The constant k in this expression is a fundamental constant of nature, the **Boltzmann constant,** named for the Austrian physicist Ludwig Boltzmann (1844–1906):

Boltzmann Constant, k

$$k = 1.38 \times 10^{-23}\ \mathrm{J/K} \qquad 17\text{–}1$$

SI unit: J/K

In general, a relationship between the thermal properties of a substance is referred to as an **equation of state.** Rearranging slightly, the equation of state for an ideal gas can be written as

Equation of State for an Ideal Gas

$$PV = NkT \qquad 17\text{–}2$$

We apply this result to the gas contained in a person's lungs in the next Example.

PROBLEM-SOLVING NOTE

Ideal Gas Law

When using the equation of state for an ideal gas, remember that the temperature must always be given in terms of the Kelvin scale.

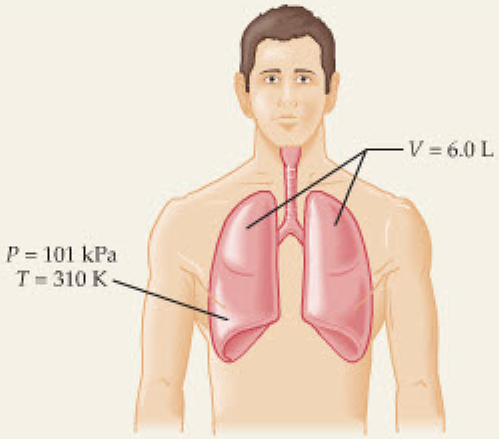
EXAMPLE 17–1 TAKE A DEEP BREATH

REAL-WORLD PHYSICS: BIO

A person's lungs might hold 6.0 L (1 L = 10^{-3} m^3) of air at body temperature (310 K) and atmospheric pressure (101 kPa). **(a)** Given that the air is 21% oxygen, find the number of oxygen molecules in the lungs. **(b)** If the person now climbs to the top of a mountain, where the air pressure is considerably less than 101 kPa, does the number of molecules in the lungs increase, decrease, or stay the same?

PICTURE THE PROBLEM

Our sketch shows a person's lungs, with their combined volume of $V = 6.0$ L. In addition, we indicate that the pressure in the lungs is $P = 101$ kPa and the temperature is $T = 310$ K.



STRATEGY

a. We will treat the air in the lungs as an ideal gas. Given the volume, temperature, and pressure of the gas, we can use the equation of state, $PV = NkT$, to solve for the number, N.

Finally, only 21% of the molecules in the air are oxygen. Therefore, we multiply N by 0.21 to find the number of oxygen molecules.

b. We apply $PV = NkT$ again, but this time with a reduced pressure P. Note that V and T remain the same, since they are determined by the person's body.

SOLUTION

Part (a)

1. Solve the equation of state for the number of molecules: $PV = NkT$ or $N = PV/kT$
2. Substitute numerical values to find the number of molecules in the lungs:
$$N = \frac{PV}{kT} = \frac{(1.01 \times 10^5\ \mathrm{Pa})(6.0 \times 10^{-3}\ \mathrm{m^3})}{(1.38 \times 10^{-23}\ \mathrm{J/K})(310\ \mathrm{K})} = 1.4 \times 10^{23}$$
3. Multiply N by 0.21 to find the number of molecules that are oxygen, O_2: $0.21N = 0.21(1.4 \times 10^{23}) = 2.9 \times 10^{22}$

Part (b)

From Step 1 we see that $N = PV/kT$. As a result, if the volume V of a person's lungs remains the same, along with the body temperature T, it follows that the number of molecules N will decrease linearly with pressure. For example, at an altitude of 10,000 ft the air pressure is about 70% of its value at sea level, meaning that the number of oxygen molecules in the lungs is only 70% of its usual value. No wonder people have a hard time "catching their breath" at high altitude.

INSIGHT

As one might expect, the number of molecules in even an average-size pair of lungs is enormous. In fact, a number this large is difficult to compare to anything we are familiar with. For example, the number of stars in the Milky Way galaxy is estimated to be "only" about 10^{11}. Thus, if each star in the Milky Way were itself a galaxy of 10^{11} stars, then all of these stars combined would just begin to approach the number of oxygen molecules in your lungs at this very moment.

PRACTICE PROBLEM

If the person takes a particularly deep breath, so that the lungs hold a total of 1.5×10^{23} molecules, what is the new volume of the lungs? [**Answer:** $V = 6.4 \times 10^{-3}\ \mathrm{m}^3 = 6.4\ \mathrm{L}$]

Some related homework problems: Problem 5, Problem 6, Problem 83

Another common way to write the ideal-gas equation of state is in terms of the number of **moles** in a gas—as opposed to using the number of molecules, N, as in Equation 17–2. In the SI system of units, the mole (mol) is defined in terms of the most abundant isotope of carbon, which is referred to as carbon-12. The definition is as follows:

> A mole is the amount of a substance that contains as many elementary entities as there are atoms in 12 g of carbon-12.

The phrase "elementary entities" refers to molecules, which may contain more than one atom, as in water (H_2O), or just a single atom, as in carbon (C) and helium (He). Experiments show that the number of atoms in a mole of carbon-12 is 6.022×10^{23}. This number is known as **Avogadro's number,** N_A, named for the Italian physicist and chemist Amedeo Avogadro (1776–1856).

Avogadro's Number, N_A

$$N_A = 6.022 \times 10^{23}\ \text{molecules/mol} \qquad \text{17–3}$$

SI unit: mol^{-1}; the number of molecules is dimensionless

Examples of one mole of various substances are shown in Figure 17–4.

Now, if we let n denote the number of moles in a gas, the number of molecules it contains is

$$N = nN_A$$

Substituting this into the ideal-gas equation of state yields

$$PV = nN_AkT$$

The constants N_A and k are combined to form the **universal gas constant,** R, defined as follows:

Universal Gas Constant, R

$$R = N_Ak = (6.022 \times 10^{23}\ \text{molecules/mol})(1.38 \times 10^{-23}\ \mathrm{J/K})$$
$$= 8.31\ \mathrm{J/(mol \cdot K)} \qquad \text{17–4}$$

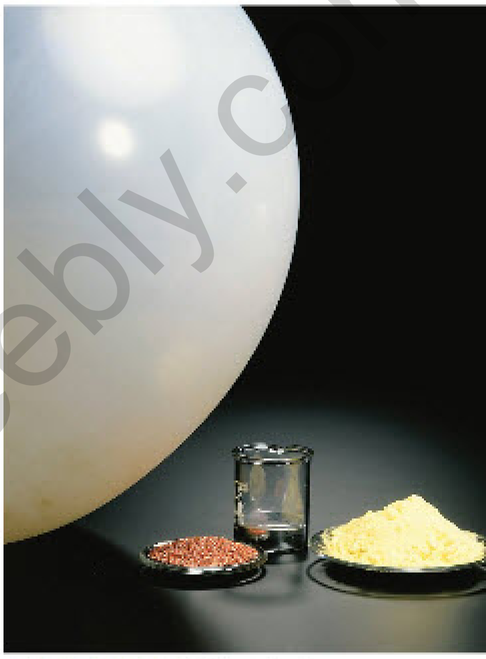
SI unit: $\mathrm{J/(mol \cdot K)}$

Thus, an alternative form of the equation of state is

Equation of State for an Ideal Gas

$$PV = nRT \qquad \text{17–5}$$

This relation is used in the following Active Example.



▲ **FIGURE 17–4 Moles of various substances**

Counting atoms or molecules is obviously hard to do, but the mole concept provides a useful way of dealing with the difficulty. A mole of any substance contains the same number of elementary entities (atoms or molecules).

This number, referred to as Avogadro's number, has the value $N_A = 6.022 \times 10^{23}$. To count out N_A molecules of a gas at standard temperature and pressure, for example, all you need do is measure out a volume of gas equal to 22.4 liters. Similarly, you can effectively count one mole of any substance by measuring out a mass in grams that is equal to the atomic mass of that substance. Thus, the mole provides a convenient bridge between the realm of atoms and molecules and the macroscopic world of observable masses and volumes. This photo shows molar amounts of four different substances: hydrogen, copper, mercury, and sulfur.

ACTIVE EXAMPLE 17–1 THE AMOUNT OF AIR IN A BASKETBALL

How many moles of air are in an inflated basketball? Assume that the pressure in the ball is 171 kPa, the temperature is 293 K, and the diameter of the ball is 30.0 cm.

SOLUTION *(Test your understanding by performing the calculations indicated in each step.)*

1. Solve $PV = nRT$ for the number of moles, n: $n = PV/RT$
2. Calculate the volume of the ball: $V = 4\pi r^3/3 = 0.0141\ \mathrm{m}^3$
3. Substitute numerical values: $n = 0.990\ \mathrm{mol}$

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INSIGHT

Thus, an inflated basketball contains approximately one mole of air.

In this calculation we used $PV = nRT$ because we were interested in the number of *moles* (n) contained in the gas. In contrast, we used $PV = NkT$ in Example 17–1 because we wanted to know the number of *molecules* (N) in the gas.

YOUR TURN

Suppose we were to halve both the temperature and the diameter of the ball. By what factor would the number of moles contained within it change?

(Answers to **Your Turn** *problems are given in the back of the book.)*

As mentioned before, a mole of anything has precisely the same number (N_A) of particles. What differs from substance to substance is the mass of one mole. For example, one mole of helium atoms has a mass of 4.00260 g, and one mole of copper atoms has a mass of 63.546 g. In general, we define the **atomic or molecular mass,** M, of a substance to be the mass in grams of one mole of that substance. Thus, the atomic mass for helium is $M = 4.00260\ \text{g/mol}$ and for copper it is $M = 63.546\ \text{g/mol}$. The periodic table in Appendix E gives the atomic masses for all elements.

Note that the atomic mass provides a convenient bridge between the macroscopic world, where we measure the mass of a substance in grams, and the microscopic world, where the number of molecules is typically 10^{23} or greater. As we have seen, if you measure out a mass of copper equal to 63.546 g you have, in effect, counted out $N_A = 6.022 \times 10^{23}$ atoms of copper. It follows that the mass of an individual copper atom, m, is the atomic mass of copper divided by Avogadro's number; that is,

$$m = \frac{M}{N_A} \qquad \text{17–6}$$

We use this relation to find the masses of a copper atom and an oxygen molecule in the following Exercise.

EXERCISE 17–1

Find the masses of **(a)** a copper atom and **(b)** a molecule of oxygen, O_2. Atomic masses are listed in Appendix E.

SOLUTION

a. The atomic mass of copper is 63.546 g/mol. Thus, a copper atom has the mass

$$m_{\text{Cu}} = \frac{M}{N_A} = \frac{63.546\ \text{g/mol}}{6.022 \times 10^{23}\ \text{atoms/mol}} = 1.055 \times 10^{-22}\ \text{g/atom}$$

b. Since the atomic mass of oxygen is 16.00 g/mol (Appendix E), the molecular mass of O_2 is 32.00 g/mol. Therefore, the mass of O_2 is

$$m_{O_2} = \frac{M}{N_A} = \frac{32.00\ \text{g/mol}}{6.022 \times 10^{23}\ \text{molecules/mol}} = 5.314 \times 10^{-23}\ \text{g/molecule}$$

Finally, we consider a common situation to further illustrate the general features of the ideal-gas equation of state.

CONCEPTUAL CHECKPOINT 17–1 AIR PRESSURE

Feeling a bit cool, you turn up the thermostat in your living room. A short time later the air is warmer. Assuming the room is well sealed, is the pressure of the air **(a)** greater than, **(b)** less than, or **(c)** the same as before you turned up the heat?

REASONING AND DISCUSSION

We assume that the number of molecules and the volume occupied by the molecules are approximately constant. Thus, increasing T, while holding N and V fixed, leads to an increased pressure, P.

ANSWER

(a) The air pressure increases.

Isotherms

Historically, the ideal-gas equation of state was arrived at piece by piece, as a result of the combined efforts of a number of researchers. For example, the English scientist Robert Boyle (1627–1691) established the fact that the pressure of a gas varies inversely with volume—as long as temperature and the number of molecules are held constant. This is known as **Boyle's law**:

$$P_iV_i = P_fV_f$$

$$(\textit{fixed number of molecules, } N\textit{; fixed temperature, } T) \qquad 17\text{–}7$$

To see that Boyle's law is consistent with $PV = NkT$, note that if N and T are constant, then so too is PV. Another way of saying that PV is a constant is to say that its initial value must be equal to its final value. This is Boyle's law.

PROBLEM-SOLVING NOTE

Pressure and Volume in an Isotherm

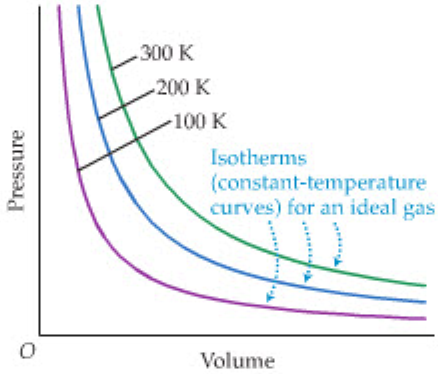
If the temperature of an ideal gas is constant, the pressure and volume change in such a way that their product has a constant value.

When N and T are constant, the equation of state $PV = NkT$ implies that

$$P = \frac{NkT}{V} = \frac{\text{constant}}{V}$$

This result is plotted in Figure 17–5, where we show P as a function of V. Note that the larger the temperature, T, the larger the constant in the numerator. Therefore, the curves farther from the origin correspond to higher temperatures, as indicated.

Each of the curves in Figure 17–5 corresponds to a different, fixed temperature. As a result, these curves are known as **isotherms**, which means, literally, "constant temperature." We shall return to the topic of isotherms in the next chapter, when we consider various thermal processes.



▶ **FIGURE 17–5 Ideal-gas isotherms**

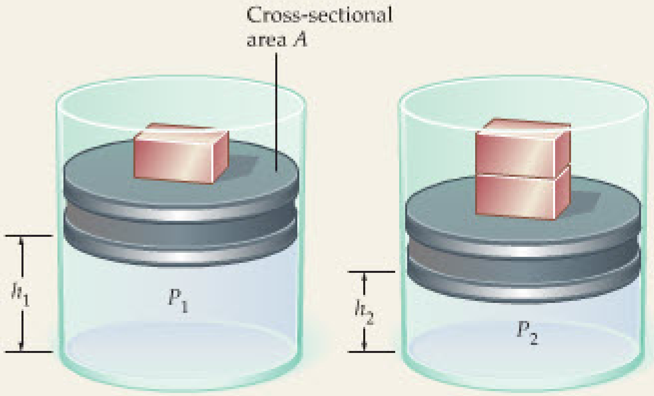
Pressure-versus-volume isotherms for an ideal gas. Each isotherm is of the form $P = NkT/V = \text{constant}/V$. The three isotherms shown are for the temperatures 100 K, 200 K, and 300 K.

EXAMPLE 17–2 UNDER PRESSURE

A cylindrical flask of cross-sectional area A is fitted with an airtight piston that is free to slide up and down. Contained within the flask is an ideal gas. Initially the pressure applied by the piston is 130 kPa and the height of the piston above the base of the flask is 25 cm. When additional mass is added to the piston, the pressure increases to 170 kPa. Assuming the system is always at the temperature 290 K, find the new height of the piston.

PICTURE THE PROBLEM

The physical system is shown in our sketch. The initial pressure in the flask is $P_1 = 130$ kPa, and the initial height of the piston is $h_1 = 25$ cm. When additional mass is placed on the piston, the pressure increases to $P_2 = 170$ kPa and the height decreases to h_2. The temperature remains constant.



STRATEGY

Since the temperature is held constant in this system, it follows that $PV = NkT$ is also constant. Thus, as in Boyle's law, $P_1V_1 = P_2V_2$. This relation can be used to find V_2 (since we are given P_1, P_2, and V_1).

Next, the cylindrical volume is related to the height by $V = Ah$. Note that the area A is the same in both cases, hence it will cancel, allowing us to solve for the height, h_2.

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SOLUTION

1. Set the initial and final values of PV equal to one another, then solve for V_2:

$$P_1V_1 = P_2V_2$$
$$V_2 = V_1(P_1/P_2)$$

2. Substitute $V_1 = Ah_1$ and $V_2 = Ah_2$, and solve for h_2:

$$Ah_2 = Ah_1(P_1/P_2)$$
$$h_2 = h_1(P_1/P_2)$$

3. Substitute numerical values:

$$h_2 = h_1\frac{P_1}{P_2} = (25\ \text{cm})\frac{130\ \text{kPa}}{170\ \text{kPa}} = 19\ \text{cm}$$

INSIGHT

In order to keep the temperature of this system constant, it is necessary to allow some heat to flow out of the cylindrical flask as the gas is compressed by the additional weight. Thus, the flask is not thermally insulated during this experiment.

PRACTICE PROBLEM

What pressure would be required to change the height of the piston to 29 cm? [**Answer:** $P = 110$ kPa]

Some related homework problems: Problem 18, Problem 19

▲ Charles's law states that the volume of a gas at constant pressure is proportional to its Kelvin temperature. For example, this balloon was fully inflated at room temperature (293 K) and atmospheric pressure. When cooled by vapors from liquid nitrogen (77 K), however, the volume of the air inside the balloon decreased markedly, causing it to shrivel.

Finally, recall that in Conceptual Checkpoint 15–2 we considered the behavior of an air bubble rising from a swimmer in shallow water. At the time, we said that the diameter of the bubble increases as it rises because the pressure of the surrounding water is decreasing. This is certainly the case, but now we can be more precise. If we assume that the water temperature is constant, and that no gas molecules are added to or removed from the bubble, then the volume of the bubble has the following dependence:

$$V = \frac{NkT}{P} = \frac{\text{constant}}{P}$$

This is just our isotherm, again, and we see that the volume indeed increases as pressure decreases.

Constant Pressure

Another aspect of ideal-gas behavior was discovered by the French scientist Jacques Charles (1746–1823) and later studied in greater detail by fellow Frenchman Joseph Gay-Lussac (1778–1850). Known today as **Charles's law,** their result is that the volume of a gas divided by its temperature is constant, as long as the pressure and number of molecules are constant:

$$\frac{V_i}{T_i} = \frac{V_f}{T_f}$$

$$(\textit{fixed number of molecules, } N\textit{; fixed pressure, } P) \qquad 17\text{–}8$$

As with Boyle's law, this result follows immediately from the ideal-gas equation of state. Solving Equation 17–2 for V/T, we have

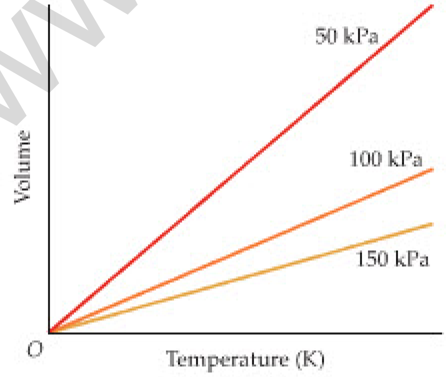
$$\frac{V}{T} = \frac{Nk}{P}$$

If N and P are constant, then so is the quantity V/T.

Charles's law can be rewritten as a linear relation between volume and temperature:

$$V = (\text{constant})T$$

The constant in this expression is Nk/P. This result is illustrated in Figure 17–6, where we see the volume of an ideal gas vanishing as the temperature approaches absolute zero.

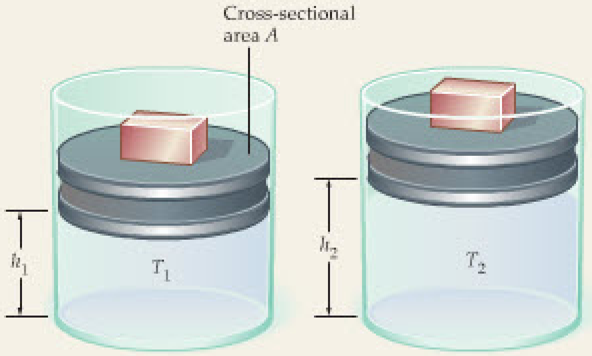


◀ **FIGURE 17–6 Volume versus temperature for an ideal gas at constant pressure**

The temperature in this plot is the Kelvin temperature, T; hence $T = 0$ corresponds to absolute zero. At absolute zero the volume attains its lowest possible value—zero.

ACTIVE EXAMPLE 17–2 FIND THE PISTON HEIGHT

Consider again the system described in Example 17–2. In this case the temperature is changed from an initial value of 290 K to a final value of 330 K. The pressure exerted on the gas remains constant at 130 kPa, and the initial height of the piston is 25 cm. Find the final height of the piston.



SOLUTION *(Test your understanding by performing the calculations indicated in each step.)*

1. Set the initial value of V/T equal to the final value of V/T: $V_1/T_1 = V_2/T_2$
2. Solve for V_2: $V_2 = V_1(T_2/T_1)$
3. Use $V = Ah$ to solve for the height, h_2: $h_2 = h_1(T_2/T_1)$
4. Substitute numerical values: $h_2 = 28$ cm

INSIGHT

The volume of the gas, and hence the height of the piston, increased in direct proportion to the increase in temperature. This is to be expected when the pressure is constant.

YOUR TURN

At what temperature is the piston height equal to 19 cm?

(Answers to **Your Turn** *problems are given in the back of the book.)*

PROBLEM-SOLVING NOTE

Volume and Temperature in an Ideal Gas

If the pressure of an ideal gas is constant, the ratio of the volume and temperature remains constant. This is true no matter what the value of the pressure.

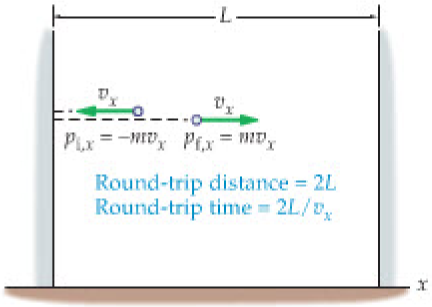
17–2 Kinetic Theory

We can readily measure the pressure and temperature of a gas using a pressure gauge and a thermometer. These are *macroscopic* quantities that apply to the gas as a whole. It is not so easy, however, to measure *microscopic* quantities, such as the position or velocity of an individual molecule. Still, there must be some connection between what happens on the microscopic level and what we observe on the macroscopic level. This connection is described by the **kinetic theory of gases.**

In the kinetic theory, we imagine a gas to be made up of a collection of molecules moving about inside a container of volume V. In particular, we assume the following:

- The container holds a very large number N of identical molecules, each of mass m. The molecules themselves can be thought of as "point particles"—that is, the volume of the molecules is negligible compared to the volume of the container, and the size of the molecules is negligible compared to the distance between them. (This is why dilute real gases are the best approximation to the ideal case.)
- The molecules move about the container in a random manner. They obey Newton's laws of motion at all times.
- When molecules hit the walls of the container or collide with one another, they bounce elastically. Other than these collisions, the molecules have no interactions.

With these basic assumptions we can relate the pressure of a gas to the behavior of the molecules themselves.



▲ **FIGURE 17–7 Force exerted by a molecule on the wall of a container**

A molecule bounces off a wall of a container, changing its momentum from $-mv_x$ to $+mv_x$; the change in momentum is $2mv_x$. A round trip will be completed in the time $\Delta t = 2L/v_x$, so the average force exerted on the molecule by the wall is $F = \Delta p/\Delta t = 2mv_x/(2L/v_x) = mv_x^2/L$.

The Origin of Pressure

As we shall see, the pressure exerted by a gas is due to the innumerable collisions between gas molecules and the walls of their container. Each collision results in a change of momentum for a given molecule, just like throwing a ball at a wall and having it bounce back. The total change in momentum of the molecules in a given time, divided by the time, is simply the force a wall must exert on the gas to contain it (see Section 9–2). The average of this force over time, and over the area of a wall, is the pressure of the gas.

To be specific, imagine a container that is a cube of length L on a side. Its volume, then, is $V = L^3$. In addition, consider a given molecule that happens to be moving in the negative x direction toward a wall, as in Figure 17–7. If its speed is v_x, its initial momentum is $p_{i,x} = -mv_x$. After bouncing from the wall, it moves in the positive x direction with the same speed (since the collision is elastic), and hence its final momentum is $p_{f,x} = +mv_x$. As a result, the molecule's change in momentum is

$$\Delta p_x = p_{f,x} - p_{i,x} = mv_x - (-mv_x) = 2mv_x$$

The wall exerts a force on the molecule to cause this momentum change.

After the bounce, the molecule travels to the other side of the container and back before bouncing off the same wall again. The time required for this round trip of length $2L$ is

$$\Delta t = 2L/v_x$$

Thus, by Newton's second law, the average force exerted by the wall on the molecule is

$$F = \frac{\Delta p_x}{\Delta t} = \frac{2mv_x}{2L/v_x} = \frac{mv_x^{\,2}}{L}$$

The average pressure exerted by this wall, then, is simply the force divided by the area. Since the area of the wall is $A = L^2$, we have

$$P = \frac{F}{A} = \frac{(mv_x^{\,2}/L)}{L^2} = \frac{mv_x^{\,2}}{L^3} = \frac{mv_x^{\,2}}{V} \qquad \text{17–9}$$

Note that we have used the fact that the volume of the container is L^3.

In this calculation we assumed that the molecule moves in the x direction. This was merely to simplify the derivation. If, instead, the molecule moves at some angle to the x axis, the calculation applies to its x component of motion. The final conclusions are unchanged.

Speed Distribution of Molecules

In deriving Equation 17–9, we considered a single molecule with a particular speed. Other molecules, of course, will have different speeds. In addition, the speed of any given molecule changes with time as it collides with other molecules in the gas. What remains constant, however, is the overall **distribution of speeds.**

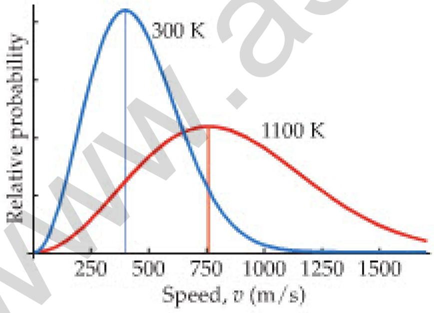
This is illustrated in Figure 17–8, where we present the results obtained by the Scottish physicist James Clerk Maxwell (1831–1879). This plot shows the relative probability that an O_2 molecule will have a given speed. For example, on the curve labeled 300 K, the most probable speed is about 390 m/s. When the temperature is increased to 1100 K, the most probable speed increases to roughly 750 m/s. Other speeds occur as well, from speeds near zero to those that are very large, but these have much lower probabilities.

Thus, in Equation 17–9, the term $v_x^{\,2}$ should be replaced with the average of $v_x^{\,2}$ over all the molecules in the gas. Writing this average as $(v_x^{\,2})_{\rm av}$, we have

$$P = \frac{m(v_x^{\,2})_{\rm av}}{V}$$

Since all N molecules in the gas follow the same distribution, the pressure exerted by the gas as a whole is N times this result:

$$P = N\frac{m(v_x^{\,2})_{\rm av}}{V} = \left(\frac{N}{V}\right)m(v_x^{\,2})_{\rm av} \qquad \text{17–10}$$



▲ **FIGURE 17–8 The Maxwell speed distribution**

The Maxwell distribution of molecular speeds for O_2 at the temperatures $T = 300$ K and $T = 1100$ K. Note that the most probable speed increases with increasing temperature.

Of course, there is nothing special about the x direction—Equation 17–10 applies equally well with $(v_y^2)_{\text{av}}$ or $(v_z^2)_{\text{av}}$ in place of $(v_x^2)_{\text{av}}$. Thus, it would be preferable to express the pressure of the gas in terms of the overall speed of the molecules, rather than in terms of a single component. The speed squared of a molecule is

$$v^2 = v_x^2 + v_y^2 + v_z^2$$

Hence, the average of v^2 is

$$(v^2)_{\text{av}} = (v_x^2)_{\text{av}} + (v_y^2)_{\text{av}} + (v_z^2)_{\text{av}}$$

Since the x, y, and z directions are equivalent, it follows that

$$(v_x^2)_{\text{av}} = (v_y^2)_{\text{av}} = (v_z^2)_{\text{av}}$$

As a result,

$$(v^2)_{\text{av}} = (v_x^2)_{\text{av}} + (v_y^2)_{\text{av}} + (v_z^2)_{\text{av}} = 3(v_x^2)_{\text{av}}$$

or, equivalently,

$$(v_x^2)_{\text{av}} = \tfrac{1}{3}(v^2)_{\text{av}}$$

Using this replacement in Equation 17–10 yields

$$P = \tfrac{1}{3}\left(\frac{N}{V}\right)m(v^2)_{\text{av}}$$

The last part of this expression, $m(v^2)_{\text{av}}$, is simply twice the average kinetic energy of a molecule. Thus, using K for the kinetic energy, as in Chapters 7 and 8, we can write the pressure as follows:

Pressure in the Kinetic Theory of Gases

$$P = \tfrac{1}{3}\left(\frac{N}{V}\right)2K_{\text{av}} = \tfrac{2}{3}\left(\frac{N}{V}\right)\left(\tfrac{1}{2}mv^2\right)_{\text{av}} \qquad \text{17–11}$$

To summarize, using the kinetic theory, we have shown that the pressure of a gas is proportional to the number of molecules and inversely proportional to the volume. We discussed both of these dependences before when considering the ideal gas. In addition, we see that

> The pressure of a gas is directly proportional to the average kinetic energy of its molecules.

This is the key connection between microscopic behavior and macroscopic observables.

Kinetic Energy and Temperature

If we compare the ideal-gas equation of state, $PV = NkT$, with the result from kinetic theory, Equation 17–11, we find

$$PV = NkT = \tfrac{2}{3}N\left(\tfrac{1}{2}mv^2\right)_{\text{av}}$$

As a result,

$$\tfrac{2}{3}\left(\tfrac{1}{2}mv^2\right)_{\text{av}} = kT$$

Equivalently,

Kinetic Energy and Temperature

$$\left(\tfrac{1}{2}mv^2\right)_{\text{av}} = K_{\text{av}} = \tfrac{3}{2}kT \qquad \text{17–12}$$

This is one of the most important results of kinetic theory. It says that the average kinetic energy of a gas molecule is directly proportional to the Kelvin temperature, T. Thus, when we heat a gas, what happens on the microscopic level is that the molecules move with speeds that are, on average, greater. Similarly, cooling a gas causes the molecules to slow.

EXERCISE 17–2

Find the average kinetic energy of oxygen molecules in the air. Assume the air is at a temperature of 21 °C.

SOLUTION

Using Equation 17–12, and the fact that 21 °C = 294 K, we find

$$K_{av} = \tfrac{3}{2}kT = \tfrac{3}{2}(1.38 \times 10^{-23}\ \mathrm{J/K})(294\ \mathrm{K}) = 6.09 \times 10^{-21}\ \mathrm{J}$$

Note that this is also the average kinetic energy of nitrogen molecules in the air. In fact, the type of molecule is not important; all that matters is the temperature of the gas.

Returning to Equation 17–12, we find with a slight rearrangement that

$$(v^2)_{av} = 3kT/m$$

Now, the square root of $(v^2)_{av}$ is given a special name—it is called the **root mean square** (rms) speed. For gas molecules, then, the rms speed, v_{rms}, is the following:

RMS Speed of a Gas Molecule

$$v_{rms} = \sqrt{(v^2)_{av}} = \sqrt{\frac{3kT}{m}} \qquad 17\text{–}13$$

SI unit: m/s

Rewriting this in terms of the molecular mass, M, we have

$$v_{rms} = \sqrt{\frac{3kT}{m}} = \sqrt{\frac{3kT}{(M/N_A)}} = \sqrt{\frac{3N_AkT}{M}}$$

Finally, using $N_Ak = R$ yields

$$v_{rms} = \sqrt{\frac{3RT}{M}} \qquad 17\text{–}14$$

The rms speed is one of the characteristic speeds of the Maxwell speed distribution. As shown in Figure 17–9, v_{rms} is slightly greater than the most probable speed, v_{mp} and the average speed, v_{av}. (Homework Problem 88 illustrates the difference between v_{av} and v_{rms} for a simple system of molecules.)

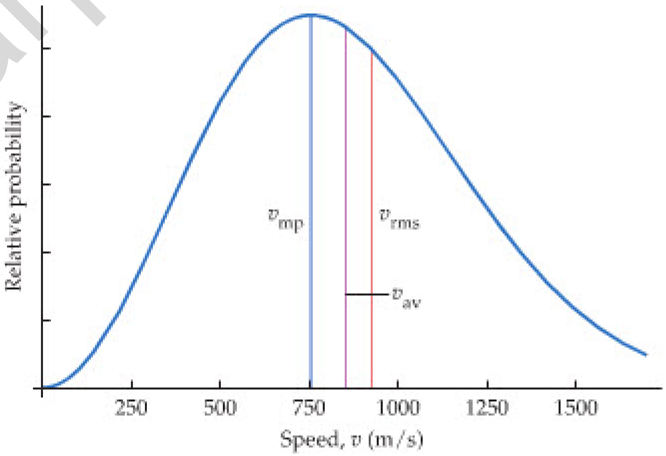


FIGURE 17–9 Most probable, average, and rms speeds

Characteristic speeds for O_2 at the temperature T = 1100 K. From left to right, the indicated speeds are the most probable speed, v_{mp}, the average speed, v_{av}, and the rms speed, v_{rms}.

EXAMPLE 17–3 FRESH AIR

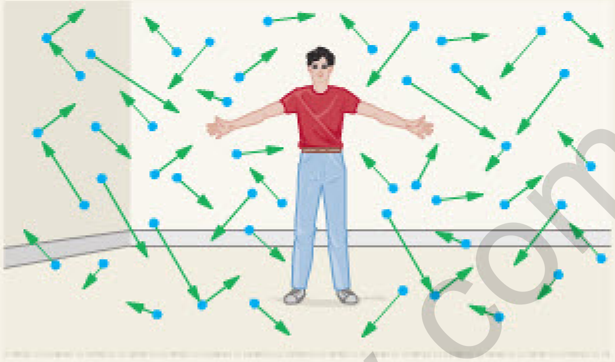
The atmosphere is composed primarily of nitrogen N_2 (78%) and oxygen O_2 (21%). **(a)** Is the rms speed of N_2 (28.0 g/mol) greater than, less than, or the same as the rms speed of O_2 (32.0 g/mol)? **(b)** Find the rms speed of N_2 and O_2 at 293 K.

PICTURE THE PROBLEM

Our sketch shows molecules in the air bouncing off the walls of a room—they also bounce off a person in the room. Note that the molecules move in random directions with a variety of speeds. Most of the molecules are N_2, but about 21% are O_2.

STRATEGY

The rms speeds are calculated by straightforward substitution into the relation $v_{\text{rms}} = \sqrt{3RT/M}$. The only point to be careful about is the molecular mass of each molecule. In particular, note that for nitrogen the molecular mass is $28.0 \text{ g/mol} = 0.0280 \text{ kg/mol}$, and similarly for oxygen, the molecular mass is $32.0 \text{ g/mol} = 0.0320 \text{ kg/mol}$.



SOLUTION

Part (a)

Since both molecules are at the same temperature, they have the same kinetic energy. The nitrogen molecule has less mass, however; thus, if it is to have the same kinetic energy, it must have a higher speed. This is also what one would expect on the basis of Equation 17–14.

Part (b)

1. To find the rms speed of N_2, substitute $M = 0.0280 \text{ kg/mol}$ in Equation 17–14:

$$v_{\text{rms}} = \sqrt{\frac{3[8.31 \text{ J/(mol}\cdot\text{K)}](293 \text{ K})}{0.0280 \text{ kg/mol}}} = 511 \text{ m/s}$$

2. To find the rms speed of O_2, substitute $M = 0.0320 \text{ kg/mol}$ in Equation 17–14:

$$v_{\text{rms}} = \sqrt{\frac{3[8.31 \text{ J/(mol}\cdot\text{K)}](293 \text{ K})}{0.0320 \text{ kg/mol}}} = 478 \text{ m/s}$$

INSIGHT

As expected, N_2 has the higher rms speed. For comparison, the speed of sound at 293 K is 343 m/s, which is just over 750 mi/h. Thus, the molecules in the air are bouncing off your skin with the speed of a supersonic jet.

Such high speeds can help to promote chemical reactions, which usually depend on collisions between molecules to take place. In general, higher temperatures mean higher molecular speeds and increased rates of chemical reactions.

PRACTICE PROBLEM

One of the substances found in air is monatomic and has an rms speed of 428 m/s, at 293 K. What is this substance? [**Answer:** $M = 0.0399 \text{ kg/mol}$; thus the substance is argon, Ar, which comprises about 0.94% of the atmosphere].

Some related homework problems: Problem 26, Problem 27

Chemical reactions usually depend on collisions between molecules to take place. Though the molecules in the air are not moving fast enough to trigger chemical reactions, high molecular speeds can help to promote chemical reactions in many circumstances. In general, higher temperatures mean higher molecular speeds and increased rates of chemical reactions. This is why the rate at which a cricket chirps increases with increasing temperature, making it a novel type of thermometer. (See the Passage Problem in Chapter 13 and Problem 75 in Chapter 16.)

REAL-WORLD PHYSICS: BIO

Chemical reactions and temperature

CONCEPTUAL CHECKPOINT 17–2 COMPARE SPEEDS

Each of two containers of equal volume holds an ideal gas. Container A has twice as many molecules as container B. If the gas pressure is the same in the two containers, is the rms speed of the molecules in container A **(a)** greater than, **(b)** less than, or **(c)** the same as the rms speed of the molecules in container B?

REASONING AND DISCUSSION

Since P and V are the same, you might think the rms speeds are the same as well. Recall, however, that *the pressure of a gas is caused by the collisions of gas molecules with the walls of the container*. If more molecules occupy a given volume, and they bounce off the walls with the same speed as in a container with fewer molecules, the pressure will be greater. Therefore, in order for the pressure in the two containers to be the same, it is necessary that the rms speed of the molecules in container A be less than the rms speed in container B.

Mathematically, we note that $P_A = P_B$ and $V_A = V_B$; hence $P_AV_A = P_BV_B$. Using the ideal-gas relation, $PV = NkT$, we can write this condition as $N_AkT_A = N_BkT_B$. Thus, if $N_A = 2N_B$, it follows that $T_A = T_B/2$. Since the temperature is lower in container A, its molecules have the smaller rms speed.

ANSWER

(b) The rms speed is less in container A.

The Internal Energy of an Ideal Gas

The internal energy of a substance is the sum of all its potential and kinetic energies. In an ideal gas there are no interactions between molecules, other than perfectly elastic collisions; hence, there is no potential energy. As a result, the total energy of the system is the sum of the kinetic energy of each of its molecules. Thus, for an ideal gas of N pointlike molecules—that is, a monatomic gas—the internal energy is simply

Internal Energy of a Monatomic Ideal Gas

$$U = \tfrac{3}{2}NkT \tag{17–15}$$

SI unit: J

In terms of moles, this result is

$$U = \tfrac{3}{2}nRT \tag{17–16}$$

We shall return to this result in the next chapter.

EXERCISE 17–3

A basketball at 290 K holds 0.95 mol of air molecules. What is the internal energy of the air in the ball?

SOLUTION

Applying Equation 17–16 we find

$$U = \tfrac{3}{2}nRT = \tfrac{3}{2}(0.95\ \text{mol})[8.31\ \text{J}/(\text{mol}\cdot\text{K})](290\ \text{K}) = 3400\ \text{J}$$

This is roughly the kinetic energy a basketball would have if you dropped it from a height of 700 m.

If an ideal gas is diatomic, there are additional contributions to the internal energy. For example, a diatomic molecule, which is shaped somewhat like a dumbbell, can have rotational kinetic energy. Diatomic molecules can also vibrate along the line joining the two atoms, which is yet another contribution to the total energy. Thus, the results in Equations 17–15 and 17–16 apply only to the simplest case, the ideal monatomic gas.

17–3 Solids and Elastic Deformation

The defining characteristic of a solid object is that it has a particular shape. For example, when Michelangelo carved the statue of David from a solid block of marble, he was confident it would retain its shape long after his work was done. Liquids and gases do not behave in this way. In contrast, they assume the shape of the container into which they are placed. On a molecular level, these differences arise from the fact that the intermolecular forces in a solid are strong enough to practically immobilize its molecules, whereas the intermolecular forces in liquids and gases are so weak the molecules can move about relatively freely.

Even so, the shape of a solid *can* be changed—though usually only slightly—if it is acted on by a force. In this section we consider various types of deformations that can occur in solids and the way these deformations are related to the forces that cause them.

Changing the Length of a Solid

A useful physical model for a solid is a lattice of small balls representing molecules connected to one another by springs representing the intermolecular forces. Pulling on a solid rod with a force F, for example, causes each "intermolecular spring" in the direction of the force to expand by an amount proportional to F. The net result is that the entire solid increases its length by an amount proportional to the force, $\Delta L \propto F$, as indicated in **Figure 17–10.**

The stretch ΔL is also proportional to the initial length of the rod, L_0. To see why, we first note that each intermolecular spring expands by the same amount, giving a total stretch that is proportional to the number of such springs. But the

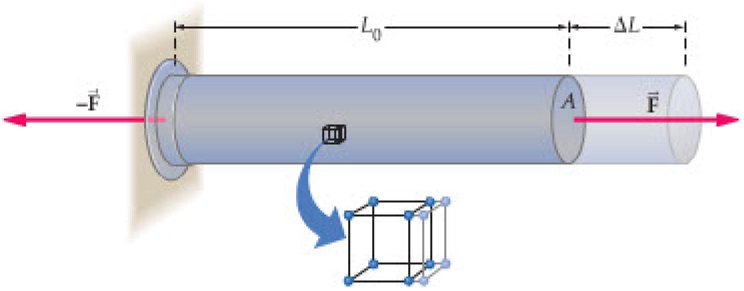


FIGURE 17–10 Stretching a rod
Equal and opposite forces applied to the ends of a rod cause it to stretch. On the atomic level, the forces stretch the "intermolecular springs" in the solid, resulting in an overall increase in length. The stretch is proportional to the force F and the initial length L_0, and inversely proportional to the cross-sectional area A.

number of intermolecular springs is proportional to the total initial length of the rod. Thus, it follows that $\Delta L \propto FL_0$.

Finally, the amount of stretch for a given force F is inversely proportional to the cross-sectional area A of the rod. For example, a rod with a cross-sectional area $2A$ is like two rods of cross-sectional area A placed side by side. Thus, applying a force F to a rod of area $2A$ is equivalent to applying a force $F/2$ to two rods of area A. The result is half the stretch when the area is doubled; that is, $\Delta L \propto FL_0/A$. Solving this relation for the amount of force F required to produce a given stretch ΔL we find $F \propto (\Delta L/L_0)A$, or as an equality

$$F = Y\left(\frac{\Delta L}{L_0}\right)A \qquad 17\text{–}17$$

The proportionality constant Y in this expression is **Young's modulus**, named for the English physicist Thomas Young (1773–1829). Comparing the two sides of Equation 17–17, we see that Young's modulus has the units of force per area ($\mathrm{N/m^2}$).

Typical values for Young's modulus are given in Table 17–1. Notice that the values vary from material to material, but are all rather large. This means that a large force is required to cause even a small stretch in a solid. Of course, Equation 17–17 applies equally well to a compression or a stretch. However, some materials have a slightly different Young's modulus for compression and stretching. For example, human bones under tension (stretching) have a Young's modulus of $1.6 \times 10^{10}\ \mathrm{N/m^2}$, while bones under compression have a slightly smaller Young's modulus of $9.4 \times 10^{9}\ \mathrm{N/m^2}$.

TABLE 17–1 Young's Modulus for Various Materials

Material	Young's modulus, Y ($\mathrm{N/m^2}$)
Tungsten	36×10^{10}
Steel	20×10^{10}
Copper	11×10^{10}
Brass	9.0×10^{10}
Aluminum	6.9×10^{10}
Pyrex glass	6.2×10^{10}
Lead	1.6×10^{10}
Bone	
Tension	1.6×10^{10}
Compression	0.94×10^{10}
Nylon	0.37×10^{10}

PROBLEM-SOLVING NOTE

Area and Young's Modulus

When using Young's modulus, remember that the area A is the area that is at right angles to the applied force.

EXAMPLE 17–4 STRETCHING A BONE

REAL-WORLD PHYSICS: BIO

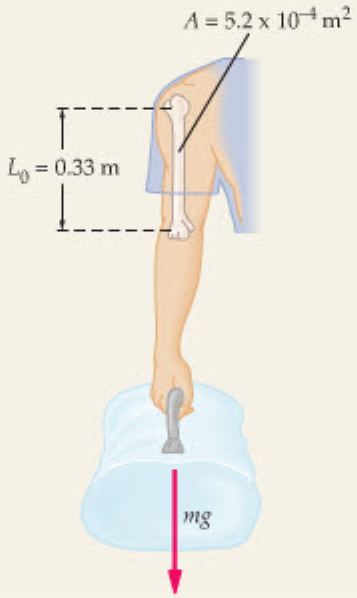
At the local airport, a person carries a 21-kg duffel bag in one hand. Assuming the humerus (the upper arm bone) supports the entire weight of the bag, determine the amount by which it stretches. (The humerus may be assumed to be 33 cm in length and to have an effective cross-sectional area of $5.2 \times 10^{-4}\ \mathrm{m^2}$.)

PICTURE THE PROBLEM

The humerus is oriented vertically, hence the weight of the duffel bag applies tension to the bone. The Young's modulus in this case is $1.6 \times 10^{10}\ \mathrm{N/m^2}$. In addition, we note that the initial length of the bone is $L_0 = 0.33\ \mathrm{m}$ and its cross-sectional area is $A = 5.2 \times 10^{-4}\ \mathrm{m^2}$.

STRATEGY

We can find the amount of stretch by solving the relation $F = Y(\Delta L/L_0)A$ for the quantity ΔL. Note that the force applied to the bone is simply the weight of the suitcase, $F = mg$, with $m = 21$ kg. (We ignore the relatively small weight of the forearm and hand.)



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SOLUTION

1. Solve Equation 17–17 for the amount of stretch, ΔL: $\Delta L = \frac{FL_0}{YA}$
2. Calculate the force applied to the humerus: $F = mg = (21\text{ kg})(9.81\text{ m/s}^2) = 210\text{ N}$
3. Substitute numerical values into the expression for ΔL:

$$\Delta L = \frac{FL_0}{YA} = \frac{(210\text{ N})(0.33\text{ m})}{(1.6 \times 10^{10}\text{ N/m}^2)(5.2 \times 10^{-4}\text{ m}^2)}$$
$$= 8.3 \times 10^{-6}\text{ m}$$

INSIGHT

We find that the amount of stretch is imperceptibly small. The reason for this, of course, is that Young's modulus is such a large number. If the bone had been compressed rather than stretched, its change in length, though still minuscule, would have been greater by a factor of 16/9.4.

PRACTICE PROBLEM

Suppose the humerus is reduced uniformly in size by a factor of 2. This means that both the length and diameter are halved. What is the amount of stretch in this case? [**Answer:** Since $L_0 \rightarrow L_0/2$ and $A \rightarrow A/4$, the stretch is doubled. Thus, $\Delta L \rightarrow 2\,\Delta L = 1.7 \times 10^{-5}\text{ m}$.]

Some related homework problems: Problem 37, Problem 38

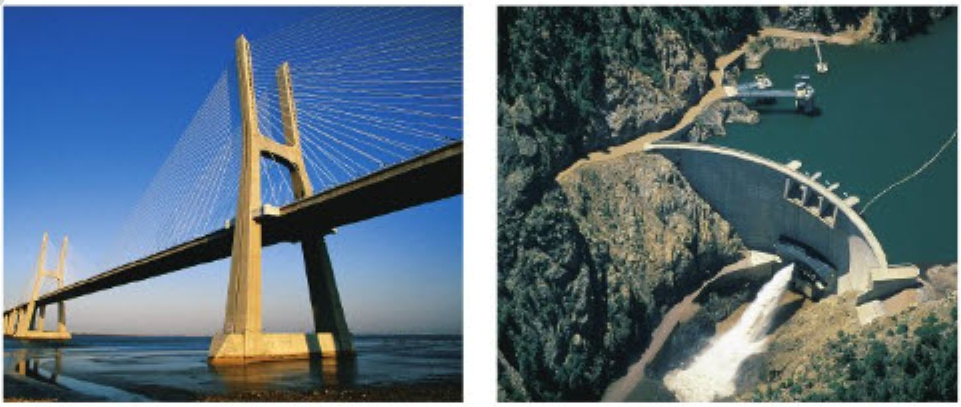
There is a straightforward connection between Equation 17–17 and Hooke's law for a spring (Equation 6–4). To see the connection, we rewrite Equation 17–17 as follows:

$$F = \left(\frac{YA}{L_0}\right)\Delta L$$

Notice that the force required to cause a certain stretch is proportional to the stretch—just as in Hooke's law. In fact, if we identify ΔL with the displacement x of a spring from equilibrium and YA/L_0 with the force constant k, we have Hooke's law:

$$F = \left(\frac{YA}{L_0}\right)\Delta L = kx$$

Thus, we see that the force constant of a spring depends on the Young's modulus, Y, of the material from which it is made, the cross-sectional area A of the wire, and the length of the wire, L_0.



▶ The cables in this suspension bridge have significant forces pulling on them from either end. As a result, they are under tension, just like the rod in Figure 17–10. It follows that the length of each cable increases by an amount that is proportional to its initial length. The dam, on the other hand, experiences forces that tend to compress it. Still, the magnitude of its change in length can be described in terms of Equation 17–17, with the appropriate value of Young's modulus.

CONCEPTUAL CHECKPOINT 17–3 COMPARE FORCE CONSTANTS

Two identical springs are connected end to end. Is the force constant of the resulting compound spring **(a)** greater than, **(b)** less than, or **(c)** equal to that of a single spring?

REASONING AND DISCUSSION

It might seem that the force constant would be the same, since we simply have twice the length of the same spring. On the other hand, it might seem that the force constant is greater, since two springs are exerting a force rather than just one. In fact, the force constant decreases.

The reason for the reduced force constant is that if we apply a force F to the compound spring, each individual spring stretches by a certain amount. The compound spring, then, stretches by twice this amount. By Hooke's law, $F = kx$, a spring that stretches twice as far for the same applied force has half the force constant.

We can also obtain this result by recalling that the force constant is $k = YA/L_0$. Thus, if the length of a spring is doubled—as in the compound spring—the force constant is halved.

ANSWER

(b) The force constant of the compound spring is less, by a factor of 2.

Changing the Shape of a Solid

Another type of deformation, referred to as a **shear deformation**, changes the *shape* of a solid. Consider a book of thickness L_0 resting on a table, as shown in Figure 17–11. A force F is applied to the right on the top cover of the book, and static friction applies a force F to the left on the bottom cover of the book. The result is that the book remains at rest but becomes slanted by the amount Δx. The force required to cause a given amount of slant is proportional to Δx, inversely proportional to the thickness of the book L_0, and proportional to the surface area A of the book's cover; that is, $F \propto A\Delta x/L_0$. Writing this as an equality, we have

$$F = S\left(\frac{\Delta x}{L_0}\right)A \qquad 17\text{–}18$$

The constant of proportionality in this case is the **shear modulus**, S. Like Young's modulus, the shear modulus has the units $\mathrm{N/m^2}$. Typical values of the shear modulus are collected in Table 17–2. As with Young's modulus, the shear modulus is large in magnitude, meaning that most solids require a large force to cause even a small amount of shear.

TABLE 17–2 Shear Modulus for Various Materials

Material	Shear modulus, S $(\mathrm{N/m^2})$
Tungsten	15×10^{10}
Steel	8.1×10^{10}
Bone	8.0×10^{10}
Copper	4.2×10^{10}
Brass	3.5×10^{10}
Aluminum	2.4×10^{10}
Lead	0.54×10^{10}

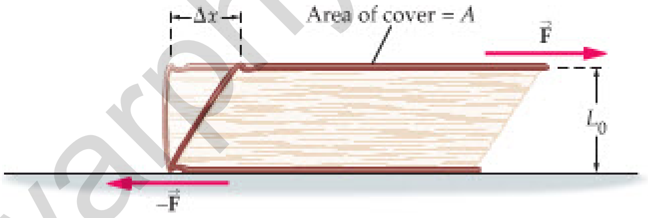


FIGURE 17–11 Shear deformation
Equal and opposite forces applied to the top and bottom of a book result in a shear deformation. The amount of deformation is proportional to the force F and the thickness of the book L_0, and inversely proportional to the area A.

Equations 17–17 and 17–18 are similar in structure, but it is important to be aware of their differences as well. For example, the term L_0 in the Young's modulus equation refers to the length of a solid measured in the direction of the applied force. In contrast, L_0 in the shear modulus equation refers to the thickness of the solid as measured in a direction perpendicular to the applied force. Similarly, the area A in Equation 17–17 is the cross-sectional area of the solid, and this area is perpendicular to the applied force. On the other hand, the area A in Equation 17–18 is the area of the solid in the plane of the applied force.

ACTIVE EXAMPLE 17–3 DEFORMING A STACK OF PANCAKES: FIND THE SHEAR MODULUS

A horizontal force of 1.2 N is applied to the top of a stack of pancakes 13 cm in diameter and 9.0 cm high. The result is a shear deformation of 2.5 cm. What is the shear modulus for these pancakes?

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PROBLEM-SOLVING NOTE

Area and the Shear Modulus

When using the shear modulus, remember that the area A is the area that is parallel to the applied force.

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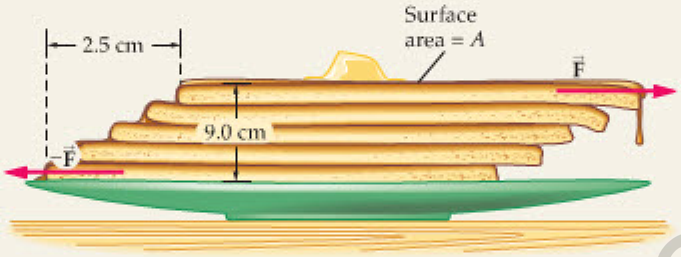


SOLUTION *(Test your understanding by performing the calculations indicated in each step.)*

1. Solve Equation 17–18 for the shear modulus: $S = FL_0/A\Delta x$
2. Calculate the area of the pancakes: $A = \pi d^2/4 = 0.013\ \text{m}^2$
3. Substitute numerical values: $S = 330\ \text{N/m}^2$

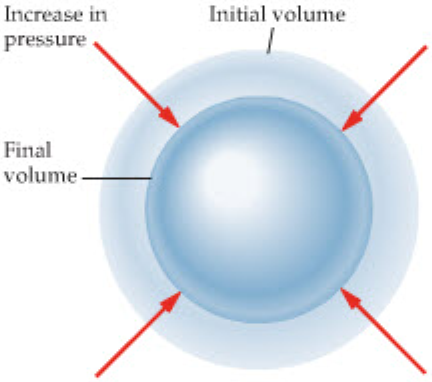
INSIGHT

Notice the small value of the pancakes' shear modulus, especially when compared to the shear modulus of a typical metal. This is a reflection of the fact that the pancake stack is easily deformed.

YOUR TURN

Suppose the stack of pancakes is doubled in height. By what factor does the shear deformation change?

(Answers to **Your Turn** *problems are given in the back of the book.)*



▲ **FIGURE 17–12 Changing the volume of a solid**

As the pressure surrounding an object increases, its volume decreases. The amount of volume change is proportional to the initial volume and to the change in pressure.

Changing the Volume of a Solid

If a piece of Styrofoam is taken deep into the ocean, the tremendous pressure of the water causes it to shrink to a fraction of its original volume. This is an extreme example of the volume change that occurs in all solids when the pressure of their surroundings is changed. The general situation is illustrated in Figure 17–12, where we show a spherical solid whose volume decreases by the amount ΔV when the pressure acting on it increases by the amount ΔP. Experiments show that the pressure difference required to cause a given change in volume, ΔV, is proportional to ΔV and inversely proportional to the initial volume of the object, V_0. Therefore, we can write ΔP as follows:

$$\Delta P = -B\left(\frac{\Delta V}{V_0}\right) \qquad 17\text{–}19$$

The constant of proportionality in this case, B, is called the **bulk modulus.** As with Young's modulus and the shear modulus, the bulk modulus is defined to be a positive quantity; hence the minus sign in Equation 17–19. For example, if the pressure increases ($\Delta P > 0$), the volume will decrease ($\Delta V < 0$) and the quantity $-\Delta V$ will be positive. Since ΔP is equal to $B(-\Delta V/V_0)$, and V_0 is always positive, it follows that B must be positive as well.

Table 17–3 gives a list of representative values of the bulk modulus. Note the large magnitudes of B, indicating that even small volume changes require large changes in pressure. Finally, the units of B are N/m², as is clear from Equation 17–19.

TABLE 17–3 Bulk Modulus for Various Materials

Material	Bulk modulus, B (N/m²)
Gold	22×10^{10}
Tungsten	20×10^{10}
Steel	16×10^{10}
Copper	14×10^{10}
Aluminum	7.0×10^{10}
Brass	6.1×10^{10}
Ice	0.80×10^{10}
Water	0.22×10^{10}
Oil	0.17×10^{10}

ACTIVE EXAMPLE 17–4 A GOLD DOUBLOON: FIND THE CHANGE IN VOLUME

A gold doubloon 6.1 cm in diameter and 2.0 mm thick is dropped over the side of a pirate ship. When it comes to rest on the ocean floor at a depth of 770 m, how much has its volume changed?

SOLUTION *(Test your understanding by performing the calculations indicated in each step.)*

1. Solve Equation 17–19 for the change in volume: $\Delta V = -V_0\,\Delta P/B$
2. Calculate the initial volume of the doubloon (a cylinder): $V_0 = 5.8 \times 10^{-6}\ \mathrm{m}^3$
3. Find the change in pressure due to the depth of seawater. Use Equation 15–7: $\Delta P = \rho g h = 7.7 \times 10^{6}\ \mathrm{N/m}^2$
4. Substitute numerical values: $\Delta V = -2.0 \times 10^{-10}\ \mathrm{m}^3$

INSIGHT

The change in volume is imperceptibly small. Clearly, enormous pressures must be applied to metals to cause a significant change in volume.

YOUR TURN

Suppose the "gold" doubloon is actually made of brass. Do you expect the change in volume in this case to be greater than or less than for gold? Verify your answer with a calculation.

(Answers to **Your Turn** *problems are given in the back of the book.)*

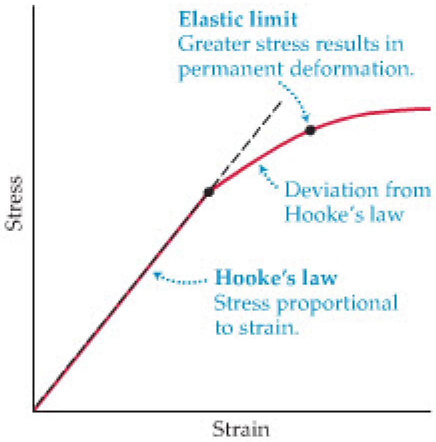
▲ Styrofoam has a very small bulk modulus, which means that even a relatively small increase in pressure can cause a large decrease in volume. The Styrofoam cup at left was immersed to a depth of 1955 m, where the water pressure is nearly 200 atm.

Stress and Strain

Equation 17–19 appears to differ from Equations 17–17 and 17–18 because of the absence of the area A on the right-hand side of the equation. When one recalls that pressure is a force per area, however, we can see that the area in Equation 17–19 is contained in the denominator of the left-hand side. Similarly, we can rewrite Equation 17–17 as $F/A = Y(\Delta L/L_0)$ and Equation 17–18 as $F/A = S(\Delta x/L_0)$. Written in this way, each of these equations states that a deformation of a particular type is proportional to a corresponding applied force per area.

In general, we refer to an applied force per area as a **stress** and the resulting deformation as a **strain.** If the stress applied to an object is not too large, the proportional relationship between the strain and stress is found to hold, and a plot of strain versus stress gives a straight line, as indicated in **Figure 17–13.** This straight-line relationship is simply a generalization of Hooke's law—extending the result for a spring to any solid object. As the stress becomes larger, the strain eventually begins to increase at a rate that is greater than the straight line.

A change in behavior occurs when the stress reaches the elastic limit. For stresses less than the elastic limit, the deformation of an object is reversible; that is, the deformation vanishes as the stress that caused it is reduced to zero. This is just like a spring returning to its equilibrium position when the applied force vanishes. When a deformation is reversible, we say that it is an **elastic deformation.** For stresses greater than the elastic limit, on the other hand, the object becomes permanently deformed, and it will not return to its original size and shape when the stress is removed. This is like a spring that has been stretched too far, or a car fender that has been dented. If the stress on an object is increased even further, the object eventually tears apart or fractures.



▲ **FIGURE 17–13 Stress versus strain**

When stress and strain are small they are proportional, as given by Hooke's law. Larger stresses can result in deviations from Hooke's law. Stresses greater than the elastic limit cause permanent deformation of the material.

17–4 Phase Equilibrium and Evaporation

To this point, we have studied the behavior of substances when they are in a single phase of matter. For example, we considered the properties of liquids in Chapter 15, and in this chapter we have turned our attention to the characteristics of gases and solids. It is common, however, for a substance to exist in more than one phase at a time. In particular, if a substance has two or more phases that coexist in a steady, stable fashion, we say that the substance exhibits **phase equilibrium.**

To see just what this means in a specific case, consider a closed container that is partially filled with a liquid, as in **Figure 17–14**. The container is kept at the constant temperature T_0, and initially the volume above the liquid is empty of

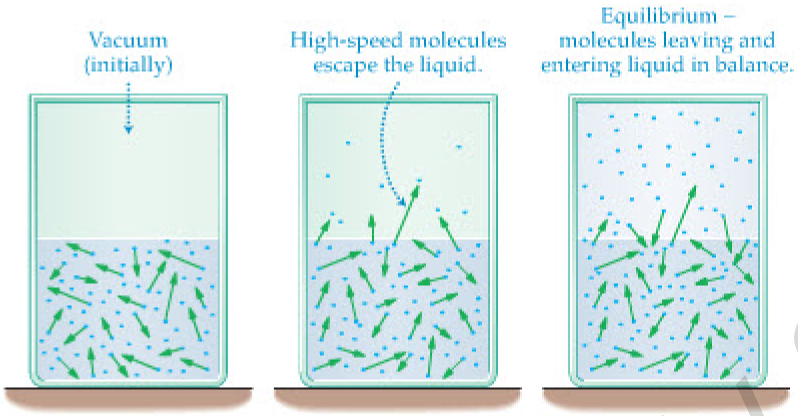


FIGURE 17–14 A liquid in equilibrium with its vapor

Initially, a liquid is placed in a container, and the volume above it is a vacuum. High-speed molecules in the liquid are able to escape into the upper region of the container, forming a low-density gas. As the gas becomes more dense, the number of molecules leaving the liquid is balanced by the number returning to the liquid. At this point the system is in equilibrium.

molecules—it is a vacuum. Soon, however, some of the faster molecules in the liquid begin to escape the relatively strong intermolecular forces of their neighbors in the liquid and start to form a low-density gas. Occasionally a gas molecule collides with the liquid and reenters it, but initially, more molecules are entering the gas than returning to the liquid.

This process continues until the gas is dense enough that the number of molecules returning to the liquid equals the number entering the gas. There is a constant "flow" of molecules in both directions, but when *phase equilibrium* is reached, these flows cancel, and the number of molecules in each phase remains constant. The pressure of the gas when equilibrium is established is referred to as the **equilibrium vapor pressure.** In **Figure 17–15** we plot the equilibrium vapor pressure of water.

What happens when we change the temperature? Well, if we increase the temperature, there will be more high-speed molecules in the liquid that can escape into the gas. Thus, to have an equal number of gas molecules returning to the liquid, it will be necessary for the pressure of the vapor to be greater. Thus, the equilibrium vapor pressure increases with temperature. This is also illustrated in Figure 17–15.

For each temperature there is just one equilibrium vapor pressure—that is, just one pressure where the precise balance between the phases is established. Thus, when we plot the equilibrium vapor pressure versus temperature, as in Figure 17–15, the result is a curve—the **vapor-pressure curve.** The significance of this curve is that it determines the boiling point of a liquid. In particular:

> A liquid boils at the temperature at which its vapor pressure equals the external pressure.

Note in Figure 17–15 that a vapor pressure equal to atmospheric pressure, $P_{\text{at}} = 101$ kPa, occurs when the temperature of the water is 100 °C, as expected.

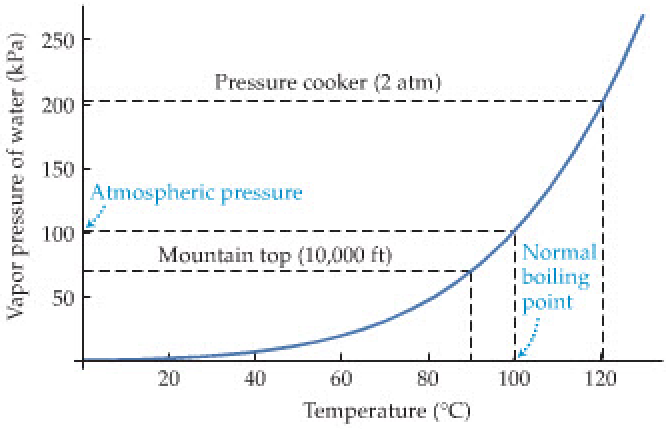


FIGURE 17–15 The vapor-pressure curve for water

The vapor pressure of water increases with increasing temperature. In particular, at $T = 100$ °C the vapor pressure is one atmosphere, 101 kPa.

CONCEPTUAL CHECKPOINT 17–4 BOILING TEMPERATURE

When water boils at the top of a mountain, is its temperature **(a)** higher than, **(b)** lower than, or **(c)** equal to 100 °C?

REASONING AND DISCUSSION

At the top of a mountain, air pressure is less than it is at sea level. Therefore, according to Figure 17–15, the boiling temperature will be lower as well.

For example, the atmospheric pressure at the top of a 10,000-ft mountain is roughly three-quarters what it is at sea level. The corresponding boiling point, as shown in Figure 17–15, is about 90 °C. Thus, cooking food in boiling water may be very difficult at such altitudes. Similarly, by measuring the boiling temperature of a pot of water on the summit of an uncharted mountain, it is possible to gain a rough estimate of its altitude.

ANSWER:

(b) The temperature of the water is lower than 100 °C.

REAL-WORLD PHYSICS: BIO

The autoclave

REAL-WORLD PHYSICS

The pressure cooker

An *autoclave* (which is basically an elaborate pressure cooker) sterilizes surgical tools by using the same principle discussed in Conceptual Checkpoint 17–4, only in the opposite direction. If surgical tools were heated in boiling water open to the atmosphere, they would experience a temperature of 100 °C. In the autoclave, which is a sealed vessel, the pressure rises to values significantly greater than atmospheric pressure. As a result, the water has a much higher boiling temperature, and the sterilization is more effective. In the pressure cooker, this elevated temperature results in a reduced cooking time.

REAL-WORLD PHYSICS

Adding salt to boiling water

Another way to increase the boiling temperature of water is to add a pinch of salt. The salt dissolves into sodium and chloride ions in the water. These ions have a strong interaction with the water molecules, making it harder for them to break free of the liquid and enter the vapor phase. As a result, the boiling temperature rises. This is why salt is often added to water when boiling eggs; the result is a higher temperature of boiling, which helps to solidify any material that might leak out of small cracks in the eggs.

The vapor-pressure curve separates areas of liquid and gas on a graph of pressure versus temperature, as shown in **Figure 17–16**. Notice that this curve comes to an end at a finite temperature and pressure. This end point is called the **critical point.** Beyond the critical point there is no longer a distinction between liquid and gas. They are one and the same, and are referred to, simply, as a fluid. Thus, as mentioned before, the liquid and gas phases are very similar—the liquid is just more dense than the gas and its molecules are somewhat less mobile.

A curve similar to the vapor-pressure curve indicates where the solid and liquid phases are in equilibrium. It is referred to as the **fusion curve.** Similarly, equilibrium between the solid and gas phases occurs along the **sublimation curve.** All

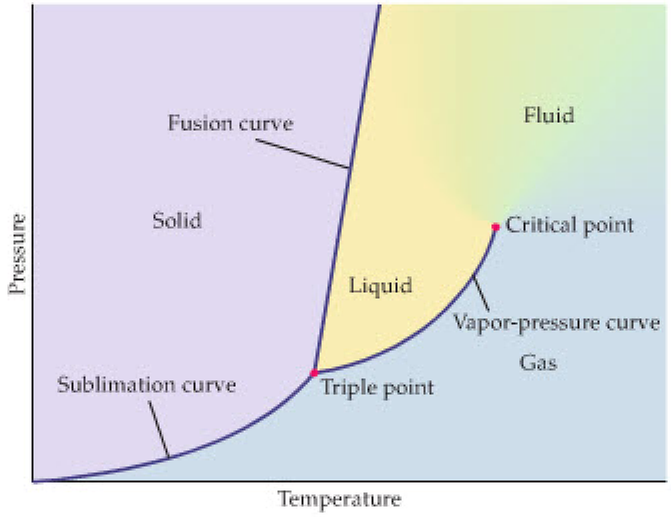
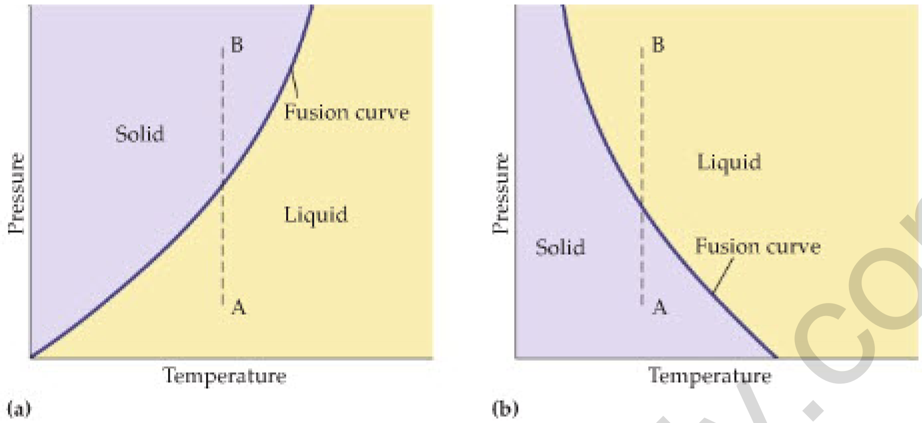


FIGURE 17–16 A typical phase diagram This phase diagram shows the regions corresponding to each of the three common phases of matter. Note that all three phases are in equilibrium at the triple point. In addition, the liquid and gas phases become indistinguishable beyond the critical point, where they are referred to as a fluid.

FIGURE 17–17 The solid–liquid phase boundary

Fusion curves for a typical substance **(a)** and for water **(b).** In (a), we note that an increase in pressure at constant temperature results in a liquid being converted to a solid. For the case of water, increasing the pressure exerted on ice at constant temperature can result in the ice melting.



three equilibrium curves are shown in Figure 17–16, on a plot that is referred to as a **phase diagram.**

Note that the phase diagram also indicates that there is one particular temperature and pressure where all three phases are in equilibrium. This point is called the **triple point.** In water, the triple point occurs at the temperature $T = 273.16\ \text{K}$ and the pressure $P = 611.2\ \text{Pa}$. At this temperature and pressure, ice, water, and steam are all in equilibrium with one another.

Finally, there is one feature of the fusion line that is of particular interest. In most substances, this line has a positive slope, as in Figure 17–16. This means that as the pressure is increased, the melting temperature of the substance also increases. This is sensible, because a solid is generally more dense than the corresponding liquid. Hence, if you apply pressure to a liquid—with the temperature held constant—the system will tend to become more dense and eventually solidify. This is indicated in Figure 17–17 (a), where we see that an increase in pressure at constant temperature (as from A to B) results in crossing the fusion curve into the solid region.

There are exceptions to this rule, however, and it will probably come as no surprise that water is one such exception. This is related to the fact that ice is less dense than water. As a result, the fusion curve for water has a negative slope coming out of the triple point. This is illustrated in Figure 17–17 (b). What this means is that if the temperature is held constant, ice will melt when the pressure applied to it is increased. Thus, the pressure due to an ice skate's blade, for example, can cause the ice beneath it to melt.

REAL-WORLD PHYSICS

Ice melting under pressure

The expansion of water as it freezes has important implications in geology as well. Suppose, for example, that water finds its way into cracks and fissures on a rocky cliff. If the temperature dips below freezing, the water in these cracks will begin to freeze and expand. This tends to split the rock farther apart in a process known as *frost wedging*. Over time, the repeated freezing and melting of water can break a "solid" rock cliff into a pile of debris, forming a talus slope at the base of the cliff. Similar effects occur in *frost heaving*, where expanding ice lifts rocks that were buried in the soil to the surface—a phenomenon well known in areas with long, cold winters.

REAL-WORLD PHYSICS

Frost wedging and frost heaving

The repeated expansion and contraction of water as it freezes and thaws over long periods of time can cause the ground to crack and buckle. Eventually, the resulting cracks may combine to form polygonal networks referred to as patterned ground (left), a common feature in arctic regions of the Earth. The same type of patterned ground has recently been observed on the surface of Mars (right), giving a strong indication that water may exist just below the surface in certain parts of the planet.

Finally, the expansion of ice can have devastating effects in living systems. If the blood cells of an organism were to freeze, for example, the resulting expansion could rupture the cells. The icefish, a transparent fish found in southern polar seas, has a high concentration of glycoproteins in its bloodstream that serve as a biological antifreeze, inhibiting the formation and growth of ice crystals.

REAL-WORLD PHYSICS: BIO

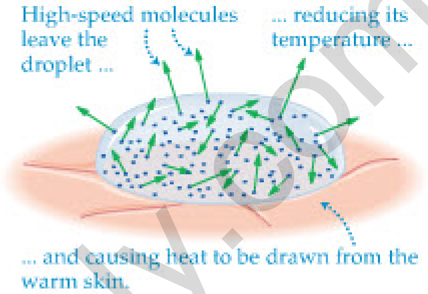
Biological antifreeze

Evaporation

If you think about it a moment, you realize that evaporation is a bit odd. After all, when you are hot and sweaty from physical exertion, steam rises from your skin. But how can this be, since water boils at 100 °C? How can a skin temperature of only 30 or 35 °C result in steam?

Recall that if a liquid is placed in a closed container with a vacuum above it, some of its fastest molecules will break loose and form a gas. When the gas becomes dense enough, its pressure rises to the equilibrium vapor pressure of the liquid, and the system attains equilibrium. But what if you open the container and let a breeze blow across it, removing much of the gas? In that case, the release of molecules from the liquid into the gas—that is, **evaporation**—continues without reaching equilibrium. As the molecules are continually removed, the liquid progressively evaporates until none is left. This is the basic mechanism of evaporation.

Let's investigate how evaporation helps to cool us when we exercise or work up a sweat. First, consider a droplet of sweat on the skin, as illustrated in **Figure 17–18**. As mentioned before, the high-speed molecules in the drop are the ones that will escape from the liquid into the surrounding air. The breeze takes these molecules away as soon as they escape; hence, the chance of their reentering the drop of sweat is very small.



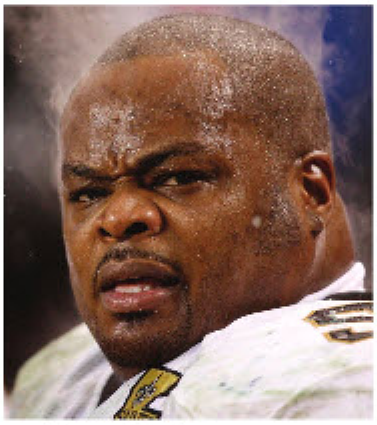
▲ **FIGURE 17–18 A droplet of sweat resting on the skin**

High-speed molecules in a droplet of sweat are able to escape the droplet and become part of the atmosphere. The average speed of the molecules that remain in the droplet is reduced, so the temperature of the droplet is reduced as well.

Now, what does this mean for the molecules that are left behind in the sweat droplet? Well, since the droplet is preferentially losing high-speed molecules, the average kinetic energy of the remaining molecules must decrease. As we know from kinetic theory, this means that the temperature of the droplet must also decrease. Since the droplet is now cooler than its surroundings, including the skin on which it rests, it draws in heat from the body. This warms the droplet, increasing the speed of its molecules, and continuing the evaporation process at more or less the same rate. Thus, sweat droplets are an effective means of drawing heat from the body and releasing it into the surrounding air in the form of high-speed water molecules.

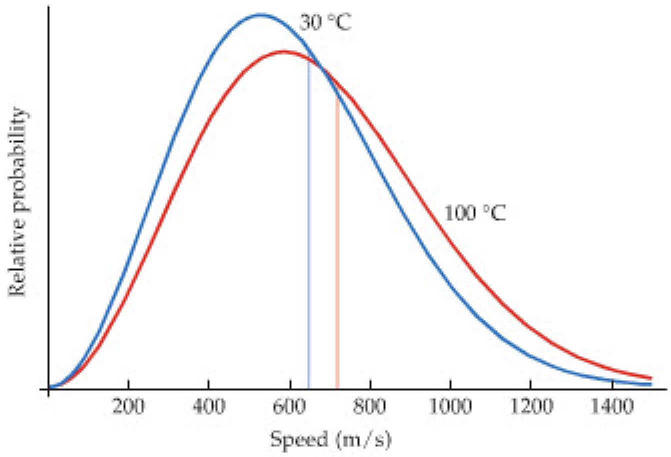
REAL-WORLD PHYSICS: BIO

Cooling the body with evaporation



▲ Although the body temperature of this athlete is well below the boiling point of water, the water contained in his sweat is evaporating rapidly from his skin. (Since water vapor is invisible, the "steam" we see in this photo, though it indicates the presence of evaporation, is not the water vapor itself. Rather, it is a cloud of tiny droplets that form when the water vapor loses heat to the cold air around it and condenses back to the liquid state.)

To look at this a bit more quantitatively, consider the Maxwell speed distribution for water molecules in a sweat droplet at 30 °C. This is shown in **Figure 17–19**. The rms speed at this temperature is 648 m/s. Also shown in Figure 17–19 is the speed distribution for water molecules at 100 °C. In this case the rms speed is 719 m/s, only slightly greater than that at 30 °C. Thus, it is clear that if water molecules at 100 °C have enough speed to escape into the gas phase, many molecules at 30 °C will be able to escape as well.



◀ **FIGURE 17–19 Speed distribution for water**

The blue curve shows the speed distribution for water at 30 °C, and the red curve corresponds to 100 °C. Note that the rms speed for 100 °C is only slightly greater than that for 30 °C.

▲ Human beings keep cool by sweating. Because the most energetic molecules are the ones most likely to escape by evaporation, significant quantities of heat are removed from the body when perspiration evaporates from the skin. Dogs, lacking sweat glands, nevertheless take advantage of the same mechanism to help regulate body temperature. In hot weather they pant to promote evaporation from the tongue. Of course, sitting in a cool pond can also help. (What mechanism is involved here?)

REAL-WORLD PHYSICS

Stability of planetary atmospheres

The Evaporating Atmosphere

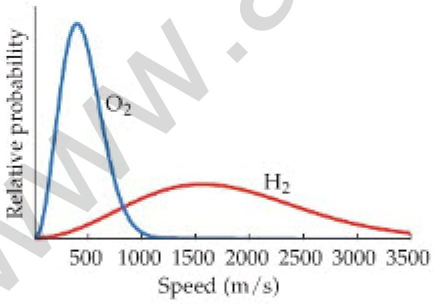
The atmosphere of a planet or a moon can evaporate in much the same way a drop of sweat evaporates from your forehead. In the case of an atmosphere, however, it is the force of gravity that must be overcome. If an astronomical body has a weak gravitational field, some molecules in its atmosphere may be moving rapidly enough to escape; that is, some molecules may have speeds in excess of the escape speed for that body.

Consider the Earth, for example. If a rocket is to escape the Earth, it must have a speed of at least 11,200 m/s. Recall from Chapter 12, however, that the escape speed is independent of the mass of the rocket. Thus, it applies equally well to molecules and rockets. As a result, molecules moving faster than 11,200 m/s may escape the Earth.

Now, let's compare the escape speed to the speeds of some of the molecules in the Earth's atmosphere. We have already seen in this chapter that the speed of nitrogen and oxygen are on the order of the speed of sound; that is, several hundred meters per second. This is much below the Earth's escape speed. Thus, the odds against a nitrogen or oxygen molecule having enough speed to escape the Earth are truly astronomical. Good thing, too, since this is why these molecules have persisted in our atmosphere for billions of years.

On the other hand, consider a lightweight molecule like hydrogen, H_2. The fact that its average kinetic energy is the same as that of all the other molecules in the air (see Equation 17–12) means that its speed will be much greater than the speed of, say, oxygen or nitrogen. This is illustrated in **Figure 17–20**, where we show the speed distribution for both O_2 and H_2. Clearly, it is very likely to find an H_2 molecule with a speed on the order of a couple thousand meters per second. Because of the higher speeds for H_2, the probability that an H_2 molecule will have enough speed to escape the Earth is about 300 orders of magnitude greater than the corresponding probability for O_2. It is no surprise, then, that Earth's atmosphere contains essentially no hydrogen.

On Jupiter, however, gravity is more intense than on Earth and the temperature is less. As a result, not even hydrogen moves quickly enough to escape. In fact, Jupiter's atmosphere is composed mostly of H_2 and He.



▲ **FIGURE 17–20 Speed distribution for O_2 and H_2 at 20 °C**

The typical speeds of an H_2 molecule (red curve) are much greater than those for an O_2 molecule (blue curve). In fact, some H_2 molecules move fast enough to escape from the Earth's atmosphere.

At the other extreme, the Moon has a rather weak gravitational field. In fact, it is unable to maintain any atmosphere at all. Whatever atmosphere it may have had early in its history has long since evaporated. You might say that the Moon's atmosphere is "lost in space."

17–5 Latent Heats

When two phases coexist something surprising happens—the temperature remains the same even when you add a small amount of heat. How can that be?

To understand this behavior, let's start by considering an ice cube initially at the temperature −10 °C. Adding an amount of heat Q results in a temperature increase ΔT given by the relation $Q = mc_{\text{ice}}\,\Delta T$, as discussed in Chapter 16. When the ice cube's temperature reaches 0 °C, however, adding more heat does not cause an additional increase in temperature. Instead, the heat goes into converting some of the ice into water. On a microscopic level, the added heat causes some of the molecules in the solid ice to vibrate with enough energy to break loose from neighboring molecules and become part of the liquid.

Thus, as long as any ice remains in a cup of water, *and* the water and ice are in equilibrium, you can be sure that both the ice and the water are at 0 °C. If heat is added to the system, the amount of ice decreases; if heat is removed, the amount of ice increases. The amount of heat required to completely convert 1 kg of ice to water is referred to as the **latent heat,** L. In general, the latent heat is defined as follows:

> The latent heat, L, is the heat that must be added to or removed from one kilogram of a substance to convert it from one phase to another.
>
> During the conversion process, the temperature of the system remains constant.

Just as with the specific heat, the latent heat is always a positive quantity.

In mathematical form, we can say that the heat Q required to convert a mass m from one phase to another is $Q = mL$. This gives the following relation:

Definition of Latent Heat, *L*

$$L = Q/m \qquad 17\text{–}20$$

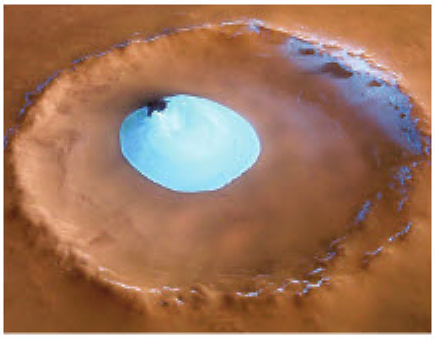
SI unit: J/kg

The value of the latent heat depends on which phases are involved. For example, the latent heat to melt (or fuse) a substance is referred to as the **latent heat of fusion,** L_f. Similarly, the latent heat required to convert a liquid to a gas is the **latent heat of vaporization,** L_v, and the latent heat needed to convert a solid directly to a gas is the **latent heat of sublimation,** L_s. Typical latent heats are given in Table 17–4.

The relationship between the temperature of a substance and the heat added to it is illustrated in Figure 17–21. Initially 1 kg of H_2O is in the form of ice at −20 °C.

TABLE 17–4 Latent Heats for Various Materials

Material	Latent heat of fusion, L_f (J/kg)	Latent heat of vaporization, L_v (J/kg)
Water	33.5×10^4	22.6×10^5
Ammonia	33.2×10^4	13.7×10^5
Copper	20.7×10^4	47.3×10^5
Benzene	12.6×10^4	3.94×10^5
Ethyl alcohol	10.8×10^4	8.55×10^5
Gold	6.28×10^4	17.2×10^5
Nitrogen	2.57×10^4	2.00×10^5
Lead	2.32×10^4	8.59×10^5
Oxygen	1.39×10^4	2.13×10^5



▲ This recently discovered ice lake on the surface of Mars lies on the floor of an impact crater. It grows or shrinks with the Martian seasons. Because atmospheric pressure is so low on Mars, however, the ice does not melt during the Martian summer—instead, it sublimates directly to the vapor phase. On Mars, water ice would have behavior similar to that of dry ice here on Earth.

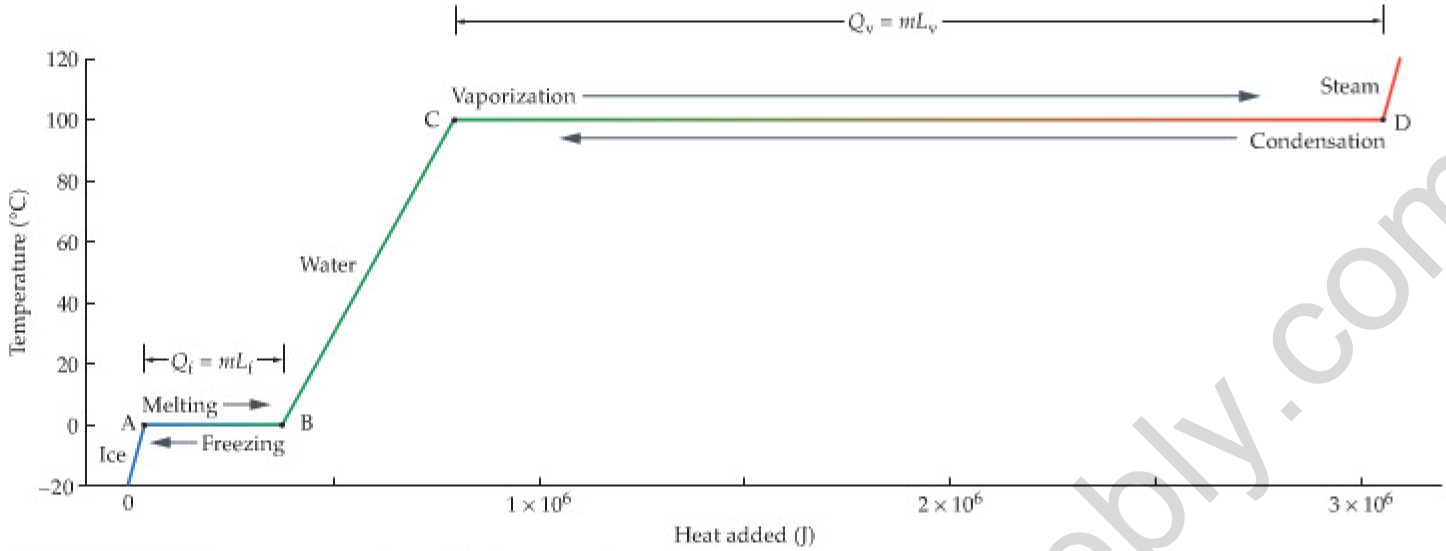


FIGURE 17–21 Temperature versus heat added or removed

The temperature of $m = 1.000$ kg of water as heat is added to or removed from the system. Note that the temperature stays the same—even as heat is added—when the system is changing from one phase to another. The points A, B, C, and D are referred to in Homework Problems 62 and 63.

As heat is added to the ice, its temperature rises until it begins to melt at 0 °C. The temperature then remains constant until the latent heat of fusion is supplied to the system. When all the ice has melted to water at 0 °C, continued heating results in a renewed increase in temperature. When the temperature of the water rises to 100 °C, boiling begins and the temperature again remains constant—this time until an amount of heat equal to the latent heat of vaporization is added to the system. Finally, with the entire system converted to steam, continued heating again produces an increasing temperature.

CONCEPTUAL CHECKPOINT 17–5 SEVERITY OF A BURN

Both water at 100 °C and steam at 100 °C can cause serious burns. Is a burn produced by steam likely to be **(a)** more severe than, **(b)** less severe than, or **(c)** the same as a burn produced by water?

REASONING AND DISCUSSION

As the water or steam comes into contact with the skin, it cools from 100 °C to a skin temperature of something like 35 °C. For the case of water, this means that a certain amount of heat is transferred to the skin, which can cause a burn. The steam, on the other hand, must first give off the heat required for it to condense to water at 100 °C. After that, the condensed water cools to body temperature, as before. Thus, the heat transferred to the skin will be larger in the case of steam, resulting in a more serious burn.

ANSWER

(a) The steam burn is worse.

As a numerical example of using latent heat, let's calculate the heat energy required to raise the temperature of 0.550 kg of ice from −20.0 °C to water at 20.0 °C; that is, from point A to point D in Figure 17–22. The way to approach a problem like this is to take it one step at a time—that is, each phase, and each conversion from one phase to another, should be treated separately.

Thus, the first step (A to B in Figure 17–22) is to find the heat necessary to warm the ice from −20.0 °C to 0 °C. Using the specific heat of ice, $c_{\text{ice}} = 2090\ \text{J/(kg}\cdot\text{C°)}$, we find

$$Q_1 = mc_{\text{ice}}\,\Delta T = (0.550\ \text{kg})[2090\ \text{J/(kg}\cdot\text{C°)}](20.0\ \text{C°}) = 23{,}000\ \text{J}$$

The second step in the process (B to C) is to melt the ice at 0 °C. The heat required for this is found using the latent heat of fusion for water ($L_f = 33.5 \times 10^4$ J/kg):

$$Q_2 = mL_f = (0.550\ \text{kg})(33.5 \times 10^4\ \text{J/kg}) = 184{,}000\ \text{J}$$

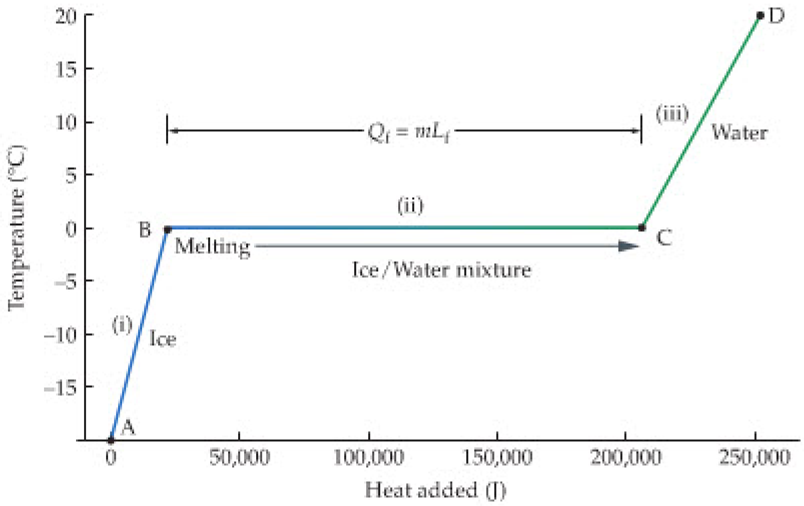


FIGURE 17–22 Heat required for a given change in temperature

The amount of heat required to raise 0.550 kg of H_2O from ice at −20.0 °C to water at 20.0 °C is the heat difference between points A and D. To calculate this heat we sum the following three heats: (i) the heat to warm the ice from −20.0 °C to 0 °C; (ii) the heat to melt all the ice; (iii) the heat to warm the water from 0 °C to 20.0 °C.

Finally, the third step (C to D) is to heat the water at 0 °C to 20.0 °C. This time we use the specific heat for water, $c_{\text{water}} = 4186\ \text{J}/(\text{kg}\cdot\text{C}°)$:

$$Q_3 = mc_{\text{water}}\,\Delta T = (0.550\ \text{kg})[4186\ \text{J}/(\text{kg}\cdot\text{C}°)](20.0\ \text{C}°) = 46{,}000\ \text{J}$$

The total heat required for this process, then, is

$$\begin{aligned} Q_{\text{total}} &= Q_1 + Q_2 + Q_3 \\ &= 23{,}000\ \text{J} + 184{,}000\ \text{J} + 46{,}000\ \text{J} = 253{,}000\ \text{J} \end{aligned}$$

We consider a similar problem in the following Example.

PROBLEM-SOLVING NOTE

Specific Heats Versus Latent Heats

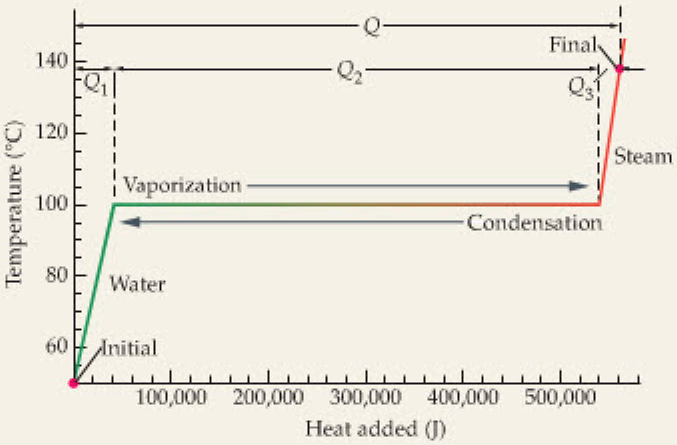
In solving problems involving specific heats and latent heats, recall that specific heats give the heat related to a *change in temperature* in a given phase, and latent heats give the heat related to a *change in phase* at a given temperature.

EXAMPLE 17–5 STEAM HEAT

To make steam, you add 5.60×10^5 J of heat to 0.220 kg of water at an initial temperature of 50.0 °C. Find the final temperature of the steam.

PICTURE THE PROBLEM

Our sketch shows the temperature–versus–heat-added curve for water. The initial point for this system, placed at the origin, is at 50.0 °C. As we shall see, adding the given amount of heat, $Q = 5.60 \times 10^5$ J, raises the temperature to the point labeled "final" in the plot.



STRATEGY

To find the final temperature, we first calculate the amount of heat that must be added to heat the water to 100 °C. If this is less than the total heat added, we continue by calculating the amount of heat needed to vaporize all the water. If the sum of these two heats is still less than the total heat added to the water, we calculate the increase in temperature when the remaining heat is added to the steam.

SOLUTION

1. Calculate the heat that must be added to the water to heat it to 100 °C. Call the result Q_1:

$$\begin{aligned} Q_1 &= mc_{\text{water}}\,\Delta T \\ &= (0.220\ \text{kg})[4186\ \text{J}/(\text{kg}\cdot\text{C}°)](50.0\ \text{C}°) \\ &= 4.60 \times 10^4\ \text{J} \end{aligned}$$

2. Next, calculate the heat that must be added to the water to convert it to steam. Let this result be Q_2:

$$\begin{aligned} Q_2 &= mL_f = (0.220\ \text{kg})(22.6 \times 10^5\ \text{J/kg}) \\ &= 4.97 \times 10^5\ \text{J} \end{aligned}$$

3. Determine the heat that is still to be added to the system. Let this remaining heat be Q_3:

$$\begin{aligned} Q_3 &= 5.60 \times 10^5\ \text{J} - Q_1 - Q_2 \\ &= 5.60 \times 10^5\ \text{J} - 4.60 \times 10^4\ \text{J} - 4.97 \times 10^5\ \text{J} \\ &= 17{,}000\ \text{J} \end{aligned}$$

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4. Use Q_3 to find the increase in temperature of the steam:

$$Q_3 = mc_{\text{steam}}\,\Delta T$$

$$\Delta T = \frac{Q_3}{mc_{\text{steam}}} = \frac{17{,}000\ \text{J}}{(0.220\ \text{kg})[(2010\ \text{J})/(\text{kg}\cdot\text{C}^\circ)]} = 38\ \text{C}^\circ$$

INSIGHT

Thus, the system ends up completely converted to steam at a temperature of 138 °C. If the amount of heat added to the system had been greater than Q_1, but less than $Q_1 + Q_2$, the final temperature of the system would have been 100 °C. In this case, the final state of the system is a mixture of liquid water and steam.

PRACTICE PROBLEM

Find the final temperature if the amount of heat added to the system is 3.40×10^5 J. [**Answer:** In this case, only 0.130 kg of water vaporizes into steam. Hence the final temperature is 100 °C, with water and steam in equilibrium].

Some related homework problems: Problem 57, Problem 60

REAL-WORLD PHYSICS

Homemade ice cream

A pleasant application of latent heat is found in the making of homemade ice cream. As you may know, it is necessary to add salt to the ice–water mixture surrounding the container holding the ingredients for the ice cream. The dissolved salt molecules interact with water molecules in the liquid, impairing their ability to interact with one another and freeze. This means that ice and water are no longer in equilibrium at 0 °C; a lower temperature is required. The result is that ice begins to melt in the ice–water mixture, and in the process of melting it draws the required latent heat from its surroundings—which include the ice cream. Thus, the salt together with the ice produces a temperature lower than the melting temperature of ice alone.

17–6 Phase Changes and Energy Conservation

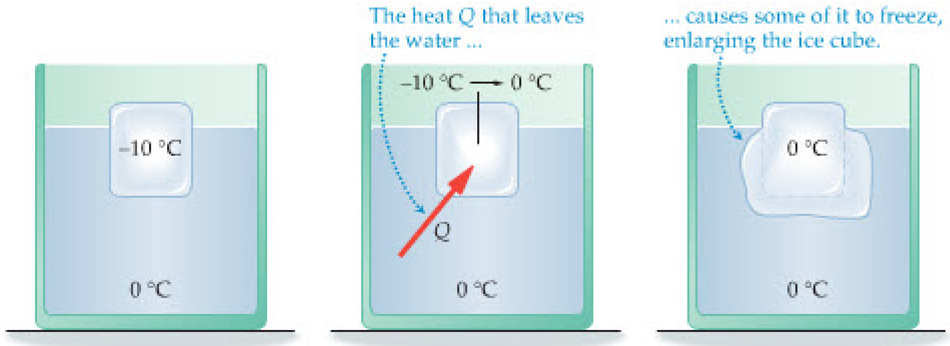
In the last section, we considered problems in which a given amount of heat is simply added to or removed from a system. We now turn to a more interesting type of problem involving energy conservation. In these problems, heat is exchanged within a system—that is, between its parts—but not with the external world. As a result, the total energy of the system is constant. Still, the heat flow within the system can cause changes in temperatures and phases.

The basic idea in solving energy conservation problems is the following:

> Set the magnitude of the heat *lost* by one part of the system equal to the magnitude of the heat *gained* by another.

For example, consider the following problem: A 0.0420-kg ice cube at −10.0 °C is placed in a Styrofoam cup containing 0.350 kg of water at 0 °C. Assuming the cup can be ignored, and that no heat is exchanged with the surroundings, find the mass of ice in the system when the equilibrium temperature of 0 °C is reached.

The initial setup for this problem is illustrated in Figure 17–23 (a). Since the water is warmer than the ice, it follows that heat flows into the ice from the water,


The heat Q that leaves the water ...

... causes some of it to freeze, enlarging the ice cube.

(a) Initial

(b) Heat goes from the warmer water into the colder ice

(c) Final

FIGURE 17–23 Water freezing to form ice

An ice cube at −10.0 °C is placed in water at 0 °C. Since the ice is colder than the water, heat flows *from* the water *into* the ice. Heat that leaves the water results in the formation of additional ice in the system.

as indicated in Figure 17–23 (b). The amount of heat will be just enough to raise the temperature of the ice from −10.0 °C to 0 °C. Thus,

$$\begin{aligned} Q &= \text{heat gained by the ice} \\ &= mc_{\text{ice}}\,\Delta T = (0.0420\text{ kg})[2090\text{ J}/(\text{kg}\cdot\text{C}^\circ)](10.0\text{ C}^\circ) = 878\text{ J} \end{aligned}$$

By energy conservation, we can say that the heat lost by the water has the same magnitude as the heat gained by the ice. (In general, in these problems it is simplest to calculate the magnitude of each heat—so that all the heats are positive—and then set the "lost" and "gained" magnitudes equal.) In this case, then, we have

$$\text{heat lost by the water} = \text{heat gained by the ice} = 878\text{ J}$$

Thus, 878 J of heat are removed from the water.

Now, since the water is already at 0 °C, removing heat from it does not lower its temperature—instead, it merely converts some of the water to ice. How much is converted? The amount is determined by the latent heat of fusion. In particular, we have

$$\begin{aligned} Q &= \text{heat lost by the water} \\ &= mL_{\text{f}} = 878\text{ J} \end{aligned}$$

In this expression, m is the mass of water that has been converted to ice. Solving for the mass yields

$$m = \frac{Q}{L_{\text{f}}} = \frac{878\text{ J}}{33.5\times 10^4\text{ J/kg}} = 0.00262\text{ kg}$$

Thus, the final amount of ice in the system is 0.0420 kg + 0.00262 kg = 0.0446 kg. This is illustrated in Figure 17–23 (c).

Finally, we consider a system in which a quantity of ice completely melts in warm water. The result is a container of liquid water at an intermediate temperature. Still, the basic idea is to apply energy conservation.

PROBLEM-SOLVING NOTE

Determining Equilibrium

In problems involving two different phases—like solid and liquid—it may not be clear in advance whether both phases or only one is present in the equilibrium state. It may be necessary to assume one case or the other and proceed with the calculation based on that assumption. If the result obtained in this way is not physical, the assumption must be changed.

EXAMPLE 17–6 WARM PUNCH

A large punch bowl holds 3.95 kg of lemonade (which is essentially just water) at 20.0 °C. A 0.0450-kg ice cube at 0 °C is placed in the lemonade. What is the final temperature of the system, and how much ice (if any) remains when the system reaches equilibrium? Ignore any heat exchange with the bowl or the surroundings.

PICTURE THE PROBLEM

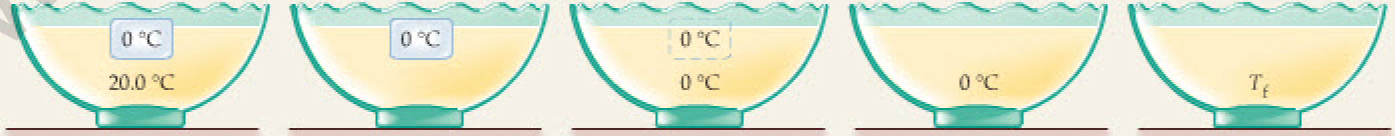
Our sketch shows the various stages imagined for this problem, starting with the initial condition in which the ice is at 0 °C and the lemonade (water) is at 20.0 °C. As one might guess from the large amount of water and the small amount of ice, all of the ice will melt. Therefore, the final condition is a container of liquid water at a final temperature, T_{f}.

STRATEGY

To apply energy conservation to this problem, we first calculate the heat that would be lost by the water, Q_{w}, if we cooled it to 0 °C. We then imagine using part of this heat, Q_{ice}, to melt the ice.

As we shall see, a great deal of heat, $Q = Q_{\text{w}} - Q_{\text{ice}}$, is left after the ice is melted. We imagine adding this heat back into the system, which now contains 3.95 kg + 0.0450 kg of water at 0 °C. Calculating the increase in temperature caused by the heat Q gives us the final temperature of the system.

INTERACTIVE FIGURE



Initial condition

Calculate heat Q_{w} lost by water, 20 °C ⟶ 0 °C

Calculate heat Q_{ice} needed to melt ice cube

Calculate remaining heat $Q = Q_{\text{w}} - Q_{\text{ice}}$

Calculate final temperature when heat Q is added to water and melted ice

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SOLUTION

1. Find the heat lost by the water, Q_w, if it is cooled to 0 °C:

$$\begin{aligned} Q_w &= m_{water} c_{water}\,\Delta T \\ &= (3.95\ \text{kg})[4186\ \text{J/(kg}\cdot\text{C}^\circ)](20.0\ \text{C}^\circ) \\ &= 3.31 \times 10^5\ \text{J} \end{aligned}$$

2. Calculate the amount of heat, Q_{ice}, needed to melt all the ice:

$$\begin{aligned} Q_{ice} &= m_{ice} L_f \\ &= (0.0450\ \text{kg})(33.5 \times 10^4\ \text{J/kg}) = 1.51 \times 10^4\ \text{J} \end{aligned}$$

3. Determine the amount of heat, Q, that is left:

$$\begin{aligned} Q &= Q_w - Q_{ice} \\ &= 3.31 \times 10^5\ \text{J} - 1.51 \times 10^4\ \text{J} = 3.16 \times 10^5\ \text{J} \end{aligned}$$

4. Use the heat Q to warm the 3.95 kg + 0.0450 kg of water at 0 °C to its final temperature:

$$\begin{aligned} Q &= (m_{water} + m_{ice}) c_{water}\,\Delta T \\ \Delta T &= \frac{Q}{(m_{water} + m_{ice}) c_{water}} \\ &= \frac{3.16 \times 10^5\ \text{J}}{(3.95\ \text{kg} + 0.0450\ \text{kg})[4186\ \text{J/(kg}\cdot\text{C}^\circ)]} \\ &= 18.9\ \text{C}^\circ \end{aligned}$$

INSIGHT

Therefore, the final temperature of the system is 18.9 °C. As expected, the relatively small ice cube did not lower the temperature of the system very much.

PRACTICE PROBLEM

What would the final temperature of the system be if the ice cube's mass were 0.0750 kg? [**Answer:** $T_f = 18.2$ °C]

Some related homework problems: Problem 67, Problem 70

THE BIG PICTURE PUTTING PHYSICS IN CONTEXT

LOOKING BACK

We use the concept of temperature (Chapter 16) throughout this chapter. We also make use of pressure (Chapter 15) in our discussion of ideal gases in Sections 17–1 and 17–2.

Notice that kinetic energy (Chapter 7) plays a central role in our understanding of an ideal gas. In particular, we make a clear and specific connection in Section 17–2 between the mechanical concept of kinetic energy and the thermodynamic concept of temperature.

We gain a deeper understanding of Hooke's law (Chapter 6) in Section 17–3 when we discuss the elastic deformation of solids.

LOOKING AHEAD

The equation of state for an ideal gas (Section 17–1) plays an important role in Chapter 18, especially when considering thermal processes (Section 18–3) and specific heats at constant pressure and volume (Section 18–4).

The concept of a phase change is used in Chapter 21 when we discuss the fact that certain materials become superconducting (have zero electrical resistance) below a critical temperature.

Phase changes appear again in Chapter 32 when we discuss the fundamental forces of nature and how they evolved as the universe cooled over time.

CHAPTER SUMMARY

17–1 IDEAL GASES

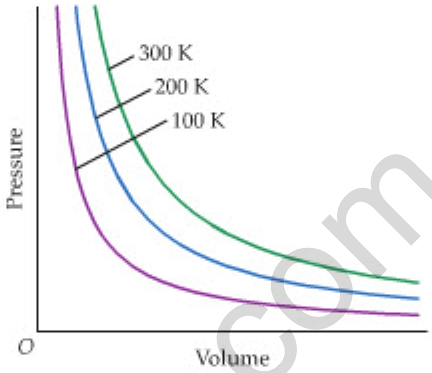
An ideal gas is a simplified model of a real gas in which interactions between molecules are ignored.

Equation of State

The equation of state for an ideal gas is

$$PV = NkT \qquad \text{17–2}$$

In this expression, N is the number of molecules, T is the Kelvin temperature, and $k = 1.38 \times 10^{-23}$ J/K is Boltzmann's constant.



In terms of the universal gas constant, $R = 8.31\ \mathrm{J/(mol \cdot K)}$, and the number of moles in the gas, n, the ideal-gas equation of state is

$$PV = nRT \tag{17–5}$$

Avogadro's Number and Moles

The number of molecules in a mole (mol) is Avogadro's number, $N_A = 6.022 \times 10^{23}$.

Molecular Mass

If the mass of an individual molecule is m, its molecular mass, M, is

$$M = N_A m \tag{17–6}$$

Isotherms and Boyle's Law

If the temperature and number of molecules are held constant, the pressure and volume of an ideal gas satisfy Boyle's law:

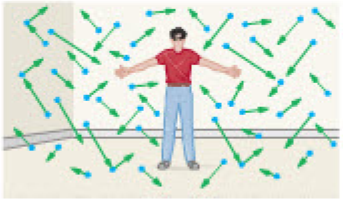
$$PV = \text{constant} \tag{17–7}$$

Constant Pressure and Charles's Law

If the pressure and number of molecules are held constant, the temperature and volume of an ideal gas satisfy Charles's law:

$$\frac{V}{T} = \text{constant} \tag{17–8}$$

17–2 KINETIC THEORY



In kinetic theory, a gas is imagined to be comprised of a large number of point-like molecules bouncing off the walls of a container.

The Origin of Pressure

The pressure exerted by a gas is a result of the momentum transfers that occur every time a molecule bounces off a wall of a container.

Speed Distribution of Molecules

The molecules in a gas have a range of speeds. The Maxwell distribution indicates which speeds are most likely to occur in a given gas.

Kinetic Energy and Temperature

Kinetic theory relates the average kinetic energy of the molecules in a gas to the Kelvin temperature of the gas, T:

$$\left(\tfrac{1}{2}mv^2\right)_{\mathrm{av}} = K_{\mathrm{av}} = \tfrac{3}{2}kT \tag{17–12}$$

RMS Speed

The rms (root mean square) speed of the molecules in a gas at the Kelvin temperature T is

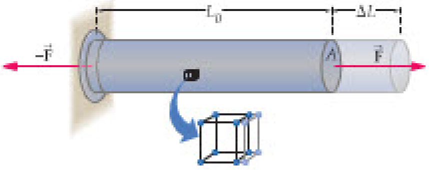
$$v_{\mathrm{rms}} = \sqrt{\frac{3kT}{m}} = \sqrt{\frac{3RT}{M}} \tag{17–13}$$

Internal Energy of an Ideal Gas

The internal energy of a monatomic ideal gas is

$$U = \tfrac{3}{2}NkT = \tfrac{3}{2}nRT \tag{17–15}$$

17–3 SOLIDS AND ELASTIC DEFORMATION



When a force is applied to a solid, its size and shape may change.

Changing the Length of a Solid

The force required to change the length of a solid by the amount ΔL is

$$F = Y\left(\frac{\Delta L}{L_0}\right)A \tag{17–17}$$

In this expression, Y is Young's modulus, L_0 is the initial length parallel to the applied force, and A is the cross-sectional area perpendicular to the applied force.

Shear Deformation

The force required to shear, or deform, a solid by the amount Δx is

$$F = S\left(\frac{\Delta x}{L_0}\right)A \tag{17–18}$$

In this expression, S is the shear modulus, L_0 is the initial length perpendicular to the applied force, and A is the cross-sectional area parallel to the applied force.

Changing the Volume of a Solid

The change in pressure required to change the volume of a solid by the amount ΔV is

$$\Delta P = -B\left(\frac{\Delta V}{V_0}\right) \qquad 17\text{–}19$$

In this expression, B is the bulk modulus and V_0 is the initial volume.

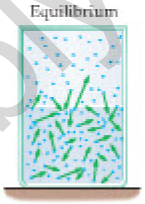
Stress and Strain

The applied force per area is the stress; the resulting deformation is the strain.

Elastic Deformation

An elastic deformation is one in which a solid returns to its original size and shape when the stress is removed.

17–4 PHASE EQUILIBRIUM AND EVAPORATION



The three most common phases of matter are the solid, liquid, and gas. Solids maintain a definite shape, whereas gases and liquids flow to take on the shape of their container.

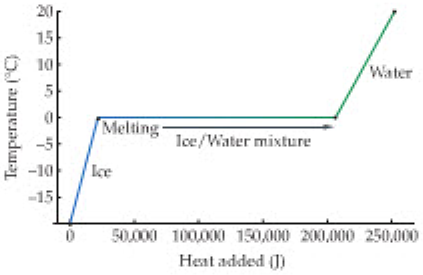
Equilibrium Between Phases

When phases are in equilibrium, the number of molecules in each phase remains constant.

Evaporation

Evaporation occurs when some molecules in a liquid have speeds great enough to allow them to escape into the gas phase.

17–5 LATENT HEATS



The latent heat, L, is the amount of heat per unit mass that must be added to or removed from a substance to convert it from one phase to another.

Latent Heat of Fusion

The heat required for melting or freezing is called the latent heat of fusion, L_f.

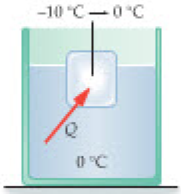
Latent Heat of Vaporization

The heat required for vaporizing or condensing is the latent heat of vaporization, L_v.

Latent Heat of Sublimation

The heat required to sublime a solid directly to a gas, or to condense a gas to a solid, is the latent heat of sublimation, L_s.

17–6 PHASE CHANGES AND ENERGY CONSERVATION



When heat is exchanged within a system, with no exchanges with the surroundings, the energy of the system is conserved.

PROBLEM-SOLVING SUMMARY

Type of Problem	Relevant Physical Concepts	Related Examples
Find pressure, volume, or temperature in an ideal gas.	These basic quantities are related by the ideal-gas equation of state, $PV = NkT = nRT$.	Examples 17–1, 17–2 Active Examples 17–1, 17–2
Relate the rms speed of a gas to the absolute temperature T.	Absolute temperature is directly related to the average kinetic energy, $3kT/2 = K_{av}$. From this connection we obtain the result $v_{rms} = \sqrt{3kT/m}$.	Example 17–3
Find the strain produced by a given stress.	Strain is proportional to stress, at least for small stress. The basic relations are $F/A = Y(\Delta L/L_0)$, $F/A = S(\Delta x/L_0)$, and $\Delta P = -B(\Delta V/V_0)$, where Y, S, and B are the Young's modulus, the shear modulus, and the bulk modulus, respectively.	Example 17–4, Active Examples 17–3, 17–4
Calculate the heat associated with a change in phase.	A certain amount of heat, called the latent heat, L, must be added to or taken away from a system to change it from one phase to another. The process occurs at constant temperature. The amount of heat involved in a change of phase is given by $Q = mL$, where m is the mass that changes phase.	Examples 17–5, 17–6

CONCEPTUAL QUESTIONS

(Answers to odd-numbered Conceptual Questions can be found in the back of the book.)

1. At the beginning of a typical airline flight you are instructed about the proper use of oxygen masks that will fall from the ceiling if the cabin pressure suddenly drops. You are advised that the oxygen masks are working properly, even if the bags do not fully inflate. In fact, the bags expand to their fullest if cabin pressure is lost at high altitude, but expand only partially if the plane is at low altitude. Explain.
2. How is the air pressure in a tightly sealed house affected by operating the furnace? Explain.
3. The average speed of air molecules in your room is on the order of the speed of sound. What is their average velocity?
4. Is it possible to change both the pressure and the volume of an ideal gas without changing the average kinetic energy of its molecules? If your answer is no, explain why not. If your answer is yes, give a specific example.
5. **An Airport at Great Elevation** One of the highest airports in the world is located in La Paz, Bolivia. Pilots prefer to take off from this airport in the morning or the evening, when the air is quite cold. Explain.
6. A camping stove just barely boils water on a mountaintop. When the stove is used at sea level, will it be able to boil water? Explain your answer.
7. An autoclave is a device used to sterilize medical instruments. It is essentially a pressure cooker that heats the instruments in water under high pressure. This ensures that the sterilization process occurs at temperatures greater than the normal boiling point of water. Explain why the autoclave produces such high temperatures.
8. As the temperature of ice is increased, it changes first into a liquid and then into a vapor. On the other hand, dry ice, which is solid carbon dioxide, changes directly from a solid to a vapor as its temperature is increased. How might one produce liquid carbon dioxide?
9. **BIO** Isopropyl alcohol is sometimes rubbed onto a patient's arms and legs to lower their body temperature. Why is this effective?
10. If you toss an ice cube into a swimming pool, is the water in the pool now at 0 °C? Explain.
11. A drop of water on a kitchen counter evaporates in a matter of minutes. However, only a relatively small fraction of the molecules in the drop move rapidly enough to escape through the drop's surface. Why, then, does the entire drop evaporate rather than just a small fraction of it?

PROBLEMS AND CONCEPTUAL EXERCISES

Note: Answers to odd-numbered Problems and Conceptual Exercises can be found in the back of the book. **IP** *denotes an integrated problem, with both conceptual and numerical parts;* **BIO** *identifies problems of biological or medical interest;* **CE** *indicates a conceptual exercise.* **Predict/Explain** *problems ask for two responses:* **(a)** *your prediction of a physical outcome, and* **(b)** *the best explanation among three provided. On all problems, red bullets (•, ••, •••) are used to indicate the level of difficulty.*

SECTION 17–1 IDEAL GASES

1. • **CE** **(a)** Is the number of molecules in one mole of N_2 greater than, less than, or equal to the number of molecules in one mole of O_2? **(b)** Is the mass of one mole of N_2 greater than, less than, or equal to the mass of one mole of O_2?
2. • **CE** Is the number of atoms in one mole of helium greater than, less than, or equal to the number of atoms in one mole of oxygen? Helium consists of individual atoms, He, and oxygen is a diatomic gas, O_2.
3. • **CE** **Predict/Explain** If you put a helium-filled balloon in the refrigerator, **(a)** will its volume increase, decrease, or stay the same? **(b)** Choose the *best explanation* from among the following:
   - I. Lowering the temperature of an ideal gas at constant pressure results in a reduced volume.
   - II. The same amount of gas is in the balloon; therefore, its volume remains the same.
   - III. The balloon can expand more in the cool air of the refrigerator, giving an increased volume.
4. • **CE** Two containers hold ideal gases at the same temperature. Container A has twice the volume and half the number of molecules as container B. What is the ratio P_A/P_B, where P_A is the pressure in container A and P_B is the pressure in container B?
5. • Standard temperature and pressure (STP) is defined as a temperature of 0 °C and a pressure of 101.3 kPa. What is the volume occupied by one mole of an ideal gas at STP?
6. • **BIO** After emptying her lungs, a person inhales 4.1 L of air at 0.0 °C and holds her breath. How much does the volume of the air increase as it warms to her body temperature of 37 °C?
7. • In the morning, when the temperature is 286 K, a bicyclist finds that the absolute pressure in his tires is 501 kPa. That afternoon he finds that the pressure in the tires has increased to 554 kPa. Ignoring expansion of the tires, find the afternoon temperature.
8. • An automobile tire has a volume of $0.0185\ \mathrm{m}^3$. At a temperature of 294 K the absolute pressure in the tire is 212 kPa. How many moles of air must be pumped into the tire to increase its pressure to 252 kPa, given that the temperature and volume of the tire remain constant?
9. • **Amount of Helium in a Blimp** The Goodyear blimp *Spirit of Akron* is 62.6 m long and contains $7023\ \mathrm{m}^3$ of helium. When the temperature of the helium is 285 K, its absolute pressure is 112 kPa. Find the mass of the helium in the blimp.
10. • A compressed-air tank holds $0.500\ \mathrm{m}^3$ of air at a temperature of 285 K and a pressure of 880 kPa. What volume would the air occupy if it were released into the atmosphere, where the pressure is 101 kPa and the temperature is 303 K?
11. • A typical region of interstellar space may contain 10^6 atoms per cubic meter (primarily hydrogen) at a temperature of 100 K. What is the pressure of this gas?
12. •• **CE** Four ideal gases have the following pressures, P, volumes, V, and mole numbers, n: gas A, $P = 100$ kPa, $V = 1\ \mathrm{m}^3$,

$n = 10$ mol; gas B, $P = 200$ kPa, $V = 2\ \text{m}^3$, $n = 20$ mol; gas C, $P = 50$ kPa, $V = 1\ \text{m}^3$, $n = 50$ mol; gas D, $P = 50$ kPa, $V = 4\ \text{m}^3$ $n = 5$ mol. Rank these gases in order of increasing temperature. Indicate ties where appropriate.

13. •• A balloon contains 3.7 liters of nitrogen gas at a temperature of 87 K and a pressure of 101 kPa. If the temperature of the gas is allowed to increase to 24 °C and the pressure remains constant, what volume will the gas occupy?

14. •• **IP** A balloon is filled with helium at a pressure of 2.4×10^5 Pa. The balloon is at a temperature of 18 °C and has a radius of 0.25 m. **(a)** How many helium atoms are contained in the balloon? **(b)** Suppose we double the number of helium atoms in the balloon, keeping the pressure and the temperature fixed. By what factor does the radius of the balloon increase? Explain.

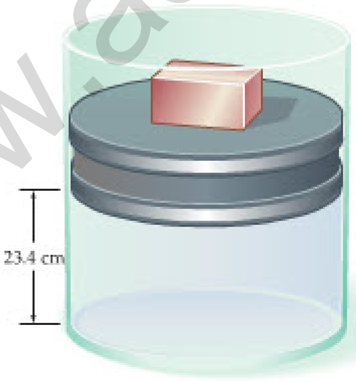
15. •• **IP** A gas has a temperature of 310 K and a pressure of 101 kPa. **(a)** Find the volume occupied by 1.25 mol of this gas, assuming it is ideal. **(b)** Assuming the gas molecules can be approximated as small spheres of diameter 2.5×10^{-10} m, determine the fraction of the volume found in part (a) that is occupied by the molecules. **(c)** In determining the properties of an ideal gas, we assume that molecules are points of zero volume. Discuss the validity of this assumption for the case considered here.

16. •• A 515-cm^3 flask contains 0.460 g of a gas at a pressure of 153 kPa and a temperature of 322 K. What is the molecular mass of this gas?

17. •• **IP The Atmosphere of Mars** On Mars, the average temperature is −64 °F and the average atmospheric pressure is 0.92 kPa. **(a)** What is the number of molecules per volume in the Martian atmosphere? **(b)** Is the number of molecules per volume on the Earth greater than, less than, or equal to the number per volume on Mars? Explain your reasoning. **(c)** Estimate the number of molecules per volume in Earth's atmosphere.

18. •• The air inside a hot-air balloon has an average temperature of 79.2 °C. The outside air has a temperature of 20.3 °C. What is the ratio of the density of air in the balloon to the density of air in the surrounding atmosphere?

19. •• A cylindrical flask is fitted with an airtight piston that is free to slide up and down, as shown in Figure 17–24. A mass rests on top of the piston. The initial temperature of the system is 313 K and the pressure of the gas is held constant at 137 kPa. The temperature is now increased until the height of the piston rises from 23.4 cm to 26.0 cm. What is the final temperature of the gas?



▲ **FIGURE 17–24** Problems 19 and 20

20. •• Consider the system described in Problem 19. Contained within the flask is an ideal gas at a constant temperature of 313 K. Initially the pressure applied by the piston and the mass is 137 kPa and the height of the piston above the base of the flask is 23.4 cm. When additional mass is added to the piston, the height of the piston decreases to 20.0 cm. Find the new pressure applied by the piston.

21. ••• One mole of a monatomic ideal gas has an initial pressure of 210 kPa, an initial volume of $1.2 \times 10^{-3}\ \text{m}^3$, and an initial temperature of 350 K. The gas now undergoes three separate processes: (i) a constant-temperature expansion that triples its volume; (ii) a constant-pressure compression to its initial volume; and (iii) a constant-volume increase in pressure to its initial pressure. At the end of these three processes, the gas is back at its initial pressure, volume, and temperature. Plot these processes on a pressure-versus-volume graph, showing the values of P and V at the end points of each process.

SECTION 17–2 KINETIC THEORY

22. • **CE Predict/Explain** The air in your room is composed mostly of oxygen (O_2) and nitrogen (N_2) molecules. The oxygen molecules are more massive than the nitrogen molecules. **(a)** Is the rms speed of the O_2 molecules greater than, less than, or equal to the rms speed of the N_2 molecules? **(b)** Choose the *best explanation* from among the following:

I. The more massive oxygen molecules have greater momentum and therefore greater speed.
II. Equal temperatures for the oxygen and nitrogen molecules imply they have equal rms speeds.
III. The temperature is the same for both molecules, and hence their average kinetic energies are equal. As a result, the more massive oxygen molecules have lower speeds.

23. • **CE** If the translational speed of molecules in an ideal gas is doubled, by what factor does the Kelvin temperature change? Explain.

24. • **CE** A piston held at the temperature T contains a gas mixture with molecules of three different types; A, B, and C. The corresponding molecular masses are $m_C > m_B > m_A$. Rank these molecular types in order of increasing **(a)** average kinetic energy and **(b)** rms speed. Indicate ties where appropriate.

25. • The molecules in a tank of hydrogen have the same rms speed as the molecules in a tank of oxygen. State whether each of the following statements is true, false, or unknowable with the given information: **(a)** the pressures are the same; **(b)** the hydrogen is at the higher temperature; **(c)** the hydrogen has the higher pressure; **(d)** the temperatures are the same; **(e)** the oxygen is at the higher temperature.

26. • At what temperature is the rms speed of H_2 equal to the rms speed that O_2 has at 313 K?

27. • Suppose a planet has an atmosphere of pure ammonia at 0.0 °C. What is the rms speed of the ammonia molecules?

28. •• **IP** Three moles of oxygen gas (that is, 3.0 mol of O_2) are placed in a portable container with a volume of 0.0035 m^3. If the temperature of the gas is 295 °C, find **(a)** the pressure of the gas and **(b)** the average kinetic energy of an oxygen molecule. **(c)** Suppose the volume of the gas is doubled, while the temperature and number of moles are held constant. By what factor do your answers to parts (a) and (b) change? Explain.

29. •• **IP** The rms speed of O_2 is 1550 m/s at a given temperature. **(a)** Is the rms speed of H_2O at this temperature greater than, less than, or equal to 1550 m/s? Explain. **(b)** Find the rms speed of H_2O at this temperature.

30. •• **IP** An ideal gas is kept in a container of constant volume. The pressure of the gas is also kept constant. **(a)** If the number of molecules in the gas is doubled, does the rms speed increase, decrease, or stay the same? Explain. **(b)** If the initial rms speed is 1300 m/s, what is the final rms speed?

31. •• What is the temperature of a gas of CO_2 molecules whose rms speed is 329 m/s?

32. •• The rms speed of a sample of gas is increased by 1%. **(a)** What is the percent change in the temperature of the gas? **(b)** What is the percent change in the pressure of the gas, assuming its volume is held constant?

33. •• **Enriching Uranium** In naturally occurring uranium atoms, 99.3% are ^{238}U (atomic mass = 238 u, where u = 1.6605×10^{-27} kg) and only 0.7% are ^{235}U (atomic mass = 235 u). Uranium-fueled reactors require an enhanced proportion of ^{235}U. Since both isotopes of uranium have identical chemical properties, they can be separated only by methods that depend on their differing masses. One such method is gaseous diffusion, in which uranium hexafluoride (UF_6) gas diffuses through a series of porous barriers. The lighter $^{235}UF_6$ molecules have a slightly higher rms speed at a given temperature than the heavier $^{238}UF_6$ molecules, and this allows the two isotopes to be separated. Find the ratio of the rms speeds of the two isotopes at 23.0 °C.

34. ••• A 350-mL spherical flask contains 0.075 mol of an ideal gas at a temperature of 293 K. What is the average force exerted on the walls of the flask by a single molecule?

SECTION 17–3 SOLIDS AND ELASTIC DEFORMATION

35. • **CE** A brick has faces with the following dimensions: face 1 is 1 cm by 2 cm; face 2 is 2 cm by 3 cm; face 3 is 1 cm by 3 cm. On which face should the brick be placed if it is to have the smallest change in dimensions due to its own weight? Explain.

36. • **CE Predict/Explain** A hollow cylindrical rod (rod 1) and a solid cylindrical rod (rod 2) are made of the same material. The two rods have the same length and the same outer radius. If the same compressional force is applied to each rod, **(a)** is the change in length of rod 1 greater than, less than, or equal to the change in length of rod 2? **(b)** Choose the *best explanation* from among the following:

I. The solid rod has the larger effective cross-sectional area, since the empty part of the hollow rod doesn't resist compression. Therefore, the solid rod has the smaller change in length.

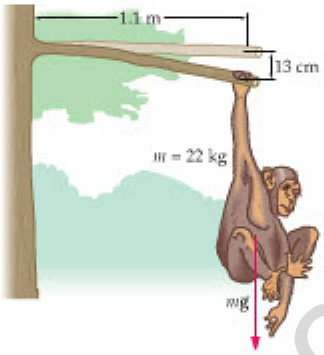
II. The rods have the same outer radius and hence the same cross-sectional area. As a result, their change in length is the same.

III. The walls of the hollow rod are hard and resist compression more than the uniform material in the solid rod. Therefore the hollow rod has the smaller change in length.

37. • A rock climber hangs freely from a nylon rope that is 14 m long and has a diameter of 8.3 mm. If the rope stretches 4.6 cm, what is the mass of the climber?

38. • **BIO** To stretch a relaxed biceps muscle 2.5 cm requires a force of 25 N. Find the Young's modulus for the muscle tissue, assuming it to be a uniform cylinder of length 0.24 m and cross-sectional area 47 cm^2.

39. • A 22-kg chimpanzee hangs from the end of a horizontal, broken branch 1.1 m long, as shown in Figure 17–25. The branch is a uniform cylinder 4.6 cm in diameter, and the end of the branch supporting the chimp sags downward through a vertical distance of 13 cm. What is the shear modulus for this branch?



▲ **FIGURE 17–25** Problem 39

40. • **The Marianas Trench** The deepest place in all the oceans is the Marianas Trench, where the depth is 10.9 km and the pressure is 1.10×10^8 Pa. If a copper ball 15.0 cm in diameter is taken to the bottom of the trench, by how much does its volume decrease?

41. •• **CE** Four cylindrical rods with various cross-sectional areas and initial lengths are stretched by an applied force, as in Figure 17–10. The resulting change in length of each rod is given in the following table. Rank these rods in order of increasing Young's modulus. Indicate ties where appropriate.

Rod	Applied Force	Cross-Sectional Area	Initial Length	Change in Length
1	F	A	L	ΔL
2	F	$2A$	$2L$	ΔL
3	$2F$	$2A$	L	$2\Delta L$
4	$3F$	A	$L/2$	ΔL

42. •• **IP** A steel wire 4.7 m long stretches 0.11 cm when it is given a tension of 360 N. **(a)** What is the diameter of the wire? **(b)** If it is desired that the stretch be less than 0.11 cm, should its diameter be increased or decreased? Explain.

43. •• **BIO Spiderweb** An orb weaver spider with a mass of 0.26 g hangs vertically by one of its threads. The thread has a Young's modulus of 4.7×10^9 N/m^2 and a radius of 9.8×10^{-6} m. **(a)** What is the fractional increase in the thread's length caused by the spider? **(b)** Suppose a 76-kg person hangs vertically from a nylon rope. What radius must the rope have if its fractional increase in length is to be the same as that of the spider's thread?

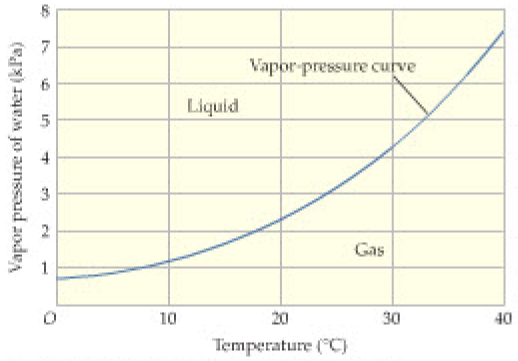
44. •• **IP** Two rods of equal length (0.55 m) and diameter (1.7 cm) are placed end to end. One rod is aluminum, the other is brass. If a compressive force of 8400 N is applied to the rods, **(a)** how much does their combined length decrease? **(b)** Which of the rods changes its length by the greatest amount? Explain.

45. •• A piano wire 0.82 m long and 0.93 mm in diameter is fixed on one end. The other end is wrapped around a tuning peg 3.5 mm in diameter. Initially the wire, whose Young's modulus is 2.4×10^{10} N/m^2, has a tension of 14 N. Find the tension in the wire after the tuning peg has been turned through one complete revolution.

SECTION 17–4 PHASE EQUILIBRIUM AND EVAPORATION

46. • The formation of ice from water is accompanied by which of the following: **(a)** an absorption of heat by the water; **(b)** an increase in temperature; **(c)** a decrease in volume; **(d)** a removal of heat from the water; **(e)** a decrease in temperature?

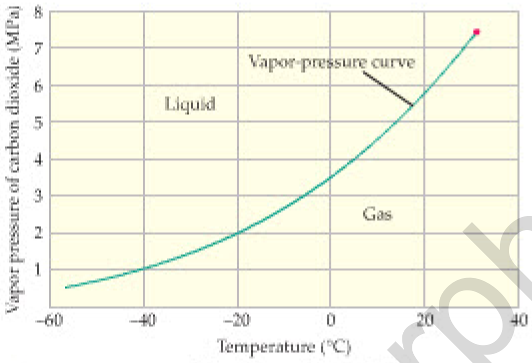
47. • **Vapor Pressure for Water** Figure 17–26 shows a portion of the vapor-pressure curve for water. Referring to the figure, estimate the pressure that would be required for water to boil at 30 °C.



▲ **FIGURE 17–26** Problems 47 and 48

48. • Using the vapor-pressure curve given in Figure 17–26, find the temperature at which water boils when the pressure is 1.5 kPa.

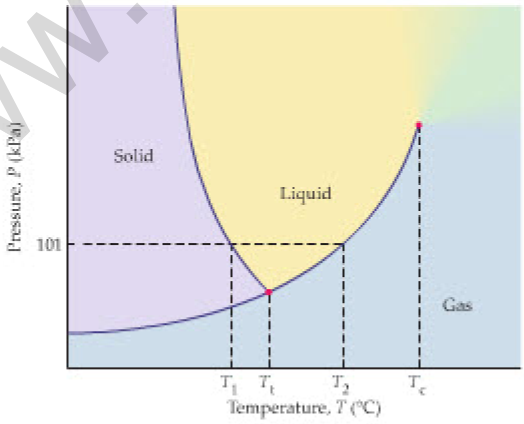
49. • **IP The Vapor Pressure of CO_2** A portion of the vapor-pressure curve for carbon dioxide is given in Figure 17–27. **(a)** Estimate the pressure at which CO_2 boils at 0 °C. **(b)** If the temperature is increased, does the boiling pressure increase, decrease, or stay the same? Explain.



▲ **FIGURE 17–27** Problems 49 and 50

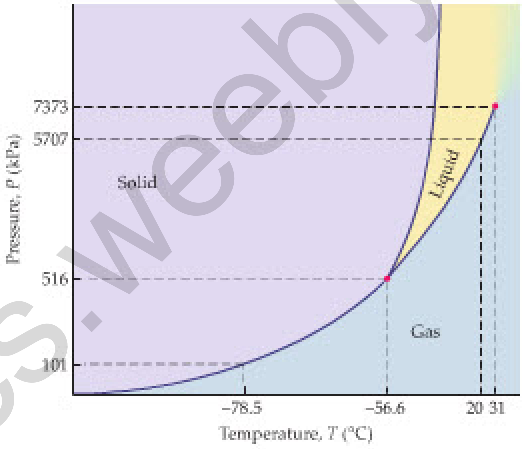
50. • **IP** Referring to the vapor-pressure curve for carbon dioxide given in Figure 17–27, **(a)** estimate the temperature at which CO_2 boils when the pressure is 1.5×10^6 Pa. **(b)** If the pressure is increased, does the boiling temperature increase, decrease, or stay the same? Explain.

51. •• **Phase Diagram for Water** The phase diagram for water is shown in Figure 17–28. **(a)** What is the temperature T_1 on the phase diagram? **(b)** What is the temperature T_2 on the phase diagram? **(c)** What happens to the melting/freezing temperature of water if atmospheric pressure is *decreased*? Justify your answer by referring to the phase diagram. **(d)** What happens to the boiling/condensation temperature of water if atmospheric pressure is *increased*? Justify your answer by referring to the phase diagram.



▲ **FIGURE 17–28** Problems 51, 53, and 54

52. •• **Phase Diagram for CO_2** The phase diagram for CO_2 is shown in Figure 17–29. **(a)** What is the phase of CO_2 at $T = 20$ °C and $P = 500$ kPa? **(b)** What is the phase of CO_2 at $T = -80$ °C and $P = 120$ kPa? **(c)** For reasons of economy and convenience, bulk CO_2 is often transported in liquid form in pressurized tanks. Using the phase diagram, determine the minimum pressure required to keep CO_2 in the liquid phase at 20 °C.



▲ **FIGURE 17–29** Problems 52 and 55

53. •• A sample of water ice at atmospheric pressure has a temperature just below the freezing point. Refer to Figure 17–28 to answer the following questions. **(a)** What phase changes occur if the temperature of the system is increased while the pressure is held constant? **(b)** Suppose, instead, that the temperature of the system is held constant just below the freezing point while the pressure is decreased. What phase changes occur now?

54. •• A sample of liquid water at atmospheric pressure has a temperature just above the freezing point. Refer to Figure 17–28 to answer the following questions. **(a)** What phase changes occur if the temperature of the system is increased while the pressure is held constant? **(b)** Suppose, instead, that the temperature of the system is held constant just above the freezing point while the pressure is decreased. What phase changes occur now?

55. •• A sample of solid carbon dioxide at a pressure of 5707 kPa has a temperature just below the freezing point. Refer to Figure 17–29 to answer the following questions. **(a)** What phase changes occur if the temperature of the system is increased while the pressure is held constant? **(b)** Suppose, instead, that the temperature of the system is held constant just below the freezing point while the pressure is decreased. What phase changes occur now?

SECTION 17–5 LATENT HEATS

56. •• **CE** Four liquids are at their freezing temperature. Heat is now removed from each of the liquids until it becomes completely solidified. The amount of heat that must be removed, Q, and the mass, m, of each of the liquids are as follows: liquid A, $Q = 33{,}500$ J, $m = 0.100$ kg; liquid B, $Q = 166{,}000$ J, $m = 0.500$ kg; liquid C, $Q = 31{,}500$ J, $m = 0.250$ kg; liquid D,

$Q = 5400$ J, $m = 0.0500$ kg. Rank these liquids in order of increasing latent heat of fusion. Indicate ties where appropriate.

57. • How much heat must be removed from 0.96 kg of water at 0 °C to make ice cubes at 0 °C?

58. • A heat transfer of 9.5×10^5 J is required to convert a block of ice at −15 °C to water at 15 °C. What was the mass of the block of ice?

59. • How much heat must be added to 1.75 kg of copper to change it from a solid at 1358 K to a liquid at 1358 K?

60. •• **IP** A 1.1-kg block of ice is initially at a temperature of −5.0 °C. **(a)** If 5.2×10^5 J of heat are added to the ice, what is the final temperature of the system? Find the amount of ice, if any, that remains. **(b)** Suppose the amount of heat added to the ice block is doubled. By what factor must the mass of the ice be increased if the system is to have the same final temperature? Explain.

61. •• **IP** Referring to the previous problem, suppose the amount of heat added to the block of ice is reduced by a factor of 2 to 2.6×10^5J. Note that this amount of heat is still sufficient to melt at least some of the ice. **(a)** Do you expect the temperature increase in this case to be one-half that found in the previous problem? Explain. **(b)** What is the final temperature of the system in this case? Find the amount of ice, if any, that remains.

62. •• Figure 17–21 shows a temperature-versus-heat plot for 1.000 kg of water. **(a)** Calculate the heat corresponding to the points A, B, C, and D. **(b)** Calculate the slope of the line from point B to point C. Show that this slope is equal to $1/c$, where c is the specific heat of liquid water.

63. •• **IP** Suppose the 1.000 kg of water in Figure 17–21 starts at point A at time zero. Heat is added to this system at the rate of 12,250 J/s. How long does it take for the system to reach **(a)** point B, **(b)** point C, and **(c)** point D? **(d)** Describe the physical state of the system at time $t = 63.00$ s.

64. •• Figure 17–22 shows a temperature-versus-heat plot for 0.550 kg of water. **(a)** Calculate the slope of the line from point A to point B. Show that the slope is equal to $1/mc$, where c is the specific heat of ice. **(b)** Calculate the slope of the line from point C to point D. Show that the slope is equal to $1/mc$, where c is the specific heat of liquid water.

65. •• **BIO** In Conceptual Checkpoint 17–5 we pointed out that steam can cause more serious burns than water at the same temperature. Here we examine this effect quantitatively, noting that flesh becomes badly damaged when its temperature reaches 50.0 °C. **(a)** Calculate the heat released as 12.5 g of liquid water at 100 °C is cooled to 50.0 °C. **(b)** Calculate the heat released when 12.5 g of steam at 100 °C is condensed and cooled to 50.0 °C. **(c)** Find the mass of flesh that can be heated from 37.0 °C (normal body temperature) to 50.0 °C for the cases considered in parts (a) and (b). (The average specific heat of flesh is 3500 J/kg · K.)

66. •• When you go out to your car one cold winter morning you discover a 0.58-cm-thick layer of ice on the windshield, which has an area of 1.6 m^2. If the temperature of the ice is −2.0 °C, and its density is 917 kg/m^3, find the heat required to melt all the ice.

SECTION 17–6 PHASE CHANGES AND ENERGY CONSERVATION

67. •• A large punch bowl holds 3.99 kg of lemonade (which is essentially water) at 20.5 °C. A 0.0550-kg ice cube at −10.2 °C is placed in the lemonade. What is the final temperature of the system, and the amount of ice (if any) remaining? Ignore any heat exchange with the bowl or the surroundings.

68. •• A 155-g aluminum cylinder is removed from a liquid nitrogen bath, where it has been cooled to −196 °C. The cylinder is immediately placed in an insulated cup containing 80.0 g of water at 15.0 °C. What is the equilibrium temperature of this system? If your answer is 0 °C, determine the amount of water that has frozen. The average specific heat of aluminum over this temperature range is 653 J/(kg · K).

69. •• An 825-g iron block is heated to 352 °C and placed in an insulated container (of negligible heat capacity) containing 40.0 g of water at 20.0 °C. What is the equilibrium temperature of this system? If your answer is 100 °C, determine the amount of water that has vaporized. The average specific heat of iron over this temperature range is 560 J/(kg · K).

70. •• **IP** A 35-g ice cube at 0.0 °C is added to 110 g of water in a 62-g aluminum cup. The cup and the water have an initial temperature of 23 °C. **(a)** Find the equilibrium temperature of the cup and its contents. **(b)** Suppose the aluminum cup is replaced with one of equal mass made from silver. Is the equilibrium temperature with the silver cup greater than, less than, or the same as with the aluminum cup? Explain.

71. •• A 48-g block of copper at −12 °C is added to 110 g of water in a 75-g aluminum cup. The cup and the water have an initial temperature of 4.1 °C. **(a)** Find the equilibrium temperature of the cup and its contents. **(b)** What mass of ice, if any, is present when the system reaches equilibrium?

72. •• A 0.075-kg ice cube at 0.0 °C is dropped into a Styrofoam cup holding 0.33 kg of water at 14 °C. **(a)** Find the final temperature of the system and the amount of ice (if any) remaining. Assume the cup and the surroundings can be ignored. **(b)** Find the initial temperature of the water that would be enough to *just barely* melt all of the ice.

73. •• To help keep her barn warm on cold days, a farmer stores 865 kg of warm water in the barn. How many hours would a 2.00-kilowatt electric heater have to operate to provide the same amount of heat as is given off by the water as it cools from 20.0 °C to 0 °C and then freezes at 0 °C?

GENERAL PROBLEMS

74. • **CE** Plastic bubble wrap is used as a protective packing material. Is the bubble wrap more effective on a cold day or on a warm day? Explain.

75. • **CE** Two adjacent rooms in a hotel are equal in size and connected by an open door. Room 1 is warmer than room 2. Which room contains more air? Explain.

76. • **CE** As you go up in altitude, do you expect the ratio of oxygen to nitrogen in the atmosphere to increase, decrease; or stay the same? Explain.

77. • **CE Predict/Explain** Suppose the Celsius temperature of an ideal gas is doubled from 100 °C to 200 °C. **(a)** Does the average kinetic energy of the molecules in this gas increase by a factor that is greater than, less than, or equal to 2? **(b)** Choose the *best explanation* from among the following:

  I. Changing the temperature from 100 °C to 200 °C goes beyond the boiling point, which will increase the kinetic energy by more than a factor of 2.

  II. The average kinetic energy is directly proportional to the temperature, so doubling the temperature doubles the kinetic energy.

III. Doubling the Celsius temperature from 100 °C to 200 °C changes the Kelvin temperature from 373.15 K to 473.15 K, which is an increase of less than a factor of 2.

78. • **CE Predict/Explain** Suppose the absolute temperature of an ideal gas is doubled from 100 K to 200 K. **(a)** Does the average speed of the molecules in this gas increase by a factor that is greater than, less than, or equal to 2? **(b)** Choose the *best explanation* from among the following:

I. Doubling the Kelvin temperature doubles the average kinetic energy, but this implies an increase in the average speed by a factor of $\sqrt{2} = 1.414\ldots$, which is less than 2.

II. The Kelvin temperature is the one we use in the ideal-gas law, and therefore doubling it also doubles the average speed of the molecules.

III. The change in average speed depends on the mass of the molecules in the gas, and hence doubling the Kelvin temperature generally results in an increase in speed that is greater than a factor of 2.

79. •**Largest Raindrops** Atmospheric scientists studying clouds in the Marshall Islands have observed what they believe to be the world's largest raindrops, with a radius of 0.52 cm. How many molecules are in these monster drops?

80. •**Cooling Computers** Researchers are developing "heat exchangers" for laptop computers that take heat from the laptop—to keep it from being damaged by overheating—and use it to vaporize methanol. Given that 5100 J of heat is removed from the laptop when 4.6 g of methanol is vaporized, what is the latent heat of vaporization for methanol?

81. •• **Scuba Tanks** In scuba diving circles, "an 80" refers to a scuba tank that holds 80 cubic feet of air, a standard amount for recreational diving. Given that a scuba tank is a cylinder 2 feet long and half a foot in diameter, determine **(a)** the volume of a tank and **(b)** the pressure in a tank when 80 cubic feet of air is compressed into its relatively small volume. **(c)** What is the mass of air in a tank that holds 80 cubic feet of air. Assume the temperature is 21 °C and that the walls of the tank are of negligible thickness.

82. •• A reaction vessel contains 8.06 g of H_2 and 64.0 g of O_2 at a temperature of 125 °C and a pressure of 101 kPa. **(a)** What is the volume of the vessel? **(b)** The hydrogen and oxygen are now ignited by a spark, initiating the reaction $2\,H_2 + O_2 \rightarrow 2\,H_2O$. This reaction consumes all the hydrogen and oxygen in the vessel. What is the pressure of the resulting water vapor when it returns to its initial temperature of 125 °C?

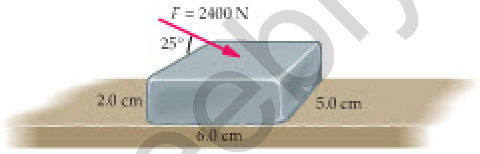
83. •• A bicycle tire with a radius of 0.68 m has a gauge pressure of 42 lb/in^2. Treating the tire as a hollow hoop with a cross-sectional area of 0.0028 m^2, find the number of air molecules in the tire when its temperature is 24 °C.

84. •• Peter catches a 4.8-kg striped bass on a fishing line 0.54 mm in diameter and begins to reel it in. He fishes from a pier well above the water, and his fish hangs vertically from the line out of the water. The fishing line has a Young's modulus of 5.1×10^9 N/m^2. **(a)** What is the fractional increase in length of the fishing line if the fish is at rest? **(b)** What is the fractional increase in the fishing line's length when the fish is pulled upward with a constant speed of 1.5 m/s? **(c)** What is the fractional increase in the fishing line's length when the fish is pulled upward with a constant acceleration of 1.5 m/s^2?

85. •• **IP** You use a steel socket wrench 28 cm long to loosen a rusty bolt, applying a force F at the end of the handle. The handle undergoes a shear deformation of 0.11 mm. **(a)** If the cross-sectional area of the handle is 2.3 cm^2, what is the magnitude of the applied force F? **(b)** If the cross-sectional area of the handle is doubled, by what factor does the shear deformation change? Explain.

86. •• A steel ball (density = 7860 kg/m^3) with a diameter of 6.4 cm is tied to an aluminum wire 82 cm long and 2.5 mm in diameter. The ball is whirled about in a vertical circle with a tangential speed of 7.8 m/s at the top of the circle and 9.3 m/s at the bottom of the circle. Find the amount of stretch in the wire **(a)** at the top and **(b)** at the bottom of the circle.

87. •• A lead brick with the dimensions shown in Figure 17–30 rests on a rough solid surface. A force of 2400 N is applied as indicated. Find **(a)** the change in height of the brick and **(b)** the amount of shear deformation.



▲ **FIGURE 17–30** Problem 87

88. •• **IP** Five molecules have the following speeds: 221 m/s, 301 m/s, 412 m/s, 44.0 m/s, and 182 m/s. **(a)** Find v_{av} for these molecules. **(b)** Do you expect $(v^2)_{av}$ to be greater than, less than, or equal to $(v_{av})^2$? Explain. **(c)** Calculate $(v^2)_{av}$ and comment on your results. **(d)** Calculate v_{rms} and compare with v_{av}.

89. •• **(a)** Find the amount of heat that must be extracted from 1.5 kg of steam at 110 °C to convert it to ice at 0.0 °C. **(b)** What speed would this 1.5-kg block of ice have if its translational kinetic energy were equal to the thermal energy calculated in part (a)?

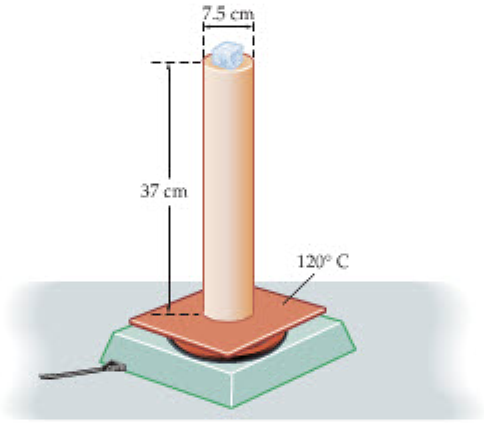
90. •• When water freezes into ice it expands in volume by 9.05%. Suppose a volume of water is in a household water pipe or a cavity in a rock. If the water freezes, what pressure must be exerted on it to keep its volume from expanding? (If the pipe or rock cannot supply this pressure, the pipe will burst and the rock will split.)

91. •• Suppose the 0.550 kg of ice in Figure 17–22 starts at point A. How much ice is left in the system after **(a)** 5.00×10^4 J, **(b)** 1.00×10^5 J, and **(c)** 1.50×10^5 J of heat are added to the system?

92. ••• Students on a spring break picnic bring a cooler that contains 5.1 kg of ice at 0.0 °C. The cooler has walls that are 3.8 cm thick and are made of Styrofoam, which has a thermal conductivity of 0.030 W/(m · C°). The surface area of the cooler is 1.5 m^2, and it rests in the shade where the air temperature is 21 °C. **(a)** Find the rate at which heat flows into the cooler. **(b)** How long does it take for the ice in the cooler to melt?

93. ••• A 5.5-kg block of ice at −1.5 °C slides on a horizontal surface with a coefficient of kinetic friction equal to 0.062. The initial speed of the block is 6.9 m/s and its final speed is 5.5 m/s. Assuming that all the energy dissipated by kinetic friction goes into melting a small mass m of the ice, and that the rest of the ice block remains at −1.5 °C, determine the value of m.

94. ••• A cylindrical copper rod 37 cm long and 7.5 cm in diameter is placed upright on a hot plate held at a constant temperature of 120 °C, as indicated in Figure 17–31. A small depression on top of the rod holds a 25-g ice cube at an initial temperature of 0.0 °C. How long does it take for the ice cube to melt? Assume there is no heat loss through the vertical surface of the rod, and that the thermal conductivity of copper is 390 W/(m · C°).

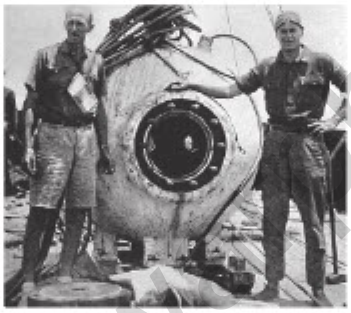


▲ **FIGURE 17–31** Problem 94

PASSAGE PROBLEMS

Diving in the Bathysphere

The American naturalist Charles William Beebe (1877–1962) set a world record in 1934 when he and Otis Barton (1899–1992) made a dive to a depth of 923 m below the surface of the ocean. The dive was made just 10 miles from Nonsuch Island, off the coast of Bermuda, in a device known as the bathysphere, designed and built by Barton. The bathysphere was basically a steel sphere 4.75 ft in diameter with three small ports made of fused quartz. Lowered into the ocean on a steel cable whose radius was 1.85 cm, the bathysphere also carried bottles of oxygen, chemicals to absorb carbon dioxide, and a telephone line to the surface.



Charles William Beebe (left) and Otis Barton with the bathysphere in 1934. (Problems 95, 96, 97, and 98)

Beebe was fascinated by the new forms of life he and Barton encountered on their numerous dives. At one point he saw a "creature, several feet long, dart toward the window, turn sideways and—explode. At the flash, which was so strong that it illumined my face . . . I saw the great red shrimp and the outpouring fluid of flame." No wonder he considered the ocean depths to be "a world as strange as that of Mars."

The dives were not without their risks, however. It was not uncommon, for example, to have the bathysphere return to the surface partially filled with water after a window seal failed. On one deep dive, water began to stream rapidly into the sphere. Beebe quickly called to the surface and asked—not to be raised quickly—but to be lowered more rapidly, in the hope that increasing water pressure would force the leaking window into its seals to stop the leak. It worked, showing that Beebe was not only an exceptional naturalist, but also a cool-headed scientist with a good knowledge of basic physics!

95. • What pressure did the bathysphere experience at its record depth?

A. 9.37 atm **B.** 89.6 atm

C. 91.9 atm **D.** 92.9 atm

96. • How many moles of air did the bathysphere contain when it was sealed at the surface, assuming a temperature of 297 K and ignoring the thickness of the metal shell? (*Note:* A resting person breathes roughly 0.5 mol of air per minute.)

A. 65.2 mol **B.** 270 mol

C. 392 mol **D.** 523 mol

97. • How much did the volume of the bathysphere decrease as it was lowered to its record depth? (For simplicity, treat the bathysphere as a solid metal sphere.)

A. $9.0 \times 10^{-5}\ \mathrm{m}^3$ **B.** $9.2 \times 10^{-5}\ \mathrm{m}^3$

C. $1.1 \times 10^{-4}\ \mathrm{m}^3$ **D.** $3.8 \times 10^{-4}\ \mathrm{m}^3$

98. • Suppose the bathysphere and its occupants had a combined mass of 12,700 kg. How much did the cable stretch when the bathysphere was at a depth of 923 m? (Neglect the weight of the cable itself, but include the effects of the bathysphere's buoyancy.)

A. 47 cm **B.** 48 cm

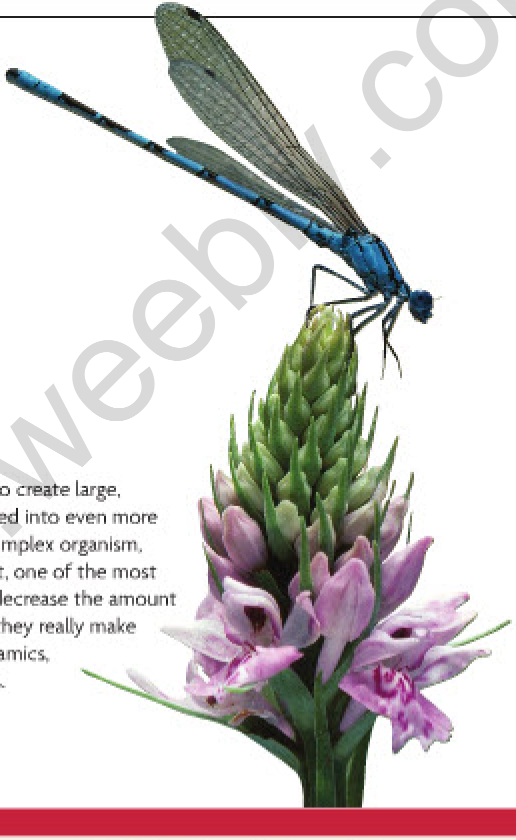
C. 52 cm **D.** 53 cm

INTERACTIVE PROBLEMS

99. •• **Referring to Example 17–6 (a)** Find the final temperature of the system if *two* 0.0450-kg ice cubes are added to the warm lemonade. The temperature of the ice is 0 °C; the temperature and mass of the warm lemonade are 20.0 °C and 3.95 kg, respectively. **(b)** How many 0.0450-kg ice cubes at 0 °C must be added to the original warm lemonade if the final temperature of the system is to be at least as cold as 15.0 °C?

100. •• **Referring to Example 17–6 (a)** Find the final temperature of the system if a single 0.045-kg ice cube at 0 °C is added to 2.00 kg of lemonade at 1.00 °C. **(b)** What initial temperature of the lemonade will be just high enough to melt all of the ice in a single ice cube and result in an equilibrium temperature of 0 °C? The mass of the lemonade is 2.00 kg and the temperature of the ice cube is 0 °C.

18 The Laws of Thermodynamics



Every day, plants and animals take small, simple molecules and use them to create large, complex molecules such as proteins and DNA. These, in turn, are assembled into even more highly-ordered structures—a damselfly grows from a formless egg to a complex organism, blooms on a plant progress from minute buds to fully formed flowers. Yet, one of the most profound and far-reaching of physical laws holds that all processes must decrease the amount of order in the universe. Are living things exempt from this principle? Do they really make the universe more ordered? This chapter explores the laws of thermodynamics, and considers the question of whether they apply to biological organisms.

In this chapter we discuss the fundamental laws of nature that govern thermodynamic processes. One of these laws—the one dealing with temperature—was first introduced in Chapter 16. The others are presented here for the first time.

Of particular interest are the first and second laws of thermodynamics. The first law extends the basic principle of energy conservation to include the type of energy transfer we call heat. The real heart of thermodynamics, however, is embodied in the second law. This law introduces a fundamentally new concept to physics—the idea that there is a directionality to the behavior of nature. Melting ice, cooling lava, and the crumbling ruins of the Parthenon all illustrate the second law in action.

Finally, the third law of thermodynamics states that absolute zero is, in fact, the lowest temperature possible. Great efforts have been made over the years to reach lower and lower temperatures, and with remarkable success, but the third law sets the ultimate limit that we can only approach, but never attain.

18–1 The Zeroth Law of Thermodynamics

Though the zeroth law of thermodynamics has already been presented in Chapter 16, we repeat it here so that all the laws of thermodynamics can be collected together in one chapter. As you recall, the zeroth law states the conditions under which objects will be in thermal equilibrium with one another. To be precise:

> **Zeroth Law of Thermodynamics**
>
> If object A is in thermal equilibrium with object C, and object B is separately in thermal equilibrium with object C, then objects A and B will be in thermal equilibrium if they are placed in thermal contact.

The physical quantity that is equal when two objects are in thermal equilibrium is the temperature. In particular, if two objects have the same temperature, we can be assured that *no heat will flow* when they are placed in thermal contact. On the other hand, if heat does flow between two objects, it follows that they are not in thermal equilibrium, and they do not have the same temperature.

Any temperature scale can be used to determine whether objects will be in thermal equilibrium. As we saw in the previous chapter, however, the Kelvin scale is particularly significant in physics. For example, the average kinetic energy of a gas molecule is directly proportional to the Kelvin temperature, as is the volume of an ideal gas. In this chapter we present additional illustrations of the special significance of the Kelvin scale.

18–2 The First Law of Thermodynamics

The first law of thermodynamics is a statement of energy conservation that specifically includes heat. For example, consider the system shown in **Figure 18–1**. The internal energy of this system—that is, the sum of all its potential and kinetic energies—has the initial value U_i. If an amount of heat Q flows into the system, its internal energy increases to the final value $U_f = U_i + Q$. Thus,

$$\Delta U = U_f - U_i = Q \qquad 18\text{–}1$$

Of course, if heat is removed from the system, its internal energy decreases. We can take this into account by giving Q a *positive* value when the system *gains* heat, and a *negative* value when it *loses* heat.

Similarly, suppose the system under consideration does a work W on the external world, as in **Figure 18–2**. If the system is insulated so that no heat can flow in or out, the energy to do the work must come from the internal energy of the system. Thus, if the initial internal energy is U_i, the final internal energy is $U_f = U_i - W$. Therefore,

$$\Delta U = U_f - U_i = -W \qquad 18\text{–}2$$

On the other hand, if work is done *on* the system, its internal energy increases. Thus, we use the following sign convention for the work: W has a *positive* value when the system *does work on the external world*, and it has a *negative* value when *work is done on the system*. These sign conventions are summarized in Table 18–1.

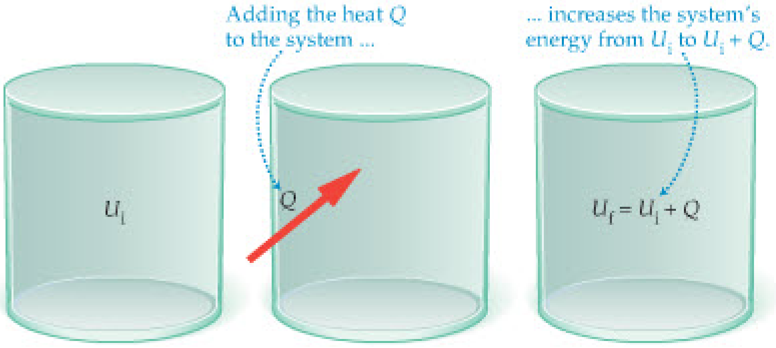


FIGURE 18–1 The internal energy of a system

A system initially has the internal energy U_i (left). After the heat Q is added, the system's new internal energy is $U_f = U_i + Q$ (right). The system has rigid walls; hence, it can do no work on the external world.

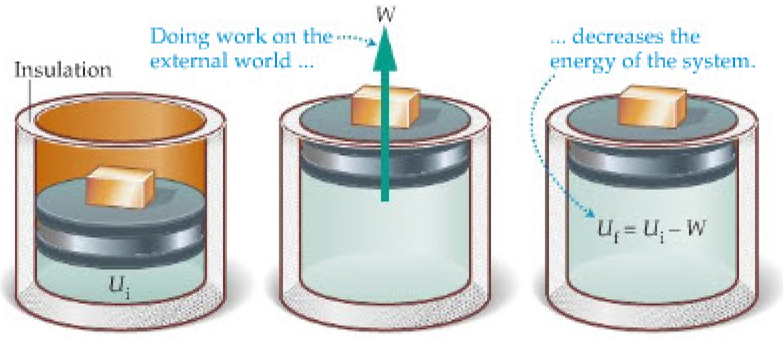


FIGURE 18–2 Work and internal energy

A system initially has the internal energy U_i (left). After the system does the work W on the external world, its remaining internal energy is $U_f = U_i - W$ (right). Note that the insulation guarantees that no heat is gained or lost by the system.

TABLE 18–1 Signs of Q and W

Q positive	System *gains* heat
Q negative	System *loses* heat
W positive	Work done *by* system
W negative	Work done *on* system

PROBLEM-SOLVING NOTE

Proper Signs for Q and W

When applying the first law of thermodynamics, it is important to determine the proper signs for Q, W, and ΔU.

Combining the results in Equations 18–1 and 18–2 yields the first law of thermodynamics:

First Law of Thermodynamics

The change in a system's internal energy, ΔU, is related to the heat Q and the work W as follows:

$$\Delta U = Q - W \qquad 18\text{–}3$$

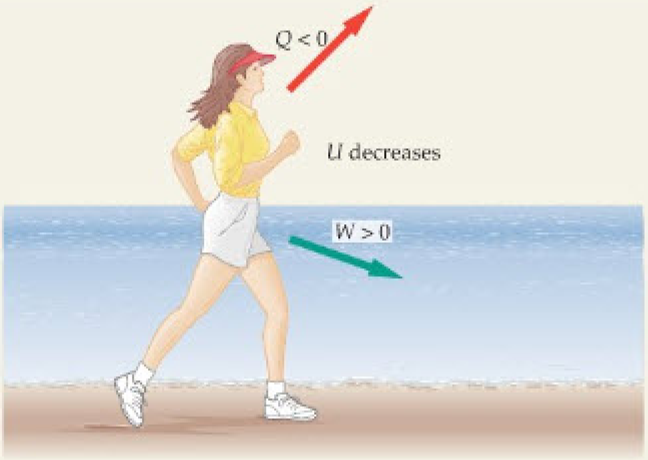
If you apply the sign conventions given here, it is straightforward to verify that adding heat to a system, and/or doing work on it, increases the internal energy. On the other hand, if the system does work, and/or heat is removed, its internal energy decreases. Example 18–1 gives a specific numerical application of the first law.

EXAMPLE 18–1 HEAT, WORK, AND INTERNAL ENERGY

(a) Jogging along the beach one day, you do 4.3×10^5 J of work and give off 3.8×10^5 J of heat. What is the change in your internal energy? **(b)** Switching over to walking, you give off 1.2×10^5 J of heat and your internal energy decreases by 2.6×10^5 J. How much work have you done while walking?

PICTURE THE PROBLEM

Our sketch shows a person jogging along the beach. The fact that the person does work on the external world means that W is positive. As for the heat, the fact that heat is given off by the person means that Q is negative.



STRATEGY

The signs of W and Q have been determined in our sketch, and the magnitudes are given in the problem statement. To find ΔU for part (a), we simply use the first law of thermodynamics, $\Delta U = Q - W$. To find the work W for part (b), we solve the first law for W, which yields $W = Q - \Delta U$.

SOLUTION

Part (a)

1. Calculate ΔU, using $Q = -3.8 \times 10^5$ J and $W = 4.3 \times 10^5$ J:

$$\Delta U = Q - W = (-3.8 \times 10^5\ \text{J}) - 4.3 \times 10^5\ \text{J} = -8.1 \times 10^5\ \text{J}$$

Part (b)

2. Solve $\Delta U = Q - W$ for W:

$$W = Q - \Delta U$$

3. Substitute $Q = -1.2 \times 10^5$ J and $\Delta U = -2.6 \times 10^5$ J:

$$W = -1.2 \times 10^5\ \text{J} - (-2.6 \times 10^5\ \text{J}) = 1.4 \times 10^5\ \text{J}$$

INSIGHT

Note the importance of using the correct signs for Q, W, and ΔU. When the proper signs are used, the first law is simply a way of keeping track of all the energy exchanges—including mechanical work and heat—that can occur in a given system.

PRACTICE PROBLEM

After walking for a few minutes you begin to run, doing 5.1×10^5 J of work and decreasing your internal energy by 8.8×10^5 J. How much heat have you given off? [**Answer:** $Q = -3.7 \times 10^5$ J. The negative sign means you have given off heat, as expected.]

Some related homework problems: Problem 3, Problem 5

Just looking at the first law, $\Delta U = Q - W$, it is easy to get the false impression that U, Q, and W are basically the same type of physical quantity. Certainly, they are all measured in the same units (J). In other respects, however, they are quite different. For example, the heat Q represents energy that flows through thermal contact. In contrast, the work W indicates a transfer of energy by the action of a force through a distance.

The most important distinction between these quantities, however, is the way they depend on the **state of a system,** which is determined by its temperature, pressure, and volume. The internal energy, for example, depends only on the state of a system, and not on how the system is brought to that state. A simple example is the ideal gas, where U depends only on the temperature T, and not on any previous values T may have had. Since U depends only on the state of a system—whether the system is an ideal gas or something more complicated—it is referred to as a **state function.**

On the other hand, Q and W are *not* state functions; they depend on the precise way—that is, on the process—by which a system is changed from one state, A, to another state, B. For example, one process connecting the states A and B may result in a heat $Q_1 = 19$ J and a work $W_1 = 32$ J. With a different process connecting the *same two states*, we may find a heat $Q_2 = -24\text{ J} \neq Q_1$, and a work $W_2 = -11\text{ J} \neq W_1$. Still, the difference in internal energy—which depends only on the initial and final states—must be the same for all processes connecting A and B. It follows that

$$\Delta U_{\text{AB}} = Q_1 - W_1 = Q_2 - W_2 = -13\text{ J}$$

Therefore, the energy of this system always decreases by 13 J when it changes from state A to state B.

We now turn our attention to specific types of thermal processes that can change the energy of a system.

18–3 Thermal Processes

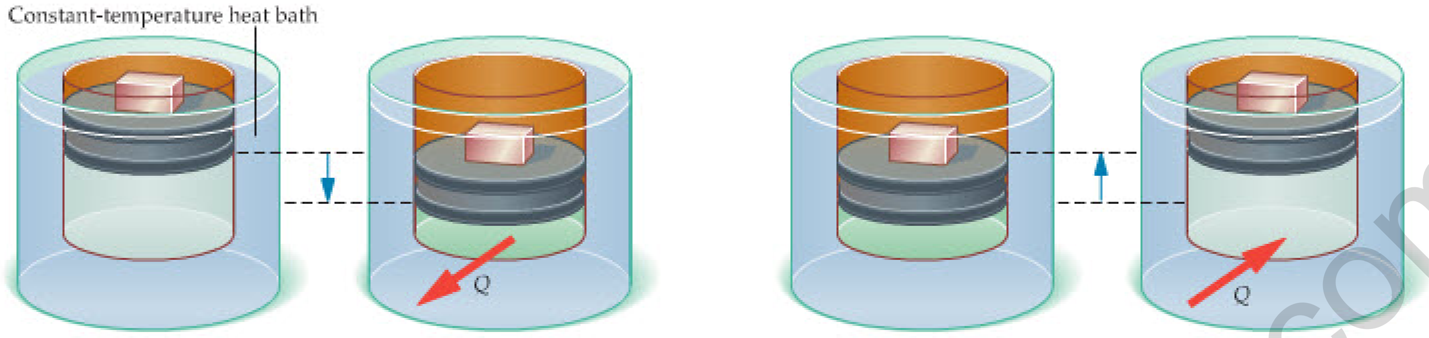
In this section we consider a variety of thermodynamic processes that can be used to change the state of a system. Among these are ones that take place at constant pressure, constant volume, or constant temperature. We also consider processes in which no heat is allowed to flow into or out of the system.

All of the processes discussed in this section are assumed to be **quasi-static,** which is a fancy way of saying they occur so slowly that at any given time the system and its surroundings are essentially in equilibrium. Thus, in a quasi-static process, the pressure and temperature are always uniform throughout the system. Furthermore, we assume that the system in question is free from friction or other dissipative forces.

These assumptions can be summarized by saying that the processes we consider are **reversible.** To be precise, a reversible process is defined by the following condition:

> For a process to be reversible, it must be possible to return both the system and its surroundings to exactly the same states they were in before the process began.

The fact that both the system and its surroundings must be returned to their initial states is the key element of this definition. For example, if there is friction between a piston and the cylinder in which it slides, a reversible process is not possible. Even if the piston is returned to its original location, the heat generated by friction



(a) Compressing at constant temperature expels heat.

(b) Expanding at constant temperature draws in heat.

▲ **FIGURE 18–3 An idealized reversible process**

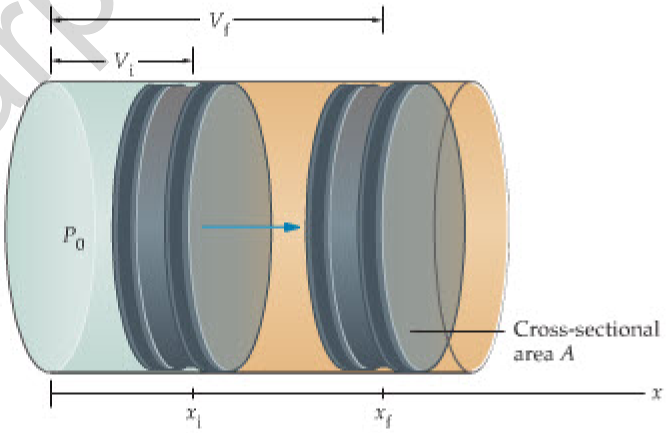
(a) A piston is slowly moved downward, compressing a gas. In order for its temperature to remain constant, a heat Q goes from the gas into the constant-temperature heat bath, which may be nothing more than a large volume of water. **(b)** As the piston is allowed to slowly rise back to its initial position, it draws the same amount of heat Q from the heat bath that it gave to the bath in (a). Hence, both the system (gas) and the surroundings (heat bath) return to their initial states.

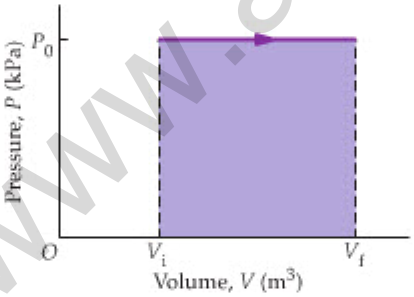
will warm the cylinder and eventually flow into the surrounding air. Thus, it is not possible to "undo" the effects of friction, and such a process is said to be **irreversible.** Even without friction, a process can be irreversible if it occurs rapidly enough to cause effects such as turbulence, or if the system is far from equilibrium.

In practice, then, all real processes are irreversible to some extent. It is still possible, however, to have a process that closely approximates a perfectly reversible process, just as we can have systems that are practically free of friction. For example, in **Figure 18–3 (a)**, we consider a "frictionless" piston that is slowly forced downward, while the gas in the cylinder is kept at constant temperature. An amount of heat, Q, goes from the gas to its surroundings in order to keep the temperature from rising. In **Figure 18–3 (b)** the piston is slowly moved back upward, drawing in the same heat Q from its surroundings to keep the temperature from dropping. In an "ideal" case like this, the process is reversible, since the system and its surroundings are left unchanged. We shall assume that all the processes described in this section are reversible.

▶ **FIGURE 18–4 Work done by an expanding gas**

A gas in a cylinder of cross-sectional area A expands with a constant pressure of P_0 from an initial volume $V_i = Ax_i$ to a final volume $V_f = Ax_f$. As it expands, it does the work $W = P_0(V_f - V_i)$.





▲ **FIGURE 18–5 A constant-pressure process**

A PV plot representing the constant-pressure process shown in Figure 18–4. The area of the shaded region, $P_0(V_f - V_i)$, is equal to the work done by the expanding gas in Figure 18–4.

Constant Pressure/Constant Volume

We begin by considering a process that occurs at **constant pressure.** To be specific, suppose that a gas with the pressure P_0 is held in a cylinder of cross-sectional area A, as in **Figure 18–4**. If the piston moves outward, so that the volume of the gas increases from an initial value V_i to a final value V_f, the process can be represented graphically as shown in **Figure 18–5**. Here we plot pressure P versus volume V in a PV plot; the process just described is the horizontal line segment.

As the gas expands, it does work on the piston. First, the gas exerts a force on the piston equal to the pressure times the area:

$$F = P_0 A$$

Second, the gas moves the piston from the position x_i to the position x_f, where $V_i = Ax_i$ and $V_f = Ax_f$, as indicated in Figure 18–4. Thus, the work done by the gas is the force times the distance through which the piston moves:

$$W = F(x_f - x_i) = P_0A(x_f - x_i) = P_0(Ax_f - Ax_i) = P_0(V_f - V_i)$$

In general, if the pressure, P, is constant, and the volume changes by the amount ΔV, the work done by the gas is

$$W = P\Delta V \qquad \textit{(constant pressure)} \qquad 18\text{–}4$$

EXERCISE 18–1

A gas with a constant pressure of 150 kPa expands from a volume of 0.76 m^3 to a volume of 0.92 m^3. How much work does the gas do?

SOLUTION

Applying Equation 18–4 we find

$$W = P\Delta V = (150 \text{ kPa})(0.92 \text{ m}^3 - 0.76 \text{ m}^3) = 24{,}000 \text{ J}$$

This is the energy required to raise the temperature of 1.0 kg of water by 5.7 C°.

Looking closely at the PV plot in Figure 18–5, we see that the work done by the gas is equal to the area under the horizontal line representing the constant-pressure process. In particular, the shaded region is a rectangle of height P_0 and width $V_f - V_i$, and therefore its area is

$$\text{area} = P_0(V_f - V_i) = W$$

Though this result was obtained for the special case of constant pressure, it applies to any process; that is, *the work done by an expanding gas is equal to the area under the curve representing the process in a PV plot.* This result is applied in the next Example.

PROBLEM-SOLVING NOTE

Work and the *PV* Diagram

When finding the work done by calculating the area on a PV diagram, recall that a pressure of 1 Pa times a volume of 1 m^3 gives an energy equal to 1 J.

EXAMPLE 18–2 WORK AREA

A gas expands from an initial volume of 0.40 m^3 to a final volume of 0.62 m^3 as the pressure increases linearly from 110 kPa to 230 kPa. Find the work done by the gas.

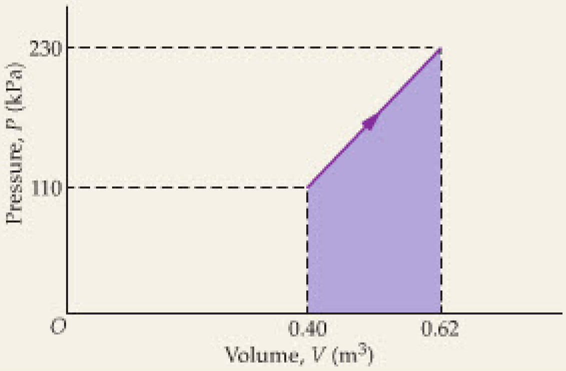
PICTURE THE PROBLEM

The sketch shows the process for this problem. As the volume increases from $V_i = 0.40 \text{ m}^3$ to $V_f = 0.62 \text{ m}^3$, the pressure increases linearly from $P_i = 110$ kPa to $P_f = 230$ kPa.

STRATEGY

The work done by this gas is equal to the shaded area in the sketch. We can calculate this area as the sum of the area of a rectangle plus the area of a triangle.

In particular, the rectangle has a height P_i and a width $(V_f - V_i)$. Similarly, the triangle has a height $(P_f - P_i)$ and a base of $(V_f - V_i)$.



SOLUTION

1. Calculate the area of the rectangular portion of the total area:

$$A_{\text{rectangle}} = P_i(V_f - V_i) = (110 \text{ kPa})(0.62 \text{ m}^3 - 0.40 \text{ m}^3) = 2.4 \times 10^4 \text{ J}$$

2. Next, calculate the area of the triangular portion of the total area:

$$A_{\text{triangle}} = \tfrac{1}{2}(P_f - P_i)(V_f - V_i) = \tfrac{1}{2}(230 \text{ kPa} - 110 \text{ kPa})(0.62 \text{ m}^3 - 0.40 \text{ m}^3) = 1.3 \times 10^4 \text{ J}$$

3. Sum these areas to find the work done by the gas:

$$W = A_{\text{rectangle}} + A_{\text{triangle}} = 2.4 \times 10^4 \text{ J} + 1.3 \times 10^4 \text{ J} = 3.7 \times 10^4 \text{ J}$$

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INSIGHT

We could also have solved this problem by noting that since the pressure varies linearly, its average value is simply $P_{av} = \frac{1}{2}(P_f + P_i) = 170$ kPa. The work done by the gas, then, is $W = P_{av}\,\Delta V = (170\text{ kPa})(0.22\text{ m}^3) = 3.7 \times 10^4$ J, as before.

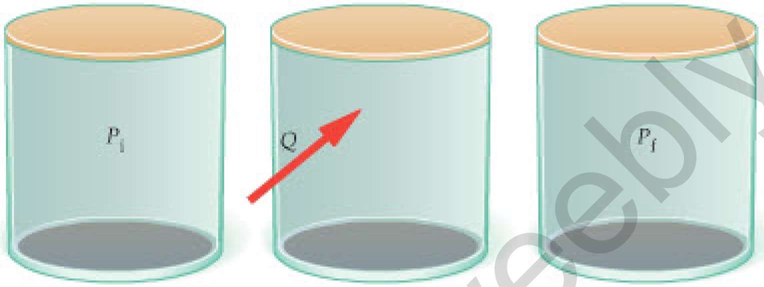
PRACTICE PROBLEM

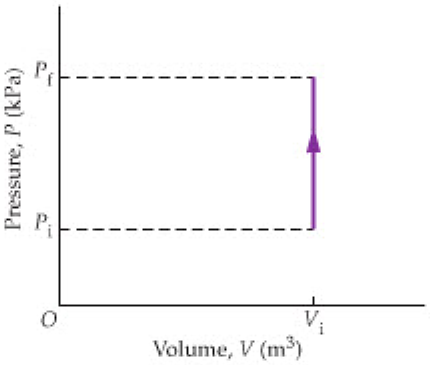
Suppose the pressure varies linearly from 110 kPa to 260 kPa. How much work does the gas do in this case?
[**Answer:** $W = 4.1 \times 10^4$ J]

Some related homework problems: Problem 18, Problem 19

▶ **FIGURE 18–6 Adding heat to a system of constant volume**

Heat is added to a system of constant volume, increasing its pressure from P_i to P_f. Since there is no displacement of the walls, there is no work done in this process. Therefore, $W = 0$ and the change in internal energy is simply equal to the heat added to or removed from the system, $\Delta U = Q$.





▲ **FIGURE 18–7 A constant-volume process**

The pressure increases from P_i to P_f, just as in Figure 18–6, while the volume remains constant at its initial value, V_i. The area under this process is zero, as is the work.

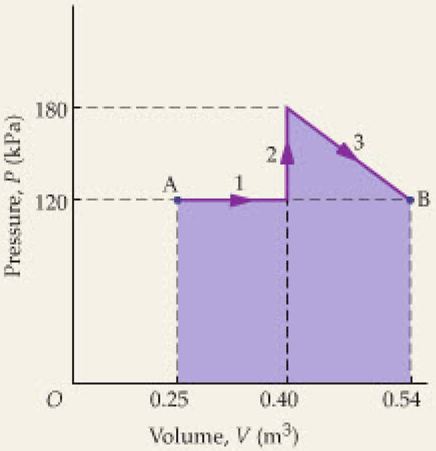
Next, we consider a **constant-volume** process. Suppose, for example, that heat is added to a gas in a container of fixed volume, as in **Figure 18–6**, causing the pressure to increase. Since there is no displacement of any of the walls, it follows that the force exerted by the gas does no work. Thus, for *any* constant-volume process, we have

$$W = 0 \qquad \textit{(constant volume)}$$

From the first law of thermodynamics, $\Delta U = Q - W$, it follows that $\Delta U = Q$; that is, the change in internal energy is equal to the amount of heat that is added to or removed from the system. Note that zero work in a constant-volume process is consistent with our earlier statement that the work is equal to the area under the curve representing the process. For example, in **Figure 18–7** we show a constant-volume process in which the pressure is increased from P_i to P_f. Since this line is vertical, the area under it is zero; that is, $W = 0$ as expected.

ACTIVE EXAMPLE 18–1 FIND THE TOTAL WORK

A gas undergoes the three-part process shown here, connecting the states A and B. Find the total work done by the gas during this process.



SOLUTION *(Test your understanding by performing the calculations indicated in each step.)*

1. Calculate the work done during part 1 of the total process: 18,000 J
2. Calculate the work done during part 2 of the total process: 0 J
3. Calculate the work done during part 3 of the total process: 21,000 J
4. Sum the results to find the total work: 39,000 J

INSIGHT

Note that the work done by the gas would be different if the process connecting A and B were a constant-pressure expansion with a pressure of 120 kPa. In that case, the total work done would be 35,000 J. Thus, the work W—which is not a state function—depends on the process, as expected.

YOUR TURN

Describe a process connecting the states A and B for which the work done by the gas is 58,000 J.

(Answers to **Your Turn** *problems are given in the back of the book.)*

Isothermal Processes

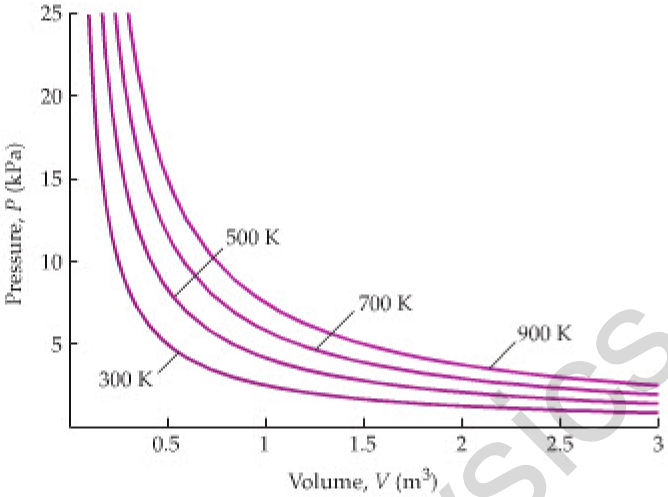
Another common process is one that takes place at constant temperature; that is, an **isothermal process.** For an ideal gas the isotherm has a relatively simple form. In particular, if T is constant, it follows that PV is a constant as well:

$$PV = NkT = \text{constant}$$

Thus, ideal-gas isotherms have the following pressure-volume relationship:

$$P = \frac{NkT}{V} = \frac{\text{constant}}{V}$$

This is illustrated in **Figure 18–8**, where we show several isotherms corresponding to different temperatures. (Recall that we've seen isotherms before, in Figure 17–5.)

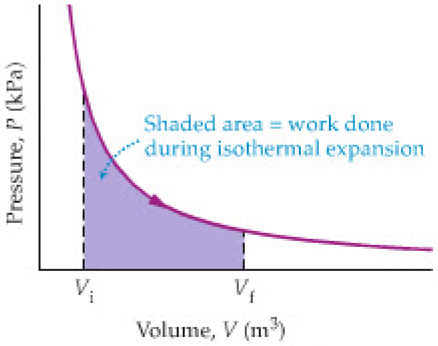


▲ **FIGURE 18–8 Isotherms on a *PV* plot**
These isotherms are for one mole of an ideal gas at the temperatures 300 K, 500 K, 700 K, and 900 K. Notice that each isotherm has the shape of a hyperbola. As the temperature is increased, however, the isotherms move farther from the origin. Thus, the pressure corresponding to a given volume increases with temperature, as one would expect.

For any given isotherm, such as the one shown in **Figure 18–9**, the work done by an expanding gas is equal to the area under the curve, as usual. In particular, the work in expanding from V_i to V_f in Figure 18–9 is equal to the shaded area. This area may be derived by using the methods of calculus. The result is found to be

$$W = NkT \ln\left(\frac{V_f}{V_i}\right) = nRT \ln\left(\frac{V_f}{V_i}\right) \qquad \text{(constant temperature)} \qquad 18\text{–}5$$

Note that "ln" stands for the natural logarithm; that is, log to the base e. This result is utilized in the next Example.



▲ **FIGURE 18–9 An isothermal expansion**
In an isothermal expansion from the volume V_i to the volume V_f, the work done is equal to the shaded area. For n moles of an ideal gas at the temperature T, the work done by the gas is $W = nRT \ln(V_f/V_i)$.

EXAMPLE 18–3 HEAT FLOW

A cylinder holds 0.50 mol of a monatomic ideal gas at a temperature of 310 K. As the gas expands isothermally from an initial volume of 0.31 m^3 to a final volume of 0.45 m^3, determine the amount of heat that must be added to the gas to maintain a constant temperature.

PICTURE THE PROBLEM

The physical process is illustrated in the drawing on the left (next page). Note that heat flows into the gas as it expands in order to keep its temperature constant at $T = 310$ K. The graph on the right (next page) is a PV plot showing the same process. The work done by the expanding gas is equal to the shaded area from $V_i = 0.31\ m^3$ to $V_f = 0.45\ m^3$.

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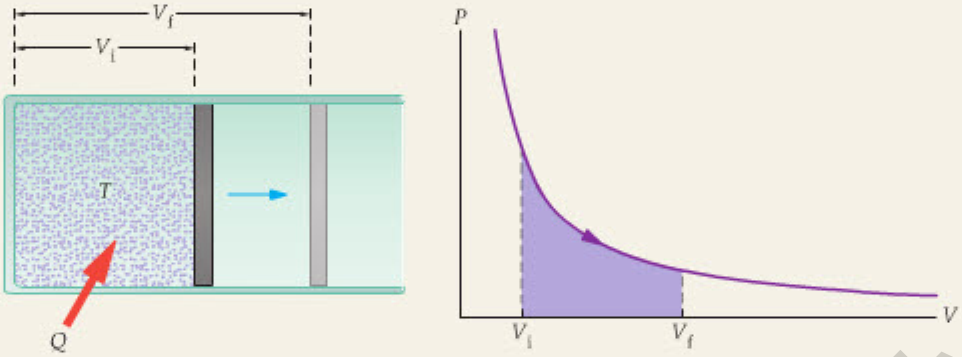


STRATEGY

We can use the first law, $\Delta U = Q - W$, to find the heat Q in terms of W and ΔU. We can find W and ΔU as follows:

First, the work W is found using $W = nRT\ln(V_f/V_i)$.

Next, recall that the internal energy of an ideal gas depends only on the temperature. For example, the internal energy of a monatomic ideal gas is $U = \frac{3}{2}nRT$. Since the temperature is constant in this process, there is no change in internal energy; that is, $\Delta U = 0$.

SOLUTION

1. Solve the first law of thermodynamics for the heat, Q:

$$\Delta U = Q - W$$
$$Q = \Delta U + W$$

2. Calculate the work done by the expanding gas:

$$W = nRT\ln\left(\frac{V_f}{V_i}\right)$$
$$= (0.50\text{ mol})[8.31\text{ J/(mol}\cdot\text{K)}](310\text{ K})\ln\left(\frac{0.45\text{ m}^3}{0.31\text{ m}^3}\right)$$
$$= 480\text{ J}$$

3. Calculate the change in the gas's internal energy:

$$\Delta U = \tfrac{3}{2}nR(T_f - T_i) = 0$$

4. Substitute numerical values to find Q:

$$Q = \Delta U + W = 0 + 480\text{ J} = 480\text{ J}$$

INSIGHT

We find, then, that when an ideal gas undergoes an isothermal expansion, the heat gained by the gas is equal to the work it does; that is, $Q = W$. This is a direct consequence of the first law of thermodynamics, $\Delta U = Q - W$, and the fact that $\Delta U = 0$ for an ideal gas at constant temperature.

PRACTICE PROBLEM

Find the final volume of this gas when it has expanded enough to do 590 J of work. [**Answer:** $V_f = 0.49\text{ m}^3$]

Some related homework problems: Problem 20, Problem 21

Note that if a gas is compressed at constant temperature, work is done *on* the gas, rather than *by* the gas. As a result, we expect the work to be negative. This is consistent with Equation 18–5, since in a compression the final volume is less than the initial volume. Thus, $V_f/V_i < 1$, and hence $W = nRT\ln(V_f/V_i)$ is negative.

Finally, we consider one last point regarding isothermal processes.

CONCEPTUAL CHECKPOINT 18–1 IDEAL OR NOT?

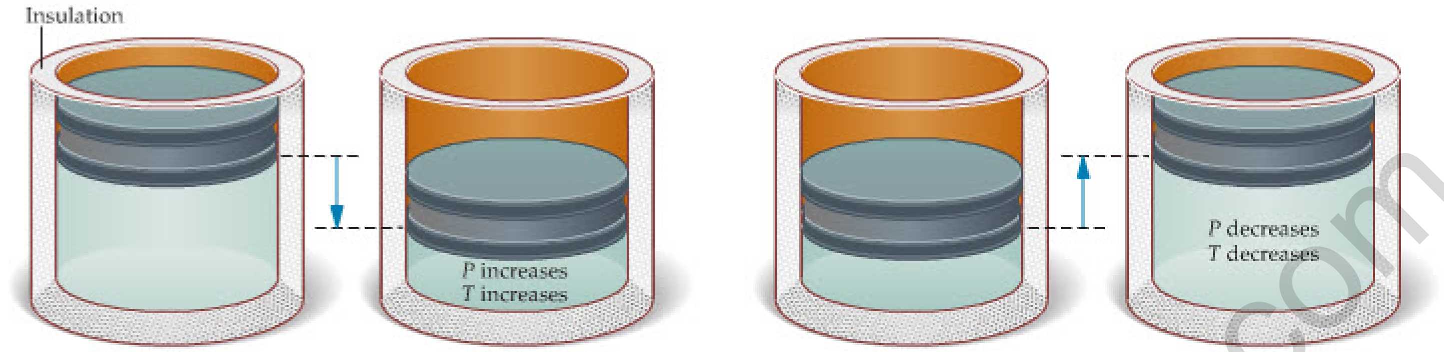
The internal energy of a certain gas increases when it is compressed isothermally. Is the gas ideal?

REASONING AND DISCUSSION

In an isothermal process, the internal energy of an ideal gas is unchanged, since the temperature is constant. Hence, if the internal energy of this gas changes during an isothermal compression, it must not be ideal.

ANSWER

No, the gas is not ideal.



(a) Compressing with no heat flow increases P and T.

(b) Expanding with no heat flow decreases P and T.

▲ **FIGURE 18–10 An adiabatic process**

In adiabatic processes, no heat flows into or out of the system. In the cases shown in this figure, heat flow is prevented by insulation. **(a)** An adiabatic compression increases both the pressure and the temperature. **(b)** An adiabatic expansion results in a decrease in pressure and temperature.

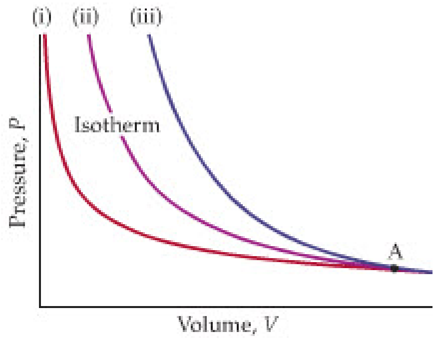
Adiabatic Processes

The final process we consider is one in which no heat flows into or out of the system. Such a process, in which $Q = 0$, is said to be **adiabatic.** One way to produce an adiabatic process is illustrated in **Figure 18–10 (a)**. Here we see a cylinder that is insulated well enough that no heat can pass through the insulation (adiabatic means, literally, "not passable"). When the piston is pushed downward in the cylinder—decreasing the volume—the gas heats up and its pressure increases. Similarly, in **Figure 18–10 (b)**, an adiabatic expansion causes the temperature of the gas to decrease, as does the pressure.

What does an adiabatic process look like on a PV plot? Certainly, its general shape must be similar to that of an isotherm; in particular, as the volume is decreased, the pressure increases. However, it can't be identical to an isotherm because, as we have pointed out, the temperature changes during an adiabatic process. The comparison between an adiabatic curve and an isothermal curve is the subject of the next Conceptual Checkpoint.

CONCEPTUAL CHECKPOINT 18–2 PRESSURE VERSUS VOLUME

A certain gas has an initial volume and pressure given by point A in the PV plot. If the gas is compressed isothermally, its pressure rises as indicated by the curve labeled "isotherm." If, instead, the gas is compressed adiabatically from point A, does its pressure follow **(a)** curve i, **(b)** curve ii, or **(c)** curve iii?



REASONING AND DISCUSSION

In an isothermal compression, some heat flows out of the system in order for its temperature to remain constant. No heat flows out of the system in the adiabatic process, however, and therefore its temperature rises. As a result, the pressure is greater for any given volume when the compression is adiabatic, as compared to isothermal. It follows, then, that the adiabatic curve, or adiabat, is represented by curve iii.

ANSWER

(c) As volume is decreased, the pressure on an adiabat rises more rapidly than on an isotherm.

As we have seen, then, an adiabatic curve is similar to an isotherm, only steeper. The precise mathematical relationship describing an adiabat will be presented in the next section.