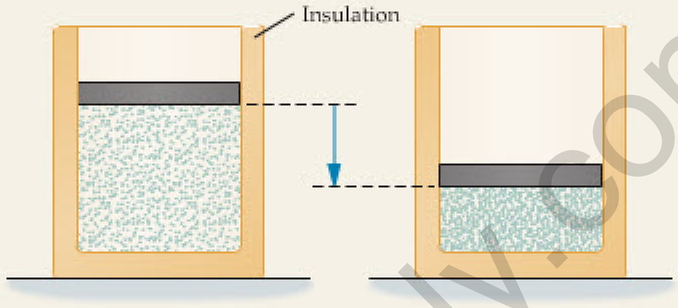
EXAMPLE 18–4 WORK INTO ENERGY

When a certain gas is compressed adiabatically, the amount of work done on it is 640 J. Find the change in internal energy of the gas.

PICTURE THE PROBLEM

Our sketch shows a piston being pushed downward, compressing a gas in an insulated cylinder. The insulation ensures that no heat can flow—as required in an adiabatic process.



STRATEGY

We know that 640 J of work is done on the gas, and we know that no heat is exchanged ($Q = 0$) in an adiabatic process. Thus, we can find ΔU by substituting Q and W into the first law of thermodynamics. One note of caution: Be careful to use the correct sign for the work. In particular, recall that work done *on* a system is negative.

SOLUTION

1. Identify the work and heat for this process: $W = -640\ \text{J}$, $Q = 0$
2. Substitute Q and W into the first law of thermodynamics to find the change in internal energy, ΔU: $\Delta U = Q - W = 0 - (-640\ \text{J}) = 640\ \text{J}$

INSIGHT

Because no energy can enter or leave the system in the form of heat, all the work done on the system goes into increasing its internal energy. This follows from the first law of thermodynamics, $\Delta U = Q - W$, which for $Q = 0$ reduces to $\Delta U = -W$. As a result, the temperature of the gas increases.

A familiar example of this type of effect is the heating that occurs when you pump air into a tire or a ball—the work done on the pump appears as an increased temperature. The effect occurs in the reverse direction as well. When air is let out of a tire, for example, it does work on the atmosphere as it expands, producing a cooling effect that can be quite noticeable. In extreme cases, the cooling can be great enough to create frost on the valve stem of the tire.

PRACTICE PROBLEM

If a system's internal energy decreases by 470 J in an adiabatic process, how much work was done by the system? [**Answer:** $W = +470\ \text{J}$]

Some related homework problems: Problem 17, Problem 24

An adiabatic process can occur when the system is thermally insulated, as in Figure 18–10, or in a system where the change in volume occurs rapidly. For example, if an expansion or compression happens quickly enough, there is no time for heat flow to occur. As a result, the process is adiabatic, even if there is no insulation.

REAL-WORLD PHYSICS
Adiabatic heating and diesel engines

An example of a rapid process is shown in **Figure 18–11**. Here, a piston is fitted into a cylinder that contains a certain volume of gas and a small piece of tissue paper. If the piston is driven downward rapidly, by a sharp impulsive blow, for example, the gas is compressed before heat has a chance to flow. As a result, the temperature of the gas rises rapidly. In fact, the rise in temperature can be enough for the paper to burst into flames.

The same principle applies to the operation of a diesel engine. As you may know, a diesel differs from a standard internal combustion engine in that it has no spark plugs. It doesn't need them. Instead of using a spark to ignite the fuel in a cylinder, it uses adiabatic heating. Fuel and air are admitted into the cylinder, then the piston rapidly compresses the air-fuel mixture. Just as with the piece of paper in Figure 18–11, the rising temperature is sufficient to ignite the fuel and run the engine.

Adiabatic heating is one of the mechanisms being considered to explain the fascinating and enigmatic phenomenon known as *sonoluminescence*. Sonoluminescence occurs when an intense, high-frequency sound wave causes a small gas bubble in water to pulsate. When the sound wave collapses the bubble to its minimum size, which is about a thousandth of a millimeter, the bubble gives off an extremely short burst of light. The light is mostly in the ultraviolet (see Chapter 25), but enough is in the visible range of light to make the bubble appear blue to the eye. For an object to give off light in the ultraviolet it must be extremely hot (see

TABLE 18–2 Thermodynamic Processes and Their Characteristics

Constant pressure	$W = P\Delta V$	$Q = \Delta U + P\Delta V$
Constant volume	$W = 0$	$Q = \Delta U$
Isothermal (constant temperature)	$W = Q$	$\Delta U = 0$
Adiabatic (no heat flow)	$W = -\Delta U$	$Q = 0$

Chapter 30). In fact, it is estimated that the temperature inside a collapsing bubble is at least 10,000 °F, about the same temperature as the surface of the Sun.

The characteristics of constant-pressure, constant-volume, isothermal, and adiabatic processes are summarized in Table 18–2.

18–4 Specific Heats for an Ideal Gas: Constant Pressure, Constant Volume

Recall that the specific heat of a substance is the amount of heat needed to raise the temperature of 1 kg of the substance by 1 Celsius degree. As we know, however, the amount of heat depends on the type of process used to raise the temperature. Thus, we should specify, for example, whether a specific heat applies to a process at constant pressure or constant volume.

If a substance is heated or cooled while open to the atmosphere, the process occurs at constant (atmospheric) pressure. This has been the case in all the specific heat discussions to this point; thus it was not necessary to make a distinction between different types of specific heats. We now wish to consider constant-volume processes as well, and the relationship between constant-volume and constant-pressure specific heats.

A constant-volume process is illustrated in Figure 18–12. Here we see an ideal gas of mass m in a container of fixed volume V. A heat Q flows into the container. As a result of this added heat, the temperature of the gas rises by the amount ΔT, and its pressure increases as well. Now, the specific heat at constant volume, c_v, is defined by the following relation:

$$Q_v = mc_v\Delta T$$

In what follows, it will be more convenient to use the **molar specific heat,** denoted by a capital letter C, which is defined in terms of the number of moles rather than the mass of the substance. Thus, if a gas contains n moles, its molar specific heat at constant volume is given by

$$Q_v = nC_v\Delta T$$

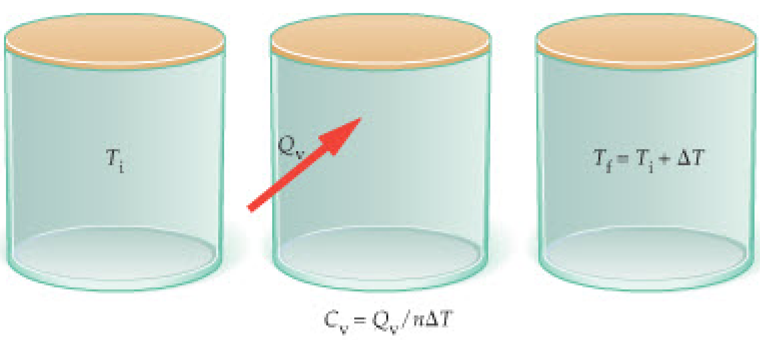
Similarly, a constant-pressure process is illustrated in Figure 18–13. In this case, the gas is held in a container with a moveable piston that applies a constant pressure P. As a heat Q is added to the gas, its temperature increases, which causes the piston to rise—after all, if the piston didn't rise, the pressure of the gas would increase. If the temperature of the gas increases by the amount ΔT, the molar specific heat at constant pressure is given by

$$Q_p = nC_p\Delta T$$

We would now like to obtain a relation between C_p and C_v.

▲ **FIGURE 18–11 Adiabatic heating**

When a piston that fits snugly inside a cylinder is pushed downward rapidly, the temperature of the gas within the cylinder increases before there is time for heat to flow out of the system. Thus, the process is essentially adiabatic. As a result, the temperature of the gas can increase enough to ignite bits of paper in the cylinder. In a diesel engine, the same principle is used to ignite an air-gasoline mixture without a spark plug.



◀ **FIGURE 18–12 Heating at constant volume**

If the heat Q_v is added to n moles of gas, and the temperature rises by ΔT, the molar specific heat at constant volume is $C_v = Q_v/n\Delta T$. No mechanical work is done at constant volume.

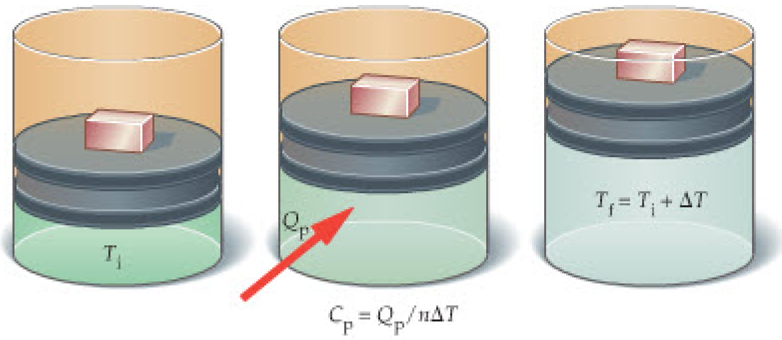


FIGURE 18–13 Heating at constant pressure
If the heat Q_p is added to n moles of gas, and the temperature rises by ΔT, the molar specific heat at constant pressure is $C_p = Q_p/n\Delta T$. Note that the heat Q_p must increase the temperature *and* do mechanical work by lifting the piston.

Before we carry out the mathematics, let's consider the qualitative relationship between these specific heats. This is addressed in the following Conceptual Checkpoint.

CONCEPTUAL CHECKPOINT 18–3 COMPARING SPECIFIC HEATS

How does the molar specific heat at constant pressure, C_p, compare with the molar specific heat at constant volume, C_v? **(a)** $C_p > C_v$; **(b)** $C_p = C_v$; **(c)** $C_p < C_v$.

REASONING AND DISCUSSION

In a constant-volume process, as in Figure 18–12, the heat that is added to a system goes entirely into increasing the temperature, since no work is done. On the other hand, at constant pressure the heat added to a system increases the temperature *and* does mechanical work. This is illustrated in Figure 18–13 where we see that the heat must not only raise the temperature, but also supply enough energy to lift the piston. Thus, more heat is required in the constant-pressure process, and hence that specific heat is greater.

ANSWER

(a) The specific heat at constant pressure is greater than the specific heat at constant volume.

We turn now to a detailed calculation of C_v for a monatomic ideal gas. To begin, rearrange the first law of thermodynamics, $\Delta U = Q - W$, to solve for the heat, Q:

$$Q = \Delta U + W$$

Recall from the previous section, however, that the work is zero, $W = 0$, for any constant-volume process. Hence, for constant volume we have

$$Q_v = \Delta U$$

Finally, noting that $U = \frac{3}{2}NkT = \frac{3}{2}nRT$ yields

$$Q_v = \Delta U = \frac{3}{2}nR\Delta T$$

Comparing with the definition of the molar specific heat, we find

Molar Specific Heat for a Monatomic Ideal Gas at Constant Volume

$$C_v = \frac{3}{2}R \qquad 18\text{–}6$$

Now we perform a similar calculation for constant pressure. In this case, referring again to the previous section, we find that $W = P\Delta V$. Since we are considering an ideal gas, in which $PV = nRT$, it follows that

$$W = P\Delta V = nR\Delta T$$

Combining this with the first law of thermodynamics yields

$$Q_p = \Delta U + W$$
$$= \frac{3}{2}nR\Delta T + nR\Delta T = \frac{5}{2}nR\Delta T$$

Applying the definition of molar specific heat yields

PROBLEM-SOLVING NOTE

Constant Volume Versus Constant Pressure

The heat required to increase the temperature of an ideal gas depends on whether the process is at constant pressure or constant volume. More heat is required when the process occurs at constant pressure.

Molar Specific Heat for a Monatomic Ideal Gas at Constant Pressure

$$C_p = \frac{5}{2}R \qquad 18\text{–}7$$

As expected, the specific heat at constant pressure is larger than the specific heat at constant volume, and the difference is precisely the extra contribution due to the work done in lifting the piston in the constant-pressure case. In particular, we see that

$$C_p - C_v = R \qquad 18\text{–}8$$

Though this relation was derived for a monatomic ideal gas, it holds for all ideal gases, regardless of the structure of their molecules. It is also a good approximation for most real gases, as can be seen in Table 18–3.

TABLE 18–3 $C_p - C_v$ **for Various Gases**

Helium	0.995 R
Nitrogen	1.00 R
Oxygen	1.00 R
Argon	1.01 R
Carbon dioxide	1.01 R
Methane	1.01 R

EXERCISE 18–2

Find the heat required to raise the temperature of 0.200 mol of a monatomic ideal gas by 5.00 C° at **(a)** constant volume and **(b)** constant pressure.

SOLUTION

Applying Equations 18–6 and 18–7, we find

a. $Q_v = \frac{3}{2}nR\Delta T = \frac{3}{2}(0.200 \text{ mol})[8.31 \text{ J}/(\text{mol}\cdot\text{K})](5.00 \text{ K}) = 12.5 \text{ J}$

b. $Q_p = \frac{5}{2}nR\Delta T = \frac{5}{2}(0.200 \text{ mol})[8.31 \text{ J}/(\text{mol}\cdot\text{K})](5.00 \text{ K}) = 20.8 \text{ J}$

Adiabatic Processes

We return now briefly to a consideration of adiabatic processes. As we shall see, the relationship between C_p and C_v is important in determining the behavior of a system undergoing an adiabatic process.

Figure 18–14 shows an adiabatic curve and two isotherms. As mentioned before, the adiabatic curve is steeper and it cuts across the isotherms. For the isotherms, we recall that the curves are described by the equation

$$PV = \text{constant} \qquad (\textit{isothermal})$$

A similar equation applies to adiabats. In this case, using calculus, it can be shown that the appropriate equation is

$$PV^{\gamma} = \text{constant} \qquad (\textit{adiabatic}) \quad 18\text{–}9$$

In this expression, the constant γ is the ratio C_p/C_v:

$$\gamma = \frac{C_p}{C_v} = \frac{\frac{5}{2}R}{\frac{3}{2}R} = \frac{5}{3}$$

This value of γ applies to monatomic ideal gases—and is a good approximation for monatomic real gases as well. The value of γ is different, however, for gases that are diatomic, triatomic, and so on.

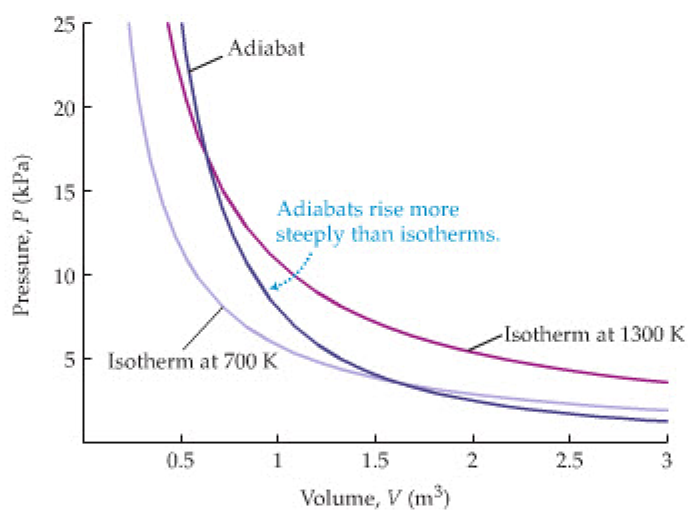


FIGURE 18–14 A comparison between isotherms and adiabats

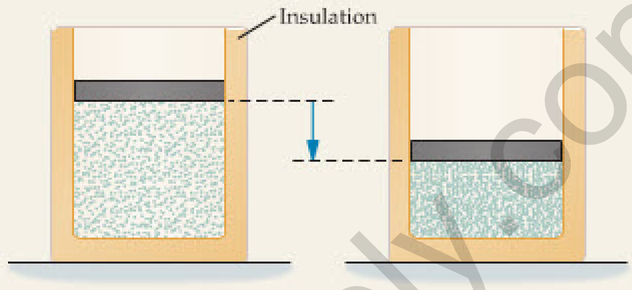
Two isotherms are shown, one for 700 K and one for 1300 K. An adiabat is also shown. Note that the adiabat is a steeper curve than the isotherms.

EXAMPLE 18–5 HOT AIR

A container with an initial volume of $0.0625\ \text{m}^3$ holds 2.50 moles of a monatomic ideal gas at a temperature of 315 K. The gas is now compressed adiabatically to a volume of $0.0350\ \text{m}^3$. Find **(a)** the final pressure and **(b)** the final temperature of the gas.

PICTURE THE PROBLEM

Our sketch shows a gas being compressed from an initial volume of $0.0625\ \text{m}^3$ to a final volume of $0.0350\ \text{m}^3$. The gas starts with a temperature of 315 K, but because no heat can flow outward through the insulation (adiabatic process), the work done on the gas results in an increased temperature.



STRATEGY

a. We can find the final pressure as follows: First, find the initial pressure using the ideal-gas equation of state, $P_iV_i = nRT_i$.

Next, let $P_iV_i^\gamma = P_fV_f^\gamma$, since this is an adiabatic process. Solve this relation for the final pressure.

b. Use the final pressure and volume to find the final temperature, using the ideal-gas relation, $P_fV_f = nRT_f$.

SOLUTION

Part (a)

1. Find the initial pressure, using $PV = nRT$:

$$P_i = \frac{nRT_i}{V_i} = \frac{(2.50\ \text{mol})[8.31\ \text{J}/(\text{mol}\cdot\text{K})](315\ \text{K})}{0.0625\ \text{m}^3} = 105\ \text{kPa}$$

2. Use $PV^\gamma = \text{constant}$ to find a relation for P_f:

$$P_iV_i^\gamma = P_fV_f^\gamma$$
$$P_f = P_i(V_i/V_f)^\gamma$$

3. Substitute numerical values:

$$P_f = (105\ \text{kPa})(0.0625\ \text{m}^3/0.0350\ \text{m}^3)^{5/3} = 276\ \text{kPa}$$

Part (b)

4. Use $PV = nRT$ to solve for the final temperature:

$$T_f = \frac{P_fV_f}{nR} = \frac{(276\ \text{kPa})(0.0350\ \text{m}^3)}{(2.50\ \text{mol})[8.31\ \text{J}/(\text{mol}\cdot\text{K})]} = 465\ \text{K}$$

INSIGHT

Thus, decreasing the volume of the gas by a factor of roughly two has increased its pressure from 105 kPa to 276 kPa and increased its temperature from 315 K to 465 K. This is a specific example of adiabatic heating. Adiabatic cooling is the reverse effect, where the temperature of a gas decreases as its volume increases. For example, if the gas in this system is expanded back to its initial volume of $0.0625\ \text{m}^3$, its temperature will drop from 465 K to 315 K.

PRACTICE PROBLEM

To what volume must the gas be compressed to yield a final pressure of 425 kPa? [**Answer:** $V_f = 0.0270\ \text{m}^3$]

Some related homework problems: Problem 39, Problem 40

Adiabatic heating and cooling can have important effects on the climate of a given region. For example, moisture-laden winds blowing from the Pacific Ocean into western Oregon are deflected upward when they encounter the Cascade Mountains. As the air rises, the atmospheric pressure decreases (see Chapter 15), allowing the air to expand and undergo adiabatic cooling. The result is that the moisture in the air condenses to form clouds and precipitation on the west side of the mountains. (In some cases, where the air holds relatively little moisture, this mechanism may result in isolated, lens-shaped clouds just above the peak of a mountain, as shown in the photograph on the next page.) When the winds continue on to the east side of the mountains they have little moisture remaining; thus, eastern Oregon is in the *rain shadow* of the Cascade Mountains. In addition, as the air descends on the east side of the mountains, it undergoes adiabatic heating. These are the primary reasons why the summers in western Oregon are moist and mild, while the summers in eastern Oregon are hot and dry.

REAL-WORLD PHYSICS
Rain shadows

18–5 The Second Law of Thermodynamics

Have you ever warmed your hands by pressing them against a block of ice? Probably not. But if you think about it, you might wonder why it doesn't work. After all, the first law of thermodynamics would be satisfied if energy simply flowed from the ice to your hands. The ice would get colder while your hands got warmer, and the energy of the universe would remain the same.

As we know, however, this sort of thing just doesn't happen—the spontaneous flow of heat is *always* from warmer objects to cooler objects, and never in the reverse direction. This simple observation, in fact, is one of many ways of expressing the **second law of thermodynamics:**

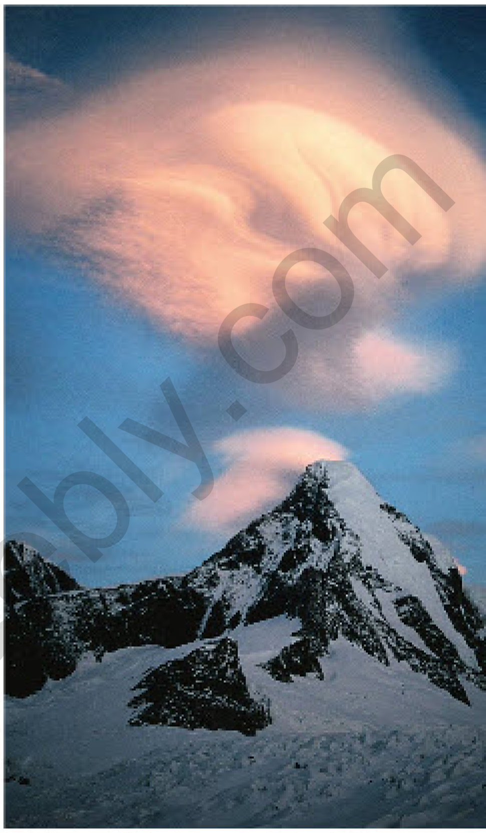
Second Law of Thermodynamics: Heat Flow

When objects of different temperatures are brought into thermal contact, the spontaneous flow of heat that results is always from the high-temperature object to the low-temperature object. Spontaneous heat flow never proceeds in the reverse direction.

Thus, the second law of thermodynamics is more restrictive than the first law; it says that of all the processes that conserve energy, only those that proceed in a certain direction actually occur. In a sense, the second law implies a definite "directionality" to the behavior of nature. For this reason, the second law is sometimes referred to as the "arrow of time."

For example, suppose you saw a movie that showed a snowflake landing on a person's hand and melting to a small drop of water. Nothing would seem particularly noteworthy about the scene from a physics point of view. But if the movie showed a drop of water on a person's hand suddenly freeze into the shape of a snowflake, then lift off the person's hand into the air, it wouldn't take long to realize the film was running backward. It is clear in which direction time should "flow."

We shall study further consequences of the second law in the next few sections. As we do so, we shall find other more precise, but equivalent, ways of stating the second law.



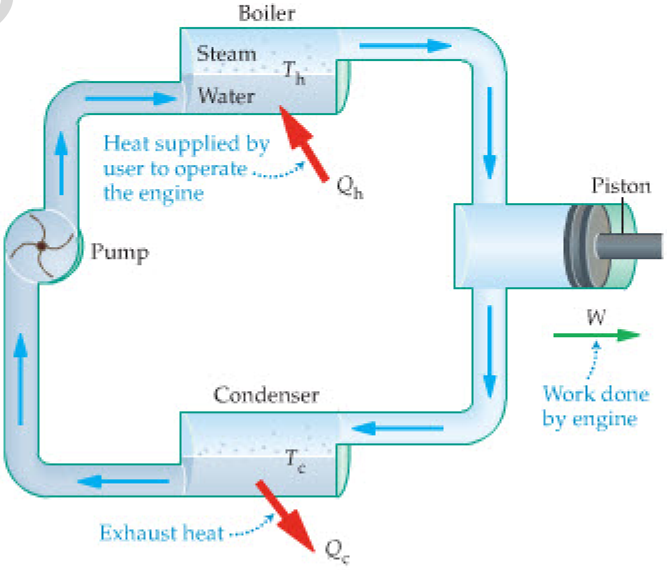
▲ A spectacular lenticular (lens-shaped) cloud floats above a mountain in Tierra del Fuego, at the southern tip of Chile. Lenticular clouds are often seen "parked" above and just downwind of high mountain peaks, even when there are no other clouds in the sky. The reason is that as moisture-laden winds are deflected upward by the mountain, the moisture they contain cools due to adiabatic expansion and condenses to form a cloud.

18–6 Heat Engines and the Carnot Cycle

A **heat engine,** simply put, is a device that converts heat into work. The classic example of this type of device is the steam engine, whose basic elements are illustrated in Figure 18–15. First, some form of fuel (oil, wood, coal, etc.) is used to vaporize water in the boiler. The resulting steam is then allowed to enter the engine itself, where it expands against a piston, doing mechanical work. As the piston moves, it causes gears or wheels to rotate, which delivers the mechanical work to

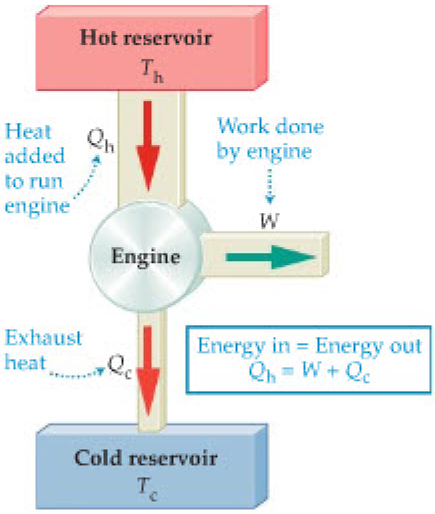
REAL-WORLD PHYSICS

The steam engine



◀ **FIGURE 18–15 A schematic steam engine**

The basic elements of a steam engine are a boiler—where heat converts water to steam—and a piston that can be displaced by the expanding steam. In some engines, the steam is simply exhausted into the atmosphere after it has expanded against the piston. More sophisticated engines send the exhaust steam to a condenser, where it is cooled and condensed back to liquid water, then recycled to the boiler.



▲ **FIGURE 18–16 A schematic heat engine** The engine absorbs a heat Q_h from the hot reservoir, performs the work W, and gives off the heat Q_c to the cold reservoir. Energy conservation gives $Q_h = W + Q_c$, where Q_h and Q_c are the magnitudes of the hot- and cold-temperature heats.

the external world. After leaving the engine, the steam proceeds to the condenser, where it gives off heat to the cool air in the atmosphere and condenses to liquid form.

What all heat engines have in common are: (i) A high-temperature region, or reservoir, that supplies heat to the engine (the boiler in the steam engine); (ii) a low-temperature reservoir where "waste" heat is released (the condenser in the steam engine); and (iii) an engine that operates in a cyclic fashion. These features are illustrated schematically in Figure 18–16. In addition, though not shown in the figure, heat engines have a working substance (steam in the steam engine) that causes the engine to operate.

To begin our analysis, we note that a certain amount of heat, Q_h, is supplied to the engine from the high temperature or "hot" reservoir during each cycle. Of this heat, a fraction appears as work, W, and the rest is given off as waste heat, Q_c, at a relatively low temperature to the "cold" reservoir. There is no change in energy for the engine, because it returns to its initial state at the completion of each cycle. Letting Q_h and Q_c denote magnitudes, so that both quantities are positive, energy conservation can be written as $Q_h = W + Q_c$, or

$$W = Q_h - Q_c \qquad \text{18–10}$$

As we shall see, the second law of thermodynamics requires that heat engines must *always* exhaust a finite amount of heat to a cold reservoir.

Of particular interest for any engine is its **efficiency**, e, which is simply the fraction of the heat supplied to the engine that appears as work. Thus, we define the efficiency to be

$$e = \frac{W}{Q_h} \qquad \text{18–11}$$

Using the energy-conservation result just derived in Equation 18–10, we find

Efficiency of a Heat Engine, *e*

$$e = \frac{W}{Q_h} = \frac{Q_h - Q_c}{Q_h} = 1 - \frac{Q_c}{Q_h} \qquad \text{18–12}$$

SI unit: dimensionless

For example, if $e = 0.20$, we say that the engine is 20% efficient. In this case, 20% of the input heat is converted to work, $W = 0.20Q_h$, and 80% goes to waste heat, $Q_c = 0.80Q_h$. Efficiency, then, can be thought of as the ratio of how much you receive (work) to how much you have to pay to run the engine (input heat).

▲ At left, a modern version of Hero's engine, invented by the Greek mathematician and engineer Hero of Alexandria. In this simple heat engine, the steam that escapes from a heated vessel of water is directed tangentially, causing the vessel to rotate. This converts the thermal energy supplied to the water into mechanical energy, in the form of rotational motion. At right, a steam engine of slightly more recent design hauls passengers up and down Mt. Washington in New Hampshire. Note in the photo that the locomotive is belching two clouds, one black and one white. Can you explain their origin?

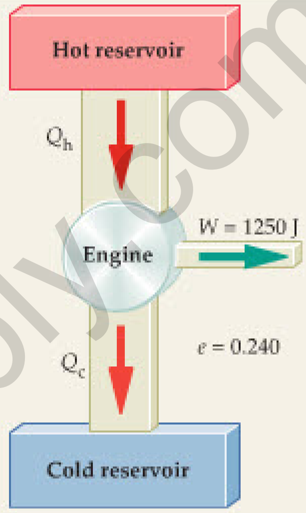
EXAMPLE 18–6 HEAT INTO WORK

A heat engine with an efficiency of 24.0% performs 1250 J of work. Find **(a)** the heat absorbed from the hot reservoir, and **(b)** the heat given off to the cold reservoir.

PICTURE THE PROBLEM

Our sketch shows a schematic of the heat engine. We know the amount of work that is done and the efficiency of the engine. We seek the heats Q_h and Q_c.

Note that an efficiency of 24.0% means that $e = 0.240$.



STRATEGY

a. We can find the heat absorbed from the hot reservoir directly from the definition of efficiency, $e = W/Q_h$.

b. We can find Q_c by using energy conservation, $W = Q_h - Q_c$, or by using the expression for efficiency in terms of the heats, $e = 1 - Q_c/Q_h$.

SOLUTION

Part (a)

1. Use $e = W/Q_h$ to solve for the heat Q_h:

$$e = W/Q_h$$
$$Q_h = \frac{W}{e} = \frac{1250\ \text{J}}{0.240} = 5210\ \text{J}$$

Part (b)

2. Use energy conservation to solve for Q_c:

$$W = Q_h - Q_c$$
$$Q_c = Q_h - W = 5210\ \text{J} - 1250\ \text{J} = 3960\ \text{J}$$

3. Use the efficiency, expressed in terms of Q_h and Q_c, to find Q_c:

$$e = 1 - Q_c/Q_h$$
$$Q_c = (1 - e)Q_h = (1 - 0.240)(5210\ \text{J}) = 3960\ \text{J}$$

INSIGHT

Note that when the efficiency of a heat engine is less than one-half (50%), as in this case, the amount of heat given off as waste to the cold reservoir is more than the amount of heat converted to work.

PRACTICE PROBLEM

What is the efficiency of a heat engine that does 1250 J of work and gives off 5250 J of heat to the cold reservoir? [**Answer:** $e = 0.192$]

Some related homework problems: Problem 45, Problem 46

A temperature difference is essential to the operation of a heat engine. As heat flows from the hot to the cold reservoir in Figure 18–16, for example, the heat engine is able to tap into that flow and convert part of it to work—the greater the efficiency of the engine, the more heat converted to work. The second law of thermodynamics imposes limits, however, on the maximum efficiency a heat engine can have. We explore these limits next.

The Carnot Cycle and Maximum Efficiency

In 1824, the French engineer Sadi Carnot (1796–1832) published a book entitled *Reflections on the Motive Power of Fire* in which he considered the following question: Under what conditions will a heat engine have maximum efficiency? To address this question, let's consider a heat engine that operates between a single hot reservoir at the fixed temperature T_h and a single cold reservoir at the fixed temperature T_c. Carnot's result, known today as **Carnot's theorem**, can be expressed as follows:

Carnot's Theorem

If an engine operating between two constant-temperature reservoirs is to have maximum efficiency, it must be an engine in which all processes are reversible.

In addition, all reversible engines operating between the same two temperatures, T_c and T_h, have the same efficiency.

We should point out that no real engine can ever be perfectly reversible, just as no surface can be perfectly frictionless. Nonetheless, the concept of a reversible engine is a useful idealization.

Carnot's theorem is remarkable for a number of reasons. First, consider what the theorem says: No engine, no matter how sophisticated or technologically advanced, can exceed the efficiency of a reversible engine. We can strive to improve the technology of heat engines, but there is an upper limit to the efficiency that can never be exceeded. Second, the theorem is just as remarkable for what it does not say. It says nothing, for example, about the working substance that is used in the engine—it is as valid for a liquid or solid working substance as for one that is gaseous. Furthermore, it says nothing about the type of reversible engine that is used, what the engine is made of, or how it is constructed. Diesel engine, jet engine, rocket engine—none of these things matter. In fact, all that *does* matter are the two temperatures, T_c and T_h.

Recall that the efficiency of a heat engine can be written as follows:

$$e = 1 - \frac{Q_c}{Q_h}$$

Since the efficiency e depends only on the temperatures T_c and T_h, according to Carnot's theorem, it follows that Q_c/Q_h must also depend only on T_c and T_h. In fact, Lord Kelvin used this observation to propose that, instead of using a thermometer to measure temperature, we measure the efficiency of a heat engine and from this determine the temperature. Thus, he suggested that we *define* the ratio of the temperatures of two reservoirs, T_c/T_h, to be equal to the ratio of the heats Q_c/Q_h:

$$\frac{Q_c}{Q_h} = \frac{T_c}{T_h}$$

If we choose the size of a degree in this temperature scale to be equal to 1 C°, then we have, in fact, defined the Kelvin temperature scale discussed in Chapter 16. Thus, if T_h and T_c are given in kelvins, the maximum efficiency of a heat engine is:

Maximum Efficiency of a Heat Engine

$$e_{max} = 1 - \frac{T_c}{T_h} \qquad 18\text{–}13$$

PROBLEM-SOLVING NOTE

Maximum Efficiency

The maximum efficiency a heat engine can have is determined solely by the temperature of the hot (T_h) and cold (T_c) reservoirs. The numerical value of this efficiency is $e = 1 - T_c/T_h$. Remember, however, that the temperatures must be expressed in the Kelvin scale for this expression to be valid.

Suppose for a moment that we *could* construct an ideal engine, perfectly reversible and free from all forms of friction. Would this ideal engine have 100% efficiency? No, it would not. From Equation 18–13 we can see that the only way the efficiency of a heat engine could be 100% (that is, $e_{max} = 1$) would be for T_c to be 0 K. As we shall see in the last section of this chapter, this is ruled out by the third law of thermodynamics. Hence, the maximum efficiency will always be less than 100%. No matter how perfect the engine, some of the input heat will always be wasted—given off as Q_c—rather than converted to work.

Since the efficiency is defined to be $e = W/Q_h$, it follows that the maximum work a heat engine can do with the input heat Q_h is

$$W_{max} = e_{max}Q_h = \left(1 - \frac{T_c}{T_h}\right)Q_h \qquad 18\text{–}14$$

If the hot and cold reservoirs have the same temperature, so that $T_c = T_h$, it follows that the maximum efficiency is zero. As a result, the amount of work that such an engine can do is also zero. As mentioned before, a heat engine requires different temperatures in order to operate. For example, for a fixed T_c, the higher the temperature of T_h the greater the efficiency.

Finally, even though Carnot's theorem may seem quite different from the second law of thermodynamics, they are, in fact, equivalent. It can be shown, for example, that if Carnot's theorem were violated, it would be possible for heat to flow spontaneously from a cold object to a hot object.

CONCEPTUAL CHECKPOINT 18–4 COMPARING EFFICIENCIES

Suppose you have a heat engine that can operate in one of two different modes. In mode 1, the temperatures of the two reservoirs are $T_c = 200$ K and $T_h = 400$ K; in mode 2, the temperatures are $T_c = 400$ K and $T_h = 600$ K. Is the efficiency of mode 1 **(a)** greater than, **(b)** less than, or **(c)** equal to the efficiency of mode 2?

REASONING AND DISCUSSION

At first, you might think that since the temperature difference is the same in the two modes, the efficiency is the same as well. This is not the case, however, since efficiency depends on the *ratio* of the two temperatures ($e = 1 - T_c/T_h$) rather than on their difference. In this case, the efficiency of mode 1 is $e_1 = 1 - 1/2 = 1/2$ and the efficiency of mode 2 is $e_2 = 1 - 2/3 = 1/3$. Thus, mode 1, even though it operates at the lower temperatures, is more efficient.

ANSWER

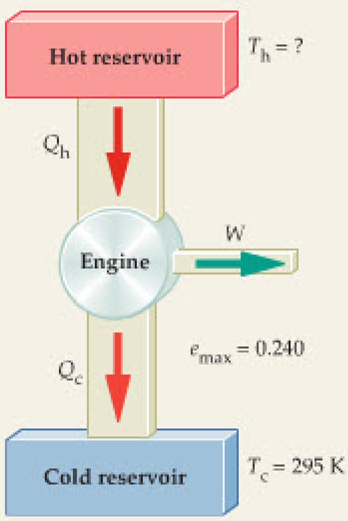
(a) The efficiency of mode 1 is greater than the efficiency of mode 2.

ACTIVE EXAMPLE 18–2 FIND THE TEMPERATURE

If the heat engine in Example 18–6 is operating at its maximum efficiency of 24.0%, and its cold reservoir is at a temperature of 295 K, what is the temperature of the hot reservoir?

SOLUTION *(Test your understanding by performing the calculations indicated in each step.)*

1. Write the efficiency, e, in terms of the hot and cold temperatures: $e = 1 - T_c/T_h$
2. Solve for T_h: $T_h = T_c/(1 - e)$
3. Substitute numerical values for T_c and e to find T_h: $T_h = 388$ K



INSIGHT

Though an efficiency of 24.0% may seem low, it is characteristic of many real engines.

YOUR TURN

What is the efficiency of this heat engine if T_h is increased by 20 K? What is the efficiency if, instead, T_c is reduced by 20 K?

(Answers to **Your Turn** *problems are given in the back of the book.)*

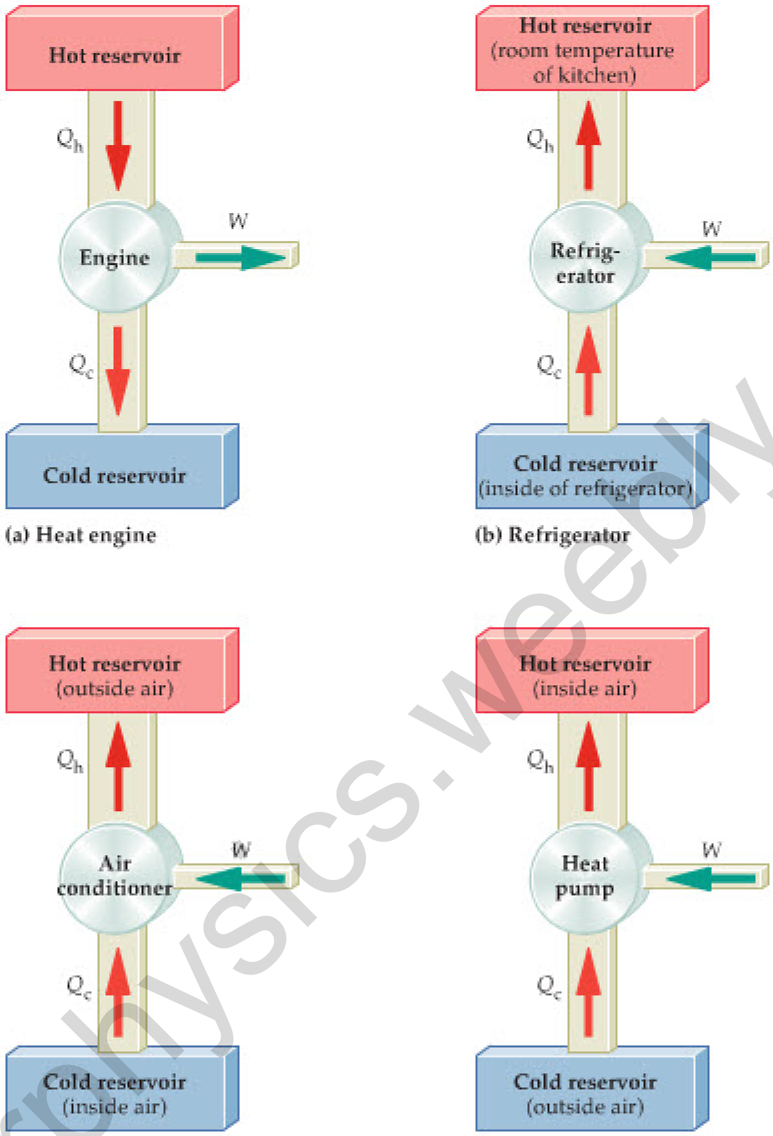
We can summarize the conclusions of this section as follows: The first law of thermodynamics states that you cannot get something for nothing. To be specific, you cannot get more work out of a heat engine than the amount of heat you put in. The best you can do is break even. The second law of thermodynamics is more restrictive than the first law; it says that you can't even break even—some of the input heat must be wasted. It's a law of nature.

18–7 Refrigerators, Air Conditioners, and Heat Pumps

When we stated the second law of thermodynamics in Section 18–5, we said that the spontaneous flow of heat is always from high temperature to low temperature. The key word here is "spontaneous." It *is* possible for heat to flow "uphill,"

FIGURE 18–17 Cooling and heating devices

Schematic comparison of a generalized heat engine **(a)** compared with a refrigerator, an air conditioner, and a heat pump. In the refrigerator **(b)**, a work W is done to remove a heat Q_c from the cold reservoir (the inside of the refrigerator). By energy conservation, the heat given off to the hot reservoir (the kitchen) is $Q_h = W + Q_c$. An air conditioner **(c)** removes heat from the cool air inside a house and exhausts it into the hot air of the atmosphere. A heat pump **(d)** moves heat from a cold reservoir to a hot reservoir, just like an air conditioner. The difference is that the reservoirs are switched, so that heat is pumped into the house rather than out to the atmosphere.



(a) Heat engine (b) Refrigerator (c) Air conditioner (d) Heat pump

REAL-WORLD PHYSICS
Refrigerators

from a cold object to a hot one, but it doesn't happen spontaneously—work must be done on the system to make it happen, just as work must be done to pump water from a well. Refrigerators, air conditioners, and heat pumps are devices that use work to transfer heat from a cold object to a hot object.

Let us first compare the operation of a heat engine, Figure 18–17 (a), and a **refrigerator**, Figure 18–17 (b). Note that all the arrows are reversed in the refrigerator; in effect, a refrigerator is a heat engine running backward. In particular, the refrigerator uses a work W to remove a certain amount of heat, Q_c, from the cold reservoir (the interior of the refrigerator). It then exhausts an even larger amount of heat, Q_h, to the hot reservoir (the air in the kitchen). By energy conservation, it follows that

$$Q_h = Q_c + W$$

Thus, as a refrigerator operates, it warms the kitchen at the same time that it cools the food stored within it.

To design an effective refrigerator, you would like it to remove as much heat from its interior as possible for the smallest amount of work. After all, the work is supplied by electrical energy that must be paid for each month. Thus, we define the **coefficient of performance, COP, for a refrigerator** as an indicator of its effectiveness:

Coefficient of Performance for a Refrigerator, COP

$$\text{COP} = \frac{Q_c}{W} \qquad 18\text{–}15$$

SI unit: dimensionless

Typical values for the coefficient of performance are in the range 2 to 6.

EXERCISE 18–3

A refrigerator has a coefficient of performance of 2.50. How much work must be supplied to this refrigerator in order to remove 225 J of heat from its interior?

SOLUTION

Solving Equation 18–15 for the work yields

$$W = \frac{Q_c}{\text{COP}} = \frac{225\ \text{J}}{2.50} = 90.0\ \text{J}$$

Thus, 90.0 J of work removes 225 J of heat from the refrigerator, and exhausts 90.0 J + 225 J = 315 J of heat into the kitchen.

REAL-WORLD PHYSICS
Air conditioners

An **air conditioner,** Figure 18–17 (c), is basically a refrigerator in which the cold reservoir is the room that is being cooled. To be specific, the air conditioner uses electrical energy to "pump" heat from the cool room to the warmer air outside. As with the refrigerator, more heat is exhausted to the hot reservoir than is removed from the cold reservoir; that is, $Q_h = Q_c + W$, as before.

CONCEPTUAL CHECKPOINT 18–5 ROOM TEMPERATURE

You haven't had time to install your new air conditioner in the window yet, so as a short-term measure you decide to place it on the dining-room table and turn it on to cool things off a bit. As a result, does the air in the dining room **(a)** get warmer, **(b)** get cooler, or **(c)** stay at the same temperature?

REASONING AND DISCUSSION

You might think the room stays at the same temperature, since the air conditioner draws heat from the room as usual, but then exhausts heat back into the room that would normally be sent outside. However, the motor of the air conditioner is doing work in order to draw heat from the room, and the heat that would normally be exhausted outdoors is equal to the heat drawn from the room *plus* the work done by the motor: $Q_h = Q_c + W$. Thus, the net effect is that the motor of the air conditioner is continually adding heat to the room, causing it to get warmer.

ANSWER

(a) The air in the dining room gets warmer.

REAL-WORLD PHYSICS
Heat pumps

Finally, a **heat pump** can be thought of as an air conditioner with the reservoirs switched. As we see in Figure 18–17 (d), a heat pump does a work W to remove an amount of heat Q_c from the cold reservoir of outdoor air, then exhausts a heat Q_h into the hot reservoir of air in the room. Just as with the refrigerator and the air conditioner, the heat going to the hot reservoir is $Q_h = Q_c + W$.

In an **ideal,** reversible heat pump with only two operating temperatures, T_c and T_h, the Carnot relationship $Q_c/Q_h = T_c/T_h$ holds, just as it does for a heat engine. Thus, if you want to add a heat Q_h to a room, the work that must be done to accomplish this is

$$W = Q_h - Q_c = Q_h\left(1 - \frac{Q_c}{Q_h}\right) = Q_h\left(1 - \frac{T_c}{T_h}\right) \qquad 18\text{–}16$$

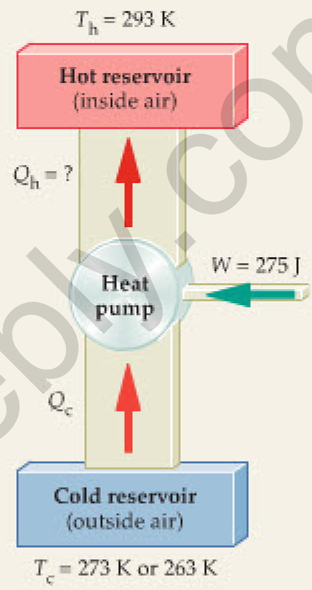
We use this result in the next Example.

EXAMPLE 18–7 PUMPING HEAT

An ideal heat pump, one that satisfies the Carnot relation given in Equation 18–16, is used to heat a room that is at 293 K. If the pump does 275 J of work, how much heat does it supply to the room if the outdoor temperature is **(a)** 273 K or **(b)** 263 K?

PICTURE THE PROBLEM

Our sketch shows the heat pump doing 275 J of mechanical work to transfer the heat Q_h to the hot reservoir at the temperature 293 K. The temperature of the cold reservoir is 273 K for part (a) and 263 K for part (b). We wish to determine Q_h for each of these cases.



STRATEGY

For an ideal heat pump, we know that $W = Q_h(1 - T_c/T_h)$. Therefore, given the hot and cold temperatures, as well as the mechanical work, it is straightforward to determine the heat delivered to the hot reservoir, Q_h.

SOLUTION

1. Solve Equation 18–16 for the heat Q_h: $Q_h = W/(1 - T_c/T_h)$

Part (a)

2. Substitute $W = 275$ J, $T_c = 273$ K, and $T_h = 293$ K into the expression for Q_h:

$$Q_h = \frac{W}{1 - \frac{T_c}{T_h}} = \frac{275\text{ J}}{1 - \frac{273\text{ K}}{293\text{ K}}} = 4030\text{ J}$$

Part (b)

3. Substitute $W = 275$ J, $T_c = 263$ K, and $T_h = 293$ K into the expression for Q_h:

$$Q_h = \frac{W}{1 - \frac{T_c}{T_h}} = \frac{275\text{ J}}{1 - \frac{263\text{ K}}{293\text{ K}}} = 2690\text{ J}$$

INSIGHT

As one might expect, the same amount of work provides less heat when the outside temperature is lower. That is, more work must be done on a colder day to provide the same heat to the inside air.

In addition, note that if 275 J of heat is supplied to an electric heater, then 275 J of heat is given to the air in the room. When that same energy is used to run a heat pump, a good deal more than 275 J of heat is added to the room.

PRACTICE PROBLEM

How much work must be done by this heat pump to supply 2550 J of heat on a day when the outside temperature is 253 K? [**Answer:** $W = 348$ J]

Some related homework problems: Problem 58, Problem 59

Since the purpose of a heat pump is to add heat to a room, and we want to add as much heat as possible for the least work, the **coefficient of performance, COP, for a heat pump** is defined as follows:

Coefficient of Performance for a Heat Pump, COP

$$\text{COP} = \frac{Q_h}{W} \qquad 18\text{–}17$$

SI unit: dimensionless

The COP for a heat pump, which is usually in the range of 3 to 4, depends on the inside and outside temperatures. We use the COP in the next Exercise.

EXERCISE 18–4

A heat pump with a coefficient of performance equal to 3.5 supplies 2500 J of heat to a room. How much work is required?

SOLUTION

Solving Equation 18–17 for the work, W, we find

$$W = \frac{Q_h}{COP} = \frac{2500\ J}{3.5} = 710\ J$$

18–8 Entropy

In this section we introduce a new quantity that is as fundamental to physics as energy or temperature. This quantity is referred to as the entropy, and it is related to the amount of disorder in a system. For example, a messy room has more entropy than a neat one, a pile of bricks has more entropy than a building constructed from the bricks, a freshly laid egg has more entropy than one that is just about to hatch, and a puddle of water has more entropy than the block of ice from which it melted. We begin by considering the connection between entropy and heat, and later develop more fully the connection between disorder and entropy.

When discussing heat engines, we saw that if an engine is reversible it satisfies the following relation:

$$\frac{Q_c}{Q_h} = \frac{T_c}{T_h}$$

Rearranging slightly, we can rewrite this as

$$\frac{Q_c}{T_c} = \frac{Q_h}{T_h}$$

Notice that the quantity Q/T is the same for both the hot and the cold reservoirs. This relationship prompted the German physicist Rudolf Clausius (1822–1888) to propose the following definition: The **entropy**, S, is a quantity whose change is given by the heat Q divided by the absolute temperature T:

Definition of Entropy Change, ΔS

$$\Delta S = \frac{Q}{T} \quad 18\text{–}18$$

SI unit: J/K

PROBLEM-SOLVING NOTE

Calculating Entropy Change

When calculating the entropy change, $\Delta S = Q/T$, be sure to convert the temperature T to kelvins. It is *only* when this is done that the correct value for ΔS can be obtained.

For this definition to be valid, the heat Q must be transferred reversibly at the fixed Kelvin temperature T. Note that if heat is added to a system ($Q > 0$), the entropy of the system increases; if heat is removed from a system ($Q < 0$), its entropy decreases.

Entropy is a state function, just like the internal energy, U. This means that the value of S depends only on the state of a system, and not on how the system gets to that state. It follows, then, that the *change* in entropy, ΔS, depends only on the initial and final states of a system. Thus, if a process is irreversible—so that Equation 18–18 *does not* hold—we can still calculate ΔS by using a reversible process to connect the same initial and final states.

EXAMPLE 18–8 MELTS IN YOUR HAND

(a) Calculate the change in entropy when a 0.125-kg chunk of ice melts at 0 °C. Assume the melting occurs reversibly. **(b)** Suppose heat is now drawn reversibly from the 0 °C meltwater, causing a decrease in entropy of 112 J/K. How much ice freezes in the process?

PICTURE THE PROBLEM

In our sketch we show a 0.125-kg chunk of ice at the temperature 0 °C. As the ice absorbs the heat Q from its surroundings, it melts to water at 0 °C. Because the system absorbs heat, its entropy increases. When heat is drawn out of the meltwater, the entropy will decrease.

CONTINUED ON NEXT PAGE

CONTINUED FROM PREVIOUS PAGE

STRATEGY

a. The entropy change is $\Delta S = Q/T$, where $T = 0\,°C = 273\,K$. To find the heat Q, we note that to melt the ice we must add to it the latent heat of fusion, L_f. Thus, the heat is $Q = mL_f$, where $L_f = 33.5 \times 10^4\,J/kg$.

b. The magnitude of heat withdrawn from the meltwater is $Q = T|\Delta S|$, where $\Delta S = -112\,J/K$. Using this heat, we can calculate the mass of re-frozen ice using $m = Q/L_f$.

SOLUTION

Part (a)

1. Find the heat that must be absorbed by the ice for it to melt: $Q = mL_f = (0.125\,kg)(33.5 \times 10^4\,J/kg) = 4.19 \times 10^4\,J$
2. Calculate the change in entropy: $\Delta S = \dfrac{Q}{T} = \dfrac{4.19 \times 10^4\,J}{273\,K} = 153\,J/K$

Part (b)

3. Find the magnitude of the heat that is removed from the meltwater: $Q = T|\Delta S| = (273\,K)(112\,J/K) = 3.06 \times 10^4\,J$
4. Use this heat to determine the amount of water that is re-frozen: $m = \dfrac{Q}{L_f} = (3.06 \times 10^4\,J)/(33.5 \times 10^4\,J/kg) = 0.0913\,kg$

INSIGHT

Note that we were careful to convert the temperature of the system from 0 °C to 273 K before we applied $\Delta S = Q/T$. This must always be done when calculating the entropy change. If we had neglected to do the conversion in this case, we would have found an infinite increase in entropy—which is clearly unphysical.

In addition, we see that the entropy change is positive in part (a) and negative in part (b). This illustrates the general rule that entropy increases when heat is added to a system, and decreases when it is removed. Finally, we could have retained the negative sign of the heat in part (b). If we had, the mass in Step 4 would have been negative. This sounds odd at first, but the *minus sign* would simply indicate that *mass is removed* from the water to become ice. As it is, we avoided the sign issue by simply noting that removing heat results in the formation of ice.

PRACTICE PROBLEM

Find the mass of ice that would be required to give an entropy change of 275 J/K. [**Answer:** $m = 0.224\,kg$]

Some related homework problems: Problem 66, Problem 67

Let's apply the definition of entropy change to the case of a reversible heat engine. First, a heat Q_h leaves the hot reservoir at the temperature T_h. Thus, the entropy of this reservoir decreases by the amount Q_h/T_h:

$$\Delta S_h = -\frac{Q_h}{T_h}$$

Recall that Q_h is the magnitude of the heat leaving the hot reservoir; hence, the minus sign is used to indicate a decrease in entropy. Similarly, heat is added to the cold reservoir; hence, its entropy increases by the amount Q_c/T_c:

$$\Delta S_c = \frac{Q_c}{T_c}$$

The total entropy change for this system is

$$\Delta S_{total} = \Delta S_h + \Delta S_c = -\frac{Q_h}{T_h} + \frac{Q_c}{T_c}$$

Since we know that $Q_h/T_h = Q_c/T_c$ it follows that the total entropy change vanishes:

$$\Delta S_{total} = -\frac{Q_h}{T_h} + \frac{Q_c}{T_c} = 0$$

What is special about a reversible engine, then, is the fact that its entropy does not change.

On the other hand, a real engine will always have a lower efficiency than a reversible engine operating between the same temperatures. This means that in a real engine less of the heat from the hot reservoir is converted to work; hence, more heat is given off as waste heat to the cold reservoir. Thus, for a given value of Q_h, the heat Q_c is greater in an irreversible engine than in a reversible one. As a result, instead of $Q_c/T_c = Q_h/T_h$, we have

$$\frac{Q_c}{T_c} > \frac{Q_h}{T_h}$$

Therefore, if an engine is irreversible, the total entropy change is positive:

$$\Delta S_{total} = -\frac{Q_h}{T_h} + \frac{Q_c}{T_c} > 0$$

In general, *any irreversible process results in an increase of entropy.*

These results can be summarized in the following general statement:

Entropy in the Universe

The total entropy of the universe *increases* whenever an *irreversible* process occurs.

The total entropy of the universe is *unchanged* whenever a *reversible* process occurs.

Since all *real* processes are irreversible (with reversible processes being a useful idealization), the total entropy of the universe continually increases. Thus, in terms of the entropy, the universe moves in only one direction—toward an ever-increasing entropy. This is quite different from the behavior with regard to energy, which remains constant no matter what type of process occurs.

In fact, this statement about entropy in the universe is yet another way of expressing the second law of thermodynamics. Recall, for example, that our original statement of the second law said that heat flows spontaneously from a hot object to a cold object. During this flow of heat the entropy of the universe increases, as we show in the next Example. Hence, the direction in which heat flows is seen to be the result of the general principle of entropy increase in the universe. Again, we see a directionality in nature—the ever-present "arrow of time."

PROBLEM-SOLVING NOTE

Entropy Change

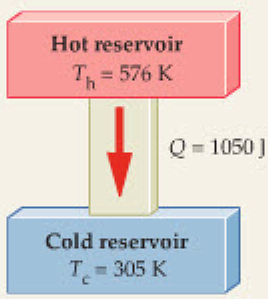
Though it is tempting to treat entropy like energy, setting the final value equal to the initial value, this is not the case in general. Only in a reversible process is the entropy unchanged—otherwise it increases. Still, the entropy of part of a system can decrease, as long as the entropy of other parts increases by the same amount or more.

EXAMPLE 18–9 ENTROPY IS NOT CONSERVED!

A hot reservoir at the temperature 576 K transfers 1050 J of heat irreversibly to a cold reservoir at the temperature 305 K. Find the change in entropy of the universe.

PICTURE THE PROBLEM

The relevant physical situation is shown in our sketch. Note that the heat $Q = 1050$ J is transferred from the hot reservoir at the temperature $T_h = 576$ K directly to the cold reservoir at the temperature $T_c = 305$ K.



STRATEGY

As the heat Q leaves the hot reservoir, its entropy *decreases* by Q/T_h. When the same heat Q enters the cold reservoir, its entropy *increases* by the amount Q/T_c. Summing these two contributions gives the entropy change of the universe.

SOLUTION

1. Calculate the entropy change of the hot reservoir: $\Delta S_h = -\frac{Q}{T_h} = -\frac{1050\ \text{J}}{576\ \text{K}} = -1.82\ \text{J/K}$

2. Calculate the entropy change of the cold reservoir: $\Delta S_c = \frac{Q}{T_c} = \frac{1050\ \text{J}}{305\ \text{K}} = 3.44\ \text{J/K}$

3. Sum these contributions to obtain the entropy change of the universe:

$$\Delta S_{universe} = \Delta S_h + \Delta S_c = -\frac{Q}{T_h} + \frac{Q}{T_c}$$
$$= -1.82\ \text{J/K} + 3.44\ \text{J/K} = 1.62\ \text{J/K}$$

CONTINUED ON NEXT PAGE

CONTINUED FROM PREVIOUS PAGE

INSIGHT

Note that the decrease in entropy of the hot reservoir is more than made up for by the increase in entropy of the cold reservoir. This is a general result.

PRACTICE PROBLEM

What amount of heat must be transferred between these reservoirs for the entropy of the universe to increase by 1.50 J/K? [**Answer:** $Q = 972$ J]

Some related homework problems: Problem 68, Problem 72

When certain processes occur, it sometimes appears as if the entropy of the universe has decreased. On closer examination, however, it can always be shown that there is a larger increase in entropy elsewhere that results in an overall increase. This issue is addressed in the next Conceptual Checkpoint.

CONCEPTUAL CHECKPOINT 18–6 ENTROPY CHANGE

You put a tray of water in the kitchen freezer, and some time later it has turned to ice. Has the entropy of the universe **(a)** increased, **(b)** decreased, or **(c)** stayed the same?

REASONING AND DISCUSSION

It might seem that the entropy of the universe has decreased. After all, heat is removed from the water to freeze it, and, as we know, removing heat from an object lowers its entropy. On the other hand, we also know that the freezer does work to draw heat from the water; hence, it exhausts more heat into the kitchen than it absorbs from the water. Detailed calculations always show that the entropy of the heated air in the kitchen increases by more than the entropy of the water decreases, thus the entropy of the universe increases—as it must for *any* real process.

ANSWER

(a) The entropy of the universe has increased.

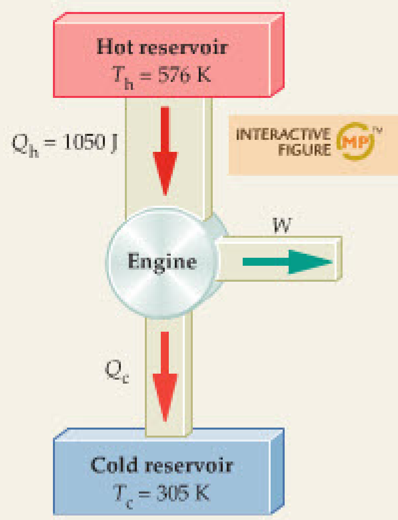
As the entropy in the universe increases, the amount of work that can be done is diminished. For example, the heat flow in Example 18–9 resulted in an increase in the entropy of the universe by the amount 1.62 J/K. If this same heat had been used in a reversible engine, however, it could have done work, and since the engine was reversible, the entropy of the universe would have stayed the same. In the next Active Example, we calculate the work that could be done with a reversible engine.

ACTIVE EXAMPLE 18–3 FIND THE WORK

Suppose a reversible heat engine operates between the two heat reservoirs described in Example 18–9. Find the amount of work done by such an engine when 1050 J of heat is drawn from the hot reservoir.

SOLUTION *(Test your understanding by performing the calculations indicated in each step.)*

1. Calculate the efficiency of this engine: $e = 1 - T_c/T_h = 0.470$
2. Multiply the efficiency by Q_h to find the work done: $W = eQ_h = 494$ J



INSIGHT

Since this engine is reversible, its total entropy change must be zero. The decrease in entropy of the hot reservoir is $-(1050\text{ J})/576\text{ K} = -1.82$ J/K. It follows that the increase in entropy of the cold reservoir must have the same magnitude. The amount of heat that flows into the cold reservoir, Q_c, is $Q_h - W = 1050\text{ J} - 494\text{ J} = 556$ J. This heat causes an entropy increase equal to $556\text{ J}/305\text{ K} = +1.82$ J/K, as expected. Thus, the reason the engine exhausts the heat Q_c is to produce zero net change in entropy. If the engine were irreversible, it would exhaust a heat greater than 556 J, and this would create a net increase in entropy and a reduction in the amount of work done. Clearly, then, a reversible engine produces the maximum amount of work.

YOUR TURN

Suppose this engine is not reversible, and that only 455 J of work is done when 1050 J of heat is drawn from the hot reservoir. What is the entropy increase of the universe in this case?

(Answers to **Your Turn** *problems are given in the back of the book.)*

Note that when 1050 J of heat is simply transferred from the hot reservoir to the cold reservoir, as in Example 18–9, the entropy of the universe increases by 1.62 J/K. When this same heat is transferred reversibly, with an ideal engine, the entropy of the universe stays the same, but 494 J of work is done. The connection between the entropy increase of the irreversible process and the work done by a reversible engine is very simple:

$$W = T_c\,\Delta S_{\text{universe}} = (305\text{ K})(1.62\text{ J/K}) = 494\text{ J}$$

To see that this expression is valid in general, recall that in Example 18–9 the total change in entropy is $\Delta S_{\text{universe}} = Q/T_c - Q/T_h$. That is, the heat $Q_h = Q$ is withdrawn from the hot reservoir (lowering the entropy by the amount Q/T_h), and the same heat $Q_c = Q$ is added to the cold reservoir (increasing the entropy by the larger amount, Q/T_c). If we multiply this increase in entropy by the temperature of the cold reservoir, T_c, we have $T_c\,\Delta S_{\text{universe}} = Q - QT_c/T_h = Q(1 - T_c/T_h)$. Recalling that the efficiency of an ideal engine is $e = 1 - T_c/T_h$, we see that $T_c\,\Delta S_{\text{universe}} = Qe$. Finally, the work done by an ideal engine is $W = eQ_h$, or in this case, $W = eQ$, since $Q_h = Q$. Therefore, we see that $W = T_c\,\Delta S_{\text{universe}}$, as expected.

In general, a process in which the entropy of the universe increases is one in which less work is done than if the process had been reversible. Thus, we lose forever the ability for that work to be done, because to restore the universe to its former state would mean lowering its entropy, which cannot be done. Thus, with every increase in entropy, there is that much less work that can be done by the universe.

For this reason, entropy is sometimes referred to as a measure of the "quality" of energy. When an irreversible process occurs, and the entropy of the universe increases, we say that the energy of the universe has been "degraded" because less of it is available to do work. This process of increasing degradation of energy and increasing entropy in the universe is a continuing aspect of nature.

18–9 Order, Disorder, and Entropy

In the previous section we considered entropy from the point of view of thermodynamics. We saw that as heat flows from a hot object to a cold object the entropy of the universe increases. In this section we show that entropy can also be thought of as a measure of the amount of **disorder** in the universe.

We begin with the situation of heat flow from a hot to a cold object. In **Figure 18–18 (a)** we show two bricks, one hot and the other cold. As we know from kinetic

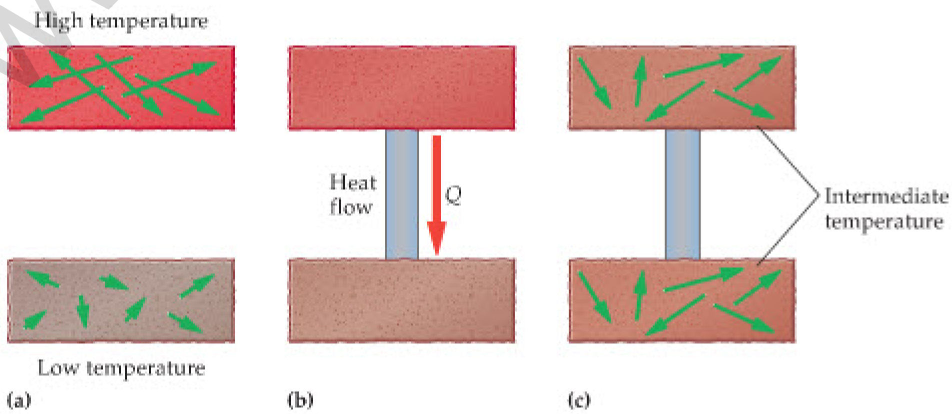


FIGURE 18–18 Heat flow and disorder
(a) Initially, two bricks have different temperatures, and hence different average kinetic energies. **(b)** Heat flows from the hot brick to the cold brick. **(c)** The final result is that both bricks have the same intermediate temperature, and all the molecules have the same average kinetic energy. Thus, the initial orderly segregation of molecules by kinetic energy has been lost.

▲ All processes that occur spontaneously increase the entropy of the universe. In this case, the movements of the water molecules become more random and chaotic when they reach the tumultuous, swirling pool at the bottom of the falls. In addition, some of their kinetic energy is converted into thermal energy—the most disordered and degraded form of energy.

theory, the molecules in the hot brick have more kinetic energy than the molecules in the cold brick. This means that the system is rather orderly, in that all the high-kinetic-energy molecules are grouped together in the hot brick, and all the low-kinetic-energy molecules are grouped together in the cold brick. There is a definite regularity, or order, to the distribution of the molecular speeds.

Now bring the bricks into thermal contact, as in **Figure 18–18 (b)**. The result is a flow of heat from the hot brick to the cold brick until the temperatures become equal. The final result is indicated in **Figure 18–18 (c)**. During the heat transfer the entropy of the universe increases, as we know, and the system loses the nice orderly distribution it had in the beginning. Now, all the molecules have the same average kinetic energy; hence, the system is randomized, or disordered. We are led to the following conclusion:

> As the entropy of a system increases, its disorder increases as well; that is, an *increase* in entropy is the same as a *decrease* in order.

Note that if heat had flowed in the opposite direction—from the cold brick to the hot brick—the ordered distribution of molecules would have been reinforced, rather than lost.

To take another example, consider the 0.125-kg chunk of ice discussed in Example 18–8. As we saw there, the entropy of the universe increases as the ice melts. Now let's consider what happens on the molecular level. To begin, the molecules are well ordered in their crystalline positions. As heat is absorbed, however, the molecules begin to free themselves from the ice and move about randomly in the growing puddle of water. Thus, the regular order of the solid is lost. Again, we see that as entropy increases, so too does the disorder of the molecules.

Thus, the second law of thermodynamics can be stated as the principle that the disorder of the universe is continually increasing. Everything that happens in the universe is simply making it a more disorderly place. And there is nothing you can do to prevent it—nothing you can do will result in the universe being more ordered. Just as freezing a tray of water to make ice actually results in more entropy—and more disorder—in the universe, so does any action you take.

Heat Death

If one carries the previous discussion to its logical conclusion, it seems that the universe is "running down." That is, the disorder of the universe constantly increases, and as it does, the amount of energy available to do work decreases. If this process continues, might there come a day when no more work can be done? And if that day does come, what then?

This is one possible scenario for the fate of the universe, and it is referred to as the "heat death" of the universe. In this scenario, heat continues to flow from hotter regions in space (like stars) to cooler regions (like planets) until, after many billions of years, all objects in the universe have the same temperature. With no temperature differences left, there can be no work done, and the universe would cease to do anything of particular interest. Not a pretty picture, but certainly a possibility. The universe may simply continue with its present expansion until the stars burn out and the galaxies fade away like the dying embers of a scattered campfire.

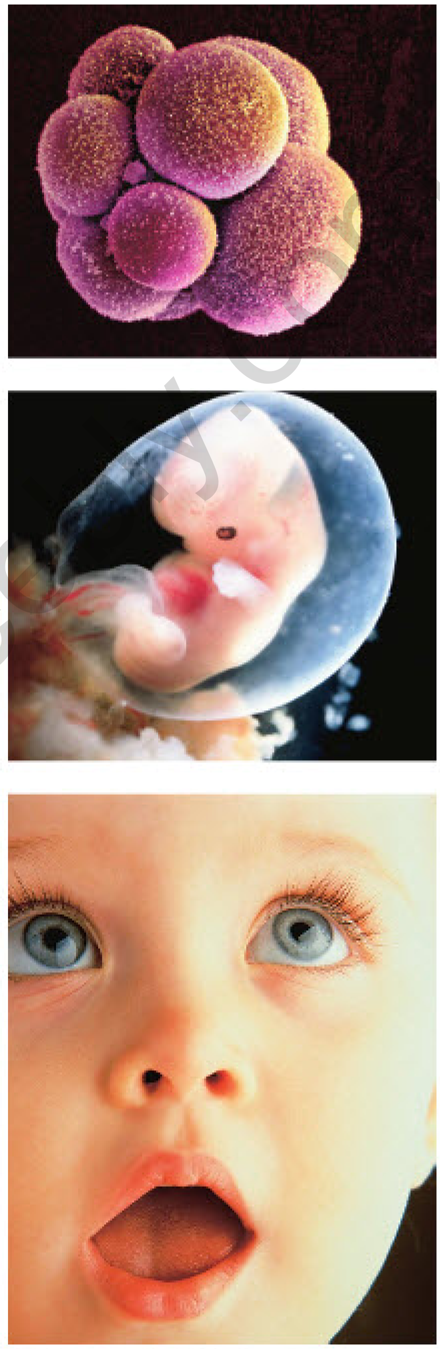
Living Systems

REAL-WORLD PHYSICS: BIO
Entropy and life

So far we have focused on the rather gloomy prospect of the universe constantly evolving toward greater disorder. Is it possible, however, that life is an exception to this rule? After all, we know that an embryo utilizes simple raw materials to produce a complex, highly ordered living organism. Similarly, the well-known biological aphorism "ontogeny recapitulates phylogeny," while not strictly correct, expresses the fact that the development of an individual organism from embryo to adult often reflects certain aspects of the evolutionary development of the species as a whole. Thus, over time, species often evolve toward more complex forms. Finally, living systems are able to use disordered raw materials in the environment to produce orderly structures in which to live. It seems, then, that there are many ways in which living systems produce increasing order.

This conclusion is flawed, however, since it fails to take into account the entropy of the environment in which the organism lives. It is similar to the conclusion that a freezer violates the second law of thermodynamics because it reduces the entropy of water as it freezes it into ice. This analysis neglects the fact that the freezer exhausts heat into the room, increasing the entropy of the air by an amount that is greater than the entropy decrease of the water. In the same way, living organisms constantly give off heat to the atmosphere as a by-product of their metabolism, increasing its entropy. Thus, if we build a house from a pile of bricks—decreasing the entropy of the bricks—the heat we give off during our exertions increases the entropy of the atmosphere more than enough to give a net increase in entropy.

Finally, all living organisms can be thought of as heat engines, tapping into the flow of energy from one place to another to produce mechanical work. Plants, for example, tap into the flow of energy from the high temperature of the Sun to the cold of deep space and use a small fraction of this energy to sustain themselves and reproduce. Animals consume plants and generate heat within their bodies as they metabolize their food. A fraction of the energy released by the metabolism is in turn converted to mechanical work. Living systems, then, obey the same laws of physics as steam engines and refrigerators—they simply produce different results as they move the universe toward greater disorder.



▲ Many species, including humans, develop from a single fertilized egg into a complex multicellular organism. In the process they create large, intricately ordered molecules such as proteins and DNA from smaller, simpler precursors. In the metabolic processes of living things, however, heat is produced, increasing the entropy of the universe as a whole. Thus, the second law is not violated.

18–10 The Third Law of Thermodynamics

Finally, we consider the **third law of thermodynamics,** which states that there is no temperature lower than absolute zero, and that absolute zero is unattainable. It is possible to cool an object to temperatures arbitrarily close to absolute zero—experiments have reached temperatures as low as 4.5×10^{-10} K—but no object can ever be cooled to precisely 0 K.

As an analogy to cooling toward absolute zero, imagine walking toward a wall, with each step half the distance between you and the wall. Even if you take an infinite number of steps, you will still not reach the wall. You can get arbitrarily close, of course, but you never get all the way there.

The same sort of thing happens when cooling. To cool an object, you can place it in thermal contact with an object that is colder. Heat transfer will occur, with your object ending up cooler and the other object ending up warmer. In particular, suppose you had a collection of objects at 0 K to use for cooling. You put your object in contact with one of the 0-K objects and your object cools, while the 0-K object warms slightly. You continue this process, each time throwing away the "warmed up" 0-K object and using a new one. Each time you cool your object it gets closer to 0 K, without ever actually getting there.

In light of this discussion, we can express the third law of thermodynamics as follows:

The Third Law of Thermodynamics
It is impossible to lower the temperature of an object to absolute zero in a finite number of steps.

As with the second law of thermodynamics, this law can be expressed in a number of different but equivalent ways. The essential idea, however, is always the same: Absolute zero is the limiting temperature and, though it can be approached arbitrarily closely, it can never be attained.

THE BIG PICTURE PUTTING PHYSICS IN CONTEXT

LOOKING BACK

The concepts of heat, work, and temperature (Chapters 16 and 17) are used throughout this chapter.

Our understanding of ideal gases (Chapter 17) is put to good use in Sections 18–3 and 18–4, where we study common thermal processes.

Specific heat (Chapter 16) is revisited in Section 18–4, where we extend the concept to processes that occur at constant pressure or constant volume.

LOOKING AHEAD

Chapter 21 shows how an electric current can generate heat, much like the heating caused by friction as one object slides against another.

In Chapter 23 we shall see how the mechanical work produced by a heat engine can be converted to electrical energy by a generator, in the form of an electric current.

The expansion of the universe is discussed in Chapter 32. During the expansion, the temperature of the universe has decreased, very much as it does for an ideal gas undergoing an adiabatic expansion.

CHAPTER SUMMARY

18–1 THE ZEROTH LAW OF THERMODYNAMICS

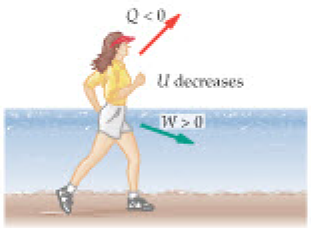
When two objects have the same temperature, they are in thermal equilibrium.

18–2 THE FIRST LAW OF THERMODYNAMICS

The first law of thermodynamics is a statement of energy conservation that includes heat.

If U is the internal energy of an object, Q is the heat added to it, and W is the work done by the object, the first law of thermodynamics can be written as follows:

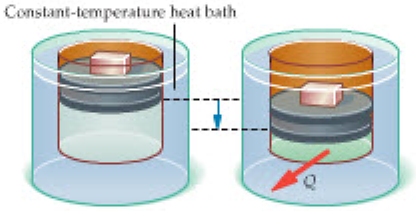
$$\Delta U = Q - W \qquad \text{18–3}$$



State Function

The internal energy U depends only on the state of a system; that is, on its temperature, pressure, and volume. The value of U is independent of how a system is brought to a certain state.

18–3 THERMAL PROCESSES



Quasi-Static

A quasi-static process is one in which a system is moved slowly from one state to another. The change of state is so slow that the system may be considered to be in equilibrium at any given time during the process.

Reversible

In a reversible process it is possible to return the system and its surroundings to their initial states.

Irreversible

Irreversible processes cannot be "undone." When the system is returned to its initial state, the surroundings have been altered.

Work

In general, the work done during a process is equal to the area under the process curve in a PV plot.

Constant Pressure

In a PV plot, a constant-pressure process is represented by a horizontal line. The work done at constant pressure is $W = P\Delta V$.

Constant Volume

In a PV plot, a constant-volume process is represented by a vertical line. The work done at constant volume is zero; $W = 0$.

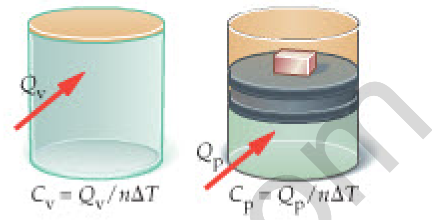
Isothermal Process

In a PV plot, an isothermal process is represented by PV = constant. The work done in an isothermal expansion from V_i to V_f is $W = nRT \ln(V_f/V_i)$.

Adiabatic Process

An adiabatic process occurs with no heat transfer; that is, $Q = 0$.

18–4 SPECIFIC HEATS FOR AN IDEAL GAS: CONSTANT PRESSURE, CONSTANT VOLUME



Specific heats have different values depending on whether they apply to a process at constant pressure or a process at constant volume.

Molar Specific Heat

The molar specific heat, C, is defined by $Q = nC\Delta T$, where n is the number of moles.

Constant Volume

The molar specific heat for an ideal monatomic gas at constant volume is

$$C_V = \tfrac{3}{2}R \qquad 18\text{–}6$$

Constant Pressure

The molar specific heat for an ideal monatomic gas at constant pressure is

$$C_P = \tfrac{5}{2}R \qquad 18\text{–}7$$

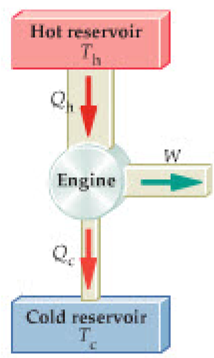
Adiabatic Process

In a PV plot, an adiabatic process is represented by PV^{γ} = constant, where γ is the ratio C_P/C_V. For a monatomic, ideal gas, $\gamma = 5/3$.

18–5 THE SECOND LAW OF THERMODYNAMICS

When objects of different temperatures are brought into thermal contact, the spontaneous flow of heat that results is always from the high-temperature object to the low-temperature object. Spontaneous heat flow never proceeds in the reverse direction.

18–6 HEAT ENGINES AND THE CARNOT CYCLE



A heat engine is a device that converts heat into work; for example, a steam engine.

Efficiency

The efficiency e of a heat engine that takes in the heat Q_h from a hot reservoir, exhausts a heat Q_c to a cold reservoir, and does the work W is

$$e = \frac{W}{Q_h} = \frac{Q_h - Q_c}{Q_h} = 1 - \frac{Q_c}{Q_h} \qquad 18\text{–}12$$

Carnot's Theorem

If an engine operating between two constant-temperature reservoirs is to have maximum efficiency, it must be an engine in which all processes are reversible. In addition, all reversible engines operating between the same two temperatures have the same efficiency.

Maximum Efficiency

The maximum efficiency of a heat engine operating between the Kelvin temperatures T_h and T_c is

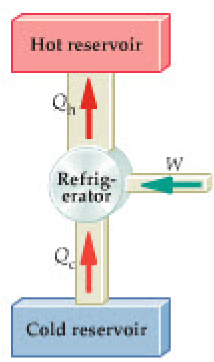
$$e_{max} = 1 - \frac{T_c}{T_h} \qquad 18\text{–}13$$

Maximum Work

If a heat engine takes in the heat Q_h from a hot reservoir, the maximum work it can do is

$$W_{max} = e_{max}Q_h = \left(1 - \frac{T_c}{T_h}\right)Q_h \qquad 18\text{–}14$$

18–7 REFRIGERATORS, AIR CONDITIONERS, AND HEAT PUMPS



Refrigerators, air conditioners, and heat pumps are devices that use work to make heat flow from a cold region to a hot region.

Coefficient of Performance

The coefficient of performance of a refrigerator or air conditioner doing the work W to remove a heat Q_c from a cold reservoir is

$$\text{COP} = \frac{Q_c}{W} \qquad 18\text{–}15$$

Heat Pump

In an ideal heat pump, the work W that must be done to deliver a heat Q_h to a hot reservoir at the temperature T_h by extracting a heat Q_c from a cold reservoir at the temperature T_c is

$$W = Q_h - Q_c = Q_h\left(1 - \frac{Q_c}{Q_h}\right) = Q_h\left(1 - \frac{T_c}{T_h}\right) \qquad 18\text{–}16$$

The coefficient of performance for a heat pump is

$$\text{COP} = \frac{Q_h}{W} \qquad 18\text{–}17$$

18–8 ENTROPY

Like the internal energy U, the entropy S is a state function.

Change in Entropy

The change in entropy during a reversible exchange of the heat Q at the Kelvin temperature T is

$$\Delta S = \frac{Q}{T} \qquad 18\text{–}18$$

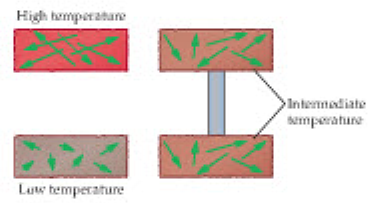
Entropy in the Universe

The total entropy of the universe increases whenever an irreversible process occurs. *Note:* Entropy is not conserved.

In an idealized reversible process the entropy of the universe is unchanged.

18–9 ORDER, DISORDER, AND ENTROPY

Entropy is a measure of the disorder of a system. As entropy increases, a system becomes more disordered.



Heat Death

A possible fate of the universe is heat death, in which everything is at the same temperature and no more work can be done.

18–10 THE THIRD LAW OF THERMODYNAMICS

It is impossible to lower the temperature of an object to absolute zero in a finite number of steps.

PROBLEM-SOLVING SUMMARY

Type of Problem	Relevant Physical Concepts	Related Examples
Relate the heat and work exchanged by a system to its change in internal energy.	The heat, work, and internal energy of a system are related by the first law of thermodynamics; $\Delta U = Q - W$.	Example 18–1
Calculate the work done by a system during a given process in terms of the corresponding PV plot.	The work done by an expanding system is equal to the area under the curve that represents the process on a PV plot. If the system is compressed, the work is equal to the negative of the area, meaning that work is done on the system rather than by it.	Example 18–2 Active Example 18–1
Find the efficiency of a heat engine.	If a heat engine takes in a heat Q_h and does the work W, its efficiency, e, is given by $e = W/Q_h$. Since W is the difference between the heat going into the engine, Q_h, and the heat leaving the engine, Q_c, the efficiency can also be written as $e = (Q_h - Q_c)/Q_h = 1 - Q_c/Q_h$. Finally, the efficiency can also be related to the absolute temperature of the hot (T_h) and cold (T_c) reservoirs as follows: $e = 1 - T_c/T_h$.	Example 18–6 Active Example 18–2
Determine the change in entropy of a system.	If a system exchanges a heat Q at the absolute temperature T, its entropy S changes by the following amount: $\Delta S = Q/T$. If heat is added to the system, its entropy increases; if heat is removed from the system, its entropy decreases.	Examples 18–8, 18–9

CONCEPTUAL QUESTIONS

(Answers to odd-numbered Conceptual Questions can be found in the back of the book.)

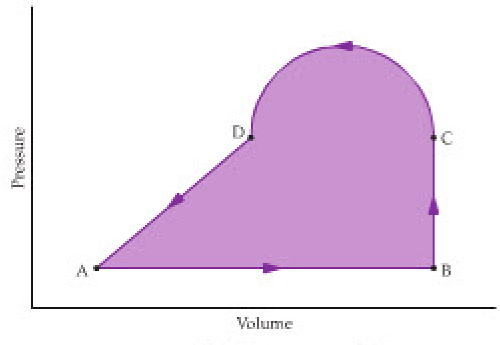
1. If an engine has a reverse gear, does this make it reversible?
2. The temperature of a substance is held fixed. Is it possible for heat to flow **(a)** into or **(b)** out of this system? For each case, give an explanation if your answer is no. If your answer is yes, give a specific example.
3. A substance is thermally insulated, so that no heat can flow between it and its surroundings. Is it possible for the temperature of this substance to **(a)** increase or **(b)** decrease? For each case, give an explanation if your answer is no. If your answer is yes, give a specific example.
4. Heat is added to a substance. Is it safe to conclude that the temperature of the substance will rise? Give an explanation if your answer is no. If your answer is yes, give a specific example.
5. The temperature of a substance is increased. Is it safe to conclude that heat was added to the substance? Give an explanation if your answer is no. If your answer is yes, give a specific example.
6. Are there thermodynamic processes in which all the heat absorbed by an ideal gas goes completely into mechanical work? If so, give an example.
7. Is it possible to convert a given amount of mechanical work completely into heat? Explain.
8. An ideal gas is held in an insulated container at the temperature T. All the gas is initially in one-half of the container, with a partition separating the gas from the other half of the container, which is a vacuum. If the partition ruptures, and the gas expands to fill the entire container, what is its final temperature?
9. Which of the following processes are approximately reversible? **(a)** Lighting a match. **(b)** Pushing a block up a frictionless inclined plane. **(c)** Frying an egg. **(d)** Swimming from one end of a pool to the other. **(e)** Stretching a spring by a small amount. **(f)** Writing a report for class.
10. Which law of thermodynamics would be violated if heat were to spontaneously flow between two objects of equal temperature?
11. Heat engines always give off a certain amount of heat to a low-temperature reservoir. Would it be possible to use this "waste" heat as the heat input to a second heat engine, and then use the "waste" heat of the second engine to run a third engine, and so on?
12. A heat pump uses 100 J of energy as it operates for a given time. Is it possible for the heat pump to deliver more than 100 J of heat to the inside of the house in this same time? Explain.
13. If you clean up a messy room, putting things back where they belong, you decrease the room's entropy. Does this violate the second law of thermodynamics? Explain.
14. Which law of thermodynamics is most pertinent to the statement that "all the king's horses and all the king's men couldn't put Humpty Dumpty back together again?"
15. Which has more entropy: **(a)** popcorn kernels, or the resulting popcorn; **(b)** two eggs in a carton, or an omelet made from the eggs; **(c)** a pile of bricks, or the resulting house; **(d)** a piece of paper, or the piece of paper after it has been burned?

PROBLEMS AND CONCEPTUAL EXERCISES

Note: Answers to odd-numbered Problems and Conceptual Exercises can be found in the back of the book. **IP** *denotes an integrated problem, with both conceptual and numerical parts;* **BIO** *identifies problems of biological or medical interest;* **CE** *indicates a conceptual exercise.* **Predict/Explain** *problems ask for two responses:* **(a)** *your prediction of a physical outcome, and* **(b)** *the best explanation among three provided. On all problems, red bullets (•, ••, •••) are used to indicate the level of difficulty.*

SECTION 18–2 THE FIRST LAW OF THERMODYNAMICS

1. • **CE** Give the change in internal energy of a system if **(a)** $W = 50$ J, $Q = 50$ J; **(b)** $W = -50$ J, $Q = -50$ J; or **(c)** $W = 50$ J, $Q = -50$ J.
2. • **CE** A gas expands, doing 100 J of work. How much heat must be added to this system for its internal energy to increase by 200 J?
3. • A swimmer does 6.7×10^5 J of work and gives off 4.1×10^5 J of heat during a workout. Determine ΔU, W, and Q for the swimmer.
4. • When 1210 J of heat are added to one mole of an ideal monatomic gas, its temperature increases from 272 K to 276 K. Find the work done by the gas during this process.
5. • Three different processes act on a system. **(a)** In process A, 42 J of work are done on the system and 77 J of heat are added to the system. Find the change in the system's internal energy. **(b)** In process B, the system does 42 J of work and 77 J of heat are added to the system. What is the change in the system's internal energy? **(c)** In process C, the system's internal energy decreases by 120 J while the system performs 120 J of work on its surroundings. How much heat was added to the system?
6. • An ideal gas is taken through the four processes shown in Figure 18–19. The changes in internal energy for three of these processes are as follows: $\Delta U_{AB} = +82$ J; $\Delta U_{BC} = +15$ J; $\Delta U_{DA} = -56$ J. Find the change in internal energy for the process from C to D.



▲ **FIGURE 18–19** Problems 6 and 80

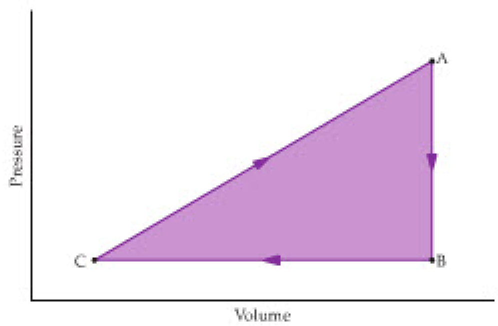
7. •• A basketball player does 2.43×10^5 J of work during her time in the game, and evaporates 0.110 kg of water. Assuming a latent heat of 2.26×10^6 J/kg for the perspiration (the same as for water), determine **(a)** the change in the player's internal energy and **(b)** the number of nutritional calories the player has converted to work and heat.
8. •• **IP** One mole of an ideal monatomic gas is initially at a temperature of 263 K. **(a)** Find the final temperature of the gas if 3280 J of heat are added to it and it does 722 J of work. **(b)** Suppose the amount of gas is doubled to two moles. Does the final temperature found in part (a) increase, decrease, or stay the same? Explain.

9. •• **IP Energy from Gasoline** Burning a gallon of gasoline releases 1.19×10^8 J of internal energy. If a certain car requires 5.20×10^5 J of work to drive one mile, **(a)** how much heat is given off to the atmosphere each mile, assuming the car gets 25.0 miles to the gallon? **(b)** If the miles per gallon of the car is increased, does the amount of heat released to the atmosphere increase, decrease, or stay the same? Explain.

10. •• A cylinder contains 4.0 moles of a monatomic gas at an initial temperature of 27 °C. The gas is compressed by doing 560 J of work on it, and its temperature increases by 130 °C. How much heat flows into or out of the gas?

11. •• An ideal gas is taken through the three processes shown in Figure 18–20. Fill in the missing entries in the following table:

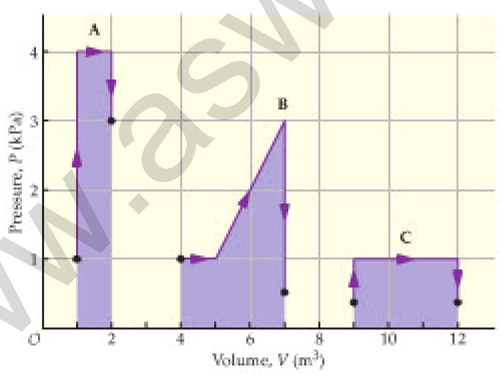
	Q	W	ΔU
A → B	−53 J	(a)	(b)
B → C	−280 J	−130 J	(c)
C → A	(e)	150 J	(d)



▲ **FIGURE 18–20** Problems 11 and 91

SECTION 18–3 THERMAL PROCESSES

12. • **CE** Figure 18–21 shows three different multistep processes, labeled A, B, and C. Rank these processes in order of increasing work done by a gas that undergoes the process. Indicate ties where appropriate.



▲ **FIGURE 18–21** Problems 12 and 76

13. • A system consisting of an ideal gas at the constant pressure of 110 kPa gains 920 J of heat. Find the change in volume of the system if the internal energy of the gas increases by **(a)** 920 J or **(b)** 360 J.

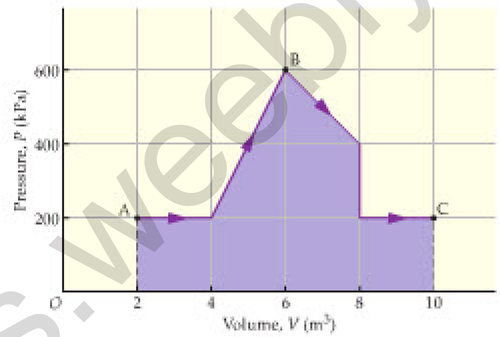
14. • An ideal gas is compressed at constant pressure to one-half its initial volume. If the pressure of the gas is 120 kPa, and 790 J of work is done on it, find the initial volume of the gas.

15. • As an ideal gas expands at constant pressure from a volume of 0.74 m^3 to a volume of 2.3 m^3 it does 93 J of work. What is the gas pressure during this process?

16. • The volume of a monatomic ideal gas doubles in an isothermal expansion. By what factor does its pressure change?

17. •• **IP** **(a)** If the internal energy of a system increases as the result of an adiabatic process, is work done on the system or by the system? **(b)** Calculate the work done on or by the system in part (a) if its internal energy increases by 670 J.

18. •• **(a)** Find the work done by a monatomic ideal gas as it expands from point A to point C along the path shown in Figure 18–22. **(b)** If the temperature of the gas is 220 K at point A, what is its temperature at point C? **(c)** How much heat has been added to or removed from the gas during this process?

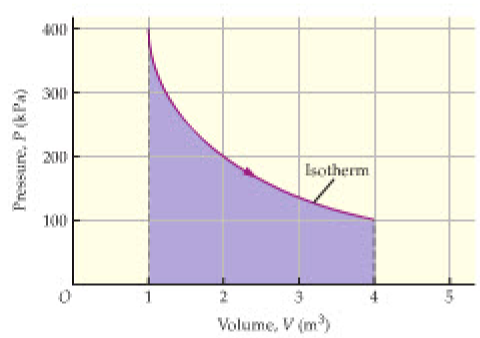


▲ **FIGURE 18–22** Problems 18 and 19

19. •• **IP** A fluid expands from point A to point B along the path shown in Figure 18–22. **(a)** How much work is done by the fluid during this expansion? **(b)** Does your answer to part (a) depend on whether the fluid is an ideal gas? Explain.

20. •• **IP** If 8.00 moles of a monatomic ideal gas at a temperature of 245 K are expanded isothermally from a volume of 1.12 L to a volume of 4.33 L, calculate **(a)** the work done and **(b)** the heat flow into or out of the gas. **(c)** If the number of moles is doubled, by what factors do your answers to parts (a) and (b) change? Explain.

21. •• Suppose 145 moles of a monatomic ideal gas undergo an isothermal expansion from 1.00 m^3 to 4.00 m^3, as shown in Figure 18–23. **(a)** What is the temperature at the beginning and at the end of this process? **(b)** How much work is done by the gas during this expansion?



▲ **FIGURE 18–23** Problems 21, 22, and 79

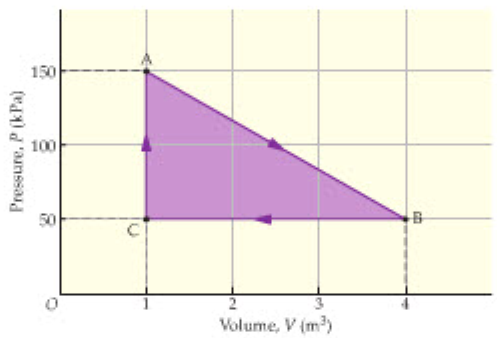
22. •• **IP** A system consisting of 121 moles of a monatomic ideal gas undergoes the isothermal expansion shown in Figure 18–23. **(a)** During this process, does heat enter or leave the system? Explain. **(b)** Is the magnitude of the heat exchanged with the gas from 1.00 m^3 to 2.00 m^3 greater than, less than, or the same as it is from 3.00 m^3 to 4.00 m^3? Explain. Calculate the heat

exchanged with the gas **(c)** from 1.00 m^3 to 2.00 m^3 and **(d)** from 3.00 m^3 to 4.00 m^3.

23. •• **IP** **(a)** A monatomic ideal gas expands at constant pressure. Is heat added to the system or taken from the system during this process? **(b)** Find the heat added to or taken from the gas in part (a) if it expands at a pressure of 130 kPa from a volume of 0.76 m^3 to a volume of 0.93 m^3.

24. •• During an adiabatic process, the temperature of 3.92 moles of a monatomic ideal gas drops from 485 °C to 205 °C. For this gas, find **(a)** the work it does, **(b)** the heat it exchanges with its surroundings, and **(c)** the change in its internal energy.

25. •• An ideal gas follows the three-part process shown in Figure 18–24. At the completion of one full cycle, find **(a)** the net work done by the system, **(b)** the net change in internal energy of the system, and **(c)** the net heat absorbed by the system.



▲ **FIGURE 18–24** Problems 25 and 27

26. •• With the pressure held constant at 210 kPa, 49 mol of a monatomic ideal gas expands from an initial volume of 0.75 m^3 to a final volume of 1.9 m^3. **(a)** How much work was done by the gas during the expansion? **(b)** What were the initial and final temperatures of the gas? **(c)** What was the change in the internal energy of the gas? **(d)** How much heat was added to the gas?

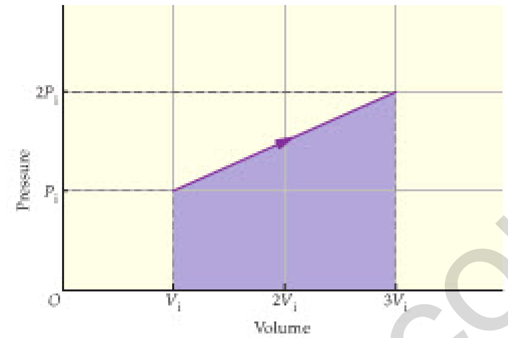
27. •• **IP** Suppose 67.5 moles of an ideal monatomic gas undergo the series of processes shown in Figure 18–24. **(a)** Calculate the temperature at the points A, B, and C. **(b)** For each process, A → B, B → C, and C → A, state whether heat enters or leaves the system. Explain in each case. **(c)** Calculate the heat exchanged with the gas during each of the three processes.

28. •• A gas is contained in a cylinder with a pressure of 140 kPa and an initial volume of 0.66 m^3. How much work is done by the gas as it **(a)** expands at constant pressure to twice its initial volume, or **(b)** is compressed to one-third its initial volume?

29. •• A system expands by 0.75 m^3 at a constant pressure of 125 kPa. Find the heat that flows into or out of the system if its internal energy **(a)** increases by 65 J or **(b)** decreases by 1850 J. In each case, give the direction of heat flow.

30. •• **IP** An ideal monatomic gas is held in a perfectly insulated cylinder fitted with a movable piston. The initial pressure of the gas is 110 kPa, and its initial temperature is 280 K. By pushing down on the piston, you are able to increase the pressure to 140 kPa. **(a)** During this process, did the temperature of the gas increase, decrease, or stay the same? Explain. **(b)** Find the final temperature of the gas.

31. ••• A certain amount of a monatomic ideal gas undergoes the process shown in Figure 18–25, in which its pressure doubles and its volume triples. In terms of the number of moles, n, the initial pressure, P_i, and the initial volume, V_i, determine **(a)** the work done by the gas W, **(b)** the change in internal energy of the gas U, and **(c)** the heat added to the gas Q.



▲ **FIGURE 18–25** Problem 31

32. ••• An ideal gas doubles its volume in one of three different ways: (i) at constant pressure; (ii) at constant temperature; (iii) adiabatically. Explain your answers to each of the following questions: **(a)** In which expansion does the gas do the most work? **(b)** In which expansion does the gas do the least work? **(c)** Which expansion results in the highest final temperature? **(d)** Which expansion results in the lowest final temperature?

SECTION 18–4 SPECIFIC HEATS FOR AN IDEAL GAS: CONSTANT PRESSURE, CONSTANT VOLUME

33. • **CE Predict/Explain** You plan to add a certain amount of heat to a gas in order to raise its temperature. **(a)** If you add the heat at constant volume, is the increase in temperature greater than, less than, or equal to the increase in temperature if you add the heat at constant pressure? **(b)** Choose the *best explanation* from among the following:

I. The same amount of heat increases the temperature by the same amount, regardless of whether the volume or the pressure is held constant.
II. All the heat goes into raising the temperature when added at constant volume; none goes into mechanical work.
III. Holding the pressure constant will cause a greater increase in temperature than simply having a fixed volume.

34. • Find the amount of heat needed to increase the temperature of 3.5 mol of an ideal monatomic gas by 23 K if **(a)** the pressure or **(b)** the volume is held constant.

35. • **(a)** If 535 J of heat are added to 45 moles of a monatomic gas at constant volume, how much does the temperature of the gas increase? **(b)** Repeat part (a), this time for a constant-pressure process.

36. • A system consists of 2.5 mol of an ideal monatomic gas at 325 K. How much heat must be added to the system to double its internal energy at **(a)** constant pressure or **(b)** constant volume?

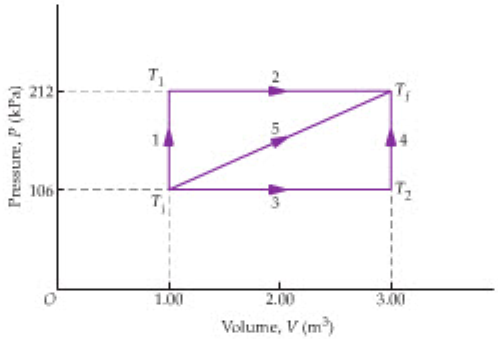
37. • Find the change in temperature if 170 J of heat are added to 2.8 mol of an ideal monatomic gas at **(a)** constant pressure or **(b)** constant volume.

38. •• **IP** A cylinder contains 18 moles of a monatomic ideal gas at a constant pressure of 160 kPa. **(a)** How much work does the gas do as it expands 3200 cm^3, from 5400 cm^3 to 8600 cm^3? **(b)** If the gas expands by 3200 cm^3 again, this time from 2200 cm^3 to 5400 cm^3, is the work it does greater than, less than, or equal to the work found in part (a)? Explain. **(c)** Calculate the work done as the gas expands from 2200 cm^3 to 5400 cm^3.

39. •• **IP** The volume of a monatomic ideal gas doubles in an adiabatic expansion. By what factor do **(a)** the pressure and **(b)** the temperature of the gas change? **(c)** Verify your answers to parts (a) and (b) by considering 135 moles of gas with an initial pressure of 330 kPa and an initial volume of 1.2 m^3. Find the pressure and temperature of the gas after it expands adiabatically to a volume of 2.4 m^3.

40. •• A monatomic ideal gas is held in a thermally insulated container with a volume of 0.0750 m^3. The pressure of the gas is 105 kPa, and its temperature is 317 K. **(a)** To what volume must the gas be compressed to increase its pressure to 145 kPa? **(b)** At what volume will the gas have a temperature of 295 K?

41. ••• Consider the expansion of 60.0 moles of a monatomic ideal gas along processes 1 and 2 in Figure 18–26. On process 1 the gas is heated at constant volume from an initial pressure of 106 kPa to a final pressure of 212 kPa. On process 2 the gas expands at constant pressure from an initial volume of 1.00 m^3 to a final volume of 3.00 m^3. **(a)** How much heat is added to the gas during these two processes? **(b)** How much work does the gas do during this expansion? **(c)** What is the change in the internal energy of the gas?



▲ **FIGURE 18–26** Problems 41, 42, and 81

42. ••• Referring to Problem 41, suppose the gas is expanded along processes 3 and 4 in Figure 18–26. On process 3 the gas expands at constant pressure from an initial volume of 1.00 m^3 to a final volume of 3.00 m^3. On process 4 the gas is heated at constant volume from an initial pressure of 106 kPa to a final pressure of 212 kPa. **(a)** How much heat is added to the gas during these two processes? **(b)** How much work does the gas do during this expansion? **(c)** What is the change in the internal energy of the gas?

SECTION 18–6 HEAT ENGINES AND THE CARNOT CYCLE

43. • **CE** A Carnot engine operates between a hot reservoir at the Kelvin temperature T_h and a cold reservoir at the Kelvin temperature T_c. **(a)** If both temperatures are doubled, does the efficiency of the engine increase, decrease, or stay the same? Explain. **(b)** If both temperatures are increased by 50 K, does the efficiency of the engine increase, decrease, or stay the same? Explain.

44. • **CE** A Carnot engine can be operated with one of the following four sets of reservoir temperatures: **A**, 400 K and 800 K; **B**, 400 K and 600 K; **C**, 800 K and 1200 K; and **D**, 800 K and 1000 K. Rank these reservoir temperatures in order of increasing efficiency of the Carnot engine. Indicate ties where appropriate.

45. • What is the efficiency of an engine that exhausts 870 J of heat in the process of doing 340 J of work?

46. • An engine receives 690 J of heat from a hot reservoir and gives off 430 J of heat to a cold reservoir. What are **(a)** the work done and **(b)** the efficiency of this engine?

47. • A Carnot engine operates between the temperatures 410 K and 290 K. **(a)** How much heat must be given to the engine to produce 2500 J of work? **(b)** How much heat is discarded to the cold reservoir as this work is done?

48. • A nuclear power plant has a reactor that produces heat at the rate of 838 MW. This heat is used to produce 253 MW of mechanical power to drive an electrical generator. **(a)** At what rate is heat discarded to the environment by this power plant? **(b)** What is the thermal efficiency of the plant?

49. • At a coal-burning power plant a steam turbine is operated with a power output of 548 MW. The thermal efficiency of the power plant is 32.0%. **(a)** At what rate is heat discarded to the environment by this power plant? **(b)** At what rate must heat be supplied to the power plant by burning coal?

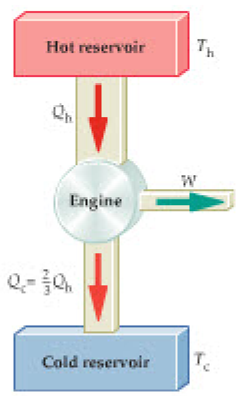
50. •• **IP** If a heat engine does 2700 J of work with an efficiency of 0.18, find **(a)** the heat taken in from the hot reservoir and **(b)** the heat given off to the cold reservoir. **(c)** If the efficiency of the engine is increased, do your answers to parts (a) and (b) increase, decrease, or stay the same? Explain.

51. •• **IP** The efficiency of a particular Carnot engine is 0.300. **(a)** If the high-temperature reservoir is at a temperature of 545 K, what is the temperature of the low-temperature reservoir? **(b)** To increase the efficiency of this engine to 40.0%, must the temperature of the low-temperature reservoir be increased or decreased? Explain. **(c)** Find the temperature of the low-temperature reservoir that gives an efficiency of 0.400.

52. •• During each cycle a reversible engine absorbs 2500 J of heat from a high-temperature reservoir and performs 2200 J of work. **(a)** What is the efficiency of this engine? **(b)** How much heat is exhausted to the low-temperature reservoir during each cycle? **(c)** What is the ratio, T_h/T_c, of the two reservoir temperatures?

53. ••• The operating temperatures for a Carnot engine are T_c and $T_h = T_c + 55$ K. The efficiency of the engine is 11%. Find T_c and T_h.

54. ••• A certain Carnot engine takes in the heat Q_h and exhausts the heat $Q_c = 2Q_h/3$, as indicated in Figure 18–27. **(a)** What is the efficiency of this engine? **(b)** Using the Kelvin temperature scale, find the ratio T_c/T_h.



▲ **FIGURE 18–27** Problem 54

SECTION 18–7 REFRIGERATORS, AIR CONDITIONERS, AND HEAT PUMPS

55. • **CE Predict/Explain (a)** If the temperature in the kitchen is decreased, is the cost (work needed) to freeze a dozen ice cubes greater than, less than, or equal to what it was before the kitchen was cooled? **(b)** Choose the *best explanation* from among the following:

- **I.** The difference in temperature between the inside and the outside of the refrigerator is decreased, and hence less work is required to freeze the ice.
- **II.** The same amount of ice is frozen in either case, which requires the same amount of heat to be removed and hence the same amount of work.
- **III.** Cooling the kitchen means that the refrigerator must do more work, both to freeze the ice cubes and to warm the kitchen.

56. • The refrigerator in your kitchen does 480 J of work to remove 110 J of heat from its interior. **(a)** How much heat does the refrigerator exhaust into the kitchen? **(b)** What is the refrigerator's coefficient of performance?

57. • A refrigerator with a coefficient of performance of 1.75 absorbs 3.45×10^4 J of heat from the low-temperature reservoir during each cycle. **(a)** How much mechanical work is required to operate the refrigerator for a cycle? **(b)** How much heat does the refrigerator discard to the high-temperature reservoir during each cycle?

58. •• To keep a room at a comfortable 21.0 °C, a Carnot heat pump does 345 J of work and supplies it with 3240 J of heat. **(a)** How much heat is removed from the outside air by the heat pump? **(b)** What is the temperature of the outside air?

59. •• An air conditioner is used to keep the interior of a house at a temperature of 21 °C while the outside temperature is 32 °C. If heat leaks into the house at the rate of 11 kW, and the air conditioner has the efficiency of a Carnot engine, what is the mechanical power required to keep the house cool?

60. •• A reversible refrigerator has a coefficient of performance equal to 10.0. What is its efficiency?

61. •• A freezer has a coefficient of performance equal to 4.0. How much electrical energy must this freezer use to produce 1.5 kg of ice at −5.0 °C from water at 15 °C?

62. •• If a Carnot engine has an efficiency of 0.23, what is its coefficient of performance if it is run backward as a heat pump?

SECTION 18–8 ENTROPY

63. • **CE Predict/Explain (a)** If you rub your hands together, does the entropy of the universe increase, decrease, or stay the same? **(b)** Choose the *best explanation* from among the following:
   I. Rubbing hands together draws heat from the surroundings, and therefore lowers the entropy.
   II. No mechanical work is done by the rubbing, and hence the entropy does not change.
   III. The heat produced by rubbing raises the temperature of your hands and the air, which increases the entropy.

64. • **CE Predict/Explain (a)** An ideal gas is expanded slowly and isothermally. Does its entropy increase, decrease, or stay the same? **(b)** Choose the *best explanation* from among the following:
   I. Heat must be added to the gas to maintain a constant temperature, and this increases the entropy of the gas.
   II. The temperature of the gas remains constant, which means its entropy also remains constant.
   III. As the gas is expanded its temperature and entropy will decrease.

65. • **CE Predict/Explain (a)** A gas is expanded reversibly and adiabatically. Does its entropy increase, decrease, or stay the same? **(b)** Choose the *best explanation* from among the following:
   I. The process is reversible, and no heat is added to the gas. Therefore, the entropy of the gas remains the same.
   II. Expanding the gas gives it more volume to occupy, and this increases its entropy.
   III. The gas is expanded with no heat added to it, and hence its temperature will decrease. This, in turn, will lower its entropy.

66. • Find the change in entropy when 1.85 kg of water at 100 °C is boiled away to steam at 100 °C.

67. • Determine the change in entropy that occurs when 3.1 kg of water freezes at 0 °C.

68. •• **CE** You heat a pan of water on the stove. Rank the following temperature increases in order of increasing entropy change. Indicate ties where appropriate: **A**, 25 °C to 35 °C; **B**, 35 °C to 45 °C; **C**, 45 °C to 50 °C; and **D**, 50 °C to 55 °C.

69. •• On a cold winter's day heat leaks slowly out of a house at the rate of 20.0 kW. If the inside temperature is 22 °C, and the outside temperature is −14.5 °C, find the rate of entropy increase.

70. •• An 88-kg parachutist descends through a vertical height of 380 m with constant speed. Find the increase in entropy produced by the parachutist, assuming the air temperature is 21 °C.

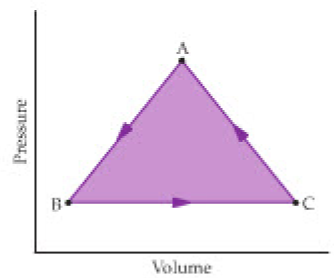
71. •• **IP** Consider the air-conditioning system described in Problem 59. **(a)** Does the entropy of the universe increase, decrease, or stay the same as the air conditioner keeps the imperfectly insulated house cool? Explain. **(b)** What is the rate at which the entropy of the universe changes during this process?

72. •• A heat engine operates between a high-temperature reservoir at 610 K and a low-temperature reservoir at 320 K. In one cycle, the engine absorbs 6400 J of heat from the high-temperature reservoir and does 2200 J of work. What is the net change in entropy as a result of this cycle?

GENERAL PROBLEMS

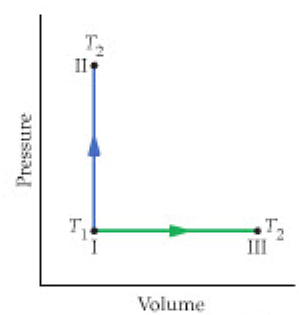
73. • **CE** An ideal gas is held in an insulated container at the temperature T. All the gas is initially in one-half of the container, with a partition separating the gas from the other half of the container, which is a vacuum. If the partition ruptures, and the gas expands to fill the entire container, is the final temperature greater than, less than, or equal to T? Explain.

74. • **CE** Consider the three-process cycle shown in Figure 18–28. For each process in the cycle, **(a)** A → B, **(b)** B → C, and **(c)** C → A, state whether the work done by the system is positive, negative, or zero.



▲ **FIGURE 18–28** Problem 74

75. • **CE** An ideal gas has the pressure and volume indicated by point I in Figure 18–29. At this point its temperature is T_1. The temperature of the gas can be increased to T_2 by using the constant-volume process, I → II, or the constant-pressure process, I → III. Is the entropy change for the process I → II greater than, less than, or equal to the entropy change on the process I → III? Explain.



▲ **FIGURE 18–29** Problem 75

76. • Find the work done by a monatomic ideal gas on each of the three multipart processes, A, B, and C, shown in Figure 18–21.

77. • Heat is added to a 0.14-kg block of ice at 0 °C, increasing its entropy by 87 J/K. How much ice melts?

78. • The heat that goes into a particular Carnot engine is 4.00 times greater than the work it performs. What is the engine's efficiency?

79. •• **IP** Consider 132 moles of a monatomic gas undergoing the isothermal expansion shown in Figure 18–23. **(a)** What is the temperature T of this expansion? **(b)** Does the entropy of the gas increase, decrease, or stay the same during the process? Explain. **(c)** Calculate the change in entropy for the gas, ΔS, if it is nonzero. **(d)** Calculate the work done by the gas during this process, and compare to $T\Delta S$.

80. •• **IP** Consider a monatomic ideal gas that undergoes the four processes shown in Figure 18–19. Is the work done by the gas positive, negative, or zero on process **(a)** AB, **(b)** BC, **(c)** CD, and **(d)** DA? Explain in each case. **(e)** If the heat added to the gas on process AB is 27 J, how much work does the gas do during that process?

81. •• **IP** Referring to Figure 18–26, suppose 60.0 moles of a monatomic ideal gas are expanded along process 5. **(a)** How much work does the gas do during this expansion? **(b)** What is the change in the internal energy of the gas? **(c)** How much heat is added to the gas during this process?

82. •• **IP** Engine A has an efficiency of 66%. Engine B absorbs the same amount of heat from the hot reservoir and exhausts twice as much heat to the cold reservoir. **(a)** Which engine has the greater efficiency? Explain. **(b)** What is the efficiency of engine B?

83. •• A freezer with a coefficient of performance of 3.88 is used to convert 1.75 kg of water to ice in one hour. The water starts at a temperature of 20.0 °C, and the ice that is produced is cooled to a temperature of −5.00 °C. **(a)** How much heat must be removed from the water for this process to occur? **(b)** How much electrical energy does the freezer use during this hour of operation? **(c)** How much heat is discarded into the room that houses the freezer?

84. •• Suppose 1800 J of heat are added to 3.6 mol of argon gas at a constant pressure of 120 kPa. Find the change in **(a)** internal energy and **(b)** temperature for this gas. **(c)** Calculate the change in volume of the gas. (Assume that the argon can be treated as an ideal monatomic gas.)

85. •• **Entropy and the Sun** The surface of the Sun has a temperature of 5500 °C and the temperature of deep space is 3.0 K. **(a)** Find the entropy increase produced by the Sun in one day, given that it radiates heat at the rate of 3.80×10^{26} W. **(b)** How much work could have been done if this heat had been used to run an ideal heat engine?

86. • The following table lists results for various processes involving n moles of a monatomic ideal gas. Fill in the missing entries.

	Q	W	ΔU
Constant pressure	$\frac{5}{2}nR\Delta T$	(a)	$\frac{3}{2}nR\Delta T$
Adiabatic	(b)	$-\frac{3}{2}nR\Delta T$	(c)
Constant volume	$\frac{3}{2}nR\Delta T$	(d)	(e)
Isothermal	(f)	$nRT\ln(V_f/V_i)$	(g)

87. •• A cylinder with a movable piston holds 2.75 mol of argon at a constant temperature of 295 K. As the gas is compressed isothermally, its pressure increases from 101 kPa to 121 kPa. Find **(a)** the final volume of the gas, **(b)** the work done by the gas, and **(c)** the heat added to the gas.

88. •• An inventor claims a new cyclic engine that uses organic grape juice as its working material. According to the claims, the engine absorbs 1250 J of heat from a 1010-K reservoir and performs 1120 J of work each cycle. The waste heat is exhausted to the atmosphere at a temperature of 302 K. **(a)** What is the efficiency that is implied by these claims? **(b)** What is the efficiency of a reversible engine operating between the same high and low temperatures used by this engine? (Should you invest in this invention?)

89. •• A nonreversible heat engine operates between a high-temperature reservoir at $T_h = 810$ K and a low-temperature reservoir at $T_c = 320$ K. During each cycle the engine absorbs 660 J of heat from the high-temperature reservoir and performs 250 J of work. **(a)** Calculate the total entropy change ΔS_{tot} for one cycle. **(b)** How much work would a reversible heat engine perform in one cycle if it operated between the same two temperatures and absorbed the same amount of heat? **(c)** Show that the difference in work between the nonreversible engine and the reversible engine is equal to $T_c\Delta S_{tot}$.

90. •• **IP** A small dish containing 530 g of water is placed outside for the birds. During the night the outside temperature drops to −5.0 °C and stays at that value for several hours. **(a)** When the water in the dish freezes at 0 °C, does its entropy increase, decrease, or stay the same? Explain. **(b)** Calculate the change in entropy that occurs as the water freezes. **(c)** When the water freezes, is there an entropy change anywhere else in the universe? If so, specify where the change occurs.

91. •• **IP** An ideal gas is taken through the three processes shown in Figure 18–20. Fill in the missing entries in the following table:

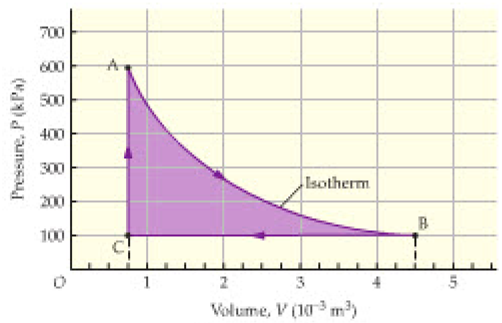
	Q	W	ΔU
A → B	(a)	(b)	−38 J
B → C	(c)	−89 J	−82 J
C → A	332 J	(d)	(e)

92. ••• Which would make the greater change in the efficiency of a Carnot heat engine: **(a)** raising the temperature of the high-temperature reservoir by ΔT, or **(b)** lowering the temperature of the low-temperature reservoir by ΔT? Justify your answer by calculating the change in efficiency for each of these cases.

93. ••• One mole of an ideal monatomic gas follows the three-part cycle shown in Figure 18–30. **(a)** Fill in the following table:

	Q	W	ΔU
A → B			
B → C			
C → A			

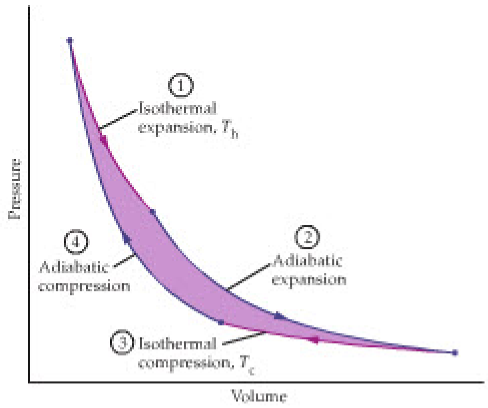
(b) What is the efficiency of this cycle?



▲ **FIGURE 18–30** Problem 93

94. ••• When a heat Q is added to a monatomic ideal gas at constant pressure, the gas does a work W. Find the ratio, W/Q.

95. ••• **The Carnot Cycle** Figure 18–31 shows an example of a Carnot cycle. The cycle consists of the following four processes: (1) an isothermal expansion from V_1 to V_2 at the temperature T_h; (2) an adiabatic expansion from V_2 to V_3 during which the temperature drops from T_h to T_c; (3) an isothermal compression from V_3 to V_4 at the temperature T_c; and (4) an adiabatic compression from V_4 to V_1 during which the temperature increases from T_c to T_h. Show that the efficiency of this cycle is $e = 1 - T_c/T_h$, as expected.



▲ **FIGURE 18–31** Problem 95

96. ••• A Carnot engine and a Carnot refrigerator operate between the same two temperatures. Show that the coefficient of performance, COP, for the refrigerator is related to the efficiency, e, of the engine by the following expression; $\text{COP} = (1 - e)/e$.

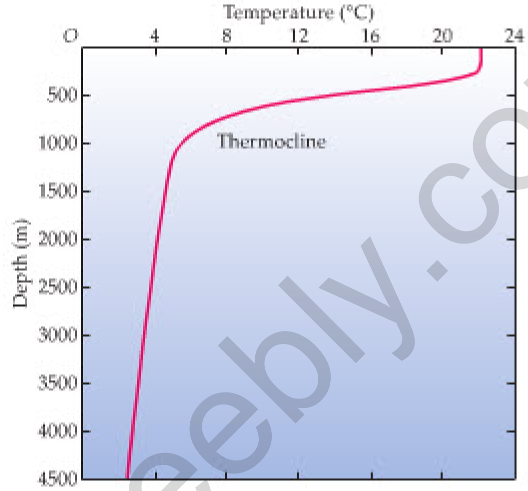
PASSAGE PROBLEMS

Energy from the Ocean

Whenever two objects are at different temperatures, thermal energy can be extracted with a heat engine. A case in point is the ocean, where one "object" is the warm water near the surface, and the other is the cold water at considerable depth. Tropical seas, in particular, can have significant temperature differences between the sun-warmed surface waters, and the cold, dark water 1000 m or more below the surface. A typical oceanic "temperature profile" is shown in Figure 18–32, where we see a rapid change in temperature—a thermocline—between depths of approximately 400 m and 900 m.

The idea of tapping this potential source of energy has been around for a long time. In 1870, for example, Captain Nemo in Jules Verne's *Twenty Thousand Leagues Under the Sea*, said, "I owe all to the ocean; it produces electricity, and electricity gives heat, light, motion, and, in a word, life to the Nautilus." Just 11 years later, the French physicist Jacques Arsene d'Arsonval proposed a practical system referred to as Ocean Thermal Energy Conversion (OTEC), and in 1930 Georges Claude, one of d'Arsonval's students, built and operated the first experimental OTEC system off the coast of Cuba.

OTEC systems, which are potentially low-cost and carbon neutral, can provide not only electricity, but also desalinated water as part of the process. In fact, an OTEC plant generating 2 MW of electricity is expected to produce over 14,000 cubic feet of desalinated water a day. Today, the governments of Hawaii, Japan, and Australia are actively pursuing plans for OTEC systems. The National Energy Laboratory of Hawaii Authority (NELHA), for example, operated a test facility near Kona, Hawaii, from 1992 to 1998, and plans further tests in the future.



▲ **FIGURE 18–32** Temperature versus depth for ocean waters in the tropics (Problems 97, 98, and 99)

97. • Suppose an OTEC system operates with surface water at 22 °C and deep water at 4.0 °C. What is the maximum efficiency this system could have?

A. 6.10% B. 8.20%
C. 9.40% D. 18.0%

98. • If 1500 kg of water at 22 °C is cooled to 4.0 °C, how much energy is released? (For comparison, the energy released in burning a gallon of gasoline is 1.3×10^8 J.)

A. 2.5×10^7 J B. 1.1×10^8 J
C. 1.4×10^8 J D. 1.6×10^8 J

99. • If we go deeper for colder water, where the temperature is only 2.0 °C, what is the maximum efficiency now?

A. 6.78% B. 9.09%
C. 9.32% D. 19.0%

INTERACTIVE PROBLEMS

100. •• **IP Referring to Active Example 18–3** Suppose we lower the temperature of the cold reservoir to 295 K; the temperature of the hot reservoir is still 576 K. **(a)** Is the new efficiency of the engine greater than, less than, or equal to 0.470? Explain. **(b)** What is the new efficiency? **(c)** Find the work done by this engine when 1050 J of heat is drawn from the hot reservoir.

101. •• **IP Referring to Active Example 18–3** Suppose the temperature of the hot reservoir is increased by 16 K, from 576 K to 592 K, *and* that the temperature of the cold reservoir is also increased by 16 K, from 305 K to 321 K. **(a)** Is the new efficiency greater than, less than, or equal to 0.470? Explain. **(b)** What is the new efficiency? **(c)** What is the change in entropy of the hot reservoir when 1050 J of heat is drawn from it? **(d)** What is the change in entropy of the cold reservoir?

PHYSICS IN PERSPECTIVE

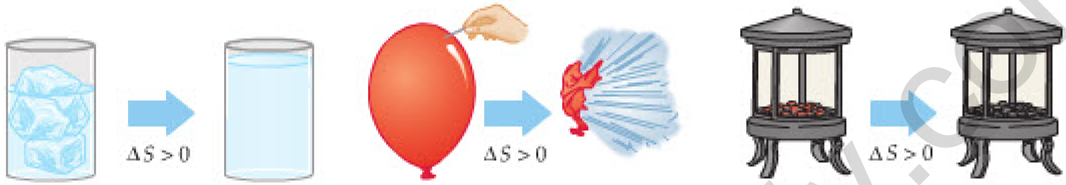
Entropy and Thermodynamics

The behavior of heat engines may seem unrelated to the fate of the universe. However, it led physicists to discover a new physical quantity: entropy. The future of the universe is shaped by the fact that the total entropy can only increase. Our fate is sealed.

1 Spontaneous processes cannot cause a decrease in entropy

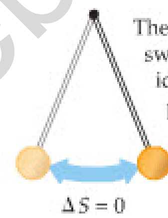
Fundamentally, entropy (S) is randomness or disorder. A process that occurs spontaneously—without a driving input of energy—cannot result in a net increase in order (decrease in entropy).

Irreversible processes: $\Delta S > 0$
An *irreversible* process runs spontaneously in just one direction—for instance, ice melts in warm water; warm water doesn't spontaneously form ice cubes. Irreversible processes always cause a net increase in entropy.



Reversible processes: $\Delta S = 0$
If a process can run spontaneously in either direction—so that a movie of it would look equally realistic run forward or backward—it is *reversible* and causes zero entropy change.

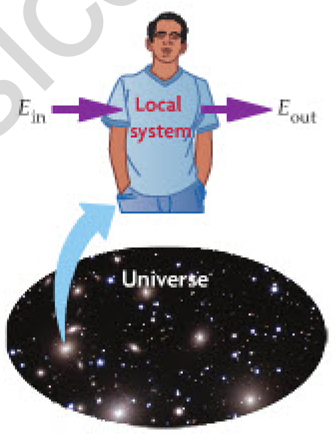
In practice, reversibility is an idealization—real processes are never completely reversible.



2 Entropy can decrease locally but must increase overall

An input of energy can be used to drive *nonspontaneous* processes that reduce disorder (entropy). That is what your body does with the energy it gains from food.

However, the universe as a whole cannot gain or lose energy, so its total entropy cannot decrease. This means that every process that decreases entropy locally must cause a larger entropy increase elsewhere.



Local system:
Input of energy can drive a decrease in entropy: $\Delta S < 0$.

Universe:
$\Delta E = 0$ (energy is conserved),
so
$\Delta S > 0$ (total entropy can only change by increasing)

3 The second law puts entropy in thermodynamic terms

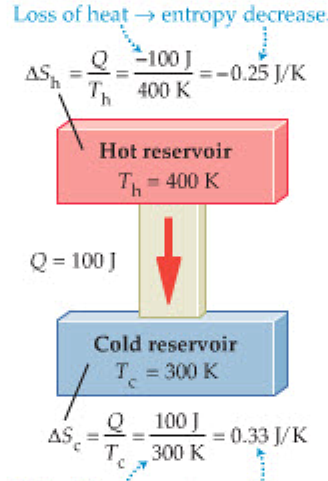
The second law of thermodynamics—that heat moves from hotter to colder objects—actually implies all that we've said about entropy. In fact, the change in entropy ΔS can be defined in terms of the thermodynamic quantities heat Q and temperature T:

$$\Delta S = \frac{Q}{T}$$

Change in system's entropy (ΔS); Heat entering or leaving system (positive if heat enters system) (Q); System's temperature (T)

As the example at right shows, the fact that temperature T is in the denominator means that the transfer of a given amount of heat Q causes a greater magnitude of entropy change for a colder object than for a hotter one.

Therefore, a flow of heat from a hotter to a colder object causes a net increase in entropy—as we would predict from the fact that this process is spontaneous and irreversible.



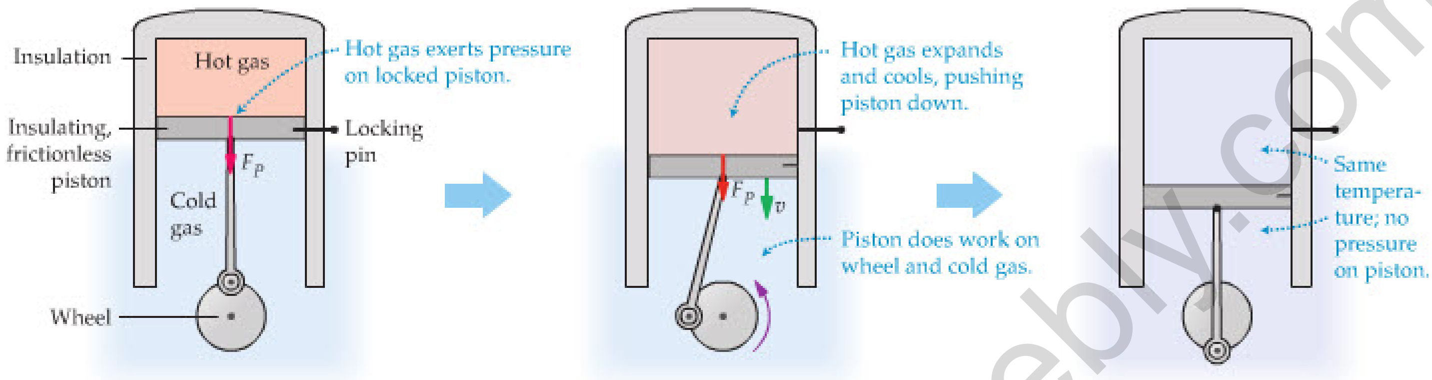
4 A temperature difference can be exploited to do work ...

The tendency of hotter and colder objects to come to the same temperature can be tapped to do work, as in this example:

Initial state: Gases at different temperatures are separated by a locked piston.

Piston unlocked: Pressure difference causes piston to move, doing work on wheel.

Final state: Gases at same temperature; no more work can be done.



The expansion shown above is a single process, not a cycle, so this piston-cylinder does not constitute a heat engine.

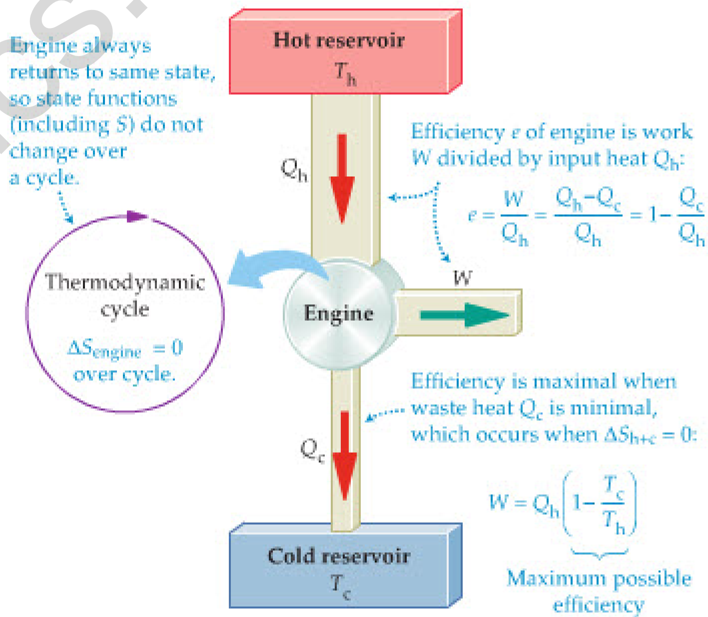
5 ... but entropy sets the limit of efficiency for a heat engine

A heat engine is a device that converts part of a heat flow into work. Entropy sets an absolute limit on the efficiency of this process.

To see why, we start with the fact that a heat engine operates on a thermodynamic cycle—it starts in a particular state, goes through a series of processes involving heat and work, and returns to its original state. (Think of the cyclic operation of a cylinder in a car engine.)

Because entropy S is a state function, the engine's entropy returns to its original value at the end of each cycle—so over the course of a cycle, the entropy change ΔS_{engine} of a heat engine is zero. Therefore, the entropy of the engine's *environment*—specifically, of the hot and cold reservoir (S_{h+c})—must increase or stay the same ($\Delta S_{h+c} \geq 0$).

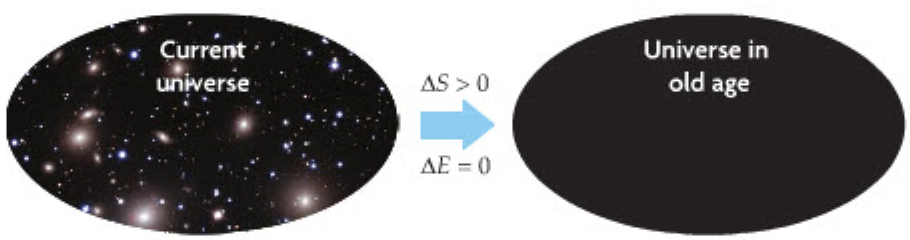
The engine will have the highest efficiency $e = W/Q_h$ when $\Delta S_{h+c} = 0$, because higher values of ΔS_{h+c} entail more waste heat (Q_c) and thus yield less work W. To be more efficient than this, an engine would have to cause a net *decrease* in entropy, which is impossible. Actual engines all have $\Delta S_{h+c} \geq 0$.



6 Entropy spells the death of the universe

The night sky shows us a universe of stars and galaxies separated by cold, nearly empty space. Over time, the inexorable growth of entropy will erase these differences, leaving a universe that is uniform in temperature and density—unable ever again to create stars or give rise to life.

Nevertheless, the energy content of the universe will remain the same as at its birth.



19 Electric Charges, Forces, and Fields

Amber, a form of fossilized tree resin long used to make beautiful beads and other ornaments, has also made contributions to two different sciences. Pieces of amber have preserved prehistoric insects and pollen grains for modern students of evolution. And over 2500 years ago, amber provided Greek scientists with their first opportunity to study electric forces—the subject of this chapter.

We are all made up of electric charges. Every atom in every human body contains both positive and negative charges held together by an attractive force that is similar to gravity—only vastly stronger. Our atoms are bound together by electric forces to form molecules; these molecules, in turn, interact with one another to produce solid bones and liquid blood. In a very real sense, we are walking, talking manifestations of electricity.

In this chapter, we discuss the basic properties of electric charge. Among these are that electric charge comes in discrete units (quantization) and that the total amount of charge in the universe remains constant (conservation). In addition, we present the force law that describes the interactions between electric charges. Finally, we introduce the idea of an electric *field*, and show how it is related to the distribution of charge.

19–1 Electric Charge

The effects of electric charge have been known since at least 600 B.C. About that time, the Greeks noticed that amber—a solid, translucent material formed from the fossilized resin of extinct coniferous trees—has a peculiar property. If a piece of amber is rubbed with animal fur, it attracts small, lightweight objects. This phenomenon is illustrated in **Figure 19–1**.

For some time, it was thought that amber was unique in its ability to become "charged." Much later, it was discovered that other materials can behave in this way as well. For example, if glass is rubbed with a piece of silk, it too can attract small objects. In this respect, glass and amber seem to be the same. It turns out, however, that these two materials have different types of charge.

To see this, imagine suspending a small, charged rod of amber from a thread, as in **Figure 19–2**. If a second piece of charged amber is brought near the rod, as shown in Figure 19–2 (a), the rod rotates away, indicating a repulsive force between the two pieces of amber. Thus, "like" charges repel. On the other hand, if a piece of charged glass is brought near the amber rod, the amber rotates toward the glass, indicating an attractive force. This is illustrated in Figure 19–2 (b). Clearly, then, the *different* charges on the glass and amber attract one another. We refer to different charges as being the "opposite" of one another, as in the familiar expression "opposites attract."

We know today that the two types of charge found on amber and glass are, in fact, the only types that exist, and we still use the purely arbitrary names—**positive** (+) charge and **negative** (−) charge—proposed by Benjamin Franklin (1706–1790) in 1747. In accordance with Franklin's original suggestion, the charge of amber is negative, and the charge of glass is positive (the opposite of negative). Calling the different charges + and − is actually quite useful mathematically; for example, an object that contains an equal amount of positive and negative charge has zero net charge. Objects with zero net charge are said to be electrically **neutral.**

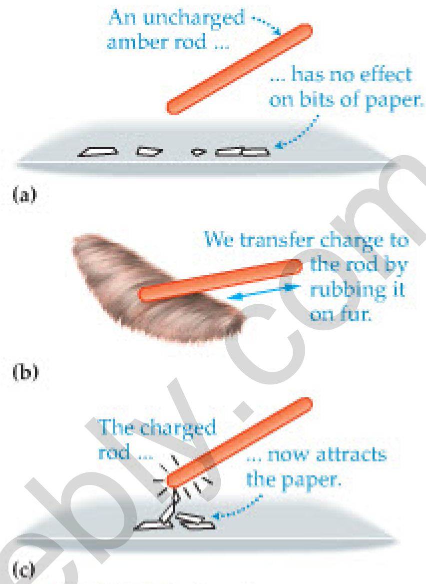


FIGURE 19–1 Charging an amber rod
An uncharged amber rod **(a)** exerts no force on scraps of paper. When the rod is rubbed against a piece of fur **(b)**, it becomes charged and then attracts the paper **(c)**.

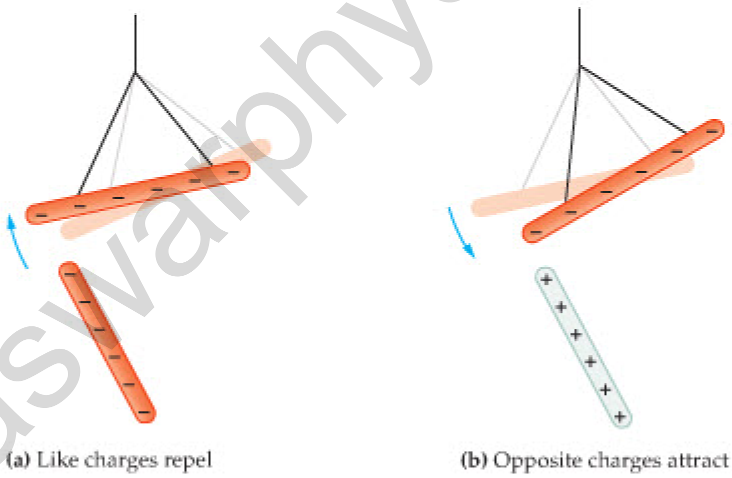


FIGURE 19–2 Likes repel, opposites attract
A charged amber rod is suspended by a string. According to the convention introduced by Benjamin Franklin, the charge on the amber is designated as negative. **(a)** When another charged amber rod is brought near the suspended rod, it rotates away, indicating a repulsive force between like charges. **(b)** When a charged glass rod is brought close to the suspended amber rod, the amber rotates toward the glass, indicating an attractive force and the existence of a second type of charge, which we designate as positive.

A familiar example of an electrically neutral object is the atom. Atoms have a small, dense nucleus with a positive charge surrounded by a cloud of negatively charged electrons (from the Greek word for amber, *elektron*). A pictorial representation of an atom is shown in **Figure 19–3**.

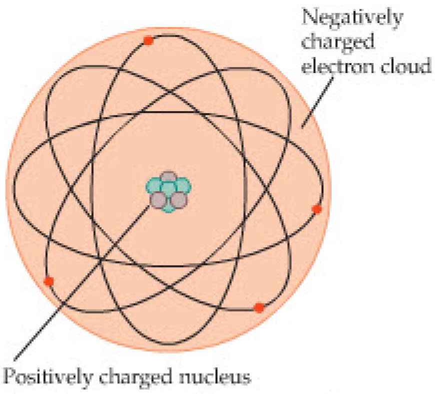
All electrons have exactly the same electric charge. This charge is very small and is defined to have a magnitude, e, given by the following:

Magnitude of an Electron's Charge, *e*

$$e = 1.60 \times 10^{-19}\ \text{C} \qquad \text{19–1}$$

SI unit: coulomb, C

In this expression, C is a unit of charge referred to as the **coulomb,** named for the French physicist Charles-Augustin de Coulomb (1736–1806). (The precise definition



▲ **FIGURE 19–3 The structure of an atom**

A crude representation of an atom, showing the positively charged nucleus at its center and the negatively charged electrons orbiting about it. More accurately, the electrons should be thought of as forming a "cloud" of negative charge surrounding the nucleus.

of the coulomb is in terms of electric current, which we shall discuss in Chapter 21.) Clearly, the charge on an electron, which is negative, is $-e$. This is one of the defining, or *intrinsic*, properties of the electron. Another intrinsic property of the electron is its mass, m_e:

$$m_e = 9.11 \times 10^{-31}\ \text{kg} \qquad \text{19–2}$$

In contrast, the charge on a proton—one of the main constituents of nuclei—is *exactly* $+e$. Therefore, the net charge on atoms, which have equal numbers of electrons and protons, is precisely zero. The mass of the proton is

$$m_p = 1.673 \times 10^{-27}\ \text{kg} \qquad \text{19–3}$$

Note that this is about 2000 times larger than the mass of the electron. The other main constituent of the nucleus is the neutron, which, as its name implies, has zero charge. Its mass is slightly larger than that of the proton:

$$m_n = 1.675 \times 10^{-27}\ \text{kg} \qquad \text{19–4}$$

Since the magnitude of the charge per electron is 1.60×10^{-19} C/electron, it follows that the number of electrons in 1 C of charge is enormous:

$$\frac{1\ \text{C}}{1.60 \times 10^{-19}\ \text{C/electron}} = 6.25 \times 10^{18}\ \text{electrons}$$

As we shall see when we consider the force between charges, a coulomb is a significant amount of charge; even a powerful lightning bolt delivers only 20 to 30 C. A more common unit of charge is the microcoulomb, μC, where 1 μC = 10^{-6} C. Still, the amount of charge contained in everyday objects is very large, even in units of the coulomb, as we show in the following Exercise.

EXERCISE 19–1

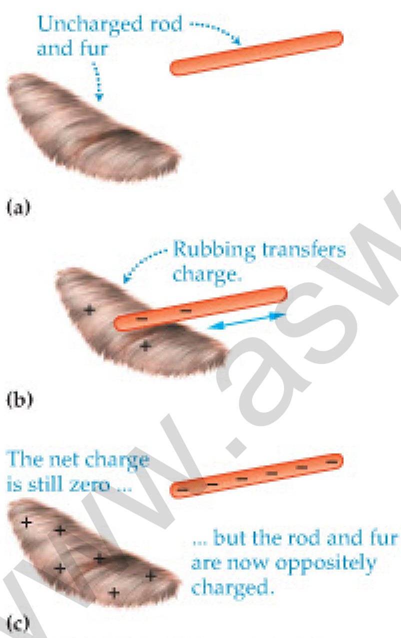
Find the amount of positive electric charge in one mole of helium atoms. (Note that the nucleus of a helium atom consists of two protons and two neutrons.)

SOLUTION

Since each helium atom contains two positive charges of magnitude e, the total positive charge in a mole is

$$N_A(2e) = (6.02 \times 10^{23})(2)(1.60 \times 10^{-19}\ \text{C}) = 1.93 \times 10^5\ \text{C}$$

Thus, a mere 4 g of helium contains almost 200,000 C of positive charge, and the same amount of negative charge, as well.



▲ **FIGURE 19–4 Charge transfer**

(a) Initially, an amber rod and a piece of fur are electrically neutral; that is, they each contain equal quantities of positive and negative charge. **(b)** As they are rubbed together, charge is transferred from one to the other. **(c)** In the end, the fur and the rod have charges of equal magnitude but opposite sign.

Charge Separation

How is it that rubbing a piece of amber with fur gives the amber a charge? Originally, it was thought that the friction of rubbing *created* the observed charge. We now know, however, that rubbing the fur across the amber simply results in a *transfer* of charge from the fur to the amber—with the total amount of charge remaining unchanged. This is indicated in Figure 19–4. Before charging, the fur and the amber are both neutral. During the rubbing process some electrons are transferred from the fur to the amber, giving the amber a net negative charge, and leaving the fur with a net positive charge. At no time during this process is charge ever created or destroyed. This, in fact, is an example of one of the fundamental conservation laws of physics:

Conservation of Electric Charge

The total electric charge of the universe is constant. No physical process can result in an increase or decrease in the total amount of electric charge in the universe.

When charge is transferred from one object to another, it is generally due to the movement of electrons. In a typical solid, the nuclei of the atoms are fixed in position. The outer electrons of these atoms, however, are often weakly bound and fairly easily separated. As a piece of fur rubs across amber, for example, some of the electrons that were originally part of the fur are separated from their atoms and deposited onto the amber. The atom that loses an electron is now a **positive ion,** and the atom that receives an extra electron becomes a **negative ion.** This is charging by separation.

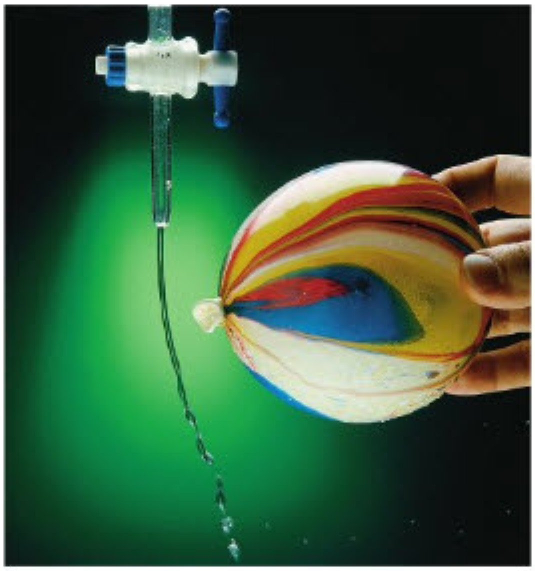
In general, when two materials are rubbed together, the magnitude *and* sign of the charge that each material acquires depend on how strongly it holds onto its electrons. For example, if silk is rubbed against glass, the glass acquires a positive charge, as was mentioned earlier in this section. It follows that electrons have moved from the glass to the silk, giving the silk a *negative* charge. If silk is rubbed against amber, however, the silk becomes *positively* charged, as electrons in this case pass from the silk to the amber.

These results can be understood by referring to Table 19–1, which presents the relative charging due to rubbing—also known as **triboelectric charging**—for a variety of materials. The more plus signs associated with a material, the more readily it gives up electrons and becomes positively charged. Similarly, the more minus signs for a material, the more readily it acquires electrons. For example, we know that amber becomes negatively charged when rubbed against fur, but a greater negative charge is obtained if rubber, PVC, or Teflon is rubbed with fur instead. In general, when two materials in Table 19–1 are rubbed together, the one higher in the list becomes positively charged, and the one lower in the list becomes negatively charged. The greater the separation on the list, the greater the magnitude of the charge.

Charge separation occurs not only when one object is rubbed against another, but also when objects collide. For example, colliding crystals of ice in a rain cloud can cause charge separation that may ultimately result in bolts of lightning to bring the charges together again. Similarly, particles in the rings of Saturn are constantly undergoing collisions and becoming charged as a result. In fact, when the *Voyager* spacecraft examined the rings of Saturn, it observed electrostatic discharges, similar to lightning bolts on Earth. In addition, ghostly radial "spokes" that extend across the rings of Saturn—which cannot be explained by gravitational forces alone—are also the result of electrostatic interactions.

TABLE 19–1 Triboelectric Charging

Material	Relative charging with rubbing
Rabbit fur	++++++
Glass	+++++
Human hair	++++
Nylon	+++
Silk	++
Paper	+
Cotton	−
Wood	−−
Amber	−−−
Rubber	−−−−
PVC	−−−−−
Teflon	−−−−−−

▲ The Van de Graaff generator (left) that these children are touching can produce very large charges of static electricity. Since they are clearly not frightened, why is their hair standing on end? On a smaller scale, if you rub a balloon against a cloth surface, the balloon acquires a negative electric charge. The balloon can then attract a stream of water (right), even though water molecules themselves are electrically neutral. This phenomenon occurs because the water molecules, though they have no net charge, are polar: one end of the molecule has a slight positive charge and the other a slight negative charge. Under the influence of the balloon's negative charge, the water molecules orient themselves so that their positive ends point toward the balloon. This alignment ensures that the electrical attraction between the balloon and the positive part of each molecule exceeds the repulsion between the balloon and the negative part of each molecule.

CONCEPTUAL CHECKPOINT 19–1 COMPARE THE MASS

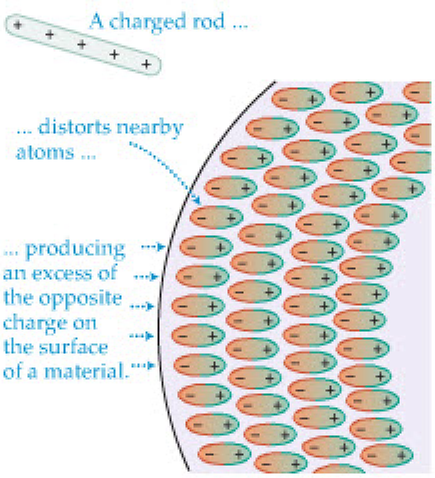
Is the mass of an amber rod after charging with fur **(a)** greater than, **(b)** less than, or **(c)** the same as its mass before charging?

REASONING AND DISCUSSION

Since an amber rod becomes negatively charged, it has acquired electrons from the fur. Each electron has a small, but nonzero, mass. Therefore, the mass of the rod increases ever so slightly as it is charged.

ANSWER

(a) The mass of the amber rod is greater after charging.



▲ FIGURE 19–5 Electrical polarization

When a charged rod is far from a neutral object, the atoms in the object are undistorted, as in Figure 19–3. As the rod is brought closer, however, the atoms distort, producing an excess of one type of charge on the surface of the object (in this case a negative charge). This induced charge is referred to as a polarization charge. Because the sign of the polarization charge is the opposite of the sign of the charge on the rod, there is an attractive force between the rod and the object.

Since electrons always have the charge $-e$, and protons always have the charge $+e$, it follows that all objects must have a net charge that is an integral multiple of e. This conclusion was confirmed early in the twentieth century by the American physicist Robert A. Millikan (1868–1953) in a classic series of experiments. He found that the charge on an object can be $\pm e$, $\pm 2e$, $\pm 3e$, and so on, but never $1.5e$ or $-9.3847e$, for example. We describe this restriction by saying that electric charge is **quantized.**

Polarization

We know that charges of opposite sign attract, but it is also possible for a charged rod to attract small objects that have zero net charge. The mechanism responsible for this attraction is called **polarization.**

To see how polarization works, consider Figure 19–5. Here we show a positively charged rod held close to an enlarged view of a neutral object. An atom near the surface of the neutral object will become elongated because the negative electrons in it are attracted to the rod while the positive protons are repelled. As a result, a net negative charge develops on the surface near the rod—the so-called polarization charge. The attractive force between the rod and this *induced* polarization charge leads to a net attraction between the rod and the entire neutral object.

Of course, the same conclusion is reached if we consider a negative rod held near a neutral object—except in this case the polarization charge is positive. Thus, the effect of polarization is to give rise to an attractive force regardless of the sign of the charged object. It is for this reason that both charged amber and charged glass attract neutral objects—even though their charges are opposite.

A potentially dangerous, and initially unsuspected, medical application of polarization occurs in endoscopic surgery. In these procedures, a tube carrying a small video camera is inserted into the body. The resulting video image is produced by electrons striking the inside surface of a computer monitor's screen, which is kept positively charged to attract the electrons. Minute airborne particles in the operating room—including dust, lint, and skin cells—are polarized by the positive charge on the screen, and are attracted to its exterior surface.

REAL-WORLD PHYSICS: BIO

Bacterial infection from endoscopic surgery

The problem comes when a surgeon touches the screen to point out an important feature to others in the medical staff. Even the slightest touch can transfer particles—many of which carry bacteria—from the screen to the surgeon's finger and from there to the patient. In fact, the surgeon's finger doesn't even have to touch the screen—as the finger approaches the screen, it too becomes polarized, and hence, it can attract particles from the screen, or directly from the air. Situations like these have resulted in infections, and surgeons are now cautioned not to bring their fingers near the video monitor.

19–2 Insulators and Conductors

Suppose you rub one end of an amber rod with fur, being careful not to touch the other end. The result is that the rubbed portion becomes charged, whereas the other end remains neutral. In particular, the negative charge transferred to the rubbed end stays put; it does not move about from one end of the rod to the other. Materials like amber, in which charges are not free to move, are referred to as **insulators.** Most insulators are nonmetallic substances, and most are also good thermal insulators.

In contrast, most metals are good **conductors** of electricity, in the sense that they allow charges to move about more or less freely. For example, suppose an uncharged metal sphere is placed on an insulating base. If a charged rod is brought into contact with the sphere, as in Figure 19–6 (a), some charge will be transferred to the sphere at the point of contact. The charge does not stay put, however. Since the metal is a good conductor of electricity, the charges are free to move about the sphere, which they do because of their mutual repulsion. The result is a uniform distribution of charge over the surface of the sphere, as shown in Figure 19–6 (b). Note that the insulating base prevents charge from flowing away from the sphere into the ground.

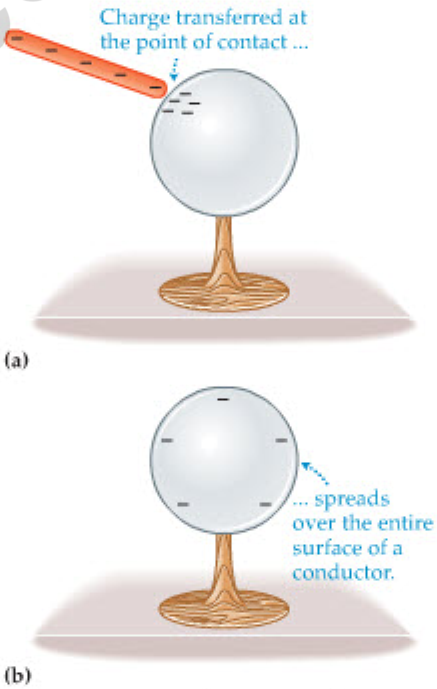
On a microscopic level, the difference between conductors and insulators is that the atoms in conductors allow one or more of their outermost electrons to become detached. These detached electrons, often referred to as "conduction electrons," can move freely throughout the conductor. In a sense, the conduction electrons behave almost like gas molecules moving about within a container. Insulators, in contrast, have very few, if any, free electrons; the electrons are bound to their atoms and cannot move from place to place within the material.

Some materials have properties that are intermediate between those of a good conductor and a good insulator. These materials, referred to as **semiconductors,** can be fine-tuned to display almost any desired degree of conductivity by controlling the concentration of the various components from which they are made. The great versatility of semiconductors is one reason they have found such wide areas of application in electronics and computers.

Exposure to light can sometimes determine whether a given material is an insulator or a conductor. An example of such a **photoconductive** material is selenium, which conducts electricity when light shines on it but is an insulator when in the dark. Because of this special property, selenium plays a key role in the production of photocopies. To see how, we first note that at the heart of every photocopier is a selenium-coated aluminum drum. Initially, the selenium is given a positive charge and kept in the dark—which causes it to retain its charge. When flash lamps illuminate a document to be copied, an image of the document falls on the drum. Where the document is light, the selenium is illuminated and becomes a conductor, and the positive charge flows away into the aluminum drum, leaving the selenium uncharged. Where the document is dark, the selenium is not illuminated, meaning that it is an insulator, and its charge remains in place. At this point, a negatively charged "toner" powder is wiped across the drum, where it sticks to those positively charged portions of the drum that were not illuminated. Next, the drum is brought into contact with paper, transferring the toner to it. Finally, the toner is fused into the paper fibers with heat, the drum is cleaned of excess toner, and the cycle repeats. Thus, a slight variation in electrical properties due to illumination is the basis of an entire technology.

The operation of a laser printer is basically the same as that of a photocopier, with the difference that in the laser printer the selenium-coated drum is illuminated with a computer-controlled laser beam. As the laser sweeps across the selenium, the computer turns the beam on and off to produce areas that will print light or dark, respectively.

▲ People who work with electricity must be careful to use gloves made of nonconducting materials. Rubber, an excellent insulator, is often used for this purpose.



▲ **FIGURE 19–6 Charging a conductor**
(a) When an uncharged metal sphere is touched by a charged rod, some charge is transferred at the point of contact. **(b)** Because like charges repel, and charges move freely on a conductor, the transferred charge quickly spreads out and covers the entire surface of the sphere.

REAL-WORLD PHYSICS
Photocopiers and laser printers

19–3 Coulomb's Law

We have already discussed the fact that electric charges exert forces on one another. The precise law describing these forces was first determined by Coulomb in the late 1780s. His result is remarkably simple. Suppose, for example, that an idealized point charge q_1 is separated by a distance r from another point charge q_2. Both charges are at rest; that is, the system is **electrostatic.** According to Coulomb's law, the magnitude of the electrostatic force between these charges is proportional to the product of the magnitude of the charges, $|q_1||q_2|$, and inversely proportional to the square of the distance, r^2, between them:

Coulomb's Law for the Magnitude of the Electrostatic Force Between Point Charges

$$F = k\frac{|q_1||q_2|}{r^2} \qquad \text{19–5}$$

SI unit: newton, N

In this expression, the proportionality constant k has the value

$$k = 8.99 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2 \qquad \text{19–6}$$

Note that the units of k are simply those required for the force F to have the units of newtons.

The direction of the force in Coulomb's law is along the line connecting the two charges. In addition, we know from the observations described in Section 19–1 that like charges repel and opposite charges attract. These properties are illustrated in Figure 19–7, where force vectors are shown for charges of various signs. Thus, when applying Coulomb's law, we first calculate the magnitude of the force using Equation 19–5, and then determine its direction with the "likes repel, opposites attract" rule.

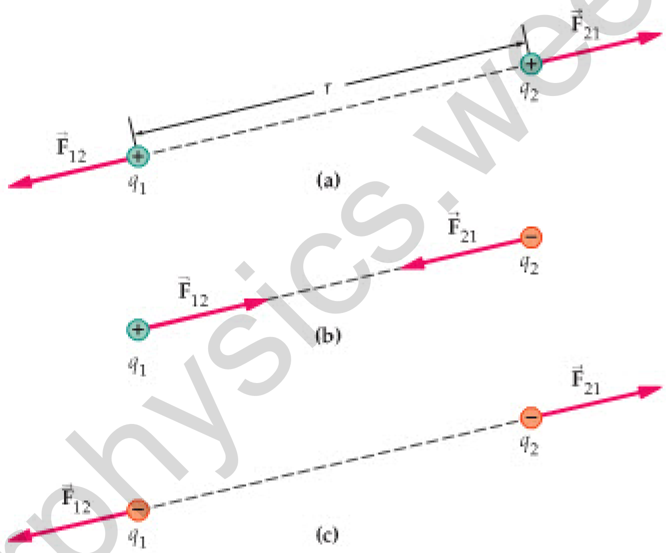


FIGURE 19–7 Forces between point charges
The forces exerted by two point charges on one another are always along the line connecting the charges. If the charges have the same sign, as in **(a)** and **(c)**, the forces are repulsive; that is, each charge experiences a force that points away from the other charge. Charges of opposite sign, as in **(b)**, experience attractive forces. Notice that in all cases the forces exerted on the two charges form an action–reaction pair. That is, $\vec{F}_{21} = -\vec{F}_{12}$.

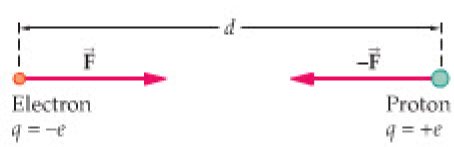
Finally, note how Newton's third law applies to each of the cases shown in Figure 19–7. For example, the force exerted on charge 1 by charge 2, $\vec{F}_{12}$, is always equal in magnitude and opposite in direction to the force exerted on charge 2 by charge 1, $\vec{F}_{21}$; that is, $\vec{F}_{21} = -\vec{F}_{12}$.

CONCEPTUAL CHECKPOINT 19–2 WHERE DO THEY COLLIDE?

An electron and a proton, initially separated by a distance d, are released from rest simultaneously. The two particles are free to move. When they collide, are they **(a)** at the midpoint of their initial separation, **(b)** closer to the initial position of the proton, or **(c)** closer to the initial position of the electron?

REASONING AND DISCUSSION

Because of Newton's third law, the forces exerted on the electron and proton are equal in magnitude and opposite in direction. For this reason, it might seem that the particles meet at the midpoint. The masses of the particles, however, are quite different. In fact, as mentioned in Section 19–1, the mass of the proton is about 2000 times greater than the mass of the electron; therefore, the proton's acceleration ($a = F/m$) is about 2000 times less than the electron's acceleration. As a result, the particles collide near the initial position of the proton. More specifically, they collide at the location of the center of mass of the system, which remains at rest throughout the process.



ANSWER

(b) The particles collide near the initial position of the proton.

It is interesting to note the similarities and differences between Coulomb's law, $F = k|q_1||q_2|/r^2$, and Newton's law of gravity, $F = Gm_1m_2/r^2$. In each case, the force decreases as the square of the distance between the two objects. In addition, both forces depend on a product of intrinsic quantities: in the case of the electric force the intrinsic quantity is the charge; in the case of gravity it is the mass.

Equally significant, however, are the differences. In particular, the force of gravity is always attractive, whereas the electric force can be attractive or repulsive. As a result, the net electric force between neutral objects, such as the Earth and the Moon, is essentially zero because attractive and repulsive forces cancel one another. Since gravity is always attractive, however, the net gravitational force between the Earth and the Moon is nonzero. Thus, in astronomy, gravity rules, and electric forces play hardly any role.

Just the opposite is true in atomic systems. To see this, let's compare the electric and gravitational forces between a proton and an electron in a hydrogen atom. Taking the distance between the two particles to be the radius of hydrogen, $r = 5.29 \times 10^{-11}$ m, we find that the gravitational force has a magnitude

$$F_g = G\frac{m_e m_p}{r^2}$$
$$= (6.67 \times 10^{-11}\ \mathrm{N \cdot m^2/kg^2})\frac{(9.11 \times 10^{-31}\ \mathrm{kg})(1.673 \times 10^{-27}\ \mathrm{kg})}{(5.29 \times 10^{-11}\ \mathrm{m})^2}$$
$$= 3.63 \times 10^{-47}\ \mathrm{N}$$

Similarly, the magnitude of the electric force between the electron and the proton is

$$F_e = k\frac{|q_1||q_2|}{r^2}$$
$$= (8.99 \times 10^9\ \mathrm{N \cdot m^2/C^2})\frac{|-1.60 \times 10^{-19}\ \mathrm{C}||1.60 \times 10^{-19}\ \mathrm{C}|}{(5.29 \times 10^{-11}\ \mathrm{m})^2}$$
$$= 8.22 \times 10^{-8}\ \mathrm{N}$$

Taking the ratio, we find that the electric force is greater than the gravitational force by a factor of

$$\frac{F_e}{F_g} = \frac{8.22 \times 10^{-8}\ \mathrm{N}}{3.63 \times 10^{-47}\ \mathrm{N}} = 2.26 \times 10^{39}$$
$$= 2{,}260{,}000{,}000{,}000{,}000{,}000{,}000{,}000{,}000{,}000{,}000{,}000{,}000$$

This huge factor explains why a small piece of charged amber can lift bits of paper off the ground, even though the entire mass of the Earth is pulling downward on the paper.

Clearly, then, the force of gravity plays essentially no role in atomic systems. The reason gravity dominates in astronomy is that, even though the force is incredibly weak, it always attracts, giving a larger net force the larger the astronomical body. The electric force, on the other hand, is very strong but cancels for neutral objects.

Next, we use the electric force to get an idea of the speed of an electron in a hydrogen atom and the frequency of its orbital motion.

PROBLEM-SOLVING NOTE

Distance Dependence of the Coulomb Force

The Coulomb force has an inverse-square dependence on distance. Be sure to divide the product of the charges, $k|q_1||q_2|$, by r^2 when calculating the force.

EXAMPLE 19–1 THE BOHR ORBIT

In an effort to better understand the behavior of atomic systems, the Danish physicist Niels Bohr (1885–1962) introduced a simple model for the hydrogen atom. In the Bohr model, as it is known today, the electron is imagined to move in a circular orbit about a stationary proton. The force responsible for the electron's circular motion is the electric force of attraction between the electron and the proton. **(a)** Given that the radius of the electron's orbit is 5.29×10^{-11} m, and its mass is $m_e = 9.11 \times 10^{-31}$ kg, find the electron's speed. **(b)** What is the frequency of the electron's orbital motion?

CONTINUED ON NEXT PAGE

CONTINUED FROM PREVIOUS PAGE

PICTURE THE PROBLEM
Our sketch shows the electron moving with a speed v in its orbit of radius r. Because the proton is so much more massive than the electron, it is essentially stationary at the center of the orbit. Note that the electron has a charge $-e$ and the proton has a charge $+e$.

STRATEGY

a. The idea behind this model is that a force is required to make the electron move in a circular path, and this force is provided by the electric force of attraction between the electron and the proton. Thus, as with any circular motion, we set the force acting on the electron equal to its mass times its centripetal acceleration. This allows us to solve for the centripetal acceleration, $a_{cp} = v^2/r$ (Equation 6–14), which in turn gives us the speed v.

b. The frequency of the electron's orbital motion is $f = 1/T$, where T is the period of the motion; that is, the time for one complete orbit. The time for an orbit, in turn, is the circumference divided by the speed, or $T = C/v = 2\pi r/v$. Taking the inverse immediately yields the frequency.

SOLUTION

Part (a)

1. Set the Coulomb force between the electron and proton equal to the centripetal force required for the electron's circular orbit:

$$k\frac{|q_1||q_2|}{r^2} = m_e a_{cp}$$

$$k\frac{e^2}{r^2} = m_e\frac{v^2}{r}$$

2. Solve for the speed of the electron, v:

$$v = e\sqrt{\frac{k}{m_e r}}$$

3. Substitute numerical values:

$$v = (1.60 \times 10^{-19}\ \text{C})\sqrt{\frac{8.99 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2}{(9.11 \times 10^{-31}\ \text{kg})(5.29 \times 10^{-11}\ \text{m})}}$$
$$= 2.19 \times 10^6\ \text{m/s}$$

Part (b)

4. Calculate the time for one orbit, T, which is the distance ($C = 2\pi r$) divided by the speed (v):

$$T = \frac{C}{v} = \frac{2\pi r}{v} = \frac{2\pi(5.29 \times 10^{-11}\ \text{m})}{2.19 \times 10^6\ \text{m/s}} = 1.52 \times 10^{-16}\ \text{s}$$

5. Take the inverse of T to find the frequency:

$$f = \frac{1}{T} = \frac{1}{1.52 \times 10^{-16}\ \text{s}} = 6.58 \times 10^{15}\ \text{Hz}$$

INSIGHT
If you could travel around the world at this speed, your trip would take only about 18 s, but your centripetal acceleration would be a more-than-lethal 75,000 times the acceleration of gravity. As it is, the centripetal acceleration of the electron in this "Bohr" orbit around the proton is about 10^{22} times greater than the acceleration of gravity on the surface of the Earth.

The frequency of the orbit is also incredibly large. We won't encounter frequencies this high again until we study light waves in Chapter 25.

PRACTICE PROBLEM
The second Bohr orbit has a radius that is four times the radius of the first orbit. What is the speed of an electron in this orbit? [**Answer:** $v = 1.09 \times 10^6$ m/s]

Some related homework problems: Problem 19, Problem 28, Problem 37

Another indication of the strength of the electric force is given in the following Exercise.

EXERCISE 19–2

Find the electric force between two 1.00-C charges separated by 1.00 m.

SOLUTION
Substituting $q_1 = q_2 = 1.00$ C and $r = 1.00$ m in Coulomb's law, we find

$$F = k\frac{|q_1||q_2|}{r^2} = (8.99 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{(1.00\ \text{C})(1.00\ \text{C})}{(1.00\ \text{m})^2} = 8.99 \times 10^9\ \text{N}$$

Exercise 19–2 shows that charges of one coulomb exert a force of about a million tons on one another when separated by a distance of a meter. If the charge in your body could be separated into a pile of positive charge on one side of the room and a pile of negative charge on the other side, the force needed to hold them apart would be roughly 10^{10} tons! Thus, everyday objects are never far from electrical neutrality, since disturbing neutrality requires such tremendous forces.

Superposition of Forces

The electric force, like all forces, is a vector quantity. Hence, when a charge experiences forces due to two or more other charges, the net force on it is simply the *vector* sum of the forces taken individually. For example, in Figure 19–8, the total force on charge 1, $\vec{F}_1$, is the vector sum of the forces due to charges 2, 3, and 4:

$$\vec{F}_1 = \vec{F}_{12} + \vec{F}_{13} + \vec{F}_{14}$$

This is referred to as the **superposition** of forces.

Notice that the total force acting on a given charge is the sum of interactions involving just *two* charges at a time, with the force between each pair of charges given by Coulomb's law. For example, the total force acting on charge 1 in Figure 19–8 is the sum of the forces between q_1 and q_2, q_1 and q_3, and q_1 and q_4. Therefore, superposition of forces can be thought of as the generalization of Coulomb's law to systems containing more than two charges. In our first numerical Example of superposition, we consider three charges in a line.

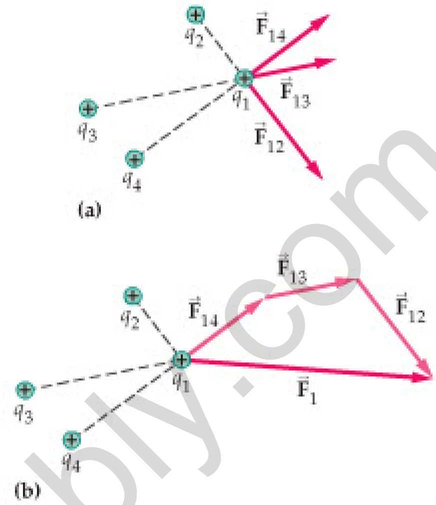


FIGURE 19–8 Superposition of forces

(a) Forces are exerted on q_1 by the charges q_2, q_3, and q_4. These forces are $\vec{F}_{12}$, $\vec{F}_{13}$, and $\vec{F}_{14}$, respectively. **(b)** The net force acting on q_1, which we label $\vec{F}_1$, is the vector sum of $\vec{F}_{12}$, $\vec{F}_{13}$, and $\vec{F}_{14}$.

EXAMPLE 19–2 NET FORCE

A charge $q_1 = -5.4\ \mu C$ is at the origin, and a charge $q_2 = -2.2\ \mu C$ is on the x axis at $x = 1.00$ m. Find the net force acting on a charge $q_3 = +1.6\ \mu C$ located at $x = 0.75$ m.

PICTURE THE PROBLEM

The physical situation is shown in our sketch, with each charge at its appropriate location. Notice that the forces exerted on charge q_3 by the charges q_1 and q_2 are in opposite directions. We give the force on q_3 due to q_1 the label $\vec{F}_{31}$, and the force on q_3 due to q_2 the label $\vec{F}_{32}$.

STRATEGY

The net force on q_3 is the vector sum of the forces due to q_1 and q_2. In particular, note that $\vec{F}_{31}$ points in the negative x direction $(-\hat{x})$, whereas $\vec{F}_{32}$ points in the positive x direction $(\hat{x})$. The magnitude of $\vec{F}_{31}$ is $k|q_1||q_3|/r^2$, with $r = 0.75$ m. Similarly, the magnitude of $\vec{F}_{32}$ is $k|q_2||q_3|/r^2$, with $r = 0.25$ m.

SOLUTION

1. Find the force acting on q_3 due to q_1. Since this force is in the negative x direction, as indicated in the sketch, we give it a negative sign:

$$\vec{F}_{31} = -k\frac{|q_1||q_3|}{r^2}\hat{x} = -(8.99 \times 10^9\ N \cdot m^2/C^2) \times \frac{(5.4 \times 10^{-6}\ C)(1.6 \times 10^{-6}\ C)}{(0.75\ m)^2}\hat{x} = -0.14\ N\,\hat{x}$$

2. Find the force acting on q_3 due to q_2. Since this force is in the positive x direction, as indicated in the sketch, we give it a positive sign:

$$\vec{F}_{32} = k\frac{|q_2||q_3|}{r^2}\hat{x} = (8.99 \times 10^9\ N \cdot m^2/C^2) \times \frac{(2.2 \times 10^{-6}\ C)(1.6 \times 10^{-6}\ C)}{(0.25\ m)^2}\hat{x} = 0.51\ N\,\hat{x}$$

3. Superpose these forces to find the total force, $\vec{F}_3$, acting on q_3:

$$\vec{F}_3 = \vec{F}_{31} + \vec{F}_{32} = -0.14\ N\,\hat{x} + 0.51\ N\,\hat{x} = 0.37\ N\,\hat{x}$$

INSIGHT

The net force acting on q_3 has a magnitude of 0.37 N, and it points in the positive x direction. As usual, notice that we use only magnitudes for the charges in the numerator of Coulomb's law.

CONTINUED ON NEXT PAGE

CONTINUED FROM PREVIOUS PAGE

PRACTICE PROBLEM

Find the net force on q_3 if it is at the location $x = 0.25$ m. [**Answer:** $\vec{\mathbf{F}}_3 = -1.2\ \mathrm{N}\hat{\mathbf{x}}$]

Some related homework problems: Problem 23, Problem 26, Problem 27

ACTIVE EXAMPLE 19–1 FIND THE LOCATION OF ZERO NET FORCE

In Example 19–2, the net force acting on the charge q_3 is to the right. To what value of x should q_3 be moved for the net force on it to be zero?

SOLUTION *(Test your understanding by performing the calculations indicated in each step.)*

1. Write the magnitude of the force due to q_1: $F_{31} = k|q_1||q_3|/x^2$
2. Write the magnitude of the force due to q_2: $F_{32} = k|q_2||q_3|/(1.00\ \mathrm{m} - x)^2$
3. Set these forces equal to one another, and cancel common terms: $|q_1|/x^2 = |q_2|/(1.00\ \mathrm{m} - x)^2$
4. Take the square root of both sides and solve for x: $x = 0.61$ m

INSIGHT

Therefore, if q_3 is placed between $x = 0.61$ m and $x = 1.00$ m, the net force acting on it is to the right, in agreement with Example 19–2. On the other hand, if q_3 is placed between $x = 0$ and $x = 0.61$ m, the net force acting on it is to the left. This agrees with the result in the Practice Problem of Example 19–2.

YOUR TURN

If the magnitude of each charge in this system is doubled, does the point of zero net force move to the right, move to the left, or remain in the same place? Explain.

(Answers to ***Your Turn*** *problems are given in the back of the book.)*

PROBLEM-SOLVING NOTE

Determining the Direction of the Electric Force

When determining the total force acting on a charge, begin by calculating the magnitude of each of the individual forces acting on it. Next, assign appropriate directions to the forces based on the principle that "opposites attract, likes repel" and perform a vector sum.

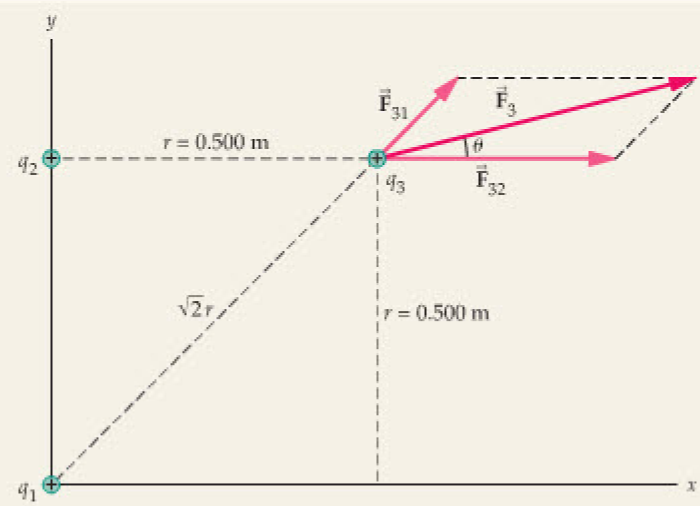
Next we consider systems in which the individual forces are not along the same line. In such cases, it is often useful to resolve the individual force vectors into components and then perform the required vector sum component by component. This technique is illustrated in the following Example and Conceptual Checkpoint.

EXAMPLE 19–3 SUPERPOSITION

Three charges, each equal to $+2.90\ \mu\mathrm{C}$, are placed at three corners of a square 0.500 m on a side, as shown in the diagram. Find the magnitude and direction of the net force on charge 3.

PICTURE THE PROBLEM

The positions of the three charges are shown in the sketch. We also show the force produced by charge 1, $\vec{\mathbf{F}}_{31}$, and the force produced by charge 2, $\vec{\mathbf{F}}_{32}$. Note that $\vec{\mathbf{F}}_{31}$ is 45.0° above the x axis and that $\vec{\mathbf{F}}_{32}$ is in the positive x direction. Also, the distance from charge 2 to charge 3 is $r = 0.500$ m, and the distance from charge 1 to charge 3 is $\sqrt{2}r$.



STRATEGY

To find the net force, we first calculate the magnitudes of $\vec{\mathbf{F}}_{31}$ and $\vec{\mathbf{F}}_{32}$ and then their components. Summing these components yields the components of the net force, $\vec{\mathbf{F}}_3$. Once we know the components of $\vec{\mathbf{F}}_3$, we can calculate its magnitude and direction in the same way as for any other vector.

SOLUTION

1. Find the magnitude of $\vec{\mathbf{F}}_{31}$:

$$F_{31} = k\frac{|q_1||q_3|}{(\sqrt{2}r)^2}$$
$$= (8.99 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{(2.90 \times 10^{-6}\ \text{C})^2}{2(0.500\ \text{m})^2}$$
$$= 0.151\ \text{N}$$

2. Find the magnitude of $\vec{\mathbf{F}}_{32}$:

$$F_{32} = k\frac{|q_2||q_3|}{r^2}$$
$$= (8.99 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{(2.90 \times 10^{-6}\ \text{C})^2}{(0.500\ \text{m})^2}$$
$$= 0.302\ \text{N}$$

3. Calculate the components of $\vec{\mathbf{F}}_{31}$ and $\vec{\mathbf{F}}_{32}$:

$$F_{31,x} = F_{31}\cos 45.0^\circ = (0.151\ \text{N})(0.707) = 0.107\ \text{N}$$
$$F_{31,y} = F_{31}\sin 45.0^\circ = (0.151\ \text{N})(0.707) = 0.107\ \text{N}$$
$$F_{32,x} = F_{32}\cos 0^\circ = (0.302\ \text{N})(1) = 0.302\ \text{N}$$
$$F_{32,y} = F_{32}\sin 0^\circ = (0.302\ \text{N})(0) = 0$$

4. Find the components of $\vec{\mathbf{F}}_3$:

$$F_{3,x} = F_{31,x} + F_{32,x} = 0.107\ \text{N} + 0.302\ \text{N} = 0.409\ \text{N}$$
$$F_{3,y} = F_{31,y} + F_{32,y} = 0.107\ \text{N} + 0 = 0.107\ \text{N}$$

5. Find the magnitude of $\vec{\mathbf{F}}_3$:

$$F_3 = \sqrt{F_{3,x}{}^2 + F_{3,y}{}^2}$$
$$= \sqrt{(0.409\ \text{N})^2 + (0.107\ \text{N})^2} = 0.423\ \text{N}$$

6. Find the direction of $\vec{\mathbf{F}}_3$:

$$\theta = \tan^{-1}\left(\frac{F_{3,y}}{F_{3,x}}\right) = \tan^{-1}\left(\frac{0.107\ \text{N}}{0.409\ \text{N}}\right) = 14.7^\circ$$

INSIGHT

Thus, the net force on charge 3 has a magnitude of 0.423 N and points in a direction 14.7° above the x axis. Note that charge 1, which is $\sqrt{2}$ times farther away from charge 3 than is charge 2, produces only half as much force as charge 2.

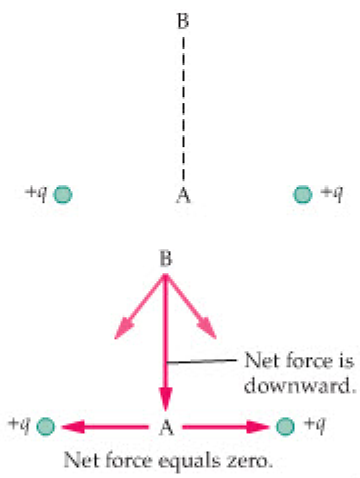
PRACTICE PROBLEM

Find the magnitude and direction of the net force on charge 3 if its magnitude is doubled to 5.80 μC. Assume that charge 1 and charge 2 are unchanged. **[Answer:** $F_3 = 2(0.423\ \text{N}) = 0.846\ \text{N}$, $\theta = 14.7^\circ$. Note that the angle is unchanged.]

Some related homework problems: Problem 31, Problem 32

CONCEPTUAL CHECKPOINT 19–3 COMPARE THE FORCE

A charge $-q$ is to be placed at either point A or point B in the accompanying figure. Assume points A and B lie on a line that is midway between the two positive charges. Is the net force experienced at point A **(a)** greater than, **(b)** equal to, or **(c)** less than the net force experienced at point B?



REASONING AND DISCUSSION

Point A is closer to the two positive charges than is point B. As a result, the force exerted by each positive charge will be greater when the charge $-q$ is placed at A. The *net* force, however, is zero at point A, since the equal attractive forces due to the two positive charges cancel, as shown in the diagram.

At point B, on the other hand, the attractive forces combine to give a net downward force. Hence, the charge $-q$ will experience a greater net force at point B.

ANSWER

(c) The net force at point A is less than the net force at point B.

Spherical Charge Distributions

Although Coulomb's law is stated in terms of point charges, it can be applied to any type of charge distribution by using the appropriate mathematics. For example, suppose a sphere has a charge Q distributed uniformly over its surface. If a point charge q is outside the sphere, a distance r from its center, the methods of calculus show that the magnitude of the force between the point charge and the sphere is simply

$$F = k\frac{|q||Q|}{r^2}$$

In situations like this, the spherical charge distribution behaves the same as if all its charge were concentrated in a point at its center. For point charges inside a charged spherical shell, the net force exerted by the shell is zero. In general, the electrical behavior of spherical *charge* distributions is analogous to the gravitational behavior of spherical *mass* distributions.

In the next Active Example, we consider a system in which a charge Q is distributed uniformly over the surface of a sphere. In such a case it is often convenient to specify the amount of *charge per area* on the sphere. This is referred to as the **surface charge density**, σ. If a sphere has an area A and a surface charge density σ, its total charge is

$$Q = \sigma A \qquad 19\text{–}7$$

Note that the SI unit of σ is C/m^2. If the radius of the sphere is R, then $A = 4\pi R^2$, and $Q = \sigma(4\pi R^2)$.

PROBLEM-SOLVING NOTE

Spherical Charge Distributions

Remember that a uniform spherical charge distribution can be replaced with a point charge only when considering points outside the charge distribution.

ACTIVE EXAMPLE 19–2 FIND THE FORCE EXERTED BY A SPHERE

An insulating sphere of radius $R = 0.10$ m has a uniform surface charge density equal to $5.9\ \mu C/m^2$. A point charge of magnitude $0.71\ \mu C$ is 0.45 m from the center of the sphere. Find the magnitude of the force exerted by the sphere on the point charge.

SOLUTION *(Test your understanding by performing the calculations indicated in each step.)*

1. Find the area of the sphere: $A = 0.13\ m^2$
2. Calculate the total charge on the sphere: $Q = 0.77\ \mu C$
3. Use Coulomb's law to calculate the magnitude of the force between the sphere and the point charge: $F = 0.024$ N

INSIGHT

As long as the point charge is outside the sphere, and the charge distribution remains spherically uniform, the sphere may be treated as a point charge.

YOUR TURN

Suppose the sphere in this problem is replaced by one with half the radius, but with the same surface charge density. Is the force exerted by this sphere greater than, less than, or the same as the force exerted by the original sphere? Explain.

(Answers to **Your Turn** *problems are given in the back of the book.)*

19–4 The Electric Field

You have probably encountered the notion of a "force field" in various science fiction novels and movies. A concrete example of a force field is provided by the force between electric charges. Consider, for example, a positive point charge q at the origin of a coordinate system, as in Figure 19–9. If a positive "test charge," q_0, is placed at point A, the force exerted on it by q is indicated by the vector $\vec{F}_A$. On the other hand, if the test charge is placed at point B, the force it experiences there is $\vec{F}_B$. At every point in space there is a corresponding force. In this sense, Figure 19–9 allows us to visualize the "force field" associated with the charge q.

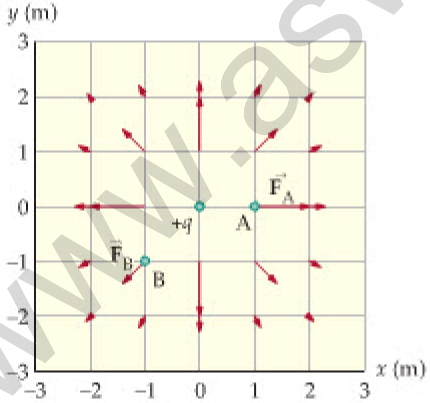


FIGURE 19–9 An electrostatic force field

The positive charge $+q$ at the origin of this coordinate system exerts a different force on a given charge at every point in space. Here we show the force vectors associated with q for a grid of points.

Since the magnitude of the force at every point in Figure 19–9 is proportional to q_0 (due to Coulomb's law), it is convenient to divide by q_0 and define a *force per*