5.3 Force and Motion in Two Dimensions

You have already worked with several situations dealing with forces in two dimensions. For example, when friction acts between two surfaces, you must take into account both the frictional force that is parallel to the surface and the normal force that is perpendicular to it. So far, you have considered only the motion along a level surface. Now you will use your skill in adding vectors to analyze situations in which the forces acting on an object are at angles other than 90°.

▶ **Objectives**

- **Determine** the force that produces equilibrium when three forces act on an object.
- **Analyze** the motion of an object on an inclined plane with and without friction.

▶ **Vocabulary**

equilibrant

Equilibrium Revisited

Recall from Chapter 4 that when the net force on an object is zero, the object is in equilibrium. According to Newton's laws, the object will not accelerate because there is no net force acting on it; an object in equilibrium is motionless or moves with constant velocity. You have already analyzed several equilibrium situations in which two forces acted on an object. It is important to realize that equilibrium can occur no matter how many forces act on an object. As long as the resultant is zero, the net force is zero and the object is in equilibrium.

Figure 5-11a shows three forces exerted on a point object. What is the net force acting on the object? Remember that vectors may be moved if you do not change their direction (angle) or length. **Figure 5-11b** shows the addition of the three forces, ***A*, *B*,** and ***C*.** Note that the three vectors form a closed triangle. There is no net force; thus, the sum is zero and the object is in equilibrium.

Suppose that two forces are exerted on an object and the sum is not zero. How could you find a third force that, when added to the other two, would add up to zero, and therefore cause the object to be in equilibrium? To find this force, first find the sum of the two forces already being exerted on the object. This single force that produces the same effect as the two individual forces added together is called the resultant force. The force that you need to find is one with the same magnitude as the resultant force, but in the opposite direction. A force that puts an object in equilibrium is called the **equilibrant. Figure 5-12** illustrates the procedure for finding this force for two vectors. Note that this general procedure works for any number of vectors.

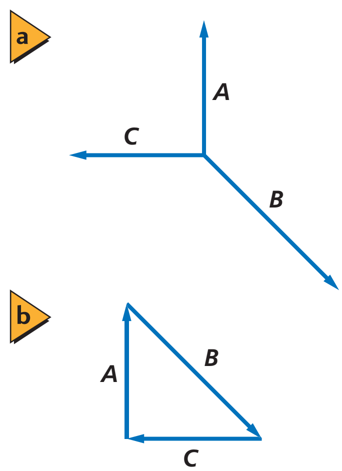
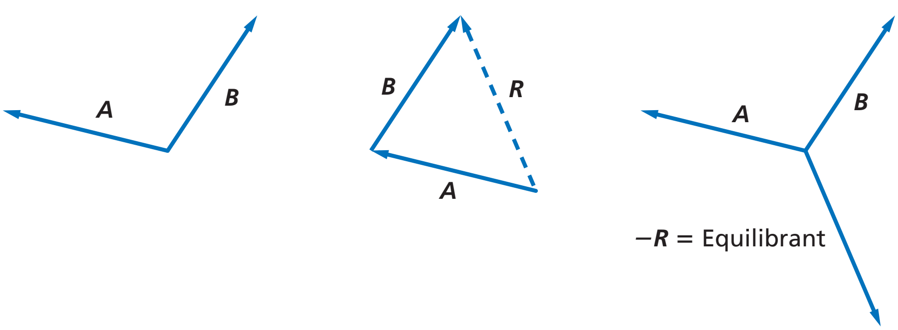


Figure 5-11 An object is in equilibrium when all the forces on it add up to zero.

Figure 5-12 The equilibrant is the same magnitude as the resultant, but opposite in direction.



CHALLENGE PROBLEM

Find the equilibrant for the following forces.

$\mathbf{F}_1$ = 61.0 N at 17.0° north of east

$\mathbf{F}_2$ = 38.0 N at 64.0° north of east

$\mathbf{F}_3$ = 54.0 N at 8.0° west of north

$\mathbf{F}_4$ = 93.0 N at 53.0° west of north

$\mathbf{F}_5$ = 65.0 N at 21.0° south of west

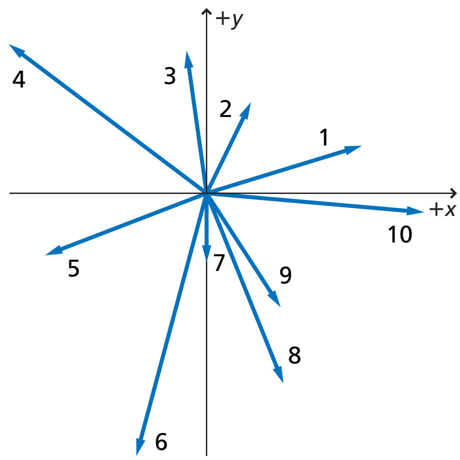
$\mathbf{F}_6$ = 102.0 N at 15.0° west of south

$\mathbf{F}_7$ = 26.0 N south

$\mathbf{F}_8$ = 77.0 N at 22.0° east of south

$\mathbf{F}_9$ = 51.0 N at 33.0° east of south

$\mathbf{F}_{10}$ = 82.0 N at 5.0° south of east

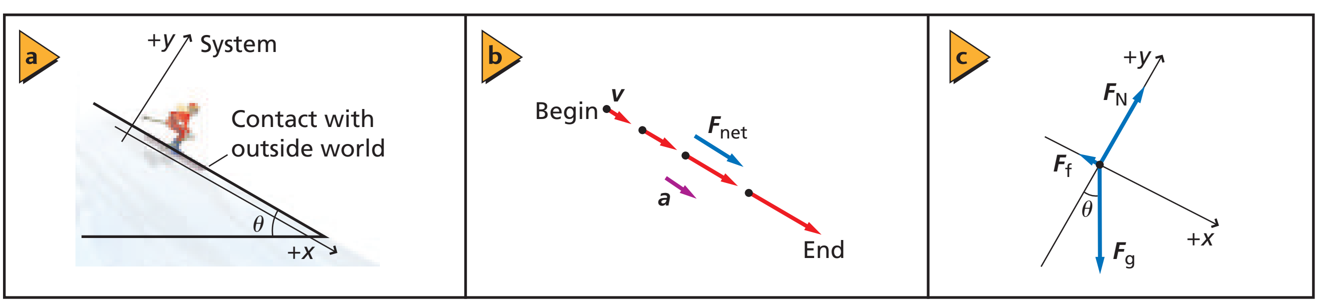


Motion Along an Inclined Plane

You have applied Newton's laws to a variety of equilibrium situations, but only to motions that were either horizontal or vertical. How would you apply them in a situation like the one in **Figure 5-13a,** in which a skier glides down a slope?

Start by identifying the forces acting on the object, the skier, as shown in **Figure 5-13b** and sketching a free-body diagram. The gravitational force on the skier is in the downward direction toward the center of Earth. There is a normal force perpendicular to the hill, and the frictional forces opposing the skier's motion are parallel to the hill. The resulting free-body diagram is shown in **Figure 5-13c.** You can see that, other than the force of friction, only one force acts horizontally or vertically, and you know from experience that the acceleration of the skier will be along the slope. How do you find the net force that causes the skier to accelerate?

Figure 5-13 A skier slides down a slope **(a).** Identify the forces that are acting upon the skier **(b)** and draw a free-body diagram describing those forces **(c).** It is important to draw the direction of the normal and the friction forces correctly in order to properly analyze these types of situations.

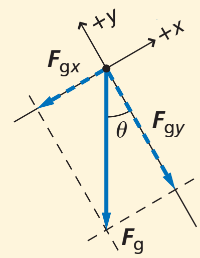


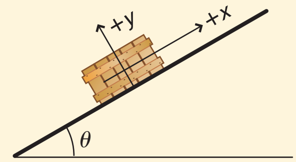
▶ EXAMPLE Problem 5

Components of Weight for an Object on an Incline A crate weighing 562 N is resting on a plane inclined 30.0° above the horizontal. Find the components of the weight forces that are parallel and perpendicular to the plane.

1 Analyze and Sketch the Problem

- Include a coordinate system with the positive *x*-axis pointing uphill.
- Draw the free-body diagram showing $\mathbf{F}_g$, the components $\mathbf{F}_{gx}$ and $\mathbf{F}_{gy}$, and the angle θ.





Known:	**Unknown:**
$F_g = 562$ N	$F_{gx} = ?$
$\theta = 30.0°$	$F_{gy} = ?$

2 Solve for the Unknown

$\mathbf{F}_{gx}$ and $\mathbf{F}_{gy}$ are negative because they point in directions opposite to the positive axes.

$$F_{gx} = -F_g(\sin\theta)$$
$$= -(562\text{ N})(\sin 30.0°) \quad \textbf{Substitute } F_g = 562,\ \theta = 30.0°$$
$$= -281\text{ N}$$
$$F_{gy} = -F_g(\cos\theta)$$
$$= -(562\text{ N})(\cos 30.0°) \quad \textbf{Substitute } F_g = 562,\ \theta = 30.0°$$
$$= -487\text{ N}$$

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3 Evaluate the Answer

- **Are the units correct?** Force is measured in newtons.
- **Do the signs make sense?** The components point in directions opposite to the positive axes.
- **Are the magnitudes realistic?** The values are less than F_g.

▶ PRACTICE Problems

Additional Problems, Appendix B

33. An ant climbs at a steady speed up the side of its anthill, which is inclined 30.0° from the vertical. Sketch a free-body diagram for the ant.

34. Scott and Becca are moving a folding table out of the sunlight. A cup of lemonade, with a mass of 0.44 kg, is on the table. Scott lifts his end of the table before Becca does, and as a result, the table makes an angle of 15.0° with the horizontal. Find the components of the cup's weight that are parallel and perpendicular to the plane of the table.

35. Kohana, who has a mass of 50.0 kg, is at the dentist's office having her teeth cleaned, as shown in **Figure 5-14.** If the component of her weight perpendicular to the plane of the seat of the chair is 449 N, at what angle is the chair tilted?

36. Fernando, who has a mass of 43.0 kg, slides down the banister at his grandparents' house. If the banister makes an angle of 35.0° with the horizontal, what is the normal force between Fernando and the banister?

37. A suitcase is on an inclined plane. At what angle, relative to the vertical, will the component of the suitcase's weight parallel to the plane be equal to half the perpendicular component of its weight?

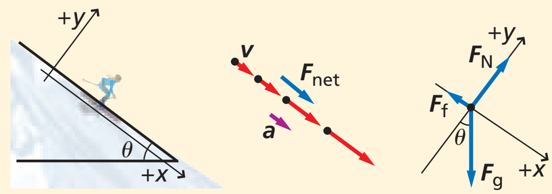
■ **Figure 5-14**

EXAMPLE Problem 6

Skiing Downhill A 62-kg person on skis is going down a hill sloped at 37°. The coefficient of kinetic friction between the skis and the snow is 0.15. How fast is the skier going 5.0 s after starting from rest?

1 Analyze and Sketch the Problem

- Establish a coordinate system.
- Draw a free-body diagram showing the skier's velocity and direction of acceleration.
- Draw a motion diagram showing increasing v, and both a and F_{net} in the $+x$ direction, like the one shown in Figure 5-13.



Known:	Unknown:
$m = 62$ kg	$a = ?$
$\theta = 37°$	$v_f = ?$
$\mu_k = 0.15$	
$v_i = 0.0$ m/s	
$t = 5.0$ s	

2 Solve for the Unknown

y-direction:

$$F_{net,\,y} = ma_y = 0.0 \text{ N}$$

There is no acceleration in the y-direction, so $a_y = 0.0 \text{ m/s}^2$.

Solve for F_N.

$$F_N - F_{gy} = F_{net,\,y}$$

F_{gy} is negative. It is in the negative direction as defined by the coordinate system.

$$F_N = F_{gy}$$

Substitute $F_{net,\,y} = 0.0$ N and rearrange

$$= mg(\cos\theta)$$

Substitute $F_{gy} = mg\cos\theta$

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x-direction:

Solve for a.

$$F_{net,\,x} = F_{gx} - F_f$$

F_f is negative because it is in the negative direction as defined by the coordinate system.

$$ma_x = mg(\sin\theta) - \mu_k F_N$$

Substitute $F_{net,\,x} = ma$, $F_{gx} = mg\sin\theta$, $F_f = \mu_k F_N$

$$= mg(\sin\theta) - \mu_k mg(\cos\theta)$$

Substitute $a = a_x$ because all the acceleration is in the x-direction; substitute $F_N = mg\cos\theta$

$$a = g(\sin\theta - \mu_k\cos\theta)$$

$$= (9.80 \text{ m/s}^2)(\sin 37° - (0.15)\cos 37°)$$

Substitute $g = 9.80 \text{ m/s}^2$, $\theta = 37°$, $\mu_k = 0.15$

$$= 4.7 \text{ m/s}^2$$

Because v_i, a, and t are all known, use the following.

$$v_f = v_i + at$$

$$= 0.0 + (4.7 \text{ m/s}^2)(5.0 \text{ s})$$

Substitute $v_i = 0.0$ m/s, $a = 4.7 \text{ m/s}^2$, $t = 5.0$ s

$$= 24 \text{ m/s}$$

3 Evaluate the Answer

- **Are the units correct?** Performing dimensional analysis on the units verifies that v_f is in m/s and a is in m/s^2.
- **Do the signs make sense?** Because v_f and a are both in the $+x$ direction, the signs do make sense.
- **Are the magnitudes realistic?** The velocity is fast, over 80 km/h (50 mph), but 37° is a steep incline, and the friction between the skis and the snow is not large.

PRACTICE Problems

Additional Problems, Appendix B

38. Consider the crate on the incline in Example Problem 5. Calculate the magnitude of the acceleration. After 4.00 s, how fast will the crate be moving?

39. If the skier in Example Problem 6 were on a 31° downhill slope, what would be the magnitude of the acceleration?

40. Stacie, who has a mass of 45 kg, starts down a slide that is inclined at an angle of 45° with the horizontal. If the coefficient of kinetic friction between Stacie's shorts and the slide is 0.25, what is her acceleration?

41. After the skier on the 37° hill in Example Problem 6 had been moving for 5.0 s, the friction of the snow suddenly increased and made the net force on the skier zero. What is the new coefficient of friction?

The most important decision in problems involving motion along a slope is what coordinate system to use. Because an object's acceleration is usually parallel to the slope, one axis, usually the *x*-axis, should be in that direction. The *y*-axis is perpendicular to the *x*-axis and perpendicular to the surface of the slope. With this coordinate system, you now have two forces, the normal and frictional forces, in the directions of the coordinate axes; however, the weight is not. This means that when an object is placed on an inclined plane, the magnitude of the normal force between the object and the plane will usually not be equal to the object's weight.

You will need to apply Newton's laws once in the *x*-direction and once in the *y*-direction. Because the weight does not point in either of these directions, you will need to break this vector into its *x*- and *y*-components before you can sum your forces in these two directions. Example Problem 5 and Example Problem 6 both showed this procedure.

MINI LAB

What's Your Angle?

Prop a board up so that it forms an inclined plane at a 45° angle. Hang a 500-g object from the spring scale.

1. Measure and record the weight of the object. Set the object on the bottom of the board and slowly pull it up the inclined plane at a constant speed.

2. Observe and record the reading on the spring scale.

Analyze and Conclude

3. Calculate the component of weight for the 500-g object that is parallel to the inclined plane.

4. Compare the spring-scale reading along the inclined plane with the component of weight parallel to the inclined plane.

5.3 Section Review

42. Forces One way to get a car unstuck is to tie one end of a strong rope to the car and the other end to a tree, then push the rope at its midpoint at right angles to the rope. Draw a free-body diagram and explain why even a small force on the rope can exert a large force on the car.

43. Mass A large scoreboard is suspended from the ceiling of a sports arena by 10 strong cables. Six of the cables make an angle of 8.0° with the vertical while the other four make an angle of 10.0°. If the tension in each cable is 1300.0 N, what is the scoreboard's mass?

44. Acceleration A 63-kg water skier is pulled up a 14.0° incline by a rope parallel to the incline with a tension of 512 N. The coefficient of kinetic friction is 0.27. What are the magnitude and direction of the skier's acceleration?

45. Equilibrium You are hanging a painting using two lengths of wire. The wires will break if the force is too great. Should you hang the painting as shown in **Figures 5-15a** or **5-15b?** Explain.

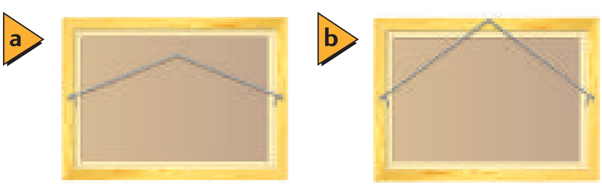


Figure 5-15

46. Critical Thinking Can the coefficient of friction ever have a value such that a skier would be able to slide uphill at a constant velocity? Explain why or why not. Assume there are no other forces acting on the skier.

PHYSICS LAB

Alternate CBL instructions can be found on the Web site.
physicspp.com

The Coefficient of Friction

Static and kinetic friction are forces that are a result of two surfaces in contact with each other. Static friction is the force that must be overcome to cause an object to begin moving, while kinetic friction occurs between two objects in motion relative to each other. The kinetic friction force, $F_{f,\,kinetic}$, is defined by $F_{f,\,kinetic} = \mu_k F_N$, where μ_k is the coefficient of kinetic friction and F_N is the normal force acting on the object. The maximum static frictional force, $F_{f,\,max\ static}$, is defined by $F_{f,\,static} = \mu_s F_N$ where μ_s is the coefficient of static friction and F_N is the normal force on the object. The maximum static frictional force that must be overcome before movement is able to begin is $\mu_s F_N$. If you apply a constant force to pull an object along a horizontal surface at a constant speed, then the frictional force opposing the motion is equal and opposite to the applied force, F_p. Therefore, $F_p = F_f$. The normal force is equal and opposite to the object's weight when the object is on a horizontal surface and the applied force is horizontal.

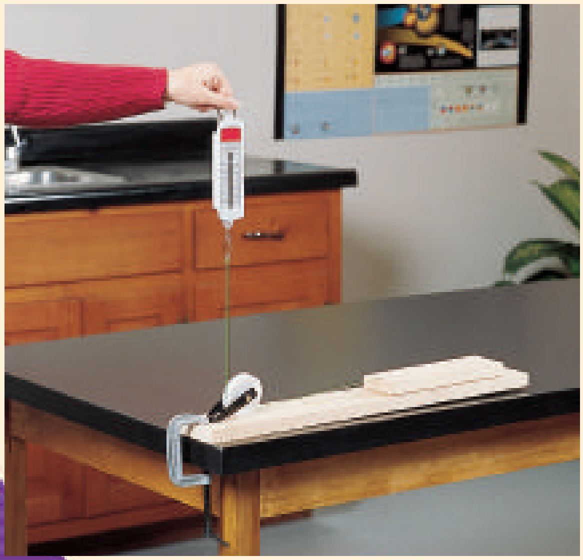
QUESTION

How can the coefficient of static and kinetic friction be determined for an object on a horizontal surface?

Objectives

- **Measure** the normal and frictional forces acting on an object starting in motion and already in motion.
- **Use numbers** to calculate μ_s and μ_k.
- **Compare and contrast** values of μ_s and μ_k.
- **Analyze** the kinetic friction results.
- **Estimate** the angle where sliding will begin for an object on an inclined plane.

Safety Precautions



Materials

pulley
C-clamp
masking tape
wood surface
string (1 m)
spring scale, 0-5 N
wood block

Procedure

1. Check your spring scale to make sure that it reads zero when it is held vertically. If necessary, follow your teacher's instructions to zero it.
2. Attach the pulley to the edge of the table with a C-clamp.
3. Attach the string to the spring scale hook and the wood block.
4. Measure the weight of the block of wood, or other small object, and record the value as the normal force, F_N, in Data Tables 1, 2, and 3.
5. Unhook the string from the spring scale and run it through the pulley. Then reattach it to the spring scale.
6. Move the wood block as far away from the pulley as the string permits, while having it remain on the wood surface.
7. With the spring scale oriented vertically so that a right angle is formed between the wood block, the pulley, and the spring scale, slowly pull up on the spring scale. Observe the force that is necessary to cause the wood block to begin sliding. Record this value for the static frictional force in Data Table 1.

Horizons Companies

CONTENTS

Material Table

Object material	
Surface material	

Data Table 1

F_N (N)	Static Friction Force, F_s (N)			
	Trial 1	Trial 2	Trial 3	Average

Data Table 2

F_N (N)	Kinetic Friction Force, F_f (N)			
	Trial 1	Trial 2	Trial 3	Average

Data Table 3

F_N(N)	F_s (N)	F_f (N)	μ_s	μ_k

Data Table 4 (Angle, θ, when sliding begins on an incline)

θ*	tan θ*

8. Repeat steps 6 and 7 for two additional trials.
9. Repeat steps 6 and 7. However, once the block begins sliding, pull just hard enough to keep it moving at a constant speed across the other horizontal surface. Record this force as the kinetic frictional force in Data Table 2.
10. Repeat step 9 for two additional trials.
11. Place the block on the end of the surface. Slowly raise one end of the surface to make an incline. Gently tap the block to cause it to move and overcome static friction. If the block stops, replace it at the top of the incline and repeat the procedure. Continue increasing the angle, θ, between the horizontal and the inclined surface, and tapping the block until it slides at a constant speed down the incline. Record the angle, θ, in Data Table 4.

Analyze

1. Average the data for the static frictional force, $F_{s,\ max}$, from the three trials and record the result in the last column of Data Table 1 and in Data Table 3.
2. Average the data for the kinetic frictional force, F_f, from the three trials and record the result in the last column of Data Table 2 and in Data Table 3.
3. Use the data in Data Table 3 to calculate the coefficient of static friction, μ_s, and record the value in Data Table 3.
4. Use the data in Data Table 3 to calculate the coefficient of kinetic friction, μ_k, and record the value in Data Table 3.
5. Calculate tan θ for your value in Data Table 4.

Conclude and Apply

1. **Compare and Contrast** Examine your values for μ_s and μ_k. Explain whether your results are reasonable or not.
2. **Use Models** Draw a free-body diagram showing the forces acting on the block if it is placed on an incline of angle θ. Make certain that you include the force due to friction in your diagram.
3. From your diagram, assuming that the angle, θ, is where sliding begins, what does tan θ represent?
4. Compare your value for tan θ (experimental), μ_s, and μ_k.

Going Further

Repeat the experiment with additional surfaces that have different characteristics.

Real-World Physics

If you were downhill skiing and wished to determine the coefficient of kinetic friction between your skis and the slope, how could you do this? Be specific about how you could find a solution to this problem.

Physics Online

To find out more about friction, visit the Web site: **physicspp.com**

CONTENTS

Technology and Society

Roller Coasters

Why are roller coasters fun? A roller-coaster ride would be no fun at all if not for the forces acting on the coaster car and the rider. What forces do riders experience as they ride a roller coaster? The force of gravity acts on the rider and the coaster car in the downward direction. The seat of the car exerts a force on the rider in the opposite direction. When the coaster car makes a turn, the rider experiences a force in the opposite direction. Also, there are forces present due to the friction between the rider and the seat, the side of the car, and the safety bar.

The Force Factor

Designers of roller coasters take into account the magnitude of the forces exerted on the rider. They design the coaster in such a way that the forces thrill the rider without causing injury or too much discomfort.

Designers measure the amount of force exerted on the rider by calculating the force factor. The force factor is equal to the force exerted by the seat on the rider divided by the weight of the rider. Suppose the rider weighs about 68 kg. When the roller coaster is at the bottom of a hill, the rider may experience a force factor of 2. That means that at the bottom of the hill, the rider will feel as though he or she weighs twice as much, or in this case 136 kg. Conversely, at the top of a hill the force factor may be 0.5 and the rider will feel as though he or she weighs half his or her normal weight. Thus, designers create excitement by designing portions that change the rider's apparent weight.

The thrill of a roller-coaster ride is produced by the forces acting on the rider and the rider's reaction to visual cues.

The Thrill Factors Roller-coaster designers manipulate the way in which the body perceives the external world to create that "thrilling" sensation. For example, the roller coaster moves up the first hill very slowly, tricking the rider into thinking that the hill is higher than it is.

The organs of the inner ear sense the position of the head both when it is still and when it is moving. These organs help maintain balance by providing information to the brain. The brain then sends nerve impulses to the skeletal muscles to contract or relax to maintain balance. The constant change in position during a roller-coaster ride causes the organs of the inner ear to send conflicting messages to the brain. As a result, the skeletal muscles contract and relax throughout the ride.

You know that you are moving at high speeds because your eyes see the surroundings move past at high speed. So, designers make use of the surrounding landscape along with twists, turns, tunnels, and loops to give the rider plenty of visual cues. These visual cues, along with the messages from the inner ear, can result in disorientation and in some cases, nausea. To enthusiasts the disorientation is part of the thrill.

In order to attract visitors, amusement parks are constantly working on designing new rides that take the rider to new thrill levels. As roller-coaster technology improves, your most thrilling roller-coaster ride may be over the next hill.

Going Further

1. **Compare and Contrast** Compare and contrast your experience as a rider in the front of a roller coaster versus the back of it. Explain your answer in terms of the forces acting on you.
2. **Critical Thinking** While older roller coasters rely on chain systems to pull the coaster up the first hill, newer ones depend on hydraulic systems to do the same job. Research each of these two systems. What do you think are the advantages and disadvantages of using each system?

Chapter 5

Study Guide

5.1 Vectors

Vocabulary

- components (p. 122)
- vector resolution (p. 122)

Key Concepts

- When two vectors are at right angles, you can use the Pythagorean theorem to determine the magnitude of the resultant vector.

$$R^2 = A^2 + B^2$$

- The law of cosines and law of sines can be used to find the magnitude of the resultant of any two vectors.

$$R^2 = A^2 + B^2 - 2AB \cos \theta$$
$$\frac{R}{\sin \theta} = \frac{A}{\sin a} = \frac{B}{\sin b}$$

- The components of a vector are projections of the component vectors.

$$\cos \theta = \frac{\text{adjacent side}}{\text{hypotenuse}} = \frac{A_x}{A}; \text{ therefore, } A_x = A \cos \theta$$
$$\sin \theta = \frac{\text{opposite side}}{\text{hypotenuse}} = \frac{A_y}{A}; \text{ therefore, } A_y = A \sin \theta$$
$$\theta = \tan^{-1}\left(\frac{R_y}{R_x}\right)$$

- Vectors can be summed by separately adding the x- and y-components.

5.2 Friction

Vocabulary

- kinetic friction (p. 126)
- static friction (p. 126)
- coefficient of kinetic friction (p. 127)
- coefficient of static friction (p. 127)

Key Concepts

- A frictional force acts when two surfaces touch.
- The frictional force is proportional to the force pushing the surfaces together.
- The kinetic friction force is equal to the coefficient of kinetic friction times the normal force.

$$F_{\text{f, kinetic}} = \mu_k F_N$$

- The static friction force is less than or equal to the coefficient of static friction times the normal force.

$$F_{\text{f, static}} \leq \mu_k F_N$$

5.3 Force and Motion in Two Dimensions

Vocabulary

- equilibrant (p. 131)

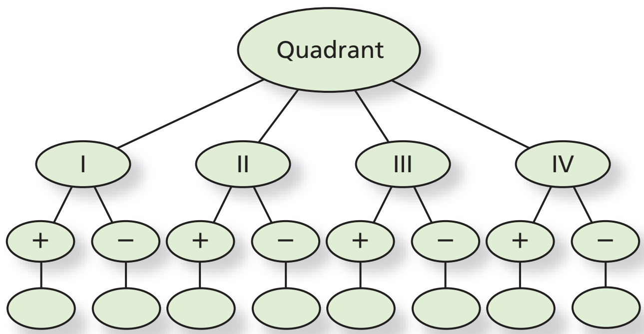
Key Concepts

- The force that must be exerted on an object to cause it to be in equilibrium is called the equilibrant.
- The equilibrant is found by finding the net force on an object, then applying a force with the same magnitude but opposite direction.
- An object on an inclined plane has a component of the force of gravity in a direction parallel to the plane; the component can accelerate the object down the plane.

Assessment

Concept Mapping

47. Complete the concept map below with the terms *sine*, *cosine*, or *tangent* to indicate whether each function is positive or negative in each quadrant. Some circles could remain blank, and others can have more than one term.



Mastering Concepts

48. How would you add two vectors graphically? (5.1)

49. Which of the following actions is permissible when you graphically add one vector to another: moving the vector, rotating the vector, or changing the vector's length? (5.1)

50. In your own words, write a clear definition of the resultant of two or more vectors. Do not explain how to find it; explain what it represents. (5.1)

51. How is the resultant displacement affected when two displacement vectors are added in a different order? (5.1)

52. Explain the method that you would use to subtract two vectors graphically. (5.1)

53. Explain the difference between **A** and *A*. (5.1)

54. The Pythagorean theorem usually is written $c^2 = a^2 + b^2$. If this relationship is used in vector addition, what do a, b, and c represent? (5.1)

55. When using a coordinate system, how is the angle or direction of a vector determined with respect to the axes of the coordinate system? (5.1)

56. What is the meaning of a coefficient of friction that is greater than 1.0? How would you measure it? (5.2)

57. **Cars** Using the model of friction described in this textbook, would the friction between a tire and the road be increased by a wide rather than a narrow tire? Explain. (5.2)

58. Describe a coordinate system that would be suitable for dealing with a problem in which a ball is thrown up into the air. (5.3)

59. If a coordinate system is set up such that the positive x-axis points in a direction 30° above the horizontal, what should be the angle between the x-axis and the y-axis? What should be the direction of the positive y-axis? (5.3)

60. Explain how you would set up a coordinate system for motion on a hill. (5.3)

61. If your textbook is in equilibrium, what can you say about the forces acting on it? (5.3)

62. Can an object that is in equilibrium be moving? Explain. (5.3)

63. What is the sum of three vectors that, when placed tip to tail, form a triangle? If these vectors represent forces on an object, what does this imply about the object? (5.3)

64. You are asked to analyze the motion of a book placed on a sloping table. (5.3)
  a. Describe the best coordinate system for analyzing the motion.
  b. How are the components of the weight of the book related to the angle of the table?

65. For a book on a sloping table, describe what happens to the component of the weight force parallel to the table and the force of friction on the book as you increase the angle that the table makes with the horizontal. (5.3)
  a. Which components of force(s) increase when the angle increases?
  b. Which components of force(s) decrease?

Applying Concepts

66. A vector that is 1 cm long represents a displacement of 5 km. How many kilometers are represented by a 3-cm vector drawn to the same scale?

67. **Mowing the Lawn** If you are pushing a lawn mower across the grass, as shown in **Figure 5-16,** can you increase the horizontal component of the force that you exert on the mower without increasing the magnitude of the force? Explain.

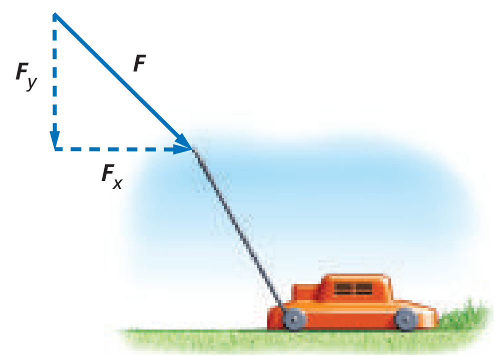


Figure 5-16

 For more problems, go to Additional Problems, Appendix B.

68. A vector drawn 15 mm long represents a velocity of 30 m/s. How long should you draw a vector to represent a velocity of 20 m/s?

69. What is the largest possible displacement resulting from two displacements with magnitudes 3 m and 4 m? What is the smallest possible resultant? Draw sketches to demonstrate your answers.

70. How does the resultant displacement change as the angle between two vectors increases from 0° to 180°?

71. A and B are two sides of a right triangle, where $\tan\theta = A/B$.
- **a.** Which side of the triangle is longer if $\tan\theta$ is greater than 1.0?
- **b.** Which side is longer if $\tan\theta$ is less than 1.0?
- **c.** What does it mean if $\tan\theta$ is equal to 1.0?

72. **Traveling by Car** A car has a velocity of 50 km/h in a direction 60° north of east. A coordinate system with the positive x-axis pointing east and a positive y-axis pointing north is chosen. Which component of the velocity vector is larger, x or y?

73. Under what conditions can the Pythagorean theorem, rather than the law of cosines, be used to find the magnitude of a resultant vector?

74. A problem involves a car moving up a hill, so a coordinate system is chosen with the positive x-axis parallel to the surface of the hill. The problem also involves a stone that is dropped onto the car. Sketch the problem and show the components of the velocity vector of the stone.

75. **Pulling a Cart** According to legend, a horse learned Newton's laws. When the horse was told to pull a cart, it refused, saying that if it pulled the cart forward, according to Newton's third law, there would be an equal force backwards; thus, there would be balanced forces, and, according to Newton's second law, the cart would not accelerate. How would you reason with this horse?

76. **Tennis** When stretching a tennis net between two posts, it is relatively easy to pull one end of the net hard enough to remove most of the slack, but you need a winch to take the last bit of slack out of the net to make the top almost completely horizontal. Why is this true?

77. The weight of a book on an inclined plane can be resolved into two vector components, one along the plane, and the other perpendicular to it.
- **a.** At what angle are the components equal?
- **b.** At what angle is the parallel component equal to zero?
- **c.** At what angle is the parallel component equal to the weight?

78. **TV Towers** The transmitting tower of a TV station is held upright by guy wires that extend from the top of the tower to the ground. The force along the guy wires can be resolved into two perpendicular components. Which one is larger?

Mastering Problems

5.1 Vectors

79. **Cars** A car moves 65 km due east, then 45 km due west. What is its total displacement?

80. Find the horizontal and vertical components of the following vectors, as shown in **Figure 5-17.**
- **a.** E
- **b.** F
- **c.** A

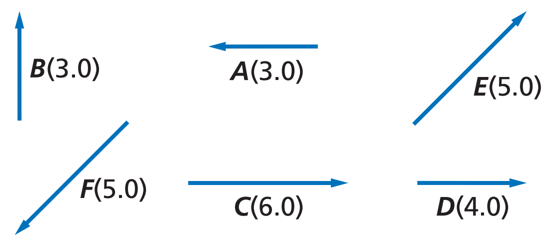


Figure 5-17

81. Graphically find the sum of the following pairs of vectors, whose lengths and directions are shown in Figure 5-17.
- **a.** D and A
- **b.** C and D
- **c.** C and A
- **d.** E and F

82. Graphically add the following sets of vectors, as shown in Figure 5-17.
- **a.** A, C, and D
- **b.** A, B, and E
- **c.** B, D, and F

83. You walk 30 m south and 30 m east. Find the magnitude and direction of the resultant displacement both graphically and algebraically.

84. **Hiking** A hiker's trip consists of three segments. Path A is 8.0 km long heading 60.0° north of east. Path B is 7.0 km long in a direction due east. Path C is 4.0 km long heading 315° counterclockwise from east.
- **a.** Graphically add the hiker's displacements in the order A, B, C.
- **b.** Graphically add the hiker's displacements in the order C, B, A.
- **c.** What can you conclude about the resulting displacements?

85. What is the net force acting on the ring in **Figure 5-18?**

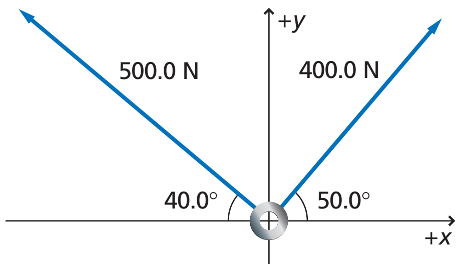


Figure 5-18

86. What is the net force acting on the ring in **Figure 5-19?**

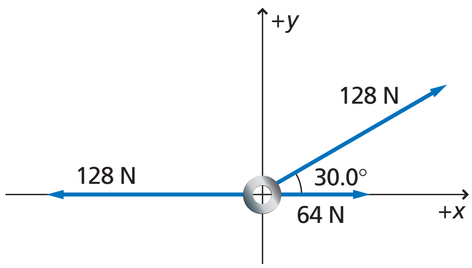


Figure 5-19

87. A Ship at Sea A ship at sea is due into a port 500.0 km due south in two days. However, a severe storm comes in and blows it 100.0 km due east from its original position. How far is the ship from its destination? In what direction must it travel to reach its destination?

88. Space Exploration A descent vehicle landing on Mars has a vertical velocity toward the surface of Mars of 5.5 m/s. At the same time, it has a horizontal velocity of 3.5 m/s.

a. At what speed does the vehicle move along its descent path?

b. At what angle with the vertical is this path?

89. Navigation Alfredo leaves camp and, using a compass, walks 4 km E, then 6 km S, 3 km E, 5 km N, 10 km W, 8 km N, and, finally, 3 km S. At the end of three days, he is lost. By drawing a diagram, compute how far Alfredo is from camp and which direction he should take to get back to camp.

5.2 Friction

90. If you use a horizontal force of 30.0 N to slide a 12.0-kg wooden crate across a floor at a constant velocity, what is the coefficient of kinetic friction between the crate and the floor?

91. A 225-kg crate is pushed horizontally with a force of 710 N. If the coefficient of friction is 0.20, calculate the acceleration of the crate.

92. A force of 40.0 N accelerates a 5.0-kg block at 6.0 m/s^2 along a horizontal surface.

a. How large is the frictional force?

b. What is the coefficient of friction?

93. Moving Appliances Your family just had a new refrigerator delivered. The delivery man has left and you realize that the refrigerator is not quite in the right position, so you plan to move it several centimeters. If the refrigerator has a mass of 180 kg, the coefficient of kinetic friction between the bottom of the refrigerator and the floor is 0.13, and the static coefficient of friction between these same surfaces is 0.21, how hard do you have to push horizontally to get the refrigerator to start moving?

94. Stopping at a Red Light You are driving a 2500.0-kg car at a constant speed of 14.0 m/s along a wet, but straight, level road. As you approach an intersection, the traffic light turns red. You slam on the brakes. The car's wheels lock, the tires begin skidding, and the car slides to a halt in a distance of 25.0 m. What is the coefficient of kinetic friction between your tires and the wet road?

5.3 Force and Motion in Two Dimensions

95. An object in equilibrium has three forces exerted on it. A 33.0-N force acts at 90.0° from the *x*-axis and a 44.0-N force acts at 60.0° from the *x*-axis. What are the magnitude and direction of the third force?

96. Five forces act on an object: (1) 60.0 N at 90.0°, (2) 40.0 N at 0.0°, (3) 80.0 N at 270.0°, (4) 40.0 N at 180.0°, and (5) 50.0 N at 60.0°. What are the magnitude and direction of a sixth force that would produce equilibrium?

97. Advertising Joe wishes to hang a sign weighing 7.50×10^2 N so that cable *A*, attached to the store, makes a 30.0° angle, as shown in **Figure 5-20.** Cable *B* is horizontal and attached to an adjoining building. What is the tension in cable *B*?

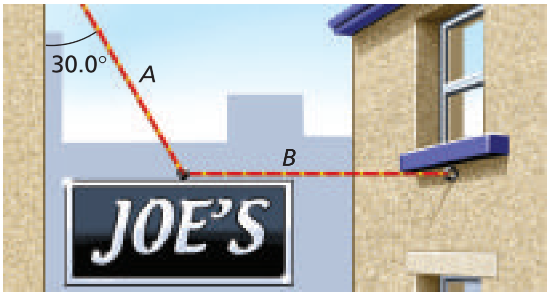


Figure 5-20

98. A street lamp weighs 150 N. It is supported by two wires that form an angle of 120.0° with each other. The tensions in the wires are equal.

a. What is the tension in each wire supporting the street lamp?

b. If the angle between the wires supporting the street lamp is reduced to 90.0°, what is the tension in each wire?

99. A 215-N box is placed on an inclined plane that makes a 35.0° angle with the horizontal. Find the component of the weight force parallel to the plane's surface.

100. **Emergency Room** You are shadowing a nurse in the emergency room of a local hospital. An orderly wheels in a patient who has been in a very serious accident and has had severe bleeding. The nurse quickly explains to you that in a case like this, the patient's bed will be tilted with the head downward to make sure the brain gets enough blood. She tells you that, for most patients, the largest angle that the bed can be tilted without the patient beginning to slide off is 32.0° from the horizontal.

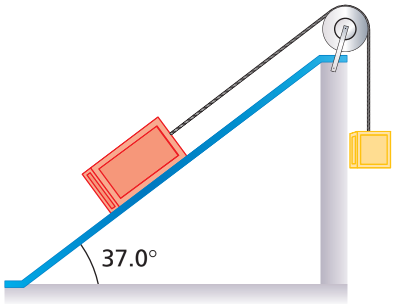
a. On what factor or factors does this angle of tilting depend?

b. Find the coefficient of static friction between a typical patient and the bed's sheets.

101. Two blocks are connected by a string over a frictionless, massless pulley such that one is resting on an inclined plane and the other is hanging over the top edge of the plane, as shown in **Figure 5-21.** The hanging block has a mass of 16.0 kg, and the one on the plane has a mass of 8.0 kg. The coefficient of kinetic friction between the block and the inclined plane is 0.23. The blocks are released from rest.

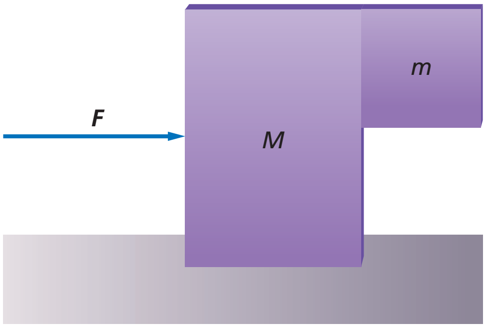
a. What is the acceleration of the blocks?

b. What is the tension in the string connecting the blocks?



■ **Figure 5-21**

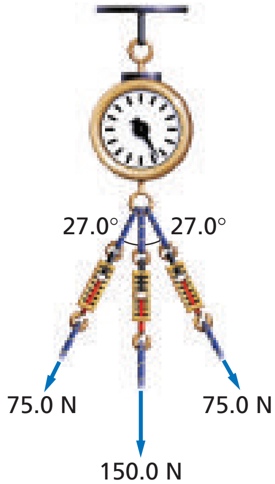
102. In **Figure 5-22,** a block of mass M is pushed with a force, F, such that the smaller block of mass m does not slide down the front of it. There is no friction between the larger block and the surface below it, but the coefficient of static friction between the two blocks is μ_s. Find an expression for F in terms of M, m, μ_s, and g.



■ **Figure 5-22**

Mixed Review

103. The scale in **Figure 5-23** is being pulled on by three ropes. What net force does the scale read?



■ **Figure 5-23**

104. **Sledding** A sled with a mass of 50.0 kg is pulled along flat, snow-covered ground. The static friction coefficient is 0.30, and the kinetic friction coefficient is 0.10.

a. What does the sled weigh?

b. What force will be needed to start the sled moving?

c. What force is needed to keep the sled moving at a constant velocity?

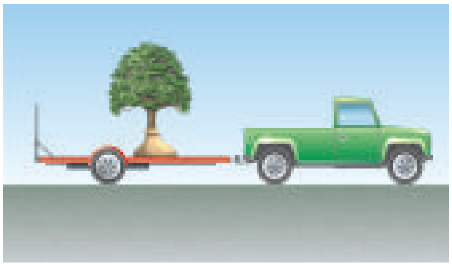
d. Once moving, what total force must be applied to the sled to accelerate it at 3.0 m/s^2?

105. Mythology Sisyphus was a character in Greek mythology who was doomed in Hades to push a boulder to the top of a steep mountain. When he reached the top, the boulder would slide back down the mountain and he would have to start all over again. Assume that Sisyphus slides the boulder up the mountain without being able to roll it, even though in most versions of the myth, he rolled it.

a. If the coefficient of kinetic friction between the boulder and the mountainside is 0.40, the mass of the boulder is 20.0 kg, and the slope of the mountain is a constant 30.0°, what is the force that Sisyphus must exert on the boulder to move it up the mountain at a constant velocity?

b. If Sisyphus pushes the boulder at a velocity of 0.25 m/s and it takes him 8.0 h to reach the top of the mountain, what is the mythical mountain's vertical height?

106. Landscaping A tree is being transported on a flatbed trailer by a landscaper, as shown in **Figure 5-24.** If the base of the tree slides on the trailer, the tree will fall over and be damaged. If the coefficient of static friction between the tree and the trailer is 0.50, what is the minimum stopping distance of the truck, traveling at 55 km/h, if it is to accelerate uniformly and not have the tree slide forward and fall on the trailer?



■ **Figure 5-24**

Thinking Critically

107. Use Models Using the Example Problems in this chapter as models, write an example problem to solve the following problem. Include the following sections: Analyze and Sketch the Problem, Solve for the Unknown (with a complete strategy), and Evaluate the Answer. A driver of a 975-kg car traveling 25 m/s puts on the brakes. What is the shortest distance it will take for the car to stop? Assume that the road is concrete, the force of friction of the road on the tires is constant, and the tires do not slip.

108. Analyze and Conclude Margaret Mary, Doug, and Kako are at a local amusement park and see an attraction called the Giant Slide, which is simply a very long and high inclined plane. Visitors at the amusement park climb a long flight of steps to the top of the 27° inclined plane and are given canvas sacks. They sit on the sacks and slide down the 70-m-long plane. At the time when the three friends walk past the slide, a 135-kg man and a 20-kg boy are each at the top preparing to slide down. "I wonder how much less time it will take the man to slide down than it will take the boy," says Margaret Mary. "I think the boy will take less time," says Doug. "You're both wrong," says Kako. "They will reach the bottom at the same time."

a. Perform the appropriate analysis to determine who is correct.

b. If the man and the boy do not take the same amount of time to reach the bottom of the slide, calculate how many seconds of difference there will be between the two times.

Writing in Physics

109. Investigate some of the techniques used in industry to reduce the friction between various parts of machines. Describe two or three of these techniques and explain the physics of how they work.

110. Olympics In recent years, many Olympic athletes, such as sprinters, swimmers, skiers, and speed skaters, have used modified equipment to reduce the effects of friction and air or water drag. Research a piece of equipment used by one of these types of athletes and the way it has changed over the years. Explain how physics has impacted these changes.

Cumulative Review

111. Add or subtract as indicated and state the answer with the correct number of significant digits. (Chapter 1)

a. 85.26 g + 4.7 g

b. 1.07 km + 0.608 km

c. 186.4 kg − 57.83 kg

d. 60.08 s − 12.2 s

112. You ride your bike for 1.5 h at an average velocity of 10 km/h, then for 30 min at 15 km/h. What is your average velocity? (Chapter 3)

113. A 45-N force is exerted in the upward direction on a 2.0-kg briefcase. What is the acceleration of the briefcase? (Chapter 4)

 For more problems, go to Additional Problems, Appendix B.

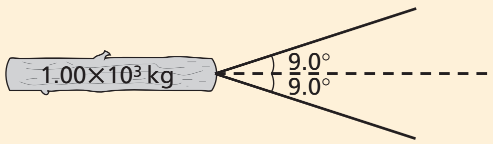
Standardized Test Practice

Multiple Choice

1. Two tractors pull against a 1.00×10^3-kg log. If the angle of the tractors' chains in relation to each other is 18.0°, and each tractor pulls with a force of 8×10^2 N, what forces will they be able to exert?

   (A) 250 N (C) 1.58×10^3 N

   (B) 1.52×10^3 N (D) 1.60×10^3 N



2. An airplane pilot tries to fly directly east with a velocity of 800.0 km/h. If a wind comes from the southwest at 80.0 km/h, what is the relative velocity of the airplane to the surface of Earth?

   (A) 804 km/h, 5.7° N of E

   (B) 858 km/h, 3.8° N of E

   (C) 859 km/h, 4.0° N of E

   (D) 880 km/h 45° N of E

3. For a winter fair, some students decide to build 30.0-kg wooden pull-carts on sled skids. If two 90.0-kg passengers get in, how much force will the puller have to exert to move a pull-cart? The coefficient of maximum static friction between the cart and the snow is 0.15.

   (A) 1.8×10^2 N (C) 2.1×10^3 N

   (B) 3.1×10^2 N (D) 1.4×10^4 N

4. It takes a minimum force of 280 N to move a 50.0-kg crate. What is the coefficient of maximum static friction between the crate and the floor?

   (A) 0.18 (C) 1.8

   (B) 0.57 (D) 5.6

5. What is the *y*-component of a 95.3-N force that is exerted at 57.1° to the horizontal?

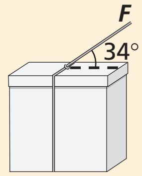
   (A) 51.8 N (C) 114 N

   (B) 80.0 N (D) 175 N

6. A string exerts a force of 18 N on a box at an angle of 34° from the horizontal. What is the horizontal component of the force on the box?

   (A) 10 N (C) 21.7 N

   (B) 15 N (D) 32 N



7. Sukey is riding her bicycle on a path when she comes around a corner and sees that a fallen tree is blocking the way 42 m ahead. If the coefficient of friction between her bicycle's tires and the gravel path is 0.36, and she is traveling at 50.0 km/h, how much stopping distance will she require? Sukey and her bicycle, together, have a mass of 95 kg.

   (A) 3.00 m (C) 8.12 m

   (B) 4.00 m (D) 27.3 m

Extended Answer

8. A man starts from a position 310 m north of his car and walks for 2.7 min in a westward direction at a constant velocity of 10 km/h. How far is he from his car when he stops?

9. Jeeves is tired of his 41.2-kg son sliding down the banister, so he decides to apply an extremely sticky paste that increases the coefficient of static friction to 0.72 to the top of the banister. What will be the magnitude of the static friction force on the boy if the banister is at an angle of 52.4° from the horizontal?

✓ Test-Taking TIP

Calculators Are Only Machines

If your test allows you to use a calculator, use it wisely. Figure out which numbers are relevant, and determine the best way to solve the problem before you start punching keys.

Chapter 6

Motion in Two Dimensions

What You'll Learn

- You will use Newton's laws and your knowledge of vectors to analyze motion in two dimensions.
- You will solve problems dealing with projectile and circular motion.
- You will solve relative-velocity problems.

Why It's Important

Almost all types of transportation and amusement-park attractions contain at least one element of projectile or circular motion or are affected by relative velocities.

Swinging Around
Before this ride starts to move, the seats hang straight down from their supports. When the ride speeds up, the seats swing out at an angle.

Think About This ▶

When the swings are moving around the circle at a constant speed, are they accelerating?

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LAUNCH Lab How can the motion of a projectile be described?

Question
Can you describe a projectile's motion in both the horizontal and the vertical directions?

Procedure

1. With a marked grid in the background, videotape a ball that is launched with an initial velocity only in the horizontal direction.
2. **Make and Use Graphs** On a sheet of graph paper, draw the location of the ball every 0.1 s (3 frames).
3. Draw two motion diagrams: one for the ball's horizontal motion and one for its vertical motion.

Analysis
How does the vertical motion change as time passes? Does it increase, decrease, or stay the same? How does the horizontal motion change as time passes? Does it increase, decrease, or stay the same?

Critical Thinking Describe the motion of a horizontally launched projectile.

6.1 Projectile Motion

If you observed the movement of a golf ball being hit from a tee, a frog hopping, or a free throw being shot with a basketball, you would notice that all of these objects move through the air along similar paths, as do baseballs, arrows, and bullets. Each path is a curve that moves upward for a distance, and then, after a time, turns and moves downward for some distance. You may be familiar with this curve, called a parabola, from math class.

An object shot through the air is called a **projectile.** A projectile can be a football, a bullet, or a drop of water. After a projectile is launched, what forces are exerted on the projectile? You can draw a free-body diagram of a launched projectile and identify all the forces that are acting on it. No matter what the object is, after a projectile has been given an initial thrust, if you ignore air resistance, it moves through the air only under the force of gravity. The force of gravity is what causes the object to curve downward in a parabolic flight path. Its path through space is called its **trajectory.** If you know the force of the initial thrust on a projectile, you can calculate its trajectory.

▶ **Objectives**

- **Recognize** that the vertical and horizontal motions of a projectile are independent.
- **Relate** the height, time in the air, and initial vertical velocity of a projectile using its vertical motion, and then determine the range using the horizontal motion.
- **Explain** how the trajectory of a projectile depends upon the frame of reference from which it is observed.

▶ **Vocabulary**

projectile
trajectory

Independence of Motion in Two Dimensions

Think about two softball players warming up for a game, tossing a ball back and forth. What does the path of the ball through the air look like? It looks like a parabola, as you just learned. Imagine that you are standing directly behind one of the players and you are watching the softball as it is being tossed. What would the motion of the ball look like? You would see it go up and back down, just like any object that is tossed straight up in the air. If you were watching the softball from a hot-air balloon high above the field, what motion would you see then? You would see the ball move from one player to the other at a constant speed, just like any object that is given an initial horizontal velocity, such as a hockey puck sliding across ice. The motion of projectiles is a combination of these two motions.

Why do projectiles behave in this way? After a softball leaves a player's hand, what forces are exerted on the ball? If you ignore air resistance, there are no contact forces on the ball. There is only the field force of gravity in the downward direction. How does this affect the ball's motion? Gravity causes the ball to have a downward acceleration.

Figure 6-1 shows the trajectories of two softballs. One was dropped and the other was given an initial horizontal velocity of 2.0 m/s. What is similar about the two paths? Look at their vertical positions. During each flash from the strobe light, the heights of the two softballs are the same. Because the change in vertical position is the same for both, their average vertical velocities during each interval are also the same. The increasingly large distance traveled vertically by the softballs, from one time interval to the next, shows that they are accelerated downward due to the force of gravity. Notice that the horizontal motion of the launched ball does not affect its vertical motion. A projectile launched horizontally has no initial vertical velocity. Therefore, its vertical motion is like that of an object dropped from rest. The downward velocity increases regularly because of the acceleration due to gravity.

MINI LAB

Over the Edge

Obtain two balls, one twice the mass of the other.

1. Predict which ball will hit the floor first when you roll them over the surface of a table and let them roll off the edge.

2. Predict which ball will hit the floor furthest from the table.

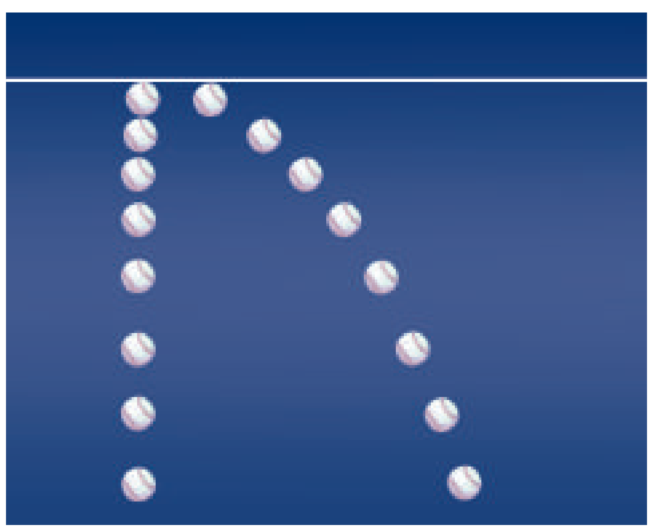
3. Explain your predictions.

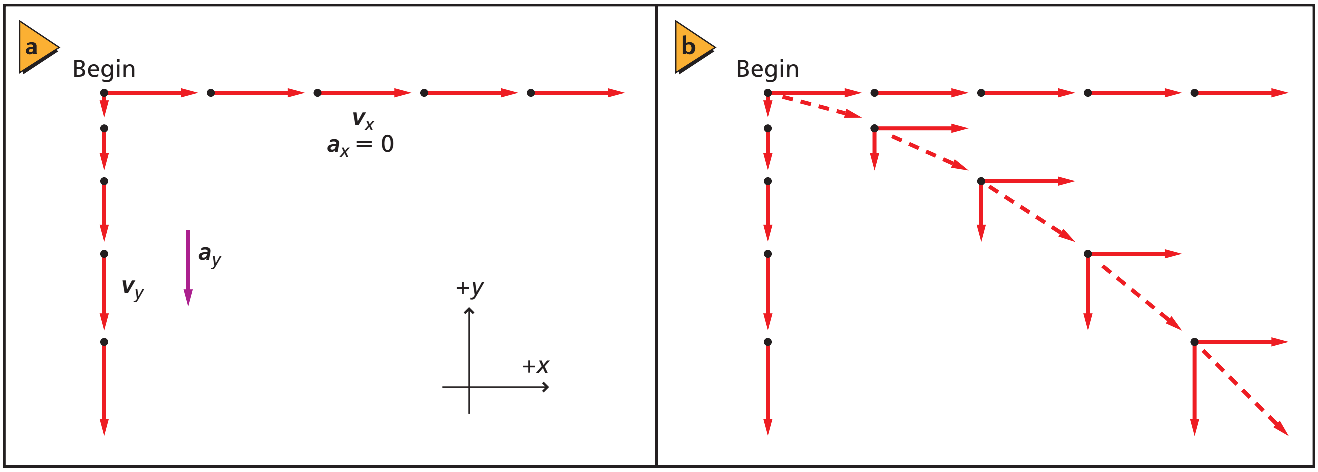
4. Test your predictions.

Analyze and Conclude

5. Does the mass of the ball affect its motion? Is mass a factor in any of the equations for projectile motion?

Figure 6-1 The ball on the right was given an initial horizontal velocity. The ball on the left was dropped at the same time from rest. Note that the vertical positions of the two objects are the same during each flash.





■ **Figure 6-2** A object's motion can be broken into its x- and y-components **(a).** When the horizontal and vertical components of the ball's velocity are combined **(b),** the resultant vectors are tangent to a parabola.

Separate motion diagrams for the horizontal and vertical motions are shown in **Figure 6-2a.** The vertical-motion diagram represents the motion of the dropped ball. The horizontal-motion diagram shows the constant velocity in the x-direction of the launched ball. This constant velocity in the horizontal direction is exactly what should be expected because there is no horizontal force acting on the ball.

In **Figure 6-2b,** the horizontal and vertical components are added to form the total velocity vector for the projectile. You can see how the combination of constant horizontal velocity and uniform vertical acceleration produces a trajectory that has a parabolic shape.

PROBLEM-SOLVING **Strategies**

Motion in Two Dimensions

Projectile motion in two dimensions can be determined by breaking the problem into two connected one-dimensional problems.

1. Divide the projectile motion into a vertical motion problem and a horizontal motion problem.
2. The vertical motion of a projectile is exactly that of an object dropped or thrown straight up or straight down. A gravitational force acts on the object and accelerates it by an amount, g. Review Section 3.3 on free fall to refresh your problem-solving skills for vertical motion.
3. Analyzing the horizontal motion of a projectile is the same as solving a constant velocity problem. No horizontal force acts on a projectile when drag due to air resistance is neglected. Consequently, there are no forces acting in the horizontal direction and therefore, no horizontal acceleration; $a_x = 0.0$ m/s. To solve, use the same methods that you learned in Section 2.4.
4. Vertical motion and horizontal motion are connected through the variable of time. The time from the launch of the projectile to the time it hits the target is the same for both vertical motion and horizontal motion. Therefore, solving for time in one of the dimensions, vertical or horizontal, automatically gives you time for the other dimension.

▶ PRACTICE Problems

Additional Problems, Appendix B

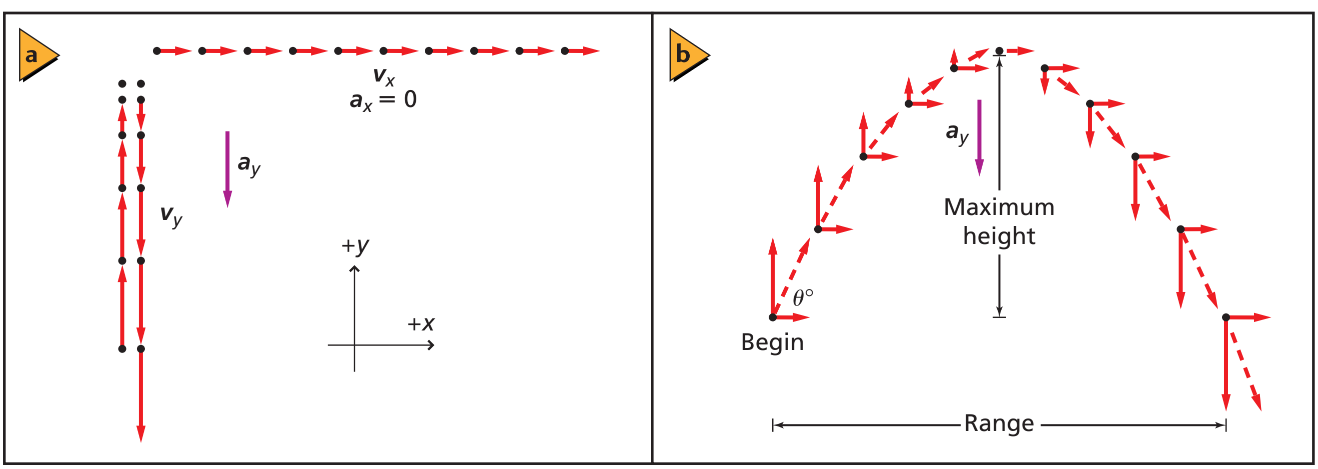
1. A stone is thrown horizontally at a speed of 5.0 m/s from the top of a cliff that is 78.4 m high.
   - **a.** How long does it take the stone to reach the bottom of the cliff?
   - **b.** How far from the base of the cliff does the stone hit the ground?
   - **c.** What are the horizontal and vertical components of the stone's velocity just before it hits the ground?
2. Lucy and her friend are working at an assembly plant making wooden toy giraffes. At the end of the line, the giraffes go horizontally off the edge of the conveyor belt and fall into a box below. If the box is 0.6 m below the level of the conveyor belt and 0.4 m away from it, what must be the horizontal velocity of giraffes as they leave the conveyor belt?
3. You are visiting a friend from elementary school who now lives in a small town. One local amusement is the ice-cream parlor, where Stan, the short-order cook, slides his completed ice-cream sundaes down the counter at a constant speed of 2.0 m/s to the servers. (The counter is kept very well polished for this purpose.) If the servers catch the sundaes 7.0 cm from the edge of the counter, how far do they fall from the edge of the counter to the point at which the servers catch them?

Projectiles Launched at an Angle

When a projectile is launched at an angle, the initial velocity has a vertical component, as well as a horizontal component. If the object is launched upward, like a ball tossed straight up in the air, it rises with slowing speed, reaches the top of its path, and descends with increasing speed. **Figure 6-3a** shows the separate vertical- and horizontal-motion diagrams for the trajectory. In the coordinate system, the positive x-axis is horizontal and the positive y-axis is vertical. Note the symmetry. At each point in the vertical direction, the velocity of the object as it is moving upward has the same magnitude as when it is moving downward. The only difference is that the directions of the two velocities are opposite.

Figure 6-3b defines two quantities associated with the trajectory. One is the maximum height, which is the height of the projectile when the vertical velocity is zero and the projectile has only its horizontal-velocity component. The other quantity depicted is the range, R, which is the horizontal distance that the projectile travels. Not shown is the flight time, which is how much time the projectile is in the air. For football punts, flight time often is called hang time.

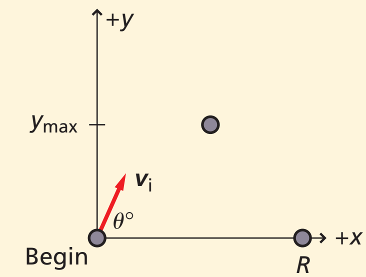
■ **Figure 6-3** The vector sum of $\boldsymbol{v}_x$ and $\boldsymbol{v}_y$ at each position points in the direction of the flight.



▶ EXAMPLE Problem 1

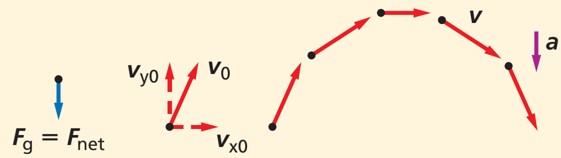
The Flight of a Ball A ball is launched at 4.5 m/s at 66° above the horizontal. What are the maximum height and flight time of the ball?

1 Analyze and Sketch the Problem



- Establish a coordinate system with the initial position of the ball at the origin.
- Show the positions of the ball at the beginning, at the maximum height, and at the end of the flight.
- Draw a motion diagram showing $\boldsymbol{v}$, $\boldsymbol{a}$, and $\boldsymbol{F}_{net}$.

Known:		**Unknown:**
$y_i = 0.0$ m	$\theta_i = 66°$	$y_{max} = ?$
$v_i = 4.5$ m/s	$a_y = -g$	$t = ?$



2 Solve for the Unknown

Find the y-component of v_i.

$$\begin{aligned} v_{yi} &= v_i(\sin\theta_i) \\ &= (4.5\text{ m/s})(\sin 66°) && \textbf{Substitute } v_i = 4.5\text{ m/s}, \theta_i = 66° \\ &= 4.1\text{ m/s} \end{aligned}$$

Find an expression for time.

$$\begin{aligned} v_y &= v_{yi} + a_y t \\ &= v_{yi} - gt && \textbf{Substitute } a_y = -g \\ t &= \frac{v_{yi} - v_y}{g} && \textbf{Solve for } t. \end{aligned}$$

Solve for the maximum height.

$$\begin{aligned} y_{max} &= y_i + v_{yi}t + \frac{1}{2}at^2 \\ &= y_i + v_{yi}\left(\frac{v_{yi} - v_y}{g}\right) + \frac{1}{2}(-g)\left(\frac{v_{yi} - v_y}{g}\right)^2 && \textbf{Substitute } t = \frac{v_{yi} - v_y}{g}, a = -g \\ &= 0.0\text{ m} + (4.1\text{ m/s})\left(\frac{4.1\text{ m/s} - 0.0\text{ m/s}}{9.80\text{ m/s}^2}\right) + \frac{1}{2}(-9.80\text{ m/s}^2)\left(\frac{4.1\text{ m/s} - 0.0\text{ m/s}}{9.80\text{ m/s}^2}\right)^2 \\ &= 0.86\text{ m} \end{aligned}$$

Substitute $y_i = 0.0$ m, $v_{yi} = 4.1$ m/s, $v_y = 0.0$ m/s at y_{max}, $g = 9.80$ m/s²

Solve for the time to return to the launching height.

$$\begin{aligned} y_f &= y_i + v_{yi}t + \frac{1}{2}at^2 \\ 0.0\text{ m} &= 0.0\text{ m} + v_{yi}t - \frac{1}{2}gt^2 && \textbf{Substitute } y_f = 0.0\text{ m}, y_i = 0.0\text{ m}, a = -g \\ t &= \frac{-v_{yi} \pm \sqrt{v_{yi}^2 - 4\left(-\frac{1}{2}g\right)(0.0\text{ m})}}{2\left(-\frac{1}{2}g\right)} && \textbf{Use the quadratic formula to solve for } t. \\ &= \frac{-v_{yi} \pm v_{yi}}{-g} \\ &= \frac{2v_{yi}}{g} && \textbf{0 is the time the ball left the launch, so use this solution.} \\ &= \frac{(2)(4.1\text{ m/s})}{(9.80\text{ m/s}^2)} && \textbf{Substitute } v_{yi} = 4.1\text{ m/s}, g = 9.80\text{ m/s}^2 \\ &= 0.84\text{ s} \end{aligned}$$

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3 Evaluate the Answer

- **Are the units correct?** Dimensional analysis verifies that the units are correct.
- **Do the signs make sense?** All should be positive.
- **Are the magnitudes realistic?** 0.84 s is fast, but an initial velocity of 4.5 m/s makes this time reasonable.

PRACTICE Problems

Additional Problems, Appendix B

4. A player kicks a football from ground level with an initial velocity of 27.0 m/s, 30.0° above the horizontal, as shown in **Figure 6-4.** Find each of the following. Assume that air resistance is negligible.

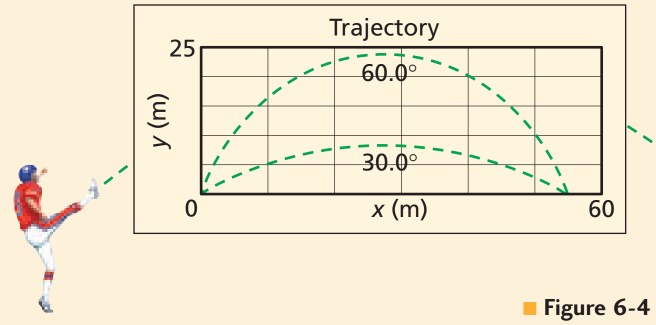
a. the ball's hang time

b. the ball's maximum height

c. the ball's range

5. The player in problem 4 then kicks the ball with the same speed, but at 60.0° from the horizontal. What is the ball's hang time, range, and maximum height?

6. A rock is thrown from a 50.0-m-high cliff with an initial velocity of 7.0 m/s at an angle of 53.0° above the horizontal. Find the velocity vector for when it hits the ground below.



■ **Figure 6-4**

Trajectories Depend upon the Viewer

Suppose you toss a ball up and catch it while riding in a bus. To you, the ball would seem to go straight up and straight down. But what would an observer on the sidewalk see? The observer would see the ball leave your hand, rise up, and return to your hand, but because the bus would be moving, your hand also would be moving. The bus, your hand, and the ball would all have the same horizontal velocity. Thus, the trajectory of the ball would be similar to that of the ball in Example Problem 1.

Air resistance So far, air resistance has been ignored in the analysis of projectile motion. While the effects of air resistance are very small for some projectiles, for others, the effects are large and complex. For example, dimples on a golf ball reduce air resistance and maximize its range. In baseball, the spin of the ball creates forces that can deflect the ball. For now, just remember that the force due to air resistance does exist and it can be important.

6.1 Section Review

7. Projectile Motion Two baseballs are pitched horizontally from the same height, but at different speeds. The faster ball crosses home plate within the strike zone, but the slower ball is below the batter's knees. Why does the faster ball not fall as far as the slower one?

8. Free-Body Diagram An ice cube slides without friction across a table at a constant velocity. It slides off the table and lands on the floor. Draw free-body and motion diagrams of the ice cube at two points on the table and at two points in the air.

9. Projectile Motion A softball is tossed into the air at an angle of 50.0° with the vertical at an initial velocity of 11.0 m/s. What is its maximum height?

10. Projectile Motion A tennis ball is thrown out a window 28 m above the ground at an initial velocity of 15.0 m/s and 20.0° below the horizontal. How far does the ball move horizontally before it hits the ground?

11. Critical Thinking Suppose that an object is thrown with the same initial velocity and direction on Earth and on the Moon, where g is one-sixth that on Earth. How will the following quantities change?

a. v_x

b. the object's time of flight

c. y_{max}

d. R

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6.2 Circular Motion

Consider an object moving in a circle at a constant speed, such as a stone being whirled on the end of a string or a fixed horse on a merry-go-round. Are these objects accelerating? At first, you might think that they are not because their speeds do not change. However, remember that acceleration is the change in velocity, not just the change in speed. Because their direction is changing, the objects must be accelerating.

▶ Objectives

- **Explain** why an object moving in a circle at a constant speed is accelerated.
- **Describe** how centripetal acceleration depends upon the object's speed and the radius of the circle.
- **Identify** the force that causes centripetal acceleration.

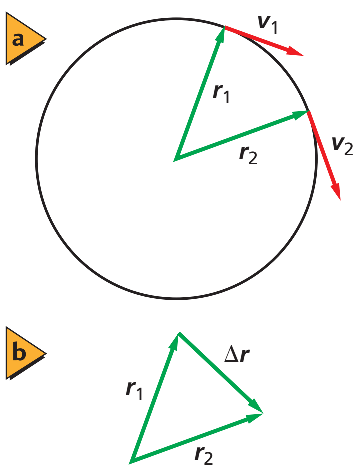
▶ Vocabulary

uniform circular motion
centripetal acceleration
centripetal force

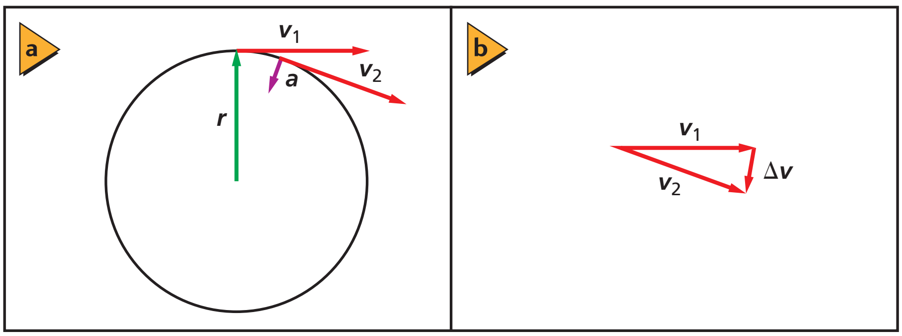
Describing Circular Motion

Uniform circular motion is the movement of an object or particle trajectory at a constant speed around a circle with a fixed radius. The positions of an object in uniform circular motion, relative to the center of the circle, are given by the position vectors $\boldsymbol{r}_1$ and $\boldsymbol{r}_2$, shown in **Figure 6-5a.** As the object moves around the circle, the length of each position vector does not change, but its direction does. To find the object's velocity, you need to find its displacement vector over a time interval. The change in position, or the object's displacement, is represented by $\Delta\boldsymbol{r}$. **Figure 6-5b** shows two position vectors: $\boldsymbol{r}_1$ at the beginning of a time interval, and $\boldsymbol{r}_2$ at the end of the time interval. Remember that a position vector is a displacement vector with its tail at the origin. In the vector diagram, $\boldsymbol{r}_1$ and $\boldsymbol{r}_2$ are subtracted to give the resultant $\Delta\boldsymbol{r}$, the displacement during the time interval. You know that a moving object's average velocity is $\Delta\boldsymbol{d}/\Delta t$, so for an object in circular motion, $\bar{v} = \Delta\boldsymbol{r}/\Delta t$. The velocity vector has the same direction as the displacement, but a different length. You can see in **Figure 6-6a** that the velocity is at right angles to the position vector, which is tangent to its circular path. As the velocity vector moves around the circle, its direction changes but its length remains the same.

What is the direction of the object's acceleration? Figure 6-6a shows the velocity vectors $\boldsymbol{v}_1$ and $\boldsymbol{v}_2$ at the beginning and end of a time interval. The difference in the two vectors, $\Delta\boldsymbol{v}$, is found by subtracting the vectors, as shown in **Figure 6-6b.** The average acceleration, $\bar{\boldsymbol{a}} = \Delta\boldsymbol{v}/\Delta t$, is in the same direction as $\Delta\boldsymbol{v}$; that is, toward the center of the circle. Repeat this process for several other time intervals when the object is in different locations on the circle. As the object moves around the circle, the direction of the acceleration vector changes, but its length remains the same. Notice that the acceleration vector of an object in uniform circular motion always points in toward the center of the circle. For this reason, the acceleration of such an object is called center-seeking or **centripetal acceleration.**



■ **Figure 6-5** The displacement, Δr, of an object in circular motion, divided by the time interval in which the displacement occurs, is the object's average velocity during that time interval.



■ **Figure 6-6** The direction of the change in velocity is toward the center of the circle, and so the acceleration vector also points to the center of the circle.

Centripetal Acceleration

What is the magnitude of an object's centripetal acceleration? Compare the triangle made from the position vectors in Figure 6-5b with the triangle made by the velocity vectors in Figure 6-6b. The angle between $\boldsymbol{r}_1$ and $\boldsymbol{r}_2$ is the same as that between $\boldsymbol{v}_1$ and $\boldsymbol{v}_2$. Therefore, the two triangles formed by subtracting the two sets of vectors are similar triangles, and the ratios of the lengths of two corresponding sides are equal. Thus, $\Delta r/r = \Delta v/v$. The equation is not changed if both sides are divided by Δt.

$$\frac{\Delta r}{r\Delta t} = \frac{\Delta v}{v\Delta t}$$

However, $v = \Delta r/\Delta t$ and $a = \Delta v/\Delta t$.

$$\frac{1}{r}\left(\frac{\Delta r}{\Delta t}\right) = \frac{1}{v}\left(\frac{\Delta v}{\Delta t}\right)$$

Substituting $v = \Delta r/\Delta t$ in the left-hand side and $a = \Delta v/\Delta t$ in the right-hand side gives the following equation.

$$\frac{v}{r} = \frac{a}{v}$$

Solve this equation for acceleration and give it the special symbol a_c, for centripetal acceleration.

Centripetal Acceleration $a_c = \frac{v^2}{r}$

Centripetal acceleration always points to the center of the circle. Its magnitude is equal to the square of the speed, divided by the radius of motion.

How can you measure the speed of an object moving in a circle? One way is to measure its period, T, the time needed for the object to make one complete revolution. During this time, the object travels a distance equal to the circumference of the circle, $2\pi r$. The object's speed, then, is represented by $v = 2\pi r/T$. If this expression is substituted for v in the equation for centripetal acceleration, the following equation is obtained.

$$a_c = \frac{\left(\frac{2\pi r}{T}\right)^2}{r} = \frac{4\pi^2 r}{T^2}$$

Because the acceleration of an object moving in a circle is always in the direction of the net force acting on it, there must be a net force toward the center of the circle. This force can be provided by any number of agents. For Earth circling the Sun, the force is the Sun's gravitational force on Earth, as you'll learn in Chapter 7. When a hammer thrower swings the hammer, as in **Figure 6-7,** the force is the tension in the chain attached to the massive ball. When an object moves in a circle, the net force toward the center of the circle is called the **centripetal force.** To accurately analyze centripetal acceleration situations, you must identify the agent of the force that causes the acceleration. Then you can apply Newton's second law for the component in the direction of the acceleration in the following way.

Newton's Second Law for Circular Motion $F_{net} = ma_c$

The net centripetal force on an object moving in a circle is equal to the object's mass, times the centripetal acceleration.

APPLYING PHYSICS

▶ **Space Elevators** Scientists are considering the use of space elevators as a low-cost transportation system to space. A cable would be anchored to a station at Earth's equator, and the cable would extend almost 35,800 km from Earth's surface. The cable would be attached to a counterweight and would stay extended due to centripetal force. Special magnetically powered vehicles would then travel along the cable. ◀

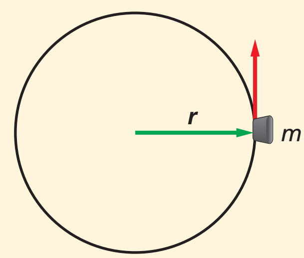
■ **Figure 6-7** When the thrower lets go, the hammer initially moves in a straight line that is tangent to the point of release. Then it follows a trajectory like that of any object released into the air with an initial horizontal velocity.

When solving problems, you have found it useful to choose a coordinate system with one axis in the direction of the acceleration. For circular motion, the direction of the acceleration is always toward the center of the circle. Rather than labeling this axis x or y, call it c, for centripetal acceleration. The other axis is in the direction of the velocity, tangent to the circle. It is labeled *tang* for tangential. You will apply Newton's second law in these directions, just as you did in the two-dimensional problems in Chapter 5. Remember that centripetal force is just another name for the net force in the centripetal direction. It is the sum of all the real forces, those for which you can identify agents that act along the centripetal axis.

In the case of the hammer thrower in Figure 6-7, in what direction does the hammer fly when the chain is released? Once the contact force of the chain is gone, there is no force accelerating the hammer toward the center of the circle, so the hammer flies off in the direction of its velocity, which is tangent to the circle. Remember, if you cannot identify the agent of the force, then it does not exist.

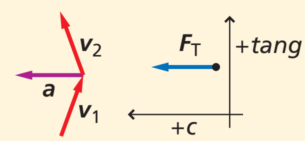
EXAMPLE Problem 2

Uniform Circular Motion **A 13-g rubber stopper is attached to a 0.93-m string. The stopper is swung in a horizontal circle, making one revolution in 1.18 s. Find the tension force exerted by the string on the stopper.**



1 Analyze and Sketch the Problem

- Draw a free-body diagram for the swinging stopper.
- Include the radius and the direction of motion.
- Establish a coordinate system labeled *tang* and c. The directions of a and F_T are parallel to c.



Known:	**Unknown:**
$m = 13$ g	$F_T = ?$
$r = 0.93$ m	
$T = 1.18$ s	

2 Solve for the Unknown

Find the centripetal acceleration.

$$a_c = \frac{4\pi^2 r}{T^2}$$

$$= \frac{4\pi^2(0.93\text{ m})}{(1.18\text{ s})^2} \qquad \textbf{Substitute } r = \textbf{0.93 m, } T = \textbf{1.18 s}$$

$$= 26\text{ m/s}^2$$

Use Newton's second law to find the tension in the string.

$$F_T = ma_c$$

$$= (0.013\text{ kg})(26\text{ m/s}^2) \qquad \textbf{Substitute } m = \textbf{0.013 kg, } a_c = \textbf{26 m/s}^2$$

$$= 0.34\text{ N}$$

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3 Evaluate the Answer

- **Are the units correct?** Dimensional analysis verifies that a is in m/s^2 and F is in N.
- **Do the signs make sense?** The signs should all be positive.
- **Are the magnitudes realistic?** The force is almost three times the weight of the stopper, and the acceleration is almost three times that of gravity, which is reasonable for such a light object.

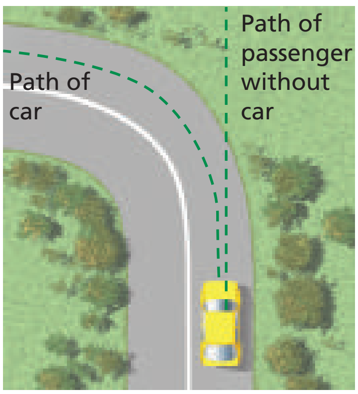
PRACTICE Problems
Additional Problems, Appendix B

12. A runner moving at a speed of 8.8 m/s rounds a bend with a radius of 25 m. What is the centripetal acceleration of the runner, and what agent exerts force on the runner?

13. A car racing on a flat track travels at 22 m/s around a curve with a 56-m radius. Find the car's centripetal acceleration. What minimum coefficient of static friction between the tires and road is necessary for the car to round the curve without slipping?

14. An airplane traveling at 201 m/s makes a turn. What is the smallest radius of the circular path (in km) that the pilot can make and keep the centripetal acceleration under 5.0 m/s^2?

15. A 45-kg merry-go-round worker stands on the ride's platform 6.3 m from the center. If her speed as she goes around the circle is 4.1 m/s, what is the force of friction necessary to keep her from falling off the platform?



■ **Figure 6-8** The passenger would move forward in a straight line if the car did not exert an inward force.

A Nonexistent Force

If a car makes a sharp left turn, a passenger on the right side might be thrown against the right door. Is there an outward force on the passenger? Consider a similar situation. If a car in which you are riding stops suddenly, you will be thrown forward into your safety belt. Is there a forward force on you? No, because according to Newton's first law, you will continue moving with the same velocity unless there is a net force acting on you. The safety belt applies the force that accelerates you to a stop. **Figure 6-8** shows a car turning to the left as viewed from above. A passenger in the car would continue to move straight ahead if it were not for the force of the door acting in the direction of the acceleration; that is, toward the center of the circle. Thus, there is no outward force on the passenger. The so-called centrifugal, or outward force, is a fictitious, nonexistent force. Newton's laws are able to explain motion in both straight lines and circles.

6.2 Section Review

16. Uniform Circular Motion What is the direction of the force that acts on the clothes in the spin cycle of a washing machine? What exerts the force?

17. Free-Body Diagram You are sitting in the backseat of a car going around a curve to the right. Sketch motion and free-body diagrams to answer the following questions.

a. What is the direction of your acceleration?

b. What is the direction of the net force that is acting on you?

c. What exerts this force?

18. Centripetal Force If a 40.0-g stone is whirled horizontally on the end of a 0.60-m string at a speed of 2.2 m/s, what is the tension in the string?

19. Centripetal Acceleration A newspaper article states that when turning a corner, a driver must be careful to balance the centripetal and centrifugal forces to keep from skidding. Write a letter to the editor that critiques this article.

20. Centripetal Force A bowling ball has a mass of 7.3 kg. If you move it around a circle with a radius of 0.75 m at a speed of 2.5 m/s, what force would you have to exert on it?

21. Critical Thinking Because of Earth's daily rotation, you always move with uniform circular motion. What is the agent that supplies the force that accelerates you? How does this motion affect your apparent weight?

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6.3 Relative Velocity

Objectives

- **Analyze** situations in which the coordinate system is moving.
- **Solve** relative-velocity problems.

Suppose that you are in a school bus that is traveling at a velocity of 8 m/s in a positive direction. You walk with a velocity of 3 m/s toward the front of the bus. If a friend of yours is standing on the side of the road watching the bus with you on it go by, how fast would your friend say that you are moving? If the bus is traveling at 8 m/s, this means that the velocity of the bus is 8 m/s, as measured by your friend in a coordinate system fixed to the road. When you are standing still, your velocity relative to the road is also 8 m/s, but your velocity relative to the bus is zero. Walking at 3 m/s toward the front of the bus means that your velocity is measured relative to the bus. The problem can be rephrased as follows: Given the velocity of the bus relative to the road and your velocity relative to the bus, what is your velocity relative to the road?

A vector representation of this problem is shown in **Figure 6-9a.** After studying it, you will find that your velocity relative to the street is 11 m/s, the sum of 8 m/s and 3 m/s. Suppose that you now walk at the same speed toward the rear of the bus. What would be your velocity relative to the road? **Figure 6-9b** shows that because the two velocities are in opposite directions, the resultant velocity is 5 m/s, the difference between 8 m/s and 3 m/s. You can see that when the velocities are along the same line, simple addition or subtraction can be used to determine the relative velocity.

Take a closer look at how these results were obtained and see if you can find a mathematical rule to describe how velocities are combined in these relative-velocity situations. For the above situation, you can designate the velocity of the bus relative to the road as $\boldsymbol{v}_{b/r}$, your velocity relative to the bus as $\boldsymbol{v}_{y/b}$, and the velocity of you relative to the road as $\boldsymbol{v}_{y/r}$. To find the velocity of you relative to the road in both cases, you vectorially added the velocities of you relative to the bus and the bus relative to the road. Mathematically, this is represented as $\boldsymbol{v}_{y/b} + \boldsymbol{v}_{b/r} = \boldsymbol{v}_{y/r}$. The more general form of this equation is as follows.

Relative Velocity $\boldsymbol{v}_{a/b} + \boldsymbol{v}_{b/c} = \boldsymbol{v}_{a/c}$

The relative velocity of object a to object c is the vector sum of object a's velocity relative to object b and object b's velocity relative to object c.

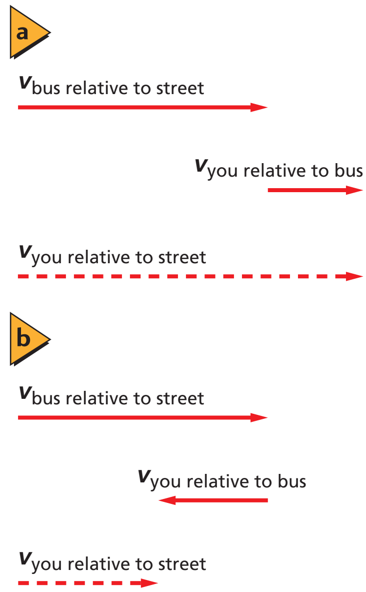


Figure 6-9 When a coordinate system is moving, two velocities are added if both motions are in the same direction and one is subtracted from the other if the motions are in opposite directions.

CHALLENGE PROBLEM

Phillipe whirls a stone of mass *m* on a rope in a perfect horizontal circle above his head such that the stone is at a height, *h*, above the ground. The circle has a radius of *r*, and the tension in the rope is *T*. Suddenly the rope breaks and the stone falls to the ground. The stone travels a horizontal distance, *s*, from the time the rope breaks until it impacts the ground. Find a mathematical expression for *s* in terms of *T*, *r*, *m*, and *h*. Does your expression change if Phillipe is walking 0.50 m/s relative to the ground?

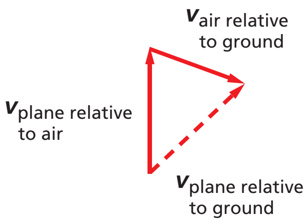


Figure 6-10 The plane's velocity relative to the ground can be obtained by vector addition.

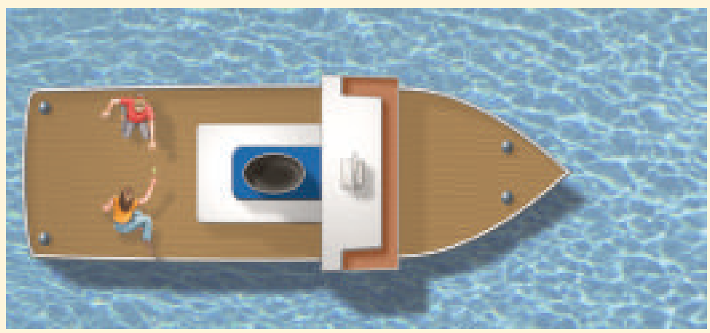
This method for adding relative velocities also applies to motion in two dimensions. For example, airline pilots cannot expect to reach their destinations by simply aiming their planes along a compass direction. They must take into account the plane's speed relative to the air, which is given by their airspeed indicators, and their direction of flight relative to the air. They also must consider the velocity of the wind at the altitude they are flying relative to the ground. These two vectors must be combined, as shown in **Figure 6-10**, to obtain the velocity of the airplane relative to the ground. The resultant vector tells the pilot how fast and in what direction the plane must travel relative to the ground to reach its destination. A similar situation occurs for boats traveling on water with a flowing current.

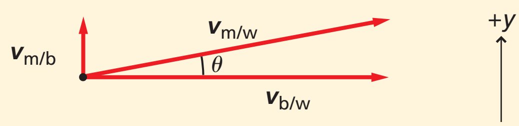
EXAMPLE Problem 3

Relative Velocity of a Marble Ana and Sandra are riding on a ferry boat that is traveling east at a speed of 4.0 m/s. Sandra rolls a marble with a velocity of 0.75 m/s north, straight across the deck of the boat to Ana. What is the velocity of the marble relative to the water?

1 Analyze and Sketch the Problem

- Establish a coordinate system.
- Draw vectors to represent the velocities of the boat relative to the water and the marble relative to the boat.





Known:

$v_{b/w} = 4.0$ m/s

$v_{m/b} = 0.75$ m/s

Unknown:

$v_{m/w} = ?$

2 Solve for the Unknown

Because the two velocities are at right angles, use the Pythagorean theorem.

$$v_{m/w}^2 = v_{b/w}^2 + v_{m/b}^2$$

$$v_{m/w} = \sqrt{v_{b/w}^2 + v_{m/b}^2}$$

$$= \sqrt{(4.0\text{ m/s})^2 + (0.75\text{ m/s})^2}$$ Substitute $v_{b/w} = 4.0$ m/s, $v_{m/b} = 0.75$ m/s

$$= 4.1\text{ m/s}$$

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Inverses of Sine, Cosine, and Tangent
page 856

Find the angle of the marble's motion.

$$\theta = \tan^{-1}\left(\frac{v_{m/b}}{v_{b/w}}\right)$$

$$= \tan^{-1}\left(\frac{0.75\text{ m/s}}{4.0\text{ m/s}}\right)$$ Substitute $v_{b/w} = 4.0$ m/s, $v_{m/b} = 0.75$ m/s

$$= 11° \text{ north of east}$$

The marble is traveling 4.1 m/s at 11° north of east.

3 Evaluate the Answer

- **Are the units correct?** Dimensional analysis verifies that the velocity is in m/s.
- **Do the signs make sense?** The signs should all be positive.
- **Are the magnitudes realistic?** The resulting velocity is of the same order of magnitude as the velocities given in the problem.

PRACTICE Problems

Additional Problems, Appendix B

22. You are riding in a bus moving slowly through heavy traffic at 2.0 m/s. You hurry to the front of the bus at 4.0 m/s relative to the bus. What is your speed relative to the street?

23. Rafi is pulling a toy wagon through the neighborhood at a speed of 0.75 m/s. A caterpillar in the wagon is crawling toward the rear of the wagon at a rate of 2.0 cm/s. What is the caterpillar's velocity relative to the ground?

24. A boat is rowed directly upriver at a speed of 2.5 m/s relative to the water. Viewers on the shore see that the boat is moving at only 0.5 m/s relative to the shore. What is the speed of the river? Is it moving with or against the boat?

25. An airplane flies due north at 150 km/h relative to the air. There is a wind blowing at 75 km/h to the east relative to the ground. What is the plane's speed relative to the ground?

Another example of combined relative velocities is the navigation of migrating neotropical songbirds. In addition to knowing in which direction to fly, a bird must account for its speed relative to the air and its direction relative to the ground. If a bird tries to fly over the Gulf of Mexico into too strong a headwind, it will run out of energy before it reaches the other shore and will perish. Similarly, the bird must account for crosswinds or it will not reach its destination. You can add relative velocities even if they are at arbitrary angles by using the graphical methods that you learned in Chapter 5.

Biology Connection

Remember that the key to properly analyzing a two-dimensional relative-velocity situation is drawing the proper triangle to represent the three velocities. Once you have this triangle, you simply apply your knowledge of vector addition from Chapter 5. If the situation contains two velocities that are perpendicular to each other, you can find the third by applying the Pythagorean theorem; however, if the situation has no right angles, you will need to use one or both of the laws of sines and cosines.

6.3 Section Review

26. Relative Velocity A fishing boat with a maximum speed of 3 m/s relative to the water is in a river that is flowing at 2 m/s. What is the maximum speed the boat can obtain relative to the shore? The minimum speed? Give the direction of the boat, relative to the river's current, for the maximum speed and the minimum speed relative to the shore.

27. Relative Velocity of a Boat A powerboat heads due northwest at 13 m/s relative to the water across a river that flows due north at 5.0 m/s. What is the velocity (both magnitude and direction) of the motorboat relative to the shore?

28. Relative Velocity An airplane flies due south at 175 km/h relative to the air. There is a wind blowing at 85 km/h to the east relative to the ground. What are the plane's speed and direction relative to the ground?

29. A Plane's Relative Velocity An airplane flies due north at 235 km/h relative to the air. There is a wind blowing at 65 km/h to the northeast relative to the ground. What are the plane's speed and direction relative to the ground?

30. Relative Velocity An airplane has a speed of 285 km/h relative to the air. There is a wind blowing at 95 km/h at 30.0° north of east relative to Earth. In which direction should the plane head to land at an airport due north of its present location? What is the plane's speed relative to the ground?

31. Critical Thinking You are piloting a boat across a fast-moving river. You want to reach a pier directly opposite your starting point. Describe how you would navigate the boat in terms of the components of your velocity relative to the water.

PHYSICS LAB • Design Your Own

On Target

In this activity, you will analyze several factors that affect the motion of a projectile and use your understanding of these factors to predict the path of a projectile. Finally, you will design a projectile launcher and hit a target a known distance away.

QUESTION

What factors affect the path of a projectile?

Objectives

- **Formulate models** and then summarize the factors that affect the motion of a projectile.
- **Use models** to predict where a projectile will land.

Safety Precautions

Possible Materials

duct tape
plastic ware
rubber bands
paper clips
paper
masking tape
wood blocks
nails
hammer
PVC tubing
handsaw
scissors
coat hanger
chicken wire
wire cutter

Procedure

1. Brainstorm and list as many factors as you are able to think of that may affect the path of a projectile.
2. Create a design for your projectile launcher and decide what object will be your projectile shot by your launcher.
3. Taking the design of your launcher into account, determine which two factors are most likely to have a significant effect on the flight path of your projectile.
4. Check the design of your launcher and discuss your two factors with your teacher and make any necessary changes to your setup before continuing.
5. Create a method for determining what effect these two factors will have on the path of your projectile.
6. Have your teacher approve your method before collecting data.

Horizons Companies

Data Table 1

Launch Angle (deg)	Distance Projectile Travels (cm)

Data Table 2

Distance Rubber Band Is Pulled Back (cm)	Distance Projectile Travels (cm)

Analyze

1. **Make and Use Graphs** Make graphs of your data to help you predict how to use your launcher to hit a target.
2. **Analyze** What are the relationships between each variable you have tested and the distance the projectile travels?

Conclude and Apply

1. What were the main factors influencing the path of the projectile?
2. Predict the conditions necessary to hit a target provided by your teacher.
3. **Explain** If you have a perfect plan and still miss the target on your first try, is there a problem with the variability of laws of physics? Explain.
4. Launch your projectile at the target. If you miss, make the necessary adjustments and try again.

Going Further

1. How might your data have varied if you did this experiment outside? Would there be any additional factors affecting the motion of your projectile?
2. How might the results of your experiment be different if the target was elevated above the height of the launcher?
3. How might your experiment differ if the launcher was elevated above the height of the target?

Real-World Physics

1. When a kicker attempts a field goal, do you think it is possible for him to miss because he kicked it too high? Explain.
2. If you wanted to hit a baseball as far as possible, what would be the best angle to hit the ball?

Physics Online

To find out more about projectile motion, visit the Web site: **physicspp.com**

Future Technology

Spinning Space Stations

There is a lot going on aboard the *International Space Station* (ISS). Scientists from different countries are conducting experiments and making observations. They have seen water drops form as floating spheres and have grown peas in space to test whether crops can be grown in weightlessness.

One goal of the ISS is to examine the effects on the human body when living in space for prolonged periods of time. If negative health effects can be identified, perhaps they can be prevented. This could give humans the option of living in space for long periods of time.

Harmful effects of weightlessness have been observed. On Earth, muscles have gravity to push and pull against. Muscles weaken from disuse if this resistance is removed. Bones can weaken for the same reason. Also, blood volume can decrease. On Earth, gravity pulls blood downward so it collects in the lower legs. In weightlessness, the blood can more easily collect in an astronaut's head. The brain senses the extra blood and sends a signal to make less of it.

Long-term life in space is hindered by the practical challenges of weightlessness as well. Imagine how daily life would change. Everything must be strapped or bolted down. You would have to be strapped down to a bed to sleep in one. Your life would be difficult in a space station unless the space station could be modified to simulate gravity. How could this be done?

The Rotating Space Station Have you ever been on a human centrifuge—a type of amusement park ride that uses centripetal force? Everyone stands against the walls of a big cylinder. Then the cylinder begins to rotate faster and faster until the riders are pressed against the walls. Because of the centripetal acceleration, the riders are held there so that even when the floor drops down they are held securely against the walls of the whirling container.

A space station could be designed that uses the effects of centripetal motion as a replacement for gravity. Imagine a space station in the form of a large ring. The space station and all the objects and occupants inside would float weightlessly inside. If the ring were made to spin, unattached objects would be held against the ring's outer edge because of the centripetal motion. If the space station spun at the right rate and if it had the right diameter, the centripetal motion would cause the occupants to experience a force of the same magnitude as gravity. The down direction in the space station would be what an observer outside the station would see as radially outward, away from the ring's center.

This is an artist's rendition of a rotating space station.

Centripetal acceleration is directly proportional to the distance from the center of a rotating object. A rotating space station could be built in the form of concentric rings, each ring experiencing a different gravity. The innermost rings would experience the smallest gravity, while outermost rings would experience the largest force. You could go from floating peacefully in a low-gravity ring to standing securely in the simulated Earth-gravity ring.

Going Further

1. **Research** What factors must engineers take into account in order to make a rotating space station that can simulate Earth's gravity?
2. **Apply** You are an astronaut aboard a rotating space station. You feel pulled by gravity against the floor. Explain what is really going on in terms of Newton's laws and centripetal force.
3. **Critical Thinking** What benefits does a rotating space station offer its occupants? What are the negative features?

Chapter 6

Study Guide

6.1 Projectile Motion

Vocabulary

- projectile (p. 147)
- trajectory (p. 147)

Key Concepts

- The vertical and horizontal motions of a projectile are independent.
- The vertical motion component of a projectile experiences a constant acceleration.
- When there is no air resistance, the horizontal motion component does not experience an acceleration and has constant velocity.
- Projectile problems are solved by first using the vertical motion to relate height, time in the air, and initial vertical velocity. Then the distance traveled horizontally is found.
- The range of a projectile depends upon the acceleration due to gravity and upon both components of the initial velocity.
- The curved flight path that is followed by a projectile is called a parabola.

6.2 Circular Motion

Vocabulary

- uniform circular motion (p. 153)
- centripetal acceleration (p. 153)
- centripetal force (p. 154)

Key Concepts

- An object moving in a circle at a constant speed accelerates toward the center of the circle, and therefore, it has centripetal acceleration.
- Centripetal acceleration depends directly on the square of the object's speed and inversely on the radius of the circle.

$$a_c = \frac{v^2}{r}$$

- The centripetal acceleration for an object traveling in a circle can also be expressed as a function of its period, T.

$$a_c = \frac{4\pi^2 r}{T^2}$$

- A net force must be exerted toward the circle's center to cause centripetal acceleration.

$$F_{net} = ma_c$$

- The velocity vector of an object with a centripetal acceleration is always tangent to the circular path.

6.3 Relative Velocity

Key Concepts

- Vector addition can be used to solve problems involving relative velocities.

$$\boldsymbol{v}_{a/b} + \boldsymbol{v}_{b/c} = \boldsymbol{v}_{a/c}$$

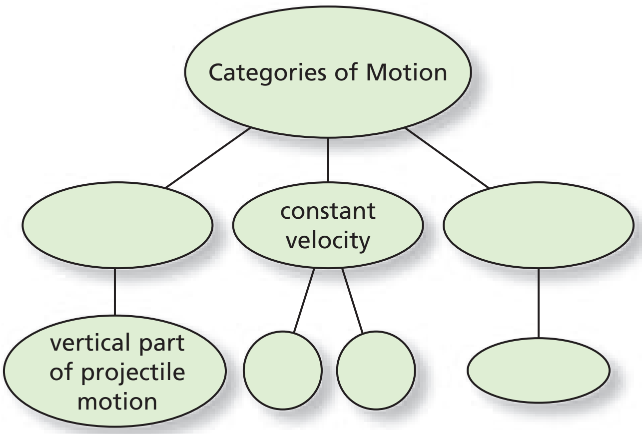
- The key to properly analyzing a two-dimensional relative-velocity problem is drawing the proper triangle to represent all three velocity vectors.

Chapter 6

Assessment

Concept Mapping

32. Use the following terms to complete the concept map below: *constant speed, horizontal part of projectile motion, constant acceleration, relative-velocity motion, uniform circular motion.*



Mastering Concepts

33. Consider the trajectory of the cannonball shown in **Figure 6-11.** (6.1)
  a. Where is the magnitude of the vertical-velocity component largest?
  b. Where is the magnitude of the horizontal-velocity component largest?
  c. Where is the vertical-velocity smallest?
  d. Where is the magnitude of the acceleration smallest?

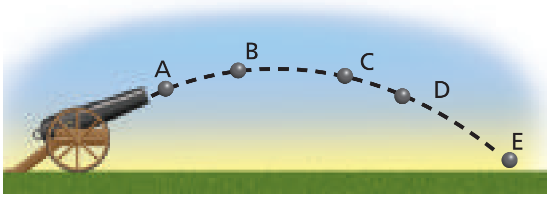


Figure 6-11

34. A student is playing with a radio-controlled race car on the balcony of a sixth-floor apartment. An accidental turn sends the car through the railing and over the edge of the balcony. Does the time it takes the car to fall depend upon the speed it had when it left the balcony? (6.1)

35. An airplane pilot flying at constant velocity and altitude drops a heavy crate. Ignoring air resistance, where will the plane be relative to the crate when the crate hits the ground? Draw the path of the crate as seen by an observer on the ground. (6.1)

36. Can you go around a curve with the following accelerations? Explain.
  a. zero acceleration
  b. constant acceleration (6.2)

37. To obtain uniform circular motion, how must the net force depend on the speed of the moving object? (6.2)

38. If you whirl a yo-yo about your head in a horizontal circle, in what direction must a force act on the yo-yo? What exerts the force? (6.2)

39. Why is it that a car traveling in the opposite direction as the car in which you are riding on the freeway often looks like it is moving faster than the speed limit? (6.3)

Applying Concepts

40. Projectile Motion Analyze how horizontal motion can be uniform while vertical motion is accelerated. How will projectile motion be affected when drag due to air resistance is taken into consideration?

41. Baseball A batter hits a pop-up straight up over home plate at an initial velocity of 20 m/s. The ball is caught by the catcher at the same height that it was hit. At what velocity does the ball land in the catcher's mitt? Neglect air resistance.

42. Fastball In baseball, a fastball takes about $\frac{1}{2}$ s to reach the plate. Assuming that such a pitch is thrown horizontally, compare the distance the ball falls in the first $\frac{1}{4}$ s with the distance it falls in the second $\frac{1}{4}$ s.

43. You throw a rock horizontally. In a second horizontal throw, you throw the rock harder and give it even more speed.
  a. How will the time it takes the rock to hit the ground be affected? Ignore air resistance.
  b. How will the increased speed affect the distance from where the rock left your hand to where the rock hits the ground?

44. Field Biology A zoologist standing on a cliff aims a tranquilizer gun at a monkey hanging from a distant tree branch. The barrel of the gun is horizontal. Just as the zoologist pulls the trigger, the monkey lets go and begins to fall. Will the dart hit the monkey? Ignore air resistance.

45. Football A quarterback throws a football at 24 m/s at a 45° angle. If it takes the ball 3.0 s to reach the top of its path and the ball is caught at the same height at which it is thrown, how long is it in the air? Ignore air resistance.

46. Track and Field You are working on improving your performance in the long jump and believe that the information in this chapter can help. Does the height that you reach make any difference to your jump? What influences the length of your jump?

For more problems, go to Additional Problems, Appendix B.

47. Imagine that you are sitting in a car tossing a ball straight up into the air.
  a. If the car is moving at a constant velocity, will the ball land in front of, behind, or in your hand?
  b. If the car rounds a curve at a constant speed, where will the ball land?

48. You swing one yo-yo around your head in a horizontal circle. Then you swing another yo-yo with twice the mass of the first one, but you don't change the length of the string or the period. How do the tensions in the strings differ?

49. **Car Racing** The curves on a race track are banked to make it easier for cars to go around the curves at high speeds. Draw a free-body diagram of a car on a banked curve. From the motion diagram, find the direction of the acceleration.
  a. What exerts the force in the direction of the acceleration?
  b. Can you have such a force without friction?

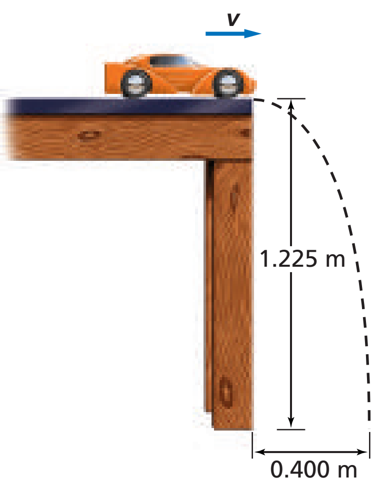
50. **Driving on the Highway** Explain why it is that when you pass a car going in the same direction as you on the freeway, it takes a longer time than when you pass a car going in the opposite direction.

Mastering Problems

6.1 Projectile Motion

51. You accidentally throw your car keys horizontally at 8.0 m/s from a cliff 64-m high. How far from the base of the cliff should you look for the keys?

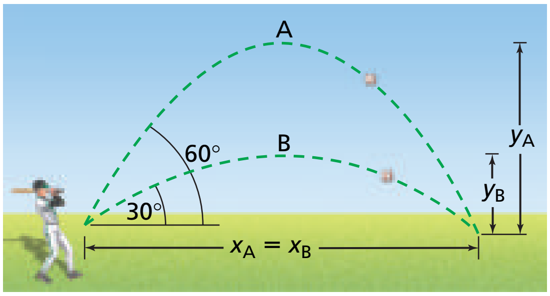
52. The toy car in **Figure 6-12** runs off the edge of a table that is 1.225-m high. The car lands 0.400 m from the base of the table.
  a. How long did it take the car to fall?
  b. How fast was the car going on the table?



■ Figure 6-12

53. A dart player throws a dart horizontally at 12.4 m/s. The dart hits the board 0.32 m below the height from which it was thrown. How far away is the player from the board?

54. The two baseballs in **Figure 6-13** were hit with the same speed, 25 m/s. Draw separate graphs of *y* versus *t* and *x* versus *t* for each ball.



■ Figure 6-13

55. **Swimming** You took a running leap off a high-diving platform. You were running at 2.8 m/s and hit the water 2.6 s later. How high was the platform, and how far from the edge of the platform did you hit the water? Ignore air resistance.

56. **Archery** An arrow is shot at 30.0° above the horizontal. Its velocity is 49 m/s, and it hits the target.
  a. What is the maximum height the arrow will attain?
  b. The target is at the height from which the arrow was shot. How far away is it?

57. **Hitting a Home Run** A pitched ball is hit by a batter at a 45° angle and just clears the outfield fence, 98 m away. If the fence is at the same height as the pitch, find the velocity of the ball when it left the bat. Ignore air resistance.

58. **At-Sea Rescue** An airplane traveling 1001 m above the ocean at 125 km/h is going to drop a box of supplies to shipwrecked victims below.
  a. How many seconds before the plane is directly overhead should the box be dropped?
  b. What is the horizontal distance between the plane and the victims when the box is dropped?

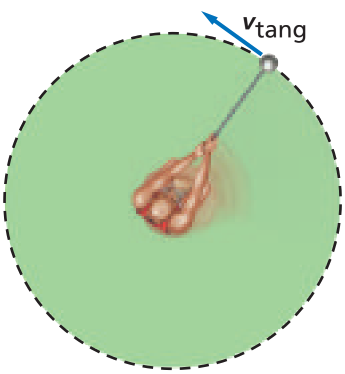
59. **Diving** Divers in Acapulco dive from a cliff that is 61 m high. If the rocks below the cliff extend outward for 23 m, what is the minimum horizontal velocity a diver must have to clear the rocks?

60. **Jump Shot** A basketball player is trying to make a half-court jump shot and releases the ball at the height of the basket. Assuming that the ball is launched at 51.0°, 14.0 m from the basket, what speed must the player give the ball?

6.2 Circular Motion

61. **Car Racing** A 615-kg racing car completes one lap in 14.3 s around a circular track with a radius of 50.0 m. The car moves at a constant speed.
a. What is the acceleration of the car?
b. What force must the track exert on the tires to produce this acceleration?

62. **Hammer Throw** An athlete whirls a 7.00-kg hammer 1.8 m from the axis of rotation in a horizontal circle, as shown in **Figure 6-14.** If the hammer makes one revolution in 1.0 s, what is the centripetal acceleration of the hammer? What is the tension in the chain?



■ **Figure 6-14**

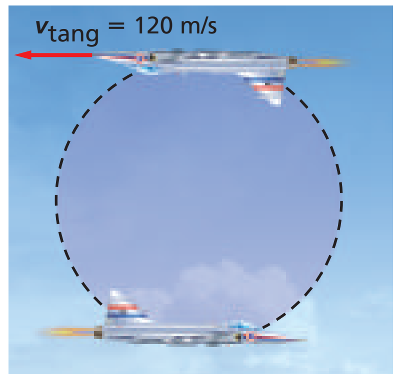
63. A coin is placed on a vinyl stereo record that is making $33\frac{1}{3}$ revolutions per minute on a turntable.
a. In what direction is the acceleration of the coin?
b. Find the magnitude of the acceleration when the coin is placed 5.0, 10.0, and 15.0 cm from the center of the record.
c. What force accelerates the coin?
d. At which of the three radii in part **b** would the coin be most likely to fly off the turntable? Why?

64. A rotating rod that is 15.3 cm long is spun with its axis through one end of the rod so that the other end of the rod has a speed of 2010 m/s (4500 mph).
a. What is the centripetal acceleration of the end of the rod?
b. If you were to attach a 1.0-g object to the end of the rod, what force would be needed to hold it on the rod?

65. Friction provides the force needed for a car to travel around a flat, circular race track. What is the maximum speed at which a car can safely travel if the radius of the track is 80.0 m and the coefficient of friction is 0.40?

66. A carnival clown rides a motorcycle down a ramp and around a vertical loop. If the loop has a radius of 18 m, what is the slowest speed the rider can have at the top of the loop to avoid falling? *Hint: At this slowest speed, the track exerts no force on the motorcycle at the top of the loop.*

67. A 75-kg pilot flies a plane in a loop as shown in **Figure 6-15.** At the top of the loop, when the plane is completely upside-down for an instant, the pilot hangs freely in the seat and does not push against the seat belt. The airspeed indicator reads 120 m/s. What is the radius of the plane's loop?



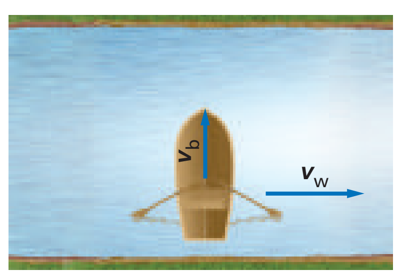
■ **Figure 6-15**

6.3 Relative Velocity

68. **Navigating an Airplane** An airplane flies at 200.0 km/h relative to the air. What is the velocity of the plane relative to the ground if it flies during the following wind conditions?
a. a 50.0-km/h tailwind
b. a 50.0-km/h headwind

69. Odina and LaToya are sitting by a river and decide to have a race. Odina will run down the shore to a dock, 1.5 km away, then turn around and run back. LaToya will also race to the dock and back, but she will row a boat in the river, which has a current of 2.0 m/s. If Odina's running speed is equal to LaToya's rowing speed in still water, which is 4.0 m/s, who will win the race? Assume that they both turn instantaneously.

70. **Crossing a River** You row a boat, such as the one in **Figure 6-16,** perpendicular to the shore of a river that flows at 3.0 m/s. The velocity of your boat is 4.0 m/s relative to the water.
a. What is the velocity of your boat relative to the shore?
b. What is the component of your velocity parallel to the shore? Perpendicular to it?



■ **Figure 6-16**

 For more problems, go to Additional Problems, Appendix B.

71. Studying the Weather A weather station releases a balloon to measure cloud conditions that rises at a constant 15 m/s relative to the air, but there is also a wind blowing at 6.5 m/s toward the west. What are the magnitude and direction of the velocity of the balloon?

72. Boating You are boating on a river that flows toward the east. Because of your knowledge of physics, you head your boat 53° west of north and have a velocity of 6.0 m/s due north relative to the shore.

a. What is the velocity of the current?

b. What is the speed of your boat relative to the water?

73. Air Travel You are piloting a small plane, and you want to reach an airport 450 km due south in 3.0 h. A wind is blowing from the west at 50.0 km/h. What heading and airspeed should you choose to reach your destination in time?

Mixed Review

74. Early skeptics of the idea of a rotating Earth said that the fast spin of Earth would throw people at the equator into space. The radius of Earth is about 6.38×10^{3} km. Show why this idea is wrong by calculating the following.

a. the speed of a 97-kg person at the equator

b. the force needed to accelerate the person in the circle

c. the weight of the person

d. the normal force of Earth on the person, that is, the person's apparent weight

75. Firing a Missile An airplane, moving at 375 m/s relative to the ground, fires a missile forward at a speed of 782 m/s relative to the plane. What is the speed of the missile relative to the ground?

76. Rocketry A rocket in outer space that is moving at a speed of 1.25 km/s relative to an observer fires its motor. Hot gases are expelled out the back at 2.75 km/s relative to the rocket. What is the speed of the gases relative to the observer?

77. Two dogs, initially separated by 500.0 m, are running towards each other, each moving with a constant speed of 2.5 m/s. A dragonfly, moving with a constant speed of 3.0 m/s, flies from the nose of one dog to the other, then turns around instantaneously and flies back to the other dog. It continues to fly back and forth until the dogs run into each other. What distance does the dragonfly fly during this time?

78. A 1.13-kg ball is swung vertically from a 0.50-m cord in uniform circular motion at a speed of 2.4 m/s. What is the tension in the cord at the bottom of the ball's motion?

79. Banked Roads Curves on roads often are banked to help prevent cars from slipping off the road. If the posted speed limit for a particular curve of radius 36.0 m is 15.7 m/s (35 mph), at what angle should the road be banked so that cars will stay on a circular path even if there were no friction between the road and the tires? If the speed limit was increased to 20.1 m/s (45 mph), at what angle should the road be banked?

80. The 1.45-kg ball in **Figure 6-17** is suspended from a 0.80-m string and swung in a horizontal circle at a constant speed such that the string makes an angle of 14.0° with the vertical.

a. What is the tension in the string?

b. What is the speed of the ball?

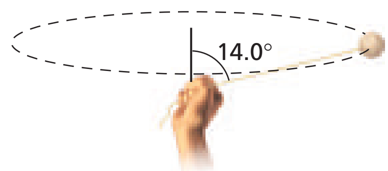


Figure 6-17

81. A baseball is hit directly in line with an outfielder at an angle of 35.0° above the horizontal with an initial velocity of 22.0 m/s. The outfielder starts running as soon as the ball is hit at a constant velocity of 2.5 m/s and barely catches the ball. Assuming that the ball is caught at the same height at which it was hit, what was the initial separation between the hitter and outfielder? *Hint: There are two possible answers.*

82. A Jewel Heist You are serving as a technical consultant for a locally produced cartoon. In one episode, two criminals, Shifty and Lefty, have stolen some jewels. Lefty has the jewels when the police start to chase him, and he runs to the top of a 60.0-m tall building in his attempt to escape. Meanwhile, Shifty runs to the convenient hot-air balloon 20.0 m from the base of the building and untethers it, so it begins to rise at a constant speed. Lefty tosses the bag of jewels horizontally with a speed of 7.3 m/s just as the balloon begins its ascent. What must the velocity of the balloon be for Shifty to easily catch the bag?

Thinking Critically

83. Apply Concepts Consider a roller-coaster loop like the one in **Figure 6-18.** Are the cars traveling through the loop in uniform circular motion? Explain.

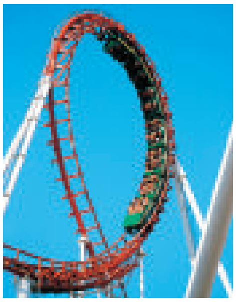


Figure 6-18

84. Use Numbers A 3-point jump shot is released 2.2 m above the ground and 6.02 m from the basket. The basket is 3.05 m above the floor. For launch angles of 30° and 60°, find the speed the ball needs to be thrown to make the basket.

85. Analyze For which angle in problem 84 is it more important that the player get the speed right? To explore this question, vary the speed at each angle by 5 percent and find the change in the range of the attempted shot.

86. Apply Computers and Calculators A baseball player hits a belt-high (1.0 m) fastball down the left-field line. The ball is hit with an initial velocity of 42.0 m/s at 26°. The left-field wall is 96.0 m from home plate at the foul pole and is 14-m high. Write the equation for the height of the ball, y, as a function of its distance from home plate, x. Use a computer or graphing calculator to plot the path of the ball. Trace along the path to find how high above the ground the ball is when it is at the wall.

a. Is the hit a home run?

b. What is the minimum speed at which the ball could be hit and clear the wall?

c. If the initial velocity of the ball is 42.0 m/s, for what range of angles will the ball go over the wall?

87. Analyze Albert Einstein showed that the rule you learned for the addition of velocities does not work for objects moving near the speed of light. For example, if a rocket moving at velocity v_A releases a missile that has velocity v_B relative to the rocket, then the velocity of the missile relative to an observer that is at rest is given by $v = (v_A + v_B)/(1 + v_A v_B/c^2)$, where c is the speed of light, 3.00×10^8 m/s. This formula gives the correct values for objects moving at slow speeds as well. Suppose a rocket moving at 11 km/s shoots a laser beam out in front of it. What speed would an unmoving observer find for the laser light? Suppose that a rocket moves at a speed $c/2$, half the speed of light, and shoots a missile forward at a speed of $c/2$ relative to the rocket. How fast would the missile be moving relative to a fixed observer?

88. Analyze and Conclude A ball on a light string moves in a vertical circle. Analyze and describe the motion of this system. Be sure to consider the effects of gravity and tension. Is this system in uniform circular motion? Explain your answer.

Writing in Physics

89. Roller Coasters If you take a look at vertical loops on roller coasters, you will notice that most of them are not circular in shape. Research why this is so and explain the physics behind this decision by the coaster engineers.

90. Many amusement-park rides utilize centripetal acceleration to create thrills for the park's customers. Choose two rides other than roller coasters that involve circular motion and explain how the physics of circular motion creates the sensations for the riders.

Cumulative Review

91. Multiply or divide, as indicated, using significant digits correctly. (Chapter 1)

a. $(5\times10^8\ \text{m})(4.2\times10^7\ \text{m})$

b. $(1.67\times10^{-2}\ \text{km})(8.5\times10^{-6}\ \text{km})$

c. $(2.6\times10^4\ \text{kg})/(9.4\times10^3\ \text{m}^3)$

d. $(6.3\times10^{-1}\ \text{m})/(3.8\times10^2\ \text{s})$

92. Plot the data in **Table 6-1** on a position-time graph. Find the average velocity in the time interval between 0.0 s and 5.0 s. (Chapter 3)

Table 6-1

Position v. Time

Clock Reading t (s)	Position d (m)
0.0	30
1.0	30
2.0	35
3.0	45
4.0	60
5.0	70

93. Carlos and his older brother Ricardo are at the grocery store. Carlos, with mass 17.0 kg, likes to hang on the front of the cart while Ricardo pushes it, even though both boys know this is not safe. Ricardo pushes the cart, with mass 12.4 kg, with his brother hanging on it such that they accelerate at a rate of 0.20 m/s². (Chapter 4)

a. With what force is Ricardo pushing?

b. What is the force the cart exerts on Carlos?

Standardized Test Practice

Multiple Choice

1. A 1.60-m-tall girl throws a football at an angle of 41.0° from the horizontal and at an initial velocity of 9.40 m/s. How far away from the girl will it land?

(A) 4.55 m (C) 8.90 m

(B) 5.90 m (D) 10.5 m

2. A dragonfly is sitting on a merry-go-round 2.8 m from the center. If the tangential velocity of the ride is 0.89 m/s, what is the centripetal acceleration of the dragonfly?

(A) 0.11 m/s^2 (C) 0.32 m/s^2

(B) 0.28 m/s^2 (D) 2.2 m/s^2

3. The centripetal force on a 0.82-kg object on the end of a 2.0-m massless string being swung in a horizontal circle is 4.0 N. What is the tangential velocity of the object?

(A) 2.8 m/s^2 (C) 4.9 m/s^2

(B) 3.1 m/s^2 (D) 9.8 m/s^2

4. A 1000-kg car enters an 80.0-m-radius curve at 20.0 m/s. What centripetal force must be supplied by friction so the car does not skid?

(A) 5.0 N (C) 5.0×10^3 N

(B) 2.5×10^2 N (D) 1.0×10^3 N

5. A jogger on a riverside path sees a rowing team coming toward him. If the jogger is moving at 10 km/h, and the boat is moving at 20 km/h, how quickly does the jogger approach the boat?

(A) 3 m/s (C) 40 m/s

(B) 8 m/s (D) 100 m/s

6. What is the maximum height obtained by a 125-g apple that is slung from a slingshot at an angle of 78° from the horizontal with an initial velocity of 18 m/s?

(A) 0.70 m (C) 32 m

(B) 16 m (D) 33 m

7. An orange is dropped at the same time a bullet is shot from a gun. Which of the following is true?

(A) The acceleration due to gravity is greater for the orange because the orange is heavier.

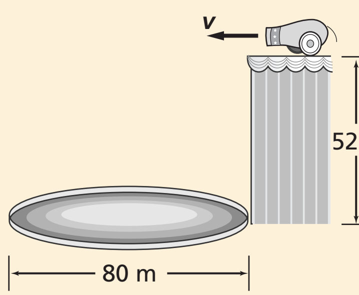
(B) Gravity acts less on the bullet than on the orange because the bullet is moving so fast.

(C) The velocities will be the same.

(D) The two objects will hit the ground at the same time.

Extended Answer

8. A colorfully feathered lead cannonball is shot horizontally out of a circus cannon 25 m/s from the high-wire platform on one side of a circus ring. If the high-wire platform is 52 m above the 80-m diameter ring, will the performers need to adjust their cannon (will the ball land inside the ring, or past it)?



(Not to scale)

9. A mythical warrior swings a 5.6-kg mace on the end of a magically massless 86-cm chain in a horizontal circle above his head. The mace makes one full revolution in 1.8 s. Find the tension in the magical chain.

✓ Test-Taking TIP

Practice Under Testlike Conditions

Answer all of the questions in the time provided without referring to your book. Did you complete the test? Could you have made better use of your time? What topics do you need to review?

Chapter 7

Gravitation

What You'll Learn

- You will learn the nature of gravitational force.
- You will relate Kepler's laws of planetary motion to Newton's laws of motion.
- You will describe the orbits of planets and satellites using the law of universal gravitation.

Why It's Important

Kepler's laws and the law of universal gravitation will help you understand the motion of planets and satellites.

Comets Comet Hale-Bopp was discovered by Alan Hale and Thomas Bopp in 1995. The comet entered the inner solar system in 1997 and was visible from Joshua Tree National Park in California, providing spectacular views of its white dust tail and blue ion tail.

Think About This ▶

Comets orbit the Sun just as planets and stars do. How can you describe the orbit of a comet such as Hale-Bopp?

Physics Online

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CONTENTS

LAUNCH Lab

Can you model Mercury's motion?

Question
Do planets in our solar system have circular orbits or do they travel in some other path?

Procedure

1. Use the data table to plot the orbit of Mercury using the scale 10 cm = 1 AU. Note that one astronomical unit, AU, is Earth's distance from the Sun. 1 AU is equal to 1.5×10^{8} km.
2. Calculate the distance in cm for each distance measured in AU.
3. Mark the center of your paper and draw a horizontal zero line and a vertical zero line going through it.
4. Place your protractor on the horizontal line and center it on the center point. Measure the degrees and place a mark.
5. Place a ruler connecting the center and the angle measurement. Mark the distance in centimeters for the corresponding angle. You will need to place the protractor on the vertical zero line for certain angle measurements.
6. Once you have marked all the data points, draw a line connecting them.

Analysis
Describe the shape of Mercury's orbit. Draw a line going through the Sun that represents the longest axis of the orbit, called the major axis.

Critical Thinking
How does the orbit of Mercury compare to the orbit of comet Hale-Bopp, shown on page 170?

Mercury's Orbit

θ (°)	d (AU)
4	0.35
61	0.31
122	0.32
172	0.38
209	0.43
239	0.46
266	0.47
295	0.44
330	0.40
350	0.37

7.1 Planetary Motion and Gravitation

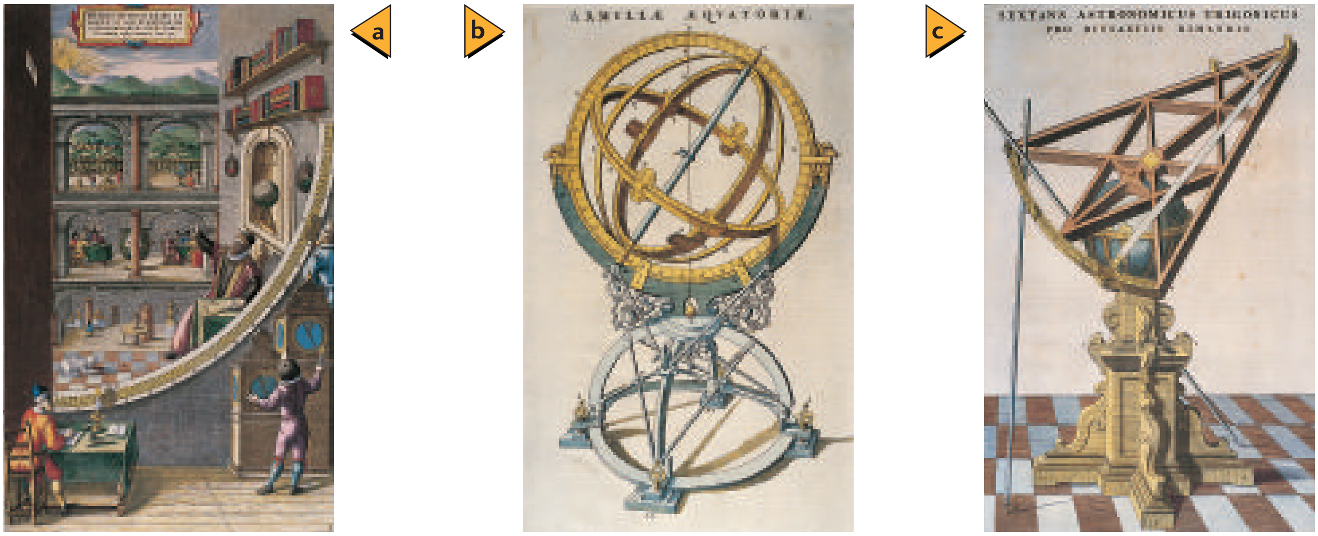
Since ancient times, the Sun, Moon, planets, and stars had been assumed to revolve around Earth. Nicholas Copernicus, a Polish astronomer, noticed that the best available observations of the movements of planets and stars did not fully agree with the Earth-centered model. The results of his many years of work were published in 1543, when Copernicus was on his deathbed. His book showed that the motion of planets is much more easily understood by assuming that Earth and other planets revolve around the Sun.

Tycho Brahe was born a few years after the death of Copernicus. As a boy of 14 in Denmark, Brahe observed an eclipse of the Sun on August 21, 1560, and vowed to become an astronomer.

Brahe studied astronomy as he traveled throughout Europe for five years. He did not use telescopes. Instead, he used huge instruments that he designed and built in his own shop on the Danish island of Hven. He spent the next 20 years carefully recording the exact positions of the planets and stars. Brahe concluded that the Sun and the Moon orbit Earth and that all other planets orbit the Sun.

▶ Objectives
- **Relate** Kepler's laws to the law of universal gravitation.
- **Calculate** orbital speeds and periods.
- **Describe** the importance of Cavendish's experiment.

▶ Vocabulary
Kepler's first law
Kepler's second law
Kepler's third law
gravitational force
law of universal gravitation

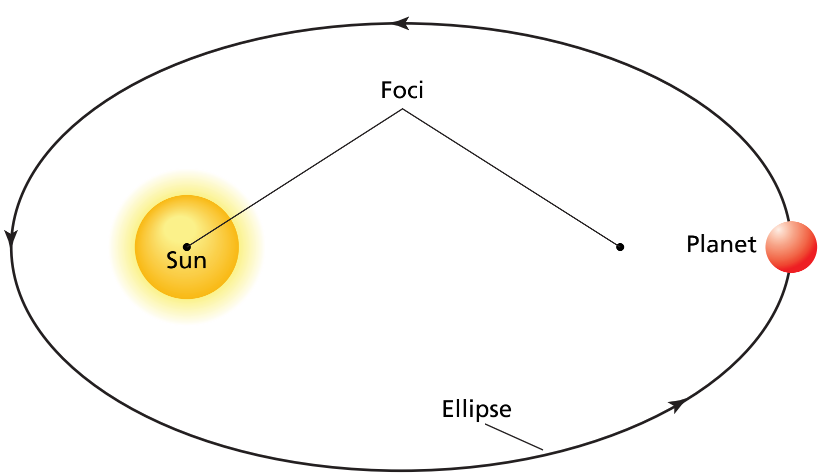


■ **Figure 7-1** Among the huge astronomical instruments that Tycho Brahe constructed to use on Hven **(a)** were an astrolabe **(b)** and a sextant **(c).**

Kepler's Laws

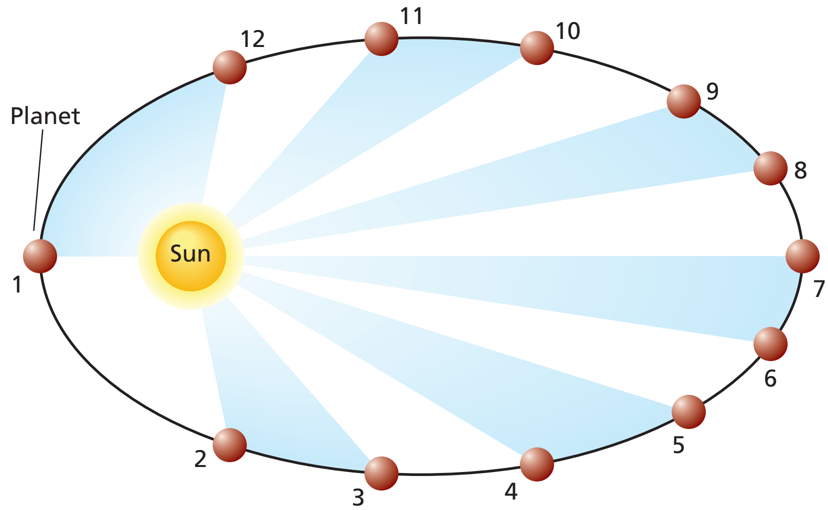
Johannes Kepler, a 29-year-old German, became one of Brahe's assistants when he moved to Prague. Brahe trained his assistants to use instruments, such as those shown in **Figure 7-1.** Upon his death in 1601, Kepler inherited 30 years' worth of Brahe's observations. He studied Brahe's data and was convinced that geometry and mathematics could be used to explain the number, distance, and motion of the planets. Kepler believed that the Sun exerted a force on the planets and placed the Sun at the center of the system. After several years of careful analysis of Brahe's data on Mars, Kepler discovered the laws that describe the motion of every planet and satellite.

Kepler's first law states that the paths of the planets are ellipses, with the Sun at one focus. An ellipse has two foci, as shown in **Figure 7-2.** Like planets and stars, comets also orbit the Sun in elliptical orbits. Comets are divided into two groups—long-period comets and short-period comets—based on orbital periods, each of which is the time it takes the comet to complete one revolution. Long-period comets have orbital periods longer than 200 years, and short-period comets have orbital periods shorter than 200 years. Comet Hale-Bopp, with a period of 2400 years, is an example of a long-period comet. Comet Halley, with a period of 76 years, is an example of a short-period comet.



■ **Figure 7-2** Planets orbit the Sun in elliptical orbits with the Sun at one focus. (Illustration not to scale)

Kepler found that the planets move faster when they are closer to the Sun and slower when they are farther away from the Sun. Thus, **Kepler's second law** states that an imaginary line from the Sun to a planet sweeps out equal areas in equal time intervals, as illustrated in **Figure 7-3.**



■ **Figure 7-3** A planet moves fastest when it is close to the Sun and slowest when it is farther from the Sun. Equal areas are swept out in equal amounts of time. (Illustration not to scale)

Kepler also found that there is a mathematical relationship between periods of planets and their mean distances away from the Sun. **Kepler's third law** states that the square of the ratio of the periods of any two planets revolving about the Sun is equal to the cube of the ratio of their average distances from the Sun. Thus, if the periods of the planets are T_A and T_B, and their average distances from the Sun are r_A and r_B, Kepler's third law can be expressed as follows.

Kepler's Third Law $\left(\frac{T_A}{T_B}\right)^2 = \left(\frac{r_A}{r_B}\right)^3$

The squared quantity of the period of object A divided by the period of object B, is equal to the cubed quantity of object A's average distance from the Sun, divided by object B's average distance from the Sun.

Note that the first two laws apply to each planet, moon, and satellite individually. The third law, however, relates the motion of several objects about a single body. For example, it can be used to compare the planets' distances from the Sun, shown in **Table 7-1,** to their periods about the Sun. It also can be used to compare distances and periods of the Moon and artificial satellites orbiting Earth.

Table 7-1

Planetary Data

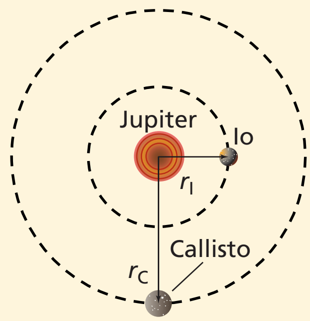
Name	Average Radius (m)	Mass (kg)	Mean Distance From Sun (m)
Sun	6.96×10^{8}	1.99×10^{30}	–
Mercury	2.44×10^{6}	3.30×10^{23}	5.79×10^{10}
Venus	6.05×10^{6}	4.87×10^{24}	1.08×10^{11}
Earth	6.38×10^{6}	5.98×10^{24}	1.50×10^{11}
Mars	3.40×10^{6}	6.42×10^{23}	2.28×10^{11}
Jupiter	7.15×10^{7}	1.90×10^{27}	7.78×10^{11}
Saturn	6.03×10^{7}	5.69×10^{26}	1.43×10^{12}
Uranus	2.56×10^{7}	8.68×10^{25}	2.87×10^{12}
Neptune	2.48×10^{7}	1.02×10^{26}	4.50×10^{12}
Pluto	1.20×10^{6}	1.25×10^{22}	5.87×10^{12}

EXAMPLE Problem 1

Callisto's Distance from Jupiter Galileo measured the orbital sizes of Jupiter's moons using the diameter of Jupiter as a unit of measure. He found that Io, the closest moon to Jupiter, had a period of 1.8 days and was 4.2 units from the center of Jupiter. Callisto, the fourth moon from Jupiter, had a period of 16.7 days. Using the same units that Galileo used, predict Callisto's distance from Jupiter.

1 Analyze and Sketch the Problem

- Sketch the orbits of Io and Callisto.
- Label the radii.



Known:	Unknown:
$T_C = 16.7$ days	$r_C = ?$
$T_I = 1.8$ days	
$r_I = 4.2$ units	

2 Solve for the Unknown

Solve Kepler's third law for r_C.

$$\left(\frac{T_C}{T_I}\right)^2 = \left(\frac{r_C}{r_I}\right)^3$$

$$r_C^{\,3} = r_I^{\,3}\left(\frac{T_C}{T_I}\right)^2$$

$$r_C = \sqrt[3]{r_I^{\,3}\left(\frac{T_C}{T_I}\right)^2}$$

$$= \sqrt[3]{(4.2\text{ units})^3\left(\frac{16.7\text{ days}}{1.8\text{ days}}\right)^2}$$

Substitute $r_I = 4.2$ units, $T_C = 16.7$ days, $T_I = 1.8$ days

$$= \sqrt[3]{6.4\times10^3\text{ units}^3}$$

$$= 19\text{ units}$$

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Isolating a Variable
page 845

3 Evaluate the Answer

- **Are the units correct?** r_C should be in Galileo's units, like r_I.
- **Is the magnitude realistic?** The period is large, so the radius should be large.

PRACTICE Problems

Additional Problems, Appendix B

1. If Ganymede, one of Jupiter's moons, has a period of 32 days, how many units are there in its orbital radius? Use the information given in Example Problem 1.
2. An asteroid revolves around the Sun with a mean orbital radius twice that of Earth's. Predict the period of the asteroid in Earth years.
3. From Table 7-1, on page 173, you can find that, on average, Mars is 1.52 times as far from the Sun as Earth is. Predict the time required for Mars to orbit the Sun in Earth days.
4. The Moon has a period of 27.3 days and a mean distance of 3.90×10^5 km from the center of Earth.
   a. Use Kepler's laws to find the period of a satellite in orbit 6.70×10^3 km from the center of Earth.
   b. How far above Earth's surface is this satellite?
5. Using the data in the previous problem for the period and radius of revolution of the Moon, predict what the mean distance from Earth's center would be for an artificial satellite that has a period of exactly 1.00 day.

Newton's Law of Universal Gravitation

In 1666, 45 years after Kepler published his work, Newton began his studies of planetary motion. He found that the magnitude of the force, F, on a planet due to the Sun varies inversely with the square of the distance, r, between the centers of the planet and the Sun. That is, F is proportional to $1/r^2$. The force, $\mathbf{F}$, acts in the direction of the line connecting the centers of the two objects.

It is quoted that the sight of a falling apple made Newton wonder if the force that caused the apple to fall might extend to the Moon, or even beyond. He found that both the apple's and Moon's accelerations agreed with the $1/r^2$ relationship. According to his own third law, the force Earth exerts on the apple is exactly the same as the force the apple exerts on Earth. The force of attraction between two objects must be proportional to the objects' masses, and is known as the **gravitational force.**

Newton was confident that the same force of attraction would act between any two objects, anywhere in the universe. He proposed his **law of universal gravitation,** which states that objects attract other objects with a force that is proportional to the product of their masses and inversely proportional to the square of the distance between them. This can be represented by the following equation.

Law of Universal Gravitation $F = G\frac{m_1 m_2}{r^2}$

The gravitational force is equal to the universal gravitational constant, times the mass of object 1, times the mass of object 2, divided by the distance between the centers of the objects, squared.

According to Newton's equation, F is directly proportional to m_1 and m_2. Thus, if the mass of a planet near the Sun were doubled, the force of attraction would be doubled. Use the Connecting Math to Physics feature below to examine how changing one variable affects another. **Figure 7-4** illustrates the inverse square law graphically.

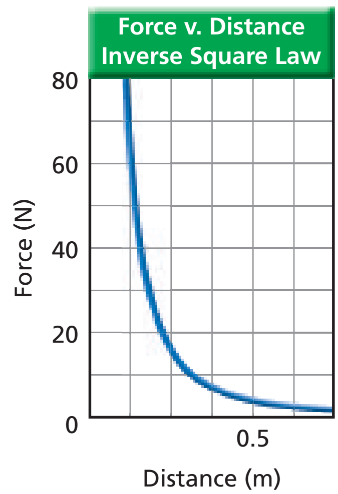
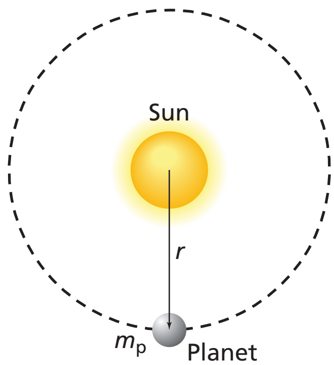


Figure 7-4 The change in gravitational force with distance follows the inverse square law.

▶ Connecting Math to Physics

Direct and Inverse Relationships Newton's law of universal gravitation has both direct and inverse relationships.

$F \propto m_1 m_2$		$F \propto \frac{1}{r^2}$	
Change	**Result**	**Change**	**Result**
$2m_1 m_2$	$2F$	$2r$	$\frac{1}{4}F$
$3m_1 m_2$	$3F$	$3r$	$\frac{1}{9}F$
$2m_1\ 3m_2$	$6F$	$\frac{1}{2}r$	$4F$
$\frac{1}{2}\ m_1 m_2$	$\frac{1}{2}F$	$\frac{1}{3}r$	$9F$



■ **Figure 7-5** A planet with mass m_p and orbital radius r orbits the Sun with mass m_S. (Illustration not to scale)

Universal Gravitation and Kepler's Third Law

Newton stated his law of universal gravitation in terms that applied to the motion of planets about the Sun. This agreed with Kepler's third law and confirmed that Newton's law fit the best observations of the day.

Consider a planet orbiting the Sun, as shown in **Figure 7-5.** Newton's second law of motion, $F_{net} = ma$, can be written as $F_{net} = m_p a_c$, where F is the gravitational force, m_p is the mass of the planet, and a_c is the centripetal acceleration of the planet. For simplicity, assume circular orbits. Recall from your study of uniform circular motion in Chapter 6 that, for a circular orbit, $a_c = 4\pi^2 r/T^2$. This means that $F_{net} = m_p a_c$ may now be written as $F_{net} = m_p 4\pi^2 r/T^2$. In this equation, T is the time required for the planet to make one complete revolution about the Sun. If you set the right side of this equation equal to the right side of the law of universal gravitation, you arrive at the following result:

$$G\frac{m_S m_p}{r^2} = \frac{m_p 4\pi^2 r}{T^2}$$

$$T^2 = \left(\frac{4\pi^2}{Gm_S}\right)r^3$$

$$\text{Thus, } T = \sqrt{\left(\frac{4\pi^2}{Gm_S}\right)r^3}$$

The period of a planet orbiting the Sun can be expressed as follows.

Period of a Planet Orbiting the Sun $T = 2\pi\sqrt{\frac{r^3}{Gm_S}}$

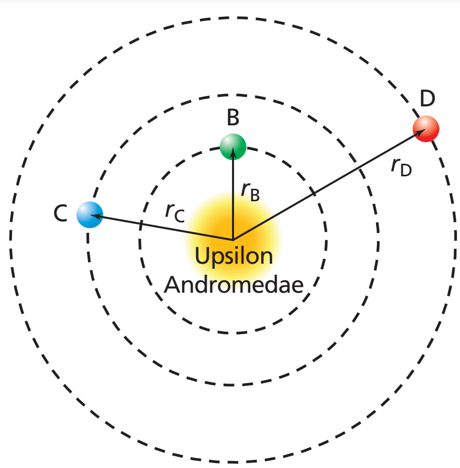
The period of a planet orbiting the Sun is equal to 2π times the square root of the orbital radius cubed, divided by the product of the universal gravitational constant and the mass of the Sun.

Squaring both sides makes it apparent that this equation is Kepler's third law of planetary motion: the square of the period is proportional to the cube of the distance that separates the masses. The factor $4\pi^2/Gm_S$ depends on the mass of the Sun and the universal gravitational constant. Newton found that this derivation applied to elliptical orbits as well.

CHALLENGE PROBLEM

Astronomers have detected three planets that orbit the star Upsilon Andromedae. Planet B has an average orbital radius of 0.059 AU and a period of 4.6170 days. Planet C has an average orbital radius of 0.829 AU and a period of 241.5 days. Planet D has an average orbital radius of 2.53 AU and a period of 1284 days. (Distances are given in astronomical units (AU)—Earth's average distance from the Sun. The distance from Earth to the Sun is 1.00 AU.)

1. Do these planets obey Kepler's third law?
2. Find the mass of the star Upsilon Andromedae in units of the Sun's mass.



Measuring the Universal Gravitational Constant

How large is the constant, G? As you know, the force of gravitational attraction between two objects on Earth is relatively small. The slightest attraction, even between two massive bowling balls, is difficult to detect. In fact, it took 100 years from the time of Newton's work for scientists to develop an apparatus that was sensitive enough to measure the force of gravitational attraction.

Cavendish's experiment In 1798, Englishman Henry Cavendish used equipment similar to the apparatus shown in **Figure 7-6** to measure the gravitational force between two objects. The apparatus had a horizontal rod with two small lead spheres attached to each end. The rod was suspended at its midpoint by a thin wire so that it could rotate. Because the rod was suspended by a thin wire, the rod and spheres were very sensitive to horizontal forces. To measure G, Cavendish placed two large lead spheres in a fixed position, close to each of the two small spheres, as shown in **Figure 7-7.** The force of attraction between the large and the small spheres caused the rod to rotate. When the force required to twist the wire equaled the gravitational force between the spheres, the rod stopped rotating. By measuring the angle through which the rod turned, Cavendish was able to calculate the attractive force between the objects. The angle through which the rod turned is measured using the beam of light that is reflected from the mirror. He measured the masses of the spheres and the distance between their centers. Substituting these values for force, mass, and distance into Newton's law of universal gravitation, he found an experimental value for G: when m_1 and m_2 are measured in kilograms, r in meters, and F in newtons, then $G = 6.67\times10^{-11}\ \mathrm{N\cdot m^2/kg^2}$.

Figure 7-6 Modern Cavendish balances are used to measure the gravitational forces between two objects.

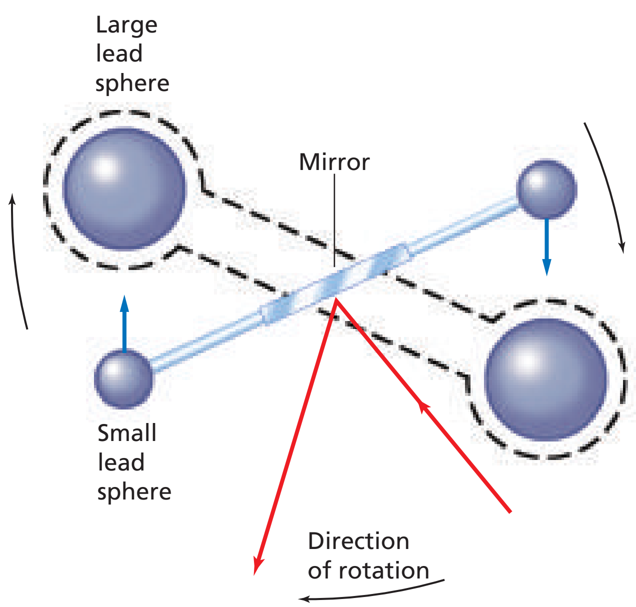


Figure 7-7 When the large lead spheres are placed near the small lead spheres, the gravitational attraction between the spheres causes the rod to rotate. The rotation is measured with the help of the reflected light ray.

The importance of *G* Cavendish's experiment often is called "weighing Earth," because his experiment helped determine Earth's mass. Once the value of G is known, not only the mass of Earth, but also the mass of the Sun can be determined. In addition, the gravitational force between any two objects can be calculated using Newton's law of universal gravitation. For example, the attractive gravitational force, F_g, between two bowling balls of mass 7.26 kg, with their centers separated by 0.30 m, can be calculated as follows:

$$F_g = \frac{(6.67\times10^{-11}\ \text{N·m}^2/\text{kg}^2)(7.26\ \text{kg})(7.26\ \text{kg})}{(0.30\ \text{m})^2} = 3.9\times10^{-8}\ \text{N}$$

You know that on Earth's surface, the weight of an object of mass m is a measure of Earth's gravitational attraction: $F_g = mg$. If Earth's mass is represented by m_E and Earth's radius is represented by r_E, the following is true:

$$F_g = G\frac{m_E m}{{r_E}^2} = mg, \text{ and so } g = G\frac{m_E}{{r_E}^2}$$

This equation can be rearranged to solve for m_E.

$$m_E = \frac{g{r_E}^2}{G}$$

Using $r_E = 6.38\times10^6$ m, $g = 9.80$ m/s^2, and $G = 6.67\times10^{-11}$ N·m^2/kg^2, the following result is obtained for Earth's mass:

$$m_E = \frac{(9.80\ \text{m/s}^2)(6.38\times10^6\ \text{m})^2}{6.67\times10^{-11}\ \text{N·m/kg}^2} = 5.98\times10^{24}\ \text{kg}$$

When you compare the mass of Earth to that of a bowling ball, you can see why the gravitational attraction between everyday objects is not easily observed. Cavendish's experiment determined the value of G, confirmed Newton's prediction that a gravitational force exists between two objects, and helped calculate the mass of Earth.

7.1 Section Review

6. Neptune's Orbital Period Neptune orbits the Sun with an orbital radius of 4.495×10^{12} m, which allows gases, such as methane, to condense and form an atmosphere, as shown in **Figure 7-8.** If the mass of the Sun is 1.99×10^{30} kg, calculate the period of Neptune's orbit.

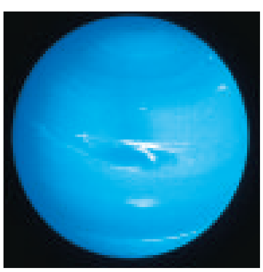


Figure 7-8

7. Gravity If Earth began to shrink, but its mass remained the same, what would happen to the value of g on Earth's surface?

8. Gravitational Force What is the gravitational force between two 15-kg packages that are 35 cm apart? What fraction is this of the weight of one package?

9. Universal Gravitational Constant Cavendish did his experiment using lead spheres. Suppose he had replaced the lead spheres with copper spheres of equal mass. Would his value of G be the same or different? Explain.

10. Laws or Theories? Kepler's three statements and Newton's equation for gravitational attraction are called "laws." Were they ever theories? Will they ever become theories?

11. Critical Thinking Picking up a rock requires less effort on the Moon than on Earth.

a. How will the weaker gravitational force on the Moon's surface affect the path of the rock if it is thrown horizontally?

b. If the thrower accidentally drops the rock on her toe, will it hurt more or less than it would on Earth? Explain.

Physics Online physicspp.com/self_check_quiz

CONTENTS

7.2 Using the Law of Universal Gravitation

The planet Uranus was discovered in 1781. By 1830, it was clear that the law of gravitation didn't correctly predict its orbit. Two astronomers proposed that Uranus was being attracted by the Sun and by an undiscovered planet. They calculated the orbit of such a planet in 1845, and, one year later, astronomers at the Berlin Observatory found the planet now called Neptune. How do planets, such as Neptune, orbit the Sun?

Objectives

- **Solve** orbital motion problems.
- **Relate** weightlessness to objects in free fall.
- **Describe** gravitational fields.
- **Compare** views on gravitation.

Vocabulary

gravitational field
inertial mass
gravitational mass

Orbits of Planets and Satellites

Newton used a drawing similar to the one shown in **Figure 7-9** to illustrate a thought experiment on the motion of satellites. Imagine a cannon, perched high atop a mountain, firing a cannonball horizontally with a given horizontal speed. The cannonball is a projectile, and its motion has both vertical and horizontal components. Like all projectiles on Earth, it would follow a parabolic trajectory and fall back to the ground.

If the cannonball's horizontal speed were increased, it would travel farther across the surface of Earth, and still fall back to the ground. If an extremely powerful cannon were used, however, the cannonball would travel all the way around Earth, and keep going. It would fall toward Earth at the same rate that Earth's surface curves away. In other words, the curvature of the projectile would continue to just match the curvature of Earth, so that the cannonball would never get any closer or farther away from Earth's curved surface. The cannonball would, therefore, be in orbit.

Newton's thought experiment ignored air resistance. For the cannonball to be free of air resistance, the mountain on which the cannon is perched would have to be more than 150 km above Earth's surface. By way of comparison, the mountain would have to be much taller than the peak of Mount Everest, the world's tallest mountain, which is only 8.85 km in height. A cannonball launched from a mountain that is 150 km above Earth's surface would encounter little or no air resistance at an altitude of 150 km, because the mountain would be above most of the atmosphere. Thus, a cannonball or any object or satellite at or above this altitude could orbit Earth for a long time.

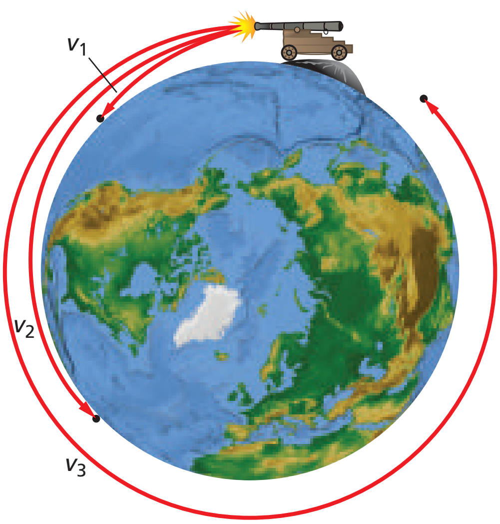


Figure 7-9 The horizontal speed v_1 is not high enough and the cannonball falls to the ground. With a higher speed, v_2, the cannonball travels farther. The cannonball travels all the way around Earth when the horizontal speed, v_3, is high enough.

APPLYING PHYSICS

Geosynchronous Orbit The *GOES-12* weather satellite orbits Earth once a day at an altitude of 35,785 km. The orbital speed of the satellite matches Earth's rate of rotation. Thus, to an observer on Earth, the satellite appears to remain above one spot on the equator. Satellite dishes on Earth can be directed to one point in the sky and not have to change position as the satellite orbits.

A satellite in an orbit that is always the same height above Earth moves in uniform circular motion. Recall that its centripetal acceleration is given by $a_c = v^2/r$. Newton's second law, $F_{net} = ma_c$, can thus be rewritten as $F_{net} = mv^2/r$. If Earth's mass is m_E, then this expression combined with Newton's law of universal gravitation produces the following equation:

$$G\frac{m_E m}{r^2} = \frac{mv^2}{r}$$

Solving for the speed of a satellite in circular orbit about Earth, v, yields the following.

Speed of a Satellite Orbiting Earth $v = \sqrt{\frac{Gm_E}{r}}$

The speed of a satellite orbiting Earth is equal to the square root of the universal gravitational constant times the mass of Earth, divided by the radius of the orbit.

Figure 7-10 *Landsat 7*, a remote sensing satellite, has a mass of about 2200 kg and orbits Earth at an altitude of about 705 km.

A satellite's orbital period A satellite's orbit around Earth is similar to a planet's orbit about the Sun. Recall that the period of a planet orbiting the Sun is expressed by the following equation:

$$T = 2\pi\sqrt{\frac{r^3}{Gm_S}}$$

Thus, the period for a satellite orbiting Earth is given by the following equation.

Period of a Satellite Orbiting Earth $T = 2\pi\sqrt{\frac{r^3}{Gm_E}}$

The period for a satellite orbiting Earth is equal to 2π times the square root of the radius of the orbit cubed, divided by the product of the universal gravitational constant and the mass of Earth.

The equations for the speed and period of a satellite can be used for any object in orbit about another. The mass of the central body will replace m_E in the equations, and r will be the distance between the centers of the orbiting body and the central body. If the mass of the central body is much greater than the mass of the orbiting body, then r is equal to the distance between the centers of the orbiting body and the central body. Orbital speed, v, and period, T, are independent of the mass of the satellite. Are there any factors that limit the mass of a satellite?

Geology Connection

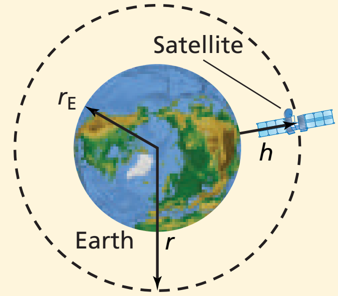
A satellite's mass *Landsat 7*, shown in **Figure 7-10,** is an artificial satellite that provides images of Earth's continental surfaces. *Landsat* images have been used to create maps, study land use, and monitor resources and global changes. The *Landsat 7* system enables researchers to monitor small-scale processes, such as deforestation, on a global scale. Satellites, such as *Landsat 7*, are accelerated to the speeds necessary for them to achieve orbit by large rockets, such as shuttle-booster rockets. Because the acceleration of any mass must follow Newton's second law of motion, $F_{net} = ma$, more force is required to launch a more massive satellite into orbit. Thus, the mass of a satellite is limited by the capability of the rocket used to launch it.

EXAMPLE Problem 2

Orbital Speed and Period Assume that a satellite orbits Earth 225 km above its surface. Given that the mass of Earth is 5.97×10^{24} kg and the radius of Earth is 6.38×10^{6} m, what are the satellite's orbital speed and period?

1 Analyze and Sketch the Problem

- Sketch the situation showing the height of the satellite's orbit.



Known:

$h = 2.25\times10^{5}$ m

$r_E = 6.38\times10^{6}$ m

$m_E = 5.97\times10^{24}$ kg

$G = 6.67\times10^{-11}\ \text{N·m}^2/\text{kg}^2$

Unknown:

$v = ?$

$T = ?$

2 Solve for the Unknown

Determine the orbital radius by adding the height of the satellite's orbit to Earth's radius.

$$r = h + r_E$$
$$= 2.25\times10^{5}\ \text{m} + 6.38\times10^{6}\ \text{m} = 6.61\times10^{6}\ \text{m}$$

Substitute $h = 2.25\times10^{5}$ m, $r_E = 6.38\times10^{6}$ m

Solve for the speed.

$$v = \sqrt{\frac{Gm_E}{r}}$$
$$= \sqrt{\frac{(6.67\times10^{-11}\ \text{N·m}^2/\text{kg}^2)(5.97\times10^{24}\ \text{kg})}{6.61\times10^{6}\ \text{m}}}$$
$$= 7.76\times10^{3}\ \text{m/s}$$

Substitute $G = 6.67\times10^{-11}\ \text{N·m}^2/\text{kg}^2$, $m_E = 5.97\times10^{24}$ kg, $r = 6.61\times10^{6}$ m

Solve for the period.

$$T = 2\pi\sqrt{\frac{r^3}{Gm_E}}$$
$$= 2\pi\sqrt{\frac{(6.61\times10^{6}\ \text{m})^3}{(6.67\times10^{-11}\ \text{N·m}^2/\text{kg}^2)(5.97\times10^{24}\ \text{kg})}}$$
$$= 5.35\times10^{3}\ \text{s}$$

Substitute $r = 6.61\times10^{6}$ m, $G = 6.67\times10^{-11}\ \text{N·m}^2/\text{kg}^2$, $m_E = 5.97\times10^{24}$ kg

This is approximately 89 min, or 1.5 h.

Math Handbook
Square and Cube Roots
pages 839–840

3 Evaluate the Answer

- **Are the units correct?** The unit for speed is m/s and the unit for period is s.

PRACTICE Problems

Additional Problems, Appendix B

For the following problems, assume a circular orbit for all calculations.

12. Suppose that the satellite in Example Problem 2 is moved to an orbit that is 24 km larger in radius than its previous orbit. What would its speed be? Is this faster or slower than its previous speed?

13. Use Newton's thought experiment on the motion of satellites to solve the following.

a. Calculate the speed that a satellite shot from a cannon must have to orbit Earth 150 km above its surface.

b. How long, in seconds and minutes, would it take for the satellite to complete one orbit and return to the cannon?

14. Use the data for Mercury in Table 7-1 on page 173 to find the following.

a. the speed of a satellite that is in orbit 260 km above Mercury's surface

b. the period of the satellite

MINI LAB

Weightless Water

This activity is best done outdoors. Use a pencil to poke two holes through a foam or paper cup: one on the bottom and the other on the side. Hold your fingers over the two holes to block them as your lab partner pours colored water into the cup until it is two-thirds full.

1. Predict what will happen as the cup is allowed to fall.

2. Test your prediction: drop the cup and watch closely.

Analyze and Conclude

3. Describe your observations.

4. Explain your results.

Acceleration Due To Gravity

The acceleration of objects due to Earth's gravity can be found by using Newton's law of universal gravitation and his second law of motion. For a free-falling object, m, the following is true:

$$F = G\frac{m_E m}{r^2} = ma, \text{ so } a = G\frac{m_E}{r^2}$$

Because $a = g$ and $r = r_E$ on Earth's surface, the following equation can be written:

$$g = G\frac{m_E}{r_E^{\,2}}, \text{ thus, } m_E = \frac{g r_E^{\,2}}{G}$$

You found above that $a = G\frac{m_E}{r^2}$ for a free-falling object. Substituting the above expression for m_E yields the following:

$$a = G\frac{\left(\frac{g r_E^{\,2}}{G}\right)}{r^2}$$

$$a = g\left(\frac{r_E}{r}\right)^2$$

This shows that as you move farther from Earth's center, that is, as r becomes larger, the acceleration due to gravity is reduced according to this inverse square relationship. What happens to your weight, m_g, as you move farther and farther from Earth's center?

Weight and weightlessness You probably have seen photos similar to the one in **Figure 7-11** in which astronauts are on the space shuttle in an environment often called "zero-g" or "weightlessness." The shuttle orbits about 400 km above Earth's surface. At that distance, $g = 8.7$ m/s^2, only slightly less than on Earth's surface. Thus, Earth's gravitational force is certainly not zero in the shuttle. In fact, gravity causes the shuttle to orbit Earth. Why, then, do the astronauts appear to have no weight?

Remember that you sense weight when something, such as the floor or your chair, exerts a contact force on you. But if you, your chair, and the floor all are accelerating toward Earth together, then no contact forces are exerted on you. Thus, your apparent weight is zero and you experience weightlessness. Similarly, the astronauts experience weightlessness as the shuttle and everything in it falls freely toward Earth.

Figure 7-11 Astronaut Chiaki Mukai experiences weightlessness on board the space shuttle *Columbia*, as the shuttle and everything in it falls freely toward Earth.

The Gravitational Field

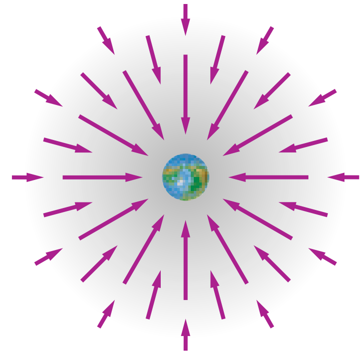
Recall from Chapter 6 that many common forces are contact forces. Friction is exerted where two objects touch, for example, when the floor and your chair or desk push on you. Gravity, however, is different. It acts on an apple falling from a tree and on the Moon in orbit. It even acts on you in midair as you jump up or skydive. In other words, gravity acts over a distance. It acts between objects that are not touching or that are not close together. Newton was puzzled by this concept. He wondered how the Sun could exert a force on planet Earth, which is hundreds of millions of kilometers away.

NASA

The answer to the puzzle arose from a study of magnetism. In the 19th century, Michael Faraday developed the concept of a field to explain how a magnet attracts objects. Later, the field concept was applied to gravity. Any object with mass is surrounded by a **gravitational field** in which another object experiences a force due to the interaction between its mass and the gravitational field, $\boldsymbol{g}$, at its location. This is expressed by the following equation.

Gravitational Field $\quad \boldsymbol{g} = \frac{GM}{r^2}$

The gravitational field is equal to the universal gravitational constant times the object's mass, divided by the square of the distance from the object's center. The direction is toward the mass's center.



■ **Figure 7-12** Vectors representing Earth's gravitational field all point toward Earth's center. The field is weaker farther from Earth.

Suppose the gravitational field is created by the Sun. Then a planet of mass m has a force exerted on it that depends on its mass and the magnitude of the gravitational field at its location. That is, $\boldsymbol{F} = m\boldsymbol{g}$, toward the Sun. The force is caused by the interaction of the planet's mass with the gravitational field at its location, not with the Sun millions of kilometers away. To find the gravitational field caused by more than one object you would calculate both gravitational fields and add them as vectors.

The gravitational field can be measured by placing an object with a small mass, m, in the gravitational field and measuring the force, F, on it. The gravitational field can be calculated using $g = F/m$. The gravitational field is measured in N/kg, which is also equal to m/s^2.

On Earth's surface, the strength of the gravitational field is 9.80 N/kg, and its direction is toward Earth's center. The field can be represented by a vector of length g pointing toward the center of the object producing the field. You can picture the gravitational field of Earth as a collection of vectors surrounding Earth and pointing toward it, as shown in **Figure 7-12.** The strength of the field varies inversely with the square of the distance from the center of Earth. The gravitational field depends on Earth's mass, but not on the mass of the object experiencing it.

Two Kinds of Mass

Recall that when the concept of mass was discussed in Chapter 4, it was defined as the slope of a graph of force versus acceleration. That is, mass is equal to the ratio of the net force exerted on an object to its acceleration. This kind of mass, related to the inertia of an object, is called **inertial mass** and is represented by the following equation.

Inertial Mass $\quad m_{\text{inertial}} = \frac{F_{\text{net}}}{a}$

Inertial mass is equal to the net force exerted on the object divided by the acceleration of the object.

■ **Figure 7-13** An inertial balance allows you to calculate the inertial mass of an object from the period (T) of the back-and-forth motion of the object. Calibration masses, such as the cylindrical ones shown here, are used to create a graph of T^2 versus the mass. The period of the unknown mass is then measured, and the inertial mass is determined from the calibration graph.

courtesy of PASCO Scientific

The inertial mass of an object is measured by exerting a force on the object and measuring the object's acceleration using an inertial balance, such as the one shown in **Figure 7-13.** The more inertial mass an object has, the less it is affected by any force—the less acceleration it undergoes. Thus, the inertial mass of an object is a measure of the object's resistance to any type of force.

■ **Figure 7-14** The platform balance shown here allows you to measure the force on a mass due to Earth's gravity.

Newton's law of universal gravitation, $F = Gm_1m_2/r^2$, also involves mass, but a different kind of mass. Mass as used in the law of universal gravitation determines the size of the gravitational force between two objects and is called **gravitational mass.** It can be measured using a simple balance, such as the one shown in **Figure 7-14.** If you measure the attractive force exerted on an object by another object of mass, *m,* at a distance, *r,* then you can define the gravitational mass in the following way.

Gravitational Mass $m_{grav} = \frac{r^2F_{grav}}{Gm}$

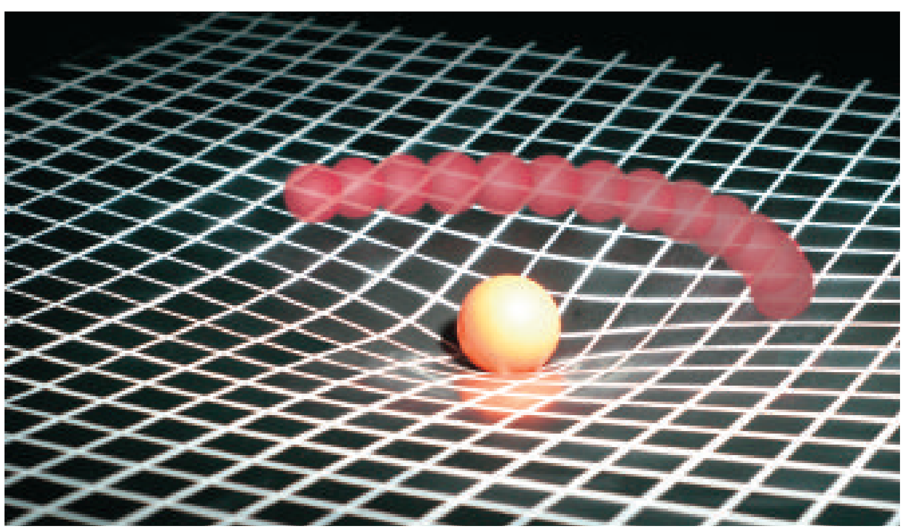
The gravitational mass of an object is equal to the distance between the objects squared, times the gravitational force, divided by the product of the universal gravitational constant, times the mass of the other object.

How different are these two kinds of mass? Suppose you have a watermelon in the trunk of your car. If you accelerate the car forward, the watermelon will roll backwards, relative to the trunk. This is a result of its inertial mass—its resistance to acceleration. Now, suppose your car climbs a steep hill at a constant speed. The watermelon will again roll backwards. But this time, it moves as a result of its gravitational mass. The watermelon is being attracted downward toward Earth. Newton made the claim that inertial mass and gravitational mass are equal in magnitude. This hypothesis is called the principle of equivalence. All experiments conducted so far have yielded data that support this principle. Albert Einstein also was intrigued by the principle of equivalence and made it a central point in his theory of gravity.

Einstein's Theory of Gravity

Newton's law of universal gravitation allows us to calculate the gravitational force that exists between two objects because of their masses. The concept of a gravitational field allows us to picture the way gravity acts on objects that are far away. Einstein proposed that gravity is not a force, but rather, an effect of space itself. According to Einstein, mass changes the space around it. Mass causes space to be curved, and other bodies are accelerated because of the way they follow this curved space.

■ **Figure 7-15** Matter causes space to curve just as an object on a rubber sheet curves the sheet around it. Moving objects near the mass follow the curvature of space. The red ball is moving clockwise around the center mass.



One way to picture how space is affected by mass is to compare space to a large, two-dimensional rubber sheet, as shown in **Figure 7-15.** The yellow ball on the sheet represents a massive object. It forms an indentation. A red ball rolling across the sheet simulates the motion of an object in space. If the red ball moves near the sagging region of the sheet, it will be accelerated. In the same way, Earth and the Sun are attracted to one another because of the way space is distorted by the two objects.

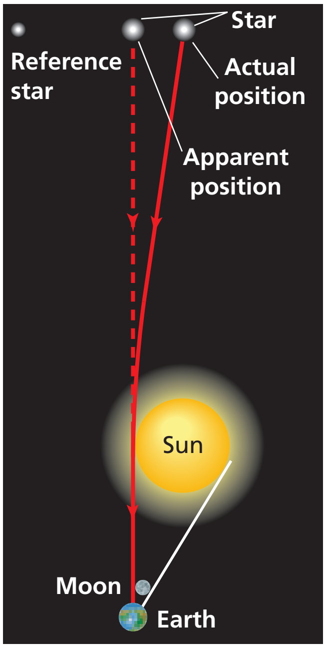
Einstein's theory, called the general theory of relativity, makes many predictions about how massive objects affect one another. In every test conducted to date, Einstein's theory has been shown to give the correct results.

Deflection of light Einstein's theory predicts the deflection or bending of light by massive objects. Light follows the curvature of space around the massive object and is deflected, as shown in **Figure 7-16.** In 1919, during an eclipse of the Sun, astronomers found that light from distant stars that passed near the Sun was deflected in agreement with Einstein's predictions.

Another result of general relativity is the effect on light from very massive objects. If an object is massive and dense enough, the light leaving it will be totally bent back to the object. No light ever escapes the object. Objects such as these, called black holes, have been identified as a result of their effects on nearby stars. The radiation produced when matter is pulled into black holes has also been helpful in their detection.

While Einstein's theory provides very accurate predictions of gravity's effects, it is still incomplete. It does not explain the origin of mass or how mass curves space. Physicists are working to understand the deeper meaning of gravity and the origin of mass itself.

Figure 7-16 The light from a distant star bends due to the Sun's gravitational field, thereby changing the apparent position of the star. (Illustration not to scale)



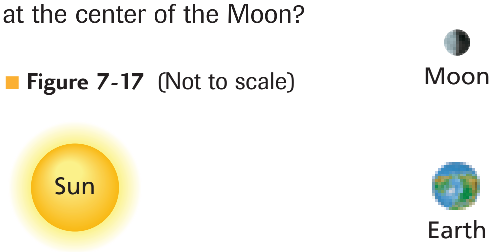
7.2 Section Review

15. Gravitational Fields The Moon is 3.9×10^5 km from Earth's center and 1.5×10^8 km from the Sun's center. The masses of Earth and the Sun are 6.0×10^{24} kg and 2.0×10^{30} kg, respectively.

a. Find the ratio of the gravitational fields due to Earth and the Sun at the center of the Moon.

b. When the Moon is in its third quarter phase, as shown in **Figure 7-17,** its direction from Earth is at right angles to the Sun's direction. What is the net gravitational field due to the Sun and Earth at the center of the Moon?

Figure 7-17 (Not to scale)



16. Gravitational Field The mass of the Moon is 7.3×10^{22} kg and its radius is 1785 km. What is the strength of the gravitational field on the surface of the Moon?

17. A Satellite's Mass When the first artificial satellite was launched into orbit by the former Soviet Union in 1957, U.S. president Dwight D. Eisenhower asked his scientific advisors to calculate the mass of the satellite. Would they have been able to make this calculation? Explain.

18. Orbital Period and Speed Two satellites are in circular orbits about Earth. One is 150 km above the surface, the other 160 km.

a. Which satellite has the larger orbital period?

b. Which one has the greater speed?

19. Theories and Laws Why is Einstein's description of gravity called a "theory," while Newton's is a "law?"

20. Weightlessness Chairs in an orbiting spacecraft are weightless. If you were on board such a spacecraft and you were barefoot, would you stub your toe if you kicked a chair? Explain.

21. Critical Thinking It is easier to launch a satellite from Earth into an orbit that circles eastward than it is to launch one that circles westward. Explain.

PHYSICS LAB

Modeling the Orbits of Planets and Satellites

In this experiment, you will analyze a model that will show how Kepler's first and second laws of motion apply to orbits of objects in space. Kepler's first law states that orbits of planets are ellipses, with the Sun at one focus. Kepler's second law states that an imaginary line from the Sun to a planet sweeps out equal areas in equal time intervals.

The shape of the elliptical orbit is defined by eccentricity, *e*, which is the ratio of the distance between the foci to the length of the major axis. When an object is at its farthest distance from the Sun along the major axis, it is at aphelion. When the object is at its closest distance from the Sun along the major axis, it is at perihelion.

QUESTION

What is the shape of the orbits of planets and satellites in the solar system?

Objectives

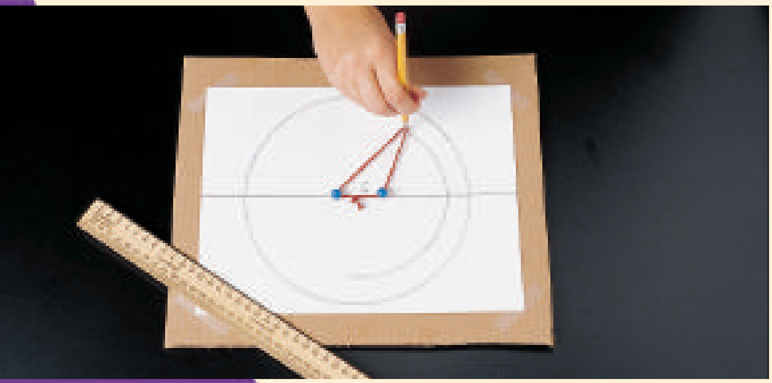
- **Formulate models** to infer the shape of orbits of planets and satellites.
- **Collect and organize data** for aphelion distances and perihelion distances of objects as they orbit the Sun.
- **Draw conclusions** about Kepler's first and second laws of motion.

Safety Precautions

- **Pins are sharp and can puncture the skin.**

Materials

piece of cardboard
sheet of blank, white paper
two push pins
metric ruler
sharp pencil or pen
four small pieces of tape
string (25 cm)



Procedure

1. Place a piece of paper on a piece of cardboard using tape at the four corners.
2. Draw a line across the center of the paper, along the length of the paper. This line represents the major axis.
3. Mark the center of the line and label it *C*.
4. Use the string to tie a loop, which, when stretched, has a length of 10 cm. For each object listed in the data table, calculate the distance between the foci, *d*, using the following equation:

$$d = \frac{2e(10.0\ \text{cm})}{e + 1}$$

5. For the circle, place a pin at C. Put the loop of string over the pin and pull it tight with your pencil. Move the pencil in a circular fashion around the center, letting the string guide it.
6. For the next object, place one pin a distance of $d/2$ from C along the major axis.
7. Place a second pin a distance of $d/2$ on the opposite side of C. The two pins represent the foci. One focus is the location of the Sun.
8. Put the loop of string over both pins and pull it tight with your pencil. Move the pencil in a circular fashion, letting the string guide it.
9. Using the same paper, repeat steps 6-8 for each of the listed objects.
10. After all of the orbits are plotted, label each orbit with the name and eccentricity of the object plotted.

Data Table

Object	Eccentricity (e)	d (cm)	Measured A	Measured P	Experimental e	% Error
Circle	0					
Earth	0.017					
Pluto	0.25					
Comet	0.70					

Analyze

1. Measure the aphelion distance, A, by measuring the distance between one focus and the farthest point in the orbit along the major axis. Record your data in the data table.
2. Measure the perihelion distance, P, by measuring the closest distance between one focus and the closest point in the orbit along the major axis. Record the data in the data table.
3. Calculate the experimental eccentricity for each of the objects and record your data in the data table. Use the following equation:
$$e = \frac{A - P}{A + P}$$
4. **Error Analysis** Calculate the percent error for each object using the experimental eccentricities compared to the known eccentricities. Record your values in the data table.
5. **Analyze** Why is the shape of the orbit with $e = 0$ a circle?
6. **Compare** How does Earth's orbit compare to a circle?
7. **Observe** Which of the orbits truly looks elliptical?

Conclude and Apply

1. Does the orbit model you constructed obey Kepler's first law? Explain.
2. Kepler studied the orbit data of Mars ($e = 0.093$) and concluded that planets move about the Sun in elliptical orbits. What would Kepler have concluded if he had been on Mars and studied Earth's orbit?
3. Where does a planet travel fastest: at aphelion or perihelion? Why?
4. Kepler's second law helps to determine the ratio between Pluto's velocity at aphelion and perihelion (v_A/v_P). To determine this ratio, first calculate the area swept out by Pluto's orbit. This area is approximately equal to the area of a triangle: Area $= \frac{1}{2}$ (distance to the Sun) current velocity $\times$ time. If the area that the orbit sweeps out in a fixed amount of time, such as 30 days, is the same at aphelion and perihelion, this relationship can be written
$$\frac{1}{2}Pv_Pt = \frac{1}{2}Av_At$$
What is the ratio v_P/v_A for Pluto?
5. Pluto's minimum orbital velocity is 3.7 km/s. What are the values for v_P and v_A?

Going Further

1. You used rough approximations to look at Kepler's second law. Suggest an experiment to obtain precise results to confirm the second law.
2. Design an experiment to verify Kepler's third law.

Real-World Physics

Does a communications or weather satellite that is orbiting Earth follow Kepler's laws? Collect data to verify your answer.

Physics Online

To find out more about gravitation, visit the Web site: **physicspp.com**

eXtreme physics

Black Holes

What would happen if you were to travel to a black hole? Your body would be stretched, flattened, and eventually pulled apart. What is a black hole? What is known about black holes?

A black hole is one of the possible final stages in the evolution of a star. When fusion reactions stop in the core of a star that is at least 20 times more massive than the Sun, the core collapses forever, compacting matter into an increasingly smaller volume. The infinitely small, but infinitely dense, object that remains is called a singularity. The force of gravity is so immense in the region around the singularity that nothing, not even light, can escape it. This region is called a black hole.

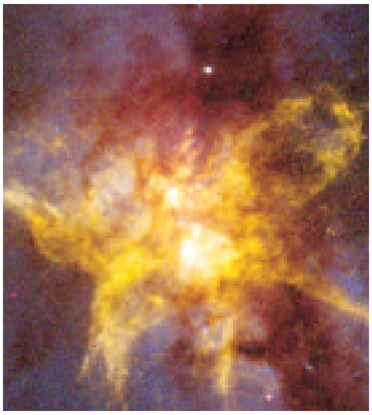
Nothing Can Escape In 1917, German mathematician Karl Schwarzschild verified, mathematically, that black holes could exist. Schwarzschild used solutions to Einstein's theory of general relativity to describe the properties of black holes. He derived an expression for a radius, called the Schwarzschild radius, within which neither light nor matter escapes the force of gravity of the singularity. The Schwarzschild radius is represented by the following equation:

$$R_s = \frac{2GM}{c^2}$$

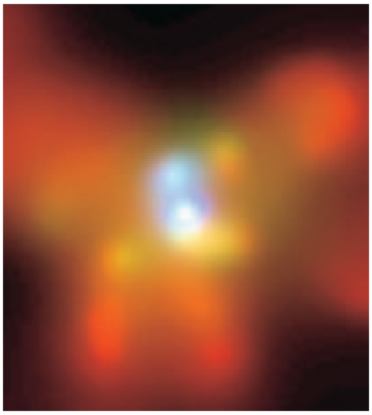
In this equation, G is Newton's universal gravitational constant, M is the mass of the black hole, and c is the speed of light. The edge of the sphere defined by the Schwarzschild radius is called the event horizon. At the event horizon, the escape velocity equals the speed of light. Because nothing travels faster than the speed of light, objects that cross the event horizon can never escape.

Indirect and Direct Evidence Black holes have three physical properties that can theoretically be measured—mass, angular momentum, and electric charge. A black hole's mass can be determined by the gravitational field it generates. Mass is calculated by using a modified form of Kepler's third law of planetary motion. Studies using NASA's *Rossi X-ray Timing Explorer* have shown that black holes spin just as stars and planets do. A black hole spins because it retains the angular momentum of the star that formed it. Even though a black hole's electric charge has not been measured, scientists hypothesize that a black hole may become charged when an excess of one type of electric charge falls into it. Super-heated gases in a black hole emit X rays, which can be detected by X-ray telescopes, such as the space-based *Chandra X-ray Observatory.*

Although not everything is known about black holes, there is direct and indirect evidence of their existence. Continued research and special missions will provide a better understanding of black holes.



***Hubble* visible image of galaxy NGC 6240.**



***Chandra* X-ray image of two black holes (blue) in NGC 6240.**

Going Further

Solve The escape velocity of an object leaving the event horizon can be represented by the following equation:

$$v = \sqrt{\frac{2GM}{R_s}}$$

In this equation, G is Newton's universal gravitational constant, M is the mass of the black hole, and R_s is the radius of the black hole. Show that the escape velocity equals the speed of light.

Chapter 7 Study Guide

7.1 Planetary Motion and Gravitation

Vocabulary

- Kepler's first law (p. 172)
- Kepler's second law (p. 173)
- Kepler's third law (p. 173)
- gravitational force (p. 175)
- law of universal gravitation (p. 175)

Key Concepts

- Kepler's first law states that planets move in elliptical orbits, with the Sun at one focus.
- Kepler's second law states that an imaginary line from the Sun to a planet sweeps out equal areas in equal times.
- Kepler's third law states that the square of the ratio of the periods of any two planets is equal to the cube of the ratio of their distances from the Sun.

$$\left(\frac{T_A}{T_B}\right)^2 = \left(\frac{r_A}{r_B}\right)^3$$

- Newton's law of universal gravitation states that the gravitational force between any two objects is directly proportional to the product of their masses and inversely proportional to the square of the distance between their centers. The force is attractive and along a line connecting their centers.

$$F = G\frac{m_1 m_2}{r^2}$$

- Newton's law of universal gravitation can be used to rewrite Kepler's third law to relate the radius and period of a planet to the mass of the Sun.

$$T^2 = \left(\frac{4\pi^2}{Gm_S}\right)r^3$$

7.2 Using the Law of Universal Gravitation

Vocabulary

- gravitational field (p. 183)
- inertial mass (p. 183)
- gravitational mass (p. 184)

Key Concepts

- The speed of an object in circular orbit is given by the following expression.

$$v = \sqrt{\frac{Gm_E}{r}}$$

- The period of a satellite in a circular orbit is given by the following expression.

$$T = 2\pi\sqrt{\frac{r^3}{Gm_E}}$$

- All objects have gravitational fields surrounding them.

$$g = \frac{Gm}{r^2}$$

- Gravitational mass and inertial mass are two essentially different concepts. The gravitational and inertial masses of an object, however, are numerically equal.

$$m_{\text{inertial}} = \frac{F_{\text{net}}}{a} \qquad m_{\text{grav}} = \frac{r^2 F_{\text{grav}}}{Gm}$$

- Einstein's general theory of relativity describes gravitational attraction as a property of space itself.

Chapter 7 Assessment

Concept Mapping

22. Create a concept map using these terms: *planets, stars, Newton's law of universal gravitation, Kepler's first law, Kepler's second law, Kepler's third law, Einstein's general theory of relativity.*

Mastering Concepts

23. In 1609, Galileo looked through his telescope at Jupiter and saw four moons. The name of one of the moons that he saw is Io. Restate Kepler's first law for Io and Jupiter. (7.1)

24. Earth moves more slowly in its orbit during summer in the northern hemisphere than it does during winter. Is it closer to the Sun in summer or in winter? (7.1)

25. Is the area swept out per unit of time by Earth moving around the Sun equal to the area swept out per unit of time by Mars moving around the Sun? (7.1)

26. Why did Newton think that a force must act on the Moon? (7.1)

27. How did Cavendish demonstrate that a gravitational force of attraction exists between two small objects? (7.1)

28. What happens to the gravitational force between two masses when the distance between the masses is doubled? (7.1)

29. According to Newton's version of Kepler's third law, how would the ratio T^2/r^3 change if the mass of the Sun were doubled? (7.1)

30. How do you answer the question, "What keeps a satellite up?" (7.2)

31. A satellite is orbiting Earth. On which of the following does its speed depend? (7.2)

a. mass of the satellite
b. distance from Earth
c. mass of Earth

32. What provides the force that causes the centripetal acceleration of a satellite in orbit? (7.2)

33. During space flight, astronauts often refer to forces as multiples of the force of gravity on Earth's surface. What does a force of $5g$ mean to an astronaut? (7.2)

34. Newton assumed that a gravitational force acts directly between Earth and the Moon. How does Einstein's view of the attractive force between the two bodies differ from Newton's view? (7.2)

35. Show that the dimensions of g in the equation $g = F/m$ are in m/s^2. (7.2)

36. If Earth were twice as massive but remained the same size, what would happen to the value of g? (7.2)

Applying Concepts

37. Golf Ball The force of gravity acting on an object near Earth's surface is proportional to the mass of the object. **Figure 7-18** shows a tennis ball and golf ball in free fall. Why does a tennis ball not fall faster than a golf ball?

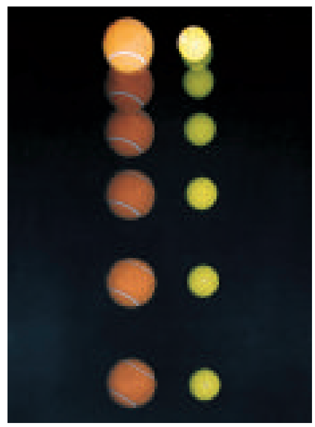


Figure 7-18

38. What information do you need to find the mass of Jupiter using Newton's version of Kepler's third law?

39. The mass of Pluto was not known until a satellite of the planet was discovered. Why?

40. Decide whether each of the orbits shown in **Figure 7-19** is a possible orbit for a planet.

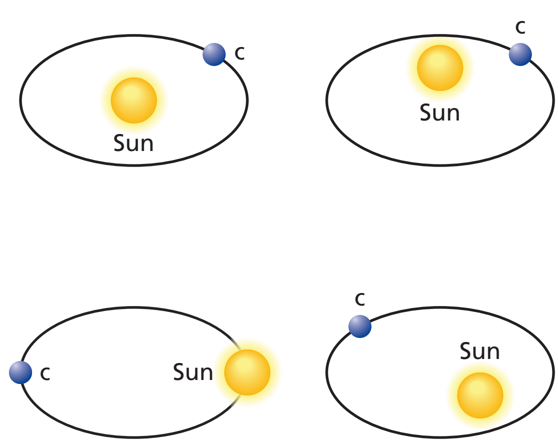


Figure 7-19

Barry Runk/Grant Heilman

 For more problems, go to Additional Problems, Appendix B.

41. The Moon and Earth are attracted to each other by gravitational force. Does the more-massive Earth attract the Moon with a greater force than the Moon attracts Earth? Explain.

42. What would happen to the value of G if Earth were twice as massive, but remained the same size?

43. **Figure 7-20** shows a satellite orbiting Earth. Examine the equation $v = \sqrt{\frac{Gm_E}{r}}$, relating the speed of an orbiting satellite and its distance from the center of Earth. Does a satellite with a large or small orbital radius have the greater velocity?

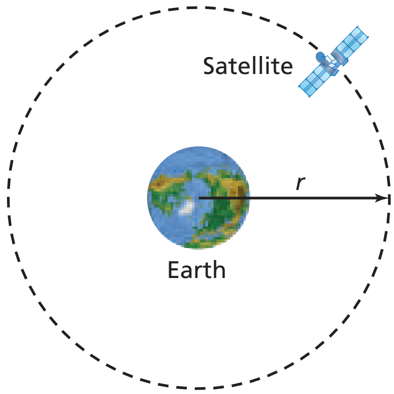


Figure 7-20 (Not to scale)

44. Space Shuttle If a space shuttle goes into a higher orbit, what happens to the shuttle's period?

45. Mars has about one-ninth the mass of Earth. **Figure 7-21** shows satellite M, which orbits Mars with the same orbital radius as satellite E, which orbits Earth. Which satellite has a smaller period?

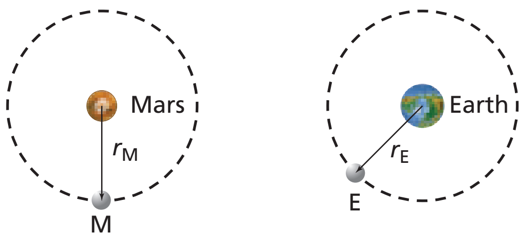


Figure 7-21 (Not to scale)

46. Jupiter has about 300 times the mass of Earth and about ten times Earth's radius. Estimate the size of g on the surface of Jupiter.

47. A satellite is one Earth radius above the surface of Earth. How does the acceleration due to gravity at that location compare to acceleration at the surface of Earth?

48. If a mass in Earth's gravitational field is doubled, what will happen to the force exerted by the field upon the mass?

49. Weight Suppose that yesterday your body had a mass of 50.0 kg. This morning you stepped on a scale and found that you had gained weight.

a. What happened, if anything, to your mass?

b. What happened, if anything, to the ratio of your weight to your mass?

50. As an astronaut in an orbiting space shuttle, how would you go about "dropping" an object down to Earth?

51. Weather Satellites The weather pictures that you see every day on TV come from a spacecraft in a stationary position relative to the surface of Earth, 35,700 km above Earth's equator. Explain how it can stay in exactly the same position day after day. What would happen if it were closer? Farther out? *Hint: Draw a pictorial model.*

Mastering Problems

7.1 Planetary Motion and Gravitation

52. Jupiter is 5.2 times farther from the Sun than Earth is. Find Jupiter's orbital period in Earth years.

53. **Figure 7-22** shows a Cavendish apparatus like the one used to find G. It has a large lead sphere that is 5.9 kg in mass and a small one with a mass of 0.047 kg. Their centers are separated by 0.055 m. Find the force of attraction between them.

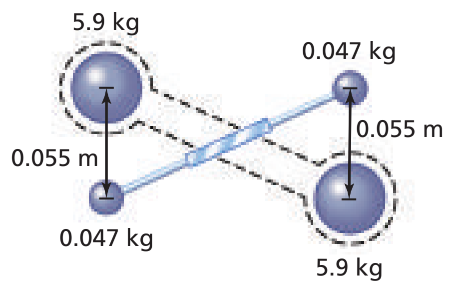


Figure 7-22

54. Use Table 7-1 on p. 173 to compute the gravitational force that the Sun exerts on Jupiter.

55. Tom has a mass of 70.0 kg and Sally has a mass of 50.0 kg. Tom and Sally are standing 20.0 m apart on the dance floor. Sally looks up and sees Tom. She feels an attraction. If the attraction is gravitational, find its size. Assume that both Tom and Sally can be replaced by spherical masses.

56. Two balls have their centers 2.0 m apart, as shown in **Figure 7-23.** One ball has a mass of 8.0 kg. The other has a mass of 6.0 kg. What is the gravitational force between them?

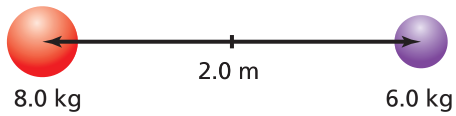


Figure 7-23

57. Two bowling balls each have a mass of 6.8 kg. They are located next to each other with their centers 21.8 cm apart. What gravitational force do they exert on each other?

58. Assume that you have a mass of 50.0 kg. Earth has a mass of 5.97×10^{24} kg and a radius of 6.38×10^{6} m.

a. What is the force of gravitational attraction between you and Earth?
b. What is your weight?

59. The gravitational force between two electrons that are 1.00 m apart is 5.54×10^{-71} N. Find the mass of an electron.

60. A 1.0-kg mass weighs 9.8 N on Earth's surface, and the radius of Earth is roughly 6.4×10^{6} m.

a. Calculate the mass of Earth.
b. Calculate the average density of Earth.

61. Uranus Uranus requires 84 years to circle the Sun. Find Uranus's orbital radius as a multiple of Earth's orbital radius.

62. Venus Venus has a period of revolution of 225 Earth days. Find the distance between the Sun and Venus as a multiple of Earth's orbital radius.

63. If a small planet, D, were located 8.0 times as far from the Sun as Earth is, how many years would it take the planet to orbit the Sun?

64. Two spheres are placed so that their centers are 2.6 m apart. The force between the two spheres is 2.75×10^{-12} N. What is the mass of each sphere if one sphere is twice the mass of the other sphere?

65. The Moon is 3.9×10^{5} km from Earth's center and 1.5×10^{8} km from the Sun's center. If the masses of the Moon, Earth, and the Sun are 7.3×10^{22} kg, 6.0×10^{24} kg, and 2.0×10^{30} kg, respectively, find the ratio of the gravitational forces exerted by Earth and the Sun on the Moon.

66. Toy Boat A force of 40.0 N is required to pull a 10.0-kg wooden toy boat at a constant velocity across a smooth glass surface on Earth. What force would be required to pull the same wooden toy boat across the same glass surface on the planet Jupiter?

67. Mimas, one of Saturn's moons, has an orbital radius of 1.87×10^{8} m and an orbital period of about 23.0 h. Use Newton's version of Kepler's third law to find Saturn's mass.

68. The Moon is 3.9×10^{8} m away from Earth and has a period of 27.33 days. Use Newton's version of Kepler's third law to find the mass of Earth. Compare this mass to the mass found in problem 60.

69. Halley's Comet Every 74 years, comet Halley is visible from Earth. Find the average distance of the comet from the Sun in astronomical units (AU).

70. Area is measured in m^2, so the rate at which area is swept out by a planet or satellite is measured in m^2/s.

a. How quickly is an area swept out by Earth in its orbit about the Sun?
b. How quickly is an area swept out by the Moon in its orbit about Earth? Use 3.9×10^{8} m as the average distance between Earth and the Moon, and 27.33 days as the period of the Moon.

7.2 Using the Law of Universal Gravitation

71. Satellite A geosynchronous satellite is one that appears to remain over one spot on Earth, as shown in **Figure 7-24.** Assume that a geosynchronous satellite has an orbital radius of 4.23×10^{7} m.

a. Calculate its speed in orbit.
b. Calculate its period.

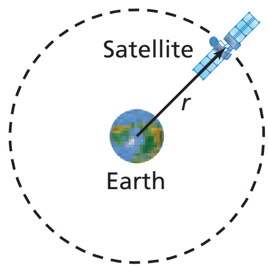


Figure 7-24 (Not to scale)

72. Asteroid The asteroid Ceres has a mass of 7×10^{20} kg and a radius of 500 km.

a. What is g on the surface of Ceres?
b. How much would a 90-kg astronaut weigh on Ceres?

73. Book A 1.25-kg book in space has a weight of 8.35 N. What is the value of the gravitational field at that location?

74. The Moon's mass is 7.34×10^{22} kg, and it is 3.8×10^{8} m away from Earth. Earth's mass is 5.97×10^{24} kg.

a. Calculate the gravitational force of attraction between Earth and the Moon.
b. Find Earth's gravitational field at the Moon.

 For more problems, go to Additional Problems, Appendix B.

75. Two 1.00-kg masses have their centers 1.00 m apart. What is the force of attraction between them?

76. The radius of Earth is about 6.38×10^3 km. A 7.20×10^3-N spacecraft travels away from Earth. What is the weight of the spacecraft at the following distances from Earth's surface?
a. 6.38×10^3 km
b. 1.28×10^4 km

77. Rocket How high does a rocket have to go above Earth's surface before its weight is half of what it is on Earth?

78. Two satellites of equal mass are put into orbit 30.0 m apart. The gravitational force between them is 2.0×10^{-7} N.
a. What is the mass of each satellite?
b. What is the initial acceleration given to each satellite by gravitational force?

79. Two large spheres are suspended close to each other. Their centers are 4.0 m apart, as shown in **Figure 7-25.** One sphere weighs 9.8×10^2 N. The other sphere has a weight of 1.96×10^2 N. What is the gravitational force between them?

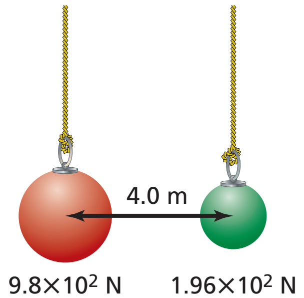


Figure 7-25

80. Suppose the centers of Earth and the Moon are 3.9×10^8 m apart, and the gravitational force between them is about 1.9×10^{20} N. What is the approximate mass of the Moon?

81. On the surface of the Moon, a 91.0-kg physics teacher weighs only 145.6 N. What is the value of the Moon's gravitational field at its surface?

82. The mass of an electron is 9.1×10^{-31} kg. The mass of a proton is 1.7×10^{-27} kg. An electron and a proton are about 0.59×10^{-10} m apart in a hydrogen atom. What gravitational force exists between the proton and the electron of a hydrogen atom?

83. Consider two spherical 8.0-kg objects that are 5.0 m apart.
a. What is the gravitational force between the two objects?
b. What is the gravitational force between them when they are 5.0×10^1 m apart?

84. If you weigh 637 N on Earth's surface, how much would you weigh on the planet Mars? Mars has a mass of 6.42×10^{23} kg and a radius of 3.40×10^6 m.

85. Using Newton's version of Kepler's third law and information from Table 7-1 on page 173, calculate the period of Earth's Moon if the orbital radius were twice the actual value of 3.9×10^8 m.

86. Find the value of g, acceleration due to gravity, in the following situations.
a. Earth's mass is triple its actual value, but its radius remains the same.
b. Earth's radius is tripled, but its mass remains the same.
c. Both the mass and radius of Earth are doubled.

87. Astronaut What would be the strength of Earth's gravitational field at a point where an 80.0-kg astronaut would experience a 25.0 percent reduction in weight?

Mixed Review

88. Use the information for Earth in Table 7-1 on page 173 to calculate the mass of the Sun, using Newton's version of Kepler's third law.

89. Earth's gravitational field is 7.83 N/kg at the altitude of the space shuttle. At this altitude, what is the size of the force of attraction between a student with a mass of 45.0 kg and Earth?

90. Use the data from Table 7-1 on page 173 to find the speed and period of a satellite that orbits Mars 175 km above its surface.

91. Satellite A satellite is placed in orbit, as shown in **Figure 7-26,** with a radius that is half the radius of the Moon's orbit. Find the period of the satellite in units of the period of the Moon.

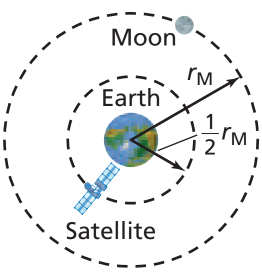


Figure 7-26

92. Cannonball The Moon's mass is 7.3×10^{22} kg and its radius is 1785 km. If Newton's thought experiment of firing a cannonball from a high mountain were attempted on the Moon, how fast would the cannonball have to be fired? How long would it take the cannonball to return to the cannon?

93. The period of the Moon is one month. Answer the following questions assuming that the mass of Earth is doubled.
  a. What would the period of the Moon be? Express your results in months.
  b. Where would a satellite with an orbital period of one month be located?
  c. How would the length of a year on Earth be affected?

94. How fast would a planet of Earth's mass and size have to spin so that an object at the equator would be weightless? Give the period of rotation of the planet in minutes.

95. Car Races Suppose that a Martian base has been established and car races are being considered. A flat, circular race track has been built for the race. If a car can achieve speeds of up to 12 m/s, what is the smallest radius of a track for which the coefficient of friction is 0.50?

96. *Apollo 11* On July 19, 1969, *Apollo 11's* revolution around the Moon was adjusted to an average orbit of 111 km. The radius of the Moon is 1785 km, and the mass of the Moon is 7.3×10^{22} kg.
  a. How many minutes did *Apollo 11* take to orbit the Moon once?
  b. At what velocity did *Apollo 11* orbit the Moon?

Thinking Critically

97. Analyze and Conclude Some people say that the tides on Earth are caused by the pull of the Moon. Is this statement true?
  a. Determine the forces that the Moon and the Sun exert on a mass, *m*, of water on Earth. Your answer will be in terms of *m* with units of N.
  b. Which celestial body, the Sun or the Moon, has a greater pull on the waters of Earth?
  c. Determine the difference in force exerted by the Moon on the water at the near surface and the water at the far surface (on the opposite side) of Earth, as illustrated in **Figure 7-27.** Again, your answer will be in terms of *m* with units of N.

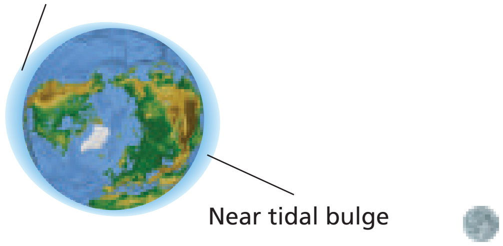


Figure 7-27 (Not to scale)

  d. Determine the difference in force exerted by the Sun on water at the near surface and on water at the far surface (on the opposite side) of Earth.
  e. Which celestial body has a greater difference in pull from one side of Earth to the other?
  f. Why is the statement that the tides result from the pull of the Moon misleading? Make a correct statement to explain how the Moon causes tides on Earth.

98. Make and Use Graphs Use Newton's law of universal gravitation to find an equation where x is equal to an object's distance from Earth's center, and y is its acceleration due to gravity. Use a graphing calculator to graph this equation, using 6400–6600 km as the range for x and 9–10 m/s^2 as the range for y. The equation should be of the form $y = c(1/x^2)$. Trace along this graph and find y for the following locations.
  a. at sea level, 6400 km
  b. on top of Mt. Everest, 6410 km
  c. in a typical satellite orbit, 6500 km
  d. in a much higher orbit, 6600 km

Writing in Physics

99. Research and describe the historical development of the measurement of the distance between the Sun and Earth.

100. Explore the discovery of planets around other stars. What methods did the astronomers use? What measurements did they take? How did they use Kepler's third law?

Cumulative Review

101. Airplanes A jet airplane took off from Pittsburgh at 2:20 P.M. and landed in Washington, DC, at 3:15 P.M. on the same day. If the jet's average speed while in the air was 441.0 km/h, what is the distance between the cities? (Chapter 2)

102. Carolyn wants to know how much her brother Jared weighs. He agrees to stand on a scale for her, but only if they are riding in an elevator. If he steps on the scale while the elevator is accelerating upward at 1.75 m/s^2 and the scale reads 716 N, what is Jared's usual weight on Earth? (Chapter 4)

103. Potato Bug A 1.0-g potato bug is walking around the outer rim of an upside-down flying disk. If the disk has a diameter of 17.2 cm and the bug moves at a rate of 0.63 cm/s, what is the centripetal force acting on the bug? What agent provides this force? (Chapter 6)

 For more problems, go to Additional Problems, Appendix B.

Standardized Test Practice

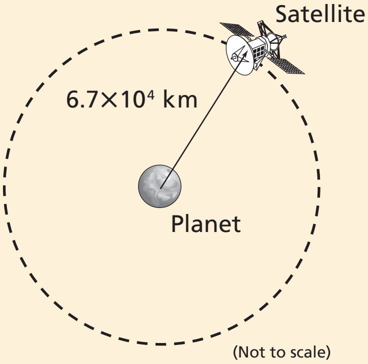
Multiple Choice

1. Two satellites are in orbit around a planet. One satellite has an orbital radius of 8.0×10^6 m. The period of rotation for this satellite is 1.0×10^6 s. The other satellite has an orbital radius of 2.0×10^7 m. What is this satellite's period of rotation?

Ⓐ 5.0×10^5 s
Ⓑ 2.5×10^6 s
Ⓒ 4.0×10^6 s
Ⓓ 1.3×10^7 s

2. The illustration below shows a satellite in orbit around a small planet. The satellite's orbital radius is 6.7×10^4 km and its speed is 2.0×10^5 m/s. What is the mass of the planet around which the satellite orbits? ($G = 6.7\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2$)

Ⓐ 2.5×10^{18} kg
Ⓑ 4.0×10^{20} kg
Ⓒ 2.5×10^{23} kg
Ⓓ 4.0×10^{28} kg



(Not to scale)

3. Two satellites are in orbit around the same planet. Satellite A has a mass of 1.5×10^2 kg, and satellite B has a mass of 4.5×10^3 kg. The mass of the planet is 6.6×10^{24} kg. Both satellites have the same orbital radius of 6.8×10^6 m. What is the difference in the orbital periods of the satellites?

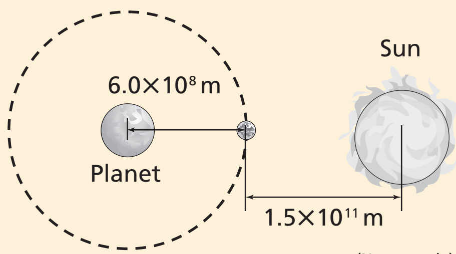
Ⓐ no difference
Ⓑ 1.5×10^2 s
Ⓒ 2.2×10^2 s
Ⓓ 3.0×10^2 s

4. A moon revolves around a planet with a speed of 9.0×10^3 m/s. The distance from the moon to the center of the planet is 5.4×10^6 m. What is the orbital period of the moon?

Ⓐ $1.2\pi\times10^2$ s
Ⓑ $6.0\pi\times10^2$ s
Ⓒ $1.2\pi\times10^3$ s
Ⓓ $1.2\pi\times10^9$ s

5. A moon in orbit around a planet experiences a gravitational force not only from the planet, but also from the Sun. The illustration below shows a moon during a solar eclipse, when the planet, the moon, and the Sun are aligned. The moon has a mass of about 3.9×10^{21} kg. The mass of the planet is 2.4×10^{26} kg, and the mass of the Sun is 2.0×10^{30} kg. The distance from the moon to the center of the planet is 6.0×10^8 m, and the distance from the moon to the Sun is 1.5×10^{11} m. What is the ratio of the gravitational force on the moon due to the planet, compared to its gravitational force due to the Sun during the solar eclipse?

Ⓐ 0.5
Ⓑ 2.5
Ⓒ 5.0
Ⓓ 7.5



(Not to scale)

Extended Answer

6. Two satellites are in orbit around a planet. Satellite S_1 takes 20 days to orbit the planet at a distance of 2×10^5 km from the center of the planet. Satellite S_2 takes 160 days to orbit the planet. What is the distance of Satellite S_2 from the center of the planet?

✓ Test-Taking TIP

Plan Your Work and Work Your Plan

Plan your workload so that you do a little work each day, rather than a lot of work all at once. The key to retaining information is repeated review and practice. You will retain more if you study one hour a night for five days in a row instead of cramming the night before a test.

Chapter 8

Rotational Motion

What You'll Learn

- You will learn how to describe and measure rotational motion.
- You will learn how torque changes rotational velocity.
- You will explore factors that determine the stability of an object.
- You will learn the nature of centrifugal and Coriolis "forces."

Why It's Important

You encounter many rotating objects in everyday life, such as CDs, wheels, and amusement-park rides.

Spin Rides Amusement-park rides that spin are designed to thrill riders using the physics of rotational motion. The thrill is produced by a "force" that is present only when the ride spins.

Think About This ▶

Why do people who ride amusement-park rides that spin in circles, such as this one, experience such strong physical reactions?

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Paul L. Ruben

LAUNCH Lab How do different objects rotate as they roll?

Question
Do different objects of similar size and mass roll at the same rate on an incline?

Procedure

1. You will need a meterstick, a piece of foam board, a ball, a solid can, and a hollow can.
2. Position the foam board on a 20° incline.
3. Place the meterstick horizontally across the foam board, near the top of the incline, and hold it.
4. Place the ball, solid can, and hollow can against the meterstick. The solid can and hollow can should be placed sideways.
5. Simultaneously, release the three objects by lifting the meterstick.
6. As each object accelerates down the incline, due to gravity, observe the order in which each object reaches the bottom.
7. Repeat steps 2–5 two more times.

Analysis
List the objects in order from the greatest to the least acceleration.

Critical Thinking Which of the objects' properties may have contributed to their behavior? List the properties that were similar and those that were different for each object.

8.1 Describing Rotational Motion

You probably have observed a spinning object many times. How would you measure such an object's rotation? Find a circular object, such as a CD. Mark one point on the edge of the CD so that you can keep track of its position. Rotate the CD to the left (counterclockwise), and as you do so, watch the location of the mark. When the mark returns to its original position, the CD has made one complete revolution. How can you measure a fraction of one revolution? It can be measured in several different ways. A grad is $\frac{1}{400}$ of a revolution, whereas a degree is $\frac{1}{360}$ of a revolution. In mathematics and physics, yet another form of measurement is used to describe fractions of revolutions. In one revolution, a point on the edge travels a distance equal to 2π times the radius of the object. For this reason, the **radian** is defined as $\frac{1}{2}\pi$ of a revolution. In other words, one complete revolution is equal to 2π radians. A radian is abbreviated "rad."

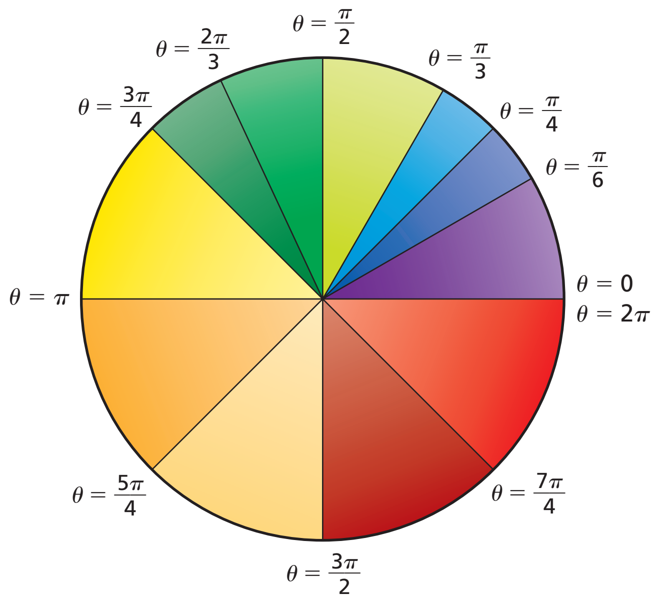
▶ **Objectives**

- **Describe** angular displacement.
- **Calculate** angular velocity.
- **Calculate** angular acceleration.
- **Solve** problems involving rotational motion.

▶ **Vocabulary**

radian
angular displacement
angular velocity
angular acceleration

■ **Figure 8-1** The pie chart shows the radian measure of the most common angles, measured in the counterclockwise direction. Each angle is measured from $\theta = 0$.



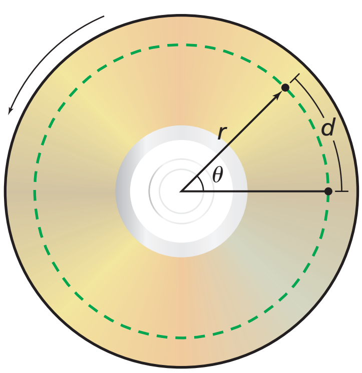
Angular Displacement

The Greek letter theta, θ, is used to represent the angle of revolution. **Figure 8-1** shows the angles in radians for several common fractions of a revolution. Note that counterclockwise rotation is designated as positive, while clockwise is negative. As an object rotates, the change in the angle is called **angular displacement.**

As you know, Earth makes one complete revolution, or 2π rad, in 24 h. In 12 h, its rotation is through π rad. Through what angle does Earth rotate in 6 h? Because 6 h is one-fourth of a day, Earth rotates through an angle of $\frac{\pi}{2}$ rad during that period. Earth's rotation as seen from the north pole is positive. Is it positive or negative when viewed from the south pole?

How far does a point on a rotating object move? You already found that a point on the edge of an object moves 2π times the radius in one revolution. In general, for rotation through an angle, θ, a point at a distance, r, from the center, as shown in **Figure 8-2,** moves a distance given by $d = r\theta$. If r is measured in meters, you might think that multiplying it by θ rad would result in d being measured in m·rad. However, this is not the case. Radians indicate the ratio between d and r. Thus, d is measured in m.

■ **Figure 8-2** The dashed line shows the path of the point on the CD as the CD rotates counterclockwise about its center.



Angular Velocity

How fast does a CD spin? How do you determine its speed of rotation? Recall from Chapter 2 that velocity is displacement divided by the time taken to make the displacement. Likewise, the **angular velocity** of an object is angular displacement divided by the time taken to make the displacement. Thus, the angular velocity of an object is given by the following equation, where angular velocity is represented by the Greek letter omega, ω.

Angular Velocity of an Object $\quad \omega = \frac{\Delta\theta}{\Delta t}$

The angular velocity is equal to the angular displacement divided by the time required to make the rotation.

Recall that if the velocity changes over a time interval, the average velocity is not equal to the instantaneous velocity at any given instant. Similarly, the angular velocity calculated in this way is actually the average angular velocity over a time interval, Δt. Instantaneous angular velocity is equal to the slope of a graph of angular position versus time.

Angular velocity is measured in rad/s. For Earth, $\omega_E = (2\pi \text{ rad})/(24.0 \text{ h})(3600 \text{ s/h}) = 7.27\times10^{-5}$ rad/s. In the same way that counterclockwise rotation produces positive angular displacement, it also results in positive angular velocity.

If an object's angular velocity is ω, then the linear velocity of a point a distance, r, from the axis of rotation is given by $v = r\omega$. The speed at which an object on Earth's equator moves as a result of Earth's rotation is given by $v = r\omega = (6.38\times10^{6} \text{ m})(7.27\times10^{-5} \text{ rad/s}) = 464$ m/s. Earth is an example of a rotating, rigid body. Even though different points on Earth rotate different distances in each revolution, all points rotate through the same angle. All parts of a rigid body rotate at the same rate. The Sun, on the other hand, is not a rigid body. Different parts of the Sun rotate at different rates. Most objects that we will consider in this chapter are rigid bodies.

Astronomy Connection

Angular Acceleration

What if angular velocity is changing? For example, if a car were accelerated from 0.0 m/s to 25 m/s in 15 s, then the angular velocity of the wheels also would change from 0.0 rad/s to 78 rad/s in the same 15 s. The wheels would undergo **angular acceleration,** which is defined as the change in angular velocity divided by the time required to make the change. Angular acceleration, α, is represented by the following equation.

Angular Acceleration of an Object $\quad \alpha = \dfrac{\Delta\omega}{\Delta t}$

Angular acceleration is equal to the change in angular velocity divided by the time required to make that change.

Angular acceleration is measured in rad/s^2. If the change in angular velocity is positive, then the angular acceleration also is positive. Angular acceleration defined in this way is also the average angular acceleration over the time interval Δt. One way to find the instantaneous angular acceleration is to find the slope of a graph of angular velocity as a function of time. The linear acceleration of a point at a distance, r, from the axis of an object with angular acceleration, α, is given by $a = r\alpha$. **Table 8-1** is a summary of linear and angular relationships.

Table 8-1

Linear and Angular Measures

Quantity	Linear	Angular	Relationship
Displacement	d (m)	θ (rad)	$d = r\theta$
Velocity	v (m/s)	ω (rad/s)	$v = r\omega$
Acceleration	a (m/s^2)	α (rad/s^2)	$a = r\alpha$

PRACTICE Problems

Additional Problems, Appendix B

1. What is the angular displacement of each of the following hands of a clock in 1 h? State your answer in three significant digits.
   a. the second hand
   b. the minute hand
   c. the hour hand
2. If a truck has a linear acceleration of 1.85 m/s^2 and the wheels have an angular acceleration of 5.23 rad/s^2, what is the diameter of the truck's wheels?
3. The truck in the previous problem is towing a trailer with wheels that have a diameter of 48 cm.
   a. How does the linear acceleration of the trailer compare with that of the truck?
   b. How do the angular accelerations of the wheels of the trailer and the wheels of the truck compare?
4. You want to replace the tires on your car with tires that have a larger diameter. After you change the tires, for trips at the same speed and over the same distance, how will the angular velocity and number of revolutions change?

Angular frequency A rotating object can make many revolutions in a given amount of time. For instance, a spinning wheel can go through several complete revolutions in 1 min. Thus, the number of complete revolutions made by the object in 1 s is called angular frequency. Angular frequency is $f = \omega/2\pi$. In the next section, you will explore the factors that cause the angular frequency to change.

8.1 Section Review

5. **Angular Displacement** A movie lasts 2 h. During that time, what is the angular displacement of each of the following?
   a. the hour hand
   b. the minute hand
6. **Angular Velocity** The Moon rotates once on its axis in 27.3 days. Its radius is 1.74×10^6 m.
   a. What is the period of the Moon's rotation in seconds?
   b. What is the frequency of the Moon's rotation in rad/s?
   c. What is the linear speed of a rock on the Moon's equator due only to the Moon's rotation?
   d. Compare this speed with the speed of a person on Earth's equator due to Earth's rotation.
7. **Angular Displacement** The ball in a computer mouse is 2.0 cm in diameter. If you move the mouse 12 cm, what is the angular displacement of the ball?
8. **Angular Displacement** Do all parts of the minute hand on a watch have the same angular displacement? Do they move the same linear distance? Explain.
9. **Angular Acceleration** In the spin cycle of a clothes washer, the drum turns at 635 rev/min. If the lid of the washer is opened, the motor is turned off. If the drum requires 8.0 s to slow to a stop, what is the angular acceleration of the drum?
10. **Critical Thinking** A CD-ROM has a spiral track that starts 2.7 cm from the center of the disk and ends 5.5 cm from the center. The disk drive must turn the disk so that the linear velocity of the track is a constant 1.4 m/s. Find the following.
    a. the angular velocity of the disk (in rad/s and rev/min) for the start of the track
    b. the disk's angular velocity at the end of the track
    c. the disk's angular acceleration if the disk is played for 76 min

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8.2 Rotational Dynamics

How do you start the rotation of an object? That is, how do you change its angular velocity? Suppose you have a soup can that you want to spin. If you wrap a string around it and pull hard, you could make the can spin rapidly. Later in this chapter, you will learn why gravity, the force of Earth's mass on the can, acts on the center of the can. The force of the string, on the other hand, is exerted at the outer edge of the can, and at right angles to the line from the center of the can, to the point where the string leaves the can's surface.

You have learned that a force changes the velocity of a point object. In the case of a soup can, a force that is exerted in a very specific way changes the angular velocity of an extended object, which is an object that has a definite shape and size. Consider how you open a door: you exert a force. How can you exert the force to open the door most easily? To get the most effect from the least force, you exert the force as far from the axis of rotation as possible, as shown in **Figure 8-3.** In this case, the axis of rotation is an imaginary vertical line through the hinges. The doorknob is near the outer edge of the door. You exert the force on the doorknob at right angles to the door, away from the hinges. Thus, the magnitude of the force, the distance from the axis to the point where the force is exerted, and the direction of the force determine the change in angular velocity.

▶ **Objectives**

- **Describe** torque and the factors that determine it.
- **Calculate** net torque.
- **Calculate** the moment of inertia.

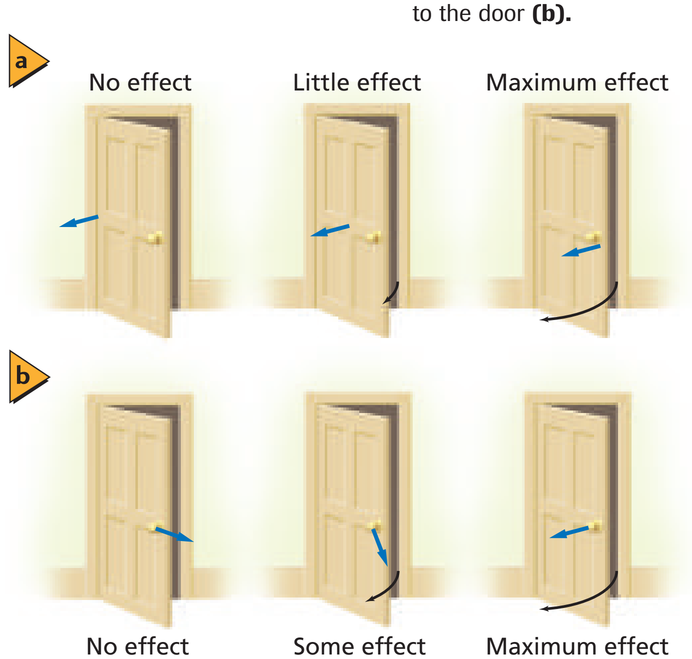
▶ **Vocabulary**

lever arm
torque
moment of inertia
Newton's second law for rotational motion

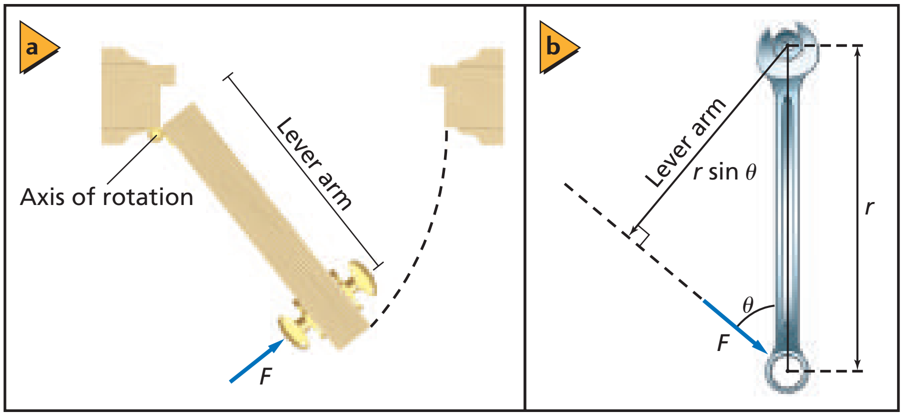
Lever arm For a given applied force, the change in angular velocity depends on the **lever arm,** which is the perpendicular distance from the axis of rotation to the point where the force is exerted. If the force is perpendicular to the radius of rotation, as it was with the soup can, then the lever arm is the distance from the axis, r. For the door, it is the distance from the hinges to the point where you exert the force, as illustrated in **Figure 8-4a,** on the next page. If the force is not perpendicular, the perpendicular component of the force must be found.

The force exerted by the string around the can is perpendicular to the radius. If a force is not exerted perpendicular to the radius, however, the lever arm is reduced. To find the lever arm, extend the line of the force until it forms a right angle with a line from the center of rotation. The distance between the intersection and the axis is the lever arm. Thus, using trigonometry, the lever arm, L, can be calculated by the equation $L = r \sin \theta$, as shown in **Figure 8-4b.** In this equation, r is the distance from the axis of rotation to the point where the force is exerted, and θ is the angle between the force and the radius from the axis of rotation to the point where the force is applied.

■ **Figure 8-3** When opening a door that is free to rotate about its hinges, the greatest torque is produced when the force is applied farthest from the hinges **(a),** at an angle perpendicular to the door **(b).**



■ **Figure 8-4** The lever arm is along the width of the door, from the hinge to the point where the force is exerted **(a).** The lever arm is equal to $r \sin \theta$, when the angle, θ, between the force and the radius of rotation is not equal to 90° **(b).**



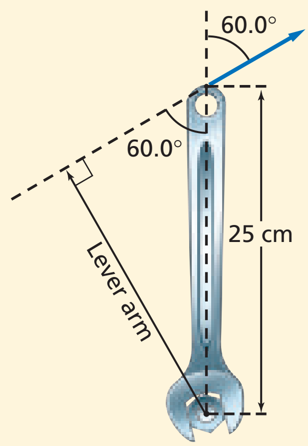
Torque is a measure of how effectively a force causes rotation. The magnitude of torque is the product of the force and the lever arm. Because force is measured in newtons, and distance is measured in meters, torque is measured in newton-meters (N·m). Torque is represented by the Greek letter tau, τ. The equation for torque is shown below.

Torque $\tau = Fr \sin \theta$

Torque is equal to the force times the lever arm.

▶ EXAMPLE Problem 1

Lever Arm **A bolt on a car engine needs to be tightened with a torque of 35 N·m. You use a 25-cm-long wrench and pull on the end of the wrench at an angle of 60.0° from the perpendicular. How long is the lever arm, and how much force do you have to exert?**



1 Analyze and Sketch the Problem

- Sketch the situation. Find the lever arm by extending the force vector backwards until a line that is perpendicular to it intersects the axis of rotation.

Known:		Unknown:
$r = 0.25$ m	$\tau = 35$ N·m	$L = ?$
$\theta = 60.0°$		$F = ?$

2 Solve for the Unknown

Solve for the length of the lever arm.

$$L = r \sin \theta$$
$$= (0.25 \text{ m})(\sin 60.0°) \quad \textbf{Substitute } r = 0.25 \text{ m}, \theta = 60.0°$$
$$= 0.22 \text{ m}$$

Solve for the force.

$$\tau = Fr \sin \theta$$
$$F = \frac{\tau}{r \sin \theta}$$
$$= \frac{35 \text{ N·m}}{(0.25 \text{ m})(\sin 60.0°)} \quad \textbf{Substitute } \tau = 35 \text{ N·m}, r = 0.25 \text{ m}, \theta = 60.0°$$
$$= 1.6 \times 10^2 \text{ N}$$

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3 Evaluate the Answer

- **Are the units correct?** Force is measured in newtons.
- **Does the sign make sense?** Only the magnitude of the force needed to rotate the wrench clockwise is calculated.

PRACTICE Problems

Additional Problems, Appendix B

11. Consider the wrench in Example Problem 1. What force is needed if it is applied to the wrench at a point perpendicular to the wrench?

12. If a torque of 55.0 N·m is required and the largest force that can be exerted by you is 135 N, what is the length of the lever arm that must be used?

13. You have a 0.234-m-long wrench. A job requires a torque of 32.4 N·m, and you can exert a force of 232 N. What is the smallest angle, with respect to the vertical, at which the force can be exerted?

14. You stand on the pedal of a bicycle. If you have a mass of 65 kg, the pedal makes an angle of 35° above the horizontal, and the pedal is 18 cm from the center of the chain ring, how much torque would you exert?

15. If the pedal in problem 14 is horizontal, how much torque would you exert? How much torque would you exert when the pedal is vertical?

Finding Net Torque

Try the following experiment. Get two pencils, some coins, and some transparent tape. Tape two identical coins to the ends of the pencil and balance it on the second pencil, as shown in **Figure 8-5.** Each coin exerts a torque that is equal to its weight, F_g, times the distance, r, from the balance point to the center of the coin, as follows:

$$\tau = F_g r$$

But the torques are equal and opposite in direction. Thus, the net torque is zero:

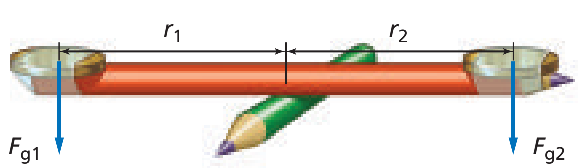
$$\tau_1 - \tau_2 = 0$$

or

$$F_{g1} r_1 - F_{g2} r_2 = 0$$

How can you make the pencil rotate? You could add a second coin on top of one of the two coins, thereby making the two forces different. You also could slide the balance point toward one end or the other of the pencil, thereby making the two distances different.

Figure 8-5 The torque exerted by the first coin, $F_{g1}r_1$, is equal and opposite in direction to the torque exerted by the second coin, $F_{g2}r_2$, when the pencil is balanced.

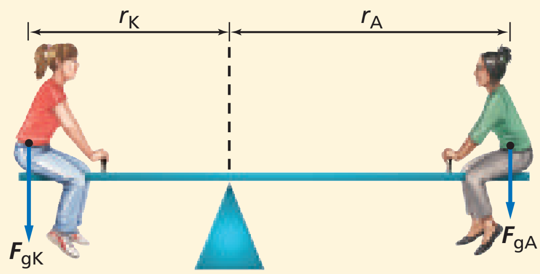


EXAMPLE Problem 2

Balancing Torques Kariann (56 kg) and Aysha (43 kg) want to balance on a 1.75-m-long seesaw. Where should they place the pivot point?

1 Analyze and Sketch the Problem

- Sketch the situation.
- Draw and label the vectors.



Known:

$m_K = 56$ kg

$m_A = 43$ kg

$r_K + r_A = 1.75$ m

Unknown:

$r_K = ?$

$r_A = ?$

2 Solve for the Unknown

Find the two forces.

Kariann:

$$F_{gK} = m_K g$$
$$= (56\text{ kg})(9.80\text{ m/s}^2) \quad \textbf{Substitute } m_K = 56\text{ kg, } g = 9.80\text{ m/s}^2$$
$$= 5.5\times10^2\text{ N}$$

Aysha:

$$F_{gA} = m_A g$$
$$= (43\text{ kg})(9.80\text{ m/s}^2) \quad \textbf{Substitute } m_A = 43\text{ kg, } g = 9.80\text{ m/s}^2$$
$$= 4.2\times10^2\text{ N}$$

Define Kariann's distance in terms of the length of the seesaw and Aysha's distance.

$$r_K = 1.75\text{ m} - r_A$$

When there is no rotation, the sum of the torques is zero.

$$F_{gK}r_K = F_{gA}r_A$$
$$F_{gK}r_K - F_{gA}r_A = 0.0\text{ N·m}$$
$$F_{gK}(1.75\text{ m} - r_A) - F_{gA}r_A = 0.0\text{ N·m} \quad \textbf{Substitute } r_K = 1.75\text{ m} - r_A$$

Solve for r_A.

$$F_{gK}(1.75\text{ m}) - F_{gK}(r_A) - F_{gA}r_A = 0.0\text{ N·m}$$
$$F_{gK}r_A + F_{gA}r_A = F_{gK}(1.75\text{ m})$$
$$(F_{gK} + F_{gA})\,r_A = F_{gK}(1.75\text{ m})$$
$$r_A = \frac{F_{gK}(1.75\text{ m})}{(F_{gK} + F_{gA})}$$
$$= \frac{(5.5\times10^2\text{ N})(1.75\text{ m})}{(5.5\times10^2\text{ N} + 4.2\times10^2\text{ N})} \quad \textbf{Substitute } F_{gK} = 5.5\times10^2\text{ N, } F_{gA} = 4.2\times10^2\text{ N}$$
$$= 0.99\text{ m}$$

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3 Evaluate the Answer

- **Are the units correct?** Distance is measured in meters.
- **Do the signs make sense?** Distances are positive.
- **Is the magnitude realistic?** Aysha is about 1 m from the center, so Kariann is about 0.75 m away from it. Because Kariann's weight is greater than Aysha's weight, the lever arm on Kariann's side should be shorter. Aysha is farther from the pivot, as expected.

PRACTICE Problems

Additional Problems, Appendix B

16. Ashok, whose mass is 43 kg, sits 1.8 m from the center of a seesaw. Steve, whose mass is 52 kg, wants to balance Ashok. How far from the center of the seesaw should Steve sit?

17. A bicycle-chain wheel has a radius of 7.70 cm. If the chain exerts a 35.0-N force on the wheel in the clockwise direction, what torque is needed to keep the wheel from turning?

18. Two baskets of fruit hang from strings going around pulleys of different diameters, as shown in **Figure 8-6.** What is the mass of basket A?

19. Suppose the radius of the larger pulley in problem 18 was increased to 6.0 cm. What is the mass of basket A now?

20. A bicyclist, of mass 65.0 kg, stands on the pedal of a bicycle. The crank, which is 0.170 m long, makes a 45.0° angle with the vertical, as shown in **Figure 8-7.** The crank is attached to the chain wheel, which has a radius of 9.70 cm. What force must the chain exert to keep the wheel from turning?

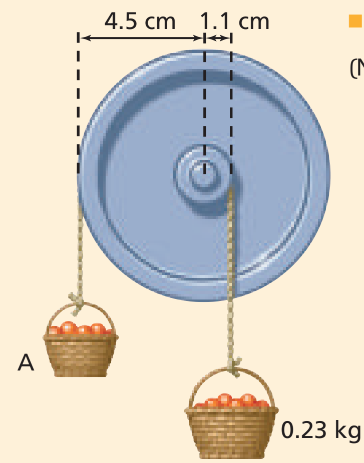


Figure 8-6

(Not to scale)

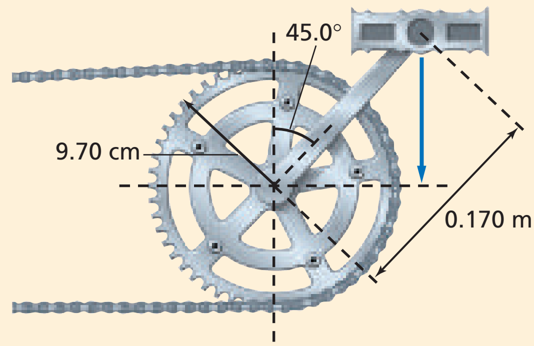


Figure 8-7

The Moment of Inertia

If you exert a force on a point mass, its acceleration will be inversely proportional to its mass. How does an extended object rotate when a torque is exerted on it? To observe firsthand, recover the pencil, the coins, and the transparent tape that you used earlier in this chapter. First, tape the coins at the ends of the pencil. Hold the pencil between your thumb and forefinger, and wiggle it back and forth. Take note of the forces that your thumb and forefinger exert. These forces create torques that change the angular velocity of the pencil and coins.

Now move the coins so that they are only 1 or 2 cm apart. Wiggle the pencil as before. Did the amount of torque and force need to be changed? The torque that was required was much less this time. Thus, the amount of mass is not the only factor that determines how much torque is needed to change angular velocity; the location of that mass also is relevant.

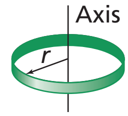
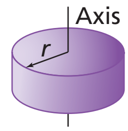
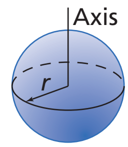
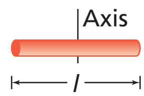
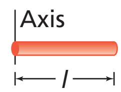
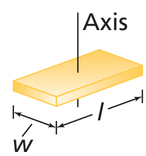
The resistance to rotation is called the **moment of inertia,** which is represented by the symbol I and has units of mass times the square of the distance. For a point object located at a distance, r, from the axis of rotation, the moment of inertia is given by the following equation.

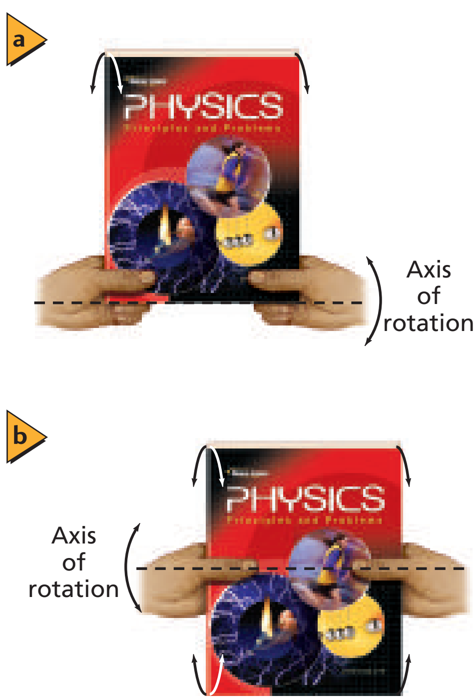
Moment of Inertia of a Point Mass $I = mr^2$

The moment of inertia of a point mass is equal to the mass of the object times the square of the object's distance from the axis of rotation.

Table 8-2

Moments of Inertia for Various Objects

Object	Location of Axis	Diagram	Moment of Inertia
Thin hoop of radius r	Through central diameter		mr^2
Solid, uniform cylinder of radius r	Through center		$\frac{1}{2}mr^2$
Uniform sphere of radius r	Through center		$\frac{2}{5}mr^2$
Long, uniform rod of length l	Through center		$\frac{1}{12}ml^2$
Long, uniform rod of length l	Through end		$\frac{1}{3}ml^2$
Thin, rectangular plate of length l and width w	Through center		$\frac{1}{12}m(l^2 + w^2)$



■ **Figure 8-8** The moment of inertia of a book depends on the axis of rotation. The moment of inertia of the book in **(a)** is larger than the moment of inertia of the book in **(b)** because the average distance of the book's mass from the rotational axis is larger.

As you have seen, the moment of inertia for complex objects, such as the pencil and coins, depends on how far the coins are from the axis of rotation. A bicycle wheel, for example, has almost all of its mass in the rim and tire. Its moment of inertia is almost exactly equal to mr^2, where r is the radius of the wheel. For most objects, however, the mass is distributed continuously and so the moment of inertia is less than mr^2. For example, as shown in **Table 8-2,** for a solid cylinder of radius r, $I = \frac{1}{2}mr^2$, while for a solid sphere, $I = \frac{2}{5}mr^2$.

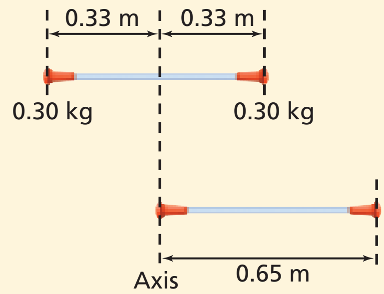
The moment of inertia also depends on the location of the rotational axis, as illustrated in **Figure 8-8.** To observe this first-hand, hold a book in the upright position, by placing your hands at the bottom of the book. Feel the torque needed to rock the book towards you, and then away from you. Now put your hands in the middle of the book and feel the torque needed to rock the book toward you and then away from you. Note that much less torque is needed when your hands are placed in the middle of the book because the average distance of the book's mass from the rotational axis is much less in this case.

▶ EXAMPLE Problem 3

Moment of Inertia A simplified model of a twirling baton is a thin rod with two round objects at each end. The length of the baton is 0.65 m, and the mass of each object is 0.30 kg. Find the moment of inertia of the baton if it is rotated about the midpoint between the round objects. What is the moment of inertia of the baton when it is rotated around one end? Which is greater? Neglect the mass of the rod.

1 Analyze and Sketch the Problem

- Sketch the situation. Show the baton with the two different axes of rotation and the distances from the axes of rotation to the masses.



Known:
$m = 0.30$ kg
$l = 0.65$ m

Unknown:
$I = ?$

2 Solve for the Unknown

Calculate the moment of inertia of each mass separately.

Rotating about the center of the rod:

$$r = \frac{1}{2}l$$
$$= \frac{1}{2}(0.65\text{ m}) \qquad \textbf{Substitute } l = 0.65\text{ m}$$
$$= 0.33\text{ m}$$

$$I_{\text{single mass}} = mr^2$$
$$= (0.30\text{ kg})(0.33\text{ m})^2 \qquad \textbf{Substitute } m = 0.30\text{ kg, } r = 0.33\text{ m}$$
$$= 0.033\text{ kg}\cdot\text{m}^2$$

Find the moment of inertia of the baton.

$$I = 2I_{\text{single mass}}$$
$$= 2(0.033\text{ kg}\cdot\text{m}^2) \qquad \textbf{Substitute } I_{\text{single mass}} = 0.033\text{ kg}\cdot\text{m}^2$$
$$= 0.066\text{ kg}\cdot\text{m}^2$$

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Rotating about one end of the rod:

$$I_{\text{single mass}} = mr^2$$
$$= (0.30\text{ kg})(0.65\text{ m})^2 \qquad \textbf{Substitute } m = 0.30\text{ kg, } r = 0.65\text{ m}$$
$$= 0.13\text{ kg}\cdot\text{m}^2$$

Find the moment of inertia of the baton.

$$I = I_{\text{single mass}}$$
$$= 0.13\text{ kg}\cdot\text{m}^2$$

The moment of inertia is greater when the baton is swung around one end.

3 Evaluate the Answer

- **Are the units correct?** Moment of inertia is measured in kg·m^2.
- **Is the magnitude realistic?** Masses and distances are small, and so are the moments of inertia. Doubling the distance increases the moment of inertia by a factor of 4. Thus, doubling the distance overcomes having only one mass contributing.

PRACTICE Problems

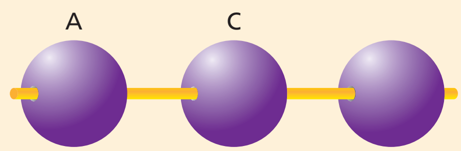
Additional Problems, Appendix B

21. Two children of equal masses sit 0.3 m from the center of a seesaw. Assuming that their masses are much greater than that of the seesaw, by how much is the moment of inertia increased when they sit 0.6 m from the center?

22. Suppose there are two balls with equal diameters and masses. One is solid, and the other is hollow, with all its mass distributed at its surface. Are the moments of inertia of the balls equal? If not, which is greater?

23. **Figure 8-9** shows three massive spheres on a rod of very small mass. Consider the moment of inertia of the system, first when it is rotated about sphere A, and then when it is rotated about sphere C. Are the moments of inertia the same or different? Explain. If the moments of inertia are different, in which case is the moment of inertia greater?

24. Each sphere in the previous problem has a mass of 0.10 kg. The distance between spheres A and C is 0.20 m. Find the moment of inertia in the following instances: rotation about sphere A, rotation about sphere C.



■ **Figure 8-9**

Newton's Second Law for Rotational Motion

Newton's second law for linear motion is expressed as $a = F_{net}/m$. If you rewrite this equation to represent rotational motion, acceleration is replaced by angular acceleration, α, force is replaced by net torque, τ_{net}, and mass is replaced by moment of inertia, I. Thus, **Newton's second law for rotational motion** states that angular acceleration is directly proportional to the net torque and inversely proportional to the moment of inertia. This law is expressed by the following equation.

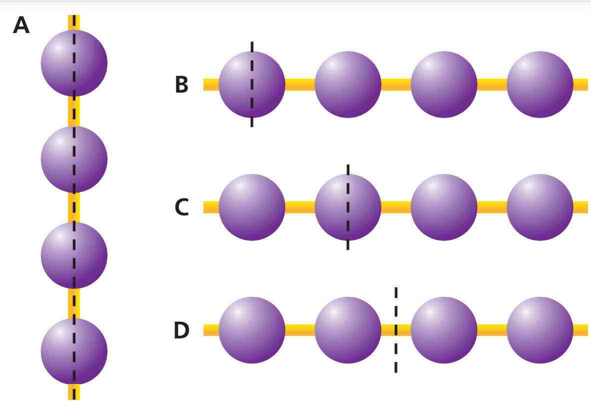
Newton's Second Law for Rotational Motion $\alpha = \frac{\tau_{net}}{I}$

The angular acceleration of an object is equal to the net torque on the object, divided by the moment of inertia.

Recall the coins taped on the pencil. To change the direction of rotation of the pencil—to give it angular acceleration—you had to apply torque to the pencil. The greater the moment of inertia, the more torque needed to produce the same angular acceleration.

CHALLENGE PROBLEM

Rank the objects shown in the diagram according to their moments of inertia about the indicated axes. All spheres have equal masses and all separations are the same.

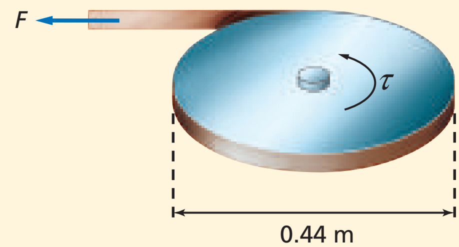


EXAMPLE Problem 4

Torque A solid steel wheel has a mass of 15 kg and a diameter of 0.44 m. It starts at rest. You want to make it rotate at 8.0 rev/s in 15 s.

a. What torque must be applied to the wheel?

b. If you apply the torque by wrapping a strap around the outside of the wheel, how much force should you exert on the strap?



1 Analyze and Sketch the Problem

- Sketch the situation. The torque must be applied in a counterclockwise direction; force must be exerted as shown.

Known:	Unknown:
$m = 15$ kg	$\alpha = ?$
$r = \frac{1}{2}(0.44\text{ m}) = 0.22$ m	$I = ?$
$\omega_i = 0.0$ rad/s	$\tau = ?$
$\omega_f = 2\pi(8.0\text{ rev/s})$	$F = ?$
$t = 15$ s	

2 Solve for the Unknown

a. Solve for angular acceleration.

$$\alpha = \frac{\Delta\omega}{\Delta t}$$

$$= \frac{2\pi(8.0\text{ rev/s}) - (0.0\text{ rad/s})}{15\text{ s}}$$ **Substitute $\omega_f = 2\pi(8.0\text{ rev/s})$, $\omega_i = 0.0$ rad/s**

$$= 3.4\text{ rad/s}^2$$

Solve for the moment of inertia.

$$I = \frac{1}{2}mr^2$$

$$= \frac{1}{2}(15\text{ kg})(0.22\text{ m})^2$$ **Substitute $m = 15$ kg, $r = 0.22$ m**

$$= 0.36\text{ kg·m}^2$$

Solve for torque.

$$\tau = I\alpha$$

$$= (0.36\text{ kg·m}^2)(3.4\text{ rad/s}^2)$$ **Substitute $I = 0.36$ kg·m^2, $\alpha = 3.4$ rad/s^2**

$$= 1.2\text{ kg·m}^2\text{/s}^2$$

$$= 1.2\text{ N·m}$$

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b. Solve for force.

$$\tau = Fr$$

$$F = \frac{\tau}{r}$$

$$= \frac{1.2\text{ N·m}}{0.22\text{ m}}$$ **Substitute $\tau = 1.2$ N·m, $r = 0.22$ m**

$$= 5.5\text{ N}$$

3 Evaluate the Answer

- **Are the units correct?** Torque is measured in N·m and force is measured in N.
- **Is the magnitude realistic?** Despite its large mass, the small size of the wheel makes it relatively easy to spin.

PRACTICE Problems

Additional Problems, Appendix B

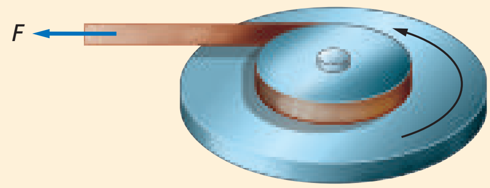
25. Consider the wheel in Example Problem 4. If the force on the strap were twice as great, what would be the speed of rotation of the wheel after 15 s?

26. A solid wheel accelerates at 3.25 rad/s^2 when a force of 4.5 N exerts a torque on it. If the wheel is replaced by a wheel with all of its mass on the rim, the moment of inertia is given by $I = mr^2$. If the same angular velocity were desired, what force would have to be exerted on the strap?

27. A bicycle wheel can be accelerated either by pulling on the chain that is on the gear or by pulling on a string wrapped around the tire. The wheel's radius is 0.38 m, while the radius of the gear is 0.14 m. If you obtained the needed acceleration with a force of 15 N on the chain, what force would you need to exert on the string?

28. The bicycle wheel in problem 27 is used with a smaller gear whose radius is 0.11 m. The wheel can be accelerated either by pulling on the chain that is on the gear or by pulling string that is wrapped around the tire. If you obtained the needed acceleration with a force of 15 N on the chain, what force would you need to exert on the string?

29. A disk with a moment of inertia of 0.26 kg·m^2 is attached to a smaller disk mounted on the same axle. The smaller disk has a diameter of 0.180 m and a mass of 2.5 kg. A strap is wrapped around the smaller disk, as shown in **Figure 8-10.** Find the force needed to give this system an angular acceleration of 2.57 rad/s^2.



■ **Figure 8-10**

In summary, changes in the amount of torque applied to an object, or changes in the moment of inertia, affect the rate of rotation. In this section, you learned how Newton's second law of motion applies to rotational motion. In the next section, you will learn how to keep objects from rotating.

8.2 Section Review

30. Torque Vijesh enters a revolving door that is not moving. Explain where and how Vijesh should push to produce a torque with the least amount of force.

31. Lever Arm You try to open a door, but you are unable to push at a right angle to the door. So, you push the door at an angle of 55° from the perpendicular. How much harder would you have to push to open the door just as fast as if you were to push it at 90°?

32. Net Torque Two people are pulling on ropes wrapped around the edge of a large wheel. The wheel has a mass of 12 kg and a diameter of 2.4 m. One person pulls in a clockwise direction with a 43-N force, while the other pulls in a counterclockwise direction with a 67-N force. What is the net torque on the wheel?

33. Moment of Inertia Refer to Table 8-2 on page 206 and rank the moments of inertia from least to greatest of the following objects: a sphere, a wheel with almost all of its mass at the rim, and a solid disk. All have equal masses and diameters. Explain the advantage of using the one with the least moment of inertia.

34. Newton's Second Law for Rotational Motion A rope is wrapped around a pulley and pulled with a force of 13.0 N. The pulley's radius is 0.150 m. The pulley's rotational speed goes from 0.0 to 14.0 rev/min in 4.50 s. What is the moment of inertia of the pulley?

35. Critical Thinking A ball on an extremely low-friction, tilted surface, will slide downhill without rotating. If the surface is rough, however, the ball will roll. Explain why, using a free-body diagram.

8.3 Equilibrium

Why are some vehicles more likely than others to roll over when involved in an accident? What causes a vehicle to roll over? The answer lies in the design of the vehicle. In this section, you will learn some of the factors that cause an object to tip over.

▶ **Objectives**

- **Define** center of mass.
- **Explain** how the location of the center of mass affects the stability of an object.
- **Define** the conditions for equilibrium.
- **Describe** how rotating frames of reference give rise to apparent forces.

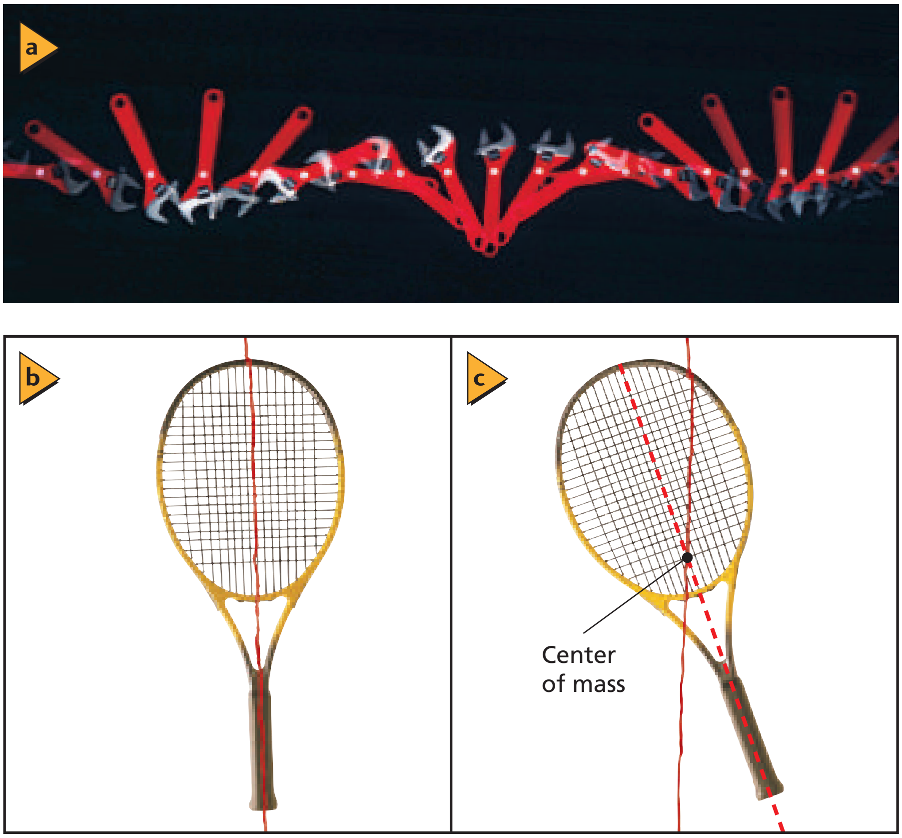
▶ **Vocabulary**

center of mass
centrifugal "force"
Coriolis "force"

The Center of Mass

How does an object rotate around its center of mass? A wrench may spin about its handle or end-over-end. Does any single point on the wrench follow a straight path? **Figure 8-11a** shows the path of the wrench. You can see that there is a single point whose path traces a straight line, as if the wrench could be replaced by a point particle at that location. The **center of mass** of an object is the point on the object that moves in the same way that a point particle would move.

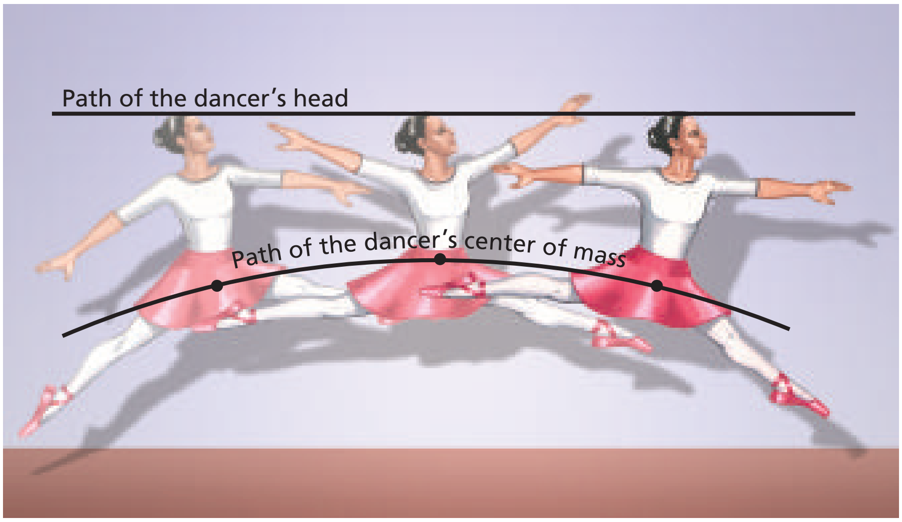
Locating the center of mass How can you locate the center of mass of an object? First, suspend the object from any point. When the object stops swinging, the center of mass is along the vertical line drawn from the suspension point as shown in **Figure 8-11b.** Draw the line. Then, suspend the object from another point. Again, the center of mass must be below this point. Draw a second vertical line. The center of mass is at the point where the two lines cross, as shown in **Figure 8-11c.** The wrench, racket, and all other freely rotating objects rotate about an axis that goes through their center of mass. Where is the center of mass of a person located?



■ **Figure 8-11** The path of the center of mass of a wrench is a straight line **(a).** The center of mass of an object such as a tennis racket can be found by first suspending it from any point **(b).** The point where the strings intersect is the location of the racket's center of mass **(c).**

(t)Richard Megna/Fundamental Photographs, (others)Hutchings Photography

■ **Figure 8-12** The upward motion of the ballet dancer's head is less than the upward motion of the center of mass. Thus, the head and torso move in a nearly horizontal path. This creates an illusion of floating.



APPLYING PHYSICS

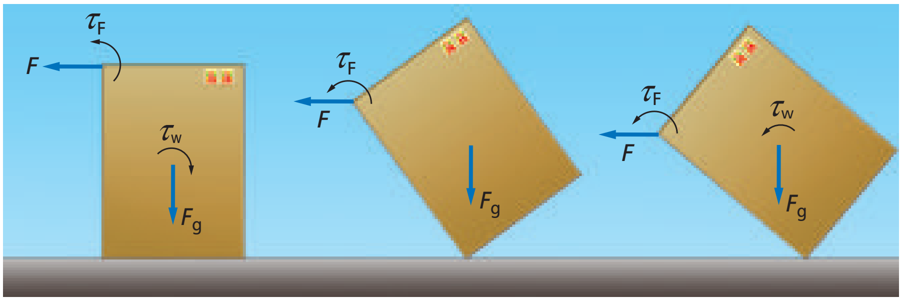
▶ **The Fosbury-Flop** In high jumping, a technique called the Fosbury-Flop allows a high jumper to clear the bar when it is placed at the highest position. This is possible because the athlete's center of mass passes below the bar as he or she somersaults over the bar, with his or her back toward it. ◀

The center of mass of a human body For a person who is standing with his or her arms hanging straight down, the center of mass is a few centimeters below the navel, midway between the front and back of the person's body. It is slightly higher in young children, because of their relatively larger heads. Because the human body is flexible, however, its center of mass is not fixed. If you raise your hands above your head, your center of mass rises 6 to 10 cm. A ballet dancer, for example, can appear to be floating on air by changing her center of mass in a leap. By raising her arms and legs while in the air, as shown in **Figure 8-12,** the dancer moves her center of mass closer to her head. The path of the center of mass is a parabola, so the dancer's head stays at almost the same height for a surprisingly long time.

Center of Mass and Stability

What factors determine whether a vehicle is stable or prone to roll over in an accident? To understand the problem, think about tipping over a box. A tall, narrow box, standing on end, tips more easily than a low, broad box. Why? To tip a box, as shown in **Figure 8-13,** you must rotate it about a corner. You pull at the top with a force, ***F***, applying a torque, τ_F. The weight of the box, acting on the center of mass, $\boldsymbol{F}_g$, applies an opposing torque, τ_w. When the center of mass is directly above the point of support, τ_w is zero. The only torque is the one applied by you. As the box rotates farther, its center of mass is no longer above its base of support, and both torques act in the same direction. At this point, the box tips over rapidly.

■ **Figure 8-13** The bent arrows show the direction of the torque produced by the force exerted to tip over a box.



Stability An object is said to be stable if an external force is required to tip it. The box in Figure 8-13 is stable as long as the direction of the torque due to its weight, τ_w tends to keep it upright. This occurs as long as the box's center of mass lies above its base. To tip the box over, you must rotate its center of mass around the axis of rotation until it is no longer above the base of the box. To rotate the box, you must lift its center of mass. The broader the base, the more stable the object is. For this reason, if you are standing on a bus that is weaving through traffic and you want to avoid falling down, you need to stand with your feet spread apart.

Why do vehicles roll over? **Figure 8-14** shows two vehicles rolling over. Note that the one with the higher center of mass does not have to be tilted very far for its center of mass to be outside its base—its center of mass does not have to be raised as much as the other vehicle's. The lower the location of an object's center of mass, the greater its stability.

You are stable when you stand flat on your feet. When you stand on tiptoe, however, your center of mass moves forward directly above the balls of your feet, and you have very little stability. A small person can use torque, rather than force, to defend himself or herself against a stronger person. In judo, aikido, and other martial arts, the fighter uses torque to rotate the opponent into an unstable position, where the opponent's center of mass does not lie above his or her feet.

In summary, if the center of mass is outside the base of an object, it is unstable and will roll over without additional torque. If the center of mass is above the base of the object, it is stable. If the base of the object is very narrow and the center of mass is high, then the object is stable, but the slightest force will cause it to tip over.

Conditions for Equilibrium

If your pen is at rest, what is needed to keep it at rest? You could either hold it up or place it on a desk or some other surface. An upward force must be exerted on the pen to balance the downward force of gravity. You must also hold the pen so that it will not rotate. An object is said to be in static equilibrium if both its velocity and angular velocity are zero or constant. Thus, for an object to be in static equilibrium, it must meet two conditions. First, it must be in translational equilibrium; that is, the net force exerted on the object must be zero. Second, it must be in rotational equilibrium; that is, the net torque exerted on the object must be zero.

MINI LAB

Spinning Tops

1. Cut out two cardboard disks of 10-cm and 15-cm diameter.

2. Use a pencil with an eraser that has rounded edges from use. If it is new, rub it on paper to round it.

3. Spin the pencil and try to make it stand on the eraser. Repeat several times and record your observations.

4. Carefully push the pencil through the center of the 10-cm disk.

5. Spin the pencil with the disk and try to make it stand on the eraser.

6. Move the disk to different points on the pencil. Spin and record your observations.

7. Repeat steps 4-6 with the 15-cm disk.

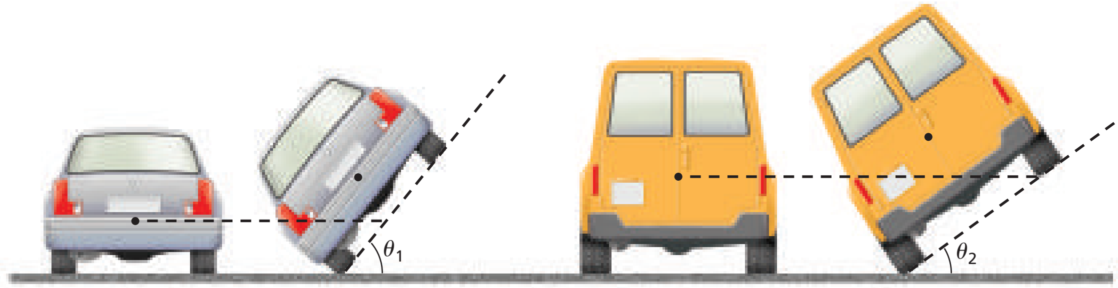
Analyze and Conclude

8. Sequence the three trials in order from least to most stable.

9. Describe the location of the pencil's center of mass.

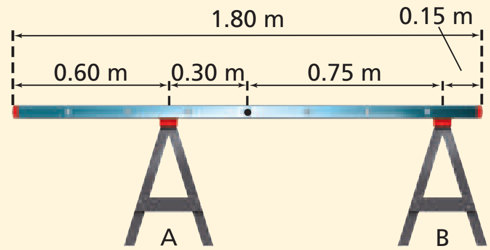
10. Analyze the placement of the disk and its effect on stability.

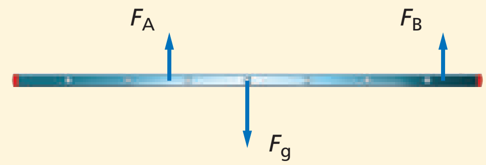
Figure 8-14 Larger vehicles have a higher center of mass than smaller ones. The higher the center of mass, the smaller the tilt needed to cause the vehicle's center of mass to move outside its base and cause the vehicle to roll over.

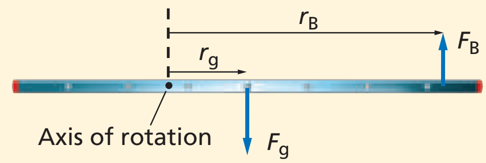


▶ EXAMPLE Problem 5

Static Equilibrium A 5.8-kg ladder, 1.80 m long, rests on two sawhorses. Sawhorse A is 0.60 m from one end of the ladder, and sawhorse B is 0.15 m from the other end of the ladder. What force does each sawhorse exert on the ladder?







1 Analyze and Sketch the Problem

- Sketch the situation.
- Choose the axis of rotation at the point where F_A acts on the ladder. Thus, the torque due to F_A is zero.

Known:	**Unknown:**
$m = 5.8$ kg	$F_A = ?$
$l = 1.80$ m	$F_B = ?$
$l_A = 0.60$ m	
$l_B = 0.15$ m	

2 Solve for the Unknown

For a ladder that has a constant density, the center of mass is at the center rung.

The net force is the sum of all forces on the ladder.

$$F_{net} = F_A + F_B + (-F_g)$$
$$0.0\text{ N} = F_A + F_B - F_g$$

The ladder is in translational equilibrium, so the net force exerted on it is zero.

Solve for F_A.

$$F_A = F_g - F_B$$

Find the torques due to F_g and F_B.

$$\tau_g = -r_g F_g$$ **τ_g is in the clockwise direction.**

$$\tau_B = +r_B F_B$$ **τ_B is in the counterclockwise direction.**

The net torque is the sum of all torques on the object.

$$\tau_{net} = \tau_B + \tau_g$$
$$0.0\text{ N·m} = \tau_B + \tau_g$$ **The ladder is in rotational equilibrium, so $\tau_{net} = 0.0$ N·m.**
$$\tau_B = \tau_g$$
$$r_B F_B = r_g F_g$$ **Substitute $\tau_B = r_B F_B$, $\tau_g = -r_g F_g$**

Solve for F_B.

$$F_B = \frac{r_g F_g}{r_B}$$
$$= \frac{r_g mg}{r_B}$$ **Substitute $F_g = mg$**

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Using the expression $F_A = F_g - F_B$, substitute in the expressions for F_B and F_g.

$$F_A = F_g - F_B$$
$$= F_g - \frac{r_g mg}{r_B}$$ **Substitute $F_B = \frac{r_g mg}{r_B}$**
$$= mg - \frac{r_g mg}{r_B}$$ **Substitute $F_g = mg$**
$$= mg\left(1 - \frac{r_g}{r_B}\right)$$

CONTENTS

Solve for r_g.

$$r_g = \frac{l}{2} - l_A$$ For a ladder, which has a constant density, the center of mass is at the center rung.

$$= 0.90 \text{ m} - 0.60 \text{ m}$$ Substitute $\frac{l}{2} = 0.90$ m, $l_A = 0.60$ m

$$= 0.30 \text{ m}$$

Solve for r_B.

$$r_B = (0.90 \text{ m} - l_B) + (0.90 \text{ m} - l_A)$$

$$= (0.90 \text{ m} - 0.15 \text{ m}) + (0.90 \text{ m} - 0.60 \text{ m})$$ Substitute $l_B = 0.15$ m, $l_A = 0.60$ m

$$= 0.75 \text{ m} + 0.30 \text{ m}$$

$$= 1.05 \text{ m}$$

Calculate F_B.

$$F_B = \frac{r_g mg}{r_B}$$

$$= \frac{(0.30 \text{ m})(5.8 \text{ kg})(9.80 \text{ m/s}^2)}{(1.05 \text{ m})}$$ Substitute $r_g = 0.30$ m, $m = 5.8$ kg, $g = 9.80$ m/s^2, $r_B = 1.05$ m

$$= 16 \text{ N}$$

Calculate F_A.

$$F_A = mg\left(1 - \frac{r_g}{r_B}\right)$$

$$= \left(1 - \frac{(0.30 \text{ m})}{(1.05 \text{ m})}\right)(5.8 \text{ kg})(9.80 \text{ m/s}^2)$$ Substitute $r_g = 0.30$ m, $m = 5.8$ kg, $g = 9.80$ m/s^2, $r_B = 1.05$ m

$$= 41 \text{ N}$$

3 Evaluate the Answer

- **Are the units correct?** Forces are measured in newtons.
- **Do the signs make sense?** Both forces are upward.
- **Is the magnitude realistic?** The forces add up to the weight of the ladder, and the force exerted by the sawhorse closer to the center of mass is greater, which is correct.

PRACTICE Problems

Additional Problems, Appendix B

36. What would be the forces exerted by the two sawhorses if the ladder in Example Problem 5 had a mass of 11.4 kg?

37. A 7.3-kg ladder, 1.92 m long, rests on two sawhorses, as shown in **Figure 8-15.** Sawhorse A, on the left, is located 0.30 m from the end, and sawhorse B, on the right, is located 0.45 m from the other end. Choose the axis of rotation to be the center of mass of the ladder.

a. What are the torques acting on the ladder?

b. Write the equation for rotational equilibrium.

c. Solve the equation for F_A in terms of F_g.

d. How would the forces exerted by the two sawhorses change if A were moved very close to, but not directly under, the center of mass?

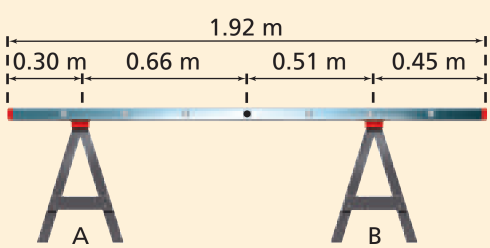


Figure 8-15

38. A 4.5-m-long wooden plank with a 24-kg mass is supported in two places. One support is directly under the center of the board, and the other is at one end. What are the forces exerted by the two supports?

39. A 85-kg diver walks to the end of a diving board. The board, which is 3.5 m long with a mass of 14 kg, is supported at the center of mass of the board and at one end. What are the forces on the two supports?

Rotating Frames of Reference

When you are on a on a rapidly spinning amusement-park ride, it feels like a strong force is pushing you to the outside. A pebble on the floor of the ride would accelerate outward without a horizontal force being exerted on it in the same direction. The pebble would not move in a straight line. In other words, Newton's laws would not apply. This is because rotating frames of reference are accelerated frames. Newton's laws are valid only in inertial or nonaccelerated frames.

Motion in a rotating reference frame is important to us because Earth rotates. The effects of the rotation of Earth are too small to be noticed in the classroom or lab, but they are significant influences on the motion of the atmosphere and therefore, on climate and weather.

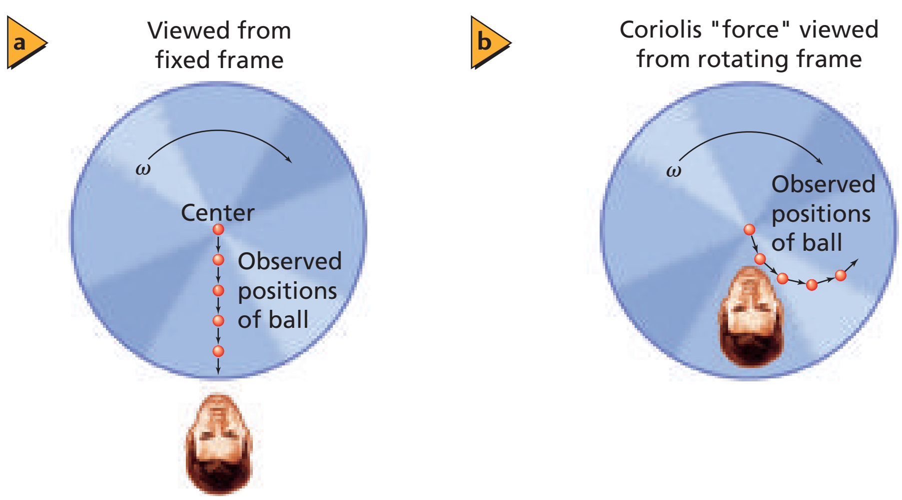
Centrifugal "Force"

Suppose you fasten one end of a spring to the center of a rotating platform. An object lies on the platform and is attached to the other end of the spring. As the platform rotates, an observer on the platform sees the object stretch the spring. The observer might think that some force toward the outside of the platform is pulling on the object. This apparent force is called **centrifugal "force."** It is not a real force because there is no physical outward push on the object. Still, this "force" seems real, as anyone who has ever been on an amusement-park ride can attest.

As the platform rotates, an observer on the ground sees things differently. This observer sees the object moving in a circle. The object accelerates toward the center because of the force of the spring. As you know, the acceleration is centripetal acceleration and is given by $a_c = v^2/r$. It also can be written in terms of angular velocity, as $a_c = \omega^2/r$. Centripetal acceleration is proportional to the distance from the axis of rotation and depends on the square of the angular velocity. Thus, if you double the rotational frequency, the acceleration increases by a factor of 4.

The Coriolis "Force"

A second effect of rotation is shown in **Figure 8-16.** Suppose a person standing at the center of a rotating disk throws a ball toward the edge of the disk. Consider the horizontal motion of the ball as seen by two observers and ignore the vertical motion of the ball as it falls.



■ **Figure 8-16** The Coriolis "force" exists only in rotating reference frames.

An observer standing outside the disk, as shown in **Figure 8-16a,** sees the ball travel in a straight line at a constant speed toward the edge of the disk. However, the other observer, who is stationed on the disk and rotating with it, as shown in **Figure 8-16b,** sees the ball follow a curved path at a constant speed. A force seems to be acting to deflect the ball. This apparent force is called the **Coriolis "force."** Like the centrifugal "force," the Coriolis "force" is not a real force. It seems to exist because we observe a deflection in horizontal motion when we are in a rotating frame of reference.

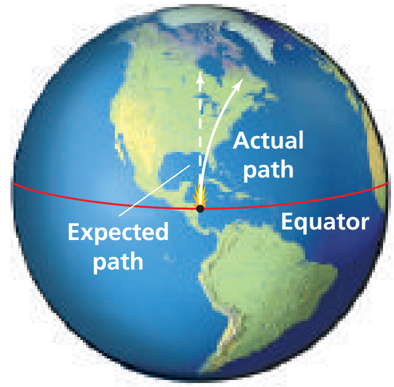


Figure 8-17 An observer on Earth sees the Coriolis "force" cause a projectile fired due north to deflect to the right of the intended target.

Coriolis "force" due to Earth Suppose a cannon is fired from a point on the equator toward a target due north of it. If the projectile were fired directly northward, it would also have an eastward velocity component because of the rotation of Earth. This eastward speed is greater at the equator than at any other latitude. Thus, as the projectile moves northward, it also moves eastward faster than points on Earth below it do. The result is that the projectile lands east of the target as shown in **Figure 8-17.** While an observer in space would see Earth's rotation, an observer on Earth could claim that the projectile missed the target because of the Coriolis "force" on the rocket. Note that for objects moving toward the equator, the direction of the apparent force is westward. A projectile will land west of the target when fired due south.

The direction of winds around high- and low-pressure areas results from the Coriolis "force." Winds flow from areas of high to low pressure. Because of the Coriolis "force" in the northern hemisphere, winds from the south go to the east of low-pressure areas. Winds from the north, however, end up west of low-pressure areas. Therefore, winds rotate counterclockwise around low-pressure areas in the northern hemisphere. In the southern hemisphere however, winds rotate clockwise around low-pressure areas.

Meteorology Connection

Most amusement-park rides thrill the riders because they are in accelerated reference frames while on the ride. The "forces" felt by roller-coaster riders at the tops and bottoms of hills, and when moving almost vertically downward, are mostly related to linear acceleration. On Ferris wheels, rotors, other circular rides, and on the curves of roller coasters, centrifugal "forces" provide most of the excitement.

8.3 Section Review

40. Center of Mass Can the center of mass of an object be located in an area where the object has no mass? Explain.

41. Stability of an Object Why is a modified vehicle with its body raised high on risers less stable than a similar vehicle with its body at normal height?

42. Conditions for Equilibrium Give an example of an object for each of the following conditions.

a. rotational equilibrium, but not translational equilibrium

b. translational equilibrium, but not rotational equilibrium

43. Center of Mass Where is the center of mass of a roll of masking tape?

44. Locating the Center of Mass Describe how you would find the center of mass of this textbook.

45. Rotating Frames of Reference A penny is placed on a rotating, old-fashioned record turntable. At the highest speed, the penny starts sliding outward. What are the forces acting on the penny?

46. Critical Thinking You have learned why the winds around a low-pressure area move in a counterclockwise direction. Would the winds move in the same or opposite direction in the southern hemisphere? Explain.

PHYSICS LAB

Horizons Companies

Translational and Rotational Equilibrium

Alternate CBL instructions can be found on the Web site.
physicspp.com

For maintenance on large buildings, scaffolding can be hung on the outside. In order for the scaffolding to support workers, it must be in translational and rotational equilibrium. If two or more forces act on the scaffolding, each can produce a rotation about either end. Scaffolding with uniform mass distribution acts as though all of the mass is concentrated at its center. In translational equilibrium the object is not accelerating; thus, the upward and downward forces are equal.

In order to achieve rotational equilibrium, the sum of all the clockwise torques must equal the sum of all the counterclockwise torques as measured from a pivot point. That is, the net torque must be zero. In this lab you will model scaffolding hung from two ropes using a meterstick and spring scales, and use numbers to measure the forces on the scaffolding.

QUESTION

What conditions are required for equilibrium when parallel forces act on an object?

Objectives

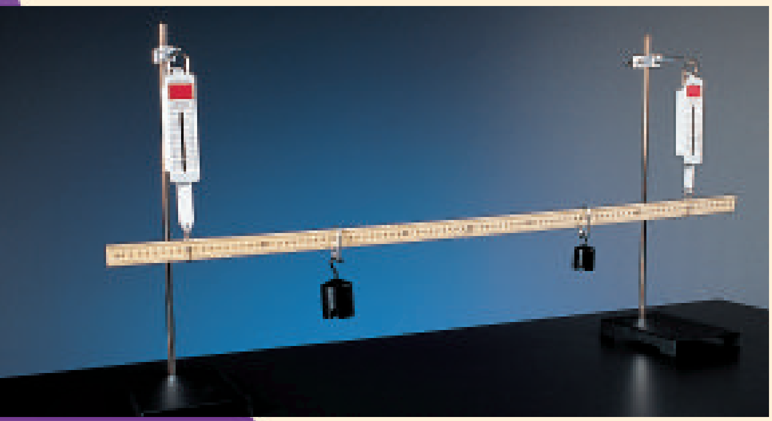
- **Collect and organize data** about the forces acting on the scaffolding.
- **Describe** clockwise and counterclockwise torque.
- **Compare and contrast** translational and rotational equilibrium.

Safety Precautions

- **Use care to avoid dropping masses.**

Materials

meterstick
two 0-5 N spring scales
two ring stands
two Buret clamps
500-g hooked mass
200-g hooked mass



Procedure

The left spring scale will be considered a pivot point for the purposes of this lab. Therefore, the lever arm will be measured from this point.

1. Place the ring stands 80 cm apart.
2. Attach a Buret clamp to each of the ring stands.
3. Verify that the scales are set to zero before use. If the scales need to be adjusted, ask your teacher for assistance.
4. Hang a spring scale from each Buret clamp attached to a ring stand.
5. Hook the meterstick onto the spring scale in such a manner that the 10-cm mark is supported by one hook and the 90-cm mark is supported by the other hook.
6. Read each spring scale and record the force in Data Table 1.
7. Hang a 500-g mass on the meterstick at the 30-cm mark. This point should be 20-cm from the left scale.
8. Read each spring scale and record the force in Data Table 1.
9. Hang a 200-g mass on the meterstick at the 70-cm mark. This point should be 60 cm from the left scale.
10. Read each spring scale and record the force in Data Table 1.

Data Table 1

Object Added	Distance From Left Scale (m)	Left Scale Reading (N)	Right Scale Reading (N)
Meterstick	0.4		
500-g mass	0.2		
200-g mass	0.6		

Data Table 2

Object Added	τ_c	τ_{cc}	Lever Arm (m)	Force (N)
Meterstick				
500-g mass				
200-g mass				
Right scale				

Data Table 3

Object Added	τ_c (N·m)	τ_{cc} (N·m)
Meterstick		
500-g mass		
200-g mass		
Right scale		
$\Sigma\tau$		

Analyze

1. **Calculate** Find the mass of the meterstick.
2. **Calculate** Find the force, or weight, that results from each object and record it in Data Table 2. For the right scale, read the force it exerts and record it in Data Table 2.
3. Using the point where the left scale is attached as a pivot point, identify the forces located elsewhere that cause the scaffold to rotate clockwise or counterclockwise. Mark these in Data Table 2 with an *x*.
4. Record the lever arm distance of each force from the pivot point in Data Table 2.
5. **Use Numbers** Calculate the torque for each object by multiplying the force and lever arm distance. Record these values in Data Table 3.

Conclude and Apply

1. Is the system in translational equilibrium? How do you know?
2. Draw a free-body diagram of your system, showing all the forces.
3. Compare and contrast the sum of the clockwise torques, $\Sigma\tau_c$, and the counterclockwise torques, $\Sigma\tau_{cc}$.
4. What is the percent difference between $\Sigma\tau_c$ and $\Sigma\tau_{cc}$?

Going Further

Use additional masses at locations of your choice with your teacher's permission and record your data.

Real-World Physics

Research the safety requirements in your area for putting up, using, and dismantling scaffolding.

Physics Online

To find out more about rotational motion, visit the Web site: **physicspp.com**

Technology and Society

The Stability of Sport-Utility Vehicles

Why are sport-utility vehicles more flippable? Many believe that the large size of the sport-utility vehicle makes it more stable and secure. But, a sport-utility vehicle, as well as other tall vehicles such as vans, is much more likely to roll over than a car.

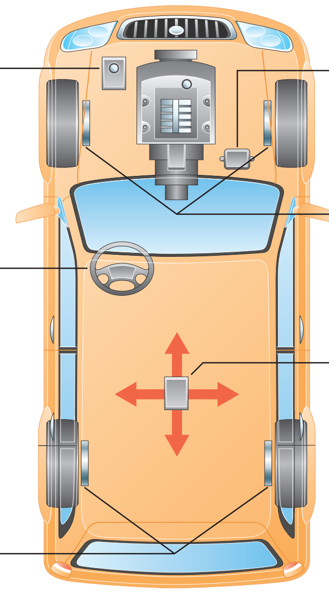
The Problem A sport-utility vehicle has a high center of mass which makes it more likely to topple. Another factor that affects rollover is the static stability factor, which is the ratio of the track width to the center of mass. Track width is defined as half the distance between the two front wheels. The higher the static stability factor, the more likely a vehicle will stay upright.

Many sport-utility vehicles have a center of mass 13 or 15 cm higher than passenger cars. Their track width, however, is about the same as that of passenger cars. Suppose the stability factor for a sport-utility vehicle is 1.06 and 1.43 for a car. Statistics show that in a single-vehicle crash, the sport-utility vehicle has a 37 percent chance of rolling over, while the car has a 10.6 percent chance of rolling over.

However, the static stability factor oversimplifies the issue. Weather and driver behavior are also contributers to rollover crashes. Vehicle factors, such as tires, suspension systems, inertial properties, and advanced handling systems all play a role as well.

It is true that most rollover crashes occur when a vehicle swerves off the road and hits a rut, soft soil, or other surface irregularity. This usually occurs when a driver is not paying proper attention or is speeding. Safe drivers greatly reduce their chances of being involved in a rollover accident by paying attention and driving at the correct speed. Still, weather and driver behavior being equal, the laws of physics indicate that sport-utility vehicles carry an increased risk.

What Is Being Done? Some models are being built with wider track widths or stronger roofs. Optional side-curtain air bags have sensors to keep the bags inflated for up to 6 s, rather than the usual fraction of a second. This will cushion passengers if the vehicle should flip several times.



An ESC system processes information from the sensors and automatically applies the brakes to individual wheels when instability is detected.

A promising new technology called Electronic Stability Control (ESC) can be used to prevent rollover accidents. An ESC system has electronic sensors that detect when a vehicle begins to spin due to oversteering, and also when it begins to slide in a plowlike manner because of understeering. In these instances, an ESC system automatically applies the brakes at one or more wheels, thereby reorienting the vehicle in the right direction.

Safe driving is the key to preventing many automobile accidents. Knowledge of the physics behind rollover accidents and the factors that affect rollover accidents may help make you an informed, safe driver.

Going Further

1. **Hypothesize** In a multi-vehicle accident, sport-utility vehicles generally fare better than the passenger cars involved in the accident. Why is this so?
2. **Debate the Issue** ESC is a life-saving technology. Should it be mandatory in all sport-utility vehicles? Why or why not?

Chapter 8

Study Guide

8.1 Describing Rotational Motion

Vocabulary

- radian (p. 197)
- angular displacement (p. 198)
- angular velocity (p. 198)
- angular acceleration (p. 199)

Key Concepts

- Angular position and its changes are measured in radians. One complete revolution is 2π rad.
- Angular velocity is given by the following equation.

$$\omega = \frac{\Delta\theta}{\Delta t}$$

- Angular acceleration is given by the following equation.

$$\alpha = \frac{\Delta\omega}{\Delta t}$$

- For a rotating, rigid object, the angular displacement, velocity, and acceleration can be related to the linear displacement, velocity, and acceleration for any point on the object.

$$d = r\theta \qquad v = r\omega \qquad a = r\alpha$$

8.2 Rotational Dynamics

Vocabulary

- lever arm (p. 201)
- torque (p. 202)
- moment of inertia (p. 205)
- Newton's second law for rotational motion (p. 208)

Key Concepts

- When torque is exerted on an object, its angular velocity changes.
- Torque depends on the magnitude of the force, the distance from the axis of rotation at which it is applied, and the angle between the force and the radius from the axis of rotation to the point where the force is applied.

$$\tau = Fr \sin\theta$$

- The moment of inertia of an object depends on the way the object's mass is distributed about the rotational axis. For a point object:

$$I = mr^2$$

- Newton's second law for rotational motion states that angular acceleration is directly proportional to the net torque and inversely proportional to the moment of inertia.

$$\alpha = \frac{\tau_{net}}{I}$$

8.3 Equilibrium

Vocabulary

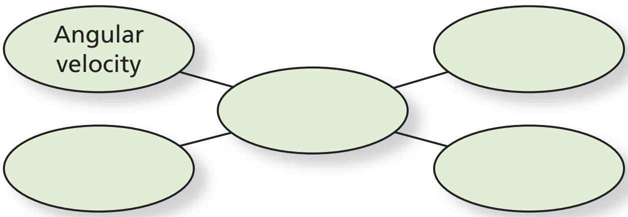
- center of mass (p. 211)
- centrifugal "force" (p. 216)
- Coriolis "force" (p. 217)

Key Concepts

- The center of mass of an object is the point on the object that moves in the same way that a point particle would move.
- An object is stable against rollover if its center of mass is above its base.
- An object is in equilibrium if there are no net forces exerted on it and if there are no net torques acting on it.
- Centrifugal "force" and the Coriolis "force" are two apparent forces that appear when a rotating object is analyzed from a coordinate system that rotates with it.

Concept Mapping

47. Complete the following concept map using the following terms: *angular acceleration, radius, tangential acceleration, centripetal acceleration.*



Mastering Concepts

48. A bicycle wheel rotates at a constant 25 rev/min. Is its angular velocity decreasing, increasing, or constant? (8.1)

49. A toy rotates at a constant 5 rev/min. Is its angular acceleration positive, negative, or zero? (8.1)

50. Do all parts of Earth rotate at the same rate? Explain. (8.1)

51. A unicycle wheel rotates at a constant 14 rev/min. Is the total acceleration of a point on the tire inward, outward, tangential, or zero? (8.1)

52. Think about some possible rotations of your textbook. Are the moments of inertia about these three axes the same or different? Explain. (8.2)

53. Torque is important when tightening bolts. Why is force not important? (8.2)

54. Rank the torques on the five doors shown in **Figure 8-18** from least to greatest. Note that the magnitude of all the forces is the same. (8.2)

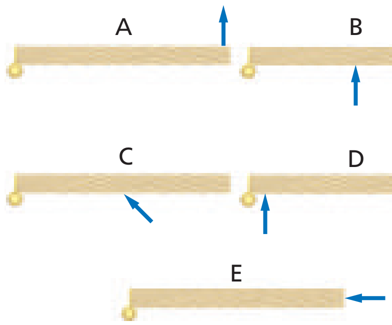


Figure 8-18

55. Explain how you can change an object's angular frequency. (8.2)

56. To balance a car's wheel, it is placed on a vertical shaft and weights are added to make the wheel horizontal. Why is this equivalent to moving the center of mass until it is at the center of the wheel? (8.3)

57. A stunt driver maneuvers a monster truck so that it is traveling on only two wheels. Where is the center of mass of the truck? (8.3)

58. Suppose you stand flat-footed, then you rise and balance on tiptoe. If you stand with your toes touching a wall, you cannot balance on tiptoe. Explain. (8.3)

59. Why does a gymnast appear to be floating on air when she raises her arms above her head in a leap? (8.3)

60. Why is a vehicle with wheels that have a large diameter more likely to roll over than a vehicle with wheels that have a smaller diameter? (8.3)

Applying Concepts

61. Two gears are in contact and rotating. One is larger than the other, as shown in **Figure 8-19.** Compare their angular velocities. Also compare the linear velocities of two teeth that are in contact.

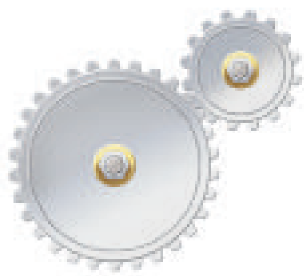


Figure 8-19

62. Videotape When a videotape is rewound, why does it wind up fastest towards the end?

63. Spin Cycle What does a spin cycle of a washing machine do? Explain in terms of the forces on the clothes and water.

64. How can you experimentally find the moment of inertia of an object?

65. Bicycle Wheels Three bicycle wheels have masses that are distributed in three different ways: mostly at the rim, uniformly, and mostly at the hub. The wheels all have the same mass. If equal torques are applied to them, which one will have the greatest angular acceleration? Which one will have the least?

66. Bowling Ball When a bowling ball leaves a bowler's hand, it does not spin. After it has gone about half the length of the lane, however, it does spin. Explain how its rotation rate increased and why it does not continue to increase.

 For more problems, go to Additional Problems, Appendix B.

67. Flat Tire Suppose your car has a flat tire. You get out your tools and find a lug wrench to remove the nuts off the bolt studs. You find it impossible to turn the nuts. Your friend suggests ways you might produce enough torque to turn them. What three ways might your friend suggest?

68. Tightrope Walkers Tightrope walkers often carry long poles that sag so that the ends are lower than the center as shown in **Figure 8-20.** How does such a pole increase the tightrope walker's stability? *Hint: Consider both center of mass and moment of inertia.*

Figure 8-20

69. Merry-Go-Round While riding a merry-go-round, you toss a key to a friend standing on the ground. For your friend to be able to catch the key, should you toss it a second or two before you reach the spot where your friend is standing or wait until your friend is directly behind you? Explain.

70. Why can you ignore forces that act on the axis of rotation of an object in static equilibrium when determining the net torque?

71. In solving problems about static equilibrium, why is the axis of rotation often placed at a point where one or more forces are acting on the object?

Mastering Problems

8.1 Describing Rotational Motion

72. A wheel is rotated so that a point on the edge moves through 1.50 m. The radius of the wheel is 2.50 m, as shown in **Figure 8-21.** Through what angle (in radians) is the wheel rotated?

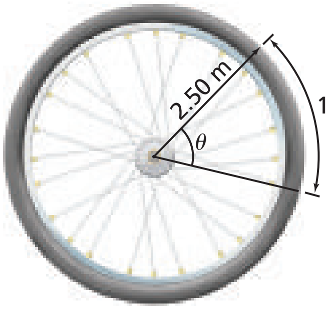


Figure 8-21

73. The outer edge of a truck tire that has a radius of 45 cm has a velocity of 23 m/s. What is the angular velocity of the tire in rad/s?

74. A steering wheel is rotated through 128°, as shown in **Figure 8-22.** Its radius is 22 cm. How far would a point on the steering wheel's edge move?

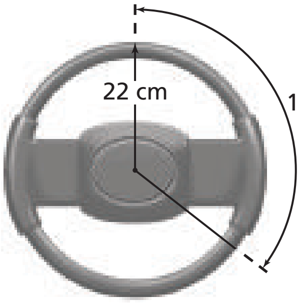


Figure 8-22

75. Propeller A propeller spins at 1880 rev/min.
a. What is its angular velocity in rad/s?
b. What is the angular displacement of the propeller in 2.50 s?

76. The propeller in the previous problem slows from 475 rev/min to 187 rev/min in 4.00 s. What is its angular acceleration?

77. An automobile wheel with a 9.00 cm radius, as shown in **Figure 8-23,** rotates at 2.50 rad/s. How fast does a point 7.00 cm from the center travel?

Figure 8-23

78. Washing Machine A washing machine's two spin cycles are 328 rev/min and 542 rev/min. The diameter of the drum is 0.43 m.
a. What is the ratio of the centripetal accelerations for the fast and slow spin cycles? Recall that $a_c = \frac{v^2}{r}$ and $v = rw$
b. What is the ratio of the linear velocity of an object at the surface of the drum for the fast and slow spin cycles?

79. Find the maximum centripetal acceleration in terms of g for the washing machine in problem 78.

80. A laboratory ultracentrifuge is designed to produce a centripetal acceleration of 0.35×10^6 g at a distance of 2.50 cm from the axis. What angular velocity in rev/min is required?

8.2 Rotational Dynamics

81. Wrench A bolt is to be tightened with a torque of 8.0 N·m. If you have a wrench that is 0.35 m long, what is the least amount of force you must exert?

82. What is the torque on a bolt produced by a 15-N force exerted perpendicular to a wrench that is 25 cm long, as shown in **Figure 8-24?**

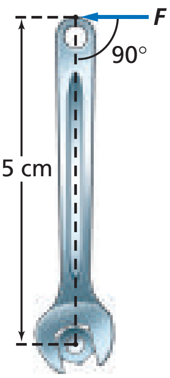


Figure 8-24

83. A toy consisting of two balls, each 0.45 kg, at the ends of a 0.46-m-long, thin, lightweight rod is shown in **Figure 8-25.** Find the moment of inertia of the toy. The moment of inertia is to be found about the center of the rod.

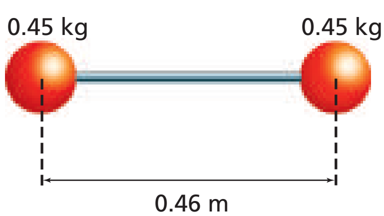


Figure 8-25

84. A bicycle wheel with a radius of 38 cm is given an angular acceleration of 2.67 rad/s^2 by applying a force of 0.35 N on the edge of the wheel. What is the wheel's moment of inertia?

85. Toy Top A toy top consists of a rod with a diameter of 8.0-mm and a disk of mass 0.0125 kg and a diameter of 3.5 cm. The moment of inertia of the rod can be neglected. The top is spun by wrapping a string around the rod and pulling it with a velocity that increases from zero to 3.0 m/s over 0.50 s.

a. What is the resulting angular velocity of the top?
b. What force was exerted on the string?

8.3 Equilibrium

86. A 12.5-kg board, 4.00 m long, is being held up on one end by Ahmed. He calls for help, and Judi responds.
a. What is the least force that Judi could exert to lift the board to the horizontal position? What part of the board should she lift to exert this force?
b. What is the greatest force that Judi could exert to lift the board to the horizontal position? What part of the board should she lift to exert this force?

87. Two people are holding up the ends of a 4.25-kg wooden board that is 1.75 m long. A 6.00-kg box sits on the board, 0.50 m from one end, as shown in **Figure 8-26.** What forces do the two people exert?

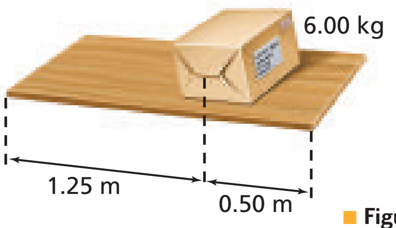


Figure 8-26

88. A car's specifications state that its weight distribution is 53 percent on the front tires and 47 percent on the rear tires. The wheel base is 2.46 m. Where is the car's center of mass?

Mixed Review

89. A wooden door of mass, m, and length, l, is held horizontally by Dan and Ajit. Dan suddenly drops his end.
a. What is the angular acceleration of the door just after Dan lets go?
b. Is the acceleration constant? Explain.

90. Topsoil Ten bags of topsoil, each weighing 175 N, are placed on a 2.43-m-long sheet of wood. They are stacked 0.50 m from one end of the sheet of wood, as shown in **Figure 8-27.** Two people lift the sheet of wood, one at each end. Ignoring the weight of the wood, how much force must each person exert?

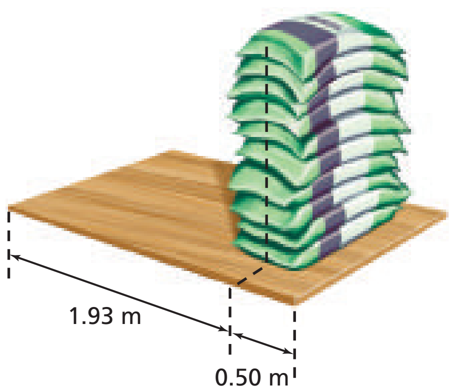


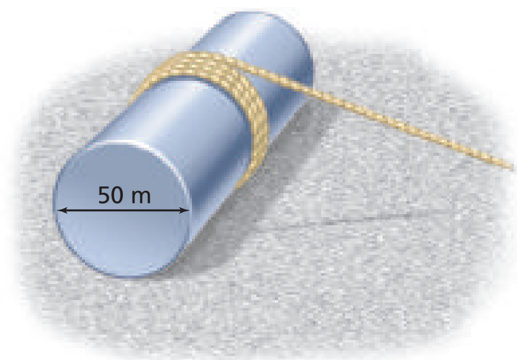
Figure 8-27

 For more problems, go to Additional Problems, Appendix B.

91. Basketball A basketball is rolled down the court. A regulation basketball has a diameter of 24.1 cm, a mass of 0.60 kg, and a moment of inertia of 5.8×10^{-3} kg·m^2. The basketball's initial velocity is 2.5 m/s.
- **a.** What is its initial angular velocity?
- **b.** The ball rolls a total of 12 m. How many revolutions does it make?
- **c.** What is its total angular displacement?

92. The basketball in the previous problem stops rolling after traveling 12 m.
- **a.** If its acceleration was constant, what was its angular acceleration?
- **b.** What torque was acting on it as it was slowing down?

93. A cylinder with a 50 m diameter, as shown in **Figure 8-28,** is at rest on a surface. A rope is wrapped around the cylinder and pulled. The cylinder rolls without slipping.
- **a.** After the rope has been pulled a distance of 2.50 m at a constant speed, how far has the center of mass of the cylinder moved?
- **b.** If the rope was pulled a distance of 2.50 m in 1.25 s, how fast was the center of mass of the cylinder moving?
- **c.** What is the angular velocity of the cylinder?



■ **Figure 8-28**

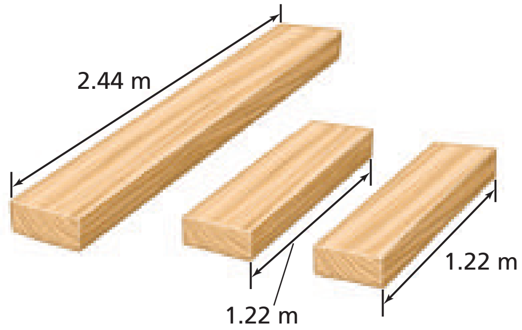
94. Hard Drive A hard drive on a modern computer spins at 7200 rpm (revolutions per minute). If the drive is designed to start from rest and reach operating speed in 1.5 s, what is the angular acceleration of the disk?

95. Speedometers Most speedometers in automobiles measure the angular velocity of the transmission and convert it to speed. How will increasing the diameter of the tires affect the reading of the speedometer?

96. A box is dragged across the floor using a rope that is a distance *h* above the floor. The coefficient of friction is 0.35. The box is 0.50 m high and 0.25 m wide. Find the force that just tips the box.

97. The second hand on a watch is 12 mm long. What is the velocity of its tip?

98. Lumber You buy a 2.44-m-long piece of 10 cm × 10 cm lumber. Your friend buys a piece of the same size and cuts it into two lengths, each 1.22 m long, as shown in **Figure 8-29.** You each carry your lumber on your shoulders.
- **a.** Which load is easier to lift? Why?
- **b.** Both you and your friend apply a torque with your hands to keep the lumber from rotating. Which load is easier to keep from rotating? Why?



■ **Figure 8-29**

99. Surfboard Harris and Paul carry a surfboard that is 2.43 m long and weighs 143 N. Paul lifts one end with a force of 57 N.
- **a.** What force must Harris exert?
- **b.** What part of the board should Harris lift?

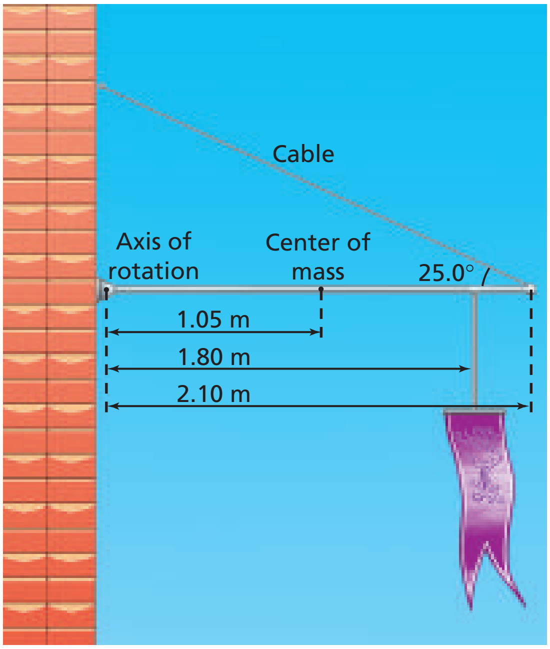
100. A steel beam that is 6.50 m long weighs 325 N. It rests on two supports, 3.00 m apart, with equal amounts of the beam extending from each end. Suki, who weighs 575 N, stands on the beam in the center and then walks toward one end. How close to the end can she come before the beam begins to tip?

Thinking Critically

101. Apply Concepts Consider a point on the edge of a rotating wheel.
- **a.** Under what conditions can the centripetal acceleration be zero?
- **b.** Under what conditions can the tangential (linear) acceleration be zero?
- **c.** Can the tangential acceleration be nonzero while the centripetal acceleration is zero? Explain.
- **d.** Can the centripetal acceleration be nonzero while the tangential acceleration is zero? Explain.

102. Apply Concepts When you apply the brakes in a car, the front end dips. Why?

103. Analyze and Conclude A banner is suspended from a horizontal, pivoted pole, as shown in **Figure 8-30.** The pole is 2.10 m long and weighs 175 N. The banner, which weighs 105 N, is suspended 1.80 m from the pivot point or axis of rotation. What is the tension in the cable supporting the pole?

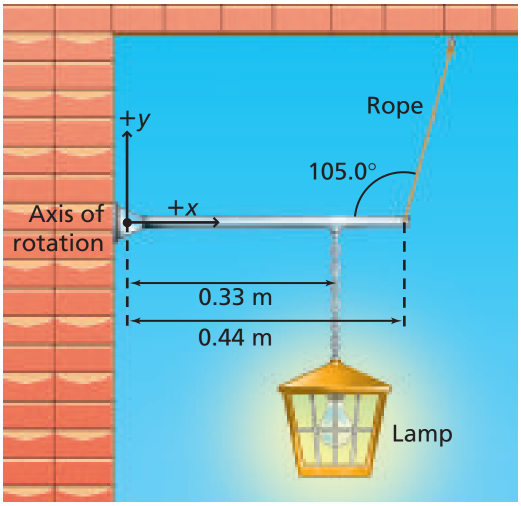


■ **Figure 8-30**

104. Analyze and Conclude A pivoted lamp pole is shown in **Figure 8-31.** The pole weighs 27 N, and the lamp weighs 64 N.

a. What is the torque caused by each force?

b. Determine the tension in the rope supporting the lamp pole.



■ **Figure 8-31**

105. Analyze and Conclude Gerald and Evelyn carry the following objects up a flight of stairs: a large mirror, a dresser, and a television. Evelyn is at the front end, and Gerald is at the bottom end. Assume that both Evelyn and Gerald exert only upward forces.

a. Draw a free-body diagram showing Gerald and Evelyn exerting the same force on the mirror.

b. Draw a free-body diagram showing Gerald exerting more force on the bottom of the dresser.

c. Where would the center of mass of the television have to be so that Gerald carries all the weight?

Writing in Physics

106. Astronomers know that if a satellite is too close to a planet, it will be torn apart by tidal forces. That is, the difference in the gravitational force on the part of the satellite nearest the planet and the part farthest from the planet is stronger than the forces holding the satellite together. Do research on the Roche limit and determine how close the Moon would have to orbit Earth to be at the Roche limit.

107. Automobile engines are rated by the torque that they produce. Research and explain why torque is an important quantity to measure.

Cumulative Review

108. Two blocks, one of mass 2.0 kg and the other of mass 3.0 kg, are tied together with a massless rope. This rope is strung over a massless, resistance-free pulley. The blocks are released from rest. Find the following. (Chapter 4)

a. the tension in the rope

b. the acceleration of the blocks.

109. Eric sits on a see-saw. At what angle, relative to the vertical, will the component of his weight parallel to the plane be equal to one-third the perpendicular component of his weight? (Chapter 5)

110. The pilot of a plane wants to reach an airport 325 km due north in 2.75 hours. A wind is blowing from the west at 30.0 km/h. What heading and airspeed should be chosen to reach the destination on time? (Chapter 6)

111. A 60.0-kg speed skater with a velocity of 18.0 m/s comes into a curve of 20.0-m radius. How much friction must be exerted between the skates and ice to negotiate the curve? (Chapter 6)

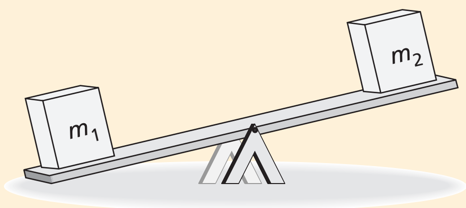
 For more problems, go to Additional Problems, Appendix B.

Standardized Test Practice

Multiple Choice

1. The illustration below shows two boxes on opposite ends of a board that is 3.0 m long. The board is supported in the middle by a fulcrum. The box on the left has a mass, m_1, of 25 kg, and the box on the right has a mass, m_2, of 15 kg. How far should the fulcrum be positioned from the left side of the board in order to balance the masses horizontally?

Ⓐ 0.38 m Ⓒ 1.1 m
Ⓑ 0.60 m Ⓓ 1.9 m



2. A force of 60 N is exerted on one end of a 1.0-m-long lever. The other end of the lever is attached to a rotating rod that is perpendicular to the lever. By pushing down on the end of the lever, you can rotate the rod. If the force on the lever is exerted at an angle of 30°, what torque is exerted on the lever? ($\sin 30° = 0.5$; $\cos 30° = 0.87$; $\tan 30° = 0.58$)

Ⓐ 30 N Ⓒ 60 N
Ⓑ 52 N Ⓓ 69 N

3. A child attempts to use a wrench to remove a nut on a bicycle. Removing the nut requires a torque of 10 N·m. The maximum force the child is capable of exerting at a 90° angle is 50 N. What is the length of the wrench the child must use to remove the nut?

Ⓐ 0.1 m Ⓒ 0.2 m
Ⓑ 0.15 m Ⓓ 0.25 m

4. A car moves a distance of 420 m. Each tire on the car has a diameter of 42 cm. Which of the following shows how many revolutions each tire makes as they move that distance?

Ⓐ $\frac{5.0\times10^1}{\pi}$ rev Ⓒ $\frac{1.5\times10^2}{\pi}$ rev
Ⓑ $\frac{1.0\times10^2}{\pi}$ rev Ⓓ $\frac{1.0\times10^3}{\pi}$ rev

5. A thin hoop with a mass of 5.0 kg rotates about a perpendicular axis through its center. A force of 25 N is exerted tangentially to the hoop. If the hoop's radius is 2.0 m, what is its angular acceleration?

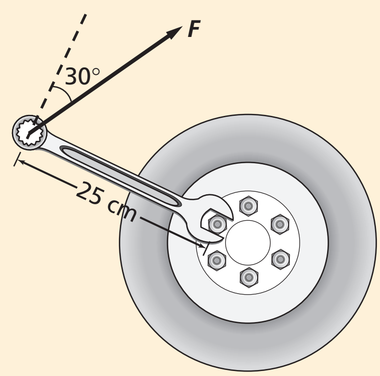
Ⓐ 1.3 rad/s Ⓒ 5.0 rad/s
Ⓑ 2.5 rad/s Ⓓ 6.3 rad/s

6. Two of the tires on a farmer's tractor have diameters of 1.5 m. If the farmer drives the tractor at a linear velocity of 3.0 m/s, what is the angular velocity of each tire?

Ⓐ 2.0 rad/s Ⓒ 4.0 rad/s
Ⓑ 2.3 rad/s Ⓓ 4.5 rad/s

Extended Answer

7. You use a 25-cm long wrench to remove the lug nuts on a car wheel, as shown in the illustration below. If you pull up on the end of the wrench with a force of 2.0×10^2 N at an angle of 30°, what is the torque on the wrench? ($\sin 30° = 0.5$, $\cos 30° = 0.87$)



✓ Test-Taking TIP

When Eliminating, Cross It Out

Consider each answer choice individually and cross out the ones you have eliminated. If you cannot write in the test booklet, use the scratch paper to list and cross off the answer choices. You will save time and stop yourself from choosing an answer you have mentally eliminated.

Chapter 9

Momentum and Its Conservation

What You'll Learn

- You will describe momentum and impulse and apply them to the interactions between objects.
- You will relate Newton's third law of motion to conservation of momentum.
- You will explore the momentum of rotating objects.

Why It's Important

Momentum is the key to success in many sporting events, including baseball, football, ice hockey, and tennis.

Baseball Every baseball player dreams of hitting a home run. When a player hits the ball, at the moment of collision, the ball and the bat are deformed by the collision. The resulting change in momentum determines the batter's success.

Think About This ▶

What is the force on a baseball bat when a home run is hit out of the park?

Physics Online

physicspp.com

LAUNCH Lab

What happens when a hollow plastic ball strikes a bocce ball?

Question

What direction will a hollow plastic ball and a bocce ball move after a head-on collision?

Procedure

1. Roll a bocce ball and a hollow plastic ball toward each other on a smooth surface.
2. Observe the direction each one moves after the collision.
3. Repeat the experiment, this time keeping the bocce ball stationary, while rolling the hollow plastic ball toward it.
4. Observe the direction each one moves after the collision.
5. Repeat the experiment one more time, but keep the hollow plastic ball stationary, while rolling the bocce ball toward it.
6. Observe the direction each one moves after the collision.

Analysis

What factors affect how fast the balls move after the collision? What factors determine the direction each one moves after the collision?

Critical Thinking What factor(s) would cause the bocce ball to move backward after colliding with the hollow plastic ball?

9.1 Impulse and Momentum

Objectives

- **Define** the momentum of an object.
- **Determine** the impulse given to an object.
- **Define** the angular momentum of an object.

Vocabulary

impulse
momentum
impulse-momentum theorem
angular momentum
angular impulse-angular momentum theorem

It is always exciting to watch a baseball player hit a home run. The pitcher fires the baseball toward the plate. The batter swings at the baseball and the baseball recoils from the impact of the bat at high speed. Rather than concentrating on the force between the baseball and bat and their resulting accelerations, as in previous chapters, you will approach this collision in a different way in this chapter. The first step in analyzing this type of interaction is to describe what happens before, during, and after the collision between the baseball and bat. You can simplify the collision between the baseball and the bat by making the assumption that all motion is in the horizontal direction. Before the collision, the baseball moves toward the bat. During the collision, the baseball is squashed against the bat. After the collision, however, the baseball moves at a higher velocity away from the bat, and the bat continues in its path, but at a slower velocity.

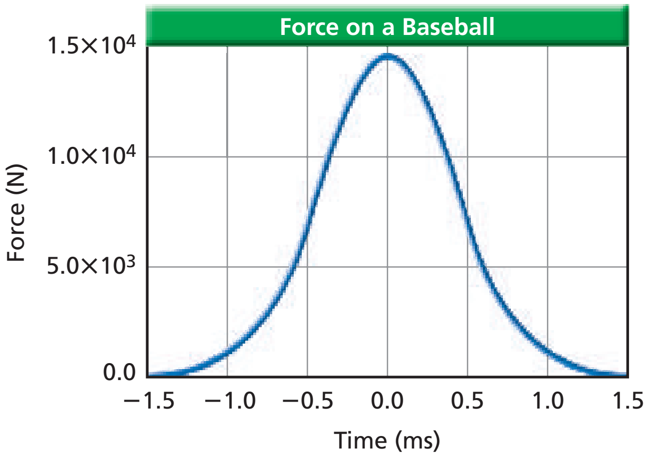


Figure 9-1 The force acting on a baseball increases, then rapidly decreases during a collision, as shown in this force-time graph.

Impulse and Momentum

How are the velocities of the ball, before and after the collision, related to the force acting on it? Newton's second law of motion describes how the velocity of an object is changed by a net force acting on it. The change in velocity of the ball must have been caused by the force exerted by the bat on the ball. The force changes over time, as shown in **Figure 9-1.** Just after contact is made, the ball is squeezed, and the force increases. After the force reaches its maximum, which is more than 10,000 times the weight of the ball, the ball recovers its shape and snaps away from the bat. The force rapidly returns to zero. This whole event takes place within about 3.0 ms. How can you calculate the change in velocity of the baseball?

Impulse Newton's second law of motion, $\boldsymbol{F} = m\boldsymbol{a}$, can be rewritten by using the definition of acceleration as the change in velocity divided by the time needed to make that change. It can be represented by the following equation:

$$\boldsymbol{F} = m\boldsymbol{a} = m\left(\frac{\Delta \boldsymbol{v}}{\Delta t}\right)$$

Multiplying both sides of the equation by the time interval, Δt, results in the following equation:

$$\boldsymbol{F}\Delta t = m\Delta\boldsymbol{v}$$

Impulse, or $\boldsymbol{F}\Delta t$, is the product of the average force on an object and the time interval over which it acts. Impulse is measured in newton-seconds. For instances in which the force varies with time, the magnitude of an impulse is found by determining the area under the curve of a force-time graph, such as the one shown in Figure 9-1.

The right side of the equation, $m\Delta\boldsymbol{v}$, involves the change in velocity: $\Delta\boldsymbol{v} = \boldsymbol{v}_f - \boldsymbol{v}_i$. Therefore, $m\Delta\boldsymbol{v} = m\boldsymbol{v}_f - m\boldsymbol{v}_i$. The product of the object's mass, m, and the object's velocity, $\boldsymbol{v}$, is defined as the **momentum** of the object. Momentum is measured in kg·m/s. An object's momentum, also known as linear momentum, is represented by the following equation.

Momentum $\boldsymbol{p} = m\boldsymbol{v}$

The momentum of an object is equal to the mass of the object times the object's velocity.

Recall the equation $\boldsymbol{F}\Delta t = m\Delta\boldsymbol{v} = m\boldsymbol{v}_f - m\boldsymbol{v}_i$. Because $m\boldsymbol{v}_f = \boldsymbol{p}_f$ and $m\boldsymbol{v}_i = \boldsymbol{p}_i$, this equation can be rewritten as follows: $\boldsymbol{F}\Delta t = m\Delta\boldsymbol{v} = \boldsymbol{p}_f - \boldsymbol{p}_i$. The right side of this equation, $\boldsymbol{p}_f - \boldsymbol{p}_i$, describes the change in momentum of an object. Thus, the impulse on an object is equal to the change in its momentum, which is called the **impulse-momentum theorem.** The impulse-momentum theorem is represented by the following equation.

Impulse-Momentum Theorem $\boldsymbol{F}\Delta t = \boldsymbol{p}_f - \boldsymbol{p}_i$

The impulse on an object is equal to the object's final momentum minus the object's initial momentum.

Color Convention

- Momentum and impulse vectors are **orange.**
- Force vectors are **blue.**
- Acceleration vectors are **violet.**
- Velocity vectors are **red.**
- Displacement vectors are **green.**

If the force on an object is constant, the impulse is the product of the force multiplied by the time interval over which it acts. Generally, the force is not constant, however, and the impulse is found by using an average force multiplied by the time interval over which it acts, or by finding the area under a force-time graph.

Because velocity is a vector, momentum also is a vector. Similarly, impulse is a vector because force is a vector. This means that signs will be important for motion in one dimension.

Using the Impulse-Momentum Theorem

What is the change in momentum of a baseball? From the impulse-momentum theorem, you know that the change in momentum is equal to the impulse acting on it. The impulse on a baseball can be calculated by using a force-time graph. In Figure 9-1, the area under the curve is approximately 13.1 N·s. The direction of the impulse is in the direction of the force. Therefore, the change in momentum of the ball also is 13.1 N·s. Because 1 N·s is equal to 1 kg·m/s, the momentum gained by the ball is 13.1 kg·m/s in the direction of the force acting on it.

Assume that a batter hits a fastball. Before the collision of the ball and bat, the ball, with a mass of 0.145 kg, has a velocity of −38 m/s. Assume that the positive direction is toward the pitcher. Therefore, the baseball's momentum is $p_i = (0.145\ \text{kg})(-38\ \text{m/s}) = -5.5\ \text{kg·m/s}$.

What is the momentum of the ball after the collision? Solve the impulse-momentum theorem for the final momentum: $p_f = p_i + F\Delta t$. The ball's final momentum is the sum of the initial momentum and the impulse. Thus, the ball's final momentum is calculated as follows.

$$\begin{aligned} p_f &= p_i + 13.1\ \text{kg·m/s} \\ &= -5.5\ \text{kg·m/s} + 13.1\ \text{kg·m/s} = +7.6\ \text{kg·m/s} \end{aligned}$$

What is the baseball's final velocity? Because $p_f = mv_f$, solving for $\boldsymbol{v}_f$ yields the following:

$$v_f = \frac{p_f}{m} = \frac{+7.6\ \text{kg·m/s}}{+0.145\ \text{kg·m/s}} = +52\ \text{m/s}$$

A speed of 52 m/s is fast enough to clear most outfield fences if the baseball is hit in the correct direction.

Using the Impulse-Momentum Theorem to Save Lives

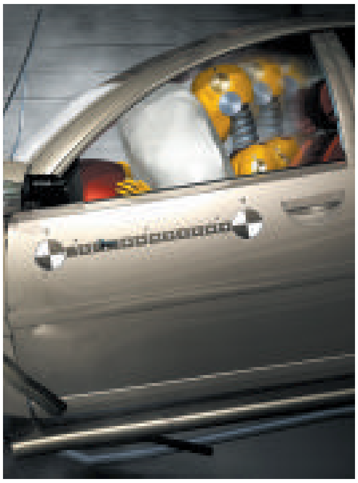
A large change in momentum occurs only when there is a large impulse. A large impulse can result either from a large force acting over a short period of time or from a smaller force acting over a long period of time.

What happens to the driver when a crash suddenly stops a car? An impulse is needed to bring the driver's momentum to zero. According to the impulse-momentum equation, $\boldsymbol{F}\Delta t = \boldsymbol{p}_f - \boldsymbol{p}_i$. The final momentum, $\boldsymbol{p}_f$, is zero. The initial momentum, $\boldsymbol{p}_i$, is the same with or without an air bag. Thus, the impulse, $\boldsymbol{F}\Delta t$, also is the same. An air bag, such as the one shown in **Figure 9-2,** reduces the force by increasing the time interval during which it acts. It also exerts the force over a larger area of the person's body, thereby reducing the likelihood of injuries.

APPLYING PHYSICS

Running Shoes Running is hard on the feet. When a runner's foot strikes the ground, the force exerted by the ground on it is as much as four times the runner's weight. The cushioning in an athletic shoe is designed to reduce this force by lengthening the time interval over which the force is exerted.

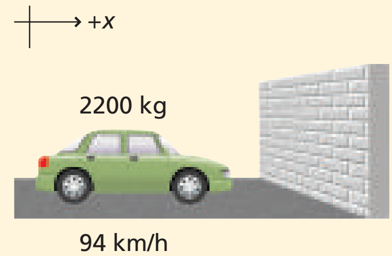
Figure 9-2 An air bag is inflated during a collision when the force due to the impact triggers the sensor. The chemicals in the air bag's inflation system react and produce a gas that rapidly inflates the air bag.



Rick Fischer/Masterfile

EXAMPLE Problem 1

Average Force A 2200-kg vehicle traveling at 94 km/h (26 m/s) can be stopped in 21 s by gently applying the brakes. It can be stopped in 3.8 s if the driver slams on the brakes, or in 0.22 s if it hits a concrete wall. What average force is exerted on the vehicle in each of these stops?

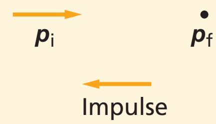


1 Analyze and Sketch the Problem

- Sketch the system.
- Include a coordinate axis and select the positive direction to be the direction of the velocity of the car.
- Draw a vector diagram for momentum and impulse.

Known:		Unknown:
$m = 2200$ kg	$\Delta t_{\text{gentle braking}} = 21$ s	$F_{\text{gentle braking}} = ?$
$v_i = +26$ m/s	$\Delta t_{\text{hard braking}} = 3.8$ s	$F_{\text{hard braking}} = ?$
$v_f = +0.0$ m/s	$\Delta t_{\text{hitting a wall}} = 0.22$ s	$F_{\text{hitting a wall}} = ?$

Vector diagram



2 Solve for the Unknown

Determine the initial momentum, p_i.

$$p_i = mv_i$$
$$= (2200\ \text{kg})(+26\ \text{m/s}) \qquad \textbf{Substitute } m = 2200\ \text{kg},\ v_i = +26\ \text{m/s}$$
$$= +5.7\times10^4\ \text{kg·m/s}$$

Determine the final momentum, p_f.

$$p_f = mv_f$$
$$= (2200\ \text{kg})(+0.0\ \text{m/s}) \qquad \textbf{Substitute } m = 2200\ \text{kg},\ v_f = +0.0\ \text{m/s}$$
$$= +0.0\ \text{kg·m/s}$$

Apply the impulse-momentum theorem to obtain the force needed to stop the vehicle.

$$F\Delta t = p_f - p_i$$
$$F\Delta t = (+0.0\ \text{kg·m/s}) - (5.7\times10^4\ \text{kg·m/s}) \qquad \textbf{Substitute } p_f = 0.0\ \text{kg·m/s},\ p_i = 5.7\times10^4\ \text{kg·m/s}$$
$$= -5.7\times10^4\ \text{kg·m/s}$$
$$F = \frac{-5.7\times10^4\ \text{kg·m/s}}{\Delta t}$$

$$F_{\text{gentle braking}} = \frac{-5.7\times10^4\ \text{kg·m/s}}{21\ \text{s}} \qquad \textbf{Substitute } \Delta t_{\text{gentle braking}} = 21\ \text{s}$$
$$= -2.7\times10^3\ \text{N}$$

$$F_{\text{hard braking}} = \frac{-5.7\times10^4\ \text{kg·m/s}}{3.8\ \text{s}} \qquad \textbf{Substitute } \Delta t_{\text{hard braking}} = 3.8\ \text{s}$$
$$= -1.5\times10^4\ \text{N}$$

$$F_{\text{hitting a wall}} = \frac{-5.7\times10^4\ \text{kg·m/s}}{0.22\ \text{s}} \qquad \textbf{Substitute } \Delta t_{\text{hitting a wall}} = 0.22\ \text{s}$$
$$= -2.6\times10^5\ \text{N}$$

Math Handbook
Operations with Significant Digits
pages 835–836

3 Evaluate the Answer

- **Are the units correct?** Force is measured in newtons.
- **Does the direction make sense?** Force is exerted in the direction opposite to the velocity of the car and thus, is negative.
- **Is the magnitude realistic?** People weigh hundreds of newtons, so it is reasonable that the force needed to stop a car would be in the thousands of newtons. The impulse is the same for all three stops. Thus, as the stopping time is shortened by more than a factor of 10, the force is increased by more than a factor of 10.

PRACTICE Problems

Additional Problems, Appendix B

1. A compact car, with mass 725 kg, is moving at 115 km/h toward the east. Sketch the moving car.
   - **a.** Find the magnitude and direction of its momentum. Draw an arrow on your sketch showing the momentum.
   - **b.** A second car, with a mass of 2175 kg, has the same momentum. What is its velocity?
2. The driver of the compact car in the previous problem suddenly applies the brakes hard for 2.0 s. As a result, an average force of 5.0×10^3 N is exerted on the car to slow it down.
   - **a.** What is the change in momentum; that is, the magnitude and direction of the impulse, on the car?
   - **b.** Complete the "before" and "after" sketches, and determine the momentum and the velocity of the car now.
3. A 7.0-kg bowling ball is rolling down the alley with a velocity of 2.0 m/s. For each impulse, shown in **Figures 9-3a** and **9-3b,** find the resulting speed and direction of motion of the bowling ball.
4. The driver accelerates a 240.0-kg snowmobile, which results in a force being exerted that speeds up the snowmobile from 6.00 m/s to 28.0 m/s over a time interval of 60.0 s.
   - **a.** Sketch the event, showing the initial and final situations.
   - **b.** What is the snowmobile's change in momentum? What is the impulse on the snowmobile?
   - **c.** What is the magnitude of the average force that is exerted on the snowmobile?
5. Suppose a 60.0-kg person was in the vehicle that hit the concrete wall in Example Problem 1. The velocity of the person equals that of the car both before and after the crash, and the velocity changes in 0.20 s. Sketch the problem.
   - **a.** What is the average force exerted on the person?
   - **b.** Some people think that they can stop their bodies from lurching forward in a vehicle that is suddenly braking by putting their hands on the dashboard. Find the mass of an object that has a weight equal to the force you just calculated. Could you lift such a mass? Are you strong enough to stop your body with your arms?

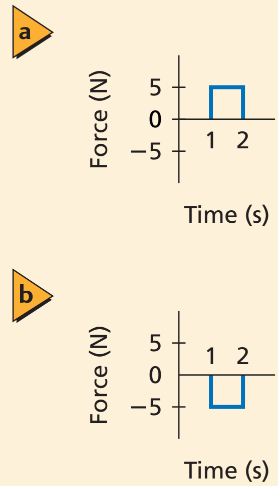


Figure 9-3

Angular Momentum

As you learned in Chapter 8, the angular velocity of a rotating object changes only if torque is applied to it. This is a statement of Newton's law for rotational motion, $\tau = I\Delta\omega/\Delta t$. This equation can be rearranged in the same way as Newton's second law of motion was, to produce $\tau\Delta t = I\Delta\omega$.

The left side of this equation, $\tau\Delta t$, is the angular impulse of the rotating object. The right side can be rewritten as $\Delta\omega = \omega_f - \omega_i$. The product of a rotating object's moment of inertia and angular velocity is called **angular momentum,** which is represented by the symbol L. The angular momentum of an object can be represented by the following equation.

Angular Momentum $L = I\omega$

The angular momentum of an object is equal to the product of the object's moment of inertia and the object's angular velocity.

Angular momentum is measured in kg·m^2/s. Just as the linear momentum of an object changes when an impulse acts on it, the angular momentum of an object changes when an angular impulse acts on it. Thus, the angular impulse on the object is equal to the change in the object's angular momentum, which is called the **angular impulse-angular momentum theorem.** The angular impulse-angular momentum theorem is represented by the following equation.

Angular Impulse-Angular Momentum Theorem $\tau \Delta t = L_f - L_i$

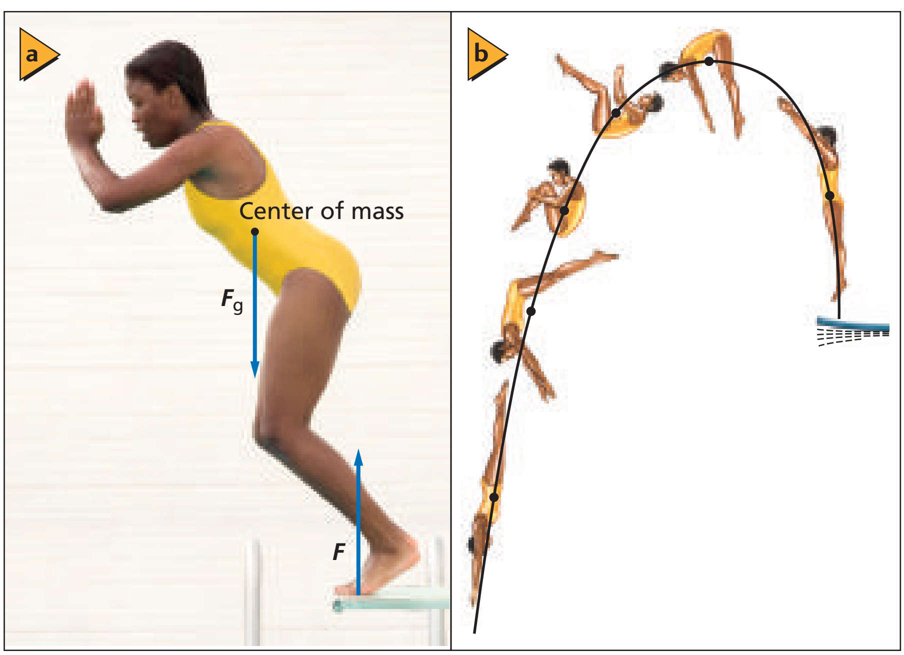
The angular impulse on an object is equal to the object's final angular momentum minus the object's initial angular momentum.

If there are no forces acting on an object, its linear momentum is constant. If there are no torques acting on an object, its angular momentum is also constant. Because an object's mass cannot be changed, if its momentum is constant, then its velocity is also constant. In the case of angular momentum, however, the object's angular velocity does not remain constant. This is because the moment of inertia depends on the object's mass and the way it is distributed about the axis of rotation or revolution. Thus, the angular velocity of an object can change even if no torques are acting on it.

Astronomy Connection

Consider, for example, a planet orbiting the Sun. The torque on the planet is zero because the gravitational force acts directly toward the Sun. Therefore, the planet's angular momentum is constant. When the distance between the planet and the Sun decreases, however, the planet's moment of inertia of revolution in orbit about the Sun also decreases. Thus, the planet's angular velocity increases and it moves faster. This is an explanation of Kepler's second law of planetary motion, based on Newton's laws of motion.

Figure 9-4 The diver's center of mass is in front of her feet as she gets ready to dive **(a).** As the diver changes her moment of inertia by moving her arms and legs to increase her angular momentum, the location of the center of mass changes, but the path of the center of mass remains a parabola **(b).**



Tim Fuller

Consider the diver in **Figure 9-4.** How does she start rotating her body? She uses the diving board to apply an external torque to her body. Then, she moves her center of mass in front of her feet and uses the board to give a final upward push to her feet. This torque acts over time, Δt, and thus increases the angular momentum of the diver.

Before the diver reaches the water, she can change her angular velocity by changing her moment of inertia. She may go into a tuck position, grabbing her knees with her hands. By moving her mass closer to the axis of rotation, the diver decreases her moment of inertia and increases her angular velocity. When she nears the water, she stretches her body straight, thereby increasing the moment of inertia and reducing the angular velocity. As a result, she goes straight into the water.

An ice-skater uses a similar method to spin. To begin rotating on one foot, the ice-skater applies an external torque to her body by pushing a portion of the other skate into the ice, as shown in **Figure 9-5.** If she pushes on the ice in one direction, the ice will exert a force on her in the opposite direction. The force results in a torque if the force is exerted some distance away from the pivot point, and in a direction that is not toward it. The greatest torque for a given force will result if the push is perpendicular to the lever arm.

The ice-skater then can control his angular velocity by changing her moment of inertia. Both arms and one leg can be extended from the body to slow the rotation, or pulled in close to the axis of rotation to speed it up. To stop spinning, another torque must be exerted by using the second skate to create a way for the ice to exert the needed force.

Figure 9-5 To spin on one foot, an ice-skater extends one leg and pushes on the ice. The ice exerts an equal and opposite force on her body and produces an external torque.

9.1 Section Review

6. Momentum Is the momentum of a car traveling south different from that of the same car when it travels north at the same speed? Draw the momentum vectors to support your answer.

7. Impulse and Momentum When you jump from a height to the ground, you let your legs bend at the knees as your feet hit the floor. Explain why you do this in terms of the physics concepts introduced in this chapter.

8. Momentum Which has more momentum, a supertanker tied to a dock or a falling raindrop?

9. Impulse and Momentum A 0.174-kg softball is pitched horizontally at 26.0 m/s. The ball moves in the opposite direction at 38.0 m/s after it is hit by the bat.

a. Draw arrows showing the ball's momentum before and after the bat hits it.

b. What is the change in momentum of the ball?

c. What is the impulse delivered by the bat?

d. If the bat and softball are in contact for 0.80 ms, what is the average force that the bat exerts on the ball?

10. Momentum The speed of a basketball as it is dribbled is the same when the ball is going toward the floor as it is when the ball rises from the floor. Is the basketball's change in momentum equal to zero when it hits the floor? If not, in which direction is the change in momentum? Draw the basketball's momentum vectors before and after it hits the floor.

11. Angular Momentum An ice-skater spins with his arms outstretched. When he pulls his arms in and raises them above his head, he spins much faster than before. Did a torque act on the ice-skater? If not, how could his angular velocity have increased?

12. Critical Thinking An archer shoots arrows at a target. Some of the arrows stick in the target, while others bounce off. Assuming that the masses of the arrows and the velocities of the arrows are the same, which arrows produce a bigger impulse on the target? *Hint: Draw a diagram to show the momentum of the arrows before and after hitting the target for the two instances.*

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CONTENTS

9.2 Conservation of Momentum

▶ **Objectives**

- **Relate** Newton's third law to conservation of momentum.
- **Recognize** the conditions under which momentum is conserved.
- **Solve** conservation of momentum problems.

▶ **Vocabulary**

closed system
isolated system
law of conservation of momentum
law of conservation of angular momentum

In the first section of this chapter, you learned how a force applied during a time interval changes the momentum of a baseball. In the discussion of Newton's third law of motion, you learned that forces are the result of interactions between two objects. The force of a bat on a ball is accompanied by an equal and opposite force of the ball on the bat. Does the momentum of the bat, therefore, also change?

Two-Particle Collisions

The bat, the hand and arm of the batter, and the ground on which the batter is standing are all objects that interact when a batter hits the ball. Thus, the bat cannot be considered a single object. In contrast to this complex system, examine for a moment the much simpler system shown in **Figure 9-6,** the collision of two balls.

During the collision of the two balls, each one briefly exerts a force on the other. Despite the differences in sizes and velocities of the balls, the forces that they exert on each other are equal and opposite, according to Newton's third law of motion. These forces are represented by the following equation: $\boldsymbol{F}_{\text{D on C}} = -\boldsymbol{F}_{\text{C on D}}$

How do the impulses imparted by both balls compare? Because the time intervals over which the forces are exerted are the same, the impulses must be equal in magnitude but opposite in direction. How did the momenta of the balls change as a result of the collision?

According to the impulse-momentum theorem, the change in momentum is equal to the impulse. Compare the changes in the momenta of the two balls.

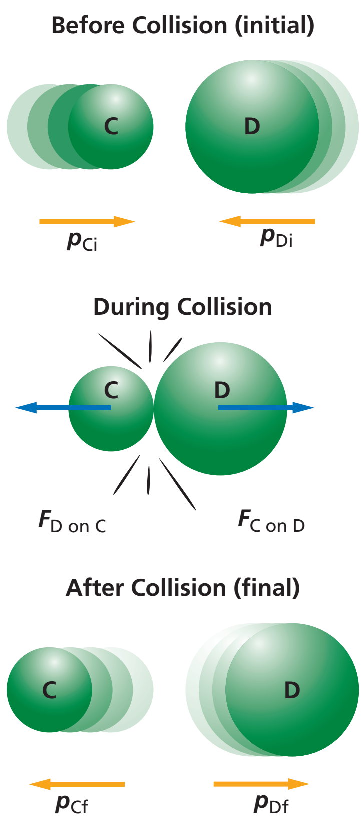
$$\text{For ball C: } \boldsymbol{p}_{\text{Cf}} - \boldsymbol{p}_{\text{Ci}} = \boldsymbol{F}_{\text{D on C}}\Delta t$$

$$\text{For ball D: } \boldsymbol{p}_{\text{Df}} - \boldsymbol{p}_{\text{Di}} = \boldsymbol{F}_{\text{C on D}}\Delta t$$

Because the time interval over which the forces were exerted is the same, the impulses are equal in magnitude, but opposite in direction. According to Newton's third law of motion, $-\boldsymbol{F}_{\text{C on D}} = \boldsymbol{F}_{\text{D on C}}$. Thus,

$$\boldsymbol{p}_{\text{Cf}} - \boldsymbol{p}_{\text{Ci}} = -(\boldsymbol{p}_{\text{Df}} - \boldsymbol{p}_{\text{Di}}), \text{ or } \boldsymbol{p}_{\text{Cf}} + \boldsymbol{p}_{\text{Df}} = \boldsymbol{p}_{\text{Ci}} + \boldsymbol{p}_{\text{Di}}.$$

This equation states that the sum of the momenta of the balls is the same before and after the collision. That is, the momentum gained by ball D is equal to the momentum lost by ball C. If the system is defined as the two balls, the momentum of the system is constant, and therefore, momentum is conserved for the system.



■ **Figure 9-6** When two balls collide, they exert forces on each other that change their momenta.

Momentum in a Closed, Isolated System

Under what conditions is the momentum of the system of two balls conserved? The first and most obvious condition is that no balls are lost and no balls are gained. Such a system, which does not gain or lose mass, is said to be a **closed system.** The second condition required to conserve the momentum of a system is that the forces involved are internal forces; that is, there are no forces acting on the system by objects outside of it.

When the net external force on a closed system is zero, the system is described as an **isolated system.** No system on Earth can be said to be absolutely isolated, however, because there will always be some interactions between a system and its surroundings. Often, these interactions are small enough to be ignored when solving physics problems.

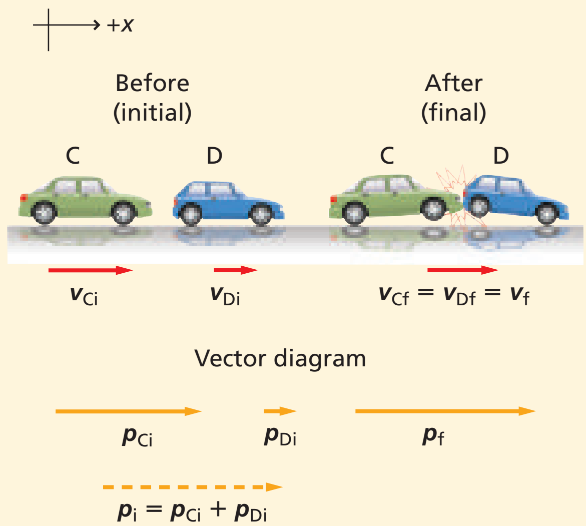
Systems can contain any number of objects, and the objects can stick together or come apart in a collision. Under these conditions, the **law of conservation of momentum** states that the momentum of any closed, isolated system does not change. This law will enable you to make a connection between conditions, before and after an interaction, without knowing any of the details of the interaction.

EXAMPLE Problem 2

Speed **A 1875-kg car going 23 m/s rear-ends a 1025-kg compact car going 17 m/s on ice in the same direction. The two cars stick together. How fast do the two cars move together immediately after the collision?**

1 Analyze and Sketch the Problem

- Define the system.
- Establish a coordinate system.
- Sketch the situation showing the "before" and "after" states.
- Draw a vector diagram for the momentum.



Known:	Unknown:
$m_C = 1875$ kg	$\mathbf{v}_f = ?$
$\mathbf{v}_{Ci} = +23$ m/s	
$m_D = 1025$ kg	
$\mathbf{v}_{Di} = +17$ m/s	

2 Solve for the Unknown

Momentum is conserved because the ice makes the total external force on the cars nearly zero.

$$p_i = p_f$$

$$p_{Ci} + p_{Di} = p_{Cf} + p_{Df}$$

$$m_C v_{Ci} + m_D v_{Di} = m_C v_{Cf} + m_D v_{Df}$$

Because the two cars stick together, their velocities after the collision, denoted as v_f, are equal.

$$v_{Cf} = v_{Df} = v_f$$

$$m_C v_{Ci} + m_D v_{Di} = (m_C + m_D) v_f$$

Solve for v_f.

$$v_f = \frac{(m_C v_{Ci} + m_D v_{Di})}{(m_C + m_D)}$$

$$= \frac{(1875\ \text{kg})(+23\ \text{m/s}) + (1025\ \text{kg})(+17\ \text{m/s})}{(1875\ \text{kg} + 1025\ \text{kg})}$$

Substitute $m_C = 1875$ kg, $v_{Ci} = +23$ m/s, $m_D = 1025$ kg, $v_{Di} = +17$ m/s

$$= +21\ \text{m/s}$$

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3 Evaluate the Answer

- **Are the units correct?** Velocity is measured in m/s.
- **Does the direction make sense?** v_i and v_f are in the positive direction; therefore, v_f should be positive.
- **Is the magnitude realistic?** The magnitude of v_f is between the initial speeds of the two cars, but closer to the speed of the more massive one, so it is reasonable.

PRACTICE Problems

Additional Problems, Appendix B

13. Two freight cars, each with a mass of 3.0×10^5 kg, collide and stick together. One was initially moving at 2.2 m/s, and the other was at rest. What is their final speed?

14. A 0.105-kg hockey puck moving at 24 m/s is caught and held by a 75-kg goalie at rest. With what speed does the goalie slide on the ice?

15. A 35.0-g bullet strikes a 5.0-kg stationary piece of lumber and embeds itself in the wood. The piece of lumber and bullet fly off together at 8.6 m/s. What was the original speed of the bullet?

16. A 35.0-g bullet moving at 475 m/s strikes a 2.5-kg bag of flour that is on ice, at rest. The bullet passes through the bag, as shown in **Figure 9-7,** and exits it at 275 m/s. How fast is the bag moving when the bullet exits?

17. The bullet in the previous problem strikes a 2.5-kg steel ball that is at rest. The bullet bounces backward after its collision at a speed of 5.0 m/s. How fast is the ball moving when the bullet bounces backward?

18. A 0.50-kg ball that is traveling at 6.0 m/s collides head-on with a 1.00-kg ball moving in the opposite direction at a speed of 12.0 m/s. The 0.50-kg ball bounces backward at 14 m/s after the collision. Find the speed of the second ball after the collision.

Figure 9-7

Recoil

It is very important to define a system carefully. The momentum of a baseball changes when the external force of a bat is exerted on it. The baseball, therefore, is not an isolated system. On the other hand, the total momentum of two colliding balls within an isolated system does not change because all forces are between the objects within the system.

Can you find the final velocities of the two in-line skaters in **Figure 9-8?** Assume that they are skating on a smooth surface with no external forces. They both start at rest, one behind the other.

Figure 9-8 The internal forces exerted by Skater C, the boy, and Skater D, the girl, cannot change the total momentum of the system.

Skater C, the boy, gives skater D, the girl, a push. Now, both skaters are moving, making this situation similar to that of an explosion. Because the push was an internal force, you can use the law of conservation of momentum to find the skaters' relative velocities. The total momentum of the system was zero before the push. Therefore, it must be zero after the push.

$$\begin{array}{ccc} \textbf{Before} & & \textbf{After} \\ \boldsymbol{p}_{\mathrm{Ci}} + \boldsymbol{p}_{\mathrm{Di}} & = & \boldsymbol{p}_{\mathrm{Cf}} + \boldsymbol{p}_{\mathrm{Df}} \\ 0 & = & \boldsymbol{p}_{\mathrm{Cf}} + \boldsymbol{p}_{\mathrm{Df}} \\ \boldsymbol{p}_{\mathrm{Cf}} & = & -\boldsymbol{p}_{\mathrm{Df}} \\ m_{\mathrm{C}}\boldsymbol{v}_{\mathrm{Cf}} & = & -m_{\mathrm{D}}\boldsymbol{v}_{\mathrm{Df}} \end{array}$$

The coordinate system was chosen so that the positive direction is to the left. The momenta of the skaters after the push are equal in magnitude but opposite in direction. The backward motion of skater C is an example of recoil. Are the skaters' velocities equal and opposite? The last equation shown above, for the velocity of skater C, can be rewritten as follows:

$$\boldsymbol{v}_{\mathrm{Cf}} = \left(\frac{-m_{\mathrm{D}}}{m_{\mathrm{C}}}\right)\boldsymbol{v}_{\mathrm{Df}}$$

The velocities depend on the skaters' relative masses. If skater C has a mass of 68.0 kg and skater D's mass is 45.4 kg, then the ratio of their velocities will be 68.0 : 45.4, or 1.50. The less massive skater moves at the greater velocity. Without more information about how hard skater C pushed skater D, however, you cannot find the velocity of each skater.

Propulsion in Space

How does a rocket in space change its velocity? The rocket carries both fuel and oxidizer. When the fuel and oxidizer combine in the rocket motor, the resulting hot gases leave the exhaust nozzle at high speed. If the rocket and chemicals are the system, then the system is a closed system. The forces that expel the gases are internal forces, so the system is also an isolated system. Thus, objects in space can accelerate by using the law of conservation of momentum and Newton's third law of motion.

A NASA space probe, called *Deep Space 1*, performed a flyby of an asteroid a few years ago. The most unusual of the 11 new technologies on board was an ion engine that exerts as much force as a sheet of paper resting on a person's hand. The ion engine shown in **Figure 9-9,** operates differently from a traditional rocket engine. In a traditional rocket engine, the products of the chemical reaction taking place in the combustion chamber are released at high speed from the rear. In the ion engine, however, xenon atoms are expelled at a speed of 30 km/s, producing a force of only 0.092 N. How can such a small force create a significant change in the momentum of the probe? Instead of operating for only a few minutes, as the traditional chemical rockets do, the ion engine can run continuously for days, weeks, or months. Therefore, the impulse delivered by the engine is large enough to increase the momentum of the 490-kg spacecraft until it reaches the speed needed to complete its mission.

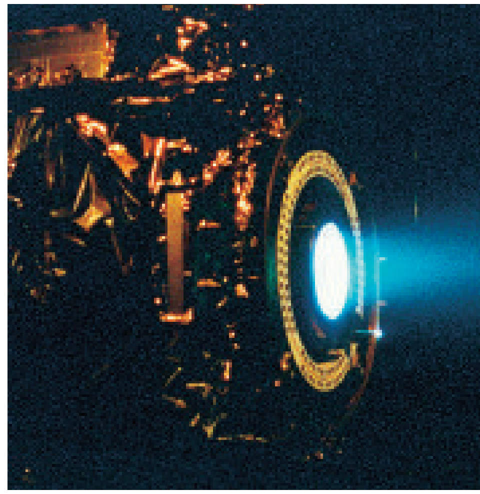

Figure 9-9 The xenon atoms in the ion engine are ionized by bombarding them with electrons. Then, the positively charged xenon ions are accelerated to high speeds.

MINI LAB

Rebound Height

An object's momentum is the product of its mass and velocity.

1. Drop a large rubber ball from about 15 cm above a table.
2. Measure and record the ball's rebound height.
3. Repeat steps 1–2 with a small rubber ball.
4. Hold the small rubber ball on top of, and in contact with, the large rubber ball.
5. Release the two rubber balls from the same height, so that they fall together.
6. Measure the rebound heights of both rubber balls.

Analyze and Conclude

7. Describe the rebound height of each rubber ball dropped by itself.
8. Compare and contrast the rebound heights from number 7 with those from number 6.
9. Explain your observations.

NASA
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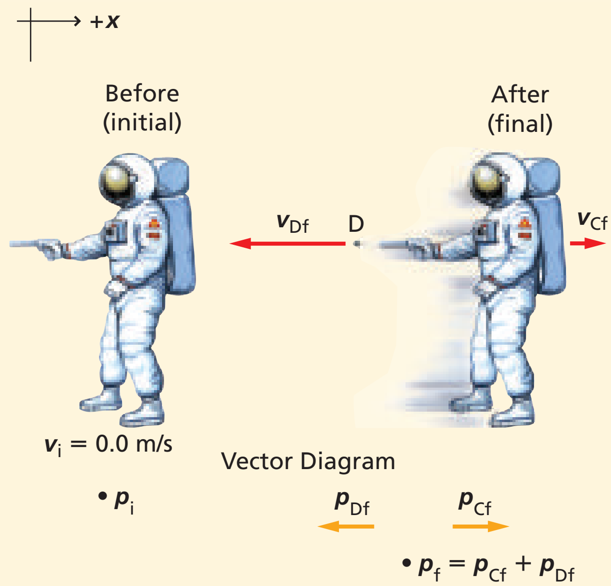
▶ EXAMPLE Problem 3

Speed An astronaut at rest in space fires a thruster pistol that expels 35 g of hot gas at 875 m/s. The combined mass of the astronaut and pistol is 84 kg. How fast and in what direction is the astronaut moving after firing the pistol?



1 Analyze and Sketch the Problem

- Define the system.
- Establish a coordinate axis.
- Sketch the "before" and "after" conditions.
- Draw a vector diagram showing momenta.

Known:	**Unknown:**
$m_C = 84$ kg	$v_{Cf} = ?$
$m_D = 0.035$ kg	
$v_{Ci} = v_{Di} = +0.0$ m/s	
$v_{Df} = -875$ m/s	

2 Solve for the Unknown

The system is the astronaut, the gun, and the chemicals that produce the gas.

$p_i = p_{Ci} + p_{Di} = +0.0$ kg·m/s — **Before the pistol is fired, all parts of the system are at rest; thus, the initial momentum is zero.**

Use the law of conservation of momentum to find p_f.

$$p_i = p_f$$
$$+0.0 \text{ kg·m/s} = p_{Cf} + p_{Df}$$
$$p_{Cf} = -p_{Df}$$

The momentum of the astronaut is equal in magnitude, but opposite in direction to the momentum of the gas leaving the pistol.

Solve for the final velocity of the astronaut, v_{Cf}.

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$$m_C v_{Cf} = -m_D v_{Df}$$
$$v_{Cf} = \left(\frac{-m_D v_{Df}}{m_C}\right)$$
$$= \frac{-(0.035 \text{ kg})(-875 \text{ m/s})}{84 \text{ kg}}$$

Substitute $m_D = 0.035$ kg, $v_{Df} = -875$ m/s, $m_C = 84$ kg

$$= +0.36 \text{ m/s}$$

3 Evaluate the Answer

- **Are the units correct?** The velocity is measured in m/s.
- **Does the direction make sense?** The velocity of the astronaut is in the opposite direction to that of the expelled gas.
- **Is the magnitude realistic?** The astronaut's mass is much larger than that of the gas, so the velocity of the astronaut is much less than that of the expelled gas.

▶ PRACTICE Problems

Additional Problems, Appendix B

19. A 4.00-kg model rocket is launched, expelling 50.0 g of burned fuel from its exhaust at a speed of 625 m/s. What is the velocity of the rocket after the fuel has burned? *Hint: Ignore the external forces of gravity and air resistance.*

20. A thread holds a 1.5-kg cart and a 4.5-kg cart together. After the thread is burned, a compressed spring pushes the carts apart, giving the 1.5-kg cart a speed of 27 cm/s to the left. What is the velocity of the 4.5-kg cart?

21. Carmen and Judi dock a canoe. 80.0-kg Carmen moves forward at 4.0 m/s as she leaves the canoe. At what speed and in what direction do the canoe and Judi move if their combined mass is 115 kg?

Two-Dimensional Collisions

Up until now, you have looked at momentum in only one dimension. The law of conservation of momentum holds for all closed systems with no external forces. It is valid regardless of the directions of the particles before or after they interact. But what happens in two or three dimensions? **Figure 9-10** shows the result of billiard ball C striking stationary billiard ball D. Consider the two billiard balls to be the system. The original momentum of the moving ball is $\boldsymbol{p}_{\mathrm{Ci}}$ and the momentum of the stationary ball is zero. Therefore, the momentum of the system before the collision is equal to $\boldsymbol{p}_{\mathrm{Ci}}$.

After the collision, both billiard balls are moving and have momenta. As long as the friction with the tabletop can be ignored, the system is closed and isolated. Thus, the law of conservation of momentum can be used. The initial momentum equals the vector sum of the final momenta, so $\boldsymbol{p}_{\mathrm{Ci}} = \boldsymbol{p}_{\mathrm{Cf}} + \boldsymbol{p}_{\mathrm{Df}}$.

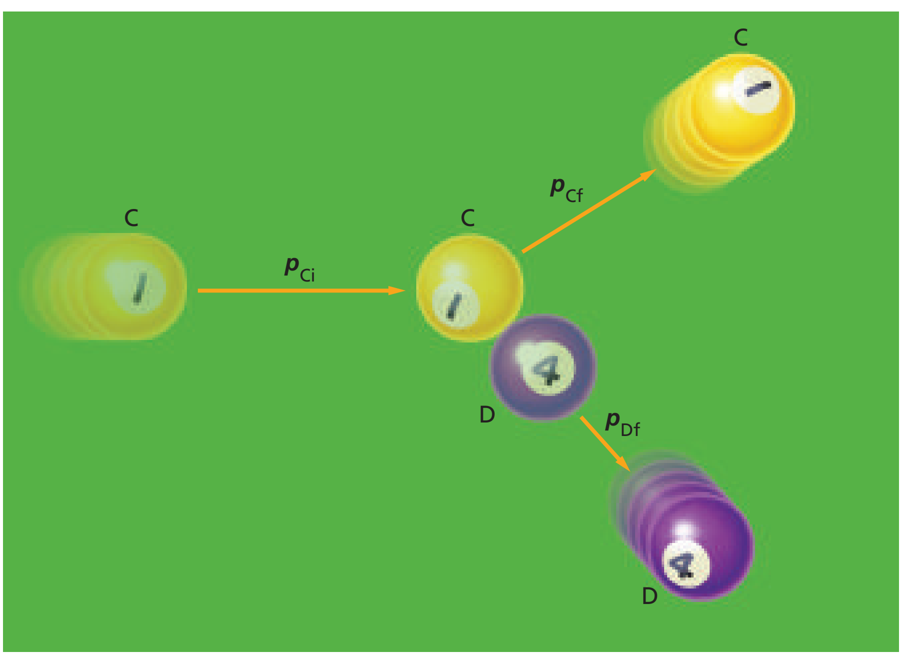
The equality of the momenta before and after the collision also means that the sum of the components of the vectors before and after the collision must be equal. Suppose the x-axis is defined to be in the direction of the initial momentum, then the y-component of the initial momentum is equal to zero. Therefore, the sum of the final y-components also must be zero:

$$\boldsymbol{p}_{\mathrm{Cf},\,y} + \boldsymbol{p}_{\mathrm{Df},\,y} = 0$$

The y-components are equal in magnitude but are in the opposite direction and, thus, have opposite signs. The sum of the horizontal components also is equal:

$$\boldsymbol{p}_{\mathrm{Ci}} = \boldsymbol{p}_{\mathrm{Cf},\,x} + \boldsymbol{p}_{\mathrm{Df},\,x}$$

■ **Figure 9-10** The law of conservation of momentum holds for all isolated, closed systems, regardless of the directions of objects before and after a collision.



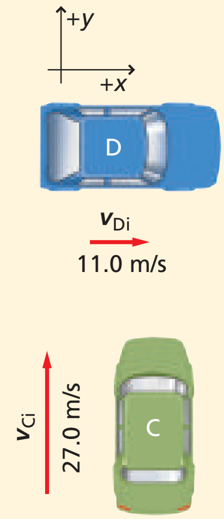
EXAMPLE Problem 4

Speed A 1325-kg car, C, moving north at 27.0 m/s, collides with a 2165-kg car, D, moving east at 11.0 m/s. The two cars are stuck together. In what direction and with what speed do they move after the collision?

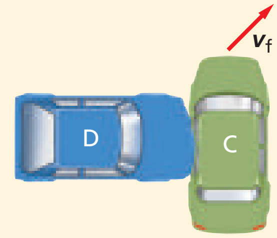
1 Analyze and Sketch the Problem

- Define the system.
- Sketch the "before" and "after" states.
- Establish the coordinate axis with the *y*-axis north and the *x*-axis east.
- Draw a momentum-vector diagram.

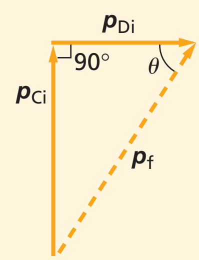
Before (initial)



After (final)



Vector Diagram



Known:	**Unknown:**
$m_C = 1325$ kg	$v_{f,\,x} = ?$
$m_D = 2165$ kg	$v_{f,\,y} = ?$
$v_{Ci,\,y} = 27.0$ m/s	$\theta = ?$
$v_{Di,\,x} = 11.0$ m/s	

2 Solve for the Unknown

Determine the initial momenta of the cars and the momentum of the system.

$$p_{Ci} = m_C v_{Ci,\,y}$$
$$= (1325\text{ kg})(27.0\text{ m/s}) \quad \textbf{Substitute } m_C = 1325\text{ kg}, v_{Ci,\,y} = 27.0\text{ m/s}$$
$$= 3.58\times10^4\text{ kg·m/s (north)}$$

$$p_{Di} = m_D v_{Di,\,x}$$
$$= (2165\text{ kg})(11.0\text{ m/s}) \quad \textbf{Substitute } m_D = 2165\text{ kg}, v_{Di,\,x} = 11.0\text{ m/s}$$
$$= 2.38\times10^4\text{ kg·m/s (east)}$$

Use the law of conservation of momentum to find p_f.

$$p_{f,\,x} = p_{i,\,x} = 2.38\times10^4\text{ kg·m/s} \quad \textbf{Substitute } p_{i,\,x} = p_{Di} = 2.38\times10^4\text{ kg·m/s}$$
$$p_{f,\,y} = p_{i,\,y} = 3.58\times10^4\text{ kg·m/s} \quad \textbf{Substitute } p_{i,\,y} = p_{Ci} = 3.58\times10^4\text{ kg·m/s}$$

Use the diagram to set up equations for $p_{f,\,x}$ and $p_{f,\,y}$.

$$p_f = \sqrt{(p_{f,\,x})^2 + (p_{f,\,y})^2}$$
$$= \sqrt{(2.38\times10^4\text{ kg·m/s})^2 + (3.58\times10^4\text{ kg·m/s})^2} \quad \textbf{Substitute } p_{f,\,x} = 2.38\times10^4\text{ kg·m/s}, p_{f,\,y} = 3.58\times10^4\text{ kg·m/s}$$
$$= 4.30\times10^4\text{ kg·m/s}$$

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page 856

Solve for θ.

$$\theta = \tan^{-1}\left(\frac{p_{f,\,y}}{p_{f,\,x}}\right)$$
$$= \tan^{-1}\left(\frac{3.58\times10^4\text{ kg·m/s}}{2.38\times10^4\text{ kg·m/s}}\right) \quad \textbf{Substitute } p_{f,\,y} = 3.58\times10^4\text{ kg·m/s}, p_{f,\,x} = 2.38\times10^4\text{ kg·m/s}$$
$$= 56.4°$$

Determine the final speed.

$$v_f = \frac{p_f}{(m_C + m_D)}$$
$$= \frac{4.30\times10^4\text{ kg·m/s}}{(1325\text{ kg} + 2165\text{ kg})} \quad \textbf{Substitute } p_f = 4.30\times10^4\text{ kg·m/s}, m_C = 1325\text{ kg}, m_D = 2165\text{ kg}$$
$$= 12.3\text{ m/s}$$

3 Evaluate the Answer

- **Are the units correct?** The correct unit for speed is m/s.
- **Do the signs make sense?** Answers are both positive and at the appropriate angles.
- **Is the magnitude realistic?** The cars stick together, so v_f must be smaller than v_{Ci}.

▶ PRACTICE Problems

Additional Problems, Appendix B

22. A 925-kg car moving north at 20.1 m/s collides with a 1865-kg car moving west at 13.4 m/s. The two cars are stuck together. In what direction and at what speed do they move after the collision?

23. A 1383-kg car moving south at 11.2 m/s is struck by a 1732-kg car moving east at 31.3 m/s. The cars are stuck together. How fast and in what direction do they move immediately after the collision?

24. A stationary billiard ball, with a mass of 0.17 kg, is struck by an identical ball moving at 4.0 m/s. After the collision, the second ball moves 60.0° to the left of its original direction. The stationary ball moves 30.0° to the right of the moving ball's original direction. What is the velocity of each ball after the collision?

25. A 1345-kg car moving east at 15.7 m/s is struck by a 1923-kg car moving north. They are stuck together and move with an initial velocity of 14.5 m/s at $\theta = 63.5°$. Was the north-moving car exceeding the 20.1 m/s speed limit?

Conservation of Angular Momentum

Like linear momentum, angular momentum can be conserved. The **law of conservation of angular momentum** states that if no net external torque acts on an object, then its angular momentum does not change. This is represented by the following equation.

Law of Conservation of Angular Momentum $L_1 = L_2$

An object's initial angular momentum is equal to its final angular momentum.

For example, Earth spins on its axis with no external torques. Its angular momentum is constant. Thus, Earth's angular momentum is conserved. As a result, the length of a day does not change. A spinning ice-skater also demonstrates conservation of angular momentum. **Figure 9-11a** shows an ice-skater spinning with his arms extended. When he pulls in his arms, as shown in **Figure 9-11b,** he begins spinning faster. Without an external torque, his angular momentum does not change; that is, $L = I\omega$ is constant. Thus, the ice-skater's increased angular velocity must be accompanied by a decreased moment of inertia. By pulling his arms close to his body, the ice-skater brings more mass closer to the axis of rotation, thereby decreasing the radius of rotation and decreasing his moment of inertia. You can calculate changes in angular velocity using the law of conservation of angular momentum.

$$L_i = L_f$$

$$\text{thus, } I_i\omega_i = I_f\omega_f$$

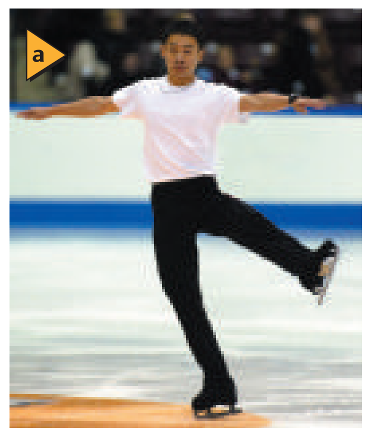
$$\frac{\omega_f}{\omega_i} = \frac{I_i}{I_f}$$

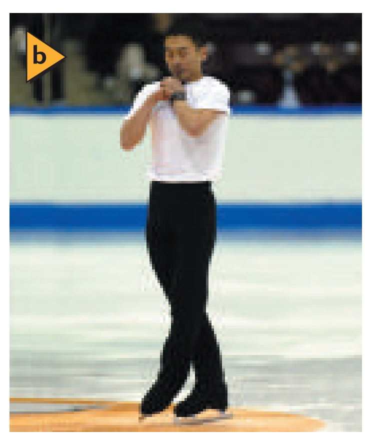
Because frequency is $f = \omega/2\pi$, the above equation can be rewritten as follows:

$$\frac{2\pi(f_f)}{2\pi(f_i)} = \frac{I_i}{I_f}$$

$$\text{thus, } \frac{f_f}{f_i} = \frac{I_i}{I_f}$$

■ **Figure 9-11** When the ice-skater's arms are extended, the moment of inertia increases and his angular velocity decreases **(a).** When his arms are closer to his body the moment of inertia decreases and results in an increased angular velocity **(b).**





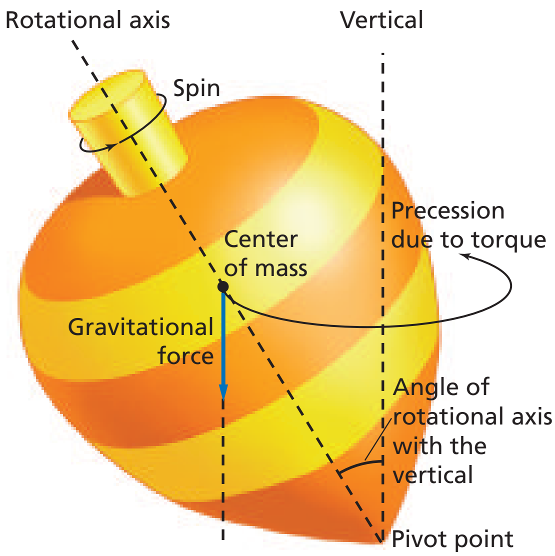


Figure 9-12 The upper end of the top precesses due to the torque acting on the top.

Notice that because f, ω, and I appear as ratios in these equations, any units may be used, as long as the same unit is used for both values of the quantity.

If a torque-free object starts with no angular momentum, it must continue to have no angular momentum. Thus, if part of an object rotates in one direction, another part must rotate in the opposite direction. For example, if you switch on a loosely held electric drill, the drill body will rotate in the direction opposite to the rotation of the motor and bit.

Consider a ball thrown at a weather vane. The ball, moving in a straight line, can start the vane rotating. Consider the ball and vane to be a system. With no external torques, angular momentum is conserved. The vane spins faster if the ball has a large mass, m, a large velocity, v, and hits at right angles as far as possible from the pivot of the vane. The angular momentum of a moving object, such as the ball, is given by $L = mvr$, where r is the perpendicular distance from the axis of rotation.

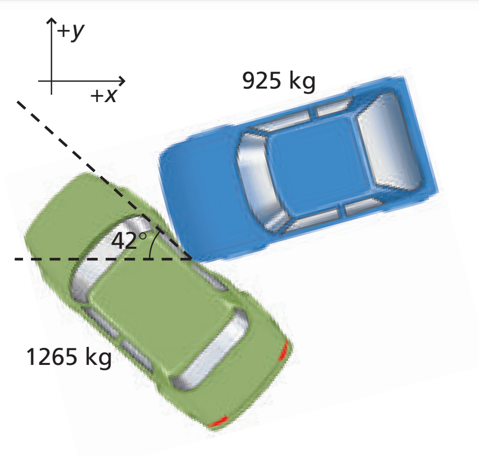
Tops and Gyroscopes

Because of the conservation of angular momentum, the direction of rotation of a spinning object can be changed only by applying a torque. If you played with a top as a child, you may have spun it by pulling the string wrapped around its axle. When a top is vertical, there is no torque on it, and the direction of its rotation does not change. If the top is tipped, as shown in **Figure 9-12,** a torque tries to rotate it downward. Rather than tipping over, however, the upper end of the top revolves, or precesses slowly about the vertical axis. Because Earth is not a perfect sphere, the Sun exerts a torque on it, causing it to precess. It takes about 26,000 years for Earth's rotational axis to go through one cycle of precession.

CHALLENGE PROBLEM

Your friend was driving her 1265-kg car north on Oak Street when she was hit by a 925-kg compact car going west on Maple Street. The cars stuck together and slid 23.1 m at 42° north of west. The speed limit on both streets is 22 m/s (50 mph). Assume that momentum was conserved during the collision and that acceleration was constant during the skid. The coefficient of kinetic friction between the tires and the pavement is 0.65.

1. Your friend claims that she wasn't speeding, but that the driver of other car was. How fast was your friend driving before the crash?
2. How fast was the other car moving before the crash? Can you support your friend's case in court?

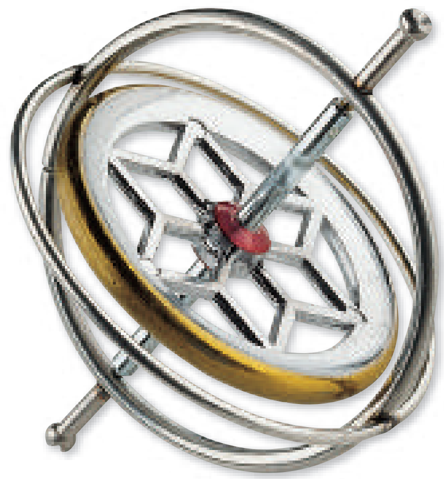


file photo

A gyroscope, such as the one shown in **Figure 9-13,** is a wheel or disk that spins rapidly around one axis while being free to rotate around one or two other axes. The direction of its large angular momentum can be changed only by applying an appropriate torque. Without such a torque, the direction of the axis of rotation does not change. Gyroscopes are used in airplanes, submarines, and spacecraft to keep an unchanging reference direction. Giant gyroscopes are used in cruise ships to reduce their motion in rough water. Gyroscopic compasses, unlike magnetic compasses, maintain direction even when they are not on a level surface.

A football quarterback uses the gyroscope effect to make an accurate forward pass. As he throws, he spins, or spirals the ball. If the quarterback throws the ball in the direction of its spin axis of rotation, the ball keeps its pointed end forward, thereby reducing air resistance. Thus, the ball can be thrown far and accurately. If its spin direction is slightly off, the ball wobbles. If the ball is not spun, it tumbles end over end.

The flight of a plastic disk also is stabilized by spin. A well-spun plastic disk can fly many meters through the air without wobbling. You are able to perform tricks with a yo-yo because its fast rotational speed keeps it rotating in one plane.



■ **Figure 9-13** Because the orientation of the spin axis of the gyroscope does not change even when it is moved, the gyroscope can be used to fix direction.

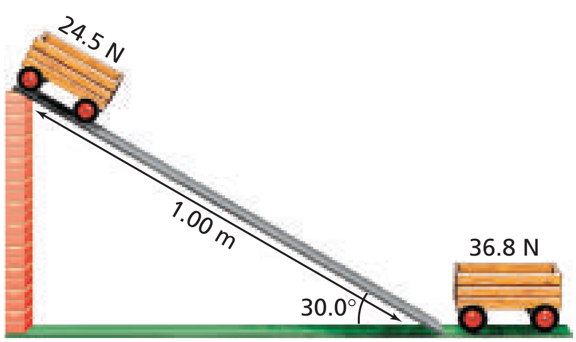
9.2 Section Review

26. Angular Momentum The outer rim of a plastic disk is thick and heavy. Besides making it easier to catch, how does this affect the rotational properties of the plastic disk?

27. Speed A cart, weighing 24.5 N, is released from rest on a 1.00-m ramp, inclined at an angle of 30.0° as shown in **Figure 9-14.** The cart rolls down the incline and strikes a second cart weighing 36.8 N.

a. Calculate the speed of the first cart at the bottom of the incline.

b. If the two carts stick together, with what initial speed will they move along?



■ **Figure 9-14**

28. Conservation of Momentum During a tennis serve, the racket of a tennis player continues forward after it hits the ball. Is momentum conserved in the collision? Explain, making sure that you define the system.

29. Momentum A pole-vaulter runs toward the launch point with horizontal momentum. Where does the vertical momentum come from as the athlete vaults over the crossbar?

30. Initial Momentum During a soccer game, two players come from opposite directions and collide when trying to head the ball. They come to rest in midair and fall to the ground. Describe their initial momenta.

31. Critical Thinking You catch a heavy ball while you are standing on a skateboard, and then you roll backward. If you were standing on the ground, however, you would be able to avoid moving while catching the ball. Explain both situations using the law of conservation of momentum. Explain which system you use in each case.

PHYSICS LAB • Internet

Alternate CBL instructions can be found on the Web site.
physicspp.com

Sticky Collisions

In this activity, one moving cart will strike a stationary cart. During the collision, the two carts will stick together. You will measure mass and velocity, both before and after the collision. You then will calculate the momentum both before and after the collision.

QUESTION

How is the momentum of a system affected by a sticky collision?

Objectives

- **Describe** how momentum is transferred during a collision.
- **Calculate** the momenta involved.
- **Interpret data** from a collision.
- **Draw conclusions** that support the law of conservation of momentum.

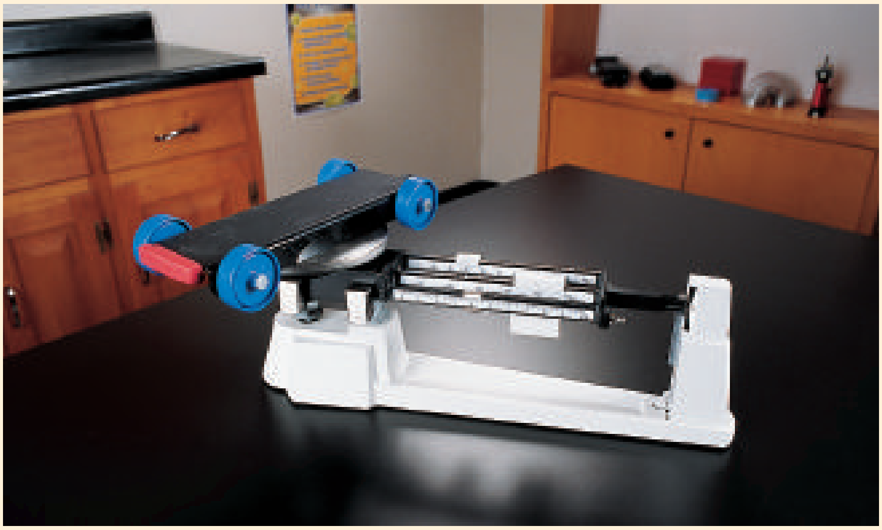
Safety Precautions

Materials

Internet access required

Procedure

1. View Chapter 9 lab video clip 1 at **physicspp.com/internet_lab** to determine the mass of the carts.
2. Record the mass of each cart.
3. Watch video clip 2: Cart 1 strikes Cart 2.
4. In the video, three frames represent 0.1 s. Record in the data table the distance Cart 1 travels in 0.1 s before the collision.
5. Observe the collision. Record in the data table the distance the Cart 1-Cart 2 system travels in 0.1 s after the collision.
6. Repeat steps 3–5 for video clip 3: Carts 1 and 3 strike Cart 2.
7. Repeat steps 3–5 for video clip 4: Carts 1, 3, and 4 strike Cart 2.
8. Repeat steps 3-5 for video clip 5: Carts 1 and 3 strike Carts 2 and 4.
9. Repeat steps 3–5 for video clip 6: Cart 1 strikes Carts 2, 3, and 4.



Horizons Companies

Data Tables

Cart	Mass (kg)
1	
2	
3	
4	

Time of Approach (s)	Distance Covered in Approach (cm)	Initial Velocity (cm/s)	Mass of Approaching Cart(s) (g)	Initial Momentum (g·cm/s)	Time of Departure (s)	Distance Covered in Departure (cm)	Final Velocity (cm/s)	Mass of Departing Cart(s) (g)	Final Momentum (g·cm/s)
0.1					0.1				
0.1					0.1				
0.1					0.1				
0.1					0.1				
0.1					0.1				

Analyze

1. Calculate the initial and final velocities for each of the cart systems.
2. Calculate the initial and final momentum for each of the cart systems.
3. **Make and Use Graphs** Make a graph showing final momentum versus initial momentum for all the video clips.

Conclude and Apply

1. What is the relationship between the initial momentum and the final momentum of the cart systems in a sticky collision?
2. In theory, what should be the slope of the line in your graph?
3. The initial and final data numbers may not be the same due to the precision of the instruments, friction, and other variables. Is the initial momentum typically greater or less than the final momentum? Explain.

Going Further

1. Describe what the velocity and momentum data might look like if the carts did not stick together, but rather, bounced off of each other?
2. Design an experiment to test the impact of friction during the collision of the cart systems. Predict how the slope of the line in your graph will change with your experiment, and then try your experiment.

Real-World Physics

1. Suppose a linebacker collides with a stationary quarterback and they become entangled. What will happen to the velocity of the linebacker-quarterback system if momentum is conserved?
2. If a car rear-ends a stationary car so that the two cars become attached, what will happen to the velocity of the first car? The second car?

Share Your Data

Interpret Data Visit **physicspp.com/internet_lab** to post your findings from the experiment testing the impact of friction during the collisions of the cart systems. Examine the data and graph of the final momentum versus initial momentum on the Web site. Notice how close to or far off the slope is from 1.00.

Physics Online

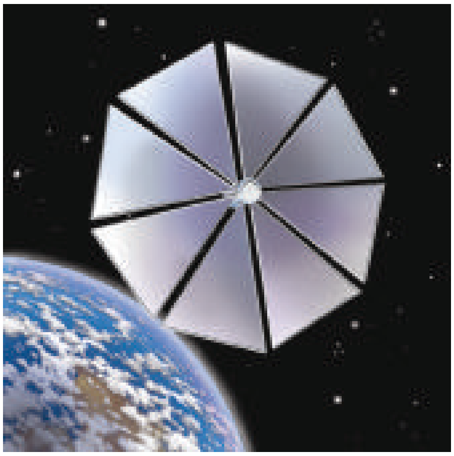
To find out more about momentum, visit the Web site: **physicspp.com**

Solar Sailing

Nearly 400 years ago, Johannes Kepler observed that comet tails appeared to be blown by a solar breeze. He suggested that ships would be able to travel in space with sails designed to catch this breeze. Thus, the idea for solar sails was born.

How Does a Solar Sail Work? A solar sail is a spacecraft without an engine. A solar sail works like a giant fabric mirror that is free to move. Solar sails usually are made of 5-micron-thick aluminized polyester film or polyimide film with a 100-nm-thick aluminum layer deposited on one side to form the reflective surface.

Reflected sunlight, rather than rocket fuel, provides the force. Sunlight is made up of individual particles called photons. Photons have momentum, and when a photon bounces off a solar sail, it transfers its momentum to the sail, which propels the spacecraft along.



This artist's rendering shows *Cosmos 1*, the first solar sail scheduled for launch in the near future.

The force of impacting photons is small in comparison to the force rocket fuel can supply. So, small sails experience only a small amount of force from sunlight, while larger sails experience a greater force. Thus, solar sails may be a kilometer or so across.

What speeds can a solar sail achieve? This depends on the momentum transferred to the sail by photons, as well as the sail's mass. To travel quickly through the solar system, a sail and the spacecraft should be lightweight.

Photons supplied by the Sun are constant. They impact the sail every second of every hour of every day during a space flight. The Sun's continuous supply of photons over time allows the sail to build up huge velocities and enables the spacecraft to travel great distances within a convenient time frame. Rockets require enormous amounts of fuel to move large masses, but solar sails only require photons from the Sun. Thus, solar sails may be a superior way to move large masses over great distances in outer space.

Future Journeys The *Cosmos 1*, the first solar-sail prototype, is scheduled for a launch in the very near future. The *Cosmos-1* mission is an international, privately funded venture. The spacecraft looks like a flower with eight huge, solar-sail petals. Being the first solar sail, goals are modest. The mission will be considered successful if the *Cosmos 1* operates for just a few days, accelerating under sunlight pressure.

Solar sails are important, not only for travel, but also for creating new types of space and Earth weather monitoring stations. These stations would be able to provide greater coverage of Earth and more advanced warning of solar storms that cause problems to communication and electric power grids. It is hoped that in the next few decades, solar sails will be used as interplanetary shuttles because of their ability to travel great distances in convenient time frames. Vast distances could someday be traversed by vehicles that do not consume any fuel.

Going Further

1. **Research** how solar sails can help provide advanced warning of solar storms.
2. **Critical Thinking** A certain solar-sail model is predicted to take more time to reach Mars than a rocket-propelled spacecraft would, but less time to go to Pluto than a rocket-propelled spacecraft would. Explain why this is so.

Chapter 9

Study Guide

9.1 Impulse and Momentum

Vocabulary

- impulse (p. 230)
- momentum (p. 230)
- impulse-momentum theorem (p. 230)
- angular momentum (p. 233)
- angular impulse-angular momentum theorem (p. 234)

Key Concepts

- When doing a momentum problem, first examine the system before and after the event.
- The momentum of an object is the product of its mass and velocity and is a vector quantity.

$$\boldsymbol{p} = m\boldsymbol{v}$$

- The impulse on an object is the average net force exerted on the object multiplied by the time interval over which the force acts.

$$\text{Impluse} = \boldsymbol{F}\Delta t$$

- The impulse on an object is equal to the change in momentum of the object.

$$\boldsymbol{F}\Delta t = \boldsymbol{p}_{\mathrm{f}} - \boldsymbol{p}_{\mathrm{i}}$$

- The angular momentum of a rotating object is the product of its moment of inertia and its angular velocity.

$$L = I\omega$$

- The angular impulse-angular momentum theorem states that the angular impulse on an object is equal to the change in the object's angular momentum.

$$\tau\Delta t = L_{\mathrm{f}} - L_{\mathrm{i}}$$

9.2 Conservation of Momentum

Vocabulary

- closed system (p. 236)
- isolated system (p. 237)
- law of conservation of momentum (p. 237)
- law of conservation of angular momentum (p. 243)

Key Concepts

- According to Newton's third law of motion and the law of conservation of momentum, the forces exerted by colliding objects on each other are equal in magnitude and opposite in direction.
- Momentum is conserved in a closed, isolated system.

$$\boldsymbol{p}_{\mathrm{f}} = \boldsymbol{p}_{\mathrm{i}}$$

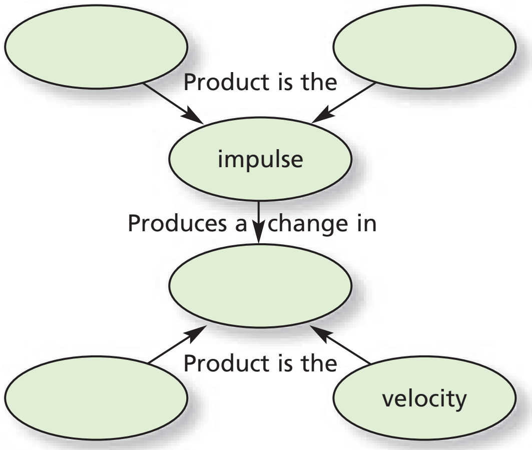
- The law of conservation of momentum can be used to explain the propulsion of rockets.
- Vector analysis is used to solve momentum-conservation problems in two dimensions.
- The law of conservation of angular momentum states that if there are no external torques acting on a system, then the angular momentum is conserved.

$$L_{\mathrm{f}} = L_{\mathrm{i}}$$

- Because angular momentum is conserved, the direction of rotation of a spinning object can be changed only by applying a torque.

Concept Mapping

32. Complete the following concept map using the following terms: *mass, momentum, average force, time over which the force is exerted.*



Mastering Concepts

33. Can a bullet have the same momentum as a truck? Explain. (9.1)

34. A pitcher throws a curve ball to the catcher. Assume that the speed of the ball doesn't change in flight. (9.1)

a. Which player exerts the larger impulse on the ball?

b. Which player exerts the larger force on the ball?

35. Newton's second law of motion states that if no net force is exerted on a system, no acceleration is possible. Does it follow that no change in momentum can occur? (9.1)

36. Why are cars made with bumpers that can be pushed in during a crash? (9.1)

37. An ice-skater is doing a spin. (9.1)

a. How can the skater's angular momentum be changed?

b. How can the skater's angular velocity be changed without changing the angular momentum?

38. What is meant by "an isolated system?" (9.2)

39. A spacecraft in outer space increases its velocity by firing its rockets. How can hot gases escaping from its rocket engine change the velocity of the craft when there is nothing in space for the gases to push against? (9.2)

40. A cue ball travels across a pool table and collides with the stationary eight ball. The two balls have equal masses. After the collision, the cue ball is at rest. What must be true regarding the speed of the eight ball? (9.2)

41. Consider a ball falling toward Earth. (9.2)

a. Why is the momentum of the ball not conserved?

b. In what system that includes the falling ball is the momentum conserved?

42. A falling basketball hits the floor. Just before it hits, the momentum is in the downward direction, and after it hits the floor, the momentum is in the upward direction. (9.2)

a. Why isn't the momentum of the basketball conserved even though the bounce is a collision?

b. In what system is the momentum conserved?

43. Only an external force can change the momentum of a system. Explain how the internal force of a car's brakes brings the car to a stop. (9.2)

44. Children's playgrounds often have circular-motion rides. How could a child change the angular momentum of such a ride as it is turning? (9.2)

Applying Concepts

45. Explain the concept of impulse using physical ideas rather than mathematics.

46. Is it possible for an object to obtain a larger impulse from a smaller force than it does from a larger force? Explain.

47. **Foul Ball** You are sitting at a baseball game when a foul ball comes in your direction. You prepare to catch it bare-handed. To catch it safely, should you move your hands toward the ball, hold them still, or move them in the same direction as the moving ball? Explain.

48. A 0.11-g bullet leaves a pistol at 323 m/s, while a similar bullet leaves a rifle at 396 m/s. Explain the difference in exit speeds of the two bullets, assuming that the forces exerted on the bullets by the expanding gases have the same magnitude.

49. An object initially at rest experiences the impulses described by the graph in **Figure 9-15.** Describe the object's motion after impulses A, B, and C.

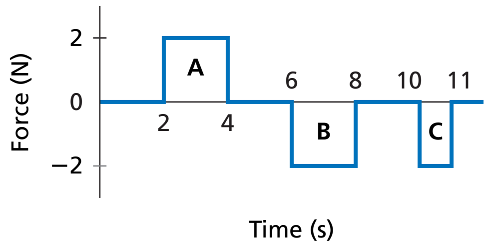


Figure 9-15

 For more problems, go to Additional Problems, Appendix B.

CONTENTS

50. During a space walk, the tether connecting an astronaut to the spaceship breaks. Using a gas pistol, the astronaut manages to get back to the ship. Use the language of the impulse-momentum theorem and a diagram to explain why this method was effective.

51. Tennis Ball As a tennis ball bounces off a wall, its momentum is reversed. Explain this action in terms of the law of conservation of momentum. Define the system and draw a diagram as a part of your explanation.

52. Imagine that you command spaceship *Zeldon*, which is moving through interplanetary space at high speed. How could you slow your ship by applying the law of conservation of momentum?

53. Two trucks that appear to be identical collide on an icy road. One was originally at rest. The trucks are stuck together and move at more than half the original speed of the moving truck. What can you conclude about the contents of the two trucks?

54. Explain, in terms of impulse and momentum, why it is advisable to place the butt of a rifle against your shoulder when first learning to shoot.

55. Bullets Two bullets of equal mass are shot at equal speeds at blocks of wood on a smooth ice rink. One bullet, made of rubber, bounces off of the wood. The other bullet, made of aluminum, burrows into the wood. In which case does the block of wood move faster? Explain.

Mastering Problems

9.1 Impulse and Momentum

56. Golf Rocío strikes a 0.058-kg golf ball with a force of 272 N and gives it a velocity of 62.0 m/s. How long was Rocío's club in contact with the ball?

57. A 0.145-kg baseball is pitched at 42 m/s. The batter hits it horizontally to the pitcher at 58 m/s.
a. Find the change in momentum of the ball.
b. If the ball and bat are in contact for 4.6×10^{-4} s, what is the average force during contact?

58. Bowling A force of 186 N acts on a 7.3-kg bowling ball for 0.40 s. What is the bowling ball's change in momentum? What is its change in velocity?

59. A 5500-kg freight truck accelerates from 4.2 m/s to 7.8 m/s in 15.0 s by the application of a constant force.
a. What change in momentum occurs?
b. How large of a force is exerted?

60. In a ballistics test at the police department, Officer Rios fires a 6.0-g bullet at 350 m/s into a container that stops it in 1.8 ms. What is the average force that stops the bullet?

61. Volleyball A 0.24-kg volleyball approaches Tina with a velocity of 3.8 m/s. Tina bumps the ball, giving it a speed of 2.4 m/s but in the opposite direction. What average force did she apply if the interaction time between her hands and the ball was 0.025 s?

62. Hockey A hockey player makes a slap shot, exerting a constant force of 30.0 N on the hockey puck for 0.16 s. What is the magnitude of the impulse given to the puck?

63. Skateboarding Your brother's mass is 35.6 kg, and he has a 1.3-kg skateboard. What is the combined momentum of your brother and his skateboard if they are moving at 9.50 m/s?

64. A hockey puck has a mass of 0.115 kg and is at rest. A hockey player makes a shot, exerting a constant force of 30.0 N on the puck for 0.16 s. With what speed does it head toward the goal?

65. Before a collision, a 25-kg object was moving at +12 m/s. Find the impulse that acted on the object if, after the collision, it moved at the following velocities.
a. +8.0 m/s
b. −8.0 m/s

66. A 0.150-kg ball, moving in the positive direction at 12 m/s, is acted on by the impulse shown in the graph in **Figure 9-16.** What is the ball's speed at 4.0 s?

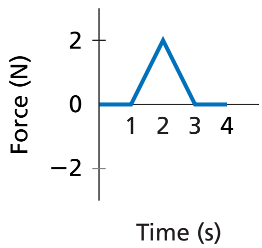


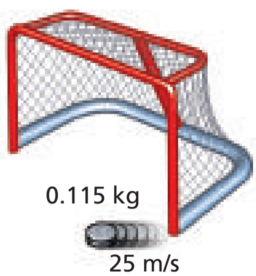
Figure 9-16

67. Baseball A 0.145-kg baseball is moving at 35 m/s when it is caught by a player.
a. Find the change in momentum of the ball.
b. If the ball is caught with the mitt held in a stationary position so that the ball stops in 0.050 s, what is the average force exerted on the ball?
c. If, instead, the mitt is moving backward so that the ball takes 0.500 s to stop, what is the average force exerted by the mitt on the ball?

68. **Hockey** A hockey puck has a mass of 0.115 kg and strikes the pole of the net at 37 m/s. It bounces off in the opposite direction at 25 m/s, as shown in **Figure 9-17.**

a. What is the impulse on the puck?

b. If the collision takes 5.0×10^{-4} s, what is the average force on the puck?



■ **Figure 9-17**

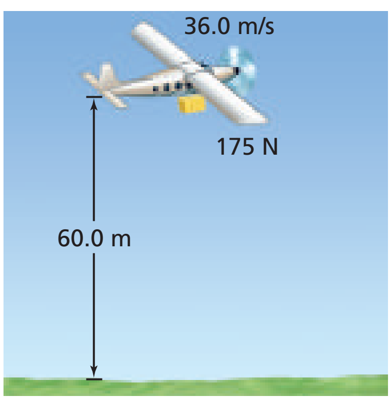
69. A nitrogen molecule with a mass of 4.7×10^{-26} kg, moving at 550 m/s, strikes the wall of a container and bounces back at the same speed.

a. What is the impulse the molecule delivers to the wall?

b. If there are 1.5×10^{23} collisions each second, what is the average force on the wall?

70. **Rockets** Small rockets are used to make tiny adjustments in the speeds of satellites. One such rocket has a thrust of 35 N. If it is fired to change the velocity of a 72,000-kg spacecraft by 63 cm/s, how long should it be fired?

71. An animal rescue plane flying due east at 36.0 m/s drops a bale of hay from an altitude of 60.0 m, as shown in **Figure 9-18.** If the bale of hay weighs 175 N, what is the momentum of the bale the moment before it strikes the ground? Give both magnitude and direction.



■ **Figure 9-18**

72. **Accident** A car moving at 10.0 m/s crashes into a barrier and stops in 0.050 s. There is a 20.0-kg child in the car. Assume that the child's velocity is changed by the same amount as that of the car, and in the same time period.

a. What is the impulse needed to stop the child?

b. What is the average force on the child?

c. What is the approximate mass of an object whose weight equals the force in part b?

d. Could you lift such a weight with your arm?

e. Why is it advisable to use a proper restraining seat rather than hold a child on your lap?

9.2 Conservation of Momentum

73. **Football** A 95-kg fullback, running at 8.2 m/s, collides in midair with a 128-kg defensive tackle moving in the opposite direction. Both players end up with zero speed.

a. Identify the "before" and "after" situations and draw a diagram of both.

b. What was the fullback's momentum before the collision?

c. What was the change in the fullback's momentum?

d. What was the change in the defensive tackle's momentum?

e. What was the defensive tackle's original momentum?

f. How fast was the defensive tackle moving originally?

74. Marble C, with mass 5.0 g, moves at a speed of 20.0 cm/s. It collides with a second marble, D, with mass 10.0 g, moving at 10.0 cm/s in the same direction. After the collision, marble C continues with a speed of 8.0 cm/s in the same direction.

a. Sketch the situation and identify the system. Identify the "before" and "after" situations and set up a coordinate system.

b. Calculate the marbles' momenta before the collision.

c. Calculate the momentum of marble C after the collision.

d. Calculate the momentum of marble D after the collision.

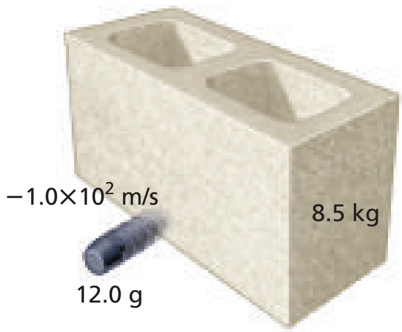
e. What is the speed of marble D after the collision?

75. Two lab carts are pushed together with a spring mechanism compressed between them. Upon release, the 5.0-kg cart repels one way with a velocity of 0.12 m/s, while the 2.0-kg cart goes in the opposite direction. What is the velocity of the 2.0-kg cart?

 For more problems, go to Additional Problems, Appendix B.

76. A 50.0-g projectile is launched with a horizontal velocity of 647 m/s from a 4.65-kg launcher moving in the same direction at 2.00 m/s. What is the launcher's velocity after the launch?

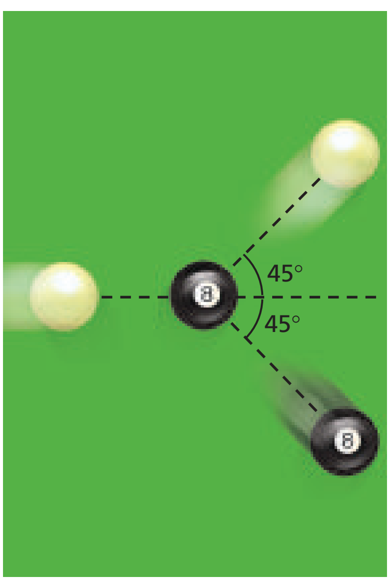
77. A 12.0-g rubber bullet travels at a velocity of 150 m/s, hits a stationary 8.5-kg concrete block resting on a frictionless surface, and ricochets in the opposite direction with a velocity of -1.0×10^2 m/s, as shown in **Figure 9-19.** How fast will the concrete block be moving?



■ **Figure 9-19**

78. **Skateboarding** Kofi, with mass 42.00 kg, is riding a skateboard with a mass of 2.00 kg and traveling at 1.20 m/s. Kofi jumps off and the skateboard stops dead in its tracks. In what direction and with what velocity did he jump?

79. **Billiards** A cue ball, with mass 0.16 kg, rolling at 4.0 m/s, hits a stationary eight ball of similar mass. If the cue ball travels 45° above its original path and the eight ball travels 45° below the horizontal, as shown in **Figure 9-20,** what is the velocity of each ball after the collision?



■ **Figure 9-20**

80. A 2575-kg van runs into the back of an 825-kg compact car at rest. They move off together at 8.5 m/s. Assuming that the friction with the road is negligible, calculate the initial speed of the van.

81. **In-line Skating** Diego and Keshia are on in-line skates and stand face-to-face, then push each other away with their hands. Diego has a mass of 90.0 kg and Keshia has a mass of 60.0 kg.

a. Sketch the event, identifying the "before" and "after" situations, and set up a coordinate axis.
b. Find the ratio of the skaters' velocities just after their hands lose contact.
c. Which skater has the greater speed?
d. Which skater pushed harder?

82. A 0.200-kg plastic ball moves with a velocity of 0.30 m/s. It collides with a second plastic ball of mass 0.100 kg, which is moving along the same line at a speed of 0.10 m/s. After the collision, both balls continue moving in the same, original direction. The speed of the 0.100-kg ball is 0.26 m/s. What is the new velocity of the 0.200-kg ball?

Mixed Review

83. A constant force of 6.00 N acts on a 3.00-kg object for 10.0 s. What are the changes in the object's momentum and velocity?

84. The velocity of a 625-kg car is changed from 10.0 m/s to 44.0 m/s in 68.0 s by an external, constant force.

a. What is the resulting change in momentum of the car?
b. What is the magnitude of the force?

85. **Dragster** An 845-kg dragster accelerates on a race track from rest to 100.0 km/h in 0.90 s.

a. What is the change in momentum of the dragster?
b. What is the average force exerted on the dragster?
c. What exerts that force?

86. **Ice Hockey** A 0.115-kg hockey puck, moving at 35.0 m/s, strikes a 0.365-kg jacket that is thrown onto the ice by a fan of a certain hockey team. The puck and jacket slide off together. Find their velocity.

87. A 50.0-kg woman, riding on a 10.0-kg cart, is moving east at 5.0 m/s. The woman jumps off the front of the cart and lands on the ground at 7.0 m/s eastward, relative to the ground.

a. Sketch the "before" and "after" situations and assign a coordinate axis to them.
b. Find the cart's velocity after the woman jumps off.

88. **Gymnastics** **Figure 9-21** shows a gymnast performing a routine. First, she does giant swings on the high bar, holding her body straight and pivoting around her hands. Then, she lets go of the high bar and grabs her knees with her hands in the tuck position. Finally, she straightens up and lands on her feet.
   a. In the second and final parts of the gymnast's routine, around what axis does she spin?
   b. Rank in order, from greatest to least, her moments of inertia for the three positions.
   c. Rank in order, from greatest to least, her angular velocities in the three positions.

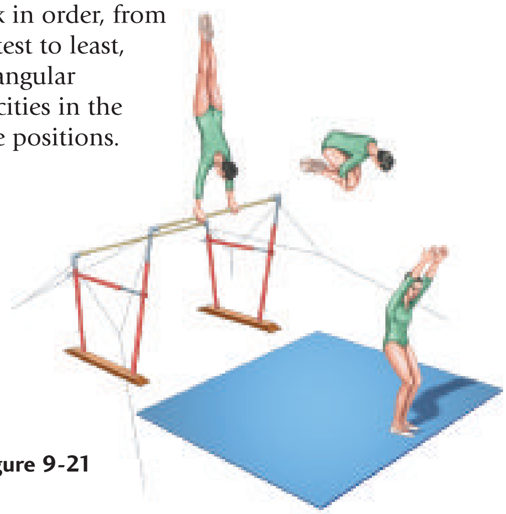


Figure 9-21

89. A 60.0-kg male dancer leaps 0.32 m high.
   a. With what momentum does he reach the ground?
   b. What impulse is needed to stop the dancer?
   c. As the dancer lands, his knees bend, lengthening the stopping time to 0.050 s. Find the average force exerted on the dancer's body.
   d. Compare the stopping force with his weight.

Thinking Critically

90. **Apply Concepts** A 92-kg fullback, running at 5.0 m/s, attempts to dive directly across the goal line for a touchdown. Just as he reaches the line, he is met head-on in midair by two 75-kg linebackers, both moving in the direction opposite the fullback. One is moving at 2.0 m/s and the other at 4.0 m/s. They all become entangled as one mass.
   a. Sketch the event, identifying the "before" and "after" situations.
   b. What is the velocity of the football players after the collision?
   c. Does the fullback score a touchdown?

91. **Analyze and Conclude** A student, holding a bicycle wheel with its axis vertical, sits on a stool that can rotate without friction. She uses her hand to get the wheel spinning. Would you expect the student and stool to turn? If so, in which direction? Explain.

92. **Analyze and Conclude** Two balls during a collision are shown in **Figure 9-22,** which is drawn to scale. The balls enter from the left, collide, and then bounce away. The heavier ball, at the bottom of the diagram, has a mass of 0.600 kg, and the other has a mass of 0.400 kg. Using a vector diagram, determine whether momentum is conserved in this collision. Explain any difference in the momentum of the system before and after the collision.

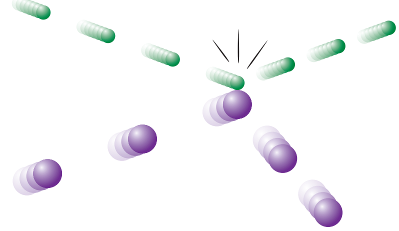


Figure 9-22

Writing in Physics

93. How can highway barriers be designed to be more effective in saving people's lives? Research this issue and describe how impulse and change in momentum can be used to analyze barrier designs.

94. While air bags save many lives, they also have caused injuries and even death. Research the arguments and responses of automobile makers to this statement. Determine whether the problems involve impulse and momentum or other issues.

Cumulative Review

95. A 0.72-kg ball is swung vertically from a 0.60-m string in uniform circular motion at a speed of 3.3 m/s. What is the tension in the cord at the top of the ball's motion? (Chapter 6)

96. You wish to launch a satellite that will remain above the same spot on Earth's surface. This means the satellite must have a period of exactly one day. Calculate the radius of the circular orbit this satellite must have. *Hint: The Moon also circles Earth and both the Moon and the satellite will obey Kepler's third law. The Moon is* 3.9×10^{8} *m from Earth and its period is 27.33 days.* (Chapter 7)

97. A rope is wrapped around a drum that is 0.600 m in diameter. A machine pulls with a constant 40.0 N force for a total of 2.00 s. In that time, 5.00 m of rope is unwound. Find α, ω at 2.00 s, and I. (Chapter 8)

 For more problems, go to Additional Problems, Appendix B.

Standardized Test Practice

Multiple Choice

1. When a star that is much larger than the Sun nears the end of its lifetime, it begins to collapse, but continues to rotate. Which of the following describes the conditions of the collapsing star's moment of inertia (I), angular momentum (L), and angular velocity (ω)?

- (A) I increases, L stays constant, ω decreases
- (B) I decreases, L stays constant, ω increases
- (C) I increases, L increases, ω increases
- (D) I increases, L increases, ω stays constant

2. A 40.0-kg ice-skater glides with a speed of 2.0 m/s toward a 10.0-kg sled at rest on the ice. The ice-skater reaches the sled and holds on to it. The ice-skater and the sled then continue sliding in the same direction in which the ice-skater was originally skating. What is the speed of the ice-skater and the sled after they collide?

- (A) 0.4 m/s
- (B) 0.8 m/s
- (C) 1.6 m/s
- (D) 3.2 m/s

3. A bicyclist applies the brakes and slows the motion of the wheels. The angular momentum of each wheel then decreases from 7.0 kg·m^2/s to 3.5 kg·m^2/s over a period of 5.0 s. What is the angular impulse on each wheel?

- (A) -0.7 kg·m^2/s
- (B) -1.4 kg·m^2/s
- (C) -2.1 kg·m^2/s
- (D) -3.5 kg·m^2/s

4. A 45.0-kg ice-skater stands at rest on the ice. A friend tosses the skater a 5.0-kg ball. The skater and the ball then move backwards across the ice with a speed of 0.50 m/s. What was the speed of the ball at the moment just before the skater caught it?

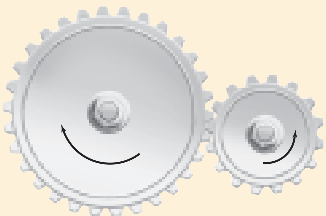
- (A) 2.5 m/s
- (B) 3.0 m/s
- (C) 4.0 m/s
- (D) 5.0 m/s

5. What is the difference in momentum between a 50.0-kg runner moving at a speed of 3.00 m/s and a 3.00×10^3-kg truck moving at a speed of only 1.00 m/s?

- (A) 1275 kg·m/s
- (B) 2550 kg·m/s
- (C) 2850 kg·m/s
- (D) 2950 kg·m/s

6. When the large gear in the diagram rotates, it turns the small gear in the opposite direction at the same linear speed. The larger gear has twice the radius and four times the mass of the smaller gear. What is the angular momentum of the larger gear as a function of the angular momentum of the smaller gear? *Hint: The moment of inertia for a disk is $\frac{1}{2}mr^2$, where m is mass and r is the radius of the disk.*

- (A) $-2L_{small}$
- (B) $-4L_{small}$
- (C) $-8L_{small}$
- (D) $-16L_{small}$



7. A force of 16 N exerted against a rock with an impulse of 0.8 kg·m/s causes the rock to fly off the ground with a speed of 4.0 m/s. What is the mass of the rock?

- (A) 0.2 kg
- (B) 0.8 kg
- (C) 1.6 kg
- (D) 4.0 kg

Extended Answer

8. A 12.0-kg rock falls to the ground. What is the impulse on the rock if its velocity at the moment it strikes the ground is 20.0 m/s?

✓ Test-Taking TIP

If It Looks Too Good To Be True

Beware of answer choices in multiple-choice questions that seem ready-made and obvious. Remember that only one answer choice for each question is correct. The rest are made up by test-makers to distract you. This means that they might look very appealing. Check each answer choice carefully before making your final selection.

Chapter 10 Energy, Work, and Simple Machines

What You'll Learn

- You will recognize that work and power describe how the external world changes the energy of a system.
- You will relate force to work and explain how machines ease the load.

Why It's Important

Simple machines and the compound machines formed from them make many everyday tasks easier to perform.

Mountain Bikes A multispeed mountain bicycle with shock absorbers allows you to match the ability of your body to exert forces, to do work, and to deliver power climbing steep hills, traversing flat terrain at high speeds, and safely descending hills.

Think About This ▶

How does a multispeed mountain bicycle enable a cyclist to ride over any kind of terrain with the least effort?

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LAUNCH Lab

What factors affect energy?

Question

What factors affect the energy of falling objects and their ability to do work?

Procedure

1. Place about 2 cm of fine sand in the bottom of a pie plate or baking pan.
2. Obtain a variety of metal balls or glass marbles of different sizes.
3. Hold a meterstick vertically in one hand, with one end just touching the surface of the sand. With the other hand, drop one of the balls into the sand. Record the height from which you dropped the ball.
4. Carefully remove the ball from the sand, so as not to disturb the impact crater it made. Measure the depth of the crater and how far sand was thrown from the crater.
5. Record the mass of the ball.
6. Smooth out the sand in the pie plate and perform steps 3–5 with different sizes of balls and drop them from varying heights. Be sure to drop different sizes of balls from the same height, as well as the same ball from different heights.

Analysis

Compare your data for the different craters. Is there an overall trend to your data? Explain.

Critical Thinking As the balls are dropped into the sand, they do work on the sand. Energy can be defined as the ability of an object to do work on itself or its surroundings. Relate the trend(s) you found in this lab to the energy of the balls. How can the energy of a ball be increased?

10.1 Energy and Work

In Chapter 9, you learned about the conservation of momentum. You learned that you could examine the state of a system before and after an impulse acted on it without knowing the details about the impulse. The law of conservation of momentum was especially useful when considering collisions, during which forces sometimes changed dramatically. Recall the discussion in Chapter 9 of the two skaters who push each other away. While momentum is conserved in this situation, the skaters continue to move after pushing each other away; whereas before the collision, they were at rest. When two cars crash into each other, momentum is conserved. Unlike the skaters, however, the cars, which were moving prior to the collision, became stationary after the crash. The collision probably resulted in a lot of twisted metal and broken glass. In these types of situations, some other quantity must have been changed as a result of the force acting on each system.

Objectives

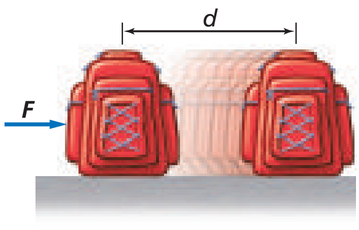
- **Describe** the relationship between work and energy.
- **Calculate** work.
- **Calculate** the power used.

Vocabulary

work
energy
kinetic energy
work-energy theorem
joule
power
watt

Work and Energy

Recall that change in momentum is the result of an impulse, which is the product of the average force exerted on an object and the time of the interaction. Consider a force exerted on an object while the object moves a certain distance. Because there is a net force, the object will be accelerated, $a = F/m$, and its velocity will increase. Examine Table 3-3 in Chapter 3, on page 68, which lists equations describing the relationships among position, velocity, and time for motion under constant acceleration. Consider the equation involving acceleration, velocity, and distance: $2ad = v_f^2 - v_i^2$. If you use Newton's second law to replace a with F/m and multiply both sides by $m/2$, you obtain $Fd = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$.



■ **Figure 10-1** Work is done when a constant force, *F*, is exerted on the backpack in the direction of motion and the backpack moves a distance, *d*.

Work The left side of the equation describes something that was done to the system by the external world (the environment). A force, *F*, was exerted on an object while the object moved a distance, *d*, as shown in **Figure 10-1**. If *F* is a constant force, exerted in the direction in which the object is moving, then **work,** *W*, is the product of the force and the object's displacement.

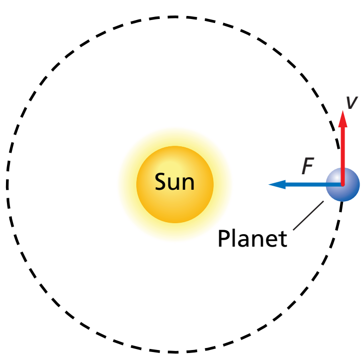
Work $W = Fd$

Work is equal to a constant force exerted on an object in the direction of motion, times the object's displacement.

You probably have used the word *work* in many other ways. For example, a computer might work well, learning physics can be hard work, and you might work at an after-school job. To physicists, however, work has a very precise meaning.

Recall that $Fd = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$. Rewriting the equation $W = Fd$ results in $W = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$. The right side of the equation involves the object's mass and its velocities after and before the force was exerted. The quantity $\frac{1}{2}mv_i^2$ describes a property of the system.

Kinetic energy What property of a system does $\frac{1}{2}mv_i^2$ describe? A massive, fast-moving vehicle can do damage to objects around it, and a baseball hit at high speed can rise high into the air. That is, an object with this property can produce a change in itself or the world around it. This property, the ability of an object to produce a change in itself or the world around it, is called **energy.** The fast-moving vehicle and the baseball possess energy that is associated with their motion. This energy resulting from motion is called **kinetic energy** and is represented by the symbol *KE*.



■ **Figure 10-2** If a planet is in a circular orbit, then the force is perpendicular to the direction of motion. Consequently, the gravitational force does no work on the planet.

Kinetic Energy $KE = \frac{1}{2}mv^2$

The kinetic energy of an object is equal to $\frac{1}{2}$ times the mass of the object multiplied by the speed of the object squared.

Substituting *KE* into the equation $W = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$ results in $W = KE_f - KE_i$. The right side is the difference, or change, in kinetic energy. The **work-energy theorem** states that when work is done on an object, the result is a change in kinetic energy. The work-energy theorem can be represented by the following equation.

Work-Energy Theorem $W = \Delta KE$

Work is equal to the change in kinetic energy.

The relationship between work done and the change in energy that results was established by nineteenth-century physicist James Prescott Joule. To honor his work, a unit of energy is called a **joule** (J). For example, if a 2-kg object moves at 1 m/s, it has a kinetic energy of 1 $kg{\cdot}m^2/s^2$, or 1 J.

Recall that a system is the object of interest and the external world is everything else. For example, one system might be a box in a warehouse and the external world might consist of yourself, Earth's mass, and anything else external to the box. Through the process of doing work, energy can move between the external world and the system.

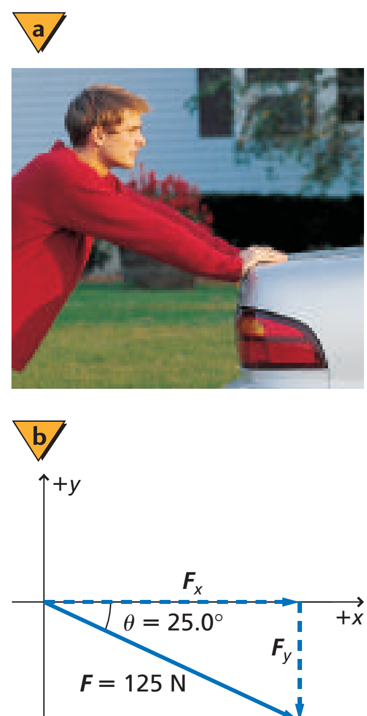
Notice that the direction of energy transfer can go both ways. If the external world does work on a system, then W is positive and the energy of the system increases. If, however, a system does work on the external world, then W is negative and the energy of the system decreases. In summary, work is the transfer of energy by mechanical means.

Calculating Work

The first equation used to calculate work is $W = Fd$. This equation, however, holds only for constant forces exerted in the direction of motion. What happens if the force is exerted perpendicular to the direction of motion? An everyday example of this is the motion of a planet around the Sun, as shown in **Figure 10-2**. If the orbit is circular, then the force is always perpendicular to the direction of motion. Recall from Chapter 6 that a perpendicular force does not change the speed of an object, only its direction. Consequently, the speed of the planet doesn't change. Therefore, its kinetic energy also is constant. Using the equation $W = \Delta KE$, you can see that when KE is constant, $\Delta KE = 0$ and thus, $W = 0$. This means that if F and d are at right angles, then $W = 0$.

Because the work done on an object equals the change in energy, work also is measured in joules. One joule of work is done when a force of 1 N acts on an object over a displacement of 1 m. An apple weighs about 1 N. Thus, when you lift an apple a distance of 1 m, you do 1 J of work on it.

Constant force exerted at an angle You've learned that a force exerted in the direction of motion does an amount of work given by $W = Fd$. A force exerted perpendicular to the motion does no work. What work does a force exerted at an angle do? For example, what work does the person pushing the car in **Figure 10-3a** do? You know that any force can be replaced by its components. If the coordinate system shown in **Figure 10-3b** is used, the 125-N force, $\boldsymbol{F}$, exerted in the direction of the person's arm, has two components. The magnitude of the horizontal component, F_x, is related to the magnitude of the force, F, by a cosine function: $\cos 25.0° = F_x/F$. By solving for F_x, you obtain $F_x = F \cos 25.0° = (125\text{ N})(\cos 25.0°) = 113$ N. Using the same method, the vertical component $F_y = -F \sin 25.0° = -(125\text{ N})(\sin 25.0°) = -52.8$ N, where the negative sign shows that the force is downward. Because the displacement is in the x direction, only the x-component does work. The y-component does no work.



■ **Figure 10-3** If a force is applied to a car at an angle, the net force doing the work is the component that acts in the direction of the displacement.

Hutchings Photography

The work you do when you exert a force on an object, at an angle to the direction of motion, is equal to the component of the force in the direction of the displacement, multiplied by the distance moved. The magnitude of the component force acting in the direction of displacement is found by multiplying the magnitude of force, F, by the cosine of the angle between F and the direction of the displacement: $F_x = F \cos \theta$. Thus, the work done is represented by the following equation.

Work (Angle Between Force and Displacement) $W = Fd \cos \theta$

Work is equal to the product of force and displacement, times the cosine of the angle between the force and the direction of the displacement.

Other agents exert forces on the pushed car as well. Which of these agents do work? Earth's gravity acts downward, the ground exerts a normal force upward, and friction exerts a horizontal force opposite the direction of motion. The upward and downward forces are perpendicular to the direction of motion and do no work. For these forces, $\theta = 90°$, which makes $\cos \theta = 0$, and thus, $W = 0$.

The work done by friction acts in the direction opposite that of motion—at an angle of 180°. Because $\cos 180° = -1$, the work done by friction is negative. Negative work done by a force exerted by something in the external world reduces the kinetic energy of the system. If the person in Figure 10-3a were to stop pushing, the car would quickly stop moving—its energy of motion would be reduced. Positive work done by a force increases the energy, while negative work decreases it. Use the problem-solving strategies below when you solve problems related to work.

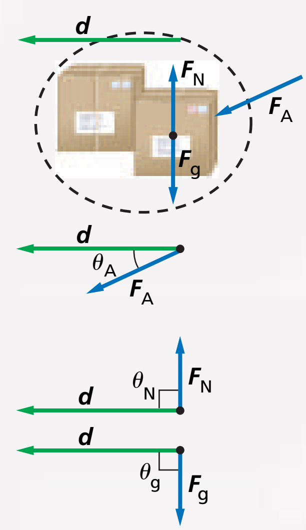
PROBLEM-SOLVING Strategies

Work

When solving work-related problems, use the following strategies.

1. Sketch the system and show the force that is doing the work.
2. Draw the force and displacement vectors of the system.
3. Find the angle, θ, between each force and displacement.
4. Calculate the work done by each force using $W = Fd \cos \theta$.
5. Calculate the net work done. Check the sign of the work using the direction of energy transfer. If the energy of the system has increased, the work done by that force is positive. If the energy has decreased, then the work done by that force is negative.

Work Diagram

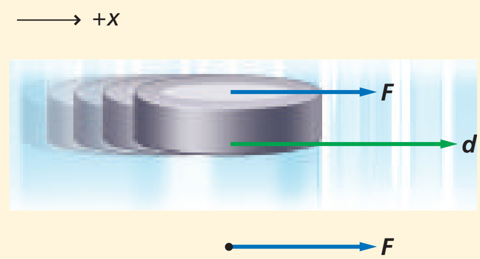


EXAMPLE Problem 1

Work and Energy A 105-g hockey puck is sliding across the ice. A player exerts a constant 4.50-N force over a distance of 0.150 m. How much work does the player do on the puck? What is the change in the puck's energy?

1 Analyze and Sketch the Problem

- Sketch the situation showing initial conditions.
- Establish a coordinate system with $+x$ to the right.
- Draw a vector diagram.



Known:	Unknown:
$m = 105$ g	$W = ?$
$F = 4.50$ N	$\Delta KE = ?$
$d = 0.150$ m	

2 Solve for the Unknown

Use the equation for work when a constant force is exerted in the same direction as the object's displacement.

$$W = Fd$$
$$= (4.50\ \text{N})(0.150\ \text{m}) \qquad \textbf{Substitute } F = 4.50\ \text{N},\ d = 0.150\ \text{m}$$
$$= 0.675\ \text{N·m}$$
$$= 0.675\ \text{J} \qquad \textbf{1 J = 1 N·m}$$

Math Handbook
Operations with Significant Digits
pages 835–836

Use the work-energy theorem to determine the change in energy of the system.

$$W = \Delta KE$$
$$\Delta KE = 0.675\ \text{J} \qquad \textbf{Substitute } W = 0.675\ \text{J}$$

3 Evaluate the Answer

- **Are the units correct?** Work is measured in joules.
- **Does the sign make sense?** The player (external world) does work on the puck (the system). So the sign of work should be positive.

PRACTICE Problems

Additional Problems, Appendix B

1. Refer to Example Problem 1 to solve the following problem.

a. If the hockey player exerted twice as much force, 9.00 N, on the puck, how would the puck's change in kinetic energy be affected?

b. If the player exerted a 9.00 N-force, but the stick was in contact with the puck for only half the distance, 0.075 m, what would be the change in kinetic energy?

2. Together, two students exert a force of 825 N in pushing a car a distance of 35 m.

a. How much work do the students do on the car?

b. If the force was doubled, how much work would they do pushing the car the same distance?

3. A rock climber wears a 7.5-kg backpack while scaling a cliff. After 30.0 min, the climber is 8.2 m above the starting point.

a. How much work does the climber do on the backpack?

b. If the climber weighs 645 N, how much work does she do lifting herself and the backpack?

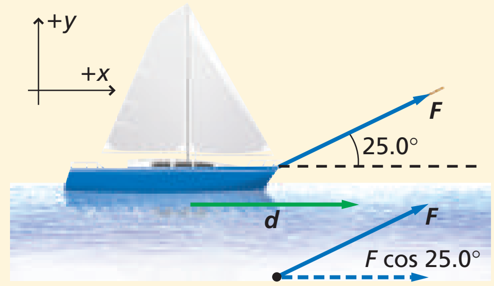
c. What is the average power developed by the climber?

EXAMPLE Problem 2

Force and Displacement at an Angle A sailor pulls a boat a distance of 30.0 m along a dock using a rope that makes a 25.0° angle with the horizontal. How much work does the sailor do on the boat if he exerts a force of 255 N on the rope?

1 Analyze and Sketch the Problem

- Establish coordinate axes.
- Sketch the situation showing the boat with initial conditions.
- Draw a vector diagram showing the force and its component in the direction of the displacement.



Known:	Unknown:
$F = 255$ N	$W = ?$
$d = 30.0$ m	
$\theta = 25.0°$	

2 Solve for the Unknown

Use the equation for work done when there is an angle between the force and displacement.

$$W = Fd \cos \theta$$

$$= (255 \text{ N})(30.0 \text{ m})(\cos 25.0°) \quad \text{Substitute } F = 255 \text{ N}, d = 30.0 \text{ m}, \theta = 25.0°$$

$$= 6.93 \times 10^{3} \text{ J}$$

3 Evaluate the Answer

- **Are the units correct?** Work is measured in joules.
- **Does the sign make sense?** The sailor does work on the boat, which agrees with a positive sign for work.

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Trigonometric Ratios
page 855

PRACTICE Problems

Additional Problems, Appendix B

4. If the sailor in Example Problem 2 pulled with the same force, and along the same distance, but at an angle of 50.0°, how much work would he do?

5. Two people lift a heavy box a distance of 15 m. They use ropes, each of which makes an angle of 15° with the vertical. Each person exerts a force of 225 N. How much work do they do?

6. An airplane passenger carries a 215-N suitcase up the stairs, a displacement of 4.20 m vertically, and 4.60 m horizontally.

a. How much work does the passenger do?

b. The same passenger carries the same suitcase back down the same set of stairs. How much work does the passenger do now?

7. A rope is used to pull a metal box a distance of 15.0 m across the floor. The rope is held at an angle of 46.0° with the floor, and a force of 628 N is applied to the rope. How much work does the force on the rope do?

8. A bicycle rider pushes a bicycle that has a mass of 13 kg up a steep hill. The incline is 25° and the road is 275 m long, as shown in **Figure 10-4.** The rider pushes the bike parallel to the road with a force of 25 N.

a. How much work does the rider do on the bike?

b. How much work is done by the force of gravity on the bike?

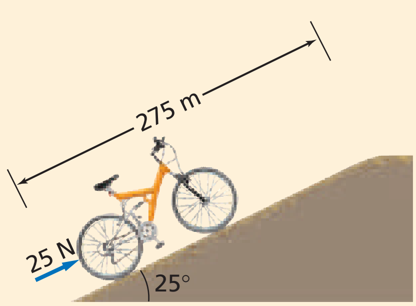


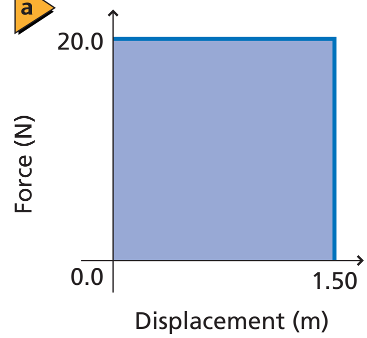
Figure 10-4 (Not to scale)

Finding work done when forces change A graph of force versus displacement lets you determine the work done by a force. This graphical method can be used to solve problems in which the force is changing. **Figure 10-5a** shows the work done by a constant force of 20.0 N that is exerted to lift an object a distance of 1.50 m. The work done by this constant force is represented by $W = Fd = (20.0\ \text{N})(1.50\ \text{m}) = 30.0\ \text{J}$. The shaded area under the graph is equal to $(20.0\ \text{N})(1.50\ \text{m})$, or 30.0 J. The area under a force-displacement graph is equal to the work done by that force, even if the force changes. **Figure 10-5b** shows the force exerted by a spring, which varies linearly from 0.0 to 20.0 N as it is compressed 1.50 m. The work done by the force that compressed the spring is the area under the graph, which is the area of a triangle, $\frac{1}{2}$(base)(altitude), or $W = \frac{1}{2}(20.0\ \text{N})(1.50\ \text{m}) = 15.0\ \text{J}$.

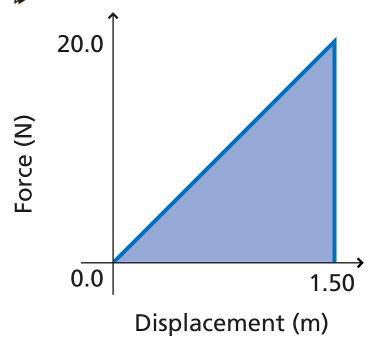
Work done by many forces Newton's second law of motion relates the net force on an object to its acceleration. In the same way, the work-energy theorem relates the net work done on a system to its energy change. If several forces are exerted on a system, calculate the work done by each force, and then add the results.

■ **Figure 10-5** Work can be obtained graphically by finding the area under a force-displacement graph.

a



b



Power

Until now, none of the discussions of work has mentioned the time it takes to move an object. The work done by a person lifting a box of books is the same whether the box is lifted onto a shelf in 2 s or each book is lifted separately so that it takes 20 min to put them all on the shelf. Although the work done is the same, the rate at which it is done is different. **Power** is the work done, divided by the time taken to do the work. In other words, power is the rate at which the external force changes the energy of the system. It is represented by the following equation.

Power $P = \frac{W}{t}$

Power is equal to the work done, divided by the time taken to do the work.

Consider the three students in **Figure 10-6.** The girl hurrying up the stairs is more powerful than the boy who is walking up the stairs. Even though the same work is accomplished by both, the girl accomplishes it in less time and thus develops more power. In the case of the two students walking up the stairs, both accomplish work in the same amount of time.

Power is measured in watts (W). One **watt** is 1 J of energy transferred in 1 s. A watt is a relatively small unit of power. For example, a glass of water weighs about 2 N. If you lift it 0.5 m to your mouth, you do 1 J of work. If you lift the glass in 1 s, you are doing work at the rate of 1 W. Because a watt is such a small unit, power often is measured in kilowatts (kW). One kilowatt is equal to 1000 W.

■ **Figure 10-6** These students are doing work at different rates while climbing the stairs.

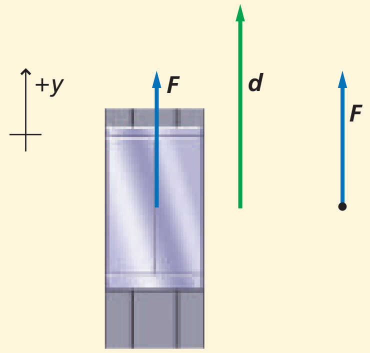
Laura Sifferlin

EXAMPLE Problem 3

Power An electric motor lifts an elevator 9.00 m in 15.0 s by exerting an upward force of 1.20×10^4 N. What power does the motor produce in kW?

1 Analyze and Sketch the Problem

- Sketch the situation showing the elevator with initial conditions.
- Establish a coordinate system with up as positive.
- Draw a vector diagram for the force and displacement.



Known:

$d = 9.00$ m

$t = 15.0$ s

$F = 1.20\times10^4$ N

Unknown:

$P = ?$

2 Solve for the Unknown

Solve for power.

$$P = \frac{W}{t}$$

$$= \frac{Fd}{t} \quad \text{Substitute } W = Fd$$

$$= \frac{(1.20\times10^4\ \text{N})(9.00\ \text{m})}{(15.0\ \text{s})} \quad \text{Substitute } F = 1.20\times10^4\ \text{N},\ d = 9.00\ \text{m},\ t = 15.0\ \text{s}$$

$$= 7.20\ \text{kW}$$

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Operations with Scientific Notation pages 842–843

3 Evaluate the Answer

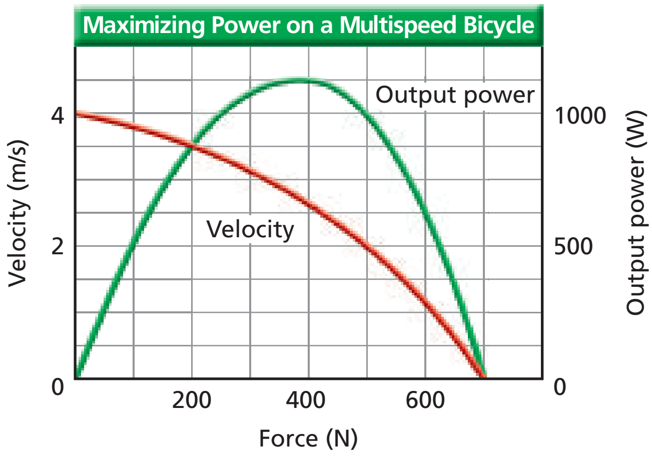
- **Are the units correct?** Power is measured in J/s.
- **Does the sign make sense?** The positive sign agrees with the upward direction of the force.

PRACTICE Problems

Additional Problems, Appendix B

9. A box that weighs 575 N is lifted a distance of 20.0 m straight up by a cable attached to a motor. The job is done in 10.0 s. What power is developed by the motor in W and kW?
10. You push a wheelbarrow a distance of 60.0 m at a constant speed for 25.0 s, by exerting a 145-N force horizontally.
    a. What power do you develop?
    b. If you move the wheelbarrow twice as fast, how much power is developed?
11. What power does a pump develop to lift 35 L of water per minute from a depth of 110 m? (1 L of water has a mass of 1.00 kg.)
12. An electric motor develops 65 kW of power as it lifts a loaded elevator 17.5 m in 35 s. How much force does the motor exert?
13. A winch designed to be mounted on a truck, as shown in **Figure 10-7,** is advertised as being able to exert a 6.8×10^3-N force and to develop a power of 0.30 kW. How long would it take the truck and the winch to pull an object 15 m?
14. Your car has stalled and you need to push it. You notice as the car gets going that you need less and less force to keep it going. Suppose that for the first 15 m, your force decreased at a constant rate from 210.0 N to 40.0 N. How much work did you do on the car? Draw a force-displacement graph to represent the work done during this period.

Figure 10-7



■ **Figure 10-8** When riding a multispeed bicycle, if the muscles in your body exert a force of 400 N and the speed is 2.6 m/s, the power output is over 1000 W.

You may have noticed in Example Problem 3 that when the force and displacement are in the same direction, $P = Fd/t$. However, because the ratio d/t is the speed, power also can be calculated using $P = Fv$.

When you are riding a multispeed bicycle, how do you choose the correct gear? You want to get your body to deliver the largest amount of power. By considering the equation $P = Fv$ you can see that either zero force or zero speed results in no power delivered. The muscles cannot exert extremely large forces, nor can they move very fast. Thus, some combination of moderate force and moderate speed will produce the largest amount of power. **Figure 10-8** shows that in this particular situation, the maximum power output is over 1000 W when the force is about 400 N and speed is about 2.6 m/s. All engines—not just humans—have these limitations. Simple machines often are designed to match the force and speed that the engine can deliver to the needs of the job. You will learn more about simple machines in the next section.

APPLYING PHYSICS

▶ **Tour de France** A bicyclist in the Tour de France rides at about 8.94 m/s for more than 6 h a day. The power output of the racer is about 1 kW. One-fourth of that power goes into moving the bike against the resistance of the air, gears, and tires. Three-fourths of the power is used to cool the racer's body. ◀

10.1 Section Review

15. Work Murimi pushes a 20-kg mass 10 m across a floor with a horizontal force of 80 N. Calculate the amount of work done by Murimi.

16. Work A mover loads a 185-kg refrigerator into a moving van by pushing it up a 10.0-m, friction-free ramp at an angle of inclination of 11.0°. How much work is done by the mover?

17. Work and Power Does the work required to lift a book to a high shelf depend on how fast you raise it? Does the power required to lift the book depend on how fast you raise it? Explain.

18. Power An elevator lifts a total mass of 1.1×10^3 kg a distance of 40.0 m in 12.5 s. How much power does the elevator generate?

19. Work A 0.180-kg ball falls 2.5 m. How much work does the force of gravity do on the ball?

20. Mass A forklift raises a box 1.2 m and does 7.0 kJ of work on it. What is the mass of the box?

21. Work You and a friend each carry identical boxes from the first floor of a building to a room located on the second floor, farther down the hall. You choose to carry the box first up the stairs, and then down the hall to the room. Your friend carries it down the hall on the first floor, then up a different stairwell to the second floor. Who does more work?

22. Work and Kinetic Energy If the work done on an object doubles its kinetic energy, does it double its velocity? If not, by what ratio does it change the velocity?

23. Critical Thinking Explain how to find the change in energy of a system if three agents exert forces on the system at once.

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CONTENTS

10.2 Machines

▶ **Objectives**

- **Demonstrate** a knowledge of the usefulness of simple machines.
- **Differentiate** between ideal and real machines in terms of efficiency.
- **Analyze** compound machines in terms of combinations of simple machines.
- **Calculate** efficiencies for simple and compound machines.

▶ **Vocabulary**

machine
effort force
resistance force
mechanical advantage
ideal mechanical advantage
efficiency
compound machine

Everyone uses machines every day. Some are simple tools, such as bottle openers and screwdrivers, while others are complex, such as bicycles and automobiles. Machines, whether powered by engines or people, make tasks easier. A **machine** eases the load by changing either the magnitude or the direction of a force to match the force to the capability of the machine or the person.

Benefits of Machines

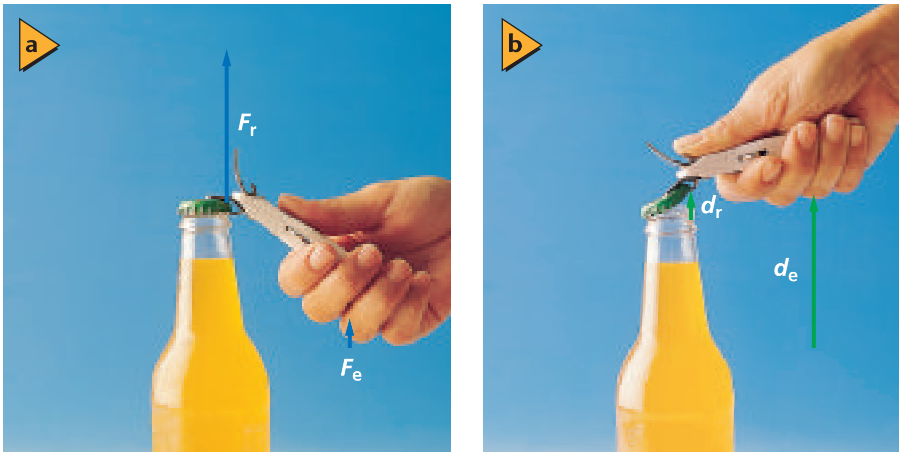
Consider the bottle opener in **Figure 10-9.** When you use the opener, you lift the handle, thereby doing work on the opener. The opener lifts the cap, doing work on it. The work that you do is called the input work, W_i. The work that the machine does is called the output work, W_o.

Recall that work is the transfer of energy by mechanical means. You put work into a machine, such as the bottle opener. That is, you transfer energy to the opener. The opener, in turn, does work on the cap, thereby transferring energy to it. The opener is not a source of energy, and therefore, the cap cannot receive more energy than the amount of energy that you put into the opener. Thus, the output work can never be greater than the input work. The machine simply aids in the transfer of energy from you to the bottle cap.

Mechanical advantage The force exerted by a person on a machine is called the **effort force,** F_e. The force exerted by the machine is called the **resistance force,** F_r. As shown in Figure 10-9a, F_e is the upward force exerted by the person using the bottle opener and F_r is the upward force exerted by the bottle opener. The ratio of resistance force to effort force, F_r/F_e, is called the **mechanical advantage,** *MA*, of the machine.

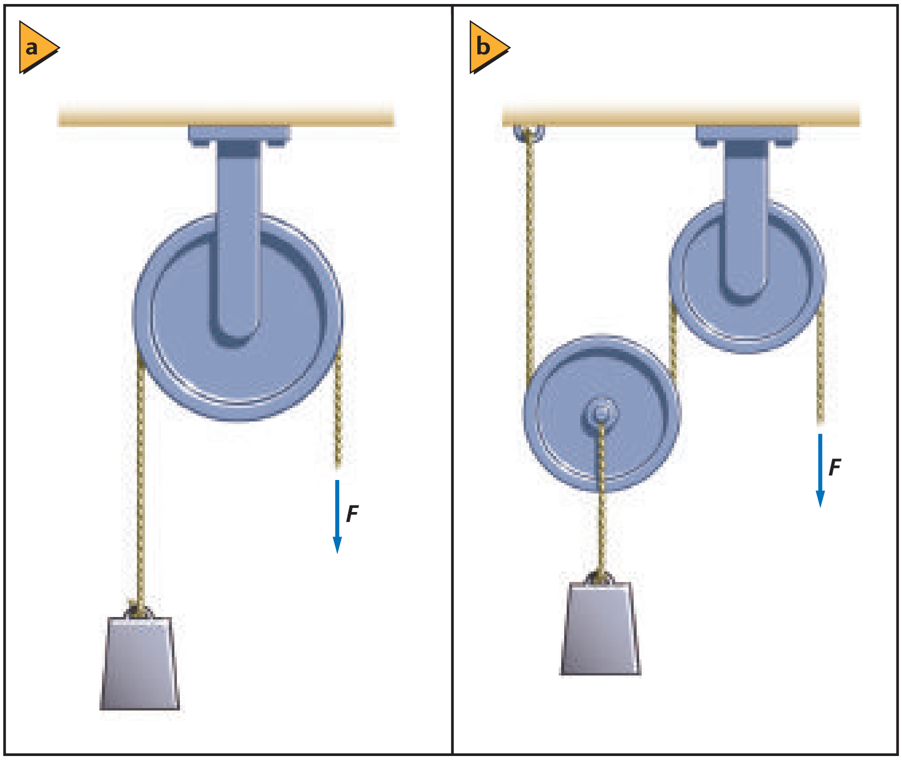
Mechanical Advantage $$MA = \frac{F_r}{F_e}$$

The mechanical advantage of a machine is equal to the resistance force divided by the effort force.



■ **Figure 10-9** A bottle opener is an example of a simple machine. It makes opening a bottle easier, but it does not lessen the work required to do so.

Hutchings Photography



■ **Figure 10-10** A fixed pulley has a mechanical advantage equal to 1 **(a).** A pulley system with a movable pulley has a mechanical advantage greater than 1 **(b).**

In a fixed pulley, such as the one shown in **Figure 10-10a,** the forces, F_e and F_r, are equal, and consequently MA is 1. What is the advantage of this machine? The fixed pulley is useful, not because the effort force is lessened, but because the direction of the effort force is changed. Many machines, such as the bottle opener shown in Figure 10-9 and the pulley system shown in **Figure 10-10b,** have a mechanical advantage greater than 1. When the mechanical advantage is greater than 1, the machine increases the force applied by a person.

You can write the mechanical advantage of a machine in another way using the definition of work. The input work is the product of the effort force that a person exerts, F_e, and the distance his or her hand moved, d_e. In the same way, the output work is the product of the resistance force, F_r, and the displacement of the load, d_r. A machine can increase force, but it cannot increase energy. An ideal machine transfers all the energy, so the output work equals the input work: $W_o = W_i$ or $F_r d_r = F_e d_e$.

This equation can be rewritten $F_r/F_e = d_e/d_r$. Recall that mechanical advantage is given by $MA = F_r/F_e$. Therefore, for an ideal machine, **ideal mechanical advantage,** IMA, is equal to the displacement of the effort force, divided by the displacement of the load. The ideal mechanical advantage can be represented by the following equation.

Ideal Mechanical Advantage $IMA = \frac{d_e}{d_r}$

The ideal mechanical advantage of an ideal machine is equal to the displacement of the effort force, divided by the displacement of the load.

Note that you measure the distances moved to calculate the ideal mechanical advantage, but you measure the forces exerted to find the actual mechanical advantage.

Efficiency In a real machine, not all of the input work is available as output work. Energy removed from the system means that there is less output work from the machine. Consequently, the machine is less efficient at accomplishing the task. The **efficiency** of a machine, e, is defined as the ratio of output work to input work.

Efficiency $e = \frac{W_o}{W_i} \times 100$

The efficiency of a machine (in %) is equal to the output work, divided by the input work, multiplied by 100.

An ideal machine has equal output and input work, $W_o/W_i = 1$, and its efficiency is 100 percent. All real machines have efficiencies of less than 100 percent.

Efficiency can be expressed in terms of the mechanical advantage and ideal mechanical advantage. Efficiency, $e = W_o/W_i$, can be rewritten as follows:

$$\frac{W_o}{W_i} = \frac{F_r d_r}{F_e d_e}$$

Because $MA = F_r/F_e$ and $IMA = d_e/d_r$, the following expression can be written for efficiency.

Efficiency $e = \frac{MA}{IMA} \times 100$

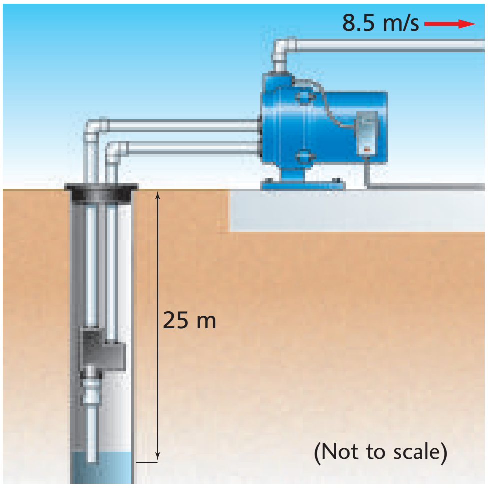
The efficiency of a machine (in %) is equal to its mechanical advantage, divided by the ideal mechanical advantage, multiplied by 100.

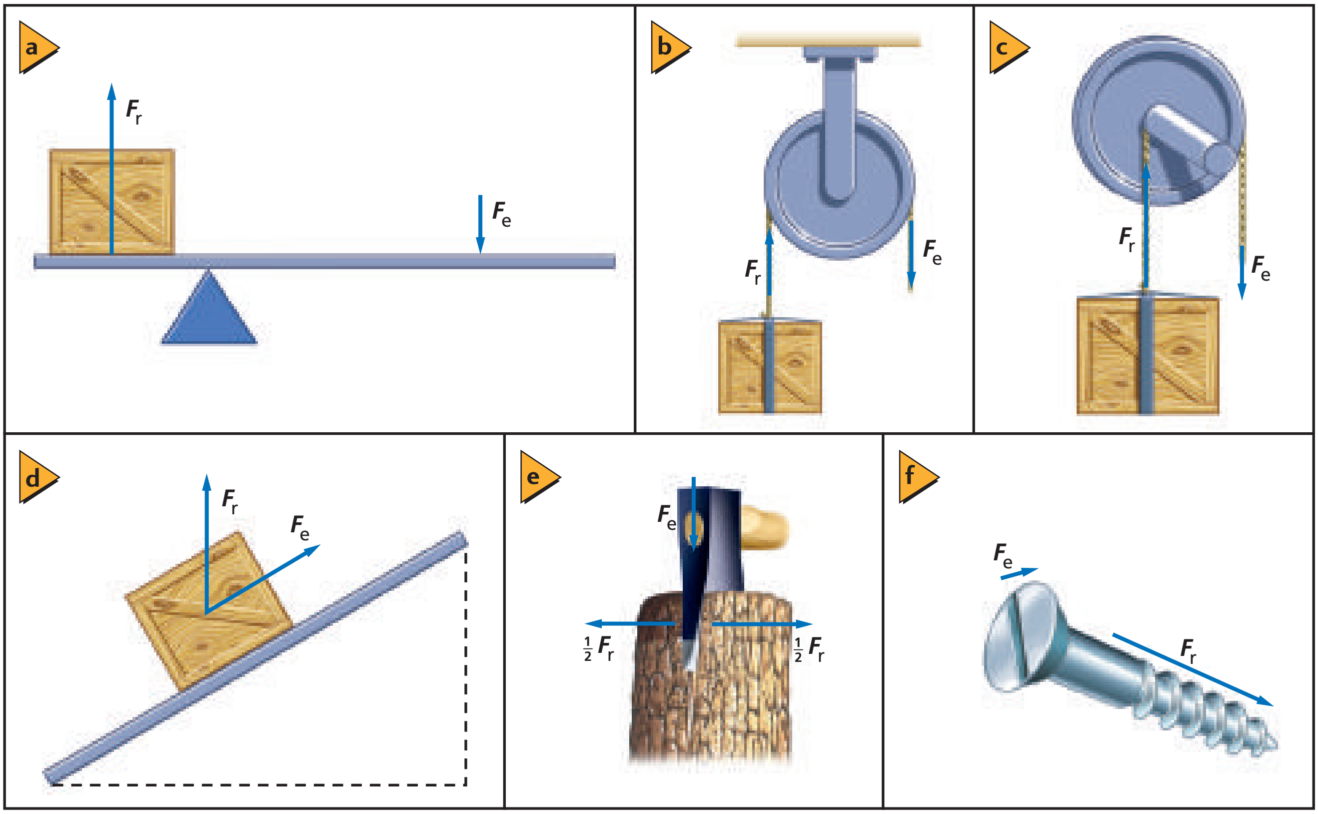
A machine's design determines its ideal mechanical advantage. An efficient machine has an MA almost equal to its IMA. A less-efficient machine has a small MA relative to its IMA. To obtain the same resistance force, a greater force must be exerted in a machine of lower efficiency than in a machine of higher efficiency.

CHALLENGE PROBLEM

An electric pump pulls water at a rate of 0.25 m^3/s from a well that is 25 m deep. The water leaves the pump at a speed of 8.5 m/s.

1. What power is needed to lift the water to the surface?
2. What power is needed to increase the pump's kinetic energy?
3. If the pump's efficiency is 80 percent, how much power must be delivered to the pump?





■ **Figure 10-11** Simple machines include the lever **(a),** pulley **(b),** wheel and axle **(c),** inclined plane **(d),** wedge **(e),** and screw **(f).**

Compound Machines

Most machines, no matter how complex, are combinations of one or more of the six simple machines: the lever, pulley, wheel and axle, inclined plane, wedge, and screw. These machines are shown in **Figure 10-11.**

The *IMA* of all the machines shown in Figure 10-11 is the ratio of distances moved. For machines, such as the lever and the wheel and axle, this ratio can be replaced by the ratio of the distance between the place where the force is applied and the pivot point. A common version of the wheel and axle is a steering wheel, such as the one shown in **Figure 10-12.** The *IMA* is the ratio of the radii of the wheel and axle.

A machine consisting of two or more simple machines linked in such a way that the resistance force of one machine becomes the effort force of the second is called a **compound machine.**

■ **Figure 10-12** The *IMA* for the steering wheel is r_e/r_r.

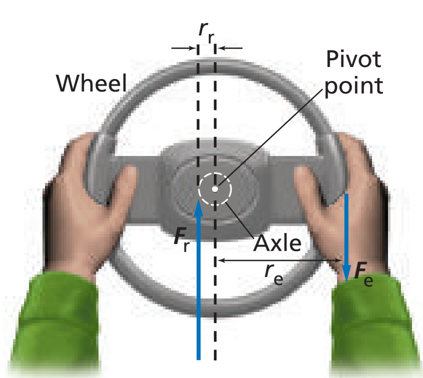


Figure 10-13 A series of simple machines combine to transmit the force that the rider exerts on the pedal to the road.

In a bicycle, the pedal and front gear act like a wheel and axle. The effort force is the force that the rider exerts on the pedal, $F_{\text{rider on pedal}}$. The resistance is the force that the front gear exerts on the chain, $F_{\text{gear on chain}}$, as shown in **Figure 10-13.** The chain exerts an effort force on the rear gear, $F_{\text{chain on gear}}$, equal to the force exerted on the chain. This gear and the rear wheel act like another wheel and axle. The resistance force is the force that the wheel exerts on the road, $F_{\text{wheel on road}}$. According to Newton's third law, the ground exerts an equal forward force on the wheel, which accelerates the bicycle forward.

The *MA* of a compound machine is the product of the *MA*s of the simple machines from which it is made. For example, in the case of the bicycle illustrated in Figure 10-13, the following is true.

$$MA = MA_{\text{machine 1}} \times MA_{\text{machine 2}}$$

$$MA = \left(\frac{F_{\text{gear on chain}}}{F_{\text{rider on pedal}}}\right)\left(\frac{F_{\text{wheel on road}}}{F_{\text{chain on gear}}}\right) = \frac{F_{\text{wheel on road}}}{F_{\text{rider on pedal}}}$$

The *IMA* of each wheel-and-axle machine is the ratio of the distances moved.

For the pedal gear, $IMA = \dfrac{\text{pedal radius}}{\text{front gear radius}}$

For the rear wheel, $IMA = \dfrac{\text{rear gear radius}}{\text{wheel radius}}$

For the bicycle, then,

$$IMA = \left(\frac{\text{pedal radius}}{\text{front gear radius}}\right)\left(\frac{\text{rear gear radius}}{\text{wheel radius}}\right)$$

$$= \left(\frac{\text{rear gear radius}}{\text{front gear radius}}\right)\left(\frac{\text{pedal radius}}{\text{wheel radius}}\right)$$

Because both gears use the same chain and have teeth of the same size, you can count the number of teeth to find the *IMA*, as follows.

$$IMA = \left(\frac{\text{teeth on rear gear}}{\text{teeth on front gear}}\right)\left(\frac{\text{pedal arm length}}{\text{wheel radius}}\right)$$

Shifting gears on a bicycle is a way of adjusting the ratio of gear radii to obtain the desired *IMA*. You know that if the pedal of a bicycle is at the top or bottom of its circle, no matter how much downward force you exert, the pedal will not turn. The force of your foot is most effective when the force is exerted perpendicular to the arm of the pedal; that is, when the torque is largest. Whenever a force on a pedal is specified, assume that it is applied perpendicular to the arm.

MINI LAB

Wheel and Axle

The gear mechanism on your bicycle multiplies the distance that you travel. What does it do to the force?

1. Mount a wheel and axle system on a sturdy support rod.

2. Wrap a 1-m-long piece of string clockwise around the axle.

3. Wrap another piece of 1-m-long string counterclockwise around the large diameter wheel.

4. Hang a 500-g mass from the end of the string on the larger wheel. ***CAUTION: Avoid dropping the mass.***

5. Pull the string from the axle down so that the mass is lifted by about 10 cm.

Analyze and Conclude

6. What did you notice about the force on the string in your hand?

7. What did you notice about the distance that your hand needed to move to lift the mass? Explain the results in terms of the work done on both strings.

▶ EXAMPLE Problem 4

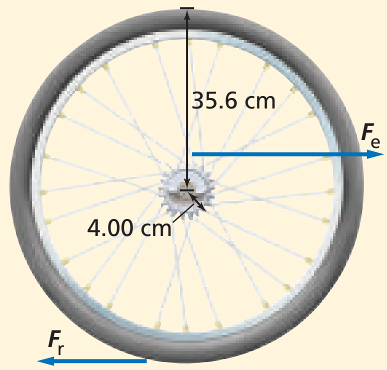
Mechanical Advantage You examine the rear wheel on your bicycle. It has a radius of 35.6 cm and has a gear with a radius of 4.00 cm. When the chain is pulled with a force of 155 N, the wheel rim moves 14.0 cm. The efficiency of this part of the bicycle is 95.0 percent.

a. What is the *IMA* of the wheel and gear?

b. What is the *MA* of the wheel and gear?

c. What is the resistance force?

d. How far was the chain pulled to move the rim 14.0 cm?



1 Analyze and Sketch the Problem

- Sketch the wheel and axle.
- Sketch the force vectors.

Known:		**Unknown:**	
r_e = 4.00 cm	e = 95.0%	IMA = ?	F_r = ?
r_r = 35.6 cm	d_r = 14.0 cm	MA = ?	d_e = ?
F_e = 155 N			

2 Solve for the Unknown

a. Solve for *IMA*.

$$IMA = \frac{r_e}{r_r}$$ For a wheel-and-axle machine, *IMA* is equal to the ratio of radii.

$$= \frac{4.00\text{ cm}}{35.6\text{ cm}}$$ Substitute r_e = 4.00 cm, r_r = 35.6 cm

$$= 0.112$$

b. Solve for *MA*.

$$e = \frac{MA}{IMA} \times 100$$

$$MA = \left(\frac{e}{100}\right) \times IMA$$

$$= \left(\frac{95.0}{100}\right) \times 0.112$$ Substitute e = 95.0%, IMA = 0.112

$$= 0.106$$

c. Solve for force.

$$MA = \frac{F_r}{F_e}$$

$$F_r = (MA)(F_e)$$

$$= (0.106)(155\text{ N})$$ Substitute MA = 0.106, F_e = 155 N

$$= 16.4\text{ N}$$

d. Solve for distance.

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$$IMA = \frac{d_e}{d_r}$$

$$d_e = (IMA)(d_r)$$

$$= (0.112)(14.0\text{ cm})$$ Substitute IMA = 0.112, d_r = 14.0 cm

$$= 1.57\text{ cm}$$

3 Evaluate the Answer

- **Are the units correct?** Force is measured in newtons and distance in centimeters.
- **Is the magnitude realistic?** *IMA* is low for a bicycle because a greater F_e is traded for a greater d_r. *MA* is always smaller than *IMA*. Because *MA* is low, F_r also will be low. The small distance the axle moves results in a large distance covered by the wheel. Thus, d_e should be very small.

PRACTICE Problems

Additional Problems, Appendix B

24. If the gear radius in the bicycle in Example Problem 4 is doubled, while the force exerted on the chain and the distance the wheel rim moves remain the same, what quantities change, and by how much?

25. A sledgehammer is used to drive a wedge into a log to split it. When the wedge is driven 0.20 m into the log, the log is separated a distance of 5.0 cm. A force of 1.7×10^4 N is needed to split the log, and the sledgehammer exerts a force of 1.1×10^4 N.

a. What is the *IMA* of the wedge?

b. What is the *MA* of the wedge?

c. Calculate the efficiency of the wedge as a machine.

26. A worker uses a pulley system to raise a 24.0-kg carton 16.5 m, as shown in **Figure 10-14.** A force of 129 N is exerted, and the rope is pulled 33.0 m.

a. What is the *MA* of the pulley system?

b. What is the efficiency of the system?

27. You exert a force of 225 N on a lever to raise a 1.25×10^3-N rock a distance of 13 cm. If the efficiency of the lever is 88.7 percent, how far did you move your end of the lever?

28. A winch has a crank with a 45-cm radius. A rope is wrapped around a drum with a 7.5-cm radius. One revolution of the crank turns the drum one revolution.

a. What is the ideal mechanical advantage of this machine?

b. If, due to friction, the machine is only 75 percent efficient, how much force would have to be exerted on the handle of the crank to exert 750 N of force on the rope?

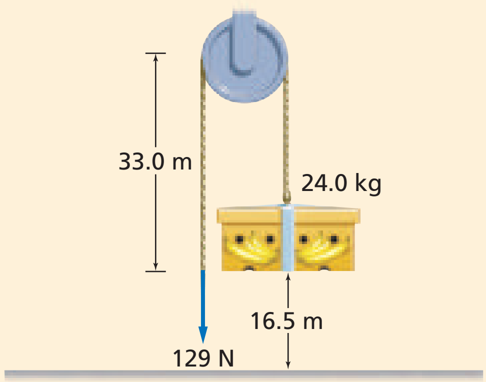


Figure 10-14

Multi-gear bicycle On a multi-gear bicycle, the rider can change the *MA* of the machine by choosing the size of one or both gears. When accelerating or climbing a hill, the rider increases the ideal mechanical advantage to increase the force that the wheel exerts on the road. To increase the *IMA*, the rider needs to make the rear gear radius large compared to the front gear radius (refer to the *IMA* equation on page 270). For the same force exerted by the rider, a larger force is exerted by the wheel on the road. However, the rider must rotate the pedals through more turns for each revolution of the wheel.

On the other hand, less force is needed to ride the bicycle at high speed on a level road. The rider needs to choose a gear that has a small rear gear and a large front gear that will result in a smaller *IMA*. Thus, for the same force exerted by the rider, a smaller force is exerted by the wheel on the road. However, in return, the rider does not have to move the pedals as far for each revolution of the wheel.

An automobile transmission works in the same way. To accelerate a car from rest, large forces are needed and the transmission increases the *IMA*. At high speeds, however, the transmission reduces the *IMA* because smaller forces are needed. Even though the speedometer shows a high speed, the tachometer indicates the engine's low angular speed.

The Human Walking Machine

Movement of the human body is explained by the same principles of force and work that describe all motion. Simple machines, in the form of levers, give humans the ability to walk and run. The lever systems of the human body are complex. However each system has the following four basic parts.

Biology Connection

1. a rigid bar (bone)
2. a source of force (muscle contraction)
3. a fulcrum or pivot (movable joints between bones)
4. a resistance (the weight of the body or an object being lifted or moved)

Figure 10-15 shows the parts of the lever system in a human leg. Lever systems of the body are not very efficient, and mechanical advantages are low. This is why walking and jogging require energy (burn calories) and help people lose weight.

When a person walks, the hip acts as a fulcrum and moves through the arc of a circle, centered on the foot. The center of mass of the body moves as a resistance around the fulcrum in the same arc. The length of the radius of the circle is the length of the lever formed by the bones of the leg. Athletes in walking races increase their velocity by swinging their hips upward to increase this radius. A tall person's body has lever systems with less mechanical advantage than a short person's does. Although tall people usually can walk faster than short people can, a tall person must apply a greater force to move the longer lever formed by the leg bones. How would a tall person do in a walking race? What are the factors that affect a tall person's performance? Walking races are usually 20 or 50 km long. Because of the inefficiency of their lever systems and the length of a walking race, very tall people rarely have the stamina to win.

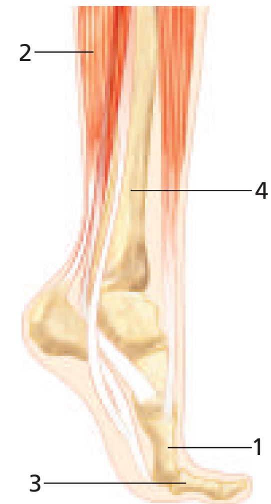


Figure 10-15 The human walking machine.

10.2 Section Review

29. Simple Machines Classify the tools below as a lever, a wheel and axle, an inclined plane, a wedge, or a pulley.

a. screwdriver
b. pliers
c. chisel
d. nail puller

30. *IMA* A worker is testing a multiple pulley system to estimate the heaviest object that he could lift. The largest downward force he could exert is equal to his weight, 875 N. When the worker moves the rope 1.5 m, the object moves 0.25 m. What is the heaviest object that he could lift?

31. Compound Machines A winch has a crank on a 45-cm arm that turns a drum with a 7.5-cm radius through a set of gears. It takes three revolutions of the crank to rotate the drum through one revolution. What is the *IMA* of this compound machine?

32. Efficiency Suppose you increase the efficiency of a simple machine. Do the *MA* and *IMA* increase, decrease, or remain the same?

33. Critical Thinking The mechanical advantage of a multi-gear bicycle is changed by moving the chain to a suitable rear gear.

a. To start out, you must accelerate the bicycle, so you want to have the bicycle exert the greatest possible force. Should you choose a small or large gear?

b. As you reach your traveling speed, you want to rotate the pedals as few times as possible. Should you choose a small or large gear?

c. Many bicycles also let you choose the size of the front gear. If you want even more force to accelerate while climbing a hill, would you move to a larger or smaller front gear?

PHYSICS LAB

Stair Climbing and Power

Can you estimate the power you develop as you climb a flight of stairs? Climbing stairs requires energy. As the weight of the body moves through a distance, work is done. Power is a measure of the rate at which work is done. In this activity you will try to maximize the power you develop by applying a vertical force up a flight of stairs over a period of time.

QUESTION

What can you do to increase the power you develop as you climb a flight of stairs?

Objectives

- **Predict** the factors that affect power.
- **Calculate** the power developed.
- **Define** power operationally.
- **Interpret** force, distance, work, time and power data.
- **Make and use graphs** of work versus time, power versus force, and power versus time.

Safety Precautions

- **Avoid wearing loose clothing.**

Materials

meterstick (or tape measure)
stopwatch
bathroom scale

Procedure

1. Measure and record the mass of each person in your group using a bathroom scale. If the scale does not have kilogram units, convert the weight in pounds to kilograms. Recall that 2.2 lbs = 1 kg.
2. Measure the distance from the floor to the top of the flight of stairs you will climb. Record it in the data table.
3. Have each person in your group climb the flight of stairs in a manner that he or she thinks will maximize the power developed.
4. Use your stopwatch to measure the time it takes each person to perform this task. Record your data in the data table.

Horizons Companies

Data Table					
Mass (kg)	Weight (N)	Distance (m)	Work Done (J)	Time (s)	Power Generated (W)

Analyze

1. **Calculate** Find each person's weight in newtons and record it in the data table.
2. Calculate the work done by each person.
3. Calculate the power developed by each person in your group as he or she climbs the flight of stairs.
4. **Make and Use Graphs** Use the data you calculated to draw a graph of work versus time and draw the best-fit line.
5. Draw a graph of power versus work and draw the best-fit line.
6. Draw a graph of power versus time and draw the best-fit line.

Conclude and Apply

1. Did each person in your group have the same power rating? Why or why not?
2. Which graph(s) showed a definite relationship between the two variables?
3. Explain why this relationship exists.
4. Write an operational definition of power.

Going Further

1. What three things can be done to increase the power you develop while climbing the flight of stairs?
2. Why were the fastest climbers not necessarily the ones who developed the most power?
3. Why were the members of your group with more mass not necessarily the ones who developed the most power?
4. Compare and contrast your data with those of other groups in your class.

Real-World Physics

1. Research a household appliance that has a power rating equal to or less than the power you developed by climbing the stairs.
2. Suppose an electric power company in your area charges $0.06/kWh. If you charged the same amount for the power you develop climbing stairs, how much money would you earn by climbing stairs for 1 h?
3. If you were designing a stair climbing machine for the local health club, what information would you need to collect? You decide that you will design a stair climbing machine with the ability to calculate the power developed. What information would you have the machine collect in order to let the climber know how much power he or she developed?

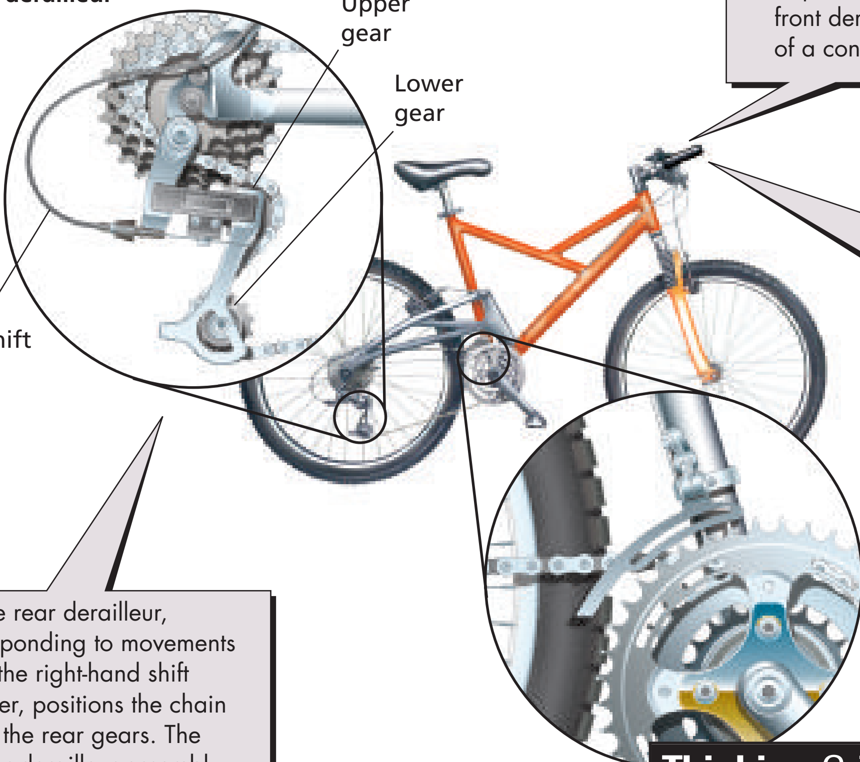
Physics Online

To find out more about energy, work, and simple machines, visit the Web site: **physicspp.com**

How it Works Bicycle Gear Shifters

In a multispeed bicycle with two or three front gears and from five to eight rear gears, front and rear derailleurs (shifters) are employed to position the chain. Changing the combination of front and rear gears varies the *IMA* of the system. A larger *IMA* reduces effort in climbing hills. A lower *IMA* allows for greater speed on level ground, but more effort is required.

$$IMA = \frac{\text{number of teeth on rear gear}}{\text{number of teeth on front gear}}$$



1 The left-hand shift lever adjusts the position of the front derailleur by means of a connecting cable.

2 The right-hand shift lever adjusts the position of the rear derailleur by means of a wire cable routed along the frame from the shift lever to the derailleur.

3 The front derailleur lifts the chain off of the gear teeth and moves it to another gear near it in response to the tightening or loosening of the cable that connects it to the left-hand shift lever.

4 The rear derailleur, responding to movements of the right-hand shift lever, positions the chain on the rear gears. The rear derailleur assembly includes two small gears. The lower gear, tensioned by a spring, keeps the chain tight as it moves from larger to smaller gears. The upper gear moves in or out to position the chain on the gears next to it.

Thinking Critically

1. **Calculate** What is the *IMA* of a multispeed bicycle in the following instances?
   a. when the chain is set on a front gear with 52 teeth and a rear gear with 14 teeth
   b. when the chain is set on a front gear with 42 teeth and a rear gear with 34 teeth
2. **Apply** Which setting in the previous problem, **a** or **b,** would you select to race with a friend on level ground? To climb a steep hill?

Chapter 10

Study Guide

10.1 Energy and Work

Vocabulary

- work (p. 258)
- energy (p. 258)
- kinetic energy (p. 258)
- work-energy theorem (p. 258)
- joule (p. 259)
- power (p. 263)
- watt (p. 263)

Key Concepts

- Work is the transfer of energy by mechanical means.

$$W = Fd$$

- A moving object has kinetic energy.

$$KE = \frac{1}{2}mv^2$$

- The work done on a system is equal to the change in energy of the system.

$$W = \Delta KE$$

- Work is the product of the force exerted on an object and the distance the object moves in the direction of the force.

$$W = Fd \cos \theta$$

- The work done can be determined by calculating the area under a force-displacement graph.
- Power is the rate of doing work, that is the rate at which energy is transferred.

$$P = \frac{W}{t}$$

10.2 Machines

Vocabulary

- machine (p. 266)
- effort force (p. 266)
- resistance force (p. 266)
- mechanical advantage (p. 266)
- ideal mechanical advantage (p. 267)
- efficiency (p. 268)
- compound machine (p. 269)

Key Concepts

- Machines, whether powered by engines or humans, do not change the amount of work done, but they do make the task easier.
- A machine eases the load, either by changing the magnitude or the direction of the force exerted to do work.
- The mechanical advantage, *MA*, is the ratio of resistance force to effort force.

$$MA = \frac{F_r}{F_e}$$

- The ideal mechanical advantage, *IMA*, is the ratio of the distances moved.

$$IMA = \frac{d_e}{d_r}$$

- The efficiency of a machine is the ratio of output work to input work.

$$e = \frac{W_o}{W_i} \times 100$$

- In all real machines, *MA* is less than *IMA*.
- The efficiency of a machine can by found from the real and ideal mechanical advantages.

$$e = \frac{MA}{IMA} \times 100$$

Chapter 10

Assessment

Concept Mapping

34. Create a concept map using the following terms: *force, displacement, direction of motion, work, change in kinetic energy.*

Mastering Concepts

35. In what units is work measured? (10.1)

36. Suppose a satellite revolves around Earth in a circular orbit. Does Earth's gravity do any work on the satellite? (10.1)

37. An object slides at constant speed on a frictionless surface. What forces act on the object? What work is done by each force? (10.1)

38. Define *work* and *power.* (10.1)

39. What is a watt equivalent to in terms of kilograms, meters, and seconds? (10.1)

40. Is it possible to get more work out of a machine than you put into it? (10.2)

41. Explain how the pedals of a bicycle are a simple machine. (10.2)

Applying Concepts

42. Which requires more work, carrying a 420-N backpack up a 200-m-high hill or carrying a 210-N backpack up a 400-m-high hill? Why?

43. Lifting You slowly lift a box of books from the floor and put it on a table. Earth's gravity exerts a force, magnitude mg, downward, and you exert a force, magnitude mg, upward. The two forces have equal magnitudes and opposite directions. It appears that no work is done, but you know that you did work. Explain what work was done.

44. You have an after-school job carrying cartons of new copy paper up a flight of stairs, and then carrying recycled paper back down the stairs. The mass of the paper does not change. Your physics teacher says that you do not work all day, so you should not be paid. In what sense is the physics teacher correct? What arrangement of payments might you make to ensure that you are properly compensated?

45. You carry the cartons of copy paper down the stairs, and then along a 15-m-long hallway. Are you working now? Explain.

46. Climbing Stairs Two people of the same mass climb the same flight of stairs. The first person climbs the stairs in 25 s; the second person does so in 35 s.

a. Which person does more work? Explain your answer.

b. Which person produces more power? Explain your answer.

47. Show that power delivered can be written as $P = Fv \cos \theta$.

48. How can you increase the ideal mechanical advantage of a machine?

49. Wedge How can you increase the mechanical advantage of a wedge without changing its ideal mechanical advantage?

50. Orbits Explain why a planet orbiting the Sun does not violate the work-energy theorem.

51. Claw Hammer A claw hammer is used to pull a nail from a piece of wood, as shown in **Figure 10-16.** Where should you place your hand on the handle and where should the nail be located in the claw to make the effort force as small as possible?

Figure 10-16

Mastering Problems

10.1 Energy and Work

52. The third floor of a house is 8 m above street level. How much work is needed to move a 150-kg refrigerator to the third floor?

53. Haloke does 176 J of work lifting himself 0.300 m. What is Haloke's mass?

54. Football After scoring a touchdown, an 84.0-kg wide receiver celebrates by leaping 1.20 m off the ground. How much work was done by the wide receiver in the celebration?

55. Tug-of-War During a tug-of-war, team A does 2.20×10^5 J of work in pulling team B 8.00 m. What force was team A exerting?

56. To keep a car traveling at a constant velocity, a 551-N force is needed to balance frictional forces. How much work is done against friction by the car as it travels from Columbus to Cincinnati, a distance of 161 km?

For more problems, go to Additional Problems, Appendix B.
Hutchings Photography

57. Cycling A cyclist exerts a force of 15.0 N as he rides a bike 251 m in 30.0 s. How much power does the cyclist develop?

58. A student librarian lifts a 2.2-kg book from the floor to a height of 1.25 m. He carries the book 8.0 m to the stacks and places the book on a shelf that is 0.35 m above the floor. How much work does he do on the book?

59. A force of 300.0 N is used to push a 145-kg mass 30.0 m horizontally in 3.00 s.
a. Calculate the work done on the mass.
b. Calculate the power developed.

60. Wagon A wagon is pulled by a force of 38.0 N exerted on the handle at an angle of 42.0° with the horizontal. If the wagon is pulled in a circle of radius 25.0 m, how much work is done?

61. Lawn Mower Shani is pushing a lawn mower with a force of 88.0 N along a handle that makes an angle of 41.0° with the horizontal. How much work is done by Shani in moving the lawn mower 1.2 km to mow the yard?

62. A 17.0-kg crate is to be pulled a distance of 20.0 m, requiring 1210 J of work to be done. If the job is done by attaching a rope and pulling with a force of 75.0 N, at what angle is the rope held?

63. Lawn Tractor A 120-kg lawn tractor, shown in **Figure 10-17,** goes up a 21° incline that is 12.0 m long in 2.5 s. Calculate the power that is developed by the tractor.

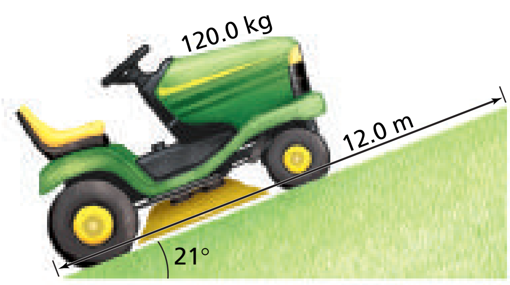


Figure 10-17

64. You slide a crate up a ramp at an angle of 30.0° by exerting a 225-N force parallel to the ramp. The crate moves at a constant speed. The coefficient of friction is 0.28. How much work did you do on the crate as it was raised a vertical distance of 1.15 m?

65. Piano A 4.2×10^3-N piano is to be slid up a 3.5-m frictionless plank at a constant speed. The plank makes an angle of 30.0° with the horizontal. Calculate the work done by the person sliding the piano up the plank.

66. Sled Diego pulls a 4.5-kg sled across level snow with a force of 225 N on a rope that is 35.0° above the horizontal, as shown in **Figure 10-18.** If the sled moves a distance of 65.3 m, how much work does Diego do?

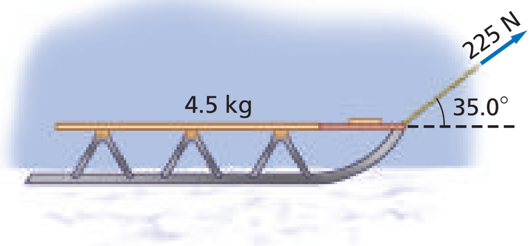


Figure 10-18

67. Escalator Sau-Lan has a mass of 52 kg. She rides up the escalator at Ocean Park in Hong Kong. This is the world's longest escalator, with a length of 227 m and an average inclination of 31°. How much work does the escalator do on Sau-Lan?

68. Lawn Roller A lawn roller is pushed across a lawn by a force of 115 N along the direction of the handle, which is 22.5° above the horizontal. If 64.6 W of power is developed for 90.0 s, what distance is the roller pushed?

69. John pushes a crate across the floor of a factory with a horizontal force. The roughness of the floor changes, and John must exert a force of 20 N for 5 m, then 35 N for 12 m, and then 10 N for 8 m.
a. Draw a graph of force as a function of distance.
b. Find the work John does pushing the crate.

70. Maricruz slides a 60.0-kg crate up an inclined ramp that is 2.0-m long and attached to a platform 1.0 m above floor level, as shown in **Figure 10-19.** A 400.0-N force, parallel to the ramp, is needed to slide the crate up the ramp at a constant speed.
a. How much work does Maricruz do in sliding the crate up the ramp?
b. How much work would be done if Maricruz simply lifted the crate straight up from the floor to the platform?

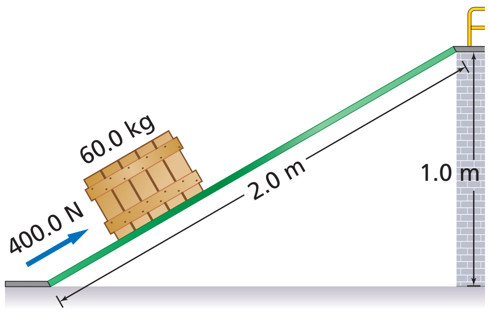


Figure 10-19

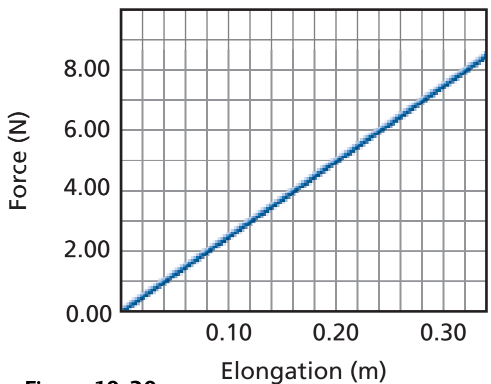
71. Boat Engine An engine moves a boat through the water at a constant speed of 15 m/s. The engine must exert a force of 6.0 kN to balance the force that the water exerts against the hull. What power does the engine develop?

72. In **Figure 10-20,** the magnitude of the force necessary to stretch a spring is plotted against the distance the spring is stretched.

a. Calculate the slope of the graph, k, and show that $F = kd$, where $k = 25$ N/m.

b. Find the amount of work done in stretching the spring from 0.00 m to 0.20 m by calculating the area under the graph from 0.00 m to 0.20 m.

c. Show that the answer to part **b** can be calculated using the formula $W = \frac{1}{2}kd^2$, where W is the work, $k = 25$ N/m (the slope of the graph), and d is the distance the spring is stretched (0.20 m).



■ **Figure 10-20**

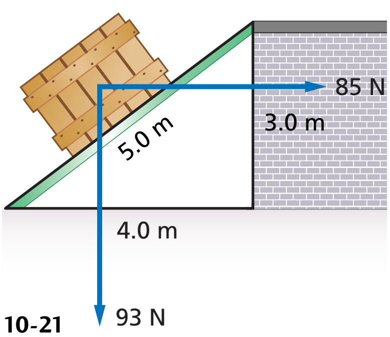
73. Use the graph in Figure 10-20 to find the work needed to stretch the spring from 0.12 m to 0.28 m.

74. A worker pushes a crate weighing 93 N up an inclined plane. The worker pushes the crate horizontally, parallel to the ground, as illustrated in **Figure 10-21.**

a. The worker exerts a force of 85 N. How much work does he do?

b. How much work is done by gravity? (Be careful with the signs you use.)

c. The coefficient of friction is $\mu = 0.20$. How much work is done by friction? (Be careful with the signs you use.)



■ **Figure 10-21**

75. Oil Pump In 35.0 s, a pump delivers 0.550 m^3 of oil into barrels on a platform 25.0 m above the intake pipe. The oil's density is 0.820 g/cm^3.

a. Calculate the work done by the pump.

b. Calculate the power produced by the pump.

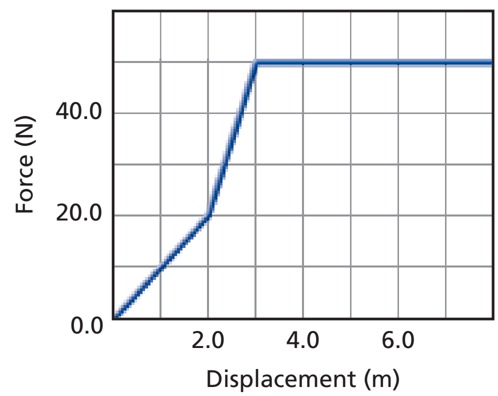
76. Conveyor Belt A 12.0-m-long conveyor belt, inclined at 30.0°, is used to transport bundles of newspapers from the mail room up to the cargo bay to be loaded onto delivery trucks. Each newspaper has a mass of 1.0 kg, and there are 25 newspapers per bundle. Determine the power that the conveyor develops if it delivers 15 bundles per minute.

77. A car is driven at a constant speed of 76 km/h down a road. The car's engine delivers 48 kW of power. Calculate the average force that is resisting the motion of the car.

78. The graph in **Figure 10-22** shows the force and displacement of an object being pulled.

a. Calculate the work done to pull the object 7.0 m.

b. Calculate the power that would be developed if the work was done in 2.0 s.



■ **Figure 10-22**

10.2 Machines

79. Piano Takeshi raises a 1200-N piano a distance of 5.00 m using a set of pulleys. He pulls in 20.0 m of rope.

a. How much effort force would Takeshi apply if this were an ideal machine?

b. What force is used to balance the friction force if the actual effort is 340 N?

c. What is the output work?

d. What is the input work?

e. What is the mechanical advantage?

80. Lever Because there is very little friction, the lever is an extremely efficient simple machine. Using a 90.0-percent-efficient lever, what input work is required to lift an 18.0-kg mass through a distance of 0.50 m?

For more problems, go to Additional Problems, Appendix B.

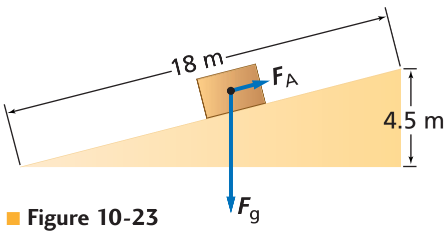
81. A pulley system lifts a 1345-N weight a distance of 0.975 m. Paul pulls the rope a distance of 3.90 m, exerting a force of 375 N.
a. What is the ideal mechanical advantage of the system?
b. What is the mechanical advantage?
c. How efficient is the system?

82. A force of 1.4 N is exerted through a distance of 40.0 cm on a rope in a pulley system to lift a 0.50-kg mass 10.0 cm. Calculate the following.
a. the *MA*
b. the *IMA*
c. the efficiency

83. A student exerts a force of 250 N on a lever, through a distance of 1.6 m, as he lifts a 150-kg crate. If the efficiency of the lever is 90.0 percent, how far is the crate lifted?

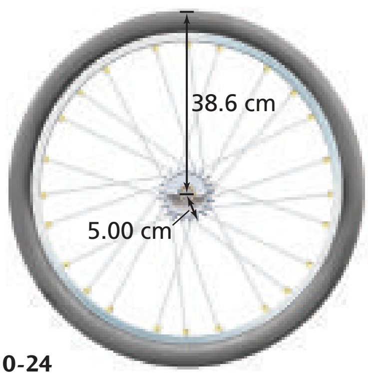
84. What work is required to lift a 215-kg mass a distance of 5.65 m, using a machine that is 72.5 percent efficient?

85. The ramp in **Figure 10-23** is 18 m long and 4.5 m high.
a. What force, parallel to the ramp (F_A), is required to slide a 25-kg box at constant speed to the top of the ramp if friction is disregarded?
b. What is the *IMA* of the ramp?
c. What are the real *MA* and the efficiency of the ramp if a parallel force of 75 N is actually required?



■ **Figure 10-23**

86. Bicycle Luisa pedals a bicycle with a gear radius of 5.00 cm and a wheel radius of 38.6 cm, as shown in **Figure 10-24.** If the wheel revolves once, what is the length of the chain that was used?



■ **Figure 10-24**

87. Crane A motor with an efficiency of 88 percent operates a crane with an efficiency of 42 percent. If the power supplied to the motor is 5.5 kW, with what constant speed does the crane lift a 410-kg crate of machine parts?

88. A compound machine is constructed by attaching a lever to a pulley system. Consider an ideal compound machine consisting of a lever with an *IMA* of 3.0 and a pulley system with an *IMA* of 2.0.
a. Show that the IMA of this compound machine is 6.0.
b. If the compound machine is 60.0 percent efficient, how much effort must be applied to the lever to lift a 540-N box?
c. If you move the effort side of the lever 12.0 cm, how far is the box lifted?

Mixed Review

89. Ramps Isra has to get a piano onto a 2.0-m-high platform. She can use a 3.0-m-long frictionless ramp or a 4.0-m-long frictionless ramp. Which ramp should Isra use if she wants to do the least amount of work?

90. Brutus, a champion weightlifter, raises 240 kg of weights a distance of 2.35 m.
a. How much work is done by Brutus lifting the weights?
b. How much work is done by Brutus holding the weights above his head?
c. How much work is done by Brutus lowering them back to the ground?
d. Does Brutus do work if he lets go of the weights and they fall back to the ground?
e. If Brutus completes the lift in 2.5 s, how much power is developed?

91. A horizontal force of 805 N is needed to drag a crate across a horizontal floor with a constant speed. You drag the crate using a rope held at an angle of 32°.
a. What force do you exert on the rope?
b. How much work do you do on the crate if you move it 22 m?
c. If you complete the job in 8.0 s, what power is developed?

92. Dolly and Ramp A mover's dolly is used to transport a refrigerator up a ramp into a house. The refrigerator has a mass of 115 kg. The ramp is 2.10 m long and rises 0.850 m. The mover pulls the dolly with a force of 496 N up the ramp. The dolly and ramp constitute a machine.
a. What work does the mover do?
b. What is the work done on the refrigerator by the machine?
c. What is the efficiency of the machine?

93. Sally does 11.4 kJ of work dragging a wooden crate 25.0 m across a floor at a constant speed. The rope makes an angle of 48.0° with the horizontal.
a. How much force does the rope exert on the crate?
b. What is the force of friction acting on the crate?
c. What work is done by the floor through the force of friction between the floor and the crate?

94. **Sledding** An 845-N sled is pulled a distance of 185 m. The task requires 1.20×10^4 J of work and is done by pulling on a rope with a force of 125 N. At what angle is the rope held?

95. An electric winch pulls a 875-N crate up a 15° incline at 0.25 m/s. The coefficient of friction between the crate and incline is 0.45.
a. What power does the winch develop?
b. If the winch is 85 percent efficient, what is the electrical power that must be delivered to the winch?

Thinking Critically

96. **Analyze and Conclude** You work at a store, carrying boxes to a storage loft that is 12 m above the ground. You have 30 boxes with a total mass of 150 kg that must be moved as quickly as possible, so you consider carrying more than one up at a time. If you try to move too many at once, you know that you will go very slowly, resting often. If you carry only one box at a time, most of the energy will go into raising your own body. The power (in watts) that your body can develop over a long time depends on the mass that you carry, as shown in **Figure 10-25.** This is an example of a power curve that applies to machines as well as to people. Find the number of boxes to carry on each trip that would minimize the time required. What time would you spend doing the job? Ignore the time needed to go back down the stairs and to lift and lower each box.

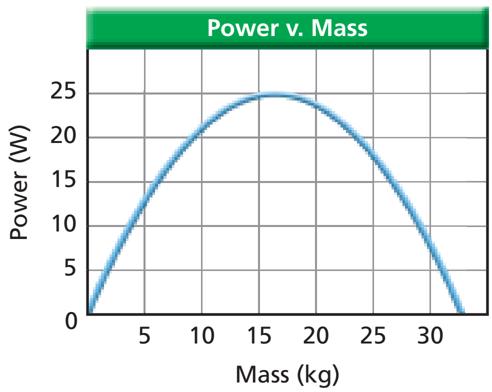


Figure 10-25

97. **Apply Concepts** A sprinter of mass 75 kg runs the 50.0-m dash in 8.50 s. Assume that the sprinter's acceleration is constant throughout the race.
a. What is the average power of the sprinter over the 50.0 m?
b. What is the maximum power generated by the sprinter?
c. Make a quantitative graph of power versus time for the entire race.

98. **Apply Concepts** The sprinter in the previous problem runs the 50.0-m dash in the same time, 8.50 s. However, this time the sprinter accelerates in the first second and runs the rest of the race at a constant velocity.
a. Calculate the average power produced for that first second.
b. What is the maximum power that the sprinter now generates?

Writing in Physics

99. Just as a bicycle is a compound machine, so is an automobile. Find the efficiencies of the component parts of the power train (engine, transmission, wheels, and tires). Explore possible improvements in each of these efficiencies.

100. The terms *force, work, power,* and *energy* often mean the same thing in everyday use. Obtain examples from advertisements, print media, radio, and television that illustrate meanings for these terms that differ from those used in physics.

Cumulative Review

101. You are helping your grandmother with some gardening and have filled a garbage can with weeds and soil. Now you have to move the garbage can across the yard and realize it is so heavy that you will need to push it, rather than lift it. If the can has a mass of 24 kg, the coefficient of kinetic friction between the can's bottom and the muddy grass is 0.27, and the static coefficient of friction between those same surfaces is 0.35, how hard do you have to push horizontally to get the can to just start moving? (Chapter 5)

102. **Baseball** If a major league pitcher throws a fastball horizontally at a speed of 40.3 m/s (90 mph) and it travels 18.4 m (60 ft, 6 in), how far has it dropped by the time it crosses home plate? (Chapter 6)

103. People sometimes say that the Moon stays in its orbit because the "centrifugal force just balances the centripetal force, giving no net force." Explain why this idea is wrong. (Chapter 8)

 For more problems, go to Additional Problems, Appendix B.

Standardized Test Practice

Multiple Choice

1. A pulley system consists of two fixed pulleys and two movable pulleys that lift a load that has a weight of 300 N. If the effort force used to lift the load is 100 N, what is the mechanical advantage of the system?

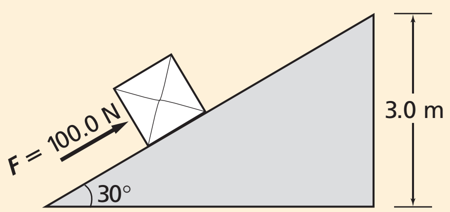
Ⓐ $\frac{1}{3}$ Ⓒ 3

Ⓑ $\frac{3}{4}$ Ⓓ 6

2. The box in the diagram is being pushed up the ramp with a force of 100.0 N. If the height of the ramp is 3.0 m, what is the work done on the box? ($\sin 30° = 0.50$, $\cos 30° = 0.87$, $\tan 30° = 0.58$)

Ⓐ 150 J Ⓒ 450 J

Ⓑ 260 J Ⓓ 600 J



3. A compound machine used to raise heavy boxes consists of a ramp and a pulley. The efficiency of pulling a 100-kg box up the ramp is 50%. If the efficiency of the pulley is 90%, what is the overall efficiency of the compound machine?

Ⓐ 40% Ⓒ 50%

Ⓑ 45% Ⓓ 70%

4. A skater with a mass of 50.0 kg slides across an icy pond with negligible friction. As he approaches a friend, both he and his friend hold out their hands, and the friend exerts a force in the direction opposite to the skater's movement, which slows the skater's speed from 2.0 m/s^2 to 1.0 m/s^2. What is the change in the skater's kinetic energy?

Ⓐ 25 J Ⓒ 100 J

Ⓑ 75 J Ⓓ 150 J

5. A 20.0-N block is attached to the end of a rope, and the rope is looped around a pulley system. If you pull the opposite end of the rope a distance of 2.00 m, the pulley system raises the block a distance of 0.40 m. What is the pulley system's ideal mechanical advantage?

Ⓐ 2.5 Ⓒ 5.0

Ⓑ 4.0 Ⓓ 10.0

6. Two people carry identical 40.0-N boxes up a ramp. The ramp is 2.00 m long and rests on a platform that is 1.00 m high. One person walks up the ramp in 2.00 s, and the other person walks up the ramp in 4.00 s. What is the difference in power the two people use to carry the boxes up the ramp?

Ⓐ 5.00 W Ⓒ 20.0 W

Ⓑ 10.0 W Ⓓ 40.0 W

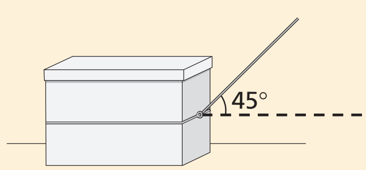
7. A 4-N soccer ball sits motionless on a field. A player's foot exerts a force of 5 N on the ball for a distance of 0.1 m, and the ball rolls a distance of 10 m. How much kinetic energy does the ball gain from the player?

Ⓐ 0.5 J Ⓒ 9 J

Ⓑ 0.9 J Ⓓ 50 J

Extended Answer

8. The diagram shows a box being pulled by a rope with a force of 200.0 N along a horizontal surface. The angle the rope makes with the horizontal is 45°. Calculate the work done on the box and the power required to pull it a distance of 5.0 m in 10.0 s. ($\sin 45° = \cos 45° = 0.7$)



✓ Test-Taking TIP

Beat the Clock and then Go Back

As you take a practice test, pace yourself to finish each section just a few minutes early so you can go back and check over your work.