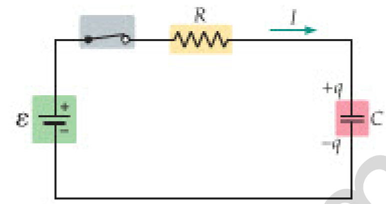
Charging a Capacitor

The charge on a capacitor in an RC circuit varies with time as follows:

$$q(t) = C\mathcal{E}(1 - e^{-t/\tau}) \qquad 21\text{–}18$$

The corresponding current is given by

$$I(t) = \left(\frac{\mathcal{E}}{R}\right)e^{-t/\tau} \qquad 21\text{–}19$$



Discharging a Capacitor

If a capacitor in an RC circuit starts with a charge Q at time $t = 0$, its charge at all later times is

$$q(t) = Qe^{-t/\tau} \qquad 21\text{–}20$$

Behavior near $t = 0$

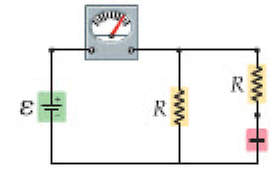
Just after the switch is closed in an RC circuit, capacitors behave like ideal wires—that is, they offer no resistance to the flow of current.

Behavior as $t \to \infty$

Long after the switch is closed in an RC circuit, capacitors behave like open circuits.

*21–8 AMMETERS AND VOLTMETERS

Ammeters and voltmeters are devices for measuring currents and voltages, respectively, in electric circuits.

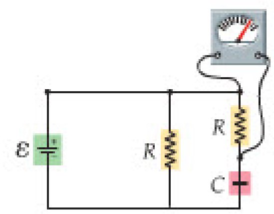


Ammeter

An ammeter is connected in series with the section of the circuit in which the current is to be measured. In the ideal case, an ammeter's resistance is zero.

Voltmeter

A voltmeter is connected in parallel with the portion of the circuit to be measured. In the ideal case, a voltmeter's resistance is infinite.



PROBLEM-SOLVING SUMMARY

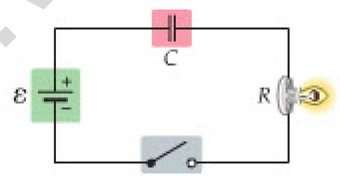
Type of Problem	Relevant Physical Concepts	Related Examples
Find the work done by a battery.	The work done by a battery is the charge that passes through the battery times the emf of the battery: $W = \Delta Q\mathcal{E}$.	Active Example 21–1
Relate resistance to resistivity.	The resistance of a wire is its resistivity, ρ, times its length, divided by its cross-sectional area: $R = \rho(L/A)$.	Example 21–2
Relate the power in an electric circuit to the current, voltage, and resistance.	The basic definition of electrical power is current times voltage: $P = IV$. Using Ohm's law when appropriate, the power can also be expressed as $P = I^2R$ and $P = V^2/R$.	Examples 21–3, 21–4
Determine the equivalent resistance of resistors in series and parallel.	Resistors in series simply add: $R_{eq} = R_1 + R_2 + \cdots$; resistors in parallel add in terms of inverses: $1/R_{eq} = 1/R_1 + 1/R_2 + \cdots$.	Examples 21–5, 21–6, 21–7
Find the current in a circuit containing resistors that are not simply in series or parallel.	Apply Kirchhoff's junction rule (the algebraic sum of currents at a junction must be zero) and loop rule (the algebraic sum of potential difference around a loop is zero).	Active Example 21–2
Determine the equivalent capacitance of capacitors in series and parallel.	Capacitors in parallel simply add: $C_{eq} = C_1 + C_2 + \cdots$; capacitors in series add in terms of inverses: $1/C_{eq} = 1/C_1 + 1/C_2 + \cdots$.	Example 21–8 Active Example 21–3
Find the charge and the current in an RC circuit as a function of time.	The charge and current in an RC circuit during charging vary exponentially with time as follows: $q(t) = C\mathcal{E}(1 - e^{-t/\tau})$; $I(t) = (\mathcal{E}/R)e^{-t/\tau}$. The characteristic time is $\tau = RC$.	Example 21–9

CONCEPTUAL QUESTIONS

For instructor-assigned homework, go to www.masteringphysics.com MP

(Answers to odd-numbered Conceptual Questions can be found in the back of the book.)

1. What is the direction of the electric current produced by an electron that falls toward the ground?
2. Your body is composed of electric charges. Does it follow, then, that you produce an electric current when you walk?
3. Suppose you charge a comb by rubbing it through your hair. Do you produce a current when you walk across the room carrying the comb?
4. Suppose you charge a comb by rubbing it through the fur on your dog's back. Do you produce a current when you walk across the room carrying the comb?
5. An electron moving through a wire has an average drift speed that is very small. Does this mean that its instantaneous velocity is also very small?
6. Are car headlights connected in series or parallel? Give an everyday observation that supports your answer.
7. Give an example of how four resistors of resistance R can be combined to produce an equivalent resistance of R.
8. Is it possible to connect a group of resistors of value R in such a way that the equivalent resistance is less than R? If so, give a specific example.
9. What physical quantity do resistors connected in series have in common?
10. What physical quantity do resistors connected in parallel have in common?
11. Explain how electrical devices can begin operating almost immediately after you throw a switch, even though individual electrons in the wire may take hours to reach the device.
12. Explain the difference between resistivity and resistance.
13. Explain why birds can roost on high-voltage wire without being electrocuted.
14. List two electrical applications that would benefit from room-temperature superconductors. List two applications for which room-temperature superconductivity would not be beneficial.
15. On what basic conservation laws are Kirchhoff's rules based?
16. What physical quantity do capacitors connected in series have in common?
17. What physical quantity do capacitors connected in parallel have in common?
18. Consider the circuit shown in Figure 21–25, in which a light of resistance R and a capacitor of capacitance C are connected in series. The capacitor has a large capacitance, and is initially uncharged. The battery provides enough power to light the bulb when connected to the battery directly. Describe the behavior of the light after the switch is closed.



▲ **FIGURE 21–25** Conceptual Question 18

PROBLEMS AND CONCEPTUAL EXERCISES

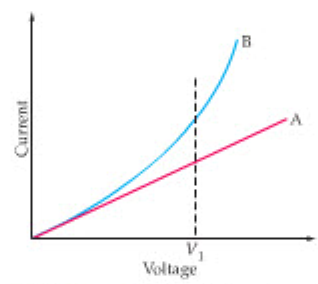
Note: Answers to odd-numbered Problems and Conceptual Exercises can be found in the back of the book. **IP** *denotes an integrated problem, with both conceptual and numerical parts;* **BIO** *identifies problems of biological or medical interest;* **CE** *indicates a conceptual exercise.* **Predict/Explain** *problems ask for two responses:* **(a)** *your prediction of a physical outcome, and* **(b)** *the best explanation among three provided. On all problems, red bullets (•, ••, •••) are used to indicate the level of difficulty.*

SECTION 21–1 ELECTRIC CURRENT

1. • How many coulombs of charge are in one ampere-hour?
2. • A flashlight bulb carries a current of 0.18 A for 78 s. How much charge flows through the bulb in this time? How many electrons?
3. • The picture tube in a particular television draws a current of 15 A. How many electrons strike the viewing screen every second?
4. • **IP** A car battery does 260 J of work on the charge passing through it as it starts an engine. **(a)** If the emf of the battery is 12 V, how much charge passes through the battery during the start? **(b)** If the emf is doubled to 24 V, does the amount of charge passing through the battery increase or decrease? By what factor?
5. • Highly sensitive ammeters can measure currents as small as 10.0 fA. How many electrons per second flow through a wire with a 10.0-fA current?
6. •• A television set connected to a 120-V outlet consumes 78 W of power. **(a)** How much current flows through the television? **(b)** How long does it take for 10 million electrons to pass through the TV?
7. •• **BIO Pacemaker Batteries** Pacemakers designed for long-term use commonly employ a lithium–iodine battery capable of supplying 0.42 A·h of charge. **(a)** How many coulombs of charge can such a battery supply? **(b)** If the average current produced by the pacemaker is 5.6 μA, what is the expected lifetime of the device?

SECTION 21–2 RESISTANCE AND OHM'S LAW

8. • **CE** A conducting wire is quadrupled in length and tripled in diameter. **(a)** Does its resistance increase, decrease, or stay the same? Explain. **(b)** By what factor does its resistance change?
9. • **CE** Figure 21–26 shows a plot of current versus voltage for two different materials, A and B. Which of these materials satisfies Ohm's law? Explain.



▲ **FIGURE 21–26** Problems 9 and 10

10. • **CE Predict/Explain** Current-versus-voltage plots for two materials, A and B, are shown in Figure 21–26. **(a)** Is the resistance of material A greater than, less than, or equal to the resistance of material B at the voltage V_1? **(b)** Choose the *best explanation* from among the following:
   I. Curve B is higher in value than curve A.
   II. A larger slope means a larger value of I/V, and hence a smaller value of R.
   III. Curve B has the larger slope at the voltage V_1 and hence the larger resistance.

11. • **CE** Two cylindrical wires are made of the same material and have the same length. If wire B is to have nine times the resistance of wire A, what must be the ratio of their radii, r_B/r_A?

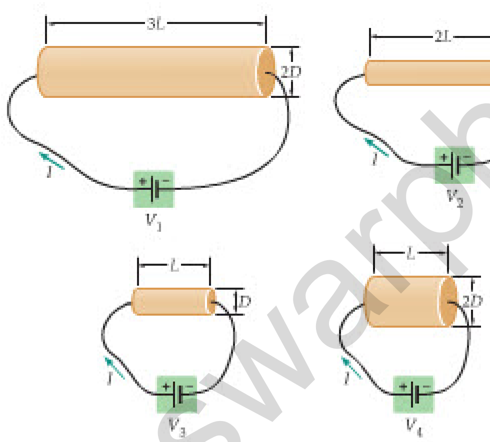
12. • A silver wire is 5.9 m long and 0.49 mm in diameter. What is its resistance?

13. • When a potential difference of 18 V is applied to a given wire, it conducts 0.35 A of current. What is the resistance of the wire?

14. • The tungsten filament of a lightbulb has a resistance of 0.07 Ω. If the filament is 27 cm long, what is its diameter?

15. • What is the resistance of 6.0 mi of copper wire with a diameter of 0.55 mm?

16. •• **CE** The four conducting cylinders shown in Figure 21–27 are all made of the same material, though they differ in length and/or diameter. They are connected to four different batteries, which supply the necessary voltages to give the circuits the same current, I. Rank the four voltages, V_1, V_2, V_3, and V_4, in order of increasing value. Indicate ties where appropriate.



▲ **FIGURE 21–27** Problem 16

17. •• **IP** A bird lands on a bare copper wire carrying a current of 32 A. The wire is 8 gauge, which means that its cross-sectional area is 0.13 cm^2. **(a)** Find the difference in potential between the bird's feet, assuming they are separated by a distance of 6.0 cm. **(b)** Will your answer to part (a) increase or decrease if the separation between the bird's feet increases? Explain.

18. •• A current of 0.96 A flows through a copper wire 0.44 mm in diameter when it is connected to a potential difference of 15 V. How long is the wire?

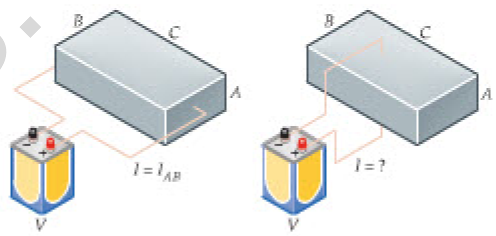
19. •• **IP BIO Current Through a Cell Membrane** A typical cell membrane is 8.0 nm thick and has an electrical resistivity of 1.3×10^7 Ω·m. **(a)** If the potential difference between the inner and outer surfaces of a cell membrane is 75 mV, how much current flows through a square area of membrane 1.0 μm on a side? **(b)** Suppose the thickness of the membrane is doubled, but the resistivity and potential difference remain the same. Does the current increase or decrease? By what factor?

20. •• When a potential difference of 12 V is applied to a wire 6.9 m long and 0.33 mm in diameter, the result is an electric current of 2.1 A. What is the resistivity of the wire?

21. •• **IP** **(a)** What is the resistance per meter of an aluminum wire with a cross-sectional area of 2.4×10^{-7} m^2. **(b)** Would your answer to part (a) increase, decrease, or stay the same if the diameter of the wire were increased? Explain. **(c)** Repeat part (a) for a wire with a cross-sectional area of 3.6×10^{-7} m^2.

22. •• **BIO Resistance and Current in the Human Finger** The interior of the human body has an electrical resistivity of 0.15 Ω·m. **(a)** Estimate the resistance for current flowing the length of your index finger. (For this calculation, ignore the much higher resistivity of your skin.) **(b)** Your muscles will contract when they carry a current greater than 15 mA. What voltage is required to produce this current through your finger?

23. ••• Consider a rectangular block of metal of height A, width B, and length C, as shown in Figure 21–28. If a potential difference V is maintained between the two $A \times B$ faces of the block, a current I_{AB} is observed to flow. Find the current that flows if the same potential difference V is applied between the two $B \times C$ faces of the block. Give your answer in terms of I_{AB}.



▲ **FIGURE 21–28** Problem 23

SECTION 21–3 ENERGY AND POWER IN ELECTRIC CIRCUITS

24. • **CE** Light A has four times the power rating of light B when operated at the same voltage. **(a)** Is the resistance of light A greater than, less than, or equal to the resistance of light B? Explain. **(b)** What is the ratio of the resistance of light A to the resistance of light B?

25. • **CE** Two lightbulbs operate on the same potential difference. Bulb A has four times the power output of bulb B. **(a)** Which bulb has the greater current passing through it? Explain. **(b)** What is the ratio of the current in bulb A to the current in bulb B?

26. • **CE** Two lightbulbs operate on the same current. Bulb A has four times the power output of bulb B. **(a)** Is the potential difference across bulb A greater than or less than the potential difference across bulb B? Explain. **(b)** What is the ratio of the potential difference across bulb A to that across bulb B?

27. • A 75-V generator supplies 3.8 kW of power. How much current does the generator produce?

28. • A portable CD player operates with a current of 22 mA at a potential difference of 4.1 V. What is the power usage of the player?

29. • Find the power dissipated in a 25-Ω electric heater connected to a 120-V outlet.

30. • The current in a 120-V reading lamp is 2.6 A. If the cost of electrical energy is \$0.075 per kilowatt-hour, how much does it cost to operate the light for an hour?

31. • It costs 2.6 cents to charge a car battery at a voltage of 12 V and a current of 15 A for 120 minutes. What is the cost of electrical energy per kilowatt-hour at this location?

32. •• **IP** A 75-W lightbulb operates on a potential difference of 95 V. Find **(a)** the current in the bulb and **(b)** the resistance of the bulb. **(c)** If this bulb is replaced with one whose resistance is half the value found in part (b), is its power rating greater than or less than 75 W? By what factor?

33. •• **Rating Car Batteries** Car batteries are rated by the following two numbers: **(1)** cranking amps = current the battery can produce for 30.0 seconds while maintaining a terminal voltage of at least 7.2 V and **(2)** reserve capacity = number of minutes the battery can produce a 25-A current while maintaining a terminal voltage of at least 10.5 V. One particular battery is advertised as having 905 cranking amps and a 155-minute reserve capacity. Which of these two ratings represents the greater amount of energy delivered by the battery?

SECTION 21–4 RESISTORS IN SERIES AND PARALLEL

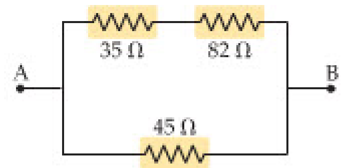
34. • **CE Predict/Explain** A dozen identical lightbulbs are connected to a given emf. **(a)** Will the lights be brighter if they are connected in series or in parallel? **(b)** Choose the *best explanation* from among the following:
   I. When connected in parallel each bulb experiences the maximum emf and dissipates the maximum power.
   II. Resistors in series have a larger equivalent resistance and dissipate more power.
   III. Resistors in parallel have a smaller equivalent resistance and dissipate less power.

35. • **CE Predict/Explain** A *fuse* is a device to protect a circuit from the effects of a large current. The fuse is a small strip of metal that burns through when the current in it exceeds a certain value, thus producing an open circuit. **(a)** Should a fuse be connected in series or in parallel with the circuit it is intended to protect? **(b)** Choose the *best explanation* from among the following:
   I. Either connection is acceptable; the main thing is to have a fuse in the circuit.
   II. The fuse should be connected in parallel, otherwise it will interrupt the current in the circuit.
   III. With the fuse connected in series, the current in the circuit drops to zero as soon as the fuse burns through.

36. • **CE** A circuit consists of three resistors, $R_1 < R_2 < R_3$, connected in series to a battery. Rank these resistors in order of increasing **(a)** current through them and **(b)** potential difference across them. Indicate ties where appropriate.

37. • **CE Predict/Explain** Two resistors are connected in parallel. **(a)** If a third resistor is now connected in parallel with the original two, does the equivalent resistance of the circuit increase, decrease, or remain the same? **(b)** Choose the *best explanation* from among the following:
   I. Adding a resistor generally tends to increase the resistance, but putting it in parallel tends to decrease the resistance; therefore the effects offset and the resistance stays the same.
   II. Adding more resistance to the circuit will increase the equivalent resistance.
   III. The third resistor gives yet another path for current to flow in the circuit, which means that the equivalent resistance is less.

38. • Find the equivalent resistance between points A and B for the group of resistors shown in Figure 21–29.



▲ **FIGURE 21–29** Problems 38 and 115

39. • What is the minimum number of 65-Ω resistors that must be connected in parallel to produce an equivalent resistance of 11 Ω or less?

40. •• Four lightbulbs (A, B, C, D) are connected together in a circuit of unknown arrangement. When each bulb is removed one at a time and replaced, the following behavior is observed:

	A	B	C	D
A removed	*	on	on	on
B removed	on	*	on	off
C removed	off	off	*	off
D removed	on	off	on	*

Draw a circuit diagram for these bulbs.

41. •• Your toaster has a power cord with a resistance of 0.020 Ω connected in series with a 9.6-Ω nichrome heating element. If the potential difference between the terminals of the toaster is 120 V, how much power is dissipated in **(a)** the power cord and **(b)** the heating element?

42. •• A hobbyist building a radio needs a 150-Ω resistor in her circuit, but has only a 220-Ω, a 79-Ω, and a 92-Ω resistor available. How can she connect these resistors to produce the desired resistance?

43. •• A circuit consists of a 12.0-V battery connected to three resistors (42 Ω, 17 Ω, and 110 Ω) in series. Find **(a)** the current that flows through the battery and **(b)** the potential difference across each resistor.

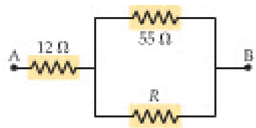
44. •• **IP** Three resistors, 11 Ω, 53 Ω, and R, are connected in series with a 24.0-V battery. The total current flowing through the battery is 0.16 A. **(a)** Find the value of resistance R. **(b)** Find the potential difference across each resistor. **(c)** If the voltage of the battery had been greater than 24.0 V, would your answer to part (a) have been larger or smaller? Explain.

45. •• A circuit consists of a battery connected to three resistors (65 Ω, 25 Ω, and 170 Ω) in parallel. The total current through the resistors is 1.8 A. Find **(a)** the emf of the battery and **(b)** the current through each resistor.

46. •• **IP** Three resistors, 22 Ω, 67 Ω, and R, are connected in parallel with a 12.0-V battery. The total current flowing through the battery is 0.88 A. **(a)** Find the value of resistance R. **(b)** Find the current through each resistor. **(c)** If the total current in the battery had been greater than 0.88 A, would your answer to part (a) have been larger or smaller? Explain.

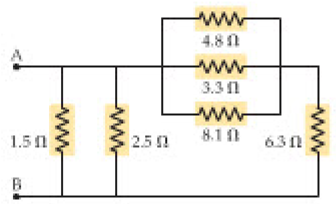
47. •• An 89-Ω resistor has a current of 0.72 A and is connected in series with a 130-Ω resistor. What is the emf of the battery to which the resistors are connected?

48. •• The equivalent resistance between points A and B of the resistors shown in Figure 21–30 is 26 Ω. Find the value of resistance R.



▲ **FIGURE 21–30** Problems 48, 52, and 98

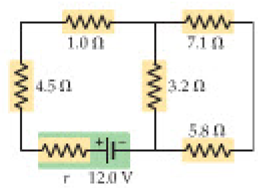
49. •• Find the equivalent resistance between points A and B shown in **Figure 21–31**.



▲ **FIGURE 21–31** Problems 49 and 53

50. •• How many 65-W lightbulbs can be connected in parallel across a potential difference of 85 V before the total current in the circuit exceeds 2.1 A?

51. •• The circuit in **Figure 21–32** includes a battery with a finite internal resistance, $r = 0.50\ \Omega$. **(a)** Find the current flowing through the 7.1-Ω and the 3.2-Ω resistors. **(b)** How much current flows through the battery? **(c)** What is the potential difference between the terminals of the battery?



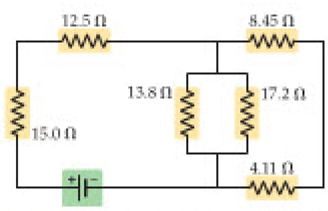
▲ **FIGURE 21–32** Problems 51 and 54

52. •• **IP** A 12-V battery is connected to terminals A and B in Figure 21–30. **(a)** Given that $R = 85\ \Omega$, find the current in each resistor. **(b)** Suppose the value of R is increased. For each resistor in turn, state whether the current flowing through it increases or decreases. Explain.

53. •• **IP** The terminals A and B in Figure 21–31 are connected to a 9.0-V battery. **(a)** Find the current flowing through each resistor. **(b)** Is the potential difference across the 6.3-Ω resistor greater than, less than, or the same as the potential difference across the 1.5-Ω resistor? Explain.

54. •• **IP** Suppose the battery in Figure 21–32 has an internal resistance $r = 0.25\ \Omega$. **(a)** How much current flows through the battery? **(b)** What is the potential difference between the terminals of the battery? **(c)** If the 3.2-Ω resistor is increased in value, will the current in the battery increase or decrease? Explain.

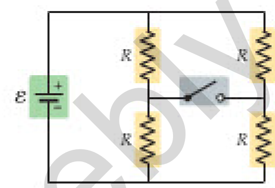
55. ••• **IP** The current flowing through the 8.45-Ω resistor in **Figure 21–33** is 1.52 A. **(a)** What is the voltage of the battery? **(b)** If the 17.2-Ω resistor is increased in value, will the current provided by the battery increase, decrease, or stay the same? Explain.



▲ **FIGURE 21–33** Problems 55 and 56

56. ••• The current in the 13.8-Ω resistor in Figure 21–33 is 0.795 A. Find the current in the other resistors in the circuit.

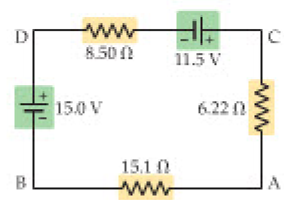
57. ••• **IP** Four identical resistors are connected to a battery as shown in **Figure 21–34**. When the switch is open, the current through the battery is I_0. **(a)** When the switch is closed, will the current through the battery increase, decrease, or stay the same? Explain. **(b)** Calculate the current that flows through the battery when the switch is closed. Give your answer in terms of I_0.



▲ **FIGURE 21–34** Problem 57

SECTION 21–5 KIRCHHOFF'S RULES

58. • Find the magnitude and direction (clockwise or counterclockwise) of the current in **Figure 21–35**.

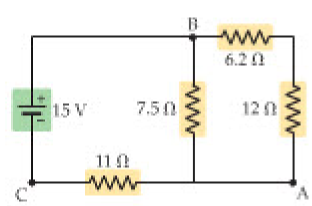


▲ **FIGURE 21–35** Problems 58, 59, and 60

59. • **IP** Suppose the polarity of the 11.5-V battery in Figure 21–35 is reversed. **(a)** Do you expect this to increase or decrease the amount of current flowing in the circuit? Explain. **(b)** Calculate the magnitude and direction (clockwise or counterclockwise) of the current in this case.

60. •• **IP** It is given that point A in Figure 21–35 is grounded ($V = 0$). **(a)** Is the potential at point B greater than or less than zero? Explain. **(b)** Is the potential at point C greater than or less than zero? Explain. **(c)** Calculate the potential at point D.

61. •• Consider the circuit shown in **Figure 21–36**. Find the current through each resistor using **(a)** the rules for series and parallel resistors and **(b)** Kirchhoff's rules.

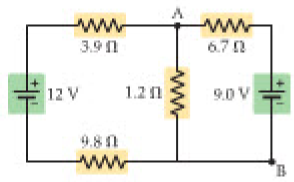


▲ **FIGURE 21–36** Problems 61 and 62

62. •• Suppose point A is grounded ($V = 0$) in Figure 21–36. Find the potential at points B and C.

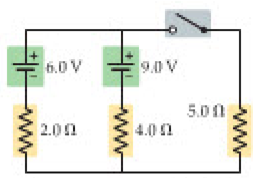
63. •• **IP** **(a)** Find the current in each resistor in **Figure 21–37**. **(b)** Is the potential at point A greater than, less than, or equal to the

potential at point B? Explain. **(c)** Determine the potential difference between the points A and B.



▲ **FIGURE 21–37** Problem 63

64\. ••• Two batteries and three resistors are connected as shown in Figure 21–38. How much current flows through each battery when the switch is **(a)** closed and **(b)** open?



▲ **FIGURE 21–38** Problem 64

SECTION 21–6 CIRCUITS CONTAINING CAPACITORS

65\. • **CE** Two capacitors, $C_1 = C$ and $C_2 = 2C$, are connected to a battery. **(a)** Which capacitor stores more energy when they are connected to the battery in series? Explain. **(b)** Which capacitor stores more energy when they are connected in parallel? Explain.

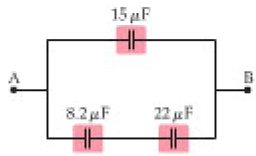
66\. • **CE Predict/Explain** Two capacitors are connected in series. **(a)** If a third capacitor is now connected in series with the original two, does the equivalent capacitance increase, decrease, or remain the same? **(b)** Choose the *best explanation* from among the following:

I. Adding a capacitor generally tends to increase the capacitance, but putting it in series tends to decrease the capacitance; therefore, the net result is no change.
II. Adding a capacitor in series will increase the total amount of charge stored, and hence increase the equivalent capacitance.
III. Adding a capacitor in series decreases the equivalent capacitance since each capacitor now has less voltage across it, and hence stores less charge.

67\. • **CE Predict/Explain** Two capacitors are connected in parallel. **(a)** If a third capacitor is now connected in parallel with the original two, does the equivalent capacitance increase, decrease, or remain the same? **(b)** Choose the *best explanation* from among the following:

I. Adding a capacitor tends to increase the capacitance, but putting it in parallel tends to decrease the capacitance; therefore, the net result is no change.
II. Adding a capacitor in parallel will increase the total amount of charge stored, and hence increase the equivalent capacitance.
III. Adding a capacitor in parallel decreases the equivalent capacitance since each capacitor now has less voltage across it, and hence stores less charge.

68\. • Find the equivalent capacitance between points A and B for the group of capacitors shown in Figure 21–39.

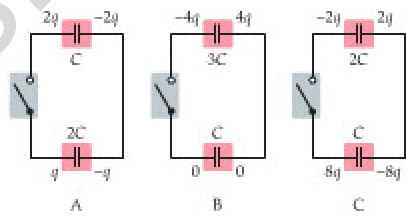


▲ **FIGURE 21–39** Problems 68 and 72

69\. • A 12-V battery is connected to three capacitors in series. The capacitors have the following capacitances: 4.5 μF, 12 μF, and 32 μF. Find the voltage across the 32-μF capacitor.

70\. •• **CE** You conduct a series of experiments in which you connect the capacitors C_1 and $C_2 > C_1$ to a battery in various ways. The experiments are as follows: **A**, C_1 alone connected to the battery; **B**, C_2 alone connected to the battery; **C**, C_1 and C_2 connected to the battery in series; **D**, C_1 and C_2 connected to the battery in parallel. Rank these four experiments in order of increasing equivalent capacitance. Indicate ties where appropriate.

71\. •• **CE** Three different circuits, each containing a switch and two capacitors, are shown in Figure 21–40. Initially, the plates of the capacitors are charged as shown. The switches are then closed, allowing charge to move freely between the capacitors. Rank the circuits in order of increasing final charge on the left plate of **(a)** the upper capacitor and **(b)** the lower capacitor. Indicate ties where appropriate.



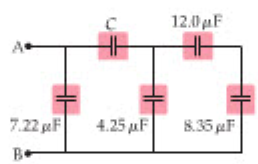
▲ **FIGURE 21–40** Problem 71

72\. •• Terminals A and B in Figure 21–39 are connected to a 9.0-V battery. Find the energy stored in each capacitor.

73\. •• **IP** Two capacitors, one 7.5 μF and the other 15 μF, are connected in parallel across a 15-V battery. **(a)** Find the equivalent capacitance of the two capacitors. **(b)** Which capacitor stores more charge? Explain. **(c)** Find the charge stored on each capacitor.

74\. •• **IP** Two capacitors, one 7.5 μF and the other 15 μF, are connected in series across a 15-V battery. **(a)** Find the equivalent capacitance of the two capacitors. **(b)** Which capacitor stores more charge? Explain. **(c)** Find the charge stored on each capacitor.

75\. •• The equivalent capacitance of the capacitors shown in Figure 21–41 is 9.22 μF. Find the value of capacitance C.

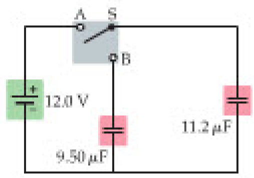


▲ **FIGURE 21–41** Problems 75 and 118

76\. ••• Two capacitors, C_1 and C_2, are connected in series and charged by a battery. Show that the energy stored in C_1 plus the energy stored in C_2 is equal to the energy stored in the equivalent capacitor, C_{eq}, when it is connected to the same battery.

77\. ••• With the switch in position A, the 11.2-μF capacitor in Figure 21–42 is fully charged by the 12.0-V battery, and the

9.50-μF capacitor is uncharged. The switch is now moved to position B. As a result, charge flows between the capacitors until they have the same voltage across their plates. Find this voltage.



▲ **FIGURE 21–42** Problem 77

SECTION 21–7 *RC* CIRCUITS

78. • The switch on an *RC* circuit is closed at $t = 0$. Given that $\mathcal{E} = 9.0$ V, $R = 150\ \Omega$, and $C = 23\ \mu$F, how much charge is on the capacitor at time $t = 4.2$ ms?

79. • The capacitor in an *RC* circuit ($R = 120\ \Omega$, $C = 45\ \mu$F) is initially uncharged. Find **(a)** the charge on the capacitor and **(b)** the current in the circuit one time constant ($\tau = RC$) after the circuit is connected to a 9.0-V battery.

80. •• **CE** Three *RC* circuits have the emf, resistance, and capacitance given in the accompanying table. Initially, the switch on the circuit is open and the capacitor is uncharged. Rank these circuits in order of increasing **(a)** initial current (immediately after the switch is closed) and **(b)** time for the capacitor to acquire half its final charge. Indicate ties where appropriate.

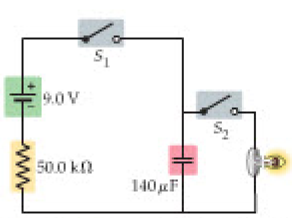
	$\mathcal{E}$ (V)	R (Ω)	C (μF)
Circuit A	12	4	3
Circuit B	9	3	1
Circuit C	9	9	2

81. •• Consider an *RC* circuit with $\mathcal{E} = 12.0$ V, $R = 175\ \Omega$, and $C = 55.7\ \mu$F. Find **(a)** the time constant for the circuit, **(b)** the maximum charge on the capacitor, and **(c)** the initial current in the circuit.

82. •• The resistor in an *RC* circuit has a resistance of 145 Ω. **(a)** What capacitance must be used in this circuit if the time constant is to be 3.5 ms? **(b)** Using the capacitance determined in part (a), calculate the current in the circuit 7.0 ms after the switch is closed. Assume that the capacitor is uncharged initially and that the emf of the battery is 9.0 V.

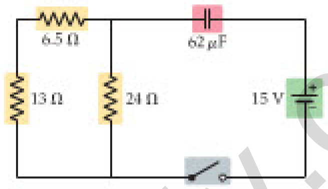
83. •• A flash unit for a camera has a capacitance of 1500 μF. What resistance is needed in this *RC* circuit if the flash is to charge to 90% of its full charge in 21 s?

84. •• Figure 21–43 shows a simplified circuit for a photographic flash unit. This circuit consists of a 9.0-V battery, a 50.0-kΩ resistor, a 140-μF capacitor, a flashbulb, and two switches. Initially, the capacitor is uncharged and the two switches are open. To charge the unit, switch S_1 is closed; to fire the flash, switch S_2 (which is connected to the camera's shutter) is closed. How long does it take to charge the capacitor to 5.0 V?



▲ **FIGURE 21–43** Problem 84

85. •• **IP** Consider the *RC* circuit shown in Figure 21–44. Find **(a)** the time constant and **(b)** the initial current for this circuit. **(c)** It is desired to increase the time constant of this circuit by adjusting the value of the 6.5-Ω resistor. Should the resistance of this resistor be increased or decreased to have the desired effect? Explain.



▲ **FIGURE 21–44** Problems 85 and 119

86. ••• The capacitor in an *RC* circuit is initially uncharged. In terms of R and C, determine **(a)** the time required for the charge on the capacitor to rise to 50% of its final value and **(b)** the time required for the initial current to drop to 10% of its initial value.

GENERAL PROBLEMS

87. • **CE** A given car battery is rated as 250 amp-hours. Is this rating a measure of energy, power, charge, voltage, or current? Explain.

88. • **CE Predict/Explain** The resistivity of tungsten increases with temperature. **(a)** When a light containing a tungsten filament heats up, does its power consumption increase, decrease, or stay the same? **(b)** Choose the *best explanation* from among the following:
    I. The voltage is unchanged, and therefore an increase in resistance implies a reduced power, as we can see from $P = V^2/R$.
    II. Increasing the resistance increases the power, as is clear from $P = I^2R$.
    III. The power consumption is independent of resistance, as we can see from $P = IV$.

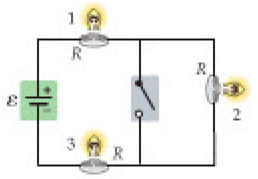
89. • **CE** A cylindrical wire is to be doubled in length, but it is desired that its resistance remain the same. **(a)** Must its radius be increased or decreased? Explain. **(b)** By what factor must the radius be changed?

90. • **CE Predict/Explain** An electric space heater has a power rating of 500 W when connected to a given voltage V. **(a)** If two of these heaters are connected in series to the same voltage, is the power consumed by the two heaters greater than, less than, or equal to 1000 W? **(b)** Choose the *best explanation* from among the following:
    I. Each heater consumes 500 W; therefore two of them will consume 500 W + 500 W = 1000 W.
    II. The voltage is the same, but the resistance is doubled by connecting the heaters in series. Therefore, the power consumed ($P = V^2/R$) is less than 1000 W.
    III. Connecting two heaters in series doubles the resistance. Since power depends on the resistance squared, it follows that the power consumed is greater than 1000 W.

91. • **CE** Two resistors, $R_1 = R$ and $R_2 = 2R$, are connected to a battery. **(a)** Which resistor dissipates more power when they are connected to the battery in series? Explain. **(b)** Which resistor dissipates more power when they are connected in parallel? Explain.

92. • **CE** Consider the circuit shown in Figure 21–45, in which three lights, each with a resistance R, are connected in series. The circuit also contains an open switch. **(a)** When the switch is closed, does the intensity of light 2 increase, decrease, or stay

the same? Explain. **(b)** Do the intensities of lights 1 and 3 increase, decrease, or stay the same when the switch is closed? Explain.

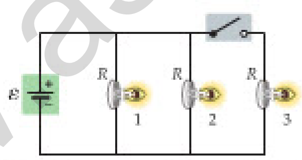


▲ **FIGURE 21–45** Problems 92, 93, and 94

93. • **CE Predict/Explain (a)** Referring to Problem 92 and the circuit in Figure 21–45, does the current supplied by the battery increase, decrease, or remain the same when the switch is closed? **(b)** Choose the *best explanation* from among the following:
   I. The current decreases because only two resistors can draw current from the battery when the switch is closed.
   II. Closing the switch makes no difference to the current since the second resistor is still connected to the battery as before.
   III. Closing the switch shorts out the second resistor, decreases the total resistance of the circuit, and increases the current.

94. • **CE Predict/Explain (a)** Referring to Problem 92 and the circuit in Figure 21–45, does the total power dissipated in the circuit increase, decrease, or remain the same when the switch is closed? **(b)** Choose the *best explanation* from among the following:
   I. Closing the switch shorts out one of the resistors, which means that the power dissipated decreases.
   II. The equivalent resistance of the circuit is reduced by closing the switch, but the voltage remains the same. Therefore, from $P = V^2/R$ we see that the power dissipated increases.
   III. The power dissipated remains the same because power, $P = IV$, is independent of resistance.

95. • **CE** Consider the circuit shown in Figure 21–46, in which three lights, each with a resistance R, are connected in parallel. The circuit also contains an open switch. **(a)** When the switch is closed, does the intensity of light 3 increase, decrease, or stay the same? Explain. **(b)** Do the intensities of lights 1 and 2 increase, decrease, or stay the same when the switch is closed? Explain.



▲ **FIGURE 21–46** Problems 95, 96, and 97

96. • **CE Predict/Explain (a)** When the switch is closed in the circuit shown in Figure 21–46, does the current supplied by the battery increase, decrease, or stay the same? **(b)** Choose the *best explanation* from among the following:
   I. The current increases because three resistors are drawing current from the battery when the switch is closed, rather than just two.
   II. Closing the switch makes no difference to the current because the voltage is the same as before.
   III. Closing the switch decreases the current because an additional resistor is added to the circuit.

97. • **CE Predict/Explain (a)** When the switch is closed in the circuit shown in Figure 21–46, does the total power dissipated in the circuit increase, decrease, or stay the same? **(b)** Choose the *best explanation* from among the following:
   I. Closing the switch adds one more resistor to the circuit. This makes it harder for the battery to supply current, which decreases the power dissipated.
   II. The equivalent resistance of the circuit is reduced by closing the switch, but the voltage remains the same. Therefore, from $P = V^2/R$ we see that the power dissipated increases.
   III. The power dissipated remains the same because power, $P = IV$, is independent of resistance.

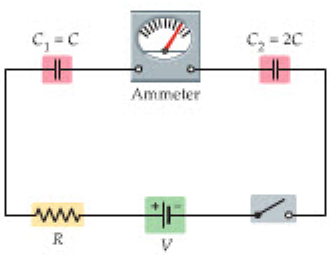
98. • Suppose that points A and B in Figure 21–30 are connected to a 12-V battery. Find the power dissipated in each of the resistors assuming that $R = 65\ \Omega$.

99. • You are given resistors of 413 Ω, 521 Ω, and 146 Ω. Describe how these resistors must be connected to produce an equivalent resistance of 255 Ω.

100. • You are given capacitors of 18 μF, 7.2 μF, and 9.0 μF. Describe how these capacitors must be connected to produce an equivalent capacitance of 22 μF.

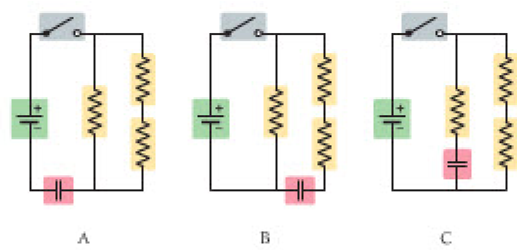
101. • Suppose your car carries a charge of 85 μC. What current does it produce as it travels from Dallas to Fort Worth (35 mi) in 0.75 h?

102. •• **CE** The circuit shown in Figure 21–47 shows a resistor and two capacitors connected in series with a battery of voltage V. The circuit also has an ammeter and a switch. Initially, the switch is open and both capacitors are uncharged. The following questions refer to a time long after the switch is closed and current has ceased to flow. **(a)** In terms of V, what is the voltage across the capacitor C_1? **(b)** In terms of CV, what is the charge on the right plate of C_2? **(c)** What is the net charge that flowed through the ammeter during charging? Give your answer in terms of CV.



▲ **FIGURE 21–47** Problem 102

103. •• **CE** The three circuits shown in Figure 21–48 have identical batteries, resistors, and capacitors. Initially, the switches are open and the capacitors are uncharged. Rank the circuits in order of increasing **(a)** final charge on the capacitor and **(b)** time for the current to drop to 90% of its initial value. Indicate ties where appropriate.



▲ **FIGURE 21–48** Problem 103

104. •• It is desired to construct a 5.0-Ω resistor from a 1.2-m length of tungsten wire. What diameter is needed for this wire?

105. •• **Electrical Safety Codes** For safety reasons, electrical codes have been established that limit the amount of current a wire of a given size can carry. For example, an 18-gauge (cross-sectional area = 1.17 mm^2), rubber-insulated extension cord with copper wires can carry a maximum current of 5.0 A. Find the voltage drop in a 12-ft, 18-gauge extension cord carrying a current of 5.0 A. (*Note:* In an extension cord, the current must flow through two lengths—down and back.)

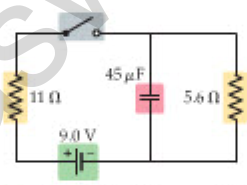
106. •• **A Three-Way Lightbulb** A three-way lightbulb has two filaments with resistances R_1 and R_2 connected in series. The resistors are connected to three terminals, as indicated in **Figure 21–49**, and the light switch determines which two of the three terminals are connected to a potential difference of 120 V at any given time. When terminals A and B are connected to 120 V the bulb uses 75.0 W of power. When terminals A and C are connected to 120 V the bulb uses 50.0 W of power. **(a)** What is the resistance R_1? **(b)** What is the resistance R_2? **(c)** How much power does the bulb use when 120 V is connected to terminals B and C?

▲ **FIGURE 21–49** Problem 106

107. •• A portable CD player uses a current of 7.5 mA at a potential difference of 3.5 V. **(a)** How much energy does the player use in 35 s? **(b)** Suppose the player has a mass of 0.65 kg. For what length of time could the player operate on the energy required to lift it through a height of 1.0 m?

108. •• An electrical heating coil is immersed in 4.6 kg of water at 22 °C. The coil, which has a resistance of 250 Ω, warms the water to 32 °C in 15 min. What is the potential difference at which the coil operates?

109. •• **IP** Consider the circuit shown in **Figure 21–50**. **(a)** Is the current flowing through the battery immediately after the switch is closed greater than, less than, or the same as the current flowing through the battery long after the switch is closed? Explain. **(b)** Find the current flowing through the battery immediately after the switch is closed. **(c)** Find the current in the battery long after the switch is closed.



▲ **FIGURE 21–50** Problems 109 and 116

110. •• A silver wire and a copper wire have the same volume and the same resistance. Find the ratio of their radii, r_{silver}/r_{copper}.

111. •• Two resistors are connected in series to a battery with an emf of 12 V. The voltage across the first resistor is 2.7 V and the current through the second resistor is 0.15 A. Find the resistance of the two resistors.

112. •• **BIO Pacemaker Pulses** A pacemaker sends a pulse to a patient's heart every time the capacitor in the pacemaker charges to a voltage of 0.25 V. It is desired that the patient receive 75 pulses per minute. Given that the capacitance of the pacemaker is 110 μF and that the battery has a voltage of 9.0 V, what value should the resistance have?

113. •• A long, thin wire has a resistance R. The wire is now cut into three segments of equal length, which are connected in parallel. In terms of R, what is the equivalent resistance of the three wire segments?

114. •• Three resistors ($R, \frac{1}{2}R, 2R$) are connected to a battery. **(a)** If the resistors are connected in series, which one has the greatest rate of energy dissipation? **(b)** Repeat part (a), this time assuming that the resistors are connected in parallel.

115. •• **IP** Suppose we connect a 12.0-V battery to terminals A and B in Figure 21–29. **(a)** Is the current in the 45-Ω resistor greater than, less than, or the same as the current in the 35-Ω resistor? Explain. **(b)** Calculate the current flowing through each of the three resistors in this circuit.

116. •• **IP** Suppose the battery in Figure 21–50 has an internal resistance of 0.73 Ω. **(a)** What is the potential difference across the terminals of the battery when the switch is open? **(b)** When the switch is closed, does the potential difference of the battery increase or decrease? Explain. **(c)** Find the potential difference across the battery after the switch has been closed a long time.

117. •• **National Electric Code** In the United States, the National Electric Code sets standards for maximum safe currents in insulated copper wires of various diameters. The accompanying table gives a portion of the code. Notice that wire diameters are identified by the *gauge* of the wire, and that 1 mil = 10^{-3} in. Find the maximum power dissipated per length in **(a)** an 8-gauge wire and **(b)** a 10-gauge wire.

Gauge	Diameter (mils)	Safe current (A)
8	129	35
10	102	25

118. ••• **IP** A 15.0-V battery is connected to terminals A and B in Figure 21–41. **(a)** Given that $C = 15.0\ \mu F$, find the charge on each of the capacitors. **(b)** Find the total energy stored in this system. **(c)** If the 7.22-μF capacitor is increased in value, will the total energy stored in the circuit increase or decrease? Explain.

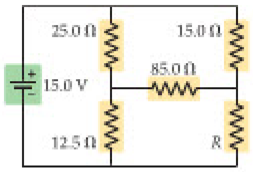
119. ••• **IP** The switch in the *RC* circuit shown in Figure 21–44 is closed at $t = 0$. **(a)** How much power is dissipated in each resistor just after $t = 0$ and in the limit $t \to \infty$? **(b)** What is the charge on the capacitor at the time $t = 0.35$ ms? **(c)** How much energy is stored in the capacitor in the limit $t \to \infty$? **(d)** If the voltage of the battery is doubled, by what factor does your answer to part (c) change? Explain.

120. ••• Two resistors, R_1 and R_2, are connected in parallel and connected to a battery. Show that the power dissipated in R_1 plus the power dissipated in R_2 is equal to the power dissipated in the equivalent resistor, R_{eq}, when it is connected to the same battery.

121. ••• A battery has an emf $\mathcal{E}$ and an internal resistance r. When the battery is connected to a 25-Ω resistor, the current through the battery is 0.65 A. When the battery is connected to a 55-Ω resistor, the current is 0.45 A. Find the battery's emf and internal resistance.

122. ••• When two resistors, R_1 and R_2, are connected in series across a 6.0-V battery, the potential difference across R_1 is 4.0 V. When R_1 and R_2 are connected in parallel to the same battery, the current through R_2 is 0.45 A. Find the values of R_1 and R_2.

123. ••• The circuit shown in **Figure 21–51** is known as a Wheatstone bridge. Find the value of the resistor R such that the current through the 85.0-Ω resistor is zero.



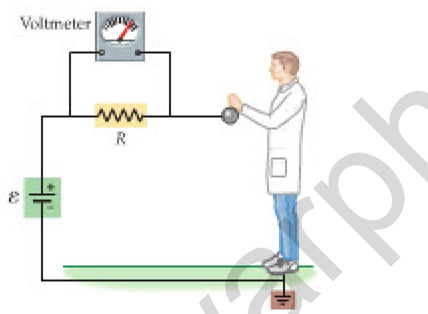
▲ **FIGURE 21–51** Problem 123

PASSAGE PROBLEMS

BIO Footwear Safety

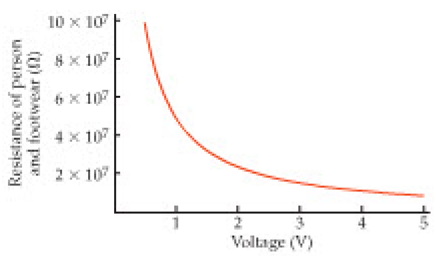
The American National Standards Institute (ANSI) specifies safety standards for a number of potential workplace hazards. For example, ANSI requires that footwear provide protection against the effects of compression from a static weight, impact from a dropped object, puncture from a sharp tool, and cuts from saws. In addition, to protect against the potentially lethal effects of an electrical shock, ANSI provides standards for the electrical resistance that a person and footwear must offer to the flow of electric current.

Specifically, regulation ANSI Z41-1999 states that the resistance of a person and his or her footwear must be tested with the circuit shown in Figure 21–52. In this circuit, the voltage supplied by the battery is $\mathcal{E} = 50.0$ V and the resistance in the circuit is $R = 1.00$ MΩ. Initially the circuit is open and no current flows. When a person touches the metal sphere attached to the battery, however, the circuit is closed and a small current flows through the person, the shoes, and back to the battery. The amount of current flowing through the person can be determined by using a voltmeter to measure the voltage drop V across the resistor R. To be safe, the current should not exceed 150 μA.



▲ **FIGURE 21–52** Problems 124, 125, 126, and 127

Notice that the experimental setup in Figure 21–52 is a dc circuit with two resistors in series—the resistance R and the resistance of the person and footwear, R_{pf}. It follows that the current in the circuit is $I = \mathcal{E}/(R + R_{pf})$. We also know that the current is $I = V/R$, where V is the reading of the voltmeter. These relations can be combined to relate the voltage V to the resistance R_{pf}, with the result shown in Figure 21–53. According to ANSI regulations, Type II footwear must give a resistance R_{pf} in the range of 0.1×10^7 Ω to 100×10^7 Ω.



▲ **FIGURE 21–53** Problems 124, 125, 126, and 127

124. • Suppose the voltmeter measures a potential difference of 3.70 V across the resistor. What is the current that flows through the person's body?

A. 3.70×10^{-6} A **B.** 5.00×10^{-5} A

C. 0.0740 A **D.** 3.70 A

125. • What is the resistance of the person and footwear when the voltmeter reads 3.70 V?

A. 1.25×10^7 Ω **B.** 1.35×10^7 Ω

C. 4.63×10^7 Ω **D.** 1.71×10^8 Ω

126. • The resistance of a given person and footwear is 4.00×10^7 Ω. What is the reading on the voltmeter when this person is tested?

A. 0.976 V **B.** 1.22 V

C. 1.25 V **D.** 50.0 V

127. • Suppose that during one test a person's shoes become wet when water spills onto the floor. When this happens, do you expect the reading on the voltmeter to increase, decrease, or stay the same?

INTERACTIVE PROBLEMS

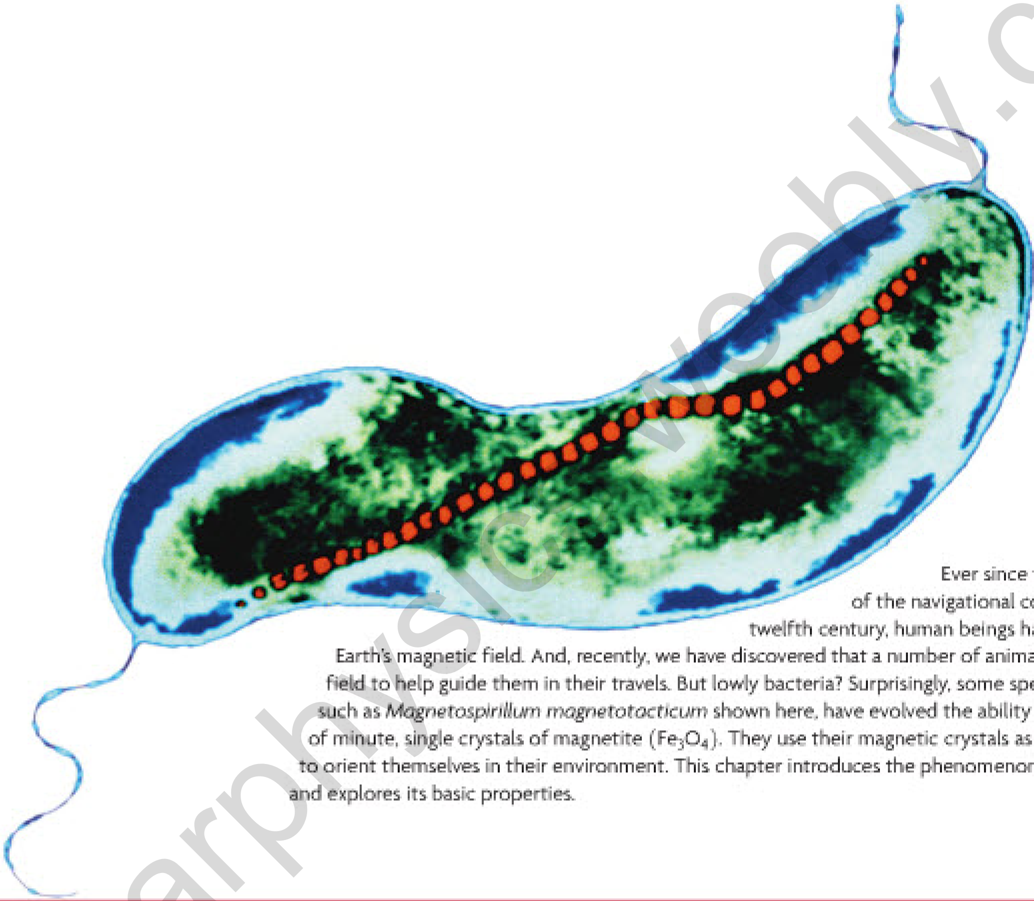
128. •• **Referring to Example 21–7** Suppose the three resistors in this circuit have the values $R_1 = 100.0$ Ω, $R_2 = 200.0$ Ω, and $R_3 = 300.0$ Ω, and that the emf of the battery is 12.0 V. (The resistor numbers are given in the Interactive Figure.) **(a)** Find the potential difference across each resistor. **(b)** Find the current that flows through each resistor.

129. •• **Referring to Example 21–7** Suppose $R_1 = R_2 = 225$ Ω and $R_3 = R$. The emf of the battery is 12.0 V. (The resistor numbers are given in the Interactive Figure.) **(a)** Find the value of R such that the current supplied by the battery is 0.0750 A. **(b)** Find the value of R that gives a potential difference of 2.65 V across resistor 2.

130. •• **IP Referring to Example 21–9** Suppose the resistance of the 126-Ω resistor is reduced by a factor of 2. The other resistor is 275 Ω, the capacitor is 182 μF, and the battery has an emf of 3.00 V. **(a)** Does the final value of the charge on the capacitor increase, decrease, or stay the same? Explain. **(b)** Does the time for the capacitor to charge to 80.0% of its final value increase, decrease, or stay the same? Explain. **(c)** Find the time referred to in part (b).

131. •• **IP Referring to Example 21–9** Suppose the capacitance of the 182-μF capacitor is reduced by a factor of 2. The two resistors are 126 Ω and 275 Ω, and the battery has an emf of 3.00 V. **(a)** Find the final value of the charge on the capacitor. **(b)** Does the time for the capacitor to charge to 80.0% of its final value increase, decrease, or stay the same? Explain. **(c)** Find the time referred to in part (b).

22 Magnetism



Ever since the development of the navigational compass in the twelfth century, human beings have steered by the Earth's magnetic field. And, recently, we have discovered that a number of animals also use this field to help guide them in their travels. But lowly bacteria? Surprisingly, some species of bacteria, such as *Magnetospirillum magnetotacticum* shown here, have evolved the ability to produce chains of minute, single crystals of magnetite (Fe_3O_4). They use their magnetic crystals as a kind of compass to orient themselves in their environment. This chapter introduces the phenomenon of magnetism and explores its basic properties.

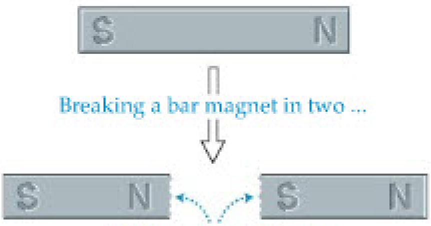
The effects of magnetism have been known since antiquity. For example, a piece of lodestone, a naturally occurring iron oxide mineral, can behave just like a modern-day bar magnet. What has been discovered only much more recently, however, is the intimate connection that exists between electricity and magnetism. The first evidence of such a connection was obtained quite by accident, in 1820, as the Danish scientist Hans Christian Oersted (1777–1851) performed a demonstration during one of his popular public lectures. His observation, and the conclusions that followed from it, are the primary focus of this chapter.

22–1 The Magnetic Field

We begin our study of magnetism with a few general observations regarding magnets and the fields they produce. These observations apply over a wide range of scales, from the behavior of small, handheld bar magnets to the global effects associated with the magnetic field of the Earth. As we shall see later in this chapter, the ultimate source of any magnetic field is nothing more than the motion of electric charge.

(a) Opposite poles attract **(b)** Like poles repel

▲ **FIGURE 22–1 The force between two bar magnets**



▲ **FIGURE 22–2 Magnets always have two poles**

When a bar magnet is broken in half, two new poles appear. Each half has both a north pole and a south pole, just like any other bar magnet.

Permanent Magnets

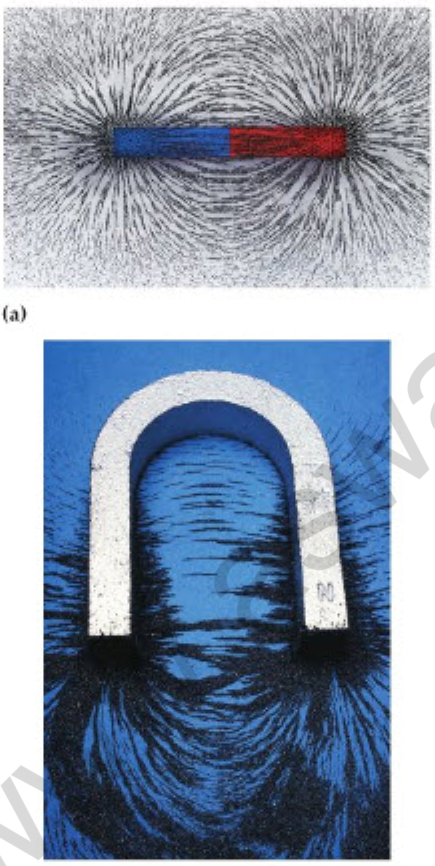
Our first direct experience with magnets is often a playful exploration of the way small permanent magnets, called bar magnets, affect one another. As we know, a bar magnet can attract another magnet or repel it, depending on which ends of the magnets are brought together. One end of a magnet is referred to as its *north pole;* the other end is its *south pole.* The poles are defined as follows: Imagine suspending a bar magnet from a string so that it is free to rotate in a horizontal plane—much like the needle of a compass. The end of the magnet that points toward the north geographic pole of the Earth we refer to as the "north-seeking pole," or simply the north pole. The opposite end of the magnet is the "south-seeking pole," or the south pole.

The rule for whether two bar magnets attract or repel each other is learned at an early age: opposites attract; likes repel. Thus, if two magnets are brought together so that opposite poles approach each other, as in **Figure 22–1 (a)**, the force they experience is attractive. Like poles brought close together, as in **Figure 22–1 (b)**, experience a repulsive force.

An interesting aspect of magnets is that they *always* have two poles. You might think that if you break a bar magnet in two, each of the halves will have just one pole. Instead, breaking a magnet in half results in the appearance of two new poles on either side of the break, as illustrated in **Figure 22–2**. This behavior is fundamentally different from that found in electricity, where the two types of charge can exist separately. Though physicists continue to look for the elusive "magnetic monopole," and speculate as to its possible properties, none has been found.

Magnetic Field Lines

Just as an electric charge creates an electric "field" in its vicinity, so too does a magnet create a magnetic field. As in the electric case, the magnetic field represents the effect a magnet has on its surroundings. For example, in Figure 19–15 we saw a visual indication of the electric field $\vec{E}$ of a point charge using grass seeds suspended in oil. Similarly, the magnetic field, which we represent with the symbol $\vec{B}$, can be visualized using small iron filings sprinkled onto a smooth surface. In **Figure 22–3 (a)**, for example, a sheet of glass or plastic is placed on top of a bar magnet. When iron filings are dropped onto the sheet they align with the magnetic field in their vicinity, giving a good representation of the overall field produced by the magnet. Similar effects are seen in **Figure 22–3 (b)**, with a magnet that has been bent to bring its poles close together. Because of its shape, this type of magnet is referred to as a horseshoe magnet.



(a) (b)

▲ **FIGURE 22–3 Magnetic field lines**

The field of a bar magnet **(a)** or horseshoe magnet **(b)** can be visualized using iron filings on a sheet of glass or paper. The filings orient themselves with the field lines, creating a "snapshot" of the magnetic field.

In both cases shown in Figure 22–3, notice that the filings are bunched together near the poles of the magnets. This is where the magnetic field is most intense. We illustrate this by drawing field lines that are densely packed near the poles, as in Figure 22–4. As one moves away from a magnet in any direction the field weakens, as indicated by the increasingly wider separations between field lines. Thus, we indicate the magnitude of the vector $\vec{B}$ by the spacing of the field lines.

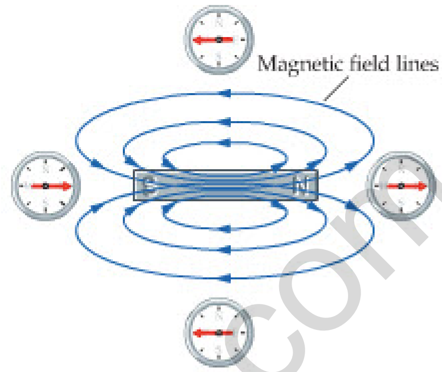
We can also assign a direction to the magnetic field. In particular, the direction of $\vec{B}$ is defined as follows:

> The direction of the magnetic field, $\vec{B}$, at a given location is the direction in which the north pole of a compass points when placed at that location.

We apply this definition to the bar magnet in Figure 22–4. Imagine, for example, placing a compass near its south pole. Because opposites attract, the north pole of the compass needle—the end with the arrowhead—will point toward the south pole of the magnet. Hence, according to our definition, the direction of the magnetic field at that location is toward the bar magnet's south pole. Similarly, one can see that the magnetic field must point away from the north pole of the bar magnet. In general,

> Magnetic field lines exit from the north pole of a magnet and enter at the south pole.

The field lines continue even within the body of a magnet. In fact, magnetic field lines always form closed loops; they never start or stop anywhere, in contrast to electric field lines. Again, this is related to the fact that there are no magnetic monopoles, where a magnetic field line could start or stop, whereas electric field lines start on positive charges and stop on negative charges.



▲ **FIGURE 22–4 Magnetic field lines for a bar magnet**
The field lines are closely spaced near the poles, where the magnetic field $\vec{B}$ is most intense. In addition, the lines form closed loops that leave at the north pole of the magnet and enter at the south pole.

REAL-WORLD PHYSICS
Refrigerator magnets

CONCEPTUAL CHECKPOINT 22–1 MAGNETIC FIELD LINES

Can magnetic field lines cross one another?

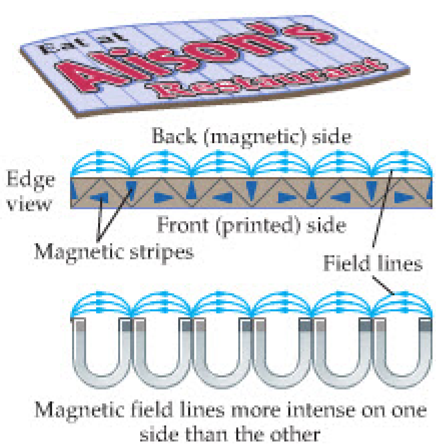
REASONING AND DISCUSSION

Recall that the direction in which a compass points at any given location is the direction of the magnetic field at that point. Since a compass can point in only one direction, there must be only one direction for the field $\vec{B}$. If field lines were to cross, however, there would be two directions for $\vec{B}$ at the crossing point, and this is not allowed.

ANSWER

No. Magnetic field lines never cross.

An interesting example of magnetic field lines is provided by the humble refrigerator magnet. The flexible variety of these popular magnets has the unusual property that one side sticks quite strongly to the refrigerator, whereas the other side (the printed side) does not stick at all. Clearly, the magnetic field produced by such a magnet is not like that of a simple bar magnet, which generates a symmetrical field. Instead, these refrigerator magnets are composed of multiple magnetic stripes of opposite polarity, as indicated in Figure 22–5. The net effect is a magnetic field similar to the field that would be produced by a large number of tiny horseshoe magnets placed side by side, pointing in the same direction. In this way, the field is intense on the side containing the poles of the tiny magnets, as in Figure 22–3 (b), and very weak on the other side.



▲ **FIGURE 22–5 Refrigerator magnet**
A flexible refrigerator magnet is made from a large number of narrow magnetic stripes with magnetic fields in different directions. The net effect is a field similar to that of a series of parallel horseshoe magnets placed side by side.

Geomagnetism

The Earth, like many planets, produces its own magnetic field. In many respects, the Earth's magnetic field is like that of a giant bar magnet, as illustrated in Figure 22–6, with a pole near each geographic pole of the Earth. The magnetic poles are not perfectly aligned with the rotational axis of the Earth, however, but are inclined at an angle that varies slowly with time. Presently, the magnetic poles are tilted away from the rotational axis by an angle of about 11.5°. The current location of the north magnetic pole is just west of Ellef Ringnes Island, one of the Queen Elizabeth Islands of extreme northern Canada.

REAL-WORLD PHYSICS
The Earth's magnetic field

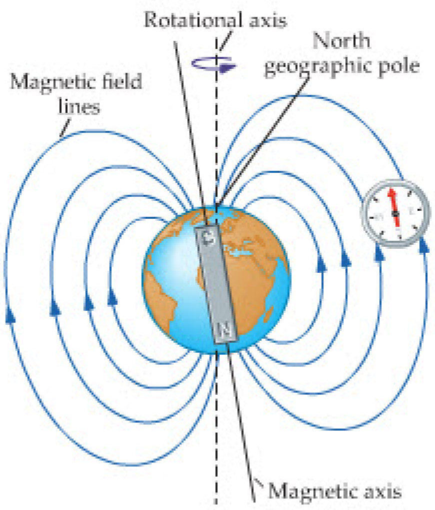


FIGURE 22–6 Magnetic field of the Earth
The Earth's magnetic field is similar to that of a giant bar magnet tilted slightly from the rotational axis. Notice that the north geographic pole is actually near the south pole of the Earth's magnetic field.

Since the north pole of a compass needle points toward the north magnetic pole of the Earth, and since opposites attract, it follows that

> the *north* geographic pole of the Earth is actually near the *south* pole of the Earth's magnetic field.

This is indicated in Figure 22–6. The figure also shows that the field lines are essentially horizontal (parallel to the Earth's surface) near the equator but enter or leave the Earth vertically near the poles. Thus, for example, if you were to stand near the north geographic pole, your compass would try to point straight down.

Although the Earth's magnetic field is similar in many respects to that of a huge bar magnet, it is far more complex in both its shape and behavior—in fact, even the mechanism by which the field is produced is still not completely understood. It seems likely, however, that flowing currents of molten material in the Earth's core are the primary cause of the field, as expressed in the **dynamo theory** of magnetism.

One of the reasons for lingering uncertainty about the Earth's magnetic field is that its behavior over time is rather complicated. For example, we have already mentioned that the magnetic poles drift about slowly with time. This is just the beginning of the field's interesting behavior, however. We have learned in the last century that the Earth's field has actually *reversed* direction many times over the ages. In fact, the last reversal occurred about 780,000 years ago—from 980,000 years ago to 780,000 years ago your compass would have pointed opposite to its direction today.

These ancient field reversals have left a permanent record in the rocks of the ocean floors, among other places. By analyzing the evidence of these reversals, geologists have found strong support for the theories of continental drift and plate tectonics and have developed a whole new branch of geology referred to as **paleomagnetism.** We shall return to these topics again briefly near the end of this chapter.

22–2 The Magnetic Force on Moving Charges

We have discussed briefly the familiar forces that act between one magnet and another. In this section we consider the force a magnetic field exerts on a moving electric charge. As we shall see, both the magnitude and the direction of this force have rather unusual characteristics. We begin with the magnitude.

Magnitude of the Magnetic Force

Consider a magnetic field $\vec{\mathbf{B}}$ that points from left to right in the plane of the page, as indicated in Figure 22–7. A particle of charge q moves through this region with a velocity $\vec{\mathbf{v}}$. Note that the angle between $\vec{\mathbf{v}}$ and $\vec{\mathbf{B}}$ has a magnitude denoted by the symbol θ. Experiment shows that the magnitude of the force $\vec{\mathbf{F}}$ experienced by this particle is given by the following:

Magnitude of the Magnetic Force, *F*

$$F = |q|vB\sin\theta \quad 22\text{–}1$$

SI unit: newton, N

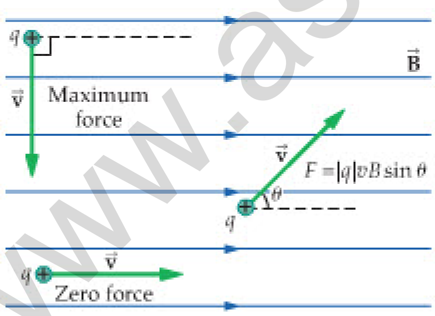


FIGURE 22–7 The magnetic force on a moving charged particle
A particle of charge q moves through a region of magnetic field $\vec{\mathbf{B}}$ with a velocity $\vec{\mathbf{v}}$. The magnitude of the force experienced by the charge is $F = |q|vB\sin\theta$. Note that the force is a maximum when the velocity is perpendicular to the field and is zero when the velocity is parallel to the field.

Thus, the magnetic force depends on several factors. Two of these are the same as for the electric force:

(i) the charge of the particle, q;
(ii) the magnitude of the field, in this case the magnetic field, B;

However, the magnetic force also depends on two factors that do not affect the strength of the electric force:

(iii) the speed of the particle, v;
(iv) the magnitude of the angle between the velocity vector and the magnetic field vector, θ.

It follows that the behavior of particles in magnetic fields is significantly different from their behavior in electric fields.

In particular, a particle must have a charge and must be moving if the magnetic field is to exert a force on it. Even then the force will vanish if the particle moves in the direction of the field (that is, $\theta = 0$), or in the direction opposite to the field ($\theta = 180°$). The maximum force is experienced when the particle moves at right angles to the field, so that $\theta = 90°$, and $\sin\theta = 1$.

Finally, the expression in Equation 22–1 gives only the magnitude of the force, and hence depends on the magnitude of the charge, $|q|$. As we shall see later in this section, the *sign* of q is important in determining the *direction* of the magnetic force. In addition, the angle θ in Equation 22–1 is the magnitude of the angle between $\vec{\mathbf{v}}$ and $\vec{\mathbf{B}}$, and always has a value in the range $0 \le \theta \le 180°$.

In practice, the magnitude—or strength—of a magnetic field is *defined* by the relation given in Equation 22–1. Thus, B is given by the following:

Definition of the Magnitude of the Magnetic Field, *B*

$$B = \frac{F}{|q|v\sin\theta} \qquad 22\text{–}2$$

SI unit: 1 tesla = 1 T = 1 N/(A · m)

The units of B are those of force divided by the product of charge and speed; that is, N/[C · (m/s)]. Rearranging slightly, we can write these units as N/[(C/s) · m]. Finally, noting that 1 A = 1 C/s, we find that 1 N/[C · (m/s)] = 1 N/(A · m). The latter combination of units is named the **tesla,** in recognition of the pioneering electrical and magnetic studies of the Croatian-born American engineer Nikola Tesla (1856–1943). In particular,

$$1\ \text{tesla} = 1\ \text{T} = 1\ \text{N/(A} \cdot \text{m)}$$

We now give an example of the application of Equation 22–1.

PROBLEM-SOLVING NOTE

The Properties of Magnetic Forces

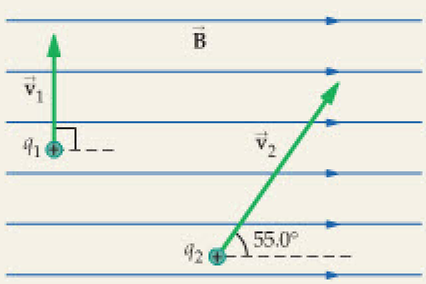
Remember that a magnetic force occurs only when a charged particle moves in a direction that is not along the line defined by the magnetic field. Stationary particles experience no magnetic force.

EXAMPLE 22–1 A TALE OF TWO CHARGES

Particle 1, with a charge $q_1 = 3.60\ \mu\text{C}$ and a speed $v_1 = 862$ m/s, travels at right angles to a uniform magnetic field. The magnetic force it experiences is 4.25×10^{-3} N. Particle 2, with a charge $q_2 = 53.0\ \mu\text{C}$ and a speed $v_2 = 1.30 \times 10^3$ m/s, moves at an angle of 55.0° relative to the same magnetic field. Find **(a)** the strength of the magnetic field and **(b)** the magnitude of the magnetic force exerted on particle 2.

PICTURE THE PROBLEM

The two charged particles are shown in our sketch, along with the magnetic field lines. Notice that $q_1 = 3.60\ \mu\text{C}$ moves at right angles to $\vec{\mathbf{B}}$; the charge $q_2 = 53.0\ \mu\text{C}$ moves at an angle of 55.0° with respect to $\vec{\mathbf{B}}$. The magnetic force also depends on the speeds of the particles, $v_1 = 862$ m/s and $v_2 = 1.30 \times 10^3$ m/s.



STRATEGY

a. We can find the strength of the magnetic field using the information given for particle 1. In particular, the particle moves at right angles to the magnetic field ($\theta_1 = 90°$), and, therefore, the magnetic force it experiences is $F_1 = |q_1|v_1B\sin\theta_1 = q_1v_1B$. Given that we know F_1, q_1, and v_1, we can solve for B.

b. Once we have determined B in part (a), it is straightforward to calculate the magnetic force on particle 2 using $F_2 = |q_2|v_2B\sin\theta_2$.

SOLUTION

Part (a)

1. Set $\theta_1 = 90°$ in $F_1 = |q_1|v_1B\sin\theta_1$, then solve for B. Use q_1 and v_1 for the magnitude of the charge and speed, respectively:

$$F_1 = |q_1|v_1B\sin 90° = q_1v_1B$$

$$B = \frac{F_1}{q_1v_1}$$

2. Substitute numerical values:

$$B = \frac{F_1}{q_1v_1} = \frac{4.25 \times 10^{-3}\ \text{N}}{(3.60 \times 10^{-6}\ \text{C})(862\ \text{m/s})} = 1.37\ \text{T}$$

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Part (b)

3. Use $B = 1.37$ T in $F_2 = |q_2|v_2 B \sin\theta_2$ to find the magnetic force exerted on particle 2:

$$F_2 = |q_2|v_2 B \sin\theta_2$$
$$= (53.0 \times 10^{-6}\ \text{C})(1.30 \times 10^3\ \text{m/s})(1.37\ \text{T}) \sin 55.0^\circ$$
$$= 0.0773\ \text{N}$$

INSIGHT

Notice that the charge and speed of a particle are not enough to determine the magnetic force acting on it—its direction of motion relative to the magnetic field is needed as well.

PRACTICE PROBLEM

At what angle relative to the magnetic field must particle 2 move if the magnetic force it experiences is to be 0.0500 N? [**Answer:** $\theta = 32.0^\circ$]

Some related homework problems: Problem 8, Problem 11

TABLE 22–1 Typical Magnetic Fields

Physical system	Magnetic field (G)
Magnetar (a magnetic neutron star formed in a supernova explosion)	10^{15}
Strongest man-made magnetic field	6×10^5
High-field MRI	15,000
Low-field MRI	2000
Sunspots	1000
Bar magnet	100
Earth	0.50

The tesla is a fairly large unit of magnetic strength, especially when compared with the magnetic field at the surface of the Earth, which is roughly 5.0×10^{-5} T. Thus, another commonly used unit of magnetism is the gauss (G), defined as follows:

$$1\ \text{gauss} = 1\ \text{G} = 10^{-4}\ \text{T}$$

In terms of the gauss, the Earth's magnetic field on the surface of the Earth is approximately 0.5 G. It should be noted, however, that the gauss is not an SI unit of magnetic field. Even so, it finds wide usage because of its convenient magnitude. The magnitudes of some typical magnetic fields are given in Table 22–1.

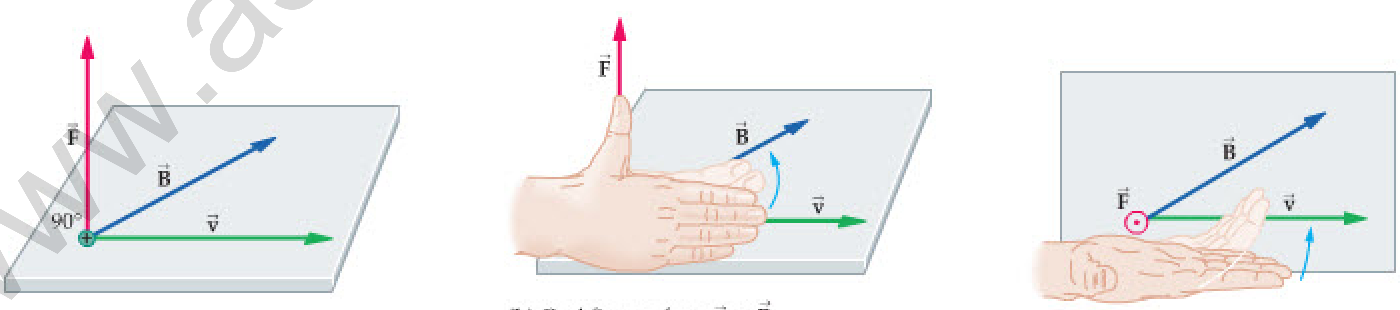
Magnetic Force Right-Hand Rule (RHR)

We now consider the *direction* of the magnetic force, which is rather interesting and unexpected. Instead of pointing in the direction of the magnetic field, $\vec{\mathbf{B}}$, or the velocity, $\vec{\mathbf{v}}$, as one might expect, the following behavior is observed:

> The magnetic force $\vec{\mathbf{F}}$ points in a direction that is perpendicular to both $\vec{\mathbf{B}}$ and $\vec{\mathbf{v}}$.

As an example, consider the vectors $\vec{\mathbf{B}}$ and $\vec{\mathbf{v}}$ in **Figure 22–8 (a)**, which lie in the indicated plane. The force on a positive charge, $\vec{\mathbf{F}}$, as shown in the figure, is perpendicular to this plane and hence to both $\vec{\mathbf{B}}$ and $\vec{\mathbf{v}}$.

Notice that $\vec{\mathbf{F}}$ could equally well point downward in Figure 22–8 (a) and still be perpendicular to the plane. The way we determine the precise direction of $\vec{\mathbf{F}}$ is with a right-hand rule—similar to the right-hand rules used in calculating torques and angular momentum in Chapter 11. To be specific, the direction of $\vec{\mathbf{F}}$ is found using the *magnetic force* right-hand rule:



(a) $\vec{\mathbf{F}}$ is perpendicular to both $\vec{\mathbf{v}}$ and $\vec{\mathbf{B}}$

(b) Curl fingers from $\vec{\mathbf{v}}$ to $\vec{\mathbf{B}}$; thumb points in direction of $\vec{\mathbf{F}}$

(c) Top view, looking down on $\vec{\mathbf{F}}$

▲ **FIGURE 22–8 The magnetic force right-hand rule**

(a) The magnetic force, $\vec{\mathbf{F}}$, is perpendicular to both the velocity, $\vec{\mathbf{v}}$, and the magnetic field, $\vec{\mathbf{B}}$. (The force vectors shown in this figure are for the case of a positive charge. The force on a negative charge would be in the opposite direction.) **(b)** As the fingers of the right hand are curled from $\vec{\mathbf{v}}$ to $\vec{\mathbf{B}}$, the thumb points in the direction of $\vec{\mathbf{F}}$. **(c)** An overhead view, looking down on the plane defined by the vectors $\vec{\mathbf{v}}$ and $\vec{\mathbf{B}}$. In this two-dimensional representation, the force vector comes out of the page and is indicated by a circle with a dot inside. If the charge was negative, the force would point into the page, and the symbol indicating $\vec{\mathbf{F}}$ would be a circle with an X inside.

Magnetic Force Right-Hand Rule (RHR)

To find the direction of the magnetic force on a positive charge, start by pointing the fingers of your right hand in the direction of the velocity, $\vec{\mathbf{v}}$. Now curl your fingers toward the direction of $\vec{\mathbf{B}}$, as illustrated in Figure 22–8 (b). Your thumb points in the direction of $\vec{\mathbf{F}}$. If the charge is negative, the force points opposite to the direction of your thumb.

Applying this rule to Figure 22–8 (a), we see that $\vec{\mathbf{F}}$ does indeed point upward, as indicated.

A mathematical way to write the magnetic force that includes both its magnitude and direction is in terms of the vector **cross product.** Details on the cross product are given in Appendix A, but basically we can write the magnetic force $\vec{\mathbf{F}}$ as follows:

$$\vec{\mathbf{F}} = q\vec{\mathbf{v}} \times \vec{\mathbf{B}}$$

In this expression, the term $\vec{\mathbf{v}} \times \vec{\mathbf{B}}$ is referred to as the cross product of $\vec{\mathbf{v}}$ and $\vec{\mathbf{B}}$. From the definition of the cross product, the magnitude of the force is $F = |q|vB \sin\theta$ where θ is the magnitude of the angle between $\vec{\mathbf{v}}$ and $\vec{\mathbf{B}}$, precisely as in Equation 22–1. In addition, the direction of $\vec{\mathbf{F}}$ is given by the magnetic force RHR. Thus, Equation 22–1 plus the magnetic force RHR are all we need to calculate magnetic forces—the cross product simply provides a more compact way of saying the same thing.

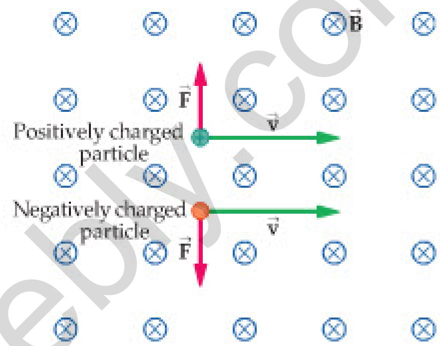
Since three-dimensional plots like the one in Figure 22–8 (b) are somewhat difficult to sketch, we often use a shorthand for indicating vectors that point into or out of the page. (See Appendix A for a discussion and examples.) Imagining a vector as an arrow, with a pointed tip at one end and crossed feathers at the other end, we say that if a vector points out of the page—toward us—we see its tip. This situation is indicated by drawing a circle with a dot inside it on the page, as in Figure 22–8 (c). If, on the other hand, the vector points into the page, we see the crossed feathers of its base; hence, we draw a circle with an X inside it to symbolize this situation.

As an example, Figure 22–9 indicates a uniform magnetic field $\vec{\mathbf{B}}$ that points into the page. A particle with a positive charge q moves to the right. Using the magnetic force RHR—extending our fingers to the right and then curling them into the page—we see that the force exerted on this particle is upward, as indicated. If the charge is negative, the direction of $\vec{\mathbf{F}}$ is reversed, as is also illustrated in Figure 22–9.

PROBLEM-SOLVING NOTE

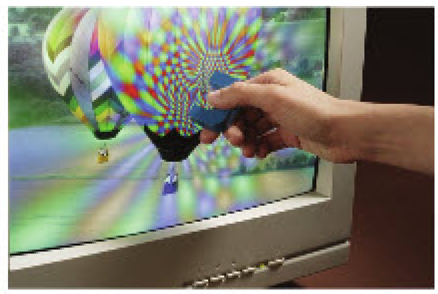
The Magnetic Force for Positive and Negative Charges

Note that the magnetic force right-hand rule is stated for the case of a positively charged particle. If the particle has a negative charge, the direction of the force is reversed.



▲ **FIGURE 22–9 The magnetic force for positive and negative charges**

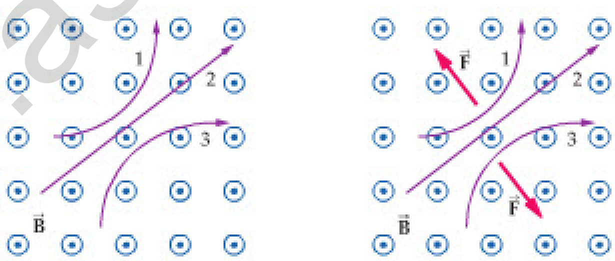
The direction of the magnetic force depends on the sign of the charge. Specifically, the force exerted on a negatively charged particle is opposite in direction to the force exerted on a positively charged particle.



▲ A cathode-ray tube (CRT) produces images by deflecting a beam of electrons with electric fields. For more details, see the discussion and figure given in the Passage Problem for Chapter 29. When a bar magnet is moved close to the screen of a CRT, as shown in this photo, the magnetic field it produces exerts a force on the moving electrons in the beam. This force changes the direction of motion of the electrons, resulting in a distorted picture. The distortion will vanish if the magnetic field is reduced by moving the bar magnet farther away from the screen.

CONCEPTUAL CHECKPOINT 22–2 CHARGE OF A PARTICLE

Three particles travel through a region of space where the magnetic field is out of the page, as shown below in the sketch to the left. For each of the three particles, state whether the particle's charge is positive, negative, or zero.



REASONING AND DISCUSSION

In the second sketch, we indicate the general direction of the force required to cause the observed motion. The force indicated for particle 3 is in the direction given by the magnetic force RHR; hence, particle 3 must have a positive charge. The force acting on particle 1 is in the opposite direction; hence, that particle must be negatively charged. Finally, particle 2 is undeflected; hence, its charge, and the force acting on it, must be zero.

ANSWER

Particle 1, negative; particle 2, zero; particle 3, positive.

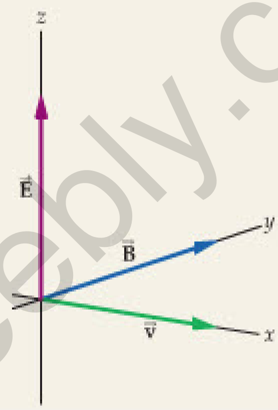
We conclude this section with an Example illustrating the magnetic force RHR.

EXAMPLE 22–2 ELECTRIC AND MAGNETIC FIELDS

A particle with a charge of 7.70 μC and a speed of 435 m/s is acted on by both an electric and a magnetic field. The particle moves along the x axis in the positive direction, the magnetic field has a strength of 3.20 T and points in the positive y direction, and the electric field points in the positive z direction with a magnitude of 8.10×10^3 N/C. Find the magnitude and direction of the net force acting on the particle.

PICTURE THE PROBLEM

The physical situation is shown in our sketch. In particular, we show a three-dimensional coordinate system with each of the three relevant vectors, $\vec{v}$, $\vec{B}$, and $\vec{E}$, indicated. Note that the charge is positive, and therefore $|q| = q$.



STRATEGY

First, the force due to the electric field is in the direction of $\vec{E}$ (positive z direction), and it has a magnitude qE. Second, the direction of the magnetic force is given by the RHR. In this case curling the fingers of a right hand from $\vec{v}$ to $\vec{B}$ results in the thumb (the direction of $\vec{F}$) pointing in the positive z direction. Thus, in this system the forces due to the electric and magnetic fields are in the same direction. Finally, because the angle between $\vec{v}$ and $\vec{B}$ is 90°, the magnitude of the magnetic force is qvB.

SOLUTION

1. Calculate the magnitude of the electric force exerted on the particle:

$$F_E = qE = (7.70 \times 10^{-6}\ \text{C})(8.10 \times 10^3\ \text{N/C}) = 6.24 \times 10^{-2}\ \text{N}$$

2. Calculate the magnitude of the magnetic force exerted on the particle:

$$F_B = qvB = (7.70 \times 10^{-6}\ \text{C})(435\ \text{m/s})(3.20\ \text{T}) = 1.07 \times 10^{-2}\ \text{N}$$

3. Since both forces are in the positive z direction, simply add them to obtain the net force:

$$F_{net} = F_E + F_B = (6.24 \times 10^{-2}\ \text{N}) + (1.07 \times 10^{-2}\ \text{N}) = 7.31 \times 10^{-2}\ \text{N}$$

INSIGHT

Note that the net force on the particle will be in the positive z direction as long as the velocity vector, $\vec{v}$, points in *any* direction in the x-y plane. The magnitude of the net force, however, will depend on the direction of $\vec{v}$.

In addition, the coordinate system shown in the Picture the Problem is a *right-handed* coordinate system. This means, for example, that the axes are set up so that $\hat{x} \times \hat{y} = \hat{z}$, as you can verify with the right-hand rule. Further details are given in Appendix A. We will always use right-handed coordinate systems in this text.

PRACTICE PROBLEM

Find the net force on the particle if its velocity is reversed and it moves in the negative x direction.
[**Answer:** $F_{net} = F_E + F_B = (6.24 \times 10^{-2}\ \text{N}) - (1.07 \times 10^{-2}\ \text{N}) = 5.17 \times 10^{-2}$ N in the positive z direction.]

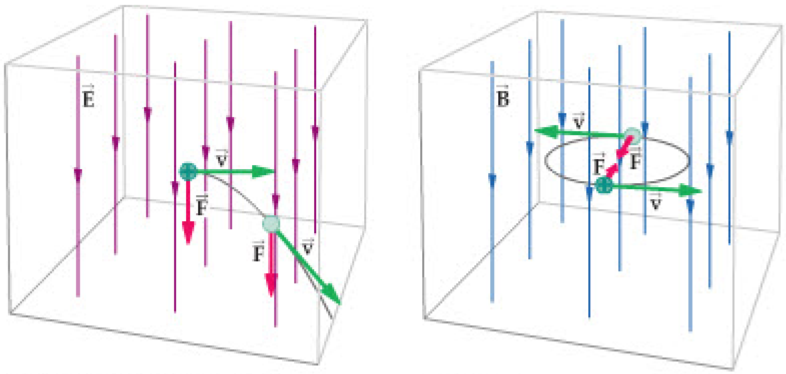
Some related homework problems: Problem 14, Problem 16

22–3 The Motion of Charged Particles in a Magnetic Field

As we have seen, the magnetic force has characteristics that set it apart from the force exerted by electric or gravitational fields. In particular, the magnetic force depends not only on the speed of a particle but on its direction of motion as well. We now explore some of the consequences that follow from these characteristics and relate them to the type of motion that occurs in magnetic fields.

Electric Versus Magnetic Forces

We begin by investigating the motion of a charged particle as it is projected into a region with either an electric or a magnetic field. For example, in Figure 22–10 (a) we consider a uniform electric field pointing downward. A positively charged particle moving horizontally into this region experiences a constant downward



(a) Motion in an electric field

(b) Motion in a magnetic field

FIGURE 22–10 Differences between motion in electric and magnetic fields

(a) A positively charged particle moving into a region with an electric field experiences a downward force that causes it to accelerate. **(b)** A positively charged particle entering a magnetic field experiences a horizontal force at right angles to its direction of motion. In this case, the speed of the particle remains constant.

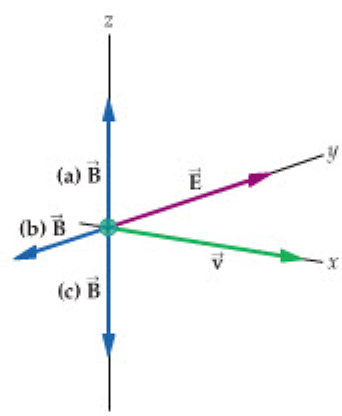
force—much like a mass in a uniform gravitational field. As a result, the particle begins to accelerate downward and follow a parabolic path.

If the same particle, moving to the right, encounters a magnetic field instead, as in **Figure 22–10 (b)**, the resulting motion is quite different. We again assume that the field is uniform and pointing downward. In this case, however, the magnetic force RHR shows that $\vec{F}$ points into the page. Our particle now begins to follow a horizontal path into the page. As we shall see, this path is circular.

Perhaps even more significant than these differences in motion is the fact that an electric field can do work on a charged particle, whereas a constant magnetic field cannot. In Figure 22–10 (a), for example, as soon as the particle begins to move downward, a component of its velocity is in the direction of the electric force. This means that the electric force does work on the particle, and its speed increases—again, just like a mass falling in a gravitational field. If the particle moves through a magnetic field, however, no work is done on it because the magnetic force is *always* at right angles to the direction of motion. Thus, the speed of the particle in the magnetic field remains constant.

CONCEPTUAL CHECKPOINT 22–3 DIRECTION OF THE MAGNETIC FIELD

In a device called a **velocity selector**, charged particles move through a region of space with both an electric and a magnetic field. If the speed of the particle has a particular value, the net force acting on it is zero. Assume that a positively charged particle moves in the positive x direction, as shown in the sketch, and the electric field is in the positive y direction. Should the magnetic field be in **(a)** the positive z direction, **(b)** the negative y direction, or **(c)** the negative z direction in order to give zero net force?



REASONING AND DISCUSSION

The force exerted by the electric field is in the positive y direction; hence, the magnetic force must be in the negative y direction if it is to cancel the electric force. If we simply try the three possible directions for $\vec{B}$ one at a time, applying the magnetic force RHR in each case, we find that only a magnetic field along the positive z axis gives rise to a force in the negative y direction, as desired.

ANSWER

(a) $\vec{B}$ should point in the positive z direction.

To follow up on Conceptual Checkpoint 22–3, let's determine the speed for which the net force is zero. First, note that the electric force has a magnitude qE. Second, the magnitude of the magnetic force is qvB. With $\vec{E}$ in the positive y direction and $\vec{B}$ in the positive z direction, these forces are in opposite directions. Setting the magnitudes of the forces equal yields $qE = qvB$. Canceling the charge q and solving for v we find

$$v = \frac{E}{B} \qquad \text{(velocity selector)}$$

A particle with this speed, regardless of its charge, passes through the velocity selector with zero net force and hence no deflection.

The physical principle illustrated in the velocity selector can be used to measure the speed of blood with a device known as an *electromagnetic flowmeter*.

REAL-WORLD PHYSICS: BIO
The electromagnetic flowmeter

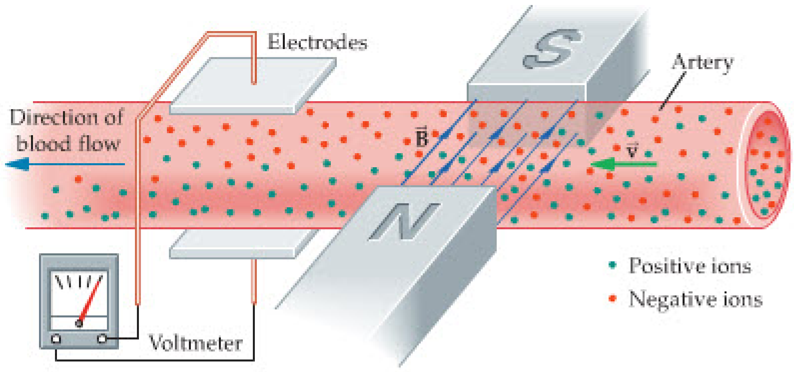


FIGURE 22–11 The electromagnetic flowmeter

As blood flows through a magnetic field, the charged ions it contains are deflected. The deflection of charged particles results in an electric field opposing the deflection. If the speed of the blood is v, the electric field generated by ions moving through the magnetic field satisfies the relation $v = E/B$.

Suppose an artery passes between the poles of a magnet, as shown in Figure 22–11. Charged ions in the blood will be deflected at right angles to the artery by the magnetic field. The resulting charge separation produces an electric field that opposes the magnetic deflection. When the electric field is strong enough, the deflection ceases and the blood flows normally through the artery. If the electric field is measured, and the magnetic field is known, the speed of the blood flow is simply $v = E/B$, as in a standard velocity selector.

Constant-Velocity, Straight-Line Motion

Recall that the magnetic force is zero if a particle's velocity $\vec{\mathbf{v}}$ is parallel (or antiparallel) to the magnetic field $\vec{\mathbf{B}}$. In such a case the particle's acceleration is zero; therefore, its velocity remains constant. Thus, the simplest type of motion in a magnetic field is a constant-velocity, straight-line drift along the magnetic field lines.

Circular Motion

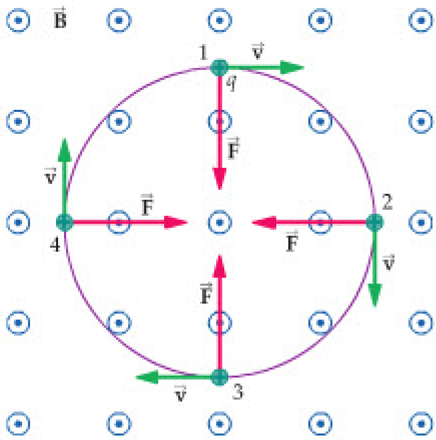


FIGURE 22–12 Circular motion in a magnetic field

A charged particle moves in a direction perpendicular to the magnetic field. At each point on the particle's path the magnetic force is at right angles to the velocity and hence toward the center of a circle.

The next simplest case is a particle with a velocity that is perpendicular to the magnetic field. Consider, for example, the situation shown in Figure 22–12. Here a particle of mass m, charge $+q$, and speed v moves in a region with a constant magnetic field $\vec{\mathbf{B}}$ pointing out of the page. Since $\vec{\mathbf{v}}$ is at right angles to $\vec{\mathbf{B}}$, the magnitude of the magnetic force is $F = |q|vB \sin 90° = |q|vB$.

Now we consider the particle's motion. At point 1 the particle is moving to the right; hence, the magnetic force is downward, causing the particle to accelerate in the downward direction. When the particle reaches point 2, it is moving downward, and now the magnetic force is to the left. This causes the particle to accelerate to the left. At point 3, the force exerted on it is upward, and so on, as the particle continues moving.

Thus, at every point on the particle's path the magnetic force is at right angles to the velocity, pointing toward a common center—but this is just the condition required for circular motion. As we saw in Section 6–5, in circular motion the acceleration is toward the center of the circle; for this reason, a centripetal force is required to cause the motion. In this case the centripetal force is supplied by the magnetic force—in the same way that a string exerts a centripetal force on a ball being whirled about in a circle.

Recall that the centripetal acceleration of a particle moving with a speed v in a circle of radius r is

$$a_{cp} = \frac{v^2}{r}$$

Therefore, setting ma_{cp} equal to the magnitude of the magnetic force, $|q|vB$, yields the following condition:

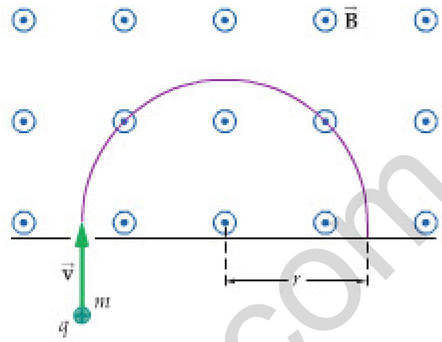
$$m\frac{v^2}{r} = |q|vB$$

Canceling one power of the speed, v, we find that the radius of the circular orbit is

$$r = \frac{mv}{|q|B} \qquad 22\text{–}3$$

▲ In addition to separating and "weighing" isotopes, mass spectrometers can be used to study the chemical composition and structure of large, biologically important molecules. Here, a researcher studies the spectrum of myoglobin, an oxygen-carrying protein found in muscle tissue. Each peak represents a fragment of the molecule with its own characteristic combination of mass and charge.

We can see, therefore, that the faster and more massive the particle, the larger the circle. Conversely, the stronger the magnetic field and the greater the charge, the smaller the circle.



▲ **FIGURE 22–13 The operating principle of a mass spectrometer**

In a mass spectrometer, a beam of charged particles enters a region with a magnetic field perpendicular to the velocity. The particles then follow a circular orbit of radius $r = mv/|q|B$. Particles of different mass will follow different paths.

EXERCISE 22–1

An electron moving perpendicular to a magnetic field of 4.60×10^{-3} T follows a circular path of radius 2.80 mm. What is the electron's speed?

SOLUTION

Solving Equation 22–3 for v, we find

$$v = \frac{r|q|B}{m} = \frac{(2.80 \times 10^{-3}\ \text{m})(1.60 \times 10^{-19}\ \text{C})(4.60 \times 10^{-3}\ \text{T})}{9.11 \times 10^{-31}\ \text{kg}} = 2.26 \times 10^{6}\ \text{m/s}$$

Thus, the speed of this electron is about 1% of the speed of light.

One of the applications of circular motion in a magnetic field is in a device known as a **mass spectrometer.** A mass spectrometer can be used to separate isotopes (atoms of the same element with different masses) and to measure atomic masses. It finds many uses in medicine (anesthesiologists use it to measure respiratory gases), biology (to determine reaction mechanisms in photosynthesis), geology (to date fossils), space science (to determine the atmospheric composition of Mars), and a variety of other fields.

REAL-WORLD PHYSICS

The mass spectrometer

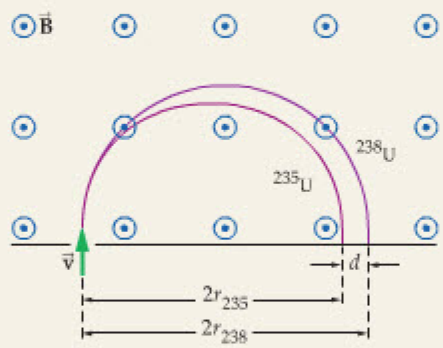
The basic principles of a mass spectrometer are illustrated in Figure 22–13. Here we see a beam of ions of mass m and charge $+q$ entering a region of constant magnetic field with a speed v. The field causes the ions to move along a circular path, with a radius that depends on the mass and charge of the ion, as described by Equation 22–3. Thus, different isotopes follow different paths and hence can be separated and identified. A specific example is considered next.

EXAMPLE 22–3 URANIUM SEPARATION

Two isotopes of uranium, ^{235}U ($m = 3.90 \times 10^{-25}$ kg) and ^{238}U ($m = 3.95 \times 10^{-25}$ kg), are sent into a mass spectrometer with a speed of 1.05×10^{5} m/s, as indicated in the accompanying sketch. Given that each isotope is singly ionized, and that the strength of the magnetic field is 0.750 T, what is the distance d between the two isotopes after they complete half a circular orbit?

PICTURE THE PROBLEM

The relevant features of the mass spectrometer are indicated in our sketch. Note that both isotopes enter at the same location with the same speed. Because they have different masses, however, they follow circular paths of different radii. This difference results in the separation d after half an orbit.



INTERACTIVE FIGURE

STRATEGY

We begin by calculating the radius of each isotope's circular path using $r = mv/|q|B$. The masses, speeds, and magnetic field are given. We can infer the charge from the fact that the isotopes are "singly ionized," which means that a single electron has been removed from each atom. Since the atoms were electrically neutral before the electron was removed, it follows that the charge of the isotopes is $q = e = 1.60 \times 10^{-19}$ C.

Finally, once we know the radii, the separation between the isotopes is given by $d = 2r_{238} - 2r_{235}$, as indicated in the sketch.

SOLUTION

1. Determine the radius of the circular path of ^{235}U:

$$r_{235} = \frac{mv}{|q|B}$$
$$= \frac{(3.90 \times 10^{-25}\ \text{kg})(1.05 \times 10^{5}\ \text{m/s})}{(1.60 \times 10^{-19}\ \text{C})(0.750\ \text{T})} = 34.1\ \text{cm}$$

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2. Determine the radius of the circular path of ^{238}U:

$$r_{238} = \frac{mv}{|q|B} = \frac{(3.95 \times 10^{-25}\ \text{kg})(1.05 \times 10^{5}\ \text{m/s})}{(1.60 \times 10^{-19}\ \text{C})(0.750\ \text{T})} = 34.6\ \text{cm}$$

3. Calculate the separation between the isotopes:

$$d = 2r_{238} - 2r_{235} = 2(34.6\ \text{cm} - 34.1\ \text{cm}) = 1\ \text{cm}$$

INSIGHT

Notice that although the difference in masses is very small, the mass spectrometer converts this small difference into an easily measurable distance of separation.

PRACTICE PROBLEM

Does the separation d increase or decrease if the magnetic field is increased? Check your answer by calculating the separation for $B = 1.00$ T, everything else remaining the same. [**Answer:** A stronger field will decrease the radii and hence decrease the separation d. In particular, if B changes by a factor x, the separation changes by the factor $1/x$. In this case, the factor is $x = 4/3$. That is, $B = (4/3)(0.750\ \text{T}) = 1.00$ T; therefore, $d = (3/4)(1\ \text{cm}) = 3/4$ cm. Note that it is not necessary to repeat the entire calculation in detail.]

Some related homework problems: Problem 19, Problem 27

ACTIVE EXAMPLE 22–1 FIND THE TIME FOR ONE ORBIT

Calculate the time T required for a particle of mass m and charge q to complete a circular orbit in a magnetic field.

SOLUTION *(Test your understanding by performing the calculations indicated in each step.)*

1. Write the speed, v, of a particle in terms of the time T to complete an orbit of radius r: $v = 2\pi r/T$
2. Substitute this expression for v into the condition for a circular orbit, $r = \dfrac{mv}{|q|B}$: $r = \dfrac{m(2\pi r/T)}{|q|B}$
3. Cancel r and solve for T: $T = \dfrac{2\pi m}{|q|B}$

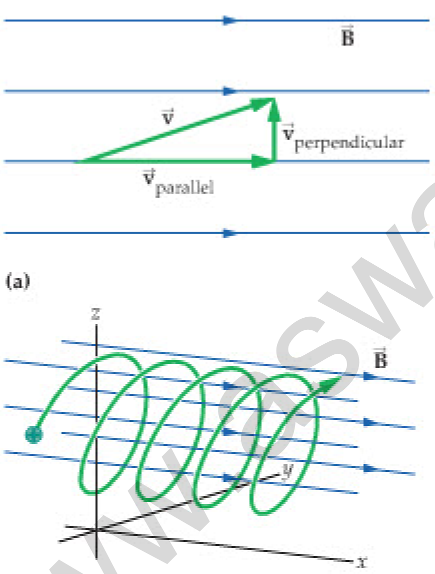
INSIGHT

Notice that the time required for an orbit is independent of its radius.

YOUR TURN

The circumference of an orbit depends on its radius, r. Why is it, then, that a particle completes an orbit in a time, T, that is independent of r?

(Answers to **Your Turn** *problems are given in the back of the book.)*



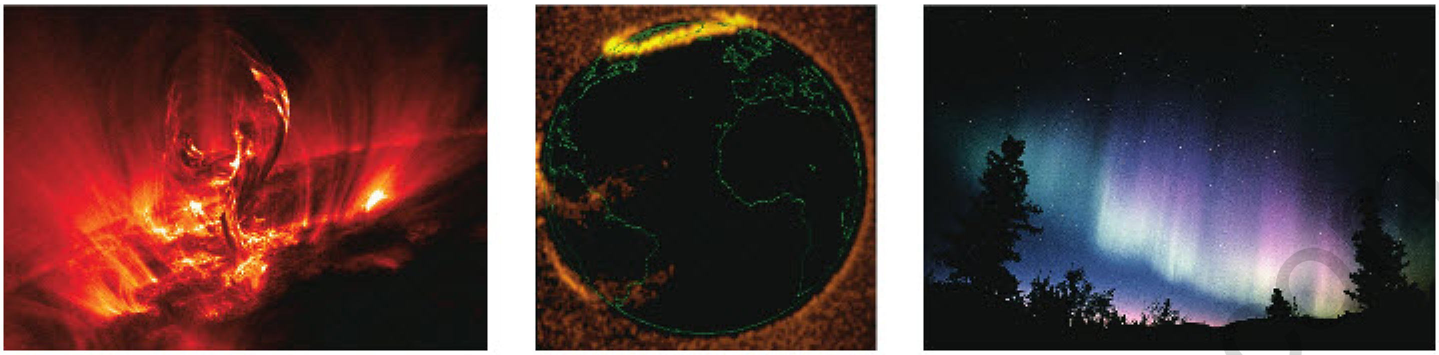
▲ **FIGURE 22–14 Helical motion in a magnetic field**

(a) A velocity at an angle to a magnetic field $\vec{B}$ can be resolved into parallel and perpendicular components. The parallel component gives a constant-velocity drift in the direction of the field. The perpendicular component gives circular motion perpendicular to the field. **(b)** Helical motion is a combination of linear motion and circular motion.

Helical Motion

The final type of motion we consider is a combination of the two motions discussed already. Suppose, for example, that a particle has an initial velocity at an angle to the magnetic field, as in Figure 22–14 (a). In this case there is a component of velocity parallel to $\vec{B}$ and a component perpendicular to $\vec{B}$. The parallel component of the velocity remains constant with time (zero force in this direction), whereas the perpendicular component results in a circular motion, as just discussed. Combining the two motions, we can see that the particle follows a helical path, as shown in Figure 22–14 (b).

If a magnetic field is curved, as in the case of a bar magnet or the magnetic field of the Earth, the helical motion of charged particles will be curved as well. Specifically, the axis of the helical motion will follow the direction of $\vec{B}$. For example, electrons and protons emitted by the "solar wind" frequently encounter the Earth's magnetic field and begin to move in helical paths following the field lines.



▲ (Left) An enormous eruption of matter from the surface of the Sun, large enough to encompass the entire Earth many times over. The loop structure is created by the Sun's own complex magnetic field. Charged particles from such eruptions often reach the Earth and become trapped in its magnetic field, producing auroras (see text). (Center) A glowing aurora borealis surrounds the Earth's north geographic pole in this photograph from space. (Right) An auroral display seen from Earth. The characteristic red and green colors are produced by ionized nitrogen and oxygen atoms, respectively.

Near the poles, where the field lines are concentrated as they approach the Earth's surface, the circulating electrons begin to collide with atoms and molecules in the atmosphere. These collisions can excite and ionize the atmospheric atoms and molecules, resulting in the emission of light known as the **aurora borealis** (northern lights) in the northern hemisphere and the **aurora australis** (southern lights) in the southern hemisphere. The most common color of the auroras is a pale green, the color of light given off by excited oxygen atoms.

REAL-WORLD PHYSICS

The aurora borealis and aurora australis

22–4 The Magnetic Force Exerted on a Current-Carrying Wire

A charged particle experiences a force when it moves across magnetic field lines. This is true whether it travels in a vacuum or in a current-carrying wire. Thus, a wire with a current will experience a force that is simply the resultant of all the forces experienced by the individual moving charges responsible for the current.

Specifically, consider a straight wire segment of length L with a current I flowing from left to right, as in Figure 22–15 (a). Also present in this region of space is a magnetic field $\vec{\mathbf{B}}$ at an angle θ to the length of the wire. If the conducting charges move through the wire with an average drift speed v, the time required for them to move from one end of the wire segment to the other is $\Delta t = L/v$. The amount of charge that flows through the wire in this time is $q = I\,\Delta t = IL/v$. Therefore, the force exerted on the wire is

$$F = qvB\sin\theta = \left(\frac{IL}{v}\right)vB\sin\theta$$

Canceling v, we find that the force on a wire segment of length L with a current I at an angle θ to the magnetic field $\vec{\mathbf{B}}$ is

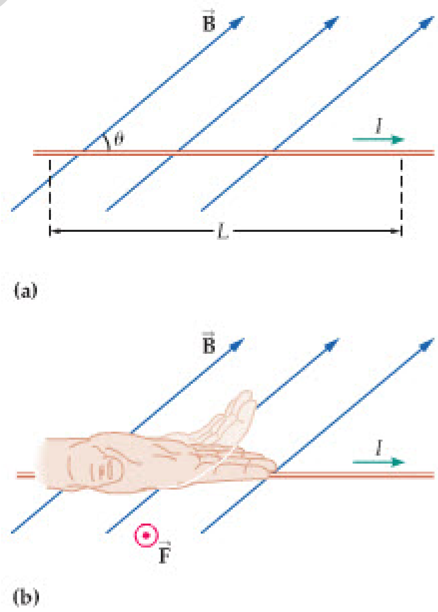
Magnetic Force on a Current-Carrying Wire

$$F = ILB\sin\theta \qquad 22\text{–}4$$

SI unit: newton, N

As with single charges, the maximum force occurs when the current is perpendicular to the magnetic field ($\theta = 90°$) and is zero if the current is in the same direction as $\vec{\mathbf{B}}$ ($\theta = 0$).

The direction of the magnetic force on a wire is given by the same right-hand rule used earlier for single charges. Thus, to find the direction of the force in Figure 22–15 (b), start by pointing the fingers of your right hand in the direction of the current I. This assumes that positive charges are flowing in the direction of I, consistent with our convention from Chapter 21. Now, curl your fingers toward the direction of $\vec{\mathbf{B}}$. Your thumb, which points out of the page, indicates the direction of $\vec{\mathbf{F}}$.



▲ **FIGURE 22–15 The magnetic force on a current-carrying wire**

A current-carrying wire in a magnetic field experiences a force, unless the current is parallel or antiparallel to the field. **(a)** For a wire segment of length L the magnitude of the force is $F = ILB\sin\theta$. **(b)** The direction of the force is given by the magnetic force RHR; the only difference is that you start by pointing the fingers of your right hand in the direction of the current I. In this case the force points out of the page.

PROBLEM-SOLVING NOTE

The Magnetic Force on a Current-Carrying Wire

Note that a current-carrying wire experiences no force when it is in the same direction as the magnetic field. The maximum force occurs when the wire is perpendicular to the magnetic field.

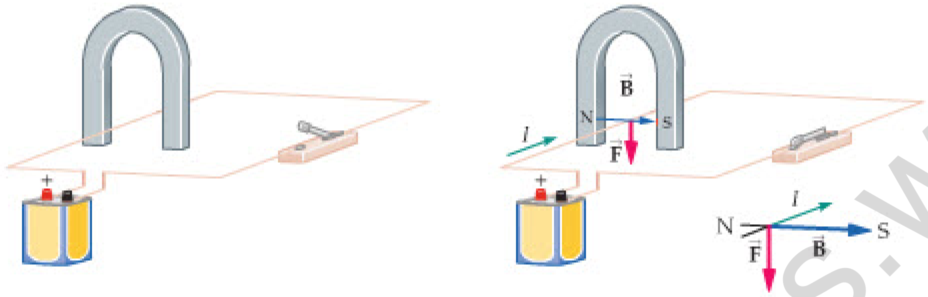
Of course, the current is actually caused by negatively charged electrons flowing in the opposite direction to the current. The magnetic force on these negatively charged particles moving in the opposite direction is the same as the force on positively charged particles moving in the direction of I. Thus, in all cases pertaining to current-carrying wires, we can simply think of the current as the direction in which positively charged particles move.

CONCEPTUAL CHECKPOINT 22–4 MAGNETIC POLES

When the switch is closed in the circuit shown in the sketch on the left, the wire between the poles of the horseshoe magnet deflects downward. Is the left end of the magnet **(a)** a north magnetic pole or **(b)** a south magnetic pole?

REASONING AND DISCUSSION

Once the switch is closed, the current in the wire is into the page, as shown in the sketch below.



Applying the magnetic force RHR, we see that the magnetic field must point from left to right in order for the force to be downward. Since magnetic field lines leave from north poles and enter at south poles, it follows that the left end of the magnet must be a north magnetic pole.

ANSWER

(a) The left end of the magnet is a north magnetic pole.

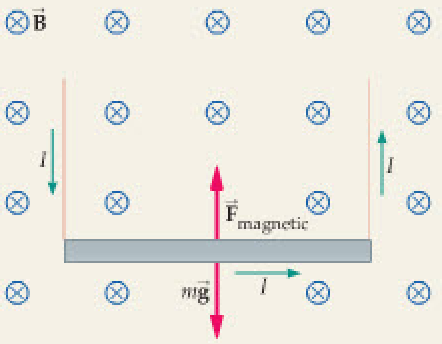
The magnetic force exerted on a current-carrying wire can be quite substantial. In the following Example, we consider the current necessary to levitate a copper rod.

EXAMPLE 22–4 MAGNETIC LEVITY

A copper rod 0.150 m long and with a mass of 0.0500 kg is suspended from two thin, flexible wires, as shown in the sketch. At right angles to the rod is a uniform magnetic field of 0.550 T pointing into the page. Find **(a)** the direction and **(b)** magnitude of the electric current needed to levitate the copper rod.

PICTURE THE PROBLEM

Our sketch shows the physical situation with relevant quantities labeled. The direction of the current has been indicated as well. Note that if the current is in this direction, the RHR gives an upward magnetic force, as required for the rod to be levitated.



STRATEGY

To find the magnitude of the current, we set the magnetic force equal in magnitude to the force of gravity. For the magnetic force we have $F = ILB \sin\theta$. In this case the field is at right angles to the current; hence, $\theta = 90°$, and the force simplifies to $F = ILB$. Thus, setting ILB equal to mg determines the current.

SOLUTION

Part (a)

1. Determine the direction of I: The current, I, points to the right. To verify, point the fingers of your right hand to the right, curl into the page, and your thumb will point upward, as desired.

Part (b)

2. Set the magnitude of the magnetic force equal to the magnitude of the force of gravity: $ILB = mg$

3. Solve for the current, I: $I = \dfrac{mg}{LB} = \dfrac{(0.0500\ \text{kg})(9.81\ \text{m/s}^2)}{(0.150\ \text{m})(0.550\ \text{T})} = 5.95\ \text{A}$

INSIGHT

Magnetic forces, like electric forces, can easily exceed the force of gravity. In fact, when we consider atomic systems in Chapter 31 we shall see that gravity plays no role in the behavior of an atom—only electric and magnetic forces are important on the atomic level.

PRACTICE PROBLEM

Suppose the rod is doubled in length, which also doubles its mass. Does the current needed to levitate the rod increase, decrease, or stay the same? [**Answer:** Since the levitation current is given by $I = mg/LB$, it is clear that doubling both m and L has no effect on I; the current needed to levitate remains the same.]

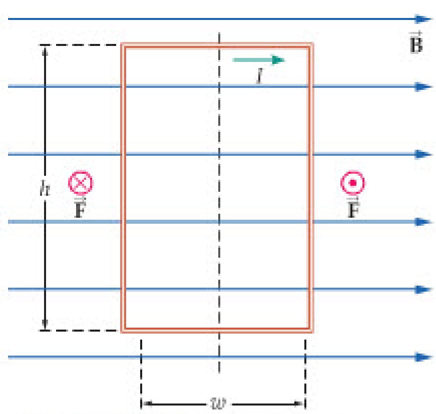
Some related homework problems: Problem 30, Problem 33

22–5 Loops of Current and Magnetic Torque

The fact that a current-carrying wire experiences a force when placed in a magnetic field is one of the fundamental discoveries that makes modern applications of electric power possible. In most of these applications, including electric motors and generators, the wire is shaped into a current-carrying loop. We will examine some of these applications further in Chapter 23; in this section we lay the groundwork by considering what happens when a simple current loop is placed in a magnetic field.

Rectangular Current Loops

Consider a rectangular loop of height h and width w carrying a current I, as shown in Figure 22–16. The loop is placed in a region of space with a uniform magnetic field $\vec{\mathbf{B}}$ that is parallel to the plane of the loop. From Figure 22–16, it is clear that the horizontal segments of the loop experience zero force, since they are parallel to the field. The vertical segments, on the other hand, are perpendicular to the field; hence, they experience forces of magnitude $F = IhB$. One of these forces is into the page (left side); the other is out of the page (right side).



▲ **FIGURE 22–16 Magnetic forces on a current loop**

A rectangular current loop in a magnetic field. Only the vertical segments of the loop experience forces, and they tend to rotate the loop about a vertical axis.

Perhaps the best way to visualize the torque caused by these forces is to use a top view and look directly down on the loop, as in Figure 22–17 (a). Here we can see more clearly that the forces on the vertical segments are equal in magnitude and opposite in direction. If we imagine an axis of rotation through the center of the loop, at the point O, it is clear that the forces exert a torque that tends to rotate the loop clockwise. The magnitude of this torque for each vertical segment is the force $(F = IhB)$ times the moment arm $(w/2)$. Noting that both vertical segments exert a torque in the same direction, we see that the total torque is simply the sum of the torque produced by each segment:

$$\tau = (IhB)\left(\frac{w}{2}\right) + (IhB)\left(\frac{w}{2}\right) = IB(hw)$$

Finally, observing that the area of the rectangular loop is $A = hw$, we can express the torque as follows:

$$\tau = IAB$$

As the loop begins to rotate, the situation will be like that shown in Figure 22–17 (b). Here we see that the forces still have the same magnitude, IhB, but now the moment arms are $(w/2)\sin\theta$ rather than $w/2$. Thus, for a general angle, the torque must include the factor $\sin\theta$:

Torque Exerted on a Rectangular Loop of Area A

$$\tau = IAB\sin\theta \qquad \text{22–5}$$

SI unit: N · m

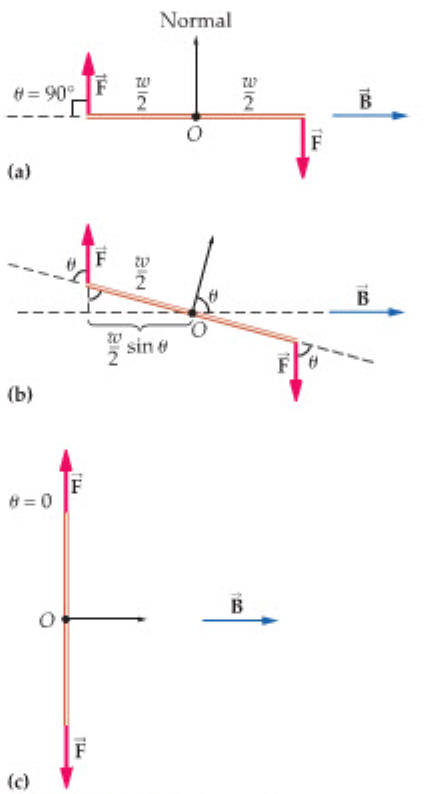


FIGURE 22–17 Magnetic torque on a current loop

A current loop placed in a magnetic field produces a torque. **(a)** The torque is greatest when the plane of the loop is parallel to the magnetic field; that is, when the normal to the loop is perpendicular to the magnetic field. **(b)** As the loop rotates, the torque decreases by a factor of $\sin\theta$. **(c)** The torque vanishes when the plane of the loop is perpendicular to the magnetic field.

Note that the angle θ is the angle between the plane of the loop and the magnetic force exerted on each side of the loop. Equivalently, θ is the angle between the normal and the magnetic field.

In the case shown in Figure 22–17 (a), the angle θ is 90°, and the normal to the plane of the loop is perpendicular to the magnetic field. In this orientation the torque attains its maximum value, $\tau = IBA$. When θ is zero, as in Figure 22–17 (c), the torque vanishes because the moment arm of the magnetic forces is zero. Thus, there is no torque when the magnetic field is perpendicular to the plane of a loop.

General Loops

We have shown that the torque exerted on a rectangular loop of area A is $\tau = IBA \sin\theta$. A more detailed derivation shows that the same relation applies to *any* planar loop, no matter what its shape. For example, a circular loop of radius r and area πr^2 experiences a torque given by the expression $\tau = IB\pi r^2 \sin\theta$.

In many applications it is desirable to produce as large a torque as possible. A simple way to increase the torque is to wrap a long wire around a loop N times, creating a coil of N "turns." Each of the N turns produces the same torque as a single loop; hence, the total torque is increased by a factor of N. In general, then, the torque produced by a loop with N turns is

Torque Exerted on a General Loop of Area *A* and *N* Turns

$$\tau = NIAB \sin\theta \qquad 22\text{–}6$$

SI unit: N · m

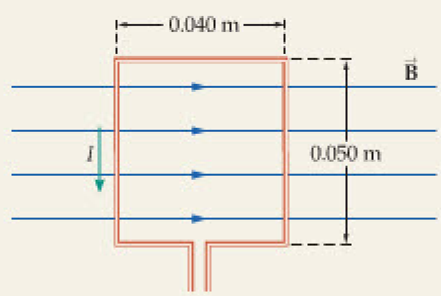
Notice that the torque depends on a number of factors in the system. First, it depends on the strength of the magnetic field, B, and on its orientation, θ, with respect to the normal of the loop. In addition, the torque depends on the current in the loop, I, the area of the loop, A, and the number of turns in the loop, N. The product of these "loop factors," NIA, is referred to as the **magnetic moment** of the loop. The magnetic moment, which has units of $A \cdot m^2$, is proportional to the amount of torque a given loop can exert.

EXAMPLE 22–5 TORQUE ON A COIL

A rectangular coil with 200 turns is 5.0 cm high and 4.0 cm wide. When the coil is placed in a magnetic field of 0.35 T, its maximum torque is 0.22 N · m. What is the current in the coil?

PICTURE THE PROBLEM

The coil, along with its dimensions, is shown in the sketch. The sketch also reflects the fact that the maximum torque is produced when the magnetic field is in the plane of the coil.



STRATEGY

The torque is given by the expression $\tau = NIAB \sin\theta$. Clearly, the maximum torque occurs when $\sin\theta = 1$; that is, $\tau_{max} = NIAB$. Solving this relation for I yields the current in the coil.

SOLUTION

1. Write an expression for the maximum torque: $\tau_{max} = NIAB$
2. Solve for the current, I: $I = \dfrac{\tau_{max}}{NAB}$
3. Substitute numerical values: $I = \dfrac{0.22\ \text{N} \cdot \text{m}}{(200)(0.050\ \text{m})(0.040\ \text{m})(0.35\ \text{T})} = 1.6\ \text{A}$

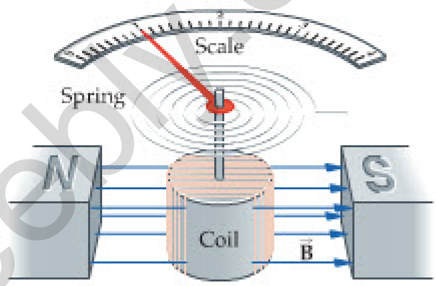
INSIGHT
Note that this calculation gives the magnitude of the current and not its direction. The direction of the current will determine whether the torque on the coil is clockwise or counterclockwise.

PRACTICE PROBLEM
If the shape of this coil were changed to circular, keeping a constant perimeter, would the maximum torque increase, decrease, or stay the same? [**Answer:** In general, a circle has the greatest area for a given perimeter. Therefore, the maximum torque would increase if the coil were made circular, because its area would be larger.]

Some related homework problems: Problem 39, Problem 40

Applications of Torque

The torque exerted by a magnetic field finds a number of useful applications. For example, if a needle is attached to a coil, as in Figure 22–18, it can be used as part of a meter. When the coil is connected to an electric circuit, it experiences a torque, and a corresponding deflection of the needle, that is proportional to the current. The result is that the current in a circuit can be indicated by the reading on the meter. A simple device of this type is referred to as a **galvanometer.**



▲ **FIGURE 22–18 The galvanometer**
The basic elements of a galvanometer are a coil, a magnetic field, a spring, and a needle attached to the coil. As current passes through the coil, a torque acts on it, causing it to rotate. The spring ensures that the angle of rotation is proportional to the current in the coil.

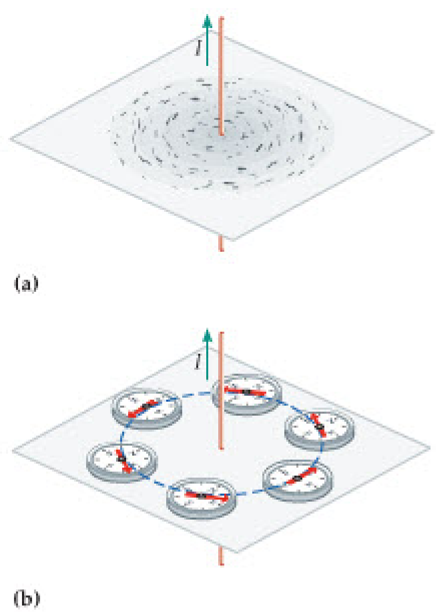
REAL-WORLD PHYSICS
The galvanometer

Notice that the galvanometer can be a very sensitive instrument for two different reasons. First, the coil can have a large number of turns. Since the torque exerted by a coil is proportional to the number of turns, it follows that a coil with many turns produces a significant torque even when the current is small. Second, the needle amplifies the motion of the coil. Specifically, if the coil turns through an angle θ, and the needle has a length L, the tip of the needle moves through a distance $L\theta$. Thus, both the number of turns and the length of the needle increase the sensitivity of the meter.

Of even greater practical importance is the fact that magnetic torque can be used to power a motor. For example, an electric current passing through the coils of a motor causes a torque that rotates the axle of the motor. As the coils rotate, a device known as the commutator reverses the direction of the current as the orientation of the coil reverses, which ensures that the torque is always in the same direction and that the coils continue to turn in the same direction. Electric motors, which will be discussed in greater detail in Chapter 23, are used in everything from CD players to electric razors to electric cars.

22–6 Electric Currents, Magnetic Fields, and Ampère's Law

In the introduction to this chapter, we mentioned that a previously unexpected connection between electricity and magnetism was discovered accidentally by Hans Christian Oersted in 1820. Specifically, Oersted was giving a public lecture on various aspects of science when, at one point, he closed a switch and allowed a current to flow through a wire. What he noticed was that a nearby compass needle deflected from its usual orientation when the switch was closed—Oersted had just discovered that electric currents can create magnetic fields. In this section we focus on the connection between electric currents and magnetic fields. In so doing, our attention shifts from the effects of magnetic fields—the subject of the previous sections—to their production.



▲ **FIGURE 22–19 The magnetic field of a current-carrying wire**
(a) An electric current flowing through a wire produces a magnetic field. In the case of a long, straight wire, the field circulates around the wire. **(b)** Compass needles point along the circumference of a circle centered on the wire.

A Long, Straight Wire

We start with the simplest possible case—a straight, infinitely long wire that carries a current I. To visualize the magnetic field such a wire produces, we shake iron filing onto a sheet of paper that is pierced by the wire, as indicated in Figure 22–19 (a). The result is that the filings form into circular patterns centered on the wire—evidently, the magnetic field "circulates" around the wire.

We can gain additional information about the field by placing a group of small compasses about the wire, as in Figure 22–19 (b). In addition to confirming the

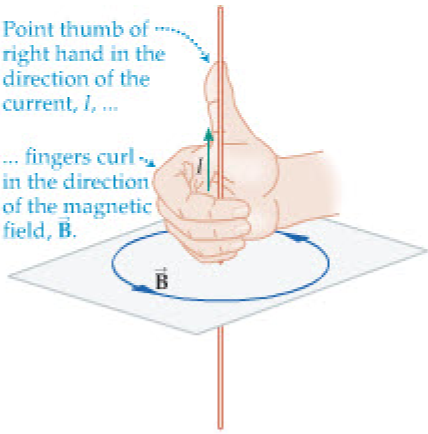


FIGURE 22–20 The magnetic-field right-hand rule
The magnetic field right-hand rule determines the direction of the magnetic field produced by a current-carrying wire. With the thumb of the right hand pointing in the direction of the current, the fingers curl in the direction of the field.

circular shape of the field lines, the compass needles show the field's direction. To understand this direction, we must again utilize a right-hand rule—this time, we refer to the rule as the *magnetic field* right-hand rule:

Magnetic Field Right-Hand Rule
To find the direction of the magnetic field due to a current-carrying wire, point the thumb of your right hand along the wire in the direction of the current I. Your fingers are now curling around the wire in the direction of the magnetic field.

This rule is illustrated in **Figure 22–20**, where we see that it predicts the same direction as that indicated by the compass needles in Figure 22–19 (b).

CONCEPTUAL CHECKPOINT 22–5 DIRECTION OF THE CURRENT

The magnetic field shown in the sketch is due to the horizontal, current-carrying wire. Does the current in the wire flow to the left or to the right?

REASONING AND DISCUSSION
If you point the thumb of your right hand along the wire to the left, your fingers curl into the page above the wire and out of the page below the wire, as shown in the figure. Thus, the current flows to the left.

ANSWER
The current in the wire flows to the left.

Experiment shows that the field produced by a current-carrying wire doubles if the current I is doubled. In addition, the field decreases by a factor of 2 if the distance from the wire, r, is doubled. Hence, we conclude that the magnetic field B must be proportional to I/r that is,

$$B = (\text{constant})\frac{I}{r}$$

The precise expression for B will now be derived using a law of nature known as Ampère's law.

Ampère's Law

Ampère's law relates the magnetic field along a closed path to the electric current enclosed by the path. Specifically, consider the current-carrying wires shown in **Figure 22–21.** These wires are enclosed by the closed path P, which can be divided into many small, straight-line segments of length ΔL. On each of these segments, the magnetic field $\vec{\mathbf{B}}$ can be resolved into a component parallel to the segment, $B_{\parallel}$, and a component perpendicular to the segment, $B_{\perp}$. Of particular interest is the product $B_{\parallel}\,\Delta L$. According to Ampère's law, the sum of $B_{\parallel}\,\Delta L$ over all segments of a closed path is equal to a constant times the current enclosed by the path. This relationship can be written symbolically as follows:

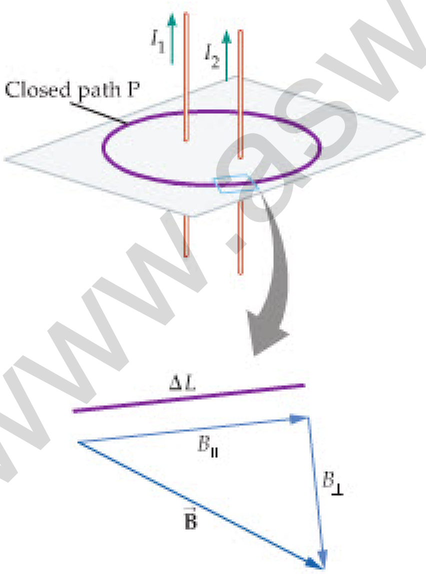


FIGURE 22–21 Illustrating Ampère's law
A closed path P encloses the currents I_1 and I_2. According to Ampère's law, the sum of $B_{\parallel}\,\Delta L$ around the path P is equal to $\mu_0 I_{\text{enclosed}}$. In this case, $I_{\text{enclosed}} = I_1 + I_2$.

Ampère's Law

$$\sum B_{\parallel}\,\Delta L = \mu_0 I_{\text{enclosed}} \qquad 22\text{–}7$$

In this expression, μ_0 is a constant called the **permeability of free space.** Its value is

$$\mu_0 = 4\pi \times 10^{-7}\ \text{T}\cdot\text{m/A} \qquad 22\text{–}8$$

We should emphasize that Ampère's law is a law of nature—it is valid for all magnetic fields and currents that are constant in time.

Let's apply Ampère's law to the case of a long, straight wire carrying a current I. We already know that the field circulates around the wire, as illustrated in Figure 22–22. It is reasonable, then, to choose a circular path of radius r (and circumference $2\pi r$) to enclose the wire. Since the magnetic field is parallel to the circular path at every point, and all points on the path are the same distance from the wire, it follows that $B_\parallel = B =$ constant. Therefore, the sum of $B_\parallel \Delta L$ around the closed path gives

$$\sum B_\parallel \Delta L = B \sum \Delta L = B(2\pi r)$$

According to Ampère's law, this sum must equal $\mu_0 I_{\text{enclosed}} = \mu_0 I$. Therefore, we have

$$B(2\pi r) = \mu_0 I$$

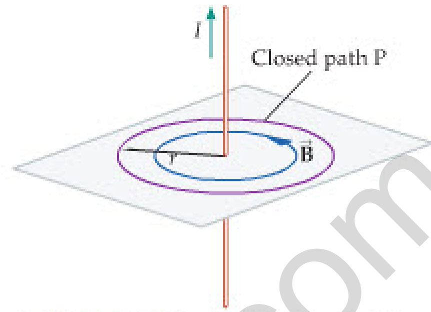
Solving for B, we obtain

Magnetic Field for a Long, Straight Wire

$$B = \frac{\mu_0 I}{2\pi r} \qquad 22\text{–}9$$

SI unit: tesla, T

As expected, the field is equal to a constant times I/r.



▲ **FIGURE 22–22 Applying Ampère's law** To apply Ampère's law to a long, straight wire, we consider a circular path centered on the wire. Since the magnetic field is everywhere parallel to this path, and has constant magnitude B at all points on it, the sum of $B_\parallel \Delta L$ over the path is $B(2\pi r)$. Setting this equal to $\mu_0 I$ yields the magnetic field of the wire: $B = \mu_0 I/2\pi r$.

EXERCISE 22–2

Find the magnitude of the magnetic field 1 m from a long, straight wire carrying a current of 1 A.

SOLUTION

Straightforward substitution in Equation 22–9 yields

$$B = \frac{\mu_0 I}{2\pi r} = \frac{(4\pi \times 10^{-7}\,\text{T}\cdot\text{m/A})(1\,\text{A})}{2\pi(1\,\text{m})} = 2 \times 10^{-7}\,\text{T}$$

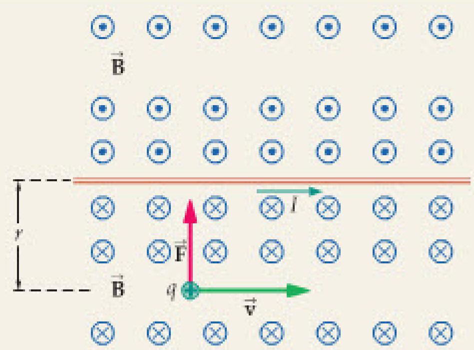
This is a weak field, less than one hundredth the strength of the Earth's magnetic field.

EXAMPLE 22–6 AN ATTRACTIVE WIRE

A 52-μC charged particle moves parallel to a long wire with a speed of 720 m/s. The separation between the particle and the wire is 13 cm, and the magnitude of the force exerted on the particle is 1.4×10^{-7} N. Find **(a)** the magnitude of the magnetic field at the location of the particle and **(b)** the current in the wire.

PICTURE THE PROBLEM

The physical situation is illustrated in our sketch. Note that the charged particle moves parallel to the current-carrying wire; hence, its velocity is at right angles to the magnetic field. This means that it will experience the maximum magnetic force. Finally, by pointing the thumb of your right hand in the direction of I, you can verify that $\vec{\mathbf{B}}$ points out of the page above the wire and into the page below it.



STRATEGY

a. The maximum magnetic force on the charged particle is $F = qvB$. This relation can be solved for B.

b. The magnetic field is produced by the current in the wire. Therefore, $B = \mu_0 I/2\pi r$. Using B from part (a), we can solve for the current I.

SOLUTION

Part (a)

1. Use $F = qvB$ to solve for the magnetic field:

$$F = qvB$$

$$B = \frac{F}{qv} = \frac{1.4 \times 10^{-7}\,\text{N}}{(5.2 \times 10^{-5}\,\text{C})(720\,\text{m/s})} = 3.7 \times 10^{-6}\,\text{T}$$

CONTINUED ON NEXT PAGE

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Part (b)

2. Use the relation for the magnetic field of a wire to solve for the current:

$$B = \frac{\mu_0 I}{2\pi r}$$

$$I = \frac{2\pi r B}{\mu_0}$$

3. Substitute the value for B found in part (a) and evaluate I:

$$I = \frac{2\pi r B}{\mu_0} = \frac{2\pi(0.13\text{ m})(3.7 \times 10^{-6}\text{ T})}{4\pi \times 10^{-7}\text{ T}\cdot\text{m/A}} = 2.4\text{ A}$$

INSIGHT

Using the magnetic force RHR, we can see that the direction of the force exerted on the charged particle is toward the wire, as illustrated in our sketch. If the particle had been negatively charged, the force would have been away from the wire.

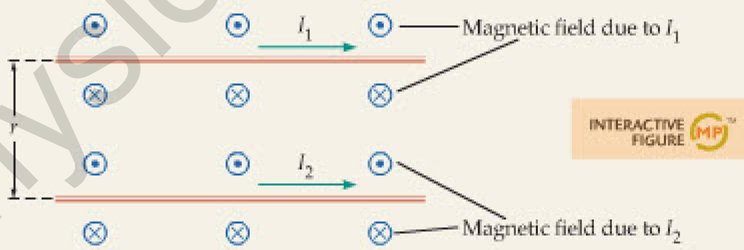
PRACTICE PROBLEM

Suppose a particle with a charge of 52 μC is 13 cm above the wire and moving with a speed of 720 m/s to the left. Find the magnitude and direction of the force acting on this particle. [**Answer:** $F = 1.4 \times 10^{-7}$ N, away from wire]

Some related homework problems: Problem 46, Problem 52

ACTIVE EXAMPLE 22–2 FIND THE MAGNETIC FIELD

Two wires separated by a distance of 22 cm carry currents in the same direction. The current in one wire is 1.5 A, and the current in the other wire is 4.5 A. Find the magnitude of the magnetic field halfway between the wires.



SOLUTION *(Test your understanding by performing the calculations indicated in each step.)*

1. Find the magnitude and direction of the magnetic field produced by wire 1: $B_1 = 2.7 \times 10^{-6}$ T, into page
2. Find the magnitude and direction of the magnetic field produced by wire 2: $B_2 = 8.2 \times 10^{-6}$ T, out of page
3. Calculate the magnitude of the net field: $B = B_2 - B_1 = 5.5 \times 10^{-6}$ T

INSIGHT

Since the field produced by I_2 has the greater magnitude, the net field is out of the page. If the currents were equal, the net field midway between the wires would be zero.

YOUR TURN

Find the magnitude and direction of the magnetic field 11 cm *below* wire 2.

(Answers to **Your Turn** *problems are given in the back of the book.)*

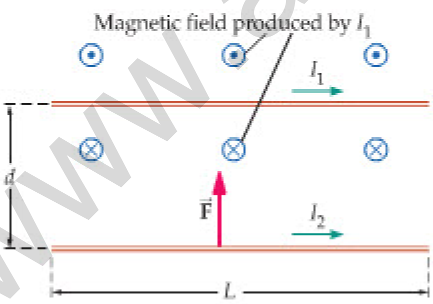


FIGURE 22–23 The magnetic force between current-carrying wires

A current in wire 1 produces a magnetic field, $B_1 = \mu_0 I_1/2\pi d$, at the location of wire 2. The result is a force exerted on a length L of wire 2 of magnitude $F = \mu_0 I_1 I_2 L/2\pi d$.

Forces Between Current-Carrying Wires

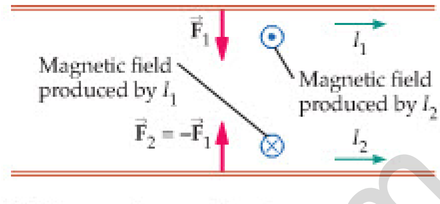
We know that a current-carrying wire in a magnetic field experiences a force. We also know that a current-carrying wire produces a magnetic field. It follows, then, that one current-carrying wire will exert a force on another.

To work this relationship out in detail, consider the two wires with parallel currents and separation d shown in Figure 22–23. The magnetic field produced by wire 1 circulates around it, coming out of the page above the wire and entering the page below the wire. Thus, wire 2 experiences a magnetic field pointing into the page with a magnitude given by Equation 22–9: $B = \mu_0 I_1/2\pi d$. The force experienced by wire 2, therefore, has a magnitude given by Equation 22–4 with $\theta = 90°$:

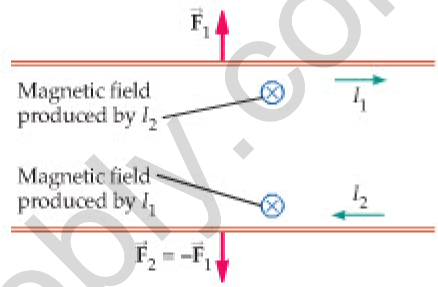
$$F = I_2 LB = I_2 L\left(\frac{\mu_0 I_1}{2\pi d}\right) = \frac{\mu_0 I_1 I_2}{2\pi d}L \qquad 22\text{–}10$$

The direction of the force acting on wire 2, as given by the magnetic force RHR, is upward, that is, toward wire 1. A similar calculation starting with the field produced by wire 2 gives a force of the same magnitude acting downward on wire 1. This is to be expected, because the forces acting on wires 1 and 2 form an action–reaction pair, as indicated in **Figure 22–24 (a)**. Hence, wires with parallel currents attract one another.

If the currents in wires 1 and 2 are in opposite directions, as in **Figure 22–24 (b)**, the situation is similar to that discussed in the preceding paragraph, except that the direction of the forces is reversed. Thus, wires with opposite currents repel one another.



(a) Currents in same direction



(b) Currents in opposite directions

▲ **FIGURE 22–24 The direction of the magnetic force between current-carrying wires**
The forces between current-carrying wires depend on the relative direction of their currents. **(a)** If the currents are in the same direction, the force is attractive. **(b)** Wires with oppositely directed currents experience repulsive forces.

22–7 Current Loops and Solenoids

We now consider the magnetic fields produced when a current-carrying wire has a circular or a helical geometry—as opposed to the straight wires considered in the previous section. As we shall see, there are many practical applications for such geometries.

Current Loop

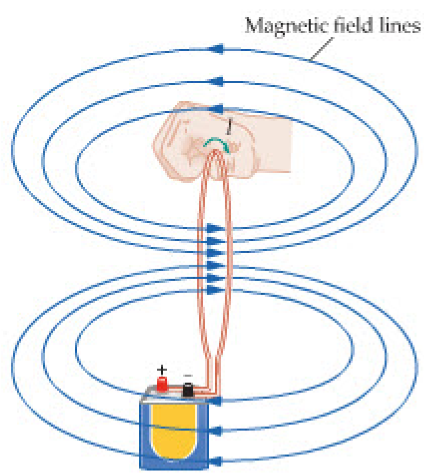
We begin by considering the magnetic field produced by a current-carrying wire that is formed into the shape of a circular loop. In **Figure 22–25 (a)** we show a wire loop connected to a battery producing a current in the direction indicated. Using the magnetic field RHR, as shown in the figure, we see that $\vec{\mathbf{B}}$ points from left to right as it passes through the loop. Notice also that the field lines are bunched together within the loop, indicating an intense field there, but are more widely spaced outside the loop.

The most interesting aspect of the field produced by the loop is its close resemblance to the field of a bar magnet. This similarity is illustrated in **Figure 22–25 (b)**, where we see that one side of the loop behaves like a north magnetic pole and the other side like a south magnetic pole. Thus, if two loops with identical currents are placed near each other, as in **Figure 22–26 (a)**, the force between them will be similar to the force between two bar magnets pointing in the same direction—that is, they will attract each other. If the loops have oppositely directed currents, as in **Figure 22–26 (b)**, the force between them is repulsive. Note the similarity of the results for straight wires, Figure 22–24, and for loops, Figure 22–26.

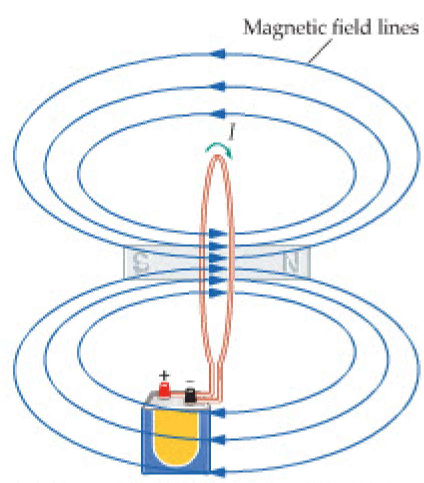
On a more fundamental level, the connection between a current loop and a permanent magnet is much more than accidental. In fact, the atomic origin of magnetic fields is due to circulating currents produced by electrons. We discuss this connection in greater detail in the next section.

The magnitude of the magnetic field produced by a circular loop of N turns, radius R, and current I varies from point to point; however, it can be shown that at the center of the loop, the field is given by the following simple expression:

$$B = \frac{N\mu_0 I}{2R} \quad \text{(center of circular loop of radius } R\text{)} \qquad 22\text{–}11$$

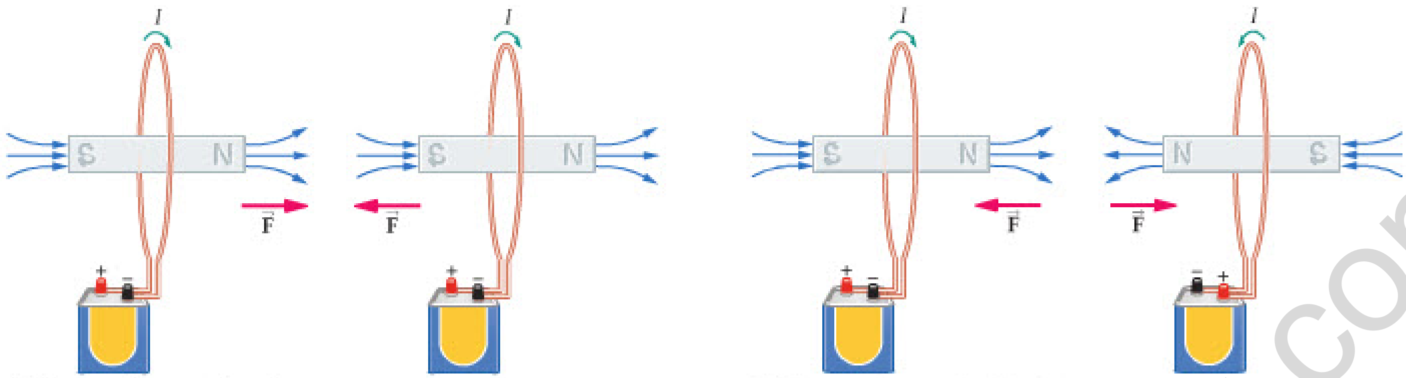


(a) Magnetic field of a current loop



(b) Magnetic field of bar magnet is similar

▶ **FIGURE 22–25 The magnetic field of a current loop**
(a) The magnetic field produced by a current loop is relatively intense within the loop and falls off rapidly outside the loop. **(b)** A permanent magnet produces a field that is very similar to the field of a current loop.

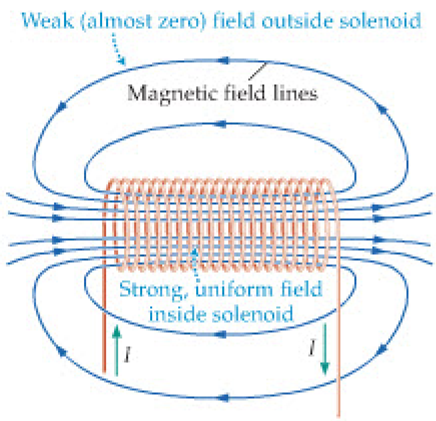


(a) Currents in same direction

(b) Currents in opposite directions

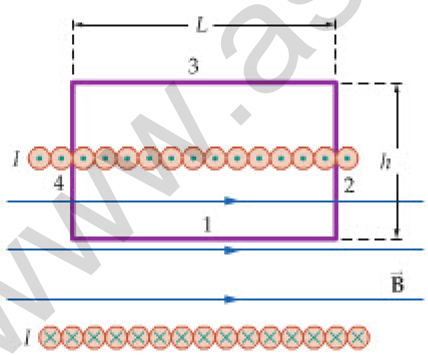
▲ **FIGURE 22–26 Magnetic forces between current loops**

To decide whether current loops will experience an attractive or repulsive force, it is useful to think in terms of the corresponding permanent magnets. **(a)** Current loops with currents in the same direction are like two bar magnets lined up in the same direction; they attract each other. **(b)** Current loops with opposite currents act like bar magnets with opposite orientations; they repel each other.



▲ **FIGURE 22–27 The solenoid**

A solenoid is formed from a long wire that is wound into a succession of current loops. The magnetic field inside the solenoid is relatively strong and uniform. Outside the solenoid the field is weak. In the ideal case, we consider the field outside the solenoid to be zero and that inside to be uniform and parallel to the solenoid's axis.



▲ **FIGURE 22–28 Ampère's law and the magnetic field in a solenoid**

To calculate the magnetic field inside a solenoid, we apply Ampère's law to the rectangular path shown here. The only side of the rectangle that has a nonzero, parallel component of $\vec{\mathbf{B}}$ is side 1.

Notice that the field is proportional to the current in the loop and inversely proportional to its radius.

Solenoid

A **solenoid** is an electrical device in which a long wire has been wound into a succession of closely spaced loops with the geometry of a helix. Also referred to as an **electromagnet**, a solenoid carrying a current produces an intense, nearly uniform, magnetic field inside the loops, as indicated in Figure 22–27. Notice that each loop of the solenoid carries a current in the same direction; therefore, the magnetic force between loops is attractive and serves to hold the loops tightly together.

The magnetic field lines in Figure 22–27 are tightly packed inside the solenoid but are widely spaced outside. In the ideal case of a very long, tightly packed solenoid, the magnetic field outside is practically zero—especially when compared with the intense field inside the solenoid. We can use this idealization, in combination with Ampère's law, to calculate the magnitude of the field inside the solenoid.

To do so, consider the rectangular path of width L and height h shown in Figure 22–28. Notice that the parallel component of the field on side 1 is simply $\vec{\mathbf{B}}$. On sides 2 and 4 the parallel component is zero, since $\vec{\mathbf{B}}$ is perpendicular to those sides. Finally, on side 3 (which is outside the solenoid) the magnetic field is zero. Using these results, we obtain the sum of $B_{\parallel}\,\Delta L$ over the rectangular loops:

$$\sum B_{\parallel}\,\Delta L = \sum_{\text{side 1}} B_{\parallel}\,\Delta L + \sum_{\text{side 2}} B_{\parallel}\,\Delta L + \sum_{\text{side 3}} B_{\parallel}\,\Delta L + \sum_{\text{side 4}} B_{\parallel}\,\Delta L$$
$$= BL + 0 + 0 + 0 = BL$$

Next, the current enclosed by the rectangular circuit is NI, where N is the number of loops in the length L. Therefore, Ampère's law gives

$$BL = \mu_0 NI$$

Solving for B and letting the number of loops per length be $n = N/L$, we find

Magnetic Field of a Solenoid

$$B = \mu_0\left(\frac{N}{L}\right)I = \mu_0 nI \qquad 22\text{–}12$$

SI unit: tesla, T

Note that this result is independent of the cross-sectional area of the solenoid.

When used as an electromagnet, a solenoid has many useful properties. First and foremost, it produces a strong magnetic field that can be turned on or off at the flip of a switch—unlike the field of a permanent magnet. In addition, the magnetic field can be further intensified by filling the core of the solenoid with an iron bar. In such a case, the magnetic field of the solenoid magnetizes the iron bar, which then adds its field to the overall field of the system. These properties and others make solenoids useful devices in a variety of electric circuits. Further examples are given in the next chapter.

PROBLEM-SOLVING NOTE

The Magnetic Field Inside a Solenoid

In an ideal solenoid, the magnetic field inside the solenoid points along the axis and is uniform. Outside the solenoid the field is zero. Therefore, when calculating the field produced by a solenoid, it is not necessary to specify a particular point inside the solenoid; all inside points have the same field.

CONCEPTUAL CHECKPOINT 22–6 MAGNETIC FIELD IN A SOLENOID

If you want to increase the strength of the magnetic field inside a solenoid, is it better to **(a)** double the number of loops, keeping the length the same, or **(b)** double the length, keeping the number of loops the same?

REASONING AND DISCUSSION

Referring to the expression $B = \mu_0(N/L)I$, we see that doubling the number of loops $(N \rightarrow 2N)$ while keeping the length the same $(L \rightarrow L)$ results in a doubled magnetic field $(B \rightarrow 2B)$. On the other hand, doubling the length $(L \rightarrow 2L)$ while keeping the number of loops the same $(N \rightarrow N)$ reduces the magnetic field by a factor of two $(B \rightarrow B/2)$. Hence, to increase the field one should pack more loops into the same length.

ANSWER

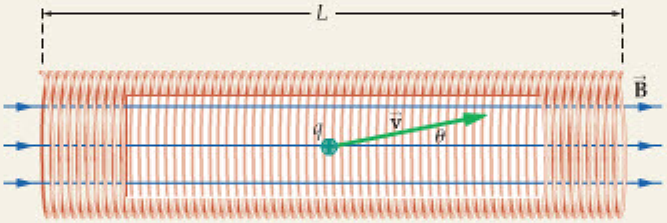
(a) Double the number of loops with the same length.

EXAMPLE 22–7 THROUGH THE CORE OF A SOLENOID

A solenoid is 20.0 cm long, has 200 loops, and carries a current of 3.25 A. Find the magnitude of the force exerted on a 15.0-μC charged particle moving at 1050 m/s through the interior of the solenoid, at an angle of 11.5° relative to the solenoid's axis.

PICTURE THE PROBLEM

Our sketch shows the solenoid, along with the uniform magnetic field it produces parallel to its axis. Inside the solenoid, a positively charged particle moves at an angle of $\theta = 11.5°$ relative to the magnetic field.



STRATEGY

The force exerted on the charged particle is magnetic; hence, we start by calculating the magnetic field produced by the solenoid, $B = \mu_0(N/L)I$. Next, we note that the magnetic field in a solenoid is parallel to its axis. It follows that the magnitude of the force exerted on the charge is given by $F = |q|vB\sin\theta$, with $\theta = 11.5°$.

SOLUTION

1. Calculate the magnetic field inside the solenoid:

$$B = \mu_0\left(\frac{N}{L}\right)I$$
$$= (4\pi \times 10^{-7}\,\mathrm{T\cdot m/A})\left(\frac{200}{0.200\,\mathrm{m}}\right)(3.25\,\mathrm{A}) = 4.08 \times 10^{-3}\,\mathrm{T}$$

2. Use B to find the force exerted on the charged particle:

$$F = |q|vB\sin\theta$$
$$= (15.0 \times 10^{-6}\,\mathrm{C})(1050\,\mathrm{m/s})(4.08 \times 10^{-3}\,\mathrm{T})\sin 11.5°$$
$$= 1.28 \times 10^{-5}\,\mathrm{N}$$

INSIGHT

Note that the magnetic field strength inside this modest solenoid is approximately 100 times greater than the magnetic field at the surface of the Earth.

PRACTICE PROBLEM

What current would be required to double the force acting on the particle to 2.56×10^{-5} N? **[Answer:** $I = 2(3.25\,\mathrm{A}) = 6.50\,\mathrm{A}$]

Some related homework problems: Problem 57, Problem 58

REAL-WORLD PHYSICS: BIO
MRI instruments

Magnetic resonance imaging (MRI) instruments utilize solenoids large enough to accommodate a person within their coils. Not only are these solenoids large in size, they are also capable of producing extremely powerful magnetic fields on the order of 1 or 2 tesla (see Table 22–1). So powerful are these fields, in fact, that metallic objects such as mop buckets and stretchers have been known to be pulled from across the room into the bore of the magnet. A metal oxygen bottle in the same room can be turned into a dangerous, high-speed projectile.

REAL-WORLD PHYSICS: BIO
Magnetic reed switches in pacemakers

In some cases, artificial pacemakers have been affected. Many types of pacemakers have what are known as *magnetic reed switches*. These switches allow a physician to change the operating mode of a pacemaker, without surgery, by simply placing a magnet at the appropriate location on a patient's chest. If a person with one of these pacemakers comes anywhere near an operating MRI instrument, the results can be serious.

22–8 Magnetism in Matter

Some materials have strong magnetic fields; others do not. To understand these differences, we must consider the behavior of matter on the atomic level.

To begin, recall that circulating electric currents can produce magnetic fields much like those produced by bar magnets. This is significant because atoms have electrons orbiting their nucleus, which means they have circulating electric currents. The fields produced by these orbiting electrons can be sizable—on the order of 10 T or so at the nucleus of the atom. In most atoms, though, the fields produced by the various individual electrons tend to cancel one another, resulting in a very small or zero net magnetic field.

Another type of circulating electric current present in atoms produces fields that occasionally do not cancel. These currents are associated with the *spin* that all electrons have. A simple model of the electron—which should *not* be taken literally—is of a spinning sphere of charge. The circulating charge associated with the spinning motion gives rise to a magnetic field. Electrons tend to "pair up" in atoms in such a way that their spins are opposite to one another, again resulting in zero net field. However, in some atoms—such as iron, nickel, and cobalt—the net field due to the spinning electrons is nonzero, and strong magnetic effects can occur as the magnetic field of one atom tends to align with the magnetic field of another. A full understanding of these types of effects requires the methods of quantum mechanics, which will be discussed in Chapters 30 and 31.

Ferromagnetism

If the tendency of magnetic atoms to self-align is strong enough, as it is in materials such as iron and nickel, the result can be an intense magnetic field—as in a bar magnet. Materials with this type of behavior are called **ferromagnets.** Counteracting the tendency of atoms to align, however, is the disorder caused by increasing temperature. In fact, all ferromagnets lose their magnetic field if the temperature is high enough to cause their atoms to orient in random directions. For example, the magnetic field of a bar magnet made of iron vanishes if its temperature exceeds 770 °C.

This type of temperature behavior shows that the simple picture of a large bar magnet within the Earth cannot be correct. As we know, temperature increases with depth below the Earth's surface. In fact, at depths of about 15 miles the temperature is already above 770 °C, so the magnetism of an iron magnet would be lost due to thermal effects. Of course, at even greater depths an iron magnet would melt to form a liquid. In fact, it appears that the magnetic field of the Earth is caused by circulating currents of molten iron, nickel, and other metals. These circulating currents create a magnetic field in much the same way as the circulating current in a solenoid.

Temperature also plays a key role in the magnetization that is observed in rocks on the ocean floor. As molten rock is extruded from mid-ocean ridges, it has no net magnetization because of its high temperature. When the rock cools, however, it becomes magnetized in the direction of the Earth's magnetic field. In effect, the Earth's magnetic field becomes "frozen" in the solidified rock. As the seafloor spreads, and more material is formed along the ridge, a continuous record of the Earth's magnetic field is formed. In particular, if the Earth's field reverses at some point in time, the field in the solidified rocks will record that fact. Figure 22–29 shows a record of the magnetization of rock that has been formed by seafloor spreading, showing the regions of oppositely magnetized rocks that indicate the field reversals.

Another important feature of ferromagnets is that their magnetism is characterized by magnetic **domains** within the material, as illustrated in Figure 22–30 (a). Each domain has a strong magnetic field, but different domains are oriented differently, so that the net effect may be small. The typical size of these domains is on the order of 10^{-4} cm to 10^{-1} cm. When an external field is applied to such a material, the magnetic domains that are pointing in the direction of the applied field often grow in size at the expense of domains with different orientations, as indicated in Figure 22–30 (b). The result is that the applied external field produces a net magnetization in the material.

Many living organisms are known to incorporate small ferromagnetic crystals, consisting of *magnetite*, in their bodies. For example, some species of bacteria use magnetite crystals to help orient themselves with respect to the Earth's magnetic field. Magnetite has also been found in the brains of bees and pigeons, where it is suspected to play a role in navigation. It is even found in human brains, though its possible function there is unclear.

Whatever the role of magnetite in people, observations show that a magnetic field can affect the way the human brain operates. In recent experiments involving people viewing optical illusions, it has been found that if the parietal lobe on one side of the brain is exposed to a highly focused magnetic field of about 1 T, a temporary interruption in much of the neural activity in that hemisphere results. Fortunately, fields of 1 T are generally not encountered in everyday life.

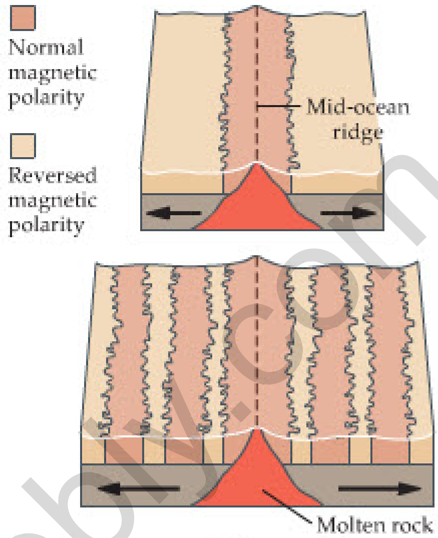
Paramagnetism and Diamagnetism

Not all magnetic materials are ferromagnetic, however. In some cases a ferromagnet has zero magnetic field simply because it is at too high a temperature. In other cases, the tendency for the self-alignment of individual atoms in a given material is too weak to produce a net magnetic field, even at low temperatures. In either case, a strong external magnetic field applied to the material can cause alignment of the atoms and result in a magnetic field. Magnetic effects of this type are referred to as **paramagnetism.**

Finally, all materials display a further magnetic effect referred to as **diamagnetism.** In the diamagnetic effect, an applied magnetic field on a material produces an oppositely directed field in response. The resulting repulsive force is usually too weak to be noticed, except in superconductors. The basic mechanism responsible for diamagnetism will be discussed in greater detail in the next chapter.

If a magnetic field is strong enough, however, even the relatively weak repulsion of diamagnetism can lead to significant effects. For example, researchers at the Nijmegen High Field Magnet Laboratory have used a field of 16 T to levitate a strawberry, a cricket, and even a frog. The diamagnetic repulsion of the water in these organisms is great enough, in a field that strong, to counteract the gravitational force of the Earth. The researchers reported that the living frog showed no visible signs of discomfort during its levitation, and that it hopped away normally after the experiment.

A similar diamagnetic effect can be used to levitate a small magnet between a person's fingertips. In the case shown in the photograph, a powerful magnet about 8 ft above the person's hand applies a magnetic field strong enough to counteract the weight of the small magnet. By itself, this system of two magnets is not stable—if the small magnet is displaced upward slightly, the increased attractive force of the upper magnet raises it farther; if it is displaced downward, the

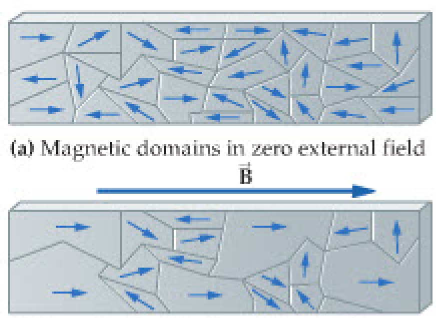


▲ **FIGURE 22–29 Mid-ocean ridge**

Molten rock extruded at a mid-ocean ridge magnetizes in the direction of the Earth's magnetic field when it cools to temperatures below about 770 °C. After cooling, the direction of the Earth's field remains "frozen" in the rocks. As these rocks move away from the ridge, due to seafloor spreading, newly extruded material near the ridge undergoes the same process. As a result, the magnetization of the rocks on either side of a mid-ocean ridge produces a geological record of the polarity of the Earth's magnetic field over time, as well as convincing confirmation of seafloor spreading.

REAL-WORLD PHYSICS: BIO
Magnetite in living organisms

REAL-WORLD PHYSICS: BIO
Magnetism and the brain



(a) Magnetic domains in zero external field

(b) Domains in direction of external magnetic field grow in size

▲ **FIGURE 22–30 Magnetic domains**

(a) A ferromagnetic material tends to form into numerous domains with magnetization pointing in different directions. **(b)** When an external field is applied, the domains pointing in the direction of the field often grow in size at the expense of domains pointing in other directions.

(Left) The repulsive diamagnetic forces produced by water molecules in the body of a frog are strong enough to levitate it in an intense magnetic field of 16 T. (Animals levitated in this way appear to suffer no harm or discomfort.) (Right) This small magnet is suspended in the strong magnetic field produced by a larger magnet above it (outside the photo). Ordinarily, such an arrangement would be highly unstable. The addition of small, repulsive diamagnetic forces due to a person's fingers, however, is sufficient to convert it into a stable equilibrium.

REAL-WORLD PHYSICS
Magnetic levitation

force of gravity pulls it down even farther. With the diamagnetic effect of the fingers, however, the small magnet is stabilized. If it is displaced upward now, the repulsive diamagnetic effect of the finger pushes it back down; if it is displaced downward, the diamagnetic repulsion of the thumb pushes it back up. The result is a stable, magnetic levitation—with no smoke or mirrors.

THE BIG PICTURE PUTTING PHYSICS IN CONTEXT

LOOKING BACK

Newton's third law (Chapter 5) is applied to the force between bar magnets in Section 22–1. It is important to note that Newton's laws apply to all types of forces, including electric and magnetic forces.

The direction of the force exerted on a moving charge by a magnetic field is given by the right-hand rule (Chapter 11), as we see in Section 22–2. The magnetic field also exerts a force on current-carrying wires. This can result in a torque (Chapter 11) on a current loop, as shown in Section 22–5.

The force exerted by a magnetic field on a moving charge is at right angles to the velocity. This can lead to circular motion (Chapter 6). Several examples of circular motion are discussed in Section 22–3.

LOOKING AHEAD

The magnetic field is central to the concept of Faraday's law of induction, which is presented in Chapter 23. It also plays a key role in alternating-current (ac) circuits, as we shall see in Chapter 24.

In Chapter 23 we introduce the idea of an inductor. In particular, we will show that an inductor can store energy in the form of a magnetic field, just as a capacitor (Chapter 20) can store energy in an electric field.

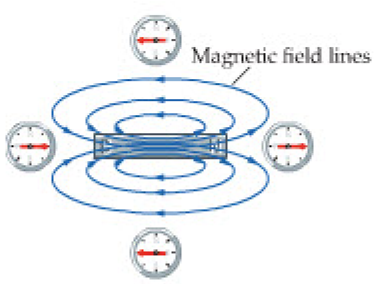
In Chapter 23 we also show that a changing magnetic field is central to the operation of electric motors and generators, as well as transformers.

We study light and the electromagnetic spectrum in Chapter 25. As we shall see, light is a wave formed of oscillating electric and magnetic fields. Both electric and magnetic fields must be present for light to exist.

CHAPTER SUMMARY

22–1 THE MAGNETIC FIELD

The magnetic field gives an indication of the effect a magnet will have in a given region.



Magnetic Poles

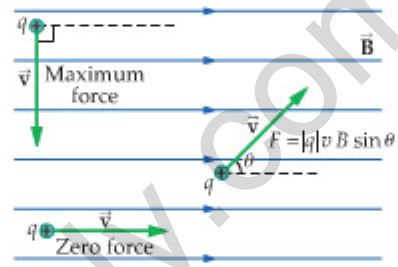
A magnet is characterized by two poles, referred to as the north pole and the south pole. These poles cannot be separated—all magnets have both poles.

Magnetic Field Lines

Magnetic fields can be represented with lines in much the same way as electric fields can be portrayed. In particular, the more closely spaced the lines, the more intense the field. Magnetic field lines, which point away from north poles and toward south poles, always form closed loops.

Geomagnetism
The Earth produces its own magnetic field, which is inclined at an angle of about 11.5° with its rotational axis. The geographic north pole of the Earth is actually the south magnetic pole of the Earth's magnetic field.

22–2 THE MAGNETIC FORCE ON MOVING CHARGES



In order for a magnetic field to exert a force on a particle, the particle must have charge and must be moving.

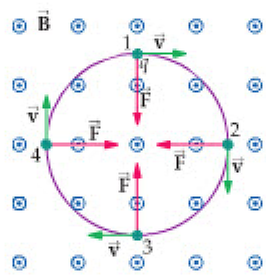
Magnitude of the Magnetic Force
The magnitude of the magnetic force is

$$F = |q|vB\sin\theta \quad 22\text{–}1$$

where q is the charge of the particle, v is its speed, B is the magnitude of the magnetic field, and θ is the angle between the velocity vector $\vec{\mathbf{v}}$ and the magnetic field vector $\vec{\mathbf{B}}$.

Magnetic Force Right-Hand Rule (RHR)
The magnetic force $\vec{\mathbf{F}}$ points in a direction that is perpendicular to both $\vec{\mathbf{B}}$ and $\vec{\mathbf{v}}$. For a positive charge, point the fingers of your right hand in the direction of $\vec{\mathbf{v}}$ and curl them toward the direction of $\vec{\mathbf{B}}$. Your thumb points in the direction of the force $\vec{\mathbf{F}}$. The force on a negative charge is in the opposite direction to that on a positive charge.

22–3 THE MOTION OF CHARGED PARTICLES IN A MAGNETIC FIELD



The motion of a charged particle in a magnetic field is quite different from that in an electric field.

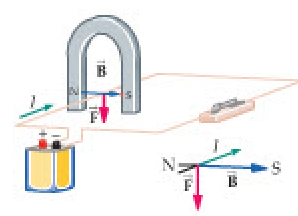
Electric Versus Magnetic Forces
A charged particle in an electric field accelerates in the direction of the field; in a magnetic field the acceleration is perpendicular to the field and to the velocity. The electric field does work on a particle and changes its speed; a magnetic field does no work on a particle, and its speed remains constant.

Constant-Velocity Motion
If a charged particle moves parallel or antiparallel to a magnetic field, it experiences no force; hence, its velocity remains constant.

Circular Motion
If a charged particle moves perpendicular to a magnetic field, it will orbit with constant speed in a circle of radius $r = mv/|q|B$.

Helical Motion
When a particle's velocity has components both parallel and perpendicular to a magnetic field, it will follow a helical path.

22–4 THE MAGNETIC FORCE EXERTED ON A CURRENT-CARRYING WIRE

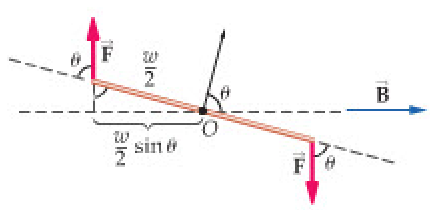


An electric current in a wire is caused by the movement of electric charges. Since moving electric charges experience magnetic forces, it follows that a current-carrying wire will as well.

Force on a Current-Carrying Wire
A wire of length L carrying a current I at an angle θ to a magnetic field B experiences a force given by

$$F = ILB\sin\theta \quad 22\text{–}4$$

22–5 LOOPS OF CURRENT AND MAGNETIC TORQUE



A current loop placed in a magnetic field experiences a torque that depends on the relative orientation of the plane of the loop and the magnetic field.

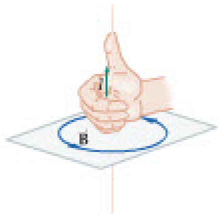
Torque on a General Loop
The magnetic torque exerted on a current loop is given by

$$\tau = NIAB\sin\theta \quad 22\text{–}6$$

where N is the number of turns around the loop, I is the current, A is the area of the loop, B is the strength of the magnetic field, and θ is the angle between the plane of the loop and the magnetic force.

22–6 ELECTRIC CURRENTS, MAGNETIC FIELDS, AND AMPÈRE'S LAW

The key observation that serves to unify electricity and magnetism is that electric currents cause magnetic fields.



Magnetic Field Right-Hand Rule

The direction of the magnetic field produced by a current is found by pointing the thumb of the right hand in the direction of the current. The fingers of the right hand curl in the direction of the field.

Ampère's Law

Ampère's law can be expressed as follows:

$$\sum B_{\|}\,\Delta L = \mu_0 I_{\text{enclosed}} \qquad 22\text{–}7$$

where $B_{\|}$ is the component of the magnetic field parallel to a segment of a closed path of length ΔL, I is the current enclosed by the path, and $\mu_0 = 4\pi \times 10^{-7}\,\text{T}\cdot\text{m/A}$ is a constant called the permeability of free space.

Magnetic Field of a Long, Straight Wire

A long, straight wire carrying a current I produces a magnetic field of magnitude B given by

$$B = \frac{\mu_0 I}{2\pi r} \qquad 22\text{–}9$$

In this expression, r is the radial distance from the wire.

Forces Between Current-Carrying Wires

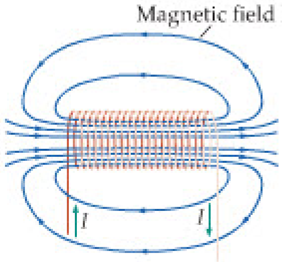
Two wires, carrying the currents I_1 and I_2, exert forces on each other. If the wires are separated by a distance d, the force exerted on a length L is

$$F = \frac{\mu_0 I_1 I_2}{2\pi d} L \qquad 22\text{–}10$$

Wires that carry current in the same direction attract one another; wires with opposite-directed currents repel one another.

22–7 CURRENT LOOPS AND SOLENOIDS

A single loop of current produces a magnetic field much like that of a permanent magnet. A succession of loops grouped together in a coil forms a solenoid.



Current Loop

The magnetic field at the center of a current loop of N turns, radius R, and current I is

$$B = \frac{N\mu_0 I}{2R} \qquad 22\text{–}11$$

Magnetic Field of a Solenoid

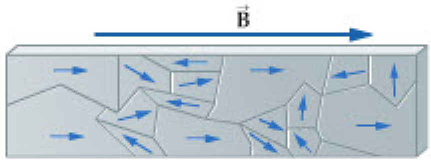
The magnetic field inside a solenoid is nearly uniform and aligned along the solenoid's axis. If the solenoid has N loops in a length L and carries a current I, its magnetic field is

$$B = \mu_0\left(\frac{N}{L}\right)I = \mu_0 n I \qquad 22\text{–}12$$

In this expression, n is the number of loops per length; $n = N/L$. The magnetic field outside a solenoid is small, and in the ideal case can be considered to be zero.

22–8 MAGNETISM IN MATTER

The ultimate origin of the magnetic fields we observe around us is circulating electric currents on the atomic level.



Paramagnetism

A paramagnetic material has no magnetic field unless an external magnetic field is applied to it. In this case, it develops a magnetization in the direction of the external field.

Ferromagnetism

A ferromagnetic material produces a magnetic field even in the absence of an external magnetic field. Permanent magnets are constructed of ferromagnetic materials.

Diamagnetism
Diamagnetism is the effect of the production by a material of a magnetic field in the opposite direction to an external magnetic field that is applied to it. All materials show at least a small diamagnetic effect.

PROBLEM-SOLVING SUMMARY

Type of Problem	Relevant Physical Concepts	Related Examples
Find the magnetic force exerted on a moving charge.	The magnitude of the magnetic force depends on the charge, the speed, the magnetic field, and the angle between the magnetic field and the velocity. The direction of the magnetic force is given by the right-hand rule.	Examples 22–1, 22–2
Determine the radius of the path followed by a charged particle in a magnetic field.	A charged particle moving at right angles to a uniform magnetic field moves on a circular path with a radius given by $r = mv/\lvert q\rvert B$.	Example 22–3
Calculate the magnetic force on a current-carrying wire.	If a wire segment of length L carries a current I at an angle θ to a magnetic field of strength B, the force it experiences is $F = ILB \sin\theta$.	Example 22–4
Find the torque exerted on a current loop.	If a current loop with current I, cross-sectional area A, and N turns is in a magnetic field B, the maximum torque is $\tau = NIAB$. The maximum torque occurs when the plane of the loop is parallel to the magnetic field ($\theta = 90°$). When the plane of the loop is at an angle θ to the magnetic force, the torque is $\tau = NIAB \sin\theta$.	Example 22–5
Determine the magnetic field produced by a long, straight wire carrying a current I.	The magnetic field produced by a long, straight wire with a current I circulates around the wire in a direction given by the right-hand rule. The magnitude of the magnetic field a radial distance r from the wire is $B = \mu_0 I/2\pi r$. This result follows from Ampère's law.	Example 22–6
Find the magnetic field inside a solenoid.	An ideal solenoid with n turns per length and current I produces a uniform magnetic field of magnitude $B = \mu_0 nI$. The field is parallel to the axis of the solenoid. In the ideal case, the magnetic field outside the solenoid is zero.	Example 22–7

CONCEPTUAL QUESTIONS

For instructor-assigned homework, go to www.masteringphysics.com

(Answers to odd-numbered Conceptual Questions can be found in the back of the book.)

1. Two charged particles move at right angles to a magnetic field and deflect in opposite directions. Can one conclude that the particles have opposite charges?
2. An electron moves with constant velocity through a region of space that is free of magnetic fields. Can one conclude that the electric field is zero in this region? Explain.
3. An electron moves with constant velocity through a region of space that is free of electric fields. Can one conclude that the magnetic field is zero in this region? Explain.
4. Describe how the motion of a charged particle can be used to distinguish between an electric and a magnetic field.
5. Explain how a charged particle moving in a circle of small radius can take the same amount of time to complete an orbit as an identical particle orbiting in a circle of large radius.
6. A current-carrying wire is placed in a region with a uniform magnetic field. The wire experiences zero magnetic force. Explain.

PROBLEMS AND CONCEPTUAL EXERCISES

Note: Answers to odd-numbered Problems and Conceptual Exercises can be found in the back of the book. **IP** *denotes an integrated problem, with both conceptual and numerical parts;* **BIO** *identifies problems of biological or medical interest;* **CE** *indicates a conceptual exercise.* **Predict/Explain** *problems ask for two responses:* **(a)** *your prediction of a physical outcome, and* **(b)** *the best explanation among three provided. On all problems, red bullets (•, ••, •••) are used to indicate the level of difficulty.*

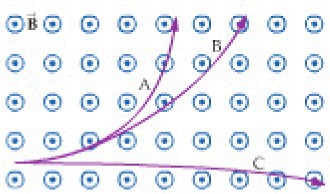
SECTION 22–2 THE MAGNETIC FORCE ON MOVING CHARGES

1. • **CE Predict/Explain** Proton 1 moves with a speed v from the east coast to the west coast in the continental United States; proton 2 moves with the same speed from the southern United States toward Canada. **(a)** Is the magnitude of the magnetic force experienced by proton 2 greater than, less than, or equal to the force experienced by proton 1? **(b)** Choose the *best explanation* from among the following:
   I. The protons experience the same force because the magnetic field is the same and their speeds are the same.
   II. Proton 1 experiences the greater force because it moves at right angles to the magnetic field.
   III. Proton 2 experiences the greater force because it moves in the same direction as the magnetic field.

2. • **CE** An electron moves west to east in the continental United States. Does the magnetic force experienced by the electron point in a direction that is generally north, south, east, west, upward, or downward? Explain.

3. • **CE** An electron moving in the positive x direction, at right angles to a magnetic field, experiences a magnetic force in the positive y direction. What is the direction of the magnetic field?

4. • **CE** Suppose particles A, B, and C in **Figure 22–31** have identical masses and charges of the same magnitude. Rank the particles in order of increasing speed. Indicate ties where appropriate.



▲ **FIGURE 22–31** Problems 4, 5, and 6

5. • **CE** Referring to Figure 22–31, what is the sign of the charge for each of the three particles? Explain.

6. • **CE** Suppose the three particles in Figure 22–31 have the same mass and speed. Rank the particles in order of increasing magnitude of their charge. Indicate ties where appropriate.

7. • What is the acceleration of a proton moving with a speed of 6.5 m/s at right angles to a magnetic field of 1.6 T?

8. • An electron moves at right angles to a magnetic field of 0.18 T. What is its speed if the force exerted on it is 8.9×10^{-15} N?

9. • A negatively charged ion moves due north with a speed of 1.5×10^{6} m/s at the Earth's equator. What is the magnetic force exerted on this ion?

10. • A proton high above the equator approaches the Earth moving straight downward with a speed of 355 m/s. Find the acceleration of the proton, given that the magnetic field at its altitude is 4.05×10^{-5} T.

11. •• A 0.32-μC particle moves with a speed of 16 m/s through a region where the magnetic field has a strength of 0.95 T. At what angle to the field is the particle moving if the force exerted on it is **(a)** 4.8×10^{-6} N, **(b)** 3.0×10^{-6} N, or **(c)** 1.0×10^{-7} N?

12. •• A particle with a charge of 14 μC experiences a force of 2.2×10^{-4} N when it moves at right angles to a magnetic field with a speed of 27 m/s. What force does this particle experience when it moves with a speed of 6.3 m/s at an angle of 25° relative to the magnetic field?

13. •• An ion experiences a magnetic force of 6.2×10^{-16} N when moving in the positive x direction but no magnetic force when moving in the positive y direction. What is the magnitude of the magnetic force exerted on the ion when it moves in the x-y plane along the line $x = y$? Assume that the ion's speed is the same in all cases.

14. •• An electron moving with a speed of 4.2×10^{5} m/s in the positive x direction experiences zero magnetic force. When it moves in the positive y direction, it experiences a force of 2.0×10^{-13} N that points in the negative z direction. What are the direction and magnitude of the magnetic field?

15. •• **IP** Two charged particles with different speeds move one at a time through a region of uniform magnetic field. The particles move in the same direction and experience equal magnetic forces. **(a)** If particle 1 has four times the charge of particle 2, which particle has the greater speed? Explain. **(b)** Find the ratio of the speeds, v_1/v_2.

16. •• A 6.60-μC particle moves through a region of space where an electric field of magnitude 1250 N/C points in the positive x direction, and a magnetic field of magnitude 1.02 T points in the positive z direction. If the net force acting on the particle is 6.23×10^{-3} N in the positive x direction, find the magnitude and direction of the particle's velocity. Assume the particle's velocity is in the x-y plane.

17. ••• When at rest, a proton experiences a net electromagnetic force of magnitude 8.0×10^{-13} N pointing in the positive x direction. When the proton moves with a speed of 1.5×10^{6} m/s in the positive y direction, the net electromagnetic force on it decreases in magnitude to 7.5×10^{-13} N, still pointing in the positive x direction. Find the magnitude and direction of **(a)** the electric field and **(b)** the magnetic field.

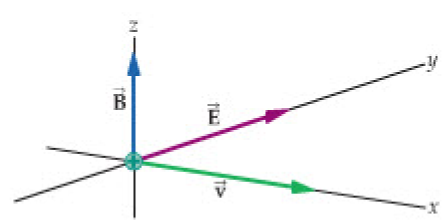
SECTION 22–3 THE MOTION OF CHARGED PARTICLES IN A MAGNETIC FIELD

18. • **CE** A velocity selector is to be constructed using a magnetic field in the positive y direction. If positively charged particles move through the selector in the positive z direction, **(a)** what must be the direction of the electric field? **(b)** Repeat part (a) for the case of negatively charged particles.

19. • Find the radius of an electron's orbit when it moves perpendicular to a magnetic field of 0.66 T with a speed of 6.27×10^{5} m/s.

20. • Find the radius of a proton's orbit when it moves perpendicular to a magnetic field of 0.66 T with a speed of 6.27×10^{5} m/s.

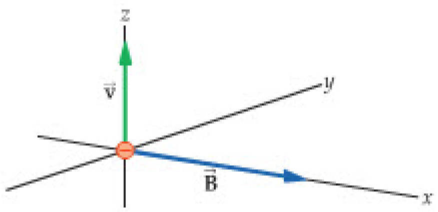
21. • Charged particles pass through a velocity selector with electric and magnetic fields at right angles to each other, as shown in **Figure 22–32**. If the electric field has a magnitude of 450 N/C and the magnetic field has a magnitude of 0.18 T, what speed must the particles have to pass through the selector undeflected?



▲ **FIGURE 22–32** Problem 21

22. • The velocity selector in **Figure 22–33** is designed to allow charged particles with a speed of 4.5×10^{3} m/s to pass through

undeflected. Find the direction and magnitude of the required electric field, given that the magnetic field has a magnitude of 0.96 T.



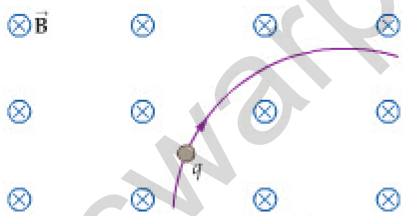
▲ **FIGURE 22–33** Problem 22

23. •• **IP BIO** The artery in Figure 22–11 has an inside diameter of 2.75 mm and passes through a region where the magnetic field is 0.065 T. **(a)** If the voltage difference between the electrodes is 195 μV, what is the speed of the blood? **(b)** Which electrode is at the higher potential? Does your answer depend on the sign of the ions in the blood? Explain.

24. •• An electron accelerated from rest through a voltage of 550 V enters a region of constant magnetic field. If the electron follows a circular path with a radius of 17 cm, what is the magnitude of the magnetic field?

25. •• A 12.5-μC particle with a mass of 2.80×10^{-5} kg moves perpendicular to a 1.01-T magnetic field in a circular path of radius 21.8 m. **(a)** How fast is the particle moving? **(b)** How long will it take the particle to complete one orbit?

26. •• **IP** When a charged particle enters a region of uniform magnetic field, it follows a circular path, as indicated in Figure 22–34. **(a)** Is this particle positively or negatively charged? Explain. **(b)** Suppose that the magnetic field has a magnitude of 0.180 T, the particle's speed is 6.0×10^6 m/s, and the radius of its path is 52.0 cm. Find the mass of the particle, given that its charge has a magnitude of 1.60×10^{-19} C. Give your result in atomic mass units, u, where 1 u = 1.67×10^{-27} kg.



▲ **FIGURE 22–34** Problem 26

27. •• A proton with a kinetic energy of 4.9×10^{-16} J moves perpendicular to a magnetic field of 0.26 T. What is the radius of its circular path?

28. •• **IP** An alpha particle (the nucleus of a helium atom) consists of two protons and two neutrons, and has a mass of 6.64×10^{-27} kg. A horizontal beam of alpha particles is injected with a speed of 1.3×10^5 m/s into a region with a vertical magnetic field of magnitude 0.155 T. **(a)** How long does it take for an alpha particle to move halfway through a complete circle? **(b)** If the speed of the alpha particle is doubled, does the time found in part (a) increase, decrease, or stay the same? Explain. **(c)** Repeat part (a) for alpha particles with a speed of 2.6×10^5 m/s.

29. ••• An electron and a proton move in circular orbits in a plane perpendicular to a uniform magnetic field $\vec{B}$. Find the ratio of the radii of their circular orbits when the electron and the proton have **(a)** the same momentum and **(b)** the same kinetic energy.

SECTION 22–4 THE MAGNETIC FORCE EXERTED ON A CURRENT-CARRYING WIRE

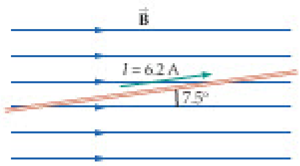
30. • What is the magnetic force exerted on a 2.15-m length of wire carrying a current of 0.899 A perpendicular to a magnetic field of 0.720 T?

31. • A wire with a current of 2.8 A is at an angle of 36.0° relative to a magnetic field of 0.88 T. Find the force exerted on a 2.25-m length of the wire.

32. • The magnetic force exerted on a 1.2-m segment of straight wire is 1.6 N. The wire carries a current of 3.0 A in a region with a constant magnetic field of 0.50 T. What is the angle between the wire and the magnetic field?

33. •• A 0.45-m copper rod with a mass of 0.17 kg carries a current of 11 A in the positive x direction. What are the magnitude and direction of the minimum magnetic field needed to levitate the rod?

34. •• The long, thin wire shown in Figure 22–35 is in a region of constant magnetic field $\vec{B}$. The wire carries a current of 6.2 A and is oriented at an angle of 7.5° to the direction of the magnetic field. **(a)** If the magnetic force exerted on this wire per meter is 0.033 N, what is the magnitude of the magnetic field? **(b)** At what angle will the force exerted on the wire per meter be equal to 0.015 N?

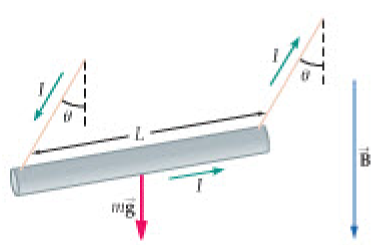


▲ **FIGURE 22–35** Problem 34

35. •• A wire with a length of 3.6 m and a mass of 0.75 kg is in a region of space with a magnetic field of 0.84 T. What is the minimum current needed to levitate the wire?

36. •• A high-voltage power line carries a current of 110 A at a location where the Earth's magnetic field has a magnitude of 0.59 G and points to the north, 72° below the horizontal. Find the direction and magnitude of the magnetic force exerted on a 250-m length of wire if the current in the wire flows **(a)** horizontally toward the east or **(b)** horizontally toward the south.

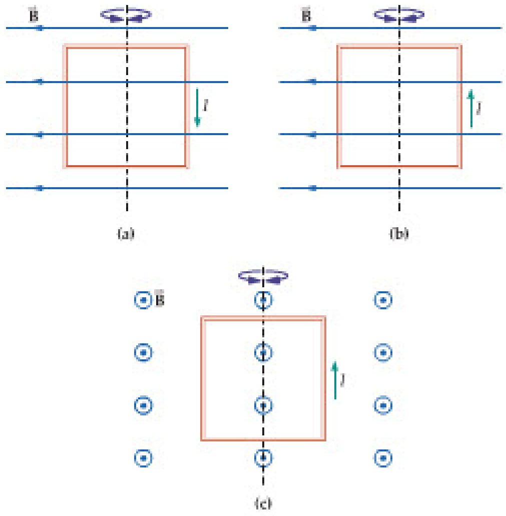
37. ••• A metal bar of mass m and length L is suspended from two conducting wires, as shown in Figure 22–36. A uniform magnetic field of magnitude B points vertically downward. Find the angle θ the suspending wires make with the vertical when the bar carries a current I.



▲ **FIGURE 22–36** Problem 37

SECTION 22–5 LOOPS OF CURRENT AND MAGNETIC TORQUE

38. • **CE** For each of the three situations shown in Figure 22–37, indicate whether there will be a tendency for the square current loop to rotate clockwise, counterclockwise, or not at all, when viewed from above the loop along the indicated axis.



▲ **FIGURE 22–37** Problems 38, 42, and 68

39\. • A rectangular loop of 260 turns is 33 cm wide and 16 cm high. What is the current in this loop if the maximum torque in a field of 0.48 T is 23 N · m?

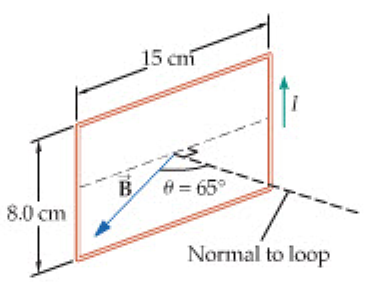
40\. • A single circular loop of radius 0.23 m carries a current of 2.6 A in a magnetic field of 0.95 T. What is the maximum torque exerted on this loop?

41\. •• In the previous problem, find the angle the plane of the loop must make with the field if the torque is to be half its maximum value.

42\. •• Consider a current loop in a region of uniform magnetic field, as shown in Figure 22–37 (a) (Problem 38). Find the magnitude of the torque exerted on the loop about the vertical axis of rotation, using the data given in Problem 68.

43\. •• **IP** Two current loops, one square the other circular, have one turn made from wires of the same length. **(a)** If these loops carry the same current and are placed in magnetic fields of equal magnitude, is the maximum torque of the square loop greater than, less than, or the same as the maximum torque of the circular loop? Explain. **(b)** Calculate the ratio of the maximum torques, $\tau_{square}/\tau_{circle}$.

44\. ••• **IP** Each of the 10 turns of wire in a vertical, rectangular loop carries a current of 0.22 A. The loop has a height of 8.0 cm and a width of 15 cm. A horizontal magnetic field of magnitude 0.050 T is oriented at an angle of $\theta = 65°$ relative to the normal to the plane of the loop, as indicated in **Figure 22–38**. Find **(a)** the magnetic force on each side of the loop, **(b)** the net magnetic force on the loop, and **(c)** the magnetic torque on the loop. **(d)** If the loop can rotate about a vertical axis with only a small amount of friction, will it end up with an orientation given by $\theta = 0$, $\theta = 90°$, or $\theta = 180°$? Explain.



▲ **FIGURE 22–38** Problem 44

SECTION 22–6 ELECTRIC CURRENTS, MAGNETIC FIELDS, AND AMPÈRE'S LAW

45\. • Find the magnetic field 6.25 cm from a long, straight wire that carries a current of 7.81 A.

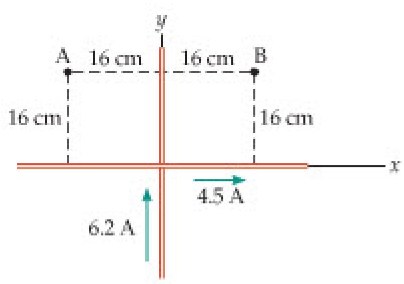
46\. • A long, straight wire carries a current of 7.2 A. How far from this wire is the magnetic field it produces equal to the Earth's magnetic field, which is approximately 5.0×10^{-5} T?

47\. • You travel to the north magnetic pole of the Earth, where the magnetic field points vertically downward. There, you draw a circle on the ground. Applying Ampère's law to this circle, show that zero current passes through its area.

48\. • Two power lines, each 270 m in length, run parallel to each other with a separation of 25 cm. If the lines carry parallel currents of 110 A, what are the magnitude and direction of the magnetic force each exerts on the other?

49\. • **BIO Pacemaker Switches** Some pacemakers employ magnetic reed switches to enable doctors to change their mode of operation without surgery. A typical reed switch can be switched from one position to another with a magnetic field of 5.0×10^{-4} T. What current must a wire carry if it is to produce a 5.0×10^{-4} T field at a distance of 0.50 m?

50\. •• **IP** Consider the long, straight, current-carrying wires shown in **Figure 22–39**. One wire carries a current of 6.2 A in the positive y direction; the other wire carries a current of 4.5 A in the positive x direction. **(a)** At which of the two points, A or B, do you expect the magnitude of the net magnetic field to be greater? Explain. **(b)** Calculate the magnitude of the net magnetic field at points A and B.



▲ **FIGURE 22–39** Problems 50 and 51

51\. •• **IP** Repeat Problem 50 for the case where the 6.2-A current is reversed in direction.

52\. •• In Oersted's experiment, suppose that the compass was 0.25 m from the current-carrying wire. If a magnetic field of half the Earth's magnetic field of 5.0×10^{-5} T was required to give a noticeable deflection of the compass needle, what current must the wire have carried?

53\. •• **IP** Two long, straight wires are separated by a distance of 9.25 cm. One wire carries a current of 2.75 A, the other carries a current of 4.33 A. **(a)** Find the force per meter exerted on the 2.75-A wire. **(b)** Is the force per meter exerted on the 4.33-A wire greater than, less than, or the same as the force per meter exerted on the 2.75-A wire? Explain.

54\. ••• Two long, straight wires are oriented perpendicular to the page, as shown in **Figure 22–40**. The current in one wire is $I_1 = 3.0$ A, pointing into the page, and the current in the other wire is $I_2 = 4.0$ A, pointing out of the page. Find the magnitude and direction of the net magnetic field at point P.