dispersion. At that time, however, there were no tests or observations that could eliminate either model, so Newton's stature in the scientific community (gained for his many and varied contributions, including the laws of motion) resulted in his winning the approval of other scientists for his less eloquent corpuscular model. What type of experiment would be necessary in order to accept or reject one of the models? What would reveal the greatest contrast between the properties of waves and particles?

Superposition of Waves

PHYSICS FILE

The superposition of waves principle holds true for linear waves. Linear waves are characterized by small amplitudes. Interestingly, the superposition principle does not apply to non-linear waves, characterized by large amplitudes. This textbook does not deal with non-linear waves.

What happens when two particles or two waves attempt to occupy the same point in space at the same time? Obviously, as two particles approach the same point, they will collide and move in a way that will obey the law of conservation of momentum. Two waves, however, *can* occupy the same space at the same time. When two waves pass through one location in the medium, the medium will oscillate in a way that resembles the sum of the effects from both waves. Waves that reach the same point simultaneously interfere with each other in terms of the displacement of the medium.

This result, called the **superposition of waves**, is formally stated as: When two or more waves propagate through the same location in a medium, the resultant displacement of the medium will be the algebraic sum of the displacements caused by each wave. Each wave behaves as though the other did not exist and, once past the area of interest, proceeds unchanged.

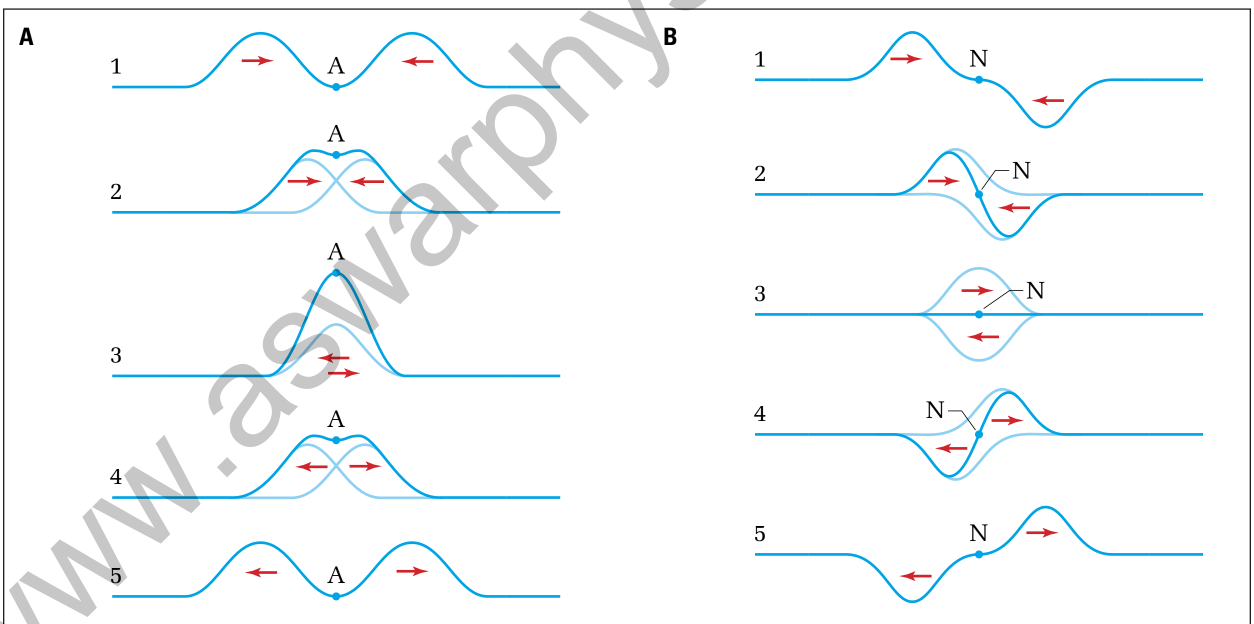


Figure 9.4 **(A)** Constructive interference results in a wave pulse that is larger than either individual pulse. **(B)** Destructive interference results in a wave pulse that is smaller than the larger of the component waves. Two identical but inverted pulses will yield a momentary amplitude of zero.

The resultant displacement of the medium caused by the superposition of two waves is unlike either of the individual waves that added together to form it. Figure 9.4 (A) illustrates two pulses travelling toward each other. The darkened wave at point A is the result of **constructive interference** of the two pulses. Constructive interference results in a wave with larger amplitude than any individual wave. Perfectly constructive interference occurs when waves are completely in phase with each other. Figure 9.4 (B) illustrates **destructive interference**. In this case, the resultant wave amplitude is smaller than the largest component wave. A **nodal point** exists in the medium when two waves with identical but inverted amplitudes exist simultaneously. The resultant displacement at a nodal point is zero.

The most definitive test of a phenomenon that could classify it as a wave is to show that it undergoes interference. In the next section, you will learn how to observe and verify that light exhibits interference and therefore must behave like a wave.

9.1 Section Review

1. MC Sound, a form of energy, can be modelled by using two distinctly different approaches.
   - **(a)** Describe the propagation of sound energy through air by discussing the motion of individual particles. Include possible mathematical equations that might apply.
   - **(b)** Describe the propagation of sound energy through air by discussing waves. Include possible mathematical equations that might apply.
   - **(c)** Does one method provide a more easily understood explanation?
   - **(d)** Does one method provide a more simple mathematical model?
2. K/U Describe Huygens' concept of wavelets.
3. **(a)** K/U Draw a series of Huygens wavelets so that they produce a circular wavefront.
   - **(b)** Describe how the wavelets that form the wavefront apparently vanish behind it.
4. **(a)** MC Do you believe that some current accepted scientific models or theories might be held in high regard because of the stature of the scientists who proposed them?
   - **(b)** Suggest one possible model or theory that is currently accepted by a majority of people that you feel might be significantly inaccurate. Explain.

UNIT PROJECT PREP

An FM transmitter produces a carrier wave with a specific frequency. A fixed frequency wave of any type is produced by periodic motion of a source.

- Hypothesize about what might experience periodic motion in the creation of FM radio waves.
- How is an understanding of frequency important in the construction of a radio transmitter?
- Apply Huygens' wave model to various waves with which you are familiar as you study this unit. Can his model predict the behaviour of all waves?

CANADIANS IN PHYSICS

The Light Fantastic

Even as a young girl, Dr. Geraldine Kenney-Wallace knew that she wanted to be a scientist. She preferred playing with crystals, minerals, and fossils rather than toys and was fascinated by electric motors, trains, and radios. In school, art, math and science were her favourite classes.

After high school, she worked as a summer research student at the Clarendon Physics Laboratories, part of England's Oxford University. Although lasers were then new, Dr. Kenney-Wallace's work at Clarendon brought her into daily contact with them. There were hints that lasers could be used in new and exciting applications to investigate atoms, molecules and semiconductors in particular.

Dr. Kenney-Wallace continued her research while working on her bachelor's degree in London. She then came to Canada for graduate work. While teaching in Toronto, she was able to indulge her passion for lasers, physics and chemistry by establishing the first University ultrafast laser laboratory in Canada.


Dr. Geraldine Kenney-Wallace

A pulsed laser works by pumping certain kinds of crystals or gases with so much energy that the electrons are pushed to a very high quantum level. Spontaneous emission occurs. In a laser cavity, multiple reflections between the end mirrors trigger *stimulated* emission so that the electrons all simultaneously drop down to a lower energy level, releasing their stored energy as photons. Then, the process begins building up in the cavity again. While they are recharging, however, lasers cannot generate any light output.

Lasers are used to investigate chemical reactions by bouncing the laser photons off the atoms and molecules. What happens, though, if the reaction takes place faster than the time it takes for the laser to recharge? Researchers realized that conventional lasers could not be used to study these fast reactions. They needed a laser that fired and recharged very quickly — an ultrafast laser. Dr. Kenney-Wallace was in the forefront of the design and application of these new ultra-fast lasers.

Dr. Kenney-Wallace's interests and talents extended beyond the frontiers of scientific research. In addition to her scientific research, she devoted part of her career to consultation on business and public policy issues, usually related to the areas dearest to her: Research and Development, and science and education.

Her ongoing work has earned her international recognition. She has been quoted in the House of Commons, interviewed by Canada's national media, and has given dozens of professional seminars throughout the world. She has 13 honorary degrees, is the former president of McMaster University, a Fellow of the Royal Society of Canada, and the first woman to hold the Chair of the Science Council of Canada. For the past few years, she has been working in England, helping to set up a number of virtual universities. She is not only at the forefront of laser research, but also at the cutting edge of e-learning education. Dr. Kenney-Wallace is living proof that, at least intellectually, you can have it all!

Going Further

1. One of Dr. Geraldine Kenney-Wallace's great strengths is her multidisciplinary background. She has interests in both chemistry and physics and also in business. Discuss other combinations of fields that might be helpful to a career in science or how a science background can help you in another career. For example, might a science background help you in business? How? Alternatively, can you think of how a knowledge of economics, for example, might be of help to a chemist? How about the combination of biology and law?
2. What do you find most interesting about Dr. Kenney-Wallace's career? Why? Report on this to your class.

9.2 Interference and the Wave Model for Light

A siren pierces the serenity of a quiet evening. Although you are unable to see the emergency vehicle generating the noise, you are certainly able to hear it. The sound is able to bend around corners, pass through doorways, and eventually reach your ears. The ability of sound energy to bend around corners and spread around barriers is not only a property of sound, but is also a property of all waves.

Figure 9.5 You can usually hear a siren long before you see the emergency vehicle, because sound can "bend" around corners.

SECTION EXPECTATIONS

- Describe the concepts related to diffraction and interference.
- Describe interference of light in qualitative and quantitative terms.
- Collect and interpret experimental data in support of a scientific model.
- Identify interference patterns produced by light.
- Describe experimental evidence supporting the wave model of light.

KEY TERMS

- coherent
- fringe
- Fraunhofer diffraction
- Fresnel diffraction

A Definitive Experiment

Bending around corners — a form of diffraction — is a property of all waves. However, scientists studying light at the time of Newton and Huygens were not able to detect any significant diffraction of light. To determine with confidence whether light behaved like a wave or a particle, scientists needed a carefully planned experiment that could clearly show evidence or lack of evidence of interference of light. To visualize the type of experiment that would be definitive, observe the pattern of water waves in Figure 9.6, which results from two point sources creating a periodic disturbance.

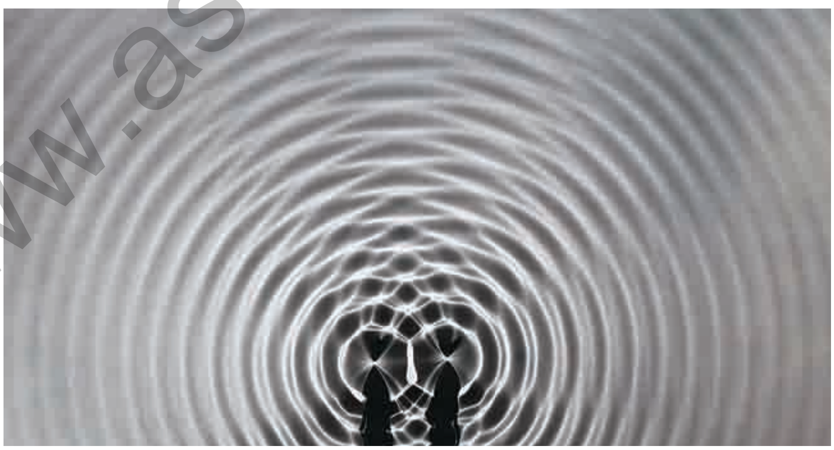


Figure 9.6 Nodal lines, resulting from total destructive interference, are clearly visible, radiating outward from between the two sources.

INVESTIGATION 9-B

Diffraction of Sound

TARGET SKILLS
- Predicting
- Performing and recording
- Analyzing and interpreting

Diffraction is readily observed using mechanical waves. In this activity, you will study some variables associated with diffraction of sound waves.

Problem

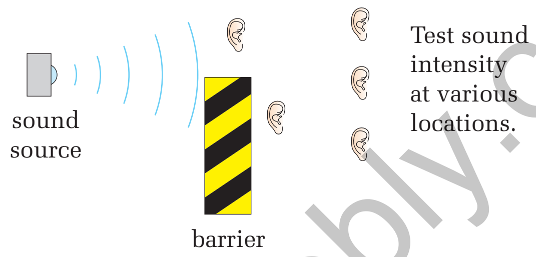
Identify some variables associated with diffraction of sound waves.

Equipment

- audio frequency generator
- speaker

Procedure

1. Using the wave model for sound, predict **(a)** how sound intensity will vary (e.g., sharply, gradually) behind the edge of a solid barrier and **(b)** how changes in wavelength will influence the results from part (a).
2. Use an audio frequency generator connected to a single speaker to act as a sound source. Recall that the frequency of sound will cause the wavelength to vary, according to the wave equation $v = f\lambda$. Use a relatively sound-proof barrier, such as a wall with a wide door, to test your predictions. A door opening into a large open space, as shown in the diagram, will reduce the amount of reflection from walls and will therefore yield the best results. Select and maintain a single, relatively low intensity (volume) to reduce effects produced by reflection of sound off nearby objects.



3. Carefully analyze how the intensity of the source varies at different locations behind the barrier, as shown in the diagram. Select an appropriate method to illustrate how the sound intensity varied.
4. Experiment to see how wavelength affects the amount of diffraction.

Analyze and Conclude

1. Were your predictions about the diffraction of sound accurate? Explain.
2. Describe and illustrate how the sound intensity varied at different locations behind the edge of the barrier.
3. Does varying the wavelength of the source affect the amount of sound wave diffraction? Explain and provide evidence.
4. Do your results validate the wave model of sound? Explain.
5. Suggest why only one speaker is used in this activity. Include the principle of superposition of waves in your answer.

In Figure 9.6, you can see lines emanating from the sources that show constructive interference — standing waves — and destructive interference — no movement of the water. If light behaves like a wave, a similar experiment should reveal bright areas — constructive interference — and dark areas — destructive interference — on a screen. Figure 9.7 shows how interference resulting from two light sources would create light and dark regions.

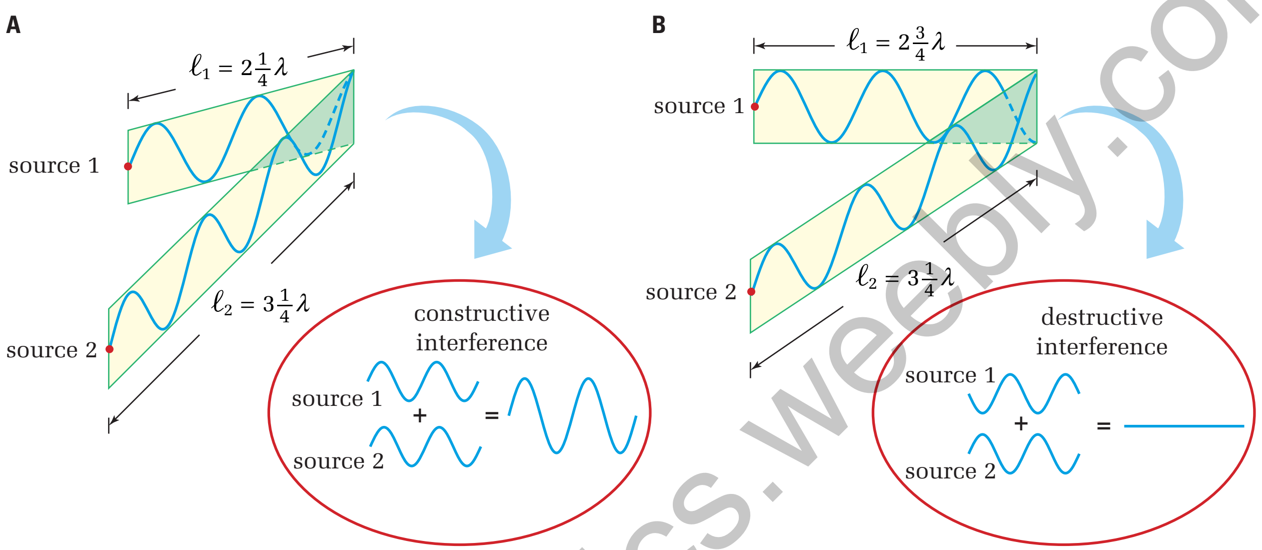


Figure 9.7 **(A)** Waves from each source with a path difference of whole-number multiples of wavelength interfere constructively. **(B)** Waves from each source with a path difference of multiples of one half wavelength interfere destructively.

Examination of Figure 9.7 reveals two important features that must be designed into the experiment. First, the sources must produce coherent waves. **Coherent** sources produce waves of the same frequency and in phase with each other. Second, the distance between the sources must be of the order of magnitude of the wavelength of the waves. If the sources are placed too far apart, the light and dark areas on a screen will be too close together to be observed. (As a rule of thumb, the sources must be no farther than 10 wavelengths apart.)

These conditions were exceedingly difficult for scientists to create in the 1700s. Physicists could produce light of one frequency by passing it through a prism but, before the invention of the laser, coherent light sources did not exist. The phases of light emanating from a source were random. Therefore, constructive or destructive interference would occur in a random way and the effects would be an average of light and dark, so that they appeared to be uniform. In addition, since physicists did not even know whether light behaved like a particle or a wave, they had no way of knowing what the wavelength might be. It took nearly 100 years after Newton and Huygens presented their models of light for the debate to be resolved.

ELECTRONIC LEARNING PARTNER

Go to your Electronic Learning Partner to enhance your understanding of interference.

Young's Double-Slit Experiment

Thomas Young (1773–1829) devised an ingenious experiment, as illustrated in Figure 9.8, that produced an interference pattern with light. Using one monochromatic light source, Young allowed the light to fall onto an opaque material with a single, narrow slit. According to Huygens' principle, this slit acted as a new source. The light passing through the single slit spread as it travelled to a second opaque barrier. The second barrier had two narrow slits placed very closely together.

In part (B) of Figure 9.8, you can see that two parts of the same wavefront from the single slit reach the double slits at the same time. Since these two parts of the same wavefront behave as new sources at the double slits, the light leaving the double slits is essentially coherent. Young experimented with this set-up for more than two years before he realized that the double slits had to be so close together that they almost appeared to be one slit to the unaided eye. The light that passed through the double-slit barrier fell on a nearby screen, producing the historic pattern of light and dark lines caused by the interference of light waves. Young's results catapulted the wave model for light into centre stage, where it would remain unchallenged for more than 100 years.

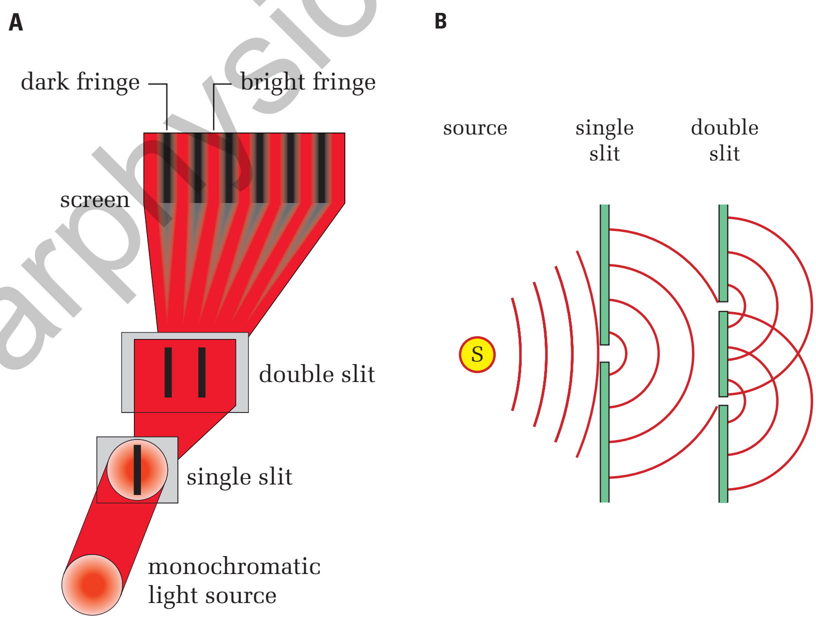


Figure 9.8 **(A)** Young's experiment used a single incandescent bulb and two narrow slits to produce coherent sources. He successfully showed that light could form an interference pattern similar to those produced with mechanical waves. **(B)** The wavefronts emanating from the double slits resemble the water waves generated by two point sources.

Young was successful where others had failed for several reasons.

- He used a monochromatic (single wavelength) light source.
- The double slits acted as two sources and were spaced much more closely together than was possible if two separate light sources were used.
- The light passing through the initial single slit acted as a point source. When a wavefront from the point source reached the double slits, two parts of the same wavefront became new sources for the double slits and were therefore coherent.

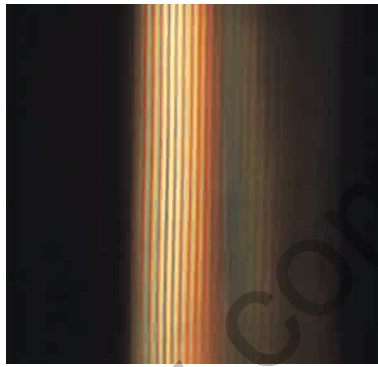


Figure 9.9 Photograph of an interference pattern from Young's experiment (notice how the intensity reduces toward the edges)

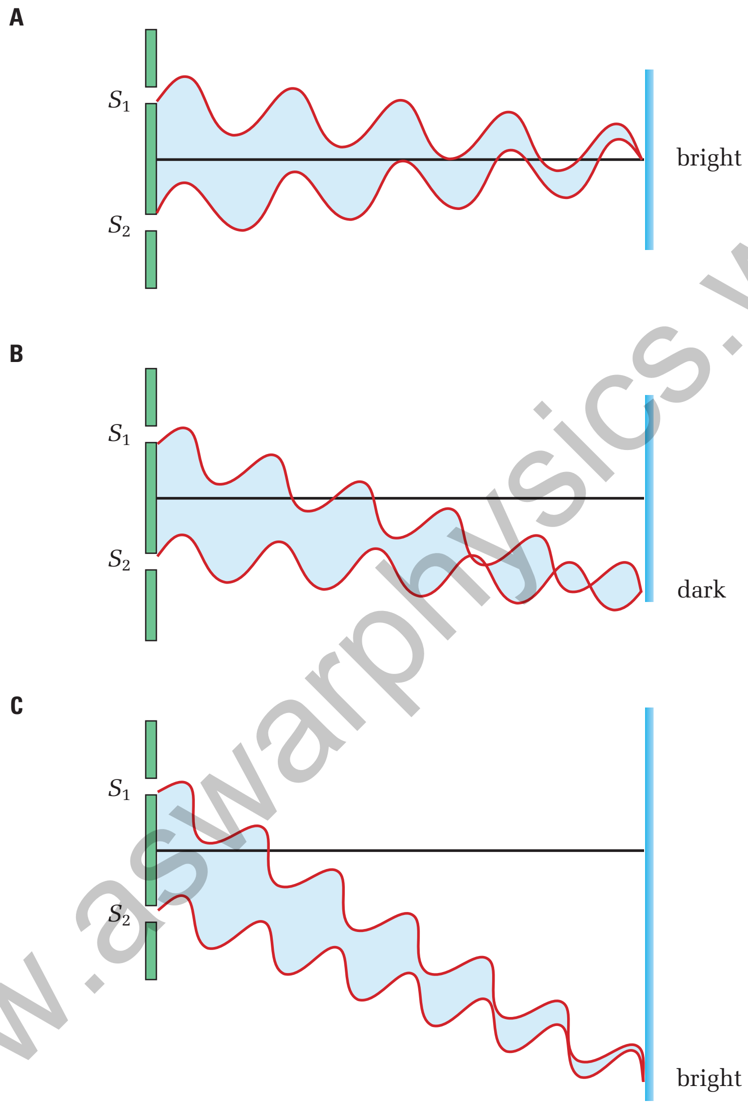


Figure 9.10 Light and dark fringes result from interfering waves.

To understand the mathematical analysis of the pattern produced by Young's double-slit experiment, examine Figure 9.11. In part (A), you see coherent light waves entering the two slits and passing through. Light leaves the slits in all directions, but you can study one direction at a time. Since the distance between the slits and the screen (labelled x) is approximately a million times larger than the distance between the slits, you can assume that the waves leaving the slits parallel to each other will hit the screen at the same point. Part (B) is drawn to the scale of the slit-to-screen distance and therefore the two parallel rays appear as one line.

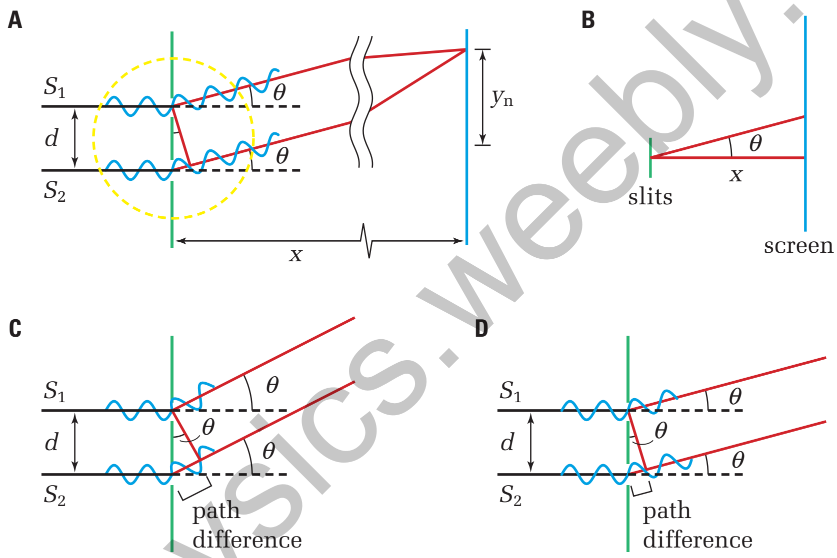


Figure 9.11 The path difference between slits that light travels to reach the screen is given by $d \sin \theta$.

Parts (C) and (D) of Figure 9.11 illustrate two special cases — constructive interference and destructive interference. Inspection of the right triangle in part (C) shows that the hypotenuse is the distance, d, between the two slits. One side is formed by a line drawn from slit 1 that is perpendicular to the ray leaving slit 2. The third side of the triangle is the distance that ray 2 must travel farther than ray 1 to reach the screen. When this path difference, PD, is exactly one wavelength, the two waves continue from the slit in phase and therefore experience constructive interference.

When the two rays reach the screen, they will produce a bright spot on the screen, called a bright **fringe**. Using trigonometry, you can see that the path difference is equal to $d \sin \theta$. In fact, if the path difference is any integer number of full wavelengths, the waves will remain in phase and will create a bright fringe on the screen. Notice that, from the geometry of the apparatus, the angle θ, formed by the slit separation and the perpendicular line between the light rays, is the same as the angle between the

PROBEWARE

If your school has probeware equipment, visit ***www.mcgrawhill.ca/links/physics12*** and click on **Web Links** for an in-depth activity about the interference effects of light.

horizontal line going to the screen and the direction of the rays going toward the screen. The result of this analysis can be expressed mathematically as shown in the following box.

CONSTRUCTIVE INTERFERENCE

A bright fringe will appear on a screen when an integer number of wavelengths of light is equal to the product of the slit separation and the sine of the angle between the slit separation and the line perpendicular to the light rays leaving the slits.

$$n\lambda = d \sin \theta \qquad \text{where } n = 0, 1, 2, 3, \ldots..$$

Quantity	Symbol	SI unit
integer number of full wavelengths	n	none
wavelength of light	λ	m (metres)
distance between slits	d	m (metres)
angle between slit separation and line perpendicular to light rays	θ	unitless (degrees are not a unit)

Unit Analysis

metre = metre   m = m

Inspection of part (D) of Figure 9.11 shows that when the path difference is a half wavelength, the light waves that leave the slits are out of phase and experience destructive interference. When the waves reach the screen, they will cancel each other and the screen will be dark. Between the bright and dark fringes, the screen will appear to be shaded. A complete analysis shows that when the path difference is exactly half a wavelength more than any number of full wavelengths, the waves will destructively interfere and a dark spot or dark fringe will appear on the screen. This condition can be described mathematically as

$$\left(n - \tfrac{1}{2}\right)\lambda = d \sin \theta \qquad \text{where } n = 1, 2, 3, \ldots$$

In a typical experiment, you would not be able to measure the path difference or the angle θ. Instead, you would measure the distance y_n between the central bright fringe and another bright fringe of your choice. You could then determine the angle θ by applying trigonometry to part (B) of Figure 9.11, which gives

$$\tan \theta = \frac{y_n}{x}$$

In this expression, the variable n has the same meaning as it does in the previous relationships. When $n = 1$, the path difference is one full wavelength and y_1 describes the distance to the first bright fringe.

For very small angles, you can make an approximation that combines the two relationships above, as shown below.

- For very small angles, the sine of an angle is approximately equal to the tangent of the angle. $\sin\theta \cong \tan\theta$
- Using this approximation, you can write the expression for the wavelength, as shown. $n\lambda \cong d\tan\theta$
- Substitute the expression for $\tan\theta$ and substitute into the equation for the wavelength. $\tan\theta = \frac{y_n}{x}$ $n\lambda \cong d\frac{y_n}{x}$

You can set n equal to 1 by using the distance between adjacent fringes and obtain the relationship shown in the following box.

APPROXIMATION OF THE WAVELENGTH OF LIGHT

The wavelength of light is approximately equal to the product of the distance between fringes and the distance between slits, divided by the slit-to-screen distance.

$$\lambda \cong \frac{\Delta y d}{x}$$

Quantity	Symbol	SI unit
wavelength	λ	m (metres)
distance separating adjacent fringes	Δy	m (metres)
distance between slits	d	m (metres)
distance from source to screen	x	m (metres)

Note 1: The distance between nodal line centres is identical to the distance between bright fringe centres. Therefore, this relationship applies equally to dark fringes (nodal lines) and bright fringes.

Note 2: This relationship is based on an approximation. Use it only for very small angles.

INVESTIGATION 9-C

Young's Double-Slit Experiment

TARGET SKILLS
- Predicting
- Identifying variables
- Communicating results

Young's ingenious double-slit experiment is readily duplicated with only simple equipment. In this investigation, you will reproduce results similar to those that Young produced — the results that convinced the scientific community that light was a wave.

Problem

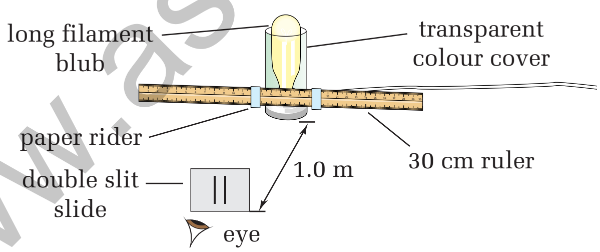
Is it possible to produce an interference pattern using light?

Prediction

- Make a prediction about the requirements of the experimental design that will be necessary to produce an interference pattern based on the wavelength of light and the nature of incandescent light sources.
- Make a second prediction about the nature of an interference pattern produced with short wavelength light (such as blue or green) compared to longer wavelength light (such as yellow or red). Which wavelength will allow you to make the most accurate measurements? Explain your prediction in detail.

Equipment

- long filament light source
- magnifying glass
- double-slit slides
- transparent colour light covers
- metre stick
- ruler with fractions of a millimetre markings
- 30 cm ruler



Procedure

1. Using a magnifying glass and finely ruled ruler, measure and record the centre-to-centre width of the slit separation.
2. Cut two paper riders for the 30 cm ruler to mark the width of the observed interference pattern.
3. Place a transparent colour cover over the portion of the light bulb with the straightest filament.
4. Place the 30 cm ruler with paper riders in front of the bulb. With your eye exactly 1 m from the bulb, observe the filament through the double slits.
5. Count the number of bright *or* dark fringes that you are able to clearly distinguish. Use the paper riders to mark off the edges of the observed fringes.
6. Repeat the experiment, varying the slit width and the wavelength of light.

Analyze and Conclude

1. Describe the effect on the observed interference pattern of **(a)** altering the slit width and **(b)** altering the wavelength of light.
2. Use your data to determine the wavelength of light used for each trial. How well did your calculated wavelength compare to expected values?
3. Was it easier to obtain data on one wavelength than on the others? If so, was your original prediction accurate? Explain.

Apply and Extend

CAUTION *Do not* look directly into the laser.

4. If time permits, use a helium-neon laser to verify the double-slit equation. Place a double-slit slide of known width in front of the laser beam. Observe the interference pattern on a screen a known distance from the laser. Calculate the wavelength of laser light. Determine the percentage deviation of the calculated value and your experimental value.

Single-Slit Diffraction

Wavelets originating at two separate but closely spaced sources produce clear interference patterns, as observed in Young's experiment. It is also possible to obtain an interference pattern from a single slit.

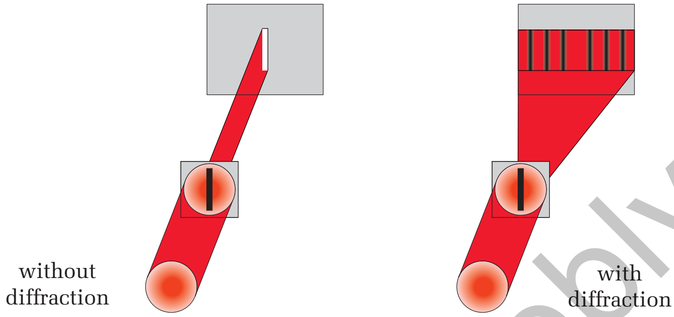


Figure 9.12 If diffraction did not occur, only a thin sliver of light would appear on the screen. In fact, an interference pattern results from the diffraction of incident light through a single slit.

The diagrams in Figure 9.13 illustrate two types of single-slit diffraction that might occur. The **Fraunhofer diffraction** pattern results when the parallel rays of light (straight or planar wavefronts) are incident on the slit. Fraunhofer diffraction produces clear interference patterns that are readily analyzed when a converging lens is used to bring the parallel rays into focus. When the incident light rays are not parallel, **Fresnel diffraction** occurs. Analysis of Fresnel diffraction requires mathematical processes that are beyond the scope of this course. However, analysis of both Fraunhofer and Fresnel diffraction is based on Huygens' principle.

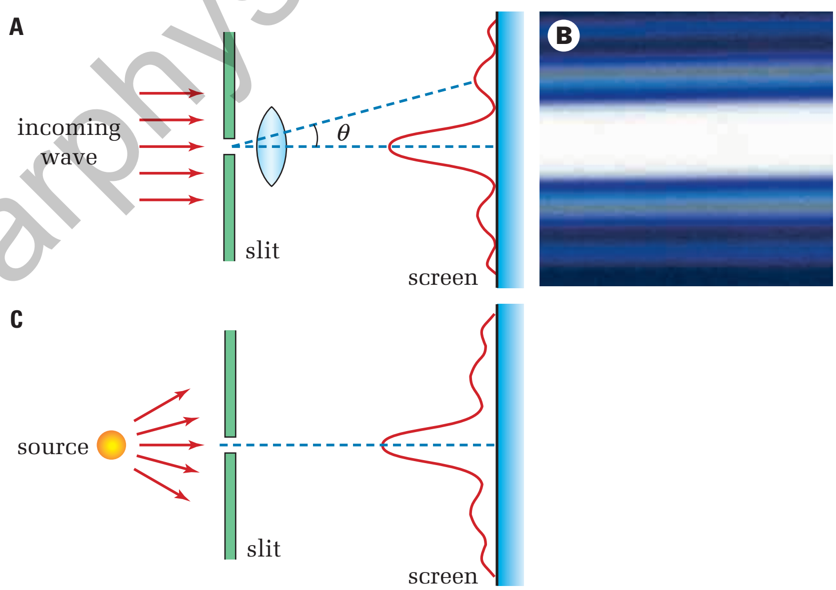


Figure 9.13 **(A)** Fraunhofer diffraction pattern created from parallel rays striking a single slit. **(B)** Photograph of Fraunhofer diffraction. Note the double-wide central maximum and the reduction of intensity of subsequent fringes. **(C)** Fresnel diffraction patterns are created when incident rays falling onto a single slit are not parallel.

To apply Huygens' principle to single-slit diffraction, imagine many point sources across the single slit. Wavelets produced by each source will interfere with each other, generating an interference pattern on a screen. Wavelets passing directly through the slit interfere constructively, producing a bright central fringe.

To understand how the destructive interference occurs, visualize the slit as two halves. Each of the infinite number of wavelets — represented and simplified by numbers 1 to 5 — are in phase as they pass through the opening. The wavelets leaving the slit at an angle θ will no longer be in phase. For example, wavelet 3, originating at the middle of the slit, will travel farther than wavelet 1 by a path difference of $\frac{1}{2}\lambda$. The wavelet just below wavelet 1 will be exactly $\frac{1}{2}\lambda$ ahead of the wavelet just below wavelet 3. In this way, all wavelets in the top half of the slit will interfere destructively with wavelets from the lower half of the slit. When the wavelets reach the screen, they will interfere destructively, as shown in Figure 9.14.

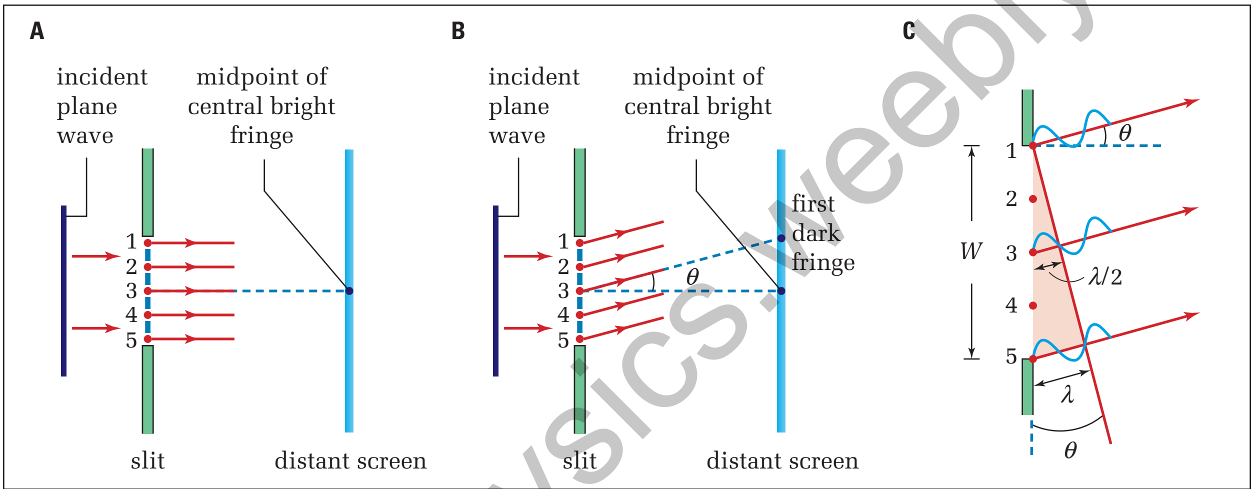


Figure 9.14 **(A)** The plane wave incident on the single slit is shown as five sources for Huygens' wavelets. The wavelets interfere constructively at the central midpoint, generating a large, bright central maximum. **(B)** The first nodal line (dark fringe) occurs when the Huygens' wavelets from each source interfere destructively. **(C)** Destructive interference requires a path difference of $\frac{1}{2}\lambda$.

The second dark fringe measured at an angle of θ is shown in Figure 9.15. Again, consider the single slit as two separate halves. This second-order dark fringe results because the path difference travelled between wavelets 1 and 2 is exactly $\frac{1}{2}\lambda$. The wavelets just under wavelet 1 and wavelet 2 will also strike the distant screen exactly $\frac{1}{2}\lambda$ apart. This process repeats for the entire top half of the slit. The wavelets in the bottom half of the slit interfere in the same way as do those in the top half. The net result is a dark fringe. Between the dark fringes, wavelets interfere constructively, forming bright fringes.

The process of destructive interference occurs repeatedly for angles that produce a path difference that is an integral multiple of the wavelength of light.

$$\sin\theta = \frac{m\lambda}{W} \qquad (m = \pm 1, \pm 2, \pm 3 \ldots)$$

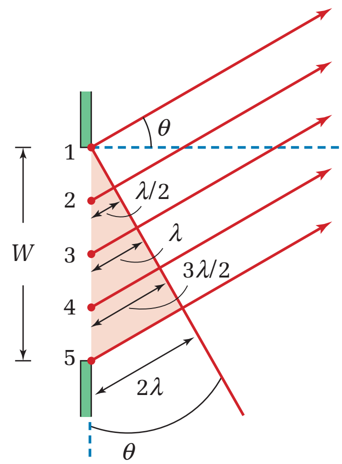


Figure 9.15 The second nodal line occurs when the path difference of every wavelet from the top half of the slit interferes with each wavelet from the bottom half.

The distance to the screen is much larger than the slit width or the separation of the dark fringes. Therefore, the perpendicular distance to the screen, L, is approximately the same length as the distance from the slit to the dark fringes, L_1, L_2, L_3, ..., as shown in Figure 9.16.

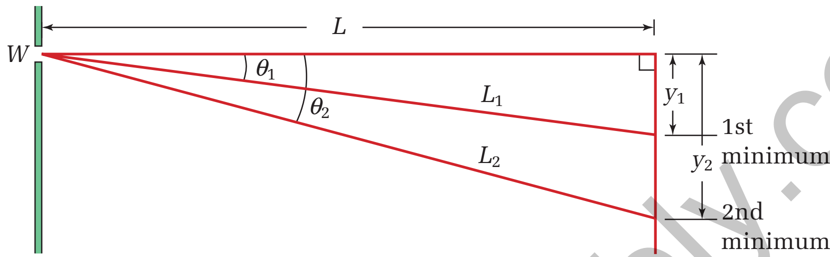


Figure 9.16 Slit width, W, is much smaller than the distance to the screen, L, allowing for the approximation that $L_1 \approx L_2 \approx L_n \approx L$.

Because $L >> y$, $\sin\theta = m\lambda/W$ $(m = \pm1, \pm2, \pm3\ldots)$ can be approximated as

$$\tan\theta = m\lambda/W \qquad (m = \pm1, \pm2, \pm3\ldots)$$

$$y_m/L = m\lambda/W \qquad (m = \pm1, \pm2, \pm3\ldots)$$

$$y_m = m\lambda L/W \qquad (m = \pm1, \pm2, \pm3\ldots)$$

SINGLE-SLIT INTERFERENCE

Dark fringes will exist on a distinct screen at regular, whole-numbered intervals

$$y_m = \frac{m\lambda L}{W} \qquad (m = \pm1, \pm2, \pm3\ldots) \qquad \text{Destructive}$$

$$y_m = \frac{\left(m + \frac{1}{2}\right)\lambda L}{W} \qquad (m = \pm1, \pm2, \pm3\ldots) \qquad \text{Constructive}$$

Quantity	**Symbol**	**SI unit**
distance to fringe from the central bisector	y_m	m (metres)
distance to screen	L	m (metres)
fringe order number ($\pm1, \pm2, \pm3\ldots$)	m	unitless
width of slit	W	m (metres)

Notice that the intensity of the bright fringes drops off dramatically in higher-order fringes. Intensity is a result of the amount of light energy striking a unit area per second. Recall that only wavelets interacting with the slit's edge are diffracted. This generates a double-wide central bright fringe, created by the light travelling directly through to the screen, unaffected by the slit's edge, plus the constructively interfering diffracted wavelets. The intensity steadily decreases as wavelets begin to destructively interfere, due to subtle changes in path difference.

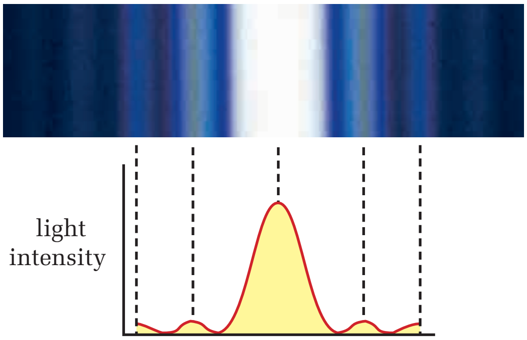


Figure 9.17 Bright and twice as wide, the central maximum appears much more intense than subsequent fringes.

SAMPLE PROBLEM

Single-Slit Diffraction

Viewing a 645 nm red light through a narrow slit cut into a piece of paper yields a series of bright and dark fringes. You estimate that five dark fringes appear in a space of 1.0 mm. If the paper is 32 cm from your eye, calculate the width of the slit.

Conceptualize the Problem

- *Light* passing through a very *narrow slit* will be *diffracted*, causing an interference pattern to be visible.
- *Dark fringes* result from *destructive interference*.
- The distance, y_1 , to the first dark fringe can be calculated by first determining the space between fringes, Δy.

Identify the Goal

The width, W, of the single slit

Identify the Variables and Constants

Known	Unknown
$m = 1$	Δy
$\lambda = 645$ m	y_m for $m = 1$
$L = 0.32$ m	W

Develop a Strategy

Determine the fringe spacing. Recall that there is always one less space between the fringes than there are fringes.

5 fringes in 1.0 mm = $4\Delta y$

$\Delta y = 0.25$ mm

continued ▶

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The distance between dark fringes, Δy, is also the distance from the central bisector to the first dark fringe.
Slit width can be determined by using the single-slit interference equation for dark fringes.

$$y_1 = 0.25\ \text{mm},\ m = 1$$
$$y_m = \frac{m\lambda L}{W}$$
$$W = \frac{m\lambda L}{y_1}$$
$$W = \frac{1(645 \times 10^{-9}\ \text{m})(0.32\ \cancel{\text{m}})}{0.25 \times 10^{-3}\ \cancel{\text{m}}}$$
$$W = 8.256 \times 10^{-4}\ \text{m}$$
$$W \cong 8.3 \times 10^{-4}\ \text{m}$$

The slit is 0.8 mm wide.

Validate the Solution

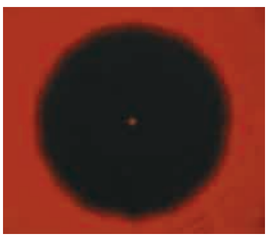
Interference fringes resulting from a single slit cut into a piece of paper would be very difficult to observe. The number, 5, must be taken as a number containing only one significant digit. The final answer is therefore provided to only one significant digit. A width of 0.8 mm is reasonable.

PRACTICE PROBLEMS

1. Determine the distance that the third bright fringe would lie from the central bisector in a single-slit diffraction pattern generated with 542 nm light incident on a 1.2×10^{-4} m slit falling onto a screen 68 cm away.
2. A special effects creator wants to generate an interference pattern on a screen 6.8 m away from a single slit. She uses 445 nm light and hopes to get the second dark fringe exactly 48 cm from the middle of the central bright maximum. What width of slit does she require?
3. Predict whether violet light ($\lambda = 404$ nm) or red light ($\lambda = 702$ nm) will have a wider central maximum when used to generate a single-slit diffraction pattern. Calculate the difference if the light is incident on a 6.9×10^{-5} m wide slit falling onto a screen 85 cm away.

9.2 Section Review

1. K/U Diffraction is defined as the spreading of light that passes by the edge of an opaque barrier. Explain why the term "spreading" is used rather than "bending."
2. I Suggest a property of light that posed the greatest difficulty for physicists attempting to observe the diffraction of light.
3. C This photograph is the result of illuminating a penny with a single point source of light. Describe what must be occurring in order to form the observed image both in the centre and around the edges.



4. K/U Describe what is meant by the term "coherent sources."
5. MC Why did the success of Young's experiment convince physicists of the time that light was some type of wave?
6. K/U Double-slit interference patterns form with equal spacing between light and dark fringes. Single-slit diffraction generates an interference pattern containing a central bright fringe that is twice as wide as any other. Explain these differing results.

9.3 Examples and Applications of Interference Effects

Not only did Young's double-slit experiment demonstrate the wave nature of light, it also paved the way for applications of interference and explained many phenomena that had been observed but not understood. If fact, Newton himself had observed some effects of interference of light, but he did not know that interference caused these effects.

SECTION EXPECTATIONS

- Describe how new technology resulted in the advancement of scientific theory.
- Outline the scientific understanding made possible through technological devices.
- Analyze thin films using diffraction, refraction, and wave interference.

KEY TERMS

- diffraction grating
- line spectrum (emission spectrum)
- resolving power
- Rayleigh criterion

Diffraction Gratings

Hold a compact disc in sunlight and a rainbow of colours will appear. Observe an Indigo snake moving through bright light and the full spectrum of colours will shimmer across its scales. The colours are separated from white light by diffraction from hundreds, even thousands, of tiny parallel ridges.

Figure 9.18 A compact disc has a thin transparent coating over a shiny metallic disk. What is the source of the rainbow colours?

If two slits are good, are 2000 slits better? For many applications, 2000 slits are definitely better. Such a device, called a **diffraction grating**, can create very fine, bright fringes that are separated by large dark fringes. A typical diffraction grating has several thousand slits or lines per centimetre. For example, a grating with 2000 lines/cm would have a slit spacing $d = (1/2000)\ \text{cm} = 5 \times 10^{-4}\ \text{cm}$. Diffraction gratings might be transmission gratings (light passes through the slits) or reflective gratings, in which light is reflected by smooth lines separated by non-reflective surfaces.

The principle on which a diffraction grating is based is the same as that of a double slit. The diffraction grating simply has thousands of pairs of double slits that all work together. As shown in Figure 9.19, constructive interference occurs when the distance travelled by a light ray from one slit is longer than that of the adjacent slit by an integral multiple of the wavelength of light.

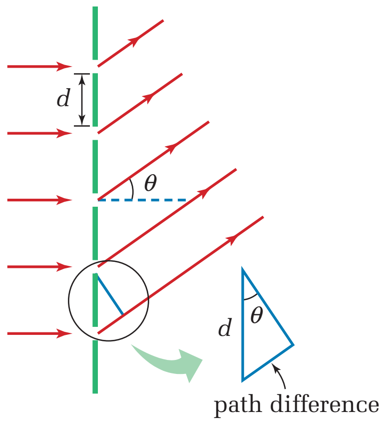


Figure 9.19 At very precise angles, the path difference travelled by waves passing through any pair of slits in the entire diffraction grating is an integer multiple of the wavelength of the light.

When $m = 0$ and the path lengths of all of the rays are the same, the rays go directly through the grating, creating a central bright fringe. The next bright fringe above or below the central fringe is called the "first-order fringe." The naming continues with second order, third order, and up to the last visible fringe.

DIFFRACTION GRATING BRIGHT FRINGES

Bright fringes will strike the screen when the path is an integer number of wavelengths of light.

$$m\lambda = d \sin \theta \qquad (m = 0, 1, 2, \ldots)$$

Quantity	Symbol	SI unit
wavelength of light	λ	m (metres)
integer number of wavelengths (0, 1, 2, …)	m	none
distance between slit centres	d	m (metres)
angle from horizontal to the ray resulting from constructive intererence	θ	unitless (degree is not a unit)

Note: Both transmission and reflection diffraction gratings can be modelled with this relationship.

The advantage of a diffraction grating over a double slit is the amount of destructive interference between the peaks of constructive interference. At the precise angles given by the equation $m\lambda = d \sin \theta$, waves from every slit interfere constructively with each other. However, at any other angle, waves from some combination of slits interfere destructively with each other. Figure 9.20 shows a typical double-slit pattern and compares it with a pattern obtained with five slits. With thousands of slits, the peaks become fine vertical lines and the space between is flat.

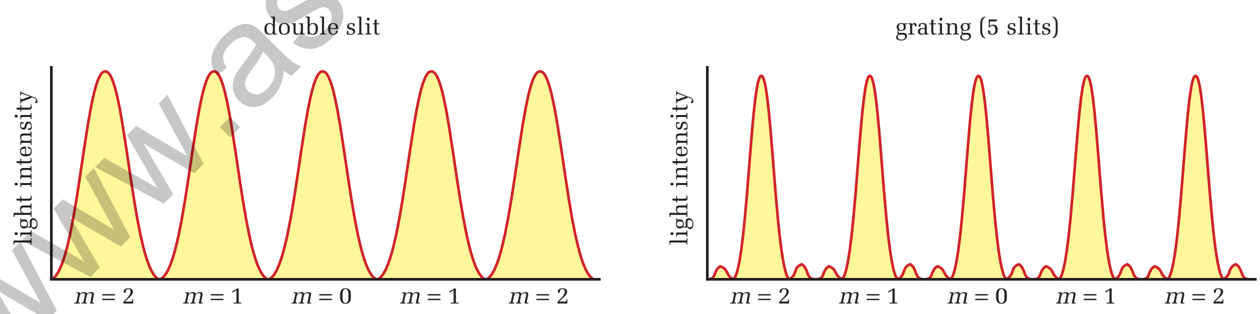


Figure 9.20 Destructive interference from distant fringes generates very narrow, bright fringes, compared to double-slit interference patterns.

For a given diffraction grating with constant slit separation, the angle that results in constructive interference depends on the wavelength of the light. Since different colours have different wavelengths, colours are separated when light passes through a grating, as shown by the rainbow of colours in Figure 9.21 (A). Figure 9.21 (B) shows what you would see if two colours passed through a diffraction grating together. This property of diffraction gratings makes them very useful in several types of instruments.

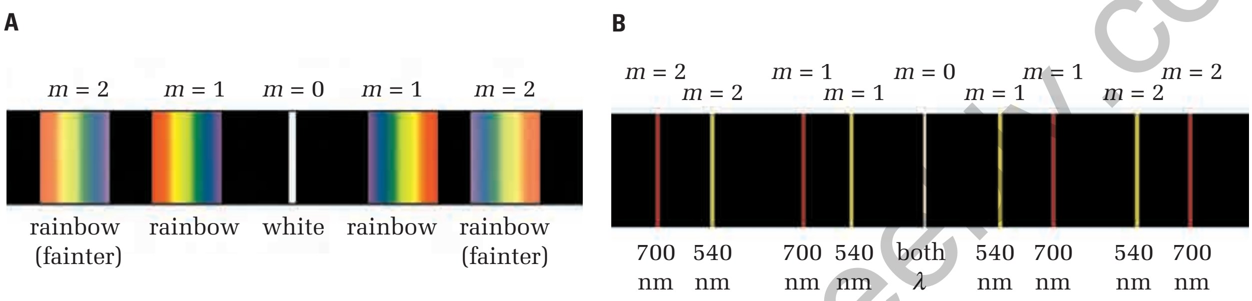


Figure 9.21 **(A)** A diffraction grating will separate white light into a rainbow of colours, because different wavelengths will be diffracted by different amounts. Higher-order fringes are more spread out. **(B)** This theoretical result would occur if a two-wavelength (700 nm, 540 nm) source was viewed through a spectroscope.

A spectroscope uses a diffraction grating to separate light into very narrow bands of specific colours (wavelengths) that you can then analyze. For example, you can identify the atoms or molecules in a gas discharge tube. When a gas is heated or has an electric discharge passed through it, it will emit light at very specific wavelengths. The set of wavelengths emitted by a pure substance is called the substance's **line spectrum** or **emission spectrum**. Figure 9.22 shows the line spectrum of several common substances.

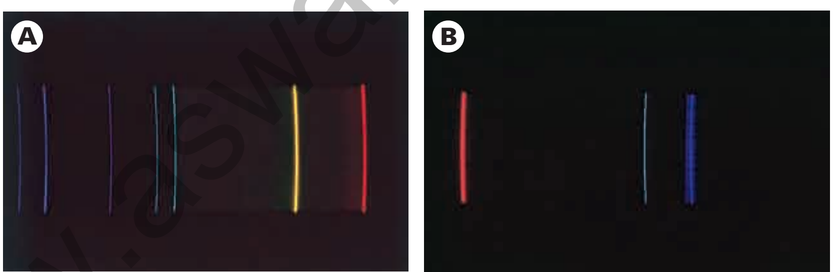


Figure 9.22 Line or emission spectrum of pure **(A)** helium, **(B)** atomic hydrogen, and **(C)** cadium as viewed through a spectroscope.

ELECTRONIC LEARNING PARTNER

Your Electronic Learning Partner contains an excellent reference source of emission and absorption spectra for every element in the periodic table.

Spectroscopes can also analyze absorption spectra. For example, the core of the Sun emits a continuous spectrum. However, atoms and molecules in the Sun's outer atmosphere absorb specific wavelengths, causing the Sun's spectrum to have several narrow black lines. Atoms and molecules also absorb light at the same wavelengths at which they emit it. Therefore, by identifying the wavelengths of light that have been absorbed by the Sun's outer atmosphere, physicists are able to identify the atoms that are present there. Careful analysis of the Sun's absorption spectrum reveals that at least two thirds of all elements present on Earth are present in the Sun. In fact, this technique is used to identify the composition of stars throughout our galaxy.

An instrument called a "spectrophotometer" is used in chemistry and biochemistry laboratories to identify and measure compounds in solutions. A spectrophotometer has a diffraction grating that separates white light into all wavelengths. You can select a specific wavelength and send it through a sample of a solution. The spectrophotometer then measures the amount of light of the wavelength that is absorbed by the sample and you can then calculate the concentration of the compound in the solution.

Interference of Thin Films

Soap bubbles always shimmer, with colour flowing across their surface. This interference phenomenon is a result of light reflecting off both surfaces of a thin film. To understand what happens when light strikes a thin film, review the process of reflection that is illustrated in Figure 9.23. In the upper left-hand photograph, a wave is moving through a "slow" medium (a heavy spring) toward an interface with a "fast" medium (a lightweight spring). On reaching the interface, both the reflected and the transmitted wave remain on the same side of the spring. In the lower left-hand photograph, a wave is moving from the right within the "fast" medium to the left toward the "slow" medium. When the wave reaches the interface, the transmitted wave remains on the same side of the medium, but the reflected wave has undergone a phase change or inversion and is on the opposite side of the medium. This change of phase or inversion that occurs when a wave reflects off an interface with a "slower" medium is a property of all waves.

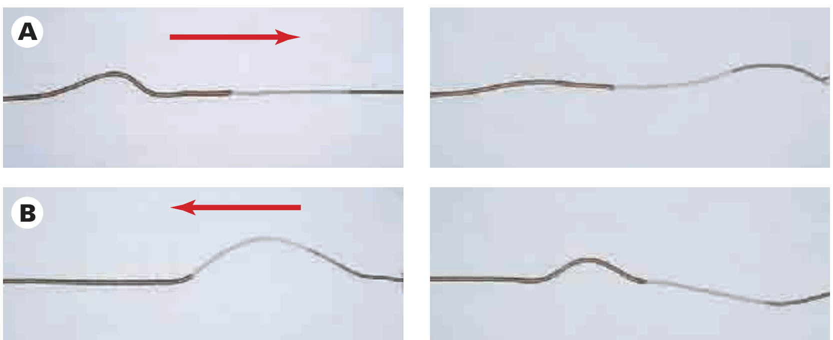


Figure 9.23 **(A)** At a slow-to-fast interface between two media, the transmitted and reflected pulses are on the same side of the spring. **(B)** At a fast-to-slow interface between two media, the transmitted pulse is on the same side of the spring, but the reflected pulse is inverted.

Very Thin Films: Destructive Interference ($t << \lambda$)

Destructive interference occurs when the film thickness, t, is much less than the wavelength, λ, of incident light, $t << \lambda$. Light travelling through air toward the very thin film of a soap bubble will behave in the same way that the wave in the spring behaves. Light incident on the surface of the soap bubble will be partially reflected and partially transmitted. In Figure 9.24, since the reflected wave (1) encounters the surface of a more-dense and therefore "slower" medium, it undergoes a phase change. When the transmitted light reaches the far surface of the soap film, it reflects off the interface with air, a "faster" medium, and therefore the reflected wave does not undergo a phase change, as indicated in Figure 9.24.

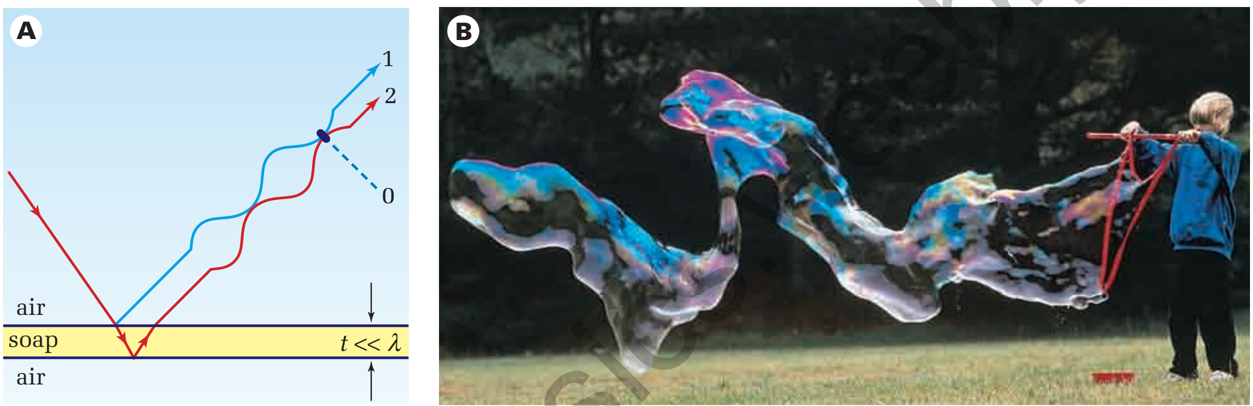


Figure 9.24 **(A)** Phase inversion causes destructive interference in very thin film when the film thickness, t, is much less than the wavelength of light, λ. **(B)** A soap bubble varies slightly in thickness, causing the destructive interference of varying colours.

In Figure 9.24, the thickness of the soap film is exaggerated relative to the wavelength of the light. In reality, the distance that wave 2 travels farther than wave 1 is negligible. The result is that when the inverted wave 1 rejoins reflected wave 2, the two waves are out of phase and undergo destructive interference. In the case of a soap bubble, the thickness varies as material flows throughout the film. The small fluctuations in thickness determine which wavelengths of light interfere destructively. The remaining wavelengths provide the colour that your eye sees.

Thin Films: Constructive Interference ($t = \lambda/4$)

Thin films that have a thickness of approximately a quarter of the incident wavelength of light cause constructive interference (see Figure 9.25). Once again, wave 1 reflects off an interface with a more-dense (slower) medium and therefore undergoes a phase change or inversion. In this case, the thickness of the film is significant. Wave 2 travels a half wavelength farther than wave 1. As a result, the two waves rejoin each other in phase and undergo constructive interference.

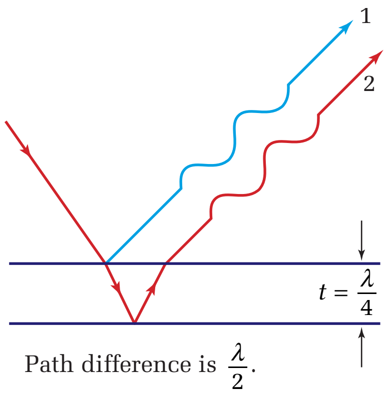


Figure 9.25 A total path difference of $\lambda/2$ combined with a phase inversion of one wave results in constructive interference.

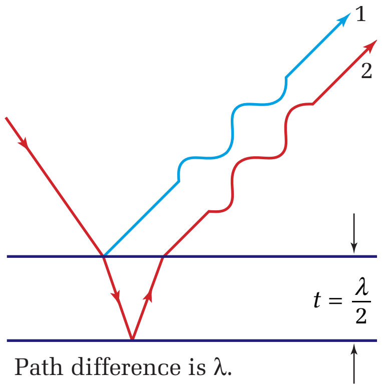


Figure 9.26 A total path difference of one wavelength, λ, combined with a phase inversion of one wave results in destructive interference.

Thin Films: Destructive Interference ($t = \lambda/2$)

A thin film with a thickness of one half of the wavelength of the incident light will cause destructive interference. Once again, wave 1 experiences a phase inversion at the top (fast-to-slow) reflecting surface. Wave 2 does not experience a phase inversion at the bottom (slow-to-fast) boundary, but does travel farther by a distance equal to one wavelength. Transmitted waves do not experience a phase shift; therefore, wave 1 and wave 2 proceed exactly out of phase and interfere destructively.

Although thin films are often just an attractive novelty, they can be useful. Eyeglass manufacturers make use of destructive interference caused by thin films to prevent reflection from lenses. By applying a coating of a carefully designed thickness to the outer surface of eyeglass lenses, specific wavelengths of reflected light can be cancelled.

Digital Videodiscs

A digital videodisc (DVD) is composed of several layers of plastic on a reflective aluminum disc. The distinctive gold colour of a DVD results from a semi-reflecting layer of gold used to separate each data layer. Data is stored in the plastic layers as series of pits or bumps that follow a spiral path from close to the centre of the disc to the outer edge. On the reflective aluminum side of a DVD, they are pits, but on the side from which the laser reads information, they are actually "bumps." Each bump has a thickness of exactly 120 nm, which is one fourth of a wavelength of the 640 nm laser light that reads the bumps.

DVD players read the information by shining laser light on the edge of the bumps and detecting the reflections. Part of the laser beam falls on the flat surface of the disc, while the other part strikes the bumps. Since the bumps are one fourth of a wavelength, the part of the beam striking the bump travels a distance that is half a wavelength shorter than the part that remains on the flat surface. When both parts of the reflected beam rejoin each other, destructive interference occurs in a way that is similar to thin film interference. The detector converts the differences in the amount of reflected laser light into the images and sound that you observe on your DVD player.

If it was possible to lift the long spiral of data-storing bumps off a DVD and stretch it into a straight line, the line would extend more than 48 km. A double-sided DVD can store up to seven times more information than a compact disc — approximately 15.9 GB. The increased data storage results from a combination of more efficient techniques to digitize the data and tighter data "bump" spacing, thanks to the use of shorter wavelength laser light. Each lap of the spiral is spaced exactly 740 nm from the previous one.

The series of tracks behaves like a diffraction grating, producing the varying colour patterns that you see when light reflects from the surface of a DVD.

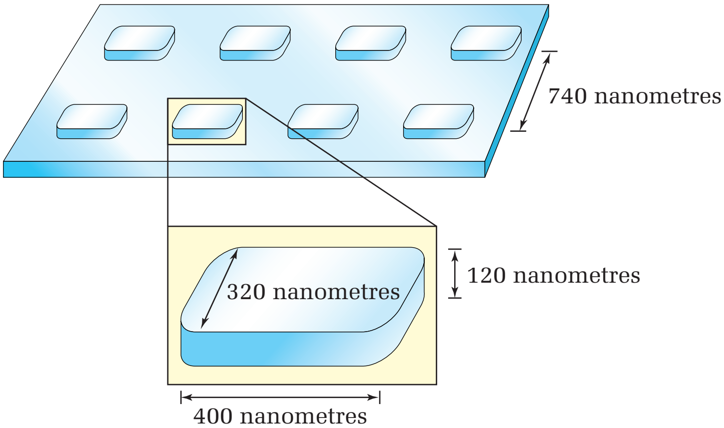


Figure 9.27 Data are stored on DVDs in the form of tiny "bumps" that are less than the wavelength of light.

Resolving Power

A light travels toward you from the distance. Is the object a motorcycle with a single headlight or a car with two separated headlights? As the object approaches, the single light slowly grows into an oval shape and then finally into two individual and distinct headlights of a car. The ability of an optical instrument, such as the human eye or a microscope, to distinguish two objects is called the **resolving power** of the instrument. An eagle has eyesight that is much better at resolving objects than is the human eye. A microscope uses lenses with resolving powers that are even greater.

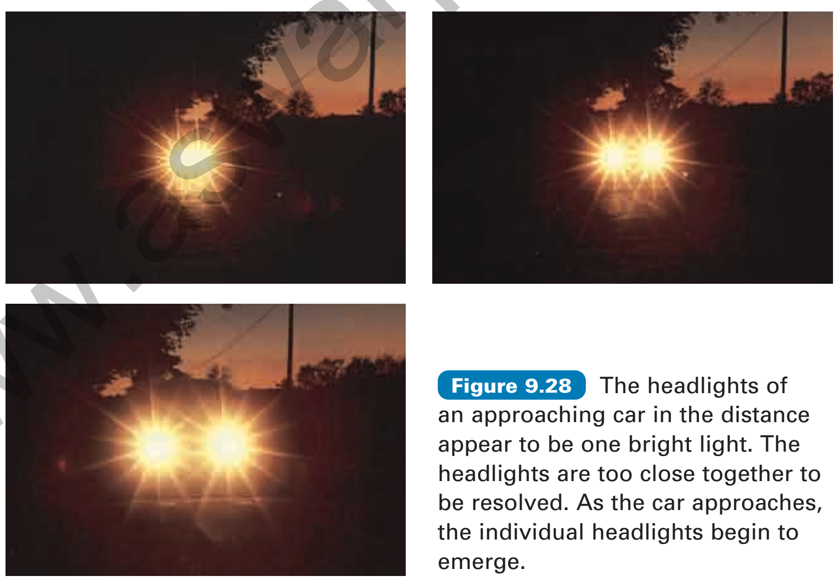


Figure 9.28 The headlights of an approaching car in the distance appear to be one bright light. The headlights are too close together to be resolved. As the car approaches, the individual headlights begin to emerge.

Light travelling through a small opening or aperture is diffracted. To distinguish between two objects through a single aperture (such as the pupil of an eye), the central bright fringes of the two sources must not overlap. Although many factors might affect the ability of an instrument to resolve two objects, the fundamental factors are the size of the aperture, the distance between the two objects, and the distance between the aperture and the objects. Lord Rayleigh (John William Strutt: 1842–1919) suggested a criterion that is still practical for use today. The **Rayleigh criterion** states that "Two points are just resolved when the first dark fringe in the diffraction pattern falls directly on the central bright fringe in the diffraction pattern of the other." Figure 9.29 illustrates the Rayleigh criterion.

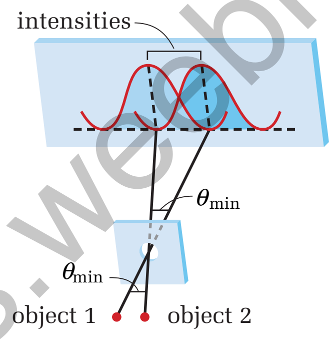


Figure 9.29 The ability to resolve two objects occurs when the first dark fringe just falls on the other object's first bright fringe. This is known as the Rayleigh criterion.

Recall that for a single slit, the first dark fringe occurs when $\lambda = W \sin \theta$. Since the Rayleigh criterion states that the central fringe of the second object must be no closer to the first object than its first dark fringe, you can use the same equation to describe the spatial relationship between the two objects. Since resolution depends on both the distance between the objects and their distance from the aperture, it is convenient to combine those distances and express them as the angle between rays coming from each object to the aperture. This is exactly the angle θ in the equation above. Therefore, the Rayleigh criterion for a single slit aperture is as follows.

$$\sin \theta = \frac{\lambda}{W}$$

For very small angles, the sine of the angle is numerically almost the same as the angle itself expressed in radians. Since the angles describing resolving power are always very small, you can express the Rayleigh criterion as shown below.

$$\theta = \frac{\lambda}{W}$$

Most optical instruments use circular apertures, rather than rectangular ones. Experimental evidence shows that the minimum angle that a circular aperture is just able to resolve is as follows.

$$\theta = \frac{1.22\lambda}{D}$$

D is the diameter of the aperture and, once again, the angle θ is expressed in radians.

RESOLVING POWER

In order to resolve two objects, the minimum angle between rays from the two objects passing through a rectangular aperture is the quotient of the wavelength and the width of the aperture. For a circular aperture, the minimum angle is the quotient of 1.22 times the wavelength and the diameter of the aperture.

$$\theta_{min} = \frac{\lambda}{W}$$ rectangular slit aperture

$$\theta_{min} = \frac{1.22\lambda}{D}$$ circular aperture

Quantity	Symbol	SI unit
minimum angle for resolution	θ	unitless (radian is not a unit)
wavelength of light	λ	m (metres)
width of rectangular aperture	W	m (metres)
diameter of circular aperture	D	m (metres)

Note: The angle measure must be provided in radians.

$360^\circ = 2\pi$ rad or 1 rad $= 57.3^\circ$

SAMPLE PROBLEM

Resolving Power

A skydiver is falling toward the ground. How close to the ground will she have to be before she is able to distinguish two yellow baseballs lying 25.0 cm apart, reflecting 625 nm light in air? Her pupil diameter is 3.35 mm. Assume that the speed of light inside the human eye is 2.21×10^8 m/s.

Conceptualize the Problem

- The human pupil is a circular opening; therefore, the circular aperture equation for resolving power applies.
- The wavelength of light will be different inside the material of the eye because the speed is less than it would be in air. The reduced speed must be used to calculate the wavelength of the light in the eye.

Identify the Goal

The maximum height, h, at which the skydiver can resolve the two objects that are 0.250 cm apart.

Identify the Variables and Constants

Known	Unknown
$s = 25.0$ cm	λ_{eye}
$\lambda_{air} = 625$ nm	θ_{min} (radians)
$D = 3.35$ mm	
$v_{eye} = 2.21 \times 10^8 \frac{m}{s}$	

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Develop a Strategy

- Determine the wavelength of the light inside her eye. Use the wave equation and the fact that the frequency of a wave does not change when it passes from one medium into another.

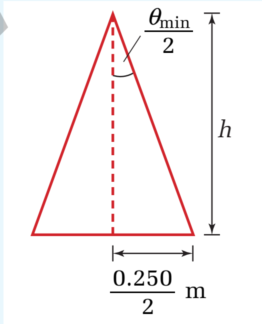
$$v = f\lambda$$
$$f = \frac{v}{\lambda}$$
$$f = \frac{v_{air}}{\lambda_{air}} = \frac{v_{eye}}{\lambda_{eye}}$$
$$\lambda_{eye} = \frac{\lambda_{air}v_{eye}}{v_{air}}$$
$$\lambda_{eye} = \frac{(625\ \text{nm})\left(2.21 \times 10^8\ \frac{\text{m}}{\text{s}}\right)}{3.00 \times 10^8\ \frac{\text{m}}{\text{s}}}$$
$$\lambda_{eye} = 460\ \text{nm}$$

- Determine the minimum angle for resolution, using the Raleigh criterion.

$$\theta_{min} = \frac{1.22\lambda}{D}$$
$$\theta_{min} = \frac{(1.22)(460 \times 10^{-9}\ \text{m})}{3.35 \times 10^{-3}\ \text{m}}$$
$$\theta_{min} = 1.6767 \times 10^{-4}\ \text{rad}$$

- Divide the isosceles triangle in half, as shown, and use the tangent function to calculate the height. (Hint: Remember that the equation gives the angle in radians. Be sure that your calculator is set to radians while performing your calculations.)

$$\tan\left(\frac{\theta_{min}}{2}\right) = \frac{\frac{0.250\ \text{m}}{2}}{h}$$
$$h = \frac{0.125\ \text{m}}{8.3837 \times 10^{-5}}$$
$$h = 1.491 \times 10^3\ \text{m}$$
$$h \cong 1.49\ \text{km}$$



If only resolving power is considered, she will just be able to distinguish the baseballs when she is 1.49 km from the ground.

Validate the Solution

Resolving objects separated by 25 cm could not possibly be accomplished from a distance of 1.5 km. Most likely, she would have to be much closer, because of the effects caused by the high-speed descent. Owls have pupils that are as much as 10 times larger than human pupils. How would such an adaptation be helpful?

PRACTICE PROBLEMS

4. **(a)** Commercial satellites are able to resolve objects separated by only 1.0 m. If these satellites orbit Earth at an altitude of 650 km, determine the size of the satellites' circular imaging aperture. Use 455 nm light for the light in the lenses of the satellites.

   (b) Describe why the value from part (a) is a theoretical best-case result. What other effects would play a role in a satellite's ability to resolve objects on the surface of Earth?

5. Calculate the resolving power of a microscope with a 1.30 cm aperture using 540 nm light. The index of refraction of the lens slows the light inside the glass to 1.98×10^8 m/s.

6. You are about to open a new business and need to select a colour scheme for your backlighted sign. You want people to be able to see your sign clearly from a highway some distance away. Assuming that brightness is not a problem for either colour, should you use blue or red lettering? Develop an answer, using resolving power arguments. Include numerical examples.

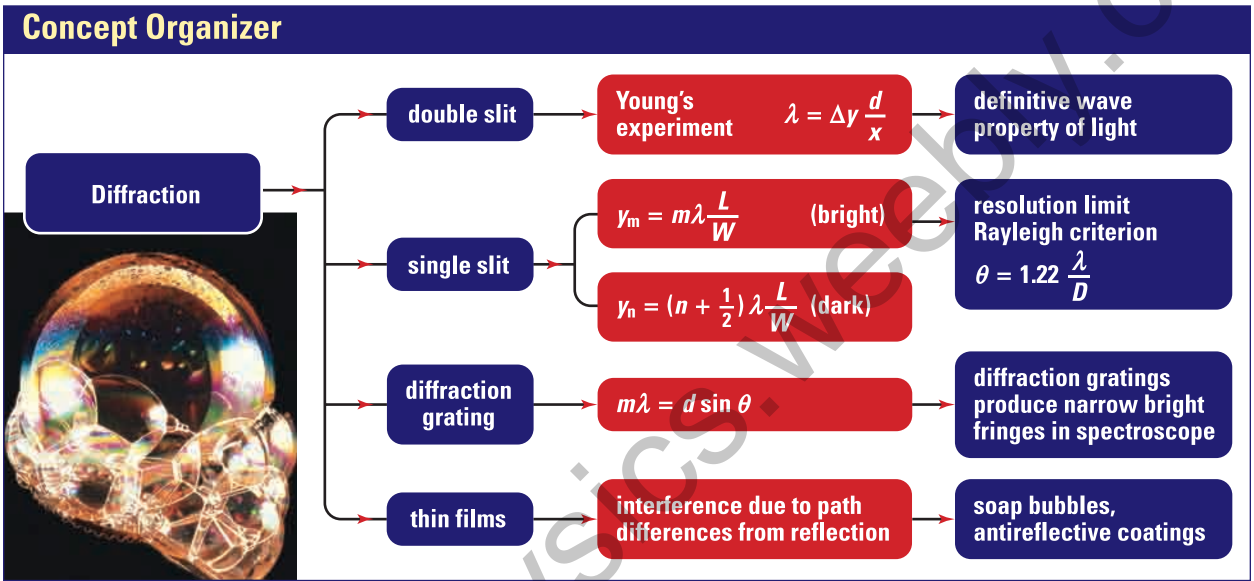


Figure 9.30 Diffraction

9.3 Section Review

1. K/U Why does a diffraction grating produce much narrower bright fringes than a double-slit interference pattern?

2. I Describe why an astronomer would pass light from a distant star through a diffraction grating? Provide the name of the instrument used and possible facts that could be learned from its use.

3. K/U Classify each of the following examples as one of **(a)** very thin film ($t << \lambda$) or **(b)** thin film ($t = \lambda/4$) or ($t = \lambda/2$) interference.
   - oil floating on water
   - antireflective coating on a television screen

4. K/U Describe how DVD technology makes use of thin film interference and partial reflection-partial refraction.

5. MC Looking through a terrestrial telescope, you see a single light shining from a distant farmhouse. The single image is the result of light streaming from two identical windows placed side by side. Describe two possible steps you could take so that the lighted windows would appear individually.

6. K/U Why is resolving power often expressed as an angle?

New Views of Earth's Surface

It's a dark and stormy night — perfect conditions to study minute changes in Earth's crust. Not from Earth's surface, but from radar satellites hundreds of kilometres away in space. Optical satellite images are used to observe weather patterns, but the data used to generate radar images contain more information than is displayed in the optical images. This additional information can be exploited to provide precision measurements of Earth's surface.

Radar satellites have two distinct advantages over optical satellites. First, they operate at longer wavelengths, which allows them to penetrate clouds. Second, they provide their own illumination instead of relying on reflected sunlight, so they can obtain images day or night and, more importantly, coherent radiation can be used. In coherent radiation, the individual waves are all emitted in step or in "phase" with one another.

The Importance of Phase Information

This phase information provides the basis for satellite radar interferometry. When the satellite views a patch of Earth's surface at an angle, the distance from the target point on the surface to each of the satellite's two antennae will be different. Coherent radiation emitted by the satellite will be received in a particular phase by the first antenna and in a different phase by the second antenna because of the different path lengths. Effectively, the phase information is like a stopwatch that indicates how long the wave has been travelling. Since the wave travels at the speed of light, this time is easily converted into a distance. The phase difference between the waves in the received signals can be used to reconstruct the height of a point on Earth's surface. A couple of snapshots taken in seconds, or even in repeat satellite passes, can provide precise topographical detail that would take geologists and surveyors years to match.

If positions and heights can be accurately measured, then by spacing observations over time, changes in Earth's surface can be detected. In two radar images taken from the same altitude but at different times, each of the corresponding picture elements, (pixels) on the two images should have the same phase. If the ground has moved toward or away from the satellite in the time between the images, this would be detected as a phase difference in the pixels. This is easily seen by constructing an "interferogram."

Displaying the Phase Information

As the name suggests, the process of constructing an interferogram involves allowing light waves to interfere with each other. When any two waves combine, they can reinforce each other, cancel each other, or do something in between, depending on the relative phases. Suppose you represent places on the resultant image where two corresponding pixels (from images of the same area taken at different times) reinforce each other with red pixels, and places where they cancel each other with blue pixels. Cases in between these extremes can be represented by colours in the spectrum between red and blue. A cycle from red to blue would then indicate a displacement on the ground equivalent to half of a wavelength. A typical interferogram will show several complete colour cycles, or fringes, because the phase between any two pixels can differ by any number of whole wavelengths (1, 2, 3,…). By counting these fringes, the total displacement can be determined.

Radar Satellites in Operation Today

Although the four radar satellites presently in operation — the Canadian RADARSAT, the European ERS-1 and ERS-2, and the Japanese JERS-1 — orbit at an altitude of several hundred kilometres, radar interferometry allows them to monitor changes in Earth's surface on the scale of half of a radar wavelength, approximately 2 to 4 cm, or smaller. The method was first applied in 1992 to examine the deformation of Earth's crust after an earthquake in Landers, California. Although the maximum displacement of the fault was 6 m, researchers were able to detect a tiny slip of 7 mm on a fault located 100 km away from where the quake had struck. Since then, applications of the method have grown rapidly.

Examples of present uses of radar interferometry include the examination of a variety of geophysical phenomena. By studying the after-effects of earthquakes worldwide, critical information can be gained that someday could be used to predict earthquakes. The subsidence of surface land due to extraction of coal or oil can be monitored. The advance and recession of glaciers and ice flow velocities can be routinely studied to improve hydrological models and assess global climate change.

Several recent studies have applied radar interferometry to volcanoes. As magma fills or drains chambers under the volcano's surface, subtle deformation of the volcano, not detected by other methods, is revealed in interferograms. Detecting uplift and swelling of volcanoes could in some cases provide early warning of an eruption.

With some promising results so far, radar interferometry is destined to become an important tool for analyzing our ever-changing Earth.

Making Connections

1. Compare and contrast the information gained from radar interferometry and from land surveying.
2. Research the limitations involved in using radar interferometry.
3. The Canadian RADARSAT has just completed an important survey of Antarctica. Report on some of the findings.

CHAPTER 9 Review

REFLECTING ON CHAPTER 9

- Mechanical waves are disturbances that transfer energy from one location to another through a medium. All waves, under appropriate conditions, are known to exhibit rectilinear propagation, reflection, refraction, partial reflection and partial refraction, and diffraction.
- Light energy reaches Earth after travelling through the void of outer space. Light, under appropriate conditions exhibits rectilinear propagation, reflection, refraction, partial reflection and partial refraction, and diffraction.
- Waves interfere with one another according to the superposition of waves, which states: When two or more waves propagate through the same location in a medium, the resultant displacement of the medium will be the algebraic sum of the displacements caused by each individual wave. When two or more waves propagate through the same location in a medium, the waves behave as though the other waves did not exist.
- Huygens' principle models light as a wave that results from the superposition of an infinite number of wavelets. The principle states: Every point on an advancing wavefront can be considered as a source of secondary waves called "wavelets." The new position of the wavefront is the envelope of the wavelets emitted from all points of the wavefront in its previous position.
- Interference is a property exhibited by waves.
- Young's double-slit experiment demonstrated that light experiences interference and forms diffraction patterns. Young's experiment can be used to determine the wavelength of a specific colour of light by the relationship $\lambda \cong \frac{\Delta y d}{x}$.
- Light passing through a single slit experiences interference and forms diffraction patterns. Single-slit interference forms distinctive patterns, according to the relationship $y_m \cong \frac{m\lambda L}{W}$ for dark fringes and $y_m \cong \frac{(m + \frac{1}{2})\lambda L}{W}$ for bright fringes.
- A diffraction grating, composed of several equally spaced slits, produces diffraction patterns with more distinct bright and dark fringes. Diffraction from each of the slits increases the degree of constructive and destructive interference.
- Spectrometers make use of the diffraction of light, splitting the incident light into fine bands of colour. The resulting spectrum is used to identify the atomic composition of the light source. Astronomers use absorption line spectra to determine the composition of stars.
- The Rayleigh criterion states that "Two points are just resolved when the first dark fringe in the diffraction pattern falls directly on the central bright fringe in the diffraction pattern of the other." Experimental evidence shows that the minimum angle that a circular aperture is just able to resolve is given by $\theta_{min} = \frac{1.22\lambda}{D}$.

Knowledge/Understanding

1. Distinguish between dispersion and diffraction.
2. What happens to the energy of light waves in which destructive interference leads to dark lines in an interference pattern?
3. How did Thomas Young's experiment support the wave model of light?
4. An interference maximum is produced on a screen by two portions of a beam originally from the same source. If the light travelled entirely in air, what can be said about the path difference of the two beams?
5. The same formula is used for the positions of light maxima produced by two slits as for a grating with a large number of finely ruled slits or lines. What is the justification for creating and using many-lined gratings?
6. **(a)** What is the difference between the first-order and the second-order spectra produced by a grating?
   (b) Which is wider?
   (c) Does a prism produce spectra of different orders?
7. Why is it important that monochromatic light be used in slit experiments?
8. How does a thin film, such as a soap bubble or gasoline on water, create an interference pattern?

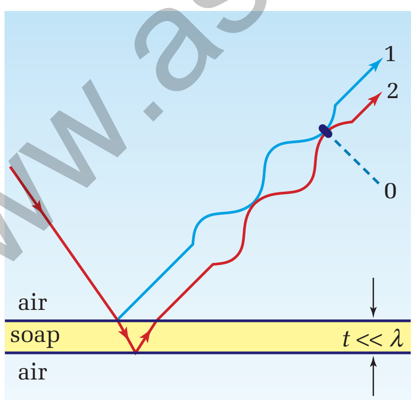
Inquiry

9. Devise a simple experiment to demonstrate the interference between sound waves from two sources.
10. Sketch the diffraction pattern produced by parallel wavefronts incident on a very wide slit. How does the pattern change as the slit size decreases?
11. Describe simple experiments to determine the following.
    (a) the resolving power of a small, backyard telescope
    (b) the wavelength of a source of monochromatic light
    (c) the separation between the rulings in a diffraction grating
12. Photographers often use small apertures to maximize depth of focus and image sharpness. However, at smaller apertures, diffraction effects become more significant. If you have access to a single-lens reflex camera, try to evaluate at which aperture diffraction effects become problematic in a particular lens. Also, try to determine the resolution of the lens. How does it compare to the manufacturer's stated value?
13. Suppose you have a source that emits light of two discrete wavelengths, one red and one blue. Assume for the sake of simplicity that each colour is emitted with the same intensity. Imagine allowing the light to pass through a diffraction grating onto a screen. Draw the appearance of the resulting line spectrum.
14. Single-slit diffraction affects the interference pattern of a double slit. Consider a double slit with slits that are 0.130 mm wide and spaced 0.390 mm apart, centre to centre.
    (a) Which orders of the double-slit pattern will be washed out by the minima of the single-slit pattern?
    (b) Sketch the interference pattern and demonstrate the above solution by superimposing the single-slit pattern on the double-slit pattern.
15. An ingenious physics student wants to remove the amount of glare reflecting from her computer screen. Describe, with the aid of a diagram, how she could make use of her knowledge of thin films to accomplish her task.

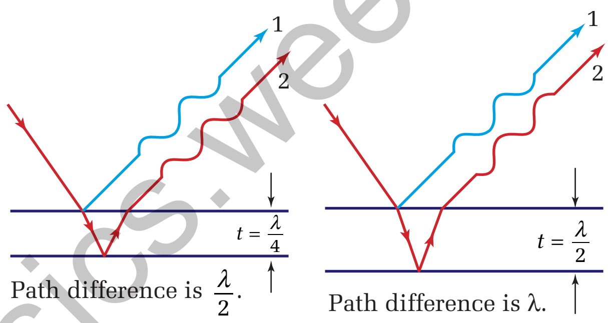
Communication

16. Explain whether a beam of light can be made increasingly narrow by passing it through narrower and narrower slits.
17. A friend who has never taken a physics course asks why light that passes through a slit produces a series of bright and dark fringes. How would you explain this phenomenon?

18. Discuss how we know that the wavelength of visible light must be very much less than a centimetre.
19. Both sound and light waves diffract on passing through an open doorway. Why does a sound wave diffract much more than a light wave? In other words, why can you hear around corners, but not see around corners?
20. Atoms have diameters of about 10^{-10} m. Visible light wavelengths are about 5×10^{-7} m. Can visible light be used to "see" an atom? Explain why or why not.
21. Suppose that, in a double-slit experiment, monochromatic blue light used to illuminate the slits was replaced by monochromatic red light. Discuss whether the fringes would be more closely or widely spaced.
22. Explain the source of colour seen on the surface of compact discs.
23. Discuss why interference fringes are not visible from thick films. (Hint: What is the effect of rays incident on thin films and thick films?)
24. The illustration depicts interference caused by very thin films.
   (a) Describe the type of interference depicted by the illustration.
   (b) Use wave model arguments to explain how the interference pattern from part (a) is caused.
   (c) Would it be possible to coat an object with a very thin film to make it invisible in white light? Explain.



25. **(a)** Analyze carefully the illustrations following this question. Write a description tracing the path of the incident light as it encounters each medium interface. Describe what is happening to the wave in each case as the wave is **(i)** reflected and **(ii)** transmitted.
   (b) Compare the resulting interference patterns from each illustration. What is the fundamental difference between the path that the light takes in each case?



Making Connections

26. Bats use echolocation to detect and locate their prey — insects. Why do they use ultrasonic vibrations for echolocation rather than audible sound?
27. CD and DVD players both utilize the effects of interference to retrieve digital information. Explain how this is done.
28. Explain how one observer's blue sky could be related to another observer's view of a red sunset.
29. Many butterflies have coloured wings due to pigmentation. In some, however, such as the Morpho butterfly, the colours do not result from pigmentation and, when the wing is viewed from different angles, the colours change. Explain how these colours are produced.
30. By studying the spectrum of a star (for example, the Sun), many physical properties can be determined in addition to the chemical composition. In fact, besides the telescope, the spectroscope is probably an astronomer's most useful tool. Write an essay discussing

the information obtained about stars from spectra, and the impact of the spectroscope on modern astronomy.

Problems for Understanding

31. In a ripple tank, two point sources that are 4.0 cm apart generate identical waves that interfere. The frequency of the waves is 10.0 Hz. A point on the second nodal line is located 15 cm from one source and 18 cm from the other. Calculate
(a) the wavelength of the waves
(b) the speed of the waves

32. Blue light is incident on two slits separated by 1.8×10^{-5} m. A first-order line appears 21.1 mm from the central bright line on a screen, 0.80 m from the slits. What is the wavelength of the blue light?

33. A sodium-vapour lamp illuminates, with monochromatic yellow light, two narrow slits that are 1.00 mm apart. If the viewing screen is 1.00 m from the slits and the distance from the central bright line to the next bright line is 0.589 mm, what is the wavelength of the light?

34. Under ordinary illumination conditions, the pupils of a person's eye are 3.0 mm in diameter and vision is generally clearest at 25 cm. Assuming the eye is limited only by diffraction, what is its resolving power? (Choose 550 nm, in the middle of the visible spectrum, for your calculation.)

35. Assuming that the eye is limited only by diffraction, how far away from your eye could you place two light sources that are 50.0 cm apart and still see them as distinct?

36. The Canada-France-Hawaii telescope has a concave mirror that is 3.6 m in diameter. If the telescope was limited only by diffraction, how many metres apart must two features on the Moon's surface be in order to be resolved by this telescope? Take the Earth-Moon distance as 385 000 km and use 550 nm for the wavelength of the light.

37. A diffraction grating with 2000 slits per cm is used to measure the wavelengths emitted by hydrogen gas. If two lines are found in the first order at angles $\theta_1 = 9.72 \times 10^{-2}$ rad and $\theta_2 = 1.32 \times 10^{-1}$ rad, what are the wavelengths of these lines?

38. The range of visible light is approximately from 4.0×10^{-7} m (violet light) to 7.0×10^{-7} m (red light).
(a) What is the angular width of the first-order spectrum (from violet to red) produced by a grating ruled with 8000 lines per cm?
(b) Will this angular width increase or decrease if the grating is replaced by one ruled with 4000 lines per cm?

39. Show that there will be yellow light but no red light in the third-order spectrum produced by a diffraction grating ruled 530 lines per mm.

40. Suppose a grating is used to examine two spectral lines. What is the ratio of the wavelengths of the lines if the second-order image of one line coincides with the third-order image of the other line?

41. **(a)** What is the largest order image of green light, 540 nm, that can be viewed with a diffraction grating ruled 4000 lines per cm?
(b) At what angle does that order appear?

42. Red light is incident normally onto a diffraction grating ruled with 4000 lines per cm, and the second-order image is diffracted 33.0° from the normal. What is the wavelength of the light?

43. Suppose you shine a light on a soap bubble that is 2.50×10^{-7} m thick. What colour will be missing from the light reflected from the soap bubble? Assume the speed of light in water is 2.25×10^{8} m/s.

CHAPTER 10

Electromagnetic Waves

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PREREQUISITE CONCEPTS AND SKILLS

- Ampère's law
- Coulomb's law
- Faraday's law
- Vibrations and waves

A camera and some rolls of film packed safely inside your favourite sweatshirt appear clearly on a monitor as your suitcase passes through the airport's security X-ray system. Various materials absorb the low-energy X rays differently, and sophisticated software analyzes the varying intensities of the X-ray signals that are transmitted. The software then interprets these signals and converts them into a colour picture for the security guard to see.

This entire airport security process is based on the production, transmission, and reception of electromagnetic radiation. The intensity of the radiation can be finely adjusted and focussed, producing clear pictures of the contents of opaque containers such as luggage without damaging even sensitive camera film.

Electromagnetic radiation and its applications provide much more than just airport security. Global communications, radar, digital videodisc players, and television remote controls also use electromagnetic radiation. This chapter explores how physicists attempt to understand electromagnetic radiation by using a wave model. You will gain a better understanding of how electromagnetic radiation is produced, how varied forms of it behave, including light, and how its properties are utilized in various communication and medical applications.

MULTI LAB

Properties of Electromagnetic Waves

TARGET SKILLS

- Analyzing and interpreting
- Identifying variables
- Communicating results

Transmission of Ultraviolet Radiation

In this lab, you will analyze the transmission of ultraviolet (UV) radiation through various substances.

Tonic water, which contains quinine, emits a blue glow when exposed to UV radiation, while pure water does not.

Fill one clear, plastic cup with tonic water and one with pure water. Shine UV radiation onto the tops of both filled glasses. From the side, observe the top centimetre of the tonic water and the pure water. Place a dark cloth behind the cups to add contrast. Record the amount and depth of the blue glow. Repeat this procedure with transparent materials such as glass, plastic, and cellulose acetate placed over the cups.

Analyze and Conclude

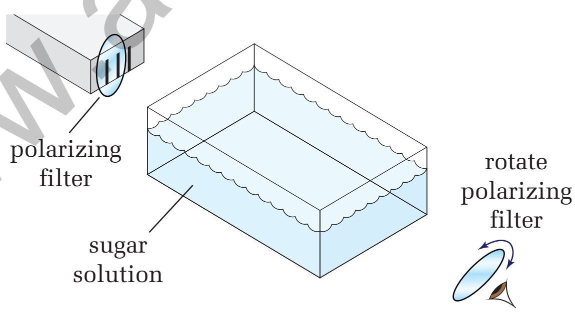
1. List the materials that you tested in the order of their effectiveness in absorbing UV radiation, starting with the material that was most effective.
2. Suggest why both the time of day and time of year affect the amount of dangerous UV radiation reaching the surface of Earth.

Apply and Extend

3. If time permits, devise an experiment to test the ability of various sunblock strengths to absorb UV radiation. (Hint: Smear the sunblock over a medium that is transparent to UV radiation.)
4. Increased amounts of UV radiation reaching Earth's surface could pose a risk to wildlife. Devise a simple experiment to verify if UV radiation is able to penetrate the surface of lakes and rivers.

Polarizing Light with Sugar

Fill a transparent, rectangular container with a supersaturated sugar solution. Using a ray box, shine three rays of light through the solution. Ensure that two rays of light pass through a polarizing filter before passing through the sugar solution. Carefully observe the rays of light through a second polarizing filter. Slowly rotate the filter closest to your eye while observing the rays of light passing through the solution.



Analyze and Conclude

1. Describe what you observed for **(a)** the two rays that passed through the first polarizing filter before entering the sugar solution and **(b)** the ray that did not pass through the first polarizing filter.
2. **(a)** Formulate an hypothesis that could explain your observations.

   (b) Is your hypothesis based on a wave or particle model of light?

10.1 The Nature of Electromagnetic Waves

SECTION EXPECTATIONS

- Describe how electromagnetic radiation is produced.
- Analyze the transmission of electromagnetic radiation.
- Define and explain the concepts related to the wave nature of electromagnetic radiation.
- Explain the underlying principle of polarizing filters.
- Identify experimental evidence for the polarization of light.
- Describe how electromagnetic radiation, as a form of energy, interacts with matter.

KEY TERMS

- Maxwell's equations
- electromagnetic wave
- electric permittivity
- magnetic permeability
- plane polarized
- photoelastic

While Huygens, Young, and others were studying the properties of light, physicists in another sector of the scientific community were exploring electric and magnetic fields. They were not yet aware of the connections among these fields of study. While a student at Cambridge, a young Scotsman, James Clerk Maxwell (1831–1879), became interested in the work of Lord Kelvin (William Thomson: 1824–1907) and Michael Faraday (1791–1867) in electric and magnetic fields and lines of force. Soon after Maxwell graduated in 1854, he gathered all of the fundamental information and publications that he could find in the fields of electricity and magnetism. After carrying out a thorough study and detailed mathematical analysis, Maxwell synthesized the work into four fundamental equations that are now known as **Maxwell's equations**. These equations form the foundation of classical electromagnetic field theory in the same way that Newton's laws form the foundation of classical mechanics.

Maxwell's Equations

Maxwell did not create the equations; he adapted and expanded mathematical descriptions of electric and magnetic fields that had been developed by others. The mathematical form of Maxwell's equations is well beyond the scope of this course, but the qualitative concepts and the implications of his equations are quite logical.

Maxwell's first two equations are based on concepts and equations developed by Carl Friedrich Gauss (1777–1855) and called "Gauss's law for electric fields" and "Gauss's law for magnetic fields" — concepts with which you are already familiar. Simply stated, Maxwell's first equation (Gauss's law for electric fields), illustrated in Figure 10.1 (A), states that for any imaginary closed surface, the number of electric field lines exiting the surface is proportional to the amount of charge enclosed inside the surface. Note that a field line entering the surface will cancel a line emerging from the surface. Fundamentally, this equation is based on the concept that electric field lines start on positive charges and end on negative charges.

Maxwell's second equation (Gauss's law for magnetic fields), illustrated in Figure 10.1 (B), states that for any imaginary closed surface, the number of magnetic field lines exiting the surface is zero. This equation simply describes the concept that magnetic field lines form closed loops and do not begin or end. If a field line enters a closed surface, it will eventually leave the surface. On first considering these equations, they do not appear to carry much significance. However, Maxwell and others after him were able to use the mathematical equations to make very significant predictions. For example, Maxwell showed that accelerating charges radiated energy in the form of electromagnetic waves.

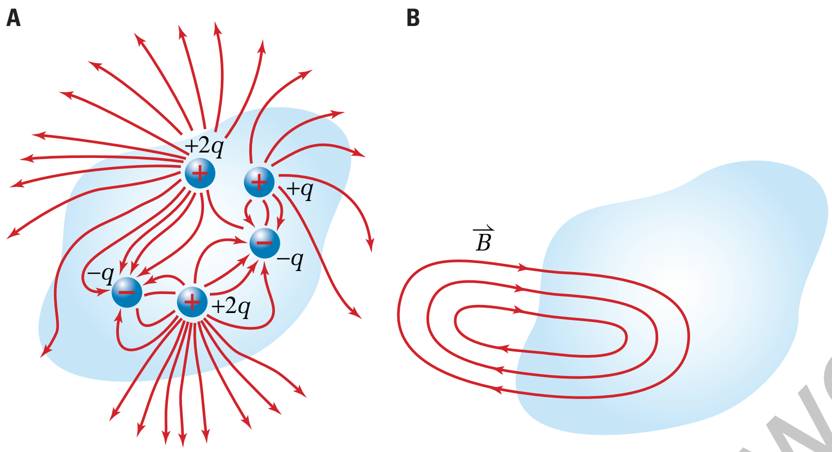


Figure 10.1 **(A)** The number of electric field lines leaving any imaginary closed surface — also called a Gaussian surface — is proportional to the amount of charge enclosed within the surface. **(B)** The number of magnetic field lines entering any imaginary closed surface is equal to the number of magnetic field lines leaving the surface.

Maxwell based his third equation on Faraday's discovery of the generator effect. You will probably recall from previous physics courses that when you move a magnet through a coil of wire, the changing magnetic field induces a current to flow in the coil, as shown in Figure 10.2 (A).

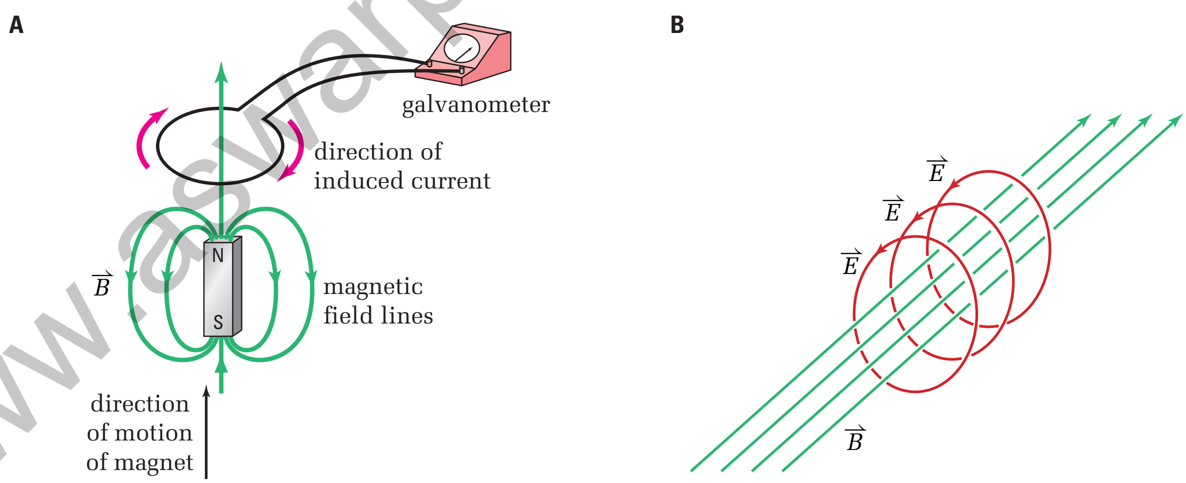


Figure 10.2 The moving magnet causes the magnetic field in and around the coil of wire to change. Since the changing magnetic field causes a current to flow in the wire, it must be generating an electric field in the region of the conductor.

HISTORY LINK

When Gauss was a child in primary school, the teacher punished the class for misbehaving by telling them to add all of the numbers from 1 to 100. The teacher noticed that, while all of the other students were writing vigorously, Gauss was staring out of the window. Then he wrote down a number. Gauss was the only student in the class who had the right answer. When the teacher asked him how he solved the problem, Gauss explained, "When I added 1 and 100, I got 101. When I added 2 plus 99, the answer was again 101. There are 50 of those combinations so the answer had to be 5050." Gauss rapidly became known for his mathematical abilities.

Maxwell expanded the concept to describe the phenomenon even when there was no coil present. To understand how he was able to make the generalization, ask yourself a few questions.

Q: What could cause the charges in the coil to move, making a current?

A: The charges must experience a force to start them moving and to overcome the frictional forces in the wire to keep the charges moving.

Q: If no visible source of a force is present, what might be providing the force?

A: An electric field exerts a force on charges that are placed in it, and if they are able to move, they will move.

These questions and answers lead to the conclusion that an electric field must exist around a changing magnetic field. If a coil is placed in the region, a current will flow. Maxwell's third equation states, in mathematical form, that a changing magnetic field induces an electric field, which is always perpendicular to the magnetic field, as illustrated in Figure 10.2 (B).

Maxwell's fourth equation is based on an observation made by Hans Christian Oersted (1777–1851) that André-Marie Ampère (1775–1836) developed into a law. Oersted observed that a current passing through a conductor produces a magnetic field around the conductor. Once again, Maxwell generalized the phenomenon to include the situation in which no wire was present. Ask yourself some more questions.

Q: What condition must exist in order for a current to flow in a wire?

A: Regardless of its origin, an electric field must exist in the wire in order for a current to flow.

Q: Can the presence of an electric field produce a magnetic field?

A: Since the current produced a magnetic field around the wire, it is probable that it was the electric field driving the current that actually produced the magnetic field.

Through mathematical derivations, Maxwell showed in his fourth equation that a changing electric field generates a magnetic field.

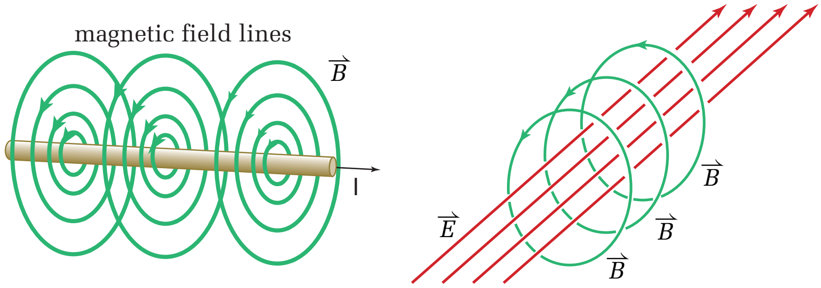


Figure 10.3 Maxwell showed mathematically that a changing electric field that exists in the absence of a conductor will produce a magnetic field around it, in the same way that a current in a conductor will produce a magnetic field.

Electromagnetic Waves

Maxwell's four equations and his excellent mathematical skills gave him exceptional tools for making predictions about electromagnetism. By applying his third and fourth equations, Maxwell was able to predict the existence of electromagnetic waves, as well as many of their properties. Think about what happens when you combine the two concepts — a changing electric field produces a magnetic field, and a changing magnetic field produces an electric field. Imagine that you generate a changing electric field. Initially, there is no magnetic field, so when the changing electric field produces a magnetic field, it has to be changing from zero intensity up to some maximum intensity. This changing magnetic field would then induce an electric field that, of course, would be changing. You have just predicted the existence of an **electromagnetic wave**.

Recall that, by applying his first equation, Maxwell showed that an accelerating charge can radiate energy. The energy that leaves the accelerating charge will be stored in the electric and magnetic fields that radiate through space. A good way to visualize this process is to envision an antenna in which electrons are oscillating up and down, as illustrated in Figure 10.4.

ELECTRONIC LEARNING PARTNER

Refer to your Electronic Learning Partner for a graphic representation of an electromagnetic wave.

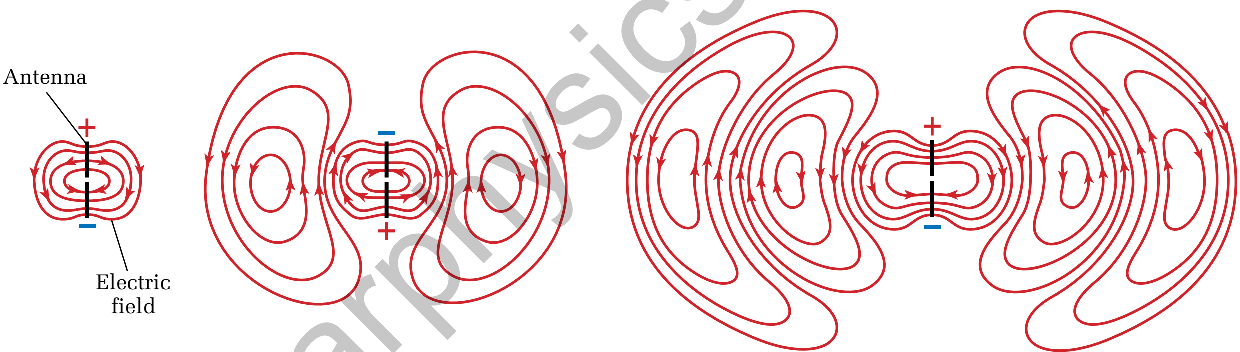


Figure 10.4 Although it is an oversimplification of the concept, it might help you to visualize the formation of an electromagnetic wave if you imagine that when the charges in the antenna reverse direction, the electric field pinches off the antenna.

In the first step, as shown in Figure 10.4, the motion of electrons made the bottom of the antenna negative, leaving the top positive. The separation of charge produced an electric field. As the electrons continued to oscillate, the antenna reversed its polarity, producing another field, with the direction of the field lines reversed from the first. Finally, the electrons moved again, producing a third field. Keep in mind that these events occur in three-dimensional space. Try to visualize each set of loops as forming a doughnut shape, coming out of the page and behind the page.

Although the magnetic field lines are not included in Figure 10.4, the changing electric fields have produced magnetic fields in which the direction of the field lines is always perpendicular to the electric field lines. To visualize these lines without making the image too complex, only one loop of electric field lines is drawn for each step. In Figure 10.5, you can see that the magnetic field lines loop into adjacent electric field lines.

The field lines in the illustrations provided so far show the direction of the electric and magnetic fields, but not their intensity. You can use a diagram, however, to estimate the relative strengths of the fields at any point in space. For example, start with the diagram in Figure 10.4 and draw a horizontal line from the centre of the antenna to the right, as shown in Figure 10.6 (A). As you move to the right of centre, the direction of the field is down. At the point where the two sets of loops meet, there are many field lines, so the field is at its greatest intensity. Farther to the right, the intensity decreases until, at the centre of the loops, it is zero. The field then changes direction and becomes stronger. As the wave propagates out into space, this pattern repeats itself over and over.

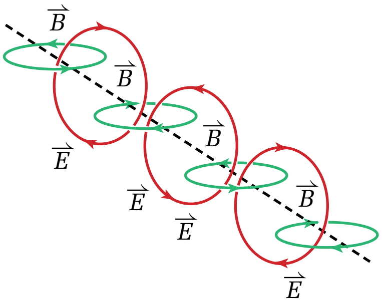


Figure 10.5 Every changing electric field generates a changing magnetic field, and every changing magnetic field generates a changing electric field. The electric and magnetic fields are always perpendicular to each other.

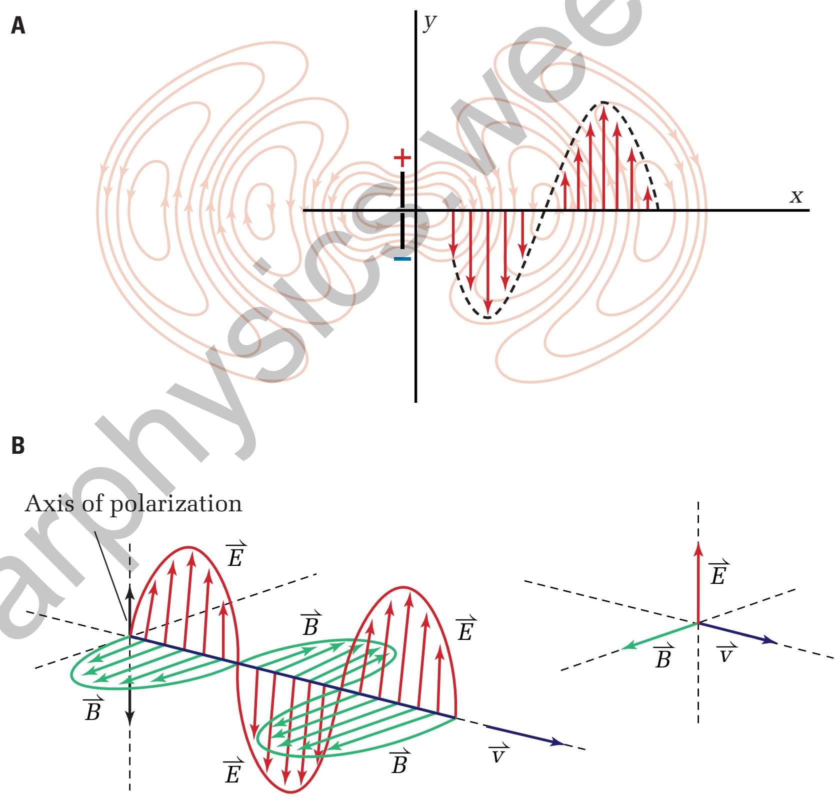


Figure 10.6 The length of each of the red arrows represents the intensity of the electric field at the point at which the base of the arrow meets the *x*-axis. The length of each of the green arrows represents the intensity of the magnetic field at the point at which the base of the arrow meets the *x*-axis.

Part (B) of Figure 10.6 duplicates the electric field lines in part (A) and adds vectors that show the magnetic field intensity. This figure shows the typical diagram for electromagnetic waves. No material objects are moving. The entities that are represented by the waves are the strengths of the electric and magnetic fields. An electromagnetic wave is a transverse wave in which electric and magnetic fields are oscillating in directions that are perpendicular to each other and perpendicular to the direction of propagation of the wave.

Experimental Evidence for Electromagnetic Waves

Although Maxwell correctly predicted the existence of electromagnetic waves and many of their properties, such as speed and the ability to reflect, refract, and undergo interference, he never saw any experimental evidence of their existence. It was not until eight years after Maxwell's death that Heinrich Hertz (1857–1894) demonstrated in his laboratory the existence of electromagnetic waves.

Hertz placed two spherical electrodes close to each other and connected them, through conductors, to the ends of an induction coil that provided short bursts of high voltage. When the voltage between the two electrodes was large enough, the air between them ionized, allowing a spark to jump from one electrode to the other. The momentary spark was evidence of electrons moving between the electrodes. The acceleration of the charges between the electrodes radiated electromagnetic energy away from the source, as predicted by Maxwell's equations.

As illustrated in Figure 10.7, Hertz used a single conducting loop with a second spark gap as a receiver. He verified the creation of electromagnetic waves by observing sparks produced in the receiver. The electromagnetic waves that Hertz produced were in the range of what is now called "radio waves."

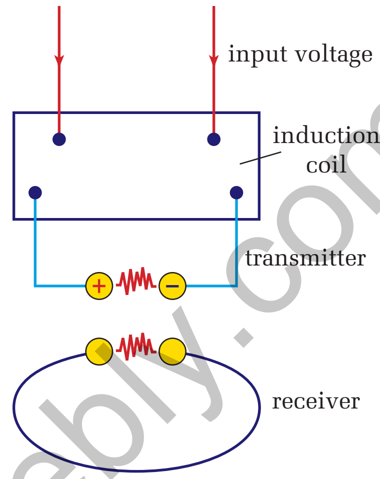


Figure 10.7 Sparks produced at the transmitter by voltage surges generate electromagnetic waves that travel to the receiver, causing a second spark to flash.

The Speed of Electromagnetic Waves in a Vacuum

As Hertz continued to study the properties of electromagnetic waves, he used the properties of interference and reflection to determine the speed of these waves.

Hertz set up a standing wave interference pattern, as illustrated in Figure 10.8. A wave of a known frequency emitted from the source and reflected back on itself, setting up a standing wave pattern. Hertz was able to detect the location of nodal points in the pattern by using a receiving antenna. Using the nodal point locations, he could determine the wavelength.

Hertz then calculated the speed of the wave, using the wave equation $v = f\lambda$. His calculated value for the speed of electromagnetic waves came very close to values of the speed of light that had been estimated and measured by several physicists in the middle 1800s.

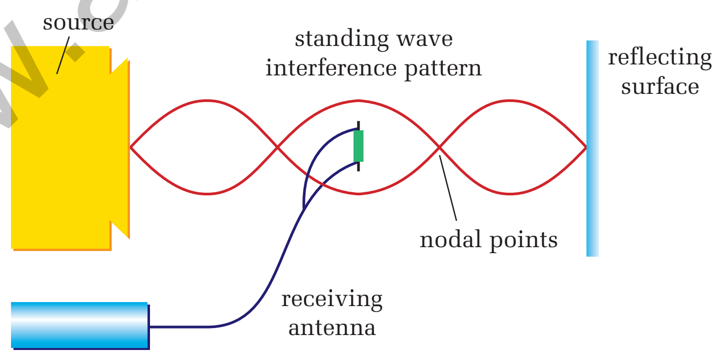


Figure 10.8 When the receiver was at an antinode, as shown here, it detected a strong signal. When it was moved to a node, it detected nothing.

In 1905, Albert A. Michelson (1852–1931) made the most accurate measurement of the speed of light of any that had made previously. In fact, it was extremely close to the value of modern measurements made with lasers. Michelson perfected a method developed by Jean Foucault (1819–1868) and illustrated in Figure 10.9. Michelson set up an apparatus on Mount Wilson in California and positioned a mirror 35 km away. A light source reflected off one side of an eight-sided mirror, then off the distant mirror, and finally off the viewing mirror. The rate of rotation (up to 32 000 times per minute) of the eight-sided mirror had to be precise for the reflection to be seen. By determining the exact rotation rate that gave a reflection and combining that with the total distance that the light travelled, he calculated the speed of light to be 2.997×10^8 m/s.

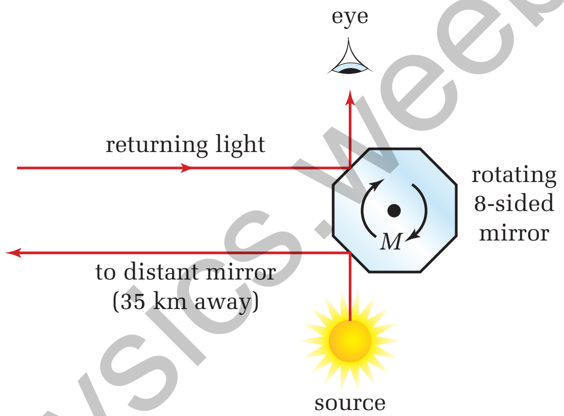


Figure 10.9 Until the laser was developed, Michelson's measurements of the speed of light using an apparatus similar to this were the best measurements of the speed of light that were available.

Maxwell's equations also provided a method for calculating the theoretical speed of electromagnetic waves. The equations include the speed as well as two constants that depend on the way in which the medium through which the waves are travelling affects electric and magnetic fields. The **electric permittivity** (ε) is a measure of the ability of a medium to resist the formation of an electric field within the medium. The constant is directly related to the Coulomb constant in Coulomb's law. The second constant, called the **magnetic permeability** (μ), is a measure of the ability of the medium to become magnetized. When electric and magnetic fields exist in a vacuum, often called "free space," the constants are written with subscript zeros: ε_0 and μ_0. Their values are known to be as follows.

$$\varepsilon_0 = 8.854\ 187\ 82 \times 10^{-12}\ \mathrm{C^2/N \cdot m^2} \quad \text{and} \quad \mu_0 = 4\pi \times 10^{-7}\ \mathrm{N/A^2}$$

Maxwell's equations show that the speed of electromagnetic waves in a vacuum or free space should be given by the equation in the following box.

PHYSICS FILE

Coulomb's law is sometimes written $F_Q = \frac{1}{4\pi\varepsilon}\frac{q_1 q_2}{r^2}$, where $\frac{1}{4\pi\varepsilon} = k$.

QUICK LAB

Producing Electromagnetic Waves

TARGET SKILLS

- Predicting
- Identifying variables
- Analyzing and interpreting

Predict whether the visible sparks between a Van de Graaff generator (set up by your teacher) and a grounded object will produce electromagnetic radiation other than light. Use a portable radio to test for electromagnetic radiation with wavelengths similar to radio waves. Clearly tune in an AM radio station before generating the sparks. Predict how the portable radio will react if the sparks generate radio waves. Generate spark discharges and observe. Repeat for an FM station. Test how the distance between the spark source and the receiver (the radio) affects observed results. If the radio has a movable antenna, test different orientations of the antenna to find out if one orientation has any greater effect than the others.

Analyze and Conclude

1. What theoretical basis exists to suggest that the sparks will produce electromagnetic radiation in the form of both light and radio waves?
2. Does the presence of small electric sparks suggest the acceleration of charged particles? Explain.
3. **(a)** Describe what happened to the portable radio when sparks were produced.
   (b) How do your results verify the production of electromagnetic radiation?
4. In terms of frequency and wavelength, how are AM and FM radio signals different?

SPEED OF ELECTROMAGNETIC RADIATION

The speed of all electromagnetic radiation is the inverse of the square root of the product of the electric permittivity of free space and the magnetic permeability of free space.

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$

Quantity	Symbol	SI unit
speed of light	c	$\frac{\text{m}}{\text{s}}$ (metres per second)
permeability of free space	μ_0	$\frac{\text{N}}{\text{A}^2}$ (newtons per ampere squared)
permittivity of free space	ε_0	$\frac{\text{C}^2}{\text{N}\cdot\text{m}^2}$ (coulombs squared per newton metre squared)

Unit Analysis

$$\frac{1}{\sqrt{\left(\frac{\text{N}}{\text{A}^2}\right)\left(\frac{\text{C}^2}{\text{N}\cdot\text{m}^2}\right)}} = \frac{1}{\sqrt{\left(\frac{\cancel{\text{N}}}{\frac{\cancel{\text{C}^2}}{\text{s}^2}}\right)\left(\frac{\cancel{\text{C}^2}}{\cancel{\text{N}}\cdot\text{m}^2}\right)}} = \frac{1}{\sqrt{\frac{\text{s}^2}{\text{m}^2}}} = \frac{\text{m}}{\text{s}}$$

Note: The symbol c, by definition, represents the speed of light in a vacuum. Since all electromagnetic waves travel at the same speed in a vacuum and light is an electromagnetic wave, it is appropriate to use c for electromagnetic waves in general.

SAMPLE PROBLEM

Speed of Electromagnetic Waves

Use the solution to Maxwell's equations for the velocity of light in free space to determine a numerical value for the speed.

Conceptualize the Problem

- Maxwell's theory predicts the velocity of light to have a theoretical speed, given by $c = \dfrac{1}{\sqrt{\mu_0 \varepsilon_0}}$.
- Free space has a constant value for electric field permittivity.
- Free space has a constant value for magnetic field permeability.

Identify the Goal

The numerical value for the theoretical speed of electromagnetic waves, including light

Identify the Variables and Constants

Known

$\mu_0 = 4\pi \times 10^{-7}\ \dfrac{\text{N}}{\text{A}^2}$

$\varepsilon_0 = 8.854\ 187\ 82 \times 10^{-12}\ \dfrac{\text{C}^2}{\text{N} \cdot \text{m}^2}$

Unknown

c

Develop a Strategy

Use Maxwell's theoretical speed equation for free space.

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$

Substitute in the values and compute the result.

$$c = \frac{1}{\sqrt{\left(4\pi \times 10^{-7}\ \dfrac{\text{N}}{\text{A}^2}\right)\left(8.854\ 187\ 82 \times 10^{-12}\ \dfrac{\text{C}^2}{\text{N} \cdot \text{m}^2}\right)}}$$

$$c = 299\ 792\ 458\ \frac{\text{m}}{\text{s}}$$

The numerical value for the theoretical speed of electromagnetic waves, including light, is 299 792 458 m/s.

Validate the Solution

The speed should be exceptionally fast, which it is. As shown on the previous page, the units cancel to give m/s which is correct for speed.

PRACTICE PROBLEMS

1. News media often conduct live interviews from locations halfway around the world. There is obviously a time-lag between when a signal is sent and when it is received.
   (a) Calculate how long the time-lag should be for a signal sent from locations on Earth separated by 2.00×10^4 km.
   (b) Suggest reasons why the actual time-lag differs from the value in (a).
2. What is the speed of light in water if, in water, $\varepsilon = 7.10 \times 10^{-10}\ \text{C}^2/\text{N} \cdot \text{m}^2$ and $\mu = 2.77 \times 10^{-8}\ \text{N/A}^2$?

Since light and electromagnetic waves all exhibit the properties of reflection, refraction, and interference, and have identical theoretical and experimental speeds in a vacuum, there is no doubt that light is no more than a form of electromagnetic waves that is detected by the human eye.

Figure 10.10 A single ray is split into two as it passes through the calcite crystal.

Polarization of Electromagnetic Waves

Polarized sunglasses eliminate the glare of reflected light from the highway and the hood of a car, while other sunglasses do not. What is unique about polarized lenses? The answer is based on a specific property of electromagnetic radiation including light. Evidence for this property was first reported by Danish scientist Erasmus Bartholinus (1625–1692) in 1669. Although he could not explain what he saw, Bartholinus observed that a single ray of light separated into two distinct rays while passing through a piece of naturally occurring calcite crystal. Figure 10.10 illustrates how a light entering a crystal from only one source (a single hole) splits while travelling through the crystal. Check it out yourself in the Quick Lab that follows.

QUICK LAB

Calcite Crystals

TARGET SKILLS
- Performing and recording
- Analyzing and interpreting
- Communicating results

Obtain a piece of calcite crystal and a piece of cardboard. Poke a small hole in the centre of the cardboard with the tip of a pencil. Place the calcite crystal tightly against the cardboard, with the hole at the crystal's centre. Hold the cardboard-crystal apparatus in front of a light source, with the crystal on the side facing you. Observe the light passing through the small hole into the crystal. Repeat the procedure, placing a single polarizing filter on the back of the card-board, over the hole. Rotate the filter while viewing the light passing into the crystal.

Analyze and Conclude

1. How many dots of light were visible exiting the crystal when the light source was viewed without the polarizing filter?
2. Describe what happened to the light passing through the crystal when the polarizing filter was being rotated.
3. What might cause a ray of light to change path?
   (a) What would happen if the electric field of an electromagnetic wave oriented vertically was able to pass through a substance at a different speed than if it was oriented horizontally?
   (b) Could your results suggest that calcite crystals have a different refractive index for light, depending on the alignment of the electric field of the wave? Explain.

To understand the principles behind polarized lenses and the splitting of a ray of light by calcite crystals, you first need to grasp the concept of polarization. As you know, electromagnetic waves are transverse waves in which both the electric and magnetic fields are perpendicular to the direction of propagation of the wave. However, the electric field might be pointing in any direction within a plane that is perpendicular to the direction of propagation, as illustrated on the left side of Figure 10.11. Make a mental note that, when examining illustrations such this, only the electric field vector is drawn, so a magnetic field exists perpendicular to the electric field.

Polarizing filters, developed in the 1920s, have the ability to selectively absorb all but one orientation of the electric fields in electromagnetic waves, as shown in Figure 10.11. After light or any electromagnetic wave has passed through such a filter, all of the electric fields lie in one plane and the wave is said to be **plane polarized**. The following Quick Lab will help you to understand polarization.

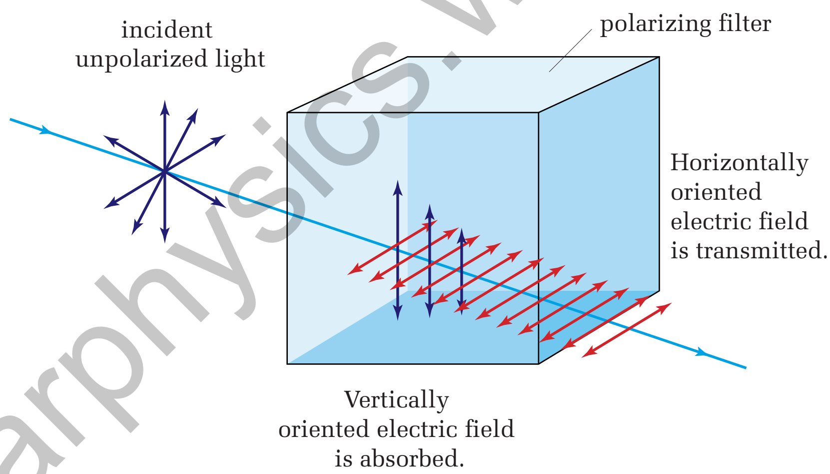


Figure 10.11 Natural light has waves with electric vectors pointing in all possible directions perpendicular to the direction of propagation of the wave. Polarizing filters absorb the energy of the waves that have electric fields in all but one orientation.

QUICK LAB

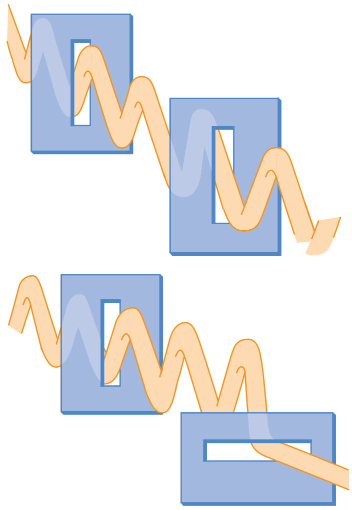
Polarization

TARGET SKILLS
- Performing and recording
- Analyzing and interpreting
- Communicating results

Part A: Modelling Polarization with Rope

A

B

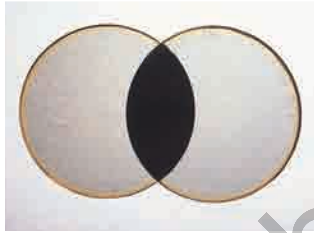


Working in groups of three, practise generating randomly polarized pulses in a length of rope held at both ends. Only one person in each group should generate the pulses. Have the third person insert a board with a horizontal slit cut into it. Observe how the pulses change after passing through the slit in the board. Rotate the board so that the slit is oriented vertically. Again, observe how the pulse changes after passing through the board. Repeat the process again, this time inserting a second board. Observe the pulses when the slits in the boards are both aligned (a) vertically and (b) at 90° to each other.

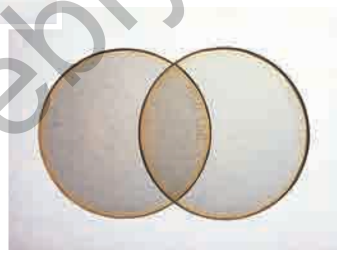
Analyze and Conclude

1. Explain the meaning of "randomly polarized pulses."
2. How do the pulses change as they pass through the **(a)** horizontally and **(b)** vertically oriented slit in the board?
3. What happens to the energy contained in pulses that are not aligned with the slit in the board?
4. Could this experiment be repeated using a spring and longitudinal pulses?

Part B: Polarization of Light



A polarizing filters with axes perpendicular



B polarizing filters with axes parallel

1. Obtain two polarizing filters. Design a simple procedure using both filters to determine whether light can be polarized.
2. Design a simple procedure to determine if reflected light (such as sunlight reflecting off a desktop) is polarized.
3. Observe the sky, preferably on a day with a bright blue sky and some fluffy white clouds. Rotate a single polarizing filter while viewing the sky to determine if the blue light from the sky is polarized.

Analyze and Conclude

1. Hypothesize what is occurring when two polarizing filters are aligned so that they **(a)** allow light through and **(b)** block all of the light.
2. Did you find any evidence for the polarization of reflected light? Explain.
3. **(a)** Describe how the image of the blue sky and white clouds changes as the polarizing filter is rotated.
   (b) Based on your observations, determine whether the blue light of the sky polarized.

You now know how polarized lenses affect light, but how do they exclusively absorb glare from the light that is reflected from a road surface or the hood of a car? When light strikes a surface such as a street or pool of water, the electric fields that are perpendicular to that surface are absorbed and the parallel or horizontal electric fields are reflected. Therefore, reflected light is polarized.

As illustrated in Figure 10.12, polarized lenses in sunglasses are oriented so that they allow only vertical electric fields to pass through and thus absorb most of the horizontally polarized reflected light. Figure 10.13 shows photos taken with and without a polarizing filter. The fish beneath the water's surface is clearly visible when the bright glare, consisting mainly of horizontally polarized light, is removed.

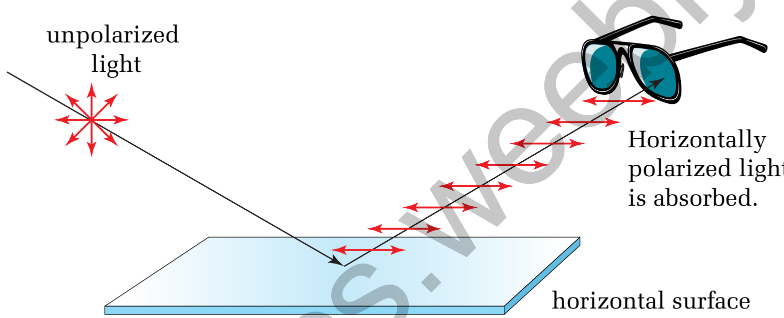


Figure 10.12 Since glare is caused by reflected light that is horizontally polarized, sunglasses with polarized lenses can eliminate glare by allowing only vertically polarized light to pass through.

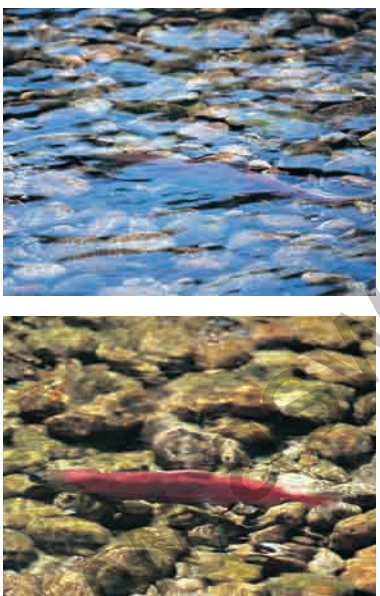


Figure 10.13 Bright sunlight reflecting off the surface of water creates a lot of glare, preventing you from seeing objects below the surface. Polarized filters allow you to clearly see the fish swimming in this pond.

How can the phenomenon of polarization explain the ability of calcite crystals to split a beam of light into two beams? Again, ask yourself some questions.

Q: What happens to light when it passes from one medium, such as air, into another medium, such as a calcite crystal?

A: Light appears to bend or refract, because the speed of light is different in the two different media.

Q: What determines the extent of bending or refraction of the light?

A: The ratio of the indices of refraction of the two media determines the angle of refraction of light. The speed of light in a medium determines its index of refraction.

Q: How can a single crystal refract a single beam of light at two different angles?

A: The crystal must have two different indices of refraction for different properties of light.

Q: How can a beam of light have different properties?

A: Natural light has electric fields pointing in different directions.

Crystals, in general, are very orderly structures. The compounds in calcite are uniquely oriented so that the speed of light polarized in one direction is different than the speed of the light polarized perpendicular to the first. As a result, a beam of light is split into two beams, because light polarized in different planes refracts to different extents. Substances such as calcite, which have a different refractive indices depending on the polarization of the light, is said to be doubly refractive.

Certain materials, such as Lucite™, exhibit double refractive properties when under mechanical stress. The stress causes molecules in the material to align and behave similar to the orderly compounds in calcite crystals. Such materials are said to be **photoelastic**. Figure 10.14 demonstrates stress patterns that become visible in the Lucite™ when it is placed between polarizing and analyzing filters.

Mechanical stress changes the refractive index of Lucite™. As the level of stress varies in a sample, so does the amount of refraction. The plane of polarization of incident plane polarized light will be rotated it travels. A changing refractive index also means that the speed of propagation will be different for each electric field orientation. Light reaching an analyzer — a second polarizing filter — will be polarized in a different plane and form a pattern highlighting the stresses in the sample.

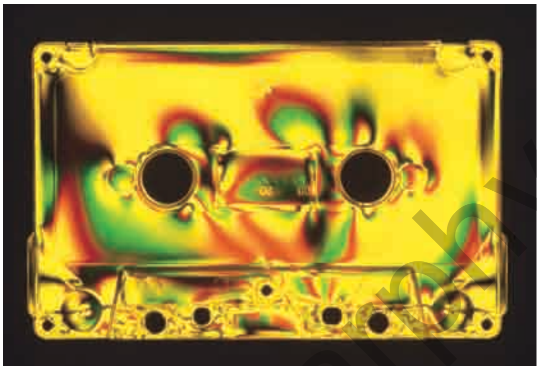


Figure 10.14 Lucite™ sandwiched between polarizing and analyzing filters yields an interference pattern showing the distribution of mechanical stress.

It is possible to produce reflective photoelastic coatings that are painted onto solid objects, allowing engineers to monitor mechanical stress and potential areas of failure. This technique is used to analyze materials for otherwise undetectable cracks and flaws.

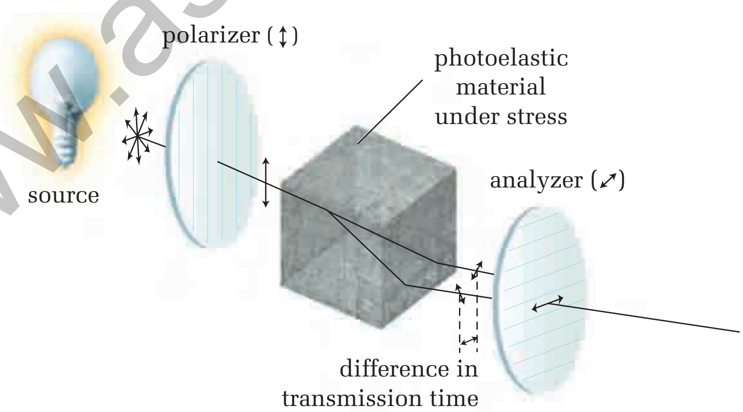


Figure 10.15 The refractive indices of doubly refractive photoelastic material varies under mechanical stress, producing interference patterns used to detect flaws.

Reflection and Absorption of Electromagnetic Waves

Light reflecting from a mirror is a common experience. So is the presence of satellite dishes used for satellite television. The satellite dishes are often a grey colour and are not nearly as smooth to the touch as a mirror. It might seem strange that the satellite dishes are not made of shiny, highly reflective material to help capture and reflect the radio waves for the receiver. In fact, although the grey coloured dishes are not highly reflective to light, they are highly reflective to radio waves. The amount of energy that is reflected depends on the wavelength of the incident wave and the material it is striking.

Light reflects off a mirror. Is the wavelength of light smaller or larger than the atoms that make up a mirror? The atoms need to be much smaller than the wavelength of light; otherwise, the mirror surface would appear bumpy. A tiny scratch in the mirror is easily visible, because it is much larger than the wavelength of light. Theoretically, if atoms were larger than the wavelength of light, it would be impossible to make a mirror that acted as a good reflector.

Conceptual Problem

- Would a mirror designed to reflect longer wavelength infrared radiation need to be smoother than a mirror designed to reflect shorter wavelength ultraviolet radiation?

Radio telescopes work by reflecting radio waves to a central receiver. The radio waves have wavelengths in the order of several metres. Therefore, the reflecting dishes can be constructed of conducting material, such as metallic fencing. To the long wavelengths, the fencing would act as a smooth surface. In general, the shorter the wavelength, the smoother the reflecting surface must be.

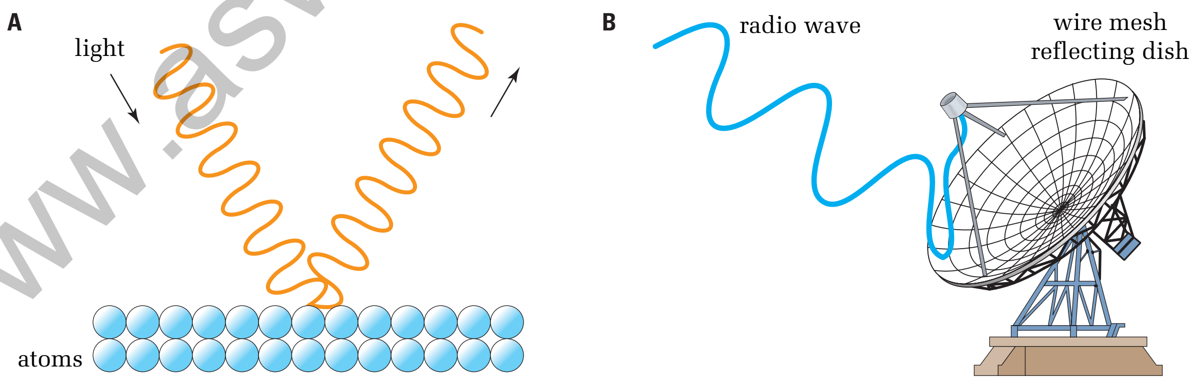
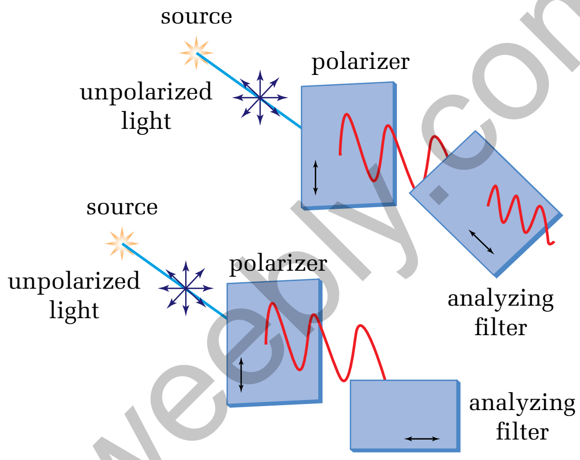


Figure 10.16 A mirror is a smooth reflector for light in the same way that wire mesh is a smooth reflector for long wavelength radio waves.

10.1 Section Review

1. K/U Qualitatively explain the meaning of Maxwell's third and fourth equations.
2. K/U What did Maxwell predict would be necessary to generate an electromagnetic wave?
3. K/U How did Hertz verify Maxwell's theory of electromagnetic waves?
4. K/U What is relationship in space between the electric and magnetic fields in an electromagnetic wave?
5. C Describe the process that allows an electromagnetic wave to exist as it radiates away from the source that created it.
6. C Describe the meaning of the terms "electric permittivity" and "magnetic permeability."
7. C Describe the apparatus that Michelson used to measure the speed of light.
8. C Describe one mechanism by which light is polarized in nature.
9. MC Assume that you are wearing polarized sunglasses while driving a car. You come to a traffic light and stop behind another car. You see that the rear window of the car ahead has a distinct pattern of light and dark areas. Explain.
10. C Define the term "photoelastic." Explain how light interacts with a photoelastic material.
11. C As illustrated in the diagram, describe the process involved as light travels from the source to the analyzing filter.



UNIT PROJECT PREP

Radio frequencies of the electromagnetic spectrum spread information around the globe at the speed of light.

- Investigate the relationship between a signal's wavelength and the length of transmitting and receiving antennas.
- Is there a relationship between the frequency of a radio signal and the range over which it might be received?
- If you could select the frequency at which your transmitter will operate, what frequency would you chose? Explain.

10.2 The Electromagnetic Spectrum

SECTION EXPECTATIONS

- Define and explain the concepts and units related to the electromagnetic spectrum.
- Describe technological applications of the electromagnetic spectrum.
- Describe and explain the design and operation of technologies related to the electromagnetic spectrum.
- Describe the development of new technologies resulting from revision of scientific theories.

KEY TERMS

- electromagnetic spectrum
- triangulation

When Hertz designed and carried out his experiments, his only intention was to test Maxwell's theories of electromagnetism. He had no idea that his success in generating and detecting electromagnetic waves would influence technology and the daily lives of the average citizen. Today, with electromagnetic radiation, people talk on cellphones, watch television that is receiving signals from satellites, and diagnose and treat disease.

There is literally no limit to the possible wavelengths and, consequently, the frequencies that an electromagnetic wave could have. Wavelengths of electromagnetic waves as long as hundreds of kilometres (10^3 Hz) to less than 10^{-13} m (3×10^{21} Hz) have been generated or detected. The **electromagnetic spectrum** has arbitrarily been divided into seven categories, based in some cases on historical situations or by their method of generation. In fact, some of the categories overlap. The following is a summary of the generation and applications of these seven categories of electromagnetic waves.

Radio Waves

By far the broadest electromagnetic wave category comprises radio waves, ranging from the longest possible wavelength or lowest frequency to about a 0.3 m wavelength or a frequency of 10^9 Hz. Radio wave frequencies are easily generated by oscillating electric circuits. They are broadcast by antennas made of electric conductors in which charges oscillate, as illustrated in Figure 10.4 on page 425. Radio waves are divided into subcategories by governments to restrict the use of certain ranges of waves to specific purposes.

Extremely low-frequency communication — 3 to 3000 Hz — is reserved for military and navigational purposes. Submarine-to-shore communications use the lowest of these frequencies when deeply submerged. Electromagnetic signals travelling through salt-water are absorbed, making communication with a deeply submerged vessel difficult. The very low frequencies are better able to penetrate the salt-water than are higher frequencies. However, such low frequencies have extremely long wavelengths that require very long antennas. In order to use these frequencies, submarines drag behind the ship a cable that can be as long as several hundred metres, to act as an antenna. Above ground, the transmission antenna consists of more than 140 km of suspended cabling.

Amplitude modulated (AM) radio — 535 to 1700 kHz — was the first widely used type of radio communication in the early part of the twentieth century. The range of frequencies that constitute the AM band was chosen arbitrarily, based primarily on the ability of the technology to generate the signals.

An AM radio station is assigned a specific "carrier" frequency on which to transmit signals. The information is carried by increasing and decreasing (modulating) the amplitude of the wave. For example, if the information is in the form of a voice or music, a microphone converts the sound waves into electric signals that are then combined or mixed with the carrier wave, as shown in Figure 10.17. A radio receiver picks up the signals by tuning an oscillating circuit in the instrument to the same frequency as the carrier frequency and then amplifies that wave. The electronic circuitry filters out the carrier wave and the "envelope" wave drives a speaker, converting the electric signal back into sound.

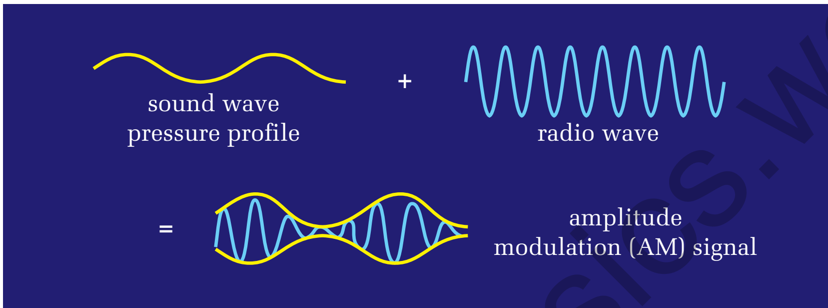


Figure 10.17 The radio wave in this figure is the carrier wave that is broadcast by the radio station. The term "modulation" refers to the changes in the amplitude of the carrier wave to match the sound wave, which contains information such as music, spoken words, and special effects.

The greatest problem with AM reception is that many machines and instruments emit random electromagnetic waves over a broad range of radio frequencies. For example, electric motors, automobile ignitions, and lightning bolts emit random "noise" signals that add to the amplitude of many AM waves. On a nearby AM radio, the signals will be picked up as static.

Short–wave radio is a range of frequencies — 5.9 MHz to 26.1 MHz — reserved for individual communication. Before satellite communications and cellular telephones, many people, called "ham operators," built their own transmitters and receivers and communicated with other ham operators around the world. Novice operators were licensed to transmit only Morse code, but they usually advanced quite rapidly and obtained licences to transmit voice. Occasionally, ham operators were the only people listening for signals when a boat or downed airplane was sending SOS calls. Ham operators were responsible for saving many lives.

Citizens' band (CB) radio frequencies — 29.96 MHz to 27.41 MHz — are reserved for communication between individuals over very short distances. The range of frequencies is divided into 40 separate channels. Because licences restrict the power of CB radios, they cannot transmit over long distances, so many people can use

PHYSICS FILE

Your calculator might be a radio wave transmitter. Turn your AM radio dial to a very low frequency and make sure that it is between stations so that there is very little sound. Turn on your electronic calculator, hold it very close to the radio, and press various calculator buttons. You might be able to play a tune on your radio.

the same band at the same time, because the ranges do not overlap. Before the advent of cellphones and other more sophisticated wireless communication systems, CB and walkie-talkie communication provided a link between homes, businesses, and people travelling in vehicles. CB radio almost created a subculture and a language among truck drivers and other long-distance travellers, who used CB radios to pass the time.

Cordless telephones use a range of frequencies between 40 and 50 MHz. The range of a cordless telephone is designed for use in a home, and is shared with garage door openers and home security systems close to 40 MHz and baby monitors close to 49 MHz. The possibility of receiving a telephone conversion from a cordless telephone over a baby monitor exists, although it is unlikely, due to the very low power of both a telephone and a baby monitor. Some cordless telephones are also designed to operate close to 900 MHz.

Television channels 2 through 6 are broadcast in the 54 to 88 MHz frequency range, which lies just below FM radio. Channels 7 through 13 are broadcast over a frequency band between 174 to 220 MHz, which lies just above FM radio. These TV signals are broadcast signals that can be received only with a TV aerial. Cable and satellite signals are quite different. The method of transmission and reception are essentially the same as radio. The television picture is transmitted as an AM signal and the sound is transmitted as an FM signal.

Wildlife tracking collars use some of the same frequencies that are used by television. Understanding complex ecological interactions sometimes involves tracking wild animal populations over long distances. Canada has become a world leader in the design and manufacturing of wildlife tagging and tracking technology.

The animal in Figure 10.18 is a cheetah, a member of an endangered species. It is wearing a tracking collar that emits an electromagnetic signal of a specific frequency, allowing researchers to follow the animal's day-to-day movements. Similar systems have been developed for various climates and conditions, including cold Arctic climates and underwater environments.

Figure 10.18 Wildlife researchers track individual animals belonging to endangered species to learn about their behaviour, in an attempt to find ways to prevent extinction of their species.

In the past, animals were tagged with identification bands. If the same animal was captured again, migration patterns could be deduced. The process required tagging a very large sample of animals and data accumulation was slow, since the recapture of tagged animals could not be guaranteed. Microchip tags on the tracking collars have dramatically increased wildlife tracking research capabilities, providing sample and log data such as ambient temperature, light, and underwater depth, as well as transmitting a tracking signal. Similar, although less sophisticated, tags are used by pet owners to identify their animals. Pet identification tags store information about the pet and its owners. The tag is inserted under the animal's skin, where it will remain for life. Placing a receiving antenna near the tag retrieves the data.

Frequency modulated (FM) radio transmits frequencies from 88 MHz to 108 MHz. FM radio was introduced in the 1940s to improve the sound quality by reducing the static that is common on AM radio. FM radio eliminates static, because a change in the amplitude of the wave has no effect on the signal. An FM carrier wave has a single frequency, as does the AM carrier. However, as shown in Figure 10.19, information is carried in the form of slight increases and decreases in the frequency of the carrier. FM frequencies are absorbed more easily by the atmosphere than are AM frequencies, so the range of an FM station is shorter than an AM station.

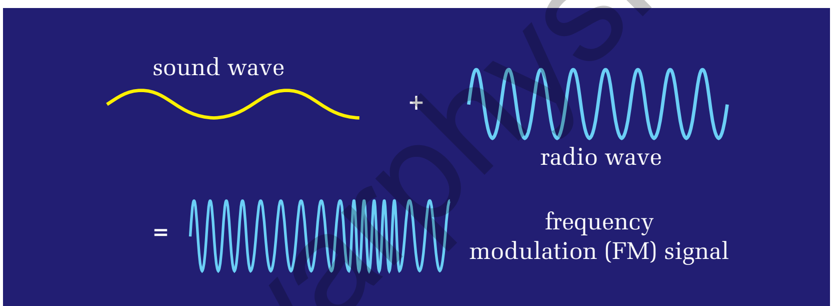


Figure 10.19 The amplitude of a carrier wave from an FM radio station never changes. In fact, if external noise from a nearby machine increases the amplitude, the radio receiver crops off the waves, accepting only a constant amplitude. The sound signal modulates, or varies, the frequency of the carrier wave.

The transmission of all radio wave frequencies is called a "line of sight" transmission, because radio waves are absorbed by the ground. Radio waves do of course penetrate walls and objects that are not extremely thick and dense, but cannot penetrate large amounts of matter. Sometimes, however, you might pick up a radio station at a distance greater than line of sight. The explanation for this phenomenon is the ability of the ionosphere to reflect radio waves, as shown in Figure 10.20. The ionosphere is a layer of charged atoms and molecules in the upper atmosphere that is created by high-energy radiation from the Sun stripping electrons from the gases. At night, the altitude of the ionosphere increases and allows signals to travel farther than during the day. You might have noticed that you can pick up radio stations at night that you cannot receive during the day.

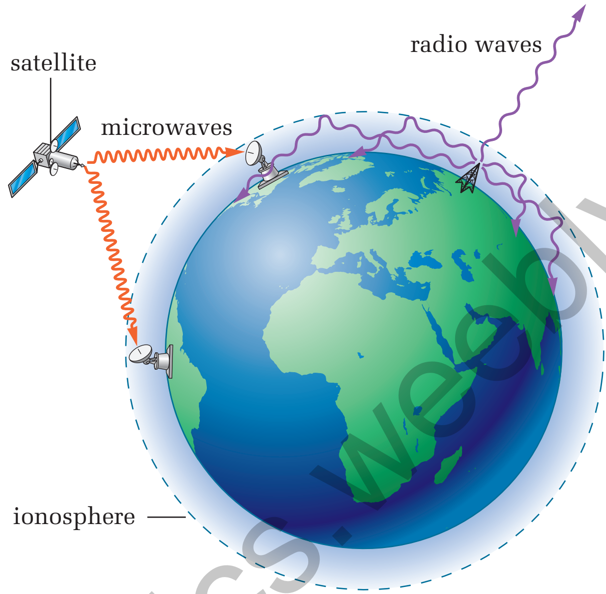


Figure 10.20 The longer wavelength radio waves reflect from the ionosphere, and most of the waves return to Earth. Although radio waves cannot be used for satellite communications, they can travel farther around Earth's surface than can the shorter microwaves. In contrast, shorter microwaves are unaffected by the ionosphere and can therefore be used in satellite communications.

The ionosphere is most effective in reflecting short-wave radio signals. Multiple reflections of short-wave radio signals between the ground and the ionosphere allow signals to travel over incredibly long distances. Ham operators are often able to communicate with others halfway around the world.

Magnetic resonance imaging (MRI) is a unique application of radio waves that is used to diagnose certain types of illnesses and injuries. MRI provides incredible detail in the study of nerves, muscles, ligaments, bones, and other body tissues by using electromagnetic signals to create image "slices" of the human body.

The largest component of an MRI system is a powerful magnet, with a tube called the "bore" running horizontally through the magnet. The patient slides on a special table into the bore. The magnetic field interacts with the nucleus of atoms of hydrogen, because these nuclei behave like tiny magnets that align themselves in the field. A pulse of radio waves is absorbed by the hydrogen, causing the hydrogen atoms to "flip" and become aligned against the external magnetic field. When the electromagnetic pulse stops, the atoms relax back to their original alignment and release absorbed energy in the form of electromagnetic waves. Sensors detect the emitted waves and send signals to a computer system that converts the electrical signals into a digital image that can be put on film.

Microwaves

Microwaves ranging from 1.0×10^{10} Hz to 3.0×10^{11} Hz have such high frequencies that there is no electronic circuitry capable of oscillating this fast. Research into the development of a device able to generate microwaves was stimulated by the development of radar. Physicist Henry Boot and biophysicist John T. Randall, both British scientists, invented an electron tube called the "resonant-cavity magnetron" that could produce microwaves. Similar tubes, called Klystron™ tubes, are the two main devices that generate microwaves today. The first application of microwave was, of course, radar, which has revolutionized safety in aviation as well as detecting weather data around the world.

Soon after the technology to generate microwaves was developed, the number of applications grew rapidly. Microwave ovens generate microwaves that have a frequency of 2450 MHz, which is close to the natural frequency of vibration of water molecules. As a result, these microwaves are efficiently absorbed by the water molecules in food, causing a dramatic increase in temperature, which cooks the food.

Microwaves have revolutionized communications for one fundamental reason — in contrast to radio waves, microwaves penetrate the ionosphere, making them useful for space-based communication. Any location, anywhere on Earth, can be reached by satellite communication.

The Global Positioning System (GPS), for example, makes it possible to determine your location and altitude anywhere on Earth through the use of geostationary satellites. Global positioning satellites, originally part of the military infrastructure, provide businesses, rescue workers, and outdoor enthusiasts with instantaneous position and tracking data. The global positioning satellite network consists of 24 geostationary satellites that are in constant communication with each other and with several ground stations.

Triangulation is the basis of the GPS. Figure 10.21 demonstrates how you could locate your exact position if you knew how far you were from three points. For example, if you knew you were exactly 65 km from Toronto, you could be anywhere in a circle with a 65 km radius around Toronto. If you also knew that you were 194 km from Windsor, you could now determine your

PHYSICS FILE

Invented by the British and shared with the U.S. military while the World War II Battle of Britain raged in 1940, the resonant-cavity magnetron was described as being capable of generating "ten kilowatts of power at ten centimetres, roughly a thousand times the output of the best U.S. [vacuum] tube on the same wavelength." From this realization flowed numerous developments, including gun-laying radar, radar-bombing systems, and air-intercept radar, as well as the first blind-landing system. As most veterans of the "Rad Lab" came to believe: "The atomic bomb only ended the war. Radar won it."

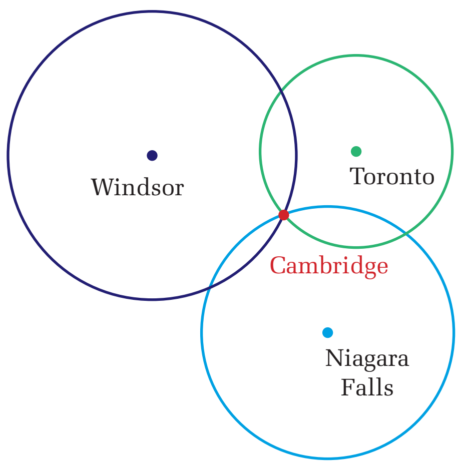


Figure 10.21 Circles generated from three known distances intersect at only one point.

location as being one of two sites where the two circles intersect. Knowing a third measure, such as that you were 79 km from Niagara Falls, you could pin-point your exact location. All three circles intersect at only one point, revealing your location to be downtown Cambridge. Global positioning technology takes this process one step further, using four spheres instead of three circles. This allows a location to be determined in three-dimensional space, including altitude.

GPS satellites continually send radio signals between each other and to Earth. The network of 24 satellites ensures that no matter where you are on Earth, at least four satellites will have a direct line of sight to your position. Hand-held GPS receivers measure the amount of time required for a microwave signal to travel from each satellite. The receivers are then able to calculate the distance from each satellite, knowing the speed of the signal ($c = 3.00 \times 10^8$ m/s). GPS hand-held systems are effective because they are an inexpensive and accurate method to determine the time the signal took to travel from the satellite to the receiver.

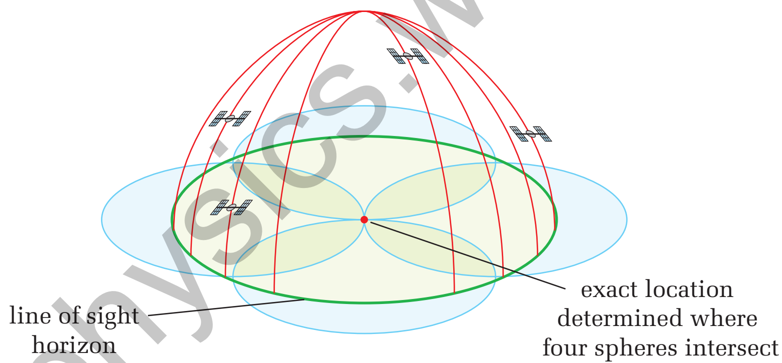


Figure 10.22 GPS receivers determine exact location by computing a single point at which imaginary spheres from each satellite intersect.

GPS technology has been made possible only through the merging of several branches of science and engineering. For example, complex mathematical models are used to calculate the speed of electromagnetic signals through our continually changing atmosphere and ionosphere. Atomic clocks on board each satellite are a product of research in atomic physics. Aerospace and rocketry advances rely on chemistry and physics.

Infrared Radiation

Infrared radiation with frequencies from 3.0×10^{11} Hz to 3.85×10^{14} Hz lies between microwaves and visible light. Infrared radiation was accidentally discovered in 1800 by Sir William Herschel (1738–1822). He was separating the colours of the visible spectrum and measuring the ability of different colours to heat the objects that were absorbing the light. He was very surprised when he placed his thermometer just beside the red light, where no visible light was falling, and discovered that the thermometer showed the greatest increase in temperature. He rightly concluded that there was some form of invisible radiant energy just beyond red light.

Any warm object, including your body, emits infrared radiation. The natural frequency of vibration and rotation of many different types of molecules lies in the infrared region. For this reason, these molecules can efficiently absorb and emit infrared radiation.

The ability to detect infrared radiation has led to several varied applications, from electronic night-scopes, which convert the heat of an animal or person into a visible image, to satellite imaging able to "see" through clouds to gather information related to the health of vegetation or the hot spots of a forest fire.

COURSE CHALLENGE

How Far Can It Go?

Scientific discoveries breed new applications with capabilities once unimagined. Page 604 of this text provides suggestions for you to consider for your *Course Challenge*.

Visible Light

Visible light encompassing frequencies between about 3.85×10^{14} Hz and 7.7×10^{14} Hz is defined as light, simply because the human eye is sensitive to electromagnetic waves within this range. In some applications, it is more common to refer to the wavelength than the frequency, so you might see the range of light waves reported as encompassing wavelengths between 400 and 700 nm.

Light technologies are too numerous to mention. You probably know more about light than any other range of the electromagnetic spectrum, because entire units in your previous science courses were based on the properties of light. Visible light is emitted from all very hot objects, due to excited electrons in molecules dropping down to lower energy levels and emitting light energy.

Ultraviolet Radiation

Ultraviolet (UV) radiation, with frequencies between 7.7×10^{14} Hz and 2.4×10^{16} Hz, has the ability to knock valence electrons free from their neutral atoms — a process known as "ionization." When electrons drop from very high energy levels in atoms to much lower levels, UV radiation is emitted. Ionization is responsible for creating the ionosphere around the globe. UV radiation activates the synthesis of vitamin D in the skin, which is very important to your health. However, large quantities of UV radiation can cause skin cancer and cataracts. Ultraviolet wavelengths are used extensively in radio astronomy.

UV radiation was discovered just one year after infrared radiation was discovered and also by "accident." J. Ritter was studying the ability of light to turn silver chloride black by releasing metallic silver. He discovered that when silver chloride was placed just beyond the violet light in a spectrum of sunlight created by a prism, it was blackened even more efficiently than when exposed to the blue or violet light.

X Rays

X rays with frequencies between 2.4×10^{16} Hz and 5.0×10^{19} Hz have great penetrating power and are very effective in ionizing atoms and molecules. X rays can be produced when electrons in outer shells of an atom fall down to a very low, empty level. Commercial generators produce X rays by directing very high energy electrons that have been accelerated by a high voltage onto a solid metal surface inside a vacuum tube as shown in Figure 10.23. When the electrons are abruptly stopped by the target electrode, X rays are emitted. Such X ray generators are used extensively in medical applications and in industry for material inspection.

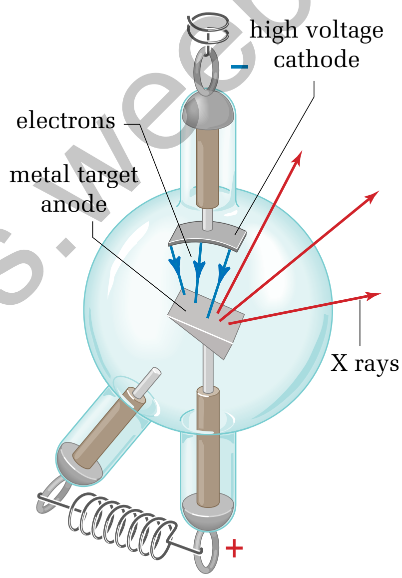


Figure 10.23 An X-ray tube must be evacuated so that the high-energy electrons are not scattered by gas molecules. The X rays produced when the electrons collide with the target and are suddenly stopped are sometimes called "Bremsstrahlung," which means "braking radiation."

X-ray images of the Sun can yield important clues about solar flares and other changes on the Sun that can affect space weather.

MISCONCEPTION

Cosmic Rays Are *Not* Rays!

Cosmic rays are not rays at all, but rather are high-energy particles ejected from stars, including the Sun, during solar flares. These particles, although they travel at very high speeds, do not travel at the speed of electromagnetic radiation. A gamma ray emitted by the Sun will arrive at Earth in approximately 8 min, whereas high-energy particles referred to as "cosmic rays" can take between several hours to several days to travel from the Sun to Earth.

Gamma Rays

Gamma rays are the highest frequency, naturally occurring electromagnetic waves, with frequencies ranging from 2.4×10^{18} Hz to 2.4×10^{21} Hz. Gamma rays are distinguished from X rays only by their source: Whereas X rays are produced by the acceleration and action of electrons, gamma rays are produced by the nuclei of certain atoms. Just as electrons in atoms can become excited by the absorption of energy, the nucleus of an atom can also be excited. An atom with an excited nucleus is said to be "radioactive." When an excited or radioactive nucleus releases energy to drop down to a more stable state, it releases gamma rays.

Due to their great penetrating and ionizing power, gamma rays are sometimes used to destroy malignant tumors deep inside the body. Radioactive atoms are also used extensively in research to track and identify specific elements. Gamma ray images of our universe provide information on the life and death of stars and on other violent processes in the universe. Gamma rays are extremely energetic and can be very harmful to life.

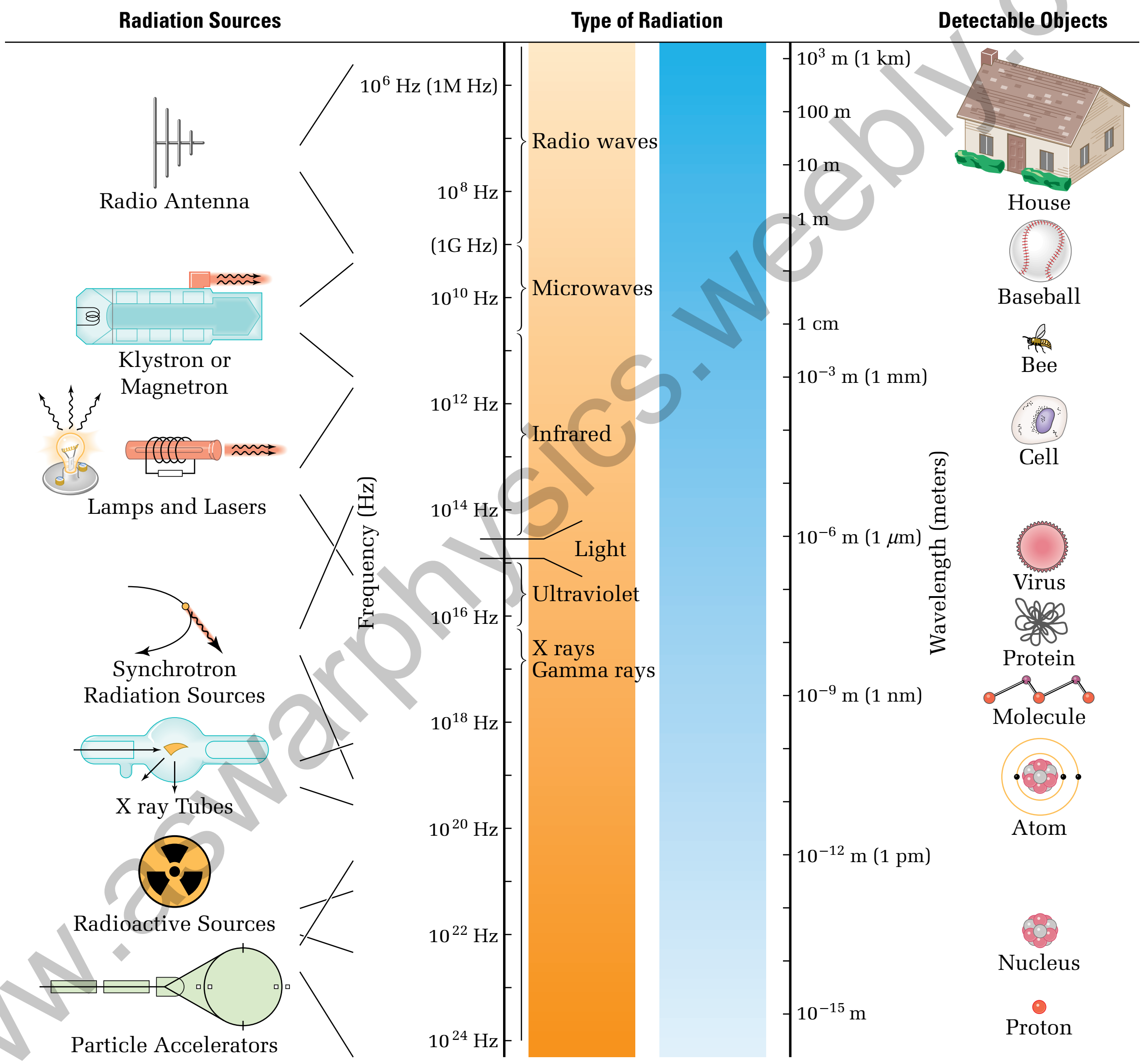


Figure 10.24 The electromagnetic spectrum is continuous throughout a range of frequencies covering more than 18 orders of magnitude (powers of 10). The subdivisions are artificial, but give scientists a way to quickly communicate the range of electromagnetic waves and the general properties of the waves of interest.

Conceptual Problems

- Describe which frequency range is best for long-distance communication. Explain.
- Suggest why gamma rays penetrate farther into matter than UV, despite the generalization that longer wavelengths have greater penetration power.

Electromagnetic Waves and the Wave Equation

The wave equation that you learned while studying mechanical waves, $v = f\lambda$, also applies to electromagnetic waves. Since electromagnetic waves always travel with the same speed in a vacuum, the wave equation can be expressed in terms of c instead of v. Since the speed of electromagnetic waves in air is almost identical to their speed in a vacuum, this equation can be used for air as well as for a vacuum or free space.

ELECTROMAGNETIC WAVE EQUATION

The speed of electromagnetic waves is the product of their frequency and wavelength.

$$c = f\lambda$$

Quantity	Symbol	SI unit
speed of electromagnetic radiation in a vacuum	$c = 3.00 \times 10^8$ m/s	$\frac{\text{m}}{\text{s}}$ (metres per second)
frequency	f	Hz (hertz or $\frac{1}{\text{s}}$)
wavelength	λ	m (metres)

Unit Analysis

$$(\text{hertz})(\text{metre}) = \left(\frac{1}{\text{s}}\right)(\text{m}) = \frac{\text{m}}{\text{s}}$$

Note: The velocity of all electromagnetic radiation in a vacuum is denoted as c. However, electromagnetic radiation travelling through air is slowed only slightly, and therefore the value of c is often used to approximate velocity values in air as well.

SAMPLE PROBLEM

Radio Waves

An FM radio station broadcasts at a frequency of 2.3×10^8 Hz. Determine the wavelength of the FM radio waves from this station.

Conceptualize the Problem

- *Radio wave* transmission is a form of *electromagnetic* radiation.
- The *speed* of electromagnetic radiation in air is approximated as c.
- *Electromagnetic waves* can be described by using the *wave equation*.

Identify the Goal

The wavelength of 2.3×10^8 Hz electromagnetic waves

Identify the Variables and Constants

Known	Implied	Unknown
$f = 2.3 \times 10^8$ Hz	$c = 3.00 \times 10^8 \frac{\text{m}}{\text{s}}$	λ

Develop a Strategy

Use the wave equation.

$$c = f\lambda$$

Manipulate the equation, solving for wavelength.

$$\lambda = \frac{c}{f}$$

Substitute and solve.

$$\lambda = \frac{3.00 \times 10^8}{2.3 \times 10^8 \text{ Hz}}$$

$$\lambda = 1.3043 \frac{\frac{\text{m}}{\text{s}}}{\text{s}^{-1}}$$

$$\lambda \cong 1.3 \text{ m}$$

The wavelength of 2.3×10^8 Hz electromagnetic radiation is 1.3 m.

Validate the Solution

Both the speed and the frequency were in the order of 10^8, which would suggest that the answer should be near unity, which it is.

PRACTICE PROBLEMS

3. (a) Determine the wavelength of an AM radio signal with a frequency of 6.40×10^6 Hz.

(b) Suggest why AM radio transmitting antennas are hundreds of metres tall.

4. Microwave oven doors have metallic screens embedded in them. Light is able to pass through these screens, but the microwaves are not. Assume that the microwave radiation is in the order of 10^{10} Hz and the light in the order of 10^{14} Hz.

(a) Calculate the wavelengths of both the microwave radiation and visible radiation.

(b) Suggest why a metallic screen is used in microwave oven doors.

10.2 Section Review

1. K/U What is the basis for naming the categories within radio waves?
2. C Explain the difference between frequency modulation (FM) and amplitude modulation (AM).
3. K/U Why does AM radio exhibit much more static than FM radio?
4. K/U Why are microwaves used for satellite communications rather than radio waves?
5. K/U How do microwaves "cook" food?
6. K/U Describe the difference between ionizing and non-ionizing electromagnetic radiation.
7. K/U In many spectra, you will see an overlap of X rays and gamma rays. What distinguishes X rays from gamma rays?
8. C Explain how GPS can help you to locate your position.
9. MC How has the application of radio tracking collars impacted wildlife research?
10. MC Magnetic resonance imaging (MRI) is not always a safe option for some patients. Suggest possible reasons that might make an MRI scan unsafe for a patient. Support your suggestions with Internet research.

UNIT PROJECT PREP

Your FM transmitter will transform your words into electromagnetic radiation to be received and heard on a typical portable radio.

- How are electromagnetic waves produced?
- Investigate the factors that determine the quality of a transmitting antenna.
- Think about the differences and similarities between the electromagnetic signal Hertz first sent and received to those used today.

CHAPTER 10 Review

REFLECTING ON CHAPTER 10

- Maxwell unified concepts from electricity and magnetism into a new field called "electromagnetism." He presented his ideas in the form of four equations.
- Maxwell's equations show that a changing electric field generates a magnetic field and a changing magnetic field generates an electric field.
- Hertz produced electromagnetic waves in the laboratory, verifying Maxwell's predictions.
- Electromagnetic waves are produced when charges are accelerated. An electromagnetic wave consists of an oscillating electric field and magnetic field at right angles to each other that propagate in a direction that is perpendicular to both fields.
- Electromagnetic waves travel with a speed of $c = 3.00 \times 10^8$ m/s through empty space.
- Electromagnetic radiation exhibits wave behaviour, undergoing diffraction and forming interference patterns.
- Electromagnetic radiation can be polarized. The electric field of plane polarized light oscillates in only one plane.
- Since the magnetic field in an electromagnetic wave is always perpendicular to the electric field, when the electric field is polarized, the magnetic field must also be polarized.
- Electromagnetic radiation can have frequencies ranging from below 1 Hz to above 10^{22} Hz, called the "electromagnetic spectrum."
- Light is one narrow section of the electromagnetic spectrum. Colour is identified by frequency or wavelength.

Colour	Wavelength (nm)
violet	400 – 450
blue	450 – 500
green	500 – 570
yellow	570 – 590
orange	590 – 610
red	610 – 750

- Radio waves are produced by oscillating charge in an antenna.
- Microwaves can be produced by Klystron™ and magnetron tubes.
- Infrared radiation and light can be produced when high-energy electrons in very hot objects drop to lower energy levels.
- Ultraviolet radiation can be produced when electrons in excited atoms drop to lower energy levels.
- X rays are produced by rapidly stopping very energetic electrons that have been accelerated by a large potential difference.
- Gamma rays are emitted by unstable nuclei when they return to a more stable state.
- A deeper understanding of electromagnetic radiation has led to several applications, including television and radio broadcasts, global positioning systems, wildlife tracking systems, and magnetic resonance imaging.

Knowledge/Understanding

1. A magnetic field in an electromagnetic wave travelling south oscillates in an east-west plane. What is the direction of the electric field vector in this wave?
2. Describe how an antenna works for transmitting and receiving radiation.
3. If you could see the electric fields in light, how would the electric fields appear if you were looking straight toward a light source?
4. How can unpolarized light be transformed into polarized light?
5. **(a)** What is the cause of glare?
   (b) How do Polaroid sunglasses reduce glare?

6. What happens if sunglasses polarized to allow vertical vibrations through are turned 90°?
7. Sketch an electromagnetic wave and label the appropriate parts.
8. Television antennas that receive broadcast stations (not cable or satellite) have the conductors oriented horizontally. What does this imply about the way signals are broadcast by the stations?
9. Why cannot a radio station transmit microwaves?
10. Why do you not see interference effects from light entering a room from two different windows?
11. Does the speed of an electromagnetic wave depend on either the frequency or wavelength?
12. Is light a longitudinal or transverse wave? How do you know?
13. Is it possible to get a sunburn through a closed window?

Inquiry

14. Before cable and satellite television were available, most people had indoor antennas called "rabbit ears" that they could manually move around to get the best reception. Often, when someone would be touching the antenna while moving it, the reception would be good. When the person walked away from the antenna, the reception became poor again. Suggest a possible reason for this phenomenon.
15. Devise one or more situations involving an electron, a proton, or a neutron in constant motion, accelerated motion, or at rest, to produce the following.
    - **(a)** an electric field only
    - **(b)** both electric and magnetic fields
    - **(c)** an electromagnetic wave
    - **(d)** none of these
16. Suppose two pairs of identical polarizing sunglasses are placed in front of each other. Explain clearly your answers to the following.
    - **(a)** What would you observe through them?
    - **(b)** If one pair is rotated 90° in relation to the other, what would you observe?
    - **(c)** If a third pair, oriented randomly, is inserted between the two pairs, what would you observe?
17. All objects, including human beings, emit electromagnetic radiation according to their temperatures. Through thought experiments, predict whether hotter objects would emit longer or shorter wavelength radiation than cooler objects.

Communication

18. Make sketches to demonstrate that a mirror's surface appears to be smooth if its atoms are smaller than the wavelength of light and that it would appear to be bumpy if its atoms were larger than a wavelength of light.
19. Find the approximate frequency and wavelength of the waves associated with the following.
    - **(a)** your favourite AM radio station
    - **(b)** your favourite FM radio station
    - **(c)** a microwave oven
    - **(d)** a conventional oven
    - **(e)** green light
    - **(f)** dental X rays
20. How can you test the light of the blue sky to determine its direction of polarization?
21. Describe how you can determine whether your sunglasses are polarizing material or tinted glass.
22. Explain why a flashlight using old batteries gives off reddish light, while light from a flashlight using new batteries is white.
23. A doctor shows a patient an X ray of a fractured bone. Explain how the image is produced. What kinds of materials can and cannot X rays penetrate?
24. At night, you can often pick up more distant radio stations than in the daytime. Explain why this is so.

Making Connections

25. Ultraviolet rays, X rays, and gamma rays can be very harmful to living things. What is unique about these forms of electromagnetic waves that could cause damage to living cells?

26. **(a)** Discuss methods to measure the speed of a race car and the speed of a bullet.
    (b) What are the largest sources of error?
    (c) What difficulties do you encounter if you try to apply these methods to measure the speed of light?
27. Some species of snakes, called "pit vipers," have sensors that can detect infrared radiation. What do you think is the function of these sensors?
28. Reflectors left on the Moon's surface by the Apollo astronauts can be used to accurately measure the Earth-Moon distance with lasers. To achieve an accuracy of 10 m, what must be the accuracy of the timing device?
29. Investigate the relationship between coloured light and coloured cloth. Make different coloured lights (with coloured glass or transparent plastic film). Explain why the colours of coloured cloths change as they are viewed under different-coloured light sources.
30. Suppose your eyes were sensitive to radio waves instead of visible light.
    (a) What size of radio dish would you need on your face to achieve the same resolution?
    (b) What things would look bright?
    (c) What things would look faint?
31. You see a lightning flash and simultaneously hear static on an AM radio. Explain why.
32. **(a)** Why does an ordinary glass dish become hot in a conventional oven but not in a microwave oven?
    (b) Why should metal not be used in a microwave oven?
    (c) Microwave ovens often have "dead spots" where food does not cook properly. Why might this occur?
33. Research the basic components required for a radio transmitter and receiver. Describe how a signal is transmitted and received.
34. Film used for modern medical and dental X rays is far more sensitive to X rays than film that was used when these forms of X rays were first developed. Why do you think it was important to increase the sensitivity of the film?

Problems for Understanding

35. If a gamma ray has a frequency of 1.21×10^{21} Hz, what is its wavelength?
36. Radio waves 300 m long have been observed on Earth from deep space. What is the frequency of these waves?
37. The announcer on an FM radio station in Toronto identifies the station as "The Edge 102.1," where the number 102.1 is the frequency in some units. What is the wavelength and frequency of the waves emitted by the radio station?
38. A 100 kW (1.00×10^5 W) radio station emits electromagnetic waves uniformly in all directions.
    (a) How much energy per second crosses a 1.0 m^2 area receiver that is 100.0 m from the transmitting antenna? (Hint: The surface area of a sphere is $4\pi r^2$.)
    (b) Repeat the above calculation for a distance of 10.0 km from the antenna.
    (c) If you double the distance between the transmitter and the receiver, by what factor will the energy per second crossing the area decrease?
39. A light-minute is the distance light travels in one minute. Calculate how many light minutes the Sun is from Earth.
40. Airplanes have radar altimeters that bounce radio waves off the ground and measure the round-trip travel time. If the measured time is 75 μs, what is the airplane's altitude?
41. If you make an intercontinental telephone call, your voice is transformed into electromagnetic waves and routed via a satellite in geosynchronous orbit at an altitude of 36 000 km. About how long does it take before your voice is heard at the other end?
42. You charge a comb by running it through your hair.
    (a) If you then shake the comb up and down, are you producing electromagnetic waves?
    (b) With what frequency would you have to shake the comb to produce visible light?

UNIT 4 PROJECT

Constructing Your Own FM Transmitter

Background

Radio waves, a form of electromagnetic radiation, play a major role in communication technology, including radio and television program transmission, cellphone service, wireless computer connections, and satellite operations. In this activity, you will construct an FM transmitter that is capable of broadcasting your voice to any nearby portable radio. You will then use the transmitter to investigate the variables that affect it.

Challenge

Construct and test an FM transmitter, using the kit provided. To accomplish this task, you will need to learn how to identify and then solder circuit components.

Design and conduct experiments to investigate relationships between the signals emitted by your transmitter and predictions made by the wave model for electromagnetic radiation.

Materials

- FM transmitter kit
- portable radio
- soldering pencil
- solder
- small wire cutters
- small screw driver
- battery
- wet sponge

Safety Precautions

- Ensure that all electrical equipment is properly grounded.
- Be extremely careful when working with a soldering pencil. The heated iron tip can cause serious burns in an instant.
- Ensure that you are wearing eye protection. Solder might splatter when you are cleaning the soldering pencil with the wet sponge.
- Avoid inhaling fumes generated by the solder. Work in a well-ventilated area.

Design Criteria

A. As a class, develop assessment criteria to address the operation of each transmitter. You might want to include some or all of the following categories.

- ability to transmit your voice from the transmitter to a portable radio
- range of reliable signal transmission
- construction quality of transmitter
- manufacture of a peaking circuit to test transmitter; a peaking circuit is a very simple circuit that allows you to tune your transmitter to its maximum output by simply measuring the potential difference across an element in the peaking circuit
- technical statistics, such as maximum output voltage radiated by your transmitter, detected using a peaking circuit

B. Develop experiment procedures that will allow you to test predictions made by the wave model for electromagnetic radiation. You might want to design and conduct experiments to investigate the relationship between

- aerial length and transmitting frequency
- range of the transmitter and the amount of input energy
- amount of signal absorption and the amount of input energy
- environmental factors and reflection or interference effects

As a class, develop criteria to assess both the experiment design and the validity of the obtained results.

ASSESSMENT

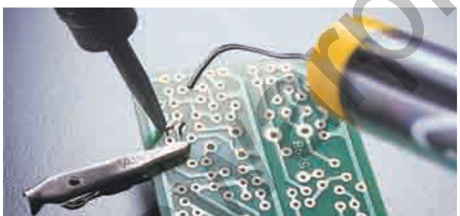
After you complete this project

- assess the development of your technical skills during the construction phase
- assess the quality of your transmitter based on established criteria
- assess the ability of your experiment design to test specified theoretical predictions

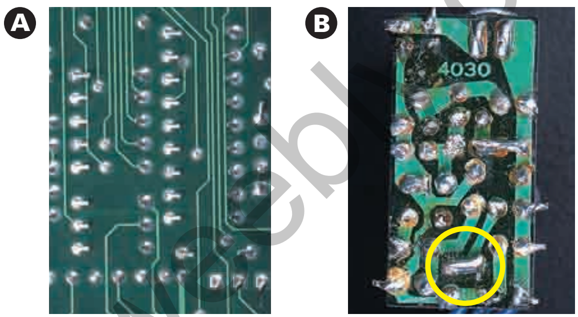
Action Plan

Part 1: Build the Transmitter

1. Investigate the proper soldering technique, referring to the Internet or other resources. Before you open your transmitter kit, practise soldering on an old circuit board and components. Place a hot, clean soldering pencil against both the conducting surface of the circuit board and the component for 1 or 2 s. Carefully dab and remove the solder at the point where the pencil is touching both the component and the circuit board. The solder must come into contact only with the component (for example, the resistor or capacitor) and the metal conducting surface of the board. Adjacent soldered connections must not touch. Always test each soldered joint by gently pushing on the component. The component should not move. Always clean excess solder from the iron by using a wet sponge between soldering attempts. Attach an alligator clip to the board to radiate away excess heat.



2. Open the FM transmitter kit. Using the kit's instructions, identify each component and its proper location on the circuit board. Begin construction by carefully soldering the shortest components first.
3. Once each component is in place, inspect each soldered joint. Ensure that they are secure and that each soldered joint is free from contact with adjacent joints.
4. Test your transmitter using a portable radio. (This might involve the construction and use of a simple peaking circuit.)



(A) Robotically soldered circuit board. **(B)** Improperly soldered connections that connect two different components.

5. Test your transmitter based on the criteria decided on by your class.

Part 2: Testing the Wave Theory

6. Design and conduct investigations relating to criteria decided on by your class. Ensure that your experiment design clearly identifies the conditions that you will control and the variables that you are testing. Record your theoretical predictions, based on the wave model, before conducting the experiment. To ensure that the appropriate safety measures are being taken, check with your teacher before conducting any experiments,.
7. Prepare a report of your findings.

Evaluate

1. Assess the success of your FM transmitter, based on your class's previously determined criteria. Compare your results with your classmates, taking careful note of differences in the construction of the device.
2. Assess your experiment design and the results you generated, based on the class's previously determined criteria. Recommend ideas for further experimentation.

UNIT 4 Review

Knowledge/Understanding

Multiple Choice

In your notebook, write the letter of the best answer for each of the following questions. Outline your reasons for your choice.

1. Two objects are just able to be resolved when the
   (a) central maximum of one falls on the first minimum of the other
   (b) central maxima of the objects do not overlap
   (c) angle separating them is greater than the wavelength of light
   (d) distance from the objects to your eyes is sufficiently reduced
2. Digital videodiscs (DVDs) use
   (a) thin-film principles
   (b) interference effects
   (c) shorter laser light than compact disc players
   (d) All of the above.
3. An electron falling through a potential difference generates
   (a) a magnetic field only
   (b) an electric field only
   (c) stationary electric and magnetic fields
   (d) no fields
   (e) an electromagnetic wave
4. Electromagnetic waves propagate in a direction
   (a) parallel to the oscillation of the electric field
   (b) parallel to the oscillation of the magnetic field
   (c) perpendicular to the oscillations of both the magnetic and electric fields
   (d) independent of the oscillations of either the magnetic or electric field
5. Which of the following phenomena leads to the interpretation that electromagnetic radiation is a transverse wave?
   (a) diffraction
   (b) partial reflection, partial refraction
   (c) linear propagation
   (d) polarization
6. Which of the following is *not* a result of the superposition of waves?
   (a) destructive interference
   (b) diffraction
   (c) refraction
   (d) constructive interference
7. Electromagnetic waves differ from mechanical waves because
   (a) they undergo diffraction
   (b) they do not require a medium in which to travel
   (c) they are transverse waves
   (d) their speed is determined by the medium through which they are travelling
8. Which of the following statements is *not* true about the properties of electromagnetic waves?
   (a) X rays are emitted by unstable nuclei of atoms.
   (b) Extremely low frequency radio waves penetrate salt-water better than higher frequencies.
   (c) Ultraviolet light was discovered by accident.
   (d) Satellites that detect infrared radiation can see through clouds.
9. Maxwell's first law, known as Gauss's law,
   (a) relates a changing magnetic field and the induced *emf*
   (b) relates electric field lines to the charges that create them
   (c) relates magnetic field lines to the charges that create them
   (d) predicts the existence of electromagnetic waves

Short Answer

10. Explain why you do not see interference effects from light entering a room from two different windows.
11. Explain the effect of turning polarized sunglasses through an angle of 90˚. What happens if sunglasses polarized to allow vertical vibrations through are turned 90˚?
12. The equation, $\lambda \cong \dfrac{\Delta y d}{x}$, relates the wavelength of light to the distance between slits in a diffraction grating, the distance from the grating to a screen, and the distance between fringes on the screen. What approximation was made in deriving this relationship and under what conditions is the approximation valid?

13. What does the Maxwell's 4th equation predict?
14. Do research on "cosmic rays" anthem. Compare the differences between cosmic rays and electromagnetic radiation.
15. **(a)** In single-slit diffraction, how does the width of the central maximum compare to the width of the other maxima?
    (b) How does the width of the maxima produced by a diffraction grating compare to those produced by a double slit?
16. Describe the necessary conditions for two light waves incident at a single location to produce a dark fringe.
17. How is it possible to determine how thick a coating should be on a pair of glasses to reduce the amount of reflection? How thick should the coating be?
18. **(a)** Define electromagnetic radiation.
    (b) What evidence is there that light is electromagnetic radiation?
    (c) What evidence is there that sound is not electromagnetic radiation?
19. Consider an electromagnetic wave propagating in the positive x-direction. At a time, t_0, the electric field points in the positive y-direction. In what direction does the magnetic field point at this time? Sketch the electromagnetic wave. In what directions will the electric and magnetic fields point half a period later?

Inquiry

20. How do magnetic resonance imaging systems make use of hydrogen atoms?
21. Two friends are hired by a telemarketing firm for the summer. Each friend has a desk that is separated from the other workers by only a thin half-wall. One friend notices a continuous humming sound when she is sitting at her desk. The other notices that there is no humming noise audible at his desk. Some investigation finds that the company has two speakers placed at one end of the working floor, 8.0 m apart. A continuous, low-frequency hum is generated to mask conversations from nearby desks. The two students conduct a survey and find that several people do not hear the humming noise at their desks. For each of these people, they measure the distance from each speaker to the desk. The table below is the data they collected.

Person	Distance from speaker A (m)	Distance from speaker B (m)
1	14	8
2	10	4
3	12	10
4	10	12
5	6	12

The friends also sample the air temperature and find that it is always 23°C.
    (a) Provide an explanation, based on the characteristics of waves, for why some workers will hear the hum while others will not.
    (b) Use the data provided in the table to determine the frequency of the low-frequency hum.
    (c) The company is unhappy to learn that five people are unable to hear the low-frequency hum intended to mask nearby conversations. Suggest how the company could mask conversations more effectively.
22. You notice that a telephone pole casts a clear shadow of the light from a distant source. Why is there no such effect for the sound of a distant car horn?
23. Design and make a simple model of a laser. Identify and explain the function of the principal components. Discuss why a laser beam is so narrow.
24. Suppose white light is used in a Young's experiment. Describe the characteristics of the resulting fringe pattern and sketch it.
25. Challenge: Many experiments and optical instruments exploit the property of rectilinear propagation of light by reflecting a light beam many times for a desired effect. In this challenge, use your knowledge of physics to

control a television or VCR from outside a room or around a corner. Construct several $10 \text{ cm} \times 10 \text{ cm}$ reflector cards by covering the cards with aluminum foil and see how many times you can reflect a remote control beam and still turn on the television or VCR. Test long distances (out in the hall, down another hall) as well.

26. Triangulation is a basic geometrical method that has been used since the time of the ancient Greeks. Surveyors use it to determine the distance to an object by sighting it from two different positions a known distance apart. With two angles and a side, or a side and two angles, the dimensions of the triangle can be determined. How could you use the method to determine the altitude of a global positioning system satellite or the distance to a planet or nearby star? What effect will a 1% error in the measured angle have on the calculated distance in each case?

Communication

27. Although the wavelengths of optical radiation are very small, they can be measured with high accuracy. Explain how this is possible.
28. You see a lightning flash and simultaneously hear static on an AM radio. Explain how these occurrences are related.
29. Explain why an optical telescope must have a smooth surface, while the surface of a radio telescope is not as highly machined.
30. The wave model of light predicted that the speed of light would slow down when travelling from one medium into another of greater optical density. Use Huygens' principle to demonstrate this prediction. Include a diagram.
31. Make a list of the characteristics of light that a model should explain: rectilinear propagation, reflection, refraction, partial reflection, partial refraction, dispersion, and diffraction. Briefly discuss how the wave model of light and the particle model of light explain these phenomena.
32. Compare the collision between two oppositely directed particles with the collision between two oppositely directed water waves. What are the similarities and differences between these interactions?
33. (a) Test light diffraction with the shadow of your hand. How can you cast the sharpest and fuzziest shadows?
    (b) Describe other examples of light diffraction.
34. (a) With a sketch that shows individual waves of light, demonstrate how Young used diffraction to create a two-point light source that was exactly in phase.
    (b) Show how these two sources interfered to produce a series of light and dark bands on a screen.
35. Due to the popularity of Newton's particle model of light, Young's work on light interference was received with scepticism by British scientists. Explain how the evidence of wave behaviour that you observed in ripple tanks models the results of Young's experiment and his conclusion that light behaves like a wave.
36. Select one of the following statements about the competing models of light and develop an argument to refute it.
    (a) Both models found support because neither model could adequately describe every observed property of light.
    (b) Both models found support because each model adequately described every observed property of light at the time.
    (c) Newton's model for light found support primarily because of his fame and respected stature.
37. (a) Explain how the definition of "one metre" was redefined in 1961.
    (b) In 1983, the metre was redefined again to be the distance light travels in 1/299 752 458 s. Why do you think this was done?
38. Thin films, such as soap bubbles or gasoline on water, often have a multicoloured appearance that sometimes changes while you are watcing. Explain the multicoloured appearance of these films and why their appearance changes with time.

Making Connections

39. In the seventeenth century, it was not known whether light travelled instantaneously or with finite speed.

(a) Early in that century, Galileo attempted to measure the speed of light by stationing one helper one kilometre away and timing how long it took a pulse of light to travel the distance. Explain why that attempt was unsuccessful.

(b) Ole Roemer made the first successful attempt to measure the speed of light around 1675. He made a long series of observations to accurately determine the period of one of Jupiter's moons, Io, around Jupiter. When he later used this information to predict when Io would be eclipsed by Jupiter, he found that his predictions were too early or too late, compared to the observations, depending on Earth's position in its orbit. Investigate Roemer's method and clearly explain, with the aid of a diagram, how he was able to successfully measure the speed of light. What factor(s) limited the accuracy of Roemer's method?

(c) An elegant and much more accurate measurement of the speed of light was made by Albert Michelson in 1880. He used a rotating octagonal mirror and sent a beam of light to a stationary mirror on a mountaintop 35 km away. Sketch Michelson's experiment set-up. He refined this experiment over many years. Discuss how he was able to measure the speed of light so accurately.

40. Cochlear implants are sometimes used to assist the hearing of deaf people. Their operation relies on the broadcasting and receiving of electromagnetic waves and the ultimate stimulation of the auditory nerve. If the auditory nerve is intact, the deaf person can learn to recognize sounds. Sketch the components of a cochlear implant and describe how it works.

41. In an interferometer, light following different paths is allowed to interfere. By measuring the interference fringes, the different path lengths can be precisely determined. Gravitational wave detectors use interferometers to search for ripples in the fabric of space and time. These ripples were predicted by Einstein's theory of general relativity and are thought to be produced by collisions of two black holes or the collapse of massive stars in supernova explosions. Research the operations of the Laser Interferometer Gravity-Wave Observatory and the proposed Laser Interferometer Space Antenna or other gravitational wave observatories. Why are the interferometers used in these observatories so long? What sensitivity do the scientists hope to achieve? What are the goals of these projects? How will detection of gravitational waves change our view of the universe?

42. Investigate and explain in detail why glass is transparent to visible light, but opaque to ultraviolet and infrared light.

43. For more than a century, photographic plates and film have been used to record light in various detectors. Now these are being replaced by devices that record light digitally, such as charged coupled devices. Contrast these two technologies in terms of sensitivity (the ability to detect faint objects) and resolution.

44. **(a)** Investigate different methods of using polarized glasses to produce 3-D films. How does the IMAX technology (see Chapter 2, The *Big* Motion Picture) differ from virtual reality technology?

(b) Some viewers complain of feeling nauseated during 3-D films. Why does this occur? How can it be avoided or minimized?

45. Astronomy is described as an observational science, not an experimental science such as physics, biology, and chemistry. Stars cannot be reproduced in the lab, so starlight and light from other sources must be studied as thoroughly as possible. Research and write an essay on how astronomers maximize the information obtained from light through the use of detectors that measure light's intensity, wavelength, direction, and polarization. Include a discussion of how astronomy has benefited from the space age by the launching of satellite observatories that observe wavelength ranges blocked by the atmosphere. What kinds of astrophysical phenomena are best studied in each wavelength range of the electromagnetic spectrum?

46. In the Search for Extraterrestrial Intelligence (SETI) program, signals in the radio wavelength region of the electromagnetic spectrum are being examined for unusual patterns that might indicate an intelligent (instead of natural) source.
   (a) Why has the radio wavelength region of the spectrum been chosen?
   (b) Are there frequencies within the radio region for which a detection might be more likely than others?
   (c) What might an intelligent signal look like?
   (d) The SETI@home program asks participants to use their home computers to scan data currently being obtained from the Arecibo radio telescope. What progress has been made in SETI's search to date?

47. The coherence of a laser beam allows it to be broken up into extremely short pulses called "bits." These bits allow information to be stored in digital form. Investigate the role of the laser in the storage, transmission, and retrieval of information in various media. Evaluate the efficiency of this process and discuss how consumers might expect it to evolve in the future. Summarize your findings in a report.

48. What is the limit of resolution of an optical microscope? How does this affect the types of things that can be studied with this instrument? To study finer detail, scanning electron microscopes are used (see Chapter 12, Quantum Mechanics and the Atom). What is the limit of resolution for these instruments?

49. Different physical processes are responsible for producing electromagnetic radiation of different wavelengths. For each wavelength region of the electromagnetic spectrum (radio, infrared, visible, ultraviolet, X ray, gamma ray), identify at least one physical process that produces radiation in that wavelength range.

Problems for Understanding

50. A detector tuned to microwave wavelengths registers 2500 wave crests in 1.0 μs. What is the wavelength, frequency, and period of the incoming wave?

51. If an electromagnetic wave has a period of 4.8 μs, what is its frequency and wavelength?

52. Calculate the wavelength of a 10^{21} Hz gamma ray.

53. How many cycles of a 5.5×10^{-9} m ultraviolet wave are registered in 1.0 s?

54. What is the colour of light that has a frequency of 7.0×10^{14} Hz?

55. The most efficient antennas have a size of half the wavelength of the radiation they are emitting. How long should an antenna be to broadcast at 980 kHz?

56. How long should a microwave antenna be for use on a frequency of 4400 MHz?

57. A light-year is the distance light travels in one year. How far is this in metres? (There are 365.25 days in one year.)

58. A concert in Halifax is simultaneously broadcast to Vancouver on FM radio. Determine whether people listening in Vancouver, approximately 5.0×10^3 km away, will hear the music just before or just after someone sitting in the back of the 82 m concert hall in Halifax. Assume that the speed of sound in the concert hall is 342 m/s.

59. Yellow light is incident on a single slit 0.0315 mm wide. On a screen 70.0 cm away, a dark band appears 13.0 mm from the centre of the bright central band. Calculate the wavelength of the light.

60. **(a)** The beam of a helium-neon laser ($\lambda = 632.8$ nm) is incident on a slit of width 0.085 mm. A screen is placed 95.0 cm away from the slit. How far from the central band is the first dark band?

(b) If the slit was two times wider, would the first dark band be closer or farther from the central band?

61. If a spectrum is to have no second order for any visible wavelength, how many lines per cm must the grating have?

62. A certain beetle has wings with a series of bands across them. When 600 nm light is incident normally and reflects off the wings, the wings appear to be bright when viewed at an angle of 49°. How far apart are the lines in the bands?

63. Radio astronomers utilize interferometry to build large arrays of radio dishes and thereby achieve much greater resolution. In fact, when the signals from two small dishes 1 km apart are properly combined, the two dishes have the same resolving power as one giant dish that is 1 km across.

(a) The Very Large Array, in New Mexico, has a maximum dish separation of 36 km. What resolution can be obtained at a wavelength of 6.0 cm?

(b) New interferometers are proposed, which would stretch across entire continents. What is the resolution of a very long baseline interferometer at this wavelength if the dishes are separated by 3600 km?

(c) What minimum separation between two radio sources at the centre of the Milky Way galaxy, 24 000 light-years away, could this interferometer distinguish? Compare this to the radius of Pluto's orbit, 5.9×10^{12} m.

(d) Would the resolution of these telescopes become better or worse if they operated at shorter radio wavelengths?

64. What size of orbiting optical space telescope would you require to measure the diameter of a star that has the same diameter as the Sun and is 10.0 light-years away? (Use a wavelength of 550 nm, and take the Sun's diameter as 1.40×10^9 m.)

65. A diffraction grating has 5000(5.000×10^3) lines per cm. Monochromatic light with a wavelength of 486 nm is incident normally on the grating. At what angles from the normal will the first, second, and fourth order maxima exit the grating? Do the angles increase linearly with the order or the maxima? Explain.

66. Cherenkov radiation is light emitted by a particle moving through a medium with a speed greater than the speed of light in the medium. (**Note:** The speed of the particle is not greater than the speed of light in a vacuum.) Consider a beam of electrons passing through water with an index of refraction of 1.33. If Cherenkov light is emitted, what is the minimum speed of the electrons?

67. If the first order maximum of He-Ne light ($\lambda = 632.8$ nm) exits a diffraction grating at an angle of 40.7°, at what angle will the first order maximum of violet light with a wavelength of 418 nm exit?

COURSE CHALLENGE

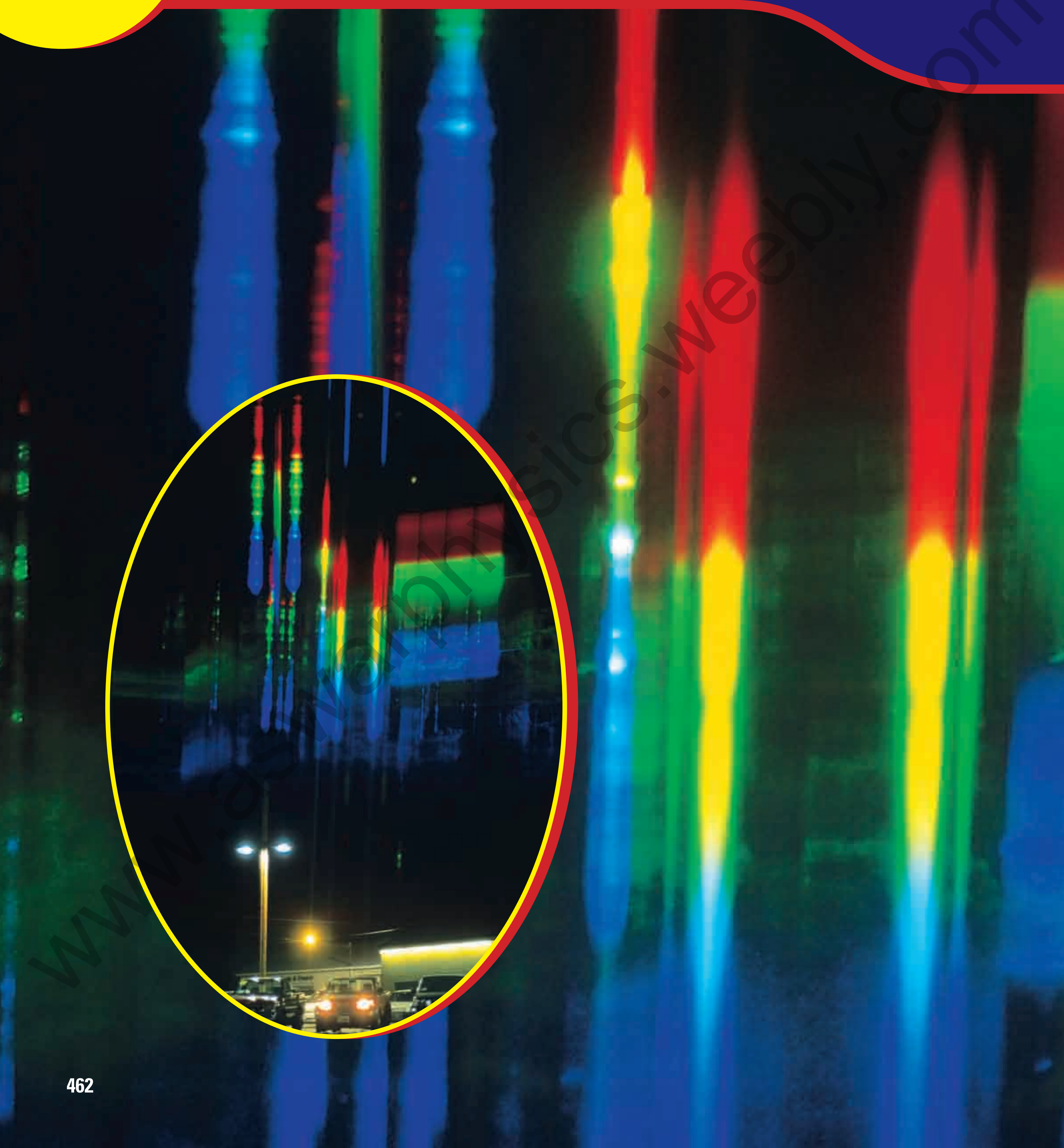
Scanning Technologies: Today and Tomorrow

Continue to plan for your end-of-course project by considering the following.

- How do the wave properties of electromagnetic radiation relate to your project?
- Are you able to incorporate newly learned skills from this unit into your project?
- Analyze the information contained in your research portfolio to identify knowledge or skills gaps that should be filled during the last unit of the course.

UNIT 5

Matter-Energy Interface



OVERALL EXPECTATIONS

DEMONSTRATE an understanding of the basic concepts of Einstein's special theory of relativity and of the development of models of matter that involve an interface between matter and energy, based on classical and early quantum mechanics.

INTERPRET data to support scientific models of matter and conduct thought experiments as a way of exploring abstract scientific ideas.

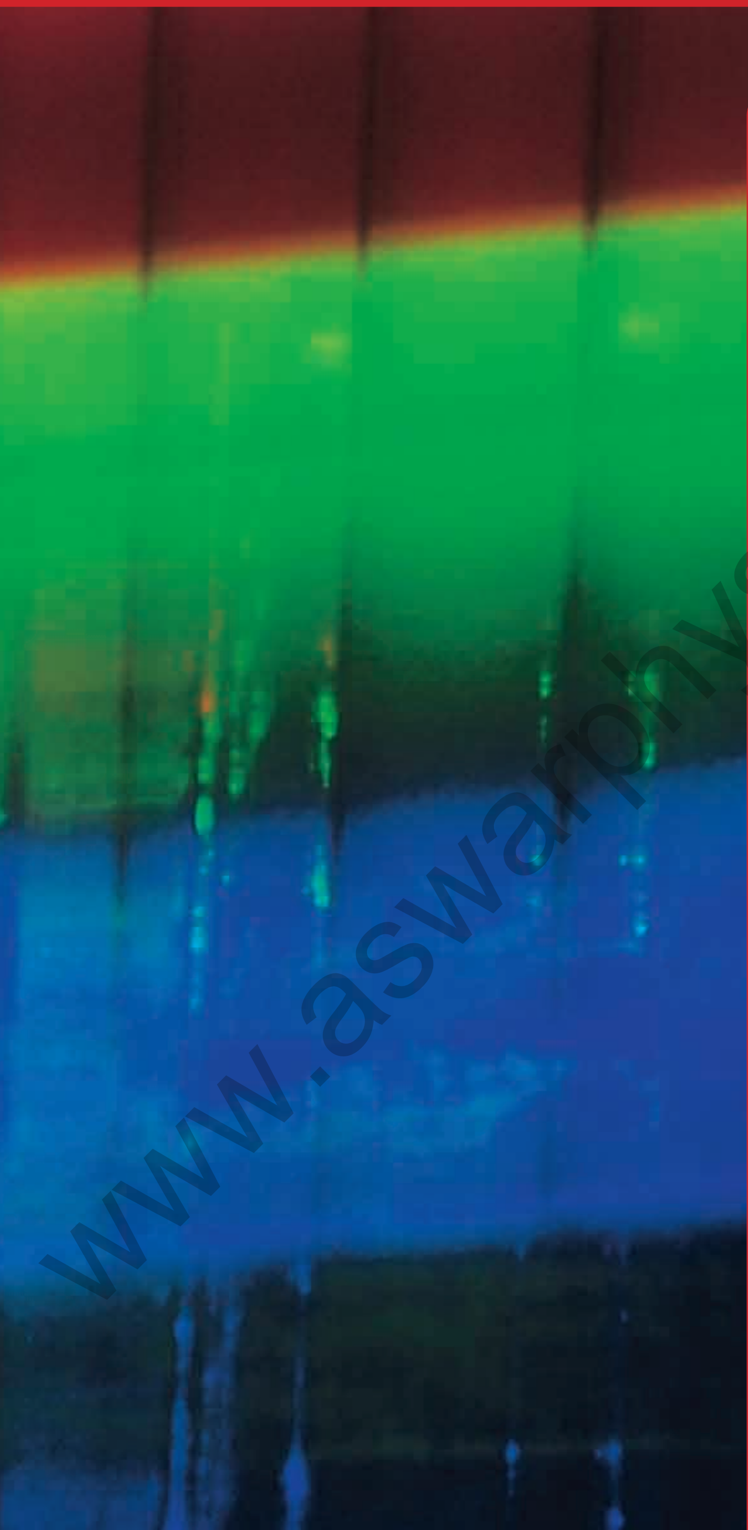
DESCRIBE how the introduction of new conceptual models and theories can influence and change scientific thought and lead to the development of new technologies.

UNIT CONTENTS

CHAPTER 11 Special Theory of Relativity

CHAPTER 12 Quantum Mechanics and the Atom

CHAPTER 13 The Nucleus and Elementary Particles



The turn of the twentieth century was a time of excitement and turmoil in science. In 1888, Heinrich Hertz demonstrated the existence of radio waves and then, in 1895, Wilhelm Conrad Röntgen discovered X rays. The following year, Antoine Henri Becquerel discovered radioactivity and, a year later, J.J. Thomson discovered the electron. Then, Philipp Lenard observed the photoelectric effect in which light ejected electrons from metals.

Along with these discoveries came a number of puzzles. How could radioactive substances emit radiation without any apparent source of energy? Why could only certain colours of light eject electrons from metals? For nearly 50 years, spectroscopists wondered why each element gave off a unique spectrum of light. Since light crossed the vacuum of space between Earth and the stars, physicists assumed that a substance called "luminiferous ether" must exist to carry light waves. However, all attempts to detect Earth's motion through it failed.

In this unit, not only will you learn more about the discoveries of some of the most outstanding scientists who ever lived, but also you will learn the answers to some of the questions that baffled them.

UNIT PROJECT PREP

Refer to pages 590–591 before beginning this unit. In this unit project, you will examine the parallels between scientists and their theories with societal pressures and realities.

- Revisit the time line on page xiv and try to remember some significant societal events that took place during the years represented.
- How closely tied do you think scientific research is to societal pressures?

CHAPTER 11

Special Theory of Relativity

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PREREQUISITE CONCEPTS AND SKILLS

- Interaction of electric and magnetic fields
- vector addition
- frames of reference
- relative velocity

The name of Albert Einstein has towered over the field of physics during the twentieth century and on into the twenty-first century. In the space of a few years, Einstein not only changed the world's way of thinking about electromagnetic radiation such as light, he also radically changed the commonly accepted picture of the universe with his two relativity theories — the special theory of relativity and the general theory of relativity.

The special theory of relativity, which you will be studying in this chapter, was not well received at first. The idea that fundamental measurements such as time, distance, and mass depended on the relative motion of the observer seemed absurd to many. In fact, Einstein was awarded the 1921 Nobel Prize for physics for his development of the concept of photons and the resulting explanation of the photoelectric effect, not for his theories of relativity.

With the advent of high-energy physics, however, Einstein's theory of special relativity became essential to the understanding of the behaviours of all high-speed subatomic particles.

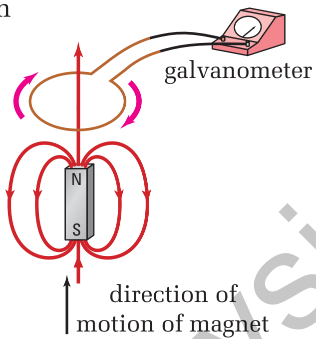
QUICK LAB

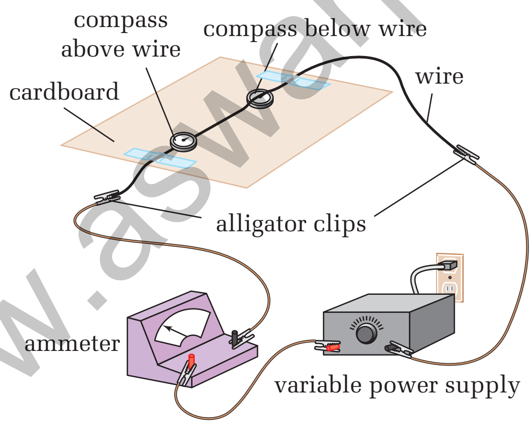
Generating Electromagnetic Fields

TARGET SKILLS
- Predicting
- Performing and recording
- Analyzing and interpreting

One of the problems that led to Einstein's special theory of relativity came from an analysis of the way in which electric and magnetic fields spread out through space as electromagnetic waves. In previous science courses, you have studied various characteristics of magnetic and electric fields; now, examine carefully the two diagrams and try to answer the questions that follow each of them. Discuss the answers with your classmates. Then, carry out the activity that follows these questions.

- Under what condition does a magnetic field generate an electric current?
- What determines the magnitude of the current?
- What determines the direction of the current?
- How do you know that an electric field must have been generated across the coil?

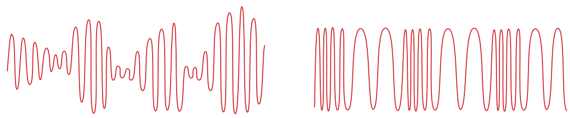




- What evidence is there that an electric current can generate a magnetic field?
- What affects the strength of the magnetic field?
- What affects the direction of the magnetic field?
- What kind of field is needed to produce an electric current?

Lab

Obtain a radio with a movable antenna. Turn on the radio and set it in the AM range. ("AM" refers to amplitude modulation, a process in which a signal is impressed on a radio carrier frequency by varying its amplitude, as shown in the diagram.) Turn on an induction coil and allow an arc (or spark) to pass between the points of the electrodes. Listen for the effect on the radio. Find the antenna orientation for which the effect is (a) greatest and (b) least. Repeat these steps with the radio tuned to an FM station. ("FM" stands for frequency modulation, in which the signal is impressed on the carrier wave through variations in its frequency, as shown.)



AM signal FM signal

Analyze and Conclude

1. What evidence is there that radio waves (electromagnetic radiation) are travelling from the arc to the radio?
2. Is there any relationship between the orientation of the arc and the orientation of the antenna for maximum and minimum effects?
3. If there is a relationship in question 2, what does that indicate about the nature of the waves produced from the arc?
4. **(a)** What difference do you notice with the FM station? **(b)** Try to explain this difference.

11.1 Troubles with the Speed of Light

SECTION EXPECTATIONS

- State Einstein's two postulates for the special theory of relativity.
- Conduct thought experiments as a way of developing an abstract understanding of the physical world.
- Outline the historical development of scientific views and models of matter and energy.

KEY TERMS

- interferometer
- Lorentz-Fitzgerald contraction

Toward the end of the nineteenth century, many scientists felt that they were close to a complete understanding of the physical world. Newton's laws described motion. Maxwell's laws described radiant energy. The chemists were learning more and more about the behaviour of atoms. No one realized that their fundamental concepts of space, time, matter, and energy were seriously limited.

The Michelson-Morley Experiment

The first indication of a difficulty came from a critical experiment performed in 1881 by Albert Michelson (1852–1931), using an **interferometer**, an instrument he had devised for measuring wavelengths of light. In this experiment, he unsuccessfully attempted to detect the motion of Earth through the luminiferous ether, the substance that was then believed to be the medium through which light waves could travel through space. The apparent failure of Michelson's first experiment to find any such motion prompted many physicists to drop the ether concept.

Later, in 1887, Michelson and Edward Williams Morley (1838–1923) performed a refined version of the experiment, using an improved version of the interferometer. They reasoned that if light behaved like a sound wave or a wave on water, if you moved toward an oncoming beam of light, it would seem to approach you at a higher speed than if you were moving away from it. These different speeds would affect the interference pattern in the interferometer. By comparing interference patterns for light beams travelling perpendicular to each other, Michelson and Morley hoped to detect and measure the speed with which Earth passed through the ether.

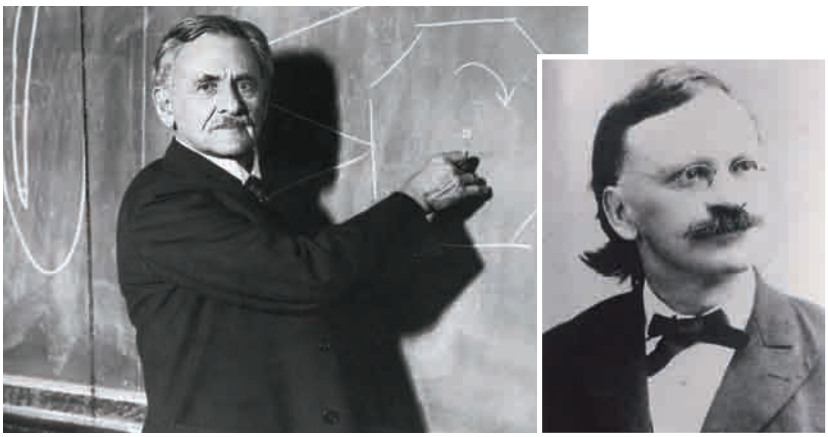


Figure 11.1 Albert Michelson (left) and Edward Morley used Michelson's interferometer (see Figure 11.2) to conduct an experiment that later became the foundation of Einstein's special theory of relativity.

To understand the basis of this experiment, consider the following scenario involving relative velocities. Two identical boats, X and Y, are about to travel in a stream. Boat Y will go straight across the stream and straight back. Boat X will travel the same distance downstream and then return to its starting point. Which boat will make the trip in the shortest time? Examine Figure 11.3 and then follow the steps below to determine the time required for boat Y to travel across the stream and back.

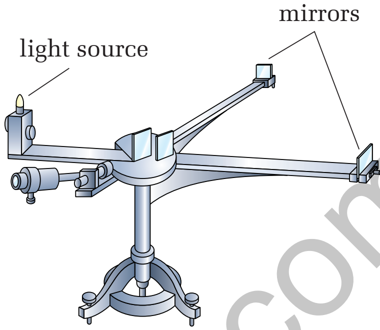


Figure 11.2 Michelson's first interferometer was designed to determine wavelengths of light. It should also be able to determine whether light travelling in directions perpendicular to each other travelled at different speeds.

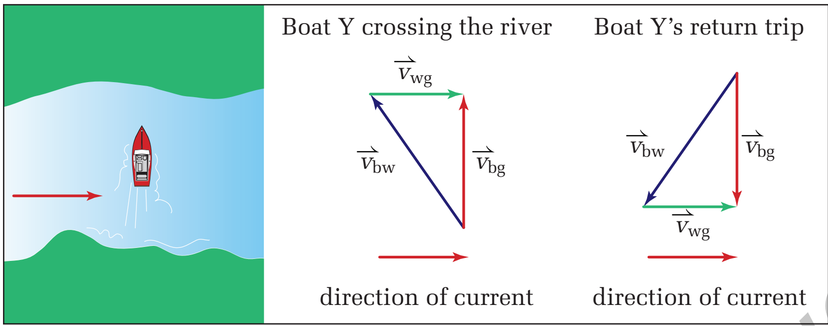


Figure 11.3 Since boat Y must go directly across the stream, the driver must angle the boat upstream while crossing either way perpendicular to the current.

- Define the symbols.

$\vec{v}_{bw}$: velocity of the boat relative to the water
$\vec{v}_{wg}$: velocity of the water relative to the ground
$\vec{v}_{bg}$: velocity of the boat relative to the ground
L : distance travelled along each leg of the trip
Δt : total time for the trip

- Write the definition for velocity and solve it for the time interval.

$$\vec{v} = \frac{\Delta\vec{d}}{\Delta t}$$
$$\Delta t = \frac{\Delta\vec{d}}{\vec{v}}$$

- Use vector addition to find the magnitude of the velocity of the boat relative to the ground. Notice in Figure 11.3 that this velocity is the same for both legs of the trip.

$$(v_{bw})^2 = (v_{bg})^2 + (v_{wg})^2$$
$$(v_{bg})^2 = (v_{bw})^2 - (v_{wg})^2$$
$$v_{bg} = \sqrt{(v_{bw})^2 - (v_{wg})^2}$$

- Substitute the total length of the trip ($2L$) and the magnitude of the velocity into the expression for the time interval to find the time required for boat Y to make the round trip.

$$\Delta t_Y = \frac{2L}{\sqrt{(v_{bw})^2 - (v_{wg})^2}}$$

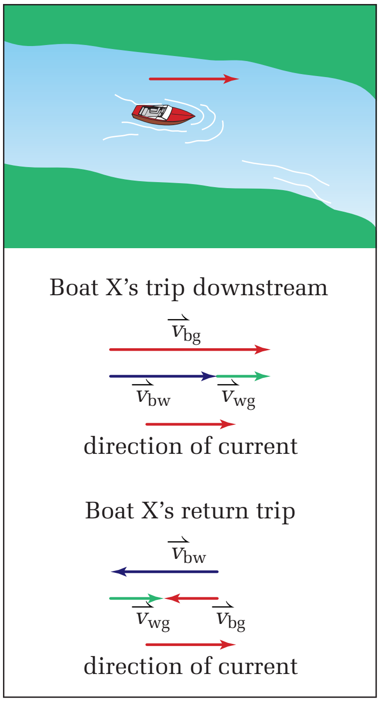


Figure 11.4 Boat X travels with the current when it is going downstream and against the current on its return trip.

Study Figure 11.4 to determine the velocities of boat X as it makes its trip downstream and back. Then, follow the steps below that determine the time for boat X to make the trip.

- Since the direction of the velocities of boat X and of the stream are in one dimension, the magnitudes can be added algebraically.

  Trip downstream: $v_{bg} = v_{bw} + v_{wg}$
  Trip upstream: $v_{bg} = v_{bw} - v_{wg}$

- Use the equation for the time interval in terms of displacement and velocity to write the time interval for boat X to travel downstream.

$$\Delta t_{down} = \frac{L}{v_{bw} + v_{wg}}$$

- Write the time interval for boat X to travel back upstream.

$$\Delta t_{up} = \frac{L}{v_{bw} - v_{wg}}$$

- To find the total time for boat X to make the round trip, add the time intervals for the two directions.

$$\Delta t_X = \frac{L}{v_{bw} + v_{wg}} + \frac{L}{v_{bw} - v_{wg}}$$

- Find a common denominator and simplify.

$$\Delta t_X = \frac{L(v_{bw} - v_{wg}) + L(v_{bw} + v_{wg})}{(v_{bw} + v_{wg})(v_{bw} - v_{wg})}$$

$$\Delta t_X = \frac{Lv_{bw} - Lv_{wg} + Lv_{bw} + Lv_{wg}}{(v_{bw})^2 - (v_{wg})^2}$$

- The time required for boat X to travel downstream and return is

$$\Delta t_X = \frac{2Lv_{bw}}{(v_{bw})^2 - (v_{wg})^2}$$

So, did boat Y or boat X complete the trip more quickly? You can find this out by dividing Δt_X by Δt_Y.

- Divide Δt_X by Δt_Y.

$$\frac{\Delta t_X}{\Delta t_Y} = \frac{\dfrac{2v_{bw}L}{(v_{bw})^2 - (v_{wg})^2}}{\dfrac{2L}{\sqrt{(v_{bw})^2 - (v_{wg})^2}}}$$

- Simplify.

$$\frac{\Delta t_X}{\Delta t_Y} = \frac{v_{bw}}{\sqrt{(v_{bw})^2 - (v_{wg})^2}}$$

- Divide the numerator and denominator by v_{bw} and simplify.

$$\frac{\Delta t_X}{\Delta t_Y} = \frac{1}{\sqrt{1 - \dfrac{(v_{wg})^2}{(v_{bw})^2}}}$$

Since the denominator is less than one, the ratio is greater than one; thus, Δt_X is greater than Δt_Y — boat Y was faster.

MATH LINK

Normally, taking a square root results in both positive and negative roots. However, since both time intervals were measured forward from a common starting point, they must both be positive, so the ratio must also be positive.

In the Michelson-Morley experiment, the speed of light through the luminiferous ether, usually represented by c, is equivalent to the speed of a boat through water. The speed of the water relative to the ground is equivalent to the speed of the ether relative to Earth. Because motion is relative, it is also the speed of Earth relative to the ether. If this speed is represented by v, the time ratio can be written as

$$\frac{\Delta t_X}{\Delta t_Y} = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

This means that light that is moving back and forth parallel to the motion of Earth should take longer to complete the trip than light that is moving back and forth perpendicular to the motion of Earth.

Conceptual Problems

- As you study the Michelson-Morley experiment, you will find similarities to this problem. Keep your answers in mind as you read further. Suppose two identical boats can travel at 5.0 m/s relative to the water. A river is flowing at 3.0 m/s. Boat Y travels 1.00×10^2 m straight across the river and then the same distance back. Boat X travels 1.00×10^2 m upstream and then returns the same distance.
  (a) Which boat makes the trip in the shortest time?
  (b) How much sooner does it arrive than the other boat?
- Imagine that both of the two identical boats in the previous problem headed out from the same point at the same time. The river flows due east. Boat Y travelled 1.00×10^2 m[NW] relative to its starting point on shore and then returned straight to its starting point. Boat X travelled 1.00×10^2 m[NE] relative to its starting point on shore and then returned straight to its starting point. Which boat will make the trip in the shortest time? Hint: Sketch the vector diagrams for each case. You might not have to do any calculations.

Michelson's Interferometer

In Michelson's interferometer, a light beam is split into two beams as it passes through the beam splitter, such as a half-silvered mirror. Beam X continues straight on, while beam Y reflects at right angles to its original path. The beams reflect from mirrors and recombine as they once again pass through the beam splitter. Since the two beams do not travel precisely the same distance before they recombine, they interfere with each other as they head toward the telescope. This combination produces an interference pattern that can be observed with the telescope. Anything that

LANGUAGE LINK

The "ether" to which the text refers is not the chemical form of ether. It stems from the Latin word *aether* and was thought to be a highly rarefied medium through which light and other electromagnetic waves travelled. The word "ethereal" comes from this concept.

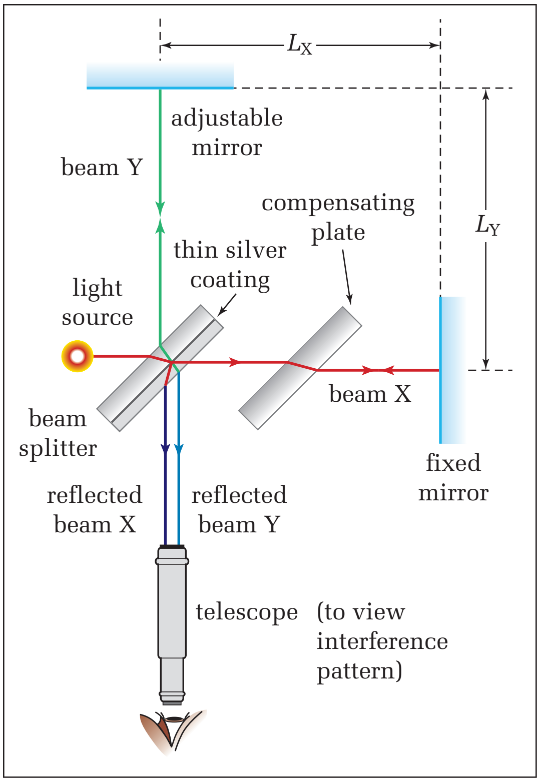


Figure 11.5 If the two beams (X and Y) are not in phase, they will interfere with each other, producing a pattern that can be seen in the telescope.

changes the time of travel of the two beams, such as moving the adjustable mirror even a small distance, produces obvious changes in the interference pattern.

When Michelson and Morley used the interferometer, they assumed that if beam X was parallel to the direction in which the planet was travelling, then that beam would take longer to reach the telescope than beam Y. This would produce a certain interference pattern. However, if the apparatus was rotated through 90˚, beam Y would lag behind. During the rotation, the interference pattern should change as the arrival time for each beam changed. Their hope was to measure this change and use it to measure the speed of Earth through the ether. The relationship between the motion of Earth and two perpendicular interferometer positions is shown in Figure 11.6.

It was an elegant experiment, and yet it seemed to be a disaster. The interference pattern refused to change. This lack of change, or nul result, greatly discouraged the two experimenters and was a source of puzzlement for other physicists. Could it be that Earth really did not move at all relative to the ether? This did not make sense, since Earth obviously orbited the Sun. Did Earth drag the luminiferous ether along with it? This did not seem likely, since that would affect the appearance of stars as seen from Earth.

One guess, which in a sense paved the way for the relativity answer, was that objects that moved through the ether were compressed, just as a spring could be compressed if it was pushed lengthwise through oil. This contraction would cause a shortening of lengths in the direction of motion, thus reducing the time required for the light to make the round trip. In this way, both light beam X and light beam Y would always arrive at the telescope at the same time. This hypothesis was known as the **Lorentz-Fitzgerald contraction**.

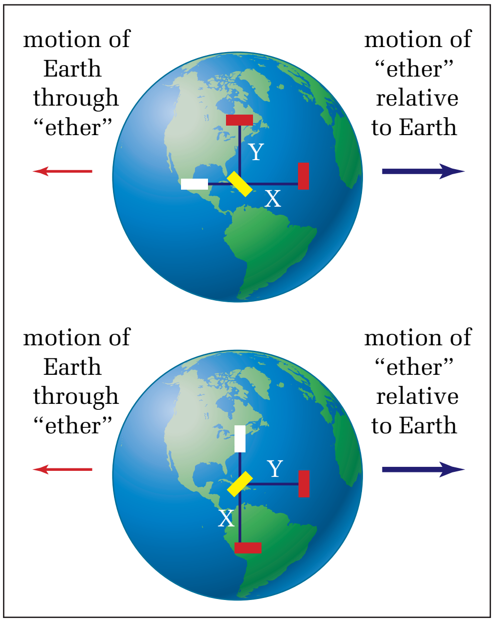


Figure 11.6 Arrival times for the light beams were expected to change when the interferometer was rotated by 90˚, but this did not happen.

The Theoretical Speed of Light

The strange results of the Michelson-Morley experiment remained a mystery for nearly two decades. Then, in 1905, the explanation came with Albert Einstein's publication of his special theory of relativity. He had developed this theory while considering the propagation of electromagnetic waves, as described by James Clerk Maxwell. Maxwell's equations showed how electromagnetic waves would spread out from accelerated charges.

In the early 1870s, Maxwell realized that a changing magnetic field could induce a changing electric field and that the changing electric field could in turn induce a changing magnetic field. Most importantly, he realized that these mutually inducing fields could spread out through space with a speed given by $c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}}$, where c represents the speed at which the fields spread out through space (the speed of light in a vacuum), ε_0 represents the electric permittivity of free space ($\varepsilon_0 = 8.85 \times 10^{-12}\ \mathrm{C^2/N \cdot m^2}$), and μ_0 represents the magnetic permeability of free space ($\mu_0 = 4\pi \times 10^{-7}\ \mathrm{T \cdot m/A}$).

This formula yields a speed for electromagnetic radiation through space of 3.00×10^8 m/s. This was a major triumph in the field of theoretical physics, since it predicted the speed of light in terms of basic properties involving the behaviour of electric and magnetic fields in space. In addition, there was now no necessity for assuming the existence of luminiferous ether — magnetic and electric fields can exist in space without such a medium.

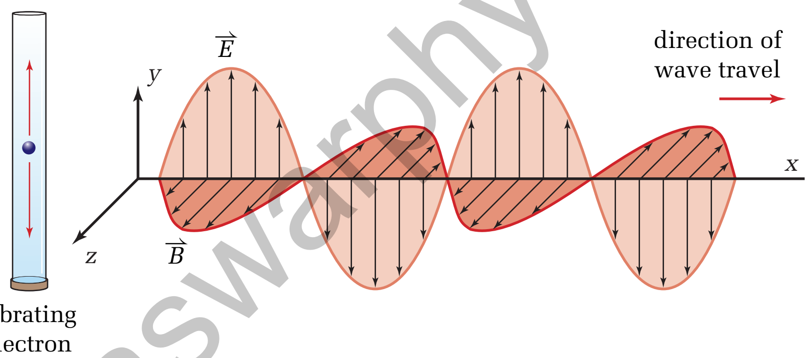


Figure 11.7 In this diagram, $\vec{E}$ represents the electric field, while $\vec{B}$ represents the magnetic field.

Einstein, however, was puzzled by an apparent inconsistency in this equation. It did not indicate any particular frame of reference. The laws of physics are expected to be valid in any inertial frame of reference. However, quantities such as speed and velocity could appear to be different from different frames of reference. For example, a race car can be seen to travel at a high speed relative to spectators in the stands. However, it might have zero velocity relative to another race car.

PHYSICS FILE

Toward the end of Michelson's life, Einstein praised him publicly for his ground-breaking experiments, which provided the first experimental confirmation for the special theory of relativity.

PHYSICS FILE

Electric permittivity is related to the Coulomb constant (k): $\varepsilon_0 = \frac{1}{4\pi k}$. Magnetic permeability comes from the expression for the strength of the magnetic field in the vicinity of a current-carrying conductor. The equation for the magnetic field, $\vec{B}$, is $\vec{B} = \frac{\mu_0 I}{2\pi r}$, where I is the current in the wire in amperes and r is the radial distance from the wire.

Apparently, there was no specified frame of reference for the speed of light in Maxwell's equation. This implied that the speed of light (and, in fact, of all members of the electromagnetic spectrum) through a vacuum should be seen as being the same in any inertial frame of reference. Einstein realized that this was indeed the case and announced his special theory of relativity.

Figure 11.8 In any race, relative velocity is all that counts.

The Special Theory of Relativity

Einstein based his special theory of relativity on two postulates.

1. All physical laws must be equally valid in all inertial frames of reference.
2. The speed of light through a vacuum will be measured to be the same in all inertial frames of reference.

The first statement had been accepted since the time of Galileo and Newton. The second one was a radical departure from the common understanding of the basics of physics, so it took scientists a long time to accept it. Eventually it was accepted, though, and the special theory of relativity is now considered to be one of the principal scientific triumphs of the twentieth century.

11.1 Section Review

1. K/U Make sketches of the velocity vectors identical to those in Figures 11.3 and 11.4 on pages 467 and 468. Label the vectors as though they represented the velocities of light through the ether, the ether relative to Earth, and light relative to Earth.
2. MC Show that the units used in Maxwell's equation for the speed of light simplify to metres per second. Note that $1\ \mathrm{T} = 1\ \mathrm{N/A \cdot m}$.
3. I In the interferometer shown in Figure 11.5, how far in wavelengths would the adjustable mirror have to move so that the interference pattern would return to its initial appearance? Hint: Review the interference relationships found in Chapter 9.
4. K/U What caused physicists to assume that space was filled with a medium that they called the "luminiferous ether"?
5. MC
   - **(a)** State the two basic postulates of the special theory of relativity.
   - **(b)** Explain why the constancy of the speed of a light beam, as seen from different inertial frames of reference, seems to be wrong. Try to use commonplace examples to make your point.

11.2 The Basics of the Special Theory of Relativity

Einstein's special theory of relativity changed our fundamental understanding of distance, time, and mass. He used his famous thought experiments to illustrate these new concepts. This section contains several thought experiments similar to the ones Einstein used.

SECTION EXPECTATIONS

- Describe Einstein's thought experiments relating to the constancy of the speed of light in all inertial frames of reference, time dilation, and length contraction.

KEY TERMS

- simultaneity
- time dilation
- proper time
- dilated time
- length contraction
- proper length
- relativistic speeds
- gamma

Thought Experiment 1: Simultaneity

Imagine that you are sitting high on a hill on Canada Day and you can see two different celebrations going on in the distance. You are startled when two sets of fireworks ignite at exactly the same time — one off to your left and the other far to your right. About 100 m behind you, a car is travelling along a highway at 95 km/h. Do the passengers in the car see the fireworks igniting simultaneously or do they think that one set ignited before the other? Your immediate reaction is probably, "Of course they saw the fireworks igniting simultaneously — they were simultaneous!"

According to Einstein's special theory of relativity, however, the answer is not quite so simple. To restate the question more precisely, are two events that are simultaneous for an observer in one inertial reference frame simultaneous for observers in all inertial reference frames? The answer is no. The constancy of the speed of light creates problems with the **simultaneity** of events, as the situation in Figure 11.9 illustrates.

In Figure 11.9 (A), observers A and B are seated equidistant from a light source (S). The light source flashes. Since the light must travel an equal distance to both observers, they would say that they received the flash at exactly the same time, that the arrival of the flash was simultaneous for both of them.

Now imagine that these two observers are actually sitting on a railway flatcar that is moving to the right with velocity $\vec{v}$ relative to the ground and to observer C in Figure 11.9 (B). Observer C makes two observations.

1. B is moving away from the point from which the light was emitted.
2. A is moving toward the point from which the light was emitted.

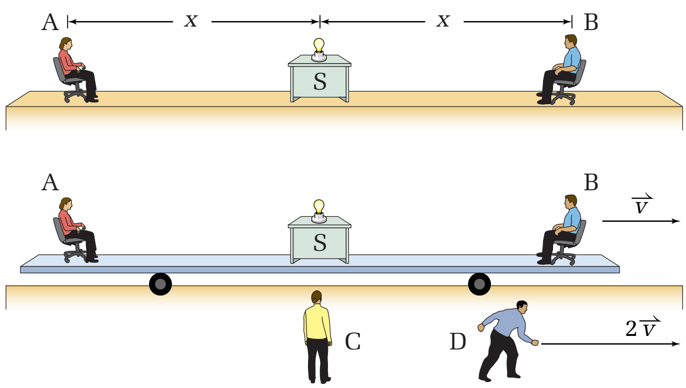


Figure 11.9 Do events that appear to be simultaneous to observers A and B also appear to be simultaneous to observers C and D?

Observer C concludes that it takes longer for light to reach B than it does to reach A. Thus, according to observer C, observer A received the flash first and B received it second. The arrivals are not simultaneous in C's frame of reference, and yet it is an inertial reference just as much as is the frame of reference of the flatcar.

In the frame of reference for observer D, who is moving to the right with a velocity of $2\vec{v}$, the flatcar is moving toward the left with a velocity of $\vec{v}$. Now, it is A who is moving away from the point from which the flash was emitted and B is moving toward that emission point. The light would take longer to reach A, so the light would arrive at observer B first.

As you can see from this example, the whole concept of simultaneity, of past, present, and future, is fuzzy in relativity. What is a future event in one frame of reference becomes a past event in another. This is due entirely to the fact that the speed of light is the same in all inertial frames of reference, regardless of their relative velocities.

Thought Experiment 2: Time Dilation

Imagine yourself back on the hilltop, watching fireworks. You look at your watch at the moment that the fireworks ignite and it says 11:23 P.M. What do the watches of the passengers in the car read? If they saw the fireworks ignite at different times, their watches cannot possibly agree with yours.

The constancy of the speed of light creates problems with time intervals. The term **time dilation** applies to situations in which time intervals appear different to observers in different inertial frames of reference. To understand the implications of this constant speed of light for time measurement, assume that an experimenter has devised a light clock. In it, a pulse of light reflects back and forth between two mirrors, A and B. The time that it takes for the pulse to travel between the mirrors is the basic tick of this clock. Figure 11.10 (A) shows such a "tick."

Now, picture this clock in a spacecraft that is speeding past Earth. An observer in the spacecraft sees the light as reflecting back and forth as it was before, so the basic tick of the clock has not changed. However, an observer on Earth would see that the mirrors moved

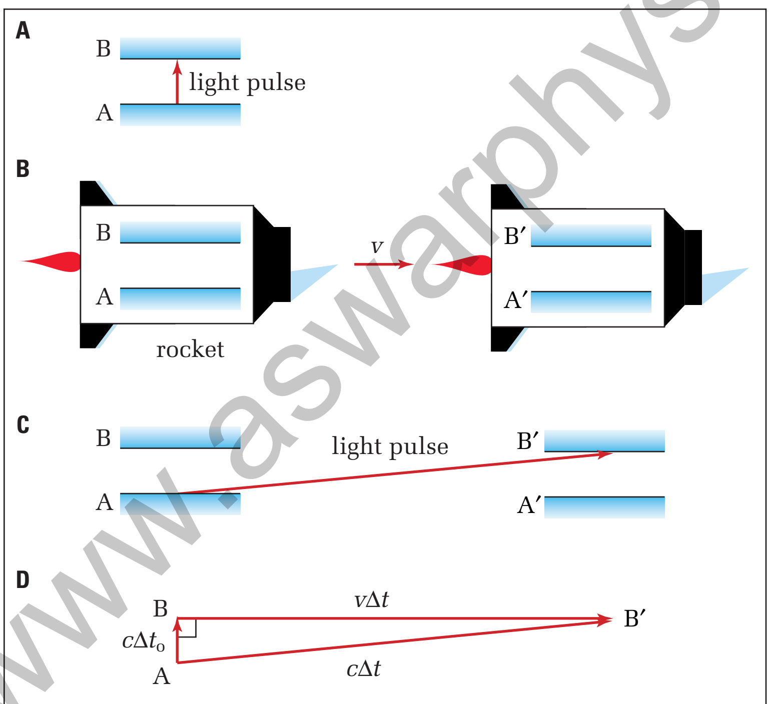


Figure 11.10 If the speed of light is the same to all observers, then light takes longer to travel from A to B′ than it does to travel from A to B.

while the light pulse travelled from A to B, as shown in Figure 11.10 (B). Since the pulse actually has to travel from A to B′, it must take longer, as indicated in Figure 11.10 (C). The tick of the clock therefore takes longer to occur in the Earth frame of reference than in the spacecraft observer's frame of reference. In fact, if the spacecraft observer was wearing a watch, the Earth observer would say that the watch was counting out the seconds too slowly. The spacecraft observer, however, would say that the watch and the light clock were working properly.

The relationship between times as measured in the spacecraft and on Earth can be deduced from Figure 11.10 (D). Assume that

- c is the speed of light, which is the same for all observers
- Δt is the time that the Earth observer says it takes for the pulse to travel between the mirrors
- Δt_o is the time that the spacecraft observer says it takes for the pulse to travel between the mirrors

The distance from A to B would be $c\Delta t_o$. The distance travelled by the spacecraft would be $v\Delta t$, since this involves a distance, speed, and time observed by the Earth observer.

The Earth observer claims that the light pulse actually travelled a distance of $c\Delta t$. These distances represent the lengths of the sides of a right-angled triangle, as seen in Figure 11.10 (D). Notice how similar this result is to the arrival-time equation in the boat X-boat Y scenario on pages 467 and 468.

- Apply the Pythagorean theorem and expand.

$$(c\Delta t)^2 = (c\Delta t_o)^2 + (v\Delta t)^2$$
$$c^2\Delta t^2 = c^2\Delta t_o^2 + v^2\Delta t^2$$

- Solve for $c^2\Delta t_o^2$.

$$c^2\Delta t_o^2 = c^2\Delta t^2 - v^2\Delta t^2$$

- Factor out a Δt^2.

$$c^2\Delta t_o^2 = \Delta t^2(c^2 - v^2)$$

- Divide by c^2.

$$\Delta t_o^2 = \frac{\Delta t^2(c^2 - v^2)}{c^2}$$

- Simplify, then take the square root of both sides of the equation.

$$\Delta t_o^2 = \Delta t^2\left(1 - \frac{v^2}{c^2}\right)$$
$$\Delta t_o = \Delta t\sqrt{1 - \frac{v^2}{c^2}}$$

- Solve for Δt.

$$\Delta t = \frac{\Delta t_o}{\sqrt{1 - \frac{v^2}{c^2}}}$$

In any question involving relativistic times, it is important to carefully identify the times.

- Δt_o is the time as measured by a person at rest relative to the object or the event. It is called the **proper time**. You could think of it as the "rest time," although this term is not generally used. Another way to picture it is as the "one-point" time, the time for an observer who sees the clock as staying at only one point.

ELECTRONIC LEARNING PARTNER

Relativity

Experiment with near-light speeds and time dilation by using your Electronic Learning Partner.

MATH LINK

Note that the negative square root has no meaning in this situation. Both times will be seen as positive. In addition, v must be less than c. If it was greater than c, the denominator would become the square root of a negative number. Although such a square root can be expressed using complex numbers, it is not expected that a time measurement would involve anything other than the set of real numbers.

- Δt is the expanded or **dilated time**. Since the denominator $\sqrt{1 - \frac{v^2}{c^2}}$ is less than one, Δt is *always* greater than Δt_o. It can also be thought of as the "two-point" time, the time as measured by an observer who sees the clock as moving between two points.

DILATED TIME

The dilated time is the quotient of the proper time and the expression: square root of one minus the velocity of the moving reference frame squared divided by the speed of light squared.

$$\Delta t = \frac{\Delta t_o}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Quantity	Symbol	SI unit
dilated time	Δt	s (seconds)
proper time	Δt_o	s (seconds)
velocity of the moving reference frame	v	$\frac{\text{m}}{\text{s}}$ (metres per second)
speed of light	c	$\frac{\text{m}}{\text{s}}$ (metres per second)

Unit Analysis

$$\text{seconds} = \frac{\text{seconds}}{\sqrt{1 - \frac{\left(\frac{\cancel{\text{metres}}}{\cancel{\text{second}}}\right)^2}{\left(\frac{\cancel{\text{metres}}}{\cancel{\text{seconds}}}\right)^2}}} = \text{seconds} \qquad \text{s} = \frac{\text{s}}{\sqrt{1 - \frac{\left(\frac{\cancel{\text{m}}}{\cancel{\text{s}}}\right)^2}{\left(\frac{\cancel{\text{m}}}{\cancel{\text{s}}}\right)^2}}} = \text{s}$$

SAMPLE PROBLEM

Relative Times

A rocket speeds past an asteroid at 0.800 *c*. If an observer in the rocket sees 10.0 s pass on her watch, how long would that time interval be as seen by an observer on the asteroid?

Conceptualize the Problem

- *Proper time*, Δt_o, and *dilated time*, Δt, are not the same. *Time intervals* appear to be *shorter* to the observer who is *moving* at a velocity close to the speed of light.
- Proper time, Δt_o, and dilated time, Δt, are related by the *speed of light*, *c*.

Identify the Goal

The amount of time, Δt, that passes for the observer on the asteroid while 10.0 s passes for the observer on the rocket

PROBLEM TIP

Since $\frac{v^2}{c^2}$ is a ratio, the speeds can have any units as long as they are the same for both the numerator and the denominator. It is often useful to express *v* in terms of *c*.

Identify the Variables and Constants

Known	**Implied**	**Unknown**
$v_{\text{rocket}} = 0.800\ c$	$c = 3.00 \times 10^8\ \dfrac{\text{m}}{\text{s}}$	Δt
$\Delta t_o = 10.0$ s		

Develop a Strategy

Select the equation that relates dilated time to proper time.

$$\Delta t = \frac{\Delta t_o}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Substitute into the equation.

$$\Delta t = \frac{10.0\ \text{s}}{\sqrt{1 - \frac{(0.800\ c)^2}{c^2}}}$$

Solve.

$$\Delta t = \frac{10.0\ \text{s}}{0.600}$$

$$\Delta t = 16.67\ \text{s}$$

$$\Delta t \cong 16.7\ \text{s}$$

The time as seen by an observer on the asteroid would be 16.7 s.

Validate the Solution

The dilated time is expected to be longer than the proper time, and it is.

PRACTICE PROBLEMS

1. A tau (τ) particle has a lifetime measured at rest in the laboratory of 1.5×10^{-13} s. If it is accelerated to 0.950 c, what will be its lifetime as measured in **(a)** the laboratory frame of reference, and **(b)** the τ particle's frame of reference?
2. A rocket passes by Earth at a speed of 0.300 c. If a person on the rocket takes 245 s to drink a cup of coffee, according to his watch, how long would that same event take according to an observer on Earth?
3. A kaon particle (κ) has a lifetime at rest in a laboratory of 1.2×10^{-8} s. At what speed must it travel to have its lifetime measured as 3.6×10^{-8} s?

Thought Experiment 3: Length Contraction

Imagine the following situation. Captain Quick is a comic book hero who can run at nearly the speed of light. In her hand, she is carrying a flare with a lit fuse set to explode in 1.50 μs (1.50×10^{-6} s). The flare must be placed into its bracket before this happens. The distance (L) between the flare and the bracket is 402 m.

PHYSICS FILE

As you will discover in Chapter 13, The Nucleus and Elementary Particles, many subatomic particles come into existence and decay into some other particles in very short periods of time. The tau and kaon particles are examples of these subatomic particles.

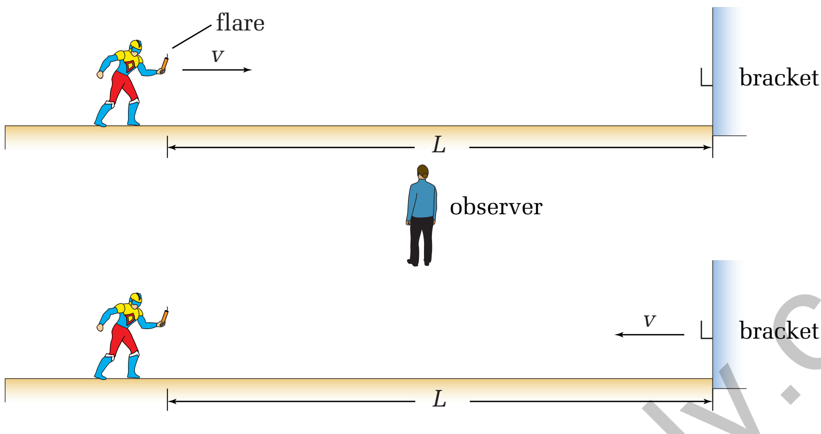


Figure 11.11 A race against time

- Captain Quick runs at $\frac{2}{3}c$ (2.00×10^8 m/s) and arrives at the bracket in time. According to classical mechanics, this would not be possible because it should take 2.01 μs as shown on the right.

$$\Delta t = \frac{L}{v}$$
$$\Delta t = \frac{402\ \text{m}}{2.00 \times 10^8\ \frac{\text{m}}{\text{s}}}$$
$$\Delta t = 2.01 \times 10^{-6}\ \text{s or } 2.01\ \mu\text{s}$$

- However, to an observer in the stationary frame of reference, the time for the fuse to burn will be dilated in relation to his own frame of reference. It will take 2.01 μs for the fuse to burn and therefore, Captain Quick will reach the bracket in time.

$$\Delta t = \frac{\Delta t_o}{\sqrt{1 - \frac{v^2}{c^2}}}$$
$$\Delta t = \frac{1.50 \times 10^{-6}\ \text{s}}{\sqrt{1 - \frac{\left(\frac{2}{3}c\right)^2}{c^2}}}$$
$$\Delta t = \frac{1.50 \times 10^{-6}\ \text{s}}{0.7454}$$
$$\Delta t = 2.01 \times 10^{-6}\ \text{s}$$

- Since Captain Quick and the fuse are in the same frame of reference, however, Captain Quick should observe the fuse burning in 1.50 μs. How did she make it in time? Then she realized that the only way she could have arrived in time was if the *distance* to the bracket in her moving frame of reference was less than the 402 m in the stationary frame. The distance must have been *multiplied* by the same factor by which the time was *divided* in the observer's frame of reference.

$$L = L_o\sqrt{1 - \frac{v^2}{c^2}}$$
$$L = (402\ \text{m})\sqrt{1 - \frac{\left(\frac{2}{3}c\right)^2}{c^2}}$$
$$L = (402\ \text{m})(0.7454)$$
$$L = 300\ \text{m}$$

- If the distance was smaller, then Captain Quick could make it to the bracket before the fuse burned out.

$$\Delta t = \frac{L}{v}$$

$$\Delta t = \frac{300\ \text{m}}{2.00 \times 10^{8}\ \frac{\text{m}}{\text{s}}}$$

$$\Delta t = 1.50 \times 10^{-6}\ \text{s or } 1.50\ \mu\text{s}$$

This thought experiment illustrates that two ideas go hand in hand. If two observers are moving relative to each other, then a time dilation from one observer's point of view will be balanced by a corresponding **length contraction** from the other observer's point of view.

In the box below, L_o represents the **proper length**, which is the length as measured by an observer at rest relative to the object or event and L is the contracted length seen by the moving observer.

LENGTH CONTRACTION

The contracted length is the product of the proper length and the expression, square root of one minus the velocity of the moving reference frame squared divided by the speed of light squared.

$$L = L_o\sqrt{1 - \frac{v^2}{c^2}}$$

Quantity	Symbol	SI unit
contracted length	L	m (metres)
proper length	L_o	m (metres)
velocity of the moving reference frame	v	$\frac{\text{m}}{\text{s}}$ (metres per second)
speed of light	c	$\frac{\text{m}}{\text{s}}$ (metres per second)

Unit Analysis

$$\text{metres} = \frac{\text{metres}}{\sqrt{1 - \frac{\left(\cancel{\frac{\text{metres}}{\text{second}}}\right)^2}{\left(\cancel{\frac{\text{metres}}{\text{seconds}}}\right)^2}}} = \text{metres}$$

$$\text{m} = \frac{\text{m}}{\sqrt{1 - \frac{\cancel{\left(\frac{\text{m}}{\text{s}}\right)^2}}{\cancel{\left(\frac{\text{m}}{\text{s}}\right)^2}}}} = \text{m}$$

Note: Length contraction applies *only* to lengths measured *parallel* to the direction of the velocity. Lengths measured perpendicular to the velocity are not affected.

This thought experiment seems to yield strange results that go against common experience. However, the results explain a phenomenon involving a tiny particle called the "mu meson" (or muon). This particle has a lifetime of 2.2×10^{-6} s and is formed about 1.0×10^4 m above the surface of Earth, speeding downward at about 0.998 c. At that speed (according to classical mechanics), it should travel only about 660 m before decaying into other particles, but it is observed in great numbers at Earth's surface. The relativistic explanation is that the muon's lifetime as measured by Earth-based observers has been dilated as follows.

$$\Delta t = \frac{\Delta t_o}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$\Delta t = \frac{2.2 \times 10^{-6}\text{ s}}{\sqrt{1 - \frac{(0.998\,c)^2}{c^2}}}$$

$$\Delta t = \frac{2.2 \times 10^{-6}\text{ s}}{0.0632}$$

$$\Delta t = 3.5 \times 10^{-5}\text{ s}$$

The distance travelled becomes

$$\Delta d = v\Delta t$$

$$\Delta d = (0.998)\left(3.00 \times 10^8\ \frac{\text{m}}{\text{s}}\right)(3.5 \times 10^{-5}\text{ s})$$

$$\Delta d = 1.0 \times 10^4\text{ m}$$

At that speed, the muon's lifetime is so expanded (according to the observers on Earth) that the particle can reach the surface. On the other hand, the muon sees its own lifetime as unchanged, and from its frame of reference, Earth's surface is rushing toward it at 0.998 c. The distance it sees to Earth's surface is given by

$$L = L_o\sqrt{1 - \frac{v^2}{c^2}}$$

$$L = (1.0 \times 10^4\text{ m})\sqrt{1 - \frac{(0.998\,c)^2}{c^2}}$$

$$L = (1.0 \times 10^4\text{ m})(0.0632)$$

$$L = 632\text{ m}$$

This reduced distance would take a shorter time, given by

$$\Delta t = \frac{\Delta x}{v}$$

$$\Delta t = \frac{632\text{ m}}{(0.998)\left(3.0 \times 10^8\ \frac{\text{m}}{\text{s}}\right)}$$

$$\Delta t = 2.1 \times 10^{-6}\text{ s}$$

The muon therefore can reach Earth's surface before decaying.

Which Is Correct?

The physicist standing on the surface of Earth claims that the lifetime of the muon is 3.5×10^{-5} s and its height above Earth's surface is 1.0×10^{4} m. From the muon's point of view, however, its lifetime is 2.2×10^{-6} s and its height is 632 m. Which is correct?

Both statements are correct. The value of any measurement is tied to the frame of reference in which that measurement is taken. Going from one inertial frame of reference to another will involve differences in the measurement of lengths and times. Normally, these differences are too small to be observed, but as relative speeds approach the speed of light, these differences become quite apparent.

Gamma Saves Time

When solving problems involving **relativistic speeds** (speeds approaching the speed of light), you will often need to calculate the value of $\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$. Physicists have assigned the symbol **gamma** (γ) to this value, or $\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$. Using the γ notation, the length and time equations become $\Delta t = \gamma \Delta t_o$ and $L = \frac{L_o}{\gamma}$.

SAMPLE PROBLEM

Relativistic Lengths

A spacecraft passes Earth at a speed of 2.00×10^{8} m/s. If observers on Earth measure the length of the spacecraft to be 554 m, how long would it be according to its passengers?

Conceptualize the Problem

- *Length* appears to be shorter, or *contracted*, to the *observer* who is *moving* relative to the object being measured.
- The amount of *length contraction* that occurs is determined by the *relative speeds* of the reference frames of the two observers.

Identify the Goal

The length of the spacecraft, L_o, as seen by its passengers

Identify the Variables and Constants

Known	Implied	Unknown
$v = 2.00 \times 10^{8}\ \frac{\text{m}}{\text{s}}$	$c = 3.00 \times 10^{8}\ \frac{\text{m}}{\text{s}}$	L_o
$L = 554$ m		

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Develop a Strategy

Calculate gamma.

$$\gamma = \sqrt{1 - \frac{v^2}{c^2}}$$

$$\gamma = \sqrt{1 - \frac{(2.00 \times 10^8\ \frac{\text{m}}{\text{s}})^2}{(3.00 \times 10^8\ \frac{\text{m}}{\text{s}})^2}}$$

$$\gamma = 1.342$$

Use the equation that describes length contraction.

$$L = \frac{L_o}{\gamma}$$

$$L_o = L\gamma$$

Solve.

$$L_o = (554\ \text{m})(1.342)$$

$$L_o = 743.2\ \text{m}$$

$$L_o \cong 743\ \text{m}$$

The length of the spacecraft as seen by its passengers is 743 m.

Validate the Solution

The proper length is expected to be longer than the contracted length, and it is.

PRACTICE PROBLEMS

4. An asteroid has a long axis of 725 km. A rocket passes by parallel to the long axis at a speed of 0.250 c. What will be the length of the long axis as measured by observers in the rocket?

5. An electron is moving at 0.95 c parallel to a metre stick. How long will the metre stick be in the electron's frame of reference?

6. A spacecraft passes a spherical space station. Observers in the spacecraft see the station's minimum diameter as 265 m and the maximum diameter as 325 m.

(a) How fast is the spacecraft travelling relative to the space station?

(b) Why does the station not look like a sphere to the observers in the spacecraft?

PHYSICS FILE

Einstein's equations allow a particle to travel faster than light if it was already travelling faster than light when it was created. For such particles (called "tachyons"), the speed of light represents the slowest speed limit. Although the equations say that tachyons can exist, there is no evidence that they do. In fact, no one knows how they would interact with normal matter.

The Universal Speed Limit

Calculation of expanded times and contracted lengths involve the expression $\sqrt{1 - \frac{v^2}{c^2}}$. Since times and lengths are measurements, they must be represented by real numbers, so the value under the square root must be a *positive* real number. For this to be true, $\frac{v^2}{c^2} < 1$. This implies that $v < c$. If v approaches c, the value of gamma approaches infinity. Consider what happens to Δt when v approaches c in $\Delta t = \frac{\Delta t_o}{\sqrt{1 - \frac{v^2}{c^2}}}$. The denominator approaches zero.

Division of a non-zero real number by zero is undefined so an object's speed must be less than the speed of light.

This speed limit applies only to material objects. Obviously, light can travel at the speed of light. Also, once a light pulse has been slowed down by passing into a medium such as water, objects can travel faster through that medium than can the pulse. The blue glow (called "Cerenkov radiation") emanating from water in which radioactive material is being stored is created by high-speed electrons (beta particles) that are travelling through the water faster than the speed of light through water. This phenomenon is sometimes compared to sonic boom, in which particles (in the form of a jet airplane) are travelling faster than the speed of sound in air.

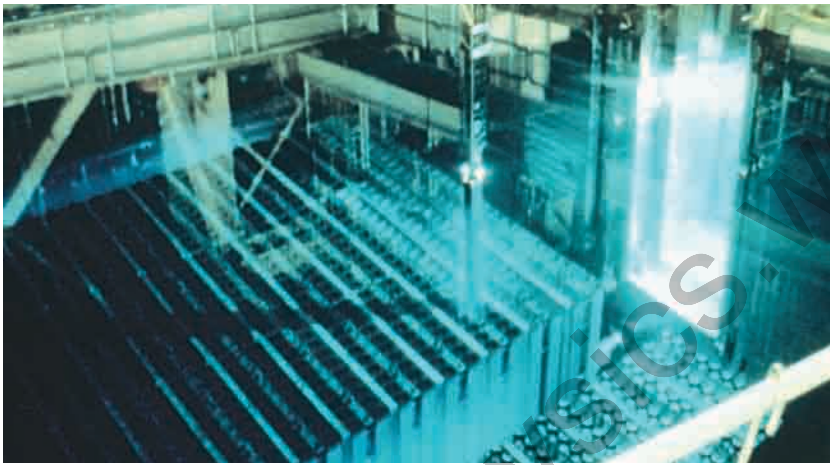


Figure 11.12 The blue glow from this storage pool in a nuclear generating station comes from particles that are travelling through the water faster than the speed of light through water.

11.2 Section Review

1. K/U
   (a) Explain what is meant by an inertial frame of reference.
   (b) Would a rotating merry-go-round be an inertial frame of reference? Give reasons for your answer.

2. K/U Explain the meaning of the terms "proper length" and "proper time."

3. I An arrow and a pipe have exactly the same length when lying side by side on a table. The arrow is then fired at a relativistic speed through the pipe, which is still lying on the table. Determine whether there is a frame of reference in which the arrow can
   (a) be completely inside the pipe with extra pipe at each end
   (b) overhang the pipe at each end

   Give reasons for your answers.

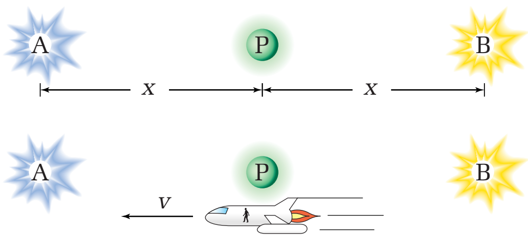
4. K/U Explain the meaning of the terms "length contraction" and "time dilation."

5. C Explain why the results of the Michelson-Morley experiment were so important.

6. K/U

(a) In the diagram, two stars (A and B) are equidistant from a planet (P) and are at rest relative to that planet. They both explode into novas at the same time, according to an observer on the planet. From the point of view of passengers in a rocket ship travelling past at relativistic speeds, however, which star went nova first? Give reasons for your answer.

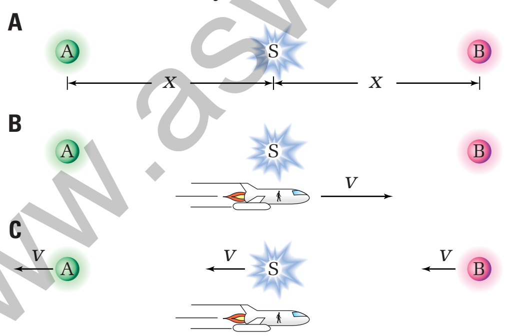
(b) Where could the observer stand on the planet in order to see both stars at the same time?



7. K/U Part (A) of the diagram shows a star (S) located at the midpoint between two planets (A) and (B), which are at rest relative to the star. The star explodes into a supernova.

(a) In the frame of reference of the planets, which planet saw the supernova first? Give reasons for your answer.

(b) A spacecraft is passing by as shown in part (B). In its frame of reference, the star and planets are moving as shown in part (C). In the spacecraft frame of reference, which planet saw the supernova first? Give reasons for your answer.



8. I

(a) Imagine that you are riding along on a motorcycle at 22 m/s and throw a ball ahead of you with a speed of 35 m/s. What will be the speed of that ball relative to the ground?

(b) If the velocity of the motorcycle relative to the ground is v_{mg}, the velocity of the ball relative to the motorcycle is v_{bm}, and the velocity of the ball relative to the ground is v_{bg}, state the vector equation for calculating the velocity of the ball relative to the ground.

(c) Apply this formula to a situation in which the motorcycle is travelling at 0.60 c and the ball is thrown forward with a speed of 0.80 c. What is the speed of the ball relative to the ground? What is wrong with this answer?

(d) In the special theory of relativity, the formula for adding these velocities is

$$v_{bg} = \frac{v_{bm} + v_{mg}}{1 + \frac{v_{bm} \cdot v_{mg}}{c^2}}$$

- What does this formula predict for the answer to (c)?
- What does this formula predict for the answer to (a)?
- Imagine that you are travelling in your car at a speed of 0.60 c and you shine a light beam ahead of you that travels away from you at a speed of c. According to this formula, what would be the speed of that light beam relative to the ground?

UNIT PROJECT PREP

How would the general public have received the new information in Einstein's special theory of relativity?

- Do you believe that at the turn of the twentieth century society had more or less faith in science than people do today? Why or why not?
- Dramatic events often steer thinking into new directions. Do you believe that Einstein was affected by any one particular event as he developed his theories?
- Are you able to link recent societal events with current changes in the direction of scientific research?

CAREERS IN PHYSICS

Not Even the Sky's the Limit!

When a signal leaves a satellite or interplanetary space probe, a special code is embedded in it to give it a time-stamp. When the signal is picked up on Earth, that time-stamp is compared to a terrestrial clock. Subtracting the two gives the travel time between the satellite or space probe and the ground station. Since the signal travels at the speed of light, all you should need to do then is multiply the time by c to determine the distance — but it's not that simple.

Gravity is part of the problem. According to Einstein's general theory of relativity, clocks run more slowly in a gravitational field. The stronger the field is, the slower the clocks run. Clocks on board spacecraft and satellites run slightly faster in interplanetary space than they do near Earth. These timing differences result in distance measurement differences between what is observed from a ground station and from a spacecraft. The situation becomes even more complicated as the spacecraft dips into and out of the gravitational fields of planets that it encounters on its voyage.

A second problem results from the relative velocity between the spacecraft and the ground station. Einstein's special theory of relativity, discussed in detail in this chapter, describes how time intervals and distance measurements vary between inertial frames of reference that are in motion relative to each other. This relative velocity is continually changing as a result of the gravity of the Sun and planets and due to Earth's orbital and rotational velocities. Relativistic corrections — numerical adjustments based on the theory of relativity — are an ongoing challenge in spacecraft instrument design.

There are "a whole suite of careers that utilize these things," Steve Lichten, manager of the Tracking Systems and Applications Section of NASA's Jet Propulsion Laboratory, says of relativity. Einsteinian physics is no longer the sole property of university researchers. Commercial satellite manufacturers must have an understanding of relativity in order for their products to work.

Theoreticians, engineers, and computer scientists must work together to help a spacecraft communicate with its ground station, so the companies that manufacture spacecraft and commercial satellites are always on the lookout for people with the necessary knowledge. Generally, an advanced graduate degree in engineering, physics, or mathematics is preferred, although a bachelor's degree in science with a demonstrated understanding of the concepts and techniques involved will go a long way.

So brush up your math skills and keep doing those thought experiments. Some day, they might take you to the stars!

Going Further

1. Describe some examples of satellites that require extremely precise distance and time measurements. Explain why such precision is necessary for those satellites.
2. Many companies that manufacture satellites or equipment for use on satellites (including space stations) offer summer internship programs for interested students. Find out if any of these companies are located near you and call them. You might be able to get a head start on a great career!
3. Research the space probes, such as the one shown in the photograph, that are currently active. Explain why precise knowledge of time intervals and distances is of extreme importance to the operation of space probes.

WEB LINK

www.mcgrawhill.ca/links/physics12

For information about the NASA Jet Propulsion Laboratory's past, current, and planned space missions, go to the above Internet site and click on **Web Links**.

11.3 Mass and Energy

SECTION EXPECTATIONS

- Conduct thought experiments as a way of developing an abstract understanding of the physical world as it relates to mass increase when an object approaches the speed of light.
- Apply quantitatively the laws of conservation of mass and energy, using Einstein's mass-energy equivalence.

KEY TERMS

- rest mass
- relativistic mass
- total energy
- rest energy

In the last section, you read a discussion based on mathematical equations that explained why no object with mass can travel at or above the speed of light. The discussion probably left you wondering why. If, for example, a spacecraft is travelling at 0.999 c, what would prevent it from burning more fuel, exerting more reaction force, and increasing its speed up to c?

The fact that no amount of extra force will provide that last change in velocity is explained when you discover that the mass of the spacecraft is also increasing. Einstein showed that, just as time dilates and length contracts when an object approaches the speed of light, its mass increases according to the equation $m = \frac{m_o}{\sqrt{1 - \frac{v^2}{c^2}}}$. In the equation, m is the mass as measured by an observer who sees the object moving with speed v, and m_o is the mass as measured by an observer at rest relative to the object. The mass, m, is sometimes called the **relativistic mass** and m_o is known as the **rest mass**.

As the speed of the object increases, the value of the denominator $\sqrt{1 - \frac{v^2}{c^2}}$ decreases. As v approaches c, the denominator approaches zero and the mass increases enormously. If v could equal c, the mass would become $\frac{m_o}{0}$, which is undefined. The speed of an object, measured from any inertial frame of reference, therefore must be less than the speed of light through space.

RELATIVISTIC MASS

Relativistic mass is the quotient of rest mass and gamma.

$$m = \frac{m_o}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Quantity	Symbol	SI unit
relativistic mass	m	kg (kilograms)
rest mass	m_o	kg (kilograms)
speed of the mass relative to observer	v	$\frac{\text{m}}{\text{s}}$ (metres per second)
speed of light	c	$\frac{\text{m}}{\text{s}}$ (metres per second)

Unit Analysis

$$\text{kilograms} = \frac{\text{kilograms}}{\sqrt{1 - \frac{\left(\cancel{\frac{\text{metres}}{\text{second}}}\right)^2}{\left(\cancel{\frac{\text{metres}}{\text{seconds}}}\right)^2}}} = \text{kilograms} \qquad \text{kg} = \frac{\text{kg}}{\sqrt{1 - \frac{\left(\cancel{\frac{\text{m}}{\text{s}}}\right)^2}{\left(\cancel{\frac{\text{m}}{\text{s}}}\right)^2}}} = \text{kg}$$

SAMPLE PROBLEM

Relativistic Masses

An electron has a rest mass of 9.11×10^{-31} kg. In a detector, it behaves as if it has a mass of 12.55×10^{-31} kg. How fast is that electron moving relative to the detector?

Conceptualize the Problem

- The *mass* of the object appears to be *much greater* to an *observer* in a frame of reference that is *moving at relativistic speeds* than it does to an observer in the *frame of reference of the object*.
- The amount of the *increase in mass* is determined by the ratio of the *object's speed* and the *speed of light*.

Identify the Goal

Determine the speed, v, of the electron relative to the detector

Identify the Variables and Constants

Known	Implied	Unknown
$m_o = 9.11 \times 10^{-31}$ kg $m = 12.55 \times 10^{-31}$ kg	$c = 3.00 \times 10^8 \frac{\text{m}}{\text{s}}$	v

Develop a Strategy

Use the equation that relates the relativistic mass, rest mass, and speed.

$$m = \frac{m_o}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Solve the equation for speed.

$$\sqrt{1 - \frac{v^2}{c^2}} = \frac{m_o}{m}$$

$$1 - \frac{v^2}{c^2} = \left(\frac{m_o}{m}\right)^2$$

$$\frac{v^2}{c^2} = 1 - \left(\frac{m_o}{m}\right)^2$$

$$v^2 = c^2\left(1 - \left(\frac{m_o}{m}\right)^2\right)$$

$$v = c\sqrt{1 - \left(\frac{m_o}{m}\right)^2}$$

Substitute numerical values and solve.

$$v = \left(3.00 \times 10^8 \frac{\text{m}}{\text{s}}\right)\sqrt{1 - \left(\frac{9.11 \times 10^{-31}\text{ kg}}{12.55 \times 10^{-31}\text{ kg}}\right)^2}$$

$$v = \left(3.00 \times 10^8 \frac{\text{m}}{\text{s}}\right)\sqrt{1 - 0.52692}$$

$$v = \pm\left(3.00 \times 10^8 \frac{\text{m}}{\text{s}}\right)(0.68780)$$

$$v = \pm 2.0634 \times 10^8 \frac{\text{m}}{\text{s}}$$

$$v \cong \pm 2.06 \times 10^8 \frac{\text{m}}{\text{s}}$$

PROBLEM TIP

In questions involving masses, the masses form a ratio. It does not matter, therefore, what the actual mass units are, as long as they are the same for both the rest mass (m_o) and the moving (relativistic) mass (m).

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Since the problem is asking only for relative speed, the negative root has no meaning. Choose the positive value.

The speed of the electron relative to the detector is $2.06 \times 10^8 \frac{\text{m}}{\text{s}}$ or $0.688c$.

Validate the Solution

Since there is an appreciable mass increase, the object must be moving at a relativistic speed.

PRACTICE PROBLEMS

7. A speck of dust in space has a rest mass of 463 μg. If it is approaching Earth with a relative speed of $0.100c$, what will be its mass as measured in the Earth frame of reference? Remember, in questions involving masses, the masses form a ratio, so it does not matter what the actual mass units are, as long as they are the same for both the rest mass and the moving, or relativistic, mass.

8. A neutron is measured to have a mass of 1.71×10^{-27} kg when travelling at 6.00×10^7 m/s. Determine its rest mass.

9. How fast should a particle be travelling relative to an experimenter in order to have a measured mass that is 20.00 times its rest mass?

Where Is the Energy?

At the start of this section, you examined the relativistic effects that occur when a spacecraft is approaching the speed of light. The conclusion was that its increasing mass must prevent it from accelerating up to the speed of light. However, while the thrusters on the spacecraft are firing, force is being exerted over a displacement, indicating that work was being done on the spacecraft. You know that, at non-relativistic speeds, the work would increase the spacecraft's kinetic energy. At relativistic speeds, however, the speed and thus the kinetic energy increase can only be very small. What, then, is happening to the energy that the work is transferring to the spacecraft?

Einstein deduced that the increased mass represented the increased energy. He expressed it in the formula $E_k = mc^2 - m_o c^2$ or $E_k = (\Delta m)c^2$. As before, m is the mass of the particle travelling at speed v, and m_o is its rest mass. The expression mc^2 is known as the **total energy** of the particle, while $m_o c^2$ is the **rest energy** of the particle. Rearranging the previous equation leads to $mc^2 = m_o c^2 + E_k$. The total energy of the particle equals the rest energy of the particle plus its kinetic energy.

TOTAL ENERGY

The total energy (relativistic mass times the square of the speed of light) of an object is the sum of the rest energy (rest mass times the square of the speed of light) and its kinetic energy.

$$mc^2 = m_o c^2 + E_k$$

Quantity	Symbol	SI unit
relativistic mass	m	kg (kilograms)
rest mass	m_o	kg (kilograms)
speed of light	c	$\frac{m}{s}$ (metres per second)
kinetic energy	E_k	J (joules)

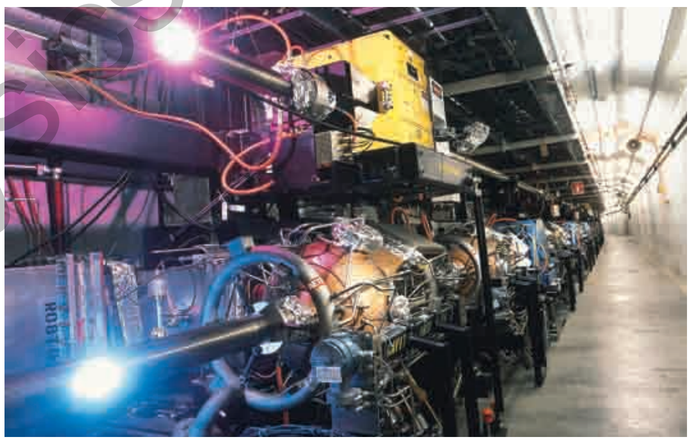
Unit Analysis

$$\text{kilogram}\left(\frac{\text{metres}}{\text{second}}\right)^2 = \text{kilogram}\left(\frac{\text{metres}}{\text{second}}\right)^2 + \text{joule} = \text{joule}$$

$$\text{kg}\left(\frac{\text{m}}{\text{s}}\right)^2 = \text{kg}\left(\frac{\text{m}}{\text{s}}\right)^2 + \text{J} = \text{J}$$

Figure 11.13 In particle accelerators, such as this one at the Stanford Linear Accelerator Center in California, particles are accelerated to speeds very close to the speed of light. Their masses are measured and are found to agree with Einstein's prediction.

No wonder physicists had difficulty accepting Einstein's theory! In these equations, he is saying that mass and energy are basically the same thing and that the conversion factor relating them is c^2, the square of the speed of light. At the time that Einstein published his work, such changes in mass could not be measured. Eventually, with the advent of high-energy physics, these measurements have become possible.



SAMPLE PROBLEMS

Kinetic Energy in a Rocket and in a Test Tube

1. **A rocket car with a mass of 2.00×10^3 kg is accelerated to 1.00×10^8 m/s. Calculate its kinetic energy**
   (a) using the classical or general equation for kinetic energy
   (b) using the relativistic equation for kinetic energy

Conceptualize the Problem

- The *classical* equation for *kinetic energy* is directly related to the object's *mass* and the *square of its velocity*.

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- The *relativistic equation* for *kinetic energy* takes into account the concept that the *object's mass changes* with its *velocity* and accounts for this *relativistic mass*.

Identify the Goal

The kinetic energy, E_k, of the rocket car, using the classical expression for kinetic energy and then the relativistic expression for kinetic energy

Identify the Variables and Constants

Known	Implied	Unknown
$m_o = 2.00 \times 10^3$ kg	$c = 3.00 \times 10^8 \frac{\text{m}}{\text{s}}$	m
$v = 1.00 \times 10^8 \frac{\text{m}}{\text{s}}$		E_k

Develop a Strategy

Select the classical equation for kinetic energy. Substitute into the equation. Solve.

(a) $E_k = \frac{1}{2}mv^2$

$$E_k = \frac{1}{2}(2.00 \times 10^3 \text{ kg})\left(1.00 \times 10^8 \frac{\text{m}}{\text{s}}\right)^2$$

$$E_k = 1.00 \times 10^{19} \text{ J}$$

Calculate gamma.

(b) $\gamma = \sqrt{1 - \frac{v^2}{c^2}}$

$$\gamma = \sqrt{1 - \frac{(1.00 \times 10^8 \frac{\text{m}}{\text{s}})^2}{(3.00 \times 10^8 \frac{\text{m}}{\text{s}})^2}}$$

$$\gamma = 1.061$$

Select the relativistic equation for E_k. Rearrange in terms of gamma.

$$E_k = mc^2 - m_oc^2$$

$$E_k = c^2\left(m_o\gamma - m_o\right)$$

$$E_k = m_oc^2\left(\gamma - 1\right)$$

Substitute into the equation.

$$E_k = \left(2.00 \times 10^3 \text{ kg}\right)\left(3.00 \times 10^8 \frac{\text{m}}{\text{s}}\right)^2\left(1.061 - 1\right)$$

Solve the equation.

$$E_k = 1.09 \times 10^{19} \text{ J}$$

(a) The classical expression for kinetic energy yields 1.00×10^{19} J.

(b) The relativistic expression for kinetic energy yields 1.09×10^{19} J.

Validate the Solution

Since gamma is not far from unity, a speed of 1.00×10^8 m/s does not provide a high degree of relativistic difference, so the two kinetic energies should not be too far apart.

---

2. A certain chemical reaction requires 13.8 J of thermal energy.

(a) What mass gain does this represent?

(b) Why would the chemist still believe in the law of conservation of mass?

Conceptualize the Problem

- *Thermal energy* is the *kinetic energy* of *molecules*.

- Einstein's equation for *relativistic kinetic energy*, which represents the difference between the *total energy* and the *rest energy*, applies to the motion of molecules as well as to rockets.
- If *thermal energy* seems to *disappear* during a chemical reaction, it must have been *converted into mass*.

Identify the Goal

The mass gain, Δm, during an absorption of 13.8 J of energy

Identify the Variables and Constants

Known	Implied	Unknown
$E_k = 13.8$ J	$c = 3.00 \times 10^8 \frac{\text{m}}{\text{s}}$	Δm

Develop a Strategy

Select the equation linking kinetic energy and mass.

$$E_k = \Delta mc^2$$

Rearrange to give the mass change.

$$\Delta m = \frac{E_k}{c^2}$$

Solve.

$$\Delta m = \frac{13.8 \text{ J}}{\left(3.00 \times 10^8 \frac{\text{m}}{\text{s}}\right)^2}$$

$$\Delta m = 1.533 \times 10^{-16} \text{ kg}$$

$$\Delta m \cong 1.53 \times 10^{-16} \text{ kg}$$

(a) The gain in mass is 1.53×10^{-16} kg.

(b) This mass change is too small for a chemist to measure with a balance, so the total mass of the products would appear to be the same as the total mass of the reactants.

Validate the Solution

The mass change at non-relativistic speeds should be extremely small.
Note: The source of the energy that is released during a chemical reaction is a loss of mass.

PRACTICE PROBLEMS

10. A physicist measures the mass of a speeding proton as being 2.20×10^{-27} kg. If its rest mass is 1.68×10^{-27} kg, how much kinetic energy does the proton possess?

11. A neutron has a rest mass of 1.68×10^{-27} kg. How much kinetic energy would it possess if it was travelling at $0.800c$?

12. How fast must a neutron be travelling relative to a detector in order to have a measured kinetic energy that is equal to its rest energy? Express your answer to two significant digits.

13. How much energy would be required to produce a kaon particle (κ) at rest with a rest mass of 8.79×10^{-28} kg?

14. If an electron and a positron (antielectron), each with a rest mass of 9.11×10^{-31} kg, met and annihilated each other, how much radiant energy would be produced? (In such a reaction involving matter and antimatter, the mass is completely converted into energy in the form of gamma rays.) Assume that the particles were barely moving before the reaction.

15. If the mass loss during a nuclear reaction is 14 μg, how much energy is released?

16. The Sun radiates away energy at the rate of 3.9×10^{26} W. At what rate is it losing mass due to this radiation?

Relativistic and Classical Kinetic Energy

It might seem odd that there are two apparently different equations for kinetic energy.

- At relativistic speeds, $E_k = mc^2 - m_o c^2$.
- At low (classical) speeds, $E_k = \frac{1}{2}mv^2$.

These equations are not as different as they appear, however. The first equation expands as follows.

$$E_k = mc^2 - m_o c^2$$
$$E_k = c^2(m - m_o)$$
$$E_k = c^2\left(\frac{m_o}{\sqrt{1 - \frac{v^2}{c^2}}} - m_o\right)$$
$$E_k = m_o c^2\left[\left(1 - \frac{v^2}{c^2}\right)^{-\frac{1}{2}} - 1\right]$$

This expression can be simplified by using an advanced mathematical approximation that reduces to the following when $v << c$.

$$E_k = c^2 m_o\left[1 - \left(-\frac{1}{2}\right)\frac{v^2}{c^2} - 1\right]$$
$$E_k = \frac{1}{2}m_o v^2$$

The relativistic expression for kinetic energy therefore becomes the classical expression for kinetic energy at normal speeds.

You have now examined the basics of the special theory of relativity and have seen how the measurement of time, length, and mass depends on the inertial frame of reference of the observer. You have also seen that mass and energy are equivalent, that matter could be considered as a condensed form of energy. What happens, though, when the frame of reference is not inertial? Such considerations are the subject of Einstein's general theory of relativity, which deals with gravitation and curved space — concepts that are beyond the scope of this course.

11.3 Section Review

1. K/U What do the terms "total energy" and "rest energy" mean?

2. K/U What term represents the lowest possible mass for an object?

3. C Using the equations involved in relativity, give two reasons why the speed of light is an unattainable speed for any material object.

4. C Imagine that the speed of light was about 400 m/s. Describe three effects that would be seen in everyday life due to relativistic effects.

5. I How might an experimenter demonstrate that high-speed (relativistic) particles have greater mass than when they are travelling at a slower speed? Assume that the experimenter has some way of measuring the speeds of these particles.

6. MC
   - (a) What must be true about the masses of the reactants and products for a combustion reaction? Why?
   - (b) Why would a chemist never notice the effect in part (a)?

CHAPTER 11 Review

REFLECTING ON CHAPTER 11

- The Michelson-Morley experiment indicated that the speed of light was the same for observers in any inertial frame of reference.
- The special theory of relativity is based on two postulates.
  1. All physical laws hold true in any inertial frame of reference.
  2. The speed of light is the same for observers in any inertial frame of reference.
- The special theory of relativity predicts that events that are simultaneous for observers in one inertial frame of reference are not necessarily simultaneous for observers in a different inertial frame of reference.
- The special theory of relativity predicts that if you are observing events in an inertial frame of reference that is moving rapidly relative to you, times in that observed frame of reference will appear to slow down. This is known as "time dilation." The effect is expressed in the formula

$$\Delta t = \frac{\Delta t_o}{\sqrt{1 - \frac{v^2}{c^2}}}$$

- The special theory of relativity predicts that if you are observing objects in an inertial frame of reference that is moving rapidly relative to you, lengths in that observed frame of reference in the direction of the motion will appear to be shorter. This is known as "length contraction." The effect is expressed in the formula

$$L = L_o\sqrt{1 - \frac{v^2}{c^2}}$$

- The special theory of relativity predicts that objects moving at a high rate of speed relative to a given inertial frame of reference will have greater mass than when they are at rest in that frame of reference. This is known as "mass increase." The effect is expressed in the formula

$$m = \frac{m_o}{\sqrt{1 - \frac{v^2}{c^2}}}$$

- The previous equations are shortened by the use of the quantity called "gamma."

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Using gamma, the equations become

$$\Delta t = \gamma \Delta t_o$$

$$L = \frac{L_o}{\gamma}$$

$$m = m_o\gamma$$

- Relativistic kinetic energy is given by

$$E_k = mc^2 - m_oc^2$$

$$\text{or } E_k = (\Delta m)c^2$$

- The speed of light through a vacuum is the fastest speed possible. Objects with mass must travel slower than this speed. Massless objects such as photons must travel at this speed.
- mc^2 represents the total energy of the particle.

Knowledge/Understanding

1. Briefly describe the Michelson-Morley experiment.
   (a) What were the predicted results?
   (b) What did Michelson and Morley actually observe?
   (c) Why did the observed results cause a complete rethinking of their basic postulates?
2. (a) In the Michelson-Morley experiment, what was the purpose of rotating the apparatus 90°?
   (b) What were the results and implications of this procedural step?
3. While riding in a streetcar in Bern, Switzerland, Einstein realized that moving clocks might not run at the same rate as

stationary clocks. He looked at a clock on a tower and realized that if the streetcar moved away from the clock at the speed of light, it would appear to him as if the clock had stopped. Describe a thought experiment to illustrate his thinking.

4. Explain, using examples, why seemingly simultaneous events might occur at different times and different places, depending on your frame of reference.
5. The speed of light in water is 2.25×10^8 m/s. Using Einstein's thinking, explain whether it is possible for a particle to travel through water at a speed greater than 2.25×10^8 m/s.
6. Explain how the behaviour of muons is used as evidence for the concepts of time dilation and length contraction.
7. Why is it postulated that electrons, protons, and other forms of matter can never travel at the speed of light?
8. A photon can be considered as a particle with a specific energy that travels at the speed of light.
   (a) According to the special theory of relativity, what is the rest mass of the photon?
   (b) If a photon has energy, does that mean it also has momentum?
9. Explain how the equation $E = mc^2$ is consistent with the law of conservation of energy.

Inquiry

10. Examine the question of when relativistic effects become important by plotting graphs of $1/\gamma$ versus velocity and γ versus velocity for speeds of 0 to $1.0c$.
    (a) At what speed would an observer experience a 1.0% time dilation effect or a 1.0% length contraction effect?
    (b) Repeat part (a) for 10%, 50%, 90%, and 99.99%.
11. Explain the first postulate of the theory of special relativity by describing how the laws of classical physics hold in an inertial frame of reference, but do not hold in a non-inertial frame of reference.
12. Describe a thought experiment to consider the effect on your everyday life if the speed of light was 3.0×10^2 m/s, rather than 3.0×10^8 m/s. Assume appropriate rates of speed and consider how much younger you would be if you flew from Toronto to Vancouver and back than if you stayed home. If the distance from Toronto to Vancouver is 2.8×10^3 km, measured at a walking pace, what distance will you have covered from the airplane's frame of reference? Assume that the airplane is flying at approximately 800 km/h.
13. Estimate the number of lights in the city of Toronto or Ottawa. Make reasonable assumptions. Suppose all of the light energy used in the city in 1 h in the evening could be captured and put into a box. Approximately how much heavier would the box become?

Communication

14. Sketch the appearance of a baseball as it flies past an observer at low speeds and at speeds that approach the speed of light.
15. A friend states that, according to Einstein, "Everything is relative." Disprove this popular statement by making a list of quantities that according to special relativity are **(a)** relative, that is, their value depends on the frame of reference, and **(b)** invariant, that is, their value is the same for all inertial observers.
16. Your lab partner is trying to convince you that a spaceship, which can travel at 0.9 c, can fit into a garage shorter than the spaceship's actual length. He suggests that if the spaceship is backed into the garage at full speed, it will undergo length contraction and thus fit into the garage. Explain to him the flaws in his thinking.

Making Connections

17. Until recently, the neutrino was thought to be massless and, therefore, to travel at the speed of light. Evidence from the Sudbury Neutrino Observatory (see the Physics Magazine in Chapter 13, The Nucleus and Elementary

Particles), published in June 2001, suggests that the neutrino has a tiny mass. Research the latest developments.

(a) What is the neutrino's mass now considered to be?

(b) Why has it been so difficult to measure?

(c) If the premise that neutrinos have mass is accepted, what are the implications in relation to setting an upper limit on their speed?

Problems for Understanding

18. How fast must a spaceship be moving for you to measure its length to be half its rest length?

19. You are speeding along in your sports car when your friend passes you on a relativistic motorcycle at 0.60*c*. You see your own car as being 4.0 m long and your friend's motorcycle as being 1.5 m long. You also notice that your friend's watch indicates that 8.5×10^{-8} s elapsed as she passed you. (It is a very large watch!)

(a) How long is your car as seen by your friend?

(b) How long is the motorcycle as seen by your friend?

(c) How much time passed on your watch while 8.5×10^{-8} s passed on your friend's watch?

20. A proton has a rest mass in a laboratory of 1.67×10^{-27} kg.

(a) What would its mass be relative to the laboratory if it was accelerated up to a speed of 0.75*c*?

(b) While the experimenter was determining the proton's mass in (a), what would be the proton's mass in its own frame of reference?

21. Create a graph showing the observed mass of an object that has a 1.0 kg rest mass as its speed goes from rest to 0.99*c*.

22. If a clock in an airplane is found to slow down by 5 parts in 10^{13}, (i.e., $\Delta t/\Delta t_o = 1.0 + 5.0 \times 10^{-13}$), at what speed is the airplane travelling? (**Hint:** You might need to use an expansion for γ.)

23. A spaceship travelling at 0.9 *c* fires a beam of light straight ahead.

(a) How fast would the crew on the spaceship measure the light beam's speed to be?

(b) How fast would a stationary observer on a spacewalk measure the light beam's speed to be?

(c) How fast would the crew on another spaceship travelling parallel to the first at the same speed of 0.9*c* measure the light beam's speed to be?

24. A pion is an unstable elementary particle that has a lifetime of 1.8×10^{-8} s. Assume that a beam of pions produced in a lab has a velocity of 0.95*c*.

(a) By what factor is the pions' lifetime increased?

(b) What will be their measured lifetimes?

(c) How far will they travel in this time?

25. What is the mass of an electron travelling at two thirds of the speed of light? Compare this to its rest mass.

26. What is the kinetic energy of an electron with the following speeds?

(a) 0.0010*c*; **(b)** 0.10*c*; **(c)** 0.50*c*; **(d)** 0.99*c*

(e) For which, if any, of these speeds, can you use the non-relativistic expression $\frac{1}{2}mv^2$ and have an error of less than 10%?

27. If an object has a mass that is 1.0% larger than its rest mass, how fast must it be moving?

28. How many 100 W light bulbs could be powered for one year by the direct conversion of 1 g of matter into energy?

29. An electron is accelerated from rest through a potential difference of 2.2 MV, so that it acquires an energy of 2.2 MeV. Calculate its mass, the ratio of its mass to its rest mass, and its speed.

30. An object at rest explodes into two fragments, each of which has a rest mass of 0.50 g.

(a) If the fragments move apart at speeds of 0.70*c* relative to the original object, what is the rest mass of the original object?

(b) How much of the object's original mass became kinetic energy of the fragments in the explosion?

CHAPTER 12

Quantum Mechanics and the Atom

CHAPTER CONTENTS

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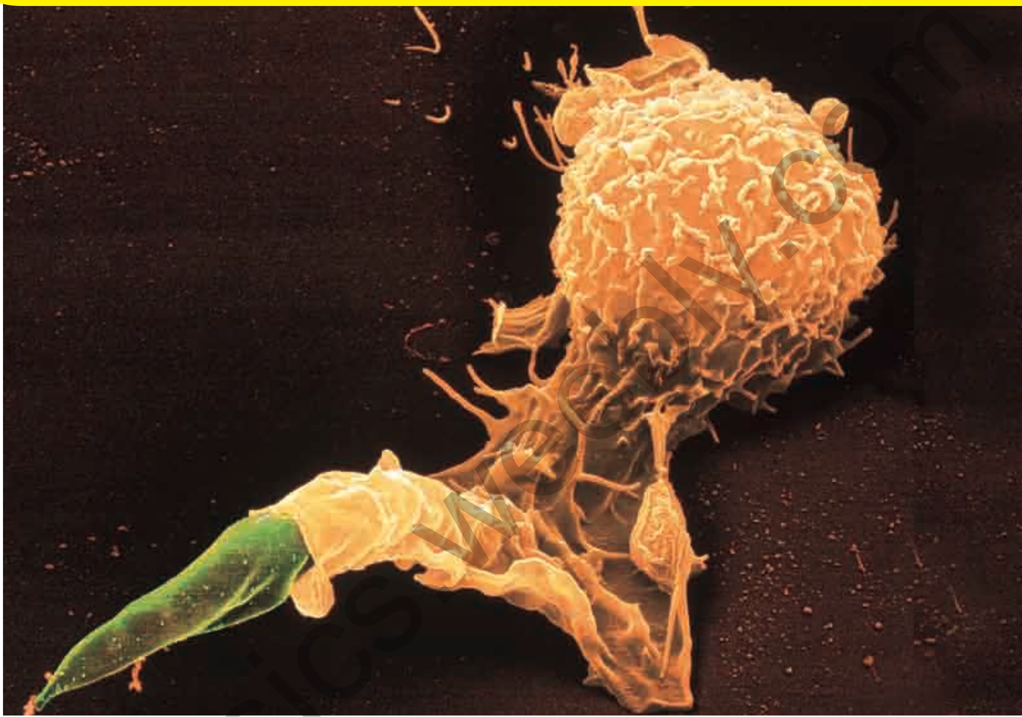
PREREQUISITE CONCEPTS AND SKILLS

- J.J. Thomson's discovery of the electron
- Rutherford's scattering experiment



In the photograph above, a white blood cell is engulfing and destroying a parasite. This process, called "phagocytosis," is one way in which your immune system protects you from disease. The image of the white blood cell was formed not by light waves, but by electrons. In previous courses, you learned in detail how light waves form images. You discovered that the wave properties of light made image formation possible. It would seem logical then, that in order for electrons to form images, they must behave like waves.

The idea that electrons, and all forms of matter, have wavelike properties was one of the concepts that shook the world of physics in the early 1900s. This discovery, along with the observation that light behaves like particles, helped form the basis of quantum theory — a theory that has permanently changed scientists' perception of the physical world. The early observations and concepts seemed so theoretical and distant from the everyday world that it was difficult to see any potential impact on the daily lives of non-scientists. However, out of quantum theory grew such technologies as electron microscopes, lasers, semiconductor electronics, light meters, and many other practical tools. In this chapter, you will follow, step by step, how and why quantum theory developed and how it influenced scientists' concept of the atom.

INVESTIGATION 12-A

Discharging an Electroscope

TARGET SKILLS
- Hypothesizing
- Performing and recording
- Analyzing and interpreting

In this investigation, you will use an electroscope to analyze the interaction between ultraviolet light and a zinc plate.

Problem

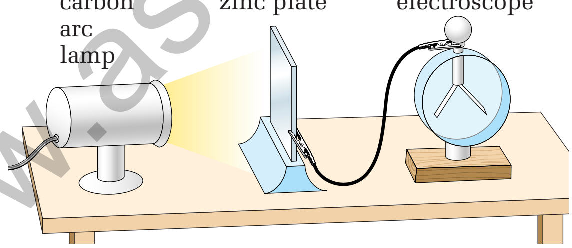
How can you discharge an electroscope when it is isolated from any source of electric grounding?

Equipment

- metal leaf electroscope
- carbon arc lamp (or source of intense ultraviolet light)
- insulating stand
- conducting wire with alligator clips
- zinc plate
- ebonite rod
- glass rod
- emery paper
- fur
- silk

Procedure

1. Polish the zinc plate with the emery paper until the plate shines.
2. Assemble the apparatus as shown in the diagram, leaving the lamp turned off. Ensure that the shiny side of the zinc plate faces the lamp.



3. Rub the ebonite rod with the fur to give the rod a negative charge.
4. Touch the ebonite rod to the sphere of the electroscope. Record the appearance of the electroscope.
5. Observe and record any changes in the electroscope over a period of 2 to 3 min.
6. Turn on the carbon arc lamp and observe and record any changes in the electroscope over a 2 to 3 min period.

CAUTION When the carbon arc lamp is on, do *not* look directly at the light or any reflected light. Ultraviolet light could damage your eyes.

7. Turn the lamp off. Touch the sphere of the electroscope with your hand to fully discharge the leaves.
8. Rub the glass rod with the silk to give it a positive charge. Touch the rod to the sphere of the electroscope.
9. Turn on the carbon arc lamp and observe and record any changes in the electroscope over a period of 2 to 3 min.
10. Turn the lamp off and discharge the electroscope.

Analyze and Conclude

1. Describe the exact conditions under which the electroscope discharged. For example, did it discharge when it was carrying a net negative charge or net positive charge? Was the carbon arc lamp on or off when this occurred?
2. Describe the conditions under which the electroscope did not discharge.
3. What entity had to escape from the electroscope in order for it to discharge?
4. Formulate a hypothesis about a mechanism that would have allowed the entity in question 3 to escape.
5. As you study this chapter, compare your hypothesis with the explanation given by physicists.

12.1 The Particle Nature of Light

SECTION EXPECTATIONS

- Describe the photoelectric effect and outline the experimental evidence that supports a particle model of light.
- Describe how the development of the quantum theory has led to technological advances such as the light meter.

KEY TERMS

- classical physics
- blackbody
- ultraviolet catastrophe
- empirical equation
- quantized
- quantum
- photoelectric effect
- stopping potential
- photon
- work function
- threshold frequency
- electron volt

In Unit 4, The Wave Nature of Light, you studied light and electromagnetic radiation. You learned that Christiaan Huygens (1629–1695) revived the wave theory of light in 1678. In 1801, Thomas Young (1773–1829) demonstrated conclusively with his famous double-slit experiment that light consisted of waves.

For more than 200 years, physicists studied electromagnetism and accumulated evidence for the wave nature of light and all forms of electromagnetic radiation. In fact, in 1873, James Clerk Maxwell (1831–1879) published his *Treatise on Electricity and Magnetism*, in which he summarized in four equations everything that was known about electromagnetism and electromagnetic waves. Maxwell's equations form the basis of electromagnetism in much the same way that Newton's laws form the basis of mechanics.

These areas of study, Newtonian mechanics and electricity and magnetism, along with thermodynamics, constitute **classical physics**. By the late 1800s, classical physics was well established. Many years of experiments and observations supported the theories of Newton and Maxwell. However, the scientific community was about to be shaken by events to come with the turn of the century.

How could observations on something as seemingly simple as a blackbody expose a flaw in these well-established theories? What, exactly, is a blackbody?

Blackbody Radiation

Based on his studies on the emission and absorption spectra of gases, Gustav Kirchhoff (1824–1887) defined the properties of a blackbody. While working with Robert Bunsen (1811–1899), Kirchhoff observed that, when heated to incandescence, gases emit certain, characteristic frequencies of light. When white light shines through the gases, they absorb the same frequencies of light that they emit, so Kirchhoff proposed that all objects absorb the same frequencies of radiation that they emit. He further reasoned that since black objects absorb all frequencies of light, they should emit all frequencies when heated to incandescence. Thus, the term **blackbody** was defined as a "perfect radiator," a body that emits a complete spectrum of electromagnetic radiation.

Fortunately for experimenters, blackbodies are not difficult to simulate in the laboratory. Any cavity with the inner walls heated to a very high temperature and with a very small hole to allow radiation to escape (see Figure 12.1) will emit a spectrum of radiation nearly identical to that of a blackbody.

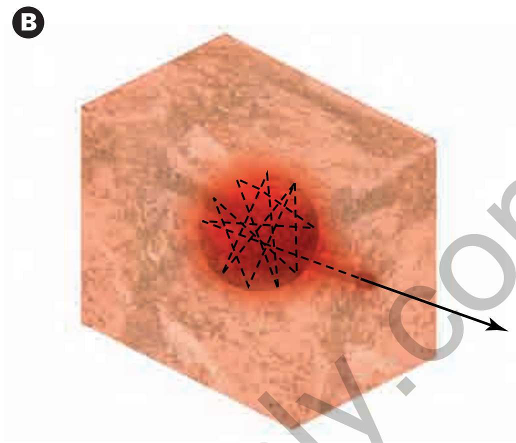


Figure 12.1 **(A)** When the temperature of a kiln surpasses 1000 K, the radiation is independent of the nature of the material in the kiln and depends only on the temperature. **(B)** A tiny hole in a very hot cavity "samples" the radiation that is being emitted and absorbed by the walls inside.

Figure 12.2 shows graphs of the blackbody radiation distribution at several different temperatures. The frequency of the radiation is plotted on the horizontal axis and the intensity of the radiation emitted at each frequency is plotted on the vertical axis. The area under the curve represents the total amount of energy emitted by a blackbody in a given time interval.

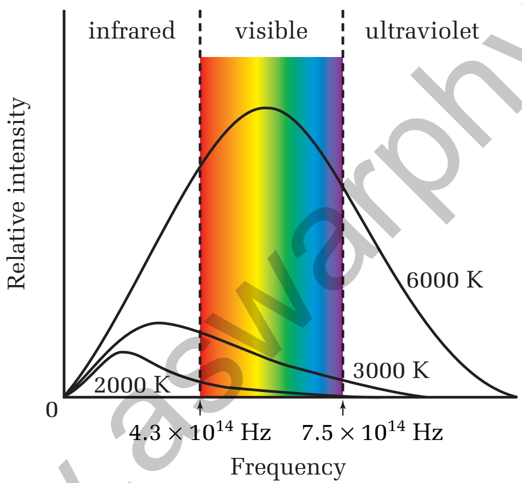


Figure 12.2 As the temperature of an incandescent body increases, the frequency that is emitted with the highest intensity (the peak of the curve) becomes higher.

Using data such as those in Figure 12.2, Kirchhoff was able to show that the power radiated by a blackbody depends on the blackbody's temperature. He also showed that the intensity of the radiation was related to the frequency in a complex way and that the distribution of intensities was different at different temperatures. Kirchhoff was unable to find the exact form of the mathematical relationships, so he challenged the scientific community to do so.

Figure 12.3 **(A)** While an object such as a crowbar emits no visible radiation at room temperature, it is actually emitting infrared radiation. **(B)** When a stove coil reaches 600 K, it emits mostly invisible, infrared radiation. The radiation appears to be red, because it emits a little visible radiation in the red end of the spectrum. **(C)** At 2000 K, a light bulb filament looks white, because it emits all frequencies in the visible range.

According to electromagnetic theory, accelerating charges emit electromagnetic radiation. Maxwell's equations describe the nature of these oscillations and the associated radiation. A blackbody therefore must have vibrating, or oscillating, charges on the surface that are emitting (or absorbing) electromagnetic energy.

Josef Stefan (1835–1893) showed experimentally in 1879 that the power (energy per unit time) emitted by a blackbody is related to the fourth power of the temperature ($P \propto T^4$). In other words, if the temperature of a blackbody doubles, the power emitted will increase by 2^4, or 16 times. Five years later, Ludwig Boltzmann (1844–1906) used Maxwell's electromagnetic theory, as well as methods Boltzmann himself had developed for thermodynamics, to provide a theoretical basis for the fourth-power relationship.

The exact mathematical relationship between frequency and intensity of radiation emitted by a blackbody is much more complex than the relationship between temperature and power. Nevertheless, Lord Rayleigh (John William Strutt, 1842–1919) and Sir James Hopwood Jeans (1877–1946) attempted to apply the same principles that Boltzmann had used for the energy-temperature relationship. When they applied these theories to blackbody radiation, they obtained the upper curve shown in Figure 12.4. The lower curve represents experimental data for the same temperature.

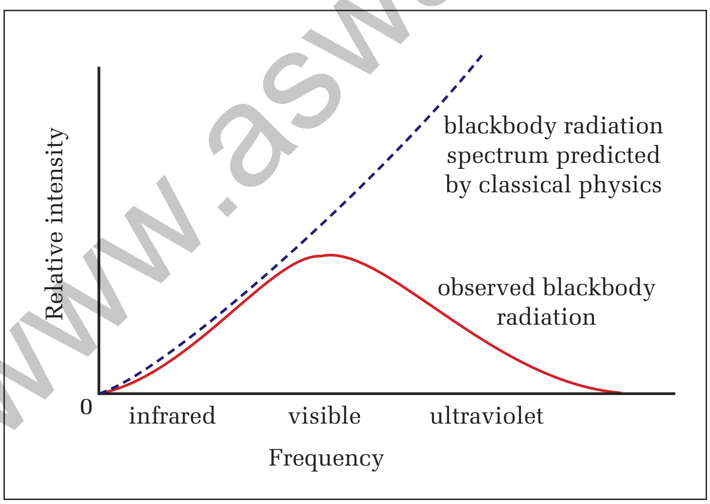


Figure 12.4 At low frequencies, predictions based on classical theory agree with observed data for the intensity of radiation from a blackbody. At high frequencies, however, theory and observation diverge quite drastically.

As you can see in Figure 12.4, classical theory applied to blackbody radiation agrees with observed data at low frequencies, but predicts that energy radiated from incandescent objects should continue to increase as the frequency increases. This discrepancy between theory and observation shocked the physicists of the day so much that they called it the **ultraviolet catastrophe**. How could a theory that had explained all of the data collected for 200 years fail to predict the emission spectrum of a blackbody? Little did they realize what was in store.

The Birth of Quantum Theory

Max Planck (1858–1947), a student of Kirchhoff, developed an empirical mathematical relationship between intensity and frequency of blackbody radiation. (An **empirical equation** is one that fits the observed data but is not based on any theory.) To develop the theory behind his empirical relationship, Planck turned to a statistical technique that Boltzmann had developed to solve certain thermodynamic problems. Planck had to make a "minor adjustment" to apply this method to energies of oscillators in the walls of a blackbody, however.

Boltzmann's statistical method required the use of discrete units, such as individual molecules of a gas. Although the energies of oscillators had always been considered to be continuous, for the sake of the mathematical method, Planck assigned discrete energy levels to the oscillators. He set the value of the allowed energies of the oscillators equal to a constant times the frequency, or $E = hf$, where h is the proportionality constant.

According to this hypothetical system, an oscillator could exist with an energy of zero or any integral multiple of hf, but not at energy levels in between, as illustrated in Figure 12.5. When the blackbody emitted radiation, it had to drop down one or more levels and emit a unit of energy equal to the difference between the allowed energy levels of the oscillator. A system such as this is said to be **quantized**, meaning that there is a minimum amount of energy, or a **quantum** of energy, that can be exchanged in any interaction.

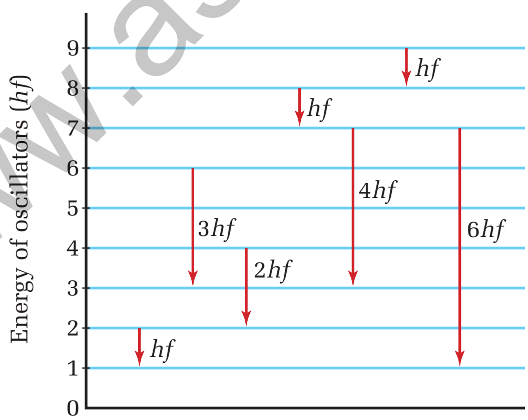


Figure 12.5 The "allowed" energy levels of the oscillators in the walls of a blackbody can be described as $E = nhf$, where n is any positive integer — 0, 1, 2, and up.

PHYSICS FILE

Shortly after Planck presented his paper on blackbody radiation, Einstein corrected one small error in the mathematics. He showed that the energy levels of the oscillators had to be $E = (n + \frac{1}{2})hf$. The addition of $\frac{1}{2}$ did not affect the *difference* between energy levels and thus did not change the prediction of the spectrum of blackbody radiation. However, it did show that the minimum possible energy of an oscillator is not zero, but $\frac{1}{2}hf$.

With discrete units of energy defined, Planck could now apply Boltzmann's statistical methods to his analysis of blackbodies. His plan was to develop an equation and then apply another mathematical technique that would allow the separation of energy levels to become smaller and smaller, until the energies were once again continuous. Planck developed the equation, but when he performed the mathematical operation to make oscillator energies continuous, the prediction reverted to the Rayleigh-Jeans curve. However, his equation fit the experimental data perfectly if the allowed energies of the oscillators remained discrete instead of continuous.

Planck was quite surprised, but he continued to analyze the equation. By matching his theoretical equation to experimental data, he was able to determine that the value of h, the proportionality constant, was approximately 6.55×10^{-34} J · s. Today, h is known as Planck's constant and its value is measured to be $6.626\,075\,5 \times 10^{-34}$ J · s.

With a theory in hand that could precisely predict the observed data for blackbody radiation, Planck presented his findings to the German Physical Society on December 14, 1900, and modern physics was born. Planck's revolutionary theory created quite a stir at the meeting, but the ideas were so new and radical that physicists — Planck included — could not readily accept them. More evidence would be needed before the scientific community would embrace the theory of the quantization of energy.

The Photoelectric Effect

The photoelectric effect, which would eventually confirm the theory of the quantization of energy, was discovered quite by accident. In 1887, Heinrich Hertz (1857–1894) was attempting to verify experimentally Maxwell's theories of electromagnetism. He assembled an electric circuit that generated an oscillating current, causing sparks to jump back and forth across a gap between electrodes, as illustrated in Figure 12.6. He showed that the sparks were generating electromagnetic waves by placing, on the far side of the room, a small coil or wire with a tiny gap. When the "transmitter" generated sparks, he observed that sparks were also forming in the gap of the "receiver" coil on the far side of the room. The electromagnetic energy had been transmitted across the room.

Hertz was able to show that these electromagnetic waves travelled with the speed of light and could be reflected and refracted, verifying Maxwell's theories. Ironically, however, Hertz made an observation that set the stage for experiments that would support the particle nature of electromagnetic radiation — the sparks were enhanced when the metal electrodes were exposed to ultraviolet light. At the time of Hertz's experiments, this phenomenon was difficult to explain.

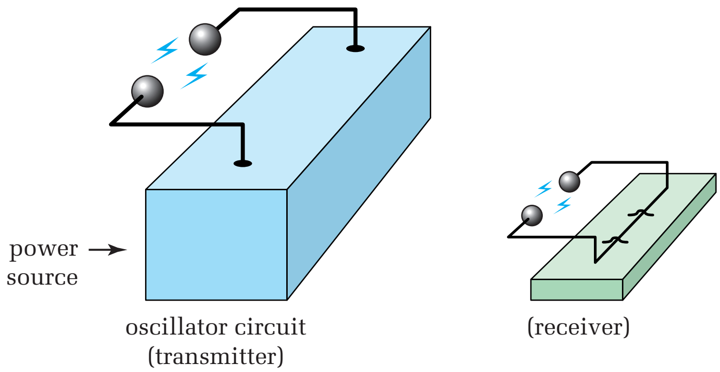


Figure 12.6 The electromagnetic waves that Hertz generated with his spark gap were in the frequency range that is now called "radio waves." Although Hertz's only intention was to verify Maxwell's theories, his experiments led to invention of the wireless telegraph, radio, television, microwave communications, and radar.

It was not until 10 years after Hertz carried out his experiments that Joseph John Thomson (1856–1940) discovered the electron. With this new knowledge, physicists suggested that the ultraviolet light had ejected electrons from Hertz's metal electrodes, thus creating a "conducting path" for the sparks to follow. The ejection of electrons by ultraviolet light became known as the **photoelectric effect**.

Early Photoelectric Effect Experiments

In 1902, physicist Philipp Lenard (1862–1947) performed more detailed experiments on the photoelectric effect. He designed an apparatus like the one shown in Figure 12.7. Electrodes are sealed in an evacuated glass tube that has a quartz window. (Ultraviolet light will not penetrate glass.) A very sensitive galvanometer detects any current passing through the circuit. Notice that the variable power supply can be connected so that it can make either electrode positive or negative.

To determine whether photoelectrons were, in fact, ejected from the "emitter," Lenard made the emitter negative and the collector positive. When he exposed the emitter to ultraviolet light, the galvanometer registered a current. The ultraviolet light had ejected electrons, which were attracted to the collector and then passed through the circuit. When Lenard increased the intensity of the ultraviolet light, he observed an increase in the current.

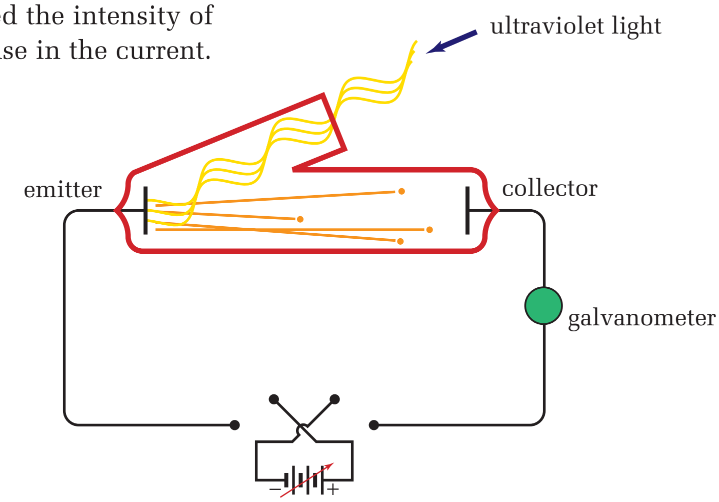


Figure 12.7 These glass tubes for experiments on the photoelectric effect had to be sealed in a vacuum so that the electrons would not collide with molecules of gas.

WEB LINK

www.mcgrawhill.ca/links/physics12

Many of the physicists who contributed to the development of quantum theory won the Nobel Prize in Physics. Find out who they were and learn more about their contributions to modern physics by going to the above Internet site and clicking on **Web Links**.

To learn more about the relative kinetic energies of photoelectrons, Lenard reversed the polarity of the power supply so that the electric field between the electrodes would oppose the motion of the photoelectrons. Starting each experiment with a very small potential difference opposing the motion of the electrons, he gradually increased the voltage and observed the effect on the current. The photoelectrons would leave the emitter with kinetic energy. He theorized that if the kinetic energy was great enough to overcome the potential difference between the plates, the electron would strike the collector. Any electrons that reached the collector would pass through the circuit, registering a current in the galvanometer. Electrons that did not have enough kinetic energy to overcome the potential difference would be forced back to the emitter.

Lenard discovered that as he increased the potential difference, the current gradually decreased until it finally stopped flowing entirely. The opposing potential had turned back even the most energetic electrons. The potential difference that stopped all photoelectrons is now called the **stopping potential**. Lenard's data indicated that ultraviolet light with a constant intensity ejected electrons with a variety of energies but that there was always a maximum kinetic energy.

In a critical study, Lenard used a prism to direct narrow ranges of frequencies of light onto the emitter. He observed that the stopping potential for higher frequencies of light was greater than it was for lower frequencies. This result means that, regardless of its intensity, light of higher frequency ejects electrons with greater kinetic energies than does light with lower frequencies. Once again, a greater *intensity* of any given frequency of light increased only the flowing current, or *number* of electrons, and had no effect on the electrons' stopping potential and, thus, no effect on their maximum kinetic energy. In summary, Lenard's investigations demonstrated the following.

- When the intensity of the light striking the emitter increases, the number of electrons ejected increases.
- The maximum kinetic energy of the electrons ejected from the metal emitter is determined *only* by the frequency of the light and is not affected by its intensity.

Lenard's first result is in agreement with the classical wave theory of light: As the intensity of the light increases, the amount of energy absorbed by the surface per unit time increases, so the number of photoelectrons should increase. However, classical theory also predicts that the kinetic energy of the photoelectrons should increase with an increase in the intensity of the light. Lenard's second finding — that the kinetic energy of the photoelectrons is determined *only* by the frequency of the light — cannot be explained by the classical wave theory of light.

Einstein and the Photoelectric Effect

Just a few years after Planck's quantum theory raised questions about the nature of electromagnetic radiation, the photoelectric effect raised even more questions. After publication, Planck's theory had been, for the most part, neglected. Now, however, because of the new evidence pointing to a flaw in the wave theory of light, Planck's ideas were revisited.

It was Albert Einstein (1879–1955) who saw the link between Planck's quantum of energy and the photoelectric effect. In 1905, Einstein published a paper in which he proposed that light must not only be emitted as quanta, or packets of energy, but it must also be absorbed as quanta. By treating light as quanta or **photons**, as they were named later, Einstein could explain Lenard's results for the photoelectric effect.

Einstein suggested that Planck's unit of energy, $E = hf$, is the energy of a photon. He proposed that when a photon strikes a metal surface, all of its energy is absorbed by one electron in one event. Since the energy of a photon is related to the frequency of the light, a photon with a higher frequency would have more energy to give an electron than would a photon with a lower frequency. This concept immediately explains why the maximum kinetic energy of photoelectrons depends only on frequency. Increasing the intensity of light of a given frequency increases only the *number* of photons and has no effect on the energy of a single photon.

Since the kinetic energy of the photoelectrons varied, some of the energy of the photons was being converted into a form of energy other than kinetic. Einstein proposed that some energy must be used to overcome the attractive forces that hold the electron onto the surface of the metal. Since some electrons are buried "deeper" in the metal, a larger amount of energy is needed to eject them from the surface. These electrons leave the emitter with less kinetic energy. The electrons with maximum kinetic energy must be the most loosely bound. Einstein gave the name **work function** (W) to this minimum amount of energy necessary to remove an electron from the metal surface. He predicted that the value would depend on the type of metal. The following mathematical expression describes the division of photon energy into the work function of the metal and the kinetic energy of the photoelectron.

$$hf = W + E_{k(max)}$$

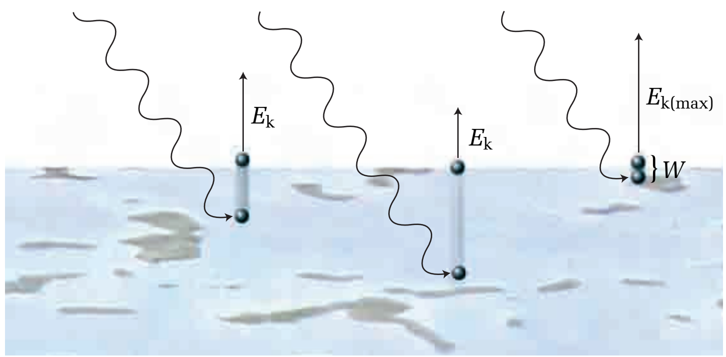


Figure 12.8 The energy of the photon must first extract the electron from the metal surface. The remainder of the energy becomes the kinetic energy of the electron.

ELECTRONIC LEARNING PARTNER

To enhance your understanding of photoelectric effect, go to your Electronic Learning Partner.

PHYSICS FILE

Albert Einstein never actually carried out any laboratory experiments. He was a genius, however, at interpreting and explaining the results of others. In addition, the technology needed to test many of his theories did not exist until many years after he published them. Einstein was truly a theoretical physicist.

While Einstein's explanation could account for all of the observations of the photoelectric effect, very few physicists, including Planck, accepted Einstein's arguments regarding the quantum (or particle) nature of light. It was very difficult to put aside the 200 years of observations that supported the wave theory. Unfortunately, when Einstein wrote his paper on the photoelectric effect, the charge on the electron was not yet known, so there was no way to prove him right or wrong.

Millikan and the Photoelectric Effect

By 1916, Robert Millikan (1868–1953) had established that the magnitude of the charge on an electron was 1.60×10^{-19} C. With this "ammunition" in hand, Millikan set out to prove that Einstein's assumptions regarding the quantum nature of light were incorrect. Like others, Millikan felt that the evidence for the wave nature of light was overwhelming.

Millikan improved on Lenard's design and built photoelectric tubes with emitters composed of various metals. For each metal, he measured the stopping potential for a variety of frequencies. Using his experimentally determined value for the charge on an electron, he calculated the values for the maximum kinetic energy for each frequency, using the familiar relation $E = qV$. In this application, E is the energy of a charge, q, that has fallen through a potential difference, V. In Millikan's case, E was the maximum kinetic energy of the photoelectrons and q was the charge on an electron. The equation becomes $E_{k(max)} = eV_{stop}$. Millikan then plotted graphs of kinetic energy versus frequency for each type of metal emitter.

To relate graphs of $E_{k(max)}$ versus f to Einstein's equation, it is convenient to solve for $E_{k(max)}$, resulting in the following equation.

$$E_{k(max)} = hf - W$$

$E_{k(max)}$ is the dependent variable, f is the independent variable, and h and W are constants for a given experiment. Notice that the equation has the form of the slope-intercept equation of a straight line.

$$y = mx + b$$

Comparing the equations, you can see that Planck's constant (h) is the slope (m), and the negative of the work function ($-W$) is the y-intercept (b).

When Millikan plotted his data, they resulted in straight lines, as shown in Figure 12.9. The slopes of the lines from all experiments were the same and were equal to Planck's constant. When Millikan extrapolated the lines to cross the vertical axis, the value gave the negative of the work function of the metal. Much to Millikan's disappointment, he had proven that Einstein's equations perfectly predicted all of his results. He begrudgingly had to concede that Einstein's assumptions about the quantum nature of light were probably correct.

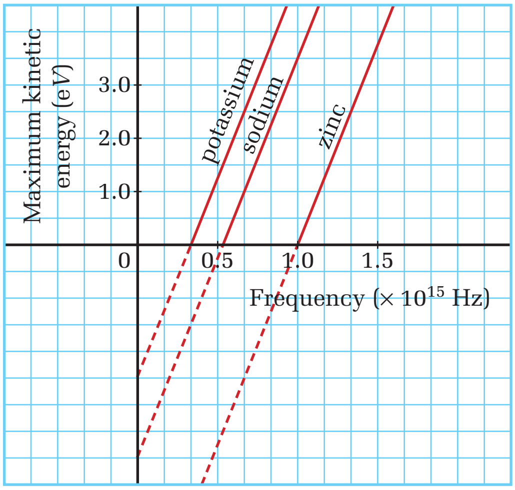


Figure 12.9 The graphs of Millikan's data were straight lines with equal slopes. The only differences were the points at which the extrapolated lines crossed the axes.

PHOTOELECTRIC EFFECT

The maximum kinetic energy of a photoelectron is the difference of the energy of the photon and the work function of the metal emitter.

$$E_{k(max)} = hf - W$$

Quantity	Symbol	SI unit
maximum kinetic energy of a photoelectron	$E_{k(max)}$	J (joules)
Planck's constant	h	J · s (joule · seconds)
frequency of electromagnetic radiation	f	Hz (hertz: equivalent to s^{-1})
work function of metal	W	J (joules)

Unit Analysis

(joule · second)(hertz) − joule = (J · s)(s^{-1}) = J

Another critical feature of a graph of maximum kinetic energy versus frequency is the point at which each line intersects the horizontal axis. On this axis, the maximum kinetic energy of the photoelectrons is zero. The frequency at this horizontal intercept is called **threshold frequency** (f_o), because it is the lowest frequency (smallest photon energy) that can eject a photoelectron from the metal. When photons with threshold frequency strike the emitter, they have just enough energy to raise the most loosely bound electrons to the surface of the emitter, but they have no energy left with which to give the photoelectrons kinetic energy. These photoelectrons are drawn back into the emitter.

Conceptual Problem

- At threshold frequency (f_o), the maximum kinetic energy of the photoelectrons is zero ($E_{k(max)} = 0$). Substitute these terms (f_o and 0) into Einstein's equation for the photoelectric effect and solve for the work function (W). Explain the meaning of the relationship that you found and how you can use it to find the work function of a metal.

You might have noticed that the unit for energy on the vertical axis in Figure 12.9 was symbolized as "eV," which in this case stands for "electron volt." The need for this new unit will become apparent when you start to use the photoelectric equation. You will find that the kinetic energy of even the most energetic electrons is an extremely small fraction of a joule. Since working with numbers such as 1.23×10^{-17} J becomes tedious and it is difficult to compare values, physicists working with subatomic particles developed the electron volt as the energy unit suitable for such particles and for photons. The **electron volt** is defined as the energy gained by one electron as it falls through the potential difference of one volt. The following calculation shows the relationship between electron volts and joules.

$$E = qV$$
$$1 \text{ eV} = (1 \text{ e})(1 \text{ V})$$
$$1 \text{ eV} = (1.60 \times 10^{-19} \text{ C})(1 \text{ V})$$
$$1 \text{ eV} = 1.60 \times 10^{-19} \text{ J}$$

Table 12.1 lists the work functions, in units of electron volts, of several common metals that have been studied as emitters in photoelectric experiments.

Like many other theoretical developments in physics, scientists soon found some practical applications for the photoelectric effect. The first light meters used the photoelectric effect to measure the intensity of light. Light meters have specialized metal emitters that are sensitive to visible light. When light strikes the metal, electrons are released and then collected by a positive electrode. The amount of current produced is proportional to the intensity of the light. The photon that physicists once had difficulty accepting is now almost a household word.

Table 12.1 Work Functions of Some Common Metals

Metal	Work function (eV)
aluminum	4.28
calcium	2.87
cesium	2.14
copper	4.65
iron	4.50
lead	4.25
lithium	2.90
nickel	5.15
platinum	5.65
potassium	2.30
tin	4.42
tungsten	4.55
zinc	4.33

12.1 Section Review

1. K/U Explain how a very hot oven can simulate a blackbody.
2. K/U Why was Planck's theory of blackbody radiation considered to be revolutionary?
3. C (a) Describe how the Hertz experiment, in which he used spark gaps to transmit and receive electromagnetic radiation, also provided early evidence for the photoelectric effect.
   (b) Name one modern technology that has its origin in the Hertz experiment. Briefly describe how it is related to this experiment.
4. C Describe how Einstein used Planck's concept of quanta of energy to explain the photoelectric effect.
5. K/U Define the terms "work function" and "threshold frequency."
6. C Describe how the quantum (photon) model for light better explains the photoelectric effect than does the classical wave theory.
7. MC What instruments have you used that rely on the photoelectric effect?
8. I Plot a graph of the following data from a photoelectric effect experiment and use the graph to determine Planck's constant, the threshold frequency, and the work function of the metal. Consult Table 12.1 and determine what metal was probably used as the target for electrons in the phototube.

Stopping potential (V)	Frequency of light (Hz)
0.91	9.0×10^{14}
1.62	10.7×10^{14}
2.35	12.4×10^{14}
3.50	15.0×10^{14}
4.21	16.5×10^{14}

12.2 Light Particles and Matter Waves

SECTION EXPECTATIONS

- Define and describe the concepts related to the understanding of matter waves.
- Describe how the development of quantum theory has led to technological advances such as the electron microscope.

KEY TERMS

- Compton effect
- de Broglie wavelength
- wave-particle duality

When Millikan's experimental results verified Einstein's interpretation of the photoelectric effect, the scientific community began to accept the particle nature of light. Physicists started to ask more questions about the extent to which particles of light, or photons, resembled particles of matter. U.S. physicist Arthur Compton (1892–1962) decided to study elastic collisions between photons and electrons. Would the law of conservation of momentum apply to such collisions? How could physicists determine the momentum (mv) of a particle that has no mass?

The Compton Effect

The ideal way to study collisions between particles is to start with free particles. Preferably, the only force acting on either particle at the moment of the collision is the impact of the other particle. However, electrons rarely exist free of atoms. So, Compton reasoned that if the photon's energy was significantly greater than the work function of the metal, the energy required to free an electron from the metal would be negligible when compared to the energy of the interaction. He needed a source of highly energetic photons.

About 30 years prior to Compton's work, Wilhelm Conrad Röntgen (1845–1923) discovered X rays and demonstrated that they are high-frequency electromagnetic waves. Thus, X-ray photons would have the amount of energy that Compton needed for his studies. In 1923, Compton carried out some very sophisticated experiments on collisions between X-ray photons and electrons. The phenomenon that he discovered is now known as the **Compton effect** and is illustrated in Figure 12.10. When a very high-energy X-ray photon collides with a "free" electron, it gives some of its energy to the electron and a lower-energy photon scatters off the electron.

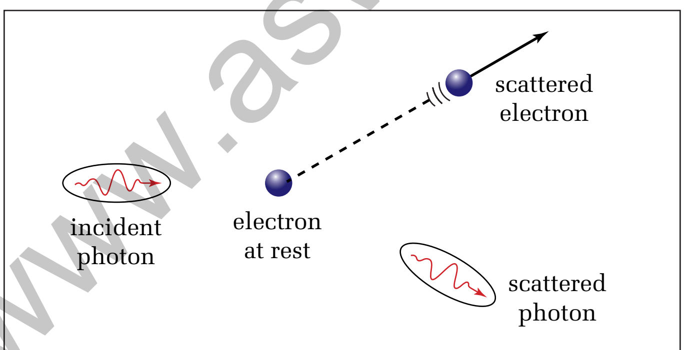


Figure 12.10 When a high-energy photon collides with a "free" electron, both energy and momentum are conserved.

You can describe mathematically the conservation of energy in a photon-electron collision as follows, where hf is the energy of the photon before the collision, hf' is the energy of the photon after the collision, and $\frac{1}{2}m_e v'^2$ is the kinetic energy of the electron after the collision.

$$hf = hf' + \frac{1}{2}m_e v'^2$$

Since the scattered photon has a lower energy, it must have a lower frequency and a longer wavelength than the original photon. Compton's measurements showed that the scattered photon had a lower frequency, and that kinetic energy gained by an electron in a collision with a photon was equal to the energy lost by the photon.

The more difficult task for Compton was finding a way to determine whether momentum had been conserved in the collision. The familiar expression for momentum, $p = mv$, contains the object's mass, but photons have no mass. So Compton turned to Einstein's now famous equation, $E = mc^2$, to find the mass equivalence of a photon. The following steps show how Compton used Einstein's relationship to derive an expression for the momentum of a photon. Since the goal is to find the magnitude of the momentum, vector notations are omitted.

- Write Einstein's equation that describes the energy equivalent of mass.
  $$E = mc^2$$
- Divide both sides of the equation by c^2 to solve for mass.
  $$m = \frac{E}{c^2}$$
- Write the equation for momentum.
  $$p = mv$$
- Substitute the energy equivalent of mass into the equation for momentum.
  $$p = \frac{E}{c^2}v$$
- Since the velocity of a photon is c, substitute c for v and simplify.
  $$P = \frac{E}{c^{\cancel{2}}}\cancel{c} = \frac{E}{c}$$
- Substitute the expression for the energy of a photon (hf) for E in the equation for momentum.
  $$p = \frac{hf}{c}$$
- The momentum of a photon is usually expressed in terms of wavelength, rather than frequency. Use the equation for the velocity of a wave to find the expression for f in terms of v. Note that the velocity of a light wave is c.
  $$f\lambda = v$$
  $$f\lambda = c$$
  $$f = \frac{c}{\lambda}$$
- Substitute the expression for frequency into the momentum equation and simplify.
  $$p = \frac{h\frac{\cancel{c}}{\lambda}}{\cancel{c}}$$
  $$p = \frac{h}{\lambda}$$

When Compton calculated the momentum of a photon using $p = \frac{h}{\lambda}$, he was able to show that momentum is conserved in collisions between photons and electrons. These collisions obey all of the laws for collisions between two masses. The line between matter and energy was becoming more and more faint.

MOMENTUM OF A PHOTON

The momentum of a photon is the quotient of Planck's constant and the wavelength of the photon.

$$p = \frac{h}{\lambda}$$

Quantity	Symbol	SI unit
momentum	p	$\frac{\text{kg}\cdot\text{m}}{\text{s}}$ (kilogram metres per seconds)
Planck's constant	h	$\text{J}\cdot\text{s}$ (joule seconds)
wavelength	λ	m (metres)

Unit Analysis

$$\frac{\text{kilogram}\cdot\text{metre}}{\text{second}} = \frac{\text{joule}\cdot\text{second}}{\text{metre}}$$

$$\frac{\text{kg}\cdot\text{m}}{\text{s}} = \frac{\text{J}\cdot\text{s}}{\text{m}} = \frac{\frac{\text{kg}\cdot\text{m}^{\cancel{2}}}{\text{s}^{\cancel{2}}}\cdot\cancel{\text{s}}}{\cancel{\text{m}}} = \frac{\text{kg}\cdot\text{m}}{\text{s}}$$

The following problem will help you to develop a feeling for the amount of momentum that is carried by photons.

SAMPLE PROBLEM

Momentum of a Photon

Calculate the momentum of a photon of light that has a frequency of 5.09×10^{14} Hz.

Conceptualize the Problem

- The *momentum* of a *photon* is related to its *wavelength*.
- A photon's *wavelength* is related to its *frequency* and the speed of light.

Identify the Goal

The momentum, p, of the photon

Identify the Variables and Constants

Known	Implied	Unknown
$f = 5.09 \times 10^{14}$ Hz	$c = 3.00 \times 10^8 \frac{\text{m}}{\text{s}}$	λ
		p

Develop a Strategy

Find the wavelength by using the equation for the speed of waves and the value for the speed of light.

$$v = f\lambda$$

$$\lambda = \frac{v}{f}$$

$$\lambda = \frac{3.00 \times 10^8 \frac{\text{m}}{\text{s}}}{5.09 \times 10^{14}\ \text{s}^{-1}}$$

$$\lambda = 5.8939 \times 10^{-7}\ \text{m}$$

Use the equation that relates the momentum of a photon to its wavelength.

$$p = \frac{h}{\lambda}$$

$$p = \frac{6.63 \times 10^{-34}\ \mathrm{J \cdot s}}{5.8939 \times 10^{-7}\ \mathrm{m}}$$

$$p = 1.1249 \times 10^{-27}\ \frac{\mathrm{kg \cdot m}}{\mathrm{s}}$$

$$p \cong 1.12 \times 10^{-27}\ \frac{\mathrm{kg \cdot m}}{\mathrm{s}}$$

The momentum of a photon with a frequency of 5.09×10^{14} Hz is $1.12 \times 10^{-27}\ \frac{\mathrm{kg \cdot m}}{\mathrm{s}}$.

Validate the Solution

You would expect the momentum of a photon to be exceedingly small, and it is. Check to see if the units cancel to give $\frac{\mathrm{kg \cdot m}}{\mathrm{s}}$.

$$\frac{\mathrm{J \cdot s}}{\mathrm{m}} = \frac{\frac{\mathrm{kg \cdot m^{\cancel{2}}}}{\mathrm{s^{\cancel{2}}}} \cdot \cancel{\mathrm{s}}}{\cancel{\mathrm{m}}} = \frac{\mathrm{kg \cdot m}}{\mathrm{s}}$$

PRACTICE PROBLEMS

1. Find the momentum of a photon with a wavelength of 1.55 m (radio wave).
2. Find the momentum of a gamma ray photon with a frequency of 4.27×10^{20} Hz.
3. What would be the wavelength of a photon that had the same momentum as a neutron travelling at 8.26×10^{7} m/s?
4. How many photons with a wavelength of 5.89×10^{-7} m would it take to equal the momentum of a 5.00 g Ping-Pong™ ball moving at 8.25 m/s?
5. What would be the frequency of a photon with a momentum of 2.45×10^{-32} kg · m/s? In what part of the electromagnetic spectrum would this photon be?

Matter Waves

By the 1920s, physicists had accepted the quantum theory of light and continued to refine the concepts. Once again, however, the scientific community was startled by the revolutionary theory proposed by a young French graduate student, who was studying at the Sorbonne. As part of his doctoral dissertation, Louis de Broglie (1892–1987) proposed that not only do light waves behave as particles, but also that particulate matter has wave properties.

De Broglie's professors at the Sorbonne thought that the concept was rather bizarre, so they sent the manuscript to Einstein and asked for his response to the proposal. Einstein read the dissertation with excitement and strongly supported de Broglie's proposal. De Broglie was promptly granted his Ph.D., and six years later, he was honoured with the Nobel Award in Physics for his theory of matter waves. The following steps lead to what is now called the **de Broglie wavelength** of matter waves.

- Write Compton's equation for the momentum of a photon. $p = \frac{h}{\lambda}$
- Solve the equation for wavelength. $\lambda = \frac{h}{p}$
- Substitute the value for the momentum of a particle for p. $\lambda = \frac{h}{mv}$

DE BROGLIE WAVELENGTH OF MATTER WAVES

The de Broglie wavelength of matter waves is the quotient of Planck's constant and the momentum of the mass.

$$\lambda = \frac{h}{mv}$$

Quantity	Symbol	SI unit
wavelength (of a matter wave)	λ	m (metres)
Planck's constant	h	J · s (joule seconds)
mass	m	kg (kilograms)
velocity	v	$\frac{\text{m}}{\text{s}}$ (metres per second)

Unit Analysis

$$\frac{\text{joule} \cdot \text{second}}{\text{kilogram}\frac{\text{metre}}{\text{second}}} = \frac{\text{J} \cdot \text{s}}{\text{kg}\frac{\text{m}}{\text{s}}} = \frac{\text{J} \cdot \text{s}}{\text{kg}} \cdot \frac{\text{s}}{\text{m}} = \frac{\frac{\cancel{\text{kg}} \cdot \text{m}^{\cancel{2}}}{\cancel{\text{s}^2}}\cancel{\text{s}^2}}{\cancel{\text{kg}} \cdot \cancel{\text{m}}} = \text{m}$$

Note: Since wavelength is a scalar quantity, vector notations are not used for velocity.

SAMPLE PROBLEM

Matter Waves

Calculate the wavelength of an electron moving with a velocity of 6.39×10^6 m/s.

Conceptualize the Problem

- *Moving particles* have *wave* properties.
- The *wavelength* of particle waves depends on *Planck's constant* and the *momentum* of the particle.

Identify the Goal

The wavelength, λ, of the electron

Identify the Variables and Constants

Known	Implied	Unknown
$v = 6.39 \times 10^6\ \frac{\mathrm{m}}{\mathrm{s}}$	$h = 6.63 \times 10^{-34}\ \mathrm{J \cdot s}$	λ
	$m_e = 9.11 \times 10^{-31}\ \mathrm{kg}$	

Develop a Strategy

Use the equation for the de Broglie wavelength.

$$\lambda = \frac{h}{mv}$$

$$\lambda = \frac{6.63 \times 10^{-34}\ \mathrm{J \cdot s}}{(9.11 \times 10^{-31}\ \mathrm{kg})\left(6.39 \times 10^6\ \frac{\mathrm{m}}{\mathrm{s}}\right)}$$

$$\lambda = 1.1389 \times 10^{-10}\ \mathrm{m}$$

$$\lambda \cong 1.14 \times 10^{-10}\ \mathrm{m}$$

The de Broglie wavelength of an electron travelling at 6.39×10^6 m/s is 1.14×10^{-10} m.

Validate the Solution

Since Planck's constant is in the numerator, you would expect that the value would be very small. Check the units to ensure that the final answer has the unit of metres.

$$\frac{\mathrm{J \cdot s}}{\mathrm{kg}\frac{\mathrm{m}}{\mathrm{s}}} = \frac{\frac{\cancel{\mathrm{kg}} \cdot \mathrm{m}^{\cancel{2}}}{\mathrm{s}^{\cancel{2}}} \cdot \cancel{\mathrm{s}}}{\cancel{\mathrm{kg}}} \cdot \frac{\cancel{\mathrm{s}}}{\cancel{\mathrm{m}}} = \mathrm{m}$$

PRACTICE PROBLEMS

6. Calculate the wavelength of a proton that is moving at 3.79×10^6 m/s.
7. Calculate the wavelength of an alpha particle that is moving at 1.28×10^7 m/s.
8. What is the wavelength of a 5.00 g Ping-Pong™ ball moving at 12.7 m/s?
9. Find the wavelength of a jet airplane with a mass of 1.12×10^5 kg that is cruising at 891 km/h.
10. Calculate the wavelength of a beta particle (electron) that has an energy of 4.35×10^4 eV.
11. What is the speed of an electron that has a wavelength of 3.32×10^{-10} m?

To verify de Broglie's hypothesis that particles have wavelike properties, an experimenter would need to show that electrons exhibit interference. A technique such as Young's double-slit experiment would be ideal. This technique is not feasible for particles such as electrons, however, because the electrons have wavelengths in the range of 10^{-10} m. It simply is not possible to mechanically cut slits this small and close together. Fortunately, a new technique for observing interference of waves with very small wavelengths had recently been devised.

PHYSICS FILE

As you know, in 1897, J.J. Thomson provided solid evidence for the existence of the electron, a subatomic particle that is contained in all atoms. Ironically, just 30 years later, his son George P. Thomson demonstrated that electrons behave like waves.

During the 10 years prior to de Broglie's proposal, physicists Max von Laue (1879–1960) and Sir Lawrence Bragg (1890–1971) were developing the theory and technique for diffraction of X rays by crystals. The spacing between atoms in crystals is in the same order of magnitude as both the wavelength of X rays and electrons, about 10^{-10} m. As illustrated in Figure 12.11, when X rays scatter from the atoms in a crystal, they form diffraction patterns in much the same way that light forms diffraction patterns when it passes through a double slit or a diffraction grating. If electrons have wave properties, then the same crystals that diffract X rays should diffract electrons and create a pattern.

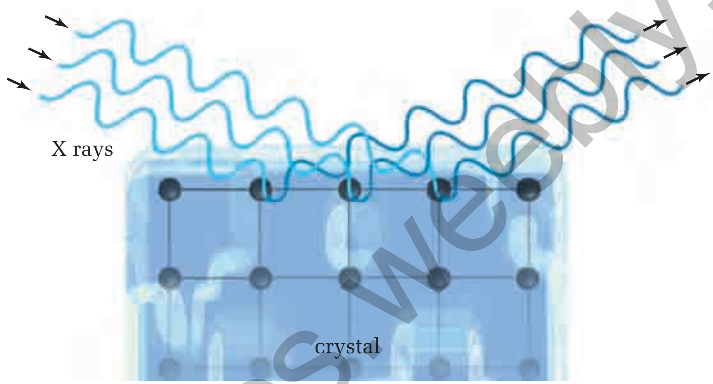


Figure 12.11 X rays scattered from regularly spaced atoms in a crystal will remain in phase only at certain scattering angles.

Within three years after de Broglie published his theory of matter waves, Clinton J. Davisson (1881–1958) and Lester H. Germer (1896–1971) of the United States and, working separately, George P. Thomson (1892–1975) of England carried out electron diffraction experiments. Both teams obtained patterns very similar to those formed by X rays. The wave nature of electrons was confirmed. In the years since, physicists have produced diffraction patterns with neutrons and other subatomic particles. Figure 12.12 shows diffraction patterns from aluminum foil formed by a beam of (A) X rays and (B) electrons.

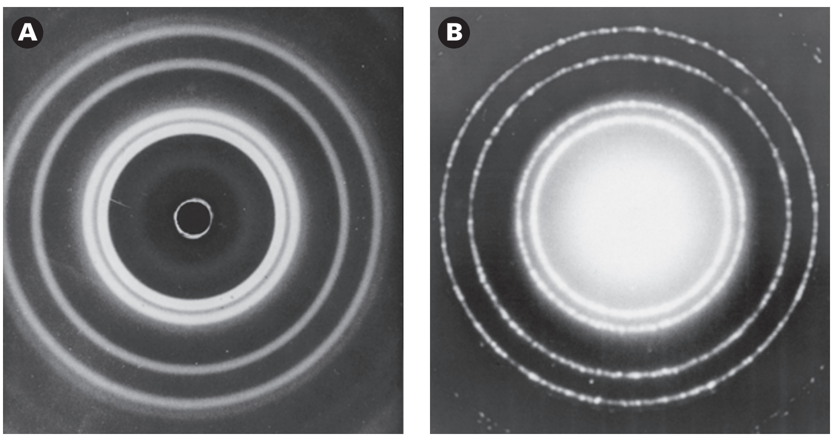


Figure 12.12 These patterns were created by diffraction of **(A)** X rays and **(B)** electrons by aluminum foil. Diffraction occurs as a result of the interference of waves. The similarity of these patterns verifies that electrons behave like waves.

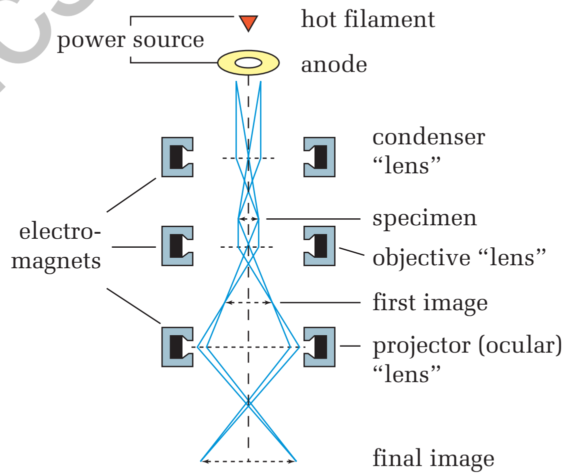
CANADIANS IN PHYSICS

University of Toronto Graduate Students Make History

Although many research groups around the world were attempting to design and build electron microscopes in the 1930s, the first high-resolution electron microscope that was practical and therefore became the prototype for the first commercial instrument was designed, built, and tested by two graduate students at the University of Toronto. James Hillier and Albert Prebus are shown in the photograph with the electron microscope that they built in 1938. Hillier continued to perfect and use the electron microscope while he completed his Ph.D. degree. In 1940, Hillier joined the staff of the Radio Corporation of America (RCA) in Camden, New Jersey, where he continued to improve the electron microscope. In 1969, Hillier became the executive vice president in charge of research and engineering for RCA. In this position, he was responsible for all of the research, development, and engineering programs.

The race to build electron microscopes was based on Davisson and Germer's verification of the wave properties of electrons. Electron microscopes have much greater resolving power than light microscopes, due to their very short wavelengths. Resolving power is the ability to distinguish two or more objects as separate entities, rather than as one large object. If the distance between two objects is much less than the wavelength, a microscope "sees" them as one particle, rather than as two. You can magnify the image to any size, but all that you will see is one large, blurred object. Since the shortest wavelength of visible light is about 400 nm and electrons can have wavelengths of 0.005 nm, electron microscopes could theoretically have a resolving power more than 10 000 times greater than light microscopes. In practice, however, electron microscopes have resolving powers about 1000 times greater than light microscopes.

The diagram shows the typical design of a transmission electron microscope. The barrel of the microscope must be evacuated, because electrons would be scattered by molecules in the air. Electrons would not penetrate glass lenses, of course, so focussing is accomplished by magnetic fields created by electromagnets. These magnetic "lenses" do not have to be moved or changed, because their focal lengths can be changed simply by adjusting the magnetic field strength of the electromagnets. Since electrons cannot penetrate glass, the extremely thin electron microscope specimens are placed on a wire mesh so that the electrons can penetrate the areas between the tiny wires.



The photograph at the beginning of this chapter was produced by a scanning electron microscope. These instruments function on a very different principle than do transmission electron microscopes. A very tiny beam of electrons sweeps back and forth across the specimen, and electrons that bounce back up from the sample are detected. Scanning electron microscopes were first developed in 1942, but they were not commercially available until 1965.

WEB LINK

www.mcgrawhill.ca/links/physics12

For more information about Hillier and Prebus and the history of the electron microscope, including diagrams and photographs, go to the above Internet site and click on **Web Links**.

The Wave-Particle Duality

Within 30 years after Planck presented his revolutionary theory to the German Physical Society, physicists had come to accept the particle nature of light and the wave nature of subatomic particles. They did not, however, forsake Maxwellian electromagnetism or Newtonian mechanics. Newton's concepts have made it possible for astronauts to travel to the Moon and back and to put satellites into orbit. Maxwell's electromagnetism permits engineers to develop the technology to send microwaves to and from these satellites. Physicists accept the dual nature of radiant energy that propagates through space as waves and interacts with matter as particles or discrete packets of energy.

Matter also has a dual nature, but only the subatomic particles have a small enough mass, and thus a large enough wavelength, to exhibit their wave nature. In 1924, Albert Einstein wrote, "There are therefore now two theories of light, both indispensable, and — as one must admit today despite twenty years of tremendous effort on the part of theoretical physicists — without any logical connection." Some physicists hope that in the future we will have a clearer picture of matter waves and quanta of energy. For now, we accept the **wave-particle duality:** Both matter and electromagnetic energy exhibit some properties of waves and some properties of particles.

12.2 Section Review

1. C Explain how Compton determined the momentum of a photon — a particle that has no mass.
2. C Describe the Compton effect.
3. K/U What was the most important result of Compton's experiments with the collisions between photons and electrons?
4. K/U Compton was able to ignore the work function of the metal in which the electrons were embedded in his momentum calculations. How was he able to justify this?
5. C Describe the reasoning that de Broglie used to come up with the idea that matter might have wave properties.
6. C When you walk through a doorway, you represent a particle having momentum and, therefore, having a wavelength. Why is it improbable that you will be "diffracted" as you pass through the doorway?
7. K/U Attempts to demonstrate the existence of de Broglie matter waves by using a beam of electrons incident on Young's double-slit apparatus proved unsuccessful. Give one possible explanation.
8. C Explain the technique that Davisson, Germer, and George P. Thomson used to verify the wave nature of electrons.
9. MC Research the production of X rays and prepare a display poster. In your display, include a diagram of the general structure of the X-ray tube, an explanation of how electrons cause the production of X rays, and an indication of the societal importance of the technology.

12.3 The Bohr Atom and Beyond

The new discoveries in quantum theory revealed phenomena that can be observed on the scale of subatomic particles, but are undetectable on a larger scale. These discoveries gave physicists the tools they needed to probe the structure of atoms in much more detail than ever before. The refinement of atomic theory grew side by side with the development of quantum theory.

SECTION EXPECTATIONS

- Describe and explain the Bohr model of the hydrogen atom.
- Collect and interpret experimental data in support of Bohr's model of the atom.
- Outline the historical development of scientific models from Bohr's model of the hydrogen atom to present-day theories of atomic structure.
- Describe how the development of quantum theory has led to technological advances such as lasers.

KEY TERMS

- nuclear model
- Balmer series
- Rydberg constant
- Bohr radius
- principal quantum number
- Zeeman effect
- Schrödinger wave equation
- wave function
- orbital
- orbital quantum number
- magnetic quantum number
- spin quantum number
- Pauli exclusion principle
- ground state

Atomic Theory before Bohr

As you have learned in previous science courses, the first significant theory of the atom was proposed by John Dalton (1766–1844) in 1808. Dalton's model could be called the "billiard ball model" because he pictured atoms as solid, indivisible spheres. According to Dalton's model, atoms of each element are identical to each other in mass and all other properties, while atoms of one element differed from atoms of each other element. Dalton's model could explain most of what was known about the chemistry of atoms and molecules for nearly a hundred years.

Figure 12.13 Dalton proposed that atoms were the smallest particles that make up matter and that they were indestructible. With his model, Dalton could predict most of what was known about chemistry at the time.

The Dalton model of the atom was replaced when J.J. Thomson established in 1897 that the atom was divisible. He discovered that the "cathode rays" in gas discharge tubes (see Figure 12.14) were negatively charged particles with a mass nearly 2000 times smaller than a hydrogen atom, the smallest known atom. These negatively charged particles, later named "electrons," appeared to have come off the metal atoms in one of the electrodes in the gas discharge tubes. Based on this new information, Thomson developed another model of the atom, which consisted of a positively charged sphere with the negatively charged electrons imbedded in it, as illustrated in Figure 12.15.

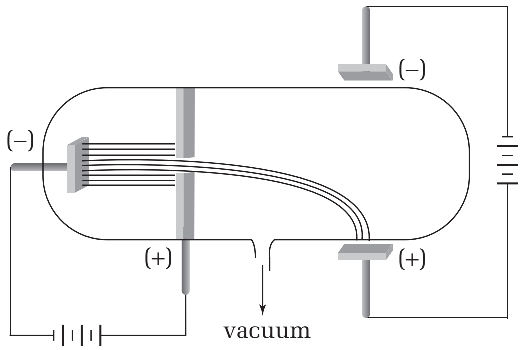


Figure 12.14 Metal electrodes were sealed in a glass tube that had been evacuated of all but a trace of a gas. A potential difference was created between the two electrodes. "Cathode rays" emanated from the negative electrode and a few passed through a hole in the positive electrode. Thomson showed that these "cathode rays" carried a negative charge by placing another set of electrodes outside the tube. The positive plate attracted the "rays."

Even as Thomson was developing his model of the atom, Ernest Rutherford (1871–1937) was beginning a series of experiments that would lead to replacement of Thomson's model. Rutherford was born and educated in New Zealand. In 1895, he went to England to continue his studies in the laboratory of J.J. Thomson. While there, he became interested in radioactivity and characterized the "rays" emitted by uranium, naming them "alpha rays" and "beta rays." He discovered that alpha rays were actually positively charged particles.

In 1898, Rutherford accepted a position in physics at McGill University in Montréal, where he continued his studies of alpha particles and published 80 scientific papers. Nine years later, Rutherford returned to England, where he accepted a position at the University of Manchester.

While in Manchester, Rutherford and his research assistant Hans Geiger (1882–1945) designed an apparatus (see Figure 12.16) to study the bombardment of very thin gold foils by highly energetic alpha particles. If Thomson's model of the atom was correct, the alpha particles would pass straight through, with little or no deflection. In their preliminary observations, most of the alpha particles did, in fact, pass straight through the gold foil. However, in a matter of days, Geiger excitedly went to Rutherford with the news that they had observed some alpha particles scatter at an angle greater than 90˚. Rutherford's famous response was, "It was quite the most incredible event that has ever happened to me in my life. It was almost as incredible as if you fired a 15 inch shell at a piece of tissue paper and it come back and hit you!" The observations were consistent: Approximately 1 in every 20 000 alpha particles was deflected more than 90˚. These results could not be explained by Thomson's model of the atom.

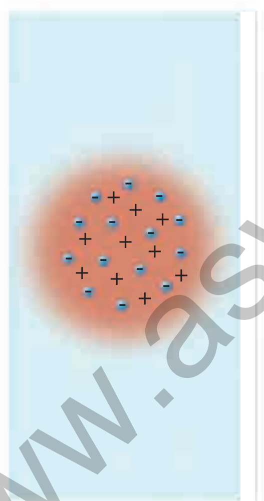


Figure 12.15 Thomson named his model the "plum pudding model" because it resembled a pudding with raisins distributed throughout.

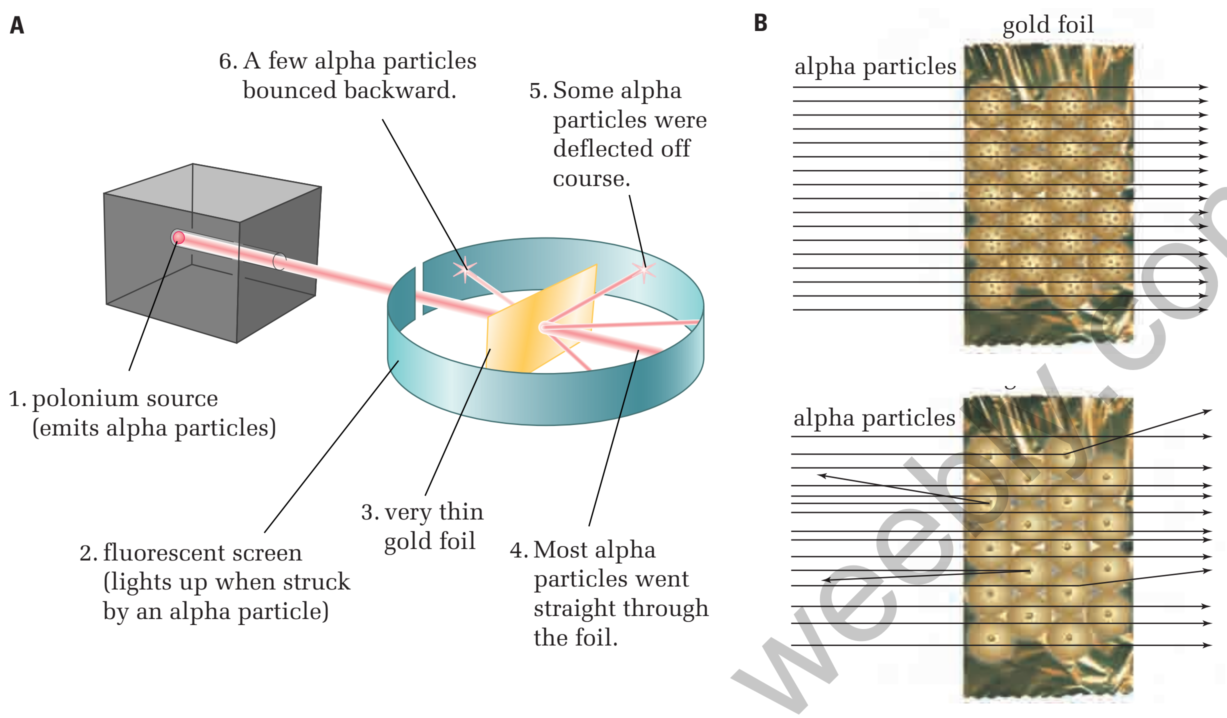


Figure 12.16 **(A)** A fine beam of alpha particles was directed at a very thin gold foil. The circular screen around the foil was coated with zinc sulfide, which emitted a flash of light when hit by an alpha particle. **(B)** If positive and negative charges were equally distributed throughout the foil, they would have little effect on the direction of the alpha particles. **(C)** If all of the positive charge in each atom was concentrated in a very tiny point, it would create a large electric field close to the point. The field would deflect alpha particles that are moving directly toward or very close to the tiny area where the positive charge is located.

What force could possibly be strong enough to repel such a highly energetic alpha particle? Rutherford searched his mind and performed many calculations. He concluded that the only force great enough to repel the alpha particles would be an extremely strong electrostatic field. The only way that a field this strong could exist was if all of the positive charge was confined in an extremely small space at the centre of the atom. Thus, Rutherford proposed his **nuclear model** of the atom. All of the positive charge and nearly all of the mass of an atom is concentrated in a very small area at the centre of the atom, while the negatively charged electrons circulate around this "nucleus," somewhat like planets around the Sun, as illustrated in Figure 12.17.

In the following Quick Lab, you will apply some of the same concepts that Rutherford used to estimate the size of the atomic nucleus.

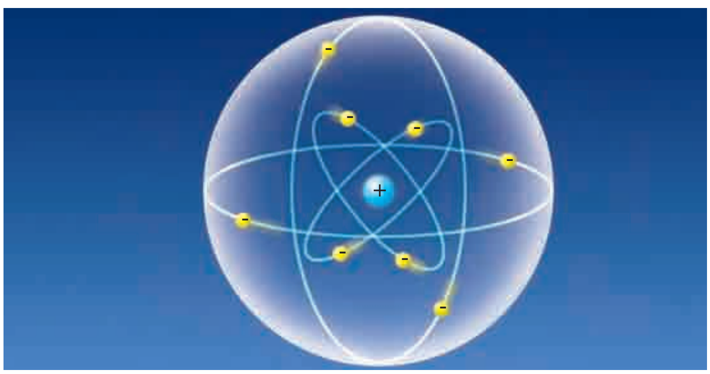


Figure 12.17 Rutherford's nuclear model resembles a solar system in which the positively charged nucleus could be likened to the Sun and the electrons are like planets orbiting the Sun.

QUICK LAB

Estimating the Size of the Nucleus

TARGET SKILLS
- Hypothesizing
- Analyzing and interpreting

Method 1

At the time that Rutherford was performing his experiments, physicists knew that the diameter of the entire atom was about 10^{-10} m.

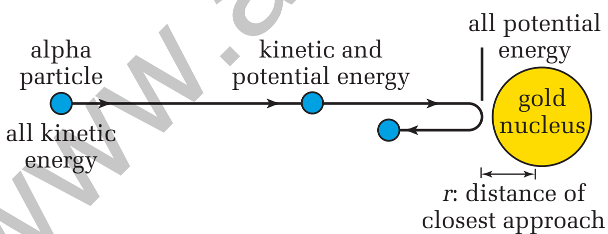
- Calculate the cross-sectional area of the atom, assuming that its diameter is 10^{-10} m.

Rutherford's students observed that about 1 in every 20 000 alpha particles scattered backward from the foil. If they were all headed toward the atom but only 1 in 20 000 was headed directly toward the nucleus, what must be the cross-sectional area of the nucleus?

- Calculate the cross-sectional area of the nucleus based on the above information.
- Using your cross-sectional area of the nucleus, calculate the diameter of the nucleus.

Method 2

Make a second estimate of the size of the nucleus based on the conservation of mechanical energy of the alpha particle. As shown in the diagram, at a large distance from the nucleus, the energy of the alpha particle is all kinetic energy. As it approaches the nucleus, its kinetic energy is converted into electric potential energy. When all of its kinetic energy is converted into electric potential energy, the alpha particle will stop. At this point, called the "distance of closest approach," the repulsive Coulomb forces will drive the alpha particle directly backward. If the alpha particle penetrated the nucleus, it would be trapped and would not scatter backward.



- The equation below states that the kinetic energy of the alpha particle at a large distance from the nucleus is equal to the electric potential energy of the alpha particle at the distance of closest approach. Substitute into the equality the mathematical expressions for kinetic energy and electric potential energy between two point charges a distance, r, apart.

$$E_k \text{ (very far from nucleus)} = E_Q \text{ (distance of closest approach)}$$

- The mass of an alpha particle is about 6.6×10^{-27} kg and those that Rutherford used had an initial velocity of 1.5×10^7 m/s. Calculate the kinetic energy of the alpha particle.
- An alpha particle has 2 positive charges and a gold nucleus has 79 positive charges. Using the magnitude of one elementary charge (1.6×10^{-19} C), calculate the magnitude of the charges needed for the determination of the electric potential energy.
- Substitute all of the known values into the equation above. You will find that r is the only unknown variable. Solve the equation for r, the distance of closest approach.

Analyze and Conclude

1. Comment on the validity of each of the two methods. What types of errors might affect the results?
2. How well do your two methods agree?
3. The accepted size of an average nucleus is in the order of magnitude of 10^{-14} m. How well do your calculations agree with the accepted value?

The Bohr Model of the Atom

Rutherford's model of the atom was based on solid experimental data, but it had one nagging problem that he did not address. According to classical electromagnetism, an accelerating charge should radiate electromagnetic waves and lose energy. If electrons are orbiting around a nucleus, then they are accelerating and they should be radiating electromagnetic waves. If the electrons lost energy through radiation, they would spiral into the nucleus. According to Rutherford's model, electrons remain permanently in orbit.

Niels Henrik David Bohr (1885–1962) addressed the problem of electrons that do not obey classical electromagnetic theory. Bohr was born and educated in Denmark, and in 1912, went to study in Rutherford's laboratory in Manchester. (Rutherford said of Bohr, "This young Dane is the most intelligent chap I've ever met.") Convinced that Rutherford was on the right track with the nuclear atom, Bohr returned home to Copenhagen, where he continued his search for an explanation for the inconsistency of the nuclear atom with classical theory.

Bohr was very aware of the recent publications of Planck and Einstein on blackbody radiation and the photoelectric effect, and that these phenomena did not appear to obey the laws of classical physics. He realized that some phenomena that are unobservable on the macroscopic level become apparent on the level of the atom. Thus, he did not hesitate to propose characteristics for the atom that appeared to contradict classical laws.

Bohr had another, very significant piece of evidence available to him — atomic spectra. When Kirchhoff defined blackbodies, he was studying very low-pressure gases in gas discharge tubes. Kirchhoff discovered that when gases of individual elements were sealed in gas discharge tubes and bombarded with "cathode rays," each element produced a unique spectrum of light. The spectrum of hydrogen is shown in Figure 12.18.

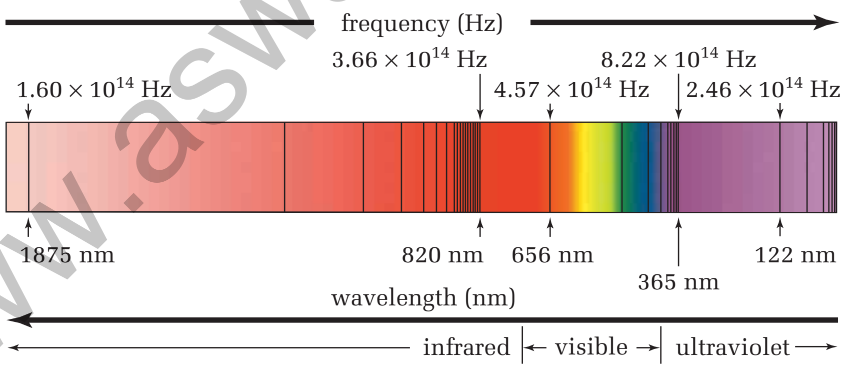


Figure 12.18 When bombarded by high-energy electrons, hydrogen atoms emit a very precise set of frequencies of electromagnetic radiation, extending from the infrared region, through the visible region, and well into the ultraviolet region of the spectrum.

Since emission spectra did not have an immediately obvious pattern, Bohr thought them too complex to be useful. However, a friend who had studied spectroscopy directed Bohr to a pattern that had been determined in 1885 by Swiss secondary school teacher Johann Jakob Balmer (1825–1898). Balmer had studied the visible range of the hydrogen spectrum and found an empirical expression that could produce the wavelength of any line in that region of the spectrum. Balmer's formula is given below. Remember that empirical equations are developed from experimental data and are not associated with any theory. Balmer could not explain why his formula had the form that it did. He could demonstrate only that it worked.

$$\frac{1}{\lambda} = R\left[\frac{1}{2^2} - \frac{1}{n^2}\right], \quad \text{where } n = 3, 4, 5, \ldots \text{ and } R = 1.097\,373\,15 \times 10^7\ \text{m}^{-1}$$

The spectral lines of hydrogen that lie in the visible range are now known as the **Balmer series**. As spectroscopists developed methods to observe lines in the infrared and ultraviolet regions of the spectrum, they found more series of lines. Swedish physicist Johannes Robert Rydberg (1854–1919) modified Balmer's formula, as shown below, to incorporate all possible lines in the hydrogen spectrum. The constant R is known as the **Rydberg constant**.

$$\frac{1}{\lambda} = R\left[\frac{1}{m^2} - \frac{1}{n^2}\right], \quad \text{where } m \text{ and } n \text{ are integers; } 1, 2, 3, 4, \ldots \text{ and } n > m$$

Bohr Postulates

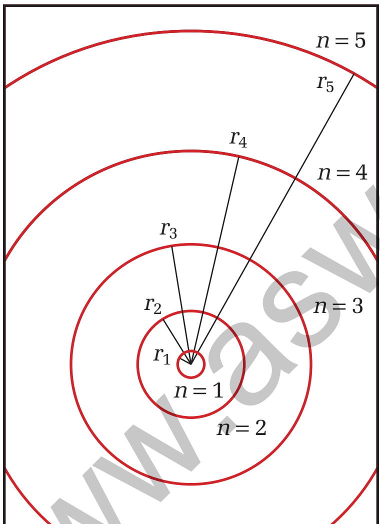


Figure 12.19 According to Bohr's model of the atom, electrons can exist in specific, allowed energy levels and can jump from one level to another by absorbing or emitting energy.

When Bohr saw these mathematical patterns, he said, "As soon as I saw Balmer's formula, the whole thing was immediately clear to me." Bohr was ready to develop his model of the atom. Bohr's model, illustrated in Figure 12.19, was based on the following postulates.

- Electrons exist in circular orbits, much like planetary orbits. However, the central force that holds them in orbit is the electrostatic force between the positive nucleus and the negative charge on the electrons, rather than a gravitational force.
- Electrons can exist only in a series of "allowed" orbits. Electrons, much like planets, have different amounts of total energy (kinetic plus potential) in each orbit, so these orbits can also be described as "energy levels." Since only certain orbits are allowed, then only certain energy levels are allowed, meaning that the energy of electrons in atoms is quantized.
- Contrary to classical theory, while an electron remains in one orbit, it does not radiate energy.
- Electrons can "jump" between orbits, or energy levels, by absorbing or emitting an amount of energy that is equal to the *difference* in the energy levels.

Electrons can "jump" to higher energy levels by absorbing thermal energy (collision with an energetic atom or molecule), by bombardment with an energetic electron (as in gas discharge tubes), or by absorbing photons of radiant energy with energies that exactly match the difference in energy levels of the electrons in the atom. Likewise, electrons can "drop" to a lower energy level by emitting a photon that has an energy equal to the *difference* between the energy levels. Since the energy of a photon is directly related to the frequency of the electromagnetic waves, you could express this relationship between energy levels and photons as follows.

$$|E_f - E_i| = hf$$

E_f is the energy of the final energy level, E_i is the energy of the initial energy level, and hf is the photon energy. This concept gives meaning to the "2" and the "n" in Balmer's formula, because the "2" represents the second energy level and "n" is any energy level above the second one. In Rydberg's more general formula, m is the final energy level, which can be any level. Likewise, n is the initial level and must therefore be higher than the final level.

To find the exact energies of these "allowed energy levels," Bohr had to determine exactly what property of the electron was quantized. An important clue comes from the units of Planck's constant — joule · seconds. First, simplify joules to base units.

$$\mathrm{J \cdot s = N \cdot m \cdot s = \frac{kg \cdot m}{s^{\cancel{2}}} \cdot m \cdot \cancel{s} = kg \cdot \frac{m}{s} \cdot m}$$

The final units, $\mathrm{kg \cdot \frac{m}{s} \cdot m}$, are the units for the quantities of mass, speed, and distance, or mvd. At one time in physics, this combination was called "action." In fact, Planck called his constant, h, the "quantum of action." If you apply these quantities to the electron in an orbit of radius r, you will get $m_e v_n 2\pi r$, where $2\pi r$ is the distance that the electron travels during one orbit around the nucleus. If this value is quantized, you would have the following.

$$2\pi m_e v_n r_n = nh$$

You might recognize the expression $m_e v_n r_n$ as the angular momentum of the electron in the n^{th} orbit. Following a similar logic, Bohr proposed that the angular momentum was quantized and then tested that hypothesis. The angular momentum of the n^{th} orbit can be written as follows.

$$m_e v_n r_n = n\frac{h}{2\pi}$$

You can test the theory by using the equation for the quantized angular momentum to find allowed radii and allowed energies of electrons. Then, you can compare these differences between energy levels to Balmer's formula and the Rydberg constant. Since you have two unknown quantities, r and v, you will need more relationships to find values for either r or v in terms of known

PHYSICS FILE

Johann Balmer's life was a contrast to that of most other contributors to the theory of the atom. Balmer taught mathematics in a secondary school for girls and lectured at the University of Basel in Switzerland. He published only two scientific papers in his career, one when he was 60 years old and one when he was 72. Balmer died 15 years before Niels Bohr provided an explanation for Balmer's now famous formula for the emission spectrum of hydrogen.

constants. Because Bohr based his concept on circular orbits, you can use the fact that the electrostatic force between the electron and the nucleus provides the centripetal force that keeps the electron in a circular orbit. The following steps will lead you through the procedure.

Deriving the Bohr Radius

- Write Coulomb's law. $F = k\frac{q_1 q_2}{r^2}$
- Let Z be the number of positive charges in the nucleus. Therefore, Ze is the charge of the nucleus. The charge on an electron is, of course, e. Let r_n be the radius of the n^{th} orbit. Substitute these values into Coulomb's law. $F = k\frac{Ze^2}{{r_n}^2}$
- Set the coulomb force equal to the centripetal force. $k\frac{Ze^2}{r_n^2} = \frac{m_e v_n^2}{r_n}$
- Multiply both sides by r_n^2. $kZe^2 = m_e v_n^2 r_n$
- Divide both sides by $m_e v_n^2$. $r_n = \frac{kZe^2}{m_e v_n^2}$
- Write Bohr's condition for quantization of angular momentum. $m_e v_n r_n = n\frac{h}{2\pi}$
- Solve for v_n. $v_n = \frac{nh}{2\pi m_e r_n}$
- Substitute this expression for v_n into the equation for r_n. $r_n = \frac{kZe^2}{m_e\left(\frac{nh}{2\pi m_e r_n}\right)^2}$
- Start the simplification by inverting the fraction in the denominator in brackets and then multiplying by the inverted fraction. $r_n = \frac{kZe^2}{m_e} \cdot \frac{4\pi^2 m_e^2 r_n^2}{n^2 h^2}$
- Divide both sides of the equation by r_n. $1 = \left(\frac{4\pi^2 kZe^2 m_e^{\cancel{2}}}{\cancel{m_e} n^2 h^2}\right) r_n$
- Invert and multiply by the expression in brackets. $r_n = \frac{n^2 h^2}{4\pi^2 kZe^2 m_e}$

The expression, $\frac{h}{2\pi}$, occurs so frequently in quantum theory that the symbol $\hbar$ is often used in place of $\frac{h}{2\pi}$. The final expression is usually written as follows.

$$r_n = n^2 \frac{\hbar^2}{m_e kZe^2}$$

For the first allowed radius of the electron in a hydrogen atom, $Z = 1$ and $n = 1$. All of the other values in the equation are constants and if you substitute them into the equation and simplify, you will obtain $r_1 = 0.052\ 917\ 7$ nm. This value is known as the **Bohr radius**.

Deriving Allowed Energy Levels

You can use the equation for the radius of the n^{th} orbit of an electron to find the energy for an electron in the n^{th} energy level in an atom as shown in the following steps.

- Write the expression for the total energy (kinetic plus potential) of a charge a distance, r, from another charge.
$$E = \frac{1}{2}mv^2 - k\frac{q_1 q_2}{r}$$
- Substitute in the values for an electron at a distance, r_n, from a nucleus.
$$E_n = \frac{1}{2}m_e v_n^2 - k\frac{Ze^2}{r_n}$$
- To eliminate the variable, v, from the equation, go back to the expression you wrote when you set the Coulomb force equal to the centripetal force.
$$k\frac{Ze^2}{r_n^2} = \frac{m_e v^2}{r_n}$$
- Multiply both sides of the expression by $\frac{r_n}{2}$ and simplify.
$$\left(k\frac{Ze^2}{r_n^2}\right)\left(\frac{r_n}{2}\right) = \left(\frac{m_e v^2}{r_n}\right)\left(\frac{r_n}{2}\right)$$
$$\frac{kZe^2}{2r_n} = \frac{1}{2}m_e v_n^2$$
- Substitute this value found in the last step for kinetic energy, $\frac{1}{2}m_e v_n^2$, in the second equation and then simplify.
$$E_n = \frac{kZe^2}{2r_n} - k\frac{Ze^2}{r_n}$$
$$E_n = -\frac{kZe^2}{2r_n}$$
- Substitute the value for r_n into the expression for energy.
$$E_n = -\frac{kZe^2}{2\left(n^2\frac{\hbar^2}{m_e kZe^2}\right)}$$
- To simplify, invert the fraction in the denominator and multiply.
$$E_n = -\frac{kZe^2}{2(n^2)} \cdot \frac{m_e kZe^2}{\hbar^2}$$
$$E_n = -\frac{k^2 e^4 m_e}{2\hbar^2} \cdot \frac{Z^2}{n^2}$$

Once again, you can write a general formula for the total energy of an electron in the n^{th} level of a hydrogen atom ($Z = 1$) by substituting the correct values for the constants. You will discover that $E_n = -\frac{13.6\ \text{eV}}{n^2}$. The integer, n, is now known as the **principal quantum number**.

Recalling Bohr's hypothesis that the difference in the energy levels would be the energies of the photons emitted from an atom, you can now use this formula to compare Bohr's model of the atom with the observed frequencies of the spectral lines for hydrogen atoms. For example, you should be able to calculate the frequency of the first line in the Balmer series by doing the following.

$$hf = E_3 - E_2$$

$$hf = \frac{-13.6\text{ eV}}{3^2} - \left(-\frac{13.6\text{ eV}}{2^2}\right)$$

$$hf = -1.511\text{ eV} + 3.40\text{ eV}$$

$$hf = 1.89\text{ eV}$$

$$f = \left(\frac{1.89\ \cancel{\text{eV}}}{h}\right)\left(\frac{1.6 \times 10^{-19}\text{ J}}{\cancel{\text{eV}}}\right)$$

$$f = \frac{3.0222 \times 10^{-19}\ \cancel{\text{J}}}{6.63 \times 10^{-34}\ \cancel{\text{J}} \cdot \text{s}}$$

$$f = 4.56 \times 10^{14}\text{ Hz}$$

This value is in excellent agreement with the observed frequency of the first line in the Balmer series. If you performed similar calculations for the other lines in the Balmer series, you would find the same excellent agreement with observations.

Spectroscopists continued to find series of lines that were matched with electrons falling from higher levels of the hydrogen atom down into the first five energy levels. These series are named and illustrated in Figure 12.20.

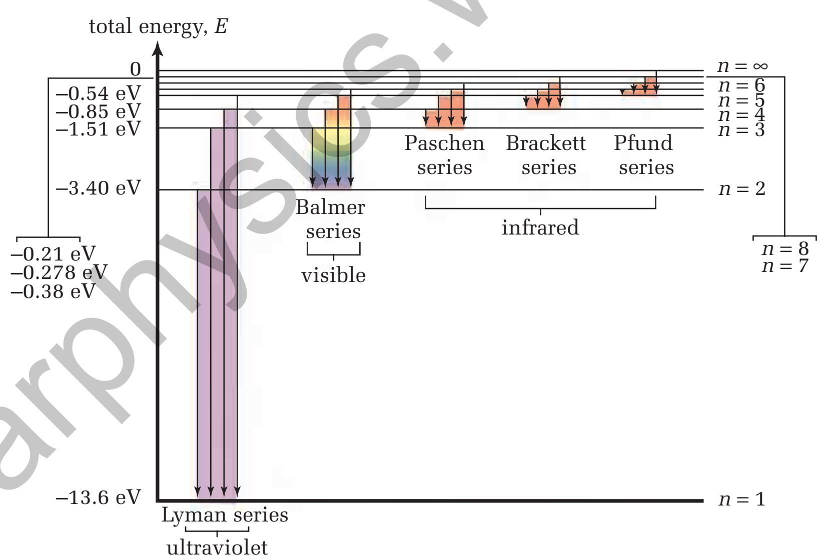


Figure 12.20 Photons from transitions that *end* at the same energy level have energies (and therefore frequencies) that are relatively close together. When inspecting a particular range of frequencies emitted by an element, therefore, an observer would find a set of spectral lines quite close together. Each set of lines is named after the person who observed and described them.

You could perform calculations such as the sample calculation of the frequency of the first line in the Balmer series for any combination of energy levels and find agreement with the corresponding line in the hydrogen spectrum. Bohr's model of the atom was thoroughly tested and was found to be in agreement with most of the data available at the time.

Conceptual Problems

- Start with the expression $hf = |E_f - E_i|$, then substitute the equation for the energy of the n^{th} level of an electron into E_f and E_i into the first expression. Finally, use the relationship $c = f\lambda$ to derive the following expression.

$$\frac{1}{\lambda} = \left| \frac{2\pi^2 k^2 e^4 m_e Z^2}{h^3 c} \left[\frac{1}{n_f^2} - \frac{1}{n_i^2} \right] \right|$$

- The equation above would be identical to the Rydberg equation if the combination of constants $\frac{2\pi^2 k^2 e^4 m_e}{h^3 c}$ was equal to the Rydberg constant for hydrogen atoms ($Z = 1$). Calculate the value of the constants and compare your answer with the Rydberg constant. What does this result tell you about Bohr's model of the atom?

ELECTRONIC LEARNING PARTNER

Enhance your understanding of the Bohr atom, modelled as a wave or as a particle, by referring to your Electronic Learning Partner.

The Quantum Mechanical Atom

Bohr's model of the atom very successfully explained many of the confusing properties of the atom — it marked a monumental first step into the quantum nature of the atom. Nevertheless, the model was incomplete. For example, a very precise examination of the spectrum of hydrogen showed that what had at first appeared to be individual lines in the spectrum were actually several lines that were extremely close together. As illustrated in Figure 12.21, this "fine structure," as it is sometimes called, could best be explained if one or more energy levels was broken up into several very closely spaced energy levels.

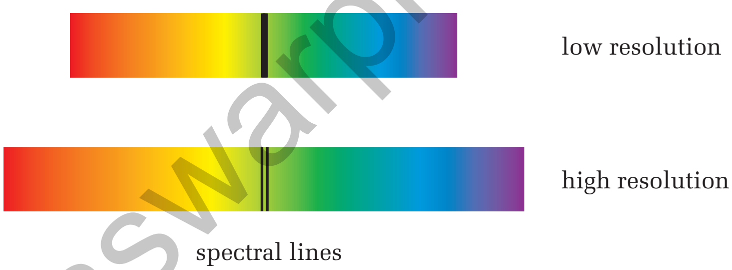


Figure 12.21 Very close examination of the lines in the hydrogen spectrum showed that some of the lines were made up of several fine lines that were very close together.

Another feature of emission spectra that the Bohr atom could not explain was observed in 1896 by Dutch physicist Pieter Zeeman (1865–1943). He placed a sodium flame in a strong magnetic field and then examined the emission spectrum of the flame with a very fine diffraction grating. He observed that the magnetic field caused certain spectral lines to "split" — what had been one line in the spectrum became two or more lines when a magnetic

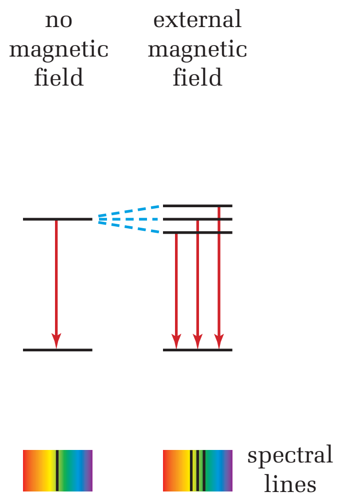


Figure 12.22 When a sample is placed in a magnetic field, some individual spectral lines become a set of closely spaced lines.

field was present. This phenomenon, illustrated in Figure 12.22, is now called the **Zeeman effect**.

Several physicists attempted with some success to modify the Bohr model to account for the fine structure and the Zeeman effect. The greatest success, however, came from an entirely different approach to modelling the atom: De Broglie's concept of matter waves paved the way to the new quantum mechanics or, as it is often called, "wave mechanics."

When de Broglie proposed his hypothesis about matter waves (about 10 years after Bohr had developed his model of the atom), he applied the ideas to the Bohr model. De Broglie suggested that when electrons were moving in circular orbits around the nucleus, the associated "pilot waves," as de Broglie named them, must form standing waves. Otherwise, destructive interference would eliminate the waves. To form a standing wave on a circular path, the length of the path would have to be an integral number of wavelengths, as shown in Figure 12.23. The procedure that follows the illustration will guide you through the first few steps of de Broglie's method for determining the radius of the orbit of the electron matter waves around the nucleus.

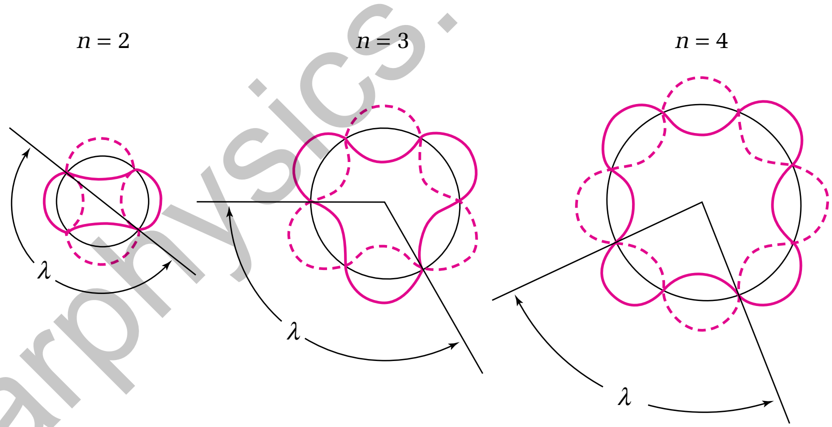


Figure 12.23 The number of wavelengths of matter waves that lie on the radius of an electron orbit is equal to the value of *n* for that energy level. No other wavelengths are allowed because they would interfere destructively with themselves.

- Write the formula for the circumference of a circle and set it equal to any integer (n) times the wavelength. $2\pi r_n = n\lambda$, where $n = 1, 2, 3, \ldots$
- Write de Broglie's formula for the wavelength of a matter wave. $\lambda = \dfrac{h}{mv}$
- Substitute de Broglie's wavelength of an electron into the first equation. $2\pi r_n = n\dfrac{h}{m_e v_n}$
- Divide both sides of the equation by 2π. $r_n = \dfrac{nh}{2\pi m_e v_n}$
- Multiply both sides of the equation by $m_e v_n$ $m_e v_n r_n = \dfrac{nh}{2\pi}$

Notice that the last equation is the same as Bohr's expression for the quantization of angular momentum. From this point on, the derivation of the equation for the radius of allowed orbits would be exactly the same as Bohr's derivation. Using two entirely different approaches to the quantization of electron orbits, Bohr's and de Broglie's results were identical.

In 1925, Viennese physicist Erwin Schrödinger (1887–1961) read de Broglie's thesis with fascination. Within a matter of weeks, Schrödinger had developed a very complex mathematical equation that can be solved to produce detailed information about matter waves and the atom. The now-famous equation, called the **Schrödinger wave equation**, forms the foundation of quantum mechanics. When you insert data describing the potential energy of an electron or electrons in an atom into the wave equation and solve the equation, you obtain mathematical expressions called "wave functions." These **wave functions**, represented by the Greek letter ψ (psi), provide information about the allowed orbits and energy levels of electrons in the atom.

Wave functions account for most of the details of the hydrogen spectra that the original Bohr model could not explain. However, Schrödinger's wave functions could not predict one small, magnetic "splitting" of energy levels. British physicist Paul Adrien Maurice Dirac (1902–1984) realized that electrons travelling in the lower orbits in an atom would be travelling at excessively high speeds, high enough to exhibit relativistic effects. In 1928, Dirac modified Schrödinger's equation to account for relativistic effects. The equation could then account for all observed properties of electrons in atomic orbits. In addition, it predicted many phenomena that had not yet been discovered when the equation was developed.

You are probably wondering, "What are wave functions and what do the amplitude and velocity of a matter wave describe?" Many physicists in the early 1900s asked the same question.

Wave functions do not describe such properties as the changing pressure of air in a sound wave or the changing electric field strength in an electromagnetic wave. In fact, wave functions cannot describe any real property, because they contain the imaginary number i ($i = \sqrt{-1}$, which does not exist). You must carry out a mathematical operation on the wave functions to eliminate the imaginary number in order to describe anything real about the atom.

The result of this operation, symbolized $\psi^*\psi$, represents the probability that the electron will occupy a certain position in the atom at a certain time. You could call the wave function a "wave of probability." You can no longer think of the electron as a solid particle that is moving in a specific path around the nucleus of an atom, but rather must try to envision a cloud such as the one shown in Figure 12.24 (A) and interpret the density of the cloud as the probability that the electron is in that location. These "regions

COURSE CHALLENGE

Waves and Particles

The process of science continues to discover truths about the nature of our universe. How far have we really come? How well are we able to describe our world? Refer to page 605 for ideas to help you incorporate philosophical debate into your *Course Challenge*.

PHYSICS FILE

Solutions to Dirac's modification of the Schrödinger wave equation predicted twice as many particles as were known to exist in the systems for which the equation was defined. Dirac realized that one stage in the solution contained a square root that yielded both positive and negative values. For example, $\sqrt{16y^4} = \pm 4y^2$. You have probably solved problems involving projectile motion or some other form of motion and found both positive and negative values for time. You simply said that a negative time had no meaning and you chose the positive value. Dirac tried this approach, but it changed the final results. Dirac's original results seemed erroneous because they predicted the existence of antiparticles, which had not yet been discovered. Soon after, antiparticles were observed experimentally by other scientists. You will learn about antiparticles in Chapter 13, The Nucleus and Elementary Particles.

in space" occupied by an electron are often called **orbitals**. If the orbital of the electron is pictured as solid in appearance, as shown in Figure 12.24 (B), it means that there is a 95% probability that the electron is within the enclosed space.

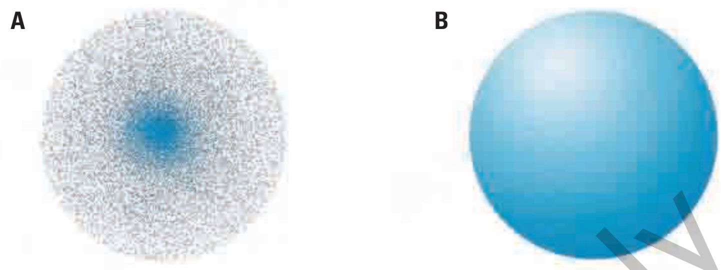


Figure 12.24 When you plot $\psi^*\psi$, you obtain orbitals such as these. **(A)** Orbitals drawn in this manner show the probability of finding the electron. **(B)** Often orbitals are drawn with solid outlines. The probability that the electron is within the enclosed space is 95%.

You might also wonder if the Bohr model was wrong and should be discarded. The answer to that question is a resounding no. The wave functions — that is, the solutions to the Schrödinger wave equation — give the same energy levels and the same principal quantum number (n) that the Bohr model gave. Also, the distance from the nucleus for which the probability of finding the electron is greatest is exactly the same as the Bohr radius. These results show that the general features of the Bohr model are correct and that it is a very useful model for general properties of the atom. The wave equation is necessary only in the finer details of structure.

Quantum Numbers

ELECTRONIC LEARNING PARTNER

Your Electronic Learning Partner contains an excellent reference source of emission and absorption spectra for every element in the periodic table.

The wave functions obtained from Schrödinger's wave equation include two more quantum numbers in addition to the principal quantum number, n. Dirac's relativistic modification of the Schrödinger equation adds another quantum number, making a total of four quantum numbers that specify the characteristics of each electron in an atom. Each quantum number represents one property of the electron that is quantized.

The principal quantum number, n, represents exactly the same property of the atom in both the Bohr model and the Schrödinger model and specifies the energy level of the electron. The value of n can be any positive integer: 1, 2, 3, 4, ... These energy levels are sometimes referred to as "shells."

The **orbital quantum number**, ℓ, specifies the shape of the orbital. The value of ℓ can be any non-negative integer less than n. For example, when $n = 1$, $\ell = 0$. When $n = 2$, ℓ can be 0 or 1. In chemistry, orbitals with different values of ℓ (0, 1, 2, 3, ...) are assigned the letters s, p, d, f, ... Figure 12.25 shows the shapes of orbitals for the first three values of ℓ.

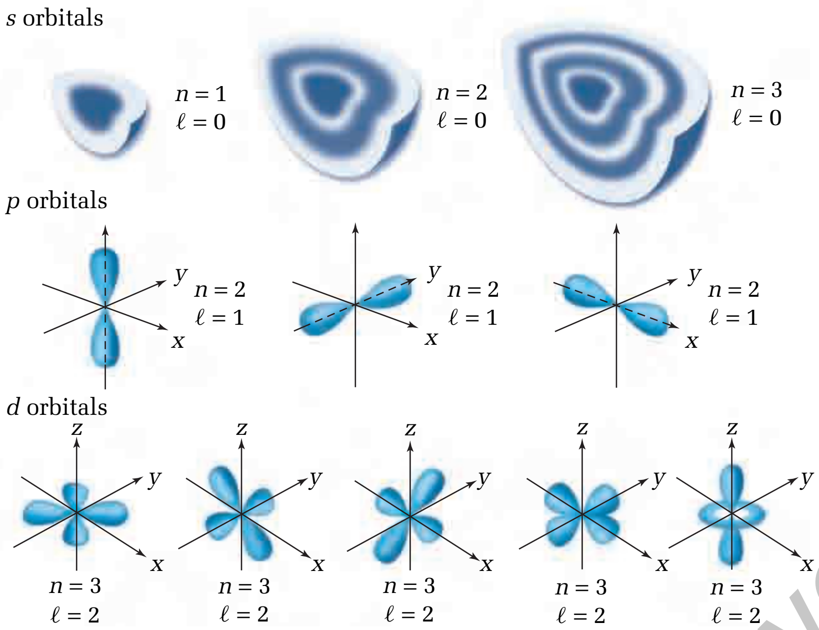


Figure 12.25 Orbitals for which $\ell = 0$ (*s* orbitals) are always spherical. When $\ell = 1$ (*p* orbitals), each orbital has two lobes. Four of the $\ell = 2$ (*d*) orbitals have four lobes and the fifth $\ell = 2$ orbital has two lobes plus a disk.

The orbital quantum number is sometimes called the "angular momentum quantum number," because it determines the angular momentum of the electron. If an electron was to move along a curved path, it would have angular momentum. Although it is not accurate to think of the electron as a tiny, solid piece of matter orbiting around the nucleus, some properties of the electron clouds of orbitals with ℓ greater than zero give angular momentum to the electron cloud. Electrons that have the same value of n but have different values of ℓ possess slightly different energies. As illustrated in Figure 12.26, these closely spaced energy levels account for the fine structure in an emission spectrum of the element.

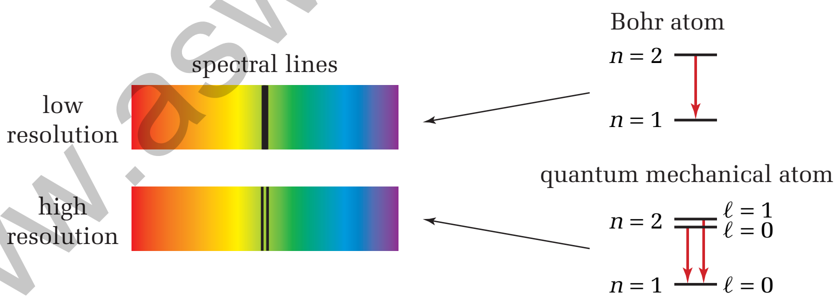


Figure 12.26 For any energy level (shell) for which $n > 0$, there is more than one value of the orbital quantum number, ℓ. The orbitals for each value of ℓ have slightly different energies. These closely spaced energies account for the fine structure, that is, the presence of more than one spectral line very close together.

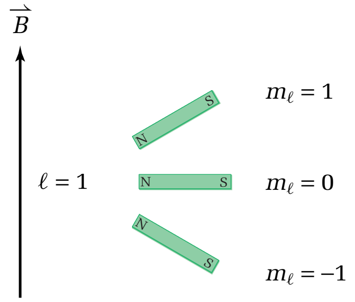


Figure 12.27 In the absence of an external magnetic field, the ℓ orbitals can take any random orientation in space. When a sample is placed in a magnetic field, the ℓ orbitals take on specific orientations in relation to the external field.

The **magnetic quantum number**, m_ℓ, determines the orientation of the orbitals when the atom is placed in an external magnetic field. To develop a sense of what this quantum number means, it is once again helpful, although not entirely accurate, to think of the electron in its cloud as an electric current flowing around the nucleus. As you know, a current flowing in a loop creates a magnetic field. The magnetic quantum number determines how this internal field is oriented if the atom is placed in an external magnetic field. In Figure 12.27, the electron's magnetic field is represented by a small bar magnet with different orientations in an external magnetic field.

The **spin quantum number**, m_s, results from the relativistic form of the wave equation. The term "spin" is used because the effect is the same as it would be if the electron was a spherical charged object that was spinning. A spinning charge creates its own magnetic field in much the same way that a circular current does. The value of m_s can be only $+\frac{1}{2}$ or $-\frac{1}{2}$. Similar to the magnetic quantum number, the spin quantum number has an effect on the energy of the electron only when the atom is placed in an external magnetic field. The two orientations in the external magnetic field are often called "spin up" and "spin down." Figure 12.28 illustrates the two possible orientations of the electron spin and its effect on the electron's energy and spectrum in an external magnetic field.

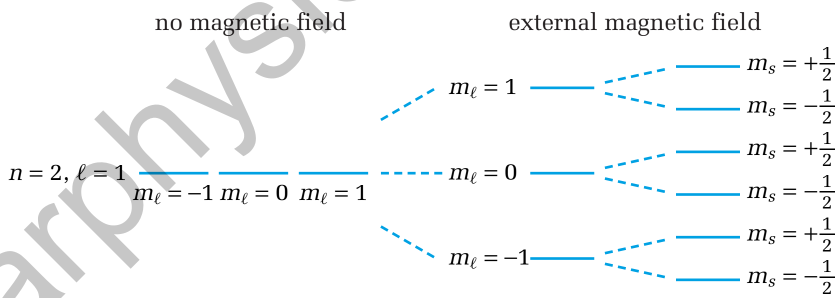
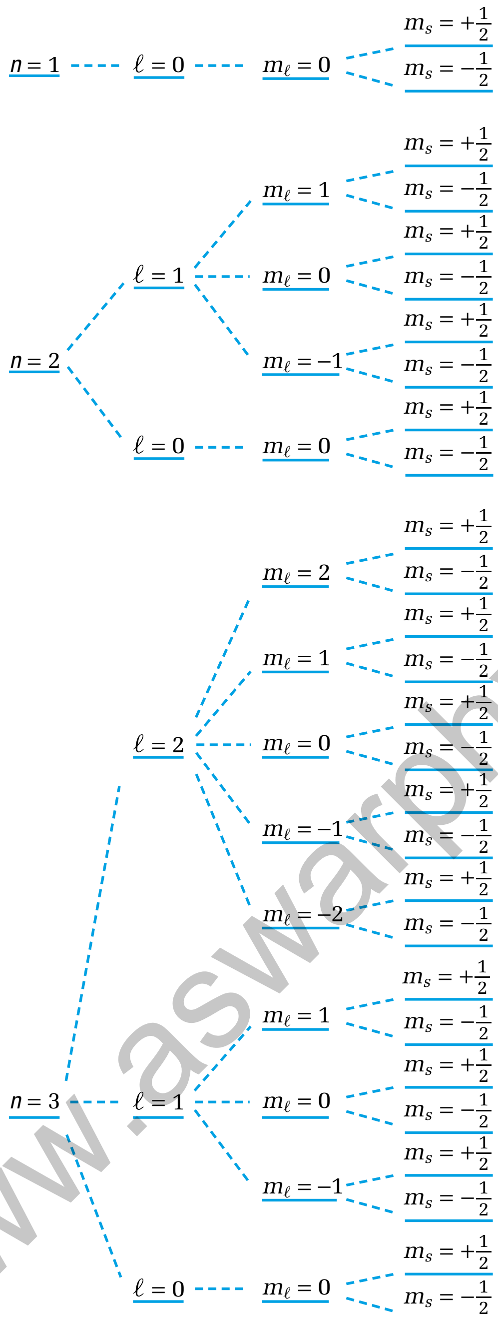


Figure 12.28 The electron spin can assume any orientation in the absence of an external magnetic field, but can take only two orientations when placed in a magnetic field — spin up or spin down.

These four quantum numbers and the associated wave functions can explain and predict essentially all of the observed characteristics of atoms. Two questions might arise, however: If almost all of the mass of an atom is confined to a very tiny nucleus and electrons, with very little mass, are in "clouds" that are enormous compared to the nucleus, why does matter seem so "solid"? Why cannot atoms be compressed into much smaller volumes?

Austrian physicist Wolfgang Pauli (1900–1958) answered those questions in 1925. According to the **Pauli exclusion principle**, *no two electrons in the same atom can occupy the same state.* An easier way of saying the same thing is that *no two electrons in the same atom can have the same four quantum numbers.* Electron clouds of atoms cannot overlap.

The Pauli exclusion principle also tells us how many electrons can fit into each energy level of an atom. The tree diagrams in Figure 12.29 show how many electrons can fit into the first three energy levels, $n = 1$, $n = 2$, and $n = 3$.



State				Total number of states
n	ℓ	m_ℓ	m_s	
1	0	0	$+\frac{1}{2}$	2
1	0	0	$-\frac{1}{2}$	
2	1	1	$+\frac{1}{2}$	8
2	1	1	$-\frac{1}{2}$	
2	1	0	$+\frac{1}{2}$	
2	1	0	$-\frac{1}{2}$	
2	1	−1	$+\frac{1}{2}$	
2	1	−1	$-\frac{1}{2}$	
2	0	0	$+\frac{1}{2}$	
2	0	0	$-\frac{1}{2}$	
3	2	2	$+\frac{1}{2}$	18
3	2	2	$-\frac{1}{2}$	
3	2	1	$+\frac{1}{2}$	
3	2	1	$-\frac{1}{2}$	
3	2	0	$+\frac{1}{2}$	
3	2	0	$-\frac{1}{2}$	
3	2	−1	$+\frac{1}{2}$	
3	2	−1	$-\frac{1}{2}$	
3	2	−2	$+\frac{1}{2}$	
3	2	−2	$-\frac{1}{2}$	
3	1	1	$+\frac{1}{2}$	
3	1	1	$-\frac{1}{2}$	
3	1	0	$+\frac{1}{2}$	
3	1	0	$-\frac{1}{2}$	
3	1	−1	$+\frac{1}{2}$	
3	1	−1	$-\frac{1}{2}$	
3	0	0	$+\frac{1}{2}$	
3	0	0	$-\frac{1}{2}$	

Figure 12.29 These tree diagrams show the energy levels, both in the absence and the presence of an external magnetic field, for the first three values of the principal quantum number, *n*.

Conceptual Problem

- Study Figure 12.29 and then draw a tree diagram for the next energy level, $n = 4$. How many electrons will fit into the fourth energy level?

Hydrogen has only one proton in the nucleus, and thus one electron in an orbital. When a hydrogen atom is not excited, the electron is in the $n = 1$, $\ell = 0$ energy level. However, the atom can absorb energy and become excited and the electron can "jump" up to any allowed orbital. All elements other than hydrogen have more than one electron. When atoms are not excited, the electrons are in the lowest possible energy levels that do not conflict with the Pauli exclusion principle. Figure 12.30 gives examples of three different elements with their electrons in the lowest possible energy levels. This condition is called the **ground state** of the atom. Similar to the electron in hydrogen, the electrons of other elements can absorb energy and rise to higher energy levels.

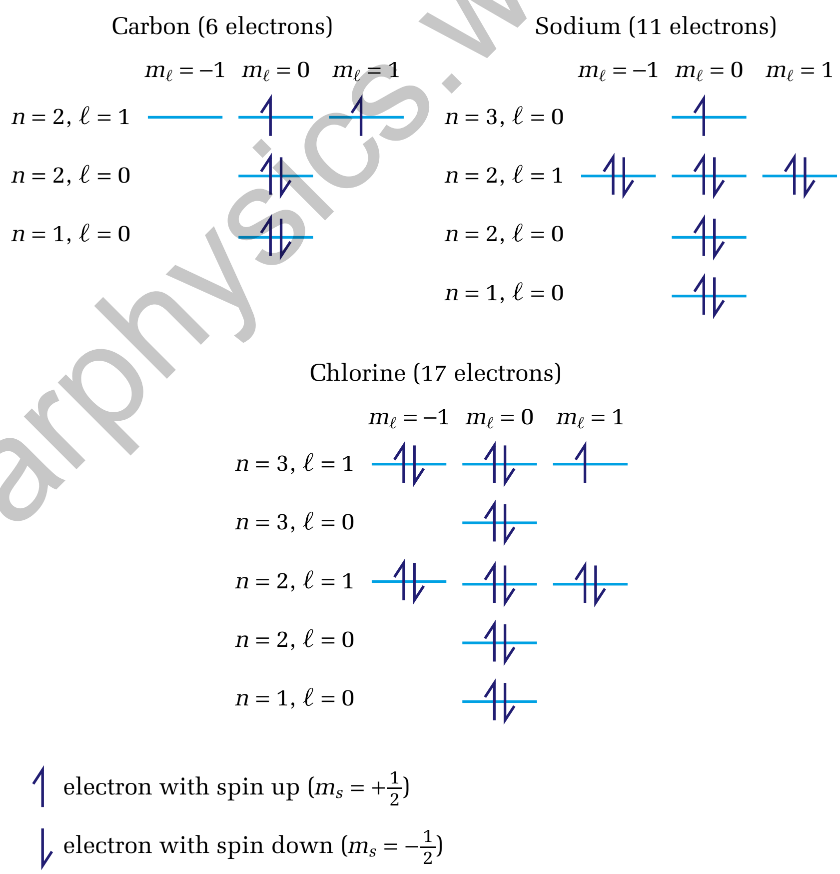


Figure 12.30 Electrons "fill" the energy levels from the lowest upward until there are as many electrons in orbitals as there are protons in the nucleus.

INVESTIGATION 12-B

Identifying Elements by Their Emission Spectra

TARGET SKILLS
- Predicting
- Performing and recording
- Analyzing and interpreting
- Communicating results

The emission spectra of atomic hydrogen gas obtained using gas discharge tubes provided Bohr with critical information that helped him to develop his model of the atom. These spectra also gave him experimental data with which to compare predictions based on his model. In this investigation, you will identify gases from observation of their emission spectra.

Problem

Identify gases from observation of their emission spectra.

Equipment

- hand-held spectroscope
- lighted incandescent bulb
- gas discharge tubes

Procedure

1. Practise using the spectroscope by observing a small incandescent light bulb. Point the slit of the spectroscope toward the bulb and move the spectroscope until you can clearly see the spectrum.
2. Record the appearance of the spectrum from the incandescent bulb.
3. Several numbered gas discharge tubes will be assembled and ready to view. Observe each tube with the spectroscope.

CAUTION A very high voltage is required to operate the gas discharge tubes. Do not come into contact with the source while viewing the tubes.

4. Make a sketch of each spectrum. Draw the relative distances between the lines as accurately as possible. Label each of the lines in each sketch with colour and wavelength to two significant figures.
5. Observe a fluorescent bulb with the spectroscope.
6. Record the appearance of the spectrum from the fluorescent bulb.

Analyze and Conclude

1. In a phrase, describe the spectrum of the incandescent bulb. Explain why the incandescent bulb emits the type of spectrum that you described.
2. Your teacher will provide you with spectra of a variety of types of gases. Compare your sketches with the spectra and attempt to identify each gas in the discharge tubes.
3. Compare your observations of the fluorescent bulb with the spectra from both the incandescent bulb and the gas discharge tubes. Which type of spectrum does the spectrum from the fluorescent bulb most resemble?
4. A fluorescent bulb is a type of gas discharge tube. However, the emissions of the gas are absorbed by a coating on the inside of the bulb and the atoms in the coating are excited and emit light. Based on this description, explain the features of the spectrum of the fluorescent bulb.
5. Is it possible to identify the gas in the fluorescent bulb? Explain why or why not.

Apply and Extend

6. Select one of the central lines in the spectrum of atomic hydrogen. Predict which transition (from which energy level to which energy level) created this line.
7. Check your prediction by using Balmer's formula to calculate the wavelength that the transition would have caused. Compare the calculated wavelength with the wavelength of the spectral line that you selected.

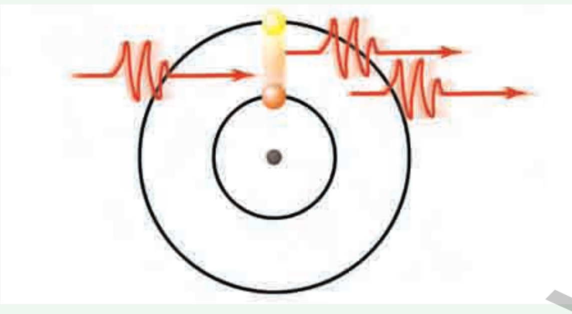
PHYSICS & TECHNOLOGY

TARGET SKILLS

- Hypothesizing
- Analyzing and interpreting

Atoms and Lasers

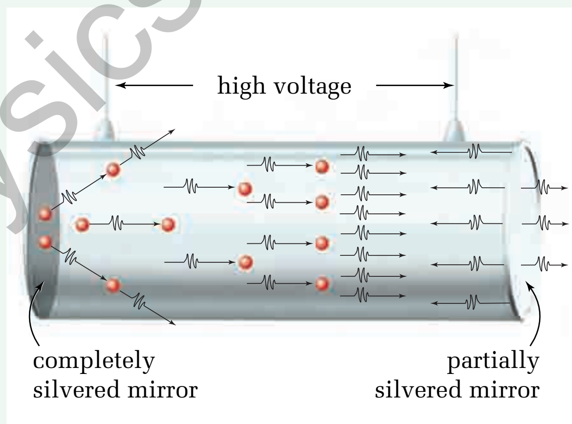
A thorough understanding of the energy levels of electrons in atoms and of transitions between these states was necessary before anyone could even imagine that a laser could be developed. Another critical property of electrons that was necessary in order to develop lasers was predicted by Einstein in 1917 — the stimulation of emission of a photon. As shown in the diagram, if an electron is in an excited state (that is, in a higher energy level), a photon with an energy level equal to the difference in allowed energy levels will stimulate the electron to drop to the lower energy level and emit another identical photon. In addition, the two photons are perfectly in phase.



If more electrons exist in the excited state than in the ground state, it is more probable that a photon will stimulate an emission instead of being absorbed. Two conditions are necessary in order to create and maintain this condition. Normally, most electrons are in the ground state at room temperature, so a stimulus is needed to excite the electrons. This stimulus can be provided by a high voltage that will accelerate free electrons, and then collisions with atoms will excite their electrons. The process is called "optical pumping."

If the excited electrons spend a longer than normal time in the excited state, stimulated emission will be more probable than spontaneous emission. This condition is met by selecting atoms of elements that have specific energy levels called the "metastable state." Electrons remain in metastable states for about 10^{-3} s, rather than the normal 10^{-8} s.

A typical gas laser tube is shown in the diagram. A high voltage excites the electrons in the gas, maintaining more atoms in an excited state than the ground state. As some photons are emitted spontaneously, they stimulate the emission of other photons. The ends of the laser tube are silvered to reflect the photons. This reflection causes more photons to stimulate the emission of a very large number of photons. Any photons that are not travelling parallel to the sides of the tube exit the tube and do not contribute to the beam. One end of the tube is only partially reflecting, and a fraction of the photons escape. These escaping photons have the same wavelength and frequency and are all in phase, creating a beam of what is called "coherent light."



Analyze

1. The word "laser" is an acronym for "light amplification by stimulated emission of radiation." Explain the significance of each term in the name.
2. Laser beams remain small and do not spread out, as does light from other sources. Based on the unique characteristics of laser light, try to explain why the beams do not spread out.
3. List as many applications of laser as you can.

The years from 1900 to 1930 were exciting ones in physics. No longer could physicists speak of waves and particles as separate entities — the boundary between the two became blurred. The long-standing Dalton model of the atom gave way to the Thomson model, which was soon usurped by the Rutherford model and, soon thereafter, by the Bohr model. Eventually, all models that represented electrons as discrete particles yielded to the quantum mechanical model described by the Schrödinger wave equation.

Today, the wave equation is still considered to be the most acceptable model. In fact, physicists have been able to show that the wave equation can give information about the nucleus and particles that was not known to exist when Schrödinger presented his equation. In the next chapter, you will learn about properties of the nucleus and particles that exist for time intervals as small as 10^{-20} s.

12.3 Section Review

1. C Discuss the similarities and differences between Dalton's model of the atom and J.J. Thomson's model of the atom.
2. K/U What surprising observation did Rutherford and Geiger make that motivated Rutherford to define a totally new model of the atom?
3. K/U In what way did Rutherford's nuclear model of the atom conflict with classical theory?
4. C Explain how experimentally observed spectra of atomic hydrogen helped Bohr develop his model of the atom.
5. K/U According to Bohr's model of the atom, what property of electrons in atoms must be quantized?
6. K/U List the four postulates on which Bohr based his model of the atom.
7. C Explain how Coulomb's law played a role in the determination of the Bohr radius.
8. C Describe the two features of the emission spectrum of atomic hydrogen that revealed a flaw in Bohr's model of the atom.
9. K/U How did Dirac improve Schrödinger's wave equation?
10. K/U What is a wave function and what type of information does a wave function provide about atoms?
11. K/U List and define the four quantum numbers.
12. C Balmer's work on the spectrum of hydrogen helped Bohr to modify Rutherford's model of the atom. Explain how he did this.
13. K/U Write down Rydberg's modification of Balmer's formula and define the terms.
14. K/U What can cause an electron in the Bohr model to "jump" to a higher energy level?
15. K/U Explain the term "principal quantum number."

UNIT PROJECT PREP

Inquisitive minds following unexpected results often lead to advances in our scientific understanding of the universe.

- Do you believe, and can you support, the idea that unexpected experimental results have contributed more to scientific discovery than any other means?
- Which theory, special relativity or quantum mechanics, was received with more skepticism by the general public of the time? Suggest reasons.

CHAPTER 12 Review

REFLECTING ON CHAPTER 12

- Oscillators on the surface of a blackbody can oscillate only with specific frequencies. When they emit electromagnetic radiation, they drop from one allowed frequency to a lower allowed frequency.
- The photoelectric effect demonstrated that electromagnetic energy can be absorbed only in discrete quanta of energy. Electromagnetic energy travels like a wave, but interacts with matter like a particle.
- When a photon ejects an electron from a metal surface, the maximum kinetic energy of the electron can be calculated from the equation $E_{k(max)} = hf - W$, where W is the work function of the metal.
- The Compton effect shows that both energy and momentum are conserved when a quantum of light energy, or a photon, collides with a free electron.
- The energy of a photon is $E = hf$.
- The momentum of a photon is $p = \frac{h}{\lambda}$.
- The diffraction of electrons by crystals demonstrated that electrons have wave properties. The wavelength of a particle of matter is $\lambda = \frac{h}{mv}$.
- Physicists accept the dual properties of matter and electromagnetic energy. Electromagnetic energy behaves like particles and particles of matter have wave properties. These concepts are called the "wave-particle duality."
- Dalton believed that atoms were the smallest, indivisible particles in nature. J.J. Thomson demonstrated that electrons could be removed from atoms and, therefore, that atoms were made up of smaller particles.
- By observing the scattering of alpha particles by a thin gold foil, Rutherford demonstrated that the positive charge in an atom must be condensed into an extremely small area at the centre of the atom.
- Bohr proposed that electrons in atoms could exist only in specific allowed energy levels. Electrons in these energy levels are in orbits with specific allowed radii.
- The energies of the photons in the observed spectra of atomic hydrogen have amounts of energy that are exactly equal to the difference in Bohr's allowed energy levels. This fact supports Bohr's concept that electrons can drop from a high energy level to a lower level by emitting a photon.
- Detailed inspection of emission spectra of gases showed that some of the spectral lines are actually made up of two or more lines that are very close together. Also, when placed in an external magnetic field, some single spectral lines split into two or more lines. These data show that Bohr's model of the atom is incomplete.
- Schrödinger's wave equation forms the foundation of quantum mechanics, or wave mechanics. Solutions to the wave equation, called "wave functions," provide information about the properties of electrons in an atom. The operation, $\psi^*\psi$ on the wave function gives the probability that an electron will be found at a specific point in space.
- Dirac modified Schrödinger's wave equation to account for relativistic effects of electrons in atoms travelling close to the speed of light. Wave functions obtained by solving this wave equation contain four quantum numbers. Each quantum number describes one property of electrons that is quantized.
- The Pauli exclusion principle states that no two electrons in the same atom can have the same four quantum numbers.

Knowledge/Understanding

1. Describe how a negatively charged electroscope can be used to provide evidence for the photoelectric effect.
2. **(a)** Describe the properties of a blackbody and explain how it is simulated in the laboratory.
   (b) How did the actual radiation spectrum emitted by a heated blackbody differ from the predictions of the classical wave theory?
3. The ultraviolet catastrophe was considered to be a flaw in the explanation of the blackbody emission spectra by the classical wave theory. In what way was it unexplained?
4. **(a)** The results of Lenard's photoelectric experiment partly correlated with the classical wave theory of light. Explain how it agreed.
   (b) In what way did Lenard's results differ from the predictions of the classical wave theory of light?
5. **(a)** Einstein saw a connection between the photoelectric effect and the Planck proposal that energy be quantized. Explain how Einstein developed an equation to describe the photoelectric effect.
   (b) Einstein's photoelectric equation is actually another example of conservation of energy. Explain how this applies.
6. A lithium surface in a photoelectric cell will emit electrons when the incident light is blue. Platinum, however, requires ultraviolet light to eject electrons from its surface.
   (a) Which of the two metals has a larger value for its work function? Explain your answer.
   (b) Which of the two metals has a higher threshold frequency? Explain your answer.
7. **(a)** How did a knowledge of the charge on an electron make it possible to calculate the numerical values of the kinetic energies of electrons emitted from a metal surface?
   (b) How did the data from Millikan's photoelectric experiments support Einstein's theory of the photoelectric effect?
8. **(a)** Explain the sequence by which Compton derived an expression for the momentum of a photon, considering that it has no mass.
   (b) In what way does a photon change "colour" after it has collided with an electron. Is "colour" always a suitable term to use?
9. Based on his premise regarding the momentum of a photon, Compton showed that momentum was conserved in collisions between photons and electrons. As a result, what can be concluded from this experiment?
10. **(a)** Explain the sequence that de Broglie used in taking Compton's expression for the momentum of a light photon and proposing that particles of matter have a corresponding matter wave and wavelength.
    (b) How can the matter wavelength of a particle be increased to make it more easily detectable?
    (c) Compare (through calculations) the de Broglie wavelength of an electron of mass 9.11×10^{-31} kg, travelling at 3.60 km/h, with that of a hockey puck of mass 0.15 kg, travelling at the same speed.
11. **(a)** What property of electromagnetic radiation represented a flaw in the Rutherford model of the atom?
    (b) Balmer's equation represents an "empirical expression." What is the significance of this term?
12. In what way did Rydberg modify Balmer's equation?
13. **(a)** Describe the key features of the Bohr model of the atom, and indicate how this model contradicts classical theory.
    (b) When electrons occupy a higher energy level, what are they likely to do? What options do they have?
14. **(a)** Write the equation linking the energy of a photon emitted from the Bohr atom to the energy levels of the atom.
    (b) How does this manifest itself in the emission spectrum of an atom?
15. **(a)** According to Bohr, what property of the electron in its orbit is quantized?

(b) In general terms, explain how Bohr used the equations for Coulomb's law, circular motion, and angular momentum to determine the "Bohr radius."

16. (a) Describe how Bohr used the equations for kinetic energy, Coulomb's law, and the Bohr radius to determine the general formula for the total energy of an electron in a hydrogen atom.
    (b) Explain how the Bohr equation for the total energy of an electron in orbit in a hydrogen atom relates to the observed emission spectrum.
17. (a) Show that the speed of an electron as it moves in an "allowed" orbit can be represented by the equation
$$v_{\text{n}} = \frac{2\pi ke^2}{nh}$$
    (b) Calculate the de Broglie wavelength associated with an electron in the first orbit of the Bohr atom.
18. Schrödinger responded to de Broglie's thesis by developing the Schrödinger wave equation. In what general way did this equation account for the discrepancies in the emission spectra?
19. (a) Do you feel Schrödinger's wave equation is just an abstract model, or is there a "real" significance?
    (b) Discuss whether you believe that the Schrödinger wave functions made the Bohr model obsolete.

Inquiry

20. Schrödinger's wave equation was a famous contribution to what is now called "quantum mechanics." At your library or through the Internet, find and record the exact text of the equation and describe, in general terms, the mathematical operations it incorporated.
21. (a) Research and describe the principle of complementarity.
    (b) Under what conditions does light tend to show its wave properties, and under what conditions does the particle (photon) nature of light predominate?
22. Design and sketch a simple door opener that will open electrically when a person passes through a light beam.

Communication

23. Explain how Lenard was able to determine the maximum kinetic energy of the electrons coming from the emitter of his photoelectric apparatus.
24. (a) Explain how Einstein was able to include the properties of different types of emitter metals in his photoelectric equation.
    (b) Initially, very few physicists accepted Einstein's claim for the quantum nature of light. Why did this opposition exist?
25. Briefly outline the key features of the model of the atom proposed by John Dalton, J.J. Thomson, and Ernest Rutherford.
26. Based on the results of the scattering experiments, Rutherford was led to believe that the atom was mainly empty space with a small charged core. Explain why he deduced this.
27. Some science fiction writers use a large sail to enable a space vehicle to move through space. It is argued that sunlight will exert a pressure on the sail, causing it to move away from the Sun. Prepare a report and/or display in which you indicate
    (a) whether the proposal has merit
    (b) what type of surface should be used for the sail
28. "Wave-particle duality" is a term used to describe the dual properties of both light and particles in motion. Has this meant that Maxwell's equations for electromagnetic wave propagation and Newton's classical mechanics have been discarded? Discuss your opinion.

Making Connections

29. A light meter is used by a photographer to ensure correct exposure for photographs. If the photocell in the meter is to operate satisfactorily up to the red light wavelength of 650 nm, what should be the work function of the emitter material?

30. Some television picture tubes emit electrons from the rear cathode and accelerate them forward through an electric potential difference of 15 000 V.

(a) What is the de Broglie wavelength of the electron just before it hits the screen?

(b) Discuss whether you think diffraction of the electron beam might pose a problem with the resulting picture.

31. Prepare a report on laser technology, including reference to the terms "spontaneous emission," "stimulated emission," "population inversion," and "metastable." In your report, include a reference to the medical applications of lasers.

Problems for Understanding

32. **(a)** The work function for a nickel surface is 5.15 eV. What is the minimum frequency of the radiation that will just eject an electron from the surface?

(b) What is the general name given to this minimum frequency?

33. **(a)** The longest wavelength of light that will just eject electrons from a particular surface is 428.7 nm. What is the work function of this surface?

(b) Use Table 12.1 to identify the material used in the surface.

34. When ultraviolet radiation was used to eject electrons from a lead surface, the maximum kinetic energy of the electrons emitted was 2.0 eV. What was the frequency of the radiation used?

35. The electrons emitted from a surface illuminated by light of wavelength 460 nm have a maximum speed of 4.2×10^5 m/s. Given that an electron has a mass of 9.11×10^{-31} kg, calculate the work function (in eV) of the surface material.

36. Assume that a particular 40.0 W light bulb emits only monochromatic light of wavelength 582 nm. If the light bulb is 5.0% efficient in converting electric energy into light, how many photons per second leave the light bulb?

37. **(a)** Calculate the momentum of a photon of light with a wavelength of 560 nm.

(b) Calculate the momentum of the photons of light with a frequency of 6.0×10^{14} Hz.

(c) A photon has an energy of 186 eV. What is its momentum?

38. An electron is moving at a speed of 4.2×10^5 m/s. What is the frequency of a photon that has an identical momentum?

39. What is the momentum of a microwave photon if the average wavelength of the microwaves is approximately 12 cm?

40. A particle has a de Broglie wavelength of 6.8×10^{-14} m. Calculate the mass of the particle if it is travelling at a speed of 1.4×10^6 m/s.

41. **(a)** Calculate the wavelength of a 4.0 eV photon.

(b) What is the de Broglie wavelength of a 4.0 eV electron?

(c) What is the momentum of an electron if its de Broglie wavelength is 1.4×10^{-10} m?

42. **(a)** Calculate the radius of the third orbit of an electron in the hydrogen atom.

(b) What is the energy level of the electron in the above orbit?

43. Calculate the wavelength of the second line in the Balmer series.

44. A photon of light is absorbed by a hydrogen atom in which the electron is already in the second energy level. The electron is lifted to the fifth energy level.

(a) What was the frequency of the absorbed photon?

(b) What was its wavelength?

(c) What is the total energy of the electron in the fifth energy level?

(d) Calculate the radius of the orbit representing the fifth energy orbit.

(e) If the electron subsequently returns to the first energy level in one "jump," calculate the wavelength of the corresponding photon to be emitted.

CHAPTER 13

The Nucleus and Elementary Particles

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PREREQUISITE CONCEPTS AND SKILLS

- Electric force and Coulomb's law
- Equivalence of mass and energy
- Conservation of mass-energy
- Potential and kinetic energy

Twentieth-century physics was ruled to a large extent by a quest for ultra-high energies. It might seem strange that such energies are required in order to investigate the tiniest, most subtle particles of matter. However, probing inside the nucleus and then inside the particles of that nucleus requires instruments capable of accelerating electrons and protons into the mega-electron volt (10^6 eV) and giga-electron volt (10^9 eV) ranges of kinetic energy.

Such instruments, called "particle accelerators," are located in university and government laboratories around the world. The above photograph shows an accelerator used at Conseil Européen pour la Recherche Nucléaire (CERN) located near Geneva, Switzerland. Along with other high-energy physics laboratories, CERN searches for new particles formed during energetic collisions between subatomic particles.

This chapter begins with the structure and properties of the nucleus and then examines the field of elementary particles, such as the ones created at CERN.

MULTI LAB

Radioactive Decay

TARGET SKILLS
- Performing and recording
- Analyzing and interpreting

Penetrating Ability of Radioactive Emissions

Obtain an end-window Geiger counter, sources of beta and gamma radiation, and shielding materials such as sheets of lead and cardboard.

CAUTION Handle the sources using tongs.

Position the tube of the Geiger counter so that either source (beta or gamma) can be placed close to the end window and so that sheets of cardboard or lead can be placed between the source and the Geiger tube. Turn on the Geiger counter and slide the beta source under it. Note the reading on the counter. Insert a sheet of cardboard between the beta source and the tube and take the reading again. Continue adding sheets until the reading is close to zero. Determine the thickness of an individual sheet and calculate the thickness of cardboard between the tube and the beta source for each radiation reading.

Repeat the process with the gamma source. If the cardboard does not provide much change to the reading, add sheets of lead instead. Continue adding sheets until the reading is close to zero (or as close as you can get). Determine the thickness of an individual sheet and calculate the total thickness of the barrier.

Analyze and Conclude

1. Plot a graph of the radioactivity reading (y-axis) against the thickness of cardboard for the beta source.
2. Plot a graph of the radioactivity reading (y-axis) against the thickness of cardboard or lead for the gamma source.
3. Which type of radioactive emission is the more penetrating?
4. Try to determine from the graphs the thickness of material that would reduce the reading to half of the unshielded value.

Half-Life

Perform this investigation as a class activity. Start with each member of the class holding a coin heads-up. Count and record the number of heads. Next, everyone should flip their coins. Count and record the number of heads. One minute later, each person who got heads on the first flip should flip again. Those who got tails are "out." Again, count and record the number of heads. Repeat the process every minute until only one or two heads remain.

Analyze and Conclude

1. Draw a graph of the number of heads remaining versus the number of flips.
2. Draw a graph of the number of heads that changed to tails versus the number of flips.
3. What are the chances that a coin will change from head to tail during a flip?
4. Why would a minute be known as the "half-life" of the coin?
5. The number of changes during each flip represents the activity. How does the activity change as time passes?
6. If you start with 160 heads, how many flips should you expect it to take to reduce the number of heads to 5? Explain your reasoning.

13.1 Structure of the Nucleus

SECTION EXPECTATIONS

- Define and describe the concepts and units related to the present-day understanding of the nature of elementary particles (e.g., mass-energy equivalence).
- Apply quantitatively the laws of conservation of mass and energy, using Einstein's mass-energy equivalence.

KEY TERMS

- proton
- neutron
- nucleon
- chemical symbol
- atomic number
- atomic mass number
- nucleon number
- strong nuclear force
- nuclide
- isotope
- mass defect
- atomic mass unit

Excitement was high in the scientific community in the early 1900s when Ernest Rutherford (1871–1937) proposed his model of the nucleus and Niels Bohr (1885–1962) developed a model of the atom that explained the spectrum of hydrogen. As is the case with many scientific breakthroughs, however, the answers to a few questions gave rise to many more.

The most obvious question arose from the realization that a great amount of positive charge was concentrated in a very small space inside the nucleus. The strength of the Coulomb repulsive force between like charges had long been established. Two positive charges located as close together as they would have to be in Rutherford's model of the nucleus would exert a mutual repulsive force of about 50 N on each other. For such tiny particles, this is a tremendous force. There had to be another, as yet unidentified, attractive force that was strong enough to overcome the repulsive Coulomb force. What is the nature of the particles that make up the nucleus and what force holds them together?

Protons and Neutrons

Again, it was Rutherford who discovered — and eventually named — the proton. When he was bombarding nitrogen gas with alpha particles, Rutherford detected the emission of positively charged particles with the same properties as the hydrogen nucleus. As evidence accumulated, it became apparent to physicists that the **proton** was identical to the hydrogen nucleus and was the fundamental particle that carried a positive charge, equal in magnitude to the charge on the electron and with a mass 1836 times as great as the mass of an electron. The positive charge of all nuclei consisted of enough protons to account for the charge.

Using the principle on which the mass spectrometer is based (refer to Chapter 8, Fields and Their Applications), several physicists discovered that the mass of most nuclei was roughly twice the size of the number of protons that would account for the charge. Rutherford encouraged the young physicists in his laboratory to search for a neutrally charged particle that could account for the excess mass of the nucleus. Finally, in 1932, English physicist James Chadwick (1891–1974) discovered such a particle. That particle, now called the **neutron**, has a mass that is nearly the same as that of a proton. The proton, neutron, and electron now account for all of the mass and charge of the atom. Since protons and neutrons have many characteristics in common, other than the charge, physicists call them **nucleons**.

Table 13.1 Properties of Particles in the Atom

Particle	Mass (kg)	Charge (C)
proton	$1.672\ 614 \times 10^{-27}$ kg	$+1.602 \times 10^{-19}$ C
neutron	$1.674\ 920 \times 10^{-27}$ kg	0 C
electron	$9.109\ 56 \times 10^{-31}$ kg	-1.602×10^{-19} C

Representing the Atom

As physicists and chemists learned more about the nucleus and atoms, they needed a way to symbolically describe them. The following symbol convention communicates much information about the particles in the atom.

$${}^{A}_{Z}X$$

X is the **chemical symbol** for the element to which the atom belongs. For example, the symbol for carbon is C, while the symbol for krypton is Kr.

Z is the **atomic number**, which represents the number of protons in the nucleus and is also the charge of the nucleus.

A is the **atomic mass number**, the total number of protons and neutrons in the nucleus. Since the particles in the nucleus are called "nucleons," the atomic mass number is sometimes called the **nucleon number**.

If N represents the number of neutrons in a nucleus, then

$$A = Z + N$$

The atomic number (Z) also indicates the number of electrons in the neutral atom, since one electron must be in orbit outside the nucleus for each proton inside the nucleus. In addition, the manner in which atoms chemically interact with each other depends on the arrangement of their outer electrons. This is influenced in turn by the atomic number. Since all atoms of an element behave the same chemically, all atoms of a given element must have the same atomic number. For example, all carbon atoms have an atomic number of 6 and all uranium atoms have an atomic number of 92.

The Strong Nuclear Force

By the end of the 1930s, physicists were beginning to accumulate data about the elusive force that holds the nucleus together. They discovered that any two protons (p↔p), two neutrons (n↔n), or a proton and a neutron (p↔n) attract each other with the most potent force known to physicists — the **strong nuclear force**.

When two protons are about 2 fm (femtometres: 2×10^{-15} m) apart, the nuclear force is roughly 100 times stronger than the repulsive Coulomb force. However, at 3 fm of separation, the nuclear force is almost non-existent. Whereas the gravitational and electrostatic forces have an unlimited range — both follow a $1/r^2$ law — the nuclear force has an exceptionally short range, which is roughly the diameter of a nucleon. Therefore, inside the nucleus, the nuclear force acts only between adjacent nucleons. When the separation distance between nucleons decreases to about 0.5 fm, the nuclear force becomes repulsive. This repulsion possibly occurs because nucleons cannot overlap. Estimates of the radius of a nucleon range from 0.3 fm to 1 fm. Figure 13.1 shows a graph of an approximated net force between two protons, relative to their separation distance.

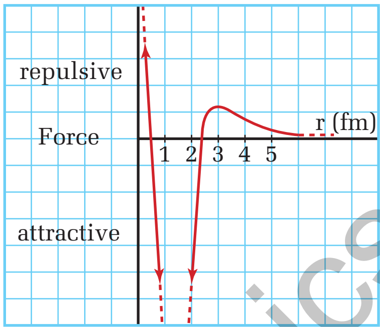


Figure 13.1 Assume that one proton is at the origin of the co-ordinate system and another proton approaches it. As the second proton approaches, it experiences a repulsive Coulomb force. If the second proton has enough energy to overcome the repulsion, it will reach a point where the net force is zero. Beyond that point, the strong nuclear force attracts it strongly.

Stability of the Nucleus

If the nuclear force is so strong, why cannot nucleons come together to form nuclei of ever-increasing size? The short range of the nuclear force accounts for this. When a nucleus contains more than approximately 20 nucleons, the nucleons on one side of the nucleus are so far from those on the opposite side that they no longer attract each other. However, the repulsive Coulomb force between protons is still very strong. Figure 13.2 shows the number of protons (Z) and neutrons (N) in all stable nuclei.

As you can see in Figure 13.2, the number of neutrons and protons is approximately equal up to a total of 40 nucleons. For example, oxygen has 8 protons and 8 neutrons and calcium has 20 protons and 20 neutrons $\left({}^{40}_{20}\text{Ca}\right)$. Beyond 40 nucleons, the ratio of neutrons to protons increases gradually up to the largest element. One form of uranium has 92 protons and 146 neutrons $\left({}^{238}_{92}\text{U}\right)$. This unbalanced ratio results in more nucleons experiencing the attractive force for each pair of protons experiencing the repulsive Coulomb forces. This combination appears to stabilize larger nuclei. Nuclei that do not lie in the range of stability will disintegrate.

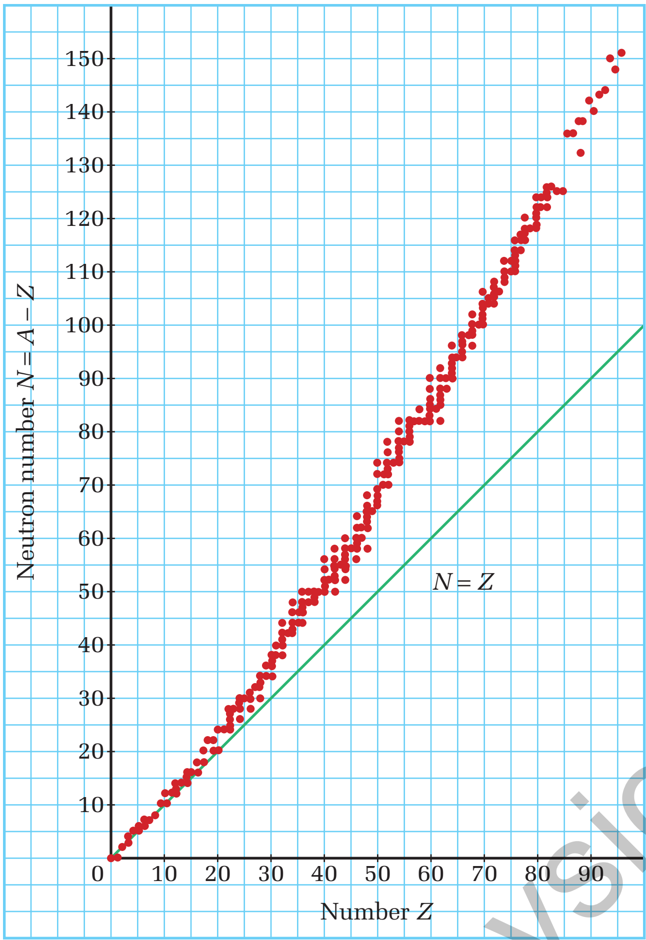


Figure 13.2 Each dot represents a stable nucleus, with the number of neutrons shown on the vertical axis and the number of protons on the horizontal axis.

Nuclides and Isotopes

Each dot in Figure 13.2 represents a unique stable nucleus with a different combination of protons and neutrons. Physicists call these unique combinations **nuclides**. The many columns of vertical dots indicate that several nuclides have the same number of protons. Since the number of protons determines the identity of the element, all of the nuclides in a vertical column are different forms of the same element, differing only in the number of neutrons. These sets of nuclides are called **isotopes**. For example, nitrogen, with 7 protons, might have 7 neutrons $\left({}^{14}_{7}\mathrm{N}\right)$ or 8 neutrons $\left({}^{15}_{7}\mathrm{N}\right)$. Figure 13.3 illustrates isotopes of hydrogen and helium.

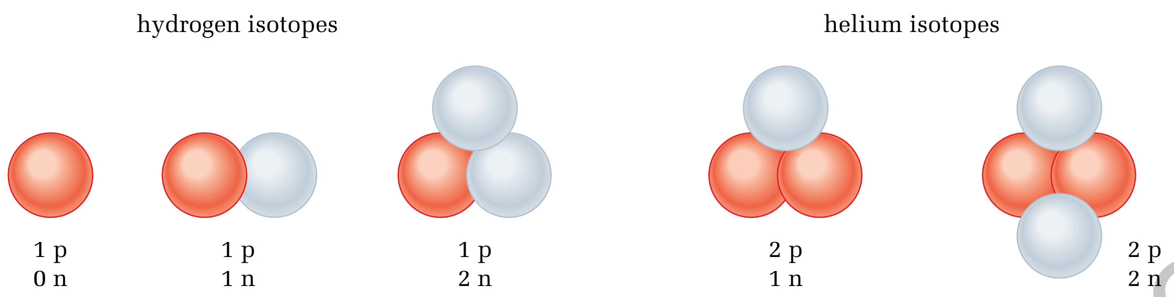


Figure 13.3 The isotopes of hydrogen are the only isotopes to which physicists have given different names: $^{2}_{1}H$ is called "deuterium" and $^{3}_{1}H$ is called "tritium." For most isotopes, physicists simply use the atomic mass number to describe the isotope: $^{3}_{2}He$ is called "helium-3" and $^{4}_{2}He$ is called "helium-4."

Nuclear Binding Energy and Mass Defect

When you consider the strength of the nuclear force, you realize that it would take a tremendous amount of energy to remove a nucleon from a nucleus. For the sake of comparison, recall that it takes 13.6 eV to ionize a hydrogen atom, which is the removal of the electron. To remove a neutron from $^{4}_{2}He$ would require more than 20 million eV (20 MeV). The amount of energy required to separate all of the nucleons in a nucleus is called the binding energy of the nucleus. Figure 13.4 shows the average binding energy per nucleon plotted against atomic mass number *A*.

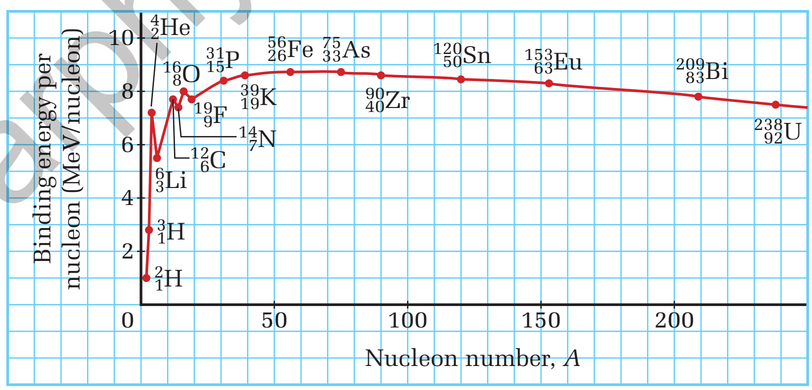


Figure 13.4 The average binding energy per nucleon is calculated by determining the total binding energy of the nucleus and dividing by the number of nuclides.

WEB LINK

www.mcgrawhill.ca/links/physics12

You can find many properties, including the mass, of all stable nuclides and many unstable nuclides in charts of the nuclides on the Internet. Just go to the above Internet site and click on **Web Links**.

Imagine that you were able to remove a neutron from $^{4}_{2}He$. What would happen to the 20 MeV of energy that you had to add in order to remove the neutron? The answer lies in Einstein's special theory of relativity: Energy is equivalent to mass. If you look up the masses of the nuclides, you would find that the mass of $^{4}_{2}He$ is smaller than the sum of the masses of ${}^{3}_{2}\text{He}$ plus a neutron $\left({}^{1}_{0}\text{n}\right)$. The energy that was added to remove a neutron from ${}^{4}_{2}\text{He}$ became mass. This difference between the mass of a nuclide and the sum of the masses of its constituents is called the **mass defect**. Einstein's equation $E = \Delta mc^2$ allows you to calculate the energy equivalent of the mass defect, Δm.

When dealing with reactions involving atoms or nuclei, expressing masses in kilograms can be cumbersome. Consequently, physicists defined a new unit — the **atomic mass unit** (u). One atomic mass unit is defined as $\frac{1}{12}$ the mass of the most common isotope of carbon $\left({}^{12}_{6}\text{C}\right)$. This gives a value for the atomic mass unit of $1\ \text{u} = 1.6605 \times 10^{-27}$ kg. Table 13.2 lists the masses of the particles in atoms.

Table 13.2 Masses of Common Elementary Particles

Particle	Mass (kg)	Mass (u)
electron	$9.109\ 56 \times 10^{-31}$ kg	0.000 549 u
proton	$1.672\ 614 \times 10^{-27}$ kg	1.007 276 u
neutron	$1.674\ 920 \times 10^{-27}$ kg	1.008 665 u

SAMPLE PROBLEM

Calculate the Binding Energy of a Nucleus

Determine the binding energy in electron volts and joules for an iron nucleus of $\left({}^{56}_{26}\text{Fe}\right)$, given that the nuclear mass is 55.9206 u.

Conceptualize the Problem

- The *energy equivalent* of the *mass defect* is the *binding energy* for the nucleus.
- The *mass defect* is the *difference* of the mass of the *nucleus* and the *sum* of the masses of the *individual particles*.

Identify the Goal

The binding energy, E, of ${}^{56}_{26}\text{Fe}$

Identify the Variables and Constants

Known	Implied	Unknown
$m_{\text{nucleus}} = 55.9206$ u	$c = 2.998 \times 10^{8}\ \frac{\text{m}}{\text{s}}$	N
$A = 56$	$m_{\text{p}} = 1.007\ 276$ u	Δm
$Z = 26$	$m_{\text{n}} = 1.008\ 665$ u	E

continued ►

continued from previous page

Develop a Strategy

Calculate the number of neutrons.

$$N = A - Z$$
$$N = 56 - 26$$
$$N = 30$$

Determine the total mass of the separate nucleons by finding the masses of the protons and neutrons and adding them together.

$$m_{p(total)} = (26)(1.007\ 276\ \text{u})$$
$$m_{p(total)} = 26.189\ 176\ \text{u}$$

Find the mass defect by subtracting the mass of the nucleus from the total nucleon mass.

$$m_{n(total)} = (30)(1.008\ 665\ \text{u})$$
$$m_{n(total)} = 30.259\ 95\ \text{u}$$
$$m_{total} = 26.189\ 176\ \text{u} + 30.259\ 95\ \text{u}$$
$$m_{total} = 56.449\ 126\ \text{u}$$
$$\Delta m = 56.449\ 126\ \text{u} - 55.9206\ \text{u}$$
$$\Delta m = 0.528\ 526\ \text{u}$$

Convert this mass into kilograms.

$$\Delta m = (0.528\ 526\ \cancel{\text{u}})\left(1.6605 \times 10^{-27}\ \frac{\text{kg}}{\cancel{\text{u}}}\right)$$
$$\Delta m = 8.7762 \times 10^{-28}\ \text{kg}$$

Find the energy equivalent of the mass defect.

$$E = \Delta mc^2$$

Find the energy in electron volts.

$$\Delta E = \left(8.7762 \times 10^{-28}\ \text{kg}\right)\left(2.998 \times 10^{8}\ \frac{\text{m}}{\text{s}}\right)^2$$
$$\Delta E = 7.888 \times 10^{-11}\ \text{J}$$
$$\Delta E = \frac{7.888 \times 10^{-11}\ \cancel{\text{J}}}{1.602 \times 10^{-19}\ \frac{\cancel{\text{J}}}{\text{eV}}}$$
$$\Delta E = 4.9239 \times 10^{8}\ \text{eV}$$

The binding energy of the nucleus is 4.924×10^8 eV, or 7.888×10^{-11} J.

Validate the Solution

The binding energy of a nucleus should be extremely small.
You would expect the binding energy per nucleon to be about 8 MeV.

$$\frac{4.93 \times 10^8\ \text{eV}}{56} = 8.79 \times 10^6\ \text{eV} = 8.79\ \text{MeV}$$

PRACTICE PROBLEMS

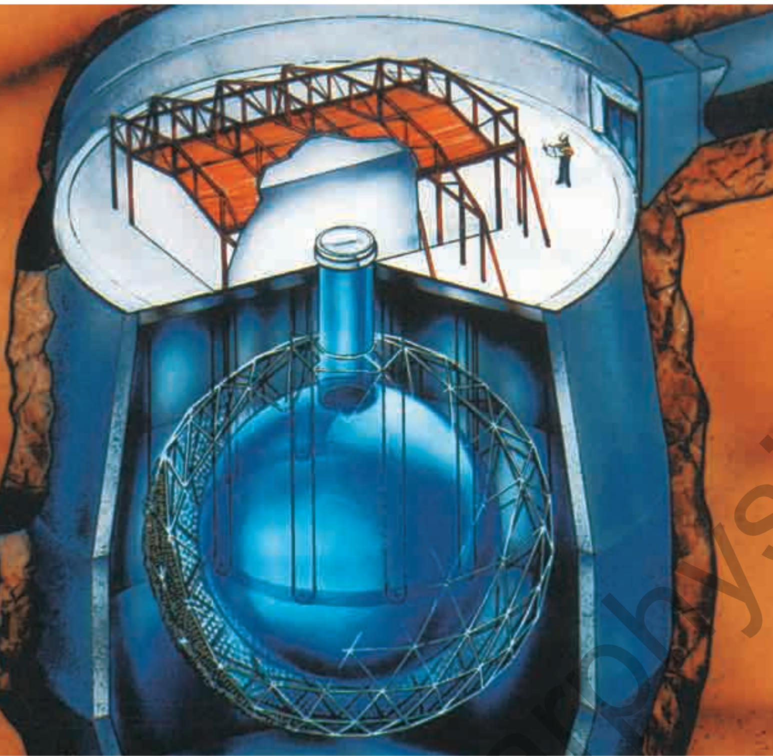
1. Determine the mass defect for $^{8}_{4}\text{Be}$ with a nuclear mass of 8.003 104 u.
2. Determine the binding energy for $^{3}_{2}\text{He}$ with a nuclear mass of 3.014 932 u.
3. Determine the binding energy for $^{235}_{92}\text{U}$ with a nuclear mass of 234.9934 u.

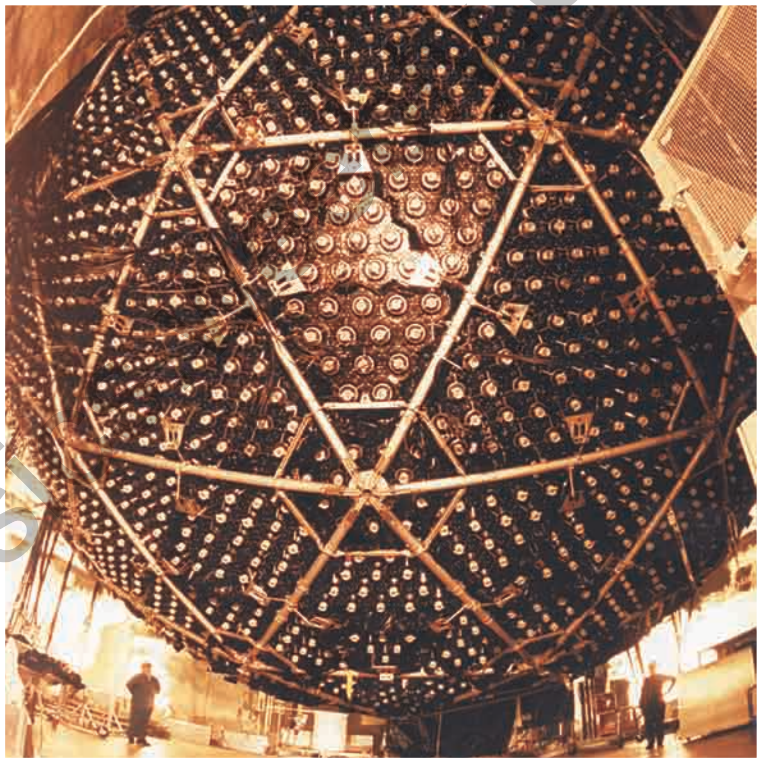
13.1 Section Review

1. K/U List the contribution of each of the following physicists to the study of the nucleus.
   (a) Ernest Rutherford
   (b) James Chadwick
2. C Why did physicists believe that a neutral particle must exist, even before the neutron was discovered?
3. K/U State the meaning of the following terms.
   (a) nucleon
   (b) atomic mass number
   (c) atomic number
4. K/U For the atom symbolized by ${}^{200}_{80}\mathrm{Hg}$, state the number of
   (a) nucleons
   (b) protons
   (c) neutrons
   (d) electrons, if the atom is electrically neutral
   (e) electrons, if the atom is a doubly charged positive ion
5. K/U How did scientists know that some force other than the electromagnetic force had to exist within the nucleus?
6. C Describe the characteristics of the nuclear force.
7. C Describe the general trend of stable nuclei in relation to the proton number and neutron number.
8. C Explain the concept of binding energy.
9. C Define the term "mass defect" and explain how to determine it for a given nucleus.
10. MC The structure of the atom is often compared to a solar system, with the nucleus as the Sun and the electrons as orbiting planets. If you were going to use this analogy, which planet should you use to represent an electron, based on its comparative distance from the Sun?

Atom	Solar system
radius of nucleus ≈ 1×10^{-15} m	radius of Sun ≈ 6.96×10^{8} m
radius of typical electron orbit ≈ 1×10^{-10} m	radii of planetary orbits $r_{\text{Mercury}} \approx 6 \times 10^{10}$ m $r_{\text{Earth}} \approx 1.49 \times 10^{11}$ m $r_{\text{Jupiter}} \approx 8 \times 10^{11}$ m $r_{\text{Pluto}} \approx 6 \times 10^{12}$ m

Not Your Average Observatory





When you think of an observatory, you probably think of a large telescope on a remote mountaintop that collects light from stars and galaxies. You're not likely to think of 1000 t of heavy water at the bottom of a mine shaft, 2 km below Earth's surface. But that's exactly what you will find at the Sudbury Neutrino Observatory — also known as SNO — where scientists are trying to detect a wily, elusive particle called the "neutrino."

The existence of the neutrino was first predicted in 1931 by Wolfgang Pauli, when certain nuclear reactions appeared to be violating the laws of conservation of energy and momentum. Rather than modify or discard the law, Pauli suggested that an unseen, chargeless and probably massless particle was carrying away some of the energy and momentum. Italian physicist Enrico Fermi later named this mysterious particle the "neutrino," which means "little neutral one" in Italian.

Not Your Average Particle

Like the photon, the neutrino is produced in enormous quantities by nuclear reactions in the centres of stars, such as the Sun. They travel at close to the speed of light and carry away substantial amounts of energy from the star's hot core. Just as photons are collected by telescopes to analyze the processes that create them, neutrinos are observed in order to understand what's happening in the centres of stars. By counting neutrinos, physicists learn about the rate of fusion reactions in stellar cores. Although there are now known to be three types of neutrinos, stars produce the type known as the "electron-neutrino."

Unlike the photon, which interacts strongly with matter, the neutrino scarcely interacts with matter at all. Some 60 billion neutrinos pass unhindered through each square centimetre of your body each second. A beam of neutrinos could sail through a shield of lead 1 light-year (10 thousand billion kilometres) thick without being reflected or absorbed. This is why it took physicists almost 30 years after Pauli's prediction to verify the neutrino's existence.

Detecting a Neutrino

Detecting a neutrino is tricky, because it passes right through photographic film and electronic detectors, the devices that register photons. This is where heavy water is useful. Heavy water is made up of oxygen and deuterium, which is a hydrogen nucleus with an added neutron. It's about 10% heavier than "light" water, and its symbol is D_2O. Occasionally, in one of several possible reactions, neutrinos will transform a deuterium nucleus into a pair of protons and an electron. Neutrinos enter the tank with super-high energies and, because energy is conserved, the electron produced in the reaction will be jettisoned at speeds faster than the speed of light in water. It's like a high-speed crash, where even the debris of the collision flies out at high speed. As the energetic electron slows down in the water, it emits a flash of light, or a shock wave — the optical equivalent of a sonic boom. About 500 to 800 photons will be generated in the flash, with a total energy proportional to that of the incident neutrino.

Outside the tank of heavy water, several of the 10 thousand photomultiplier tubes will detect this tiny light flash. These photomultipliers comprise the main part of the detector. Together, they are about 200 000 times more sensitive than the human eye. This sensitivity is required because the flash of light is only as bright as the flash of a camera seen from the distance of the Moon! So, even though the neutrino is not detected directly, the product of its interaction is. Despite the 1000 t of heavy water, only about 10 neutrinos are detected per day.

One Mystery Solved

One longstanding problem that the Sudbury Neutrino Observatory investigated was the significant difference between the number of predicted neutrinos based on solar models and the number of neutrinos that were actually observed. Was the difference due to observation errors in the experiments and detectors, which were begun in the 1970s, or to errors in the scientific model calculations? Despite refinements in both over nearly 30 years, the difference persisted. Why?

The unexpected answer, which SNO helped provide in June 2001, was due to the neutrinos themselves. Because the Sun generates only electron-neutrinos, the earlier experiments were designed to detect only this type of neutrino, and not the other two types. Recent results from SNO, together with those of another detector in Japan, indicate that after neutrinos are generated in the Sun, they oscillate between the three types while they travel. Because of its ability to detect different types of neutrino reactions, SNO is sensitive to all three types of neutrinos, and it was able to show that earlier experiments simply missed the transformed neutrinos — but they were there all along.

For such transformations to take place, the neutrino must have a tiny mass, contrary to what was originally thought. SNO has measured this mass to be roughly 60 000 times less than that of an electron. What this new result means for elementary particle theory remains to be seen. How the neutrinos change type is also still not understood, but with continued monitoring of incoming neutrinos, SNO hopes to shed light on that too.

Making Connections

1. Could neutrinos enter the Sudbury Neutrino Observatory after passing through the far side of Earth, that is, on their way back out into space?
2. How do the laws of conservation of energy and momentum apply to reactions between particles?
3. How does the SNO differ from other neutrino observatories worldwide?

13.2 Radioactivity and Nuclear Reactions

SECTION EXPECTATIONS

- Define and describe the concepts and units related to radioactivity.
- Describe the principal forms of nuclear decay.
- Compare the properties of alpha particles, beta particles, and gamma rays in terms of mass, charge, speed, penetrating power, and ionizing ability.
- Compile, organize, and display data or simulations to determine and display the half-lives for radioactive decay of isotopes.

KEY TERMS

- radioactive material
- alpha particle
- beta particle
- gamma ray
- radioactive isotope (radioisotope)
- parent nucleus
- daughter nucleus
- transmutation
- ionizing radiation
- neutrino
- antineutrino
- positron
- half-life
- nuclear fission
- nuclear fusion

Observation of the effects of cathode ray tubes carried out by J.J. Thomson and others stimulated many other scientists to perform related studies in which a material was bombarded with "rays" of various types. When Wilhelm Conrad Röntgen (1845–1923) was using a cathode ray tube, he was surprised to see a fluorescent screen glowing on the far side of the room. Because he did not know the nature of these rays, he called them "X rays." French physicist Henri Becquerel (1852–1908) became curious about the emission of these X rays and wondered if luminescent materials, when exposed to light, might also emit X rays.

At first, Becquerel's experiment seemed to confirm his hypothesis. He wrapped photographic film to shield it from natural light and placed it under phosphorescent uranium salts. When he exposed the phosphorescent salts to sunlight, silhouettes of the crystals appeared when he developed the film. The salts appeared to absorb sunlight and reemit the energy as X rays that then passed through the film's wrapping. However, during a cloudy period, Becquerel stored the uranium salts and wrapped film in a drawer. When he later developed the film, he discovered that it had been exposed while in the drawer. This is the first recorded observation of the effects of radioactivity.

Radioactive Isotopes

Physicists discovered, studied, and used **radioactive materials** (materials that emit high-energy particles and rays) long before they learned the reason for these emissions. As you know, Rutherford discovered **alpha particles** (α) and used them in many of his famous experiments. He examined the nature of alpha particles by passing some through an evacuated glass tube and then performing a spectral analysis of the tube's contents. The trapped alpha particles displayed the characteristic spectrum of helium; alpha particles are simply helium nuclei.

Rutherford also discovered **beta particles**, and other scientists studied their charge-to-mass ratio and showed that beta (β) particles were identical to electrons. French physicist Paul Villard discovered that, in addition to beta particles, radium emitted another form of very penetrating radiation, which was given the name "gamma (γ) rays." **Gamma rays** are a very high-frequency electromagnetic wave. Figure 13.5 shows the separation of these radioactive emissions as they pass between oppositely charged plates.

In Section 13.1, you learned about the nuclei of atoms and about many of the characteristics that made them stable. Did you wonder what would happen to a nucleus if it was not stable? The answer is that it would disintegrate by emitting some form of radiation and transform into a more stable nucleus. Unstable nuclei are called **radioactive isotopes** (or "radioisotopes"). When a nucleus disintegrates or decays, the process obeys several conservation laws — conservation of mass-energy, conservation of momentum, conservation of nucleon number, and conservation of charge. The following subsections summarize the important characteristics of alpha, beta, and gamma radiation.

PHYSICS FILE

Pierre and Marie Curie once gave Henri Becquerel a sample of radium that they had prepared. When Becquerel carried the sample in his vest pocket, it burned his skin slightly. This observation triggered interest among physicians and eventually led to the use of radioactivity for medical purposes. Becquerel shared the 1903 Nobel Prize in Physics with the Curies.

Alpha Decay

When a radioactive isotope emits an alpha particle, it loses two protons and two neutrons. As a result, the atomic number (Z) decreases by two and the atomic mass number (A) decreases by four. Physicists describe this form of decay as shown below, where P represents the original nucleus or **parent nucleus** and D represents the resulting nucleus or **daughter nucleus**.

$$^{A}_{Z}\text{P} \rightarrow {}^{4}_{2}\text{He} + {}^{A-4}_{Z-2}\text{D}$$

Only very large nuclei emit alpha particles. One such reaction would be the alpha emission from radium-223 ($^{223}_{88}\text{Ra}$). To determine the identity of the daughter nucleus, write as much as you know about the reaction.

$$^{223}_{88}\text{Ra} \rightarrow {}^{4}_{2}\text{He} + {}^{219}_{86}?$$

Then look up the identity of an element with an atomic number of 86, and you will find that it is radon. The final equation becomes

$$^{223}_{88}\text{Ra} \rightarrow {}^{4}_{2}\text{He} + {}^{219}_{86}\text{Rn}$$

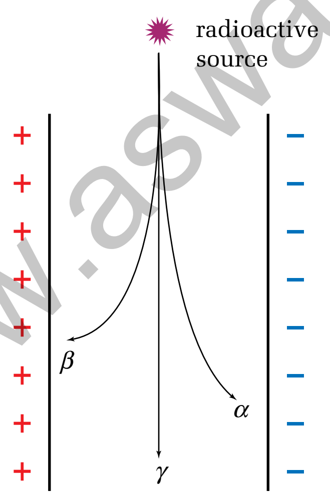


Figure 13.5 Positive alpha particles are attracted to the negative plate, while negatively charged beta particles are attracted to the positive plate. Gamma rays are not attracted to either plate, indicating that they do not carry a charge.

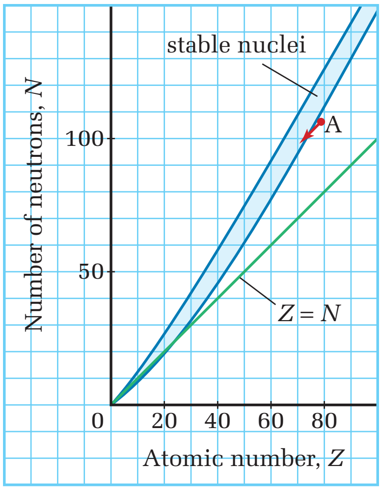


Figure 13.6 The emission of an alpha particle is represented here as a diagonal arrow going down and to the left. This process brings the tip of the arrow to a nucleus that has two fewer neutrons and two fewer protons than the nucleus at the tail of the arrow.

During this reaction, one element is converted into a different element. Such a change is called **transmutation**. Why would such a transmutation result in a more stable nucleus? You can find the answer by studying the simplified representation of stable nuclei in Figure 13.6. The point labelled "A" represents a nuclide that lies outside of the range of stability. The arrow shows the location of the daughter nucleus when the unstable parent loses two neutrons and two protons. As you can see, the daughter nucleus lies within the range of stability. In addition, the helium nucleus — alpha particle — is one of the most stable nuclei of all. Since you now have two nuclei that are more stable than the parent nucleus, the total binding energy increased. The mass defect becomes kinetic energy of the alpha particle and daughter nucleus. Typical alpha particle energies are between 4 MeV and 10 MeV.

Conceptual Problem

- Write the nuclear reaction for the alpha decay of the following nuclei.

(a) $^{222}_{86}\text{Rn}$

(b) $^{210}_{84}\text{Po}$

(c) $^{214}_{83}\text{Bi}$

(d) $^{230}_{90}\text{Th}$

Alpha particles do not penetrate materials very well. A thick sheet of paper or about 5 cm of air can stop an alpha particle. In stopping, it severely affects the atoms and molecules that are in its way. With the alpha particle's positive charge, relatively large mass, and very high speed (possibly close to 2×10^7 m/s), it gives some of the electrons in the atoms enough energy to break free, leaving a charged ion behind. For this reason, alpha particles are classified as **ionizing radiation**. These ions can disrupt biological molecules. Because of its low penetrating ability, alpha radiation is not usually harmful, unless the radioactive material is inhaled or ingested.

Beta Decay

When a radioactive isotope emits a beta particle, it appears to lose an electron from within the nucleus. However, electrons as such do not exist in the nucleus — a transformation of a nucleon had to take place to create the electron. In fact, in the process, a neutron becomes a proton, so the total nucleon number (A) remains the same, but the atomic number (Z) increases by one. You can write the general reaction for beta decay as follows, where $^{0}_{-1}\text{e}$ represents the beta particle, which is a high-energy electron. The superscript zero does not mean zero mass, because an electron has mass. The zero means that there are no nucleons.

$$^{A}_{Z}\text{P} \rightarrow {}^{0}_{-1}\text{e} + {}^{A}_{Z+1}\text{D}$$

Many common elements such as carbon have isotopes that are beta emitters.

$$^{14}_{6}\text{C} \rightarrow {}^{0}_{-1}\text{e} + {}^{14}_{7}?$$

When you look up the identity of an element with an atomic number of 7, you will find that it is nitrogen. The final equation becomes

$$^{14}_{6}\text{C} \rightarrow {}^{0}_{-1}\text{e} + {}^{14}_{7}\text{N}$$

When physicists were doing some of the original research on beta decay, they made some very puzzling observations. Linear momentum of the beta particle and daughter nucleus was not conserved. As well, they determined the spin of each particle and observed that angular momentum was not conserved. To add to the puzzle, the physicists calculated the mass defect and discovered that mass-energy was not conserved.

Some physicists were ready to accept that these subatomic particles did not follow the conservation laws. However, Wolfgang Pauli (1900–1958) proposed an explanation for these apparent violations of the fundamental laws of physics. He proposed the existence of an as yet unknown, undiscovered particle that would account for all of the missing momentum and energy. It was more than 25 years before this elusive particle, the **neutrino** (ν_e), was discovered.

In reality, the particle that is emitted with a beta particle is an **antineutrino**, a form of antimatter. The antineutrino has a very small or zero rest mass and so can travel at or near the speed of light. It accounts for all of the "missing pieces" of beta decay. The correct reaction for beta decay should be written as follows. The bar above the symbol ν_e for the neutrino indicates that it is an antiparticle.

$$^{A}_{Z}\text{P} \rightarrow {}^{0}_{-1}\text{e} + {}_{Z+1}^{\;\;A}\text{D} + \overline{\nu_e}$$

Physicists soon discovered a different form of beta decay — the emission of a "positive electron" that is, in fact, an antielectron. It has properties identical to those of electrons, except that it has a positive charge. The more common name for the antielectron is **positron**. Since the parent nucleus loses a positive charge but does not lose any nucleons, the value of A does not change, but Z decreases by one. A proton in the parent nucleus is transformed into a neutron. As you might suspect, the emission of a neutrino accompanies the positron. The reaction for positive electron or positron emission is written as follows.

$$^{A}_{Z}\text{P} \rightarrow {}^{0}_{+1}\text{e} + {}_{Z-1}^{\;\;A}\text{D} + \nu_e$$

You can understand why beta emission produces a more stable nucleus by examining Figure 13.7. The emission of an electron changes a neutron to a proton; in the chart, this is represented by an arrow going diagonally down to the right. Emission of a positron changes a proton into a neutron and the arrow in the chart goes diagonally upward and to the left.

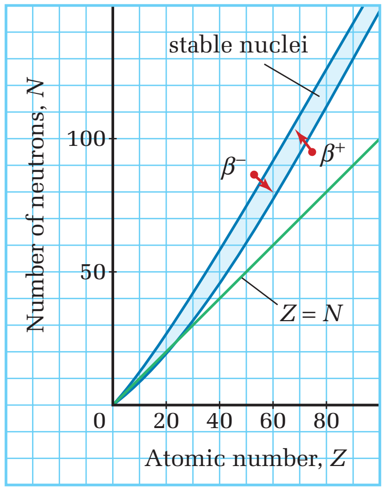
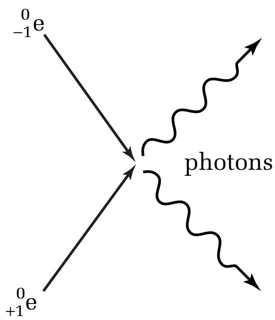


Figure 13.7 If a nucleus lies above the range of stability, it can transform into a more stable nucleus by beta emission. If it lies below the range of stability, it can transform into a more stable ion by emitting a positron. (Arrows are not drawn to scale.)

PHYSICS FILE

The positron is the antimatter particle for the electron. When they meet, they annihilate each other and release their mass-energy as a gamma photon.

$$^{0}_{1}e + ^{0}_{-1}e \rightarrow \gamma$$



The collision of a particle with its own antimatter particle results in the annihilation of the particles and the creation of two gamma ray photons.

Beta particles penetrate matter to a far greater extent than do alpha particles, mainly due to their much smaller mass, size, and charge. They can penetrate about 0.1 mm of lead or about 10 m of air. Although they can penetrate better than alpha particles, they are only about 5% to 10% as biologically destructive. Like alpha particles, they do their damage by ionizing atoms and molecules, and so are classified as ionizing radiation.

Conceptual Problems

- Free neutrons $\left(^{1}_{0}n\right)$ decay by beta minus emission. Write the reaction.
- Free protons $\left(^{1}_{1}p\right)$ can decay by beta plus emission. Write the reaction.
- Tritium, the isotope of hydrogen that consists of a proton and two neutrons, decays by beta minus emission. Write the reaction.
- Carbon-10 decays by positron emission. Write the reaction.
- Calcium-39 $\left(^{39}_{20}Ca\right)$ decays into potassium-39 $\left(^{39}_{19}K\right)$. Write the equation and identify the emitted particle.
- Plutonium-240 $\left(^{240}_{94}Pu\right)$ decays into uranium-236 $\left(^{236}_{92}U\right)$. Write the equation and identify the emitted particle.
- Lead-109 $\left(^{109}_{46}Pb\right)$ decays into silver-109 $\left(^{109}_{47}Ag\right)$. Write the equation and identify the emitted particle.
- Write the equation for the alpha decay of fermium-252 $\left(^{252}_{100}Fm\right)$.
- Write the equation for the beta positive decay of vanadium-48 $\left(^{48}_{23}V\right)$.
- Write the equation for the beta negative decay of gold-198 $\left(^{198}_{79}Au\right)$.

Gamma Decay

When a nucleus decays by alpha or beta emission, the daughter nucleus is often left in an excited state. The nucleus then emits a gamma ray to drop down to its ground state. This process can be compared to an electron in an atom that is in a high-energy level. When it drops to its ground state, it emits a photon. However, a gamma ray photon has much more energy than a photon emitted by an atom. The decay process can be expressed as follows, where the star indicates that the nucleus is in an excited state.

$$^{A}_{Z}P^{*} \rightarrow ^{A}_{Z}P + ^{0}_{0}\gamma$$

The following is an example of gamma decay:

$$^{192}_{77}Ir^{*} \rightarrow ^{192}_{77}Ir + ^{0}_{0}\gamma.$$

Gamma radiation is the most penetrating of all. It can pass through about 10 cm of lead or about 2 km of air. The penetrating ability of gamma radiation is due to two factors. First, it carries no electric charge and therefore does not tend to disrupt electrons as it passes by. Second, its photon energy is far beyond any electron energy level in the atoms. Consequently, it cannot be absorbed through electron jumps between energy levels.

However, when gamma radiation is absorbed, it frees an electron from an atom, leaving behind a positive ion and producing an electron with the same range of kinetic energy as a beta particle — often called "secondary electron emission." For this reason, gamma radiation is found to be just as biologically damaging as beta radiation. As in the case of alpha and beta radiation, gamma is classified as ionizing radiation.

Decay Series

When a large nucleus decays by the emission of an alpha or beta particle, the daughter nucleus is more stable than the parent is; however, the daughter nucleus might still be unstable. Consequently, a nucleus can tumble through numerous transmutations before it reaches stability. Figure 13.8 shows one such decay sequence for uranium-238. Notice that the end product is lead-82, then go back to Figure 13.4 on page 550. You will find lead at the peak of the curve of binding energy per nucleon. Lead is one of the most stable nuclei of all of the elements.

Notice that during the progress of the transmutations the following occurs.

- An alpha decay decreases the atomic number by 2 and decreases the atomic mass number by 4.
- A beta negative decay increases the atomic number by 1, while leaving the atomic mass number unchanged.

Knowledge of decay sequences such as the one in Figure 13.8 gives scientists information about the history of materials that contain lead. For example, if a rock contains traces of lead-82, that isotope of lead probably came from the decay of uranium-238 that was trapped in crystals as molten rock solidified in the past. A geologist can determine the original amount of uranium-238 in the rock and compare it to the amount of uranium-238 that remains. Knowing the disintegration rate of the isotopes in the series, a geologist can determine the age of the rock. This method was used to determine that the Canadian Shield contains some of the most ancient rock in the world, aged close to 4 billion years.

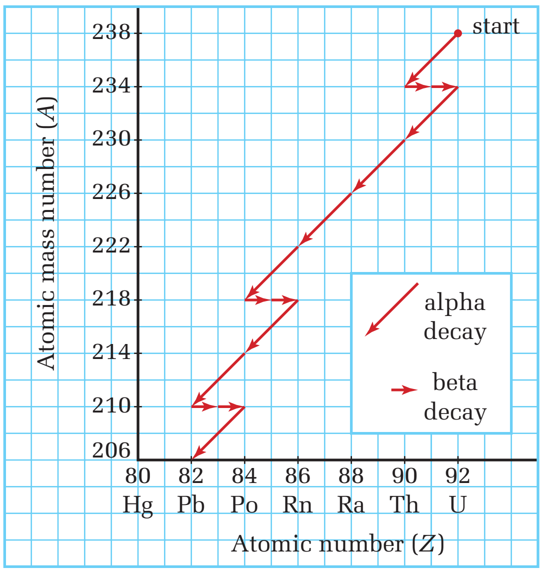


Figure 13.8 This series represents only one of several possible pathways of decay for uranium-238.

Rate of Radioactive Decay

You cannot predict exactly when a specific nucleus will disintegrate. You can only state the probability that it will disintegrate within a given time interval. Using probabilities might seem to be very imprecise, but if you have an exceedingly large number of atoms of the same isotope, you can state very precisely when half of them will have disintegrated. Physicists use the term **half-life**, symbolized by $T_{\frac{1}{2}}$, to describe the decay rate of radioactive isotopes. One half-life is the time during which the nucleus has a 50% probability of decaying. The half-life is also the time interval over which half of the nuclei in a large sample will disintegrate.

Imagine that you had a sample of polonium-218 $\left({}^{218}_{84}\mathrm{Po}\right)$. It decays by alpha emission with a half-life of 3.0 min. If you started with 160.0 μg of the pure substance, it would decay as shown in Table 13.3.

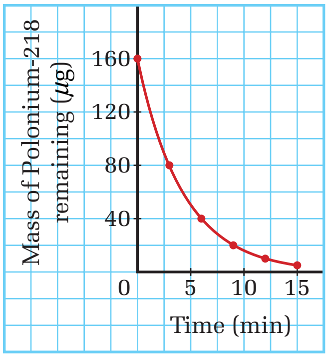


Figure 13.9 A graph of the decay of polonium and all other radioactive isotopes is an exponential curve.

Table 13.3 Decay of Polonium-218

Time (min)	Mass of Po-218 remaining (μg)
0	160.0
3.0	80.0
6.0	40.0
9.0	20.0
12.0	10.0
15.0	5.0

From Figure 13.9 we can estimate the following.

- After 7.0 min, there should be about 32 μg of polonium-218 remaining.
- It would take about 13 min to reduce the mass of polonium-218 to 8.0 μg.

You can obtain more accurate values by using a mathematical equation that relates the mass of the isotope and time interval. You can derive such an equation as follows.

- Let N represent the amount of the original sample remaining after any given time interval.
- Let N_o represent the original amount in the sample; must be given in the same units as N.
- Let Δt represent the time interval, and $T_{\frac{1}{2}}$ represent the half-life.
- After 1 half-life, $N = \frac{1}{2}N_o$.
- After 2 half-lives, $N = \frac{1}{2}\left(\frac{1}{2}N_o\right) = \left(\frac{1}{2}\right)^2 N_o$.

ELECTRONIC LEARNING PARTNER

To enhance your understanding of radioactive decay and half-life, go to your Electronic Learning Partner.

- After 3 half-lives, $N = \frac{1}{2}\left(\frac{1}{2}\right)^2 N_o = \left(\frac{1}{2}\right)^3 N_o$.
- After 4 half-lives, $N = \frac{1}{2}\left(\frac{1}{2}\right)^3 N_o = \left(\frac{1}{2}\right)^4 N_o$.
- You can now see a pattern emerging and can state the general expression in which "n" is the number of half-lives. $N = \left(\frac{1}{2}\right)^n N_o$
- However, the number, n, of half-lives is equal to the time interval divided by the time for 1 half-life. $n = \frac{\Delta t}{T_{\frac{1}{2}}}$
- Substituting the value for n, you obtain the final equation. $N = \left(\frac{1}{2}\right)^{\frac{\Delta t}{T_{\frac{1}{2}}}} N_o$

The amount of sample, N, can be expressed as the number of nuclei, the number of moles of the isotope, the mass in grams, the decay rate, or any measurement that describes an amount of a sample. The unit for decay rate in disintegrations per second is the becquerel, symbolized as Bq in honour of Henri Becquerel.

RADIOACTIVE DECAY

The amount of a sample remaining is one half to the exponent time interval divided by the half-life, all times the amount of the original sample.

$$N = N_o\left(\frac{1}{2}\right)^{\frac{\Delta t}{T_{\frac{1}{2}}}}$$

Quantity	Symbol	SI unit
amount of sample remaining	N	kilograms, moles, or Bq (might also be in number of atoms)
amount in original sample	N_o	kilograms, moles, or Bq (might also be in number of atoms)
elapsed time	Δt	s (often reported in min, days, years, etc.)
half life	$T_{\frac{1}{2}}$	s (often reported in min, days, years, etc.)

Unit Analysis

kilograms = kilograms kg = kg

Note: The elapsed time and the half-life must be given in the same units so that they will cancel, making the exponent of one half a pure number. Also, the amount of the sample remaining and in the original sample at time zero must be given in the same units.

SAMPLE PROBLEM

Decay of Polonium-218

You have a 160.0 μg sample of polonium-218 that has a half-life of 3.0 min.

(a) How much will remain after 7.0 min?

(b) How long will it take to decrease the mass of the polonium-218 to 8.0 micrograms?

Conceptualize the Problem

- The *half-life* of a radioactive isotope determines the *amount* of a sample at any given *time*.

Identify the Goal

Amount of polonium-218 remaining after 7.0 min

Length of time required for the mass of the sample to decrease to 8.0 μg

Identify the Variables and Constants

Known	Unknown
$m_o = 160.0\ \mu g$	m (at 7.0 min)
$T_{\frac{1}{2}} = 3.0$ min	Δt (at 8.0 μg)
$\Delta t = 7.0$ min	
$m = 8.0\ \mu g$	

Develop a Strategy

Write the decay relationship

$$N = N_o\left(\frac{1}{2}\right)^{\frac{\Delta t}{T_{\frac{1}{2}}}}$$

Substitute and solve.

$$N = 160.0\ \mu g\left(\frac{1}{2}\right)^{\frac{7.0\ \cancel{\text{min}}}{3.0\ \cancel{\text{min}}}}$$

$$N = 160.0\ \mu g(0.198\ 425)$$

$$N = 31.748\ \mu g$$

$$N \cong 32\ \mu g$$

(a) The mass remaining after 7.0 min will be 32 μg.

Write the decay equation.

$$N = N_o\left(\frac{1}{2}\right)^{\frac{\Delta t}{T_{\frac{1}{2}}}}$$

Rearrange the equation to solve for the ratio N to N_o.

$$\frac{N}{N_o} = \left(\frac{1}{2}\right)^{\frac{\Delta t}{T_{\frac{1}{2}}}}$$

Substitute numerical values.

$$\left(\frac{1}{2}\right)^{\frac{\Delta t}{3.0\ \text{min}}} = \frac{8.0\ \cancel{\mu g}}{160.0\ \cancel{\mu g}}$$

PROBLEM TIPS

The data for amounts of a sample, time intervals, and half-lives in decay rate problems can be given in a variety of units. Always be sure that, in your calculations, the amounts of a sample, N and N_o, are in the same units and that the time interval and the half-life are in the same units.

Solve by taking logarithms on both sides.

$$\log\left(\frac{1}{2}\right)^{\frac{\Delta t}{3.0\text{ min}}} = \log\frac{8.0}{160.0}$$

$$\frac{\Delta t}{3.0\text{ min}}\log\left(\frac{1}{2}\right) = \log 0.050$$

$$\Delta t = (3.0\text{ min})\frac{\log 0.050}{\log\left(\frac{1}{2}\right)}$$

$$\Delta t = (3.0\text{ min})\left(\frac{-1.301\ 003}{-0.301\ 03}\right)$$

$$\Delta t = 12.965\ 78\text{ min}$$

$$\Delta t \cong 13\text{ min}$$

(b) The time interval after which only 8.0 μg of polonium-218 will remain is 13 min.

Validate the Solution

These answers are the same as the answers estimated from the graph in Figure 13.9.

PRACTICE PROBLEMS

4. When a sample of lava solidified, it contained 27.4 mg of uranium-238, which has a half-life of 4.5×10^9 a (annum or year). If that lava sample was later found to contain only 18.3 mg of U-238, how many years had passed since the lava solidified?

5. Carbon-14 has a half-life of 5730 a. Every gram of living plant or animal tissue absorbs enough radioactive C-14 to provide an activity of 0.23 Bq. Once the plant or animal dies, no more C-14 is taken in. If ashes from a fire (equivalent to 1 g of tissue) have an activity of 0.15 Bq, how old are they? Assume that all of the radiation comes from the remaining C-14.

6. Radioactive iodine-128, with a half-life of 24.99 min, is sometimes used to treat thyroid problems. If 40.0 mg of I-128 is injected into a patient, how much will remain after 12.0 h?

Nuclear Reactions

When you were solving the problems above, you encountered radioactive isotopes that have half-lives of 3.0 min and 25 min. Did you wonder how any such isotopes could exist and why they had not decayed entirely? Most of the radioactive isotopes that are used in medicine and research are produced artificially. One of the first observations of artificial production of a radioisotope was accomplished by bombarding aluminum-27 with alpha particles as follows.

$$^{4}_{2}\text{He} + ^{27}_{13}\text{Al} \rightarrow ^{31}_{15}\text{P}^* \rightarrow ^{30}_{15}\text{P} + ^{1}_{0}\text{n}$$

The star on the phosphorus-31 indicates that it is very unstable and decays into phosphorus-30 and a neutron. Phosphorus-30 is a radioisotope that emits a positron. Today, many artificial isotopes are produced by bombarding stable isotopes with neutrons in nuclear reactors. For example, stable sodium-23 can absorb a neutron and become radioactive sodium-24.

Nuclear Fission

One of the most important reactions that is stimulated by absorbing a neutron is **nuclear fission**, the reaction in which a very large nucleus splits into two large nuclei plus two or more neutrons. The two most common isotopes that can undergo fission are $^{235}_{92}U$ and $^{239}_{94}Pu$. When a nucleus fissions, or splits, a tremendous amount of energy is released in the form of kinetic energy of the fission products — the resulting smaller nuclei. Since the kinetic energy of atoms and molecules is thermal energy, the temperature of the material rises dramatically. You can understand why such large amounts of energy are released by examining the graph of binding energy per nucleon in Figure 13.10.

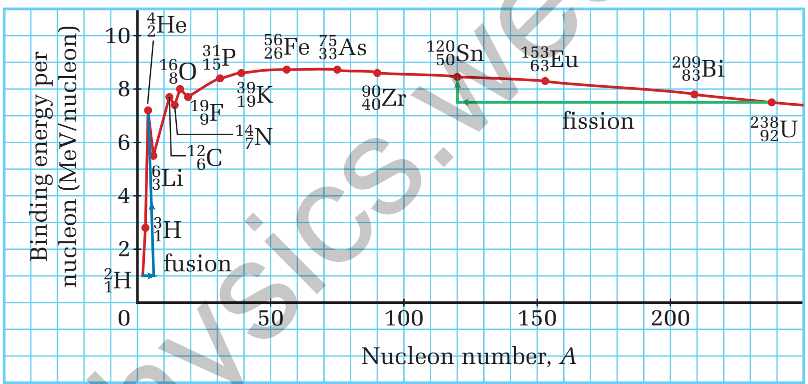


Figure 13.10 The binding energy of mid-range nuclei is greater than that of either very large or very small nuclei.

As you can see in Figure 13.10, when a large nucleus fissions, the smaller nuclei have much larger binding energies than the original nucleus did. Consequently, the sum of the masses of the fission products is much smaller than the mass of the original nucleus. This large mass defect yields the large amount of energy.

Nuclear fission is the reaction that occurs in nuclear reactors. The thermal energy that is released is then used to produce steam to drive electric generators. In this reaction, uranium-235 captures a slow neutron, producing a nucleus of uranium-236. This nucleus is quite unstable and will rapidly split apart. One possible result of this splitting or fission is shown in Figure 13.11. Notice that several neutrons are ejected during the fission. These neutrons can then cause further fissions, causing a chain reaction. However, the neutrons must be slowed down, or the uranium-235 nuclei cannot absorb them. In most Canadian reactors, heavy water is used, since neutrons are slowed down when they collide with the deuterium ($^{2}_{1}H$ or $^{2}_{1}D$) nuclei in the water. The reaction portrayed in

Figure 13.11, $^{235}_{92}\text{U} + ^{1}_{0}\text{n} \rightarrow ^{141}_{56}\text{Ba} + ^{92}_{36}\text{Kr} + 3^{1}_{0}\text{n}$, is only one of a large number of possibilities. Many different fission products are formed.

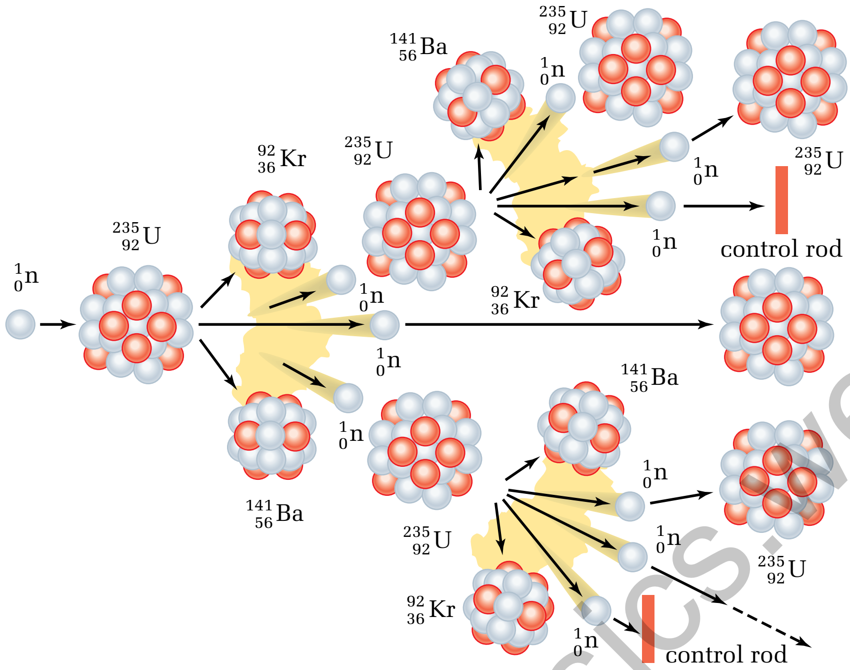


Figure 13.11 The neutrons given off by this nuclear fission reaction must be slowed down before they can be captured by other uranium-235 nuclei and produce further fission.

You have probably heard about the hazards of nuclear energy and the problems with the disposal of the products. Uranium-235 is an alpha emitter with a very long half-life, so it is not a serious danger itself. Alpha radiation is not very penetrating and the long half-life implies a low activity. The fission products cause the hazards. The reason becomes obvious when you examine Figure 13.12, which represents the stable nuclides. Fission products have about the same neutron-to-proton ratio as does the parent uranium nucleus. The fission products therefore lie far outside of the range of stability and so are highly radioactive.

Nuclear Fusion

Nuclear fusion is the opposite reaction to nuclear fission. In this process, small nuclei combine together to create larger nuclei. One such fusion reaction involves the combining of two isotopes of hydrogen, deuterium ($^{2}_{1}\text{H}$) and tritium ($^{3}_{1}\text{H}$). During the process, a neutron is released. The equation for the fusion reaction illustration in Figure 13.13 is $^{2}_{1}\text{H} + ^{3}_{1}\text{H} \rightarrow ^{4}_{2}\text{He} + ^{1}_{0}\text{n}$.

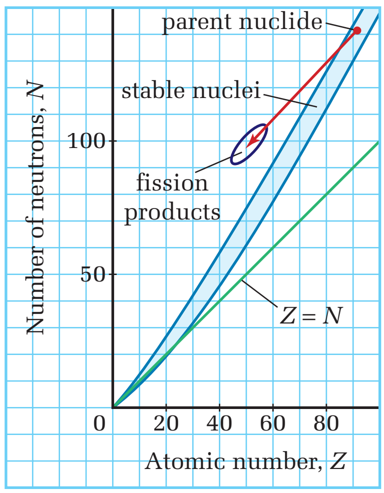


Figure 13.12 Fission products lie above the range of stability and are therefore mostly beta emitters.

PHYSICS FILE

Most stars generate energy through a process often called "hydrogen burning." This is not a good term, since no combustion is involved. Instead, single protons or hydrogen nuclei are fused together through a sequence of steps to form helium. Once the amount of hydrogen has diminished to the point where it no longer emits enough radiation to support the outer layers of the star against the inward pull of gravity, the star begins to collapse. This compression of the core causes its temperature to rise to the point at which helium begins to fuse. The star now swells up to become a Red Giant. This process of successive partial collapses and new fusion can continue until the core tries to fuse iron. At this point, the fusion reaction requires energy to continue, rather than releasing energy. The collapse is now catastrophic and the star blazes into a supernova.

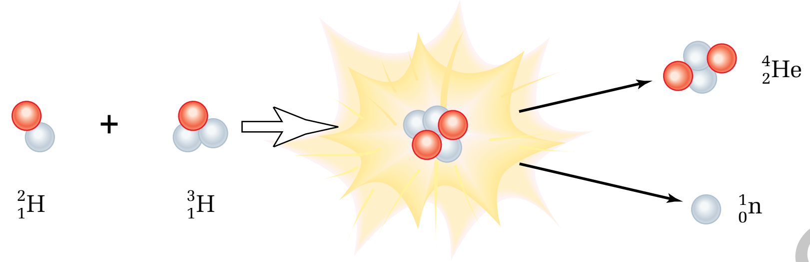


Figure 13.13 The helium nucleus has a much larger binding energy than either deuterium or tritium, so large amounts of energy are released in this nuclear fusion reaction.

Since nuclei repel each other due to their positive charges, they must be travelling at an extremely high speed for them to get close enough for the nuclear force to pull them together. An extremely high temperature can produce such speeds. As long as the product comes before iron in the periodic table, this reaction releases energy (is exothermic). After iron, the reaction requires an input of energy (is endothermic).

Fusion reactions occur in the cores of stars and in hydrogen bombs. Eventually it might be possible to control the fusion reaction so that it can be used to provide reasonably safe energy on Earth for many centuries to come. After all, the oceans contain vast amounts of hydrogen isotopes. Unfortunately, controlled fusion reactions have yet to provide a net output of energy.

SAMPLE PROBLEM

Energy from Nuclear Reactions

Determine the mass defect in the fission reaction given in the text and the amount of energy released due to each fission.

Data

Particle	Nuclear mass (u)
$^{235}_{92}U$	234.993
$^{1}_{0}n$	1.008
$^{141}_{56}Ba$	140.883
$^{92}_{36}Kr$	91.905

Conceptualize the Problem

- *Mass defect* is the *difference* between the total *mass* of the *reactants* and the total *mass* of the *fission products*.
- The *energy* released is the *energy equivalent* of the *mass defect*.

Identify the Goal

The mass defect, Δm, and the energy, E, released during each fission reaction

Identify the Variables and Constants

Known	**Implied**	**Unknown**
A, Z and m for all particles	$c = 2.998 \times 10^8 \frac{\text{m}}{\text{s}}$	Δm E

Develop a Strategy

Find the total mass of reactants.

$$m_{\text{neutron}} = 1.008\,665\text{ u}$$
$$m({}^{235}_{92}\text{U}) = 234.993\text{ u}$$
$$m_{\text{reactants}} = 1.008\,665\text{ u} + 234.993\text{ u}$$
$$m_{\text{reactants}} = 236.002\text{ u}$$
$$m({}^{141}_{56}\text{Ba}) = 140.883\text{ u}$$
$$m({}^{92}_{36}\text{Kr}) = 91.905\text{ u}$$
$$m_{3\text{ neutrons}} = 3 \times 1.008\,665\text{ u}$$
$$m_{3\text{ neutrons}} = 3.025\,995\text{ u}$$

Find the total mass of the products.

$$m_{\text{products}} = 140.883\text{ u} + 91.905\text{ u} + 3.026\text{ u}$$
$$m_{\text{products}} = 235.814\text{ u}$$

Find the mass defect by subtraction.

$$\Delta m = 236.002\text{ u} - 235.814\text{ u}$$
$$\Delta m = 0.18767\text{ u}$$

Convert the mass defect into kilograms.

$$\Delta m = (0.18767\,\cancel{\text{u}})\left(1.6605 \times 10^{-27}\,\frac{\text{kg}}{\cancel{\text{u}}}\right)$$
$$\Delta m = 3.1163 \times 10^{-28}\text{ kg}$$

Convert the mass into energy, using $\Delta E = \Delta mc^2$.

$$\Delta E = \Delta mc^2$$
$$\Delta E = (3.1163 \times 10^{-28}\text{ kg})\left(2.998 \times 10^8\,\frac{\text{m}}{\text{s}}\right)^2$$
$$\Delta E = 2.8009 \times 10^{-11}\text{ J}$$

The mass defect is 0.1877 u or 3.116×10^{-28} kg. This is equivalent to an energy of 2.801×10^{-11} J.

Validate the Solution

The mass defect is positive, indicating an energy release.

PRACTICE PROBLEMS

7. Another possible fission reaction involving uranium-235 would proceed as follows.

$${}^{1}_{0}\text{n} + {}^{235}_{92}\text{U} \rightarrow {}^{90}_{38}\text{Sr} + {}^{135}_{54}\text{Xe} + 11\,{}^{1}_{0}\text{n}$$

Determine the mass loss and the amount of energy released in this reaction.

Particle	Mass (u)
${}^{1}_{0}\text{n}$	1.008 665
${}^{235}_{92}\text{U}$	234.993
${}^{90}_{38}\text{Sr}$	89.886
${}^{135}_{54}\text{Xe}$	134.879

continued ►

continued from previous page

8. Determine the energy that would be released by the fusion of the nuclei of deuterium and tritium as indicated by the equation $^{2}_{1}H + ^{3}_{1}H \rightarrow ^{4}_{2}He + ^{1}_{0}n$.

Particle	Mass (u)
$^{2}_{1}H$	2.013 553
$^{3}_{1}H$	3.015 500
$^{4}_{2}He$	4.001 506
$^{1}_{0}n$	1.008 665

9. In the Sun, four hydrogen nuclei are combined into a single helium nucleus by a series of reactions. The overall effect is given by the following equation.

$$4^{1}_{1}H \rightarrow ^{4}_{2}He + 2^{0}_{1}e$$

(a) Calculate the mass defect for the reaction and the energy produced by this fusion.

(b) If 4.00 g of helium contain 6.02×10^{23} nuclei, determine how much energy is released by the production of 1.00 g of helium.

Particle	Mass (u)
$^{1}_{1}H$	1.007 276
$^{4}_{2}He$	4.001 506
$^{0}_{1}e$	0.000 549

Detecting Radiation

Most people have heard of a Geiger counter, which is used to detect ionizing radiation; however, it is only one of a wide variety of instruments used for measuring radiation. Each is designed for a specific purpose, but they all function in the way that alpha, beta, and gamma radiation interacts with matter — by ionizing or exciting atoms or molecules in the object. Some possible interactions, summarized in Figure 13.14, are exposing film, ionizing atoms, or exciting atoms or molecules and causing them to fluoresce or phosphoresce.

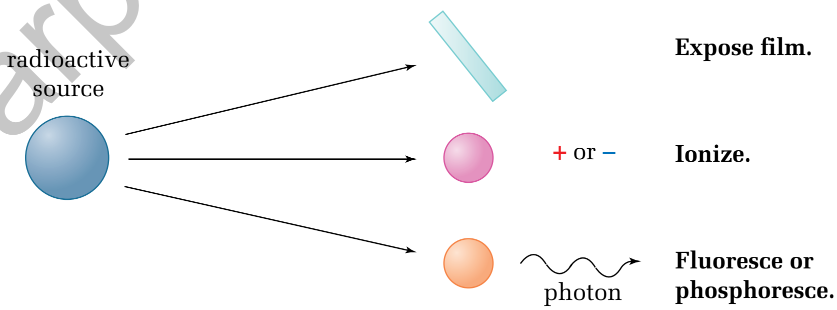


Figure 13.14 Radiation detectors commonly use one of three effects of radiation — the exposing of film, the ionization of matter, or the fluorescence of matter.

Radioactivity was discovered because it darkened film that was wrapped to protect it from light. For many years, people who worked in the nuclear industry or in laboratories where radioisotopes were used wore film badges. Technicians would develop the film, determine the degree of darkening, and calculate the amount of radiation to which the wearer of the badge had been exposed. Currently, many personnel badges contain lithium fluoride, a compound that enters an excited state when it absorbs energy from radiation. The material is thermoluminescent, meaning that when it becomes excited, it cannot return to the ground state unless it is heated. When heated, it emits light as it returns to its stable state. The technician collects the badges, puts the lithium fluoride in a device that heats it, and reads the amount of light emitted.

Geiger counters and other similar instruments detect the ions created by radiation as it passes through a probe that contains a gas at low pressure. When ionizing radiation passes through the gas, it ionizes some of the atoms in the gas. A high voltage between the wire and the cylinder accelerates the ions, giving them enough kinetic energy to collide with other gas molecules and ionize them. The process continues until an avalanche of electrons arrives at the central wire. The electronic circuitry registers the current pulse.

Geiger counters work well for low levels of radiation, but become saturated by higher levels. Ionization chambers are similar to Geiger counters, but they do not accelerate the ions formed by the radiation. They simply collect the primary ions formed by the radiation, which creates a current in the detector that is proportional to the amount of radiation present in the vicinity of the instrument. One such detector for high levels of radiation is called a "cutie pie."

Figure 13.15 This badge indicates the amount of radiation received by its wearer.

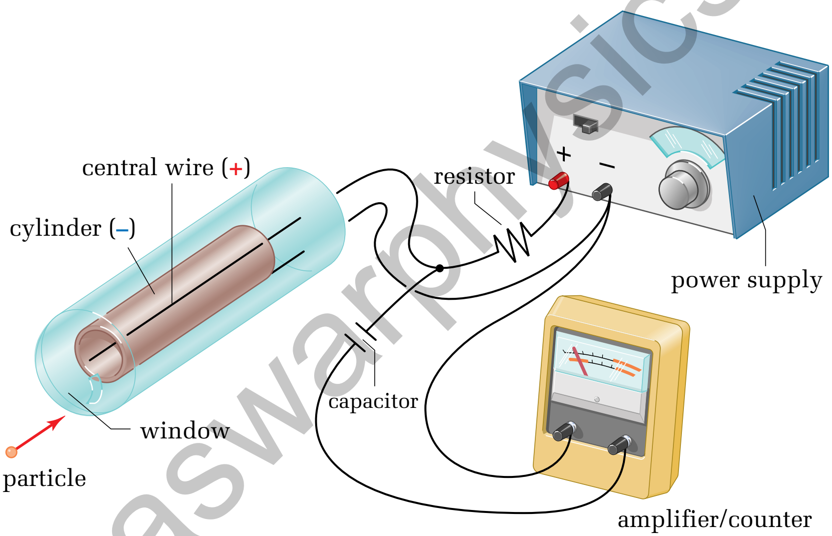


Figure 13.16 The passage of ionizing radiation through this tube creates an avalanche of electrons.

For accurately counting very small amounts of radioactivity, you would probably choose a scintillation counter. A crystal or a liquid consists of a material that, when excited by the absorption of radiation, will emit a pulse of light. Photomultiplier tubes that function on the principle of the photoelectric effect will detect the light and generate an electrical signal that is registered by electronic circuitry.

WEB LINK

www.mcgrawhill.ca/links/physics12

For in-depth information about radiation detection and protection, visit the TRIUMF Internet site. TRIUMF is Canada's national laboratory for particle and nuclear physics, located at the University of British Columbia in Vancouver. Just go to the above Internet site and click on **Web Links**.

INVESTIGATION 13-A

Half-Life of a Radioactive Isotope

TARGET SKILLS

- Performing and recording
- Analyzing and interpreting
- Communicating results

In the second Multi-Lab at the start of this chapter (Half-Life), you investigated the concept of half-life by flipping coins. This investigation will allow you to actually determine the half-life of a radioactive isotope. A common type of generator for a half-life investigation contains cesium-137, which slowly decays into an excited nucleus, barium-137m. This in turn emits gamma radiation as it drops to its ground state, barium-137. The excited nucleus is leached from the system to provide a slightly radioactive solution.

Problem

The object of this investigation is to determine the half-life of barium-137m.

Equipment

- barium-137m
- Geiger counter
- small test tube and holder
- gloves
- tongs

CAUTION This investigation should be performed as a class demonstration. The experimenter should wear gloves and wash up at the end of the demonstration. All radioactive materials must be safely secured and locked up at the end of the demonstration.

Dispose of the barium solution according to WHMIS procedures.

Procedure

1. Prepare a table with the following headings: Time (min), Measured activity (Bq), Background radiation (Bq), and Net activity (Bq). **Note:** 1 Bq is one count per second.
2. Turn on the Geiger counter and place the tube of the counter close to the test tube holder. Measure an average value for the background radiation.
3. Prepare the barium solution and pour it into the test tube.
4. Take activity readings every half minute until the activity of the source is close to zero.
5. Subtract the background radiation from each reading in order to obtain the activity from the source.

Analyze and Conclude

1. Draw a graph of actual activity against time with the activity on the *y*-axis.
2. From the graph, determine the time interval during which the actual activity decreases by 50%. Repeat this determination in several regions of the graph. How constant is this time interval?
3. What is the half-life of this radioisotope?

Apply and Extend

4. From your graph, how would you determine the rate at which the activity is changing? Perform this determination at two different locations along the curve.
5. Brainstorm a number of possible uses for knowledge of the half-life of a radioisotope.

Applications of Radioactive Isotopes

Exposure to radiation can cause cancer, but it also can destroy cancerous tumours. How can radiation do both?

As you have learned, alpha, beta, and gamma radiation ionize atoms and molecules in their paths. In living cells, the resulting ions cause chemical reactions that can damage critical biological molecules. If that damage occurs in a few very precise regions of the genetic material, the result can be a mutation that destroys the cell's ability to control growth and cell division. Then, the cell divides over and over, out of control, and becomes a cancerous tumour.

On the other hand, if the amount of radiation is much higher, the damage to the molecules that maintain the cell functions will be too great, and the cell will die. If a few healthy cells die, they can usually be replaced, so little or no harm is caused to the individual. If cancerous cells die, the tumour could be destroyed and the person would be free of the cancer.

Great care must be taken when treating tumours with radiation, since healthy cells in the area are exposed to radiation and might themselves become cancerous. If the amount of irradiation is excessive (in a nuclear accident, for example) and the entire body is exposed, too many cells could die at the same time, seriously affecting the ability of the organs to function. Death would result.

Irradiating tumours with gamma radiation is sometimes the only feasible way to treat a tumour, however, and it can be very successful. Figure 13.17 shows one method of treating a tumour with radiation from the radioisotope cobalt-60. A thin beam of gamma rays is aimed at the tumour and then the unit rotates so that the beam is constantly aimed at the tumour. In this way the tumour is highly irradiated, while the surrounding tissue receives much less radiation.

Figure 13.17 Gamma radiation from cobalt-60 is used to destroy tumours.

COURSE CHALLENGE

"Seeing" with Radioisotopes

Techniques exist by which radioisotopes, injected into living bodies, will accumulate in infected areas or other diseased tissues. Observing the location of the radioisotopes provides critical information. Refer to page 605 for ideas to help you include these scanning techniques in your *Course Challenge*.

Radioactive Tracers

Because traces of radioactivity can be detected and identified, scientists can use very small quantities of radioactive substances to follow the chemical or physical activity of specific compounds. For example, iodine-131 is useful for investigating the heart and the thyroid gland. Phosphorus-32 accumulates in cancerous tumours, identifying their location. Technetium-99 portrays the structure of organs. Other applications include the following.

- Slight amounts of a radioisotope added to a fluid passing through an underground pipe allows technicians to locate leaks.
- Gamma radiation is used to sterilize food so that it will stay fresh longer.
- Exposing plants to radioactive carbon dioxide allows researchers to determine the long series of chemical reactions that convert carbon dioxide and water into glucose.
- Radioisotopes are a common tool in biochemistry research.

Smoke Detectors

Many smoke detectors contain a small amount of a radioisotope that emits alpha radiation. Because the gas in the detector is ionized, a current can pass through and be measured. When soot and ash particles in smoke enter the detector, they tend to collect these ions and neutralize them. The resulting drop in current triggers the smoke detector alarm.

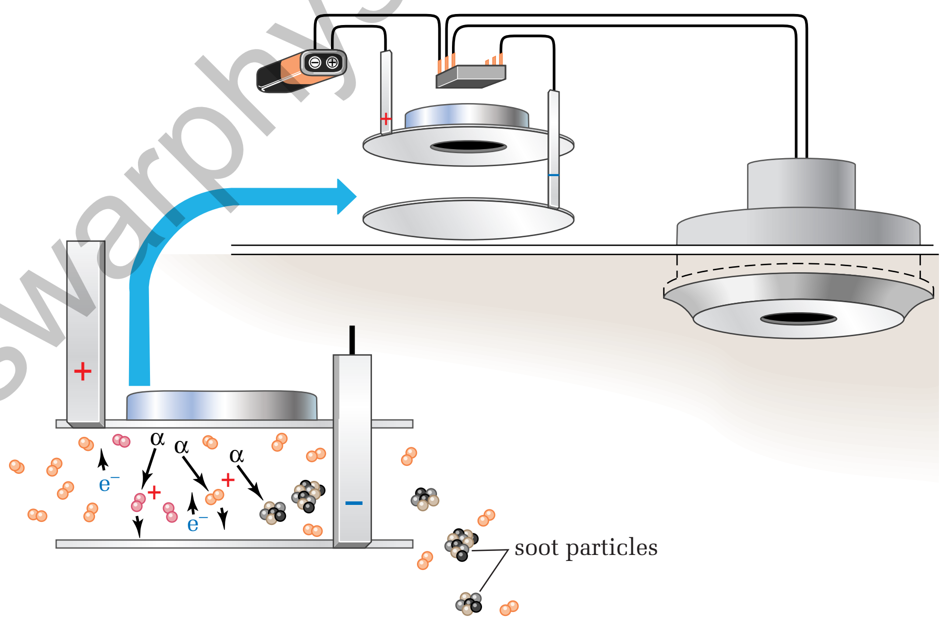


Figure 13.18 When alpha particles ionize molecules in the air, the positive ions are attracted to the negative electrode and the electrons are attracted to the positive electrode and a current passes through the circuit. Soot particles absorb and neutralizes some of the ions and the current decreases.

13.2 Section Review

1. K/U Explain why beta negative radiation tends to do less biological damage than an equal amount of alpha radiation.

2. K/U Which type of particle would you expect to penetrate best through lead, a beta positive particle (positron) or a beta negative particle? Give a reason for your choice.

3. K/U Why is gamma radiation much more penetrating than beta negative radiation?

4. K/U State the conservation laws used in writing nuclear reactions.

5. C Prepare a table for alpha radiation, beta negative radiation, and gamma radiation, comparing them with respect to mass, charge, relative penetrating ability, and relative biological damage.

6. C Draw a graph to illustrate the decay of carbon-14 in a wooden relic. Assume that the initial mass of the isotope in the wood was 240 mg.

7. C Draw a decay sequence similar to the one shown in Figure 13.8 on page 561. Begin with $\left({}^{255}_{101}\mathrm{Md}\right)$. It emits four alpha particles in succession, then a beta negative particle, followed by two alpha particles and then a beta negative particle. Another alpha emission is followed by another beta emission. (There are more, but this is enough for this question.)

8. C Fission is a process in which a nucleus splits into two parts that are roughly half the size of the original nucleus. In fusion, two nuclei fuse, or combine, to form one nucleus. These reactions seem to be opposite to each other, yet they both release large amounts of energy. Explain why this is not really a contradiction. Use the graph of binding energy per nucleon versus atomic mass number in your explanation.

9. MC Give a possible reason why a smoke detector uses an alpha source rather than a beta or gamma emitter.

10. MC Suggest an equation to represent the transformation of nitrogen-14 into carbon-14.

11. I Research the use of radioisotopes for medical or non-medical purposes and prepare a poster to illustrate your findings.

UNIT PROJECT PREP

The eventual identification of radioactivity began as a "mysterious" laboratory result at the turn of the nineteenth century.

- While working on your unit project, have you found information about people who could be called "visionaries" because of their belief that radioactivity would eventually play a role in everyone's daily life?
- Can you identify emerging scientific discoveries that you believe will result in wide-ranging applications by the end of the twenty-first century, as radioactivity has over the past 100 years?

13.3 Elementary Particles

SECTION EXPECTATIONS

- Define and describe the concepts and units related to the present-day understanding of elementary particles.
- Analyze images of the trajectories of elementary particles to determine the mass-versus-charge ratio
- Describe the standard model of elementary particles in terms of the characteristic properties of quarks, leptons, and bosons.
- Identify the quarks that form familiar particles such as the proton and the neutron.

KEY TERMS

- lepton
- hadron
- quark
- standard model

By the 1930s, scientists believed that they knew the particles on which all matter was built — the proton, the neutron, and the electron. These were the "elementary" particles, in that nothing was more basic. Nature, however, was not that simple. The first hint of a greater complexity came with the missing energy and momentum in beta decay and Pauli's proposal of the neutrino.

Today, even the proposition that these particles are massless is being challenged. Several difficulties with our understanding of the nuclear reactions in the Sun would be cleared up if neutrinos actually had mass. Neutrinos are extremely tiny and neutral, so they barely interact with matter. Consequently, during the day, neutrinos rain down through us from the Sun. At night, they pour through the planet and stream upward through us.

In 1935, Japanese physicist, Hideki Yukawa (1907–1981) proposed that the strong nuclear force that bound the nuclei together was carried by a particle with a mass between that of the electron and the proton. This particle came to be known as a "meson," because of its intermediate mass. Eventually, it was discovered in 1947, when it was termed the "π meson" (or simply, the "pion"). In fact, though, these particles are not the ones that hold the nuclei together — they never interact with other particles or nuclei by means of the strong force. Elementary particle physics is one of the most active fields in theoretical physics.

The Search for New Particles

In Section 13.2, you learned that research into beta decay revealed the existence of neutrinos and positrons. Physicists discovered that positrons were, in fact, antielectrons. When a particle (positron) and its antiparticle (electron) interact, they annihilate each other and transform into two gamma rays. With the realization that the proton, neutron, and electron were not the only subatomic particles, physicists began to search in earnest for more elementary particles.

In Chapter 8, Fields and Their Applications, you learned how electric and magnetic fields are used in powerful instruments to accelerate protons and electrons to close to the speed of light. When these extremely high-energy particles collide with other particles and nuclei, new, very short-lived particles are produced. For example, when a very high-energy proton collides with another proton, a neutral particle called a "neutral pion" (π^0) is produced. The particle exists for only about 0.8×10^{-16} s, and then decays into two gamma rays.

You are probably wondering how physicists can study a particle that disappears within 10^{-16} s after it is produced. One instrument that physicists use is called a "cloud chamber," which was invented in 1894 by C.T.R. Wilson (1869–1959). A cloud chamber contains a cold, supersaturated gas. When any form of ionizing radiation passes through, the ions formed in the path of the particle form condensation nuclei and the cold gas liquefies, creating a visible droplet. If the cloud chamber is placed in a magnetic field, charged particles will follow curved paths. Physicists analyze photographs of the "tracks" in the cloud chamber and can determine their size and speed. In the following investigation, you will make observations using a cloud chamber.

During the 1930s and 1940s, several more particles were discovered, and physicists also realized that every elementary particle has an antiparticle. U.S. physicists S.H. Neddermeyer and C. D. Anderson discovered positive and negative muons (μ^+ and μ^-) that have a mass about 207 times that of an electron. Muons have a lifetime of 2.2×10^{-6} s and decay as shown below.

$$\mu^- \rightarrow e^- + \nu_\mu + \overline{\nu}_e$$
$$\mu^+ \rightarrow e^+ + \overline{\nu}_\mu + \nu_e$$

These discoveries also revealed the existence of muon neutrinos (ν_μ) and muon antineutrinos ($\overline{\nu}_\mu$). The subscript e must now be used to indicate electron neutrinos.

In the 1940s, two more pions were discovered, one having a positive charge and the other a negative charge. These pions have a lifetime of 2.6×10^{-8} s and decay as shown below.

$$\pi^- \rightarrow \mu^- + \overline{\nu}_\mu$$
$$\pi^+ \rightarrow \mu^+ + \nu_\mu$$

Since these early discoveries, physicists have continued to identify many more extraordinary particles. Eventually, a pattern evolved and physicists were able to start classifying these elementary particles according to the types of forces through which they interact with other particles. For example, protons and neutrons interact through the strong nuclear force, whereas electrons do not experience the strong nuclear force. Particles might be affected by one or more of the four fundamental forces — gravitational, electromagnetic, strong nuclear, and weak nuclear forces.

Families of Particles

The smallest family of particles is the photon family, consisting only of the photon itself. Photons interact only through the electromagnetic force and interact only with charged particles. The photon is its own antiparticle.

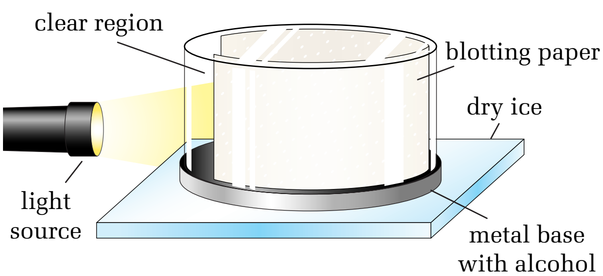
The **lepton** family of particles interacts by means of the weak nuclear force. Leptons can interact through the gravitational force,

INVESTIGATION 13-B

The Wilson Cloud Chamber

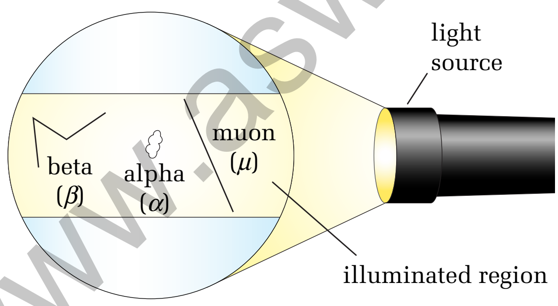
TARGET SKILLS
- Performing and recording
- Analyzing and interpreting

The cloud chamber that you will use consists of a short, transparent cylinder with a glass top and a metal base. The sides are lined with blotting paper, except for a section through which you will shine an intense light beam, as shown in the diagram. The light will make the liquid drops visible against the dark background.



The next diagram shows the general appearance of some tracks that you might see. Alpha radiation appears as a short (1 cm to 2 cm long), thick puff of white “cloud.” Beta particles (high-speed electrons) appear as long, thin strands that bend gradually or zigzag from collisions with atoms.

Because the muon is much more massive than the beta particle, it appears as a thin, extremely straight strand that goes across the chamber. Many muons angle downward and are difficult to observe.



Since Earth is constantly bombarded by cosmic rays (which are really high-energy particles), you can nearly always observe tracks in a cloud chamber.

Equipment

- cloud chamber
- radioactive source (optional)
- light source
- alcohol
- dry ice

CAUTION Do not touch dry ice unless you are wearing thick gloves.

Procedure

1. Place the cloud chamber on the block of dry ice and pour in the alcohol to a depth of about 1 cm. Put on the glass cover and let the chamber stand for a few minutes, until the alcohol has a chance to reach equilibrium.
2. Working in groups of three or four, take turns watching the cloud chamber carefully for a total of at least 15 min. Make a sketch of every track that you see.
3. Obtain similar data from all of the groups that are performing the observations.
4. (Optional) If your cloud chamber has a small access hole in the side and if you have a small radioactive source on the end of a pin, insert it into the hole.
5. Make a sketch of the tracks that you observe emanating from the source.

Analyze and Conclude

1. Try to identify the tracks that you observed.
2. List the types of radiation observed in this investigation, from the most common to the least common.
3. What type of radioactive source did you use? Were the tracks consistent with the nature of the radiation emitted by the source? Explain.

and if they are charged, through the electromagnetic force, but are immune to the strong nuclear force. Once called the "beta decay interaction," the weak nuclear force is involved in beta decay.

As you would probably expect, electrons and electron neutrinos are leptons. Muons and their neutrinos are also leptons. A more recently discovered particle, the tau (τ) particle and its neutrino, also fit into the lepton family. As previously stated, for every particle, there is an antiparticle. The antiparticles always have the same mass as the particle, and if the particle has a charge, the antiparticle has the opposite charge. When the particles are neutrally charged, the antiparticle is also neutral but opposite in some other property. In such cases, the antiparticles are denoted with a bar over the symbol. Leptons and their antiparticles appear to be true elementary particles. There is no indication that they consist of any more fundamental particles.

Particles of the **hadron** family interact through the strong and weak nuclear forces. Hadrons can also interact through the gravitational force, and if they are charged, through the electromagnetic force. The hadron family is the largest family and is subdivided into the groups, mesons and baryons. The common proton and neutron and their antiparticles are baryons, while pions are mesons. Pions were at one time called "pi mesons." As larger and more powerful particle accelerators were built, more and more hadrons were discovered.

Table 13.4 summarizes the properties of most of the subatomic particles that have been discovered. However, the list of hadrons is incomplete and will certainly continue to grow as physicists continue their search. Since most of the particles are very short-lived and are eventually transformed back into energy, Table 13.4 reports the energy equivalent of the rest masses of the particles in units of MeV, rather than reporting in units of mass. If you calculated the energy equivalent of 1 u, you would find that it is about 931.5 MeV.

Quarks

As the number of hadrons that had been discovered grew, physicists became suspicious that hadrons might not really be elementary particles. Some physicists were studying the scattering of electrons off protons and neutrons and saw evidence that there were three "centres" of some type within the nucleons. At the same time, theoretical physicists Murray Gell-Mann (1929–) and George Zweig (1937–), working independently, proposed the existence of truly elementary particles that made up hadrons. Gell-Mann somewhat jokingly called these particles **quarks**, from a line in *Finnegan's Wake* by James Joyce — "Three quarks for Muster Mark." The name stuck. Today, physicists accept that quarks are the elementary particles of which all hadrons consist.

Table 13.4 Some Particles and Their Properties

Family	Particle	Particle Symbol	Antiparticle symbol	Rest energy (MeV)	Lifetime (s)
Photon	photon	γ	self*	0	stable
Lepton	electron	e^-	e^+	0.511	stable
	muon	μ^-	μ^+	105.7	2.2×10^{-6}
	tau	τ^-	τ^+	1784	10^{-13}
	electron neutrino	ν_e	$\overline{\nu}_e$	≈0	stable
	muon neutrino	ν_μ	$\overline{\nu}_\mu$	≈0	stable
	tau neutrino	ν_τ	$\overline{\nu}_\tau$	≈0	stable
Hadron					
Mesons	pion	π^+	π^-	139.6	2.6×10^{-8}
		π^0	self*	135.0	0.8×10^{-16}
	kaon	K^+	K^-	493.7	1.2×10^{-8}
		K_S^0	$\overline{K}_S^0$	497.7	0.9×10^{-10}
		K_L^0	$\overline{K}_L^0$	497.7	5.2×10^{-8}
	eta	η^0	self*	548.8	$<10^{-18}$
Baryons	proton	p	$\overline{p}$	938.8	stable
	neutron	n	$\overline{n}$	939.6	900
	lambda	Λ^0	$\overline{\Lambda}^0$	1116	2.6×10^{-10}
	sigma	Σ^+	$\overline{\Sigma}^-$	1189	0.8×10^{-10}
		Σ^0	$\overline{\Sigma}^0$	1192	6×10^{-20}
		Σ^-	$\overline{\Sigma}^+$	1197	1.5×10^{-10}
	omega	Ω^-	Ω^+	1672	0.8×10^{-10}

*The particle is its own antiparticle.

At the time that quarks were proposed, three quarks and their antiquarks could account for all known hadrons. Mesons consisted of two quarks, and baryons consisted of three quarks, given the names "up" (u), "down" (d), and "strange" (s). Uniquely, quarks have fractional charges of $+\frac{2}{3}e$, $-\frac{1}{3}e$, $-\frac{1}{3}e$, respectively, while the antiquarks have charges of the same size but opposite charge. Figure 13.19 gives examples of the quarks that make up the common neutron, proton, and positive and negative pions. Notice that the baryons consist of three quarks and the mesons consist of two.

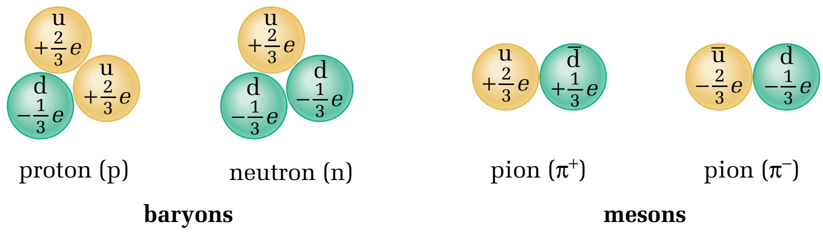


Figure 13.19 The combination of quarks in hadrons always results in a neutral charge or in a unit charge.

The quark model worked very well in explaining the properties of hadrons until about 1974, when more hadrons were discovered. Eventually, physicists discovered that six quarks were necessary in order to account for all of the newly discovered hadrons. The three new quarks were given the names "charmed" (c), "top" (t), and "bottom" (b), although some physicists, particularly in Europe, prefer to call the last two quarks, "truth" and "beauty." The quarks and some of their properties are summarized in Table 13.5.

Table 13.5 The Quarks

Quark name	Rest energy (GeV)	Quark		Antiquark	
		Symbol	Charge	Symbol	Charge
up	0.004	u	$+\frac{2}{3}e$	$\bar{u}$	$-\frac{2}{3}e$
down	0.008	d	$-\frac{1}{3}e$	$\bar{d}$	$+\frac{1}{3}e$
strange	0.15	s	$-\frac{1}{3}e$	$\bar{s}$	$+\frac{1}{3}e$
charm	1.5	c	$+\frac{2}{3}e$	$\bar{c}$	$-\frac{2}{3}e$
top (or truth)	176	t	$+\frac{2}{3}e$	$\bar{t}$	$-\frac{2}{3}e$
bottom (or beauty)	4.7	b	$-\frac{1}{3}e$	$\bar{b}$	$+\frac{1}{3}e$

As physicists collected more and more details about hadrons and their quarks, they discovered that quarks have more properties than just charge. A property that physicists call "colour" explains many of their observations, as well as placing the quark in agreement with the Pauli exclusion principle.

Exchange Particles

Physicists' current view of the structure of matter is summarized in Figure 13.20, but this summary does not present the complete picture. You have read many times about the four fundamental forces of nature, the properties of these forces, and how elementary particles are even categorized according to the forces that they experience. The question remains: How do these forces work? While studying elementary particles, physicists also discovered some basic information about the fundamental forces of nature. The **standard model** refers to the currently accepted mechanisms of the strong, weak, and electromagnetic forces. Physicists hope to bring the gravitational force into the model, but so far, it has been elusive.

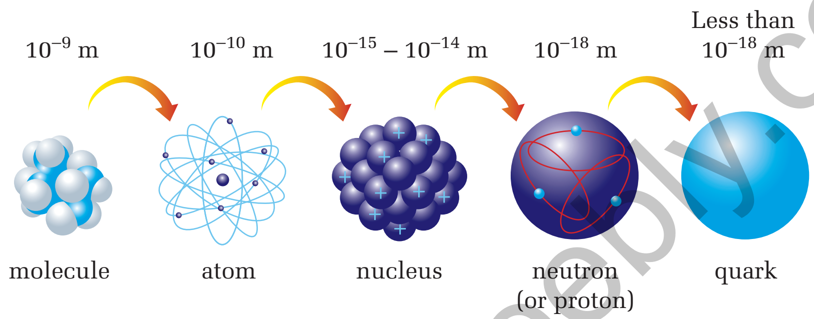


Figure 13.20 Scientists' view of the smallest indivisible piece of matter has changed greatly over the past century — going from Dalton's model of the atom to the current view of the quark.

Physicists have found particles that are exchanged by the elementary particles that account for the interactions between them. Some of the properties of these exchange particles are listed in Table 13.6.

When charged particles interact through the electromagnetic force, they exchange a photon. Because photons have no mass and travel at the speed of light, the range of the force is unlimited. In the opposite extreme, the weak nuclear force is mediated by bosons that have a large mass and such a short lifetime that the range of the interaction is extremely short.

Table 13.6 Force Carriers

Force	Name of Particle	Symbol	Mass (GeV)	Charge	Range (m)
electromagnetic	photon	γ	0	0	unlimited
weak nuclear	weak boson	W^+ W^- Z^o	80.2 80.2 91.2	$+e$ $-e$ 0	10^{-17}
strong nuclear	gluon	g	0	0	10^{-15}
gravitational	graviton*	G	0	0	unlimited

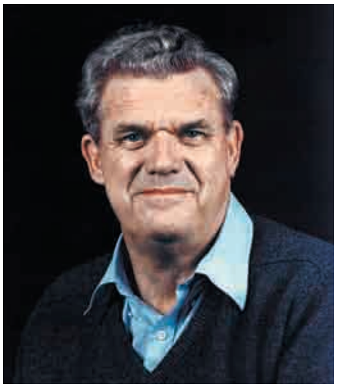
*The graviton has been proposed as a carrier of gravitational force. However, its existence has yet to be confirmed.

The exchange of gluons holds quarks together in hadrons. The theory is that when quarks exchange gluons, they change colour. Physicists have proposed the existence of a graviton as an exchange particle for the gravitational force and have determined some of the properties that such a particle would have to have. However, they have never observed any indication that gravitons exist. As you can see, the story is far from complete and there are many more challenges ahead for elementary particle physicists.

CANADIANS IN PHYSICS

"Not the Brightest Student" — But Wins Nobel Prize

There is no greater prize for a scientist than the Nobel Prize, and in 1990, Dr. Richard Taylor became the first Canadian to win the prestigious award in physics. He and two U.S. colleagues shared the award for proving the existence of quarks. The team used a powerful linear accelerator, operating at 21 GeV, to bombard protons and neutrons with electrons. They discovered that protons and neutrons, once thought to be indivisible, are made of these quarks, the existence of which had been theorized but never proven.



Dr. Richard Taylor
Courtesy Stanford Linear Accelerator Center

Born and raised in Medicine Hat, Alberta, Dr. Taylor was interested in experimental science from an early age, and this interest resulted in an accident that could have prematurely ended his science career. Several older boys showed him a formula for a better type of gunpowder than was available at that time. His attempt to follow the formula resulted in a powerful explosion that amputated three fingers of his left hand.

Dr. Taylor has said in interviews that he wasn't the brightest student in high school. "I did reasonably well in mathematics and science, thanks to some talented and dedicated teachers," he commented, "but I wasn't an outstanding student, although I did read quite a bit and high school mathematics came quite easily to me. You don't necessarily have to be a great student to do well later in life, although it is always important to work hard."

After completing his undergraduate work at the University of Alberta, Dr. Taylor was accepted into the graduate program at Stanford University in California, where he has spent much of his working life. "I found I had to work hard to keep up with my fellow students," said Dr. Taylor, "but learning physics was great fun in those surroundings."

Dr. Taylor stresses the importance of an inquiring mind. "It's fun to understand things and you should learn all you can. Reading gives you independence and a sense of freedom," he said, adding that he believes it is important to be educated in a broad range of subjects. Curiosity and a love of experimentation drive Dr. Taylor. While he has a great respect for theoretical physicists, calling them "smarter" than experimental physicists, he feels that "in experimental science, you can make contributions more easily."

Still a resident of California, Dr. Taylor works at the Stanford Linear Accelerator Center, also spending time in Europe at the HERA laboratories in Germany. After his prize-winning work to discover the quark, he is now interested in searching for gravitational waves and is involved with a new satellite experiment to detect high-energy gamma rays from sources in outer space.

Although at age 71 Dr. Taylor considers most of his scientific contributions to be behind him, much more work lies ahead in the field of particle physics. To the next generation of physicists, he says, "What the young people have to deal with is the fact that there are three generations of quarks. There are the quarks that everything we know of is made of, and then there are two more sets of quarks. The question is: Why are they there?" Dr. Taylor expects this question to occupy the physicists of tomorrow "for the next 50 years."

INVESTIGATION 13-C

Measuring the Mass-to-Charge Ratio for Electrons

TARGET SKILLS

- Identifying variables
- Performing and recording
- Conducting research

In this investigation, you will perform an experiment very similar to the one in which J.J. Thomson discovered and characterized the electron. You will accelerate electrons by means of a large potential difference and then deflect them in a cathode ray tube by means of a known magnetic field.

Problem

(1) Determine the speed of electrons that pass through a cathode ray tube and **(2)** measure the ratio of the mass to the charge for the electron.

Equipment

- DC power supply for heated cathode tubes
- Helmholtz coils
- DC power supply for Helmholtz coils
- ammeter
- Thomson deflection tube

CAUTION Avoid touching the high voltage connections.

A cathode ray tube emits a small amount of X rays, so stay in front of it very briefly.

Procedure

1. With all power supplies turned off, set the anode voltage to zero.
2. Connect the Thomson deflection tube to the power supply according to the instructions in the manual for the tube. Check that all connections are secure and correct.
3. Set the power supply for the Helmholtz coils to zero. Connect the ammeter in series with the power supply and the coils.
4. Measure the radius of the Helmholtz coils (or record the value provided with the coils).
5. Turn on the deflection tube power supply. Make sure that the filament voltage is set correctly, according the manual (probably 6.3 V).
6. Increase the anode voltage to 5000 V and observe the glowing trace of the cathode rays across the screen.
7. Gradually increase the voltage of the Helmholtz coils until the electron beam has been strongly deflected by the time the beam leaves the screen. Ensure that the maximum current for the coils is not exceeded. Record the value of the current.
8. Record the coordinates for two grid points along the trajectory of the beam.
9. Reduce all voltages to zero and turn off the power supplies.

Calculating the Radius of the Circular Trajectory

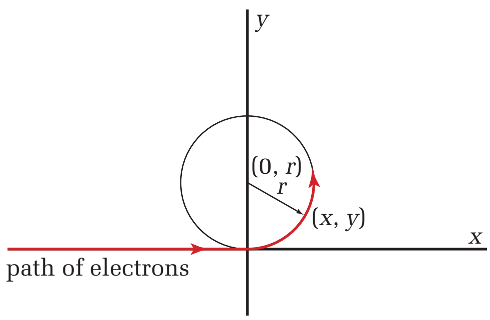
In general, the distance between any two points with known coordinates is given by

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

For the circular trajectory, the distance between any point on the circle and the centre is the radius, r.

Since the deflection begins when the beam passes throught the origin of the graph, the centre of the circle must be at $(0, r)$

Thus, $r = \sqrt{(x - 0)^2 + (y - r)^2}$ (3)



Analyze and Conclude

1. **(a)** State the coordinates for the two observed points on the electron beam's trajectory.
   (b) Write an expression for r based on the equation given above.
   (c) Calculate the value of r for each point and find the average. Use this average in your further calculations.
2. The magnetic field between the two Helmholtz coils is given by the equation
$$B = \frac{32\pi nI}{5\sqrt{5}(R_c)} \times 10^{-7}\ \mathrm{T},$$
where n is the number of turns in the coils (as indicated on the coils), I is the current in amperes, and R_c is the radius of the coils. Calculate the magnetic field (B) between the coils.
3. Use the equation $v = \frac{2V}{Br}$ to determine the speed of the electrons. Substitute the value for v into an expression, $\frac{m}{e} = \frac{2V}{v^2}$, to find the charge-to-mass ratio.
4. Review the information in Chapter 8, Fields and Their Applications, about the motion of charged particles moving through a magnetic field and derive the equations above.

13.3 Section Review

1. K/U How do physicists know of the existence of particles with lifetimes that are as short as 10^{-10} s, and how can they determine any properties of these particles?
2. K/U Why are the electrons in the lowest energy level of an atom not affected by the strong nuclear force?
3. K/U
   (a) State two ways in which leptons differ from hadrons.
   (b) In what ways are mesons similar to baryons?
   (c) How are mesons different from baryons?
4. MC Using quark notation, how could you represent **(a)** a negative pion and **(b)** an antiproton?
5. MC An antineutron must be neutral and have exactly the same mass as the neutron. What should its quark composition be?
6. I If neutrinos barely interact with matter, how can they be detected? Research the question and provide a diagram to explain the process.

CHAPTER 13 Review

REFLECTING ON CHAPTER 13

- The neutron was discovered by James Chadwick.
- The particles in the nucleus are called "nucleons" and consist of protons and neutrons. Their number is indicated as the atomic mass number (A).
- The number of protons in the nucleus is indicated by the atomic number (Z).
- In a neutral atom, the number of electrons orbiting the nucleus equals the number of protons in the nucleus.
- A common mass unit for atoms and nuclei is the atomic mass unit (u).
$$1 \text{ u} = 1.6605 \times 10^{-27} \text{ kg}$$
- The mass defect is the difference between the separate total mass of the nucleons and the mass of the nucleus. It represents the binding energy for that nucleus.
- Nuclear fission is the splitting apart of a very large nucleus to produce two smaller nuclei plus several neutrons and energy.
- Nuclear fusion is the joining of two low-mass nuclei to form a larger nucleus.
- Henri Becquerel discovered radioactivity.
- Radioactivity consists of the emission of alpha particles (helium nuclei), beta negative particles (high-speed electrons), beta positive particles (high-speed positrons), and gamma rays (photons).
- Alpha, beta, and gamma radiation vary in their mass, charge, penetrating ability, and possible biological damage. The passage of any of these rays through matter leaves ions behind, so the radiation is called "ionizing radiation."
- Radiation can be detected by exposing film; causing ionization in matter by using, for example, the Geiger counter; and identifying the fluorescence or phosphorescence that radiation creates in some substances.
- Radioactivity has many uses, both medical and non-medical. For example, it is commonly used in smoke detectors.
- During any nuclear reaction the total atomic mass number (A) and the total atomic number (Z) remain unchanged.
- Transmutation is the conversion of one element into another.
- The rate of radioactive decay is indicated by the half-life of the radioisotope.
- Radioactive decay rates can be used to determine the age of ancient materials.
- The amount of a radioactive isotope remaining after a given time interval can be determined by using the following equation.
$$N = N_0\left(\frac{1}{2}\right)^{\frac{\Delta t}{T_{\frac{1}{2}}}}$$
- A common unit in the field of radioactivity and radiation is the becquerel.
- Exposure to radiation can lead to various levels of sickness and, if severe enough, to death.
- Subatomic particles are grouped into three families — photons, leptons and hadrons. Hadrons consist of particles that are built from quarks.
- According to the standard model, forces are the result of the exchange of particles.
- The model of matter that involves particles as force carriers and the concept that all hadrons, such as protons and neutrons, are composed of quarks is known as the standard model.

Knowledge/Understanding

1. Use Einstein's theory to explain how the term "mass defect" refers to an amount of energy.
2. Outline the rationale for postulating the existence of a strong nuclear force as one of the fundamental forces of nature.
3. Compare the range of the field of influence of a strong nuclear force with that of an electromagnetic force when considering the effect of each on a proton near or in the nucleus of an atom.
4. Explain, with the aid of a series of sketches, the relative effects of an electromagnetic force and a strong nuclear force at several stages as a proton is propelled toward a nucleus in a fusion reaction.
5. Describe the characteristics of the three common forms of radioactivity.
6. Explain, based on our scientific understanding of radiation, why it is now useful to use the concept of a nucleon rather than a proton as a basic particle located in an atom's nucleus.
7. Explain why the daughter nuclei from fission reactions are likely to be radioactive.
8. Describe the concept of a *force carrier*. Outline how this concept is an explanatory device for outlining a scientific model in which mass and energy are simply different forms of the same phenomena.

Inquiry

9. The concept of antimatter has stimulated the imagination of many science fiction writers. Research and prepare a report of the scientific discoveries that led to the inclusion of antimatter particles in scientific models of matter.
10. Insight into nuclear structure can be gained by considering the binding energy per nucleon, $\Delta E/A$, for different elements. **(a)** Describe how the calculation of $\Delta E/A$ is used to indicate nucleons in a specific nucleus are tightly bound or loosely bound. **(b)** Calculate the binding energy, in both joules and MeV, for the following 12 elements: helium, carbon, neon, oxygen, chlorine, manganese, iron, cobalt, silver, gold, cesium, and uranium. In each case, divide the binding energy by the mass number, A.
(c) Write the equation for a common fusion reaction. Locate the position of the initial nuclei, by their nucleon number, on the graph on page 550. Locate the position of the fused nuclei on the graph. Describe the effect of fusion on the binding-energy-per-nucleon ratio.
(d) Write the equation for a common fission reaction. Locate on the graph on page 550 the position of the initial nuclei, by their nucleon number. Locate the position of the daughter nuclei on the graph. Describe the effect of fission on the binding energy per nucleon ratio.
(e) Locate on the graph the range of nucleon numbers of those elements that are more likely to undergo fusion and the range of nucleon numbers for those that are more likely to undergo fission.
11. Suppose an experiment is designed to allow continuous observation of a single atom of a certain radioactive material. If the half-life is 1.5 h, can the observer predict when the atom will decay?
12. Use conservation laws to determine which of the following reactions are possible. Explain your reasoning in each case.
(a) $p + p \rightarrow p + n + \pi^+$
(b) $p + p \rightarrow p + p + n$
(c) $p + p \rightarrow p + \pi^+$
(d) $p + p \rightarrow p + p + \pi^0$

Communication

13. Explain why neutrons are said to make better "nuclear bullets" than either protons or electrons.
14. Use the concepts of fission, fusion, and binding energies to provide a scientific explanation of what limits the size a stable nucleus.

Making Connections

15. Explain why, to date, nuclear reactors have been constructed to use fission, but none have been constructed to use fusion.

16. Food and surgical supplies are sometimes sterilized by radiation. What are the advantages and disadvantages of using this procedure rather than sterilization by heating?

17. In 1989, two scientists at the University of Utah announced to the public that they had produced excess energy in a fusion-like experiment at room temperatures. The experiment was dubbed "cold fusion" and the scientists thought they had identified a new, cheap energy source. However, other experimenters failed to reproduce the results of this experiment, so even today, most of the scientific community does not consider cold fusion as a real possibility. Research this episode of physics history and use it to discuss the roles of peer review and reproducing results in scientific methodology.

18. Prepare a report on how radioactive tracers are used to either **(a)** follow the path of rainwater through groundwater reservoirs to lakes, streams, and wells or **(b)** map ocean currents.

19. The word "radiation" strikes fear into the hearts of many people. In fact, many would not live anywhere near a nuclear power station. Gather information about common concerns and misconceptions about "radiation" by interviewing people and generating a file of newspaper articles. Identify four or five of the common issues. Write a scientific perspective on each. Make a recommendation of the safety features that you consider essential for operating a nuclear power station in such a way that you would feel comfortable living within a one kilometre radius of it.

Problems for Understanding

20. Determine the number of protons, neutrons and electrons in **(a)** a doubly ionized calcium ion $^{40}_{20}Ca^{++}$ **(b)** an iron atom $^{56}_{26}Fe$ **(c)** a singly charged chlorine ion $^{35}_{17}Cl^{-}$.

21. Calculate the binding energy for **(a)** $^{12}_{6}C$ with a atomic mass of 12.000 000 u **(b)** $^{133}_{55}Cs$ with a atomic mass of 132.905 429 u.

22. Write the equation for the alpha decay of thorium: $^{230}_{90}Th$.

23. What fraction of the original number of nuclei in a sample are left after **(a)** two half-lives, **(b)** four half-lives, and **(c)** 12 half-lives?

24. **(a)** How much energy is released when radium-226 (nuclear mass 225.977 09 u) alpha decays and becomes radon-222 (nuclear mass 221.970 356 u)? Answer in MeV.
(b) If the nucleus was initially at rest, calculate the velocities of the alpha particle and the radon-222 nucleus in part (a).
(c) What percentage of the total kinetic energy does the alpha particle carry away?

25. Hafnium-173 has a half-life of 24.0 h. If you begin with 0.25 g, how much will be left after 21 days?

26. How long will it take a 125 mg sample of krypton-89, which has a half-life of 3.16 min, to decrease to 10.0 μg?

27. A scientist at an archeological dig finds a bone that has a carbon-14 activity of 5.70×10^{-2} Bq. If the half-life of carbon-14 is 5.73×10^{3} a, what is the age of the bone? (Assume that the initial activity was 0.23 Bq.)

28. Suppose you began with a sample of 800 radioactive atoms with a half-life of 5 min.
(a) How many atoms of the parent nucleus would be left after 10 min?
(b) How many atoms of the daughter nucleus would be present after 10 min?
(c) How many atoms of the parent nucleus would be left after 25 min?
(d) How many atoms of the daughter nucleus would be left after 25 min?
(e) Write an equation to determine the number of daughter nuclei present at any time.

29. In radioactive dating, ratios of the numbers of parent and daughter nuclei from the same decay chain, such as uranium-238 and lead-206, are determined. Assume that when the sample formed, it contained no daughter nuclei. Consider the analyses of three different rock samples that have been determined to have ratios of uranium-238 to lead-206 of 1.08:1, 1.22:1, and 1.75:1.
   (a) Using the results of the previous question, write an equation for the ratio of the number of uranium-238 atoms to lead-206 atoms present at any time. (Hint: the initial number of uranium-238 atoms will divide out.)
   (b) Solve the above equation for time, and determine the ages of the three samples. (The half-life of uranium-238 is 4.5×10^9 a.)
   (c) Explain whether these samples could have been taken from an area where the rock solidified all at once.
   (d) Intuitively, what conclusion can you draw if you measure a ratio of less than one?

30. What is the wavelength of each of the two photons produced in electron-positron annihilation?

31. Heavy water used in the Sudbury Neutrino Observatory is made up of oxygen and deuterium, a radioactive isotope of hydrogen (see Not Your Average Observatory, page 554). One of the reactions that physicists at the observatory are trying to detect is $\nu_e + {}^2_1\mathrm{H} \rightarrow p + p + e^-$, where ν_e is an electron-neutrino. For this reaction to be observed, the neutrino's energy must be greater than the binding energy of a deuterium atom.
   (a) Given that the nuclear mass of deuterium is 2.013 553 u, calculate the minimum neutrino energy for this reaction to occur.
   (b) If 95.0% of the neutrino's kinetic energy goes into the kinetic energy of the produced electron, calculate the speed of the electron. (Hint: The electron's speed is relativistic.)
   (c) Compare the electron's speed with the speed of light in water.

32. Analyze the following reactions in terms of their constituent quarks.
   (a) $n \rightarrow p + e^- + \overline{\nu}_e$
   (b) $\gamma + n \rightarrow \pi^- + p$

UNIT 5 PROJECT

Decades of Triumph and Turmoil

Background

From the late 1800s to the mid-1900s, the world saw change at a rate it had never experienced before. Science progressed rapidly as understanding of the atom deepened. Molecules were mapped, and from that mapping came new products — plastics, pharmaceuticals, stronger alloys. Harnessing the nucleus gave promise of bountiful energy in peacetime and mass destruction in time of war. Through study of the electromagnetic spectrum came an ever-increasing ability to probe inward to understand the workings of our body cells and outward to observe the workings of the universe. Some parts of the spectrum became crowded with use as radio and television stations staked their claims to frequencies.

Along with scientific and technological change came societal change. Two world wars left their legacy of broken lives, shattered countries and economies, and radical changes in social outlooks and value systems. Warriors returned to very different homelands. Changes in production techniques also had a huge impact on society. Augmented by new technologies, the assembly line became the backbone of many huge industries, and the need for unskilled workers plummeted.


A World War I battle scene

It is easy to forget that the scientists whose contributions you have studied during this course lived and worked in the midst of these changes. They, too, were affected, and sometimes even caused or influenced these changes. The goal of this project is to examine the parallel between these scientists' professional lives and what their lives were like when they stepped outside of their offices and laboratories.

Plan and Present

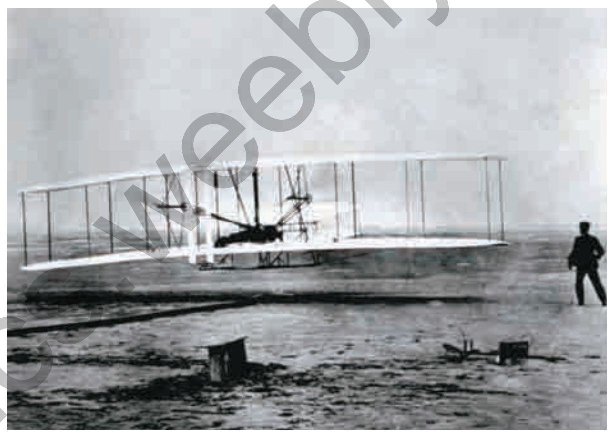
1. As a class, establish clear guidelines for evaluating the finished project. Discuss specifics such as
   - deadlines
   - expectations for the diary or letter: Will there be a minimum length, a minimum number of societal factors to be included, a specified presentation format?
   - expectations for the poster: Will presentation attractiveness and organization be assessed, as well as the content? Will there be a minimum amount of biographical and scientific material that must be included?
   - expectations for the time line: How do you intend to assess a group's contribution to the overall historical time line?
2. As a class, prepare an initial time line for the period of 1881–1950, listing major scientific advances and discoveries alongside major events in society, such as World Wars I and II, the Depression, the birth of jazz, the first automobiles, the introduction of radio and then television shows, and aviation, from the Wright brothers' first experiment on a North Carolina seashore to space exploration.
3. Divide the study up into six time spans: the two-decade period of 1881 to 1900 and the five individual decades between 1901 and 1950. Assign a team to each era. (You could perhaps allocate the number of members per team according to the number of events in each era.)

ASSESSMENT

After you complete this project

- **assess your ability to conduct research: Did you find relevant information?**
- **assess your teamwork skills: How effectively were you able to share your ideas with other members of your group and contribute to the group effort?**
- **assess your communication skills: How effectively did you communicate your ideas in the poster and written portions of this task?**

4. Each team is to research three major scientific or technological events that occurred during its designated time period and prepare a poster on each event. This presentation must include biographical data for the people involved and an outline of the nature and importance of the event. The team is then to research the major societal events and changes that might have affected those scientists.



The famous reconnaissance aircraft SR71 *Blackbird* is a descendent of the Wright brothers' *Wright Flyer*, which made history on December 17, 1903, when Orville Wright piloted the first powered, manned, controlled flight. For its debut, the *Wright Flyer* was in the air for 12 s and covered a distance of 37 m. By contrast, the Blackbird flew 3500 missions and was so fast that a missile had to be fired 48 km ahead of the plane to reach it in time.

5. As a class, construct an overall time line. This could perhaps be a horizontal version of the time line shown on page xiv, and could be posted around the classroom near the top of two or three of the walls. The names of the scientists along with brief outlines of their contributions or applications of these contributions could be placed on one side of the line, with a listing of the corresponding major societal and world events on the other side of the line. The posters could also be displayed.

6. Working individually or in pairs, you will write letters or diaries that represent what one or more of the featured scientists might have written about their everyday lives.
   - What type of transportation did the scientist probably use, locally and for long-distance travel?
   - What type of lighting was available in that scientist's time?
   - What were the major newspaper stories at the time the scientist did his or her most notable work?
   - What type of medical treatment was available at that time?

Evaluate

1. Evaluate the extent to which your group met the expectations for the project in relation to the
   - posters
   - timeline
   - written material

2. **(a)** Which items prepared by your group do you feel were most effective? Explain.
   (b) Which items prepared by your group do you feel were least effective? How might they have been improved?

UNIT 5 Review

Knowledge/Understanding

Multiple Choice

In your notebook, choose the most correct answer for each of the following questions. Outline your reasons for your choice.

1. Of the following quantities, which, if any, have the same value to all observers?
   (a) mass of the muon
   (b) average lifetime of the muon
   (c) charge of the muon
   (d) energy of a photon
   (e) speed of light
2. The kinetic energy of a particle travelling near the speed of light is
   (a) always less than the rest energy
   (b) equal to mc^2
   (c) equal to $\frac{1}{2}mv^2$
   (d) equal to $(m - m_0)c^2$
   (e) equal to $(m_0 - m)c^2$
3. When an object with a rest mass of 2.0 kg approaches the speed of light, its mass approaches
   (a) 0 (b) 0.5 kg (c) 1.0 kg (d) c^2 (e) ∞
4. If you direct light at a metal surface, the energies of the emitted electrons
   (a) are random
   (b) vary with the speed of light
   (c) vary with the intensity of light
   (d) vary with the frequency of light
   (e) are constant
5. In the Bohr model of the atom, an electron emits energy when it
   (a) accelerates in its orbit
   (b) decelerates in its orbit
   (c) jumps from a higher energy level to a lower energy level
   (d) jumps from a lower energy level to a higher energy level
   (e) is in the ground state
6. The strong nuclear force has a limited range. A consequence of this is the
   (a) magnitude of nuclear binding energies
   (b) instability of large nuclei
   (c) ratio of atomic size to nuclear size
   (d) existence of isotopes
   (e) existence of neutrinos
7. The number of elementary charge units in a nucleus determine the atomic
   (a) size (b) weight (c) mass (d) number (e) density
8. The half-life of ^{28}Ni is six days. What fraction of a sample of this nuclide will remain after 30 days?
   (a) $\frac{1}{4}$ (b) $\frac{1}{8}$ (c) $\frac{1}{16}$ (d) $\frac{1}{32}$ (e) $\frac{1}{64}$
9. After 4 h, $\frac{1}{16}$ of the initial amount of a certain radioactive isotope remains undecayed. The half-life of the isotope is
   (a) 15 min (b) 30 min (c) 45 min (d) 1 h (e) 2 h
10. A particle that will not leave a curved track in a bubble chamber is the
    (a) proton (b) positron (c) electron (d) neutron (e) alpha particle

Short Answer

11. If you were travelling in a spaceship at 0.9 c, would you notice any time dilation effects for clocks in the spaceship? Explain your reasoning.
12. A clock on a flying carpet streaks past an Earthling who is looking at her watch. What does the Earthling notice about the passage of time on the moving clock, compared with her watch? What would a wizard on the flying carpet notice about the passage of time on the Earthling's watch, compared to the clock on the carpet? Does it matter which timepiece is considered to be in motion and which is considered to be at rest?
13. Explain the following statement: The speed of light is a constant.
14. Max Planck introduced an hypothesis regarding the energy of vibration of the molecules in order to satisfy the observed spectrum emitted

from a hot body. What was this hypothesis and on whose work did he reportedly base his idea?

15. **(a)** Describe the relationship Phillip Lenard found between the energy of photoelectrons and the frequency of the incident light.
(b) Describe how increasing the light intensity affects the electron flow.

16. **(a)** Use the photon theory of light to explain why a photographer might use a red safety light in a darkroom for black and white photography.
(b) Sunburn is caused by the ultraviolet component of sunlight, not by the infrared component. How does the photon theory account for this?

17. Does it take more or less energy to remove a photoelectron from lead than from aluminum? (See Table 12.1 on page 509.) Explain your reasons.

18. Describe the technique that was used successfully to demonstrate the existence of de Broglie matter waves.

19. **(a)** Some features of the emission spectrum could still not properly be explained by the Bohr model. Name two such features.
(b) Paul Dirac modified Erwin Schrödinger's equation. What was he seeking to include and how successful was he?

20. Differentiate between a transmutation and a radioactive decay.

21. Describe how a knowledge of electromagnetism has been used to develop technologies to probe matter for indirect evidence of its elementary particles.

22. Explain, with the aid of a series of sketches, the relative effects of an electromagnetic force and a strong nuclear force at several stages as a proton is propelled toward a nucleus in a fusion reaction. Build on your explanation to suggest why "cold fusion" is not considered to be scientifically possible.

Inquiry

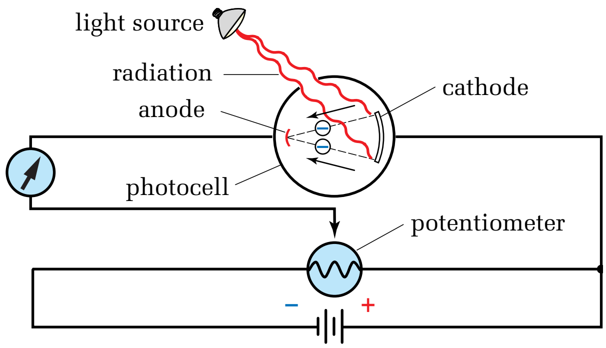
23. Suppose you had a rod of length L aligned parallel to the y-axis of an x-y reference frame labelled S and an identical rod of length L' aligned parallel to the y'-axis of an x'-y' reference frame labelled S' When the two frames are aligned, it is seen that the rods are the same length. Allow the frames to be offset in the x-direction and then set one of them in motion so that the rods move past each other. Argue that the length of either rod will not be seen to change. What would be the physical implications if one of the rods was observed to change?

24. Some people thought that they had disproved Einstein's special theory of relativity by describing the twin paradox. According to this thought experiment, identical twins Al and Bert grow up on Earth. Al rides a rocket, which travels close the speed of light, to Alpha Centauri and then returns. Consider the following points.

- From Bert's point of view of Earth, Al has been travelling at a high rate of speed, so his clock would have slowed down. When Al returns, he should be younger than Bert.
- However, from Al's point of view, it was Bert who was travelling at a high rate of speed. It was Bert's clock that slowed down, so Bert would be younger than Al. Since these two results are contradictory, the special theory of relativity must be wrong.

Explain why the special theory of relativity does not fully describe what is happening in this example. (Hint: Are both frames of reference equivalent?)

25. The phototube shown in the diagram was used to determine the stopping potential (also called "cut-off voltage") for electrons emitted from the cathode (emitter) when different wavelengths of light were incident on its surface. The table that follows the diagram records the values of the wavelengths used and the corresponding stopping potential.



Colour	Wave-length (nm)	Stopping potential (V)	Maximum E_k of photoelectrons (J)	Frequency (Hz)
green	530.0	0.045		
green	500.0	0.244		
blue	460.0	0.402		
violet	410.0	0.731		

(a) Prepare a table similar to the one above and complete the remaining two columns by calculating the maximum kinetic energy of the emitted electrons (using $E = qV$) and the frequency of the light.

(b) Draw a graph with maximum E_k on the vertical axis and frequency on the horizontal axis.

(c) From your graph, determine the work function for the particular emitter material used.

(d) Identify the metal used in the emitter (see Table 12.1 on page 509).

(e) Calculate the slope of the graph and compare it with Planck's constant.

(f) Explain how you feel that the graph would or would not be different if
- the emitter had been made from a different material
- the intensity of the light was doubled in each case

26. When a charged particle passes through a magnetic field that is perpendicular to its motion, its path is deflected into a circular path. If the strength of the field (B) is known and you assume that the particles are singly charged, prove that the radius of the path indicates the momentum of the particle.

27. Assume that a pure sample of a radioisotope contains exactly 1.6×10^4 nuclei with a half-life of 10.0 s.

(a) Determine the expected number of nuclei remaining after time intervals of 10 s, 20 s, 30 s, 40 s, 50 s, and 60 s.

(b) Draw an accurate graph of the data with the time interval (t) on the x-axis and the number of nuclei remaining (N) on the y-axis.

(c) The activity at any given time is given by $A = \Delta N/\Delta t$. What property of the graph does this ratio represent?

(d) Determine the activity of the sample at 10 s, 20 s, 30 s, 40 s, and 50 s.

(e) Draw an accurate graph of the data with the time interval (t) on the x-axis and the activity (A) on the y-axis.

(f) Compare the two graphs.

28. In the initial form of the quark model in the 1960s, three quarks were proposed: up, down, and strange.

(a) Make a table to show the charges on these quark-antiquark combinations.. Include a column of "strangeness," determined as follows: If the particle contains a "strange" quark, assign it a strangeness of −1. If the particle contains an "anti-strange" quark, assign it a strangeness of +1. Sum strangeness the same way in which charge is summed.

(b) Evidence for an underlying simplicity in matter was shown by plotting strangeness versus charge. Why are there apparent holes in the graph?

(c) Repeat (a) for all three quark combinations. Include columns for charge and strangeness.

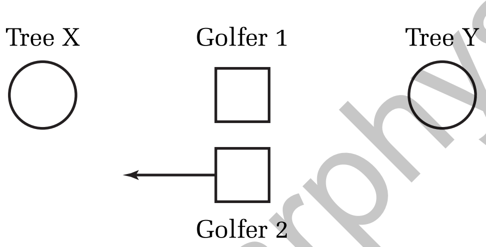
(d) Plot strangeness versus charge for the three quark particles. Comment on the symmetry of the plot.

29. Use conservation laws to determine which of the following reactions are possible. Explain your reasoning in each case.
(a) $p + p \rightarrow p + n + \pi^{+}$
(b) $p + p \rightarrow p + p + n$
(c) $p + p \rightarrow p + \pi^{+}$
(d) $p + p \rightarrow p + p + \pi^{0}$

Communication

30. In your own words, explain the term "relativity."

31. **(a)** Several golfers are out on a golf course when two trees are struck by lightning. The arrangement is as shown in the diagram. Golfer 1 is at rest relative to the two trees and observes that both trees were struck simultaneously. Golfer 2 is driving a relativistic golf cart. In the golf cart frame of reference, which tree was struck first? Give reasons for your answer.



(b) During a storm, a passenger in a stretch limousine, travelling close to the speed of light (the ultimate speed limit), noticed that two large hailstones struck the limousine simultaneously, one on the hood of the car and one on the trunk. According to a pedestrian who was standing on the sidewalk as the car sped past, which hailstone struck first? Give reasons for your answer.

32. Explain what it means to say that a certain quantity is quantized.

33. Describe the evidence that matter behaves as a wave.

34. Explain what limits the size of a stable nucleus.

35. Develop a graphic organizer to show how the elementary particles are related to other groups of subatomic particles.

Making Connections

36. Find examples in this textbook where the study of a particular area of physics was advanced by new experimental results that led to a new theory, and vice versa. Express your thoughts in writing about the manner in which science advances, using these examples.

37. Despite the complexity of some observed phenomena and some equations, the following statement is true: The basic ideas underlying all science are simple. Prove this to yourself by examining the chapters in this textbook. For each chapter, write down at least three simple but scientifically correct statements that summarize one or more of the concepts in the chapter. For example, one of the sentences for Chapter 11, Special Theory of Relativity, could be "Energy and matter are equivalent" or "Energy and matter are interchangeable." Make some of your statements general (to apply to an entire unit, for example) and relate some to specific concepts.

38. The Cavendish Laboratory for experimental physics at the University of Cambridge, England, has been responsible for many significant discoveries and inventions in the history of physics. These include the discoveries of the electron and neutron and the inventions of the mass spectrometer, cloud chamber, and the Cockcroft-Walton proton accelerator. Between 1879 and 1937, the chair of the laboratory was occupied by James Clerk Maxwell, Lord Rayleigh, J.J. Thomson, and Ernest Rutherford. Write an essay that examines the research done in this famous laboratory. Identify and discuss some of the factors that have enabled members of the Cavendish Laboratory to be so productive.

39. Draw a circuit diagram for a smoke detector and explain how it works. What determines its sensitivity?

40. Distinguish between fission and fusion. Research and prepare a report on why some elements are most likely to be involved in

nuclear fission reactions while others are most likely to be involved in nuclear fusion reactions.

41. Although physics has come a long way in its understanding of matter and energy, much work remains and it is uncertain that a full understanding is even possible. Write an essay to discuss the status of the standard model. What are its present weaknesses? Express your own views on whether a full understanding of the interactions between matter and energy is possible. Popular books that explore this topic have been written by Stephen Weinberg, Murray Gell-Mann, and Leon Lederman, and will help you to frame your argument.

Problems for Understanding

42. Relativistic speeds are speeds at which relativistic effects become noticeable. Just how fast is this? To answer the question, determine the following.
- **(a)** At what speed relative to your frame of reference would a particle have to travel so that you would see that its length in the direction of motion had decreased by 1.0%?
- **(b)** At what speed relative to your frame of reference would a particle have to travel for you to detect that its mass had increased by 0.10%?

43. **(a)** How much energy would be released if a 1.0 kg brick was converted directly into energy?
- **(b)** For how long could this amount of energy power a 100 W light bulb?

44. The star Alpha Centauri is 4.2 light-years away (a light-year is the distance light travels in one year: 365.25 days).
- **(a)** If you travelled in a spaceship at a speed of 2.0×10^8 m/s, how long would this distance appear to be?
- **(b)** How long would a one-way trip take you?
- **(c)** How much time would pass for someone back on Earth?

45. **(a)** Calculate the energy required to give an electron a speed of $0.90c$, starting from rest.
- **(b)** Compare this to its rest mass energy.
- **(c)** In terms of its rest mass, what is the mass of an electron travelling at this speed?

46. Suppose you allowed a 100 W light bulb to burn continuously for one year.
- **(a)** How much energy would it radiate in this time?
- **(b)** To what change in mass does this correspond?

47. Radiation of wavelength 362 nm is incident on a potassium surface. What will be the maximum kinetic energy of the electrons emitted from this surface? (Refer to Table 12.1 on page 509.)

48. Calculate the maximum kinetic energy of the electrons emitted from the cathode emitter of a photocell if the stopping potential is 4.7 V.

49. **(a)** A zinc surface is used on the emitter of a photocell. What will be the threshold frequency necessary for a photocurrent to flow? (See Table 12.1 on page 509)
- **(b)** What is the threshold wavelength for zinc?

50. **(a)** Calculate the de Broglie wavelength of an electron moving with a speed of 5.82×10^5 m/s.
- **(b)** An electron is accelerated across an electric potential difference of 64.0 V. Calculate the de Broglie wavelength of this electron.

51. An electron drops from the second energy level of the hydrogen atom to the first energy level.
- **(a)** Calculate the frequency of the photon emitted.
- **(b)** Calculate the wavelength of the photon.
- **(c)** In which series does the spectral line belong?

52. Calculate the wavelength of the second line in the Balmer series.

53. A typical classroom helium-neon laser has a power of 0.80 mW and emits a monochromatic beam of red light of wavelength 670 nm.
- **(a)** Calculate the energy (in J) of each photon in the beam.
- **(b)** If the laser is left on for 5.0 min, how many photons will be emitted?

54. A photon of light is absorbed by a hydrogen atom in which the electron is already in the second energy level. The electron is lifted to the fifth energy level.

(a) What was the frequency of the absorbed photon?

(b) What was its wavelength?

(c) What is the total energy of the electron in the fifth energy level?

(d) Calculate the radius of the orbit representing the fifth energy orbit.

(e) If the electron subsequently returns to the first energy level in one "jump," calculate the wavelength of the corresponding photon to be emitted.

(f) In which region of the electromagnetic spectrum would the radiation be found?

55. A prediction of the lifetime of the Sun can be calculated by analyzing its observed rate of energy emission, 3.90×10^{26} J/s. (Hint: In making the following calculations, pay close attention to unit analysis.)

(a) Calculate the amount of energy released in the conversion of four protons to one helium nucleus: $4{}^{1}_{1}\text{H} \rightarrow {}^{4}_{2}\text{He} + 2{}^{0}_{1}\text{e}$.

(b) If the above is considered as one reaction, how many reactions must occur each second to produce the observed rate of energy emission?

(c) How much helium is produced during each reaction?

(d) How much helium is produced per second?

(e) Let the lifetime of the Sun be defined as the time it takes 10.0% of the Sun's total mass to be converted into helium. (You can make this assumption, since it is accepted that only the reactions in the Sun's core need be considered.) Calculate the Sun's lifetime in years.

56. The Sun's lifetime can also be determined by calculating the total energy available and dividing by the energy radiated per second.

(a) Calculate the mass defect for converting four protons into one helium nucleus.

(b) What fraction of the mass of the initial four protons does this mass defect represent? This is the fraction of the mass of each proton that is converted into energy.

(c) Suppose the Sun's entire mass (1.99×10^{30} kg) was composed of protons. What is the total energy available?

(d) Assume that only 10.0% of the Sun's mass of protons are available to undergo fusion and calculate the lifetime of the Sun in years. (The Sun radiates 3.90×10^{26} J/s.)

57. Consider a sample of rock that solidified with Earth 4.55×10^{9} years ago. If it contains N atoms of uranium-235 (half-life: 7.04×10^{8} a), how many atoms were in the rock when it solidified?

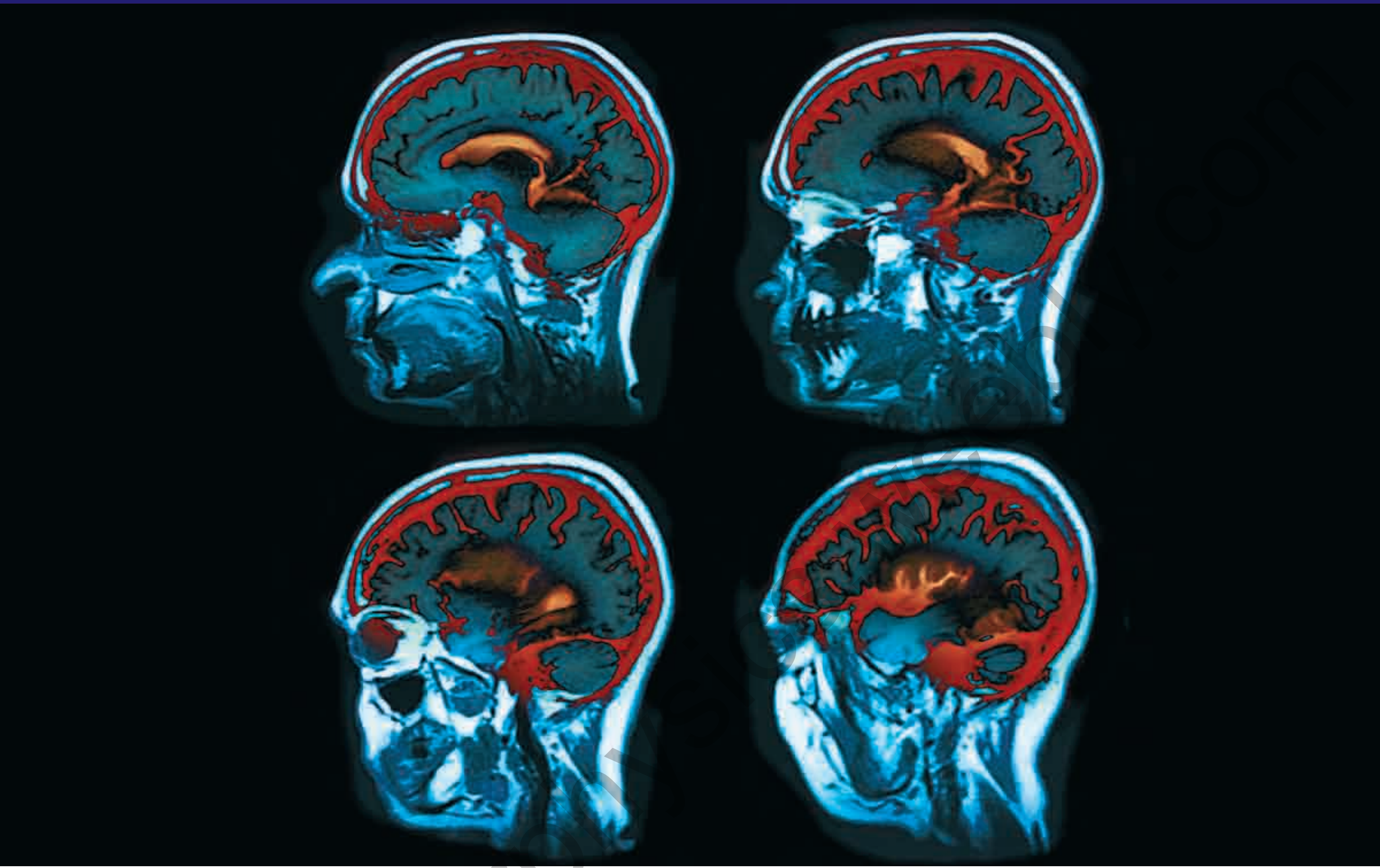
58. In the very early universe, protons and antiprotons existed with gamma rays. What is the minimum gamma ray energy required to create a proton-antiproton pair? To what wavelength does this amount of energy correspond?

COURSE CHALLENGE

Scanning Technologies: Today and Tomorrow

Consider the following as you complete the final information-gathering stage for your end-of-course project.

- Attempt to combine concepts from this unit with relevant topics from previous units.
- Verify that you have a variety of information items, including concept organizers, useful Internet sites, experimental data, and unanswered questions to help you create an effective final presentation.
- Scan magazines, scientific journals, and the Internet for interesting information to validate previously identified content and to enhance your project.



These magnetic resonance imaging (MRI) scans reveal four profile views at different depths of a healthy human brain. The folded cerebral cortex — associated with thought processes — is highlighted in red.

Scanning Technologies: Today and Tomorrow

An X-ray image of a tooth or broken bone is commonplace, and ultrasound images of a developing fetus are a regular part of prenatal care. Without the need for a single incision, various forms of non-invasive imaging technology provide clear images of the soft tissues of our bodies. Imaging technology also exposes the contents of locked luggage during airport security checks. Satellites circle Earth, relaying data about geological changes, volcanoes, hurricanes, and crop and vegetation densities.

Understanding the fundamental properties of matter, fields, waves, and energy has opened the door to hundreds of scanning technologies, and continuing research results in yet more scanning methods and continues to push the capabilities of these technologies to new heights. Research costs money, however, and is very time-consuming. Are these new scanning techniques worth the expense and time involved?

ASSESSMENT

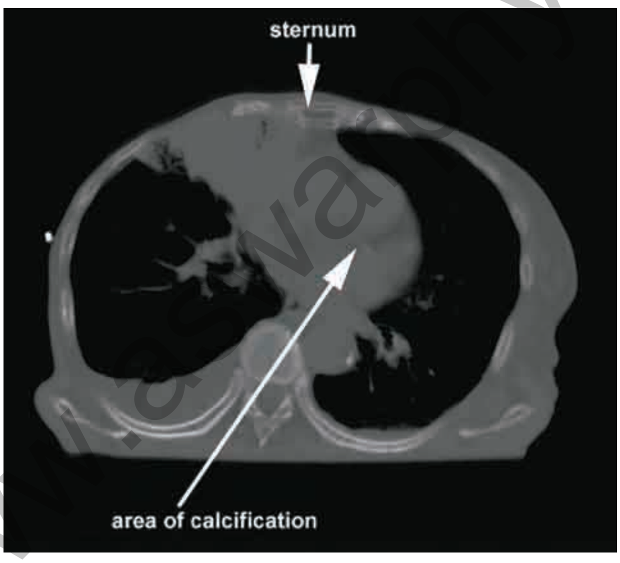
After you complete this Course Challenge, you will be assessed on

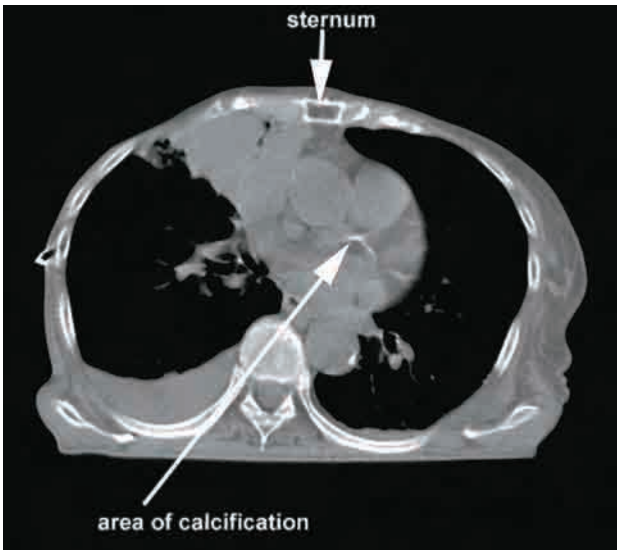
- **the quality of your research**
- **the accuracy and depth of your understanding**
- **your presentation**
- **other criteria you decide on as a class**

This Course Challenge prompts you to examine the costs and benefits of imaging technologies to both the scientific community and society. To help you get started, three fields of scanning technology and some associated issues are presented here.

Medical Issues

Doctors and politicians are often criticized when professional athletes gain access to magnetic resonance imaging (MRI) diagnosis immediately after sustaining an injury, while the general public must often wait months. Questions arise about the real expense of MRI equipment, its availability, and the value of the results as compared to other methods. How does an MRI machine work? What fundamental principles of nature does it exploit? Why is MRI scanning so expensive? Will the costs reduce with time? Will the technology improve with time? Are there better, less expensive options that should be pursued? Will this technology ever be made available to citizens of developing nations? To develop an argument supporting continued use of and research into MRI technology, you need to be able to answer these and other questions.





Motion, such as a beating heart or breathing, causes a blurring of conventional computerized tomography (CT) scan images. New computer technology, involving millions of frames of reference calculations, is able to remove the blur and produce much clearer images.

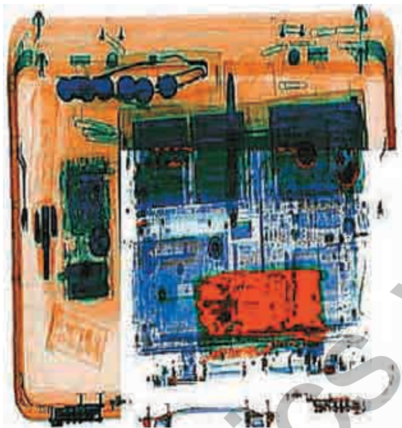
Photos courtesy of Dr. S. Stergiopoulos, Defence R&D Canada - Human Sciences, Toronto, Canada

PHYSICS FILE

A remote mountaintop in western North America rose 10 cm in four years, from 1996 to 2000. The 10 cm bulge is at the centre of a circle with a 12 km radius, and is only a few kilometres from the South Sister, a volcano that erupted 2000 years ago. The bulge is believed to be the result of growing pressure in an enormous chamber of magma beneath the mountain, and a telltale sign of potential volcanic activity. The 10 cm shift is a very small indicator of the tremendous amount of energy behind it, and would never have been detected without the remote-sensing ability of Earth-orbiting satellites.

Security Issues

Border-crossing and airport security often rely on technology to solve problems associated with screening large numbers of people and baggage in an efficient way. Some debate the effectiveness of the technological solutions compared to the enormous costs required to install and maintain the equipment. Opponents suggest that a human work force could do a more thorough and efficient job. Developing an argument supporting either side of this debate requires an in-depth understanding of the technology and its capabilities, and perhaps even a sense of its future potential.



A security imaging system can be set to detect the presence of explosives, narcotics, currency, or gold. In this case, the computer analyzed the contents of a laptop computer case and identified explosive material, indicated by the bright red area in this scan image.

Space Issues

Earth-orbiting, satellite-scanning technologies are used for environmental data collection, which is required for the development of sustainable agricultural, industrial, and even population-settlement plans. Weather satellites have allowed meteorologists to dramatically improve their forecasts. Surveillance satellites provide governments with information about covert operations.

A wealth of information comes from space, but the launching and maintaining of satellites is extremely expensive. World citizens need to be convinced that the economic costs associated with space-based research and related technologies are worth the rewards.

The Canadian Space Agency (CSA) and the U.S. National Aeronautics and Space Agency (NASA) devote a substantial amount of effort to global education, providing evidence of the benefits of space-related research. These agencies also work diligently to include other nations in large projects, such as the International Space Station. The CSA and NASA also recognize that projects must offer the global business community financial opportunities, as well as knowledge, to be successful in the long term. Does the commercialization of space fit with your vision of the future?

Debating which technologies are worth the investment of monetary and human resources can be accomplished only when all of the facts are known. Think about these questions as you undertake this Course Challenge.

Challenge

Develop and present a case either for or against the use of a particular scanning technology. You will use the knowledge and concepts you have acquired throughout this course, along with additional research, to develop your presentation about the economic, social, or environmental viability of a medical, industrial, or environmental scanning technology. Your class will decide together whether the presentations will be made through

- a formal debate
- research report presentations (either as a written report, an audiovisual presentation, or an information billboard)
- another format of your choice

Materials

All presentations are to be supported by your portfolio of research findings, the results of supporting experiments conducted, and a complete bibliography of references used.

Design Criteria

A. You need to develop a system to collect and organize information that will include data, useful mathematical relationships, and even questions that you use to formulate your final presentation near the end of the course. You can collect your own rough notes in a research portfolio.

B. Building a Research Portfolio
Your individual creativity will shape the amount, type, and organization of the material that will eventually fill your portfolio. Do not limit yourself to the items mentioned in the Course Challenge cues scattered throughout textbook; if something seems to fit, include it. The following are suggested items for your research portfolio.

- experiments you have designed yourself, and their findings
- useful equations
- specific facts
- interesting facts
- disputed facts
- conceptual explanations
- diagrams
- graphical organizers
- useful Internet site URLs
- experimental data
- unanswered questions
- pertinent economic or social statistics (Canadian or global)

C. As a class, decide on the type(s) of assessment you will use for your portfolio and for its presentation. Working with your teacher and classmates, select which type of presentation you will use to present your scanning technology arguments.

Action Plan

1. As a class, have a brainstorming session to establish what you already know and to raise questions about various scanning technologies that are currently being used or researched today. For example, what medical value does an MRI offer over other diagnostic methods, and is that difference worth the economic price? How widely available is MRI technology in **(a)** Canada or **(b)** other parts of the developed or underdeveloped world?
2. As a class, design an evaluation scheme, such as a rubric or rubrics for assessing the task. You could decide to assess specific components leading up to the final presentation, as well as the presentation itself.
3. Decide on the grouping, or assessment categories, for this task.
4. Familiarize yourself with what you need to know about the task that you choose. For example, if you choose a debate, it is important to research the proper rules of debating in order to carry out the debate effectively.
5. Develop a plan to find, collect, and organize in your research portfolio the information that is critical to your presentation.
6. Carry out the Course Challenge recommendations that are interspersed throughout the textbook wherever the Course Challenge logo and heading appear, and keep an accurate record of these in your portfolio.
7. When researching concepts, designing experiments or surveys, or following a Course Challenge suggestion in the textbook, you might find that the McGraw-Hill Ryerson Internet site is a good place to begin: www.mcgrawhill.ca/links/physics12
8. Carry out your plan, making necessary modifications throughout the course.
9. Present your arguments to your class. Review each presentation against the assessment criteria that you decided on as a class.

Evaluate Your Challenge

1. Using the assessment criteria you have prepared, evaluate your work and presentation. How effectively did your portfolio and presentation support your arguments? Were others able to follow your line of reasoning, based on the evidence, results, and conclusions you presented? How would you revise your presentation?

2. Evaluate your classmates' Course Challenge presentations.
3. After analyzing the presentations of your classmates, what changes would you make to your own project if you had the opportunity to do it again? Provide reasons for your proposed changes.
4. How did the process required to complete this challenge help you to think about what you have learned in this course?

Background Information

The following sections provide ideas to consider. They are linked to topics covered in the course and relate to the Course Challenge cues in your textbook. Your arguments will be both strengthened and redirected as you gain knowledge from each unit in this course.

Unit 1 Forces and Motion: Dynamics

Frames of Reference

Chapter 1, page 11

Describing motion in two and three dimensions requires the use of vector quantities. Consider the scanning technology that you have selected for investigation. How is an image obtained? Does the scanning machinery move, or does the item that is being scanned move? Does the technology detect motion or the change in orientation of atomic and subatomic particles? Analyze the scanning technology you are investigating from the perspective of frames of reference. Develop a comprehensive description detailing how an image is formed based on the location of particles in a two- or three-dimensional space.

Unit 2 Energy and Momentum

Momentum

Chapter 4, page 150

The conservation of momentum is the principle that allows navigation in space. Conservation of momentum is a fundamental property of our universe. Conservation of momentum applies to planetary, human, and subatomic levels. Investigate possible applications of momentum conservation used in the scanning technology that you are investigating. If the conservation of momentum applies only to atomic and subatomic interactions, you might want to complete your analysis during your study of Unit 5, Matter-Energy Interface, in the textbook.

Energy Transformations

Chapter 5, page 217

Producing scanned images requires very controlled energy transformations. Investigate the energy path used by the technology you have chosen to investigate. Answer questions such as: What energy is directed at the item to be scanned? Is energy absorbed, transmitted, or both? What energy transformations occur within the scanned item? What energy transformations occur at the scanning receiver? Support your presentation with quantitative energy transformation analysis. Is there an economic, social, or safety aspect relating your technology to energy transformation issues?

Unit 3 Electric, Gravitational, and Magnetic Fields

Contact versus Non-Contact

Chapter 7, page 275

You might want to compare contact versus non-contact forces. A century ago, a medical examination conducted to identify an abnormal growth would have involved physical contact, because the doctor used touch to assess the patient. Current medical examinations are able to obtain a much clearer picture of an abnormal growth inside the body without ever coming into direct contact with the patient. Consider the scanning technology you have chosen in these terms.

Field Energy

Chapter 8, page 356

Ultimately, the energy stored in fields will be the basis for the operation of any scanning technology. Satellite-based technologies orbit Earth, held in position by the gravitational field. Medical scans employ powerful magnetic fields to obtain diagnostic imagery. Investigate how fields play a role in the production of images in the technology that you are investigating. You might want to consider your technology in terms of a quantitative application of Coulomb's law.

Unit 4 The Wave Nature of Light

How Far Can It Go?

Chapter 10, page 445

Energy transported in the form of oscillating electric and magnetic fields is the fundamental method used in most scanning technologies. This textbook provides an introduction to some of these applications in Chapter 10, Section 10.2, The Electromagnetic Spectrum. Consider those discussions while you complete your analysis. You might want to direct your arguments in terms of past and future scientific developments. What has been accomplished? What new research is taking place? Are you able to predict how scanning technology might change in the next five years? Monetary and social arguments fit naturally into discussions based on possible changes in the field.

Unit 5 Matter-Energy Interface

Waves and Particles

Chapter 12, page 531

Scientific models evolve when theories are modified and validated by new experimental results. Physicists realize that electromagnetic radiation can be fully described only by using two completely different scientific models. Models are made by humans and therefore change as more knowledge is acquired. You might be able to demonstrate that a complete description of your chosen scanning technology requires both the wave and particle nature of electromagnetic radiation.

Nuclear Energy

Chapter 13, page 574

Nuclear energy provides electrical power not only to our homes, but also to most of the satellites orbiting overhead. Nuclear energy is used to probe living tissue in a variety of medical scanning technologies. Investigate nuclear decay rates of various materials and how they relate to your scanning technology. You might want to introduce safety and societal issues related to the use of nuclear material in the technology that you are investigating.

Wrap-Up

These ideas and questions are provided to help you develop your arguments related to a specific scanning technology. The ultimate shape of your presentation will be determined by the technology you choose to investigate, the issues you choose to address, and your own creativity. In order to prepare a high-quality, in-depth presentation, you will need to limit the amount of information that you attempt to present, focussing on the key points. Attempt to support your ideas with experimental evidence, mathematical verification, and comparisons to accepted scientific models. Give your project added relevance by relating your topic to key societal issues, such as economic or safety considerations.

Use your Course Challenge presentation to assist your learning by drawing together topics from each unit of study. As is often the case with any issue, the quality of discussion improves when knowledgeable links are made between topics.

Precision, Error, and Accuracy

A major component of the scientific inquiry process is the comparison of experimental results with predicted or accepted theoretical values. In conducting experiments, you must realize that all measurements have a maximum degree of certainty, beyond which there is uncertainty. The uncertainty, often referred to as "error," is not a result of a mistake, but rather, it is caused by the limitations of the equipment or the experimenter. The best scientist, using all possible care, could not measure the height of a doorway to a fraction of a millimetre accuracy using a metre stick. The uncertainty introduced through measurement must be communicated using specific vocabulary. Experimental results can be characterized by both their accuracy and their precision.

Precision describes the exactness and repeatabilty of a value or set of values. A set of data could be grouped very tightly, demonstrating good precision, but not necessarily be accurate. The darts in illustration (A) missed the bull's-eye and yet are tightly grouped, demonstrating precision without accuracy.

A B C

Differentiating between accuracy and precision

Accuracy describes the degree to which the result of an experiment or calculation approximates the true value. The darts in illustration (B) missed the bull's-eye in different directions, but are all relatively the same distance away from the centre. The darts demonstrate three throws that share approximately the same accuracy, with limited precision.

The darts in illustration (C) demonstrate accuracy and precision.

Random Error

- Random error results from small variations in measurements due to randomly changing conditions (weather, humidity, quality of equipment, level of care, etc.).
- Repeating trials will reduce but never eliminate random error.
- Random error is unbiased.
- Random error affects precision.

Systematic Error

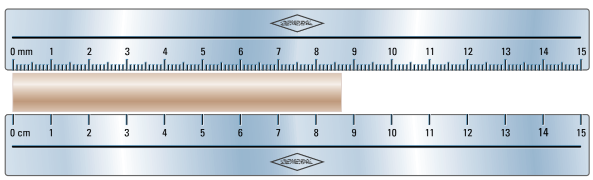
- Systematic error results from consistent bias in observation.
- Repeating trials will not reduce systematic error.
- Three types of systematic error are natural error, instrument-calibration error, and personal error.
- Systematic error affects accuracy.

Error Analysis

Error exists in every measured or experimentally obtained value. The error could deal with extremely tiny values, such as wavelengths of light, or with large values, such as the distances between stars. A practical way to illustrate the error is to compare it to the specific data as a percentage.

Relative Uncertainty

Relative uncertainty calculations are used to determine the error introduced by the natural limitations of the equipment used to collect the data. For instance, measuring the width of your textbook will have a certain degree of error due to the quality of the equipment used. This error, called "estimated uncertainty," has been deemed by the scientific community to be half of the smallest division of the measuring device. A metre stick with only centimetres marked would have an error of ±0.5 cm. A ruler that includes millimetre divisions would have a smaller error of ±0.5 mm. The measure should be recorded showing the estimated uncertainty, such as 21.00 ± 0.5 cm. Use the relative uncertainty equation to convert the estimated uncertainty into a percentage of the actual measured value.



Estimated uncertainty is accepted to be half of the smallest visible division. In this case, the estimated uncertainty is ±0.5 mm for the top ruler and ±0.5 cm for the bottom ruler.

$$\text{relative uncertainty} = \frac{\text{estimated uncertainty}}{\text{actual measurement}} \times 100\%$$

Example:

Converting the error represented by 21.00 ± 0.5 cm to a percentage

$$\text{relative uncertainty} = \frac{0.05\ \cancel{\text{cm}}}{21.00\ \cancel{\text{cm}}} \times 100\%$$

$$\text{relative uncertainty} = 0.2\%$$

Percent Deviation

In conducting experiments, it frequently is unreasonable to expect that accepted theoretical values can be verified, because of the limitations of available equipment. In such cases, percent deviation calculations are made. For instance, the standard value for acceleration due to gravity on Earth is 9.81 m/s^2 toward the centre of Earth in a vacuum. Conducting a crude experiment to verify this value might yield a value of 9.6 m/s^2. This result deviates from the accepted standard value. It is not necessarily due to error. The deviation, as with most high school experiments, might be due to physical differences in the actual lab (for example, the experiment might not have been conducted in a vacuum). Therefore, deviation is not necessarily due to error, but could be the result of experimental conditions that should be explained as part of the error analysis. Use the percent deviation equation to determine how close the experimental results are to the accepted or theoretical value.

$$\text{percent deviation} = \left|\frac{\text{experimental value} - \text{theoretical value}}{\text{theoretical value}}\right| \times 100\%$$

Example:

$$\text{percent deviation} = \frac{\left|9.6\ \cancel{\frac{\text{m}}{\text{s}^2}} - 9.8\ \cancel{\frac{\text{m}}{\text{s}^2}}\right|}{9.8\ \cancel{\frac{\text{m}}{\text{s}^2}}} \times 100\%$$

$$\text{percent deviation} = 2\%$$

Percent Difference

Experimental inquiry does not always involve an attempt at verifying a theoretical value. For instance, measurements made in determining the width of your textbook do not have a theoretical value based on a scientific theory. You still might want to know, however, how precise your measurements were. Suppose you measured the width 100 times and found that the smallest width measurement was 20.6 cm, the largest was 21.4 cm, and the average measurement of all 100 trials was 21.0 cm. The error contained in your ability to measure the width of the textbook can be estimated using the percent difference equation.

$$\text{percent difference} = \frac{\text{maximum difference in measurements}}{\text{average measurement}} \times 100\%$$

Example:

$$\text{percent difference} = \frac{(21.4\ \cancel{\text{cm}} - 20.6\ \cancel{\text{cm}})}{21.0\ \cancel{\text{cm}}} \times 100\%$$

$$\text{percent difference} = 4\%$$

SET 1 Skill Review

1. In Sèvres, France, a platinum–iridium cylinder is kept in a vacuum under lock and key. It is the standard kilogram with mass 1.0000 kg. Imagine you were granted the opportunity to experiment with this special mass, and obtained the following data: 1.32 kg, 1.33 kg, and 1.31 kg. Describe your results in terms of precision and accuracy.
2. You found that an improperly zeroed triple-beam balance affected the results obtained in question 1. If you used this balance for each measure, what type of error did it introduce?
3. Describe a fictitious experiment with obvious random error.
4. Describe a fictitious experiment with obvious systematic error.
5. **(a)** Using common scientific practice, find the estimated uncertainty of a stopwatch that displays up to a hundredth of a second.
   (b) If you were to use the stopwatch in part (a) to time repeated events that lasted less than 2.0 s, could you argue that the estimated uncertainty from part (a) is not sufficient? Explain.

Rounding, Scientific Notation, and Significant Digits

When working with experimental data, follow basic rules to ensure that accuracy and precision are not either overstated or compromised. Consider the 100 m sprint race. Several people using different equipment could have timed the winner of the race. The times might not agree, but would all be accurate within the capability of the equipment used.

Sprinter's Time with Different Devices

Time (s)	Estimated error of device (s)	Device
11.356	±0.000 5	photogate timer
11.36	±0.005	digital stopwatch
11.4	±0.05	digital stopwatch
11	±0.5	second hand of a dial watch

Using the example of the 100 m race, you will solidify ideas you need to know about exact numbers, number precision, number accuracy, and significant digits.

Exact Numbers If there were eight competitors in the race, then the number 8 is considered to be an exact number. Whenever objects are counted, number accuracy and significant digits are not involved.

Number Precision If our race winner wants a very precise value of her time, she would want to see the photogate result. The electronic equipment is able to provide a time value accurate to 1/1000th of a second. The time recorded using the second hand on a dial watch is not able to provide nearly as precise a value.

Number Accuracy and Significant Digits The race winner goes home to share the good news. She decides to share the fastest time with her family. What timing method does she share? She would share the 11 s time recorded using the second hand of a dial watch. All of the other methods provide data that has her taking a longer time to cross the finish line. Is the 11 s value accurate?

The 11 s value is accurate to within ±0.5 s, following common scientific practice of estimating error. The 11.356 s time is accurate to within 0.0005 s. The photogate time is simply more precise. It would be inaccurate to write the photogate time as 11.356 00 s. In that case, you would be adding precision that goes beyond the ability of the equipment used to collect the data, as the photogate method can measure time only to the thousandths of a second. Scientists have devised a system to help ensure that number accuracy and number precision are maintained. It is a system of significant digits, which requires that the precision of a value does not exceed either (a) the precision of the equipment used to obtain it or (b) the least precise number used in a calculation to determine the value. The table on the left provides the number of significant digits for each measurement of the sprinter's times.

There are strict rules used to determine the number of significant digits in a given value.

When Digits Are Significant ✔

1. All non-zero digits are significant (159 — three significant digits).
2. Any zeros between two non-zero digits are significant (109 — three significant digits).
3. Any zeros to the right of *both* the decimal point and a non-zero digit are significant (1.900 — four significant digits).
4. All digits (zero or non-zero) used in scientific notation are significant.

When Digits Are Not Significant ✘

1. Any zeros to the right of the decimal point but preceding a non-zero digit are not significant; they are placeholders. For example, 0.000 19 kg = 0.19 g (two significant digits).
2. Ambiguous case: Any zeros to the right of a non-zero digit are not significant; they are placeholders (2500 — two significant digits). If the zeros are intended to be significant, then scientific notation must be used. For example, 2.5×10^3 (two significant digits) and 2.500×10^3 (four significant digits).

Calculations and Accuracy As a general rule, accuracy is maintained though mathematical calculations by ensuring that the final answer has the same number of significant digits as the least precise number used during the calculations.

Example:

Find the product of these lengths.

12.5 m 16 m 15.88 m

Product = 12.5 m × 16 m × 15.88 m

Product = 3176 m^3

Considering each data point, notice that 16 has only two significant digits; therefore the answer must be shown with only two significant digits.

Total length = 3.2×10^3 m (two significant digits)

Rounding to Maintain Accuracy It would seem that rounding numbers would introduce error, but in fact, proper rounding is required to help maintain accuracy. This point can be illustrated by multiplying two values with differing numbers of significant digits. As you know, the right-most digit in any data point contains some uncertainty. It follows that any calculations using these uncertain digits will yield uncertain results.

> Multiply 32 and 13.55. The last digit, being the most uncertain, is highlighted.
>
> 13.55
> × 32
> 2710 Each digit in this line is obtained using an uncertain digit.
> 4065 In this line only the 5 is obtained using uncertain digits.
> 433.60
>
> The product 433.60 should be rounded so that the last digit shown is the only one with uncertainty. Therefore, 4.3×10^2.

Notice that this value contains two significant digits, which follows the general rule.

Showing results of calculations with every digit obtained actually introduces inaccuracy. The number would be represented as having significantly more precision than it really has. It is necessary to round numbers to the appropriate number of significant digits.

Rounding Rules When extra significant digits exist in a result, rounding is required to maintain accuracy. Rounding is not simply removing the extra digits. There are three distinct rounding rules.

1. **Rounding Down**
   When the digits dropped are less than 5, 50, 500, etc., the remaining digit is left unchanged.

   Example:

   4.123 becomes
   4.12 rounding based on the "3"
   4.1 rounding based on the "23"

2. **Rounding Up**
   When the digits dropped are greater than 5, 50, 500, etc., the remaining digit is increased or rounded up.

   Example:

   4.756 becomes
   4.76 rounding based on the "6"
   4.8 rounding based on the "56"

3. **Rounding with 5, 50, 500, etc.**
   When the digits dropped are exactly equal to 5, 50, 500, etc., the remaining digit is rounded to the *closest even number*.

   Example:

   4.850 becomes
   4.8 rounding based on "50"
   4.750 becomes
   4.8 rounding based on "50"

Always carry extra digits throughout a calculation, rounding only the final answer.

Scientific Notation Numbers in science are sometimes very large or very small. For example, the distance from Earth to the Sun is approximated as 150 000 000 000 m and the wavelength of red light is 0.000 000 65 m. Scientific notation allows a more efficient method of writing these types of numbers.

- Scientific notation requires that a single digit between 1 and 9 be followed by the decimal and all remaining significant digits.
- The number of places the decimal must move determines the exponent.
- Numbers greater than 1 require a positive exponent.
- Numbers less than 1 require a negative exponent.
- Only significant digits are represented in scientific notation.

Example:

1 5 0 0 0 0 0 0 0 0 0 0 . becomes 1.5×10^{11} m

0.0 0 0 0 0 0 6.5 becomes 6.5×10^{-7} m

continued ►

Skill Set 2 continued from previous page

SET 2 Skill Review

1. There are a dozen apples in a bowl. In this case, what type of number is 12?
2. Put the following numbers in order from most precise to least precise.
   (a) 3.2, 5.88, 8, 8.965, 1.000 08
   (b) 6.22, 8.5, 4.005, 1.2000×10^{-8}
3. How many significant digits are represented by each value?
   (a) 215
   (b) 31
   (c) 3.25
   (d) 0.56
   (e) 1.06
   (f) 0.002
   (g) 0.006 04
   (h) 1.250 000
   (i) 1×10^{6}
   (j) 3.8×10^{4}
   (k) 6.807×10^{58}
   (l) 3.000×10^{8}
4. Round the following values to two significant digits.
   (a) 1.23
   (b) 2.348
   (c) 5.86
   (d) 6.851
   (e) 6.250
   (f) 4.500
   (g) 5.500
   (h) 9.950
5. Complete the following calculations. Provide the final answer to the correct number of significant digits.
   (a) 2.358×4.1
   (b) $102 \div 0.35$
   (c) $2.1 + 5.88 + 6.0 + 8.526$
   (d) $12.1 - 4.2 - 3$
6. Write each of the following in scientific notation.
   (a) 2.5597
   (b) 1000
   (c) 0.256
   (d) 0.000 050 8
7. Write each value from question 6 in scientific notation accurate to three significant digits.

Skill Set 3

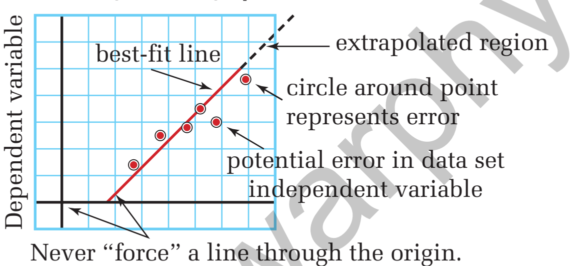
Drawing and Interpreting Graphs

Graphical analysis of scientific data is used to determine trends. Good communication requires that graphs be produced using a standard method. Careful analysis of a graph could reveal more information than the data alone.

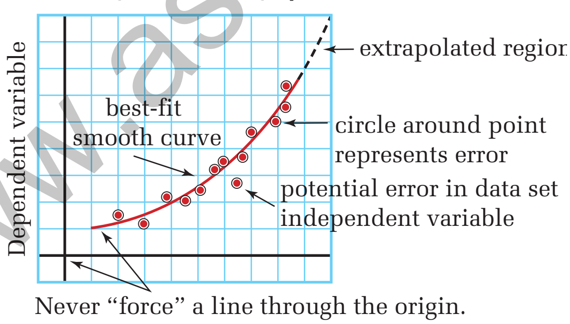
Standards for Drawing a Graph

- Independent variable is plotted along the horizontal axis (include units).
- Dependent variable is plotted along the vertical axis (include units).
- Decide whether the origin (0,0) is a valid data point.
- Select convenient scaling on the graph paper that will spread the data out as much as possible.
- A small circle is drawn around each data point to represent possible error.
- Determine a trend in the data — draw a best-fit line or best-fit smooth curve. Data points should never be connected directly when finding a trend.
- Select a title that clearly identifies what the graph represents.

Constructing a linear graph



Constructing a non-linear graph



Interpolation and Extrapolation

A best-fit line or best-fit smooth curve that is extended beyond the size of the data set should be shown as a dashed line. You are extrapolating values when you read them from the dashed-line region of the graph. You are interpolating values when you read them from the solid-line region of the graph.

Find a Trend

The best-fit line or smooth curve provides insight into the type of relationship between the variables represented in a graph.

A *best-fit line* is drawn so that it matches the general trend of the data. You should try to have as many points above the line as are below it. Do not cause the line to change slope dramatically to include only one data point that does not seem to be in line with all of the others.

A *best-fit smooth curve* should be drawn so that it matches the general trend of the data. You should try to have as many points above the line as are below it, but ensure that the curve changes smoothly. Do not cause the curve to change direction dramatically to include only one data point that does not seem to be in line with all of the others.

Definition of a Linear Relationship

A data set that is most accurately represented with a *straight line* is said to be linear. Data related by a linear relationship can be written in the form

$$y = mx + b$$

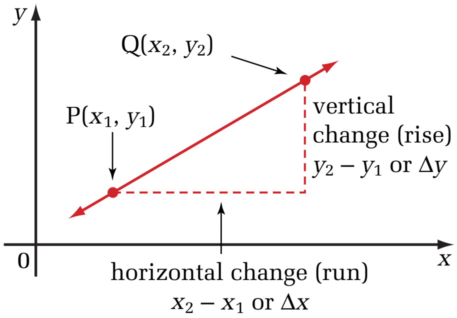
Quantity	Symbol	SI unit
y value (dependent variable)	y	obtained from the vertical axis
x value (independent variable)	x	obtained from the horizontal axis
slope of the line	m	rise/run
y-intercept	b	obtained from the vertical axis when x is zero

continued ►

Skill Set 3 continued from previous page

Slope (m)

Calculating the slope of a line



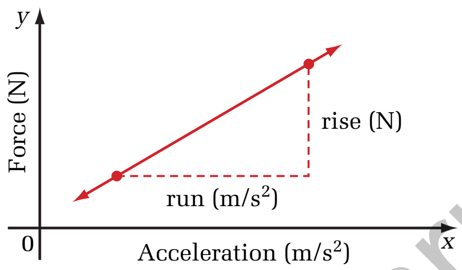
$$\text{slope } (m) = \frac{\text{vertical change (rise)}}{\text{horizontal change (run)}}$$

$$m = \frac{\Delta y}{\Delta x}$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}, x_2 \neq x_1$$

Mathematically, slope provides a measure of the steepness of a line by dividing the vertical change (rise) by the horizontal change (run). In scientific situations, it is also very important to include units of the slope. The units will provide physical significance to the slope value.

For example:



Including the units throughout the calculation helps verify the physical quantity that the slope represents.

$$m = \frac{\text{rise (N)}}{\text{run (m/s}^2\text{)}} \quad \text{Recall : 1 N = 1 kg} \cdot \text{m/s}^2$$

$$m = \frac{\text{kg} \cdot \cancel{\text{m/s}^2}}{\cancel{\text{m/s}^2}}$$

$$m = \text{kg}$$

In this example, the slope of the line represents the physical quantity of mass.

Definition of a Non-Linear Relationship

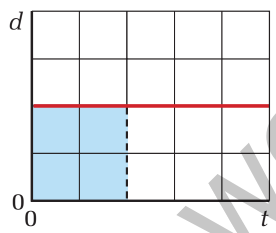
A data set that is most accurately represented with a smooth curve is said to be non-linear. Data related by a non-linear relationship can take several different forms. Two common non-linear relationships are as follows.

(a) parabolic $y = ax^2 + k$

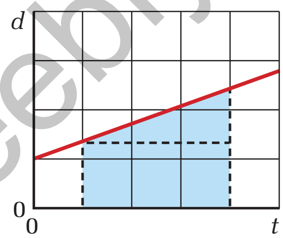
(b) inverse $y = \dfrac{1}{x}$

Area Under a Curve

Mathematically, the area under a curve can be obtained without the use of calculus by finding the area using geometric shapes.

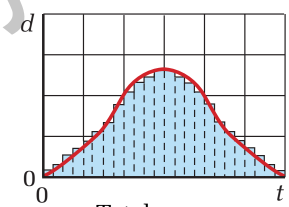


Total area = length × width

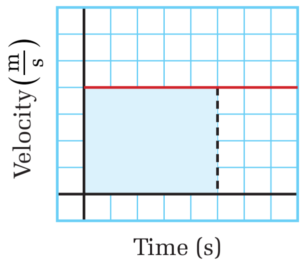


Total area =
area of the rectangle +
area of the triangle

Always include units in area calculations. The units will provide physical significance to the area value. For example, see below.



Total area =
area 1 + area 2 + area 3 …



Including the units throughout the calculation helps verify the physical quantity that the area represents.

Area = (length)(width)
Area = (velocity)(time)
Area = (m/s)(s)
Area = m (base unit for displacement)

The units verify that the area under a speed-versus-time curve represents displacement (m).

SET 3 Skill Review

1. (a) Plot the data in Table 1 by hand, ensuring that it fills at least two thirds of the page and has clearly labelled axes that include the units.
   (b) Draw a best-fit line through the plotted data.
   (c) Based on the data trend and the best-fit line, which data point seems to be most in error?
   (d) Interpolate the time it would take to travel 14 m.
   (e) Extrapolate to find how far the object would travel in 20 s.

Table 1

Time (s)	Distance (m)	Time (s)	Distance (m)
0	2	8	17
1	4	9	20
2	7	10	23
3	8	11	24
4	5	12	26
5	12	13	29
6	16	14	28
7	16	15	33

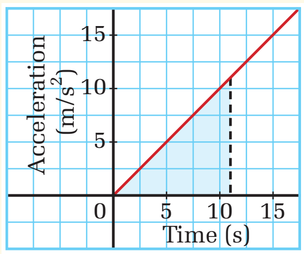
2. (a) Plot the data in Table 2 by hand, ensuring that it fills at least two thirds of the page and has clearly labelled axes that include the units.
   (b) Draw a best-fit smooth curve through the plotted data.
   (c) Does this smooth curve represent a linear or non-linear relationship?
   (d) At what force is the position at the greatest value?

Table 2

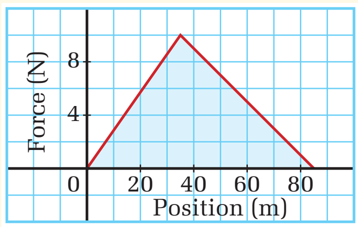
Force (N)	Position (m)	Force (N)	Position (m)
0	0.0	1.1	2.5
0.1	0.5	1.2	2.5
0.2	0.9	1.3	2.4
0.3	1.3	1.4	2.2
0.4	1.6	1.5	2.0
0.5	1.9	1.6	1.7
0.6	2.1	1.7	1.4
0.7	2.3	1.8	1.1
0.8	2.4	1.9	0.7
0.9	2.5	2	0.2
1	2.6		

3. Find the area of the shaded regions under the following graphs. Use the units to determine the physical quantity that the area represents.

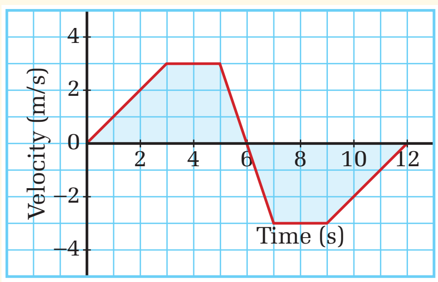
(a)



(b)



(c)



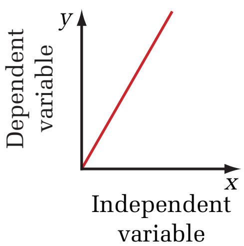
Mathematical Modelling and Curve Straightening

Patterns in quantitative data can be expressed in the form of mathematical equations. These relationships form a type of *mathematical model* of the phenomenon being studied. You can use the model to examine trends and to make testable numerical predictions.

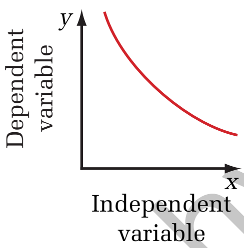
Identifying Types of Relationships

Graphing data is a common way of revealing patterns. Simply by drawing a best-fit curve through the data points, it might be possible to identify a general type of mathematical relationship expressed in the observations. Four common patterns are illustrated below. Each pattern can be expressed algebraically as a proportionality statement ($a \propto b$) or as an equation. In mathematics courses, you might also have studied the graphs and equations of logarithmic, sinusoidal, or other types of relationships.

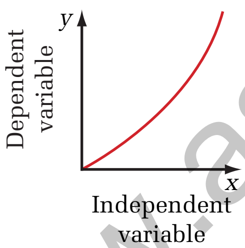
Basic mathematical relationships



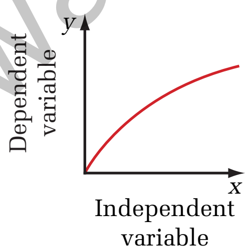
A linear
$y \propto x$
$y = kx$



B inverse
$y \propto \frac{1}{x^n}$ or $y \propto x^{-n}$
$y = kx^{-n}$



C exponential
$y \propto x^n$
$y = kx^n$

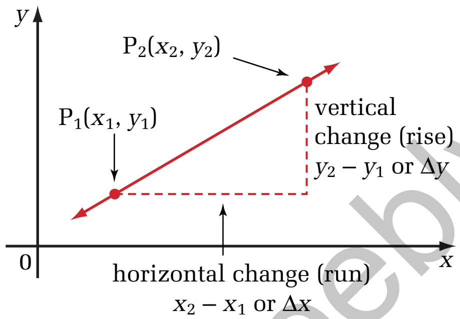


D root
$y \propto \sqrt[n]{x}$ or $y \propto x^{\frac{1}{n}}$
$y = kx^{\frac{1}{n}}$

Linear Relationships

In previous studies, you have used the straight-line graph of a linear relationship to produce a specific mathematical equation that represents the graph. The equation is completely determined by the slope, m, of the graph and its y-intercept, b.

The equation of a straight-line graph



$$\text{slope } (m) = \frac{\text{vertical change (rise)}}{\text{horizontal change (run)}} = \frac{\Delta y}{\Delta x}$$

$$m = \frac{y_2 - y_1}{x_2 - x_1},\ x_2 \neq x_1$$

Equation of the line: $y = mx + b$

Straightening Non-Linear Graphs

You can often produce a straight-line graph from a non-linear relationship by making an appropriate choice of independent variables for the graph. By analyzing the resulting straight line, you can obtain an equation that fits the data. This procedure, which is called "curve straightening," produces equations of the form

(quantity on the vertical axis =
m (quantity on the horizontal axis) + b

You can straighten a curve by selecting the quantity graphed on the horizontal axis to match the general type of variation shown by the data. If the independent variable is x and you suspect

- inverse variation: plot $\frac{1}{x}$ or $\frac{1}{x^2}$ or $\frac{1}{x^3}$ on the horizontal axis
- exponential variation: plot x^2 or x^3 on the horizontal axis
- root variation: plot $x^{\frac{1}{2}}$ or $x^{\frac{1}{3}}$ on the horizontal axis

There is no mathematical reason why other exponents could not be used. Most phenomena examined in this course, however, are best modelled using integer exponents or roots no greater than three.

Procedure

1. From a table of raw data for two variables, x and y, produce an initial graph of y versus x.
2. Identify the general type of relationship shown by the graph.
3. Modify the independent variable to suit the proposed type of relationship. Add the new quantity to your data table and then draw a new graph of y against this quantity derived from x.
4. If the new graph is a straight line, calculate its slope and y-intercept. Use these values to write and simplify an equation to represent the data.
5. If the new graph is not a straight line, repeat steps 3 and 4, using a different modification of the independent variable until you obtain a straight-line graph.

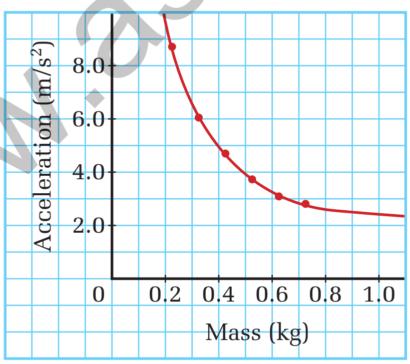
Example

A force of 1.96 N was used to accelerate a lab cart with mass 0.225 kg. The mass of the cart was then systematically increased, producing the accelerations shown below. Find an equation that represents this data.

Mass (kg)	Acceleration $\left(\frac{m}{s^2}\right)$
0.225	8.71
0.325	6.05
0.425	4.70
0.525	3.73
0.625	3.09
0.725	2.81

1. Graph the raw data.

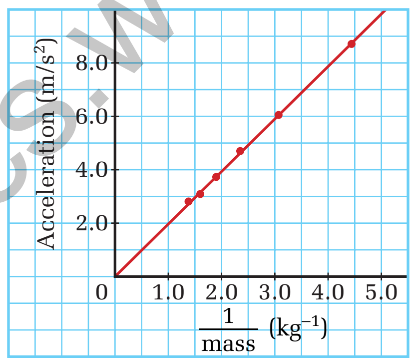
Acceleration against mass



2. Identify the type of variation. This graph shows inverse variation.
3. Modify the independent variable, extend the data table, and regraph the data. Choose the most simple possibility, $\frac{1}{\text{mass}}$, to investigate.

Mass (kg)	Acceleration $\left(\frac{m}{s^2}\right)$	$\frac{1}{\text{mass}}\left(\frac{1}{kg}\right)$
0.225	8.71	4.44
0.325	6.05	3.07
0.425	4.70	2.35
0.525	3.73	1.90
0.625	3.09	1.60
0.725	2.81	1.38

Acceleration against $\frac{1}{\text{mass}}$



4. Since the graph is a straight line, use its slope and y-intercept to obtain its equation.

   slope (m) = 1.96

   y-intercept (b) 0.0500

   Equation of the line

   (quantity on the vertical axis) = m (quantity on the horizontal axis) + b

   $$\text{acceleration} = 1.96\left(\frac{1}{\text{mass}}\right) + 0.0500$$

   $$\text{acceleration} = \frac{1.96}{m} + 0.0500$$

5. If the graph had not been a straight line, the next most simple variation of the independent variable would have been considered: $\frac{1}{(\text{mass})^2}$.
6. The equation determined from the data is reasonable, because the situation is an example of Newton's second law, $F = ma$. Solved for acceleration, this becomes $a = \frac{F}{m}$,

Skill Set 4 continued from previous page

or $a = F\left(\frac{1}{m}\right)$, which has the same form as the equation for the graph. The slope of the graph represents the force applied to the cart.
The y-intercept, 0.0500, is probably due to experimental error, as the graph should pass through the origin.

Points to Remember

Make sure that the scales on your graph axes start at zero. Otherwise, you will see a magnified view of only a small portion of the graph. The overall shape of the graph might not be shown, so it will be difficult to identify the type of variation in the data. Part of a gentle curve, for example, can appear to be a straight line.

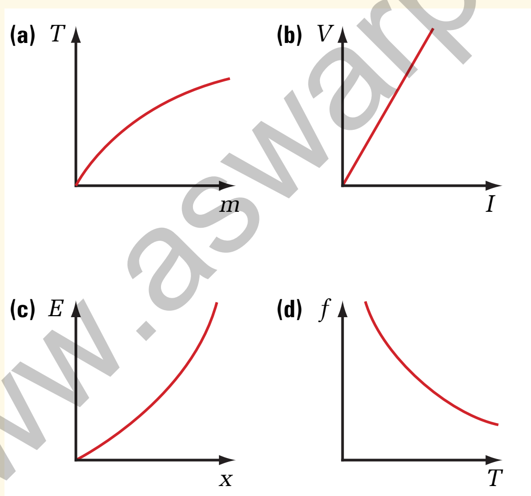
Sometimes it is a good idea to look at only part of a data set. The relationship between force applied to a spring and its extension (Hooke's law), for example, is linear, as long as the force does not exceed a certain value. Rather than trying to find a relationship that fits the entire graph, only the linear portion is usually considered. An initial graph of your data will show parts that are easy to model and will also reveal data points that are far from the best-fit line. Obvious errors are often excluded when developing equations to fit data.

Some computer programs are capable of automatic curve fitting. The resulting equations might fit experimental data well, without being very helpful. If you suspect that a certain phenomenon follows an inverse square law, for example, it would be sensible to choose $\frac{1}{(\text{dependent variable})^2}$ as the quantity to graph. Then you can determine how closely your observations approach an ideal model.

You might want to investigate other aspects of mathematical modelling. If you are familiar with logarithms, for example, consider the advantages and disadvantages of curve straightening by graphing, or the use of log or semi-log graph paper. You might also look into correlation coefficients, statistical measures that can express how well a given equation models a set of data. Finally, you might explore the use of power series to produce approximations of complex relationships.

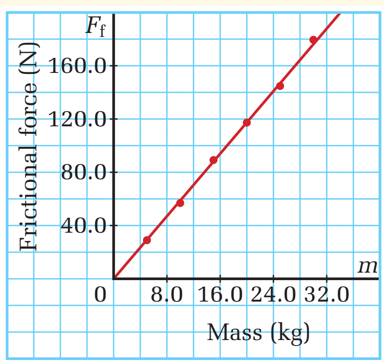
SET 4 Skill Review

1. Name the type of relationship represented by each graph below and write the relationship as a proportion and as a general equation.



2. The graph in the next column shows the force of static friction when you push different masses placed on a horizontal carpet and they start to move.
   (a) Determine the slope and intercept of the graph below.
   (b) Write an equation that represents the data.
   (c) Explain the physical meaning of each numerical coefficient in the equation.

Static friction between leather-soled shoes and a carpet



3. For each of the following relationships, use curve-straightening techniques to determine an equation that represents the data. If possible, validate your solution by giving physical reasons why the relationship must have the form it does.

(a) The gravitational attraction between two lead spheres in a Cavendish apparatus depends on the separation between their centres.

Separation (cm)	Gravitational force ($N \times 10^{-9}$)
55	3.13
70	1.93
85	1.31
100	0.946
115	0.715
130	0.560

(b) The buoyant force on a spherical weather balloon depends on how much the balloon is inflated (the volume of the balloon).

Radius of balloon (m)	Force (N)
2.285	569
2.616	855
2.879	1135
3.102	1422
3.296	1709
3.470	1993
3.628	2281

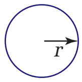
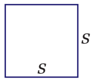
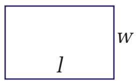
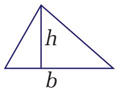
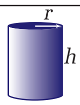
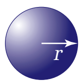
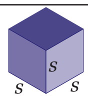
(c) The power dissipated by a light bulb is related to the electric current flowing through the light bulb.

Electrical current (A)	Power (W)
16.0	15
20.6	25
26.3	40
32.0	60
35.7	75
41.2	100

(d) The kinetic energy of a moving car depends on the car's velocity.

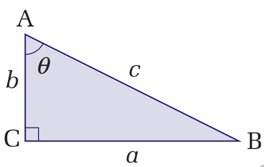
Velocity $\left(\frac{km}{h}\right)$	Kinetic energy (kJ)
15	15.4
25	42.7
35	83.8
45	139.4
55	208.9
65	289.8

A Math Toolbox

	Circumference/ perimeter	Area	Surface area	Volume
	$C = 2\pi r$	$A = \pi r^2$		
	$P = 4s$	$A = s^2$		
	$P = 2l + 2w$	$A = lw$		
		$A = \frac{1}{2}bh$		
			$SA = 2\pi rh + 2\pi r^2$	$V = \pi r^2 h$
			$SA = 4\pi r^2$	$V = \frac{4}{3}\pi r^3$
			$SA = 6s^2$	$V = s^3$

Trigonometric Ratios

The ratios of side lengths from a right-angle triangle can be used to define the basic trigonometric function sine (sin), cosine (cos), and tangent (tan).



$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin\theta = \frac{a}{c}$$

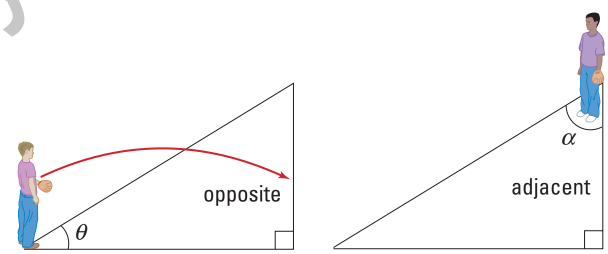
$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\cos\theta = \frac{b}{c}$$

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan\theta = \frac{a}{b}$$

The angle selected determines which side will be called the opposite side and which the adjacent side. The hypotenuse is always the side across from the 90˚ angle. Picture yourself standing on top of the angle you select. The side that is directly across from your position is called the *opposite* side. The side that you could touch and is not the hypotenuse is the *adjacent* side.



A scientific calculator or trigonometry tables can be used to obtain an angle value from the ratio result. Your calculator performs a complex calculation (Maclaurin series summation) when the $\sin^{-1}$, or $\cos^{-1}$, or $\tan^{-1}$ operation is used to determine the angle value. Sin^{-1} is not simply a 1/sin operation.

Definition of the Pythagorean Theorem

The Pythagorean theorem is used to determine side lengths of a right-angle (90°) triangle. Given a right-angle triangle ABC, the Pythagorean theorem states

$$c^2 = a^2 + b^2$$

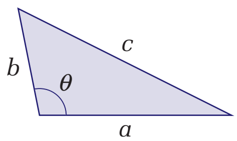
Quantity	Symbol	SI unit
hypotenuse side is opposite the 90°angle	c	m (metres)
side a	a	m (metres)
side b	b	m (metres)

Note: The hypotenuse is always the side across from the right (90°) angle. The Pythagorean theorem is a special case of a more general mathematical law called the "cosine law." The cosine law works for all triangles.

Definition of the Cosine Law

The cosine law is useful when

- determining the length of an unknown side given two side lengths and the contained angle between them
- determining an unknown angle given all side lengths



Angle θ is contained between sides a and b.

The cosine law states $\boldsymbol{c^2 = a^2 + b^2 - 2ab\cos\theta}$.

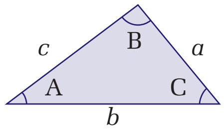
Quantity	Symbol	SI unit
unknown length side c opposite angle θ	c	m (metres)
length side a	a	m (metres)
length side b	b	m (metres)
angle θ opposite unknown side c	θ	(radians)

Note: Applying the cosine law to a right angle triangle, setting $\theta = 90°$, yields the special case of the Pythagorean theorem.

Definition of the Sine Law

The sine law is useful when

- two angles and any one side length are known
- two side lengths and any one angle are known



Given any triangle ABC the sine law states

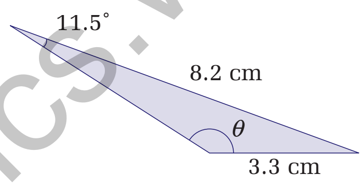
$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Quantity	Symbol	SI unit
length side a opposite angle A	a	m (metres)
length side b opposite angle B	b	m (metres)
length side c opposite angle C	c	m (metres)
angle A opposite side a	A	(radians)
angle B opposite side b	B	(radians)
angle C opposite side c	C	(radians)

Note: The sine law generates ambiguous results in some situations because it does not discriminate between obtuse and acute triangles. An example of the ambiguous case is shown below.

Example

Use the sine law to solve for θ.



Sine law: ambiguous case

$$\frac{\sin\theta}{8.2} = \frac{\sin 11.5°}{3.3}$$

$$\sin\theta = 0.5$$

$$\theta = 30°$$

Clearly, angle θ is much greater than 30°. In this case, the supplementary angle is required (180° – 30° = 150°). It is important to recognize when dealing with obtuse angles (> 90°) that the supplementary angle might be required. Application of the cosine law in these situations will help reduce the potential for error.

Algebra

In some situations, it might be preferable to use algebraic manipulation of equations to solve for a specific variable before substituting numbers. Algebraic manipulation of variables follows the same rules that are used to solve equations after substituting values. In both cases, to maintain equality, whatever is done to one side must be done to the other.

Skill Set 5 continued from previous page

Solving for "*x*" before Numerical Substitution

(a)

$A = kx$	x is multiplied by k, so divide by k to isolate x.
$\frac{A}{k} = \frac{\cancel{k}x}{\cancel{k}}$	Divide both sides of the equation by k.
$\frac{A}{k} = x$	Simplify.
$x = \frac{A}{k}$	Rewrite with x on the left side.

(b)

$B = \frac{x}{g}$	x is divided by g, so multiply by g to isolate x.
$Bg = \frac{x\cancel{g}}{\cancel{g}}$	Multiply both sides of the equation by g.
$Bg = x$	Simplify.
$x = Bg$	Rewrite with x on the left side.

(c)

$W = x + f$	x is added to f, so
$W - f = x + f - f$	subtract f to isolate x. Subtract f on both sides of the equation.
$W - f = x$	Simplify.
$x = W - f$	Rearrange for x.

(d)

$W = \sqrt{x}$	x is under a square root,
$W^2 = (\sqrt{x})^2$	so square both sides of the equation.
$W^2 = x$	Simplify.
$x = W^2$	Rearrange for x.

Solving for "*x*" after Numerical Substitution

(a)

$8 = 2x$	x is multiplied by 2, so divide by 2 to isolate x.
$\frac{8}{2} = \frac{\cancel{2}x}{\cancel{2}}$	Divide both sides of the equation by 2.
$4 = x$	Simplify.
$x = 4$	Rewrite with x on the left side.

(b)

$8 = \frac{x}{4}$	x is divided by 4, so multiply by 4 to isolate x.
$(10)(4) = \frac{4x}{4}$	Multiply both sides of the equation by 4.
$40 = x$	Simplify.
$x = 40$	Rewrite with x on the left-hand side.

(c)

$25 = x + 13$	x is added to 13, so subtract 13 to isolate x.
$25 - 13 = x + 13 - 13$	Subtract 13 from both sides of the equation.
$12 = x$	Simplify.
$x = 12$	Rewrite with x on the left-hand side.

(d)

$6 = \sqrt{x}$	x is under a square root, so
$6^2 = (\sqrt{x})^2$	square both sides of the equation.
$36 = x$	Simplify.
$x = 36$	Rewrite with x on the left-hand side.

Definition of the Quadratic Formula

The quadratic equation is used to solve for the roots of a quadratic function. Given a quadratic equation in the form $ax^2 + bx + c = 0$, where a, b, and c are real numbers and $a \neq 0$, the roots of it can be found using

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Statistical Analysis

In science, data are collected until a trend is observed. Three statistical tools that assist in determining if a trend is developing are *mean*, *median*, and *mode*.

Mean: The sum of the numbers divided by the number of values. It is also called the "average."

Median: When a set of numbers is organized in order of size, the median is the middle number. When the data set contains an even number of values, the median is the average of the two middle numbers.

Mode: The number that occurs most often in a set of numbers. Some data sets will have more than one mode.

See examples of these on the following page.

Example 1:
Odd number of data points

Data Set 1: 12, 11, 15, 14, 11, 16, 13

$$\text{mean} = \frac{12 + 11 + 15 + 14 + 11 + 16 + 13}{7}$$
$$\text{mean} = 13$$
$$\text{reorganized data} = 11, 11, 12, 13, 14, 15, 16$$
$$\text{median} = 13$$
$$\text{mode} = 11$$

Example 2:
Even number of data points

Data Set 2: 87, 95, 85, 63, 74, 76, 87, 64, 87, 64, 92, 64

$$\text{mean} = \frac{(87 + 95 + 85 + 63 + 74 + 76 + 87 + 64 + 87 + 64 + 92 + 64)}{12}$$
$$\text{mean} = 78$$
$$\text{reorganized data} = 63, 64, 64, 64, 74, 76, 85, 87, 87, 87, 92, 95$$
$$\text{median} = \frac{(76 + 85)}{2}$$
$$\text{median} = 80$$

An even number of data points requires that the middle two numbers be averaged.

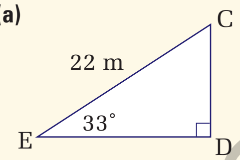
$$\text{mode} = 64, 87$$

In this example, the data set is bimodal (contains two modes).

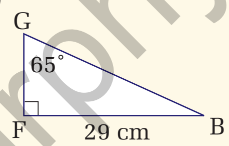
SET 5 Skill Review

1. Calculate the area of a circle with radius 6.5 m.
2. By how much does the surface area of a sphere increase when the radius is doubled?
3. By how much does the volume of a sphere increase when the radius is doubled?
4. Find all unknown angles and side lengths.

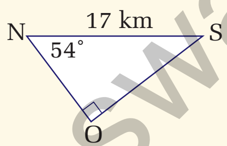
(a)



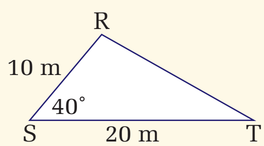
(b)



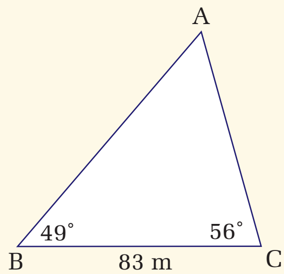
(c)



5. Use the cosine law to solve for the unknown side.



6. Use the sine law to solve for the unknown sides.



7. Solve for x in each of the following.
   (a) $42 = 7x$
   (b) $30 = x/5$
   (c) $12 = x \sin 30^\circ$
   (d) $8 = 2x - 12^4$
8. Solve for x in each of the following.
   (a) $F = kx$
   (b) $G = hk + x$
   (c) $a = bx \cos\theta$
   (d) $b = d \cos x$
   (e) $a = bc + x^2$
   (f) $T = 2\pi\sqrt{\frac{1}{x}}$
9. Use the quadratic equation to find the roots of the function.
   $4x^2 + 15x + 13 = 0$
10. Find the mean, median, and mode of each data set.
   (a) 25, 38, 55, 58, 60, 61, 61, 65, 70, 74, 74, 74, 78, 79, 82, 85, 90
   (b) 13, 14, 16, 17, 18, 20, 20, 22, 26, 30, 31, 32, 32, 35

The Metric System: Fundamental and Derived Units

Metric System Prefixes

Prefix	Symbol	Factor
tera	T	1 000 000 000 000 = 10^{12}
giga	G	1 000 000 000 = 10^{9}
mega	M	1 000 000 = 10^{6}
kilo	k	1000 = 10^{3}
hecto	h	100 = 10^{2}
deca	da	10 = 10^{1}
		1 = 10^{0}
deci	d	0.1 = 10^{-1}
centi	c	0.01 = 10^{-2}
milli	m	0.001 = 10^{-3}
micro	μ	0.000 001 = 10^{-6}
nano	n	0.000 000 001 = 10^{-9}
pico	p	0.000 000 000 001 = 10^{-12}
femto	f	0.000 000 000 000 001 = 10^{-15}
atto	a	0.000 000 000 000 000 001 = 10^{-18}

Fundamental Physical Quantities and Their SI Units

Quantity	Symbol	Unit	Symbol
length	l	metre	m
mass	m	kilogram	kg
time	t	second	s
absolute temperature	T	Kelvin	K
electric current	I	ampère (amp)	A
amount of substance	mol	mole	mol

Derived SI Units

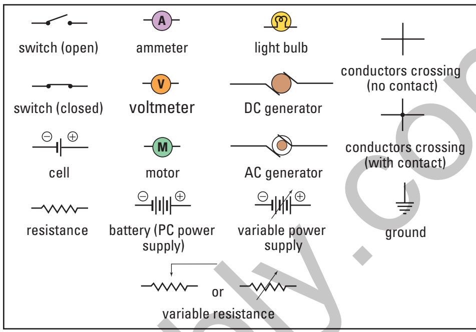
Quantity	Quantity symbol	Unit	Unit symbol	Equivalent unit(s)
area	A	square metre	m^2	
volume	V	cubic metre	m^3	
velocity	v	metre per second	m/s	
acceleration	a	metre per second per second	m/s^2	
force	F	newton	N	$kg \cdot m/s^2$
work	W	joule	J	$N \cdot m$, $kg \cdot m^2/s^2$
energy	E	joule	J	$N \cdot m$, $kg \cdot m^2/s^2$
power	P	watt	W	J/s, $kg \cdot m^2/s^3$
density	ρ	kilogram per cubic metre	kg/m^3	
pressure	p	pascal	Pa	N/m^2, $kg/(m \cdot s^2)$
frequency	f	hertz	Hz	s^{-1}
period	T	second	s	
wavelength	λ	metre	m	
electric charge	Q	coulomb	C	$A \cdot s$
electric potential	V	volt	V	W/A, J/C, $kg \cdot m^2/(C \cdot s^2)$
resistance	R	ohm	Ω	V/A, $kg \cdot m^2/(C^2 \cdot s)$
magnetic field intensity	B	tesla	T	$N \cdot s/(C \cdot m)$, $N/(A \cdot m)$
magnetic flux	Φ	weber	Wb	$V \cdot s$, $T \cdot m^2, m^2 \cdot kg/(C \cdot s)$
radioactivity	$\Delta N/\Delta t$	becquerel	Bq	s^{-1}
radiation dose		gray	Gy	$J/kg \cdot m^2/s^2$
temperature (Celsius)	T	degree Celsius	°C	T°C = (T + 273.15) K
		atomic mass unit	u	$1u = 1.660\ 566 \times 10^{-27}$ kg
		electron volt	eV	$1\ eV = 1.602 \times 10^{-19}$ J

Physical Constants and Data

Fundamental Physical Constants

Quantity	Symbol	Accepted value
speed of light in a vacuum	c	2.998×10^{8} m/s
gravitational constant	G	6.673×10^{-11} N · m²/kg²
Coulomb's constant	k	8.988×10^{9} N · m²/C²
charge on an electron	e	1.602×10^{-19} C
rest mass of an electron	m_e	9.109×10^{-31} kg
rest mass of a proton	m_p	1.673×10^{-27} kg
rest mass of a neutron	m_n	1.675×10^{-27} kg
Planck's constant	h	6.626×10^{-34} J · s

Electric Circuit Symbols

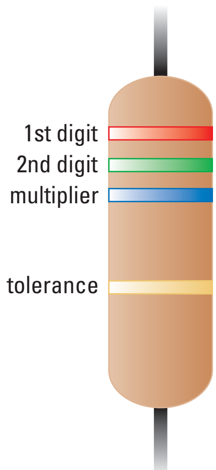


Other Physical Data

Quantity	Symbol	Accepted value
standard atmospheric pressure	P	1.013×10^{5} Pa
speed of sound in air		343 m/s (at 20°C)
water: density (4°C)		1.000×10^{3} kg/m³
latent heat of fusion		3.34×10^{5} J/kg
latent heat of vaporization		2.26×10^{6} J/kg
specific heat capacity (15°C)		4186 J/(kg°C)
kilowatt hour	E	3.6×10^{6} J
acceleration due to Earth's gravity	g	9.81 m/s² (standard value; at sea level)
mass of Earth	m_E	5.98×10^{24} kg
mean radius of Earth	r_E	6.38×10^{6} m
mean radius of Earth's orbit	R_E	1.49×10^{11} m
period of Earth's orbit	T_E	365.25 days or 3.16×10^{7} s
mass of Moon	m_M	7.36×10^{22} kg
mean radius of Moon	r_M	1.74×10^{6} m
mean radius of Moon's orbit	R_M	3.84×10^{8} m
period of Moon's orbit	T_M	27.3 days or 2.36×10^{6} s
mass of Sun	m_s	1.99×10^{30} kg
radius of Sun	r_s	6.96×10^{8} m

Resistor Colour Codes

Colour	Digit represented	Multiplier	Tolerance
black	0	$\times 1$	
brown	1	$\times 1.0 \times 10^{1}$	
red	2	$\times 1.0 \times 10^{2}$	
orange	3	$\times 1.0 \times 10^{3}$	
yellow	4	$\times 1.0 \times 10^{4}$	
green	5	$\times 1.0 \times 10^{5}$	
blue	6	$\times 1.0 \times 10^{6}$	
violet	7	$\times 1.0 \times 10^{7}$	
gray	8	$\times 1.0 \times 10^{8}$	
white	9	$\times 1.0 \times 10^{9}$	
gold		$\times 1.0 \times 10^{-1}$	5%
silver		$\times 1.0 \times 10^{-2}$	10%
no colour			20%



Mathematical Equations

Equations in Unit 1 — Forces and Motion: Dynamics

Equation	Variable	Name
$\vec{F} = m\vec{a}$	$\vec{F}$ = net force m = mass $\vec{a}$ = acceleration	Newton's second law
$\lvert\vec{F}_f\rvert = \mu_s\lvert\vec{F}_N\rvert$ $\lvert\vec{F}_f\rvert = \mu_k\lvert\vec{F}_N\rvert$	$\vec{F}_f$ = force of friction μ_s = coefficient of static friction μ_k = coefficient of kinetic friction $\vec{F}_N$ = normal force	friction
$\vec{F}_g = m\vec{g}$	$\vec{F}$ = force of gravity at Earth's surface m = mass $\vec{g}$ = acceleration due to gravity (on Earth)	weight
$\Delta d = v\Delta t$ $v_2 = v_1 + a\Delta t$ $\Delta d = v_1\Delta t + \frac{1}{2}a\Delta t^2$ $v_2^2 = v_1^2 + 2a\Delta d$	Δd = displacement v = velocity v_1 = initial velocity v_2 = final velocity a = acceleration Δt = time interval	motion equations (constant acceleration)
$R = \frac{v_i^2 \sin 2\theta}{g}$ $H_{max} = \frac{v_i \sin^2 \theta}{2g}$	R = range H_{max} = maximum height v_i = initial velocity θ = launch angle g = acceleration due to gravity	projectile range projectile maximum height
$a_c = \frac{v^2}{r}$	a_c = centripetal acceleration v = velocity r = radius	centripetal acceleration
$F_c = \frac{mv^2}{r}$	F_c = centripetal force v = velocity r = radius	centripetal force
$\frac{T^2}{r^3} = k$ $\frac{T_A^2}{r_A^3} = \frac{T_B^2}{r_B^3}$	T = period of planet r = average distance from planet to Sun k = constant T_A = period of planet A T_B = period of planet B r_A = average distance of planet A to Sun r_B = average distance of planet B to Sun	Kepler's third law
$F_g = G\frac{m_1 m_2}{r^2}$	F_g = force of gravity between 2 point masses G = universal gravitational constant m_1 = first mass m_2 = second mass r = distance between the centres of the masses	Newton's law of universal gravitation

Equations in Unit 2 — Energy and Momentum

$\vec{p} = m\vec{v}$	$\vec{p}$ = momentum m = mass $\vec{v}$ = velocity	momentum
$\vec{J} = \vec{F}\Delta t = m\vec{v}_2 - m\vec{v}_1 = \Delta\vec{p}$	$\vec{J}$ = impulse $\vec{F}$ = force Δt = time interval m = mass $\vec{v}_1$ = initial velocity $\vec{v}_2$ = final velocity $\Delta\vec{p}$ = change in momentum	impulse
$m_A\vec{v}_A + m_B\vec{v}_B = m_A\vec{v}'_A + m_B\vec{v}'_B$	m_A = mass of object A m_B = mass of object B $\vec{v}_A$ = velocity of object A *before* collision $\vec{v}_B$ = velocity of object B *before* collision $\vec{v}'_A$ = velocity of object A *after* collision $\vec{v}'_B$ = velocity of object B *after* collision	conservation of momentum
$v'_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)v_1$ $v'_2 = \left(\frac{2m_1}{m_1 + m_2}\right)v_1$	v'_1 = velocity of object 1 *after* collision v'_2 = velocity of object 2 *after* collision m_1 = mass of object 1 m_2 = mass of object 2 v_1 = velocity of object 1 *before* collision	perfectly elastic, head-on collision, using a frame of reference such that $v_2 = 0$
$W = F\Delta d\cos\theta$	W = work F = applied force Δd = displacement θ = angle between force and displacement	work
$E_k = \frac{1}{2}mv^2$	E_k = mechanical kinetic energy m = mass v = velocity	mechanical kinetic energy
$E_g = mg\Delta h$	E_g = gravitational potential energy m = mass Δh = change in height	gravitational potential energy
$E_k + E_g + E_e = E'_k + E'_g + E'_e$	E_k = initial kinetic energy E_g = initial gravitational energy E_e = initial elastic energy E'_k = final kinetic energy E'_g = final gravitational energy E'_e = final elastic energy	conservation of mechanical energy
$F_a = kx$	F_a = applied force k = spring constant x = extension or compression of spring	Hooke's law
$E_e = \frac{1}{2}kx^2$	E_e = elastic potential energy k = spring constant x = extension or compression of spring	elastic potential energy
$v = \sqrt{\frac{2GM_p}{r_p}}$	v = escape speed G = universal gravitational constant M_p = mass of planet r_p = radius of planet	escape speed

Appendix C continued from previous page

$F_c = \frac{mv^2}{r} = \frac{4\pi^2 rm}{T^2}$	F_c = centripetal force m = mass of object v = speed of object r = radius of orbit T = period of orbit	centripetal force
$v = \sqrt{\frac{GM}{r}}$	v = speed of orbiting object G = universal gravitational constant M = mass of planet or star r = radius of orbit	speed of satellite
$a_c = \frac{v^2}{r} = \frac{4\pi^2}{T^2} r$	a_c = centripetal acceleration v = speed of orbiting object r = radius of orbit T = period of orbit	centripetal acceleration
$T = \sqrt{\frac{4\pi^2 r^3}{GM}}$	T = period of orbit r = radius of orbit G = universal gravitational constant M = mass of planet or star	orbital period (squared)
$E_g = -\frac{GMm}{r}$ $E_k = \frac{GMm}{2r}$ $E_{total} = -\frac{GMm}{2r}$ $E_{binding} = \frac{GMm}{r}$	E_g = gravitational potential energy E_k = kinetic energy E_{total} = total orbital energy $E_{binding}$ = binding energy	orbital energies
$F_{thrust} = \left(\frac{m_{gas}}{\Delta t}\right)\Delta v_{gas}$	F_{thrust} = thrust force m_1 = mass of expelled gas Δv_{gas} = speed of expelled gas Δt = time interval	rocket thrust
Equations in Unit 3 — Electric, Gravitational, and Magnetic Fields		
$F_Q = k\frac{q_1 q_2}{r^2}$	F_Q = electrostatic force between charges k = Coulomb's constant q_1 = electric charge on object 1 q_2 = electric charge on object 2 r = distance between object centres	Coulomb's law
$\vec{E} = \frac{\vec{F}_Q}{q}$	$\vec{E}$ = electric field intensity $\vec{F}_Q$ = electric force q = electric charge	electric field intensity
$\vec{g} = \frac{\vec{F}_g}{m}$ $g = \frac{Gm}{r^2}$	$\vec{g}$ = gravitational field intensity $\vec{F}_g$ = force of gravity m = mass G = universal gravitational constant r = distance from centre of object	gravitational field intensity
$\lvert\vec{E}\rvert = k\frac{q}{r^2}$	$\vec{E}$ = electric field intensity k = Coulomb's constant q = source charge r = distance from centre of charge	Coulombic electrostatic field

$V = \frac{E_Q}{q}$ $V = k\frac{q}{r}$	V = electric potential difference E_Q = electric potential energy q = electric charge r = distance k = Coulomb's constant	electric potential electric potential due to a point charge
$\Delta V = \frac{W}{q}$	ΔV = electric potential difference W = work done q = electric charge	electric potential difference
$\lvert\vec{E}\rvert = \frac{\Delta V}{\Delta d}$	$\lvert\vec{E}_Q\rvert$ = electric field intensity ΔV = electric potential difference Δd = component of displacement between points, parallel to field	electric field and potential difference
$F_M = qvB\sin\theta$	F_M = magnitude of the magnetic force on moving charged particle q = electric charge on particle v = speed of particle B = magnetic field intensity θ = angle between velocity vector and magnetic field vector	force on a moving charge in a magnetic field
$F_M = IlB\sin\theta$	F_M = magnitude of the magnetic force on moving charged particle B = magnetic field intensity I = electric current in conductor l = length of conductor in magnetic field θ = angle between conductor and magnetic field	force on a current-carrying conductor in a magnetic field
Equations in Unit 4 — The Wave Nature of Light		
$T = \frac{\Delta t}{N}$ $f = \frac{1}{T}$ $f = \frac{N}{\Delta t}$	T = period f = frequency Δt = time interval N = number of cycles	period and frequency
$PD = (n - \frac{1}{2})\lambda = d\sin\theta$ $n = 1, 2, 3, \ldots$ for dark fringes $PD = n\lambda = d\sin\theta$ $n = 0, 1, 2, 3, \ldots$ for light fringes $\lambda \cong \frac{d}{(n - \frac{1}{2})}\left(\frac{y_n}{x}\right)$ $\lambda \cong \left(\frac{d}{n}\right)\left(\frac{y_n}{x}\right)$	PD = path difference n = nodal line number λ = wavelength d = slit separation θ = angle between central bisector and line formed from slit to nodal point y_n = distance from central maximum fringe to fringe n x = distance from slits to screen	path difference dark fringes light fringes
$\lambda \cong \frac{\Delta y d}{x}$	λ = wavelength Δy = space between fringes d = slit separation x = distance from slits to screen	Young's double-slit experiment

Appendix C continued from previous page

$y_m \cong \frac{m\lambda L}{w}$ $(m = \pm 1, \pm 2, \pm 3 \ldots)$ $y_m \cong \frac{(m + \frac{1}{2})\lambda L}{w}$ $(m = \pm 1, \pm 2, \pm 3 \ldots)$	y_m = distance from fringe m to central bisector m = fringe order number L = distance to screen w = width of slit	single-slit interference destructive constructive
$\theta_{min} = \frac{\lambda}{w}$ $\theta_{min} = \frac{1.22\lambda}{D}$	θ_{min} = minimum angle for resolution λ = wavelength of light w = width of rectangular aperture D = diameter of circular aperture	resolution slit aperture circular aperture
$m\lambda = d \sin \theta$ $(m = 0, 1, 2, \ldots)$	m = fringe order number λ = wavelength of light d = distance between slit centres θ = angle from central bisector	diffraction grating bright fringes
$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$	c = speed of light μ_0 = permeability of free space ε_0 = permittivity of free space	speed of electromagnetic radiation
$E = hf$	E = energy of photon h = Planck's constant f = frequency of wave	energy of photon
$c = f\lambda$	c = speed of light f = frequency of wave λ = wavelength	wave equation
Equations in Unit 5 — Matter-Energy Interface		
$\Delta t = \frac{\Delta t_o}{\sqrt{1 - \frac{v^2}{c^2}}}$ $L = L_o\sqrt{1 - \frac{v^2}{c^2}}$ $m = \frac{m_o}{\sqrt{1 - \frac{v^2}{c^2}}}$ $\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$	Δt = dilated time Δt_0 = proper time v = speed of object c = speed of light L = relativistic length L_0 = proper length m = relativistic mass m_0 = proper mass γ = gamma	time dilation length contraction mass increase variable substitution for simplicity
$E_k = mc^2 - m_oc^2$ $E_{k(max)} = hf - W$	E_k = relativistic kinetic energy m = relativistic mass m_0 = proper mass c = speed of light $E_{k(max)}$ = maximum kinetic energy of photoelectron h = Planck's constant f = frequency of electromagnetic radiation W = work function of metal	kinetic energy at relativistic speeds photoelectric effect

$p = \frac{h}{\lambda}$	p = momentum h = Planck's constant λ = wavelength	momentum of a photon
$\lambda = \frac{h}{mv}$	λ = wavelength m = mass v = velocity h = Planck's constant	de Broglie wavelength
$\mathrm{A} = \mathrm{Z} + \mathrm{N}$	A = atomic mass number Z = atomic number N = number of neutrons	atomic mass number
$N = N_0\left(\frac{1}{2}\right)^{\frac{\Delta t}{T_{\frac{1}{2}}}}$	N_0 = original amount of radioactive material N = amount of radioactive material remaining after Δt Δt = time interval $T_{\frac{1}{2}}$ = half life	amount of radioactive material remaining
$v = \frac{2V}{Br}$ $\frac{m}{e} = \frac{2V}{v^2}$	v = speed of electron V = accelerating voltage B = magnetic field strength r = radius of trajectory m = mass of electron e = charge of electron	speed of electrons electron mass to charge ratio

Achieving in Physics

WEB LINK

www.mcgrawhill.ca/links/physics12

This feature directs you to interesting and informative Internet sites. Access is easy when you use the Physics 12 Internet page links.

PROBEWARE

This logo indicates where electronic probes could be used as part of the procedure or as a separate lab.

The following Achievement Chart identifies the four categories of knowledge and skills in science that will be used in all science courses to assess and evaluate your achievement. The chart is provided to help you in assessing your own learning and in planning strategies for improvement with the help of your teacher.

You will find that all written text, problems, investigations, activities, and questions throughout this textbook have been developed to encompass the curriculum expectations of your course. The expectations are encompassed by these general categories: Knowledge/Understanding K/U, Inquiry I, Communication C, and Making Connections MC. You will find, for example, that questions in the textbook have been designated under these categories so that you can determine if you are able to achieve well in each category. Some questions could easily fall under a different category; for each question, the category chosen is the one with which it best complies. (In addition, problems that involve calculation have been designated either as practice problems or, in chapter and unit reviews, as Problems for Understanding.) Keep a copy of this chart in your notebook as a reminder of the expectations of you as you proceed through the course.

Achievement Chart

Knowledge/ Understanding	Inquiry	Communication	Making Connections
■ Understanding of concepts, principles, laws, and theories ■ Knowledge of facts and terms ■ Transfer of concepts to new contexts ■ Understanding of relationships between concepts	■ Application of the skills and strategies of scientific inquiry ■ Application of technical skills and procedures ■ Use of tools, equipment, and materials	■ Communication of information and ideas ■ Use of scientific terminology, symbols, conventions, and standard (SI) units ■ Communication for different audiences and purposes ■ Use of various forms of communication ■ Use of information technology for scientific purposes	■ Understanding of connections among science, technology, society, and the environment ■ Analysis of social and economic issues involving science and technology ■ Assessment of impacts of science and technology on the environment ■ Proposing courses of practical action in relation to science- and technology-based problems

Safety Symbols

The following safety symbols are used in *Physics 12* to alert you to possible dangers. Make sure that you understand each symbol that appears in a lab or investigation before you begin.

	Thermal Safety This symbol appears as a reminder to be careful when handling hot objects.
	Sharp Object Safety This symbol appears when there is danger of cuts or punctures caused by the use of sharp objects.
	Fume Safety This symbol appears when chemicals or chemical reactions could cause dangerous fumes.
	Electrical Safety This symbol appears as a reminder to be careful when using electrical equipment.
	Skin Protection Safety This symbol appears when the use of caustic chemicals might irritate the skin or when contact with micro-organisms might transmit infection.
	Clothing Protection Safety A lab apron should be worn when this symbol appears.
	Fire Safety This symbol appears as a reminder to be careful around open flames.
	Eye Safety This symbol appears when there is danger to the eyes and safety glasses should be worn.
	Chemical Safety This symbol appears when chemicals could cause burns or are poisonous if absorbed through the skin.

Safety symbols used in *Physics 12*

Look carefully at the WHMIS (Workplace Hazardous Materials Information System) safety symbols shown below. These symbols are used throughout Canada to identify dangerous materials in all workplaces, including schools. Make sure that you understand what these symbols mean. When you see these symbols on containers in your classroom, at home, or in a workplace, use safety precautions.

Compressed Gas	Flammable and Combustible Material
Oxidizing Material	Corrosive Material
Poisonous and Infectious Material Causing Immediate and Serious Toxic Effects	Poisonous and Infectious Material Causing Other Toxic Effects
Biohazardous Infectious Material	Dangerously Reactive Material

WHMIS symbols

Practice Problems and Chapter and Unit Review Problems

Chapter 1

Practice Problems

1. 9.6×10^{-13} N
2. 9.3 m/s
3. 0.61 m/s^2
4. (a) 0.249 N (b) 0.00127
5. 78 N
6. (a) 58 N (b) 16 m/s^{-2}
7. 6.7 m
8. 1.6×10^3 N, 9.1×10^2 N
9. (a) 21 N (b) 15 N
10. (a) 74 N (b) 34 N
11. negative; 5.9×10^2 N
12. down (negative); 6.9×10^2 N
13. up (positive); 5.9×10^2 N
14. -1.9 m/s^2
15. No, the climber must limit his descent to $a = -2.5$ m/s^2.
16. (a) downward (b) -1.1 m/s^2 (c) 87 N
17. 1.7×10^2 N
18. 1.8 m/s^2
19. 0.49 m/s^2; 39 N
20. 14 kg; 75 N
21. 62 kg; 1.6 m/s^2
22. 17 N
23. Both of them will rise, with $a = +1.0$ m/s^2.
24. (a) 3.88 N (b) 2.04 m/s^2
25. 0.67 s
26. 15 m/s
27. (a) 1.2 m/s^2 (b) 0.16 m/s^2 (c) 12 s
28. 0.061
29. 0.34 m
30. 0.37

Chapter 1 Review

Problems for Understanding

16. 3.0 m/s[N]
17. 11 kg
18. (a) $v = 0$; $a = -9.8$ m/s^2 (b) 3.5 m/s; -9.8 m/s^2
19. (a) 1.34 m/s^2 (b) 334 N
20. 1.1 m/s^2[W]
21. 1.2 N
22. (a) 0.062 m/s^2 (b) 0.40 m/s^2 (c) A friction force of magnitude 3.4 N operates to reduce the ideal acceleration ($a = F/m$)
23. 5.4 m
24. 11 m
25. (a) 5.4 m/s[down] (b) 3.8×10^4 N[up]
26. 49 N
27. 1.3 m/s^2
28. (a) $a_2 = 2.5a_1$ (b) $d_2 = 2.5d_1$
29. (a) 9.00 N (b) 132 N (c) 293 N (d) 0.451
30. 3.3 m/s^2; 13 N
31. (a) 4.6 m/s^2 (b) 0.70 N

Chapter 2

Practice Problems

1. -677 m
2. 4.67 m/s
3. 89.6 m, 45.2 m/s [60.3° below the horizontal]
4. 0.156 m
5. 3.05 m/s
6. 0.55 m
7. 74 m
8. (a) 153 m (b) 5.00 m/s
9. 85 m
10. 4.0×10^1 m
11. 18 m/s [52° below the horizontal]
12. 2.8 m/s
13. (a) 58.9 m (b) 21.0 m (c) 4.14 s
14. 33.2°; 2.39 m; 1.40 s
15. 47.0 m/s
16. 8.3×10^{-8} N
17. (a) 48.6 N (b) 54.2 N (c) 9.62 m/s
18. 5.9×10^3 N
19. 84 m
20. 103 m
21. 13 m/s (47 km/h)
22. 19.1 m/s (68.8 km/h)
23. 20.1°

Chapter 2 Review

Problems for Understanding

20. (a) 3.0×10^1 m (b) 3.7 s
21. (a) 0.78 s (b) at the same position (c) 4.7 m (d) 9.7 m/s
22. (a) 42 m (b) 62 m
23. 2.7×10^2 m
24. (a) 2.1 s (b) 34 m (c) 8.5 m (d) $v_x = 16$ m/s; $v_y = +3.8$ m/s or -3.8 m/s (e) 38.2°
25. 52 m/s
26. Yes. It travels 330 m.
27. (a) 7.4 s (b) 67 m (c) 1.2×10^2 m (d) x: 34 m, y: 53 m (e) $v_x = 17$ m/s; $v_y = -23$ m/s
28. (a) 193 m/s (b) 843 m (c) $v_x = 162$ m/s, $v_y = 156.3$ m/s (d) 44.0°
29. (a) 2.0 m/s (b) 1.2 m/s^2
30. 7.1×10^2 N
31. (a) 1.33×10^{14} m/s^2 (b) 1.21×10^{-16} N
32. 0.33
33. 8.9 m/s
34. 33°
35. (a) 9.90 m/s (b) A factor of $\sqrt{2}$
36. 0.62
37. (a) 4.6×10^2 m/s (b) 2.0 N (for $m = 60.0$ kg) (c) Toward the centre of Earth; gravity (d) $mg = 589$ N (for $m = 60.0$ kg) (e) $N = mg - mv^2/r = 587$ N (f) $mg - N = ma_c$; because $mg > N$, there is a net acceleration toward the centre of Earth.

Chapter 3

Practice Problems

1. 3.57×10^{22} N
2. 1.99×10^{20} N
3. 5.1×10^{-3} m. This is much smaller than the radii of the bowling balls.
4. 3.61×10^{-47} N
5. 5.0×10^{24} kg
6. 0.25 m
7. $F_{Uranus} = 0.80 \times F_{Earth}$
8. $0.9 \times$ Earth-Moon distance
9. 1.899×10^{27} kg
10. 1.472×10^{22} kg
11. 2.74×10^5 m
12. 1.02×10^3 m/s
13. (a) 6.18×10^4 s (17.2 h) (b) 7.93×10^2 m/s
14. 4×10^{41} kg $= 2 \times 10^{11} \times M_{Sun}$

15. 7.42×10^3 m/s; 8.59×10^5 m
16. 7.77×10^3 m/s; 5.34×10^3 s (89.0 min)
17. (a) 5.21×10^9 s (165 years); 5.43×10^3 m/s
(b) It will complete one orbit, after its discovery, in the year 2011.

Chapter 3 Review

Problems for Understanding

22. 1/8
23. (c) F
24. (b) $a/3$
25. (a) 3.0×10^4 m/s
(b) 6.0×10^{-3} m/s^2
26. 1.8×10^{-8} m/s^{-2}
27. 9.03 m/s^2 = 92% of acceleration due to gravity at Earth's surface
28. 4.1×10^{36} kg = $2.0 \times 10^6 \times m_{\text{Sun}}$
29. 2.67×10^{-10} N
30. (a) 5.3×10^5 m
(b) 5.7×10^3 s = 95 min
31. 1.02×10^3 m/s; 2.37×10^6 s = 27.4 days
32. (a) Yes.
(b) 5.69×10^{26} kg
33. (a) 4×10^{15} kg
(b) 4×10^{27} kg
(c) $m_{\text{Oort}} = 700 m_{\text{Earth}} = 2 m_{\text{Jupiter}}$

Unit 1 Review

Problems for Understanding

29. 1.4 m/s^2
30. (a) 2.00 (b) 2.00
31. 1.6×10^4 N. The acceleration remains constant.
32. (a) 3.1×10^3 N (b) 4.5 m
33. 1.1×10^4 N
34. (a) 7.00×10^3 N (b) $9.16 \times$ true
35. (a) 1.5×10^4 N
(b) 3.8×10^3 N
(c) 2.5 m/s^2
(d) 22 m/s = 81 km/h
(e) 9.0 s
36. 17°
37. (a) 9.8×10^2 N (b) 13 km
38. 3.3 m/s^2; 23 N
39. (a) 1.4 s (c) 5.0×10^1 m
(b) 1.8 s
40. (a) 21.3 m/s (c) down
(b) 1.54 m
41. 2.40 m
42. 0.084 m
43. (a) 4.4×10^2 N; $1.0 \times$ her weight
(b) 2.0×10^2 N; $0.45 \times$ her weight
(c) same as (a)
(d) 6.8×10^2 N; $1.6 \times$ her weight
44. 2.0×10^2 N
45. (a) 6.9×10^3 N (b) 64 km/h
46. (a) 612 N (c) 786 N
(b) 437 N (d) 612 N
47. 29 m/s^2
48. (a) 5.1×10^2 N (b) 5.6×10^2 N
49. (a) 1.7×10^2 N (b) 29 m/s
50. (a) 8.0 m/s^2
(b) 6.9 m/s^2
(c) 6.0×10^1 m/s[down]; 52 m/s[down]
51. (a) 0 m/s^2; 2.0×10^1 N
(b) 2.0 m/s^2; 16 N
(c) 0.50
52. 2.4 m/s^2; 0.61 m/s^2
53. Swift-Tuttle: (a) 51.69 AU; (b) 26.32 AU; (c) 135.5 a
Hale-Bopp: (a) 369.2 AU; (b) 185.1 AU; (c) 2511 a
Encke: (a) 4.096 AU; (b) 2.218 AU; (c) 3.303 a
Kopff: (a) 5.351 AU; (b) 3.467 AU; (c) 6.456 a
Hyakutake: (a) 1918 AU; (b) 959.1 AU; (c) 2.970×10^4 a
(d) student sketch
(e) Swift-Tuttle, Hale-Bopp, and Hyakutake
54. (a) 4.6×10^2 m/s
(b) 7.9×10^3 m/s
55. (a) 0.7445 AU (c) 1.732 a
(b) 1.442 AU
56. 2×10^{42} kg; 1×10^{12} m_{Sun}

Chapter 4

Practice Problems

1. (a) 11.5 kg m/s[E]
(b) 2.6×10^8 kg m/s[W]
(c) 8.39×10^7 kg m/s[S]
(d) 5.88×10^{-24} kg m/s[N]
2. 43.6 N s[down]
3. 2.58×10^5 N · s[S]
4. 4.52×10^6 N[S]
5. 2.6 kg m/s[horizontal]
6. −38 kg m/s
7. 8.8 kg m/s[up]
8. 2.7 m/s[in the original direction]
9. 0.11 m/s[in the direction that car A was travelling]
10. 2.10 m/s[S]
11. 0.11 m/s[E]
12. -2.43×10^2 m/s
13. 6.4 m/s[40.0° counterclockwise]
14. 1.16 m/s[6.1° clockwise from original direction]
15. v_A = 34.3 km/h[S]; v_B = 67.3 km/h[E]
16. v_2 = 6.32 m/s[41.5° counterclockwise from the original direction of the first ball]; the collision is not elastic: E_k = 12.1 J; E_k' = 10.2 J.
17. 1.24×10^5 kg km/h = 3.44×10^4 kg m/s[N39.5°W]; the collision was not elastic: $E_k = 3.60 \times 10^6$ kg km^2/h^2; $E_k' = 1.80 \times 10^6$ kg km^2/h^2
18. 261 m/s
19. The cart will stop at 0.018 m; therefore, it will not reach the end of the track.
20. 55.5 km/h = 15.4 m/s
21. 18.2 m/s
22. 3.62 m/s; 1.71 m

Chapter 4 Review

Problems for Understanding

28. 18 kg m/s[N]
29. 1.5×10^3 kg
30. 1.20 m/s[S]
31. 6.0×10^3 m/s[forward]
32. (a) 0.023 N · s[E]
(b) 0.036 N · s[S]
33. 3.8×10^3 N
34. 3.6×10^{-2} s
35. (a) −16 kg m/s
(b) 6.4×10^{-3} s
36. 2.5×10^4 N[E]
37. 2.9×10^4 N
38. 134 m/s[E]
39. 3.1 m/s[E]
40. −2.3 m/s
41. 1.3 m/s[forward]
42. 0.17 m/s[forward]
43. 4.4 m/s[35.2° clockwise]
44. 5.6×10^6 m/s[26.6° with respect to the $+x$ direction]
45. (b) (i) $v_1' = -v_1$; $v_2' \approx 0$;
(ii) $v_1' \approx 0$; $v_2' = v_1$;
(iii) $v_1' = v_1$; $v_2' = 2v_1$
(c) (i) is the limiting case of a small object hitting a wall: it bounces back with the same speed and opposite direction. In (ii), all of the momentum is transferred to the other particle. In (iii), the massive object continues as if the light object had not been there, while the light object flies off with twice the speed of the massive object.

46. $v_1' = 0.86$ m/s[S]; $v_2' = 1.25$ m/s[N]. In a perfectly elastic head-on collision between identical masses, the two bodies simply exchange velocities.

47. (a) 0.29 m/s[W21°N]
(b) 70%

48. (a) 0.21 m/s (c) 95%
(b) 13 kg m/s

Chapter 5

Practice Problems

1. 1.810×10^4 J
2. 1.22×10^4 m
3. 31.5°
4. 61.6 m
5. 34.6 m/s
6. -2.6×10^2 N
7. 515 kg
8. 15.0 m
9. 4.9×10^{-2} J
10. 13 m/s
11. 7.7 m
12. 4.8 m
13. 0.25 J
14. 250 J
15. $v_A = 2.0$ m/s; $v_B = 2.8$ m/s
16. 5×10^2 N/m
17. (a) 0.414 m (b) -455 N
18. 0.0153 kg
19. 1.0 J
20. 0.30 m
21. 1.4 J
22. (a) 0.28 m (b) 1.3 m/s
(c) 17m/s^2
23. 1.4×10^3 N/m
24. 6.59×10^3 N/m
25. 0.42 m
26. (a) 405 N/m (b) 44.1 m/s^2
27. 11 m/s
28. 14 m/s
29. 7.4×10^2 J

Chapter 5 Review

Problems for Understanding

18. (a) 0.035 N (c) 0.025 J
(b) -0.025 J
19. (a) 16 J (b) 16 J
20. (a) 7.7×10^3 J
(b) 6.7×10^3 J
(c) 9.4 m/s; 8.7 m/s
(d) infinity (no friction); 1.3×10^2 m

21. 3.2×10^2 N·m
22. (a) 9.0 m/s
(b) $E_k = W = 2750$ J (2.8×10^3 J)
(c) 4.1 m
23. 57 N
24. 4.6 m/s
25. 4.5×10^2 N/m
26. (a) 0.38 J (b) 9.6 N
27. 0.19 m
28. $k = m_1 g/x$
29. 3.6 m/s
30. 4.1°C
31. 0.28°C
32. (a) 2.3 m/s (b) 5.3 N
33. 1.3 m/s
34. 0.77 m/s; 0.031 m
35. 5.0 m/s
36. 0.15 m
37. 0.45 m
38. 0.096 m
39. (a) -8.7×10^2 J (b) -1.8 m

Chapter 6

Practice Problems

1. 4.0×10^6 J; 1.16×10^3 m/s
2. 1.9×10^5 J; 5.0×10^3 m/s
3. 1.85×10^4 m/s
4. (a) 1.5×10^9 J (c) -1.5×10^9 J
(b) -3.0×10^9 J (d) 1.5×10^9 J
5. (a) 3.32×10^9 J (c) 7.51×10^6 m
(b) 7.29×10^3 m/s
6. (a) 4.12×10^9 J
(b) thermal energy, acoustic energy
7. 1.57×10^3 m/s; 649 m/s
8. (a) 4.87×10^7 J; 1.27×10^3 m/s
(b) -9.74×10^7 J (d) 4.87×10^7 J
(c) -4.87×10^7 J (e) 528 m/s
9. (a) 1.7×10^8 J; 1.8×10^3 m/s
(b) -3.4×10^8 J (d) 1.7×10^8 J
(c) -1.7×10^8 J (e) 7.7×10^2 m/s
10. 1.4×10^{31} kg, or 7.1 times the mass of the Sun
11. 6.00×10^6 N[forward]
12. 5.01×10^3 kg/s
13. (a) 0.33 m/s; 0.69 m/s; 1.1 m/s; 1.5 m/s; 1.9 m/s; 2.4 m/s
(b) 3.0 m/s, a difference of 0.6 m/s. Throwing all of the boxes at once contributes more to the momentum of the cart, because the cart is lighter without the boxes on it.

Chapter 6 Review

Problems for Understanding

14. 3.13×10^9 J
15. (a) 1.1×10^{11} J (b) 39%
16. -1.78×10^{32} J
17. $0.488 \times v_{Earth}$
18. (a) 6.18×10^5 m/s
(b) 4.22×10^4 m/s
(c) 6.71×10^3 m/s
19. (a) 7.0×10^7 m (b) 650 km
20. (a) 1.6×10^2 m/s. No.
(b) 1.2×10^2 m/s. No.
(c) 12 m/s. Yes.
(d) 2.9×10^8 m/s. No — in fact, the poor pitcher would be crushed by the strong gravity before he could even wind up for the throw!
21. 11.1 km/s; 99.4% of Earth's escape speed
22. 7.9×10^{11} m. This is just past Jupiter's orbit.
23. (a) -4.1×10^{10} J (c) 3.7×10^{10} J
(b) -3.1×10^9 J (d) 3.1×10^9 J
24. (a) $v_{200} = 7.78 \times 10^3$ m/s; $v_{100} = 7.84 \times 10^3$ m/s
(b) $E(r = 200 \text{ km}) = -1.52 \times 10^{10}$ J; $E(r = 100 \text{ km}) = -1.54 \times 10^{10}$ J
25. (a) 2.3×10^7 m/s; $0.077c$
(b) 0.14 s
26. 4.89×10^6 kg
27. (a) 3.4×10^6 N (b) 1.2×10^5 m/s

Unit 2 Review

Problems for Understanding

37. 3.5×10^4 kg m/s[N]
38. (a) 6.6 kg m/s
(b) 4.0×10^1 kg m/s
(c) 3.0×10^3 kg m/s
39. (a) 9.6 kg m/s[N]
(b) -17 kg m/s[N]
(c) 17 kg m/s[S]
(d) 2.6×10^2 N[N]
(e) 2.6×10^2 N[S]
40. (a) 45 N (b) 42 m/s
41. 36 m/s
42. (a) 1.3×10^4 kg m/s
(b) -1.3×10^4 kg m/s
(c) -1.3×10^4 kg m/s
(d) 19 m/s
43. 2.6×10^2 m/s[forward]
44. 1.5 m/s[N27°E]
45. (a) 0.76 m/s[E24°N]
(b) 17%

46. (a) 780 J
(b) It loses 780 J.

47. (a) 3×10^{11} J (b) 5 GW

48. (a) 66 m
(b) 74 m
(c) No change; the result is independent of mass.

49. -7.9×10^3 N

50. (a) 0.24 J (b) 48 J

51. (a) 0.32 m (b) 12 J

52. 15 kg

53. 60.0 m

54. (a) 1.46×10^4 J
(b) 1.46×10^4 J; 12.5 m/s
(c) Needed: coefficient of friction, μ, and slope of hill, θ: $E_k = mgh(1 - \mu/\tan\theta)$; $v = \sqrt{2gh(1 - \mu/\tan\theta)}$. For $\mu = 0.45$ and $\theta = 30.0°$, $E_k = 3.2 \times 10^3$ J, $v = 5.9$ m/s.

55. 3.1 m/s

56. (a) 0.47 m (b) 0.47 m

57. (a) 6.0 N (c) 0.023 J
(b) 0.15 J

58. 1.16×10^3 J. No, work is done by friction forces.

59. (a) 4.4 m/s (b) 3.5 m/s

60. (a) 11.2 km/s
(b) 7.91 km/s
(c) 6×10^{10} m or 10 000 Earth radii

61. 7.3×10^3 m/s

62. At Earth's distance from the Sun, the escape velocity is 42 km/s. Thus, the first comet is bound (it has negative total energy) and the second one is not bound (it has positive total energy).

63. 4.2×10^3 m/s; 1.0×10^4 m/s

64. 6.8 km/s; 15 km/s

65. (a) 1.3×10^{10} J
(b) -1.3×10^{10} J
(c) 6.1×10^3 J
(d) 1.3×10^{10} J
(e) 2.2×10^9 J; 3.1 km/s

66. (a) -7.64×10^{28} J
(b) -5.33×10^{33} J

67. 6.2×10^5 m/s

68. 2.6×10^2 m/s[forward]

Chapter 7

Practice Problems

1. 0.34 N
2. 0.80 m
3. 5.1×10^{-7} C
4. 0.50 N
5. 0.17 N (repulsive)
6. 0.12 m (directly above the first proton)
7. $F_A = 1.2 \times 10^{-2}$ N[W73°S]; $F_B = 1.6 \times 10^{-2}$ N[E63°N]; $F_C = 4.6 \times 10^{-3}$ N[W36°S]
8. 8.74 N[E18.2°N]
9. 2.0×10^{-8} C
10. 7.9×10^{-8} C
11. 1.5×10^5 N/C (to the right)
12. 0.019 N[W]
13. 2.5×10^4 N/C (to the left)
14. -4.0×10^{-4} C
15. 3.8 N/kg
16. 52 N
17. 3.46 kg
18. 2.60 N/kg
19. 2.60 m/s^2
20. -7.8×10^5 N/C (toward the sphere)
21. -1.2×10^{-5} C
22. 0.32 m
23. 5.80×10^9 electrons
24. -1.5×10^6 N/C (toward the sphere)
25. 0.080 m
26. 5.3×10^8 N/C[81.4° above the $+x$-axis]
27. 1.9×10^4 N/C[86.7° above the $+x$-axis]
28. 3.4×10^6 N/C[23.7° above the $-x$-axis]
29. 2.25×10^{14} N/C (toward the negative charge)
30. 2.9×10^7 N/C[73.6° above the $+x$-axis]
31. 5.7×10^{-2} N/kg
32. 3.81×10^7 m
33. 8.09 N/kg
34. 5.82×10^{23} kg
35. 5.0×10^{-11} N/kg
36. 8.09 N/kg
37. 1.03×10^{26} kg
38. -4.7×10^{-2} J
39. 0.18 J
40. 5.1×10^2 m
41. 1.55×10^{-4} C. The signs of the two charges must be the same, either both positive or both negative.
42. 4.8×10^6 N/C
43. 1.5×10^{10} m
44. 2.9×10^{-5} J
45. -4.7×10^{-12} C
46. If the positive charge is placed at 0.0 cm and the negative charge is placed at 10.0 cm, there are two locations where the electric potential will be zero: 6.2 cm and 27 cm.
47. 1.1×10^6 V
48. 8.0 V
49. -2.1×10^6 V
50. 1.6×10^6 V
51. 1.4×10^{-6} C
52. 2.0 V
53. 12 J
54. -2.4×10^4 V
55. (a) 1.9×10^5 V
(b) 1.2×10^{-3} J
(c) A. It takes positive work to move a positive test charge to a higher potential. Since in this case, you invest positive work to move your positive test charge from B to A, A must be at a higher potential.
56. 5.3 cm and 16 cm to the right of the positive charge.
57. any point lying on a line midway between the two charges and perpendicular to the line that connects them
58. The potential is zero 3.4 cm above the origin and 24 cm below the origin.
59. If the distances of the first and second charges, q_1 and q_2, from the point of zero potential are d_1 and d_2, then d_2 must satisfy $d_2 = (-q_2/q_1)d_1$, with $q_2 < 0$. For example, if $q_2 = -8.0$ μC, then $d_2 = 16$ cm and the charge would be located either 24 cm to the right of q_1 or 8.0 cm to the left of q_1. Other solutions can be similarly determined.
60. 4.0 cm to the right of the -4.0 μC charge

Chapter 7 Review

Problems for Understanding

18. 9×10^3 N
19. 2.3×10^8 N
20. 5.6 cm
21. $F_A = 4.5 \times 10^{-2}$ N to the left; $F_B = 0.29$ N to the right; $F_C = 0.24$ N to the left
22. $F_A = 3.8$ N[N3.0°E]; $F_B = 4.4$ N[E23°S]; $F_C = 4.7$ N[W26°S]
23. $F_Q = 8.2 \times 10^{-8}$ N; $F_g = 3.6 \times 10^{-47}$ N
24. The charges on Earth (Q_E) and the Moon (Q_{Moon}) must satisfy

$|Q_E| \times |Q_M| = 3.3 \times 10^{27}$ C^2, and they must have opposite signs.

25. 4.2×10^{42}

26. -57 C

27. 5.2×10^{-3} N

28. **(a)** 8.65×10^{25} kg
(b) 8.81 N/kg
(c) 881 N

29. $2/9\ g_{\text{Earth}} = 2.18$ N/kg

30. **(a)** 8.24×10^{-8} N
(b) 2.19×10^6 m/s
(c) 5.14×10^{11} N/C
(d) 27.2 V

31. 1.86×10^{-9} kg $= 2.04 \times 10^{21} \times m_{\text{actual}}$

32. 9×10^{-5} N[W]

33. 0.51 m

34. 6.0×10^4 N/C[E37°N]

35. **(a)** -7.5×10^{-8} J
(b) It loses energy.

36. -2.9×10^{-6} J

37. 2.8×10^2 C

38. **(a)** 4.5×10^3 V
(b) Yes; the spheres have to be at equal potential, because the same point cannot have two different potentials.
(c) big sphere: 52 nC; small sphere: 23 nC

39. **(a)** $E = 0$; $V = 2.2 \times 10^5$ V
(b) $E = 4.3 \times 10^5$ N/C; $V = 0$
(c) When the two charges have the same sign, the electric fields at the midpoint have the same magnitude but opposite directions, so they cancel. The potential is the algebraic sum of the potentials due to the individual charges; it is a scalar and, in this case, adds to be greater than zero. When the signs are different, the electric fields point in the same direction and the magnitudes add. However, the potentials have opposite signs and cancel.

40. **(a)** 2.3 J **(c)** X
(b) 1.2×10^6 V

41. **(a)** 4.0×10^5 V **(b)** R

Chapter 8

Practice Problems

1. **(a)** 3.0×10^2 N/C[W]
(b) 3.0×10^2 N/C[W]
(c) 3.0×10^2 N/C[W]
(d) double the charge on each plate; halve the area of each plate

2. **(a)** 5.0×10^3 N/C[E]
(b) The area of the plates was decreased by a factor of 4.

3. 8.0×10^2 N[N]

4. 1.4×10^3 N/C

5. **(a)** 3.0×10^3 N/C
(b) 60.0 V

6. **(a)** 0.222 m
(b) 1.44×10^{-3} N[toward + plate]
(c) 1.44×10^{-3} N[toward + plate]

7. 26 V

8. **(a)** 4.0×10^1 V **(b)** 2.0×10^3 N/C

9. **(a)** 9.62×10^{-19} C **(b)** 6.00

10. **(a)** 3.7×10^{-15} kg **(b)** 3.6×10^3 V

11. 1.13×10^4 V

12. 1.4×10^{-13} N[toward the bottom of the page]

13. 2.7×10^{-14} N[left 28° up]

14. 30°

15. 9.8×10^{-3} T[N]

16. 4.2×10^{-3} T[up out of page]

17. 3.0×10^2 N

18. 9.2×10^{-2} T[into the page]

19. 1.8 m

20. **(a)** 6.4×10^2 A
(b) If such a large current could be passed through the wire, Earth's magnetic field could be used to levitate the wire. However, this is such a large current that it is probably not practical to do so.

21. **(a)** 2.884×10^{-17} J
(b) 1.86×10^5 m/s

22. **(a)** up (that is, opposite to gravity)
(b) 59 V

23. **(a)** 1.4×10^7 m/s
(b) 2.0×10^6 V

24. 2.8×10^{-2} m

25. 6.8×10^3 m/s

26. **(a)** 5.01×10^{-27} kg
(b) tritium

Chapter 8 Review

Problems for Understanding

22. 20 V

23. **(a)** 1.60×10^2 V
(b) $V_A = 0.0$ V; $V_C = 8.0 \times 10^1$ V; $V_D = 1.2 \times 10^2$ V
(c) $V_B - V_A = 4.0 \times 10^1$ V; $V_C - V_B = 4.0 \times 10^1$ V; $V_D - V_A = 1.2 \times 10^2$ V
(d) 2.0×10^3 N/C
(e) 2.0×10^{-3} N in both cases [toward negative plate]
(f) 4.0×10^{-3} N[toward negative plate]

24. **(a)** -8.0×10^{-19} C
(b) five electrons

25. 3.1×10^{10} electrons

26. 0.63 N

27. 3.2×10^5 m/s

28. **(a)** 1.8×10^{-3} N[up]
(b) 0.18 g

29. 1.0×10^{-26} kg

30. **(a)** $r(\text{slow}) = 1.4 \times 10^{-4}$ m; $r(\text{fast}) = 2.8 \times 10^{-4}$ m
(b) $T(\text{slow}) = T(\text{fast}) = 8.9 \times 10^{-11}$ s
(c) $f(\text{slow}) = f(\text{fast}) = 1.1 \times 10^{10}$ Hz
(d) The period and frequency is independent of the particle's velocity and the radius of its orbit. The faster electron completes an orbit of larger radius in the same time in which the slower electron completes an orbit of smaller radius.

31. **(a)** $T_e/T_p = m_e/m_p = 5.4 \times 10^{-4}$
(b) $r_e/r_p = \sqrt{\dfrac{m_e}{m_p}} = 0.023$

32. **(a)** 3.0×10^2 Hz; 6.3×10^3 m
(b) 3.0×10^2 Hz; 3.1×10^3 m

33. **(a)** 1.1×10^{-17} T
(b) [E]

Unit 3 Review

Problems for Understanding

49. 1.8×10^8 C

50. 8.23×10^{-8} N

51. 2.3×10^{-9} N

52. $\pm 14\ \mu$C

53. 1.5×10^4 electrons

54. **(a)** -50.0 N **(c)** 3.33 N
(b) 5.56 N **(d)** -3.70 N

55. $\pm 0.14\ \mu$C

56. 1.8×10^{13} C

57. -1.0×10^4 C

58. 0.12 m

59. 8×10^{27} N

60. 9.2×10^{-26} N

61. 1.1×10^{-5} C

62. **(a)** 9 N
(b) An additional nuclear force — the strong force — holds the nucleus together.

63. 2000 N/C

64. 6.2×10^{12} electrons

65. **(a)** 0 J
(b) 8.6×10^{-10} J
(c) equipotential surfaces

66. (a) 4.8×10^{-19} C
(b) three electrons (deficit)
(c) 1.2×10^4 V

67. 0.10 T

68. (a) 3.7 nC
(b) It will be reduced by one half.

69. (a) 2.2×10^{-13} N
(b) 1.3×10^{14} m/s^2[up]

70. 0.9 m

72. (a) 1.8×10^{-10} s (b) 2.8×10^{-4} m

73. 330 N/C

74. (a) $v_{\parallel} = 5.6 \times 10^6$ m/s; $v_{\perp} = 3.2 \times 10^6$ m/s
(b) 0.13 m
(c) 2.2×10^{-7} s
(d) 1.4 m
(e) From the side, a helical path will be seen; two places on consecutive orbits of the proton will be separated by 1.4 m. Face on, the orbit will appear to be circular, with a radius of 0.13 m.

75. 2.5 cm

Chapter 9

Practice Problems

1. 0.011 m
2. 1.3×10^{-5} m
3. red light will have a wider maximum; 7.3×10^{-3} m
4. (a) 0.36 m
(b) The value ignores changes in the speed of light due to the lenses of the imaging system and changes in the density of the atmosphere.
5. 0.00192°
6. Angular resolution improves with shorter wavelengths, so you should use blue lettering.

Chapter 9 Review

Problems for Understanding

31. (a) 0.020 m (b) 0.20 m/s

32. 4.8×10^2 nm

33. 589 nm

34. 2.1×10^{-5} m

35. 3.0 km

36. 72 m

37. 485 nm; 658 nm

38. (a) 15° (b) decrease

39. sin θ will be greater than 1 for $\lambda > 629$ nm.

40. $\lambda_1/\lambda_2 = 3/2$

41. (a) 4th (b) 60°

42. 681 nm

43. 667 nm; red light

Chapter 10

Practice Problems

1. (a) 0.0667 s
(b) The distance that the signal passes is usually larger than the geographic separation between the two points (due to satellite networks); also, the speed of light depends on the medium.
2. 2.25×10^8 m/s
3. (a) 46.8 m
(b) The antenna must be larger than the wavelength of the radiation.
4. (a) $\lambda_{micro} = 0.03$ m; $\lambda_{light} = 3 \times 10^{-6}$ m
(b) The metallic screen is used to stop the microwave radiation by working as an antenna for microwave wavelengths.

Chapter 10 Review

Problems for Understanding

35. 2.48×10^{-13} m

36. 1×10^6 Hz or 1 MHz

37. 2.938 m; 102.1 MHz

38. (a) 0.80 J/s (c) 1/4
(b) 8.0×10^{-5} J/s

39. 8.3 light-minutes

40. 1.1×10^4 m

41. 0.24 s

42. (a) Yes, but with very low frequency.
(b) Greater than 4×10^{14} Hz

Unit 4 Review

Problems for Understanding

50. 0.12 m; 2.5×10^9 Hz; 4.0×10^{-10} s

51. 2.1×10^5 Hz; 1.4×10^3 m

52. 3×10^{-13} m

53. 5.4×10^{16}

54. 4.3×10^{-7} m = violet

55. 1.5×10^2 m

56. 3.4×10^{-2} m

57. 9.4610×10^{15} m

58. Listeners in Vancouver will hear a particular sound after 1.7×10^{-2} s, while listeners in the back of the concert hall will hear the same sound after 0.24 s, so listeners in Vancouver will hear it first.

59. 585 nm

60. (a) 7.1 mm (b) closer

61. 1.2×10^4 lines/cm

62. 8×10^{-7} m

63. (a) 0.42″ = 2.0×10^{-6} rad
(b) 0.0042″ = 2.0×10^{-8} rad
(c) 4.6×10^{12} m apart, or, it could distinguish objects that are $0.77 \times$ Pluto's distance from the Sun apart
(d) better (resolution is proportional to wavelength)

64. 45 m

65. (a) 5.8×10^{-19} J (c) 9.0×10^{-20} J
(b) 1.3×10^{-17} J (d) 4.0×10^{-26} J

66. 2.26×10^8 m/s

67. (a) 5.0×10^{-9} m (b) X ray

Chapter 11

Practice Problems

1. (a) 4.8×10^{-13} s (b) 1.5×10^{-13} s
2. 257 s
3. $0.94c = 2.8 \times 10^8$ m/s
4. 702 km
5. 0.31 m
6. (a) 1.74×10^8 m/s
(b) The sphere's diameter appears contracted only in the direction parallel to the spacecraft's motion. Therefore, the sphere appears to be distorted.
7. 465 μg
8. 1.68×10^{-27} kg
9. $0.9987c = 2.994 \times 10^8$ m/s
10. 4.68×10^{-11} J
11. 1.01×10^{-10} J
12. 2.6×10^8 m/s
13. 7.91×10^{-11} J
14. 1.64×10^{-13} J
15. 1.3×10^9 J
16. 4.3×10^9 kg/s

Chapter 11 Review

Problems for Understanding

18. $0.87c$

19. (a) 3.2 m (c) 6.8×10^{-8} s
(b) 1.9 m

20. (a) 2.5×10^{-27} kg (b) 1.7×10^{-27} kg

21. plot

22. 3.0×10^2 m/s

23. (a) c (c) c
(b) c

24. (a) 3.2 (c) 16 m
(b) 5.8×10^{-8} s

25. 1.2×10^{-30} kg, which is 1.3 times its rest mass

26. (a) 4.1×10^{-20} J (b) 4.1×10^{-16} J (c) 1.3×10^{-14} J (d) 5.0×10^{-13} J (e) (a) and (b)

27. $0.14c = 4.2 \times 10^{7}$ m/s

28. 3×10^{4} light bulbs

29. 4.8×10^{-30} kg; $m/m_o = 5.3$; $0.98c = 2.9 \times 10^{8}$ m/s

30. (a) 1.4 g (b) 29% or 0.40 g

Chapter 12

Practice Problems

1. 4.28×10^{-34} kg m/s
2. 9.44×10^{-22} kg m/s
3. 4.59×10^{-15} m
4. 3.66×10^{25} photons
5. 1.11×10^{10} Hz; radio
6. 1.05×10^{-13} m
7. 7.80×10^{-15} m
8. 1.04×10^{-32} m
9. 2.39×10^{-41} m
10. 5.77×10^{-12} m
11. 2.19×10^{6} m/s

Chapter 12 Review

Problems for Understanding

32. (a) 1.24×10^{15} Hz (b) threshold frequency

33. (a) 2.900 eV (b) lithium

34. 1.5×10^{15} Hz

35. 2.2 eV

36. 5.8×10^{18} photons/s

37. (a) 1.2×10^{-27} kg m/s (b) 1.3×10^{-27} kg m/s (c) 9.92×10^{-26} kg m/s

38. 1.7×10^{17} Hz

39. 5.5×10^{-33} kg m/s

40. 7.0×10^{-27} kg

41. (a) 3.1×10^{-7} m (b) 6.14×10^{-10} m (c) 4.7×10^{-24} kg m/s

42. (a) 4.8×10^{-10} m (b) -1.5 eV

43. 486 nm

44. (a) 3.08×10^{15} Hz (b) 97.3 nm (c) -0.850 eV $= -1.36 \times 10^{-19}$ J (d) 0.847 nm (e) 487 nm

Chapter 13

Practice Problems

1. 0.060 660 00 u $= 1.0073 \times 10^{-28}$ kg
2. 1.237×10^{-12} J
3. 2.858×10^{-10} J
4. 2.6×10^{9} a
5. 3.5×10^{3} a
6. 8.49×10^{-8} mg
7. 0.141 68 u $= 2.3527 \times 10^{-28}$ kg; 2.114×10^{-11} J
8. 2.818×10^{-12} J
9. (a) 0.0265 u $= 4.40 \times 10^{-29}$ kg; 3.96×10^{-12} J (b) 5.96×10^{11} J

Chapter 13 Review

Problems for Understanding

20. (a) $20p$, $20n$, $18e$ (b) $26p$, $30n$, $26e$ (c) $16p$, $18n$, $17e$

21. (a) 1.477×10^{-11} J (b) 1.793×10^{-10} J

22. ${}^{230}_{90}\mathrm{Th} \rightarrow {}^{4}_{2}\mathrm{He} + {}^{226}_{88}\mathrm{Ra}$

23. (a) 1/4 (b) 1/16 (c) 1/4096

24. (a) 4.876 MeV (b) $v_{He} = 1.520 \times 10^{7}$ m/s; $v_{Rn} = 2.740 \times 10^{5}$ m/s (c) 98%

25. 1.2×10^{-7} kg

26. 11.5 min

27. 1.2×10^{3} a

28. (a) 200 (b) 600 (c) 25 (d) 775 (e) ${}^{D}N(t) = {}^{P}N(0)(1 - (\frac{1}{2})^{t/T_{1/2}})$, where ${}^{D}N(t)$ is the number of daughter nuclei at any time, t, ${}^{P}N(0)$ is the number of parent nuclei at time, $t = 0$, and $T_{\frac{1}{2}}$ is the half-life of the parent nucleus.

29. (a) $R = 1/[2^{t/T_{1/2}} - 1]$ where R is the ratio of parent to daughter nuclei at any time. (b) 4.25×10^{9} a; 3.89×10^{9} a; 2.93×10^{9} a (c) Assuming that the samples were not polluted by having daughter nuclei present in the beginning, considering the large differences in ages, it is unlikely that these samples came from the same place. (d) More than one half-life has elapsed.

30. 2.4×10^{-12} m (assuming the electron and positron are at rest initially)

31. (a) 3.56×10^{-13} J $= 2.23$ MeV (b) $0.981c = 2.94 \times 10^{8}$ m/s (c) The electron travels faster than the speed of light in water, which is 2.25×10^{8} m/s, and consequently emits Cherenkov radiation, a phenomenon analogous to a sonic boom.

32. (a) $udd \rightarrow uud + e^{-} + \bar{\nu}_e$ or $d \rightarrow u + e^{-} + \bar{\nu}_e$ (b) $\gamma + udd \rightarrow \bar{u}d + uud$

Unit 5 Review

Problems for Understanding

42. (a) $0.14c$ (b) $0.045c$

43. (a) 9×10^{16} J (b) 3×10^{7} a

44. (a) 3.1 light-year (b) 4.7 a (c) 6.3 a

45. (a) 1.1×10^{-13} J (b) $1.3 \times$ rest mass energy (c) 2.1×10^{-30} kg or $2.3 \times$ rest mass

46. (a) 3×10^{9} J (b) 4×10^{-8} kg

47. 1.12eV $= 1.80 \times 10^{-19}$ J

48. 4.7eV $= 7.5 \times 10^{-19}$ J

49. (a) 1.05×10^{15} Hz (b) 287 nm

50. (a) 1.25 nm (b) 0.153 nm

51. (a) 2.47×10^{15} Hz (b) 1.22×10^{-7} m (c) Lyman

52. 486 nm

53. (a) 3.0×10^{-19} J (b) 8.1×10^{17} photons

54. (a) 6.91×10^{14} Hz (b) 4.34×10^{-7} m (c) -0.544 eV $= -8.70 \times 10^{-20}$ J (d) 1.32 nm (e) 9.49×10^{-8} m (f) UV

55. (a) 3.96×10^{-12} J/reaction (b) 9.68×10^{37} reactions/s (c) 6.64×10^{-27} kg/reaction (d) 6.43×10^{11} kg/s (e) 9.82×10^{9} a

56. (a) 4.40×10^{-29} kg (b) 0.658% (c) 1.18×10^{45} J (d) 9.59×10^{9} a

57. 88.2 N

58. 1.9 GeV, 6.6×10^{-16} m

A

action at a distance the force between two objects not in contact (7.2)

air resistance friction due to the motion of an object through air; proportional to the object's velocity (1.3)

alpha particle one or more helium nuclei ejected from a radioactive nucleus (13.2)

antimatter matter composed of antiparticles, which have the same mass but opposite charge, and/or other properties, compared to particles (13.2)

antineutrino a chargeless, very low-mass particle involved in weak interactions (13.2)

apparent weight the weight measured by a scale; same as true weight, unless the object is accelerating (1.3)

atomic mass number the number (A) that represents the total number of protons and neutrons in an atomic nucleus (13.1)

atomic mass unit the value of mass equal to mass of the most common carbon isotope ($^{12}_{6}C$) divided by 12; 1 u = 1.6605×10^{-27} kg (13.1)

atomic number the number (Z) that represents the number of protons in the nucleus; also represents the charge of the nucleus in units of e (13.1)

B

Balmer series spectral lines of hydrogen that lie in the visible wavelength range (12.3)

baryon a subset of the hadron family, such as the proton and neutron, that are composed of combinations of three quarks (13.3)

beta particle high-speed electrons or positrons ejected from a radioactive nucleus (13.2)

betatron a cyclotron modified to accelerate electrons through magnetic induction, instead of using electric fields (8.3)

binding energy 1. the amount of additional energy an object needs to escape from a planet or star (6.1) 2. the amount of energy that must be supplied to nuclear particles in order to separate them (13.1)

blackbody an object that absorbs and emits all radiation of all possible frequencies (12.1)

Bohr radius the distance from the nucleus of the lowest allowed energy level in the hydrogen atom: $r = 0.0529177$ nm (12.3)

C

centripetal acceleration the centre-directed acceleration of a body moving continuously along a circular path; the quotient of the square of the object's velocity and the radius of the circle (2.2)

centripetal force the centre-directed force required for an object to move in a circular path (2.2)

charge density the charge per unit area (8.1)

chemical symbol a shorthand symbol for an element (13.1)

circular orbit an orbit produced by a centripetal force (6.2)

classical physics the long-established parts of physics, including Newtonian mechanics electricity and magnetism, and thermodynamics, studied before the twentieth century (12.1)

closed system a system that can exchange energy with its surroundings, but not with matter (4.2)

coefficient of kinetic friction for two specific materials in contact, the ratio of the frictional force to the normal force between the surfaces when they are in relative motion (1.2)

coefficient of static friction for two specific materials in contact, the ratio of the frictional force to the normal force between the surfaces when they are not moving relative to each other (1.2)

coherent light that is in phase (the maxima and minima occur at the same time and place) (9.2)

combustion chamber the part of an engine where gases are burned (e.g., a jet engine) (6.3)

Compton effect a phenomenon involving the scattering of an X-ray photon with a "free" electron, in which, through conservation of energy and momentum, some of the photon's energy is transferred to the electron (12.2)

conservation of mechanical energy the change in the total mechanical energy (kinetic plus potential) of an isolated system is zero (5.1)

conservation of momentum the total momentum of two objects before a collision is the same as the total momentum of the same two objects after they collide (4.2)

conservative force a force that does work on an object in such a way that the amount of work done is independent of the path taken (5.3)

constructive interference a situation in which a combined or resultant wave has a larger amplitude than either of its component waves (9.1)

coordinate system consists of perpendicular axes that define an origin or zero position and dimensions (1.1)

Coulomb's constant the proportionality constant in Coulomb's law: $k = 9 \times 10^9\ \mathrm{N \bullet m^2/C^2}$ (7.1)

Coulomb's law the force between charges at rest, proportional to the magnitudes of the charges and inversely proportional to the square of the distance between their centres (7.1)

counterweight a heavy, movable mass that balances another mass (1.3)

cyclotron a particle accelerator that subjects particles in a circular path to a large number of small increases in potential in order to accelerate them (8.3)

D

daughter nucleus the nucleus remaining after a transmutation reaction (13.2)

de Broglie wavelength the wavelength associated with a particle; the quotient of Planck's constant and the momentum of the particle (12.2)

destructive interference a situation in which a combined or resultant wave has a smaller amplitude than at least one of its component waves (9.1)

deuterium an isotope of hydrogen, consisting of a proton and neutron in the nucleus (13.1)

diffraction the bending of waves around a barrier (9.1)

diffraction grating a device for producing spectra by diffraction and for the measurement of wavelength (9.3)

dilated time the time measured by an observer who sees a clock that is in a frame of reference that is moving relative to the observer (11.2)

dispersion the separation of light into its range of colours (9.1)

doubly refractive having a different refractive index, depending on the polarization of the light (10.1)

dynamics the study of the motions of bodies while considering their masses and the responsible forces; simply, the study of *why* objects move the way they do (1.2)

E

elastic collision a collision in which both momentum and kinetic energy are conserved (4.3)

elastic potential energy a form of energy that accumulates when an elastic object is bent, stretched, or compressed (5.2)

electric field intensity the quotient of the electric force on a unit charge located at that point (7.2)

electric field a region in space that influences electric charges in that region (7.2)

electric field lines imaginary directed lines that indicate the direction a tiny point charge with zero mass would follow if free to move in the electric field; these lines radiate away from positive charges and toward negative charges (7.2)

electric permittivity a number that characterizes a material's ability to resist the formation of an electric field in it (10.1)

electric potential difference the work done per unit charge between two locations (7.3)

electromagnetic force an infinite range force that operates between all charged particles (13.3)

electromagnetic spectrum the range of frequencies of electromagnetic waves (10.3)

electromagnetic wave a wave consisting of changing electric and magnetic fields (10.1)

electron an elementary particle with negative charge and a mass of 9.11×10^{-31} kg (13.1)

electron volt the energy gained by one electron as it falls through a potential difference of one volt: $1\ \mathrm{eV} = 1.60 \times 10^{-19}\ \mathrm{J}$ (12.1)

electrostatic force the force between charges at rest; see also: Coulomb's law (7.1)

electroweak force the fundamental force from which the electromagnetic and weak nuclear forces are derived (13.3)

elementary charge the basic unit of charge: $e = \pm 1.602 \times 10^{-19}$ C (13.1)

elementary particle a stable particle that cannot be subdivided into smaller particles (13.3)

empirical equation an equation based on observed data and not on any theory (12.1)

energy the ability to do work (5.1)

equipotential surface a surface in which all points have the same electric or gravitational potential (7.3)

escape energy the amount of energy required for an object to escape from the gravitational force of a planet or star and not return (6.1)

escape speed the minimum speed at the surface of a planet that will allow an object to leave the planet (6.1)

exhaust velocity the backward velocity of the gas ejected from the combustion chamber of a rocket relative to the combustion chamber (6.3)

external force any force exerted by an object that is not part of the system on an object within the system (4.2)

F

Faraday cage a metal screen that is used to shield a region from an external electric field (8.2)

fictitious force a force that must be invoked to explain motion in a non-inertial frame of reference (1.1)

field a region in space that influences a mass, charge, or magnet placed in the region (7.2)

frame of reference a subset of the physical world defined by an observer in which positions or motions can be discussed or compared (1.1)

Fraunhofer diffraction diffraction produced by plane wavefronts of a parallel beam of light (9.2)

free-body diagram a diagram in which all of the forces acting on an object are shown as acting on a point representing the object (1.2)

free fall a situation in which gravity is the only force acting on an object (1.3)

Fresnel diffraction diffraction produced by curved wavefronts, such as that produced by a point source of light (9.2)

frictional forces forces that oppose motion between two surfaces in contact (1.2)

fringe a bright or dark band produced by interference of light (9.2)

G

gamma (γ) an abbreviation of an expression that is used in equations for length contraction and time dilation: $\gamma = \sqrt{1 - \frac{v^2}{c^2}}$ (11.2)

gamma ray high-frequency radiation emitted from a radioactive nucleus (13.2)

geostationary orbit the orbit of a satellite around Earth's equator, which gives the satellite the appearance of hovering over the same spot on Earth's surface at all times (3.2)

gluons exchange particles responsible for holding quarks together (13.3)

gradient a change in a quantity relative to a change in position, or displacement (8.1)

gravitational assist or gravitational slingshot interaction typically between a spacecraft and a planet in which the planet loses a small amount of energy and the spacecraft gains a large amount of energy (6.3)

gravitational field intensity the quotient of the gravitational force and the magnitude of the test mass at a given point in a field; the product of the universal gravitation constant and mass, divided by the square of the distance of a given location from the centre of the object (7.2)

gravitational field lines imaginary directed lines that indicate the direction a tiny test mass would follow if free to move in the gravitational field; these lines radiate inward toward the mass that generates them (7.2)

gravitational force infinite range force that operates between all massive particles (13.3)

gravitational mass the property of matter that determines the strength of the gravitational force; compare to: inertial mass (1.1)

gravitational potential the gravitational potential energy per unit mass (7.3)

graviton exchange particle postulated to be responsible for the gravitational force (13.3)

ground state the lowest possible state that an electron can occupy in an atom (13.3)

H

hadron particles that contain quarks (13.3)

half-life the time in which the amount of a radioactive nuclide decays to half its original amount (13.2)

heat the transfer of thermal energy from one system to another due to their different temperatures (5.3)

heavy water water composed of molecules of oxygen and deuterium instead of oxygen and hydrogen (13.2)

Hooke's law states that the applied force is directly proportional to the amount of extension or compression of a spring (5.2)

Huygens' principle each point on a wavefront can be considered to be a source of a secondary wave, called a "wavelet," that spreads out in front of the wave at the same speed as the wave itself (9.1)

I

impulse the product of the force exerted on an object and the time interval over which the force acts (4.1)

impulse-momentum theorem states that the impulse is equal to the change in momentum of an object involved in an interaction (4.1)

inelastic collision a collision in which momentum is conserved, but kinetic energy is not conserved (4.3)

inertia the natural tendency of an object to stay at rest or in uniform motion in the absence of outside forces; proportional to an object's mass (1.1)

inertial frame of reference a frame of reference in which the law of inertia is valid; it is a non-accelerating frame of reference (1.1)

inertial mass the property of matter that resists a change in motion; compare to: gravitational mass (1.1)

interferometer an instrument for measuring wavelengths of light by allowing light beams to interfere with each other (11.1)

internal force any force exerted on an object in the system due to another object in the system (4.2)

inverse square law the relationship in which the force between two objects is inversely proportional to the square of the distance that separates the centres of the objects; for example, the gravitational and electrostatic forces (7.1)

ion an electrically charged atom or molecule (13.1)

ionizing radiation radiation of sufficient energy to liberate the electrons from the atoms or molecules (13.2)

isolated system a system that does not exchange either matter or energy with its surroundings (4.2)

isotope two or more atoms of an element that have the same number of protons but a different number of neutrons in their nuclei (13.1)

K

Kepler's laws three empirical relationships that describe the motion of planets (3.1)

kinematics the study of the motions of bodies without reference to mass or force; the study of *how* objects move in terms of displacement, velocity, and acceleration (1.2)

L

law of universal gravitation the force of gravity between any two objects is proportional to the product of their masses and inversely proportional to the square of the distance between their centres (3.1)

length contraction a consequence of special relativity, in which an object at rest in one frame of reference will appear to be shorter in the direction parallel to its motion in another frame of reference (11.2)

lepton particles, such as electrons and neutrinos, that do not contain quarks and do not take part in strong nuclear force interactions (13.3)

line spectrum (emission spectrum) a spectrum consisting of bright lines at specific wavelengths, produced by atoms of heated elements (9.3)

linear accelerator a particle accelerator that uses alternating electric fields to accelerate particles in stages (8.3)

Lorentz-Fitzgerald contraction contraction of an object in the direction of its motion (11.1)

M

magnetic field intensity the magnetic force acting on a unit length of a current-carrying wire placed at right angles to the magnetic field, measured in tesla (T) (7.2)

magnetic field lines imaginary directed lines that indicate the direction in which the N-pole of a compass would point when placed at that location; these lines radiate out of the magnet's N-pole and into its S-pole and form closed loops in the magnet (7.2)

magnetic permeability a number that characterizes a material's ability to become magnetized (10.1)

magnetic quantum number determines the orientation of the electron orbitals when the atom is placed in an external magnetic field (13.3)

magnetic resonance imaging a medical imaging technique for obtaining pictures of internal parts of the body in a non-invasive manner (10.2)

mass defect the difference between the mass of a nucleus and the sum of the masses of its constituent particles; the mass equivalent of the *binding energy* of a nucleus (13.1)

mass spectrometer an instrument that can separate streams of particles by mass and measure that mass by application of electric and magnetic deflecting fields (8.3)

mass-to-charge ratio the quotient of a particle's mass to its charge, which is easier to measure than either quantity individually (13.3)

Maxwell's equations a series of four related equations that summarize the behaviour of electric and magnetic fields and their interactions (10.1)

meson a particle composed of a quark and an antiquark (13.3)

microgravity the condition of apparent weightlessness (3.2)

modulation a process of adding data to an electromagnetic wave by changing the amplitude or frequency (10.2)

momentum the product of an object's mass and velocity (4.1)

N

neutrino a chargeless, very low-mass particle involved in weak interactions (13.2)

neutron a particle with zero charge, found in the nucleus of all atoms except the hydrogen atom (13.1)

nodal point a stationary point in a medium produced by destructive interference of two waves travelling in opposite directions (9.1)

non-conservative force a force that does work on an object in such a way that the amount of work done is dependent on the path taken (5.3)

non-elastic or plastic the description of a material that does not return precisely to its original form after the applied force is removed (5.2)

non-inertial frame of reference an accelerating frame of reference (1.1)

nuclear fission the splitting of a large nucleus into two or more lighter nuclei; usually caused by the impact of a neutron and accompanied by the release of energy (13.2)

nuclear fusion the formation of a larger nucleus from two or more lighter nuclei, accompanied by the release of energy (13.2)

nuclear model a model for the atom in which all of the positive charge and most of the mass are concentrated in the centre of the atom, while negatively charged electrons circulate well beyond this "nucleus" (12.3)

nucleon the collective term for a particle (proton and/or neutron) in the atomic nucleus (13.1)

nucleon number the total number of nucleons (protons and neutrons) in the nucleus; also called the "atomic mass number" (13.1)

nuclide the nucleus of a particular atom, as characterized by its atomic number and atomic mass number (13.1)

O

open system a system that can exchange both matter and energy with its surroundings (4.2)

orbital quantum number specifies the shape of an electron's orbital or energy level; has integer values of one less than the principal quantum number (12.3)

P

parabola a geometric figure formed by slicing a cone with a plane that is parallel to the axis of the cone (2.1)

parent nucleus the initial nucleus involved in a transmutation reaction (13.2)

particle accelerator an instrument capable of emitting beams of high-speed, subatomic-sized particles, such as protons and electrons (8.3)

Pauli exclusion principle states that no two electrons in the same atom can occupy the same state; alternatively, no two electrons in the same atom can have the same four quantum numbers (13.3)

periodic motion the motion of an object in a repeated pattern over regular time intervals (5.2)

perturbation deviation of a body in orbit from its regular path, caused by the presence of one or more other bodies (3.2)

photoelastic materials that exhibit doubly refractive properties while under mechanical stress (10.1)

photoelectric effect the emission of electrons from matter by radiation of certain frequencies (12.1)

photon a quantum of light or electromagnetic radiation (12.1)

pion a type of meson (13.3)

plane polarized light or an electromagnetic wave in which the vibrations of the electric field lie in one plane and are perpendicular to the direction of travel (10.1)

polarization the orientation of the oscillations in a transverse wave (10.1)

positron a particle with the same mass as the electron, but with a positive charge; an antielectron (13.2)

potential gradient the quotient of the electric potential difference between two points and the component of the displacement between the points that is parallel to the field (8.1)

principal quantum number describes the orbital or energy level of an electron in an atom (12.3)

projectile an object that is given an initial thrust and allowed to move through space under the force of gravity only (2.1)

proper length the length of an object measured by an observer at rest relative to the object (11.2)

proper time the duration of an event measured by an observer at rest relative to the event (11.2)

proton a positively charged particle found in the nucleus of all atoms (13.1)

Q

quantized a property of a system that occurs only in multiples of a minimum amount (12.1)

quantum a discrete amount of energy, given by the product of Planck's constant (h) and the frequency of the radiation (f): hf (12.1)

quark the family of six types of particles with charges of $\frac{1}{3}$ or $\frac{2}{3}$ of the elementary charge, which comprise all hadrons (13.3)

R

radioactive isotope (radioisotope) an isotope of an element that has an unstable nucleus and therefore disintegrates, emitting alpha, beta, or gamma radiation (13.2)

radioactive material material that contains radioactive nuclei (13.2)

radioactivity the spontaneous disintegration of the nuclei of certain elements, accompanied by the emission of alpha, beta, or gamma radiation (13.2)

range the horizontal distance a projectile travels (2.1)

Rayleigh criterion the criterion for resolution of two point sources, which states that the inner dark ring of one diffraction pattern should coincide with the centre of the second bright fringe (9.3)

reaction mass matter ejected backward from a rocket in order to propel it forward (6.3)

recoil the interaction that occurs when two stationary objects push against each other and then move apart (4.2)

relativistic speeds speeds close to the speed of light (11.2)

resolving power the ability of a telescope or microscope to distinguish objects that are close together (9.3)

rest mass the mass of an object measured by an observer at rest relative to the object (11.3)

restoring force the force exerted by a spring on an object; proportional to the amount of extension or compression of the spring (5.2)

Rydberg constant the constant of proportionality that relates the wavelength of a spectral line in the hydrogen atom and the difference of energy level numbers that produce it: $R = 1.09737315 \times 10^7 \text{m}^{-1}$ (12.3)

S

Schrödinger wave equation the basic quantum mechanical equation used to determine the properties of a particle (13.3)

simultaneity a concept that describes events that occur at the same time and in the same inertial reference frame (11.2)

spin quantum number specifies the orientation, up or down, of the electron's "spin"; has values $+\frac{1}{2}$ or $-\frac{1}{2}$ when placed in a magnetic field (13.3)

spring constant the amount of force a spring can exert per unit distance of extension or compression (5.2)

standard model a comprehensive model that describes subatomic particles, their properties, and the force particles that govern their interactions (13.3)

Stoke's law states that the drag force on a sphere moving through a liquid is proportional to the radius of the sphere and its velocity (8.1)

stopping potential in the photoelectric effect, the potential difference required to stop the emission of photoelectrons from the surface of a metal (12.1)

strong nuclear force the fundamental force that holds the parts of the nucleus together (13.1)

superposition of waves when two or more waves propagate through the same location in a medium, the resultant displacement of the medium will be the algebraic sum of the displacements caused by each wave (9.1)

synchrocyclotron a modified cyclotron, in which the frequency of the accelerating electric field is adjusted to allow for the relativistic mass increase of the particles (8.3)

synchrotron a cyclic particle accelerator that uses a series of magnets around the circular path and several high-frequency accelerating cavities (8.3)

system of particles an arbitrarily assigned group of objects (4.2)

T

tension the magnitude of the force exerted on and by a cable, rope, or string (1.3)

terminal velocity the velocity of a falling object at which the force of friction is equal in magnitude to the force of gravity (1.3)

test charge a charge of a magnitude that is small enough that it will not affect the field being measured; it is used to determine the strength of an electric field (7.2)

threshold frequency the lowest frequency of light (smallest photon energy) that can eject a photoelectron from a particular metal (12.1)

thrust the force with which gases ejected from a rocket push back on the rocket (6.3)

time dilation a consequence of special relativity in which two observers moving at constant velocity relative to each other will each observe the other's clock to have slowed down (11.2)

torsion balance a sensitive instrument for measuring the twisting forces in metal wires, consisting of an arm suspended from a fibre (7.1)

total energy the sum of the rest mass energy of a particle and its kinetic energy (11.3)

total orbital energy the sum of the mechanical (gravitational potential and kinetic) energies of an orbiting body (6.2)

trajectory the path described by an object moving due to a force or forces (2.1)

transmutation the conversion of one element into another, usually as a result of radioactive decay (13.2)

triangulation a geometrical method for determining distances through the measurement of one side and two angles of a right triangle (10.4)

tritium an isotope of hydrogen, consisting of a proton and two neutrons in the nucleus (13.1)

Tychonic system a planetary model in which the Sun and Moon revolve around Earth, but the other planets revolve around the Sun (3.1)

U

ultraviolet catastrophe the significant discrepancy at ultraviolet and higher frequencies between the predictions based on classical physics and observations of blackbody radiation (12.1)

uniform circular motion motion with constant speed in a circle (2.2)

uniform motion motion at a constant velocity (1.2)

uniformly accelerated motion motion under constant acceleration (1.2)

W

W^+, W^-, Z° bosons exchange particles responsible for the behaviour of the weak nuclear force (13.3)

wave function a mathematical expression that is a solution of the Schrödinger wave equation; describes the behaviour of a particle (13.3)

wave-particle duality both matter and radiation have wave-like properties and particle-like properties (12.2)

weak nuclear force a short-range interaction between elementary particles that is much weaker than the strong nuclear force and governs the process of beta decay; one of the four fundamental forces (13.3)

work the transfer of mechanical energy from one system to another; equivalent to a force acting through a distance (5.1)

work function in the photoelectric effect, the minimum amount of energy necessary to remove an electron from a metal surface (12.1)

work-kinetic energy theorem the relationship between the work done by a force on an object and the resulting change in kinetic energy: $W = \Delta E_k$ (5.1)

work-energy theorem the relationship between the work done on an object by a force and the resulting change in the object's potential and kinetic energy: $W = \Delta E_k + \Delta E_p$ (5.1)

Z

Zeeman effect the splitting of the spectral lines of an atom when it is placed in a magnetic field (12.3)

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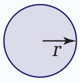
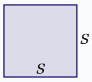
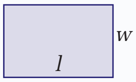
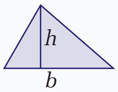
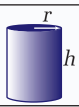
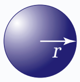
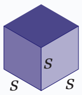
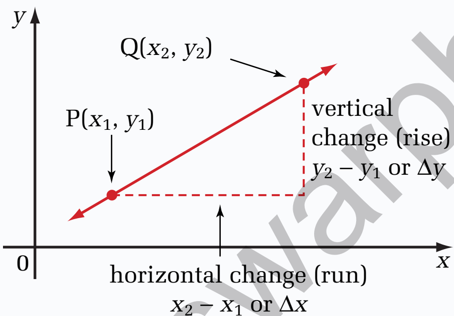
	Circumference/ perimeter	Area	Surface area	Volume
	$C = 2\pi r$	$A = \pi r^2$		
	$P = 4s$	$A = s^2$		
	$P = 2l + 2w$	$A = lw$		
		$A = \frac{1}{2}bh$		
			$SA = 2\pi rh + 2\pi r^2$	$V = \pi r^2 h$
			$SA = 4\pi r^2$	$V = \frac{4}{3}\pi r^3$
			$SA = 6s^2$	$V = s^3$

Slope (m)

Calculating the slope of a line

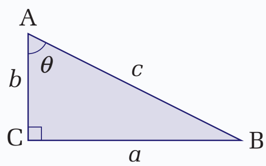


$$\text{slope } (m) = \frac{\text{vertical change (rise)}}{\text{horizontal change (run)}}$$

$$m = \Delta y/\Delta x$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}, x_2 \neq x_1$$

Trigonometric Ratios



$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{a}{c}$$

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{b}{c}$$

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{a}{b}$$

The Greek Alphabet

alpha	Α	α	iota	Ι	ι	rho	Ρ	ρ
beta	Β	β	kappa	Κ	κ	sigma	Σ	σ
gamma	Γ	γ	lambda	Λ	λ	tau	Τ	τ
delta	Δ	δ	mu	Μ	μ	upsilon	Υ	υ
epsilon	Ε	ε	nu	Ν	ν	phi	Φ	ϕ
zeta	Ζ	ζ	xi	Ξ	ξ	chi	Χ	χ
eta	Η	η	omicron	Ο	o	psi	Ψ	ψ
theta	Θ	θ	pi	Π	π	omega	Ω	ω

Fundamental Physical Constants

Quantity	Symbol	Accepted value
speed of light in a vacuum	c	2.998×10^8 m/s
gravitational constant	G	6.673×10^{-11} N · m^2/kg^2
Coulomb's constant	k	8.988×10^9 N · m^2/C^2
charge on an electron	e	1.602×10^{-19} C
rest mass of an electron	m_e	9.109×10^{-31} kg
rest mass of a proton	m_p	1.673×10^{-27} kg
rest mass of a neutron	m_n	1.675×10^{-27} kg
atomic mass unit	u	1.661×10^{-27} kg
Planck's constant	h	6.626×10^{-34} J · s

Metric System Prefixes

Prefix	Symbol	Factor
tera	T	1 000 000 000 000 = 10^{12}
giga	G	1 000 000 000 = 10^9
mega	M	1 000 000 = 10^6
kilo	k	1000 = 10^3
hecto	h	100 = 10^2
deca	da	10 = 10^1
		1 = 10^0
deci	d	0.1 = 10^{-1}
centi	c	0.01 = 10^{-2}
milli	m	0.001 = 10^{-3}
micro	μ	0.000 001 = 10^{-6}
nano	n	0.000 000 001 = 10^{-9}
pico	p	0.000 000 000 001 = 10^{-12}
femto	f	0.000 000 000 000 001 = 10^{-15}
atto	a	0.000 000 000 000 000 001 = 10^{-18}

Other Physical Data

Quantity	Symbol	Accepted value
standard atmospheric pressure	P	1.013×10^5 Pa
speed of sound in air		343 m/s (at 20°C)
water: density (4°C)		1.000×10^3 kg/m^3
latent heat of fusion		3.34×10^5 J/kg
latent heat of vaporization		2.26×10^6 J/kg
specific heat capacity (15°C)		4186 J/(kg°C)
kilowatt hour	E	3.6×10^6 J
acceleration due to Earth's gravity	g	9.81 m/s^2 (standard value; at sea level)
mass of Earth	m_E	5.98×10^{24} kg
mean radius of Earth	r_E	6.38×10^6 m
mean radius of Earth's orbit	R_E	1.49×10^{11} m
period of Earth's orbit	T_E	365.25 days or 3.16×10^7 s
mass of Moon	m_M	7.36×10^{22} kg
mean radius of Moon	r_M	1.74×10^6 m
mean radius of Moon's orbit	R_M	3.84×10^8 m
period of Moon's orbit	T_M	27.3 days or 2.36×10^6 s
mass of Sun	m_s	1.99×10^{30} kg
radius of Sun	r_s	6.96×10^8 m

Derived Units

Quantity	Quantity symbol	Unit	Unit symbol	Equivalent unit(s)
area	A	square metre	m^2	
volume	V	cubic metre	m^3	
velocity	v	metre per second	m/s	
acceleration	a	metre per second per second	m/s^2	
force	F	newton	N	$kg \cdot m/s^2$
work	W	joule	J	$N \cdot m$, $kg \cdot m^2/s^2$
energy	E	joule	J	$N \cdot m$, $kg \cdot m^2/s^2$
power	P	watt	W	J/s, $kg \cdot m^2/s^3$
density	ρ	kilogram per cubic metre	kg/m^3	
pressure	p	pascal	Pa	N/m^2, kg/s^2
frequency	f	hertz	Hz	s^{-1}
period	T	second	s	
wavelength	λ	metre	m	
electric charge	Q	coulomb	C	$A \cdot s$
electric potential difference	V	volt	V	W/A, J/C, $kg \cdot m^2/(C \cdot s^2)$
resistance	R	ohm	Ω	V/A, $kg \cdot m^2/(C^2 \cdot s)$
magnetic field strength	B	tesla	T	$N \cdot s/(C \cdot m)$, $N/A \cdot m$
magnetic flux	Φ	weber	Wb	$V \cdot s$, $T \cdot m^2$, $m^2 \cdot kg/(C \cdot s)$
radioactivity	$\Delta N/\Delta t$	becquerel	Bq	s^{-1}
radiation dose		gray	Gy	$J/kg \cdot m^2/s^2$
radiation dose equivalent		sievert	Sv	$J/kg \cdot m^2/s^2$
temperature (Celsius)	T	degree Celsius	°C	T°C = (T + 273.15) K
		atomic mass unit	u	$1u = 1.660\,566 \times 10^{-27}$ kg
		electron volt	eV	$1\ eV = 1.602 \times 10^{-19}$ J

Electromagnetic Spectrum