REASONING AND DISCUSSION

Since the capacitance of a parallel-plate capacitor is $C = \varepsilon_0 A/d$, increasing the separation, d, decreases the capacitance. With a smaller value of C, and a constant value for V, the charge $Q = CV$ will decrease. The same general behavior can be expected with any capacitor.

ANSWER

(b) The charge on the plates decreases.

Sometimes a capacitor is first connected to a battery to be charged and is then disconnected. In this case, the charge on the plates is "trapped"—it has no place to go—and hence Q must remain constant. If the capacitance is changed now, the result is a different potential difference, $V = Q/C$, between the plates.

Dielectrics

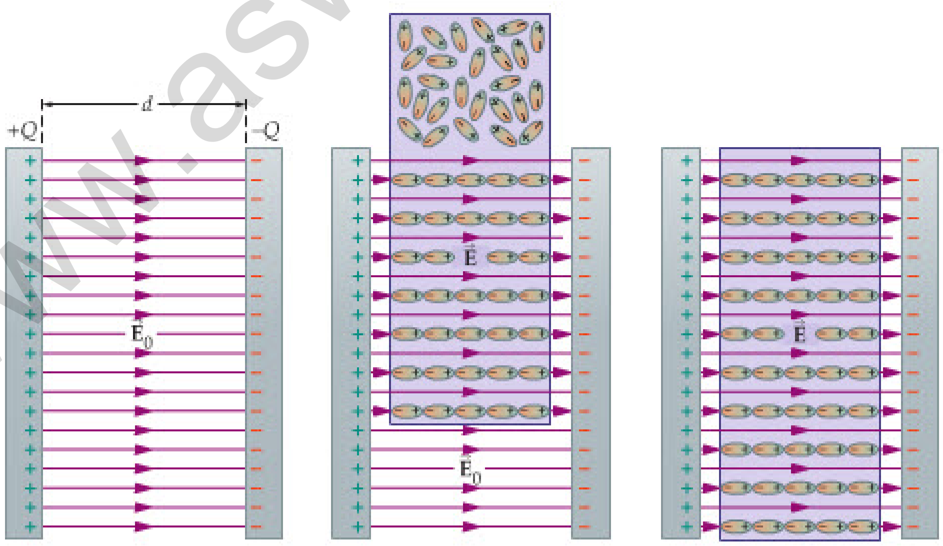
One way to increase the capacitance of a capacitor is to insert an insulating material, referred to as a **dielectric**, between its plates. With a dielectric in place, a capacitor can store more charge or energy in the same volume. Thus, dielectrics play an important role in miniaturizing electronic devices.

To see how this works, consider the parallel-plate capacitor shown in Figure 20–15 (a). Initially the plates are separated by a vacuum and connected to a battery, giving the plates the charges $+Q$ and $-Q$. The battery is now removed, and the charge on the plates remains constant. The electric field between the plates is uniform and has a magnitude E_0. If the distance between the plates is d, the corresponding potential difference is $V_0 = E_0 d$, and the capacitance is

$$C_0 = \frac{Q}{V_0}$$

Now, insert a dielectric slab, as illustrated in Figures 20–15 (b) and (c). If the molecules in the dielectric have a permanent dipole moment, they will align with the field, as shown in Figures 20–15 (b). Even without a permanent dipole moment, however, the molecules will become polarized by the field (see Section 19–1). This polarization leads to the same type of alignment, although the effect is weaker. The result of this alignment is a positive charge on the surface of the slab near the negative plate and a negative charge on the surface near the positive plate.

Recalling that electric field lines terminate on negative charges and start on positive charges, we can see from Figure 20–15 (c) that fewer field lines exist within



(a) Capacitor without dielectric

(b) Inserting a dielectric

(c) A dielectric reduces the electric field within a capacitor

FIGURE 20–15 The effect of a dielectric on the electric field of a capacitor
When a dielectric is placed in the electric field between the plates of a capacitor, the molecules of the dielectric tend to become oriented with their positive ends pointing toward the negatively charged plate and their negative ends pointing toward the positively charged plate. The result is a buildup of positive charge on one surface of the dielectric and of negative charge on the other. Since field lines start on positive charges and end on negative charges, we see that the number of field lines within the dielectric is reduced. Thus, within the dielectric the applied electric field $\vec{E}_0$ is partially canceled. Because the strength of the electric field is less, the voltage between the plates is less as well. Since V is smaller while Q remains the same, the capacitance, $C = Q/V$, is increased by the dielectric.

the dielectric. Consequently, there is a reduced field, E, in a dielectric, which we characterize with a dimensionless **dielectric constant**, κ, as follows:

$$E = \frac{E_0}{\kappa} \qquad \text{20–13}$$

In the case of a vacuum, $\kappa = 1$, and $E = E_0$, as before. For an insulating material, however, the value of κ is greater than one. For example, paper has a dielectric constant of roughly 4, which means that the electric field within paper is about one-quarter what it would be in a vacuum. Typical values of κ are listed in Table 20–1.

Thus, a dielectric reduces the field between the plates of a capacitor by a factor of κ. This, in turn, decreases the *potential difference* between the plates by the same factor:

$$V = Ed = \left(\frac{E_0}{\kappa}\right)d = \frac{E_0 d}{\kappa} = \frac{V_0}{\kappa}$$

Finally, since the potential difference is smaller, the capacitance must be larger:

$$C = \frac{Q}{V} = \frac{Q}{(V_0/\kappa)} = \kappa\frac{Q}{V_0} = \kappa C_0 \qquad \text{20–14}$$

In effect, the dielectric partially shields one plate from the other, making it easier to build up a charge on the plates. If the space between the plates of a capacitor is filled with paper, for example, the capacitance will be about four times larger than if the space had been a vacuum.

The relation $C = \kappa C_0$ applies to any capacitor. For the special case of a parallel-plate capacitor filled with a dielectric, we have

Capacitance of a Parallel-Plate Capacitor Filled with a Dielectric

$$C = \frac{\kappa\varepsilon_0 A}{d} \qquad \text{20–15}$$

We apply this relation in the next Example.

TABLE 20–1 Dielectric Constants

Substance	Dielectric constant, κ
Water	80.4
Neoprene rubber	6.7
Pyrex glass	5.6
Mica	5.4
Paper	3.7
Mylar	3.1
Teflon	2.1
Air	1.00059
Vacuum	1

PROBLEM-SOLVING NOTE

The Effects of a Dielectric

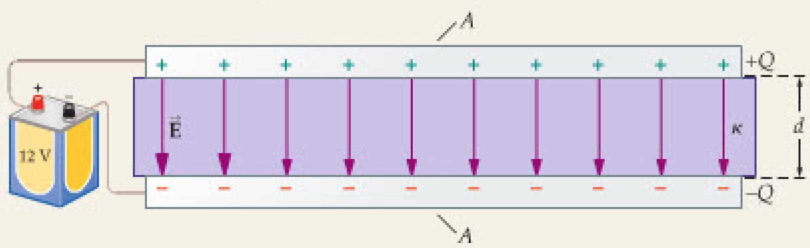
Dielectrics reduce the electric field in a capacitor, which results in a reduced potential difference between the plates. As a result, a dielectric always increases the capacitance.

EXAMPLE 20–6 EVEN MORE CHARGED UP

A parallel-plate capacitor is constructed with plates of area 0.0280 m^2 and separation 0.550 mm. The space between the plates is filled with a dielectric with dielectric constant κ. When the capacitor is connected to a 12.0-V battery, each of the plates has a charge of magnitude 3.62×10^{-8} C. What is the value of the dielectric constant, κ?

PICTURE THE PROBLEM

The sketch shows the capacitor with a dielectric material inserted between the plates. In other respects, the capacitor is the same as the one considered in Example 20–5.



STRATEGY

Since we are given the potential difference V and the charge Q, we can find the capacitance using $C = Q/V$. Next, we relate the capacitance to the physical characteristics of the capacitor with $C = \kappa\varepsilon_0 A/d$. Using the given values for A and d, we solve for κ.

SOLUTION

1. Determine the value of the capacitance: $C = \dfrac{Q}{V} = \dfrac{(3.62 \times 10^{-8}\ \text{C})}{12.0\ \text{V}} = 3.02 \times 10^{-9}\ \text{F}$

2. Solve $C = \kappa\varepsilon_0 A/d$ for the dielectric constant, κ: $C = \kappa\varepsilon_0 A/d$; $\kappa = Cd/\varepsilon_0 A$

3. Substitute numerical values to find κ:

$$\kappa = \frac{Cd}{\varepsilon_0 A} = \frac{(3.02 \times 10^{-9}\ \text{F})(0.550 \times 10^{-3}\ \text{m})}{(8.85 \times 10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(0.0280\ \text{m}^2)} = 6.70$$

INSIGHT
Comparing our result with the dielectric constants given in Table 20–1, we see that the dielectric may be neoprene rubber.

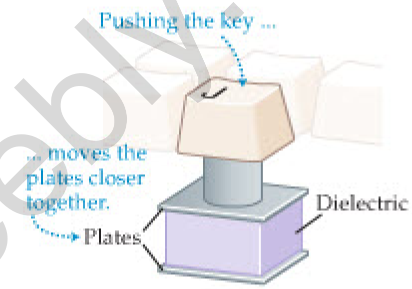
PRACTICE PROBLEM
If a different dielectric with a smaller dielectric constant is inserted into the capacitor, does the charge on the plates increase, decrease, or remain the same? Find the charge on the plates for $\kappa = 3.5$. [**Answer:** The charge decreases to $Q = 1.89 \times 10^{-8}$ C.]

Some related homework problems: Problem 53, Problem 54

The fact that the capacitance of a capacitor depends on the separation of its plates finds a number of interesting applications. For example, if you have ever typed on a computer keyboard, you have probably been utilizing the phenomenon of capacitance without realizing it. Many computer keyboards are designed in such a way that each key is connected to the upper plate of a parallel-plate capacitor, as illustrated in **Figure 20–16**. When you depress a given key, the separation between the plates of that capacitor decreases, and the corresponding capacitance increases. The circuitry of the computer can detect this change in capacitance, thereby determining which key you have pressed.

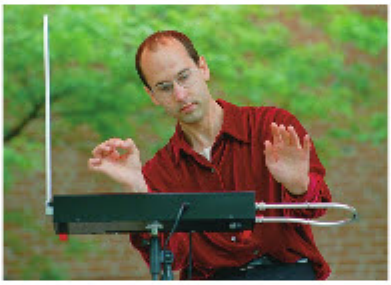
Another, less well-known application of capacitance is the theremin, a musical instrument that you play without touching! Two antennas on the theremin are used to control the sound it makes; one antenna adjusts the volume, the other adjusts the pitch. When a person places a hand near one of the antennas, the effect is similar to that of a parallel-plate capacitor, with the hand playing the role of one plate and the antenna playing the role of the other plate. Changing the separation between hand and antenna changes the capacitance, which the theremin's circuitry then converts into a corresponding change of volume or pitch. Theremins have been used to provide "ethereal" music for a number of science fiction films, and some popular bands use theremins in their musical arrangements.

REAL-WORLD PHYSICS
Computer keyboards



▲ **FIGURE 20–16 Capacitance and the computer keyboard**
The keys on many computer keyboards form part of a parallel-plate capacitor. Depressing the key changes the plate separation. The corresponding change in capacitance can be detected by the computer's circuitry.

REAL-WORLD PHYSICS
The theremin—a musical instrument you play without touching



▲ A musician plays a theremin at an outdoor concert.

Dielectric Breakdown

If the electric field applied to a dielectric is large enough, it can literally tear the atoms apart, allowing the dielectric to conduct electricity. This condition is referred to as **dielectric breakdown.** The maximum field a dielectric can withstand before breakdown is called the **dielectric strength.** Typical values are given in Table 20–2.

For example, if the electric field in air exceeds about 3,000,000 V/m, dielectric breakdown will occur, leading to a spark on a small scale or a bolt of lightning on a larger scale. Next time you walk across a carpet and get a shock when reaching for the doorknob, think about the fact that you have just produced an electric field of roughly *3 million volts per meter!* The sharp tip of a lightning rod, which has a high electric field in its vicinity, helps initiate and guide lightning to the ground, or to dissipate charge harmlessly so that no lightning occurs at all. Saint Elmo's fire—the glow of light around the rigging of a ship in a storm—is another example of dielectric breakdown in air.

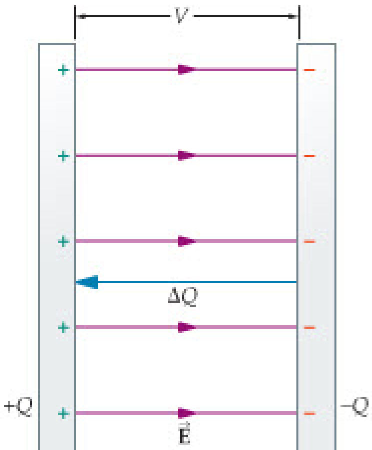
20–6 Electrical Energy Storage

As mentioned in the previous section, capacitors store more than just charge—they also store energy. To see how, consider a capacitor that has charges of magnitude Q on its plates, and a potential difference of V. Now, imagine transferring a small amount of charge, ΔQ, from one plate to the other, as in **Figure 20–17**. Since this charge must be moved across a potential difference of V, the change in electric potential energy is $\Delta U = (\Delta Q)V$. Thus, the potential energy of the capacitor increases by $(\Delta Q)V$ when the magnitude of the charge on its plates is increased from Q to $Q + \Delta Q$. As more charge is transferred from one plate to the other, more electric potential energy is stored in the capacitor.

To find the total electric energy stored in a capacitor, we must take into account the fact that the potential difference between the plates increases as the charge on

TABLE 20–2 Dielectric Strengths

Substance	Dielectric Strength (V/m)
Mica	100×10^6
Teflon	60×10^6
Paper	16×10^6
Pyrex glass	14×10^6
Neoprene rubber	12×10^6
Air	3.0×10^6



▲ **FIGURE 20–17 The energy required to charge a capacitor**

A capacitor has a charge of magnitude Q on its plates and a potential difference V between the plates. Transferring a small charge increment, $+\Delta Q$, from the negative plate to the positive plate increases the electric potential energy of the capacitor by the amount $\Delta U = (\Delta Q)V$.

the plates increases. In fact, recalling that the potential difference is given by $V = Q/C$, it is clear that V increases linearly with the charge, as illustrated in Figure 20–18. In particular, if the final potential difference is V, the average potential during charging is $\frac{1}{2}V$. Therefore, the total energy U stored in a capacitor with charge Q and potential difference V can be written as follows:

$$U = QV_{av} = \tfrac{1}{2}QV \qquad 20\text{–}16$$

Equivalently, since $Q = CV$, the energy stored in a capacitor of capacitance C and voltage V is

$$U = \tfrac{1}{2}CV^2 \qquad 20\text{–}17$$

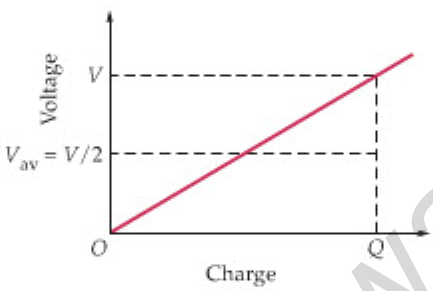
Finally, using $V = Q/C$, we find that the energy stored in a capacitor of charge Q and capacitance C is

$$U = \frac{Q^2}{2C} \qquad 20\text{–}18$$

All these expressions are equivalent; they simply give the energy in terms of different variables.

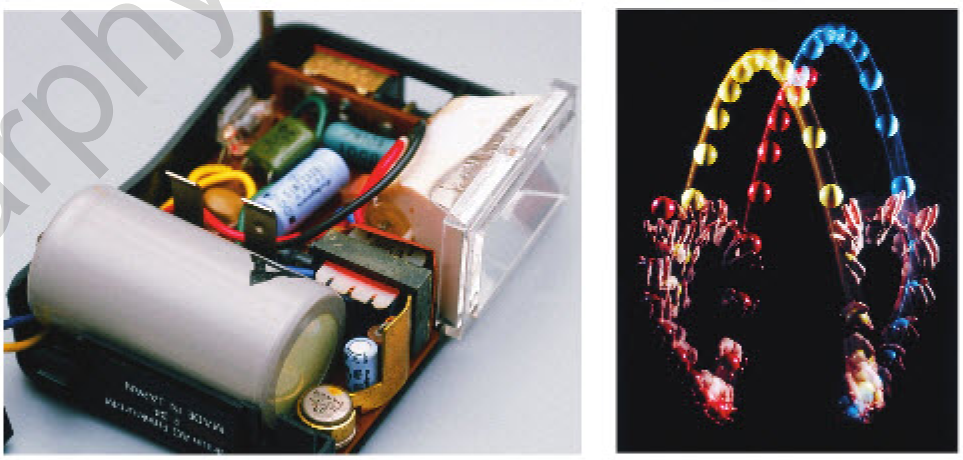
REAL-WORLD PHYSICS
The electronic flash

The energy stored in a capacitor can be put to a number of practical uses. Any time you take a flash photograph, for example, you are triggering the rapid release of energy from a capacitor. The flash unit typically contains a capacitor with a capacitance of 100 to 400 μF. When fully charged to a voltage of about 300 V, the capacitor contains roughly 15 J of energy. Activating the flash causes the stored energy, which took several seconds to accumulate, to be released in less than a millisecond. Because of the rapid release of energy, the power output of a flash unit is impressively large—about 10 to 20 kW. This is far in excess of the power provided by the battery that operates the unit. Similar considerations apply to the defibrillator used in the treatment of heart attack victims, as we show in the next Example.



▲ **FIGURE 20–18 The voltage of a capacitor being charged**

The voltage V between the plates of a capacitor increases linearly with the charge Q on the plates, $V = Q/C$. Therefore, if a capacitor is charged to a final voltage of V, the average voltage during charging is $V_{av} = \frac{1}{2}V$.



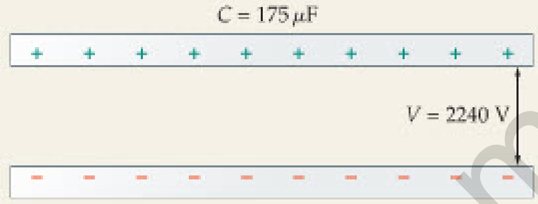
▲ An electronic flash unit like the one at left includes a capacitor (gray) that can store a large amount of charge. When the charge is released, the resulting flash can be as brief as a millisecond or less, allowing photographers to "freeze" motion, as in the photo at right. Even faster strobe units can be used to photograph explosions, shock waves, or speeding bullets.

EXAMPLE 20–7 THE DEFIBRILLATOR: DELIVERING A SHOCK TO THE SYSTEM

REAL WORLD PHYSICS: BIO

When a person's heart undergoes ventricular fibrillation—a rapid, uncoordinated twitching of the heart muscles—it often takes a strong jolt of electrical energy to restore the heart's regular beating and save the person's life. The device that delivers this jolt of energy is known as a defibrillator, and it uses a capacitor to store the necessary energy. In a typical defibrillator, a 175-μF capacitor is charged until the potential difference between the plates is 2240 V. **(a)** What is the magnitude of the charge on each plate of the fully charged capacitor? **(b)** Find the energy stored in the charged-up defibrillator.

PICTURE THE PROBLEM
Our sketch shows a simplified representation of a capacitor. The values of the capacitance and the potential difference are indicated.



STRATEGY

a. We can find the charge stored on the capacitor plates using $Q = CV$.

b. The energy stored in the capacitor can be determined immediately using $U = \frac{1}{2}CV^2$. In addition, now that we know the charge on each plate of the capacitor, the energy can also be found with the relations $U = \frac{1}{2}QV$ and $U = Q^2/2C$.

SOLUTION

Part (a)

1. Use $Q = CV$ to find the charge on the plates: $Q = CV = (175 \times 10^{-6}\ \text{F})(2240\ \text{V}) = 0.392\ \text{C}$

Part (b)

2. Find the stored energy using $U = \frac{1}{2}CV^2$: $U = \frac{1}{2}CV^2 = \frac{1}{2}(175 \times 10^{-6}\ \text{F})(2240\ \text{V})^2 = 439\ \text{J}$

3. As a check, use $U = \frac{1}{2}QV$: $U = \frac{1}{2}QV = \frac{1}{2}(0.392\ \text{C})(2240\ \text{V}) = 439\ \text{J}$

4. Finally, use the relation $U = Q^2/2C$: $U = \frac{Q^2}{2C} = \frac{(0.392\ \text{C})^2}{2(175 \times 10^{-6}\ \text{F})} = 439\ \text{J}$

INSIGHT
Of the 439 J stored in the defibrillator's capacitor, typically about 200 J will actually pass through the person's body in a pulse lasting about 2 ms. The power delivered by the pulse is approximately $P = U/t = (200\ \text{J})/(0.002\ \text{s}) = 100\ \text{kW}$. This is significantly larger than the power delivered by the battery, which can take up to 30 s to fully charge the capacitor.

PRACTICE PROBLEM
Suppose the defibrillator is "fired" when the voltage is only half its maximum value of 2240 V. How much energy is stored in this case? [**Answer:** $E = (439\ \text{J})/4 = 110\ \text{J}$]

Some related homework problems: Problem 64, Problem 69

A defibrillator uses a capacitor to deliver a shock to a person's heart, restoring it to normal function. Capacitors can have the opposite effect as well, and it is for this reason that they can be quite dangerous, even in electrical devices that are turned off and unplugged from the wall. For example, a television set contains a number of capacitors, some of which store significant amounts of charge and energy. When a TV is unplugged, the capacitors retain their charge for long periods of time. Therefore, if you reach into the back of an unplugged television set there is a danger that you may come in contact with the terminals of a capacitor, which would then discharge its stored energy through your body. The resulting shock could be harmful or even fatal.

REAL-WORLD PHYSICS: BIO
Capacitor hazards

▲ A jolt of electric current from a defibrillator can restore normal heartbeat when the heart muscle has begun to twitch irregularly or has stopped beating altogether. A capacitor is used to store electricity, discharging it in a burst lasting only a couple of milliseconds.

Finally, we have discussed many examples of energy stored in a capacitor, but where exactly is the energy located? The answer is that the energy can be thought of as stored in the electric field, E, between the plates. To be specific, consider the relation

$$\text{energy} = \tfrac{1}{2}QV$$

In the case of a parallel-plate capacitor of area A and separation d, we know that $Q = \varepsilon_0 EA$ (Equation 20–11) and $V = Ed$. Thus, the energy stored in the capacitor can be written as

$$U = \text{energy} = \tfrac{1}{2}(\varepsilon_0 EA)(Ed) = \tfrac{1}{2}\varepsilon_0 E^2(Ad)$$

We have grouped A and d together because the product Ad is simply the total volume between the plates. Therefore, the **energy density** (energy per volume) is given by the following:

$$u_E = \text{electric energy density} = \frac{\text{electric energy}}{\text{volume}} = \tfrac{1}{2}\varepsilon_0 E^2 \qquad 20\text{–}19$$

This result, though derived for a capacitor, is valid for any electric field, whether it occurs within a capacitor or anyplace else.

THE BIG PICTURE PUTTING PHYSICS IN CONTEXT

LOOKING BACK

In Section 20–1 we introduce the concepts of the electric potential and the electric potential energy. The development of these concepts parallels that of the gravitational potential energy for a uniform gravitational field (Chapter 8).

Energy conservation (Chapter 8) is used in Section 20–2, this time with the electric potential energy.

We also return to the idea of equipotential curves—curves along which the potential energy is constant—in direct analogy to the contour maps discussed at the end of Chapter 8.

The electric potential of a point charge is developed in Section 20–3. The results are almost identical to those obtained for the gravitational potential energy of a point mass in Chapter 12.

LOOKING AHEAD

Electrical energy is generalized to direct-current (dc) electric circuits in Chapter 21.

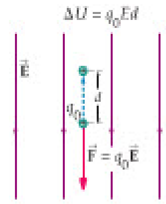
We will also see the important role that capacitors play in dc circuits in Chapter 21, and in alternating-current (ac) circuits in Chapter 24.

The concept of electrical energy is generalized yet again in Chapter 23, where we show how an electric motor can convert electrical energy to mechanical energy. We also show that the reverse process is possible, with a generator converting mechanical energy to electrical energy.

The electric potential energy for a point charge is applied to Bohr's model of the hydrogen atom in Chapter 31. With this energy we can determine the colors of light that hydrogen atoms emit.

CHAPTER SUMMARY

20–1 ELECTRIC POTENTIAL ENERGY AND THE ELECTRIC POTENTIAL



The electric force is conservative, just like the force of gravity. As a result, there is a potential energy U associated with the electric force.

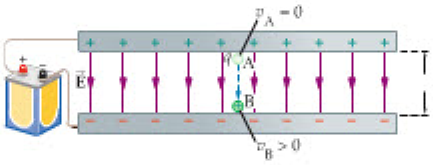
Electric Potential Energy, *U*
The change in electric potential energy is defined by $\Delta U = -W$, where W is the work done by the electric field.

Electric Potential, *V*
The change in electric potential is defined to be $\Delta V = \Delta U/q_0$.

Relation Between the Electric Field and the Electric Potential
The electric field is related to the rate of change of the electric potential. In particular, if the electric potential changes by the amount ΔV with a displacement Δs, the electric field in the direction of the displacement is

$$E = -\frac{\Delta V}{\Delta s} \qquad 20\text{–}4$$

20–2 ENERGY CONSERVATION



Another consequence of the fact that the electric force is conservative is that the total energy of an object is conserved—as long as nonconservative forces like friction can be ignored.

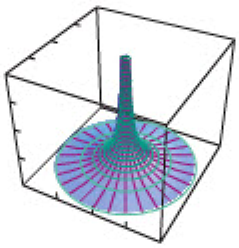
Energy Conservation
As usual, energy conservation can be expressed as follows:

$$\tfrac{1}{2}mv_A^2 + U_A = \tfrac{1}{2}mv_B^2 + U_B$$

In the case of the electric force, the potential energy is $U = q_0V$.

Direction of Acceleration
Positive charges accelerate in the direction of decreasing electric potential; negative charges accelerate in the direction of increasing electric potential.

20–3 THE ELECTRIC POTENTIAL OF POINT CHARGES



Electric potential of a positive point charge

If we define the electric potential of a point charge q to be zero at an infinite distance from the charge, the electric potential at a distance r is

$$V = \frac{kq}{r} \qquad 20\text{–}7$$

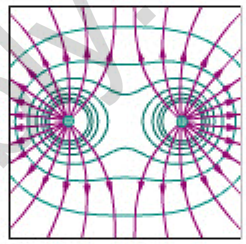
Electric Potential Energy
We define the electric potential energy of two charges, q_0 and q, to be zero when the separation between them is infinite. When the charges are separated by a distance r, the potential energy of the system is

$$U = \frac{kq_0q}{r} \qquad \text{20–8}$$

Superposition
The electric potential of two or more point charges is simply the algebraic sum of the potentials due to each charge separately.

The total electric potential energy of two or more point charges is the sum of the potential energies due to each pair of charges.

20–4 EQUIPOTENTIAL SURFACES AND THE ELECTRIC FIELD

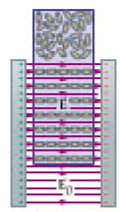

Equipotentials for two positive charges

Equipotential surfaces are defined as surfaces on which the electric potential is constant. Different equipotential surfaces correspond to different values of the potential.

Electric Field
The electric field is always perpendicular to the equipotential surfaces, and it points in the direction of decreasing electric potential.

Ideal Conductors
Ideal conductors are equipotential surfaces; every point on or within an ideal conductor is at the same potential. The electric field, therefore, is perpendicular to the surface of a conductor.

20–5 CAPACITORS AND DIELECTRICS



A capacitor is a device that stores electric charge.

Capacitance
Capacitance is defined as the amount of charge Q stored in a capacitor per volt of potential difference V between the plates of the capacitor. Thus,

$$C = \frac{Q}{V} \qquad \text{20–9}$$

Parallel-Plate Capacitor
The capacitance of a parallel-plate capacitor, with plates of area A and separation d, is

$$C = \frac{\varepsilon_0 A}{d} \qquad \text{20–12}$$

Dielectrics
A dielectric is an insulating material that increases the capacitance of a capacitor.

Dielectric Constant
A dielectric is characterized by the dimensionless dielectric constant, κ. In particular, the electric field in a dielectric is reduced by the factor κ, $E = E_0/\kappa$; the potential difference between capacitor plates is decreased by the factor κ, $V = V_0/\kappa$; and the capacitance is increased by the factor κ:

$$C = \kappa C_0 \qquad \text{20–14}$$

Dielectric Breakdown/Dielectric Strength
A large electric field can cause a dielectric material to conduct electricity. This condition is referred to as dielectric breakdown. The strength of electric field required for dielectric breakdown is called the dielectric strength of the material.

20–6 ELECTRICAL ENERGY STORAGE

A capacitor, in addition to storing charge, also stores electrical energy.

Energy Stored in a Capacitor
The electrical energy stored in a capacitor can be expressed as follows:

$$U = \tfrac{1}{2}QV = \tfrac{1}{2}CV^2 = Q^2/2C \qquad \text{20–16, 17, 18}$$

Electric Energy Density of an Electric Field
Electric energy can be thought of as stored in the electric field. The electrical energy per volume, referred to as the electric energy density, is given by the following relation:

$$u_E = \text{electric energy density} = \tfrac{1}{2}\varepsilon_0 E^2 \qquad 20\text{–}19$$

PROBLEM-SOLVING SUMMARY

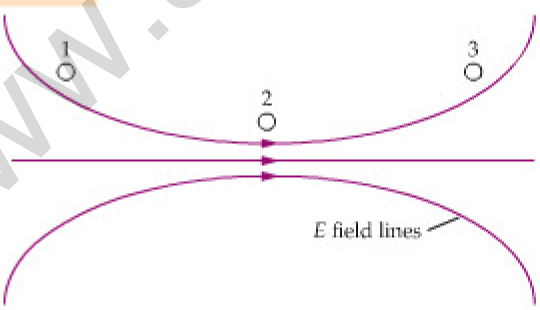
Type of Problem	Relevant Physical Concepts	Related Examples
Find the electric field corresponding to a change in electric potential.	The electric field is related to the change of electric potential with distance. The precise relation is $E = -\Delta V/\Delta s$.	Example 20–1 Active Example 20–1
Find the kinetic energy or speed of a particle moving in an electric field.	Apply energy conservation, including the electric potential energy, $U = qV$.	Examples 20–2, 20–4 Active Example 20–2
Calculate the electric potential due to a system of point charges.	The electric potential of a single point charge q at a distance r is $V = kq/r$. For a system of point charges, the total electric potential is the algebraic sum of the potentials calculated for each charge separately.	Examples 20–3, 20–4 Active Example 20–3
Determine the charge on the plates of a capacitor, or the potential difference between the plates.	The charge Q and potential difference V are related to the capacitance C by the expression $C = Q/V$.	Examples 20–5, 20–6
Determine the amount of energy stored in a capacitor.	The energy stored in a capacitor is given by three equivalent expressions: $U = \frac{1}{2}QV$; $U = \frac{1}{2}CV^2$; $U = Q^2/2C$.	Example 20–7

CONCEPTUAL QUESTIONS

For instructor-assigned homework, go to www.masteringphysics.com

(Answers to odd-numbered Conceptual Questions can be found in the back of the book.)

1. In one region of space the electric potential has a positive constant value. In another region of space the potential has a negative constant value. What can be said about the electric field within each of these two regions of space?
2. Two like charges a distance r apart have a positive electric potential energy. Conversely, two unlike charges a distance r apart have a negative electric potential energy. Explain the physical significance of these observations.
3. If the electric field is zero in some region of space is the electric potential zero there as well? Explain.
4. Sketch the equipotential surface that goes through point 1 in Figure 20–19. Repeat for point 2 and for point 3.



▲ **FIGURE 20–19** Conceptual Question 4 and Problems 43 and 44

5. How much work is required to move a charge from one location on an equipotential to another point on the same equipotential? Explain.
6. It is known that the electric potential is constant on a given two-dimensional surface. What can be said about the electric field on this surface?
7. Explain why equipotentials are always perpendicular to the electric field.
8. Two charges are at locations that have the same value of the electric potential. Is the electric potential energy the same for these charges? Explain.
9. A capacitor is connected to a battery and fully charged. What becomes of the charge on the capacitor when it is disconnected from the battery? What becomes of the charge when the two terminals of the capacitor are connected to one another?
10. It would be unwise to unplug a television set, take off the back, and reach inside. The reason for the danger is that if you happen to touch the terminals of a high-voltage capacitor you could receive a large electrical shock—even though the set is unplugged. Why?
11. On which of the following quantities does the capacitance of a capacitor depend: **(a)** the charge on the plates; **(b)** the separation of the plates; **(c)** the voltage difference between the plates; **(d)** the electric field between the plates; or **(e)** the area of the plates?
12. We say that a capacitor stores charge, yet the total charge in a capacitor is zero; that is, $Q + (-Q) = 0$. In what sense does a capacitor store charge if the net charge within it is zero?
13. The plates of a particular parallel-plate capacitor are uncharged. Is the capacitance of this capacitor zero? Explain.

PROBLEMS AND CONCEPTUAL EXERCISES

Note: Answers to odd-numbered Problems and Conceptual Exercises can be found in the back of the book. **IP** *denotes an integrated problem, with both conceptual and numerical parts;* **BIO** *identifies problems of biological or medical interest;* **CE** *indicates a conceptual exercise.* **Predict/Explain** *problems ask for two responses:* **(a)** *your prediction of a physical outcome, and* **(b)** *the best explanation among three provided. On all problems, red bullets (•, ••, •••) are used to indicate the level of difficulty.*

SECTION 20–1 ELECTRIC POTENTIAL ENERGY AND THE ELECTRIC POTENTIAL

1\. • **CE** An electron is released from rest in a region of space with a nonzero electric field. As the electron moves, does it experience an increasing or decreasing electric potential? Explain.

2\. • A uniform electric field of magnitude 4.1×10^5 N/C points in the positive x direction. Find the change in electric potential energy of a 4.5-μC charge as it moves from the origin to the points **(a)** (0, 6.0 m); **(b)** (6.0 m, 0); and **(c)** (6.0 m, 6.0 m).

3\. • A uniform electric field of magnitude 6.8×10^5 N/C points in the positive x direction. Find the change in electric potential between the origin and the points **(a)** (0, 6.0 m); **(b)** (6.0 m, 0); and **(c)** (6.0 m, 6.0 m).

4\. • **BIO Electric Potential Across a Cell Membrane** In a typical living cell, the electric potential inside the cell is 0.070 V lower than the electric potential outside the cell. The thickness of the cell membrane is 0.10 μm. What are the magnitude and direction of the electric field within the cell membrane?

5\. • A computer monitor accelerates electrons and directs them to the screen in order to create an image. If the accelerating plates are 1.05 cm apart, and have a potential difference of 25,500 V, what is the magnitude of the uniform electric field between them?

6\. • Find the change in electric potential energy for an electron that moves from one accelerating plate to the other in the computer monitor described in the previous problem.

7\. • A parallel-plate capacitor has plates separated by 0.75 mm. If the electric field between the plates has a magnitude of **(a)** 1.2×10^5 V/m or **(b)** 2.4×10^4 N/C, what is the potential difference between the plates?

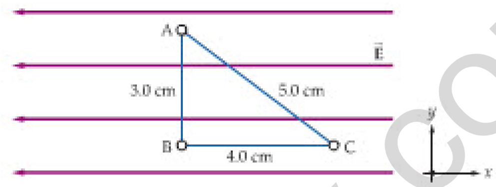
8\. • When an ion accelerates through a potential difference of 2140 V, its electric potential energy decreases by 1.37×10^{-15} J. What is the charge on the ion?

9\. • **The Electric Potential of the Earth** The Earth has a vertical electric field with a magnitude of approximately 100 V/m near its surface. What is the magnitude of the potential difference between a point on the ground and a point on the same level as the top of the Washington Monument (555 ft high)?

10\. •• A uniform electric field with a magnitude of 6350 N/C points in the positive x direction. Find the change in electric potential energy when a +12.5-μC charge is moved 5.50 cm in **(a)** the positive x direction, **(b)** the negative x direction, and **(c)** the positive y direction.

11\. ••**IP** A spark plug in a car has electrodes separated by a gap of 0.025 in. To create a spark and ignite the air-fuel mixture in the engine, an electric field of 3.0×10^6 V/m is required in the gap. **(a)** What potential difference must be applied to the spark plug to initiate a spark? **(b)** If the separation between electrodes is increased, does the required potential difference increase, decrease, or stay the same? Explain. **(c)** Find the potential difference for a separation of 0.050 in.

12\. •• A uniform electric field with a magnitude of 1200 N/C points in the negative x direction, as shown in Figure 20–20.



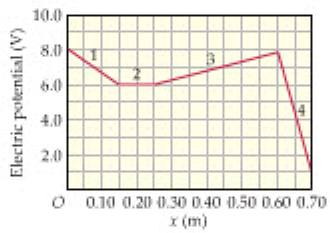
▲ **FIGURE 20–20** Problems 12 and 21

(a) What is the difference in electric potential, $\Delta V = V_B - V_A$, between points A and B? **(b)** What is the difference in electric potential, $\Delta V = V_B - V_C$, between points B and C? **(c)** What is the difference in electric potential, $\Delta V = V_C - V_A$, between points C and A? **(d)** From the information given in this problem, is it possible to determine the value of the electric potential at point A? If so, determine V_A; if not, explain why.

13\. •• **A Charged Battery** A typical 12-V car battery can deliver 7.5×10^5 C of charge. If the energy supplied by the battery could be converted entirely to kinetic energy, what speed would it give to a 1400-kg car that is initially at rest?

14\. •• **IP BIO The Sodium Pump** Living cells actively "pump" positive sodium ions (Na^+) from inside the cell to outside the cell. This process is referred to as pumping because work must be done on the ions to move them from the negatively charged inner surface of the membrane to the positively charged outer surface. Given that the electric potential is 0.070 V higher outside the cell than inside the cell, and that the cell membrane is 0.10 μm thick, **(a)** calculate the work that must be done (in joules) to move one sodium ion from inside the cell to outside. **(b)** If the thickness of the cell membrane is increased, does your answer to part (a) increase, decrease, or stay the same? Explain. (It is estimated that as much as 20% of the energy we consume in a resting state is used in operating this "sodium pump.")

15\. •• **IP** The electric potential of a system as a function of position along the x axis is given in Figure 20–21. **(a)** In which of the regions, 1, 2, 3, or 4, do you expect E_x to be greatest? In which region does E_x have its greatest magnitude? Explain. **(b)** Calculate the value of E_x in each of the regions, 1, 2, 3, and 4.



▲ **FIGURE 20–21** Problems 15 and 94

16\. ••• Points A and B have electric potentials of 332 V and 149 V, respectively. When an electron released from rest at point A arrives at point C, its kinetic energy is K_A. When the electron is released from rest at point B, however, its kinetic energy when it reaches point C is $K_B = 2K_A$. What are **(a)** the electric potential at point C and **(b)** the kinetic energy K_A?

SECTION 20–2 ENERGY CONSERVATION

17\. • **CE Predict/Explain** An electron is released from rest in a region of space with a nonzero electric field. **(a)** As the electron moves, does the electric potential energy of the system increase, decrease, or stay the same? **(b)** Choose the *best explanation* from among the following:

I. Because the electron has a negative charge its electric potential energy doesn't decrease, as one might expect, but increases instead.
II. As the electron begins to move, its kinetic energy increases. The increase in kinetic energy is equal to the decrease in the electric potential energy of the system.
III. The electron will move perpendicular to the electric field, and hence its electric potential energy will remain the same.

18. • Calculate the speed of **(a)** a proton and **(b)** an electron after each particle accelerates from rest through a potential difference of 275 V.

19. • The electrons in a TV picture tube are accelerated from rest through a potential difference of 25 kV. What is the speed of the electrons after they have been accelerated by this potential difference?

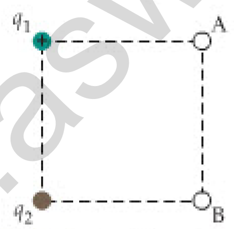
20. • Find the potential difference required to accelerate protons from rest to 10% of the speed of light. (At this point, relativistic effects start to become significant.)

21. •• **IP** A particle with a mass of 3.8 g and a charge of $+0.045\ \mu C$ is released from rest at point A in Figure 20–20. **(a)** In which direction will this charge move? **(b)** What speed will it have after moving through a distance of 5.0 cm? The electric field has a magnitude of 1200 N/C. **(c)** Suppose the particle continues moving for another 5.0 cm. Will its increase in speed for the second 5.0 cm be greater than, less than, or equal to its increase in speed in the first 5.0 cm? Explain.

22. •• A proton has an initial speed of 4.0×10^5 m/s. **(a)** What potential difference is required to bring the proton to rest? **(b)** What potential difference is required to reduce the initial speed of the proton by a factor of 2? **(c)** What potential difference is required to reduce the initial kinetic energy of the proton by a factor of 2?

SECTION 20–3 THE ELECTRIC POTENTIAL OF POINT CHARGES

23. • In Figure 20–22, it is given that $q_1 = +Q$. **(a)** What value must q_2 have if the electric potential at point A is to be zero? **(b)** With the value for q_2 found in part (a), is the electric potential at point B positive, negative, or zero? Explain.



▲ **FIGURE 20–22** Problems 23, 24, and 25

24. • **CE** The charge q_1 in Figure 20–22 has the value $+Q$. **(a)** What value must q_2 have if the electric potential at point B is to be zero? **(b)** With the value for q_2 found in part (a), is the electric potential at point A positive, negative, or zero? Explain.

25. • **CE** It is given that the electric potential is zero at the center of the square in Figure 20–22. **(a)** If $q_1 = +Q$, what is the value of the charge q_2? **(b)** Is the electric potential at point A positive, negative, or zero? Explain. **(c)** Is the electric potential at point B positive, negative, or zero? Explain.

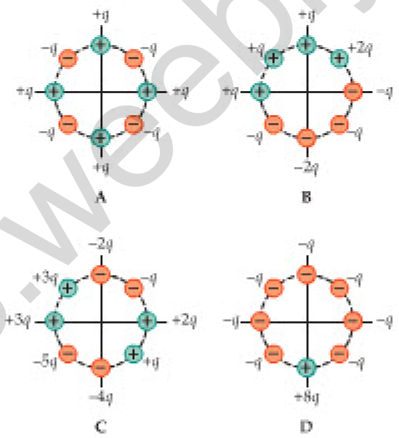
26. • The electric potential 1.1 m from a point charge q is 2.8×10^4 V. What is the value of q?

27. • A point charge of $-7.2\ \mu C$ is at the origin. What is the electric potential at **(a)** (3.0 m, 0); **(b)** (−3.0 m, 0); and **(c)** (3.0 m, −3.0 m)?

28. • **The Bohr Atom** The hydrogen atom consists of one electron and one proton. In the Bohr model of the hydrogen atom, the electron orbits the proton in a circular orbit of radius 0.529×10^{-10} m. What is the electric potential due to the proton at the electron's orbit?

29. • How far must the point charges $q_1 = +7.22\ \mu C$ and $q_2 = -26.1\ \mu C$ be separated for the electric potential energy of the system to be −126 J?

30. •• **CE** Four different arrangements of point charges are shown in Figure 20–23. In each case the charges are the same distance from the origin. Rank the four arrangements in order of increasing electric potential at the origin, taking the potential at infinity to be zero. Indicate ties where appropriate.



▲ **FIGURE 20–23** Problem 30

31. •• **IP** Point charges $+4.1\ \mu C$ and $-2.2\ \mu C$ are placed on the x axis at (11 m, 0) and (−11 m, 0), respectively. **(a)** Sketch the electric potential on the x axis for this system. **(b)** Your sketch should show one point on the x axis between the two charges where the potential vanishes. Is this point closer to the $+4.1$-μC charge or closer to the -2.2-μC charge? Explain. **(c)** Find the point referred to in part (b).

32. •• **IP** **(a)** In the previous problem, find the point to the left of the negative charge where the electric potential vanishes. **(b)** Is the electric field at the point found in part (a) positive, negative, or zero? Explain.

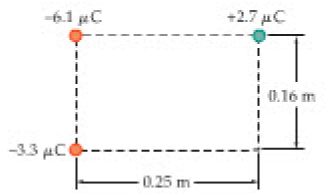
33. •• A dipole is formed by point charges $+3.6\ \mu C$ and $-3.6\ \mu C$ placed on the x axis at (0.25 m, 0) and (−0.25 m, 0), respectively. **(a)** Sketch the electric potential on the x axis for this system. **(b)** At what positions on the x axis does the potential have the value 7.5×10^5 V?

34. •• A charge of $3.05\ \mu C$ is held fixed at the origin. A second charge of $3.05\ \mu C$ is released from rest at the position (1.25 m, 0.570 m). **(a)** If the mass of the second charge is 2.16 g, what is its speed when it moves infinitely far from the origin? **(b)** At what distance from the origin does the second charge attain half the speed it will have at infinity?

35. •• **IP** A charge of $20.2\ \mu C$ is held fixed at the origin. **(a)** If a -5.25-μC charge with a mass of 3.20 g is released from rest at the position (0.925 m, 1.17 m), what is its speed when it is halfway to the origin? **(b)** Suppose the -5.25-μC charge is released from rest at the point $x = \frac{1}{2}(0.925\text{ m})$ and $y = \frac{1}{2}(1.17\text{ m})$. When it is halfway to the origin, is its speed greater than, less than, or equal to the speed found in part (a)? Explain. **(c)** Find the speed of the charge for the situation described in part (b).

36. •• A charge of $-2.205\ \mu C$ is located at (3.055 m, 4.501 m), and a charge of $1.800\ \mu C$ is located at (−2.533 m, 0). **(a)** Find the electric potential at the origin. **(b)** There is one point on the line connecting these two charges where the potential is zero. Find this point.

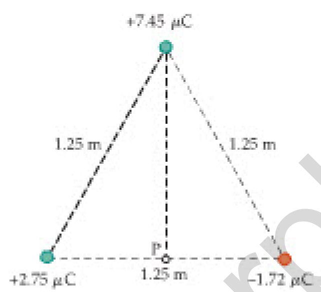
37. •• **IP** Figure 20–24 shows three charges at the corners of a rectangle. **(a)** How much work must be done to move the $+2.7\text{-}\mu C$ charge to infinity? **(b)** Suppose, instead, that we move the $-6.1\text{-}\mu C$ charge to infinity. Is the work required in this case greater than, less than, or the same as when we moved the $+2.7\text{-}\mu C$ charge to infinity? Explain. **(c)** Calculate the work needed to move the $-6.1\text{-}\mu C$ charge to infinity.



▲ **FIGURE 20–24** Problems 37 and 38

38. •• How much work must be done to move the three charges in Figure 20–24 infinitely far from one another?

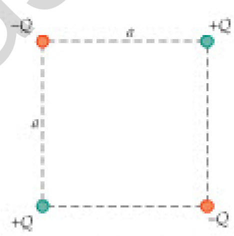
39. •• **(a)** Find the electric potential at point P in Figure 20–25. **(b)** Suppose the three charges shown in Figure 20–25 are held in place. A fourth charge, with a charge of $+6.11\ \mu C$ and a mass of 4.71 g, is released from rest at point P. What is the speed of the fourth charge when it has moved infinitely far away from the other three charges?



▲ **FIGURE 20–25** Problems 39 and 91

40. ••• A square of side a has a charge $+Q$ at each corner. What is the electric potential energy of this system of charges?

41. ••• A square of side a has charges $+Q$ and $-Q$ alternating from one corner to the next, as shown in Figure 20–26. Find the electric potential energy for this system of charges.



▲ **FIGURE 20–26** Problems 41 and 100

SECTION 20–4 EQUIPOTENTIAL SURFACES AND THE ELECTRIC FIELD

42. • **CE Predict/Explain** A positive charge is moved from one location on an equipotential to another point on the same equipotential. **(a)** Is the work done on the charge positive, negative, or zero? **(b)** Choose the *best explanation* from among the following:
   I. The electric field is perpendicular to an equipotential, therefore the work done in moving along an equipotential is zero.
   II. Because the charge is positive the work done on it is also positive.
   III. It takes negative work to keep the positive charge from accelerating as it moves along the equipotential.

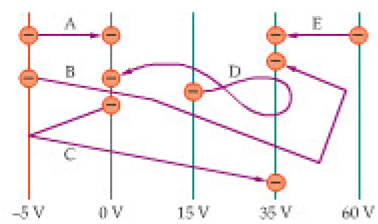
43. • **CE Predict/Explain** **(a)** Is the electric potential at point 1 in Figure 20–19 greater than, less than, or equal to the electric potential at point 3? **(b)** Choose the *best explanation* from among the following:
   I. The electric field lines point to the right, indicating that the electric potential is greater at point 3 than at point 1.
   II. The value of the electric potential is large where the electric field lines are close together, and small where they are widely spaced. Therefore, the electric potential is the same at points 1 and 3.
   III. The electric potential decreases as we move in the direction of the electric field, as shown in Figure 20–3. Therefore, the electric potential is greater at point 1 than at point 3.

44. • **CE Predict/Explain** Imagine sketching a large number of equipotential surfaces in Figure 20–19, with a constant difference in electric potential between adjacent surfaces. **(a)** Would the equipotentials at point 2 be more closely spaced, be less closely spaced, or have the same spacing as equipotentials at point 1? **(b)** Choose the *best explanation* from among the following:
   I. When electric field lines are close together, the corresponding equipotentials are far apart.
   II. Equipotential surfaces, by definition, always have equal spacing between them.
   III. The electric field is more intense at point 2 than at point 1, which means the equipotential surfaces are more closely spaced in that region.

45. • Two point charges are on the x axis. Charge 1 is $+q$ and is located at $x = -1.0$ m; charge 2 is $-2q$ and is located at $x = 1.0$ m. Make sketches of the equipotential surfaces for this system **(a)** out to a distance of about 2.0 m from the origin and **(b)** far from the origin. In each case, indicate the direction in which the potential increases.

46. • Two point charges are on the x axis. Charge 1 is $+q$ and is located at $x = -1.0$ m; charge 2 is $+2q$ and is located at $x = 1.0$ m. Make sketches of the equipotential surfaces for this system **(a)** out to a distance of about 2.0 m from the origin and **(b)** far from the origin. In each case, indicate the direction in which the potential increases.

47. •• **CE** Figure 20–27 shows a series of equipotentials in a particular region of space, and five different paths along which an electron is moved. **(a)** Does the electric field in this region point to the right, to the left, up, or down? Explain. **(b)** For each path, indicate whether the work done on the electron by the electric field is positive, negative, or zero. **(c)** Rank the paths in order of increasing amount of work done on the electron by the electric field. Indicate ties where appropriate. **(d)** Is the electric field near path A greater than, less than, or equal to the electric field near path E? Explain.

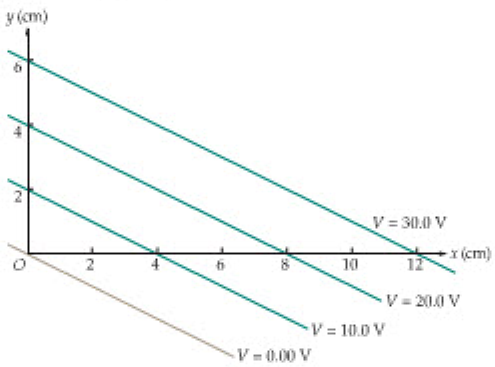


▲ **FIGURE 20–27** Problem 47

48. •• **IP** Consider a region in space where a uniform electric field $E = 6500$ N/C points in the negative x direction. **(a)** What is the orientation of the equipotential surfaces? Explain. **(b)** If you

move in the positive x direction, does the electric potential increase or decrease? Explain. **(c)** What is the distance between the +14-V and the +16-V equipotentials?

49. •• A given system has the equipotential surfaces shown in Figure 20–28. **(a)** What are the magnitude and direction of the electric field? **(b)** What is the shortest distance one can move to undergo a change in potential of 5.00 V?



▲ **FIGURE 20–28** Problems 49 and 92

SECTION 20–5 CAPACITORS AND DIELECTRICS

50. • A 0.40-μF capacitor is connected to a 9.0-V battery. How much charge is on each plate of the capacitor?

51. • It is desired that 5.8 μC of charge be stored on each plate of a 3.2-μF capacitor. What potential difference is required between the plates?

52. • To operate a given flash lamp requires a charge of 32 μC. What capacitance is needed to store this much charge in a capacitor with a potential difference between its plates of 9.0 V?

53. •• A parallel-plate capacitor is made from two aluminum-foil sheets, each 6.3 cm wide and 5.4 m long. Between the sheets is a Teflon strip of the same width and length that is 0.035 mm thick. What is the capacitance of this capacitor? (The dielectric constant of Teflon is 2.1.)

54. •• A parallel-plate capacitor is constructed with circular plates of radius 0.056 m. The plates are separated by 0.25 mm, and the space between the plates is filled with a dielectric with dielectric constant κ. When the charge on the capacitor is 1.2 μC the potential difference between the plates is 750 V. Find the value of the dielectric constant, κ.

55. •• **IP** A parallel-plate capacitor has plates with an area of 0.012 m^2 and a separation of 0.88 mm. The space between the plates is filled with a dielectric whose dielectric constant is 2.0. **(a)** What is the potential difference between the plates when the charge on the capacitor plates is 4.7 μC? **(b)** Will your answer to part (a) increase, decrease, or stay the same if the dielectric constant is increased? Explain. **(c)** Calculate the potential difference for the case where the dielectric constant is 4.0.

56. •• **IP** Consider a parallel-plate capacitor constructed from two circular metal plates of radius R. The plates are separated by a distance of 1.5 mm. **(a)** What radius must the plates have if the capacitance of this capacitor is to be 1.0 μF? **(b)** If the separation between the plates is increased, should the radius of the plates be increased or decreased to maintain a capacitance of 1.0 μF? Explain. **(c)** Find the radius of the plates that gives a capacitance of 1.0 μF for a plate separation of 3.0 mm.

57. •• A parallel-plate capacitor has plates of area 3.45×10^{-4} m^2. What plate separation is required if the capacitance is to be 1630 pF? Assume that the space between the plates is filled with **(a)** air or **(b)** paper.

58. •• **IP** A parallel-plate capacitor filled with air has plates of area 0.0066 m^2 and a separation of 0.45 mm. **(a)** Find the magnitude of the charge on each plate when the capacitor is connected to a 12-V battery. **(b)** Will your answer to part (a) increase, decrease, or stay the same if the separation between the plates is increased? Explain. **(c)** Calculate the magnitude of the charge on the plates if the separation is 0.90 mm.

59. •• Suppose that after walking across a carpeted floor you reach for a doorknob and just before you touch it a spark jumps 0.50 cm from your finger to the knob. Find the minimum voltage needed between your finger and the doorknob to generate this spark.

60. •• **(a)** What plate area is required if an air-filled, parallel-plate capacitor with a plate separation of 2.6 mm is to have a capacitance of 22 pF? **(b)** What is the maximum voltage that can be applied to this capacitor without causing dielectric breakdown?

61. •• **Lightning** As a crude model for lightning, consider the ground to be one plate of a parallel-plate capacitor and a cloud at an altitude of 550 m to be the other plate. Assume the surface area of the cloud to be the same as the area of a square that is 0.50 km on a side. **(a)** What is the capacitance of this capacitor? **(b)** How much charge can the cloud hold before the dielectric strength of the air is exceeded and a spark (lightning) results?

62. ••• A parallel-plate capacitor is made from two aluminum-foil sheets, each 3.00 cm wide and 10.0 m long. Between the sheets is a mica strip of the same width and length that is 0.0225 mm thick. What is the maximum charge that can be stored in this capacitor? (The dielectric constant of mica is 5.4, and its dielectric strength is 1.00×10^8 V/m.)

SECTION 20–6 ELECTRICAL ENERGY STORAGE

63. • Calculate the work done by a 3.0-V battery as it charges a 7.8-μF capacitor in the flash unit of a camera.

64. • **BIO Defibrillator** An automatic external defibrillator (AED) delivers 125 J of energy at a voltage of 1050 V. What is the capacitance of this device?

65. •• **IP BIO Cell Membranes** The membrane of a living cell can be approximated by a parallel-plate capacitor with plates of area 4.75×10^{-9} m^2, a plate separation of 8.5×10^{-9} m, and a dielectric with a dielectric constant of 4.5. **(a)** What is the energy stored in such a cell membrane if the potential difference across it is 0.0725 V? **(b)** Would your answer to part (a) increase, decrease, or stay the same if the thickness of the cell membrane is increased? Explain.

66. •• A 0.22-μF capacitor is charged by a 1.5-V battery. After being charged, the capacitor is connected to a small electric motor. Assuming 100% efficiency, **(a)** to what height can the motor lift a 5.0-g mass? **(b)** What initial voltage must the capacitor have if it is to lift a 5.0-g mass through a height of 1.0 cm?

67. •• Find the electric energy density between the plates of a 225-μF parallel-plate capacitor. The potential difference between the plates is 345 V, and the plate separation is 0.223 mm.

68. •• What electric field strength would store 17.5 J of energy in every 1.00 mm^3 of space?

69. •• An electronic flash unit for a camera contains a capacitor with a capacitance of 890 μF. When the unit is fully charged and ready for operation, the potential difference between the capacitor plates is 330 V. **(a)** What is the magnitude of the charge on each plate of the fully charged capacitor? **(b)** Find the energy stored in the "charged-up" flash unit.

70. ••• A parallel-plate capacitor has plates with an area of 405 cm^2 and an air-filled gap between the plates that is 2.25 mm thick. The capacitor is charged by a battery to 575 V and then is disconnected from the battery. (a) How much energy is stored in the capacitor? (b) The separation between the plates is now increased to 4.50 mm. How much energy is stored in the capacitor now? (c) How much work is required to increase the separation of the plates from 2.25 mm to 4.50 mm? Explain your reasoning.

GENERAL PROBLEMS

71. • **CE** A proton is released from rest in a region of space with a nonzero electric field. As the proton moves, does it experience an increasing or decreasing electric potential? Explain.

72. • **CE Predict/Explain** A proton is released from rest in a region of space with a nonzero electric field. **(a)** As the proton moves, does the electric potential energy of the system increase, decrease, or stay the same? **(b)** Choose the *best explanation* from among the following:

I. As the proton begins to move, its kinetic energy increases. The increase in kinetic energy is equal to the decrease in the electric potential energy of the system.

II. Because the proton has a positive charge, its electric potential energy will always increase.

III. The proton will move perpendicular to the electric field, and hence its electric potential energy will remain the same.

73. • **CE** In the Bohr model of the hydrogen atom, a proton and an electron are separated by a constant distance r. **(a)** Would the electric potential energy of the system increase, decrease, or stay the same if the electron is replaced with a proton? Explain. **(b)** Suppose, instead, that the proton is replaced with an electron. Would the electric potential energy of the system increase, decrease, or stay the same? Explain.

74. • **CE** The plates of a parallel-plate capacitor have constant charges of $+Q$ and $-Q$. Do the following quantities increase, decrease, or remain the same as the separation of the plates is increased? **(a)** The electric field between the plates; **(b)** the potential difference between the plates; **(c)** the capacitance; **(d)** the energy stored in the capacitor.

75. • **CE** A parallel-plate capacitor is connected to a battery that maintains a constant potential difference V between the plates. If the plates of the capacitor are pulled farther apart, do the following quantities increase, decrease, or remain the same? **(a)** The electric field between the plates; **(b)** the charge on the plates; **(c)** the capacitance; **(d)** the energy stored in the capacitor.

76. • **CE** The plates of a parallel-plate capacitor have constant charges of $+Q$ and $-Q$. Do the following quantities increase, decrease, or remain the same as a dielectric is inserted between the plates? **(a)** The electric field between the plates; **(b)** the potential difference between the plates; **(c)** the capacitance; **(d)** the energy stored in the capacitor.

77. • **CE** A parallel-plate capacitor is connected to a battery that maintains a constant potential difference V between the plates. If a dielectric is inserted between the plates of the capacitor, do the following quantities increase, decrease, or remain the same? **(a)** The electric field between the plates; **(b)** the charge on the plates; **(c)** the capacitance; **(d)** the energy stored in the capacitor.

78. • Find the difference in electric potential, $\Delta V = V_B - V_A$, between the points A and B for the following cases: **(a)** The electric field does 0.052 J of work as you move a +5.7-μC charge from A to B. **(b)** The electric field does −0.052 J of work as you move a −5.7-μC charge from A to B. **(c)** You perform 0.052 J of work as you slowly move a +5.7-μC charge from A to B.

79. • The separation between the plates of a parallel-plate capacitor is doubled and the area of the plates is halved. How is the capacitance affected?

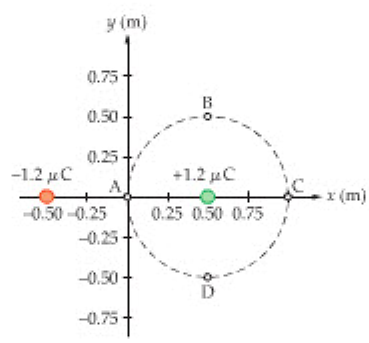
80. • A parallel-plate capacitor is connected to a battery that maintains a constant potential difference between the plates. If the spacing between the plates is doubled, how is the magnitude of charge on the plates affected?

81. •• **CE** Two point charges are placed on the x axis. The charge $+2q$ is at $x = 1.5$ m, and the charge $-q$ is at $x = -1.5$ m. **(a)** There is a point on the x axis between the two charges where the electric potential is zero. Where is this point? **(b)** The electric potential also vanishes at a point in one of the following regions: region 1, x between 1.5 m and 5.0 m; region 2, x between −1.5 m and −3.0 m; region 3, x between −3.5 m and −5.0 m. Identify the appropriate region. **(c)** Find the value of x referred to in part (b).

82. •• A charge of 24.5 μC is located at (4.40 m, 6.22 m), and a charge of −11.2 μC is located at (−4.50 m, 6.75 m). What charge must be located at (2.23 m, −3.31 m) if the electric potential is to be zero at the origin?

83. •• **The Bohr Model** In the Bohr model of the hydrogen atom (see Problem 28) what is the smallest amount of work that must be done on the electron to move it from its circular orbit, with a radius of 0.529×10^{-10} m, to an infinite distance from the proton? This value is referred to as the ionization energy of hydrogen.

84. •• **IP** A +1.2-μC charge and a −1.2-μC charge are placed at (0.50 m, 0) and (−0.50 m, 0), respectively. **(a)** In **Figure 20–29**, at which of the points A, B, C, or D is the electric potential smallest in value? At which of these points does it have its greatest value? Explain. **(b)** Calculate the electric potential at points A, B, C, and D.



▲ **FIGURE 20–29** Problems 84, 85, and 103

85. •• Repeat Problem 84 for the case where both charges are +1.2 μC.

86. •• How much work is required to bring three protons, initially infinitely far apart, to a configuration where each proton is 1.5×10^{-15} m from the other two? (This is a typical separation for protons in a nucleus.)

87. •• A point charge $Q = +87.1\ \mu$C is held fixed at the origin. A second point charge, with mass $m = 0.0576$ kg and charge $q = -2.87\ \mu$C, is placed at the location (0.323 m, 0). **(a)** Find the electric potential energy of this system of charges. **(b)** If the second charge is released from rest, what is its speed when it reaches the point (0.121 m, 0)?

88. •• **Electron Escape Speed** An electron is at rest just above the surface of a sphere with a radius of 2.7 mm and a uniformly distributed positive charge of 1.8×10^{-15} C. Like a rocket blasting off from the Earth, the electron is given an initial speed v_e radially outward from the sphere. If the electron coasts to infinity, where its kinetic energy drops to zero, what is the escape speed, v_e?

89. •• **Quark Model of the Neutron** According to the quark model of fundamental particles, neutrons—the neutral particles in an atom's nucleus—are composed of three quarks. Two of these quarks are "down" quarks, each with a charge of $-e/3$; the third quark is an "up" quark, with a charge of $+2e/3$. This gives the neutron a net charge of zero. What is the electric potential energy of these three quarks, assuming they are equidistant from one another, with a separation distance of 1.3×10^{-15} m? (Quarks are discussed in Chapter 32.)

90. •• A parallel-plate capacitor is charged to an electric potential of 325 V by moving 3.75×10^{16} electrons from one plate to the other. How much work is done in charging the capacitor?

91. •• **IP** The three charges shown in Figure 20–25 are held in place as a fourth charge, q, is brought from infinity to the point P. The charge q starts at rest at infinity and is also at rest when it is placed at the point P. **(a)** If q is a positive charge, is the work required to bring it to the point P positive, negative, or zero? Explain. **(b)** Find the value of q if the work needed to bring it to point P is -1.3×10^{-11} J.

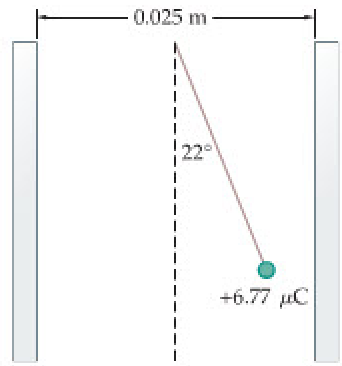
92. •• **(a)** In Figure 20–28 we see that the electric potential increases by 10.0 V as one moves 4.00 cm in the positive x direction. Use this information to calculate the x component of the electric field. (Ignore the y direction for the moment.) **(b)** Apply the same reasoning as in part (a) to calculate the y component of the electric field. **(c)** Combine the results from parts (a) and (b) to find the magnitude and direction of the electric field for this system.

93. •• **IP BIO Electric Catfish** The electric catfish (*Malapterurus electricus*) is an aggressive fish, 1.0 m in length, found today in tropical Africa (and depicted in Egyptian hieroglyphics). The catfish is capable of generating jolts of electricity up to 350 V by producing a positively charged region of muscle near the head and a negatively charged region near the tail. **(a)** For the same amount of charge, can the catfish generate a higher voltage by separating the charge from one end of its body to the other, as it does, or from one side of the body to the other? Explain. **(b)** Estimate the charge generated at each end of a catfish as follows: Treat the catfish as a parallel-plate capacitor with plates of area 1.8×10^{-2} m^2, separation 1.0 m, and filled with a dielectric with a dielectric constant $\kappa = 95$.

94. •• As a $+6.2$-μC charge moves along the x axis from $x = 0$ to $x = 0.70$ m, the electric potential it experiences is shown in Figure 20–21. Find the approximate location(s) of the charge when its electric potential energy is **(a)** 2.6×10^{-5} J and **(b)** 4.3×10^{-5} J.

95. •• **IP Computer Keyboards** Many computer keyboards operate on the principle of capacitance. As shown in Figure 20–16, each key forms a small parallel-plate capacitor whose separation is reduced when the key is depressed. **(a)** Does depressing a key increase or decrease its capacitance? Explain. **(b)** Suppose the plates for each key have an area of 47.5 mm^2 and an initial separation of 0.550 mm. In addition, let the dielectric have a dielectric constant of 3.75. If the circuitry of the computer can detect a change in capacitance of 0.425 pF, what is the minimum distance a key must be depressed to be detected?

96. •• **IP** A point charge of mass 0.081 kg and charge $+6.77$ μC is suspended by a thread between the vertical parallel plates of a parallel-plate capacitor, as shown in Figure 20–30. **(a)** If the charge deflects to the right of vertical, as indicated in the figure, which of the two plates is at the higher electric potential? **(b)** If the angle of deflection is 22°, and the separation between the plates is 0.025 m, what is the potential difference between the plates?



▲ **FIGURE 20–30** Problems 96 and 101

97. •• **BIO Cell Membranes and Dielectrics** Many cells in the body have a cell membrane whose inner and outer surfaces carry opposite charges, just like the plates of a parallel-plate capacitor. Suppose a typical cell membrane has a thickness of 8.1×10^{-9} m, and its inner and outer surfaces carry charge densities of -0.58×10^{-3} C/m^2 and $+0.58 \times 10^{-3}$ C/m^2, respectively. In addition, assume that the material in the cell membrane has a dielectric constant of 5.5. **(a)** Find the direction and magnitude of the electric field within the cell membrane. **(b)** Calculate the potential difference between the inner and outer walls of the membrane, and indicate which wall of the membrane has the higher potential.

98. •• Long, long ago, on a planet far, far away, a physics experiment was carried out. First, a 0.250-kg ball with zero net charge was dropped from rest at a height of 1.00 m. The ball landed 0.552 s later. Next, the ball was given a net charge of 7.75 μC and dropped in the same way from the same height. This time the ball fell for 0.680 s before landing. What is the electric potential at a height of 1.00 m above the ground on this planet, given that the electric potential at ground level is zero? (Air resistance can be ignored.)

99. •• **Rutherford's Planetary Model of the Atom** In 1911, Ernest Rutherford developed a planetary model of the atom, in which a small positively charged nucleus is orbited by electrons. The model was motivated by an experiment carried out by Rutherford and his graduate students, Geiger and Marsden. In this experiment, they fired alpha particles with an initial speed of 1.75×10^7 m/s at a thin sheet of gold. (Alpha particles are obtained from certain radioactive decays. They have a charge of $+2e$ and a mass of 6.64×10^{-27} kg.) How close can the alpha particles get to a gold nucleus (charge $= +79e$), assuming the nucleus remains stationary? (This calculation sets an upper limit on the size of the gold nucleus. See Chapter 31 for further details.)

100. ••• **IP (a)** One of the $-Q$ charges in Figure 20–26 is given an outward "kick" that sends it off with an initial speed v_0 while the other three charges are held at rest. If the moving charge has a mass m, what is its speed when it is infinitely far from the other charges? **(b)** Suppose the remaining $-Q$ charge, which also has a mass m, is now given the same initial speed, v_0. When it is infinitely far away from the two $+Q$ charges, is its speed greater than, less than, or the same as the speed found in part (a)? Explain.

101. ••• Figure 20–30 shows a charge $q = +6.77$ μC with a mass $m = 0.071$ kg suspended by a thread of length $L = 0.022$ m between the plates of a capacitor. **(a)** Plot the electric potential energy of the system as a function of the angle θ the thread makes with the vertical. (The electric field between the plates has a magnitude $E = 4.16 \times 10^4$ V/m.) **(b)** Repeat part (a) for

the case of the gravitational potential energy of the system. **(c)** Show that the total potential energy of the system (electric plus gravitational) is a minimum when the angle θ satisfies the equilibrium condition for the charge, $\tan\theta = qE/mg$. This relation implies that $\theta = 22°$.

102. ••• The electric potential a distance r from a point charge q is 2.70×10^4 V. One meter farther away from the charge the potential is 6140 V. Find the charge q and the initial distance r.

103. ••• Referring to Problem 84, calculate and plot the electric potential on the circle centered at (0.50 m, 0). Give your results in terms of the angle θ, defined as follows: θ is the angle measured counterclockwise from a vertex at the center of the circle, with $\theta = 0$ at point C.

104. ••• When the potential difference between the plates of a capacitor is increased by 3.25 V, the magnitude of the charge on each plate increases by 13.5 μC. What is the capacitance of this capacitor?

105. ••• The electric potential a distance r from a point charge q is 155 V, and the magnitude of the electric field is 2240 N/C. Find the values of q and r.

PASSAGE PROBLEMS

BIO The Electric Eel

Of the many unique and unusual animals that inhabit the rainforests of South America, including howler monkeys, freshwater dolphins, and deadly piranhas, one stands out because of its mastery of electricity. The electric eel (*Electrophorus electricus*), one of the few creatures on Earth able to generate, store, and discharge electricity, can deliver a powerful series of high-voltage discharges reaching 650 V. These jolts of electricity are so strong, in fact, that electric eels have been known to topple a horse crossing a stream 20 feet away, and to cause respiratory paralysis, cardiac arrhythmia, and even death in humans.

Though similar in appearance to an eel, the electric "eel" is actually more closely related to catfish. They are found primarily in the Amazon and Orinoco river basins, where they navigate the slow-moving, muddy water with low-voltage electric organ discharges (EOD), saving the high-voltage EODs for stunning prey and defending against predators. Obligate air breathers, electric eels obtain about 80% of their oxygen by gulping air at the water's surface. Even so, they are able to attain lengths of 2.5 m and a mass of 20 kg.

The organs that produce the eel's electricity take up most of its body, and consist of thousands of modified muscle cells—called electroplaques—stacked together like the cells in a battery. Each electroplaque is capable of generating a voltage of 0.15 V, and together they produce a positive charge near the head of the eel and a negative charge near its tail.

106. • Electric eels produce an electric field within their body. In which direction does the electric field point?

A. toward the head B. toward the tail
C. upward D. downward

107. • As a rough approximation, consider an electric eel to be a parallel-plate capacitor with plates of area 1.8×10^{-2} m^2 separated by 2.0 m and filled with a dielectric whose dielectric constant is $\kappa = 95$. What is the capacitance of the eel in this model?

A. 8.0×10^{-14} F B. 7.6×10^{-12} F
C. 1.5×10^{-11} F D. 9.3×10^{-8} F

108. • In terms of the parallel-plate model of the previous problem, how much charge does an electric eel generate at each end of its body when it produces a voltage of 650 V?

A. 1.2×10^{-14} C B. 5.2×10^{-11} C
C. 4.9×10^{-9} C D. 6.1×10^{-5} C

109. • How much energy is stored by an electric eel when it is charged up to 650 V. Use the same parallel-plate model discussed in the previous two problems.

A. 1.8×10^{-17} J B. 1.7×10^{-8} J
C. 1.6×10^{-6} J D. 2.0×10^{-2} J

INTERACTIVE PROBLEMS

110. •• IP **Referring to Example 20–3** Suppose the charge $-2q$ at $x = 1.00$ m is replaced with a charge $-3q$, where $q = 4.11 \times 10^{-9}$ C. The charge $+q$ is at the origin. **(a)** Is the electric potential positive, negative, or zero at the point $x = 0.333$ m? Explain. **(b)** Find the point between $x = 0$ and $x = 1.00$ m where the electric potential vanishes. **(c)** Is there a point in the region $x < 0$ where the electric potential passes through zero?

111. •• **Referring to Example 20–3** Suppose we can change the location of the charge $-2q$ on the x axis. The charge $+q$ (where $q = 4.11 \times 10^{-9}$C) is still at the origin. **(a)** Where should the charge $-2q$ be placed to ensure that the electric potential vanishes at $x = 0.500$ m? **(b)** With the location of $-2q$ found in part (a), where does the electric potential pass through zero in the region $x < 0$?

112. •• IP **Referring to Example 20–3** Suppose the charge $+q$ at the origin is replaced with a charge $+5q$, where $q = 4.11 \times 10^{-9}$ C. The charge $-2q$ is still at $x = 1.00$ m. **(a)** Is there a point in the region $x < 0$ where the electric potential passes through zero? **(b)** Find the location between $x = 0$ and $x = 1.00$ m where the electric potential passes through zero. **(c)** Find the location in the region $x > 1.00$ m where the electric potential passes through zero.

21 Electric Current and Direct-Current Circuits

A battery is a device that uses chemical energy to separate positive and negative charges, producing a potential difference between its terminals. In this case, the chemical energy comes from reactions that take place between the metal electrodes and the acid in the lemon juice. The potential difference causes a current to flow in the wires, which is measured by the attached meter. This chapter explores simple electric circuits, like the one seen here, and shows how to analyze more complex ones as well.

As you read this paragraph, your heart is pumping blood through the arteries and veins in your body. In a way, your heart is acting like a battery in an electric circuit: A battery causes electric charge to flow through a closed circuit of wires; your heart causes blood to flow through your body. Just as the flow of blood is important to life, the flow of electric charge is of central importance to modern technology. In this chapter we consider some of the basic properties of moving electric charges, and we apply these results to simple electric circuits.

21–1 Electric Current

A flow of electric charge from one place to another is referred to as an **electric current.** Often, the charge is carried by electrons moving through a metal wire. Though the analogy should not be pushed too far, the electrons flowing through a wire are much like water molecules flowing through a garden hose or blood cells flowing through an artery.

To be specific, suppose a charge ΔQ flows past a given point in a wire in a time Δt. In such a case, we say that the electric current, I, in the wire is:

Definition of Electric Current, *I*

$$I = \frac{\Delta Q}{\Delta t} \qquad 21\text{–}1$$

SI unit: coulomb per second, C/s = ampere, A

The unit of current, the ampere (A) or *amp* for short, is named for the French physicist André-Marie Ampère (1775–1836) and is defined simply as 1 coulomb per second:

$$1\ \text{A} = 1\ \text{C/s}$$

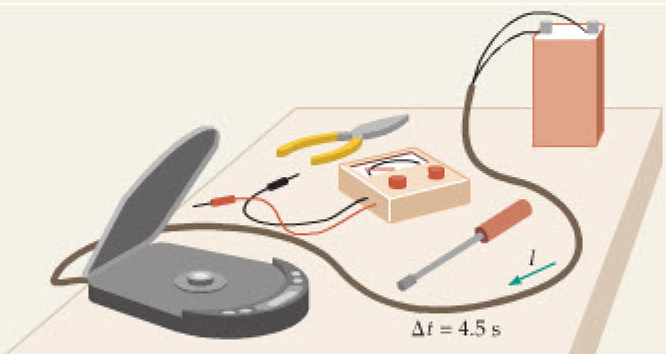
The following Example shows that the number of electrons involved in typical electric circuits, with currents of roughly an amp, is extremely large—not unlike the large number of water molecules flowing through a garden hose.

EXAMPLE 21–1 MEGA BLASTER

The disk drive in a portable CD player is connected to a battery that supplies it with a current of 0.22 A. How many electrons pass through the drive in 4.5 s?

PICTURE THE PROBLEM

Our sketch shows the CD drive with a current $I = 0.22$ A flowing through it. Also indicated is the time $\Delta t = 4.5$ s during which the current flows.



STRATEGY

Since we know both the current, I, and the length of time, Δt, we can use the definition of current, $I = \Delta Q/\Delta t$, to find the charge, ΔQ, that flows through the player. Once we know the charge, the number of electrons, N, is simply ΔQ divided by the magnitude of the electron's charge: $N = \Delta Q/e$.

SOLUTION

1. Calculate the charge, ΔQ, that flows through the drive: $\Delta Q = I\,\Delta t = (0.22\ \text{A})(4.5\ \text{s}) = 0.99\ \text{C}$
2. Divide by the magnitude of the electron's charge, e, to find the number of electrons: $N = \dfrac{\Delta Q}{e} = \dfrac{0.99\ \text{C}}{1.60 \times 10^{-19}\ \text{C/electron}} = 6.2 \times 10^{18}$ electrons

INSIGHT

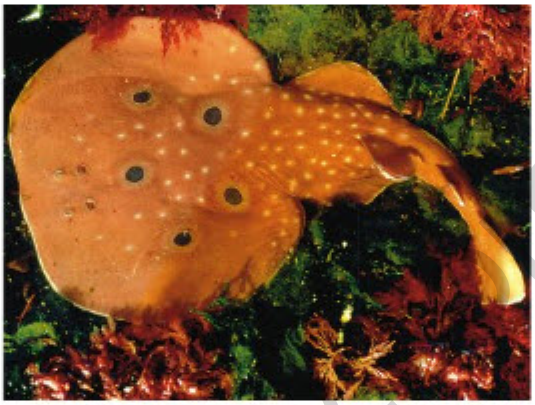
Thus, even a modest current flowing for a brief time corresponds to the transport of an extremely large number of electrons.

PRACTICE PROBLEM

How long must this current last if 7.5×10^{18} electrons are to flow through the disk drive? [**Answer:** 5.5 s]

Some related homework problems: Problem 1, Problem 2

When charge flows through a closed path and returns to its starting point, we refer to the closed path as an *electric circuit*. In this chapter we consider **direct-current circuits,** also known as dc circuits, in which the current always flows in the same direction. Circuits with currents that periodically reverse their direction

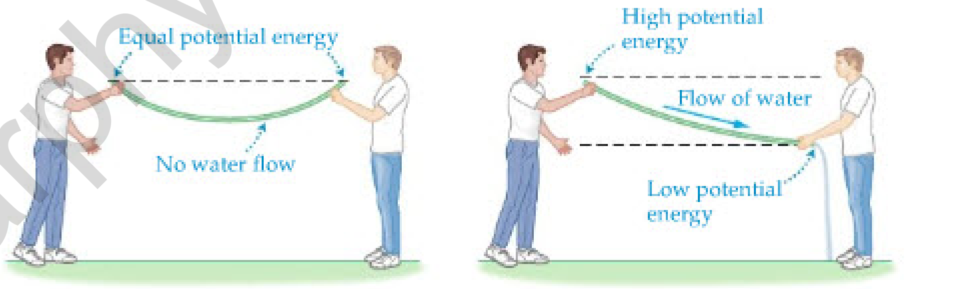


▲ Electric currents are not confined to the wires in our houses and machines, but occur in nature as well. A lightning bolt is simply an enormous, brief current. It flows when the difference in electric potential between cloud and ground (or cloud and cloud) becomes so great that it exceeds the breakdown strength of air. An enormous quantity of charge then leaps across the gap in a fraction of a second. Some organisms, such as this electric torpedo ray, have internal organic "batteries" that can produce significant electric potentials. The resulting current is used to stun their prey.

are referred to as **alternating-current circuits.** These AC circuits are considered in detail in Chapter 24.

Batteries and Electromotive Force

Although electrons move rather freely in metal wires, they do not flow unless the wires are connected to a source of electrical energy. A close analogy is provided by water in a garden hose. Imagine that you and a friend each hold one end of a garden hose filled with water. If the two ends are held at the same level, as in Figure 21–1 (a), the water does not flow. If, however, one end is raised above the other, as in Figure 21–1 (b), water flows from the high end—where the gravitational potential energy is high—to the low end.



(a) Equal potential energy → no flow (b) Water flows from high potential energy to low

▶ **FIGURE 21–1 Water flow as an analogy for electric current**

Water can flow quite freely through a garden hose, but if both ends are at the same level **(a)**, there is no flow. If the ends are held at different levels **(b)**, the water flows from the region where the gravitational potential energy is high to the region where it is low.

A **battery** performs a similar function in an electric circuit. To put it simply, a battery uses chemical reactions to produce a difference in electric potential between its two ends, or **terminals.** The symbol for a battery is ⊣⊢ . The terminal corresponding to a high electric potential is denoted by a +, and the terminal corresponding to a low electric potential is denoted by a −. When the battery is connected to a circuit, electrons move in a closed path from the negative terminal of the battery, through the circuit, and back to the positive terminal.

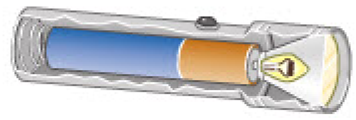
A simple example of an electrical system is shown in Figure 21–2 (a), where we show a battery, a switch, and a lightbulb as they might be connected in a flashlight. In the schematic circuit shown in Figure 21–2 (b), the switch is "open"—creating an **open circuit**—which means there is no closed path through which the electrons can flow. As a result, the light is off. When the switch is closed—which "closes" the circuit—charge flows around the circuit, causing the light to glow.

A mechanical analog to the flashlight circuit is shown in Figure 21–3. In this system, the person raising the water from a low to a high level is analogous to the battery, the paddle wheel is analogous to the lightbulb, and the water is analogous to the electric charge. Notice that the person does work in raising the water; later, as the water falls to its original level, it does work on the external world by turning the paddle wheel.

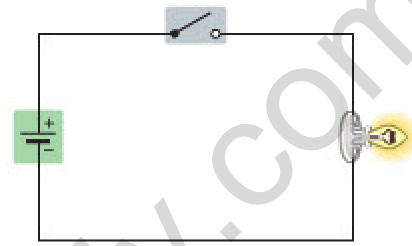
When a battery is disconnected from a circuit and carries no current, the difference in electric potential between its terminals is referred to as its *electromotive force*, or *emf* ($\mathcal{E}$). It follows that the units of emf are the same as those of electric potential, namely, volts. Clearly, then, the electromotive force is not really a force at all. Instead, the emf determines the amount of work a battery does to move a certain amount of charge around a circuit (like the person lifting water in Figure 21–3). To be specific, the magnitude of the work done by a battery of emf $\mathcal{E}$ as a charge ΔQ moves from one of its terminals to the other is given by Equation 20–2:

$$W = \Delta Q \mathcal{E}$$

We apply this relation to a flashlight circuit in the following Active Example.



(a) A simple flashlight



(b) Circuit diagram for flashlight

▲ **FIGURE 21–2 The flashlight: A simple electric circuit**

(a) A simple flashlight, consisting of a battery, a switch, and a lightbulb. **(b)** When the switch is in the open position, the circuit is "broken," and no charge can flow. When the switch is closed, electrons flow through the circuit and the light glows.

ACTIVE EXAMPLE 21–1 OPERATING A FLASHLIGHT: FIND THE CHARGE AND THE WORK

A battery with an emf of 1.5 V delivers a current of 0.44 A to a flashlight bulb for 64 s (see Figure 21–2). Find **(a)** the charge that passes through the circuit and **(b)** the work done by the battery.

SOLUTION *(Test your understanding by performing the calculations indicated in each step.)*

Part (a)

1. Use the definition of current, $I = \Delta Q/\Delta t$, to find the charge that flows through the circuit: $\Delta Q = 28$ C

Part (b)

2. Once we know ΔQ, we can use $W = \Delta Q\mathcal{E}$ to find the work: $W = 42$ J

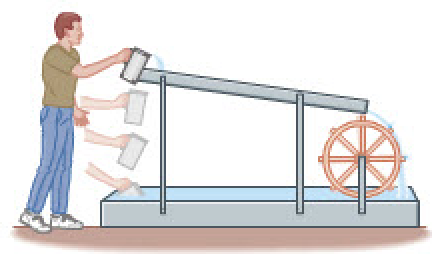
INSIGHT

Note that the more charge a battery moves through a circuit, the more work it does. Similarly, the greater the emf, the greater the work. We can see, then, that a car battery that operates at 12 volts and delivers several amps of current does much more work than a flashlight battery—as expected.

YOUR TURN

How long must the flashlight battery operate to do 150 J of work?

(Answers to **Your Turn** *problems are given in the back of the book.)*



▲ **FIGURE 21–3 A mechanical analog to the flashlight circuit**

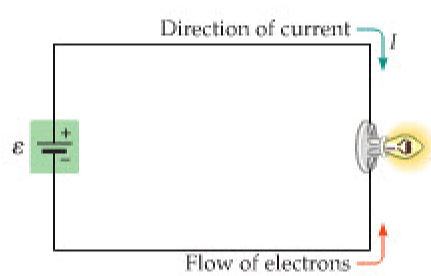
The person lifting the water corresponds to the battery in Figure 21–2, and the paddle wheel corresponds to the lightbulb.

The emf of a battery is the potential difference it can produce between its terminals under ideal conditions. In real batteries, however, there is always some internal loss, leading to a potential difference that is less than the ideal value. In fact, the greater the current flowing through a battery, the greater the reduction in potential difference between its terminals, as we shall see in Section 21–4. Only when the current is zero can a real battery produce its full emf. Because most batteries have relatively small internal losses, we shall treat batteries as ideal—always producing a potential difference precisely equal to $\mathcal{E}$—unless specifically stated otherwise.

When we draw an electric circuit, it will be useful to draw an arrow indicating the flow of current. By convention, the direction of the current arrow is given in terms of a positive test charge, in much the same way that the direction of the electric field is determined:

> The direction of the current in an electric circuit is the direction in which a *positive* test charge would move.

Of course, in typical circuits the charges that flow are actually *negatively* charged electrons. As a result, the flow of electrons and the current arrow point in opposite directions, as indicated in Figure 21–4. Notice that a positive charge will flow from



▲ **FIGURE 21–4 Direction of current and electron flow**

In the flashlight circuit, electrons flow from the negative terminal of the battery to the positive terminal. The direction of the current, I, is just the opposite: from the positive terminal to the negative terminal.

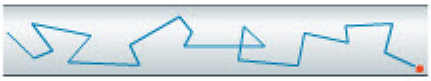


FIGURE 21–5 Path of an electron in a wire

Typical path of an electron as it bounces off atoms in a metal wire. Because of the tortuous path the electron follows, its average velocity is rather small.

a region of high electric potential, near the positive terminal of the battery, to a region of low electric potential, near the negative terminal, as one would expect.

Finally, surprising as it may seem, electrons move rather slowly through a typical wire. They suffer numerous collisions with the atoms in the wire, and hence their path is rather tortuous and roundabout, as indicated in Figure 21–5. Like a car contending with a series of speed bumps, the electron's average speed, or **drift speed** as it is often called, is limited by the repeated collisions—in fact, their average speed is commonly about 10^{-4} m/s. Thus, if you switch on the headlights of a car, for example, an electron leaving the battery will take about an hour to reach the lightbulb, yet the lights seem to shine from the instant the switch is turned on. How is this possible?

The answer is that as an electron begins to move away from the battery, it exerts a force on its neighbors, causing them to move in the same general direction and, in turn, to exert a force on their neighbors, and so on. This process generates a propagating influence that travels through the wire at nearly the speed of light. The phenomenon is analogous to a bowling ball hitting one end of a line of balls; the effect of the colliding ball travels through the line at roughly the speed of sound, although the individual balls have very little displacement. Similarly, the electrons in a wire move with a rather small average velocity as they collide with and bounce off the atoms making up the wire, whereas the influence they have on one another races ahead and causes the light to shine.

21–2 Resistance and Ohm's Law

Electrons flow through metal wires with relative ease. In the ideal case, nothing about the wire would prevent their free motion. Real wires, however, under normal conditions, always affect the electrons to some extent, creating a **resistance** to their motion in much the same way that friction slows a box sliding across the floor.

In order to cause electrons to move against the resistance of a wire, it is necessary to apply a potential difference between its ends. For a wire with constant resistance, R, the potential difference, V, necessary to create a current, I, is given by Ohm's law:

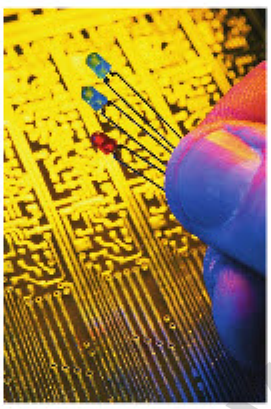
Ohm's Law

$$V = IR \qquad (21\text{–}2)$$

SI unit: volt, V

Ohm's law is named for the German physicist Georg Simon Ohm (1789–1854).

It should be noted at the outset that Ohm's law is not a law of nature but more on the order of a useful rule of thumb—like Hooke's law for springs or the ideal-gas laws that approximate the behavior of real gases. Materials that are well approximated by Ohm's law are said to be "ohmic" in their behavior; they show a simple linear relationship between the voltage applied to them and the current that results. In particular, if one plots current versus voltage for an ohmic material, the result is a straight line, with a constant slope equal to $1/R$. Nonohmic materials, on the other hand, have more complex relationships between voltage and current. A plot of current versus voltage for a nonohmic material is nonlinear; hence, the material does not have a constant resistance. (As an example, see Problem 9.) It is precisely these "nonlinearities," however, that can make such materials so useful in the construction of electronic devices, including the ubiquitous light-emitting diodes (LEDs).



A light-emitting diode (LED) is a relatively small, nonohmic device (top), but groups of LEDs can be used to form displays of practically any size (bottom). Because LEDs are extremely durable, and predicted to last 20 years or more, they are becoming the illumination of choice in high-reliability applications such as traffic lights, emergency exit signs, and brake lights. You'll probably see several on your way home today.

Solving Ohm's law for the resistance, we find

$$R = \frac{V}{I}$$

From this expression it is clear that the units of resistance are volts per amp. In particular, we define 1 volt per amp to be 1 **ohm.** Letting the Greek letter omega, Ω, designate the ohm, we have

$$1\ \Omega = 1\ \text{V/A}$$

A device for measuring resistance is called an ohmmeter. We describe the operation of an ohmmeter in Section 21–8.

EXERCISE 21–1

A potential difference of 24 V is applied to a 150-Ω resistor. How much current flows through the resistor?

SOLUTION

Solving Ohm's law for the current, I, we find

$$I = \frac{V}{R} = \frac{24\ \text{V}}{150\ \Omega} = \frac{24\ \text{V}}{150\ \text{V/A}} = 0.16\ \text{A}$$

In an electric circuit a resistor is signified by a zigzag line:  . The straight lines in a circuit indicate ideal wires of zero resistance. To indicate the resistance of a real wire or device, we simply include a resistor of the appropriate value in the circuit.

Resistivity

Suppose you have a piece of wire of length L and cross-sectional area A. The resistance of this wire depends on the particular material from which it is constructed. If the wire is made of copper, for instance, its resistance will be less than if it is made from iron. The quantity that characterizes the resistance of a given material is its **resistivity**, ρ. For a wire of given dimensions, the greater the resistivity, the greater the resistance.

The resistance of a wire also depends on its length and area. To understand the dependence on L and A, consider again the analogy of water flowing through a hose. If the hose is very long, the resistance it presents to the water will be correspondingly large, whereas a wider hose—one with a greater cross-sectional area—will offer less resistance to the water. After all, water flows more easily through a short fire hose than through a long soda straw; hence, the resistance of a hose—and similarly a piece of wire—should be proportional to L and inversely proportional to A; that is, proportional to (L/A).

Combining these observations, we can write the resistance of a wire of length L, area A, and resistivity ρ in the following way:

Definition of Resistivity, ρ

$$R = \rho\left(\frac{L}{A}\right) \qquad 21\text{–}3$$

Since the units of L are m and the units of A are m^2, it follows that the units of resistivity are $(\Omega \cdot \text{m})$. Typical values for ρ are given in Table 21–1. Notice the enormous range in values of ρ, with the resistivity of an insulator like rubber about 10^{21} times greater than the resistivity of a good conductor like silver.

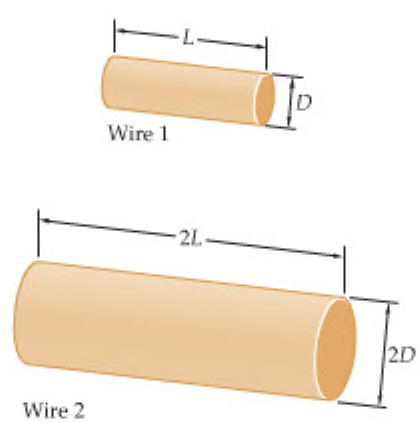
TABLE 21–1 Resistivities

Substance	Resistivity, ρ $(\Omega \cdot \text{m})$
Insulators	
Quartz (fused)	7.5×10^{17}
Rubber	$1 \text{ to } 100 \times 10^{13}$
Glass	$1 \text{ to } 10{,}000 \times 10^{9}$
Semiconductors	
Silicon*	0.10 to 60
Germanium*	0.001 to 0.5
Conductors	
Lead	22×10^{-8}
Iron	9.71×10^{-8}
Tungsten	5.6×10^{-8}
Aluminum	2.65×10^{-8}
Gold	2.20×10^{-8}
Copper	1.68×10^{-8}
Silver	1.59×10^{-8}

*The resistivity of a semiconductor varies greatly with the type and amount of impurities it contains. This property makes them particularly useful in electronic applications.

CONCEPTUAL CHECKPOINT 21–1 COMPARE THE RESISTANCE

Wire 1 has a length L and a circular cross section of diameter D. Wire 2 is constructed from the same material as wire 1 and has the same shape, but its length is $2L$, and its diameter is $2D$. Is the resistance of wire 2 **(a)** the same as that of wire 1, **(b)** twice that of wire 1, or **(c)** half that of wire 1?



REASONING AND DISCUSSION

First, the resistance of wire 1 is

$$R_1 = \rho\left(\frac{L}{A}\right) = \rho\frac{L}{(\pi D^2/4)}$$

Note that we have used the fact that the area of a circle of diameter D is $\pi D^2/4$. For wire 2 we replace L with $2L$ and D with $2D$:

$$R_2 = \rho\frac{2L}{[\pi(2D)^2/4]} = \left(\tfrac{1}{2}\right)\rho\frac{L}{(\pi D^2/4)} = \tfrac{1}{2}R_1$$

CONTINUED ON NEXT PAGE

Thus, increasing the length by a factor of 2 increases the resistance by a factor of 2; on the other hand, increasing the diameter by a factor of 2 increases the area, and decreases the resistance, by a factor of 4. Overall, then, the resistance of wire 2 is half that of wire 1.

ANSWER

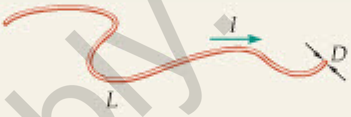
(c) The resistance of wire 2 is half that of wire 1; $R_2 = R_1/2$.

EXAMPLE 21–2 A CURRENT-CARRYING WIRE

A current of 1.82 A flows through a copper wire 1.75 m long and 1.10 mm in diameter. Find the potential difference between the ends of the wire. (The value of ρ for copper may be found in Table 21–1.)

PICTURE THE PROBLEM

The wire carries a current $I = 1.82$ A, and its total length L is 1.75 m. We assume that the wire has a circular cross section, with a diameter $D = 1.10$ mm.



STRATEGY

We know from Ohm's law that the potential difference associated with a current I and a resistance R is $V = IR$. We are given the current in the wire, but not the resistance. The resistance is easily determined, however, using $R = \rho(L/A)$ with $A = \pi D^2/4$. Thus, we first calculate R and then substitute the result into $V = IR$ to obtain the potential difference.

SOLUTION

1. Calculate the resistance of the wire:

$$R = \rho\left(\frac{L}{A}\right) = \rho\left(\frac{L}{\pi D^2/4}\right)$$

$$= (1.72 \times 10^{-8}\ \Omega \cdot \mathrm{m})\left[\frac{1.75\ \mathrm{m}}{\pi(0.00110\ \mathrm{m})^2/4}\right] = 0.0317\ \Omega$$

2. Multiply R by the current, I, to find the potential difference:

$$V = IR = (1.82\ \mathrm{A})(0.0317\ \Omega) = 0.0577\ \mathrm{V}$$

INSIGHT

Copper is an excellent conductor; therefore, both the resistance and the potential difference are quite small.

PRACTICE PROBLEM

What diameter of copper wire is needed for there to be a potential difference of 0.100 V? Assume that all other quantities remain the same. [**Answer:** 0.835 mm]

Some related homework problems: Problem 17, Problem 18

Temperature Dependence and Superconductivity

We know from everyday experience that a wire carrying an electric current can become warm—even quite hot, as in the case of a burner on a stove or the filament in an incandescent lightbulb. This follows from our earlier discussion of the fact that electrons collide with the atoms in a wire as they flow through an electric circuit. These collisions cause the atoms to jiggle with greater kinetic energy about their equilibrium positions. As a result, the temperature of the wire increases (see Section 17–2, and Equation 17–21 in particular). For example, the wire filament in an incandescent lightbulb can reach temperatures of roughly 2800 °C (in comparison, the surface of the Sun has a temperature of about 5500 °C), and the heating coil on a stove has a temperature of about 750 °C.

As a wire is heated, its resistivity tends to increase. This is because atoms that are jiggling more rapidly are more likely to collide with electrons and slow their progress through the wire. In fact, many metals show an approximately linear increase of ρ over a wide range of temperature. Once the dependence of ρ on T is known for a given material, the change in resistivity can be used as a means of measuring temperature.

REAL-WORLD PHYSICS
The bolometer

The first practical application of this principle was in a device known as the **bolometer.** Invented in 1880, the bolometer is an extremely sensitive thermometer that uses the temperature variation in the resistivity of platinum, nickel, or bismuth as a means of detecting temperature changes as small as 0.0001 C°. Soon after its invention, a bolometer was used to detect infrared radiation from the stars.

Some materials, like semiconductors, actually show a drop in resistivity as temperature is increased. This is because the resistivity of a semiconductor is strongly dependent on the number of electrons that are free to move about and conduct a current. As the temperature is increased in a semiconductor, more electrons are able to break free from their atoms, leading to an increased current and a reduced resistivity. Electronic devices incorporating such temperature-dependent semiconductors are known as **thermistors.** The digital fever thermometer so common in today's hospitals uses a thermistor to provide accurate measurements of a patient's temperature.

REAL-WORLD PHYSICS

Thermistors and fever thermometers

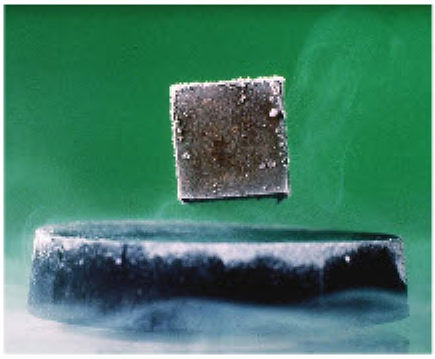
Since resistivity typically increases with temperature, it follows that if a wire is cooled below room temperature, its resistivity will *decrease*. Quite surprising, however, was a discovery made in the laboratory of Heike Kamerlingh-Onnes in 1911. Measuring the resistance of a sample of mercury at temperatures just a few degrees above absolute zero, researchers found that at about 4.2 K the resistance of the mercury suddenly dropped to zero—not just to a very small value, but to *zero*. At this temperature, we say that the mercury becomes **superconducting**, a hitherto unknown phase of matter. Since that time many different superconducting materials have been discovered, with various different **critical temperatures,** T_c, at which superconductivity begins. Today we know that superconductivity is a result of quantum effects (Chapter 30).

REAL-WORLD PHYSICS

Superconductors and high-temperature superconductivity

When a material becomes superconducting, a current can flow through it with absolutely no resistance. In fact, if a current is initiated in a superconducting ring of material, it will flow undiminished for as long as the ring is kept cool enough. In some cases, circulating currents have been maintained for years, with absolutely no sign of diminishing.

In 1986 a new class of superconductors was discovered that has zero resistance at temperatures significantly greater than those of any previously known superconducting materials. At the moment, the highest temperature at which superconductivity has been observed is about 125 K. Since the discovery of these "high-T_c" superconductors, hopes have been raised that it may one day be possible to produce room-temperature superconductors. The practical benefits of such a breakthrough, including power transmission with no losses, improved MRI scanners, and magnetically levitated trains, could be immense.



▲ When cooled below their critical temperature, superconductors not only lose their resistance to current flow but also exhibit new magnetic properties, such as repelling an external magnetic field. Here, a superconductor (bottom) levitates a small permanent magnet.

21–3 Energy and Power in Electric Circuits

When a charge ΔQ moves across a potential difference V, its electrical potential energy, U, changes by the amount

$$\Delta U = (\Delta Q)V$$

Recalling that power is the rate at which energy changes, $P = \Delta U/\Delta t$, we can write the electrical power as follows:

$$P = \frac{\Delta U}{\Delta t} = \frac{(\Delta Q)V}{\Delta t}$$

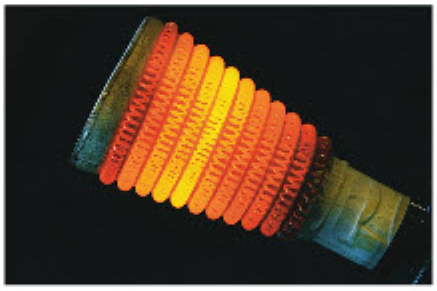
Since the electric current is given by $I = \Delta Q/\Delta t$, we have:

Electrical Power

$$P = IV \qquad 21\text{–}4$$

SI unit: watt, W

Thus, a current of 1 amp flowing across a potential difference of 1 V produces a power of 1 W.



▲ The heating element of an electric space heater is nothing more than a length of resistive wire coiled up for compactness. As electric current flows through the wire, the power it dissipates ($P = I^2R$) is converted to heat and light. The coils near the center are the hottest, and hence they glow with a higher-frequency, yellowish light.

EXERCISE 21–2

A handheld electric fan operates on a 3.00-V battery. If the power generated by the fan is 2.24 W, what is the current supplied by the battery?

SOLUTION

Solving $P = IV$ for the current, we obtain

$$I = \frac{P}{V} = \frac{2.24\text{ W}}{3.00\text{ V}} = 0.747\text{ A}$$

The expression $P = IV$ applies to any electrical system. In the special case of a resistor, the electrical power is dissipated in the form of heat. Applying Ohm's law ($V = IR$) to this case, we can write the power dissipated in a resistor as

$$P = IV = I(IR) = I^2R \qquad 21\text{–}5$$

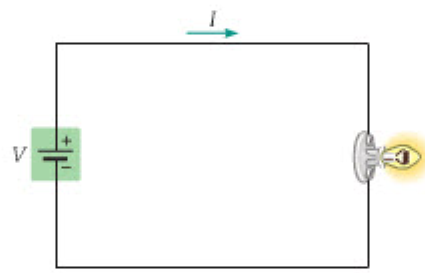
Similarly, using Ohm's law to solve for the current, $I = V/R$, we have

$$P = IV = \left(\frac{V}{R}\right)V = \frac{V^2}{R} \qquad 21\text{–}6$$

These relations also apply to incandescent lightbulbs, which are basically resistors that become hot enough to glow.

CONCEPTUAL CHECKPOINT 21–2 COMPARE LIGHTBULBS

A battery that produces a potential difference V is connected to a 5-W lightbulb. Later, the 5-W lightbulb is replaced with a 10-W lightbulb. **(a)** In which case does the battery supply more current? **(b)** Which lightbulb has the greater resistance?



REASONING AND DISCUSSION

a. To compare the currents, we need consider only the relation $P = IV$. Solving for the current yields $I = P/V$. When the voltage V is the same, it follows that the greater the power, the greater the current. In this case, then, the current in the 10-W bulb is twice the current in the 5-W bulb.

b. We now consider the relation $P = V^2/R$, which gives resistance in terms of voltage and power. In fact, $R = V^2/P$. Again, with V the same, it follows that the smaller the power, the greater the resistance. Thus, the resistance of the 5-W bulb is twice that of the 10-W bulb.

ANSWER

(a) When the battery is connected to the 10-W bulb, it delivers twice as much current as when it is connected to the 5-W bulb. **(b)** The 5-W bulb has twice as much resistance as the 10-W bulb.

On a microscopic level, the power dissipated by a resistor is the result of incessant collisions between electrons moving through the circuit and atoms making up the resistor. Specifically, the electric potential difference produced by the battery causes electrons to accelerate until they bounce off an atom of the resistor. At this point the electrons transfer energy to the atoms, causing them to jiggle more rapidly. The increased kinetic energy of the atoms is reflected in an increased temperature of the resistor (see Section 17–2). After each collision, the potential difference accelerates the electrons again and the process repeats—like a car bouncing through a series of speed bumps—resulting in a continuous transfer of energy from the electrons to the atoms.

EXAMPLE 21–3 HEATED RESISTANCE

A battery with an emf of 12 V is connected to a 545-Ω resistor. How much energy is dissipated in the resistor in 65 s?

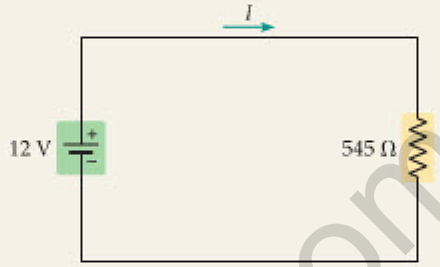
PICTURE THE PROBLEM

The circuit, consisting of a battery and a resistor, is shown in our sketch. We show the current flowing from the positive terminal of the 12-V battery, through the 545-Ω resistor, and into the negative terminal of the battery.

STRATEGY

We know that a current flowing through a resistor dissipates power (energy per time), which means that the energy it dissipates in a given time is simply the power multiplied by the time: $\Delta U = P\,\Delta t$. The time is given ($\Delta t = 65$ s), and the power can be found using $P = IV$, $P = I^2R$, or $P = V^2/R$. The last expression is most convenient in this case, because the problem statement gives us the voltage and resistance.

To summarize, we first calculate the power, then multiply by the time.



SOLUTION

1. Calculate the power dissipated in the resistor: $P = V^2/R = (12\text{ V})^2/(545\ \Omega) = 0.26\text{ W}$
2. Multiply the power by the time to find the energy dissipated: $\Delta U = P\,\Delta t = (0.26\text{ W})(65\text{ s}) = 17\text{ J}$

INSIGHT

The current in this circuit is $I = V/R = 0.022$ A. Using this result, we find that the power is $P = I^2R = IV = 0.26$ W, as expected.

PRACTICE PROBLEM

How much energy is dissipated in the resistor if the voltage is doubled to 24 V? [**Answer:** 4(17 J) = 68 J]

Some related homework problems: Problem 29, Problem 32

The battery testers now often built into battery packages (left) employ a tapered graphite strip. The narrow end (at bottom in the right-hand photo) has the highest resistance, and thus produces the most heat when a current flows through the strip. The heat is used to produce the display on the front that indicates the strength of the battery—if the current is sufficient to warm even the top of the strip, where the resistance is lowest, the battery is fresh.

REAL-WORLD PHYSICS

"Battery check" meters

A commonly encountered application of resistance heating is found in the "battery check" meters often included with packs of batteries. To operate one of these meters, you simply press the contacts on either end of the meter against the corresponding terminals of the battery to be checked. This allows a current to flow through the main working element of the meter—a tapered strip of graphite.

The reason the strip is tapered is to provide a variation in resistance. According to the relation given in Equation 21–3, $R = \rho(L/A)$, the smaller the cross-sectional area A of the strip, the larger the resistance R. It follows that the narrow end has a higher resistance than the wide end. Because the same current I flows through all parts of the strip, the power dissipated is expressed most conveniently in the form $P = I^2R$. It follows that at the narrow end of the strip, where R is largest, the heating due to the current will be the greatest. Pressing the meter against the terminals of the battery, then, results in an overall warming of the graphite strip, with the narrow end warmer than the wide end.

The final element in the meter is a thin layer of liquid crystal (similar to the material used in LCD displays) that responds to small increases in temperature. In particular, this liquid crystal is black and opaque at room temperature but transparent when heated slightly. The liquid crystal is placed in front of a colored background, which can be seen in those regions where the graphite strip is warm enough to make the liquid crystal transparent. If the battery is weak, only the narrow portion of the strip becomes warm enough, and the meter shows only a small stripe of color. A strong battery, on the other hand, heats the entire strip enough to make the liquid crystal transparent, resulting in a colored stripe the full length of the meter.

Energy Usage

When you get a bill from the local electric company, you will find the number of kilowatt-hours of electricity that you have used. Notice that a kilowatt-hour (kWh) has the units of energy:

$$1 \text{ kilowatt-hour} = (1000 \text{ W})(3600 \text{ s}) = (1000 \text{ J/s})(3600 \text{ s}) = 3.6 \times 10^6 \text{ J}$$

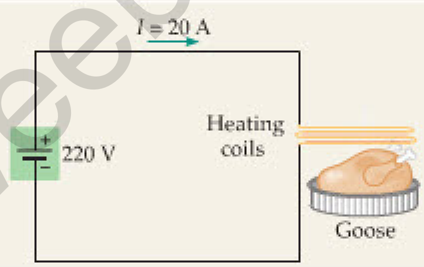
Thus, the electric company is charging for the amount of energy you use—as one would expect—and not for the rate at which you use it. The following Example considers the energy and monetary cost for a typical everyday situation.

EXAMPLE 21–4 YOUR GOOSE IS COOKED

A holiday goose is cooked in the kitchen oven for 4.00 h. Assume that the stove draws a current of 20.0 A, operates at a voltage of 220.0 V, and uses electrical energy that costs \$0.068 per kWh. How much does it cost to cook your goose?

PICTURE THE PROBLEM

We show a schematic representation of the stove cooking the goose in our sketch. The current in the circuit is 20.0 A, and the voltage difference across the heating coils is 220 V.



STRATEGY

The cost is simply the energy usage (in kWh) times the cost per kilowatt-hour (\$0.068). To find the energy used, we note that energy is power multiplied by time. The time is given, and the power associated with a current I and a voltage V is $P = IV$.

Thus, we find the power, multiply by the time, and then multiply by \$0.068 to find the cost.

SOLUTION

1. Calculate the power delivered to the stove: $P = IV = (20.0 \text{ A})(220.0 \text{ V}) = 4.40 \text{ kW}$
2. Multiply power by time to determine the total energy supplied to the stove during the 4.00 h of cooking: $\Delta U = P\,\Delta t = (4.40 \text{ kW})(4.00 \text{ h}) = 17.6 \text{ kWh}$
3. Multiply by the cost per kilowatt-hour to find the total cost of cooking: $\text{cost} = (17.6 \text{ kWh})(\$0.068/\text{kWh}) = \$1.20$

INSIGHT

Thus, your goose can be cooked for just over a dollar.

PRACTICE PROBLEM

If the voltage and current are reduced by a factor of 2 each, how long must the goose be cooked to use the same amount of energy? [**Answer:** $4(4.00 \text{ h}) = 16.0$ h. *Note:* You should be able to answer a question like this by referring to your previous solution, without repeating the calculation in detail.]

Some related homework problems: Problem 30, Problem 31

21–4 Resistors in Series and Parallel

Electric circuits often contain a number of resistors connected in various ways. In this section we consider simple circuits containing only resistors and batteries. For each type of circuit considered, we calculate the **equivalent resistance** produced by a group of individual resistors.

Resistors in Series

When resistors are connected one after the other, end to end, we say that they are in *series*. **Figure 21–6 (a)** shows three resistors, R_1, R_2, and R_3, connected in series. The three resistors acting together have the same effect—that is, they draw the same current—as a single resistor, referred to as the equivalent resistor, R_{eq}. This equivalence is illustrated in **Figure 21–6 (b)**. We now calculate the value of the equivalent resistance.

The first thing to notice about the circuit in Figure 21–6 (a) is that the same current I must flow through each of the resistors—there is no other place for the current to go. As a result, the potential differences across the three resistors are

$V_1 = IR_1$, $V_2 = IR_2$, and $V_3 = IR_3$, respectively. Since the total potential difference from point A to point B must be the emf of the battery, $\mathcal{E}$, it follows that

$$\mathcal{E} = V_1 + V_2 + V_3$$

Writing each of the potentials in terms of the current and resistance, we find

$$\mathcal{E} = IR_1 + IR_2 + IR_3 = I(R_1 + R_2 + R_3)$$

Now, let's compare this expression with the result we obtain for the equivalent circuit in Figure 21–6 (b). In this circuit, the potential difference across the battery is $V = IR_{eq}$. Since this potential must be the same as the emf of the battery, we have

$$\mathcal{E} = IR_{eq}$$

Comparing this expression with $\mathcal{E} = I(R_1 + R_2 + R_3)$, we see that the equivalent resistance is simply the sum of the individual resistances:

$$R_{eq} = R_1 + R_2 + R_3$$

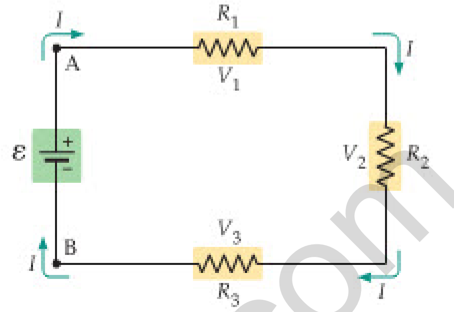
In general, for any number of resistors in series, the equivalent resistance is

Equivalent Resistance for Resistors in Series

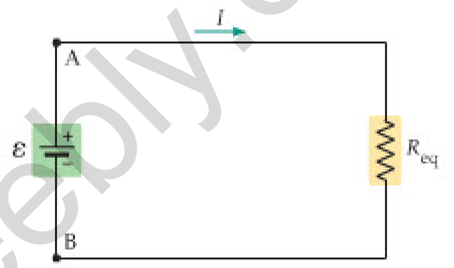
$$R_{eq} = R_1 + R_2 + R_3 + \cdots = \sum R \qquad \text{21–7}$$

SI unit: ohm, Ω

Note that the equivalent resistance is greater than the greatest resistance of any of the individual resistors. Connecting the resistors in series is like making a single resistor increasingly longer; as its length increases so does its resistance.



(a) Three resistors in series



(b) Equivalent resistance has the same current

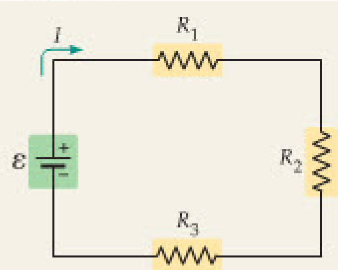
▲ **FIGURE 21–6 Resistors in series**
(a) Three resistors, R_1, R_2, and R_3, connected in series. Note that the same current I flows through each resistor. **(b)** The equivalent resistance, $R_{eq} = R_1 + R_2 + R_3$, has the same current I flowing through it as the current I in the original circuit.

EXAMPLE 21–5 THREE RESISTORS IN SERIES

A circuit consists of three resistors connected in series to a 24.0-V battery. The current in the circuit is 0.0320 A. Given that $R_1 = 250.0\ \Omega$ and $R_2 = 150.0\ \Omega$, find **(a)** the value of R_3 and **(b)** the potential difference across each resistor.

PICTURE THE PROBLEM

The circuit is shown in our sketch. Note that the 24.0-V battery delivers the same current, $I = 0.0320$ A, to each of the three resistors. This is the key characteristic of a series circuit.



STRATEGY

a. First, we can obtain the equivalent resistance of the circuit using Ohm's law (as in Equation 21–2); $R_{eq} = \mathcal{E}/I$. Since the resistors are in series, we also know that $R_{eq} = R_1 + R_2 + R_3$. We can solve this relation for the only unknown, R_3.

b. We can then calculate the potential difference across each resistor using Ohm's law, $V = IR$.

SOLUTION

Part (a)

1. Use Ohm's law to find the equivalent resistance of the circuit: $R_{eq} = \dfrac{\mathcal{E}}{I} = \dfrac{24.0\ \text{V}}{0.0320\ \text{A}} = 7.50 \times 10^2\ \Omega$

2. Set R_{eq} equal to the sum of the individual resistances, and solve for R_3:

$$R_{eq} = R_1 + R_2 + R_3$$
$$R_3 = R_{eq} - R_1 - R_2$$
$$= 7.50 \times 10^2\ \Omega - 250.0\ \Omega - 150.0\ \Omega = 3.50 \times 10^2\ \Omega$$

Part (b)

3. Use Ohm's law to determine the potential difference across R_1: $V_1 = IR_1 = (0.0320\ \text{A})(250.0\ \Omega) = 8.00\ \text{V}$

4. Find the potential difference across R_2: $V_2 = IR_2 = (0.0320\ \text{A})(150.0\ \Omega) = 4.80\ \text{V}$

5. Find the potential difference across R_3: $V_3 = IR_3 = (0.0320\ \text{A})(3.50 \times 10^2\ \Omega) = 11.2\ \text{V}$

CONTINUED ON NEXT PAGE

CONTINUED FROM PREVIOUS PAGE

INSIGHT

Note that the greater the resistance, the greater the potential difference. In addition, the sum of the individual potential differences is 8.00 V + 4.80 V + 11.2 V = 24.0 V, as expected.

PRACTICE PROBLEM

Find the power dissipated in each resistor. **[Answer:** $P_1 = 0.256$ W, $P_2 = 0.154$ W, $P_3 = 0.358$ W]

Some related homework problems: Problem 43, Problem 44

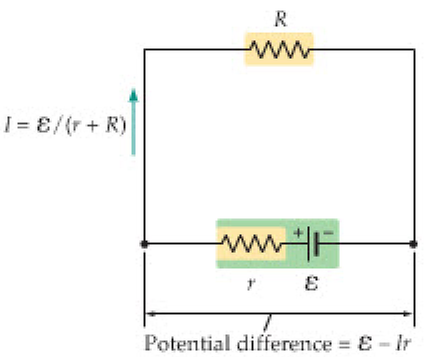


FIGURE 21–7 The internal resistance of a battery

Real batteries always dissipate some energy in the form of heat. These losses can be modeled by a small "internal" resistance, r, within the battery. As a result, the potential difference between the terminals of a real battery is less than its ideal emf, $\mathcal{E}$. For example, if a battery produces a current I, the potential difference between its terminals is $\mathcal{E} - Ir$. In the case shown here, a battery is connected in series with the resistor R. Instead of producing the current $I = \mathcal{E}/R$, as in the ideal case, it produces the current $I = \mathcal{E}/(r + R)$.

An everyday example of resistors in series is the **internal resistance,** r, of a battery. As was mentioned in Section 21–1, real batteries have internal losses that cause the potential difference between their terminals to be less than $\mathcal{E}$ and to depend on the current in the battery. The simplest way to model a real battery is to imagine it to consist of an ideal battery of emf $\mathcal{E}$ in series with an internal resistance r, as shown in Figure 21–7. If this battery is then connected to an external resistance, R, the equivalent resistance of the circuit is $r + R$. As a result, the current flowing through the circuit is $I = \mathcal{E}/(r + R)$, and the potential difference between the terminals of the battery is $\mathcal{E} - Ir$. Thus, we see that the potential difference produced by the battery is less than $\mathcal{E}$ by an amount that is proportional to the current I. Only in the limit of zero current, or zero internal resistance, will the battery produce its full emf. (See Problems 51, 54, 116, and 121 for examples of batteries with internal resistance.)

Another application of resistors in series is the three-way lightbulb circuit shown in Figure 21–8. In this circuit, the two resistors represent two different filaments within a single bulb that are connected to a constant potential difference V. At the "high" setting, the lower-resistance filament, R_1, is connected to the electrical outlet via terminals A and B, and the brightest light is obtained ($P = V^2/R$). At the "middle" setting, the higher-resistance filament, R_2, is connected to the outlet via terminals B and C, resulting in a dimmer light. Finally, at the "low" setting, both filaments are connected in series via terminals A and C. This setting gives the greatest equivalent resistance, and thus the lowest light output.

An alternative method of producing a three-way lightbulb is to connect the resistors in parallel. This will be discussed in the next subsection.

REAL-WORLD PHYSICS

Three-way lightbulbs

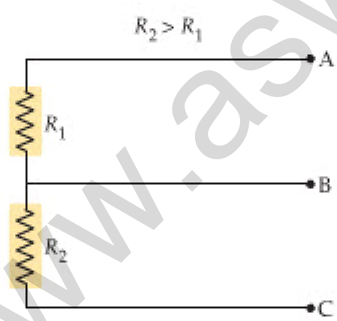


FIGURE 21–8 A three-way bulb

The circuit diagram for a three-way lightbulb. For the brightest light, terminals A and B are connected to the household electrical line, so the current passes through the low-resistance filament R_1. For intermediate brightness, terminals B and C are used, so the current passes through the higher-resistance filament R_2. For the lowest light output, terminals A and C are used, so the current passes through both R_1 and R_2 in series.

Resistors in Parallel

Resistors are in *parallel* when they are connected across the same potential difference, as in Figure 21–9 (a). In a case like this, the current has parallel paths through which it can flow. As a result, the total current in the circuit, I, is equal to the sum of the currents through each of the three resistors:

$$I = I_1 + I_2 + I_3$$

Since the potential difference is the same for each of the resistors, it follows that the currents flowing through them are as follows:

$$I_1 = \frac{\mathcal{E}}{R_1}, \quad I_2 = \frac{\mathcal{E}}{R_2}, \quad I_3 = \frac{\mathcal{E}}{R_3}$$

Summing these three currents, we find

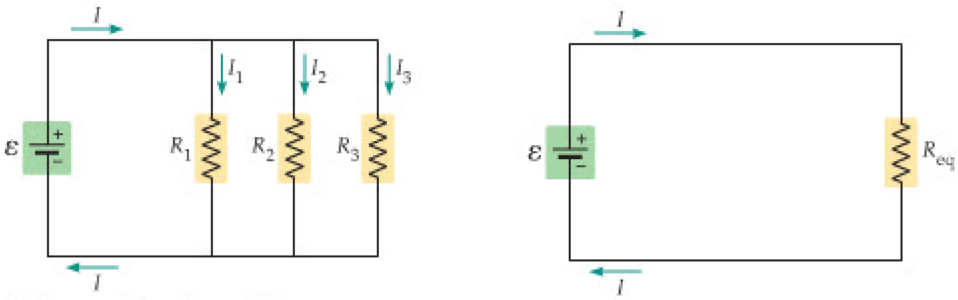
$$I = \frac{\mathcal{E}}{R_1} + \frac{\mathcal{E}}{R_2} + \frac{\mathcal{E}}{R_3} = \mathcal{E}\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right) \qquad 21\text{–}8$$

Now, in the equivalent circuit shown in Figure 21–9 (b), Ohm's law gives $\mathcal{E} = IR_{eq}$, or

$$I = \mathcal{E}\left(\frac{1}{R_{eq}}\right) \qquad 21\text{–}9$$

Comparing Equations 21–8 and 21–9, we find that the equivalent resistance for three resistors in parallel is

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$



(a) Three resistors in parallel

(b) Equivalent resistance has the same current

FIGURE 21–9 Resistors in parallel

(a) Three resistors, R_1, R_2, and R_3, connected in parallel. Note that each resistor is connected across the same potential difference $\mathcal{E}$. **(b)** The equivalent resistance, $1/R_{eq} = 1/R_1 + 1/R_2 + 1/R_3$, has the same current flowing through it as the total current I in the original circuit.

In general, for any number of resistors in parallel, we have:

Equivalent Resistance for Resistors in Parallel

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots = \sum \frac{1}{R} \qquad \text{21–10}$$

SI unit: ohm, Ω

PROBLEM-SOLVING NOTE

The Equivalent Resistance of Resistors in Parallel

After summing the inverse of resistors in parallel, remember to take one more inverse at the end of your calculation to find the equivalent resistance.

As a simple example, consider a circuit with two identical resistors R connected in parallel. The equivalent resistance in this case is given by

$$\frac{1}{R_{eq}} = \frac{1}{R} + \frac{1}{R} = \frac{2}{R}$$

Solving for R_{eq}, we find $R_{eq} = \frac{1}{2}R$. If we connect three such resistors in parallel, the corresponding result is

$$\frac{1}{R_{eq}} = \frac{1}{R} + \frac{1}{R} + \frac{1}{R} = \frac{3}{R}$$

In this case, $R_{eq} = \frac{1}{3}R$. Thus, the more resistors we connect in parallel, the smaller the equivalent resistance. Each time we add a new resistor in parallel with the others, we give the battery a new path through which current can flow—analogous to opening an additional lane of traffic on a busy highway. Stated another way, giving the current multiple paths through which it can flow is equivalent to using a wire with a greater cross-sectional area. From either point of view, the fact that more current flows with the same potential difference means that the equivalent resistance has been reduced.

Finally, if any one of the resistors in a parallel connection is equal to zero, the equivalent resistance is also zero. This situation is referred to as a **short circuit**, and is illustrated in Figure 21–10. In this case, all of the current flows through the path of zero resistance.

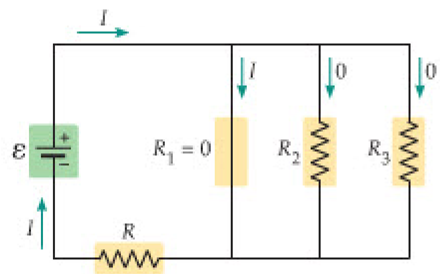


FIGURE 21–10 A short circuit

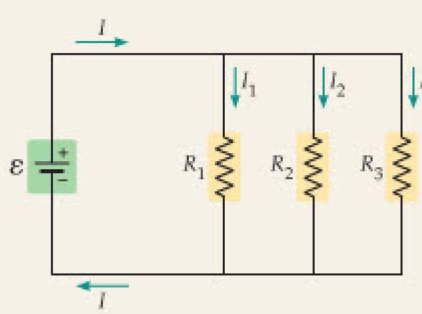
If one of the resistors in parallel with others is equal to zero, all the current flows through that portion of the circuit, giving rise to a short circuit. In this case, resistors R_2 and R_3 are "shorted out," and the current in the circuit is $I = \mathcal{E}/R$.

EXAMPLE 21–6 THREE RESISTORS IN PARALLEL

Consider a circuit with three resistors, $R_1 = 250.0\ \Omega$, $R_2 = 150.0\ \Omega$, and $R_3 = 350.0\ \Omega$, connected in parallel with a 24.0-V battery. Find **(a)** the total current supplied by the battery and **(b)** the current through each resistor.

PICTURE THE PROBLEM

The accompanying sketch indicates the parallel connection of the resistors with the battery. Notice that each of the resistors experiences precisely the same potential difference; namely, the 24.0 V produced by the battery. This is the feature that characterizes parallel connections.



STRATEGY

a. We can find the total current from $I = \mathcal{E}/R_{eq}$, where $1/R_{eq} = 1/R_1 + 1/R_2 + 1/R_3$.

b. For each resistor, the current is given by Ohm's law, $I = \mathcal{E}/R$.

CONTINUED ON NEXT PAGE

CONTINUED FROM PREVIOUS PAGE

SOLUTION

Part (a)

1. Find the equivalent resistance of the circuit:

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

$$= \frac{1}{250.0\ \Omega} + \frac{1}{150.0\ \Omega} + \frac{1}{350.0\ \Omega} = 0.01352\ \Omega^{-1}$$

$$R_{eq} = (0.01352\ \Omega^{-1})^{-1} = 73.96\ \Omega$$

2. Use Ohm's law to find the total current:

$$I = \frac{\mathcal{E}}{R_{eq}} = \frac{24.0\ \text{V}}{73.96\ \Omega} = 0.325\ \text{A}$$

Part (b)

3. Calculate I_1 using $I_1 = \mathcal{E}/R_1$ with $\mathcal{E} = 24.0$ V:

$$I_1 = \frac{\mathcal{E}}{R_1} = \frac{24.0\ \text{V}}{250.0\ \Omega} = 0.0960\ \text{A}$$

4. Repeat the preceding calculation for resistors 2 and 3:

$$I_2 = \frac{\mathcal{E}}{R_2} = \frac{24.0\ \text{V}}{150.0\ \Omega} = 0.160\ \text{A}$$

$$I_3 = \frac{\mathcal{E}}{R_3} = \frac{24.0\ \text{V}}{350.0\ \Omega} = 0.0686\ \text{A}$$

INSIGHT

As expected, the smallest resistor, R_2, carries the greatest current. The three currents combined yield the total current, as they must; that is, $I_1 + I_2 + I_3 = 0.0960\ \text{A} + 0.160\ \text{A} + 0.0686\ \text{A} = 0.325\ \text{A} = I$.

PRACTICE PROBLEM

Find the power dissipated in each resistor. [**Answer:** $P_1 = 2.30$ W, $P_2 = 3.84$ W, $P_3 = 1.65$ W]

Some related homework problems: Problem 45, Problem 46

In comparing Examples 21–5 and 21–6 note the differences in the power dissipated in each circuit. First, the total power dissipated in the parallel circuit is much greater than that dissipated in the series circuit. This is due to the fact that the equivalent resistance of the parallel circuit is smaller than the equivalent resistance of the series circuit, and the power delivered by a voltage V to a resistance R is inversely proportional to the resistance ($P = V^2/R$). In addition, note that the smallest resistor, R_2, has the smallest power in the series circuit but the largest power in the parallel circuit. These issues are explored further in the following Conceptual Checkpoint.

CONCEPTUAL CHECKPOINT 21–3 SERIES VERSUS PARALLEL

Two identical lightbulbs are connected to a battery, either in series or in parallel. Are the bulbs in series **(a)** brighter than, **(b)** dimmer than, or **(c)** the same brightness as the bulbs in parallel?

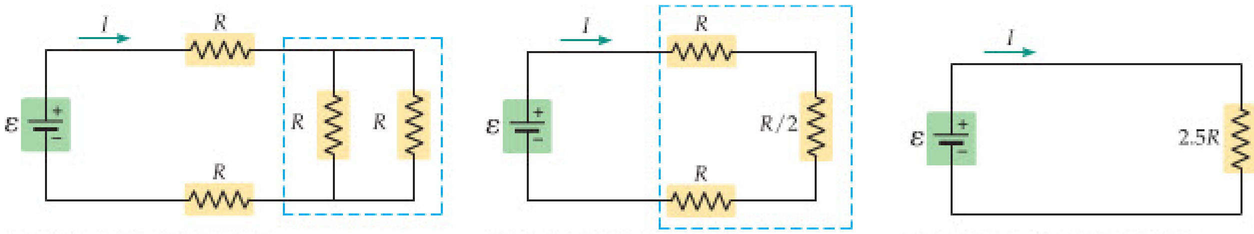
REASONING AND DISCUSSION

Both sets of lightbulbs are connected to the same potential difference, V; hence, the power delivered to the bulbs is V^2/R_{eq}, where R_{eq} is twice the resistance of a bulb in the series circuit and half the resistance of a bulb in the parallel circuit. As a result, more power is converted to light in the parallel circuit.

ANSWER

(b) The bulbs connected in series are dimmer than the bulbs connected in parallel.

Finally, note that a three-way lightbulb can also be produced by simply wiring two filaments in parallel. For example, one filament might have a power of 50 W and the second filament a power of 100 W. One setting of the switch sends current through the 50-W filament, the next setting sends current through the 100-W filament, and the third setting connects the two filaments in parallel. With the third connection, each filament produces the same power as before—since each is connected to the same potential difference—giving a total power of 50 W + 100 W = 150 W.



(a) Replace parallel resistors (b) Replace series resistors (c) Final equivalent resistance

▲ **FIGURE 21–11 Analyzing a complex circuit of resistors**

(a) The two vertical resistors are in parallel with one another; hence, they can be replaced with their equivalent resistance, $R/2$. **(b)** Now the circuit consists of three resistors in series. The equivalent resistance of these three resistors is 2.5 R. **(c)** The original circuit reduced to a single equivalent resistance.

Combination Circuits

The rules we have developed for series and parallel resistors can be applied to more complex circuits as well. For example, consider the circuit shown in Figure 21–11 (a), where four resistors, each equal to R, are connected in a way that combines series and parallel features. To analyze this circuit, we first note that the two vertically oriented resistors are connected in parallel with one another. Therefore, the equivalent resistance of this unit is given by $1/R_{eq} = 1/R + 1/R$, or $R_{eq} = R/2$. Replacing these two resistors with $R/2$ yields the circuit shown in Figure 21–11 (b), which consists of three resistors in series. As a result, the equivalent resistance of the entire circuit is $R + R/2 + R = 2.5R$, as indicated in Figure 21–11 (c). Similar methods can be applied to a wide variety of circuits.

PROBLEM-SOLVING NOTE

Analyzing a Complex Circuit

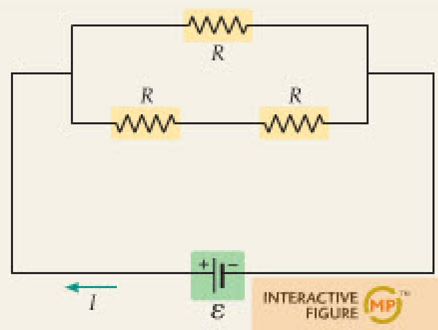
When considering an electric circuit with resistors in series and parallel, work from the smallest units of the circuit outward to ever larger units.

EXAMPLE 21–7 COMBINATION SPECIAL

In the circuit shown in the diagram, the emf of the battery is 12.0 V, and each resistor has a resistance of 200.0 Ω. Find **(a)** the current supplied by the battery to this circuit and **(b)** the current through the lower two resistors.

PICTURE THE PROBLEM

The circuit for this problem has three resistors connected to a battery. Note that the lower two resistors are in series with one another, and in parallel with the upper resistor. The battery has an emf of 12.0 V.



STRATEGY

a. The current supplied by the battery, I, is given by Ohm's law, $I = \mathcal{E}/R_{eq}$, where R_{eq} is the equivalent resistance of the three resistors. To find R_{eq}, we first note that the lower two resistors are in series, giving a net resistance of $2R$. Next, the upper resistor, R, is in parallel with $2R$. Calculating this equivalent resistance yields the desired R_{eq}.

b. Because the voltage across the lower two resistors is $\mathcal{E}$, the current through them is $I_{lower} = \mathcal{E}/R_{eq,lower} = \mathcal{E}/2R$.

SOLUTION

Part (a)

1. Calculate the equivalent resistance of the lower two resistors: $R_{eq,lower} = R + R = 2R$

2. Calculate the equivalent resistance of R in parallel with $2R$:

$$\frac{1}{R_{eq}} = \frac{1}{R} + \frac{1}{2R} = \frac{3}{2R}$$

$$R_{eq} = \tfrac{2}{3}R = \tfrac{2}{3}(200.0\ \Omega) = 133.3\ \Omega$$

3. Find the current supplied by the battery, I:

$$I = \frac{\mathcal{E}}{R_{eq}} = \frac{12.0\ \text{V}}{133.3\ \Omega} = 0.0900\ \text{A}$$

Part (b)

4. Use $\mathcal{E}$ and $R_{eq,lower}$ to find the current in the lower two resistors:

$$I_{lower} = \frac{\mathcal{E}}{R_{eq,lower}} = \frac{12.0\ \text{V}}{2(200.0\ \Omega)} = 0.0300\ \text{A}$$

INSIGHT

Note that the total resistance of the three 200.0-Ω resistors is less than 200.0 Ω—in fact, it is only 133.3 Ω. We also see that 0.0300 A flows through the lower two resistors, and therefore twice that much—0.0600 A—flows through the upper resistor.

CONTINUED ON NEXT PAGE

CONTINUED FROM PREVIOUS PAGE

PRACTICE PROBLEM

Suppose the upper resistor is changed from R to $2R$, and the lower two resistors remain the same. **(a)** Will the current supplied by the battery increase, decrease, or stay the same? **(b)** Find the new current. [**Answer:** (a) The current will decrease because there is greater resistance to its flow; (b) 0.0600 A.]

Some related homework problems: Problem 48, Problem 49, Problem 51

▶ The electric circuit in these photos starts with two identical lightbulbs (1 and 2) in series with a battery, as we see on the left. The bulbs are equally bright. Now, *before you examine the photo to the right*, consider the effect of adding a third identical bulb (3) to the circuit by placing it in the empty socket. What happens to the brightness of bulbs 1 and 2? As you can see, adding bulb 3 creates a new path for the current and increases the total current in the circuit by a factor of 4/3 (check this yourself). The current passing through bulb 1 is equally split between bulbs 2 and 3, however, and the new current in bulb 2 is now only $\frac{1}{2}(4/3) = 2/3$ of its original value. Thus, bulb 1 brightens and bulb 2 becomes dimmer.

21–5 Kirchhoff's Rules

To find the currents and voltages in a general electric circuit, we use two rules first introduced by the German physicist Gustav Kirchhoff (1824–1887). The *Kirchhoff rules* are simply ways of expressing charge conservation (the junction rule) and energy conservation (the loop rule) in a closed circuit. Since these conservation laws are always obeyed in nature, the Kirchhoff rules are completely general.

The Junction Rule

The junction rule follows from the observation that the current entering any point in a circuit must equal the current leaving that point. If this were not the case, charge would either build up or disappear from a circuit.

As an example, consider the circuit shown in Figure 21–12. At point A, three wires join to form a **junction.** (In general, a *junction* is any point in a circuit where three or more wires meet.) The current carried by each of the three wires is indicated in the figure. Notice that the current entering the junction is I_1; the current leaving the junction is $I_2 + I_3$. Setting the incoming and outgoing currents equal, we have $I_1 = I_2 + I_3$, or equivalently

$$I_1 - I_2 - I_3 = 0$$

This is Kirchhoff's junction rule applied to the junction at point A.

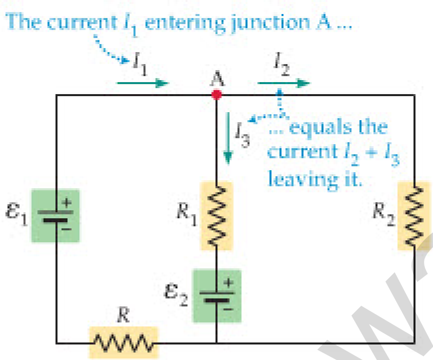
In general, if we associate a + sign with currents entering a junction and a − sign with currents leaving a junction, Kirchhoff's junction rule can be stated as follows:

> The algebraic sum of all currents meeting at any junction in a circuit must equal zero.

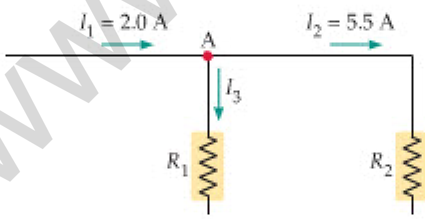
In the example just discussed, I_1 enters the junction (+), I_2 and I_3 leave the junction (−); hence, the algebraic sum of currents at the junction is $I_1 - I_2 - I_3$. Setting this sum equal to zero recovers our previous result.

In some cases we may not know the direction of all the currents meeting at a junction in advance. When this happens, we simply choose a direction for the unknown currents, apply the junction rule, and continue as usual. If the value we obtain for a given current is negative, it simply means that the direction we chose was wrong; the current actually flows in the opposite direction.

For example, suppose we know both the direction and magnitude of the currents I_1 and I_2 in Figure 21–13. To find the third current, we apply the junction



▲ **FIGURE 21–12 Kirchhoff's junction rule**
Kirchhoff's junction rule states that the sum of the currents entering a junction must equal the sum of the currents leaving the junction. In this case, for the junction labeled A, $I_1 = I_2 + I_3$, or $I_1 - I_2 - I_3 = 0$.



▲ **FIGURE 21–13 A specific application of Kirchhoff's junction rule**
Applying Kirchhoff's junction rule to the junction A, $I_1 - I_2 - I_3 = 0$, yields the result $I_3 = -3.5$ A. The minus sign indicates that I_3 flows opposite to the direction shown; that is, I_3 is actually upward.

rule—but first we must choose a direction for I_3. If we choose I_3 to point downward, as shown in the figure, the junction rule gives

$$I_1 - I_2 - I_3 = 0$$

Solving for I_3, we have

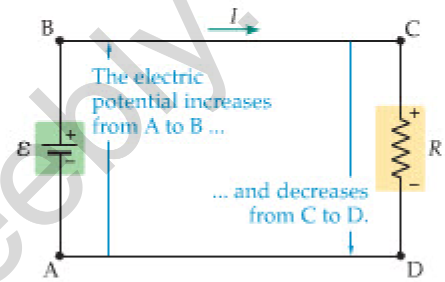
$$I_3 = I_1 - I_2 = 2.0\text{ A} - 5.5\text{ A} = -3.5\text{ A}$$

Since I_3 is negative, we conclude that the actual direction of this current is upward; that is, the 2.0-A current and the 3.5-A current enter the junction and combine to yield the 5.5-A current that leaves the junction.

The Loop Rule

Imagine taking a day hike on a mountain path. First, you gain altitude to reach a scenic viewpoint; later you descend below your starting point into a valley; finally, you gain altitude again and return to the trailhead. During the hike you sometimes increase your gravitational potential energy, and sometimes you decrease it, but the net change at the end of the hike is zero—after all, you return to the same altitude from which you started. Kirchhoff's loop rule is an application of the same idea to an electric circuit.

For example, consider the simple circuit shown in **Figure 21–14.** The electric potential increases by the amount $\mathcal{E}$ in going from point A to point B, since we move from the low-potential (−) terminal of the battery to the high-potential (+) terminal. This is like gaining altitude in the hiking analogy. Next, there is no potential change as we go from point B to point C, since these points are connected by an ideal wire. As we move from point C to point D, however, the potential does change—recall that a potential difference is required to force a current through a resistor. We label the potential difference across the resistor ΔV_{CD}. Finally, there is no change in potential between points D and A, since they too are connected by an ideal wire.



▲ **FIGURE 21–14 Kirchhoff's loop rule**
Kirchhoff's loop rule states that as one moves around a closed loop in a circuit, the algebraic sum of all potential differences must be zero. The electric potential increases as one moves from the − to the + plate of a battery; it decreases as one moves through a resistor in the direction of the current.

We can now apply the idea that the net change in electric potential (the analog to gravitational potential energy in the hike) must be zero around any closed loop. In this case, we have

$$\mathcal{E} + \Delta V_{CD} = 0$$

Thus, we find that $\Delta V_{CD} = -\mathcal{E}$; that is, the electric potential *decreases* as one moves across the resistor *in the direction of the current*. To indicate this drop in potential, we label the side where the current enters the resistor with a + (indicating high potential) and the side where the current leaves the resistor with a − (indicating low potential). Finally, we can use Ohm's law to set the magnitude of the potential drop equal to IR and find the current in the circuit:

$$|\Delta V_{CD}| = \mathcal{E} = IR$$

$$I = \frac{\mathcal{E}}{R}$$

This, of course, is the expected result.

In general, Kirchhoff's loop rule can be stated as follows:

> The algebraic sum of all potential differences around any closed loop in a circuit is zero.

We now consider a variety of applications in which both the junction rule and the loop rule are used to find the various currents and potentials in a circuit.

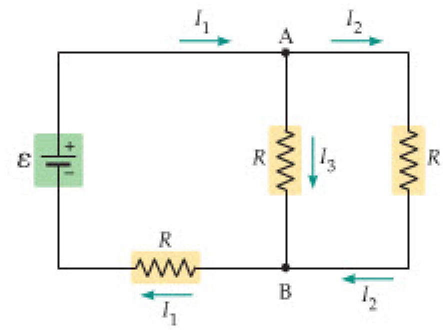
PROBLEM-SOLVING NOTE

Applying Kirchhoff's Rules

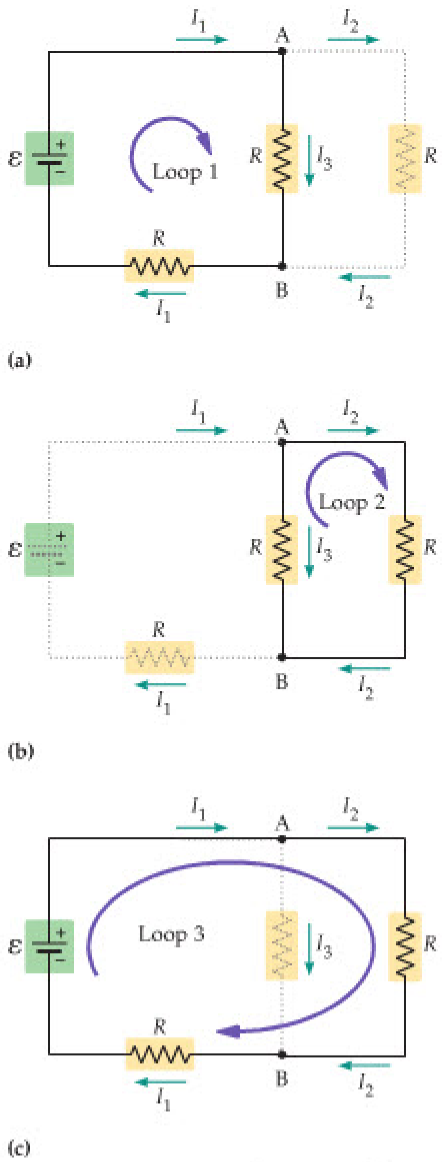
When applying Kirchhoff's rules, be sure to use the appropriate sign for currents and potential differences.

Applications

We begin by considering the relatively simple circuit shown in **Figure 21–15.** The currents and voltages in this circuit can be found by considering various parallel and series combinations of the resistors, as we did in the previous section. Thus, Kirchhoff's rules are not strictly needed in this case. Still, applying the rules to this circuit illustrates many of the techniques that can be used when studying more complex circuits.



▲ **FIGURE 21–15 Analyzing a simple circuit**
A simple circuit that can be studied using either equivalent resistance or Kirchhoff's rules.



▲ **FIGURE 21–16 Using loops to analyze a circuit**

Three loops associated with the circuit in Figure 21–15.

Let's suppose that all the resistors have the value $R = 100.0\ \Omega$, and that the emf of the battery is $\mathcal{E} = 15.0$ V. The equivalent resistance of the resistors can be obtained by noting that the vertical resistors are connected in parallel with one another and in series with the horizontal resistor. The vertical resistors combine to give a resistance of $R/2$, which, when added to the horizontal resistor, gives an equivalent resistance of $R_{eq} = 3R/2 = 150.0\ \Omega$. The current in the circuit, then, is $I = \mathcal{E}/R_{eq} = 15.0\ \text{V}/150.0\ \Omega = 0.100$ A.

Now we approach the same problem from the point of view of Kirchhoff's rules. First, we apply the junction rule to point A:

$$I_1 - I_2 - I_3 = 0 \qquad \text{(junction A)} \qquad 21\text{–}11$$

Note that current I_1 splits at point A into currents I_2 and I_3, which combine again at point B to give I_1 flowing through the horizontal resistor. We can apply the junction rule to point B, which gives $-I_1 + I_2 + I_3 = 0$, but since this differs from Equation 21–11 by only a minus sign, no new information is gained.

Next, we apply the loop rule. Since there are three unknowns, I_1, I_2, and I_3, we need three independent equations for a full solution. One has already been given by the junction rule; thus, we expect that two loop equations will be required to complete the solution. To begin, we consider loop 1, which is shown in **Figure 21–16 (a)**. We choose to move around this loop in the clockwise direction. (If we were to choose the counterclockwise direction instead, the same information would be obtained.) For loop 1, then, we have an increase in potential as we move across the battery, a drop in potential across the vertical resistor of I_3R, and another drop in potential across the horizontal resistor, this time of magnitude I_1R. Applying the loop rule, we find the following:

$$\mathcal{E} - I_3R - I_1R = 0 \qquad \text{(loop 1)} \qquad 21\text{–}12$$

Similarly, we can apply the loop rule to loop 2, shown in **Figure 21–16 (b)**. In this case we cross the right-hand vertical resistor in the direction of the current, implying a drop in potential, and we cross the left-hand vertical resistor against the current, implying an increase in potential. Therefore, the loop rule gives

$$I_3R - I_2R = 0 \qquad \text{(loop 2)} \qquad 21\text{–}13$$

There is a third possible loop, shown in **Figure 21–16 (c)**, but the information it gives is not different from that already obtained. In fact, *any two of the three loops* complete our solution.

Note that R cancels in Equation 21–13; hence, we see that $I_3 - I_2 = 0$, or $I_3 = I_2$. Substituting this result into the junction rule (Equation 21–11), we obtain

$$I_1 - I_2 - I_3 = I_1 - I_2 - I_2$$
$$= I_1 - 2I_2 = 0$$

Solving this equation for I_2 gives us $I_2 = I_1/2 = I_3$. Finally, using the first loop equation (Equation 21–12), we find

$$\mathcal{E} - (I_1/2)R - I_1R = \mathcal{E} - \tfrac{3}{2}I_1R = 0$$

Note that the only unknown in this equation is current I_1. Solving for this current, we find

$$I_1 = \frac{\mathcal{E}}{\frac{3}{2}R} = \frac{15.0\ \text{V}}{\frac{3}{2}(100.0\ \Omega)} = 0.100\ \text{A}$$

As expected, our result using Kirchhoff's rules agrees with the result obtained previously. Finally, the other two currents in the circuit are $I_2 = I_3 = I_1/2 = 0.0500$ A.

EXERCISE 21–3

Write the loop equation for loop 3 in Figure 21–16 (c).

SOLUTION

Proceeding in a clockwise direction, as indicated in the figure, we find

$$\mathcal{E} - I_2R - I_1R = 0$$

Since I_2 and I_3 are equal (loop 2), it follows that loop 1 ($\mathcal{E} - I_3R - I_1R = 0$) and loop 3 ($\mathcal{E} - I_2R - I_1R = 0$) give the same information. If we proceed in a counterclockwise direction around loop 3, we find

$$-\mathcal{E} + I_2R + I_1R = 0$$

Notice that this result is the same as the clockwise result except for an overall minus sign, and, therefore, it contains no new information. In general, it does not matter in which direction we choose to go around a loop.

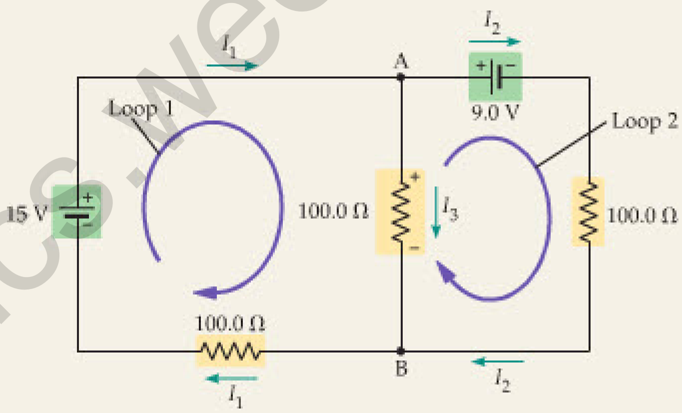
Clearly, the Kirchhoff approach is more involved than the equivalent-resistance method. However, it is not possible to analyze all circuits in terms of equivalent resistances. In such cases, Kirchhoff's rules are the only option, as illustrated in the next Active Example.

ACTIVE EXAMPLE 21–2 TWO LOOPS, TWO BATTERIES: FIND THE CURRENTS

Find the currents in the circuit shown.

SOLUTION *(Test your understanding by performing the calculations indicated in each step.)*

1. Apply the junction rule to point A: $I_1 - I_2 - I_3 = 0$
2. Apply the loop rule to loop 1 (let $R = 100.0\ \Omega$): $15\text{ V} - I_3R - I_1R = 0$
3. Apply the loop rule to loop 2 (let $R = 100.0\ \Omega$): $-9.0\text{ V} - I_2R + I_3R = 0$
4. Solve for I_1, I_2, and I_3: $I_1 = 0.070\text{ A}$, $I_2 = -0.010\text{ A}$, $I_3 = 0.080\text{ A}$



INSIGHT

Note that I_2 is negative. This means that its direction is opposite to that shown in the circuit diagram.

YOUR TURN

Suppose the polarity of the 9.0-V battery is reversed. What are the currents in this case?

(Answers to **Your Turn** *problems are given in the back of the book.)*

21–6 Circuits Containing Capacitors

To this point we have considered only resistors and batteries in electric circuits. Capacitors, which can also play an important role, are represented by a set of parallel lines (reminiscent of a parallel-plate capacitor): ⊣⊢. We now investigate simple circuits involving batteries and capacitors, leaving for the next section circuits that combine all three circuit elements.

Capacitors in Parallel

The simplest way to combine capacitors, as we shall see, is by connecting them in parallel. For example, Figure 21–17 (a) shows three capacitors connected in parallel with a battery of emf ε. As a result, each capacitor has the same potential difference, ε, between its plates. The magnitudes of the charges on each capacitor are as follows:

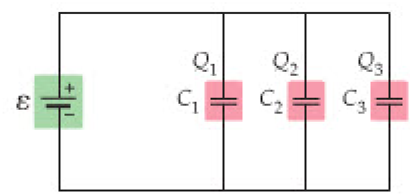
$$Q_1 = C_1\mathcal{E}, \qquad Q_2 = C_2\mathcal{E}, \qquad Q_3 = C_3\mathcal{E}$$

As a result, the total charge on the three capacitors is

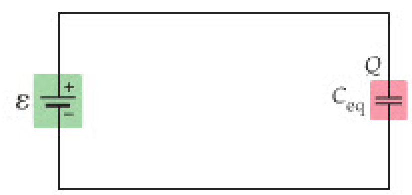
$$Q = Q_1 + Q_2 + Q_3 = \mathcal{E}C_1 + \mathcal{E}C_2 + \mathcal{E}C_3 = (C_1 + C_2 + C_3)\mathcal{E}$$

If an equivalent capacitor is used to replace the three in parallel, as in Figure 21–17 (b), the charge on its plates must be the same as the total charge on the individual capacitors:

$$Q = C_{eq}\mathcal{E}$$



(a) Three capacitors in parallel



(b) Equivalent capacitance with same total charge

▲ **FIGURE 21–17 Capacitors in parallel**

(a) Three capacitors, C_1, C_2, and C_3, connected in parallel. Note that each capacitor is connected across the same potential difference, $\mathcal{E}$. **(b)** The equivalent capacitance, $C_{eq} = C_1 + C_2 + C_3$, has the same charge on its plates as the total charge on the three original capacitors.

Comparing $Q = C_{eq}\mathcal{E}$ with $Q = (C_1 + C_2 + C_3)\mathcal{E}$, we see that the equivalent capacitance is simply

$$C_{eq} = C_1 + C_2 + C_3$$

In general, the equivalent capacitance of capacitors connected in parallel is the sum of the individual capacitances:

Equivalent Capacitance for Capacitors in Parallel

$$C_{eq} = C_1 + C_2 + C_3 + \cdots = \sum C \qquad \text{21–14}$$

SI unit: farad, F

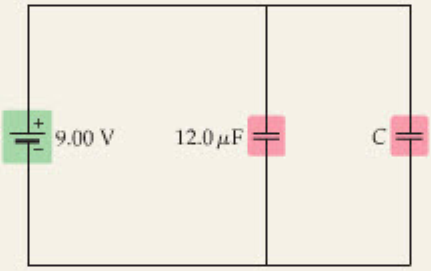
Thus, connecting capacitors in parallel produces an equivalent capacitance greater than the greatest individual capacitance. It is as if the plates of the individual capacitors are connected together to give one large set of plates, with a correspondingly large capacitance.

EXAMPLE 21–8 ENERGY IN PARALLEL

Two capacitors, one 12.0 μF and the other of unknown capacitance C, are connected in parallel across a battery with an emf of 9.00 V. The total energy stored in the two capacitors is 0.0115 J. What is the value of the capacitance C?

PICTURE THE PROBLEM

The circuit, consisting of one 9.00-V battery and two capacitors, is illustrated in the diagram. The total energy of 0.0115 J stored in the two capacitors is the same as the energy stored in the equivalent capacitance for this circuit.



STRATEGY

Recall from Chapter 20 that the energy stored in a capacitor can be written as $U = \frac{1}{2}CV^2$. It follows, then, that for an equivalent capacitance, C_{eq}, the energy is $U = \frac{1}{2}C_{eq}V^2$. Since we know the energy and voltage, we can solve this relation for the equivalent capacitance. Finally, the equivalent capacitance is the sum of the individual capacitances, $C_{eq} = 12.0\ \mu\text{F} + C$. We use this relation to solve for C.

SOLUTION

1. Solve $U = \frac{1}{2}C_{eq}V^2$ for the equivalent capacitance:

$$U = \frac{1}{2}C_{eq}V^2$$
$$C_{eq} = \frac{2U}{V^2}$$

2. Substitute numerical values to find C_{eq}:

$$C_{eq} = \frac{2U}{V^2} = \frac{2(0.0115\ \text{J})}{(9.00\ \text{V})^2} = 284\ \mu\text{F}$$

3. Solve for C in terms of the equivalent capacitance:

$$C_{eq} = 12.0\ \mu\text{F} + C$$
$$C = C_{eq} - 12.0\ \mu\text{F} = 284\ \mu\text{F} - 12.0\ \mu\text{F} = 272\ \mu\text{F}$$

INSIGHT

The energy stored in the 12.0-μF capacitor is $U = \frac{1}{2}CV^2 = 0.000486$ J. In comparison, the 272-μF capacitor stores an energy equal to 0.0110 J. Thus, the larger capacitor stores the greater amount of energy. Though this may seem only natural, one needs to be careful. When we examine capacitors in *series* later in this section, we shall find exactly the opposite result.

PRACTICE PROBLEM

What is the total charge stored on the two capacitors? [**Answer:** $Q = C_{eq}\mathcal{E} = 2.56 \times 10^{-3}$ C]

Some related homework problems: Problem 72, Problem 73

REAL-WORLD PHYSICS
"Touch-sensitive" lamps

Although you probably haven't realized it, when you turn on a "touch sensitive" lamp, you are part of a circuit with capacitors in parallel. In fact, you are one of the capacitors! When you touch such a lamp, a small amount of charge moves onto your body—your body is like the plate of a capacitor. Because you have effectively increased the plate area—as always happens when capacitors are connected in parallel—the capacitance of the circuit increases. The electronic circuitry in the lamp senses this increase in capacitance and triggers the switch to turn the light on or off.

Capacitors in Series

You have probably noticed from Equation 21–14 that capacitors connected in *parallel* combine in the same way as resistors connected in *series*. Similarly, capacitors connected in *series* obey the same rules as resistors connected in *parallel*, as we now show.

Consider three capacitors—initially uncharged—connected in series with a battery, as in **Figure 21–18 (a)**. The battery causes the left plate of C_1 to acquire a positive charge, $+Q$. This charge, in turn, attracts a negative charge $-Q$ onto the right plate of the capacitor. Because the capacitors start out uncharged, there is zero net charge between C_1 and C_2. As a result, the negative charge $-Q$ on the right plate of C_1 leaves a corresponding positive charge $+Q$ on the upper plate of C_2. The charge $+Q$ on the upper plate of C_2 attracts a negative charge $-Q$ onto its lower plate, leaving a corresponding positive charge $+Q$ on the right plate of C_3. Finally, the positive charge on the right plate of C_3 attracts a negative charge $-Q$ onto its left plate. The result is that all three capacitors have charge of the same magnitude on their plates.

With the same charge Q on all the capacitors, the potential difference for each is as follows:

$$V_1 = \frac{Q}{C_1}, \qquad V_2 = \frac{Q}{C_2}, \qquad V_3 = \frac{Q}{C_3}$$

Since the total potential difference across the three capacitors must equal the emf of the battery, we have

$$\mathcal{E} = V_1 + V_2 + V_3 = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3} = Q\left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right) \qquad \text{21–15}$$

An equivalent capacitor connected to the same battery, as in **Figure 21–18 (b)**, will satisfy the relation $Q = C_{eq}\mathcal{E}$, or

$$\mathcal{E} = Q\left(\frac{1}{C_{eq}}\right) \qquad \text{21–16}$$

A comparison of Equations 21–15 and 21–16 yields the result

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

Thus, in general, we have the following rule for combining capacitors in series:

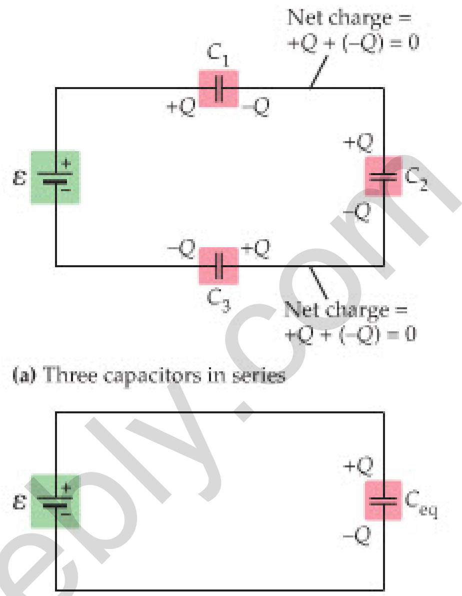
Equivalent Capacitance for Capacitors in Series

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots = \sum \frac{1}{C} \qquad \text{21–17}$$

SI unit: farad, F

It follows, then, that the equivalent capacitance of a group of capacitors connected in series is less than the smallest individual capacitance. In this case, it is as if the plate separations of the individual capacitors add to give a larger effective separation, and a correspondingly smaller capacitance.

More complex circuits, with some capacitors in series and others in parallel, can be handled in the same way as was done earlier with resistors. This is illustrated in the following Active Example.



(a) Three capacitors in series

(b) Equivalent capacitance with same total charge

▲ **FIGURE 21–18 Capacitors in series**
(a) Three capacitors, C_1, C_2, and C_3, connected in series. Note that each capacitor has the same magnitude charge on its plates. **(b)** The equivalent capacitance, $1/C_{eq} = 1/C_1 + 1/C_2 + 1/C_3$, has the same charge as the original capacitors.

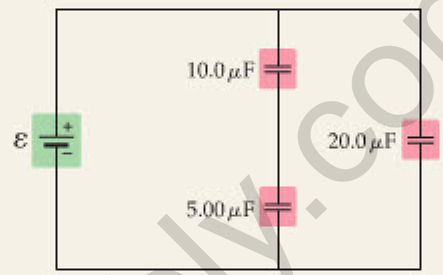
PROBLEM-SOLVING NOTE

Finding the Equivalent Capacitance of a Circuit

When calculating the equivalent capacitance of capacitors in series, be sure to take one final inverse at the end of your calculation to find C_{eq}. Also, when considering circuits with capacitors in both series and parallel, start with the smallest units of the circuit and work your way out to the larger units.

ACTIVE EXAMPLE 21–3 FIND THE EQUIVALENT CAPACITANCE AND THE STORED ENERGY

Consider the electric circuit shown here, consisting of a 12.0-V battery and three capacitors connected partly in series and partly in parallel. Find **(a)** the equivalent capacitance of this circuit and **(b)** the total energy stored in the capacitors.



SOLUTION *(Test your understanding by performing the calculations indicated in each step.)*

Part (a)

1. Find the equivalent capacitance of a 10.0-μF capacitor in series with a 5.00-μF capacitor: — 3.33 μF
2. Find the equivalent capacitance of a 3.33-μF capacitor in parallel with a 20.0-μF capacitor: — $C_{eq} = 23.3\ \mu F$

Part (b)

3. Calculate the stored energy using $U = \frac{1}{2}C_{eq}V^2$: — $U = 1.68 \times 10^{-3}$ J

INSIGHT

Notice that the 10.0-μF capacitor and the 5.00-μF capacitor are connected in series. As you might expect, one of these capacitors stores twice as much energy as the other. Which is it? Check the Your Turn question for the answer.

YOUR TURN

Is the energy stored in the 10.0-μF capacitor greater than or less than the energy stored in the 5.0-μF capacitor? Explain. Check your answer by calculating the energy stored in each of the capacitors.

(Answers to **Your Turn** *problems are given in the back of the book.)*

21–7 *RC* Circuits

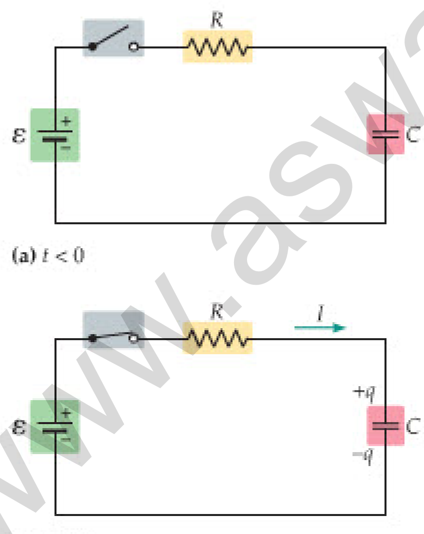
When the switch is closed on a circuit containing only batteries and capacitors, the charge on the capacitor plates appears almost instantaneously—essentially at the speed of light. This is not the case, however, in circuits that also contain resistors. In these situations, the resistors limit the rate at which charge can flow, and an appreciable amount of time may be required before the capacitors acquire a significant charge. A useful analogy is the amount of time needed to fill a bucket with water. If you use a fire hose, which has little resistance to the flow of water, the bucket fills almost instantly. If you use a garden hose, which presents a much greater resistance to the water, filling the bucket may take a minute or more.

The simplest example of such a circuit, a so-called ***RC* circuit,** is shown in Figure 21–19. Initially (before $t = 0$) the switch is open, and there is no current in the resistor or charge on the capacitor. At $t = 0$ the switch is closed and current begins to flow. If the resistor was not present, the capacitor would immediately take on the charge $Q = C\mathcal{E}$. The effect of the resistor, however, is to slow the charging process—in fact, the larger the resistance, the longer it takes for the capacitor to charge. One way to think of this is to note that as long as a current flows in the circuit, as in Figure 21–19 (b), there is a potential drop across the resistor; hence, the potential difference between the plates of the capacitor is less than the emf of the battery. With less voltage across the capacitor there will be less charge on its plates compared with the charge that would result if the plates were connected directly to the battery.

The methods of calculus can be used to show that the charge on the capacitor in Figure 21–19 varies with time as follows:

$$q(t) = C\mathcal{E}(1 - e^{-t/\tau}) \qquad 21\text{–}18$$

In this expression, e is Euler's number ($e = 2.718\ldots$) or, more precisely, the base of natural logarithms (see Appendix A). The quantity τ is referred to as the **time constant** of the circuit. The time constant is related to the resistance and capacitance of a circuit by the following simple relation: $\tau = RC$. As we shall see, τ can be thought of as a characteristic time for the behavior of an RC circuit.



▲ **FIGURE 21–19 A typical *RC* circuit**
(a) Before the switch is closed ($t < 0$) there is no current in the circuit and no charge on the capacitor. **(b)** After the switch is closed ($t > 0$), current flows and the charge on the capacitor builds up over a finite time. As $t \to \infty$ the charge on the capacitor approaches $Q = C\mathcal{E}$.

For example, at time $t = 0$ the exponential term is $e^{-0/\tau} = e^0 = 1$; therefore, the charge on the capacitor is zero at $t = 0$, as expected:

$$q(0) = C\mathcal{E}(1 - 1) = 0$$

In the opposite limit, $t \to \infty$, the exponential vanishes: $e^{-\infty/\tau} = 0$. Thus the charge in this limit is $C\mathcal{E}$:

$$q(t \to \infty) = C\mathcal{E}(1 - 0) = C\mathcal{E}$$

This is just the charge Q the capacitor would have had from $t = 0$ on if there had been no resistor in the circuit. Finally, at time $t = \tau$ the charge on the capacitor is $q = C\mathcal{E}(1 - e^{-1}) = C\mathcal{E}(1 - 0.368) = 0.632C\mathcal{E}$, which is 63.2% of its final charge. The charge on the capacitor as a function of time is plotted in Figure 21–20.

Before we continue, let's check to see that the quantity $\tau = RC$ is in fact a time. Suppose, for example, that the resistor and capacitor in an RC circuit have the values $R = 120\ \Omega$ and $C = 3.5\ \mu F$, respectively. Multiplying R and C we find

$$\tau = RC = (120\text{ ohm})(3.5 \times 10^{-6}\text{ farad})$$
$$= \left(\frac{120\text{ volt}}{\text{coulomb/second}}\right)\left(\frac{3.5 \times 10^{-6}\text{ coulomb}}{\text{volt}}\right) = 4.2 \times 10^{-4}\text{ second}$$

The tick marks on the horizontal axis in Figure 21–20 indicate the times τ, 2τ, 3τ, and 4τ. Notice that the capacitor is almost completely charged by the time $t = 4\tau$.

Figure 21–20 also shows that the charge on the capacitor increases rapidly initially, indicating a large current in the circuit. Eventually, the charging slows down, because the greater the charge on the capacitor, the harder it is to transfer additional charge against the electrical repulsive force. Later, the charge barely changes with time, which means that the current is essentially zero. In fact, the mathematical expression for the current—again derived from calculus—is the following:

$$I(t) = \left(\frac{\mathcal{E}}{R}\right)e^{-t/\tau} \qquad 21\text{–}19$$

This expression is plotted in Figure 21–21, where we see that significant variation in the current occurs over times ranging from $t = 0$ to $t \sim 4\tau$. At time $t = 0$ the current is $I(0) = \mathcal{E}/R$, which is the value it would have if the capacitor were replaced by an ideal wire. As $t \to \infty$, the current approaches zero, as expected: $I(t \to \infty) \to 0$. In this limit, the capacitor is essentially fully charged, so that no more charge can flow onto its plates. Thus, in this limit, the capacitor behaves like an open switch.

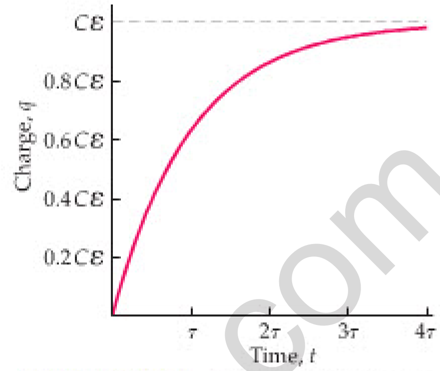


FIGURE 21–20 Charge versus time for the RC circuit in Figure 21–19
The horizontal axis shows time in units of the characteristic time, $\tau = RC$. The vertical axis shows the magnitude of the charge on the capacitor in units of $C\mathcal{E}$.

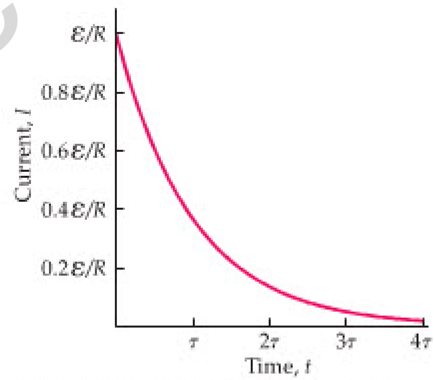


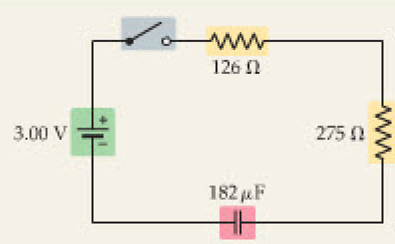
FIGURE 21–21 Current versus time for the RC circuit in Figure 21–19
Initially the current is $\mathcal{E}/R$, the same as if the capacitor were not present. The current approaches zero after a period equal to several time constants, $\tau = RC$.

EXAMPLE 21–9 CHARGING A CAPACITOR

A circuit consists of a 126-Ω resistor, a 275-Ω resistor, a 182-μF capacitor, a switch, and a 3.00-V battery all connected in series. Initially the capacitor is uncharged and the switch is open. At time $t = 0$ the switch is closed. **(a)** What charge will the capacitor have a long time after the switch is closed? **(b)** At what time will the charge on the capacitor be 80.0% of the value found in part (a)?

PICTURE THE PROBLEM

The circuit described in the problem statement is shown with the switch in the open position. Once the switch is closed at $t = 0$, current will flow in the circuit and charge will begin to accumulate on the capacitor plates.



INTERACTIVE FIGURE

STRATEGY

a. A long time after the switch is closed, the current stops and the capacitor is fully charged. At this point, the voltage across the capacitor is equal to the emf of the battery. Therefore, the charge on the capacitor is $Q = C\mathcal{E}$.

b. To find the time when the charge will be 80.0% of the full charge, $Q = C\mathcal{E}$, we can set $q(t) = C\mathcal{E}(1 - e^{-t/\tau}) = 0.800C\mathcal{E}$ and solve for the desired time, t.

CONTINUED ON NEXT PAGE

CONTINUED FROM PREVIOUS PAGE

SOLUTION

Part (a)

1. Evaluate $Q = C\mathcal{E}$ for this circuit: $Q = C\mathcal{E} = (182\ \mu\text{F})(3.00\ \text{V}) = 546\ \mu\text{C}$

Part (b)

2. Set $q(t) = 0.800C\mathcal{E}$ in $q(t) = C\mathcal{E}(1 - e^{-t/\tau})$ and cancel $C\mathcal{E}$: $q(t) = 0.800C\mathcal{E} = C\mathcal{E}(1 - e^{-t/\tau})$
$0.800 = 1 - e^{-t/\tau}$

3. Solve for t in terms of the time constant τ: $e^{-t/\tau} = 1 - 0.800 = 0.200$
$t = -\tau \ln(0.200)$

4. Calculate τ and use the result to find the time t: $\tau = RC = (126\ \Omega + 275\ \Omega)(182\ \mu\text{F}) = 73.0\ \text{ms}$
$t = -(73.0\ \text{ms}) \ln(0.200)$
$= -(73.0\ \text{ms})(-1.61) = 118\ \text{ms}$

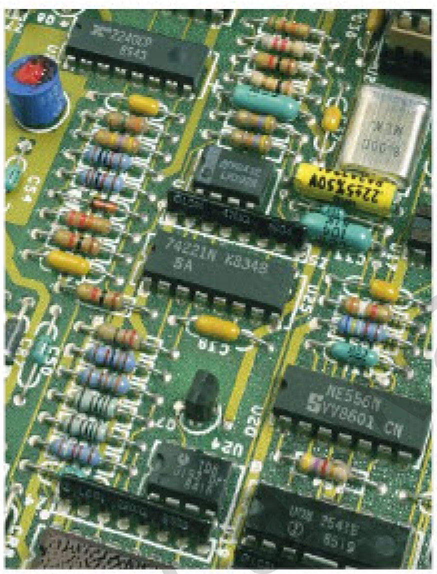
INSIGHT

Note that the time required for the charge on a capacitor to reach 80.0% of its final value is 1.61 time constants. This result is independent of the values of R and C in an RC circuit.

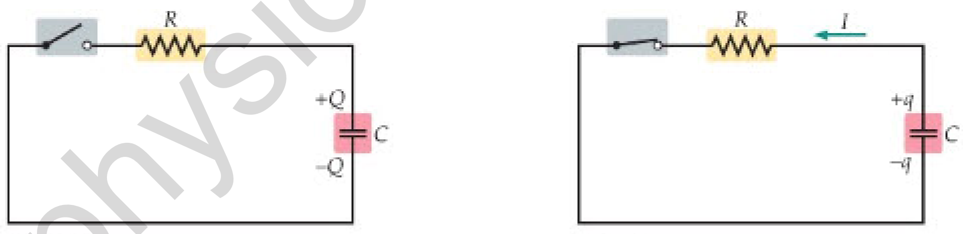
PRACTICE PROBLEM

What is the current in this circuit at the time found in part (b)? [**Answer:** $I(t) = (\mathcal{E}/R)e^{-t/\tau} = [(3.00\ \text{V})/(126\ \Omega + 275\ \Omega)](0.200) = (7.48\ \text{mA})(0.200) = 1.50\ \text{mA}$]

Some related homework problems: Problem 79, Problem 82



▲ A modern-day circuit board incorporates numerous resistors (cylinders with colored bands) and capacitors (yellow cylinders and metal container).



(a) $t < 0$ (b) $t > 0$

▲ **FIGURE 21–22 Discharging a capacitor**

(a) A charged capacitor is connected to a resistor. Initially the circuit is open, and no current can flow. **(b)** When the switch is closed, current flows from the + plate of the capacitor to the − plate. The charge remaining on the capacitor approaches zero after several time units, RC.

Similar behavior occurs when a charged capacitor is allowed to discharge, as in Figure 21–22. In this case, the initial charge on the capacitor is Q. If the switch is closed at $t = 0$, the charge for later times is

$$q(t) = Qe^{-t/\tau} \qquad 21\text{–}20$$

Like charging, the discharging of a capacitor occurs with a characteristic time $\tau = RC$.

To summarize, circuits with resistors and capacitors have the following general characteristics:

- Charging and discharging occur over a finite, characteristic time given by the time constant, $\tau = RC$.
- At $t = 0$ current flows freely through a capacitor being charged; it behaves like a short circuit.
- As $t \to \infty$ the current flowing into a capacitor approaches zero. In this limit, a capacitor behaves like an open switch.

We explore these features further in the following Conceptual Checkpoint.

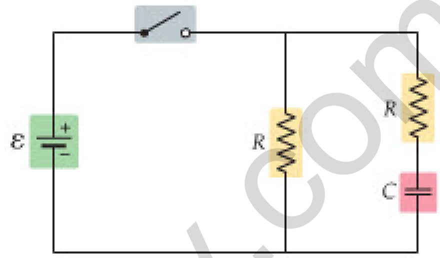
PROBLEM-SOLVING NOTE

The Limiting Behavior of Capacitors

Capacitors in dc circuits act like short circuits at $t = 0$ and open circuits as $t \to \infty$.

CONCEPTUAL CHECKPOINT 21–4 CURRENT IN AN *RC* CIRCUIT

What current flows through the battery in this circuit **(a)** immediately after the switch is closed and **(b)** a long time after the switch is closed?



REASONING AND DISCUSSION

a. Immediately after the switch is closed, the capacitor acts like a short circuit; that is, as if the battery were connected to two resistors R in parallel. The equivalent resistance in this case is $R/2$; therefore, the current is $I = \mathcal{E}/(R/2) = 2\mathcal{E}/R$.

b. After current has been flowing in the circuit for a long time, the capacitor acts like an open switch. Now current can flow only through the one resistor, R; hence, the current is $I = \mathcal{E}/R$, half of its initial value.

ANSWER

(a) The current is $2\mathcal{E}/R$; **(b)** the current is $\mathcal{E}/R$.

REAL-WORLD PHYSICS

Delay circuits in windshield wipers and turn signals

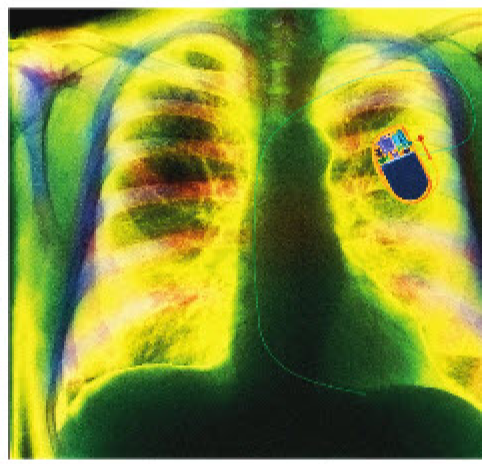
The fact that *RC* circuits have a characteristic time makes them useful in a variety of different applications. On a rather mundane level, *RC* circuits are used to determine the time delay on windshield wipers. When you adjust the delay knob in your car, you change a resistance or a capacitance, which in turn changes the time constant of the circuit. This results in a greater or a smaller delay. The blinking rate of turn signals is also determined by the time constant of an *RC* circuit.

REAL-WORLD PHYSICS: BIO

Pacemakers

A more critical application of *RC* circuits is the heart pacemaker. In the simplest case, these devices use an *RC* circuit to deliver precisely timed pulses directly to the heart. The more sophisticated pacemakers available today can even "sense" when a patient's heart rate falls below a predetermined value. The pacemaker then begins sending appropriate pulses to the heart to increase its rate. Many pacemakers can even be reprogrammed after they are surgically implanted to respond to changes in a patient's condition.

Normally, the heart's rate of beating is determined by its own natural pacemaker, the sinoatrial or SA node, located in the upper right chamber of the heart. If the SA node is not functioning properly, it may cause the heart to beat slowly or irregularly. To correct the problem, a pacemaker is implanted just under the collarbone, and an electrode is introduced intravenously via the cephalic vein. The distal end of the electrode is positioned, with the aid of fluoroscopic guidance, in the right ventricular apex. From that point on, the operation of the pacemaker follows the basic principles of electric circuits, as described in this chapter.



▲ An X-ray showing a pacemaker installed in a person's chest. The timing of the electrical pulses that keep the heart beating regularly is determined by an *RC* circuit powered by a small, long-lived battery.

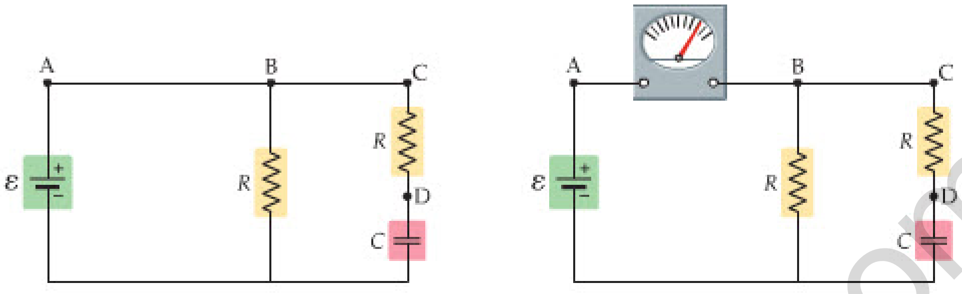
*21–8 Ammeters and Voltmeters

Devices for measuring currents and voltages in a circuit are referred to as **ammeters** and **voltmeters**, respectively. In each case, the ideal situation is for the meter to measure the desired quantity without altering the characteristics of the circuit being studied. This is accomplished in different ways for these two types of meters, as we shall see.

First, the ammeter is designed to measure the flow of current through a particular portion of a circuit. For example, we may want to know the current flowing between points A and B in the circuit shown in **Figure 21–23 (a)**. To measure this current, we insert the ammeter into the circuit in such a way that all the current flowing from A to B must also flow through the meter. This is done by connecting the meter "in series" with the other circuit elements between A and B, as indicated in **Figure 21–23 (b)**.

If the ammeter has a finite resistance—which must be the case for real meters—the presence of the meter in the circuit will alter the current it is intended to measure. Thus, an *ideal* ammeter would be one with zero resistance. In practice, if the resistance of the ammeter is much less than the other resistances in the circuit, its reading will be reasonably accurate.

▲ A typical digital multimeter, which can measure resistance (teal settings), current (yellow settings), or voltage (red settings). This meter is measuring the voltage of a "9 volt" battery.



(a) Typical electric circuit (b) Measuring the current between A and B

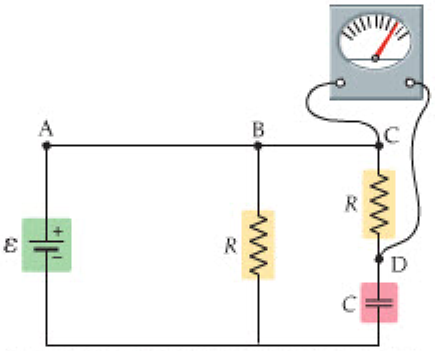
▲ **FIGURE 21–23 Measuring the current in a circuit**

To measure the current flowing between points A and B in **(a)**, an ammeter is inserted into the circuit, as shown in **(b)**. An ideal ammeter would have zero resistance.

Second, a voltmeter measures the potential drop between any two points in a circuit. Referring again to the circuit in Figure 21–23 (a), we may wish to know the difference in potential between points C and D. To measure this voltage, we connect the voltmeter "in parallel" to the circuit at the appropriate points, as in Figure 21–24.

A real voltmeter always allows some current to flow through it, which means that the current flowing through the circuit is less than before the meter was connected. As a result, the measured voltage is altered from its ideal value. An *ideal* voltmeter, then, would be one in which the resistance is infinite, so that the current it draws from the circuit is negligible. In practical situations it is sufficient that the resistance of the meter be much greater than the resistances in the circuit.

Sometimes the functions of an ammeter, voltmeter, and ohmmeter are combined in a single device called a **multimeter.** Adjusting the settings on a multimeter allows a variety of circuit properties to be measured.



Measuring the voltage between C and D

◀ **FIGURE 21–24 Measuring the voltage in a circuit**

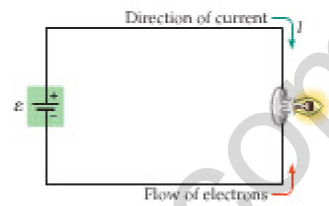
The voltage difference between points C and D can be measured by connecting a voltmeter in parallel to the original circuit. An ideal voltmeter would have infinite resistance.

THE BIG PICTURE PUTTING PHYSICS IN CONTEXT

LOOKING BACK	LOOKING AHEAD
The concept of electric potential energy (Chapter 20) is used in Section 21–3, where we talk about the energy associated with an electric circuit. We also discuss the power of an electric circuit in Section 21–3. For this we refer back to mechanics, where power was originally introduced in Chapter 7. Capacitors, first introduced in Chapter 20, are used in dc circuits in Section 21–6.	A dc circuit with a current flowing through it will play an important role in our discussion of magnetism in Chapter 22. We will also consider the magnetic force exerted on a current-carrying wire in Chapter 22. In Chapter 24 we extend our discussion of electric circuits from those in which the current flows in only one direction (dc) to circuits in which the current alternates in direction (ac, or alternating current). We will again use resistors and capacitors in the ac circuits. A simple dc circuit appears in Chapter 30, where we discuss the photoelectric effect and its importance in the development of quantum mechanics.

CHAPTER SUMMARY

21–1 ELECTRIC CURRENT



Electric current is the flow of electric charge.

Definition
If a charge ΔQ passes a given point in the time Δt, the corresponding electric current is

$$I = \frac{\Delta Q}{\Delta t} \qquad \text{21–1}$$

Ampere
The unit of current is the ampere, or amp for short. By definition, 1 amp is one coulomb per second; $1\ \text{A} = 1\ \text{C/s}$.

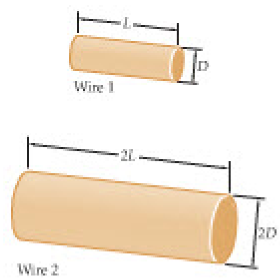
Battery
A battery is a device that uses chemical reactions to produce a potential difference between its two terminals.

Electromotive Force
The electromotive force, or emf, $\mathcal{E}$, is the potential difference between the terminals of a battery under ideal conditions.

Work Done by a Battery
As a battery moves a charge ΔQ around a circuit, it does the work $W = (\Delta Q)\mathcal{E}$.

Direction of Current
By definition, the direction of the current I in a circuit is the direction in which *positive* charges would move. The actual charge carriers, however, are generally electrons; hence, they move in the opposite direction to I.

21–2 RESISTANCE AND OHM'S LAW



When electrons move through a wire, they encounter resistance to their motion. In order to move electrons against this resistance, it is necessary to apply a potential difference between the ends of the wire.

Ohm's Law
To produce a current I through a wire with resistance R the following potential difference, V, is required:

$$V = IR \qquad \text{21–2}$$

Resistivity
The resistivity ρ of a material determines how much resistance it gives to the flow of electric current.

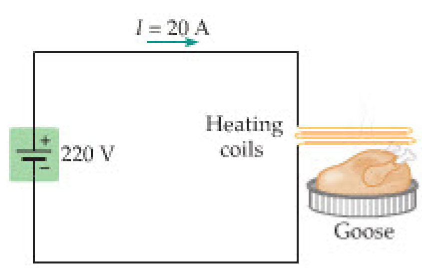
Resistance of a Wire
The resistance of a wire of length L, cross-sectional area A, and resistivity ρ is

$$R = \rho\left(\frac{L}{A}\right) \qquad \text{21–3}$$

Temperature Dependence
The resistivity of most metals increases approximately linearly with temperature.

Superconductivity
Below a certain critical temperature, T_c, certain materials lose all electrical resistance. A current flowing in a superconductor can continue undiminished as long as its temperature is maintained below T_c.

21–3 ENERGY AND POWER IN ELECTRIC CIRCUITS



In general, energy is required to cause an electric current to flow through a circuit. The rate at which the energy must be supplied is the power.

Electrical Power
If a current I flows across a potential difference V, the corresponding electrical power is

$$P = IV \qquad \text{21–4}$$

Power Dissipation in a Resistor

If a potential difference V produces a current I in a resistor R, the electrical power converted to heat is

$$P = I^2R = V^2/R \qquad \text{21–5, 21–6}$$

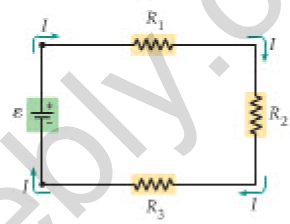
Energy Usage and the Kilowatt-Hour

The energy equivalent of one kilowatt-hour (kWh) is

$$1\text{ kWh} = 3.6 \times 10^6\text{ J}$$

21–4 RESISTORS IN SERIES AND PARALLEL

Resistors connected end to end—so that the same current flows through each one—are said to be in series. Resistors connected across the same potential difference—allowing parallel paths for the current to flow—are said to be connected in parallel.



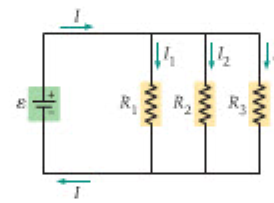
Series

The equivalent resistance, R_{eq}, of resistors connected in series is equal to the sum of the individual resistances:

$$R_{eq} = R_1 + R_2 + R_3 + \cdots = \sum R \qquad \text{21–7}$$

Parallel

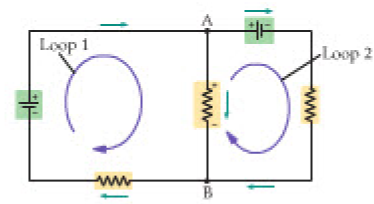
The equivalent resistance, R_{eq}, of resistors connected in parallel is given by the following:



$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots = \sum \frac{1}{R} \qquad \text{21–10}$$

21–5 KIRCHHOFF'S RULES

Kirchhoff's rules are statements of charge conservation and energy conservation as applied to closed electric circuits.



Junction Rule (Charge Conservation)

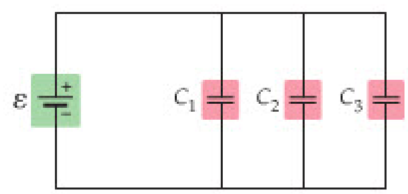
The algebraic sum of all currents meeting at a junction must equal zero. Currents entering the junction are taken to be positive; currents leaving are taken to be negative.

Loop Rule (Energy Conservation)

The algebraic sum of all potential differences around a closed loop is zero. The potential increases in going from the − to the + terminal of a battery and decreases when crossing a resistor in the direction of the current.

21–6 CIRCUITS CONTAINING CAPACITORS

Capacitors connected end to end—so that the same charge is on each one—are said to be in series. Capacitors connected across the same potential difference are said to be connected in parallel.



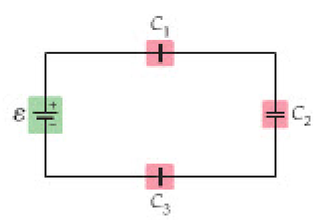
Parallel

The equivalent capacitance, C_{eq}, of capacitors connected in parallel is equal to the sum of the individual capacitances:

$$C_{eq} = C_1 + C_2 + C_3 + \cdots = \sum C \qquad \text{21–14}$$

Series

The equivalent capacitance, C_{eq}, of capacitors connected in series is given by



$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots = \sum \frac{1}{C} \qquad \text{21–17}$$

21–7 *RC* CIRCUITS

In circuits containing both resistors and capacitors, there is a characteristic time, $\tau = RC$, during which significant changes occur. This time is referred to as the time constant. The simplest such circuit, known as an *RC* circuit, consists of one resistor and one capacitor connected in series.