## Standardized Test Practice

## Multiple Choice

1. How does sound travel from its source to your ear?
(A) by changes in air pressure
(B) by vibrations in wires or strings
(D) by electromagnetic waves
(D) by infrared waves
2. Paulo is listening to classical music in the speakers installed in his swimming pool. A note with a frequency of 327 Hz reaches his ears while he is under water. What is the wavelength of the sound that reaches Paulo's ears? Use $1493 \mathrm{~m} / \mathrm{s}$ for the speed of sound in water.
(A) 2.19 nm
(C) $2.19 \times 10^{-1} \mathrm{~m}$
(B) $4.88 \times 10^{-5} \mathrm{~m}$
(D) 4.57 m
3. The sound from a trumpet travels at $351 \mathrm{~m} / \mathrm{s}$ in air. If the frequency of the note is 298 Hz , what is the wavelength of the sound wave?
```
(A) }9.93\times1\mp@subsup{0}{}{-4}\textrm{m
    (C) }1.18\textrm{m
(B) }0.849\textrm{m
(D) \(1.05 \times 10^{5} \mathrm{~m}\)
```

4. The horn of a car attracts the attention of a stationary observer. If the car is approaching the observer at $60.0 \mathrm{~km} / \mathrm{h}$ and the horn has a frequency of 512 Hz , what is the frequency of the sound perceived by the observer? Use $343 \mathrm{~m} / \mathrm{s}$ for the speed of sound in air.
```
(A) }488\textrm{Hz
(C) 538 Hz
(B) 512 Hz
(D) 600 Hz
```

5. As shown in the diagram below, a car is receding at $72 \mathrm{~km} / \mathrm{h}$ from a stationary siren. If the siren is wailing at 657 Hz , what is the frequency of the sound perceived by the driver? Use $343 \mathrm{~m} / \mathrm{s}$ for the speed of sound.
```
(A) }543\textrm{Hz
(C) }647\textrm{Hz
(B) }620\textrm{Hz
(D) }698\textrm{Hz
```


6. Reba hears 20 beats in 5.0 s when she plays two notes on her piano. She is certain that one note has a frequency of 262 Hz . What are the possible frequencies of the second note?
(A) 242 Hz or 282 Hz
(B) 258 Hz or 266 Hz
(C) 260 Hz or 264 Hz
(D) 270 Hz or 278 Hz
7. Which of the following pairs of instruments have resonant frequencies at each wholenumber multiple of the lowest frequency?
(A) a clamped string and a closed pipe
(B) a clamped string and an open pipe
(C) an open pipe and a closed pipe
(D) an open pipe and a reed instrument

## Extended Answer

8. The figure below shows the first resonance length of a closed air column. If the frequency of the sound is 488 Hz , what is the speed of the sound?


## Test-Taking TIP

## Write It Down

Most tests ask you a large number of questions in a small amount of time. Write down your work whenever possible. Do math on paper, not in your head. Underline and reread important facts in passages and diagrams-do not try to memorize them.
nline

## Chapter <br> 16 <br> Fundamentals of Light

## What You'll Learn

- You will understand sources of light and how light illuminates the universe around us.
- You will be able to describe the wave nature of light and some phenomena that reveal this nature.

Why It's Important
Light is a primary source of information about how the universe behaves. We all use information such as color, brightness, and shadow every day to interpret the events occurring around us.
Balloon Race You can tell the difference between the competing balloons because of the different colors visible in the sunlight. You can distinguish the balloons from the backgrounds because of color differences in the grass and sky.

Think About This >
What causes these differences in color? How are these colors related?

## Physics

 physicspp.com
## LAUNCH Lab

## How can you determine

 the path of light through air?
## Question

What path does light take as it travels through the air?

## Procedure 11 Fan 을

1. Punch a hole with a pushpin in the center of an index card.
2. Using clay, stand the index card so that its longer edge is on the table top.
3. Turn on a lamp and have one lab partner hold the lamp so that the lightbulb shines through the hole in the card. CAUTION: Lamp can get hot over time.
4. Hold a mirror on the opposite side of the index card so that light coming through the hole strikes the mirror. Darken the room.
5. Angle the mirror so that it reflects the beam of light onto the back of the card. CAUTION: Be careful not to reflect the light beam into someone's eyes.
6. Write down your observations.

## Analysis

Describe the image of the reflected light beam that you see on the index card. Describe the path that the light beam takes.
Critical Thinking Can you see the light beam in the air? Why or why not?


### 16.1 Illumination

Light and sound are two methods by which you can receive information. Of the two, light seems to provide the greater variety of information. The human eye can detect tiny changes in the size, position, brightness, and color of an object. Our eyes usually can distinguish shadows from solid objects and sometimes distinguish reflections of objects from the objects themselves. In this section, you will learn where light comes from and how it illuminates the universe around you.

One of the first things that you ever discovered about light, although you may not have been conscious of your discovery, is that it travels in a straight line. How do you know this? When a narrow beam of light, such as that of a flashlight or sunlight streaming through a small window, is made visible by dust particles in the air, you see the path of the light as a straight line. When your body blocks sunlight, you see your outline in a shadow. Also, whenever you locate an object with your eyes and walk toward it, you most likely walk in a straight path. These things are possible only because light travels in straight lines. Based on this knowledge of how light travels, models have been developed that describe how light works.

- Objectives
- Develop the ray model of light.
- Predict the effect of distance on light's illumination.
- Solve problems involving the speed of light.
- Vocabulary
ray model of light
luminous source
illuminated source
opaque
transparent
translucent
luminous flux illuminance


Figure 16-1 A ray is a straight line that represents the linear path of a narrow beam of light (a). A light ray can change direction if it is reflected (b) or refracted (c).

## Color Convention

- Light rays are red.
- Figure 16-2 The Sun acts as a luminous source to Earth and the Moon. The Moon acts as an illuminated source to Earth. (Illustration not to scale)



## Ray Model of Light

Isaac Newton, whose laws of motion you studied in Chapter 6, believed that light is a stream of fast-moving, unimaginably tiny particles, which he called corpuscles. However, his model could not explain all of the properties of light. Experiments showed that light also behaves like a wave. In the ray model of light, light is represented as a ray that travels in a straight path, the direction of which can be changed only by placing an obstruction in the path, as shown in Figure 16-1. The ray model of light was introduced as a way to study how light interacts with matter, regardless of whether light is a particle or a wave. This study of light is called ray optics or geometric optics.

Sources of light Rays of light come from sources of light. Our major source of light is the Sun. Other natural sources of light include flames, sparks, and even fireflies. In the past 100 years, humans have been able to produce several other kinds of light sources. Incandescent bulbs, fluorescent lamps, television screens, lasers, and tiny, light-emitting diodes (LEDs) are each a result of humans using electricity to produce light.

What is the difference between sunlight and moonlight? Sunlight, of course, is much, much brighter. There also is an important fundamental difference between the two. The Sun is a luminous source, an object that emits light. In contrast, the Moon is an illuminated source, an object that becomes visible as a result of the light reflecting off it, as shown in Figure 16-2. An incandescent lamp, such as a common lightbulb, is luminous because electrical energy heats a thin tungsten wire in the bulb and causes it to glow. An incandescent source emits light as a result of its high temperature. A bicycle reflector, on the other hand, works as an illuminated source. It is designed to become highly visible when it is illuminated by luminous automobile headlights.



Figure 16-3 The transparent glass allows objects to be seen through it (a). The translucent lamp shade allows light to pass through, although the lightbulb source itself is not visible (b). The opaque tarp covers the statue, preventing the statue from being seen (c).

Illuminated sources are visible to you because light is reflecting off or transmitting (passing) through the object to your eyes. Media, such as brick, that do not transmit light, but reflect some light, are opaque media. Media that transmit light, such as air and glass, are transparent media. Media that transmit light, but do not permit objects to be seen clearly through them, are translucent media. Lamp shades and frosted lightbulbs are examples of objects that are made of translucent media. All three types of media are illustrated in Figure 16-3. Transparent or translucent media not only transmit light, but they also can reflect a fraction of the light. For example, you often can see your reflection in a glass window.

Quantity of light The rate at which light energy is emitted from a luminous source is called the luminous flux, $P$. The unit of luminous flux is the lumen (lm). A typical 100-W incandescent lightbulb emits approximately 1750 lm . You can think of the luminous flux as a measure of the rate at which light rays come out of a luminous source. Imagine placing a lightbulb at the center of a 1-m-radius sphere, as shown in Figure 16-4. The lightbulb emits light in almost all directions. The 1750 lm of luminous flux characterizes all of the light that strikes the inside surface of the sphere in a given unit of time. Even if the sphere was 2 m in radius, the luminous flux of the lightbulb would be the same as for the 1 -m-radius sphere, because the total number of light rays does not increase.

Once you know the quantity of light being emitted by a luminous source, you can determine the amount of illumination that the luminous source provides to an object, such as a book. The illumination of a surface, or the rate at which light strikes the surface, is called the illuminance, $E$. You can think of this as a measure of the number of light rays that strike a surface. Illuminance is measured in lux, lx , which is equivalent to lumens per square meter, $\mathrm{lm} / \mathrm{m}^{2}$.

Consider the setup shown in Figure 16-4. What is the illuminance of the sphere's inside surface? The equation for the surface area of a sphere is $4 \pi r^{2}$, so the surface area of this sphere is $4 \pi(1.00 \mathrm{~m})^{2}=4 \pi \mathrm{~m}^{2}$. The luminous flux striking each square meter of the sphere is $1750 \mathrm{~lm} /\left(4 \pi \mathrm{~m}^{2}\right)=139 \mathrm{~lx}$. At a distance of 1.00 m from the bulb, 139 lm strikes each square meter. The illuminance of the inside of the sphere is 139 lx .


- Figure 16-5 The illuminance, $E$, produced by a point source of light varies inversely as the square of the distance from the light source.
- Figure 16-6 The illuminance is the same on both sides of the screen, though the lightbulb is brighter than the candle.


An inverse-square relationship What would happen if the sphere surrounding the lamp were larger? If the sphere had a radius of 2.00 m , the luminous flux still would total 1750 lm , but the area of the sphere would be $4 \pi(2.00 \mathrm{~m})^{2}=16.0 \pi \mathrm{~m}^{2}$, four times larger than the $1.00-\mathrm{m}$ sphere, as shown in Figure 16-5. The illuminance of the inside of the $2.00-\mathrm{m}$ sphere is $1750 \mathrm{~lm} /\left(16.0 \pi \mathrm{~m}^{2}\right)=34.8 \mathrm{~lx}$, so 34.8 lm strikes each square meter.

The illuminance on the inside surface of the $2.00-\mathrm{m}$ sphere is one-fourth the illuminance on the inside of the $1.00-\mathrm{m}$ sphere. In the same way, the inside of a sphere with a $3.00-\mathrm{m}$ radius has an illuminance only $(1 / 3)^{2}$, or $1 / 9$, as large as the $1.00-\mathrm{m}$ sphere. Figure $16-5$ shows that the illuminance produced by a point source is proportional to $1 / r^{2}$, an inverse-square relationship. As the light rays spread out in straight lines in all directions from a point source, the number of light rays available to illuminate a unit of area decreases as the square of the distance from the point source.

Luminous intensity Some luminous sources are specified in candela, cd. A candela is not a measure of luminous flux, but of luminous intensity. The luminous intensity of a point source is the luminous flux that falls on $1 \mathrm{~m}^{2}$ of the inside of a $1-\mathrm{m}$-radius sphere. Thus, luminous intensity is luminous flux divided by $4 \pi$. A bulb with 1750 lm of flux has an intensity of $1750 \operatorname{lm} / 4 \pi=139 \mathrm{~cd}$.

In Figure 16-6, the lightbulb is twice as far away from the screen as the candle. For the lightbulb to provide the same illuminance on the lightbulb side of the screen as the candle does on the candle side of the screen, the lightbulb would have to be four times brighter than the candle, and, therefore, the luminous intensity of the lightbulb would have to be four times the luminous intensity of the candle.


## How to Illuminate a Surface

How would you increase the illuminance of your desktop? You could use a brighter bulb, which would increase the luminous flux, or you could move the light source closer to the surface of your desk, thereby decreasing the distance between the light source and the surface it is illuminating. To make the problem easier, you can use the simplification that the light source is a point source. Thus, the illuminance and distance will follow the inverse-square relationship. The problem is further simplified if you assume that light from the source strikes perpendicular to the surface of the desk. Using these simplifications, the illuminance caused by a point light source is represented by the following equation.

> Point Source Illuminance $E=\frac{P}{4 \pi r^{2}}$
> If an object is illuminated by a point source of light, then the illuminance at the object is equal to the luminous flux of the light source, divided by the surface area of the sphere, whose radius is equal to the distance the object is from the light source.

Remember that the luminous flux of the light source is spreading out spherically in all directions, so only a fraction of the luminous flux is available to illuminate the desk. Use of this equation is valid only if the light from the luminous source strikes perpendicular to the surface it is illuminating. It is also only valid for luminous sources that are small enough or far enough away to be considered point sources. Thus, the equation does not give accurate values of illuminance for long, fluorescent lamps or incandescent lightbulbs that are close to the surfaces that they illuminate.

## Connecting Math to Physics

Direct and Inverse Relationships The illuminance provided by a source of light has both a direct and an inverse relationship.

| Math | Physics |
| :--- | :--- |
| $y=\frac{x}{a z^{2}}$ | $E=\frac{P}{4 \pi r^{2}}$ |
| If $z$ is constant, then $y$ is directly <br> proportional to $x$. | If $r$ is constant, then $E$ is directly <br> proportional to $P$. |
| • When $x$ increases, $y$ increases. |  |
| • When $x$ decreases, $y$ decreases. | • When $P$ increases, $E$ increases. $P$ decreases, $E$ decreases. <br> If $x$ is constant, then $y$ is inversely <br> proportional to $z^{2}$. <br> - When $z^{2}$ increases, $y$ decreases. <br> inversely proportional to $r^{2}$. <br> • When $z^{2}$ decreases, $y$ increases. • When $r^{2}$ increases, $E$ decreases. $r^{2}$ decreases, $E$ increases. |

## APPLYING PHYSICS

- Illuminated Minds When deciding how to achieve the correct illuminance on students' desktops, architects must consider the luminous flux of the lights as well as the distance of the lights above the desktops. In addition, the efficiencies of the light sources are an important economic factor.


## EXAMPLE Problem 1

Illumination of a Surface What is the illuminance at on your desktop if it is lighted by a 1750 - Im lamp that is 2.50 m above your desk?
1 Analyze and Sketch the Problem

- Assume that the lightbulb is the point source.
- Diagram the position of the bulb and desktop. Label $P$ and $r$.

$$
\begin{array}{ll}
\text { Known: } & \text { Unknown: } \\
P=1.75 \times 10^{3} \mathrm{Im} & E=? \\
r=2.50 \mathrm{~m} &
\end{array}
$$

## 2 Solve for the Unknown

The surface is perpendicular to the direction in which the light ray is traveling, so you can use the point source illuminance equation.


$$
\begin{aligned}
E & =\frac{P}{4 \pi r^{2}} \\
& =\frac{1.75 \times 10^{3} \mathrm{~m}}{4 \pi(2.50 \mathrm{~m})^{2}} \quad \text { Substitue } P=1.75 \times 10^{3} \mathrm{Im}, r=2.50 \mathrm{~m} \\
& =22.3 \mathrm{~lm} / \mathrm{m}^{2} \\
& =22.3 \mathrm{~lx}
\end{aligned}
$$

Math Handbook
Operations with Significant Digits pages 835-836

3 Evaluate the Answer

- Are the units correct? The units of luminance are $\mathrm{Im} / \mathrm{m}^{2}=\mathrm{lx}$, which the answer agrees with.
- Do the signs make sense? All quantities are positive, as they should be.
- Is the magnitude realistic? The illuminance is less than the luminous flux, which it should be at this distance.


## PRACTICE Problems

## Additional Problems, Appendix B

1. A lamp is moved from 30 cm to 90 cm above the pages of a book. Compare the illumination on the book before and after the lamp is moved.
2. What is the illumination on a surface that is 3.0 m below a $150-\mathrm{W}$ incandescent lamp that emits a luminous flux of 2275 Im?
3. Draw a graph of the illuminance produced by a 150-W incandescent lamp between 0.50 m and 5.0 m .
4. A $64-\mathrm{cd}$ point source of light is 3.0 m above the surface of a desk. What is the illumination on the desk's surface in lux?
5. A public school law requires a minimum illuminance of 160 lx at the surface of each student's desk. An architect's specifications


Figure 16-7 (Not to scale)

Engineers who design lighting systems must understand how the light will be used. If an even illumination is needed to prevent dark areas, the common practice is to evenly space normal lights over the area to be illuminated, as was most likely done with the lights in your classroom. Because such light sources do not produce truly even light, however, engineers also design special light sources that control the spread of the light, such that they produce even illuminations over large surface areas. Much work has been done in this field with automobile headlights.

## The Speed of Light

For light to travel from a source to an object to be illuminated, it must travel across some distance. According to classical mechanics, if you can measure the distance and the time it takes to travel that distance, you can calculate a speed. Before the seventeenth century, most people believed that light traveled instantaneously. Galileo was the first to hypothesize that light has a finite speed, and to suggest a method of measuring its speed using distance and time. His method, however, was not precise enough, and he was forced to conclude that the speed of light is too fast to be measured over a distance of a few kilometers.

Danish astronomer Ole Roemer was the first to determine that light does travel with a measurable speed. Between 1668 and 1674, Roemer made 70 measurements of the 1.8-day orbital period of Io, one of Jupiter's moons. He recorded the times when Io emerged from Jupiter's shadow, as shown in Figure 16-8. He made his measurements as part of a project to improve maps by calculating the longitude of locations on Earth. This is an early example of the needs of technology driving scientific advances.

After making many measurements, Roemer was able to predict when the next eclipse of Io would occur. He compared his predictions with the actual measured times and found that Io's orbital period increased on average by about 13 s per orbit when Earth was moving away from Jupiter and decreased on average by about 13 s per orbit when Earth was approaching Jupiter. Roemer believed that Jupiter's moons were just as regular in their orbits as Earth's moon; thus, he wondered what might cause this discrepancy in the measurement of Io's orbital period.


- Figure 16-8 Roemer measured the time between eclipses as lo emerged from Jupiter's shadow. During successive eclipses, Io's measured orbital period became increasingly smaller or larger depending on whether Earth was moving toward (from position 3 to 1) or away from (from position 1 to 3) Jupiter. (Illustration not to scale)

Measurements of the speed of light Roemer concluded that as Earth moved away from Jupiter, the light from each new appearance of Io took longer to travel to Earth because of the increasing distance to Earth. Likewise, as Earth approached Jupiter, Io's orbital period seemed to decrease. Roemer noted that during the 182.5 days that it took Earth to travel from position 1 to position 3, as shown in Figure 16-8, there were ( 185.2 days)( 1 Io eclipse/ 1.8 days) $=103$ Io eclipses. Thus, for light to travel the diameter of Earth's orbit, he calculated that it takes $(103$ eclipses $)(13 \mathrm{~s} /$ eclipse $)=1.3 \times 10^{3} \mathrm{~s}$, or 22 min .

Using the presently known value of the diameter of Earth's orbit $\left(2.9 \times 10^{11} \mathrm{~m}\right)$, Roemer's value of 22 min gives a value for the speed of light of $2.9 \times 10^{11} \mathrm{~m} /((22 \mathrm{~min})(60 \mathrm{~s} / \mathrm{min}))=2.2 \times 10^{8} \mathrm{~m} / \mathrm{s}$. Today, the speed of light is known to be closer to $3.0 \times 10^{8} \mathrm{~m} / \mathrm{s}$. Thus, light takes 16.5 min , not 22 min, to cross Earth's orbit. Nevertheless, Roemer had successfully proved that light moves at a finite speed.

Although many measurements of the speed of light have been made, the most notable were performed by American physicist Albert A. Michelson. Between 1880 and the 1920s, he developed Earth-based techniques to measure the speed of light. In 1926, Michelson measured the time required for light to make a round-trip between two California mountains 35 km apart. Michelson used a set of rotating mirrors to measure such small time intervals. Michelson's best result was $(2.997996 \pm 0.00004) \times 10^{8} \mathrm{~m} / \mathrm{s}$. For this work, he became the first American to receive a Nobel prize in science.

The speed of light in a vacuum is a very important and universal value; thus it has its own special symbol, $c$. Based on the wave nature of light, which you will study in the next section, the International Committee on Weights and Measurements has measured and defined the speed of light in a vacuum to be $c=299,792,458 \mathrm{~m} / \mathrm{s}$. For many calculations, the value $c=3.00 \times 10^{8} \mathrm{~m} / \mathrm{s}$ is precise enough. At this speed, light travels $9.46 \times 10^{12} \mathrm{~km}$ in a year. This amount of distance is called a light-year.

### 16.1 Section Review

7. Use of Material Light Properties Why might you choose a window shade that is translucent? Opaque?
8. Illuminance Does one lightbulb provide more illuminance than two identical lightbulbs at twice the distance? Explain.
9. Luminous Intensity Two lamps illuminate a screen equally-lamp A at 5.0 m , lamp B at 3.0 m . If lamp A is rated 75 cd , what is lamp B rated?
10. Distance of a Light Source Suppose that a lightbulb illuminating your desk provides only half the illuminance that it should. If it is currently 1.0 m away, how far should it be to provide the correct illuminance?
11. Distance of Light Travel How far does light travel in the time it takes sound to travel 1 cm in air at $20^{\circ} \mathrm{C}$ ?
12. Distance of Light Travel The distance to the Moon can be found with the help of mirrors left on the Moon by astronauts. A pulse of light is sent to the Moon and returns to Earth in 2.562 s. Using the measured value of the speed of light to the same precision, calculate the distance from Earth to the Moon.
13. Critical Thinking Use the correct time taken for light to cross Earth's orbit, 16.5 min , and the diameter of Earth's orbit, $2.98 \times 10^{11} \mathrm{~m}$, to calculate the speed of light using Roemer's method. Does this method appear to be accurate? Why or why not?

### 16.2 The Wave Nature of Light

You probably have heard that light is composed of waves, but what evidence do you have that this is so? Suppose that you walk by the open door of the band-rehearsal room at school. You hear the music as you walk toward the rehearsal-room door long before you can see the band members through the door. Sound seems to have reached you by bending around the edge of the door, whereas the light that enables you to see the band members has traveled only in a straight line. If light is composed of waves, why doesn't light seem to act in the same way as sound does? In fact, light does act in the same way; however, the effect is much less obvious with light than with sound.

## Diffraction and the Wave Model of Light

In 1665, Italian scientist Francesco Maria Grimaldi observed that the edges of shadows are not perfectly sharp. He introduced a narrow beam of light into a dark room and held a rod in front of the light such that it cast a shadow on a white surface. The shadow cast by the rod on the white surface was wider than the shadow should have been if light traveled in a straight line past the edges of the rod. Grimaldi also noted that the shadow was bordered by colored bands. Grimaldi recognized this phenomenon as diffraction, which is the bending of light around a barrier.

In 1678, Dutch scientist Christiaan Huygens argued in favor of a wave model to explain diffraction. According to Huygens' principle, all the points of a wave front of light can be thought of as new sources of smaller waves. These wavelets expand in every direction and are in step with one another. A flat, or plane, wave front of light consists of an infinite number of point sources in a line. As this wave front passes by an edge, the edge cuts the wave front such that each circular wavelet generated by each Huygens' point will propagate as a circular wave in the region where the original wave front was bent, as shown in Figure 16-9. This is diffraction.

- Figure 16-9 According to Huygens' principle, the crest of each wave can be thought of as a series of point sources. Each point source creates a circular wavelet. All the wavelets combine to make a flat wave front, except at the edge where circular wavelets of the Huygens' points move away from the wave front.

- Objectives
- Describe how diffraction demonstrates that light is a wave.
- Predict the effect of combining colors of light and mixing pigments.
- Explain phenomena such as polarization and the Doppler effect.
- Vocabulary
diffraction primary color secondary color complementary color primary pigment secondary pigment polarization Malus's law


Figure 16-10 White light, when passed through a prism, is separated into a spectrum of colors.

- Figure 16-12 Different combinations of blue, green, and red light can produce yellow, cyan, magenta, or white light.



## Color

In 1666, possibly prompted by Grimaldi's publication of his diffraction results, Newton performed experiments on the colors produced when a narrow beam of sunlight passed through a glass prism, as shown in Figure 16-10. Newton called the ordered arrangement of colors a spectrum. Using his corpuscle model of light, he believed that particles of light were interacting with some unevenness in the glass to produce the spectrum.

To test this assumption, Newton allowed the spectrum from one prism to fall on a second prism. If the spectrum was caused by irregularities in the glass, he reasoned that the second prism would increase the spread in colors. Instead, the second prism reversed the spreading of colors and recombined them to form white light. After more experiments, Newton concluded that white light is composed of colors, and that a property of the glass other than unevenness caused the light to separate into colors.

Based on the work of Grimaldi, Huygens, and others, we know that light has wave properties and that each color of light is associated with a wavelength. Light falls within the range of wavelengths from about 400 nm $\left(4.00 \times 10^{-7} \mathrm{~m}\right)$ to $700 \mathrm{~nm}\left(7.00 \times 10^{-7} \mathrm{~m}\right)$, as shown in Figure 16-11. The longest wavelengths are seen as red light. As wavelength decreases, the color changes to orange, yellow, green, blue, indigo, and finally, violet.


Figure 16-11 The spectrum of light ranges from the long, red wavelength to the short, violet wavelength.

As white light crosses the boundary from air into glass and back into air in Figure 16-11, its wave nature causes each different color of light to be bent, or refracted, at a different angle. This unequal bending of the different colors causes the white light to be spread into a spectrum. This reveals that different wavelengths of light interact in different but predictable ways with matter.

Color by addition of light White light can be formed from colored light in a variety of ways. For example, when the correct intensities of red, green, and blue light are projected onto a white screen, as in Figure 16-12, the region where these three colors overlap on the screen will appear to be white. Thus, red, green, and blue light form white light when they are combined. This is called the additive color process, which is used in color-television tubes. A color-television tube contains tiny, dotlike sources of red, green, and blue light. When all three colors of light have the correct intensities, the screen appears to be white. For this reason, red, green, and blue are each called a primary color. The primary colors can be mixed in pairs to form three additional colors, as shown in Figure 16-12. Red and green light together produce yellow light, blue and green light produce cyan, and red and blue light produce magenta. The colors yellow, cyan, and magenta are each called a secondary color, because each is a combination of two primary colors.

As shown in Figure 16-12, yellow light can be made from red light and green light. If yellow light and blue light are projected onto a white screen with the correct intensities, the surface will appear to be white. Complementary colors are two colors of light that can be combined to make white light. Thus, yellow is a complementary color of blue, and vice versa, because the two colors of light combine to make white light. In the same way, cyan and red are complementary colors, as are magenta and green. Yellowish laundry can be whitened with a bluing agent added to detergent.

Color by subtraction of light As you learned in the first section of this chapter, objects can reflect and transmit light. They also can absorb light. Not only does the color of an object depend on the wavelengths present in the light that illuminates the object, but an object's color also depends on what wavelengths are absorbed by the object and what wavelengths are reflected. The natural existence or artificial placement of dyes in the material of an object, or pigments on its surface, give the object color.

A dye is a molecule that absorbs certain wavelengths of light and transmits or reflects others. When light is absorbed, its energy is taken into the object that it strikes and is turned into other forms of energy. A red shirt is red because the dyes in it reflect red light to our eyes. When white light falls on the red object shown in Figure 16-13, the dye molecules in the object absorb the blue and green light and reflect the red light. When only blue light falls on the red object, very little light is reflected and the object appears to be almost black.

The difference between a dye and a pigment is that pigments usually are made of crushed minerals, rather than plant or insect extracts. Pigment particles can be seen with a microscope. A pigment that absorbs only one primary color and reflects two from white light is called a primary pigment. Yellow pigment absorbs blue light and reflects red and green light. Yellow, cyan, and magenta are the colors of primary pigments. A pigment that absorbs two primary colors and reflects one color is called a secondary pigment. The colors of secondary pigments are red (which absorbs green and blue light), green (which absorbs red and blue light), and blue (which absorbs red and green light). Note that the primary pigment colors are the secondary colors. In the same way, the secondary pigment colors are the primary colors.

## -MINI LAB

## Color by Temperature Fir

Some artists refer to red and orange as hot colors and green and blue as cool colors. Do colors really relate to temperature in this way?

1. Obtain a glass prism from your teacher.
2. Obtain a lamp with a dimmer switch from your teacher. Turn on the lamp and turn off the room light. Set the dimmer to minimum brightness of the lamp.
3. Slowly increase the brightness of the lamp. CAUTION: Lamp can get hot and burn skin.
4. Observe the color of light produced by the prism and how it relates to the warmth of the lightbulb on your hand.

## Analyze and Conclude

5. What colors appeared first when the light was dim?
6. What colors were the last to appear as you brightened the light?
7. How do these colors relate to the temperature of the filament?

- Figure 16-13 The dyes in the dice selectively absorb and reflect various wavelengths of light. The dice are illuminated by white light (a), red light (b), and blue light (c).


Figure 16-14 The primary pigments are magenta, cyan, and yellow. Mixing these in pairs produces the secondary pigments: red, green, and blue.

## Bology Connection

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The primary and secondary pigments are shown in Figure 16-14. When the primary pigments yellow and cyan are mixed, the yellow absorbs blue light and the cyan absorbs red light. Thus, Figure 16-14 shows yellow and cyan combining to make green pigment. When yellow pigment is mixed with the secondary pigment, blue, which absorbs green and red light, all of the primary colors are absorbed, so the result is black. Thus, yellow and blue are complementary pigments. Cyan and red, as well as magenta and green, are also complementary pigments.

A color printer uses yellow, magenta, and cyan dots of pigment to make a color image on paper. Often, pigments that are used are finely ground compounds, such as titanium(IV) oxide (white), chromium(III) oxide (green), and cadmium sulfide (yellow). Pigments mix to form suspensions rather than solutions. Their chemical form is not changed in a mixture, so they still absorb and reflect the same wavelengths.

Results in color You can now begin to understand the colors that you see in the photo at the beginning of this chapter. The plants on the hillside look green because of the chlorophyll in them. One type of chlorophyll absorbs red light and the other absorbs blue light, but they both reflect green light. The energy in the red and blue light that is absorbed is used by the plants for photosynthesis, which is the process by which green plants make food.

In the same photo, the sky is bluish. Violet and blue light are scattered (repeatedly reflected) much more by molecules in the air than are other wavelengths of light. Green and red light are not scattered much by the air, which is why the Sun looks yellow or orange, as shown in Figure 16-15. However, violet and blue light from the Sun are scattered in all directions, illuminating the sky in a bluish hue.



## Polarization of Light

Have you ever looked at light reflected off a road through polarizing sunglasses? As you rotate the glasses, the road first appears to be dark, then light, and then dark again. Light from a lamp, however, changes very little as the glasses are rotated. Why is there a difference? Normal lamplight is not polarized. However, the light that is coming from the road is reflected and has become polarized. Polarization is the production of light in a single plane of oscillation.

Polarization by filtering Polarization can be understood by considering a rope model of light waves, as shown in Figure 16-16. The transverse mechanical waves in the rope represent transverse light waves. The slot represents what is referred to as the polarizing axis of a polarizing medium. When the rope waves are parallel to the slot, they pass through. When they are perpendicular to the slot, the waves are blocked. Polarizing media contain long molecules in which electrons can oscillate, or move back and forth, all in the same direction. As light travels past the molecules, the electrons can absorb light waves that oscillate in the same direction as the electrons. This process allows light waves vibrating in one direction to pass through, while the waves vibrating in the other direction are absorbed. The direction of a polarizing medium perpendicular to the long molecules is called the polarizing axis. Only waves oscillating parallel to that axis can pass through.

Ordinary light actually contains waves vibrating in every direction perpendicular to its direction of travel. If a polarizing medium is placed in a beam of ordinary light, only the components of the waves in the same direction as the polarizing axis can pass through. On average, half of the total light amplitude passes through, thereby reducing the intensity of the light by half. The polarizing medium produces light that is polarized. Such a medium is called a polarizing filter.
Polarization by reflection When you look through a polarizing filter at the light reflected by a sheet of glass and rotate the filter, you will see the light brighten and dim. The light is partially polarized along the plane of the glass when it is reflected. That is, the reflected ray contains a great deal of light vibrating parallel to the surface of the glass. The polarization of light reflected by roads is the reason why polarizing sunglasses reduce glare. The fact that the intensity of light reflected off a road varies as polarizing sunglasses are rotated suggests that the reflected light is partially polarized. Photographers can use polarizing filters over camera lenses to block reflected light, as shown in Figure 16-17.

■ Figure 16-16 In this rope model of light, light is a single wave oriented in relation to the vertical plane and thus passes through a vertical polarizer (a). It cannot pass through a horizontal polarizer (b).


- Figure 16-17 This photo of a music store, taken without a polarizing filter, contains the glare of light off of the surface of the window (a). This photo of the same scene was taken with a polarizing filter (b).
- Figure 16-18 When two polarizing filters are arranged with their polarizing axes in parallel, a maximum amount of light passes through (a). When two polarizing filters are arranged with perpendicular axes, no light passes through (b).


Polarization analysis Suppose that you produce polarized light with a polarizing filter. What would happen if you place a second polarizing filter in the path of the polarized light? If the polarizing axis of the second filter is parallel to that of the first, the light will pass through, as shown in Figure 16-18a. If the polarizing axis of the second filter is perpendicular to that of the first, no light will pass through, as shown in Figure 16-18b.

The law that explains the reduction of light intensity as it passes through a second polarizing filter is called Malus's law. If the light intensity after the first polarizing filter is $I_{1}$, then a second polarizing filter, with its polarizing axis at an angle, $\theta$, relative to the polarizing axis of the first, will result in a light intensity, $I_{2}$, that is equal to or less than $I_{1}$.

## Malus's Law $I_{2}=I_{1} \cos ^{2} \theta$

The intensity of light coming out of a second polarizing filter is equal to the intensity of polarized light coming out of a first polarizing filter multiplied by the cosine, squared, of the angle between the polarizing axes of the two filters.

Using Malus's law, you can compare the light intensity coming out of the second polarizing filter to the light intensity coming out of the first polarizing filter, and thereby determine the orientation of the polarizing axis of the first filter relative to the second filter. A polarizing filter that uses Malus's law to accomplish this is called an analyzer. Analyzers can be used to determine the polarization of light coming from any source.

## CHALLENGE PROBLEM

You place an analyzer filter between the two cross-polarized filters, such that its polarizing axis is not parallel to either of the two filters, as shown in the figure to the right.

1. You observe that some light passes through filter 2, though no light passed through filter 2 previous to inserting the analyzer filter. Why does this happen?
2. The analyzer filter is placed at an angle of $\theta$ relative to the polarizing axes of filter 1. Derive an equation for the intensity of light coming out of filter 2 compared to the intensity of light coming out of filter 1 .


## The Speed of a Light Wave

As you learned in Chapter 14, the wavelength, $\lambda$, of a wave is a function of its speed in the medium in which it is traveling and its constant frequency, $f$. Because light has wave properties, the same mathematical models used to describe waves in general can be used to describe light. For light of a given frequency traveling through a vacuum, wavelength is a function of the speed of light, $c$, which can be written as $\lambda_{0}=c / f$. The development of the laser in the 1960s provided new methods of measuring the speed of light. The frequency of light can be counted with extreme precision using lasers and the time standard provided by atomic clocks. Measurements of wavelengths of light, however, are much less precise.

Different colors of light have different frequencies and wavelengths, but in a vacuum, they all travel at $c$. Because all wavelengths of light travel at the same speed in a vacuum, when you know the frequency of a light wave in a vacuum, you can calculate its wavelength, and vice versa. Thus, using precise measurements of light frequency and light speed, you can calculate a precise value of light wavelength.

Relative motion and light What happens if a source of light is traveling toward you or you are moving toward the light source? You learned in Chapter 15 that the frequency of a sound heard by the listener changes if either the source or the listener of the sound is moving. The same is true for light. However, when you consider the velocities of a source of sound and the observer, you are really considering each one's velocity relative to the medium through which the sound travels.

Because light waves are not vibrations of the particles of a mechanical medium, unlike sound waves, the Doppler effect of light can involve only the velocities of the source and the observer relative to each other. The magnitude of the difference between the velocities of the source and observer is called the relative speed. Remember that the only factors in the Doppler effect are the velocity components along the axis between the source and observer, as shown in Figure 16-19.

- Figure 16-19 The observer and the light source have different velocities (a). The magnitude of the vector subtraction of the velocity components along the axis between the source of light and the observer of the light is referred to as the relative speed along the axis between the source and observer, $v$ (b). (Illustration not to scale)


The Doppler effect To study the Doppler effect for light, the problem can be simplified by considering axial relative speeds that are much less than the speed of light $(v \ll c)$. This simplification is used to develop the equation for the obseved light frequency, $f_{\text {obs }}$, which is the frequency of light as seen by an observer.

## Observed Light Frequency $f_{\text {obs }}=f\left(1 \pm \frac{v}{c}\right)$

The observed frequency of light from a source is equal to the actual frequency of the light generated by the source, times the quantity 1 plus the relative speed along the axis between the source and the observer if they are moving toward each other, or 1 minus the relative speed if they are moving away from each other.

Because most observations of the Doppler effect for light have been made in the context of astronomy, the equation for the Doppler effect for light generally is written in terms of wavelength rather than frequency. Using the relationship $\lambda=c / f$ and the $v \ll c$ simplification, the following equation can be used for the Doppler shift, $\Delta \lambda$, which is the difference between the observed wavelength of light and the actual wavelength.

$$
\text { Doppler Shift } \quad\left(\lambda_{\text {obs }}-\lambda\right)=\Delta \lambda= \pm \frac{v}{c} \lambda
$$

The difference between the observed wavelength of light and the actual wavelength of light generated by a source is equal to the actual wavelength of light generated by the source, times the relative speed of the source and observer, divided by the speed of light. This quantity is positive if they are moving away from each other or negative if they are moving toward each other.

A positive change in wavelength means that the light is red-shifted. This occurs when the relative velocity of the source is in a direction away from the observer. A negative change in wavelength means that the light is blueshifted. This occurs when the relative velocity of the source is in a direction toward the observer. When the wavelength is red-shifted (lengthens), the observed frequency is lower as a result of the inverse relationship between the two variables, because the speed of light remains constant. When the wavelength is blue-shifted, the observed frequency is higher.

Researchers can determine how astronomical objects, such as galaxies, are moving relative to Earth by observing the Doppler shift of light. This is done by observing the spectrum of light coming from stars in the galaxy using a spectrometer, as shown in Figure 16-20. Elements that are present in the stars of galaxies emit specific wavelengths in the lab. A spectrometer is able to measure the Doppler shift of these wavelengths.

- Figure 16-20 Three emission lines of hydrogen are visibly redshifted in the spectrum of quasar 3C 273, as indicated by the taglines outside the spectra. Their wavelengths are shifted approximately $16 \%$ of what they are in a laboratory setting.都



## PRACTICE Problems

14. What is the frequency of oxygen's spectral line if its wavelength is 513 nm ?
15. A hydrogen atom in a galaxy moving with a speed of $6.55 \times 10^{6} \mathrm{~m} / \mathrm{s}$ away from Earth emits light with a frequency of $6.16 \times 10^{14} \mathrm{~Hz}$. What frequency of light from that hydrogen atom would be observed by an astronomer on Earth?
16. A hydrogen atom in a galaxy moving with a speed of $6.55 \times 10^{16} \mathrm{~m} / \mathrm{s}$ away from Earth emits light with a wavelength of $4.86 \times 10^{-7} \mathrm{~m}$. What wavelength would be observed on Earth from that hydrogen atom?
17. An astronomer is looking at the spectrum of a galaxy and finds that it has an oxygen spectral line of 525 nm , while the laboratory value is measured at 513 nm . Calculate how fast the galaxy would be moving relative to Earth. Explain whether the galaxy is moving toward or away from Earth and how you know.

In 1929, Edwin Hubble suggested that the universe is expanding. Hubble reached his conclusion of the expanding universe by analyzing. emission spectra from many galaxies. Hubble noticed that the spectral lines of familiar elements were at longer wavelengths than expected. The lines were shifted toward the red end of the spectrum. No matter what area of the skies he observed, the galaxies were sending red-shifted light to Earth. What do you think caused the spectral lines to be red-shifted? Hubble concluded that all galaxies are moving away from Earth.

You have learned that some characteristics of light can be explained with a simple ray model of light, whereas others require a wave model of light. In Chapters 17 and 18, you will use both of these models to study how light interacts with mirrors and lenses. In Chapter 19, you will learn about other aspects of light that can be understood only through the use of the wave model of light. is slower in air and water than in a vacuum. The frequency, however, does not change when red light enters water. Does the wavelength change? If so, how?
22. Polarization Describe a simple experiment that you could do to determine whether sunglasses in a store are polarizing.
23. Critical Thinking Astronomers have determined that Andromeda, a neighboring galaxy to our own galaxy, the Milky Way, is moving toward the Milky Way. Explain how they determined this. Can you think of a possible reason why Andromeda is moving toward our galaxy?

## Polarization of Light

A light source that produces transverse light waves that are all in the same fixed

Alternate CBL instructions can be found on the Web site.
physicspp.com plane is said to be polarized in that plane. A polarizing filter can be used to find light sources that produce polarized light. Some media can rotate the plane of polarization of light as it transmits the light. Such media are said to be optically active. In this activity, you will investigate these concepts of polarized light.

## QUESTION

What types of luminous and illuminated light sources produce polarized light?

## Objectives

■ Experiment with various sources of light and polarizing filters.

- Describe the results of your experiment.

■ Recognize possible uses of polarizing filters in everyday life.

## Safety Precautions



Minimize the length of time you look directly at bright light sources.
Do not do this lab with laser light sources.
Do not look at the Sun, even if you are using polarizing filters.
Light sources can get hot and burn skin.

## Materials

two polarizing filter sheets incandescent light source fluorescent light source pieces of white and black paper calculator with a liquid crystal display clear, plastic protractor mirror

## Procedure

1. Take a polarizing filter and look at an incandescent light source. Rotate the filter. Write your observations in the data table.
2. Use a polarizing filter to look at a fluorescent light source. Rotate the filter. Write your observations in the data table.
3. Use a polarizing filter to observe light reflected off the surface of a mirror at approximately a $45^{\circ}$ angle. Rotate the filter. Record your observations in the data table.


## Data Table

| Light Source | Observations |
| :--- | :--- |
| 1 |  |
| 2 |  |
| 3 |  |
| 4 |  |
| 5 |  |
| 6 |  |
| 7 |  |
| 8 |  |

4. Use a polarizing filter to observe light reflected off a white piece of paper at approximately a $45^{\circ}$ angle. Rotate the filter. Record your observations in the data table.
5. Use a polarizing filter to observe light reflected off a piece of black paper at approximately a $45^{\circ}$ angle. Rotate the filter. Record your observations in the data table.
6. Use a polarizing filter to observe a liquid crystal display on a calculator. Rotate the filter. Write your observations in the data table.
7. Place one polarizing filter on top of the other filter. Look at an incandescent light source through this set of the filters. Rotate one of the filters with respect to the other. Make a complete rotation. Record your observations in the data table.
8. Place a clear, plastic protractor between the two polarizing filters. Look at an incandescent light source with this. Do a complete rotation of one of the filters. Position the two filters the same way that produced no light in step 7. Record your observations in the data table.

## Analyze

1. Interpret Data Does incandescent light produce polarized light? How do you know?
2. Interpret Data Does fluorescent light produce polarized light? How do you know?
3. Interpret Data Does reflection from a mirrored surface produce polarized light? How do you know?
4. Compare and Contrast How does reflected light from white paper compare to reflected light from black paper in terms of polarized light? Why are they different?
5. Interpret Data Is the light from liquid crystal displays polarized? How do you know?

## Conclude and Apply

1. Analyze and Conclude How can two polarizing filters be used to prevent any light from passing through them?
2. Analyze and Conclude Why can the clear, plastic protractor between the polarizing filters be seen even though nothing else can be seen through the polarizing filters?
3. Draw Conclusions In general, what types of situations produce polarized light?

## Going Further

1. On a sunny day, look at the polarization of blue sky near and far from the Sun using a polarizing filter. CAUTION: Do not look directly at the Sun. What characteristics of polarized light do you observe?
2. Is reflected light from clouds polarized? Make an observation to confirm your answer.

## Real-World Physics

1. Why are high quality sunglasses made with polarizing lenses?
2. Why are polarizing sunglasses a better option than tinted sunglasses when driving a car?

## Physics nline

To find out more about light, visit the Web site: physicspp.com

## Advances In Lighting

History has recorded the use of oil, candles, and gas to provide illumination in the dark hours of the night. However, there has always been inherent danger with the use of open flames to provide light. The invention of electric lighting in the nineteenth century provided brighter light and improved safety to the public.

The original form of electric light, which is still in common use, is the incandescent bulb. A tungsten filament is heated by electricity until it glows white. The tungsten does not burn, but it vaporizes, which eventually breaks the filament. Because the light is a result of heating the tungsten, this is not very efficient. Recent pursuits in electric lighting have produced longer-lasting, lower-heat light sources.

## Quartz-Halogen

Lamps To prevent a filament from breaking, the bulb can be made very small and filled with bromine or iodine gas. Tungsten ions from the filament combine with the gas molecules in the cooler parts of the lamp to make a compound, which circulates through the lamp and recombines with the filament. The light is very white and bright, but it also is very hot. An ordinary glass bulb would melt, so fused quartz, which has a very high melting point, is used.

Gas-Discharge Lamps This type of lamp is made of a glass tube with a wire electrode sealed into each end. All of the air is extracted and replaced by a very small amount of a specially chosen gas. A high voltage is applied across the electrodes. The electricity ionizes, or strips, some electrons from the gas atoms. An ionized gas is a good conductor, so electric current flows through it, causing the gas to glow brightly.


Clockwise from the upper left, the photos show LEDs, a fluorescent light, a halogen light, and gas-discharge lamps in the form of neon lights.

Fluorescent Lamps The glow produced by mercury vapor is almost invisible because most of its spectrum is in the ultraviolet region, which is not visible. A fluorescent lamp is made by painting the inside of a mercury-discharge lamp with phosphor, a chemical that glows brightly when ultraviolet light strikes it. Fluorescent lights can be made in any color by changing the mixture of red, green, and blue phosphors. They have a long life and are economical to use, because they produce little heat and a great deal of light.

## Light-Emitting Diodes

 The light of the future may be the white lightemitting diode, or LED. The LED produces white light by illuminating a tiny phosphor screen inside the LED with blue light. LEDs are bright enough to read by and produce almost no heat as they operate. They are so efficient that a car battery could power the lamps in a home for days without being recharged.
## Going Further

1. Observe Novelty stores sell many devices that use lights. Examine some of them to see what types of lamp technology are used.
2. Research Find out about the inner construction, characteristic color, and typical uses of a few types of gasdischarge lamps.

### 16.1 Illumination

## Vocabulary

- ray model of light (p. 432)
- luminous source (p. 432)
- illuminated source (p. 432)
- opaque (p. 433)
- transparent (p. 433)
- translucent (p. 433)
- luminous flux (p. 433)
- illuminance (p. 433)


## Key Concepts

- Light travels in a straight line through any uniform medium.
- Materials can be characterized as being transparent, translucent, or opaque, depending on the amount of light that they reflect, transmit, or absorb.
- The luminous flux of a light source is the rate at which light is emitted. It is measured in lumens (lm).
- Illuminance is the luminous flux per unit area. It is measured in lux (lx), or lumens per square meter $\left(\mathrm{lm} / \mathrm{m}^{2}\right)$.
- For a point source, illuminance follows an inverse-square relationship with distance and a direct relationship with luminous flux.

$$
E=\frac{P}{4 \pi r^{2}}
$$

- In a vacuum, light has a constant speed of $c=3.00 \times 10^{8} \mathrm{~m} / \mathrm{s}$.


### 16.2 The Wave Nature of Light

## Vocabulary

- diffraction (p. 439)
- primary color (p. 440)
- secondary color (p. 440)
- complementary colors (p. 441)
- primary pigment (p. 441)
- secondary pigment (p. 441)
- polarization (p. 443)
- Malus's law (p. 444)


## Key Concepts

- Light can have wavelengths between 400 and 700 nm .
- White light is a combination of the spectrum of colors, each color having a different wavelength.
- Combining the primary colors, red, blue, and green, forms white light. Combinations of two primary colors form the secondary colors, yellow, cyan, and magenta.
- The primary pigments, cyan, magenta, and yellow, are used in combinations of two to produce the secondary pigments, red, blue, and green.
- Polarized light consists of waves oscillating in the same plane.
- When two polarizing filters are used to polarize light, the intensity of the light coming out of the last filter is dependent on the angle between the polarizing axes of the two filters.

$$
I_{2}=I_{1} \cos ^{2} \theta
$$

- Light waves traveling through a vacuum can be characterized in terms of frequency, wavelength, and the speed of light.

$$
\lambda_{0}=\frac{c}{f}
$$

- Light waves are Doppler shifted based upon the relative speed along the axis of the observer and the source of light.

$$
\begin{gathered}
f_{\mathrm{obs}}=f\left(1 \pm \frac{v}{c}\right) \\
\Delta \lambda=\left(\lambda_{\mathrm{obs}}-\lambda\right)= \pm \frac{v}{c} \lambda
\end{gathered}
$$

## Concept Mapping

24. Complete the following concept map using the following terms: wave, $c$, Doppler effect, polarization.


## Mastering Concepts

25. Sound does not travel through a vacuum. How do we know that light does? (16.1)
26. Distinguish between a luminous source and an illuminated source. (16.1)
27. Look carefully at an ordinary, frosted, incandescent bulb. Is it a luminous or an illuminated source? (16.1)
28. Explain how you can see ordinary, nonluminous classroom objects. (16.1)
29. Distinguish among transparent, translucent, and opaque objects. (16.1)
30. To what is the illumination of a surface by a light source directly proportional? To what is it inversely proportional? (16.1)
31. What did Galileo assume about the speed of light? (16.1)
32. Why is the diffraction of sound waves more familiar in everyday experience than is the diffraction of light waves? (16.2)
33. What color of light has the shortest wavelength? (16.2)
34. What is the range of the wavelengths of light, from shortest to longest? (16.2)
35. Of what colors does white light consist? (16.2)
36. Why does an object appear to be black? (16.2)
37. Can longitudinal waves be polarized? Explain. (16.2)
38. If a distant galaxy were to emit a spectral line in the green region of the light spectrum, would the observed wavelength on Earth shift toward red light or toward blue light? Explain. (16.2)
39. What happens to the wavelength of light as the frequency increases? (16.2)

## Applying Concepts

40. A point source of light is 2.0 m from screen A and 4.0 m from screen B, as shown in Figure 16-21. How does the illuminance at screen B compare with the illuminance at screen A ?


Figure 16-21
41. Reading Lamp You have a small reading lamp that is 35 cm from the pages of a book. You decide to double the distance.
a. Is the illuminance at the book the same?
b. If not, how much more or less is it?
42. Why are the insides of binoculars and cameras painted black?
43. Eye Sensitivity The eye is most sensitive to yellowgreen light. Its sensitivity to red and blue light is less than 10 percent as great. Based on this knowledge, what color would you recommend that fire trucks and ambulances be painted? Why?
44. Streetlight Color Some very efficient streetlights contain sodium vapor under high pressure. They produce light that is mainly yellow with some red. Should a community that has these lights buy dark blue police cars? Why or why not?

Refer to Figure 16-22 for problems 45 and 46.


Figure 16-22
45. What happens to the illuminance at a book as the lamp is moved farther away from the book?
46. What happens to the luminous intensity of the lamp as it is moved farther away from the book?
47. Polarized Pictures Photographers often put polarizing filters over camera lenses to make clouds in the sky more visible. The clouds remain white, while the sky looks darker. Explain this based on your knowledge of polarized light.
48. An apple is red because it reflects red light and absorbs blue and green light.
a. Why does red cellophane look red in reflected light?
b. Why does red cellophane make a white lightbulb look red when you hold the cellophane between your eye and the lightbulb?
c. What happens to the blue and green light?
49. You put a piece of red cellophane over one flashlight and a piece of green cellophane over another. You shine the light beams on a white wall. What color will you see where the two flashlight beams overlap?
50. You now put both the red and green cellophane pieces over one of the flashlights in Problem 49. If you shine the flashlight beam on a white wall, what color will you see? Explain.
51. If you have yellow, cyan, and magenta pigments, how can you make a blue pigment? Explain.
52. Traffic Violation Suppose that you are a traffic officer and you stop a driver for going through a red light. Further suppose that the driver draws a picture for you (Figure 16-23) and explains that the light looked green because of the Doppler effect when he went through it. Explain to him using the Doppler shift equation just how fast he would have had to be going for the red light $(\lambda=645 \mathrm{~nm})$, to appear green $(\lambda=545 \mathrm{~nm})$. Hint: For the purpose of this problem, assume that the Doppler shift equation is valid at this speed.


Figure 16-23

## Mastering Problems

### 16.1 Illumination

53. Find the illumination 4.0 m below a 405-lm lamp.
54. Light takes 1.28 s to travel from the Moon to Earth. What is the distance between them?
55. A three-way bulb uses 50, 100, or 150 W of electrical power to deliver 665, 1620, or 2285 lm in its three settings. The bulb is placed 80 cm above a sheet of paper. If an illumination of at least 175 lx is needed on the paper, what is the minimum setting that should be used?
56. Earth's Speed Ole Roemer found that the average increased delay in the disappearance of Io from one orbit around Jupiter to the next is 13 s .
a. How far does light travel in 13 s ?
b. Each orbit of Io takes 42.5 h . Earth travels the distance calculated in part a in 42.5 h . Find the speed of Earth in km/s.
c. Check to make sure that your answer for part b is reasonable. Calculate Earth's speed in orbit using the orbital radius, $1.5 \times 10^{8} \mathrm{~km}$, and the period, 1.0 yr .
57. A student wants to compare the luminous flux of a lightbulb with that of a $1750-\mathrm{lm}$ lamp. The lightbulb and the lamp illuminate a sheet of paper equally. The $1750-\mathrm{lm}$ lamp is 1.25 m away from the sheet of paper; the lightbulb is 1.08 m away. What is the lightbulb's luminous flux?
58. Suppose that you wanted to measure the speed of light by putting a mirror on a distant mountain, setting off a camera flash, and measuring the time it takes the flash to reflect off the mirror and return to you, as shown in Figure 16-24. Without instruments, a person can detect a time interval of about 0.10 s . How many kilometers away would the mirror have to be? Compare this distance with that of some known distances.


Figure 16-24

### 16.2 The Wave Nature of Light

59. Convert 700 nm , the wavelength of red light, to meters.
60. Galactic Motion How fast is a galaxy moving relative to Earth if a hydrogen spectral line of 486 nm is red-shifted to 491 nm ?

## Chapter 16 Assessment

61. Suppose that you are facing due east at sunrise. Sunlight is reflected off the surface of a lake, as shown in Figure 16-25. Is the reflected light polarized? If so, in what direction?


- Figure 16-25

62. Polarizing Sunglasses In which direction should the transmission axis of polarizing sunglasses be oriented to cut the glare from the surface of a road: vertically or horizontally? Explain.
63. Galactic Motion A hydrogen spectral line that is known to be 434 nm is red-shifted by 6.50 percent in light coming from a distant galaxy. How fast is the galaxy moving away from Earth?
64. For any spectral line, what would be an unrealistic value of the apparent wavelength for a galaxy moving away from Earth. Why?

## Mixed Review

65. Streetlight Illumination A streetlight contains two identical bulbs that are 3.3 m above the ground. If the community wants to save electrical energy by removing one bulb, how far from the ground should the streetlight be positioned to have the same illumination on the ground under the lamp?
66. An octave in music is a doubling of frequency. Compare the number of octaves that correspond to the human hearing range to the number of octaves in the human vision range.
67. A 10.0-cd point-source lamp and a 60.0-cd pointsource lamp cast equal intensities on a wall. If the $10.0-\mathrm{cd}$ lamp is 6.0 m from the wall, how far from the wall is the 60.0-cd lamp?
68. Thunder and Lightning Explain why it takes 5 s to hear thunder when lightning is 1.6 km away.
69. Solar Rotation Because the Sun rotates on its axis, one edge of the Sun moves toward Earth and the other moves away. The Sun rotates approximately once every 25 days, and the diameter of the Sun is $1.4 \times 10^{9} \mathrm{~m}$. Hydrogen on the Sun emits light of frequency $6.16 \times 10^{14} \mathrm{~Hz}$ from the two sides of the Sun. What changes in wavelength are observed?

## Thinking Critically

70. Research Why did Galileo's method for measuring the speed of light not work?
71. Make and Use Graphs A 110-cd light source is 1.0 m from a screen. Determine the illumination on the screen originally and for every meter of increasing distance up to 7.0 m . Graph the data.
a. What is the shape of the graph?
b. What is the relationship between illuminance and distance shown by the graph?
72. Analyze and Conclude If you were to drive at sunset in a city filled with buildings that have glass-covered walls, the setting Sun reflected off the building's walls might temporarily blind you. Would polarizing glasses solve this problem?

## Writing in Physics

73. Write an essay describing the history of human understanding of the speed of light. Include significant individuals and the contribution that each individual made.
74. Look up information on the SI unit candela, $c d$, and explain in your own words the standard that is used to set the value of 1 cd .

## Cumulative Review

75. A $2.0-\mathrm{kg}$ object is attached to a $1.5-\mathrm{m}$ long string and swung in a vertical circle at a constant speed of $12 \mathrm{~m} / \mathrm{s}$. (Chapter 7)
a. What is the tension in the string when the object is at the bottom of its path?
b. What is the tension in the string when the object is at the top of its path?
76. A space probe with a mass of $7.600 \times 10^{3} \mathrm{~kg}$ is traveling through space at $125 \mathrm{~m} / \mathrm{s}$. Mission control decides that a course correction of $30.0^{\circ}$ is needed and instructs the probe to fire rockets perpendicular to its present direction of motion. If the gas expelled by the rockets has a speed of $3.200 \mathrm{~km} / \mathrm{s}$, what mass of gas should be released? (Chapter 9)
77. When a $60.0-\mathrm{cm}$-long guitar string is plucked in the middle, it plays a note of frequency 440 Hz . What is the speed of the waves on the string? (Chapter 14)
78. What is the wavelength of a sound wave with a frequency of $17,000 \mathrm{~Hz}$ in water at $25^{\circ} \mathrm{C}$ ? (Chapter 15)

## Standardized Test Practice

## Multiple Choice

1. In 1987, a supernova was observed in a neighboring galaxy. Scientists believed the galaxy was $1.66 \times 10^{21} \mathrm{~m}$ away. How many years prior to the observation did the supernova explosion actually occur?
(A) $5.53 \times 10^{3} \mathrm{yr}$
(C) $5.53 \times 10^{12} \mathrm{yr}$
(B) $1.75 \times 10^{5} \mathrm{yr}$
(D)
$1.74 \times 10^{20} \mathrm{yr}$
2. A galaxy is moving away at $5.8 \times 10^{6} \mathrm{~m} / \mathrm{s}$. Its light appears to observers to have a frequency of $5.6 \times 10^{14} \mathrm{~Hz}$. What is the emitted frequency of the light?
```
(A)
    1.1\times10\mp@subsup{0}{}{13}\textrm{Hz}
    (C) }5.7\times1\mp@subsup{0}{}{14}\textrm{Hz
(B) }5.5\times1\mp@subsup{0}{}{14}\textrm{Hz
(D) }6.2\times1\mp@subsup{0}{}{14}\textrm{Hz
```

3. Which of the following light color combinations is incorrect?
(A) Red plus green produces yellow.
(B) Red plus yellow produces magenta.
(C) Blue plus green produces cyan.
(D) Blue plus yellow produces white.
4. The illuminance of direct sunlight on Earth is about $1 \times 10^{5} \mathrm{~lx}$. A light on a stage has an intensity in a certain direction of $5 \times 10^{6} \mathrm{~cd}$. At what distance from the stage does a member of the audience experience an illuminance equal to that of sunlight?

| (A) $1.4 \times 10^{-1} \mathrm{~m}$ | (C) 10 m |
| :--- | :--- |
| (B) 7 m | (D) $5 \times 10^{1} \mathrm{~m}$ |

5. What is meant by the phrase color by subtraction of light?
(A) Adding green, red, and blue light produces white light.
(B) Exciting phosphors with electrons in a television produces color.
(C) Paint color is changed by subtracting certain colors, such as producing blue paint from green by removing yellow.
(D) The color that an object appears to be is a result of the material absorbing specific light wavelengths and reflecting the rest.
6. The illuminance due to a $60.0-\mathrm{W}$ lightbulb at 3.0 m is 9.35 lx . What is the total luminous flux of the bulb?
(A) $8.3 \times 10^{-2} \mathrm{~lm}$
(C) $1.2 \times 10^{2} \mathrm{~lm}$
(B) $7.4 \times 10^{-1} \mathrm{~lm}$
(D) $1.1 \times 10^{3} \mathrm{~lm}$
7. Light from the Sun takes about 8.0 min to reach Earth. How far away is the Sun?
(A) $2.4 \times 10^{9} \mathrm{~m}$
(C) $1.4 \times 10^{8} \mathrm{~km}$
(B) $1.4 \times 10^{10} \mathrm{~m}$
(D) $2.4 \times 10^{9} \mathrm{~km}$
8. What is the frequency of 404 nm of light in a vacuum?
(A)
$2.48 \times 10^{-3} \mathrm{~Hz}$
(C) $2.48 \times 10^{6} \mathrm{~Hz}$
(B) $7.43 \times 10^{5} \mathrm{~Hz}$
(D) $7.43 \times 10^{14} \mathrm{~Hz}$

## Extended Answer

9. A celestial object is known to contain an element that emits light at a wavelength of 525 nm . The observed spectral line for this element is at 473 nm . Is the object approaching or receding, and at what speed?
10. Nonpolarized light of intensity $I_{\mathrm{o}}$ is incident on a polarizing filter, and the emerging light strikes a second polarizing filter, as shown in the figure. What is the light intensity emerging from the second polarizing filter?


## Test-Taking TIP

## Ask Questions

When you have a question about what will be on a test, the way a test is scored, the time limits placed on each section, or anything else, ask the instructor or the person giving the test.

## Chapter 17

## Reflection and Mirrors

## What You'll Learn

- You will learn how light reflects off different surfaces.
- You will learn about the different types of mirrors and their uses.
- You will use ray tracing and mathematical models to describe images formed by mirrors.


## Why It's Important

How light reflects off a surface into your eyes determines the reflection that you see. When you look down at the surface of a lake, you see an upright reflection of yourself.
Mountain Scene When you look across a lake, you might see a scene like the one in this photo. The image of the trees and mountains in the lake appears to you to be upside-down.

## Think About This >

Why would the image you see of yourself in the lake be upright, while the image of the mountain is upside-down?


## LAUNCH Lab How is an image shown on a screen?

## Question

What types of mirrors are able to reflect an image onto a screen?

## 

1. Obtain an index card, a plane mirror, a concave mirror, a convex mirror, and a flashlight from your teacher.
2. Turn off the room lights and stand near the window.
3. Hold the index card in one hand. Hold the flat, plane mirror in the other hand.
4. Reflect the light coming through the window onto the index card. CAUTION: Do not look directly at the Sun or at the reflection of the Sun in a mirror. Slowly move the index card closer to and then farther away from the mirror and try to make a clear image of objects outside the window.
5. If you can project a clear image, this is called a real image. If you only see a fuzzy light on the index card then no real image is formed. Record your observations.
6. Repeat steps $3-5$ with the concave and convex mirror.
7. Perform step 4 for each mirror with a flashlight and observe the reflection on the index card.

## Analysis

Which mirror(s) produced a real image?
What are some things you notice about the imagess) you see?
Critical Thinking Based upon your observation of the flashlight images, propose an explanation of how a real image is formed.


### 17.1 Reflection from Plane Mirrors

Undoubtedly, as long as there have been humans, they have seen their faces reflected in the quiet water of lakes and ponds. When you look at the surface of a body of water, however, you don't always see a clear reflection. Sometimes, the wind causes ripples in the water, and passing boats create waves. Disturbances on the surface of the water prevent the light from reflecting in a manner such that a clear reflection is visible.

Almost 4000 years ago, Egyptians understood that reflection requires smooth surfaces. They used polished metal mirrors to view their images. Sharp, well-defined, reflected images were not possible until 1857, however, when Jean Foucault, a French scientist, developed a method of coating glass with silver. Modern mirrors are produced using ever-increasing precision. They are made with greater reflecting ability by the evaporation of aluminum or silver onto highly polished glass. The quality of reflecting surfaces is even more important in applications such as lasers and telescopes. More than ever before, clear reflections in modern, optical instruments require smooth surfaces.


- Figure 17-1 The incident ray and the reflected ray are in the same plane of travel.


## Color Convention

- Light rays and wave fronts are red.
- Mirrors are light blue.


## The Law of Reflection

What happens to the light that is striking this book? When you hold the book up to the light, you will see that no light passes through it. Recall from Chapter 16 that an object like this is called opaque. Part of the light is absorbed, and part is reflected. The absorbed light spreads out as thermal energy. The behavior of the reflected light depends on the type of surface and the angle at which the light strikes the surface.

Recall from Chapter 14 that when a wave traveling in two dimensions encounters a barrier, the angle of incidence is equal to the angle of reflection of the wave. This same two-dimensional reflection relationship applies to light waves. Consider what happens when you bounce-pass a basketball. The ball bounces in a straight line, as viewed from above, to the other player. Light reflects in the same way as a basketball. Figure 17-1 shows a ray of light striking a reflecting surface. The normal is an imaginary line that is perpendicular to a surface at the location where light strikes the surface. The reflected ray, the incident ray, and the normal to the surface always will be in the same plane. Although the light is traveling in three dimensions, the reflection of the light is planar (two-dimensional). The planar and angle relationships are known together as the law of reflection.

## Law of Reflection $\quad \theta_{\mathrm{r}}=\theta_{\mathrm{i}}$

The angle that a reflected ray makes as measured from the normal to a reflective surface equals the angle that the incident ray makes as measured from the same normal.

This law can be explained in terms of the wave model of light. Figure 17-2 shows a wave front of light approaching a reflective surface. As each point along the wave front reaches the surface, it reflects off at the same angle as the preceding point. Because all points are traveling at the same speed, they all travel the same total distance in the same time. Thus, the wave front as a whole leaves the surface at an angle equal to its incident angle. Note that the wavelength of the light does not affect this process. Red, green, and blue light all follow this law.

- Figure 17-2 A wave front of light approaches a reflective surface. Point $P$ on the wave front strikes the surface first (a). Point $Q$ strikes the surface after point $P$ reflects at an angle equal to the incident angle (b). The process continues with all points reflecting off at angles equal to their incident angles, resulting in a reflected wave front (c).



Smooth and rough surfaces Consider the beam of light shown in Figure 17-3a. All of the rays in this beam of light reflect off the surface parallel to one another, as shown in Figure 17-3b. This occurs only if the reflecting surface is not rough on the scale of the wavelength of light. Such a surface is considered to be smooth relative to the light. A smooth surface, such as a mirror, causes specular reflection, in which parallel light rays are reflected in parallel.

What happens when light strikes a surface that appears to be smooth, but actually is rough on the scale of the wavelength of light, such as the page of this textbook or a white wall? Is light reflected? How could you demonstrate this? Figure 17-3c shows a beam of light reflecting off a sheet of paper, which has a rough surface. All of the light rays that make up the beam are parallel before striking the surface, but the reflected rays are not parallel, as shown in Figure 17-3d. This is diffuse reflection, the scattering of light off a rough surface.

The law of reflection applies to both smooth and rough surfaces. For a rough surface, the angle that each incident ray makes with the normal equals the angle that its reflected ray makes with the normal. However, on a microscopic scale, the normals to the surface locations where the rays strike are not parallel. Thus, the reflected rays cannot be parallel. The rough surface prevents them from being parallel. In this case, a reflected beam cannot be seen because the reflected rays are scattered in different directions. With specular reflection, as with a mirror, you can see your face. But no matter how much light is reflected off a wall or a sheet of paper, you will never be able to use them as mirrors.

- Figure 17-3 When a beam of light strikes a mirrored surface (a), the parallel rays in the beam reflect in parallel and maintain the light as a beam (b). When the light beam strikes a rough surface (c), the parallel rays that make up the beam are reflected from different microscopic surfaces, thereby diffusing the beam (d).


## EXAMPLE Problem 1

Changing the Angle of Incidence A light ray strikes a plane mirror at an angle of $52.0^{\circ}$ to the normal. The mirror then rotates $35.0^{\circ}$ around the point where the beam strikes the mirror so that the angle of incidence of the light ray decreases. The axis of rotation is perpendicular to the plane of the incident and the reflected rays. What is the angle of rotation of the reflected ray?

1 Analyze and Sketch the Problem

- Sketch the situation before the rotation of the mirror.
- Draw another sketch with the angle of rotation applied to the mirror.

$$
\begin{array}{ll}
\text { Known: } & \text { Unknown: } \\
\theta_{\mathrm{i}, \text { initial }}=52.0^{\circ} & \Delta \theta_{\mathrm{r}}=? \\
\Delta \theta_{\text {mirror }}=35.0^{\circ} &
\end{array}
$$



## 2 Solve for the Unknown

For the angle of incidence to reduce, rotate clockwise.

$$
\begin{aligned}
\theta_{\mathrm{i}, \text { final }} & =\theta_{\mathrm{i}, \text { initial }}-\Delta \theta_{\text {mirror }} \\
& =52.0^{\circ}-35.0^{\circ} \quad \text { Substitute } \theta_{\mathrm{i}, \text { initial }}=52 . \mathbf{0}^{\circ}, \Delta \theta_{\text {mirror }}=\mathbf{3 5 . 0 ^ { \circ }} \\
& =17.0^{\circ} \text { clockwise from the new normal }
\end{aligned}
$$

Apply the law of reflection.

$$
\begin{aligned}
\theta_{\mathrm{r}, \text { final }} & =\theta_{\mathrm{i}, \text { final }} \\
& =17.0^{\circ} \text { counterclockwise } \quad \text { Substitute } \theta_{\mathrm{i}, \text { final }}=17.0^{\circ}
\end{aligned}
$$



Using the two sketches, determine the angle through which the reflected ray has rotated.

$$
\begin{aligned}
\Delta \theta_{\mathrm{r}} & =52.0^{\circ}+35.0^{\circ}-17.0^{\circ} \\
& =70.0^{\circ} \text { clockwise from the original angle }
\end{aligned}
$$

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## 3 Evaluate the Answer

- Is the magnitude realistic? Comparing the final sketch with the initial sketch shows that the angle the light ray makes with the normal decreases as the mirror rotates clockwise toward the light ray. It makes sense, then, that the reflected ray also rotates clockwise.


## PRACIICE Problems

1. Explain why the reflection of light off ground glass changes from diffuse to specular if you spill water on it.
2. If the angle of incidence of a ray of light is $42^{\circ}$, what is each of the following?
a. the angle of reflection
b. the angle the incident ray makes with the mirror
c. the angle between the incident ray and the reflected ray
3. If a light ray reflects off a plane mirror at an angle of $35^{\circ}$ to the normal, what was the angle of incidence of the ray?
4. Light from a laser strikes a plane mirror at an angle of $38^{\circ}$ to the normal. If the laser is moved so that the angle of incidence increases by $13^{\circ}$, what is the new angle of reflection?
5. Two plane mirrors are positioned at right angles to one another. A ray of light strikes one mirror at an angle of $30^{\circ}$ to the normal. It then reflects toward the second mirror. What is the angle of reflection of the light ray off the second mirror?

## Objects and Plane-Mirror Images

If you looked at yourself in a mirror this morning you saw your reflection in a plane mirror. A plane mirror is a flat, smooth surface from which light is reflected by specular reflection. To understand reflection from a mirror, you must consider the object of the reflection and the type of image that is formed. In Chapter 16, the word object was used to refer to sources of light. In describing mirrors, the word object is used in the same way, but with a more specific application. An object is a source of light rays that are to be reflected by a mirrored surface. An object can be a luminous source, such as a lightbulb, or an illuminated source, such as a girl, as shown in Figure 17-4. Most objects that you will work with in this chapter are a source of light that diverges, or spreads out from the source.

Consider a single point on the bird in Figure 17-5. Light reflects diffusely from the crest of the bird, the object point. What happens to the light? Some of the light travels from the bird to the mirror and reflects. What does the boy see? Some of the reflected light from the bird hits his eyes. Because his brain processes this information as if the light has traveled a straight path, it seems to the boy as if the light had followed the dashed lines. The light seems to have come from a point behind the mirror, the image point.

The boy in Figure 17-5 sees rays of light that come from many points on the bird. The combination of the image points produced by reflected light rays forms the image of the bird. It is a virtual image, which is a type of image formed by diverging light rays. A virtual image is always on the opposite side of the mirror from the object. Images of real objects produced by plane mirrors are always virtual images.

## Properties of Plane-Mirror Images

Looking at yourself in a mirror, you can see that your image appears to be the same distance behind the mirror as you are in front of the mirror. How could you test this? Place a ruler between you and the mirror. Where does the image touch the ruler? You also see that your image is oriented as you are, and it matches your size. This is where the expression mirror image originates. If you move toward the mirror, your image moves toward the mirror. If you move away, your image also moves away.



Figure 17-4 The lightbulb is a luminous source that produces diverging light by shining in all directions. The girl is an illuminated source that produces diverging light by the diffused reflection from her body of light that comes from the lightbulb.

- Figure 17-5 The reflected rays that enter the eye appear to originate at a point behind the mirror.


Figure 17-6 Light rays (two are shown) leave a point on the object. Some strike the mirror and are reflected into the eye. Sight lines, drawn as dashed lines, extend from the location on the mirror where the rays are reflected back to where they converge. The image is located where the sight lines converge: $d_{\mathrm{i}}=-d_{\mathrm{o}}$.

## MINI LAB

## Virtual Image Position

Suppose you are looking at your image in a plane mirror. Can you measure the location of the image?

1. Obtain a camera from your teacher with a focusing ring that has distances marked on it.
2. Stand 1.0 m from a mirror and focus on the edge of the mirror. Check the reading on the focusing ring. It should be 1.0 m .
3. Measure the position of the image by focusing the camera on your image. Check the reading on the focusing ring.

## Analyze and Conclude

4. How far is the image behind the mirror?
5. Why is the camera able to focus on a virtual image that is behind the mirror even though there is no real object behind the mirror?

Image position and height The geometric model of Figure 17-6 demonstrates why the distances are the same. Two rays from point O at the tip of the candle strike the mirror at point $\mathrm{P}_{1}$ and point $\mathrm{P}_{2}$, respectively. Both rays are reflected according to the law of reflection. The reflected rays are extended behind the mirror as sight lines, converging at point I , which is the image of point O. Ray 1, which strikes the mirror at an angle of incidence of $0^{\circ}$, is reflected back on itself, so the sight line is at $90^{\circ}$ to the mirror, just as ray 1 . Ray 2 is also reflected at an angle equal to the angle of incidence, so the sight line is at the same angle to the mirror as ray 2.

This geometric model reveals that line segments $\overline{\mathrm{OP}_{1}}$ and $\overline{\mathrm{IP}_{1}}$ are corresponding sides of two congruent triangles, $\mathrm{OP}_{1} \mathrm{P}_{2}$ and $\mathrm{IP}_{1} \mathrm{P}_{2}$. The position of the object with respect to the mirror, or the object position, $d_{o^{\prime}}$, has a length equal to the length of $\overline{\mathrm{OP}}_{1}$. The apparent position of the image with respect to the mirror, or the image position, $d_{\mathrm{i}}$, has a length equal to the length of $\overline{\mathrm{IP}_{1}}$. Using the convention that image position is negative to indicate that the image is virtual, the following is true.

## Plane-Mirror Image Position $d_{\mathrm{i}}=-d_{\mathrm{o}}$

With a plane mirror, the image position is equal to the negative of the object position. The negative sign indicates that the image is virtual.

You can draw rays from the object to the mirror to determine the size of the image. The sight lines of two rays originating from the bottom of the candle in Figure 17-6 will converge at the bottom of the image. Using the law of reflection and congruent-triangle geometry, the following is true of the object height, $h_{\mathrm{o}^{\prime}}$, and image height, $h_{\mathrm{i}^{\prime}}$, and any other dimension of the object and image.

Plane-Mirror Image Height $\quad h_{\mathrm{i}}=h_{\mathrm{o}}$
With a plane mirror, image height is equal to object height.


Image orientation A plane mirror produces an image with the same orientation as the object. If you are standing on your feet, a plane mirror produces an image of you standing on your feet. If you are doing a headstand, the mirror shows you doing a headstand. However, there is a difference between you and the appearance of your image in a mirror. Follow the sight lines in Figure 17-7. The ray that diverges from the right hand of the boy converges at what appears to be the left hand of his image. Left and right appear to be reversed by a plane mirror. Why, then, are top and bottom not also reversed? This does not happen because a plane mirror does not really reverse left and right. The mirror in Figure 17-7 only reverses the boy's image such that it is facing in the opposite direction as the boy, or, in other words, it produces a front-to-back reversal.

Consider the image of the mountain in the photo at the beginning of the chapter. In this case, the image of the mountain can be described as upside down, but the image is actually a front-to-back reversal of your view of the actual mountain. Because the mirror (the lake surface) is horizontal, rather than vertical, your perspective, or angle of view, makes the image look upside down. Turn your book $90^{\circ}$ counterclockwise and look at Figure 17-7 again. The actual boy is now facing down, and his image is facing up, upside down relative to the actual boy, just like the image of the mountain. The only thing that has changed is your perspective.

- Figure 17-7 The image formed in a plane mirror is the same size as the object and is the same distance behind the mirror as the object is in front. However, when the boy blinks his right eye, the left eye of his image blinks.


### 17.1 Section Review

6. Reflection A light ray strikes a flat, smooth, reflecting surface at an angle of $80^{\circ}$ to the normal. What is the angle that the reflected ray makes with the surface of the mirror?
7. Law of Reflection Explain how the law of reflection applies to diffuse reflection.
8. Reflecting Surfaces Categorize each of the following as a specular or a diffuse reflecting surface: paper, polished metal, window glass, rough metal, plastic milk jug, smooth water surface, and ground glass.
9. Image Properties A $50-\mathrm{cm}$-tall dog stands 3 m from a plane mirror and looks at its image. What is the image position, height, and type?
10. Image Diagram A car is following another car down a straight road. The first car has a rear window tilted $45^{\circ}$. Draw a ray diagram showing the position of the Sun that would cause sunlight to reflect into the eyes of the driver of the second car.
11. Critical Thinking Explain how diffuse reflection of light off an object enables you to see an object from any angle.

### 17.2 Curved Mirrors

- Objectives
- Explain how concave and convex mirrors form images.
- Describe properties and uses of spherical mirrors.
- Determine the locations and sizes of spherical mirror images.
- Vocabulary
concave mirror
principal axis
focal point
focal length
real image spherical aberration magnification convex mirror

Figure 17-8 The focal point of a spherical concave mirror is located halfway between the center of curvature and the mirror surface. Rays entering parallel to the principal axis are reflected to converge at the focal point, F .
f you look at the surface of a shiny spoon, you will notice that your reflection is different from what you see in a plane mirror. The spoon acts as a curved mirror, with one side curved inward and the other curved outward. The properties of curved mirrors and the images that they form depend on the shape of the mirror and the object's position.

## Concave Mirrors

The inside surface of a shiny spoon, the side that holds food, acts as a concave mirror. A concave mirror has a reflective surface, the edges of which curve toward the observer. Properties of a concave mirror depend on how much it is curved. Figure 17-8 shows how a spherical concave mirror works. In a spherical concave mirror, the mirror is shaped as if it were a section of a hollow sphere with an inner reflective surface. The mirror has the same geometric center, C , and radius of curvature, $r$, as a sphere of radius, $r$. The line that includes line segment CM is the principal axis, which is the straight line perpendicular to the surface of the mirror that divides the mirror in half. Point $M$ is the center of the mirror where the principal axis intersects the mirror.


When you point the principal axis of a concave mirror toward the Sun, all the rays are reflected through a single point. You can locate this point by moving a sheet of paper toward and away from the mirror until the smallest and sharpest spot of sunlight is focused on the paper. This spot is called the focal point of the mirror, the point where incident light rays that are parallel to the principal axis converge after reflecting from the mirror. The Sun is a source of parallel light rays because it is very far away. All of the light that comes directly from the Sun must follow almost parallel paths to Earth, just as all of the arrows shot by an archer must follow almost parallel paths to hit within the circle of a bull's-eye.

When a ray strikes a mirror, it is reflected according to the law of reflection. Figure $17-8$ shows that a ray parallel to the principal axis is reflected and crosses the principal axis at point F , the focal point. F is at the halfway point between M and C . The focal length, $f$, is the position of the focal point with respect to the mirror along the principal axis and can be expressed as $f=r / 2$. The focal length is positive for a concave mirror.


## Graphical Method of Finding the Image

You have already drawn rays to follow the path of light that reflects off plane mirrors. This method is even more useful when applied to curved mirrors. Not only can the location of the image vary, but so can the orientation and size of the image. You can use a ray diagram to determine properties of an image formed by a curved mirror. Figure $\mathbf{1 7 - 9}$ shows the formation of a real image, an image that is formed by the converging of light rays. The image is inverted and larger than the object. The rays actually converge at the point where the image is located. The point of intersection, I, of the two reflected rays determines the position of the image. You can see the image floating in space if you place your eye so that the rays that form the image fall on your eye, as in Figure 17-9a. As Figure 17-9b shows, however, your eye must be oriented so as to see the rays coming from the image location. You cannot look at the image from behind. If you were to place a movie screen at this point, the image would appear on the screen, as shown in Figure 17-9c. You cannot do this with virtual images.

To more easily understand how ray tracing works with curved mirrors, you can use simple, one-dimensional objects, such as the arrow shown in Figure 17-10a. A spherical concave mirror produces an inverted real image if the object position, $d_{\mathrm{o}^{\prime}}$ is greater than twice the focal length, $f$. The object is then beyond the center of curvature, C. If the object is placed between the center of curvature and the focal point, F, as shown in Figure 17-10b, the image is again real and inverted. However, the size of the image is now greater than the size of the object.


Figure 17-9 The real image, as seen by the unaided eye (a). The unaided eye cannot see the real image if it is not in a location to catch the rays that form the image (b). The real image as seen on a white opaque screen (c).

- Figure 17-10 When the object is farther from the mirror than C , the image is a real image that is inverted and smaller compared to the object (a). When the object is located between $C$ and $F$, the real image is inverted, larger than the object, and located beyond C (b).


- Figure 17-11 A Gregorian telescope produces a real image that is upright.

Astronomy Connection


Primary concave mirror

How can the inverted real image created by a concave mirror be turned right-side up? In 1663, Scottish astronomer James Gregory developed the Gregorian telescope, shown in Figure 17-11, to resolve this problem. It is composed of a large concave mirror and a small concave mirror arranged such that the smaller mirror is outside of the focal point of the larger mirror. Parallel rays of light from distant objects strike the larger mirror and reflect toward the smaller mirror. The rays then reflect off the smaller mirror and form a real image that is oriented exactly as the object is.

## PROBLEM-SOLVING Strategies

## Using Ray Tracing to Locate Images Formed by Spherical Mirrors

Use the following strategies for spherical-mirror problems. Refer to Figure 17-10.

1. Using lined or graph paper, draw the principal axis of the mirror as a horizontal line from the left side to the right side of your paper, leaving six blank lines above and six blank lines below.
2. Place a point and a label on the principal axis the object, C , and F , as follows.
a. If the mirror is a concave mirror and the object is beyond C , away from the mirror, place the mirror at the right side of the page, place the object at the left side of the page, and place C and F to scale.
b. If the mirror is a concave mirror and the object is between C and F , place the mirror at the right side of the page, place C at the center of the paper, F halfway between the mirror and C , and the object to scale.
c. For any other situation, place the mirror in the center of the page. Place the object or F (whichever is the greatest distance from the mirror) at the left side of the page, and place the other to scale.
3. To represent the mirror, draw a vertical line at the mirror point that extends the full 12 lines of space. This is the principal plane.
4. Draw the object as an arrow and label its top $\mathrm{O}_{1}$. For concave mirrors, objects inside of $C$ should not be higher than three lines high. For all other situations, the objects should be six lines high. The scale for the height of the object will be different from the scale along the principal axis.
5. Draw ray 1 , the parallel ray. It is parallel to the principal axis and reflects off the principal plane and passes through F.
6. Draw ray 2 , the focus ray. It passes through F, reflects off the principal plane, and is reflected parallel to the principal axis.
7. The image is located where rays 1 and 2 (or their sight lines) cross after reflection. Label the point $I_{1}$. The image is an arrow perpendicular from the principal axis to $I_{1}$.


Real image defects in concave mirrors In tracing rays, you have reflected the rays from the principal plane, which is a vertical line representing the mirror. In reality, rays are reflected off the mirror itself, as shown in Figure 17-12a. Notice that only parallel rays that are close to the principal axis, or paraxial rays, are reflected through the focal point. Other rays converge at points closer to the mirror. The image formed by parallel rays reflecting off a spherical mirror with a large mirror diameter and a small radius of curvature is a disk, not a point. This effect, called spherical aberration, makes an image look fuzzy, not sharp.

A mirror ground to the shape of a parabola, as in Figure 17-12b, suffers no spherical aberration. Because of the cost of manufacturing large, perfectly parabolic mirrors, many of the newest telescopes use spherical mirrors and smaller, specially-designed secondary mirrors or lenses to correct for spherical aberration. Also, spherical aberration is reduced as the ratio of the mirror's diameter, shown in Figure 17-12a, to its radius of curvature is reduced. Thus, lower-cost spherical mirrors can be used in lower-precision applications.

## Mathematical Method of Locating the Image

The spherical mirror model can be used to develop a simple equation for spherical mirrors. You must use the paraxial ray approximation, which states that only rays that are close to and almost parallel with the principal axis are used to form an image. Using this, in combination with the law of reflection, leads to the mirror equation, relating the focal length, $f$, object position, $d_{\mathrm{o}^{\prime}}$ and image position, $d_{\mathrm{i}^{\prime}}$ of a spherical mirror.

## Mirror Equation $\frac{1}{f}=\frac{1}{d_{\mathrm{i}}}+\frac{1}{d_{\mathrm{o}}}$

The reciprocal of the focal length of a spherical mirror is equal to the sum of the reciprocals of the image position and the object position.

When using this equation to solve problems, it is important to remember that it is only approximately correct. It does not predict spherical aberration, because it uses the paraxial ray approximation. In reality, light coming from an object toward a mirror is diverging, so not all of the light is close to or parallel to the axis. When the mirror diameter is small relative to the radius of curvature to minimize spherical aberration, this equation predicts image properties more precisely.

- Figure 17-12 A concave spherical mirror reflects some rays, such that they converge at points other than the focus (a). A parabolic mirror focuses all parallel rays at a point (b).


## APPLYING PHYSICS

- Hubble Trouble In 1990, NASA launched the Hubble Space Telescope into orbit around Earth. Hubble was expected to provide clear images without atmospheric distortions. However, soon after it was deployed, Hubble was found to have a spherical aberration. In 1993, corrective optics, called COSTAR, were installed on Hubble to enable it to produce clear images.


## Connecting Math to Physics

Adding and Subtracting Fractions When using the mirror equation, you first use math to move the fraction that contains the quantity you are seeking to the left-hand side of the equation and everything else to the right. Then you combine the two fractions on the right-hand side using a common denominator that results from multiplying the denominators.

| Math | Physics |
| :---: | :---: |
| $\frac{1}{x}=\frac{1}{y}+\frac{1}{z}$ | $\frac{1}{f}=\frac{1}{d_{\mathrm{i}}}+\frac{1}{d_{0}}$ |
| $\frac{1}{y}=\frac{1}{x}-\frac{1}{z}$ | $\frac{1}{d_{\mathrm{i}}}=\frac{1}{f}-\frac{1}{d_{\mathrm{o}}}$ |
| $\frac{1}{y}=\left(\frac{1}{x}\right)\left(\frac{z}{z}\right)-\left(\frac{1}{z}\right)\left(\frac{x}{x}\right)$ | $\frac{1}{d_{\mathrm{i}}}=\left(\frac{1}{f}\right)\left(\frac{d_{0}}{d_{0}}\right)-\left(\frac{1}{d_{0}}\right)\left(\frac{f}{f}\right)$ |
| $\frac{1}{y}=\frac{z-x}{x z}$ | $\frac{1}{d_{\mathrm{i}}}=\frac{d_{0}-f}{f d_{\mathrm{o}}}$ |
| $y=\frac{x z}{z-x}$ | $d_{\mathrm{i}}=\frac{f d_{0}}{d_{\mathrm{o}}-f}$ |

Using this approach, the following relationships can be derived for image position, object position, and focal length:

$$
d_{\mathrm{i}}=\frac{f d_{\mathrm{o}}}{d_{\mathrm{o}}-f} \quad d_{\mathrm{o}}=\frac{f d_{\mathrm{i}}}{d_{\mathrm{i}}-f} \quad f=\frac{d_{\mathrm{i}} d_{\mathrm{o}}}{d_{\mathrm{o}}+d_{\mathrm{i}}}
$$

Magnification Another property of a spherical mirror is magnification, $m$, which is how much larger or smaller the image is relative to the object. In practice, this is a simple ratio of the image height to the object height. Using similar-triangle geometry, this ratio can be written in terms of image positon and object position.

Magnification $m \equiv \frac{h_{\mathrm{i}}}{h_{\mathrm{o}}}=\frac{-d_{\mathrm{i}}}{d_{\mathrm{o}}}$
The magnification of an object by a spherical mirror, defined as the image height divided by the object height, is equal to the negative of the image position, divided by the object position.

Image position is positive for a real image when using the preceding equations. Thus, the magnification is negative, which means that the image is inverted compared to the object. If the object is beyond point C , the absolute value of the magnification for the real image is less than 1 . This means that the image is smaller than the object. If the object is placed between point C and point F , the absolute value of the magnification for the real image is greater than 1 . Thus, the image is larger than the object.

## EXAMPLE Problem 2

Real Image Formation by a Concave Mirror A concave mirror has a radius of 20.0 cm . A $2.0-\mathrm{cm}$-tall object is 30.0 cm from the mirror. What is the image position and image height?

## 1 Analyze and Sketch the Problem

- Draw a diagram with the object and the mirror.
- Draw two principal rays to locate the image in the diagram.

$$
\begin{array}{ll}
\text { Known: } & \text { Unknown: } \\
h_{\mathrm{o}}=2.0 \mathrm{~cm} & d_{\mathrm{i}}=? \\
d_{\mathrm{o}}=30.0 \mathrm{~cm} & h_{\mathrm{i}}=? \\
r=20.0 \mathrm{~cm} &
\end{array}
$$

## 2 Solve for the Unknown

Focal length is half the radius of curvature.

$$
\begin{aligned}
f & =\frac{r}{2} \\
& =\frac{20.0 \mathrm{~cm}}{2} \\
& =10.0 \mathrm{~cm}
\end{aligned}
$$



Substitute $\boldsymbol{r}=\mathbf{2 0 . 0} \mathbf{~ c m}$

Use the mirror equation and solve for image position.

$$
\begin{aligned}
\frac{1}{f} & =\frac{1}{d_{\mathrm{i}}}+\frac{1}{d_{\mathrm{o}}} \\
d_{\mathrm{i}} & =\frac{f d_{\mathrm{o}}}{d_{\mathrm{o}}-f} \\
& =\frac{(10.0 \mathrm{~cm})(30.0 \mathrm{~cm})}{30.0 \mathrm{~cm}-10.0 \mathrm{~cm}} \quad \text { Substitute } f=10.0 \mathrm{~cm}, d_{\mathrm{o}}=30.0 \mathrm{~cm} \\
& =15.0 \mathrm{~cm} \text { (real image, in front of the mirror) }
\end{aligned}
$$

Use the magnification equation and solve for image height.

$$
\begin{aligned}
m & \equiv \frac{h_{\mathrm{i}}}{h_{\mathrm{o}}}=\frac{-d_{\mathrm{i}}}{d_{\mathrm{o}}} \\
h_{\mathrm{i}} & =\frac{-d_{\mathrm{i}} h_{\mathrm{o}}}{d_{\mathrm{o}}} \\
& =\frac{-(15.0 \mathrm{~cm})(2.0 \mathrm{~cm})}{30.0 \mathrm{~cm}} \quad \text { Substitute } d_{\mathrm{i}}=15.0 \mathrm{~cm}, h_{\mathrm{o}}=2.0 \mathrm{~cm}, d_{\mathrm{o}}=30.0 \mathrm{~cm} \\
& =-1.0 \mathrm{~cm} \text { (inverted, smaller image) }
\end{aligned}
$$

## 3 Evaluate the Answer

- Are the units correct? All positions are in centimeters.
- Do the signs make sense? Positive position and negative height agree with the drawing.


## PRACTICE Problems

12. Use a ray diagram, drawn to scale, to solve Example Problem 2.
13. An object is 36.0 cm in front of a concave mirror with a $16.0-\mathrm{cm}$ focal length. Determine the image position.
14. A $3.0-\mathrm{cm}$-tall object is 20.0 cm from a 16.0 -cm-radius concave mirror. Determine the image position and image height.
15. A concave mirror has a $7.0-\mathrm{cm}$ focal length. A $2.4-\mathrm{cm}$-tall object is 16.0 cm from the mirror. Determine the image height.
16. An object is near a concave mirror of $10.0-\mathrm{cm}$ focal length. The image is 3.0 cm tall, inverted, and 16.0 cm from the mirror. What are the object position and object height?


Figure 17-13 When an object is located between the focal point and a spherical concave mirror, a virtual image that is upright and larger compared to the object is formed behind the mirror (a), as shown with the stack of blocks (b). What else do you see in this picture?

## Virtual Images with Concave Mirrors

You have seen that as an object approaches the focal point, F, of a concave mirror, the image moves farther away from the mirror. If the object is at the focal point, all reflected rays are parallel. They never meet, therefore, and the image is said to be at infinity, so the object could never be seen. What happens if the object is moved even closer to the mirror?

What do you see when you move your face close to a concave mirror? The image of your face is right-side up and behind the mirror. A concave mirror produces a virtual image if the object is located between the mirror and the focal point, as shown in the ray diagram in Figure 17-13a. Again, two rays are drawn to locate the image of a point on an object. As before, ray 1 is drawn parallel to the principal axis and reflected through the focal point. Ray 2 is drawn as a line from the point on the object to the mirror, along a line defined by the focal point and the point on the object. At the mirror, ray 2 is reflected parallel to the principal axis. Note that ray 1 and ray 2 diverge as they leave the mirror, so there cannot be a real image. However, sight lines extended behind the mirror converge, showing that the virtual image forms behind the mirror.

When you use the mirror equation to solve problems involving concave mirrors for which an object is between the mirror and the focal point, you will find that the image position is negative. The magnification equation gives a positive magnification greater than 1 , which means that the image is upright and larger compared to the object, like the image in Figure 17-13b.

## - CHALLENGE PROBLEM

An object of height $h_{0}$ is located at $d_{0}$ relative to a concave mirror with focal length $f$.

1. Draw and label a ray diagram showing the focal length and location of the object if the image is located twice as far from the mirror as the object. Prove your answer mathematically. Calculate the focal length as a function of object position for this placement.
2. Draw and label a ray diagram showing the location of the object if the image is located twice as far from the mirror as the focal point. Prove your answer mathematically. Calculate the image height as a function of the object height for this placement.
3. Where should the object be located so that no image is formed?


## Convex Mirrors

In the first part of this chapter, you learned that the inner surface of a shiny spoon acts as a concave mirror. If you turn the spoon around, the outer surface acts as a convex mirror, a reflective surface with edges that curve away from the observer. What do you see when you look at the back of a spoon? You see an upright, but smaller image of yourself.

Properties of a spherical convex mirror are shown in Figure 17-14. Rays reflected from a convex mirror always diverge. Thus, convex mirrors form virtual images. Points F and C are behind the mirror. In the mirror equation, $f$ and $d_{\mathrm{i}}$ are negative numbers because they are both behind the mirror.

The ray diagram in Figure 17-14 represents how an image is formed by a spherical convex mirror. The figure uses two rays, but remember that there are an infinite number of rays. Ray 1 approaches the mirror parallel to the principal axis. The reflected ray is drawn along a sight line from F through the point where ray 1 strikes the mirror. Ray 2 approaches the mirror on a path that, if extended behind the mirror, would pass through F . The reflected part of ray 2 and its sight line are parallel to the principal axis. The two reflected rays diverge, and the sight lines intersect behind the mirror at the location of the image. An image produced by a single convex mirror is a virtual image that is upright and smaller compared to the object.

The magnification equation is useful for determining the apparent dimensions of an object as seen in a spherical convex mirror. If you know the diameter of an object, you can multiply by the magnification fraction to see how the diameter changes. You will find that the diameter is smaller, as are all other dimensions. This is why the objects appear to be farther away than they actually are for convex mirrors.
Field of view It may seem that convex mirrors would have little use because the images that they form are smaller than the objects. However, this property of convex mirrors does have practical uses. By forming smaller images, convex mirrors enlarge the area, or field of view, that an observer sees, as shown in Figure 17-15. Also, the center of this field of view is visible from any angle of an observer off the principal axis of the mirror; thus, the field of view is visible from a wide perspective. For this reason, convex mirrors often are used in cars as passenger-side rearview mirrors.

- Figure 17-14 A convex mirror always produces virtual images that are upright and smaller compared to the object.


Figure 17-15 Convex mirrors produce images that are smaller than the objects. This increases the field of view for observers.

## EXAMPLE Problem 3

Image in a Security Mirror A convex security mirror in a warehouse has a $-0.50-\mathrm{m}$ focal length. A $2.0-\mathrm{m}$-tall forklift is 5.0 m from the mirror. What are the image position and image height?

## 1 Analyze and Sketch the Problem

- Draw a diagram with the mirror and the object.
- Draw two principal rays to locate the image in the diagram.

Known: Unknown:
$h_{\mathrm{o}}=2.0 \mathrm{~m} \quad d_{\mathrm{i}}=$ ?
$d_{\mathrm{o}}=5.0 \mathrm{~m} \quad h_{\mathrm{i}}=$ ?
$f=-0.50 \mathrm{~m}$

## 2 Solve for the Unknown

Use the mirror equation and solve for image position.

$$
\begin{aligned}
\frac{1}{f} & =\frac{1}{d_{\mathrm{i}}}+\frac{1}{d_{\mathrm{o}}} \\
d_{\mathrm{i}} & =\frac{f d_{\mathrm{o}}}{d_{\mathrm{o}}-f} \\
& =\frac{(-0.50 \mathrm{~m})(5.0 \mathrm{~m})}{5.0 \mathrm{~m}-0.50 \mathrm{~m}} \quad \text { Substitute } f=-0.50 \mathrm{~m}, \boldsymbol{d}_{\mathrm{o}}=5.0 \mathrm{~m} \\
& =-0.45 \mathrm{~m} \text { (virtual image, behind the mirror) }
\end{aligned}
$$



$$
\begin{aligned}
m & \equiv \frac{h_{\mathrm{i}}}{h_{\mathrm{o}}}=\frac{-d_{\mathrm{i}}}{d_{\mathrm{o}}} \\
h_{\mathrm{i}} & =\frac{-d_{\mathrm{i}} h_{\mathrm{o}}}{d_{\mathrm{o}}} \\
& =\frac{-(-0.45 \mathrm{~m})(2.0 \mathrm{~m})}{(5.0 \mathrm{~m})} \quad \text { Substitute } d_{\mathrm{i}}=-0.45 \mathrm{~m}, h_{\mathrm{o}}=2.0 \mathrm{~m}, d_{\mathrm{o}}=5.0 \mathrm{~m} \\
& =0.18 \mathrm{~m} \text { (upright, smaller image) }
\end{aligned}
$$

Use the magnification equation and solve for image height.

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## 3 Evaluate the Answer

- Are the units correct? All positions are in meters.
- Do the signs make sense? A negative position indicates a virtual image; a positive height indicates an image that is upright. These agree with the diagram.


## PRACTICE Problems

17. An object is located 20.0 cm in front of a convex mirror with a $-15.0-\mathrm{cm}$ focal length. Find the image position using both a scale diagram and the mirror equation.
18. A convex mirror has a focal length of -13.0 cm . A lightbulb with a diameter of 6.0 cm is placed 60.0 cm from the mirror. What is the lightbulb's image position and diameter?
19. A convex mirror is needed to produce an image that is three-fourths the size of an object and located 24 cm behind the mirror. What focal length should be specified?
20. A 7.6 -cm-diameter ball is located 22.0 cm from a convex mirror with a radius of curvature of 60.0 cm . What are the ball's image position and diameter?
21. A 1.8 -m-tall girl stands 2.4 m from a store's security mirror. Her image appears to be 0.36 m tall. What is the focal length of the mirror?

Table 17-1

| Single-Mirror System Properties |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Mirror Type | $f$ | $d_{0}$ | $d_{i}$ | m | Image |
| Plane | N/A | $d_{0}>0$ | $\begin{aligned} & \left\|d_{\mathrm{i}}\right\|=d_{\mathrm{o}} \\ & \text { (negative) } \end{aligned}$ | Same size | Virtual |
| Concave | + | $d_{0}>r$ | $r>d_{\mathrm{i}}>f$ | Reduced, inverted | Real |
|  |  | $r>d_{0}>f$ | $d_{i}>r$ | Enlarged, inverted | Real |
|  |  | $f>d_{0}>0$ | $\begin{aligned} & \left\|d_{\mathrm{i}}\right\|>d_{\mathrm{o}} \\ & \text { (negative) } \end{aligned}$ | Enlarged | Virtual |
| Convex | - | $d_{0}>0$ | $\underset{\text { (negative) }}{\|f\|}\left\|d_{a}\right\|>0$ | Reduced | Virtual |

## Mirror Comparison

How do the various types of mirrors compare? Table 17-1 compares the properties of single-mirror systems with objects that are located on the principal axis of the mirror. Virtual images are always behind the mirror, which means that the image position is negative. When the absolute value of a magnification is between zero and one, the image is smaller than the object. A negative magnification means the image is inverted relative to the object. Notice that the single plane mirror and convex mirror produce only virtual images, whereas the concave mirror can produce real images or virtual images. Plane mirrors give simple reflections, and convex mirrors expand the field of view. A concave mirror acts as a magnifier when an object is within the focal length of the mirror.

### 17.2 Section Review

22. Image Properties If you know the focal length of a concave mirror, where should you place an object so that its image is upright and larger compared to the object? Will this produce a real or virtual image?
23. Magnification An object is placed 20.0 cm in front of a concave mirror with a focal length of 9.0 cm . What is the magnification of the image?
24. Object Position The placement of an object in front of a concave mirror with a focal length of 12.0 cm forms a real image that is 22.3 cm from the mirror. What is the object position?
25. Image Position and Height A $3.0-\mathrm{cm}$-tall object is placed 22.0 cm in front of a concave mirror having a focal length of 12.0 cm . Find the image position and height by drawing a ray diagram to scale. Verify your answer using the mirror and magnification equations.
26. Ray Diagram A 4.0-cm-tall object is located 14.0 cm from a convex mirror with a focal length of -12.0 cm . Draw a scale ray diagram showing the image position and height. Verify your answer using the mirror and magnification equations.
27. Radius of Curvature A $6.0-\mathrm{cm}$-tall object is placed 16.4 cm from a convex mirror. If the image of the object is 2.8 cm tall, what is the radius of curvature of the mirror?
28. Focal Length A convex mirror is used to produce an image that is two-thirds the size of an object and located 12 cm behind the mirror. What is the focal length of the mirror?
29. Critical Thinking Would spherical aberration be less for a mirror whose height, compared to its radius of curvature, is small or large? Explain.

## Concave Mirror Images

A concave mirror reflects light rays that arrive parallel to the principal axis through the focal point. Depending on the object position, different types of images can be formed. Real images can be projected onto a screen while virtual images cannot. In this experiment you will investigate how changing the object position affects the image location and type.

## QUESTION

What are the conditions needed to produce real and virtual images using a concave mirror?

## Objectives

$■$ Collect and organize data of object and image positions.

- Observe real and virtual images.

■ Summarize conditions for production of real and virtual images with a concave mirror.

## Safety Precautions

## 

Do not look at the reflection of the Sun in a mirror or use a concave mirror to focus sunlight.

## Materials

concave mirror
flashlight
screen support
mirror holder
two metersticks
four meterstick supports screen
lamp with a 15-W lightbulb


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## Procedure

1. Determine the focal length of your concave mirror by using the following procedure. CAUTION: Do not use the Sun to perform this procedure. Reflect light from a flashlight onto a screen and slowly move the screen closer or farther away from the mirror until a sharp, bright image is visible. Measure the distance between the screen and the mirror along the principal axis. Record this value as the actual focal length of the mirror, $f$.
2. On the lab table, set up two metersticks on supports in a V orientation. Place the zero measurement ends at the apex of the two metersticks.
3. Place the mirror in a mirror holder and place it at the apex of the two metersticks.
4. Using the lamp as the object of the reflection, place it on one meterstick at the opposite end from the apex. Place the mirror and the screen, supported by a screen support, on the other meterstick at the opposite end from the apex.
5. Turn the room lights off.
6. Turn on the lamp. CAUTION: Do not touch the hot lightbulb. Measure object position, $d_{0}$, and record this as Trial 1. Measure the object height, $h_{0}$, and record it as Trial 1. This is measured as the actual height of the lightbulb, or glowing filament if the bulb is clear.
7. Adjust the mirror or metersticks, as necessary, such that the reflected light shines on the screen. Slowly move the screen back and forth along the meterstick until a sharp image is seen. Measure image position, $d_{\mathrm{i}}$, and the image height, $h_{\mathrm{i}}$, and record these as Trial 1.

## Data Table

| Trial | $d_{0}(\mathrm{~cm})$ | $d_{\mathrm{i}}(\mathrm{cm})$ | $\boldsymbol{h}_{\mathbf{0}}(\mathrm{cm})$ | $\boldsymbol{h}_{\mathrm{i}}(\mathrm{cm})$ |
| :---: | :--- | :--- | :--- | :--- |
| 1 |  |  |  |  |
| 2 |  |  |  |  |
| 3 |  |  |  |  |
| 4 |  |  |  |  |
| 5 |  |  |  |  |

## Calculation Table

| Trial | $\frac{1}{d_{0}}\left(\mathrm{~cm}^{-1}\right)$ | $\frac{1}{d_{\mathrm{i}}}\left(\mathrm{cm}^{-1}\right)$ | $\frac{1}{d_{0}}+\frac{1}{d_{\mathrm{i}}}\left(\mathrm{cm}^{-1}\right)$ | $f_{\text {calc }}(\mathrm{cm})$ | \% error |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 |  |  |  |  |  |
| 2 |  |  |  |  |  |
| 3 |  |  |  |  |  |
| 4 |  |  |  |  |  |
| 5 |  |  |  |  |  |

8. Move the lamp closer to the mirror so that $d_{0}$ is twice the focal length, $f$. Record this as Trial 2. Move the screen until an image is obtained on the screen. Measure $d_{\mathrm{i}}$ and $h_{\mathrm{i}}$, and record these as Trial 2.
9. Move the lamp closer to the mirror so that $d_{0}$ is a few centimeters larger than $f$. Record this as Trial 3. Move the screen until an image is obtained on the screen. Measure $d_{\mathrm{i}}$ and $h_{\mathrm{i}}$, and record these as Trial 3.
10. Move the lamp so that $d_{0}$ is equal to $f$. Record this as Trial 4 data. Move the screen back and forth and try to obtain an image. What do you observe?
11. Move the lamp so that $d_{0}$ is less than $f$ by a few centimeters. Record this as Trial 5. Move the screen back and forth and try to obtain an image. What do you observe?

## Analyze

1. Use Numbers Calculate $1 / d_{0}$ and $1 / d_{\mathrm{i}}$ and enter the values in the calculation table.
2. Use Numbers Calculate the sum of $1 / d_{0}$ and $1 / d_{\mathrm{i}}$ and enter the values in the calculation table. Calculate the reciprocal of this number and enter it in the calculation table as $f_{\text {calc }}$.
3. Error Analysis Compare the experimental focal length, $f_{\text {calc }}$, with $f$, the accepted focal length, by finding the percent error.
percent error $=\frac{\left|f-f_{\text {calc }}\right|}{f} \times 100$

## Conclude and Apply

1. Classify What type of image was observed in each of the trials?
2. Analyze What conditions cause real images to be formed?
3. Analyze What conditions cause virtual images to be formed?

## Going Further

1. What are the conditions needed for the image to be larger than the object?
2. Review the methods used for data collection. Identify sources of error and what might be done to improve accuracy.

## Real-World Physics

What advantage would there be in using a telescope with a concave mirror?

## Physics nline

To find out more about reflection, visit the Web site: physicspp.com

## Future Jechnology

Adaptive Optical Systems

Objects in space are difficult to observe from Earth because they twinkle. Our moving, unevenlyheated atmosphere refracts their light in a chaotic manner. It's like trying to look at a small object through the bottom of an empty, clear, glass jar while rotating the jar.

## Flexible Adaptive Mirror

An adaptive optical system (AOS) continuously compensates for the distortion of the atmosphere, removing the twinkle from star images to allow astronomers to view and photograph steady images of the most distant objects in the visible universe.

An AOS directs the magnified image of the stars from the telescope onto a flexible adaptive mirror made of thin glass. This mirror is stretched across 20-30 movable pistons that can poke or pull the surface of the mirror into many complicated shapes. Each piston is driven by a fast, computer-controlled motor. When the mirror surface is shaped into just the right pattern at just the right time, it will compensate for the convective movement of the atmosphere between the telescope and the star, and reflect a clear image to the observer or camera.

Wave-Front Sensor To detect the atmospheric distortion at each instant of time, a wave-front sensor looks at a single star through the telescope. This device has an array of tiny lenses (lenslets) in several rows. Each lenslet forms an image of the star on a sensitive screen behind it. The position of each image can be read by the computer.

If each image is not directly behind its lenslet, then the computer knows that the star's light waves are being distorted by the atmosphere. A star is a distant point source of light, so it should produce plane waves. Distorted images of a star are non-planar light waves, and these uneven waves cause the images of the star behind some of the lenslets to be displaced.


AOS compensates for distortion when viewing Titan, Saturn's largest moon.


The computer looks at this error and calculates how the adaptive mirror should be wrinkled to bring each of the lenslet images back into place. The star image reflected to the observer then will be correct, and a clear image of all other objects (like galaxies and planets) in the vicinity also will be seen clearly. The adaptive mirror is re-shaped about 1000 times per second.

## Going Further

1. Research What is done if there is not a suitable star for the wave-front sensor to analyze in a region of space under observation?
2. Apply Research how adaptive optics will be used in the future to correct vision.

### 17.1 Reflection from Plane Mirrors

## Vocabulary

- specular reflection (p. 459)
- diffuse reflection (p. 459)
- plane mirror (p. 461)
- object (p. 461)
- image (p. 461)
- virtual image (p. 461)


## Key Concepts

- According to the law of reflection, the angle that an incident ray makes with the normal equals the angle that the reflected ray makes with the normal.

$$
\theta_{\mathrm{r}}=\theta_{\mathrm{i}}
$$

- The law of reflection applies to both smooth and rough surfaces. A rough surface, however, has normals that go in lots of different directions, which means that parallel incident rays will not be reflected in parallel.
- A smooth surface produces specular reflection. A rough surface produces diffuse reflection.
- Specular reflection results in the formation of images that appear to be behind plane mirrors.
- An image produced by a plane mirror is always virtual, is the same size as the object, has the same orientation, and is the same distance from the mirror as the object.

$$
d_{\mathrm{i}}=-d_{\mathrm{o}} \quad h_{\mathrm{i}}=h_{\mathrm{o}}
$$

### 17.2 Curved Mirrors

## Vocabulary

- concave mirror (p. 464)
- principal axis (p. 464)
- focal point (p. 464)
- focal length (p. 464)
- real image (p. 465)
- spherical aberration (p. 467)
- magnification (p. 468)
- convex mirror (p. 471)


## Key Concepts

- You can locate the image created by a spherical mirror by drawing two rays from a point on the object to the mirror. The intersection of the two reflected rays is the image of the object point.
- The mirror equation gives the relationship among image position, object position, and focal length of a spherical mirror.

$$
\frac{1}{f}=\frac{1}{d_{\mathrm{i}}}=\frac{1}{d_{\mathrm{o}}}
$$

- The magnification of a mirror image is given by equations relating either the positions or the heights of the image and the object.

$$
m \equiv \frac{h_{\mathrm{i}}}{h_{\mathrm{o}}}=\frac{-d_{\mathrm{i}}}{d_{\mathrm{o}}}
$$

- A single concave mirror produces a real image that is inverted when the object position is greater than the focal length.
- A single concave mirror produces a virtual image that is upright when the object position is less than the focal length.
- A single convex mirror always produces a virtual image that is upright and smaller compared to the object.
- By forming smaller images, convex mirrors make images seem farther away and produce a wide field of view.
- Mirrors can be used in combinations to produce images of any size, orientation, and location desired. The most common use of combinations of mirrors is as telescopes.


## Concept Mapping

30. Complete the following concept map using the following terms: convex, upright, inverted, real, virtual.


## Mastering Concepts

31. How does specular reflection differ from diffuse reflection? (17.1)
32. What is meant by the phrase "normal to the surface"? (17.1)
33. Where is the image produced by a plane mirror located? (17.1)
34. Describe the properties of a plane mirror. (17.1)
35. A student believes that very sensitive photographic film can detect a virtual image. The student puts photographic film at the location of a virtual image. Does this attempt succeed? Explain. (17.1)
36. How can you prove to someone that an image is a real image? (17.1)
37. An object produces a virtual image in a concave mirror. Where is the object located? (17.2)
38. What is the defect that all concave spherical mirrors have and what causes it? (17.2)
39. What is the equation relating the focal point, object position, and image position? (17.2)
40. What is the relationship between the center of curvature and the focal length of a concave mirror? (17.2)
41. If you know the image position and object position relative to a curved mirror, how can you determine the mirror's magnification? (17.2)
42. Why are convex mirrors used as rearview mirrors? (17.2)
43. Why is it impossible for a convex mirror to form a real image? (17.2)

## Applying Concepts

44. Wet Road A dry road is more of a diffuse reflector than a wet road. Based on Figure 17-16, explain why a wet road appears blacker to a driver than a dry road does.


Figure 17-16
45. Book Pages Why is it desirable that the pages of a book be rough rather than smooth and glossy?
46. Locate and describe the physical properties of the image produced by a concave mirror when the object is located at the center of curvature.
47. An object is located beyond the center of curvature of a spherical concave mirror. Locate and describe the physical properties of the image.
48. Telescope You have to order a large concave mirror for a telescope that produces high-quality images. Should you order a spherical mirror or a parabolic mirror? Explain.
49. Describe the properties of the image seen in the single convex mirror in Figure 17-17.


- Figure 17-17

50. List all the possible arrangements in which you could use a spherical mirror, either concave or convex, to form a real image.
51. List all possible arrangements in which you could use a spherical mirror, either concave or convex, to form an image that is smaller compared to the object.
52. Rearview Mirrors The outside rearview mirrors of cars often carry the warning "Objects in the mirror are closer than they appear." What kind of mirrors are these and what advantage do they have?

## Mastering Problems

### 17.1 Reflection from Plane Mirrors

53. A ray of light strikes a mirror at an angle of $38^{\circ}$ to the normal. What is the angle that the reflected angle makes with the normal?
54. A ray of light strikes a mirror at an angle of $53^{\circ}$ to the normal.
a. What is the angle of reflection?
b. What is the angle between the incident ray and the reflected ray?
55. A ray of light incident upon a mirror makes an angle of $36^{\circ}$ with the mirror. What is the angle between the incident ray and the reflected ray?
56. Picture in a Mirror Penny wishes to take a picture of her image in a plane mirror, as shown in Figure $\mathbf{1 7 - 1 8}$. If the camera is 1.2 m in front of the mirror, at what distance should the camera lens be focused?


Figure 17-18
57. Two adjacent plane mirrors form a right angle, as shown in Figure 17-19. A light ray is incident upon one of the mirrors at an angle of $30^{\circ}$ to the normal.
a. What is the angle at which the light ray is reflected from the other mirror?
b. A retroreflector is a device that reflects incoming light rays back in a direction opposite to that of the incident rays. Draw a diagram showing the angle of incidence on the first mirror for which the mirror system acts as a retroreflector.


Figure 17-19
58. Draw a ray diagram of a plane mirror to show that if you want to see yourself from your feet to the top of your head, the mirror must be at least half your height.
59. Two plane mirrors are connected at their sides so that they form a $45^{\circ}$ angle between them. A light ray strikes one mirror at an angle of $30^{\circ}$ to the normal and then reflects off the second mirror. Calculate the angle of reflection of the light ray off the second mirror.
60. A ray of light strikes a mirror at an angle of $60^{\circ}$ to the normal. The mirror is then rotated $18^{\circ}$ clockwise, as shown in Figure 17-20. What is the angle that the reflected ray makes with the mirror?


Figure 17-20

## Chapter 17 Assessment

### 17.2 Curved Mirrors

61. A concave mirror has a focal length of 10.0 cm . What is its radius of curvature?
62. An object located 18 cm from a convex mirror produces a virtual image 9 cm from the mirror. What is the magnification of the image?
63. Fun House A boy is standing near a convex mirror in a fun house at a fair. He notices that his image appears to be 0.60 m tall. If the magnification of the mirror is $\frac{1}{3}$, what is the boy's height?
64. Describe the image produced by the object in

Figure 17-21 as real or virtual, inverted or upright, and smaller or larger than the object.


Figure 17-21
65. Star Image Light from a star is collected by a concave mirror. How far from the mirror is the image of the star if the radius of curvature is 150 cm ?
66. Find the image position and height for the object shown in Figure 17-22.


Figure 17-22
67. Rearview Mirror How far does the image of a car appear behind a convex mirror, with a focal length of -6.0 m , when the car is 10.0 m from the mirror?
68. An object is 30.0 cm from a concave mirror of 15.0 cm focal length. The object is 1.8 cm tall. Use the mirror equation to find the image position. What is the image height?
69. Dental Mirror A dentist uses a small mirror with a radius of 40 mm to locate a cavity in a patient's tooth. If the mirror is concave and is held 16 mm from the tooth, what is the magnification of the image?
70. A $3.0-\mathrm{cm}$-tall object is 22.4 cm from a concave mirror. If the mirror has a radius of curvature of 34.0 cm , what are the image position and height?
71. Jeweler's Mirror A jeweler inspects a watch with a diameter of 3.0 cm by placing it 8.0 cm in front of a concave mirror of $12.0-\mathrm{cm}$ focal length.
a. Where will the image of the watch appear?
b. What will be the diameter of the image?
72. Sunlight falls on a concave mirror and forms an image that is 3.0 cm from the mirror. An object that is 24 mm tall is placed 12.0 cm from the mirror.
a. Sketch the ray diagram to show the location of the image.
b. Use the mirror equation to calculate the image position.
c. How tall is the image?
73. Shiny spheres that are placed on pedestals on a lawn are convex mirrors. One such sphere has a diameter of 40.0 cm . A $12-\mathrm{cm}$-tall robin sits in a tree that is 1.5 m from the sphere. Where is the image of the robin and how tall is the image?

## Mixed Review

74. A light ray strikes a plane mirror at an angle of $28^{\circ}$ to the normal. If the light source is moved so that the angle of incidence increases by $34^{\circ}$, what is the new angle of reflection?
75. Copy Figure 17-23 on a sheet of paper. Draw rays on the diagram to determine the height and location of the image.

76. An object is located 4.4 cm in front of a concave mirror with a $24.0-\mathrm{cm}$ radius. Locate the image using the mirror equation.
77. A concave mirror has a radius of curvature of 26.0 cm . An object that is 2.4 cm tall is placed 30.0 cm from the mirror.
a. Where is the image position?
b. What is the image height?
78. What is the radius of curvature of a concave mirror that magnifies an object by a factor of +3.2 when the object is placed 20 cm from the mirror?
79. A convex mirror is needed to produce an image one-half the size of an object and located 36 cm behind the mirror. What focal length should the mirror have?
80. Surveillance Mirror A convenience store uses a surveillance mirror to monitor the store's aisles. Each mirror has a radius of curvature of 3.8 m .
a. What is the image position of a customer who stands 6.5 m in front of the mirror?
b. What is the image height of a customer who is 1.7 m tall?
81. Inspection Mirror A production-line inspector wants a mirror that produces an image that is upright with a magnification of 7.5 when it is located 14.0 mm from a machine part.
a. What kind of mirror would do this job?
b. What is its radius of curvature?
82. The object in Figure 17-24 moves from position 1 to position 2. Copy the diagram onto a sheet of paper. Draw rays showing how the image changes.


Figure 17-24
83. A ball is positioned 22 cm in front of a spherical mirror and forms a virtual image. If the spherical mirror is replaced with a plane mirror, the image appears 12 cm closer to the mirror. What kind of spherical mirror was used?
84. A $1.6-\mathrm{m}$-tall girl stands 3.2 m from a convex mirror. What is the focal length of the mirror if her image appears to be 0.28 m tall?
85. Magic Trick A magician uses a concave mirror with a focal length of 8.0 m to make a $3.0-\mathrm{m}$-tall hidden object, located 18.0 m from the mirror, appear as a real image that is seen by his audience. Draw a scale ray diagram to find the height and location of the image.
86. A $4.0-\mathrm{cm}$-tall object is placed 12.0 cm from a convex mirror. If the image of the object is 2.0 cm tall, and the image is located at -6.0 cm , what is the focal length of the mirror? Draw a ray diagram to answer the question. Use the mirror equation and the magnification equation to verify your answer.

## Thinking Critically

87. Apply Concepts The ball in Figure 17-25 slowly rolls toward the concave mirror on the right. Describe how the size of the ball's image changes as it rolls along.


Figure 17-25
88. Analyze and Conclude The object in Figure 17-26 is located 22 cm from a concave mirror. What is the focal length of the mirror?


Figure 17-26

## Chapter 17 Assessment

89. Use Equations Show that as the radius of curvature of a concave mirror increases to infinity, the mirror equation reduces to the relationship between the object position and the image position for a plane mirror.
90. Analyze and Conclude An object is located 6.0 cm from a plane mirror. If the plane mirror is replaced with a concave mirror, the resulting image is 8.0 cm farther behind the mirror. Assuming that the object is located between the focal point and the concave mirror, what is the focal length of the concave mirror?
91. Analyze and Conclude The layout of the twomirror system shown in Figure 17-11 is that of a Gregorian telescope. For this question, the larger concave mirror has a radius of curvature of 1.0 m , and the smaller mirror is located 0.75 m away. Why is the secondary mirror concave?
92. Analyze and Conclude An optical arrangement used in some telescopes is the Cassegrain focus, shown in Figure 17-27. This telescope uses a convex secondary mirror that is positioned between the primary mirror and the focal point of the primary mirror.


Figure 17-27
a. A single convex mirror produces only virtual images. Explain how the convex mirror in this telescope functions within the system of mirrors to produce real images.
b. Are the images produced by the Cassegrain focus upright or inverted? How does this relate to the number of times that the light crosses?

## Writing in Physics

93. Research a method used for grinding, polishing, and testing mirrors used in reflecting telescopes. You may report either on methods used by amateur astronomers who make their own telescope optics, or on a method used by a project at a national laboratory. Prepare a one-page report describing the method, and present it to the class.
94. Mirrors reflect light because of their metallic coating. Research and write a summary of one of the following:
a. the different types of coatings used and the advantages and disadvantages of each
b. the precision optical polishing of aluminum to such a degree of smoothness that no glass is needed in the process of making a mirror

## Cumulative Review

95. A child runs down the school hallway and then slides on the newly waxed floor. He was running at $4.7 \mathrm{~m} / \mathrm{s}$ before he started sliding and he slid 6.2 m before stopping. What was the coefficient of friction of the waxed floor? (Chapter 11)
96. A 1.0 g piece of copper falls from a height of $1.0 \times 10^{4} \mathrm{~m}$ from an airplane to the ground. Because of air resistance it reaches the ground moving at a velocity of $70.0 \mathrm{~m} / \mathrm{s}$. Assuming that half of the energy lost by the piece was distributed as thermal energy to the copper, how much did it heat during the fall? (Chapter 12)
97. It is possible to lift a person who is sitting on a pillow made from a large sealed plastic garbage bag by blowing air into the bag through a soda straw. Suppose that the cross-sectional area of the person sitting on the bag is $0.25 \mathrm{~m}^{2}$ and the person's weight is 600 N . The soda straw has a cross-sectional area of $2 \times 10^{-5} \mathrm{~m}^{2}$. With what pressure must you blow into the straw to lift the person that is sitting on the sealed garbage bag? (Chapter 13)
98. What would be the period of a $2.0-\mathrm{m}$-long pendulum on the Moon's surface? The Moon's mass is $7.34 \times 10^{22} \mathrm{~kg}$, and its radius is $1.74 \times 10^{6} \mathrm{~m}$. What is the period of this pendulum on Earth? (Chapter 14)
99. Organ pipes An organ builder must design a pipe organ that will fit into a small space. (Chapter 15)
a. Should he design the instrument to have open pipes or closed pipes? Explain.
b. Will an organ constructed with open pipes sound the same as one constructed with closed pipes? Explain.
100. Filters are added to flashlights so that one shines red light and the other shines green light. The beams are crossed. Explain in terms of waves why the light from both flashlights is yellow where the beams cross, but revert back to their original colors beyond the intersection point. (Chapter 16)

## Standardized Test Practice

## Multiple Choice

1. Where is the object located if the image that is produced by a concave mirror is smaller than the object?
(A) at the mirror's focal point
(B) between the mirror and the focal point
(C) between the focal point and center of curvature
(D) past the center of curvature
2. What is the focal length of a concave mirror that magnifies, by a factor of +3.2 , an object that is placed 30 cm from the mirror?
```
(A) }23\textrm{cm
(C) }44\textrm{cm
(B) }32\textrm{cm
(D) }46\textrm{cm
```

3. An object is placed 21 cm in front of a concave mirror with a focal length of 14 cm . What is the image position?
(A) -42 cm
(C) 8.4 cm
(B) -8.4 cm
(D) 42 cm
4. The light rays in the illustration below do not properly focus at the focal point. This problem occurs with $\qquad$ _.
(A) all spherical mirrors
(B) all parabolic mirrors
(C) only defective spherical mirrors
(D) only defective parabolic mirrors

5. A ray of light strikes a plane mirror at an angle of $23^{\circ}$ to the normal. What is the angle between the reflected ray and the mirror?
```
(A) }2\mp@subsup{3}{}{\circ
(C) }6\mp@subsup{7}{}{\circ
(B) }4\mp@subsup{6}{}{\circ
(D) 134
```

6. A concave mirror produces an inverted image that is 8.5 cm tall, located 34.5 cm in front of the mirror. If the focal point of the mirror is 24.0 cm , then what is the height of the object that is reflected?
(A) 2.3 cm
(C) 14 cm
(B) 3.5 cm
(D) 19 cm
7. A concave mirror with a focal length of 16.0 cm produces an image located 38.6 cm from the mirror. What is the distance of the object from the front of the mirror?
(A) 2.4 cm
(C) 22.6 cm
(B) 11.3 cm
(D) 27.3 cm
8. A convex mirror is used to produce an image that is three-fourths the size of an object and located 8.4 cm behind the mirror. What is the focal length of the mirror?
(A) -34 cm
(C) -6.3 cm
(B) -11 cm
(D) -4.8 cm
9. A cup sits 17 cm from a concave mirror. The image of the book appears 34 cm in front of the mirror. What are the magnification and orientation of the cup's image?
(A) 0.5, inverted
(C) 2.0, inverted
(B) 0.5 , upright
(D) 2.0, upright

## Extended Answer

10. A $5.0-\mathrm{cm}$-tall object is located 20.0 cm from a convex mirror with a focal length of -14.0 cm . Draw a scale-ray diagram showing the image height.

## Test-Taking TIP

## Your Answers Are Better Than the Test's

When you know how to solve a problem, solve it before looking at the answer choices. Often, more than one answer choice will look good, so do the calculations first, and arm yourself with your answer before looking.

## Chapter 18

## Refraction and Lenses

## What You'll Learn

- You will learn how light changes direction and speed when it travels through different materials.
- You will compare properties of lenses and the images that they form.
- You will learn about different applications of lenses, including how lenses in your eyes enable you to see.

Why It's Important
Some light travels in a straight path from objects to your eyes. Some light is reflected before it reaches you. Other light follows a path that appears to be bent.
Wavy Trees If you swim underwater, you will notice that things underwater look normal, but objects above the surface of the water appear to be distorted by the waves on the surface.

Think About This $>$
What causes the images of the trees to be wavy?

## Physics inline physicspp.com

## LAUNCH Lab <br> What does a straw in a liquid look like from the side view?

## Question

Does a straw look different when observed through water, oil, and corn syrup?

## Procedure 일 푼

1. Fill one $400-\mathrm{mL}$ beaker with water.
2. Fill a second $400-\mathrm{mL}$ beaker halfway with corn syrup and the rest with water (pour slowly as to avoid mixing the two liquids).
3. Fill a third $400-\mathrm{mL}$ beaker halfway with water and the rest with cooking oil (pour slowly as to avoid mixing the two liquids).
4. Place a straw gently in each beaker and lean it on the spout.
5. Observe each straw through the side of the beaker as you slowly turn the beaker.
6. Make a data table to record descriptions of the straws' appearance in each solution.

## Analysis

In which containers does the straw appear to be broken? Are all amounts of break the same? When does the straw not appear to be broken? Explain.

Critical Thinking Form a hypothesis as to when a solid object appears to be broken and when it does not. Be sure to include an explanation of the amount of break.


### 18.1 Refraction of Light

Looking at the surface of a swimming pool on a summer day, you can see sunlight reflecting off the water. You can see objects that are in the pool because some of the sunlight travels into the water and reflects off the objects. When you look closely at objects in the water, however, you will notice that they look distorted. For example, things beneath the surface look closer than normal, the feet of a person standing still in the pool appear to move back and forth, and lines along the bottom of the pool seem to sway with the movement of the water. These effects occur because light changes direction as it passes from water to air.

As you learned in Chapter 16, the path of light is bent as it crosses the boundary between two media due to refraction. The amount of refraction depends on properties of the two media and on the angle at which the light strikes the boundary. As waves travel along the surface of the water, the boundary between the air and water moves up and down, and tilts back and forth. The path of light leaving the water shifts as the boundary moves, causing objects under the surface to appear to waver.

## - Objectives

- Solve problems involving refraction.
- Explain total internal reflection.
- Explain some optical effects caused by refraction.
- Vocabulary
index of refraction
Snell's law of refraction critical angle total internal reflection dispersion
- Figure 18-1 Light bends toward the normal as it moves from air to glass and bends away from the normal as it moves from glass to air (a). The bending of light makes objects appear to be shifted from their actual locations (b).


## Color Convention

- Refracting media and lenses are light blue.

| Table 18-1 |  |
| :--- | :---: |
| Indices of Refraction <br> for Yellow Light <br> $\mathbf{( \lambda = 5 8 9 ~ n m ~ i n ~ v a c u u m ) ~}$ |  |
| Medium | $\boldsymbol{n}$ |
| Vacuum | 1.00 |
| Air | 1.0003 |
| Water | 1.33 |
| Ethanol | 1.36 |
| Crown glass | 1.52 |
| Quartz | 1.54 |
| Flint glass | 1.62 |
| Diamond | 2.42 |



## Snell's Law of Refraction

What happens when you shine a narrow beam of light at the surface of a piece of glass? As you can see in Figure 18-1, it bends as it crosses the boundary from air to glass. The bending of light, called refraction, was first studied by René Descartes and Willebrord Snell around the time of Kepler and Galileo.

To discuss the results of Descartes and Snell, you have to define two angles. The angle of incidence, $\theta_{1}$, is the angle at which the light ray strikes the surface. It is measured from the normal to the surface. The angle of refraction, $\theta_{2}$, is the angle at which the transmitted light leaves the surface. It also is measured with respect to the normal. In 1621, Snell found that when light passed from air into a transparent substance, the sines of the angles were related by the equation $\sin \theta_{1} / \sin \theta_{2}=n$. Here $n$ is a constant that depends on the substance, not on the angles, and is called the index of refraction. The indices of refraction for some substances are listed in Table 18-1. The relationship found by Snell is valid when light goes across a boundary between any two materials. This more general equation is known as Snell's law of refraction.

Snell's Law of Refraction $\quad n_{1} \sin \theta_{1}=n_{2} \sin \theta_{2}$
The product of the index of refraction of the first medium and the sine of the angle of incidence is equal to the product of the index of refraction of the second medium and the sine of the angle of refraction.

Figure 18-1 shows how Snell's law applies when light travels through a piece of glass with parallel surfaces, such as a windowpane. The light is refracted both when it enters the glass and again when it leaves the glass. When light goes from air to glass it moves from material with a lower index of refraction to one with a higher index of refraction. That is, $n_{1}<n_{2}$. To keep the two sides of the equation equal, one must have $\sin \theta_{1}>\sin \theta_{2}$. The light beam is bent toward the normal to the surface.

When light travels from glass to air it moves from material having a higher index of refraction to one with a lower index. In this case, $n_{1}>n_{2}$. To keep the two sides of the equation equal one must have $\sin \theta_{1}^{\prime}<\sin \theta_{2}^{\prime}$. That is, the light is bent away from the normal. Note that the direction of the ray when it leaves the glass is the same as it was before it struck the glass, but it is shifted from its original position.

## EXAMPLE Problem 1

Angle of Refraction A light beam in air hits a sheet of crown glass at an angle of $30.0^{\circ}$. At what angle is the light beam refracted?

## 1 Analyze and Sketch the Problem

- Make a sketch of the air and crown glass boundary.
- Draw a ray diagram.

Known: Unknown:
$\theta_{1}=30.0^{\circ} \quad \theta_{2}=$ ?
$n_{1}=1.00$
$n_{2}=1.52$


## 2 Solve for the Unknown

Use Snell's law to solve for the sine of the angle of refraction.

$$
\begin{aligned}
n_{1} \sin \theta_{1} & =n_{2} \sin \theta_{2} \\
\sin \theta_{2} & =\left(\frac{n_{1}}{n_{2}}\right) \sin \theta_{1} \\
\theta_{2} & =\sin ^{-1}\left(\left(\frac{n_{1}}{n_{2}}\right) \sin \theta_{1}\right) \\
& =\sin ^{-1}\left(\left(\frac{1.00}{1.52}\right) \sin 30.0^{\circ}\right) \quad \text { Substitute } n_{1}=1.00, n_{2}=1.52, \theta_{1}=30.0^{\circ} \\
& =19.2^{\circ}
\end{aligned}
$$

## 3 Evaluate the Answer

- Are the units correct? Angles are expressed in degrees.
- Is the magnitude realistic? The index of refraction, $n_{2}$, is greater than the index of refraction, $n_{1}$. Therefore, the angle of refraction, $\theta_{2}$, must be less than the angle of incidence, $\theta_{1}$.


## PRACTICE Problems

## Additional Problems, Appendix B

1. A laser beam in air is incident upon ethanol at an angle of incidence of $37.0^{\circ}$. What is the angle of refraction?
2. Light in air is incident upon a piece of crown glass at an angle of incidence of $45.0^{\circ}$. What is the angle of refraction?
3. Light passes from air into water at $30.0^{\circ}$ to the normal. Find the angle of refraction.
4. Light is incident upon a diamond facet at $45.0^{\circ}$. What is the angle of refraction?
5. A block of unknown material is submerged in water. Light in the water is incident on the block at an angle of incidence of $31^{\circ}$. The angle of refraction of the light in the block is $27^{\circ}$. What is the index of refraction of the material of the block?

Refraction is responsible for the Moon appearing red during a lunar eclipse. A lunar eclipse occurs when Earth blocks sunlight from the Moon. As a result, you might expect the Moon to be completely dark. Instead, light refracts through Earth's atmosphere and bends around Earth toward the Moon. Recall that Earth's atmosphere scatters most of the blue and green light. Thus, mostly red light illuminates the Moon. Because the Moon reflects most colors of light equally well, it reflects the red light back to Earth, and therefore the Moon appears to be red.


Figure 18-2 Light moves from air to glass to air again (a). Light slows down and bends toward the normal when it enters a region of a higher index of refraction (b).

## Wave Model of Refraction

The wave model of light was developed almost 200 years after Snell published his research. An understanding that light interacts with atoms when traveling through a medium, such that it moves more slowly than in a vacuum, was achieved three hundred years after Snell's work. The wave relationship that you learned in Chapter 16 for light traveling through a vacuum, $\lambda_{0}=c / f$, can be rewritten as $\lambda=v / f$, where $v$ is the speed of light in any medium and $\lambda$ is the wavelength. The frequency of light, $f$, does not change when it crosses a boundary. That is, the number of oscillations per second that arrive at a boundary is the same as the number that leave the boundary and transmit through the refracting medium. So, the wavelength of light, $\lambda$, must decrease when light slows down. Wavelength in a medium is shorter than wavelength in a vacuum.

What happens when light travels from a region with a high speed into one with a low speed, as shown in Figure 18-2a? The diagram in Figure $\mathbf{1 8}-\mathbf{2 b}$ shows a beam of light as being made up of a series of parallel, straight wave fronts. Each wave front represents the crest of a wave and is perpendicular to the direction of the beam. The beam strikes the surface at an angle, $\theta_{1}$. Consider the triangle PQR. Because the wave fronts are perpendicular to the direction of the beam, $\angle \mathrm{PQR}$ is a right angle and $\angle \mathrm{QRP}$ is equal to $\theta_{1}$. Therefore, $\sin \theta_{1}$ is equal to the distance between P and Q divided by the distance between P and R .

$$
\sin \theta_{1}=\frac{\overline{\mathrm{PQ}}}{\overline{\mathrm{PR}}}
$$

The angle of refraction, $\theta_{2}$, can be related in a similar way to the triangle PSR. In this case

$$
\sin \theta_{2}=\frac{\overline{\mathrm{RS}}}{\overline{\mathrm{PR}}}
$$

By taking the ratio of the sines of the two angles, $\overline{\mathrm{PR}}$ is canceled, leaving the following equation:

$$
\frac{\sin \theta_{2}}{\sin \theta_{1}}=\frac{\overline{\mathrm{RS}}}{\overline{\mathrm{PQ}}}
$$

Figure $18-2 \mathrm{~b}$ is drawn such that the distance between P and Q is equal to the length of three wavelengths of light in medium 1 , or $\overline{\mathrm{PQ}}=3 \lambda_{1}$. In a similar way, $\overline{\mathrm{RS}}=3 \lambda_{2}$. Substituting these two values into the previous equation and canceling the common factor of 3 provides an equation that relates the angles of incidence and refraction with the wavelength of the light in each medium.

$$
\frac{\sin \theta_{2}}{\sin \theta_{1}}=\frac{3 \lambda_{2}}{3 \lambda_{1}}=\frac{\lambda_{2}}{\lambda_{1}}
$$

Using $\lambda=v / f$ in the above equation and canceling the common factor of $f$, the equation is rewritten as follows:

$$
\frac{\sin \theta_{2}}{\sin \theta_{1}}=\frac{v_{2}}{v_{1}}
$$

Snell's law also can be written as a ratio of the sines of the angles of incidence and refraction.

$$
\frac{\sin \theta_{2}}{\sin \theta_{1}}=\frac{n_{1}}{n_{2}}
$$

Index of refraction Using the transitive property of equality, the previous two equations lead to the following equation:

$$
\frac{n_{1}}{n_{2}}=\frac{v_{2}}{v_{1}}
$$

In a vacuum, $n=1$ and $v=c$. If either medium is a vacuum, then the equation is simplified to an equation that relates the index of refraction to the speed of light in a medium.

Index of Refraction $n=\frac{c}{v}$
The index of refraction of a medium is equal to the speed of light in a vacuum divided by the speed of light in the medium.

This definition of the index of refraction can be used to find the wavelength of light in a medium compared to the wavelength the light would have in a vacuum. In a medium with an index of refraction $n$, the speed of light is given by $v=c / n$. The wavelength of the light in a vacuum is $\lambda_{0}=c / f$. Solve for frequency, and substitute $f=c / \lambda_{0}$ and $v=c / n$ into $\lambda=v / f . \lambda=(c / n) /\left(c / \lambda_{0}\right)=\lambda_{0} / n$, and thus the wavelength of light in a medium is smaller than the wavelength in a vacuum.

## Total Internal Reflection

The angle of refraction is larger than the angle of incidence when light passes into a medium of a lower index of refraction, as shown in Figure 18-3a. This leads to an interesting phenomenon. As the angle of incidence increases, the angle of refraction increases. At a certain angle of incidence known as the critical angle, $\theta_{\mathrm{c}^{\prime}}$ the refracted light ray lies along the boundary of the two media, as shown in Figure 18-3b.

Recall from Chapter 16 that when light strikes a transparent boundary, even though much of the light is transmitted, some is reflected. Total internal reflection occurs when light traveling from a region of a higher index of refraction to a region of a lower index of refraction strikes the boundary at an angle greater than the critical angle such that all light reflects back into the region of the higher index of refraction, as shown in Figure 18-3c. To construct an equation for the critical angle of any boundary, you can use Snell's law and substitute $\theta_{1}=\theta_{\mathrm{c}}$ and $\theta_{2}=90.0^{\circ}$.

## Critical Angle for Total Internal Reflection $\sin \theta_{c}=\frac{n_{2}}{n_{1}}$

The sine of the critical angle is equal to the index of refraction of the refracting medium divided by the index of refraction of the incident medium.

Total internal reflection causes some curious effects. Suppose that you are looking up at the surface from underwater in a calm pool. You might see an upside-down reflection of another nearby object that also is underwater or a reflection of the bottom of the pool itself. The surface of the water acts like a mirror. Likewise, when you are standing on the side of a pool, it is possible for things below the surface of the water to not be visible to you. When a swimmer is underwater, near the surface, and on the opposite side of the pool from you, you might not see him or her. This is because the light from his or her body is reflected.

- Figure 18-3 Ray $A$ is partially refracted and partially reflected (a). Ray $B$ is refracted along the boundary of the medium and forms the critical angle (b). An angle of incidence greater than the critical angle results in the total internal reflection of Ray C, which follows the law of reflection (c).



Figure 18-4 Light impulses from a source enter one end of the optical fiber. Each time the light strikes the surface, the angle of incidence is larger than the crictical angle, and, therefore, the light is kept within the fiber.

- Figure 18-5 A mirage is seen on the surface of a road (a). Light from the car bends upward into the eye of the observer (b). The bottom of the wave front moves faster than the top (c).

Optical fibers are an important technical application of total internal reflection. As shown in Figure 18-4, the light traveling through the transparent fiber always hits the internal boundary of the optical fiber at an angle greater than the critical angle, so all of the light is reflected and none of the light is transmitted through the boundary. Thus, the light maintains its intensity over the distance of the fiber.

## Mirages

On a hot summer day, you sometimes can see the mirage effect shown in Figure 18-5a. As you drive down a road, you see what appears to be the reflection of an oncoming car in a pool of water. The pool, however, disappears as you approach it. The mirage is the result of the Sun heating the road. The hot road heats the air above it and produces a thermal layering of air that causes light traveling toward the road to gradually bend upward. This makes the light appear to be coming from a reflection in a pool, as shown in Figure 18-5b.

Figure 18-5c shows how this occurs. As light from a distant object travels downward toward the road, the index of refraction of the air decreases as the air gets hotter, but the temperature change is gradual. Recall from Chapter 16 that light wave fronts are comprised of Huygens' wavelets. In the case of a mirage, the Huygens' wavelets closer to the ground travel faster than those higher up, causing the wave fronts to gradually turn upward. A similar phenomenon, called a superior mirage, occurs when a reflection of a distant boat appears above the boat. The water keeps the air that is closer to its surface cooler.


## Dispersion of Light

The speed of light in a medium is determined by interactions between the light and the atoms that make up the medium. Recall from Chapters 12 and 13 that temperature and pressure are related to the energy of particles on the atomic level. The speed of light, and therefore, the index of refraction for a gaseous medium, can change slightly with temperature. In addition, the speed of light and the index of refraction vary for different wavelengths of light in the same liquid or solid medium.

You learned in Chapter 16 that white light separates into a spectrum of colors when it passes through a glass prism, as shown in Figure 18-6a. This phenomenon is called dispersion. If you look carefully at the light that passes through a prism, you will notice that violet is refracted more than red, as shown in Figure 18-6b. This occurs because the speed of violet light through glass is less than the speed of red light through glass. Violet light has a higher frequency than red light, which causes it to interact differently with the atoms of the glass. This results in glass having a slightly higher index of refraction for violet light than it has for red light.

Rainbows A prism is not the only means of dispersing light. A rainbow is a spectrum formed when sunlight is dispersed by water droplets in the atmosphere. Sunlight that falls on a water droplet is refracted. Because of dispersion, each color is refracted at a slightly different angle, as shown in Figure 18-7a. At the back surface of the droplet, some of the light undergoes internal reflection. On the way out of the droplet, the light once again is refracted and dispersed.

Although each droplet produces a complete spectrum, an observer positioned between the Sun and the rain will see only a certain wavelength of light from each droplet. The wavelength depends on the relative positions of the Sun, the droplet, and the observer, as shown in Figure 18-7b. Because there are many droplets in the sky, a complete spectrum is visible. The droplets reflecting red light make an angle of $42^{\circ}$ in relation to the direction of the Sun's rays; the droplets reflecting blue light make an angle of $40^{\circ}$.



Figure 18-6 White light directed through a prism is dispersed into bands of different colors (a). Different colors of light bend different amounts when they enter a medium (b).

- Figure 18-7 Rainbows form because white light is dispersed as it enters, reflects at the inside boundary, and exits the raindrops (a). Because of dispersion, only one color from each raindrop reaches an observer (b). (Illustration not to scale)
- Figure 18-8 A mist across your view allows for light comprising the entire spectrum of colors to reach your eyes in the form of a rainbow. Reflection from the raindrops sometimes enables you to see a second rainbow with the colors reversed.


Sometimes, you can see a faint second-order rainbow like the one shown in Figure 18-8. The second rainbow is outside of the first, is fainter, and has the order of the colors reversed. Light rays that are reflected twice inside water droplets produce this effect. Very rarely, a third rainbow is visible outside the second. What is your prediction about how many times light is reflected in the water droplets and the order of appearance of the colors for the third rainbow?

### 18.1 Section Review

6. Index of Refraction You notice that when a light ray enters a certain liquid from water, it is bent toward the normal, but when it enters the same liquid from crown glass, it is bent away from the normal. What can you conclude about the liquid's index of refraction?
7. Index of Refraction A ray of light has an angle of incidence of $30.0^{\circ}$ on a block of unknown material and an angle of refraction of $20.0^{\circ}$. What is the index of refraction of the material?
8. Speed of Light Could an index of refraction ever be less than 1 ? What would this imply about the speed of light in that medium?
9. Speed of Light What is the speed of light in chloroform ( $n=1.51$ )?
10. Total Internal Reflection If you were to use quartz and crown glass to make an optical fiber, which would you use for the cladding layer? Why?
11. Angle of Refraction $A$ beam of light passes from water into polyethylene with $n=1.50$. If $\theta_{\mathrm{i}}=57.5^{\circ}$, what is the angle of refraction in the polyethylene?
12. Critical Angle Is there a critical angle for light traveling from glass to water? From water to glass?
13. Dispersion Why can you see the image of the Sun just above the horizon when the Sun itself has already set?
14. Critical Thinking In what direction can you see a rainbow on a rainy late afternoon? Explain.

### 18.2 Convex and Concave Lenses

The refraction of light in nature that forms rainbows and red lunar eclipses is beautiful, but refraction also is useful. In 1303, French physician Bernard of Gordon wrote of the use of lenses to correct eyesight. Around 1610, Galileo used two lenses to make a telescope, with which he discovered the moons of Jupiter. Since Galileo's time, lenses have been used in many instruments, such as microscopes and cameras. Lenses are probably the most useful of all optical devices.

## Types of Lenses

A lens is a piece of transparent material, such as glass or plastic, that is used to focus light and form an image. Each of a lens's two faces might be either curved or flat. The lens in Figure 18-9a is called a convex lens because it is thicker at the center than at the edges. A convex lens often is called a converging lens because when surrounded by material with a lower index of refraction it refracts parallel light rays so that the rays meet at a point. The lens in Figure 18-9b is called a concave lens because it is thinner in the middle than at the edges. A concave lens often is called a diverging lens because when surrounded by material with a lower index of refraction rays passing through it spread out.

When light passes through a lens, refraction occurs at the two lens surfaces. Using Snell's law and geometry, you can predict the paths of rays passing through lenses. To simplify such problems, assume that all refraction occurs on a plane, called the principal plane, that passes through the center of the lens. This approximation, called the thin lens model, applies to all the lenses that you will learn about in this chapter section.
Lens equations The problems that you will solve involve spherical thin lenses, lenses that have faces with the same curvature as a sphere. Based on the thin lens model, as well as the other simplifications used in solving problems for spherical mirrors, equations have been developed that look exactly like the equations for spherical mirrors. The thin lens equation relates the focal length of a spherical thin lens to the object position and the image position.

## Thin Lens Equation $\frac{1}{f}=\frac{1}{d_{\mathrm{i}}}+\frac{1}{d_{\mathrm{o}}}$

The inverse of the focal length of a spherical lens is equal to the sum of the inverses of the image position and the object position.

The magnification equation for spherical mirrors used in Chapter 17 also can be used for spherical thin lenses. It is used to determine the height and orientation of the image formed by a spherical thin lens.

$$
\text { Magnification } \quad m \equiv \frac{h_{\mathrm{i}}}{h_{\mathrm{o}}}=\frac{-d_{\mathrm{i}}}{d_{\mathrm{o}}}
$$

The magnification of an object by a spherical lens, defined as the image height divided by the object height, is equal to the negative of the image position divided by the object position.

## - Objectives

- Describe how real and virtual images are formed by single convex and concave lenses.
- Locate images formed by lenses using ray tracing and equations.
- Explain how chromatic aberration can be reduced.
- Vocabulary
lens
convex lens
concave lens
thin lens equation chromatic aberration achromatic lens


Figure 18-9 A convex lens causes rays of light to converge (a). A concave lens causes rays of light to diverge (b).

| Properties of a Single Spherical Lens System |  |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Lens Type | $\boldsymbol{f}$ | $\boldsymbol{d}_{\mathbf{o}}$ | $\boldsymbol{d}_{\mathbf{i}}$ | $\boldsymbol{m}$ | Image |
| Convex | + | $d_{0}>2 f$ | $2 f>d_{\mathrm{i}}>f$ | Reduced <br> Inverted | Real |
|  |  | $2 f>d_{\mathrm{o}}>f$ | $d_{\mathrm{i}}>2 f$ | Enlarged <br> Inverted | Real |
|  | $f>d_{\mathrm{o}}>0$ | $\left\|d_{\mathrm{i}}\right\|>d_{\mathrm{o}}$ <br> (negative) | Enlarged | Virtual |  |
|  | - | $d_{\mathrm{o}}>0$ | $\|f\|>\left\|d_{\mathrm{i}}\right\|>0$ <br> (negative) | Reduced | Virtual |

Using the equations for lenses It is important that you use the proper sign conventions when using these equations. Table 18-2 shows a comparison of the image position, magnification, and type of image formed by single convex and concave lenses when an object is placed at various object positions, $d_{\mathrm{o}^{\prime}}$ relative to the lens. Notice the similarity of this table to Table 17-1 for mirrors. As with mirrors, the distance from the principal plane of a lens to its focal point is the focal length, $f$. The focal length depends upon the shape of the lens and the index of refraction of the lens material. Focal lengths and image positions can be negative.

For lenses, virtual images are always on the same side of the lens as the object, which means that the image position is negative. When the absolute value of a magnification is between zero and one, the image is smaller than the object. Magnifications with absolute values greater than one represent images that are larger than the objects. A negative magnification means the image is inverted compared to the object. Notice that a concave lens produces only virtual images, whereas a convex lens can produce real images or virtual images.

## Convex Lenses and Real Images

As shown in Figure 18-10a, paper can be ignited by producing a real image of the Sun on the paper. Recall from Chapter 17 that the rays of the Sun are almost exactly parallel when they reach Earth. After being refracted by the lens, the rays converge at the focal point, F, of the lens. Figure 1810b shows two focal points, one on each side of the lens. You could turn the lens around, and it will work the same.

- Figure 18-10 A converging lens can be used to ignite paper (a). Light entering parallel to the principal axis converges at the focal point of the lens, concentrating solar energy (b).



Ray diagrams In Figure 18-11, rays are traced from an object located far from a convex lens. For the purpose of locating the image, you only need to use two rays. Ray 1 is parallel to the principal axis. It refracts and passes through F on the other side of the lens. Ray 2 passes through F on its way to the lens. After refraction, its path is parallel to the principal axis. The two rays intersect at a point beyond F and locate the image. Rays selected from other points on the object converge at corresponding points to form the complete image. Note that this is a real image that is inverted and smaller compared to the object.

You can use Figure 18-11 to locate the image of an object that is closer to the lens than the object in the figure. If a refracted ray is reversed in direction, it will follow its original path in the reverse direction. This means that the image and object may be interchanged by changing the direction of the rays. Imagine that the path of light through the lens in Figure $18-11$ is reversed and the object is at a distance of 15 cm from the right side of the lens. The new image, located 30 cm from the left side of the lens, is a real image that is inverted and larger compared to the object.

If the object is placed at twice the focal length from the lens at the point 2F, as shown in Figure 18-12, the image also is found at 2F. Because of symmetry, the image and object have the same size. Thus, you can conclude that if an object is more than twice the focal length from the lens, the image is smaller than the object. If the object is between F and 2 F , then the image is larger than the object.

- Figure 18-12 When an object is placed at a distance equal to twice the focal length from the lens, the image is the same size as the object.

- Figure 18-11 When an object is placed at a distance greater than twice the focal length of the lens, a real image is produced that is inverted and smaller compared to the object. If the object is placed at the location of the image, you could locate the new image by tracing the same rays in the opposite direction.


## OMINI LAAB

## Lens Masking Effects <br> ETCN

What happens when you mask, or cover, part of a lens? Does this cause only part of a real image to be formed by the lens?

1. Stick the edge of a convex lens into a ball of clay and place the lens on a tabletop. CAUTION:
Lenses have sharp edges. Handle carefully.
2. Use a small lamp on one side and a screen on the other side to get a sharp image of the lamp's lightbulb. CAUTION: Lamps get hot and can burn skin.
3. Predict what will happen to the image if you place your hand over the top half of the lens. This is called masking.
4. Observe the effects of masking more of the lens and masking less of the lens.

## Analyze and Conclude

5. How much of the lens is needed for a complete image?
6. What is the effect of masking the lens?

## EXAMPLE Problem 2

An Image Formed by a Convex Lens An object is placed 32.0 cm from a convex lens that has a focal length of 8.0 cm .
a. Where is the image?
b. If the object is 3.0 cm high, how tall is the image?
c. What is the orientation of the image?

1 Analyze and Sketch the Problem

- Sketch the situation, locating the object and the lens.
- Draw the two principal rays.

$$
\begin{array}{ll}
\text { Known: } & \text { Unknown: } \\
d_{\mathrm{o}}=32.0 \mathrm{~cm} & d_{\mathrm{i}}=? \\
h_{\mathrm{o}}=3.0 \mathrm{~cm} & h_{\mathrm{i}}=? \\
f=8.0 \mathrm{~cm} &
\end{array}
$$

## 2 Solve for the Unknown

a. Use the thin lens equation to determine $d_{i}$.

$$
\begin{aligned}
\frac{1}{f} & =\frac{1}{d_{\mathrm{i}}}+\frac{1}{d_{\mathrm{o}}} \\
d_{\mathrm{i}} & =\frac{f d_{\mathrm{o}}}{d_{\mathrm{o}}-f} \\
& =\frac{(8.0 \mathrm{~cm})(32.0 \mathrm{~cm})}{32.0 \mathrm{~cm}-8.0 \mathrm{~cm}} \quad \text { Substitute } f=8.0 \mathrm{~cm}, d_{\mathrm{o}}=32.0 \mathrm{~cm} \\
& =11 \mathrm{~cm}(11 \mathrm{~cm} \text { away from the lens on the side opposite the object) }
\end{aligned}
$$

b. Use the magnification equation and solve for image height.

$$
\begin{aligned}
m & \equiv \frac{h_{\mathrm{i}}}{h_{\mathrm{o}}}=\frac{-d_{\mathrm{i}}}{d_{\mathrm{o}}} \\
h_{\mathrm{i}} & =\frac{-d_{\mathrm{i}} h_{\mathrm{o}}}{d_{\mathrm{o}}} \\
& =\frac{-(11 \mathrm{~cm})(3.0 \mathrm{~cm})}{32.0 \mathrm{~cm}} \quad \text { Substitute } d_{\mathrm{i}}=11 \mathrm{~cm}, h_{\mathrm{o}}=3.0 \mathrm{~cm}, d_{\mathrm{o}}=32.0 \mathrm{~cm} \\
& =-1.0 \mathrm{~cm}(1.0 \mathrm{~cm} \text { tall })
\end{aligned}
$$

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Operations with Significant Digits pages 835-836
c. The negative sign in part $\mathbf{b}$ means that the image is inverted.

3 Evaluate the Answer

- Are the units correct? All are in centimeters.
- Do the signs make sense? Image position is positive (real image) and image height is negative (inverted compared to the object), which make sense for a convex lens.


## PRACTICE Problems

Additional Problems, Appendix B
15. A $2.25-\mathrm{cm}$-tall object is 8.5 cm to the left of a convex lens of $5.5-\mathrm{cm}$ focal length. Find the image position and height.
16. An object near a convex lens produces a $1.8-\mathrm{cm}$-tall real image that is 10.4 cm from the lens and inverted. If the focal length of the lens is 6.8 cm , what are the object position and height?
17. An object is placed to the left of a convex lens with a $25-\mathrm{mm}$ focal length so that its image is the same size as the object. What are the image and object positions?
18. Use a scale ray diagram to find the image position of an object that is 30 cm to the left of a convex lens with a $10-\mathrm{cm}$ focal length.
19. Calculate the image position and height of a $2.0-\mathrm{cm}$-tall object located 25 cm from a convex lens with a focal length of 5.0 cm . What is the orientation of the image?


## Convex Lenses and Virtual Images

When an object is placed at the focal point of a convex lens, the refracted rays will emerge in a parallel beam and no image will be seen. When the object is brought closer to the lens, the rays will diverge on the opposite side of the lens, and the rays will appear to an observer to come from a spot on the same side of the lens as the object. This is a virtual image that is upright and larger compared to the object.

Figure 18-13 shows how a convex lens forms a virtual image. The object is located between F and the lens. Ray 1, as usual, approaches the lens parallel to the principal axis and is refracted through the focal point, F. Ray 2 travels from the tip of the object, in the direction it would have if it had started at F on the object side of the lens. The dashed line from F to the object shows you how to draw ray 2. Ray 2 leaves the lens parallel to the principal axis. Rays 1 and 2 diverge as they leave the lens. Thus, no real image is possible. Drawing sight lines for the two rays back to their apparent intersection locates the virtual image. It is on the same side of the lens as the object, and it is upright and larger compared to the object. Note that the actual image is formed by light that passes through the lens, but you can still determine the location of the image by drawing rays that do not have to pass through the lens.

## PRACTICE Problems Additional Problems, Appendix B

20. A newspaper is held 6.0 cm from a convex lens of $20.0-\mathrm{cm}$ focal length. Find the image position of the newsprint image.
21. A magnifying glass has a focal length of 12.0 cm . A coin, 2.0 cm in diameter, is placed 3.4 cm from the lens. Locate the image of the coin. What is the diameter of the image?
22. A convex lens with a focal length of 22.0 cm is used to view a
$15.0-\mathrm{cm}$-long pencil located 10.0 cm away. Find the height and
23. A convex lens with a focal length of 22.0 cm is used to view a
$15.0-\mathrm{cm}$-long pencil located 10.0 cm away. Find the height and orientation of the image.
24. A stamp collector wants to magnify a stamp by 4.0 when the stamp is 3.5 cm from the lens. What focal length is needed for the lens?
25. A magnifier with a focal length of 30 cm is used to view a $1-\mathrm{cm}$-tall object. Use ray tracing to determine the location and size of the image when the magnifier is positioned 10 cm from the object.

- Figure 18-13 The two principal rays show that a convex lens forms a virtual image that is upright and larger compared to the object when the object is located between the lens and the focal point. Because the principal rays are simply part of a model to help locate an image, they do not have to pass through the picture of the lens in a diagram. In reality, the image is formed only by the light that passes through the actual lens.
- Figure 18-14 Concave lenses produce only virtual images that are upright and smaller compared to their objects.



## Concave Lenses

A concave lens causes all rays to diverge. Figure 18-14 shows how such a lens forms a virtual image. Ray 1 approaches the lens parallel to the principal axis. It leaves the lens along a line that extends back through the focal point. Ray 2 approaches the lens as if it is going to pass through the focal point on the opposite side, and leaves the lens parallel to the principal axis. The sight lines of rays 1 and 2 intersect on the same side of the lens as the object. Because the rays diverge, they produce a virtual image. The image is located at the point from where the two rays apparently diverge. The image also is upright and smaller compared to the object. This is true no matter how far from the lens the object is located. The focal length of a concave lens is negative.

When solving problems for concave lenses using the thin lens equation, you should remember that the sign convention for focal length is different from that of convex lenses. If the focal point for a concave lens is 24 cm from the lens, you should use the value $f=-24 \mathrm{~cm}$ in the thin lens equation. All images for a concave lens are virtual. Thus, if an image distance is given as 20 cm from the lens, then you should use $d_{\mathrm{i}}=-20 \mathrm{~cm}$. The object position always will be positive.

## Defects of Spherical Lenses

Throughout this section, you have studied lenses that produce perfect images at specific positions. In reality, spherical lenses, just like spherical mirrors, have intrinsic defects that cause problems with the focus and color of images. Spherical lenses exhibit an aberration associated with their spherical design, just as mirrors do. In addition, the dispersion of light through a spherical lens causes an aberration that mirrors do not exhibit.

Spherical aberration The model that you have used for drawing rays through spherical lenses suggests that all parallel rays focus at the same position. However, this is only an approximation. In reality, parallel rays that pass through the edges of a spherical lens focus at positions different from those of parallel rays that pass through the center. This inability of a spherical lens to focus all parallel rays to a single point is called spherical aberration. Making lens surfaces aspherical, such as in cameras, eliminates spherical aberration. In high-precision instruments, many lenses, often five or more, are used to form sharp, well-defined images.

Chromatic aberration Lenses have a second defect that mirrors do not have. A lens is like a prism, so different wavelengths of light are refracted at slightly different angles, as you can see in Figure 18-15a. Thus, the light that passes through a lens, especially near the edges, is slightly dispersed. An object viewed through a lens appears to be ringed with color. This effect is called chromatic aberration. The term chromatic comes from the Greek word chromo, which means "color."

Chromatic aberration is always present when a single lens is used. However, this defect can be greatly reduced by an achromatic lens, which is a system of two or more lenses, such as a convex lens with a concave lens, that have different indices of refraction. Such a combination of lenses is shown in Figure 18-15b. Both lenses in the figure disperse light, but the dispersion caused by the convex lens is almost canceled by the dispersion caused by the concave lens. The index of refraction of the convex lens is chosen so that the combination of lenses still converges the light.


Figure 18-15 All simple lenses have chromatic aberration, in which light of different wavelengths is focused at different points (a). An achromatic lens is a combination of lenses, which minimizes the chromatic defect (b).

### 18.2 Section Review

25. Magnification Magnifying glasses normally are used to produce images that are larger than the related objects, but they also can produce images that are smaller than the related objects. Explain.
26. Image Position and Height A $3.0-\mathrm{cm}$-tall object is located 2.0 cm from a convex lens having a focal length of 6.0 cm . Draw a ray diagram to determine the location and size of the image. Use the thin lens equation and the magnification equation to verify your answer.
27. Types of Lenses The cross sections of four different thin lenses are shown in Figure 18-16.
a. Which of these lenses, if any, are convex, or converging, lenses?
b. Which of these lenses, if any, are concave, or diverging, lenses?


Figure 18-16
28. Chromatic Aberration All simple lenses have chromatic aberration. Explain, then, why you do not see this effect when you look through a microscope.
29. Chromatic Aberration You shine white light through a convex lens onto a screen and adjust the distance of the screen from the lens to focus the red light. Which direction should you move the screen to focus the blue light?
30. Critical Thinking An air lens constructed of two watch glasses is placed in a tank of water. Copy Figure 18-17 and draw the effect of this lens on parallel light rays incident on the lens.


Figure 18-17

### 18.3 Applications of Lenses

## Objectives

- Describe how the eye focuses light to form an image.
- Explain nearsightedness and farsightedness and how eyeglass lenses correct these defects.
- Describe the optical systems in some common optical instruments.
- Vocabulary nearsightedness farsightedness


## Bology Connection

- Figure 18-18 The human eye is complex and has many components that must work together.

The properties that you have learned for the refraction of light through lenses are used in almost every optical instrument. In many cases, a combination of lenses and mirrors is used to produce clear images of small or faraway objects. Telescopes, binoculars, cameras, microscopes, and even your eyes contain lenses.

## Lenses in Eyes

The eye is a remarkable optical device. As shown in Figure 18-18, the eye is a fluid-filled, almost spherical vessel. Light that is emitted or reflected off an object travels into the eye through the cornea. The light then passes through the lens and focuses onto the retina that is at the back of the eye. Specialized cells on the retina absorb this light and send information about the image along the optic nerve to the brain.

Focusing images Because of its name, you might assume that the lens of an eye is responsible for focusing light onto the retina. In fact, light entering the eye is primarily focused by the cornea because the air-cornea surface has the greatest difference in indices of refraction. The lens is responsible for the fine focus that allows you to clearly see both distant and nearby objects. Using a process called accommodation, muscles surrounding the lens can contract or relax, thereby changing the shape of the lens. This, in turn, changes the focal length of the eye. When the muscles are relaxed, the image of distant objects is focused on the retina. When the muscles contract, the focal length is shortened, and this allows images of closer objects to be focused on the retina.



Nearsightedness and farsightedness The eyes of many people do not focus sharp images on the retina. Instead, images are focused either in front of the retina or behind it. External lenses, in the form of eyeglasses or contact lenses, are needed to adjust the focal length and move images to the retina. Figure 18-19a shows the condition of nearsightedness, or myopia, whereby the focal length of the eye is too short to focus light on the retina. Images are formed in front of the retina. As shown in Figure 18-19b, concave lenses correct this by diverging light, thereby increasing images' distances from the lens, and forming images on the retina.

You also can see in Figure 18-19c that farsightedness, or hyperopia, is the condition in which the focal length of the eye is too long. Images are therefore formed past the retina. A similar result is caused by the increasing rigidity of the lenses in the eyes of people who are more than about 45 years old. Their muscles cannot shorten the focal length enough to focus images of close objects on the retina. For either defect, convex lenses produce virtual images farther from the eye than the associated objects, as shown in Figure 18-19d. The images then become the objects for the eye lens and can be focused on the retina, thereby correcting the defect.
_ Figure 18-19 A nearsighted person cannot see distant objects clearly because images are focused in front of the retina (a). A concave lens corrects this defect (c). A farsighted person cannot see close objects clearly because images are focused behind the retina (b). A convex lens corrects this defect (d).

## APPLYING PHYSICS

- Contacts Contact lenses produce the same results as eyeglasses do. These small, thin lenses are placed directly on the corneas. A thin layer of tears between the cornea and lens keeps the lens in place. Most of the refraction occurs at the air-lens surface, where the difference in indices of refraction is greatest.


## - CHALLENGE PROBLEM

As light enters the eye, it first encounters the air/cornea interface. Consider a ray of light that strikes the interface between the air and a person's cornea at an angle of $30.0^{\circ}$ to the normal. The index of refraction of the cornea is approximately 1.4.

1. Use Snell's law to calculate the angle of refraction.
2. What would the angle of refraction be if the person was swimming underwater?
3. Is the refraction greater in air or in water? Does this mean that objects under water seem closer or more distant than they would in air?
4. If you want the angle of refraction for the light ray in water to be the same as it is for air, what should the new angle of incidence be?


- Figure 18-20 An astronomical refracting telescope creates a virtual image that is inverted compared to the object. (Illustration not to scale)
- Figure 18-21 Binoculars are like two side-by-side refracting telescopes.



## Refracting Telescopes

An astronomical refracting telescope uses lenses to magnify distant objects. Figure 18-20 shows the optical system for a Keplerian telescope. Light from stars and other astronomical objects is so far away that the rays can be considered parallel. The parallel rays of light enter the objective convex lens and are focused as a real image at the focal point of the objective lens. The image is inverted compared to the object. This image then becomes the object for the convex lens of the eyepiece. Notice that the eyepiece lens is positioned so that the focal point of the objective lens is between the eyepiece lens and its focal point. This means that a virtual image is produced that is upright and larger than the first image. However, because the first image was already inverted, the final image is still inverted. For viewing astronomical objects, an image that is inverted is acceptable.

In a telescope, the convex lens of the eyepiece is almost always an achromatic lens. Recall that an achromatic lens is a combination of lenses that function as one lens. The combination of lenses eliminates the peripheral colors, or chromatic aberration, that can form on images.


## Binoculars

Binoculars, like telescopes, produce magnified images of faraway objects. Figure $\mathbf{1 8} \mathbf{- 2 1}$ shows a typical binocular design. Each side of the binoculars is like a small telescope: light enters a convex objective lens, which inverts the image. The light then travels through two prisms that use total internal reflection to invert the image again, so that the viewer sees an image that is upright compared to the object. The prisms also extend the path along which the light travels and direct it toward the eyepiece of the binoculars. Just as the separation of your two eyes gives you a sense of three dimensions and depth, the prisms allow a greater separation of the objective lenses, thereby improving the three-dimensional view of a distant object.


## Cameras

Figure 18-22a shows the optical system used in a single-lens reflex camera. As light enters the camera, it passes through an achromatic lens. This lens system refracts the light much like a single convex lens would, forming an image that is inverted on the reflex mirror. The image is reflected upward to a prism that inverts and redirects the light to the viewfinder. When the person holding the camera takes a photograph, he or she presses the shutter-release button, which briefly raises the mirror, as shown in Figure 18-22b. The light, instead of being diverted upward to the prism, then travels along a straight path to form an image on the film.

## Microscopes

Like a telescope, a microscope has both an objective convex lens and a convex eyepiece. However, microscopes are used to view small objects. Figure 18-23 shows the optical system used in a simple compound microscope. The object is located between one and two focal lengths from the objective lens. A real image is produced that is inverted and larger than the object. As with a telescope, this image then becomes the object for the eyepiece. This image is between the eyepiece and its focal point. A virtual image is produced that is upright and larger than the image of the objective lens. Thus, the viewer sees an image that is inverted and greatly larger than the original object.

Figure 18-22 The single-lens reflex camera shown here can divert the image formed by the lens through a prism for viewing (a) or directly to the film (b).

- Figure 18-23 The objective lens and the eyepiece in this simple microscope produce an image that is inverted and larger compared to the object.



### 18.3 Section Review

31. Refraction Explain why the cornea is the primary focusing element in the eye.
32. Lens Types Which type of lens, convex or concave, should a nearsighted person use? Which type should a farsighted person use?
33. Focal Length Suppose your camera is focused on a person who is 2 m away. You now want to focus it on a tree that is farther away. Should you move the lens closer to the film or farther away?
34. Image Why is the image that you observe in a refracting telescope inverted?
35. Prisms What are three benefits of having prisms in binoculars?
36. Critical Thinking When you use the highest magnification on a microscope, the image is much darker than it is at lower magnifications. What are some possible reasons for the darker image? What could you do to obtain a brighter image?

## Convex Lenses and Focal Length

The thin lens equation states that the inverse of the focal length is equal to the sum of the inverses of the image position from the lens and the object position from the lens.

## QUESTION

How is the image position with a thin convex lens related to the object position and the focal length?

## Objectives

■ Make and use graphs to describe the relationship between the image position with a thin convex lens and the object position.
Use models to show that no matter the image position, the focal length is a constant.

## Safety Precautions

## 

■ Ensure the lamp is turned off before plugging and unplugging it from the electrical outlet. Use caution when handling lamps. They get hot and can burn the skin.
$\square$ Lenses have sharp edges. Handle carefully.


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## Materials

25-W straight-line filament bulb
lamp base
thin convex lens
meterstick
lens holder
index card

## Procedure

1. Place a meterstick on your lab table so that it is balancing on the thin side and the metric numbers are right side up.
2. Place a convex lens in a lens holder and set it on the meterstick on or between the $10-\mathrm{cm}$ and $40-\mathrm{cm}$ marks on the meterstick. (Distances will vary depending on the focal length of the lens used.)
3. Turn on the lamp and set it next to the meterstick so that the center of the lightbulb is even with the $0-\mathrm{cm}$ end of the meterstick.
4. Hold an index card so that the lens is between the lamp and the index card.
5. Move the index card back and forth until an upside-down image of the lightbulb is as sharp as possible.
6. Record the distance of the lightbulb from the lens $\left(d_{0}\right)$ and the distance of the image from the lens $\left(d_{\mathrm{i}}\right)$.
7. Move the lens to another spot between 10 cm and 40 cm and repeat steps 5 and 6. (Distances will vary depending on the focal length of the lens used.)
8. Repeat step 7 three more times.

## Data Table

| Trial | $\boldsymbol{d}_{\mathbf{0}}(\mathrm{cm})$ | $\boldsymbol{d}_{\mathrm{i}}(\mathrm{cm})$ |
| :---: | :--- | :--- |
| 1 |  |  |
| 2 |  |  |
| 3 |  |  |
| 4 |  |  |
| 5 |  |  |

## Calculation Table

| Trial | $\frac{1}{d_{0}}\left(\mathrm{~cm}^{-1}\right)$ | $\frac{1}{d_{i}}\left(\mathrm{~cm}^{-1}\right)$ | $\frac{1}{d_{0}}+\frac{1}{d_{\mathrm{i}}}\left(\mathrm{cm}^{-1}\right)$ | $f(\mathrm{~cm})$ |
| :---: | :---: | :---: | :---: | :---: |
| 1 |  |  |  |  |
| 2 |  |  |  |  |
| 3 |  |  |  |  |
| 4 |  |  |  |  |
| 5 |  |  |  |  |

## Analyze

1. Make and Use Graphs Make a scatter-plot graph of the image position (vertical axis) versus the object position (horizontal axis). Use a computer or calculator to construct the graph if possible.
2. Use Numbers Calculate $1 / d_{0}$ and $1 / d_{\mathrm{i}}$ and enter the values in the calculation table.
3. Use Numbers Calculate the sum of $1 / d_{0}$ and $1 / d_{\mathrm{i}}$ and enter the values in the calculation table. Calculate the reciprocal of this number and enter it in the calculation table as $f$.

## Conclude and Apply

1. Interpret Data Looking at the graph, describe the relationship between $d_{0}$ and $d_{\mathrm{i}}$.
2. Interpret Data Find out the actual focal length of the lens from your teacher. How accurate are your calculations of $f$ ?
3. Interpret Data Compare the results of your focal length calculations of the five trials. Are your results similar?
4. Lab Techniques Why do you suppose you were instructed not to hold your lens closer than 10 cm or farther than 40 cm ?

## Going Further

1. Which measurement is more precise: $d_{0}$ or $d_{i}$ ? Why do you think so?
2. What can you do to make either (or both) measurement(s) more accurate?

## Real-World Physics

1. If you were to take a picture with a camera, first of a distant scene, then of an object less than a meter away, how should the distance between the lens and film be changed?
2. What are two ways in which the image projected onto your retina differs from the object you look at? (Remember the lens in your eye is also convex.)

## Physics nline

To find out more about lenses and refraction, visit the Web site: physicspp.com

# BXreme phusics 

## Gravitational Lenses

In 1979, astronomers at the Jodrell Bank Observatory in Great Britain discovered two quasars that were separated by only 7 arc seconds (seven 0.36th's of a degree). Measurements showed they should have been 500,000 light years apart. The two quasars seemed to fluctuate in brightness and in rhythm with each other. The most amazing thing, however, was that the quasars had identical spectra. There appeared to be two different objects, but the two objects were the same.


The blue shapes are multiple images of the same galaxy produced by gravitational lensing from galaxy cluster $0024+1654$ in the center of the photo.

Further work by astronomers around the world confirmed that there was just a single quasar, and that its light was being distorted by a cluster of galaxies dominated by a massive elliptical galaxy lying in the line of sight between the quasar and Earth. The astronomers realized that they were seeing two images of one quasar. The galaxy acted like an imperfect convex lens, focusing the deflected light in such a way that two images were formed from one object. Why would they think that the light was bent?

Gravity and Light The astronomers remembered the work of Albert Einstein and his theory of relativity. Einstein proposed that light would be bent by the gravitational fields of massive objects. In the classical theory of space, known as Euclidean space, light travels in a straight line. According to Einstein, light bends when it comes near a massive object.

In 1919, comparison of starlight before and during a solar eclipse proved Einstein's theory to be true.

In 1936, Einstein proposed the phenomenon of the gravitational lens. Because light can be bent by the gravitational fields of massive objects, virtual images of rings should be seen by observers on Earth when a massive object is between Earth and the object being observed. Einstein never observed such a phenomenon, but his theory of relativity supported the possible existence of gravitational lenses.

The illustration shows how light from a distant galaxy is bent around a galaxy cluster before reaching Earth.


The Evidence As often occurs in science, once someone discovers something for the first time, many more supporting discoveries are made soon after. Since Einstein's proposals, and the discovery in 1979 of the double-image quasar, many more gravitational lenses have been observed. Both Einstein's rings and multiple images have been observed. Einstein's rings result when the gravitational lens and the light from the object are in near-perfect alignment. Multiple images are formed when the gravitational lens and the light from the object are not in perfect alignment. Over 50 gravitational lenses have been discovered.

## Going Further

1. Infer Why was the discovery of gravitational lenses important?
2. Compare and Contrast How are gravitational lenses similar to convex lenses? How are they different?

## Study Guide

### 18.1 Refraction of Light

## Vocabulary

- index of refraction (p. 486)
- Snell's law of refraction (p. 486)
- critical angle (p. 489)
- total internal reflection (p. 489)
- dispersion (p. 491)


## Key Concepts

- The path of travel of light bends when it passes from a medium with an index of refraction, $n_{1}$, into a medium with a different index of refraction, $n_{2}$.

$$
n_{1} \sin \theta_{1}=n_{2} \sin \theta_{2}
$$

- The ratio of the speed of light in a vacuum, $c$, to the speed of light in any medium, $v$, is the index of refraction, $n$, of the medium.

$$
n=\frac{c}{v}
$$

- When light traveling through a medium hits a boundary of a medium with a smaller index of refraction, if the angle of incidence exceeds the critical angle, $\theta_{c^{\prime}}$ the light will be reflected back into the original medium by total internal reflection.

$$
\sin \theta_{\mathrm{c}}=\frac{n_{2}}{n_{1}}
$$

### 18.2 Convex and Concave Lenses

## Vocabulary

- lens (p. 493)
- convex lens (p. 493)
- concave lens (p. 493)
- thin lens equation (p. 493)
- chromatic aberration (p. 499)
- achromatic lens (p. 499)


## Key Concepts

- The focal length, $f$; the object position, $d_{\mathrm{o}}$; and the image position, $d_{\mathrm{i}}$, for a lens are related by the thin lens equation.

$$
\frac{1}{f}=\frac{1}{d_{\mathrm{i}}}+\frac{1}{d_{\mathrm{o}}}
$$

- The magnification, $m$, of an image by a lens is defined in the same way as the magnification of an image by a mirror.

$$
m \equiv \frac{h_{\mathrm{i}}}{h_{\mathrm{o}}}=\frac{-d_{\mathrm{i}}}{d_{\mathrm{o}}}
$$

- A single convex lens produces a real image that is inverted when the object position is greater than the focal length. The image is reduced or enlarged, depending on the object position.
- A single convex lens produces a virtual image that is upright and larger than the object when the object is located between the lens and the focal point.
- A single concave lens always produces a virtual image that is upright and smaller than the object.
- All simple lenses have chromatic aberration. All lenses made with spherical surfaces have spherical aberration.


### 18.3 Applications of Lenses

## Vocabulary

- nearsightedness (p. 501)
- farsightedness (p. 501)


## Key Concepts

- Differences in indices of refraction between air and the cornea are primarily responsible for focusing light in the eye.
- Optical instruments use combinations of lenses to obtain clear images of small or distant objects.


## Concept Mapping

37. Complete the following concept map using the following terms: inverted, larger, smaller, virtual.


## Mastering Concepts

38. How does the angle of incidence compare with the angle of refraction when a light ray passes from air into glass at a nonzero angle? (18.1)
39. How does the angle of incidence compare with the angle of refraction when a light ray leaves glass and enters air at a nonzero angle? (18.1)
40. Regarding refraction, what is the critical angle? (18.1)
41. Although the light coming from the Sun is refracted while passing through Earth's atmosphere, the light is not separated into its spectrum. What does this indicate about the speeds of different colors of light traveling through air? (18.1)
42. Explain why the Moon looks red during a lunar eclipse. (18.1)
43. How do the shapes of convex and concave lenses differ? (18.2)
44. Locate and describe the physical properties of the image produced by a convex lens when an object is placed some distance beyond 2F. (18.2)
45. What factor, other than the curvature of the surfaces of a lens, determines the location of the focal point of the lens? (18.2)
46. To project an image from a movie projector onto a screen, the film is placed between F and 2 F of a converging lens. This arrangement produces an image that is inverted. Why does the filmed scene appear to be upright when the film is viewed? (18.2)
47. Describe why precision optical instruments use achromatic lenses. (18.2)
48. Describe how the eye focuses light. (18.3)
49. What is the condition in which the focal length of the eye is too short to focus light on the retina? (18.3)
50. What type of image is produced by the objective lens in a refracting telescope? (18.3)
51. The prisms in binoculars increase the distance between the objective lenses. Why is this useful? (18.3)
52. What is the purpose of a camera's reflex mirror? (18.3)

## Applying Concepts

53. Which substance, A or B, in Figure 18-24 has a larger index of refraction? Explain.


Figure 18-24
54. A light ray strikes the boundary between two transparent media. What is the angle of incidence for which there is no refraction?
55. How does the speed of light change as the index of refraction increases?
56. How does the size of the critical angle change as the index of refraction increases?
57. Which pair of media, air and water or air and glass, has the smaller critical angle?
58. Cracked Windshield If you crack the windshield of your car, you will see a silvery line along the crack. The glass has separated at the crack, and there is air in the crack. The silvery line indicates that light is reflecting off the crack. Draw a ray diagram to explain why this occurs. What phenomenon does this illustrate?
59. Legendary Mirage According to legend, Eric the Red sailed from Iceland and discovered Greenland after he had seen the island in a mirage. Describe how the mirage might have occurred.
60. A prism bends violet light more than it bends red light. Explain.
61. Rainbows Why would you never see a rainbow in the southern sky if you were in the northern hemisphere? In which direction should you look to see rainbows if you are in the southern hemisphere?
62. Suppose that Figure 18-14 is redrawn with a lens of the same focal length but a larger diameter. Explain why the location of the image does not change. Would the image be affected in any way?
63. A swimmer uses a magnifying glass to observe a small object on the bottom of a swimming pool. She discovers that the magnifying glass does not magnify the object very well. Explain why the magnifying glass is not functioning as it would in air.
64. Why is there chromatic aberration for light that goes through a lens but not for light that reflects from a mirror?
65. When subjected to bright sunlight, the pupils of your eyes are smaller than when they are subjected to dimmer light. Explain why your eyes can focus better in bright light.
66. Binoculars The objective lenses in binoculars form real images that are upright compared to their objects. Where are the images located relative to the eyepiece lenses?

## Mastering Problems

### 18.1 Refraction of Light

67. A ray of light travels from air into a liquid, as shown in Figure 18-25. The ray is incident upon the liquid at an angle of $30.0^{\circ}$. The angle of refraction is $22.0^{\circ}$.
a. Using Snell's law, calculate the index of refraction of the liquid.
b. Compare the calculated index of refraction to those in Table 18-1. What might the liquid be?


Figure 18-25
Physic
nline
68. Light travels from flint glass into ethanol. The angle of refraction in the ethanol is $25.0^{\circ}$. What is the angle of incidence in the glass?
69. A beam of light strikes the flat, glass side of a waterfilled aquarium at an angle of $40.0^{\circ}$ to the normal. For glass, $n=1.50$.
a. At what angle does the beam enter the glass?
b. At what angle does the beam enter the water?
70. Refer to Table 18-1. Use the index of refraction of diamond to calculate the speed of light in diamond.
71. Refer to Table 18-1. Find the critical angle for a diamond in air.
72. Aquarium Tank A thick sheet of plastic, $n=1.500$, is used as the side of an aquarium tank. Light reflected from a fish in the water has an angle of incidence of $35.0^{\circ}$. At what angle does the light enter the air?
73. Swimming-Pool Lights A light source is located 2.0 m below the surface of a swimming pool and 1.5 m from one edge of the pool, as shown in

Figure 18-26. The pool is filled to the top with water.
a. At what angle does the light reaching the edge of the pool leave the water?
b. Does this cause the light viewed from this angle to appear deeper or shallower than it actually is?


Figure 18-26 (Not to scale)
74. A diamond's index of refraction for red light, 656 nm , is 2.410, while that for blue light, 434 nm , is 2.450 . Suppose that white light is incident on the diamond at $30.0^{\circ}$. Find the angles of refraction for red and blue light.
75. The index of refraction of crown glass is 1.53 for violet light, and it is 1.51 for red light.
a. What is the speed of violet light in crown glass?
b. What is the speed of red light in crown glass?

## Chapter 18 Assessment

76. The critical angle for a special glass in air is $41.0^{\circ}$. What is the critical angle if the glass is immersed in water?
77. A ray of light in a tank of water has an angle of incidence of $55.0^{\circ}$. What is the angle of refraction in air?
78. The ray of light shown in Figure 18-27 is incident upon a $60^{\circ}-60^{\circ}-60^{\circ}$ glass prism, $n=1.5$.
a. Using Snell's law of refraction, determine the angle, $\theta_{2}$, to the nearest degree.
b. Using elementary geometry, determine the value of $\theta_{1}{ }^{\prime}$.
c. Determine $\theta_{2}{ }^{\prime}$.


Figure 18-27
79. The speed of light in a clear plastic is $1.90 \times 10^{8} \mathrm{~m} / \mathrm{s}$. A ray of light strikes the plastic at an angle of $22.0^{\circ}$. At what angle is the ray refracted?
80. A light ray enters a block of crown glass, as illustrated in Figure 18-28. Use a ray diagram to trace the path of the ray until it leaves the glass.


Figure 18-28

### 18.2 Convex and Concave Lenses

81. The focal length of a convex lens is 17 cm . A candle is placed 34 cm in front of the lens. Make a ray diagram to locate the image.
82. A converging lens has a focal length of 25.5 cm . If it is placed 72.5 cm from an object, at what distance from the lens will the image be?
83. If an object is 10.0 cm from a converging lens that has a focal length of 5.00 cm , how far from the lens will the image be?
84. A convex lens is needed to produce an image that is 0.75 times the size of the object and located 24 cm from the lens on the other side. What focal length should be specified?
85. An object is located 14.0 cm from a convex lens that has a focal length of 6.0 cm . The object is 2.4 cm high.
a. Draw a ray diagram to determine the location, size, and orientation of the image.
b. Solve the problem mathematically.
86. A $3.0-\mathrm{cm}$-tall object is placed 22 cm in front of a converging lens. A real image is formed 11 cm from the lens. What is the size of the image?
87. A $3.0-\mathrm{cm}$-tall object is placed 15.0 cm in front of a converging lens. A real image is formed 10.0 cm from the lens.
a. What is the focal length of the lens?
b. If the original lens is replaced with a lens having twice the focal length, what are the image position, size, and orientation?
88. A diverging lens has a focal length of 15.0 cm . An object placed near it forms a $2.0-\mathrm{cm}$-high image at a distance of 5.0 cm from the lens.
a. What are the object position and object height?
b. The diverging lens is now replaced by a converging lens with the same focal length. What are the image position, height, and orientation? Is it a virtual image or a real image?

### 18.3 Applications of Lenses

89. Camera Lenses Camera lenses are described in terms of their focal length. A $50.0-\mathrm{mm}$ lens has a focal length of 50.0 mm .
a. A camera with a $50.0-\mathrm{mm}$ lens is focused on an object 3.0 m away. What is the image position?
b. A $1000.0-\mathrm{mm}$ lens is focused on an object 125 m away. What is the image position?
90. Eyeglasses To clearly read a book 25 cm away, a farsighted girl needs the image to be 45 cm from her eyes. What focal length is needed for the lenses in her eyeglasses?
91. Copy Machine The convex lens of a copy machine has a focal length of 25.0 cm . A letter to be copied is placed 40.0 cm from the lens.
a. How far from the lens is the copy paper?
b. How much larger will the copy be?
92. Camera A camera lens with a focal length of 35 mm is used to photograph a distant object. How far from the lens is the real image of the object? Explain.
93. Microscope A slide of an onion cell is placed 12 mm from the objective lens of a microscope. The focal length of the objective lens is 10.0 mm .
a. How far from the lens is the image formed?
b. What is the magnification of this image?
c. The real image formed is located 10.0 mm beneath the eyepiece lens. If the focal length of the eyepiece is 20.0 mm , where does the final image appear?
d. What is the final magnification of this compound system?
94. Telescope The optical system of a toy refracting telescope consists of a converging objective lens with a focal length of 20.0 cm , located 25.0 cm from a converging eyepiece lens with a focal length of 4.05 cm . The telescope is used to view a $10.0-\mathrm{cm}$-high object, located 425 cm from the objective lens.
a. What are the image position, height, and orientation as formed by the objective lens? Is this a real or virtual image?
b. The objective lens image becomes the object for the eyepiece lens. What are the image position, height, and orientation that a person sees when looking into the telescope? Is this a real or virtual image?
c. What is the magnification of the telescope?

## Mixed Review

95. A block of glass has a critical angle of $45.0^{\circ}$. What is its index of refraction?
96. Find the speed of light in antimony trioxide if it has an index of refraction of 2.35 .
97. A $3.0-\mathrm{cm}$-tall object is placed 20 cm in front of a converging lens. A real image is formed 10 cm from the lens. What is the focal length of the lens?
98. Derive $n=\sin \theta_{1} / \sin \theta_{2}$ from the general form of Snell's law of refraction, $n_{1} \sin \theta_{1}=n_{2} \sin \theta_{2}$. State any assumptions and restrictions.
99. Astronomy How many more minutes would it take light from the Sun to reach Earth if the space between them were filled with water rather than a vacuum? The Sun is $1.5 \times 10^{8} \mathrm{~km}$ from Earth.
100. What is the focal length of the lenses in your eyes when you read a book that is 35.0 cm from them? The distance from each lens to the retina is 0.19 mm .
101. Apparent Depth Sunlight reflects diffusively off the bottom of an aquarium. Figure 18-29 shows two of the many light rays that would reflect diffusively from a point off the bottom of the tank and travel to the surface. The light rays refract into the air as shown. The red dashed line extending back from the refracted light ray is a sight line that intersects with the vertical ray at the location where an observer would see the image of the bottom of the tank.
a. Compute the direction that the refracted ray will travel above the surface of the water.
b. At what depth does the bottom of the tank appear to be if you look into the water? Divide this apparent depth into the true depth and compare it to the index of refraction.


Figure 18-29
102. It is impossible to see through adjacent sides of a square block of glass with an index of refraction of 1.5. The side adjacent to the side that an observer is looking through acts as a mirror. Figure 18-30 shows the limiting case for the adjacent side to not act like a mirror. Use your knowledge of geometry and critical angles to show that this ray configuration is not achievable when $n_{\text {glass }}=1.5$.


## Chapter 18 Assessment

103. Bank Teller Window A $25-\mathrm{mm}$-thick sheet of plastic, $n=1.5$, is used in a bank teller's window. A ray of light strikes the sheet at an angle of $45^{\circ}$. The ray leaves the sheet at $45^{\circ}$, but at a different location. Use a ray diagram to find the distance between the ray that leaves and the one that would have left if the plastic were not there.

## Thinking Critically

104. Recognize Spatial Relationships White light traveling through air $(n=1.0003)$ enters a slab of glass, incident at exactly $45^{\circ}$. For dense flint glass, $n=1.7708$ for blue light $(\lambda=435.8 \mathrm{~nm})$ and $n=1.7273$ for red light $(\lambda=643.8 \mathrm{~nm})$. What is the angular dispersion of the red and blue light?
105. Compare and Contrast Find the critical angle for ice ( $n=1.31$ ). In a very cold world, would fiberoptic cables made of ice or those made of glass do a better job of keeping light inside the cable? Explain.
106. Recognize Cause and Effect Your lab partner used a convex lens to produce an image with $d_{\mathrm{i}}=25 \mathrm{~cm}$ and $h_{\mathrm{i}}=4.0 \mathrm{~cm}$. You are examining a concave lens with a focal length of -15 cm . You place the concave lens between the convex lens and the original image, 10 cm from the image. To your surprise, you see a real image on the wall that is larger than the object. You are told that the image from the convex lens is now the object for the concave lens, and because it is on the opposite side of the concave lens, it is a virtual object. Use these hints to find the new image position and image height and to predict whether the concave lens changed the orientation of the original image.
107. Define Operationally Name and describe the effect that causes the rainbow-colored fringe commonly seen at the edges of a spot of white light from a slide or overhead projector.
108. Think Critically A lens is used to project the image of an object onto a screen. Suppose that you cover the right half of the lens. What will happen to the image?

## Writing in Physics

109. The process of accommodation, whereby muscles surrounding the lens in the eye contract or relax to enable the eye to focus on close or distant objects, varies for different species. Investigate this effect for different animals. Prepare a report for the class showing how this fine focusing is accomplished for different eye mechanisms.
110. Investigate the lens system used in an optical instrument such as an overhead projector or a particular camera or telescope. Prepare a graphics display for the class explaining how the instrument forms images.

## Cumulative Review

111. If you drop a 2.0 kg bag of lead shot from a height of 1.5 m , you could assume that half of the potential energy will be converted into thermal energy in the lead. The other half would go to thermal energy in the floor. How many times would you have to drop the bag to heat it by $10^{\circ} \mathrm{C}$ ? (Chapter 12)
112. A blacksmith puts an iron hoop or tire on the outer rim of a wooden carriage wheel by heating the hoop so that it expands to a diameter greater than the wooden wheel. When the hoop cools, it contracts to hold the rim in place. If a blacksmith has a wooden wheel with a $1.0000-\mathrm{m}$ diameter and wants to put a rim with a $0.9950-\mathrm{m}$ diameter on the wheel, what is the minimum temperature change the iron must experience? $\left(\alpha_{\text {iron }}=12 \times 10^{-6} /{ }^{\circ} \mathrm{C}\right)$ (Chapter 13)
113. A car sounds its horn as it approaches a pedestrian in a crosswalk. What does the pedestrian hear as the car brakes to allow him to cross the street? (Chapter 15)
114. Suppose that you could stand on the surface of the Sun and weigh yourself. Also suppose that you could measure the illuminance on your hand from the Sun's visible spectrum produced at that position. Next, imagine yourself traveling to a position 1000 times farther away from the center of the Sun as you were when standing on its surface. (Chapter 16)
a. How would the force of gravity on you from the Sun at the new position compare to what it was at the surface?
b. How would the illuminance on your hand from the Sun at the new position compare to what it was when you were standing on its surface? (For simplicity, assume that the Sun is a point source at both positions.)
c. Compare the effect of distance upon the gravitational force and illuminance.
115. Beautician's Mirror The nose of a customer who is trying some face powder is $3.00-\mathrm{cm}$ high and is located 6.00 cm in front of a concave mirror having a $14.0-\mathrm{cm}$ focal length. Find the image position and height of the customer's nose by means of the following. (Chapter 17)
a. a ray diagram drawn to scale
b. the mirror and magnification equations

## Standardized Test Practice

## Multiple Choice

1. A flashlight beam is directed at a swimming pool in the dark at an angle of $46^{\circ}$ with respect to the normal to the surface of the water. What is the angle of refraction of the beam in the water? (The refractive index for water is 1.33.)
```
(A) 188
    (C) }3\mp@subsup{3}{}{\circ
(B) }3\mp@subsup{0}{}{\circ
(D) }4\mp@subsup{4}{}{\circ
```

2. The speed of light in diamond is $1.24 \times 10^{8} \mathrm{~m} / \mathrm{s}$. What is the index of refraction of diamond?
(A) 0.0422
(C) 1.24
(B) 0.413
(D) 2.42
3. Which one of the items below is not involved in the formation of rainbows?
(A) diffraction
(C) reflection
(B) dispersion
(D) refraction
4. George's picture is being taken by Cami, as shown in the figure, using a camera which has a convex lens with a focal length of 0.0470 m . Determine George's image position.
(A) 1.86 cm
(C) 4.82 cm
(B) 4.70 cm
(D) 20.7 cm

5. What is the magnification of an object that is 4.15 m in front of a camera that has an image position of 5.0 cm ?
```
(A) -0.83
    (C) 0.83
(B) -0.012
(D) }1.
```

6. Which one of the items below is not involved in the formation of mirages?
(A) heating of air near the ground
(B) Huygens' wavelets
(C) reflection
(D) refraction
7. What is the image position for the situation shown in the figure?
```
(A) }-6.00\textrm{m
(C) }0.167\textrm{m
(B) }-1.20\textrm{m
(D) }0.833\textrm{m
```


8. What is the critical angle for total internal reflection when light travels from glass ( $n=1.52$ ) to water $(n=1.33)$ ?

```
(A) 29.0
(C) 48.8
(B) }41.\mp@subsup{2}{}{\circ
(D) }61.\mp@subsup{0}{}{\circ
```

9. What happens to the image formed by a convex lens when half of the lens is covered?
(A) half of the image disappears
(B) the image dims
(C) the image gets blurry
(D) the image inverts

## Extended Answer

10. The critical angle for total internal reflection at a diamond-air boundary is $24.4^{\circ}$. What is the angle of refraction in the air if light is incident on the boundary at an angle of $20.0^{\circ}$ ?
11. An object that is 6.98 cm from a lens produces an image that is 2.95 cm from the lens on the same side of the lens. Determine the type of lens that is producing the image and explain how you know.

## Test-Taking TIP

## Use as Much Time as You Can

You will not get extra points for finishing a test early. Work slowly and carefully to prevent careless errors that can occur when you are hurrying to finish.

## Chapter

 19
# Interference and Diffraction 

What You'll Learn

- You will learn how interference and diffraction patterns demonstrate that light behaves like a wave.
- You will learn how interference and diffraction patterns occur in nature and how they are used.


## Why It's Important

Interference and diffraction can be seen all around you. Compact discs demonstrate diffraction, bubbles show interference, and the wings of a Morpho butterfly show both.

Bubble Solution Bubble solution in a container is transparent. However, if you suspend the solution in a grid of plastic, swirls of color can be seen. These colors are not caused by pigments or dyes in the soap, but by an effect of the wave nature of light.

Think About This > How does bubble solution produce a rainbow of colors?

## Physjos inline

 physicspp.com

## LAUNCH Lab

## Why does a compact disc reflect

 a rainbow of light?
## Question

How is light affected when it reflects off a compact disc?

## Procedure 절 푸눙

1. Obtain a compact disc (CD or DVD), a light projector, and color light filters from your teacher.
2. Lay the compact disc on a table with the reflective surface facing up.
3. Place a color filter on the light projector.
4. Turn the light on and shine it on the compact disc, causing the light to reflect off it onto a white surface. CAUTION: Do not look directly into the projector light.
5. Record your observations of the light on the screen.
6. Turn the light off and change to a different color filter.
7. Repeat steps $4-5$ with the new color filter.
8. Repeat steps $4-5$ with white light.


#### Abstract

Analysis Does the color of light affect the pattern? How is the reflection of white light different from that of singlecolor light?

Critical Thinking Look closely at your observations of white light reflecting from the discs. Suggest possible sources for the bands of color.




### 19.1 Interference

In Chapter 16, you learned that light sometimes acts like a wave. Light can be diffracted when it passes by an edge, just like water waves and sound waves can. In Chapters 17 and 18, you learned that reflection and refraction can be explained when light is modeled as a wave. What led scientists to believe that light has wave properties? They discovered that light could be made to interfere, which you will learn about in this section.

When you look at objects that are illuminated by a white light source such as a nearby lightbulb, you are seeing incoherent light, which is light with unsynchronized wave fronts. The effect of incoherence in waves can be seen in the example of heavy rain falling on a swimming pool. The surface of the water is choppy and does not have any regular pattern of wave fronts or standing waves. Because light waves have such a high frequency, incoherent light does not appear choppy to you. Instead, as light from an incoherent white light source illuminates an object, you see the superposition of the incoherent light waves as an even, white light.

- Objectives
- Explain how light falling on two slits produces an interference pattern.
- Calculate light wavelengths from interference patterns.
- Apply modeling techniques to thin-film interference.
- Vocabulary
incoherent light coherent light interference fringes monochromatic light thin-film interference
- Figure 19-1 Smooth wave fronts of light are created by point sources (a) and by lasers (b).
$\square$ Figure 19-2 These are doubleslit interference patterns for blue light (a), for red light (b), and for white light (c).


Circular wave fronts


Straight line wave fronts


## Interference of Coherent Light

The opposite of incoherent light is coherent light, which is light from two or more sources that add together in superposition to produce smooth wave fronts. A smooth wave front can be created by a point source, as shown in Figure 19-1a. A smooth wave front also can be created by multiple point sources when all point sources are synchronized, such as with a laser, as shown in Figure 19-1b. Only the superposition of light waves from coherent light sources can produce the interference phenomena that you will examine in this section.

English physician Thomas Young proved that light has wave properties when he produced an interference pattern by shining light from a single coherent source through two slits. Young directed coherent light at two closely spaced, narrow slits in a barrier. When the overlapping light from the two slits fell on an observing screen, the overlap did not produce even illumination, but instead created a pattern of bright and dark bands that Young called interference fringes. He explained that these bands must be the result of constructive and destructive interference of light waves from the two slits in the barrier.

In a double-slit interference experiment that uses monochromatic light, which is light of only one wavelength, constructive interference produces a bright central band of the given color on the screen, as well as other bright bands of near-equal spacing and near-equal width on either side, as shown in Figures 19-2a and 19-2b. The intensity of the bright bands decreases the farther the band is from the central band, as you can easily see in Figure 19-2a. Between the bright bands are dark areas where destructive interference occurs. The positions of the constructive and destructive interference bands depend on the wavelength of the light. When white light is used in a double-slit experiment, however, interference causes the appearance of colored spectra instead of bright and dark bands, as shown in Figure 19-2c. All wavelengths interfere constructively in the central bright band, and thus that band is white. The positions of the other colored bands result from the overlap of the interference fringes that occur where wavelengths of each separate color interfere constructively.


Double-slit interference To create coherent light from incoherent light, Young placed a light barrier with a narrow slit in front of a monochromatic light source. Because the width of the slit is very small, only a coherent portion of the light passes through and is diffracted by the slit, producing nearly cylindrical diffracted wave fronts, as shown in Figure 19-3. Because a cylinder is symmetrical, the two portions of the wave front arriving at the second barrier with two slits will be in phase. The two slits at the second barrier then produce coherent, nearly cylindrical wave fronts that can then interfere, as shown in Figure 19-3. Depending on their phase relationship, the two waves can undergo constructive or destructive interference, as shown in Figure 19-4.

- Figure 19-3 The coherent source that is created by a narrow single slit produces coherent, nearly cylindrical waves that travel to the two slits in the second barrier. Two coherent, nearly cylindrical waves leave the double slit.
- Figure 19-4 A pair of in-phase waves is created at the two slits. At some locations, the waves might undergo constructive interference to create a bright band (a), or destructive interference to create a dark band (b).


- Figure 19-5 The interference of monochromatic light that passes through the double slit produces bright and dark bands on a screen (a). This diagram (b) represents an analysis of the first bright band. The distance from the slits to the screen, $L$, is about $10^{5}$ times longer than the separation, $d$, between the two slits. (Illustrations not to scale)

Measuring the wavelength of light A top view of nearly cylindrical wave fronts and Young's double slit experiment are shown in Figure 19-5a. The wave fronts interfere constructively and destructively to form a pattern of light and dark bands. A typical diagram that is used to analyze Young's experiment is shown in Figure 19-5b. Light that reaches point $\mathrm{P}_{0}$ travels the same distance from each slit. Because the waves are in phase, they interfere constructively on the screen to create the central bright band at $\mathrm{P}_{0}$. There is also constructive interference at the first bright band, $\mathrm{P}_{1}$, on either side of the central band, because line segment $P_{1} S_{1}$ is one wavelength, $\lambda$, longer than the line segment $\mathrm{P}_{1} \mathrm{~S}_{2}$. Thus, the waves arrive at $\mathrm{P}_{1}$ in phase.

There are two triangles shaded in the figure. The larger triangle is a right triangle, so $\tan \theta=x / L$. In the smaller triangle $\mathrm{RS}_{1} \mathrm{~S}_{2}$, the side $\mathrm{S}_{1} \mathrm{R}$ is the length difference of the two light paths, which is one wavelength. There are now two simplifications that make the problem easier to solve.

1. If $L$ is much larger than $d$, then line segments $S_{1} P_{1}$ and $S_{2} P_{1}$ are nearly parallel to each other and to line segment $\mathrm{QP}_{1}$, and triangle $R S_{1} S_{2}$ is very nearly a right triangle. Thus, $\sin \theta \approx \lambda / d$.
2. If the angle $\theta$ is small, then $\sin \theta$ is very nearly equal to $\tan \theta$.

With the above simplifications, the relationships $\tan \theta=x / L, \sin \theta \approx \lambda / d$, and $\sin \theta \approx \tan \theta$ combine to form the equation $x / L=\lambda / d$. Solving for $\lambda$ gives the following.

## Wavelength from Double-Slit Experiment $\lambda=\frac{x d}{L}$

The wavelength of light, as measured by a double slit, is equal to the distance on the screen from the central bright band to the first bright band, multiplied by the distance between the slits, divided by the distance to the screen.

Constructive interference from two slits occurs at locations, $x_{m^{\prime}}$ on either side of the central bright band, which are determined using the equation $m \lambda=x_{\mathrm{m}} d / L$, where $m=0,1,2$, etc., as limited by the small angle simplification. The central bright band occurs at $m=0$. Frequently, the band given by $m=1$ is called the first-order band, and so on.

## EXAMPLE Problem 1

Wavelength of Light A double-slit experiment is performed to measure the wavelength of red light. The slits are 0.0190 mm apart. A screen is placed 0.600 m away, and the first-order bright band is found to be 21.1 mm from the central bright band. What is the wavelength of the red light?

## 1 Analyze and Sketch the Problem

- Sketch the experiment, showing the slits and the screen.
- Draw the interfence pattern with bands in appropriate locations.

$$
\begin{array}{ll}
\text { Known: } & \text { Unknown: } \\
d=1.90 \times 10^{-5} \mathrm{~m} & \lambda=? \\
x=2.11 \times 10^{-2} \mathrm{~m} & \\
L=0.600 \mathrm{~m} &
\end{array}
$$



2 Solve for the Unknown

Math Handbook
Operations with Scientific Notation pages 842-843

$$
\begin{aligned}
\lambda & =\frac{x d}{L} \\
& =\frac{\left(2.11 \times 10^{-2} \mathrm{~m}\right)\left(1.90 \times 10^{-5} \mathrm{~m}\right)}{(0.600 \mathrm{~m})} \\
& =6.68 \times 10^{-7} \mathrm{~nm}=668 \mathrm{~nm}
\end{aligned}
$$

Substitute $x=2.11 \times 10^{-2} \mathrm{~m}, d=1.90 \times 10^{-5} \mathrm{~m}, L=0.600 \mathrm{~m}$

3 Evaluate the Answer

- Are the units correct? The answer is in units of length, which is correct for wavelength.
- Is the magnitude realistic? The wavelength of red light is near 700 nm , and that of blue is near 400 nm . Thus, the answer is reasonable for red light.


## PRACTICE Problems

1. Violet light falls on two slits separated by $1.90 \times 10^{-5} \mathrm{~m}$. A first-order bright band appears 13.2 mm from the central bright band on a screen 0.600 m from the slits. What is $\lambda$ ?
2. Yellow-orange light from a sodium lamp of wavelength 596 nm is aimed at two slits that are separated by $1.90 \times 10^{-5} \mathrm{~m}$. What is the distance from the central band to the first-order yellow band if the screen is 0.600 m from the slits?
3. In a double-slit experiment, physics students use a laser with $\lambda=632.8 \mathrm{~nm}$. A student places the screen 1.000 m from the slits and finds the first-order bright band 65.5 mm from the central line. What is the slit separation?
4. Yellow-orange light with a wavelength of 596 nm passes through two slits that are separated by $2.25 \times 10^{-5} \mathrm{~m}$ and makes an interference pattern on a screen. If the distance from the central line to the first-order yellow band is $2.00 \times 10^{-2} \mathrm{~m}$, how far is the screen from the slits?

Young presented his findings in 1803, but was ridiculed by the scientific community. His conclusions did not begin to gain acceptance until 1820 after Jean Fresnel proposed a mathematical solution for the wave nature of light in a competition. One of the judges, Siméon Denis Poisson, showed that, if Fresnel was correct, a shadow of a circular object illuminated with coherent light would have a bright spot at the center of the shadow. Another judge, Jean Arago, proved this experimentally. Before this, both Poisson and Arago were skeptics of the wave nature of light.

- Figure 19-6 Each wavelength is reinforced where the soap film thickness is $\lambda / 4,3 \lambda / 4,5 \lambda / 4$ (a). Because each color has a different wavelength, a series of color bands is reflected from the soap film (b).


## APPLYING PHYSICS

- Nonreflective Eyeglasses A thin film can be placed on the lenses of eyeglasses to keep them from reflecting wavelengths of light that are highly visible to the human eye. This prevents the glare of reflected light. 4



## Thin-Film Interference

Have you ever seen a spectrum of colors produced by a soap bubble or by the oily film on a water puddle in a parking lot? These colors were not the result of separation of white light by a prism or of absorption of colors in a pigment. The spectrum of colors was a result of the constructive and destructive interference of light waves due to reflection in a thin film, a phenomenon called thin-film interference.

If a soap film is held vertically, as in Figure 19-6, its weight makes it thicker at the bottom than at the top. The thickness varies gradually from top to bottom. When a light wave strikes the film, it is partially reflected, as shown by ray 1 , and partially transmitted. The reflected and transmitted waves have the same frequency as the original. The transmitted wave travels through the film to the back surface, where, again, part is reflected, as shown by ray 2 . This act of splitting each light wave from an incoherent source into a matched pair of waves means that the reflected light from a thin film is coherent.

Color reinforcement How is the reflection of one color enhanced? This happens when the two reflected waves are in phase for a given wavelength. If the thickness of the soap film in Figure 19-6 is one-fourth of the wavelength of the wave in the film, $\lambda / 4$, then the round-trip path length in the film is $\lambda / 2$. In this case, it would appear that ray 2 would return to the front surface one-half wavelength out of phase with ray 1 , and that the two waves would cancel each other based on the superposition principle. However, when a transverse wave is reflected from a medium with a slower wave speed, the wave is inverted. With light, this happens at a medium with a larger index of refraction. As a result, ray 1 is inverted on reflection; whereas ray 2 is reflected from a medium with a smaller index of refraction (air) and is not inverted. Thus, ray 1 and ray 2 are in phase.

If the film thickness, $d$, satisfies the requirement, $d=\lambda / 4$, then the color of light with that wavelength will be most strongly reflected. Note that because the wavelength of light in the film is shorter than the wavelength in air, $d=\lambda_{\text {film }} / 4$, or, in terms of the wavelength in air, $d=\lambda_{\text {vacuum }} / 4 n_{\text {film }}$. The two waves reinforce each other as they leave the film. Light with other wavelengths undergoes destructive interference.

As you know, different colors of light have different wavelengths. For a film of varying thickness, such as the one shown in Figure 19-6, the wavelength requirement will be met at different thicknesses for different colors. The result is a rainbow of color. Where the film is too thin to produce constructive interference for any wavelength of color, the film appears to be black. Notice in Figure 19-6b that the spectrum repeats. When the thickness of the film is $3 \lambda / 4$, the round-trip distance is $3 \lambda / 2$, and constructive interference occurs again. Any thickness equal to $1 \lambda / 4,3 \lambda / 4,5 \lambda / 4$, and so on, satisfies the conditions for constructive interference for a given wavelength.
Applications of thin-film interference The example of a film of soapy water in air involves constructive interference with one of two waves inverted upon reflection. In the chapter opener example of bubble solution, as the thickness of the film changes, the wavelength undergoing constructive interference changes. This creates a shifting spectrum of color on the surface of the film soap when it is under white light. In other examples of thinfilm interference, neither wave or both waves might be inverted. You can develop a solution for any problem involving thin-film interference using the following strategies.

## PROBLEM-SOLVING Strategies

## Connecting Math to Physics

## Thin-Film Interference

When solving thin-film interference problems, construct an equation that is specific to the problem by using the following strategies.

1. Make a sketch of the thin film and the two coherent waves. For simplicity, draw the waves as rays.
2. Read the problem. Is the reflected light enhanced or reduced? When it is enhanced, the two reflected waves undergo constructive interference. When the reflected light is reduced, the waves undergo destructive interference.
3. Are either or both waves inverted on reflection? If the index of refraction changes from a lower to a higher value, then the wave is inverted. If it changes from a higher to a lower value, there is no inversion.
4. Find the extra distance that the second wave must travel through the thin film to create the needed interference.
a. If you need constructive interference and one wave is inverted OR you need destructive interference and either both waves or none are inverted, then the difference in distance is an odd number of half wavelengths: $(m+1 / 2) \lambda_{\text {film }}$, where $m=0,1,2$, etc.
b. If you need constructive interference and either both waves or none are inverted OR you need destructive interference and one wave is inverted, then the difference is an integer number of wavelengths: $m \lambda_{\text {film }}$, where $m=1,2,3$, etc.
5. Set the extra distance traveled by the second ray to twice the film thickness, $2 d$.
6. Recall from Chapter 18 that $\lambda_{\text {film }}=\lambda_{\text {vacuum }} / n_{\text {film }}$.

Reflection from a Thin Film


## EXAMPLE Problem 2

Oil and Water You observe colored rings on a puddle and conclude that there must be an oil slick on the water. You look directly down on the puddle and see a yellow-green ( $\lambda=555 \mathrm{~nm}$ ) region. If the refractive index of oil is 1.45 and that of water is 1.33 , what is the minimum thickness of oil that could cause this color?

## 1 Analyze and Sketch the Problem

- Sketch the thin film and layers above and below it.
- Draw rays showing reflection off the top of the film as well as the bottom.

$$
\begin{array}{lll}
\text { Known: } & \text { Unknown: } \\
n_{\text {water }} & =1.33 & d=? \\
n_{\text {oil }} & =1.45 & \\
\lambda & =555 \mathrm{~nm} &
\end{array}
$$



2 Solve for the Unknown
Because $n_{\text {oil }}>n_{\text {air }}$, there is a phase inversion on the first reflection. Because $n_{\text {water }}<n_{\text {oil }}$, there is no phase inversion on the second reflection. Thus, there is one wave inversion.
The wavelength in oil is less than it is in air.
Follow the problem-solving strategy to construct the equation.

$$
2 d=\left(m+\frac{1}{2}\right) \frac{\lambda}{n_{\mathrm{oil}}}
$$

Because you want the minimum thickness, $m=0$.

$$
\begin{aligned}
d & =\frac{\lambda}{4 n_{\text {oil }}} & & \text { Substitute } m=0 \\
& =\frac{555 \mathrm{~nm}}{(4)(1.45)} & & \text { Substitute } \lambda=555 \mathrm{~nm}, n_{\text {oil }}=1.45 \\
& =95.7 \mathrm{~nm} & &
\end{aligned}
$$

## 3 Evaluate the Answer

- Are the units correct? The answer is in nm, which is correct for thickness.
- Is the magnitude realistic? The minimum thickness is smaller than one wavelength, which is what it should be.


## PRACTICE Problems

5. In the situation in Example Problem 2, what would be the thinnest film that would create a reflected red ( $\lambda=635 \mathrm{~nm}$ ) band?
6. A glass lens has a nonreflective coating placed on it. If a film of magnesium fluoride, $n=1.38$, is placed on the glass, $n=1.52$, how thick should the layer be to keep yellowgreen light from being reflected?
7. A silicon solar cell has a nonreflective coating placed on it. If a film of sodium monoxide, $n=1.45$, is placed on the silicon, $n=3.5$, how thick should the layer be to keep yellowgreen light $(\lambda=555 \mathrm{~nm})$ from being reflected?
8. You can observe thin-film interference by dipping a bubble wand into some bubble solution and holding the wand in the air. What is the thickness of the thinnest soap film at which you would see a black stripe if the light illuminating the film has a wavelength of 521 nm ? Use $n=1.33$.
9. What is the thinnest soap film $(n=1.33)$ for which light of wavelength 521 nm will constructively interfere with itself?


■ Figure 19-7 The Morpho butterfly has a blue color that shimmers iridescently (a). An electron microscope is used to view a cross section of the wing looking down the length of the ridges (b). The ridges are steplike structures. Matched pairs of light rays from a single step and from multiple steps can interfere (c).


Thin-film interference also occurs naturally on the wings of the Morpho butterfly, shown in Figure 19-7a. The shimmering blue of the butterfly is caused by ridges that project up from the ground scales of the wings, as shown in Figure 19-7b. Light is reflected from and refracted through a series of steplike structures, as diagrammed in Figure 19-7c, forming a blue interference pattern that appears to shimmer to those who see the butterfly.

### 19.1 Section Review

10. Film Thickness Lucien is blowing bubbles and holds the bubble wand up so that a soap film is suspended vertically in the air. What is the second thinnest width of the soap film at which he could expect to see a bright stripe if the light illuminating the film has a wavelength of 575 nm ? Assume the soap solution has an index of refraction of 1.33.
11. Bright and Dark Patterns Two very narrow slits are cut close to each other in a large piece of cardboard. They are illuminated by monochromatic red light. A sheet of white paper is placed far from the slits, and a pattern of bright and dark bands is seen on the paper. Describe how a wave behaves when it encounters a slit, and explain why some regions are bright while others are dark.
12. Interference Patterns Sketch the pattern described in problem 11.
13. Interference Patterns Sketch what happens to the pattern in problem 11 when the red light is replaced by blue light.
14. Film Thickness A plastic reflecting film ( $n=1.83$ ) is placed on an auto glass window ( $n=1.52$ ).
a. What is the thinnest film that will reflect yellowgreen light?
b. Unfortunately, a film this thin cannot be manufactured. What is the next-thinnest film that will produce the same effect?
15. Critical Thinking The equation for wavelength from a double-slit experiment uses the simplification that $\theta$ is small so that $\sin \theta \approx \tan \theta$. Up to what angle is this a good approximation when your data has two significant figures? Would the maximum angle for a valid approximation increase or decrease as you increase the precision of your angle measurement?

### 19.2 Diffraction

- Objectives
- Explain how diffraction gratings form diffraction patterns.
- Describe how diffraction gratings are used in grating spectrometers.
- Discuss how diffraction limits the ability to distinguish two closely spaced objects with a lens.
- Vocabulary
diffraction pattern diffraction grating Rayleigh criterion
- Figure 19-8 Notice the wide central band and the narrower bands on either side. A single-slit diffraction pattern for red light would have a wider central band than blue light as long as the same size slit is used for both colors.

- Figure 19-9 A slit of width $w$ is divided into pairs of lines that form Huygens' wavelets, each pair separated by w/2.

In Chapter 16, you learned that smooth wave fronts of light spread when they are diffracted around an edge. Diffraction was explained using Huygens' principle that a smooth wave front is made up of many small point-source wavelets. The cutting of coherent light on two edges spaced closely together produces a diffraction pattern, which is a pattern on a screen of constructive and destructive interference of Huygens' wavelets.

## Single-Slit Diffraction

When coherent, blue light passes through a single, small opening that is larger than the wavelength of the light, the light is diffracted by both edges, and a series of bright and dark bands appears on a distant screen, as shown in Figure 19-8. Instead of the nearly equally spaced bands produced by two coherent sources in Young's double-slit experiment, this pattern has a wide, bright central band with dimmer, narrower bands on either side. When using red light instead of blue, the width of the bright central band increases. With white light, the pattern is a mixture of patterns of all the colors of the spectrum.

To see how Huygens' wavelets produce the diffraction pattern, imagine a slit of width $w$ as being divided into an even number of Huygens' points, as shown in Figure 19-9. Each Huygens' point acts as a point source of Huygens' wavelets. Divide the slit into two equal parts and choose one source from each part so that the pair is separated by a distance $w / 2$. This pair of sources produces coherent, cylindrical waves that will interfere.

For any Huygens' wavelet produced in the top half, there will be another Huygens' wavelet in the bottom half, a distance $w / 2$ away, that it will interfere with destructively to create a dark band on the screen. All similar pairings of Huygens' wavelets interfere destructively at dark bands. Conversely, a bright band on the screen is where pairings of Huygens' wavelets interfere constructively. In the dim regions between bright and dark bands, partial destructive interference occurs.



Diffraction pattern When the single slit is illuminated, a central bright band appears at location $\mathrm{P}_{0}$ on the screen, as shown in Figure 19-10. The first dark band is at position $\mathrm{P}_{1}$. At this location, the path lengths $r_{1}$ and $r_{2}$ of the two Huygens' wavelets differ by one-half wavelength, thereby producing a dark band by destructive interference. This model is mathematically similar to that of double-slit interference. A comparison of a single-slit diffraction pattern with a double-slit interference pattern using slits of the same width reveals that all of the bright interference fringes of the doubleslit interference pattern fit within the width of the central bright band of the single-slit diffraction pattern. Double-slit interference results from the interference of the single-slit diffraction patterns of the two slits.

An equation now can be developed for the diffraction pattern produced by a single slit using the same simplifications that were used for double-slit interference, assuming that the distance to the screen is much larger than $w$. The separation distance between the sources of the two interfering waves is now $w / 2$. To find the distance measured on the screen to the first dark band, $x_{1}$, note that the path length difference is now $\lambda / 2$ because at the dark band there is destructive interference. As a result $x_{1} / L=\lambda / w$.

You can see from Figure 19-10 that it might be difficult to measure the distance to the first dark band from the center of the central bright band. A better method of determining $x_{1}$ is to measure the width of the central bright band, $2 x_{1}$. The following equation gives the width of the central bright band from single-slit diffraction.

## Width of Bright Band in Single-Slit Diffraction $2 x_{1}=\frac{2 \lambda L}{w}$

The width of the central bright band is equal to the product of twice the wavelength times the distance to the screen, divided by the width of the slit.

Canceling the 2's out of the above equation gives you the distance from the center of the central bright band to where the first dark band occurs. The location of additional dark bands can be found where the path lengths differ by $3 \lambda / 2,5 \lambda / 2$, and so on. This can be expressed as $x_{\mathrm{m}}=m \lambda L / w$, where $m=1,2,3$, etc., as limited by the small angle simplication. When $m=1$, this equation provides the position of the first-order dark band. The second-order dark band occurs at $m=2$, and so forth.

- Figure 19-10 This diagram represents an analysis of the first dark band. The distance to the screen, $L$, is much larger than the slit width, $w$. (Illustration not to scale)


## PRACTICE Problems

16. Monochromatic green light of wavelength 546 nm falls on a single slit with a width of 0.095 mm . The slit is located 75 cm from a screen. How wide will the central bright band be?
17. Yellow light with a wavelength of 589 nm passes through a slit of width 0.110 mm and makes a pattern on a screen. If the width of the central bright band is $2.60 \times 10^{-2} \mathrm{~m}$, how far is it from the slits to the screen?
18. Light from a He-Ne laser $(\lambda=632.8 \mathrm{~nm})$ falls on a slit of unknown width. A pattern is formed on a screen 1.15 m away, on which the central bright band is 15 mm wide. How wide is the slit?
19. Yellow light falls on a single slit 0.0295 mm wide. On a screen that is 60.0 cm away, the central bright band is 24.0 mm wide. What is the wavelength of the light?
20. White light falls on a single slit that is 0.050 mm wide. A screen is placed 1.00 m away. A student first puts a blue-violet filter ( $\lambda=441 \mathrm{~nm}$ ) over the slit, then a red filter $(\lambda=622 \mathrm{~nm})$. The student measures the width of the central bright band.
a. Which filter produced the wider band?
b. Calculate the width of the central bright band for each of the two filters.

Single-slit diffraction patterns make the wave nature of light noticeable when the slits are 10 to 100 times the wavelength of the light. Larger openings, however, cast sharp shadows, as Isaac Newton first observed. While the single-slit pattern depends on the wavelength of light, it is only when a large number of slits are put together that diffraction provides a useful tool for measuring wavelength.

## - CHALLENGE PROBLEM

You have several unknown substances and wish to use a single-slit diffraction apparatus to determine what each one is. You decide to place a sample of an unknown substance in the region between the slit and the screen and use the data that you obtain to determine the identity of each substance by calculating its index of refraction.

1. Come up with a general formula for the index of refraction of an unknown substance in terms of the wavelength of the light, $\lambda_{\text {vacuum }}$, the width of the slit, $w$, the distance from the slit to the screen, $L$, and the distance between the central bright band and the first dark band, $x_{1}$.
2. If the source you used had a wavelength of 634 nm , the slit width was 0.10 mm , the distance from the slit to the screen was 1.15 m , and you immersed the apparatus in water ( $n_{\text {substance }}=1.33$ ), then what would you expect the width of the center band to be?


## Diffraction Gratings

Although double-slit interference and single-slit diffraction depend on the wavelength of light, diffraction gratings, such as those shown in Figure 19-11, are used to make precision measurements of wavelength. A diffraction grating is a device made up of many single slits that diffract light and form a diffraction pattern that is an overlap of singleslit diffraction patterns. Diffraction gratings can have as many as 10,000 slits per centimeter. That is, the spacing between the slits can be as small as $10^{-6} \mathrm{~m}$, or 1000 nm .

One type of diffraction grating is called a transmission grating. A transmission grating can be made by scratching very fine lines with a diamond point on glass that trans-
 mits light. The spaces between the scratched lines act like slits. A less expensive type of grating is a replica grating. A replica grating is made by pressing a thin plastic sheet onto a glass grating. When the plastic is pulled away, it contains an accurate imprint of the scratches. Jewelry made from replica gratings, shown in Figure 19-12a, produces a spectrum.

Reflection gratings are made by inscribing fine lines on metallic or reflective glass surfaces. The color spectra produced when white light reflects off the surface of a CD or DVD is the result of a reflection grating, as shown in Figure 19-12b. If you were to shine monochromatic light on a DVD, the reflected light would produce a diffraction pattern on a screen. Transmission and reflection gratings produce similar diffraction patterns, which can be analyzed in the same manner.

Holographic diffraction gratings produce the brightest spectra. They are made by using a laser and mirrors to create an interference pattern consisting of parallel bright and dark lines. The pattern is projected on a piece of metal that is coated with a light-sensitive material. The light produces a chemical reaction that hardens the material. The metal is then placed in acid, which attacks the metal wherever it was not protected by the hardened material. The result is a series of hills and valleys in the metal identical to the original interference pattern. The metal can be used as a reflection grating or a replica transmission grating can be made from it. Because of the sinusoidal shape of the hills and valleys, the diffraction patterns are exceptionally bright.


- Figure 19-12 A transmission grating spectrum is created by jewelry made with replica gratings (a). Compact discs act as reflection gratings, creating a spectrum diffraction pattern when they are placed under white light (b).
- Figure 19-13 A spectroscope is used to measure the wavelengths of light emitted by a light source.

Figure 19-14 A grating was used to produce diffraction patterns for red light (a) and white light (b).


Measuring wavelength An instrument used to measure light wavelengths using a diffraction grating is called a grating spectroscope, as shown in the diagram in Figure 19-13. The source to be analyzed emits light that is directed to a slit. The light from the slit passes through a diffraction grating. The grating produces a diffraction pattern that is viewed through a telescope.

The diffraction pattern produced by a diffraction grating has narrow, equally spaced, bright lines, as shown in Figure 19-14. The larger the number of slits per unit length of the grating, the narrower the lines in the diffraction pattern. As a result, the distance between the bright lines can be measured much more precisely with a grating spectroscope than with a double slit.

Earlier in this chapter, you found that the interference pattern produced by a double slit could be used to calculate wavelength. An equation for the diffraction grating can be developed in the same way as for the double slit. However, with a diffraction grating, $\theta$ could be large, so the small angle simplification does not apply. Wavelength can be found by measuring the angle, $\theta$, between the central bright line and the first-order bright line.

## Wavelength from a Diffraction Grating $\lambda=d \sin \theta$

The wavelength of light is equal to the slit separation distance times the sine of the angle at which the first-order bright line occurs.

Constructive interference from a diffraction grating occurs at angles on either side of the central bright line given by the equation $m \lambda=d \sin \theta$, where $m=0,1,2$, etc. The central bright line occurs at $m=0$.


## EXAMPLE Problem 3

Using a DVD as a Diffraction Grating A student noticed the beautiful spectrum reflected off a rented DVD. She directed a beam from her teacher's green laser pointer at the DVD and found three bright spots reflected on the wall. The label on the pointer indicated that the wavelength was 532 nm . The student found that the spacing between the spots was 1.29 m on the wall, which was 1.25 m away. What is the spacing between the rows on the DVD?

## 1 Analyze and Sketch the Problem

- Sketch the experiment, showing the DVD as a grating and the spots on the wall.
- Identify and label the knowns.

$$
\begin{array}{lc}
\text { Known: } & \text { Unknown: } \\
x=1.29 \mathrm{~m} & d=? \\
L=1.25 \mathrm{~m} & \\
\lambda=532 \mathrm{~nm} &
\end{array}
$$

## 2 Solve for the Unknown

Find the angle between the central bright spot and one next
to it using $\tan \theta=x / L$.


$$
\begin{aligned}
\theta & =\tan ^{-1}\left(\frac{x}{L}\right) \\
& =\tan ^{-1}\left(\frac{1.29 \mathrm{~m}}{1.25 \mathrm{~m}}\right) \quad \text { Substitute } x=1.29 \mathrm{~m}, L=1.25 \mathrm{~m} \\
& =45.9^{\circ}
\end{aligned}
$$

3 Evaluate the Answer

- Are the units correct? The answer is in m , which is correct for separation.
- Is the magnitude realistic? With $x$ and $L$ almost the same size, $d$ is close to $\lambda$.


## PRACTICE Problems

21. White light shines through a grating onto a screen. Describe the pattern that is produced.
22. If blue light of wavelength 434 nm shines on a diffraction grating and the spacing of the resulting lines on a screen that is 1.05 m away is 0.55 m , what is the spacing between the slits in the grating?
23. A diffraction grating with slits separated by $8.60 \times 10^{-7} \mathrm{~m}$ is illuminated by violet light with a wavelength of 421 nm . If the screen is 80.0 cm from the grating, what is the separation of the lines in the diffraction pattern?
24. Blue light shines on the DVD in Example Problem 3. If the dots produced on a wall that is 0.65 m away are separated by 58.0 cm , what is the wavelength of the light?
25. Light of wavelength 632 nm passes through a diffraction grating and creates a pattern on a screen that is 0.55 m away. If the first bright band is 5.6 cm from the central bright band, how many slits per centimeter does the grating have?


Figure 19-15 The diffraction pattern of a circular aperture produces alternating dark and bright rings. (Illustration not to scale)

## Astronomy Connection

- Figure 19-16 Similar-triangle geometry allows you to calculate the actual separation distance of objects. The blue and red colors are used only for the purpose of illustration. (Illustration not to scale)

In thin-film interference, the interference pattern is visible only within a narrow angle of view straight over the film. This would be the case for the Morpho butterfly's blue, shimmering interference pattern, if not for the layer of glass-like scales on top of the layer of ground scales. This glass-like scale layer acts as a diffraction grating and causes the blue, shimmering interference pattern to be spread to a diffraction pattern with a wider angle of view. Scientists believe that this makes the Morpho butterfly more visible to potential mates.

## Resolving Power of Lenses

The circular lens of a telescope, a microscope, and even your eye acts as a hole, called an aperture, through which light is allowed to pass. An aperture diffracts the light, just as a single slit does. Alternating bright and dark rings occur with a circular aperture, as shown in Figure 19-15. The equation for an aperture is similar to that for a single slit. However, an aperture has a circular edge rather than the two edges of a slit, so slit width, $w$, is replaced by aperture diameter, $D$, and an extra geometric factor of 1.22 enters the equation, resulting in $x_{1}=1.22 \lambda L / D$.

When light from a distant star is viewed through the aperture of a telescope, the image is spread out due to diffraction. If two stars are close enough together, the images may be blurred together, as shown in Figure 19-16. In 1879, Lord Rayleigh, a British physicist, mathematician, and Nobel prize winner, established a criterion for determining whether there is one or two stars in such an image. The Rayleigh criterion states that if the center of the bright spot of one star's image falls on the first dark ring of the second, the two images are at the limit of resolution. That is, a viewer will be able to tell that there are two stars rather than only one.

If two images are at the limit of resolution, how far apart are the objects? Using the Rayleigh criterion, the centers of the bright spots of the two images are a distance of $x_{1}$ apart. Figure 19-16 shows that similar triangles can be used to find that $x_{\mathrm{obj}} / L_{\mathrm{obj}}=x_{1} / L$. Combining this with $x_{1}=1.22 \lambda L / D$ to eliminate $x_{1} / L$, and solving for the separation distance between objects, $x_{\text {obj }}$, the following equation can be derived.

$$
\text { Rayleigh Criterion } \quad x_{\mathrm{obj}}=\frac{1.22 \lambda L_{\mathrm{obj}}}{D}
$$

The separation distance between objects that are at the limit of resolution is equal to 1.22 , times the wavelength of light, times the distance from the circular aperture to the objects, divided by the diameter of the circular aperture.


Diffraction in the eye In bright light the eye's pupil is about 3 mm in diameter. The eye is most sensitive to yellow-green light where $\lambda=550 \mathrm{~nm}$. So the Rayleigh criterion applied to the eye gives $x_{\mathrm{obj}}=2 \times 10^{-4} L_{\mathrm{obj}}$. The distance between the pupil and retina is about 2 cm , so two barely resolved point sources would be separated by about $4 \mu \mathrm{~m}$ on the retina. The spacing between the light detectors, the cones, in the most sensitive part of the eye, the fovea, is about $2 \mu \mathrm{~m}$. Thus, in the ideal case, the three adjacent cones would record light, dark, and light. It would seem that the eye is ideally constructed. If cones were closer together, they would see details of the diffraction pattern, not of the sources. If cones were farther apart, they would not be able to resolve all possible detail.

Applying the Rayleigh criterion to find the ability of the eye to separate two distance sources shows that the eye could separate two automobile headlamps ( 1.5 m apart) at a distance of 7 km . In practice, however, the eye is not limited by diffraction. Imperfections in the lens and the liquid that fills the eye reduce the eye's resolution to about five times that set by the Rayleigh criterion. Most people use their eyes for purposes other than resolving point sources. For example, the eye seems to have a built-in ability to detect straight edges.

Many telescope manufacturers advertise that their instruments are diffraction limited. That is, they claim that their telescopes can separate two point sources at the Rayleigh criterion. To reach this limit they must grind the mirrors and lenses to an accuracy of one-tenth of a wavelength, or about 55 nm . The larger the diameter of the mirror, the greater the resolution of the telescope. Unfortunately, light from planets or stars must go through Earth's atmosphere. The same variations in the atmosphere that cause stars to twinkle keep telescopes from reaching the diffraction limit. Because the Hubble Space Telescope is above Earth's atmosphere, the resolution of its images is much better than those of larger telescopes on Earth's surface.

OMINI LAAB

## Retinal

Projection Screen
Did you know that you can use the retina of your eyeball as a screen? CAUTION: Do not do the following with a laser or the Sun.

1. Plug in and turn on an incandescent lamp with a straight filament. Stand about 2 m from the lamp.
2. Hold a diffraction grating in front of your eye so the color spectra are oriented horizontally.
3. Observe the color spectra patterns and draw your observations using colored pencils.

## Analyze and Conclude

4. Which color is closest to the central bright line (the light filament)? Which is farthest?
5. How many spectra are you able to see on either side of the light?
6. Interpret Data Are your observations consistent with the equation for the wavelength from a diffraction grating?

### 19.2 Section Review

26. Distance Between First-Order Dark Bands Monochromatic green light of wavelength 546 nm falls on a single slit of width 0.080 mm . The slit is located 68.0 cm from a screen. What is the separation of the first dark bands on each side of the central bright band?
27. Diffraction Patterns Many narrow slits are close to each other and equally spaced in a large piece of cardboard. They are illuminated by monochromatic red light. A sheet of white paper is placed far from the slits, and a pattern of bright and dark bands is visible on the paper. Sketch the pattern that would be seen on the screen.
28. Line Spacing You shine a red laser light through one diffraction grating and form a pattern of red dots on a screen. Then you substitute a second diffraction grating for the first one, forming a different
pattern. The dots produced by the first grating are spread out more than those produced by the second. Which grating has more lines per millimeter?
29. Rayleigh Criterion The brightest star in the winter sky in the northern hemisphere is Sirius. In reality, Sirius is a system of two stars that orbit each other. If the Hubble Space Telescope (diameter 2.4 m ) is pointed at the Sirius system, which is 8.44 light-years from Earth, what is the minimum separation there would need to be between the stars in order for the telescope to be able to resolve them? Assume that the average light coming from the stars has a wavelength of 550 nm .
30. Critical Thinking You are shown a spectrometer, but do not know whether it produces its spectrum with a prism or a grating. By looking at a whitelight spectrum, how could you tell?

# PHYSICS LAB•Design Your Own <br> <br> Double-Slit Interference of Light <br> <br> Double-Slit Interference of Light <br> Alternate CBL instructions 

can be found on the Web site.
physicspp.com

Light sometimes behaves as a wave. As coherent light strikes a pair of slits that are close together, the light passing through the slits will create a pattern of constructive and destructive interference on a screen. In this investigation you will develop a procedure and measure the wavelength of a monochromatic light source using two slits.

## QUESTION

How can a double-slit interference pattern of light be used to measure the light's wavelength?

## Objectives

■ Observe a double-slit interference pattern of monochromatic light.
■ Calculate the wavelength of light using a double-slit interference pattern.

## Safety Precautions

## ए चर <br> Use laser protective eyewear approved by ANSI.

Never look directly into the light of a laser.

## Possible Materials

laser pointer or laser to be tested double-slit plate
laser pointer or laser of known wavelength clothes pin to hold a laser pointer clay ball to hold the double-slit plate meterstick

## Procedure

1. Determine which equation applies to doubleslit interference.
2. Use a double slit of known slit-separation distance, $d$, or develop a method to determine $d$.
3. Sketch how light passes through a double slit to help you determine how $x$ and $L$ can be measured.
4. Using your sketch from step 3 and the list of possible materials provided in this lab, design the lab setup and write a procedure for performing the experiment.
5. Determine the values of $m$ that would be invalid for the equation.
6. CAUTION: Looking directly into laser light could damage your eyes.
7. Be sure to check with your teacher and have approval before you implement your design.
8. Perform your experiment. Write your data in a data table similar to the one on the next page.


Data Table

| Source | Color | Accepted $\lambda$ <br> $(\mathbf{m})$ | $\boldsymbol{d}$ <br> $(\mathbf{m})$ | $\boldsymbol{m}$ | $\boldsymbol{x}$ <br> $(\mathbf{m})$ | $\boldsymbol{L}$ <br> $(\mathbf{m})$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  | 1 |  |  |
|  |  |  |  | 2 |  |  |
|  |  |  |  | 3 |  |  |
|  |  |  |  | 4 |  |  |
|  |  |  |  | 5 |  |  |

## Analyze

1. Adjust the distance of your slits from the screen. Is there a distance that allows you to collect the most data with the best precision?
2. Calculate the wavelength, $\lambda$, of your light source using $m$ and measurements of $x, d$, and $L$.
3. Error Analysis Compare your calculated wavelength to the accepted value by determining the percentage of error.

## Conclude and Apply

1. Conclude Did your procedure enable you to use a double-slit interference pattern to measure the wavelength of light? Explain.
2. Estimate what results you would get if you used a plate with a smaller slit separation distance, $d$, and performed the experiment exactly the same?
3. Infer How would your observations change if you used green light, but used the same double-slit plate and screen distance? What would you observe?

## Going Further

1. Use a Scientific Explanation Describe why the double-slit interference pattern dims, brightens, and dims again as distance from the center of the pattern increases.
2. Error Analysis Describe several things you could do in the future to reduce systematic error in your experiment.
3. Evaluate Examine the measuring equipment you used and determine which equipment limited you the most on the precision of your calculations and which equipment gave you more precision than you needed, if any.
4. Lab Techniques What might be done to an experimental setup to use white light from a normal lightbulb to produce a double-slit interference pattern?

## Real-World Physics

1. When white light shines through slits in a screen door, why is a pattern not visible in the shadow on a wall?
2. Would things look different if all of the light that illuminated your world was coherent? Explain.

## Physics nline

To find out more about interference patterns, visit the Web site: physicspp.com

## How its WV rks <br> Holography

Holography is a form of photography that produces a threedimensional image. Dennis Gabor made the first hologram in 1947, but holography was impractical until the invention of the gas laser in 1960. Holograms are used on credit cards to help prevent counterfeiting, and they may one day be used for

The reference and object beams are directed by mirrors, and are made to diverge by lenses.




When a transparent film of the developed plate is placed in a diverging laser beam, light passing through the film creates a three-dimensional virtual image of the original object with rainbowlike bands of color.

## Thinking Critically

1. Infer A hologram records a complex pattern of constructive and destructive interference fringes. Why do you suppose a vibration-isolated surface is needed for good results?
2. Use Scientific Explanations Identify and explain where the following wave properties occur in the diagrams: reflection, refraction, and interference.

### 19.1 Interference

## Vocabulary

- incoherent light (p. 515)
- coherent light (p. 516)
- interference fringes (p. 516)
- monochromatic light (p. 516)
- thin-film interference (p. 520)


### 19.2 Diffraction

## Vocabulary

- diffraction pattern (p. 524)
- diffraction grating (p. 527)
- Rayleigh criterion (p. 530)


## Key Concepts

- Incoherent light illuminates an object evenly, just as a lightbulb illuminates your desk.
- Only the superposition of light waves from coherent light sources can produce an interference pattern.
- Interference demonstrates that light has wave properties.
- Light passing through two closely spaced, narrow slits produces a pattern on a screen of dark and light bands called interference fringes.
- Interference patterns can be used to measure the wavelength of light.

$$
\lambda=\frac{x d}{L}
$$

- Interference patterns can result from the creation of coherent light at the refractive boundary of a thin film.


## Key Concepts

- Light passing through a narrow slit is diffracted, or spread out from a straight-line path, and produces a diffraction pattern on a screen.
- The diffraction pattern from a single slit has a bright central band that has a width equal to the distance between the first dark bands on either side of the bright central band.

$$
2 x_{1}=\frac{2 \lambda L}{w}
$$

- Diffraction gratings consist of large numbers of slits that are very close together and produce narrow lines that result from an overlap of the singleslit diffraction patterns of all of the slits in the grating.
- Diffraction gratings can be used to measure the wavelength of light precisely or to separate light composed of different wavelengths.

$$
\lambda=d \sin \theta
$$

- Diffraction limits the ability of an aperture to distinguish two closely spaced objects.

$$
x_{\mathrm{obj}}=\frac{1.22 \lambda L_{\mathrm{obj}}}{D}
$$

- If the central bright spot of one image falls on the first dark ring of the second image, the images are at the limit of resolution.


## Concept Mapping

31. Monochromatic light of wavelength $\lambda$ illuminates two slits in a Young's double-slit experiment setup that are separated by a distance, $d$. A pattern is projected onto a screen a distance, $L$, away from the slits. Complete the following concept map using $\lambda$, $L$, and $d$ to indicate how you could vary them to produce the indicated change in the spacing between adjacent bright bands, $x$.


## Mastering Concepts

32. Why is it important that monochromatic light was used to make the interference pattern in Young's interference experiment? (19.1)
33. Explain why the position of the central bright band of a double-slit interference pattern cannot be used to determine the wavelength of the light waves. (19.1)
34. Describe how you could use light of a known wavelength to find the distance between two slits. (19.1)
35. Describe in your own words what happens in thinfilm interference when a dark band is produced by light shining on a soap film suspended in air. Make sure you include in your explanation how the wavelength of the light and thickness of the film are related. (19.1)
36. White light shines through a diffraction grating. Are the resulting red lines spaced more closely or farther apart than the resulting violet lines? Why? (19.2)
37. Why do diffraction gratings have large numbers of slits? Why are these slits so close together? (19.2)
38. Why would a telescope with a small diameter not be able to resolve the images of two closely spaced stars? (19.2)
39. For a given diffraction grating, which color of visible light produces a bright line closest to the central bright band? (19.2)

## Applying Concepts

40. For each of the following examples, indicate whether the color is produced by thin-film interference, refraction, or the presence of pigments.
a. soap bubbles
c. oil films
b. rose petals
d. a rainbow
41. How can you tell whether a pattern is produced by a single slit or a double slit?
42. Describe the changes in a single-slit diffraction pattern as the width of the slit is decreased.
43. Science Fair At a science fair, one exhibition is a very large soap film that has a fairly consistent width. It is illuminated by a light with a wavelength of 432 nm , and nearly the entire surface appears to be a lovely shade of purple. What would you see in the following situations?
a. the film thickness was doubled
b. the film thickness was increased by half a wavelength of the illuminating light
c. the film thickness was decreased by one quarter of a wavelength of the illuminating light
44. What are the differences in the characteristics of the diffraction patterns formed by diffraction gratings containing $10^{4}$ lines $/ \mathrm{cm}$ and $10^{5}$ lines $/ \mathrm{cm}$ ?
45. Laser Pointer Challenge You have two laser pointers, a red one and a green one. Your friends Mark and Carlos disagree about which has the longer wavelength. Mark insists that red light has a longer wavelength, while Carlos is sure that green has the longer wavelength. You have a diffraction grating handy. Describe what demonstration you would do with this equipment and how you would explain the results to Carlos and Mark to settle their disagreement.
46. Optical Microscope Why is blue light used for illumination in an optical microscope?

## Mastering Problems

### 19.1 Interference

47. Light falls on a pair of slits $19.0 \mu \mathrm{~m}$ apart and 80.0 cm from a screen, as shown in Figure 19-17. The first-order bright band is 1.90 cm from the central bright band. What is the wavelength of the light?


Figure 19-17 (Not to scale)
48. Oil Slick After a short spring shower, Tom and Ann take their dog for a walk and notice a thin film of oil ( $n=1.45$ ) on a puddle of water, producing different colors. What is the minimum thickness of a place where the oil creates constructive interference for light with a wavelength of 545 nm ?
49. Light of wavelength 542 nm falls on a double slit. First-order bright bands appear 4.00 cm from the central bright band. The screen is 1.20 m from the slits. How far apart are the slits?
50. Insulation Film Winter is approaching and Alejandro is helping to cover the windows in his home with thin sheets of clear plastic $(n=1.81)$ to keep the drafts out. After the plastic is taped up around the windows such that there is air between the plastic and the glass panes, the plastic is heated with a hair dryer to shrink-wrap the window. The thickness of the plastic is altered during this process. Alejandro notices a place on the plastic where there is a blue stripe of color. He realizes that this is created by thin-film interference. What are three possible thicknesses of the portion of the plastic where the blue stripe is produced if the wavelength of the light is $4.40 \times 10^{2} \mathrm{~nm}$ ?
51. Samir shines a red laser pointer through three different double-slit setups. In setup A, the slits are separated by 0.150 mm and the screen is 0.60 m away from the slits. In setup B, the slits are separated by 0.175 mm and the screen is 0.80 m away. Setup C has the slits separated by 0.150 mm and the screen a distance of 0.80 m away. Rank the three setups according to the separation between the central bright band and the first-order bright band, from least to most separation. Specifically indicate any ties.

### 19.2 Diffraction

52. Monochromatic light passes through a single slit with a width of 0.010 cm and falls on a screen 100 cm away, as shown in Figure 19-18. If the width of the central band is 1.20 cm , what is the wavelength of the light?


■ Figure 19-18 (Not to scale)
Physics nline
physicspp.com/chapter_test
53. A good diffraction grating has $2.5 \times 10^{3}$ lines per cm . What is the distance between two lines in the grating?
54. Light with a wavelength of $4.5 \times 10^{-5} \mathrm{~cm}$ passes through a single slit and falls on a screen 100 cm away. If the slit is 0.015 cm wide, what is the distance from the center of the pattern to the first dark band?
55. Hubble Space Telescope Suppose the Hubble Space Telescope, 2.4 m in diameter, is in orbit $1.0 \times 10^{5} \mathrm{~m}$ above Earth and is turned to view Earth, as shown in Figure 19-19. If you ignore the effect of the atmosphere, how large an object can the telescope resolve? Use $\lambda=5.1 \times 10^{-7} \mathrm{~m}$.


Figure 19-19
56. Monochromatic light with a wavelength of 425 nm passes through a single slit and falls on a screen 75 cm away. If the central bright band is 0.60 cm wide, what is the width of the slit?
57. Kaleidoscope Jennifer is playing with a kaleidoscope from which the mirrors have been removed. The eyehole at the end is 7.0 mm in diameter. If she can just distinguish two bluishpurple specks on the other end of the kaleidoscope separated by $40 \mu \mathrm{~m}$, what is the length of the kaleidoscope? Use $\lambda=650 \mathrm{~nm}$ and assume that the resolution is diffraction limited through the eyehole.
58. Spectroscope A spectroscope uses a grating with 12,000 lines $/ \mathrm{cm}$. Find the angles at which red light, 632 nm , and blue light, 421 nm , have first-order bright lines.

## Mixed Review

59. Record Marie uses an old $33 \frac{1}{3} \mathrm{rpm}$ record as a diffraction grating. She shines a laser, $\lambda=632.8 \mathrm{~nm}$, on the record, as shown in Figure 19-20. On a screen 4.0 m from the record, a series of red dots 21 mm apart are visible.
a. How many ridges are there in a centimeter along the radius of the record?
b. Marie checks her results by noting that the ridges represent a song that lasts 4.01 minutes and takes up 16 mm on the record. How many ridges should there be in a centimeter?


- Figure 19-20 (Not to scale)

60. An anti-reflective coating, $n=1.2$, is applied to a lens. If the thickness of the coating is 125 nm , what is (are) the color(s) of light for which complete destructive interference will occur? Hint: Assume the lens is made out of glass.
61. Camera When a camera with a $50-\mathrm{mm}$ lens is set at $\mathrm{f} / 8$, its aperture has an opening 6.25 mm in diameter. a. For light with $\lambda=550 \mathrm{~nm}$, what is the resolution of the lens? The film is 50.0 mm from the lens.
b. The owner of a camera needs to decide which film to buy for it. The expensive one, called finegrained film, has 200 grains $/ \mathrm{mm}$. The less costly, coarse-grained film has only 50 grains $/ \mathrm{mm}$. If the owner wants a grain to be no smaller than the width of the central bright spot calculated in part $\mathbf{a}$, which film should he purchase?

## Thinking Critically

62. Apply Concepts Yellow light falls on a diffraction grating. On a screen behind the grating, you see three spots: one at zero degrees, where there is no diffraction, and one each at $+30^{\circ}$ and $-30^{\circ}$. You now add a blue light of equal intensity that is in the same direction as the yellow light. What pattern of spots will you now see on the screen?
63. Apply Concepts Blue light of wavelength $\lambda$ passes through a single slit of width $w$. A diffraction pattern appears on a screen. If you now replace the blue light with a green light of wavelength $1.5 \lambda$, to what width should you change the slit to get the original pattern back?
64. Analyze and Conclude At night, the pupil of a human eye has an aperture diameter of 8.0 mm . The diameter is smaller in daylight. An automobile's headlights are separated by 1.8 m .
a. Based upon Rayleigh's criterion, how far away can the human eye distinguish the two headlights at night? Hint: Assume a wavelength of 525 nm .
b. Can you actually see a car's headlights at the distance calculated in part a? Does diffraction limit your eyes' sensing ability? Hypothesize as to what might be the limiting factors.

## Writing in Physics

65. Research and describe Thomas Young's contributions to physics. Evaluate the impact of his research on the scientific thought about the nature of light.
66. Research and interpret the role of diffraction in medicine and astronomy. Describe at least two applications in each field.

## Cumulative Review

67. How much work must be done to push a $0.5-\mathrm{m}^{3}$ block of wood to the bottom of a 4 -m-deep swimming pool? The density of wood is $500 \mathrm{~kg} / \mathrm{m}^{3}$. (Chapter 13)
68. What are the wavelengths of microwaves in an oven if their frequency is 2.4 GHz ? (Chapter 14)
69. Sound wave crests that are emitted by an airplane are 1.00 m apart in front of the plane, and 2.00 m apart behind the plane. (Chapter 15)
a. What is the wavelength of the sound in still air?
b. If the speed of sound is $330 \mathrm{~m} / \mathrm{s}$, what is the frequency of the source?
c. What is the speed of the airplane?
70. A concave mirror has a $48.0-\mathrm{cm}$ radius. A $2.0-\mathrm{cm}-$ tall object is placed 12.0 cm from the mirror. Calculate the image position and image height. (Chapter 17)
71. The focal length of a convex lens is 21.0 cm . A $2.00-\mathrm{cm}$-tall candle is located 7.50 cm from the lens. Use the thin-lens equation to calculate the image position and image height. (Chapter 18)

## Standardized Test Practice

## Multiple Choice

1. What is the best possible explanation for why the colors of a thin film, such as a soap bubble or oil on water, appear to change and move as you watch?
(A) because convective heat waves in the air next to the thin film distort the light
(B) because the film thickness at any given location changes over time
(C) because the wavelengths in sunlight vary over time
(D) because your vision varies slightly over time
2. Light at 410 nm shines through a slit and falls on a flat screen as shown in the figure below. The width of the slit is $3.8 \times 10^{-6} \mathrm{~m}$. What is the width of the central bright band?
```
(A) }0.024\textrm{m
(D) 0.048 m
(B) 0.031 m
(D) 0.063 m
```


3. For the situation in problem 2, what is the angle, $\theta$, of the first dark band?

```
(A) 3.10
(C) 12.4
(B) }6.\mp@subsup{2}{}{\circ
(D) }1\mp@subsup{7}{}{\circ
```

4. Two stars $6.2 \times 10^{4}$ light-years from Earth are 3.1 light-years apart. What is the smallest diameter telescope that could resolve them using 610 nm light?
(A) $5.0 \times 10^{-5} \mathrm{~m}$
(C) $1.5 \times 10^{-2} \mathrm{~m}$
(B) $6.1 \times 10^{-5} \mathrm{~m}$
(D) $1.5 \times 10^{7} \mathrm{~m}$
5. A grating has slits that are 0.055 mm apart. What is the angle of the first-order bright line for light with a wavelength of 650 nm ?
```
(A) 0.012}\mp@subsup{}{}{\circ
(C) }1.\mp@subsup{0}{}{\circ
(B) 0.68
(D) 11员
```

6. A laser beam at 638 nm illuminates two narrow slits. The third-order band of the resulting pattern is 7.5 cm from the center bright band. The screen is 2.475 m from the slits. How far apart are the slits?
(A) $5.8 \times 10^{-8} \mathrm{~m}$
(C) $2.1 \times 10^{-5} \mathrm{~m}$
(B) $6.3 \times 10^{-7} \mathrm{~m}$
(D) $6.3 \times 10^{-5} \mathrm{~m}$
7. A flat screen is placed 4.200 m from a pair of slits that are illuminated by a beam of monochromatic light. On the screen, the separation between the central bright band and the second-order bright band is 0.082 m . The distance between the slits is $5.3 \times 10^{-5} \mathrm{~m}$. Determine the wavelength of the light.
(A) $2.6 \times 10^{-7} \mathrm{~m}$
(C) $6.2 \times 10^{-7} \mathrm{~m}$
(B) $5.2 \times 10^{-7} \mathrm{~m}$
(D) $1.0 \times 10^{-6} \mathrm{~m}$
8. A clown is blowing soap bubbles and you notice that the color of one region of a particularly large bubble matches the color of his nose. If the bubble is reflecting $6.5 \times 10^{-7} \mathrm{~m}$ red light waves, and the index of refraction of the soap film is 1.41 , what is the minimum thickness of the soap bubble at the location where it is reflecting red?
(A) $1.2 \times 10^{-7} \mathrm{~m}$
(C) $9.2 \times 10^{-7} \mathrm{~m}$
(B) $3.5 \times 10^{-7} \mathrm{~m}$
(D) $1.9 \times 10^{-6} \mathrm{~m}$

## Extended Answer

9. A diffraction grating that has 6000 slits per cm produces a diffraction pattern that has a firstorder bright line at $20^{\circ}$ from the central bright line. What is the wavelength of the light?

## Test-Taking TIP <br> Don't Be Afraid To Ask For Help

If you are practicing for a test and you are having difficulty understanding why you got a question wrong or you are having difficulty even arriving at an answer, ask someone for help. It is important to ask for help before a test because you cannot ask for help during a test.

## Chapter <br> 20

## Static Electricity

## What You'll Learn

- You will observe the behavior of electric charges and analyze how these charges interact with matter.
- You will examine the forces that act between electric charges.


## Why It's Important

Static electricity enables the operation of devices such as printers and copiers, but it has harmful effects on electronic components and in the form of lightning.
Lightning The tiny spark that you experience when you touch a doorknob and the dazzling display of lightning in a storm are both examples of the discharge of static electricity. The charging processes and the means of discharging are vastly different in scale, but they are similar in their fundamental nature.

Think About This >
What causes charge to build up in a thundercloud, and how does it discharge in the form of a spectacular lightning bolt?

## Physios inline physicspp.com

## LAUNCH Lab

## Which forces act over a distance?

## Question

What happens when a plastic ruler is rubbed with wool and then brought near a pile of paper scraps?

## Procedure 든

1. Place $15-20$ scraps of paper from a hole punch on the table.
2. Take a plastic ruler and rub it with a piece of wool.
3. Bring the ruler close to the pieces of paper. Observe the effect the ruler has on the scraps of paper.

## Analysis

What happens to the pieces of paper when the ruler is brought close to them? What happens to the pieces of paper that come in contact with the ruler? Did you observe any unexpected results when the ruler was brought close to the paper scraps? If so, describe these results.

## Critical Thinking

What forces are acting on the pieces of paper before the ruler is brought close to them? What can you infer about the forces on the paper after the ruler is brought near?
Based on your answers to the previous questions, form a hypothesis that explains the effect the ruler has on the scraps of paper.


### 20.1 Electric Charge

You may have had the experience of rubbing your shoes on a carpet to create a spark when you touched someone. In 1752, Benjamin Franklin set off a flurry of research in the field of electricity when his famous kite experiment showed that lightning is similar to the sparks caused by friction. In his experiment, Franklin flew a kite with a key attached to the string. As a thunderstorm approached, the loose threads of the kite string began to stand up and repel one another, and when Franklin brought his knuckle close to the key, he experienced a spark. Electric effects produced in this way are called static electricity.

In this chapter, you will investigate electrostatics, the study of electric charges that can be collected and held in one place. The effects of electrostatics are observable over a vast scale, from huge displays of lightning to the submicroscopic world of atoms and molecules. Current electricity, which is produced by batteries and generators, will be explored in later chapters.

## - Objectives

- Demonstrate that charged objects exert forces, both attractive and repulsive.
- Recognize that charging is the separation, not the creation, of electric charges.
- Describe the differences between conductors and insulators.
- Vocabulary
electrostatics
neutral
insulator
conductor
- Figure 20-1 Rubbing a plastic ruler with wool produces a new force of attraction between the ruler and bits of paper. When the ruler is brought close to bits of paper, the attractive electric force accelerates the paper bits upward against the force of gravity.


Have you ever noticed the way that your hair is attracted to the comb when you comb your hair on a dry day or the way that your hair stands on end after it is rubbed with a balloon? Perhaps you also have found that socks sometimes stick together when you take them out of a clothes dryer. If so, you will recognize the attraction of the bits of paper to a plastic ruler demonstrated by the Launch Lab and shown in Figure 20-1. You might have noticed the way the paper pieces jumped up to the ruler as you worked through the Launch Lab. There must be a new, relatively strong force causing this upward acceleration because it is larger than the downward acceleration caused by the gravitational force of Earth.

There are other differences between this new force and gravity. Paper is attracted to a plastic ruler only after the ruler has been rubbed; if you wait a while, the attractive property of the ruler disappears. Gravity, on the other hand, does not require rubbing and does not disappear. The ancient Greeks noticed effects similar to that of the ruler when they rubbed amber. The Greek word for amber is elektron, and today this attractive property is called electric. An object that exhibits electric interaction after rubbing is said to be charged.
Like charges You can explore electric interactions with simple objects, such as transparent tape. Fold over about 5 mm of the end of a strip of tape for a handle, and then stick the remaining 8 - to $12-\mathrm{cm}$-long part of the tape strip on a dry, smooth surface, such as your desktop. Stick a second, similar piece of tape next to the first. Quickly pull both strips off the desk and bring them near each other. A new property causes the strips to repel each other: they are electrically charged. Because they were prepared in the same way, they must have the same type of charge. Thus, you have demonstrated that two objects with the same type of charge repel each other.

You can learn more about this charge by doing some simple experiments. You may have found that the tape is attracted to your hand. Are both sides attracted, or just one? If you wait a while, especially in humid weather, you will find that the electric charge disappears. You can restore it by again sticking the tape to the desk and pulling it off. You also can remove its charge by gently rubbing your fingers down both sides of the tape.

Opposite charges Now, stick one strip of tape on the desk and place the second strip on top of the first. As shown in Figure 20-2a, use the handle of the bottom strip of tape to pull the two off the desk together. Rub them with your fingers until they are no longer attracted to your hand. You now have removed all the electric charge. With one hand on the handle of one strip and the other on the handle of the second strip, quickly pull the two strips apart. You will find that they are now both charged. They once again are attracted to your hands. Do they still repel each other? No, they now attract each other. They are charged, but they are no longer charged alike. They have opposite charges and therefore attract each other.

Is tape the only object that you can charge? Once again, stick one strip of tape to the desk and the second strip on top. Label the bottom strip $B$ and the top strip $T$. Pull the pair off together. Discharge them, then pull them apart. Stick the handle end of each strip to the edge of a table, the bottom of a lamp shade, or some similar object. The two should hang down a short distance apart. Finally, rub a comb or pen on your clothing and bring it near one strip of tape and then the other. You will find that one strip will be attracted to the comb, while the other will be repelled by it, as shown in Figure 20-2b. You now can explore the interactions of charged objects with the strips of tape.

Experimenting with charge Try to charge other objects, such as glasses and plastic bags. Rub them with different materials, such as silk, wool, and plastic wrap. If the air is dry, scuff your shoes on carpet and bring your finger near the strips of tape. To test silk or wool, slip a plastic bag over your hand before holding the cloth. After rubbing, take your hand out of the bag and bring both the bag and cloth near the strips of tape.

Most charged objects will attract one strip and repel the other. You will never find an object that repels both strips of tape, although you might find some that attract both. For example, your finger will attract both strips. You will explore this effect later in this chapter.
Types of charge From your experiments, you can make a list of objects labeled $B$, for bottom, which have the same charge as the tape stuck on the desk. Another list can be made of objects labeled $T$, which have the same charge as the top strip of tape. There are only two lists, because there are only two types of charge. Benjamin Franklin called them positive and negative charges. Using Franklin's convention, when hard rubber and plastic are rubbed, they become negatively charged. When materials such as glass and wool are rubbed, they become positively charged.

Just as you showed that an uncharged pair of tape strips became oppositely charged, you probably were able to show that if you rubbed plastic with wool, the plastic became negatively charged and the wool positively charged. The two kinds of charges were not created alone, but in pairs. These experiments suggest that matter normally contains both charges, positive and negative. Contact in some way separates the two. To explore this further, you must consider the microscopic picture of matter.

## A Microscopic View of Charge

Electric charges exist within atoms. In 1897, J.J. Thomson discovered that all materials contain light, negatively charged particles that he called electrons. Between 1909 and 1911, Ernest Rutherford, a student of Thomson from New Zealand, discovered that the atom has a massive, positively charged nucleus. When the positive charge of the nucleus equals the negative charge of the surrounding electrons, then the atom is neutral.
$\square$ Figure 20-2 Strips of tape can be given opposite charges (a) and then be used to demonstrate the interactions of like and opposite charges (b).

## Color Convention

- Positive charges are shown in red.
- Negative charges are shown in blue.

Figure 20-3 When wool is used to charge a rubber rod, electrons are removed from the wool atoms and cling to the rubber atoms. In this way, both objects become charged.


With the addition of energy, the outer electrons can be removed from atoms. An atom missing electrons has an overall positive charge, and consequently, any matter made of these electron-deficient atoms is positively charged. The freed electrons can remain unattached or become attached to other atoms, resulting in negatively charged particles. From a microscopic viewpoint, acquiring charge is a process of transferring electrons.

Separation of charge If two neutral objects are rubbed together, each can become charged. For instance, when rubber and wool are rubbed together, electrons from atoms on the wool are transferred to the rubber, as shown in Figure 20-3. The extra electrons on the rubber result in a net negative charge. The electrons missing from the wool result in a net positive charge. The combined total charge of the two objects remains the same. Charge is conserved, which is one way of saying that individual charges never are created or destroyed. All that happens is that the positive and negative charges are separated through a transfer of electrons.

Complex processes that affect the tires of a moving car or truck can cause the tires to become charged. Processes inside a thundercloud can cause the cloud bottom to become negatively charged and the cloud top to become positively charged. In both these cases, charge is not created, but separated.

## Conductors and Insulators

Hold a plastic rod or comb at its midpoint and rub only one end. You will find that only the rubbed end becomes charged. In other words, the charges that you transferred to the plastic stayed where they were put; they did not move. A material through which a charge will not move easily is called an electric insulator. The strips of tape that you charged earlier in this chapter acted in this way. Glass, dry wood, most plastics, cloth, and dry air are all good insulators.

Suppose that you support a metal rod on an insulator so that it is isolated, or completely surrounded by insulators. If you then touch the charged comb to one end of the metal rod, you will find that the charge spreads very quickly over the entire rod. A material that allows charges to move about easily is called an electric conductor. Electrons carry, or conduct, electric charge through the metal. Metals are good conductors because at least one electron on each atom of the metal can be removed easily. These electrons act as if they no longer belong to any one atom, but to the metal as a whole; consequently, they move freely throughout the piece of metal. Figure 20-4 contrasts how charges behave when they are

placed on a conductor with how they behave on an insulator. Copper and aluminum are both excellent conductors and are used commercially to carry electricity. Plasma, a highly ionized gas, and graphite also are good conductors of electric charge.

When air becomes a conductor Air is an insulator; however, under certain conditions, charges move through air as if it were a conductor. The spark that jumps between your finger and a doorknob after you have rubbed your feet on a carpet discharges you. In other words, you have become neutral because the excess charges have left you. Similarly, lightning discharges a thundercloud. In both of these cases, air became a conductor for a brief moment. Recall that conductors must have charges that are free to move. For a spark or lightning to occur, freely moving charged particles must be formed in the normally neutral air. In the case of lightning, excess charges in the cloud and on the ground are great enough to remove electrons from the molecules in the air. The electrons and positively or negatively charged atoms form a plasma, which is a conductor. The discharge of Earth and the thundercloud by means of this conductor forms a luminous arc called lightning. In the case of your finger and the doorknob, the discharge is called a spark.

Figure 20-4 Charges placed on a conductor will spread over the entire surface (a). Charges placed on an insulator will remain where they are placed (b).

### 20.1 Section Review

1. Charged Objects After a comb is rubbed on a wool sweater, it is able to pick up small pieces of paper. Why does the comb lose that ability after a few minutes?
2. Types of Charge In the experiments described earlier in this section, how could you find out which strip of tape, B or T , is positively charged?
3. Types of Charge A pith ball is a small sphere made of a light material, such as plastic foam, often coated with a layer of graphite or aluminum paint. How could you determine whether a pith ball that is suspended from an insulating thread is neutral, is charged positively, or is charged negatively?
4. Charge Separation A rubber rod can be charged negatively when it is rubbed with wool. What happens to the charge of the wool? Why?
5. Conservation of Charge An apple contains trillions of charged particles. Why don't two apples repel each other when they are brought together?
6. Charging a Conductor Suppose you hang a long metal rod from silk threads so that the rod is isolated. You then touch a charged glass rod to one end of the metal rod. Describe the charges on the metal rod.
7. Charging by Friction You can charge a rubber rod negatively by rubbing it with wool. What happens when you rub a copper rod with wool?
8. Critical Thinking It once was proposed that electric charge is a type of fluid that flows from objects with an excess of the fluid to objects with a deficit. Why is the current two-charge model better than the single-fluid model?

### 20.2 Electric Force

- Objectives
- Summarize the relationships between electric forces, charges, and distance.
- Explain how to charge objects by conduction and induction.
- Develop a model of how charged objects can attract a neutral object.
- Apply Coulomb's law to problems in one and two dimensions.
- Vocabulary
electroscope charging by conduction charging by induction grounding Coulomb's law coulomb elementary charge
- Figure 20-5 A charged rod, when brought close to another charged and suspended rod, will attract or repel the suspended rod.

Electric forces must be strong because they can easily produce accelerations larger than the acceleration caused by gravity. You also have learned that they can be either repulsive or attractive, while gravitational forces always are attractive. Over the years, many scientists made attempts to measure electric forces. Daniel Bernoulli, best known for his work with fluids, made some crude measurements in 1760. In the 1770s, Henry Cavendish showed that electric forces must obey an inverse square force law, but being extremely shy, he did not publish his work. His manuscripts were discovered over a century later, after all his work had been duplicated by others.

## Forces on Charged Bodies

The forces that you observed on tape strips also can be demonstrated by suspending a negatively charged, hard rubber rod so that it turns easily, as shown in Figure 20-5. If you bring another negatively charged rod near the suspended rod, the suspended rod will turn away. The negative charges on the rods repel each other. It is not necessary for the rods to make contact. The force, called the electric force, acts at a distance. If a positively charged glass rod is suspended and a similarly charged glass rod is brought close, the two positively charged rods also will repel each other. If a negatively charged rod is brought near a positively charged rod, however, the two will attract each other, and the suspended rod will turn toward the oppositely charged rod. The results of your tape experiments and these actions of charged rods can be summarized in the following way:

- There are two kinds of electric charges: positive and negative.
- Charges exert forces on other charges at a distance.
- The force is stronger when the charges are closer together.
- Like charges repel; opposite charges attract.

Neither a strip of tape nor a large rod that is hanging in open air is a very sensitive or convenient way of determining charge. Instead, a device called an electroscope is used. An electroscope consists of a metal knob connected by a metal stem to two thin, lightweight pieces of metal foil, called leaves. Figure 20-6 shows a neutral electroscope. Note that the leaves hang loosely and are enclosed to eliminate stray air currents.


Charging by conduction When a negatively charged rod is touched to the knob of an electroscope, electrons are added to the knob. These charges spread over all the metal surfaces. As shown in Figure 20-7a, the two leaves are charged negatively and repel each other; therefore, they spread apart. The electroscope has been given a net charge. Charging a neutral body by touching it with a charged body is called charging by conduction. The leaves also will spread apart if the electroscope is charged positively. How, then, can you find out whether the electroscope is charged positively or negatively? The type of charge can be determined by observing the leaves when a rod of known charge is brought close to the knob. The leaves will spread farther apart if the rod and the electroscope have the same charge, as shown in Figure 20-7b. The leaves will fall slightly if the electroscope's charge is opposite that of the rod, as in Figure 20-7c.

Separation of charge on neutral objects Earlier in this chapter, when you brought your finger near either charged strip of tape, the tape was attracted to your finger. Your finger, however, was neutral-it had equal amounts of positive and negative charge. You know that in conductors, charges can move easily, and that in the case of sparks, electric forces can change insulators into conductors. Given this information, you can develop a plausible model for the force that your finger exerted on the strips of tape.

Suppose you move your finger, or any uncharged object, close to a positively charged object. The negative charges in your finger will be attracted to the positively charged object, and the positive charges in your finger will be repelled. Your finger will remain neutral, but the positive and negative charges will be separated. The electric force is stronger for charges that are closer together; therefore, the separation results in an attractive force between your finger and the charged object. The force that a charged ruler exerts on neutral pieces of paper is the result of the same process, the separation of charges.

The negative charges at the bottom of thunderclouds also can cause charge separation in Earth. Positive charges in the ground are attracted to Earth's surface under the cloud. The forces of the charges in the cloud and those on Earth's surface can break molecules into positively and negatively charged particles. These charged particles are free to move, and they establish a conducting path from the ground to the cloud. The lightning that you observe occurs when a bolt travels at speeds on the order of $500,000 \mathrm{~km} / \mathrm{h}$ along the conducting path and discharges the cloud.



Figure 20-6 An electroscope is a device used for detecting charges. In a neutral electroscope, the leaves hang loosely, almost touching one another.

- Figure 20-7 A negatively charged electroscope will have its leaves spread apart (a). A negatively charged rod pushes electrons down to the leaves, causing them to spread farther apart (b). A positively charged rod attracts some of the electrons, causing the leaves to spread apart less (c).


Figure 20-8 One method of charging by induction begins with neutral spheres that are touching (a). A charged rod is brought near them (b), then the spheres are separated and the charged rod is removed (c). The charges on the separated spheres are equal in magnitude, but opposite in sign.

- Figure 20-9 A negatively charged rod induces a separation of charges in an electroscope (a). The electroscope is grounded, and negative charges are pushed from the electroscope to the ground (b). The ground is removed before the rod, and the electroscope is left with a positive charge (c).


Charging by induction Suppose that two identical, insulated metal spheres are touching, as shown in Figure 20-8a. When a negatively charged rod is brought close to one, as in Figure 20-8b, electrons from the first sphere will be forced onto the sphere farther from the rod and will make it negatively charged. The closer sphere is now positively charged. If the spheres are separated while the rod is nearby, each sphere will have a charge, and the charges will be equal but opposite, as shown in Figure 20-8c. This process of charging an object without touching it is called charging by induction.

A single object can be charged by induction through grounding, which is the process of connecting a body to Earth to eliminate excess charge. Earth is a very large sphere, and it can absorb great amounts of charge without becoming noticeably charged itself. If a charged body is touched to Earth, almost any amount of charge can flow to Earth.

If a negatively charged rod is brought close to the knob of an electroscope, as in Figure 20-9a, electrons are repelled onto the leaves. If the knob is then grounded on the side opposite the charged rod, electrons will be pushed from the electroscope into the ground until the leaves are neutral, as in Figure 20-9b. Removing the ground before the rod leaves the electroscope with a deficit of electrons, and it will be positively charged, as in Figure 20-9c. Grounding also can be used as a source of electrons. If a positive rod is brought near the knob of a grounded electroscope, electrons will be attracted from the ground, and the electroscope will obtain a negative charge. When this process is employed, the charge induced on the electroscope is opposite that of the object used to charge it. Because the rod never touches the electroscope, its charge is not transferred, and it can be used many times to charge objects by induction.


## Coulomb's Law

You have seen that a force acts between two or more charged objects. In your experiments with tape, you found that the force depends on distance. The closer you brought the charged comb to the tape, the stronger the force was. You also found that the more you charged the comb, the stronger the force was. How can you vary the quantity of charge in a controlled way? This problem was solved in 1785 by French physicist Charles Coulomb. The type of apparatus used by Coulomb is shown in Figure 20-10. An insulating rod with small conducting spheres, A and $\mathrm{A}^{\prime}$, at each end was suspended by a thin wire. A similar sphere, B, was placed in contact with sphere A. When they were touched with a charged object, the charge spread evenly over the two spheres. Because they were the same size, they received equal amounts of charge. The symbol for charge is $q$. Therefore, the amount of charge on the spheres can be represented by the notation $q_{\mathrm{A}}$ and $q_{\mathrm{B}}$.

Force depends on distance Coulomb found how the force between the two charged spheres depended on the distance. First, he carefully measured the amount of force needed to twist the suspending wire through a given angle. He then placed equal charges on spheres A and B and varied the distance, $r$, between them. The force moved A , which twisted the suspending wire. By measuring the deflection of $\mathrm{A}, \mathrm{Coulomb}$ could calculate the force of repulsion. He showed that the force, $F$, varied inversely with the square of the distance between the centers of the spheres.

$$
F \propto \frac{1}{r^{2}}
$$

Force depends on charge To investigate the way in which the force depended on the amount of charge, Coulomb had to change the charges on the spheres in a measured way. He first charged spheres A and B equally, as before. Then he selected an uncharged sphere, C , of the same size as sphere B. When C was placed in contact with B, the spheres shared the charge that had been on $B$ alone. Because the two were the same size, $B$ then had only half of its original charge. Therefore, the charge on B was only one-half the charge on A. After Coulomb adjusted the position of B so that the distance, $r$, between A and B was the same as before, he found that the force between $A$ and $B$ was half of its former value. That is, he found that the force varied directly with the charge of the bodies.

$$
F \propto q_{\mathrm{A}} q_{\mathrm{B}}
$$

After many similar measurements, Coulomb summarized the results in a law now known as Coulomb's law: the magnitude of the force between charge $q_{\mathrm{A}}$ and charge $q_{\mathrm{B}^{\prime}}$, separated by a distance $r$, is proportional to the magnitude of the charges and inversely proportional to the square of the distance between them.

$$
F \propto \frac{q_{\mathrm{A}} q_{\mathrm{B}}}{r^{2}}
$$

The unit of charge: the coulomb The amount of charge that an object has is difficult to measure directly. Coulomb's experiments, however, showed that the quantity of charge could be related to force. Thus, Coulomb could define a standard quantity of charge in terms of the amount of force that it produces. The SI standard unit of charge is called the coulomb (C).


Figure 20-10 Coulomb used a similar type of apparatus to measure the force between two spheres, $A$ and $B$. He observed the deflection of $A$ while varying the distance between A and B .

## OMINI LIAB

## Investigating Induction and Conduction <br> 

Use a balloon and an electroscope to investigate charging by induction and charging by conduction.

1. Predict what will happen if you charge a balloon by rubbing it with wool and bring it near a neutral electroscope.
2. Predict what will happen if you touch the balloon to the electroscope.
3. Test your predictions.

Analyze and Conclude
4. Describe your results.
5. Explain the movements of the leaves in each step of the experiment. Include diagrams.
6. Describe the results if the wool had been used to charge the electroscope.


- Figure 20-11 The rule for determining the direction of force is: like charges repel; unlike charges attract.

One coulomb is the charge of $6.24 \times 10^{18}$ electrons or protons. A typical lightning bolt can carry 5 C to 25 C of charge. The charge on a single electron is $1.60 \times 10^{-19} \mathrm{C}$. The magnitude of the charge of an electron is called the elementary charge. Even small pieces of matter, such as coins, contain up to $10^{6} \mathrm{C}$ of negative charge. This enormous amount of negative charge produces almost no external effects because it is balanced by an equal amount of positive charge. If the charge is unbalanced, even as small a charge as $10^{-9} \mathrm{C}$ can result in large forces.

According to Coulomb's law, the magnitude of the force on charge $q_{\mathrm{A}}$ caused by charge $q_{\mathrm{B}}$ a distance $r$ away can be written as follows.

$$
\text { Coulomb's Law } \quad F=K \frac{q_{\mathrm{A}} q_{\mathrm{B}}}{r^{2}}
$$

The force between two charges is equal to Coulomb's constant, times the product of the two charges, divided by the square of the distance between them.

When the charges are measured in coulombs, the distance in meters, and the force in newtons, the constant, $K$, is $9.0 \times 10^{9} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{C}^{2}$.

The Coulomb's law equation gives the magnitude of the force that charge $q_{\mathrm{A}}$ exerts on $q_{\mathrm{B}}$ and also the force that $q_{\mathrm{B}}$ exerts on $q_{\mathrm{A}}$. These two forces are equal in magnitude but opposite in direction. You can observe this example of Newton's third law of motion in action when you bring two strips of tape with like charges together. Each exerts forces on the other. If you bring a charged comb near either strip of tape, the strip, with its small mass, moves readily. The acceleration of the comb and you is, of course, much less because of the much greater mass.

The electric force, like all other forces, is a vector quantity. Force vectors need both a magnitude and a direction. However, the Coulomb's law equation above gives only the magnitude of the force. To determine the direction, you need to draw a diagram and interpret charge relations carefully. If two positively charged objects, A and B, are brought near, the forces they exert on each other are repulsive, as shown in Figure 20-11a. If, instead, $B$ is negatively charged, the forces are attractive, as shown in Figure 20-11b.

## PROBLEM-SOLVING Strategies

## Electric Force Problems

Use these steps to find the magnitude and direction of the force between charges.

1. Sketch the system showing all distances and angles to scale.
2. Diagram the vectors of the system.
3. Use Coulomb's law to find the magnitude of the force.
4. Use your diagram along with trigonometric relations to find the direction of the force.
5. Perform all algebraic operations on both the numbers and the units. Make sure that the units match the variables in question.
6. Consider the magnitude of your answer. Is it reasonable?

## EXAMPLE Problem 1

Coulomb's Law in Two Dimensions Sphere A, with a charge of $+6.0 \mu \mathrm{C}$, is located near another charged sphere, B. Sphere B has a charge of $-3.0 \mu \mathrm{C}$ and is located 4.0 cm to the right of $A$.
a. What is the force of sphere $B$ on sphere $A$ ?
b. A third sphere, C , with a $+1.5-\mu \mathrm{C}$ charge, is added to the configuration. If it is located 3.0 cm directly beneath A , what is the new net force on sphere $A$ ?

1 Analyze and Sketch the Problem

- Establish coordinate axes and sketch the spheres.
- Show and label the distances between the spheres.
- Diagram and label the force vectors.

Known:
$q_{\mathrm{A}}=+6.0 \mu \mathrm{C} \quad r_{\mathrm{AB}}=4.0 \mathrm{~cm}$
$q_{\mathrm{B}}=-3.0 \mu \mathrm{C} \quad r_{\mathrm{AC}}=3.0 \mathrm{~cm}$
$q_{\mathrm{C}}=+1.5 \mu \mathrm{C}$

Unknown:
$\boldsymbol{F}_{\mathrm{B} \text { on } \mathrm{A}}=$ ?
$\boldsymbol{F}_{\mathrm{C} \text { on } \mathrm{A}}=$ ?
$\boldsymbol{F}_{\text {net }}=$ ?

2 Solve for the Unknown
a. Find the force of sphere $B$ on sphere $A$.

$$
\begin{aligned}
F_{\mathrm{B} \text { on } \mathrm{A}} & =K \frac{q_{\mathrm{A}} q_{\mathrm{B}}}{r_{\mathrm{AB}}{ }^{2}} & & \\
& =\left(9.0 \times 10^{9} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{C}^{2}\right) \frac{\left(6.0 \times 10^{-6} \mathrm{C}\right)\left(3.0 \times 10^{-6} \mathrm{C}\right)}{\left(4.0 \times 10^{-2} \mathrm{~m}\right)^{2}} & & \text { Substitute } q_{\mathrm{A}}=6.0 \mu \mathrm{C}, \\
& =1.0 \times 10^{2} \mathrm{~N} & & q_{\mathrm{B}}=3.0 \mu \mathrm{C}, r_{\mathrm{AB}}=4.0 \mathrm{~cm}
\end{aligned}
$$

b. Find the force of sphere C on sphere A .

$$
\begin{aligned}
F_{\mathrm{C} \text { on } \mathrm{A}} & =K \frac{q_{\mathrm{A}} q_{\mathrm{C}}}{r_{\mathrm{AC}}{ }^{2}} \\
& =\left(9.0 \times 10^{9} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{C}^{2}\right) \frac{\left(6.0 \times 10^{-6} \mathrm{C}\right)\left(1.5 \times 10^{-6} \mathrm{C}\right)}{\left(3.0 \times 10^{-2} \mathrm{~m}\right)^{2}} \\
& =9.0 \times 10^{1} \mathrm{~N}
\end{aligned}
$$

Substitute $\boldsymbol{q}_{\mathrm{A}}=\mathbf{6 . 0} \boldsymbol{\mu} \mathrm{C}$, $q_{\mathrm{C}}=1.5 \mu \mathrm{C}, r_{\mathrm{AC}}=\mathbf{3 . 0} \mathrm{cm}$

Spheres $A$ and $C$ have like charges, which repel. The force of $C$ on $A$ is upward.
Find the vector sum of $\boldsymbol{F}_{\mathrm{B} \text { on A }}$ and $\boldsymbol{F}_{\mathrm{C} \text { on } \mathrm{A}}$ to find $\boldsymbol{F}_{\text {net }}$ on sphere A.

$$
\begin{aligned}
& =\sqrt{\left(1.0 \times 10^{2} \mathrm{~N}\right)^{2}+\left(9.0 \times 10^{1} \mathrm{~N}\right)^{2}} \quad \text { Substitute } F_{\mathrm{B} \text { on } \mathrm{A}}=1.0 \times 10^{2} \mathrm{~N}, F_{\mathrm{C} \text { on } \mathrm{A}}=9.0 \times 10^{1} \mathrm{~N} \\
& =130 \mathrm{~N} \\
& \tan \theta=\frac{F_{\mathrm{C} \text { on } \mathrm{A}}}{F_{\mathrm{B} \text { on } \mathrm{A}}} \\
& \theta=\tan ^{-1}\left(\frac{F_{\mathrm{C} \text { on } \mathrm{A}}}{F_{\mathrm{B} \text { on } \mathrm{A}}}\right) \\
& \text { Math Handbook } \\
& \text { Inverses of Sine, } \\
& \text { Cosine, and Tangent } \\
& \text { page } 856 \\
& =\tan ^{-1}\left(\frac{9.0 \times 10^{1} \mathrm{~N}}{1.0 \times 10^{2} \mathrm{~N}}\right) \\
& \text { Substitute } F_{\mathrm{C} \text { on } \mathrm{A}}=\mathbf{9 . 0 \times 1 0 ^ { 1 }} \mathrm{N}, \boldsymbol{F}_{\mathrm{B} \text { on } \mathrm{A}}=1.0 \times 10^{2} \mathrm{~N} \\
& =42^{\circ} \\
& \boldsymbol{F}_{\text {net }}=130 \mathrm{~N}, 42^{\circ} \text { above the } x \text {-axis }
\end{aligned}
$$

## 3 Evaluate the Answer

- Are the units correct? $\left(\mathrm{N} \cdot \mathrm{m}^{2} / \mathrm{C}^{2}\right)(\mathrm{C})(\mathrm{C}) / \mathrm{m}^{2}=\mathrm{N}$. The units work out to be newtons.
- Does the direction make sense? Like charges repel; unlike charges attract.
- Is the magnitude realistic? The magnitude of the net force is in agreement with the magnitudes of the component forces.

9. A negative charge of $-2.0 \times 10^{-4} \mathrm{C}$ and a positive charge of $8.0 \times 10^{-4} \mathrm{C}$ are separated by 0.30 m . What is the force between the two charges?
10. A negative charge of $-6.0 \times 10^{-6} \mathrm{C}$ exerts an attractive force of 65 N on a second charge that is 0.050 m away. What is the magnitude of the second charge?
11. The charge on $B$ in Example Problem 1 is replaced by a charge of $+3.00 \mu \mathrm{C}$. Diagram the new situation and find the net force on A .
12. Sphere $A$ is located at the origin and has a charge of $+2.0 \times 10^{-6} \mathrm{C}$. Sphere B is located at +0.60 m on the $x$-axis and has a charge of $-3.6 \times 10^{-6} C$. Sphere C is located at +0.80 m on the $x$-axis and has a charge of $+4.0 \times 10^{-6} \mathrm{C}$. Determine the net force on sphere $A$.
13. Determine the net force on sphere $B$ in the previous problem.

As you use the Coulomb's law equation, keep in mind that Coulomb's law is valid only for point charges or uniform spherical charge distributions. That is, a charged sphere may be treated as if all the charge were located at its center if the charge is spread evenly across its entire surface or throughout its volume. If a sphere is a conductor and another charge is brought near it, the charges on the sphere will be attracted or repelled, and the charge no longer will act as if it were at the sphere's center. Therefore, it is important to consider how large and how far apart two charged spheres are before applying Coulomb's law. The problems in this textbook assume that charged spheres are small enough and far enough apart to be considered point charges unless otherwise noted. When shapes such as long wires or flat plates are considered, Coulomb's law must be modified to account for the nonpoint charge distributions.

## Application of Electrostatic Forces

There are many applications of electric forces on particles. For example, these forces can collect soot in smokestacks, thereby reducing air pollution, as shown in Figure 20-12. Tiny paint droplets, charged by

## CHALLENGE PROBLEM

As shown in the figure on the right, two spheres of equal mass, $m$, and equal positive charge, $q$, are a distance, $r$, apart.

1. Derive an expression for the charge, $q$, that must be on each sphere so that the spheres are in equilibrium; that is, so that the attractive and repulsive forces between them are balanced.
2. If the distance between the spheres is doubled, how will that affect the expression for the value of $q$ that you determined in the previous problem? Explain.

3. If the mass of each sphere is 1.50 kg , determine the charge on each sphere needed to maintain the equilibrium.


Figure 20-12 The fly ash being released by these smokestacks is a by-product of burning coal. Static-electricity precipitators can be used to reduce fly ash emissions.
induction, can be used to paint automobiles and other objects very uniformly. Photocopy machines use static electricity to place black toner on a page so that a precise reproduction of the original document is made. In other instances, applications are concerned with the control of static charge. For example, static charge can ruin film if it attracts dust, and electronic equipment can be damaged by the discharge of static charge. In these cases, applications are designed to avoid the buildup of static charge and to safely eliminate any charge that does build up.

### 20.2 Section Review

14. Force and Charge How are electric force and charge related? Describe the force when the charges are like charges and the force when the charges are opposite charges.
15. Force and Distance How are electric force and distance related? How would the force change if the distance between two charges were tripled?
16. Electroscopes When an electroscope is charged, the leaves rise to a certain angle and remain at that angle. Why do they not rise farther?
17. Charging an Electroscope Explain how to charge an electroscope positively using
a. a positive rod.
b. a negative rod.
18. Attraction of Neutral Objects What two properties explain why a neutral object is attracted to both positively and negatively charged objects?
19. Charging by Induction In an electroscope being charged by induction, what happens when the charging rod is moved away before the ground is removed from the knob?
20. Electric Forces Two charged spheres are held a distance, $r$, apart. One sphere has a charge of $+3 \mu \mathrm{C}$, and the other sphere has a charge of $+9 \mu \mathrm{C}$. Compare the force of the $+3 \mu \mathrm{C}$ sphere on the $+9 \mu \mathrm{C}$ sphere with the force of the $+9 \mu \mathrm{C}$ sphere on the $+3 \mu \mathrm{C}$ sphere.
21. Critical Thinking Suppose that you are testing Coulomb's law using a small, positively charged plastic sphere and a large, positively charged metal sphere. According to Coulomb's law, the force depends on $1 / r^{2}$, where $r$ is the distance between the centers of the spheres. As the spheres get close together, the force is smaller than expected from Coulomb's law. Explain.

# PHYSICS LAB•Design Your Own <br> <br> Charged Objects 

 <br> <br> Charged Objects}

In this chapter, you observed and studied phenomena that result from the separation of electric charges. You learned that hard rubber and plastic tend to become negatively charged when they are rubbed, while glass and wool tend to become positively charged. But what happens if two objects that tend to become negatively charged are rubbed together? Will electrons be transferred? If so, which material will gain electrons, and which will lose them? In this physics lab, you will design a procedure to further your investigations of positive and negative charges.

## QUESTION

How can you test materials for their ability to hold positive and negative charges?

## Objectives

■ Observe that different materials tend to become positively or negatively charged.

- Compare and contrast the ability of materials to acquire and hold positive and negative charges.
■ Interpret data to order a list of materials from strongest tendency to be negatively charged to strongest tendency to be positively charged.


## Safety Precautions



## Materials

$15-\mathrm{cm}$ plastic ruler thread ring stand with ring masking tape materials to be charged, such as rubber rods, plastic rods, glass rods, PVC pipe, copper pipe, steel pipe, pencils, pens, wool, silk, plastic wrap, plastic sandwich bags, waxed paper, and aluminum foil

## Procedure

1. Use the lab photo as a guide to suspend a $15-\mathrm{cm}$ plastic ruler. It is advisable to wash the ruler in soapy water, then rinse and dry it thoroughly before each use, especially if it is a humid day. The thread should be attached at the midpoint of the ruler with two or three wraps of masking tape between the thread and ruler.
2. Use the following situations as a reference for types of charges a material can have: 1) a plastic ruler rubbed with wool gives the plastic ruler an excess negative charge and the wool an excess positive charge, and 2) a plastic ruler rubbed with plastic wrap gives the plastic ruler an excess positive charge and the plastic wrap an excess negative charge.

## Data Table

| Material 1 | Material 2 | Charge <br> on Ruler <br> $(+,-, 0)$ | Observation <br> of Ruler's <br> Movements | Charge on <br> Material 1 <br> $(+,-, 0)$ | Charge on <br> Material 2 <br> $(+,-, 0)$ |
| :--- | :--- | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |
|  |  |  |  |  |  |
|  |  |  |  |  |  |
|  |  |  |  |  |  |
|  |  |  |  |  |  |

3. Design a procedure to test which objects tend to become negatively charged and which tend to become positively charged. Try various combinations of materials and record your observations in the data table.
4. Develop a test to see if an object is neutral. Remember that a charged ruler may be attracted to a neutral object if it induces a separation of charge in the neutral object.
5. Be sure to check with your teacher and have your procedure approved before you proceed with your lab.

## Analyze

1. Observe and Infer As you brought charged materials together, could you detect a force between the charged materials? Describe this force.
2. Formulate Models Make a drawing of the charge distribution on the two materials for one of your trials. Use this drawing to explain why the materials acted the way they did during your experiments with them.
3. Draw Conclusions Which materials hold an excess charge? Which materials do not hold a charge very well?
4. Draw Conclusions Which materials tend to become negatively charged? Which tend to become positively charged?
5. Interpret Data Use your data table to list the relative tendencies of materials to be positively or negatively charged.

## Conclude and Apply

1. Explain what is meant by the phrases excess charge and charge imbalance when referring to static electricity.
2. Does excess charge remain on a material or does it dissipate over time?
3. Could you complete this physics lab using a metal rod in place of the suspended plastic ruler? Explain.
4. Clear plastic wrap seals containers of food. Why does plastic wrap cling to itself after it is pulled from its container?

## Going Further

Review the information in your textbook about electroscopes. Redesign the lab using an electroscope, rather than a suspended ruler, to test for the type of charge on an object.

## Real-World Physics

Trucks often have a rubber strap or a chain that drags along the road. Why are they used?

## Physics Oline

To find out more about static charge, visit the Web site: physicspp.com

## Future Technology

## Spacecraft and Static Electricity

Most objects on Earth do not build up substantial static-electric charges because a layer of moisture clings to surfaces, allowing charges to migrate to or from the ground. As you learned in this chapter, Earth can absorb almost any amount of charge. However, there is no moisture in space, and Earth is far away. Charged particles ejected from the Sun, or in the ionosphere, strike and cling to spacecraft, charging their surfaces to thousands of volts.

## Plasma and Charging

 In Chapter 13, you learned that plasma consists of free electrons and positive ions. Orbiting spacecraft are surrounded by a thin cloud of this plasma. The electrons in plasma can move far more easily than more massive positive ions. Thus, spacecraft surfaces tend to attract electrons and develop a negative charge. This negative charge eventually attracts some heavy positive ions, which strike the spacecraft and can damage its surface.On the International Space Station, an additional difficulty stems from the array of solar panels that convert energy from the Sun into electricity. When the arrays are powering the space station, the voltage on the surface of the craft tends to be close to the voltage of the solar array. As a result, it is possible that an electric arc could form between the space station and the plasma that surrounds it.

Consequences of an Arc Arcs are extremely hot and carry a great deal of current. They can prematurely ignite retro-rockets or explosive bolts and interfere with the operation of the spacecraft's electronic equipment. The solar panels are particularly susceptible to arc damage. In addition to damage to the


Plasma Contactor Unit (PCU)

spacecraft's components, there is a remote chance that the buildup of charge might endanger astronauts on space walks.

To discharge the potential difference and protect craft and crew, the space station's skin must be connected by a conductor, called a plasma contactor, to the plasma cloud surrounding it. The connection begins on board the station, where a stream of xenon gas from a tank in the Plasma Contactor Unit (PCU) is ionized by an electric current. This ionization takes place in the cathode assembly. The ionized xenon, now in the plasma state, passes out of the craft through the cathode assembly. It is this stream of conductive plasma that connects the craft to the surrounding plasma cloud, thereby reducing the potential difference to safe levels.

## Future Applications

Future spacecraft might integrate the plasma contactor into the propulsion system. For example, the Variable Specific Impulse Magnetoplasma Rocket (VASIMR) could use the plasma exhaust that it produces to provide an electric connection between the spacecraft and the surrounding plasma. Scientists think that this type of rocket could be used in the future to travel between planets.

### 20.1 Electric Charge

## Vocabulary

- electrostatics (p. 541)
- neutral (p. 543)
- insulator (p. 544)
- conductor (p. 544)


## Key Concepts

- There are two kinds of electric charge, positive and negative. Interactions of these charges explain the attraction and repulsion that you observed in the strips of tape.
- Electric charge is not created or destroyed; it is conserved. Charging is the separation, not creation, of electric charges.
- Objects can be charged by the transfer of electrons. An area with excess electrons has a net negative charge; an area with a deficit of electrons has a net positive charge.
- Charges added to one part of an insulator remain on that part. Insulators include glass, dry wood, plastics, and dry air.
- Charges added to a conductor quickly spread over the surface of the object. In general, examples of conductors include graphite, metals, and matter in the plasma state.
- Under certain conditions, charges can move through a substance that is ordinarily an insulator. Lightning moving through air is one example.


### 20.2 Electric Force

## Vocabulary

- electroscope (p. 546)
- charging by conduction (p. 547)
- charging by induction (p. 548)
- grounding (p. 548)
- Coulomb's law (p. 549)
- coulomb (p. 549)
- elementary charge (p. 550)


## Key Concepts

- When an electroscope is charged, electric forces cause its thin metal leaves to spread apart.
- An object can be charged by conduction by touching it with a charged object.
- A charged object will induce a separation of charges within a neutral conductor. This process will result in an attractive force between the charged object and the neutral conductor.
- To charge a conductor by induction, a charged object is first brought near it, causing a separation of charges. Then, the conductor to be charged is separated, trapping opposite charges on the two halves.
- Grounding is the removal of excess charge by touching an object to Earth. Grounding can be used in the process of charging an electroscope by induction.
- Coulomb's law states that the force between two charged particles varies directly with the product of their charges and inversely with the square of the distance between them.

$$
F=K \frac{q_{\mathrm{A}} q_{\mathrm{B}}}{r^{2}}
$$

To determine the direction of the force, remember the following rule: like charges repel; unlike charges attract.

- The SI unit of charge is the coulomb. One coulomb (C) is the magnitude of the charge of $6.24 \times 10^{18}$ electrons or protons. The elementary charge, the charge of a proton or electron, is $1.60 \times 10^{-19} \mathrm{C}$.


## Concept Mapping

22. Complete the concept map below using the following terms: conduction, distance, elementary charge.


## Mastering Concepts

23. If you comb your hair on a dry day, the comb can become positively charged. Can your hair remain neutral? Explain. (20.1)
24. List some insulators and conductors. (20.1)
25. What property makes metal a good conductor and rubber a good insulator? (20.1)
26. Laundry Why do socks taken from a clothes dryer sometimes cling to other clothes? (20.2)
27. Compact Discs If you wipe a compact disc with a clean cloth, why does the CD then attract dust? (20.2)
28. Coins The combined charge of all electrons in a nickel is hundreds of thousands of coulombs. Does this imply anything about the net charge on the coin? Explain. (20.2)
29. How does the distance between two charges impact the force between them? If the distance is decreased while the charges remain the same, what happens to the force? (20.2)
30. Explain how to charge a conductor negatively if you have only a positively charged rod. (20.2)

## Applying Concepts

31. How does the charge of an electron differ from the charge of a proton? How are they similar?
32. Using a charged rod and an electroscope, how can you find whether or not an object is a conductor?
33. A charged rod is brought near a pile of tiny plastic spheres. Some of the spheres are attracted to the rod, but as soon as they touch the rod, they are flung off in different directions. Explain.
34. Lightning Lightning usually occurs when a negative charge in a cloud is transported to Earth. If Earth is neutral, what provides the attractive force that pulls the electrons toward Earth?
35. Explain what happens to the leaves of a positively charged electroscope when rods with the following charges are brought close to, but not touching, the electroscope.
a. positive
b. negative
36. As shown in Figure 20-13, Coulomb's law and Newton's law of universal gravitation appear to be similar. In what ways are the electric and gravitational forces similar? How are they different?

## Law of Universal Gravitation

$$
F=G \frac{m_{A} m_{B}}{r^{2}}
$$

$$
F=K \frac{q_{A} q_{B}}{r^{2}}
$$



Figure 20-13 (Not to scale)
37. The constant, $K$, in Coulomb's equation is much larger than the constant, $G$, in the universal gravitation equation. Of what significance is this?
38. The text describes Coulomb's method for charging two spheres, A and B, so that the charge on B was exactly half the charge on A. Suggest a way that Coulomb could have placed a charge on sphere B that was exactly one-third the charge on sphere $A$.
39. Coulomb measured the deflection of sphere $A$ when spheres A and B had equal charges and were a distance, $r$, apart. He then made the charge on B one-third the charge on A . How far apart would the two spheres then have had to be for A to have had the same deflection that it had before?
40. Two charged bodies exert a force of 0.145 N on each other. If they are moved so that they are onefourth as far apart, what force is exerted?
41. Electric forces between charges are enormous in comparison to gravitational forces. Yet, we normally do not sense electric forces between us and our surroundings, while we do sense gravitational interactions with Earth. Explain.

## Mastering Problems

### 20.2 Electric Force

42. Two charges, $q_{\mathrm{A}}$ and $q_{\mathrm{B}^{\prime}}$, are separated by a distance, $r$, and exert a force, $F$, on each other. Analyze Coulomb's law and identify what new force would exist under the following conditions.
a. $q_{\mathrm{A}}$ is doubled
b. $q_{\mathrm{A}}$ and $q_{\mathrm{B}}$ are cut in half
c. $r$ is tripled
d. $r$ is cut in half
e. $q_{\mathrm{A}}$ is tripled and $r$ is doubled
43. Lightning A strong lightning bolt transfers about 25 C to Earth. How many electrons are transferred?
44. Atoms Two electrons in an atom are separated by $1.5 \times 10^{-10} \mathrm{~m}$, the typical size of an atom. What is the electric force between them?
45. A positive and a negative charge, each of magnitude $2.5 \times 10^{-5} \mathrm{C}$, are separated by a distance of 15 cm . Find the force on each of the particles.
46. A force of $2.4 \times 10^{2} \mathrm{~N}$ exists between a positive charge of $8.0 \times 10^{-5} \mathrm{C}$ and a positive charge of $3.0 \times 10^{-5} \mathrm{C}$. What distance separates the charges?
47. Two identical positive charges exert a repulsive force of $6.4 \times 10^{-9} \mathrm{~N}$ when separated by a distance of $3.8 \times 10^{-10} \mathrm{~m}$. Calculate the charge of each.
48. A positive charge of $3.0 \mu \mathrm{C}$ is pulled on by two negative charges. As shown in Figure 20-14, one negative charge, $-2.0 \mu \mathrm{C}$, is 0.050 m to the west, and the other, $-4.0 \mu \mathrm{C}$, is 0.030 m to the east. What total force is exerted on the positive charge?


Figure 20-14
49. Figure 20-15 shows two positively charged spheres, one with three times the charge of the other. The spheres are 16 cm apart, and the force between them is 0.28 N . What are the charges on the two spheres?


Figure 20-15
50. Charge in a Coin How many coulombs of charge are on the electrons in a nickel? Use the following method to find the answer.
a. Find the number of atoms in a nickel. A nickel has a mass of about 5 g . A nickel is 75 percent Cu and 25 percent Ni , so each mole of the coin's atoms will have a mass of about 62 g .
b. Find the number of electrons in the coin. On average, each atom has 28.75 electrons.
c. Find the coulombs on the electrons.
51. Three particles are placed in a line. The left particle has a charge of $-55 \mu \mathrm{C}$, the middle one has a charge of $+45 \mu \mathrm{C}$, and the right one has a charge of $-78 \mu \mathrm{C}$. The middle particle is 72 cm from each of the others, as shown in Figure 20-16.
a. Find the net force on the middle particle.
b. Find the net force on the right particle.


Figure 20-16

## Mixed Review

52. A small metal sphere with charge $1.2 \times 10^{-5} \mathrm{C}$ is touched to an identical neutral sphere and then placed 0.15 m from the second sphere. What is the electric force between the two spheres?
53. Atoms What is the electric force between an electron and a proton placed $5.3 \times 10^{-11} \mathrm{~m}$ apart, the approximate radius of a hydrogen atom?
54. A small sphere of charge $2.4 \mu \mathrm{C}$ experiences a force of 0.36 N when a second sphere of unknown charge is placed 5.5 cm from it. What is the charge of the second sphere?
55. Two identically charged spheres placed 12 cm apart have an electric force of 0.28 N between them. What is the charge of each sphere?
56. In an experiment using Coulomb's apparatus, a sphere with a charge of $3.6 \times 10^{-8} \mathrm{C}$ is 1.4 cm from a second sphere of unknown charge. The force between the spheres is $2.7 \times 10^{-2} \mathrm{~N}$. What is the charge of the second sphere?
57. The force between a proton and an electron is $3.5 \times 10^{-10} \mathrm{~N}$. What is the distance between these two particles?

## Chapter 20 Assessment

## Thinking Critically

58. Apply Concepts Calculate the ratio of the electric force to the gravitational force between the electron and the proton in a hydrogen atom.
59. Analyze and Conclude Sphere A , with a charge of $+64 \mu \mathrm{C}$, is positioned at the origin. A second sphere, B , with a charge of $-16 \mu \mathrm{C}$, is placed at +1.00 m on the $x$-axis.
a. Where must a third sphere, C , of charge $+12 \mu \mathrm{C}$ be placed so there is no net force on it?
b. If the third sphere had a charge of $+6 \mu \mathrm{C}$, where should it be placed?
c. If the third sphere had a charge of $-12 \mu \mathrm{C}$, where should it be placed?
60. Three charged spheres are located at the positions shown in Figure 20-17. Find the total force on sphere B.


Figure 20-17
61. The two pith balls in Figure 20-18 each have a mass of 1.0 g and an equal charge. One pith ball is suspended by an insulating thread. The other is brought to 3.0 cm from the suspended ball. The suspended ball is now hanging with the thread forming an angle of $30.0^{\circ}$ with the vertical. The ball is in equilibrium with $F_{\mathrm{E}^{\prime}} \boldsymbol{F}_{\mathrm{g}^{\prime}}$ and $\boldsymbol{F}_{\mathrm{T}}$. Calculate each of the following.
a. $F_{\mathrm{g}}$ on the suspended ball
b. $F_{\mathrm{E}}^{\mathrm{g}}$
c. the charge on the balls


Figure 20-18
62. Two charges, $q_{\mathrm{A}}$ and $q_{\mathrm{B}^{\prime}}$, are at rest near a positive test charge, $q_{\mathrm{T}^{\prime}}$ of $7.2 \mu \mathrm{C}$. The first charge, $q_{\mathrm{A}^{\prime}}$ is a positive charge of $3.6 \mu \mathrm{C}$ located 2.5 cm away from $q_{\mathrm{T}}$ at $35^{\circ} ; q_{\mathrm{B}}$ is a negative charge of $-6.6 \mu \mathrm{C}$ located 6.8 cm away at $125^{\circ}$.
a. Determine the magnitude of each of the forces acting on $q_{\mathrm{T}}$.
b. Sketch a force diagram.
c. Graphically determine the resultant force acting on $q_{\mathrm{T}}$.

## Writing in Physics

63. History of Science Research several devices that were used in the seventeenth and eighteenth centuries to study static electricity. Examples that you might consider include the Leyden jar and the Wimshurst machine. Discuss how they were constructed and how they worked.
64. In Chapter 13, you learned that forces exist between water molecules that cause water to be denser as a liquid between $0^{\circ} \mathrm{C}$ and $4^{\circ} \mathrm{C}$ than as a solid at $0^{\circ} \mathrm{C}$. These forces are electrostatic in nature. Research electrostatic intermolecular forces, such as van der Waals forces and dipole-dipole forces, and describe their effects on matter.

## Cumulative Review

65. Explain how a pendulum can be used to determine the acceleration of gravity. (Chapter 14)
66. A submarine that is moving $12.0 \mathrm{~m} / \mathrm{s}$ sends a sonar ping of frequency $1.50 \times 10^{3} \mathrm{~Hz}$ toward a seamount that is directly in front of the submarine. It receives the echo 1.800 s later. (Chapter 15)
a. How far is the submarine from the seamount?
b. What is the frequency of the sonar wave that strikes the seamount?
c. What is the frequency of the echo received by the submarine?
67. Security Mirror A security mirror is used to produce an image that is three-fourths the size of an object and is located 12.0 cm behind the mirror. What is the focal length of the mirror? (Chapter 17)
68. A $2.00-\mathrm{cm}$-tall object is located 20.0 cm away from a diverging lens with a focal length of 24.0 cm . What are the image position, height, and orientation? Is this a real or a virtual image? (Chapter 18)
69. Spectrometer A spectrometer contains a grating of 11,500 slits $/ \mathrm{cm}$. Find the angle at which light of wavelength 527 nm has a first-order bright band. (Chapter 19)

## Standardized Test Practice

## Multiple Choice

1. How many electrons have been removed from a positively charged electroscope if it has a net charge of $7.5 \times 10^{-11} \mathrm{C}$ ?
(A) $7.5 \times 10^{-11}$ electrons
(B) $2.1 \times 10^{-9}$ electrons
(C) $1.2 \times 10^{8}$ electrons
(D) $4.7 \times 10^{8}$ electrons
2. The force exerted on a particle with a charge of $5.0 \times 10^{-9} \mathrm{C}$ by a second particle that is 4 cm away is $8.4 \times 10^{-5} \mathrm{~N}$. What is the charge of the second particle?
(A) $4.2 \times 10^{-13} \mathrm{C}$
(C) $3.0 \times 10^{-9} \mathrm{C}$
(B) $2.0 \times 10^{-9} \mathrm{C}$
(D) $6.0 \times 10^{-5} \mathrm{C}$
3. Three charges, A, B, and C, are located in a line, as shown below. What is the net force on charge B ?
```
(A) 78 N toward A
(C) 130 N toward A
```

(B) 78 N toward C
(D) 210 N toward C

$$
+8.5 \times 10^{-6} \mathrm{C} \quad+3.1 \times 10^{-6} \mathrm{C}+6.4 \times 10^{-6} \mathrm{C}
$$


4. What is the charge on an electroscope that has an excess of $4.8 \times 10^{10}$ electrons?

$$
\begin{array}{ll}
\text { (A) } 3.3 \times 10^{-30} \mathrm{C} & \text { (C) } 7.7 \times 10^{-9} \mathrm{C} \\
\text { (B) } 4.8 \times 10^{-10} \mathrm{C} & \text { (D) } 4.8 \times 10^{10} \mathrm{C}
\end{array}
$$

5. Two charged bodies exert a force of 86 N on each other. If they are moved so that they are six times farther apart, what is the new force that they will exert on each other?
```
(A)
    2.4 N
(C) 86 N
(B) 14 N
(D) \(5.2 \times 10^{2} \mathrm{~N}\)
```

6. Two equally charged bodies exert a force of 90 N on each other. If one of the bodies is exchanged for a body of the same size, but three times as much charge, what is the new force that they will exert on each other?
(A) 10 N
(C) $2.7 \times 10^{2} \mathrm{~N}$
(B) 30 N
(D) $8.1 \times 10^{2} \mathrm{~N}$
7. An alpha particle has a mass of $6.68 \times 10^{-27} \mathrm{~kg}$ and a charge of $3.2 \times 10^{-19} \mathrm{C}$. What is the ratio of the electrostatic force to the gravitational force between two alpha particles?
```
(A) }
(C) }2.3\times1\mp@subsup{0}{}{15
(B) }4.8\times1\mp@subsup{0}{}{7
(D) }3.1\times1\mp@subsup{0}{}{35
```

8. Charging a neutral body by touching it with a charged body is called charging by $\qquad$ .
(A) conduction
(C) grounding
(B) induction
(D) discharging
9. Macy rubs a balloon with wool, giving the balloon a charge of $-8.9 \times 10^{-14} \mathrm{C}$. What is the force between the balloon and a metal sphere that is charged to 25 C and is 2 km away?
(A) $8.9 \times 10^{-15} \mathrm{~N}$
(C) $2.2 \times 10^{-12} \mathrm{~N}$
(B) $5.0 \times 10^{-9} \mathrm{~N}$
(D) $5.6 \times 10^{4} \mathrm{~N}$

## Extended Answer

10. According to the diagram, what is the net force exerted by charges $A$ and $B$ on charge $C$ ? In your answer, include a diagram showing the force vectors $\boldsymbol{F}_{\mathrm{A} \text { on } \mathrm{C}^{\prime}} \boldsymbol{F}_{\mathrm{B} \text { on } \mathrm{C}^{\prime}}$ and $\boldsymbol{F}_{\text {net }}$.


## Test-Taking TIP

## Slow Down

Check to make sure you are answering the question that each problem is posing. Read the questions and answer choices very carefully. Remember that doing most of the problems and getting them right is always preferable to doing all of the problems and getting a lot of them wrong.

## 3 <br> 21

## Electric Fields

## What You'll Learn

- You will relate electric fields to electric forces and distinguish between them.
- You will relate electric potential difference to work and energy.
- You will describe how charges are distributed on conductors.
- You will explain how capacitors store electric charges.

Why It's Important
Electricity is an essential form of energy for modern societies.

High-Energy Discharge A high-voltage generator produces the glow you see inside these discharge spheres.

## Think About This >

 Why doesn't an ordinary lightbulb glow in the same way as these discharge spheres connected to a high-voltage generator?

## LAUNCH Lab <br> How do charged objects interact at a distance?

## Question

How is a charged object affected by interaction with other charged objects at a distance?

## Procedure $\underbrace{2 \times 1}$

1. Inflate and tie off two balloons. Attach a $\frac{1}{2}-\mathrm{m}$ length of string to each balloon.
2. Rub one balloon back and forth on your shirt six to eight times, causing it to become charged. Hang it from a cabinet, table, or other support by the string with a piece of tape.
3. Rub the second balloon the same way and then suspend it from its string.
4. Observe Slowly bring the second balloon toward the suspended one. How do the balloons behave? Tape the second balloon so it hangs by its string next to the first balloon.
5. Observe Bring your hand toward the charged balloons. What happens?

## Analysis

What did you observe as the two balloons were brought near each other? What happened as your hand was brought near the balloons?
Critical Thinking With what two objects have you previously observed similar behaviors of action at a distance?


### 21.1 Creating and Measuring Electric Fields

Electric force, like gravitational force, which you studied in Chapter 8, varies inversely as the square of the distance between two point objects. Both forces can act from great distances. How can a force be exerted across what seems to be empty space? Michael Faraday suggested that because an electrically charged object, A, creates a force on another charged object, B , anywhere in space, object A must somehow change the properties of space. Object B somehow senses the change in space and experiences a force due to the properties of the space at its location. We call the changed property of space an electric field. An electric field means that the interaction is not between two distant objects, but between an object and the field at its location.

The forces exerted by electric fields can do work, transferring energy from the field to another charged object. This energy is something you use on a daily basis, whether you plug an appliance into an electric outlet or use a battery-powered, portable device. In this chapter, you will learn more about electric fields, forces, and electric energy.

## - Objectives

- Define an electric field.
- Solve problems relating to charge, electric fields, and forces.
- Diagram electric field lines.
- Vocabulary electric field electric field line


## Color Convention

- Electric field lines are indigo.
- Positive charges are red.
- Negative charges are blue.


Figure 21-1 Arrows can be used to represent the magnitude and direction of the electric field about an electric charge at various locations.

## The Electric Field

How can you measure an electric field? Place a small charged object at some location. If there is an electric force on it, then there is an electric field at that point. The charge on the object that is used to test the field, called the test charge, must be small enough that it doesn't affect other charges.

Consider Figure 21-1, which illustrates a charged object with a charge of q. Suppose you place the positive test charge at some point, A, and measure a force, $\boldsymbol{F}$. According to Coulomb's law, the force is directly proportional to the strength of the test charge, $q^{\prime}$. That is, if the charge is doubled, so is the force. Therefore, the ratio of the force to the charge is a constant. If you divide the force, $\boldsymbol{F}$, by the test charge, $q^{\prime}$, you obtain a vector quantity, $\boldsymbol{F} / q^{\prime}$. This quantity does not depend on the test charge, only on the force, $\boldsymbol{F}$, and the location of point A . The electric field at point A , the location of $q^{\prime}$, is represented by the following equation.

Electric Field Strengh $\quad \boldsymbol{E}=\frac{\boldsymbol{F}_{\text {on } q^{\prime}}}{q^{\prime}}$
The strength of an electric field is equal to the force on a positive test charge divided by the strength of the test charge.

The direction of an electric field is the direction of the force on a positive test charge. The magnitude of the electric field strength is measured in newtons per coulomb, N/C.

A picture of an electric field can be made by using arrows to represent the field vectors at various locations, as shown in Figure 21-1. The length of the arrow is used to show the strength of the field. The direction of the arrow shows the field direction. To find the field from two charges, the fields from the individual charges are added vectorially. A test charge can be used to map out the field resulting from any collection of charges. Typical electric field strengths produced by charge collections are shown in Table 21-1.

An electric field should be measured only by a very small test charge. This is because the test charge also exerts a force on $q$. It is important that the force exerted by the test charge does not cause charge to be redistributed on a conductor, thereby causing $q$ to move to another location and thus, changing the force on $q^{\prime}$ as well as the electric field strength being measured. A test charge always should be small enough so that its effect on $q$ is negligible.

Table 21-1

| Approximate Values of Typical Electric Fields |  |
| :--- | :---: |
| Field | Value (N/C) |
| Near a charged, hard-rubber rod | $1 \times 10^{3}$ |
| In a television picture tube | $1 \times 10^{5}$ |
| Needed to create a spark in air | $3 \times 10^{6}$ |
| At an electron's orbit in a hydrogen atom | $5 \times 10^{11}$ |

## EXAMPLE Problem 1

Electric Field Strength An electric field is measured using a positive test charge of $3.0 \times 10^{-6} \mathrm{C}$. This test charge experiences a force of 0.12 N at an angle of $15^{\circ}$ north of east. What are the magnitude and direction of the electric field strength at the location of the test charge?

## 1 Analyze and Sketch the Problem

- Draw and label the test charge, $q^{\prime}$.
- Show and label the coordinate system centered on the test charge.
- Diagram and label the force vector at $15^{\circ}$ north of east.

$$
\begin{array}{ll}
\text { Known: } & \text { Unknown: } \\
q^{\prime}=+3.0 \times 10^{-6} \mathrm{C} & \boldsymbol{E}=? \\
\boldsymbol{F}=0.12 \mathrm{~N} \text { at } 15^{\circ} \mathrm{N} \text { of } \mathrm{E} &
\end{array}
$$

2 Solve for the Unknown

$$
\begin{aligned}
E & =\frac{F}{q^{\prime}} \\
& =\frac{0.12 \mathrm{~N}}{3.0 \times 10^{-6} \mathrm{~N} / \mathrm{C}} \quad \text { Substitute } F=0.12 \mathrm{~N}, \boldsymbol{q}^{\prime}=3.0 \times 10^{-6} \mathrm{C} \\
& =4.0 \times 10^{4} \mathrm{~N} / \mathrm{C}
\end{aligned}
$$



The force on the test charge and the electric field are in the same direction.

## Math Handbook

Operations with Significant Digits pages 835-836
$\boldsymbol{E}=4.0 \times 10^{4} \mathrm{~N} / \mathrm{C}$ at $15^{\circ} \mathrm{N}$ of E

## 3 Evaluate the Answer

- Are the units correct? Electric field strength is correctly measured in N/C.
- Does the direction make sense? The field direction is in the direction of the force because the test charge is positive.
- Is the magnitude realistic? This field strength is consistent with the values listed in Table 21-1.


## PRACTICE Problems Additional Problems, Appendix B

1. A positive test charge of $5.0 \times 10^{-6} \mathrm{C}$ is in an electric field that exerts a force of $2.0 \times 10^{-4} \mathrm{~N}$ on it. What is the magnitude of the electric field at the location of the test charge?
2. A negative charge of $2.0 \times 10^{-8} \mathrm{C}$ experiences a force of 0.060 N to the right in an electric field. What are the field's magnitude and direction at that location?
3. A positive charge of $3.0 \times 10^{-7} \mathrm{C}$ is located in a field of $27 \mathrm{~N} / \mathrm{C}$ directed toward the south. What is the force acting on the charge?
4. A pith ball weighing $2.1 \times 10^{-3} \mathrm{~N}$ is placed in a downward electric field of $6.5 \times 10^{4} \mathrm{~N} / \mathrm{C}$. What charge (magnitude and sign) must be placed on the pith ball so that the electric force acting on it will suspend it against the force of gravity?
5. You are probing the electric field of a charge of unknown magnitude and sign. You first map the field with a $1.0 \times 10^{-6}-\mathrm{C}$ test charge, then repeat your work with a $2.0 \times 10^{-6}-\mathrm{C}$ test charge.
a. Would you measure the same forces at the same place with the two test charges? Explain.
b. Would you find the same field strengths? Explain.

## EXAMPLE Problem 2

Electric Field Strength What is the electric field strength at a point that is 0.30 m to the right of a small sphere with a charge of $-4.0 \times 10^{-6} \mathrm{C}$ ?

## 1 Analyze and Sketch the Problem

- Draw and label the sphere and its charge, $q$, and the test charge, $q^{\prime}$.
- Show and label the distance between the charges.
- Diagram and label the force vector acting on $q^{\prime}$.

$$
q=-4.0 \times 10^{-6} C
$$

| Known: | Unknown: |
| :--- | :--- |
| $q=-4.0 \times 10^{-6} \mathrm{C}$ | $\boldsymbol{E}=$ ? |
| $d=0.30 \mathrm{~m}$ |  |


$q^{\prime}$
$q=-4.0 \times 10^{-6} \mathrm{C}$
known:
$d=0.30 \mathrm{~m}$


2 Solve for the Unknown
The force and the magnitude of the test charge are unknown, so use Coulomb's law in combination with the electric
field strength.

Math Handbook

$$
\begin{aligned}
E & =\frac{F}{q^{\prime}} \\
& =K \frac{q q^{\prime}}{d^{2} q^{\prime}}
\end{aligned}
$$

$$
\text { Substitute } \boldsymbol{F}=\boldsymbol{K} \frac{q q^{\prime}}{\boldsymbol{d}^{2}}
$$

$$
=K \frac{q}{d^{2}}
$$

$$
=\left(9.0 \times 10^{9} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{C}^{2}\right) \frac{\left(-4.0 \times 10^{-6} \mathrm{C}\right)}{(0.30 \mathrm{~m})^{2}} \quad \text { Substitute } K=9.0 \times 10^{9} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{C}^{2}, q=-4.0 \times 10^{-6} \mathrm{C}, d=0.30 \mathrm{~m}
$$

$$
=-4.0 \times 10^{5} \mathrm{~N} / \mathrm{C}
$$

$\boldsymbol{E}=4.0 \times 10^{5} \mathrm{~N} / \mathrm{C}$ toward the sphere, or to the left
3 Evaluate the Answer

- Are the units correct? $\left(\mathrm{N} \cdot \mathrm{m}^{2} / \mathrm{C}^{2}\right)(\mathrm{C}) / \mathrm{m}^{2}=\mathrm{N} / \mathrm{C}$. The units work out to be N/C, which is correct for electric field strength.
- Does the direction make sense? The negative sign indicates that the positive test charge is attracted toward the negative point charge.
- Is the magnitude realistic? This field strength is consistent with the values listed in Table 21-1.


## PRACTICE Problems

Additional Problems, Appendix B
6. What is the magnitude of the electric field strength at a position that is 1.2 m from a point charge of $4.2 \times 10^{-6} \mathrm{C}$ ?
7. What is the magnitude of the electric field strength at a distance twice as far from the point charge in problem 6 ?
8. What is the electric field at a position that is 1.6 m east of a point charge of $+7.2 \times 10^{-6} \mathrm{C}$ ?
9. The electric field that is 0.25 m from a small sphere is $450 \mathrm{~N} / \mathrm{C}$ toward the sphere. What is the charge on the sphere?
10. How far from a point charge of $+2.4 \times 10^{-6} \mathrm{C}$ must a test charge be placed to measure a field of $360 \mathrm{~N} / \mathrm{C}$ ?


So far, you have measured an electric field at a single point. Now, imagine moving the test charge to another location. Measure the force on it again and calculate the electric field. Repeat this process again and again until you assign every location in space a measurement of the vector quantity of the electric field strength associated with it. The field is present even if there is no test charge to measure it. Any charge placed in an electric field experiences a force on it resulting from the electric field at that location. The strength of the force depends on the magnitude of the field, $\boldsymbol{E}$, and the magnitude of the charge, $q$. Thus, $\boldsymbol{F}=\boldsymbol{E} q$. The direction of the force depends on the direction of the field and the sign of the charge.

## Picturing the Electric Field

A picture of an electric field is shown in Figure 21-2. Each of the lines used to represent the actual field in the space around a charge is called an electric field line. The direction of the field at any point is the tangent drawn to a field line at that point. The strength of the electric field is indicated by the spacing between the lines. The field is strong where the lines are close together. It is weaker where the lines are spaced farther apart. Although only two-dimensional models can be shown here, remember that electric fields exist in three dimensions.

The direction of the force on a positive test charge near another positive charge is away from the other charge. Thus, the field lines extend radially outward like the spokes of a wheel, as shown in Figure 21-2a. Near a negative charge, the direction of the force on the positive test charge is toward the negative charge, so the field lines point radially inward, as shown in Figure 21-2b. When there are two or more charges, the field is the vector sum of the fields resulting from the individual charges. The field lines become curved and the pattern is more complex, as shown in Figure 21-2c. Note that field lines always leave a positive charge and enter a negative charge, and that they never cross each other.


Figure 21-2 Lines of force are drawn perpendicularly away from a positively charged object (a) and perpendicularly into a negatively charged object (b). Electric field lines are shown between like charged and oppositely charged objects (c).

- Figure 21-3 In the Van de Graaff generator (a), charge is transferred onto a moving belt at A, and from the belt to the metal dome at $B$. An electric motor does the work needed to increase the electric potential energy. When a person touches a Van de Graaff generator, the results can be dramatic (b).

$\square$ Figure 21-4 Lines of force between unlike charges ( $\mathbf{a}, \mathbf{c}$ ) and between like charges (b, d) describe the behavior of a positively charged object in a field. The top figures are computer tracings of electric field lines.


Robert Van de Graaff devised the high-voltage electrostatic generator in the 1930s. Van de Graaff's machine, shown in Figure 21-3a on the previous page, is a device that transfers large amounts of charge from one part of the machine to a metal terminal at the top of the device. Charge is transferred onto a moving belt at the base of the generator, position $A$, and is transferred off the belt at the metal dome at the top, position B. An electric motor does the work needed to increase the electric potential energy. A person touching the terminal of a Van de Graaff machine is charged electrically. The charges on the person's hairs repel each other, causing the hairs to follow the field lines, as shown in Figure 21-3b.

Another method of visualizing field lines is to use grass seed in an insulating liquid, such as mineral oil. The electric forces cause a separation of charge in each long, thin grass seed. The seeds then turn so that they line up along the direction of the electric field. The seeds thus form a pattern of the electric field lines, as in Figure 21-4. Field lines do not really exist. They are simply a means of providing a model of an electric field. Electric fields, on the other hand, do exist. Although they provide a method of calculating the force on a charged body, they do not explain why charged bodies exert forces on each other.

### 21.1 Section Review

11. Measuring Electric Fields Suppose you are asked to measure the electric field in space. How do you detect the field at a point? How do you determine the magnitude of the field? How do you choose the magnitude of the test charge? What do you do next?
12. Field Strength and Direction A positive test charge of magnitude $2.40 \times 10^{-8} \mathrm{C}$ experiences a force of $1.50 \times 10^{-3} \mathrm{~N}$ toward the east. What is the electric field at the position of the test charge?
13. Field Lines In Figure 21-4, can you tell which charges are positive and which are negative? What would you add to complete the field lines?
14. Field Versus Force How does the electric field, $\boldsymbol{E}$, at the test charge differ from the force, $\boldsymbol{F}$, on it?
15. Critical Thinking Suppose the top charge in Figure $21-2 \mathrm{c}$ is a test charge measuring the field resulting from the two negative charges. Is it small enough to produce an accurate measure? Explain.

### 21.2 Applications of Electric Fields

As you have learned, the concept of energy is extremely useful in mechanics. The law of conservation of energy allows us to solve motion problems without knowing the forces in detail. The same is true in the study of electrical interactions. The work performed moving a charged particle in an electric field can result in the particle's gaining potential, or kinetic energy, or both. Because this chapter investigates charges at rest, only changes in potential energy will be discussed.

## Energy and Electric Potential

Recall the change in gravitational potential energy of a ball when it is lifted, as shown in Figure 21-5. Both the gravitational force, $\boldsymbol{F}$, and the
 force of gravity, you do work on it, thereby increasing its potential energy.

The situation is similar with two unlike charges: they attract each other, and so you must do work to pull one charge away from the other. When you do the work, you transfer energy to the charge where that energy is stored as potential energy. The larger the test charge, the greater the increase in its potential energy, $\triangle P E$.

Although the force on the test charge depends on its magnitude, $q^{\prime}$, the electric field it experiences does not. The electric field, $\boldsymbol{E}=\boldsymbol{F} / q^{\prime}$, is the force per unit charge. In a similar way, the electric potential difference, $\Delta V$, is defined as the work done moving a positive test charge between two points in an electric field divided by the magnitude of the test charge.

## Electric Potential Difference $\quad \Delta V=\frac{W_{\text {on } q^{\prime}}}{q^{\prime}}$

The difference in electrical potential is the ratio of the work needed to move a charge to the strength of that charge.

Electric potential difference is measured in joules per coulomb. One joule per coulomb is called a volt $(\mathrm{J} / \mathrm{C}=\mathrm{V})$.

Consider the situation shown in Figure 21-6 on the next page. The negative charge creates an electric field toward itself. Suppose you place a small positive test charge in the field at position A . It will experience a force in the direction of the field. If you now move the test charge away from the negative charge to position B, as in Figure 21-6a, you will have to exert a force, $\boldsymbol{F}$, on the charge. Because the force that you exert is in the same direction as the displacement, the work that you do on the test charge is positive. Therefore, there also will be a positive change in the electric potential difference. The change in potential difference does not depend on the magnitude of the test charge. It depends only on the field and the displacement.


## Objectives

- Define electric potential difference.
- Calculate potential difference from the work required to move a charge.
- Describe how charges are distributed on solid and hollow conductors.
- Solve problems pertaining to capacitance.
- Vocabulary
electric potential difference volt
equipotential
capacitor
capacitance
$\square$ Figure 21-5 Work is needed to move an object against the force of gravity (a) and against the electric force (b). In both cases, the potential energy of the object is increased.

- Figure 21-6 Electric potential difference is determined by measuring the work per unit charge. If you move unlike charges apart, you increase the electric potential difference (a). If you move unlike charges closer together, you reduce the electric potential difference (b).


## APPLYING PHYSICS

- Static Electricity Modern electronic devices, such as personal computers, contain components that are easily damaged by static electric discharges. To prevent damage to these sensitive components during repair, a technician will wear a conductive strap around his or her wrist. The other end of this strap is clipped to a grounded piece of metal. The strap conducts charge away from the technician and eliminates any possible potential difference with the grounded equipment.
$\square$ Figure 21-7 Electric potential is smaller when two unlike charges are closer together (a) and larger when two like charges are closer together (b).


Suppose you now move the test charge back to position A from position B, as in Figure 21-6b. The force that you exert is now in the direction opposite the displacement, so the work that you do is negative. The electric potential difference is also negative. In fact, it is equal and opposite to the potential difference for the move from position A to position B. The electric potential difference does not depend on the path used to go from one position to another. It does depend on the two positions.

Is there always an electric potential difference between the two positions? Suppose you move the test charge in a circle around the negative charge. The force that the electric field exerts on the test charge is always perpendicular to the direction in which you moved it, so you do no work. Therefore, the electric potential difference is zero. Whenever the electric potential difference between two or more positions is zero, those positions are said to be at equipotential.

Only differences in potential energy can be measured. The same is true of electric potential; thus, only differences in electric potential are important. The electric potential difference from point A to point B is defined as $\Delta V=V_{\mathrm{B}}-V_{\mathrm{A}}$. Electric potential differences are measured with a voltmeter. Sometimes, the electric potential difference is simply called the voltage. Do not confuse electric potential difference, $\Delta V$, with the unit for volts, V .


You have seen that electric potential difference increases as a positive test charge is separated from a negative charge. What happens when a positive test charge is separated from a positive charge? There is a repulsive force between these two charges. Potential energy decreases as the two charges are moved farther apart. Therefore, the electric potential is smaller at points farther from the positive charge, as shown in Figure 21-7.

As you learned in Chapter 11, the potential energy of a system can be defined as zero at any reference point. In the same way, the electric potential of any point can be defined as zero. No matter what reference point is chosen, the value of the electric potential difference from point A to point $B$ always will be the same.

## The Electric Potential in a Uniform Field

A uniform electric force and field can be made by placing two large, flat, conducting plates parallel to each other. One is charged positively and the other is charged negatively. The electric field between the plates is constant, except at the edges of the plates, and its direction is from the positive to the negative plate. The pattern formed by the grass seeds pictured in Figure 21-8 represents the electric field between parallel plates.

If a positive test charge, $q^{\prime}$, is moved a distance, $d$, in the direction opposite the electric field direction, the work done is found by the relationship $W_{\text {on } q^{\prime}}=F d$. Thus, the electric potential difference, the work done per unit charge, is $\Delta V=F d / q^{\prime}=\left(F / q^{\prime}\right) d$. Now, the electric field intensity is the force per unit charge, $E=F / q^{\prime}$. Therefore, the electric potential difference, $\Delta V$, between two points a distance, $d$, apart in a uniform field, $E$, is represented by the following equation.

## Electric Potential Difference in a Uniform Field $\Delta V=E d$

The electrical potential difference in a uniform field is equal to the product of electric field intensity and the distance moved by a charge.

The electric potential increases in the direction opposite the electric field direction. That is, the electric potential is higher near the positively charged plate. By dimensional analysis, the product of the units of $E$ and $d$ is $(\mathrm{N} / \mathrm{C})(\mathrm{m})$. This is equivalent to one $\mathrm{J} / \mathrm{C}$, which is the definition of 1 V .

$\square$ Figure 21-8 A representation of an electric field between parallel plates is shown.

## PRACTICE Problems

Additional Problems, Appendix B
16. The electric field intensity between two large, charged, parallel metal plates is $6000 \mathrm{~N} / \mathrm{C}$. The plates are 0.05 m apart. What is the electric potential difference between them?
17. A voltmeter reads 400 V across two charged, parallel plates that are 0.020 m apart. What is the electric field between them?
18. What electric potential difference is applied to two metal plates that are 0.200 m apart if the electric field between them is $2.50 \times 10^{3} \mathrm{~N} / \mathrm{C}$ ?
19. When a potential difference of 125 V is applied to two parallel plates, the field between them is $4.25 \times 10^{3} \mathrm{~N} / \mathrm{C}$. How far apart are the plates?
20. A potential difference of 275 V is applied to two parallel plates that are 0.35 cm apart. What is the electric field between the plates?

## EXAMPLE Problem 3

Work Required to Move a Proton Between Charged Parallel Plates Two charged parallel plates are 1.5 cm apart. The magnitude of the electric field between the plates is $1800 \mathrm{~N} / \mathrm{C}$.
a. What is the electric potential difference between the plates?
b. What work is required to move a proton from the negative plate to the positive plate?

## 1 Analyze and Sketch the Problem

- Draw the plates separated by 1.5 cm .
- Label one plate with positive charges and the other with negative charges.
- Draw uniformly spaced electric field lines from the positive plate to the negative plate.
- Indicate the electric field strength between the plates.

- Place a proton in the electric field.


## Known: Unknown:

$E=1800 \mathrm{~N} / \mathrm{C} \quad \Delta V=$ ?
$d=1.5 \mathrm{~cm} \quad W=$ ?
$q=1.60 \times 10^{-19} \mathrm{C}$
2 Solve for the Unknown

Math Handbook
Operations with Scientific Notation pages 842-843

$$
\begin{aligned}
\Delta V & =E d \\
& =(1800 \\
& =27 \mathrm{~V}
\end{aligned}
$$

$$
=(1800 \mathrm{~N} / \mathrm{C})(0.015 \mathrm{~m}) \quad \text { Substitute } E=1800 \mathrm{~N} / \mathrm{C}, \boldsymbol{d}=0.015 \mathrm{~m}
$$

$$
\Delta V=\frac{W}{q}
$$

$$
W=q \Delta V
$$

$$
=\left(1.60 \times 10^{-19} \mathrm{C}\right)(27 \mathrm{~V}) \quad \text { Substitute } q=1.60 \times 10^{-19} \mathrm{C}, \Delta \mathrm{~V}=27 \mathrm{~V}
$$

$$
=4.3 \times 10^{-18} \mathrm{~J}
$$

## 3 Evaluate the Answer

- Are the units correct? $(\mathrm{N} / \mathrm{C})(\mathrm{m})=\mathrm{N} \cdot \mathrm{m} / \mathrm{C}=\mathrm{J} / \mathrm{C}=\mathrm{V}$. The units work out to be volts. $\mathrm{C} \cdot \mathrm{V}=\mathrm{C}(\mathrm{J} / \mathrm{C})=\mathrm{J}$, the unit for work.
- Does the sign make sense? Positive work must be done to move a positive charge toward a positive plate.
- Is the magnitude realistic? With such a small charge moved through a potential difference of a few volts, the work performed will be small.


## PRACTICE Problems

## Additional Problems, Appendix B

21. What work is done when 3.0 C is moved through an electric potential difference of 1.5 V ?
22. A $12-\mathrm{V}$ car battery can store $1.44 \times 10^{6} \mathrm{C}$ when it is fully charged. How much work can be done by this battery before it needs recharging?
23. An electron in a television picture tube passes through a potential difference of $18,000 \mathrm{~V}$. How much work is done on the electron as it passes through that potential difference?
24. If the potential difference in problem 18 is between two parallel plates that are 2.4 cm apart, what is the magnitude of the electric field between them?
25. The electric field in a particle-accelerator machine is $4.5 \times 10^{5} \mathrm{~N} / \mathrm{C}$. How much work is done to move a proton 25 cm through that field?

## Millikan's Oil-Drop Experiment

One important application of the uniform electric field between two parallel plates is the measurement of the charge of an electron. This first was determined by American physicist Robert A. Millikan in 1909. Figure 21-9 shows the method used by Millikan to measure the charge carried by a single electron. First, fine oil drops were sprayed from an atomizer into the air. These drops were charged by friction with the atomizer as they were sprayed. Gravity acting on the drops caused them to fall, and a few of them entered the hole in the top plate of the apparatus. An electric potential difference then was placed across the two plates. The resulting electric field between the plates exerted a force on the charged drops. When the top plate was made positive enough, the electric force caused negatively charged drops to rise. The electric potential difference between the plates was adjusted to suspend a charged drop between the plates. At this point, the downward force of Earth's gravitational field and the upward force of the electric field were equal in magnitude.

The magnitude of the electric field, $E$, was determined from the electric potential difference between the plates. A second measurement had to be made to find the weight of the drop using the relationship $m g$, which was too tiny to measure by ordinary methods. To make this measurement, a drop first was suspended. Then, the electric field was turned off, and the rate of the fall of the drop was measured. Because of friction with the air molecules, the oil drop quickly reached terminal velocity, which was related to the mass of the drop by a complex equation. Using the measured terminal velocity to calculate $m g$ and knowing $E$, the charge, $q$, could be calculated.

Charge on an electron Millikan found that there was a great deal of variation in the charges of the drops. When he used X rays to ionize the air and add or remove electrons from the drops, he noted, however, that the changes in the charge on the drops were always a multiple of $1.60 \times 10^{-19} \mathrm{C}$. The changes were caused by one or more electrons being added to or removed from the drops. Millikan concluded that the smallest change in charge that could occur was the amount of charge of one electron. Therefore, Millikan proposed that each electron always has the same charge, $1.60 \times 10^{-19} \mathrm{C}$. Millikan's experiment showed that charge is quantized. This means that an object can have only a charge with a magnitude that is some integral multiple of the charge of an electron.


## Electric Fields

Tie a pith ball on the end of a $20-\mathrm{cm}$ nylon thread and tie the other end to a plastic straw. Holding the straw horizontally, notice that the ball hangs straight down. Now rub a piece of wool on a $30 \mathrm{~cm} \times 30 \mathrm{~cm}$ square of plastic foam to charge both objects. Stand the foam vertically. Hold the straw and touch the pith ball to the wool.

1. Predict what will happen when the ball is close to the foam.
2. Test your prediction by slowly bringing the hanging ball toward the charged plastic foam.
3. Predict the ball's behavior at different locations around the foam, and test your prediction.
4. Observe the angle of the thread as you move the pith ball to different regions around the foam.
Analyze and Conclude
5. Explain, in terms of the electric field, why the ball swings toward the charged plastic.
6. Compare the angle of the thread at various points around the foam. Why did it change?
7. Infer what the angle of the thread indicates about the strength and the direction of the electric field.

- Figure 21-9 This illustration shows a cross-sectional view of the apparatus that Millikan used to determine the charge on an electron.


## EXAMPLE Problem 4

Finding the Charge on an Oil Drop In a Millikan oil-drop experiment, a drop has been found to weigh $2.4 \times 10^{-14} \mathrm{~N}$. The parallel plates are separated by a distance of 1.2 cm . When the potential difference between the plates is 450 V , the drop is suspended, motionless.
a. What is the charge on the oil drop?
b. If the upper plate is positive, how many excess electrons are on the oil drop?

1 Analyze and Sketch the Problem

- Draw the plates with the oil drop suspended between them.
- Draw and label vectors representing the forces.
- Indicate the potential difference and the distance between the plates.

Known:

$$
\begin{aligned}
\Delta V & =450 \mathrm{~V} \\
F_{\mathrm{g}} & =2.4 \times 10^{-14} \mathrm{~N} \\
d & =1.2 \mathrm{~cm}
\end{aligned}
$$

## Unknown:

charge on drop, $q=$ ?
number of electrons, $n=$ ?


## 2 Solve for the Unknown

To be suspended, the electric force and gravitational force must be balanced.

$$
\begin{array}{lll}
F_{\mathrm{e}} & =F_{\mathrm{g}} & \\
q E & =F_{\mathrm{g}} & \\
\frac{q \Delta V}{d} & =F_{\mathrm{g}} & \\
\text { Substitute } F_{\mathrm{e}}=q E \\
\text { Substitute } E=\frac{\Delta V}{d}
\end{array}
$$

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Isolating a Variable page 845

Solve for $q$.

$$
\begin{aligned}
q & =\frac{F_{g} d}{\Delta V} \\
& =\frac{\left(2.4 \times 10^{-14} \mathrm{~N}\right)(0.012 \mathrm{~m})}{450 \mathrm{~V}} \quad \text { Substitute } F_{g}=2.4 \times 10^{-14} \mathrm{~N}, \boldsymbol{d}=0.012 \mathrm{~m}, \Delta \mathrm{~V}=450 \mathrm{~V} \\
& =6.4 \times 10^{-19} \mathrm{C}
\end{aligned}
$$

Solve for the number of electrons on the drop.

$$
\begin{aligned}
n & =\frac{q}{e} \\
& =\frac{6.4 \times 10^{-19} \mathrm{C}}{1.6 \times 10^{-19} \mathrm{C}} \\
& =4
\end{aligned}
$$

## 3 Evaluate the Answer

- Are the units correct? $\mathrm{N} \cdot \mathrm{m} / \mathrm{V}=\mathrm{J} /(\mathrm{J} / \mathrm{C})=\mathrm{C}$, the unit for charge.
- Is the magnitude realistic? This is a small whole number of elementary charges.


## PRACTICE Problems

## Additional Problems, Appendix B

26. A drop is falling in a Millikan oil-drop apparatus with no electric field. What forces are acting on the oil drop, regardless of its acceleration? If the drop is falling at a constant velocity, describe the forces acting on it.
27. An oil drop weighs $1.9 \times 10^{-15} \mathrm{~N}$. It is suspended in an electric field of $6.0 \times 10^{3} \mathrm{~N} / \mathrm{C}$. What is the charge on the drop? How many excess electrons does it carry?
28. An oil drop carries one excess electron and weighs $6.4 \times 10^{-15} \mathrm{~N}$. What electric field strength is required to suspend the drop so it is motionless?
29. A positively charged oil drop weighing $1.2 \times 10^{-14} \mathrm{~N}$ is suspended between parallel plates separated by 0.64 cm . The potential difference between the plates is 240 V . What is the charge on the drop? How many electrons is the drop missing?

## Sharing of Charge

All systems come to equilibrium when the energy of the system is at a minimum. For example, if a ball is placed on a hill, it finally will come to rest in a valley where its gravitational potential energy is smallest. This also would be the location where its gravitational potential has been reduced by the largest amount. This same principle explains what happens when an insulated, positively charged metal sphere, such as the one shown in Figure 21-10, touches a second, uncharged sphere.

The excess charges on sphere A repel each other, so when the neutral sphere, $B$, touches sphere A, there is a net force on the charges on A toward B. Suppose that you were to physically move the charges, individually, from A to B. When you move the first charge, the other charges on A would push it toward B, so, to control its speed, you would have to exert a force in the opposite direction. Therefore, you do negative work on it, and the electric potential difference from A to B is negative. When the next few charges are moved, they feel a small repulsive force from the charges already on B , but there is still a net positive force in that direction. At some point, the force pushing a charge off A will equal the repulsive force from the charges on $B$, and the electric potential difference is zero. After this point of equilibrium, work would have to be done to move the next charge to B, so this would not happen by itself and would require an increase in the energy of the system. However, if you did continue to move charges, the electric potential difference from A to B would then be positive. Thus, you can see that charges would move from A to B without external forces until there is no electric potential difference between the two spheres.

Different sizes of spheres Suppose that the two spheres have different sizes, as in Figure 21-11. Although the total numbers of charges on the two spheres are the same, the larger sphere has a larger surface area, so the charges can spread farther apart, and the repulsive force between them is reduced. Thus, if the two spheres now are touched together, there will be a net force that will move charges from the smaller to the larger sphere. Again, the charges will move to the sphere with the lower electric potential until there is no electric potential difference between the two spheres. In this case, the larger sphere will have a larger charge when equilibrium is reached.

## Metal Spheres of Unequal Size





Figure 21-10 A charged sphere shares charge equally with a neutral sphere of equal size when they are placed in contact with each other.

- Figure 21-11 Charges are transferred from a sphere with high potential to a sphere with lower potential when they touch. The charges move to create no potential difference.
- Figure 21-12 The ground wire on a fuel truck prevents ignition of the gasoline vapors.


A


B


- Figure 21-13 On a conducting sphere, ( $\mathbf{a}$ ), the charge is evenly distributed around the surface. The charges on the hollow sphere, (b), are entirely on the outer surface. In irregular shapes, (c), the charges will be closest together at sharp points.


The same principle explains how charges move on the individual spheres, or on any conductor. They distribute themselves so that the net force on each charge is zero. With no force, there is no electric field along the surface of the conductor. Thus, there is no electric potential difference anywhere on the surface. The surface of a conductor is, therefore, an equipotential surface.

If a charged body is grounded by touching Earth, almost any amount of charge can flow to Earth until the electric potential difference between that body and Earth is reduced to zero. Gasoline trucks, for example, can become charged by friction. If the charge on a gasoline truck were to jump to Earth through gasoline vapor, it could cause an explosion. To prevent this, a metal wire on the truck safely conducts the charge to the ground, as shown in Figure 21-12. Similarly, if a computer is not grounded, an electric potential difference between the computer and Earth can occur. If a person then touches the computer, charges could flow through the computer to the person and damage the equipment or hurt the person.

## Electric Fields Near Conductors

The charges on a conductor are spread as far apart as they can be to make the energy of the system as low as possible. The result is that all charges are on the surface of a solid conductor. If the conductor is hollow, excess charges will move to the outer surface. If a closed metal container is charged, there will be no charges on the inside surfaces of the container. In this way, a closed metal container shields the inside from electric fields. For example, people inside a car are protected from the electric fields generated by lightning. Likewise, on an open coffee can, there will be very few charges inside and none near the bottom. Even if the inner surface of an object is pitted or bumpy, giving it a larger surface area than the outer surface, the charge still will be entirely on the outside.

On the outside of a conductor, however, the electric field often is not zero. Even though the surface of a conductor is at an equipotential, the electric field around the outside of it depends on the shape of the conductor, as well as on the electric potential difference between it and Earth. The charges are closer together at sharp points of a conductor, as indicated in Figure 21-13. Therefore, the field lines are closer together and the field is stronger. This field can become so strong that when electrons are knocked off of atoms by passing cosmic rays, the electrons and resulting ions are accelerated by the field, causing them to strike other atoms, resulting in more ionization of atoms. This chain reaction is what results in the pink glow,
such as that seen inside a gas-discharge sphere. If the field is strong enough, when the particles hit other molecules they will produce a stream of ions and electrons that form a plasma, which is a conductor. The result is a spark, or, in extreme cases, lightning. To reduce discharges and sparking, conductors that are highly charged or that operate at high potentials are made smooth in shape to reduce the electric fields.

In contrast, a lightning rod is pointed so that the electric field will be strong near the end of the rod. As the field accelerates electrons and ions, they form the start of a conducting path from the rod to the clouds. As a result of the rod's sharply pointed shape, charges in the clouds spark to the rod, rather than to a chimney or other high point on a house or other building. From the rod, a conductor takes the charges safely to the ground.

Lightning usually requires a potential difference of millions of volts between Earth and the clouds. Even a small gas-discharge tube operates at several thousand volts. Household wiring, on the other hand, does not normally carry a high enough potential difference to cause such discharges.

## Storing Charges: The Capacitor

When you lift a book, you increase its gravitational potential energy. This can be interpreted as storing energy in a gravitational field. In a similar way, you can store energy in an electric field. In 1746, Dutch physician and physicist Pieter Van Musschenbroek invented a small device that could store a large electric charge. In honor of the city in which he worked, it was called a Leyden jar. Benjamin Franklin used a Leyden jar to store the charge from lightning and in many other experiments. A version of the Leyden jar is still in use today in electric equipment. This new device for storing a charge has a new form, is much smaller in size, and is called a capacitor.

As charge is added to an object, the electric potential difference between that object and Earth increases. For a given shape and size of an object, the ratio of charge stored to electric potential difference, $q / \Delta V$, is a constant called the capacitance, $C$. For a small sphere far from the ground, even a small amount of added charge will increase the electric potential difference. Thus, $C$ is small. A larger sphere can hold more charge for the same increase in electric potential difference, and its capacitance is larger.

Capacitors are designed to have specific capacitances. All capacitors are made up of two conductors that are separated by an insulator. The two conductors have equal and opposite charges. Capacitors are used today in electric circuits to store charge. Commercial capacitors, such as those shown in Figure 21-14, typically contain strips of aluminum foil separated by thin plastic that are tightly rolled up to conserve space.

The capacitance of a capacitor is independent of the charge on it, and can be measured by first placing charge $q$ on one plate and charge $-q$ on the other, and then measuring the electric potential difference, $\Delta V$, that results. The capacitance is found by using the following equation, and is measured in farads, F.

## Capacitance $C=\frac{q}{\Delta V}$

Capacitance is the ratio of charge on one plate to potential difference.

- Figure 21-14 Various types of capacitors are pictured below.


The farad as a unit of measure One farad, F, named after Michael Faraday, is one coulomb per volt, $\mathrm{C} / \mathrm{V}$. Just as 1 C is a large amount of charge, 1 F is also a fairly large capacitance. Most capacitors used in modern electronics have capacitances between 10 picofarads ( $10 \times 10^{-12} \mathrm{~F}$ ) and 500 microfarads ( $500 \times 10^{-6}$ F). However, memory capacitors that are used to prevent loss of memory in some computers can have capacitance from 0.5 F to 1.0 F . Note that if the charge is increased, the electric potential difference also increases. The capacitance depends only on the construction of the capacitor, not on the charge, $q$.

## EXAMPLE Problem 5

Finding Capacitance A sphere has an electric potential difference between it and Earth of 40.0 V when it has been charged to $2.4 \times 10^{-6} \mathrm{C}$. What is its capacitance?

1 Analyze and Sketch the Problem

- Draw a sphere above Earth and label the charge and potential difference.


$$
\begin{array}{lll}
\text { Known: } & & \text { Unknown: } \\
\begin{aligned}
\Delta V & =40.0 \mathrm{~V}
\end{aligned} & C=? \\
q=2.4 \times 10^{-6} \mathrm{C} & &
\end{array}
$$

2 Solve for the Unknown

$$
\begin{aligned}
C & =\frac{q}{\Delta V} \\
& =\frac{2.4 \times 10^{-6} \mathrm{C}}{40.0 \mathrm{~V}} \quad \text { Substitute } \Delta V=40.0 \mathrm{~V}, q=2.4 \times 10^{-6} \mathrm{C} \\
& =6.0 \times 10^{-8} \mathrm{~F} \\
& =0.060 \mu \mathrm{~F}
\end{aligned}
$$

Math Handbook
Operations with Significant Digits pages 835-836

## 3 Evaluate the Answer

- Are the units correct? $C / V=F$. The units are farads.
- Is the magnitude realistic? A small capacitance would store a small charge at a low voltage.


## PRACTICE Problems

## Additional Problems, Appendix B

30. A $27-\mu \mathrm{F}$ capacitor has an electric potential difference of 45 V across it. What is the charge on the capacitor?
31. Both a $3.3-\mu \mathrm{F}$ and a $6.8-\mu \mathrm{F}$ capacitor are connected across a $24-\mathrm{V}$ electric potential difference. Which capacitor has a greater charge? What is it?
32. The same two capacitors as in problem 31 are each charged to $3.5 \times 10^{-4} \mathrm{C}$. Which has the larger electric potential difference across it? What is it?
33. A $2.2-\mu \mathrm{F}$ capacitor first is charged so that the electric potential difference is 6.0 V . How much additional charge is needed to increase the electric potential difference to 15.0 V ?
34. When a charge of $2.5 \times 10^{-5} \mathrm{C}$ is added to a capacitor, the potential difference increases from 12.0 V to 14.5 V . What is the capacitance of the capacitor?

## CHALLENGE PROBLEM

The plates of a capacitor attract each other because they carry opposite charges. A capacitor consisting of two parallel plates that are separated by a distance, $d$, has capacitance, $C$.

1. Derive an expression for the force between the two plates when the capacitor has charge, $q$.
2. What charge must be stored on a $22-\mu \mathrm{F}$ capacitor to have a force of 2.0 N between the plates if they are separated by 1.5 mm ?

Varieties of capacitors Capacitors have many shapes and sizes, as shown in Figure 21-14. Some are large enough to fill whole rooms and can store enough charge to create artificial lightning or power giant lasers that release thousands of joules of energy in a few billionths of a second. Capacitors in television sets can store enough charge at several hundred volts to be very dangerous if they are touched. These capacitors can remain charged for hours after the televisions have been turned off. This is why you should not open the case of a television or a computer monitor even if it is unplugged.

The capacitance of a capacitor is controlled by varying the surface area of the two conductors, or plates, within a capacitor, by the distance between the plates, and by the nature of the insulating material. Capacitors are named for the type of insulator, or dielectric, used to separate the plates, and include ceramic, mica, polyester, paper, and air. Higher capacitance is obtained by increasing the surface area and decreasing the separation of the plates. Certain dielectrics have the ability to effectively offset some of the charge on the plates and allow more charge to be stored.

### 21.2 Section Review

35. Potential Difference What is the difference between electric potential energy and electric potential difference?
36. Electric Field and Potential Difference Show that a volt per meter is the same as a newton per coulomb.
37. Millikan Experiment When the charge on an oil drop suspended in a Millikan apparatus is changed, the drop begins to fall. How should the potential difference on the plates be changed to bring the drop back into balance?
38. Charge and Potential Difference In problem 37, if changing the potential difference has no effect on the falling drop, what does this tell you about the new charge on the drop?
39. Capacitance How much charge is stored on a $0.47-\mu \mathrm{F}$ capacitor when a potential difference of 12 V is applied to it?
40. Charge Sharing If a large, positively charged, conducting sphere is touched by a small, negatively charged, conducting sphere, what can be said about the following?
a. the potentials of the two spheres
b. the charges on the two spheres
41. Critical Thinking Referring back to Figure 21-3a, explain how charge continues to build up on the metal dome of a Van de Graaff generator. In particular, why isn't charge repelled back onto the belt at point $B$ ?

## Charging of Capacitors

A capacitor is an electric device that is made from two conductors, or plates, that are separated by an insulator. It is designed to have a specific capacitance. The capacitance depends on the physical characteristics and geometric arrangement of the conductors and the insulator. In the circuit schematic, the capacitor appears to create an open circuit, even when the switch is in the closed position. However, because capacitors store charge, when the switch is closed, charge from the battery will move to the capacitor. The equal, but opposite charges on the two plates within the capacitor establishes a potential difference, or voltage. As charge is added to the capacitor, the electric potential difference increases. In this laboratory activity you will examine the charging of several different capacitors.

## QUESTION

## How do the charging times of different capacitors vary with capacitance?

## Objectives

Collect and organize data on the rate of charge of different capacitors.
$\square$ Compare and contrast the rate of charging for different capacitances.
Make and use graphs of potential difference versus time for several capacitors.

## Safety Precautions

## NㅡㄴN

## Materials

9-V battery voltmeter 9-V battery clip hook-up wires switch

47-k $\Omega$ resistor stopwatch capacitors: $1000 \mu \mathrm{~F}$, $500 \mu \mathrm{~F}, 240 \mu \mathrm{~F}$

## Procedure

1. Before you begin, leave the switch open (off). Do not attach the battery at this time.
CAUTION: Be careful to avoid a short circuit, especially by permitting the leads from the battery clip to touch each other. Connect the circuit, as illustrated. Do this by connecting either end of the resistor to one side of the switch. The resistor is used to reduce the charging of the capacitor to a measurable rate. Connect the other end of the resistor to the negative side of the 9-V battery clip. Inspect your $1000-\mu \mathrm{F}$ capacitor to determine whether either end is marked with a negative sign, or an arrow with negative signs on it, that points to the lead that is to be connected to the negative side of the battery. Connect this negative lead to the other side of the switch. Attach the unconnected (positive) lead of the capacitor to the positive lead from the battery clip.
2. Connect the positive terminal of the voltmeter to the positive side of the capacitor and the negative terminal to the negative side of the capacitor. Compare your circuit to the photo to verify your connections. Attach the battery after your teacher has inspected the circuit.
3. Prepare a data table having columns for time and potential difference on each of the three different capacitors.
4. One person should watch the time and another should record potential difference at the designated times. Close the switch and measure the voltage at $5-\mathrm{s}$ intervals. Open the switch after you have collected data.


## Data Table

| Time (s) | Voltage (V) <br> across $1000 \mu \mathrm{~F}$ | Voltage (V) <br> across $500 \mu \mathrm{~F}$ | Voltage (V) <br> across $240 \mu \mathrm{~F}$ | Time (s) | Voltage (V) <br> across $1000 \mu \mathrm{~F}$ | Voltage (V) <br> across $500 \mu \mathrm{~F}$ | Voltage (V) <br> across $240 \mu \mathrm{~F}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 |  |  |  | 55 |  |  |  |
| 5 |  |  |  | 60 |  |  |  |
| 10 |  |  |  | 65 |  |  |  |
| 15 |  |  |  | 70 |  |  |  |
| 20 |  |  |  | 80 |  |  |  |
| 25 |  |  |  | 85 |  |  |  |
| 30 |  |  |  | 95 |  |  |  |
| 35 |  |  |  | 100 |  |  |  |
| 40 |  |  |  | 105 |  |  |  |
| 45 |  |  |  |  |  |  |  |
| 50 |  |  |  |  |  |  |  |

## Going Further

5. When you have completed the trial, take a short piece of wire and place it across both ends of the capacitor. This will cause the capacitor to discharge.
6. Replace the $1000-\mu \mathrm{F}$ capacitor with a $500-\mu \mathrm{F}$ capacitor. Repeat steps 4-5 and enter data into the appropriate columns of your data table for the $500-\mu \mathrm{F}$ capacitor.
7. Replace the $500-\mu \mathrm{F}$ capacitor with a $240-\mu \mathrm{F}$ capacitor. Repeat steps 4-5 and enter data into the appropriate column of your data table for this last capacitor.

## Analyze

1. Observe and Infer Does each capacitor charge to 9 V ? Propose an explanation for the observed behavior.
2. Make and Use Graphs Prepare a graph that plots the time horizontally and the potential difference vertically. Make a separate labeled line for each capacitor.

## Conclude and Apply

1. Interpret Data Does the voltage on the capacitor immediately jump to the battery's potential difference (9-V)? Explain the reason for the observed behavior.
2. Infer Does the larger capacitor require a longer time to become fully charged? Explain why or why not.
3. The time for a capacitor to charge to the voltage of the battery depends upon its capacitance and the opposition to the flow of charge in the circuit. In this lab, the opposition to the flow of charge was controlled by the 47-k $\Omega$ resistor that was placed in the circuit. In circuits with a capacitor and resistance, such as in this activity, the time in seconds to charge the capacitor to 63.3 percent of the applied voltage is equal to the product of the capacitor and resistance. This is called the time constant. Therefore, $T=R C$, where $T$ is in seconds, $R$ is in ohms, and $C$ is in microfarads. Calculate the time constant for each of the capacitors with the $47-\mathrm{k} \Omega$ resistor.
4. Compare your time constants to the values from your graph.

## Real-World Physics

Explain Small, disposable, flash cameras, as well as regular electronic flash units, require time before the flash is ready to be used. A capacitor stores the energy for the flash. Explain what might be going on during the time you must wait to take your next picture.

## Physics $\quad$ nline

To find out more about electric fields, visit the Web site: physicspp.com

## How it WOrks <br> Lightning Rods

Lightning can be very destructive because it creates huge currents in materials that are poor conductors and generates a great deal of heat. In addition to protecting a structure by dissipating some of the charge before lightning strikes, lightning rods are excellent conductors that provide a safe path for the current. Benjamin Franklin is credited with inventing the lightning rod in the 1750s.

3 Positive charges spark out from the lightning rod, meeting the step leader. The conducting path is complete and current neutralizes the separation of charges. Even if the strike does not hit the lightning rod directly, the massive current still can leap to the rod, which is the path of least resistance to the ground.


### 21.1 Creating and Measuring Electric Fields

## Vocabulary

- electric field (p. 563)
- electric field line (p. 567)


## Key Concepts

- An electric field exists around any charged object. The field produces forces on other charged objects.
- The electric field is the force per unit charge.

$$
\boldsymbol{E}=\frac{\boldsymbol{F}}{q^{\prime}}
$$

- The direction of the electric field is the direction of the force on a tiny, positive test charge.
- Electric field lines provide a picture of the electric field. They are directed away from positive charges and toward negative charges. They never cross, and their density is related to the strength of the field.


### 21.2 Applications of Electric Fields

## Vocabulary

- electric potential difference (p. 569)
- volt (p. 569)
- equipotential (p. 570)
- capacitor (p. 577)
- capacitance (p. 577)


## Key Concepts

- Electric potential difference is the change in potential energy per unit charge in an electric field.

$$
\Delta V=\frac{W}{q^{\prime}}
$$

- Electric potential differences are measured in volts.
- The electric field between two parallel plates is uniform between the plates, except near the edges. In a uniform field, the potential difference is related to the field strength by the following.

$$
\Delta V=E d
$$

- Robert Millikan's experiments showed that electric charge is quantized.
- Robert Millikan also showed that the negative charge carried by an electron is $1.60 \times 10^{-19} \mathrm{C}$.
- Charges will move in a conductor until the electric potential is the same everywhere on the conductor.
- Grounding makes the potential difference between an object and Earth equal to zero.
- Grounding can prevent sparks resulting from a neutral object making contact with objects that have built-up charge on them.
- Electric fields are strongest near sharply pointed conductors.
- Capacitance is the ratio of the charge on an object to its electric potential difference.

$$
C=\frac{q}{\Delta V}
$$

- Capacitance is independent of the charge on an object and the electric potential difference across it.
- Capacitors are used to store charge.


## Concept Mapping

42. Complete the concept map below using the following terms: capacitance, field strength, J/C, work.


## Mastering Concepts

43. What are the two properties that a test charge must have? (21.1)
44. How is the direction of an electric field defined? (21.1)
45. What are electric field lines? (21.1)
46. How is the strength of an electric field indicated with electric field lines? (21.1)
47. Draw some of the electric field lines between each of the following. (21.1)
a. two like charges of equal magnitude
b. two unlike charges of equal magnitude
c. a positive charge and a negative charge having twice the magnitude of the positive charge
d. two oppositely charged parallel plates
48. In Figure 21-15, where do the electric field lines leave the positive charge end? (21.1)


Figure 21-15
49. What SI unit is used to measure electric potential energy? What SI unit is used to measure electric potential difference? (21.2)
50. Define volt in terms of the change in potential energy of a charge moving in an electric field. (21.2)
51. Why does a charged object lose its charge when it is touched to the ground? (21.2)
52. A charged rubber rod that is placed on a table maintains its charge for some time. Why is the charged rod not discharged immediately? (21.2)
53. A metal box is charged. Compare the concentration of charge at the corners of the box to the charge concentration on the sides of the box. (21.2)
54. Computers Delicate parts in electronic equipment, such as those pictured in Figure 21-16, are contained within a metal box inside a plastic case. Why? (21.2)


Figure 21-16

## Applying Concepts

55. What happens to the strength of an electric field when the charge on the test charge is halved?
56. Does it require more energy or less energy to move a constant positive charge through an increasing electric field?
57. What will happen to the electric potential energy of a charged particle in an electric field when the particle is released and free to move?
58. Figure 21-17 shows three spheres with charges of equal magnitude, with their signs as shown. Spheres $y$ and $z$ are held in place, but sphere $x$ is free to move. Initially, sphere $x$ is equidistant from spheres $y$ and $z$.
 Choose the path that sphere $x$ will begin to follow. Assume that no
 other forces are acting on the spheres.
59. What is the unit of electric potential difference in terms of $\mathrm{m}, \mathrm{kg}$, s , and C ?
60. What do the electric field lines look like when the electric field has the same strength at all points in a region?
61. Millikan Oil-Drop Experiment When doing a Millikan oil-drop experiment, it is best to work with drops that have small charges. Therefore, when the electric field is turned on, should you try to find drops that are moving rapidly or slowly? Explain.
62. Two oil drops are held motionless in a Millikan oil-drop experiment.
a. Can you be sure that the charges are the same?
b. The ratios of which two properties of the oil drops have to be equal?
63. José and Sue are standing on an insulating platform and holding hands when they are given a charge, as in Figure 21-18. José is larger than Sue. Who has the larger amount of charge, or do they both have the same amount?


Figure 21-18
64. Which has a larger capacitance, an aluminum sphere with a $1-\mathrm{cm}$ diameter or one with a $10-\mathrm{cm}$ diameter?
65. How can you store different amounts of charge in a capacitor?

## Mastering Problems

### 21.1 Creating and Measuring Electric Fields

The charge of an electron is $-1.60 \times 10^{-19} \mathrm{C}$.
66. What charge exists on a test charge that experiences a force of $1.4 \times 10^{-8} \mathrm{~N}$ at a point where the electric field intensity is $5.0 \times 10^{-4} \mathrm{~N} / \mathrm{C}$ ?
67. A positive charge of $1.0 \times 10^{-5} \mathrm{C}$, shown in

Figure 21-19, experiences a force of 0.30 N when it is located at a certain point. What is the electric field intensity at that point?


Figure 21-19
68. A test charge experiences a force of 0.30 N on it when it is placed in an electric field intensity of $4.5 \times 10^{5} \mathrm{~N} / \mathrm{C}$. What is the magnitude of the charge?
69. The electric field in the atmosphere is about $150 \mathrm{~N} / \mathrm{C}$ downward.
a. What is the direction of the force on a negatively charged particle?
b. Find the electric force on an electron with charge $-1.6 \times 10^{-19} \mathrm{C}$.
c. Compare the force in part $\mathbf{b}$ with the force of gravity on the same electron (mass $\left.=9.1 \times 10^{-31} \mathrm{~kg}\right)$.
70. Carefully sketch each of the following.
a. the electric field produced by a $+1.0-\mu \mathrm{C}$ charge
b. the electric field resulting from $\mathrm{a}+2.0-\mu \mathrm{C}$ charge (Make the number of field lines proportional to the change in charge.)
71. A positive test charge of $6.0 \times 10^{-6} \mathrm{C}$ is placed in an electric field of 50.0-N/C intensity, as in Figure 21-20. What is the strength of the force exerted on the test charge?

72. Charges $\mathrm{X}, \mathrm{Y}$, and Z all are equidistant from each other. X has a $+1.0-\mu \mathrm{C}$ charge, Y has a $+2.0-\mu \mathrm{C}$ charge, and $Z$ has a small negative charge.
a. Draw an arrow representing the force on charge Z .
b. Charge Z now has a small positive charge on it. Draw an arrow representing the force on it.

## Chapter 21 Assessment

73. In a television picture tube, electrons are accelerated by an electric field having a value of $1.00 \times 10^{5} \mathrm{~N} / \mathrm{C}$.
a. Find the force on an electron.
b. If the field is constant, find the acceleration of the electron (mass $=9.11 \times 10^{-31} \mathrm{~kg}$ ).
74. What is the electric field strength 20.0 cm from a point charge of $8.0 \times 10^{-7} \mathrm{C}$ ?
75. The nucleus of a lead atom has a charge of 82 protons.
a. What are the direction and magnitude of the electric field at $1.0 \times 10^{-10} \mathrm{~m}$ from the nucleus?
b. What are the direction and magnitude of the force exerted on an electron located at this distance?

### 21.2 Applications of Electric Fields

76. If 120 J of work is performed to move 2.4 C of charge from the positive plate to the negative plate shown in Figure 21-21, what potential difference exists between the plates?


Figure 21-21
77. How much work is done to transfer 0.15 C of charge through an electric potential difference of 9.0 V ?
78. An electron is moved through an electric potential difference of 450 V . How much work is done on the electron?
79. A $12-\mathrm{V}$ battery does 1200 J of work transferring charge. How much charge is transferred?
80. The electric field intensity between two charged plates is $1.5 \times 10^{3} \mathrm{~N} / \mathrm{C}$. The plates are 0.060 m apart. What is the electric potential difference, in volts, between the plates?
81. A voltmeter indicates that the electric potential difference between two plates is 70.0 V . The plates are 0.020 m apart. What electric field intensity exists between them?
82. A capacitor that is connected to a $45.0-\mathrm{V}$ source contains $90.0 \mu \mathrm{C}$ of charge. What is the capacitor's capacitance?
83. What electric potential difference exists across a $5.4-\mu \mathrm{F}$ capacitor that has a charge of $8.1 \times 10^{-4} \mathrm{C}$ ?
84. The oil drop shown in Figure 21-22 is negatively charged and weighs $4.5 \times 10^{-15} \mathrm{~N}$. The drop is suspended in an electric field intensity of $5.6 \times 10^{3} \mathrm{~N} / \mathrm{C}$.
a. What is the charge on the drop?
b. How many excess electrons does it carry?


- Figure 21-22

85. What is the charge on a $15.0-\mathrm{pF}$ capacitor when it is connected across a $45.0-\mathrm{V}$ source?
86. A force of 0.065 N is required to move a charge of $37 \mu \mathrm{C}$ a distance of 25 cm in a uniform electric field, as in Figure 21-23. What is the size of the electric potential difference between the two points?

87. Photoflash The energy stored in a capacitor with capacitance $C$, and an electric potential difference, $\Delta V$, is represented by $W=\frac{1}{2} C \Delta V^{2}$. One application of this is in the electronic photoflash of a strobe light, like the one in Figure 21-24. In such a unit, a capacitor of $10.0 \mu \mathrm{~F}$ is charged to $3.0 \times 10^{2} \mathrm{~V}$. Find the energy stored.


- Figure 21-24

88. Suppose it took 25 s to charge the capacitor in problem 87.
a. Find the average power required to charge the capacitor in this time.
b. When this capacitor is discharged through the strobe lamp, it transfers all its energy in $1.0 \times 10^{-4} \mathrm{~s}$. Find the power delivered to the lamp.
c. How is such a large amount of power possible?
89. Lasers Lasers are used to try to produce controlled fusion reactions. These lasers require brief pulses of energy that are stored in large rooms filled with capacitors. One such room has a capacitance of $61 \times 10^{-3} \mathrm{~F}$ charged to a potential difference of 10.0 kV .
a. Given that $W=\frac{1}{2} C \Delta V^{2}$, find the energy stored in the capacitors.
b. The capacitors are discharged in 10 ns $\left(1.0 \times 10^{-8} \mathrm{~s}\right)$. What power is produced?
c. If the capacitors are charged by a generator with a power capacity of 1.0 kW , how many seconds will be required to charge the capacitors?

## Mixed Review

90. How much work does it take to move $0.25 \mu \mathrm{C}$ between two parallel plates that are 0.40 cm apart if the field between the plates is $6400 \mathrm{~N} / \mathrm{C}$ ?
91. How much charge is stored on a $0.22-\mu \mathrm{F}$ parallel plate capacitor if the plates are 1.2 cm apart and the electric field between them is $2400 \mathrm{~N} / \mathrm{C}$ ?
92. Two identical small spheres, 25 cm apart, carry equal but opposite charges of $0.060 \mu \mathrm{C}$, as in Figure 21-25. If the potential difference between them is 300 V , what is the capacitance of the system?


- Figure 21-25

93. The plates of a $0.047 \mu \mathrm{~F}$ capacitor are 0.25 cm apart and are charged to a potential difference of 120 V . How much charge is stored on the capacitor?
94. What is the strength of the electric field between the plates of the capacitor in Problem 93 above?
95. An electron is placed between the plates of the capacitor in Problem 93 above, as in Figure 21-26. What force is exerted on that electron?


- Figure 21-26

96. How much work would it take to move an additional $0.010 \mu \mathrm{C}$ between the plates at 120 V in Problem 93?
97. The graph in Figure 21-27 represents the charge stored in a capacitor as the charging potential increases. What does the slope of the line represent?

98. What is the capacitance of the capacitor represented by Figure 21-27?
99. What does the area under the graph line in Figure 21-27 represent?
100. How much work is required to charge the capacitor in problem 98 to a potential difference of 25 V ?
101. The work found in Problem 100 above is not equal to $q \Delta \mathrm{~V}$. Why not?
102. Graph the electric field strength near a positive point charge as a function of distance from it.
103. Where is the field of a point charge equal to zero?
104. What is the electric field strength at a distance of zero meters from a point charge? Is there such a thing as a true point charge?

## Thinking Critically

105. Apply Concepts Although a lightning rod is designed to carry charge safely to the ground, its primary purpose is to prevent lightning from striking in the first place. How does it do that?
106. Analyze and Conclude In an early set of experiments in 1911, Millikan observed that the following measured charges could appear on a single oil drop. What value of elementary charge can be deduced from these data?
a. $6.563 \times 10^{-19} \mathrm{C}$
b. $8.204 \times 10^{-19} \mathrm{C}$
c. $11.50 \times 10^{-19} \mathrm{C}$
d. $13.13 \times 10^{-19} \mathrm{C}$
e. $16.48 \times 10^{-19} \mathrm{C}$
f. $18.08 \times 10^{-19} \mathrm{C}$
g. $19.71 \times 10^{-19} \mathrm{C}$
h. $22.89 \times 10^{-19} \mathrm{C}$
i. $26.13 \times 10^{-19} \mathrm{C}$

## Chapter 21 Assessment

107. Analyze and Conclude Two small spheres, A and B, lie on the $x$-axis, as in Figure 21-28. Sphere A has a charge of $+3.00 \times 10^{-6} \mathrm{C}$. Sphere B is 0.800 m to the right of sphere A and has a charge of $-5.00 \times 10^{-6} \mathrm{C}$. Find the magnitude and direction of the electric field strength at a point above the $x$-axis that would form the apex of an equilateral triangle with spheres A and B.


Figure 21-28
108. Analyze and Conclude In an ink-jet printer, drops of ink are given a certain amount of charge before they move between two large, parallel plates. The purpose of the plates is to deflect the charges so that they are stopped by a gutter and do not reach the paper. This is shown in Figure 21-29. The plates are $1.5-\mathrm{cm}$ long and have an electric field of $E=1.2 \times 10^{6} \mathrm{~N} / \mathrm{C}$ between them. Drops with a mass $m=0.10 \mathrm{ng}$, and a charge $q=1.0 \times 10^{-16} \mathrm{C}$, are moving horizontally at a speed, $v=15 \mathrm{~m} / \mathrm{s}$, parallel to the plates. What is the vertical displacement of the drops when they leave the plates? To answer this question, complete the following steps.
a. What is the vertical force on the drops?
b. What is their vertical acceleration?
c. How long are they between the plates?
d. How far are they displaced?


Figure 21-29
109. Apply Concepts Suppose the Moon had a net negative charge equal to $-q$, and Earth had a net positive charge equal to $+10 q$. What value of $q$ would yield the same magnitude of force that you now attribute to gravity?

## Writing in Physics

110. Choose the name of an electric unit, such as coulomb, volt, or farad, and research the life and work of the scientist for whom it was named. Write a brief essay on this person and include a discussion of the work that justified the honor of having a unit named for him.

## Cumulative Review

111. Michelson measured the speed of light by sending a beam of light to a mirror on a mountain 35 km away. (Chapter 16)
a. How long does it take light to travel the distance to the mountain and back?
b. Assume that Michelson used a rotating octagon with a mirror on each face of the octagon. Also assume that the light reflects from one mirror, travels to the other mountain, reflects off of a fixed mirror on that mountain, and returns to the rotating mirrors. If the rotating mirror has advanced so that when the light returns, it reflects off of the next mirror in the rotation, how fast is the mirror rotating?
c. If each mirror has a mass of $1.0 \times 10^{1} \mathrm{~g}$ and rotates in a circle with an average radius of $1.0 \times 10^{1} \mathrm{~cm}$, what is the approximate centripetal force needed to hold the mirror while is it rotating?
112. Mountain Scene You can see an image of a distant mountain in a smooth lake just as you can see a mountain biker next to the lake because light from each strikes the surface of the lake at about the same angle of incidence and is reflected to your eyes. If the lake is about 100 m in diameter, the reflection of the top of the mountain is about in the middle of the lake, the mountain is about 50 km away from the lake, and you are about 2 m tall, then approximately how high above the lake does the top of the mountain reach? (Chapter 17)
113. A converging lens has a focal length of 38.0 cm . If it is placed 60.0 cm from an object, at what distance from the lens will the image be? (Chapter 18)
114. A force, $F$, is measured between two charges, $Q$ and $q$, separated by a distance, $r$. What would the new force be for each of the following? (Chapter 20)
a. $r$ is tripled
b. $Q$ is tripled
c. both $r$ and $Q$ are tripled
d. both $r$ and $Q$ are doubled
e. all three, $r, Q$, and $q$, are tripled

## Standardized Test Practice

## Multiple Choice

1. Why is an electric field measured only by a small test charge?
(A) so the charge doesn't disturb the field
(B) because small charges have small momentum
(C) so its size doesn't nudge the charge to be measured aside
(D) because an electron always is used as the test charge and electrons are small
2. A force of 14 N exists on charge $q$, which is $2.1 \times 10^{-9} \mathrm{C}$. What is the magnitude of the electric field?
$\begin{array}{lll}\text { (A) } & 0.15 \times 10^{-9} \mathrm{~N} / \mathrm{C} & \text { © } 29 \times 10^{-9} \mathrm{~N} / \mathrm{C} \\ \text { (B) } & 6.7 \times 10^{-9} \mathrm{~N} / \mathrm{C} & \text { (D) } 6.7 \times 10^{9} \mathrm{~N} / \mathrm{C}\end{array}$
3. A positive test charge of $8.7 \mu \mathrm{C}$ experiences a force of $8.1 \times 10^{-6} \mathrm{~N}$ at an angle of $24^{\circ} \mathrm{N}$ of E . What are the magnitude and direction of the electric field strength at the location of the test charge?
(A) $7.0 \times 10^{-8} \mathrm{~N} / \mathrm{C}, 24^{\circ} \mathrm{N}$ of E
(B) $1.7 \times 10^{-6} \mathrm{~N} / \mathrm{C}, 24^{\circ} \mathrm{S}$ of W
(C) $1.1 \times 10^{-3} \mathrm{~N} / \mathrm{C}, 24^{\circ} \mathrm{W}$ of S
(D) $9.3 \times 10^{-1} \mathrm{~N} / \mathrm{C}, 24^{\circ} \mathrm{N}$ of E
4. What is the potential difference between two plates that are 18 cm apart with a field of $4.8 \times 10^{3} \mathrm{~N} / \mathrm{C}$ ?
(A) 27 V
(C) 0.86 kV
(B) 86 V
(D) 27 kV
5. How much work is done on a proton to move it from the negative plate to a positive plate 4.3 cm away if the field is $125 \mathrm{~N} / \mathrm{C}$ ?
(A)
$5.5 \times 10^{-23} \mathrm{~J}$
(C) $1.1 \times 10^{-16} \mathrm{~J}$
(B) $8.6 \times 10^{-19} \mathrm{~J}$
(D) 5.4 J

6. How was the magnitude of the field in Millikan's oil-drop experiment determined?
(A) using a measurable electromagnet
(B) from the electric potential between the plates
(C) from the magnitude of the charge
(D) by an electrometer
7. In an oil drop experiment, a drop with a weight of $1.9 \times 10^{-14} \mathrm{~N}$ was suspended motionless when the potential difference between the plates that were 63 mm apart was 0.78 kV . What was the charge on the drop?

$$
\begin{array}{ll}
\text { (A) } & -1.5 \times 10^{-18} \mathrm{C} \\
\text { (B) } & -3.9 \times 10^{-16} \mathrm{C} \\
\text { (D) } & -9.2 \times 10^{-15} \mathrm{C} \\
\hline 10^{-13} \mathrm{C}
\end{array}
$$


8. A capacitor has a capacitance of $0.093 \mu \mathrm{~F}$. If the charge on the capacitor is $58 \mu \mathrm{C}$, what is the electrical potential difference?
(A) $5.4 \times 10^{-12} \mathrm{~V}$
(C) $6.2 \times 10^{2} \mathrm{~V}$
(B) $1.6 \times 10^{-6} \mathrm{~V}$
(D) $5.4 \times 10^{3} \mathrm{~V}$

## Extended Answer

9. Assume 18 extra electrons are on an oil drop. Calculate the charge of the oil drop, and calculate the potential difference needed to suspend it if it has a weight of $6.12 \times 10^{-14} \mathrm{~N}$ and the plates are 14.1 mm apart.

## Test-Taking TIP

## Use the Buddy System

Study in a group. A small study group works well because it allows you to draw from a broader base of skills and content knowledge. Keep your group small, question each other, and stay on target.

# Chapter <br> Current Electricity 

## What You'll Learn

- You will explain energy transfer in circuits.
- You will solve problems involving current, potential difference, and resistance.
- You will diagram simple electric circuits.


## Why It's Important

The electric tools and appliances that you use are based upon the ability of electric circuits to transfer energy resulting from potential difference, and thus, perform work.

## Power Transmission

Lines Transmission lines crisscross our country to transfer energy to where it is needed. This transfer is accomplished at high potential differences, often as high as $500,000 \mathrm{~V}$.

Think About This $>$ Transmission line voltages are too high to use safely in homes and businesses. Why are such high voltages used in transmission lines?

## Physjos inline

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## LAUNCH Lab

## Question

Given a wire, a battery, and a lightbulb, can you get the bulb to light?

## Procedure 氚 푼

1. Obtain a lightbulb, a wire, and a battery. Try to find as many ways as possible to get the lightbulb to light. Caution: Wire is sharp and can cut skin. Wire can also get hot if connected across the battery.
2. Diagram two ways in which you are able to get the lightbulb to work. Be sure to label the battery, the wire, and the bulb.
3. Diagram at least three ways in which you are not able to get the bulb to light.

## Analysis

How did you know if electric current was flowing? What do your diagrams of the lit bulb
have in common? What do your diagrams of the unlit bulb have in common? From your observations, what conditions seem to be necessary in order for the bulb to light?
Critical Thinking What causes electricity to flow through the bulb?


### 22.1 Current and Circuits

As you learned in Chapter 11, flowing water at the top of a waterfall has both potential and kinetic energy. However, the large amount of natural potential and kinetic energy available from resources such as Niagara Falls are of little use to people or manufacturers who are 100 km away, unless that energy can be transported efficiently. Electric energy provides the means to transfer large quantities of energy great distances with little loss. This transfer usually is done at high potential differences through power lines, such as those shown in the photo on the left. Once this energy reaches the consumer, it can easily be converted into another form or combination of forms, including sound, light, thermal energy, and motion.

Because electric energy can so easily be changed into other forms, it has become indispensable in our daily lives. Even quick glances around you will likely generate ample examples of the conversion of electric energy. Inside, lights to help you read at night, microwaves and electric ranges to cook food, computers, and stereos all rely on electricity for power. Outside, street lamps, store signs, advertisements, and the starters in cars all use flowing electric charges. In this chapter, you will learn how potential differences, resistance, and current are related. You also will learn about electric power and energy transfer.

## - Objectives

- Describe conditions that create current in an electric circuit.
- Explain Ohm's law.
- Design closed circuits.
- Differentiate between power and energy in an electric circuit.
- Vocabulary
electric current conventional current battery electric circuit
ampere
resistance
resistor
parallel connection
series connection
- Figure 22-1 Conventional current is defined as positive charges flowing from the positive plate to the negative plate (a). A generator pumps the positive charges back to the positive plate and maintains the current (b). In most metals, negatively-charged electrons actually flow from the negative to the positive plate, creating the appearance of positive charges that are moving in the opposite direction.



## Producing Electric Current

In Chapter 21, you learned that when two conducting spheres touch, charges flow from the sphere at a higher potential to the one at a lower potential. The flow continues until there is no potential difference between the two spheres.

A flow of charged particles is an electric current. In Figure 22-1a, two conductors, A and B , are connected by a wire conductor, C . Charges flow from the higher potential difference of $B$ to $A$ through $C$. This flow of positive charge is called conventional current. The flow stops when the potential difference between $\mathrm{A}, \mathrm{B}$, and C is zero. You could maintain the electric potential difference between $B$ and $A$ by pumping charged particles from A back to B, as illustrated in Figure 22-1b. Since the pump increases the electric potential energy of the charges, it requires an external energy source to run. This energy could come from a variety of sources. One familiar source, a voltaic or galvanic cell (a common dry cell), converts chemical energy to electric energy. Several galvanic cells connected together are called a battery. A second source of electric energy-a photovoltaic cell, or solar cell—changes light energy into electric energy.

## Electric Circuits

The charges in Figure 22-1b move around a closed loop, cycling from the pump to B , through C , to A and back to the pump. Any closed loop or conducting path allowing electric charges to flow is called an electric circuit. A circuit includes a charge pump, which increases the potential energy of the charges flowing from A to B , and a device that reduces the potential energy of the charges flowing from $B$ to $A$. The potential energy lost by the charges, $q V$, moving through the device is usually converted into some other form of energy. For example, electric energy is converted to kinetic energy by a motor, to light energy by a lamp, and to thermal energy by a heater.

A charge pump creates the flow of charged particles that make up a current. Consider a generator driven by a waterwheel, such as the one pictured in Figure 22-2a. The water falls and rotates the waterwheel and generator. Thus, the kinetic energy of the water is converted to electric energy by the generator. The generator, like the charge pump, increases the electric potential difference, $V$. Energy in the amount $q V$ is needed to increase the potential difference of the charges. This energy comes from the change in energy of the water. Not all of the water's kinetic energy, however, is converted to electric energy, as shown in Figure 22-2b.

If the generator attached to the waterwheel is connected to a motor, the charges in the wire flow into the motor. The flow of charges continues through the circuit back to the generator. The motor converts electric energy to kinetic energy.

Conservation of charge Charges cannot be created or destroyed, but they can be separated. Thus, the total amount of charge-the number of negative electrons and positive ions-in the circuit does not change. If one coulomb flows through the generator in 1 s , then one coulomb also will flow through the motor in 1 s . Thus, charge is a conserved quantity. Energy also is conserved. The change in electric energy, $\Delta E$, equals $q V$. Because $q$ is conserved,

the net change in potential energy of the charges going completely around the circuit must be zero. The increase in potential difference produced by the generator equals the decrease in potential difference across the motor.

If the potential difference between two wires is 120 V , the waterwheel and the generator must do 120 J of work on each coulomb of charge that is delivered. Every coulomb of charge moving through the motor delivers 120 J of energy to the motor.

## Rates of Charge Flow and Energy Transfer

Power, which is defined in watts, W , measures the rate at which energy is transferred. If a generator transfers 1 J of kinetic energy to electric energy each second, it is transferring energy at the rate of $1 \mathrm{~J} / \mathrm{s}$, or 1 W . The energy carried by an electric current depends on the charge transferred, $q$, and the potential difference across which it moves, $V$. Thus, $E=q V$. Recall from Chapter 20 that the unit for the quantity of electric charge is the coulomb. The rate of flow of electric charge, $q / t$, called electric current, is measured in coulombs per second. Electric current is represented by $I$, so $I=q / t$. A flow of $1 \mathrm{C} / \mathrm{s}$ is called an ampere, A .

The energy carried by an electric current is related to the voltage, $E=q V$. Since current, $I=q / t$, is the rate of charge flow, the power, $P=E / t$, of an electric device can be determined by multiplying voltage and current. To derive the familiar form of the equation for the power delivered to an electric device, you can use $P=E / t$ and substitute $E=q V$ and $q=I t$.

$$
\text { Power } P=I V
$$

Power is equal to the current times the potential difference.

If the current through the motor in Figure 22-2a is 3.0 A and the potential difference is 120 V , the power in the motor is calculated using the expression $P=(3.0 \mathrm{C} / \mathrm{s})(120 \mathrm{~J} / \mathrm{C})=360 \mathrm{~J} / \mathrm{s}$, which is 360 W .

- Figure 22-2 The potential energy of the waterfall is eventually converted into work done on the bucket (a). The production and use of electric current is not 100 percent efficient. Some thermal energy is produced by the splashing water, friction, and electric resistance (b).


## EXAMPLE Problem 1

Electric Power and Energy A 6.0-V battery delivers a $0.50-\mathrm{A}$ current to an electric motor connected across its terminals.
a. What power is delivered to the motor?
b. If the motor runs for 5.0 min , how much electric energy is delivered?

1 Analyze and Sketch the Problem

- Draw a circuit showing the positive terminal of a battery connected to a motor and the return wire from the motor connected to the negative terminal of the battery.
- Show the direction of conventional current.

| Known: | Unknown: |
| :--- | :--- |
| $V=6.0 \mathrm{~V}$ | $P=?$ |
| $I=0.50 \mathrm{~A}$ | $E=?$ |
| $t=5.0 \mathrm{~min}$ |  |



2 Solve for the Unknown
a. Use $P=I V$ to find the power.

$$
\begin{aligned}
P & =I V \\
P & =(0.50 \mathrm{~A} \\
& =3.0 \mathrm{~W}
\end{aligned}
$$

$$
P=(0.50 \mathrm{~A})(6.0 \mathrm{~V}) \quad \text { Substitue } I=0.50 \mathrm{~A}, \mathrm{~V}=6.0 \mathrm{~V}
$$

b. In Chapter 10, you learned that $P=E / t$. Solve for $E$ to find the energy.

$$
\begin{array}{rlr}
E & =P t & \\
& =(3.0 \mathrm{~W})(5.0 \mathrm{~min}) \quad \text { Substitute } P=3.0 \mathrm{~W}, t=5.0 \mathrm{~min} \\
& =(3.0 \mathrm{~J} / \mathrm{s})(5.0 \mathrm{~min})\left(\frac{60 \mathrm{~s}}{1 \mathrm{~min}}\right) \\
& =9.0 \times 10^{2} \mathrm{~J} &
\end{array}
$$

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3 Evaluate the Answer

- Are the units correct? Power is measured in watts, and energy is measured in joules.
- Is the magnitude realistic? With relatively low voltage and current, a few watts of power is reasonable.


## PRACTICE Problems Additional Problems, Appendix B

1. The current through a lightbulb connected across the terminals of a $125-\mathrm{V}$ outlet is 0.50 A . At what rate does the bulb convert electric energy to light? (Assume 100 percent efficiency.)
2. A car battery causes a current of 2.0 A through a lamp and produces 12 V across it. What is the power used by the lamp?
3. What is the current through a $75-\mathrm{W}$ lightbulb that is connected to a 125-V outlet?
4. The current through the starter motor of a car is 210 A . If the battery maintains 12 V across the motor, how much electric energy is delivered to the starter in 10.0 s ?
5. A flashlight bulb is rated at 0.90 W . If the lightbulb drops 3.0 V , how much current goes through it?

| Table 22-1 |  |  |  |
| :---: | :---: | :---: | :---: |
| Changing Resistance |  |  |  |
| Factor | How resistance changes | Example |  |
| Length | Resistance increases as length increases. |  | $R_{\mathrm{L} 1}>R_{\mathrm{L} 2}$ |
| Cross-sectional area | Resistance increases as cross-sectional area decreases. |  | $R_{\text {A } 1}>R_{\text {A } 2}$ |
| Temperature | Resistance increases as temperature increases. |  | $R_{\mathrm{T} 1}>R_{\mathrm{T} 2}$ |
| Material | Keeping length, cross-sectional area, and temperature constant, resistance varies with the material used. |  |  |

## Resistance and Ohm's Law

Suppose two conductors have a potential difference between them. If they are connected with a copper rod, a large current is created. On the other hand, putting a glass rod between them creates almost no current. The property determining how much current will flow is called resistance.
Table 22-1 lists some of the factors that impact resistance. Resistance is measured by placing a potential difference across a conductor and dividing the voltage by the current. The resistance, $R$, is defined as the ratio of electric potential difference, $V$, to the current, $I$.

$$
\text { Resistance } R=\frac{V}{I}
$$

Resistance is equal to voltage divided by current.

- Figure 22-3 One ohm, $\Omega$, is defined as $1 \mathrm{~V} / \mathrm{A}$. In a circuit with a $3-\Omega$ resistance and a $12-\mathrm{V}$ battery, there is a $4-\mathrm{A}$ current.

The resistance of the conductor, $R$, is measured in ohms. One ohm $(1 \Omega)$ is the resistance permitting an electric charge of 1 A to flow when a potential difference of 1 V is applied across the resistance. A simple circuit relating resistance, current, and voltage is shown in Figure 22-3. A $12-\mathrm{V}$ car battery is connected to one of the car's $3-\Omega$ brake lights. The circuit is completed by a connection to an ammeter, which is a device that measures current. The current carrying the energy to the lights will measure 4 A .



Figure 22-4 The current through a simple circuit (a) can be regulated by removing some of the dry cells (b) or by increasing the resistance of the circuit (c).

- Figure 22-5 A potentiometer can be used to change current in an electric circuit.


The unit for resistance is named for German scientist Georg Simon Ohm, who found that the ratio of potential difference to current is constant for a given conductor. The resistance for most conductors does not vary as the magnitude or direction of the potential applied to it changes. A device having constant resistance independent of the potential difference obeys Ohm's law.

Most metallic conductors obey Ohm's law, at least over a limited range of voltages. Many important devices, however, do not. A radio and a pocket calculator contain many devices, such as transistors and diodes, that do not obey Ohm's law. Even a lightbulb has resistance that depends on its temperature and does not obey Ohm's law.

Wires used to connect electric devices have low resistance. A 1-m length of a typical wire used in physics labs has a resistance of about $0.03 \Omega$. Wires used in home wiring offer as little as $0.004 \Omega$ of resistance for each meter of length. Because wires have so little resistance, there is almost no potential drop across them. To produce greater potential drops, a large resistance concentrated into a small volume is necessary. A resistor is a device designed to have a specific resistance. Resistors may be made of graphite, semiconductors, or wires that are long and thin.

There are two ways to control the current in a circuit. Because $I=V / R$, $I$ can be changed by varying $V, R$, or both. Figure 22-4a shows a simple circuit. When $V$ is 6 V and $R$ is $30 \Omega$, the current is 0.2 A . How could the current be reduced to 0.1 A ? According to Ohm's law, the greater the voltage placed across a resistor, the larger the current passing through it. If the current through a resistor is cut in half, the potential difference also is cut

in half. In Figure 22-4b, the voltage applied across the resistor is reduced from 6 V to 3 V to reduce the current to 0.1 A . A second way to reduce the current to 0.1 A is to replace the $30-\Omega$ resistor with a $60-\Omega$ resistor, as shown in Figure 22-4c.

Resistors often are used to control the current in circuits or parts of circuits. Sometimes, a smooth, continuous variation of the current is desired. For example, the speed control on some electric motors allows continuous, rather than step-by-step, changes in the rotation of the motor. To achieve this kind of control, a variable resistor, called a potentiometer, is used. A circuit containing a potentiometer is shown in Figure 22-5. Some variable resistors consist of a coil of resistance wire and a sliding contact point. Moving the contact point to various positions along the coil varies the amount of wire in the circuit. As more wire is placed in the circuit, the resistance of the circuit increases; thus, the current changes in accordance with the equation $I=V / R$. In this way, the speed of a motor can be adjusted from fast, with little wire in the circuit, to slow, with a lot of wire in the circuit. Other examples of using variable resistors to adjust the levels of electrical energy can be found on the front of a TV: the volume, brightness, contrast, tone, and hue controls are all variable resistors.

The human body The human body acts as a variable resistor. When dry, skin's resistance is high enough to keep currents that are produced by small and moderate voltages low. If skin becomes wet, however, its resistance is lower, and the electric current can rise to dangerous levels. A current as low as 1 mA can be felt as a mild shock, while currents of 15 mA can cause loss of muscle control and currents of 100 mA can cause death.

## Diagramming Circuits

A simple circuit can be described in words. It can also be depicted by photographs or artists' drawings of the parts. Most frequently, however, an electric circuit is drawn using standard symbols for the circuit elements. Such a diagram is called a circuit schematic. Some of the symbols used in circuit schematics are shown in Figure 22-6.

## APPLYING PHYSICS

- Resistance The resistance of an operating $100-\mathrm{W}$ lightbulb is about $140 \Omega$. When the lightbulb is turned off and at room temperature, its resistance is only about $10 \Omega$. This is because of the great difference between room temperature and the lightbulb's operating temperature.
- Figure 22-6 These symbols commonly are used to diagram electric circuits.



## EXAMPLE Problem 2

Current Through a Resistor A 30.0-V battery is connected to a $10.0-\Omega$ resistor. What is the current in the circuit?

## 1 Analyze and Sketch the Problem

- Draw a circuit containing a battery, an ammeter, and a resistor.
- Show the direction of the conventional current.

$$
\begin{array}{ll}
\text { Known: } & \text { Unknown: } \\
V=30.0 \mathrm{~V} & I=? \\
R=10.0 \Omega &
\end{array}
$$

2 Solve for the Unknown


Use $I=V / R$ to determine the current.

$$
\begin{aligned}
I & =\frac{V}{R} \\
& =\frac{30.0 \mathrm{~V}}{10.0 \Omega} \quad \text { Substitute } V=30.0 \mathrm{~V}, R=10.0 \Omega \\
& =3.00 \mathrm{~A}
\end{aligned}
$$

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3 Evaluate the Answer

- Are the units correct? Current is measured in amperes.
- Is the magnitude realistic? There is a fairly large voltage and a small resistance, so a current of 3.00 A is reasonable.


## PRACTICE Problems Additional Problems, Appendix B

For all problems, assume that the battery voltage and lamp resistances are constant, no matter what current is present.
6. An automobile panel lamp with a resistance of $33 \Omega$ is placed across a $12-\mathrm{V}$ battery. What is the current through the circuit?
7. A motor with an operating resistance of $32 \Omega$ is connected to a voltage source. The current in the circuit is 3.8 A . What is the voltage of the source?
8. A sensor uses $2.0 \times 10^{-4} \mathrm{~A}$ of current when it is operated by a 3.0-V battery. What is the resistance of the sensor circuit?
9. A lamp draws a current of 0.50 A when it is connected to a 120-V source.
a. What is the resistance of the lamp?
b. What is the power consumption of the lamp?
10. A $75-\mathrm{W}$ lamp is connected to 125 V .
a. What is the current through the lamp?
b. What is the resistance of the lamp?
11. A resistor is added to the lamp in the previous problem to reduce the current to half of its original value.
a. What is the potential difference across the lamp?
b. How much resistance was added to the circuit?
c. How much power is now dissipated in the lamp?


[^0]:    $\square$ Figure 16-15 The Sun can appear to be a shade of yellow or orange because of the scattering of violet and blue light.

