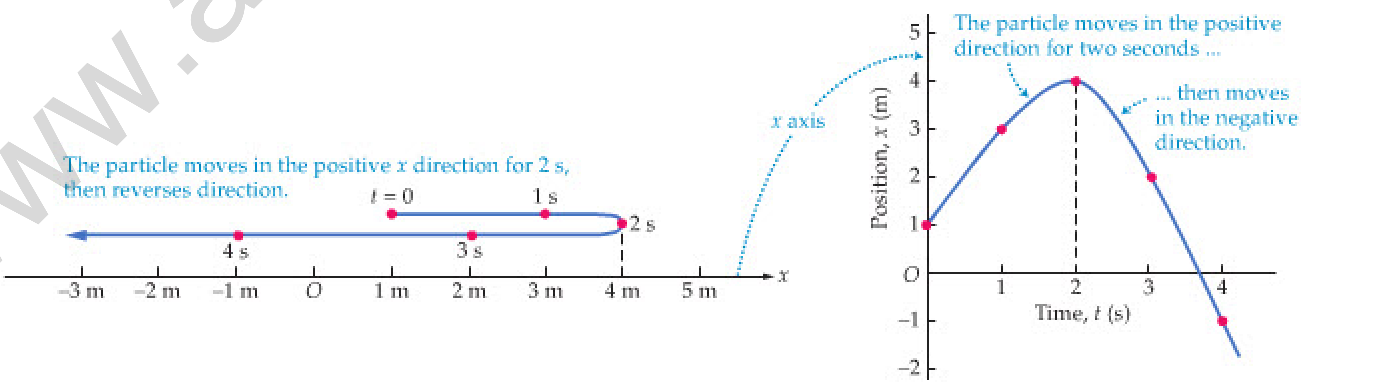
Some related homework problems: Problem 9, Problem 17, Problem 18

Graphical Interpretation of Average Velocity

It is often useful to "visualize" a particle's motion by sketching its position as a function of time. For example, consider a particle moving back and forth along the x axis, as shown in Figure 2–3 (a). In this plot, we have indicated the position of a particle at a variety of times.

This way of keeping track of a particle's position and the corresponding time is a bit messy, though, so let's replot the same information with a different type of graph. In Figure 2–3 (b) we again plot the motion shown in Figure 2–3 (a), but this time with the vertical axis representing the position, x, and the horizontal axis representing time, t. An **x-versus-t graph** like this makes it considerably easier to visualize a particle's motion.

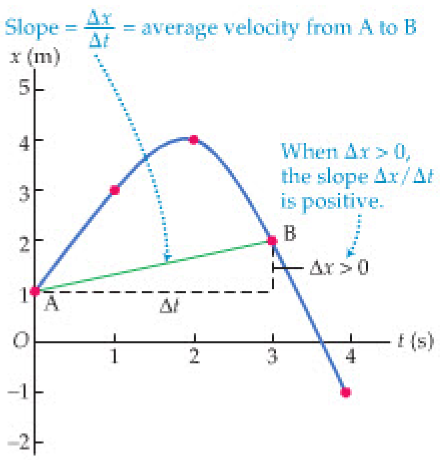
An x-versus-t plot also leads to a particularly useful interpretation of average velocity. To see how, suppose you would like to know the average velocity of the particle in Figures 2–3 (a) and 2–3 (b) from $t = 0$ to $t = 3\text{ s}$. From our definition of average velocity in Equation 2–3, we know that $v_{av} = \Delta x/\Delta t = (2\text{ m} - 1\text{ m})/(3\text{ s} - 0) = +0.3\text{ m/s}$. To relate this to the



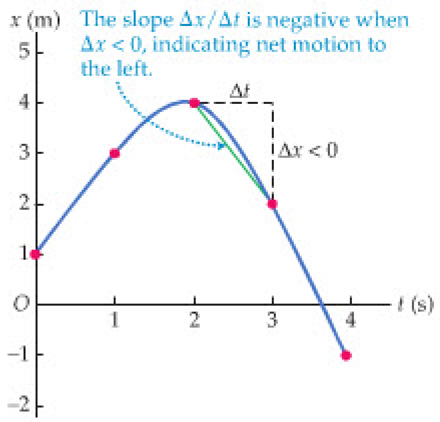
(a) The particle's path shown on a coordinate axis

(b) The same path as a graph of position x versus time t

▲ **FIGURE 2–3 Two ways to visualize one-dimensional motion**
Although the path in (a) is shown as a "U" for clarity, the particle actually moves straight back and forth along the x axis.



(a) Average velocity between $t = 0$ and $t = 3$ s



(b) Average velocity between $t = 2$ s and $t = 3$ s

▲ **FIGURE 2–4 Average velocity on an *x*-versus-*t* graph**

The slope of a straight line between any two points on an x-versus-t graph equals the average velocity between those points. Positive slopes indicate net motion to the right; negative slopes indicate net motion to the left.

▲ A speedometer indicates the instantaneous speed of a car. Note that the speedometer gives no information about the *direction* of motion. Thus, the speedometer is truly a "speed meter," not a velocity meter.

x-versus-t plot, draw a straight line connecting the position at $t = 0$ (call this point A) and the position at $t = 3$ s (point B). The result is shown in **Figure 2–4 (a)**.

The slope of the straight line from A to B is equal to the rise over the run, which in this case is $\Delta x/\Delta t$. But $\Delta x/\Delta t$ is the average velocity. Thus we see that:

- The slope of a line connecting two points on an x-versus-t plot is equal to the average velocity during that time interval.

As an additional example, let's calculate the average velocity between times $t = 2$ s and $t = 3$ s in Figure 2–3 (b). A line connecting the corresponding points is shown in **Figure 2–4 (b)**.

The first thing we notice about this line is that it has a negative slope; thus $v_{av} < 0$ and the particle is moving to the left. We also note that it is inclined more steeply than the line in Figure 2–4 (a), hence the magnitude of its slope is greater. In fact, if we calculate the slope of this line we find that $v_{av} = -2$ m/s for this time interval.

Thus, connecting points on an x-versus-t plot gives an immediate "feeling" for the average velocity over a given time interval. This type of graphical analysis will be particularly useful in the next section.

2–3 Instantaneous Velocity

Though average velocity is a useful way to characterize motion, it can miss a lot. For example, suppose you travel by car on a long, straight highway, covering 92 mi in 2.0 hours. Your average velocity is 46 mi/h. Even so, there may have been only a few times during the trip when you were actually driving at 46 mi/h. You may have sped along at 65 mi/h during most of the time, except when you stopped to have a bite to eat at a roadside diner, during which time your average velocity was zero.

To have a more accurate representation of your trip, you should average your velocity over shorter periods of time. If you calculate your average velocity every 15 minutes, you have a better picture of what the trip was like. An even better, more realistic picture of the trip is obtained if you calculate the average velocity every minute or every second. Ideally, when dealing with the motion of any particle, it is desirable to know the velocity of the particle at each instant of time.

This idea of a velocity corresponding to an instant of time is just what is meant by the **instantaneous velocity.** Mathematically, we define the instantaneous velocity as follows:

Definition: Instantaneous Velocity, *v*

$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} \qquad 2\text{–}4$$

SI unit: meter per second, m/s

In this expression the notation $\lim_{\Delta t \to 0}$ means "evaluate the average velocity, $\Delta x/\Delta t$, over shorter and shorter time intervals, approaching zero in the limit." Note that the instantaneous velocity can be positive, negative, or zero, just like the average velocity—and just like the average velocity, the instantaneous velocity is a one-dimensional vector. The magnitude of the instantaneous velocity is called the **instantaneous speed.** In a car, the speedometer gives a reading of the vehicle's instantaneous speed.

As Δt becomes smaller, Δx becomes smaller as well, but the ratio $\Delta x/\Delta t$ approaches a constant value. To see how this works, consider first the simple case of a particle moving with a constant velocity of +1 m/s. If the particle starts at $x = 0$ at $t = 0$, then its position at $t = 1$ s is $x = 1$ m, its position at $t = 2$ s is $x = 2$ m, and so on. Plotting this motion in an x-versus-t plot gives a straight line, as shown in **Figure 2–5**.

Now, suppose we want to find the instantaneous velocity at $t = 3$ s. To do so, we calculate the average velocity over small intervals of time centered at 3 s, and let the time intervals become arbitrarily small, as shown in the Figure. Since x-versus-t is a straight line, it is clear that $\Delta x/\Delta t = \Delta x_1/\Delta t_1$, no matter how small the time interval Δt. As Δt becomes smaller, so does Δx, but the ratio $\Delta x/\Delta t$ is simply the slope of the line, 1 m/s. Thus, the instantaneous velocity at $t = 3$ s is 1 m/s.

Of course, in this example the instantaneous velocity is 1 m/s for any instant of time, not just $t = 3$ s. Therefore:

- When velocity is constant, the average velocity over any time interval is equal to the instantaneous velocity at any time.

In general, a particle's velocity varies with time, and the x-versus-t plot is not a straight line. An example is shown in Figure 2–6, with the corresponding numerical values of x and t given in Table 2–1.

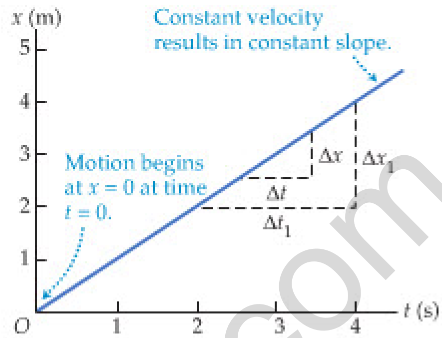


FIGURE 2–5 Constant velocity corresponds to constant slope on an x-versus-t graph

The slope $\Delta x_1/\Delta t_1$ is equal to $(4\text{ m} - 2\text{ m})/(4\text{ s} - 2\text{ s}) = (2\text{ m})/(2\text{ s}) = 1$ m/s. Because x-versus-t is a straight line, the slope $\Delta x/\Delta t$ is also equal to 1 m/s for any value of Δt.

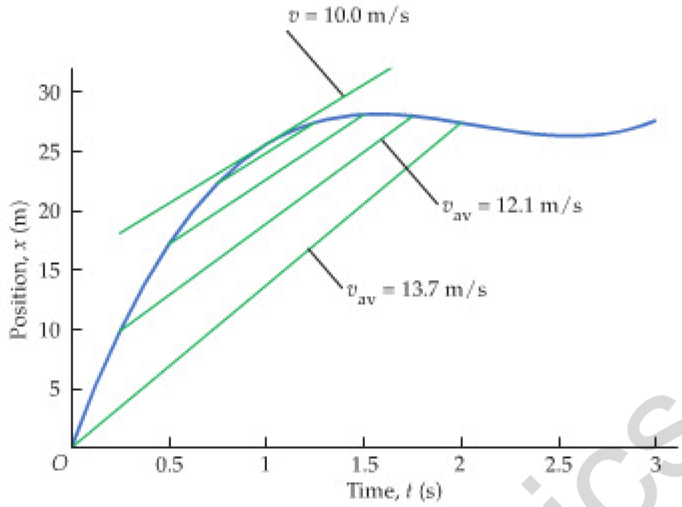


FIGURE 2–6 Instantaneous velocity

An x-versus-t plot for motion with variable velocity. The instantaneous velocity at $t = 1$ s is equal to the slope of the tangent line at that time. The average velocity for a small time interval centered on $t = 1$ s approaches the instantaneous velocity at $t = 1$ s as the time interval goes to zero.

In this case, what is the instantaneous velocity at, say, $t = 1.00$ s? As a first approximation, let's calculate the average velocity for the time interval from $t_i = 0$ to $t_f = 2.00$ s. Note that this time interval is centered at $t = 1.00$ s. From Table 2–1 we see that $x_i = 0$ and $x_f = 27.4$ m, thus $v_{av} = 13.7$ m/s. The corresponding straight line connecting these two points is the lowest straight line in Figure 2–6.

The next three lines, in upward progression, refer to time intervals from 0.250 s to 1.75 s, 0.500 s to 1.50 s, and 0.750 s to 1.25 s, respectively. The corresponding average velocities, given in Table 2–2, are 12.1 m/s, 10.9 m/s, and 10.2 m/s. Table 2–2 also gives results for even smaller time intervals. In particular, for the interval from 0.900 s to 1.10 s the average velocity is 10.0 m/s. Smaller intervals also give 10.0 m/s. Thus, we can conclude that the instantaneous velocity at $t = 1.00$ s is $v = 10.0$ m/s.

The uppermost straight line in Figure 2–6 is the tangent line to the x-versus-t curve at the time $t = 1.00$ s; that is, it is the line that touches the curve at just a single point. Its slope is 10.0 m/s. Clearly, the average-velocity lines have slopes that

TABLE 2–1
x-versus-t Values for Figure 2–6

t (s)	x (m)
0	0
0.25	9.85
0.50	17.2
0.75	22.3
1.00	25.6
1.25	27.4
1.50	28.1
1.75	28.0
2.00	27.4

TABLE 2–2 Calculating the Instantaneous Velocity at $t = 1$ s

t_i (s)	t_f (s)	Δt (s)	x_i (m)	x_f (m)	Δx (m)	$v_{av} = \Delta x/\Delta t$ (m/s)
0	2.00	2.00	0	27.4	27.4	13.7
0.250	1.75	1.50	9.85	28.0	18.2	12.1
0.500	1.50	1.00	17.2	28.1	10.9	10.9
0.750	1.25	0.50	22.3	27.4	5.10	10.2
0.900	1.10	0.20	24.5	26.5	2.00	10.0
0.950	1.05	0.10	25.1	26.1	1.00	10.0

approach the slope of the tangent line as the time intervals become smaller. This is an example of the following general result:

- The instantaneous velocity at a given time is equal to the slope of the tangent line at that point on an x-versus-t graph.

Thus, a visual inspection of an x-versus-t graph gives information not only about the location of a particle, but also about its velocity.

CONCEPTUAL CHECKPOINT 2–2 INSTANTANEOUS VELOCITY

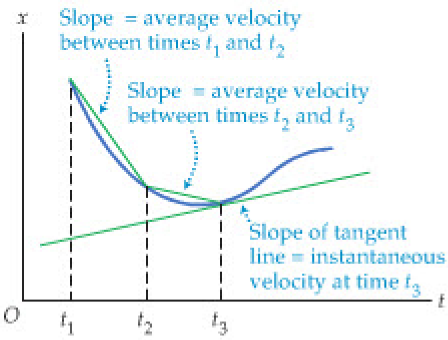
Referring to Figure 2–6, is the instantaneous velocity at $t = 0.500$ s **(a)** greater than, **(b)** less than, or **(c)** the same as the instantaneous velocity at $t = 1.00$ s?

REASONING AND DISCUSSION

From the x-versus-t graph in Figure 2–6 it is clear that the slope of a tangent line drawn at $t = 0.500$ s is greater than the slope of the tangent line at $t = 1.00$ s. It follows that the particle's velocity at 0.500 s is greater than its velocity at 1.00 s.

ANSWER

(a) The instantaneous velocity is greater at $t = 0.500$ s.



▲ **FIGURE 2–7 Graphical interpretation of average and instantaneous velocity**

Average velocities correspond to the slope of straight-line segments connecting different points on an x-versus-t graph. Instantaneous velocities are given by the slope of the tangent line at a given time.

In the remainder of the book, when we say velocity it is to be understood that we mean *instantaneous* velocity. If we want to refer to the average velocity, we will specifically say average velocity.

Graphical Interpretation of Average and Instantaneous Velocity

Let's summarize the graphical interpretations of average and instantaneous velocity on an x-versus-t graph:

- Average velocity is the slope of the straight line connecting two points corresponding to a given time interval.
- Instantaneous velocity is the slope of the tangent line at a given instant of time.

These relations are illustrated in **Figure 2–7**.

2–4 Acceleration

Just as velocity is the rate of change of *displacement* with time, **acceleration** is the rate of change of *velocity* with time. Thus, an object accelerates whenever its velocity *changes*, no matter what the change—it accelerates when its velocity increases, it accelerates when its velocity decreases. Of all the concepts discussed in this chapter, perhaps none is more central to physics than acceleration. Galileo, for example, showed that falling bodies move with constant acceleration. Newton showed that acceleration and force are directly related, as we shall see in Chapter 5. Thus, it is particularly important to have a clear, complete understanding of acceleration before leaving this chapter.

We begin, then, with the definition of **average acceleration:**

Definition: Average Acceleration, a_{av}

$$a_{av} = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{t_f - t_i} \qquad \text{2–5}$$

SI unit: meter per second per second, m/s^2

▲ The space shuttle *Discovery* accelerates upward on the initial phase of its journey into orbit. During this time the astronauts on board the shuttle experience an approximately linear acceleration that may be as great as 20 m/s^2.

Note that the dimensions of average acceleration are the dimensions of velocity per time, or (meters per second) per second:

$$\frac{\text{meters per second}}{\text{second}} = \frac{m/s}{s} = \frac{m}{s^2}$$

This is generally expressed as meters per second squared. For example, the acceleration of gravity on the Earth's surface is approximately 9.81 m/s^2, which means that the velocity of a falling object changes by 9.81 meters per second (m/s) every second (s). In addition, we see that the average acceleration can be positive, negative, or zero. In fact, it is a one-dimensional vector, just like displacement, average velocity, and instantaneous velocity. Typical magnitudes of acceleration are given in Table 2–3.

TABLE 2–3 Typical Accelerations (m/s^2)

Ultracentrifuge	3×10^6
Bullet fired from a rifle	4.4×10^5
Batted baseball	3×10^4
Click beetle righting itself	400
Acceleration required to deploy airbags	60
Bungee jump	30
High jump	15
Acceleration of gravity on Earth	9.81
Emergency stop in a car	8
Airplane during takeoff	5
An elevator	3
Acceleration of gravity on the Moon	1.62

EXERCISE 2–1

a. Saab advertises a car that goes from 0 to 60.0 mi/h in 6.2 s. What is the average acceleration of this car?

b. An airplane has an average acceleration of $5.6\ m/s^2$ during takeoff. How long does it take for the plane to reach a speed of 150 mi/h?

SOLUTION

a. average acceleration $= a_{av} = (60.0\ mi/h)/(6.2\ s)$

$= (26.8\ m/s)/(6.2\ s) = 4.3\ m/s^2$

b. $\Delta t = \Delta v/a_{av} = (150\ mi/h)/(5.6\ m/s^2) = (67.0\ m/s)/(5.6\ m/s^2) = 12\ s$

Next, just as we considered the limit of smaller and smaller time intervals to find an instantaneous velocity, we can do the same to define an **instantaneous acceleration:**

Definition: Instantaneous Acceleration, a

$$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} \qquad 2\text{–}6$$

SI unit: meter per second per second, m/s^2

As you might expect, the instantaneous acceleration is a one-dimensional vector, just like the average acceleration, and its direction is given by its sign. For simplicity, when we say acceleration in the future we are referring to the instantaneous acceleration.

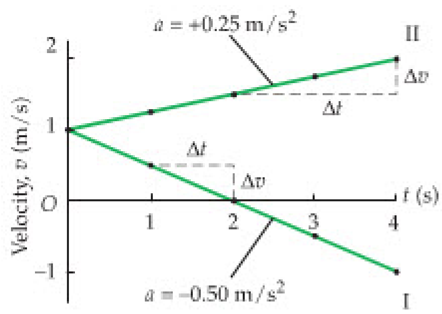
One final note before we go on to some examples. If the acceleration is constant, it has the same value at all times. Therefore:

- When acceleration is constant, the instantaneous and average accelerations are the same.

We shall make use of this fact when we return to the special case of constant acceleration in the next section.

Graphical Interpretation of Acceleration

To see how acceleration can be interpreted graphically, suppose that a particle has a constant acceleration of $-0.50\ m/s^2$. This means that the velocity of the particle *decreases* by 0.50 m/s each second. Thus, if its velocity is 1.0 m/s at $t = 0$, then at $t = 1$ s its velocity is 0.50 m/s, at $t = 2$ s its velocity is 0, at $t = 3$ s its velocity is -0.50 m/s, and so on. This is illustrated by curve I in **Figure 2–8**, where we see that a plot of v-versus-t results in a straight line with a negative slope. Curve II in Figure 2–8 has a positive slope, corresponding to a constant acceleration of $+0.25\ m/s^2$. Thus, in terms of a v-versus-t plot, a constant acceleration results in a straight line with a slope equal to the acceleration.



▲ **FIGURE 2–8 *v*-versus-*t* plots for motion with constant acceleration**
Curve I represents the movement of a particle with constant acceleration $a = -0.50\ m/s^2$. Curve II represents the motion of a particle with constant acceleration $a = +0.25\ m/s^2$.

CONCEPTUAL CHECKPOINT 2–3 SPEED AS A FUNCTION OF TIME

The speed of a particle with the v-versus-t graph shown by curve II in Figure 2–8 increases steadily with time. Consider, instead, a particle whose v-versus-t graph is given by curve I in Figure 2–8. As a function of time, does the speed of this particle **(a)** increase, **(b)** decrease, or **(c)** decrease and then increase?

REASONING AND DISCUSSION

Recall that speed is the *magnitude* of velocity. In curve I of Figure 2–8 the speed starts out at 1.0 m/s, then *decreases* to 0 at $t = 2$ s. After $t = 2$ s the speed *increases* again. For example, at $t = 3$ s the speed is 0.50 m/s, and at $t = 4$ s the speed is 1 m/s.

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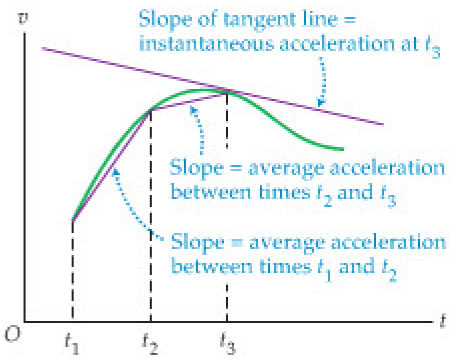


▲ **FIGURE 2–9 Graphical interpretation of average and instantaneous acceleration**
Average accelerations correspond to the slope of straight-line segments connecting different points on a v-versus-t graph. Instantaneous accelerations are given by the slope of the tangent line at a given time.

CONTINUED FROM PREVIOUS PAGE

Did you realize that the particle represented by curve I in Figure 2–8 changes direction at $t = 2$ s? It certainly does. Before $t = 2$ s the particle moves in the positive direction; after $t = 2$ s it moves in the negative direction. At precisely $t = 2$ s the particle is momentarily at rest. However, regardless of whether the particle is moving in the positive direction, moving in the negative direction, or instantaneously at rest, it still has the same constant acceleration. Acceleration has to do only with the way the velocity is *changing* at a given moment.

ANSWER

(c) The speed decreases and then increases.

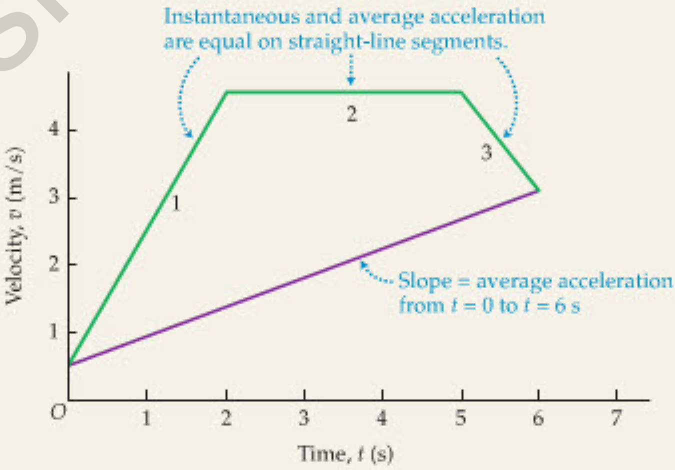
The graphical interpretations for velocity presented in Figure 2–7 apply equally well to acceleration, with just one small change: Instead of an x-versus-t graph, we use a v-versus-t graph, as in **Figure 2–9**. Thus, the average acceleration in a v-versus-t plot is the slope of a straight line connecting points corresponding to two different times. Similarly, the instantaneous acceleration is the slope of the tangent line at a particular time.

EXAMPLE 2–3 AN ACCELERATING TRAIN

A train moving in a straight line with an initial velocity of 0.50 m/s accelerates at 2.0 m/s^2 for 2.0 seconds, coasts with zero acceleration for 3.0 seconds, and then accelerates at -1.5 m/s^2 for 1.0 second. **(a)** What is the final velocity of the train? **(b)** What is the average acceleration of the train?

PICTURE THE PROBLEM

We begin by sketching a v-versus-t plot for the train. The basic idea is that each interval of constant acceleration is represented by a straight line of the appropriate slope. Therefore, we draw a straight line with the slope 2.0 m/s^2 from $t = 0$ to $t = 2.0$ s, a line with zero slope from $t = 2.0$ s to $t = 5.0$ s, and a line with the slope -1.5 m/s^2 from $t = 5.0$ s to $t = 6.0$ s. The line connecting the initial and final points determines the average acceleration.



STRATEGY

During each period of constant acceleration the change in velocity is $\Delta v = a_{av}\Delta t = a\Delta t$.

a. Adding the individual changes in velocity gives the total change, $\Delta v = v_f - v_i$. Since v_i is known, this expression can be solved for the final velocity, v_f.

b. The average acceleration can be calculated using Equation 2–5, $a_{av} = \Delta v/\Delta t$. Note that Δv has been obtained in part (a), and that the total time interval is $\Delta t = 6.0$ s, as is clear from the graph.

SOLUTION

Part (a)

1. Find the change in velocity during each of the three periods of constant acceleration:

$$\Delta v_1 = a_1\Delta t_1 = (2.0\ \text{m/s}^2)(2.0\ \text{s}) = 4.0\ \text{m/s}$$
$$\Delta v_2 = a_2\Delta t_2 = (0)(3.0\ \text{s}) = 0$$
$$\Delta v_3 = a_3\Delta t_3 = (-1.5\ \text{m/s}^2)(1.0\ \text{s}) = -1.5\ \text{m/s}$$

2. Sum the change in velocity for each period to obtain the total Δv:

$$\Delta v = \Delta v_1 + \Delta v_2 + \Delta v_3 = 4.0\ \text{m/s} + 0 - 1.5\ \text{m/s} = 2.5\ \text{m/s}$$

3. Use Δv to find v_f, recalling that $v_i = 0.50$ m/s:

$$\Delta v = v_f - v_i$$
$$v_f = \Delta v + v_i = 2.5\ \text{m/s} + 0.50\ \text{m/s} = 3.0\ \text{m/s}$$

Part (b)

4. The average acceleration is $\Delta v/\Delta t$:
$$a_{av} = \frac{\Delta v}{\Delta t} = \frac{2.5 \text{ m/s}}{6.0 \text{ s}} = 0.42 \text{ m/s}^2$$

INSIGHT

Note that the average acceleration for these six seconds is not simply the average of the individual accelerations, 2.0 m/s^2, 0 m/s^2, and -1.5 m/s^2. The reason is that different amounts of time are spent with each acceleration. In addition, the average acceleration can be found graphically, as indicated in the v-versus-t sketch on the previous page. Specifically, the graph shows that Δv is 2.5 m/s for the time interval from $t = 0$ to $t = 6.0$ s.

PRACTICE PROBLEM

What is the average acceleration of the train between $t = 2.0$ s and $t = 6.0$ s?
[**Answer:** $a_{av} = \Delta v/\Delta t = (3.0 \text{ m/s} - 4.5 \text{ m/s})/(6.0 \text{ s} - 2.0 \text{ s}) = -0.38 \text{ m/s}^2$]

Some related homework problems: Problem 36, Problem 38

In one dimension, nonzero velocities and accelerations are either positive or negative, depending on whether they point in the positive or negative direction of the coordinate system chosen. Thus, the velocity and acceleration of an object may have the same or opposite signs. (Of course, in two or three dimensions the relationship between velocity and acceleration can be much more varied, as we shall see in the next several chapters.) This leads to the following two possibilities:

- When the velocity and acceleration of an object have the same sign, the speed of the object increases. In this case, the velocity and acceleration point in the same direction.
- When the velocity and acceleration of an object have opposite signs, the speed of the object decreases. In this case, the velocity and acceleration point in opposite directions.

These two possibilities are illustrated in Figure 2–10. Notice that when a particle's speed increases, it means either that its velocity becomes more positive, as in Figure 2–10 (a), or more negative, as in Figure 2–10 (d). In either case, it is the magnitude of the velocity—the speed—that increases.

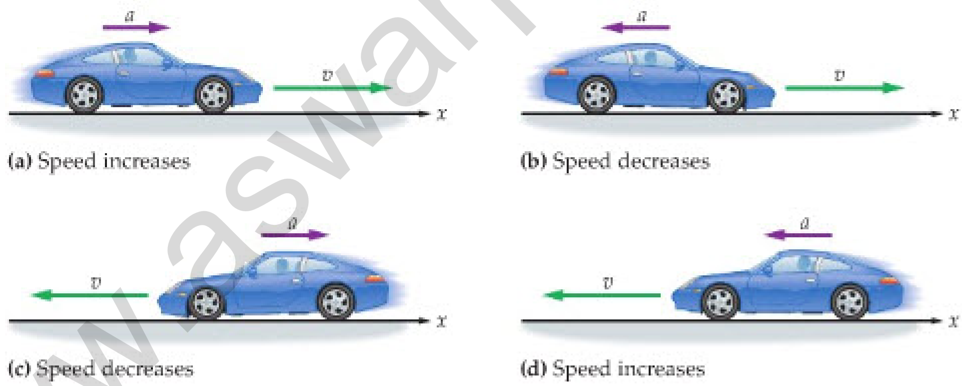
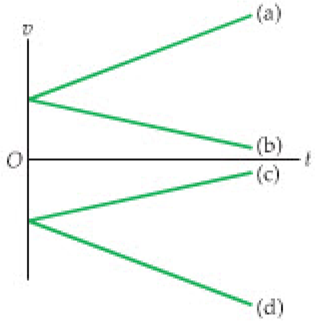


FIGURE 2–10 Cars accelerating or decelerating
A car's speed increases when its velocity and acceleration point in the same direction, as in cases **(a)** and **(d).** When the velocity and acceleration point in opposite directions, as in cases **(b)** and **(c),** the car's speed decreases.

The winner of this race was traveling at a speed of 313.91 mi/h at the end of the quarter-mile course. Since the winning time was just 4.607 s, the *average* acceleration during this race was approximately three times the acceleration of gravity (Section 2–7).



▲ **FIGURE 2–11** ***v*-versus-*t* plots with constant acceleration**

Four plots of v versus t corresponding to the four situations shown in Figure 2–10. Note that the speed increases in cases **(a)** and **(d)**, but decreases in cases **(b)** and **(c)**.

When a particle's speed decreases, it is often said to be *decelerating*. A common misconception is that deceleration implies a negative acceleration. This is not true. Deceleration can be caused by a positive or a negative acceleration, depending on the direction of the initial velocity. For example, the car in **Figure 2–10 (b)** has a positive velocity and a negative acceleration, while the car in **Figure 2–10 (c)** has a negative velocity and a positive acceleration. In both cases, the speed of the car decreases. Again, all that is required for deceleration in one dimension is that the velocity and acceleration have *opposite signs*; that is, they must point in *opposite directions*, as in parts (b) and (c) of Figure 2–10.

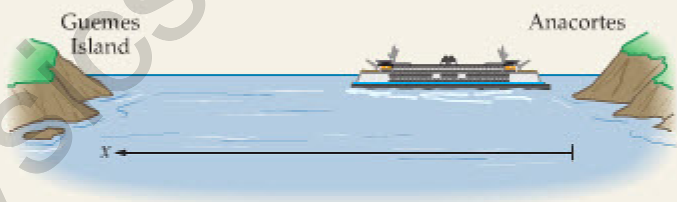
Velocity-versus-time plots for the four situations shown in Figure 2–10 are presented in **Figure 2–11**. In each of the four plots in Figure 2–11 we assume constant acceleration. Be sure to understand clearly the connection between the v-versus-t plots in Figure 2–11 and the corresponding physical motions indicated in Figure 2–10.

EXAMPLE 2–4 THE FERRY DOCKS

A ferry makes a short run between two docks; one in Anacortes, Washington, the other on Guemes Island. As the ferry approaches Guemes Island (traveling in the positive x direction), its speed is 7.4 m/s. **(a)** If the ferry slows to a stop in 12.3 s, what is its average acceleration? **(b)** As the ferry returns to the Anacortes dock, its speed is 7.3 m/s. If it comes to rest in 13.1 s, what is its average acceleration?

PICTURE THE PROBLEM

Our sketch shows the locations of the two docks and the positive direction indicated in the problem. Note that the distance between docks is not given, nor is it needed.



STRATEGY

We are given the initial and final velocities (the ferry comes to a stop in each case, so its final speed is zero) and the relevant times. Therefore, we can find the average acceleration using $a_{av} = \Delta v/\Delta t$, being careful to get the signs right.

SOLUTION

Part (a)

1. Calculate the average acceleration, noting that $v_i = 7.4$ m/s and $v_f = 0$:

$$a_{av} = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{\Delta t} = \frac{0 - 7.4\text{ m/s}}{12.3\text{ s}} = -0.60\text{ m/s}^2$$

Part (b)

2. In this case, $v_i = -7.3$ m/s and $v_f = 0$:

$$a_{av} = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{\Delta t} = \frac{0 - (-7.3\text{ m/s})}{13.1\text{ s}} = 0.56\text{ m/s}^2$$

INSIGHT

In each case, the acceleration of the ferry is opposite in sign to its velocity; therefore the ferry decelerates.

PRACTICE PROBLEM

When the ferry leaves Guemes Island, its speed increases from 0 to 5.8 m/s in 9.25 s. What is its average acceleration? [**Answer:** $a_{av} = -0.63\text{ m/s}^2$]

Some related homework problems: Problem 34, Problem 35

2–5 Motion with Constant Acceleration

In this section, we derive equations describing the motion of particles moving with **constant acceleration.** These "equations of motion" can be used to describe a wide range of everyday phenomena. For example, in an idealized world with no air resistance, falling bodies have constant acceleration.

As mentioned in the previous section, if a particle has constant acceleration—that is, the same acceleration at every instant of time—then its instantaneous acceleration, a, is equal to its average acceleration, a_{av}. Recalling the definition of average acceleration, Equation 2–5, we have

$$a_{av} = \frac{v_f - v_i}{t_f - t_i} = a$$

where the initial and final times may be chosen arbitrarily. For example, let $t_i = 0$ for the initial time, and let $v_i = v_0$ denote the velocity at time zero. For the final time and velocity we drop the subscripts to simplify notation; thus we let $t_f = t$ and $v_f = v$. With these identifications we have

$$a_{av} = \frac{v - v_0}{t - 0} = a$$

Therefore,

$$v - v_0 = a(t - 0) = at$$

or

Constant-Acceleration Equation of Motion: Velocity as a Function of Time

$$v = v_0 + at \qquad 2\text{–}7$$

Note that Equation 2–7 describes a straight line on a v-versus-t plot. The line crosses the velocity axis at the value v_0 and has a slope a, in agreement with the graphical interpretations discussed in the previous section. For example, in curve I of Figure 2–8, the equation of motion is $v = v_0 + at = (1 \text{ m/s}) + (-0.5 \text{ m/s}^2)t$. Also, note that $(-0.5 \text{ m/s}^2)t$ has the units $(\text{m/s}^2)\ (\text{s}) = \text{m/s}$; thus each term in Equation 2–7 has the same dimensions (as it must to be a valid physical equation).

EXERCISE 2–2

A ball is thrown straight upward with an initial velocity of $+8.2$ m/s. If the acceleration of the ball is -9.81 m/s^2, what is its velocity after

a. 0.50 s, and **b.** 1.0 s?

SOLUTION

a. Substituting $t = 0.50$ s in Equation 2–7 yields

$$v = 8.2 \text{ m/s} + (-9.81 \text{ m/s}^2)(0.50 \text{ s}) = 3.3 \text{ m/s}$$

b. Similarly, using $t = 1.0$ s in Equation 2–7 gives

$$v = 8.2 \text{ m/s} + (-9.81 \text{ m/s}^2)(1.0 \text{ s}) = -1.6 \text{ m/s}$$

Next, how far does a particle move in a given time if its acceleration is constant? To answer this question, recall the definition of average velocity:

$$v_{av} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i}$$

Using the same identifications given previously for initial and final times, and letting $x_i = x_0$ and $x_f = x$, we have

$$v_{av} = \frac{x - x_0}{t - 0}$$

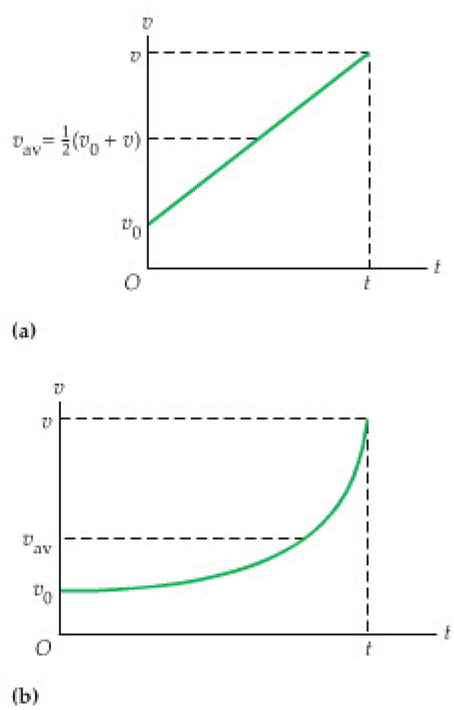
Thus,

$$x - x_0 = v_{av}(t - 0) = v_{av}t$$

or

$$x = x_0 + v_{av}t \qquad 2\text{–}8$$

Now, Equation 2–8 is fine as it is. In fact, it applies whether the acceleration is constant or not. A more useful expression, for the case of constant acceleration, is obtained by writing v_{av} in terms of the initial and final velocities. This can be done by referring to **Figure 2–12 (a)**. Here the velocity changes linearly (since a is



▲ **FIGURE 2–12 The average velocity**
(a) When acceleration is constant, the velocity varies linearly with time. As a result, the average velocity, v_{av}, is simply the average of the initial velocity, v_0, and the final velocity, v. **(b)** The velocity curve for nonconstant acceleration is nonlinear. In this case, the average velocity is no longer midway between the initial and final velocities.

constant) from v_0 at $t = 0$ to v at some later time t. The average velocity during this period of time is simply the average of the initial and final velocities; that is, the sum of the two velocities divided by two:

Constant-Acceleration Equation of Motion: Average Velocity

$$v_{av} = \tfrac{1}{2}(v_0 + v) \qquad 2\text{–}9$$

The average velocity is indicated in the figure. Note that if the acceleration is not constant, as in **Figure 2–12 (b)**, this simple averaging of initial and final velocities is no longer valid.

Substituting the expression for v_{av} from Equation 2–9 into Equation 2–8 yields

Constant-Acceleration Equation of Motion: Position as a Function of Time

$$x = x_0 + \tfrac{1}{2}(v_0 + v)t \qquad 2\text{–}10$$

This equation, like Equation 2–7, is valid *only* for constant acceleration. The utility of Equations 2–7 and 2–10 is illustrated in the next Example.

PROBLEM-SOLVING NOTE

"Coordinate" the Problem

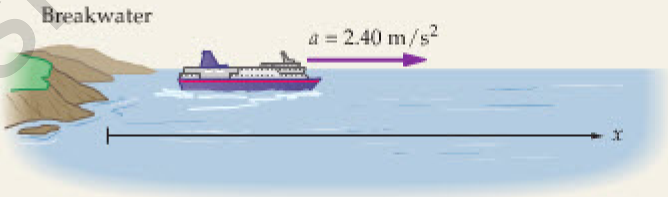
The first step in solving a physics problem is to produce a simple sketch of the system. Your sketch should include a coordinate system, along with an origin and a positive direction. Next, you should identify quantities that are given in the problem, such as initial position, initial velocity, acceleration, and so on. These preliminaries will help in producing a mathematical representation of the problem.

EXAMPLE 2–5 FULL SPEED AHEAD

A boat moves slowly inside a marina (so as not to leave a wake) with a constant speed of 1.50 m/s. As soon as it passes the breakwater, leaving the marina, it throttles up and accelerates at 2.40 m/s^2. **(a)** How fast is the boat moving after accelerating for 5.00 s? **(b)** How far has the boat traveled in this time?

PICTURE THE PROBLEM

In our sketch we choose the origin to be at the breakwater, and the positive x direction to be the direction of motion. With this choice the initial position is $x_0 = 0$, and the initial velocity is $v_0 = 1.50$ m/s.



STRATEGY

The acceleration is constant, so we can use Equations 2–7 to 2–10. In part (a) we want to relate velocity to time, so we use Equation 2–7, $v = v_0 + at$. In part (b) our knowledge of the initial and final velocities allows us to relate position to time using Equation 2–10, $x = x_0 + \frac{1}{2}(v_0 + v)t$.

SOLUTION

Part (a)

1. Use Equation 2–7 with $v_0 = 1.50$ m/s and $a = 2.40$ m/s^2:

$$v = v_0 + at = 1.50\text{ m/s} + (2.40\text{ m/s}^2)(5.00\text{ s})$$
$$= 1.50\text{ m/s} + 12.0\text{ m/s} = 13.5\text{ m/s}$$

Part (b)

2. Apply Equation 2–10, using the result for v obtained in part (a):

$$x = x_0 + \tfrac{1}{2}(v_0 + v)t$$
$$= 0 + \tfrac{1}{2}(1.50\text{ m/s} + 13.5\text{ m/s})(5.00\text{ s})$$
$$= (7.50\text{ m/s})(5.00\text{ s}) = 37.5\text{ m}$$

INSIGHT

Since the boat has a constant acceleration between $t = 0$ and $t = 5.00$ s, its velocity-versus-time curve is linear during this time interval. As a result, the average velocity for these 5.00 seconds is the average of the initial and final velocities, $v_{av} = \frac{1}{2}(1.50\text{ m/s} + 13.5\text{ m/s}) = 7.50$ m/s. Multiplying the average velocity by the time, 5.00 s, gives the distance traveled—which is exactly what Equation 2–10 does in Step 2.

PRACTICE PROBLEM

At what time is the boat's speed equal to 10.0 m/s? **[Answer:** $t = 3.54$ s]

Some related homework problems: Problem 47, Problem 48

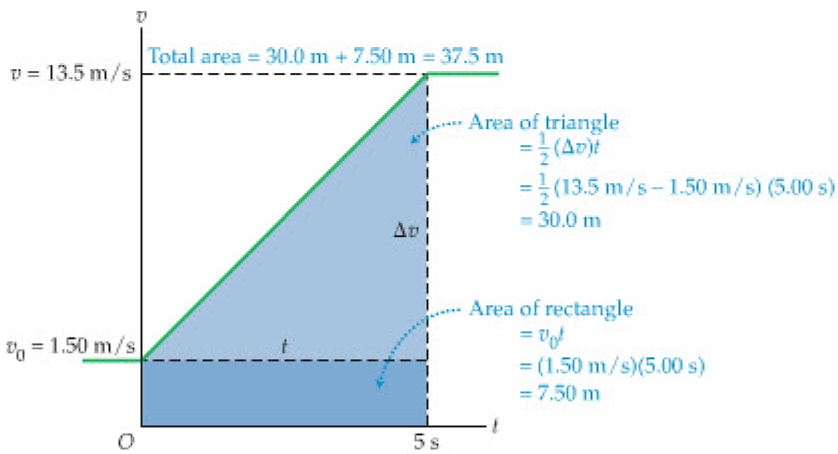


FIGURE 2–13 Velocity versus time for the boat in Example 2–5 The distance traveled by the boat between $t = 0$ and $t = 5.00$ s is equal to the corresponding area under the velocity curve.

The velocity of the boat in Example 2–5 is plotted as a function of time in **Figure 2–13**, with the acceleration starting at time $t = 0$ and ending at $t = 5.00$ s. We will now show that the *distance* traveled by the boat from $t = 0$ to $t = 5.00$ s *is equal to the corresponding area under the velocity-versus-time curve*. This is a general result, valid for any velocity curve and any time interval:

- The distance traveled by an object from a time t_1 to a time t_2 is equal to the area under the velocity curve between those two times.

In this case, the area is the sum of the areas of a rectangle and a triangle. The rectangle has a base of 5.00 s and a height of 1.50 m/s, which gives an area of $(5.00\text{ s})(1.50\text{ m/s}) = 7.50\text{ m}$. Similarly, the triangle has a base of 5.00 s and a height of $(13.5\text{ m/s} - 1.50\text{ m/s}) = 12.0\text{ m/s}$, for an area of $\frac{1}{2}(5.00\text{ s})(12.0\text{ m/s}) = 30.0\text{ m}$. Clearly, the total area is 37.5 m, just as found in Example 2–5.

Staying with Example 2–5 for a moment, let's repeat the calculation of part (b), only this time for the general case. First, we use the final velocity from part (a), calculated with $v = v_0 + at$, in the expression for the average velocity, $v_{av} = \frac{1}{2}(v_0 + v)$. Symbolically, this gives the following:

$$\tfrac{1}{2}(v_0 + v) = \tfrac{1}{2}[v_0 + (v_0 + at)] = v_0 + \tfrac{1}{2}at \qquad \textit{(constant acceleration)}$$

Next, we substitute this result into Equation 2–10, which yields

$$x = x_0 + \tfrac{1}{2}(v_0 + v)t = x_0 + \left(v_0 + \tfrac{1}{2}at\right)t$$

Multiplying through by t gives the following result:

Constant-Acceleration Equation of Motion: Position as a Function of Time

$$x = x_0 + v_0 t + \tfrac{1}{2}at^2 \qquad \text{2–11}$$

Here we have an expression for position versus time that is explicitly in terms of the acceleration, a.

Note that each term in Equation 2–11 has the same dimensions, as they must. For example, the velocity term, $v_0 t$, has the units $(\text{m/s})(\text{s}) = \text{m}$. Similarly, the acceleration term, $\frac{1}{2}at^2$, has the units $(\text{m/s}^2)(\text{s}^2) = \text{m}$.

EXERCISE 2–3

Repeat part (b) of Example 2–5 using Equation 2–11.

SOLUTION

$$x = x_0 + v_0 t + \tfrac{1}{2}at^2 = 0 + (1.50\text{ m/s})(5.00\text{ s}) + \tfrac{1}{2}(2.40\text{ m/s}^2)(5.00\text{ s})^2 = 37.5\text{ m}$$

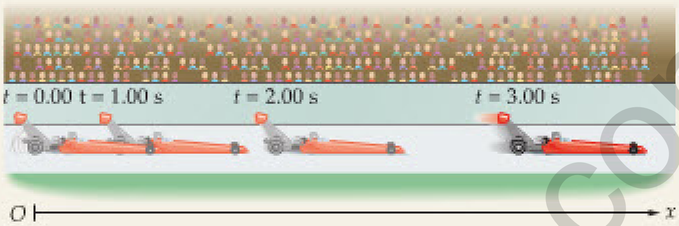
The next Example gives further insight into the physical meaning of Equation 2–11.

EXAMPLE 2–6 PUT THE PEDAL TO THE METAL

A drag racer starts from rest and accelerates at 7.40 m/s^2. How far has it traveled in **(a)** 1.00 s, **(b)** 2.00 s, **(c)** 3.00 s?

PICTURE THE PROBLEM

We set up a coordinate system in which the drag racer starts at the origin and accelerates in the positive x direction. With this choice, it follows that $x_0 = 0$ and $a = +7.40 \text{ m/s}^2$. Also, since the racer starts from rest, its initial velocity is zero, $v_0 = 0$. Incidentally, the positions of the racer in the sketch have been drawn to scale.



STRATEGY

Since this problem gives the acceleration, which is constant, and asks for a relationship between position and time, we use Equation 2–11.

SOLUTION

Part (a)

1. Evaluate Equation 2–11 with $a = 7.40 \text{ m/s}^2$ and $t = 1.00$ s:

$$x = x_0 + v_0t + \tfrac{1}{2}at^2 = 0 + 0 + \tfrac{1}{2}at^2 = \tfrac{1}{2}at^2$$
$$x = \tfrac{1}{2}(7.40 \text{ m/s}^2)(1.00 \text{ s})^2 = 3.70 \text{ m}$$

Part (b)

2. From the calculation in part (a), Equation 2–11 reduces to $x = \frac{1}{2}at^2$ in this situation. Evaluate $x = \frac{1}{2}at^2$ at $t = 2.00$ s:

$$x = \tfrac{1}{2}at^2$$
$$= \tfrac{1}{2}(7.40 \text{ m/s}^2)(2.00 \text{ s})^2 = 14.8 \text{ m} = 4(3.70 \text{ m})$$

Part (c)

3. Repeat with $t = 3.00$ s:

$$x = \tfrac{1}{2}at^2$$
$$= \tfrac{1}{2}(7.40 \text{ m/s}^2)(3.00 \text{ s})^2 = 33.3 \text{ m} = 9(3.70 \text{ m})$$

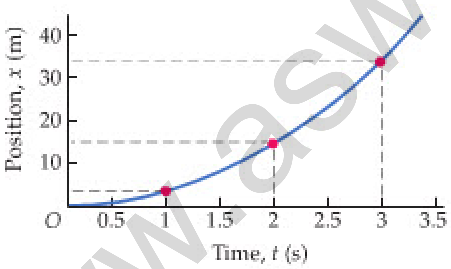
INSIGHT

This Example illustrates one of the key features of accelerated motion—position does not change uniformly with time when an object accelerates. In this case, the distance traveled in the first two seconds is 4 times the distance traveled in the first second, and the distance traveled in the first three seconds is 9 times the distance traveled in the first second. This kind of behavior is a direct result of the fact that x depends on t^2 when the acceleration is nonzero.

PRACTICE PROBLEM

In one second the racer travels 3.70 m. How long does it take for the racer to travel $2(3.70 \text{ m}) = 7.40$ m? [**Answer:** $t = \sqrt{2}$ s $= 1.41$ s]

Some related homework problems: Problem 49, Problem 64



▲ FIGURE 2–14 Position versus time for Example 2–6
The upward-curving, parabolic shape of this x-versus-t plot indicates a positive, constant acceleration. The dots on the curve show the position of the drag racer in Example 2–6 at the times 1.00 s, 2.00 s, and 3.00 s.

Figure 2–14 shows a graph of x-versus-t for Example 2–6. Notice the parabolic shape of the x-versus-t curve, which is due to the $\frac{1}{2}at^2$ term, and is characteristic of constant acceleration. In particular, if acceleration is positive ($a > 0$), then a plot of x-versus-t curves upward; if acceleration is negative ($a < 0$), a plot of x-versus-t curves downward. The greater the magnitude of a, the greater the curvature. In contrast, if a particle moves with constant velocity ($a = 0$) the t^2 dependence vanishes, and the x-versus-t plot is a straight line.

Our final equation of motion with constant acceleration relates velocity to position. We start by solving for the time, t, in Equation 2–7:

$$v = v_0 + at \quad \text{or} \quad t = \frac{v - v_0}{a}$$

Next, we substitute this result into Equation 2–10, thus eliminating t:

$$x = x_0 + \tfrac{1}{2}(v_0 + v)t = x_0 + \tfrac{1}{2}(v_0 + v)\left(\frac{v - v_0}{a}\right)$$

Noting that $(v_0 + v)(v - v_0) = v_0v - v_0^2 + v^2 - vv_0 = v^2 - v_0^2$, we have

$$x = x_0 + \frac{v^2 - v_0^2}{2a}$$

Finally, a straightforward rearrangement of terms yields

Constant-Acceleration Equation of Motion: Velocity in Terms of Displacement

$$v^2 = v_0^2 + 2a(x - x_0) = v_0^2 + 2a\Delta x \qquad 2\text{–}12$$

This equation allows us to relate the velocity at one position to the velocity at another position, without knowing how much time is involved. The next Example shows how Equation 2–12 can be used.

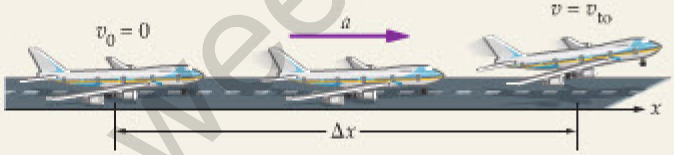
EXAMPLE 2–7 TAKEOFF DISTANCE FOR AN AIRLINER

REAL-WORLD PHYSICS

Jets at JFK International Airport accelerate from rest at one end of a runway, and must attain takeoff speed before reaching the other end of the runway. **(a)** Plane A has acceleration a and takeoff speed v_{to}. What is the minimum length of runway, Δx_A, required for this plane? Give a symbolic answer. **(b)** Plane B has the same acceleration as plane A, but requires twice the takeoff speed. Find Δx_B and compare with Δx_A. **(c)** Find the minimum runway length for plane A if $a = 2.20\ \text{m/s}^2$ and $v_{to} = 95.0\ \text{m/s}$. (These values are typical for a 747 jetliner.)

PICTURE THE PROBLEM

In our sketch, we choose the positive x direction to be the direction of motion. With this choice, it follows that the acceleration of the plane is positive, $a = +2.20\ \text{m/s}^2$. Similarly, the takeoff velocity is positive as well, $v_{to} = +95.0\ \text{m/s}$.



STRATEGY

From the sketch it is clear that we want to express Δx, the distance the plane travels in attaining takeoff speed, in terms of the acceleration, a, and the takeoff speed, v_{to}. Equation 2–12, which relates distance to velocity, allows us to do this.

SOLUTION

Part (a)

1. Solve Equation 2–12 for Δx. To find Δx_A, set $v_0 = 0$ and $v = v_{to}$:

$$\Delta x = \frac{v^2 - v_0^2}{2a} \qquad \Delta x_A = \frac{v_{to}^2}{2a}$$

Part (b)

2. To find Δx_B, simply change v_{to} to $2v_{to}$ in part (a):

$$\Delta x_B = \frac{(2v_{to})^2}{2a} = \frac{4v_{to}^2}{2a} = 4\Delta x_A$$

Part (c)

3. Substitute numerical values into the result found in part (a):

$$\Delta x_A = \frac{v_{to}^2}{2a} = \frac{(95.0\ \text{m/s})^2}{2(2.20\ \text{m/s}^2)} = 2050\ \text{m}$$

INSIGHT

For purposes of comparison, the shortest runway at JFK International Airport is 04R/22L, which has a length of 2560 m.

This Example illustrates the fact that there are many advantages to obtaining symbolic results before substituting numerical values. In this case, we find that the takeoff distance is proportional to v^2; hence, we conclude immediately that doubling v results in a fourfold increase of Δx.

PRACTICE PROBLEM

Find the minimum acceleration needed for a takeoff speed of $v_{to} = (95.0\ \text{m/s})/2 = 47.5\ \text{m/s}$ on a runway of length $\Delta x = (2050\ \text{m})/4 = 513\ \text{m}$. [**Answer:** $a = v_{to}^2/2\Delta x = 2.20\ \text{m/s}^2$]

Some related homework problems: Problem 55, Problem 57

Finally, all of our constant-acceleration equations of motion are collected for easy reference in Table 2–4.

TABLE 2–4 Constant-Acceleration Equations of Motion

Variables Related	Equation	Number
velocity, time, acceleration	$v = v_0 + at$	2–7
initial, final, and average velocity	$v_{av} = \frac{1}{2}(v_0 + v)$	2–9
position, time, velocity	$x = x_0 + \frac{1}{2}(v_0 + v)t$	2–10
position, time, acceleration	$x = x_0 + v_0 t + \frac{1}{2}at^2$	2–11
velocity, position, acceleration	$v^2 = v_0^2 + 2a(x - x_0) = v_0^2 + 2a\Delta x$	2–12

2–6 Applications of the Equations of Motion

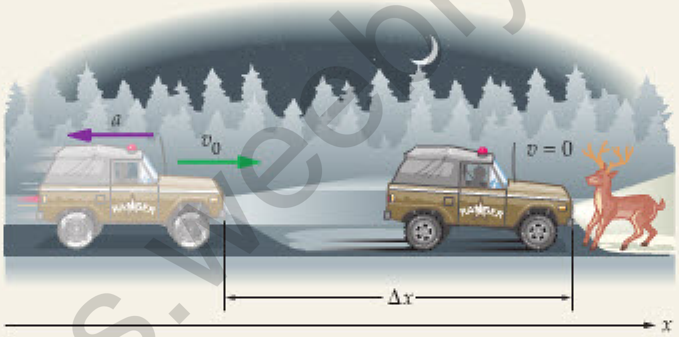
We devote this section to a variety of examples further illustrating the use of the constant-acceleration equations of motion. In our first Example, we consider the distance and time needed to brake a vehicle to a complete stop.

EXAMPLE 2–8 HIT THE BRAKES!

A park ranger driving on a back country road suddenly sees a deer "frozen" in the headlights. The ranger, who is driving at 11.4 m/s, immediately applies the brakes and slows with an acceleration of $3.80\ \mathrm{m/s^2}$. **(a)** If the deer is 20.0 m from the ranger's vehicle when the brakes are applied, how close does the ranger come to hitting the deer? **(b)** How much time is needed for the ranger's vehicle to stop?

PICTURE THE PROBLEM

We choose the positive x direction to be the direction of motion. With this choice it follows that $v_0 = +11.4$ m/s. In addition, the fact that the ranger's vehicle is slowing down means its acceleration points in the *opposite* direction to that of the velocity [see Figure 2–10 (b) and (c)]. Therefore, the vehicle's acceleration is $a = -3.80\ \mathrm{m/s^2}$. Finally, when the vehicle comes to rest its velocity is zero, $v = 0$.



STRATEGY

The acceleration is constant, so we can use the equations listed in Table 2–4. In part (a) we want to find a distance when we know the velocity and acceleration, so we use a rearranged version of Equation 2–12. In part (b) we want to find a time when we know the velocity and acceleration, so we use a rearranged version of Equation 2–7.

SOLUTION

Part (a)

1. Solve Equation 2–12 for Δx: $$\Delta x = \frac{v^2 - {v_0}^2}{2a}$$

2. Set $v = 0$, and substitute numerical values: $$\Delta x = -\frac{{v_0}^2}{2a} = -\frac{(11.4\ \mathrm{m/s})^2}{2(-3.80\ \mathrm{m/s^2})} = 17.1\ \mathrm{m}$$

3. Subtract Δx from 20.0 m to find the distance between the stopped vehicle and the deer: $$20.0\ \mathrm{m} - 17.1\ \mathrm{m} = 2.9\ \mathrm{m}$$

Part (b)

4. Set $v = 0$ in Equation 2–7 and solve for t: $$v = v_0 + at = 0$$ $$t = -\frac{v_0}{a} = -\frac{11.4\ \mathrm{m/s}}{(-3.80\ \mathrm{m/s^2})} = 3.00\ \mathrm{s}$$

INSIGHT

Note the difference in the way t and Δx depend on the initial speed. If the initial speed is doubled, for example, the time needed to stop also doubles, but the distance needed to stop increases by a factor of four. This is one reason why speed on the highway has such a great influence on safety.

PRACTICE PROBLEM

Show that using $t = 3.00$ s in Equation 2–11 results in the same distance needed to stop.
[**Answer:** $x = x_0 + v_0 t + \frac{1}{2}at^2 = 0 + (11.4\ \mathrm{m/s})(3.00\ \mathrm{s}) + \frac{1}{2}(-3.80\ \mathrm{m/s^2})(3.00\ \mathrm{s})^2 = 17.1\ \mathrm{m}$, as expected.]

Some related homework problems: Problem 57, Problem 58

In Example 2–8, we calculated the distance necessary for a vehicle to come to a complete stop. But how does v vary with distance as the vehicle slows down? The next Conceptual Checkpoint deals with this topic.

CONCEPTUAL CHECKPOINT 2–4 STOPPING DISTANCE

The ranger in Example 2–8 brakes for 17.1 m. After braking for only half that distance, $\frac{1}{2}(17.1\text{ m}) = 8.55\text{ m}$, is the ranger's speed **(a)** equal to $\frac{1}{2}v_0$, **(b)** greater than $\frac{1}{2}v_0$, or **(c)** less than $\frac{1}{2}v_0$?

REASONING AND DISCUSSION

As pointed out in the Insight for Example 2–8, the fact that the stopping distance, Δx, depends on $v_0{}^2$ means that this distance increases by a factor of four when the speed is doubled. For example, the stopping distance with an initial speed of v_0 is four times the stopping distance when the initial speed is $v_0/2$.

To apply this observation to the ranger, suppose that the stopping distance with an initial speed of v_0 is Δx. It follows that the stopping distance for an initial speed of $v_0/2$ is $\Delta x/4$. This means that as the ranger slows from v_0 to 0, it takes a distance $\Delta x/4$ to slow from $v_0/2$ to 0, and the remaining distance, $3\Delta x/4$, to slow from v_0 to $v_0/2$. Thus, at the halfway point the ranger has not yet slowed to half of the initial velocity—the speed at this point is greater than $v_0/2$.

ANSWER

(b) The ranger's speed is greater than $\frac{1}{2}v_0$.

REAL-WORLD PHYSICS

The stopping distance of a car

Clearly, v does not decrease uniformly with distance. A plot showing v as a function of x for Example 2–8 is shown in **Figure 2–15**. As we can see from the graph, v changes more in the second half of the braking distance than in the first half.

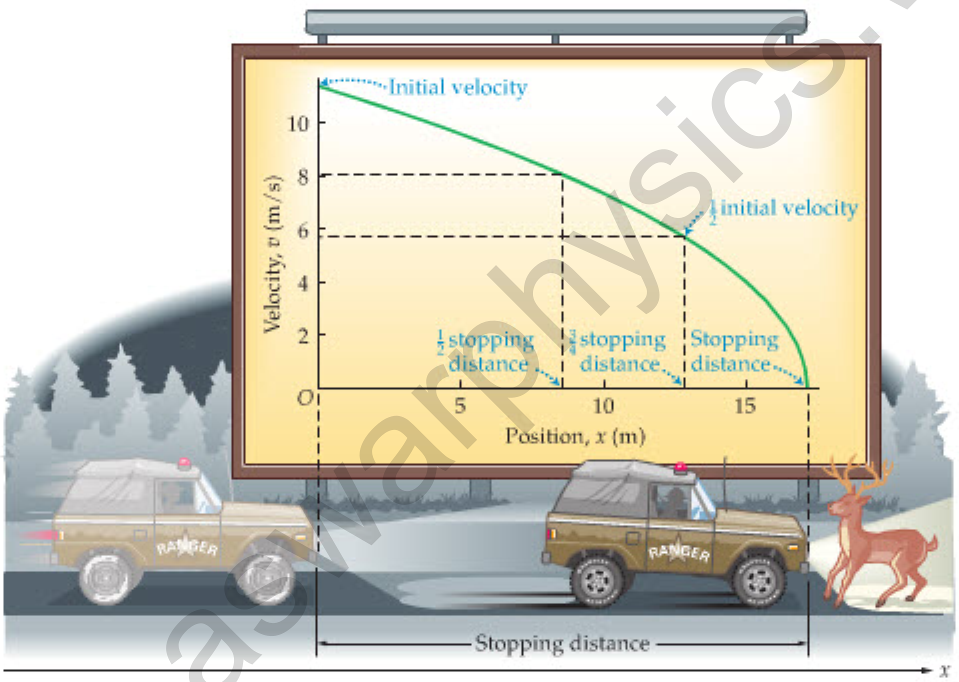


FIGURE 2–15 Velocity as a function of position for the ranger in Example 2–8

The ranger's vehicle in Example 2–8 comes to rest with constant acceleration, which means that its velocity decreases uniformly with time. The velocity *does not* decrease uniformly with distance, however. In particular, note how rapidly the velocity decreases in the final one-quarter of the stopping distance.

We close this section with a familiar, everyday example: a police car accelerating to overtake a speeder. This is the first time that we use two equations of motion for two different objects to solve a problem—but it won't be the last. Problems of this type are often more interesting than problems involving only a single object, and they relate to many types of situations in everyday life.

PROBLEM-SOLVING NOTE

Strategize

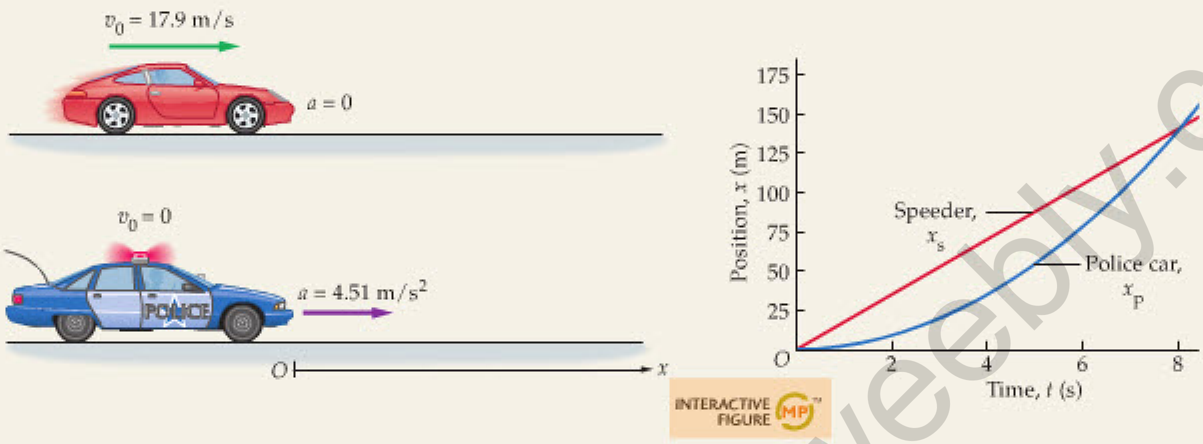
Before attempting to solve a problem, it is a good idea to have some sort of plan, or "strategy," for how to proceed. It may be as simple as saying, "The problem asks me to relate velocity and time, therefore I will use Equation 2–7." In other cases the strategy is a bit more involved. Producing effective strategies is one of the most challenging—and creative—aspects of problem solving.

EXAMPLE 2–9 CATCHING A SPEEDER

A speeder doing 40.0 mi/h (about 17.9 m/s) in a 25 mi/h zone approaches a parked police car. The instant the speeder passes the police car, the police begin their pursuit. If the speeder maintains a constant velocity, and the police car accelerates with a constant acceleration of 4.51 m/s^2, **(a)** how long does it take for the police car to catch the speeder, **(b)** how far have the two cars traveled in this time, and **(c)** what is the velocity of the police car when it catches the speeder?

PICTURE THE PROBLEM

Our sketch shows the two cars at the moment the speeder passes the resting police car. At this instant, which we take to be $t = 0$, both the speeder and the police car are at the origin, $x = 0$. In addition, we choose the positive x direction to be the direction of motion; therefore, the speeder's initial velocity is given by $v_s = +17.9\ \text{m/s}$, and the police car's initial velocity is zero. The speeder's acceleration is zero, but the police car has an acceleration given by $a = +4.51\ \text{m/s}^2$. Finally, our plot shows the linear x-versus-t plot for the speeder, and the parabolic x-versus-t plot for the police car.



STRATEGY

To solve this problem, first write down a position-versus-time equation for the police car, x_p, and a separate equation for the speeder, x_s. Next, we find the time it takes the police car to catch the speeder by setting $x_p = x_s$ and solving the resulting equation for t. Once the catch time is determined, it is straightforward to calculate the distance traveled and the velocity of the police car.

SOLUTION

Part (a)

1. Write equations of motion for the two vehicles. For the police car, $v_0 = 0$ and $a = 4.51\ \text{m/s}^2$. For the speeder, $v_0 = 17.9\ \text{m/s} = v_s$ and $a = 0$:

$$x_p = \tfrac{1}{2}at^2$$
$$x_s = v_s t$$

2. Set $x_p = x_s$ and solve for the time:

$$\tfrac{1}{2}at^2 = v_s t \text{ or } (\tfrac{1}{2}at - v_s)t = 0$$

$$\text{two solutions: } t = 0 \quad \text{or} \quad t = \frac{2v_s}{a}$$

3. Clearly, $t = 0$ corresponds to the initial conditions, because both vehicles started at $x = 0$ at that time. The time of interest is obtained by substituting numerical values into the other solution:

$$t = \frac{2v_s}{a} = \frac{2(17.9\ \text{m/s})}{4.51\ \text{m/s}^2} = 7.94\ \text{s}$$

Part (b)

4. Substitute $t = 7.94$ s into the equations of motion for x_p and x_s. Note that $x_p = x_s$, as expected:

$$x_p = \tfrac{1}{2}at^2 = \tfrac{1}{2}(4.51\ \text{m/s}^2)(7.94\ \text{s})^2 = 142$$
$$x_s = v_s t = (17.9\ \text{m/s})(7.94\ \text{s}) = 142\ \text{m}$$

Part (c)

5. To find the velocity of the police car use Equation 2–7, which relates velocity to time:

$$v_p = v_0 + at = 0 + (4.51\ \text{m/s}^2)(7.94\ \text{s}) = 35.8\ \text{m/s}$$

INSIGHT

When the police car catches up with the speeder, its velocity is 35.8 m/s, which is exactly twice the velocity of the speeder. A coincidence? Not at all. When the police car catches the speeder, both have traveled the same distance (142 m) in the same time (7.94 s), therefore they have the same average velocity. Of course, the average velocity of the speeder is simply v_s. The average velocity of the police car is $\frac{1}{2}(v_0 + v)$, since its acceleration is constant, and thus $\frac{1}{2}(v_0 + v) = v_s$. Since $v_0 = 0$ for the police car, it follows that $v = 2v_s$. Notice that this result is independent of the acceleration of the police car, as we show in the following Practice Problem.

PRACTICE PROBLEM

Repeat this Example for the case where the acceleration of the police car is $a = 3.25\ \text{m/s}^2$. [**Answer:** **(a)** $t = 11.0$ s, **(b)** $x_p = x_s = 197$ m, **(c)** $v_p = 35.8$ m/s]

Some related homework problems: Problem 54, Problem 65

2–7 Freely Falling Objects

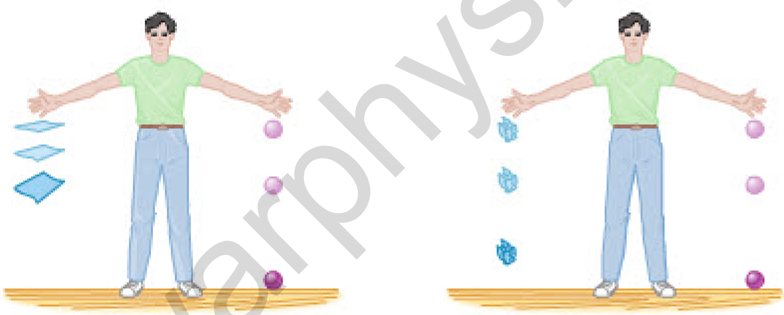
The most famous example of motion with constant acceleration is **free fall**—the motion of an object falling freely under the influence of gravity. It was Galileo (1564–1642) who first showed, with his own experiments, that falling bodies move with constant acceleration. His conclusions were based on experiments done by rolling balls down inclines of various steepness. By using an incline, Galileo was able to reduce the acceleration of the balls, thus producing motion slow enough to be timed with the rather crude instruments available.

Galileo also pointed out that objects of different weight fall with the *same* constant acceleration—provided air resistance is small enough to be ignored. Whether he dropped objects from the Leaning Tower of Pisa to demonstrate this fact, as legend has it, will probably never be known for certain, but we do know that he performed extensive experiments to support his claim.

Today it is easy to verify Galileo's assertion by dropping objects in a vacuum chamber, where the effects of air resistance are essentially removed. In a standard classroom demonstration, a feather and a coin are dropped in a vacuum, and both fall at the same rate. In 1971, a novel version of this experiment was carried out on the Moon by astronaut David Scott. In the near-perfect vacuum on the Moon's surface he dropped a feather and a hammer and showed a worldwide television audience that they fell to the ground in the same time.

▲ In the absence of air resistance, all bodies fall with the same acceleration, regardless of their mass.

To illustrate the effect of air resistance in everyday terms, consider dropping a sheet of paper and a rubber ball (**Figure 2–16**). The paper drifts slowly to the ground, taking much longer to fall than the ball. Now, wad the sheet of paper into a tight ball and repeat the experiment. This time the ball of paper and the rubber ball reach the ground in nearly the same time. What was different in the two experiments? Clearly, when the sheet of paper was wadded into a ball, the effect of air resistance on it was greatly reduced, so that both objects fell almost as they would in a vacuum.



(a) Dropping a sheet of paper and a rubber ball

(b) Dropping a wadded-up sheet of paper and a rubber ball

◀ **FIGURE 2–16 Free fall and air resistance**

Before considering a few examples, let's first discuss exactly what is meant by "free fall." To begin, the word *free* in free fall means free from any effects other than gravity. For example, in free fall we assume that an object's motion is not influenced by any form of friction or air resistance.

- Free fall is the motion of an object subject *only* to the influence of gravity.

Though free fall is an idealization—which does not apply to many real-world situations—it is still a useful approximation in many other cases. In the following examples we assume that the motion may be considered as free fall.

Next, it should be realized that the word *fall* in free fall does not mean the object is necessarily moving downward. By free fall, we mean *any* motion under the influence of gravity alone. If you drop a ball, it is in free fall. If you throw a ball upward or downward, it is in free fall as soon as it leaves your hand.

- An object is in free fall as soon as it is released, whether it is dropped from rest, thrown downward, or thrown upward.

Finally, the acceleration produced by gravity on the Earth's surface (sometimes called the gravitational strength) is denoted with the symbol g. As a shorthand

▲ Whether she is on the way up, at the peak of her flight, or on the way down, this girl is in free fall, accelerating downward with the acceleration of gravity. Only when she is in contact with the blanket does her acceleration change.

TABLE 2–5 Values of g at Different Locations on Earth (m/s^2)

Location	Latitude	g
North Pole	90° N	9.832
Oslo, Norway	60° N	9.819
Hong Kong	30° N	9.793
Quito, Ecuador	0°	9.780

name, we will frequently refer to g simply as "the acceleration due to gravity." In fact, as we shall see in Chapter 12, the value of g varies according to one's location on the surface of the Earth, as well as one's altitude above it. Table 2–5 shows how g varies with latitude.

In all the calculations that follow in this book, we shall use $g = 9.81\ m/s^2$ for the acceleration due to gravity. Note, in particular, that g always stands for $+9.81\ m/s^2$, never $-9.81\ m/s^2$. For example, if we choose a coordinate system with the positive direction upward, the acceleration in free fall is $a = -g$. If the positive direction is downward, then free-fall acceleration is $a = g$.

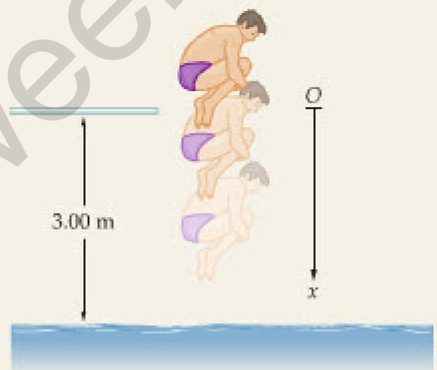
With these comments, we are ready to explore a variety of free-fall examples.

EXAMPLE 2–10 DO THE CANNONBALL!

A person steps off the end of a 3.00-m-high diving board and drops to the water below. **(a)** How long does it take for the person to reach the water? **(b)** What is the person's speed on entering the water?

PICTURE THE PROBLEM

In our sketch we choose the origin to be at the height of the diving board, and we let the positive direction be downward. With these choices, $x_0 = 0$, $a = g$, and the water is at $x = 3.00$ m. Of course, $v_0 = 0$ since the person simply steps off the board.



STRATEGY

We can neglect air resistance in this case and model the motion as free fall. This means we can assume a constant acceleration equal to g and use the equations of motion in Table 2–4. For part (a) we want to find the time of fall when we know the distance and acceleration, so we use Equation 2–11. For part (b) we can relate velocity to time by using Equation 2–7, or we can relate velocity to position by using Equation 2–12. We will implement both approaches and show that the results are the same.

SOLUTION

Part (a)

1. Write Equation 2–11 with $x_0 = 0$, $v_0 = 0$, and $a = g$: $x = x_0 + v_0 t + \frac{1}{2}at^2 = 0 + 0 + \frac{1}{2}gt^2 = \frac{1}{2}gt^2$

2. Solve for the time, t, and set $x = 3.00$ m: $t = \sqrt{\frac{2x}{g}} = \sqrt{\frac{2(3.00\ m)}{9.81\ m/s^2}} = 0.782\ s$

Part (b)

3. Use the time found in part (a) in Equation 2–7: $v = v_0 + gt = 0 + (9.81\ m/s^2)(0.782\ s) = 7.67\ m/s$

4. We can also find the velocity without knowing the time by using Equation 2–12 with $\Delta x = 3.00$ m: $v^2 = v_0^2 + 2a\Delta x = 0 + 2g\Delta x$

$v = \sqrt{2g\Delta x} = \sqrt{2(9.81\ m/s^2)(3.00\ m)} = 7.67\ m/s$

INSIGHT

Let's put these results in more common, everyday units. If you step off a diving board 9.84 ft (3.00 m) above the water, you enter the water with a speed of 17.2 mi/h (7.67 m/s).

PRACTICE PROBLEM

What is your speed on entering the water if you step off a 10.0-m diving tower? [**Answer:** $v = \sqrt{2(9.81\ m/s^2)(10.0\ m)} = 14.0\ m/s = 31.4\ mi/h$]

Some related homework problems: Problem 71, Problem 83

The special case of free fall from rest occurs so frequently, and in so many different contexts, that it deserves special attention. If we take x_0 to be zero, and positive to be downward, then position as a function of time is $x = x_0 + v_0 t + \frac{1}{2}gt^2 = 0 + 0 + \frac{1}{2}gt^2$, or

$$x = \frac{1}{2}gt^2 \qquad 2\text{–}13$$

Similarly, velocity as a function of time is

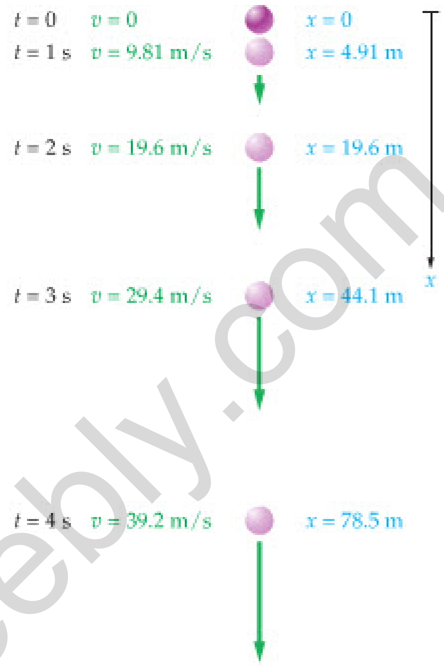
$$v = gt \qquad 2\text{–}14$$

and velocity as a function of position is

$$v = \sqrt{2gx} \qquad \text{2–15}$$

The behavior of these functions is illustrated in Figure 2–17. Note that position increases with time squared, whereas velocity increases linearly with time.

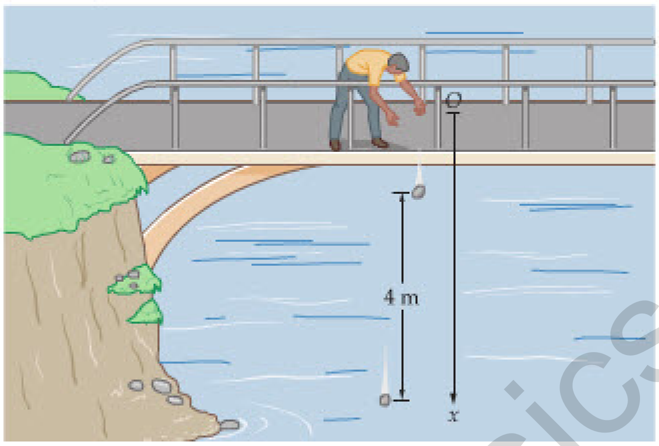
Next we consider two objects dropped from rest, one after the other, and discuss how their separation varies with time.



▲ **FIGURE 2–17 Free fall from rest**
Position and velocity are shown as functions of time. It is apparent that velocity depends linearly on t, whereas position depends on t^2.

CONCEPTUAL CHECKPOINT 2–5
FREE-FALL SEPARATION

You drop a rock from a bridge to the river below. When the rock has fallen 4 m, you drop a second rock. As the rocks continue their free fall, does their separation **(a)** increase, **(b)** decrease, or **(c)** stay the same?



REASONING AND DISCUSSION
It might seem that since both rocks are in free fall, their separation remains the same. This is not so. The rock that has a head start always has a greater velocity than the later one; thus it covers a greater distance in any interval of time. As a result, the separation between the rocks increases.

ANSWER
(a) The separation between the rocks increases.

An erupting volcano shooting out fountains of lava is an impressive sight. In the next Example we show how a simple timing experiment can determine the initial velocity of the erupting lava.

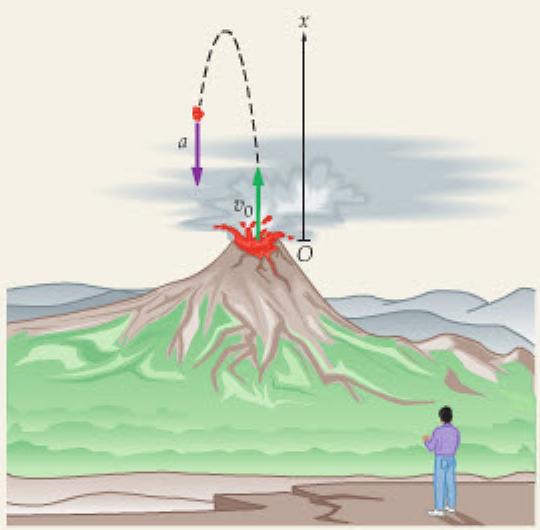
PROBLEM-SOLVING NOTE
Check Your Solution

Once you have a solution to a problem, check to see whether it makes sense. First, make sure the units are correct; m/s for speed, m/s^2 for acceleration, and so on. Second, check the numerical value of your answer. If you are solving for the speed of a diver dropping from a 3.0-m diving board and you get an unreasonable value like 200 m/s (≈450 mi/h), chances are good that you've made a mistake.

EXAMPLE 2–11 BOMBS AWAY: CALCULATING THE SPEED OF A LAVA BOMB

REAL-WORLD PHYSICS

A volcano shoots out blobs of molten lava, called lava bombs, from its summit. A geologist observing the eruption uses a stopwatch to time the flight of a particular lava bomb that is projected straight upward. If the time for it to rise and fall back to its launch height is 4.75 s, and its acceleration is 9.81 m/s^2 downward, what is its initial speed?



PICTURE THE PROBLEM
Our sketch shows a coordinate system with upward as the positive x direction. For clarity, we offset the upward and downward trajectories slightly. In addition, we choose $t = 0$ to be the time at which the lava bomb is launched. With these choices it follows that $x_0 = 0$ and the acceleration is $a = -g = -9.81\ m/s^2$. The initial speed to be determined is v_0.

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STRATEGY

Once again, we can neglect air resistance and model the motion of the lava bomb as free fall—this time with an initial upward velocity. We know that the lava bomb starts at $x = 0$ at the time $t = 0$ and returns to $x = 0$ at the time $t = 4.75$ s. This means that we know the bomb's position, time, and acceleration ($a = -g$), from which we would like to determine the initial velocity. A reasonable approach is to use Equation 2–11 and solve it for the one unknown it contains, v_0.

SOLUTION

1. Write out $x = x_0 + v_0t + \frac{1}{2}at^2$ with $x_0 = 0$ and $a = -g$. Factor out a time, t, from the two remaining terms:

$$x = x_0 + v_0t + \tfrac{1}{2}at^2 = v_0t - \tfrac{1}{2}gt^2 = \left(v_0 - \tfrac{1}{2}gt\right)t$$

2. Set x equal to zero, since this is the position of the lava bomb at $t = 0$ and $t = 4.75$ s:

$x = \left(v_0 - \frac{1}{2}gt\right)t = 0$ two solutions:

(i) $t = 0$

(ii) $v_0 - \frac{1}{2}gt = 0$

3. The first solution is simply the initial condition; that is, $x = 0$ at $t = 0$. Solve the second solution for the initial speed:

$$v_0 - \tfrac{1}{2}gt = 0 \quad \text{or} \quad v_0 = \tfrac{1}{2}gt$$

4. Substitute numerical values for g and the time the lava bomb lands:

$$v_0 = \tfrac{1}{2}gt = \tfrac{1}{2}(9.81\ \text{m/s}^2)(4.75\ \text{s}) = 23.3\ \text{m/s}$$

INSIGHT

A geologist can determine a lava bomb's initial speed by simply observing its flight time. Knowing the lava bomb's initial speed can help geologists determine how severe a volcanic eruption will be, and how dangerous it might be to people in the surrounding area.

PRACTICE PROBLEM

A second lava bomb is projected straight upward with an initial speed of 25 m/s. How long is it in the air? [**Answer:** $t = 5.1$ s]

Some related homework problems: Problem 73, Problem 86

▲ In the absence of air resistance, these lava bombs from the Kilauea volcano on the big island of Hawaii would strike the water with the same speed they had when they were blasted into the air.

What is the speed of a lava bomb when it returns to Earth; that is, when it returns to the same level from which it was launched? Physical intuition might suggest that, in the absence of air resistance, it should be the same as the initial speed. To show that this hypothesis is indeed correct, write out Equation 2–7 for this case:

$$v = v_0 - gt$$

Substituting numerical values, we find

$$v = v_0 - gt = 23.3\ \text{m/s} - (9.81\ \text{m/s}^2)(4.75\ \text{s}) = -23.3\ \text{m/s}$$

Thus, the velocity of the lava when it lands is just the negative of the velocity it had when launched upward. Or put another way, when the lava lands, it has the same speed as when it was launched; it's just traveling in the opposite direction.

It is instructive to verify this result symbolically. Recall from Example 2–11 that $v_0 = \frac{1}{2}gt$, where t is the time the bomb lands. Substituting this result into Equation 2–7 we find

$$v = \tfrac{1}{2}gt - gt = -\tfrac{1}{2}gt = -v_0$$

The advantage of the symbolic solution lies in showing that the result is not a fluke—no matter what the initial velocity, no matter what the acceleration, the bomb lands with the velocity $-v_0$.

These results hint at a symmetry relating the motion on the way up to the motion on the way down. To make this symmetry more apparent, we first solve for the time when the lava bomb lands. Using the result $v_0 = \frac{1}{2}gt$ from Example 2–11, we find

$$t = \frac{2v_0}{g} \qquad \text{(time of landing)}$$

Next, we find the time when the velocity of the lava is zero, which is at its highest point. Setting $v = 0$ in Equation 2–7, we have $v = v_0 - gt = 0$, or

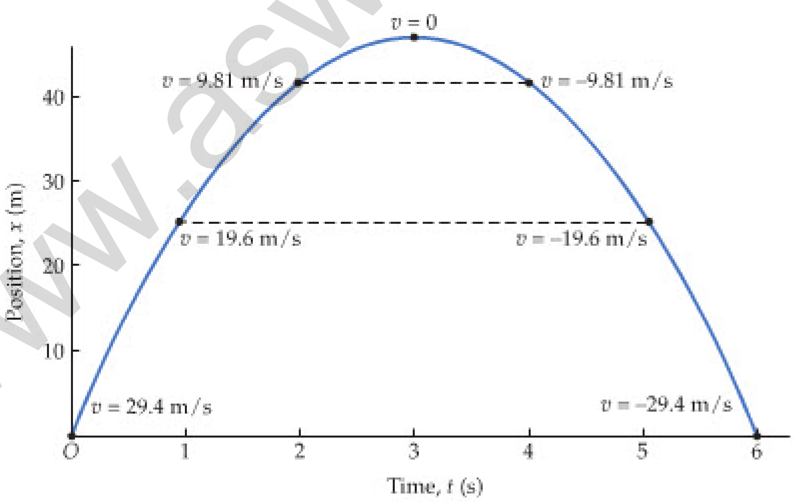
$$t = \frac{v_0}{g} \qquad \text{(time when } v = 0\text{)}$$

Note that this is exactly half the time required for the lava to make the round trip. Thus, the velocity of the lava is zero and the height of the lava is greatest exactly halfway between launch and landing.

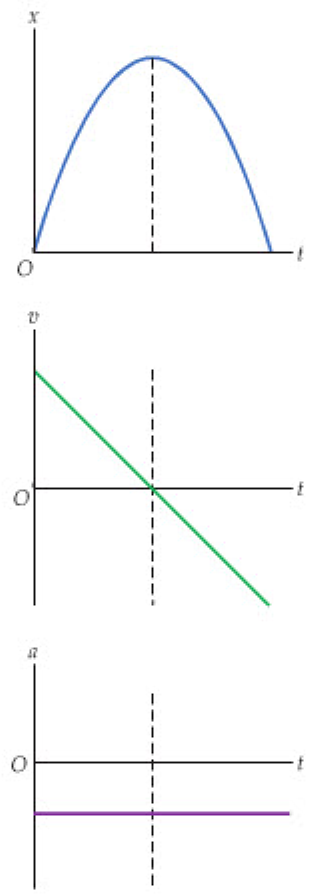
This symmetry is illustrated in **Figure 2–18**. In this case we consider a lava bomb that is in the air for 6.00 s, moving without air resistance. Note that at $t = 3.00$ s the lava is at its highest point and its velocity is zero. At times equally spaced before and after $t = 3.00$ s, the lava is at the same height, has the same speed, but is moving in opposite directions. As a result of this symmetry, a movie of the lava bomb's flight would look the same whether run forward or in reverse.

Figure 2–19 shows the time dependence of position, velocity, and acceleration for an object in free fall without air resistance after being thrown upward. As soon as the object is released, it begins to accelerate downward—as indicated by the negative slope of the velocity-versus-time plot—though it isn't necessarily moving downward. For example, if you throw a ball *upward* it begins to accelerate *downward* the moment it leaves your hand. It continues moving upward, however, until its speed diminishes to zero. Since gravity is causing the downward acceleration, and gravity doesn't turn off just because the ball's velocity goes through zero, the ball continues to accelerate downward even when it is momentarily at rest.

Similarly, in the next Example we consider a sandbag that falls from an ascending hot-air balloon. This means that before the bag is in free fall it was moving upward—just like a ball thrown upward. And just like the ball, the sandbag continues moving upward for a brief time before momentarily stopping and then moving downward.



▲ **FIGURE 2–18 Position and velocity of a lava bomb**
This lava bomb is in the air for 6 seconds. Note the symmetry about the midpoint of the bomb's flight.



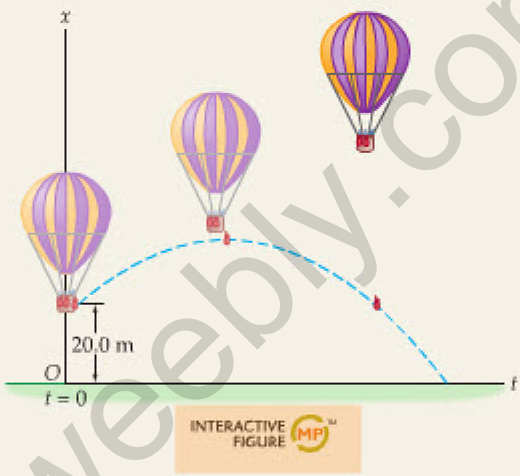
▲ **FIGURE 2–19 Position, velocity, and acceleration of a lava bomb as functions of time**
The fact that x versus t is curved indicates an acceleration; the downward curvature shows that the acceleration is negative. This is also clear from v versus t, which has a negative slope. The constant slope of the straight line in the v-versus-t plot indicates a constant acceleration, as shown in the a-versus-t plot.

EXAMPLE 2–12 LOOK OUT BELOW! A SANDBAG IN FREE FALL

A hot-air balloon is rising straight upward with a constant speed of 6.5 m/s. When the basket of the balloon is 20.0 m above the ground, a bag of sand tied to the basket comes loose. **(a)** How long is the bag of sand in the air before it hits the ground? **(b)** What is the greatest height of the bag of sand during its fall to the ground?

PICTURE THE PROBLEM

We choose the origin to be at ground level and positive to be upward. This means that, for the bag, we have $x_0 = 20.0$ m, $v_0 = 6.5$ m/s, and $a = -g$. Our sketch also shows snapshots of the balloon and bag of sand at three different times, starting at $t = 0$ when the bag comes loose. Note that the bag is moving upward with the balloon at the time it comes loose. It therefore continues to move upward for a short time after it separates from the basket, exactly as if it had been thrown upward.



STRATEGY

The effects of air resistance on the sandbag can be ignored. As a result, we can use the equations in Table 2–4 with a constant acceleration $a = -g$.

In part (a) we want to relate position and time—knowing the initial position and initial velocity—so we use Equation 2–11. To find the time the bag hits the ground, we set $x = 0$ and solve for t.

For part (b) we have no expression that gives the maximum height of a particle—so we will have to come up with something on our own. We can start with the fact that $v = 0$ at the greatest height, since it is there the bag momentarily stops as it changes direction. Therefore, we can find the time t when $v = 0$ by using Equation 2–7, and then substitute t into Equation 2–11 to find x_{max}.

SOLUTION

Part (a)

1. Apply Equation 2–11 to the bag of sand, where x_0 and v_0 have the values given. Set $x = 0$:

$$x = x_0 + v_0 t - \tfrac{1}{2}gt^2 = 0$$

2. Note that we have a quadratic equation for t in the form $v_f = v_i$ where $A = -\frac{1}{2}g$, $B = v_0$, and $C = x_0$. Solve this equation for t. The positive solution, 2.78 s, applies to this problem: (Quadratic equations and their solutions are discussed in Appendix A. In general, one can expect two solutions to a quadratic equation.)

$$t = \frac{-v_0 \pm \sqrt{v_0^2 - 4\left(-\frac{1}{2}g\right)(x_0)}}{2\left(-\frac{1}{2}g\right)}$$

$$= \frac{-(6.5\text{ m/s}) \pm \sqrt{(6.5\text{ m/s})^2 + 2(9.81\text{ m/s}^2)(20.0\text{ m})}}{(-9.81\text{ m/s}^2)}$$

$$= \frac{-(6.5\text{ m/s}) \pm 20.8\text{ m/s}}{(-9.81\text{ m/s}^2)} = 2.78\text{ s}, -1.46\text{ s}$$

Part (b)

3. Apply Equation 2–7 to the bag of sand, then find the time when the velocity equals zero:

$$v = v_0 + at = v_0 - gt$$

$$v_0 - gt = 0 \quad \text{or} \quad t = \frac{v_0}{g} = \frac{6.5\text{ m/s}}{9.81\text{ m/s}^2} = 0.66\text{ s}$$

4. Use $t = 0.66$ s in Equation 2–11 to find the maximum height:

$$x_{max} = 20.0\text{ m} + (6.5\text{ m/s})(0.66\text{ s}) - \tfrac{1}{2}(9.81\text{ m/s}^2)(0.66\text{ s})^2 = 22\text{ m}$$

INSIGHT

The positive solution to the quadratic equation is certainly the one that applies here, but the negative solution is not completely without meaning. What physical meaning might it have? Well, if the balloon had been *descending* with a speed of 6.5 m/s, instead of rising, then the time for the bag to reach the ground would have been 1.46 s. Try it! Let $v_0 = -6.5$ m/s and repeat the calculation given in part (a).

PRACTICE PROBLEM

What is the velocity of the bag of sand just before it hits the ground? [**Answer:** $v = v_0 - gt = (6.5\text{ m/s}) - (9.81\text{ m/s}^2)(2.78\text{ s}) = -20.8$ m/s; the minus sign indicates the bag is moving downward.]

Some related homework problems: Problem 90, Problem 107

THE BIG PICTURE PUTTING PHYSICS IN CONTEXT

LOOKING BACK

In this chapter we have made extensive use of the sign conventions for one-dimensional vectors—positive for one direction, negative for the opposite direction—as introduced in Chapter 1. See, for example, the positive and negative velocities in Figure 2–18.

We have been careful to check the dimensional consistency of our equations in this chapter. For example, the discussion following Equation 2–11 shows that all the terms in that equation have the dimensions of length.

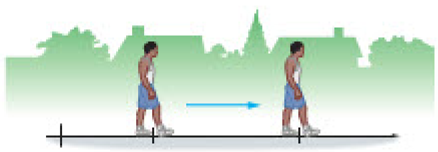
LOOKING AHEAD

The distinctions developed in this chapter between velocity and acceleration will play a key role in our understanding of Newton's laws of motion in Chapters 5 and 6, and everywhere else that Newton's laws are used throughout the text.

The equations developed for motion with constant acceleration in this chapter (Equations 2–7, 2–10, 2–11, and 2–12) will be used again with slightly different symbols when we study rotational motion in Chapter 10. See, in particular, Equations 10–8, 10–9, 10–10, and 10–11.

CHAPTER SUMMARY

2–1 POSITION, DISTANCE, AND DISPLACEMENT



Distance

Total length of travel, from beginning to end. The distance is always positive.

Displacement

Displacement, Δx, is the change in position:

$$\Delta x = x_f - x_i \qquad 2\text{–}1$$

When calculating displacement, it is important to remember that it is always *final* position minus *initial* position—never the other way. Displacement can be positive, negative, or zero.

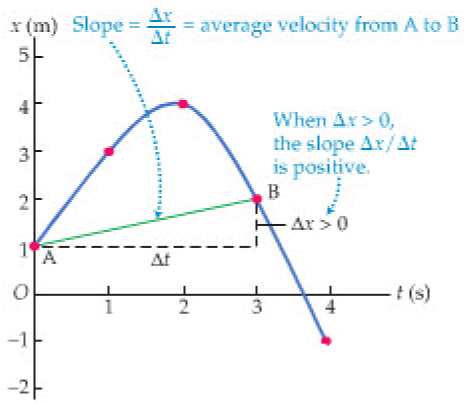
Positive and Negative Displacement

The *sign* of the displacement indicates the *direction* of motion. For example, suppose we choose the positive direction to be to the right. Then $\Delta x > 0$ means motion to the right, and $\Delta x < 0$ means motion to the left.

Units

The SI unit of distance and displacement is the meter, m.

2–2 AVERAGE SPEED AND VELOCITY



Average Speed

Average speed is *distance* divided by elapsed time:

$$\text{average speed} = \text{distance/time} \qquad 2\text{–}2$$

Average speed is never negative.

Average Velocity

Average velocity, v_{av}, is *displacement* divided by time:

$$v_{av} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i} \qquad 2\text{–}3$$

Average velocity is positive if motion is in the positive direction, and negative if motion is in the negative direction.

Graphical Interpretation of Velocity

In an x-versus-t plot, the average velocity is the slope of a line connecting two points.

Units

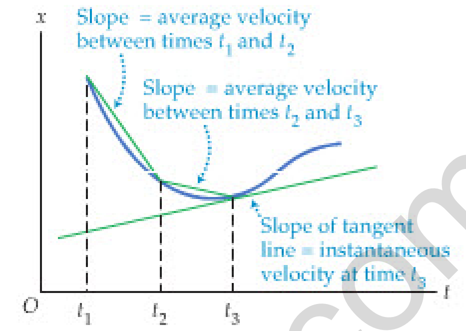
The SI unit of speed and velocity is meters per second, m/s.

2–3 INSTANTANEOUS VELOCITY

The velocity at an instant of time is the limit of the average velocity over shorter and shorter time intervals:

$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} \qquad \text{2–4}$$

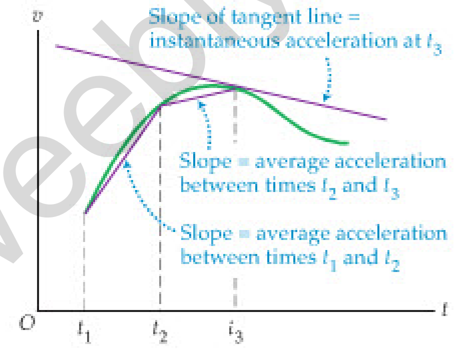
Instantaneous velocity can be positive, negative, or zero, with the sign indicating the direction of motion.



Constant Velocity
When velocity is constant, the instantaneous velocity is equal to the average velocity.

Graphical Interpretation
In an x-versus-t plot, the instantaneous velocity at a given time is equal to the slope of the tangent line at that time.

2–4 ACCELERATION



Average Acceleration
Average acceleration is the change in velocity divided by the change in time:

$$a_{av} = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{t_f - t_i} \qquad \text{2–5}$$

Average acceleration is positive if $v_f > v_i$, is negative if $v_f < v_i$, and is zero if $v_f = v_i$.

Instantaneous Acceleration
Instantaneous acceleration is the limit of the average acceleration as the time interval goes to zero:

$$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} \qquad \text{2–6}$$

Instantaneous acceleration can be positive, negative, or zero, depending on whether the velocity is becoming more positive, more negative, or is staying the same. Knowing the sign of the acceleration *does not* tell you whether an object is speeding up or slowing down, and it *does not* give the direction of motion.

Constant Acceleration
When acceleration is constant, the instantaneous acceleration is equal to the average acceleration.

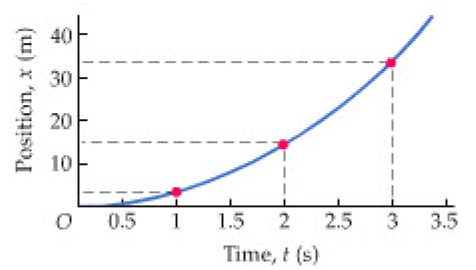
Deceleration
An object whose speed is decreasing is said to be decelerating. Deceleration occurs whenever the velocity and acceleration have opposite signs.

Graphical Interpretation
In a v-versus-t plot, the instantaneous acceleration is equal to the slope of the tangent line at a given time.

Units
The SI unit of acceleration is meters per second per second, or m/s^2.

2–5 MOTION WITH CONSTANT ACCELERATION

Several different "equations of motion" describe particles moving with constant acceleration. Each equation relates a different set of variables:



Velocity as a Function of Time

$$v = v_0 + at \qquad \text{2–7}$$

Initial, Final, and Average Velocity

$$v_{av} = \tfrac{1}{2}(v_0 + v) \qquad \text{2–9}$$

Position as a Function of Time and Velocity

$$x = x_0 + \tfrac{1}{2}(v_0 + v)t \qquad \text{2–10}$$

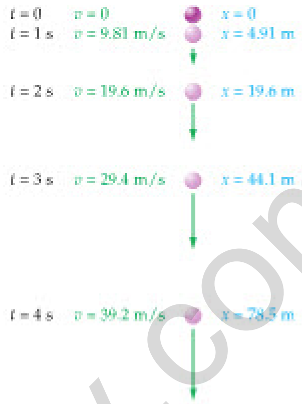
Position as a Function of Time and Acceleration

$$x = x_0 + v_0 t + \tfrac{1}{2}at^2 \qquad \text{2–11}$$

Velocity as a Function of Position

$$v^2 = v_0^2 + 2a(x - x_0) = v_0^2 + 2a\Delta x \qquad \text{2–12}$$

2–7 FREELY FALLING OBJECTS



Objects in free fall move under the influence of gravity alone. An object is in free fall as soon as it is released, whether it is thrown upward, thrown downward, or released from rest.

Acceleration Due to Gravity

The acceleration due to gravity on the Earth's surface varies slightly from place to place. In this book we shall define the acceleration of gravity to have the following magnitude:

$$g = 9.81 \text{ m/s}^2$$

Note that g is always a positive quantity. If we choose the positive direction of our coordinate system to be downward (in the direction of the acceleration of gravity), it follows that the acceleration of an object in free fall is $a = +g$. On the other hand, if we choose our positive direction to be upward, the acceleration of a freely falling object is in the negative direction; hence $a = -g$.

PROBLEM-SOLVING SUMMARY

Type of Calculation	Relevant Physical Concepts	Related Examples
Relate velocity to time.	In motion with uniform acceleration a, the velocity changes with time as $v = v_0 + at$ (Equation 2–7).	Examples 2–5, 2–8, 2–9, 2–10, 2–11, 2–12
Relate velocity to position.	If an object with an initial velocity v_0 accelerates with a uniform acceleration a for a distance Δx, the final velocity, v, is given by $v^2 = v_0{}^2 + 2a\Delta x$ (Equation 2–12).	Examples 2–7, 2–8, 2–10
Relate position to time.	The position of an object moving with constant acceleration a varies with time as follows: $x = x_0 + \frac{1}{2}(v_0 + v)t$ (Equation 2–10) or equivalently $x = x_0 + v_0 t + \frac{1}{2}at^2$ (Equation 2–11).	Examples 2–5, 2–6, 2–9, 2–10, 2–11, 2–12

CONCEPTUAL QUESTIONS

For instructor-assigned homework, go to www.masteringphysics.com MP™

(Answers to odd-numbered Conceptual Questions can be found in the back of the book.)
(The effects of air resistance are to be ignored in this chapter.)

1. You and your dog go for a walk to a nearby park. On the way, your dog takes many short side trips to chase squirrels, examine fire hydrants, and so on. When you arrive at the park, do you and your dog have the same displacement? Have you traveled the same distance? Explain.
2. Does an odometer in a car measure distance or displacement? Explain.
3. Can you drive your car in such a way that the distance it covers is **(a)** greater than, **(b)** equal to, or **(c)** less than the magnitude of its displacement? In each case, give an example if your answer is yes, explain why not if your answer is no.
4. An astronaut orbits Earth in the space shuttle. In one complete orbit, is the magnitude of the displacement the same as the distance traveled? Explain.
5. After a tennis match the players dash to the net to congratulate one another. If they both run with a speed of 3 m/s, are their velocities equal? Explain.
6. Does a speedometer measure speed or velocity? Explain.
7. Is it possible for a car to circle a race track with constant velocity? Can it do so with constant speed? Explain.
8. Friends tell you that on a recent trip their average velocity was +20 m/s. Is it possible that their instantaneous velocity was negative at any time during the trip? Explain.
9. For what kind of motion are the instantaneous and average velocities equal?
10. If the position of an object is zero, does its speed have to be zero? Explain.
11. Assume that the brakes in your car create a constant deceleration, regardless of how fast you are going. If you double your driving speed, how does this affect **(a)** the time required to come to a stop, and **(b)** the distance needed to stop?
12. The velocity of an object is zero at a given instant of time. **(a)** Is it possible for the object's acceleration to be zero at this time? Explain. **(b)** Is it possible for the object's acceleration to be nonzero at this time? Explain.
13. If the velocity of an object is nonzero, can its acceleration be zero? Give an example if your answer is yes, explain why not if your answer is no.
14. Is it possible for an object to have zero average velocity over a given interval of time, yet still be accelerating during the interval? Give an example if your answer is yes, explain why not if your answer is no.
15. A batter hits a pop fly straight up. **(a)** Is the acceleration of the ball on the way up different from its acceleration on the way down? **(b)** Is the acceleration of the ball at the top of its flight different from its acceleration just before it lands?

16. A person on a trampoline bounces straight upward with an initial speed of 4.5 m/s. What is the person's speed when she returns to her initial height?

17. After winning a baseball game, one player drops a glove, while another tosses a glove into the air. How do the accelerations of the two gloves compare?

18. A volcano shoots a lava bomb straight upward. Does the displacement of the lava bomb depend on **(a)** your choice of origin for your coordinate system, or **(b)** your choice of a positive direction? Explain in each case.

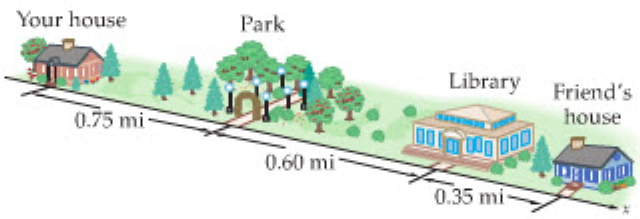
PROBLEMS AND CONCEPTUAL EXERCISES

Note: Answers to odd-numbered Problems and Conceptual Exercises can be found in the back of the book. **IP** *denotes an integrated problem, with both conceptual and numerical parts;* **BIO** *identifies problems of biological or medical interest;* **CE** *indicates a conceptual exercise.* **Predict/Explain** *problems ask for two responses:* **(a)** *your prediction of a physical outcome, and* **(b)** *the best explanation among three provided. On all problems, red bullets (•, ••, •••) are used to indicate the level of difficulty.*

(The effects of air resistance are to be ignored in this chapter.)

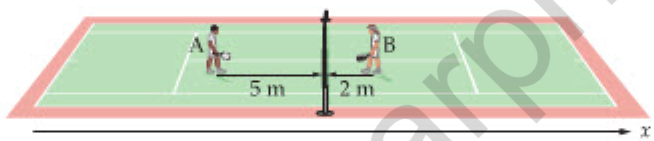
SECTION 2–1 POSITION, DISTANCE, AND DISPLACEMENT

1. • Referring to Figure 2–20, you walk from your home to the library, then to the park. **(a)** What is the distance traveled? **(b)** What is your displacement?



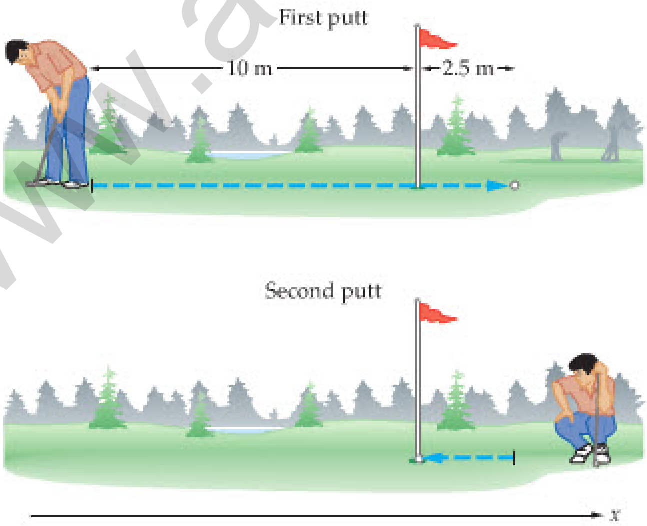
▲ **FIGURE 2–20** Problems 1 and 4

2. • The two tennis players shown in Figure 2–21 walk to the net to congratulate one another. **(a)** Find the distance traveled and the displacement of player A. **(b)** Repeat for player B.



▲ **FIGURE 2–21** Problem 2

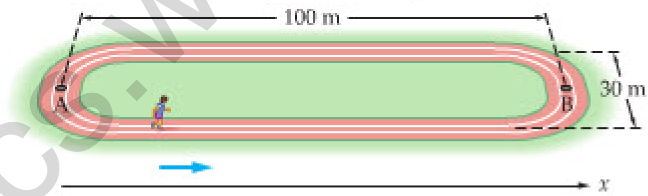
3. • The golfer in Figure 2–22 sinks the ball in two putts, as shown. What are **(a)** the distance traveled by the ball, and **(b)** the displacement of the ball?



▲ **FIGURE 2–22** Problem 3

4. • In Figure 2–20, you walk from the park to your friend's house, then back to your house. What is your **(a)** distance traveled, and **(b)** displacement?

5. • A jogger runs on the track shown in Figure 2–23. Neglecting the curvature of the corners, **(a)** what is the distance traveled and the displacement in running from point A to point B? **(b)** Find the distance and displacement for a complete circuit of the track.



▲ **FIGURE 2–23** Problem 5

6. •• **IP** A child rides a pony on a circular track whose radius is 4.5 m. **(a)** Find the distance traveled and the displacement after the child has gone halfway around the track. **(b)** Does the distance traveled increase, decrease, or stay the same when the child completes one circuit of the track? Explain. **(c)** Does the displacement increase, decrease, or stay the same when the child completes one circuit of the track? Explain. **(d)** Find the distance and displacement after a complete circuit of the track.

SECTION 2–2 AVERAGE SPEED AND VELOCITY

7. • **CE Predict/Explain** You drive your car in a straight line at 15 m/s for 10 kilometers, then at 25 m/s for another 10 kilometers. **(a)** Is your average speed for the entire trip more than, less than, or equal to 20 m/s? **(b)** Choose the *best* explanation from among the following:

I. More time is spent at 15 m/s than at 25 m/s.
II. The average of 15 m/s and 25 m/s is 20 m/s.
III. Less time is spent at 15 m/s than at 25 m/s.

8. • **CE Predict/Explain** You drive your car in a straight line at 15 m/s for 10 minutes, then at 25 m/s for another 10 minutes. **(a)** Is your average speed for the entire trip more than, less than, or equal to 20 m/s? **(b)** Choose the *best* explanation from among the following:

I. More time is required to drive at 15 m/s than at 25 m/s.
II. Less distance is covered at 25 m/s than at 15 m/s.
III. Equal time is spent at 15 m/s and 25 m/s.

9. • Joseph DeLoach of the United States set an Olympic record in 1988 for the 200-meter dash with a time of 19.75 seconds. What was his average speed? Give your answer in meters per second and miles per hour.

10. • In 1992 Zhuang Yong of China set a women's Olympic record in the 100-meter freestyle swim with a time of 54.64 seconds. What was her average speed in m/s and mi/h?

11. • **BIO** Kangaroos have been clocked at speeds of 65 km/h. **(a)** How far can a kangaroo hop in 3.2 minutes at this speed? **(b)** How long will it take a kangaroo to hop 0.25 km at this speed?

12. • **Rubber Ducks** A severe storm on January 10, 1992, caused a cargo ship near the Aleutian Islands to spill 29,000 rubber ducks and other bath toys into the ocean. Ten months later hundreds of rubber ducks began to appear along the shoreline near Sitka, Alaska, roughly 1600 miles away. What was the approximate average speed of the ocean current that carried the ducks to shore in **(a)** m/s and **(b)** mi/h? (Rubber ducks from the same spill began to appear on the coast of Maine in July 2003.)

13. • Radio waves travel at the speed of light, approximately 186,000 miles per second. How long does it take for a radio message to travel from Earth to the Moon and back? (See the inside back cover for the necessary data.)

14. • It was a dark and stormy night, when suddenly you saw a flash of lightning. Three-and-a-half seconds later you heard the thunder. Given that the speed of sound in air is about 340 m/s, how far away was the lightning bolt?

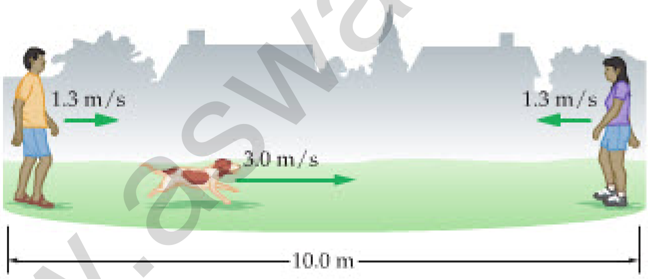
15. • **BIO Nerve Impulses** The human nervous system can propagate nerve impulses at about 10^2 m/s. Estimate the time it takes for a nerve impulse generated when your finger touches a hot object to travel to your brain.

16. • Estimate how fast your hair grows in miles per hour.

17. •• A finch rides on the back of a Galapagos tortoise, which walks at the stately pace of 0.060 m/s. After 1.2 minutes the finch tires of the tortoise's slow pace, and takes flight in the same direction for another 1.2 minutes at 12 m/s. What was the average speed of the finch for this 2.4-minute interval?

18. •• You jog at 9.5 km/h for 8.0 km, then you jump into a car and drive an additional 16 km. With what average speed must you drive your car if your average speed for the entire 24 km is to be 22 km/h?

19. •• A dog runs back and forth between its two owners, who are walking toward one another (Figure 2–24). The dog starts running when the owners are 10.0 m apart. If the dog runs with a speed of 3.0 m/s, and the owners each walk with a speed of 1.3 m/s, how far has the dog traveled when the owners meet?



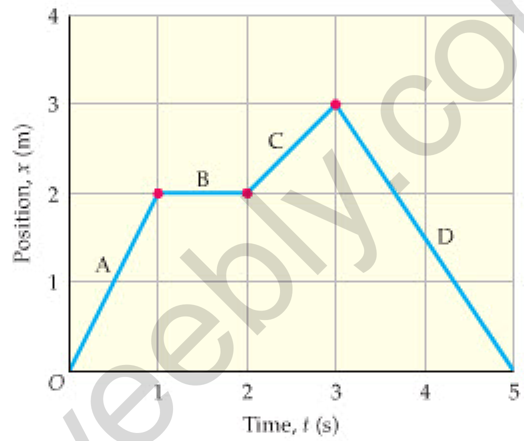
▲ **FIGURE 2–24** Problem 19

20. •• **IP** You drive in a straight line at 20.0 m/s for 10.0 minutes, then at 30.0 m/s for another 10.0 minutes. **(a)** Is your average speed 25.0 m/s, more than 25.0 m/s, or less than 25.0 m/s? Explain. **(b)** Verify your answer to part (a) by calculating the average speed.

21. •• In heavy rush-hour traffic you drive in a straight line at 12 m/s for 1.5 minutes, then you have to stop for 3.5 minutes, and finally you drive at 15 m/s for another 2.5 minutes. **(a)** Plot a position-versus-time graph for this motion. Your plot should extend from $t = 0$ to $t = 7.5$ minutes. **(b)** Use your plot from part (a) to calculate the average velocity between $t = 0$ and $t = 7.5$ minutes.

22. •• **IP** You drive in a straight line at 20.0 m/s for 10.0 miles, then at 30.0 m/s for another 10.0 miles. **(a)** Is your average speed 25.0 m/s, more than 25.0 m/s, or less than 25.0 m/s? Explain. **(b)** Verify your answer to part (a) by calculating the average speed.

23. •• **IP** An expectant father paces back and forth, producing the position-versus-time graph shown in Figure 2–25. Without performing a calculation, indicate whether the father's velocity is positive, negative, or zero on each of the following segments of the graph: **(a)** A, **(b)** B, **(c)** C, and **(d)** D. Calculate the numerical value of the father's velocity for the segments **(e)** A, **(f)** B, **(g)** C, and **(h)** D, and show that your results verify your answers to parts (a)–(d).

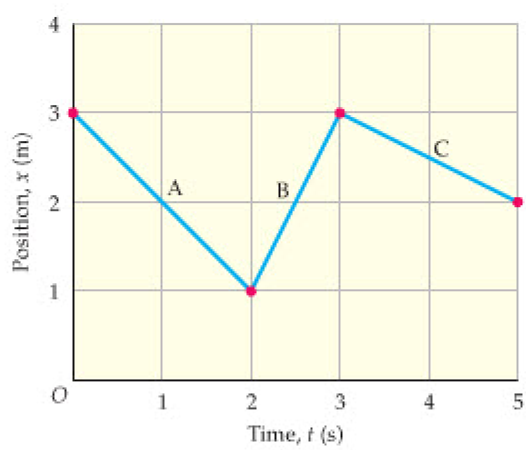


▲ **FIGURE 2–25** Problem 23

24. •• The position of a particle as a function of time is given by $x = (-5\text{ m/s})t + (3\text{ m/s}^2)t^2$. **(a)** Plot x versus t for $t = 0$ to $t = 2$ s. **(b)** Find the average velocity of the particle from $t = 0$ to $t = 1$ s. **(c)** Find the average speed from $t = 0$ to $t = 1$ s.

25. •• The position of a particle as a function of time is given by $x = (6\text{ m/s})t + (-2\text{ m/s}^2)t^2$. **(a)** Plot x versus t for $t = 0$ to $t = 2$ s. **(b)** Find the average velocity of the particle from $t = 0$ to $t = 1$ s. **(c)** Find the average speed from $t = 0$ to $t = 1$ s.

26. •• **IP** A tennis player moves back and forth along the baseline while waiting for her opponent to serve, producing the position-versus-time graph shown in Figure 2–26. **(a)** Without performing a calculation, indicate on which of the segments of the graph, A, B, or C, the player has the greatest speed. Calculate the player's speed for **(b)** segment A, **(c)** segment B, and **(d)** segment C, and show that your results verify your answers to part (a).

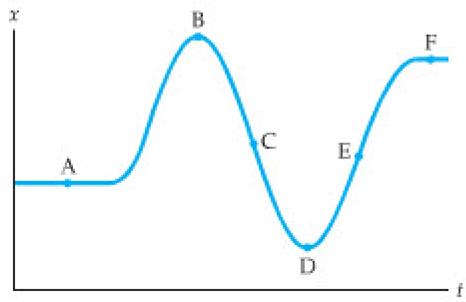


▲ **FIGURE 2–26** Problem 26

27. ••• On your wedding day you leave for the church 30.0 minutes before the ceremony is to begin, which should be plenty of time since the church is only 10.0 miles away. On the way, however, you have to make an unanticipated stop for construction work on the road. As a result, your average speed for the first 15 minutes is only 5.0 mi/h. What average speed do you need for the rest of the trip to get you to the church on time?

SECTION 2–3 INSTANTANEOUS VELOCITY

28. • **CE** The position-versus-time plot of a boat positioning itself next to a dock is shown in Figure 2–27. Rank the six points indicated in the plot in order of increasing value of the velocity v, starting with the most negative. Indicate a tie with an equal sign.



▲ **FIGURE 2–27** Problem 28

29. •• The position of a particle as a function of time is given by $x = (2.0\text{ m/s})t + (-3.0\text{ m/s}^3)t^3$. **(a)** Plot x versus t for time from $t = 0$ to $t = 1.0$ s. **(b)** Find the average velocity of the particle from $t = 0.35$ s to $t = 0.45$ s. **(c)** Find the average velocity from $t = 0.39$ s to $t = 0.41$ s. **(d)** Do you expect the instantaneous velocity at $t = 0.40$ s to be closer to 0.54 m/s, 0.56 m/s, or 0.58 m/s? Explain.

30. •• The position of a particle as a function of time is given by $x = (-2.00\text{ m/s})t + (3.00\text{ m/s}^3)t^3$. **(a)** Plot x versus t for time from $t = 0$ to $t = 1.00$ s. **(b)** Find the average velocity of the particle from $t = 0.150$ s to $t = 0.250$ s. **(c)** Find the average velocity from $t = 0.190$ s to $t = 0.210$ s. **(d)** Do you expect the instantaneous velocity at $t = 0.200$ s to be closer to -1.62 m/s, or -1.66 m/s? Explain.

SECTION 2–4 ACCELERATION

31. • **CE Predict/Explain** Two bows shoot identical arrows with the same launch speed. To accomplish this, the string in bow 1 must be pulled back farther when shooting its arrow than the string in bow 2. **(a)** Is the acceleration of the arrow shot by bow 1 greater than, less than, or equal to the acceleration of the arrow shot by bow 2? **(b)** Choose the *best explanation* from among the following:
   I. The arrow in bow 2 accelerates for a greater time.
   II. Both arrows start from rest.
   III. The arrow in bow 1 accelerates for a greater time.

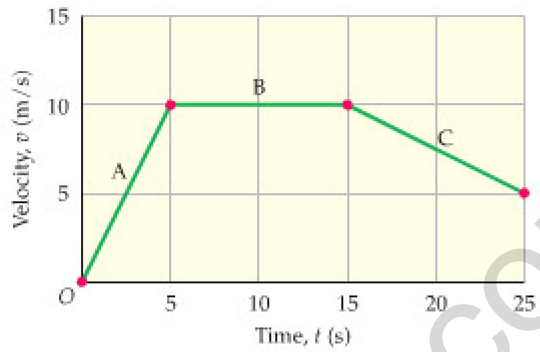
32. • A 747 airliner reaches its takeoff speed of 173 mi/h in 35.2 s. What is the magnitude of its average acceleration?

33. • At the starting gun, a runner accelerates at 1.9 m/s^2 for 5.2 s. The runner's acceleration is zero for the rest of the race. What is the speed of the runner **(a)** at $t = 2.0$ s, and **(b)** at the end of the race?

34. • A jet makes a landing traveling due east with a speed of 115 m/s. If the jet comes to rest in 13.0 s, what are the magnitude and direction of its average acceleration?

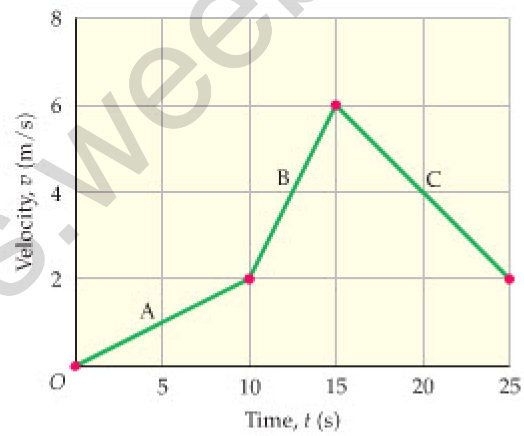
35. • A car is traveling due north at 18.1 m/s. Find the velocity of the car after 7.50 s if its acceleration is **(a)** 1.30 m/s^2 due north, or **(b)** 1.15 m/s^2 due south.

36. •• A motorcycle moves according to the velocity-versus-time graph shown in Figure 2–28. Find the average acceleration of the motorcycle during each of the following segments of the motion: **(a)** A, **(b)** B, and **(c)** C.



▲ **FIGURE 2–28** Problem 36

37. •• A person on horseback moves according to the velocity-versus-time graph shown in Figure 2–29. Find the displacement of the person for each of the following segments of the motion: **(a)** A, **(b)** B, and **(c)** C.



▲ **FIGURE 2–29** Problem 37

38. •• Running with an initial velocity of +11 m/s, a horse has an average acceleration of -1.81 m/s^2. How long does it take for the horse to decrease its velocity to +6.5 m/s?

39. •• **IP** Assume that the brakes in your car create a constant deceleration of 4.2 m/s^2 regardless of how fast you are driving. If you double your driving speed from 16 m/s to 32 m/s, **(a)** does the time required to come to a stop increase by a factor of two or a factor of four? Explain. Verify your answer to part (a) by calculating the stopping times for initial speeds of **(b)** 16 m/s and **(c)** 32 m/s.

40. •• **IP** In the previous problem, **(a)** does the distance needed to stop increase by a factor of two or a factor of four? Explain. Verify your answer to part (a) by calculating the stopping distances for initial speeds of **(b)** 16 m/s and **(c)** 32 m/s.

41. •• As a train accelerates away from a station, it reaches a speed of 4.7 m/s in 5.0 s. If the train's acceleration remains constant, what is its speed after an additional 6.0 s has elapsed?

42. •• A particle has an acceleration of +6.24 m/s^2 for 0.300 s. At the end of this time the particle's velocity is +9.31 m/s. What was the particle's initial velocity?

SECTION 2–5 MOTION WITH CONSTANT ACCELERATION

43. • Landing with a speed of 81.9 m/s, and traveling due south, a jet comes to rest in 949 m. Assuming the jet slows with constant acceleration, find the magnitude and direction of its acceleration.

44. • When you see a traffic light turn red, you apply the brakes until you come to a stop. If your initial speed was 12 m/s, and

you were heading due west, what was your average velocity during braking? Assume constant deceleration.

45. **CE** •• A ball is released at the point $x = 2$ m on an inclined plane with a nonzero initial velocity. After being released, the ball moves with constant acceleration. The acceleration and initial velocity of the ball are described by one of the following four cases: case 1, $a > 0$, $v_0 > 0$; case 2, $a > 0$, $v_0 < 0$; case 3, $a < 0$, $v_0 > 0$; case 4, $a < 0$, $v_0 < 0$. **(a)** In which of these cases will the ball definitely pass $x = 0$ at some later time? **(b)** In which of these cases is more information needed to determine whether the ball will cross $x = 0$? **(c)** In which of these cases will the ball come to rest momentarily at some time during its motion?

46. •• Suppose the car in Problem 44 comes to rest in 35 m. How much time does this take?

47. •• Starting from rest, a boat increases its speed to 4.12 m/s with constant acceleration. **(a)** What is the boat's average speed? **(b)** If it takes the boat 4.77 s to reach this speed, how far has it traveled?

48. •• **IP BIO** A cheetah can accelerate from rest to 25.0 m/s in 6.22 s. Assuming constant acceleration, **(a)** how far has the cheetah run in this time? **(b)** After sprinting for just 3.11 s, is the cheetah's speed 12.5 m/s, more than 12.5 m/s, or less than 12.5 m/s? Explain. **(c)** What is the cheetah's average speed for the first 3.11 s of its sprint? For the second 3.11 s of its sprint? **(d)** Calculate the distance covered by the cheetah in the first 3.11 s and the second 3.11 s.

SECTION 2–6 APPLICATIONS OF THE EQUATIONS OF MOTION

49. • A child slides down a hill on a toboggan with an acceleration of 1.8 m/s^2. If she starts at rest, how far has she traveled in **(a)** 1.0 s, **(b)** 2.0 s, and **(c)** 3.0 s?

50. • **The Detonator** On a ride called the Detonator at Worlds of Fun in Kansas City, passengers accelerate straight downward from rest to 45 mi/h in 2.2 seconds. What is the average acceleration of the passengers on this ride?

The Detonator (Problem 50)

51. • **Air Bags** Air bags are designed to deploy in 10 ms. Estimate the acceleration of the front surface of the bag as it expands. Express your answer in terms of the acceleration of gravity g.

52. • **Jules Verne** In his novel *From the Earth to the Moon* (1866), Jules Verne describes a spaceship that is blasted out of a cannon, called the *Columbiad*, with a speed of 12,000 yards/s. The *Columbiad* is 900 ft long, but part of it is packed with powder, so the spaceship accelerates over a distance of only 700 ft. Estimate the acceleration experienced by the occupants of the spaceship during launch. Give your answer in m/s^2. (Verne realized that the "travelers would . . . encounter a violent recoil," but he probably didn't know that people generally lose consciousness if they experience accelerations greater than about $7g \sim 70$ m/s^2.)

53. •• **BIO Bacterial Motion** Approximately 0.1% of the bacteria in an adult human's intestines are *Escherichia coli*. These bacteria have been observed to move with speeds up to 15 μm/s and maximum accelerations of 166 μm/s^2. Suppose an *E. coli* bacterium in your intestines starts at rest and accelerates at 156 μm/s^2. How much **(a)** time and **(b)** distance are required for the bacterium to reach a speed of 12 μm/s?

54. •• Two cars drive on a straight highway. At time $t = 0$, car 1 passes mile marker 0 traveling due east with a speed of 20.0 m/s. At the same time, car 2 is 1.0 km east of mile marker 0 traveling at 30.0 m/s due west. Car 1 is speeding up with an acceleration of magnitude 2.5 m/s^2, and car 2 is slowing down with an acceleration of magnitude 3.2 m/s^2. **(a)** Write x-versus-t equations of motion for both cars, taking east as the positive direction. **(b)** At what time do the cars pass next to one another?

55. •• **A Meteorite Strikes** On October 9, 1992, a 27-pound meteorite struck a car in Peekskill, NY, leaving a dent 22 cm deep in the trunk. If the meteorite struck the car with a speed of 130 m/s, what was the magnitude of its deceleration, assuming it to be constant?

56. •• A rocket blasts off and moves straight upward from the launch pad with constant acceleration. After 3.0 s the rocket is at a height of 77 m. **(a)** What are the magnitude and direction of the rocket's acceleration? **(b)** What is its speed at this time?

57. •• **IP** You are driving through town at 12.0 m/s when suddenly a ball rolls out in front of you. You apply the brakes and begin decelerating at 3.5 m/s^2. **(a)** How far do you travel before stopping? **(b)** When you have traveled only half the distance in part (a), is your speed 6.0 m/s, greater than 6.0 m/s, or less than 6.0 m/s? Support your answer with a calculation.

58. •• **IP** You are driving through town at 16 m/s when suddenly a car backs out of a driveway in front of you. You apply the brakes and begin decelerating at 3.2 m/s^2. **(a)** How much time does it take to stop? **(b)** After braking half the time found in part (a), is your speed 8.0 m/s, greater than 8.0 m/s, or less than 8.0 m/s? Support your answer with a calculation. **(c)** If the car backing out was initially 55 m in front of you, what is the maximum reaction time you can have before hitting the brakes and still avoid hitting the car?

59. •• **IP BIO A Tongue's Acceleration** When a chameleon captures an insect, its tongue can extend 16 cm in 0.10 s. **(a)** Find the magnitude of the tongue's acceleration, assuming it to be constant. **(b)** In the first 0.050 s, does the tongue extend 8.0 cm, more than 8.0 cm, or less than 8.0 cm? Support your conclusion with a calculation.

It's not polite to reach! (Problem 59)

60. •• **IP** Coasting due west on your bicycle at 8.4 m/s, you encounter a sandy patch of road 7.2 m across. When you leave the sandy patch your speed has been reduced by 2.0 m/s to 6.4 m/s. **(a)** Assuming the sand causes a constant acceleration, what was the bicycle's acceleration in the sandy patch? Give both magnitude and direction. **(b)** How long did it take to cross the sandy patch? **(c)** Suppose you enter the sandy patch with a speed of only 5.4 m/s. Is your final speed in this case 3.4 m/s, more than 3.4 m/s, or less than 3.4 m/s? Explain.

61. •• **BIO Surviving a Large Deceleration** On July 13, 1977, while on a test drive at Britain's Silverstone racetrack, the throttle on David Purley's car stuck wide open. The resulting crash subjected Purley to the greatest "g-force" ever survived by a human—he decelerated from 173 km/h to zero in a distance of only about 0.66 m. Calculate the magnitude of the acceleration experienced by Purley (assuming it to be constant), and express your answer in units of the acceleration of gravity, $g = 9.81\ m/s^2$.

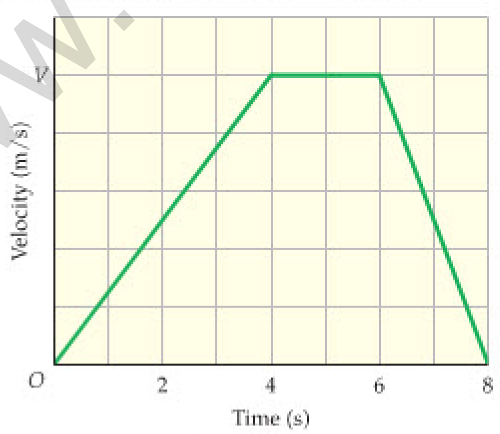
62. •• **IP** A boat is cruising in a straight line at a constant speed of 2.6 m/s when it is shifted into neutral. After coasting 12 m the engine is engaged again, and the boat resumes cruising at the reduced constant speed of 1.6 m/s. Assuming constant acceleration while coasting, **(a)** how long did it take for the boat to coast the 12 m? **(b)** What was the boat's acceleration while it was coasting? **(c)** When the boat had coasted for 6.0 m, was its speed 2.1 m/s, more than 2.1 m/s, or less than 2.1 m/s? Explain.

63. •• A model rocket rises with constant acceleration to a height of 3.2 m, at which point its speed is 26.0 m/s. **(a)** How much time does it take for the rocket to reach this height? **(b)** What was the magnitude of the rocket's acceleration? **(c)** Find the height and speed of the rocket 0.10 s after launch.

64. •• The infamous chicken is dashing toward home plate with a speed of 5.8 m/s when he decides to hit the dirt. The chicken slides for 1.1 s, just reaching the plate as he stops (safe, of course). **(a)** What are the magnitude and direction of the chicken's acceleration? **(b)** How far did the chicken slide?

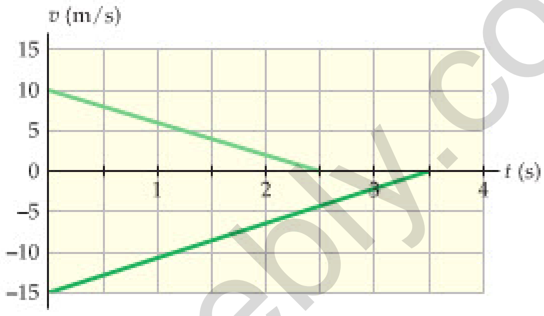
65. •• A bicyclist is finishing his repair of a flat tire when a friend rides by with a constant speed of 3.5 m/s. Two seconds later the bicyclist hops on his bike and accelerates at 2.4 m/s^2 until he catches his friend. **(a)** How much time does it take until he catches his friend? **(b)** How far has he traveled in this time? **(c)** What is his speed when he catches up?

66. •• A car in stop-and-go traffic starts at rest, moves forward 13 m in 8.0 s, then comes to rest again. The velocity-versus-time plot for this car is given in **Figure 2–30**. What distance does the car cover in **(a)** the first 4.0 seconds of its motion and **(b)** the last 2.0 seconds of its motion? **(c)** What is the constant speed V that characterizes the middle portion of its motion?



▲ **FIGURE 2–30** Problem 66

67. ••• A car and a truck are heading directly toward one another on a straight and narrow street, but they avoid a head-on collision by simultaneously applying their brakes at $t = 0$. The resulting velocity-versus-time graphs are shown in **Figure 2–31**. What is the separation between the car and the truck when they have come to rest, given that at $t = 0$ the car is at $x = 15$ m and the truck is at $x = -35$ m? (Note that this information determines which line in the graph corresponds to which vehicle.)



▲ **FIGURE 2–31** Problem 67

68. ••• In a physics lab, students measure the time it takes a small cart to slide a distance of 1.00 m on a smooth track inclined at an angle θ above the horizontal. Their results are given in the following table.

θ	**10.0°**	**20.0°**	**30.0°**
time, s	1.08	0.770	0.640

(a) Find the magnitude of the cart's acceleration for each angle. **(b)** Show that your results for part (a) are in close agreement with the formula, $a = g \sin\theta$. (We will derive this formula in Chapter 5.)

SECTION 2–7 FREELY FALLING OBJECTS

(Problems 71 and 72)

69. • **CE** At the edge of a roof you throw ball 1 upward with an initial speed v_0; a moment later you throw ball 2 downward with the same initial speed. The balls land at the same time. Which of the following statements is true for the instant just before the balls hit the ground? **A.** The speed of ball 1 is greater than the speed of ball 2; **B.** The speed of ball 1 is equal to the speed of ball 2; **C.** The speed of ball 1 is less than the speed of ball 2.

70. • Legend has it that Isaac Newton was hit on the head by a falling apple, thus triggering his thoughts on gravity. Assuming the story to be true, estimate the speed of the apple when it struck Newton.

71. • The cartoon shows a car in free fall. Is the statement made in the cartoon accurate? Justify your answer.

72. • Referring to the cartoon in Problem 71, how long would it take for the car to go from 0 to 30 mi/h?

73. • **Jordan's Jump** Michael Jordan's vertical leap is reported to be 48 inches. What is his takeoff speed? Give your answer in meters per second.

74. • **BIO** Gulls are often observed dropping clams and other shellfish from a height to the rocks below, as a means of opening the shells. If a seagull drops a shell from rest at a height of 14 m, how fast is the shell moving when it hits the rocks?

75. • A volcano launches a lava bomb straight upward with an initial speed of 28 m/s. Taking upward to be the positive direction, find the speed and direction of motion of the lava bomb **(a)** 2.0 seconds and **(b)** 3.0 seconds after it is launched.

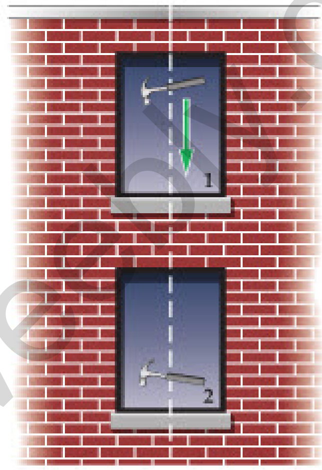
76. • **An Extraterrestrial Volcano** The first active volcano observed outside the Earth was discovered in 1979 on Io, one of the moons of Jupiter. The volcano was observed to be ejecting material to a height of about 2.00×10^5 m. Given that the acceleration of gravity on Io is 1.80 m/s^2, find the initial velocity of the ejected material.

77. • **BIO Measure Your Reaction Time** Here's something you *can* try at home—an experiment to measure your reaction time. Have a friend hold a ruler by one end, letting the other end hang down vertically. At the lower end, hold your thumb and index finger on either side of the ruler, ready to grip it. Have your friend release the ruler without warning. Catch it as quickly as you can. If you catch the ruler 5.2 cm from the lower end, what is your reaction time?

How fast are your reactions? (Problem 77)

78. •• **CE Predict/Explain** A carpenter on the roof of a building accidentally drops her hammer. As the hammer falls it passes two windows of equal height, as shown in Figure 2–32. **(a)** Is the *increase* in speed of the hammer as it drops past window 1 greater than, less than, or equal to the *increase* in speed as it drops past window 2? **(b)** Choose the *best explanation* from among the following:

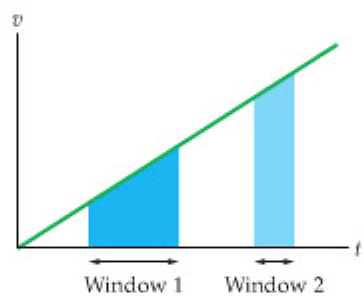
I. The greater speed at window 2 results in a greater increase in speed.
II. Constant acceleration means the hammer speeds up the same amount for each window.
III. The hammer spends more time dropping past window 1.



▲ **FIGURE 2–32** Problem 78

79. •• **CE Predict/Explain** Figure 2–33 shows a v-versus-t plot for the hammer dropped by the carpenter in Problem 78. Notice that the times when the hammer passes the two windows are indicated by shaded areas. **(a)** Is the area of the shaded region corresponding to window 1 greater than, less than, or equal to the area of the shaded region corresponding to window 2? **(b)** Choose the *best explanation* from among the following:

I. The shaded area for window 2 is higher than the shaded area for window 1.
II. The windows are equally tall.
III. The shaded area for window 1 is wider than the shaded area for window 2.



▲ **FIGURE 2–33** Problem 79

80. •• **CE** A ball is thrown straight upward with an initial speed v_0. When it reaches the top of its flight at height h, a second ball is thrown straight upward with the same initial speed. Do the balls cross paths at height $\frac{1}{2}h$, above $\frac{1}{2}h$, or below $\frac{1}{2}h$?

81. •• Bill steps off a 3.0-m-high diving board and drops to the water below. At the same time, Ted jumps upward with a speed of 4.2 m/s from a 1.0-m-high diving board. Choosing the origin to be at the water's surface, and upward to be the positive x direction, write x-versus-t equations of motion for both Bill and Ted.

82. •• Repeat the previous problem, this time with the origin 3.0 m above the water, and with downward as the positive x direction.

83. •• On a hot summer day in the state of Washington while kayaking, I saw several swimmers jump from a railroad bridge into the Snohomish River below. The swimmers stepped off the bridge, and I estimated that they hit the water 1.5 s later. **(a)** How high was the bridge? **(b)** How fast were the swimmers moving when they hit the water? **(c)** What would the swimmers' drop time be if the bridge were twice as high?

84. •• **Highest Water Fountain** The world's highest fountain of water is located, appropriately enough, in Fountain Hills, Arizona. The fountain rises to a height of 560 ft (5 feet higher than the Washington Monument). **(a)** What is the initial speed of the water? **(b)** How long does it take for water to reach the top of the fountain?

85. •• Wrongly called for a foul, an angry basketball player throws the ball straight down to the floor. If the ball bounces straight up and returns to the floor 2.8 s after first striking it, what was the ball's greatest height above the floor?

86. •• To celebrate a victory, a pitcher throws her glove straight upward with an initial speed of 6.0 m/s. **(a)** How long does it take for the glove to return to the pitcher? **(b)** How long does it take for the glove to reach its maximum height?

87. •• **IP** Standing at the edge of a cliff 32.5 m high, you drop a ball. Later, you throw a second ball downward with an initial speed of 11.0 m/s. **(a)** Which ball has the greater *increase* in speed when it reaches the base of the cliff, or do both balls speed up by the same amount? **(b)** Verify your answer to part (a) with a calculation.

88. •• You shoot an arrow into the air. Two seconds later (2.00 s) the arrow has gone straight upward to a height of 30.0 m above its launch point. **(a)** What was the arrow's initial speed? **(b)** How long did it take for the arrow to first reach a height of 15.0 m above its launch point?

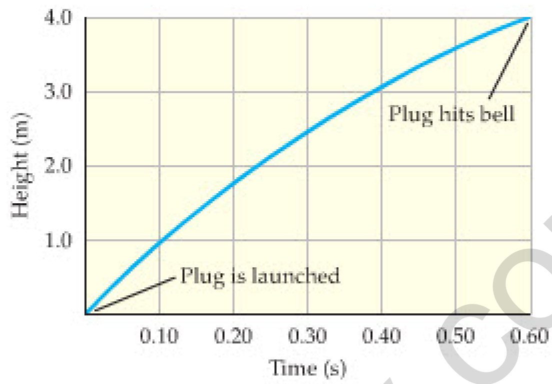
89. •• While riding on an elevator descending with a constant speed of 3.0 m/s, you accidentally drop a book from under your arm. **(a)** How long does it take for the book to reach the elevator floor, 1.2 m below your arm? **(b)** What is the book's speed relative to you when it hits the elevator floor?

90. •• A hot-air balloon is descending at a rate of 2.0 m/s when a passenger drops a camera. If the camera is 45 m above the ground when it is dropped, **(a)** how long does it take for the camera to reach the ground, and **(b)** what is its velocity just before it lands? Let upward be the positive direction for this problem.

91. •• **IP** Standing side by side, you and a friend step off a bridge at different times and fall for 1.6 s to the water below. Your friend goes first, and you follow after she has dropped a distance of 2.0 m. **(a)** When your friend hits the water, is the separation between the two of you 2.0 m, less than 2.0 m, or more than 2.0 m? **(b)** Verify your answer to part (a) with a calculation.

92. •• A model rocket blasts off and moves upward with an acceleration of 12 m/s^2 until it reaches a height of 26 m, at which point its engine shuts off and it continues its flight in free fall. **(a)** What is the maximum height attained by the rocket? **(b)** What is the speed of the rocket just before it hits the ground? **(c)** What is the total duration of the rocket's flight?

93. ••• **Hitting the "High Striker"** A young woman at a carnival steps up to the "high striker," a popular test of strength where the contestant hits one end of a lever with a mallet, propelling a small metal plug upward toward a bell. She gives the mallet a mighty swing and sends the plug to the top of the striker, where it rings the bell. Figure 2–34 shows the corresponding position-versus-time plot for the plug. Using the information given in the plot, answer the following questions: **(a)** What is the average speed of the plug during its upward journey? **(b)** By how much does the speed of the plug decrease during its upward journey? **(c)** What is the initial speed of the plug? (Assume the plug to be in free fall during its upward motion, with no effects of air resistance or friction.)



▲ **FIGURE 2–34** Problem 93

94. ••• While sitting on a tree branch 10.0 m above the ground, you drop a chestnut. When the chestnut has fallen 2.5 m, you throw a second chestnut straight down. What initial speed must you give the second chestnut if they are both to reach the ground at the same time?

GENERAL PROBLEMS

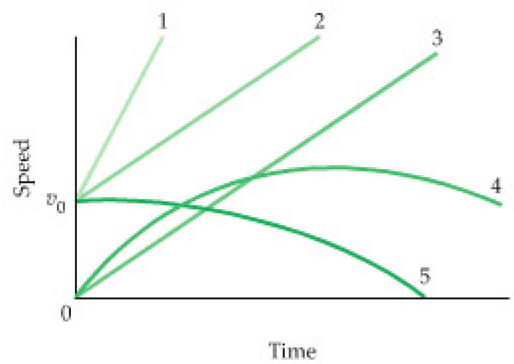
95. • In a well-known Jules Verne novel, Phileas Fogg travels around the world in 80 days. What was Mr. Fogg's approximate average speed during his adventure?

96. • An astronaut on the Moon drops a rock straight downward from a height of 1.25 m. If the acceleration of gravity on the Moon is 1.62 m/s^2, what is the speed of the rock just before it lands?

97. • You jump from the top of a boulder to the ground 1.5 m below. Estimate your deceleration on landing.

98. • **A Supersonic Waterfall** Geologists have learned of periods in the past when the Strait of Gibraltar closed off, and the Mediterranean Sea dried out and become a desert. Later, when the strait reopened, a massive saltwater waterfall was created. According to geologists, the water in this waterfall was supersonic; that is, it fell with speeds in excess of the speed of sound. Ignoring air resistance, what is the minimum height necessary to create a supersonic waterfall? (The speed of sound may be taken to be 340 m/s.)

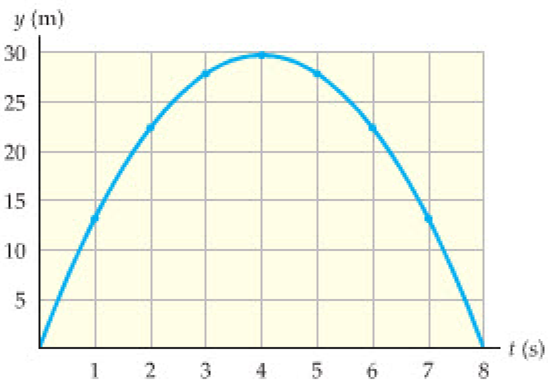
99. •• **CE** At the edge of a roof you drop ball A from rest, and then throw ball B downward with an initial velocity of v_0. Is the increase in speed just before the balls land more for ball A, more for ball B, or the same for each ball?



▲ **FIGURE 2–35** Problem 100

100. •• **CE** Suppose the two balls described in Problem 99 are released at the same time, with ball A dropped from rest and ball B thrown downward with an initial speed v_0. Identify which of the five plots shown in Figure 2–35 corresponds to **(a)** ball A and **(b)** ball B.

101. •• Astronauts on a distant planet throw a rock straight upward and record its motion with a video camera. After digitizing their video, they are able to produce the graph of height, y, versus time, t, shown in Figure 2–36. **(a)** What is the acceleration of gravity on this planet? **(b)** What was the initial speed of the rock?



▲ **FIGURE 2–36** Problem 101

102. •• **Drop Tower** NASA operates a 2.2-second drop tower at the Glenn Research Center in Cleveland, Ohio. At this facility, experimental packages are dropped from the top of the tower, on the 8th floor of the building. During their 2.2 seconds of free fall, experiments experience a microgravity environment similar to that of a spacecraft in orbit. **(a)** What is the drop distance of a 2.2-s tower? **(b)** How fast are the experiments traveling when they hit the air bags at the bottom of the tower? **(c)** If the experimental package comes to rest over a distance of 0.75 m upon hitting the air bags, what is the average stopping acceleration?

103. •• **IP** A youngster bounces straight up and down on a trampoline. Suppose she doubles her initial speed from 2.0 m/s to 4.0 m/s. **(a)** By what factor does her time in the air increase? **(b)** By what factor does her maximum height increase? **(c)** Verify your answers to parts (a) and (b) with an explicit calculation.

104. •• At the 18th green of the U.S. Open you need to make a 20.5-ft putt to win the tournament. When you hit the ball, giving it an initial speed of 1.57 m/s, it stops 6.00 ft short of the hole. **(a)** Assuming the deceleration caused by the grass is constant, what should the initial speed have been to just make the putt? **(b)** What initial speed do you need to make the remaining 6.00-ft putt?

105. •• **IP** A popular entertainment at some carnivals is the blanket toss (see photo, p. 39). **(a)** If a person is thrown to a maximum height of 28.0 ft above the blanket, how long does she spend in the air? **(b)** Is the amount of time the person is above a height of 14.0 ft more than, less than, or equal to the amount of time the person is below a height of 14.0 ft? Explain. **(c)** Verify your answer to part (b) with a calculation.

106. •• Referring to Conceptual Checkpoint 2–5, find the separation between the rocks at **(a)** $t = 1.0$ s, **(b)** $t = 2.0$ s, and **(c)** $t = 3.0$ s, where time is measured from the instant the second rock is dropped. **(d)** Verify that the separation increases linearly with time.

107. •• **IP** A glaucous-winged gull, ascending straight upward at 5.20 m/s, drops a shell when it is 12.5 m above the ground. **(a)** What are the magnitude and direction of the shell's acceleration just after it is released? **(b)** Find the maximum height above the ground reached by the shell. **(c)** How long does it take for the shell to reach the ground? **(d)** What is the speed of the shell at this time?

108. •• A doctor, preparing to give a patient an injection, squirts a small amount of liquid straight upward from a syringe. If the liquid emerges with a speed of 1.5 m/s, **(a)** how long does it take for it to return to the level of the syringe? **(b)** What is the maximum height of the liquid above the syringe?

109. •• A hot-air balloon has just lifted off and is rising at the constant rate of 2.0 m/s. Suddenly one of the passengers realizes she has left her camera on the ground. A friend picks it up and tosses it straight upward with an initial speed of 13 m/s. If the passenger is 2.5 m above her friend when the camera is tossed, how high is she when the camera reaches her?

110. ••• In the previous problem, what is the minimum initial speed of the camera if it is to just reach the passenger? (*Hint:* When the camera is thrown with its minimum speed, its speed on reaching the passenger is the same as the speed of the passenger.)

111. ••• **Old Faithful** Watching Old Faithful erupt, you notice that it takes a time t for water to emerge from the base of the geyser and reach its maximum height. **(a)** What is the height of the geyser, and **(b)** what is the initial speed of the water? Evaluate your expressions for **(c)** the height and **(d)** the initial speed for a measured time of 1.65 s.

112. ••• **IP** A ball is thrown upward with an initial speed v_0. When it reaches the top of its flight, at a height h, a second ball is thrown upward with the same initial velocity. **(a)** Sketch an x-versus-t plot for each ball. **(b)** From your graph, decide whether the balls cross paths at $h/2$, above $h/2$, or below $h/2$. **(c)** Find the height where the paths cross.

113. ••• Weights are tied to each end of a 20.0-cm string. You hold one weight in your hand and let the other hang vertically a height h above the floor. When you release the weight in your hand, the two weights strike the ground one after the other with audible thuds. Find the value of h for which the time between release and the first thud is equal to the time between the first thud and the second thud.

114. ••• A ball, dropped from rest, covers three-quarters of the distance to the ground in the last second of its fall. **(a)** From what height was the ball dropped? **(b)** What was the total time of fall?

115. ••• A stalactite on the roof of a cave drips water at a steady rate to a pool 4.0 m below. As one drop of water hits the pool, a second drop is in the air, and a third is just detaching from the stalactite. **(a)** What are the position and velocity of the second drop when the first drop hits the pool? **(b)** How many drops per minute fall into the pool?

116. ••• You drop a ski glove from a height h onto fresh snow, and it sinks to a depth d before coming to rest. **(a)** In terms of g and h, what is the speed of the glove when it reaches the snow? **(b)** What are the magnitude and direction of the glove's acceleration as it moves through the snow, assuming it to be constant? Give your answer in terms of g, h, and d.

117. ••• To find the height of an overhead power line, you throw a ball straight upward. The ball passes the line on the way up after 0.75 s, and passes it again on the way down 1.5 s after it was tossed. What are the height of the power line and the initial speed of the ball?

118. ••• Suppose the first rock in Conceptual Checkpoint 2–5 drops through a height h before the second rock is released from rest. Show that the separation between the rocks, S, is given by the following expression:

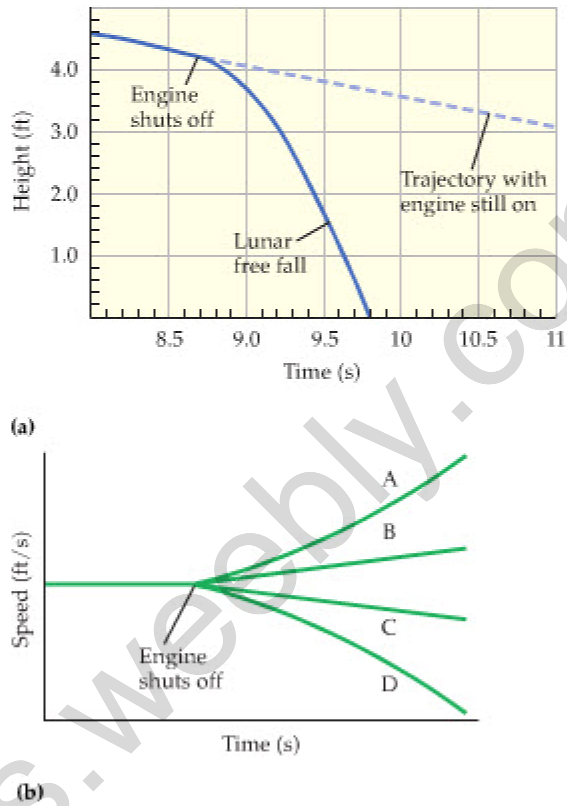
$$S = h + (\sqrt{2gh})t$$

In this result, the time t is measured from the time the second rock is dropped.

119. ••• An arrow is fired with a speed of 20.0 m/s at a block of Styrofoam resting on a smooth surface. The arrow penetrates a certain distance into the block before coming to rest relative to it. During this process the arrow's deceleration has a magnitude of 1550 m/s^2 and the block's acceleration has a magnitude of 450 m/s^2. **(a)** How long does it take for the arrow to stop moving with respect to the block? **(b)** What is the common speed of the arrow and block when this happens? **(c)** How far into the block does the arrow penetrate?

120. ••• Sitting in a second-story apartment, a physicist notices a ball moving straight upward just outside her window. The ball is visible for 0.25 s as it moves a distance of 1.05 m from the bottom to the top of the window. **(a)** How long does it take before the ball reappears? **(b)** What is the greatest height of the ball above the top of the window?

121. ••• **The Quadratic Formula from Kinematics** In this problem we show how the kinematic equations of motion can be used to derive the quadratic formula. First, consider an object with an initial position x_0, an initial velocity v_0, and an acceleration a. To find the time when this object reaches the position $x = 0$ we can use the quadratic formula, or apply the following two-step procedure: **(a)** Use Equation 2–12 to show that the velocity of the object when it reaches $x = 0$ is given by $v = \pm\sqrt{v_0^2 - 2ax_0}$. **(b)** Use Equation 2–7 to show that the time corresponding to the velocity found in part (a) is $t = \dfrac{-v_0 \pm \sqrt{v_0^2 - 2ax_0}}{a}$. **(c)** To complete our derivation, show that the result of part (b) is the same as applying the quadratic formula to $x = x_0 + v_0t + \frac{1}{2}at^2 = 0$.



▲ **FIGURE 2–37** Problems 122, 123, 124, and 125

PASSAGE PROBLEMS

Bam!—*Apollo 15* Lands on the Moon

The first word spoken on the surface of the Moon after *Apollo 15* landed on July 30, 1971, was "Bam!" This was James Irwin's involuntary reaction to their rather bone-jarring touchdown. "We did hit harder than any of the other flights!" says Irwin. "And I was startled, obviously, when I said, 'Bam!'"

The reason for the "firm touchdown" of *Apollo 15*, as pilot David Scott later characterized it, was that the rocket engine was shut off a bit earlier than planned, when the lander was still 4.30 ft above the lunar surface and moving downward with a speed of 0.500 ft/s. From that point on the lander descended in lunar free fall, with an acceleration of 1.62 m/s^2. As a result, the landing speed of *Apollo 15* was by far the largest of any of the *Apollo* missions. In comparison, Neil Armstrong's landing speed on *Apollo 11* was the lowest at 1.7 ft/s—he didn't shut off the engine until the footpads were actually on the surface. *Apollos 12, 14,* and *17* all landed with speeds between 3.0 and 3.5 ft/s.

To better understand the descent of *Apollo 15*, we show its trajectory during the final stages of landing in Figure 2–37 (a). In Figure 2–37 (b) we show a variety of speed-versus-time plots.

122. • How long did it take for the lander to drop the final 4.30 ft to the Moon's surface?

A. 1.18 s **B.** 1.37 s

C. 1.78 s **D.** 2.36 s

123. •• What was the impact speed of the lander when it touched down? Give your answer in feet per second (ft/s), the same units used by the astronauts.

A. 2.41 ft/s **B.** 6.78 ft/s

C. 9.95 ft/s **D.** 10.6 ft/s

124. • Which of the speed-versus-time plots in Figure 2–37 (b) correctly represents the speed of the *Apollo 15* lander?

A B C D

125. • Suppose, instead of shutting off the engine, the astronauts had increased its thrust, giving the lander a small, but constant, upward acceleration. Which speed-versus-time plot in Figure 2–37 (b) would describe this situation?

A B C D

INTERACTIVE PROBLEMS

126. •• **Referring to Example 2–9** Suppose the speeder (red car) is traveling with a constant speed of 25 m/s, and that the maximum acceleration of the police car (blue car) is 3.8 m/s^2. If the police car is to start from rest and catch the speeder in 15 s or less, what is the maximum head-start distance the speeder can have? Measure time from the moment the police car starts.

127. •• **Referring to Example 2–9** The speeder passes the position of the police car with a constant speed of 15 m/s. The police car immediately starts from rest and pursues the speeder with constant acceleration. What acceleration must the police car have if it is to catch the speeder in 7.0 s? Measure time from the moment the police car starts.

128. •• **IP Referring to Example 2–12 (a)** In Example 2–12, the bag of sand is released at 20.0 m and reaches a maximum height of 22 m. If the bag had been released at 30.0 m instead, with everything else remaining the same, would its maximum height be 32 m, greater than 32 m, or less than 32 m? **(b)** Find the speed of the bag just before it lands when it is released from 30.0 m.

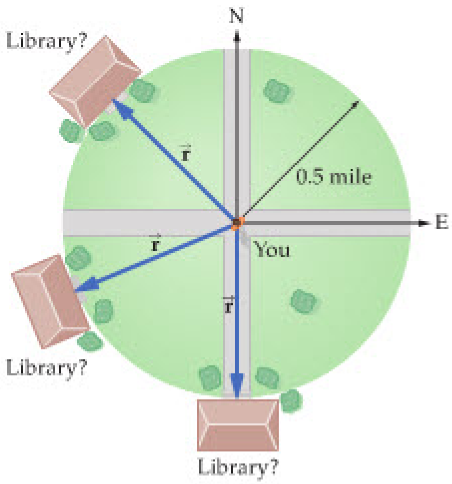
129. •• **Referring to Example 2–12** Suppose the balloon is descending with a constant speed of 4.2 m/s when the bag of sand comes loose at a height of 35 m. **(a)** How long is the bag in the air? **(b)** What is the speed of the bag when it is 15 m above the ground?

3 Vectors in Physics

The points of the compass have long been used as a framework for indicating directions. The compass shown here was produced by Gowin Knight (1713–1772), whose improved designs were adopted by the Royal Navy in 1752. In physics, we more frequently indicate directions with x and y rather than N, S, E, and W. Either way, specifying a direction as well as a magnitude is essential to defining one of the physicist's basic tools, the vector.

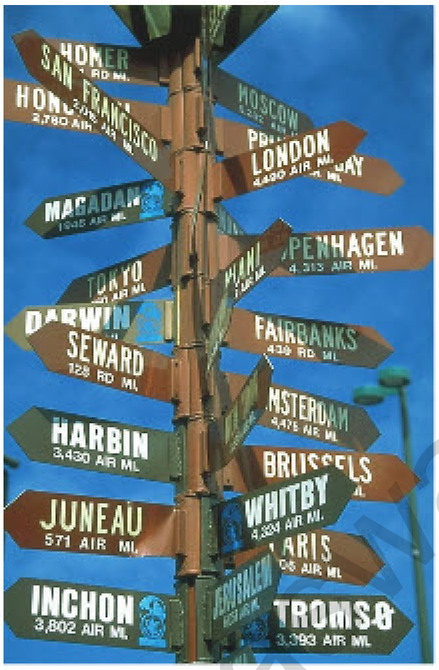
Of all the mathematical tools used in this book, perhaps none is more important than the vector. In the next chapter, for example, we use vectors to extend our study of motion from one dimension to two dimensions. More generally, vectors are *indispensable* when a physical quantity has a direction associated with it. Suppose, for example, that a pilot wants to fly from Denver to Dallas. If the air is still, the pilot can simply head the plane toward the destination. If there is a wind blowing from west to east, however, the pilot must use vectors to determine the correct heading so that the plane and its passengers will arrive in Dallas and not Little Rock.

In this chapter we discuss what a vector is, how it differs from a scalar, and how it can represent a physical quantity. We also show how to find the components of a vector and how to add and subtract vectors. All of these techniques are used time and again throughout the book. Other useful aspects of vectors, such as how to multiply them, will be presented in later chapters when the need arises.



▲ **FIGURE 3–1 Distance and direction**

If you know only that the library is 0.5 mi from you, it could lie anywhere on a circle of radius 0.5 mi. If, instead, you are told the library is 0.5 mi northwest, you know its precise location.



▲ The information given by this sign includes both a distance and a direction for each city. In effect, the sign defines a displacement vector for each of these destinations.

3–1 Scalars Versus Vectors

Numbers can represent many quantities in physics. For example, a numerical value, together with the appropriate units, can specify the volume of a container, the temperature of the air, or the time of an event. In physics, a number with its units is referred to as a **scalar:**

- A scalar is a number with units. It can be positive, negative, or zero.

Sometimes, however, a scalar isn't enough to adequately describe a physical quantity—in many cases, a direction is needed as well. For example, suppose you're walking in an unfamiliar city and you want directions to the library. You ask a passerby, "Do you know where the library is?" If the person replies "Yes," and walks on, he hasn't been too helpful. If he says, "Yes, it is half a mile from here," that is more helpful, but you still don't know where it is. The library could be anywhere on a circle of radius one-half mile, as shown in Figure 3–1. To pin down the location, you need a reply such as, "Yes, the library is half a mile northwest of here." With both a distance *and* a direction, you know the location of the library.

Thus, if you walk northwest for half a mile you arrive at the library, as indicated by the upper left arrow in Figure 3–1. The arrow points in the direction traveled, and its **magnitude,** 0.5 mi in this case, represents the distance covered. In general, a quantity that is specified by both a *magnitude* and a *direction* is represented by a **vector:**

- A vector is a mathematical quantity with both a direction and a magnitude.

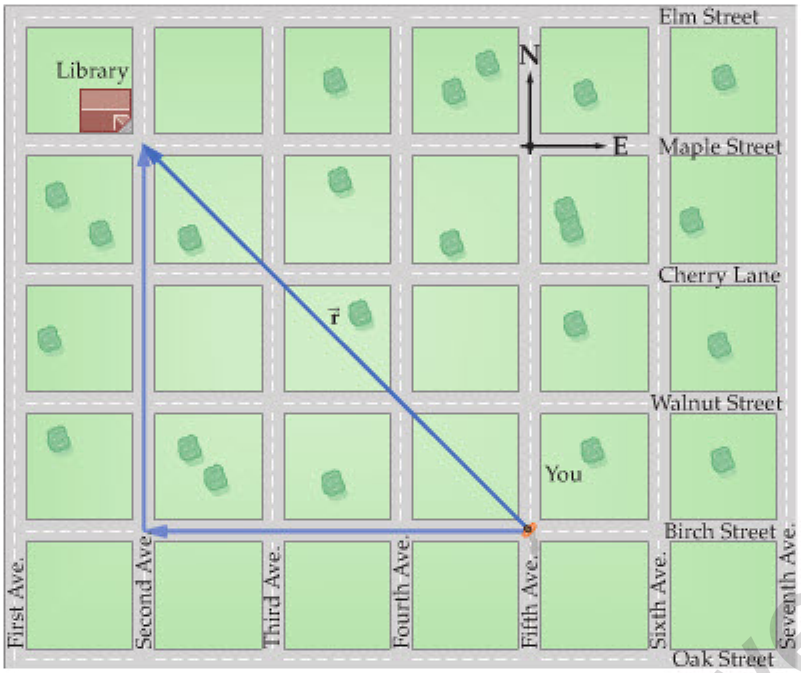
In the example of walking to the library, the vector corresponding to the trip is the displacement vector. Other examples of vector quantities are the velocity and the acceleration of an object. For example, the magnitude of a velocity vector is its speed, and its direction is the direction of motion, as we shall see later in this chapter.

When we indicate a vector on a diagram or a sketch, we draw an arrow, as in Figure 3–1. To indicate a vector with a written symbol, we use **boldface** for the vector itself, with a small arrow above it to remind us of its vector nature, and *italic* for its magnitude. Thus, for example, the upper-left vector in Figure 3–1 is designated by the symbol $\vec{\mathbf{r}}$, and its magnitude is $r = 0.5$ mi. (When we represent a vector in a graph, we sometimes label it with the corresponding boldface symbol, and sometimes with the appropriate magnitude.) It is common in handwritten material to draw a small arrow over the vector's symbol, which is very similar to the way vectors are represented in this text.

3–2 The Components of a Vector

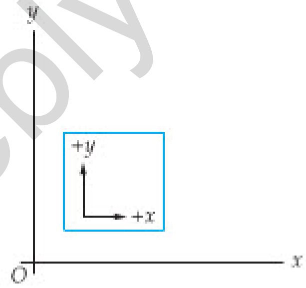
When we discussed directions for finding a library in the previous section, we pointed out that knowing the magnitude and direction angle—0.5 mi northwest—gives its precise location. We left out one key element in actually *getting* to the library, however. In most cities it would not be possible to simply walk in a straight line for 0.5 mi directly to the library, since to do so would take you through buildings where there are no doors, through people's backyards, and through all kinds of other obstructions. In fact, if the city streets are laid out along north–south and east–west directions, you might instead walk west for a certain distance, then turn and proceed north an equal distance until you reach the library, as illustrated in Figure 3–2. What you have just done is "resolved" displacement vector $\vec{\mathbf{r}}$ between you and the library into east–west and north–south "components."

In general, to find the components of a vector we need to set up a coordinate system. In two dimensions we choose an origin, *O*, and a positive direction for both the *x* and the *y* axes, as in Figure 3–3. If the system were three-dimensional, we would also indicate a *z* axis.



▲ **FIGURE 3–2 A walk along city streets to the library**

By taking the indicated path, we have "resolved" the vector $\vec{\mathbf{r}}$ into east–west and north–south components.



▲ **FIGURE 3–3 A two-dimensional coordinate system**

The positive x and y directions are indicated in this shorthand form.

Now, a vector is defined by its magnitude (indicated by the length of the arrow representing the vector) and its direction. For example, suppose an ant leaves its nest at the origin and, after foraging for some time, is at the location given by the vector $\vec{\mathbf{r}}$ in **Figure 3–4 (a)**. This vector has a magnitude $r = 1.50$ m and points in a direction $\theta = 25.0°$ above the x axis. Equivalently, $\vec{\mathbf{r}}$ can be defined by saying that it extends a distance r_x in the x direction and a distance r_y in the y direction, as shown in **Figure 3–4 (b)**. The quantities r_x and r_y are referred to as the x and y **scalar components** of the vector $\vec{\mathbf{r}}$.

We can find r_x and r_y by using standard trigonometric relations, as summarized in the Problem-Solving Note on this page. Referring to Figure 3–4 (b), we see that

$$r_x = r\cos 25.0° = (1.50\text{ m})(0.906) = 1.36\text{ m}$$

and

$$r_y = r\sin 25.0° = (1.50\text{ m})(0.423) = 0.634\text{ m}$$

Thus, we can say that the ant's final displacement is equivalent to what it would be if the ant had simply walked 1.36 m in the x direction and then 0.634 m in the y direction.

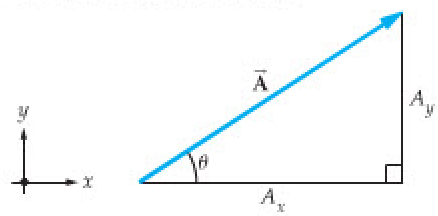
To show the equivalence of these two ways of describing a vector, let's start with the components of $\vec{\mathbf{r}}$, as determined previously, and use them to calculate the magnitude r and the angle θ. First, note that r_x, r_y, and r form a right triangle with r as the hypotenuse. Thus, we can use the Pythagorean theorem (Appendix A) to find r in terms of r_x and r_y. This gives

$$r = \sqrt{r_x^2 + r_y^2} = \sqrt{(1.36\text{ m})^2 + (0.634\text{ m})^2} = \sqrt{2.25\text{ m}^2} = 1.50\text{ m}$$

PROBLEM-SOLVING NOTE

A Vector and Its Components

Given the magnitude and direction of a vector, find its components:

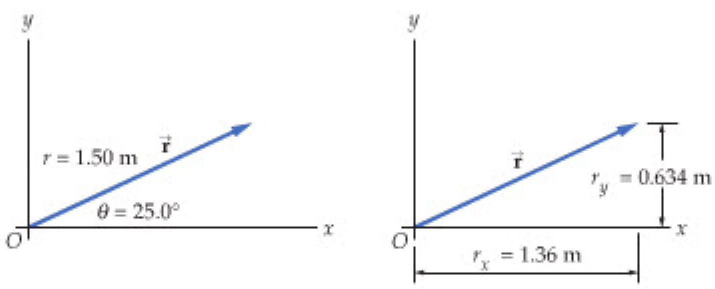


$$A_x = A\cos\theta$$
$$A_y = A\sin\theta$$

Given the components of a vector, find its magnitude and direction:

$$A = \sqrt{A_x^2 + A_y^2}$$

$$\theta = \tan^{-1}\frac{A_y}{A_x}$$



(a) A vector defined in terms of its length and direction angle

(b) The same vector defined in terms of its x and y components

◀ **FIGURE 3–4 A vector and its scalar components**

(a) The vector $\vec{\mathbf{r}}$ is defined by its length ($r = 1.50$ m) and its direction angle ($\theta = 25.0°$) measured counterclockwise from the positive x axis. **(b)** Alternatively, the vector $\vec{\mathbf{r}}$ can be defined by its x component, $r_x = 1.36$ m, and its y component, $r_y = 0.634$ m.

as expected. Second, we can use any two sides of the triangle to obtain the angle θ, as shown in the next three calculations:

$$\theta = \sin^{-1}\left(\frac{0.634 \text{ m}}{1.50 \text{ m}}\right) = \sin^{-1}(0.423) = 25.0^\circ$$

$$\theta = \cos^{-1}\left(\frac{1.36 \text{ m}}{1.50 \text{ m}}\right) = \cos^{-1}(0.907) = 25.0^\circ$$

$$\theta = \tan^{-1}\left(\frac{0.634 \text{ m}}{1.36 \text{ m}}\right) = \tan^{-1}(0.466) = 25.0^\circ$$

In some situations we know a vector's magnitude and direction; in other cases we are given the vector's components. You will find it useful to be able to convert quickly and easily from one description of a vector to the other using trigonometric functions and the Pythagorean theorem.

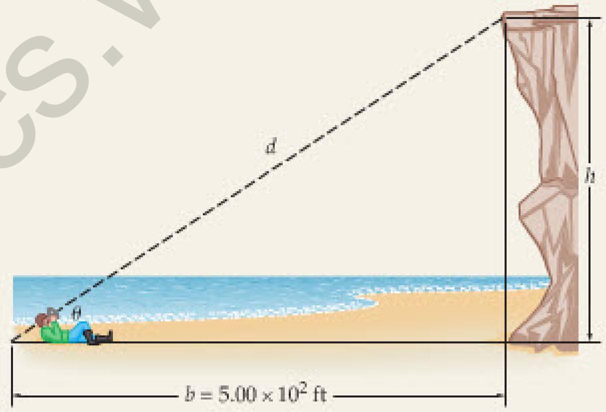
EXAMPLE 3–1 DETERMINING THE HEIGHT OF A CLIFF

REAL-WORLD PHYSICS

In the Jules Verne novel *Mysterious Island*, Captain Cyrus Harding wants to find the height of a cliff. He stands with his back to the base of the cliff, then marches straight away from it for 5.00×10^2 ft. At this point he lies on the ground and measures the angle from the horizontal to the top of the cliff. If the angle is 34.0°, **(a)** how high is the cliff? **(b)** What is the straight-line distance from Captain Harding to the top of the cliff?

PICTURE THE PROBLEM

Our sketch shows Cyrus Harding making his measurement of the angle, $\theta = 34.0^\circ$, to the top of the cliff. The relevant triangle for this problem is also indicated. Note that the opposite side of the triangle is the height of the cliff, h; the adjacent side is the distance from the base of the cliff to Harding, $b = 5.00 \times 10^2$ ft; and finally, the hypotenuse is the distance, d, from Harding to the top of the cliff.



STRATEGY

The tangent of θ is the height of the triangle divided by the base: $\tan\theta = h/b$. Since we know both θ and the base, we can find the height using this relation. Similarly, the distance from Harding to the top of the cliff can be obtained by solving $\cos\theta = b/d$ for d.

SOLUTION

Part (a)

1. Use $\tan\theta = h/b$ to solve for the height of the cliff, h: $h = b\tan\theta = (5.00 \times 10^2 \text{ ft})\tan 34.0^\circ = 337 \text{ ft}$

Part (b)

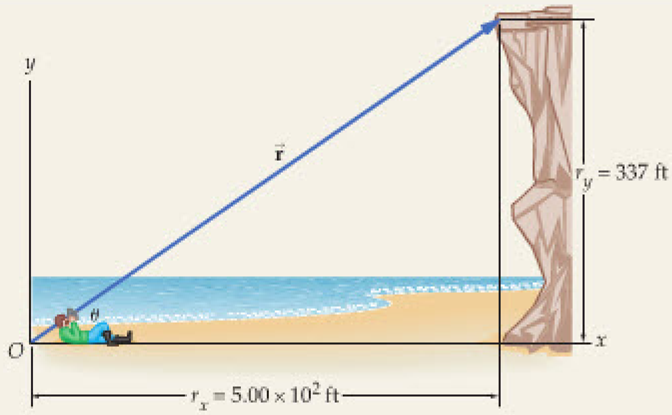
2. Similarly, use $\cos\theta = b/d$ to solve for the distance d from Captain Harding to the top of the cliff: $d = \dfrac{b}{\cos\theta} = \dfrac{5.00 \times 10^2 \text{ ft}}{\cos 34.0^\circ} = 603 \text{ ft}$

INSIGHT

An alternative way to solve part (b) is to use the Pythagorean theorem:

$$d = \sqrt{h^2 + b^2} = \sqrt{(337 \text{ ft})^2 + (5.00 \times 10^2 \text{ ft})^2} = 603 \text{ ft}$$

Thus, if we let $\vec{\mathbf{r}}$ denote the vector from Cyrus Harding to the top of the cliff, as shown here, its magnitude is 603 ft and its direction is 34.0° above the x axis. Alternatively, the x component of $\vec{\mathbf{r}}$ is 5.00×10^2 ft and its y component is 337 ft.



PRACTICE PROBLEM

What angle would Cyrus Harding have found if he had walked 6.00×10^2 ft from the cliff to make his measurement? [**Answer:** $\theta = 29.3^\circ$]

Some related homework problems: Problem 5, Problem 17

EXERCISE 3–1

a. Find A_x and A_y for the vector $\vec{\mathbf{A}}$ with magnitude and direction given by $A = 3.5$ m and $\theta = 66°$, respectively.

b. Find B and θ for the vector $\vec{\mathbf{B}}$ with components $B_x = 75.5$ m and $B_y = 6.20$ m.

SOLUTION

a. $A_x = 1.4$ m, $A_y = 3.2$ m

b. $B = 75.8$ m, $\theta = 4.69°$

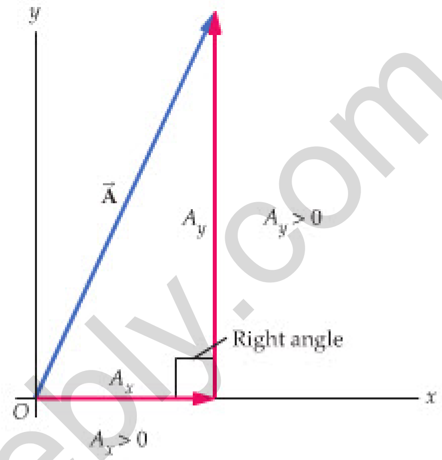


FIGURE 3–5 A vector whose *x* and *y* components are positive

Next, how do you determine the correct signs for the x and y components of a vector? This can be done by considering the right triangle formed by A_x, A_y, and $\vec{\mathbf{A}}$, as shown in **Figure 3–5**. To determine the sign of A_x, start at the tail of the vector and move along the x axis toward the right angle. If you are moving in the positive x direction, then A_x is positive ($A_x > 0$); if you are moving in the negative x direction, then A_x is negative ($A_x < 0$). For the y component, start at the right angle and move toward the tip of the arrow. A_y is positive or negative depending on whether you are moving in the positive or negative y direction.

For example, consider the vector shown in **Figure 3–6 (a)**. In this case, $A_x > 0$ and $A_y < 0$, as indicated in the figure. Similarly, the signs of A_x and A_y are given in **Figure 3–6 (b, c, d)** for the vectors shown there. Be sure to verify each of these cases by applying the rules just given. As we continue our study of physics, it is important to be able to find the components of a vector *and* to assign to them the correct signs.

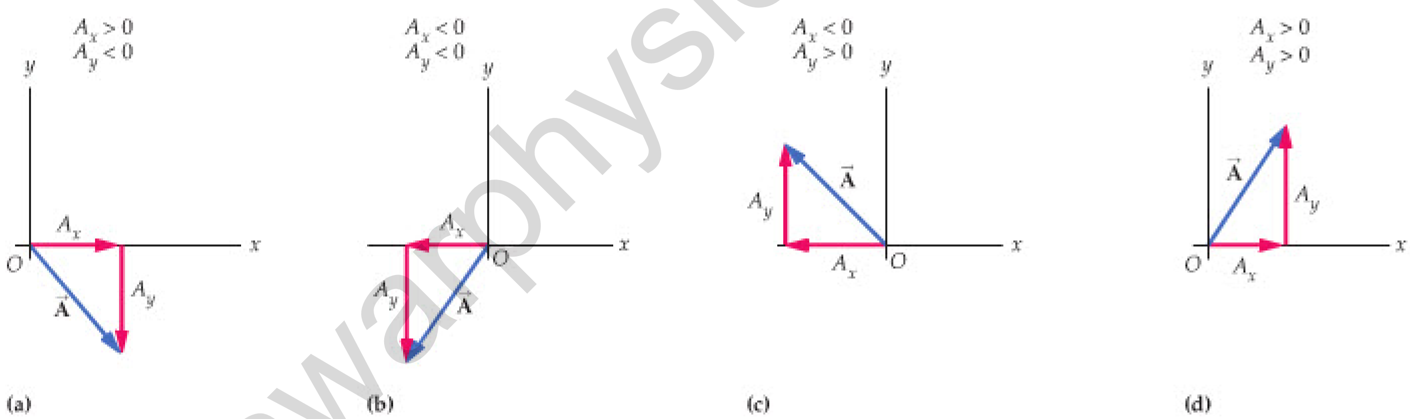


FIGURE 3–6 Examples of vectors with components of different signs

To determine the signs of a vector's components, it is only necessary to observe the direction in which they point. For example, in part (a) the x component points in the positive direction; hence $A_x > 0$. Similarly, the y component in part (a) points in the negative y direction; therefore $A_y < 0$.

EXERCISE 3–2

The vector $\vec{\mathbf{A}}$ has a magnitude of 7.25 m. Find its components for direction angles of

a. $\theta = 5.00°$ **c.** $\theta = 245°$

b. $\theta = 125°$ **d.** $\theta = 335°$

SOLUTION

a. $A_x = 7.22$ m, $A_y = 0.632$ m

b. $A_x = -4.16$ m, $A_y = 5.94$ m

c. $A_x = -3.06$ m, $A_y = -6.57$ m

d. $A_x = 6.57$ m, $A_y = -3.06$ m

Be careful when using your calculator to determine the direction angle, θ, because you may need to add 180° to get the correct angle, as measured counterclockwise from the positive x axis. For example, if $A_x = -0.50$ m and $A_y = 1.0$ m, your calculator will give the following result:

$$\theta = \tan^{-1}\left(\frac{1.0 \text{ m}}{-0.50 \text{ m}}\right) = \tan^{-1}(-2.0) = -63°$$

Does this angle correspond to the specified vector? The way to check is to sketch $\vec{\mathbf{A}}$. When you do, your drawing is similar to Figure 3–6 (c), and thus the direction angle of $\vec{\mathbf{A}}$ should be between 90° and 180°. To obtain the correct angle, add 180° to the calculator's result:

$$\theta = -63° + 180° = 117°$$

This, in fact, is the direction angle for the vector $\vec{\mathbf{A}}$.

EXERCISE 3–3

The vector $\vec{\mathbf{B}}$ has components $B_x = -2.10$ m and $B_y = -1.70$ m. Find the direction angle, θ, for this vector.

SOLUTION

$\tan^{-1}[(-1.70 \text{ m})/(-2.10 \text{ m})] = \tan^{-1}(1.70/2.10) = 39.0°, \theta = 39.0 + 180° = 219°$

Finally, in many situations the direction of a vector $\vec{\mathbf{A}}$ is given by the angle θ, measured relative to the x axis, as in Figure 3–7 (a). In these cases we know that

$$A_x = A \cos\theta$$

and

$$A_y = A \sin\theta$$

On the other hand, we are sometimes given the angle between the vector and the y axis, as in Figure 3–7 (b). If we call this angle θ', then it follows that

$$A_x = A \sin\theta'$$

and

$$A_y = A \cos\theta'$$

These two seemingly different results are actually in complete agreement. Note that $\theta + \theta' = 90°$, or $\theta' = 90° - \theta$. If we use the trigonometric identities given in Appendix A, we find

$$A_x = A \sin\theta' = A \sin(90° - \theta) = A \cos\theta$$

and

$$A_y = A \cos\theta' = A \cos(90° - \theta) = A \sin\theta$$

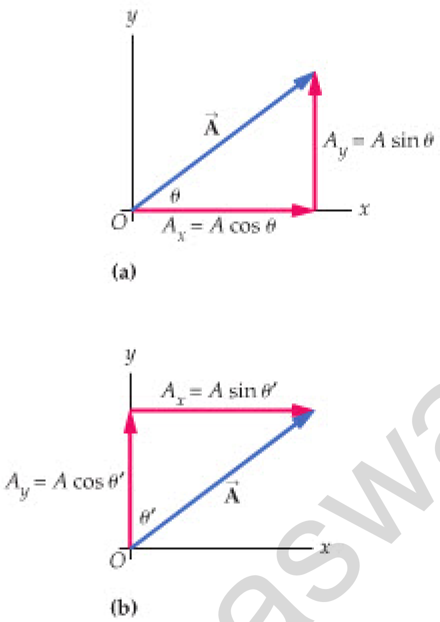


FIGURE 3–7 Vector direction angles
Vector $\vec{\mathbf{A}}$ and its components in terms of **(a)** the angle relative to the x axis and **(b)** the angle relative to the y axis.

EXERCISE 3–4

If a vector's direction angle relative to the x axis is 35°, then its direction angle relative to the y axis is 55°. Find the components of a vector $\vec{\mathbf{A}}$ of magnitude 5.2 m in terms of

a. its direction relative to the x axis, and
b. its direction relative to the y axis.

SOLUTION

a. $A_x = (5.2 \text{ m}) \cos 35° = 4.3$ m, $A_y = (5.2 \text{ m}) \sin 35° = 3.0$ m
b. $A_x = (5.2 \text{ m}) \sin 55° = 4.3$ m, $A_y = (5.2 \text{ m}) \cos 55° = 3.0$ m

3–3 Adding and Subtracting Vectors

One important reason for determining the components of a vector is that they are useful in adding and subtracting vectors. In this section we begin by defining vector addition graphically, and then show how the same addition can be performed more concisely and accurately with components.

Adding Vectors Graphically

One day you open an old chest in the attic and find a treasure map inside. To locate the treasure, the map says that you must "Go to the sycamore tree in the backyard, march 5 paces north, then 3 paces east." If these two displacements are represented by the vectors $\vec{A}$ and $\vec{B}$ in **Figure 3–8**, the total displacement from the tree to the treasure is given by the vector $\vec{C}$. We say that $\vec{C}$ is the *vector sum* of $\vec{A}$ and $\vec{B}$; that is, $\vec{C} = \vec{A} + \vec{B}$. In general, vectors are added graphically according to the following rule:

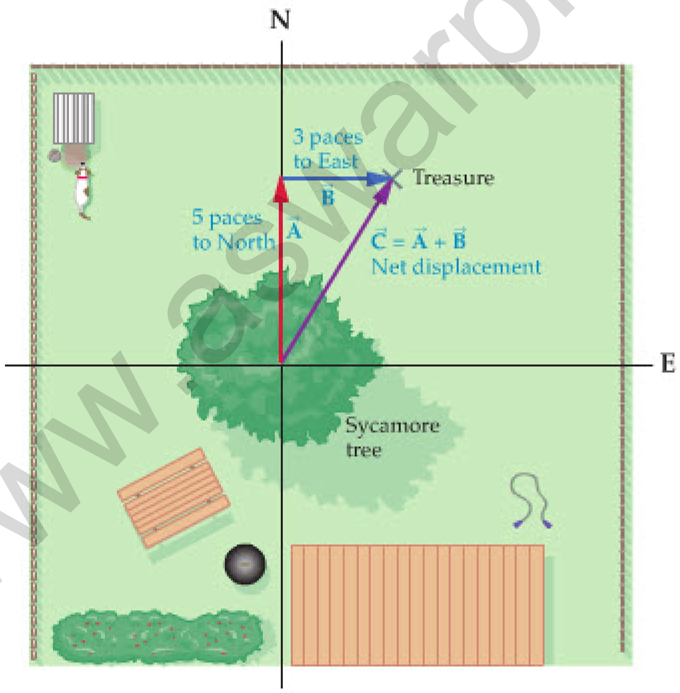
- To add the vectors $\vec{A}$ and $\vec{B}$, place the tail of $\vec{B}$ at the head of $\vec{A}$. The sum, $\vec{C} = \vec{A} + \vec{B}$, is the vector extending from the tail of $\vec{A}$ to the head of $\vec{B}$.

If the instructions to find the treasure were a bit more complicated—5 paces north, 3 paces east, then 4 paces southeast, for example—the path from the sycamore tree to the treasure would be like that shown in **Figure 3–9**. In this case, the total displacement, $\vec{D}$, is the sum of the three vectors $\vec{A}$, $\vec{B}$, and $\vec{C}$; that is, $\vec{D} = \vec{A} + \vec{B} + \vec{C}$. It follows that to add more than two vectors, we just keep placing the vectors head-to-tail, head-to-tail, and then draw a vector from the tail of the first vector to the head of the last vector, as in Figure 3–9.

In order to place a given pair of vectors head-to-tail, it may be necessary to move the corresponding arrows. This is fine, as long as you don't change their length or their direction. After all, a vector is defined by its length and direction; if these are unchanged, so is the vector.

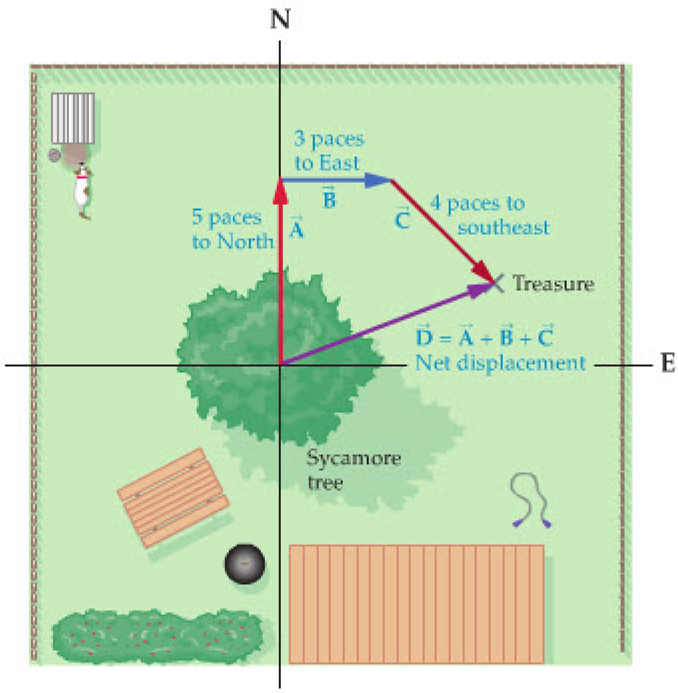
- A vector is defined by its magnitude and direction, regardless of its location.

▲ To a good approximation, these snow geese are all moving in the same direction with the same speed. As a result, their velocity vectors are equal, even though their positions are different.



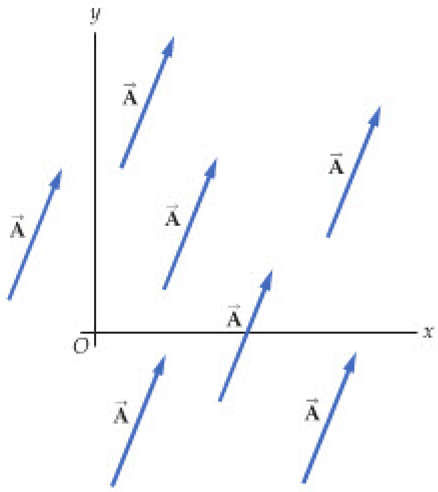
▲ **FIGURE 3–8 The sum of two vectors**

To go from the sycamore tree to the treasure, one must first go 5 paces north ($\vec{A}$) and then 3 paces east ($\vec{B}$). The net displacement from the tree to the treasure is $\vec{C} = \vec{A} + \vec{B}$.



▲ **FIGURE 3–9 Adding several vectors**

Searching for a treasure that is 5 paces north ($\vec{A}$), 3 paces east ($\vec{B}$), and 4 paces southeast ($\vec{C}$) of the sycamore tree. The net displacement from the tree to the treasure is $\vec{D} = \vec{A} + \vec{B} + \vec{C}$.



▲ **FIGURE 3–10 Identical vectors $\vec{A}$ at different locations**

A vector is defined by its direction and length; its location is immaterial.

For example, in Figure 3–10 all of the vectors are the same, even though they are at different locations on the graph.

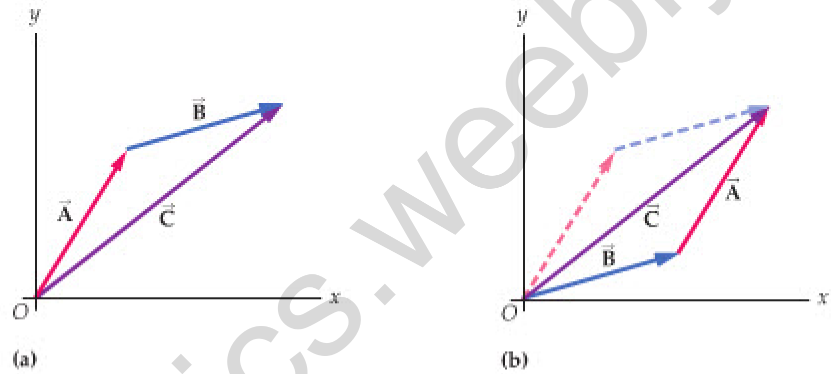
As an example of moving vectors, consider two vectors, $\vec{A}$ and $\vec{B}$, and their vector sum, $\vec{C}$:

$$\vec{C} = \vec{A} + \vec{B}$$

as illustrated in Figure 3–11 (a). By moving the arrow representing $\vec{B}$ so that its tail is at the origin, and moving the arrow for $\vec{A}$ so that its tail is at the head of $\vec{B}$, we obtain the construction shown in Figure 3–11 (b). From this graph we see that $\vec{C}$, which is $\vec{A} + \vec{B}$, is also equal to $\vec{B} + \vec{A}$:

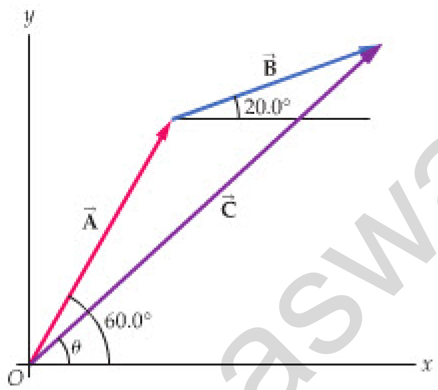
$$\vec{C} = \vec{A} + \vec{B} = \vec{B} + \vec{A}$$

That is, the sum of vectors is independent of the order in which the vectors are added.



▲ **FIGURE 3–11 $\vec{A} + \vec{B} = \vec{B} + \vec{A}$**

The vector $\vec{C}$ is equal to **(a)** $\vec{A} + \vec{B}$ and **(b)** $\vec{B} + \vec{A}$. Note also that $\vec{C}$ is the diagonal of the parallelogram formed by the vectors $\vec{A}$ and $\vec{B}$. For this reason, this method of vector addition is referred to as the "parallelogram method."



▲ **FIGURE 3–12 Graphical addition of vectors**

The vector $\vec{A}$ has a magnitude of 5.00 m and a direction angle of 60.0°; the vector $\vec{B}$ has a magnitude of 4.00 m and a direction angle of 20.0°. The magnitude and direction of $\vec{C} = \vec{A} + \vec{B}$ can be measured on the graph with a ruler and a protractor.

Now, suppose that $\vec{A}$ has a magnitude of 5.00 m and a direction angle of 60.0° above the x axis, and that $\vec{B}$ has a magnitude of 4.00 m and a direction angle of 20.0° above the x axis. These two vectors and their sum, $\vec{C}$, are shown in Figure 3–12. The question is: What are the length and direction angle of $\vec{C}$?

A graphical way to answer this question is to simply measure the length and direction of $\vec{C}$ in Figure 3–12. With a ruler, we find the length of $\vec{C}$ to be approximately 1.75 times the length of $\vec{A}$, which means that $\vec{C}$ is roughly 1.75 (5.00 m) = 8.75 m. Similarly, with a protractor we measure the angle θ to be about 45.0° above the x axis.

Adding Vectors Using Components

The graphical method of adding vectors yields approximate results, limited by the accuracy with which the vectors can be drawn and measured. In contrast, exact results can be obtained by adding $\vec{A}$ and $\vec{B}$ in terms of their components. To see how this is done, consider Figure 3–13 (a), which shows the components of $\vec{A}$ and $\vec{B}$, and Figure 3–13 (b), which shows the components of $\vec{C}$. Clearly,

$$C_x = A_x + B_x$$

and

$$C_y = A_y + B_y$$

Thus, to add vectors, you simply add the components.

Returning to our example in Figure 3–12, the components of $\vec{A}$ and $\vec{B}$ are

$$A_x = (5.00 \text{ m}) \cos 60.0° = 2.50 \text{ m} \quad A_y = (5.00 \text{ m}) \sin 60.0° = 4.33 \text{ m}$$

and

$$B_x = (4.00 \text{ m}) \cos 20.0° = 3.76 \text{ m} \quad B_y = (4.00 \text{ m}) \sin 20.0° = 1.37 \text{ m}$$

Adding component by component yields the components of $\vec{\mathbf{C}} = \vec{\mathbf{A}} + \vec{\mathbf{B}}$:

$$C_x = A_x + B_x = 2.50 \text{ m} + 3.76 \text{ m} = 6.26 \text{ m}$$

and

$$C_y = A_y + B_y = 4.33 \text{ m} + 1.37 \text{ m} = 5.70 \text{ m}$$

With these results, we can now find *precise* values for C, the magnitude of vector $\vec{\mathbf{C}}$, and its direction angle θ. In particular,

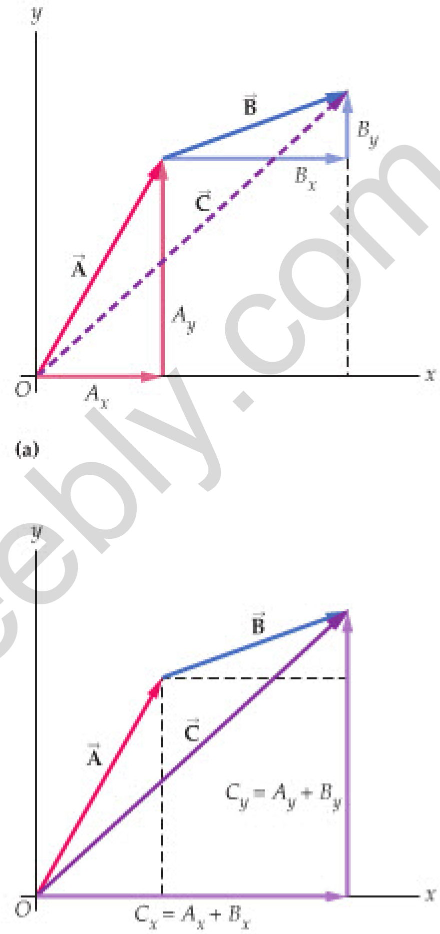
$$C = \sqrt{C_x^2 + C_y^2} = \sqrt{(6.26 \text{ m})^2 + (5.70 \text{ m})^2} = \sqrt{71.7 \text{ m}^2} = 8.47 \text{ m}$$

and

$$\theta = \tan^{-1}\left(\frac{C_y}{C_x}\right) = \tan^{-1}\left(\frac{5.70 \text{ m}}{6.26 \text{ m}}\right) = \tan^{-1}(0.911) = 42.3^\circ$$

Note that these exact values are in rough agreement with the approximate results found by graphical addition.

In the future, we will always add vectors using components—graphical addition is useful primarily as a rough check on the results obtained with components.



▲ **FIGURE 3–13 Component addition of vectors**
(a) The x and y components of $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$. **(b)** The x and y components of $\vec{\mathbf{C}}$. Notice that $C_x = A_x + B_x$ and $C_y = A_y + B_y$.

ACTIVE EXAMPLE 3–1 TREASURE HUNT: FIND THE DIRECTION AND MAGNITUDE

What are the magnitude and direction of the total displacement for the treasure hunt illustrated in Figure 3–9? Assume each pace is 0.750 m in length.

SOLUTION *(Test your understanding by performing the calculations indicated in each step.)*

To define a convenient notation, let the first 5 paces be represented by $\vec{\mathbf{A}}$, the next 3 paces by $\vec{\mathbf{B}}$, and the final 4 paces by $\vec{\mathbf{C}}$. The total displacement, then, is $\vec{\mathbf{D}} = \vec{\mathbf{A}} + \vec{\mathbf{B}} + \vec{\mathbf{C}}$.

1. Find the components of $\vec{\mathbf{A}}$: $A_x = 0$, $A_y = 3.75$ m
2. Find the components of $\vec{\mathbf{B}}$: $B_x = 2.25$ m, $B_y = 0$
3. Find the components of $\vec{\mathbf{C}}$: $C_x = 2.12$ m, $C_y = -2.12$ m
4. Sum the components of $\vec{\mathbf{A}}$, $\vec{\mathbf{B}}$, and $\vec{\mathbf{C}}$ to find the components of $\vec{\mathbf{D}}$: $D_x = 4.37$ m, $D_y = 1.63$ m
5. Determine D and θ: $D = 4.66$ m, $\theta = 20.5^\circ$

YOUR TURN

If the length of each pace is decreased by a factor of two, to 0.375 m, by what factors do you expect D and θ to change? Verify your answers with a numerical calculation.

(Answers to **Your Turn** *problems are given in the back of the book.)*

Subtracting Vectors

Next, how do we subtract vectors? Suppose, for example, that we would like to determine the vector $\vec{\mathbf{D}}$, where

$$\vec{\mathbf{D}} = \vec{\mathbf{A}} - \vec{\mathbf{B}}$$

and $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ are the vectors shown in Figure 3–12. To find $\vec{\mathbf{D}}$, we start by rewriting it as follows:

$$\vec{\mathbf{D}} = \vec{\mathbf{A}} + (-\vec{\mathbf{B}})$$

That is, $\vec{\mathbf{D}}$ is the sum of $\vec{\mathbf{A}}$ and $-\vec{\mathbf{B}}$. Now the negative of a vector has a very simple graphical interpretation:

- The negative of a vector is represented by an arrow of the same length as the original vector, but pointing in the opposite direction. That is, multiplying a vector by minus one *reverses its direction*.