d is the grating spacing of the object. The minimum value of d occurs when $i = \alpha$ and $d(\sin i + \sin \alpha) = \lambda$.

$$\therefore \quad 2d \sin \alpha = \lambda$$

$$\therefore \quad d = \frac{\lambda}{2 \sin \alpha}$$

This expression for d gives the grating spacing of the finest regular structure of the object which can just be resolved. If a medium such as oil of refractive index n is used in the object space beneath the objective,

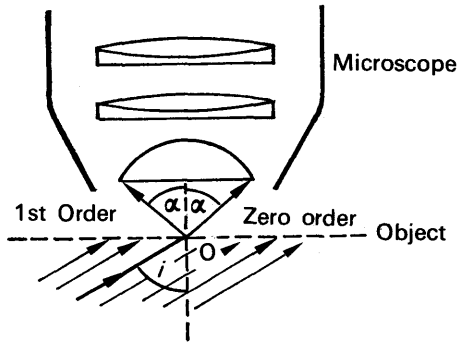


FIG. 29.30. Revolving power of microscope.

the least distance d or *limit of resolution* (also called the "*resolving power*") is:

$$\text{limit of resolution} = \frac{\lambda}{2n \sin \alpha}$$

The use of an oil-immersion objective was suggested by Abbe. The limit of resolution for the best optical microscopes is about 2×10^{-5} cm. The eye can resolve about 0·01 cm. The largest useful magnifying power of a microscope is one which magnifies the limit of resolution of the objective to that of the eye, and is about 1000 with glass lenses and visible light. Higher resolving powers may be obtained with ultra-violet light, from $\lambda/2n \sin \alpha$. An *electron microscope*, which contains electron lenses and utilises electrons in place of light, has a limit of resolution less than 10^{-7} cm owing to the much shorter wavelength of moving electrons compared with that of light (p. 1077). Much larger useful magnifying powers, such as 100 000, are thus obtained by using electron microscopes in place of optical microscopes.

Wavelengths of Electromagnetic Waves

In this book we have encountered rays which affect the sensation of vision (*visible rays*), rays which cause heat (*infra-red rays*, p. 456), and rays which cause chemical action (ultra-violet rays, p. 456). As these rays are all due to electric and magnetic vibrations they are examples of **electromagnetic** waves (see p. 719). Scientists have measured the wavelengths of these waves by a diffraction grating method, and results show a gradual transition in the magnitudes of the wavelength from one type of ray to another. Thus infra-red rays have a longer wavelength

than visible rays, which in turn have a longer wavelength than ultra-violet rays. Radio waves are electromagnetic waves of longer wavelength

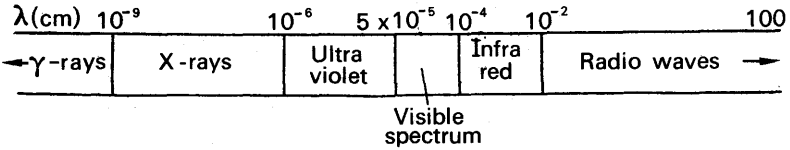


FIG. 29.31. Spectrum of electromagnetic waves (not to scale).

than infra-red rays, while X-rays and γ-rays are due to waves of shorter wavelength than ultra-violet waves. The whole spectrum of electromagnetic waves are shown in Fig. 29.31; this gives only an approximate value of the limits of the wavelength in the various parts of the spectrum, because these limits are themselves vague.

EXAMPLE

What is meant in optics by (*a*) interference, (*b*) diffraction? What part do each of these phenomena play in the production of spectra by a diffraction grating? A parallel beam of sodium light is incident normally on a diffraction grating. The angle between the two first order spectra on either side of the normal is 27° 42′. Assuming that the wavelength of the light is 5893×10^{-8} cm, find the number of rulings per cm on the grating. (*N.*)

First part. Briefly, interference is the name given to the phenomena obtained by the combined effect of light waves from two separate coherent sources; diffraction is the name given to the phenomena due to the combined effect of light waves from secondary sources on the same wavefront. In the diffraction grating, production of spectra is due to the interference between secondary sources on the same wavefront which are separated by a multiple of d, where d is the spacing of the grating rulings (p. 707).

Second part. The first order spectrum occurs at an angle $\theta = \frac{1}{2} \times 27° 42' = 13° 51'$.

But $$d \sin \theta = \lambda$$

$$\therefore \quad d = \frac{\lambda}{\sin \theta} = \frac{5893 \times 10^{-8}}{\sin 13° 51'} \text{ cm}$$

$$\therefore \text{ number of rulings per cm } = \frac{1}{d} = \frac{\sin 13° 51'}{5893 \times 10^{-8}}$$

$$= 4062$$

POLARISATION OF LIGHT

We have shown that light is a wave-motion of some kind, i.e., that it is a travelling vibration. For a long time after the wave-theory was revived it was thought that the vibrations of light occurred in the same direction as the light wave travelled, analogous to sound waves. Thus light waves were thought to be longitudinal waves (p. 584). Observations and experiments, however, to be described shortly, showed that the vibrations of light occur in planes *perpendicular* to the direction along which the light wave travels, and thus light waves are *transverse* waves.

Polarisation of Transverse Waves

Suppose that a rope ABCD passes through two parallel slits, B, C, and is attached to a fixed point at D, Fig. 29.32 (i). Transverse waves can be set up along AB by holding the end A in the hand and moving it up and down in all directions perpendicular to AB, as illustrated by the arrows in the plane X. A wave then emerges along BC, but unlike the

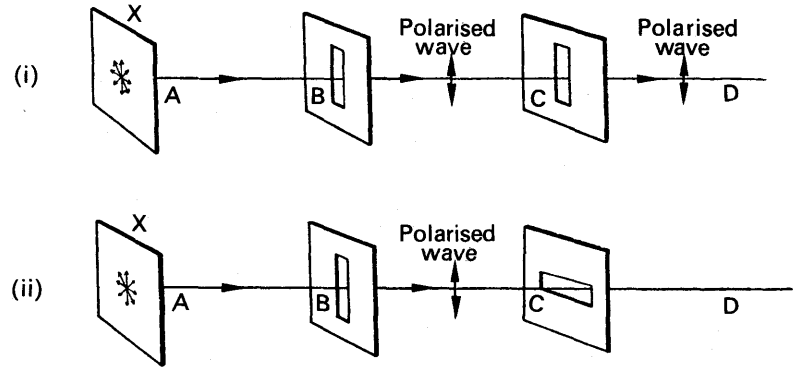


FIG. 29.32. Formation of plane-polarised waves.

waves along AB, which are due to transverse vibrations in every plane, it is due only to transverse vibrations parallel to the slit at B. This type of wave is called a *plane-polarised* wave. It shows a *lack of symmetry about the direction of propagation*, because a slit C allows the wave to pass through when it is parallel to B, but prevents it from passing when C is perpendicular to B, Fig. 29.32 (i), (ii). If B is turned so that it is perpendicular to the position shown in Fig. 29.32 (i), a polarised wave is again obtained along BC; but the vibrations which produce it are perpendicular to those shown between B and C in Fig. 29.32 (i).

Polarised Light

Years ago it was discovered accidentally that certain natural crystals affect light passing through them. *Tourmaline* is an example of such a crystal, *quartz* and *calcite* or *Iceland spar* are others (p. 720). Suppose two tourmaline crystals, P, Q, are placed with their axes, *a*, *b*, parallel, Fig. 29.33 (i). If a beam of light is incident on P, the light emerging from Q appears slightly darker. If Q is rotated slowly about the line of vision, with its plane parallel to P, the emergent light becomes darker and darker, and at one stage it disappears. In the latter case the axes *a*, *b* of the crystals are perpendicular, Fig. 29.33 (ii). When Q is rotated further the light reappears, and becomes brightest when the axes *a*, *b* are again parallel.

This simple experiment leads to the conclusion that light waves are *transverse* waves; otherwise the light emerging from Q could never be extinguished by simply rotating this crystal. The experiment, in fact, is analogous to that illustrated in Fig. 29.32, where transverse waves were set up along a rope and plane-polarised waves were obtained by means of a slit B. Tourmaline is a crystal which, because of its internal molecular

structure, transmits only those vibrations of light parallel to its axis. Consequently plane-polarised light is obtained beyond the crystal P, and no light emerges beyond Q when its axis is perpendicular to P. Fig. 29.33 should be compared with Fig. 29.32.

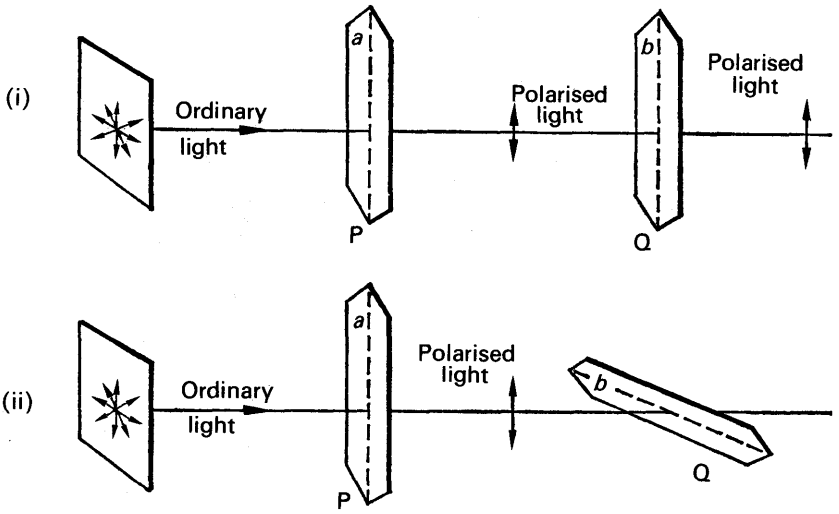


FIG. 29.33. Formation of plane-polarised light waves.

Vibrations in Unpolarised and Polarised Light

Fig. 29.34 (i) is an attempt to represent diagrammatically the vibrations of ordinary or unpolarised light at a point A when a ray travels in a direction AB. X is a plane perpendicular to AB, and ordinary (unpolarised) light may be imagined as due to vibrations which occur in

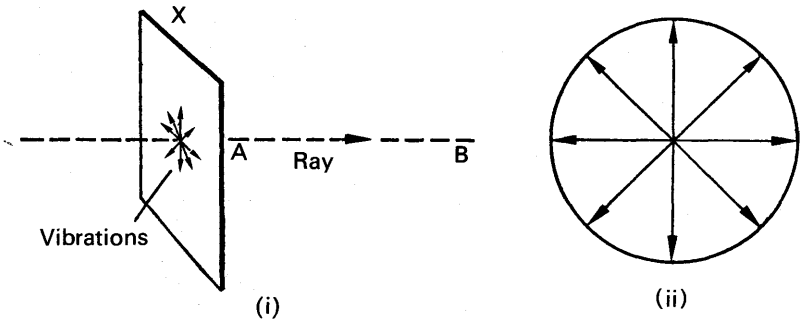


FIG. 29.34. (i). Vibrations occur in every plane perpendicular to AB.
(ii). Vibrations in ordinary light.

every one of the millions of planes which pass through AB and are perpendicular to X. As represented in Fig. 29.34 (ii), the amplitudes of the vibrations are all equal.

Consider the vibrations in ordinary light when it is incident on the tourmaline P in Fig. 29.33 (i). Each vibrations can be resolved into two components, one in a direction parallel to the axis *a* of the tourmaline P and the other in a direction *m* perpendicular to *a*, Fig. 29.35. Tourmaline absorbs the light due to the latter vibrations, known as the *ordinary* rays, allowing the light due to the former vibrations, known as the *extraordinary* rays, to pass through it. Thus plane-polarised light, due to the extraordinary rays, is produced by the tourmaline. Polaroid is a crystaline material, used in sun-glasses for example, which also has selective absorption.

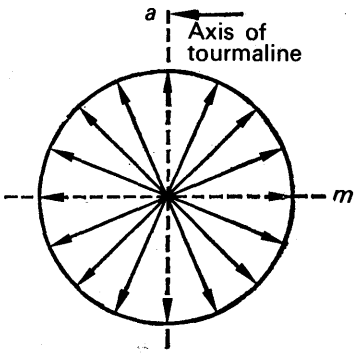


FIG. 29.35. Production of plane-polarised waves by tourmaline.

Light waves are electromagnetic waves. Theory and experiment show that the vibrations of light are *electromagnetic* in origin; a varying electric vector *E* is present, with a varying magnetic vector *B* which has the same frequency and phase. *E* and *B* are perpendicular to each other, and are in a plane at right angles to the ray of light, Fig. 29.36. Experiments have shown that the electric force in a light wave affects a photographic plate and causes fluorescence, while the magnetic force, though present, plays no part in this effect of a light wave. On this account the vibrations of the electric force, *E*, are now chosen as the "vibrations of light", and the planes containing the vibrations shown in Fig. 29.35 (i), (ii) are those in which only the electric forces are present.

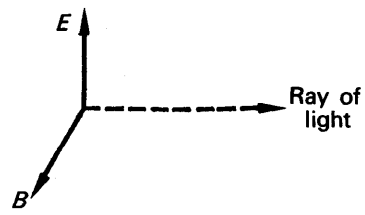


FIG. 29.36. Electromagnetic wave.

Polarised Light by Reflection

The production of polarised light by tourmaline is due to *selective absorption* of the "ordinary" rays. In 1808 MALUS discovered that polarised light is obtained when ordinary light is reflected by a plane sheet of glass (p. 719). The most suitable angle of incidence is about 56°, Fig. 29.37. If the reflected light is viewed through a tourmaline crystal which is slowly rotated about the line of vision, the light is practically extinguished at one position of the crystal. This proves that the light reflected by the glass is plane-polarised. Malus also showed that the light reflected by water is plane-polarised.

The production of the polarised light by the glass is explained as follows. Each of the vibrations of the incident (ordinary) light can be resolved into a component parallel to the glass surface and a component perpendicular to the surface. The light due to the components

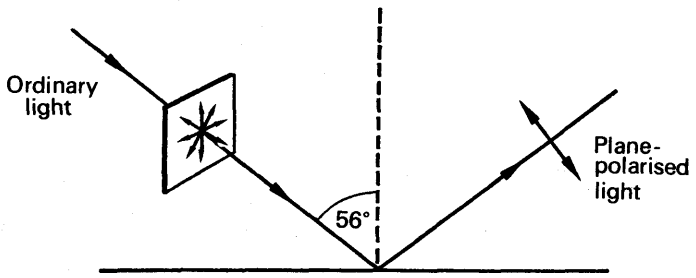


FIG. 29.37. Plane-polarised waves by reflection.

parallel to the glass is reflected, but the remainder of the light, due to the components perpendicular to the glass, is *refracted* into the glass. Thus the light reflected by the glass is plane-polarised.

Brewster's Law. Polarisation by Pile of Plates

The particular angle of incidence i on a transparent medium when the reflected light is almost completely plane-polarized is called the *polarising angle*. BREWSTER found that, in this case, $\tan i = n$, where n is the refractive index of the medium (*Brewster's law*). Since $\sin i/\sin r$, where r is the angle of refraction, it then follows that $\cos i = \sin r$, or $i + r = 90°$. Thus the reflected and refracted beams are at 90° to each other.

The refracted beam contains light mainly due to vibrations perpendicular to that reflected and is therefore partially plane-polarised. Since refraction and reflection occur at both sides of a glass plate, the transmitted beam contains a fair percentage of plane-polarised light. A *pile of plates* increases the percentage, and thus provides a simple method of producing plane-polarised light. They are mounted inclined in a tube so that the ordinary (unpolarised) light is incident at the polarising angle, and the transmitted light it then fairly plane-polarised.

Polarisation by Double Refraction

We have already considered two methods of producing polarised light. The first observation of polarised light, however, was made by BARTHOLINUS in 1669, who placed a crystal of iceland spar on some words on a sheet of paper. To his surprise, two images were seen through the crystal. Bartholinus therefore gave the name of *double refraction* to the phenomenon, and experiments more than a century later showed that the crystal produced plane-polarised light when ordinary light was incident on it. See Fig. 29.38.

Iceland spar is a crystalline form of calcite (calcium carbonate) which cleaves in the form of a "rhomboid" when it is lightly tapped; this is a solid whose opposite faces are parallelograms. When a beam of unpolarised light is incident on one face of the crystal, its internal molecular structure produces two beams of polarised light, E, O, whose vibrations are perpendicular to each other, Fig. 29.39. If the incident direction AB is parallel to a plane known as the "principal section" of the crystal, one

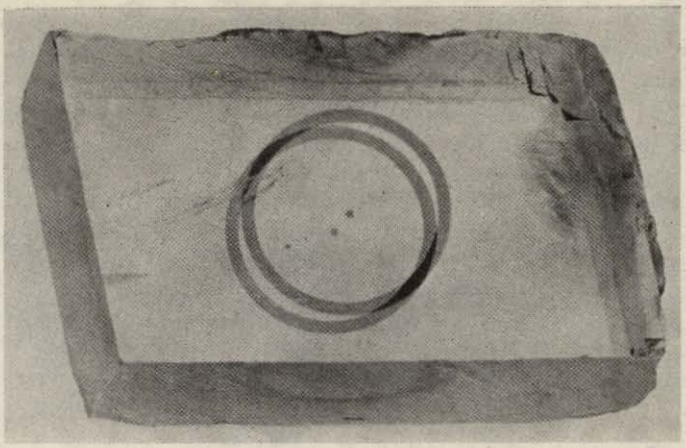


FIG. 29.38. DOUBLE REFRACTION. A ring with a spot in the centre, photographed through a crystal of Iceland spar. The light forms two rings and two spots.

beam O emerges parallel to AB, while the other beam E emerges displaced in a different direction. As the crystal is rotated about the line of vision the beam E revolves round O. On account of this abnormal behaviour the rays in E are called "extraordinary" rays; the rays in O are known as "ordinary" rays (p. 719). Thus two images of a word on a paper, for example, are seen when an Iceland spar crystal is placed on top of it; one image is due to the ordinary rays, while the other is due to the extraordinary rays.

With the aid of an Iceland spar crystal Malus discovered the polarisation of light by reflection (p. 719). While on a visit to Paris he gazed

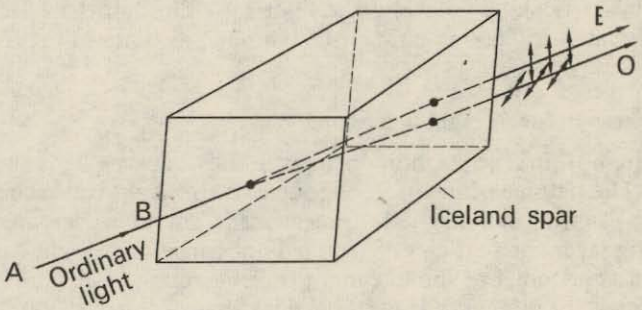


FIG. 29.39. Action of Iceland spar.

through the crystal at the light of the sun reflected from the windows of the Palace of Luxemburg, and observed that only one image was obtained for a particular position of the crystal when it was rotated slowly. The light reflected from the windows could not therefore be ordinary (unpolarised) light, and Malus found it was plane-polarised.

Nicol Prism

We have seen that a tourmaline crystal produces polarised light, and that the crystal can be used to detect such light (p. 717). NICOL designed a crystal of Iceland spar which is widely used for producing and detecting

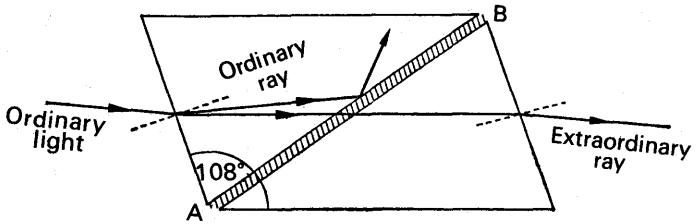


FIG. 29.40. Action of Nicol prism.

polarised light, and it is known as a *Nicol prism*. A crystal whose faces contain angles of 72° and 108° is broken into two halves along the diagonal AB, and the halves are cemented together by a layer of Canada balsam, Fig 29. 40. The refractive index of the crystal for the ordinary rays is 1·66, and is 1·49 for the extraordinary rays; the refractive index of the Canada balsam is about 1·55 for both rays, since Canada balsam does not produce polarised light. A critical angle thus exists between the crystal and Canada balsam for the ordinary rays, but not for the extraordinary rays. Hence total reflection of the former rays takes place at the canada balsam if the angle of incidence is large enough, as it is with the Nicol prism. The emergent light is then due to the extraordinary rays, and is polarised.

The prism is used like a tourmaline crystal to detect plane-polarised light, namely, the prism is held in front of the beam of light and is rotated. If the beam is plane-polarised the light seen through the Nicol prism varies in intensity, and is extinguished at one position of the prism.

Differences Between Light and Sound Waves

We are now in a position to distinguish fully between light and sound waves. The physical difference, of course, is that light waves are due to varying electric and magnetic forces, while sound waves are due to vibrating layers or particles of the medium concerned. Light can travel through a vacuum, but sound cannot travel through a vacuum. Another very important difference is that the vibrations of the particles in sound waves are in the same direction as that along which the sound travels, whereas the vibrations in light waves are perpendicular to the direction along which the light travels. Sound waves are therefore *longitudinal* waves, whereas light waves are *transverse* waves. As we have seen, sound waves can be reflected and refracted, and can give rise to interference phenomena; but no polarisation phenomena can be obtained with sound waves since they are longitudinal waves, unlike the case of light waves.

EXERCISES 29

Interference

1. Describe how to set up apparatus to observe and make measurements on the interference fringes produced by Young's slits. Explain how (i) the wavelengths of two monochromatic light sources could be compared, (ii) the separation of the slits could be deduced using a source of known wavelength. Establish any formula required.

State, giving reasons, what you would expect to observe (*a*) if a white light source were substituted for a monochromatic source, (*b*) if the source slit were then displaced slightly at right angles to its length in the plane parallel to the plane of the Young's slits. (*L.*)

2. Explain the formation of interference fringes by an air wedge and describe how the necessary apparatus may be arranged to demonstrate them.

Fringes are formed when light is reflected between the flat top of a crystal resting on a fixed base and a sloping glass plate. The lower end of the plate rests on the crystal and the upper end on a fixed knife-edge. When the temperature of the crystal is raised the fringe separation changes from 0·96 mm to 1·00 mm. If the length of the glass plate from knife-edge to crystal is 5·00 cm, and the light of wavelength $6{\cdot}00 \times 10^{-5}$ cm is incident normally on the wedge, calculate the expansion of the crystal. (*L.*)

3. Describe in detail how the radius of curvature of the spherical face of a planoconvex lens may be found by observations made on Newton's rings.

Two plane glass plates which are in contact at one edge are separated by a piece of metal foil 12·50 cm from that edge. Interference fringes parallel to the line of contact are observed in reflected light of wavelength 5460 Å and are found to be 1·50 mm apart. Find the thickness of the foil. (*L.*)

4. Describe, with the aid of a labelled diagram, how the wavelength of monochromatic light may be found using Young's slits. Give the theory of the experiment.

State, and give physical reasons for the features which are common to this method and to *either* the method based on Lloyd's mirror *or* that based on Fresnel's biprism.

In an experiment using Young's slits the distance between the centre of the interference pattern and the tenth bright fringe on either side is 3·44 cm and the distance between the slits and the screen is 2·00 m. If the wavelength of the light used is $5{\cdot}89 \times 10^{-7}$ m determine the slit separation. (*N.*)

5. Explain what is meant by the term *path-difference* with reference to the interference of two wave-motions.

Why is it not possible to see interference where the light beams from the headlamps of a car overlap?

Interference fringes were produced by the Young's slits method, the wavelength of the light being 6×10^{-5} cm. When a film of material $3{\cdot}6 \times 10^{-3}$ cm thick was placed over *one* of the slits, the fringe pattern was displaced by a distance equal to 30 times that between two adjacent fringes. Calculate the refractive index of the material. To which side are the fringes displaced?

(When a layer of transparent material whose refractive index is n and whose thickness is d is placed in the path of a beam of light, it introduces a path difference equal to $(n-1)d$.) (*O. & C.*)

6. Show how, with the aid of Huygens' idea of secondary wavelets, the wave theory of light will account for the laws of refraction and of reflexion at a plane surface.

Describe briefly Young's two-slit experiment and explain how it confirms the wave nature of light. (*L.*)

7. Describe, giving both theory and experimental detail, how you would find the radius of curvature of one surface of a convex lens by means of Newton's rings. You may assume that monochromatic light of a known wavelength is available.

Newton's rings are formed by reflexion between an equiconvex lens of focal length 100 cm made of glass of refractive index 1·50 and in contact with a plane glass plate of refractive index 1·60. Find the radius of the 5th bright ring using monochromatic light of wavelength 6000 Å.

Explain the changes which occur when oil of refractive index 1·55 fills the space between the lens and plate. (1 Å = 10^{-8} cm.) (*N.*)

8. Define *velocity*, *frequency* and *wavelength* for any wave motion, and deduce a relation between them. What do you understand by 'interference between waves' and 'coherent wave trains'? Explain why interference is not observed between the beams of two electric torches.

Deduce the relation connecting the refractive index of a material with the velocities of light in vacuo and in the material. State clearly the assumptions you make about wave fronts in order to do this.

Light passes through a single crystal of ruby 10·0 cm long and emerges with a wavelength of $6{\cdot}94 \times 10^{-5}$ cm. If the critical angle of ruby for light of this wavelength if 34° 50′, calculate the number of wavelengths inside the crystal. (*C.*)

9. State the conditions necessary for the production of interference effects by two overlapping beams of light.

Describe fully *one* method for the production of interference fringes using light from a given monochromatic source. Show how with the aid of suitable measurements the wavelength of light emitted by the source may be determined with your apparatus.

Describe how the fringes produced by your apparatus would appear if a source of white light were employed instead of a monochromatic one. (*O. & C.*)

10. Explain how Newton's rings are formed, and describe how you would demonstrate them experimentally. How is it possible to predict the appearance of the centre of the ring pattern when (*a*) the surfaces are touching, and (*b*) the surfaces are not touching?

In a Newton's rings experiment one surface was fixed and the other movable along the axis of the system. As the latter surface was moved the rings appeared to contract and the centre of the pattern, initially at its darkest, became alternately bright and dark, passing through 26 bright phases and finishing at its darkest again. If the wavelength of the light was 5461 Å, how far was the surface moved and did it approach, or recede from, the fixed surface? Suggest one possible application of this experiment. (*O. & C.*)

11. Explain the formation of Newton's rings and describe how you would use them to measure the radius of curvature of the convex surface of a long-focus planoconvex lens.

The diameters of the mth and $(m + 10)$th bright rings formed by such a lens resting on a plane glass surface are respectively 0·14 cm and 0·86 cm. When the space between lens and glass is filled with water the diameters of the qth and $(q + 10)$th bright rings are respectively 0·23 cm and 0·77 cm. What is the refractive index of water? (*L.*)

12. What are the conditions essential for the production of optical interference fringes?

Explain how these conditions are satisfied in the case of (*a*) Young's fringes, and (*b*) thin film interference fringes. (*N.*)

13. Describe, in detail how you would arrange apparatus to observe, in monochromatic light, interference fringes formed by light reflected from two glass plates enclosing an air wedge. Show how the angle of the wedge could be obtained from measurements on the fringes.

Newton's rings are formed with light of wavelength $5{\cdot}89 \times 10^{-5}$ cm between the curved surface of a planoconvex lens and a flat glass plate, in perfect contact. Find the radius of the 20th dark ring from the centre if the radius of curvature of the lens surface is 100 cm. How will this ring move and what will its radius become if the lens and the plate are slowly separated to a distance apart of $5{\cdot}00 \times 10^{-4}$ cm? (*L.*)

14. What are the necessary conditions for interference of light to be observable? Describe with the aid of a labelled diagram how optical interference may be demonstrated using Young's slits. Indicate suitable values for all the distances shown.

How are the colours observed in thin films explained in terms of the wave nature of light? Why does a small oil patch on the road often show approximately circular coloured rings? (*L.*)

Diffraction

15. Describe and give the theory of an experiment to compare the wavelengths of yellow light from a sodium and red light from a cadmium discharge lamp, using a diffraction grating. Derive the required formula from first principles.

White light is reflected normally from a soap film of refractive index 1·33 and then directed upon the slit of a spectrometer employing a diffraction grating at normal incidence. In the first-order spectrum a dark band is observed with minimum intensity at an angle of 18° 0′ to the normal. If the grating has 5000 lines per cm, determine the thickness of the soap film assuming this to be the minimum value consistent with the observations. (*L.*)

16. Describe the phenomena which occur when plane waves pass (*a*) through a wide aperture, (*b*) through an aperture whose width is comparable with the wavelength of the waves.

How does the wave theory of light account for the apparent rectilinear propagation of light?

A diffraction grating has 6000 lines per cm. Calculate the angular separation between wavelengths $5{\cdot}896 \times 10^{-5}$ cm and $5{\cdot}461 \times 10^{-5}$ cm respectively after transmission through it at normal incidence, in the first-order spectrum. (*O. & C.*)

17. Describe two experiments to show the diffraction of light.

Describe how a diffraction grating may be used to measure the wavelength of sodium light, deriving any formulae employed. (*L.*)

18. What are the advantages and disadvantages of a diffraction grating as compared with a prism for the study of spectra?

A rectangular piece of glass 2 cm × 3 cm has 18000 evenly spaced lines ruled across its whole surface, parallel to the shorter side, to form a diffraction grating. Parallel rays of light of wavelength 5×10^{-5} cm fall normally on the grating. What is the highest order of spectrum in the transmitted light?

What is the minimum diameter of a camera lens which can accept all the light of this wavelength in this order which leaves the grating on one side of the normal? *(O. & C.)*

19. In an experiment using a spectrometer in normal adjustment fitted with a plane transmission grating and using monochromatic light of wavelength $5{\cdot}89 \times 10^{-5}$ cm, diffraction maxima are obtained with telescope settings of 153° 44′, 124° 5′, 76° 55′ and 47° 16′, the central maximum being at 100° 30′. Show that these observations are consistent with normal incidence and calculate the number of rulings per cm of the grating.

If this grating is replaced by an opaque plate having a single vertical slit $2{\cdot}00 \times 10^{-2}$ cm wide, describe and explain the diffraction pattern which may now be observed. Contrast the appearance of this pattern with that produced by the grating. *(N.)*

20. *(a)* What is meant by (i) *diffraction*, (ii) *superposition* of waves? Describe *one* phenomenon to illustrate each in the case of *sound waves*.

(b) The floats of two men fishing in a lake from boats are 22·5 metres apart. A disturbance at a point in line with the floats sends out a train of waves along the surface of the water, so that the floats bob up and down 20 times per minute. A man in a third boat observes that when the float of one of his colleagues is on the crest of a wave that of the other is in a trough, and that there is then one crest between them. What is the velocity of the waves? *(O. & C.)*

21. Give an account of the theory of the production of a spectrum by means of a plane diffraction grating. How does it differ from the spectrum produced by means of a prism?

Parallel light consisting of two monochromatic radiations of wavelengths 6×10^{-5} cm and 4×10^{-5} cm falls normally on a plane transmission grating ruled with 5000 lines per cm. What is the angular separation of the second-order spectra of the two wavelengths? *(C.)*

22. A pure spectrum is one in which there is no overlapping of light of different wavelengths. Describe how you would set up a diffraction grating to display on a screen as close an approximation as possible to a pure spectrum. Explain the purpose of each optical component which you would use.

A grating spectrometer is used at normal incidence to observe the light from a sodium flame. A strong yellow line is seen in the first order when the telescope axis is at an angle of 16° 26′ to the normal to the grating. What is the highest order in which the line can be seen?

The grating has 4800 lines per cm; calculate the wavelength of the yellow radiation.

What would you expect to observe in the spectrometer set to observe the first-order spectrum if a small but very bright source of white light is placed close to the sodium flame so that the flame is between it and the spectrometer? *(O. & C.)*

23. Describe how you would determine the wavelength of monochromatic light using a diffraction grating and a spectrometer. Give the theory of the method.

A filter which transmits only light between 6300 Å and 6000 Å is placed between a source of white light and the slit of a spectrometer; the grating has 5000 lines to the centimetre; and the telescope has an objective of focal length 15 cm with an eyepiece of focal length 3 cm. Find the width in millimetres of the first-order spectrum formed in the focal plane of the objective. Find also the angular width of this spectrum seen through the eyepiece. *(O.)*

Polarisation

24. What is meant by *plane of polarisation*? Explain why the phenomenon of polarisation is met with in dealing with light waves, but not with sound waves.

Describe and explain the action of (*a*) a nicol prism, (*b*) a sheet of Polaroid.

How can a pair of Polaroid sheets and a source of natural light be used to produce a beam of light the intensity of which may be varied in a calculable manner? (*L.*)

25. Explain what is meant by the statement that a beam of light is *plane polarised.* Describe *one* experiment in each instance to demonstrate (*a*) polarisation by reflexion, (*b*) polarisation by double refraction, (*c*) polarisation by scattering.

The refractive index of diamond for sodium light is 2·417. Find the angle of incidence for which the light reflected from diamond is completely plane polarised. (*L.*)

26. Give an account of the action of (*a*) a single glass plate. (*b*) a Nicol prism, in producing plane-polarised light. State *one* disadvantage of *each* method.

Mention *two* practical uses of polarising devices. (*N.*)

27. Describe how, using a long, heavy rope, you would demonstrate (*a*) a plane-polarised wave, and (*b*) a stationary wave.

Give a short account, with diagrams, of *three* ways in which plane-polarised light is obtained (other than by using 'polaroid'). State some uses of polarised light.

Two polaroid sheets are placed close together in front of a lamp so that no light passes through them. Describe and explain what happens when one sheet is slowly rotated, the other remaining in its original position. (*C.*)

28. Answer *two* of the following:

(i) How may it be shown that the radiation from (*a*) a sodium lamp, and (*b*) a radio transmitter (such as a broadcasting station or a microwave source) consists of waves?

(ii) Explain what is meant by the polarisation of light, and describe how you would demonstrate it. Why is light from most light sources unpolarised?

(iii) When a diffraction grating is illuminated normally by monochromatic light an appreciable amount of light leaves the grating in certain directions. Explain this phenomenon, and show how these directions may be predicted. (*O. & C.*)

29. What is meant by (*a*) polarised light, (*b*) polarising angle? Describe and explain two methods for producing plane-polarised light.

Calculate the polarising angle for light travelling from water, of refractive index 1·33, to glass, of refractive index 1·53. (*L.*)

30. A beam of plane-polarised light falls normally on a sheet of Polaroid, which is at first set so that the intensity of the transmitted light, as estimated by a photographer's light-meter, is a maximum. (The meter is suitably shielded from all other illumination.) Describe and explain the way in which you would expect the light-meter readings to vary as the Polaroid is rotated in stages through 180° about an axis at right angles to its plane.

How would you show experimentally (*a*) that calcite is doubly refracting, (*b*) that the two refracted beams are plane polarised, in planes at right angles to one another, and (*c*) that in general the two beams travel through the crystal with different velocities? (*O.*)

31. What is plane-polarised light?

Explain why two images of an object are seen through a crystal of Iceland Spar. What would be seen if the object were viewed through two crystals, one of which was slowly rotated about the line of vision?

How would you produce a plane-polarised beam of light by reflection from a glass surface? (*C.*)

32. What is meant by the polarisation of light? How is polarisation explained on the hypothesis that light has wave properties?

Describe how polarisation can be produced and detected by reflexion. Mention another way of obtaining polarised light and describe how you would determine which of the two methods is the more effective.

Describe briefly *two* uses of polarised light. (*N.*)

33. Give an account of the evidence for believing that light is a wave motion. What reason is there for believing light waves to be transverse waves?

Two dishes *A* and *B* each contain liquid to a depth of 3·000 cm. *A* contains alcohol, *B* a layer of water on which is a layer of transparent oil. The depths of the oil and water are adjusted so that for monochromatic light passing vertically through them, the number of wavelengths is the same in *A* and *B*. Find the depth of the water layer, if the refractive indices of alcohol, water and oil are respectively 1·363, 1·333 and 1·475. (*L.*)

PART FOUR

Electricity and Atomic Physics

chapter thirty

Electrostatics

GENERAL PHENOMENA

IF a rod of ebonite is rubbed with fur, or a fountain-pen with a coat-sleeve, it gains the power to attract light bodies, such as pieces of paper or tin-foil or a suspended pith-ball. The discovery that a body could be made attractive by rubbing is attributed to Thales (640–548 B.C.). He seems to have been led to it through the Greeks' practice of spinning silk with an amber spindle; the rubbing of the spindle in its bearings caused the silk to adhere to it. The Greek word for amber is *elektron*, and a body made attractive by rubbing is said to be 'electrified'. This branch of Electricity, the earliest discovered, is called *Electrostatics*.

Conductors and Insulators

Little progress was made in the study of electrification until the sixteenth century A.D. Then Gilbert (1540–1603), who was physician-in-ordinary to Queen Elizabeth, found that other substances besides amber could be electrified: for example, glass when rubbed with silk. He failed to electrify metals, however, and concluded that to do so was impossible.

More than 100 years later—in 1734—he was shown to be wrong, by du Fay; du Fay found that a metal could be electrified by rubbing with fur or silk, but only if it were held in a handle of glass or amber; it could not be electrified if it were held directly in the hand. His experiments followed the discovery, by Gray in 1729, that electric charges could be transmitted through the human body, water, and metals. These are examples of *conductors*; glass and amber are examples of *insulators*.

Positive and Negative Electricity

In the course of his experiments du Fay also discovered that there were two kinds of electrification: he showed that electrified glass and amber tended to oppose one another's attractiveness. To illustrate how he did so, we may use ebonite instead of amber, which has the same electrical properties. We suspend a pith-ball, and attract it with an electrified ebonite rod E (Fig. 30.1(i)); we then bring an electrified glass rod G towards the ebonite rod, and the pith-ball falls away (Fig. 30.1(ii)). Benjamin Franklin, a pioneer of electrostatics, gave the name of 'positive electricity' to the charge on a glass rod rubbed with silk, and 'negative electricity' to that on an ebonite rod rubbed with fur.

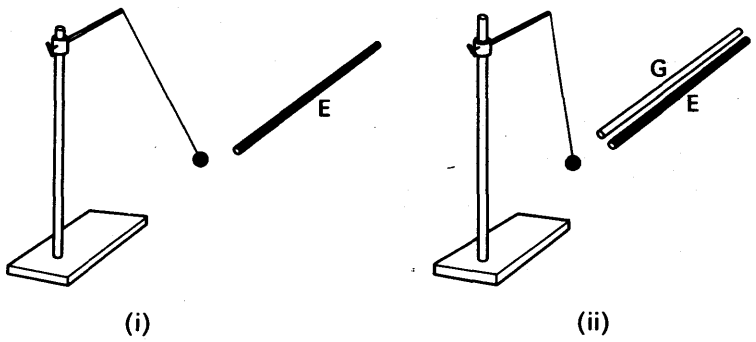


FIG. 30.1. Demonstrating that electrified glass tends to oppose effect of electrified ebonite.

Electrons and Electrostatics

Towards the end of the nineteenth century Sir J. J. Thomson discovered the existence of the *electron* (p. 1002). This is the lightest particle known—it is about 1/1840th of the mass of the hydrogen atom—and experiments show that it carries a tiny quantity of *negative* electricity. Later experiments showed that electrons are present in all atoms.

The detailed structure of atoms is complicated, but, generally, electrons exist round a minute core or nucleus carrying positive electricity. Normally, atoms are electrically neutral, that is, there is no surplus of charge on them. Consequently the total negative charge on the electrons is equal to the positive charge on the nucleus. In insulators, all the electrons appear to be firmly 'bound' to the nucleus under the attraction of the unlike charges. In metals, however, some of the electrons appear to be relatively 'free'. These electrons play an important part in electrical phenomena concerning metals.

The theory of electrons (negatively charged particles) gives simple explanations of electrification by friction, and of the attraction of uncharged bodies by charged ones. If the silk on which a glass rod has been rubbed is brought near to a charged and suspended ebonite rod it repels it; the silk must therefore have a negative charge. We know that the glass has a positive charge, and therefore we suppose that when the two were rubbed together electrons from the surface atoms were transferred from the glass to the silk. Likewise we suppose that when fur and ebonite are rubbed together, electrons go from the fur to the ebonite.

Attraction of Charged Body for Uncharged Bodies

To explain the attraction of a charged body for an uncharged one, we shall suppose that the uncharged body is a conductor—a metal. If it is brought near to a charged ebonite rod, say, then the negative charge on the rod repels the free electrons in the metal to its remote end (Fig. 30.2). A positive charge is thus left on the near end of the metal; this, being nearer than the negative charge on the far end, is attracted more strongly than the negative charge is repelled. On the

whole, therefore, the metal is attracted. If the uncharged body is not a conductor, the mechanism by which it is attracted is more complicated; we shall postpone its description to a later chapter.

FIG. 30.2. Attraction by charged body.

Electrostatics Today

The discovery of the electron has led, in the last twenty or thirty years, to a great increase in the practical importance of electrostatics. In devices such as radio valves and cathode-ray tubes, for example, electrons are moving under the influence of electrostatic forces. The problems of preventing sparks and the breakdown of insulators are essentially electrostatic. There are also difficulties in making measurements at very high voltages. These problems occur in high-voltage electrical engineering. Later, we shall also describe a modern electrostatic generator, of the type used to provide a million volts or more for X-ray work and nuclear bombardment. Such generators work on principles of electrostatics discovered over a hundred years ago.

Gold-leaf Electroscope

One of the earliest instruments used for testing positive and negative charges consisted of a metal rod A to which gold leaves L were

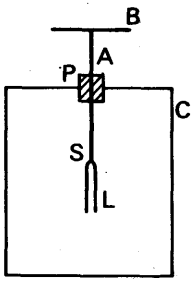


FIG. 30.3. A gold-leaf electroscope.

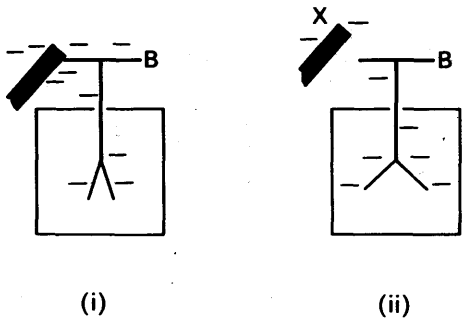


FIG. 30.4. Testing charge with electroscope.

attached (Fig. 30.3). The rod was fitted with a circular disc or cap B, and was insulated with a plug P from a metal case C which screened L from outside influences other than those brought near to B.

When B is touched by an ebonite rod rubbed with fur, some of the negative charge on the rod passes to the cap and L; and since like charges repel, the leaves diverge (Fig. 30.4(i)). If an unknown charge X is now brought near to B, an increased divergence implies that X is negative (Fig. 30.4(ii)). A positive charge is tested in a similar way; the electroscope is first given a positive charge and an increased divergence indicates a positive charge.

Induction

We shall now show that it is possible to obtain charges, called *induced charges*, without any contact with another charge. An experiment on electrostatic induction, as the phenomenon is called, is shown in Fig. 30.5(i). Two insulated metal spheres A, B are arranged so that they

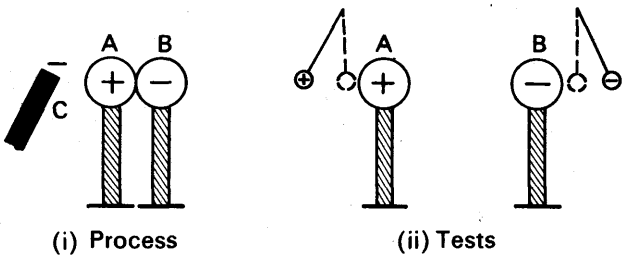


FIG. 30.5. Charges induced on a conductor.

touch one another, and a negatively charged ebonite rod C is brought near to A. The spheres are now separated, and then the rod is taken away. Tests with a charged pith-ball now show that A has a positive charge and B a negative charge (Fig. 30.5(ii)). If the spheres are placed together so that they touch, it is found that they now have no effect on a pith-ball held near. Their charges must therefore have neutralized each other completely, thus showing that the induced positive and negative charges are equal. This is explained by the movement of electrons from A to B when the rod is brought near. B has then a negative charge and A an equal positive charge.

Charging by Induction

Fig. 30.6 shows how a conductor can be given a permanent charge by induction, without dividing it in two. We first bring a charged ebonite rod, say, near to the conductor, (i); next we connect the conductor to

earth by touching it momentarily (ii); finally we remove the ebonite. We then find that the conductor is left with a positive charge (iii). If we use a charged glass rod, we find that the conductor is left with a negative charge; the charge left, called the induced charge, has always the opposite sign to the inducing charge.

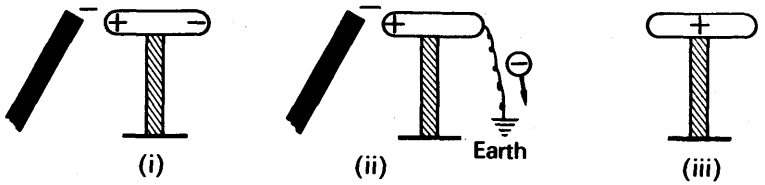


FIG. 30.6. Charging permanently by induction.

This phenomenon of induction can again be explained by the movement of electrons. If the inducing charge is negative, then, when we touch the conductor, electrons are repelled from it to earth, as shown in Fig. 30.6(ii), and a positive charge is left on the conductor. If the inducing charge is positive, then the electrons are attracted up from the earth to the conductor, which then becomes negatively charged.

Induction and the Electroscope

It is always observed that the leaves of an electroscope diverge when a charged body is brought near its cap, without touching it. This we can now easily understand; if, for example, we bring a negatively charged rod near the cap, it induces a positive charge on the cap, and a negative one on the leaves: the leaves then repel each other. Further, the negative charge on the leaves induces a positive one on the inside of the case, the corresponding negative charge running to the earth, on which the case rests. The positive charge on the case attracts the negative charge on the leaves, and makes them diverge further.

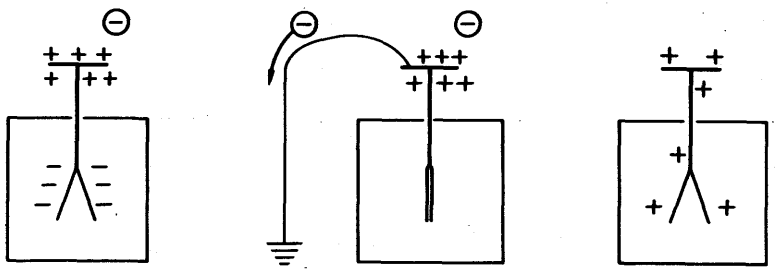


FIG. 30.7. Charging electroscope by induction.

We can use induction to give a permanent charge to the cap and leaves of an electroscope, by momentarily earthing the cap while holding an inducing charge near it. This is illustrated in Fig. 30.7.

The Electrophorus

A device which provides an almost unlimited supply of charge, by induction, was invented by Volta about 1800; it is called an *electrophorus*. It consists of an ebonite or perspex base, E in Fig. 30.8, and a metal disc D on an insulating handle. The ebonite is charged negatively by rubbing it—or, much better, beating it— with fur. The disc is then laid upon it, and acquires induced charges, positive underneath and negative on top, (i). Very little negative charge escapes from the ebonite to the disc, because the natural unevenness of their surfaces prevents them touching at more than a few points; charge escapes from these points only, because the ebonite is a non-conductor. After it has been placed on the ebonite, the disc is earthed with the finger, and the negative charge on its upper surface flows away, (ii). The disc can then be removed, and carries with it the positive charge which was on its underside, (iii).

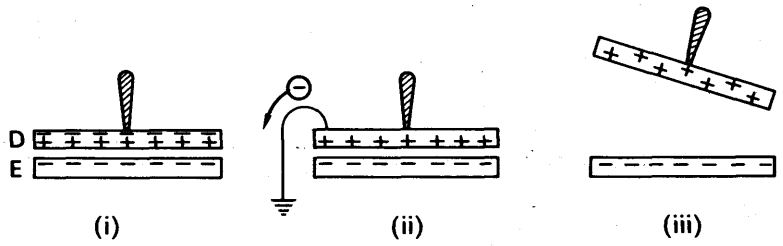


FIG. 30.8. The electrophorus.

An electrophorus produces sufficient charge to give an audible—and sometimes a visible—spark. The disc can be discharged and charged again repeatedly, until the charge on the ebonite has disappeared by leakage. Apparently, therefore, it is in principle an inexhaustible source of energy. However, work is done in raising the disc from the ebonite, against the attraction of their opposite charges, and this work must be done each time the disc is charged; the electrophorus is therefore not a source of energy, but a device for converting it from a mechanical into an electrical form.

The action of the electrophorus illustrates the advantages of charging by induction. First, the supply of charge is almost inexhaustible, because the original charge is not carried away. Second, a great charge —nearly equal to the charge on the whole of the ebonite—can be concentrated on to the conducting disc. As we have seen, only a very small charge could be transferred by contact, because the ebonite is not a conductor.

The Action of Points, Van de Graaff Generator

Sometimes in experiments with an electroscope connected to other apparatus by a wire, the leaves of the electroscope gradually collapse, as though its charge were leaking away. This behaviour can often be traced to a sharp point on the wire—if the point is blunted, the leakage stops. Charge leaks away from a sharp point through the air, being carried by molecules away from the point. This is explained later (p. 749).

Points are used to collect the charges produced in *electrostatic generators.* These are machines for induction, and thus building up very great charges and potential differences. Fig. 30.9 is a simplified diagram of one such machine, due to Van de Graff. A hollow metal sphere S is supported on an insulating tube T, many feet high. A silk belt B runs over the pulleys shown, of which the lower is driven by an electric motor. Near the bottom and top of its run, the belt passes close to the elctrodes *E*, which are sharply pointed combs, pointing towards the belt. The electrode E_1 is made about 10000 volts positive with respect to the earth by a battery. Its point then sprays the lower part of the belt with positive charge, which is carried up into the sphere. There it induces a negative charge on the points of electrode E_2 and a positive charge on the sphere to which the blunt end of E_2 is connected. The point sprays the belt with negative charge, and discharges it before it passes over the pulley. The sphere gradually charges up positively, until its potential is about a million volts relative to the earth.

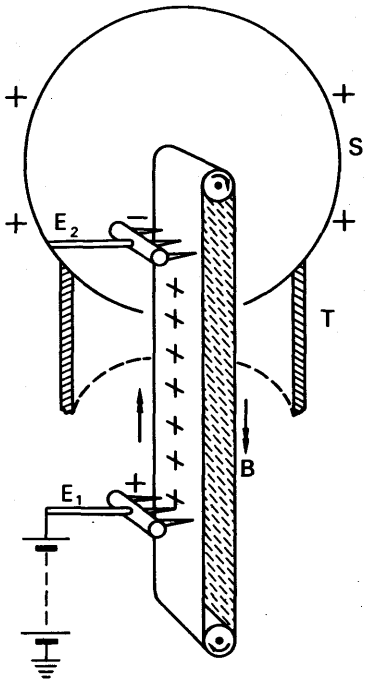


FIG. 30.9. Principle of Van de Graaff generator.

Large machines of this type are used with high-voltage X-ray tubes, and for atom-splitting experiments. They have more elaborate electrode systems, stand about 15 m high, and have 4 m spheres. They can produce potential differences up to 5 000 000 volts and currents of about 50 microamperes. The electrical energy which they deliver comes from the work done by the motor in drawing the positively charges belt towards the positively charged sphere, which repels it.

In all types of high-voltage equipment sharp corners and edges must be avoided, except where points are deliberately used as electrodes. Otherwise, corona discharges may break out from the sharp places. All such places are therefore enlarged by metal globes, these are called stress-distributors. See also p. 749.

FIG. 30.10. Van de Graaff Electrostatic Generator at Aldermaston, England. The dome is the high-voltage terminal. The insulated rings are equipotentials, and provide a uniform potential gradient down the column. Beams of protons or deuterons, produced in the dome, are accelerated down the column to bombard different materials at the bottom, thereby producing nuclear reactions which can be studied.

Ice-pail Experiment

A famous experiment on electrostatic induction was made by Faraday in 1843. In it he used the ice-pail from which it takes its name; but it was a modest pail, 27 cm high—not a bucket. He stood the pail on an insulator, and connected it to a gold-leaf electroscope, as in Fig. 30.11(i). He next held a metal ball on the end of a long silk thread, and charged it positively by a spark from an electrophorus. Then he lowered the ball into the pail, without letting it touch the sides or bottom (Fig. 30.11(ii)).

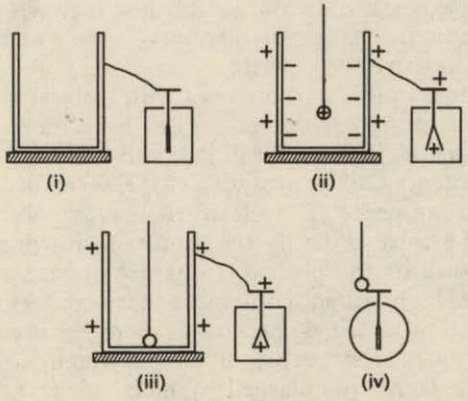


FIG. 30.11. Faraday's ice-pail experiment.

A positive charge was induced on the outside of the pail and the leaves, and made the leaves diverge. Once the ball was well inside the pail, Faraday found that the divergence of the leaves did not change when he moved the ball about—nearer to or farther from the walls or the bottom. This showed that the amount of the induced positive charge did not depend on the position of the ball, once it was well inside the pail. Faraday then allowed the ball to touch the pail, and noticed that the leaves of the electrocope still did not move (Fig. 30.11(iii)). When the ball touched the pail, therefore, no charge was given to, or taken from, the outside of the pail. Faraday next lifted the ball out of the pail, and tested it for charge with another electroscope. He found that the ball had no charge whatever (Fig. 30.11(iv)). The induced negative charge on the inside of the pail, must therefore have been equal in magnitude to the original positive charge on the ball.

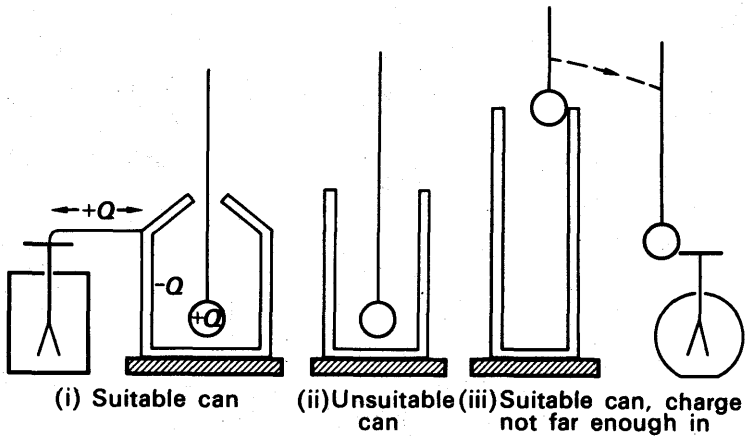


FIG. 30.12. Referring to Faraday's ice-pail experiment.

Faraday's experiment does not give these simple results unless the pail—or whatever is used in place of it—very nearly surrounds the charged ball (Fig. 30.12(i)(ii)). If, for example, the ball is allowed to touch the pail before it is well inside, as in Fig. 30.12(iii), then it does not lose all its charge.

Conclusions

The conclusions to be drawn from the experiment therefore apply, strictly, to a *hollow closed conductor*. They are:

(*i*) When a charged body is enclosed in a hollow conductor it induces on the inside of that conductor a charge equal but opposite to its own; and on the outside a charge equal and similar to its own (Fig. 30.11(i)).

(*ii*) The *total* charge inside a hollow conductor is always zero: either there are equal and opposite charges on the inside walls and within the volume (before the ball touches), or there is no charge at all (after the ball has touched).

Comparison and Collection of Charges

Faraday's ice-pail experiment gives us a method of comparing quantities of electric charges. The experiment shows that if a charged body is lowered well inside a tall, narrow can then it gives to the outside of the can a charge equal to its own. If the can is connected to the cap of an electroscope, the divergence of the leaves is a measure of the charge on the body. Thus we can compare the magnitudes of charges, without removing them from the bodies which carry them: we merely lower those bodies, in turn, into a tall insulated can, connected to an electroscope

Sometimes we may wish to discharge a conductor completely, without letting its charge run to earth. We can do this by letting the conductor touch the bottom of a tall can on an insulating stand. The whole of the body's charge is then transferred to the outside of the can.

Charges Produced by Separation; Lines of Force

The ice-pail experiment suggests that a positive electric charge, for example, is always accompanied by an equal negative charge. Faraday repeated his experiment with a nest of hollow conductors, insulated

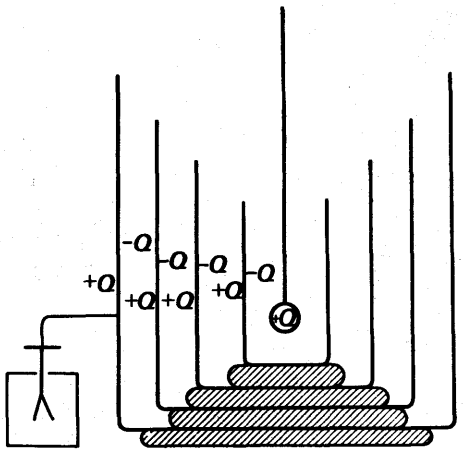


FIG. 30.13. Extension of the ice-pail experiment.

from one another, and showed that equal and opposite charges were induced on the inner and outer walls of each (Fig. 30.13).

Faraday also showed that equal and opposite charges are produced when a body if electrified by rubbing. He fitted an ebonite rod with a fur cap, which he rotated by a silk thread or string wrapped round it (Fig. 30.14(i)); he then compared the charges produced with an ice-pail and electroscope (Fig. 30.14(i)(ii)(iii)(iv)).

In describing the conclusions from this last experiment, we now say, as indeed we have done already, that electrons flow from the fur to the

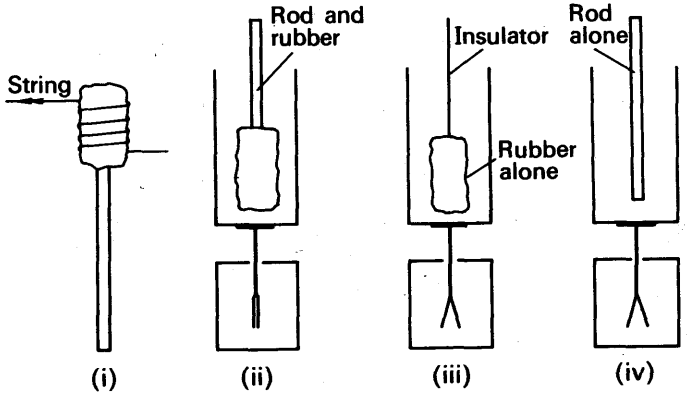


FIG. 30.14. Stages in showing that equal and opposite charges are produced by friction.

ebonite, carrying to it a negative charge, and leaving on the fur a positive charge. It appears, therefore, that free charges are always produced by separating equal amounts of the opposite kinds of electricity.

The idea that charges always occur in equal opposite pairs affects our drawing of lines of force diagrams. Lines of force radiate outwards

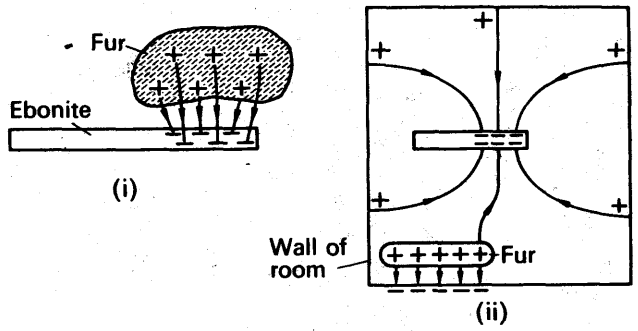


FIG. 30.15. Charging by friction—lines of force.

from a positive charge, and inwards to a negative one; from any positive charge, therefore, we draw lines of force ending on an equal negative charge. Figs. 30.15, 30.16 give some illustrations of this procedure.

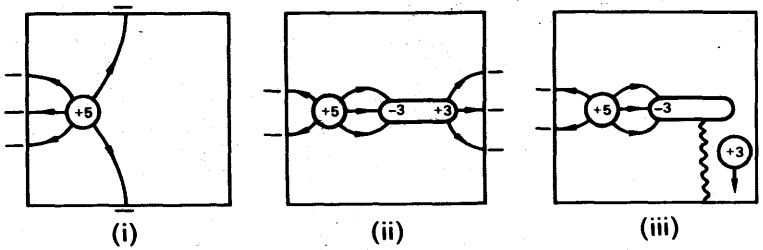


FIG. 30.16. Charging by induction—lines of force.

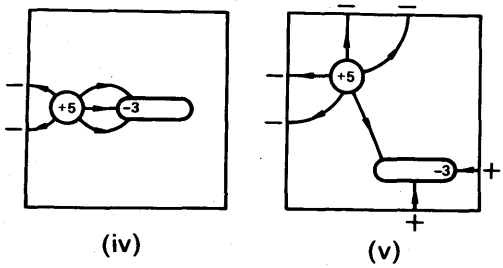


FIG. 30.16. Charging by induction—lines of force.

Distribution of Charge; Surface Density

By using a can connected to an electroscope we can find how electricity is distributed over a charged conductor of any form—pear-shaped, for example. We take a number of small leaves of tin-foil, all of the same area, but differently shaped to fit closely over the different parts of the conductor, and mounted on ebonite handles (Fig. 30.17(i)).

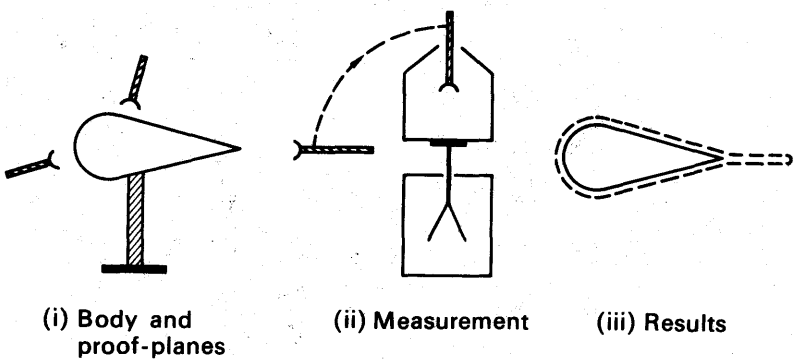


FIG. 30.17. Investigating charge distribution.

These are called *proof-planes*. We charge the body from an electrophorus, press a proof-plane against the part which it fits, and then lower the proof-plane into a can connected to an electroscope (Fig. 30.17(ii)). After noting the divergence of the leaves we discharge the can and electroscope by touching one of them, and repeat the observation with a proof-plane fitting a different part of the body. Since the proof-planes have equal areas, each of them carries away a charge proportional to the charge per unit area of the body, over the region which it touched. The charge per unit area over a region of the body is called the *surface-density* of the charge in that region. We find that the surface-density increases with the curvature of the body, as shown in Fig. 30.17(iii); the distance of the dotted line from the outline of the body is proportional to the surface-density of charge.

THE ELECTROSTATIC FIELD

Law of Force between two Charges

The magnitude of the force between two electrically charged bodies was studied by Coulomb in 1875. He showed that, if the bodies were small compared with the distance between them, then the force F was inversely proportional to the square of the distance r, i.e.

$$F \propto \frac{1}{r^2}. \quad . \quad . \quad . \quad . \quad (1)$$

This result is known as the *inverse square law*, or Coulomb's law.

It is not possible to verify the law accurately by direct measurement of the force between two charged bodies. In 1936 Plimton and Lawton showed, by an indirect method, that the power in the law cannot differ from 2 by more than $\pm 2 \times 10^{-9}$. We have no reason to suppose, therefore, that the inverse square law is other than exactly true.

Quantity of Charge

The SI unit of charge is the *coulomb* (C). The *ampere* (A), the unit of current, is defined later (p. 939). The coulomb is defined as that quantity of charge which passes a section of a conductor in one second when the current flowing is one ampère.

By measuring the force F between two charges when their respective magnitudes Q and Q' are varied, it is found that F is proportional to the product QQ'. Thus

$$F \propto QQ' \quad . \quad . \quad . \quad . \quad (2)$$

Law of Force

Combining (1) and (2), we have

$$F \propto \frac{QQ'}{r^2}$$

$$\therefore F = k\frac{QQ'}{r^2}, \quad . \quad . \quad . \quad . \quad (3)$$

where k is a constant. For reasons explained later, k is written as $1/4\pi\varepsilon_0$, where ε_0 is a constant called the *permittivity* of free space if we suppose the charges are situated in a vacuum. Thus

$$F = \frac{1}{4\pi\varepsilon_0}\frac{QQ'}{r^2} \quad . \quad . \quad . \quad (4)$$

In this expression, F is measured in newtons (N), Q in coulombs (C) and r in metres (m). Now, from (4),

$$\varepsilon_0 = \frac{QQ'}{4\pi F r^2}.$$

Hence the units of ε_0 are coulomb^2 newton^{-1} metre^{-2} ($C^2\,N^{-1}m^{-2}$). Another unit of ε_0, more widely used, is *farad metre*$^{-1}$ ($F\,m^{-1}$). See p. 774.

We shall see later that ε_0 has the numerical value of $8{\cdot}854 \times 10^{-12}$, and $1/4\pi\varepsilon_0$ then has the value 9×10^9 approximately.

Permittivity

So far we have considered charges in a vacuum. If charges are situated in other media such as water, then the force between the charges is reduced. Equation (4) is true only in a vacuum. In general, we write

$$F = \frac{1}{4\pi\varepsilon}\frac{QQ'}{r^2} \quad . \quad . \quad . \quad . \quad (1)$$

where ε is the *permittivity* of the medium. The permittivity of air at normal pressure is only about 1·005 times that, ε_0, of a vacuum. For most purposes, therefore, we may assume the value of ε_0 for the permittivity of air. The permittivity of water is about eighty times that of a vacuum. Thus the force between charges situated in water is eighty times less than if they were situated the same distance apart in a vacuum.

EXAMPLE

(a) Calculate the value of two equal charges if they repel one another with a force of 0·1 N when situated 50 cm apart in a vacuum.

(b) What would be the size of the charges if they were situated in an insulating liquid whose permittivity was ten times that of a vacuum?

(a) From (4),

$$F = \frac{1}{4\pi\varepsilon_0}\frac{QQ'}{r^2}.$$

Since $Q = Q'$ here,

$$0{\cdot}1 = \frac{9 \times 10^9\, Q^2}{(0{\cdot}5)^2}$$

or

$$Q^2 = \frac{0{\cdot}1 \times (0{\cdot}5)^2}{9 \times 10^9}$$

$$Q = 1{\cdot}7 \times 10^{-6}\ \text{C (coulomb), approx.}$$

$$= 1{\cdot}7\ \mu\text{C (microcoulomb).}$$

(b) The permittivity of the liquid $\varepsilon = 10\,\varepsilon_0$.

$$F = \frac{1}{4\pi\varepsilon}\frac{QQ'}{r^2}$$

$$= \frac{1}{10(4\pi\varepsilon_0)}\frac{Q^2}{r^2}$$

$$Q^2 = \frac{(0{\cdot}1) \times (0{\cdot}5)^2 \times 10}{9 \times 10^9}$$

$$Q = 5{\cdot}3 \times 10^{-6}\ \text{C} = 5{\cdot}3\ \mu\text{C}.$$

Electric Intensity or Field-strength. Lines of Force

An 'electric field' can be defined as a region where an electric force is experienced. As in magnetism, electric fields can be mapped out by

electrostatic lines of force, which may be defined as a line such that the tangent to it is in the direction of the force on a small positive charge at that point. Arrows on the lines of force show the direction of the force on a positive charge; the force on a negative charge is in the opposite direction. Fig 30.18 shows the lines of force, also called *electric flux*, in some electrostatic fields.

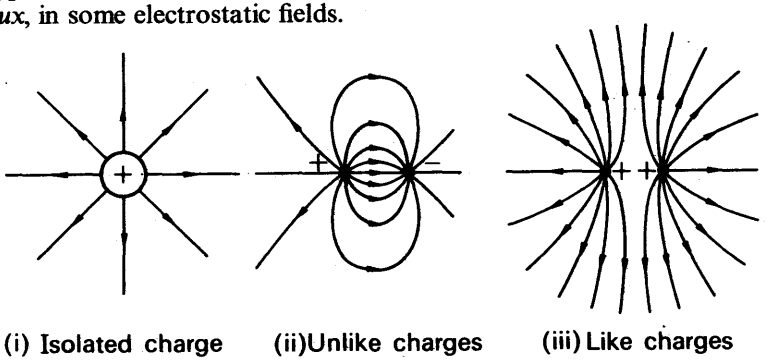


FIG. 30.18. Lines of electrostatic force.

The force exerted on a charged body in an electric field depends on the charge of the body and on the *intensity* or *strength* of the field. If we wish to explore the variation in intensity of an electric field, then we must place a test charge Q' at the point concerned which is small enough not to upset the field by its introduction. The intensity E of an electrostatic field at any point is defined as *the force per unit charge* which it exerts at that point. Its direction is that of the force exerted on a positive charge.

From this definition,

$$E = \frac{F}{Q'}$$

$$F = EQ' \quad . \quad . \quad . \quad . \quad (1)$$

Since F is measured in newtons and Q' in coulombs, it follows that *intensity E has units of newton per coulomb* ($N\,C^{-1}$). We shall see later that a more practical unit of E is *volt metre*$^{-1}$ ($V\,m^{-1}$) (see p. 755).

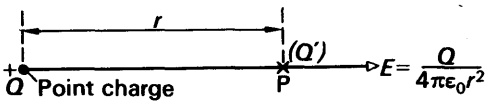


FIG. 30.19. Electric field intensity due to point charge.

We can easily find an expression for the strength E of the electric field due to a point charge Q situated in a vacuum (Fig. 30.19). We start from equation (4), p. 743, for the force between two such charges:

$$F = \frac{1}{4\pi\varepsilon_0}\frac{QQ'}{r^2}$$

If the test charge Q' is situated at the point P in Fig. 30.19, the electric field strength at that point is given by (1).

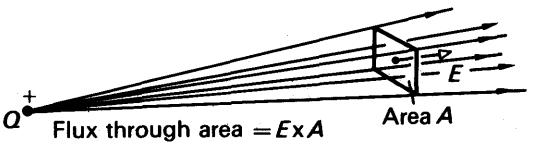
$$\therefore E = \frac{F}{Q'} = \frac{Q}{4\pi\varepsilon_0 r^2} \quad . \quad . \quad . \quad (2)$$

The direction of the field is radially outward if the charge Q is positive (Fig. 30.18(i)); it is radially inward if the charge Q is negative. If the charge were surrounded by a material of permittivity ε then,

$$E = \frac{Q}{4\pi\varepsilon r^2} \quad . \quad . \quad . \quad . \quad (3)$$

Flux from a Point Charge

We have already shown how electric fields can be described by lines of force. From Fig. 30.18(i) it can be seen that the density of the lines increases near the charge where the field intensity is high. The intensity E at a point can thus be represented by *the number of lines per unit area* through a surface perpendicular to the lines of force at the point considered. The *flux* through an area perpendicular to the lines of force is the name given to the product of $E \times$ *area*, where E is the intensity at that place. This is illustrated in Fig. 30.20(i).



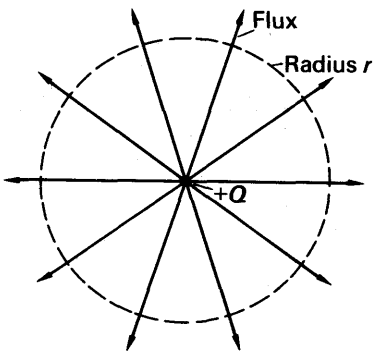


FIG. 30.20. Flux from a point charge.

Consider a sphere of radius r drawn in space concentric with a point charge (Fig. 30.20(ii)). The value of E at this place is given by (3),

p. 746. The total flux through the sphere is,

$$E \times \text{area} = E \times 4\pi r^2$$

$$= \frac{Q}{4\pi\varepsilon r^2} \times 4\pi r^2$$

$$= \frac{Q}{\varepsilon}$$

$$= \frac{\text{charge inside sphere}}{\text{permittivity}} \quad . \quad . \quad (1)$$

This demonstrates the important fact that the total flux crossing any sphere drawn outside and concentrically around a point charge is a constant. It does not depend on the distance from the charged sphere. It should be noted that this result is only true if the inverse square law is true.

To see this, suppose some other force law were valid, i.e. $E = Q/4\pi\varepsilon r^n$. Then the total flux through the area

$$= \frac{Q}{4\pi\varepsilon r^n} \times 4\pi r^2$$

$$= \frac{Q}{\varepsilon} r^{(2-n)}$$

This is only independent of r if $n = 2$.

Field due to Charged Sphere and Plane Conductor

Equation (1) can be shown to be generally true. Thus the flux passing through any *closed* surface whatever its shape, is always equal to Q/ε, where Q is the total charge enclosed by the surface. This relation, called *Gauss's Theorem*, can be used to find the value of E in other common cases.

(1) *Outside a charged sphere*

The flux across a spherical surface of radius r, concentric with a small sphere carrying a charge Q (Fig. 30.21), is given by,

$$\text{Flux} = \frac{Q}{\varepsilon}$$

$$\therefore E \times 4\pi r^2 = \frac{Q}{\varepsilon}$$

$$\therefore E = \frac{Q}{4\pi\varepsilon r^2}$$

This is the same answer as that for a point charge. This means that *outside* a charged sphere, the field behaves as if all the charge on the sphere were concentrated at the centre.

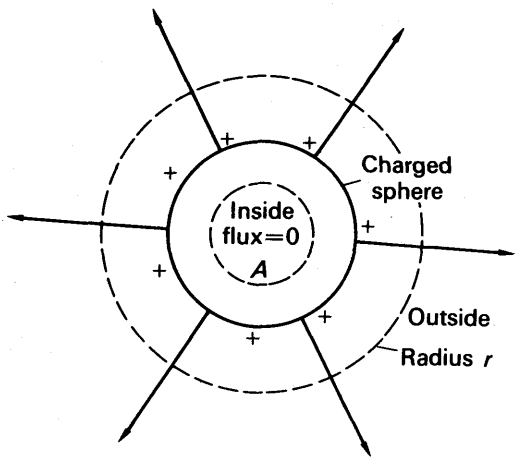


FIG. 30.21. Electric field of a charged sphere.

(2) *Inside a charged empty sphere*

Suppose a spherical surface A is drawn *inside* a charge sphere, as shown in Fig. 30.21. Inside this sphere there are no charges and so Q in equation (1), p. 747, is zero. This result is independent of the radius drawn, provided that it is less than that of the charged sphere. Hence from (1), p. 747, *E must be zero everywhere inside a charged sphere.*

(3) *Outside a Charged Plane Conductor*

Now consider a charged *plane* conductor S, with a surface charge density of σ coulomb metre^{-2}. Fig. 30.22 shows a plane surface P, drawn outside S, which is parallel to S and has an area A metre^2. Applying equation (1),

$$\therefore E \times \text{area} = \frac{\text{Charge inside surface}}{\varepsilon}$$

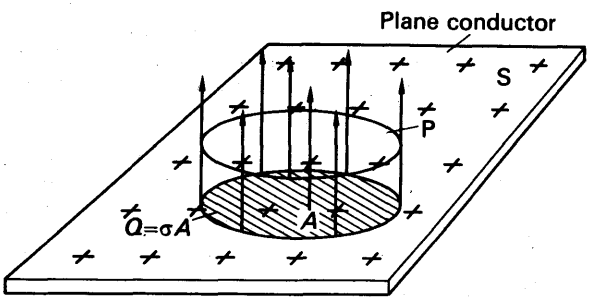


FIG. 30.22. Field of a charged plane conductor.

Now by symmetry, the intensity in the field must be perpendicular to the surface. Further, the charges which produce this field are those in the projection of the area P on the surface S, i.e. those within the shaded

area A in Fig. 30.22. The total charge here is thus σA coulomb.

$$\therefore E.A = \frac{\sigma A}{\varepsilon}$$

$$\therefore E = \frac{\sigma}{\varepsilon}$$

Field Round Points

On p. 742 we saw that the surface-density of charge (charge per unit area) round a point of a conductor is very great. Consequently, the strength of the electric field near the point is very great. The intense electric field breaks down the insulation of the air, and sends a stream of charged molecules away from the point. The mechanism of the breakdown, which is called a 'corona discharge', is complicated, and we shall not discuss it here; some of the processes in it are similar to those in conduction through a gas at low pressure, which we shall describe in Chapter 40. Corona breakdown starts when the electric field strength is about 3 million volt metre^{-1}. The corresponding surface-density is about $2{\cdot}7 \times 10^{-5}$ coulomb metre^{-2}.

EXAMPLE

An electron of charge $1{\cdot}6 \times 10^{-19}$ C is situated in a uniform electric field of intensity 1200 volt cm^{-1}. Find the force on it, its acceleration, and the time it takes to travel 2 cm from rest (electronic mass, m, $= 9{\cdot}10 \times 10^{-31}$ kg).

Force on electron $F = eE$.
Now $E = 1200$ volt cm^{-1} $= 120\,000$ volt m^{-1}.

$$\therefore \quad F = 1{\cdot}6 \times 10^{-19} \times 1{\cdot}2 \times 10^{5}$$

$$= 1{\cdot}92 \times 10^{-14} \text{ N (newton).}$$

Acceleration, $$a = \frac{F}{m} = \frac{1{\cdot}92 \times 10^{-14}}{9{\cdot}1 \times 10^{-31}}$$

$$= 2{\cdot}12 \times 10^{16} \text{ m s}^{-2} \text{ (metre second}^{-2}\text{)}$$

Time for 2 cm travel is given by

$$s = \tfrac{1}{2}at^2$$

$$\therefore \quad t = \sqrt{\frac{2s}{a}} = \sqrt{\frac{2 \times 0{\cdot}02}{2{\cdot}12 \times 10^{16}}}$$

$$= 1{\cdot}37 \times 10^{-9} \text{ seconds.}$$

The extreme shortness of this time is due to the fact that the ratio of charge-to-mass for an electron is very great:

$$\frac{e}{m} = \frac{1{\cdot}6 \times 10^{-19}}{9{\cdot}1 \times 10^{-31}} = 1{\cdot}8 \times 10^{11} \text{ C kg}^{-1}.$$

In an electric field, the charge e determines the force on an electron, while the mass m determines its inertia. Because of the large ratio e/m, the electron moves almost instantaneously, and requires very little energy to displace it. Also it can respond to changes in an electric field which take place even millions of times per second. Thus it is the large value of e/m for electrons which makes electronic tubes, for example, useful in electrical communication and remote control.

ELECTRIC POTENTIAL

Potential in Fields

When an object is held at a height above the earth it is said to have potential energy. A heavy body tends to move under the force of attraction of the earth from a point of great height to one of less, and we say that points in the earth's gravitational field have potential values depending on their height.

Electric potential is analogous to gravitational potential, but this time we think of points in an electric field. Thus in the field round a positive charge, for example, a positive charge moves from points near the charge to points further away. Points round the charge are said to have an 'electric potential'.

Potential Difference

In mechanics we are always concerned with differences of height; if a point A on a hill is h metre higher than a point B, and our weight is w newton, then we do wh joule of work in climbing from B to A (Fig. 30.23 (i)). Similarly in electricity we are often concerned with differences of potential; and we define these also in terms of work.

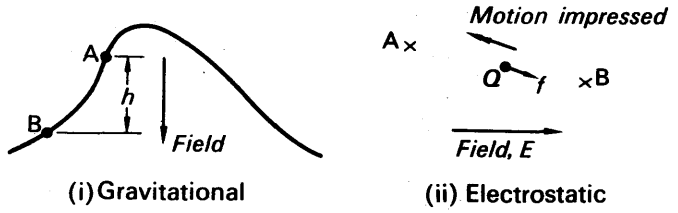


FIG. 30.23. Work done, in gravitational and electrostatic fields.

Let us consider two points A and B in an electrostatic field, and let us suppose that the force on a positive charge Q has a component f in the direction AB (Fig. 30.23 (ii)). Then if we move a positively charged body from B to A, we do work against this component of the field E. *We define the potential difference between A and B as the work done in moving a unit positive charge from B to A.* We denote it by the symbol V_{AB}.

The work done will be measured in joules (J). The unit of potential difference is called the volt and may be defined as follows: *The potential difference between two points A and B is one volt if the work done in taking one coulomb of positive charge from B to A is one joule.*

From this definition, if a charge of Q coulombs is moved through a p.d. of V volt, then the work done W in joules is given by

$$W = QV \quad . \quad . \quad . \quad . \quad (1)$$

Potential and Energy

Let us consider two points A and B in an electrostatic field, A being at a higher potential than B. The potential difference between A and

B we denote as usual by V_{AB}. If we take a positive charge Q from B to A, we do work on it of amount QV_{AB}: the charge gains this amount of potential energy. If we now let the charge go back from A to B, it loses that potential energy: work is done on it by the electrostatic force, in the same way as work is done on a falling stone by gravity. This work may become kinetic energy, if the charge moves freely, or external work if the charge is attached to some machine, or a mixture of the two.

The work which we must do in first taking the charge from B to A does not depend on the path along which we carry it, just as the work done in climbing a hill does not depend on the route we take. If this were not true, we could devise a perpetual motion machine, in which we did less work in carrying a charge from B to A via X than it did for us in returning from A to B via Y (Fig. 30.24).

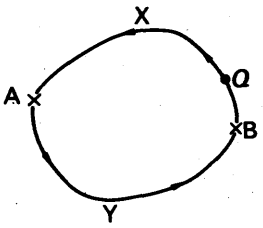


FIG. 30.24. A closed path in an electrostatic field.

The fact that the potential differences between two points is a constant, independent of the path chosen between the points, is the most important property of potential in general; we shall see why later on. This property can be conveniently expressed by saying that the work done in carrying a charge round a closed path in an electrostatic field, such as BXAYB in Fig. 30.24 is zero.

Potential Difference Formula

To obtain a formula for potential difference, let us calculate the potential difference between two points in the field of a single point positive charge, Q in Fig. 30.25. For simplicity we will assume that the points, A and B, lie on a line of force at distances a and b respectively

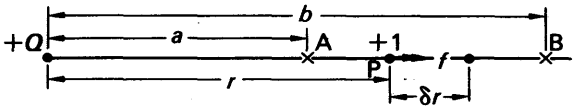


FIG. 30.25. Calculation of potential.

from the charge. When a unit positive charge is at a distance r from the charge Q in free space the force on it is

$$f = \frac{Q \times 1}{4\pi\varepsilon_0 r^2}$$

The work done in taking the charge from B to A, against the force f, is equal to the work which the force f would do if the charge were allowed to go from A to B. Over the short distance δr, the work done by the force f is

$$\delta W = f\delta r.$$

Over the whole distance AB, therefore, the work done by the force on the unit charge is

$$\int_A^B \delta W = \int_{r=a}^{r=b} f dr = \int_a^b \frac{Q}{4\pi\varepsilon_0 r^2} dr$$

$$= -\left[\frac{Q}{4\pi\varepsilon_0 r}\right]_a^b = \frac{Q}{4\pi\varepsilon_0 a} - \frac{Q}{4\pi\varepsilon_0 b}$$

This, then, is the value of the work which an external agent must do to carry a unit positive charge from B to A. The work per coulomb is the potential difference V_{AB} between A and B.

$$\therefore V_{AB} = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{a} - \frac{1}{b}\right) \quad . \quad . \quad . \quad (1)$$

V_{AB} will be in volts if Q is in coulombs, a and b are in metres and ε_0 is taken as $8{\cdot}85 \times 10^{-12}$ or $1/4\pi\varepsilon_0$ as 9×10^9 approximately (see p. 744).

EXAMPLE

Two positive point charges, of 12 and 8 microcoulomb respectively, are 10 cm apart. Find the work done in bringing them 4 cm closer. (Assume $1/4\pi\varepsilon_0 = 9 \times 10^9$.)

Suppose the 12 μC (microcoulomb) charge is fixed in position. Since 6 cm = 0·06 m and 10 cm = 0·1 m, then the potential difference between points 6 and 10 cm from it is given by (1).

$$\therefore \quad V = \frac{12 \times 10^{-6}}{4\pi\varepsilon_0}\left(\frac{1}{0{\cdot}06} - \frac{1}{0{\cdot}1}\right)$$

$$= 12 \times 10^{-6} \times 9 \times 10^9 \,(16\tfrac{2}{3} - 10)$$

$$= 720\,000 \text{ V}.$$

(Note the very high potential difference due to quite small charges.)

The work done in moving the 8 μC charge from 10 cm to 6 cm away from the other is given by, using $W = QV$,

$$W = 8 \times 10^6 \times V$$

$$= 8 \times 10^{-6} \times 720\,000$$

$$= 5{\cdot}8 \text{ J}.$$

Zero Potential

Instead of speaking continually of potential differences between pairs of points, we may speak of the potential at a single point—provided we always refer it to some other, agreed, reference point. This procedure is analogous to referring the heights of mountains to sea-level.

For practical purposes we generally choose as our reference point the electric potential of the surface of the earth. Although the earth is large it is all at the same potential, because it is a good conductor of electricity; if one point on it were at a higher potential than another, electrons would flow from the lower to the higher potential. As a result,

the higher potential would fall, and the lower would rise; the flow of electricity would cease only when the potentials became equalized.

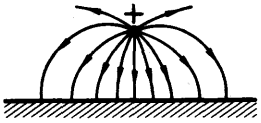


FIG. 30.26. Electric field of positive charge near earth.

In general it is difficult to calculate the potential of a point relative to the earth. This is because the electric field due to a charged body near a conducting surface is complicated, as shown by the lines of force diagram in Fig. 30.26. In theoretical calculations, therefore, we often find it convenient to consider charges so far from the earth that the effect of the earth on their field is negligible; we call these 'isolated' charges.

Thus we define the potential at a point A as V volts if V joules of work is done in bringing one coulomb of positive charge from infinity to A.

Potential Formula

Equation (1), p. 752, gives the potential difference between two points in the field of an isolated point charge Q. If we let the point B retreat to infinity, then $b \gg a$, and the equation gives for the potential at A:

$$V_A = \frac{Q}{4\pi\varepsilon_0 a} \quad . \quad . \quad . \quad . \quad (1)$$

The derivation of this equation shows us what we mean by the word 'infinity': the distance b is infinite if $1/b$ is negligible compared with $1/a$. If a is 1 cm, and b is 1 m, we make an error of only 1 per cent. in ignoring it; if b is 100 m, then for all practical purposes the point B is at infinity. In atomic physics, where the distances concerned have the order of 10^{-8} cm, a fraction of a millimetre is infinite.

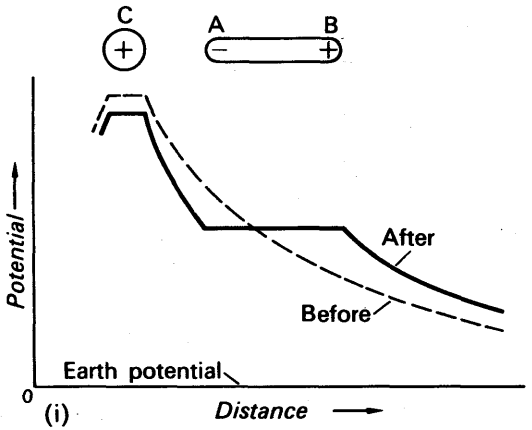


FIG. 30.27 (i). Potential distribution near a positive charge before and after bringing up an uncharged conductor.

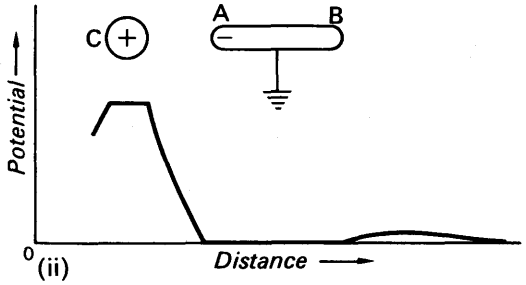


FIG. 30.27 (ii). Potential distribution near a positive charge in the presence of an earthed conductor.

In the neighbourhood of an isolated negative charge, the potential is negative, because Q in equation (1) is negative. The potential is also negative in the neighbourhood of a negative charge near the earth: the earth is at zero potential, and a positive charge will tend to move from it towards the negative charge. A negative potential is analogous to the depth of a mine below sea-level. Fig. 30.27(i) shows the potential variation near a positive charge C before and after a conductor AB is brought near. Fig. 30.27(ii) shows the potential variation when AB is earthed.

Potential Difference and Intensity

We shall now see how potential difference is related to intensity or field-strength. Suppose A, B are two neighbouring points on a line of force, so close together that the electric field-intensity between them is constant and equal to E (Fig. 30.28). If V is the potential at A, $V+\delta V$ is that at B, and the respective distances of A, B from the origin are x and $x+\delta x$, then

$$V_{AB} = \text{potential difference between A, B}$$
$$= V_A - V_B = V-(V+\delta V) = -\delta V.$$

The work done in taking a unit charge from B to A

$$= \text{force} \times \text{distance} = E \times \delta x = V_{AB} = -\delta V.$$

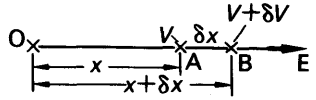


FIG. 30.28. Field strength and potential gradient.

Hence
$$E = -\frac{\delta V}{\delta x},$$

or, in the limit,

$$E = -\frac{dV}{dx}. \quad . \quad . \quad . \quad . \quad (1)$$

The quantity dV/dx is the rate at which the potential rises with distance, and is called the potential gradient. Equation (1) shows that the strength of the electric field is equal to the negative of the potential gradient, and strong and weak fields in relation to potential are illustrated in Fig. 30.29.

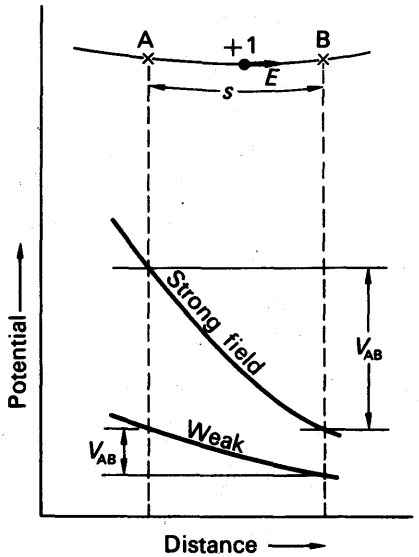


FIG. 30.29. Relationship between potential and field strength.

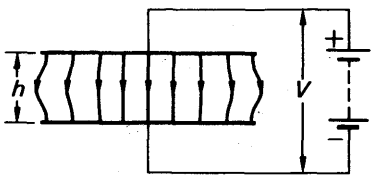


FIG. 30.30. Electric field between parallel plates.

In Fig. 30.30 the electric intensity $= V/h$, the potential gradient, and this is uniform in magnitude in the middle of the plates. At the edge of the plates the field becomes non-uniform.

We can now see why E is usually given in units of 'volt per metre' ($V\ m^{-1}$).

From (1), $E = -(dV/dx)$. Since V is measured in volts and x in metres, then E will be in volt per metre ($V\ m^{-1}$). From the original definition of E, summarized by equation (1) on p. 745, the units of E were newton coulomb^{-1}. To show that these are equivalent, from (1),

$$1 \text{ volt} = 1 \text{ joule coulomb}^{-1}$$
$$= 1 \text{ newton metre coulomb}^{-1}$$

Since $$1 \text{ joule} = 1 \text{ newton} \times 1 \text{ metre}$$

$$\therefore 1 \text{ volt metre}^{-1} = 1 \text{ newton coulomb}^{-1}$$

EXAMPLES

1 An electron is liberated from the lower of two large parallel metal plates separated by a distance $h = 2$ cm. The upper plate has a potential of 2400 volts relative to the lower. How long does the electron take to reach it?

Between large parallel plates, close together, the electric field is uniform except near the edges of the plates, as shown in Fig. 30.30. Except near the edges, therefore, the potential gradient between the plates is uniform; its magnitude is V/h,

$$\begin{aligned}\text{electric intensity } E &= \text{potential gradient}\\ &= 2400/0{\cdot}02 \text{ V m}^{-1}\\ &= 1{\cdot}2 \times 10^5 \text{ V m}^{-1}.\end{aligned}$$

The rest of this problem may now be worked out exactly as the example on p. 749.

2 An electron is liberated from a hot filament, and attracted by an anode, of potential 1200 volts positive with respect to the filament. What is the speed of the electron when it strikes the anode?

e = electronic charge $= 1{\cdot}6 \times 10^{-19}$ C.

$V = 1200$ V, m = mass of electron $= 9{\cdot}1 \times 10^{-31}$ kg.

The energy which the electron gains from the field $= QV = eV$.

Kinetic energy gained

$$= \tfrac{1}{2}mv^2 = eV,$$

where v is the speed gained from rest.

$$v = \sqrt{\frac{2eV}{m}} = \sqrt{\frac{2 \times 1{\cdot}6 \times 10^{-19} \times 1200}{9{\cdot}1 \times 10^{-31}}}$$

$$= 2{\cdot}1 \times 10^7 \text{ m s}^{-1}.$$

The Electron-Volt

The kinetic energy gained by an electron which has been accelerated through a potential difference of 1 volt is called an *electron-volt* (eV). Since the energy gained in moving a charge Q through a p.d. $V = QV$,

$\therefore$ 1 eV = electronic charge $\times$ 1 $= 1{\cdot}6 \times 10^{-19} \times 1$ joule $= 1{\cdot}6 \times 10^{-19}$ J.

The electron-volt is a useful unit of energy in atomic physics. For example, the work necessary to extract a conduction electron from tungsten is 4·52 electron-volt. This quantity determines the magnitude of the thermionic emission from the metal at a given temperature (p. 1026); it is analogous to the latent heat of evaporation of a liquid.

EQUIPOTENTIALS

Equipotentials

We have already said that the earth must have the same potential all over, because it is a conductor. In a conductor there can be no differences of potential, because these would set up a potential gradient or electric field; electrons would then redistribute themselves throughout the conductor, under the influence of the field, until they had

destroyed the field. This is true whether the conductor has a net charge, positive or negative, or whether it is uncharged; it is true whatever the actual potential of the conductor, relative to any other body.

Any surface or volume over which the potential is constant is called an *equipotential.* The volume or surface may be that of a material body, or simply a surface or volume in space. For example, as we shall see later, the space inside a hollow charged conductor is an equipotential volume. Equipotential surfaces can be drawn throughout any space in which there is an electric field, as we shall now explain.

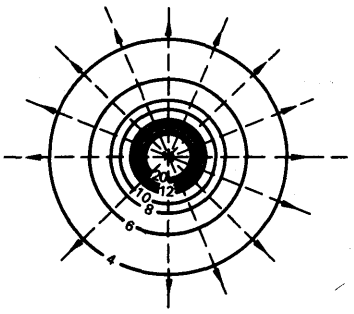


FIG. 30.31. Equipotentials and lines of force around a point charge.

Let us consider the field of an isolated point-charge Q. At a distance a from the charge, the potential is $Q/4\pi\varepsilon_0 a$; a sphere of radius a and centre at Q is therefore an equipotential surface, of potential $Q/4\pi\varepsilon_0 a$. In fact, all spheres centred on the charge are equipotential surfaces, whose potentials are inversely proportional to their radii (Fig. 30.31). An equipotential surface has the property that, along any direction lying in the surface, there is no electric field; for there is no potential gradient. *Equipotential surfaces are therefore always at right angles to lines of force,* as shown in Fig. 30.31. This also shows numerical values proportional to their potentials. Since conductors are always equipotentials, if any conductors appear in an electric-field diagram the lines of force must always be drawn to meet them at right angles.

Potential due to a System of Charges

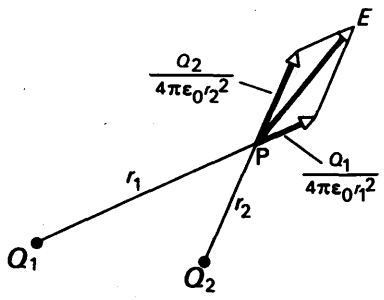
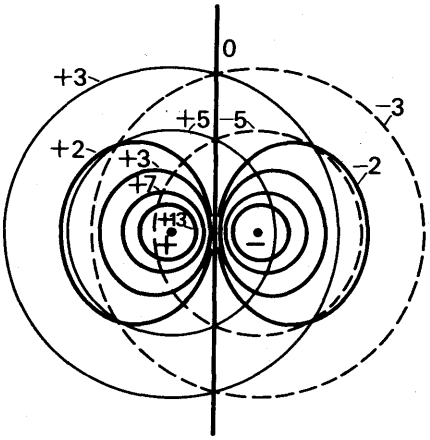


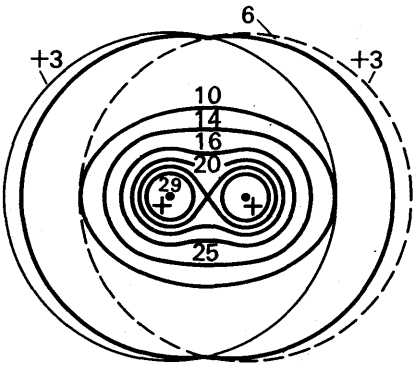
FIG. 30.32. Finding resultant field of two point-charges.

When we set out to consider the electric field due to more charges than one, then we see the advantages of the idea of potential over the idea of field-strength. If we wish to find the field-strength E at the point P in Fig. 30.32, due to the two charges Q_1 and Q_2, we have first to find the force exerted by each on a unit charge at P, and then to compound these forces by the parallelogram method. See Fig. 30.32. On the other hand, if we wish to find the potential at P, we merely calculate the potential due to each charge, and *add the potentials algebraically.*

Quantities which can be added algebraically are called 'scalars'; they may have signs—positive or negative, like a bank balance— but



(i) Opposite charges



(ii) Like charges

FIG. 30.33. Equipotentials in the field of two point charges.

they have no direction: they do not point north, east, south, or west. Quantities which have direction, like forces, are called 'vectors'; they have to be added either by resolution into components, or by the parallelogram method. Either way is slow and clumsy, compared with the addition of scalars. For example, we can draw the equipotentials round a point-charge with compasses; if we draw two sets of them, as in Fig. 30.33(i) or (ii), then by simple addition we can rapidly sketch the equipotentials around the two charges together.

And when we have plotted the equipotentials, they turn out to be more useful than lines of force. A line of force diagram appeals to the imagination, and helps us to see what would happen to a charge in the field. But it tells us little about the strength of the field—at the best, if it is more carefully drawn than most, we can only say that the field is strongest where the lines are closest. But equipotentials can

be labelled with the values of potential they represent; and from their spacing we can find the actual value of the potential gradient, and hence the field-strength. The only difficulty in interpreting equipotential diagrams lies in visualizing the direction of the force on a charge; this is always at right angles to the curves.

Field inside Hollow Conductor. Potential Difference and Gold-leaf Electroscope

If a hollow conductor contains no charged bodies, then, whatever charge there may be on its outside, there is none on its inside. Inside it, therefore, there is no electric field; the space within the conductor is an equipotential volume. If the conductor has an open end, like a can, then most of the space inside it is equipotential, but near its mouth there is a weak field (Fig. 30.34).

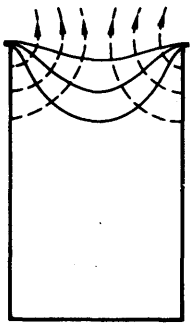


FIG. 30.34. Equipotentials and lines of force near mouth of an open charged can.

The behaviour of the *gold-leaf electroscope* illustrates this point. If we stand the case on an insulator, and connect the cap to it with a wire, then, no matter what charge we give to the cap, the leaves do not diverge (Fig. 30.35). Any charge we give to the cap spreads over the case

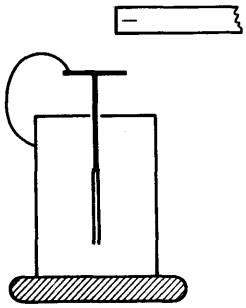


FIG. 30.35. Electroscope cap joined to case.

of the electroscope, but none appears on the leaves, and there is no force acting to diverge them. When, as usual, the cap is insulated and

the case earthed, charging the cap sets up a potential difference between it and the case. Charges appear on the leaves, and the field between them and the case makes them diverge (p. 733). If the case is insulated from earth, as well as from the cap, the leaves diverge less; the charge on them and the cap raises the potential of the case and reduces the potential difference between it and the leaves. The field acting on the leaves is thus made weaker, and the force on the leaves less. We can sum up these observations by saying that *the electroscope indicates the potential difference between its leaves and its case.*

Potential of Pear-shaped Conductor

On p. 742 we saw that the surface-density of the charge on a pear-shaped conductor was greatest where the curvature was greatest. The potential of the conductor at various points can be examined by means of the gold-leaf electroscope, the case being earthed. One end of a wire is connected to the cap; some of the wire is then wrapped round an insulating rod, and the free end of the wire is placed on the conductor. As the free end is moved over the conductor, it is observed that the divergence of the leaf remains constant. This result was explained on pp. 756, 757.

Electrostatic Shielding

The fact that there is no electric field inside a close conductor, when it contains no charged bodies, was demonstrated by Faraday in a spectacular manner. He made for himself a large wire cage, supported it on insulators, and sat inside it with his electroscopes. He then had the cage charged by an induction machine—a forerunner of the type we described on p. 737—until painful sparks could be drawn from its outside. Inside the cage Faraday sat in safety and comfort, and there was no deflection to be seen on even his most sensitive electroscope.

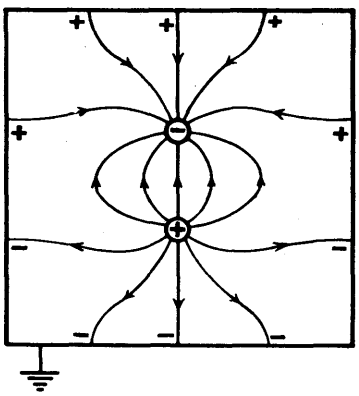


FIG. 30.36. Lines of force round charges.

If we wish to protect any persons or instruments from intense electric fields, therefore, we enclose them in hollow conductors; these are called 'Faraday cages', and are widely used in high-voltage measurements in industry.

We may also wish to prevent charges in one place from setting up an electric field beyond their immediate neighbourhood. To do this we surround the charges with a Faraday cage, and connect the cage to earth (Fig. 30.36). The charge induced on the outside of the cage then runs to earth, and there is no external field. (When a cage is used to shield something *inside* it, it does not have to be earthed.)

Comparison of Static and Current Phenomena

Broadly speaking, we may say that in electrostatic phenomena we meet small quantities of charge, but great differences of potential. On the other hand in the phenomena of current electricity discussed later, the potential differences are small but the amounts of charge transported by the currents are great. Sparks and shocks are common in electrostatics, because they require great potential differences; but they are rarely dangerous, because the total amount of energy available is usually small. On the other hand, shocks and sparks in current electricity are rare, but, when the potential difference is great enough to cause them, they are likely to be dangerous.

These quantitative differences make problems of insulation much more difficult in electrostatic apparatus than in apparatus for use with currents. The high potentials met in electrostatics make leakage currents relatively great, and the small charges therefore tend to disappear rapidly. Any wood, for example, ranks as an insulator for current electricity, but a conductor in electrostatics. In electrostatic experiments we sometimes wish to connect a charged body to earth; all we have then to do is to touch it.

EXAMPLE

Three charges $-1\ \mu C$, $2\ \mu C$ and $3\ \mu C$ are placed respectively at the corners A, B, C of an equilateral triangle of side 2 metres. Calculate (*a*) the potential, (*b*) the electric field, at a point X which is half-way along BC. Fig. 30.37.

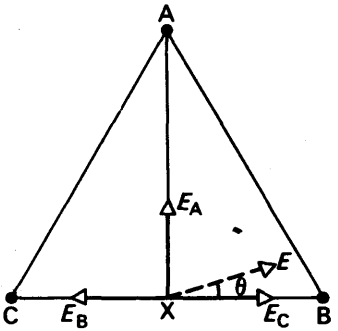


FIG. 30.37. Example.

(*a*) Potential at X due to charge at B is

$$\frac{Q}{4\pi\varepsilon_0 r} = \frac{2\times10^{-6}}{4\pi\varepsilon_0\times 1}$$

$$= 18\times10^3\ \text{V}.$$

Similarly potential at X due to the charge at C $= 27\times10^3$ V, and the potential due to A is

$$V_A = \frac{Q}{4\pi\varepsilon_0 r} = \frac{-10^{-6}}{4\pi\varepsilon_0\times\sqrt{3}}$$

$$= -5\times10^3\ \text{V (approx.)}.$$

Since potential is a scalar quantity and can be added algebraically, the net potential at X

$$= (18+27-5)\times10^3\ \text{V}$$

$$= 40\times10^3\ \text{V}.$$

(*b*) The resultant field at X is due to the three electric fields from the three charges.

The field due to B, E_B has magnitude given by

$$E_B = \frac{Q}{4\pi\varepsilon_0 r^2} = \frac{2\times10^{-6}}{4\pi\varepsilon_0\times 1^2}$$

$$= 18\times10^3\ \text{V m}^{-1}.$$

Similarly $E_C = 27\times10^3\ \text{V m}^{-1}$.

Since these act along the same straight line the resultant of E_B and $E_C = 9 \times 10^3$ V m^{-1} directed from C to B.

Also,
$$E_A = \frac{Q}{4\pi\varepsilon_0 r^2} = \frac{10^{-6}}{4\pi\varepsilon_0 \times (\sqrt{3})^2}$$
$$= 3 \times 10^3 \text{ V m}^{-1}.$$

The resultant field has magnitude, E, given by
$$E^2 = E_A^2 + (E_C - E_B)^2$$
$$= (9+81) \times 10^6$$
$$= 90 \times 10^6$$
$$E = 3\sqrt{10} \times 10^3 \text{ V m}^{-1} = 9{\cdot}5 \times 10^3 \text{ V m}^{-1}.$$

This makes an angle θ with CB, in the direction shown by the dotted line, where
$$\tan\theta = \frac{E_A}{E_C - E_B} = \frac{3 \times 10^3}{9 \times 10^3} = \frac{1}{3}$$
$$\therefore \quad \theta = 18° 25'.$$

EXERCISES 30

1. Describe experiments with a gold leaf electroscope:
(*a*) to demonstrate that this instrument indicates the potential difference between its leaves and its case and not necessarily the total charge on its leaves and cap;
(*b*) to compare the quantities of electricity on two conductors of unequal size;
(*c*) to investigate the distribution of electricity on a charged conductor.
In case (*c*) state the results you would expect if the conductor were spherical with a pointed rod attached to it, and describe and explain two practical applications of pointed conductors. (*L.*)

2. Describe, with the aid of a labelled diagram, a Van de Graaff generator, explaining the physical principles of its action.

The high voltage terminal of such a generator consists of a spherical conducting shell of radius 50 cm. Estimate the maximum potential to which it can be raised in air for which electrical breakdown occurs when the electric intensity exceeds 30000 volt cm^{-1}.

State two ways in which this maximum potential could be increased. (*N.*)

3. Define *potential at a point* in an electric field.

Sketch a graph illustrating the variation of potential along a radius from the centre of a charged isolated conducting sphere to infinity.

Assuming the expression for the potential of a charged isolated conducting sphere in air, determine the change in the potential of such a sphere caused by surrounding it with an earthed concentric thin conducting sphere having three times its radius. (*N.*)

4. What is meant by the terms (*a*) *potential* and (*b*) *field strength* in electrostatics? State whether each quantity is a scalar or a vector.

Write down the law which gives the force between two point charges q_1 and q_2 at a distance r apart and use it to derive the electric field strength and the potential due to a point charge q at a distance x from it.

The points A, B and C form an equilateral triangle of side z. Point charges

of equal magnitude q are placed at A and B. Find the electric field strength and the potential at C due to these charges when (i) both charges are positive and (ii) the charge at A is positive and the charge at B is negative.

O is the midpoint of AB and POQ is the perpendicular bisector of AB; PO and OQ are very large distances compared with AB. Draw rough graphs to show how the magnitude of the electric field strength and the potential vary along the line POQ in cases (i) and (ii) above. (*O. & C.*)

5. Describe simple electrostatic experiments to illustrate *two* of the following: (*a*) the production of equal and opposite charges by induction; (*b*) the action of points; (*c*) the effect on a charged conductor of the approach of an earthed conductor. (*L.*)

6. Discuss the following, giving examples: *Electrostatic induction, electric discharge from points* and *dielectric strength.*

Describe a modern form of apparatus for obtaining a small current at a very high voltage. (*L.*)

7. An isolated conducting spherical shell of radius 10 cm, in vacuo, carries a positive charge of $1{\cdot}0 \times 10^{-7}$ coulomb. Calculate (*a*) the electric field intensity, (*b*) the potential, at a point on the surface of the conductor. Sketch a graph to show how one of these quantities varies with distance along a radius from the centre to a point well outside the spherical shell. Point out the main features of the graph. (*N.*)

8. Define (*a*) electric intensity, (*b*) difference of potential. How are these quantities related?

A charged oil-drop of radius 0·00013 cm is prevented from falling under gravity by the vertical field between two horizontal plates charged to a difference of potential of 8340 volts. The distance between the plates is 1·6 cm, and the density of oil is 920 kg m^{-3}. Calculate the magnitude of the charge on the drop ($g = 9{\cdot}81$ m s^{-2}). (*O. & C.*)

9. Two plane parallel conducting plates 1·50 cm apart are held horizontal, one above the other, in air. The upper plate is maintained at a positive potential of 1500 volts while the lower plate is earthed. Calculate the number of electrons which must be attached to a small oil drop of mass $4{\cdot}90 \times 10^{-12}$ g, if it remains stationary in the air between the plates. (Assume that the density of air is negligible in comparison with that of oil.)

If the potential of the upper plate is suddenly changed to -1500 volts what is the initial acceleration of the charged drop? Indicate, giving reasons, how the acceleration will change.

10. Show how (i) the surface density, (ii) the intensity of electric field, (iii) the potential, varies over the surface of an elongated conductor charged with electricity. Describe experiments you would perform to support your answer in cases (i) and (iii).

Describe and explain the action of points on a charged conductor; and give two practical applications of the effect. (*L.*)

11. Describe carefully Faraday's ice-pail experiments and discuss the deductions to be drawn from them. How would you investigate experimentally the charge distribution over the surface of a conductor? (*C.*)

12. What is an *electric field*? With reference to such a field define *electric potential.*

Two plane parallel conducting plates are held horizontal, one above the other, in a vacuum. Electrons having a speed of $6{\cdot}0 \times 10^{8}$ cm s^{-1} and moving normally

to the plates enter the region between them through a hole in the lower plate which is earthed. What potential must be applied to the other plate so that the electrons just fail to reach it? What is the subsequent motion of these electrons? Assume that the electrons do not interact with one another.

(Ratio of charge to mass of electron is $1{\cdot}8 \times 10^{11}$ coulomb kg^{-1}.) (*N.*)

13. Define the *electric potential V* and the *electric field strength E* at a point in an electrostatic field. How are they related? Write down an expression for the electric field strength at a point close to a charged conducting surface, in terms of the surface density of charge.

Corona discharge into the air from a charged conductor takes place when the potential gradient at its surface exceeds 3×10^6 volt $metre^{-1}$; a potential gradient of this magnitude also breaks down the insulation afforded by a solid dielectric. Calculate the greatest charge that can be placed on a conducting sphere of radius 20 cm supported in the atmosphere on a long insulating pillar; also calculate the corresponding potential of the sphere. Discuss whether this potential could be achieved if the pillar of insulating dielectric was only 50 cm long. (Take ε_0 to be $8{\cdot}85 \times 10^{-12}$ farad $metre^{-1}$.) (*O.*)

14. (i) A needle is mounted vertically, point upwards, on the plate (cap) of a gold leaf electroscope, the blunt end being in metallic contact with the plate. When a negatively charged body is brought close to the needle point, without touching it, and is then withdrawn the gold leaf is left with a permanent deflection. What is the sign of the charge causing this deflection, and how was this charge produced?

(ii) A gold leaf electroscope is so constructed that for a few degrees deflection the gold leaf touches the case and is thereby earthed. Describe and explain the behaviour of the leaf when:

(*a*) a positively charged, insulated body is brought towards the plate (cap) of the electroscope until the leaf touches the case;

(*b*) the positively charged body is then moved slowly closer to the plate;

(*c*) the positively charged body is fixed close to the plate and the air between them is feebly ionized. (*O. & C.*)

chapter thirty-one

Capacitors

A capacitor (or 'condenser'), is a device for storing electricity. The earliest capacitor was invented—almost accidentally—by van Musschenbroek of Leyden, in about 1746, and became known as a Leyden jar. A present-day form of it is shown in Fig. 31.1(i), J is a glass jar, FF are tin-foil coatings over the lower parts of its walls, and T is a knob connected to the inner coating. Modern forms of capacitor are shown at (ii) and (iv) in the figure. Essentially, all capacitors consist of two metal plates separated by an insulator. The insulator is called the dielectric; in some capacitors it is oil or air. Fig. 31.1(iii) shows the conventional symbol for a capacitor.

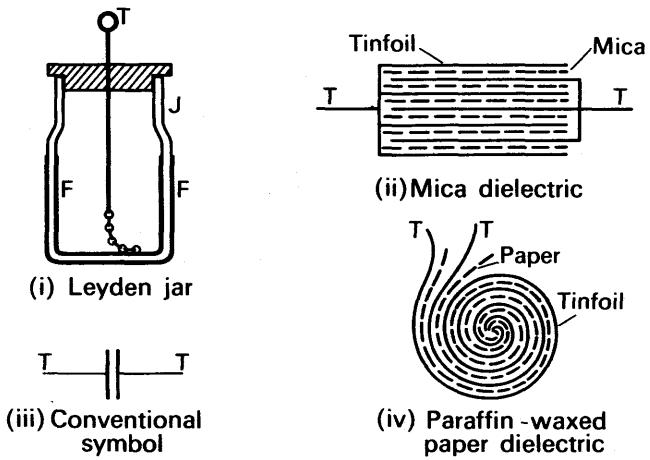


FIG. 31.1. Types of capacitor.

Charging a Capacitor

To study the action of a capacitor we need a paper one of about 4 microfarad capacitance (see later), a couple of high-tension batteries D and a high impedance voltmeter V such as a valve voltmeter reading to about 300 volts. We also need a two-way key (K in Fig. 31.2) and a poor conductor (R). The latter is a short stick of powdered and compressed carbon; it is called a radio resistor, and should have a resistance of about 5 megohms (p. 790). We connect the batteries in series, and measure their total voltage, V_0, with the voltmeter (Fig. 31.2(i)). We then connect up all the apparatus as shown in Fig. 31.2(ii). If we close

the key at A, the capacitor is connected via the resistor to the battery, and the potential difference across the capacitor, V, which is measured

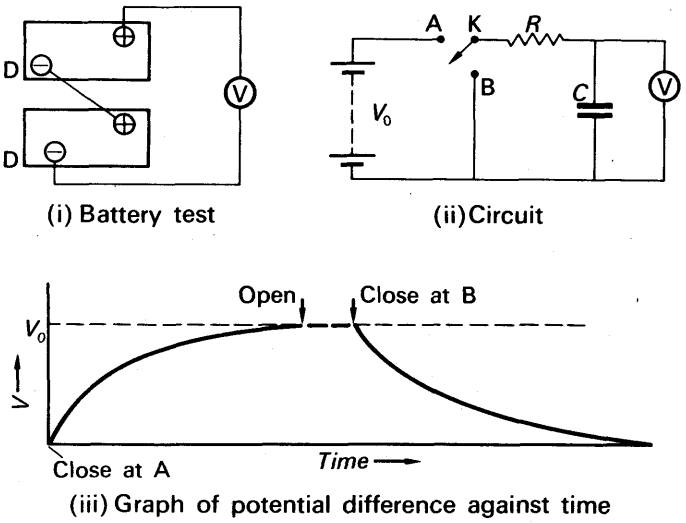


FIG. 31.2. Charging and discharging a capacitor through a resistor.

by the voltmeter, begins to rise (Fig. 31.2(iii)). The potential difference becomes steady when it is equal to the battery voltage V_0. If we now open the key, the voltmeter reading stays unchanged (unless the capacitor is leaky). The capacitor is said to be charged, to the battery voltage; its condition does not depend at all on the resistor, whose only purpose was to slow down the charging process, so that we could follow it on the voltmeter.

Discharging a Capacitor

We can show that the charged capacitor is storing electricity by discharging it: if we put a piece of wire across its terminals, a fat spark passes just as the wire makes contact, and the voltmeter reading falls to zero.

If we now recharge the capacitor and then close the key at B, in Fig 31.2(ii), we allow the capacitor to discharge through the resistor R. The potential difference across it now falls to zero as slowly as it rose during charging.

Charging and Discharging Processes

When we connect a capacitor to a battery, electrons flow from the negative terminal of the battery on to the plate A of the capacitor

connected to it (Fig. 31.3); and, at the same rate, electrons flow from the other plate B of the capacitor towards the positive terminal of the battery. Positive and negative charges thus appear on the plates, and oppose the flow of electrons which causes them. As the charges accumulate, the potential difference between the plates increases, and the charging current falls to zero when the potential difference becomes equal to the battery voltage V_0.

When the battery is disconnected and the plates are joined together by a wire, electrons flow back from plate A to plate B until the positive charge on B is completely neutralized. A current thus flows for a time in the wire, and at the end of the time the charges on the plates become zero.

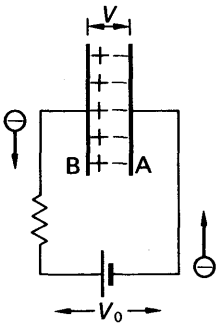


FIG. 31.3. A capacitor charging. (Resistance is shown because some is always present, even if only that of the connecting wires.)

Capacitors in A.C. Circuits

Capacitors are widely used in alternating current and radio circuits, because they can transmit alternating currents. To see how they do so, let us consider the circuit of Fig. 31.4, in which the capacitor may be connected across either of the batteries X, Y. When the key is closed at A, current flows from the battery X, and charges the plate D of the capacitor positively. If the key is now closed at B instead, current flows from the battery Y; the plate D loses its positive charge and becomes negatively charged. Thus if the key is rocked rapidly between A and B, current surges backwards and forwards along the wires connected to the capacitor. An alternating voltage, as we shall see in Chapter 39, is one which reverses many times a second; when such a voltage is applied to a capacitor, therefore, an alternating current flows in the connecting wires.

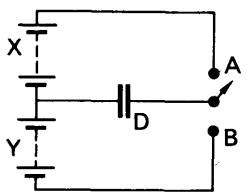


FIG. 31.4. Reversals of voltage applied to capacitor.

Capacitance Definition, and Units

Experiments with a ballistic galvanometer, which measures quantity of electricity, show that, when a capacitor is charged to a potential difference V, the charges stored on its plates, $\pm Q$, are proportional to V. The ratio of the charge on either plate to the potential difference between the plates is called the *capacitance*, C, of the capacitor:

$$C = \frac{Q}{V}. \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Thus

$$Q = CV, \quad . \quad . \quad . \quad . \quad . \quad (2)$$

and

$$V = \frac{Q}{C}. \quad . \quad . \quad . \quad . \quad . \quad (3)$$

When Q is in coulombs (C) and V is in volts (V), then capacitance C is in farads (F). One farad (1F) is the capacitance of an extremely large capacitor. In practical circuits, such as in radio receivers, the capacitance of capacitors used are therefore expressed in *microfarads* (μF). One microfarad is one millionth part of a farad, that is, $1\mu F = 10^{-6}F$. It is also quite usual to express small capacitors, such as those used on radiograms for altering tone, in *picofarads* (pF). A picofarad is one millionth part of a microfarad, that is, $1pF = 10^{-6}\mu F = 10^{-12}F$.

Comparison of Capacitances

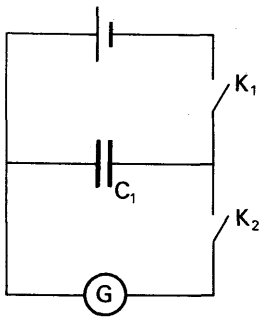


FIG. 31.5. Comparison of capacitances.

Large capacitances, of the order of microfarads, can be compared with the aid of a ballistic galvanometer. In this instrument, as explained on p. 920, the first 'throw' or deflection is proportional to the quantity of electricity discharged through it.

The circuit required is shown in Fig. 31.5. The capacitor of capacitance C_1 is charged by a battery of e.m.f. V, and then discharged through the ballistic galvanometer G. The corresponding first deflection θ_1 is observed. The capacitor is now replaced by another of capacitance C_2, charged again by the battery, and the new deflection θ_2 is observed when the capacitor is discharged.

Now

$$\frac{Q_1}{Q_2} = \frac{\theta_1}{\theta_2}.$$

$$\therefore \frac{C_1 V}{C_2 V} = \frac{\theta_1}{\theta_2}.$$

$$\therefore \frac{C_1}{C_2} = \frac{\theta_1}{\theta_2}. \quad . \quad . \quad . \quad . \quad (4)$$

If C_2 is a standard capacitor, whose value is known, then the capacitance of C_1 can be found.

Factors determining Capacitance. Variable Capacitor

We shall now find out by experiment what factors influence capacitance. To interpret our observations we shall require the formula for potential difference:

$$V = \frac{Q}{C}.$$

This shows that, when a capacitor is given a fixed charge, the potential difference between its plates is inversely proportional to its capacitance.

Distance between plates. In Fig. 31.6 (i), A and B are two metal plates, B being earthed, while A is insulated and connected to an electroscope.

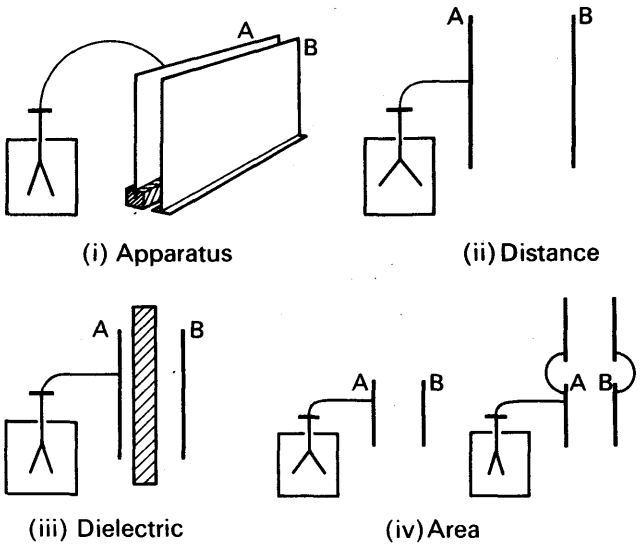


FIG. 31.6. Factors determining capacitance.

We set the plates close together, but not touching, and charge A from an electrophorus. The leaves of the electroscope diverge by an amount which measures the potential difference between the plates. If we move the plates further apart the leaves diverge further, showing that the potential difference has increased (Fig. 31.6 (ii)). Since we have done nothing to increase the charge on the plates, the increase in potential difference must be due to a decrease in capacitance (see above equation). The capacitance of a capacitor therefore decreases when the separation of its plates is increased; we shall see in the next chapter that the capacitance is inversely proportional to the separation.

Dielectric. Let us now put a dielectric—a sheet of glass or ebonite—between the plates. The leaves diverge less, showing that the potential difference has decreased (Fig. 31.6 (iii)). The capacitance has therefore increased, and the increase is due to the dielectric. By using several sheets of it we can show that the effect of the dielectric increases with its thickness. In practical capacitors the dielectric completely fills the space between the plates.

Area of plates. To see how the capacitance depends on the area of the plates, we set them at a known distance apart. We then take another pair of plates, at the same distance apart, and connect them to the first pair by wires held on insulating handles (Fig. 31.6 (iv)). The leaves diverge less, showing that the capacitance has increased; it is, in fact, directly proportional to the area of the plates.

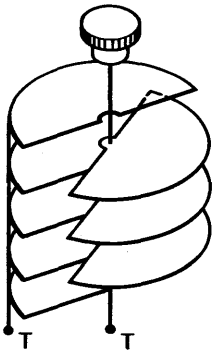


FIG. 31.7. Variable air capacitor.

A capacitor in which the effective area of the plates can be adjusted is called a *variable capacitor*. In the type shown in Fig. 31.7, the plates are semicircular and one set can be swung into or out of the other. The capacitance is proportional to the area of overlap of the plates. The plates are made of brass or aluminium, and the dielectric may be air, oil, or mica.

We shall see shortly that a capacitor with parallel plates, having a vacuum (or air, if we assume the permittivity of air is the same as a vacuum) between them, has a capacitance given by

$$C = \frac{\varepsilon_0 A}{d}$$

where C = capacitance in farads (F), A = area of overlap of plates in m^2, d = distance between plates in m and $\varepsilon_0 = 8{\cdot}854 \times 10^{-12}$ farad metre^{-1}.

If a material of permittivity ε completely fills the space between the plates, then the capacitance becomes:

$$C = \frac{\varepsilon A}{d}$$

Capacitance Values. Isolated Sphere

Suppose a sphere of radius r metre situated in air is given a charge of Q coulombs. We assume, as on p. 747, that the charge on a sphere gives rise to potentials *on and outside* the sphere as if all the charge were concentrated at the *centre*. From p. 757, the surface of the sphere has a potential given by:

$$V = \frac{Q}{4\pi\varepsilon_0 r}$$

$$\therefore \frac{Q}{V} = 4\pi\varepsilon_0 r$$

$$\therefore \text{Capacitance, } C = 4\pi\varepsilon_0 r \quad . \quad . \quad . \quad . \quad (1)$$

The other 'plate' of the capacitor is the earth.

Suppose $r = 10$ cm $= 0{\cdot}1$ m. Then,

$$C = 4\pi\varepsilon_0 r = 4\pi \times 8{\cdot}85 \times 10^{-12} \times 0{\cdot}1 \text{ F}$$

$$= 11 \times 10^{-12} \text{ F (approx.)} = 11 \text{ pF}.$$

Concentric Spheres

Faraday used two concentric spheres to investigate the dielectric constant of liquids. Suppose a, b are the respective radii of the inner

and outer spheres (Fig. 31.8). Let $+Q$ be the charge given to the inner sphere and let the outer sphere be earthed, with air between them.

The induced charge on the outer sphere is $-Q$ (see p. 740). The potential V_a of the inner sphere = potential due to $+Q$ and potential due to $-Q$ = $+\frac{Q}{4\pi\varepsilon_0 a} - \frac{Q}{4\pi\varepsilon_0 b}$, since the potential due to the charge $-Q$ is $-Q/4\pi\varepsilon_0 b$ everywhere inside the larger sphere (see p. 757).

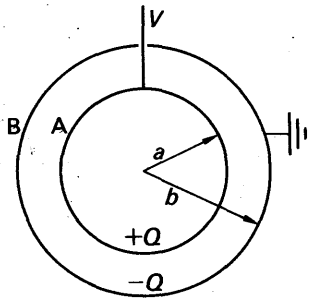


FIG. 31.8. Concentric spherical capacitor.

But $V_b = 0$, as the outer sphere is earthed.

$$\therefore \text{ potential difference, } V = V_a - V_b = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q}{a} - \frac{Q}{b}\right)$$

$$\therefore V = \frac{Q}{4\pi\varepsilon_0}\left(\frac{b-a}{ab}\right).$$

$$\therefore \frac{Q}{V} = \frac{4\pi\varepsilon_0 ab}{b-a},$$

$$C = \frac{4\pi\varepsilon_0 ab}{b-a} \quad . \quad . \quad . \quad . \quad . \quad (1)$$

As an example, suppose $b = 10$ cm and $a = 9$ cm

$$\therefore C = \frac{4\pi\varepsilon_0 ab}{b-a}$$

$$= \frac{4\pi \times 8{\cdot}85 \times 10^{-12} \times 0{\cdot}1 \times 0{\cdot}09}{(0{\cdot}1 - 0{\cdot}09)}$$

$$= 100 \text{ pF (approx).}$$

Note that the inclusion of a nearby second plate to the capacitor increases the capacitance. For an *isolated* sphere of radius 10 cm, the capacitance was 11 pF (p. 770).

Parallel Plate Capacitor

Suppose two parallel plates of a capacitor each have a charge numerically equal to Q, Fig. 31.9. The surface density σ is then Q/A

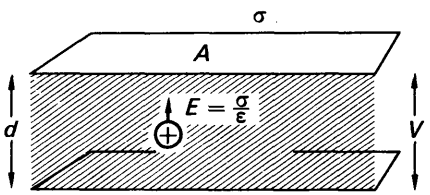


FIG. 31.9. Parallel-plate capacitor.

where A is the area of either plate, and the intensity between the plates, E, is given, from p. 749, by

$$E = \frac{\sigma}{\varepsilon} = \frac{Q}{\varepsilon A}$$

Now the work done in taking a unit charge from one plate to the other = force × distance = $E \times d$ where d is the distance between the plates. But the work done per unit charge = V, the p.d. between the plates.

$$\therefore V = \frac{\sigma}{\varepsilon} d = \frac{Q}{\varepsilon A} d$$

$$\therefore \frac{Q}{V} = \frac{\varepsilon A}{d}$$

$$\therefore C = \frac{\varepsilon A}{d} \quad . \quad . \quad . \quad . \quad (1)$$

It should be noted that this formula for C is approximate, as the field becomes non-uniform at the edges. See Fig. 30.30, p. 755.

Dielectric Constant (Relative Permittivity) and Strength

To study the effect of the dielectric in a capacitor, the capacitance of a given capacitor must be measured: first without a dielectric, and then with one. We shall see later how this can be done (p. 776).

The ratio of the capacitance with and without the dielectric between the plates is called the *dielectric constant* (or *relative permittivity*) of the material used. The expression 'without a dielectric' strictly means 'with the plates in a vacuum'; but the effect of air on the capacitance of a capacitor is so small that for most purposes it may be neglected. The dielectric constant of a substance is denoted by the letter ε: thus

$$\varepsilon = \frac{\text{capacitance of given capacitor, with space between plates filled with dielectric}}{\text{capacitance of same capacitor with plates } \textit{in vacuo}}$$

If we take the case of a parallel plate capacitor as an example, then

$$\varepsilon_r = \frac{\varepsilon A/d}{\varepsilon_0 A/d} = \frac{\varepsilon}{\varepsilon_0}$$

Thus the dielectric constant is the ratio of the permittivity of the substance to that of free space. It is for this reason that dielectric constant is also known as 'relative permittivity'. Note that the dielectric constant is a pure number and has no dimensions, unlike ε or ε_0.

The following table gives the value of dielectric constant, and also of *dielectric strength*, for various substances. The strength of a dielectric is the potential gradient at which its insulation breaks down, and a spark passes through it. A solid dielectric is ruined by such a breakdown, but a liquid or gaseous one heals up as soon as the applied potential difference is reduced.

Water is not suitable for a dielectric in practice, because it is a good

insulator only when it is very pure, and to remove all matter dissolved in it is almost impossible.

PROPERTIES OF DIELECTRICS

Substance	Dielectric constant	Dielectric strength, kilovolts per mm
Glass	5–10	30–150
Mica	6	80–200
Ebonite	2·8	30–110
Ice*	94	—
Paraffin wax	2	15–50
Paraffined paper	2	40–60
Paraffin oil	4·7	—
Methyl alcohol*	32	—
Water*	81	—
Air	1·0005	—
Sulphur dioxide*	1·01	—

* Polar molecules (see p. 774).

Action of Dielectric

The explanation of dielectric action which we shall now give is similar in principle to Faraday's, but expressed in modern terms—there was no knowledge of electrons in his day.

We regard a molecule as a collection of atomic nuclei, positively charged, and surrounded by a cloud of negative electrons. When a dielectric is in a charged capacitor, its molecules are in an electric field; the nuclei are urged in the direction of the field, and the electrons in the opposite direction (Fig. 31.10 (i)). Thus each molecule is distorted, or polarized: one end has an excess of positive charge, the other an excess of negative. At the surfaces of the dielectric, therefore, charges appear, as shown in Fig. 31.10 (ii). These charges are of opposite sign to the charges on the plates, and so reduce the potential difference between the plates.

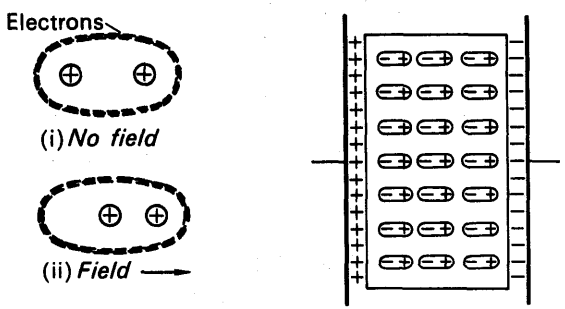


FIG. 31.10. Polarization of a dielectric.

If the capacitor is connected to a battery, then its potential difference is constant; but the surface charges on the dielectric still increase its capacitance. They do so because they offset the mutual repulsions of the charges on the plates, and so enable greater charges to accumulate there before the potential difference rises to the battery voltage.

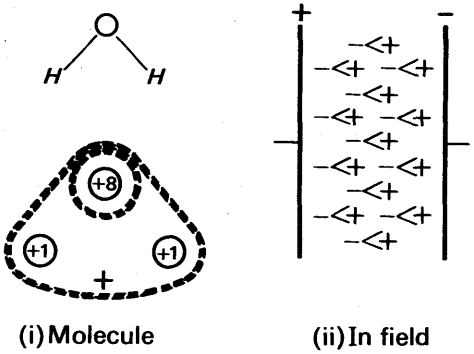


FIG. 31.11. Water as a dielectric.

Some molecules, we believe, are permanently polarized: they are called *polar molecules*. For example, the water molecule consists of an oxygen atom, *O*, with two hydrogen atoms *H*, making roughly a right-angled structure (Fig. 31.11 (i)). Oxygen has a nuclear charge of $+8e$, where e is the electronic charge, and has eight electrons. Hydrogen has a nuclear charge of $+e$, and one electron. In the water molecule, the two electrons from the hydrogen atom move in paths which surround the oxygen nucleus. Thus they are partly added to the oxygen atom, and partly withdrawn from the hydrogen atoms. On the average, therefore, the apex of the triangle is negatively charged, and its base is positively charged. In an electric field, water molecules tend to orient themselves as shown in Fig. 31.11 (ii). The effect of this, in a capacitor, is to increase the capacitance in the way already described. The increase is, in fact, much greater than that obtained with a dielectric which is polarized merely by the action of the field.

ε_0 and its measurement

We can now see how the units of ε_0 may be stated in a more convenient manner and how its magnitude may be measured.

Units. From $C = \dfrac{\varepsilon_0 A}{d}$, we have $\varepsilon_0 = \dfrac{Cd}{A}$

Thus the unit of $\varepsilon_0 = \dfrac{\text{farad} \times \text{metre}}{\text{metre}^2}$

$= \text{farad metre}^{-1}$ (see also p. 743).

Measurement. In order to find the magnitude of ε_0, the circuit in Fig. 31.12 is used.

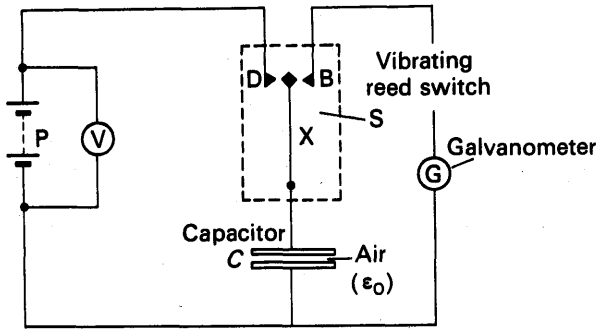


FIG. 31.12. Measurement of ε_0.

C is a parallel plate capacitor, which may be made of sheets of glass or perspex coated with aluminium foil or aquadag. The two conducting surfaces are placed facing inwards, so that only air is present between these plates. The area A of the plates in metre^2, and the separation d in metres, are measured. P is a high tension supply capable of delivering about 200 V, and G is a calibrated sensitive galvanometer, such as a 'Scalamp' type. S is a *vibrating switch* unit, energized by a low a.c. voltage from the mains. When operating, the vibrating bar X touches and then B, and the motion is repeated at the mains frequency, fifty times a second. When the switch is in contact with D, the capacitor is charged from the supply P to a potential difference of V volt, measured on the voltmeter. When the contact moves over to B, the capacitor discharges through the galvanometer. The galvanometer thus receives fifty pulses of charge per second. This gives an average steady reading on the galvanometer, corresponding to a mean current I.

Now from previous, $C = \varepsilon_0 A/d$. Thus on charging, the charge stored, Q, is given by

$$Q = CV = \frac{\varepsilon_0 VA}{d}.$$

The capacitor is discharged fifty times per second. Since the current is the charge flowing per second,

$$\therefore \qquad I = \frac{\varepsilon_0 VA \,.\, 50}{d} \text{ ampere}$$

$$\therefore \qquad \varepsilon_0 = \frac{Id}{50VA} \text{ farad metre}^{-1}$$

The following results were obtained in one experiment:

$A = 0{\cdot}0317\ \text{m}^2, d = 1{\cdot}0\ \text{cm} = 0{\cdot}010\ \text{m}, V = 150\ \text{V}, I = 0{\cdot}21 \times 10^{-6}\text{A}$

$$\therefore \varepsilon_0 = \frac{Id}{50\,VA}$$

$$= \frac{0{\cdot}21 \times 10^{-6} \times 0{\cdot}01}{50 \times 150 \times 0{\cdot}0317}$$

$$= 8{\cdot}8 \times 10^{-12} \text{ farad metre}^{-1}$$

As very small currents are concerned, care must be taken to make

the apparatus of high quality insulating material, otherwise leakage currents will lead to serious error.

This method can also be used to find the permittivity of various materials. Thus if the experiment is repeated with a material of permittivity ε completely filling the space between the plates, then $\varepsilon = I'd/50\,VA$, where I' is the new current.

If only the relative permittivity or dielectric constant, ε_r, is required, there is no need to know the p.d. supplied or the dimensions of the capacitor. In this case,

$$\varepsilon_r = \frac{\varepsilon}{\varepsilon_0} = \frac{I'd/50\,VA}{Id/50\,VA} = \frac{I'}{I}$$

Thus ε_r is the ratio of the respective currents in G with and without the dielectric between the plates.

Measurement of Capacitance

If a standard capacitor is available, an unknown capacitance can be measured as described on p. 768.

If no standard capacitor is available, the method of the last section can be employed. The unknown capacitor replaces C in the circuit of Fig. 31.12. The current I in the galvanometer, and the p.d. supplied, V, are then measured. Now the charge $Q = CV$. This is discharged fifty times per second. Since the current is the charge flowing per second,

$$\therefore\ I = 50\,CV$$

$$\therefore\ C = \frac{I}{50\,V}\ \text{farad.}$$

Arrangements of Capacitors

In radio circuits, capacitors often appear in arrangements whose resultant capacitances must be known. To derive expressions for these, we need the equation defining capacitance in its three possible forms:

$$C = \frac{Q}{V}, \qquad V = \frac{Q}{C}, \qquad Q = CV.$$

In Parallel. Fig. 31.13 shows three capacitors, having all their left-hand plates connected together, and all their right-hand plates likewise.

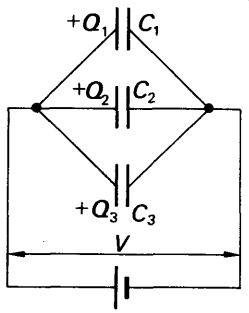


FIG. 31.13. Capacitors in parallel.

They are said to be connected in parallel. If a cell is now connected across them, they all have the same potential difference V. (For, if they had not, current would flow from one to another until they had.) The charges on the individual capacitors are respectively

$$\left.\begin{aligned} Q_1 &= C_1V \\ Q_2 &= C_2V \\ Q_3 &= C_3V \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad (1)$$

The total charge on the system of capacitors is

$$Q = Q_1+Q_2+Q_3 = (C_1+C_2+C_3)V.$$

And the system is therefore equivalent to a single capacitor, of capacitance

$$C = \frac{Q}{V} = C_1+C_2+C_3.$$

Thus when capacitors are connected in parallel, their resultant capacitance is the sum of their individual capacitances. It is greater than the greatest individual one.

In Series. Fig. 31.14 shows three capacitors having the right-hand plate of one connected to the left-hand of the next, and so on— connected in series. When a cell is connected across the ends of the system, a charge Q is transferred from the plate H to the plate A, a charge $-Q$ being left on H. This charge induces a charge $+Q$ on plate G; similarly, charges

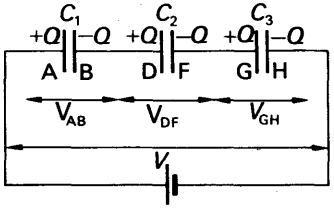


FIG. 31.14. Capacitors in series.

appear on all the other capacitor plates, as shown in the figure. (The induced and inducing charges are equal because the capacitor plates are very large and very close together, in effect, either may be said to enclose the other.) The potential differences across the individual capacitors are, therefore, given by

$$\left.\begin{aligned} V_{AB} &= \frac{Q}{C_1} \\ V_{DF} &= \frac{Q}{C_2} \\ V_{GH} &= \frac{Q}{C_3} \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad (2)$$

The sum of these is equal to the applied potential difference V because the work done in taking a unit charge from H to A is the sum of the work done in taking it from H to G, from F to D, and from B to A. Therefore

$$V = V_{AB} + V_{DF} + V_{GH}$$
$$= Q\left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right). \quad . \quad . \quad . \quad (3)$$

The resultant capacitance of the system is the ratio of the charge stored to the applied potential difference, V. The charge stored is equal to Q, because, if the battery is removed, and the plates HA joined by a wire, a charge Q will pass through that wire, and the whole system will be discharged. The resultant capacitance is therefore given by

$$C = \frac{Q}{V}, \quad \text{or} \quad \frac{1}{C} = \frac{V}{Q},$$

whence, by equation (3),

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}. \quad . \quad . \quad . \quad (4)$$

Thus, to find the resultant capacitance of capacitors in series, we must add their reciprocals of the individual capacitances. The resultant is less than the smallest individual.

Comparison of Series and Parallel Arrangements. Let us compare Figs. 31.13 and 13.14. In Fig. 31.14, where the capacitors are in series, all the capacitors carry the same charge, which is equal to the charge carried by the system as a whole, Q. The potential difference applied to the system, however, is divided amongst the capacitors, in inverse proportion to their capacitances (equations (2)). In Fig. 31.13, where the capacitors are in parallel, they all have the same potential difference; but the charge stored is divided amongst them, in direct proportion to the capacitances (equations (1)).

EXAMPLE

Find the charges on the capacitors in Fig. 31.15, and the potential differences across them.

Capacitance between A and B,

$$C' = C_2 + C_3 = 3\,\mu\text{F}.$$

Overall capacitance A to D,

$$C = \frac{C_1 C'}{C_1 + C'} = \frac{2 \times 3}{2+3} = 1{\cdot}2\,\mu\text{F}.$$

Charge stored in this

$$= Q_1 = Q_2 + Q_3 = CV = 1{\cdot}2 \times 10^{-6} \times 120$$
$$= 144 \times 10^{-6} \text{ coulomb},$$

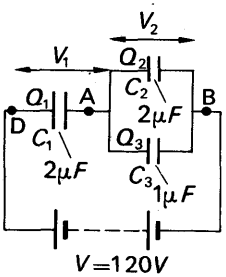


FIG. 31.15. Example.

$$\therefore \quad V_1 = \frac{Q_1}{C_1} = \frac{144 \times 10^{-6}}{2 \times 10^{-6}} = 72 \text{ volt},$$

$$V_2 = V - V_1 = 120 - 72 = 48 \text{ volt},$$

$$Q_2 = C_2 V_2 = 2 \times 10^{-6} \times 48 = 96 \times 10^{-6} \text{ coulomb},$$

$$Q_3 = C_3 V_2 = 10^{-6} \times 48 = 48 \times 10^{-6} \text{ coulomb}.$$

Energy of a Charged Capacitor

A charged capacitor is a store of electrical energy, as we may see from the vigorous spark it can give on discharge. To find the energy stored, let us suppose that the capacitor, of capacitance C, is already charged to a potential difference V. And let us suppose that we wish to increase the charge on its plates from Q to $Q + \delta Q$, where δQ is very small. Then we must transfer a charge δQ from the negative plate to the positive. In doing so, we shall increase the potential difference by the amount

$$\delta V = \frac{\delta Q}{C}.$$

But if δQ is very small compared with Q, δV will be very small compared with V, and the potential difference will be almost constant at the value V. Then the work done in displacing the charge δQ will be

$$\delta W = V \delta Q.$$

from the definition of potential difference. But

$$V = \frac{Q}{C},$$

and therefore

$$\delta W = \frac{Q}{C} \delta Q.$$

If we now suppose that the capacitor is at first completely discharged, and the charged until the final charge on the plates has some definite value Q_1, then the work done in charging it is

$$\int_{Q=0}^{Q=Q_1} dW = \int_0^{Q_1} \frac{Q dQ}{C} = \left[\frac{1}{2}\frac{Q^2}{C}\right]_0^{Q_1} = \frac{1}{2}\frac{Q_1^2}{C}.$$

In general, therefore, the energy stored by a capacitor of capacitance C, carrying a charge Q, at a potential difference V, is

$$\begin{aligned} W &= \tfrac{1}{2}(Q^2/C) \\ &= \tfrac{1}{2}QV \qquad \ldots \qquad (1) \\ &= \tfrac{1}{2}CV^2 \end{aligned}$$

If C is measured in farad, Q in coulomb and V in volt, then the expressions derived in (1) will give the energy W in joules.

EXAMPLES

1. Define a *capacitance* of a capacitor. Explain how, using a capacitor in conjunction with a gold-leaf electroscope, the voltage sensitivity of the electroscope may be increased.

Two capacitors, of capacitance 4 μF and 2 μF respectively, are joined in series with a battery of e.m.f. 100 volts. The connexions are broken and the like terminals of the capacitors are then joined. Find the final charge on each capacitor. (*L.*)

The combined capacitance, C, of the capacitors is given by

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2},$$

$$\therefore \quad \frac{1}{C} = \frac{1}{4} + \frac{1}{2} = \frac{3}{4}, \quad \text{or} \quad C = \frac{4}{3}\mu\text{F}.$$

$\therefore$ charge on each capacitor = charge on 'equivalent' capacitor

$$= CV = \frac{4}{3} \times 100 = \frac{400}{3} \text{ microcoulomb}.$$

When like terminals are joined together, the p.d. across each capacitor, which is different at first, becomes equalized. Suppose it reaches a p.d. V. Then, as the total charge remains constant,

$$\text{initial total charge} = \frac{400}{3} + \frac{400}{3} = \text{final total charge} = 4V + 2V.$$

$$\therefore \quad 6V = \frac{800}{3},$$

$$\therefore \quad V = \frac{400}{9} \text{ volt},$$

$\therefore$ charge on larger capacitor $= 4 \times \frac{400}{9} = \frac{1600}{9}$ microcoulomb,

and charge on smaller capacitor $= 2 \times \frac{400}{9} = \frac{800}{9}$ microcoulomb.

2. Define the *electrostatic potential* of an isolated conductor. Obtain an expression relating the energy of a charged conductor to the charge on it and its capacity.

Two insulated spherical conductors of radii 5·00 cm and 10·00 cm are charged to potentials of 600 volts and 300 volts respectively. Calculate the total energy of the system. Also calculate the energy after the spheres have been connected by a fine wire. Comment on the difference between the two results. (*N.*)

Capacitance C of a sphere of radius $r = 4\pi\varepsilon_0 r$.

For 5 cm radius, or 5×10^{-2} m, $C_1 = 4\pi\varepsilon_0 \times 5 \times 10^{-2} = \frac{5 \times 10^{-2}}{9 \times 10^9} = \frac{5}{9} \times 10^{-11}$ F,

using $4\pi\varepsilon_0 = 1/(9 \times 10^9)$ approx.

For 10 cm radius, $C_2 = \frac{10}{9} \times 10^{-11}$ F.

Since energy, $W = \frac{1}{2}CV^2$,

$$\therefore \text{ total energy} = \frac{1}{2} \times \frac{5}{9} \times 10^{-11} \times 600^2 + \frac{1}{2} \times \frac{15}{9} \times 10^{-11} \times 300^2$$

$$= 15 \times 10^{-7} \text{ J}. \qquad \text{(i)}$$

When the spheres are connected by a fine wire, the potentials become equalized. Suppose this is V. Then, since $Q = CV$ *and the total charge is constant,*

$$\text{original total charge} = \text{total final charge}.$$

$$\therefore\ 4\pi\varepsilon_0(5\times10^{-2}\times600+10\times10^{-2}\times300) = 4\pi\varepsilon_0(5\times10^{-2}\,V+10\times10^{-2}\,V).$$

Solving, $$V = 400 \text{ volt}$$

$$\therefore\ \text{total final energy} = \tfrac{1}{2}C_1V^2+\tfrac{1}{2}C_2V^2 = \tfrac{1}{2}(C_1+C_2)V^2$$

$$= \tfrac{1}{2}\times\frac{1}{9\times10^9}(5\times10^{-2}+10\times10^{-2})\times400^2$$

$$= 13\tfrac{1}{3}\times10^{-7}\text{ J}. \quad . \quad . \quad . \quad . \quad \text{(ii)}$$

From (i) and (ii), $1\frac{2}{3}\times10^{-7}$ J of electrical energy have been converted into heat. This occurred because a current flows in the connecting wire when the two spheres are connected.

3. Define *electrical capacitance.* Describe experiments to demonstrate the factors which determine its value for a parallel plate capacitance.

The plates of a parallel plate air capacitor consisting of two circular plates, each of 10 cm radius, placed 2 mm apart, are connected to the terminals of an electrostatic voltmeter. The system is charged to give a reading of 100 on the voltmeter scale. The space between the plates is then filled with oil of dielectric constant 4·7 and the voltmeter reading falls to 25. Calculate the capacitance of the voltmeter. You may assume that the voltage recorded by the voltmeter is proportional to the scale reading. (*N.*)

Suppose V is the initial p.d. across the air capacitor and voltmeter, and let C_1 be the voltmeter capacitance.

Then $$\text{total charge} = CV+C_1V = (C+C_1)V. \quad . \quad . \quad . \quad \text{(i)}$$

When the plates are filled with oil the capacitance increases to $4{\cdot}7C$, and the p.d. falls to V_1. But the total charge remains constant.

$$\therefore\ 4{\cdot}7CV_1+C_1V_1 = (C+C_1)V, \qquad \text{from (i).}$$

$$\therefore\ (4{\cdot}7C+C_1)V_1 = (C+C_1)V,$$

$$\therefore\ \frac{4{\cdot}7C+C_1}{C+C_1} = \frac{V}{V_1} = \frac{100}{25} = 4,$$

$$\therefore\ 0{\cdot}7C = 3C_1,$$

$$\therefore\ C_1 = \frac{0{\cdot}7C}{3} = \frac{7}{30}C.$$

Now $C = \varepsilon_0 A/d$, where A is in metre^2 and d is in metres.

$$\therefore\ C = \frac{8{\cdot}85\times10^{-12}\times\pi\times(10\times10^{-2})^2}{2\times10^{-3}}\text{ F}$$

$$= 1{\cdot}4\times10^{-10}\text{ F (approx.)}.$$

$$\therefore\ C_1 = \frac{7}{30}\times1{\cdot}4\times10^{-10}\text{ F} = 3{\cdot}3\times10^{-11}\text{ F}.$$

EXERCISES 31

1. Derive an expression for the capacitance of a parallel plate capacitor. Describe an experiment to demonstrate the variation of capacitance with *one* of the physical quantities involved.

A 2·5 μF capacitor is charged to a potential difference of 100 volts and is disconnected from the supply. Its terminals are then connected to those of an uncharged 10 μF capacitor. Find (*a*) the resulting potential difference across the two capacitors and (*b*) the total energy stored in them. Compare the result in (*b*) with the energy originally stored in the 2·5 μF capacitor and comment on the difference. (*L.*)

2. Explain the meaning of the term *capacitance* as used in electrostatics.

A potential difference of 90 V is applied across uncharged capacitors of 2 μF, 3 μF and 1·5 μF connected in series. Across which capacitor is the potential difference least? Explain this and find the numerical value of this potential difference. (*N.*)

3. Define *capacitance* of a conductor. State the factors on which its value depends and describe simple experiments to justify your answers.

The circular plates, *A* and *B*, of a parallel plate air capacitor have each an effective diameter of 10·0 cm and are 2·0 mm apart. The plates, *C* and *D*, of a similar capacitor have each an effective diameter of 12·0 cm and are 3·0 mm apart. *A* is earthed, *B* and *C* are connected together and *D* is connected to the positive pole of a 120-volt battery whose negative pole is earthed. Calculate (*a*) the combined capacitance of the arrangement, (*b*) the energy stored in it, (*c*) the energy stored in each capacitor. (*L.*)

4. Explain what is meant by *dielectric constant* (*relative permittivity*). State two physical properties desirable in a material to be used as the dielectric in a capacitor.

A sheet of paper 4·0 cm wide and $1{\cdot}5 \times 10^{-3}$ cm thick between metal foil of the same width is used to make a 2·0 μF capacitor. If the dielectric constant (relative permittivity) of the paper is 2·5, what length of paper is required? ($\varepsilon_0 = 8{\cdot}85 \times 10^{-12}$ farad m^{-1}.) (*N.*)

5. Define *capacitance*. Obtain from first principles a formula for the capacitance of a parallel-plate capacitor.

Outline, without full experimental details, a method of determining the relative permittivity (dielectric constant) of a dielectric material.

A sensitive moving-coil meter with a suspended system that can swing freely is calibrated both as a ballistic galvanometer and as a galvanometer to record steady currents. Its sensitivity is 8×10^{-8} coulombs per division ballistically, and $2{\cdot}5 \times 10^{-7}$ amperes per division for steady currents. When a capacitor is charged from a 24 volt battery and discharged through the meter, the ballistic throw is 30 divisions. When a 2 volt battery is used, and the capacitor is charged and discharged through the meter many times per second by means of a vibrating-reed switch, the steady deflection is 80 divisions. Find (*a*) the value of the capacitance, and (*b*) the number of times the capacitor is discharged per second. (*O.*)

6. Explain what is meant by the capacitance of a capacitor and define a unit in which it is measured.

Explain the principle of the guard ring capacitor. How could it be used to pass a charge of known value through a galvanometer?

A voltmeter of high internal resistance is connected across the terminals of a 10 μF capacitor which is initially charged to 410 V. When the charging source is removed the voltmeter reading falls and reaches 390 V after 10·0 seconds.

Calculate the internal resistance of the voltmeter. (You may use an approximate method of calculation, but must explain why it is approximate.) (*O. & C.*)

7. Derive an expression for the energy stored in a capacitor C when there is a potential difference V between the plates. If C is in microfarads and V is in volts, express the result in joules.

Show that when a battery is used to charge a capacitor through a resistor, the heat dissipated in the circuit is equal to the energy stored in the capacitor.

Describe the structure of a 1 microfarad capacitor and describe an experiment to compare the capacitances of two capacitors of this type. (*N.*)

8. Define *electrostatic potential* and *capacitance*. Assuming that the electric intensity between the plates of a parallel-plate air capacitor is $1/\varepsilon_0$ times the charge density on one of the plates, obtain an expression for its capacitance. If you were provided with a capacitor of unknown capacitance and a standard capacitor of comparable capacitance, how would you find the value of the unknown capacitance? A parallel-plate air capacitor is charged to a potential difference of 300 volts and is then connected in parallel with another capacitor of equal dimensions with ebonite as dielectric. The potential of the combination is found to be 75 volts. Calculate the dielectric constant of the ebonite. (*N.*)

9. Deduce expressions for the combined capacitance of two capacitors (*a*) connected in series, (*b*) connected in parallel. Describe how *one* of these expressions may be verified by experiment.

A fixed capacitor of capacitance 10^{-4} microfarad is connected in series with a variable capacitor the capacitance of which may be varied from zero to 10^{-4} microfarad in steps of 10^{-6} microfarad. These two in series are connected in parallel with a third capacitor of capacitance 5×10^{-4} microfarad. Calculate (i) the maximum capacitance of the whole combination, (ii) the smallest change in this capacitance which can be produced by the arrangement. (*N.*)

10. Explain how the strength of the electric field at any point is related to the electric potential at and near the point.

A parallel plate capacitor consists of two large plates 2 cm apart, the dielectric on one side of the middle plane between the plates consisting of air, and on the other side of an insulating material of dielectric constant 4·2. Calculate the strength of the electric field in the half occupied by air, if the difference of potential between the plates is 500 volts. (*L.*)

11. Define *capacitance* (or capacity), *relative permittivity* (or dielectric constant), microfarad (μF).

Derive an approximate expression for the capacitance of an air capacitor consisting of two parallel opposite circular plates of radius r cm at a distance t cm apart. Explain on what the degree of approximation depends.

Two capacitors of capacitances respectively 2 μF and 3 μF are joined in series between points A and B. What capacitance must be placed in parallel with the 2 μF capacitor in order to increase the capacitance from A to B by 0·8 μF? (*L.*)

12. State the law of force between electric charges. Write down expressions for (*a*) the electric field strength, and (*b*) the electric potential, at a point in air situated at a distance r from a point charge Q. State the units in which each of the quantities is measured.

Describe the Faraday ice-pail experiment. Illustrate the successive steps in the experiment by a series of simple diagrams, and state the conclusions that can be drawn from the experiment.

The sensitivity of a ballistic galvanometer is found by charging a 0·001 microfarad capacitor from a 12-volt battery, and discharging it through the meter,

which gives a corrected swing of 16·8 divisions. A large hollow metal can is then connected to one terminal of the galvanometer, while the other is earthed; an insulated conducting sphere, of 5 cm radius, is charged to an unknown potential V, thrust right inside the can, and allowed to touch its inner surface; as a result, the galvanometer gives a corrected swing of 21·0 divisions. Find, in coulombs, the charge on the sphere, and also the value in V in volts. (*O.*)

13. Define *potential, capacitance.*

Obtain from first principles a formula for the capacitance of a parallel-plate capacitor.

The plates of such a capacitor are each 0·4 m square, and separated by 10^{-3} m, the space between being filled with a medium of relative permittivity 5. A vibrating contact, with frequency 50 seconds^{-1}, repeatedly connects the capacitor across a 120-volt battery and then discharges it through a galvanometer whose resistance is of the order of 50 ohm. Calculate the current recorded, and explain why this is independent of the actual value of the galvanometer resistance. (Take the permittivity of space to be $8{\cdot}85 \times 10^{-12}$ F m^{-1}.) (*O.*)

14. Define 'the capacitance of a capacitor' and show how it leads to a practical unit of capacitance. Why is it necessary to specify 'capacitor' in this definition?

State the physical factors which affect the magnitude of a capacitance, and describe an experiment to demonstrate the effect of one of them.

A charged 20 microfarad capacitor A is connected to an electrostatic voltmeter of infinite resistance and negligible capacitance which reads 500 volts. A 0·25 microfarad capacitor B is now connected in parallel with A. The capacitor B is then disconnected from A, discharged, and the process of connection and discharge repeated ten times in all. Calculate (*a*) what fraction of the charge on A remains after the first connexion, (*b*) what fraction remains eventually, and (*c*) the final reading of the voltmeter (*C.*)

chapter thirty-two

Current Electricity

OHM'S AND JOULE'S LAWS: RESISTANCE AND POWER

Discovery and Electric Current

By the middle of the eighteenth century, electrostatics was a well-established branch of physics. Machines had been invented which could produce by friction great amounts of charge, giving sparks and electric shocks. The momentary current (as we would now call it) carried by the spark or the body was called a 'discharge'.

In 1786 Galvani, while dissecting a frog, noticed that its leg-muscle twitched when one of his assistants produced an electric spark in another part of the room. He also found that, when a frog's leg-muscle was hung by a copper hook from an iron stand, the muscle twitched whenever it swung so as to touch the stand. Galvani supposed that the electricity which caused the twitching was generated within the muscle, but his fellow-Italian Volta believed that it arose from the contact of the two different metals. Volta turned out to be right, and

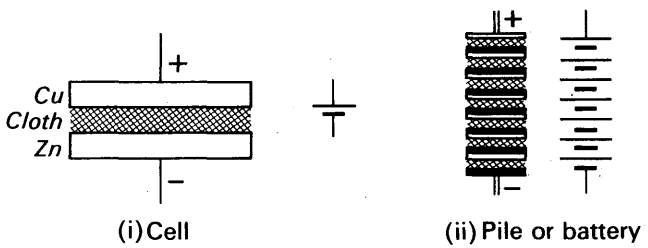


FIG. 32.1. Voltaic cell and pile, with conventional symbols.

in 1799 he discovered how to obtain from two metals a continuous supply of electricity: he placed a piece of cloth soaked in brine between copper and zinc plates (Fig. 32.1 (i)). The arrangement is called a *voltaic cell*, and the metal plates its 'poles'; the copper is known as the positive pole, the zinc as the negative cell. Volta increased the power by building a pile of cells, with the zinc of one cell resting on the copper of the other (Fig. 32.1 (ii)). From this pile he obtained sparks and shocks similar to those given by electrostatic machines.

Shortly after, it was found that water was decomposed into hydrogen and oxygen when connected to a voltaic pile. This was the earliest discovery of the chemical effect of an electric current. The heating

effect was also soon found, but the magnetic effect, the most important effect, was discovered some twenty years later.

Ohm's Law

The properties of an electric circuit, as distinct from the effects of a current, were first studied by Ohm in 1826. He set out to find how the length of wire in a circuit affected the current through it—in modern language, he investigated electrical resistance. In his first experiments he used voltaic piles as sources of current, but he found that the current which they gave fluctuated considerably, and he later replaced them by thermocouples (p. 803). He passed the currents through various lengths of brass wire, 0·037 cm in diameter, and observed them on a torsion balance galvanometer; this is represented by G in Fig. 32.2 (i). No unit of current had been defined at the time of these experiments, but physicists had agreed to take the strength of a current as proportional to its magnetic field. Ohm found that the

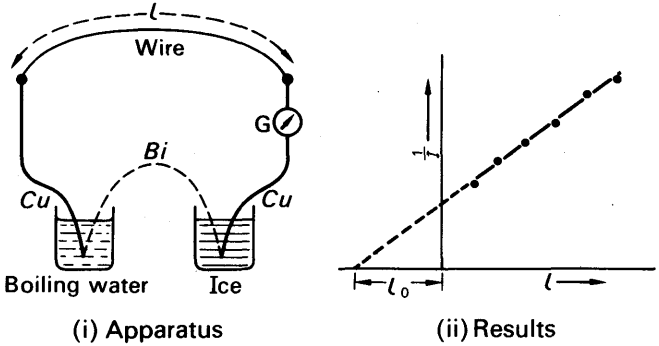


FIG. 32.2. Ohm's experiment.

current I in his experiments was almost inversely proportional to the length of wire, l, in the circuit. He plotted the reciprocal of the current (in arbitrary units) against the length l, and got a straight line, as shown in Fig. 32.2 (ii). Thus

$$I \propto \frac{1}{l_0 + l},$$

where l_0 is the intercept of the line on the axis of length. Ohm explained this result by supposing, naturally, that the thermocouples and galvanometer, as well as the wire, offered resistance to the current. He interpreted the constant l_0 as the length of wire equal in resistance to the galvanometer and thermocouples. The *ohm*, symbol Ω, is the unit of electrical resistance (see p. 790).

Mechanism of Metallic Conduction

The conduction of electricity in metals is due to free electrons. Free electrons have thermal energy, and wander randomly through the

metal from atom to atom. When a battery is connected across the ends of the metal, an electric field is set up. The electrons are now accelerated by the field, so they gain velocity and energy. When they 'collide' with an atom vibrating about its fixed mean position (called a 'lattice site'), they give up some of their energy to it. The amplitude of the vibrations is then increased and the temperature of the metal rises. The electrons are then again accelerated by the field and again give up some energy. Although their movement is erratic, on the average the electrons drift in the direction of the field with a mean speed we calculate shortly. This drift constitutes an 'electric current'. It will be noted that heat is generated by the collision of electrons whichever way they flow. Thus the heating effect of a current—called *Joule heating* (p. 790)—is irreversible, that is, it still occurs when the current in a wire is reversed.

A simple calculation enables the average drift speed to be estimated. Fig. 32.3 shows a portion of a copper wire of cross-sectional area A

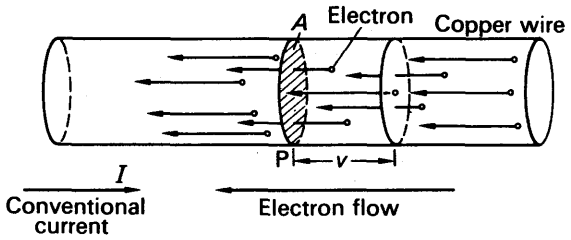


FIG. 32.3. Theory of metallic conduction.

through which a current I is flowing. We suppose that there are n electrons per unit volume, and that each electron carries a charge e. Now in one second all those electrons within a distance v to the right of the plane at P, that is, in a volume Av, will flow through this plane, as shown. This volume contains nAv electrons and hence a charge $nAve$. Thus a charge of $nAve$ per second passes P, and so the current I is given by

$$I = nAve \quad . \quad . \quad . \quad . \quad (1)$$

To find the order of magnitude of v, suppose $I = 10\text{A}$, $A = 1\ \text{mm}^2 = 10^{-6}\text{m}^2$, $e = 1{\cdot}6 \times 10^{-19}\text{C}$, and $n = 10^{28}$ electrons m^{-3}. Then, from (1),

$$v = \frac{I}{nAe} = \frac{10}{10^{28} \times 10^{-6} \times 1{\cdot}6 \times 10^{-19}}$$

$$= \frac{1}{160}\ \text{m s}^{-1}\ \text{(approx.)}$$

This is a surprisingly slow drift compared with the average thermal speeds, which are of the order of several hundred metres per second (p. 234).

Resistivity

Ohm showed, by using wires of different length and diameter, that the resistance of a wire, R, is proportional to its length, l, and inversely proportional to its cross-sectional area A. The truth of this can easily be demonstrated today by experiments with a Wheatstone bridge (see p. 829) and suitable lengths of wire. We have, then, for a given wire,

$$R \propto \frac{l}{A};$$

we may therefore write

$$R = \rho\frac{l}{A}, \qquad . \quad . \quad . \quad . \quad (1)$$

where ρ is a constant for the material of the wire. It is called the *resistivity* of that material.

To define it in words, we imagine a rectangular prism of the material, of unit length and unit cross-section. Then $l = 1$, $A = 1$, and $R = \rho$. Thus the resistivity of a substance is the resistance between the faces of a rectangular prism of the substance, which is 1 cm long and whose cross-sectional area is 1 cm^2. One unit of resistivity is 1 ohm cm, because, from equation (1),

$$\rho = R\frac{A}{l}, \qquad . \quad . \quad . \quad . \quad (2)$$

which has the units

$$\text{ohms} \times \frac{\text{cm}^2}{\text{cm}} = \text{ohms} \times \text{cm}.$$

Resistivities are thus often expressed in microhm centimetres; 1 microhm = 10^{-6} ohm. The SI unit is the *ohm metre* (Ω m). Using it, R is in ohms when l is in metres and A is in $metre^2$ in equation (2).

RESISTIVITIES

Substance	Resistivity ρ, ohm m (at 20°C)	Temperature coefficient α, K^{-1}
Aluminium	$2{\cdot}82 \times 10^{-8}$	0·0039
Brass	*c.* 8×10^{-8}	*c.* 0·0015
Constantan[1]	*c.* 49×10^{-8}	0·00001
Copper	$1{\cdot}72 \times 10^{-8}$	0·0043
Iron	*c.* $9{\cdot}8 \times 10^{-8}$	0·0056
Manganin[2]	*c.* 44×10^{-8}	*c.* 0·00001
Mercury	$95{\cdot}77 \times 10^{-8}$	0·00091
Nichrome[3]	*c.* 100×10^{-8}	0·0004
Silver	$1{\cdot}62 \times 10^{-8}$	0·0039
Tungsten[4]	$5{\cdot}5 \times 10^{-8}$	0·0058
Carbon (graphite)	33 to 185×10^{-8}	−0·0006 to −0·0012

[1] Also called Eureka; 60 per cent Cu, 40 per cent Ni.
[2] 84 per cent Cu, 12 per cent Mn, 4 per cent Ni; used for resistance boxes and shunts.
[3] Ni–Cu–Cr; used for electric fires—does not oxidize at 1000°C.
[4] Used for lamp filaments—melts at 3380°C.

The resistivity of a metal is increased by even small amounts of impurity; and alloys, such as Constantan, may have resistivities far greater than any of their constituents.

Ohm's Theory of the Circuit

We have seen that Ohm abandoned the use of voltaic piles in his experiments, because the currents which they gave were not steady. He attributed this to fluctuations in their 'exciting force'—electromotive force, as we now call it. Similarly, when he used thermocouples, he found that the current through a given circuit increased when the difference in temperature between the couples was increased. He was thus led to propose a 'mathematical law of the galvanic circuit':

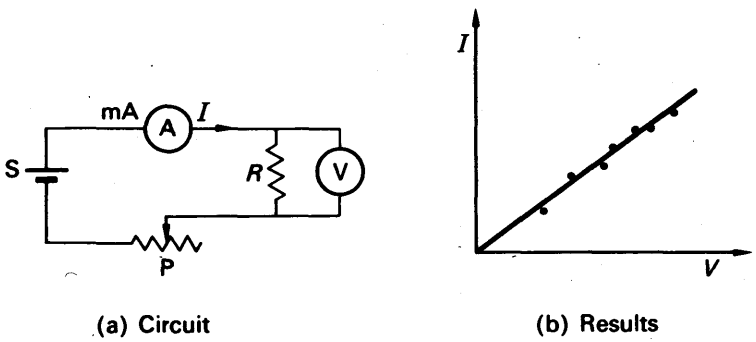
$$I = \frac{E}{R}. \qquad . \quad . \quad . \quad . \quad (3)$$

Here I stands for the strength of the current, E for the exciting force (e.m.f.), and R for the total resistance of the circuit.

Demonstration of Ohm's Law

Ohm showed that his law applied not only to a complete circuit but to any part of it. To understand this, let us consider an experiment which can easily be done with modern apparatus. As shown in Fig. 32.4 (*a*), we connect in series the following apparatus:

(i) one or more accumulators, S;
(ii) a milliammeter reading to 15 milliamperes;
(iii) a wire-wound resistor R of the order of 50 ohms;
(iv) a suitable variable resistance or *rheostat P* of the same order of resistance.



(a) Circuit (b) Results

FIG. 32.4. Demonstration of Ohm's law.

Across the resistor R we connect a voltmeter to measure the potential difference V across R. This must be a voltmeter such as a potentiometer (p. 817) whose calibration does not depend on Ohm's law, otherwise the experiment would not be valid. The milliammeter calibration likewise must not depend on Ohm's law. By adjusting the resistor P we vary the current I through the circuit, and at each value of I we

measure V. On plotting V against I we get a straight line through the origin, as in Fig. 32.4 (*b*); this shows that the potential difference across the resistor R is proportional to the current through it:

$$V \propto I. \quad . \quad . \quad . \quad . \quad (4)$$

Thus, taking into account that the resistance of a conductor depends on its temperature and on other physical conditions such as mechanical strain, Ohm's law can be stated as follows:

Under constant physical conditions, the potential difference across a conductor is proportional to the current through it.

The law is obeyed by the most important class of conductors—metals—and by some others, such as carbon. It is not obeyed by some crystals, such as silicon carbide, nor by some conducting solutions, nor by diode valves, nor—as in a neon lamp— by gases.

Resistance

From Ohm's law, it follows that

$$\frac{V}{I} = R, \text{ a constant.} \quad . \quad . \quad . \quad (5)$$

R is *defined* as the 'resistance' of the conductor.

The unit of potential difference, V, is the *volt*, symbol V; the unit of current, I, is the *ampere*, symbol A; the unit of resistance, R, is the *ohm*, symbol Ω. The ohm is thus the resistance of a conductor through which a current of one ampere flows when a potential difference (p.d.) of one volt is across it.

From the above equation, it also follows that

$$V = IR, \quad \text{and} \quad I = \frac{V}{R}. \quad . \quad . \quad . \quad (6)$$

Smaller units of current are the milliampere (one-thousandth of an ampere), symbol mA and the micro-ampere (one-millionth of an ampere), symbol μA. Smaller units of p.d. are the millivolt (1/1000 V) and the microvolt ($1/10^6$ V). A small unit of resistance is the microhm ($1/10^6$ ohm); larger units are the kilohm (1,000 ohms) and the megohm (10^6 ohms).

HEAT AND POWER

Electrical Heating. Joule's Laws

In 1841 Joule studied the heating effect of an electric current by passing it through a coil of wire in a jar of water (Fig. 32.5). He used various currents, measured by an early form of galvanometer G, and various lengths of wire, but always the same mass of water. The rise in temperature of the water, in a given time, was then proportional to the heat developed by the current in that time. Joule found that the heat produced in a given time, with a given wire, was proportional to

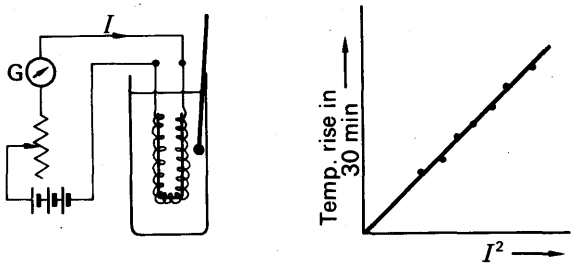


FIG. 32.5. Joule's experiment on heating effect of current.

I^2, where I is the current flowing. If H is the heat produced per second, then

$$H \propto I^2. \quad . \quad . \quad . \quad . \quad (7)$$

Joule also made experiments on the heat produced by a given current in different wires. He used wires of different lengths, but of the same diameter, and of the same material; he found that the rate at which heat was produced, by a given current, was proportional to the length of the wire. That is to say, he found that the rate of heat production was proportional to what Ohm had already called the resistance of the wire:

$$H \propto R. \quad . \quad . \quad . \quad . \quad (8)$$

Relationships (7) and (8) together give

$$H \propto I^2R. \quad . \quad . \quad . \quad . \quad (9)$$

Mechanism of the Heating Effect

Heat is a form of energy. The heat produced per second by a current in a wire is therefore a measure of the energy which it liberates in one second, as it flows through the wire. The heat is produced, we suppose, by the free electrons as they move through the metal. On their way they collide frequently with atoms; at each collision they lose some of their kinetic energy, and give it to the atoms which they strike. Thus, as the current flows through the wire, it increases the kinetic energy of vibration of the metal atoms: it generates heat in the wire. The electrical resistance of the metal is due, we say, to its atoms obstructing the drift of the electrons past them: it is analogous to mechanical friction. As the current flows through the wire, the energy lost per second by the electrons is the electrical power supplied by the battery which maintains the current. That power comes, as we shall see later, from the chemical energy liberated by these actions within the battery.

Potential Difference and Energy

On p. 750 we defined the potential difference V_{AB} between two points, A and B, as the work done by an external agent in taking a unit

positive charge from B to A (Fig. 32.6 (i)). This definition applies equally well to points in an electrostatic field and to points on a conductor carrying a current.

In Fig. 32.6 (ii), D represents any electrical device or circuit element: a lamp, motor, or battery on charge, for example. A current of I amperes flows through it from the terminal A to the terminal B; if it flows for t seconds, the charge Q which it carries from A to B is, since a current is the quantity of electricity per second flowing,

$$Q = It \text{ coulombs.} \quad . \quad (10)$$

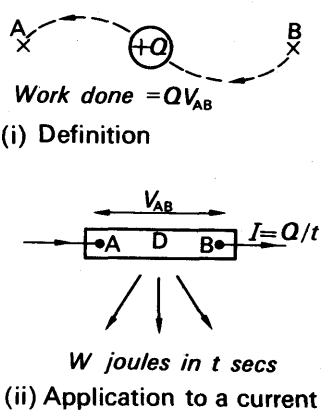


FIG. 32.6. Potential difference and energy.

Let us suppose that the device D liberates a total amount of energy W joules in the time t; this total may be made up of heat, light, sound, mechanical work, chemical transformation, and any other forms of energy. Then W is the amount of electrical energy given up by the charge Q in passing through the device D from A to B.

$$\therefore W = QV_{AB}, \quad . \quad . \quad . \quad . \quad (11)$$

where V_{AB} is the potential difference between A and B in volts.

The work, in all its forms, which the current I does in t seconds as it flows through the device, is therefore

$$W = IV_{AB}t, \quad . \quad . \quad . \quad . \quad (12)$$

by equations (10) and (11).

Electrical Power

The energy liberated per second in the device is defined as its electrical *power*. The electrical power, P, supplied is given, from above, by

$$P = \frac{W}{t} = \frac{IV_{AB}t}{t}$$

or

$$P = IV_{AB}. \quad . \quad . \quad . \quad . \quad (13)$$

When an electric current flows through a wire or 'passive' resistor, all the power which it conveys to the wire appears as heat. If I is the current, R is the resistance, then $V_{AB} = IR$, Fig. 32.7.

$$\therefore P = I^2R. \quad . \quad . \quad . \quad . \quad (14)$$

Also,

$$P = \frac{V_{AB}^2}{R}. \quad . \quad . \quad . \quad . \quad (15)$$

The power, P, is in *watts* (W) when I is in amp, R is in ohms, and V_{AB} is in volts. 1 kilowatt (kW) = 1000 watts.

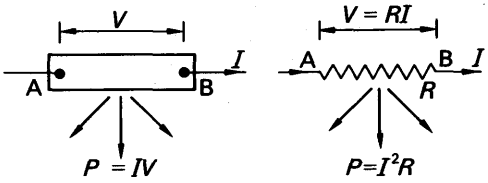


FIG. 32.7. Power equations.

The formulae for power, $P = I^2R$ or V^2/R, is true only when all the electrical power supplied is dissipated as heat. As we shall see, the formulae do not hold when part of the electrical energy supplied is converted into mechanical work, as in a motor, or into chemical energy, as in an accumulator being charged. A device which converts all the electrical energy supplied to it into heat is called a 'passive' resistor; it may be a wire, or a strip of carbon, or a liquid which conducts electricity but is not decomposed by it. Since the joule (J) is the unit of heat, it follows that, for a resistor, the heat H in it in joules is given by

$$H = IVt$$

or by

$$H = I^2Rt \quad . \quad . \quad . \quad (16)$$

or by

$$H = \frac{V^2t}{R}$$

The units of I, V, R are amperes (A), volts (V), ohms(Ω) respectively.

High-tension Transmission

When electricity has to be transmitted from a source, such as a power station, to a distant load, such as a factory, the two must be connected by cables. These cables have resistance, which is in effect added to the internal resistance of the generator; power is wasted in them as heat. If r is the total resistance of the cables, and I the supply current, the power wasted is I^2r. The power delivered to the factory is IV, where V is the potential difference at the factory. Economy requires the waste power, I^2r, to be small; but it also requires the cables to be thin, and therefore cheap to buy and erect. The thinner the cables, however, the higher their resistance r. Thus the most economical way to transmit the power is to make the current, I, as small as possible; this means making the potential difference V as high as possible. When large amounts of power are to be transmitted, therefore, very high voltages are used: 132000 volts on the main lines of the British grid, 6000 volts on subsidiary lines. These voltages are much too high to be brought into a house, or even a factory. They are stepped down by transformers, in a way which we shall describe later; stepping-down in that way is possible only with alternating current, which is one of the main reasons why alternating current is so widely used.

EXAMPLE

An electric heating element to dissipate 480 watts on 240 V mains is to be made from Nichrome ribbon 1 mm wide and thickness 0·05 mm. Calculate the length of ribbon required if the resistivity of Nichrome is $1{\cdot}1 \times 10^{-6}$ ohm metre.

Power, $$P = \frac{V^2}{R}$$

$$\therefore R = \frac{V^2}{P} = \frac{240^2}{480} = 120\Omega$$

The area A of cross-section of the ribbon $= 1 \times 0{\cdot}05\ \text{mm}^2 = 0{\cdot}05 \times 10^{-6}\ \text{m}^2$.

From $$R = \frac{\rho l}{A}$$

$$\therefore l = \frac{R.A}{\rho} = \frac{120 \times 0{\cdot}05 \times 10^{-6}}{1{\cdot}1 \times 10^{-6}} = 5{\cdot}45 \text{ metre}$$

Summary of Formulae Related to Power and Ohm's Law

In any device whatever (Fig. 32.7 (i)):

Electrical power consumed = power developed in other forms,

$$P = IV,$$

watts = amperes × volts.

In a passive resistor (Fig. 32.7 (ii)):

(i) $$V = IR; \quad I = \frac{V}{R}; \quad R = \frac{V}{I},$$

volts = ohms × amperes.

(ii) Power consumed = heat developed per second, in watts.

$$P = I^2R = IV = \frac{V^2}{R}.$$

(iii) Heat developed in time t:

Electrical energy consumed = heat developed in joules

$$I^2Rt = IVt = \frac{V^2}{R}t.$$

Board of Trade (commercial) unit = kilowatt hour (kWh) = kilowatt × hour

$$= 3{\cdot}6 \times 10^6 \text{ joule.}$$

ELECTROMOTIVE FORCE

E.M.F. Internal Resistance

If we take a high-tension battery, and connect a high resistance voltmeter across it, the meter reads about 120 volts. Across two batteries

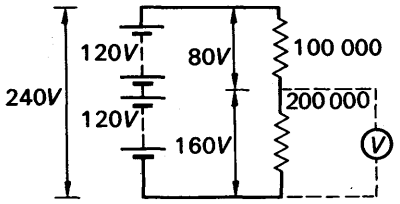
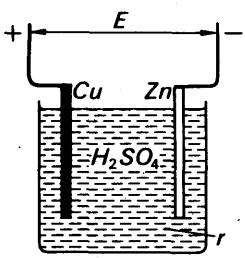


FIG. 32.8. P.D. with moving-coil voltmeter.

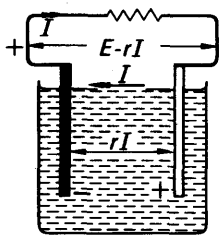
in series it reads 240 volts. Let us now connect the two batteries in series with two resistors, of resistance 200000 and 100000 ohms, as in Fig. 32.8. By using the voltmeter, we find that the potential difference across the 200000-ohm resistor is 160 volts, and across the 100000-ohm resistor 80 volts. These add up to 240 volts; if we inserted a third resistor, we should find that the potential difference across all three added up to the same value. It appears that the batteries always maintain a total potential difference of 240 volts across any circuit to which they are joined. This constant potential difference represents what Ohm called the 'exciting force' of the batteries. Since it is the property which enables the batteries to maintain a flow of electricity in a circuit, we may call it their electromotive force.

Now let us connect a lower resistance across the batteries of Fig. 32.8—say 10000 ohms. We find the potential difference across their terminals falls slightly. And if we use a still lower resistance—say 1000 ohms—then the potential difference falls greatly, perhaps to about $\frac{2}{3}$ of its open-circuit value.

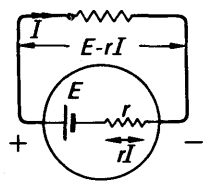
We suppose, therefore, that the batteries have some internal resistance—the resistance r of the chemical solutions between the plates (Fig. 32.9 (i)). This is analogous to the resistance of the wires of Ohm's thermocouples (p. 780). When an appreciable current I flows from the battery, it sets up a potential difference rI across the internal resistance, and by that amount makes the external potential difference less than the electromotive force, E (Fig. 32.9 (ii)). In a rough-and-ready way we may represent the battery as a source of constant potential difference E, in series with a passive resistor r (Fig. 32.9 (iii)).



(i) A voltaic cell on open circuit



(ii) On load



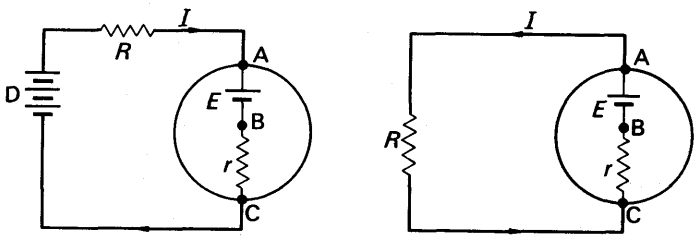
(iii) Representation

FIG. 32.9. E.m.f. and internal resistance.

Electromotive Force and Energy

To get a rigorous definition of electromotive force, let us first imagine that we pass a current through the device of Fig. 32.10 in opposition to its e.m.f. We can do this by connecting its terminals AC, via a resistance

R, to a battery D which has a greater e.m.f. (Fig. 32.10 (i)). If a charge Q passes round the circuit in a given time, then the work done in carrying it from A to B, against the potential difference E, is QE joules. This work appears as chemical changes in the source of E. Now suppose that we remove the battery D, and connect a resistor across the terminals AC (Fig. 32.10 (ii)). The potential difference will now send a current round the circuit in the opposite direction to the previous current. And when a



(i) Current in opposition to e.m.f.

(ii) Current in direction of e.m.f.

FIG. 32.10. Illustrating definition of e.m.f.

charge Q has passed, the energy delivered by the source of e.m.f. E will be QE joules. The chemical changes in the source will have been reversed, and will have given up this amount of energy, as electrical energy. The current, in passing round the circuit, will have converted this energy into heat. Some of the heat will have been dissipated in the external resistance R, some in the internal resistance r.

Our picture of a source of current as a constant potential difference in series with a resistance is over-simplified, but it has brought us to the point where we can make a definition of e.m.f. which is both rigorous and intelligible. We shall make it first in terms of charge, later in terms of current.

In terms of charge: *if a device has an electromotive force E, then, in passing a charge Q round a circuit joined to it, it liberates an amount of electrical energy equal to QE.* If a charge Q is passed through the source against its e.m.f., then the work done against the e.m.f. is QE. The above definition of e.m.f. does not depend on any assumptions about the nature of its source.

If a device of e.m.f. E passes a steady current I for a time t, then the charge that it circulates is

$$Q = It.$$

Thus:

$$\text{electrical energy liberated, } W, = QE = IEt, \quad . \quad (17)$$

and

$$\text{electrical power generated, } P, = \frac{W}{t} = EI. \quad . \quad (18)$$

We can now define e.m.f. in terms of power and current, and therefore

in a way suitable for dealing with circuit problems. From equation (18)

$$P = EI,$$

or

$$E = \frac{P}{I}.$$

Thus *the e.m.f. of a device is the ratio of the electrical power which it generates, to the current which it delivers.* If current is forced through a device in opposition to its e.m.f., then equation (18) gives the power consumed in overcoming the e.m.f.

Electromotive force resembles potential difference in that both can be defined as the ratio of power to current. The unit of e.m.f. is therefore 1 watt per ampere, or 1 volt; and *the e.m.f. of a source, in volts, is numerically equal to the power which it generates when it delivers a current of* 1 *ampere.*

Representation of an E.M.F. and Internal Resistance

Sources of e.m.f. differ widely in their nature. In a thermocouple, the e.m.f. arises at the junction of the two metals—this point is sometimes called the seat of the e.m.f. In a voltaic cell, we believe, the e.m.f. arises at the interfaces of the plates and solutions—part at one interface, the rest at the other. In each of these sources we can distinguish between the seat, or seats, of the e.m.f., and that of the internal resistance, which is in the bulk of the solution or the wires of the thermocouple. The e.m.f. of a dynamo, however, does not arise at a point: it acts along the wires of the armature coil as they move in the magnetic field (p. 915). Here we cannot distinguish between the seats of the e.m.f. and the internal resistance. In solving circuit problems, however, it is helpful to show the e.m.f. and internal resistance separately, although as a rule they are physically inextricable.

Ohm's Law for Complete Circuit

Fig. 32.11 shows a source of current connected to a passive resistor—called the load—of resistance R. To find the current I, we equate the power generated by the source to the heat developed per second in the resistances:

$$EI = I^2r + I^2R.$$

Thus

$$E = Ir + IR, \quad . \quad . \quad . \quad (19)$$

whence

$$I = \frac{E}{R+r}. \quad . \quad . \quad . \quad (20)$$

Equation (20) asserts that the current is equal to the e.m.f. of the source divided by the total resistance of the circuit; it is Ohm's original statement of his law (p. 786). Equation (19) asserts that the sum of the potential differences across the resistances is equal to the e.m.f. The potential difference Ir appears across the internal resistance, and is often called the voltage drop; because of it, the

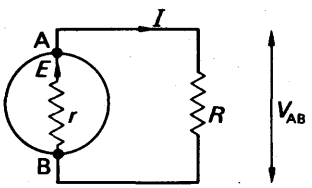


FIG. 32.11. A complete circuit.

potential difference between the terminals of the cell, V_{AB}, falls when the current taken, I, is increased:

$$V_{AB} = E - Ir. \quad . \quad . \quad . \quad . \quad (21)$$

Terminal Potential Difference

The quantity V_{AB} is often called the terminal potential difference of the cell; it is also the potential difference across the load. Equations (19) and (21) give

$$V_{AB} = IR, \quad . \quad . \quad . \quad . \quad (22)$$

which is Ohm's law for the load alone; we could have written it down directly. Equations (20) and (22) together give the terminal potential difference in terms of the e.m.f. and the resistances:

$$V_{AB} = IR = \frac{E}{R+r}R. \quad . \quad . \quad . \quad (23)$$

Output and Efficiency

The power delivered to the load in Fig. 32.10 is called the output power, P_{out}; its value is

$$P_{out} = IV_{AB}$$
$$= I^2R. \quad . \quad . \quad . \quad . \quad (24)$$

The power generated by the source of current is

$$P_{gen} = IE. \quad . \quad . \quad . \quad . \quad (25)$$

The difference between the power generated and the output is the power wasted as heat in the source: I^2r. The ratio of the power output to the power generated is the efficiency, η, of the circuit as a whole:

$$\eta = \frac{P_{out}}{P_{gen}}. \quad . \quad . \quad . \quad . \quad (26)$$

By equations (24) and (25), therefore,

$$\eta = \frac{P_{out}}{P_{gen}} = \frac{IV_{AB}}{IE}.$$
$$= \frac{V_{AB}}{E}.$$

Equation (23) now gives

$$\eta = \frac{R}{R+r}. \quad . \quad . \quad . \quad . \quad (27)$$

This shows that the efficiency tends to unity (or 100 per cent) as the load resistance R tends to infinity. For high efficiency the load resistance must be several times the internal resistance of the source. When the load resistance is equal to the internal resistance, the efficiency is 50 per cent. (See Fig. 32.12 (i).)

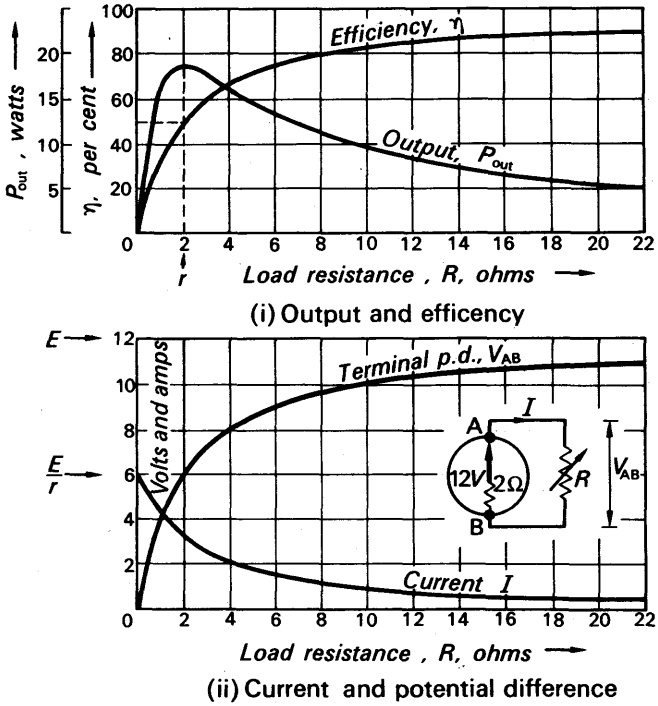


FIG. 32.12. Effect of varying load resistance.

Power Variation

Now let us consider how the power output varies with the load resistance. Equations (24) and (20) give

$$P_{out} = I^2R,$$

and

$$I = \frac{E}{R+r},$$

whence

$$P_{out} = \frac{E^2R}{(R+r)^2}.$$

If we take fixed values of E and r, and plot P_{out} as a function of R, we find that it passes through a maximum when $R = r$ (Fig. 32.12 (i)). We can get the same result in a more general way by differentiating P_{out} with respect to R, and equating the differential coefficient to zero. Physically, this result means that the power output is very small when R is either very large or very small, compared with r. When R is very large, the terminal potential difference, V_{AB}, approaches a constant value equal to the e.m.f. E (Fig. 32.12 (ii)); as R is increased the current I falls, and the power IV_{AB} falls with it. When R is very small, the current approaches the constant value E/r, but the potential difference (which is equal to IR) falls steadily with R; the power output therefore

falls likewise. Consequently the power output is greatest for a moderate value of R; the mathematics show that this value is actually $R = r$.

To prove $R = r$, differentiate the expression for P_{out}, given on p. 799, with respect to R. Then

$$E^2\frac{(R+r)^2 - R \cdot 2(R+r)}{(R+r)^4} = 0, \text{ for a maximum.}$$

From the numerator, $r^2 - R^2 = 0$, or $R = r$.

Examples of Loads in Electrical Circuits

The loading on a dynamo or battery is generally adjusted for high efficiency, because that means greatest economy. Also, if a large dynamo were used with a load not much greater than its internal resistance, the current would be so large that the heat generated in the internal resistance would ruin the machine. With batteries and dynamos, therefore, the load resistance is made many times greater than the internal resistance.

Loading for greatest power output is common in communication engineering. For example, the last transistor in a receiver delivers electrical power to the loudspeaker, which the speaker converts into mechanical power as sound-waves (p. 596). Because it converts electrical energy into mechanical energy, and not heat, the loudspeaker is not a passive resistor, and the simple equations above do not apply to it. Nevertheless, circuit conditions can be specified which enable the transistor to deliver the greatest power to the speaker; these are similar to the condition of equal load and internal resistances, and are usually satisfied in practice.

Load not a Passive Resistor

As an example of a load which is not a passive resistor, we shall take an accumulator being charged. The charging is done by connecting the accumulator X in opposition to a source of greater e.m.f., Y in Fig. 32.13,

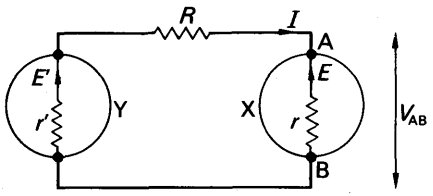


FIG. 32.13. Accumulator charging.

via a controlling resistor R. If E, E' and r, r' are the e.m.f. and internal resistances of X and Y respectively, then the current I is given by the equation:

$$\left.\begin{matrix}\text{power generated} \\ \text{in Y}\end{matrix}\right\} = \left\{\begin{matrix}\text{power converted to} \\ \text{chemical energy} \\ \text{in X}\end{matrix}\right\} + \left\{\begin{matrix}\text{power dissipated} \\ \text{as heat in all} \\ \text{resistances}\end{matrix}\right.$$

$$E'I \quad = \quad EI \quad + \quad I^2R + I^2r' + I^2r. \qquad (28)$$

Thus $$(E'-E)I = I^2(R+r'+r),$$

whence $$I = \frac{E'-E}{R+r'+r}. \quad . \quad . \quad . \quad (29)$$

The potential difference across the accumulator, V_{AB}, is given by

$$\left.\begin{matrix}\text{power delivered}\\ \text{to X}\end{matrix}\right\} = \left\{\begin{matrix}\text{power converted to}\\ \text{chemical energy}\end{matrix}\right\} + \left\{\begin{matrix}\text{power dissipated}\\ \text{as heat}\end{matrix}\right.$$

$$IV_{AB} \quad = \quad IE \quad + \quad I^2r.$$

Hence $$V_{AB} = E+Ir. \quad . \quad . \quad . \quad . \quad (30)$$

Equation (30) shows that, when current is driven through a generator in opposition to its e.m.f., then the potential difference across the generator is equal to the *sum* of its e.m.f. and the voltage drop across its internal resistance. This result follows at once from energy considerations, as we have just seen.

Cells in Series and Parallel

When cells or batteries are in series and assist each other, then the total e.m.f.

$$E = E_1+E_2+E_3+\ldots, \quad . \quad . \quad . \quad (31)$$

and the total internal resistance

$$r = r_1+r_2+r_3+\ldots, \quad . \quad . \quad . \quad (32)$$

where E_1, E_2 are the individual e.m.f.s and r_1, r_2 are the corresponding internal resistances. If one cell, e.m.f. E_2 say, is turned round 'in opposition' to the others, then $E = E_1-E_2+E_3+\ldots$; but the total internal resistance remains unaltered.

When *similar* cells are in parallel, the total e.m.f. $= E$, the e.m.f. of any one of them. The internal resistance r is here given by

$$\frac{1}{r} = \frac{1}{r_1}+\frac{1}{r_1}+\ldots, \quad . \quad . \quad . \quad (33)$$

where r_1 is the internal resistance of each cell. If different cells are in parallel, there is no simple formula for the total e.m.f. and the total internal resistance, and any calculations involving circuits with such cells are dealt with by applying Kirchhoff's laws (see p. 827).

Summary of Formulae Involving E.M.F.

(i) *Any load*:

$$(a)\ \left.\begin{matrix}\text{power generated}\end{matrix}\right\} = \left\{\begin{matrix}\text{power supplied}\\ \text{to load}\end{matrix}\right\} + \left\{\begin{matrix}\text{power dissipated}\\ \text{in internal}\\ \text{resistance}\end{matrix}\right.$$

$$EI \quad = \quad IV_{AB} \quad + \quad I^2r,$$

(*b*) terminal p.d. = e.m.f. − voltage drop in internal resistance

$$V_{AB} = E-rI.$$

(ii) *Passive resistance load*:

power equation: $EI = I^2R + I^2r$,

current: $I = \dfrac{E}{R+r}$,

terminal p.d.: $V_{AB} = E - rI = RI$

$= E\dfrac{R}{R+r}$.

EXAMPLES

1. What is meant by the *electromotive force* of a cell?

A voltmeter is connected in parallel with a variable resistance, R, which is in series with an ammeter and a cell. For one value of R the meters read 0·3 amp and 0·9 volt. For another value of R the readings are 0·25 amp and 1·0 volt. Find the values of R, the e.m.f. of the cell, and the internal resistance of the cell. What assumptions are made about the resistance of the meters in the calculation?

If in this experiment the ammeter had a resistance of 10 ohms and the voltmeter a resistance of 100 ohms and R was 2 ohms, what would the meters read? (*L.*)

First part (see p. 796).

Second part. The voltmeter reads the p.d. across the cell if the resistances of the meters are neglected. Thus, with the usual notation,

$$E - Ir = 0{\cdot}9, \quad \text{or} \quad E - 0{\cdot}3r = 0{\cdot}9 \quad . \quad . \quad . \quad \text{(i)}$$

and

$$E - 0{\cdot}25r = 1{\cdot}0. \quad . \quad . \quad . \quad . \quad \text{(ii)}$$

Subtracting (i) from (ii),

$$0{\cdot}05r = 0{\cdot}1, \quad \text{i.e.} \quad r = 2 \text{ ohms}.$$

Also, from (i),

$$E = 0{\cdot}3r + 0{\cdot}9 = 0{\cdot}6 + 0{\cdot}9 = 1{\cdot}5 \text{ volts}.$$

Further,

$$R_1 = \frac{V}{I} = \frac{0{\cdot}9}{0{\cdot}3} = 3 \text{ ohms}.$$

and

$$R_2 = \frac{1{\cdot}0}{0{\cdot}25} = 4 \text{ ohms}.$$

If the voltmeter has 100 ohms resistance and is in parallel with the 2 ohms resistance, the combined resistance R is given by

$$\frac{1}{R} = \frac{1}{2} + \frac{1}{100} = \frac{51}{100}, \quad \text{or} \quad R = \frac{100}{51} \text{ ohms}.$$

$$\therefore \text{ current, } I = \frac{E}{\text{Total resistance}}$$

$$= \frac{1{\cdot}5}{\frac{100}{51} + 10 + 2} = 0{\cdot}11 \text{ A}.$$

$$\text{Also, voltmeter reading} = IR = 0{\cdot}11 \times \frac{100}{51} = 0{\cdot}21 \text{ volt}.$$

2. Define *internal resistance* of a voltaic cell. Describe one method of finding by experiment the internal resistance of a primary cell.

Two Daniell cells A and B are connected in series with a coil of resistance 9·8 ohms. A voltmeter of very high resistance connected to the terminals of A reads 0·96 volt and when connected to the terminals of B it reads 1·00 volt. Find the internal resistance of each cell. (Take the e.m.f. of a Daniell cell as 1·08 volts.) (*L.*)

First part. The 'internal resistance' is the resistance of the chemicals inside the cell between the poles, and is given by (drop in terminal p.d.)/current, when the cell is used. The potentiometer may be used to measure the internal resistance. See p. 820.

Second part. The p.d. across both cells $= 0{\cdot}96 + 1{\cdot}00 = 1{\cdot}96$ volts
$=$ p.d. across 9·8 ohms.

$$\therefore \text{ current flowing, } I, = \frac{V}{R} = \frac{1{\cdot}96}{9{\cdot}8} = 0{\cdot}2 \text{ A.}$$

Now terminal p.d. across each cell $= E - Ir$.

$\therefore$ for cell A, $0{\cdot}96 = 1{\cdot}08 - 0{\cdot}2r$, or $r = 0{\cdot}6$ ohm.

for cell B, $1{\cdot}00 = 1{\cdot}08 - 0{\cdot}2r$, or $r = 0{\cdot}4$ ohm.

THE THERMOELECTRIC EFFECT

Seebeck Effect

The heating effect of the current converts electrical energy into heat, but we have not so far described any mechanism which converts heat into electrical energy. This was discovered by Seebeck in 1822. In his experiments he connected a plate of bismuth between copper wires leading to a galvanometer, as shown in Fig. 32.14 (i). He found that if

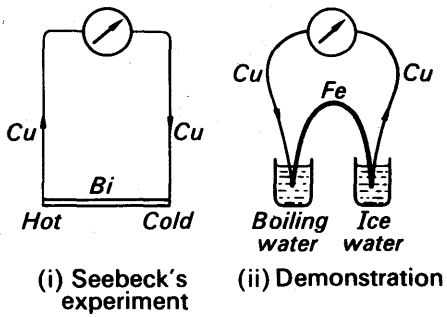


FIG. 32.14. The thermo-electric effect.

one of the bismuth-copper junctions was heated, while the other was kept cool, then a current flowed through the galvanometer. The direction of the current was from the copper to the bismuth at the cold junction. We can easily repeat Seebeck's experiment, using copper and iron wires and a galvanometer capable of indicating a few micro-amperes (p. 882) (Fig. 32.14 (ii)).

Thermocouples

Seebeck went on to show that a current flowed, without a battery, in any circuit containing two different metals, with their two junctions

at different temperatures. Currents obtained in this way are called thermo-electric currents, and a pair of metals, with their junctions at different temperatures, are said to form a thermocouple. The following is a list of metals, such that if any two of them form a thermocouple, then the current will flow from the higher to the lower in the list, across the cold junction:

Antimony, Iron, Zinc, Lead, Copper, Platinum, Bismuth.

Thermo-electric currents often appear when they are not wanted; they may arise from small differences in purity of two samples of the same metal, and from small differences of temperature—due, perhaps, to the warmth of the hand. They can cause a great deal of trouble in circuits used for precise measurements, or for detecting other small currents, not of thermal origin. As sources of electrical energy, thermo-electric currents are neither convenient nor economical, but they have been used—in gas-driven radio sets. Their only wide application is in the measurement of temperature, and of other quantities, such as radiant energy, which can be measured by a temperature rise.

Variation of Thermoelectric E.M.F. with Temperature

On p. 825 we shall see how thermo-electric e.m.f.s are measured. When the cold junction of a given thermocouple is kept constant at 0°C, and the hot junction temperature t°C is varied, the e.m.f. E is found to vary as $E = at + bt^2$, where a, b are constants. This is a parabola-shaped curve (Fig. 32.15). The temperature A corresponding to the maximum e.m.f. is known as the *neutral temperature*; it is about 250°C for a copper-iron thermocouple. Beyond the temperature B, known as the *inversion temperature*, the e.m.f. reverses. Thermo-electric thermometers, which utilize thermocouples, are used only as far as the neutral temperature, as the same e.m.f. is obtained at two different temperatures, from Fig. 32.15.

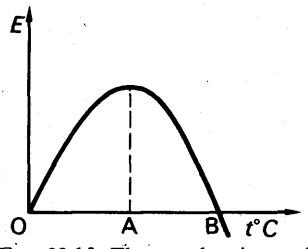


FIG. 32.15. Thermo-electric e.m.f. variation with temperature.

Peltier and Thomson Effects

When a current flows along the junction A of two metals in series, heat is evolved or absorbed at A depending on the current direction. This is known as the *Peltier effect*. It has no connexion with the usual heating or Joule effect of a current, discussed on p. 791. The Joule effect is irreversible, that is, heat is obtained in both directions of the current. In the Peltier effect, however, the effect is reversed when the current is reversed; that is, a cooling is produced at the junction of two metals in one direction, and an evolution of heat in the other direction.

Sir William Thomson, later Lord Kelvin, also found that heat was evolved or absorbed when a current flows along a metal whose ends are

kept at different temperatures. The *Thomson effect*, like the Peltier effect, is also reversible.

EXERCISES 32

1. State the laws of the development of heat when an electric current flows (*a*) through a wire of uniform material, (*b*) across the junction between two metals.

An electric heating coil is connected in series with a resistance of X ohms across the 240-volt mains, the coil being immersed in a kilogramme of water at 20°C. The temperature of the water rises to boiling-point in 10 minutes. When a second heating experiment is made with the resistance X short-circuited, the time required to develop the same quantity of heat is reduced to 6 minutes. Calculate the value of X. (Heat losses may be neglected.) (*L.*)

2. Define *electromotive force* and explain with the help of an example the difference between electromotive force and potential difference.

A thermocouple whose junctions are maintained at constant temperatures has a resistance of 5 ohms and its e.m.f. as measured using a potentiometer is 3·9 mV. What will be the reading on a millivoltmeter of resistance 60 ohms connected directly to the thermocouple? (*N.*)

3. Deduce an expression for the heat developed in a wire by the passage of an electric current.

The temperature of 300 g of paraffin oil in a vacuum flask rises 1·0°C per minute with an immersion heater of 12·3 watts input. On repeating with 400 g of oil the temperature rises by 1·20°C per minute for an input of 19·2 watts. Find the specific heat of the oil and the thermal capacity (assumed constant) of the flask. (*L.*)

4. Describe an experiment to determine the resistance of a wire by a calorimetric method.

It is desired to construct a 5-amp fuse from tin wire which has a melting-point of 230°C and resistivity 22×10^{-8} ohm m at that temperature. Estimate the diameter of the wire required if the emissivity of its surface is 88×10^{-5} J per sq. cm per second per °C excess temperature above the surroundings whose temperature is 20°C. Neglect the heat loss by conduction along the wire. (*N.*)

5. Describe the chief thermo-electric effects which occur in a circuit which includes two metals such as copper and iron.

Make a labelled diagram showing clearly the arrangement of a potentiometer circuit suitable for measuring a thermo-electric e.m.f. of about 2 mV. (*L.*)

6. Indicate, by means of graphs, the relation between the current and voltage (*a*) for a uniform manganin wire; (*b*) for a water voltameter; (*c*) for a diode valve. How do you account for the differences between the three curves?

An electric hot plate has two coils of manganin wire, each 20 metres in length and 0·23 mm^2 cross-sectional area. Show that it will be possible to arrange for three different rates of heating, and calculate the wattage in each case when the heater is supplied from 200-volt mains. The resistivity of manganin is $4{\cdot}6 \times 10^{-7}$ ohm m. (*O. & C.*)

7. Describe an experiment for determining the variation of the resistance of a coil of wire with temperature.

An electric fire dissipates 1 kW when connected to a 250-volt supply. Calculate to the nearest whole number the percentage change that must be made in the resistance of the heating element in order that it may dissipate 1 kW on a 200-volt supply. What percentage change in the length of the heating element will produce this change of resistance if the consequent increase in the temperature

of the wire causes its resistivity to increase by a factor 1·05? The cross-sectional area may be assumed constant. (*N.*)

8. What is a thermocouple? Explain the use of a potentiometer to measure the small electromotive forces developed by a thermocouple.

What are the relative advantages and disadvantages of a thermocouple used as a thermometer as compared with the resistance thermometer? (*L.*)

9. Give a general account of the thermo-electric effect. Describe how you would calibrate a thermocouple for use over the range of 0°–100°C on the mercury-in-glass scale of temperature. (*C.*)

10. Derive, from first principles, an expression for the rate at which heat is generated in a resistance R by the passage of a current I.

An electric lamp takes 60 watts on a 240-volt circuit. How many dry cells, each of e.m.f. 1·45 volts and internal resistance 1·0 ohm, would be required to light the lamp? How much zinc would be consumed by the battery in 1 hour? (1 faraday = 96,500 coulombs; equivalent weight of zinc = 32·5.) (*O. & C.*)

11. Describe an experimental method of producing a thermo-electric e.m.f.

How may a thermojunction be used to measure temperatures?

Why is a copper-iron junction not used to measure temperatures above 250°C, although a copper-constantan junction is often so employed? (*L.*)

12. Describe an instrument which measures the strength of an electric current by making use of its heating effect. State the advantages and disadvantages of this method.

A surge suppressor is made of a material whose conducting properties are such that the current passing through is directly proportional to the fourth power of the applied voltage. If the suppressor dissipates energy at a rate of 6·0 watts when the potential difference across it is 240 volts, estimate the power dissipated when the potential difference rises to 1200 volts. (*C.*)

13. Give a short account of the thermoelectric effect first discovered by Seebeck.

If you were given ice, boiling water, a thermocouple, a variable resistor and a sensitive galvanometer (with a linear scale) but no thermometer, describe how you would determine the temperature inside a domestic refrigerator. How would you test the assumption you are making? No other apparatus is available, but you may assume body temperature is 36·9°C.

Describe how you would measure, in the laboratory, the resistance of a pair of headphones, which is damaged if the current through it exceeds 100 milli-amp. (*C.*)

chapter thirty-three

Applications of Ohm's Law

MEASUREMENTS. NETWORKS

IN this chapter we shall apply Ohm's law to circuits more complicated than those of Chapter 32. We shall see that some special types of circuits can be used to make electrical measurements more accurately than with pointer instruments.

RESISTORS AND THEIR ARRANGEMENTS

Series Resistors

The resistors of an electric circuit may be arranged in series, so that the charges carrying the current flow through each in turn (Fig. 33.1); or they may be arranged in parallel, so that the flow of charge divides between them (Fig. 33.2), p. 808.

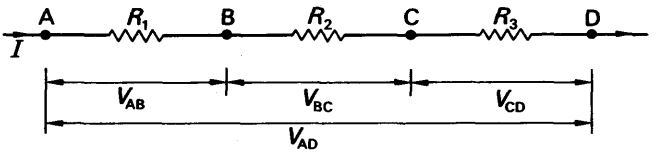


FIG. 33.1. Resistances in series.

Fig. 33.1 shows three passive resistors in series, carrying a current I. If V_{AD} is the potential difference across the whole system, the electrical energy supplied to the system per second is IV_{AD}. This is equal to the electrical energy dissipated per second in all the resistors; therefore

$$IV_{AD} = IV_{AB} + IV_{BC} + IV_{CD},$$

whence

$$V_{AD} = V_{AB} + V_{BC} + V_{CD}, \quad . \quad . \quad . \quad (1)$$

The individual potential differences are given by Ohm's law:

$$\left.\begin{aligned} V_{AB} &= IR_1 \\ V_{BC} &= IR_2 \\ V_{CD} &= IR_3 \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad (2)$$

and

Hence, by equation (1),

$$\begin{aligned} V_{AD} &= IR_1 + IR_2 + IR_3 \\ &= I(R_1 + R_2 + R_3). \quad . \quad . \quad . \quad (3) \end{aligned}$$

And the effective resistance of the system is

$$R = \frac{V_{AD}}{I} = R_1 + R_2 + R_3. \quad . \quad . \quad (4)$$

The physical facts are:

(i) *Current same through all resistors.*
(ii) *Total potential difference = sum of individual potential difference* (equation (1)).
(iii) *Individual potential differences directly proportional to individual resistances* (equation (2)).
(iv) *Total resistance greater than greatest individual resistance* (equation (4)).
(v) *Total resistance = sum of individual resistances.*

Resistors in Parallel

Fig. 33.2 shows three passive resistors connected in parallel, between the points A, B. A current I enters the system at A and leaves at B,

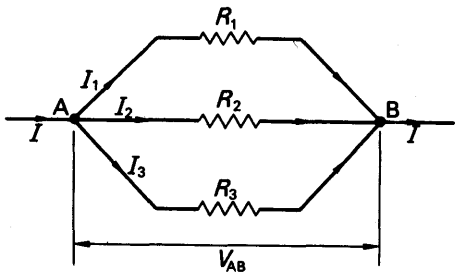


FIG. 33.2. Resistances in parallel.

setting up a potential difference V_{AB} between those points. The current branches into I_1, I_2, I_3, through the three elements, and

$$I = I_1 + I_2 + I_3. \quad . \quad . \quad . \quad (5)$$

Now $$I_1 = \frac{V_{AB}}{R_1}, \quad I_2 = \frac{V_{AB}}{R_2}, \quad I_3 = \frac{V_{AB}}{R_3}.$$

$$\therefore I = V_{AB}\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right).$$

$$\therefore \frac{I}{V_{AB}} = \frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}. \quad . \quad . \quad (6)$$

where R is the effective resistance (V_{AB}/I) of the system.

The physical facts about resistors in parallel may be summarized as follows:

(i) *Potential difference same across each resistor.*
(ii) *Total current = sum of individual currents* (equation (5)).

(iii) *Individual currents inversely proportional to individual resistances.*
(iv) *Effective resistance less than least individual resistance* (equation (6)).

Resistance Boxes

In many electrical measurements variable known resistances are required; they are called resistance boxes. As shown in Fig. 33.3 (i) ten coils, each of resistance 1 ohm, for example, are connected in series. A rotary switch with eleven contacts enables any number of these coils to be connected between the terminals AA'. A resistance box contains several sets of coils and switches, the first giving resistances 0–10 ohms in steps of 1 ohm, the next 0–100 ohms in steps of 10 ohms, and so on. These are called decade boxes.

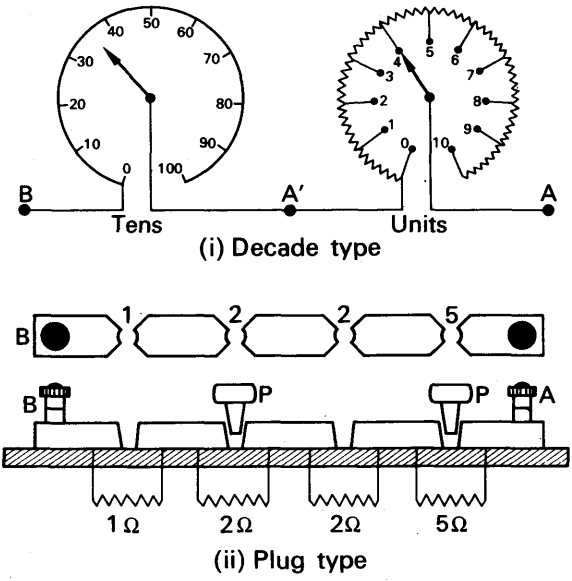


FIG. 33.3. Resistance boxes.

The switches used in a decade box are of very high quality; their contact resistances are negligible compared with the resistances of the coils which they select. Switches of this kind have been developed only in the last twenty years or so: in older boxes no switches are used. Instead, the resistances are varied by means of plugs. As shown in Fig. 33.3 (ii), the resistance coils are joined across gaps in a thick brass bar, and the gaps are formed into tapered sockets to receive short-circuiting plugs P. The resistance between the terminals A and B in Fig. 33.3 (ii) is the sum of the unplugged resistances between them—3 ohms in this example.

The coils of a resistance box are wound in a particular way, which we shall describe and explain later (p. 924). They are not intended to carry large currents, and must not be allowed to dissipate more than one watt. Therefore, since $P = I^2R$, the greatest safe current for a 1-ohm coil is 1 amp, and for a 10-ohm coil about 0·3 amp. If the one-watt limit is exceeded, the insulation will be damaged, or the wire burnt out.

The Potential Divider

Two resistance boxes in series are often used in the laboratory to provide a known fraction of a given potential difference—for example, of one which is too large to measure easily. Fig. 33.4 (i) shows the

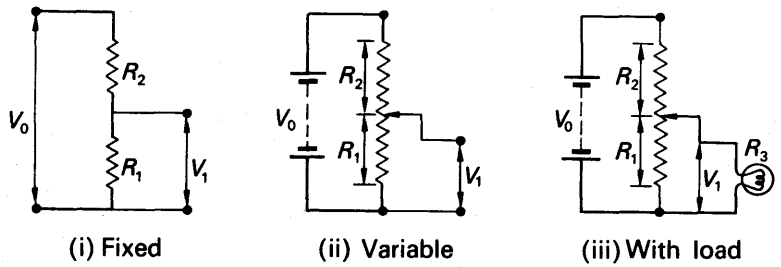


FIG. 33.4. Potential divider.

arrangement, which is called a resistance 'potential divider'. The current flowing, I, is given by

$$I = \frac{V_0}{R_1 + R_2},$$

$$\therefore V_1 = IR_1 = \frac{R_1}{R_1 + R_2} V_0. \quad . \quad . \quad . \quad (7)$$

A resistor with a sliding contact can similarly be used, as shown in Fig. 33.4 (ii), to provide a continuously variable potential difference, from zero to the full supply value V_0. This is a convenient way of controlling the voltage applied to a load, such as a lamp (Fig. 33.4 (iii)). The resistance of the load, R_3, however, acts in parallel with the resistance R_1; equation (7) is therefore no longer true, and the voltage V_1 must be measured with a voltmeter. It can be calculated, as in the following example, if R_3 is known; but if the load is a lamp its resistance varies greatly with the current through it, because its temperature varies.

EXAMPLE

A load of 2000 ohms is connected, via a potential divider of resistance 4000 ohms, to a 10-volt supply (Fig. 33.5). What is the potential difference across the load when the slider is (*a*) one-quarter, (*b*) half-way up the divider?

Since $\frac{1}{R} = \frac{1}{2000} + \frac{1}{1000}$,

(*a*) $R_{BC} = \frac{2000 \times 1000}{2000+1000} = \frac{2000}{3}$ ohms.

$\therefore R_{AC} = R_{AB} + R_{BC} = 3000 + \frac{2000}{3}$ ohms,

$\therefore V_{BC} = \frac{R_{BC}}{R_{AC}} V_C$

$= \frac{2000/3}{11000/3} \times 10 = \frac{2}{11} \times 10$

$= 1{\cdot}8$ volts.

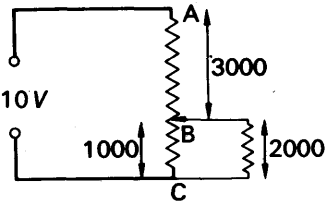


FIG. 33.5. A loaded potential divider.

If the load were removed, V_{BC} would be 2·5 volts.

(*b*) It is left for the reader to show similarly that $V_{BC} = 3{\cdot}3$ volts. Without the load it would be 5 volts.

MEASURING INSTRUMENTS

Conversion of a Milliammeter into a Voltmeter

Ohm's law enables us to use a milliammeter as a voltmeter. Let us suppose that we have a moving-coil instrument which requires 5 milliamperes for full-scale deflection (f.s.d.). And let us suppose that the resistance of its coil, r, is 20 ohms (Fig. 33.6). Then, when it is fully deflected, the potential difference across it is

$V = rI$

$= 20 \times 5 \times 10^{-3} = 100 \times 10^{-3}$ volt

$= 0{\cdot}1$ volt.

Since the coil obeys Ohm's law, the current through it is proportional to the potential difference across it; and since the deflection of the pointer is proportional to the current it is therefore also proportional to the potential difference. Thus the instrument can be used as a voltmeter, giving full-scale deflection for a potential difference of 0·1 volt, or 100 millivolts. Its scale could be engraved as shown at the top of Fig. 33.6.

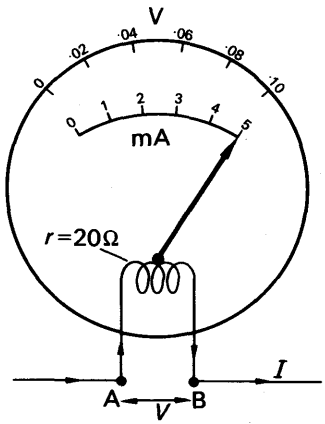
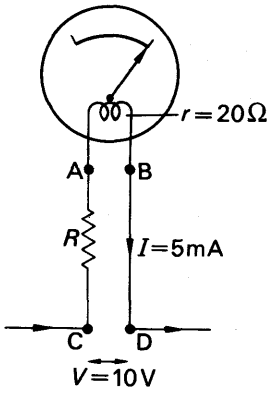


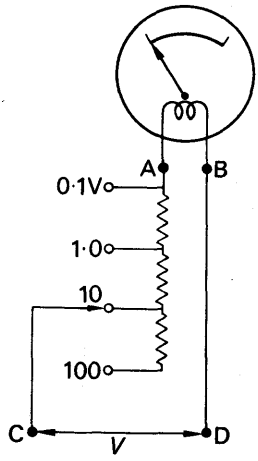
FIG. 33.6. P.D. across moving-coil meter.

The potential differences to be measured in the laboratory are usually greater than 100 millivolts, however. To measure such a potential difference, we insert a resistor R in series with the coil, as shown in Fig. 33.7. If we wish to measure up to 10 volts we must choose the resistance R so that, when 10 volts are applied between the



Single-range

FIG. 33.7. Single-range voltmeter.



Multi-range

FIG. 33.8. Multi-range voltmeter.

terminals CD, then a current of 5 milliamperes flows through the moving coil. By Ohm's law

$$V = (R+r)I,$$

$$\therefore\ 10 = (R+20)\times 5\times 10^{-3}$$

or
$$R+20 = \frac{10}{5\times 10^{-3}} = 2\times 10^{3} = 2000 \text{ ohms.}$$

$$\therefore\ R = 2000-20$$

$$= 1980 \text{ ohms.} \quad . \quad . \quad . \quad . \quad . \quad (8)$$

The resistance R is called a *multiplier.* Many voltmeters contain a series of multipliers of different resistances, which can be chosen by a switch or plug-and-socket arrangement (Fig. 33.8).

Conversion of a Milliammeter into an Ammeter

Moving-coil instruments give full-scale deflection for currents smaller than those generally encountered in the laboratory. If we wish to measure a current of the order of an ampere or more we connect a low resistance S, called a shunt, across the terminals of a moving-coil meter (Fig. 33.9). The shunt diverts most of the current to be measured, I, away from the coil—hence its name. Let us suppose that, as before,

the coil of the meter has a resistance r of 20 ohms and is fully deflected by a current, I_C, of 5 milliamperes. And let us suppose that we wish to shunt it so that it gives f.s.d. for 5 amperes to be measured. Then the current through the shunt is

$$I_S = I - I_C$$
$$= 5 - 0{\cdot}005$$
$$= 4{\cdot}995 \text{ amp.}$$

The potential difference across the shunt is the same as that across the coil, which is

$$V = rI_C = 20 \times 0{\cdot}005 = 0{\cdot}1 \text{ volt.}$$

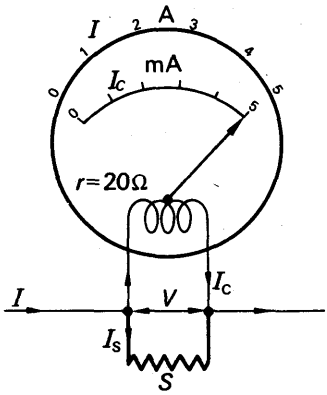


FIG. 33.9. Conversion of milliammeter to ammeter.

The resistance of the shunt must therefore be

$$S = \frac{V}{I_S} = \frac{0{\cdot}1}{4{\cdot}995} = 0{\cdot}02002 \text{ ohms.} \quad . \quad . \quad (9)$$

The ratio of the current measured to the current through the coil is

$$\frac{I}{I_C} = \frac{5}{5 \times 10^{-3}} = 1000.$$

This ratio is the same whatever the current I, because it depends only on the resistances S and r; the reader may easily show that its value is $(S+r)/S$. The deflection of the coil is therefore proportional to the measured current, as indicated in the figure, and the shunt is said to have a 'power' of 1000 when used with this instrument.

The resistance of shunts and multipliers are always given with four-figure accuracy. The moving-coil instrument itself has an error of the order of 1 per cent.; a similar error in the shunt or multiplier would therefore double the error in the instrument as a whole. On the other hand, there is nothing to be gained by making the error in the shunt less than about 0·1 per cent., because at that value it is swamped by the error of the moving system.

Multimeters

A *multimeter* instrument is one which is adapted for measuring both current and voltage. It has a shunt R as shown, and a series of voltage multipliers R' (Fig. 33.10). The shunt is connected permanently across the coil, and the resistances in R' are adjusted to give the desired full-scale voltages with the shunt in position. A switch or plug enables the various full-scale values of current or voltage to be chosen, but the user does the mental arithmetic. The instrument shown in the figure is reading 1·7 volts; if it were on the 10-volt range, it would be reading 6·4.

The terminals of a meter, multimeter or otherwise, are usually marked + and −; the pointer is deflected to the right when current passes through the meter from + to −.

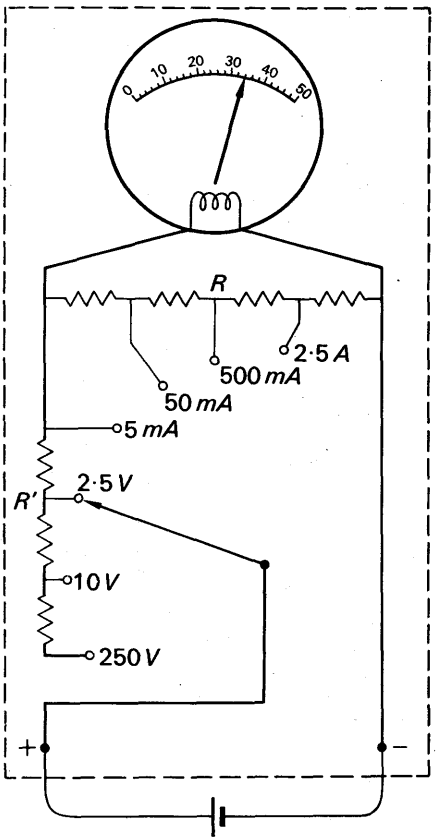


FIG. 33.10. A multimeter.

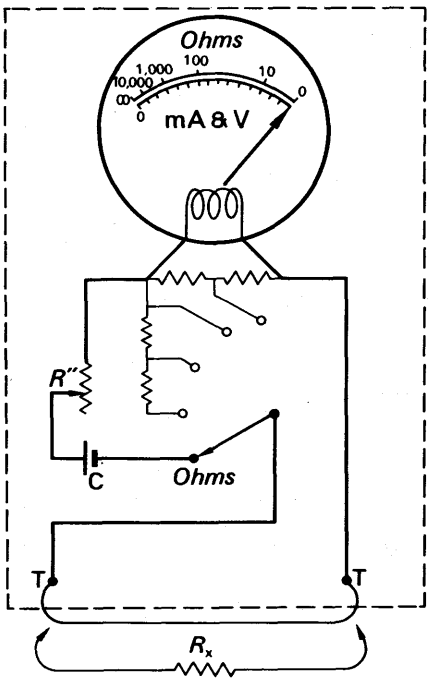


FIG. 33.11. Measurement of resistance with multimeter.

The multimeters are generally arranged to measure resistance as well as current and voltage. An extra position on the switch, marked 'R' or 'ohms', puts a dry cell C and a variable resistor R'' in series with the moving coil (Fig. 33.11). Before the instrument is used to measure a resistance, its terminals TT are short-circuited, and R'' is adjusted until the pointer is fully deflected. As shown in the figure, it is then opposite the zero on the ohms scale. The short-circuit is next removed, and the unknown resistance R_x is connected across the terminals. The current falls, and the pointer moves to the left, indicating on the ohms scale the value of R_x. The ohms scale is calibrated by the makers with known resistances.

Use of Voltmeter and Ammeter

A moving-coil voltmeter is a current-operated instrument. It can be used to measure potential differences only because the current which it

draws is proportional to the potential difference applied to it, from Ohm's law. Since its action depends on Ohm's law, a moving-coil voltmeter cannot be used in any experiment to demonstrate that law; that is why, when describing such an experiment on p. 789, we specified measuring instruments whose readings are not dependent on Ohm's law.

Having once established Ohm's law, however, we can use moving-coil voltmeters freely; they are both more sensitive and more accurate than other forms of voltmeters. The current which they take does, however, sometimes complicate their use. To see how it may do so, let us suppose that we wish to measure a resistance R of about 100 ohms. As shown in Fig. 33.12, we connect it in series with a cell, a milliammeter, and a

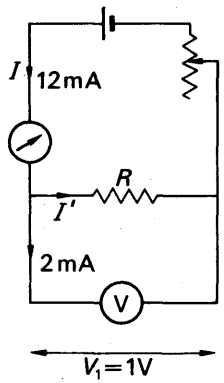


FIG. 33.12

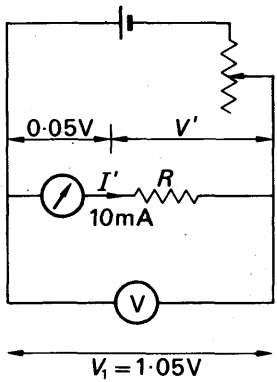


FIG. 33.13.

Use of ammeter and voltmeter.

variable resistance; across it we place the voltmeter. We adjust the current until the voltmeter reads, say, $V_1 = 1$ volt; let us suppose that the milliammeter then reads $I = 12$ mA. The value of the resistance then appears to be

$$R = \frac{V_1}{I} = \frac{1}{12 \times 10^{-3}} = \frac{10^3}{12}$$

$$= 83 \text{ ohms (approx.)}.$$

But the milliammeter reading includes the current drawn by the voltmeter. If that is 2 mA, then the current through the resistor, I', is only 10 mA and its resistance is actually

$$R = \frac{V_1}{I'} = \frac{1}{10 \times 10^{-3}} = \frac{1}{10^{-2}}$$

$$= 100 \text{ ohms}.$$

The current drawn by the voltmeter has made the resistance appear 17 per cent. lower than its true value.

In an attempt to avoid this error, we might connect the voltmeter as shown in Fig. 33.13: across both the resistor and the milliammeter. But

its reading would then include the potential difference across the milliammeter. Let us suppose that this is 0·05 volt when the current through the milliammeter is 10 mA. Then the potential difference V' across the resistor would be 1 volt, and the voltmeter would read 1·05 volt. The resistance would appear to be

$$R = \frac{1{\cdot}05}{10 \times 10^{-3}} = \frac{1{\cdot}05}{10^{-2}}$$

$$= 105 \text{ ohms.}$$

Thus the voltage drop across the milliammeter would make the resistance appear 5 per cent. higher than its true value.

Errors of this kind are negligible only when the voltmeter current is much less than the current through the resistor, or when the voltage drop across the ammeter is much less than the potential difference across the resistor. If we were measuring a resistance of about 1 ohm, for example, the current I' in Fig. 33.12 would be 1 amp, and I would be 1·002 amp. The error in measuring R would then be only 0·2 per cent—less than the intrinsic error of the meter. But the circuit of Fig. 33.13 would give the same error as before. It could do so because, as we saw when considering shunts, the shunt across the milliammeter would have been chosen to make the voltage drop still 0·05 volt. Thus V_1 would still be 1·05 volt when V' was 1 volt, and the error would be 5 per cent as before.

In low-resistance circuits, therefore, the voltmeter should be connected as in Fig. 33.12, so that its reading does not include the voltage drop across the ammeter. But in high-resistance circuits the voltmeter should be connected as in Fig. 33.13, so that the ammeter does not carry its current.

If a moving-coil voltmeter is connected across a cell, it will not read its true e.m.f., because the current which it draws will set up a voltage drop across the internal resistance of the cell. The drop will be negligible only if the resistance of the voltmeter is very high compared with the internal resistance. E.m.f.s are thus compared by a potentiometer method, discussed shortly.

Figure of Merit of a Voltmeter

If a milliammeter of 1 mA f.s.d. (full scale deflection) is converted into a voltmeter, then if it is to have 1 volt f.s.d. its total resistance—coil plus multiplier—must be 1000 ohms. (One volt across its terminals will send through it a current of 1/1000 amp = 1 mA.) If it is to have 10 volts f.s.d., then its total resistance must be 10000 ohms; for 20 volts f.s.d., 20000 ohms, and so on. It will have a resistance of 1000 ohms for every volt of its full-scale deflection. Such a meter is said to have a *figure of merit* of 1000 ohms per volt. Similarly, a voltmeter which takes 10 mA, or 1/100 amp, for full-scale deflection has a figure of merit of 100 ohms per volt. The greater the figure of merit of a voltmeter, expressed in this way, the less will it disturb any circuit to which

it is connected, and the less error will its current cause in any measurements made with it. On the other hand, the greater the figure of merit, the more delicate the moving system of the meter and the greater its intrinsic error. First-grade, and particularly 'sub-standard', meters therefore have medium or low figures of merit: from 500 to 66·7 ohms per volt.

When a voltmeter of low figure of merit is being used, it may be necessary to allow for the current which it draws. The allowance is made in the way indicated on p. 815, where the use of a voltmeter and ammeter together was discussed.

THE POTENTIOMETER

Pointer instruments are useless for very accurate measurements: the best of them have an intrinsic error of abour 1 per cent of full scale. Where greater accuracy than this is required, elaborate measuring circuits are used.

One of the most versatile of these, due to Poggendorf, is the potentiometer. It consists of a uniform wire, AB in Fig. 33.13(i), about a metre long; through it an accumulator X maintains a steady current I. Since the wire is uniform, its resistance per centimetre, R, is constant; the voltage drop across 1 cm of the wire, RI, is therefore also constant.

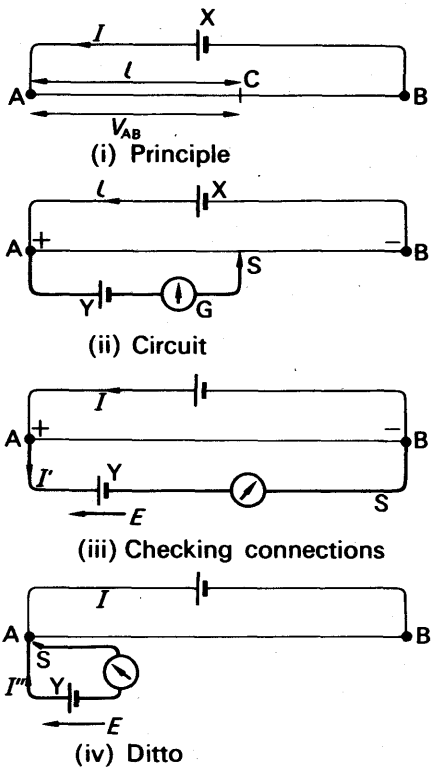


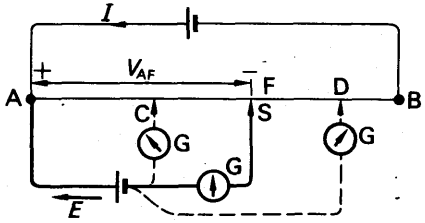
FIG. 33.14. The potentiometer.

The potential difference between the end A of the wire, and any point C upon it, is thus proportional to the length of wire l between A and C:

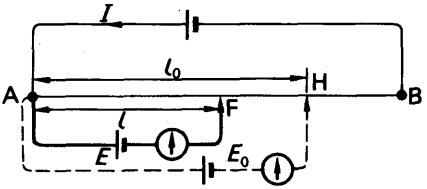
$$V_{AC} \propto l. \quad . \quad . \quad . \quad . \quad (10)$$

Comparison of E.M.F.s

To illustrate the use of the potentiometer, let us suppose that we take a cell, Y in Fig. 33.14(ii), and join its positive terminal to the point A (to which the positive terminal of X is also joined). We connect the negative terminal of Y, via a sensitive galvanometer, to a slider S, which we can press on to any point in the wire. Let us suppose that the cell Y has an e.m.f. E, which is less than the potential difference V_{AB} across the whole of the wire. Then if we press the slider on B, a current I' will flow through Y in opposition to its e.m.f. (Fig. 33.14(iii)). This current will deflect the galvanometer G—let us say to the right. If we now press the slider on A, the cell Y will be connected straight across the galvanometer, and will deliver a current I'' in the direction of its e.m.f. (Fig. 33.14(iv)). The galvanometer will therefore show a deflection to the left. If the deflections at A and B are not opposite, then either the e.m.f. of Y is greater than the potential difference across the whole wire, or we have connected the circuit wrongly. The commonest mistake in connecting up is not joining both positives to A.



(i) Finding balance point



(ii) Comparison of e.m.f.

FIG. 33.15. Use of potentiometer.

Now let us suppose that we place the slider on to the wire at a point a few centimetres from A, then at a point a few centimetres farther on, and so forth. (We do not run the slider continuously along the wire, because the scraping would destroy the uniformity.) When the slider is at a point C near A (Fig. 33.15(i)) the potential difference V_{AC} is less than the e.m.f. E of Y; current therefore flows through G in the direction

of E, and G may deflect to the left. When the slider is at D near B, V_{AD} is greater than E, current flows through G in opposition to E, and G deflects to the right. By trial and error (but no scraping of the slider) we can find a point F such that, when the slider is pressed upon it, the galvanometer shows no deflection. The potential difference V_{AF} is then equal to the e.m.f. E; no current flows through the galvanometer because E and V_{AF} act in opposite directions in the galvanometer circuit (Fig. 33.15(i)). Because no current flows, the resistance of the galvanometer, and the internal resistance of the cell, cause no voltage drop; the full e.m.f. E therefore appears, between the points, A and S, and is balanced by V_{AF}:

$$E = V_{AF}.$$

If we now take another cell of e.m.f. E_0, and balance it in the same way, at a point H (Fig. 33.15(ii)), then

$$E_0 = V_{AH}.$$

Therefore

$$\frac{E}{E_0} = \frac{V_{AF}}{V_{AH}}.$$

The potential differences V_{AF}, V_{AH} are proportional to the lengths l, l_0 from A to F, and from A to H, respectively. Therefore

$$\frac{E}{E_0} = \frac{l}{l_0}. \quad . \quad . \quad . \quad . \quad . \quad (11)$$

Accuracy

When the potentiometer is used to compare the e.m.f.s of cells, no errors are introduced by the internal resistances, because no current flows at the balance-points.

The potentiometer is more accurate than an electrometer instrument, which, like a moving-coil voltmeter, has an intrinsic error of about 1 per cent of full-scale. The accuracy of a potentiometer is limited by the non-uniformity of the slide-wire, the uncertainty of the balance-point, and the error in measuring the length l of wire from the balance-point to the end A. With even crude apparatus, the balance-point can be located to within about 0·5 mm; if the length l is 50 cm, or 500 mm, then the error in locating the balance-point is 1:1000. If the wire has been carefully treated, its non-uniformity may introduce an error of about the same magnitude. The overall error is then about ten times less than that of a pointer instrument. A refined potentiometer has a still smaller error.

The precision with which the balance-point of a potentiometer can be found depends on the sensitivity of the galvanometer—the smallness of the current which will give a just-discernible deflection. A moving-coil galvanometer must be protected by a series resistance R of several thousand ohms, which is shorted out when the balance is nearly reached (Fig. 33.16). A series resistance is preferable to a shunt, because it re-

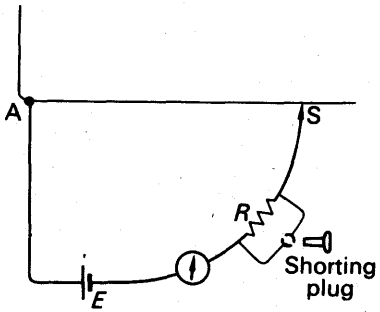


FIG. 33.16. Use of protective resistance with galvanometer.

duces the current drawn from the cell under test, when the potentiometer is unbalanced. The process of seeking the balance-point then causes less change in the chemical condition of the cell, and therefore in its e.m.f.

It is important to realize that the accuracy of a potentiometer does not depend on the accuracy of the galvanometer, but only on its sensitivity. The galvanometer is used not to measure a current but merely to show one when the potentiometer is off balance. It is said to be used as a null-indicator, and the potentiometer method of measurement, like the bridge methods which we shall describe shortly, is called a null method.

The current through the potentiometer wire must be steady—it must not change appreciably between the finding of one balance-point and the next. The accumulator which provides it should therefore be neither freshly charged nor nearly run-down; when an accumulator is in either of those conditions its e.m.f. falls with time. Errors in potentiometer measurements may be caused by non-uniformity of the wire, and by the resistance of its connexion to the terminal at A. This resistance is added to the resistance of the length l of the wire between A and the balance-point, and if it is appreciable it makes equation (11) invalid. Both these sources of error are eliminated in the Rayleigh potentiometer, which we shall describe later (p. 825).

Uses of the Potentiometer. E.M.F. and Internal Resistance

All the uses of the potentiometer depend on the fact that it can measure potential difference accurately, and without drawing current from the circuit under test.

If one of the cells in Fig. 33.15 (ii) has a known e.m.f., say E_0 then the e.m.f. of the other, E, is given by equation:

$$\frac{E}{E_0} = \frac{l}{l_0}. \quad . \quad . \quad . \quad . \quad (12)$$

A cell of known e.m.f. is called a standard cell. The e.m.f.s of standard cells are determined absolutely—that is to say, without reference to the e.m.f.s of any other cell—by methods which depend, in principle, on the definition of e.m.f. (power/current, p. 797). Standard cells are described on p. 865, along with the precautions which must be taken in their use. For simple experiments a Daniell cell (p. 860), whose e.m.f. is about 1·1 volt, may be used as a standard.

Equation (12) is true only if the current I through the potentiometer wire has remained constant. The easiest way to check that it has done so is to balance the standard cell against the wire before and after balancing the unknown cell. If the lengths to the balance-point are

equal—within the limits of experimental error— then the current I may be taken as constant. A check of this kind should be made in each of the experiments to be described.

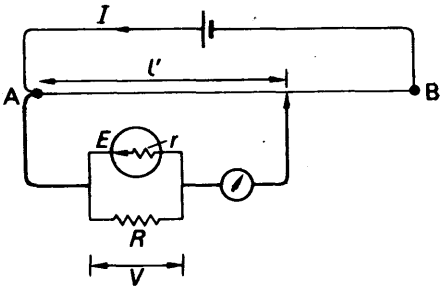


FIG. 33.17. Measurement of internal resistance.

The internal resistance of a cell, r, can be found with a potentiometer by balancing first its e.m.f., E, and the its terminal potential difference, V, when a known resistance R is connected across it (Fig. 33.17). Ohm's law for the complete circuit gives

$$\frac{V}{E} = \frac{R}{R+r} \quad . \quad . \quad . \quad . \quad . \quad (13)$$

But

$$\frac{V}{E} = \frac{l'}{l} \quad . \quad . \quad . \quad . \quad . \quad (14)$$

where l and l' are the lengths of potentiometer wire required to balance E and V. From equations (13) and (14), r can be found from

$$r = \left(\frac{l}{l'} - 1\right) R.$$

Calibration of Voltmeter

Fig. 33.18 shows how a potentiometer can be used to calibrate a voltmeter. A standard cell is first used to find the p.d. per cm or volts

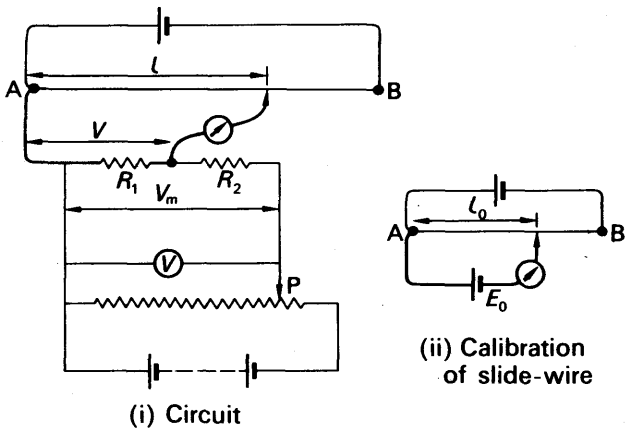


FIG. 33.18. Calibration of voltmeter with potentiometer.

per cm of the wire (Fig. 33.18(ii)): if its e.m.f. E_0 is balanced by a length l_0, then

$$\text{volts per cm} = \frac{E_0}{l_0}. \quad . \quad . \quad . \quad (15)$$

Different voltages V_m are now applied to the voltmeter by the adjustable potential divider P (Fig. 33.18(i)). The fixed potential divider, comprising R_1R_2, gives a known fraction V of each value of V_m, which is then balanced on the potentiometer:

$$\frac{V}{V_m} = \frac{R_1}{R_1+R_2}. \quad . \quad . \quad . \quad (16)$$

(The resistances R_1 and R_2 are high—of the order of 1000 to 10000 ohms, so that the voltage adjustment by P is fairly uniform. Their ratio is chosen so that the greatest value of V is measurable on the potentiometer—about 1·5 volts.) If l is the lengths of potentiometer wire which balances a given value of V, then

$$V = l \times (\text{volt/cm of wire})$$
$$= l\frac{E_0}{l_0}.$$

From each value of V, the value of V_m is calculated by equation (16). If the voltmeter reading is V_{obs}, then the correction to be added to it is $V_m - V_{obs}$. This is plotted against V_{obs}, as in Fig. 33.19.

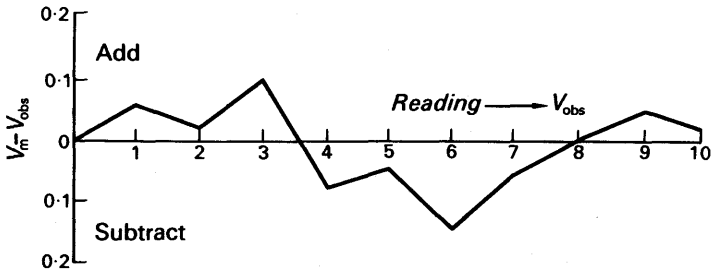


FIG. 33.19. Correction curve of voltmeter.

Measurement of Current

A current can be measured on a potentiometer by means of the potential difference which it sets up across a known resistance, R in Fig. 33.20(i). The resistance is low, being chosen so that the potential difference across it is of the order of 0·1 volt. (A higher value is not chosen, because the voltage drop across the resistor disturbs the circuit in which it is inserted.) Fig. 33.20(ii) shows in detail the kind of resistor used, which is often called a standard shunt. It consists of a broad strip of alloy, such as manganin, whose resistance varies very little with temperature (p. 837). The current is led in and out as the terminals i, i. The terminals v, v are connected to fine wires soldered to points PP

on the strip; they are called the potential terminals. The marked value R of the resistance is the value between the points PP; it is adjusted by making hack-saw cuts into the edges of the strip.

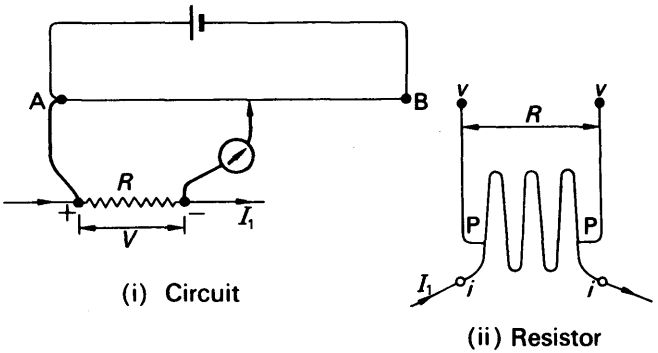


FIG. 33.20. Measurement of current with potentiometer.

As shown in Fig. 33.20(i), the current to be measured, I_1, is passed through the shunt, and the potential difference between its potential terminals, V, is balanced on the potentiometer wire. If l is the length of wire to the balance-point, then

$$\frac{V}{E_0} = \frac{l}{l_0}, \qquad . \quad . \quad . \quad . \quad (17)$$

where E_0 and l_0 refer to a standard cell as before. Equation (17) enables the current to be found in terms of E_0, l_0, l, and R, since

$$\mathrm{V} = I_1 R.$$

The resistance of the wires connecting the potential terminals to the points PP, and to the potentiometer circuit, do not affect the result, because at the balance-point the current through them is zero.

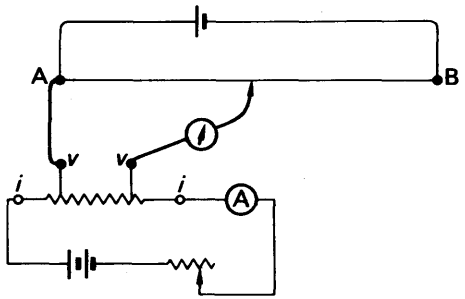


FIG. 33.21. Calibration of ammeter with potentiometer.

This method of measuring a current can be used to calibrate an ammeter, A. The circuit is shown in Fig. 33.21; its principle and use

should explain themselves. The results are treated in the same way as in the calibration of a voltmeter.

Comparison of Resistances

A potentiometer can be used to compare resistances, by comparing the potential differences across them when they are carrying the same current I_1 (Fig. 33.22). This method is particularly useful for very low resistances, because, as we have just seen, the resistances of the connecting wires do not affect the result of the experiment. It can, however, be used for higher resistances if desired. With low resistances the ammeter A′ and rheostat P are necessary to adjust the current to a value which will neither exhaust the accumulator Y, nor overheat the resistors. No standard cell is required. The potential difference across the first resistor, $V_1 = R_1 I_1$, is balanced against a length l_1 of the potentiometer wire, as shown by the full lines in the figure. Both potential terminals

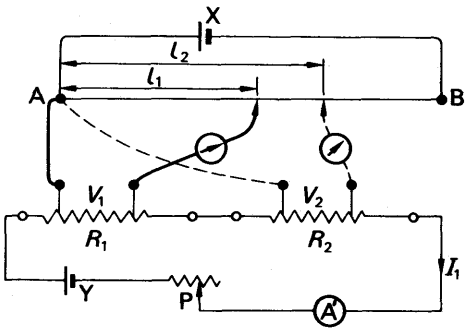


FIG. 33.22.
Comparison of resistances with potentiometer.

of R_1 are then disconnected from the potentiometer, and those of R_2 are connected in their place. If l_2 is the length to the new balance-point, then

$$\frac{l_1}{l_2} = \frac{V_1}{V_2} = \frac{R_1 I_1}{R_2 I_1} = \frac{R_1}{R_2}.$$

This result is true only if the current I_1 is constant; as well as the potentiometer current. The accumulator Y, as well as X, must therefore be in good condition. To check the constancy of the current I_1, the ammeter A′ is not accurate enough. The reliability of the experiment as a whole can be checked by balancing the potential V_1 a second time, after V_2. If the new value of l_1 differs from the original, then at least one of the accumulators is running down and must be replaced.

The Rayleigh Potentiometer

Fig. 33.23 shows a potentiometer devised by Lord Rayleigh (1842–1919), which is free from errors due to non-uniformity of the wire and to contact resistance at the end A (p. 820). It consists of two plug-type resistance boxes, R_1, R_2, joined in series. (These boxes may well be the R-sections of two similar Post Office boxes (p. 834). At the start of a measurement all the plugs of R_1 are inserted, and all of R_2 taken out. Then R_1 is zero, and the main current I sets up no potential difference across it; but when the key K is pressed, the unknown e.m.f. E deflects the galvanometer. R_1 is now increased by, say, 100 ohms, and R_2 is

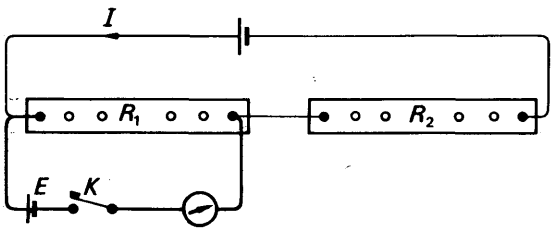


FIG. 33.23. Rayleigh potentiometer.

decreased by the same amount. In this way R_1+R_2 is kept constant, and the current I does not change. But there is now a potential difference across R_1, which opposes E. Plugs are taken out of R_1 and put into R_2, so as to keep R_1+R_2 constant, until the galvanometer shows no deflection when K is pressed. If R_1' is the value of R_1 at this point, then

$$E = R_1' I.$$

The procedure is now repeated with a standard cell of e.m.f. E_0, in place of E. Since R_1+R_2 has been kept constant, the current I is the same as before; hence, if R_1'' is the new value of R_1 at balance,

$$E_0 = R_1'' I.$$

Consequently,

$$\frac{E}{E_0} = \frac{R_1' I}{R_1'' I} = \frac{R_1'}{R_1''}.$$

Measurement of Thermal E.M.F.

The e.m.f.s of thermojunctions (p. 803) are small—of the order of a millivolt. If we attempted to measure such an e.m.f. on a simple potentiometer we should find the balance-point very near one end of the wire, so that the end-error would be serious. The Rayleigh potentiometer, although it is free from end-errors, is not suitable for measuring small e.m.f.s; if, in Fig. 33.23, $R_1+R_2 = 10000\ \Omega$, and the e.m.f.

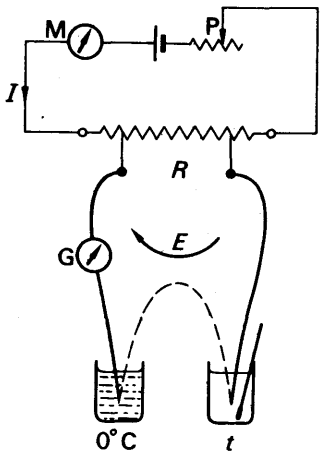


FIG. 33.24. Measurement of thermal e.m.f.

of the accumulator = 2 volts, then $I = 2/10000 = 2 \times 10^{-4}$ amp = 0·2 mA. To balance a thermal e.m.f. of 2 mV, R_1 would therefore have to be 10 Ω; and since R_1 cannot be adjusted in steps smaller than 1 Ω, the e.m.f. cannot be measured to a greater accuracy than 10 per cent.

For accurate measurement of thermal e.m.f.s special potentiometers have been devised, but the simple circuit of Fig. 33.24 will do for a laboratory experiment. The e.m.f. E is applied via a sensitive galvanometer G across a standard shunt R of about 1 ohm. A current I, of a few milliamperes, is passed through the shunt, and measured on the milliammeter M. Its value is adjusted by the rheostat P until G shows no deflection. The potential difference RI is then equal and opposite to the thermal e.m.f.

$$E = RI.$$

If a balance cannot be found, the connexions of the junction to R should be reversed.

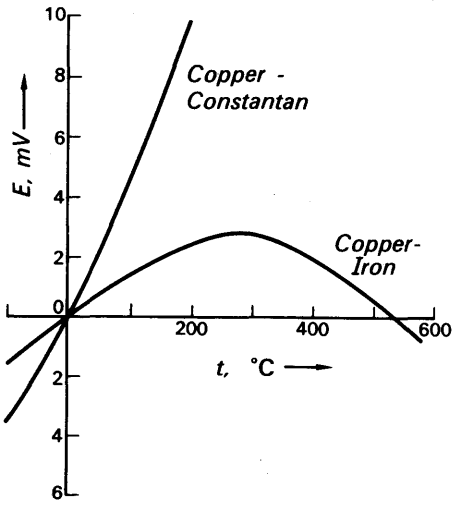


FIG. 33.25. E.m.f.s of thermocouples.
(Reckoned positive when into copper at the cold junction.)

Fig. 33.25 shows the results of measuring the e.m.f. E when the cold junction is at 0°C and the hot is at various temperatures t. The curves approximate to parabolas:

$$E = at + bt^2. \quad . \quad . \quad . \quad (18)$$

THERMO-ELECTRIC E.M.F.s

(E in micro-volts when t is in °C
and cold junction at 0°C)

Junction	a	b	Range for a and b, °C	Limits of use, °C
Cu/Fe . . .	14	−0·02	0–100	*See* 1
Cu/Constantan[2] .	41	0·04	−50 to +300	−200 to +300
Pt/Pt−Rh[3] . .	6·4	0·006	0–200	0–1700
Chromel[4]/Alumel[5] .	41	0·00	0–900	0–1300

[1] Simple demonstrations.
[2] See p. 788.
[3] 10 per cent Rh; used only for accurate work or very high temperatures.
[4] 90 per cent Ni, 10 per cent Cr.
[5] 94 per cent Ni, 3 per cent Mn, 2 per cent Al, 1 per cent Si.

NETWORKS

Kirchhoff's Laws

A 'network' is usually a complicated system of electrical conductors. Kirchhoff (1824–87) extended Ohm's law to networks, and gave two laws, which together enabled the current in any part of the network to be calculated.

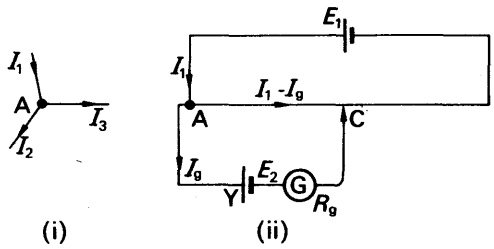


FIG. 33.26. Kirchhoff's laws.

The *first law* refers to any point in the network, such as A in Fig. 33.26 (*a*); it states that the total current flowing into the point is equal to the total current flowing out of it:

$$I_1 = I_2 + I_3.$$

The law follows from the fact that electric charges do not accumulate at the points of a network. It is often put in the form that *the algebraic sum of the currents at a junction of a circuit is zero*, or

$$\Sigma I = 0,$$

where a current, I, is reckoned positive if it flows towards the point, and negative if it flows away from it. Thus at A in Fig. 33.26 (i),

$$I_1 - I_2 - I_3 = 0.$$

Kirchhoff's first law gives a set of equations which contribute towards the solving of the network; in practice, however, we can shorten the work by putting the first law straight into the diagram, as shown in Fig. 33.26 (ii) for example, since

$$\text{current along AC} = I_1 - I_g.$$

Kirchhoff's *second law* is a generalization of Ohm's law for the complete circuit. It refers to any closed loop, such as AYCA in Fig. 33.26 (ii); and it states that, round such a loop, the algebraic sum of the voltage drops is equal to the algebraic sum of the e.m.f.s:

$$\Sigma RI = \Sigma E.$$

Thus, clockwise round the loop,

$$R_{AC}(I_1 - I_g) - R_g I_g = E_2.$$

We have used the potentiometer to illustrate Kirchhoff's laws merely because it is already familiar to us; we shall not go on and solve it as a network, because we have already dealt with as much of the theory of it as we need.

EXAMPLE

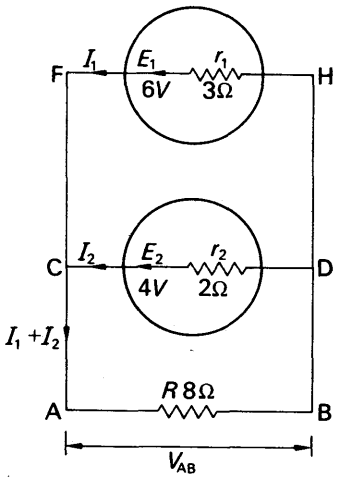


FIG. 33.27. Load across cells in parallel.

Fig. 33.27 shows a network which can be solved by Kirchhoff's laws. From the first law, the current in the 8-ohm wire is (I_1+I_2), assuming I_1, I_2 are the currents through the cells. Taking closed circuits formed by each cell with the 8-ohm wire, we have, from the second law,

$$E_1 = 6 = 3I_1 + 8(I_1+I_2) = 11I_1 + 8I_2$$

and

$$E_2 = 4 = 2I_2 + 8(I_1+I_2) = 8I_1 + 10I_2.$$

Solving the two equations, we find $I_1 = \frac{14}{23} = 0{\cdot}61$ amp, $I_2 = -\frac{2}{23} = -0{\cdot}09$ amp.

The minus sign indicates that the current I_2 flows in the sense opposite to that shown in the diagram; i.e. it flows against the e.m.f. of the generator E_2. It does so because the potential difference V_{AB} is greater than E_2:

$$V_{AB} = \left(R\,I_1 + I_2\right) = 8\left(\frac{14}{23} - \frac{2}{23}\right)$$

$$= 8 \times \frac{12}{23} = \frac{96}{23} = 4{\cdot}2 \text{ volts.}$$

This is equal to the e.m.f. E_2 *plus* the drop across the internal resistance r_2 (p. 828):

$$V_{CD} = 4+2\times\frac{2}{23} = 4+\frac{4}{23}$$

$$= \frac{96}{23}\text{ volts} = V_{AB}.$$

It is also equal to the e.m.f. E_1 *minus* the drop across r_1, because the current flows through the upper generator in the direction of its e.m.f.:

$$V_{FH} = 6-3\times\frac{14}{23} = 6-\frac{42}{23} = \frac{138-42}{23}$$

$$= \frac{96}{23}\text{ volts} = V_{AB}.$$

WHEATSTONE BRIDGE MEASUREMENT OF RESISTANCE

Wheatstone Bridge Circuit

About 1843 Wheatstone designed a circuit called a 'bridge circuit' which gave an accurate method for measuring resistance. We shall deal later with the practical aspects. In Fig. 33.28, X is the unknown resistance, and P, Q, R are resistance boxes. One of these—usually R— is adjusted until the galvanometer between A, C, represented by its resistance R_g, shows no deflection: that is to say,

$$I_g = 0.$$

Then, as we shall show,

$$\frac{P}{Q} = \frac{R}{X},$$

whence

$$X = \frac{Q}{P}R.$$

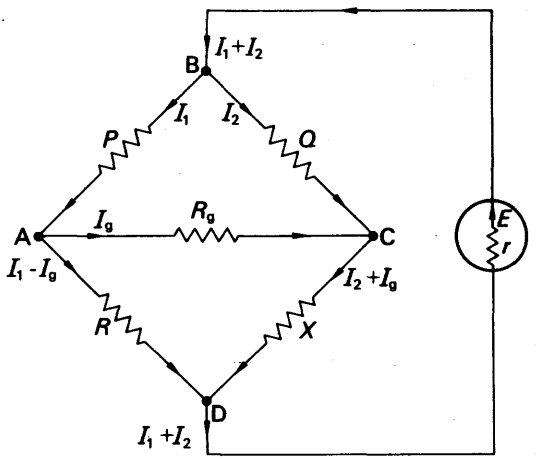


FIG. 33.28. Analysis of Wheatstone bridge.

Fig 33.28 shows Kirchhoff's first law applied to the circuit. From the second law, we have:

$$\text{loop ACBA:} \quad R_g I_g - QI_2 + PI_1 = 0, \quad . \quad . \quad \text{(i)}$$

$$\text{loop ACDA:} \quad R_g I_g + X(I_2 + I_g) - R(I_1 - I_g) = 0,$$

or
$$I_g(R_g + X + R) + XI_2 + RI_1 = 0. \quad . \quad . \quad \text{(ii)}$$

If we wished to find I_g, we would have to set up a third equation, by going round one of the loops, including the battery (p. 831). But if we wish only to find the condition for no deflection of the galvanometer, we have merely to put $I_g = 0$ in equations (i) and (ii). Then

$$-QI_2 + PI_1 = 0, \quad \text{or} \quad PI_1 = QI_2,$$

whence
$$\frac{P}{Q} = \frac{I_2}{I_1};$$

and
$$IX_2 - RI_1 = 0, \quad \text{or} \quad XI_2 = RI_1,$$

whence
$$\frac{R}{X} = \frac{I_2}{I_1}$$

Therefore, as already stated,

$$\frac{P}{Q} = \frac{R}{X}. \quad . \quad . \quad . \quad \text{(19)}$$

This is the condition for balance of the bridge. It is the same, as the reader may easily show, if the battery and galvanometer are interchanged in the citcuit.

Alternative Wheatstone Bridge Proof

Equation (19) for the balance condition can be got without the use of Kirchhoff's laws. At balance, since no current flows through the galvanometer, the points A and C must be at the same potential (Fig. 33.29). Therefore

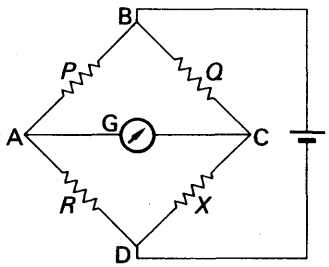


FIG. 33.29. Wheatstone bridge.

$$V_{AB} = V_{CB}$$

and
$$V_{AD} = V_{CD},$$

whence
$$\frac{V_{AB}}{V_{AD}} = \frac{V_{CB}}{V_{CD}}. \quad . \quad . \quad . \quad \text{(i)}$$

Also, since $I_g = 0$, P and R carry the same current, I_1, and X and Q carry the same current, I_2. Therefore

$$\frac{V_{AB}}{V_{AD}} = \frac{I_1 P}{I_1 R} = \frac{P}{R}$$

and
$$\frac{V_{CB}}{V_{CD}} = \frac{I_2 Q_2}{I_2 X} = \frac{Q}{X} \quad . \quad . \quad . \quad \text{(ii)}$$

Hence by equations (i) and (ii),

$$\frac{P}{R} = \frac{Q}{X}$$

or

$$\frac{P}{Q} = \frac{R}{X}$$

Galvanometer Position

We shall now show, by taking a numerical example, how the galvanometer in a bridge circuit can best be positioned.

Fig. 33.30 shows an unbalanced Wheatstone bridge, fed from a cell

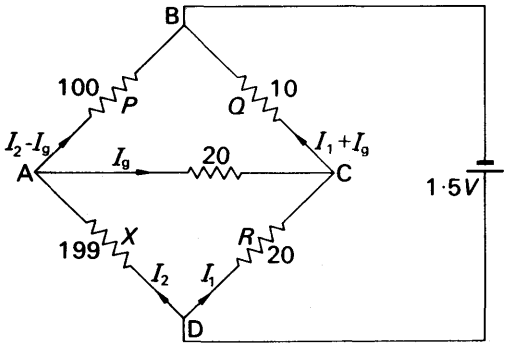


FIG. 33.30. An unbalanced Wheatstone bridge.

of negligible internal resistance. The figures give the resistance in ohms, and I_g is to be found. Applying Kirchhoff's laws:

$$\text{Loop ACBA:} \quad 20I_g + 10(I_1 + I_g) - 100(I_2 - I_g) = 0$$

or

$$130I_g + 10I_1 - 100I_2 = 0,$$

whence

$$I_1 = 10I_2 - 13I_g. \qquad \text{(i)}$$

$$\text{Loop ADCA:} \quad -139I_2 + 20I_1 - 20I_g = 0.$$

Substituting for I_1: $-199I_2 + 200I_2 - 260I_g - 20^9{}_g = 0,$

or

$$I_2 - 280I_g = 0,$$

whence

$$I_2 = 280I_g.$$

$$\therefore \text{ by (i)}, \quad I_1 = 2800I_g - 13I_g = 2787I_g. \qquad \text{(ii)}$$

$$\text{Loop DCBXD:} \quad 20I_1 + 10(I_1 + I_g) = 1{\cdot}5,$$

or

$$30I_1 + 10I_g = 1{\cdot}5.$$

Substituting from (ii) for I_1: $30 \times 2787I_g + 10I_g = 1{\cdot}5$

or

$$83620I_g = 1{\cdot}5,$$

whence

$$I_g = \frac{1{\cdot}5}{83620} = 1{\cdot}79 \times 10^{-5} \text{ A}$$

$$= 17{\cdot}9 \text{ microamperes.}$$

The reader should now show that, if the battery and galvanometer were interchanged, the current I_g would be 13·2 microamperes. This result illustrates an important point in the use of the Wheatstone bridge: with a given unbalance, the galvanometer current is greatest when the galvanometer is connected from the junction of the highest resistances to the junction of the lowest. Therefore, unless $P = Q$, which is unusual, the galvanometer should be connected across PQ.

Practical Arrangement

A practical form of Wheatstone bridge is shown in Fig. 33.31. The resistances P and Q can be given values of 10, 100, or 1000 ohms by three-point switches. The resistance R has four decade dials by which it can be varied from 1 ohm to more than 10000 ohms. Pairs of terminals are provided for connecting the unknown resistance, the battery, and the galvanometer, X, B, G; and keys K_1 and K_2 are fitted in the battery and galvanometer circuits.

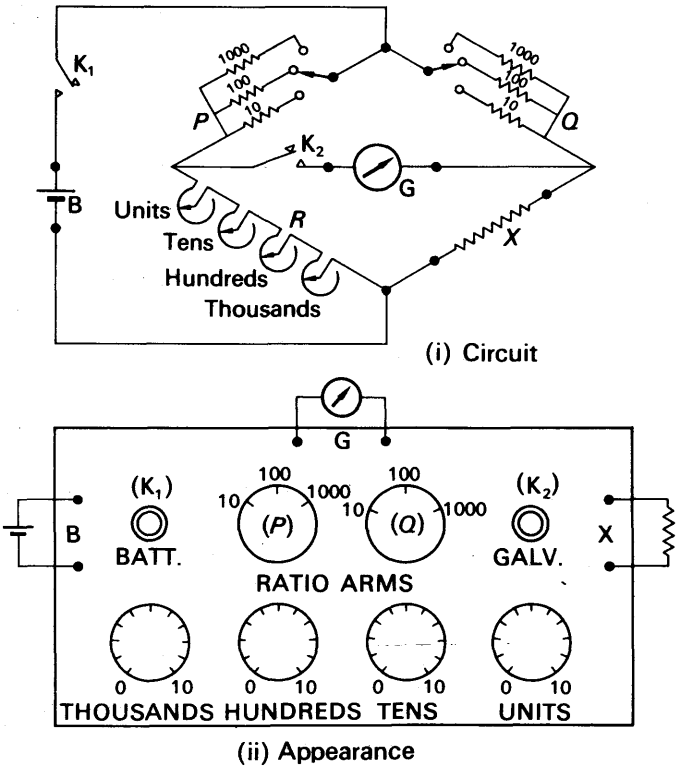


FIG. 33.31. Practical form of Wheatstone bridge.

To measure a resistance, we first set $P = Q = 10\,\Omega$. We set $R = 0$ and press first K_1 then K_2; the small interval between pressing K_1 and K_2 gives time for the currents in the bridge to become steady (Chapter

31). Let us suppose that, when we press K_2, the galvanometer deflects to the right. We then set $R = 10000\ \Omega$ and again press K_1, K_2. If the galvanometer deflects to the left we can proceed; if it deflects again to the right, then either we have made a wrong connexion —which with this form of bridge is almost impossible— or X is greater than 10000 ohms. If the galvanometer deflects to the left, we try again with $R = 1000\ \Omega$; and so on with 100 Ω and 10 Ω, if necessary. Let us suppose that the galvanometer deflects to the left with $R = 100\ \Omega$, but to the right with 10 Ω. We then adjust the 10's dial until we get, say, a leftward deflection with 40 Ω and a rightward with 30 Ω. With the unit's dial we now narrow the limits to, let us say, 36 Ω (left) and 35 Ω (right). We have

$$\frac{X}{R} = \frac{Q}{P} = \frac{10}{10} = 1.$$

$$\therefore X = R.$$

It follows that X lies between 35 and 36 Ω.

We now set $P = 100\ \Omega$, so that

$$\frac{X}{R} = \frac{Q}{P} = \frac{10}{100} = \frac{1}{10}$$

or

$$X = \frac{R}{10}.$$

The balance-point now lies between $R = 350\ \Omega$ (right) and $R = 360\ \Omega$ (left); by using the unit's dial we can now locate it between, say, 353 and 354. Then, from the equation, X lies between 35·3 and 35·4 ohms. If we finally make $P = 1000\ \Omega$, we have

$$\frac{X}{R} = \frac{Q}{P} = \frac{10}{1000} = \frac{1}{100}$$

or

$$X = \frac{R}{100}.$$

Only a sensitive galvanometer will give considerable deflections near the balance-point in this condition; if it locates the balance-point between $R = 3536\ \Omega$ and 3537 Ω then X lies between 35·36 and 35·37 ohms. If a moving-coil galvanometer is used, it must be protected by a high series resistance while balance is being sought.

Range of Measurable Resistance

The resistors P and Q are often called the ratio arms of the bridge, because their resistances determine the ratio of R to X. If X is greater than the greatest value of R, it can be measured by making $Q = 100$, $P = 10$. Then

$$\frac{X}{R} = \frac{Q}{P} = \frac{100}{10} = 10$$

and

$$X = 10R.$$

A balance-point between $R = 14620$ and 14630, say, would mean that R lay between 146 200 and 146 300 ohms. Similarly, by making $Q = 1000$, $P = 10$, resistances can be measured up to 100 times the greatest value of R: that is to say, up to a little more than 1 000 000 ohms. With these high resistances, however, the near-balance currents are very small, and a sensitive galvanometer is necessary.

The lowest resistance which a bridge of this type can measure with reasonable accuracy is about 1 ohm; R can be adjusted in steps of 1 ohm, and P/Q can be made 1/100, so that measurements can be made to within 1/100 ohm. Resistances lower than about 1 ohm cannot be measured accurately on a Wheatstone bridge, whatever the ratios available, or the smallest steps in R. They cannot because of the resistances of the wires connecting them to the X terminals, and of the contacts between those wires and the terminals to which they are, at each end, attached. This is the reason why the potentiometer method is more satisfactory for low resistances.

The Post Office Box

An old-fashioned type of Wheatstone bridge, with plugs instead of switches, is called the Post Office box, and is illustrated in Fig. 33.32. It is connected up and used in the same way as the dial type of bridge,

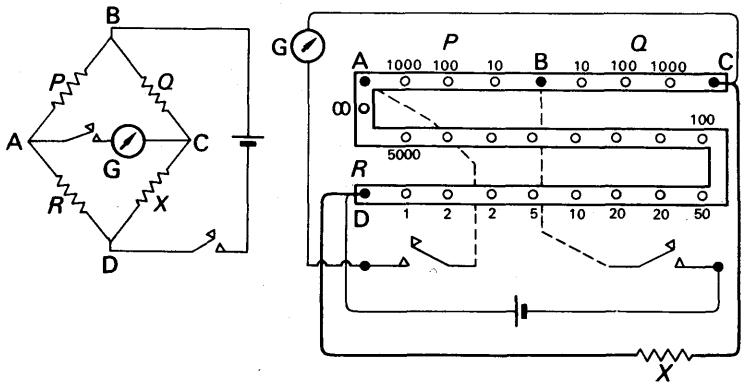


FIG. 33.32. Post Office box.

but requires far more skill by its operator. Anyone who has to use a Post Office box should observe the following rules:

(i) do not attempt to memorize the wiring-up; the circuit should be worked out from the Wheatstone bridge diagram (Fig. 33.29);
(ii) take the 10 Ω plugs out of each ratio arm P, Q before testing the circuit;
(iii) test for correct connexions by seeing whether the galvanometer gives opposite deflections with $R = 0$ and $R = \infty$ (for the latter an 'infinity' plug is provided, whose gap is not bridged by any resistor—Fig. 33.32);

(iv) press plugs home firmly, with a half-turn to the right;
(v) never mix the plugs from different boxes—always put loose plugs in the lid of their box, never on the bench.

The Slide-wire (Metre) Bridge

Fig. 33.33 shows a simple and cheap form of Wheatstone bridge; it is sometimes called a metre bridge, for no better reason than that the wire AB is often a metre long. The wire is uniform, as in a potentiometer, and can be explored by a slider S. The unknown resistance X and a known resistance R are connected as shown in the figure; heavy brass or copper strip is used for the connexions AD, FH, KB, whose resistances are generally negligible. When the slider is at a point C in

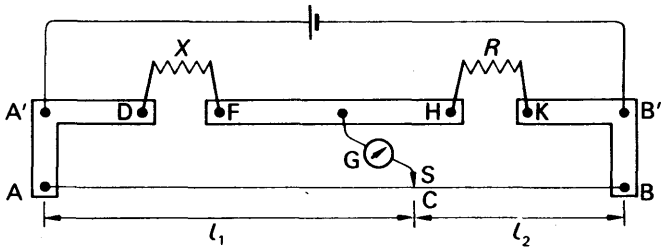


FIG. 33.33. Slide-wire (metre) bridge.

the wire it divides the wire into two parts, of resistances R_{AC} and R_{CB}; these, with X and R, form a Wheatstone bridge. (The galvanometer and battery are interchanged relative to the circuits we have given earlier; that enables the slider S to be used as the galvanometer key. We have already seen that the interchange does not affect the condition for balance (p. 830).) The connexions are checked by placing S first on A, then on B. The balance-point is found by trial and error—not by scraping S along AB. At balance,

$$\frac{X}{R} = \frac{R_{AC}}{R_{CB}}.$$

Since the wire is uniform, the resistances R_{AC} and R_{CB} are proportional to the lengths of wire, l_1 and l_2. Therefore

$$\frac{X}{R} = \frac{l_1}{l_2}. \qquad . \quad . \quad . \quad (20)$$

The resistance R should be chosen so that the balance-point C comes fairly near to the centre of the wire—within, say, its middle third. If either l_1 or l_2 is small, the resistance of its end connexion AA' or BB' in Fig. 33.33 is not negligible in comparison with its own resistance; equation (20) then does not hold. Some idea of the accuracy of a particular measurement can be got by interchanging R and X, and balancing again. If the new ratio agrees with the old within about 1 per cent, then their average may be taken as the value of X.

Resistance by Substitution

Fig. 33.34 illustrates a simple way of measuring a resistance X. It is connected in series with a rheostat S, an ammeter A, and a cell. S is adjusted until A gives a large deflection. X is then replaced by a box of known resistances, R, which can be selected by plugs or dials. R is varied until the ammeter gives the same reading as before. Then, if the e.m.f. of the cell has not fallen, $R = X$. The accuracy of this method is limited, by the inherent error of the ammeter, to about 1 per cent. It does not depend on the accuracy of *calibration* of the ammeter, but on the accuracy with which it reproduces a given deflection for a given current. The ammeter is used simple as a 'transfer instrument'—to indicate when the current in the second part of the experiment is the same as in the first. This principle is very useful in measurements more difficult than that of resistance by direct method. For example, the power output of a small radio transmitter can be measured by making it light a lamp, which is placed near to a lightmeter (p. 571). The lamp is then connected to a source of direct current, and the current through it is adjusted until the light-meter gives the same reading. Simple measurements of the current, and the voltage across the lamp, then give the power supplied to it—which is equal to the power output of the radio transmitter.

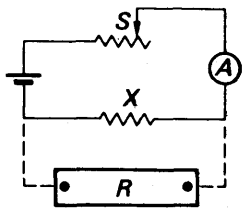


FIG. 33.34. Measurement of resistance by substitution.

Resistance and Sensitivity of a Galvanometer

It is often necessary to know the resistance, R_g, and sensitivity of a suspended-coil galvanometer. To find them, we may use the circuit of Fig. 33.35 (i). S is a rheostat of about 1000 ohms maximum, r is a standard shunt of about 0·01 ohm, M is a milliammeter, and R is a resistance box. The current in the main circuit, I, is adjusted to a value which can be accurately read on M—say 10 milliamperes. Since r is very

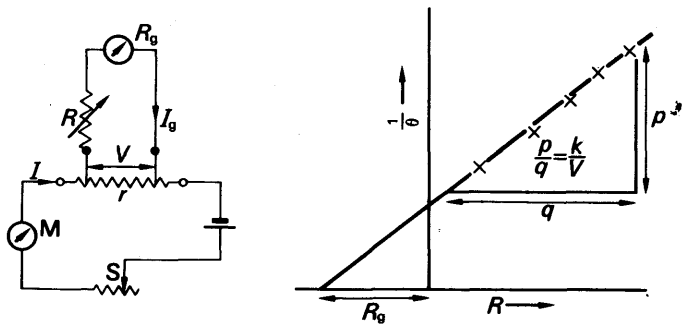


FIG. 33.35. Galvanometer calibration.

small compared with $R+R_g$, the galvanometer current I_g is negligible compared with I, and we may say that the potential difference across r is

$$V = rI.$$

In this example it would be $0{\cdot}01 \times 10 \times 10^{-3} = 10^{-4}$ volt $= 0{\cdot}1$ millivolt. The galvanometer current is therefore

$$I_g = \frac{V}{R+R_g}.$$

If $R+R_g$ is 1000 ohms, then $I_g = 10^{-4}/10^3 = 10^{-7}$ amp $= 0{\cdot}1$ microampere, which is a reasonable value for a galvanometer of moderate sensitivity. If θ is the deflection of the galvanometer, then

$$I_g = k\theta,$$

where k is a constant (the reduction factor) which we wish to find.

From the equation for I_g above.

$$\frac{V}{R+R_g} = k\theta,$$

whence

$$\frac{1}{\theta} = \frac{k}{V}(R+R_g).$$

Therefore, if we vary R and plot $1/\theta$ against it, we get a straight line, as shown in Fig. 33.35 (ii). The line makes an intercept on the R axis, which gives the value of R_g. And its slope p/q is k/V. Since we know V from above, we can hence find k; and $1/k$, the deflection per unit current, is the sensitivity.

Temperature Coefficient of Resistance

We have seen that the resistance of a given wire increases with its temperature. If we put a coil of fine copper wire into a water bath, and use a Wheatstone bridge to measure its resistance at various moderate temperatures t, we find that the resistance, R, increases

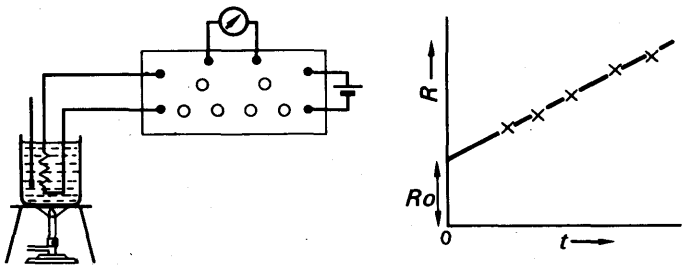


FIG. 33.36. Measurement of temperature coefficient.

uniformly with the temperature (Fig. 33.36). We may therefore define a temperature coefficient of resistance, α, such that

$$R = R_0(1+\alpha t), \quad . \quad . \quad . \quad (21)$$

where R_0 is the resistance at 0°C. In words,

$$\alpha = \frac{\text{increase of resistance per deg. C rise of temperature}}{\text{resistance at 0°C}}$$

If R_1 and R_2 are the resistances at t_1°C and t_2°C, then

$$\frac{R_1}{R_2} = \frac{1+\alpha t_1}{1+\alpha t_2}. \quad . \quad . \quad . \quad (22)$$

Values of α for pure metals are of the order of 0·004 per deg. C. They are much less for alloys than for pure metals, a fact which enhances the value of alloys as materials for resistance boxes and shunts.

Equation (21) represents the change of resistance with temperature fairly well, but not as accurately as it can be measured. More accurate equations are given on p. 370 in the Heat section of this book, where resistance thermometers are discussed.

EXAMPLES

1. How would you compare the resistances of two wires A and B, using (*a*) a Wheatstone bridge method and (*b*) a potentiometer? For each case draw a circuit diagram and indicate the method of calculating the result.

In an experiment carried out at 0°C, A was 120 cm of Nichrome wire of resistivity 100×10^{-6} ohm cm and diameter 1·20 mm, and B a German silver wire 0·80 mm diameter and resistivity 28×10^{-6} ohm cm. The ratio of the resistances A/B was 1·20. What was the length of the wire B?

If the temperature coefficient of resistance of Nichrome is 0·00040 per deg. C and of German silver is 0·00030 per deg. C, what would the ratio of the resistances become if the temperature were raised by 100 deg. C? (*L.*)

First part (see pp. 830, 824).

Second part. With usual notation,

for A, $$R_1 = \frac{\rho_1 l_1}{a_1},$$

and for B, $$R_2 = \frac{\rho_2 l_2}{a_2}.$$

$$\therefore \frac{R_1}{R_2} = \frac{\rho_1}{\rho_2} \cdot \frac{l_1}{l_2} \cdot \frac{a_2}{a_1} = \frac{\rho_1}{\rho_2} \cdot \frac{l_1}{l_2} \cdot \frac{d_2^2}{d_1^2},$$

where d_2, d_1 are the respective diameters of B and A.

$$\therefore 1{\cdot}20 = \frac{100}{28} \times \frac{120}{l_2} \times \frac{0{\cdot}8^2}{10{\cdot}2^2}.$$

$$\therefore l_2 = \frac{100 \times 120 \times 0{\cdot}64}{1{\cdot}20 \times 28 \times 1{\cdot}44} = 159 \text{ cm} \quad . \quad . \quad \text{(i)}$$

When the temperature is raised by 100°C, the resistance increases according to the relation $R_t = R_0(1+\alpha t)$. Thus

$$\text{new Nichrome resistance, } R_A = R_1(1+\alpha . 100) = R_1 \times 1{\cdot}04,$$

and new German silver resistance, $R_B = R_2(1+\alpha' . 100) = R_2 \times 1{\cdot}03$.

$$\therefore \frac{R_A}{R_B} = \frac{R_1}{R_2} \times \frac{1{\cdot}04}{1{\cdot}03} = 1{\cdot}20 \times \frac{1{\cdot}04}{1{\cdot}03} = 1{\cdot}21 \quad . \quad . \quad . \quad \text{(ii)}$$

2. State Kirchhoff's laws relating to the currents in network of conductors. Two cells of e.m.f. 1·5 volts and 2 volts respectively and internal resistances of 1 ohm and 2 ohms respectively are connected in parallel to an external resistance of 5 ohms. Calculate the currents in each of the three branches of the network. (*N.*)

First part (see p. 828).

Second part. Suppose x, y amp are the respective currents through the cells (Fig. 33.37). Then, from Kirchhoff's first law, the current through the 5-ohm wire is $(x+y)$ amp.

Applying Kirchhoff's second law to the complete circuit with the cell of e.m.f. 1·5 volts and external resistance 5 ohms, we have

$$1{\cdot}5 = x+5(x+y) = 6x+5y \quad . \quad . \quad . \quad \text{(i)}$$

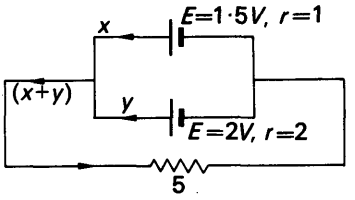


FIG. 33.37. Example.

Applying the law to the complete circuit with the cell of e.m.f. 2 volts and external resistance 5 ohms, then

$$2 = 2y+5(x+y) = 5x+7y. \quad . \quad . \quad . \quad \text{(ii)}$$

Solving (i) and (ii), we find $y = 9/34$ amp, $x = 1/34$ amp. Thus the current through the 5-ohm resistor $= x+y = 10/34 = 5/17$ amp.

EXERCISES 33

Circuit Calculations

1. State Ohm's law and describe an experiment to verify it.

A resistance of 1000 ohms and one of 2000 ohms are placed in series with a 100-volt mains supply. What will be the reading on a voltmeter of internal resistance 2000 ohms when placed across (*a*) the 1000 ohms resistance, (*b*) the 2000 ohms resistance? (*L.*)

2. Define the practical unit of potential difference and hence show that the rate of production of heat in a wire of constant resistance is proportional to the square of the current passing through it.

A cell A, e.m.f. 1·1 volts and internal resistance 3 ohms, is joined in parallel with another cell B, e.m.f. 1·4 volts and internal resistance 1 ohm, similar poles being connected together. The ends of a wire, of resistance 4 ohms, are joined to the terminals of A. Find (*a*) the current through the wire, (*b*) the rate of dissipation of energy in watts in each of the cells A and B. (*L.*)

3. Twelve cells each of e.m.f. 2 volts and of internal resistance $\frac{1}{2}$ ohm are arranged in a battery of n rows and an external resistance of $\frac{3}{8}$ ohm is connected to the poles of the battery. Determine the current flowing through the resistance in terms of n.

Obtain numerical values of the current for the possible values which n may take and draw a graph of current against n by drawing a smooth curve through the points. Give the value of the current corresponding to the maximum of the curve and find the internal resistance of the battery when the maximum current is produced. (*L.*)

4. Describe with full experimental details an experiment to test the validity of Ohm's law for a metallic conductor.

An accumulator of e.m.f. 2 volts and of negligible internal resistance is joined in series with a resistance of 500 ohms and an unknown resistance X ohms. The readings of a voltmeter successively across the 500-ohm resistance and X are 2/7 and 8/7 volts respectively. Comment on this and calculate the value of X and the resistance of the voltmeter. (*N.*)

5. State with reasons the essential requirement for the resistance of (*a*) an ammeter, (*b*) a voltmeter.

A voltmeter having a resistance of 1800 ohms is used to measure the potential difference across a 200 ohm resistance which is connected to the terminals of a d.c. power supply having an e.m.f. of 50 volts and an internal resistance of 20 ohms. Determine the percentage change in the potential difference across the 200 ohm resistor as a result of connecting the voltmeter across it. (*N.*)

6. State Ohm's law and describe how you would test its validity. Why would an experiment involving the use of a moving-coil ammeter and a moving-coil voltmeter be unsatisfactory?

In order to calibrate a galvanometer an accumulator of e.m.f. 200 volts and negligible resistance is connected in series with two resistances P and Q. A resistance R and the galvanometer, resistance G, are joined in series and then connected to the ends of P. The galvanometer is shunted by a resistance S. If $P = 200$ ohms, $Q = 1880$ ohms, $R = 291$ ohms, $S = 10$ ohms, $G = 90$ ohms, and the deflection is 20 divisions, calculate the current sensitivity in micro-amperes per division.

7. State Kirchhoff's laws for flow of electricity through a network containing sources of e.m.f.

Wheatstone bridge with slide wire of 0·5 ohm and length 50 cm is used to compare two resistances, each of 2 ohms. The cell has an e.m.f. of 2 volts and no internal resistance, and the galvanometer has resistance of 100 ohms. Find the current through the galvanometer when the bridge is 1 cm off balance. Compare the result with that of approximate calculation. (*L.*)

8. State Ohm's law, and describe the experiments you would make in order to verify it. The positive poles A and C of two cells are connected by a uniform wire of resistance 4 ohms and their negative poles B and D by a uniform wire of resistance 6 ohms. The middle point of BD is connected to earth. The e.m.f.s of the cells AB and CD are 2 volts and 1 volt respectively, their resistances 1 ohm and 2 ohms respectively. Find the potential at the middle point of AC. (*O. & C.*)

9. From the adoption of either the fundamental units cm, g, second, or the fundamental units m, kg, second, trace the steps necessary to define the *volt* and the *ohm* in terms of the ampere.

Discuss the suitability of (*a*) a moving-coil voltmeter, and (*b*) a slide-wire potentiometer for determining the potential differences in an experiment designed to verify Ohm's law.

Four resistors AB, BC, CD and DA of resistance 4 ohms, 8 ohms, 4 ohms and 8 ohms respectively are connected to form a closed loop, and a 6-volt battery of negligible resistance is connected between A and C. Calculate (i) the potential

difference between B and D, and (ii) the value of the additional resistance which must be connected between A and D so that no current flows through a galvanometer connected between B and D. (*O. & C.*)

Measurements

10. Describe and explain how you would use a potentiometer (*a*) to compare the electromotive forces of two cells, (*b*) to test the accuracy of the 1-amp reading of an ammeter.

The e.m.f. of a cell is balanced by the fall in potential along 150 cm of a potentiometer wire. When the cell is shunted by a resistance of 14 ohms the length required is 140 cm. What is the internal resistance of the cell? (*N.*)

11. Describe a metre bridge and the method of using it to compare the resistances of two conductors, giving the theory of the method. Why is the method unsatisfactory if the two resistances (*a*) differ widely from one another, (*b*) are very small?

A metre bridge is balanced with a piece of aluminium wire of resistance 7·30 ohm in the left-hand gap, the slide contact being 42·6 cm from the left-hand end of the bridge wire and the temperature 17°C. If the temperature of the aluminium wire is raised to 57°C, how may the balance be restored (*a*) by adjusting the slide contact, (*b*) by keeping the contact at 42·6 cm and connecting a conductor in parallel with the aluminium wire? (The temperature coefficient of resistance for aluminium may be taken as $3{\cdot}8 \times 10^{-3}\ K^{-1}$) (*L.*)

12. Describe and explain how a potentiometer is used to test the accuracy of the 1 volt reading of a voltmeter.

A potentiometer consists of a fixed resistance of 2030 ohms in series with a slide wire of resistance 4 ohm metre^{-1}. When a constant current flows in the potentiometer circuit a balance is obtained when (*a*) a Weston cell of e.m.f. 1·018 volt is connected across the fixed resistance and 150 cm of the slide wire and also when (*b*) a thermocouple is connected across 125 cm of the slide wire only. Find the current in the potentiometer circuit and the e.m.f. of the thermocouple.

Find the value of the additional resistance which must be present in the above potentiometer circuit in order that the constant current shall flow through it, given that the driver cell is a lead accumulator of e.m.f. 2 volt and of negligible resistance and the length of the slide wire is 2 metres. (*L.*)

13. Describe the Wheatstone bridge circuit and deduce the condition for 'balance'. State clearly the fundamental electrical principles on which you base your argument. Upon what factors do (*a*) the sensitivity of the bridge, (*b*) the accuracy of the measurement made with it, depend?

Using such a circuit, a coil of wire was found to have a resistance of 5 ohms in melting ice. When the coil was heated to 100°C, a 100 ohm resistor had to be connected in parallel with the coil in order to keep the bridge balanced at the same point. Calculate the temperature coefficient of resistance of the coil. (*C.*)

14. Explain the use of the Weston cell and the precautions to be taken when using it.

A steady current is passed through a manganin potentiometer wire AB of length 10 metres and diameter 0·56 mm connected to a resistance box BC with a resistance of 1001 ohms. A Weston cell of e.m.f. 1·018 volts, with a sensitive galvanometer in series with it, is connected in parallel between the points A and C. It is seen that no deflection is produced in the galvanometer. The Weston cell is removed and a thermocouple is connected *via* the galvanometer to points on the potentiometer wire 524 cm apart. Again there is no deflection. Draw the two

circuits and calculate the e.m.f. of the thermocouple. (Resistivity of manganin $= 41{\cdot}87 \times 10^{-6}$ ohm cm.) (*L.*)

15. Describe, giving full experimental details, how you would compare the values of two unknown resistances each of the order of 0·1 ohm using a potentiometer. Draw a circuit diagram and give the theory of the method.

Suppose that, having set up the circuit to carry out this experiment, you found that no balance point could be obtained along the potentiometer. Discuss three possible reasons for this and the procedure you would adopt in order to trace it. (*N.*)

16. A two-metre potentiometer wire is used in an experiment to determine the internal resistance of a voltaic cell. The e.m.f. of the cell is balanced by the fall of potential along 90·6 cm of wire. When a standard resistance of 10 ohms is connected across the cell the balance length is found to be 75·5 cm. Draw a labelled circuit diagram and calculate, from first principles, the internal reistance of the cell.

How may the accuracy of this determination be improved? Assume that other electrical components are available if required. (*N.*)

17. Define *resistivity* and *temperature coefficient of resistance.*

Explain, with the help of a clear circuit diagram, how you would use a Post Office box to determine the resistance of an electric lamp filament at room temperature.

A carbon lamp filament was found to have a resistance 375 ohms at the laboratory temperature of 20°C. The lamp was them connected in series with an ammeter and a d.c. supply, and a voltmeter of resistance 1050 ohms was connected in parallel with the lamp. The ammeter and voltmeter indicated 0·76 amp and 100 volt respectively. The temperature of the carbon filament was estimated to be 1200°C. Estimate the mean value of the temperature coefficient of resistance of carbon between 20°C and 1200°C, and comment on your result. (*L.*)

18. Explain the principle of the potentiometer.

If a slide wire potentiometer of total resistance 5 ohms and a Weston standard cell of e.m.f. 1·0187 volt were available, together with the necessary auxiliary apparatus, describe in detail how you would (*a*) determine the variation of the e.m.f. of a thermocouple with the temperature difference between its junctions (maximum e.m.f. 3 millivolt), (*b*) compare the values of two resistances nominally 0·010 ohm and 0·005 ohm. (*L.*)

19. State Ohm's law. Deduce a formula for the resistance of a number of resistors connected in parallel.

If you were given a cell of constant internal resistance, an ammeter of negligible resistance and various wires of different lengths but otherwise identical, describe how you would determine how many cm of wire had the same resistance as the internal resistance of the cell.

Describe how you would compare the resistances of two resistors each about 10^6 ohms using only an accumulator of e.m.f. of about 2 volts, a low-resistance uncalibrated galvanometer with a linear scale and full-scale deflection for about 10^{-7} amp and a three-terminal slider rheostat of about 20 ohms resistance. Justify the calculation of the resistance ratio from the readings you would take. (*C.*)

20. Describe how you would calibrate an ammeter using a standard resistor, a rheostat, accumulators, potentiometer slide wire with usual accessories, and a standard cell.

A standard low resistor is accidentally connected across a 100 volt d.c. main. It emits a momentary flash of light, vapourizes, and immediately breaks the

circuit without further sparking. Estimate the duration of the flash if the wire becomes incandescent at 550°C, melts at 900°C, has specific heat 0·42 kJ kg^{-1} K^{-1}, density 5500 kg m^{-3}, is 10 cm long and has constant resistivity of 40 microhm cm. Assume that the rate of energy loss by the wire is small compared with the rate of heat development within it.

Give a physical explanation of the fact that this time is independent of the cross-sectional area of the wire. (*C.*)

chapter thirty-four

The Chemical Effect of the Current

In this chapter we shall deal both with the effects of an electric current when it is passed through a chemical solution, and with chemical generators of electric current, or cells.

ELECTROLYSIS

The chemical effect of the electric current was first studied quantitatively by Faraday, who introduced most of the technical terms which are now used in describing it. A conducting solution is called an *electrolyte* and the chemical changes which occur when a current passes through it are called *electrolysis* (*lysis* = decomposition). Solutions in water of acids, bases, and salts are electrolytes, and so are their solutions in some other solvents, such as alcohol. The plates or wires which dip into the electrolyte to connect it to the circuit are called electrodes; the one by which the current enters the solution is called the *anode*, and the one by which it leaves is called the *cathode* (Fig. 34.1 (i)).

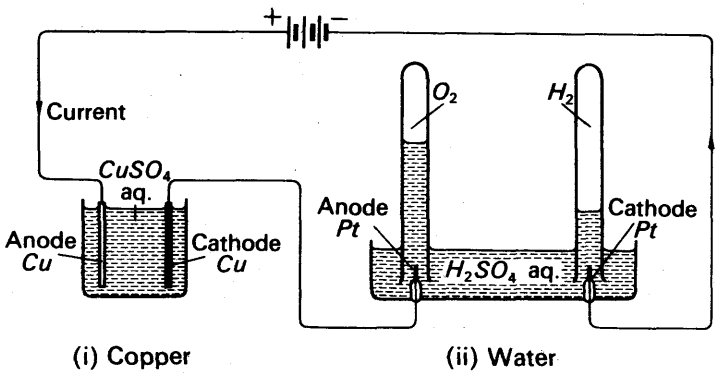
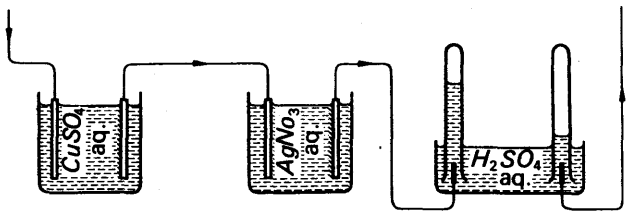


Fig. 34.1. Voltameters.

The whole arrangement is called a *voltameter*, presumably because it can be used to measure the current delivered by a voltaic cell; if the electrolyte is a solution of a copper or silver salt, the voltameter is called a copper or silver voltameter. If the electrolyte is acidulated water, then the voltameter is called a water voltameter, because when a current passes through it, the water, not the acid, is decomposed (Fig. 34.2 (ii)). We shall see why later.

Faraday's Laws of Electrolysis

When a current is passed through copper sulphate solution with copper electrodes, copper is deposited on the cathode and lost from the anode. Faraday showed that the mass dissolved off the anode by a given current in a given time is equal to the mass deposited on the cathode. He also showed that the mass is proportional to the product of the current, and the time for which it flows: that is to say, to the quantity of charge which passes through the voltameter. When he studied the electrolysis of water, he found that the masses of hydrogen and oxygen, though not equal, were each proportional to the quantity of charge that flowed. He therefore put forward his first law of electrolysis: *the mass of any substance liberated in electrolysis is proportional to the quantity of electric charge that liberated it.*



Element liberated	*Cu*	*Ag*	O_2	H_2
Relative mass	31·8	107·9	8·00	1·008
Chemical equivalent	31·8	107·9	8·00	1·008

FIG. 34.2. Voltameters in series (same quantity of charge passes through each).

Faraday's second law of electrolysis concerns the masses of different substances liberated by the same quantity of charge. An experiment to illustrate it is in Fig. 34.2. The experiment shows that *the masses of different substances, liberated in electrolysis by the same quantity of electric charge, are proportional to the ratio of the relative atomic mass to the valency* (also called the *chemical equivalent*). This is Faraday's second law; it implies that the same quantity of charge is required to liberate one mole divided by the valency (also called the 'gramme-equivalent') of any substance. Recent measurements give this quantity as 96500 coulombs; it is called Faraday's constant, symbol F. It is also known as the *faraday*.

Electrochemical Equivalent

The mass of a substance which is liberated by one coulomb is called its *electrochemical equivalent.* It is expressed in kilogrammes per coulomb (kg C^{-1}) in SI units. If z is the electrochemical equivalent of a substance, the mass of it in kilogrammes liberated by I amperes in t seconds is

$$m = zIt. \quad . \quad . \quad . \quad . \quad (1)$$

Since the chemical equivalent of hydrogen is 1·008 then 1·008 g of hydrogen is liberated by 96500 coulombs, and the electrochemical equivalent of hydrogen is

$$z_H = \frac{1{\cdot}008}{06{,}500} = 10{\cdot}5 \times 10^{-5} \text{ g C}^{-1} = 1{\cdot}05 \times 10^{-8} \text{ kg C}^{-1}$$

And similarly, since the chemical equivalent of copper is 31·8, its electrochemical equivalent is

$$z_{Cu} = \frac{31{\cdot}8}{96{,}500} = 3{\cdot}29 \times 10^{-4} \text{ g C}^{-1} = 3{\cdot}29 \times 10^{-7} \text{ kg C}^{-1}$$

Measurement of Current by Electrolysis

In the past, the chemical effect of the current was used to define the ampere, because measurements with the current balance (p. 940) could not be made as accurately as simple weighings. Nowadays they can, and the ampere is defined as on p. 939. In those days the ampere was defined as the current which, when flowing steadily, would deposit 0·001118 g of silver per second.

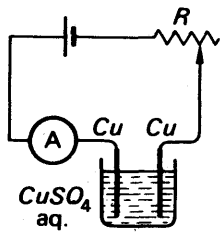


FIG. 34.3.
Current measurement by chemical effect.

In the absence of a standard ammeter, the chemical effect can be used to find the error in a particular reading on an ammeter A (Fig. 34.3). A copper voltameter is connected in series with the ammeter, and a steady current passed for a known time t. The current is kept constant by adjusting the rheostat R to keep the deflection constant. The cathode is weighed before and after the experiment. Its increase in mass, m, gives the current I, in terms of the electrochemical equivalent of copper, z:

$$m = zIt,$$

$$I = \frac{m}{zt}. \quad . \quad . \quad . \quad . \quad (2)$$

The error in the ammeter is then the difference in the reading on A and the current calculated from (2).

Great care must be taken in this experiment over the cleanliness of the electrodes. They must be cleaned with sandpaper at the start; and, at the finish, the cathode must be rinsed with water and dried with alcohol, or over a gentle spirit flame: strong heating will oxidize the copper deposit.

The Mechanism of Conduction; Ions

The theory of electrolytic conduction is generally attributed to Arrhenius (1859–1927), although Faraday had stated some of its essentials in 1834. Faraday suggested that the current through an electrolyte was carried by charged particles, which he called ions (Greek

ion = go). A solution of silver nitrate, he supposed, contained silver ions and 'nitrate' ions. The silver ions were silver atoms with a positive charge; they were positive because silver was deposited at the cathode, or negative electrode (Fig. 34.4). The nitrate ions were groups of

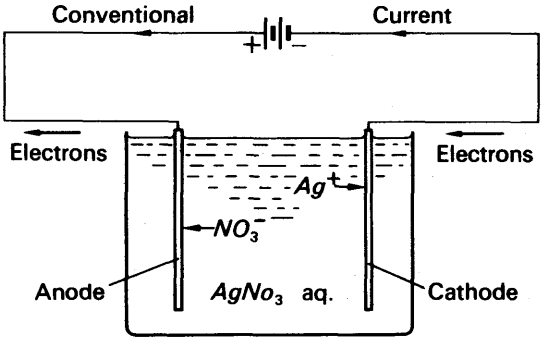


FIG. 34.4. Ions in electrolysis.

atoms—NO_3 groups—with a negative charge; they travelled towards the anode, or positive electrode, and, when silver electrodes are used, formed silver nitrate.

Nowadays, we consider that a silver ion is a silver atom which has lost an electron; this electron transfers itself to the NO_3 group when the silver nitrate molecule is formed, and gives the nitrate ion its negative charge. We denote nitrate and silver ions, respectively, by the symbols NO_3^- and Ag^+. When the ions appear at the electrodes of a voltameter they are discharged. The current in the external circuit brings electrons to the cathode, and takes them away from the anode (Fig. 34.4). At the anode silver atoms lose electrons and go into solution as positive ions. *In effect*, the negative charges carried across the cell by the NO_3^- ions flow away through the external circuit. At the cathode, each silver ions gains an electron, and becomes a silver atom, which is deposited upon the electrode.

Ionization

The splitting up of a compound into ions in solution is called ionization, or ionic dissociation. Faraday does not seem to have paid much attention to how it took place, and the theory of it was given by Arrhenius in 1887. For a reason which we will consider later, Arrhenius suggested that an electrolyte ionized as soon as it was dissolved: that its ions were not produced by the current through it, but were present as such in the solution, before ever the current was passed.

We now consider that salts of strong bases and acids, such as silver nitrate, copper sulphate, sodium chloride, ionize completely as soon as they are dissolved in water. That is to say, a solution contains no molecules of these salts, but only their ions. Such salts are called strong electrolytes; so are the acids and bases from which they are formed, for these also ionize completely when dissolved in water.

Other salts, such as sodium carbonate, do not appear to ionize completely on solution in water. They are the salts of weak acids, and are called weak electrolytes. The weak acids themselves are also incompletely ionized in water.

Formation of Ions; Mechanism of Ionization

In the Heat section of this book we described the structure of the solid state (p. 294). In solid crystalline salts such as sodium chloride the structure is made up of sodium and chlorine ions: not of atoms, nor of NaCl molecules, but of Na^+ and Cl^- ions. In other words, we think today that ions exist in solid crystalline salts, as well as in their solutions. We do so for the reason that the idea enables us to build up a consistent theory of chemical combination, of the solid state, and of electrolytic dissociation.

A sodium atom contains eleven electrons, ten of which move in orbits close to the nucleus, and one of which ranges much more widely; for our present purposes we may represent it as in Fig. 34.5 (i). A chlorine atom has ten inner electrons and seven outer ones; for our present purposes we may lump these into two groups, as in Fig. 34.5 (ii). The outer electron of the sodium atom is weakly attracted to its nucleus, but the outer electrons of the chlorine atom are strongly attracted (because the ten inner electrons are a more effective shield round the +11 nucleus of sodium than round the +17 nucleus of chlorine). Therefore, when a sodium and a chlorine atom approach one another, the outer electron of the sodium atom is attracted more strongly by the chlorine nucleus than by the sodium nucleus. It leaves the sodium atom, and joins the outer electrons of

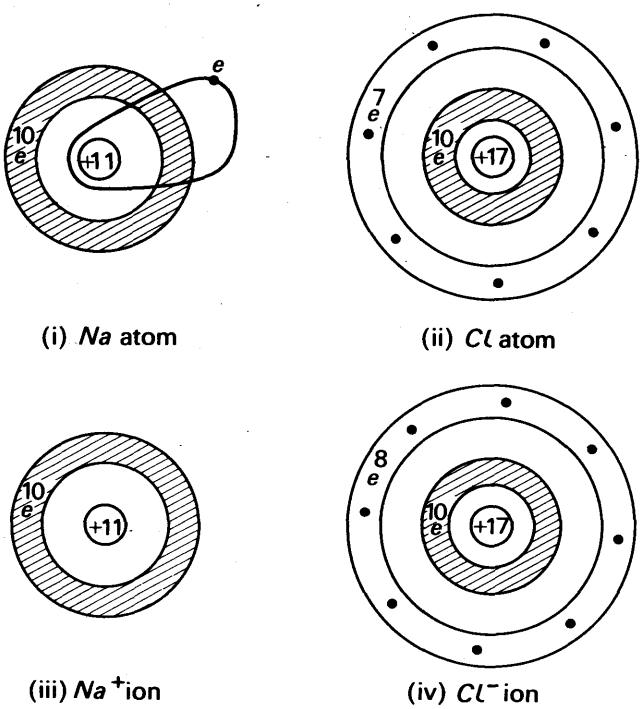


FIG. 34.5. Sodium and chlorine, atoms and ions.

the chlorine; the sodium atom becomes a positively charged sodium ion. Fig. 34.5 (iii), and the chlorine atom a negatively charged chlorine ion, Fig. 34.5 (iv). Between these two ions there now appears a strong electrostatic attraction, which holds them together as a molecule of NaCl. In the solid state, the ions are arranged alternately positive and negative; the forces between them bind the whole into a rigid crystal.

When such a crystal is dropped into water, it dissolves and ionizes. We can readily understand this when we remember that water has a very high dielectric constant: 81 (p. 774). It therefore reduces the forces between the ions 81 times, and the crystal falls apart into ions. In the same way we explain the ionization of other salts, and bases and acids. The idea that these dissociate because they are held together by electrostatic forces, which the solvent weakens, is supported by the fact that they ionize in some other solvents as well as water. These solvents also are liquids which have a high dielectric constant, such as methyl and ethyl alcohols (32 and 26 respectively). In these liquids, however, it seems that strong electrolytes behave as weak ones do in water: only a fraction of the dissolved molecules, not all of them, dissociate. In the electronic theory of atomic structure, the chemical behaviour of an element is determined by the number of its outer electrons. If it can readily lose one or two it is metallic, and forms positive ions; if it can readily gain one or two, it is acidic, and forms negative ions. Ions are not chemically active in the way that atoms are. Sodium atoms, in the form of a lump of the metal, react violently with water; but the hydroxide which they form ionizes ($NaOH \rightarrow Na^+ + OH^-$), and the sodium ions drift peaceably about in the solution—which is still mainly water.

Pure water is a feeble conductor of electricity, and we consider that it is but feebly ionized into H^+ and OH^- ions. These, we believe, are continually joining up to form water molecules, and then dissociating again in a dynamical equilibrium:

$$H_2O \rightleftharpoons H^+ + OH^-.$$

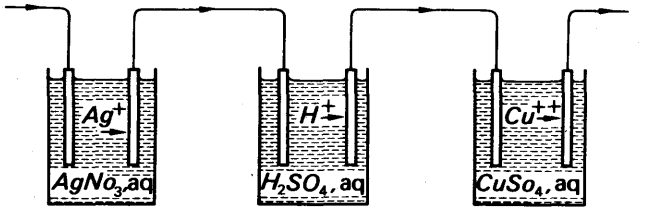
If, as we shall find in the electrolysis of water, H^+ and OH^- ions are removed from water, then more molecules dissociate, to restore the equilibrium.

The concentrations of H^+ and OH^- in water are so small that they do not contribute appreciably to the conduction of electricity when an electrolyte is dissolved in the water; but, as we shall see, they sometimes take part in reactions at the electrodes.

Explanation of Faraday's Laws

The theory of dissociation neatly explains Faraday's laws and some other phenomena of electrolysis. If an $AgNO_3$ molecule splits up into Ag^+ and NO_3^- ions, then each NO_3^- ion that reaches the anode dissolves one silver atom off it. At the same time, one silver atom is deposited on the cathode. Thus the gain in mass of the cathode is equal to the loss in mass of the anode. Also the total mass of silver nitrate in solution is unchanged; experiment shows that this is true. The mass of silver deposited is proportional to the number of ions reaching the cathode; if all the ions carry the same charge—a reasonable assumption—then the number deposited is proportional to the quantity of charge which deposits them. This is Faraday's first law.

To see how the ionic theory explains Faraday's second law, let us again consider a number of voltameters in series (Fig. 34.6). When a



Relative mass liberated	107·9	1·008	31·8
Atomic weight	107·9	1·008	63·6

FIG. 34.6. Illustrating Faraday's second law.

current flows through them all, the same quantity of charge passes through each in a given time. Experiment shows that

$$\frac{\text{mass of silver deposited}}{\text{mass of hydrogen liberated}} = 107{\cdot}0.$$

From experiments on chemical combination, we know that

$$\frac{\text{mass of silver atom}}{\text{mass of hydrogen atom}} = \frac{107{\cdot}9}{1{\cdot}008} = 107{\cdot}0.$$

Therefore we may say that, each time a silver ion is discharged and deposited as an atom, a hydrogen ion is also discharged and becomes an atom. The hydrogen atoms thus formed join up in pairs, and escape as molecules of hydrogen gas. The theory fits the facts, on the simple assumption that the hydrogen and silver atoms carry equal charges: we now say that each is an atom which has lost one electron.

But when we consider the copper voltameter in Fig. 34.6, we find a complication. For

$$\frac{\text{mass of copper deposited}}{\text{mass of hydrogen liberated}} = \frac{31{\cdot}8}{1{\cdot}008},$$

whereas

$$\frac{\text{mass of copper atom}}{\text{mass of hydrogen atom}} = \frac{63{\cdot}6}{1{\cdot}008}.$$

To explain this result we must suppose that only one copper atom is deposited for every two hydrogen atoms liberated. In terms of the ionic theory, therefore, only one copper ion is discharged for every two hydrogen ions. It follows that a copper ion must have twice as great a charge as a hydrogen ion: it must be an atom which has lost two electrons.

This conclusion fits in with our knowledge of the chemistry of copper. One atom of copper can replace two of hydrogen, as it does, for example, in the formation of copper sulphate, $CuSO_4$, from sulphuric acid, H_2SO_4. We therefore suppose that the sulphate ion also is doubly charged: SO_4^{2-}. When sulphuric acid is formed, two hydrogen atoms

each lose an electron, and the SO_4 group gains two. When copper sulphate is formed, each copper atom gives up two electrons to an SO_4 group. And when copper sulphate ionizes, each molecule splits into two doubly charged ions:

$$CuSO_4 \rightarrow Cu^{2+} + SO_4^{2-}. \quad . \quad . \quad . \quad (3)$$

In general, if we express the charge on an ion in units of the electronic charge, we find that it is equal to the valency of the atom from which the ion was formed. That is to say, it is equal to the number of hydrogen atoms which the atom can combine with or replace.

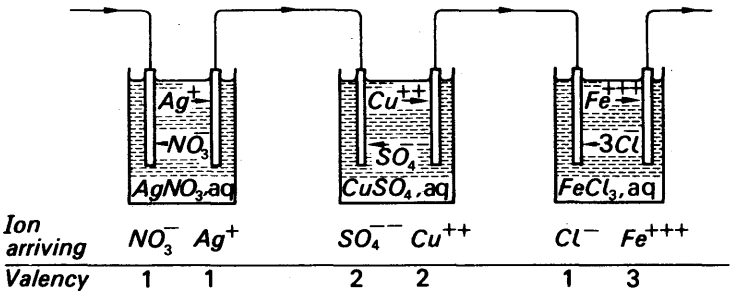


FIG. 34.7. Movement of ions in electrolysis.

This assertion, which is illustrated in Fig. 34.7, explains Faraday's second law (mass deposited $\propto$ chemical equivalent). For if a current I passes through a voltameter for a time t, the total charge carried through it is It. And if q is the charge on an ion, the number of ions liberated is It/q. If M is the mass of an ion, the mass liberated is $M(It/q)$, and is therefore proportional to M/q. But M is virtually equal to the relative atomic mass, since the mass of an electron is negligible. And q, we have just seen, is equal, in electronic units, to the valency. Therefore the mass liberated is proportional to the ratio of relative atomic mass to valency. See also p. 845.

Electrolysis of Copper Sulphate Solution

If copper sulphate solution is electrolysed with platinum or carbon electrodes, copper is deposited on the cathode, but the anode is not dissolved away: instead, oxygen is evolved from it (Fig. 34.8). The SO_4^{2-} ions which approach the anode do not attack it; neither carbon nor platinum forms a sulphate, and each is said to be insoluble, in the electrolysis of copper sulphate. As the electrolysis proceeds, the solution becomes paler in colour; chemical tests show that it is gradually losing copper sulphate, but gaining sulphuric acid—Cu^{2+} ions are disappearing from the solution, but the SO_4^{2-} are remaining in it.

The oxygen which is evolved comes from the water of the solution. We have already seen that water is always slightly ionized, into H^+ and OH^- ions. When copper sulphate is electrolyzed with platinum or

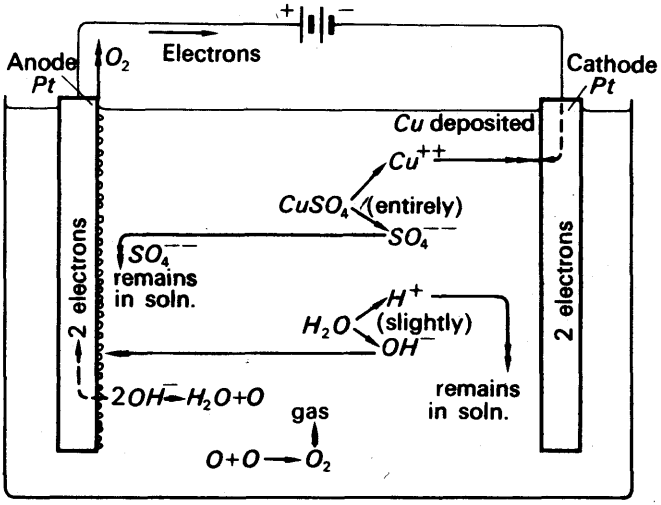


FIG. 34.8. Electrolysis of copper sulphate with insoluble electrodes.

carbon electrodes, the OH^- ions of the water are discharged at the anode. Each gives up an electron; they then combine in pairs to give a water molecule and an oxygen atom:

$$2OH^- - 2\,\text{electrons} \rightarrow H_2O + O.$$

The oxygen atoms combine in pairs and come off as molecules:

$$2O \rightarrow O_2\uparrow$$

As the OH^- ions disappear, an excess of H^+ ions appear in the solution. If the electrolysis were carried to the point where all the copper originally in the solution was deposited on the cathode, the solution would become simply one of sulphuric acid. This would be ionized into H^+ and SO_4^{2-} ions, in the proportion two H^+ to one SO_4^{2-}.

Electrolysis of Water

When a current is passed through water acidulated with dilute sulphuric acid, and platinum electrodes are used, oxygen and hydrogen are produced at the anode and cathode respectively. The amount of acid in solution remains unaltered, and the net effect is thus the electrolysis of water. The sulphuric acid ionizes into hydrogen and sulphate ions (Fig. 34.9):

$$H_2SO_4 \rightarrow 2H^+ + SO_4^{2-}.$$

The hydrogen ions from the acid greatly outnumber those from the water, but we cannot distinguish between them. All we can say is that, for every SO_4^{2-} ion that approaches the anode, two H^+ ions approach the cathode. At the cathode, the H^+ ions collect electrons, join up in pairs, and come off as molecules of hydrogen gas, H_2. At the anode,

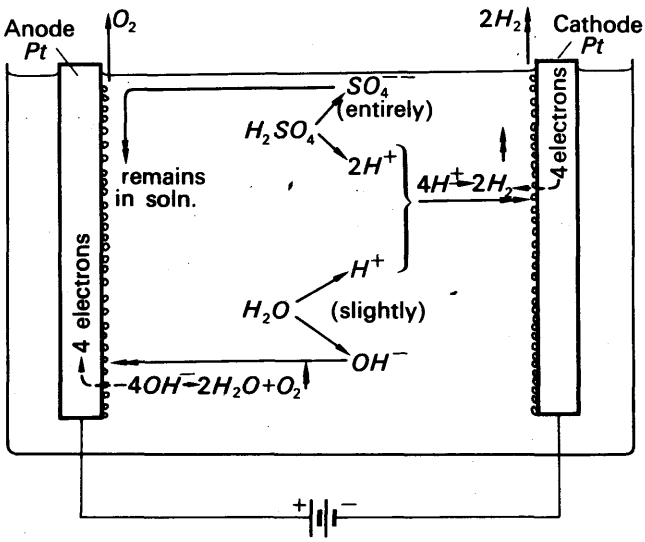


FIG. 34.9. Electrolysis of water.

however, the SO_4^{2-} ions remain in solution, and OH^- ions are discharged; as before, they form water and oxygen molecules, and the oxygen comes off as gas.

To produce one molecule of oxygen, four OH^- ions must be discharged:

$$4OH^- \rightarrow 2H_2O + O_2\uparrow + 4 \text{ electrons to circuit.} \quad . \quad (4)$$

And every time four OH^- ions are discharged at the anode, four H^+ ions are discharged at the cathode. (If this were not so, a net positive or negative charge would accumulate in the solution, and we could draw sparks from it.) Thus the reaction is accompanied by

$$4H^+ + 4 \text{ electrons} \rightarrow 2H_2\uparrow \quad . \quad . \quad . \quad (5)$$

Equations (4) and (5) agree with the experimental fact that hydrogen and oxygen come off in the proportions in which they are found in water: 2 to 1 by volume. This can be shown by collecting the gases in tubes filled with electrolyte and inverted over the electrodes (Fig. 34.1 (ii)). The electrolysis decomposes the water only, and leaves the acid unchanged. Equal numbers of H^+ and OH^- ions are discharged, and in the solution H^+ ions remain, in the proportion of two H^+ to one SO_4^{2-}.

The only function of the sulphuric acid is to increase the concentration of ions in the solution, and so to enable it to carry a greater current with a given potential difference than would pure water. The greater current discharges H^+ ions at a greater rate, and so causes the water to dissociate faster into H^+ and OH^-. Thus more ions are formed to carry the current.

The Electrolytic Capacitor

An electrolytic capacitor is one in which the dielectric is formed by electrolysis—by a secondary reaction at an insoluble electrode. It is made from two coaxial aluminium tubes, A and K in Fig. 34.10, with a solution or paste of ammonium borate between them. A current is passed through from A (anode) to K (cathode) and a secondary reaction at A liberates oxygen. The oxygen does not come off as a gas, however, but combines with the aluminium to form a layer of aluminium oxide over the electrode A. This layer is about 1/400 cm thick, and is an insulator. When the layer has been formed, the whole system can be used as a capacitor, one of whose electrodes is the cylinder A, and the other the surface of the liquid or paste adjacent to A. Because the dielectric layer is so thin, the capacitance is much greater than that of a paper capacitor of the same size.

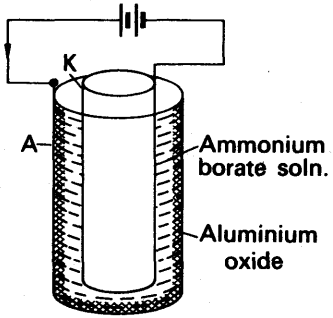


FIG. 34.10. Forming an electrolytic capacitor.

In the use of an electrolytic capacitor, some precautions must be taken. The voltage applied to it must not exceed a value determined by the thickness of the dielectric, and marked on the condenser; otherwise the layer of aluminium oxide will break down (see 'dielectric strength', p. 772). And the voltage must always be applied in the same sense as when the layer was being formed. If the plate A is made negative with respect to K, the oxide layer is rapidly dissolved away. Consequently an alternating voltage must never be applied to an electrolytic capacitor. This condition limits the usefulness of these capacitors.

Electrolytic capacitors are not very reliable, because the oxide layer is apt to break down with age. Domestic radio receivers abound in them, but in high-grade apparatus they are avoided.

Application of Ohm's Law to Electrolytes

Fig. 34.11 (i) shows how the current through an electrolyte, and the potential difference across it, may be measured. If the electrodes are soluble—copper in copper sulphate, for example—then the current is proportional to the potential difference (Fig. 34.11 (ii)). The best results in this experiment are obtained with very small currents. If the current is large, the solution becomes non-uniform: it becomes stronger near the anode, where copper is dissolved by the attack of the SO_4^{2-} ions, and weaker near the cathode, where copper is deposited, and SO_4^{2-} ions drift away. Near the cathode the solution becomes paler in colour; near the anode it becomes deeper. The total amount of copper sulphate in solution remains constant, but is gradually transferred to the neighbourhood of the anode. As the solution round the cathode becomes weaker, its resistance increases, and more than offsets the decreasing

resistance of the solution round the anode; with a given potential difference, therefore, the current gradually falls. A small current makes

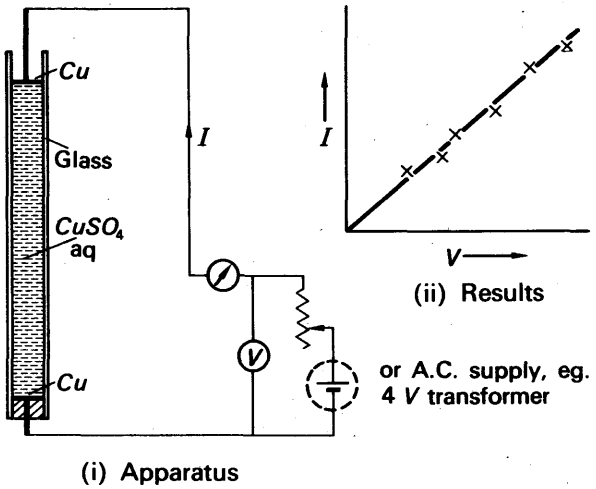


FIG. 34.11. Current/Voltage characteristic of electrolyte with soluble electrodes.

this effect unimportant, because it makes the electrolysis very slow. But the best way to do the experiment is to use an alternating current; the electrolysis reverses with the current fifty or more times per second, and no changes of concentration build up. Rectifier-type meters (p. 1011) are most suitable for measuring the current and potential difference in this case.

When the measurements are properly made they show, as we have said, that the current is proportional to the potential difference: for example, copper sulphate solution, with copper electrodes, obeys Ohm's law. The voltameter behaves as a passive resistor; all the electrical energy delivered to it by the current appears as heat (pp. 791–2); no electrical energy is converted into mechanical or chemical work. In particular, therefore, no electrical energy is used to break up the molecules of copper sulphate into ions. This is the argument which led Arrhenius to suggest that the electrolyte dissociates into ions as soon as it is dissolved; dissociation is a result of solution, not of electrolysis.

Measurement of Resistance of Electrolyte

The resistance of an electrolyte can be measured on a Wheatstone bridge, preferably with an alternating current supply (Fig. 34.12). A telephone earpiece T is used as the detector, in place of the galvanometer—it gives minimum sound at the balance-point. 50 Hz mains

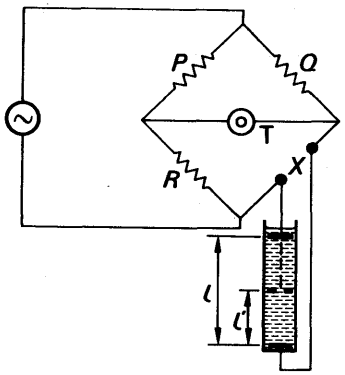


FIG. 34.12. Measurement of electrolyte resistance.

give an uncomfortably low-pitched note for listening, and a supply of frequency 400 to 1000 Hz is more satisfactory. This may be a small induction coil of a type sold for the purpose, which has fewer secondary turns than the common spark-coil, because a high voltage is not required. By sliding the upper electrode of the cell, the resistances of two lengths of electrolyte, l and l' are measured; their difference, r, is the resistance of a length $l-l'$, and is free from end-errors. If A is the cross-section of the tube, then the specific resistance of the electrolyte, ρ, is given by

$$r = \frac{(l-l')\rho}{A}.$$

It is usual, however, not to give the resistivity of an electrolyte, but to give instead its conductivity, σ. This is defined as the reciprocal of its resistivity, and is expressed in mho per centimetre:

$$\sigma = \frac{1}{\rho} = \frac{l-l'}{Ar}.$$

Electrical Behaviour with Insoluble Electrodes

If we set out to find the current/voltage relationship for a water voltameter, using a d.c. supply, we find that the voltameter does not obey Ohm's law. If we apply to it a voltage E less than 1·7 volts, the current flows only for a short time and then stops, as though the voltameter were charged like a capacitor. To study the matter further, we may arrange a two-way key and a second galvanometer, G, as in Fig. 34.13. We first press the key at Y, and pass a current through the voltameter until it stops; then we press the key at X, and connect the galvanometer G straight across the voltameter. A brief current I flows through G, whose direction shows that the anode of the voltameter is acting as the positive pole of a current supply. It appears, therefore, that the voltameter is setting up a back-e.m.f. which prevents a steady current from flowing, unless the supply voltage is greater than 1·7 volts.

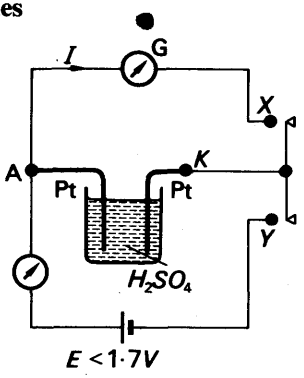
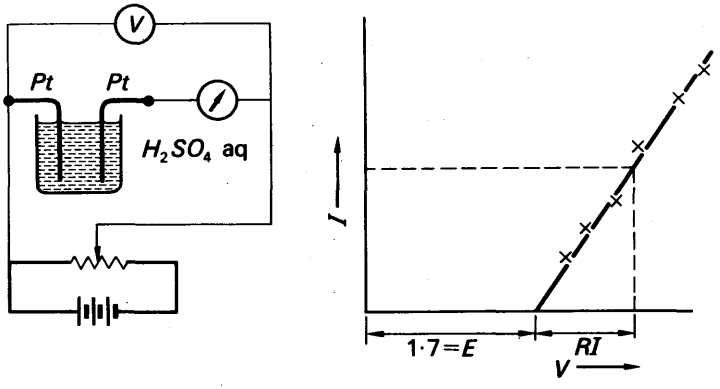


FIG. 34.13. Demonstrating back-e.m.f. of water voltameter.

To investigate this behaviour further, let us gradually increase the potential difference across the voltameter, as in Fig. 34.14 (i). We then



(i) Apparatus (ii) Results

FIG. 34.14. Current/Voltage characteristic of water voltameter.

find that, if we work with small currents, the current increases linearly with the potential difference V, when the latter is greater than 1·7 volts (Fig. 34.14 (ii)). It follows that the voltameter does indeed exert a back-e.m.f., E, equal to 1·7 volts. If V is the potential difference applied to it, then the current through it obeys the relationship

$$I \propto (V-E).$$

We may write this as

$$I = \frac{V-E}{R},$$

where R is the resistance of the electrolyte.

Electrical Energy Consumed in Decomposition

We have not yet explained the origin of the back-e.m.f. E, but we shall try to do so later. Meanwhile let us notice that the behaviour of the voltameter is somewhat like that of an electric motor. When the armature of a motor rotates, a back-e.m.f. is induced in it, and the current through it is given by an equation similar to that above. The back-e.m.f. in the motor, we shall see, represents the electrical power converted into mechanical work. So here the back-e.m.f. E represents this electrical power converted into chemical work—used in breaking up the water molecules. The potential difference across the voltameter is

$$V = IR + E$$

from the equation for I; the power equation is therefore

$$IV = I^2R + EI.$$

The left-hand term is the electrical power input; the first term on the right is the heat produced per second in the electrolyte; and the second term is the work done per second in decomposing the water.

Chemists tell us that when oxygen and hydrogen combine to form one mole of water (18 g) then 286 000 joules of heat are evolved. The energy set free is therefore 286 000 joules.

When one mole of water is decomposed, this much work must be done. In the process two moles of hydrogen are liberated (because the formula for water is H_2O). The quantity of electricity required to decompose one mole of water is therefore 2×96500 coulombs. If the back-e.m.f. is E, the corresponding amount of energy is $2 \times 96500 \times E$ joules. Therefore

$$2 \times 96500 \times E = 286\,000$$

whence

$$E = \frac{286\,000}{2 \times 96500}$$

$$= 1{\cdot}48 \text{ volts.}$$

The lowest value of E which anyone has ever got by experiment is 1·67 volts—from which it appears that we have something yet to learn about what happens in a water voltameter.

CELLS

If we put plates of copper and zinc into a beaker of dilute sulphuric acid, we have a voltaic cell (p. 785). It is often called a simple cell. If we join its plates via a galvanometer, current flows through the galvanometer from the copper to the zinc (Fig. 34.15); the cell sets up an e.m.f. which acts, in the external circuit, from copper to zinc. Its value is about one volt. The copper plate is at a higher potential than the zinc plate, and is the positive terminal of the cell; the zinc is the negative terminal. Within the cell there must be some agency which

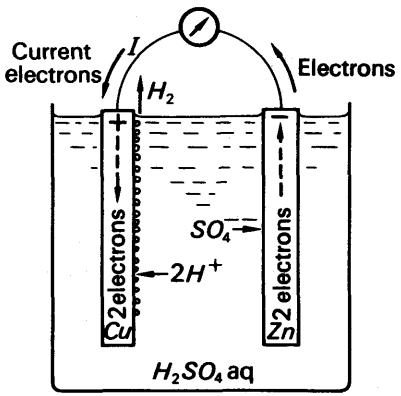


FIG. 34.15. A simple cell.

carries the current from the zinc to the copper. This is the agency which gives rise to the e.m.f. of the cell; it is analogous to the force exerted by the magnetic field on the moving electrons in the armature winding of a dynamo.

In a voltaic cell, the agency which gives rise to the e.m.f. is not so easy to track down as in a dynamo; there has been much argument about what is called the 'seat' of the e.m.f. We may start to seek it by placing a penknife blade into a strong solution of copper sulphate: a pink film of metallic copper is deposited on the blade. It appears, then, that copper ions have a tendency to go out of solution on to iron. In the same way we can show that they tend to go out on to zinc.

Do metal ions ever tend to go the other way—from solid metal into solution? They certainly do if the solution is sulphuric acid and the metal zinc or iron: the metal enters the solution in the form of ions, displaces the hydrogen ions, which are discharged and come off as gas, and turns the solution into one of iron or zinc sulphate. But this happens only if the zinc or iron is impure. Pure zinc in sulphuric acid gives no action at all—no zinc sulphate, no hydrogen.

Action in a Simple Cell

If we want to make pure zinc react with sulphuric acid, we must make it into part of a voltaic cell: we must connect it to a plate of a different metal, such as copper which also dips in the acid. Then the zinc is eaten away, and hydrogen bubbles off; but the hydrogen appears at the copper plate, and not at the zinc (Fig. 34.15). At the same time the solution becomes one of zinc sulphate—which simply means that it contains zinc ions in place of hydrogen ones. We can now form a picture of what happens when a stick of pure zinc is put alone into dilute sulphuric acid (Fig. 34.16). At first zinc ions leave the metal, and go into the liquid. But they leave negative charges on the zinc rod, which attract the zinc ions, and prevent any more from leaving. Nothing further happens. But if we introduce a plate of copper, and connect it to the zinc, electrons can flow from the zinc to the copper (Fig. 34.15). At the copper they can neutralize the charges on hydrogen ions, and to enable molecules to form and come off in bubbles. As the electrons flow away from the zinc, more zinc ions can go into solution, and so the zinc can continuously dissolve in the acid. In doing so, it maintains a continuous electric current in the wire connecting it to the copper. (When the zinc is impure each speck of impurity acts as the other plate of a minute cell, and enables the zinc around it to react with the acid. This 'local action', as it is called, makes impure zinc undesirable in voltaic cells, for it consumes the zinc without giving any useful current. It can be prevented by rubbing the zinc with mercury, which dissolves it and presents a surface of pure zinc to the acid. The process is called amalagamating the zinc.)

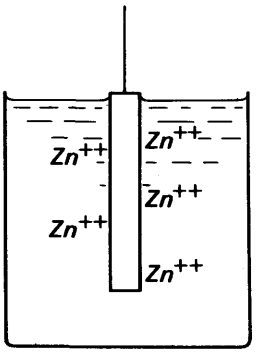


FIG. 34.16.
Pure zinc in dilute sulphuric acid.

To explain the voltaic cell, therefore, we must suppose that zinc ions tend to dissolve

from zinc into sulphuric acid, but copper ions do not. This is consistent with the fact that copper does not react chemically with cold dilute sulphuric acid. The passage of ions from metals to solutions, and oppositely, was studied by Nernst about 1889; we shall give a slight account of his theory later.

Daniell's Cell

Fig 34.17 shows a cell, developed by Daniell about 1850, which has some advantages over Volta's. Daniell's cell consists of a zinc rod, Zn, in a porous pot, P, containing sulphuric acid; this in turn stands in a strong solution of copper sulphate in a copper vessel, Cu. (Sometimes the copper is just a thin sheet in a glass vessel.) When the copper and

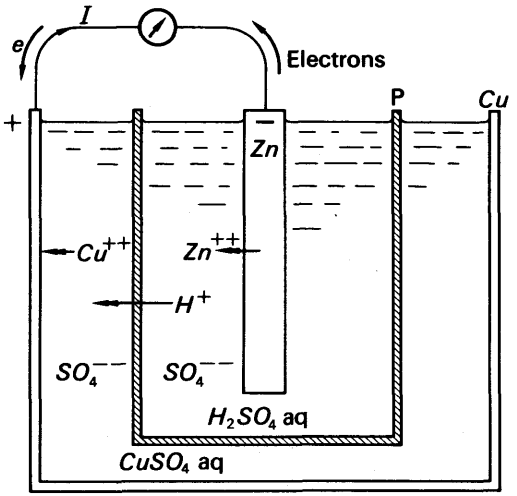


FIG. 34.17. A Daniell cell.

zinc are connected by a wire, current flows through the wire from copper to zinc. The copper is therefore the positive terminal of the cell, and the zinc the negative. As in Volta's cell, zinc ions go into solution at the zinc rod, leaving electrons on it. But at the copper plate, copper ions go out of solution—a metallic film of copper is deposited on the vessel. When the zinc and copper are joined by a wire, electrons from the zinc can go along it to the copper vessel, and discharge the copper ions as they reach it. To complete the action of the cell hydrogen ions from the sulphuric acid pass through the porous pot into the copper sulphate solution (Fig. 34.17). Thus zinc is dissolved, the acid gradually changes to zinc sulphate, the copper sulphate gradually changes to sulphuric acid, and copper is deposited on the copper vessel.

The e.m.f. of a Daniell cell is about 1·08 volts. Its internal resistance depends on its size and condition—the size is usually about that of a plant-pot, and the internal resistance is of the order of several ohms.

Polarization

The great disadvantage of the simple cell is that it does not give a steady current; from the moment of making the circuit, the current starts to fall, and after a minute or two it almost ceases to flow. The current decays because a layer of hydrogen gas forms over the copper plate; scraping the plate enables the current to start once more, but it soon decays again. The hydrogen layer increases the internal resistance of the cell, but we do not believe that this is the main reason for the decay of the current. If the copper plate is replaced by one of platinum black (platinum with a finely grained surface), bubbles form on it very easily, and escape readily. The hydrogen layer may then be no more than one molecule thick; but the current decays as before.

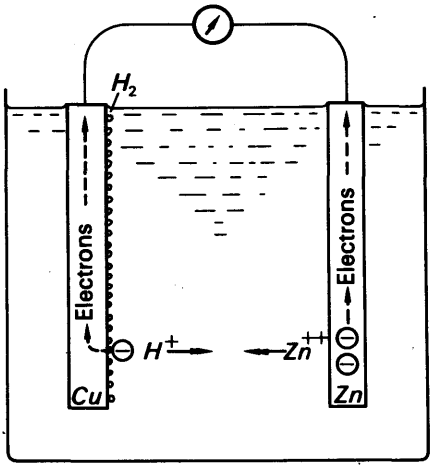


FIG. 34.18. Polarization in simple cell.

To explain this decay of the current we think that the hydrogen layer replaces the copper *as an electrode of the cell.* We suppose that the hydrogen tends to go back into solution as positive hydrogen ions (Fig. 34.18). In other words, it tends to behave in the same way as the zinc rod, on the other side of the cell, which also goes into solution as positive ions. Thus the hydrogen sets up an e.m.f. which opposes the original e.m.f. of the cell, and cuts down the current; the hydrogen thus sets up a 'back-e.m.f.' in the circuit. This behaviour is called polarization of the cell.

Depolarization

The advantage of Daniell's cell over Volta's is that it does not polarize. Hydrogen ions drift from the acid compartment into the copper sulphate compartment, but they are never discharged, no hydrogen molecules are formed, and no layer of hydrogen appears on the copper electrode. The copper sulphate solution is often called the depolarizer, because it prevents the formation of hydrogen gas.

Polarization in Water Voltameter

We can find support for the idea of polarization in the behaviour of the water voltameter (p. 856). When the potential difference across the voltameter is less than 1·7 volts, the current through it falls to zero in a minute or less. The voltameter itself can then deliver a current for a short time. Its positive terminal, as a source, is that which was its anode, and its negative terminal is that which was its cathode. While current was being sent through the voltameter, the cathode became covered with hydrogen, and the anode with oxygen. When the voltameter acts as a source of current it has, in effect, electrodes of oxygen and hydrogen and the current through the external circuit flows from the oxygen plate to the hydrogen (Fig. 34.19). In the simple cell, when it is polarized, there is no oxygen plate, but there is a hydrogen one, and this lies over the copper plate. We may therefore suppose

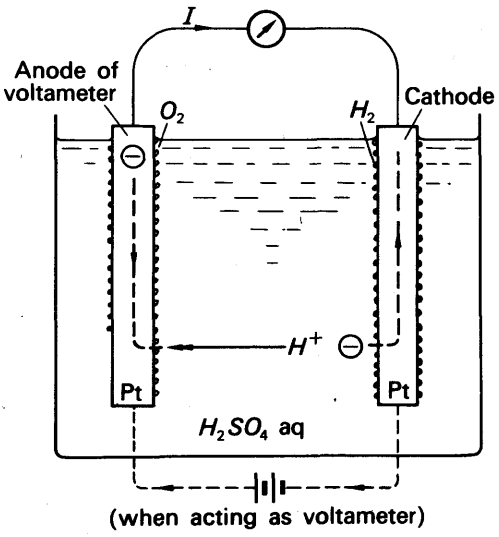


FIG. 34.19. Polarization of water voltameter.

that it tends to drive a current, through the external circuit, from zinc to copper; that is to say, it sets up an e.m.f. opposing that of the cell with the copper plate clean.

Nernst's Theory of the Voltaic cell; Electrode Potentials

If a metal is in contact with a solution of one of its own salts, it is surrounded by its own ions. Whether the ions deposit themselves on the metal, or the metal goes into solution, depends partly on the particular metal concerned, and partly on the strength of the solution: the stronger the solution the greater its tendency to deposit ions on the metal. If the solution deposits ions, the metal comes to a positive potential with respect to it; if the metal goes into solution as ions, it

becomes negative with respect to the solution (Fig. 34.20 (i)). By methods beyond the scope of this book, the potential difference between a metal and a solution can be measured. These show that copper in normal copper sulphate solution (1 g-equivalent weight per litre) becomes 0·08 volt positive with respect to the solution.

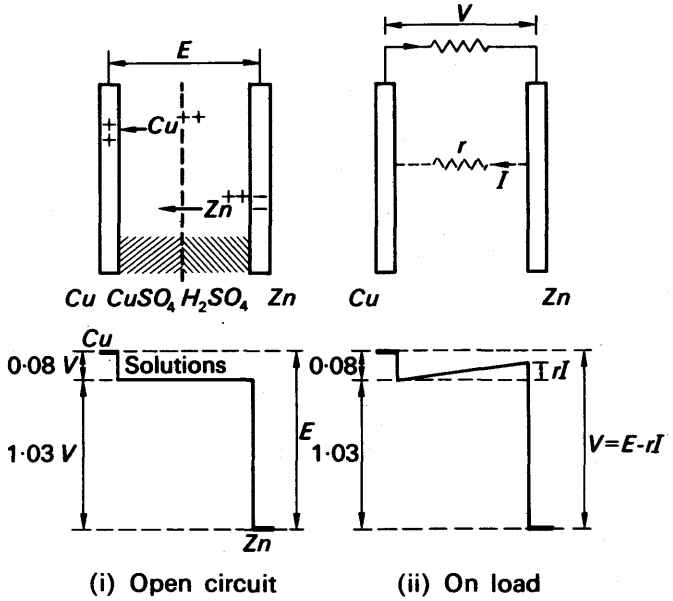


FIG. 34.20. Potential differences in voltaic cell.

Zinc in normal zinc sulphate solution becomes 1·03 volts negative. Now let us imagine a Daniell cell in which zinc sulphate replaces the sulphuric acid, and both solutions are normal. If we suppose that the solutions themselves set up no appreciable potential difference at their interface, then we get a potential distribution like that shown in Fig. 33.20 (i). The difference in potential between the copper and zinc is very nearly equal to the e.m.f. of a Daniell cell: 1·11 volts compared with 1·08. We may attribute the difference to the fact that a Daniell cell in practice has sulphuric acid, not zinc sulphate solution, in contact with the zinc; also the solutions are not normal: the acid is usually 1 to 4 of water, and the copper sulphate is saturated.

These considerations explain a striking experimental fact about all cells: the e.m.f. depends only on the nature and concentration of the constituent chemicals. The size of a cell affects only its internal resistance.

When a current I is drawn from a cell, there is a voltage drop across the internal resistance r, that is to say, across the solution or solutions. This modifies the potential diagram as shown in Fig. 34.20 (ii). The terminal voltage V, which is the observed potential difference between the copper and zinc, is now less than its open-circuit value, which is the e.m.f. of the cell.

The Leclanché Cell

Daniell's cell has the great practical disadvantage that it cannot be left set up; the solutions gradually mix by diffusion through the porous pot. It is now used only in teaching laboratories as a simple standard of e.m.f.; its e.m.f. is more nearly constant than that of any other

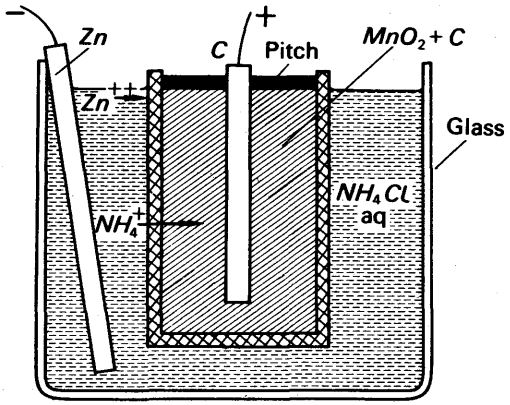


FIG. 34.21. Leclanché cell.

cheap and robust type of cell, and it is remarkably free from polarization.

A practically more useful cell is that devised by Leclanché. Its negative electrode is a zinc rod in a strong solution of ammonium chloride (Fig. 34.21). Its positive electrode is a carbon plate in a porous pot packed with manganese dioxide, which acts as the depolarizer. Manganese dioxide is a poor conductor of electricity, and powdered carbon is therefore packed in the pot with it.

The ammonium chloride ionizes into ammonium ions and chlorine ions:

$$NH_4Cl \rightarrow NH_4^+ \text{ and } Cl^-.$$

The zinc goes into solution, as zinc ions, and the ammonium ions drift through the porous pot towards the carbon plate. When a current is drawn from the cell, electrons flow from the zinc to the carbon, and discharge the NH_4^+ ions. The chemical action in this cell is complicated, but may be crudely represented as

$$2NH_4^+ + 2 \text{ electrons} \rightarrow 2NH_3 + H_2.$$

The hydrogen tends to polarize the cell, but is gradually oxidized by the manganese dioxide; again the action is complicated, but it reduces to

$$2MnO_2 + 2H_2O + H_2 \rightarrow 2Mn(OH)_3.$$

The depolarizing action is slow, and a Leclanché cell is therefore not suitable for giving a large current for a long time. A short rest, however, enables the manganese dioxide to remove the hydrogen and restore

the e.m.f. of the cell. Thus Leclanché cells are suitable for giving intermittent currents: they are widely used, for example, with electric bells. They are also suitable for Wheatstone bridges, because they cannot give a current large enough to burn out the resistance coils when a wrong connexion is made. The e.m.f. of a Leclanché cell, before polarization sets in, is about 1·5 volts, and its internal resistance about 1 ohm.

Dry Cell

Fig. 34.22 shows a dry form of Leclanché cell, which has the obvious advantage that it is portable. The ammonium chloride is made into a paste with water, zinc chloride, flour, and gum; and the porous pot is replaced by a muslin bag. A cardboard spacer prevents the bag from touching the zinc and short-circuiting the cell. A dry Leclanché cell has the same e.m.f. as a wet one, but, for a given size, a lower internal resistance, because the thickness of solution between the zinc and carbon is less. It depolarizes better, because the volume of manganese dioxide is greater in relation to the overall size of the cell: the cycle-lamp size will give a useful light continuously for two hours or more.

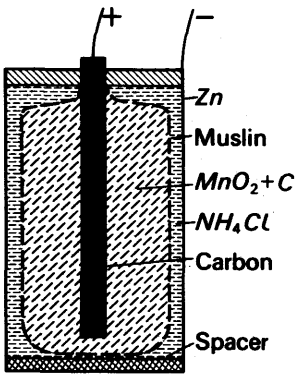


FIG. 34.22. A dry cell.

Standard Cells

A standard cell is one whose e.m.f. varies very little with time, and with temperature, so that it can be used as a standard of potential difference in potentiometer experiments. The commonest type is the Weston cadmium cell (Fig. 34.23). It is housed in an H-shaped glass

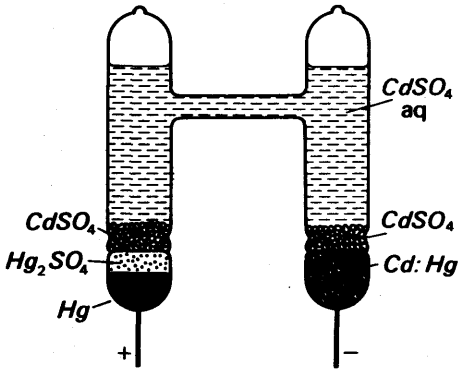


FIG. 34.23. A Weston cadmium cell.

tube because its electrodes are liquid or semi-liquid. The negative electrode is an amalgam of cadmium in mercury; the solution is of cadmium sulphate; the depolarizer is a paste of mercurous sulphate; and the positive electrode is mercury. In some cells, crystals of cadmium sulphate are placed on top of the electrodes to keep the solution saturated. The e.m.f. of one of these, in volts at a temperature t°C, is

$$E = 1{\cdot}01830 - 0{\cdot}0000406\,(t-20) + 0{\cdot}00000095\,(t-20)^2 + 0{\cdot}00000001\,(t-20)^3.$$

The e.m.f. of the type without crystals is about 1·0186 volts between 0°C and 40°C.

A standard cell without crystals of cadmium sulphate is called an unsaturated cell; one with crystals is called a saturated cell, because the crystals keep the solution saturated. Saturated cells give an accurately reproducible e.m.f., because the concentration of the solution is sharply defined at any given temperature. Unsaturated cells do not agree among one another so well, because the solution may vary a little from one to the other. But the e.m.f. of a given unsaturated cell varies less with temperature than that of a saturated cell, because the concentration of the solution is constant.

The depolarizer of a standard cell is effective only for very small currents, and the e.m.f. of the cell will change appreciably if more than about 10 microamperes are drawn from it. *A standard cell must not, in any circumstances, be used as a source of current.* In the early stages of balancing a standard cell against a potentiometer wire, a protective resistance of about 100 000 ohms should be connected in series with the cell.

Primary and Secondary Cells

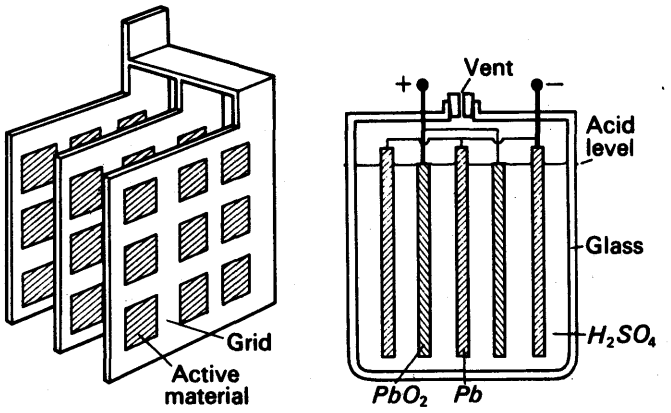
The cells which we have so far described are called primary cells. When they are run-down, their active materials must be renewed; the cells cannot be recharged by passing a current through them from another source. A secondary cell is one which can be recharged in this way.

SECONDARY CELLS

The Lead Accumulator

The commonest secondary cell is the lead-acid accumulator. Its active materials are spongy lead, Pb (for the negative plate), lead dioxide, PbO_2 (the for positive plate), and sulphuric acid. The active materials of the plates are supported in grids of hard lead-antimony alloy (Fig. 34.24 (i)). These are assembled in interchanging groups, closely spaced to give a low internal resistance, and often held apart by strips of wood or celluloid (Fig. 34.24 (ii)).

When the cell is discharging—giving a current—hydrogen ions drift to the positive plate, and SO_4^{2-} ions to the negative. As they give up their charges they attack the plates, and reduce the active materials of each to lead sulphate.



(i) Structure of plates (ii) Assembly of cell

FIG. 34.24. Lead-acid accumulator.

At the negative plate the reaction is

$$Pb + SO_4^{2-} - 2 \text{ electrons} \rightarrow PbSO_4. \quad . \quad . \quad (6)$$

The chemical action at the positive plate is generally given as

(i) $PbO_2 + 2H^+ + 2 \text{ electrons} \rightarrow PbO + H_2O$;
(ii) $PbO + H_2SO_4 \rightarrow PbSO_4 + H_2O$;

whence, altogether

$$PbO_2 + H_2SO_4 + 2H^+ + 2 \text{ electrons} \rightarrow PbSO_4 + 2H_2O. \quad . \quad (7)$$

However, H_2SO_4 molecules do not exist in the solution—they are dissociated into $2H^+$ and SO_4^- ions. We may therefore write equation (7) as

$$PbO_2 + SO_4^{2-} + 4H^+ + 2 \text{ electrons} \rightarrow PbSO_4 + 2H_2O. \quad . \quad (8)$$

The lead sulphate produced in these reactions is a soft form, which is chemically more active than the hard, insoluble lead sulphate familiar in the general chemistry of lead. In the discharging reactions water is formed and sulphuric acid consumed: the concentration of the acid, and therefore its specific gravity, fall.

Charging the Accumulator

When the cell is to be charged it is connected, in opposition, to a supply of greater e.m.f., via a rheostat and ammeter (Fig. 34.25). The

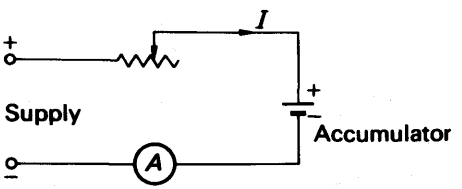


FIG. 34.25 Charging an accumulator.

supply forces a current I through the cell in the opposite direction to the discharging current, so that hydrogen ions are carried to the negative plate, and SO_4^{2-} ions to the positive. The chemical reactions are as follows.

At the negative plate:

$$PbSO_4 + 2H^+ + 2\text{ electrons} \rightarrow Pb + H_2SO_4. \quad (9)$$

At the positive plate:

(i) $PbSO_4 + SO_4^{2-} - 2\text{ electrons} \rightarrow PbO_2 + 2SO_3$;
(ii) $2SO_3 + 2H_2O \rightarrow 2H_2SO_4$;

altogether:

$$PbSO_4 + 2H_2O + SO_4^{2-} - 2\text{ electrons} \rightarrow PbO_2 + 2H_2SO_4. \quad (10)$$

The active materials are converted back to lead and lead dioxide, water is consumed, and sulphuric acid is formed. The acid therefore becomes more concentrated during charge, and its specific gravity rises.

Properties and Care of the Lead Accumulator

The e.m.f. of a freshly charged lead accumulator is about 2·2 volts, and the specific gravity of the acid about 1·25. When the cell is being discharged its e.m.f. falls rapidly to about 2 volts, and then becomes steady (Fig. 34.26); but towards the end of the discharge the e.m.f. begins to fall again. When the terminal voltage load has dropped below about 1·9 volts, or the specific gravity of the acid below about 1·15, the cell should be recharged. If the cell is discharged too far, or left in a discharged condition, hard lead sulphate forms on its plates, and it becomes useless.

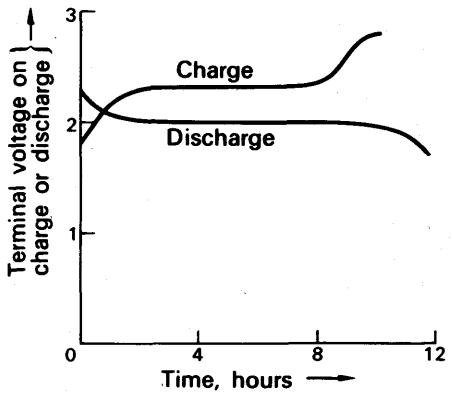


FIG. 34.26. Voltage-time curves of lead accumulator.

The internal resistance of a lead accumulator, like that of any other cell, depends on the area and spacing of its plates. It is much lower

than that of any primary cell, however, being usually of the order of 1/10 to 1/100 ohm. The amount of electricity which an accumulator can store is called its *capacity*. It is a vague quantity, but a particular accumulator may give, for example, 4 amperes for 20 hours before needing a recharge. The capacity of this accumulator would be 80 ampere-hours. (One ampere-hour = 3600 coulombs.) If the accumulator were discharged faster—at 8 amperes, say—then it would probably need recharging after rather less than 10 hours; and if it were discharged more slowly—say at 2 amperes—it might hold out for more than 40 hours. The capacity of an accumulator therefore depends on its rate of discharge; it is usually specified at the '10-hour' or '20-hour' rate. Discharging an accumulator faster than at about the 10-hour rate causes the active material to fall out of the plates.

Accumulators are usually charged at about the '8-hour' rate—say 5 amperes for the cell discussed above. The charging is continued until gas is bubbling freely off the plates. When the plates are gassing, the chemical reactions (9) and (10) have been completed, and the current through the cell is simply decomposing the water in it. Before the charge is started the vent-plugs in the cell-case must be removed to let the gases out; the gases are hydrogen and oxygen and naked lights near are dangerous. The water lost at the end of each charge must be made up by pouring in distilled water until the acid rises to the level marked on the case. If the specific gravity of the acid is then less than 1·25, the charging must be continued. Near the end of the charging, the back-e.m.f. of the cell rises sharply to about 2·6 volts (Fig. 34.26). It never gives a forward e.m.f. as great as this: as soon as it is put on discharge, its e.m.f. falls to about 2·2 volts.

Efficiency of Accumulator

The number of ampere-hours put into an accumulator on charge is greater than the number which can be got out of it without discharging it too far. The ratio

$$\frac{\text{ampere-hours on discharge}}{\text{ampere-hours on charge}}$$

is called the ampere-hour efficiency of the cell; its value is commonly about 90 per cent. However, to judge an accumulator by its ampere-hour efficiency is to flatter it; not only does it take in more ampere-hours on charge than it gives out on discharge, but it takes them in at a higher voltage. The electrical energy put into an accumulator on charge is the integral of current, e.m.f., and time:

$$W = \int I\,E\,dt.$$

For simplicity we may say

$$\text{energy put in} = \text{quantity of electricity put in} \times \text{average e.m.f. on charge.}$$

If the quantity of electricity is measured in ampere-hours, the energy is in watt-hours instead of joules. Similarly, on discharge,

$$\text{energy given out} = \text{quantity of electricity given out} \times \text{average e.m.f. on discharge.}$$

The energy efficiency of the cell is

$$\frac{\text{energy given out on discharge}}{\text{energy taken in on charge}}$$

$$= \frac{\text{amp-hours} \times \text{average e.m.f. on discharge}}{\text{amp-hours} \times \text{average e.m.f. on charge}}$$

$$= \text{amp-hour efficiency} \times \frac{\text{average e.m.f. on discharge}}{\text{average e.m.f. on charge}}$$

$$\simeq \text{amp-hour efficiency} \times \frac{2{\cdot}0}{2{\cdot}2}.$$

The energy efficiency is more often called the watt-hour efficiency of the cell; it is about 80 per cent.

The Nickel-iron Accumulator

The nickel-iron (NIFE) accumulator has active materials of nickel hydroxide (positive), iron (negative) and caustic potash solution. Its e.m.f. varies from 1·3 volt to 1·0 on discharge; it has a higher internal resistance than a lead accumulator of similar size; and it is less efficient. Its advantages are that it is more rugged, both mechanically and electrically. Very rapid charging and discharging do not harm it, nor do overdischarging and overcharging. Vibration does not make the active materials fall out of the plates, as it does with a lead cell. Nickel-iron accumulators are therefore used in electric trucks and at sea.

EXAMPLES

1. A copper refining cell consists of two parallel copper plate electrodes, 6 cm apart and 1 metre square, immersed in a copper sulphate solution of resistivity $1{\cdot}2 \times 10^{-2}$ ohm metre. Calculate the potential difference which must be established between the plates to provide a constant current to deposit 480 g of copper on the cathode in one hour (e.c.e. of copper = $3{\cdot}29 \times 10^{-7}$ kg C^{-1}).

From $m = zIt$, since 480 g = 0·48 kg,

$$I = \frac{m}{zt} = \frac{0{\cdot}48}{3{\cdot}29 \times 10^{-7} \times 3600} \text{ A} \quad . \quad . \quad . \quad \text{(i)}$$

The resistance of the cell, $R = \dfrac{\rho l}{A}$

$$= \frac{1{\cdot}2 \times 10^{-2} \times 6 \times 10^{-2}}{1^2}$$

Hence, from (i), the p.d. $V = IR = \dfrac{0{\cdot}48 \times 1{\cdot}2 \times 10^{-2} \times 6 \times 10^{-2}}{3{\cdot}29 \times 10^{-7} \times 3600 \times 1^2}$

$$= 0{\cdot}3 \text{ V (approx.).}$$

2. State Faraday's laws of electrolysis and show that the ionic dissociation theory offers an explanation of them. Acidulated water is electrolyzed between platinum electrodes. Sketch a graph showing the relation between the strength of the current and the reading of a voltmeter connected to the electrodes. Comment on the nature of the graph.

Give a circuit diagram showing how you would charge a series battery of 12 lead accumulators, each of e.m.f. 2 volts and internal resistance 1/24 ohm, from 240-volt d.c. mains, if the charging current is not to exceed 3 amp. What percentage of the energy taken from the mains would be wasted? (*L.*)

First part (see text). When the water is electrolyzed, no current flows until the p.d. is greater than about 1·7 volts, when the back-e.m.f. of the liberated product is overcome. After this, a straight-line graph is obtained between V and I.

Second part. A series resistance R is required, given by

$$I = 3 = \frac{240 - 12 \times 2}{R + \frac{12}{24}}.$$

$$\therefore 3R + 1{\cdot}5 = 216.$$

$$\therefore R = \frac{214{\cdot}5}{3} = 71{\cdot}5 \text{ ohms}.$$

Energy taken from mains $= EIt = 240 \times 3t = 720t$, where t is the time.

Energy wasted $= (I^2R + I^2r)t = (3^2 \times 71{\cdot}5 + 3^2 \times 0{\cdot}5)t$
$= 648t.$

$$\therefore \text{percentage wasted} = \frac{648t}{720t} \times 100\% = 90\%.$$

3. State Faraday's laws of electrolysis. How would you verify the laws experimentally? Discuss briefly the phenomenon of polarization in electrolysis and how it is overcome in the Daniell cell.

Calculate a value for the e.m.f. of a Daniell cell from energy considerations, using the following data: 1 g of zinc dissolved in copper sulphate solution liberates 3327 J (e.c.e. of zinc = 0·000340 g C^{-1}). (*L.*)

The e.m.f. E of a cell can be defined as the energy per coulomb delivered by the cell (p. 797).

When 1 g of zinc is dissolved, number of coulombs flowing, $Q = \frac{1}{0{\cdot}00034}$.

Energy liberated, $W = 3327$ J.

$$\therefore \text{energy liberated per coulomb} = \frac{W}{Q}$$
$$= 3327 \times 0{\cdot}00034 \text{ joules/coulomb}$$
$$= 1{\cdot}13 \text{ volts}.$$

EXERCISES 34

1. State Faraday's laws of electrolysis and show how they can be interpreted in terms of the ionic theory.

A voltameter with large platinum plates contains dilute sulphuric acid in which 10^{-2} g of metallic copper has been dissolved. A constant current of 10^{-2} amp is then passed through the solution for 25 minutes. Calculate (*a*) the mass of copper deposited on the cathode, (*b*) the volume of oxygen liberated at the anode if the pressure is 750 mm Hg and the temperature is 20°C and (*c*) the additional time for which the same current must be passed before hydrogen is liberated at the cathode. (The chemical equivalents of copper and oxygen are 31·8 and 8

respectively; the Faraday is 95600 coulombs; 32 g of oxygen occupy 22·4 litres at S.T.P.) (*O. & C.*)

2. Describe the electrolytic processes which occur in a Daniell cell when its terminals are joined through a small resistance.

A steady current of 5 A is passed through a silver voltameter in series with a coil of wire of 10 ohms resistance immersed in 200 g of water. What will be the rise of temperature of the latter when 0·10 g of silver has been deposited? (Assume that the e.c.e. of silver = $1{\cdot}118 \times 10^{-6}$ kg C^{-1}; thermal capacity of the coil and vessel = 42 J K^{-1}.) (*L.*)

3. State Faraday's laws of electrolysis. Explain why it is necessary to have a potential greater than about 1·5 volts in order to maintain a large steady current through acidulated water.

In the electrolysis of water 83·7 cm^3 of hydrogen were collected at a pressure of 68 cm of mercury at 25°C when a current of 0·5 A had been passed for 20 minutes. What is the electrochemical equivalent of copper in copper sulphate ($CuSO_4$)? (Atomic weight of copper = 63·57, atomic weight of hydrogen = 1·008, density of hydrogen at S.T.P. = 0·08987 kg m^{-3}.) (*L.*)

4. What do you understand by (*a*) a *cathode*, and (*b*) an *ion*? Describe how the simple ionic theory accounts for the fact that the liberation of 1 g of hydrogen in the electrolysis of dilute sulphuric acid, using platinum electrodes, always requires the passage of a definite quantity of charge.

If the heat produced when 1 g of hydrogen burns to form water is $1{\cdot}44 \times 10^5$ joules, calculate the minimum e.m.f. which must be applied in such electrolysis before a continuous current will flow. Justify and explain your calculation by reference to the principle of conservation of energy. (1 Faraday = $9{\cdot}65 \times 10^4$ coulombs.)

Describe an experiment to determine the actual value of the back e.m.f. generated by a gas at the electrodes in such electrolysis. (*C.*)

5. State (*a*) Faraday's laws of electrolysis, (*b*) the main features of the theory of ionic dissociation, showing that the theory is in accordance with these laws.

A potential difference of 14 volts is applied to a water voltameter whose total thermal capacity is 2121 J K^{-1}. The temperature of the voltameter rises 1°C in the time taken to liberate 20 cm^3 of hydrogen at S.T.P. Calculate the back e.m.f. of the voltameter and state the source of this e.m.f. (Assume the electrochemical equivalent of hydrogen to be $1{\cdot}044 \times 10^{-8}$ kg C^{-1} and its density at S.T.P. to be 9×10^{-2} kg m^{-3}.) (*L.*)

6. To answer this question you are asked to assume that no galvanometer of any kind is available and that you cannot take for granted that the same current flows continuously round a series circuit.

A circuit consists only of a straight horizontal length of wire and a copper voltmeter connected in series to a suitable d.c. supply. How would you show that a current is flowing through the wire as well as through the voltmeter?

How would you show that the direction of the current through the wire is the same as that of the current through the voltameter?

Outline the measurements you would make to discover whether the strength of the current in the wire is the same as that of the current through the voltameter.

Give an account of the actual physical processes by which a current passes through a metal wire. Explain briefly why the resistance normally increases as the temperature rises. (*O.*)

7. Explain the general nature of the chemical changes that take place in a lead accumulator during charging and discharging.

A battery of accumulators, of e.m.f. 50 volts and internal resistance 2 ohms, is charged on a 100-volt direct-current mains. What series resistance will be required to give a charging current of 2 A? If the price of electrical energy is 1*d.* per kilowatt-hour, what will it cost to charge the battery for 8 hours, and what percentage of the energy supplied will be wasted in the form of heat? (*C.*)

8. State the laws of electrolysis and give a concise account of an elementary theory of electrolysis which is consistent with the laws.

If an electric current passes through a copper voltameter and a water voltameter in series, calculate the volume of hydrogen which will be liberated in the latter, at 25°C and 78 cm of mercury pressure, whilst 0·05 g of copper is deposited in the former. (Take e.c.e. of hydrogen as $1{\cdot}04 \times 10^{-8}$ kg C^{-1}, e.c.e. of copper as $3{\cdot}3 \times 10^{-7}$ kg C^{-1}, density of hydrogen as 9×10^{-2} kg m^{-3} at S.T.P.)(*L.*)

9. State Faraday's laws of electrolysis and describe experiments to verify them.

A difference of potential of 60 volts is maintained between two electrodes 12 cm apart in a solution of common salt. How long will it take a chlorine ion to travel 3 cm in the solution? (The mobility of chlorine ions may be taken as 0·00053 cm s^{-1} per V cm^{-1}.) (*L.*)

10. Explain what happens when an e.m.f. is applied to platinum electrodes immersed in dilute sulphuric acid. What is the relation between the e.m.f. and the current in such a cell?

If the electrochemical equivalent of hydrogen is $1{\cdot}04 \times 10^{-8}$ kg coulomb^{-1}, and if 1 g of hydrogen on burning to form water liberates 147000 joules, calculate the back-e.m.f. produced in a water voltameter when it is connected to a 2-volt accumulator. (*C.*)

chapter thirty-five

Magnetic Field and Force on Currents

NATURAL magnets were known some thousands of years ago, and in the eleventh century A.D. the Chinese invented the magnetic compass. This consisted of a magnet, floating on a buoyant support in a dish of water. The respective ends of the magnet, where iron filings are attracted most, are called the north and south poles.

In the thirteenth century the properties of magnets were studied by Peter Peregrinus. He showed that

like poles repel and *unlike poles attract*

His work was forgotten, however, and his results were rediscovered in the sixteenth century by Dr. Gilbert, who is famous for his researches in magnetism and electrostatics.

Ferromagnetism

About 1823 Sturgeon placed an iron core into a coil carrying a current, and found that the magnetic effect of the current was increased enormously. On switching off the current the iron lost nearly all its magnetism. Iron, which can be magnetized strongly, is called a *ferromagnetic* material. Steel, made by adding a small percentage of carbon to iron, is also ferromagnetic. It retains its magnetism, however, after removal from a current-carrying coil, and is more difficult to magnetize than iron. Nickel and cobalt are the only other ferromagnetic elements in addition to iron, and are widely used for modern magnetic apparatus. A modern alloy for permanent magnets, called *alnico*, has the composition 54 per cent iron, 18 per cent nickel, 12 per cent cobalt, 6 per cent copper, 10 per cent aluminium. It retains its magnetism extremely well, and, by analogy with steel, is therefore said to be magnetically very hard. Alloys which are very easily magnetized, but do not retain their magnetism, are said to be magnetically soft. An example is *mumetal*, which contains 76 per cent nickel, 17 per cent iron, 5 per cent copper, 2 per cent chromium.

Magnetic Fields

The region round a magnet, where a magnetic force is experienced, is called a *magnetic field*. The appearance of a magnetic field is quickly obtained by iron filings, and accurately plotted with a small compass, as the reader knows. The *direction* of a magnetic field is taken as the direction which a north pole would move if placed in the field.

Fig. 35.1 shows a few typical fields. The field round a bar-magnet is 'non-uniform', that is, its strength and direction vary from place to

place (Fig. 35.1 (i)). The earth's field locally, however, is uniform (Fig. 35.1 (ii)). A bar of soft iron placed north–south becomes magnetized by

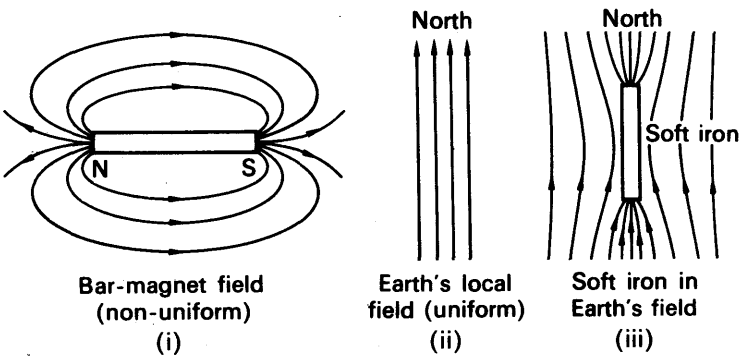


FIG. 35.1. Magnetic fields.

induction by the earth's field, and the lines of force become concentrated in the soft iron (Fig. 35.1 (iii)). The tangent to a line of force at a point gives the direction of the magnetic field at that point.

Oersted's Discovery

The magnetic effect of the electric current was discovered by Oersted in 1820. Like many others, Oersted suspected a relationship between electricity and magnetism, and was deliberately looking for it. In the course of his experiments, he happened to lead a wire carrying a current

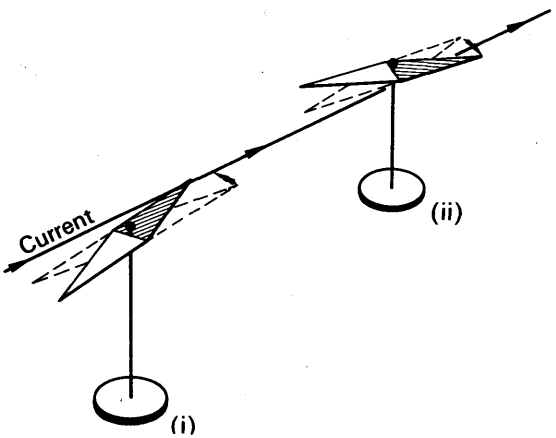


FIG. 35.2. Deflection of compass-needle by electric current.

over, but parallel to, a compass-needle, as shown in Fig. 35.2(i); the needle was deflected. Oersted then found that if the wire was led under

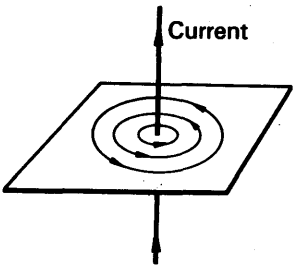


FIG. 35.3. Magnetic field of long straight conductor.

the needle, it was deflected in the opposite sense (Fig. 35.2(ii)). From these observations he concluded that 'the electric conflict performs gyrations'. What he meant by this we can see by plotting the lines of force of a long vertical wire, as shown in Fig. 35.3. To get a clear result a strong current is needed, and we must work close to the wire, so that the effect of the earth's field is negligible. It is then seen that the lines of force are circles, concentric with the wire.

Directions of Current and Field; Corkscrew Rule

The relationship between the direction of the lines of force and of the current is expressed in Maxwell's corkscrew rule: if we imagine ourselves driving a corkscrew in the direction of the current, then the direction of rotation of the corkscrew is the direction of the lines of force. Fig. 35.4 illustrates this rule, the small, heavy circle representing the wire, and the large light one a line of force. At (i) the current is flowing into the paper; its direction is indicated by a cross, which stands for the tail of a retreating arrow. At (ii) the current is flowing out of the paper; the dot in the centre of the wire stands for the point of an approaching arrow.

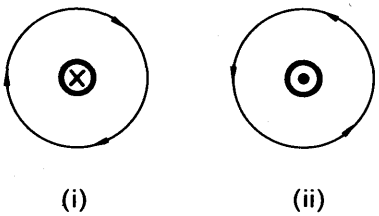


FIG. 35.4. Illustrating corkscrew rule.

If we plot the magnetic field of a circular coil carrying a current, we get the result shown in Fig. 35.5. Near the circumference of the coil the lines of force are closed loops, which are not circular, but whose

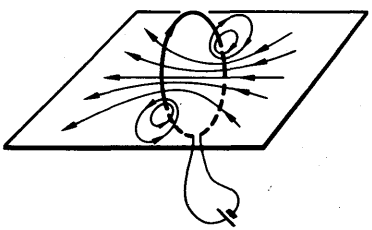


FIG. 35.5. Magnetic field of narrow coil.

directions are still given by the corkscrew rule, as in Fig. 35.5. Near the centre of the coil, the lines are almost straight and parallel. Their

direction here is again given by the corkscrew rule, but the current and the lines of force are interchanged: if we turn the screw in the direction of the current, then it travels in the direction of the lines.

The Solenoid

The same is true of the magnetic field of a long cylindrical coil, shown in Fig. 35.6. Such a coil is called a solenoid; it has a field similar to that of a bar-magnet, whose poles are indicated in the figure. If an iron or steel core were put into the coil, it would become magnetized, with the polarity shown.

If the terminals of a battery are joined by a wire which is simply

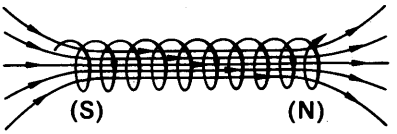


FIG. 35.6. Magnetic field of solenoid.

doubled back on itself, as in Fig. 35.7, there is no magnetic field at all: each element of the outward run, such as AB, in effect cancels the field of the corresponding element of the inward run, CD. But as soon as the wire is opened out into a loop, its magnetic field appears (Fig. 35.8).

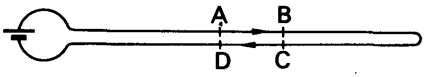


FIG. 35.7. A double-back current has no magnetic field.

Within the loop, the field is strong, because all the elements of the loop give magnetic fields in the same sense, as we can see by applying the corkscrew rule to each side of the square ABCD. Outside the loop, for example at the point P, corresponding elements of the loop give

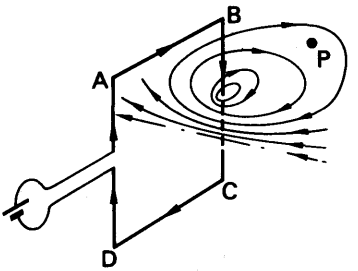


FIG. 35.8. An open loop of current has magnetic field.

opposing fields (for example, DA opposes BC); but these elements are at different distances from P (DA is farther away than BC). Thus

there is a resultant field at P, but it is weak compared with the field inside the loop. A magnetic field can thus be set up either by wires carrying a current, or by the use of permanent magnets.

Force on Conductor. Fleming's Rule

When a conductor carrying a current is placed in a magnetic field due to some source other than itself, it experiences a mechanical force.

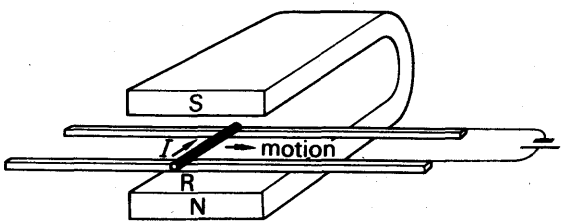


FIG. 35.9. Force on current in magnetic field.

To demonstrate this, a short brass rod R is connected across a pair of brass rails, as shown in Fig. 35.9. A horseshoe magnet is placed so that the rod lies in the field between its poles. When we pass a current through the rod, from an accumulator, the rod rolls along the rails. The relative directions of the current, the applied field, and the motion are shown in Fig. 35.10; they are the same as those of the middle finger, the fore-finger, and the thumb of the left hand when held all at right angles to one another. If we place the horseshoe magnet so that its field lies along the rod carrying the current, then the rod experiences no force.

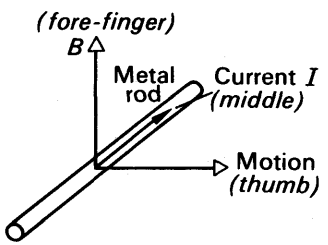


FIG. 35.10. Left-hand rule.

Experiments like this were first made by Ampère in 1820. As a result of them, he concluded that the force on a conductor is always *at right angles to the plane which contains both the conductor and the direction of the field in which it is placed.* He also showed that, if the conductor makes an angle α with the field, the force on it is proportional to sin α, so that the maximum force is exerted when the conductor is perpendicular to the field.

Dependence of Force on Physical Factors

Since the magnitude of the force on a current-carrying conductor is given by

$$F \propto \sin \alpha, \quad . \quad . \quad . \quad . \quad (1)$$

where α is the angle between the conductor and the field, it follows that F is zero when the conductor is parallel to the field direction. This defines the direction of the magnetic field. To find which way it points,

we can apply Fleming's rule to the case when the conductor is placed at right angles to the field. The sense of the field then corresponds to the direction of the forefinger.

Variation of *F* with *I* and *l*

To investigate how the magnitude of the force *F* depends on the current *I* and the length *l* of the conductor, we may use the apparatus of Fig. 35.11.

Here the conductor AC is situated in the field of a solenoid S. The current flows into, and out of, the wire via the pivot points X and Y. The scale pan T is placed at the same distance from the pivot as the straight wire AC, which is perpendicular to the axis of the coil. The

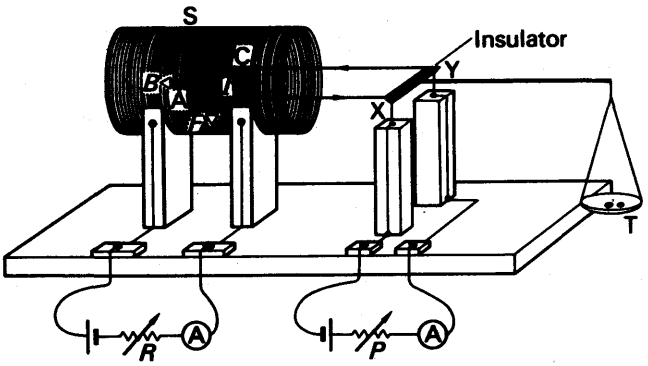


FIG. 35.11. Experiment to show how *F* varies with *I* and *B*.

frame is first balanced with no current flowing in AC. A current is then passed, and the extra weight needed to restore the frame to a horizontal position is equal to the force on the wire AC. By varying the current in AC with the rheostat *P*, it may be shown that:

$$F \propto I \qquad . \quad . \quad . \quad . \quad (2)$$

If different frames are used so that the length, *l*, of AC is changed, it can be shown that

$$F \propto l \qquad . \quad . \quad . \quad . \quad (3)$$

Effect of *B*

The magnetic field due to the solenoid will depend on the current flowing in it. If this current is varied by adjusting the rheostat *R*, it can be shown that the larger the current *in the solenoid*, S, the larger is the force *F*. It is reasonable to suppose that a larger current in S produces a stronger magnetic field. Thus the force *F* increases if the magnetic field strength is increased. The magnetic field is represented by a vector quantity which is given the symbol *B*. This is called the *magnetic induction* or *flux density* in the field. We assume that:

$$F \propto B \qquad . \quad . \quad . \quad . \quad (4)$$

Magnitude of F

From the results expressed in equations (1) to (4), we obtain

$$F \propto BIl \sin \alpha,$$

or

$$F = \kappa BIl \sin \alpha \quad . \quad . \quad . \quad . \quad (5)$$

where κ is a constant.

In the SI system of units, the unit of B is the tesla (T). 1 T = 1 weber metre^{-2} (Wb m^{-2}). One tesla may be defined as the flux density in a field when the force on a conductor 1 metre long, placed perpendicular

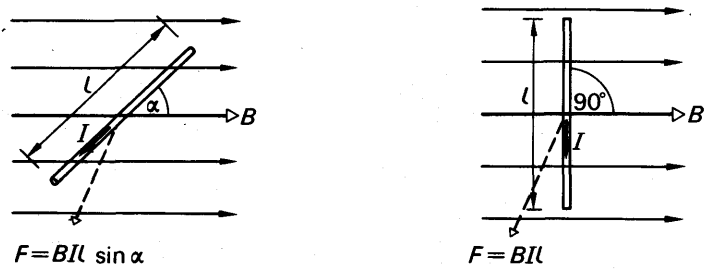


FIG. 35.12. Magnitude of F.

to the field and carrying a current of 1 ampere, is 1 newton. Substituting $F = 1$, $B = 1$, $l = 1$ and $\sin \alpha = \sin 90° = 1$ in (5), then $k = 1$. Thus in Fig. 35.12 (i), with the above units,

$$F = BIl \sin \alpha \quad . \quad . \quad . \quad . \quad (5)$$

When the whole length of the conductor is perpendicular to the field B, Fig. 35.12 (ii), then, since $\alpha = 90°$ in this case,

$$F = BIl \quad . \quad . \quad . \quad . \quad (6)$$

It may be noted that the apparatus of Fig. 35.11 can be used to determine the flux density B of the field in the solenoid. In this case, $\alpha = 90°$ and $\sin \alpha = 1$, so that measurement of F, I and l enables B to be found from (6).

EXAMPLE

A wire carrying a current of 10 A and 2 metres in length is placed in a field of flux density 0·15 T (Wb m^{-2}). What is the force on the wire if it is placed (*a*) at right angles to the field, (*b*) at 45° to the field, (*c*) along the field.

From (5) $F = BIl \sin \alpha$

(*a*) $\therefore F = 0{\cdot}15 \times 10 \times 2 \times \sin 90°$
$= 3$ newtons.

(*b*) $\therefore F = 0{\cdot}15 \times 10 \times 2 \times \sin 45°$
$= 2{\cdot}12$ newtons.

(*c*) $\therefore F = 0$, since $\sin 0° = 0$.

Force on Moving Charges

It was explained in Chapter 32 that an electric current in a wire can be regarded as a drift of electrons in the wire, superimposed on their random thermal motions. If the electrons in the wire drift with average velocity v, and the wire lies at right angles to the field, then the force on *each* electron is given by

$$F = Bev \quad . \quad . \quad . \quad . \quad (1)$$

If B is in tesla (T) or weber metre^{-2} (Wb m^{-2}), e is in coulomb (C) and v in metre second^{-1} (m s^{-1}), then F will be in newtons (N) (Fig. 35.13(i)).

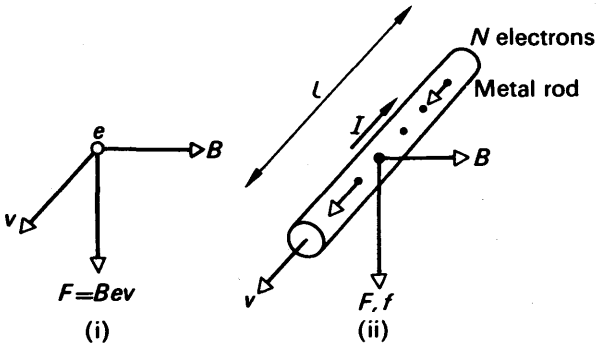


FIG. 35.13. Force on moving electron in magnetic field (v at right angles to page).

The proof of equation (1) is easily obtained. If there are N free electrons in a wire of length l, their total charge is Ne, Fig. 35.13 (ii). If they have a drift velocity v, the time which any one of them takes to travel the distance l is

$$t = \frac{l}{v}.$$

In this time, therefore, the N electrons are swept out of the wire by the current and are replaced by another N electrons. The rate at which charge flows along the wire, which is the current through it, is therefore

$$I = \frac{Ne}{t} = \frac{Nev}{l}.$$

If the wire is at right angles to a magnetic field B, the force on it is

$$f = BIl = BNev.$$

Therefore the force on a single electron is

$$F = \frac{f}{N} = Bev.$$

An electron moving across a magnetic field experiences a force whether it is in a wire or not—for example, it may be one of a beam of electrons in a vacuum tube. Because of this force, a magnetic field can be used to focus or deflect an electron beam, instead of an electrostatic field as on p. 1002. Magnetic deflection and focusing are common in cathode ray tubes used for television.

The Moving-coil Galvanometer

All except the most accurate of current measurements are made today with the moving-coil galvanometer. In this instrument a coil of fine insulated copper wire, ABDF in Fig. 35.14 (i), hangs in a strong magnetic field. The field is set up between soft iron pole-pieces, NS, attached to a powerful permanent magnet.

The pole-pieces are curved to form parts of a cylinder coaxial with the suspension of the coil. And between them lies a cylindrical core of soft iron, C; it is supported on a brass pin, T in Fig. 35.14(iii), which is placed so that it does not foul the coil. As this figure shows, the magnetic field B is radial to the core and pole-pieces, over the region in which the coil can swing.

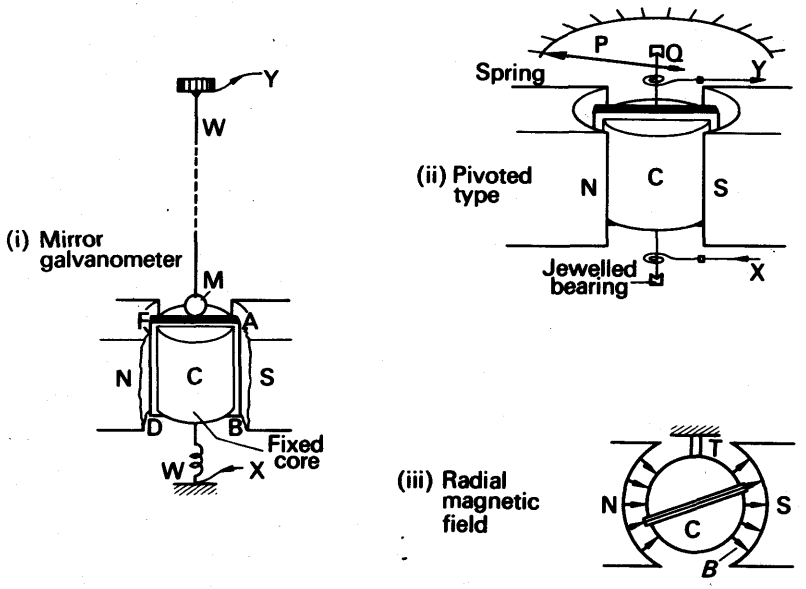


FIG. 35.14. Moving-coil galvanometers.

In the more sensitive instruments, the coil is suspended on a phosphor-bronze wire, WW, which is kept taut. The current is led into and out of the coil through the suspension, at X and Y, and the deflection of the coil is shown by a mirror, M. This is known as a *mirror galvanometer*. More robust but less sensitive forms of the galvanometer have hair-springs and jewelled bearings, instead of the phosphor-bronze suspension (Fig. 35.14 (ii)). The coil is wound on a rigid but light former, of bamboo or aluminium, which also carries the pivots. The pivots are insulated from the former if it is aluminium, and the current is led in and out through the springs. The framework, which

carries the springs and jewels, is made from brass or aluminium—if it were steel it would affect the magnetic field. An aluminium pointer, P, shows the deflection of the coil; it is balanced by a counterweight, Q.

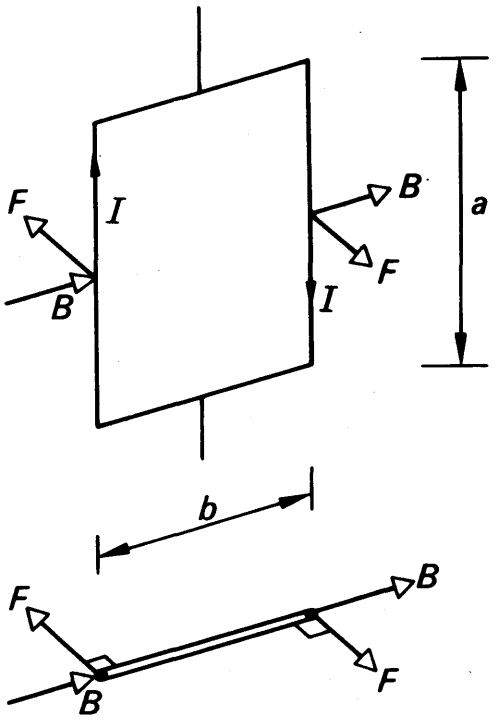


FIG. 35.15. Couple on coil in radial field.

Theory of Moving-coil Instrument

Fig. 35.15 shows a current I flowing through the coil of a moving-coil instrument. The magnetic field exerts forces F on the vertical limbs of each turn of the coil. These forces tend to rotate the coil about the suspension. Since the magnetic field B is at right angles to the vertical limbs, the forces are given by

$$F = BIl,$$

where l is the height of the coil; F acts at right angles to its plane because the magnetic field is radial. Therefore, if b is the width of the coil, the couple which the forces F exert on each turn is:

$$C = Fb = BIlb = BIA,$$

If the coil has N turns, then the total couple which the current exerts on it is

$$C' = NC = BIAN \quad . \quad . \quad . \quad (1)$$

The coil will turn until the restoring couple due to the twist in the suspension is equal to C'. This couple is proportional to the twist θ, which is also the deflection of the coil; thus

$$C' = k\theta,$$

where k is a constant of the suspension. Hence

$$BIAN = k\theta$$

or

$$I = \frac{k}{BAN}\theta. \quad . \quad . \quad . \quad . \quad (2)$$

Equation (2) shows that the deflection is proportional to the current. The pointer type of instrument (Fig. 35.15 (ii)) usually has a scale calibrated directly in milliamperes or microamperes. Full-scale reading on such an instrument corresponds to a deflection θ of 90° to 120°; it may represent a current of 50 microamperes to 15 milliamperes, according to the strength of the hair-springs, the geometry of the coil, and the strength of the magnetic field. The less sensitive models are more accurate, because their pivots and springs are more robust, and therefore are less affected by dust, vibration, and hard use. Models known as 'first grade' (FG) have an error not greater than 1 per cent of full-scale deflection. This error is constant over the scale, so that the inaccuracy may be 10 per cent when the reading is one-tenth of full scale. Readings less than about half full scale must therefore be avoided in accurate work. The best moving-coil instruments have an error of about 0·2 per cent of full-scale deflection.

When a galvanometer is of the suspended-coil type (Fig. 35.15 (i)), its sensitivity is generally expressed in terms of the displacement of the spot of light reflected from the mirror on to the scale. At a scale distance of 1 metre a moderately sensitive instrument of this type will give a deflection of 100 cm per microampere.

All forms of moving-coil galvanometer have one disadvantage: they are easily damaged by overload. A current much greater than that which the instrument is intended to measure will burn out its hair-springs or suspension.

Couple on a Coil in a Uniform Field

So far we have considered a coil in a radial magnetic field. Now let us consider one in a uniform field. Fig. 35.16 shows a rectangular coil of one turn, whose plane makes an angle α with a uniform magnetic field B. If it carries a current I amperes, the forces F_1 on its vertical limbs are given by

$$F_1 = BIl = BIa,$$

where a is the height of the coil. And the forces F_2 on its horizontal limbs are given by

$$F_2 = BIb \sin \alpha,$$

where b is the width of the coil. The forces F_2 merely compress the coil, and are resisted by its rigidity. The forces F_1 set up a couple, whose moment is, from the lower figure,

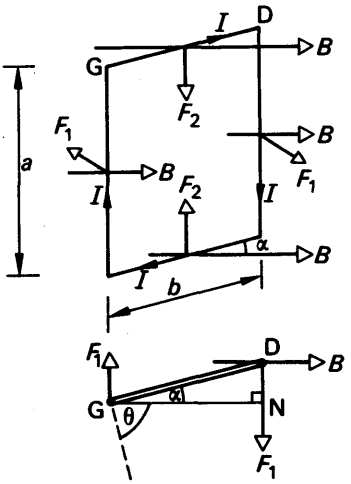


FIG. 35.16. Couple on coil in uniform field.

$$C = F_1 \times \text{GN}$$

$$= F_1 b \cos \alpha.$$

Hence

$$C = BIab \cos \alpha$$

$$= BIA \cos \alpha, \quad . \quad . \quad . \quad . \quad (1)$$

where A is the area of the coil. If θ is the angle between the field and the normal to the plane of the coil, then $\theta = 90° - \alpha$, and

$$C = BIA \sin \theta. \quad . \quad . \quad . \quad . \quad (2)$$

With a coil of N turns,

$$C = BNIA \quad . \quad . \quad . \quad . \quad (3)$$

Magnetic Moment

The couple on the coil thus depends on the magnitude of the flux density B, the current I in it and the Area A. We can write equation (3) as

$$C = mB \sin \theta \quad . \quad . \quad . \quad . \quad (1)$$

where $m = NIA$. m is a property of the coil and the current in it and is called the *magnetic moment* of the coil. (Fig. 35.17 (i)). In general, the magnetic moment of a coil is defined as the couple exerted on it when it is placed with its plane parallel to a field of unit induction. In this case $B = 1$, $\sin \theta = \sin 90° = 1$ and hence $C = m$ (Fig. 35.17 (ii)). It should be noted that the magnetic moment of a coil can be calculated from

$$m = NIA \quad . \quad . \quad . \quad . \quad (2)$$

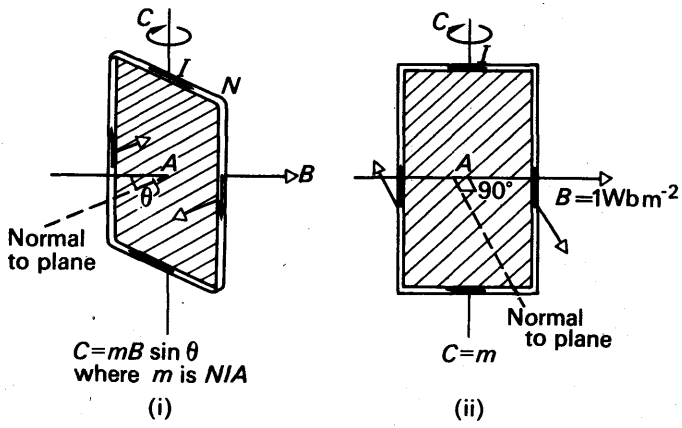


FIG. 35.17. Magnetic moment and couple.

whatever the shape of the coil. From this expression the unit of m is ampere metre^2 (A m^2).

EXAMPLE

What couple is needed to hold a small single-turn coil of area 5 cm^2 in equilibrium, when it carries a current of 10 A and is placed with its axis at right angles to a field of flux density 0·15 T? What couple would be required if the coil had 20 turns?

Area $A = 5 \text{ cm}^2 = 5 \times 10^{-4} \text{ m}^2$.

$\therefore$ Magnetic moment of single-turn coil $= m = IA$

$= 10 \times (5 \times 10^{-5}) \text{ A m}^2$.

$\therefore C = mB \sin \theta$

$= (10 \times 5 \times 10^{-5}) \times 0{\cdot}15 \times \sin 90°$

$= 0{\cdot}75 \times 10^{-4}$ newton metre (N m).

If the coil has 20 turns, each will have the same magnetic moment. Hence

total couple $= 20 \times 0{\cdot}75 \times 10^{-4}$

$= 1{\cdot}5 \times 10^{-3}$ N m.

Vibration Magnetometer

Magnetism is due to circulating and spinning electrons inside atoms (see p. 948). The moving charges are equivalent to electric currents. Consequently, like a current-carrying coil, permanent magnets also have a couple acting on them when they are placed with their axis at an angle to a magnetic field. Like the coil, they turn and settle in equilibrium with their axis along the field direction.

Consider a magnet suspended from a torsionless silk thread in a draught-shield. If it is gently disturbed, for example by momentarily bringing a piece of iron towards it, the magnet *vibrates* about the

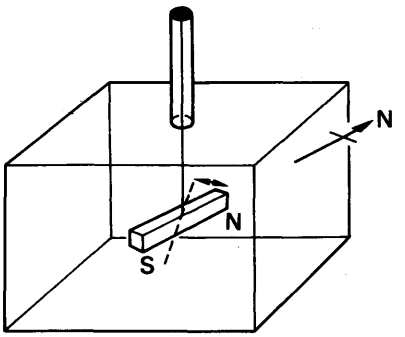


FIG. 35.18. Oscillation of bar-magnet.

magnetic meridian (Fig. 35.18).

The oscillation occurs because, when the magnet is deflected from the meridian, a couple acts on it, tending to bring it back into the meridian. On p. 885 it was shown that the couple had a moment $mB \sin \theta$, where m is the moment of the magnet, B is the field intensity, and θ is the angle between the magnet and the field. If the angular displacement, θ, is small—say not more than 10° —then, to a close approximation, $\sin \theta = \theta$ in radians. The restoring couple C is then given by

$$C = mB\theta. \quad . \quad . \quad . \quad . \quad (1)$$

Thus the restoring couple is proportional to the angular displacement θ, and the magnet therefore makes simple harmonic vibrations (see p. 89).

Period of Vibration

The period of vibration, T, depends on the restoring couple per unit deflection of the magnet; from (1),

$$\frac{C}{\theta} = mB.$$

The period also depends on the moment of inertia K of the magnet, which is determined by its mass, size, and shape. Now from equation (1), p. 89,

$$T = 2\pi\sqrt{\frac{K}{\text{restoring couple per unit deflection}}},$$

$$\therefore T = 2\pi\sqrt{\frac{K}{mB}}. \quad . \quad . \quad . \quad (1)$$

The number of vibrations made per second, n, or the *frequency* of the vibration, is given by

$$n = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{mB}{K}},$$

$$\therefore n^2 = \frac{mB}{4\pi^2 K},$$

or

$$B \propto n^2. \quad . \quad . \quad . \quad . \quad (2)$$

This relationship shows that a vibrating magnet may be used to compare magnetic flux densities, for example, to explore the earth's field over the laboratory. When used in this way it is called a *vibration magnetometer.*

It should be noted that the relations (1) and (2) are true only when the fibre suspending the magnet is torsionless, that is, it exerts a negligible restoring couple when it is twisted. The restoring couple on the magnet is then simply that due to the magnetic field ($mB \sin \theta$), as already assumed. A common material used for the suspension is unspun silk. The magnet must be small so that the whole of it is in the uniform field, and it must be protected from draughts, which would upset the oscillation.

Comparison of Flux Densities

By means of a vibration magnetometer, the values of B in two magnetic fields can easily be compared (see also p. 892). The magnetometer is set up in one field of magnitude B_1 say and the time period T_1 of small swings is measured. The new time period T_2 is then measured in the other field of magnitude B_2. Then, from previous,

$$T_1 = 2\pi\sqrt{\frac{K}{mB_1}} \quad \text{and} \quad T_2 = 2\pi\sqrt{\frac{K}{mB_2}}$$

Dividing the two equations,

$$\therefore \frac{T_1}{T_2} = \sqrt{\frac{B_2}{B_1}}, \quad \text{or} \quad \frac{B_1}{B_2} = \frac{T_2^2}{T_1^2}.$$

Hence the flux densities in the two fields may be compared.

Comparison of Moments

The method of vibration can also be used to compare the magnetic moments of two magnets. The magnets are rigidly fixed together, with similar poles adjacent, and allowed to vibrate in the earth's field (Fig. 35.19 (i)). If m_1, m_2, K_1, K_2, are their magnetic moments and moments

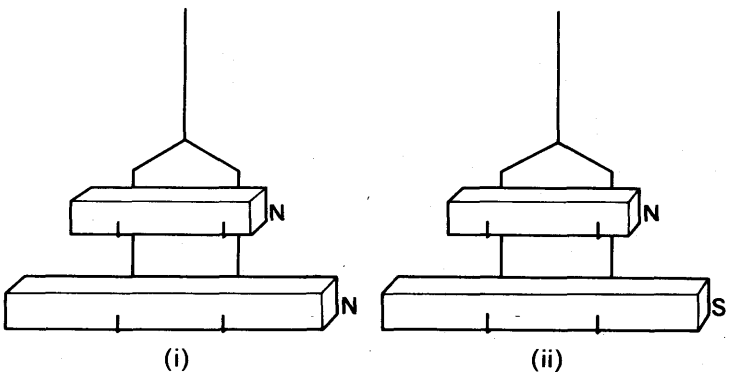


FIG. 35.19. Comparison of moments by vibration.

of inertia respectively, and if B_H is the earth's horizontal field, then the period of the magnets is

$$T_1 = 2\pi\sqrt{\frac{K_1+K_2}{(m_1+m_2)B_H}}.$$

The magnets are now fixed together with opposite poles adjacent, and again allowed to vibrate. Fig. 35.19 (ii). Their period is now

$$T_2 = 2\pi\sqrt{\frac{K_1+K_2}{(m_1-m_2)B_H}}.$$

Therefore

$$\frac{T_2}{T_1} = \sqrt{\frac{m_1+m_2}{m_1-m_2}},$$

whence

$$\frac{m_1}{m_2} = \frac{T_2^2+T_1^2}{T_2^2-T_1^2} \quad . \quad . \quad . \quad . \quad (1)$$

The Wattmeter

The wattmeter is an instrument for measuring electrical power. In construction and appearance it resembles a moving-coil voltmeter or ammeter, but it has no permanent magnet. Instead it has two fixed coils, FF in Fig. 35.20; these set up the magnetic field in which the suspended coil, M, moves. When the instrument is in use, the coils FF are connected in series with the device X whose power consumption is

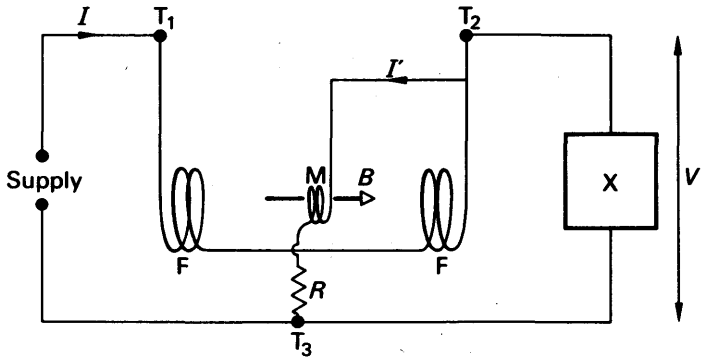


FIG. 35.20. Principle of wattmeter.

to be measured. The magnetic field B, set up by FF, is then proportional to the current I drawn by X:

$$B \propto I.$$

The moving coil M is connected across the device X. In series with M is a high resistance R, similar to the multiplier of a voltmeter; M is, indeed, often called the volt-coil. The current I' through the volt-coil

is small compared with the main current I, and is proportional to the potential difference V across the device X:

$$I' \propto V.$$

The couple acting on the moving coil is proportional to the current through it, and to the magnetic field in which it is placed:

$$C \propto BI'.$$

Consequently $$C \propto IV.$$

That is to say, the couple on the coil is proportional to the product of the current through the device X, and the voltage across it. The couple is therefore proportional to the power consumed by X, and the power can be measured by the deflection of the coil.

The diagram shows that, because the volt-coil draws current, the current through the fixed coils is a little greater than the current through X. As a rule, the error arising from this is negligible; if not, it can be allowed for as when a voltmeter and ammeter are used separately.

Hall Effect

In 1879, Hall found that an e.m.f. is set up *transversely* or *across* a current-carrying conductor when a perpendicular magnetic field is applied. This is called the *Hall effect*.

To explain the Hall effect, consider a slab of metal carrying a current (Fig. 35.21). The flow of electrons is in the opposite direction to the conventional current. If the metal is placed in a magnetic field B at

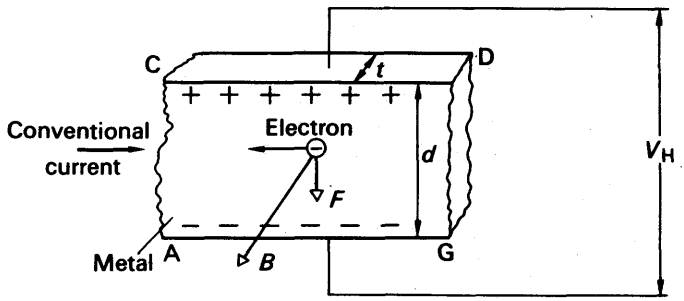


FIG. 35.21. Hall voltage.

right angles to the face AGDC of the slab and directed out of the plane of the paper, a force Bev then acts on each electron in the direction from CD to AG. Thus electrons accumulate along the side AG of the metal, which will make AG negatively charged and lower its potential with respect to CD. Thus a potential difference or e.m.f. opposes the electron flow. The flow ceases when the e.m.f. reaches a particular value V_H called the *Hall voltage*, which may be measured by using a high impedance voltmeter as shown in Fig. 35.22.

Magnitude of Hall Voltage

Suppose V_H is the magnitude of the Hall voltage and d is the width of the slab. Then the electric field intensity E set up across the slab is numerically equal to the potential gradient and hence $E = V_H/d$. Hence the force on each electron $= Ee = V_H e/d$.

This force, which is directed upwards from AG to CD, is equal to the force produced by the magnetic field when the electrons are in equilibrium.

$$\therefore Ee = Bev$$

$$\therefore \frac{V_H e}{d} = Bev$$

$$\therefore V_H = Bvd \quad . \quad . \quad . \quad . \quad . \quad (1)$$

From p. 787, the drift velocity of the electrons is given by

$$I = NevA, \quad . \quad . \quad . \quad . \quad . \quad (2)$$

where N is the number of electrons per unit volume and A is the area of cross section of the conductor. In this case $A = td$ where t is the thickness. Hence, from (2),

$$v = \frac{I}{Netd}$$

Substituting in (1),

$$\therefore V_H = \frac{BI}{Net} \quad . \quad . \quad . \quad . \quad . \quad (3)$$

We now take some typical values for copper to see the order of magnitude of V_H. Suppose $B = 1$ T, a field obtained by using a large laboratory electromagnet. For copper, $N \simeq 10^{29}$ electrons per metre^3, and the charge on the electron is $1{\cdot}6 \times 10^{-19}$ coulomb. Suppose the specimen carries a current of 10 A and that its thickness is about 1 mm or 10^{-3} m. Then

$$V_H = \frac{1 \times 10}{10^{29} \times 1{\cdot}6 \times 10^{-19} \times 10^{-3}} = 0{\cdot}6\ \mu\text{V (approx.)}.$$

This e.m.f. is very small and would be difficult to measure. The importance of the Hall effect becomes apparent when semiconductors are used, as we now see.

Hall Effect in Semiconductors

In semiconductors, the charge carriers which produce a current when they move may be positively or negatively charged (see p. 1024). The Hall effect helps us to find the sign of the charge carried. In Fig. 35.21, p. 890, suppose that electrons were not responsible for carrying the current, and that the current was due to the movement of positive charges in the *same* direction as the conventional current. The magnetic force on these charges would also be downwards, in the same direction as if the current were carried by electrons. This is because the sign *and* the direction of movement of the charge carriers have both been reversed. Thus AB would now become positively charged, and the polarity of the Hall voltage would be reversed. Experimental investigation of the polarity of the Hall voltage hence tells us whether the current is predominantly due to the drift of positive charges or to the drift of negative

charges. In this way it was shown that the current in a metal such as copper is due to movement of negative charges, but that in impure semiconductors such as germanium or silicon, the current may be predominantly due to movement of either negative or positive charges (p. 945).

The magnitude of the Hall voltage V_H in metals was shown on p. to be very small. In semiconductors it is much larger because the number N of charge carriers per metre^3 is much *less* than in a metal and $V_H = BI/Net$. Suppose that N is about 10^{25} per metre^3 in a semiconductor, and $B = 1$ T (Wb m^{-2}), $t = 10^{-3}$ m, $e = 1{\cdot}6 \times 10^{-19}$ C, as on p. 891. Then

$$V_H = \frac{1 \times 10}{10^{25} \times 1{\cdot}6 \times 10^{-19} \times 10^{-3}} = 6 \times 10^{-3} \text{ V (approx.)} = 6 \text{ mV}.$$

The Hall voltage is thus much more measurable in semiconductors than in metals.

Use of Hall Effect

An instrument called a *Hall probe* may now be used to measure the flux density B of a magnetic field. A simple Hall probe is shown in Fig. 35.22. Here a wafer of semiconductor has two contacts on opposite

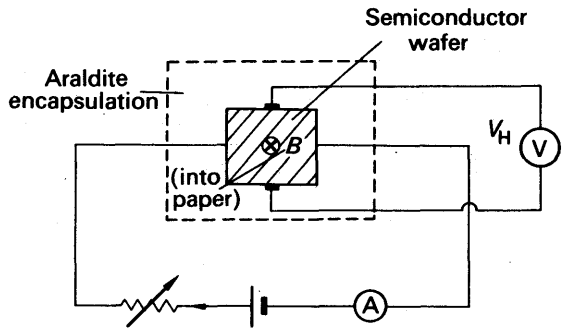


FIG. 35.22. B by Hall voltage.

sides which are connected to a high impedance voltmeter, V. A current, generally less than one ampere, is passed through the semiconductor and is measured on the ammeter, A. The 'araldite' encapsulation prevents the wires from being detached from the wafer. Now, from (3) on p. 891.

$$V_H = \frac{BI}{Net}$$

$$\therefore B = \frac{V_H Net}{I}$$

Now Net is a constant for the given semiconductor, which can be determined previously. Thus from the measurement of V_H and I, B can be found.

EXERCISES 35

1. With the aid of clear labelled diagrams describe the structure of a suspended moving-coil galvanometer and explain its mode of action. Derive an expression for its current sensitivity.

A certain galvanometer when shunted with a resistance 0·0500 ohm gives a full-scale deflection for 2 A, while if placed in series with a resistance 495·05 ohms it gives a full-scale deflection for 10 volts. Deduce the resistance of the galvanometer and the current required to produce a full-scale deflection when it is used alone. (*L.*)

2. A rectangular coil of wire of n turns and area A is suspended at the midpoint of one side by a fibre of torsional constant c so that its plane is parallel to a horizontal uniform field of magnetic induction (flux density) B. Derive an expression for the deflection of the coil when a steady current I flows through it.

Explain, with reasons, how a modern moving coil galvanometer of the suspended type has been developed from this simple arrangement.

Distinguish between the accuracy and sensitivity of such a galvanometer and explain the factors on which the sensitivity depends. (*N.*)

3. Describe a moving-coil type of galvanometer and deduce a relation between its deflection and the steady current passing through it.

A galvanometer, with a scale divided into 150 equal divisions, has a current sensitivity of 10 divisions per milliampere and a voltage sensitivity of 2 divisions per millivolt. How can the instrument be adapted to serve (*a*) as an ammeter reading to 6 A, (*b*) as a voltmeter in which each division represents 1 volt? (*L.*)

4. Describe an experiment to show that a force is exerted on a conductor carrying a current when it is placed in a magnetic field. Give a diagram showing the directions of the current, the field, and the force.

A rectangular coil of 50 turns hangs vertically in a uniform magnetic field of 10^{-2} T (Wb m^{-2}), so that the plane of the coil is parallel to the field. The mean height of the coil is 5 cm and its mean width 2 cm. Calculate the strength of the current that must pass through the coil in order to deflect it 30° if the torsional constant of the suspension is 10^{-9} newton metre per degree. Give a labelled diagram of a moving-coil galvanometer. (*L.*)

5. Explain the terms *magnetic flux, magnetic flux density (magnetic induction), magnetic flux linkage.*

Write down an expression for the force experienced by a long straight wire of length L carrying a current I when it is situated at right angles to a uniform field of flux-density B. State the units in which each of the quantities in the expression is measured, and show with the aid of a diagram the directions of the current, the magnetic field and the force.

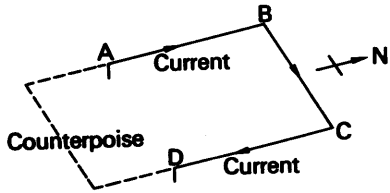


FIG. 35.23.

Fig. 35.23 represents a rigid rectangular wire frame with three members AB = 30 cm, BC = 60 cm, and CD = 30 cm. It is pivoted at the contacts A and D

and suitably counterpoised. When BC is at right angles to the magnetic meridian and a current of 5 A passes along ABCD as shown, the frame is found to be in equilibrium when its plane is horizontal. Find the magnitude and direction of the force on each of the members AB, BC, and CD, and also the moment of the deflecting couple which results. (Take the flux density of the earth's magnetic field to be 6×10^{-5} T (Wb m^{-2}) and the angle of dip to be 70°.)

State, and explain briefly without attempting any numerical calculation, what would be obtained (*a*) if the current through the frame were reversed, (*b*) if the whole system were turned through 90° about a vertical axis, the counterpoising couple in each case remaining the same as before. (*O.*)

6. Define *magnetic moment.*

A magnetized uniform rod of length 0·15 m is pivoted about a horizontal axis passing through its centre of mass. The axis of rotation is perpendicular to a uniform magnetic field of 64 A m^{-1} inclined at 60° to the horizontal. When a small mass of 0·2 g is fixed to one end of the rod, the rod sets horizontally. Draw a diagram to show the couples acting on the rod in this position. Calculate the magnetic moment of the rod. (*N.*)

7. Draw a clear labelled diagram showing the essential features of a moving-coil ballistic galvanometer. What differences in structure would be necessary and for what reasons, to make the galvanometer dead beat?

Describe, with a circuit diagram, how a ballistic galvanometer may be used to compare the capacitances of two capacitors and state the precautions necessary to obtain a reliable result. (*L.*)

8. Describe the construction of a sensitive moving-coil galvanometer. How could the instrument be adapted for use a millivoltmeter?

A standard cell of e.m.f. 1·018 volts and internal resistance 1000 ohms is joined to two resistances in series of values 149000 and 2 ohms respectively. The ends of the 2-ohm resistance are also connected to the terminals of a galvanometer of resistance 8 ohms, when a scale deflection of 100 mm is recorded. What is the sensitivity of the instrument expressed in microamperes per scale division? (*L.*)

9. With the help of a labelled diagram or diagrams describe the construction and explain the action of a pivoted moving-coil galvanometer. Indicate on diagrams how the direction of deflection is related to the direction of current flow and the polarity of the magnet.

If such an instrument has a resistance of 10 ohms and gives a full-scale deflection when a current of 20 milliamps is passing, how would it be converted into a voltmeter with 3- and 150-volt ranges? What reading would this voltmeter give when connected to a battery of e.m.f. 120 volts and internal resistance 300 ohms? (*L.*)

chapter thirty-six

Electromagnetic Induction

Faraday's Discovery

AFTER Ampere and others had investigated the magnetic effect of a current Faraday attempted to find its converse: he tried to produce a current by means of a magnetic field. He began work on the problem in 1825 but did not succeed until 1831.

The apparatus with which he worked is represented in Fig. 36.1; it consists of two coils of insulated wire, A, B, wound on a wooden core.

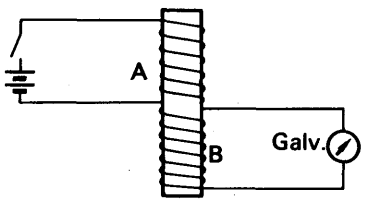


FIG. 36.1. Faraday's experiment on induction.

One coil was connected to a galvanometer, and the other to a battery. No current flowed through the galvanometer, as in all Faraday's previous attempts. But when he disconnected the battery Faraday happened to notice that the galvanometer needle gave a kick. And when he connected the battery back again, he noticed a kick in the opposite direction. However often he disconnected and reconnected the battery, he got the same results. The 'kicks' could hardly be all accidental—they must indicate momentary currents. Faraday had been looking for a steady current, but the effect he sought turned out to be a transient one—that was why it took him six years to find it.

Conditions for Generation of Induced Current

The results of Faraday's experiments showed that a current flowed in coil B of Fig. 36.1 only while the magnetic field due to coil A was changing—the field building up as the current in A was switched on, decaying as the current in A was switched off. And the current which flowed in B while the field was decaying was in the opposite direction to the current which flowed while the field was building up. Faraday called the current in B an induced current. He found that it could be made much greater by winding the two coils on an iron core, instead of a wooden one.

Once he had realized that an induced current was produced only by a change in the magnetic field inducing it, Faraday was able to find induced currents wherever he had previously sought them. In place of the coil A he used a magnet, and showed that as long as the coil and the magnet were at rest, there was no induced current (Fig. 36.2 (i)). But when he moved either the coil or the magnet an induced current flowed as long as the motion continued (Fig. 36.2 (ii)). If the current flowed one way when the north pole of the magnet was approaching

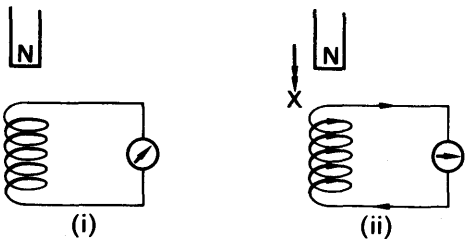


FIG. 36.2. Induction of current by moving magnet.

the end X of the coil, it flowed the other way when the north pole was retreating from X, or the south pole approached X.

Since a flow of current implies the presence of an e.m.f., Faraday's experiments showed that an e.m.f. could be induced in a coil by moving it relatively to a magnetic field. In discussing induction it is more fundamental to deal with the e.m.f. than the current, because the current depends on both the e.m.f. and the resistance.

Direction of E.M.F.; Lenz's Law

Before considering the magnitude of an induced e.m.f., let us investigate its direction. To do so we must first see which way the galvanometer deflects when a current passes through it in a known direction: we can find this out with a battery and a megohm resistor (Fig. 36.3 (i)). We then take a coil whose direction of winding we know, and connect this to the galvanometer. In turn we plunge each pole of a magnet into and out of the coil; and we get the results shown in Fig. 36.3 (ii), (iii), (iv). These results were generalized most elegantly into a rule by Lenz in 1835. He said that *the induced current flows always in such a direction as to oppose the change which is giving rise to it.* If the reader will sketch with a pencil on Fig. 36.3 the magnetic fields of the induced currents, then he will see what Lenz meant: when the magnet is approaching the coil, the coil repels it; when the magnet is retreating from the coil, the coil attracts it.

Lenz's law is a beautiful example of the conservation of energy: the induced current sets up a force on the magnet, which the mover of the

magnet must overcome: the work done in overcoming this force provides the electrical energy of the current. (This energy is dissipated as heat in the coil.) If the induced current flowed in the opposite direction to

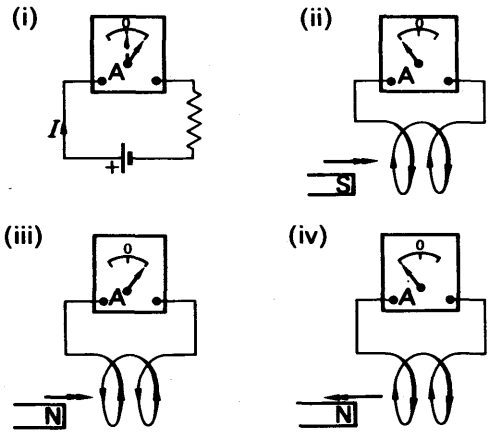


FIG. 36.3. Direction of induced currents.

that which it actually takes, then it would aid—it would speed up—the motion of the magnet. It would enhance its own cause, and grow indefinitely; at the same time, it would continuously increase the kinetic energy of the magnet. Thus both mechanical and electrical energy would be produced, without any agent having to do work. The system would be a perpetual motion machine.

The direction of the induced e.m.f., E, is specified by that of the current, as in Fig. 36.4. If we wished to reword Lenz's law, substituting e.m.f. for current, we would have to speak of the e.m.f.s *tending* to oppose the change... etc., because there can be no opposing force unless the circuit is closed and a current can flow.

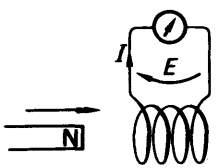


FIG. 36.4. Direction of induced e.m.f.

Magnitude of E.M.F.

Accurate experiments on induction are difficult to contrive with simple apparatus; but rough-and-ready experiments will show on what factors the magnitude depends. We require coils of the same diameter but different numbers of turns, coils of the same number of turns but different diameters, and two similar magnets, which we can use singly or together. If we use a high-resistance galvanometer, the current will not vary much with the resistance of the coil in which the e.m.f. is induced, and we can take the deflection as a measure of the e.m.f. There is no need to plunge the magnet into and out of the coil: we can

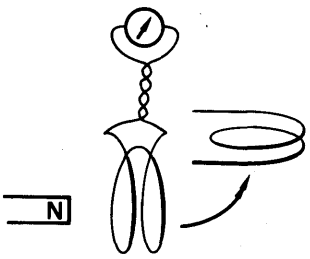


FIG. 36.5. E.m.f. induced by turning coil.

get just as great a deflection by simply turning the coil through a right angle, so that its plane changes from parallel to perpendicular to the magnet, or vice versa (Fig. 36.5). We find that the induced e.m.f. increases with:

(i) the speed with which we turn the coil;
(ii) the area of the coil;
(iii) the strength of the magnetic field (two magnets give a greater e.m.f. than one);
(iv) the number of turns in the coil.

To generalize these results and to build up useful formulae, we use the idea of *magnetic flux*, or field lines passing through a coil. Fig. 36.6 shows a coil, of area A, whose normal makes an angle θ with a uniform magnetic field of induction B. The component of the field at right angles to the plane of the coil is $B\cos\theta$, and we say that the magnetic flux Φ through the coil is

$$\Phi = AB\cos\theta \quad . \quad . \quad . \quad . \quad (1)$$

(We get the same result if we multiply the field-strength B by the area projected at right angles to the field, $A\cos\theta$.) If either the strength of the field is changed, or the coil is turned so as to change the angle θ, then the flux through the coil changes.

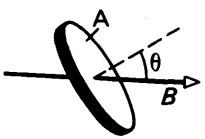


FIG. 36.6. Magnetic flux.

Results (i) to (iii) above, therefore, show that the e.m.f. induced in a coil increases with the *rate of change of the magnetic flux* through it. More accurate experiments show that the induced e.m.f. is actually proportional to the rate of change of flux through the coil; this result is sometimes called *Faraday's*, or *Neumann's, law.*

The unit of magnetic flux Φ is called the *weber* (Wb). Hence the unit of B is the *weber per metre*^{2} (Wb m^{-2}) or *tesla* (T).

Flux Linkage

If a coil has more than one turn, then the flux through the whole coil is the sum of the fluxes through the individual turns. We call this the *flux linkage* through the whole coil. If the magnetic field is uniform, the flux through one turn is given, from (1), by $AB\cos\theta$. If the coil has N turns, the total flux linkage Φ is given by

$$\Phi = NAB\cos\theta \quad . \quad . \quad . \quad . \quad (2)$$

From Faraday's or Neumann's law, the e.m.f. induced in a coil is proportional to the rate of change of the flux linkage, Φ. Hence

$$E \propto \frac{d\Phi}{dt},$$

or

$$E = -k\frac{d\Phi}{dt}, \quad . \quad . \quad . \quad . \quad (3)$$

where k is a positive constant. The minus sign expresses Lenz's law. It means that the induced e.m.f. is in such a direction that, if the circuit is closed, the induced current *opposes* the change of flux. Note that an induced e.m.f. exists across the terminals of a coil when the flux linkage changes, even though the coil is on 'open circuit'. A current, of course, does not flow in the latter case.

On p. 901, it is shown that $E = -kd\Phi/dt$ is consistent with the expression $F = BIl$ for the force on a conductor only if $k = 1$. We may therefore say that

$$E = -\frac{d\Phi}{dt}, \quad . \quad . \quad . \quad . \quad (4)$$

where Φ is the flux linkage in webers, t is in seconds, and E is in volts.

From (4), it follows that one weber is the flux linking a circuit if the induced e.m.f. is one volt when the flux is reduced uniformly to zero in one second.

E.M.F. Induced in Moving Rod

Generators at power stations produce high induced voltages by rotating long *straight conductors*. Fig. 36.7 (i) shows a simple apparatus

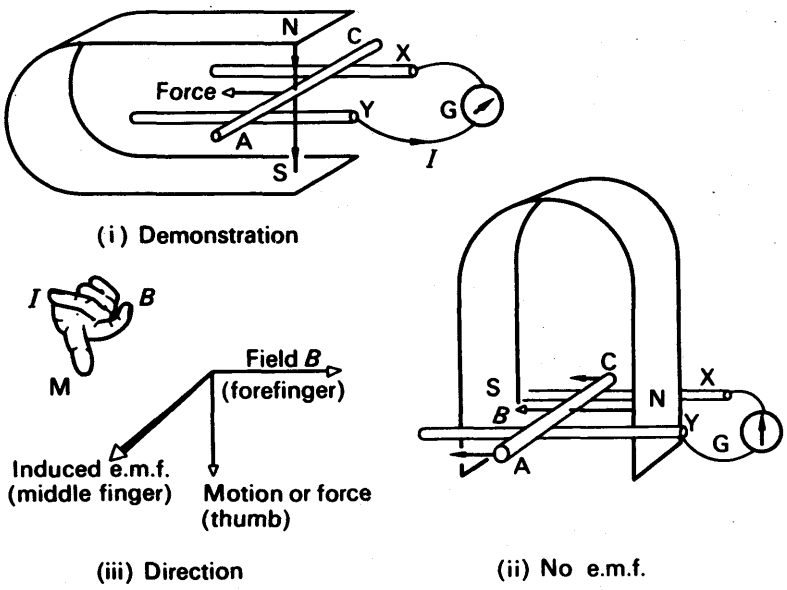


FIG. 36.7. E.m.f. induced in moving rod.

for demonstrating that an e.m.f. may be induced in a straight rod or wire, when it is moved across a magnetic field. The apparatus consists of a rod AC resting on rails XY, and lying between the poles NS of a permanent magnet. The rails are connected to a galvanometer G.

If we move the rod to the left, so that it cuts across the field B of the magnet, a current I flows as shown. If we move the rod to the right, the current reverses. We notice that the current flows only while the rod is moving, and we conclude that the motion of the rod AC induces an e.m.f. in it.

By turning the magnet into a vertical position (Fig. 36.7 (ii)) we can show that no e.m.f. is induced in the rod when it moves parallel to the field B. We conclude that an e.m.f. is induced in the rod only when it *cuts across* the field. And, whatever the direction of the field, no e.m.f. is induced when we slide the rod parallel to its own length. The induced e.m.f. is greatest when we move the rod at right angles, both to its own length and to the magnetic field. These results may be summarized in *Fleming's right-hand rule*:

If we extend the thumb and first two fingers of the right hand, so that they are all at right angles to one another, then the directions of field, motion, and induced e.m.f. are related as in Fig. 36.7 (iii).

To show E.M.F. $\propto$ Rate of Change

The variation of the magnitude of the e.m.f. in a rod with the speed of 'cutting' magnetic flux can be demonstrated with the apparatus in Fig. 36.8 (i).

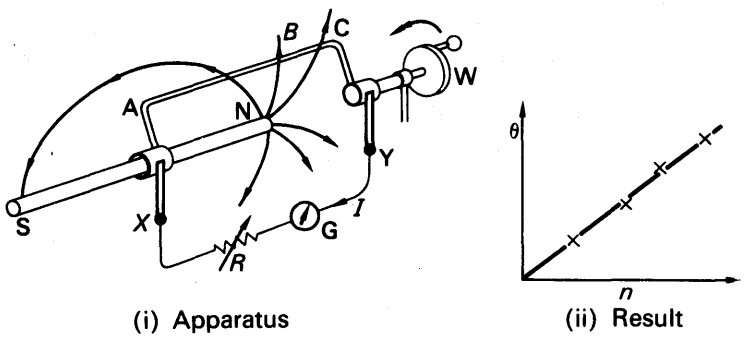


FIG. 36.8. Induced e.m.f.

Here AC is a copper rod, which can be rotated by a wheel W around one pole N of a long magnet. Brushing contacts at X and Y connect the rod to a galvanometer G and a series resistance R. When we turn the wheel, the rod AC cuts across the field B of the magnet, and an e.m.f. is induced in it. If we turn the wheel steadily, the galvanometer gives a steady deflection, showing that a steady current is flowing round the circuit.

To find how the current and e.m.f. depends on the speed of the rod, we keep the circuit resistance constant, and vary the rate at which we

turn the wheel. We time the revolutions with a stop-watch, and find that the deflection θ is proportional to the number of revolutions per second, n (Fig. 36.8 (ii)). It follows that the induced e.m.f. is proportional to the speed of the rod.

Calculation of E.M.F. in Rod

Consider the circuit shown in Fig. 36.9. PQ is a straight wire touching the two connected parallel wires QR, PS and free to move over them. All the conductors are situated in a uniform magnetic field of induction B, perpendicular to the plane of PQRS.

Suppose the rod PQ is pulled with a uniform velocity v by an external force F. There will then be a change of flux linkage in the area PQRS and so an e.m.f. will be induced in the circuit. This produces a current I which flows round the circuit. A *force* will now act on the wire PQ

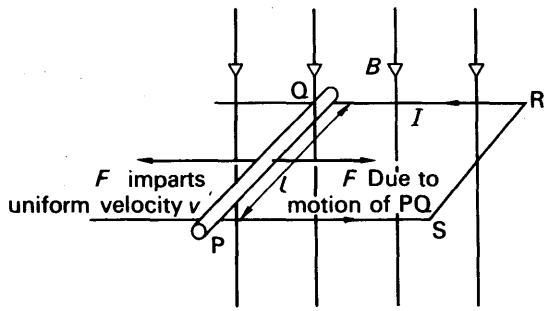


FIG. 36.9 Calculation of induced e.m.f.

due to the current flowing and to the presence of the magnetic field (p. 878). By Lenz's law, the direction of this force will oppose the movement of PQ. If the current flowing is I, and the length of PQ is l, the force on PQ is BIl. This is equal to the external force F, since PQ is not accelerating.

Because energy is conserved, the rate of working by the external force is equal to the rate at which energy is supplied to the electrical circuit. Now in one second, PQ moves a distance v. Hence

$$\text{work done per second} = \text{force} \times \text{distance moved per second.}$$

$$= BIlv$$

If the induced e.m.f. is E, the electrical energy used in one second, or power, $= EI$.

$$\therefore EI = BIlv$$

$$\therefore E = Blv. \quad . \quad . \quad . \quad (6)$$

This result has been derived without using the relation $E = -d\Phi/dt$. To see if the same result as (6) can be obtained, consider the flux changes. In one second the area of PQRS changes by vl. Hence the

change in flux linkage per second, $d\Phi/dt = B \times$ area change per second $= Blv$. Hence, numerically,

$$\therefore E = Blv.$$

This means that the relation $E = -d\Phi/dt$ may be used to find the induced e.m.f. in a straight wire.

Induced E.M.F. and Force on Moving Electrons

We have seen that an electron moving across a magnetic field experiences a mechanical force (p. 881). This explains neatly the e.m.f. induced in a wire: when we move the wire across the field, we move each free electron in it, likewise across the field. As Fig. 36.10 shows, the force on the electrons, F, is at right angles to the plane containing the velocity v of the wire, and the magnetic field B. Thus it tends to drive the electrons along the wire. The direction in which it does so agrees with the direction of the conventional e.m.f., which is the direction of the force on a positive charge.

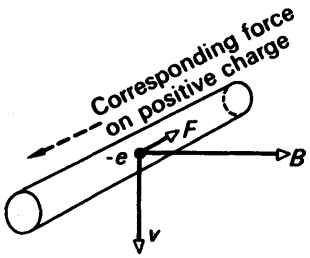


FIG. 36.10. Force on a moving electron.

When a wire AC is swept, as shown in Fig. 36.11, across a magnetic field B, the force on the electrons in it acts from A to C. Therefore, if the wire is not connected to a closed circuit, electrons will pile up at C: the end C will gain a negative

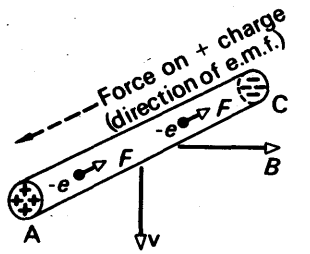


FIG. 36.11. Induced e.m.f. arising from force on moving electrons.

charge, and A will be left with a positive charge. The end A will therefore be at a higher potential than C.

If we now clear our minds of electrons, we see that the conventional e.m.f. acts from C to A. If positive electricity were free to move, it would accumulate at A;

in other words, the tendency of the e.m.f., acting from C to A, is to give A a higher potential than C. The *potential difference* between A and C tends to drive positive electricity the other way. Equilibrium is reached when the potential difference V_{AC} is equal to the e.m.f. acting from C to A.

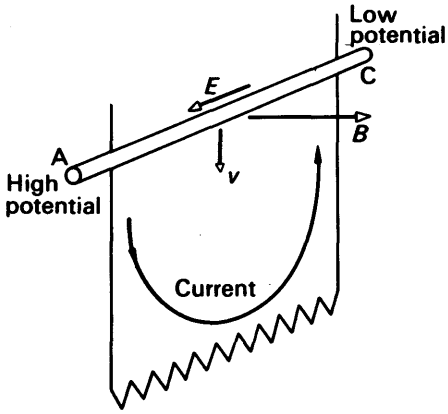


FIG. 36.12. E.m.f. and potential difference.

When the wire is connected to a closed circuit, current flows from A to C round the external circuit (Fig. 36.12). Within the source of current—the wire AC—the e.m.f. drives the current (of positive charge) from C to A: from low potential to high. *This is the essential function of an e.m.f.*; an e.m.f. is an agency which can drive an electric current *against* a potential difference. When the e.m.f. arises in a wire moving across a magnetic field, this agency is the force on the electrons moving with the wire.

The e.m.f. induced in a wire can easily be calculated from the force on a moving electron. If the wire moves with a velocity v at right angles to a field B, then so do the electrons in it. Each of them therefore experiences a force

$$F = Bev,$$

(equation (1), p. 881), where e is the electronic charge. The work which this force does in carrying the electron along the length l of the wire is Fl. But it is also, by definition, equal to the product of the e.m.f. E, and the charge e. Therefore

$$Ee = Fl = Bevl,$$

whence

$$E = Blv.$$

APPLICATIONS OF INDUCTION

The Induction Coil

The induction coil is a device for getting a high voltage from a low one. It was at one time used for X-ray tubes (p. 1067), and is nowadays used in car radios. It consists of a core of iron wires, around which is wrapped a coil of about a hundred turns of thick insulated wire, called the primary (Fig. 36.13 (i)). Around the primary is wound the secondary coil, which has many thousands of turns of fine insulated wire. The

primary is connected to a battery of accumulators, via a make-and-break M, which works in the same way as the contact-breaker of an electric bell: it switches the current on and off many times a second, thus varying the magnetic flux.

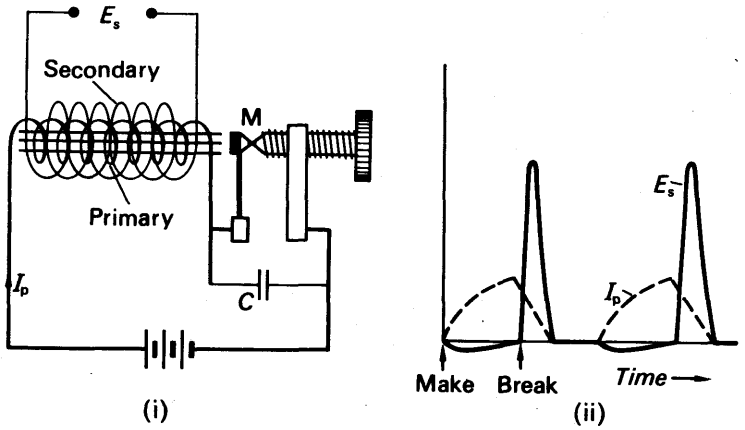


FIG. 36.13. Induction coil.

When the primary current I_p is switched on, the rise of its magnetic field induces an e.m.f. E_s in the secondary. A similar e.m.f., but in the opposite sense, is induced in the secondary when the primary current is switched off, by the collapse of the magnetic field. The secondary e.m.f.s are determined by the number of turns in the secondary coil, and by the rate of change of the magnetic flux through the iron core. Because of the great number of secondary turns, the secondary e.m.f.s may be high and of the order of thousands of volts (Fig. 36.13 (ii)).

In practice, an induction coil such as we have described—consisting simply of primary, secondary, and contact-breaker—would not give high secondary e.m.f.s. For, at the make of the primary current, the current would rise slowly, because of the self-inductance (see p. 924) of the primary winding. The rate of change of flux linked with the secondary would therefore be small, and the secondary e.m.f. low. And at the break of the primary current a spark would pass between the contacts of the make-and-break. The spark would allow primary current to continue to flow, and the primary current would fall slowly. At the instant of break, before the spark began, the primary current would be falling rapidly and the secondary e.m.f. would be high; but the e.m.f. would remain high for only a very short time: as soon as the spark passed the secondary e.m.f. would fall to a value about as low as at make.

Nothing can be done about the low secondary e.m.f. at make. But the secondary e.m.f. at break can be made high, by preventing sparking at the contact-breaker. To prevent sparking, a capacitor, C in Fig. 36.13, is connected across the contacts.

As we shall see on p. 926, the capacitor actually *slows down* the fall of the primary current at the instant of break; but in doing so it prevents the induced e.m.f. in the primary from rising high enough to start a spark. And the rate at which the primary current falls, in charging the capacitor, is greater than the rate at which it would fall if a spark were passing. Thus, with a capacitor, the secondary e.m.f. is less at the instant of break than without one, but it is greater throughout the rest of the fall of the primary current. Consequently the average secondary voltage at break is higher with a capacitor than without; in practice it is much higher. To get the greatest possible secondary voltage, the capacitance of the capacitor is chosen so that it just suppresses sparking at the contacts. The secondary voltage is then a series of almost unidirectional pulses, as shown in Fig. 36.13 (ii).

The iron core of an induction coil is made from a bundle of wires, to minimize eddy-currents (p. 913). If eddy-currents were to flow they would, by Lenz's law, set up a flux opposing the change of primary current. Thus they would reduce the secondary e.m.f.

The Dynamo and Generator

Faraday's discovery of electromagnetic induction was the beginning of electrical engineering. Nearly all the electric current used today is generated by induction, in machines which contain coils moving continuously in a magnetic field. Fig. 36.14 illustrates the principle of such a machine, which is called a dynamo, or generator. A coil DEFG, shown for simplicity as having only one turn, rotates on a shaft, which is not shown, between the poles NS of a horseshoe magnet. The ends of the coil are connected to flat brass rings R, which are supported on the shaft by discs of insulating material, also not shown. Contact with the rings is made by small blocks of carbon B, supported on springs, and shown connected to a lamp L. As the coil rotates, the flux linking it changes, and a current is induced in it which flows, via the carbon blocks, through the lamp. The magnitude (which we study shortly) and the direction of the current are not constant. Thus when the coil is in the position shown, the limb ED is moving downwards through the lines of force, and GF is moving upwards. Half a revolution later, ED and GF will have interchanged their positions, and ED will be moving upwards. Consequently, applying Fleming's right-hand rule, the current round the coil must reverse as ED changes from downward to upward motion. The

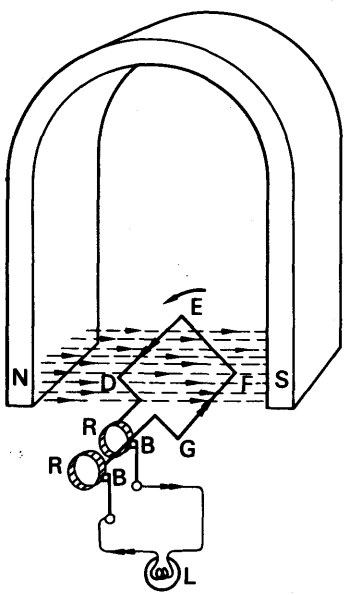


FIG. 36.14. A simple dynamo.

actual direction of the current at the instant shown on the diagram is indicated by the double arrows, from Fleming's rule. By applying this rule, it can be seen that the current reverses every time the plane of the coil passes the vertical position.

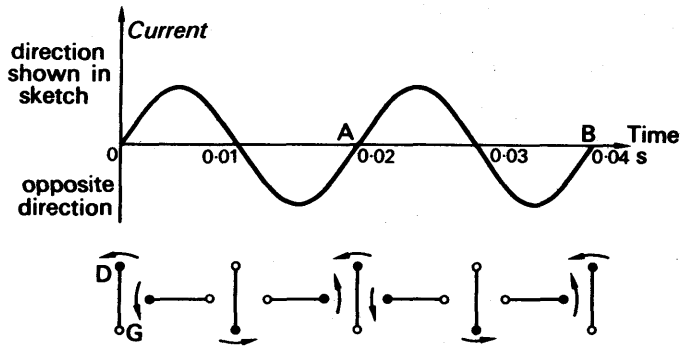


FIG. 36.15. Current generated by dynamo of Fig. 36.14, plotted against time and coil position.

We shall see shortly that the magnitude of the e.m.f. and current varies with time as shown in Fig. 36.15; this diagram also shows the corresponding position of DG. This type of current is called an *alternating current* (*A.C.*). A complete alternation, such as from A to B in the figure, is called a 'cycle'; and the number of cycles which the current goes through in one second is called its 'frequency'. The frequency of the current represented in the figure is that of most domestic supplies in Britain—50 Hz (cycles per second).

E.M.F. in Dynamo

We can now calculate the e.m.f. in the rotating coil. If the coil has an area A, and its normal makes an angle θ with the magnetic field B, as in Fig. 36.16, then the flux through the coil

$= AB \cos \theta$ (see p. 898).

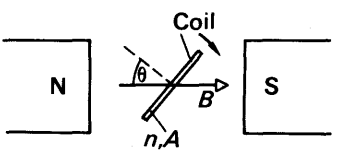


FIG. 36.16. Coil inclined to magnetic field.

The flux linkages with the coil, if it has N turns, are

$$\Phi = NAB \cos \theta.$$

If the coil turns with a steady angular velocity ω or $d\theta/dt$, then the e.m.f. induced in volts in the coil is

$$E = -\frac{d\Phi}{dt}$$

$$= -NAB\frac{d}{dt}(\cos \theta)$$

$$= NAB \sin \theta \frac{d\theta}{dt} \quad . \quad . \quad . \quad . \quad (1)$$

In terms of the number of revolutions per second, f, which the coil makes, we have

$$\frac{d\theta}{dt} = 2\pi f$$

and $$\theta = 2\pi ft,$$

$$\therefore E = 2\pi f NAB \sin 2\pi ft. \quad . \quad . \quad (2)$$

Thus the e.m.f. varies sinusoidally with time, like the pressure in a sound-wave, the frequency being f cycles per second.

The maximum (peak) value or amplitude of E occurs when $\sin 2\pi ft$ reaches the value 1. If the maximum value is denoted by E_0, it follows that

$$E_0 = 2\pi f NAB,$$

and $$E = E_0 \sin 2\pi ft. \quad . \quad . \quad . \quad (3)$$

The e.m.f. E sends an alternating current, of a similar sine equation, through a resistor connected across the coil.

Alternators

Generators of alternating current are often called *alternators*. In all but the smallest, the magnetic field of an alternator is provided by an electromagnet called a field-magnet or *field*, as shown in Fig. 36.17; it has a core of cast steel, and is fed with direct current from a separate d.c.

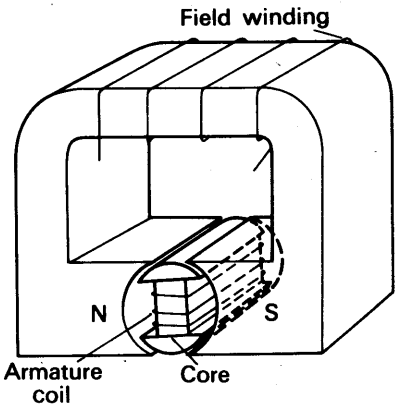


FIG. 36.17. Field magnet and armature.

generator. The rotating coil, called the armature, is wound on an iron core, which is shaped so that it can turn within the pole-pieces of the field-magnet. With the field-magnet, the armature core forms a system which is almost wholly iron, and can be strongly magnetized by a small current through the field winding. The field in which the armature turns is much stronger than if the coil had no iron core, and the e.m.f. is

proportionately greater. In the small alternators used for bicycle lighting the armature is stationary, and the field is provided by permanent magnets, which rotate around it. In this way rubbing contacts, for leading the current into and out of the armature, are avoided.

When no current is being drawn from a generator, the horse-power required to turn its armature is merely that needed to overcome friction, since no electrical energy is produced. But when a current is drawn, the horse-power required increases, to provide the electrical power. The current, flowing through the armature winding, causes the magnetic field to set up a couple which opposes the rotation of the armature, and so demands the extra horse-power. The reader should check the truth of this statement by marking the direction of the e.m.f., current, and force on the limbs of the coil in Fig. 36.17.

The Transformer

A transformer is a device for stepping up—or down—an alternating voltage. It has primary and secondary windings, as in an induction coil, but no make-and-break (Fig. 36.18). It has an iron core, which is made from E-shaped laminations, interleaved so that the magnetic flux does not pass through air at all; in this way the greatest flux is obtained

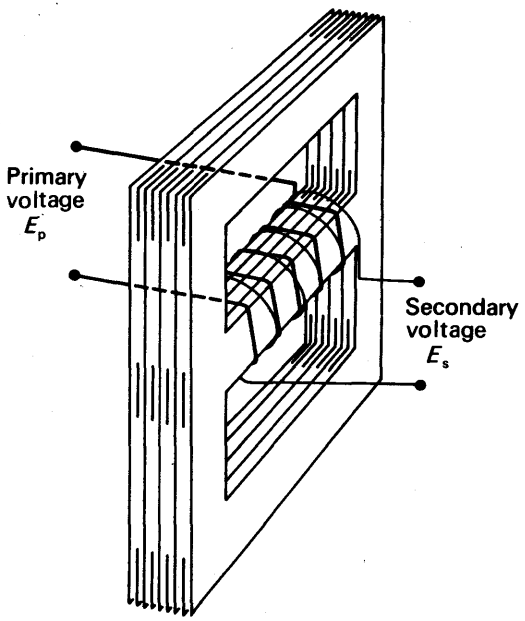


FIG. 36.18. Transformer.

with a given current. When an alternating e.m.f. E_p is impressed on the primary winding, it sends an alternating current through it, which sets up an alternating flux in the core of magnitude BA, where B is

the induction and A is the cross-sectional area. This induces an alternating e.m.f. in the secondary E_s. If N_p, N_s are the number of turns in the primary and secondary coils, their linkages with the flux Φ are:

$$\Phi_p = N_p AB \qquad \Phi_s = N_s AB$$

The magnitude of the e.m.f. induced in the secondary is, from the formula on p. 899:

$$E_s = \frac{d\Phi_s}{dt} = N_s A \frac{dB}{dt}$$

The changing flux also induces a back-e.m.f. in the primary, whose magnitude is

$$E_p = \frac{d\Phi_p}{dt} = N_p A \frac{dB}{dt}$$

Because the primary winding has inevitably some resistance, the current flowing through it sets up a voltage drop across the resistance. But in practice this is negligible compared with the back-e.m.f. due to the changing flux. Consequently we may say that the voltage applied to the primary, from the source of current, is used simply in overcoming the back-e.m.f. E_p. Therefore it is equal in magnitude to E_p. (This is analogous to saying, in mechanics, that action and reaction are equal and opposite.) Consequently we have

$$\frac{\textit{e.m.f. induced in secondary}}{\textit{voltage applied to primary}} = \frac{E_s}{E_p} = \frac{N_s}{N_p} \quad . \quad . \quad (1)$$

Thus the transformer steps voltage up or down according to its '*turns-ratio*':

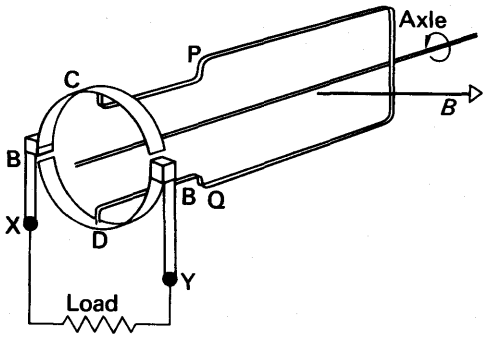
$$\frac{\text{secondary voltage}}{\text{primary voltage}} = \frac{\text{secondary turns}}{\text{primary turns}}.$$

When a load is connected to the secondary winding, a current flows in it. This current flows in such a direction as to reduce the flux in the core. At the instant that the load is connected, therefore, the back-e.m.f. in the primary falls. The primary current then increases. The increase in primary current increases the flux through the core, and continues until the flux is restored to its original value. The back-e.m.f. in the primary is then again equal to the applied voltage, and equilibrium is restored. But now a greater primary current is flowing than before the secondary was loaded. Thus the power drawn from the secondary is drawn, in turn, from the supply to which the primary is connected.

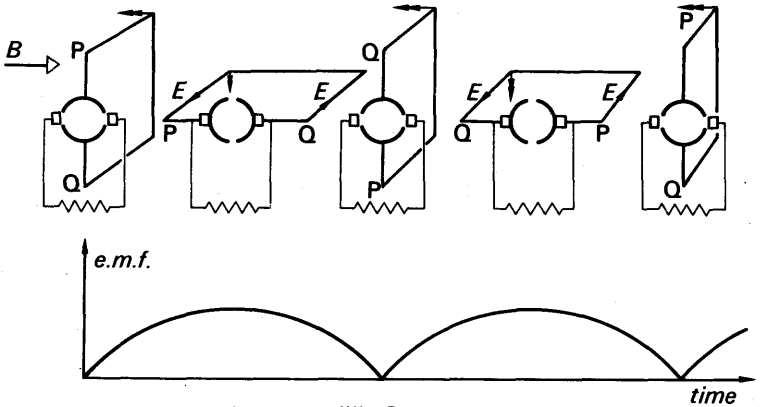
Transformers are used to step up the voltage generated at a power station, from 11000 to 132 000 volts for high-tension transmission (p. 793). After transmission they are used to step it down again to a value safer for distribution (240 volts in houses). Inside a house a transformer may be used to step the voltage down from 240 to 4, for ringing bells. Transformers with several secondaries are used in, for example, radio-receivers, where several different voltages are required.

D.C. Generators

Fig. 36.19 (i) is a diagram of a direct-current generator or dynamo. Its essential difference from an alternator is that the armature winding is connected to a *commutator* instead of slip-rings. The commutator consists of two half-rings of copper C, D, insulated from one another, and turning with the coil. Brushes BB, with carbon tips, press against the commutator and are connected to the external circuit. The commutator



(i) Principle



(ii) Output

FIG. 36.19. D.C. generator.

is oriented so that it reverses the connexions from the coil to the circuit at the instant when the e.m.f. reverses in the coil. Fig. 36.19 (ii) shows several positions of the coil and commutator, and the e.m.f. observed at the terminals XY. This e.m.f. pulsates in magnitude, but it acts always in the same sense round the circuit connected to XY. It is a pulsating direct e.m.f. The average value in this case can be shown to be $2/\pi$ of the maximum e.m.f. E_0, given in equation (3), p. 907.

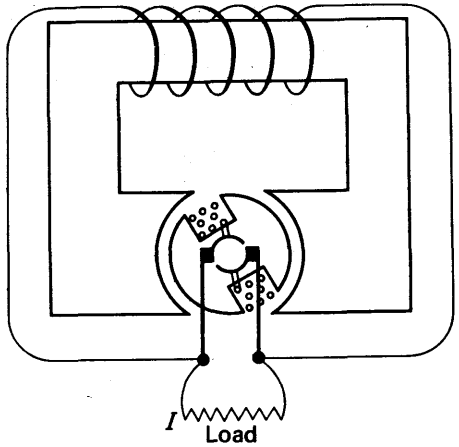


FIG. 36.20. D.C. generator with energized field.

In practice, as in an alternator, the armature coil is wound with insulated wire on a soft iron core, and the field-magnet is energized by a current (Fig. 36.20). This current is provided by the dynamo itself. The steel of the field-magnet has always a small residual magnetism, so that as soon as the armature is turned an e.m.f. is induced in it. This then sends a current through the field winding, which increases the field and the e.m.f.; the e.m.f. rapidly builds up to its working value.

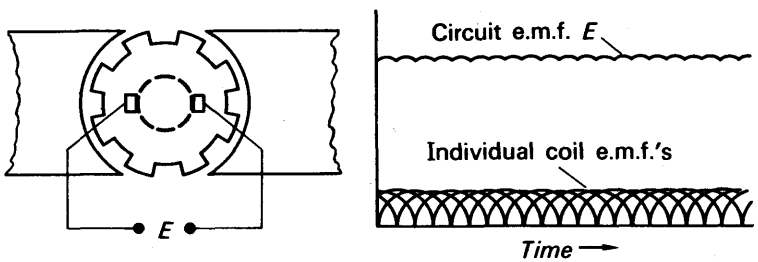


FIG. 36.21. D.C. generator with segment commutator.

Most consumers of direct current wish it to be steady, not pulsating as in Fig. 36.19. A reasonably steady e.m.f. is given by an armature with many coils, inclined to one another, and a commutator with a correspondingly large number of segments. The coils are connected to the commutator in such a way that their e.m.f.s add round the external circuit (Fig. 36.21).

Homopolar Generator

Another type of generator, which gives a very steady e.m.f., is illustrated in Fig. 36.22. It consists of a copper disc which rotates between

the poles of a magnet; connexions are made to its axle and circumference. If we assume (as is not true) that the magnetic field B is uniform over the radius XY, then we can calculate the induced e.m.f. E. In one revolution the radius XY sweeps out an area $(\pi(r_1^2-r_2^2)$, where r_1 and r_2 are the radii of the wheel and the axle. If T is the time for one revolution, then the rate at which XY sweeps out are is $\pi(r_1^2-r_2^2)/T$.

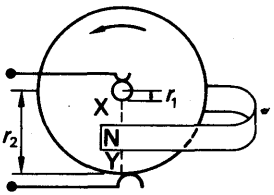


FIG. 36.22.
E.m.f. induced in a disc.

The rate at which it sweeps out flux is therefore

$$\frac{\pi(r_1^2-r_2^2)}{T}B = \pi(r_1^2-r_2^2)Bf,$$

where f denotes the revolutions of the wheel per second. Thus

$$E = \pi(r_1^2-r_2^2)Bf$$
$$= \pi(r_1^2-r_2^2)Bf$$

Generators of this kind are called *homopolar* because the e.m.f. induced in the moving conductor is always in the same sense. They are sometimes used for electroplating, where only a small voltage is required, but they are not useful for most purposes, because they give too small an e.m.f. The e.m.f. of a commutator dynamo can be made large by having many turns in the coil; but the e.m.f. of a homopolar dynamo is limited to that induced in one radius of the disc.

Applications of Alternating and Direct Currents

Direct currents are less easy to generate than alternating currents, and alternating e.m.f.s are more convenient to step up and to step down, and to distribute over a wide area. The national grid system, which supplies electricity to the whole country, is therefore fed with alternating current. Alternating current is just as suitable for heating as is direct current, because the heating effect of a current is independent of its direction. It is also equally suitable for lighting, because filament lamps depend on the heating effect, and gas-discharge lamps —neon, sodium, mercury—run as well on alternating current as on direct. Small motors, of the size used in vacuum-cleaners and common machine-tools, run satisfactorily on alternating current, but large ones, as a general rule, do not. Direct current is therefore used on most electric railway and tramway systems. These systems either have their own generating stations, or convert alternating current from the grid into direct current. One way of conveying alternating current into direct is to use a valve rectifier, whose principle we shall describe later.

For electro-chemical processes alternating current is useless. The chemical effect of a current reverses with its direction, and if, therefore, we tried to deposit a metal by alternating current, we would merely cause a small amount of the metal to be alternately deposited and

dissolved. For electroplating, and for battery charging, alternating current must be rectified.

Eddy-currents

The core of the armature of a dynamo is built up from thin sheets of soft iron insulated from one another by an even thinner film of oxide, as shown in Fig. 36.23 (i). These are called laminations, and the armature is said to be laminated. If the armature were solid, then, since iron is a conductor, currents would be induced in it by its motion across the magnetic field (Fig. 36.23 (ii)). These currents would absorb power by opposing the rotation of the armature, and they would dissipate that power as heat, which would damage the insulation of the winding; but

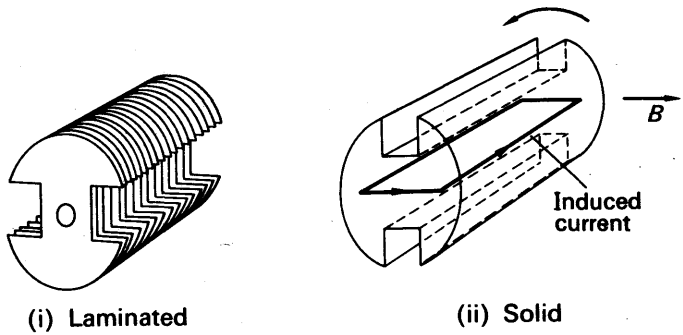


FIG. 36.23. Armature cores.

when the armature is laminated, these currents cannot flow, because the induced e.m.f. acts at right angles to the laminations, and therefore to the insulation between them. The magnetization of the core, however, is not affected, because it acts along the laminations. Thus the eddy-currents are suppressed, while the desired e.m.f.—in the armature coil—is not.

Eddy-currents, by Lenz's law, always tend to oppose the motion of a solid conductor in a magnetic field. The opposition can be shown in many ways. One of the most impressive is to make a chopper with a thick copper blade, and to try to slash it between the poles of a strong electromagnet; then to hold it delicately and allow it to drop between them. The resistance to the motion in the former case can be felt.

Sometimes eddy-currents can be made use of—for example, in damping a galvanometer. When a current is passed through the coil of a galvanometer, it applies a couple to the coil which sets it swinging. If the swings are opposed only by the viscosity of the air, they decay very slowly and are said to be naturally damped (Fig. 36.24). The pointer or light-spot takes a long time to come to its final steady deflection θ. To bring the spot or pointer more rapidly to rest, the damping must be increased. One way of increasing the damping is to wind the coil on a metal former. Then, as the coil swings, the field of

the permanent magnet induces eddy-currents in it; and these, by Lenz's law, oppose its motion. They therefore slow down the turning of the coil towards its eventual position, but they also suppress its swings about that position; in the end the coil comes to rest sooner than if it

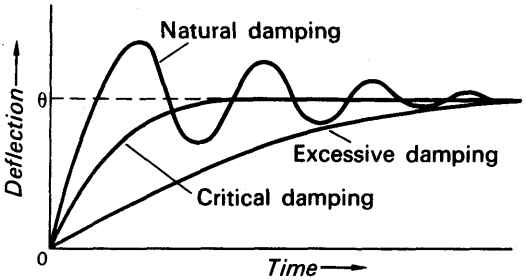


FIG. 36.24. Damping of galvanometer.

were not damped. Galvanometer coils which are wound on insulating formers can be damped by short-circuiting a few of their turns, or by connecting an external shunt across the whole coil. With a shunt the eddy-currents circulate round the coil and shunt, independently of the current to be measured. The smaller the shunt, the greater the eddy-currents and the damping; if the coil is overdamped, as shown in Fig. 36.24, it may take almost as long to come to rest as when it is undamped. The damping which is just sufficient to prevent overshoot is called 'critical' damping.

Electric Motors

If a simple direct-current dynamo, of the kind described on p. 910, is connected to a battery it will run as a motor (Fig. 36.25). Current flows round the armature coil, and the magnetic field exerts a couple on this, as in a moving-coil galvanometer. The commutator reverses the current just as the limbs of the coil are changing from upward to

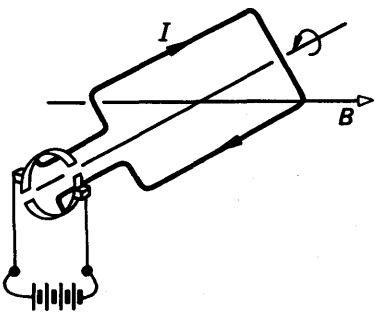


FIG. 36.25. Principle of D.C. motor.

downward movement and vice versa. Thus the couple on the armature is always in the same sense, and the shaft turns continuously. (The reader should verify these statements with the help of Fig. 36.25.)

The armature of a motor is laminated, in the same way and for the same reason, as the armature of a dynamo.

Back-e.m.f. in Motor

When the armature of a motor rotates, an e.m.f. is induced in its windings; by Lenz's law this e.m.f. opposes the current which is making the coil turn. It is therefore called a back-e.m.f. If its magnitude is E, and V is the potential difference applied to the armature by the supply, then the armature current is

$$I_a = \frac{V-E}{R_a}. \qquad . \quad . \quad . \quad (1)$$

Here R_a is the resistance of the armature, which is generally small—of the order of 1 ohm.

The back-e.m.f. E is proportional to the strength of the magnetic field, and the speed of rotation of the armature. When the motor is first switched on, the back-e.m.f. is zero: it rises as the motor speeds up. In a large motor the starting current would be ruinously great; to limit it, a variable resistance is inserted in series with the armature, which is gradually reduced to zero as the motor gains speed.

When a motor is running, the back-e.m.f. in its armature E is not much less than the supply voltage V. For example, a motor running off the mains ($V = 230$ volts) might develop a back-e.m.f. $E = 220$ volts. If the armature had a resistance of 1 ohm, the armature current would then be 10 amp (equation (1)). When the motor was switched on, the armature current would be 230 amp if no starting resistor were used.

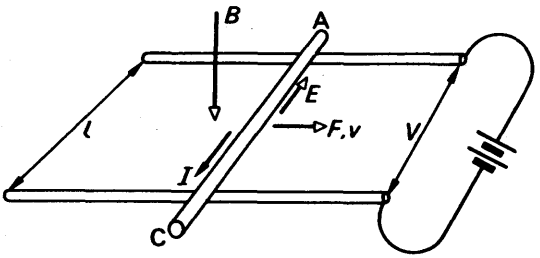


FIG. 36.26. Back-e.m.f. and mechanical power.

Back-e.m.f. and Power

The back-e.m.f. in the armature of a motor represents the mechanical power which it develops. To see that this is so, we use an argument similar to that which we used in finding an expression for the e.m.f. induced in a conductor. We consider a rod AC, able to slide along rails, in a plane at right angles to a magnetic field B (Fig. 36.26). But we now suppose that a current I is maintained in the rod by a battery, which sets up a potential difference V between the rails. The magnetic field then exerts a force F on the rod, given by

$$F = BIl.$$

The force F makes the rod move; if its velocity is v, the mechanical power developed by the force F is

$$P_m = Fv = BIlv \quad . \quad . \quad . \quad . \quad (1)$$

As the rod moves, a back-e.m.f. is induced in it, whose magnitude E is given by the expression for the e.m.f. in a moving rod (p. 901):

$$E = Blv. \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Equations (1) and (2) together give

$$P_m = EI. \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Thus the mechanical power developed is equal to the product of the back-e.m.f. and the current.

Before returning to consider motors, we may complete the analysis of the action represented in Fig. 36.26. If R is the resistance of the rails and rod, the heat developed in them is I^2R. The power supplied by the battery is IV, and the battery is the only source of power in the whole system. Therefore

$$IV = I^2R + P_m; \quad . \quad . \quad . \quad . \quad (4)$$

the power supplied by the battery goes partly into heat, and partly into useful mechanical power. Also, by equation (3),

$$IV = I^2R + EI, \quad . \quad . \quad . \quad . \quad (5)$$

whence $V = IR + E$

or $I = \dfrac{V-E}{R}$.

This is equation (1), p. 915, which we previously obtained simply from Ohm's law.

Let us apply this theory to the example which we were considering. We had:

supply voltage, V, = 230 volts;

back-e.m.f., E, = 220 volts;

armature resistance, R_a, = 1 ohm;

armature current, I_a, = 10 amp.

The power dissipated as heat in the armature is $I_a^2R_a = 100 \times 1 = 100$ watts. The power supplied to the armature is $I_aV = 10 \times 230 = 2300$ watts, and the mechanical power is $I_aE = 10 \times 220 = 2200$ watts. Of the power supplied to the armature, the fraction which appears as mechanical power is 2200/2300 = 96 per cent. This is not, however, the efficiency of the motor as a whole, because current is taken by the winding on the field magnet.

The Field Winding

The field winding of a motor may be connected in series or in parallel with the armature. If it is connected in series, it carries the armature current, which is large (Fig. 36.27). The field winding therefore has few turns of thick wire, to keep down its resistance, and so the power wasted in it as heat. The few turns are enough to magnetize the iron, because the current is large. If the field coil is connected in parallel with the armature, as in Fig. 36.28, the motor is said to be 'shunt-wound'. The field winding has many turns of fine wire to keep down

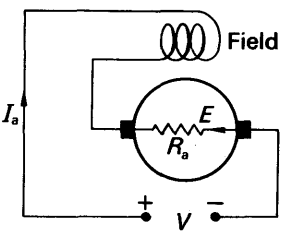


FIG. 36.27. Series-wound motor.

the current which it consumes. In the above example, if the motor is shunt-wound and the field current is 0·5 A, then the power dissipated as heat in the field is $0{\cdot}5 \times 230 = 115$ watts. The power consumption of the motor is therefore $2300 + 115 = 2415$ watts, and its efficiency is

$$\frac{\text{mechanical power developed}}{\text{electrical power consumed}} = \frac{2220}{2415}$$

$$= 92 \text{ per cent.}$$

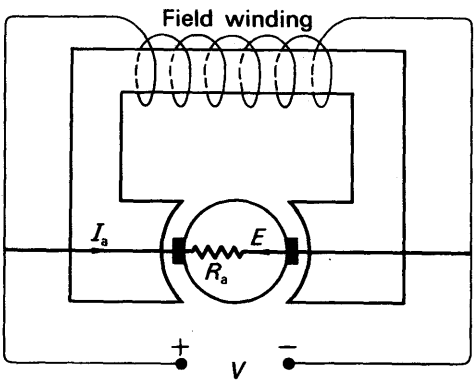


FIG. 36.28. Current and voltages in shunt-wound motor.

The working efficiency of the motor will be a little less than this, because some of the mechanical power will be used in overcoming friction in the bearings.

Shunt Field

Shunt-wound motors are used for driving machine-tools, and in other jobs where a steady speed is required. A shunt motor keeps a nearly steady speed for the following reason. If the load is increased, the speed falls a little; the back-e.m.f. then falls in proportion to the speed, and the current rises, enabling the motor to develop more power to overcome the increased load. In the example, p. 916, if the speed falls by 1 part in 220, the back-e.m.f. falls from 220 to 219 V. The current then rises from $\frac{230-220}{1} = 10$ A to $\frac{230-219}{1} = 11$ A. And the mechanical power increases from $220 \times 10 = 2200$ watts to $219 \times 11 = 2409$ watts ($\simeq 2400$). Thus an increase in load of $\frac{2400-2200}{2200} = 9$ per cent causes a fall in speed of 1 part in 220—less than $\frac{1}{2}$ per cent.

Series Field

Series motors are used where great torque is required in starting—for example, in cranes.

They develop a great starting torque because the armature current flows through the field coil. At the start the armature back-e.m.f. is small, and the current is great—as great as the starting resistance will allow. The field-magnet is therefore very strongly magnetized. The torque on the armature is proportional to the field and to the armature current; since both are great at the start, the torque is very great.

A series motor does not keep such a steady speed as a shunt motor. Just as in a shunt motor, when the load increases the speed falls; and the fall in speed decreases the back-e.m.f., and allows more current to flow. But, as we will see in a moment, the back-e.m.f., in a series motor, does not fall with the speed as sharply as it does in a shunt motor. To meet a given increase in load, the armature current must increase by a definite amount. And therefore the back-e.m.f. must fall by a definite amount. But it falls less with the speed than it does in a shunt motor. Consequently, to meet a given increase in load, the speed of a series motor must fall more than that of a shunt motor.

We now show that the back-e.m.f. in a series motor falls less with the speed than in a shunt one. The argument is best given in steps:

(i) when the speed falls, the back-e.m.f. falls;
(ii) the current through both armature and field winding increases;
(iii) the field becomes stronger;
(iv) the increase in the field tends to *increase* the back-e.m.f., i.e. to offset its initial fall;
(v) thus the very fall of the back-e.m.f., by permitting a greater current and strengthening the field, tends to offset itself;
(vi) therefore the back-e.m.f. falls slowly with the speed—more slowly than in a shunt motor, where the field is constant;
(vii) as we have already seen, this means that the speed must fall further, to meet a given increase in load.

CHARGE AND FLUX LINKAGE

Flux and Charge relation

We have already seen that an electromotive force is induced in a circuit when the magnetic flux linked with it changes. If the circuit is closed, a current flows, and electric charge is carried round the circuit. As we shall now show, there is a simple relationship between the charge and the change of flux.

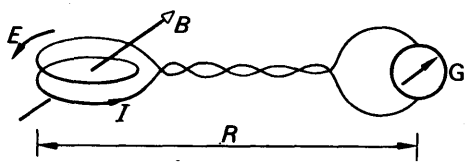


FIG. 36.29. Coil with changing flux.

Consider a closed circuit of total resistance R ohm, which has a total flux linkage Φ with a magnetic field (Fig. 36.29). If the flux

linkages start to change,

$$\text{induced e.m.f., } E = -\frac{d\Phi}{dt}.$$

$$\therefore \text{ current, } I = \frac{E}{R} = -\frac{1}{R}\frac{d\Phi}{dt} \quad . \quad . \quad (1)$$

In general, the flux linkage will not change at a steady rate, and the current will not be constant. But, throughout its change, charge is being carried round the circuit. If a time t seconds is taken to reach a new constant value, the charge carried round the circuit in that time is

$$Q = \int_0^t I dt.$$

From (1),

$$\therefore Q = -\frac{1}{R}\int_0^t \frac{d\Phi}{dt}.dt$$

$$= -\frac{1}{R}\int_{\Phi_0}^{\Phi_t} d\Phi,$$

where Φ_0 is the number of linkages at $t = 0$, and Φ_t is the number of linkages at time t. Thus

$$Q = -\frac{\Phi_t - \Phi_0}{R} = \frac{\Phi_0 - \Phi_t}{R}.$$

The quantity $\Phi_0 - \Phi_t$ is positive if the linkages Φ have decreased, and negative if they have increased. But as a rule we are interested only in the magnitude of the charge, and we may write

$$Q = \frac{\textit{change of flux linkage}}{R}. \quad . \quad . \quad (2)$$

Equation (2) shows that the charge circulated is proportional to the change of flux-linkages, and independent of the time.

Ballistic Galvanometer

It can be seen from the last section that the charge which flows round a given circuit is directly proportional to the change of flux linkage. If the charge flowing is measured by a *ballistic galvanometer* G, as shown in Fig. 36.29, then we have a measure of the change in flux linkage, Φ.

Ballistics is the study of the motion of a body, such as a projectile, which is set off by a blow, and then allowed to move freely. By freely, we mean without friction. A ballistic galvanometer is one used to measure an electrical blow, or impulse: for example, the charge Q which circulates when a capacitor is discharged through it. A galvanometer which is intended to be used ballistically has a heavier coil than one which is not; and it has as little damping as possible—an insulating former, no short-circuited turns, no shunt. The mass of its coil makes it swing slowly; in the example above, for instance, the capacitor has

discharged, and the charge has finished circulating, while the galvanometer coil is just beginning to turn. The galvanometer coil continues to turn, however; and as it does so it twists the suspension. The coil stops turning when its kinetic energy, which it gained from the forces set up by the current, has been converted into potential energy of the suspending fibre. The coil then swings back, as the suspension untwists itself, and it continues to swing back and forth for some time. Eventually it comes to rest, but only because of the damping due to the viscosity of the air, and to the internal friction of the fibre. Theory shows that, if the damping is negligible, *the first deflection of the galvanometer is proportional to the quantity of electricity, Q, that passed through its coil, as it began to move.* This first deflection, θ, is often called the 'throw' of the galvanometer; we have, then,

$$Q = k\theta, \quad . \quad . \quad . \quad . \quad (1)$$

where k is a constant of the galvanometer.

Equation (1) is true only if all the energy given to the coil is spent in twisting the suspension. If an appreciable amount of energy is used to overcome damping—i.e. dissipated as heat by eddy currents—then the galvanometer is not ballistic, and θ is not proportional to Q.

To calibrate the ballistic galvanometer, a capacitor of known capacitance, e.g. 2 μF, is charged by a battery of known e.m.f., e.g. 50 volt, and then discharged through the instrument. See p. 768. Suppose the deflection is 200 divisions. The charge $Q = CV = 100$ microcoulomb, and thus the galvanometer sensitivity is 2 divisions per microcoulomb.

Measurement of Induction

Fig. 36.30 illustrates the principle of measuring the induction B in the field between the poles of a powerful magnet. A small coil, called a *search coil*, with a known area and number of turns, is connected to a ballistic galvanometer G. It is positioned at right angles to the field to be measured, as shown, so that the flux enters the coil face normally.

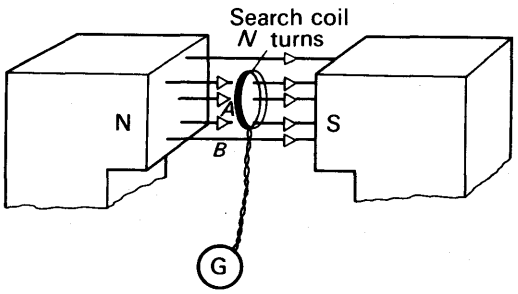


FIG. 36.20. Induction by ballistic galvanometer.

The coil is then pulled completely out of the field by moving it smartly downwards, for example, and the throw θ produced in the galvanometer is observed. The charge q which passes round the circuit is proportional to θ, from above.

Suppose B is the field-strength in Wb m^{-2} or tesla (T), A is the area of the coil in m^2 and N is the number of turns. Then

$$\text{change of flux-linkages} = NAB$$

$$\therefore \text{ quantity, } Q, \text{ through galvanometer} = \frac{NAB}{R},$$

where R is the *total* resistance of the galvanometer and search coil. But

$$Q = c\theta,$$

where c is the quantity per unit deflection of the ballistic galvanometer.

$$\therefore \frac{NAB}{R} = c\theta$$

$$\therefore B = \frac{Rc\theta}{NA}. \quad . \quad . \quad . \quad . \quad (1)$$

The constant c is found by discharging a capacitor through the galvanometer (see p. 768). If C is the capacitance in farads, V the p.d. in volts of the battery originally charging it, and α the deflection of the galvanometer, then $c = CV/\alpha$ coulomb per unit deflection.

The Earth Inductor

As another example of the use of a search coil and ballistic galvanometer, we describe a method that has been used for measuring the angle of dip of the earth's magnetic field (p. 944). The earth's field is so nearly uniform that the search coil may be large, usually about 30 cm square; but the field is also so weak that even a large coil must have many turns—of the order of 100. The coil, which is called an earth inductor, is pivoted in a wooden frame, and this is fitted with stops so that the coil can be turned rapidly through 180° (Fig. 36.31). To find

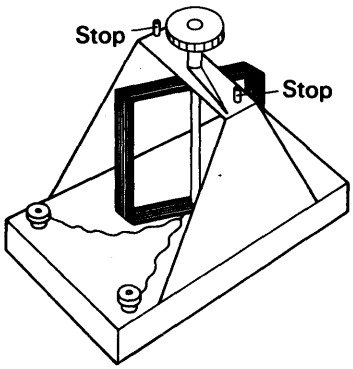


FIG. 36.31. Earth inductor.

the angle of dip, we connect the coil to a ballistic galvanometer, and set it with its plane horizontal, as shown at (i) in Fig. 36.32. The flux linking the coil at N turns is then

$$\Phi = NAB_V,$$

where A is the area of the coil, and B_V is the vertical component of the earth's field. If we were to turn the coil through 90°, the flux would fall to zero; and if we were to turn it through a further 90°, the flux linkage would become NAB_V once more, but it would thread the coil in the opposite direction. Therefore we turn the coil through 180°, and change the flux linkage by $2NAB_V$; at the same time we observe the throw, θ, of the galvanometer.

If R is the total resistance of galvanometer and search coil, the circulated charge is, by equation (2) on p. 919,

$$Q = \frac{\Phi}{R} = \frac{2NAB_V}{R}.$$

But

$$Q = c\theta,$$

where c is the constant of the galvanometer. Therefore

$$\frac{2NAB_V}{R} = c\theta. \quad . \quad . \quad . \quad (1)$$

We now set the frame of the earth inductor so that the axis of the coil is vertical, and so that, when the coil is held by one of the stops, its plane lies East–West. See Fig. 36.32 (ii). The flux threading the coil is now NAB_H, where B_H is the horizontal component of the earth's field. Therefore, when we turn the coil through 180°, the throw θ' of the galvanometer is given by

$$\frac{2NAB_H}{R} = c\theta' \quad . \quad (2)$$

Now the angle of dip, δ, is given by

$$\tan \delta = \frac{B_V}{B_H}.$$

Therefore, from equations (1) and (2),

$$\tan \delta = \frac{\theta}{\theta'}.$$

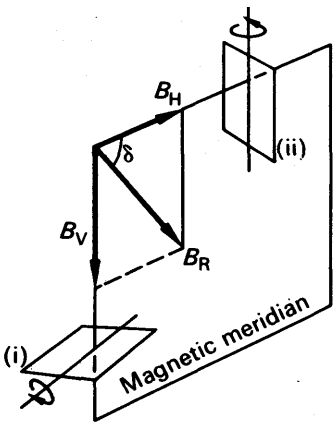


FIG. 36.32. Measurement of dip.

Self-induction

The phenomenon which we call self-induction was discovered by the American, Joseph Henry, in 1832. He was led to it by a theoretical argument, starting from the phenomena of induced e.m.f., which he had discovered at about the same time as Faraday.

When a current flows through a coil, it sets up a magnetic field. And that field threads the coil which produces it. Fig. 36.33 (i). If the current through the coil is changed—by means of a variable resistance, for example—the flux linked with the turn of the coil changes. An e.m.f. is therefore induced in the coil. By Lenz's law the direction of the induced e.m.f. will be such as to oppose the change of curent; the e.m.f. will be

against the current if it is increasing, with it if it is decreasing (Fig. 36.33 (ii)).

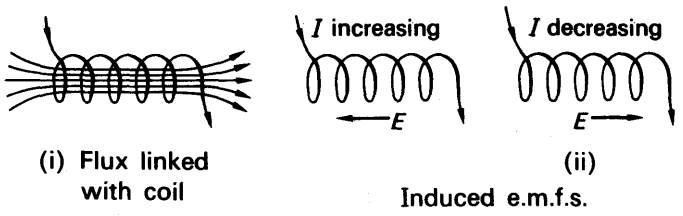


FIG. 36.33. Self-induction.

Back-E.M.F.

When an e.m.f. is induced in a circuit by a change in the current through that circuit, the process of induction is called self-induction. The e.m.f. induced is called a back-e.m.f. Self-induction opposes the growth of current in a coil, and so makes it gradual. This effect can be demonstrated by connecting an iron-cored coil of many turns in series with an ammeter and a few accumulators (Fig. 36.34 (i)). (The ammeter

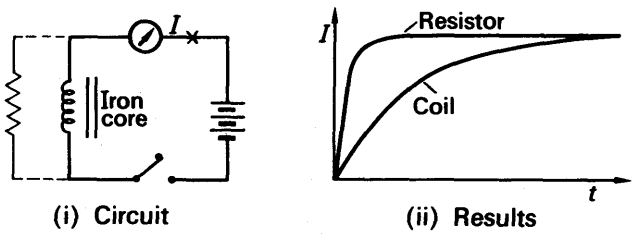


FIG. 36.34. Self-induction experiment.

should be of the 'short-period' type and critically damped.) When the current is switched on, the pointer of the ammeter moves slowly over to its final position. If the coil is now replaced by a rheostat of the same *resistance*, the pointer moves much more swiftly to the same reading (Fig. 36.34 (ii)).

Just as self-induction opposes the rise of an electric current when it is switched on, so also it opposes the decay of the current when it is switched off. When the circuit is broken, the current starts to fall very rapidly, and a correspondingly great e.m.f. is induced, which tends to maintain the current. This e.m.f. is often great enough to break down the insulation of the air between the switch contacts, and produce a spark. To do so, the e.m.f. must be about 350 volts or more, because air will not break down—not over any gaps, narrow or wide—when the voltage is less than that value. The e.m.f. at break may be much greater than the e.m.f. of the supply which maintained the current: a spark can easily be obtained, for example, by breaking a circuit consisting of an iron-cored coil and an accumulator.

Non-inductive Coils

In bridge circuits, such as are used for resistance measurements, self-induction is a nuisance. When the galvanometer key of a bridge is closed, the currents in the arms of the bridge are redistributed, unless the bridge happens to be balanced. While the currents are being redistributed they are changing, and self-induction delays the reaching of a new equilibrium. Thus the galvanometer deflection at the instant of closing the key, does not correspond to the steady state which the bridge will eventually reach. It may therefore be misleading. To minimize their self-inductance, the coils of bridges and resistance boxes are wound so as to set up extremely small magnetic fields: as shown in Fig. 36.35, the wire is doubled-back on itself before being coiled up. Every part of the coil is then traversed by the same current travelling in opposite directions, and its magnetic field is negligible. Such a coil is said to be non-inductive.

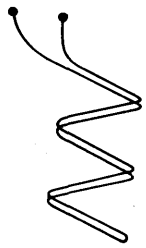


FIG. 36.35. Non-inductive winding.

When describing the use of a bridge, we said that the battery key should be pressed before the galvanometer key. Doing so gives time for the currents in the arms of the bridge to become steady before the galvanometer key is pressed. It therefore minimizes any possible effects of self-induction.

Self-inductance

To discuss the effects of self-induction we must define the property of a coil which gives rise to them. This property is called the self-inductance of the coil, and is defined as follows:

$$\textit{self-inductance} = \frac{\textit{back-e.m.f. induced in coil by a changing current}}{\textit{rate of change of current through coil}}$$

Self-inductance is denoted by the symbol L; we may therefore write its definition as

$$L = \frac{E_{back}}{dI/dt}.$$

or

$$E_{back} = L\frac{dI}{dt}. \quad . \quad . \quad . \quad . \quad (1)$$

Equation (1) is the simplest form in which to remember the definition.

The unit of self-inductance is the henry (H). It is defined by making each term in equation (1) equal to unity; thus *a coil has a self-inductance of* 1 *henry if the back-e.m.f. in it is* 1 *volt, when the current through it is changing at the rate of* 1 *ampere per second.* Equation (1) then becomes:

$$E_{back}\,(\text{volts}) = L\,(\text{henrys}) \times \frac{dI}{dt}\,(\text{ampere/second}).$$

The iron-cored coils used for smoothing the rectified supply current to a radio receiver (p. 1011) are usually very large and have an inductance of about 30 henrys.

L for Coil

Since the induced e.m.f. $E = d\Phi/dt = L = dI/dt$, numerically, it follows by integration from a limit of zero that

$$\Phi = LI.$$

Thus $L = \Phi/I$. Hence the self-inductance may be defined as the *flux linkage per unit current*. When Φ is in webers and I in amperes, then L is in henrys. Thus if a current of 2A produces a flux linkage of 4 Wb in a coil, the inductance $L = 4$ Wb/2A $= 2$H.

We shall see later that when a long coil of N turns and length l carries a current I, (i) the magnetizing field $H = NI/l$ and (ii) this produces a flux-density or induction B inside the coil given by $B = \mu H$, where μ is the permeability of the material inside the coil (p. 941). Hence

$$\text{flux linkage } \Phi = NAB = NA\mu H = \frac{\mu N^2 AI}{l}$$

$$\therefore L = \frac{\Phi}{I} = \frac{\mu N^2 A}{l} \quad . \quad . \quad . \quad (1)$$

This formula may be used to find the approximate value of the inductance of a coil. L is in henrys when A is in metre^2, l in metre and μ is in henry metre^{-1}.

Energy Stored; E.M.F. at Break

The spark which passes when the current in a coil is interrupted liberates energy in the form of heat and light. This energy has been stored in the magnetic field of the coil, just as the energy of a charged capacitor is stored in the electrostatic field between its plates (p. 779). When the current in the coil is first switched on, the back-e.m.f. opposes the rise of current; the current flows against the back-e.m.f. and therefore does work against it (p. 795). When the current becomes steady, there is no back-e.m.f. and no more work done against it. The total work done in bringing the current to its final value is stored in the magnetic field of the coil. It is liberated when the current collapses; for then the induced e.m.f. tends to maintain the current, and to do external work of some kind.

To calculate the energy stored in a coil, we suppose that the current through it is rising at a rate dI/dt ampere per second. Then, if L is its self-inductance in henrys, the back-e.m.f. across it is given by

$$E = L\frac{dI}{dt} \text{ volt.}$$

If the value of the current, at the instant concerned, is I amperes, then the rate at which work is being done against the back-e.m.f. is

$$P = EI = LI\frac{dI}{dt} \text{ watt.}$$

The total work done in bringing the current from zero to a steady value I_0 is therefore

$$W = \int P dt = \int_0^{I_0} LI \frac{dI}{dt} dt = \int_0^{I_0} LI dt$$
$$= \tfrac{1}{2} L I_0^2 \text{ joule.}$$

This is the energy stored in the coil.

To calculate the e.m.f. induced at break is, in general, a complicated business. But we can easily do it for one important practical circuit. To prevent sparking

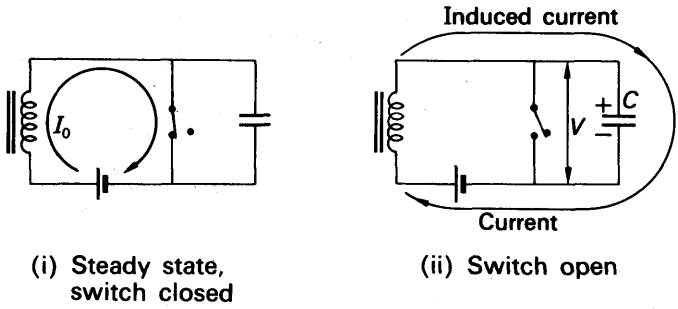


FIG. 36.36. Prevention of sparking by capacitor.

at the contacts of the switch in an inductive circuit, a capacitor is often connected across them (Fig. 36.36 (i)). When the circuit is broken, the collapsing flux through the coil tends to maintain the current; but now the current can continue to flow for a brief time; it can flow by charging the capacitor (Fig. 36.36 (ii)). Consequently the current does not decay as rapidly as it would without the capacitor, and the back-e.m.f. never rises as high. If the capacitance of the capacitor is great enough, the potential difference across it (and therefore across the switch) never rises high enough to cause a spark.

To find the value to which the potential difference does rise, we assume that all the energy originally stored in the magnetic field of the coil is now stored in the electrostatic field of the capacitor.

If C is the capacitance of the capacitor in farad, and V_0 the final value of potential difference across it in volt, then the energy stored in it is $\frac{1}{2}CV_0^2$ joule (p. 779). Equating this to the original value of the energy stored in the coil, we have

$$\tfrac{1}{2}CV_0^2 = \tfrac{1}{2}LI_0^2.$$

Let us suppse that a current of 1 ampere is to be broken, without sparking, in a circuit of self-inductance 1 henry. To prevent sparking, the potential difference across the capacitor must not rise above 350 volt. The least capacitance that must be connected across the switch is therefore given by

$$\tfrac{1}{2}C \times 350^2 = \tfrac{1}{2} \times 1 \times 1^2.$$

Hence
$$C = \frac{1}{350^2} = 8 \times 10^{-6} \text{ farad} = 8\ \mu\text{F}.$$

A paper capacitor of capacitance 8 μF, and able to withstand 350 volts, would therefore be required.

Mutual Induction

We have already seen than an e.m.f. may be induced in one circuit by a changing current in another (Fig. 36.1, p. 895). The phenomenon is often called mutual induction, and the pair of circuits which show it are said to have mutual inductance. The mutual inductance, M, between two circuits is defined by the equation:

$$\left.\begin{matrix}\text{e.m.f. induced in B, by}\\ \text{changing current in A}\end{matrix}\right\} = M \times \left\{\begin{matrix}\text{rate of change of}\\ \text{current in A.}\end{matrix}\right.$$

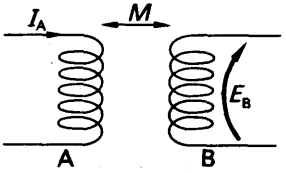


FIG. 36.37. Mutual induction.

See Fig. 36.37. In symbols,

$$E_B = M\frac{dI_A}{dt}.$$

Mutual inductance is truly mutual; it is the same from B to A as A to B. Its unit is the same as that of self-inductance, the henry.

EXERCISES 36

1. State Lenz's law of electromagnetic induction and describe, with explanation, an experiment which illustrates its truth.

Describe the structure of a transformer suitable for supplying 12 volts from 240-volt mains and explain its action. Indicate the energy losses which occur in the transformer and explain how they are reduced to a minimum.

When the primary of a transformer is connected to the a.c. mains the current in it (*a*) is very small if the secondary circuit is open, but (*b*) increases when the secondary circuit is closed. Explain these facts. (*L.*)

2. Describe, with the aid of a large labelled diagram, the structure of a simple form of a.c. generator.

Explain (*a*) how it could be modified to produce direct current;
(*b*) the features that enable is to produce a high e.m.f. (compared with a cell);
(*c*) the features that minimize the heat wasted. (*L.*)

3. Define *electromotive force* and state the *laws of electromagnetic induction.* Using the definition and the laws, derive an expression for the e.m.f. induced in a conductor moving in a magnetic field.

When a wheel with metal spokes 120 cm long is rotated in a magnetic field of flux density $0{\cdot}5 \times 10^{-4}$ Wb m^{-2} normal to the plane of the wheel, an e.m.f. of 10^{-2} volt is induced between the rim and the axle. Find the rate of rotation of the wheel. (*L.*)

4. Describe the differences in structure and action between a non-ballistic and a ballistic moving-coil galvanometer.

A corrected deflection of 24 scale divisions of a ballistic galvanometer is obtained *either* by charging a capacitor of 3 μF capacitance to a potential difference of 2 volt and discharging it through the galvanometer, or by connecting the ballistic galvanometer in series with a flat circular coil of 80 turns each of diameter 1 cm, the combined resistance of coil and galvanometer being 2000 ohms, and quickly thrusting the coil into a strong magnetic field so that the plane of the coil is perpendicular to the direction of the field. State the sensitivity of the galvanometer and calculate the strength of the magnetic field. (The strength of the earth's magnetic field may be neglected.) (*L.*)

5. Explain what is meant by *self inductance* and define the practical unit in which it is measured.

Describe and explain an experiment which demonstrates the phenomenon of self-induction. (*N.*)

6. State the laws relating to (*a*) the direction, (*b*) the magnitude of an electromagnetically induced electromotive force, and describe very briefly an experiment illustrating each.

Deduce the relation between the quantity of electricity flowing through a circuit and the flux change producing it.

A flat coil of 150 turns, each of area 300 cm^2 and of total resistance 50 ohms, is connected to a circuit whose resistance is 40 ohms. Starting with its plane horizontal, the coil is rotated quickly through a half-turn about a diametral axis pointing along the magnetic meridian. If the quantity of electricity which then flows round the circuit is 4 microcoulombs, find the intensity of the vertical component of the earth's magnetic field. (*N.*)

7. What are eddy currents?

Describe and explain an experiment in which eddy currents are produced.

Describe one useful application of eddy currents. (*N.*)

8. Define *self inductance* and *mutual inductance.*

Explain the differences in structure and action between a ballistic and an aperiodic galvanometer.

A ballistic galvanometer of resistance 15 ohms and sensitivity 5 divisions per microcoulomb is connected in series with a resistance of 100 ohms and a secondary coil of 500 turns and of resistance 50 ohms. This coil is wound round the middle of a long solenoid of radius 3 cm having 10 turns cm^{-1} and carrying a current of 0·6 A. Assuming no damping, calculate the deflection produced in the galvanometer when the current in the solenoid is switched off. (*L.*)

9. Describe experiments (one in each case) involving the use of a moving-coil ballistic galvanometer to (*a*) compare two capacitances of approximately the same magnitude, (*b*) compare the magnetic induction (flux density) between the poles of one electromagnet with that between the poles of another. In each case justify the method used to calculate the result.

Explain two special features of a galvanometer suitable for use in these experiments. (*N.*)

10. State Lenz's law and describe how you would demonstrate it using a solenoid with two separate superimposed windings with clearly visible turns, a cell with marked polarity, and a centre-zero galvanometer. Illustrate your answer with diagrams.

A metal aircraft with a wing span of 40 m flies with a ground speed of 1000 km h^{-1} in a direction due east at constant altitude in a region of the northern hemisphere where the horizontal component of the earth's magnetic field is $1{\cdot}6 \times 10^{-5}$ T (Wb m^{-2}) and the angle of dip is 71·6°. Find the potential difference in volts that exists between the wing tips and state, with reasons, which tip is at the higher potential. (*N.*)

11. State the laws of electromagnetic induction and describe briefly experiments to show their validity.

A coil *A* passes a current of 1·25 A when a steady potential difference of 5 V is maintained across it, and an r.m.s. current of 1 A when it has across it a sinusoidal potential difference of 5 V r.m.s. at a frequency of 60 Hz (cycles per second). Explain why the current is less in the second case, and calculate the resistance and the inductance of the coil.

The same coil *A*, which has 100 turns, has a second coil *B* with 500 turns wound on it so that all the magnetic flux produced by *A* is linked by *B*. Find the r.m.s. value of the e.m.f. that appears across the open-circuit ends of *B* when a sinusoidal alternating current of 1 A r.m.s. at a frequency of 50 Hz is passed through *A*. Why is the ratio of this e.m.f. to the r.m.s. potential difference across *A* not the same as the ratio of the number of turns in *B* and *A*, i.e. 5:1?

Explain why the insertion of an iron core into the coils would decrease the current in *A* and increase the e.m.f. across *B*, if the alternating potential difference across *A* were kept unchanged; the effects of hysteresis and eddy currents in the iron may be neglected. (*O. & C.*)

12. A choke of large self inductance and small resistance, a battery and a switch are connected in series. Sketch and explain a graph illustrating how the current varies with time after the switch is closed. If the self inductance and resistance of the coil are 10 henrys and 5 ohms respectively and the battery has an e.m.f. of 20 volts and negligible resistance, what are the greatest values after the switch is closed of (*a*) the current, (*b*) the rate of change of current? (*N.*)

13. State the laws of electromagnetic induction, and describe an experiment by which one of them can be verified.

A piece of wire 8 cm long, of resistance 0·020 ohm and mass 22 mg, is bent to form a closed square *ABCD*. It is mounted so as to turn without friction about a horizontal axis through *AB*; a uniform horizontal magnetic field of flux-density 0·50 Wb m^{-2} is applied at right angles to this axis. The side *CD* is raised until the plane of the square is horizontal and then released. Calculate approximately the time taken for the plane of the square to become vertical. You can assume that, during the falling, the couple due to gravity is equal and opposite to that due to electromagnetic forces. (*C.*)

14. State the laws of electromagnetic induction. Hence derive an expression for the time variation of the electromotive force induced in a single turn of wire rotating about an axis in its plane, the axis being perpendicular to a uniform magnetic field. Explain the action of a simple alternating current generator.

What modification of this generator is required to produce a direct current? Indicate by a sketch how the e.m.f. across the output terminals of a single coil would vary with time in the case of (*a*) an a.c. and (*b*) a d.c. generator. How may a more uniform output e.m.f. be obtained in the latter case?

A rectangular coil of wire having 100 turns, of dimensions 30 cm × 30 cm, is rotated at a constant speed of 600 r.p.m. in a magnetic field of 0·1 Wb m^{-2}, the axis of rotation being in the plane of the coil and perpendicular to the field. Calculate the induced e.m.f. (*L.*)

15. Describe with the aid of diagrams (*a*) a transformer and (*b*) an induction coil. Explain the action of each.

Draw diagrams to show, in a general way, how the voltage output from each of these appliances varies with the time. (*N.*)

16. What are eddy-currents? Give two examples of the practical use of such currents.

A metal disc of diameter 20 cm rotates at a constant speed of 600 r.p.m. about an axis through its centre and perpendicular to its plane in a uniform magnetic field of 5×10^{-3} Wb m^{-2} established parallel to the axis of rotation. Calculate the e.m.f. in volts between the centre and rim of the disc. Show clearly on a diagram the direction of rotation of the disc and the direction of the magnetic field and of the e.m.f. induced. (*L.*)

17. How would you show that a change in the number of lines of magnetic force, however produced, threading through a circuit produces an induced e.m.f.?

A magnet is suspended by a thin wire so that its axis is horizontal and its centre is above the centre of a circular copper disc, mounted horizontally. Explain what happens to the magnet when the disc is rotated.

What would be the effect of replacing the disc by one of identical dimensions but made of a substance of high resistivity? (*L.*)

18. State Lenz's law and describe fully a method by which you could verify this law experimentally.

A horizontal metal disc of radius 10 cm is rotated about a central vertical axis at a region where the value of the earth's magnetic flux density is $5{\cdot}3 \times 10^{-5}$ T (Wb m^{-2}) and the angle of dip is 70°. A sensitive galvanometer of resistance 150 ohms is connected between the centre of the disc and a brush pressing on the rim. Assuming the resistance of the disc to be negligible, what will be the current through the galvanometer when the disc is rotated at 1500 rev. $min.^{-1}$? If the system is frictionless, calculate the power required to maintain the motion. (*C.*)

19. State the laws of electromagnetic induction and describe experiments you would perform to illustrate the factors which determine the magnitude of the induced current set up in a closed circuit.

A simple electric motor has an armature of 0·1 ohm resistance. When the motor is running on a 50-volt supply the current is found to be 5 amp. Explain this and show what bearing it has on the method of starting large motors. (*L.*)

20. State the laws relating to the electromotive force induced in a conductor which is moving in a magnetic field.

Describe the mode of action of a simple dynamo.

Find in volts the e.m.f. induced in a straight conductor of length 20 cm, on the armature of a dynamo and 10 cm from the axis when the conductor is moving in a uniform radial field of 0·5 Wb m^{-2} and the armature is rotating at 1000 r.p.m. (*L.*)

chapter thirty-seven

Magnetic Fields of Current-Carrying Conductors

In the previous chapters, the induction or flux density B in a magnetic field was used to find the force on conductors and the e.m.f. induced in conductors. In this chapter we shall see how the magnitude of B is calculated. This depends on the geometry of the conductor, that is, whether it is a straight wire, or a coil, or a solenoid. The geometry also determines the pattern of the lines of force in the field.

Law of Biot and Savart

To calculate B for any shape of conductor, Biot and Savart gave a law which can now be stated as follows: The induction or flux density δB at a point P due to a small element δl of a conductor carrying a current is given by

$$\delta B \propto \frac{I \delta l \sin \alpha}{r^2}, \quad . \quad . \quad . \quad (1)$$

where r is the distance from the point P to the element and α is the angle between the element and the line joining it to P (Fig. 37.1).

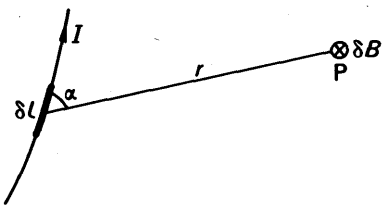


FIG. 37.1. Biot and Savart law.

The formula in (1) cannot be proved directly, as we cannot experiment with an infinitesimally small conductor. We believe in its truth because the deductions for large practical conductors turn out to be true.

The constant of proportionality in equation (1) depends on the medium in which the conductor is situated. In air (or, more exactly, in a vacuum), we write

$$\delta B = \frac{\mu_0}{4\pi} \frac{I \delta l \sin \alpha}{r^2} \quad . \quad . \quad . \quad (2)$$

The value of μ_0 is defined to be,

$$\mu_0 = 4\pi \times 10^{-7},$$

and its unit is '*henry per metre*' (H m^{-1}) as will be shown later.

Induction Formula for Narrow Coil

The formula for the induction B at the centre of a narrow circular coil can be immediately deduced from (2). Here the radius r is constant for all the elements δl, and the angle α is constant and equal to 90° (Fig. 37.2 (i)). If the coil has N turns, the length of wire in it is $2\pi rN$, and the field at its centre is therefore given, if the current is I, by

$$B = \int dB = \frac{\mu_0}{4\pi}\int_0^{2\pi rn} \frac{Idl \sin 90°}{r^2}$$

$$= \frac{\mu_0 I}{4\pi r^2}\int_0^{2\pi rn} dl = \frac{\mu_0 I}{4\pi r^2} 2\pi rN$$

$$= \frac{\mu_0 NI}{2r} \quad . \quad . \quad . \quad . \quad . \quad (1)$$

From (1), $B \propto I$ when r and N are constant, $B \propto 1/r$ when I and N are constant, and $B \propto N$ when I and r are constant. Any one of these relations may be verified with the apparatus shown in Fig. 37.2 (ii). This

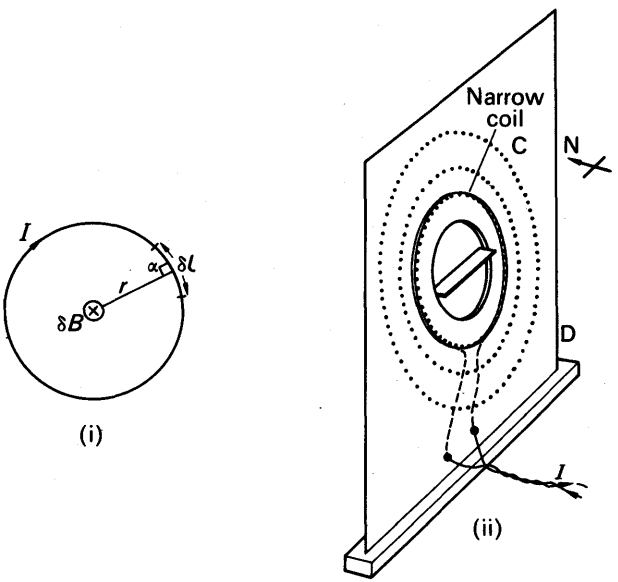


FIG. 37.2. Field of circular coil.

is a board D with sets of pegs, round which wire can be wound to form narrow coils of various radii. If a vibration magnetometer is used to measure B, the board is turned so that the axis of the coils C is in the direction of the magnetic meridian. The number of vibrations per minute is then found with the current in one direction and again when the current is reversed.

Suppose n_1 is the number when the field B of the current assists the field B_H due to the earth, so that $(B+B_H)$ is the resultant field. Suppose n_2 is the number when the field B opposes B_H and B is stronger than B_H, so that $(B-B_H)$ is the resultant field. Now on p. 887, it was shown that the flux density in a field was directly proportional to the square of the frequency of the vibration magnetometer. Hence

$$B+B_H = kn_1^2 \quad \text{and} \quad B-B_H = kn_2^2.$$

Adding,

$$\therefore 2B = k(n_1^2+n_2^2), \quad \text{or} \quad B \propto (n_1^2+n_2^2).$$

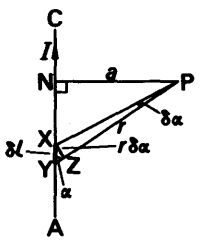
To verify $B \propto 1/r$ when I and N are constant, $(n_1^2+n_2^2)$ is evaluated when the wire is wound round other sets of pegs and I and N are kept constant each time. A graph of $(n_1^2+n_2^2)$ against $1/r$ is then plotted. This is found to be a straight line passing through the origin. Hence $B \propto 1/r$ when I and N are constant.

In a similar way, by varying the current I it can be shown that $B \propto I$ when N and r are constant. Similar experiments show that $B \propto N$ when I and r are constant. Thus $B \propto NI/r$ for a narrow circular coil.

The induction B in a magnetic field may also be measured by means of a *ballistic galvanometer* or by an *a.c. method.* See pp. 920, 934.

Field due to Long Straight Wire

We now deduce the induction at a point outside a long straight wire.



(i)

FIG. 37.3 (i). Field of a long, straight wire.

In Fig. 37.3 (i), AC represents part of a long straight wire. P is taken as a point so near it that, from P, the wire looks infinitely long—it subtends very nearly 180°. An element XY of this wire, of length δl, makes an angle α with the radius vector, r, from P. It therefore contributes to the magnetic field at P an amount

$$\delta B = \frac{\mu_0 I \delta l \sin \alpha}{4\pi r^2} \quad . \quad \text{(i)}$$

when the wire carries a current I. If α is the perpendicular distance, PN, from P to the wire, then

$$\text{PN} = \text{PX} \sin \alpha$$

or

$$a = r \sin \alpha,$$

whence

$$r = \frac{a}{\sin \alpha}. \quad . \quad . \quad . \quad \text{(ii)}$$

Also, if we draw XZ perpendicular to PY, we have

$$\text{XZ} = \text{XY} \sin \alpha = \delta l \sin \alpha.$$

If δl subtends an angle $\delta\alpha$ at P, then

$$\text{XZ} = r\delta\alpha = \delta l \sin \alpha.$$

From (i),

$$\therefore \delta B = \frac{\mu_0 I \delta l \sin \alpha}{4\pi r^2} = \frac{\mu_0 I r \delta \alpha}{4\pi r^2} = \frac{\mu_0 I \delta \alpha}{4\pi r}$$

From (ii),

$$\therefore \delta B = \frac{\mu_0 I \sin \alpha \delta \alpha}{4\pi a}$$

When the point Y is at the bottom end A of the wire, $\alpha = 0$; and when Y is at the top C of the wire, $\alpha = \pi$. Therefore the total magnetic field at P is

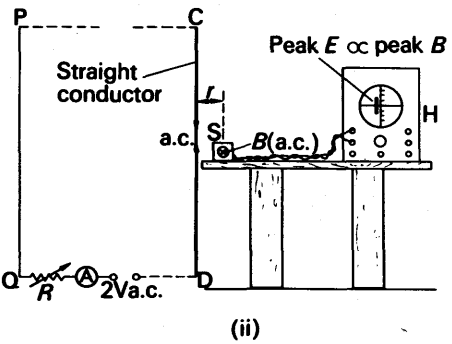
$$B = \frac{\mu_0}{4\pi}\int_0^{\pi} \frac{I \sin \alpha \delta \alpha}{a} = \frac{\mu_0 I}{4\pi a}\left[-\cos \alpha\right]_0^{\pi}$$

$$\therefore B = \frac{\mu_0 I}{2\pi a} \quad . \quad . \quad . \quad (1)$$

Equation (1) shows that the magnetic field of a long straight wire, at a point near it, is inversely proportional to the distance of the point from the wire. The result was discovered experimentally by Biot and Savart using a vibration magnetometer method, and led to their general formula in (i) which we used to derive (1).

Variation of *B* with Distance—A.C. Method

An apparatus suitable for finding the variation of *B* with distance from a long straight wire CD is shown in Fig. 37.3 (ii). Alternating current (a.c.) of the order of 10 A, from a low voltage mains transformer, is passed through CD by using another long wire PQ at least one metre away, a rheostat B and an a.c. ammeter A. A small search coil S, with thousands of turns of wire, such as the coil from an output transformer, is placed near CD. It is positioned with its axis at a small distance *r* from CD



(ii)

FIG. 37.3 (ii). Investigation of *B* due to long straight conductor.

and so that the flux from CD enters its face normally. S is joined by long twin flex to the Y-plates of an oscilloscope H and the greatest sensitivity, such as 5 mV/cm, is used.

When the a.c. supply is switched on, the varying flux through S produces an induced alternating e.m.f. E. The peak value of E can be determined by switching off the time-base and measuring the length of the line trace, Fig. 37.3 (ii). See p. 1014. Now the peak value of B, the magnetic induction, is proportional to the peak value of E, as shown in the case of the simple dynamo on p. 907. Thus the length of the trace gives a measure of the peak value of B.

The distance r of the coil from CD is then increased and the corresponding length of the trace is measured. The length of the trace plotted against $1/r$ gives a straight line graph passing through the origin. Hence $B \propto 1/r$. A similar method can be used for investigating the induction B for the case of a narrow circular coil or for a solenoid (p. 939).

EXAMPLE

Calculate the flux density at a distance of 1 cm or 0·01 m from a very long vertical straight wire carrying a current of 10 A. At what distance from the wire will the field induction neutralize that due to the earth's horizontal component flux-density, $0{\cdot}2 \times 10^{-4}$ T?

(i) $$B = \frac{\mu_0 I}{2\pi a} = \frac{4\pi \times 10^{-7} \times 10}{2\pi \times 10^{-2}}$$

$$= 2 \times 10^{-4} \text{ T}.$$

(ii) 1 cm from the wire, the induction due to the current is 2×10^{-4} T.

Now B is inversely-proportional to a, the distance from the wire. Thus B is $0{\cdot}2 \times 10^{-4}$ Wb m^{-2}, or ten times smaller than at 1 cm, at a distance 10 times as great. Thus the distance is 10 cm.

Note that the actual position of the point where the two fields neutralize must take account of the fact that B is a *vector*, that is, it has direction and magnitude. For a downward current of 10 A, the point concerned is due east of the wire. It is called a *neutral* point.

Field along Axis of a Narrow Circular Coil

We will now find the magnetic field at a point anywhere on the axis of a narrow circular coil (P in Fig. 37.4). We consider an element δl of the coil, at right angles to the plane of the paper. This sets up a field δB at P, in the plane of the paper, and at right angles to the radius vector r. If β is the angle between r and the axis of the coil, then the field δB has components $\delta B \sin \beta$ along the axis, and

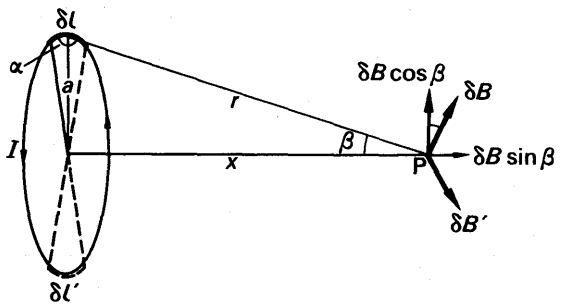


FIG. 37.4. Field on axis of flat coil.

$\delta B \cos\beta$ at right angles to the axis. If we now consider the element $\delta l'$ diametrically opposite to δl, we see that it sets up a field $\delta B'$ equal in magnitude to δB. This also has a component, $\delta B' \cos\beta$, at right angles to the axis; but this component acts in the opposite direction to $\delta B \cos\beta$ and therefore cancels it. By considering elements such as δl and $\delta l'$ all round the circumference of the coil, we see that the field at P can have no component at right angles to the axis. Its value along the axis is

$$B = \int dB \sin\beta.$$

From Fig. 37.4, we see that the length of the radius vector r is the same for all points on the circumference of the coil, and that the angle α is also constant, being 90°. This, if the coil has a single turn, and carries a current I,

$$\delta B = \frac{\mu_0 I \delta l \sin\alpha}{4\pi r^2} = \frac{\mu_0 I}{4\pi r^2}\delta l.$$

And, if the coil has a radius a, then

$$B = \int dB \sin\beta = \int_0^{2\pi a} \frac{\mu_0 I}{4\pi r^2} dl \sin\beta$$

$$= \frac{\mu_0 I a \sin\beta}{2r^2}. \qquad \text{(i)}$$

When the coil has more than one turn, the distance r varies slightly from one turn to the next. But if the width of the coil is small compared with all its other dimensions, we may neglect it, and write,

$$B = \frac{\mu_0 N I a \sin\beta}{2r^2}, \qquad \text{(ii)}$$

where N is the number of turns.

Equation (i) can be put into a variety of forms, by using the facts that

$$\sin\beta = \frac{a}{r},$$

and

$$r^2 = x^2 + a^2,$$

where x is the distance from P to the centre of the coil. Thus

$$B = \frac{\mu_0 N I a^2}{2r^3} = \frac{\mu_0 N A I}{2\pi(x^2+a^2)^{3/2}}, \qquad \text{(1)}$$

When the distance x is large compared with a, the expression (1) reduced to:

$$B = \frac{\mu_0 N A I}{2\pi x^3} = \frac{\mu_0 m}{2\pi x^3}, \qquad \text{(2)}$$

where $m = NAI$ = the *magnetic moment* of the circular coil, p. 885.

Helmholtz Coils

The field along the axis of a single coil varies with the distance x from the coil. In order to obtain a *uniform* field, Helmholtz used two coaxial parallel coils of equal radius R, separated by a distance R. In this case, when the same current flows round each coil in the same direction, the resultant field B is uniform for some distance on either side of the point on their axis midway between the coils. This may be seen roughly by adding the fields due to each coil alone. Helmholtz coils were used in Thomson's determination of e/m (p. 1003).

The magnitude of the resultant field B at the midpoint can be found from our previous formula for a single coil. We now have $a = R$ and $x = R/2$. Thus, for the two coils,

$$B = 2 \times \frac{\mu_0 NIR^2}{2(R^2/4 + R^2)^{3/2}} = \left(\frac{4}{5}\right)^{3/2} \times \frac{\mu_0 NI}{R}$$

$$= 0{\cdot}72 \frac{\mu_0 NI}{R} \text{ (approx.)}.$$

Field on Axis of a Long Solenoid

We may regard a solenoid as a long succession of narrow coils; if it has n turns per metre, then in an element δx of it there are $n\delta x$ coils (Fig. 37.5). At a point P on the axis of the solenoid, the field due to these is, by equation (ii),

$$\delta B = \frac{\mu_0 Ia \sin \beta}{2r^2} n\delta x,$$

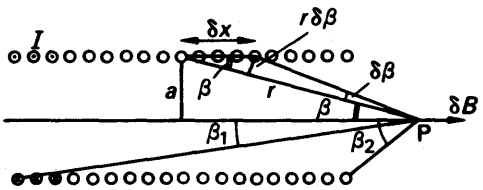


FIG. 37.5. Field on axis of solenoid.

in the notation which we have used for the flat coil. If the element δx subtends an angle $\delta\beta$ at P, then, from the figure,

$$r\delta\beta = \delta x \sin \beta;$$

whence

$$\delta x = \frac{r\delta\beta}{\sin \beta}.$$

Also,

$$a = r \sin \beta.$$

Thus

$$\delta B = \frac{\mu_0 Ir \sin^2 \beta}{2r^2} n \frac{r\delta\beta}{\sin \beta}.$$

$$= \frac{\mu_0 nI}{2} \sin \beta \delta\beta.$$

If the radii of the coil, at its ends, subtend the angles β_1 and β_2 at P, then the field at P is

$$H = \int_{\beta_1}^{\beta_2} \frac{\mu_0 nI}{2} \sin \beta d\beta$$

$$= \frac{\mu_0 nI}{2} \left[-\cos \beta\right]_{\beta_2}^{\beta_2},$$

$$= \frac{\mu_0 nI}{2} (\cos \beta_1 - \cos \beta_2). \quad . \quad . \quad . \quad . \quad (1)$$

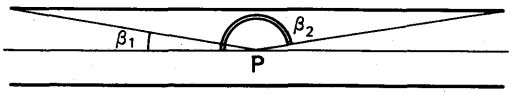


FIG. 37.6. A very long solenoid.

If the point P is inside a very long solenoid—so long that we may regard it as infinite—then $\beta_1 = 0$ and $\beta_2 = \pi$, as shown in Fig. 37.6. Then, by equation (1):

$$B = \frac{\mu_0 nI}{2}\Big[-\cos\beta\Big]_0^{\pi}.$$

whence

$$B = \mu_0 nI \quad . \quad . \quad . \quad . \quad . \quad (1)$$

The quantity nI is often called the 'ampere-turns per metre'.

Very Long Solenoid or Toroid

Equation (1) shows that the field along the axis of an infinite solenoid is constant: it depends only on the number of turns per centimetre, and the current. By methods beyond the scope of this book, it can also be shown that the field is the same at points not on the axis. An infinite solenoid therefore gives us a means of producing a uniform magnetic field.

In practice, solenoids cannot be made infinitely long. But if the length of a solenoid is about ten times its diameter, the field near its middle is fairly uniform, and has the value given by equation (1).

A form of coil which gives a very nearly uniform field is shown in Fig. 37.7. It is a solenoid of N turns and length L metre wound on a

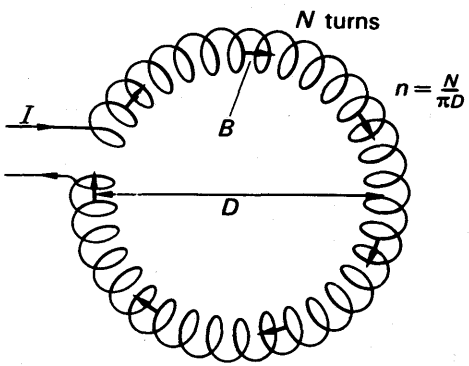


FIG. 37.7. A toroid.

circular support instead of a straight one, and is called a toroid. If its average diameter D is several times its core diameter d, then the turns of wire are almost equally spaced around its inside and outside circumferences; their number per metre is therefore

$$n = \frac{N}{L} = \frac{N}{\pi D} \quad . \quad . \quad . \quad . \quad (1)$$

The magnetic field within a toroid is very nearly uniform, because the coil has no ends. The coil is equivalent to an infinitely long solenoid, and the field-strength at all points within it is given by

$$B = \mu_0 nI \quad . \quad . \quad . \quad . \quad (2)$$

A 'Slinky' is a coil which can be stretched to provide simply a solenoid with a varying number of turns per metre, n. A small search coil with many thousands of turns, placed coaxially inside the solenoid, can be connected to an oscilloscope to provide a measure of B when alternating current is passed into the solenoid. See p. 934. Since $n \propto 1/L$, where L is the length of the coil, a graph of B against $1/L$ can be plotted for various values of L, the current being the same each time. A straight line through the origin is obtained, showing that $B \propto n$. A search coil connected to a ballistic galvanometer, and a direct current which is reversed in the coil, may also be used to provide a measure of B (p. 919).

Forces between Currents

Ampère carried out many experiments on the forces of attraction and repulsion between two current-carrying conductors. Each was acted on by the field of the other, as shown in Fig. 37.8. Currents flowing in the same direction ('like' currents) attracted each other,

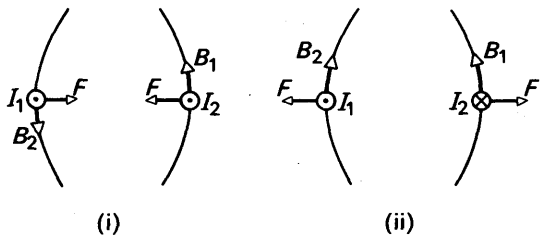


FIG. 37.8. Forces between currents.

Fig. 37.8 (i), while 'unlike' currents (in opposite directions) repelled each other, Fig. 37.8 (ii). This difference from the laws governing poles and charges greatly impressed Ampère.

If two long straight conductors lie parallel and close together at a distance r apart, and carry currents I, I' respectively, then the current I is in a magnetic field of flux density B equal to $\mu_0 I'/2\pi r$ due to the current I (p. 934). The force per metre length F is hence given by

$$F = BIl = BI \times 1 = \frac{\mu_0 I'}{2\pi r} \times I \times 1$$

$$\therefore F = \frac{\mu_0 II'}{2\pi r} \quad . \quad . \quad . \quad . \quad (1)$$

Nowadays the ampere is *defined* in terms of the force between conductors. It is *that current, which flowing in each of two infinitely-long*

parallel straight wires of negligible cross-sectional area separated by a distance of 1 metre in vacuo, produces a force between the wires of 2×10^{-7} *newton metre*$^{-1}$.

Taking $I = I' = 1$ A, $r = 1$ metre, $F = 2 \times 10^{-7}$ newton metre^{-1}, then, from (1),

$$2 \times 10^{-7} = \frac{\mu_0 \times 1 \times 1}{2\pi \times 1}$$

$$\therefore \mu_0 = 4\pi \times 10^{-7} \text{ henry metre}^{-1},$$

which is the value quoted above.

Unit of μ_0

The permeability of free space μ_0 was defined previously from the relation

$$\delta B = \frac{\mu_0 I \delta s \sin \theta}{4\pi r^2}$$

From this relation, the unit of μ_0 is

$$\frac{\text{weber metre}^{-2} \times \text{metre}^2}{\text{ampere} \times \text{metre}} \text{ or Wb A}^{-1}\text{ m}^{-1} \quad . \quad (2)$$

Now the unit of inductance L is the henry (H), which can be defined from the relation $\Phi = LI$ (p. 925). Thus, since $L = \Phi/I$

$$1 \text{ H} = 1 \text{ Wb A}^{-1}.$$

From (2), it follows that the unit of μ_0 can be written as

$$\text{H m}^{-1} \text{ (henry per metre)},$$

and this is the SI unit of μ_0 and of permeability μ generally.

Absolute Determination of Current

A laboratory form of an *ampere balance*, which measures current by measuring the force between current-carrying conductors, is shown in Fig. 37.9.

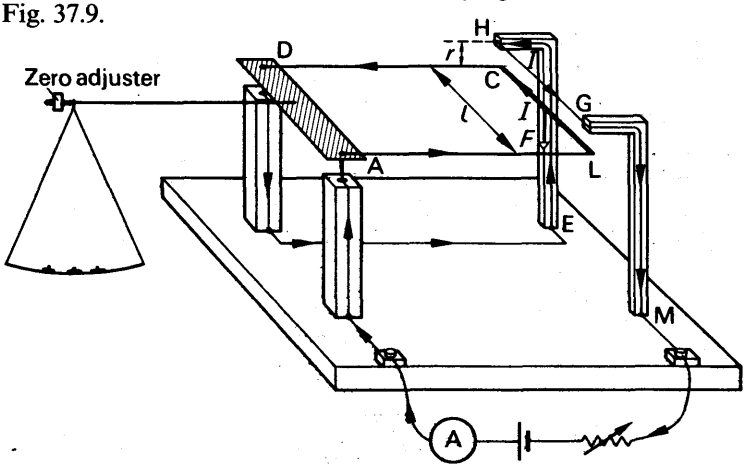


FIG. 37.9. Laboratory form of Ampere Balance.

With no current flowing, the zero screw is adjusted until the plane of ALCD is horizontal. The current I to be measured is then switched on so that it flows through ALCD and EHGM in series and HG repels CL. The mass m necessary to restore balance is then measured, and mg is the force between the conductors since the respective distances of CL and the scale pan from the pivot are equal. The equal lengths l of the straight wires CL and HG, and their separation r, are all measured.

From equation (1) on p. 939,

$$\text{force per metre} = \frac{4\pi \times 10^{-7} I^2}{2\pi r}$$

$$\therefore mg = \frac{4\pi \times 10^{-7} I^2 l}{2\pi r}$$

$$\therefore I = \sqrt{\frac{mgr}{2 \times 10^{-7} l}}$$

In this expression, I will be in amperes if m is in kilogrammes, $g = 9{\cdot}8\ \text{m s}^{-2}$ and l and r are measured in metres.

Magnetizing Force, or Intensity, H

Biot and Savart's law has been stated as

$$\delta B = \frac{\mu_0 I \delta l \sin \alpha}{4\pi r^2}$$

This is only true in air or a vacuum. In other materials the flux density may be altered, even though the currents remain the same. To take account of this we write

$$\delta B = \frac{\mu I \delta l \sin \alpha}{4\pi r^2}$$

where μ is the *permeability* of the medium. μ_0 is called the permeability of 'free space' or vacuum.

So far we have only used the flux density, B, in a field. Another field quantity, symbol H, is also used. It is called the *magnetizing force or intensity*, and is defined by the relation

$$H = \frac{B}{\mu}.$$

We may thus write δH arising from a current I in an element of length δl as

$$\delta H = \frac{I \delta l \sin \alpha}{4\pi r^2} \quad . \quad . \quad . \quad . \quad (1)$$

From this it can be seen that the unit of H is

$$\frac{\text{ampere metre}}{\text{metre}^2} \quad \text{or} \quad \text{ampere metre}^{-1}\ (\text{A m}^{-1});$$

whereas the unit of B in the tesla (T) or weber metre^{-2} (Wb m^{-2}).

In any medium δB has a value depending on the permeability of the medium. From (1), it can be seen that H does not depend on μ but

only the currents and their geometry. H is *independent*, therefore, of the medium in which the conductors are situated. It is for this reason that H is regarded as being due directly to the currents. H is then a 'cause' which gives rise to a flux density B given by μH, and so B is dependent on the medium used. Because of this interpretation, H is often called the 'magnetizing force', or 'magnetizing intensity'.

From equation (1) on p. 934, the magnetizing force due to a straight wire is given by

$$H = \frac{I}{2\pi a}.$$

Ampère's Theorem

In the calculation of magnetic fields, we have used so far only the Biot and Savart law. Another law useful for calculating magnetic field strengths is *Ampère's theorem.*

Consider Fig. 37.10, in which a continuous closed line or loop L is drawn round the wires P, Q, R which carry currents of I_1, I_2, I_3 respectively. The total current enclosed by L is $(I_1+I_2+I_3)$. Now if H is the magnetizing force or *magnetic field intensity* at any element dl

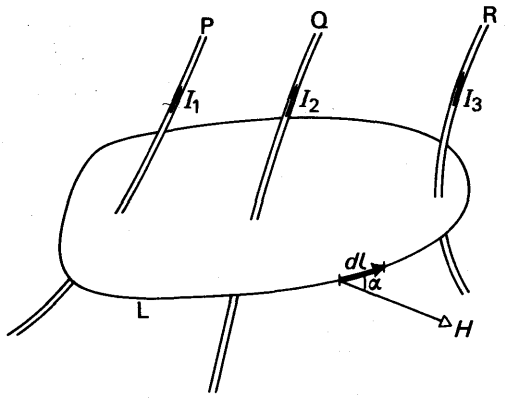


FIG. 37.10. Ampere's theorem.

in the loop L, obtained from the Biot and Savart law, it is possible to calculate the quantity $H.dl.\cos\alpha$, where α is the angle between the element and the field H. When the quantity $H.dl.\cos\alpha$ is summed for every element in the closed loop L, calculation beyond the scope of this book shows that the result is $(I_1+I_2+I_3)$. This is expressed by

$$\oint H\,.dl\,.\cos\alpha = I_1+I_2+I_3,$$

where the symbol $\oint$ represents the integral taken completely round the closed loop. Ampère's theorem is the general statement

$$\oint H\,.\,dl\,.\cos\alpha = I,$$

where I is the total current enclosed by the loop.

We now apply the theorem to two special cases of current-carrying conductors.

1. *Straight wire*

Fig. 37.11 shows a circular loop L of radius r, drawn concentrically round a straight wire carrying a current I. The lines of force are circles and hence, at *every* part of a closed line, H is directed along the line itself. Thus $\alpha = 0°$ all round the line. Further, by symmetry, H has the same value everywhere on the line.

$$\therefore \oint H \,.\, dl \,.\, \cos\alpha = H \oint dl = H \,.\, 2\pi r,$$

since $\alpha = 0°$ and H is constant. Hence, from Ampère's theorem,

$$H \,.\, 2\pi r = I$$

$$\therefore H = \frac{I}{2\pi r}, \qquad \text{and} \qquad B = \mu_0 H = \frac{\mu_0 I}{2\pi r}.$$

This agrees with the result for B on p. 934.

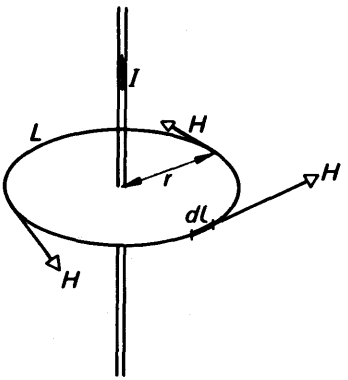


FIG. 37.11. Field intensity of straight wire.

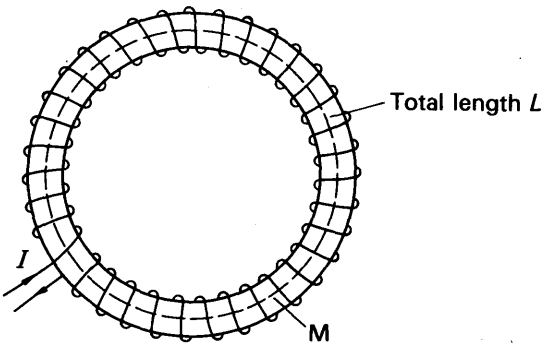


FIG. 37.12. Field intensity of toroid.

2. *Toroid*

Consider the closed loop M indicated by the broken line in Fig. 37.12. Again H is everywhere the same on M and is directed along the loop.

$$\therefore \oint H \,.\, dl \cos\alpha = H \oint dl$$
$$= HL,$$

where L is the total length of the loop M. Hence, from Ampere's theorem,

$$HL = NI$$

$$\therefore H = \frac{NI}{L} = nI,$$

where N is the total number of turns, and n is the number of turns per metre. This agrees with p. 938.

Earth's Magnetism

It was Dr. Gilbert who first showed that a magnetized needle, when freely suspended about its centre of gravity, dipped downwards towards the north at about 70° to the horizontal in England. He also found that this *angle of dip* increased with latitude, as shown in Fig. 37.13, and concluded that the earth itself was, or contained, a magnet. The points where the angle of dip is 90° are called the earth's magnetic poles; they

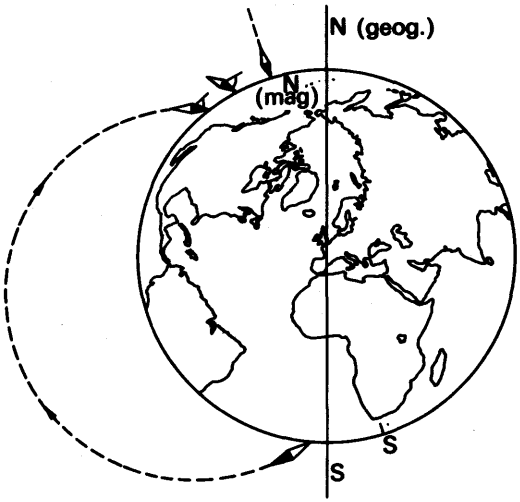


FIG. 37.13. Illustrating the angle of dip.

are fairly near to the geographic poles, but their positions are continuously, though slowly, changing. Gilbert's simple idea of the earth as a magnet has had to be rejected. The earth's crust does not contain enough magnetic material to make a magnet of the required strength; the earth's core is, we believe, molten—and molten iron is non-magnetic. The origin of the earth's magnetism is, in fact, one of the great theoretical problems of the present day.

Horizontal and Vertical Components. Variation and Dip

Since a freely suspended magnetic needle dips downward at some angle δ to the horizontal, the earth's *resultant magnetic field*, B_R acts at an angle δ to the horizontal. The 'angle of dip', or *inclination*, can

thus be defined as the angle between the resultant earth's field and the horizontal. The earth's field has a *vertical component*, B_V, given by

$$B_V = B_R \sin \delta, \quad . \quad . \quad . \quad . \quad (1)$$

and a *horizontal component*, B_H, given by

$$B_H = B_R \cos \delta. \quad . \quad . \quad . \quad . \quad (2)$$

Also,

$$\frac{B_V}{B_H} = \tan \delta. \quad . \quad . \quad . \quad . \quad (3)$$

To specify the earth's magnetic field at any point, we must state its

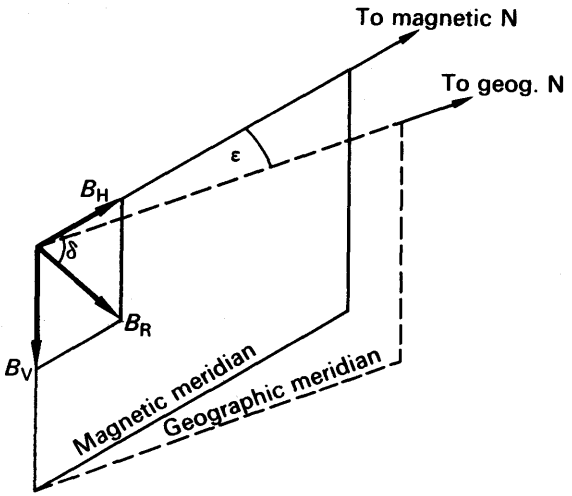


FIG. 37.14. Magnetic and geographic meridians. Dip.

strength and direction. To specify its direction we must give the direction of the magnetic meridian, and the angle of dip δ (Fig. 37.14). In most parts of the world the magnetic meridian does not lie along the geographic meridian (the vertical plane running geographically north–south). The angle between the magnetic and geographic meridians, ε, is called the magnetic variation, or sometimes the declination, at the place concerned; it is shown on the margins of maps. The horizontal and vertical components of the earth's field, and the angle of dip, can be measured by a large coil or 'earth inductor' (see p. 921).

EXERCISES 37

1. Define the *ampere*. Write down expressions for (i) the magnetic field strength (magnetizing force) at a distance of d from a very long straight conductor carrying a current I, and (ii) the mechanical force acting on a straight conductor of length l carrying a current I at right angles to a uniform magnetic field of flux density B.

Show how these two expressions may be used to deduce a formula for the force per unit length between two long straight parallel conductors in vacuo carrying currents I_1 and I_2 separated by a distance d.

A horizontal straight wire 5 cm long weighing 1·2 g m^{-1} is placed perpendicular to a uniform horizontal magnetic field of flux density 0·6 Wb m^{-2}. If the resistance of the wire is 3·8 ohm m^{-1}, calculate the p.d. that has to be applied between the ends of the wire to make it just self-supporting. Draw a diagram showing the direction of the field and the direction in which the current would have to flow in the wire. (*C.*)

2. State the law of force acting on a conductor carrying an electric current in a magnetic field. Indicate the direction of the force and show how its magnitude depends on the angle between the conductor and the direction of the field.

Sketch the magnetic field due solely to two long parallel conductors carrying respectively currents of 12 and 8 A in the same direction. If the wires are 10 cm apart, find where a third parallel wire also carrying a current must be placed so that the force experienced by it shall be zero. (*L.*)

3. Define the *ampere*.

Two long vertical wires, set in a plane at right angles to the magnetic meridian, carry equal currents flowing in opposite directions. Draw a diagram showing the pattern, in a horizontal plane, of the magnetic flux due to the currents alone—that is, for the moment ignoring the earth's magnetic field.

Next, taking into account the earth's magnetic field, discuss the various situations that can give rise to neutral points in the plane of the diagram.

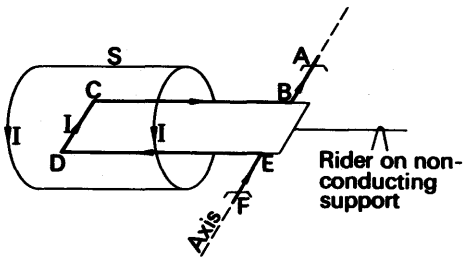


FIG. 37.15.

Fig. 37.15 shows a simple form of current balance. The 'long' solenoid S, which has 2000 turns per metre, is in series with the horizontal rectangular copper loop ABCDEF, where BC = 10 cm and CD = 3 cm. The loop, which is freely pivoted on the axis AF, goes well inside the solenoid, and CD is perpendicular to the axis of the solenoid. When the current is switched on, a rider of mass 0·2 g placed 5 cm from the axis is needed to restore equilibrium. Calculate the value of the current, I. (*O.*)

4. Define *magnetic moment*.

A small magnet, suspended with its axis horizontal so as to be able to rotate freely about a vertical axis, is situated at the centre of a long horizontal solenoid,

the axis of which lies at right angles to the magnetic meridian. If the solenoid has 20 turns per cm, determine the value of the current passing through it which would cause the magnet to rotate through 50°. (Horizontal component of earth's magnetic field intensity = 14 A m^{-1}.) (*N.*)

5. Describe with experimental details how you would carry out any *two* of the following in the laboratory:

(*a*) Determine the current sensitivity of a moving-coil galvanometer.

(*b*) Determine the internal resistance of a dry cell using a potentiometer.

(*c*) Investigate, using a vibration magnetometer, how the magnetic field due to a long straight wire carrying a steady current varies with distance from the wire. (*L.*)

6. Define the *ampere.*

Draw a labelled diagram of an instrument suitable for measuring a current absolutely in terms of the ampere, and describe the principle of it.

A very long straight wire PQ of negligible diameter carries a steady current I_1. A square coil ABCD of side l with n turns of wire also of negligible diameter is set up with sides AB and DC parallel to and coplanar with PQ; the side AB is nearest to PQ and is at a distance d from it. Derive an expression for the resultant force on the coil when a steady current I_2 flows in it, and indicate on a diagram the direction of this force when the current flows in the same direction in PQ and AB.

Calculate the magnitude of the force when $I_1 = 5$ A, $I_2 = 3$ A, $d = 3$ cm, $n = 48$ and $l = 5$ cm. (*O. & C.*)

7. (i) Draw a diagram showing the pattern of the magnetic field lines in a horizontal plane passing through the centre of a plane vertical circular coil carrying a steady current. Indicate on your diagram possible positions for neutral points if the plane of the coil were to be set in the magnetic meridian.

(ii) How would you show experimentally that the value of B at the centre of such a coil is proportional to nI/r, where n is the number of turns and r the radius?

8. A long straight wire carries a steady current I. Write down a formula for the magnetic intensity (magnetizing force) H due to the current at a point distant y from the wire. Give a consistent set of units for the quantities in your formula.

Show on a diagram the lines of magnetic force due to the currents when two parallel wires separated by a distance $2a$ carry equal currents in the same direction. Use your formula to derive an expression for the magnetic intensity at a point P in a plane perpendicular to the wires at a distance x from the point midway between the wires and along the right bisector of the line joining them in this plane. Hence derive the condition that the intensity at P shall be a maximum. (*N.*)

chapter thirty-eight

Magnetic Properties of Materials

THE magnetic properties of materials require investigation to decide whether they are suitable for permanent magnets such as loudspeaker magnets, for temporary magnets such as electromagnets, or for cores of electromagnetic induction machines such as transformers.

Induction or Flux Density in Magnetic Material

Consider a toroid of length L, wound with N turns each carrying a current I round a ring of magnetic material, Fig. 38.1.

The *total flux density* B in the material is partly due to the currents

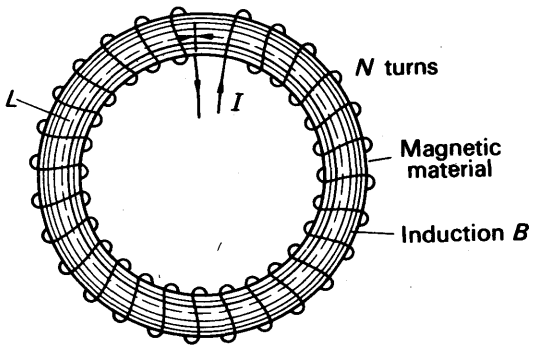


FIG. 38.1. Induction in magnetic material.

flowing in the wire and partly due to the magnetization of the material. We thus write

$$B = B_0 + B_M, \quad . \quad . \quad . \quad . \quad (1)$$

where B_0 is the flux density due directly to the current in the wire, and B_M is the flux density due to the magnetization of the material.

We now assume that the induction B_M is produced by many small circulating currents inside the magnetic material, due to the circulating and spinning electrons in the atoms. Fig. 38.2 shows that the effect of many small adjacent current loops may be thought of as one current loop. In the same way, the internal circulating and spinning currents can be replaced by a single current I_M flowing in the coil wound round

the core. This theoretical surface or magnetization current is additional to the real or actual current I flowing in the coil.

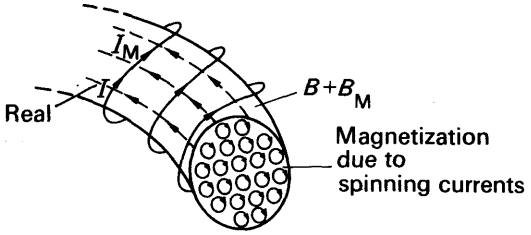


FIG. 38.2. Magnetizing surface current.

By itself, the real current I produces a flux-density B_0. If the toroid has n turns per unit length ($n = N/L$), then, from p. 938,

$$B_0 = \mu_0 nI.$$

The surface current I_M may be imagined to flow in n turns per metre of the solenoid, as the real current does. The flux density or induction B_M is then given by

$$B_M = \mu_0 n I_M.$$

This is the induction which would be produced by the current I_M if the material were not present in the toroid, that is, a current I_M in the coil would produce a flux density equal to that due to the magnetization in the material.

$$\therefore \text{ total induction } B = B_0 + B_M = \mu_0 n(I + I_M) \quad . \quad (2)$$

Intensity of Magnetization

It is possible to write B_M, the induction due to the magnetization of the material, in a different way. The magnetic moment of each turn due to this imaginary surface current $= A \times I_M$, where A is the area of each turn. See p. 885. The magnetic moment of the whole toroid is then $nLAI_M$, since nL is the total number of turns. Hence the *magnetic moment per unit volume*

$$= \frac{nLAI_M}{\text{volume}} = \frac{nLAI_M}{LA} = nI_M$$

The 'magnetic moment per unit volume' is called the *intensity of magnetization*, M, of the magnetic core in the toroid. Thus $B_M = \mu_0 nI_M = \mu_0 M$. Further, nI is the magnetic field intensity, H, in the toroid due to the current I.

Thus the *total induction or flux-density* B in the core when the coil carries a current is given by

$$B = B_0 + B_M = \mu_0 nI + \mu_0 nI_M$$

$$\therefore B = \mu_0(H + M) \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Equation (3) relates the total induction B to the magnetizing field intensity H due to the current and to the intensity of magnetization M of the material produced. Note that H and M have the same units from (3). *Thus the unit of M is ampere per metre* (A m^{-1}).

EXAMPLE

A solenoid 1 m long is wound with 10^4 turns of copper wire each carrying a current of 10 A. If the cross-sectional area of the coil is 10 cm^2, calculate (*a*) the magnetizing force or intensity in the solenoid, (*b*) the couple exerted on the solenoid when it is placed at right angles to an external field of induction 10^{-2} T.

If the solenoid is now filled with certain magnetic material, the induction in the material is 1·5 T. Calculate (*c*) the intensity of magnetization in the material, (*d*) the total couple exerted when the solenoid and material inside is placed at right angles to a field of induction $10^{-}2$ T.

(*a*) We have $H = nI$, where n is the number of turns per metre,

$$= 10^4 \times 10$$

$$= 10^5 \text{ ampere metre}^{-1} \text{ (A m}^{-1}\text{)}.$$

(*b*) Magnetic moment of each solenoid turn $= IA$.

$\therefore$ magnetic moment of empty solenoid, m, $= NIA$.

Since $N = 10^4$, $I = 10$ A, $A = 10$ cm^2 $= 10 \times 10^{-4}$ m^2,

$$\therefore m = 10^4 \times 10 \times 10 \times 10^{-4}$$

$$= 100 \text{ A m}^2.$$

Now couple $C = mB \sin \alpha$, where α is the angle between the solenoid axis and the external field B.

$$\therefore C = mB \sin 90°$$

$$= 100 \times 10^{-2}$$

$$= 1 \text{ newton metre (N m)}$$

(*c*) We have $B = \mu_0(H + M)$

$$\therefore M = \frac{B - \mu_0 H}{\mu_0}$$

$$= \frac{1{\cdot}5 - 4\pi \times 10^{-7} \times 10^5}{4\pi \times 10^{-7}}$$

$$= 10{\cdot}9 \times 10^5 \text{ ampere metre}^{-1}$$

(*d*) Total magnetic moment, m, of the specimen is given by,

$$m = MV$$

$$= 10{\cdot}9 \times 10^5 \times (1 \times 10^{-3})$$

$$= 10{\cdot}9 \times 10^2 \text{ A m}^2$$

$\therefore$ Total magnetic moment of the solenoid and the material inside

$$= 100 + 10{\cdot}9 \times 10^2$$

$$= 11{\cdot}9 \times 10^2 \text{ A m}^2$$

$$\therefore \text{ Couple} = mB \sin \alpha \text{ newton metre}$$
$$= 11{\cdot}9 \times 10^2 \times 10^{-2} \times \sin 90°$$
$$= 11{\cdot}9 \text{ N m}.$$

Relative Permeability

The permeability of a material is defined by the relation $\mu = B/H$. Now, in general, $B = \mu_0(H+M)$

$$\therefore \mu H = \mu_0(H+M)$$

or
$$\frac{\mu}{\mu_0} = 1+\frac{M}{H}$$

The ratio μ/μ_0 is called the *relative permeability*, μ_r, of the material. The ratio M/H is called the *susceptibility*, χ, of the material. Hence, from above,

$$\mu_r = 1+\chi. \quad . \quad . \quad . \quad . \quad (4)$$

It should be noted that μ_r and χ are dimensionless; each is the ratio of two quantities having the same dimensions. Permeability, μ, however, which is the ratio B/H, has dimensions; its unit is *henry per metre* (H m^{-1}). See p. 940.

We shall describe shortly how the variation of B with H may be measured. Here we shall anticipate the results. As a magnetic material, originally unmagnetized, is subjected to an increasing field, the intensity of magnetization M increases until it reaches a maximum value (Fig. 38.3 (*a*)). The material is then 'saturated'; that is, its magnetic

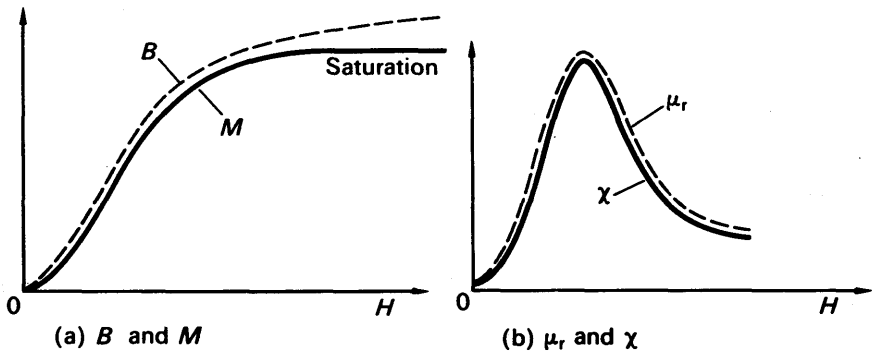


FIG. 38.3. Variation of B, M, μ_r, χ.

'domains' are completely aligned with the field H. B, however, continues to increase with H, since $B = \mu_0(H+M)$. Fig. 38.3 (*b*) also shows how the relative permeability μ_r and the susceptibility χ varies with H. It increases at first, and passes through a maximum value. As the material approaches saturation the domains cannot yield much further, and the susceptibility falls to a low value.

Variation of B with H

Ferromagnetic materials are those which have a high susceptibility. This is generally very much greater than 1, for example, 3000. Some materials are capable of retaining their magnetization and forming strong permanent magnets. Others form temporary magnets (p. 954).

The relationship between B, the flux density in a material, and the applied magnetizing field or force H, is best investigated experimentally by using a toroid shaped specimen. This eliminates the reduction in the field due to the effect of poles at the ends of a cylindrical rod. However, in the experiment described shortly, a specimen is placed inside a long current-carrying solenoid. This does not produce as large a value B as that obtained with a toroid, but the essential features of the variation of B with H are still observed.

One form of apparatus is shown in Fig. 38.4. The current I through the solenoid is measured on the ammeter A, and since $H = nI$, the current I is directly proportional to the magnetizing force H. C is a

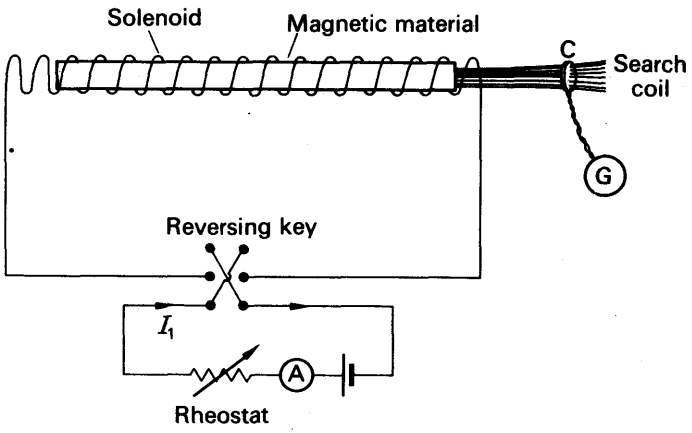


FIG. 38.4. Variation of B with H by experiment.

small search coil, which is placed in contact with the end of the specimen, and which is connected to a ballistic galvanometer G. When the coil is sharply removed from the vicinity of the specimen the throw on the galvanometer gives a measure of the flux change (see p. 920). Since the number of turns and area of the coil is constant, this throw will be proportional to the flux density B.

Ths specimen is first demagnetized (see p. 958) and placed in the solenoid. The current I is now increased in steps from zero, and the corresponding deflections θ on the ballistic galvanometer are observed. The specimen is taken through a *magnetic cycle* of magnetization. This is done by increasing I until θ is nearly constant, when the specimen has become saturated, then reducing I to zero and reversing it until saturation is reached in the opposite direction, and finally reducing I to zero and increasing it once more in the opposite direction.

H can be calculated from the relation $H = nI$. If the search coil has N turns of area A, and the ballistic galvanometer is calibrated, then B can be found from:

$$\text{Charge} = \frac{\text{Change in flux}}{\text{Resistance of circuit, } R}$$

or

$$Q = \frac{BAN}{R}$$

If R and Q are measured, then B can be found.

Sometimes it is required to plot M against H rather than B against H. In this case M is calculated, using the relation (3) on p. 949, from

$$M = \frac{B - \mu_0 H}{\mu_0}.$$

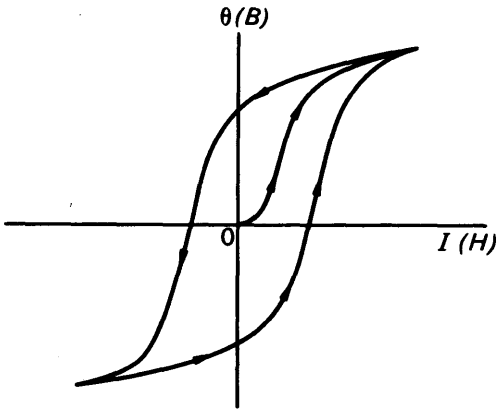


FIG. 38.5. *B-H* variation.

If only the general form of the $B-H$ curve is wanted, it is sufficient to plot θ against I. Fig. 38.5 illustrates a typical graph obtained; the arrows show the sequence in going round the magnetic cycle.

Hysteresis. Remanence. Coercive Force

Fig. 38.6 shows the variation of magnetic induction, B, with the applied field, H, when the specimen is taken through a complete cycle. After the specimen has become saturated, and the field is reduced to zero, the iron is still quite strongly magnetized, setting up a flux-density B_r. This flux-density is called the *remanence*; it is due to the tendency of groups of molecules, or domains, to stay put once they have been aligned.

When the field is reversed, the residual magnetism is opposed. Each increase of magnetizing field now causes a decrease of flux-density, as the domains are twisted farther out of alignment. Eventually, the flux-density is reduced to zero, when the opposing field H has the value

H_c. This value of H is called the *coercive force* of the iron; it is a measure of the difficulty of breaking up the alignment of the domains.

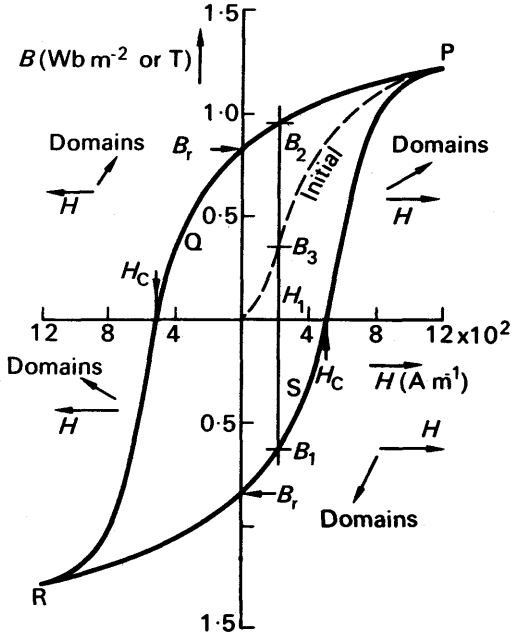


FIG. 38.6. Hysteresis loop.

We now see that, when once the iron has been magnetized, its magnetization curve never passes through the origin again. Instead, it forms the closed loop PQRS, which is called a *hysteresis loop*. Hysteresis, which comes from a Greek work meaning 'delayed', can be defined as *the lagging of the magnetic induction, B, behind the magnetizing field, H*, when the specimen is taken through a magnetic cycle.

Properties of Magnetic Materials

Fig 38.7 shows the hysteresis loops of iron and steel. Steel is more suitable for permanent magnets, because its high coercivity means that it is not easily demagnetized by shaking. The fact that the remanence of iron is a little greater than that of steel is completely outweighed by its much smaller coercivity, which makes it very easy to demagnetize. On the other hand, iron is much more suitable for electromagnets, which have to be switched on and off, as in relays. Iron is also more suitable for the cores of transformers and the armatures of machines. Both of these go through complete magnetizing cycles continuously: transformer cores because they are magnetized by alternating current, armatures because they are turning round and round in a constant field. In each cycle the iron passes through two parts of its hysteresis loop (near Q and S in Fig. 38.6), where the magnetizing field is having to demagnetize the iron. There the field is doing work against the internal

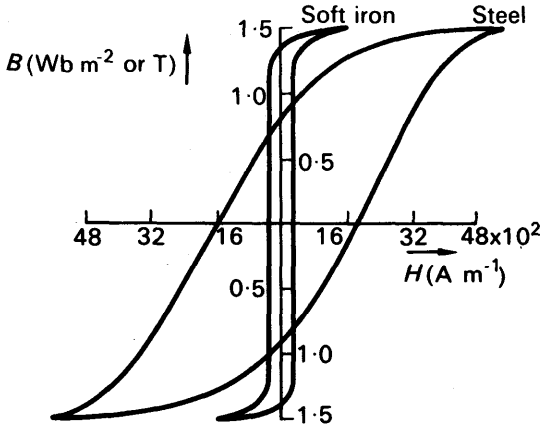


FIG. 38.7. Hysteresis curves for iron and steel.

friction of the domains. This work, like all work that is done against friction, is dissipated as heat. The energy dissipated in this way, per cycle, is less for iron than for steel, because iron is easier to demagnetize. It is called the *hysteresis loss*; we will show soon that it is proportional to the area of the *B-H* loop (p. 957).

In a large transformer the hysteresis loss, together with the heat developed by the current in the resistance of the windings, liberates so much heat that the transformer must be artificially cooled. The cooling is done by circulating oil, which itself is cooled by the atmosphere: it passes through pipes which can be seen outside the transformer, running from top to bottom.

The table on p. 956 gives the properties of some typical magnetic materials; mumetal and ticonal are inventions of the last twenty years, the results of deliberate attempts to develop materials with extreme properties.

Hysteresis Loss; Area of Loop

To calculate the work done in carrying a piece of iron round a hysteresis loop, we adopt the method which we used to calculate the energy stored in the magnetic field of a coil: we consider the back-e.m.f. induced during a change of flux.

Fig 38.8 shows a ring of iron, wound with a uniform magnetizing coil of N turns, and mean length l. If the current through the coil is I, the magnetizing field is

$$H = \frac{NI}{l} \quad . \quad . \quad . \quad \text{(i)}$$

And if B is the flux density in the iron, and A its cross-sectional area, the flux through it $= AB$. The flux linkages with the magnetizing coil are therefore

$$\Phi = NAB.$$

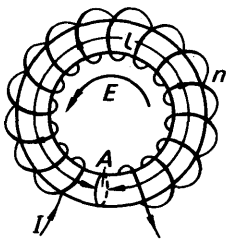


FIG. 38.8. Work done in magnetization of iron.

PROPERTIES OF MAGNETIC MATERIALS

Material	Relative Permeability μ_r	Saturation B_m 10^{-4} T	Remanence B_r 10^{-4} T	Coercivity H_c A m^{-1}	Hysteresis loss W J m^{-3} per cycle	Critical temp. t,* °C	Resistivity† ρ (approx.) 10^{-8} Ω-m	Applications
	SOFT MATERIALS (iron-like, used for electromagnets with varying currents)							
Iron (99·94%)	5500	21500	13000	81·0	500	770	10	
Nickel	600	6100	3600	272	30	360	7	
Cobalt	240	18000	5000	800	200	1120	10	
Silicon Iron (Stalloy) (Fe 96, Si 4)	6700	20000	12000	40	350	690	55	Armatures, relays, large transformer cores, telephone diaphragms
Mumetal (Ni 74, Fe 20, Cu 5, Mn 1)	80000	8500	6000	4	20	—	—	Small transformer cores, magnetic shields
	HARD MATERIALS (steel-like, used for permanent magnets)							
Carbon Steel	—	10000	8000	4800	20000	—	—	
Cobalt Steel	—	—	8000–10000	12000–19200	—	—	—	Moving-coil instruments, loudspeakers, microphones, telephone earpiece magnets
Ticonal (Fe 51, Co 24, Ni 14, Al 8, Cu 3)	—	—	12500	44000	—	—	—	

* See page 963.

† The higher the resistivity of a magnetic material, the less the eddy-currents in it when the flux through it is changed; and therefore the less the energy lost as heat.

Now let us suppose that, in a brief time δt, we increase the current by a small amount, and so increase the flux and flux linkages. During the change a back-e.m.f. will be induced in the coil, of magnitude

$$E = \frac{d\Phi}{dt}$$

Hence, from above,

$$E = NA\frac{dB}{dt}$$

To overcome this back-e.m.f., the source of the current I must supply energy to the coil at the rate

$$P = EI$$

Thus the total energy supplied to the coil, in increasing the flux through the iron, is

$$\begin{aligned}\delta W = P\delta t &= EI\delta t \\ &= INA\frac{dB}{dt}\delta t \\ &= INA\delta B\end{aligned}$$

where δB is the increase in flux-density.

Let us now substitute for the current I in terms of the magnetizing field H. From equation (i) we have

$$I = \frac{Hl}{N}$$

Hence

$$\begin{aligned}\delta W &= \frac{lHNA}{N}\delta B \\ &= VH\delta B \quad . \quad . \quad . \quad . \quad . \quad \text{(ii)}\end{aligned}$$

where V, $= lA$, is the volume of the iron.

Equation (ii) shows that, in taking the flux from any value B_1 to any other B_2, the work done is

$$W = V\int_{B_1}^{B_2} HdB \quad . \quad . \quad . \quad . \quad \text{(iii)}$$

On unit volume of the iron, the work is

$$\int_{B_1}^{B_2} HdB$$

The integral in this expression is the area between the B-H curve and the axis of B. Round the complete hysteresis loop, the work done per unit volume is

$$W = \oint HdB$$

Here the symbol $\oint$ denotes integration round the closed loop (PQRSP in Fig. 38.6); the integral is proportional to the area of the loop.

Subsidiary Hysteresis Loops

When a piece of iron is magnetized, first one way and then the other, it goes round a hysteresis loop even if it is not magnetized to saturation

at any point (Fig. 38.9). The subsidiary loops, *ab* for example, may represent the magnetization of a transformer core by an alternating

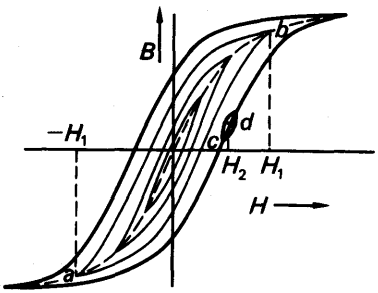


FIG. 38.9. Subsidiary hysteresis loops.

current in the primary winding: the amplitude H_1 of the magnetizing field is proportional to the amplitude of the current. A transformer core is designed so that it is never saturated under working conditions. For, if it were saturated, the flux through it would not follow the changes in primary current; and the e.m.f. induced in the secondary would be less than it should.

The energy dissipated as heat in going round a subsidiary hysteresis loop is proportional to the area of the loop, just as in going round the main one.

Another kind of subsidiary loop is shown at *cd* in the figure. The iron goes round such a loop when the field is varied above and below the value H_2. This happens in a transformer when the primary carries a fluctuating direct current.

Demagnetization

The only satisfactory way to demagnetize a piece of iron or steel is to carry it round a series of hysteresis loops, shrinking gradually to the origin. If the iron is the core of a toroid, we can do this by connecting the winding to an a.c. supply via a potential divider, as in Fig. 38.10, and reducing the current to zero. Since the iron goes through fifty loops per second, we do not have to reduce the current very slowly.

To demagnetize a loose piece of iron or steel, such as a watch, we merely put it into, and take it out of, a coil of many turns connected to an a.c. supply. As we draw the watch out, it moves into an ever-weakening field, and is demagnetized.

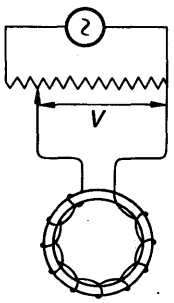


FIG. 38.10. Demagnetization.

Electromagnets and Magnetic Circuits

Consider the electromagnet shown in Fig. 38.11. The magnetic material, which is almost a complete toroid, is wound with N turns

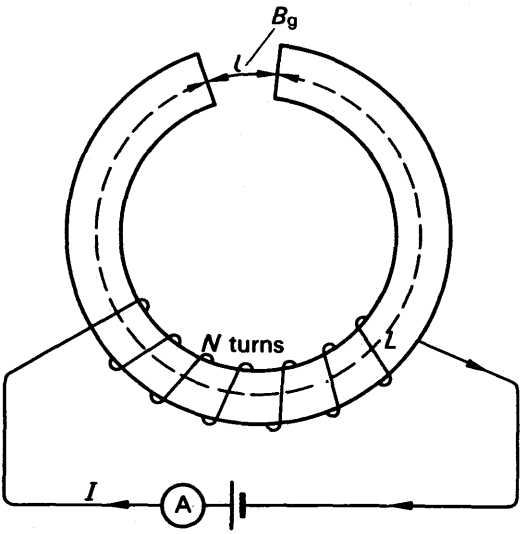


FIG. 38.11. Magnetic circuit.

each carrying a current I. The length of the magnetic material is L and of the small air gap l. We now calculate the induction B_g in the gap between the faces of magnetic material. From $B = \mu H$,

$$B_g = \mu_0 H_g \quad . \quad . \quad . \quad . \quad (1)$$

and

$$B_m = \mu H_m, \quad . \quad . \quad . \quad . \quad (2)$$

where H_g is the value of the magnetizing force in the gap, and H_m and B_m are respectively the magnetizing force and flux density inside the material. μ is the permeability of the material.

Taking the closed loop indicated in Fig. 38.11, by Ampere's theorem,

$$H_g l + H_m L = NI.$$

Thus from (1) and (2):

$$\frac{B_g l}{\mu_0} + \frac{B_m L}{\mu} = NI.$$

The total flux Φ round the magnetic circuit is constant. Since the area of cross-section of the air gap is the same as that of the material, it follows that

$$B_g = B_m.$$

$$\therefore B_g \left(\frac{l}{\mu_0} + \frac{L}{\mu} \right) = NI$$

or
$$B_g = \frac{NI}{(l/\mu_0 + L/\mu)} \quad . \quad . \quad . \quad (3)$$

$$\therefore \Phi = B_g A = \frac{NI}{(l/\mu_0 A + L/\mu A)}$$

In general, the value of μ depends on the current flowing in the electromagnet coil.

EXAMPLE

A toroid made from an iron bar of length 6 cm and of cross-sectional area 4 cm^2, has an air gap of length 1 cm. If it is wound with 500 turns of wire carrying a current of 20 A, find the flux density in the gap. The material may be assumed to have a permeability of 3000 times that of free space under these conditions (that is $\mu_r = 3000$). If there were no air gap, what would be the induction or flux density in the iron?

We have $N = 500$, $I = 20$ A, $l = 1$ cm $= 0{\cdot}01$ m, $\mu = \mu_r\mu_0 = 3000 \times 4\pi \times 10^{-7}$. From (3) above,

$$\therefore B_g = \frac{500 \times 20}{1/\mu_0(l + L/3000)} = \frac{500 \times 20 \times 4\pi \times 10^{-7}}{(0{\cdot}01 + 0{\cdot}0002)}$$

$$= 1{\cdot}3 \text{ Wb m}^{-2} \text{ (approx.)}.$$

If $l = 0$, so that there is no air gap, and assuming the value of μ does not alter, then the flux density in the iron is now given by

$$B = \frac{NI}{L/\mu} = \frac{500 \times 20 \times 3000 \times 4\pi \times 10^{-7}}{0{\cdot}6}$$

$$= 53 \text{ Wb m}^{-2}.$$

Note that even a small air gap considerably reduces the value of the flux density obtained.

Magnetomotive Force. Reluctance.

In equation (3), NI is called the *magnetomotive force*, M.M.F., in the magnetic circuit comprising the iron and air gap. It is analogous to the e.m.f. in an electric circuit. The flux Φ through a cross-section of the circuit, which is given by BA, is analogous to the current.

The quantity $(l/A\mu_0 + L/A\mu)$ in the denominator in (3) is called the *reluctance* of the magnetic circuit. It is analogous to electrical resistance R, since $R = \rho l/A$ (p. 788). Each term in the expression for the reluctance is due to a separate part of the magnetic circuit. Thus $l/\mu_0 A$ is the reluctance of the air gap and $L/\mu A$ is that of the iron. In the above example, the reluctance of the air gap was $0{\cdot}01/(4\pi \times 10^{-7} \times A)$ whilst that of the iron was $0{\cdot}6/(3000 \times 4\pi \times 10^{-7} \times A)$. Thus the reluctance of the air gap is fifty times greater than that of the iron. This accounts for the considerable reduction in the value of B when the air gap is introduced. It should be noted that the reluctances are in series in Fig. 38.11. They are added together, as are series resistances in electric circuits.

Permanent Magnets

A permanent magnet is shown in Fig. 38.12 (i). We apply Ampère's theorem to the circuit shown by the dotted line. Here there are no external currents

flowing so that, using the same notation as in the last section,

$$H_m L + H_g L = 0$$

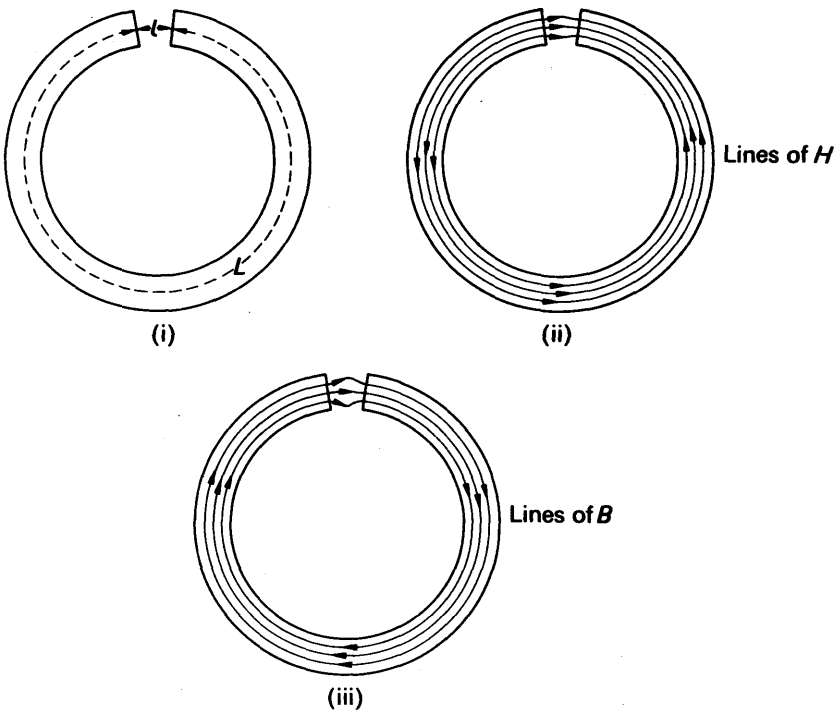


FIG. 38.12. Permanent magnet. B and H.

Also,

$$B_g = \mu H_g \quad \text{and} \quad B_g = B_m,$$

for the same reasons as in the case of electromagnets.
Combining these equations,

$$\therefore \frac{B_m}{H_m} = -\mu_0 \frac{L}{l} \quad . \quad . \quad . \quad . \quad (1)$$

This result expresses the fact that *B and H are oppositely directed inside the medium.* The lines of B and H are shown in Fig. 38.12 (ii) and (iii) respectively. Since $B = \mu H$, the value of μ in the case of this permanent magnet is $-\mu_0 L/l$. Fig. 38.13 shows the points, A or C, representing the magnetic state of the material of a permanent magnet.

The relationship between B and H is provided by the hysteresis loop. Also, B and H inside the material satisfy equation (1), which is represented by the line AOC. The only two points which satisfy both these conditions are A and C, the points of intersection of the line with the curve. At A or C, H is oppositely directed to B and hence μ is negative. For any given shape of toroidal magnet, one can predict, using (1), the greatest possible induction in the air gap.

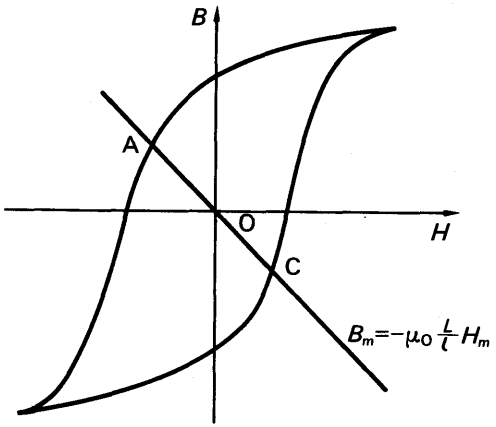


FIG. 38.13. Permanent magnet.

Diamagnetism, Paramagnetism, Ferromagnetism

We have already mentioned that the magnetic properties of materials are due to circulating and spinning electrons within the atoms.

Diamagnetism

If a magnetic field is produced in the neighbourhood of a magnetic material, a changing flux occurs in the current loops within the atoms. An e.m.f. or electric field will then be set up which causes the electrons to alter their motions, so that an extra or induced current is produced. By Lenz's law, this current gives rise to a magnetic field which *opposes* the applied magnetic field H. Thus the induced magnetization will be in the opposite direction to H, that is, M/H is negative. Hence the susceptibility χ is negative. This phenomenon is called *diamagnetism*. For a diamagnetic material, χ is generally very small, about $-0{\cdot}000015$ for bismuth, for example. The relative permeability, μ, which is given by $\mu_r = 1+\chi$, is thus generally slightly less than 1. All substances have a diamagnetic contribution to their susceptibility, since the induced currents always oppose the applied field. In many substances, the diamagnetism is completely masked by another magnetic phenomenon (p. 953).

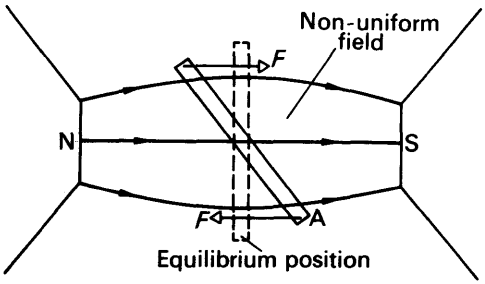


FIG. 38.14. Rod of diamagnetic material in strong field.

If a rod of diamagnetic material is placed in a non-uniform magnetic field, it will settle at right angles to the field. Fig. 38.14 shows the specimen slightly

displaced from this position. The magnetization will oppose the applied field so that the end A will now effectively be a weak S pole. It will then experience a force as shown by the arrow, so that a restoring couple turns the specimen back to its position at right angles to the field.

It should be noted that diamagnetism is a natural 'reaction' to an applied magnetic field and that it is independent of temperature.

Paramagnetism

In contrast to bismuth, a rod of a material such as platinum will settle along the same direction as the applied magnetic field. Further, the induced magnetism will be in the same direction as the field. Fig. 38.15. Platinum is an example of a *paramagnetic* material. The susceptibility, χ, of a paramagnetic substance is very small and positive, +0·0001 for example, so that its relative permeability μ_r is very slightly greater than 1 from $\mu_r = 1+\chi$.

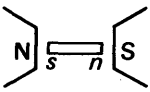


FIG. 38.15. Rod of paramagnetic material in strong field.

As we have already mentioned, atoms contain circulating and spinning electrons. Each electron possesses a resultant magnetic moment on account of its orbital motion and its spin motion. In a diamagnetic atom, all these contributions to the magnetic moment cancel. In a paramagnetic atom, however, there is a resultant magnetic moment. Generally, the thermal motions of the atoms will cause these magnetic moments to be oriented purely at random and there will be no resultant magnetization. If, however, a field is applied, each atomic moment will try to set in the direction of the field but the thermal motions will prevent complete alignment. In this case there will be overall weak magnetization in the direction of the applied field. This accounts for the phenomenon of paramagnetization.

It is clear that paramagnetism is temperature dependent. At low temperatures, the thermal motions will be less successful at preventing the alignment of the atomic moments and so the susceptibility will be larger. At higher temperatures thermal motion will make alignment difficult. At very high temperatures, the material may become diamagnetic, for the diamagnetic contribution to χ is not affected by temperature whilst the paramagnetic contribution falls.

Ferromagnetism. Magnetic Domain Theory

A ferromagnetic material has a very high value of susceptibility, χ, and hence of relative permeability, μ_r. The value of μ_r can be several thousands. Like a paramagnetic material, the magnetization is in the direction of the applied field and a rod of a ferromagnetic material will align itself along the field.

In a paramagnetic substance which is not subjected to a magnetic field, the magnetic moments are oriented purely at random due to the thermal vibrations. In a ferromagnetic material, however, strong 'interactions' are present between the moments, the nature of which requires quantum theory to understand it and is outside the scope of this work. These cause neighbouring moments to align, even in the absence of an applied field, with the result that tiny regions of very strong magnetism are obtained inside the unmagnetized material called *magnetic domains*. Above a critical temperature called the *Curie point*, ferromagnetics become paramagnetics (see Table, p. 956).

Domain Formation

A crystal of ferromagnetic material is shown in Fig. 38.16 (i). If all the domains were aligned completely, the material would behave like one enormous domain and the energy in the magnetic field outside is then considerable, as represented by the flux shown. Now all physical systems settle in equilibrium when their energy is a minimum. Fig. 38.16(ii) is therefore more stable than Fig. 38.16(i)

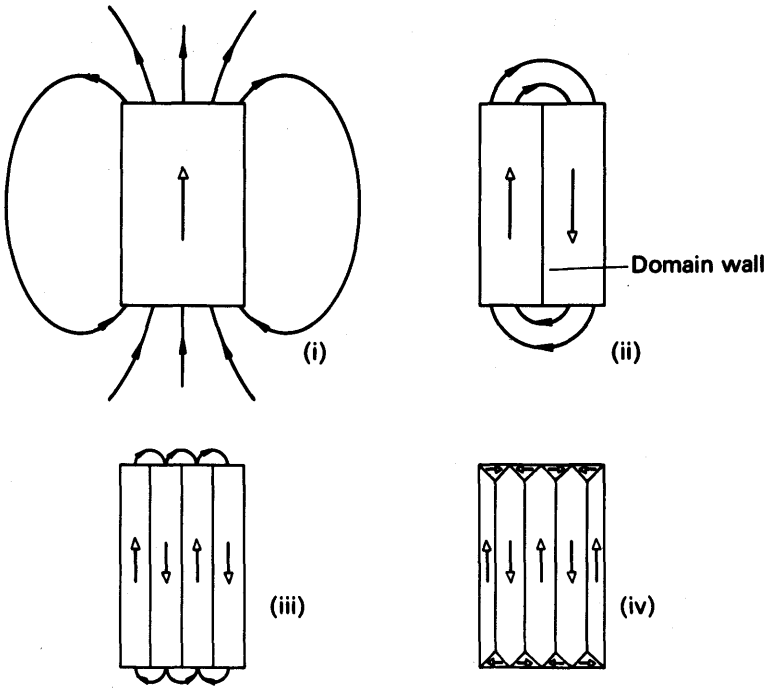


FIG. 38.16. Magnetic domains.

because the external magnetic field energy is less. Thus the domains grow in the material, as shown in Fig. 38.16 (iii) and (iv). The region between two domains, where the magnetization changes direction, is called a *domain wall* and also contains energy. When the formation of a new domain wall requires more energy than is gained by the reduction in the external magnetic field, no more domains are formed. Thus there is a limit to the number of domains formed. This occurs when the volume of the domains is of the order 10^{-4} cm^3 or less.

Domains and Magnetization

Some of the phenomena in magnetization of ferromagnetic material can now be explained. In an unmagnetized specimen, the domains are oriented in different directions. The net magnetization is then zero. If a small magnetic field H is applied, there is some small rotation of the magnetization within the domains, which produces an overall component of magnetization in the direction of H. This occurs in the region AB of the magnetization-field (M-H) curve shown in Fig. 38.17.

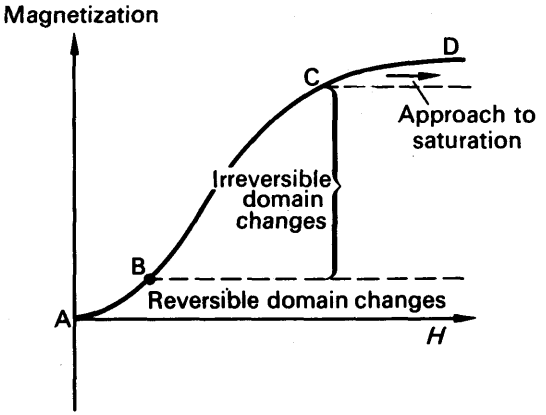


FIG. 38.17. Domain movement in magnetization.

If the field H is removed, the domain magnetization returns to its original direction. Thus the magnetization returns to zero. The changes in the part AB of the curve are hence reversible. If the field H is increased beyond B in the region BC, the magnetization becomes greater. On removal of the field the magnetization does not return to zero, and so remanence occurs. Along BC, then, irreversible changes take place; the domains grow in the direction of the field, by movement of domain walls, at the expense of those whose magnetization is in the opposite direction. At very high applied fields H there is complete alignment of the domains and so the magnetization M approaches 'saturation' along CD.

EXERCISES 38

1. Define *intensity of magnetization*, *susceptibility* and *permeability*. Two substances, A and B, have relative permeabilities slightly greater and slightly less than unity respectively. What does this signify about their magnetic properties? To what group of magnetic substances fo A and B each belong?

A soft iron ring of cross-sectional diameter 8 cm and mean circumference 200 cm has 400 turns of wire wound uniformly on it. Calculate the current necessary to produce magnetic flux to the value of 5×10^{-4} Wb if the relative permeability of the iron in the condition stated is 1800. Why is it not possible to say from this information what the flux would be if the current were reduced to 1/10 of its calculated value? (*L.*)

2. Define *permeability* (*relative permeability*) of a magnetic substance.

Indicate the orders of magnetide of the relative permeabilities of ferromagnetic, paramagnetic and diamagnetic substances respectively. Discuss in relation to its (relative) permeability one aspect of the behaviour of a specimen of each of these substances when placed in turn in the same magnetic field.

An iron ring of mean circumference 30 cm and of area of cross-section 1·5 cm^2 has 240 turns of wire uniformly wound on it, through which passes a current of 2 A. If the flux in the iron is found to be $7{\cdot}5 \times 10^{-4}$ Wb, find the relative permeability of the iron. (*L.*)

3. Give diagrams to illustrate the distribution of the lines of induction when a sphere of (*a*) soft iron, (*b*) bismuth (which is diamagnetic) is placed in a magnetic

field, initially uniform. Compare and contrast the magnetic properties of these materials.

The magnetic induction (flux density) in a uniformly magnetized specimen of cast iron is 0·3 Wb m^{-2} when the strength of the magnetizing field is 1000 A m^{-1}. Find (i) the intensity of magnetization, (ii) the relative permeability, (iii) the magnetic susceptibility of the specimen. (*L.*)

4. What is meant by *intensity of magnetization* and *hysteresis*? Describe an experiment from the observations of which a graph of intensity of magnetization (M) against magnetizing field strength (H) over a complete hysteresis cycle may be plotted for a ferromagnetic material. What information may be obtained from the graph by measuring (*a*) the area enclosed by the loop, (*b*) the intercept on the M axis, (*c*) the intercept on the H axis, (*d*) the slope of a line joining a point on the graph to the origin? (*L.*)

5. Define *intensity of magnetization* M and *magnetic susceptibility* χ_m, and with the aid of a diagram explain what is meant by *hysteresis.*

Sketch a graph showing how M varies with the applied magnetizing field as the field applied to a specimen of soft iron, initially unmagnetized, is slowly increased from zero. On the same diagram sketch the corresponding graph for steel.

By reference to your earlier account state, with reasons, desirable magnetic properties of materials to be used as (*a*) the core of an electromagnet, (*b*) the core of a transformer, (*c*) a permanent magnet. (*L.*)

6. Define *magnetic moment* and explain why this concept is useful in the study of magnetism.

A bar of magnetic material is magnetized in the uniform field inside a solenoid. How would you study experimentally the changes in its magnetic moment when the magnetizing current is varied in magnitude and direction?

How would you represent the results of your measurements graphically? Give freehand sketches of the graphs you would expect to obtain for (i) material suitable for the core of a transformer, (ii) material suitable for a permanent magnet. Point out in each case the features of the graph which indicate that the material is suitable for its purpose. (*O. & C.*)

7. Define *susceptibility, permeability,* and state the relation between them.

Describe briefly how you would test if a small rod is diamagnetic, paramagnetic, or ferromagnetic. Draw intensity of magnetization graphs for each type of material and comment on their special features. (*N.*)

8. Explain what is meant by a *ferromagnetic material.* Give a general account of the wide range of magnetic properties exhibited by different ferromagnetic materials and indicate the practical applications of these properties. (*N.*)

chapter thirty-nine

A.C. Circuits. Electromagnetic Waves

Measurement of A.C.

IF an alternating current (a.c.) is passed through a moving-coil meter, the pointer does not move. The coil is urged clockwise and anticlockwise at the frequency of the current—50 times per second if it is drawn from the British grid—and does not move at all. In a delicate instrument the pointer may be seen to vibrate, with a small amplitude. Instruments for measuring alternating currents must be so made that the pointer deflects the same way when the current flows through the instrument in either direction.

Moving-iron Instrument

A fairly common type of such instrument, called the moving-iron type, is shown in Fig. 39.1. It consists of two iron rods XY, PQ, surrounded by a coil which carries the current. The coil is fixed to the framework of the meter, and so is one of the rods PQ. The other rod is attached to an axle, which also carries the pointer, its motion is controlled by hair-springs. When a current flows through the coil, it

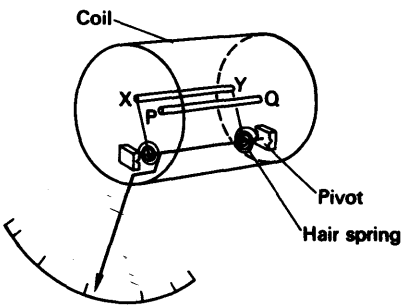


FIG. 39.1. Moving-iron meter, repulsion type.

magnetizes the rods, so that their adjacent ends have the same polarity. The polarity of each pair of ends reverses with the current, but whichever direction the current has, the iron rods repel each other. The force on the pivoted rod is therefore always in the same direction, and the pointer is deflected through an angle which is proportional to the average force. To a fair approximation, the magnetization of the rods at any instant is proportional to the current at that instant; the force

between the rods is therefore roughly proportional to the square of the current. The deflection of the pointer is therefore roughly proportional to the average value of the square of the current.

Hot-wire Instrument

Another type of 'square law' instrument is the hot-wire ammeter (Fig. 39.2). In it the current flows through a fine resistance-wire XY, which it heats. The wire warms up to such a temperature that it loses heat—mainly by convection—at a rate equal to the average rate at which heat is developed in the wire. The rise in temperature of the wire makes it expand and sag; the sag is taken up by a second fine wire PQ, which is held taut by a spring. The wire PQ passes round a pulley attached to the pointer of the instrument, which rotates as the wire XY sags. The deflection of the pointer is roughly proportional to the average rate at which heat is developed in the wire XY; it is therefore roughly proportional to the average value of the square of the alternating current, and the scale is a square-law one.

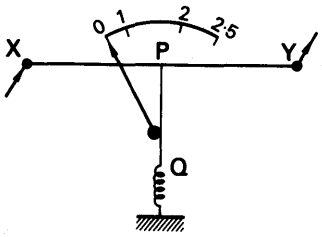


FIG. 39.2. Hot wire meter.

Root-mean-square value of A.C.

On p. 907 we saw that an alternating current I varied sinusoidally; that is, it could be represented by the equation $I = I_m \sin \omega t$, where I_m was the peak (maximum) value of the current. In commercial practice, alternating currents are always measured and expressed in terms of their *root-mean-square* (*r.m.s.*) value, which we shall now consider.

Consider two resistors of equal resistance R, one carrying an alternating current and the other a direct current. Suppose both are dissipating the same power, P, as heat. The root-mean-square (r.m.s.) value of the alternating current, I_r, is defined as equal to the direct current, I_d. Thus:

the root-mean-square value of an alternating current is defined as that value of steady current which would dissipate heat at the same rate in a given resistance.

Since the power dissipated by the direct current is

$$P = I_d^2 R,$$

our definition means that, in the a.c. circuit,

$$P = I_r^2 R. \quad . \quad . \quad . \quad . \quad (1)$$

Whatever the wave-form of the alternating current, if I is its value at any instant, the power which it delivers to the resistance R at that instant is I^2R. Consequently, the average power P is given by

$$P = \text{average value of } (I^2R)$$
$$= R \times \text{average value of } (I^2),$$

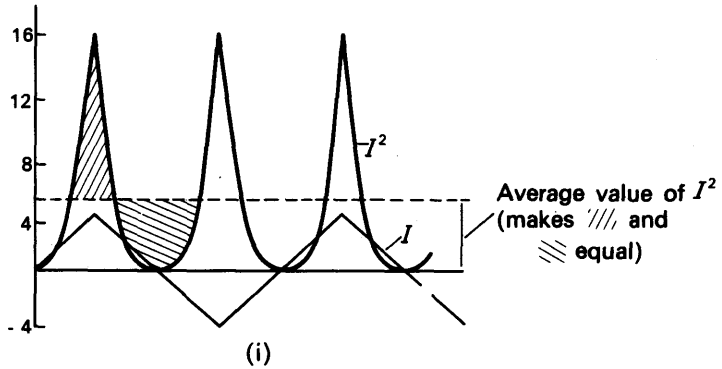
since R is a constant. Therefore, by equation (1),

$$I_r^2 R = R \times \text{average value of } (I^2)$$

or

$$I_r^2 = \text{average value of } (I^2). \quad . \quad . \quad . \quad (2)$$

The average value of (I^2) is called the mean-square current; the meaning of the term is illustrated for a non-sinusoidal current in Fig. 39.3 (i).



(i)

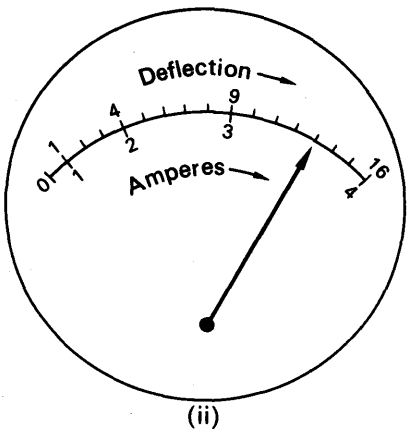
FIG. 39.3 (i). Mean-square values.

$$\therefore I_r = \sqrt{\text{average value of } (I^2)}. \quad . \quad . \quad (3)$$

For a sinusoidal current, the average value of I^2 is $I_m^2/2$, where I_m is the maximum value of the current

$$\therefore I_r = \frac{I_m}{\sqrt{2}}. \quad . \quad . \quad . \quad . \quad (4)$$

Equation (3) shows the origin of the term 'root-mean-square'. We therefore require a meter whose deflection measures not the current through it but the average value of the square of the current. As we have already seen, moving-iron and hot-wire meters have just this property (p. 968).



(ii)

FIG. 39.3 (ii). Scale of A.C. ammeter.

For convenience, such meters are scaled to read amperes, not (amperes)^2, as in Fig. 39.3 (ii). The scale reading is then proportional to the square-root of the deflection, and indicates directly the root-mean-square value of the current I_r. An a.c. meter of the moving-iron or hot-wire type can be calibrated in direct current, as shown in Fig. 39.3 (ii). This follows at once from the definition of the r.m.s. value of current.

A.C. through a Capacitor

In many radio circuits, resistors, capacitors, and coils are present. An alternating current can flow through a resistor, but it is not obvious at first that it can flow through a capacitor. This can be demonstrated, however, by wiring a capacitor, of capacitance one or more microfarads, in series with a neon lamp, and connecting them to the a.c. mains via a plug. In place of the neon lamp we could use a mains filament lamp of low rating, such as 25 watts. The lamp lights up, showing that a current is flowing through it.

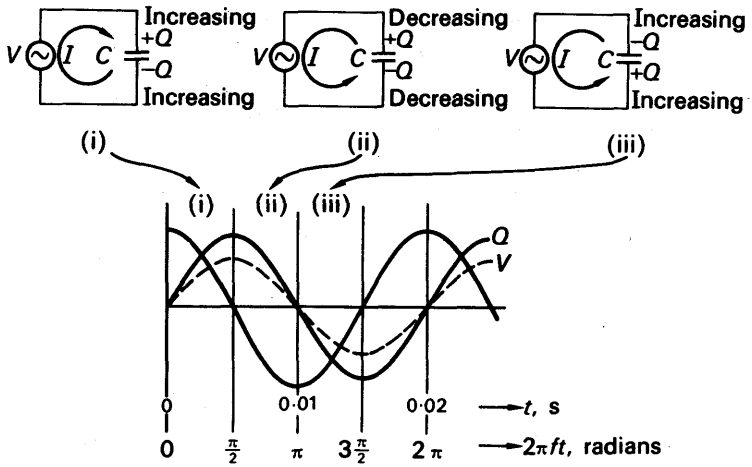


FIG. 39.4. Flow of A.C. through capacitor, frequency 50 Hz.

The current flows because the capacitor plates are being continually charged, discharged, and charged the other way round by the alternating voltage of the mains (Fig. 39.4). The current thus flows round the circuit, and can be measured by an a.c. milliammeter inserted in any of the connecting wires.

To find the amplitude of the current, let us denote the amplitude of the voltage by V_m, and its frequency by f. Then, as in equation (3), p.907, the instantaneous voltage at any time t is

$$V = V_m \sin 2\pi ft.$$

If C is the capacitance of the capacitor, then the charge Q on its plates is

$$Q = CV,$$

whence

$$Q = CV_m \sin 2\pi ft.$$