McGraw-Hill Ryerson

Physics 12

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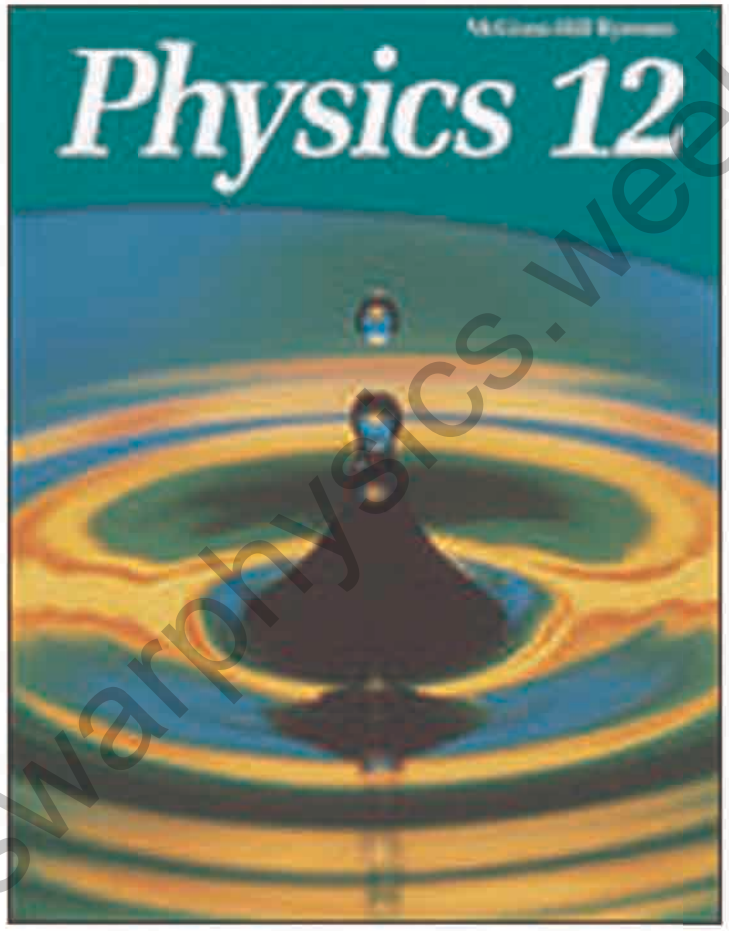


UNIT 1

Forces and Motion: Dynamics

OVERALL EXPECTATIONS

ANALYZE, predict, and explain the motion of selected objects in vertical, horizontal, and inclined planes.

INVESTIGATE, represent, and analyze motion and forces in linear, projectile, and circular motion.

RELATE your understanding of dynamics to the development and use of motion technologies.

UNIT CONTENTS

Spectators are mesmerized by trapeze artists making perfectly timed releases, gliding through graceful arcs, and intersecting the paths of their partners. An error in timing and a graceful arc could become a trajectory of panic. Trapeze artists know that tiny differences in height, velocity, and timing are critical. Swinging from a trapeze, the performer forces his body from its natural straight-line path. Gliding freely through the air, he is subject only to gravity. Then, the outstretched hands of his partner make contact, and the performer is acutely aware of the forces that change his speed and direction.

In this unit, you will explore the relationship between motion and the forces that cause it and investigate how different perspectives of the same motion are related. You will learn how to analyze forces and motion, not only in a straight line, but also in circular paths, in parabolic trajectories, and on inclined surfaces. You will discover how the motion of planets and satellites is caused, described, and analyzed.

UNIT PROJECT PREP

Refer to pages 126–127 before beginning this unit. In the unit project, you will design and build a working catapult to launch small objects through the air.

- What launching devices have you used, watched, or read about? How do they develop and control the force needed to propel an object?
- What projectiles have you launched? How do you direct their flight so that they reach a maximum height or stay in the air for the longest possible time?

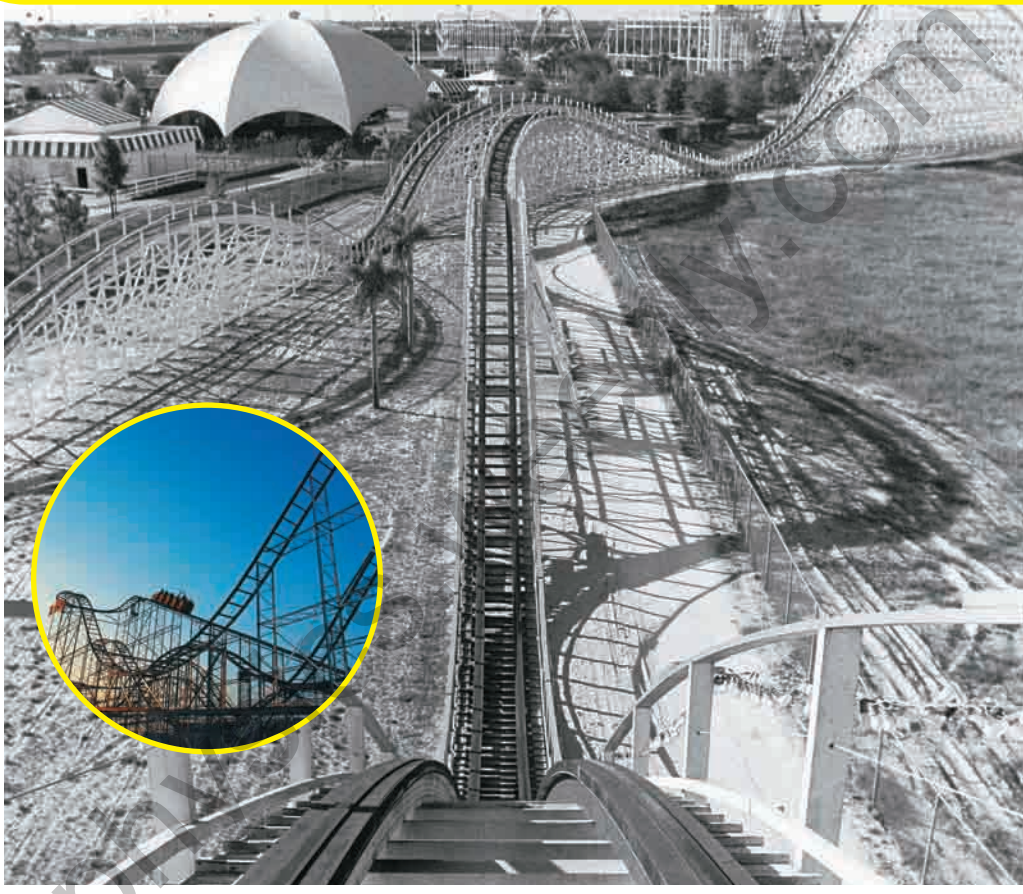
CHAPTER 1

Fundamentals of Dynamics

CHAPTER CONTENTS

PREREQUISITE CONCEPTS AND SKILLS

- Using the kinematic equations for uniformly accelerated motion.



How many times have you heard the saying, "It all depends on your perspective"? The photographers who took the two pictures of the roller coaster shown here certainly had different perspectives. When you are on a roller coaster, the world looks and feels very different than it does when you are observing the motion from a distance. Now imagine doing a physics experiment from these two perspectives, studying the motion of a pendulum, for example. Your results would definitely depend on your perspective or frame of reference. You can describe motion from any frame of reference, but some frames of reference simplify the process of describing the motion and the laws that determine that motion.

In previous courses, you learned techniques for measuring and describing motion, and you studied and applied the laws of motion. In this chapter, you will study in more detail how to choose and define frames of reference. Then, you will extend your knowledge of the dynamics of motion in a straight line.

MULTI LAB

Thinking Physics

TARGET SKILLS
- Predicting
- Identifying variables
- Analyzing and interpreting

Suspended Spring

Tape a plastic cup to one end of a short section of a large-diameter spring, such as a Slinky™. Hold the other end of the spring high enough so that the plastic cup is at least 1 m above the floor. Before you release the spring, predict the exact motion of the cup from the instant that it is released until the moment that it hits the floor. While your partner watches the cup closely from a kneeling position, release the top of the spring. Observe the motion of the cup.

Analyze and Conclude

1. Describe the motion of the cup and the lower end of the spring. Compare the motion to your prediction and describe any differences.
2. Is it possible for any unsupported object to be suspended in midair for any length of time? Create a detailed explanation to account for the behaviour of the cup at the moment at which you released the top of the spring.
3. Athletes and dancers sometimes seem to be momentarily suspended in the air. How might the motion of these athletes be related to the spring's movement in this lab?

Thought Experiments

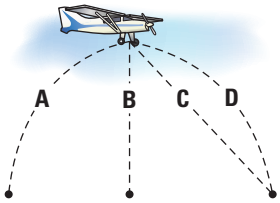
Without discussing the following questions with anyone else, write down your answers.

1. Student A and Student B sit in identical office chairs facing each other, as illustrated. Student A, who is heavier than Student B, suddenly pushes with his feet, causing both chairs to move. Which of the following occurs?

   (a) Neither student applies a force to the other.
   (b) A exerts a force that is applied to B, but A experiences no force.
   (c) Each student applies a force to the other, but A exerts the larger force.
   (d) The students exert the same amount of force on each other.

2. A golf pro drives a ball through the air. What force(s) is/are acting on the golf ball for the *entirety* of its flight?
   (a) force of gravity only
   (b) force of gravity and the force of the "hit"
   (c) force of gravity and the force of air resistance
   (d) force of gravity, the force of the "hit," and the force of air resistance

3. A photographer accidentally drops a camera out of a small airplane as it flies horizontally. As seen from the ground, which path would the camera most closely follow as it fell?



Analyze and Conclude

Tally the class results. As a class, discuss the answers to the questions.

1.1 Inertia and Frames of Reference

SECTION EXPECTATIONS

- Describe and distinguish between inertial and non-inertial frames of reference.
- Define and describe the concept and units of mass.
- Investigate and analyze linear motion, using vectors, graphs, and free-body diagrams.

KEY TERMS

- inertia
- inertial mass
- gravitational mass
- coordinate system
- frame of reference
- inertial frame of reference
- non-inertial frame of reference
- fictitious force

Imagine watching a bowling ball sitting still in the rack. Nothing moves; the ball remains totally at rest until someone picks it up and hurls it down the alley. Galileo Galilei (1564–1642) and later Sir Isaac Newton (1642–1727) attributed this behaviour to the property of matter now called **inertia**, meaning resistance to changes in motion. Stationary objects such as the bowling ball remain motionless due to their inertia.

Now picture a bowling ball rumbling down the alley. Experience tells you that the ball might change direction and, if the alley was long enough, it would slow down and eventually stop. Galileo realized that these changes in motion were due to factors that interfere with the ball's "natural" motion. Hundreds of years of experiments and observations clearly show that Galileo was correct. Moving objects continue moving in the same direction, at the same speed, due to their inertia, unless some external force interferes with their motion.

Figure 1.1 You assume that an inanimate object such as a bowling ball will remain stationary until someone exerts a force on it. Galileo and Newton realized that this "lack of motion" is a very important property of matter.

Analyzing Forces

Newton refined and extended Galileo's ideas about inertia and straight-line motion at constant speed — now called "uniform motion."

NEWTON'S FIRST LAW: THE LAW OF INERTIA

An object at rest or in uniform motion will remain at rest or in uniform motion unless acted on by an external force.

Newton's first law states that a force is required to *change* an object's uniform motion or velocity. Newton's second law then permits you to determine how great a force is needed in order to change an object's velocity by a given amount. Recalling that acceleration is defined as the change in velocity, you can state Newton's second law by saying, "The *net* force ($\vec{F}$) required to accelerate an object of mass m by an amount ($\vec{a}$) is the product of the mass and acceleration."

LANGUAGE LINK

The Latin root of *inertia* means "sluggish" or "inactive." An *inertial guidance system* relies on a gyroscope, a "sluggish" mechanical device that resists a change in the direction of motion. What does this suggest about the chemical properties of an *inert gas*?

NEWTON'S SECOND LAW

The word equation for Newton's second law is: Net force is the product of mass and acceleration.

$$\vec{F} = m\vec{a}$$

Quantity	Symbol	SI unit
force	$\vec{F}$	N (newtons)
mass	m	kg (kilograms)
acceleration	$\vec{a}$	$\frac{\text{m}}{\text{s}^2}$ (metres per second squared)

Unit analysis

$$\text{(mass)(acceleration)} = \text{(kilogram)}\left(\frac{\text{metres}}{\text{second}^2}\right) \text{kg}\frac{\text{m}}{\text{s}^2} = \frac{\text{kg}\cdot\text{m}}{\text{s}^2} = \text{N}$$

Note: The force ($\vec{F}$) in Newton's second law refers to the vector sum of all of the forces acting on the object.

Inertial Mass

When you compare the two laws of motion, you discover that the first law identifies inertia as the property of matter that resists a change in its motion; that is, it resists acceleration. The second law gives a quantitative method of finding acceleration, but it does not seem to mention inertia. Instead, the second law indicates that the property that relates force and acceleration is mass.

Actually, the mass (m) used in the second law is correctly described as the **inertial mass** of the object, the property that resists a change in motion. As you know, matter has another property — it experiences a gravitational attractive force. Physicists refer to this property of matter as its **gravitational mass.** Physicists never assume that two seemingly different properties are related without thoroughly studying them. In the next investigation, you will examine the relationship between inertial mass and gravitational mass.

INVESTIGATION 1-A

Measuring Inertial Mass

TARGET SKILLS

- Hypothesizing
- Performing and recording
- Analyzing and interpreting

Problem

Is there a direct relationship between an object's inertial mass and its gravitational mass?

Hypothesis

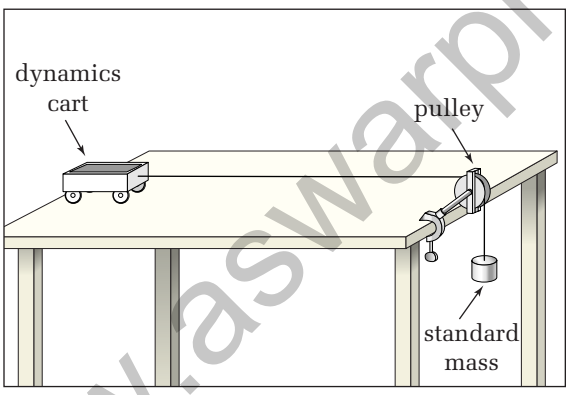
Formulate an hypothesis about the relationship between inertial mass and its gravitational mass.

Equipment

- dynamics cart
- pulley and string
- laboratory balance
- standard mass (about 500 g)
- metre stick and stopwatch *or* motion sensor
- unit masses (six identical objects, such as small C-clamps)
- unknown mass (measuring between one and six unit masses, such as a stone)

Procedure

1. Arrange the pulley, string, standard mass, and dynamics cart on a table, as illustrated.



2. Set up your measuring instruments to determine the acceleration of the cart when it is pulled by the falling standard mass. Find the acceleration directly by using computer software, or calculate it from measurements of displacement and time.
3. Measure the acceleration of the empty cart.
4. Add unit masses one at a time and measure the acceleration several times after each addition. Average your results.
5. Graph the acceleration versus the number of unit inertial masses on the cart.
6. Remove the unit masses from the cart and replace them with the unknown mass, then measure the acceleration of the cart.
7. Use the graph to find the inertial mass of the unknown mass (in unit inertial masses).
8. Find the gravitational mass of one unit of inertial mass, using a laboratory balance.
9. Add a second scale to the horizontal axis of your graph, using standard gravitational mass units (kilograms).
10. Use the second scale on the graph to predict the gravitational mass of the unknown mass.
11. Verify your prediction: Find the unknown's gravitational mass on a laboratory balance.

Analyze and Conclude

1. Based on your data, are inertial and gravitational masses equal, proportional, or independent?
2. Does your graph fit a linear, inverse, exponential, or radical relationship? Write the relationship as a proportion ($a \propto ?$).
3. Write Newton's second law. Solve the expression for acceleration. Compare this expression to your answer to question 2. What inferences can you make?
4. Extrapolate your graph back to the vertical axis. What is the significance of the point at which your graph now crosses the axis?
5. Verify the relationship you identified in question 2 by using curve-straightening techniques (see Skill Set 4, Mathematical Modelling and Curve Straightening). Write a specific equation for the line in your graph.

Over many years of observations and investigations, physicists concluded that inertial mass and gravitational mass were two different manifestations of the same property of matter. Therefore, when you write *m* for mass, you do not have to specify what type of mass it is.

Action-Reaction Forces

Newton's first and second laws are sufficient for explaining and predicting motion in many situations. However, you will discover that, in some cases, you will need Newton's third law. Unlike the first two laws that focus on the forces acting on one object, Newton's third law considers two objects exerting forces on each other. For example, when you push on a wall, you can feel the wall pushing back on you. Newton's third law states that this condition always exists — when one object exerts a force on another, the second force always exerts a force on the first. The third law is sometimes called the "law of action-reaction forces."

NEWTON'S THIRD LAW

For every action force on an object (B) due to another object (A), there is a reaction force, equal in magnitude but opposite in direction, on object A, due to object B.

$$\vec{F}_{\text{A on B}} = -\vec{F}_{\text{B on A}}$$

To avoid confusion, be sure to note that the forces described in Newton's third law refer to two different objects. When you apply Newton's second law to an object, you consider only one of these forces — the force that acts *on* the object. You do *not* include any forces that the object itself exerts on something else. If this concept is clear to you, you will be able to solve the "horse-cart paradox" described below.

Conceptual Problem

- The famous horse-cart paradox asks, "If the cart is pulling on the horse with a force that is equal in magnitude and opposite in direction to the force that the horse is exerting on the cart, how can the horse make the cart move?" Discuss the answer with a classmate, then write a clear explanation of the paradox.

QUICK LAB

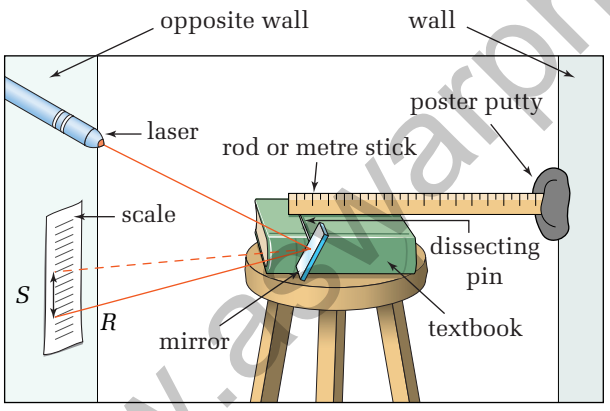
Bend a Wall

TARGET SKILLS
- Initiating and planning
- Performing and recording
- Analyzing and interpreting

Sometimes it might not seem as though an object on which you are pushing is exhibiting any type of motion. However, the proper apparatus might detect some motion. Prove that you can move — or at least, bend — a wall.

CAUTION Do not look into the laser.

Glue a small mirror to a 5 cm T-head dissecting pin. Put a textbook on a stool beside the wall that you will attempt to bend. Place the pin-mirror assembly on the edge of the textbook. As shown in the diagram, attach a metre stick to the wall with putty or modelling clay and rest the other end on the pin-mirror assembly. The pin-mirror should act as a roller, so that any movement of the metre stick turns the mirror slightly. Place a laser pointer so that its beam reflects off the mirror and onto the opposite wall. Prepare a linear scale on a sheet of paper and fasten it to the opposite wall, so that you can make the required measurements.



Push hard on the wall near the metre stick and observe the deflection of the laser spot. Measure
- the radius of the pin (r)
- the deflection of the laser spot (S)
- the distance from the mirror to the opposite wall (R)

Analyze and Conclude

1. Calculate the extent of the movement (s) — or how much the wall "bent" — using the formula $s = \frac{rS}{2R}$.
2. If other surfaces behave as the wall does, list other situations in which an apparently inflexible surface or object is probably moving slightly to generate a resisting or supporting force.
3. Do your observations "prove" that the wall bent? Suppose a literal-minded observer questioned your results by claiming that you did not actually see the wall bend, but that you actually observed movement of the laser spot. How would you counter this objection?
4. Is it scientifically acceptable to use a mathematical formula, such as the one above, without having derived or proved it? Justify your response.
5. If you have studied the arc length formula in mathematics, try to derive the formula above. (Hint: Use the fact that the angular displacement of the laser beam is actually twice the angular displacement of the mirror.)

Apply and Extend

6. Imagine that you are explaining this experiment to a friend who has not yet taken a physics course. You tell your friend that "When I pushed on the wall, the wall pushed back on me." Your friend says, "That's silly. Walls don't push on people." Use the laws of physics to justify your original statement.
7. Why is it logical to expect that a wall will move when you push on it?
8. Dentists sometimes check the health of your teeth and gums by measuring tooth mobility. Design an apparatus that could be used to measure tooth mobility.

Frames of Reference

In order to use Newton's laws to analyze and predict the motion of an object, you need a reference point and definitions of distance and direction. In other words, you need a **coordinate system.** One of the most commonly used systems is the Cartesian coordinate system, which has an origin and three mutually perpendicular axes to define direction.

Once you have chosen a coordinate system, you must decide where to place it. For example, imagine that you were studying the motion of objects inside a car. You might begin by gluing metre sticks to the inside of the vehicle so you could precisely express the positions of passengers and objects relative to an origin. You might choose the centre of the rearview mirror as the origin and then you could locate any object by finding its height above or below the origin, its distance left or right of the origin, and its position in front of or behind the origin. The metre sticks would define a coordinate system for measurements within the car, as shown in Figure 1.2. The car itself could be called the **frame of reference** for the measurements. Coordinate systems are always attached to or located on a frame of reference.

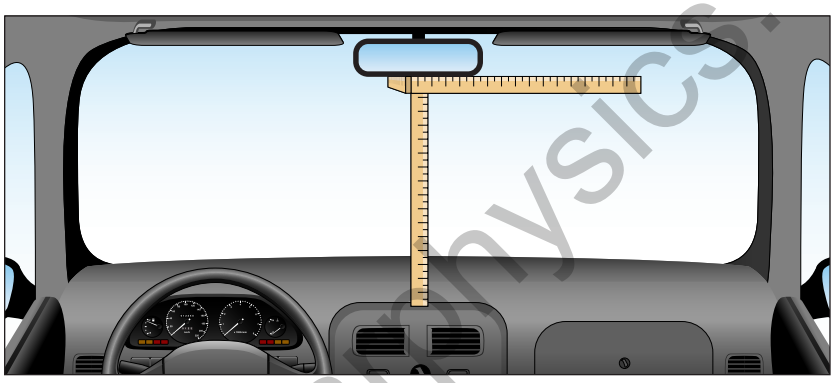


Figure 1.2 Establishing a coordinate system and defining a frame of reference are fundamental steps in motion experiments.

An observer in the car's frame of reference might describe the motion of a person in the car by stating that "The passenger did not move during the entire trip." An observer who chose Earth's surface as a frame of reference, however, would describe the passenger's motion quite differently: "During the trip, the passenger moved 12.86 km." Clearly, descriptions of motion depend very much on the chosen frame of reference. Is there a right or wrong way to choose a frame of reference?

The answer to the above question is no, there is no right or wrong choice for a frame of reference. However, some frames of reference make calculations and predictions much easier than do others. Think again about the coordinate system in the car. Imagine that you are riding along a straight, smooth road at a constant velocity. You are almost unaware of any motion. Then

COURSE CHALLENGE

Reference Frames

A desire to know your location on Earth has made GPS receivers very popular. Discussion about location requires the use of frames of reference concepts. Ideas about frames of reference and your *Course Challenge* are cued on page 603 of this text.

PHYSICS FILE

Albert Einstein used the equivalence of inertial and gravitational mass as a foundation of his general theory of relativity, published in 1916. According to Einstein's principle of equivalence, if you were in a laboratory from which you could not see outside, you could not make any measurements that would indicate whether the laboratory (your frame of reference) was stationary on Earth's surface or in space and accelerating at a value that was locally equal to *g*.

the driver suddenly slams on the brakes and your upper body falls forward until the seat belt stops you. In the frame of reference of the car, you were initially at rest and then suddenly began to accelerate.

According to Newton's first law, a force is necessary to cause a mass — your body — to accelerate. However, in this situation you cannot attribute your acceleration to any observable force: No object has exerted a force on you. The seat belt stopped your motion relative to the car, but what started your motion? It would appear that your motion relative to the car did not conform to Newton's laws.

The two stages of motion during the ride in a car — moving with a constant velocity or accelerating — illustrate two classes of frames of reference. A frame of reference that is at rest or moving at a constant velocity is called an **inertial frame of reference**.

When you are riding in a car that is moving at a constant velocity, motion inside the car seems similar to motion inside a parked car or even in a room in a building. In fact, imagine that you are in a laboratory inside a truck's semitrailer and you cannot see what is happening outside. If the truck and trailer ran perfectly smoothly, preventing you from feeling any bumps or vibrations, there are no experiments that you could conduct that would allow you to determine whether the truck and trailer were at rest or moving at a constant velocity. The law of inertia and Newton's second and third laws apply in exactly the same way in all inertial frames of reference.

Now think about the point at which the driver of the car abruptly applied the brakes and the car began to slow. The velocity was changing, so the car was accelerating. An accelerating frame of reference is called a **non-inertial frame of reference**. Newton's laws of motion do *not* apply to a non-inertial frame of reference. By observing the motion of the car and its occupant from outside the car (that is, from an inertial frame of reference, as shown in Figure 1.3), you can see why the law of inertia cannot apply.

Figure 1.3 The crosses on the car seat and the dots on the passenger's shoulder represent the changing locations of the car and the passenger at equal time intervals. In the first three frames, the distances are equal, indicating that the car and passenger are moving at the same velocity. In the last two frames, the crosses are closer together, indicating that the car is slowing. The passenger, however, continues to move at the same velocity until stopped by a seat belt.

In the first three frames, the passenger's body and the car are moving at the same velocity, as shown by the cross on the car seat and the dot on the passenger's shoulder. When the car first begins to slow, no force has yet acted on the passenger. Therefore, his

body continues to move with the same constant velocity until a force, such as a seat belt, acts on him. When you are a passenger, you feel as though you are being thrown forward. In reality, the car has slowed down but, due to its own inertia, your body tries to continue to move with a constant velocity.

Since a change in direction is also an acceleration, the same situation occurs when a car turns. You feel as though you are being pushed to the side, but in reality, your body is attempting to continue in a straight line, while the car is changing its direction.

INERTIAL AND NON-INERTIAL FRAMES OF REFERENCE

An inertial frame of reference is one in which Newton's first and second laws are valid. Inertial frames of reference are at rest or in uniform motion, but they are not accelerating.

A non-inertial frame of reference is one in which Newton's first and second laws are not valid. Accelerating frames of reference are always non-inertial.

Clearly, in most cases, it is easier to work in an inertial frame of reference so that you can use Newton's laws of motion. However, if a physicist chooses to work in a non-inertial frame of reference and still apply Newton's laws of motion, it is necessary to invoke hypothetical quantities that are often called **fictitious forces:** inertial effects that are perceived as "forces" in non-inertial frames of reference, but do not exist in inertial frames of reference.

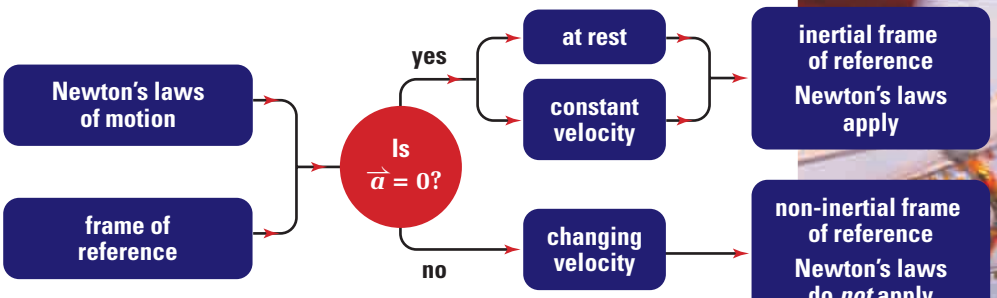
Conceptual Problem

- Passengers in a high-speed elevator feel as though they are being pressed heavily against the floor when the elevator starts moving up. After the elevator reaches its maximum speed, the feeling disappears.
  - **(a)** When do the elevator and passengers form an inertial frame of reference? A non-inertial frame of reference?
  - **(b)** Before the elevator starts moving, what forces are acting on the passengers? How large is the external (unbalanced) force? How do you know?
  - **(c)** Is a person standing outside the elevator in an inertial or non-inertial frame of reference?
  - **(d)** Suggest the cause of the pressure the passengers feel when the elevator starts to move upward. Sketch a free-body diagram to illustrate your answer.
  - **(e)** Is the pressure that the passengers feel in part (d) a fictitious force? Justify your answer.

PHYSICS FILE

Earth and everything on it are in continual circular motion. Earth is rotating on its axis, travelling around the Sun and circling the centre of the galaxy along with the rest of the solar system. The direction of motion is constantly changing, which means the motion is accelerated. Earth is a non-inertial frame of reference, and large-scale phenomena such as atmospheric circulation are greatly affected by Earth's continual acceleration. In laboratory experiments with moving objects, however, the effects of Earth's rotation are usually not detectable.

Concept Organizer



Some amusement park rides make you feel as though you are being thrown to the side, although no force is pushing you outward from the centre. Your frame of reference is moving rapidly along a curved path and therefore it is accelerating. You are in a non-inertial frame of reference, so it seems as though your motion is not following Newton's laws of motion.

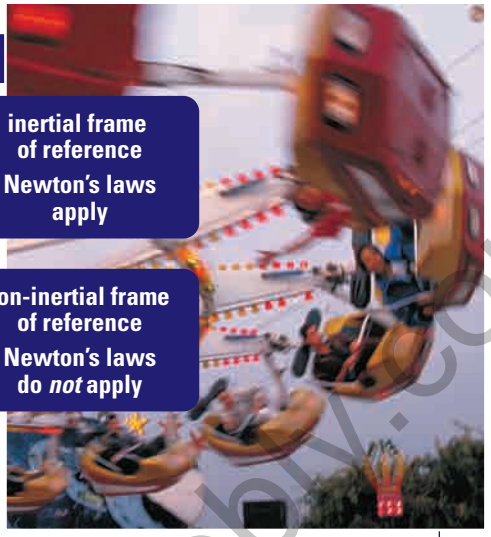


Figure 1.4 You can determine the nature of a frame of reference by analyzing its acceleration.

1.1 Section Review

1. **K/U** State Newton's first law in two different ways.
2. **C** Identify the two basic situations that Newton's first law describes and explain how one statement can cover both situations.
3. **K/U** State Newton's second law in words and symbols.
4. **MC** A stage trick involves covering a table with a smooth cloth and then placing dinnerware on the cloth. When the cloth is suddenly pulled horizontally, the dishes "magically" stay in position and drop onto the table.
   (a) Identify all forces acting on the dishes during the trick.
   (b) Explain how inertia and frictional forces are involved in the trick.
5. **K/U** Give an example of an unusual frame of reference used in a movie or a television program. Suggest why this viewpoint was chosen.
6. **K/U** Identify the defining characteristic of inertial and non-inertial frames of reference. Give an example of each type of frame of reference.
7. **C** In what circumstances is it necessary to invoke ficticious forces in order to explain motion? Why is this term appropriate to describe these forces?
8. **C** Compare inertial mass and gravitational mass, giving similarities and differences.
9. **C** Why do physicists, who take pride in precise, unambiguous terminology, usually speak just of "mass," rather than distinguishing between inertial and gravitational mass?

UNIT PROJECT PREP

- What frame of reference would be the best choice for measuring and analyzing the performance of your catapult?
- What forces will be acting on the payload of your catapult when it is being accelerated? When it is flying through the air?
- How will the inertia of the payload affect its behaviour? How will the mass of the payload affect its behaviour?

Test your ideas using a simple elastic band or slingshot.

CAUTION Take appropriate safety precautions before any tests. Use eye protection.

1.2 Analyzing Motion

The deafening roar of the engine of a competitor's tractor conveys the magnitude of the force that is applied to the sled in a tractor-pull contest. As the sled begins to move, weights shift to increase frictional forces. Despite the power of their engines, most tractors are slowed to a standstill before reaching the end of the 91 m track. In contrast to the brute strength of the tractors, dragsters "sprint" to the finish line. Many elements of the two situations are identical, however, since forces applied to masses change the linear (straight-line) motion of a vehicle.

In the previous section, you focussed on basic **dynamics** — the cause of changes in motion. In this section, you will analyze **kinematics** — the motion itself — in more detail. You will consider objects moving horizontally in straight lines.

SECTION EXPECTATIONS

- Analyze, predict, and explain linear motion of objects in horizontal planes.
- Analyze experimental data to determine the net force acting on an object and its resulting motion.

KEY TERMS

- dynamics
- kinematics
- uniform motion
- uniformly accelerated motion
- free-body diagram
- frictional forces
- coefficient of static friction
- coefficient of kinetic friction

Kinematic Equations

To analyze the motion of objects quantitatively, you will use the kinematic equations (or equations of motion) that you learned in previous courses. The two types of motion that you will analyze are **uniform motion** — motion with a constant velocity — and **uniformly accelerated motion** — motion under constant acceleration. When you use these equations, you will apply them to only one dimension at a time. Therefore, vector notations will not be necessary, because positive and negative signs are all that you will need to indicate direction. The kinematic equations are summarized on the next page, and apply only to the type of motion indicated.

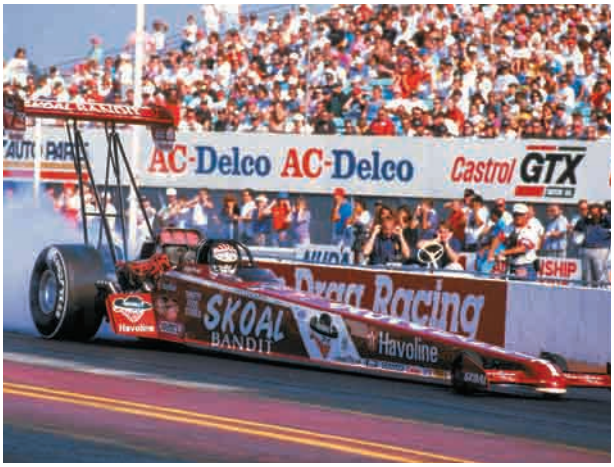


Figure 1.5 In a tractor pull, vehicles develop up to 9000 horsepower to accelerate a sled, until they can no longer overcome the constantly increasing frictional forces. Dragsters, on the other hand, accelerate right up to the finish line.

Uniform motion

- definition of velocity $v = \frac{\Delta d}{\Delta t}$
- Solve for displacement in terms of velocity and time. $\Delta d = v\Delta t$

Uniformly accelerated motion

- definition of acceleration $a = \frac{\Delta v}{\Delta t}$ or $a = \frac{v_2 - v_1}{\Delta t}$
- Solve for final velocity in terms of initial velocity, acceleration, and time interval. $v_2 = v_1 + a\Delta t$
- displacement in terms of initial velocity, final velocity, and time interval $\Delta d = \frac{(v_1 + v_2)}{2}\Delta t$
- displacement in terms of initial velocity, acceleration, and time interval $\Delta d = v_1\Delta t + \frac{1}{2}a\Delta t^2$
- final velocity in terms of initial velocity, acceleration, and displacement $v_2^2 = v_1^2 + 2a\Delta d$

Conceptual Problem

- The equations above are the most fundamental kinematic equations. You can derive many more equations by making combinations of the above equations. For example, it is sometimes useful to use the relationship $\Delta d = v_2\Delta t - \frac{1}{2}a\Delta t^2$. Derive this equation by manipulating two or more of the equations above. (Hint: Notice that the equation you need to derive is very similar to one of the equations in the list, with the exception that it has the final velocity instead of the initial velocity. What other equation can you use to eliminate the initial velocity from the equation that is similar to the desired equation?)

Combining Dynamics and Kinematics

When analyzing motion, you often need to solve a problem in two steps. You might have information about the forces acting on an object, which you would use to find the acceleration. In the next step, you would use the acceleration that you determined in order to calculate some other property of the motion. In other cases, you might analyze the motion to find the acceleration and then use the acceleration to calculate the force applied to a mass. The following sample problem will illustrate this process.

ELECTRONIC LEARNING PARTNER

Refer to your Electronic Learning Partner to enhance your understanding of acceleration and velocity.

SAMPLE PROBLEM

Finding Velocity from Dynamics Data

In television picture tubes and computer monitors (cathode ray tubes), light is produced when fast-moving electrons collide with phosphor molecules on the surface of the screen. The electrons (mass 9.1×10^{-31} kg) are accelerated from rest in the electron "gun" at the back of the vacuum tube. Find the velocity of an electron when it exits the gun after experiencing an electric force of 5.8×10^{-15} N over a distance of 3.5 mm.

Conceptualize the Problem

- The electrons are moving *horizontally*, from the back to the front of the tube, under an *electric force*.
- The *force of gravity* on an electron is exceedingly small, due to the electron's small mass. Since the electrons move so quickly, the time interval of the entire flight is very short. Therefore, the *effect of the force of gravity is too small to be detected* and you can consider the electric force to be the only force affecting the electrons.
- Information about *dynamics data* allows you to find the electrons' *acceleration*.
- Each electron is *initially at rest*, meaning that the *initial velocity is zero*.
- Given the acceleration, the equations of motion lead to other variables of motion.
- Let the direction of the force, and therefore the direction of the acceleration, be positive.

Identify the Goal

The final velocity, v_2, of an electron when exiting the electron gun

Identify the Variables and Constants

Known	Implied	Unknown
$m_e = 9.1 \times 10^{-31}$ kg	$v_1 = 0\ \frac{\text{m}}{\text{s}}$	a
$F = 5.8 \times 10^{-15}$ N		v_2
$\Delta d = 3.5 \times 10^{-3}$ m		

Develop a Strategy

Apply Newton's second law to find the net force.

$$\vec{F} = m\vec{a}$$

Write Newton's second law in terms of acceleration.

$$\vec{a} = \frac{\vec{F}}{m}$$

Substitute and solve.

$$\vec{a} = \frac{+5.8 \times 10^{-15}\ \text{N}}{9.1 \times 10^{-31}\ \text{kg}}$$

$\frac{\text{N}}{\text{kg}}$ is equivalent to $\frac{\text{m}}{\text{s}^2}$.

$$\vec{a} = 6.374 \times 10^{15}\ \frac{\text{m}}{\text{s}^2}\text{[toward the front of tube]}$$

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Apply the kinematic equation that relates initial velocity, acceleration, and displacement to final velocity.

$$v_2^2 = v_1^2 + 2a\Delta d$$
$$v_2^2 = 0 + 2\left(6.374 \times 10^{15}\ \frac{\text{m}}{\text{s}^2}\right)(3.5 \times 10^{-3}\ \text{m})$$
$$v_2 = 6.67\ 967 \times 10^6\ \frac{\text{m}}{\text{s}}$$
$$v_2 \cong 6.7 \times 10^6\ \frac{\text{m}}{\text{s}}$$

The final velocity of the electrons is about 6.7×10^6 m/s in the direction of the applied force.

Validate the Solution

Electrons, with their very small inertial mass, could be expected to reach high speeds. You can also solve the problem using the concepts of work and energy that you learned in previous courses. The work done on the electrons was converted into kinetic energy, so $W = F\Delta d = \frac{1}{2}mv^2$. Therefore,

$$v = \sqrt{\frac{2F\Delta d}{m}} = \sqrt{\frac{2(5.8 \times 10^{-15}\ \text{N})(3.5 \times 10^{-3}\ \text{m})}{9.1 \times 10^{-31}\ \text{kg}}} = 6.679 \times 10^6\ \frac{\text{m}}{\text{s}} \cong 6.7 \times 10^6\ \frac{\text{m}}{\text{s}}.$$

Obtaining the same answer by two different methods is a strong validation of the results.

PRACTICE PROBLEMS

1. A linear accelerator accelerated a germanium ion ($m = 7.2 \times 10^{-25}$ kg) from rest to a velocity of 7.3×10^6 m/s over a time interval of 5.5×10^{-6} s. What was the magnitude of the force that was required to accelerate the ion?

2. A hockey stick exerts an average force of 39 N on a 0.20 kg hockey puck over a displacement of 0.22 m. If the hockey puck started from rest, what is the final velocity of the puck? Assume that the friction between the puck and the ice is negligible.

Determining the Net Force

In almost every instance of motion, more than one force is acting on the object of interest. To apply Newton's second law, you need to find the resultant force. A free-body diagram is an excellent tool that will help to ensure that you have correctly identified and combined the forces.

To draw a **free-body diagram**, start with a dot that represents the object of interest. Then draw one vector to represent each force acting on the object. The tails of the vector arrows should all start at the dot and indicate the direction of the force, with the arrowhead pointing away from the dot. Study Figure 1.6 to see how a free-body diagram is constructed. Figure 1.6 (A) illustrates a crate being pulled across a floor by a rope attached to the edge of the crate. Figure 1.6 (B) is a free-body diagram representing the forces acting on the crate.

Two of the most common types of forces that influence the motion of familiar objects are frictional forces and the force of gravity. You will probably recall from previous studies that the

magnitude of the force of gravity acting on objects on or near Earth's surface can be expressed as $F = mg$, where g (which is often called the acceleration due to gravity) has a value 9.81 m/s^2. Near Earth's surface, the force of gravity always points toward the centre of Earth.

Whenever two surfaces are in contact, **frictional forces** oppose any motion between them. Therefore, the direction of the frictional force is always opposite to the direction of the motion. You might recall from previous studies that the magnitudes of frictional forces can be calculated by using the equation $F_f = \mu F_N$. The normal force in this relationship (F_N) is the force perpendicular to the surfaces in contact. You might think of the normal force as the force that is pressing the two surfaces together. The nature of the surfaces and their relative motion determines the value of the coefficient of friction (μ). These values must be determined experimentally. Some typical values are listed in Table 1.1.

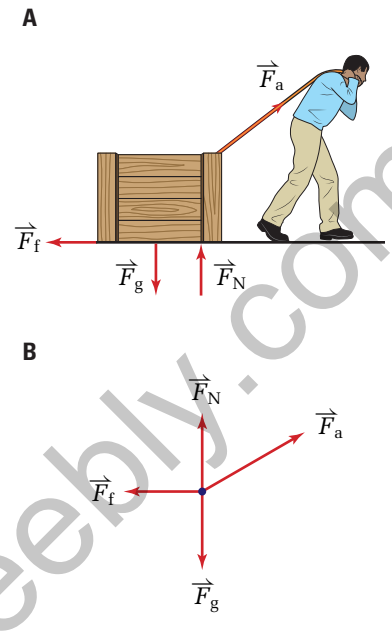


Figure 1.6 **(A)** The forces of gravity ($\vec{F}_g$), friction ($\vec{F}_f$), the normal force of the floor ($\vec{F}_N$), and the applied force of the rope ($\vec{F}_a$) all act on the crate at the same time. **(B)** The free-body diagram includes *only* those forces acting *on* the crate and none of the forces that the crate exerts on other objects.

Table 1.1 Coefficients of Friction for Some Common Surfaces

Surface	Coefficient of static friction (μ_s)	Coefficient of kinetic friction (μ_k)
rubber on dry, solid surfaces	1–4	1
rubber on dry concrete	1.00	0.80
rubber on wet concrete	0.70	0.50
glass on glass	0.94	0.40
steel on steel (unlubricated)	0.74	0.57
steel on steel (lubricated)	0.15	0.06
wood on wood	0.40	0.20
ice on ice	0.10	0.03
Teflon™ on steel in air	0.04	0.04
ball bearings (lubricated)	<0.01	<0.01
joint in humans	0.01	0.003

If the objects are not moving relative to each other, you would use the **coefficient of static friction** (μ_s). If the objects are moving, the somewhat smaller **coefficient of kinetic friction** (μ_k) applies to the motion.

As you begin to solve problems involving several forces, you will be working in one dimension at a time. You will select a coordinate system and resolve the forces into their components in each dimension. Note that the components of a force are not vectors themselves. Positive and negative signs completely describe the motion in one dimension. Thus, when you apply Newton's laws to the components of the forces in one dimension, you will not use vector notations.

ELECTRONIC LEARNING PARTNER

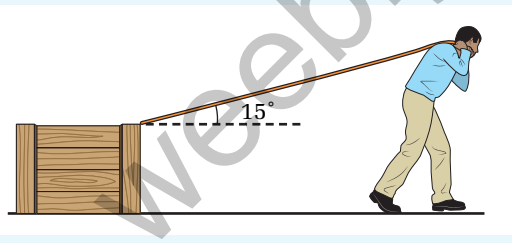
Refer to your Electronic Learning Partner to enhance your understanding of forces and vectors.

Another convention used in this textbook involves writing the sum of all of the forces in one dimension. In the first step, when the forces are identified as, for example, gravitational, frictional, or applied, only plus signs will be used. Then, when information about that specific force is inserted into the calculation, a positive or negative sign will be included to indicate the direction of that specific force. Watch for these conventions in sample problems.

SAMPLE PROBLEM

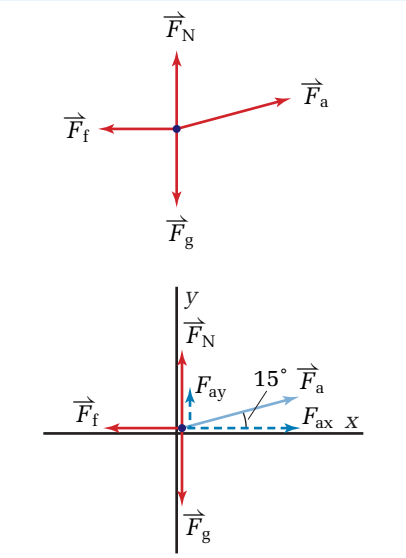
Working with Three Forces

To move a 45 kg wooden crate across a wooden floor (μ = 0.20), you tie a rope onto the crate and pull on the rope. While you are pulling the rope with a force of 115 N, it makes an angle of 15° with the horizontal. How much time elapses between the time at which the crate just starts to move and the time at which you are pulling it with a velocity of 1.4 m/s?



Conceptualize the Problem

- To start framing this problem, draw a free-body diagram.
- *Motion* is in the horizontal direction, so the net *horizontal force* is causing the crate to *accelerate*.
- Let the *direction* of the motion be the *positive* horizontal direction.
- There is no motion in the vertical direction, so the *vertical acceleration is zero*. If the acceleration is zero, the *net vertical force* must be *zero*. This information leads to the value of the *normal force*. Let "up" be the *positive* vertical direction.
- Since the beginning of the time interval in question is the instant at which the crate begins to move, the *coefficient of kinetic friction* applies to the motion.
- Once the *acceleration* is found, the *kinematic equations* allow you to determine the values of other quantities involved in the motion.



Identify the Goal

The time, Δt, required to reach a velocity of 1.4 m/s

Identify the Variables

Known		Implied	Unknown	
$\vec{F}_a = +115$ N	$m = 45$ kg	$v_i = 0 \frac{m}{s}$	$\vec{F}_N$	$\vec{a}$
$\theta = 15°$	$v_f = 1.4 \frac{m}{s}$	$g = 9.81 \frac{m}{s^2}$	$\vec{F}_g$	Δt
$\mu = 0.20$			$\vec{F}_f$	

Develop a Strategy

To find the normal force, apply Newton's second law to the vertical forces. Analyze the free-body diagram to find all of the vertical forces that act on the crate.

$$\vec{F} = m\vec{a}$$
$$F_{a(vertical)} + F_g + F_N = ma$$
$$F_g = -mg$$
$$F_{a(vertical)} - mg + F_N = ma$$
$$F_N = ma + mg - F_{a(vertical)}$$
$$F_N = 0 + (45\text{ kg})\left(9.81\ \frac{\text{m}}{\text{s}^2}\right) - (115\text{ N})\sin 15°$$
$$F_N = 441.45\text{ N} - 29.76\text{ N}$$
$$F_N = 411.69\text{ N}$$

To find the acceleration, apply Newton's second law to the horizontal forces. Analyze the free-body diagram to find all of the horizontal forces that act on the crate.

$$\vec{F} = m\vec{a}$$
$$F_{a(horizontal)} + F_f = ma$$
$$F_f = -\mu F_N$$
$$a = \frac{F_{a(horizontal)} - \mu F_N}{m}$$
$$a = \frac{(115\text{ N})\cos 15° - (0.20)(411.69\text{ N})}{45\text{ kg}}$$
$$a = \frac{111.08\text{ N} - 82.34\text{ N}}{45\text{ kg}}$$
$$a = 0.6387\ \frac{\text{m}}{\text{s}^2}$$

To find the time interval, use the kinematic equation that relates acceleration, initial velocity, final velocity, and time.

$$a = \frac{v_f - v_i}{\Delta t}$$
$$\Delta t = \frac{v_f - v_i}{a}$$
$$\Delta t = \frac{1.4\ \frac{\text{m}}{\text{s}} - 0\ \frac{\text{m}}{\text{s}}}{0.6387\ \frac{\text{m}}{\text{s}^2}}$$
$$\Delta t = 2.19\text{ s}$$
$$\Delta t \cong 2.2\text{ s}$$

You will be pulling the crate at 1.4 m/s at 2.2 s after the crate begins to move.

Validate the Solution

Check the units for acceleration: $\frac{\text{N}}{\text{kg}} = \frac{\frac{\cancel{\text{kg}}\cdot\text{m}}{\text{s}^2}}{\cancel{\text{kg}}} = \frac{\text{m}}{\text{s}^2}$. The units are correct. A velocity of 1.4 m/s is not very fast, so you would expect that the time interval required to reach that velocity would be short. The answer of 2.2 s is very reasonable.

PRACTICE PROBLEMS

3. In a tractor-pull competition, a tractor applies a force of 1.3 kN to the sled, which has mass 1.1×10^4 kg. At that point, the coefficient of kinetic friction between the sled and the ground has increased to 0.80. What is the acceleration of the sled? Explain the significance of the sign of the acceleration.

4. A curling stone with mass 20.0 kg leaves the curler's hand at a speed of 0.885 m/s. It slides 31.5 m down the rink before coming to rest.

(a) Find the average force of friction acting on the stone.

(b) Find the coefficient of kinetic friction between the ice and the stone.

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5. Pushing a grocery cart with a force of 95 N, applied at an angle of 35° down from the horizontal, makes the cart travel at a constant speed of 1.2 m/s. What is the frictional force acting on the cart?

6. A man walking with the aid of a cane approaches a skateboard (mass 3.5 kg) lying on the sidewalk. Pushing with an angle of 60° down from the horizontal with his cane, he applies a force of 115 N, which is enough to roll the skateboard out of his way.
   (a) Calculate the horizontal force acting on the skateboard.
   (b) Calculate the initial acceleration of the skateboard.

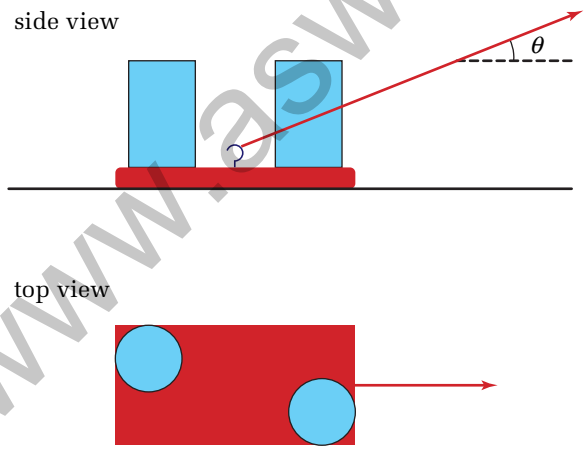
7. A mountain bike with mass 13.5 kg, with a rider having mass 63.5 kg, is travelling at 32 km/h when the rider applies the brakes, locking the wheels. How far does the bike travel before coming to a stop if the coefficient of friction between the rubber tires and the asphalt road is 0.60?

QUICK LAB

Best Angle for Pulling a Block

TARGET SKILLS
- Predicting
- Performing and recording
- Analyzing and interpreting

Set two 500 g masses on a block of wood. Attach a rope and drag the block along a table. If the rope makes a steeper angle with the surface, friction will be reduced (why?) and the block will slide more easily. Predict the angle at which the block will move with least effort. Attach a force sensor to the rope and measure the force needed to drag the block at a constant speed at a variety of different angles. Graph your results to test your prediction.



Analyze and Conclude

1. Identify from your graph the "best" angle at which to move the block.
2. How close did your prediction come to the experimental value?
3. Identify any uncontrolled variables in the experiment that could be responsible for some error in your results.
4. In theory, the "best" angle is related to the coefficient of static friction between the surface and the block: $\tan \theta_{best} = \mu_s$. Use your results to calculate the coefficient of static friction between the block and the table.
5. What effect does the horizontal component of the force have on the block? What effect does the vertical component have on the block?
6. Are the results of this experiment relevant to competitors in a tractor pull, such as the one described in the text and photograph caption at the beginning of this section? Explain your answer in detail.

Applying Newton's Third Law

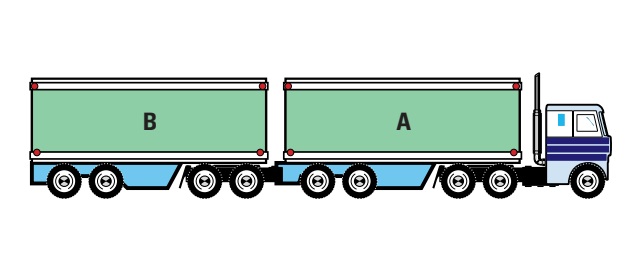
Examine the photograph of the tractor-trailer in Figure 1.7 and think about all of the forces exerted on each of the three sections of the vehicle. Automotive engineers must know how much force each trailer hitch needs to withstand. Is the hitch holding the second trailer subjected to as great a force as the hitch that attaches the first trailer to the truck?

Figure 1.7 This truck and its two trailers move as one unit. The velocity and acceleration of each of the three sections are the same. However, each section is experiencing a different net force.

To analyze the individual forces acting on each part of a train of objects, you need to apply Newton's third law to determine the force that each section exerts on the adjacent section. Study the following sample problem to learn how to determine all of the forces on the truck and on each trailer. These techniques will apply to any type of train problem in which the first of several sections of a moving set of objects is pulling all of the sections behind it.

SAMPLE PROBLEM

Forces on Connected Objects



A tractor-trailer pulling two trailers starts from rest and accelerates to a speed of 16.2 km/h in 15 s on a straight, level section of highway. The mass of the truck itself (T) is 5450 kg, the mass of the first trailer (A) is 31 500 kg, and the mass of the second trailer (B) is 19 600 kg. What magnitude of force must the truck generate in order to accelerate the entire vehicle? What magnitude of force must each of the trailer hitches withstand while the vehicle is accelerating? (Assume that frictional forces are negligible in comparison with the forces needed to accelerate the large masses.)

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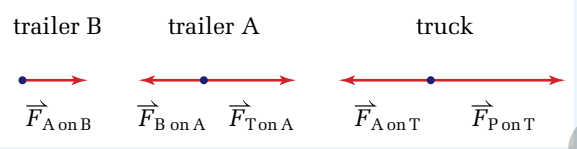
Conceptualize the Problem

- The truck engine generates energy to turn the wheels. When the wheels turn, they exert a *frictional force on the pavement*. According to Newton's third law, the *pavement* exerts a reaction *force* that is equal in magnitude and opposite in direction to the force exerted by the tires. The force of the pavement on the truck tires, $\vec{F}_{\text{P on T}}$, *accelerates* the entire system.
- The truck exerts a *force* on trailer A. According to Newton's third law, the trailer exerts a *force of equal magnitude* on the truck.
- Trailer A exerts a *force* on trailer B, and trailer B therefore must exert a force of equal magnitude on trailer A.
- Summarize all of the *forces* by drawing *free-body diagrams* of each section of the vehicle.



- The *kinematic equations* allow you to calculate the *acceleration* of the system.
- Since each section of the system has the *same acceleration*, this value, along with the masses and *Newton's second law*, lead to all of the *forces*.
- Since the motion is in a straight line and the question asks for only the magnitudes of the forces, vector notations are not needed.

Identify the Goal

The force, $F_{\text{P on T}}$, that the pavement exerts on the truck tires; the force, $F_{\text{T on A}}$, that the truck exerts on trailer A; the force, $F_{\text{A on B}}$, that trailer A exerts on trailer B

Identify the Variables

Known		Implied	Unknown			
$v_f = 16.2\ \frac{\text{km}}{\text{h}}$	$m_T = 5450\ \text{kg}$	$v_i = 0\ \frac{\text{km}}{\text{h}}$	a	$F_{\text{T on A}}$	$F_{\text{A on T}}$	m_{total}
$\Delta t = 15\ \text{s}$	$m_A = 31\ 500\ \text{kg}$		$F_{\text{P on T}}$	$F_{\text{A on B}}$	$F_{\text{B on A}}$	
	$m_B = 19\ 600\ \text{kg}$					

Develop a Strategy

Use the kinematic equation that relates the initial velocity, final velocity, time interval, and acceleration to find the acceleration.

$$a = \frac{v_2 - v_1}{\Delta t}$$

$$a = \frac{\left(16.2\ \frac{\cancel{\text{km}}}{\cancel{\text{h}}} - 0\ \frac{\cancel{\text{km}}}{\cancel{\text{h}}}\right)\left(\frac{1\ \cancel{\text{h}}}{3600\ \text{s}}\right)\left(\frac{1000\ \text{m}}{1\ \cancel{\text{km}}}\right)}{15\ \text{s}}$$

$$a = 0.30\ \frac{\text{m}}{\text{s}^2}$$

Find the total mass of the truck plus trailers.

$$m_{\text{total}} = m_T + m_A + m_B$$

$$m_{\text{total}} = 5450\ \text{kg} + 31\ 500\ \text{kg} + 19\ 600\ \text{kg}$$

$$m_{\text{total}} = 56\ 550\ \text{kg}$$

Use Newton's second law to find the force required to accelerate the total mass. This will be the force that the pavement must exert on the truck tires.

$$\vec{F} = m\vec{a}$$

$$F_{\text{P on T}} = (56\ 550\ \text{kg})\left(0.30\ \frac{\text{m}}{\text{s}^2}\right)$$

$$F_{\text{P on T}} = 16\ 965\ \frac{\text{kg}\cdot\text{m}}{\text{s}^2}$$

$$F_{\text{P on T}} \cong 1.7 \times 10^4\ \text{N}$$

The pavement exerts 1.7×10^4 N on the truck tires.

Use Newton's second law to find the force necessary to accelerate trailer B at 0.30 m/s^2. This is the force that the second trailer hitch must withstand.

$$F_{\text{A on B}} = m_{\text{B}}a$$
$$F_{\text{A on B}} = (19\ 600\ \text{kg})\left(0.30\ \frac{\text{m}}{\text{s}^2}\right)$$
$$F_{\text{A on B}} = 5.88 \times 10^3\ \frac{\text{kg}\cdot\text{m}}{\text{s}^2}$$
$$F_{\text{A on B}} \cong 5.9 \times 10^3\ \text{N}$$

The force that the second hitch must withstand is 5.9×10^3 N.

Use Newton's second law to find the total force necessary to accelerate trailer A at 0.30 m/s^2.

$$F_{\text{total on A}} = m_{\text{A}}a$$
$$F_{\text{total on A}} = (31\ 500\ \text{kg})\left(0.30\ \frac{\text{m}}{\text{s}^2}\right)$$
$$F_{\text{total on A}} = 9.45 \times 10^3\ \frac{\text{kg}\cdot\text{m}}{\text{s}^2}$$
$$F_{\text{total on A}} \cong 9.5 \times 10^3\ \text{N}$$

Use the free-body diagram to help write the expression for total (horizontal) force on trailer A.

$$F_{\text{total}} = F_{\text{T on A}} + F_{\text{B on A}}$$

The force that the first hitch must withstand is the force that the truck exerts on trailer A. Solve the force equation above for $F_{\text{T on A}}$ and calculate the value. According to Newton's third law, $F_{\text{B on A}} = -F_{\text{A on B}}$.

$$F_{\text{T on A}} = F_{\text{total on A}} - F_{\text{B on A}}$$
$$F_{\text{T on A}} = 9.45 \times 10^3\ \text{N} - (-5.88 \times 10^3\ \text{N})$$
$$F_{\text{T on A}} = 1.533 \times 10^4\ \text{N}$$
$$F_{\text{T on A}} \cong 1.5 \times 10^4\ \text{N}$$

The force that the first hitch must withstand is 1.5×10^4 N.

Validate the Solution

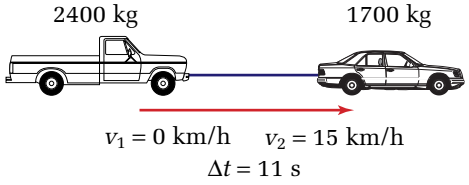
You would expect that $F_{\text{P on T}} > F_{\text{T on A}} > F_{\text{A on B}}$. The calculated forces agree with this relationship. You would also expect that the force exerted by the tractor on trailer A would be the force necessary to accelerate the sum of the masses of trailers A and B at 0.30 m/s^2.

$$F_{\text{T on A}} = (31\ 500\ \text{kg} + 19\ 600\ \text{kg})\left(0.30\ \frac{\text{m}}{\text{s}^2}\right) = 15\ 330\ \text{N} \cong 1.5 \times 10^4\ \text{N}$$

This value agrees with the value above.

PRACTICE PROBLEMS

8. A 1700 kg car is towing a larger vehicle with mass 2400 kg. The two vehicles accelerate uniformly from a stoplight, reaching a speed of 15 km/h in 11 s. Find the force needed to accelerate the connected vehicles, as well as the minimum strength of the rope between them.



9. An ice skater pulls three small children, one behind the other, with masses 25 kg, 31 kg, and 35 kg. Assume that the ice is smooth enough to be considered frictionless.

(a) Find the total force applied to the "train" of children if they reach a speed of 3.5 m/s in 15 s.

(b) If the skater is holding onto the 25 kg child, find the tension in the arms of the next child in line.

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10. A solo Arctic adventurer pulls a string of two toboggans of supplies across level, snowy ground. The toboggans have masses of 95 kg and 55 kg. Applying a force of 165 N causes the toboggans to accelerate at 0.61 m/s^2.

(a) Calculate the frictional force acting on the toboggans.

(b) Find the tension in the rope attached to the second (55 kg) toboggan.

1.2 Section Review

1. K/U How is direction represented when analyzing linear motion?

2. K/U When you pull on a rope, the rope pulls back on you. Describe how the rope creates this reaction force.

3. K/U Explain how to calculate

(a) the horizontal component (F_x) of a force F

(b) the vertical component (F_y) of a force F

(c) the coefficient of friction (μ) between two surfaces

(d) the gravitational force (F_g) acting on an object

4. K/U Define (a) a *normal* force and (b) the weight of an object.

5. K/U An object is being propelled horizontally by a force F. If the force doubles, use Newton's second law and kinematic equations to determine the change in

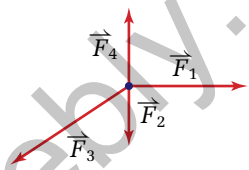
(a) the acceleration of the object

(b) the velocity of the object after 10 s

6. K/U A 0.30 kg lab cart is observed to accelerate twice as fast as a 0.60 kg cart. Does that mean that the net force on the more massive cart is twice as large as the force on the smaller cart? Explain.

7. K/U A force F produces an acceleration a when applied to a certain body. If the mass of the body is doubled and the force is increased fivefold, what will be the effect on the acceleration of the body?

8. K/U An object is being acted on by forces pictured in the diagram.



(a) Could the object be accelerating horizontally? Explain.

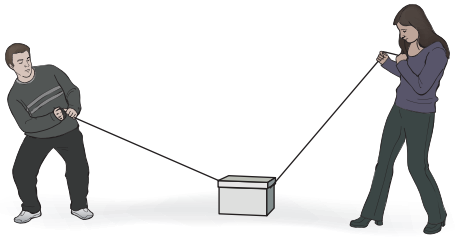
(b) Could the object be moving horizontally? Explain.

9. C Three identical blocks, fastened together by a string, are pulled across a frictionless surface by a constant force, F.

(a) Compare the tension in string A to the magnitude of the applied force, F.

(b) Draw a free-body diagram of the forces acting on block 2.

10. K/U A tall person and a short person pull on a load at different angles but with equal force, as shown.



(a) Which person applies the greater *horizontal* force to the load? What effect does this have on the motion of the load?

(b) Which person applies the greater *vertical* force to the load? What effect does this have on frictional forces? On the motion of the load?

1.3 Vertical Motion

Catapulting a diver high into the air requires a force. How large a force? How hard must the board push up on the diver to overcome her weight and accelerate her upward? After the diver leaves the board, how long will it take before her ascent stops and she turns and plunges toward the water? In this section, you will investigate the dynamics of diving and other motions involving rising and falling or straight-line motion in a vertical plane.

Figure 1.8 After the diver leaves the diving board and before she hits the water, the most important force acting on her is the gravitational force directed downward. Gravity affects all forms of vertical motion.

Weight versus Apparent Weight

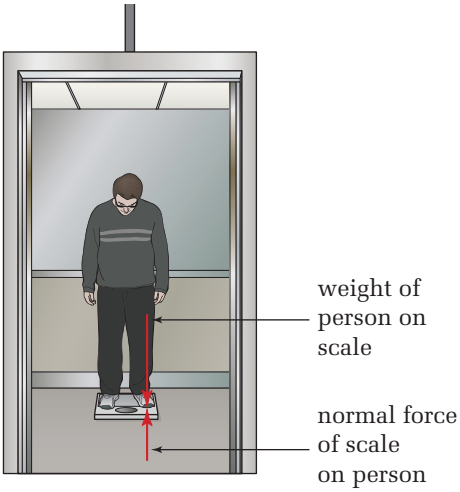
One of the most common examples of linear vertical motion is riding in an elevator. You experience some strange sensations when the elevator begins to rise or descend or when it slows and comes to a stop. For example, if you get on at the first floor and start to go up, you feel heavier for a moment. In fact, if you are carrying a book bag or a suitcase, it feels heavier, too. When the elevator slows and eventually stops, you and anything you are carrying feels lighter. When the elevator is moving at a constant velocity, however, you feel normal. Are these just sensations that living organisms feel or, if you were standing on a scale in the elevator, would the scale indicate that you *were* heavier? You can answer that question by applying Newton's laws of motion to a person riding in an elevator.

SECTION EXPECTATIONS

- Analyze the motion of objects in vertical planes.
- Explain linear vertical motion in terms of forces.
- Solve problems and predict the motion of objects in vertical planes.

KEY TERMS

- apparent weight
- tension
- counterweight
- free fall
- air resistance
- terminal velocity



Imagine that you are standing on a scale in an elevator, as shown in Figure 1.9. When the elevator is standing still, the reading on the scale is your weight. Recall that your weight is the force of gravity acting on your mass. Your weight can be calculated by using the equation $F_g = mg$, where g is the acceleration due to gravity. Vector notations are sometimes omitted because the force due to gravity is always directed toward the centre of Earth. Find out what happens to the reading on the scale by studying the following sample problem.

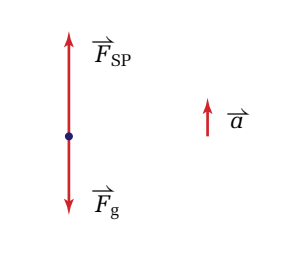
Figure 1.9 When you are standing on a scale, you exert a force on the scale. According to Newton's third law, the scale must exert an equal and opposite force on you. Therefore, the reading on the scale is equal to the force that you exert on it.

SAMPLE PROBLEM

Apparent Weight

A 55 kg person is standing on a scale in an elevator. If the scale is calibrated in newtons, what is the reading on the scale when the elevator is not moving? If the elevator begins to accelerate upward at 0.75 m/s^2, what will be the reading on the scale?

Conceptualize the Problem



- Start framing the problem by drawing a *free-body diagram* of the person on the scale. A free-body diagram includes all of the *forces acting on the person*.
- The *forces* acting on the person are *gravity* ($\vec{F}_g$) and the *normal force* of the scale.
- According to *Newton's third law*, when the person exerts a *force* ($\vec{F}_{PS}$) on the scale, it exerts an *equal and opposite force* ($\vec{F}_{SP}$) on the person. Therefore, the reading on the scale is the same as the force that the person exerts on the scale.
- When the elevator is *standing still*, the person's *acceleration is zero*.
- When the elevator begins to *rise*, the person is *accelerating* at the same rate as the elevator.
- Since the motion is in one dimension, use only positive and negative signs to indicate direction. Let "*up*" *be positive* and "*down*" *be negative*.
- Apply *Newton's second law* to find the magnitude of $\vec{F}_{SP}$.
- By *Newton's third law*, the magnitudes of $\vec{F}_{PS}$ and $\vec{F}_{SP}$ are equal to each other, and therefore to the reading on the scale.

Identify the Goal

The reading on the scale, $|\vec{F}_{SP}|$, when the elevator is standing still and when it is accelerating upward

Identify the Variables

Known	Implied	Unknown
$m = 55\ \text{kg}$	$g = 9.81\ \frac{\text{m}}{\text{s}^2}$	$\vec{F}_{PS}$ $\vec{F}_{SP}$
$\vec{a} = +0.75\ \frac{\text{m}}{\text{s}^2}$		$\vec{F}_g$

Develop a Strategy

Apply Newton's second law and solve for the force that the scale exerts on the person. The force in Newton's second law is the vector sum of all of the forces acting *on* the person.

$$\vec{F} = m\vec{a}$$
$$\vec{F}_g + \vec{F}_{SP} = m\vec{a}$$
$$\vec{F}_{SP} = -\vec{F}_g + m\vec{a}$$

In the first part of the problem, the acceleration is zero.

$$\vec{F}_{SP} = -(-mg) + m\vec{a}$$
$$\vec{F}_{SP} = (55\ \text{kg})\left(9.81\ \frac{\text{m}}{\text{s}^2}\right) + 0$$
$$\vec{F}_{SP} = 539.55\ \frac{\text{kg}\cdot\text{m}}{\text{s}^2}$$
$$\vec{F}_{SP} \cong 5.4 \times 10^2\ \text{N}$$

When the elevator is not moving, the reading on the scale is 5.4×10^2 N, which is the person's weight.

Apply Newton's second law to the case in which the elevator is accelerating upward. The acceleration is positive.

$$\vec{F} = m\vec{a}$$
$$\vec{F}_g + \vec{F}_{SP} = m\vec{a}$$
$$\vec{F}_{SP} = -\vec{F}_g + m\vec{a}$$
$$\vec{F}_{SP} = -(-mg) + m\vec{a}$$
$$\vec{F}_{SP} = (55\ \text{kg})\left(9.81\ \frac{\text{m}}{\text{s}^2}\right) + (55\ \text{kg})\left(+0.75\ \frac{\text{m}}{\text{s}^2}\right)$$
$$\vec{F}_{SP} = 580.8\ \text{N}$$
$$\vec{F}_{SP} \cong 5.8 \times 10^2\ \text{N[up]}$$

When the elevator is accelerating upward, the reading on the scale is 5.8×10^2 N.

Validate the Solution

When an elevator first starts moving upward, it must exert a force that is greater than the person's weight so that, as well as supporting the person, an additional force causes the person to accelerate. The reading on the scale should reflect this larger force. It does. The acceleration of the elevator was small, so you would expect that the increase in the reading on the scale would not increase by a large amount. It increased by only about 7%.

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PRACTICE PROBLEMS

11. A 64 kg person is standing on a scale in an elevator. The elevator is rising at a constant velocity but then begins to slow, with an acceleration of 0.59 m/s^2. What is the sign of the acceleration? What is the reading on the scale while the elevator is accelerating?

12. A 75 kg man is standing on a scale in an elevator when the elevator begins to descend with an acceleration of 0.66 m/s^2. What is the direction of the acceleration? What is the reading on the scale while the elevator is accelerating?

13. A 549 N woman is standing on a scale in an elevator that is going down at a constant velocity. Then, the elevator begins to slow and eventually comes to a stop. The magnitude of the acceleration is 0.73 m/s^2. What is the direction of the acceleration? What is the reading on the scale while the elevator is accelerating?

As you saw in the problems, when you are standing on a scale in an elevator that is accelerating, the reading on the scale is not the same as your true weight. This reading is called your **apparent weight**.

When the direction of the acceleration of the elevator is positive — it starts to ascend or stops while descending — your apparent weight is greater than your true weight. You feel heavier because the floor of the elevator is pushing on you with a greater force than it is when the elevator is stationary or moving with a constant velocity.

When the direction of the acceleration is negative — when the elevator is rising and slows to a stop or begins to descend — your apparent weight is smaller than your true weight. The floor of the elevator is exerting a force on you that is smaller than your weight, so you feel lighter.

Figure 1.10 Mobile construction cranes can withstand the tension necessary to lift loads of up to 1000 t.

Tension in Ropes and Cables

While an elevator is supporting or lifting you, what is supporting the elevator? The simple answer is cables — exceedingly strong steel cables. Construction cranes such the one in Figure 1.10 also use steel cables to lift building materials to the top of skyscrapers under construction. When a crane exerts a force on one end of a cable, each particle in the cable exerts an equal force on the next particle in the cable, creating tension throughout the cable. The cable then exerts a force on its load. **Tension** is the magnitude of the force exerted on and by a cable, rope, or string. How do engineers determine the amount of tension that these cables must be able to withstand? They apply Newton's laws of motion.

To avoid using complex mathematical analyses, you can make several assumptions about cables and ropes that support loads. Your results will be quite close to the values calculated by computers that are programmed to take into account all of the non-ideal conditions. The simplifying assumptions are as follows.

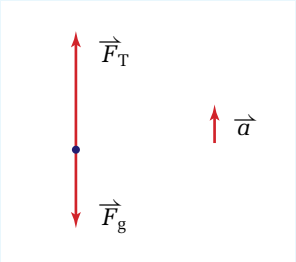
- The mass of the rope or cable is so much smaller than the mass of the load that it does not significantly affect the motion or forces involved.
- The tension is the same at every point in the rope or cable.
- If a rope or cable passes over a pulley, the direction of the tension forces changes, but the magnitude remains the same. This statement is the same as saying that the pulley is frictionless and its mass is negligible.

SAMPLE PROBLEM

Tension in a Cable

An elevator filled with people has a total mass of 2245 kg. As the elevator begins to rise, the acceleration is 0.55 m/s^2. What is the tension in the cable that is lifting the elevator?

Conceptualize the Problem



- To begin framing the problem, draw a free-body diagram.
- The *tension* in the cable has the *same magnitude* as the *force* it exerts on the elevator.
- *Two forces* are acting on the elevator: the *cable* ($\vec{F}_T$) and gravity ($\vec{F}_g$).
- The elevator is *rising* and speeding up, so the *acceleration* is *upward*.
- *Newton's second law* applies to the problem.
- The motion is in *one dimension*, so let positive and negative *signs* indicate *direction*. Let "*up*" *be positive* and "*down*" *be negative*.

Identify the Goal

The tension, F_T, in the rope

Identify the Variables

Known	Implied	Unknown
$m = 2245$ kg	$g = 9.81 \frac{\text{m}}{\text{s}^2}$	$\vec{F}_T$
$\vec{a} = 0.55 \frac{\text{m}}{\text{s}^2}$[up]		$\vec{F}_g$

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Develop a Strategy

Apply Newton's second law and insert all of the forces acting on the elevator. Then solve for the tension.

$$\vec{F} = m\vec{a}$$
$$\vec{F}_T + \vec{F}_g = m\vec{a}$$
$$\vec{F}_T = -\vec{F}_g + m\vec{a}$$
$$\vec{F}_T = -(-mg) + m\vec{a}$$

Substitute values and solve.

$$\vec{F} = (2245\ \text{kg})\left(9.81\ \frac{\text{m}}{\text{s}^2}\right) + (2245\ \text{kg})\left(0.55\ \frac{\text{m}}{\text{s}^2}\right)$$
$$\vec{F}_T = 23\ 258.2\ \frac{\text{kg}\cdot\text{m}}{\text{s}^2}$$
$$\vec{F}_T \cong 2.3 \times 10^4\ \text{N[up]}$$

The magnitude of the tension in the cable is 2.3×10^4 N[up].

Validate the Solution

The weight of the elevator is $(2245\ \text{kg})(9.81\ \text{m/s}^2) \cong 2.2 \times 10^4$ N. The tension in the cable must support the weight of the elevator and exert an additional force to accelerate the elevator. Therefore, you would expect the tension to be a little larger than the weight of the elevator, which it is.

PRACTICE PROBLEMS

14. A 32 kg child is practising climbing skills on a climbing wall, while being belayed (secured at the end of a rope) by a parent. The child loses her grip and dangles from the belay rope. When the parent starts lowering the child, the tension in the rope is 253 N. Find the acceleration of the child when she is first being lowered.

15. A 92 kg mountain climber rappels down a rope, applying friction with a figure eight (a piece of climbing equipment) to reduce his downward acceleration. The rope, which is damaged, can withstand a tension of only 675 N. Can the climber limit his descent to a constant speed without breaking the rope? If not, to what value can he limit his downward acceleration?

16. A 10.0 kg mass is hooked on a spring scale fastened to a hoist rope. As the hoist starts moving the mass, the scale momentarily reads 87 N. Find

(a) the direction of motion

(b) the acceleration of the mass

(c) the tension in the hoist rope

17. Pulling on the strap of a 15 kg backpack, a student accelerates it upward at 1.3 m/s^2. How hard is the student pulling on the strap?

18. A 485 kg elevator is rated to hold 15 people of average mass (75 kg). The elevator cable can withstand a maximum tension of 3.74×10^4 N, which is twice the maximum force that the load will create (a 200% safety factor). What is the greatest acceleration that the elevator can have with the maximum load?

Connected Objects

Imagine how much energy it would require to lift an elevator carrying 20 people to the main deck of the CN Tower in Toronto, 346 m high. A rough calculation using the equation for gravitational potential energy ($E_g = mg\Delta h$), which you learned in previous science courses, would yield a value of about 10 million joules of energy. Is there a way to avoid using so much energy?

Elevators are not usually simply suspended from cables. Instead, the supporting cable passes up over a pulley and then back down to a heavy, movable **counterweight**, as shown in Figure 1.11. Gravitational forces acting *downward* on the counterweight create tension in the cable. The cable then exerts an *upward* force on the elevator cage. Most of the weight of the elevator and passengers is balanced by the counterweight. Only relatively small additional forces from the elevator motors are needed to raise and lower the elevator and its counterweight. Although the elevator and counterweight move in different directions, they are connected by a cable, so they accelerate at the same rate.

Elevators are only one of many examples of machines that have large masses connected by a cable that runs over a pulley. In fact, in 1784, mathematician George Atwood (1745–1807) built a machine similar to the simplified illustration in Figure 1.12. He used his machine to test and demonstrate the laws of uniformly accelerated motion and to determine the value of g, the acceleration due to gravity. The acceleration of Atwood's machine depended on g, but was small enough to measure accurately. In the following investigation, you will use an Atwood machine to measure g.

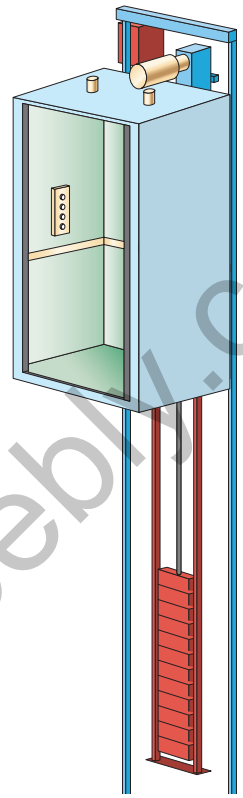


Figure 1.11 Most elevators are connected by a cable to a counterweight that moves in the opposite direction to the elevator. A typical counterweight has a mass that is the same as the mass of the empty elevator plus about half the mass of a full load of passengers.

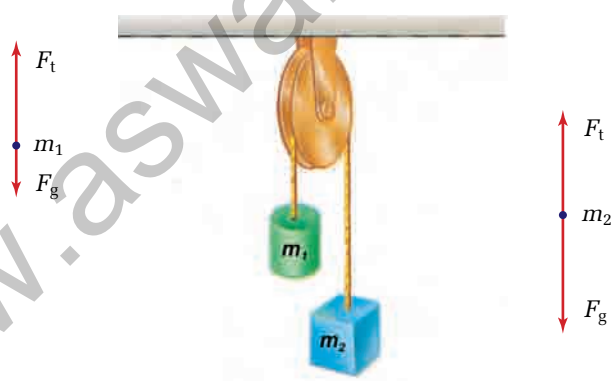


Figure 1.12 An Atwood machine uses a counterweight to reduce acceleration due to gravity.

INVESTIGATION 1-B

Atwood's Machine

TARGET SKILLS
- Predicting
- Performing and recording
- Analyzing and interpreting

George Atwood designed his machine to demonstrate the laws of motion. In this investigation, you will demonstrate those laws and determine the value of g.

Problem

How can you determine the value of g, the acceleration due to gravity, by using an Atwood machine?

Prediction

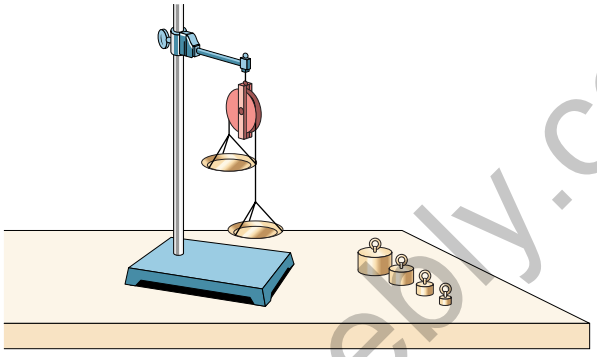
- Predict how changes in the *difference* between the two masses will affect the acceleration of the Atwood machine if the sum of the masses is held constant.
- When the difference between the two masses in an Atwood machine is held constant, predict how increasing the total mass (sum of the two masses) will affect their acceleration.

Equipment

- retort stand
- clamps
- masses: 100 g (2), 20 g (1), 10 g (10), or similar identical masses, such as 1 inch plate washers
- 2 plastic cups to hold masses
- light string

Traditional instrumentation
lab pulley
lab timer
metre stick

Probeware
Smart Pulley® or photogates or ultrasonic range finder
motion analysis software
computer



Procedure

Constant Mass Difference

1. Set up a data table to record m_1, m_2, total mass, Δd and Δt (if you use traditional equipment), and a.
2. Set up an Atwood machine at the edge of a table, so that m_1 = 120 g and m_2 = 100 g.
3. Lift the heavier mass as close as possible to the pulley. Release the mass and make the measurements necessary for finding its downward acceleration. Catch the mass before it hits the floor.
   - Using traditional equipment, find displacement (Δd) and the time interval (Δt) while the mass descends smoothly.
   - Using probeware, measure velocity (v) and graph velocity versus time. Find acceleration from the slope of the line during an interval when velocity was increasing steadily.
4. Increase each mass by 10 g and repeat the observations. Continue increasing mass and finding acceleration until you have five total mass-acceleration data pairs.
5. Graph acceleration versus total mass. Draw a best-fit line through your data points.

Constant Total Mass

6. Set up a data table to record m_1, m_2, mass difference (Δm), Δd and Δt (if you use traditional equipment), and a.
7. Make $m_1 = 150$ g and $m_2 = 160$ g. Make observations to find the downward acceleration, using the same method as in step 3.
8. Transfer one 10 g mass from m_1 to m_2. The mass difference will now be 30 g, but the total mass will not have changed. Repeat your measurements.
9. Repeat step 8 until you have data for five mass difference-acceleration pairs.
10. Graph acceleration versus mass difference. Draw a best-fit line or curve through your data points.

Analyze and Conclude

1. Based on your graphs for step 5, what type of relationship exists between total mass and acceleration in an Atwood machine? Use appropriate curve-straightening techniques to support your answer (see Skill Set 4, Mathematical Modelling and Curve Straightening). Write the relationship symbolically.
2. Based on your graphs for step 10, what type of relationship exists between mass difference and acceleration in an Atwood machine? Write the relationship symbolically.
3. How well do your results support your prediction?
4. String that is equal in length to the string connecting the masses over the pulley is sometimes tied to the bottoms of the two masses, where it hangs suspended between them. Explain why this would reduce experimental errors. Hint: Consider the mass of the string as the apparatus moves and how that affects m_1 and m_2.
5. Mathematical analysis shows that the acceleration of an ideal (frictionless) Atwood machine is given by $a = g\frac{m_1 - m_2}{m_1 + m_2}$. Use this relationship and your experimental results to find an experimental result for g.
6. Calculate experimental error in your value of g. Suggest the most likely causes of experimental error in your apparatus and procedure.

Apply and Extend

7. Start with Newton's second law in the form $\vec{a} = \frac{\vec{F}}{m}$ and derive the equation for a in question 5 above. Hint: Write $\vec{F}$ and m in terms of the forces and masses in the Atwood machine.
8. Using the formula $a = g\frac{m_1 - m_2}{m_1 + m_2}$ for an Atwood machine, find the acceleration when $m_1 = 2m_2$.
9. Under what circumstances would the acceleration of the Atwood machine be zero?
10. What combination of masses would make the acceleration of an Atwood machine equal to $\frac{1}{2}g$?

WEB LINK

www.mcgrawhill.ca/links/physics12

For some interactive activities involving the Atwood machine, go to the above Internet site and click on **Web Links**.

Assigning Direction to the Motion of Connected Objects

When two objects are connected by a flexible cable or rope that runs over a pulley, such as the masses in an Atwood machine, they are moving in different directions. However, as you learned when working with trains of objects, connected objects move as a unit. For some calculations, you need to work with the forces acting on the combined objects and the acceleration of the combined objects. How can you treat the pair of objects as a unit when two objects are moving in different directions?

Since the connecting cable or rope changes only the direction of the forces acting on the objects and has no effect on the magnitude of the forces, you can assign the direction of the motion as being from one end of the cable or rope to the other. You can call one end "negative" and the other end "positive," as shown in Figure 1.13.

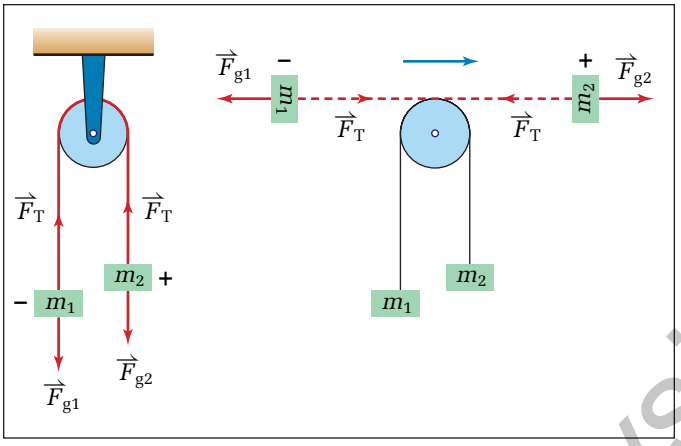


Figure 1.13 You can assign the bottom of the left-hand side of the machine to be negative and the bottom of the right-hand side to be positive. You can then imagine the connected objects as forming a straight line, with left as negative and right as positive. When you picture the objects as a linear train, make sure that you keep the force arrows in the same *relative* directions in relation to the individual objects.

When you have assigned the directions to a pair of connected objects, you can apply Newton's laws to the objects as a unit or to each object independently. When you treat the objects as one unit, you must ignore the tension in the rope because it does not affect the movement of the combined objects. Notice that the force exerted by the rope on one object is equal in magnitude and opposite in direction to the force exerted on the other object. However, when you apply the laws of motion to one object at a time, you must include the tension in the rope, as shown in the following sample problem.

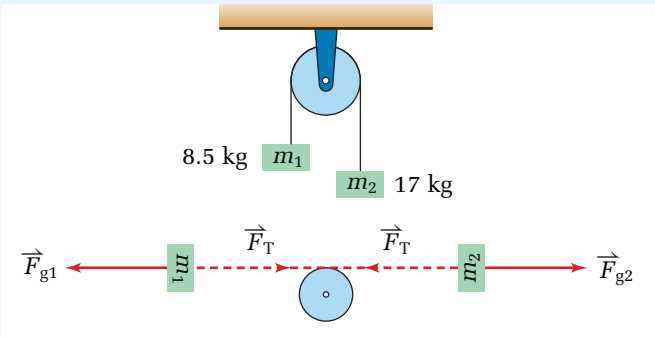
SAMPLE PROBLEM

Motion of Connected Objects

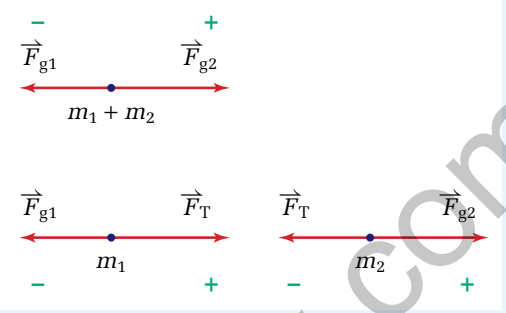
An Atwood machine is made of two objects connected by a rope that runs over a pulley. The object on the left (m_1) has a mass of 8.5 kg and the object on the right (m_2) has a mass of 17 kg.

(a) What is the acceleration of the masses?

(b) What is the tension in the rope?



Conceptualize the Problem



- To start framing the problem, draw free-body diagrams. Draw one diagram of the system moving as a unit and diagrams of each of the two individual objects.
- Let the *negative* direction point from the centre to the 8.5 kg mass and the *positive* direction point from the centre to the 17 kg mass.
- Both objects move with the same acceleration.
- The force of gravity acts on both objects.
- The tension is constant throughout the rope.
- The rope exerts a force of equal magnitude and opposite direction on each object.
- When you isolate the individual objects, the tension in the rope is one of the forces acting on the object.
- Newton's second law applies to the combination of the two objects and to each individual object.

Identify the Goal

(a) The acceleration, $\vec{a}$, of the two objects

(b) The tension, $|\vec{F}_T|$, in the rope

Identify the Variables

Known	Implied	Unknown
$m_1 = 8.5$ kg	$g = 9.81 \frac{\text{m}}{\text{s}^2}$	$\vec{F}_{g1}$ $\vec{F}_T$
$m_2 = 17$ kg		$\vec{F}_{g2}$

Develop a Strategy

Apply Newton's second law to the combination of masses to find the acceleration.

The mass of the combination is the sum of the individual masses.

$$\vec{F} = m\vec{a}$$

$$\vec{F}_{g1} + \vec{F}_{g2} = (m_1 + m_2)\vec{a}$$

$$-m_1 g + m_2 g = (m_1 + m_2)\vec{a}$$

$$\vec{a} = \frac{(m_2 - m_1)g}{m_1 + m_2}$$

$$\vec{a} = \frac{(17\text{ kg} - 8.5\text{ kg})9.8\ \frac{\text{m}}{\text{s}^2}}{8.5\text{ kg} + 17\text{ kg}}$$

$$\vec{a} = 3.27\ \frac{\text{m}}{\text{s}^2}$$

$$\vec{a} \cong 3.3\ \frac{\text{m}}{\text{s}^2}\text{[to the right]}$$

(a) The acceleration of the combination of objects is 3.3 m/s^2 to the right.

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Apply Newton's second law to m_1 and solve for tension.

$$\vec{F} = m\vec{a}$$
$$\vec{F}_{g1} + \vec{F}_T = m_1\vec{a}$$
$$-m_1g + \vec{F}_T = m_1\vec{a}$$
$$\vec{F}_T = m_1g + m_1\vec{a}$$
$$\vec{F}_T = (8.5\text{ kg})\left(9.81\ \frac{\text{m}}{\text{s}^2}\right) + (8.5\text{ kg})\left(3.27\ \frac{\text{m}}{\text{s}^2}\right)$$
$$\vec{F}_T = 111.18\ \frac{\text{kg}\cdot\text{m}}{\text{s}^2}$$
$$\vec{F}_T \cong 1.1 \times 10^2\text{ N}$$

(b) The tension in the rope is 1.1×10^2 N.

Validate the Solution

You can test your solution by applying Newton's second law to the second mass.

$$\vec{F}_{g2} + \vec{F}_T = m_2\vec{a}$$
$$m_2g + \vec{F}_T = m_2\vec{a}$$
$$\vec{F}_T = m_2\vec{a} - m_2g$$
$$\vec{F}_T = (17\text{ kg})\left(3.27\ \frac{\text{m}}{\text{s}^2}\right) - (17\text{ kg})\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)$$
$$\vec{F}_T = -111.18\text{ N}$$
$$\vec{F}_T = -1.1 \times 10^2\text{ N}$$

The magnitudes of the tensions calculated from the two masses independently agree. Also, notice that the application of Newton's second law correctly gave the direction of the force on the second mass.

PRACTICE PROBLEMS

19. An Atwood machine consists of masses of 3.8 kg and 4.2 kg. What is the acceleration of the masses? What is the tension in the rope?

20. The smaller mass on an Atwood machine is 5.2 kg. If the masses accelerate at 4.6 m/s^2, what is the mass of the second object? What is the tension in the rope?

21. The smaller mass on an Atwood machine is 45 kg. If the tension in the rope is 512 N, what is the mass of the second object? What is the acceleration of the objects?

22. A 3.0 kg counterweight is connected to a 4.5 kg window that freely slides vertically in its frame. How much force must you exert to start the window opening with an acceleration of 0.25 m/s^2?

23. Two gymnasts of identical 37 kg mass dangle from opposite sides of a rope that passes over a frictionless, weightless pulley. If one of the gymnasts starts to pull herself up the rope with an acceleration of 1.0 m/s^2, what happens to her? What happens to the other gymnast?

Objects Connected at Right Angles

In the lab, a falling weight is often used to provide a constant force to accelerate dynamics carts. Gravitational forces acting *downward* on the weight create tension in the connecting string. The pulley changes the direction of the forces, so the string exerts a *horizontal* force on the cart. Both masses experience the same acceleration because they are connected, but the cart and weight move at right angles to each other.

You can approach problems with connected objects such as the lab cart and weight in the same way that you solved problems involving the Atwood machine. Even if a block is sliding, with friction, over a surface, the mathematical treatment is much the same. Study Figure 1.14 and follow the directions below to learn how to treat connected objects that are moving both horizontally and vertically.

- Analyze the forces on each individual object, then label the diagram with the forces.
- Assign a direction to the motion.
- Draw the connecting string or rope as though it was a straight line. Be sure that the force vectors are in the same direction relative to each mass.
- Draw a free-body diagram of the combination and of each individual mass.
- Apply Newton's second law to each free-body diagram.

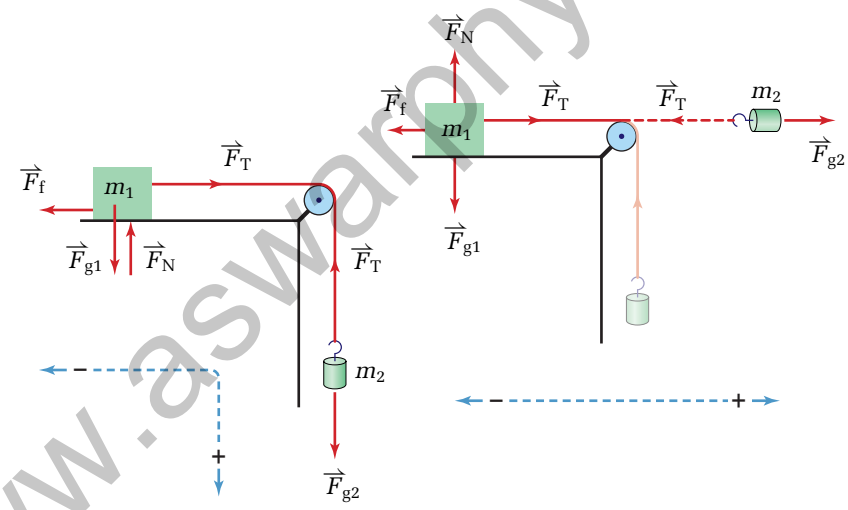


Figure 1.14 When you visualize the string "straightened," the force of gravity appears to pull down on mass 1, but to the side on mass 2. Although it might look strange, be assured that these directions are correct regarding the way in which the forces affect the motion of the objects.

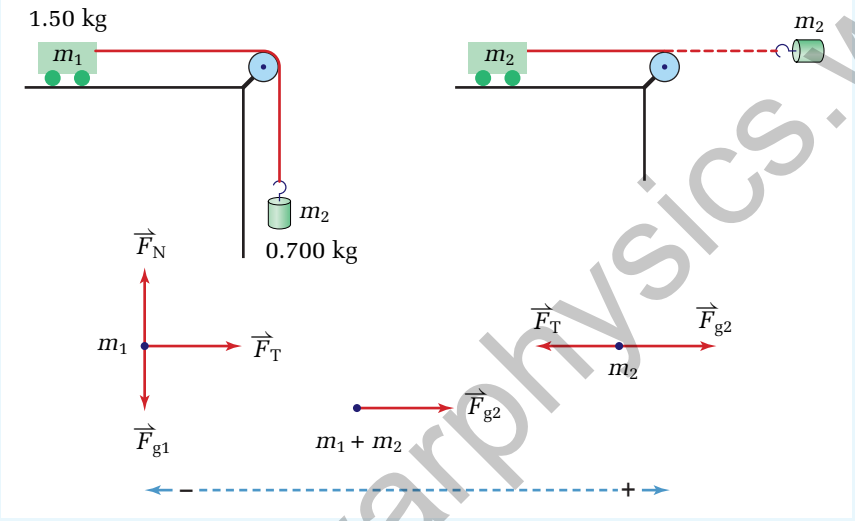
SAMPLE PROBLEM

Connected Objects

A 0.700 kg mass is connected to a 1.50 kg lab cart by a lightweight cable passing over a low-friction pulley. How fast does the cart accelerate and what is the tension in the cable? (Assume that the cart rolls without friction.)

Conceptualize the Problem

- Make a simplified diagram of the connected masses and assign forces.
- Visualize the cable in a straight configuration.
- Sketch free-body diagrams of the forces acting on each object and of the forces acting on the combined objects.



- The force causing the *acceleration* of both masses is the *force* of *gravity* acting on mass 2.
- Newton's second law applies to the combined masses and to each individual mass.
- Let left be the *negative* direction and right be the *positive* direction.

Identify the Goal

The acceleration of the cart, $\vec{a}$, and the magnitude of the tension force in the cable, F_T

Identify the Variables and Constants

Known	Implied	Unknown	
$m_1 = 1.50$ kg	$g = 9.81 \frac{m}{s^2}$	$\vec{a}$	$\vec{F}_{g1}$
$m_2 = 0.700$ kg		$\vec{F}_T$	$\vec{F}_{g2}$

Develop a Strategy

Apply Newton's second law to the combined masses and solve for acceleration.

$$\vec{F} = m\vec{a}$$
$$\vec{F}_{g2} = (m_1 + m_2)\vec{a}$$
$$m_2 g = (m_1 + m_2)\vec{a}$$
$$\vec{a} = \frac{m_2 g}{m_1 + m_2}$$

Substitute values and solve.

$$\vec{a} = \frac{(0.700\ \text{kg})\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)}{0.700\ \text{kg} + 1.5\ \text{kg}}$$
$$\vec{a} = 3.121\ 36\ \frac{\text{m}}{\text{s}^2}$$
$$\vec{a} \cong 3.1\ \frac{\text{m}}{\text{s}^2}$$

The cart accelerates at about 3.1 m/s^2. Since the sign is positive, it accelerates to the right.

Apply Newton's second law to mass 1 to find the tension in the rope.

$$\vec{F} = m\vec{a}$$
$$\vec{F}_T = m_1\vec{a}$$
$$\vec{F}_T = (1.5\ \text{kg})\left(3.121\ 36\ \frac{\text{m}}{\text{s}^2}\right)$$
$$\vec{F}_T = 4.682\ 04\ \text{N}$$
$$\vec{F}_T \cong 4.7\ \text{N}$$

The tension in the cable is about 4.7 N.

Validate the Solution

The acceleration of the combined masses is less than 9.81 m/s^2, which is reasonable since only part of the mass is subject to unbalanced gravitational forces. Also, the tension calculated at m_2 is also about 4.7 N.

$$\vec{F} = m\vec{a}$$
$$\vec{F}_g + \vec{F}_T = m_2\vec{a}$$
$$\vec{F}_T = m_2\vec{a} - \vec{F}_g$$
$$\vec{F}_T = (0.700\ \text{kg})\left(3.121\ 36\ \frac{\text{m}}{\text{s}^2}\right) - (0.700\ \text{kg})\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)$$
$$\vec{F}_T \cong -4.7\ \text{N}$$

PRACTICE PROBLEMS

24. A Fletcher's trolley apparatus consists of a 1.90 kg cart on a level track attached to a light string passing over a pulley and holding a 0.500 kg mass suspended in the air. Neglecting friction, calculate

(a) the tension in the string when the suspended mass is released

(b) the acceleration of the trolley

25. A 40.0 g glider on an air track is connected to a suspended 25.0 g mass by a string passing over a frictionless pulley. When the mass is released, how long will it take the glider to travel the 0.85 m to the other end of the track? (Assume the mass does not hit the floor, so there is constant acceleration during the experiment.)

Free Fall

Have you ever dared to take an amusement park ride that lets you fall with almost no support for a short time? A roller coaster as it drops from a high point in its track can bring you close to the same feeling of **free fall**, a condition in which gravity is the only force acting on you. To investigate free fall quantitatively, imagine, once again, that you are standing on a scale in an elevator. If the cable was to break, there were no safety devices, and friction was negligible, what would be your apparent weight?

Figure 1.15 When you are on a free-fall amusement park ride, you feel weightless.

If gravity is the only force acting on the elevator, it will accelerate downward at the acceleration due to gravity, or g. Substitute this value into Newton's second law and solve for your apparent weight.

- Write Newton's second law. $\vec{F} = m\vec{a}$
- Let "up" be positive and "down" be negative. The total force acting on you is the downward force of gravity and the upward normal force of the scale. Your acceleration is g downward. $F_N + F_g = -mg$
- The force of gravity is $-mg$. $F_N - mg = -mg$
- Solve for the normal force. $F_N = mg - mg$ $F_N = 0$

The reading on the scale is zero. Your apparent weight is zero. This condition is often called "weightlessness." Your mass has not changed, but you feel weightless because nothing is pushing up on you, preventing you from accelerating at the acceleration due to gravity.

Conceptual Problem

- How would a person on a scale in a freely falling elevator analyze the forces that were acting? Make a free-body analysis similar to the one in the sample problem (Apparent Weight) on page 28, using the elevator as your frame of reference. Consider these points.
  - **(a)** To an observer in the elevator, the person on the scale would not appear to be moving.
  - **(b)** The reading on the scale (the normal force) would be zero.

Close to Earth's surface, weightlessness is rarely experienced, due to the resistance of the atmosphere. As an object collides with molecules of the gases and particles in the air, the collisions act as a force opposing the force of gravity. **Air resistance** or air friction is quite different from the surface friction that you have studied. When an object moves through a fluid such as air, the force of friction increases as the velocity of the object increases.

A falling object eventually reaches a velocity at which the force of friction is equal to the force of gravity. At that point, the net force acting on the object is zero and it no longer accelerates but maintains a constant velocity called **terminal velocity**. The shape and orientation of an object affects its terminal velocity. For example, skydivers control their velocity by their position, as illustrated in Figure 1.16. Table 1.2 lists the approximate terminal velocities for some common objects.

Figure 1.16 Gravity is not the only force affecting these sky-divers, who have become experts at manipulating air friction and controlling their descent.

Table 1.2 Approximate Terminal Velocities

Object	Terminal velocity (m/s downward)
large feather	0.4
fluffy snowflake	1
parachutist	7
penny	9
skydiver (spread-eagled)	58

PHYSICS FILE

In 1942, Soviet air force pilot I. M. Chisov was forced to parachute from a height of almost 6700 m. To escape being shot by enemy fighters, Chisov started to free fall, but soon lost consciousness and never opened his parachute. Air resistance slowed his descent, so he probably hit the ground at about 193 km/h, plowing through a metre of snow as he skidded down the side of a steep ravine. Amazingly, Chisov survived with relatively minor injuries and returned to work in less than four months.

TECHNOLOGY LINK

Air resistance is of great concern to vehicle designers, who can increase fuel efficiency by using body shapes that reduce the amount of air friction or drag that is slowing the vehicle. Athletes such as racing cyclists and speed skaters use body position and specially designed clothing to minimize drag and gain a competitive advantage. Advanced computer hardware and modelling software are making computerized simulations of air resistance a practical alternative to traditional experimental studies using scale models in wind tunnels.

Father of the Canadian Space Program

Can you imagine sending one of the very first satellites into space? How about writing a report that changed the entire direction of Canada's space efforts, or being involved in a telecommunications program that won an Emmy award? These are just a few of the accomplishments that earned John H. Chapman the nickname "Father of the Canadian Space Program."

Chapman, who was born in 1921, was a science graduate of McGill University in Montréal. In 1951, the London, Ontario, native became section leader of the Defence Research Board's unit at Shirley's Bay, Ontario. While there, he played a key role in several ground-breaking projects.

John Herbert Chapman

Lift Off!

Early in the history of space exploration, Canadian space scientists focussed on the study of Earth's upper atmosphere and ionosphere. They wanted to understand the behaviour of radio waves in these lofty regions, especially above Canada's North. As head of the government team researching this area, Chapman was a moving force in the Alouette/International Satellites for Ionospheric Studies (ISIS) program.

With the successful launch of *Alouette I* in 1962, Canada became the third nation to reach space, following the Soviet Union and the United States. Designed to last for one year, *Alouette I* functioned for ten. It has been hailed as one of the greatest achievements in Canadian engineering in the past century. The ISIS satellites lasted for 20 years, earning Canada an international reputation for excellence in satellite design and engineering.

During this time, Chapman brought Canadian industry into the space age. He argued that private companies, not just government laboratories, had the "right stuff" to design and build space hardware. As a result, Canadian industry was given a steadily increasing role in the manufacture of *Alouette II* and the ISIS satellites.

Connecting Canada and the World

Chapman also influenced the very purpose for which Canada's satellites were built. *The Chapman Report*, issued in 1967, helped turn Canada's space program away from space science and toward telecommunications. Chapman believed that satellites could deliver signals to rural and remote regions of the country. This was achieved in 1972, when Canada placed the *Anik A1* satellite into stationary orbit above the equator and became the first country to have its own communications satellite system of this type.

Today, live news reports can be delivered from remote locations, due to technology that Chapman and his team helped pioneer in co-operation with NASA and the European Space Agency. Before the *Hermes* satellite was launched in 1976, videotapes of news events were flown to a production centre and distributed. This was a time-consuming process. With *Hermes* in place, a telecommunications dish on location could beam news up to the satellite and, from there, to anywhere in the world. *Hermes* was also revolutionary because it sent and received television signals on high frequencies that did not interfere with frequencies already in use. For this innovation, the *Hermes* satellite program won an Emmy in 1987.

At the time of his death in 1979, John Chapman was the Assistant Deputy Minister for Space in the Canadian Department of Communications. On October 2, 1996, in recognition of his distinguished career, the headquarters of the Canadian Space Agency was dedicated as the John H. Chapman Space Centre.

WEB LINK

www.mcgrawhill.ca/links/physics12

For more information about the Canadian Space Agency and the *Alouette, Hermes*, and ISIS space programs, visit the above Internet site and click on **Web Links**.

QUICK LAB

Descending Drops

TARGET SKILLS

- Performing and recording
- Modelling concepts
- Analyzing and interpreting

You can observe drops of water falling at terminal velocity through cooking oil in a test tube. Use an eyedropper to carefully "inject" drops of cold water below the surface of the cooking oil. Measure the diameter of the drops and the speed of their descent.

Analyze and Conclude

1. Assume that the drops are spherical and are pure water with density 1.0 g/cm^3. Using the formulas for volume of a sphere $\left(V = \frac{4}{3}\pi r^3\right)$ and density $\left(D = \frac{m}{V}\right)$, calculate the mass of each drop.
2. Calculate the gravitational force and the retarding force on each drop.
3. What force(s) are retarding the downward force of gravity acting on the drops? Compare these forces to those acting on an object falling through air.
4. The curved sides of the test tube act like a lens, producing some optical magnification of objects inside. Describe in detail how this might be affecting your results.
5. How well does this activity model the movement of an object through air and the phenomenon of terminal velocity? Justify your answer.

1.3 Section Review

1. K/U Explain why your apparent weight is sometimes not the same as your true weight.
2. K/U Explain how Newton's third law applies to connected objects that are all pulled by one end.
3. C How does an Atwood machine make it easier to determine g (the acceleration due to gravity), rather than by just measuring the acceleration of a free-falling object?
4. C Suppose you are standing on a scale in a moving elevator and notice that the scale reading is *less* than your true weight.
   (a) Draw a free-body diagram to represent the forces acting on you.
   (b) Describe the elevator motion that would produce the effect.
5. K/U List the simplifying assumptions usually made about supporting cables and ropes. Why are simplifying assumptions needed?
6. K/U Two objects are moving in different directions. Under what circumstances can you treat this as a one-dimensional problem?
7. MC By the mid-1800s, steam-driven elevators with counterweights had been developed. However, they were not in common use until 1852, when Elisha Otis invented an elevator with a safety device that prevented the elevator from falling if the cable broke. How do you think that the invention of a safe elevator changed modern society?
8. C Describe a situation in which you could be standing on a scale and the reading on the scale would be zero. (**Note:** The scale is functioning properly and is accurate.) What is the name of this condition?

1.4 Motion along an Incline

SECTION EXPECTATIONS

- Analyze the motion of objects along inclined planes.
- Use vector and free-body diagrams to analyze forces.
- Predict and explain motion along inclined planes.

When you watch speed skiers, it appears as though there is no limit to the rate at which they can accelerate. In reality, their acceleration is always less that that of a free-falling object, because the skier is being accelerated by only a component of the force of gravity and not by the total force. Using the principles of dynamics and the forces affecting the motion, you can predict details of motion along an inclined plane.

Figure 1.17 Gravitational forces acting on downhill skiers have produced speeds greater than 241 km/h, even though only part of the total gravitational force accelerates a skier.

Choosing a Coordinate System for an Incline

The key to analyzing the dynamics and motion of objects on an inclined plane is choosing a coordinate system that simplifies the procedure. Since all of the motion is along the plane, it is convenient to place the *x*-axis of the coordinate system parallel to the plane, making the *y*-axis perpendicular to the plane, as shown in Figure 1.18.

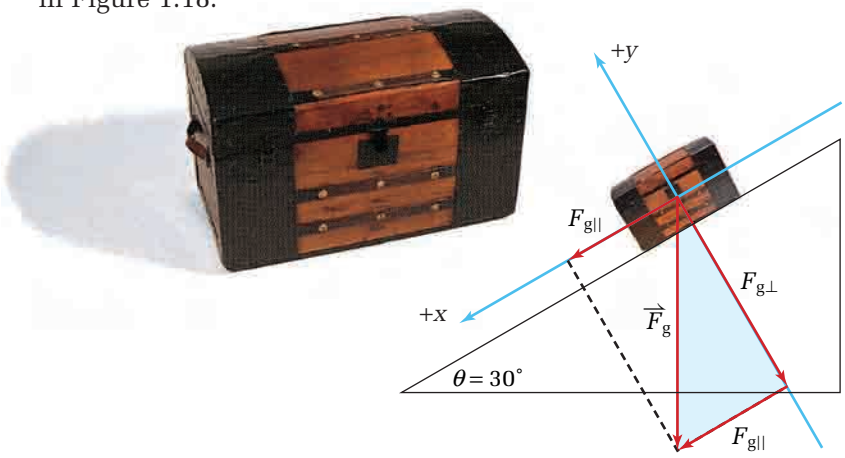


Figure 1.18 To find the components of the gravitational force vector, use the shaded triangle. Note that $\vec{F}_g$ is perpendicular to the horizontal line at the bottom and $F_{g\perp}$ is perpendicular to the plane of the ramp. Since the angles between two sets of perpendicular lines must be equal, the angle (θ) in the triangle is equal to the angle that the inclined plane makes with the horizontal.

The force of gravity affects motion on inclined planes, but the force vector is at an angle to the plane. Therefore, you must resolve the gravitational force vector into components parallel to and perpendicular to the plane, as shown in Figure 1.18. The component of force parallel to the plane influences the acceleration of the object and the perpendicular component affects the magnitude of the friction. Since several forces in addition to the gravitational force can affect the motion on an inclined plane, free-body diagrams are essential in solving problems, as shown in the sample problem below.

SAMPLE PROBLEM

Sliding Down an Inclined Plane

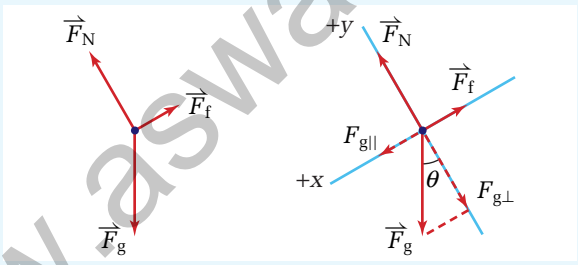
You are holding an 85 kg trunk at the top of a ramp that slopes from a moving van to the ground, making an angle of 35° with the ground. You lose your grip and the trunk begins to slide.

(a) If the coefficient of friction between the trunk and the ramp is 0.42, what is the acceleration of the trunk?

(b) If the trunk slides 1.3 m before reaching the bottom of the ramp, for what time interval did it slide?

Conceptualize the Problem

- To start framing the problem, draw a *free-body diagram*.
- Beside the free-body diagram, draw a coordinate system with the *x-axis parallel* to the ramp. On the coordinate system, draw the forces and *components of forces* acting on the trunk.



- Let the direction pointing *down* the slope be the *positive* direction.
- To find the *normal force* that is needed to determine the magnitude of the *frictional force*, apply *Newton's second law* to the forces or components of forces that are *perpendicular* to the ramp.
- The *acceleration perpendicular* to the ramp is *zero*.
- The *component of gravity parallel* to the trunk causes the trunk to *accelerate* down the ramp.
- *Friction* between the trunk and the ramp *opposes* the motion.
- If the *net force* along the ramp is *positive*, the trunk will *accelerate* down the ramp.
- To find the *acceleration* of the trunk down the ramp, apply *Newton's second law* to the forces or components of forces *parallel* to the ramp.
- Given the *acceleration* of the trunk, you can use the *kinematic equations* to find *other quantities* of motion.

Identify the Goal

(a) The acceleration, $a_{||}$, of the trunk along the ramp

(b) The time interval, Δt, for the trunk to reach the end of the ramp

continued ▶

continued from previous page

Identify the Variables

Known		Implied	Unknown		
$m = 85\ \text{kg}$	$\theta = 35^\circ$	$g = 9.81\ \frac{\text{m}}{\text{s}^2}$	$\vec{F}_g$	$\vec{F}_f$	v_f
$\mu = 0.42$	$\Delta d = 1.3\ \text{m}$	$v_i = 0$	$F_{g\parallel}$	$\vec{F}_N$	$a_\parallel$
		$a_\perp = 0$	$F_{g\perp}$		

Develop a Strategy

Apply Newton's second law to the forces perpendicular to the ramp. Refer to the diagram to find all of the forces that are perpendicular to the ramp. Solve for the normal force.

$$\vec{F} = m\vec{a}$$
$$F_N + F_{g\perp} = ma_\perp$$
$$F_N - mg\cos\theta = ma_\perp$$
$$F_N = mg\cos\theta + ma_\perp$$

Insert values and solve. Note that the acceleration perpendicular to the ramp ($a_\perp$) is zero.

$$F_N = (85\ \text{kg})\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)\cos 35^\circ + 0$$
$$F_N = 683.05\ \text{N}$$

Apply Newton's second law to the forces parallel to the ramp. Refer to the diagram to find all of the forces that are parallel to the ramp. Solve for the acceleration parallel to the ramp.

$$\vec{F} = m\vec{a}$$
$$F_{g\parallel} + F_f = ma_\parallel$$
$$F_f = \mu F_N \text{ in negative direction}$$
$$mg\sin\theta - \mu F_N = ma_\parallel$$
$$a_\parallel = \frac{mg\sin\theta - \mu F_N}{m}$$

Insert values and solve.

$$a_\parallel = \frac{(85\ \text{kg})\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)\sin 35^\circ - (0.42)(683.05\ \text{N})}{85\ \text{kg}}$$
$$a_\parallel = 2.251\ 71\ \frac{\text{m}}{\text{s}^2}$$
$$a_\parallel \cong 2.3\ \frac{\text{m}}{\text{s}^2}$$

(a) The acceleration of the trunk down the ramp is $2.3\ \text{m/s}^2$.

Apply the kinematic equation that relates displacement, acceleration, initial velocity, and time interval. Given that the initial velocity was zero, solve the equation for the time interval.

$$\Delta d = v_i\Delta t + \frac{1}{2}a\Delta t^2$$
$$\Delta t^2 = \frac{2\Delta d}{a}$$
$$\Delta t = \sqrt{\frac{2\Delta d}{a}}$$

Insert values and solve.

$$\Delta t = \sqrt{\frac{2(1.3\ \text{m})}{2.251\ 71\ \frac{\text{m}}{\text{s}^2}}}$$
$$\Delta t = 1.075\ \text{s}$$
$$\Delta t \cong 1.1\ \text{s}$$

(b) The trunk slid for 1.1 s before reaching the end of the ramp.

Validate the Solution

(a) Since the ramp is not at an extremely steep slope and since there is a significant amount of friction, you would expect that the acceleration would be much smaller than $9.81\ \text{m/s}^2$, which it is.

(b) The ramp is very short, so you would expect that it would not take long for the trunk to reach the bottom of the ramp. A time of 1.1 s is quite reasonable.

26. A 1975 kg car is parked at the top of a steep 42 m long hill inclined at an angle of 15°. If the car starts rolling down the hill, how fast will it be going when it reaches the bottom of the hill? (Neglect friction.)

27. Starting from rest, a cyclist coasts down the starting ramp at a professional biking track. If the ramp has the minimum legal dimensions (1.5 m high and 12 m long), find

(a) the acceleration of the cyclist, ignoring friction

(b) the acceleration of the cyclist if all sources of friction yield an effective coefficient of friction of $\mu = 0.11$

(c) the time taken to reach the bottom of the ramp, if friction acts as in (b)

28. A skier coasts down a 3.5° slope at constant speed. Find the coefficient of kinetic friction between the skis and the snow covering the slope.

QUICK LAB

The Slippery Slope

TARGET SKILLS

- **Performing and recording**
- **Analyzing and interpreting**

You can determine the coefficients of static and kinetic friction experimentally. Use a coin or small block of wood as the object and a textbook as a ramp. Find the mass of the object. Experiment to find the maximum angle of inclination possible before the object begins to slide down the ramp (θ_1). Then, use a slightly greater angle (θ_2), so that the object slides down the ramp. Make appropriate measurements of displacement and time, so that you can calculate the average acceleration. If the distance is too short to make accurate timings, use a longer ramp, such as a length of smooth wood or metal.

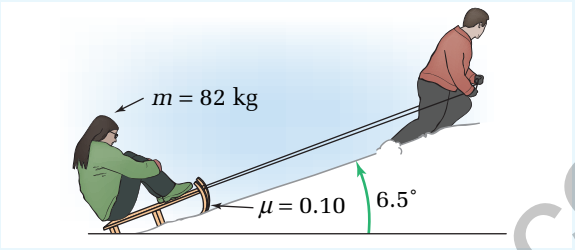
Analyze and Conclude

1. Calculate the gravitational force on the object (weight). Resolve the gravitational force into parallel and perpendicular components.
2. Draw a free-body diagram of the forces acting on the object and use it to find the magnitude of all forces acting on the object just before it started to slide (at angle θ_1). **Note:** If the object is not accelerating, no net force is acting on it, so every force must be balanced by an equal and opposite force.
3. Calculate the coefficient of static friction, μ_s, between the object and the ramp, using your answer to question 2.
4. Use the data you collected when the ramp was inclined at θ_2 to calculate the acceleration of the object. Find the net force necessary to cause this acceleration.
5. Use the net force and the parallel component of the object's weight to find the force of friction between the object and the ramp.
6. Calculate the coefficient of kinetic friction, μ_k, between the object and the ramp.
7. Compare μ_s and μ_k. Are they in the expected relationship to each other? How well do your experimental values agree with standard values for the materials that you used for your object and ramp? (Obtain coefficients of friction from reference materials.)

SAMPLE PROBLEM

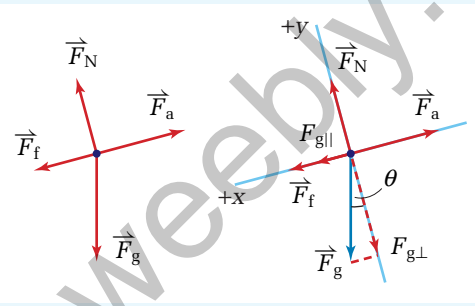
Pushing or Pulling an Object Up an Incline

You are pulling a sled and rider with combined mass of 82 kg up a 6.5° slope at a steady speed. If the coefficient of kinetic friction between the sled and snow is 0.10, what is the tension in the rope?



Conceptualize the Problem

- Sketch a *free-body diagram* of the forces acting on the sled. Beside it, sketch the *components of the forces* that are *parallel* and *perpendicular* to the slope.
- Since the sled is moving at a *constant velocity*, the *acceleration is zero*.
- The *parallel component* of the sled's weight and the *force of friction* are acting *down* the slope (*positive* direction).
- The *applied force* of the rope acts up the slope on the sled (*negative* direction).
- The *tension* in the rope is the magnitude of the *force* that the *rope exerts* on the sled.
- *Newton's second law* applies independently to the forces perpendicular and parallel to the slope.



Identify the Goal

The magnitude of the tension, $|\vec{F}_a|$, in the rope

Identify the Variables and Constants

Known	Implied	Unknown		
$m = 82$ kg	$g = 9.81 \frac{m}{s^2}$	$\vec{F}_g$	$F_{g\|\|}$	$F_{g\perp}$
$\mu = 0.10$	$a_{\|\|} = 0 \frac{m}{s^2}$	$\vec{F}_N$	$\vec{F}_f$	$\vec{F}_a$
$\theta = 6.5°$				
v = constant				

Develop a Strategy

Apply Newton's second law to the forces perpendicular to the slope. Refer to the diagram to find all of the forces that are perpendicular to the slope. Solve for the normal force.

$$\vec{F} = m\vec{a}$$
$$F_N + F_{g\perp} = ma_\perp$$
$$F_N - mg\cos\theta = ma_\perp$$
$$F_N = mg\cos\theta + ma_\perp$$

Insert values and solve. Note that the acceleration perpendicular to the slope ($a_\perp$) is zero.

$$F_N = (82\text{ kg})\left(9.81\ \frac{m}{s^2}\right)\cos 6.5° + 0$$
$$F_N = 799.25\text{ N}$$

Apply Newton's second law to the forces parallel to the slope. Refer to the diagram to find all of the forces that are parallel to the slope. Solve for the force that the rope exerts on the sled.

$$\vec{F} = m\vec{a}$$
$$F_f + F_a + F_{g||} = ma_{||}$$
$$\mu F_N + F_a + mg\sin\theta = ma_{||}$$
$$F_a = ma_{||} - \mu F_N - mg\sin\theta$$

Insert values and solve.

$$F_a = (82\text{ kg})\left(0\ \frac{\text{m}}{\text{s}^2}\right) - (0.10)(799.25\text{ N}) - (82\text{ kg})\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)\sin 6.5^\circ$$

$$F_a = -79.925\text{ N} - 91.063\text{ N}$$

$$F_a = -170.988\text{ N}$$

$$|\vec{F}_a| \cong 1.7 \times 10^2\text{ N}$$

The tension force in the rope is about 1.7×10^2 N.

Validate the Solution

The tension is much less than the force of gravity on the sled, since most of the weight of the sled is being supported by the ground. The tension is also greater than the parallel component of the sled's weight, because the rope must balance both the force of friction and the component of the force of gravity parallel to the slope.

PRACTICE PROBLEMS

29. You flick a 5.5 g coin up a smooth board propped at an angle of 25° to the floor. If the initial velocity of the coin is 2.3 m/s up the board and the coefficient of kinetic friction between the coin and the board is 0.40, how far does the coin travel before stopping?

30. You are pushing a 53 kg crate at a constant velocity up a ramp onto a truck. The ramp makes an angle of 22° with the horizontal. If your applied force is 373 N, what is the coefficient of friction between the crate and the ramp?

1.4 Section Review

1. K/U Sketch a free-body diagram and an additional diagram showing the parallel and perpendicular components of gravitational force acting on an object on a ramp inclined at an angle of θ to the horizontal. State the equation used to calculate each force component.

2. K/U Which component of gravitational force affects each of the following?

(a) acceleration down a frictionless incline

(b) the force of friction acting on an object on a ramp

(c) the tension in a rope holding the object motionless

(d) the tension in a rope pulling the object up the ramp

3. C Why is it necessary to use two coefficients (kinetic and static) to describe the frictional forces between two surfaces? How do you decide which coefficient to use when solving a problem?

4. C Suppose you are pulling a heavy box up a ramp into a moving van. Why is it much harder to *start* the box moving than it is to *keep it moving*?

CHAPTER 1 Review

REFLECTING ON CHAPTER 1

- Dynamics relates the motion of objects to the forces acting on them.
- Inertia is the tendency of objects to resist changes in motion.
- In an inertial frame of reference, Newton's laws of motion describe motion correctly. Inertial frames of reference might be stationary or moving at constant velocity.
- In non-inertial frames of reference, Newton's laws of motion do not accurately describe motion. Accelerating frames of reference are non-inertial.
- Fictitious forces are needed to explain motion in non-inertial frames of reference. If the same motion is observed from an inertial frame of reference, the motion can be explained without the use of fictitious forces.
- Inertial mass is equivalent to gravitational mass.
- Frictional forces are described by the equation $F_f = \mu F_N$, where μ is the coefficient of friction between two surfaces and F_N is the normal force pressing the surfaces together. The coefficient of kinetic friction (μ_k) applies when the object is moving. The coefficient of static friction (μ_s) applies when the object is motionless.
- The weight of an object is the gravitational force on it ($F_g = mg$).
- Free fall is vertical motion that is affected by gravitational forces only. In free fall, all objects accelerate at the same rate.
- Terminal velocity is the maximum downward speed reached by a falling object when the force of air friction becomes equal to the force of gravity.
- Air resistance depends on the surface area, shape, and speed of an object relative to the air around it.

Knowledge/Understanding

1. Identify and provide examples of what physicists consider to be the two "natural" types of motion.
2. What is the term used to describe the tendency for objects to have differing amounts of "persistence" in maintaining their natural motion?
3. What concept is used to quantify the inertia of an object?
4. Distinguish between, and provide examples of, inertial and non-inertial frames of reference.
5. Imagine that you are looking sideways out of a car that is stopped at a stoplight. The light turns green and your driver accelerates until the car is travelling with uniform motion at the speed limit.
   (a) Sketch a velocity-time graph of your motion, illustrating the time intervals during which you were stopped, accelerating, and travelling with a constant velocity.
   (b) Identify the time interval(s) during which you were observing objects at the side of the road from an inertial frame of reference or from a non-inertial frame of reference.
   (c) Use this example to explain why Newton's first and second laws do not accurately predict the motion of the objects you are observing at the side of the road while you are accelerating.
6. You know that if you drop two balls from rest from the top of a building, they will accelerate uniformly and strike the ground at the same time (ignoring air resistance). Consider these variations.
   (a) Suppose you drop a ball from rest from the top of a building and it strikes the ground with a final velocity v_f. At the same time that the first ball is dropped, your friend launches a second ball from the ground with a velocity v_f, the same velocity with which the first ball strikes the ground. Will the second ball reach the top of the building at

the same time that the first ball strikes the ground? Explain where the balls cross paths, at half the height of the building, above the halfway point or below the halfway point. Ignore air resistance.

(b) You launch a ball from the edge of the top of a building with an initial velocity of 25 m/s [upward]. The ball rises to a certain height and then falls down and strikes the ground next to the building. Your friend on the ground measures the velocity with which the ball strikes the ground. Next, you launch a second ball from the edge of the building with a velocity of 25 m/s [downward]. Ignoring air resistance, will the second ball strike the ground with greater, smaller, or the same velocity as the first ball?

Hint: what is the velocity of the first ball when it is at the height of the top of the building (after falling from its maximum height) and on its way down?

7. You are having a debate with your lab partner about the correct solution to a physics problem. He says that the normal force acting on an object moving along a surface is *always* equal and opposite to the force of gravity. You disagree with this definition.

(a) Provide the proper definition for the normal force acting on an object.

(b) Describe, with the aid of free-body diagrams, three situations in which the normal force acting on an object cannot be determined using your lab partner's definition.

(c) Describe, with the aid of a free-body diagram, a situation in which your lab partner's definition could apply.

Inquiry

8. You are given two bowling balls. One is pure wood, while the other has an iron core. Your task is to verify Newton's laws. Accordingly, you set up an inclined plane in such a way that you can let the balls roll down the plane and along the floor.

(a) Design an experiment to determine which ball has more inertia.

(b) Sketch a velocity-time graph to illustrate your predictions of the motion of each ball.

(c) Explain how Newton's first law affects the motion of the ball during each phase of its motion. Explain your reasoning.

(d) Draw a free-body diagram for each ball as it descends the ramp. Write equations to predict the acceleration of each. Provide an analysis of the equations to show that each ball's acceleration down the ramp should be the same.

(e) The analysis in **(d)** seems to defy Newton's first law. Initially, you might predict that the ball with more inertia would have a different acceleration. Provide an explanation, based on Newton's laws, of why the ball with more inertia does not experience a greater acceleration.

9. Recall the apparatus set-up you used for Investigation 1-A to explore inertial mass. In this case, you are given a dynamics cart that has a mass of 500 g and you use a falling mass of 200 g.

(a) Assume that the coefficient of friction between the cart and the ramp is 0.12. Calculate theoretical predictions for the acceleration of the system when incremental masses of 100 g, 200 g, 300 g, 400 g, and 500 g are added to the cart.

(b) Plot an acceleration-versus-incremental-mass graph for your theoretical values.

(c) Does the line on this graph pass through the origin? Explain your reasoning.

(d) What acceleration is indicated at the point where the line on the graph crosses the y-axis?

(e) Describe two different modifications you could make to this set-up so that the cart would have zero acceleration.

Communication

10. Draw a free-body diagram of a diver being lowered into the water from a hovering helicopter to make a sea rescue. His downward speed is decreasing. Label all forces and show them with correct scale lengths.

11. Use free-body diagrams to show that the tension in the rope is the same for both of the following situations.

(a) Two horses are pulling in opposite directions on the same rope, with equal and opposite forces of 800 N.

(b) One horse is pulling on a rope, which is tied to a tree, with a force of 800 N.

12. A toy rocket is shot straight into the air and reaches a height of 162 m. It begins its descent in free fall for 2 s before its parachute opens. The rocket then quickly reaches terminal velocity.

(a) Sketch a velocity-time graph for the descent.

(b) Draw a free-body diagram for each of the three passes of the descent: the free fall, the parachute opening and slowing the descent, and terminal velocity

13. A large crate sits on the floor of an elevator. The force of static friction keeps the crate from moving. However, the magnitude of this force changes when the elevator **(a)** is stationary, **(b)** accelerates downward and **(c)** accelerates upward. Explain how the three forces should be ranked from weakest to strongest.

14. Two blocks, of mass M and mass m, are in contact on a horizontal frictionless table (with the block of mass M on the left and the block of mass m on the right). A force F_1 is applied to the block of mass M and the two blocks accelerate together to the right.

(a) Draw a free-body diagram for each block.

(b) Suppose the larger block M exerts a force F_2 on the smaller mass m. By Newton's third law, the smaller block m exerts a force F_2 on the larger block M. Argue whether $F_1 = F_2$ or not. Justify your reasoning.

(c) Derive an expression for the acceleration of the system.

(d) Derive an expression for the magnitude of the force F_2 that the larger block exerts on the smaller block.

(e) Choose different values of M and m (e.g. $M = 2m$, $M = 5m$, including the case $M = m$) and compare the magnitudes of F_1 and F_2.

(f) Comment on the above results.

Making Connections

15. Car tires are designed to optimize the amount of friction between the tire surface and the road. If there is too little friction, the car will be hard to control. Too much friction will negatively affect the car's performance and fuel efficiency.

(a) List the different types of road conditions under which cars are operated. Research the different types of tread designs that have been developed to respond to these conditions. Explain how the different designs are intended to increase or decrease the coefficient of friction between a car's tires and the road.

(b) Compare the positive and negative factors of using "all-season" tires rather than changing car tires to suit the season (e.g., changing to special winter tires). Do a cost analysis of the two systems and recommend your choice for the climatic conditions in your own community.

Problems for Understanding

16. Suppose a marble is rolling with a velocity of 3.0 m/s[N] and no horizontal force is acting on it. What will be its velocity at 10.0 s?

17. What is the mass of a sack of potatoes that weighs 110 N (1.1×10^2 N)?

18. A physics teacher is in an elevator moving upward at a velocity of 3.5 m/s when he drops his watch. What are the initial velocity and acceleration of the watch in a frame that is attached to **(a)** the elevator and **(b)** the building?

19. **(a)** What is the acceleration of a 68.0 kg crate that is pushed across the floor by a 425 N force, if the coefficient of kinetic friction between the box and floor is 0.500?
    (b) What force would be required to push the crate across the floor with constant velocity?

20. A red ball that weighs 24.5 N and a blue ball that weighs 39.2 N are connected by a piece of elastic of negligible mass. The balls are pulled apart, stretching the elastic. If the balls are released at exactly the same time, the initial acceleration of the red ball is 1.8 m/s^2 eastward. What is the initial acceleration of the blue ball?

21. If a 0.24 kg ball is accelerated at 5.0 m/s^2, what is the magnitude of the force acting on it?

22. A 10.0 kg brick is pulled from rest along a horizontal bench by a constant force of 4.0 N. It is observed to move a distance of 2.0 m in 8.0 s.
    (a) What is the acceleration of the brick?
    (b) What is the ratio of the applied force to the mass?
    (c) Explain why your two answers above do not agree. Use numerical calculations to support your explanation.

23. A football is thrown deep into the end zone for a touchdown. If the ball was in the air for 2.1 s and air friction is neglected, to what vertical height must it have risen?

24. A 2200 kg car is travelling at 45 km/h when its brakes are applied and it skids to a stop. If the coefficient of friction between the road and the tires is 0.70, how far does the car go before stopping?

25. A 55.0 kg woman jumps to the floor from a height of 1.5 m.
    (a) What is her velocity at the instant before her feet touch the floor?
    (b) If her body comes to rest during a time interval of 8.00×10^{-3} s, what is the force of the floor on her feet?

26. You are pushing horizontally on a 3.0 kg block of wood, pressing it against a wall. If the coefficient of static friction between the block and the wall is 0.60, how much force must you exert on the block to prevent it from sliding down?

27. The maximum acceleration of a truck is 2.6 m/s^2. If the truck tows another truck with a mass the same as its own, what is its maximum acceleration?

28. A force F produces an acceleration a when applied to a certain body. If the mass of the body is doubled and the force is increased five-fold, what will be the effect on the following?
    (a) the acceleration of the body
    (b) the distance travelled by the body in a given time

29. A 45.0 kg box is pulled with a force of 205 N by a rope held at an angle of 46.5° to the horizontal. The velocity of the box increases from 1.00 m/s to 1.50 m/s in 2.50 s. Calculate
    (a) the net force acting horizontally on the box.
    (b) the frictional force acting on the box.
    (c) the horizontal component of the applied force.
    (d) the coefficient of kinetic friction between the box and the floor.

30. A Fletcher's trolley apparatus consists of a 4.0 kg cart and a 2.0 kg mass attached by a string that runs over a pulley. Find the acceleration of the trolley and the tension in the string when the suspended mass is released.

31. You are a passenger on an airplane and you decide to measure its acceleration as it travels down the runway before taking off. You take out a yo-yo and notice that when you suspend it, it makes an angle of 25° with the vertical. Assume the plane's mass is 4.0×10^3 kg.
    (a) What is the acceleration of the airplane?
    (b) If the yo-yo mass is 65 g, what is the tension in the string?

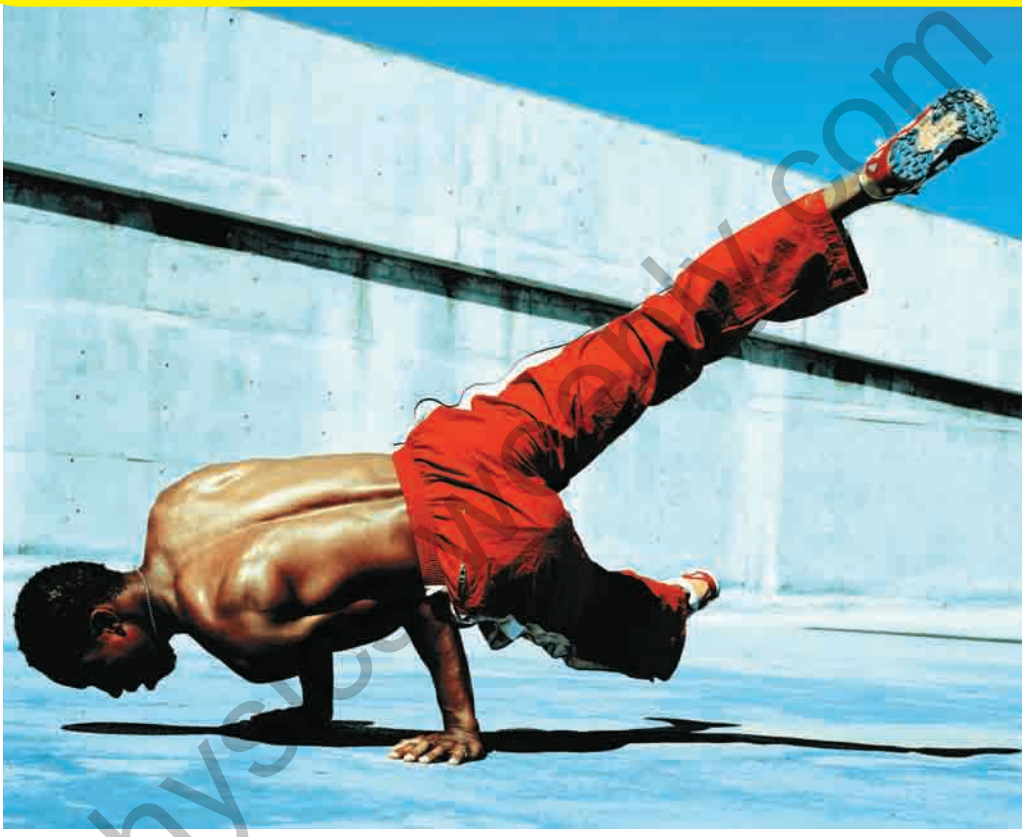
CHAPTER 2

Dynamics in Two Dimensions

CHAPTER CONTENTS

PREREQUISITE CONCEPTS AND SKILLS

- Resolving vectors into components
- Combining vector components



Dancers spin, twist, and swing through the air. Athletes move in constantly changing directions. The details of these complex motions are studied by kinesiologists by tracking the position of sensors fastened to knees, elbows, or other joints. Position-time data gathered from such experiments can be used to produce photo-realistic animations for movies and video games. This data can also be used to study the details of the motion from a physicist's point of view, providing a basis for measurement of changes in speed and direction — accelerations and the forces that cause them.

When you turn while you run, walk, or dance, you are moving in two dimensions. You might change direction suddenly or do it over several steps, following a curved path. In either case, changes in direction are accelerations, and accelerations require an unbalanced force. In this chapter, you will examine the accelerations and forces involved in two types of two-dimensional motion — objects following a curved path after being launched into the air and objects moving in a circle or part of a circle.

MULTI LAB

Motion in Two Dimensions

TARGET SKILLS

- Performing and recording
- Analyzing and interpreting

CAUTION Wear impact-resistant safety goggles. Also, do not stand close to other people or equipment while doing these activities.

Race to the Ground

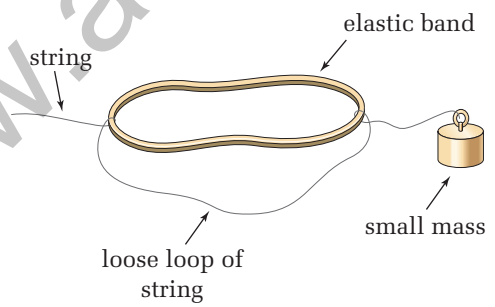
If your school has a vertical acceleration demonstrator, set it up to make observations. If you do not have a demonstrator, devise a method for launching one object, such as a small metal ball, in the horizontal direction, while at the same instant dropping a second object from exactly the same height. Perform several trials, observing the paths of the objects very carefully.

Analyze and Conclude

1. Describe in detail the paths of the two objects. Compare the motion of the two objects.
2. Which object hit the floor first?
3. Did the horizontal motion of the first object appear to affect its vertical motion? Explain your reasoning for your conclusion.

Feel the Force

According to the law of inertia, objects must experience an unbalanced force to change the direction of their motion. What does this suggest about an object moving in a circle? Assemble the apparatus as shown in the diagram to obtain information on the forces involved in circular motion. Gently swing the mass in a horizontal circle. Carefully increase the speed of rotation and observe the effect on the elastic band and the path of the object. Change the angle so that the object moves first in an inclined plane and then in a vertical plane and repeat your observations.



Analyze and Conclude

1. How does the force exerted on the object by the elastic band change as the elastic band stretches?
2. How does the force exerted on the object change as the speed of the mass increases?
3. Sketch free-body diagrams showing the forces acting on the object as it moves in a
   (a) horizontal plane
   (b) vertical plane (at the top of the swing, the bottom of the swing, and when it is at one side of the circle)
4. Describe and attempt to explain any other changes you observed in the object's motion as its speed varied.
5. Was there any difference in the force exerted by the elastic band at the highest and lowest points of the mass's path when it moved in a vertical plane? If so, suggest an explanation.

2.1 Projectile Motion

SECTION EXPECTATIONS

- Describe qualitatively and quantitatively the path of a projectile.
- Analyze, predict, and explain projectile motion in terms of horizontal and vertical components.
- Design and conduct experiments to test the predictions about the motion of a projectile.

KEY TERMS

- trajectory
- projectile
- range
- parabola

Figure 2.1 After the water leaves the pipes in this fountain, the only forces acting on the water are gravity and air friction.

A tourist visiting Monte Carlo, Monaco, would probably stand and admire the beauty of the fountain shown in the photograph, and might even toss a coin into the fountain and make a wish. A physics student, however, might admire the symmetry of the water jets. He or she might estimate the highest point that the water reaches and the angle at which it leaves the fountain, and then mentally calculate the initial velocity the water must have in order to reach that height.

The student might then try to think of as many examples of this type of motion as possible. For example, a golf ball hit off the tee, a leaping frog, a punted football, and a show-jumping horse all follow the same type of path or **trajectory** as the water from a fountain. Any object given an initial thrust and then allowed to soar through the air under the force of gravity only is called a **projectile**. The horizontal distance that the projectile travels is called its **range**.

Air friction does, of course, affect the trajectory of a projectile and therefore the range of the projectile, but the mathematics needed to account for air friction is complex. You can learn a great deal about the trajectory of projectiles by neglecting friction, while keeping in mind that air friction will modify the actual motion.

You do not need to learn any new concepts in order to analyze and predict the motion of projectiles. All you need are data that will provide you with the velocity of the projectile at the moment it is launched and the kinematic equations for uniformly accelerated motion. You observed projectile motion in the Race

HISTORY LINK

Calculating actual trajectories of artillery shells was an enormous task before the advent of electronic computers. Human experts required up to 20 h to do the job, even with mechanical calculators. ENIAC, the first electronic computer that stored its program instructions, was built in 1946 and could calculate the trajectory of an artillery shell in about 30 s. The vacuum tubes in ENIAC's processing unit required 174 kW of electric power, so the energy required to calculate one trajectory was comparable to the energy needed to actually fire the artillery shell!

to the Ground segment of the Multi-Lab and identified a feature of the motion that simplifies the analysis. The horizontal motion of the projectile does not influence the vertical motion, nor does the vertical motion affect the horizontal motion. *You can treat the motion in the two directions independently.* The following points will help you analyze all instances of projectile motion.

- Gravity is the only force influencing ideal projectile motion. (Neglect air friction.)
- Gravity affects only the vertical motion, so equations for uniformly accelerated motion apply.
- No forces affect horizontal motion, so equations for uniform motion apply.
- The horizontal and vertical motions are taking place during the same time interval, thus providing a link between the motion in these dimensions.

ELECTRONIC LEARNING PARTNER

To enhance your understanding of two-dimensional motion, go to your Electronic Learning Partner for an interactive activity.

Projectiles Launched Horizontally

If you had taken a picture with a strobe light of your Race to the Ground lab, you would have obtained a photograph similar to the one in Figure 2.2. The ball on the right was given an initial horizontal velocity while, at the same moment, the ball on the left was dropped. As you can see in the photograph, the two balls were the same distance from the floor at any given time — the vertical motion of the two balls was identical. This observation verifies that horizontal motion does not influence vertical motion. Examine the following sample problem to learn how to make use of this feature of projectile motion.

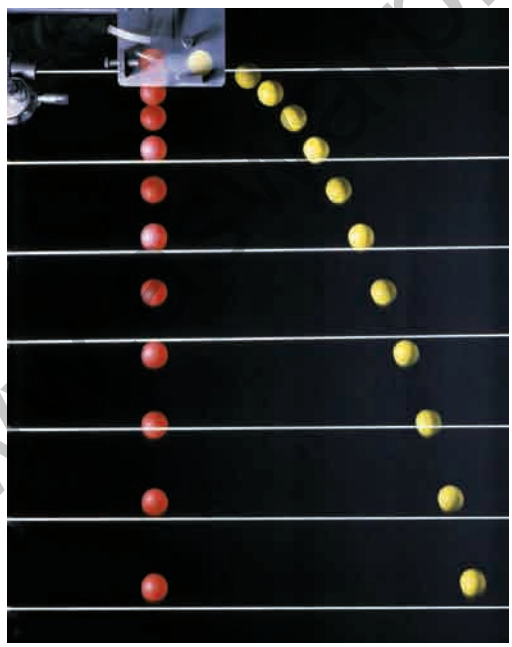


Figure 2.2 You can see that the balls are accelerating downward, because the distances they have travelled between flashes of the strobe light are increasing. If you inspected the horizontal motion of the ball on the right, you would find that it travelled the same horizontal distance between each flash of the strobe light.

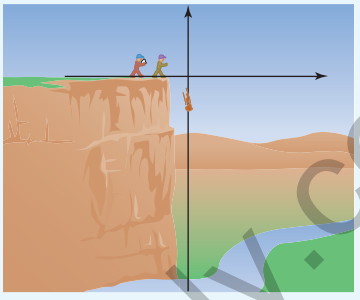
SAMPLE PROBLEM

Analyzing a Horizontal Projectile

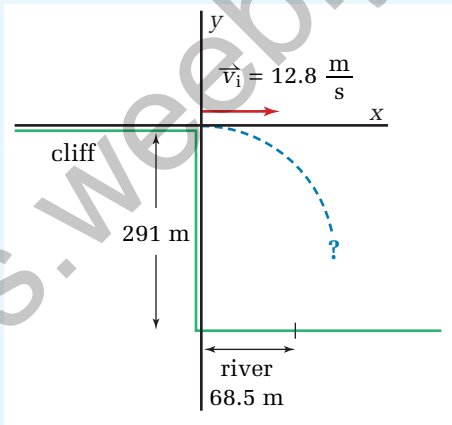
While hiking in the wilderness, you come to a cliff overlooking a river. A topographical map shows that the cliff is 291 m high and the river is 68.5 m wide at that point. You throw a rock directly forward from the top of the cliff, giving the rock a horizontal velocity of 12.8 m/s.

(a) Did the rock make it across the river?

(b) With what velocity did the rock hit the ground or water?



Conceptualize the Problem



- Start to frame the problem by making a rough sketch of the cliff with a *coordinate system* superimposed on it. Write the initial conditions on the sketch.
- The rock *initially* has *no vertical velocity*. It falls, from *rest*, with the *acceleration due to gravity*. Since "down" was chosen as negative, the *acceleration* of the rock is *negative*. (Neglect air friction.)
- Since the coordinate system was placed at the top of the cliff, the *vertical component* of the *displacement* of the rock is *negative*.
- The *displacement* that the rock falls determines the *time interval* during which it falls, according to the kinematic equations.
- The rock moves *horizontally* with a *constant velocity* until it hits the ground or water at the end of the time interval.
- The *final velocity* of the rock at the instant before it hits the ground or water is the *vector sum* of the horizontal velocity and the final vertical velocity.
- Use x to represent the horizontal component of displacement and y for the vertical component of displacement. Use x and y subscripts to identify the horizontal and vertical components of the velocity.

Identify the Goal

(a) Whether the horizontal distance, Δx, travelled by the rock was greater than 68.5 m, the width of the river

(b) The final velocity, $\vec{v}_f$, of the rock the instant before it hit the ground

Identify the Variables

Known		Implied	Unknown
$\Delta y = -291$ m	river width = 68.5 m	$a_y = -9.81 \frac{m}{s^2}$	Δx
$v_x = 12.8 \frac{m}{s}$		$v_{iy} = 0.0 \frac{m}{s}$	$\vec{v}_f$

Develop a Strategy

Find the time interval during which the rock was falling by using the kinematic equation that relates displacement, initial velocity, acceleration, and time interval. Note that the vertical component of the initial velocity is zero and solve for the time interval.

$$\Delta y = v_{yi}\Delta t + \frac{1}{2}a\Delta t^2$$
$$\Delta y = \frac{1}{2}a\Delta t^2$$
$$\frac{2\Delta y}{a} = \Delta t^2$$
$$\Delta t = \sqrt{\frac{2\Delta y}{a}}$$

Insert numerical values and solve.

$$\Delta t = \sqrt{\frac{2(-291\ \cancel{\text{m}})}{-9.81\ \frac{\cancel{\text{m}}}{\text{s}^2}}}$$
$$\Delta t = \sqrt{59.327\ \text{s}^2}$$
$$\Delta t = \pm 7.7024\ \text{s}$$

Find the horizontal displacement of the rock by using the equation for uniform motion (constant velocity) that relates velocity, distance, and time interval. Solve for displacement.

$$v_x = \frac{\Delta x}{\Delta t}$$
$$\Delta x = v_x\Delta t$$

Use the time calculated above and initial velocity to calculate the horizontal distance travelled by the rock. Choose the positive value for time, since negative time has no meaning in this application.

$$\Delta x = \left(12.8\ \frac{\text{m}}{\cancel{\text{s}}}\right)(7.7024\ \cancel{\text{s}})$$
$$\Delta x = 98.591\ \text{m}$$
$$\Delta x \cong 98.6\ \text{m}$$

(a) Since the horizontal distance travelled by the rock (98.6 m) was much greater than the width of the river (68.5 m), the rock hit the ground on the far side of the river.

Find the vertical component of the final velocity by using the kinematic equation that relates initial velocity, final velocity, acceleration, and time.

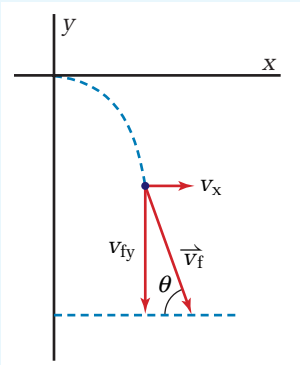
$$v_{fy} = v_{iy} + a\Delta t$$

Insert the numerical values and solve.

$$v_{fy} = 0.0\ \frac{\text{m}}{\text{s}} + \left(-9.81\ \frac{\text{m}}{\text{s}^2}\right)(7.7024\ \text{s})$$
$$v_{fy} = -75.561\ \frac{\text{m}}{\text{s}}$$

Use the Pythagorean theorem to find the magnitude of the resultant velocity.

$$|\vec{v}_f| = \sqrt{(v_x)^2 + (v_{fy})^2}$$
$$|\vec{v}_f| = \sqrt{\left(12.8\ \frac{\text{m}}{\text{s}}\right)^2 + \left(-75.561\ \frac{\text{m}}{\text{s}}\right)^2}$$
$$|\vec{v}_f| = \sqrt{5873.30\ \frac{\text{m}^2}{\text{s}^2}}$$
$$|\vec{v}_f| = 76.637\ \frac{\text{m}}{\text{s}}$$
$$|\vec{v}_f| \cong 76.6\ \frac{\text{m}}{\text{s}}$$



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Use trigonometry to find the angle that the rock made with the horizontal when it struck the ground.

$$\tan\theta = \frac{v_{fy}}{v_x}$$

$$\theta = \tan^{-1}\frac{v_{fy}}{v_x}$$

$$\theta = \tan^{-1}\frac{75.561\ \frac{m}{s}}{12.8\ \frac{m}{s}}$$

$$\theta = \tan^{-1} 5.9032$$

$$\theta = 80.385^\circ$$

$$\theta \cong 80.4^\circ$$

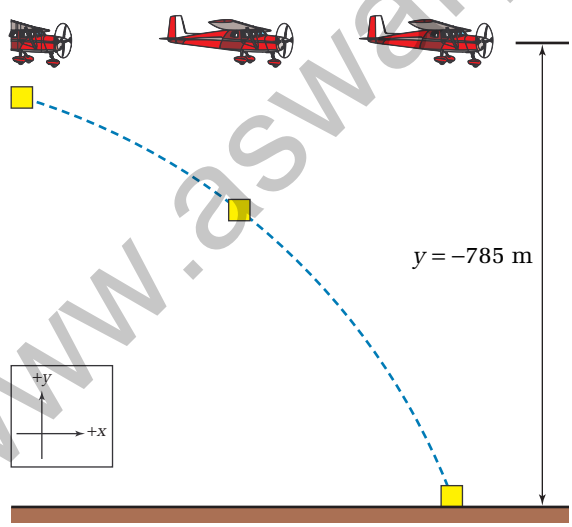
(b) The rock hit the ground with a velocity of 76.6 m/s at an angle of 80.4° with the horizontal.

Validate the Solution

The distance that the rock fell vertically was very large, so you would expect that the rock would be travelling very fast and that it would hit the ground at an angle that was nearly perpendicular to the ground. Both conditions were observed.

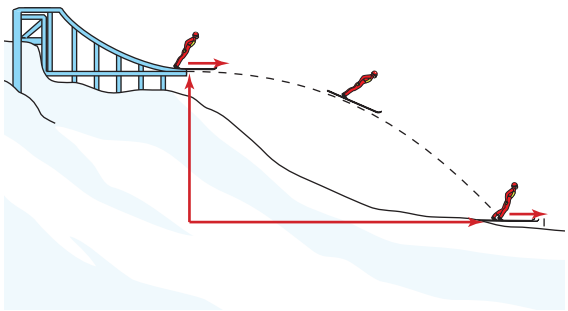
PRACTICE PROBLEMS

1. An airplane is dropping supplies to northern villages that are isolated by severe blizzards and cannot be reached by land vehicles. The airplane is flying at an altitude of 785 m and at a constant horizontal velocity of 53.5 m/s. At what horizontal distance before the drop point should the co-pilot drop the supplies so that they will land at the drop point? (Neglect air friction.)



2. A cougar is crouched on the branch of a tree that is 3.82 m above the ground. He sees an unsuspecting rabbit on the ground, sitting 4.12 m from the spot directly below the branch on which he is crouched. At what horizontal velocity should the cougar jump from the branch in order to land at the point at which the rabbit is sitting?

3. A skier leaves a jump with a horizontal velocity of 22.4 m/s. If the landing point is 78.5 m lower than the end of the ski jump, what horizontal distance did the skier jump? What was the skier's velocity when she landed? (Neglect air friction.)



4. An archer shoots an arrow toward a target, giving it a horizontal velocity of 70.1 m/s. If the target is 12.5 m away from the archer, at what vertical distance below the point of release will the arrow hit the target? (Neglect air friction.)

5. In a physics experiment, you are rolling a golf ball off a table. If the tabletop is 1.22 m above the floor and the golf ball hits the floor 1.52 m horizontally from the table, what was the initial velocity of the golf ball?

6. As you sit at your desk at home, your favourite autographed baseball rolls across a shelf at 1.0 m/s and falls 1.5 m to the floor. How far does it land from the base of the shelf?

7. A stone is thrown horizontally at 22 m/s from a canyon wall that is 55 m high. At what distance from the base of the canyon wall will the stone land?

8. A sharpshooter shoots a bullet horizontally over level ground with a velocity of 3.00×10^2 m/s. At the instant that the bullet leaves the barrel, its empty shell casing falls vertically and strikes the ground with a vertical velocity of 5.00 m/s.

   (a) How far does the bullet travel?

   (b) What is the vertical component of the bullet's velocity at the instant before it hits the ground?

Projectiles Launched at an Angle

Most projectiles, including living ones such as the playful dolphins in Figure 2.3, do not start their trajectory horizontally. Most projectiles, from footballs to frogs, start at an angle with the horizontal. Consequently, they have an initial velocity in both the horizontal and vertical directions. These trajectories are described mathematically as **parabolas**. The only additional step required to analyze the motion of projectiles launched at an angle is to determine the magnitude of the horizontal and vertical components of the initial velocity.

Mathematically, the path of any ideal projectile lies along a parabola. In the following investigation, you will develop some mathematical relationships that describe parabolas. Then, the sample problems that follow will help you apply mathematical techniques for analyzing projectiles.

Figure 2.3 Dolphins have been seen jumping as high as 4.9 m from the surface of the water in a behaviour called a "breach."

INVESTIGATION 2-A

The Components of Projectile Motion

TARGET SKILLS

- Analyzing and interpreting
- Modelling concepts
- Communicating results

A heavy steel ball rolling up and down a ramp follows the same type of trajectory that a projectile follows. You will obtain a permanent record of the steel ball's path by placing a set of white paper and carbon paper in its path. You will then analyze the vertical and horizontal motion of the ball and find mathematical relationships that describe the path.

Problem

What patterns exist in the horizontal and vertical components of projectile velocity?

Hypothesis

Formulate a hypothesis about the relationships between time and the vertical distance travelled by the steel ball.

Equipment

- large sheet of plywood
- very heavy steel ball
- metre stick, graph paper, tape
- set of white paper and carbon paper (or pressure-sensitive paper)

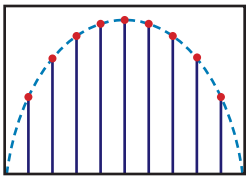
Procedure

1. Set up the apparatus as illustrated.
   CAUTION Wear impact-resistant safety goggles. Also, do not stand close to other people or equipment while doing these activities.

2. Practise rolling the steel ball up the slope at an angle, so that it follows a curved path that will fit the size of your set of white paper and carbon paper.
3. Tape the carbon paper and white paper onto the plywood so that, when the steel ball rolls over it, the carbon paper will leave marks on the white paper.
4. Roll the steel ball up the slope at an angle, as you practised, so that it will roll over the paper and leave a record of its path.
5. Remove the white paper from the plywood. Draw approximately nine or more equally spaced lines vertically through the trajectory.

Analyze and Conclude

1. Measure the vertical displacement in each segment of the path of the steel ball, as shown in the diagram.



2. Assuming that the motion of the ball was uniform in the horizontal direction, each equally spaced vertical line represents the same amount of time. Call it one unit of time.
3. Separate your data into two parts: (a) the period of time that the ball was rolling upward and (b) the period of time that the ball was rolling downward. For each set of data, make a graph of vertical-distance-versus-time units.
4. Use curve-straightening techniques to convert your graphs to straight lines. (See Skill Set 4.)
5. Write equations to describe your graphs.
6. Is the vertical motion of the steel ball uniform or uniformly accelerated?
7. How does it compare to the vertical motion of a freely falling object?
8. Was your hypothesis valid or invalid?
9. Is this lab an appropriate model for actual projectile motion? Explain why or why not.

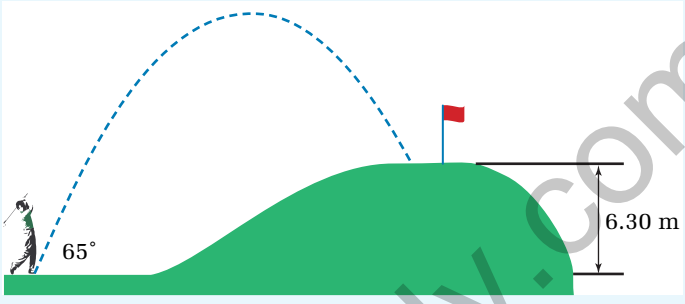
Analyzing Parabolic Trajectories

1. A golfer hits the golf ball off the tee, giving it an initial velocity of 32.6 m/s at an angle of 65° with the horizontal. The green where the golf ball lands is 6.30 m higher than the tee, as shown in the illustration. Find

(a) the time interval during which the golf ball was in the air

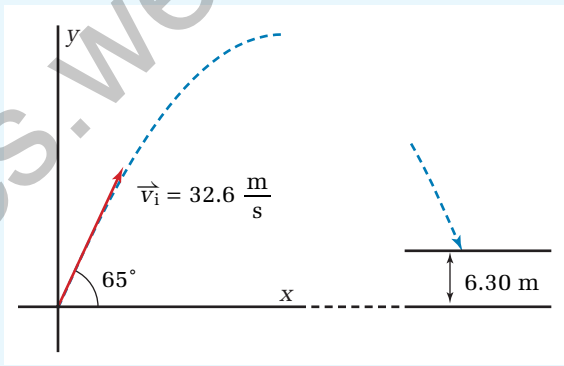
(b) the horizontal distance that it travelled

(c) the velocity of the ball just before it hit the ground (neglect air friction)



Conceptualize the Problem

- Start to frame the problem by making a sketch that includes a *coordinate system*, the *initial conditions*, and all of the known information.
- The golf ball has a *positive initial velocity* in the *vertical* direction. It will rise and then fall according to the kinematic equations.
- The *vertical acceleration* of the golf ball is *negative* and has the *magnitude* of the acceleration due to *gravity*.
- The *time interval* is determined by the *vertical* motion. The *time interval ends* when the golf ball is at a height equal to the *height of the green*.
- The golf ball will be at the height of the green *twice*, once while it is *rising* and once while it is *falling*.
- Motion in the *horizontal* direction is *uniform*; that is, it has a *constant velocity*.
- The horizontal *displacement of the ball* depends on the *horizontal component* of the *initial velocity* and on the duration of the flight.



Identify the Goal

(a) The time interval, Δt, that the golf ball was in the air

(b) The horizontal distance, Δx, that the golf ball travelled

(c) The final velocity of the golf ball, $\vec{v}_f$

Identify the Variables

Known		Implied	Unknown	
$\lvert\vec{v}_i\rvert = 32.6\ \frac{m}{s}$	$\Delta y = 6.30$ m	$a_y = -9.81\ \frac{m}{s^2}$	Δt	$\vec{v}_f$
$\theta_i = 65°$			Δx	θ_f
			v_{ix}	v_{iy}

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Develop a Strategy

Find the horizontal and vertical components of the initial velocity.

$$v_{ix} = |\vec{v}_i| \cos\theta \qquad v_{iy} = |\vec{v}_i| \sin\theta$$

$$v_{ix} = 32.6\ \frac{\text{m}}{\text{s}} \cos 65^\circ \qquad v_{iy} = 32.6\ \frac{\text{m}}{\text{s}} \sin 65^\circ$$

$$v_{ix} = 13.78\ \frac{\text{m}}{\text{s}} \qquad v_{iy} = 29.55\ \frac{\text{m}}{\text{s}}$$

Find the time interval at which the ball is at a vertical position of 6.30 m by using the kinematic equation that relates displacement, initial velocity, acceleration, and the time interval. You cannot solve directly for the time interval, because you have a quadratic equation. Substitute in the numerical values.

$$\Delta y = v_{iy}\Delta t + \frac{1}{2}a_y\Delta t^2$$

$$6.30\ \text{m} = 29.55\ \frac{\text{m}}{\text{s}}\Delta t + \frac{1}{2}\left(-9.81\ \frac{\text{m}}{\text{s}^2}\right)\Delta t^2$$

Rearrange the equation into the general form of a quadratic equation and solve using the quadratic formula $\Delta t = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

$$4.905\Delta t^2 - 29.55\Delta t + 6.30 = 0$$

$$\Delta t = \frac{29.55 \pm \sqrt{(29.55)^2 - 4(4.905)(6.30)}}{2(4.905)}$$

$$\Delta t = \frac{29.55 \pm \sqrt{749.597}}{9.81}$$

$$\Delta t = 0.2213\ \text{s (or) } 5.803\ \text{s}$$

$$\Delta t \cong 5.8\ \text{s}$$

(a) The smaller value is the time that the ball reached a height of 6.30 m when it was rising. The golf ball hit the green 5.8 s after it was hit off the tee.

Use 5.803 s and the equation for constant velocity to determine the horizontal distance travelled by the golf ball.

$$v = \frac{\Delta x}{\Delta t}$$

$$\Delta x = v\Delta t$$

$$\Delta x = \left(13.78\ \frac{\text{m}}{\cancel{\text{s}}}\right)(5.803\ \cancel{\text{s}})$$

$$\Delta x = 79.965\ \text{m}$$

$$\Delta x \cong 8.0 \times 10^1\ \text{m}$$

(b) The golf ball travelled 80 m in the horizontal direction.

Find the vertical component of the final velocity by using the kinematic equation that relates the initial and final velocities to the acceleration and the time interval.

$$v_{fy} = v_{iy} + a_y\Delta t$$

$$v_{fy} = 29.55\ \frac{\text{m}}{\text{s}} + \left(-9.81\ \frac{\text{m}}{\text{s}^{\cancel{2}}}\right)(5.803\ \cancel{\text{s}})$$

$$v_{fy} = -27.38\ \frac{\text{m}}{\text{s}}$$

Use the Pythagorean theorem to find the magnitude of the final velocity.

$$|\vec{v}_f| = \sqrt{\left(13.78\ \frac{m}{s}\right)^2 + \left(-27.38\ \frac{m}{s}\right)^2}$$

$$|\vec{v}_f| = \sqrt{939.55\ \frac{m^2}{s^2}}$$

$$|\vec{v}_f| = 30.65\ \frac{m}{s}$$

$$|\vec{v}_f| \cong 31\ \frac{m}{s}$$

Use trigonometry to find the angle that the final velocity makes with the horizontal.

$$\tan\theta = \left(\frac{v_{fy}}{v_x}\right)$$

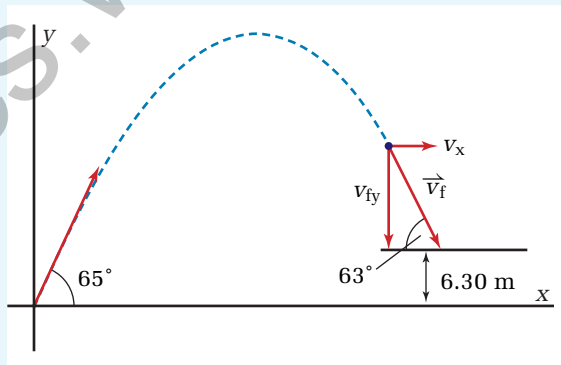
$$\theta = \tan^{-1}\left(\frac{v_{fy}}{v_x}\right)$$

$$\theta = \tan^{-1}\frac{|-27.38\ \frac{m}{s}|}{|13.78\ \frac{m}{s}|}$$

$$\theta = \tan^{-1} 1.9869$$

$$\theta = 63.28^\circ$$

$$\theta \cong 63^\circ$$



(c) The final velocity of the golf ball just before it hit the ground was 31 m/s at 63° with the horizontal.

Validate the Solution

Since the golf ball hit the ground at a level slightly higher than the level at which it started, you would expect the final velocity to be slightly smaller than the initial velocity and the angle to be a little smaller than the initial angle. These results were obtained. All of the units cancelled properly.

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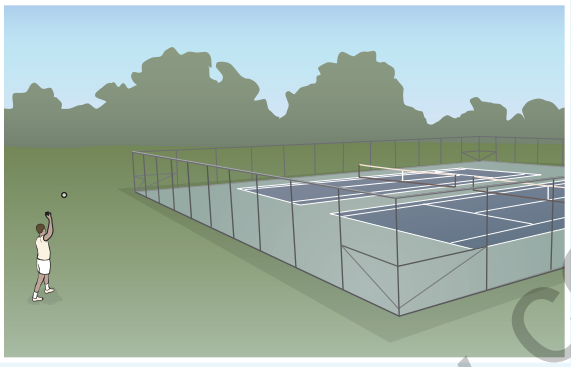
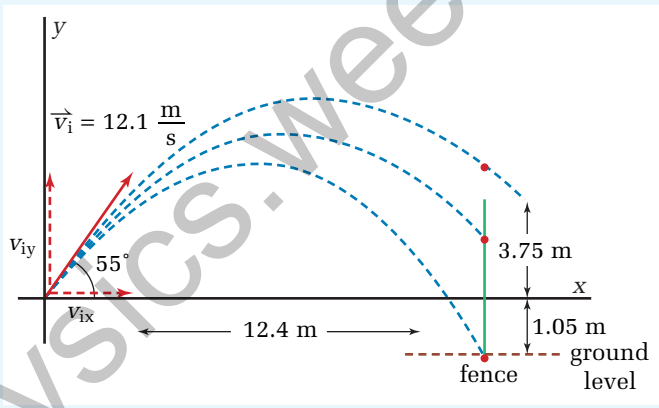
2. You are playing tennis with a friend on tennis courts that are surrounded by a 4.8 m fence. Your opponent hits the ball over the fence and you offer to retrieve it. You find the ball at a distance of 12.4 m on the other side of the fence. You throw the ball at an angle of 55.0° with the horizontal, giving it an initial velocity of 12.1 m/s. The ball is 1.05 m above the ground when you release it. Did the ball go over the fence, hit the fence, or hit the ground before it reached the fence? (Ignore air friction.)



Conceptualize the Problem

- Make a sketch of the *initial conditions* and the three options listed in the question.
- Choose the *origin* of the coordinate system to be at the point at which the *ball left your hand*.
- The equations for *uniformly accelerated motion* apply to the *vertical* motion.
- The definition for *constant velocity* applies to the *horizontal* motion.
- Because the *x*-axis is above ground level, you will have to determine where the top of the fence is relative to the *x*-axis.
- The *time interval* is the *link* between the *vertical* motion and the *horizontal* motion. Finding the time interval required for the ball to reach the position of the fence will allow you to determine the *height* of the ball when it reaches the fence.



Identify the Goal

Whether the ball went over the fence, hit the fence, or hit the ground before reaching the fence

Identify the Variables

Known		Implied	Unknown	
$\lvert\vec{v}_i\rvert = 12.1 \frac{m}{s}$	$\Delta x = 12.4$ m	$a_y = -9.81 \frac{m}{s^2}$	Δt	$\vec{v}_{iy}$
$\theta = 55°$	$h = 4.8$ m		$\vec{v}_{ix}$	Δy

Develop a Strategy

Find the *x*- and *y*-components of the initial velocity.

$$v_{ix} = \lvert\vec{v}_i\rvert \cos\theta \qquad v_{iy} = \lvert\vec{v}_i\rvert \sin\theta$$

$$v_{ix} = 12.1 \frac{m}{s} \cos 55° \qquad v_{iy} = 12.1 \frac{m}{s} \sin 55°$$

$$v_{ix} = 6.940 \frac{m}{s} \qquad v_{iy} = 9.912 \frac{m}{s}$$

To find the time interval, use the equation for the definition of constant velocity and the data for motion in the horizontal direction.

$$v_x = \frac{\Delta x}{\Delta t}$$

$$\Delta t = \frac{\Delta x}{v_x}$$

$$\Delta t = \frac{12.4\ \cancel{\text{m}}}{6.940\ \frac{\cancel{\text{m}}}{\text{s}}}$$

$$\Delta t = 1.787\ \text{s}$$

To find the height of the ball at the time that it reaches the fence, use the kinematic equation that relates displacement, acceleration, initial velocity, and time interval.

$$\Delta y = v_{iy}\Delta t + \frac{1}{2}a_y\Delta t^2$$

$$\Delta y = \left(9.912\ \frac{\text{m}}{\cancel{\text{s}}}\right)(1.787\ \cancel{\text{s}}) + \frac{1}{2}\left(-9.81\ \frac{\text{m}}{\text{s}^{\cancel{2}}}\right)(1.787\ \cancel{\text{s}})^2$$

$$\Delta y = 2.05\ \text{m}$$

Determine the position of the top of the fence in the chosen coordinate system.

$$y_{\text{fence}} = h - y_{\text{ground to } x\text{–axis}}$$
$$y_{\text{fence}} = 4.8\ \text{m} - 1.05\ \text{m}$$
$$y_{\text{fence}} = 3.75\ \text{m}$$

The ball hit the fence. The fence is 3.75 m above the horizontal axis of the chosen coordinate system, but the ball was only 2.05 m above the horizontal axis when it reached the fence.

Validate the Solution

The units all cancel correctly. The time of flight (about 1.8 s) and the height of the ball (about 2 m) are reasonable values.

PRACTICE PROBLEMS

9. While hiking in the wilderness, you come to the top of a cliff that is 60.0 m high. You throw a stone from the cliff, giving it an initial velocity of 21 m/s at 35° above the horizontal. How far from the base of the cliff does the stone land?

10. A batter hits a baseball, giving it an initial velocity of 41 m/s at 47° above the horizontal. It is a home run, and the ball is caught by a fan in the stands. The vertical component of the velocity of the ball when the fan caught it was –11 m/s. How high is the fan seated above the field?

11. During baseball practice, you go up into the bleachers to retrieve a ball. You throw the ball back into the playing field at an angle of 42° above the horizontal, giving it an initial velocity of 15 m/s. If the ball is 5.3 m above the level of the playing field when you throw it, what will be the velocity of the ball when it hits the ground of the playing field?

12. Large insects such as locusts can jump as far as 75 cm horizontally on a level surface. An entomologist analyzed a photograph and found that the insect's launch angle was 55°. What was the insect's initial velocity?

You have learned to make predictions about projectile motion by doing calculations, but can you make any predictions about patterns of motion without doing calculations? In the following Quick Lab, you will make and test some qualitative predictions.

QUICK LAB

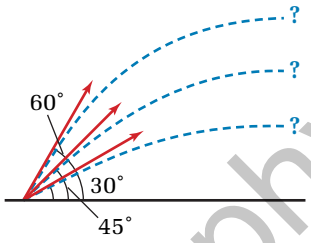
Maximum Range of a Projectile

TARGET SKILLS

- Initiating and planning
- Predicting
- Performing and recording

Football punters try to maximize "hang time" to give their teammates an opportunity to rush downfield while the ball is in the air. Small variations in the initial velocity, especially the angle, make the difference between a great kick and good field position for the opposition.

What launch angle above the horizontal do you predict would maximize the range of an ideal projectile? Make a prediction and then, if your school has a projectile launcher, test your prediction by launching the same projectile several times at the same speed, but at a variety of different angles. If you do not have a projectile launcher, try to devise a system that will allow you to launch a projectile consistently with the same speed but at different angles. Carry out enough trials so you can be confident that you have found the launch angle that gives the projectile the longest range. Always consult with your teacher before using a launch system.



Analyze and Conclude

1. What effect do very large launch angles have on the following quantities?
   - (a) maximum height
   - (b) vertical velocity component
   - (c) horizontal velocity component
   - (d) range
2. What effect do very small launch angles have on the above quantities?
3. Did you see any patterns in the relationship between the launch angle and the range of the projectile? If so, describe these patterns.
4. How well did your experimental results match your prediction?
5. What factors might be causing your projectile to deviate from the ideal?
6. Suppose your experimental results were quite different from your prediction. In which number would you place more confidence, your theoretical prediction or your experimental results? Why?

Symmetrical Trajectories

If a projectile lands at exactly the same level from which it was launched and air friction is neglected, the trajectory is a perfectly symmetrical parabola, as shown in Figure 2.4. You can derive some general relationships that apply to all symmetrical trajectories and use them to analyze these trajectories. Follow the steps in the next series of derivations to see how to determine the time of flight, the range, and the maximum height for projectiles that have symmetrical trajectories.

ELECTRONIC LEARNING PARTNER

Enhance your knowledge and test your predictive skills by doing the projectile motion interactive activity provided by your Electronic Learning Partner.

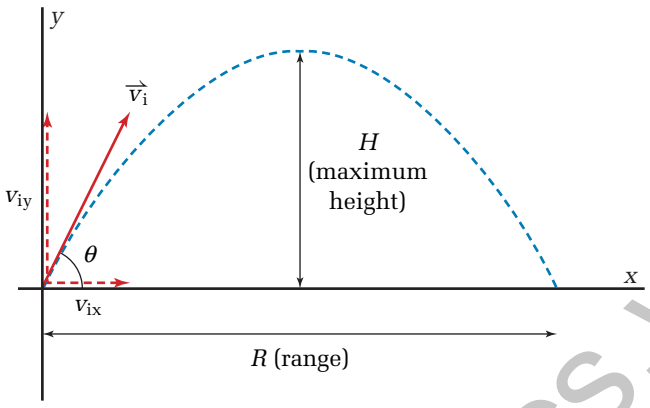


Figure 2.4 The maximum height, H, and the range, R, as well as the time of flight, T, are functions of the initial velocity, $\vec{v}_i$, the angle, θ, and the acceleration due to gravity, g.

Time of flight

- The time of flight ends when the projectile hits the ground. Since the height of the projectile is zero when it hits the ground, you can express this position as $\Delta y = 0$. Write the kinematic equation for vertical displacement and set $\Delta y = 0$.

$$\Delta y = v_{iy}\Delta t + \frac{1}{2}a\Delta t^2$$
$$0 = v_{iy}\Delta t + \frac{1}{2}a\Delta t^2$$

- Write the vertical component of the velocity in terms of the initial velocity and the angle θ. Then, substitute the expression into the equation above. Also, substitute $-g$ for the acceleration, a.

$$v_{iy} = |\vec{v}_i| \sin\theta$$
$$0 = (|\vec{v}_i| \sin\theta)\Delta t + \frac{1}{2}(-g)\Delta t^2$$

- Rearrange the equation to put the zero on the right-hand side and factor out a Δt.

$$\frac{1}{2}g\Delta t^2 - |\vec{v}_i|\Delta t \sin\theta = 0$$
$$\Delta t\left(\frac{1}{2}g\Delta t - |\vec{v}_i| \sin\theta\right) = 0$$

- If either factor is zero, the equation above is satisfied. Write the two solutions.

$$\Delta t = 0 \quad \text{or} \quad \frac{1}{2}g\Delta t = |\vec{v}_i| \sin\theta$$
$$\Delta t = \frac{2|\vec{v}_i| \sin\theta}{g}$$

- $\Delta t = 0$ represents the instant that the projectile was launched. Therefore, the second expression represents the time of flight, T, that the projectile spent in the air before it landed. Since T is a scalar, write the initial velocity without a vector symbol.

$$T = \frac{2v_i \sin\theta}{g}$$

Range

- The range is the horizontal distance that the projectile has travelled when it hits the ground. Write the equation for displacement in the horizontal direction.

$$\Delta x = v_{ix}\Delta t$$

- Write the expression for the horizontal component of velocity in terms of the initial velocity and the launch angle θ. Substitute this expression into the equation for the displacement above.

$$v_{ix} = |\vec{v}_i| \cos\theta$$
$$\Delta x = (|\vec{v}_i| \cos\theta)\Delta t$$

- Since the projectile is at the endpoint of its range, R, when $\Delta t = T$, substitute the expression for T into the equation and simplify. Since R is always in one dimension, omit the vector symbol for the initial velocity.

$$R = \frac{(v_i \cos\theta)(2v_i \sin\theta)}{g}$$
$$R = \frac{v_i^2 2 \sin\theta \cos\theta}{g}$$

- Write the trigonometric identity for $2 \sin\theta \cos\theta$ and substitute the simpler form into the equation.

$$2 \sin\theta \cos\theta = \sin 2\theta$$
$$R = \frac{v_i^2 \sin 2\theta}{g}$$

Maximum height

- As a projectile rises, it slows its upward motion, stops, and then starts downward. Therefore, at its maximum height, its vertical component of velocity is zero. Write the kinematic equation that relates initial and final velocities, acceleration, and displacement and solve for displacement, Δy.

$$v_{fy}^2 = v_{iy}^2 + 2a\Delta y$$
$$0 = v_{iy}^2 + 2a\Delta y$$
$$2a\Delta y = -v_{iy}^2$$
$$\Delta y = \frac{-v_{iy}^2}{2a}$$

- Substitute the expression for initial vertical velocity in terms of the initial velocity and the launch angle, θ. Substitute $-g$ for a. Now Δy is the maximum height, H. Since H is always in one dimension, omit the vector symbol for the initial velocity.

$$H = \frac{-(|\vec{v}_i| \sin\theta)^2}{2(-g)}$$
$$H = \frac{v_i^2 \sin^2\theta}{2g}$$

These three relationships — time of flight, range, and the maximum height — allow you to make important predictions about projectile motion without performing calculations. For example, you can determine the launch angle that will give you the maximum range by inspecting the equation for range. Study the logic of the following steps.

- Inspect the equation for range.

$$R = \frac{v_i^2 \sin 2\theta}{g}$$

- For a given initial velocity on the surface of Earth, the only variable is θ. Therefore, the term "$\sin 2\theta$" determines the maximum range. The largest value that the sine of any angle can achieve is 1.

$$\sin 2\theta = 1$$
$$R_{max} = \frac{v_i^2(1)}{g}$$

- For what angle, θ, is $\sin 2\theta = 1$? Recall that the angle for which the sine is 1 is 90°. Use this information to find θ.

$$\sin 2\theta = 1$$
$$2\theta = \sin^{-1} 1$$
$$2\theta = 90^\circ$$
$$\theta = 45^\circ$$

For any symmetrical trajectory, neglecting air friction, the launch angle that yields the greatest range is 45°. Draw some conclusions of your own by answering the questions in the Conceptual Problems that follow.

Conceptual Problems

- Examine the equation for maximum height. For a given initial velocity, what launch angle would give a projectile the greatest height? What would be the shape of its trajectory?
- Examine the equation for time of flight. For a given initial velocity, what launch angle would give a projectile the greatest time of flight? Would this be a good angle for a football punter? Why?
- Consider the equation for range and a launch angle of 30°. What other launch angle would yield a range exactly equal to that of the range for an angle of 30°?
- Find another pair of launch angles (in addition to your answer to the above question) that would yield identical ranges.
- The acceleration due to gravity on the Moon is roughly one sixth of that on Earth ($g_{moon} = \frac{1}{6}g$). For a projectile with a given initial velocity, determine the time of flight, range, and maximum height on the Moon relative to those values on Earth.
- The general equation for a parabola is $y = Ax^2 + Bx + C$, where A, B, and C are constants. Start with the following equations of motion for a projectile and develop one equation in terms of Δx and Δy by eliminating Δt. Show that the resulting equation, in which Δy is a function of Δx, describes a parabola. Note that the values for the initial velocity (v_i) and launch angle (θ) are constants for a given trajectory.

$$\Delta x = v_i \Delta t \cos\theta$$
$$\Delta y = v_i \Delta t \sin\theta - \frac{1}{2} g \Delta t^2$$

SAMPLE PROBLEM

Analyzing a Kickoff

A player kicks a football for the opening kickoff. He gives the ball an initial velocity of 29 m/s at an angle of 69° with the horizontal. Neglecting friction, determine the ball's maximum height, hang time, and range.

Conceptualize the Problem

- A football field is level, so the trajectory of the ball is a *symmetrical parabola*.
- You can use the equations that were developed for symmetrical trajectories.
- "Hang time" is the time of flight of the ball.

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Identify the Goal

The maximum height, H, of the football
The time of flight, T, of the football
The range, R, of the football

Identify the Variables and Constants

Known	Implied	Unknown
$\lvert\vec{v}_i\rvert = 29\ \frac{m}{s}$	$g = 9.81\ \frac{m}{s^2}$	H
$\theta = 69°$		T
		R

Develop a Strategy

Use the equation for the maximum height of a symmetrical trajectory.

$$H = \frac{v_i^2 \sin^2 \theta}{2g}$$

Substitute the numerical values and solve.

$$H = \frac{\left(29\ \frac{m}{s}\right)^2 (\sin 69°)^2}{2\left(9.81\ \frac{m}{s^2}\right)}$$

$$H = \frac{\left(841\ \frac{m^2}{s^2}\right)(0.871\ 57)}{19.62\ \frac{m}{s^2}}$$

$$H = 37.359\ m$$

$$H \cong 37\ m$$

The maximum height the football reached was 37 m.

Use the equation for the time of flight of a symmetrical trajectory.

$$T = \frac{2v_i \sin \theta}{g}$$

Substitute the numerical values and solve.

$$T = \frac{2\left(29\ \frac{m}{s}\right)(\sin 69°)}{\left(9.81\ \frac{m}{s^2}\right)}$$

$$T = \frac{\left(58\ \frac{m}{s}\right)(0.933\ 58)}{9.81\ \frac{m}{s^2}}$$

$$T = 5.5196\ s$$

$$T \cong 5.5\ s$$

The time of flight, or hang time, of the football was 5.5 s.

Use the equation for the range of a symmetrical trajectory.

$$R = \frac{v_i^2 \sin 2\theta}{g}$$

Substitute the numerical values and solve.

$$R = \frac{\left(29\ \frac{m}{s}\right)^2 (\sin 2(69°))}{9.81\ \frac{m}{s^2}}$$

$$R = \frac{\left(841\ \frac{m^2}{s^2}\right)(0.669\ 13)}{9.81\ \frac{m}{s^2}}$$

$$R = 57.3637\ m$$

$$R \cong 57\ m$$

The football travelled 57 m.

Validate the Solution

All of the values are reasonable for a football kickoff. In every case, the units cancel properly to give metres for the range and maximum height and seconds for the time of flight, or hang time.

PRACTICE PROBLEMS

13. A circus stunt person was launched as a human cannon ball over a Ferris wheel. His initial velocity was 24.8 m/s at an angle of 55°. (Neglect friction)

(a) Where should the safety net be positioned?

(b) If the Ferris wheel was placed halfway between the launch position and the safety net, what is the maximum height of the Ferris wheel over which the stunt person could travel?

(c) How much time did the stunt person spend in the air?

14. You want to shoot a stone with a slingshot and hit a target on the ground 14.6 m away. If you give the stone an initial velocity of 12.5 m/s, neglecting friction, what must be the launch angle in order for the stone to hit the target? What would be the maximum height reached by the stone? What would be its time of flight?

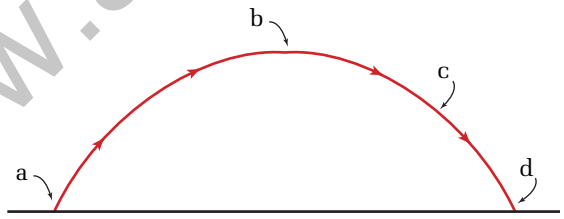
2.1 Section Review

1. K/U Projectiles travel in two dimensions at the same time. Why is it possible to apply kinematic equations for one dimension to projectile motion?

2. K/U How does the analysis of projectiles launched at an angle differ from the analysis of projectiles launched horizontally?

3. C Explain why time is a particularly significant parameter when analyzing projectile motion.

4. C What can you infer about the velocity at each labelled point on the trajectory in this diagram?



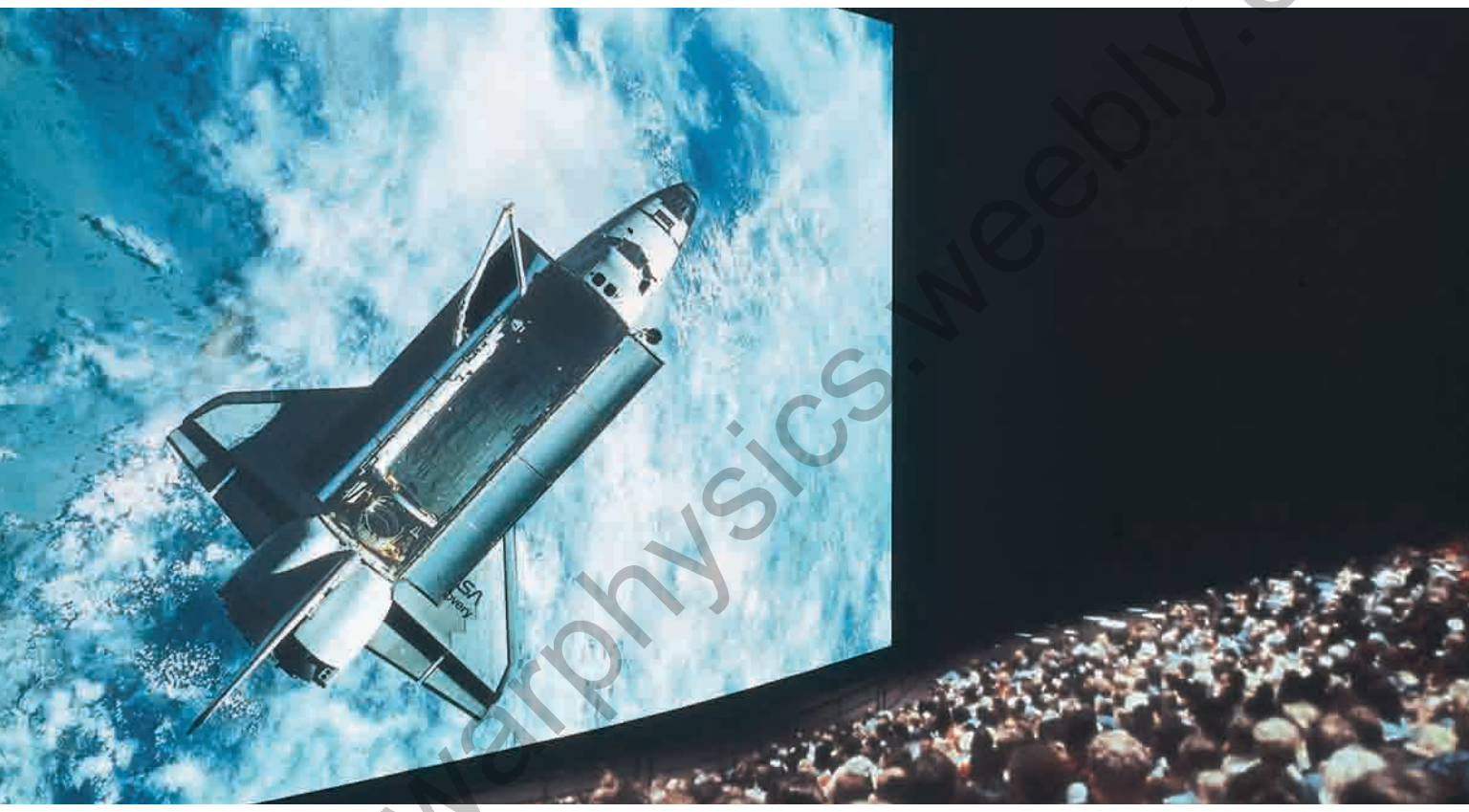
5. C Imagine that you are solving a problem in projectile motion in which you are asked to find the time at which a projectile reaches a certain vertical position. When you solve the problem, you find two different positive values for time that both satisfy the conditions of the problem. Explain how this result is not only possible, but also logical.

6. K/U What properties of projectile motion must you apply when deriving an equation for the maximum height of a projectile?

7. K/U What properties of projectile motion must you apply when deriving an equation for the range of a projectile?

8. I Suppose you knew the maximum height reached by a projectile. Could you find its launch angle from this information alone? If not, what additional information would be required?

The BIG Motion Picture: An IMAX Interview



"Filling people's peripheral vision with image to the point that they lose the sense of actually watching a picture and become totally absorbed in the medium" is the goal of the IMAX Corporation, which has been making and screening large-format films since 1970. Former IMAX executive vice-president of technology Michael Gibbon went on to say in a recent interview, "If you give people a very large image, you can almost disconnect them from reality. They become very involved with the 'thing' they are seeing."

IMAX develops and supplies all of the equipment used by filmmakers and theatres to create an exciting and enthralling film experience — the camera, the projector, and even the enormous movie screen.

The roots of the IMAX system go back to Montréal's Expo '67, where films shown simultaneously on multiple wide screens by several standard 35 mm movie theatre projectors became very popular. A small group of Canadians involved in making some of those films decided to design a new system using a single, powerful projector, rather than the cumbersome multiple projectors. The resulting IMAX system premiered at Expo '70 in Osaka, Japan, and the first permanent IMAX projection system was installed at Toronto's Ontario Place in 1971. In 1997, IMAX Corporation won an Oscar, the highest award of the Academy of Motion Picture Arts and Sciences, for scientific and technical achievement.

We spoke to Gibbon, who joined IMAX in 1986 and is now a consultant to the corporation, about the technical challenges IMAX faces when producing its large-screen films.

Q: How did IMAX create the technology to give people this sense of total immersion in the image?

A: Sensibly, IMAX chose the largest film format that was commercially available, rather than have Kodak produce something new. It was 70 mm, but IMAX turned it on its side and advanced it 15 perforations at a time.

Q: Can you explain a bit more about the film stock and film frames?

A: A filmstrip is a series of individual frames with perforations that run along the sides to help feed film through the projector. Today's cinemas show films with a frame size of 35 mm and advance it four perforations at a time. 70 mm existed when our corporation was starting up, but IMAX's choice of advancing it 15 perforations at a time was fairly revolutionary.

Q: What was the first challenge?

A: There were a number of 70 mm projectors in existence from quite early in the history of cinema, advancing five perforations of film at a time. There were many more 35 mm projectors advancing four perforations of film at a time. Our challenge was to move a format three times larger than 70 mm/5 perforations, and to do that in such a way that the film and film frame not only survived the process, but also were steady when projected. We needed steadiness because we were going to sit people very close to a very large image.

Q: In terms of film motion, what was the problem, exactly?

A: It's the sheer dynamics of the film. You're trying to advance the film quickly. You need to run it at 24 frames per second. That's the standard rate of film advancement. Also, the 35 mm mechanism is fairly rough on a film. It has a high acceleration rate, so the stresses on the perforations are not minor — you can damage the film. When the frame comes to a rest, it can deform, particularly around the perforations, so that you're not absolutely sure where you're going to finish up.

Q: So the frame "overshoots" too far or "undershoots" not far enough?

A: Yes, this was a known problem in the 35 mm projectors. The IMAX Rolling Loop projector was created to solve the problem of advancing more film quickly, yet making sure that the film was firmly in place for exposure in the aperture. The fundamental advantage it has over the 35 mm projectors is fixed registration pins.

Q: Where are registration pins? How do they work?

A: They are pins that are fixed on either side of the aperture. They simply hold the film in place when it is being illuminated.

Q: How does the Rolling Loop work?

A: There is a rotor and on the periphery of that rotor are a total of eight gaps. The film is induced to build or loop up into the gaps, and the rotor rotates. Essentially, what it's doing is lifting up the film, putting it into the gap, and rolling it along.

Q: Did you experiment with different interior motions inside the projector?

A: What evolved after a number of experiments was a deceleration cam. This takes the film, which is coming in at almost two metres per second, grabs it in the last part of its travel, slowly brings it down in a controlled manner and then puts it onto the registration pins at a very low final velocity. The pins hold the frame in a very precise location and then it's vacuumed up against the lens to keep the entire frame in focus.

Q: What are you looking at for the future?

A: Digital is an obvious consideration, but the image quality is not at the level of film. It doesn't yet have the ability to depict fine detail or to produce the same amount of light. But we're working on it.

Making Connections

1. IMAX films are known for creating a sense of motion, instead of simply showing a picture of it. How does IMAX do this?
2. Research the differences between the IMAX Rolling Loop projector and a standard 35 mm projector.

2.2 Uniform Circular Motion

SECTION EXPECTATIONS

- Analyze, predict, and explain uniform circular motion.
- Explain forces involved in uniform circular motion in horizontal and vertical planes.
- Investigate relationships between period and frequency of an object in uniform circular motion.

KEY TERMS

- uniform circular motion
- centripetal acceleration
- centripetal force

Have you ever ridden on the Round Up at the Canadian National Exhibition, the ride shown in the photograph? From a distance, it might not look exciting, but the sensations could surprise you.

Everyone lines up around the outer edge and the ride slowly begins to turn. Not very exciting yet, but soon, the ride is spinning quite fast and you feel as though you are being pressed tightly against the wall. The rotations begin to make you feel disoriented and your stomach starts to feel a little queasy. Then, suddenly, the floor drops away, but you stay helplessly "stuck" to the wall. Just as you realize that you are not going to fall, the entire ride begins to tilt. At one point during each rotation, you find yourself looking toward the ground, which is almost directly in front of you. You do not feel as though you are going to fall, though, because you are literally stuck to the wall.

Figure 2.5 If this ride stopped turning, the people would start to fall. What feature of circular motion prevents people from falling when the ride is in motion and they are facing the ground?

What is unique about moving in a circle that allows you to apparently defy gravity? What causes people on the Round Up to stick to the wall? As you study this section, you will be able to answer these questions and many more.

Centripetal Acceleration

Amusement park rides are only one of a very large number of examples of circular motion. Motors, generators, vehicle wheels, fans, air in a tornado or hurricane, or a car going around a curve are other examples of circular motion. When an object is moving in a circle and its speed — the magnitude of its velocity — is

constant, it is said to be moving with **uniform circular motion**. The direction of the object's velocity is always tangent to the circle. Since the direction of the motion is always changing, the object is always accelerating.

Figure 2.6 shows the how the velocity of the object changes when it is undergoing uniform circular motion. As an object moves from point P to point Q, its velocity changes from $\vec{v}_1$ to $\vec{v}_2$. Since the direction of the acceleration is the same as the direction of the *change* in the velocity, you need to find $\Delta\vec{v}$ or $\vec{v}_2 - \vec{v}_1$. Vectors and $\vec{v}_1$ and $\vec{v}_2$ are subtracted graphically under the circle. To develop an equation for centripetal acceleration, you will first need to show that the triangle OPQ is similar to the triangle formed by the velocity vectors, as shown in the following points.

- $r_1 = r_2$ because they are radii of the same circle. Therefore, triangle OPQ is an isosceles triangle.
- $|\vec{v}_1| = |\vec{v}_2|$ because the speed is constant. Therefore, the triangle formed by $-\vec{v}_1$, $\vec{v}_2$, and $\Delta\vec{v}$ is an isosceles triangle.
- $r_1 \perp \vec{v}_1$ and $r_2 \perp \vec{v}_2$ because the radius of a circle is perpendicular to the tangent to the point where the radius contacts the circle.
- $\theta_r = \theta_v$ because the angle between corresponding members of sets of perpendicular lines are equal.
- Since the angles between the equal sides of two isosceles triangles are equal, the triangles are similar.

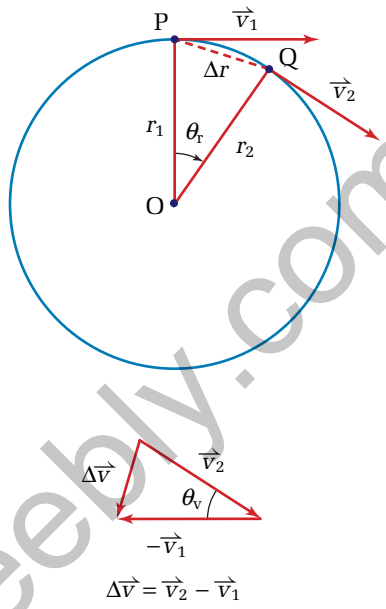


Figure 2.6 The direction of the change in velocity is found by defining the vector $-\vec{v}_1$ and then adding $\vec{v}_2$ and $-\vec{v}_1$. Place the tail of $-\vec{v}_1$ at the tip of $\vec{v}_2$ and draw the resultant vector, $\Delta\vec{v}$, from the tail of $\vec{v}_2$ to the tip of $-\vec{v}_1$.

Now use the two similar triangles to find the magnitude of the acceleration. Since the derivation involves only magnitudes, omit vector notations.

- The ratios of the corresponding sides of similar triangles are equal. There is no need to distinguish between the sides r_1 and r_2 or v_1 and v_2, because the radii are equal and the magnitudes of the velocities are equal.

$$\frac{\Delta r}{r} = \frac{\Delta v}{v}$$

- The object travelled from point P to point Q in the time interval Δt. Therefore, the magnitude of the object's displacement along the arc from P to Q is

$$\Delta d = v\Delta t$$

- The length of the arc from point P to point Q is almost equal to Δr. As the angle becomes very small, the lengths become more nearly identical.

$$\Delta r = v\Delta t$$

- Substitute this value of Δr into the first equation.

$$\frac{v\Delta t}{r} = \frac{\Delta v}{v}$$

MATH LINK

Mathematicians have developed a unique system for defining components of vectors such as force, acceleration, and velocity for movement on curved paths, even when the magnitude of the velocity is changing. Any curve can be treated as an arc of a circle. So, instead of using the x- and y-components of the typical Cartesian coordinate system, the vectors are divided into tangential and radial components. The tangential component is the component of the vector that is tangent to the curved path at the point at which the object is momentarily located. The radial component is perpendicular to the path and points to the centre of the circle defined by the arc or curved section of the path. Radial components are the same as centripetal components.

- Divide both sides of the equation by Δt. $\frac{v}{r} = \frac{\Delta v}{v\Delta t}$
- Recall the definition of acceleration. $a = \frac{\Delta v}{\Delta t}$
- Substitute a into the equation for $\frac{\Delta v}{\Delta t}$. $\frac{v}{r} = \frac{a}{v}$
- Multiply both sides of the equation by v. $a = \frac{v^2}{r}$

The magnitude of the acceleration of an object moving with uniform circular motion is $a = v^2/r$. To determine its direction, again inspect the triangle formed by the velocity vectors in Figure 2.6. The acceleration is changing constantly, so imagine a vector $\vec{v}_2$ as close to $\vec{v}_1$ as possible. The angle θ is extremely small. In this case, $\Delta\vec{v}$ is almost exactly perpendicular to both $\vec{v}_1$ and $\vec{v}_2$. Since $\vec{v}_1$ and $\vec{v}_2$ are tangent to the circle and therefore are perpendicular to the associated radii of the circle, the acceleration vector points directly toward the centre of the circle.

Describing the acceleration vector in a typical Cartesian coordinate system would be extremely difficult, because the direction is always changing and, therefore, the magnitude of the x- and y-components would always be changing. It is much simpler to specify only the magnitude of the acceleration, which is constant for uniform circular motion, and to note that the direction is always toward the centre of the circle. To indicate this, physicists speak of a "centre-seeking acceleration" or **centripetal acceleration**, which is denoted as a_c, without a vector notation.

CENTRIPETAL ACCELERATION

Centripetal acceleration is the quotient of the square of the velocity and the radius of the circle.

$$a_c = \frac{v^2}{r}$$

Quantity	Symbol	SI unit
centripetal acceleration	a_c	$\frac{\text{m}}{\text{s}^2}$ (metres per second squared)
velocity (magnitude)	v	$\frac{\text{m}}{\text{s}}$ (metres per second)
radius (of circle)	r	m (metres)

Unit Analysis

$$\frac{\text{metre}}{\text{second}^2} = \frac{\left(\frac{\text{metre}}{\text{second}}\right)^2}{\text{metre}} \qquad \frac{\left(\frac{\text{m}}{\text{s}}\right)^2}{\text{m}} = \frac{\frac{\text{m}^{\cancel{2}}}{\text{s}^2}}{\cancel{\text{m}}} = \frac{\text{m}}{\text{s}^2}$$

Note: The direction of the centripetal acceleration is always along a radius pointing toward the centre of the circle.

Centripetal Force

According to Newton's laws of motion, an object will accelerate only if a force is exerted on it. Since an object moving with uniform circular motion is always accelerating, there must always be a force exerted on it in the same direction as the acceleration, as illustrated in Figure 2.7. If at any instant the force is withdrawn, the object will stop moving along the circular path and will proceed to move with uniform motion, that is, in a straight line that is tangent to the circular path on which it had been moving.

Since the force causing a centripetal acceleration is always pointing toward the centre of the circular path, it is called a **centripetal force**. The concept of centripetal force differs greatly from that of other forces that you have encountered. It is not a type of force such as friction or gravity. It is, instead, a force that is *required* in order for an object to move in a circular path.

A centripetal force can be supplied by any type of force. For example, as illustrated in Figure 2.8, gravity provides the centripetal force that keeps the Moon on a roughly circular path around Earth, friction provides a centripetal force that causes a car to move in a circular path on a flat road, and the tension in a string tied to a ball will cause the ball to move in a circular path when you twirl it around. In fact, two different types of force could act together to provide a centripetal force.

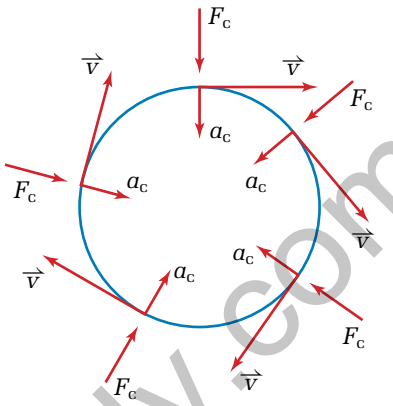


Figure 2.7 A force acting perpendicular to the direction of the velocity is always required in order for any object to move continuously along a circular path.

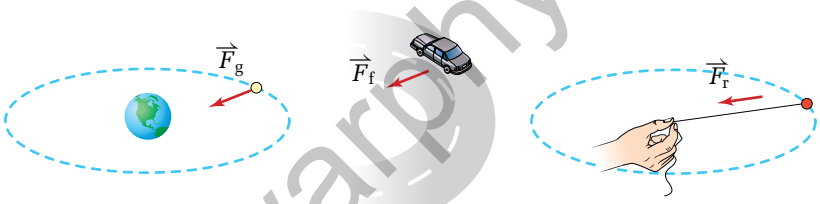


Figure 2.8 Any force that is directed toward the centre of a circle can provide a centripetal force.

You can determine the magnitude of a centripetal force required to cause an object to travel in a circular path by applying Newton's second law to a mass moving with a centripetal acceleration.

- Write Newton's second law. $\vec{F} = m\vec{a}$
- Write the equation describing centripetal acceleration. $a_c = \frac{v^2}{r}$
- Substitute into Newton's second law. Omit vector notations because the force and acceleration always point toward the centre of the circular path. $F_c = \frac{mv^2}{r}$

The equation for the centripetal force required to cause a mass m moving with a velocity v to follow a circular path of radius r is summarized in the following box.

CENTRIPETAL FORCE

The magnitude of the centripetal force is the quotient of the mass times the square of the velocity and the radius of the circle.

$$F_c = \frac{mv^2}{r}$$

Quantity	Symbol	SI unit
centripetal force	F_c	N (newtons)
mass	m	kg (kilograms)
velocity	v	$\frac{\text{m}}{\text{s}}$ (metres per second)
radius of circular path	r	m (metres)

Unit Analysis

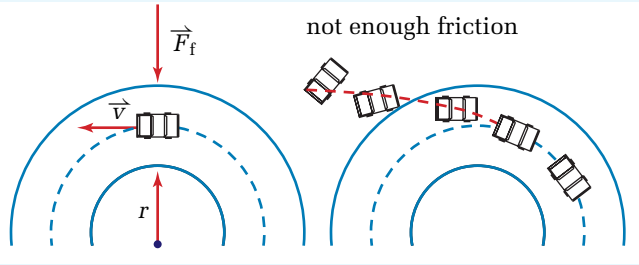
$$(\text{newtons}) = \left(\frac{\text{kilogram}\left(\frac{\text{metres}}{\text{second}}\right)^2}{\text{metres}}\right)$$

$$N = \frac{\text{kg}\left(\frac{\text{m}}{\text{s}}\right)^2}{\text{m}} = \frac{\text{kg}\frac{\text{m}^{\cancel{2}}}{\text{s}^2}}{\cancel{\text{m}}} = \frac{\text{kg}\cdot\text{m}}{\text{s}^2} = \text{N}$$

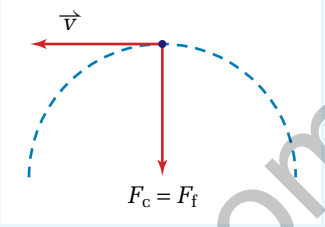
SAMPLE PROBLEMS

Centripetal Force in a Horizontal and a Vertical Plane

1. **A car with a mass of 2135 kg is rounding a curve on a level road. If the radius of curvature of the road is 52 m and the coefficient of friction between the tires and the road is 0.70, what is the maximum speed at which the car can make the curve without skidding off the road?**



Conceptualize the Problem



- Make a sketch of the motion of the car and the forces acting on it.
- The *force of friction* must provide a sufficient *centripetal force* to cause the car to follow the curved road.
- The magnitude of *force* required to keep the car on the road depends on the *velocity* of the car, its *mass*, and the *radius of curvature* of the road.
- Since r is in the denominator of the expression for centripetal force, as the *radius* becomes *smaller*, the amount of *force* required becomes *greater*.
- Since v is in the numerator, as the *velocity* becomes *larger*, the *force* required to keep the car on the road becomes *greater*.

Identify the Goal

The maximum speed, v, at which the car can make the turn

Identify the Variables

Known		Implied	Unknown	
$m = 2135$ kg	$\mu = 0.70$	$g = 9.81\ \frac{\text{m}}{\text{s}^2}$	F_f	F_N
$r = 52$ m			v	

Develop a Strategy

Set the frictional force equal to the centripetal force.

$$F_f = F_c$$
$$\mu F_N = \frac{mv^2}{r}$$

Since the car is moving on a level road, the normal force of the road is equal to the weight of the car. Substitute mg for F_N.

$$\mu mg = \frac{mv^2}{r}$$

Solve for the velocity.

$$v^2 = \mu \cancel{m} g\left(\frac{r}{\cancel{m}}\right)$$
$$v = \sqrt{\mu r g}$$

Substitute in the numerical values and solve.

$$v = \sqrt{(0.70)(52\ \text{m})\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)}$$
$$v = \sqrt{357.08\ \frac{\text{m}^2}{\text{s}^2}}$$
$$v = 18.897\ \frac{\text{m}}{\text{s}}$$
$$v \cong 19\ \frac{\text{m}}{\text{s}}$$

If the car is going faster than 19 m/s, it will skid off the road.

Validate the Solution

A radius of curvature of 52 m is a sharp curve. A speed of 19 m/s is equivalent to 68 km/h, which is a high speed at which to take a sharp curve. The answer is reasonable. The units cancelled properly to give metres per second for velocity.

continued ▶

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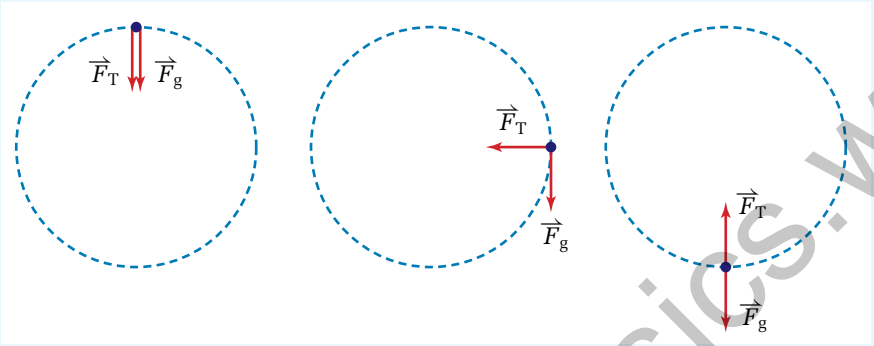
2. You are playing with a yo-yo with a mass of 225 g. The full length of the string is 1.2 m. You decide to see how slowly you can swing it in a vertical circle while keeping the string fully extended, even when the yo-yo is at the top of its swing.

(a) Calculate the minimum speed at which you can swing the yo-yo while keeping it on a circular path.

(b) At the speed that you determine in part (a), find the tension in the string when the yo-yo is at the side and at the bottom of its swing.

Conceptualize the Problem

- Draw *free-body diagrams* of the yo-yo at the *top, bottom, and one side* of the swing.



- At the *top* of the swing, both *tension and the force of gravity* are acting *toward the centre* of the circle.
- If the required *centripetal force* is *less than the force of gravity*, the yo-yo will *fall* away from the circular path.
- If the required *centripetal force* is *greater than the force of gravity*, the *tension* in the string *will* have to *contribute* to the centripetal force.
- Therefore, the *smallest possible velocity* would be the case where the required *centripetal force* is exactly *equal* to the *force of gravity*.
- At the *side* of the swing, the force of *gravity is perpendicular* to the direction of the required centripetal force and therefore contributes *nothing*. The centripetal force must all be supplied by the tension in the string.
- At the *bottom* of the swing, the force of *gravity* is in the *opposite* direction from the required *centripetal force*. Therefore, the *tension* in the string must *balance* the force of *gravity* and *supply* the required *centripetal force*.

Identify the Goal

The minimum speed, v, at which the yo-yo will stay on a circular path
The tension, F_T, in the string when the yo-yo is at the side of its circular path
The tension, F_T, in the string when the yo-yo is at the bottom of its circular path

Identify the Variables

Known	Implied	Unknown
$m = 225\ \text{kg}$	$g = 9.81\ \frac{\text{m}}{\text{s}^2}$	v_{min}
$r = 1.2\ \text{m}$		$F_{\text{T(side)}}$
		$F_{\text{T(bottom)}}$

Develop a Strategy

Set the force of gravity on the yo-yo equal to the centripetal force and solve for the velocity.

$$F_g = F_c$$

$$mg = \frac{mv^2}{r}$$

$$\cancel{m}g\left(\frac{r}{\cancel{m}}\right) = v^2$$

$$v = \sqrt{gr}$$

Substitute numerical values and solve.

$$v = \sqrt{\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)(1.2\ \text{m})}$$

$$v = \sqrt{11.772\ \frac{\text{m}^2}{\text{s}^2}}$$

A negative answer has no meaning in this application.

$$v = \pm 3.431\ \frac{\text{m}}{\text{s}}$$

$$v \cong 3.4\ \frac{\text{m}}{\text{s}}$$

(a) The minimum speed at which the yo-yo can move is 3.4 m/s.

Set the force of tension in the string equal to the centripetal force. Insert numerical values and solve.

$$F_T = F_c$$

$$F_T = \frac{mv^2}{r}$$

$$F_T = \frac{(225\ \cancel{g})\left(\frac{1\ \text{kg}}{1000\ \cancel{g}}\right)\left(3.431\ \frac{\text{m}}{\text{s}}\right)^2}{1.2\ \text{m}}$$

$$F_T = 2.207\ \frac{\text{kg}\frac{\text{m}^{\cancel{2}}}{\text{s}^2}}{\cancel{\text{m}}}$$

$$F_T \cong 2.2\ \text{N}$$

(b): Side – When the yo-yo is at the side of its swing, the tension in the string is 2.2 N.

Set the centripetal force equal to the vector sum of the force of tension in the string and the gravitational force. Solve for the force due to the tension in the string.

$$F_c = F_T + F_g$$

$$\frac{mv^2}{r} = F_T - mg$$

$$F_T = \frac{mv^2}{r} + mg$$

Substitute numerical values and solve.

$$F_T = \frac{(225\ \cancel{g})\left(\frac{1\ \text{kg}}{1000\ \cancel{g}}\right)\left(3.431\ \frac{\text{m}}{\text{s}}\right)^2}{1.2\ \text{m}} + (225\ \cancel{g})\left(\frac{1\ \text{kg}}{1000\ \cancel{g}}\right)\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)$$

$$F_T = 2.207\ \frac{\frac{\text{kg}\cdot\text{m}^{\cancel{2}}}{\text{s}^2}}{\cancel{\text{m}}} + 2.207\ \frac{\text{kg}\cdot\text{m}}{\text{s}^2}$$

$$F_T = 4.414\ \text{N}$$

$$F_T \cong 4.4\ \text{N}$$

(b): Bottom – When the yo-yo is at the bottom of its swing, the tension in the string is 4.4 N.

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Validate the Solution

The force of gravity (weight) of the yo-yo is 2.2 N. At the top of the swing, the weight supplies the entire centripetal force and the speed of the yo-yo is determined by this value. At the side of the swing, the tension must provide the centripetal force and the problem was set up so that the centripetal force had to be equal to the weight of the yo-yo, or 2.2 N. At the bottom of the swing, the tension must support the weight (2.2 N) and, in addition, provide the required centripetal force (2.2 N). You would therefore expect that the tension would be twice the weight of the yo-yo. The units cancel properly to give newtons for force.

PRACTICE PROBLEMS

15. A boy is twirling a 155 g ball on a 1.65 m string in a horizontal circle. The string will break if the tension reaches 208 N. What is the maximum speed at which the ball can move without breaking the string?

16. An electron (mass 9.11×10^{-31} kg) orbits a hydrogen nucleus at a radius of 5.3×10^{-11} m at a speed of 2.2×10^{6} m/s. Find the centripetal force acting on the electron. What type of force supplies the centripetal force?

17. A stone of mass 284 g is twirled at a constant speed of 12.4 m/s in a vertical circle of radius 0.850 m. Find the tension in the string **(a)** at the top and **(b)** at the bottom of the revolution. **(c)** What is the maximum speed the stone can have if the string will break when the tension reaches 33.7 N?

18. You are driving a 1654 kg car on a level road surface and start to round a curve at 77 km/h. If the radius of curvature is 129 m, what must be the frictional force between the tires and the road so that you can safely make the turn?

19. A stunt driver for a movie needs to make a 2545 kg car begin to skid on a large, flat, parking lot surface. The force of friction between his tires and the concrete surface is 1.75×10^{4} N and he is driving at a speed of 24 m/s. As he turns more and more sharply, what radius of curvature will he reach when the car just begins to skid?

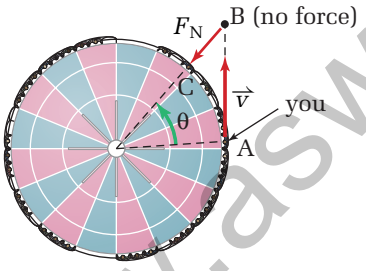


Figure 2.9 Assume that the Round Up ride is rotating at a constant speed and you are at point A. After a short time interval, in the *absence* of a force acting on you, you would move to point B, radially outward from point C. A centripetal force is required to change the direction of your velocity and place you at point C.

Centripetal Force versus Centrifugal Force

You read in Chapter 1, Fundamentals of Dynamics, that a centrifugal force is a fictitious force. Now that you have learned about centripetal forces, you can understand more clearly why a centrifugal force is classed as fictitious.

Analyze the motion of and the force on a person who is riding the Round Up. Imagine that Figure 2.9 is a view of the Round Up ride from above and at some instant you are at point A on the ride. At that moment, your velocity ($\vec{v}$) is tangent to the path of the ride. If no force was acting on you at all, you would soon be located at point B. However, the solid cylindrical structure of the ride exerts a normal force on you, pushing you to point C. There is no force pushing you outward, just a centripetal force pushing you toward the centre of the circular ride.

CAREERS IN PHYSICS

An Amusing Side of Physics

Your roller-coaster car is heading up the first and highest incline. As you turn around to wave to your faint-of-heart friends on the ground, you realize that six large players from the Hamilton Tiger Cats football team have piled into the three cars immediately behind you! Now you're poised at the top, ready to drop, and hoping fervently that the roller-coaster manufacturer designed the cars to stay on the track, even when carrying exceptionally heavy loads.

Perhaps your anxiety will lessen if you are aware that a highly qualified mechanical engineer, such as Matthew Chan, has checked out the performance specifications of the roller coaster, as well as all of the other rides at the amusement park. They meet Canada's safety codes, or the park is not allowed to operate them.

Chan has worked for the Technical Standards and Safety Authority (TSSA) for the past 11 years. As special devices engineer for the Elevating and Amusement Devices Safety Division, he spends most of his time in the office, verifying design submissions from elevator, ski lift, and amusement park ride manufacturers.

When some unique piece of machinery comes along or when he wants to verify how equipment will perform in reality versus on paper, Chan goes into the field. He confesses a special fondness for going on-site to test amusement park rides, including at Canada's Wonderland, north of Toronto, the Western Fair in London, Ontario, and Toronto's Canadian National Exhibition.

His favourite challenge is analyzing the performance of reverse bungees, amusement park rides that look like giant slingshots and that hurl people hundreds of feet into the air. Chan says that most reverse bungees are unique devices — rarely are two designed and made exactly the same. "It takes all of your engineering knowledge to analyze these devices," claims Chan. "There are no hard and fast rules for how reverse bungees are made, and the industry is always changing."

A reverse bungee ride

A university degree in mechanical engineering is required to become a special devices engineer, and Chan says that an understanding of the dynamics of motion is a must. You have to be able to predict how a device will behave in a variety of "what if" scenarios. The details of balance, mass placement, overload situations, restraint systems, and footings are all worked through carefully.

In short, you are in good hands. Now, sit back, relax, and enjoy the ride!

Going Further

1. According to Chan, one of the best ways to get a hands-on feel for the complex dynamic forces at work in amusement park rides is to build and operate scale models. Form a study group with some of your friends and investigate designs for your favourite ride. Build a scale model and test it to determine the smallest and greatest weights it can carry safely.
2. Visit a science centre that has exhibits demonstrating the principles of dynamics.

WEB LINK

www.mcgrawhill.ca/links/physics12

The Internet has sites that allow you to design and "run" your own amusement park rides. Go to the Internet site shown above and click on **Web Links**.

PROBEWARE

If your school has probeware equipment, visit ***www.mcgrawhill.ca/links/physics12*** and follow the links for an in-depth activity on circular motion.

Describing Rotational Motion

When an object is constantly rotating, physicists sometimes find it more convenient to describe the motion in terms of the frequency — the number of complete rotations per unit time — or the period — the time required for one complete rotation — instead of the velocity of the object. You can express the centripetal acceleration and the centripetal force in these terms by finding the relationship between the magnitude of the velocity of an object in uniform circular motion and its frequency and period.

- Write the definition of velocity. Since period and frequency are scalar quantities, omit vector notations.

$$v = \frac{\Delta d}{\Delta t}$$

- The distance that an object travels in one rotation is the circumference of the circle.

$$\Delta d = 2\pi r$$

- The time interval for one cycle is the period, T.

$$\Delta t = T$$

- Substitute the distance and period into the equation for velocity, v.

$$v = \frac{2\pi r}{T}$$

- Substitute the above value for v into the equation for centripetal acceleration, a, and simplify.

$$a_c = \frac{v^2}{r}$$

$$a_c = \frac{\left(\frac{2\pi r}{T}\right)^2}{r}$$

$$a_c = \frac{\frac{4\pi^2 r^{\cancel{2}}}{T^2}}{\cancel{r}}$$

$$a_c = \frac{4\pi^2 r}{T^2}$$

- Substitute the above value for a into the equation for centripetal force and simplify.

$$F_c = ma_c$$

$$F_c = m\left(\frac{4\pi^2 r}{T^2}\right)$$

$$F_c = \frac{4\pi^2 mr}{T^2}$$

- The frequency is the inverse of the period.

$$f = \frac{1}{T} \text{ or } T = \frac{1}{f}$$

- Substitute the above value for the period into the equation for centripetal acceleration and simplify.

$$a_c = \frac{4\pi^2 r}{\left(\frac{1}{f}\right)^2}$$

$$a_c = 4\pi^2 r f^2$$

- Substitute the above value for acceleration into the equation for the centripetal force.

$$F_c = m(4\pi^2 r f^2)$$

$$F_c = 4\pi^2 m r f^2$$

INVESTIGATION 2-B

Verifying the Circular Motion Equation

TARGET SKILLS

- Performing and recording
- Analyzing and interpreting
- Communicating results

You have seen the derivation of the equation for circular motion and solved problems by using it. However, it is always hard to accept a theoretical concept until you test it for yourself. In this investigation, you will obtain experimental data for uniform circular motion and compare your data to the theory.

Problem

How well does the equation describe actual experimental results?

Equipment

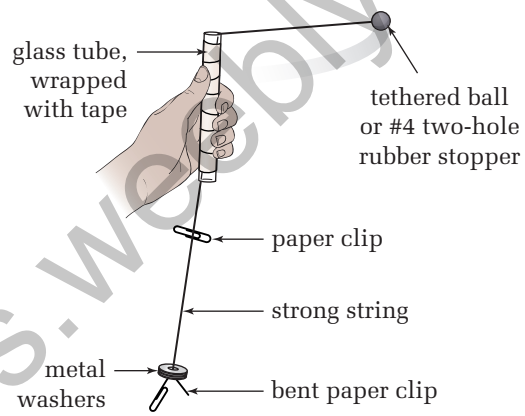
- laboratory balance
- force probeware or stopwatch
- ball on the end of a strong string
- glass tube (15 cm long with fire-polished ends, wrapped in tape)
- metre stick
- 12 metal washers
- tape
- paper clips

CAUTION Wear impact-resistant safety goggles. Also, do not stand close to other people and equipment while doing this activity.

Procedure

Alternative A: Using Traditional Apparatus

1. Measure the mass of the ball.
2. Choose a convenient radius for swinging the ball in a circle. Use the paper clip or tape as a marker, as shown in the diagram at the top of the next column, so you can keep the ball circling within your chosen radius.
3. Measure the mass of one washer.
4. Fasten three washers to the free end of the string, using a bent paper clip to hold them in place. Swing the string at a velocity that will maintain the chosen radius. Measure the time for several revolutions and use it to calculate the period of rotation.



5. Calculate the gravitational force on the washers (weight), which creates tension in the string. This force provides the centripetal force to keep the ball moving on the circular path.
6. Repeat for at least four more radii.

Alternative B: Using Probeware

1. Measure the mass of the ball.
2. Attach the free end of the string to a swivel on a force probe, as shown in the diagram on the next page.
3. Set the software to collect force-time data approximately 50 times per second. Start the ball rotating at constant velocity, keeping the radius at the proper value, and collect data for at least 10 revolutions.

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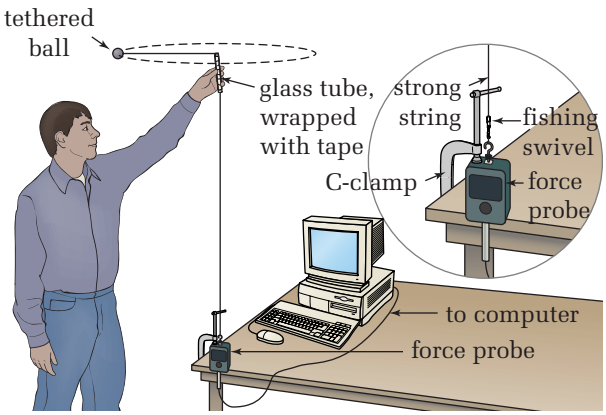
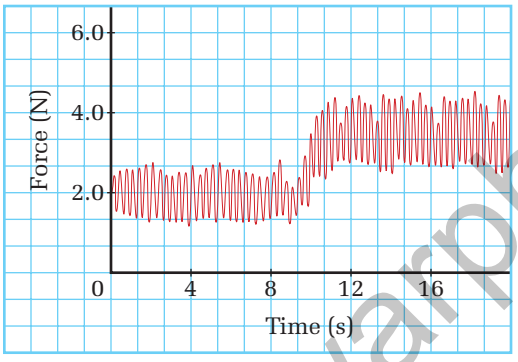
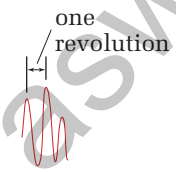


4. Examination of the graph will show regular variations from which you can calculate the period of one revolution, as well as the average force.





5. Repeat for at least five different radii.

Analyze and Conclude

1. For each radius, calculate and record in your data table the velocity of the ball. Use the period and the distance the ball travels in one revolution (the circumference of its circular path).
2. For each radius, calculate and record in your data table $\frac{mv^2}{r}$.
3. Graph F_c against $\frac{mv^2}{r}$. Each radius will produce one data point on your graph.
4. Draw the best-fit line through your data points. How can you tell from the position of the points whether the relationship being tested, $F_c = \frac{mv^2}{r}$, actually describes the data reasonably well?
5. Calculate the slope of the line. What does the slope tell you about the validity of the mathematical relationship?
6. Identify the most likely sources of error in the experiment. That is, what facet of the experiment might have been ignored, even though it could have a significant effect on the results?

Apply and Extend

Based on the experience you have gained in this investigation and the theory that you have learned, answer the following questions about circular motion. Support your answers in each case by describing how you would experimentally determine the answer to the question and how you would use the equations to support your answer.

7. How is the required centripetal force affected when everything else remains the same but the frequency of rotation increases?
8. How is the required centripetal force affected when everything else remains the same but the period of rotation increases?
9. If the radius of the circular path of an object increases and the frequency remains the same, how will the centripetal force change?
10. How can you keep the velocity of the object constant while the radius of the circular path decreases?

Banked Curves

Have you ever wondered why airplanes tilt or bank so much when they turn, as the airplanes in the photograph are doing? Now that you have learned that a centripetal force is required in order to follow a curved path or turn, you probably realize that banking the airplane has something to do with creating a centripetal force. Land vehicles can use friction between the tires and the road surface to obtain a centripetal force, but air friction (or drag) acts opposite to the direction of the motion of the airplane and cannot act perpendicular to the direction of motion. What force could possibly be used to provide a centripetal force for an airplane?

Figure 2.10 When an airplane follows a curved path, it must tilt or bank to generate a centripetal force.

When an airplane is flying straight and horizontally, the design of the wings and the flow of air over them creates a lift force (*L*) that keeps the airplane in the air, as shown in Figure 2.11. The lift must be equal in magnitude and opposite in direction to the weight of the airplane in order for the airplane to remain on a level path. When an airplane banks, the lift force is still perpendicular to the wings. The vertical component of the lift now must balance the gravitational force, while the horizontal component of the lift provides a centripetal force. The free-body diagram on the right-hand side of Figure 2.11 helps you to see the relationship of the forces more clearly.

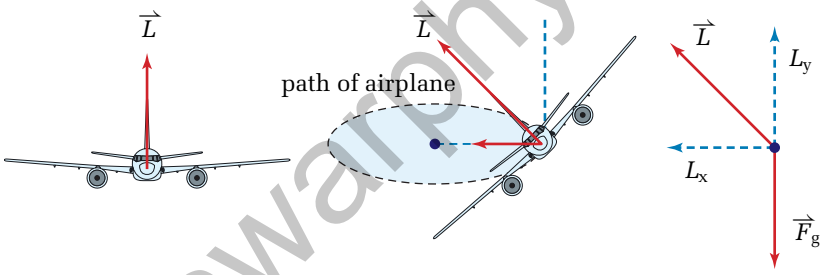


Figure 2.11 When a pilot banks an airplane, the forces of gravity and lift are not balanced. The resultant force is perpendicular to the direction that the airplane is flying, thus creating a centripetal force.

Cars and trucks can use friction as a centripetal force. However, the amount of friction changes with road conditions and can become very small when the roads are icy. As well, friction causes wear and tear on tires and causes them to wear out faster. For these reasons, the engineers who design highways where speeds are high and large centripetal forces are required incorporate another source of a centripetal force — banked curves. Banked curves on a road function in a way that is similar to the banking of airplanes.

Figure 2.12 shows you that the normal force of the road on a car provides a centripetal force when the road is banked, since a normal force is always perpendicular to the road surface.

You can use the following logic to develop an equation relating the angle of banking to the speed of a vehicle rounding a curve. Since an angle is a scalar quantity, omit vector notations and use only magnitudes. Assume that you wanted to know what angle of banking would allow a vehicle to move around a curve with a radius of curvature r at a speed v, without needing any friction to supply part of the centripetal force.

- Since a car does not move in a vertical direction, the vertical component of the normal force must be equal in magnitude to the force of gravity.

$$F_N \cos\theta = F_g$$
$$F_N \cos\theta = mg$$

- The horizontal component of the normal force must supply the centripetal force.

$$F_N \sin\theta = F_c$$
$$F_N \sin\theta = \frac{mv^2}{r}$$

- Divide the second equation by the first and simplify.

$$\frac{\cancel{F_N} \sin\theta}{\cancel{F_N} \cos\theta} = \frac{\frac{\cancel{m}v^2}{r}}{\cancel{m}g}$$
$$\frac{\sin\theta}{\cos\theta} = \frac{\frac{v^2}{r}}{g}$$
$$\tan\theta = \frac{v^2}{rg}$$

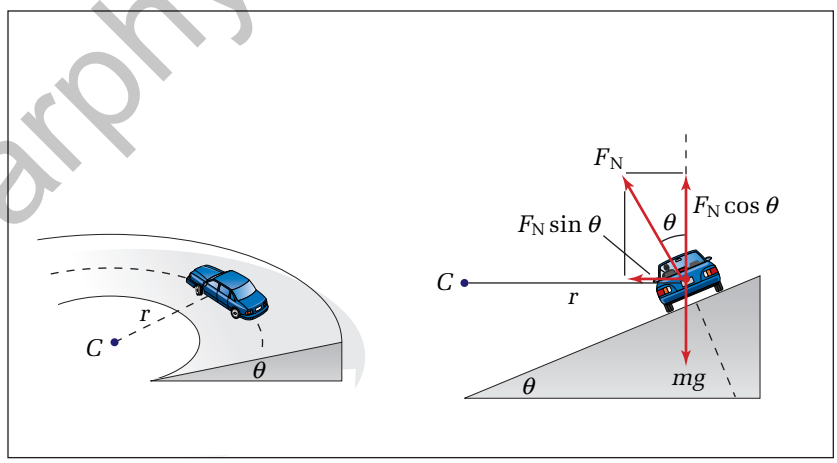
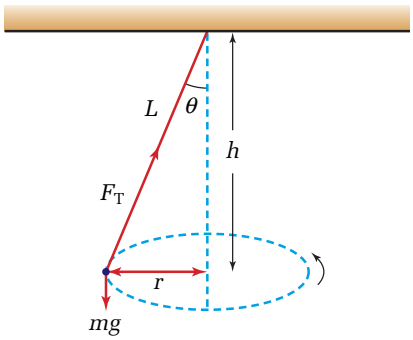


Figure 2.12 When you look at a cross section of a car rounding a curve, you can see that the only two forces in a vertical plane that are acting on the car are the force of gravity and the normal force of the road. The resultant force is horizontal and perpendicular to the direction in which the car is moving. This resultant force supplies a centripetal force that causes the car to follow a curved path.

Notice that the mass of the vehicle does not affect the amount of banking that is needed to drive safely around a curve. A semitrailer and truck could take a curve at the same speed as a motorcycle without relying on friction to supply any of the required centripetal force. Apply what you have learned about banking to the following problems.

Conceptual Problem

- A conical pendulum swings in a circle, as shown in the diagram. Show that the form of the equation relating the angle that the string of the pendulum makes with the vertical to the speed of the pendulum bob is identical to the equation for the banking of curves. The pendulum has a length L, an angle θ with the vertical, a force of tension F_T in the string, a weight mg, and swings in a circular path of radius r. The plane of the circle is a distance h from the ceiling from which the pendulum hangs.



SAMPLE PROBLEM

Banked Curves and Centripetal Force

Canadian Indy racing car driver Paul Tracy set the speed record for time trials at the Michigan International Speedway (MIS) in the year 2000. Tracy averaged 378.11 km/h in the time trials. The ends of the 3 km oval track at MIS are banked at 18.0° and the radius of curvature is 382 m.

(a) At what speed can the cars round the curves without needing to rely on friction to provide a centripetal force?

(b) Did Tracy rely on friction for some of his required centripetal force?

Conceptualize the Problem

- The *normal force* of a *banked curve provides* a *centripetal force* to help cars turn without requiring an excessive amount of friction.
- For a given *radius of curvature* and *angle of banking*, there is *one speed* at which the normal force provides precisely the amount of centripetal force that is needed.

Identify the Goal

(a) The speed, v, for which the normal force provides exactly the required amount of centripetal force for driving around the curve

(b) Whether Tracy needed friction to provide an additional amount of centripetal force

Identify the Variables and Constant

Known	Implied	Unknown
$r = 382$ m	$g = 9.81 \frac{\text{m}}{\text{s}^2}$	v
$\theta = 18.0°$		
$v_{PT} = 378.11 \frac{\text{km}}{\text{h}}$		

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Develop a Strategy

Write the equation that relates angle of banking, speed, and radius of curvature, and solve for speed, v.

$$\tan\theta = \frac{v^2}{rg}$$
$$v^2 = rg\tan\theta$$
$$v = \sqrt{rg\tan\theta}$$

Substitute the numerical values and solve.

$$v = \sqrt{(382\text{ m})\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)(\tan 18.0^\circ)}$$
$$v = \sqrt{1217.61\ \frac{\text{m}^2}{\text{s}^2}}$$
$$v = 34.894\ \frac{\text{m}}{\text{s}}$$
$$v \cong 34.9\ \frac{\text{m}}{\text{s}}$$

(a) A vehicle driving at 34.9 m/s could round the curve without needing any friction for centripetal force.

Convert the velocity in m/s into km/h.

$$v = \left(34.894\ \frac{\cancel{\text{m}}}{\cancel{\text{s}}}\right)\left(\frac{3600\ \cancel{\text{s}}}{\text{h}}\right)\left(\frac{1\text{ km}}{1000\ \cancel{\text{m}}}\right)$$
$$v = 125.619\ \frac{\text{km}}{\text{h}}$$
$$v \cong 126\ \frac{\text{km}}{\text{h}}$$

(b) Tracy was driving three times as fast as the speed of 126 km/h at which the normal force provides the needed centripetal force. Paul had to rely on friction for a large part of the needed centripetal force.

Validate the Solution

An angle of banking of 18° is very large compared to the banking on normal highway curves. You would expect that it was designed for speeds much higher than the highway speed limit. A speed of 126 km/h is higher than highway speed limits.

PRACTICE PROBLEMS

20. An engineer designed a turn on a road so that a 1225 kg car would need 4825 N of centripetal force when travelling around the curve at 72.5 km/h. What is the radius of curvature of the road?

21. A car exits a highway on a ramp that is banked at 15° to the horizontal. The exit ramp has a radius of curvature of 65 m. If the conditions are extremely icy and the driver cannot depend on any friction to help make the turn, at what speed should the driver travel so that the car will not skid off the ramp?

22. An icy curve with a radius of curvature of 175 m is banked at 12°. At what speed must a car travel to ensure that it does not leave the road?

23. An engineer must design a highway curve with a radius of curvature of 155 m that can accommodate cars travelling at 85 km/h. At what angle should the curve be banked?

You have studied just a few examples of circular motion that you observe or experience nearly every day. Although you rarely think about it, you have been experiencing several forms of circular motion every minute of your life. Simply existing on Earth's surface places you in uniform circular motion as Earth rotates. In addition, Earth is revolving around the Sun. In the next chapter, you will apply many of the concepts you have just learned about force and motion to the motion of planets, moons, and stars, as well as to artificial satellites.

2.2 Section Review

1. K/U Define uniform circular motion and describe the type of acceleration that is associated with it.
2. K/U Study the diagram in Figure 2.6 on page 79. Explain what approximation was made in the derivation that requires you to imagine what occurs as the angle becomes smaller and smaller.
3. C What are the benefits of using the concept of centripetal acceleration rather than working on a traditional Cartesian coordinate system?
4. K/U Explain how centripetal force differs from common forces, such as the forces of friction and gravity.
5. K/U If you were swinging a ball on a string around in a circle in a vertical plane, at what point in the path would the string be the most likely to break? Explain why. In what direction would the ball fly when the string broke?
6. C Explain why gravity does *not* affect circular motion in a horizontal plane, and why it *does* affect a similar motion in a vertical plane.
7. C Describe three examples in which different forces are contributing the centripetal force that is causing an object to follow a circular path.
8. MC When airplane pilots make very sharp turns, they are subjected to very large *g* forces. Based on your knowledge of centripetal force, explain why this occurs.
9. C A centrifugal force, if it existed, would be directed radially outward from the centre of a circle during circular motion. Explain why it feels as though you are being thrown outward when you are riding on an amusement park ride that causes you to spin in a circle.
10. K/U On a highway, why are sharp turns banked more steeply than gentle turns? Use vector diagrams to clarify your answer.
11. I Imagine that you are in a car on a major highway. When going around a curve, the car starts to slide sideways down the banking of the curve. Describe conditions that could cause this to happen.

UNIT PROJECT PREP

Parts of your catapult launch mechanism will move in part of a circle. The payload, once launched, will be a projectile.

- How will your launch mechanism apply enough centripetal force to the payload to move it in a circle, while still allowing the payload to be released?
- How will you ensure that the payload is launched at the optimum angle for maximum range?
- What data will you need to gather from a launch to produce the most complete possible analysis of the payload's actual path and flight parameters?

CHAPTER 2 Review

REFLECTING ON CHAPTER 2

- Ideal projectiles move in a parabolic trajectory in a vertical plane under the influence of gravity alone.
- The path of a projectile is determined by the initial velocity.
- The range of a projectile is its horizontal displacement.
- For a given magnitude of velocity, the maximum range of an ideal projectile occurs when the projectile is launched at an angle of 45°.
- Projectile trajectories are computed by separately analyzing horizontal and vertical components of velocity during a common time interval.
- Objects in uniform circular motion experience a centripetal (centre-seeking) acceleration: $a_c = v^2/r$.
- A centripetal (centre-seeking) force is required to keep an object in uniform circular motion: $F_c = mv^2/r$.
- Centripetal force can be supplied by any type of force, such as tension, gravitational forces, friction, and electrostatic force, or by a combination of forces.
- The force of gravity has no effect on circular motion in a horizontal plane, but does affect circular motion in a vertical plane.
- Objects moving around banked curves experience a centripetal force due to the horizontal component of the normal force exerted by the surfaces on which they travel.

Knowledge/Understanding

1. Differentiate between the terms "one-dimensional motion" and "two-dimensional motion." Provide examples of each.
2. Explain what physicists mean by the "two-dimensional nature of motion in a plane," when common sense suggests that an object can be travelling in only one direction at any particular instant in time.
3. Describe and explain two specific examples that illustrate how the vertical and horizontal components of projectile motion are independent of each other.
4. When analyzing ideal projectiles, what type of motion is the horizontal component? What type of motion is the vertical component?
5. Standing on the school roof, a physics student swings a rubber stopper tied to a string in a circle in a vertical plane. He releases the string so that the stopper flies outward in a horizontal direction.
   (a) Draw a sketch of this situation. Draw and label the velocity and acceleration vectors at the instant at which he releases the string in order to produce the horizontal motion.
   (b) Explain at what point the horizontal component of the stopper's motion becomes uniform.
6. Explain why an object with uniform circular motion is accelerating.
7. Draw a free-body diagram of a ball on the end of a string that is in uniform circular motion in a horizontal plane. Explain why the weight of the ball *does not* affect the value of the tension of the string that is providing the centripetal force required to maintain the motion.
8. Draw a free-body diagram of a car rounding a banked curve. Explain why the weight of the car *does* affect the value of the centripetal force required to keep the car in a circular path.
9. A rubber stopper tied to a string is being swung in a vertical loop.
   (a) Draw free-body diagrams of the stopper at its highest and lowest points.
   (b) Write equations to show the relationships among the centripetal force, the tension in

the string, and the weight of the stopper for each location.

10. Outline the conditions under which an object will travel in uniform circular motion and explain why a centripetal force is considered to be the *net* force required to maintain this motion.

Inquiry

11. Design and conduct a simple experiment to test the independence of the horizontal and vertical components of projectiles. Analyze your data to determine the percent deviation between your theoretical predictions and your actual results. Identify factors that could explain any deviations.

12. Design and construct a model of a vertical loop-the-loop section of a roller coaster. Refine your model, and your skill at operating it, until the vehicle will consistently round the loop without falling. Determine the minimum speed at which the vehicle must travel in order to complete the vertical loop without falling. Given the radius of your loop, calculate the theoretical value of the speed at which your vehicle would need to be travelling. Explain any deviation between the theoretical prediction and your actual results.

Communication

13. A stone is thrown off a cliff that has a vertical height of 45 m above the ocean. The initial horizontal velocity component is 15 m/s. The initial vertical velocity component is 10 m/s upward. Draw a scale diagram of the stone's trajectory by locating its position at one-second time intervals. At each of these points, draw a velocity vector to show the horizontal velocity component, the vertical velocity component, and the resultant velocity in the frame of reference of a person standing on the cliff. Assume that the stone is an ideal projectile and use 10 m/s^2 for the value of g to simplify calculations.

14. Draw a diagram to represent an object moving with uniform circular motion by constructing a rectangular x–y-coordinate system and drawing a circle with radius of 5 cm centred on the origin. Label the point where the circle crosses the positive y-axis, A; the positive x-axis, B; the negative y-axis, C; and the negative x-axis, D.

(a) At each of the four labelled points, draw a 2 cm vector to represent the object's instantaneous velocity.

(b) Construct a series of scale vector diagrams to determine the average acceleration between A and B, B and C, C and D, and D and A. Assume the direction of motion to be clockwise.

(c) Designate on the diagram at which points the average accelerations would occur and draw in the respective acceleration vectors.

(d) Write a general statement about the direction of the acceleration of an object in circular motion.

15. The mass of an object does not affect the angle at which a curve must be banked. The law of inertia, however, states that the motion of any object is affected by its inertia, which depends on its mass. How can objects rounding banked curves obey the law of inertia if the amount of banking required for a curve of a given radius of curvature and speed is independent of mass?

16. You are facing north, twirling a tethered ball in a horizontal circle above your head. At what point in the circle must you release the string in order to hit a target directly to the east? Sketch the situation, indicating the correct velocity vector.

17. A transport truck is rounding a curve in the highway. The curve is banked at an angle of 10° to the horizontal.

(a) Draw a free-body diagram to show all of the forces acting on the truck.

(b) Write an equation in terms of the weight of a truck, that will express the value of the centripetal force needed to keep the truck turning in a circle.

Making Connections

18. A pitched baseball is subject to the forces of gravity, air resistance, and lift. The lift force is produced by the ball's spin. Do research to find out

(a) how a spinning baseball creates lift

(b) how a pitcher can create trajectories for different types of pitches, such as fastballs, curve balls, knuckle balls, and sliders

(c) why these pitches are often effective in tricking the batter

19. Discuss similarities between a banked curve in a road and the tilt or banking of an airplane as it makes a turn. Draw free-body diagrams for each situation. What is the direction of the lift force on the airplane before and during the turn? Explain how tilting the airplane creates a centripetal force. What must the pilot do to make a sharper turn?

Problems for Understanding

20. You throw a rock off a 68 m cliff, giving it a horizontal velocity of 8.0 m/s.

(a) How far from the base of the cliff will it land?

(b) How long will the rock be in the air?

21. A physics student is demonstrating how the horizontal and vertical components of projectile motion are independent of each other. At the same instant as she rolls a wooden ball along the floor, her lab partner rolls an identical wooden ball from the edge of a platform directly above the first ball. Both balls have an initial horizontal velocity of 6.0 m/s. The platform is 3.0 m above the ground.

(a) When will the second ball strike the ground?

(b) Where, relative to the first ball, will the second ball hit the ground?

(c) At what distance from the base of the platform will the second ball land?

(d) With what velocity will the second ball land?

22. **(a)** A 350 g baseball is thrown horizontally at 22 m/s[forward] from a roof that is 18 m high. How far does it travel before hitting the ground?

(b) If the baseball is thrown with the same velocity but at an angle of 25° above the horizontal, how far does it travel? (Neglect air friction.)

23. A rescue plane flying horizontally at 175 km/h[N], at an altitude of 150 m, drops a 25 kg emergency package to a group of explorers. Where will the package land relative to the point above which it was released? (Neglect friction.)

24. You throw a ball with a velocity of 18 m/s at 24° above the horizontal from the top of your garage, 5.8 m above the ground. Calculate the

(a) time of flight

(b) horizontal range

(c) maximum height

(d) velocity when the ball is 2.0 m above the roof

(e) angle at which the ball hits the ground

25. Using a slingshot, you fire a stone horizontally from a tower that is 27 m tall. It lands 122 m from the base of the tower. What was its initial velocity?

26. At a ballpark, a batter hits a baseball at an angle of 37° to the horizontal with an initial velocity of 58 m/s. If the outfield fence is 3.15 m high and 323 m away, will the hit be a home run?

27. An archer shoots a 4.0 g arrow into the air, giving it a velocity of 40.0 m/s at an elevation angle of 65°. Find

(a) its time of flight

(b) its maximum height

(c) its range

(d) its horizontal and vertical distance from the starting point at 2.0 s after it leaves the bow

(e) the horizontal and vertical components of its velocity at 6.0 s after it leaves the bow

(f) its direction at 6.0 s after leaving the bow

Plot the trajectory on a displacement-versus-time graph.

28. A hang-glider, diving at an angle of 57.0° with the vertical, drops a water balloon at an altitude of 680.0 m. The water balloon hits the ground 5.20 s after being released.
   (a) What was the velocity of the hang-glider?
   (b) How far did the water balloon travel during its flight?
   (c) What were the horizontal and vertical components of its velocity just before striking the ground?
   (d) At what angle does it hit the ground?

29. A ball moving in a circular path with a constant speed of 3.0 m/s changes direction by 40.0° in 1.75 s.
   (a) What is its change in velocity?
   (b) What is the acceleration during this time?

30. You rotate a 450 g ball on the end of a string in a horizontal circle of radius 2.5 m. The ball completes eight rotations in 2.0 s. What is the centripetal force of the string on the ball?

31. A beam of electrons is caused to move in a circular path of radius 3.00 m at a velocity of 2.00×10^7 m/s. The electron mass is 9.11×10^{-31} kg.
   (a) What is the centripetal acceleration of one of the electrons?
   (b) What is the centripetal force on one electron?

32. A car travelling on a curved road will skid if the road does not supply enough friction. Calculate the centripetal force required to keep a 1500 kg car travelling at 65 km/h on a flat curve of radius 1.0×10^2 m. What must be the coefficient of friction between the car's wheels and the ground?

33. Consider an icy curved road, banked 6.2° to the horizontal, with a radius of curvature of 75.0 m. At what speed must a 1200 kg car travel to stay on the road?

34. You want to design a curve, with a radius of curvature of 350 m, so that a car can turn at a velocity of 15 m/s on it without depending on friction. At what angle must the road be banked?

35. (a) A motorcycle stunt rider wants to do a loop-the-loop within a vertical circular track. If the radius of the circular track is 10.0 m, what minimum speed must the motorcyclist maintain to stay on the track?
   (b) Suppose the radius of the track was doubled. By what factor will the motorcyclist need to increase her speed to loop-the-loop on the new track?

36. An amusement park ride consists of a large cylinder that rotates around a vertical axis. People stand on a ledge inside. When the rotational speed is high enough, the ledge drops away and people "stick" to the wall. If the period of rotation is 2.5 s and the radius is 2.5 m, what is the minimum coefficient of friction required to keep the riders from sliding down?

37. Use your understanding of the physics of circular motion to explain why we are not thrown off Earth like heavy particles in a centrifuge or mud off a tire, even though Earth is spinning at an incredible rate of speed. To make some relevant calculations, assume that you are standing in the central square of Quito, a city in Ecuador that is located on Earth's equator.
   (a) Calculate your average speed around the centre of Earth.
   (b) Determine the centripetal force needed to move you in a circle with Earth's radius at the speed that you calculated in part (a).
   (c) In what direction does the centripetal force act? What actual force is providing the amount of centripetal force that is required to keep you in uniform circular motion on Earth's surface?
   (d) What is your weight?
   (e) What is the normal force exerted on you by Earth's surface?
   (f) Use the calculations just made and other concepts about circular motion that you have been studying to explain why you are not thrown off Earth as it spins around its axis.

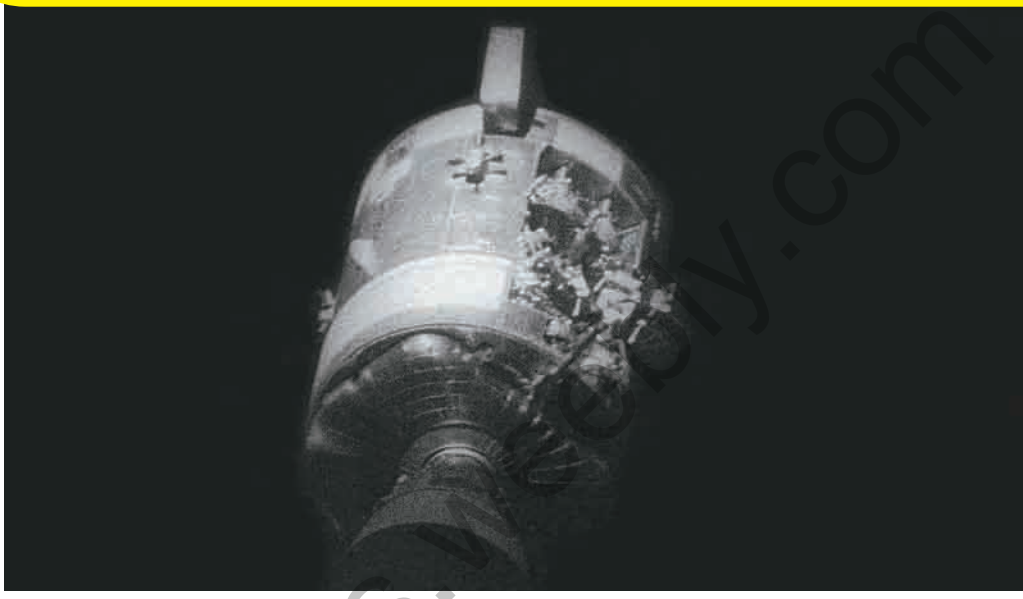
CHAPTER 3

Planetary and Satellite Dynamics

CHAPTER CONTENTS

PREREQUISITE CONCEPTS AND SKILLS

- Centripetal force
- Centripetal acceleration



On April 13, 1970, almost 56 h and 333 000 km into their flight to the Moon, the crew of *Apollo 13* heard a loud bang and felt the spacecraft shudder. Astronaut Jack Swigert radioed NASA Ground Control: "Houston, we've had a problem here." The above photograph, taken by the astronauts after they jettisoned the service module, shows how serious that problem was — an oxygen tank had exploded and damaged the only other oxygen tank. After assessing the situation, the astronauts climbed into the lunar landing module, where the oxygen and supplies were designed to support two people for two days. They would have to support the three astronauts for four days.

The spacecraft was still hurtling toward the Moon at more than 5000 km/h, and the engines of the lunar landing module could certainly not provide the force necessary to turn the craft back toward Earth. The only available force that could send the astronauts home was the gravitational force of the Moon, which swung the crippled spacecraft around behind the Moon and hurled it back toward Earth. With the engines of the lunar landing module, the crew made two small course corrections that prevented the craft from careening past Earth into deep space. Exactly 5 days, 22 h, and 54 min after lift-off, the astronauts, back inside the command module, landed in the Pacific Ocean, less than 800 m from the rescue ship.

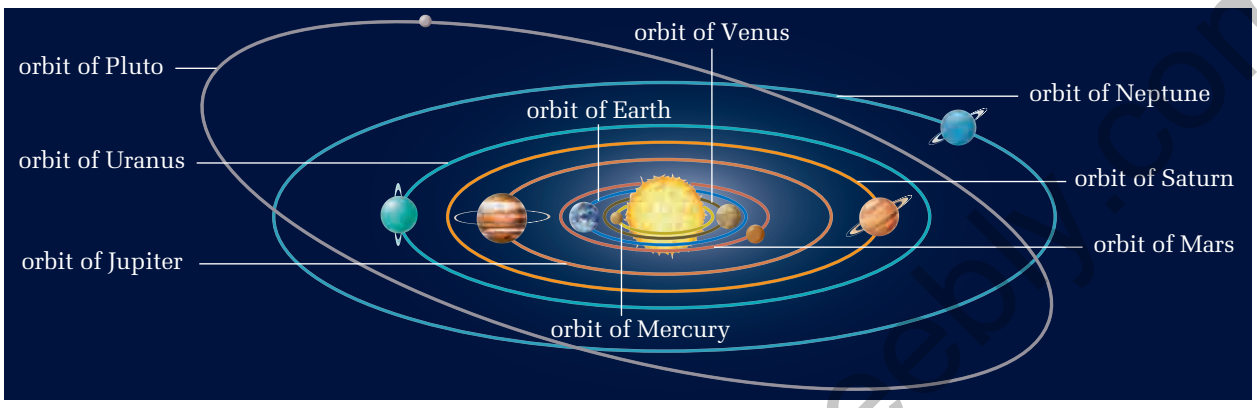
In this chapter, you will learn about Newton's law of universal gravitation and how it guides the motion of planets and satellites — and damaged spacecraft.

QUICK LAB

Kepler's Empirical Equations

TARGET SKILLS

- Initiating and planning
- Analyzing and interpreting
- Communicating results



The famous German astronomer Johannes Kepler (1571–1630) studied a vast amount of detailed astronomical data and found three empirical mathematical relationships within these data. Empirical equations are based solely on data and have no theoretical foundation. Often, however, an empirical equation will provide scientists with insights that will lead to a hypothesis that can be tested further.

In this chapter, you will learn the significance of Kepler's empirical equations. First, however, you will examine the data below, which is similar to the data that Kepler used, and look for a relationship.

Planet	Orbital radius R (AU)*	Orbital period T (days)
Mercury	0.389	87.77
Venus	0.724	224.70
Earth	1.000	365.25
Mars	1.524	686.98
Jupiter	5.200	4332.62
Saturn	9.150	10 759.20

* One astronomical unit (AU) is the average distance from Earth to the Sun, so distances expressed in AU are fractions or multiples of the Earth's average orbital radius.

From the data, make a graph of radius (R) versus period (T). Study the graph. Does the curve look like an inverse relationship, a logarithmic relationship, or an exponential relationship? Choose the type of mathematical relationship that you think is the most likely. Review Skill Set 4, Mathematical Modelling and Curve Straightening, and make at least four attempts to manipulate the data and plot the results. If you found a relationship that gives you a straight-line plot, write the mathematical relationship between radius and period. If you did not find the correct relationship, confer with your classmates to see if anyone found the correct relationship. As a class, agree on the final mathematical relationship.

Analyze and Conclude

1. When Kepler worked with astronomical data, he did not know whether a relationship existed between specific pairs of variables. In addition, Kepler had no calculator — he had to do all of his calculations by hand. Comment on the effort that he exerted in order to find his relationships.
2. Think about the relationship between the radius of an orbit and the period of an orbit on which your class agreed. Try to think of a theoretical basis for this relationship.
3. What type of additional information do you think that you would need in order to give a physical meaning to your mathematical relationship?

3.1 Newton's Law of Universal Gravitation

SECTION EXPECTATIONS

- Describe Newton's law of universal gravitation.
- Apply Newton's law of universal gravitation quantitatively.

KEY TERMS

- Tychonic system
- Kepler's laws
- law of universal gravitation

In previous science courses, you learned about the Ptolemaic system for describing the motion of the planets and the Sun. The system developed by Ptolemy (151–127 B.C.E.) was very complex because it was geocentric, that is, it placed Earth at the centre of the universe. In 1543, Nicholas Copernicus (1473–1543) proposed a much simpler, heliocentric system for the universe in which Earth and all of the other planets revolved around the Sun. The Copernican system was rejected by the clergy, however, because the religious belief system at the time placed great importance on humans and Earth as being central to a physically perfect universe. You probably remember learning that the clergy put Galileo Galilei (1564–1642) on trial for supporting the Copernican system.

Have you ever heard of the **Tychonic system**? A famous Danish nobleman and astronomer, Tycho Brahe (1546–1601), proposed a system, shown in Figure 3.1, that was intermediate between the Ptolemaic and Copernican systems. In Brahe's system, Earth is still and is the centre of the universe; the Sun and Moon revolve around Earth, but the other planets revolve around the Sun. Brahe's system captured the interest of many scientists, but never assumed the prominence of either the Ptolemaic or Copernican systems. Nevertheless, Tycho Brahe contributed a vast amount of detailed, accurate information to the field of astronomy.

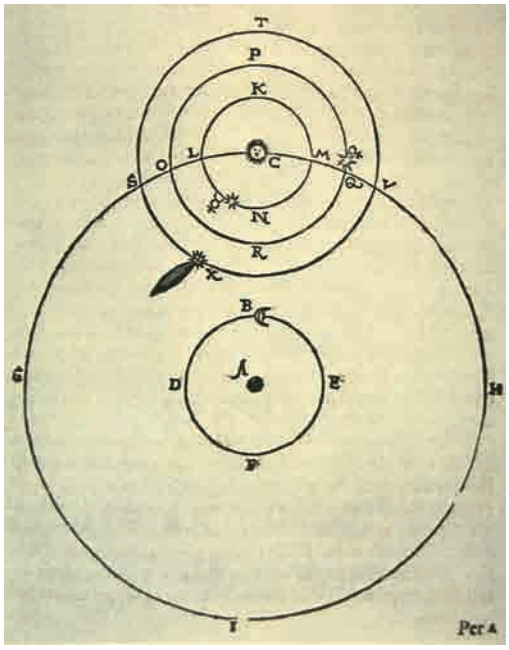


Figure 3.1 The Tychonic universe was acceptable to the clergy, because it maintained that Earth was the centre of the universe. The system was somewhat satisfying for scientists, because it was simpler than the Ptolemaic system.

Laying the Groundwork for Newton

Astronomy began to come of age as an exact science with the detailed and accurate observations of Tycho Brahe. For more than 20 years, Brahe kept detailed records of the positions of the planets and stars. He catalogued more than 777 stars and, in 1572, discovered a new star that he named "Nova." Brahe's star was one of very few supernovae ever found in the Milky Way galaxy.

In 1577, Brahe discovered a comet and demonstrated that it was not an atmospheric phenomenon as some scientists had believed, but rather that its orbit lay beyond the Moon. In addition to making observations and collecting data, Brahe designed and built the most accurate astronomical instruments of the day (see Figure 3.2). In addition, he was the first astronomer to make corrections for the refraction of light by the atmosphere.

HISTORY LINK

Tycho Brahe was a brilliant astronomer who led an unusual and tumultuous life. At age 19, he was involved in a duel with another student and part of his nose was cut off. For the rest of his life, Brahe wore an artificial metal nose.

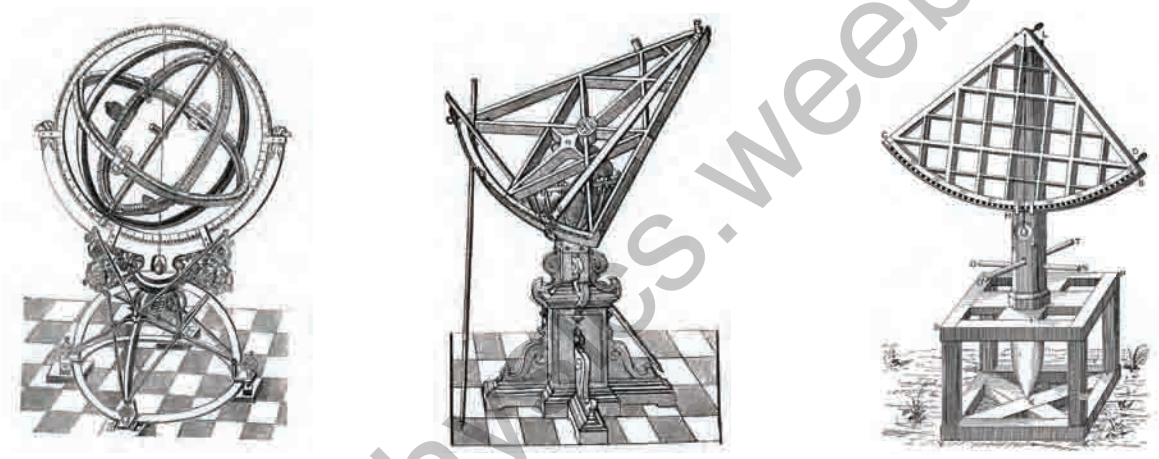


Figure 3.2 Brahe's observatory in Hveen, Denmark, contained gigantic instruments that, without magnification, were precise to 1/30 of a degree.

In 1600, Brahe invited Kepler to be one of his assistants. Brahe died suddenly the following year, leaving all of his detailed data to Kepler. With this wealth of astronomical data and his ability to perform meticulous mathematical analyses, Kepler discovered three empirical relationships that describe the motion of the planets. These relationships are known today as **Kepler's laws**.

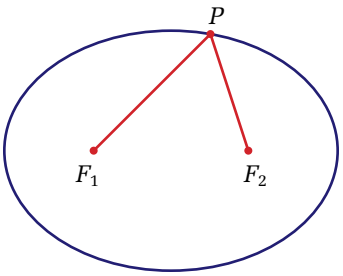
KEPLER'S LAWS

1. Planets move in elliptical orbits, with the Sun at one focus of the ellipse.
2. An imaginary line between the Sun and a planet sweeps out equal areas in equal time intervals.
3. The quotient of the square of the period of a planet's revolution around the Sun and the cube of the average distance from the Sun is constant and the same for all planets.

$$\frac{T^2}{r^3} = k \quad \text{or} \quad \frac{T_A^2}{r_A^3} = \frac{T_B^2}{r_B^3}, \quad \text{where A and B are two planets.}$$

MATH LINK

A circle is a special case of an ellipse. An ellipse is defined by two focuses and the relationship $\overline{F_1P} + \overline{F_2P} = k$, where k is a constant and is the same for every point on the ellipse. If the two focuses of an ellipse are brought closer and closer together until they are superimposed on each other, the ellipse becomes a circle.



Kepler's first law does not sound terribly profound, but he was contending not only with scientific observations of the day, but also with religious and philosophical views. For centuries, the perfection of "celestial spheres" was of extreme importance in religious beliefs. Ellipses were not considered to be "perfect," so many astronomers resisted accepting any orbit other than a "perfect" circle that fit on the surface of a sphere. However, since Kepler published his laws, there has never been a case in which the data for the movement of a satellite, either natural or artificial, did not fit an ellipse.

Kepler's second law is illustrated in Figure 3.3. Each of the shaded sections of the ellipse has an equal area. According to Kepler's second law, therefore, the planet moves along the arc of each section in the same period of time. Since the arcs close to the Sun are longer than the arcs more distant from the Sun, the planet must be moving more rapidly when it is close to the Sun.

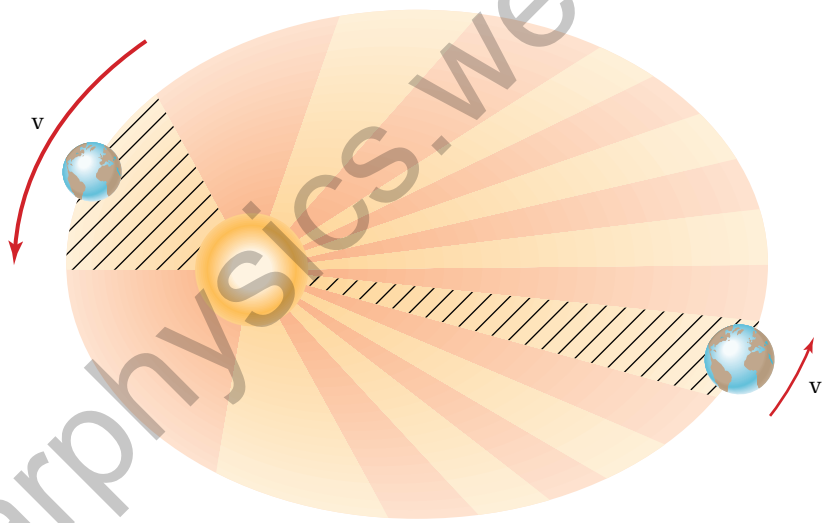


Figure 3.3 According to Kepler's second law, the same length of time was required for a planet to move along each of the arcs at the ends of the segments of the ellipse. Kepler could not explain why planets moved faster when they were close to the Sun than when they were farther away.

When Kepler published his third law, he had no way of knowing the significance of the constant in the mathematical expression $T^2/r^3 = k$. All he knew was that the data fit the equation. Kepler suspected that the Sun was in some way influencing the motion of the planets, but he did not know how or why this would lead to the mathematical relationship. The numerical value of the constant in Kepler's third law and its relationship to the interaction between the Sun and the planets would take on significance only when Sir Isaac Newton (1642–1727) presented his law of universal gravitation.

Universal Gravitation

Typically in research, the scientist makes some observations that lead to an hypothesis. The scientist then tests the hypothesis by planning experiments, accumulating data, and then comparing the results to the hypothesis. The development of Newton's law of universal gravitation happened in reverse. Brahe's data and Kepler's analysis of the data were ready and waiting for Newton to use to test his hypothesis about gravity.

Newton was not the only scientist of his time who was searching for an explanation for the motion, or orbital dynamics, of the planets. In fact, several scientists were racing to see who could find the correct explanation first. One of those scientists was astronomer Edmond Halley (1656–1742). Halley and others, based on their calculations, had proposed that the force between the planets and the Sun decreased with the square of the distance between a planet and the Sun. However, they did not know how to apply that concept to predict the shape of an orbit.

Halley decided to put the question to Newton. Halley first met Newton in 1684, when he visited Cambridge. He asked Newton what type of path a planet would take if the force attracting it to the Sun decreased with the square of the distance from the Sun. Newton quickly answered, "An elliptical path." When Halley asked him how he knew, Newton replied that he had made that calculation many years ago, but he did not know where his calculations were. Halley urged Newton to repeat the calculations and send them to him.

Three months later, Halley's urging paid off. He received an article from Newton entitled "De Motu" ("On Motion"). Newton continued to improve and expand his article and in less than three years, he produced one of the most famous and fundamental scientific works: *Philosophiae Naturalis Principia Mathematica* (*The Mathematical Principles of Natural Philosophy*). The treatise contained not only the law of universal gravitation, but also Newton's three laws of motion.

Possibly, Newton was successful in finding the law of universal gravitation because he extended the concept beyond the motion of planets and applied it to all masses in all situations. While other scientists were looking at the motion of planets, Newton was watching an apple fall from a tree to the ground. He reasoned that the same attractive force that existed between the Sun and Earth was also responsible for attracting the apple to Earth. He also reasoned that the force of gravity acting on a falling object was proportional to the mass of the object. Then, using his own third law of action-reaction forces, if a falling object such as an apple was attracted to Earth, then Earth must also be attracted to the apple, so the force of gravity must also be proportional to the mass of Earth. Newton therefore proposed that *the force of gravity between any two objects is proportional to the product of their*

HISTORY LINK

Sir Edmond Halley, the astronomer who prompted Newton to publish his work on gravitation, is the same astronomer who discovered the comet that was named in his honour — Halley's Comet. Without Halley's urging, Newton might never have published his famous *Principia*, greatly slowing the progress of physics.

masses and inversely proportional to the square of the distance between their centres — the **law of universal gravitation**. The mathematical equation for the law of universal gravitation is given in the following box.

NEWTON'S LAW OF UNIVERSAL GRAVITATION

The force of gravity is proportional to the product of the two masses that are interacting and inversely proportional to the square of the distance between their centres.

$$F_g = G\frac{m_1 m_2}{r^2}$$

Quantity	Symbol	SI unit
force of gravity	F_g	N (newtons)
first mass	m_1	kg (kilograms)
second mass	m_2	kg (kilograms)
distance between the centres of the masses	r	m (metres)
universal gravitational constant	G	$\frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}$ (newton · metre squared per kilogram squared)

Unit Analysis

$$\text{newton} = \left(\frac{\text{newton}\cdot\ \text{metre}^2}{\text{kilogram}^2}\right)\left(\frac{\text{kilogram}\cdot\text{kilogram}}{\text{metre}^2}\right)$$

$$\left(\frac{\text{N}\cdot\cancel{\text{m}^2}}{\cancel{\text{kg}^2}}\right)\left(\frac{\cancel{\text{kg}}\cdot\cancel{\text{kg}}}{\cancel{\text{m}^2}}\right) = \text{N}$$

Note: The value of the universal gravitational constant is

$$G = 6.67 \times 10^{-11}\ \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}.$$

Conceptual Problem

- You have used the equation $F_g = mg$ many times to calculate the weight of an object on Earth's surface. Now, you have learned that the weight of an object on Earth's surface is $F_g = G\frac{m_E m_o}{r_{E\text{-}o}^2}$, where m_E is the mass of Earth, m_o is the mass of the object, and $r_{E\text{-}o}$ is the distance between the centres of Earth and the object. Explain how the two equations are related. Express g in terms of the variables and constant in Newton's law of universal gravitation.

SAMPLE PROBLEM

Weighing an Astronaut

A 65.0 kg astronaut is walking on the surface of the Moon, which has a mean radius of 1.74×10^3 km and a mass of 7.35×10^{22} kg. What is the weight of the astronaut?

Conceptualize the Problem

- The weight of the astronaut is the gravitational force on her.
- The relationship $F_g = mg$, where $g = 9.81 \frac{\text{m}}{\text{s}^2}$, *cannot* be used in this problem, since the astronaut is not on Earth's surface.
- The law of universal gravitation applies to this problem.

Identify the Goal

The gravitational force, F_g, on the astronaut

Identify the Variables and Constants

Known	Implied	Unknown
$m_M = 7.35 \times 10^{22}$ kg	$G = 6.67 \times 10^{-11} \frac{\text{N} \cdot \text{m}^2}{\text{kg}^2}$	F_g
$m_a = 65.0$ kg		
$r = 1.74 \times 10^3$ km $(1.74 \times 10^6$ m$)$		

Develop a Strategy

Apply the law of universal gravitation.

$$F_g = G\frac{m_1 m_2}{r^2}$$

Substitute the numerical values and solve.

$$F_g = \left(6.67 \times 10^{-11} \frac{\text{N} \cdot \text{m}^2}{\text{kg}^2}\right)\frac{(7.35 \times 10^{22}\text{ kg})(65.0\text{ kg})}{(1.74 \times 10^6\text{ m})^2}$$

$$F_g = 105.25\text{ N}$$

$$F_g \cong 105\text{ N}$$

The weight of the astronaut is approximately 105 N.

Validate the Solution

Weight on the Moon is known to be much less than that on Earth. The astronaut's weight on the Moon is about one sixth of her weight on Earth $(65.0\text{ kg} \times 9.81 \frac{\text{m}}{\text{s}^2} \cong 638\text{ N})$, which is consistent with this common knowledge.

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PRACTICE PROBLEMS

1. Find the gravitational force between Earth and the Sun. (See Appendix B, Physical Constants and Data.)
2. Find the gravitational force between Earth and the Moon. (See Appendix B, Physical Constants and Data.)
3. How far apart would you have to place two 7.0 kg bowling balls so that the force of gravity between them would be 1.25×10^{-4} N? Would it be possible to place them at this distance? Why or why not?
4. Find the gravitational force between the electron and the proton in a hydrogen atom if they are 5.30×10^{-11} m apart. (See Appendix B, Physical Constants and Data.)
5. On Venus, a person with mass 68 kg would weigh 572 N. Find the mass of Venus from this data, given that the planet's radius is 6.31×10^{6} m.
6. In an experiment, an 8.0 kg lead sphere is brought close to a 1.5 kg mass. The gravitational force between the two objects is 1.28×10^{-8} N. How far apart are the centres of the objects?
7. The radius of the planet Uranus is 4.3 times the radius of earth. The mass of Uranus is 14.7 times Earth's mass. How does the gravitational force on Uranus' surface compare to that on Earth's surface?
8. Along a line connecting Earth and the Moon, at what distance from Earth's centre would an object have to be located so that the gravitational attractive force of Earth on the object was equal in magnitude and opposite in direction from the gravitational attractive force of the Moon on the object?

Gravity and Kepler's Laws

The numerical value of G, the universal gravitational constant, was not determined experimentally until more than 70 years after Newton's death. Nevertheless, Newton could work with concepts and proportionalities to verify his law.

Newton had already shown that the inverse square relationship between gravitational force and the distance between masses was supported by Kepler's first law — that planets follow elliptical paths.

Kepler's second law showed that planets move more rapidly when they are close to the Sun and more slowly when they are farther from the Sun. The mathematics of elliptical orbits in combination with an inverse square relationship to yield the speed of the planets is somewhat complex. However, you can test the concepts graphically by completing the following investigation.

INVESTIGATION 3-A

Orbital Speed of Planets

TARGET SKILLS

- **Modelling concepts**
- **Analyzing and interpreting**
- **Communicating results**

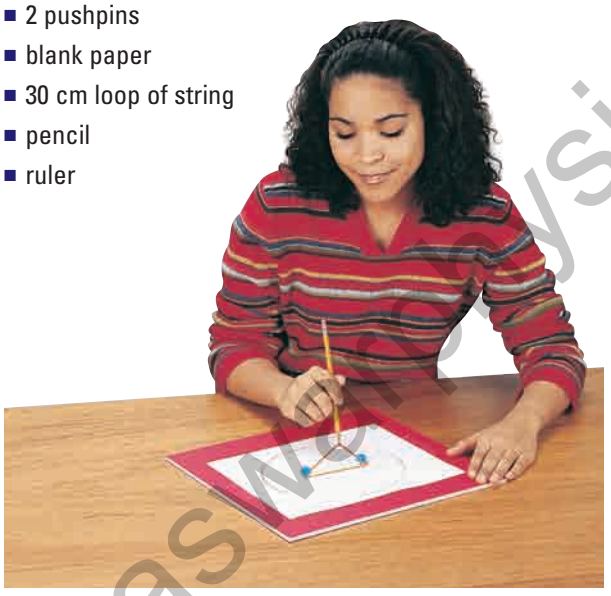
Can you show diagrammatically that a force directed along the line between the centres of the Sun and a planet would cause the planet's speed to increase as it approached the Sun and decrease as it moved away? If you can, you have demonstrated that Kepler's second law supports Newton's proposed law of universal gravitation.

Problem

How does a force that follows an inverse square relationship affect the orbital speed of a planet in an elliptical orbit?

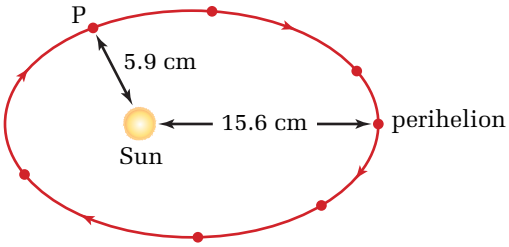
Equipment

- corkboard or large, thick piece of cardboard
- 2 pushpins
- blank paper
- 30 cm loop of string
- pencil
- ruler



Procedure

1. Place the paper on the corkboard or cardboard. Insert two pushpins into the paper about 8 to 10 cm apart.
2. Loop the string around the pushpins, as shown in the illustration. With your pencil, pull the string so that it is taut and draw an ellipse by pulling the string all the way around the pushpins.
3. Remove the string and pushpins and label one of the pinholes "Sun."
4. Choose a direction around the elliptical orbit in which your planet will be moving. Make about four small arrowheads on the ellipse to indicate the direction of motion of the planet.
5. Make a dot for the planet at the point that is most distant from the Sun (the perihelion). Measure and record the distance on the paper from the perihelion to the Sun. From that point, draw a 1 cm vector directed straight toward the Sun.
6. This vector represents the force of gravity on the planet at that point: $F_{g(per)} = 1$ unit. ($F_{g(per)}$ is the force of gravity when the planet is at perihelion.)
7. Select and label at least three more points on each side of the ellipse at which you will analyze the force and motion of the planet.
8. For each point, measure and record, on a separate piece of paper, the distance from the Sun to point P, as indicated in the diagram. Do not write on your diagram, because it will become too cluttered.



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9. Follow the steps in the table to see how to determine the length of the force vector at each point.

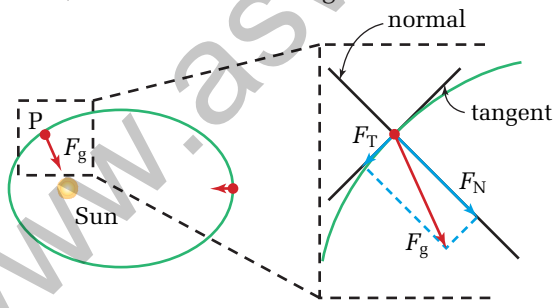
Procedure	Equation
■ The masses of the Sun and planet remain the same, so the value Gm_Sm_p is constant. Therefore, the expression F_gr^2 for any point on the orbit is equal to the same value.	$F_g = G\frac{m_Sm_p}{r^2}$ $F_gr^2 = Gm_Sm_p$
■ Consequently, you can set the expression F_gr^2 for any one point equal to F_gr^2 for any other point. Use the values at perihelion as a reference and set $F_{g(P)}r^2$ equal to $F_{g(peri)}r^2_{peri}$. Then solve for the $F_{g(P)}$.	$F_{g(peri)}\,r^2_{peri} = F_{g(P)}\,r^2_P$ $F_{g(P)} = \frac{F_{g(peri)}\,r^2_{peri}}{r^2_P}$
■ You can now find the relative magnitude of the gravitational force on the planet at any point on the orbit by substituting the magnitudes of the radii into the above equation. For example, the magnitude of the force at point P in step 8 is 6.99 units.	$F_{g(P)} = \frac{(1\text{ unit})(15.6\text{ cm})^2}{(5.9\text{ cm})^2}$ $F_{g(P)} = 6.99\text{ units}$

10. Calculate the length of the force vector from each of the points that you have selected on your orbit.

11. On your diagram, draw force vectors from each point directly toward the Sun, making the lengths of the vectors equal to the values that you calculated in step 10.

12. At each point at which you have a force vector, draw a very light pencil line tangent to the ellipse. Then, draw a line that is perpendicular (normal) to the tangent line.

13. Graphically draw components of the force vector along the tangent (F_T) and normal (F_N) lines, as shown in the diagram.



Analyze and Conclude

1. The tangential component of the force vector (F_T) is parallel to the direction of the velocity of the planet when it passes point P. What effect will the tangential component of force have on the velocity of the planet?

2. The normal component of the force vector (F_N) is perpendicular to the direction of the velocity of the planet when it passes point P. What effect will the normal component of force have on the velocity of the planet?

3. Analyze the change in the motion of the planet caused by the tangential and normal components of the gravitational force at each point where you have drawn force vectors. Be sure to note the direction of the velocity of the planet as you analyze the effect of the components of force at each point.

4. Summarize the changes in the velocity of the planet as it makes one complete orbit around the Sun.

5. The force vectors and components that you drew were predictions based on Newton's law of universal gravitation. How well do these predictions agree with Kepler's observations as summarized in his second law? Would you say that Kepler's data supports Newton's predictions?

Kepler's third law simply states that the ratio T^2/r^3 is constant and the same for each planet orbiting the Sun. At first glance, it would appear to have little relationship to Newton's law of universal gravitation, but a mathematical analysis will yield a relationship. To keep the mathematics simple, you will consider only circular orbits. The final result obtained by considering elliptical orbits is the same, although the math is more complex. Follow the steps below to see how Newton's law of universal gravitation yields the same ratio as given by Kepler's third law.

- Write Newton's law of universal gravitation, using m_S for the mass of the Sun and m_p for the mass of a planet.

$$F_g = G\frac{m_S m_p}{r^2}$$

- Since the force of gravity must provide a centripetal force for the planets, set the gravitational force equal to the required centripetal force.

$$G\frac{m_S m_p}{r^2} = \frac{m_p v^2}{r}$$

Simplify the equation.

$$G\frac{m_S}{r} = v^2$$

- Since Kepler's third law includes the period, T, as a variable, find an expression for the velocity, v, of the planet in terms of its period.

$$v = \frac{\Delta d}{\Delta t}$$

$$\Delta d = 2\pi r$$

$$\Delta t = T$$

A planet travels a distance equal to the circumference of the orbit during a time interval equal to its period.

$$v = \frac{2\pi r}{T}$$

- Substitute the expression for the velocity of the planet into the above equation.

$$G\frac{m_S}{r} = \left(\frac{2\pi r}{T}\right)^2$$

$$G\frac{m_S}{r} = \frac{4\pi^2 r^2}{T^2}$$

- Multiply each side of the equation by T^2/r^2.

$$\left(G\frac{m_S}{r}\right)\left(\frac{T^2}{r^2}\right) = \left(\frac{4\pi^2 \cancel{r^2}}{\cancel{T^2}}\right)\left(\frac{\cancel{T^2}}{\cancel{r^2}}\right)$$

$$\frac{G m_S T^2}{r^3} = 4\pi^2$$

- Solve for T^2/r^3.

$$\frac{T^2}{r^3} = \frac{4\pi^2}{G m_S}$$

As you can see, Newton's law of universal gravitation indicates not only that the ratio T^2/r^3 is constant, but also that the constant is $4\pi^2/Gm_S$. All of Kepler's laws, developed prior to the time when

Newton did his work, support Newton's law of universal gravitation. Kepler had focussed only on the Sun and planets, but Newton proposed that the laws applied to all types of orbital motion, such as moons around planets. Today, we know that all of the artificial satellites orbiting Earth, as well as the Moon, follow Kepler's laws.

Mass of the Sun and Planets

Have you ever looked at tables that contain data for the mass of the Sun and planets and wondered how anyone could "weigh" the Sun and planets or determine their masses? English physicist and chemist Henry Cavendish (1731–1810) realized that if he could determine the universal gravitational constant, G, he could use the mathematical relationship in Kepler's third law to calculate the mass of the Sun. A brilliant experimentalist, Cavendish designed a torsion balance, similar to the system in Figure 3.4, that allowed him to measure G.

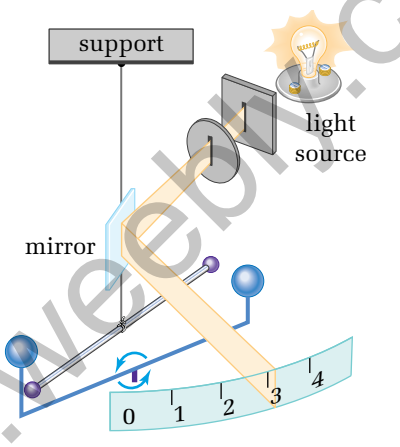


Figure 3.4 In the torsion balance that Cavendish designed and used, the spheres were made of lead. The small spheres were about 5 cm in diameter and were attached by a thin but rigid rod about 1.83 m long. The large spheres were about 20 cm in diameter.

A torsion balance can measure extremely small amounts of the rotation of a wire. First, the torsion balance must be calibrated to determine the amount of force that causes the wire to twist by a specific amount. Then, the large spheres are positioned so that the bar supporting them is perpendicular to the rod supporting the small spheres. In this position, the large spheres are exerting equal gravitational attractive forces on each of the small spheres. The system is in equilibrium and the scale can be set to zero. The large spheres are then moved close to the small spheres and the amount of twisting of the wire is determined. From the amount of twisting and the calibration, the mutual attractive force between the large and small spheres is calculated.

Using his torsion balance, Cavendish calculated the value of G to be $6.75 \times 10^{-11}\ \mathrm{N \cdot m^2/kg^2}$. The best-known figure today is $6.672\ 59 \times 10^{-11}\ \mathrm{N \cdot m^2/kg^2}$. Cavendish's measurement was within approximately 1% of the correct value. As Cavendish did, you can now calculate the mass of the Sun and other celestial bodies.

HISTORY LINK

Henry Cavendish was a very wealthy and brilliant man, but he also was very reclusive. He was rarely seen in public places, other than at scientific meetings. His work was meticulous, yet he published only a very small part of it. After his death, other scientists discovered his notebooks and finally published his results. Cavendish had performed the same experiments and obtained the same results for some experiments that were later done by Coulomb, Faraday, and Ohm, who received the credit for the work.

PHYSICS FILE

The value of G shows that the force of gravity is extremely small. For example, use unit amounts of each of the variables and substitute them into Newton's law of universal gravitation. You will find that the mutual attractive force between two 1 kg masses that are 1 m apart is $6.672\ 59 \times 10^{-11}$ N.

SAMPLE PROBLEM

The Mass of the Sun

Find the mass of the Sun, using Earth's orbital radius and period of revolution.

Conceptualize the Problem

- Kepler's third law, combined with Newton's law of universal gravitation, yields an equation that relates the period and orbital radius of a satellite to the mass of the body around which the satellite is orbiting.
- Earth orbits the Sun once per year.
- Let R_E represent the radius of Earth's orbit around the Sun. This value can be found in Appendix B, Physical Constants and Data.

Identify the Goal

The mass of the Sun, m_S

Identify the Variables and Constants

Known	Implied	Unknown
Sun	$G = 6.67 \times 10^{-11}\ \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}$	m_S
	$T = 365.25$ days	
	$R_{E(orbit)} = 1.49 \times 10^{11}$ m	

Develop a Strategy

Write Kepler's third law, using the constant derived from Newton's law of universal gravitation.

$$\frac{T^2}{r^3} = \frac{4\pi^2}{Gm_S}$$

Solve for the mass of the Sun.

$$\frac{T^2}{r^3}m_S = \frac{4\pi^2}{G\cancel{m_S}}\cancel{m_S}$$

$$m_S = \left(\frac{4\pi^2}{G}\right)\left(\frac{r^3}{T^2}\right)$$

Convert the period into SI units.

$$365.25\ \cancel{\text{days}}\left(\frac{24\ \cancel{\text{h}}}{\cancel{\text{day}}}\right)\left(\frac{60\ \cancel{\text{min}}}{\cancel{\text{h}}}\right)\left(\frac{60\ \text{s}}{\cancel{\text{min}}}\right) = 3.1558 \times 10^7\ \text{s}$$

Substitute the numerical values into the equation and solve.

$$m_S = \left(\frac{4\pi^2}{6.67 \times 10^{-11}\ \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}}\right)\frac{(1.49 \times 10^{11}\ \text{m})^3}{(3.1558 \times 10^7\ \text{s})^2}$$

$$m_S = 1.9660 \times 10^{30}\ \text{kg}$$

$$m_S \cong 1.97 \times 10^{30}\ \text{kg}$$

The mass of the Sun is approximately 1.97×10^{30} kg.

Validate the Solution

The Sun is much more massive than any of the planets. The value sounds reasonable.

Check the units: $\left(\frac{1}{\frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}}\right)\left(\frac{\text{m}^3}{\text{s}^2}\right) = \left(\frac{\text{kg}^2}{\text{N}\cdot\cancel{\text{m}^2}}\right)\left(\frac{\cancel{\text{m}^3}}{\text{s}^2}\right) = \left(\frac{\text{kg}^{\cancel{2}}}{\frac{\cancel{\text{kg}}\cdot\cancel{\text{m}}}{\cancel{\text{s}^2}}\cdot\cancel{\text{m}^2}}\right)\left(\frac{\cancel{\text{m}^3}}{\cancel{\text{s}^2}}\right) = \text{kg}$.

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PRACTICE PROBLEMS

9. Jupiter's moon Io orbits Jupiter once every 1.769 days. Its average orbital radius is 4.216×10^8 m. What is Jupiter's mass?

10. Charon, the only known moon of the planet Pluto, has an orbital period of 6.387 days at an average distance of 1.9640×10^7 m from Pluto. Use Newton's form of Kepler's third law to find the mass of Pluto from this data.

11. Some weather satellites orbit Earth every 90.0 min. How far above Earth's surface is their orbit? (Hint: Remember that the centre of the orbit is the centre of Earth.)

12. How fast is the moon moving as it orbits Earth at a distance of 3.84×10^5 km?

13. On each of the *Apollo* lunar missions, the command module was placed in a very low, approximately circular orbit above the Moon. Assume that the average height was 60.0 km above the surface and that the Moon's radius is 7738 km.
    - **(a)** What was the command module's orbital period?
    - **(b)** How fast was the command module moving in its orbit?

14. A star at the edge of the Andromeda galaxy appears to be orbiting the centre of that galaxy at a speed of about 2.0×10^2 km/s. The star is about 5×10^9 AU from the centre of the galaxy. Calculate a rough estimate of the mass of the Andromeda galaxy. Earth's orbital radius (1 AU) is 1.49×10^8 km.

Newton's law of universal gravitation has stood the test of time and the extended limits of space. As far into space as astronomers can observe, celestial bodies move according to Newton's law. As well, the astronauts of the crippled *Apollo 13* spacecraft owe their lives to the dependability and predictability of the Moon's gravity. Although Albert Einstein (1879–1955) took a different approach in describing gravity in his general theory of relativity, most calculations that need to be made can use Newton's law of universal gravitation and make accurate predictions.

3.1 Section Review

1. K/U Explain the meaning of the term "empirical" as it applies to empirical equations.

2. K/U What did Tycho Brahe contribute to the development of the law of universal gravitation?

3. K/U Describe how Newton used each of the following phenomena to support the law of universal gravitation.
    - **(a)** the orbit of the moon
    - **(b)** Kepler's third law

4. K/U How did Newton's concepts about gravity and his development of the law of universal gravitation differ from the ideas of other scientists and astronomers who were attempting to find a relationship that could explain the motion of the planets?

5. K/U Describe the objective, apparatus, and results of the Cavendish experiment.

6. C Explain how you can "weigh" a planet.

7. I Suppose the distance between two objects is doubled and the mass of one is tripled. What effect does this have on the gravitational force between the objects?

3.2 Planetary and Satellite Motion

A perfectly executed football or hockey pass is an amazing achievement. A fast-moving player launches an object toward the place where a fast-moving receiver will most likely be when the ball or puck arrives. The direction and force of the pass are guided by intuition and skill acquired from long practice.

Launching a spacecraft has the same objective, but extreme precision is required — the outcome is more critical than that of an incomplete pass. Scientists and engineers calculate every detail of the trajectories and orbits in advance. The magnitude and direction of the forces and the time interval for firing the rockets are analyzed and specified in minute detail. Even last-minute adjustments are calculated exhaustively. The process can be extremely complex, but it is based on principles that you have already studied — the dynamics of circular motion and the law of universal gravitation.

SECTION EXPECTATION

- Use Newton's law of universal gravitation to explain planetary and satellite motion.

KEY TERMS

- geostationary orbit
- microgravity
- perturbation

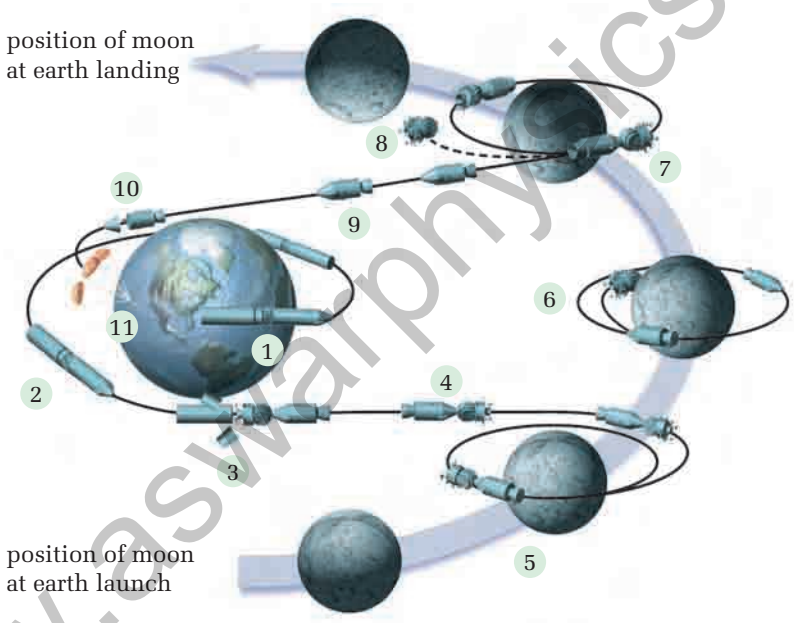

Figure 3.5 The steps in a typical *Apollo* lunar mission are: **(1)** lift off and enter Earth orbit, **(2)** leave Earth orbit, **(3)** release booster rocket, turn, and dock with lunar module that is stored between the booster rocket and the service module, **(4)** make a mid-course correction, **(5)** enter lunar orbit, **(6)** command module continues to orbit the Moon, while the lunar module descends to the lunar surface, carries out tasks, ascends, and reconnects with the command module, **(7)** leave lunar orbit, **(8)** eject lunar module, **(9)** mid-course correction, **(10)** eject service module, and **(11)** command module lands in the Pacific Ocean.

Newton's Mountain

The planets in our solar system appear to have been "orbiting" the Sun while they were forming. Great swirling dust clouds in space began to condense around a newly formed Sun until they finally became the planets. How, then, do artificial satellites begin orbiting Earth?

Soon after Newton formulated his law of universal gravitation, he began thought experiments about artificial satellites. He reasoned that you could put a cannon at the top of an extremely high mountain and shoot a cannon ball horizontally, as shown in Figure 3.6. The cannon ball would certainly fall toward Earth. If the cannon ball travelled far enough horizontally while it fell, however, the curvature of Earth would be such that Earth's surface would "fall away" as fast as the cannon ball fell.

You can determine how far the cannon ball will fall in one second by using the kinematic equation $\Delta d = v_i\Delta t + \frac{1}{2}a\Delta t^2$. If a cannon ball had zero vertical velocity at time zero, in one second it would fall a distance $\Delta d = 0 + \frac{1}{2}\left(-9.81\ \frac{\text{m}}{\text{s}^2}\right)(1\ \text{s})^2 = -4.9\ \text{m}$. From the size and curvature of Earth, Newton knew that Earth's surface would drop by 4.9 m over a horizontal distance of 8 km.

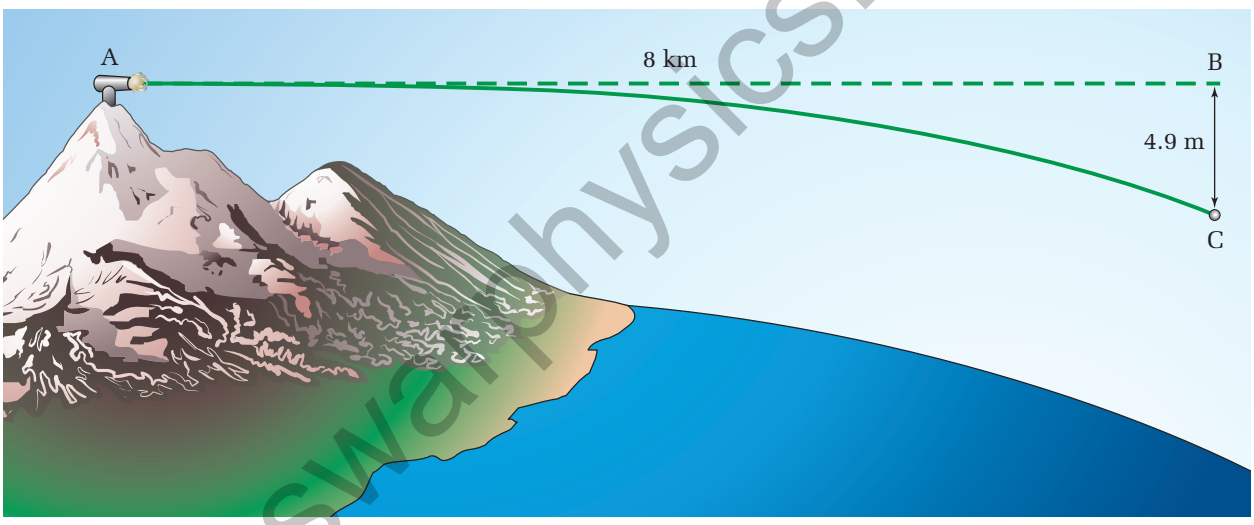

Figure 3.6 The values shown here represent the distance that the cannon ball would have to go in one second in order to go into orbit.

Newton's reasoning was absolutely correct, but he did not account for air friction. Although the air is too thin to breathe easily on top of Mount Everest, Earth's highest mountain, it would still exert a large amount of air friction on an object moving at 8 km/s. If you could take the cannon to 150 km above Earth's surface, the atmosphere would be so thin that air friction would be negligible. Newton understood how to put an artificial satellite into orbit, but he did not have the technology.

Today, launching satellites into orbit is almost routine, but the scientists and engineers must still carefully select an orbit and perform detailed calculations to ensure that the orbit will fulfil the purpose of the satellite. For example, some weather satellites orbit over the Poles at a relatively low altitude in order to collect data in detail. Since a satellite is constantly moving in relation to a ground observer, the satellite receiver has to track the satellite continually so that it can capture the signals that the satellite is sending. In addition, the satellite is on the opposite side of Earth for long periods of time, so several receivers must be located around the globe to collect data at all times.

Communication satellites and some weather satellites travel in a **geostationary orbit** over the equator, which means that they appear to hover over one spot on Earth's surface at all times. Consequently, a receiver can be aimed in the same direction at all times and constantly receive a signal from the satellite. The following problem will help you to find out how these types of orbits are attained.

PHYSICS FILE

Arthur C. Clarke (1917–), scientist and science fiction writer, wrote a technical paper in 1945, setting out the principles of geostationary satellites for communications. Many scientists in the field at the time did not believe that it was possible. Today, geostationary orbits are sometimes called "Clarke orbits." Clarke also co-authored the book and movie *2001: A Space Odyssey*.

SAMPLE PROBLEM

Geostationary Orbits

At what velocity and altitude must a satellite orbit in order to be geostationary?

Conceptualize the Problem

- A satellite in a *geostationary orbit* must remain over the *same point* on Earth at all times.
- To be geostationary, the satellite must make *one complete orbit* in exactly the same time that Earth rotates on its axis. Therefore, the period must be *24 h*.
- The *period* is related to the *velocity* of the satellite.
- The *velocity* and *altitude* of the satellite are determined by the amount of *centripetal force* that is causing the satellite to remain on a circular path.
- Earth's *gravity* provides the *centripetal force* for satellite motion.
- The values for the mass and radius of Earth are listed in Appendix B, Physical Constants and Data.

Identify the Goal

(a) The velocity, v, of a geostationary satellite

(b) The altitude, h, of a geostationary satellite

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Identify the Variables and Constants

Known	Implied		Unknown
Orbit is geostationary.	$T = 24\ \text{h}$	$m_E = 5.98 \times 10^{24}\ \text{kg}$	r (radius of satellite's orbit)
	$G = 6.67 \times 10^{-11}\ \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}$	$r_E = 6.38 \times 10^6\ \text{m}$	h

Develop a Strategy

To find the velocity, start by setting the gravitational force equal to the centripetal force.

$$F_g = F_c$$

$$G\frac{m_E m_s}{r^2} = \frac{m_s v^2}{r}$$

Simplify the expression.

$$G\frac{m_E}{r} = v^2$$

To eliminate r from the equation, use the equation for the definition of velocity and solve for r. Recall that the period, T, is known.

$$v = \frac{\Delta d}{\Delta t}$$

$$\Delta d = 2\pi r$$

$$\Delta t = T$$

$$v = \frac{2\pi r}{T}$$

$$r = \frac{vT}{2\pi}$$

Substitute the expression for r into the equation for v above and solve for v.

$$G\frac{m_E}{\frac{vT}{2\pi}} = v^2$$

$$v^2 = \frac{2\pi G m_E}{vT}$$

$$v^3 = \frac{2\pi G m_E}{T}$$

$$v = \sqrt[3]{\frac{2\pi G m_E}{T}}$$

Convert T to SI units.

$$T = (24\ \cancel{\text{h}})\left(\frac{60\ \cancel{\text{min}}}{\cancel{\text{h}}}\right)\left(\frac{60\ \text{s}}{\cancel{\text{min}}}\right) = 8.64 \times 10^4\ \text{s}$$

Substitute numerical values and solve.

$$v = \sqrt[3]{\frac{2\pi\left(6.67 \times 10^{-11}\ \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)(5.98 \times 10^{24}\ \text{kg})}{8.64 \times 10^4\ \text{s}}}$$

$$v = \sqrt[3]{2.9006 \times 10^{10}\ \frac{\text{m}^3}{\text{s}^3}}$$

$$v = 3.0724 \times 10^3\ \frac{\text{m}}{\text{s}}$$

$$v \cong 3.07 \times 10^3\ \frac{\text{m}}{\text{s}}$$

(a) The orbital velocity of the satellite is about 3.07×10^3 m/s, which is approximately 11 000 km/h.

To find the altitude of the satellite, substitute the value for velocity into the expression above that you developed to find r in terms of v.

$$r = \frac{vT}{2\pi}$$

$$r = \frac{\left(3.0724 \times 10^3 \ \frac{\text{m}}{\text{s}}\right)(8.64 \times 10^4 \text{ s})}{2\pi}$$

$$r = 4.2249 \times 10^7 \text{ m}$$

The calculated value for r is the distance from Earth's centre to the satellite. To find the altitude of the satellite, you must subtract Earth's radius from r.

$$r = r_E + h$$

$$h = r - r_E$$

$$h = 4.2249 \times 10^7 \text{ m} - 6.38 \times 10^6 \text{ m}$$

$$h = 3.5869 \times 10^7 \text{ m}$$

$$h \cong 3.59 \times 10^7 \text{ m}$$

(b) The altitude of all geostationary satellites must be 3.59×10^7 m, or 3.59×10^4 km, above Earth's surface.

Validate the Solution

A velocity of 11 000 km/h seems extremely fast to us, but the satellite is circling Earth once per day, so the velocity is reasonable.

Check the units for velocity. $\sqrt[3]{\dfrac{\frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\text{kg}}{\text{s}}} = \sqrt[3]{\dfrac{\text{N}\cdot\text{m}^2\cdot\cancel{\text{kg}}}{\text{s}\cdot\text{kg}^{\cancel{2}}}} = \sqrt[3]{\dfrac{\frac{\cancel{\text{kg}}\cdot\text{m}^3}{\text{s}^2}}{\text{s}\cdot\cancel{\text{kg}}}} = \sqrt[3]{\dfrac{\text{m}^3}{\text{s}^3}} = \dfrac{\text{m}}{\text{s}}$

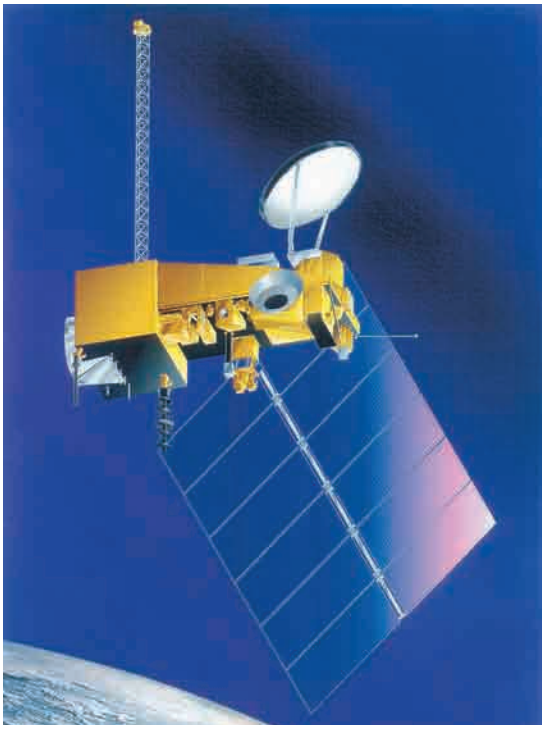
PRACTICE PROBLEMS

15. The polar-orbiting environmental satellites (POES) and some military satellites orbit at a much lower level in order to obtain more detailed information. POES complete an Earth orbit 14.1 times per day. What are the orbital speed and the altitude of POES?

16. The International Space Station orbits at an altitude of approximately 226 km. What is its orbital speed and period?

17. **(a)** The planet Neptune has an orbital radius around the Sun of about 4.50×10^{12} m. What are its period and its orbital speed?

(b) Neptune was discovered in 1846. How many orbits has it completed since its discovery?



NASA operates two polar-orbiting environmental satellites (POES) designed to collect global data on cloud cover; surface conditions such as ice, snow, and vegetation; atmospheric temperatures; and moisture, aerosol, and ozone distributions.

"Weightlessness" in Orbit

You have probably seen pictures of astronauts in a space capsule, space shuttle, or space station similar to Figure 3.7. The astronauts appear to float freely in the spacecraft and they describe the condition as being "weightless." Is a 65 kg astronaut in a space station weightless? Check it out, using the following calculation.

Figure 3.7 If an astronaut in a spacecraft dropped an apple, it would fall toward Earth, but it would not look as though it was falling.

- The space station orbits at an altitude of approximately 226 km. Find the radius of its orbit by adding the altitude to Earth's radius.

$$r = r_E + h$$

$$r = 6.38 \times 10^6 \text{ m} + 226 \cancel{\text{km}} \left(\frac{1000 \text{ m}}{\cancel{\text{km}}}\right)$$

$$r = 6.606 \times 10^6 \text{ m}$$

- Use Newton's law of universal gravitation to find the astronaut's weight.

$$F_g = G\frac{m_1 m_2}{r^2}$$

$$F_g = \left(6.67 \times 10^{-11} \frac{\text{N} \cdot \text{m}^2}{\text{kg}^2}\right)\frac{(5.98 \times 10^{24} \text{ kg})(65 \text{ kg})}{(6.606 \times 10^6 \text{ m})^2}$$

$$F_g = 594 \text{ N}$$

The astronaut's weight on Earth would be $F_g = mg = (65 \text{ kg})(9.81 \frac{\text{kg} \cdot \text{m}}{\text{s}^2}) \cong 638 \text{ N}$. There is very little difference between the astronaut's weight on Earth and in the space station. Why do astronauts appear "weightless" in space? Think back to your calculations of apparent weight in Chapter 1, Fundamentals of Dynamics. When the imaginary elevator that you were riding in was falling with an acceleration of 9.81 m/s^2, the scale you were standing on read zero newtons. Your apparent weight was zero because you, the scale, and the elevator were falling with the same acceleration. You and the scale were not exerting any force on each other.

The same situation exists in the space station and all orbiting spacecraft all of the time. As in the case of Newton's cannon ball, everything falls to Earth with the same acceleration, but Earth is "falling away" equally as fast. You could say that a satellite is "falling around Earth." Some physicists object to the term "weightlessness" because, as you saw, there is no such condition. NASA coined the term **microgravity** to describe the condition of apparent weightlessness.

WEB LINK

www.mcgrawhill.ca/links/physics12

To learn about the effects of weightlessness on the human body, go to the above Internet site and click on **Web Links**.

PHYSICS & TECHNOLOGY

TARGET SKILLS

- Analyzing and interpreting
- Communicating results

Simulating Microgravity

In preparation for space flights, astronauts benefit by practising movements in microgravity conditions. As you recall from your study of projectile motion, an object will follow a parabolic trajectory if the only force acting on it is gravity. Large jet aircraft can fly on a perfect parabolic path by exerting a force to overcome air friction. Thus, inside the aircraft, all objects move as though they were following a path determined only by gravity. Objects inside the aircraft, including people, exert no forces on each other, because they are all "falling" with the same acceleration. Astronauts can experience 20 s of microgravity on each parabolic trajectory.

Scientists have also found that certain chemical and physical reactions occur in a different way in microgravity. They believe that some manufacturing processes can be carried out more efficiently under these conditions — a concept that might lead to manufacturing in space. To test some of these reactions without going into orbit, researchers sometimes use drop towers, as shown in the photograph. A drop tower has a very long shaft that can be evacuated to eliminate air friction. A sample object or, sometimes, an entire experiment is dropped and the reaction can proceed for up to 10 s in microgravity conditions. These experiments provide critical information for future processing in space.

Analyze

1. Explain in detail why an airplane must use energy to follow an accurate parabolic trajectory, but everything inside the airplane appears to be weightless.
2. Do research to learn about some chemical or physical processes that might be improved by carrying them out in microgravity.

WEB LINK

www.mcgrawhill.ca/links/physics12

To learn more about experiments carried out in drop towers, go to the above Internet site and click on **Web Links**.

Perturbing Orbits

In all of the examples that you have studied to this point, you have considered only perfectly circular or perfectly elliptical orbits. Such perfect orbits would occur only if the central body and satellite were totally isolated from all other objects. Since this is essentially never achieved, all orbits, such as those of planets around the Sun and moons and satellites around planets, are slightly distorted ellipses. For example, when an artificial satellite is between the Moon and Earth, the Moon's gravity pulls in an opposite direction to that of Earth's gravity. When a satellite is on the side of Earth opposite to the Moon, the Moon and Earth exert their forces in the same direction. The overall effect is a very slight change in the satellite's orbit. Engineers must take these effects into account.

In the solar system, each planet exerts a gravitational force on every other planet, so each planet perturbs the orbit of the other planets. In some cases, the effects are so small that they cannot be measured. Astronomers can, however, observe these **perturbations** in the paths of the planets when the conditions are right. In fact, in 1845, two astronomers in two different countries individually observed perturbations in the orbit of the planet Uranus. British astronomer and mathematician John Couch Adams (1819–1892) and French astronomer Urbain John Joseph Le Verrier (1811–1877) could not account for their observed perturbations of the planet's orbit, even by calculating the effects of the gravitational force of the other planets. Both astronomers performed detailed calculations and predicted both the existence and the position of a new, as yet undiscovered planet. In September of 1846, at the Berlin Observatory, astronomer J.G. Galle (1812–1910) searched the skies at the location predicted by the two mathematical astronomers. Having excellent star charts for comparison, Galle almost immediately observed the new planet, which is now called "Neptune."

About 50 years later, U.S. astronomer Percival Lowell (1855–1916) performed calculations on the orbits of both Neptune and Uranus, and discovered that these orbits were again perturbed, probably by yet another undiscovered planet. About 14 years after Lowell's death, astronomer Clyde Tombaugh (1906–1997), working in the Lowell Observatory, discovered the planet now called "Pluto."

Since the discovery of two planets that were predicted mathematically by the perturbations of orbits of known planets, several more predictions about undiscovered planets have been made. None have been discovered and most astronomers believe that no more planets exist in our solar system. The laws of Newton and Kepler, however, have provided scientists and astronomers with a solid foundation on which to explain observations and make predictions about planetary motion, as well as send space probes out to observe all of the planets in our solar system.

3.2 Section Review

1. C Explain Newton's thought experiment about "launching a cannon-ball satellite."
2. I Why must a geostationary satellite orbit over the equator? To answer that question, think about the point that is the centre of the orbit. If you launched a satellite that had a period of 24 hr, but it did not start out over the equator, what path would it follow? If you were at the spot on Earth just below the point where the satellite started to orbit, how would the path of the satellite appear to you?
3. K/U What conditions create apparent weightlessness when an astronaut is in an orbiting spacecraft?
4. K/U How could you discover a planet without seeing it with a telescope?

CHAPTER 3 Review

REFLECTING ON CHAPTER 3

- Tycho Brahe collected detailed astronomical data for 20 years.
- Johannes Kepler analyzed Brahe's data and developed three empirical equations that are now called "Kepler's laws."
  1. Planets follow elliptical paths.
  2. The areas swept out by a line from the Sun to a planet during a given time interval are always the same.
  3. $\frac{T^2}{r^3} = k$ or $\frac{T_A^2}{r_A^3} = \frac{T_B^2}{r_B^3}$
- Kepler's laws support Newton's law of universal gravitation.
- Newton extended the concept of gravity and showed that not only does it cause celestial bodies to attract each other, but also that all masses exert a mutually attractive force on each other.
- Newton's law of universal gravitation is written mathematically as $F_g = G\frac{m_1 m_2}{r^2}$.
- Newton's law of universal gravitation shows that the constant in Kepler's third law is given by $\frac{T^2}{r^3} = \frac{4\pi^2}{Gm_s}$.
- You can use the combination of Newton's law of universal gravitation and Kepler's third law to determine the mass of any celestial body that has one or more satellites.
- Cavendish used a torsion balance to determine the universal gravitational constant, G, in Newton's law of universal gravitation: $G = 6.67 \times 10^{-11}\ \mathrm{N \cdot m^2/kg^2}$.
- Newton reasoned that if you shot a cannon ball from the top of a very high mountain and it went fast enough horizontally, it would fall exactly the same distance that Earth's surface would drop, due to the spherical shape. This is the correct theory about satellite motion.
- A satellite in a geostationary orbit appears to hover over one spot on Earth's surface, because it completes one cycle as Earth makes one revolution on its axis.
- Apparent weightlessness in orbiting spacecraft is due to the fact that the spacecraft and everything in it are falling toward Earth with exactly the same acceleration.
- The planets Neptune and Pluto were both discovered because their gravity perturbed the path of other planets.

Knowledge/Understanding

1. Distinguish between mass and weight.
2. Define **(a)** heliocentric and **(b)** geostationary.
3. State Kepler's laws.
4. Several scientists and astronomers had developed the concept that the attractive force on planets orbiting the Sun decreased with the square of the distance between a planet and the Sun. What was Newton's reasoning for including in his law of universal gravitation the magnitude of the masses of the planets? In what other ways did Newton's law of universal gravitation differ conceptually from the ideas of other scientists of his time?
5. Explain how Kepler's third law supports Newton's law of universal gravitation.
6. Is Kepler's constant a universal constant? That is, can it be applied to Jupiter's system of satellites or to other planetary systems? Explain.
7. How does a torsion balance work?
8. Explain whether it is possible to place a satellite into geosynchronous orbit above Earth's North Pole.

Inquiry

9. No reliable evidence supports the astrological claim that the motions of the stars and planets affect human activities. However, belief in astrology remains strong. Create an astrology defence or opposition kit. Include any or all of the following: arguments based on Newton's laws to refute or support astrological claims,

an experiment that tests the validity of birth horoscopes, a report on scientific studies of astrological claims and any findings, a summary of the success and failure of astrological predictions and a comparison of these to the success of predictions based on chance.

10. Devise an observational test (which will require a telescope) that will convince a doubting friend that Earth orbits the Sun.

11. Demonstrate the inverse square law form of the universal law of gravitation by calculating the force on a 100.0 kg astronaut who is placed at a range of distances from Earth's surface, out to several Earth radii. Make a graph of force versus position and comment on the results.

12. You often hear that the Moon's gravity, as opposed to the Sun's gravity, is responsible for the tides.

(a) Calculate the force of gravity that the Moon and the Sun exert on Earth. How does this appear to conflict with the concept stated above?

(b) Calculate the force of the Moon's gravity on 1.00×10^4 kg of water at the surface of an ocean on the same side of Earth as the Moon and on the opposite side of Earth from the Moon. Also, calculate the Moon's gravity on 1.00×10^4 kg of matter at Earth's centre. (Assume that the distance between the Moon's centre and Earth's centre is 3.84×10^8 m and that Earth's radius is 6.38×10^6 m.)

(c) Perform the same calculations for the Sun's gravity on these masses. (Use 1.49×10^{11} m for the distance between the centres of Earth and the Sun.)

(d) Examine your results from parts (b) and (c) and use them to justify the claim that the Moon's gravity is responsible for tides.

13. Many comets have been identified and the regularity of their return to the centre of the solar system is very predictable.

(a) From what you have learned about satellite motion, provide a logical explanation for the disappearance and reappearance of comets.

(b) Make a rough sketch of the solar system and add to it a probable comet path.

(c) What is the nature of the path taken by comets?

Communication

14. Consider a marble of mass m accelerating in free fall in Earth's gravity. Neglect air resistance and show that the marble's acceleration due to gravity is independent of its mass. (That is, you could use a bowling ball, a feather, or any other object in free fall and obtain the same result.) Hint: Equate Newton's universal law of gravitation to Newton's second law. Look up the values for Earth's mass and radius and use them in your acceleration equation to calculate the marble's acceleration. This number should be familiar to you!

15. Suppose that the Sun's mass was four times greater than it is now and that the radius of Earth's orbit was unchanged. Explain whether a year would be longer or shorter. By what factor would the period change? Explain in detail how you determined your answer.

16. **(a)** At what distance from Earth would an astronaut have to travel to actually experience a zero gravitational force, or "zero g"?

(b) Are astronauts in a space shuttle orbiting Earth subject to a gravitational force?

(c) How can they appear to be "weightless"?

17. A cow attempted to jump over the Moon but ended up in orbit around the Moon, instead. Describe how the cow could be used to determine the mass of the Moon.

18. Discuss what would happen to Earth's motion if the Sun's gravity was magically turned off.

19. The Sun gravitationally attracts Earth. Explain why Earth does not fall into the Sun.

Making Connections

20. Examine some Olympic records, such as those for the long-jump, shot-put, weightlifting, high-jump, javelin throw, 100 m dash, 400 m hurdles, and the marathon. How would you expect these records to change if the events

were performed under an athletic dome on the Moon?

21. Einstein once recalled his inspiration for the theory of general relativity from a sudden thought that occurred to him: "If a person falls freely, he will not feel his own weight." He said he was startled by the simple thought and that it impelled him toward a theory of gravitation. Although the mathematics of the general theory of relativity is advanced, its concepts are fascinating and have been described in several popular books. Research and write a short essay on the general theory of relativity, including a discussion of its predictions and tests, and how it supersedes (but does not replace) Newton's theory of gravitation.

Problems for Understanding

22. The gravitational force between two objects is 80.0 N. What would the force become if the mass of one object was halved and the distance between the two objects was doubled?

23. Two stars of masses $m*$ and $3m*$ are 7.5×10^{11} m apart. If the force on the large star is F, which of the following is the force on the small star?

(a) $F/9$ **(b)** $F/3$ **(c)** F **(d)** $3F$ **(e)** $9F$

24. For the above situation, if the acceleration of the small star is a, what is the acceleration of the large star?

(a) $a/9$ **(b)** $a/3$ **(c)** a **(d)** $3a$ **(e)** $9a$

25. **(a)** Use Newton's law of universal gravitation and the centripetal force of the Sun to determine Earth's orbital speed. Assume that Earth orbits in a circle.

(b) What is Earth's centripetal acceleration around the Sun?

26. Calculate the Sun's acceleration caused by the force of Earth.

27. A space shuttle is orbiting Earth at an altitude of 295 km. Calculate its acceleration and compare it to the acceleration at Earth's surface.

28. Orbital motions are routinely used by astronomers to calculate masses. A ring of high-velocity gas, orbiting at approximately 3.4×10^4 m/s at a distance of 25 light-years from the centre of the Milky Way, is considered to be evidence for a black hole at the centre. Calculate the mass of this putative black hole. How many times greater than the Sun's mass is it?

29. In a Cavendish experiment, two 1.0 kg spheres are placed 50.0 cm apart. Using the known value of G, calculate the gravitational force between these spheres. Compare this force to the weight of a flea.

30. The Hubble space telescope orbits Earth with an orbital speed of 7.6×10^3 m/s.

(a) Calculate its altitude above Earth's surface.

(b) What is its period?

31. The Moon orbits Earth at a distance of 3.84×10^8 m. What are its orbital velocity and period?

32. The following table gives orbital information for five of Saturn's largest satellites.

Satellite	Mean orbital radius (m)	Period (days)
Tethys	2.95×10^8	1.888
Dione	3.78×10^8	2.737
Rhea	5.26×10^8	4.517
Titan	1.221×10^9	15.945
Iapetus	3.561×10^9	79.331

(a) Determine whether these satellites obey Kepler's third law.

(b) If they obey Kepler's third law, use the data for the satellites to calculate an average value for the mass of Saturn.

33. Suppose the Oort cloud of comets contains 10^{12} comets, which have an average diameter of 10 km each.

(a) Assume that a comet is composed mostly of water-ice with a density of 1.00 g/cm^3 and calculate the mass of a comet.

(b) Calculate the total mass of the Oort cloud.

(c) Compare your mass of the Oort cloud to the mass of Earth and of Jupiter.

UNIT 1 PROJECT

Catapult Machine

Background

Humans have built machines for launching projectiles since ancient times. The Romans constructed *ballista* to hurl stones and *catapulta* to shoot arrows. In one design, stretched or twisted ropes were suddenly released to launch the projectile. Other machines bent and then released wooden beams. The medieval *trebuchet* harnessed the energy of a falling counterweight. More recently, catapults powered by compressed air provided the first effective method of launching aircraft from ships.

A medieval catapult

Challenge

In a small group, design, construct, test, and evaluate a catapult that launches a standard projectile to meet specified flight criteria. Your class as a whole will decide on the criteria for the flight and any restrictions on building materials and cost. Each catapult will be evaluated by comparing its performance to the expected results for an ideal projectile. As part of the project, you will prepare a report that outlines the design features of your catapult, provides an analysis of its operation, and makes recommendations for its improvement.

Materials

- construction materials, such as wood, plastic, cardboard, metal
- elastic materials, such as elastic bands, springs, or a mousetrap
- materials to attach parts together, such as fasteners, tape, and glue
- materials for the projectile
  - foam plastic egg carton
  - plastic sandwich bags
  - sand

Safety Precautions

- Wear eye protection when using power tools.
- Ensure that electrical equipment, such as power tools, is properly grounded.
- Take appropriate precautions when using electrical equipment.
- Take appropriate precautions when using knives, saws, and other sharp tools.
- Handle glue guns with care to avoid burns. Hot glue guns take several minutes to cool after they are disconnected.
- Wear eye protection at all times when testing your catapult.
- Test your catapult in a large, clear space, away from other people and from equipment and windows that could be damaged.
- Ensure that all spectators are behind the catapult before firing it.

Project Criteria

A. As a class, decide on the criteria for evaluating your catapults. Possible challenges are to construct catapults that launch projectiles to hit a specific target, or to achieve maximum range or a specified flight time, or to reach a particular height or go over a wall.

B. As a class, decide whether your catapults are to be made entirely from recycled materials or to be constructed from other materials within a set cost limit.

C. Research, design, and construct your catapult to launch egg-carton projectiles. These projectiles are to be made from a single section of a foam plastic egg carton, filled with 25 grams of sand in a plastic bag. Tape the bag of sand securely inside the egg-carton section. The total mass of the projectile is not to exceed 30 grams.

D. Prepare a written report about your project that includes

- an appropriate title and the identification of group members
- a labelled design drawing of the catapult
- an overview of the physics involved that includes calculations of
  - the average force applied to the projectile
  - the distance through which the force is applied
  - the time for which the force is applied
- a theoretical prediction for the performance of your projectile that includes
  - its range
  - its maximum height
  - its flight time
- an analysis of the catapult's performance that includes calculations or measurements of the
  - average launch velocity
  - launch angle
  - flight time
  - range
  - maximum height achieved
- an evaluation of the catapult's performance and recommendations for its refinement

ASSESSMENT

After you complete this project

- **assess the performance of your catapult. How closely did the projectile meet your challenge criteria?**
- **assess the design of your catapult. What physics and engineering principles did you incorporate into its design?**
- **assess the problem-solving effectiveness of your group. What design and construction obstacles did you face during this project? How did you overcome them?**

Action Plan

1. Work in groups of two to four people.
2. Establish a time line for the design, construction, testing, and evaluation phases of this project.
3. Research possible designs and energy sources for your catapult. Select one that can be adapted to be feasible and meet the design criteria.
4. Construct and test your catapult, measuring the quantities specified above.
5. Prepare the written report and enter the competition.

Evaluate

1. Compare the average performance of your catapult with your theoretical predictions and the challenge criteria set out by the class.
2. Recommend refinements to your catapult. Indicate specifically how performance was affected by each design feature that you have identified for improvement.

WEB LINK

www.mcgrawhill.ca/links/physics12

For a lot of pictures and some stories about "leverage artillery" used in times past and for plans for various types of catapults, go to the above Internet site and click on **Web Links**.

UNIT 1 Review

Knowledge/Understanding

Multiple Choice

In your notebook, choose the most correct answer for each of the following questions. Outline your reasons for your choice.

1. A ball is thrown upward. After it is released, its acceleration
   (a) is zero
   (b) increases
   (c) decreases
   (d) remains constant
   (e) increases, then decreases
2. You drop a 1.0 kg stone off the roof of a 10-storey building. Just as the stone passes the fifth floor, your friend drops a 1.0 kg ball out of a fifth-floor window. If air resistance is neglected, which of the following statements is true? Explain your reasoning.
   (a) The stone and the ball hit the ground at the same time and at the same speed.
   (b) The stone hits the ground first and with a greater speed than the ball does.
   (c) The stone and the ball hit the ground at the same time, but the speed of the stone is greater.
   (d) The ball hits the ground before the stone.
3. A football is thrown by a quarterback to a receiver deep in the end zone. The acceleration of the football during the flight
   (a) depends on how the ball was thrown
   (b) depends on whether the ball is going up or coming down
   (c) is the same during the entire flight
   (d) is greatest at the top of its trajectory
   (e) is greatest at the beginning and end of its trajectory
4. A ball of mass m_1 is dropped from the roof of a 10-storey building. At the same instant, another ball of mass m_2 is dropped out of a ninth-storey window, 10 m below the roof. The distance between the balls during the flight
   (a) remains at 10 m throughout
   (b) decreases
   (c) increases
   (d) depends on the ratio m_1/m_2
5. On a position-time graph, a straight horizontal line corresponds to motion at
   (a) zero speed
   (b) constant speed
   (c) increasing speed
   (d) decreasing speed

Short Answer

6. **(a)** Define what is meant by a *net* or *unbalanced force* acting on an object.
   (b) Explain, with the aid of a free-body diagram, how an object can be experiencing no net force when it has at least three forces acting on it.
   (c) Describe, with the aid of a free-body diagram, an object that is experiencing a net force. Identify in which direction the object will move and with what type of motion. Relate the direction and type of motion to the direction of the net force.
7. A child is riding a merry-go-round that is travelling at a constant speed.
   (a) Is he viewing the world from an inertial or non-inertial frame of reference? Explain your reasoning.
   (b) What type of force does his horse exert on him to keep him travelling in a circle? In which direction does this force act?
   (c) In what direction does the child feel that a force is pushing him? Explain why this perceived force is called a "fictitious force."
8. A football is kicked into the air. Where in its trajectory is the velocity at a minimum? Where is it at a maximum?
9. A bright orange ball is dropped from a hot-air balloon that is travelling with a constant velocity.
   (a) Draw a sketch of the path the ball will travel from the perspective of a person standing on the ground from the instant in time at which the ball was dropped until the instant it lands.
   (b) From the ground, what type of motion is observed in the horizontal dimension? Identify the mathematical equations that can be used to model this motion.

(c) From the ground, what type of motion is observed in the vertical dimension? Identify the mathematical equations that can be used to model this motion.

(d) Identify the variable that is common to the equations that describe the horizontal motion and those that describe the vertical motion.

(e) Describe, with the aid of sketches, how motion on a plane can be modelled by considering its component motion along two directions that are perpendicular to each other.

Inquiry

10. A rope and pulley are often used to assist in lifting heavy loads. Demonstrate with the use of free-body diagrams and equations that, using the same force, a heavier load can be lifted with a rope and pulley system than with a rope alone.

11. A wooden T-bar attached to a cable is used at many ski hills to tow skiers and snowboarders up the hill in pairs. Design a T-bar lift for a ski hill. Estimate how much tension the cable for an individual T-bar should be able to withstand, assuming that it transports two adults, the slope is 10.0°, and the T-bar cable pulls the people at an angle of 25.0° to the slope. Determine how the tension is affected when the steepness of the slope, the angle of the T-bar cable to the slope, or the coefficient of friction of the snow changes.

12. Examine three different ways of suspending signs (for example, for stores) in front of buildings or above sidewalks by using cables or rods (that is, the sign is not attached directly to the building). Determine which method can support the heaviest sign.

13. Review the meaning of the kinematics equations for constant acceleration by deriving them for yourself. Begin with the following situation. In a time interval, Δt, a car accelerates uniformly from an initial velocity, v_i, to a final velocity, v_f. Sketch the situation in a velocity-versus-time graph. By determining the slope of the graph and the area under the graph (Hint: What quantities do these represent?), see how many of the kinematics equations you can derive.

Communication

14. According to Newton's third law, for every action force, there is an equal and opposite reaction force. How, then, can a team win a tug-of-war contest?

15. Consider a block of wood on an incline. Determine the acceleration of the block and the normal force of the incline on the block for the two extreme cases where $\theta = 0°$ and $\theta = 90°$, and for the general case of $0° < \theta < 90°$. Discuss the results, particularly why an inclined plane could be described as a way of "diluting" gravity. (**Note:** Galileo recognized this.)

16. You probably have a working understanding of mass and velocity, but what about force and acceleration? At what rate can a person accelerate on a bicycle? What average force does a tennis racquet exert on a tennis ball?

(a) Construct examples of everyday situations involving accelerations of approximately $0.5\ m/s^2$, $2.0\ m/s^2$, $5.0\ m/s^2$, and $20\ m/s^2$.

(b) Construct examples of everyday situations involving forces of 1 N, 10 N, 50 N, 100 N, 1000 N, and 1.0×10^4 N.

17. A ball rolls down an inclined plane, across a horizontal surface, and then up another inclined plane. Assume there is no friction.

(a) What forces act on the ball at the beginning, middle, and end of its roll?

(b) If the angles of the inclined planes are equal and the ball begins its roll from a vertical height of 10 cm, how high will the ball roll up the second inclined plane?

(c) If the first inclined plane is twice as steep as the second and the ball begins its roll from a vertical height of 10 cm, to what height will the ball roll up the second inclined plane?

(d) If the ball begins its roll from a vertical height of 10 cm on the first inclined plane and the second inclined plane is removed, how far will the ball roll across the horizontal surface?

(e) Explain how the above four situations are explained by using the law of inertia.

18. A car turns left off the highway onto a curved exit ramp.
    (a) What type of motion does the passengers' frame of reference experience relative to the ground?
    (b) Explain why the passengers feel a force to the right as the car turns.
    (c) How would an observer on an overpass describe the motion of the passengers and the car at the beginning of the curve?
    (d) Suppose that in the middle of the turn, the car hits a patch of ice. Sketch the path of the car as it slides.
    (e) Determine the magnitude and direction of the force that the road exerts in dry and icy road conditions and discuss the results for the two situations.

19. Suppose you could place a satellite above Earth's atmosphere with a gigantic crane. In which direction would the satellite travel when the crane released it? Explain your answer.

20. Explain why the kinematics equations, which describe the motion of an object that has constant acceleration, cannot be applied to uniform circular motion.

Making Connections

21. Choose an Olympic sport and estimate the magnitude of realistic accelerations and forces involved in the motion. For example, approximately how fast do Olympic athletes accelerate during the first 10 m of the 100 m dash? What average force is applied during this time? What average force do shot-putters exert on the shot-put as they propel it? How does this compare to the force exerted by discus throwers?

22. In automobiles, antilock braking systems were developed to slow down a car without letting the wheels skid and thus reduce the stopping distance, as compared to a braking system in which the wheels lock and skid. Explain the physics behind this technology, using the concepts of static friction and kinetic friction. Develop and solve a problem that demonstrates this situation. In which case, stopping without skidding or stopping with skidding, do you use the coefficient of kinetic friction and in which case do you use the coefficient of static friction?

23. "Natural motion" is difficult to explore experimentally on Earth because of the inherent presence of friction. Research the history of friction experiments. Examine the relationship between static and kinetic friction. Explain why a fundamental theory of friction eludes physicists.

24. Aristotle's theory of dynamics differed from Newton's and Galileo's theories partly because Aristotle tried to develop common sense explanations for real-life situations, whereas Newton and Galileo imagined ideal situations and tested them by experiment. Outline in an essay the differences in these two approaches and their results.

25. Railroads are typically built on level land, but in mountainous regions, inclines are unavoidable. In 1909, to improve the Canadian Pacific Railway through the Rocky Mountains, near Field, British Columbia, engineers significantly reduced the grade of the old track by building a spiral tunnel through a mountain. Research this engineering feat and answer the following questions.
    (a) What is the grade of the incline?
    (b) How much force must the train exert going up through the tunnel, as compared to when it goes down or travels on level track, or as compared to what it required for travel on the old track?
    (c) What is the elevation of the train before entering and after leaving the tunnel?
    (d) What is the typical acceleration of the train in the tunnel? Make some rough calculations, if necessary, to support your answers.

26. Tycho Brahe built two observatories and had his assistants observe the same things independently. He also repeated observations in order to understand his errors. He is recognized as the greatest astronomical observer prior to development of the telescope. Research the

contribution he made to observational astronomy and the role his methods played in developing the scientific method.

27. In the solar system, objects at greater distances from the Sun have slower orbital velocities because of the decrease in the gravitational force from the Sun. This pattern is expected to be observed in the Milky Way galaxy also. However, some objects that are more distant from the centre of the galaxy than the Sun (such as star clusters) have higher orbital velocities than the Sun. This is considered to be evidence for dark matter in the galaxy. Review some recent articles in astronomy magazines and research the nature of this problem. Why are the above observations considered to be evidence for dark matter? How strong is this evidence? What are some of the candidates?

28. Volcanoes on Mars, such as Olympus Mons, are much taller than those on Earth. Compare the sizes of volcanoes on different bodies in the solar system and discuss the role that gravity plays in determining the size of volcanoes.

Problems for Understanding

29. A 1.2×10^3 kg car is pulled along level ground by a tow rope. The tow rope will break if the tension exceeds 1.7×10^3 N. What is the largest acceleration the rope can give to the car? Assume that there is no friction.

30. Two objects, m_1 and m_2, are accelerated independently by forces of equal magnitude. Object m_1 accelerates at 10.0 m/s^2 and m_2 at 20.0 m/s^2. What is the ratio of **(a)** their inertial masses? **(b)** their gravitational masses?

31. A 720 kg rocket is to be launched vertically from the surface of Earth. What force is needed to give the rocket an initial upward acceleration of 12 m/s^2? Explain what happens to the acceleration of the rocket during the first few minutes after lift-off if the force propelling it remains constant.

32. A 42.0 kg girl jumps on a trampoline. After stretching to its bottom limit, the trampoline exerts an average upward force on the girl over a displacement of 0.50 m. During the time that the trampoline is pushing her up, she experiences an average acceleration of 65.0 m/s^2. Her velocity at the moment that she leaves the trampoline is 9.4 m/s[up].

(a) What is the average force that the trampoline exerts on the girl?

(b) How high does she bounce?

33. An 8.0 g bullet moving at 350 m/s penetrates a wood beam to a distance of 4.5 cm before coming to rest. Determine the magnitude of the average force that the bullet exerts on the beam.

34. Soon after blast-off, the acceleration of the *Saturn V* rocket is 80.0 m/s^2[up].

(a) What is the apparent weight of a 78.0 kg astronaut during this time?

(b) What is the ratio of the astronaut's apparent weight to true weight?

35. A 1500 kg car stands at rest on a hill that has an incline of 15°. If the brakes are suddenly released, describe the dynamics of the car's motion by calculating the following: **(a)** the car's weight, **(b)** the component of the weight parallel to the incline, **(c)** the car's acceleration, **(d)** the velocity acquired after travelling 100.0 m (in m/s and km/h), and **(e)** the time for the car to travel 100.0 m.

36. A 2.5 kg brick is placed on an adjustable inclined plane. If the coefficient of static friction between the brick and the plane is 0.30, calculate the maximum angle to which the plane can be raised before the brick begins to slip.

37. Superman tries to stop a speeding truck before it crashes through a store window. He stands in front of it and extends his arm to stop it. If the force he exerts is limited only by the frictional force between his feet and the ground, and $\mu_s = \mu_k = 1.0$, **(a)** what is the maximum force he can exert? (Let Superman's mass be 1.00×10^2 kg, the truck's mass 4.0×10^4 kg, and the truck's velocity 25 m/s.) **(b)** What is the minimum distance over which he can stop the truck?

38. Two bricks, with masses 1.75 kg and 3.5 kg, are suspended from a string on either side of a pulley. Calculate the acceleration of the masses and the tension in the string when the masses are released. Assume that the pulley is massless and frictionless.

39. A helicopter is flying horizontally at 8.0 m/s when it drops a package.
    (a) How much time elapses before the velocity of the package doubles?
    (b) How much additional time is required for the velocity of the package to double again?
    (c) At what altitude is the helicopter flying if the package strikes the ground just as its velocity doubles the second time?

40. A soccer player redirects a pass, hitting the ball toward the goal 21.0 m in front of him. The ball takes off with an initial velocity of 22.0 m/s at an angle of 17.0° above the ground.
    (a) With what velocity does the goalie catch the ball in front of the goal line?
    (b) At what height does the goalie catch the ball?
    (c) Is the ball on its way up or down when it is caught?

41. A wheelchair basketball player made a basket by shooting the ball at an angle of 62°, with an initial velocity of 6.87 m/s. The ball was 1.25 m above the floor when the player released it and the basket was 3.05 m above the floor. How far from the basket was the player when making the shot?

42. A 10.0 g arrow is fired horizontally at a target 25 m away. If it is fired from a height of 2.0 m with an initial velocity of 40.0 m/s, at what height should the target be placed above the ground for the arrow to hit it?

43. A Ferris wheel of radius 10.0 m rotates in a vertical circle of 7.0 rev/min. A 45.0 kg girl rides in a car alone. What (vertical) normal force would she experience when she is:
    (a) halfway towards the top, on her way up?
    (b) at the top?
    (c) halfway towards the bottom?
    (d) at the bottom?
    Compare this to her weight in each case.

44. How much force is needed to push a 75.0 kg trunk at constant velocity across a floor, if the coefficient of friction between the floor and the crate is 0.27?

45. A car can accelerate from rest to 100 km/h (1.00×10^2 km/h) in 6.0 s. If its mass is 1.5×10^3 kg, what is the magnitude and direction of the applied force?

46. A 62.4 kg woman stands on a scale in an elevator. What is the scale reading (in newtons) for the following situations.
    (a) The elevator is at rest.
    (b) The elevator has a downward acceleration of 2.80 m/s^2.
    (c) The elevator has an upward acceleration of 2.80 m/s^2.
    (d) The elevator is moving upward with a constant velocity of 2.80 m/s.

47. Suppose you attach a rope to a 5.0 kg brick and lift it straight up. If the rope is capable of holding a 20.0 kg mass at rest, what is the maximum upward acceleration you can give to the brick?

48. A 52.0 kg parachutist is gliding to Earth with a constant velocity of 6.0 m/s[down]. The parachute has a mass of 5.0 kg.
    (a) How much does the parachutist weigh?
    (b) How much upward force does the air exert on the parachutist and parachute?

49. **(a)** If you want to give an 8.0 g bullet an acceleration of 2.1×10^4 m/s^2, what average net force must be exerted on the bullet as it is propelled through the barrel of the gun?
(b) With this acceleration, how fast will the bullet be travelling after it has moved 2.00 cm from rest?

50. A snowboarder, whose mass including the board is 51 kg, stands on a steep 55° slope and wants to go straight down without turning. What will be his acceleration if **(a)** there is no friction and **(b)** the coefficient of kinetic friction is 0.20? **(c)** In each case, starting from rest, what will be his velocity after 7.5 s?

51. Replace the cart in a Fletcher's trolley apparatus (see page 8) with a block of wood of mass 4.0 kg and use a suspended mass of 2.0 kg. Calculate the acceleration of the system and the tension in the string when the mass is released, if the coefficient of friction between the block of wood and the table is
(a) 0.60
(b) 0.20
(c) What is the maximum value of the coefficient of friction that will allow the system to move?

52. Calculate the acceleration of two different satellites that orbit Earth. One is located at 2.0 Earth radii and the other at 4.0 Earth radii.

53. **(a)** Calculate your velocity on the surface of Earth (at the equator) due to Earth's rotation.
(b) What velocity would you require to orbit Earth at this distance? (Neglect air resistance and obstructions.)

54. Two galaxies are orbiting each other at a separation of 1×10^{11} AU and the orbital period is estimated to be 30 billion years. Use Kepler's third law to find the total mass of the pair of galaxies. Calculate how many times larger the mass of the pair of galaxies is than the Sun's mass, which is 1.99×10^{30} kg.

COURSE CHALLENGE

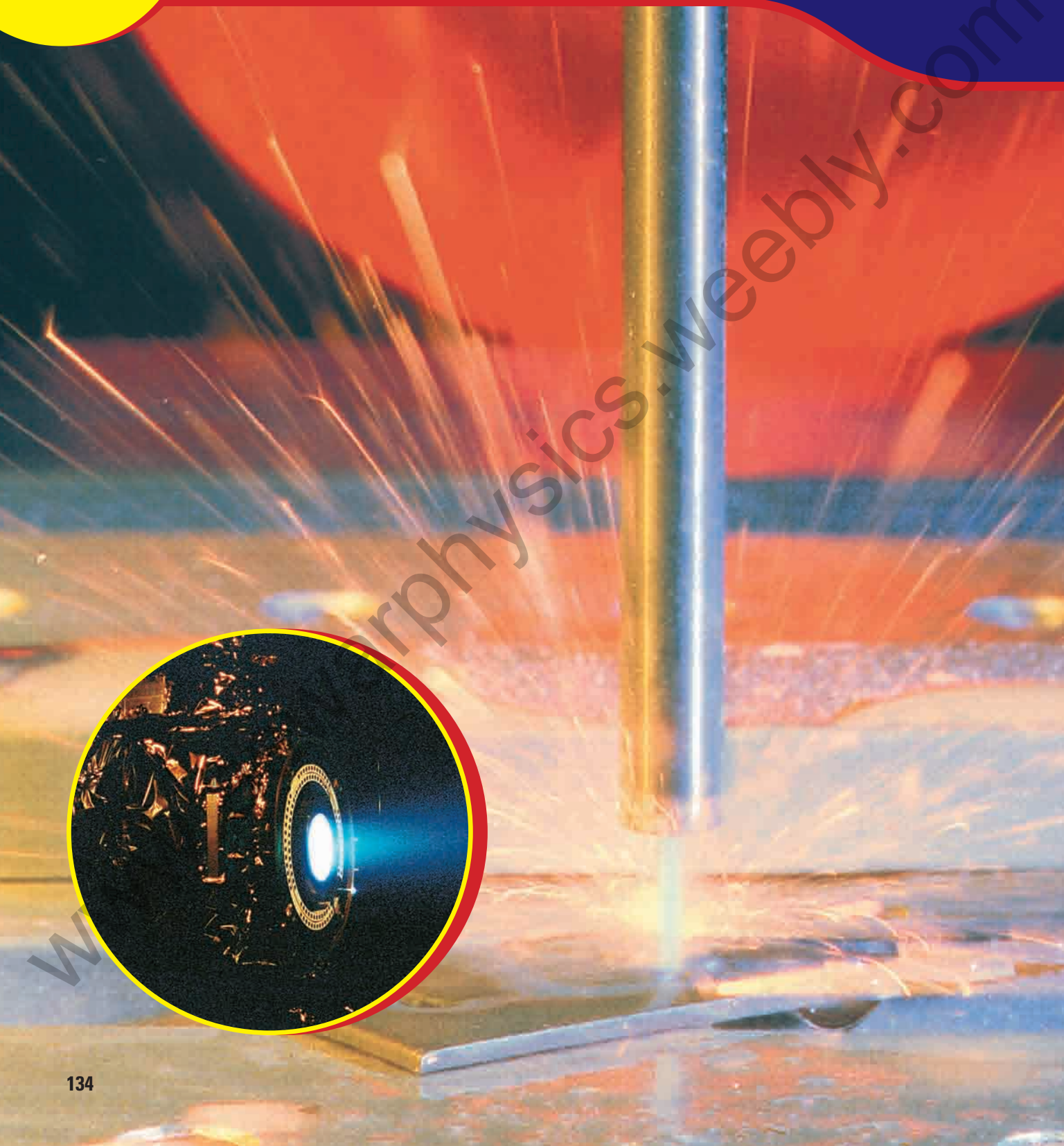
Scanning Technologies: Today and Tomorrow

Consider the following as you begin gathering information for your end-of-course project.

- Analyze the contents of this unit and begin recording concepts, diagrams, and equations that might be useful.
- Collect information in a variety of ways, including concept organizers, useful Internet sites, experimental data, and perhaps unanswered questions to help you create your final presentation.
- Scan magazines, newspapers, and the Internet for interesting information to enhance your project.

UNIT 2

Energy and Momentum



OVERALL EXPECTATIONS

DEMONSTRATE an understanding of work, energy, impulse, momentum, and conservation of energy and of momentum.

INVESTIGATE and analyze two-dimensional situations involving conservation of energy and of momentum.

DESCRIBE and analyze how common impact-absorbing devices apply concepts of energy and momentum.

UNIT CONTENTS

The motion of water, subjected to 4000 kPa of pressure, has sufficient energy to cut through steel. The motion of electrically charged particles, propelled at speeds in excess of 3.0×10^4 m/s, could provide the energy needed to power space probes in the near future. In the small photograph of an experimental ion engine, the blue glow is composed of electrically charged ions of xenon gas, travelling at speeds in excess of 3.0×10^4 m/s. An ion engine emits even smaller high-speed particles than a water-jet cutting tool.

In this unit, you will build on previous studies of energy to include another concept: momentum. Momentum considers the amount of motion in an object and the effect of moving objects — large or small, solid, liquid, or gas — on each other. You will use the concepts of energy and momentum to analyze physical interactions, such as collisions and propulsion systems. You will also examine two great theoretical foundations of physics: the law of conservation of momentum and the law of conservation of energy.

UNIT PROJECT PREP

Refer to pages 262–263 before beginning this unit. In this unit project, you will create a presentation that explores the importance of scientific theories.

- What theories do you already know that are helpful in analyzing changes in the motion of an object?
- In what career fields are ideas, principles, or mathematical techniques used to analyze the motion and interactions of objects?

CHAPTER 4

Momentum and Impulse

CHAPTER CONTENTS

PREREQUISITE CONCEPTS AND SKILLS

- Newton's laws of motion
- Kinetic energy
- Gravitational potential energy
- Conservation of mechanical energy

The driver of the race car in the above photograph walked away from the crash without a scratch. Luck had little to do with this fortunate outcome, though — a practical application of Newton's laws of motion by the engineers who designed the car and its safety equipment protected the driver from injury.

You learned in Unit 1 that Newton's laws can explain and predict a wide variety of patterns of motion, such as the motion of a projectile and the orbits of planets. How can some of the same laws that guide the stars and planets protect a race car driver who is in a crash?

When Newton originally formulated his laws of motion, he expressed them in a somewhat different form than you see in most textbooks today. Newton emphasized a concept called a "quantity of motion," which is defined as the product of an object's mass and its velocity. Today, we call this quantity "momentum." In this chapter, you will see how the use of momentum allows you to analyze and predict the motion of objects in countless situations that you might not yet have encountered in your study of physics.

INVESTIGATION 4-A

Newton's Cradle

TARGET SKILLS

- Hypothesizing
- Performing and recording
- Analyzing and interpreting

Newton's cradle, also called a "Newtonian demonstrator," looks like a simple child's toy. However, explaining the motion of Newton's cradle requires the application of more than one important physical principle.

Problem

Explain the motion of a Newton's cradle.

Equipment

- Newton's cradle
- modelling clay

Procedure

1. Pull to the side one sphere at the end of the row of the Newton's cradle, keeping the supporting cords taut. Then, release the sphere. Observe and record the resulting motion of the spheres.
2. Pull two spheres to the side, keeping all of the supporting cords taut and keeping the spheres in contact. Release the spheres and observe and record the resulting motion.
3. Repeat step 2, using first three spheres and then four spheres.
4. Pull back two spheres from one end and one sphere from the other end. Release all of the spheres at the same time. Observe and record the motion.
5. Pull one of the end spheres aside and put a small piece of modelling clay on the second sphere at the point where the first sphere will hit it. Release the first sphere and observe and record the motion of the spheres.
6. Leaving the clay in place between the two spheres, pull back one sphere from the opposite end of the row. Release the sphere and observe and record the resulting motion.

Analyze and Conclude

1. Summarize any patterns of motion that you observed for the various trials with the Newton's cradle.
2. Imagine that an end sphere was moving at 0.16 m/s when it hit the row and that two spheres bounced off the other end. What would the speed of the two spheres have to be in order to conserve kinetic energy? Assume that each sphere has a mass of 0.050 kg.
3. Could kinetic energy be conserved in the pattern described in question 2? During your trials, did you ever observe the pattern described in question 2?
4. Did you ever observe a pattern in which more than one sphere was released and only one sphere bounced off the far end?
5. Propose a possible explanation for the motion you observed in Procedure steps 5 and 6.
6. Momentum is involved in the motion of the spheres. Write a definition of momentum as you now understand it.
7. Formulate an hypothesis that could explain why some patterns that would *not* violate the law of conservation of energy were, however, *not* observed.
8. As you study this chapter, look for explanations for the patterns of motion that you observed. Reread your hypothesis and make any necessary corrections.

4.1 Defining Momentum and Impulse

SECTION EXPECTATIONS

- Define and describe the concepts and units related to momentum and impulse.
- Analyze and describe practical applications of momentum, using the concepts of momentum.
- Identify and analyze social issues that relate to the development of safety devices for automobiles.

KEY TERMS

- momentum
- impulse
- impulse-momentum theorem

By now, you have become quite familiar with a wide variety of situations to which Newton's laws apply. Frequently, you have been cautioned to remember that when you apply Newton's second law, you must use only the forces acting on one specific object. Then, by applying Newton's laws, you can predict precisely the motion of that object. However, there are a few types of interactions for which it is difficult to determine or describe the forces acting on an object or on a group of objects. These interactions include collisions, explosions, and recoil. For these more complex scenarios, it is easier to observe the motion of the objects before and after the interaction and then analyze the interaction by using Newton's concept of a quantity of motion.

Defining Momentum

Although you have not used the mathematical expression for momentum, you probably have a qualitative sense of its meaning. For example, when you look at the photographs in Figure 4.1, you could easily list the objects in order of their momentum. Becoming familiar with the mathematical expression for momentum will help you to analyze interactions between objects.

Momentum is the product of an object's mass and its velocity, and is symbolized by $\vec{p}$. Since it is the product of a vector and a scalar, momentum is a vector quantity. The direction of the momentum is the same as the direction of the velocity.

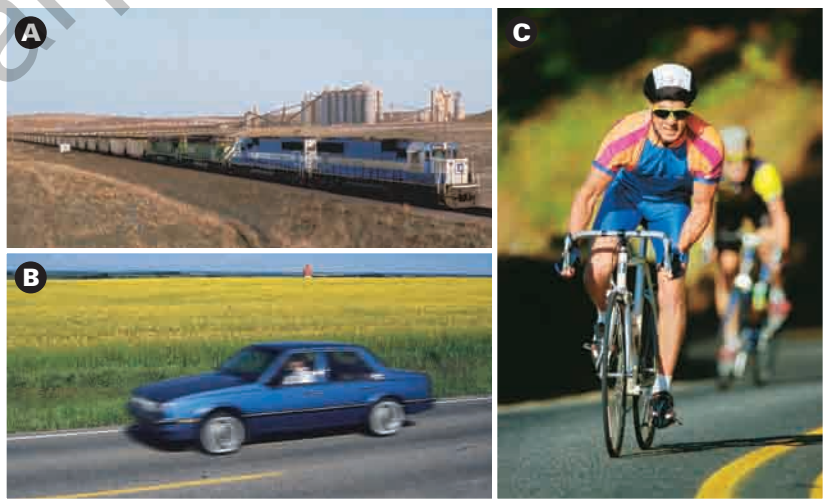


Figure 4.1 If the operator of each of these vehicles was suddenly to slam on the brakes, which vehicle would take the longest time to stop?

DEFINITION OF MOMENTUM

Momentum is the product of an object's mass and its velocity.

$$\vec{p} = m\vec{v}$$

Quantity	Symbol	SI unit
momentum	$\vec{p}$	$\frac{\text{kg} \cdot \text{m}}{\text{s}}$ (kilogram metres per second)
mass	m	kg (kilograms)
velocity	$\vec{v}$	$\frac{\text{m}}{\text{s}}$ (metres per second)

Unit Analysis

$$(\text{mass})(\text{velocity}) = \text{kg} \cdot \frac{\text{m}}{\text{s}} = \frac{\text{kg} \cdot \text{m}}{\text{s}}$$

Note: Momentum does not have a unique unit of its own.

SAMPLE PROBLEM

Momentum of a Hockey Puck

Determine the momentum of a 0.300 kg hockey puck travelling across the ice at a velocity of 5.55 m/s[N].

Conceptualize the Problem

- The *mass* is *moving*; therefore, it has *momentum*.
- The *direction* of an object's *momentum* is the same as the *direction* of its *velocity*.

Identify the Goal

The momentum, $\vec{p}$, of the hockey puck

Identify the Variables and Constants

Known	Unknown
$m = 0.300$ kg	$\vec{p}$
$\vec{v} = 5.55 \frac{\text{m}}{\text{s}}$[N]	

Develop a Strategy

Use the equation that defines momentum.

$$\vec{p} = m\vec{v}$$
$$\vec{p} = (0.300 \text{ kg}) \times \left(5.55 \frac{\text{m}}{\text{s}}[\text{N}]\right)$$
$$\vec{p} = 1.665 \frac{\text{kg} \cdot \text{m}}{\text{s}}[\text{N}]$$
$$\vec{p} \cong 1.67 \frac{\text{kg} \cdot \text{m}}{\text{s}}[\text{N}]$$

The momentum of the hockey puck was $1.67 \frac{\text{kg} \cdot \text{m}}{\text{s}}$[N].

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Validate the Solution

Approximate the solution by multiplying 0.3 kg times 6 m/s. The magnitude of the momentum should be slightly less than this product, which is 1.8 kg · m/s. The value, 1.67 kg · m/s, fits the approximation very well. The direction of the momentum is always the same as the velocity of the object.

PRACTICE PROBLEM

1. Determine the momentum of the following objects.
   - **(a)** 0.250 kg baseball travelling at 46.1 m/s[E]
   - **(b)** 7.5×10^6 kg train travelling west at 125 km/h
   - **(c)** 4.00×10^5 kg jet travelling south at 755 km/h
   - **(d)** electron (9.11×10^{-31} kg) travelling north at 6.45×10^6 m/s

Defining Impulse

Originally, Newton expressed his second law by stating that the change in an object's motion (rate of change of momentum) is proportional to the force impressed on it. Expressed mathematically, his second law can be written as follows.

$$\vec{F} = \frac{\Delta\vec{p}}{\Delta t}$$

To show that this expression is fundamentally equivalent to the equation that you have learned in the past, take the following steps.

- Write the change in momentum as the difference of the final and initial momenta.

$$\vec{F} = \frac{\vec{p}_f - \vec{p}_i}{\Delta t}$$

- Write momentum in terms of mass and velocity.

$$\vec{F} = \frac{m\vec{v}_f - m\vec{v}_i}{\Delta t}$$

- If you assume that m is constant (that is, does not change for the duration of the time interval), you can factor out the mass, m.

$$\vec{F} = \frac{m(\vec{v}_f - \vec{v}_i)}{\Delta t}$$

$$\vec{F} = \frac{m\Delta\vec{v}}{\Delta t}$$

- Recall that the definition of average acceleration is the rate of change of velocity, and substitute an $\vec{a}$ into the above expression.

$$\vec{a} = \frac{\Delta\vec{v}}{\Delta t}$$

$$\vec{F} = m\vec{a}$$

Knowing that $\vec{F} = \frac{\Delta\vec{p}}{\Delta t}$ is a valid expression of Newton's second law, you can mathematically rearrange the expression to demonstrate some very useful relationships involving momentum. When you multiply both sides of the equation by the time interval, you derive a new quantity, $\vec{F}\Delta t$, called "impulse."

$$\vec{F}\Delta t = \Delta\vec{p}$$

MATH LINK

In reality, Newton expressed his second law using the calculus that he invented. The procedure involves allowing the time interval to become smaller and smaller, until it becomes "infinitesimally small." The result allows you to find the instantaneous change in momentum at each instant in time. The formulation of Newton's second law using calculus looks like this.

$$\vec{F} = \frac{d\vec{p}}{dt}$$

Impulse is the product of the force exerted on an object and the time interval over which the force acts, and is often given the symbol $\vec{J}$. Impulse is a vector quantity, and the direction of the impulse is the same as the direction of the force that causes it.

DEFINITION OF IMPULSE

Impulse is the product of force and the time interval.

$$\vec{J} = \vec{F}\Delta t$$

Quantity	Symbol	SI unit
impulse	$\vec{J}$	N · s (newton seconds)
force	$\vec{F}$	N (newtons)
time interval	Δt	s (seconds)

Unit Analysis

(impulse) = (force)(time interval) = N · s

Note: Impulse is equal to the change in momentum, which has units of $\frac{\text{kg} \cdot \text{m}}{\text{s}}$. To show that these units are equivalent to the N · s, express N in terms of the base units.

$$\text{N} \cdot \text{s} = \frac{\text{kg} \cdot \text{m}}{\text{s}^{\cancel{2}}} \cdot \cancel{\text{s}} = \frac{\text{kg} \cdot \text{m}}{\text{s}}$$

MISCONCEPTION

$\vec{F} = m\vec{a}$ Is Correct!

When students read the sentence "If you assume that *m* is constant (that is, does not change for the duration of the time interval), you can factor out the mass, *m*," they sometimes think that the result of the derivation, $\vec{F} = m\vec{a}$, is wrong. However, this equation is a special case of Newton's second law that is correct for all cases in which the mass, *m*, is constant. Since the mass is constant in a very large number of situations, it is acceptable to consider $\vec{F} = m\vec{a}$ as a valid statement of Newton's second law.

SAMPLE PROBLEM

Impulse on a Golf Ball

If a golf club exerts an average force of 5.25×10^3 N[W] on a golf ball over a time interval of 5.45×10^{-4} s, what is the impulse of the interaction?

Conceptualize the Problem

- The golf club exerts an *average force* on the golf ball for a period of *time*. The product of these quantities is defined as *impulse*.
- Impulse is a *vector* quantity.
- The *direction* of the impulse is the same as the *direction of its average force*.

Identify the Goal

The impulse, $\vec{J}$, of the interaction

Identify the Variables and Constants

Known	Unknown
$\vec{F} = 5.25 \times 10^3$ N[W]	$\vec{J}$
$\Delta t = 5.45 \times 10^{-4}$ s	

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Develop a Strategy

Apply the equation that defines impulse.

$$\vec{J} = \vec{F}\Delta t$$
$$\vec{J} = (5.25 \times 10^3 \text{ N[W]})(5.45 \times 10^{-4} \text{ s})$$
$$\vec{J} = 2.8612 \text{ N} \cdot \text{s[W]}$$
$$\vec{J} \cong 2.86 \text{ N} \cdot \text{s[W]}$$

When the golf club strikes the golf ball, the impulse to drive the ball down the fairway is 2.86 N · s[W].

Validate the Solution

Round the values in the data to 5000 N[W] and 0.0006 s and do mental multiplication. The product is 3 N · s[W]. The answer, 2.86 N · s[W], is very close to the estimate.

PROBLEM TIP

When you want to use mental math to approximate an answer to validate your calculations, you can usually find the best approximation by rounding one value up and the other value down before multiplying.

PRACTICE PROBLEMS

2. A sledgehammer strikes a spike with an average force of 2125 N[down] over a time interval of 0.0205 s. Calculate the impulse of the interaction.

3. In a crash test, a car strikes a wall with an average force of 1.23×10^7 N[S] over an interval of 21.0 ms. Calculate the impulse.

4. In a crash test similar to the one described in problem 3, another car, with the same mass and velocity as the first car, experiences an impulse identical to the value you calculated in problem 3. However, the second car was designed to crumple more slowly than the first. As a result, the duration of the interaction was 57.1 ms. Determine the average force exerted on the second car.

The Impulse-Momentum Theorem

You probably noticed that the sample and practice problems above always referred to "average force" and not simply to "force." Average force must be used to calculate impulse in these short, intense interactions, because the force changes continually throughout the few milliseconds of contact of the two objects. For example, when a golf club first contacts a golf ball, the force is very small. Within milliseconds, the force is great enough to deform the ball. The ball then begins to move and return to its original shape and the force soon drops back to zero. Figure 4.2 shows how the force changes with time. You could find the impulse by determining the area under the curve of force versus time.

In many collisions, it is exceedingly difficult to make the precise measurements of force and time that you need in order to calculate the impulse. The relationship between impulse and momentum provides an alternative approach to analyzing such collisions, as well as other interactions. By analyzing the momentum before and after an interaction between two objects, you can determine the impulse.

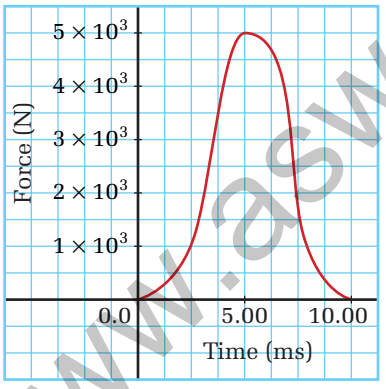

Figure 4.2 You can find the impulse of an interaction (area under the curve) by using the same mathematical methods that you used to find work done from a force-versus-position curve.

When you first rearranged the expression for Newton's second law, you focussed only on the concept of impulse, $\vec{F}\Delta t$. By taking another look at the equation $\vec{F}\Delta t = \Delta\vec{p}$, you can see that impulse is equal to the *change* in the momentum of an object. This relationship is called the **impulse-momentum theorem** and is often expressed as shown in the box below.

ELECTRONIC LEARNING PARTNER

Refer to your Electronic Learning Partner to enhance your understanding of momentum.

IMPULSE-MOMENTUM THEOREM

Impulse is the difference of the final momentum and initial momentum of an object involved in an interaction.

$$\vec{F}\Delta t = m\vec{v}_2 - m\vec{v}_1$$

Quantity	Symbol	SI unit
force	$\vec{F}$	N (newtons)
time interval	Δt	s (seconds)
mass	m	kg (kilograms)
initial velocity	$\vec{v}_1$	$\frac{\text{m}}{\text{s}}$ (metres per second)
final velocity	$\vec{v}_2$	$\frac{\text{m}}{\text{s}}$ (metres per second)

Unit Analysis

(force)(time interval) = (mass)(velocity)

$\text{N} \cdot \text{s} = \text{kg}\frac{\text{m}}{\text{s}}$ $\qquad \frac{\text{kg} \cdot \text{m}}{\text{s}^{\cancel{2}}}\cancel{\text{s}} = \frac{\text{kg} \cdot \text{m}}{\text{s}}$

Note: Impulse is a vector quantity. The direction of the impulse is the same as the direction of the *change* in the momentum.

SAMPLE PROBLEM

Impulse and Average Force of a Tennis Ball

A student practises her tennis volleys by hitting a tennis ball against a wall.

(a) If the 0.060 kg ball travels 48 m/s before hitting the wall and then bounces directly backward at 35 m/s, what is the impulse of the interaction?

(b) If the duration of the interaction is 25 ms, what is the average force exerted on the ball by the wall?

Conceptualize the Problem

- The *mass* and *velocities* before and after the interaction are known, so it is possible to calculate the *momentum* before and after the interaction.
- Momentum is a *vector* quantity, so all calculations must include *directions*.

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- Since the motion is all in *one dimension*, use plus and minus to denote direction. Let the *initial direction* be the *positive direction*.
- You can find the impulse from the *change* in momentum.

Identify the Goal

The impulse, $\vec{J}$, of the interaction

The average force, $\vec{F}$, on the tennis ball

PROBLEM TIP

Whenever you use a result from one step in a problem as data for the next step, use the unrounded form of the data.

Identify the Variables and Constants

Known

$m = 0.060\ \text{kg}$

$\Delta t = 25\ \text{ms} = 0.025\ \text{s}$

$\vec{v}_1 = 48\ \frac{\text{m}}{\text{s}}$

$\vec{v}_2 = -35\ \frac{\text{m}}{\text{s}}$

Unknown

$\vec{J}$

$\vec{F}$

Develop a Strategy

Use the impulse-momentum theorem to calculate the impulse.

$$\vec{F}\Delta t = m\vec{v}_2 - m\vec{v}_1$$
$$\vec{F}\Delta t = 0.060\ \text{kg}\left(-35\ \frac{\text{m}}{\text{s}}\right) - 0.060\ \text{kg}\left(48\ \frac{\text{m}}{\text{s}}\right)$$
$$\vec{F}\Delta t = -2.1\ \frac{\text{kg}\cdot\text{m}}{\text{s}} - 2.88\ \frac{\text{kg}\cdot\text{m}}{\text{s}}$$
$$\vec{F}\Delta t = -4.98\ \frac{\text{kg}\cdot\text{m}}{\text{s}}$$
$$\vec{F}\Delta t \cong -5.0\ \frac{\text{kg}\cdot\text{m}}{\text{s}}$$

(a) The impulse was $5.0\ \text{kg}\cdot\text{m/s}$ in a direction opposite to the initial direction of the motion of the ball.

Use the definition of impulse to find the average force.

$$\vec{F}\Delta t = -4.98\ \frac{\text{kg}\cdot\text{m}}{\text{s}}$$
$$\vec{F} = \frac{-4.98\ \frac{\text{kg}\cdot\text{m}}{\text{s}}}{\Delta t}$$
$$\vec{F} = \frac{-4.98\ \frac{\text{kg}\cdot\text{m}}{\text{s}}}{0.025\ \text{s}}$$
$$\vec{F} = -199.2\ \text{N}$$
$$\vec{F} \cong -2.0 \times 10^2\ \text{N}$$

(b) The average force of the wall on the tennis ball was 2.0×10^2 N in the direction opposite to the initial direction of the ball.

Validate the Solution

Use an alternative mathematical technique for the impulse calculation by factoring out the mass, subtracting the velocities, then multiplying to see if you get the same answer.

$$\vec{F}\Delta t = m(\vec{v}_2 - \vec{v}_1)$$
$$\vec{F}\Delta t = 0.060\ \text{kg}\left(-35\frac{\text{m}}{\text{s}} - 48\frac{\text{m}}{\text{s}}\right)$$
$$\vec{F}\Delta t = (0.060\ \text{kg})\left(-83\ \frac{\text{m}}{\text{s}}\right)$$
$$\vec{F}\Delta t = -4.98\ \frac{\text{kg}\cdot\text{m}}{\text{s}} \cong -5.0\ \frac{\text{kg}\cdot\text{m}}{\text{s}}$$

Check the units for the second part of the problem.

$$\frac{\frac{\text{kg}\cdot\text{m}}{\text{s}}}{\text{s}} = \frac{\text{kg}\cdot\text{m}}{\text{s}^2} = \text{N}$$

PRACTICE PROBLEMS

5. The velocity of the serve of some professional tennis players has been clocked at 43 m/s horizontally. (Hint: Assume that any vertical motion of the ball is negligible and consider only the horizontal direction of the ball after it was struck by the racquet.) If the mass of the ball was 0.060 kg, what was the impulse of the racquet on the ball?

6. A 0.35 kg baseball is travelling at 46 m/s toward the batter. After the batter hits the ball, it is travelling 62 m/s in the opposite direction. Calculate the impulse of the bat on the ball.

7. A student dropped a 1.5 kg book from a height of 1.75 m. Determine the impulse that the floor exerted on the book when the book hit the floor.

Impulse and Auto Safety

One of the most practical and important applications of impulse is in the design of automobiles and their safety equipment. When a car hits another car or a solid wall, little can be done to reduce the change in momentum. The mass of the car certainly does not change, while the velocity changes to zero at the moment of impact. Since you cannot reduce the change in momentum, you cannot reduce the impulse. However, since impulse ($\vec{F}\Delta t$) depends on both force and time, engineers have found ways to reduce the force exerted on car occupants by extending the time interval of the interaction. Think about how the design of a car can expand the duration of a crash.

In the early days of auto manufacturing, engineers and designers thought that a very strong, solid car would be ideal. As the number of cars on the road and the speed of the cars increased, the number and seriousness of accident injuries made it clear that the very sturdy cars were not protecting car occupants. By the late 1950s and early 1960s, engineers were designing cars with very rigid passenger cells that would not collapse onto the passengers, but with less rigid "crumple zones" in the front and rear, as shown in Figure 4.3.

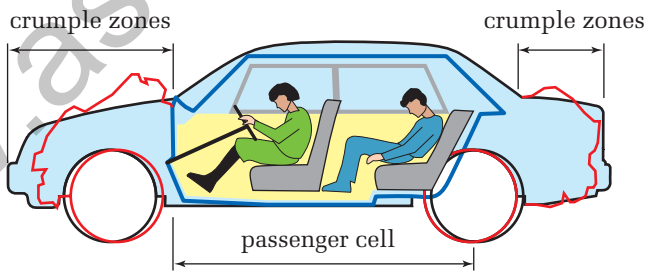


Figure 4.3 Although a car crash seems almost instantaneous, the time taken for the front or rear of the car to "crumple" is great enough to significantly reduce the average force of the impact and, therefore, the average force on the passenger cell and the passengers.

PROBEWARE

If your school has probeware equipment, visit ***www.mcgrawhill.ca/links/physics12*** and follow the links for an in-depth activity on impulse and momentum.

ELECTRONIC LEARNING PARTNER

Use the crash test provided by your Electronic Learning Partner to enhance your understanding of momentum.

QUICK LAB

Designing Crumple Zones

TARGET SKILLS

- Hypothesizing
- Performing and recording
- Analyzing and interpreting
- Communicating results

How soft is too soft and how rigid is too rigid for an effective vehicle crumple zone? In this lab, you will design and test several materials to determine the optimum conditions for passengers in a vehicle.

Obtain a rigid (preferably metal) toy vehicle to simulate the passenger cell of an automobile. The vehicle must have an open space in the centre for the "passenger." Make a passenger out of putty, modelling clay, or some material that will easily show "injuries" in the form of dents and deformations.

Design and build some type of device that will propel your vehicle rapidly into a solid wall (or stack of bricks) with nearly the same speed in all trials. The wall must be solid, but you will need to ensure that you do not damage the wall. Perform several crash tests with your vehicle and passenger and observe the types of injuries and the extent of injuries caused by the collision.

Select a variety of materials, from very soft to very hard, from which to build crumple zones. For example, you could use very soft foam rubber for the soft material. The thickness of each crumple zone must be approximately one third the length of your vehicle.

One at a time, attach your various crumple zones to your vehicle and test the effectiveness of the material in reducing the severity of injury to the passenger. Be sure that the vehicle travels at the same speed with the crumple zone attached as it did in the original crash tests without a crumple zone. Also, be sure that the materials you use to attach the crumple zones do not influence the performance of the crumple zones. Formulate an hypothesis about the relative effectiveness of each of the various crumple zones that you designed.

Analyze and Conclude

1. How do the injuries to the passenger that occurred with a very soft crumple zone compare to the injuries in the original crash tests?
2. How do the injuries to the passenger that occurred with a very rigid crumple zone compare to the injuries in the original crash tests?
3. Describe the difference in the passenger's injuries between the original crash tests and the test using the most effective crumple zone material.

Apply and Extend

4. The optimal crumple zone for a very massive car would be much more rigid than one for a small, lightweight car. However, a crash between a large and a small car would result in much greater damage to the small car. Write a paragraph responding to the question "Should car manufacturers consider other cars on the road when they design their own cars, or should they ignore what might happen to other manufacturers' cars?"
5. Crumple zones are just one of many types of safety systems designed for cars. Should the government regulate the incorporation of safety systems into cars? Give a rationale for your answer.
6. Some safety systems are very costly. Who should absorb the extra cost — the buyer, the manufacturer, or the government? For example, should the government provide a tax break or some other monetary incentive for manufacturers to build or consumers to buy cars with highly effective safety systems? Give a rationale for your answer.

When a rigid car hits a wall, a huge force stops the car almost instantaneously. The car might even look as though it was only slightly damaged. However, parts of the car, such as the steering wheel, windshield, or dashboard, exert an equally large force on the passengers, stopping them exceedingly rapidly and possibly causing very serious injuries.

When a car with well-designed crumple zones hits a wall, the force of the wall on the car causes the front of the car to collapse over a slightly longer time interval than it would in the absence of a crumple zone. Since $\vec{F}\Delta t$ is constant and Δt is larger, the average force, $\vec{F}$, is smaller than it would be for a rigid car. Although many other factors must be considered to reduce injury in collisions, the presence of crumple zones has had a significant effect in reducing the severity of injuries in automobile accidents.

The concept of increasing the duration of an impact applies to many forms of safety equipment. For example, the linings of safety helmets are designed to compress relatively slowly. If the lining was extremely soft, it would compress so rapidly that the hard outer layer of the helmet would impact on the head very quickly. If the lining did not compress at all, it would collide with the head over an extremely short time interval and cause serious injury. Each type of sport helmet is designed to compress in a way that compensates for the type of impacts expected in that sport.

WEB LINK

www.mcgrawhill.ca/links/physics12

To learn more about the design and testing of helmets and other safety equipment in sports, go to the above Internet site and click on **Web Links**.

4.1 Section Review

1. **K/U** Define momentum qualitatively and quantitatively.
2. **K/U** What assumption do you have to make in order to show that the two forms of Newton's second law ($\vec{F} = \frac{\Delta\vec{p}}{\Delta t}$ and $\vec{F} = m\vec{a}$) are equivalent?
3. **I** Try to imagine a situation in which the form $\vec{F} = m\vec{a}$ would not apply, but the form $\vec{F} = \frac{\Delta\vec{p}}{\Delta t}$ could be used. Describe that situation. How could you test your prediction?
4. **C** State the impulse-momentum theorem and give one example of its use.
5. **MC** A bungee jumper jumps from a very high tower with bungee cords attached to his ankles. As he reaches the end of the bungee cord, it begins to stretch. The cord stretches for a relatively long period of time and then it recoils, pulling him back up. After several bounces, he dangles unhurt from the bungee cord (if he carried out the jump with all of the proper safety precautions). If he jumped from the same point with an ordinary rope attached to his ankles, he would be very severely injured. Use the concept of impulse to explain the difference in the results of a jump using a proper bungee cord and a jump using an ordinary rope.

UNIT PROJECT PREP

Can environmentally responsible transportation be the product of properly applying scientific models and theories?

- Is the theory of momentum and impulse currently used in vehicle design?
- Can you envision using momentum and impulse theory to design more environmentally responsible transportation systems?

4.2 Conservation of Momentum

SECTION EXPECTATIONS

- Define and describe the concepts related to elastic and inelastic collisions and to open and closed energy systems.
- Analyze situations involving the conservation of momentum and apply them quantitatively.
- Investigate the law of conservation of momentum in one and two dimensions.

KEY TERMS

- conservation of momentum
- system of particles
- internal force
- external force
- open system
- closed system
- isolated system
- recoil

When the cue ball hits the eight ball in billiards, the eight ball hits the cue ball. When a rock hits the ground, the ground hits the rock. In any collision, two objects exert forces on each other. You can learn more about momentum by analyzing the motion of both objects in a collision.

Figure 4.4 The game of billiards offers many excellent examples of collisions.

Newton's Third Law and Momentum

Newton's third law states that "For every action force on object B due to object A, there is a reaction force, equal in magnitude but opposite in direction, acting on object A due to object B." Unlike Newton's second law, which focusses on the motion of one specific object, his third law deals with the interaction between *two* objects. When you apply Newton's third law to collisions, you discover one of the most important laws of physics — the law of conservation of momentum. The following steps, along with the diagram in Figure 4.5, show you how to derive the law of conservation of momentum by applying Newton's third law to a collision between two objects.

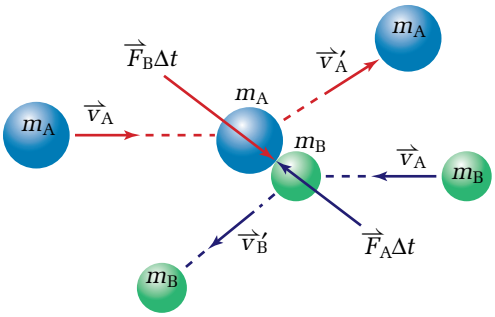


Figure 4.5 Object A exerts a force on object B, causing a change in B's momentum. At the same time, object B exerts a force equal in size and opposite in direction on object A, changing A's momentum.

- Write the impulse-momentum theorem for each of two objects, A and B, that collide with each other.

$$\vec{F}_A\Delta t = m_A\vec{v}_{A2} - m_A\vec{v}_{A1}$$
$$\vec{F}_B\Delta t = m_B\vec{v}_{B2} - m_B\vec{v}_{B1}$$

- Apply Newton's third law to the forces that A and B exert on each other.

$$\vec{F}_A = -\vec{F}_B$$

- The duration of the collision is the same for both objects. Therefore, you can multiply both sides of the equation above by Δt.

$$\vec{F}_A\Delta t = -\vec{F}_B\Delta t$$

- Substitute the expressions for change in momentum in the first step into the equation in the third step and then simplify.

$$m_A\vec{v}_{A2} - m_A\vec{v}_{A1} = -(m_B\vec{v}_{B2} - m_B\vec{v}_{B1})$$

- Algebraically rearrange the last equation so that (1) the terms representing the before-collision conditions precede the equals sign and (2) the terms for the after-collision conditions follow the equals sign.

$$m_A\vec{v}_{A2} - m_A\vec{v}_{A1} = -m_B\vec{v}_{B2} + m_B\vec{v}_{B1}$$
$$m_A\vec{v}_{A1} + m_B\vec{v}_{B1} = m_A\vec{v}_{A2} + m_B\vec{v}_{B2}$$

The last equation is a mathematical expression of the law of **conservation of momentum,** which states that the total momentum of two objects before a collision is the same as the total momentum of the same two objects after they collide.

LAW OF CONSERVATION OF MOMENTUM

The sum of the momenta of two objects before collision is equal to the sum of their momenta after they collide.

$$m_A\vec{v}_A + m_B\vec{v}_B = m_A\vec{v}'_A + m_B\vec{v}'_B$$

Quantity	Symbol	SI unit
mass of object A	m_A	kg (kilograms)
mass of object B	m_B	kg (kilograms)
velocity of object A before the collision	$\vec{v}_A$	$\frac{m}{s}$ (metres per second)
velocity of object B before the collision	$\vec{v}_B$	$\frac{m}{s}$ (metres per second)
velocity of object A after the collision	$\vec{v}'_A$	$\frac{m}{s}$ (metres per second)
velocity of object B after the collision	$\vec{v}'_B$	$\frac{m}{s}$ (metres per second)

PHYSICS FILE

When working with collisions, instead of using subscripts such as "2," physicists often use a superscript symbol called a "prime," which looks like an apostrophe, to represent the variables *after* a collision. The variable is said to be "primed." Look for this notation in the box on the left.

MISCONCEPTION

A Closed System Is Not Isolated

Many people confuse the terms "closed" and "isolated" as they apply to systems. Although it might sound as though closed systems would not exchange anything with their surroundings, they do allow energy to enter or leave. Only isolated systems prevent the exchange of energy with the surroundings.

The law of conservation of momentum can be broadened to more than two objects by defining a **system of particles**. Any group of objects can be defined as a system of particles. Once a system is defined, forces are classified as internal or external forces. An **internal force** is any force exerted on any object in the system due to another object in the system. An **external force** is any force exerted by an object that is not part of the system on an object within the system.

Scientists classify systems according to their interaction with their surroundings, as illustrated in Figure 4.6. An **open system** can exchange both matter and energy with its surroundings. Matter does not enter or leave a **closed system,** but energy can enter or leave. Neither matter nor energy can enter or leave an **isolated system.**

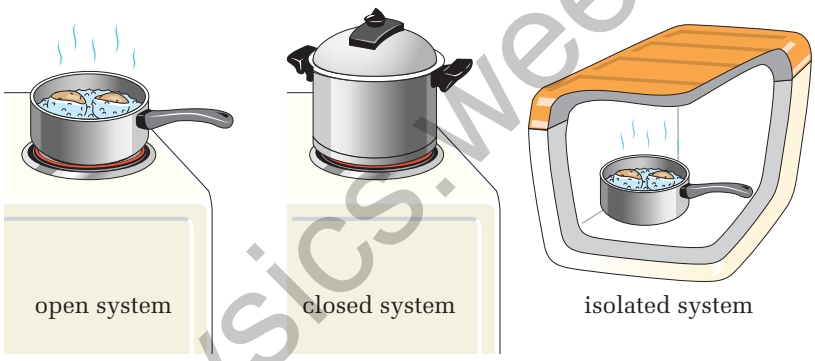


Figure 4.6 An open pot of potatoes boiling on the stove represents an open system, because heat is entering the pot and water vapour is leaving the system. A pressure cooker prevents any matter from escaping but heat is entering, so the pressure cooker represents a closed system. If the pot is placed inside a perfect insulator, neither heat nor water can enter or leave the system, making it an isolated system.

A force can do work on a closed system, thus increasing the energy of the system. Clearly, if no external forces can act on a system, it is isolated. To demonstrate that the momentum of an isolated system is conserved, start with the impulse-momentum theorem, where $\vec{p}_{sys}$ represents the total momentum of all of the objects within the system.

- An impulse on a system due to an external force causes a change in the momentum of the system. $\vec{F}_{ext}\Delta t = \Delta\vec{p}_{sys}$
- If a system is isolated, the net external force acting on the system is zero. $(0.0\ \text{N})\ \Delta t = |\Delta\vec{p}_{sys}|$
- If the impulse is zero, the *change* in momentum must be zero. $|\Delta\vec{p}_{sys}| = 0.0\ \frac{\text{kg}\cdot\text{m}}{\text{s}}$

COURSE CHALLENGE

Momentum

A moving planet, bowling ball, or an electron share the property of momentum. Learn more about momentum conservation as it relates to your *Course Challenge* on page 603 of this text.

The last expression is an alternative form of the equation for the conservation of momentum. The equation states that the change in momentum of an isolated system is zero. The particles or objects within the system might interact with each other and exchange momentum, but the total momentum of the isolated system does not change.

In reality, systems are rarely perfectly isolated. In nearly all real situations, immediately after a collision, frictional forces and interactions with other objects change the momentum of the objects involved in the collision. Therefore, it might appear that the law of conservation of momentum is not very useful. However, the law always applies to a system from the instant before to the instant after a collision. If you know the conditions just before a collision, you can always use conservation of momentum to determine the momentum and, thus, velocity of an object at the instant after a collision. Often, these values are all that you need to know.

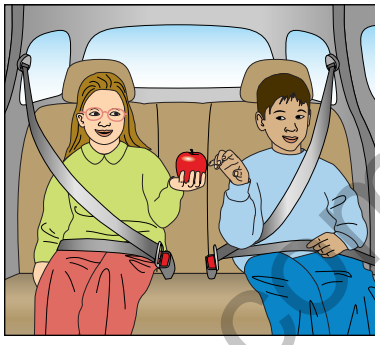


Figure 4.7 A moving car and its occupants can be defined as being a system. The children in the car might be exerting forces on each other or on objects that they are handling. Although they are exchanging momentum between themselves and the objects, these changes have no effect on the total momentum of the system.

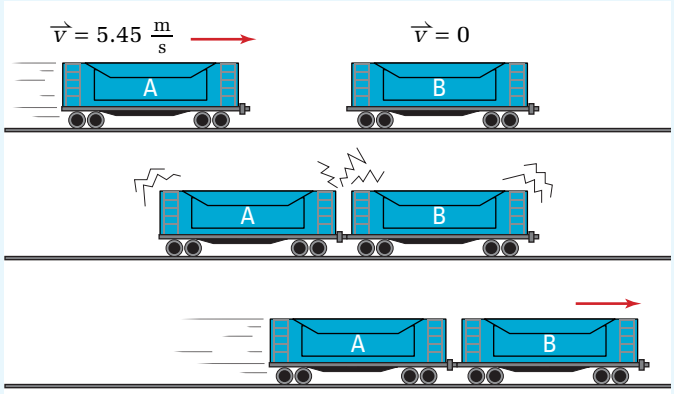
Collisions in One Dimension

Since momentum is a vector quantity, both the magnitude and the direction of the momentum must be conserved. Therefore, momentum is conserved in each dimension, *independently*. For complex situations, it is often convenient to separate the momentum into its components and work with each dimension separately. Then you can combine the results and find the resultant momentum of the objects in question. Solving problems that involve only one dimension is good practice for tackling more complex problems.

SAMPLE PROBLEM

Analyzing a Collision between Boxcars

A 1.75×10^4 kg boxcar is rolling down a track toward a stationary boxcar that has a mass of 2.00×10^4 kg. Just before the collision, the first boxcar is moving east at 5.45 m/s. When the boxcars collide, they lock together and continue down the track. What is the velocity of the two boxcars immediately after the collision?



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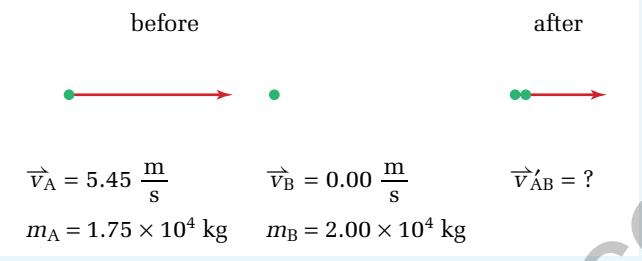
Conceptualize the Problem

- Make a sketch of the *momentum vectors* representing conditions just *before* and just *after* the collision.
- Before the collision, only *one boxcar* (A) is *moving* and therefore has *momentum*.
- At the instant of the collision, *momentum is conserved*.
- After the collision, the *two boxcars* (A and B) *move as one mass*, with the *same velocity*.



Identify the Goal

The velocity, $\vec{v}'_{AB}$, of the combined boxcars immediately after the collision

Identify the Variables and Constants

Known	Implied	Unknown
$m_A = 1.75 \times 10^4$ kg	$\vec{v}_B = 0.00\ \frac{m}{s}$	$\vec{v}'_{AB}$
$m_B = 2.00 \times 10^4$ kg		
$\vec{v}_A = 5.45\ \frac{m}{s}$[E]		

Develop a Strategy

Apply the law of conservation of momentum.

$$m_A\vec{v}_A + m_B\vec{v}_B = m_A\vec{v}'_A + m_B\vec{v}'_B$$

After the collision, the two masses act as one, with one velocity. Rewrite the equation to show this condition.

$$m_A\vec{v}_A + m_B\vec{v}_B = (m_A + m_B)\vec{v}'_{AB}$$

Solve for $\vec{v}'_{AB}$.

$$\vec{v}'_{AB} = \frac{m_A\vec{v}_A + m_B\vec{v}_B}{(m_A + m_B)}$$

Substitute values and solve.

$$\vec{v}'_{AB} = \frac{(1.75 \times 10^4\ \text{kg})(5.45\ \frac{m}{s}[E]) + (2.00 \times 10^4\ \text{kg})(0.00\ \frac{m}{s})}{(1.75 \times 10^4\ \text{kg} + 2.00 \times 10^4\ \text{kg})}$$

$$\vec{v}'_{AB} = \frac{9.53\ 75 \times 10^4\ \frac{\cancel{kg}\cdot m}{s}[E]}{3.75 \times 10^4\ \cancel{kg}}$$

$$\vec{v}'_{AB} = 2.543\ \frac{m}{s}[E]$$

$$\vec{v}'_{AB} \cong 2.54\ \frac{m}{s}[E]$$

The locked boxcars were rolling *east* down the track at 2.54 m/s.

Validate the Solution

The combined mass of the boxcars was nearly double the mass of the boxcar that was moving before the collision. Since the exponents of mass and velocity are always one, making the relationships linear, you would expect that the velocity of the combined boxcars would be just under half of the velocity of the single boxcar before the collision. Half of 5.45 m/s is approximately 2.7 m/s. The calculated value of 2.54 m/s is very close to what you would expect.

8. Claude and Heather are practising pairs skating for a competition. Heather (47 kg) is skating with a velocity of 2.2 m/s. Claude (72 kg) is directly behind her, skating with a velocity of 3.1 m/s. When he reaches her, he holds her waist and they skate together. At the instant after he takes hold of her waist, what is their velocity?

9. Two amusement park "wrecker cars" are heading directly toward each other. The combined mass of car A plus driver is 375 kg and it is moving with a velocity of +1.8 m/s. The combined mass of car B plus driver is 422 kg and it is moving with a velocity of –1.4 m/s. When they collide, they attach and continue moving along the same straight line. What is their velocity immediately after they collide?

Recoil

Imagine yourself in the situation illustrated in Figure 4.8. You are in a small canoe with a friend and you decide to change places. Assume that the friction between the canoe and the water is negligible. While the canoe is not moving in the water, you very carefully stand up and start to take a step. You suddenly have the sense that the boat is moving under your feet. Why?

Figure 4.8 If you start to step forward in a canoe, the canoe recoils under your feet.

When you stepped forward, your foot pushed against the bottom of the canoe and you started to move. You gained momentum due to your velocity. Momentum of the system — you, your friend, and the canoe — must be conserved, so the canoe started to move in the opposite direction. The interaction that occurs when two stationary objects push against each other and then move apart is called **recoil.** You can use the equation for conservation of momentum to solve recoil problems, as the following problem illustrates.

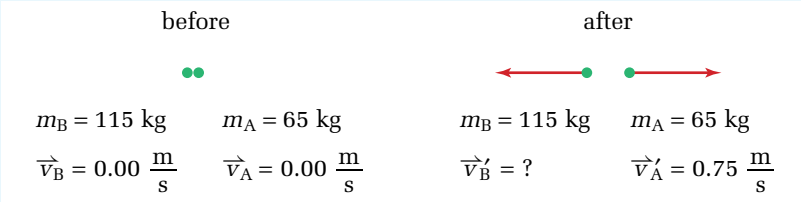
SAMPLE PROBLEM

Recoil of a Canoe

For the case described in the text, find the velocity of the canoe and your friend at the instant that you start to take a step, if your velocity is 0.75 m/s[forward]. Assume that your mass is 65 kg and the combined mass of the canoe and your friend is 115 kg.

Conceptualize the Problem

- Make a simple sketch of the conditions before and after you took a step.



- The canoe was *not moving* when you started to take a step.
- You gained *momentum* when you started to *move*. Label yourself "A" and consider the direction of your motion to be *positive*.
- The canoe had to *move* in a *negative direction* in order to conserve momentum. Label the canoe and your friend "B."

Identify the Goal

The initial velocity, $\vec{v}'_B$ of the canoe and your friend

Identify the Variables and Constants

Known		Implied	Unknown
$m_A = 65$ kg	$\vec{v}'_A = 0.75 \frac{m}{s}$	$\vec{v}_A = 0.00 \frac{m}{s}$	$\vec{v}'_B$
$m_B = 115$ kg		$\vec{v}_B = 0.00 \frac{m}{s}$	

Develop a Strategy

Apply conservation of momentum.

$$m_A\vec{v}_A + m_B\vec{v}_B = m_A\vec{v}'_A + m_B\vec{v}'_B$$

Velocities before the interaction were zero; therefore, the total momentum before the interaction was zero. Set these values equal to zero and solve for the velocity of B after the reaction.

$$0.0 \frac{kg \cdot m}{s} = m_A\vec{v}'_A + m_B\vec{v}'_B$$

$$m_A\vec{v}'_A = -m_B\vec{v}'_B$$

$$\vec{v}'_B = -\frac{m_A\vec{v}'_A}{m_B}$$

Substitute values and solve.

$$\vec{v}'_B = -\frac{(65\ \cancel{kg})(0.75\ \frac{m}{s})}{115\ \cancel{kg}}$$

$$\vec{v}'_B = -0.4239 \frac{m}{s}$$

$$\vec{v}'_B \cong -0.42 \frac{m}{s}$$

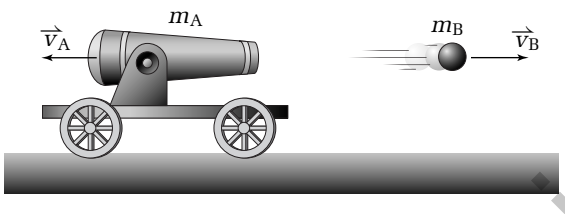
The velocity of the canoe and your friend, immediately after you started moving, was –0.42 m/s.

Validate the Solution

Since the mass of the canoe plus your friend was larger than your mass, you would expect that the magnitude of their velocity would be smaller, which it was. Also, the direction of the velocity of the canoe plus your friend must be negative, that is, in a direction opposite to your direction. Again, it was.

PRACTICE PROBLEMS

10. A 1385 kg cannon containing a 58.5 kg cannon ball is on wheels. The cannon fires the cannon ball, giving it a velocity of 49.8 m/s north. What is the initial velocity of the cannon the instant after it fires the cannon ball?



11. While you are wearing in-line skates, you are standing still and holding a 1.7 kg rock. Assume that your mass is 57 kg. If you throw the rock directly west with a velocity of 3.8 m/s, what will be your recoil velocity?

12. The mass of a uranium-238 atom is 3.95×10^{-25} kg. A stationary uranium atom emits an alpha particle with a mass of 6.64×10^{-27} kg. If the alpha particle has a velocity of 1.42×10^{4} m/s, what is the recoil velocity of the uranium atom?

Collisions in Two Dimensions

Very few collisions are confined to one dimension, as anyone who has played billiards knows. Nevertheless, you can work in one dimension at a time, because momentum is conserved in each dimension independently. For example, consider the car crash illustrated in Figure 4.9. Car A is heading north and car B is heading east when they collide at the intersection. The cars lock together and move off at an angle. You can find the total momentum of the entangled cars because the component of the momentum to the north must be the same as car A's original momentum. The eastward component of the momentum must be the same as car B's original momentum. You can use the Pythagorean theorem to find the resultant momentum, as shown in the following problems.

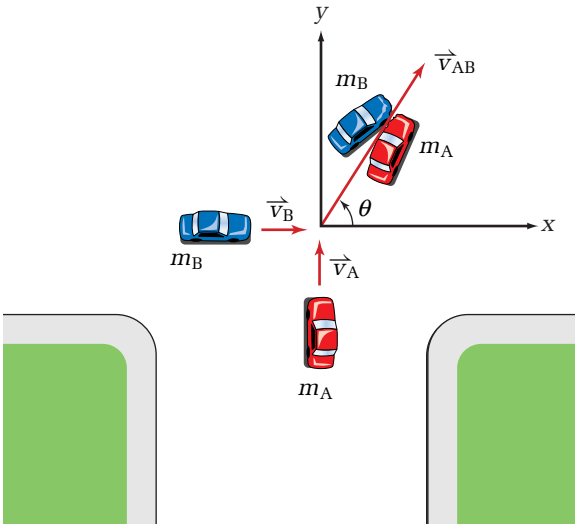
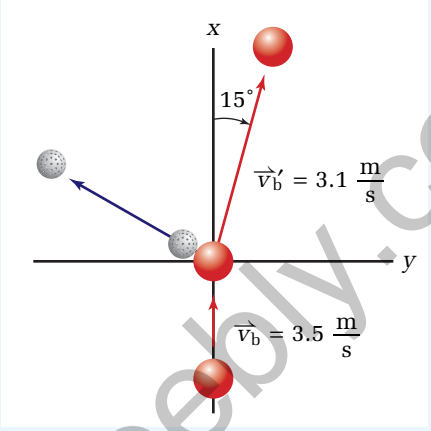
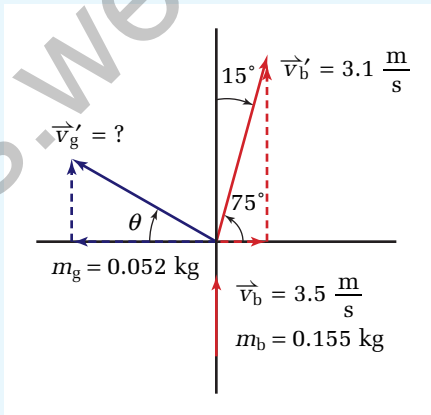


Figure 4.9 Momentum is conserved independently in both the north-south dimension and the east-west dimension.

Applying Conservation of Momentum in Two Dimensions

1. A billiard ball of mass 0.155 kg is rolling directly away from you at 3.5 m/s. It collides with a stationary golf ball of mass 0.052 kg. The billiard ball rolls off at an angle of 15° clockwise from its original direction with a velocity of 3.1 m/s. What is the velocity of the golf ball?





Conceptualize the Problem

- Sketch the vectors representing the momentum of the billiard ball and the golf ball immediately before and just after the collision. It is always helpful to superimpose an x–y-coordinate system on the vectors so that the origin is at the point of the contact of the two balls. For calculations, use the angles that the vectors make with the x-axis.
- Momentum is *conserved* in the x and y directions *independently*.
- The *total momentum* of the system (billiard ball and golf ball) *before* the collision is carried by the *billiard ball* and is all in the positive y direction.
- *After* the collision, both balls have *momentum* in both the y direction and the x direction.
- Since the *momentum* in the x *direction* was *zero* before the collision, it must be *zero* after the collision. Therefore, the *x-components of the momentum* of the two balls after the collision must be *equal in magnitude* and *opposite in direction*.
- The *sum* of the y-components of the two balls *after* the collision must equal the *momentum* of the billiard ball *before* the collision.
- Use subscript "b" for the billiard ball and subscript "g" for the golf ball.

Identify the Goal

The velocity, $\vec{v}_g'$, of the golf ball after the collision

Identify the Variables and Constants

Known		Implied	Unknown
$m_b = 0.155$ kg	$\vec{v}_b = 3.5 \frac{m}{s}$ [forward]	$\vec{v}_g = 0.00 \frac{m}{s}$	$\vec{v}_g'$
$m_g = 0.052$ kg	$\vec{v}_b' = 3.1 \frac{m}{s}$ [15° clockwise from original]		

PROBLEM TIP

When you are working with many bits of data in one problem, it is often helpful to organize the data in a table such as the one shown here.

Object		P_x	P_y
before	A		
	B		
	total		
after	A		
	B		
	total		

Develop a Strategy

Write the expression for the conservation of momentum in the x direction.

$$m_b v_{bx} + m_g v_{gx} = m_b v'_{bx} + m_g v'_{gx}$$

Note that the x-component of the momentum of both balls was zero before the collision. Then solve for the x-component of the velocity of the golf ball after the collision.

$$0.0\ \frac{\text{kg}\cdot\text{m}}{\text{s}} = m_b v'_{bx} + m_g v'_{gx}$$

$$m_g v'_{gx} = -m_b v'_{bx}$$

$$v'_{gx} = -\frac{m_b v'_{bx}}{m_g}$$

Substitute values and solve.

$$v'_{gx} = -\frac{(0.155\ \text{kg})(3.1\ \frac{\text{m}}{\text{s}}\cos 75^\circ)}{0.052\ \text{kg}}$$

$$v'_{gx} = -2.3916\ \frac{\text{m}}{\text{s}}$$

Carry out the same procedure for the y-components.

$$m_b v_{by} + m_g v_{gy} = m_b v'_{by} + m_g v'_{gy}$$

$$m_b v_{by} + 0.0\ \frac{\text{kg}\cdot\text{m}}{s} = m_b v'_{by} + m_g v'_{gy}$$

$$m_g v'_{gy} = m_b v'_{by} - m_b v_{by}$$

$$v'_{gy} = \frac{m_b v_{by} - m_b v'_{by}}{m_g}$$

$$v'_{gy} = \frac{(0.155\ \text{kg})(3.5\ \frac{\text{m}}{\text{s}}) - (0.155\ \text{kg})(3.1\ \frac{\text{m}}{\text{s}}\sin 75^\circ)}{0.052\ \text{kg}}$$

$$v'_{gy} = 1.507\ \frac{\text{m}}{\text{s}}$$

Use the Pythagorean theorem to find the magnitude of the resultant velocity vector of the golf ball.

$$|\vec{v}'_g|^2 = v'^2_{gx} + v'^2_{gy}$$

$$|\vec{v}'_g|^2 = \left(-2.3916\ \frac{\text{m}}{\text{s}}\right)^2 + \left(1.507\ \frac{\text{m}}{\text{s}}\right)^2$$

$$|\vec{v}'_g|^2 = 5.7198\ \frac{\text{m}^2}{\text{s}^2} + 2.271\ \frac{\text{m}^2}{\text{s}^2}$$

$$|\vec{v}'_g|^2 = 7.9908\ \frac{\text{m}^2}{\text{s}^2}$$

$$|\vec{v}'_g| = 2.8268\ \frac{\text{m}}{\text{s}}$$

$$|\vec{v}'_g| \cong 2.8\ \frac{\text{m}}{\text{s}}$$

Use the tangent function to find the direction of the velocity vector.

$$\tan\theta = \frac{v_{gy}}{v_{gx}}$$

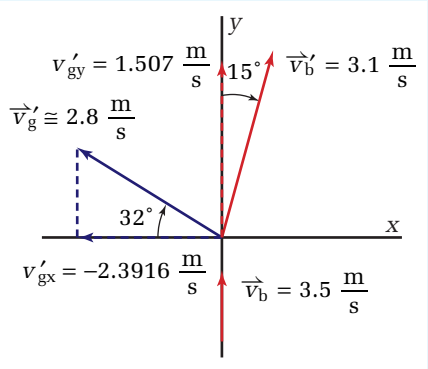
$$\tan\theta = \frac{1.507\ \frac{\text{m}}{\text{s}}}{2.3916\ \frac{\text{m}}{\text{s}}}$$

$$\theta = \tan^{-1} 0.6301$$

$$\theta = 32.22^\circ$$

$$\theta \cong 32^\circ$$

Since the x-component is negative and the y-component is positive, the vector is in the second quadrant. Use positive values to find the magnitude of the angle from the x-axis.



Since the x-component is negative and the y-component is positive, the resultant vector lies in the second quadrant and the angle is measured clockwise from the x-axis.

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The velocity of the golf ball after the collision is 2.8 m/s at 32° clockwise from the negative *x*-axis. (At more advanced levels, you will be expected to report angles counterclockwise from the positive *x*-axis. In this case, the angle would be 180° − 32° = 148° counterclockwise from the *x*-axis.)

Validate the Solution

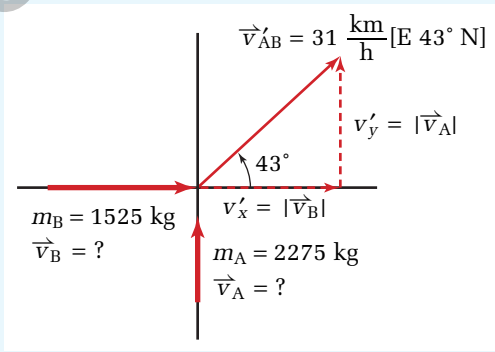
Since all of the momentum before the collision was in the positive *y* direction, the *y*-component of momentum after the collision had to be in the positive *y* direction, which it was. Since there was no momentum in the *x* direction before the collision, the *x*-components of the momentum after the collision had to be in opposite directions, which they were.

---

2. The police are investigating an accident similar to the one pictured in Figure 4.9. Using data tables, they have determined that the mass of car A is 2275 kg and the mass of car B is 1525 kg. From the skid marks and data for the friction between tires and concrete, the police determined that the cars, when they were locked together, had a velocity of 31 km/h at an angle of 43° north of the eastbound street. If the speed limit was 35 km/h on both streets, should one or both cars be ticketed for speeding? Which car had the right of way at the intersection? Was one driver or were both drivers at fault for the accident?

Conceptualize the Problem

- Sketch a vector diagram of the momentum before and after the collision.
- Consider the two cars to be a "system." Before the collision, the *north component* of the *momentum* of the system was carried by car A and the *east component* was carried by car B.
- Momentum is conserved in the north-south direction and in the east-west direction independently.
- After the collision, the cars form *one mass* with all of the *momentum*.



Identify the Goal

The velocities, $\vec{v}_A$ and $\vec{v}_B$, of the two cars before the collision (in order to determine who should be ticketed)

Identify the Variables and Constants

Known		Unknown
$m_A = 2275$ kg	$\vec{v}'_{AB} = 31\ \frac{km}{h}$[E43°N]	$\vec{v}_A$
$m_B = 1525$ kg		$\vec{v}_B$

PROBLEM TIP

In this problem, you have two unknown values, the velocity of car A and the velocity of car B before the collision. To find two unknown values, you need at least two equations. Since momentum is a vector quantity, conservation of momentum provides three equations, one for each dimension. Remember, use as many dimensions as you have unknowns and you will be able to solve momentum problems with as many as three unknowns.

Develop a Strategy

Write the equation for conservation of momentum.

$$m_A\vec{v}_A + m_B\vec{v}_B = m_A\vec{v}'_A + m_B\vec{v}'_B$$

Work with the north-south direction only. Modify the equation to show that car B was moving directly east before the crash; its north-south momentum was zero. After the crash, the cars were combined.

$$m_A v_A[\text{N}] = (m_A + m_B)v'_{A/B}[\text{N}]$$

(Note that vector notations are not included, because you are considering only the north-south component of the velocities.)

Solve the equation for the original velocity of car A.

$$v_A[\text{N}] = \frac{(m_A + m_B)v'_{AB}[\text{N}]}{m_A}$$

Substitute the values and solve.

$$v_A[\text{N}] = \frac{(2275\text{ kg} + 1525\text{ kg})(31\ \frac{\text{km}}{\text{h}}\sin 43^\circ)}{2275\text{ kg}}$$

$$v_A[\text{N}] = \frac{(3800\text{ kg})(31\ \frac{\text{km}}{\text{h}})(0.681\ 998\ 4)}{2275\text{ kg}}$$

$$v_A[\text{N}] = 35.3\ \frac{\text{km}}{\text{h}}$$

$$\vec{v}_A \cong 35\ \frac{\text{km}}{\text{h}}[\text{N}]$$

(Note that the north component of car A's velocity before the crash was the total velocity.)

Carry out the same procedure for the east-west direction of the momentum.

$$m_B v_B[\text{E}] = (m_A + m_B)v'_{AB}[\text{E}]$$

$$v_B[\text{E}] = \frac{(m_A + m_B)v'_{AB}[\text{E}]}{m_B}$$

$$v_B[\text{E}] = \frac{(2275\text{ kg} + 1525\text{ kg})(31\ \frac{\text{km}}{\text{h}}\cos 43^\circ)}{1525\text{ kg}}$$

$$v_B[\text{E}] = 56.49\ \frac{\text{km}}{\text{h}}$$

$$\vec{v}_B \cong 56\ \frac{\text{km}}{\text{h}}[\text{E}]$$

Car A was travelling 35 km/h north and car B was travelling 56 km/h east at the instant before the crash. Therefore, car B was speeding and the driver should be ticketed. As well, the driver on the right has the right of way, giving car A the right of way at the intersection. The driver of car B was at fault for the collision. Nevertheless, the driver of car A would have benefited if he or she could have prevented the crash.

Validate the Solution

The angle at which the locked cars moved after the crash was very close to 45°, which means that the momentum of the two cars before the crash was nearly the same. Car B had a smaller mass than car A, so car B must have moving at a greater speed (magnitude of the velocity), which agrees with the results. Also, the units all cancelled to give km/h, which is correct for velocity.

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PRACTICE PROBLEMS

13. A 0.150 kg billiard ball (A) is rolling toward a stationary billiard ball (B) at 10.0 m/s. After the collision, ball A rolls off at 7.7 m/s at an angle of 40.0° clockwise from its original direction. What is the speed and direction of ball B after the collision?

14. A bowling ball with a mass of 6.00 kg rolls with a velocity of 1.20 m/s toward a single standing bowling pin that has a mass of 0.220 kg. When the ball strikes the bowling pin, the pin flies off at an angle of 70.0° counterclockwise from the original direction of the ball, with a velocity of 3.60 m/s. What was the velocity of the bowling ball after it hit the pin?

15. Car A (1750 kg) is travelling due south and car B (1450 kg) is travelling due east. They reach the same intersection at the same time and collide. The cars lock together and move off at 35.8 km/h[E31.6°S]. What was the velocity of each car before they collided?

Angular Momentum

Why is a bicycle easy to balance when you are riding, but falls over when you come to a stop? Why does a toy gyroscope, like the one in Figure 4.10, balance on a pointed pedestal when it is spinning, but falls off the pedestal when it stops spinning? The answer lies in the conservation of angular momentum.

Figure 4.10 When a spinning object begins to fall, its angular momentum resists the direction of the fall.

When an object is moving on a curved path or rotating, it has angular momentum. Angular momentum and linear (or translational) momentum are similar in that they are both dependent on an object's mass and velocity. Analyze Figure 4.11 to find the third quantity that affects angular momentum.

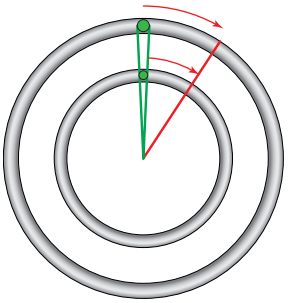


Figure 4.11 As the distance from the centre of rotation increases, a unit of mass must move faster in order to maintain a constant rate of rotation.

WEB LINK

www.mcgrawhill.ca/links/physics12

For information on current accident-investigation research topics and technological developments related to vehicle safety, go to the above Internet site and click on **Web Links**.

PHYSICS & TECHNOLOGY

TARGET SKILLS

- **Predicting**
- **Hypothesizing**
- **Communicating results**
- **Conducting research**

The Physics of a Car Crash

Skid marks, broken glass, mangled pieces of metal — these telltale signs of an automobile crash are stark reminders of the dangers of road travel. For accident investigators, sometimes referred to as "crash analysts" or "reconstructionists," these remnants of a collision can also provide valuable clues that will help in under-standing the cause and the nature of an accident.

The reasons for studying a car crash can vary, depending on who is conducting the investigation. Police officers might be interested in determining how fast a vehicle was being driven prior to a collision in order to know whether to lay criminal charges. Insurance companies might require proof that the occupants of a car were wearing seat belts to make decisions on insurance claims.

Government agencies, meanwhile, conduct large-scale research projects (based on both real-world accidents and staged collisions) that guide in the establishment of safety standards and regulations for the manufacture of automobiles.

Regardless of the purpose, however, most car crash investigations share some common elements. First, they typically draw on the same fundamental concepts and principles of physics that you are learning in this chapter — especially those related to energy and momentum. As well, these investigations often incorporate an array of technological resources to help with both the data-collection and data-analysis phases of the process. Investigators make use of a variety of data-collection tools, ranging from everyday hardware, such as a measuring tape and a camera, to more sophisticated instruments, such as brake-activated chalk guns and laser-operated surveyors' transits.

Customized computer programs, designed with algorithms based on Newtonian mechanics, are used to analyze data collected from the scene of the accident. The length and direction of skid marks, "crush" measurements and stiffness coefficients associated with the damaged vehicles, coefficients of friction specific to the tires and road surface — known or estimated values of these and other relevant parameters are fed into the computer programs. The programs then generate estimates of important variables, such as the speed at the time of impact or the change in speed over the duration of a collision. Quantitative and qualitative data obtained from car accidents is also often coded and added to large computerized databases that can be accessed for future investigations and for research purposes.

Recently, some automobile manufacturers have started installing event data recorders (EDRs) in the vehicles they build. Like the cockpit data recorder or "black box" commonly used in the aviation industry, EDRs in road vehicles record valuable data such as vehicle speed, engine revolutions per minute, brake-switch status, throttle position, and seat-belt use. This information is processed by a vehicle's central computer system to monitor and regulate the operation of such safety components as air bags and antilock brakes. It also provides accident investigators with an additional source of data about the conditions that existed immediately before and during a collision. As a result, EDRs promise to provide significant enhancement to the field of automobile accident investigation.

Analyze

1. Why is it important for car accident investigators to take into account the weather conditions that existed at the time of a car crash? List some specific weather conditions and predict the effects that they might have on the calculations carried out as part of an investigation.
2. Explore the Internet to learn about current research topics and technological developments related to vehicle safety. Prepare a one-page report on your research results.

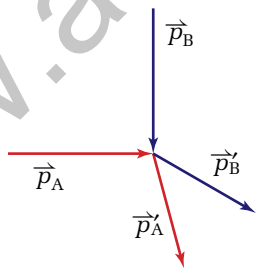
PHYSICS FILE

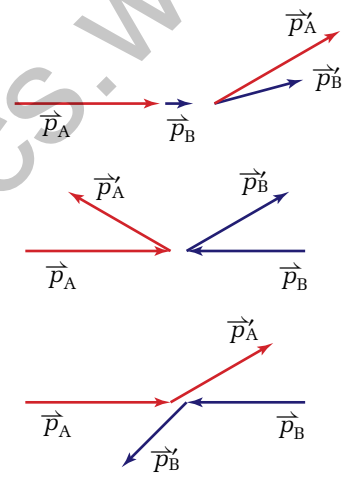
Although Kepler knew nothing about angular momentum, his second law, the law of areas, is an excellent example of the conservation of angular momentum. With somewhat complex mathematics, it is possible to write the law of conservation of angular momentum for a planet in orbit and show that it is equivalent to Kepler's second law.

Picture the movement of a unit of mass in each of the two wheels illustrated in Figure 4.11. If the two wheels are rotating at the same rate, each unit of mass in the large wheel is moving faster than a unit of mass in the small wheel. Thus, r, the distance of a mass from the centre of rotation, affects the angular momentum. The magnitude of the angular momentum, L, of a particle that is moving in a circle is equal to the product of its mass, velocity, and distance from the centre of rotation, or $L = mvr$. You will not pursue a quantitative study of angular momentum any further in this course, but it is essential to be aware of the law of conservation of angular momentum in order to have a complete picture of the important conservation laws of physics. Similar to conservation of linear momentum, the angular momentum of an isolated system is conserved.

4.2 Section Review

1. **C** Explain qualitatively how Newton's third law is related to the law of conservation of momentum.
2. **K/U** What is the difference between an internal force and an external force?
3. **K/U** How does a closed system differ from an isolated system?
4. **K/U** Under what circumstances is the change in momentum of a system equal to zero?
5. **K/U** Define and give an example of recoil.
6. **I** The vectors in the following diagrams represent the momentum of objects before and after a collision. Which of the diagrams (there might be more than one) does *not* represent real collisions? Explain your reasoning.





7. **C** Some collision problems have two unknown variables, such as the velocities of two cars before a collision. Explain how it is possible to find two unknowns by using only the law of conservation of momentum.
8. **MC** Two cars of identical mass are approaching the same intersection, one from the south and one from the west. They reach the intersection at the same time and collide. The cars lock together and move away at an angle of 22° counterclockwise from the road, heading east. Which car was travelling faster than the other before the collision? Explain your reasoning.

4.3 Elastic and Inelastic Collisions

Momentum is conserved in the two collisions pictured in Figure 4.12, but the two cases are quite different. When the metal spheres in the Newton's cradle collided, both momentum and kinetic energy were conserved. When the cars in the photograph crashed, kinetic energy was *not* conserved. This feature divides all collisions into two classes. Collisions in which kinetic energy is conserved are said to be **elastic.** When kinetic energy is *not* conserved, the collisions are **inelastic.**

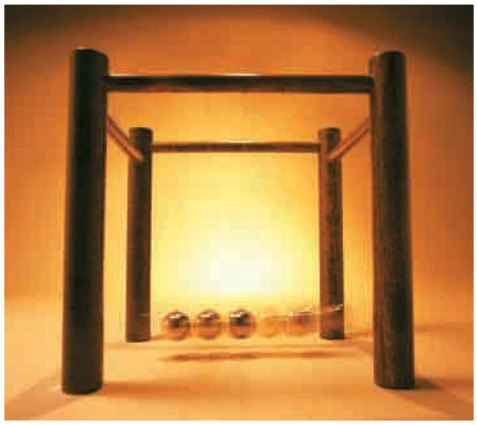


Figure 4.12
How do the collisions pictured here differ from each other?

SECTION EXPECTATIONS

- Distinguish between elastic and inelastic collisions.
- Define and describe the concepts related to momentum, energy, and elastic and inelastic collisions.
- Investigate the laws of conservation of momentum and of energy in one and two dimensions.

KEY TERMS

- elastic
- inelastic

Analyzing Collisions

You can determine whether a collision is elastic or inelastic by calculating both the momentum and the kinetic energy before and after the collision. Since momentum is always conserved at the instant of the collision, you can use the law of conservation of momentum to find unknown values for velocity. Then, use the known and calculated values for velocity to calculate the total kinetic energy before and after the collision. You will probably recall that the equation for kinetic energy is $E = \frac{1}{2}mv^2$.

INVESTIGATION 4-B

Examining Collisions

TARGET SKILLS

- Predicting
- Performing and recording
- Analyzing and interpreting

You have learned the definition of elastic and inelastic collisions, but are there characteristics that allow you to predict whether a collision will be elastic? In this investigation, you will observe and analyze several collisions and draw conclusions regarding whether a type of collision will be elastic or inelastic.

Problem

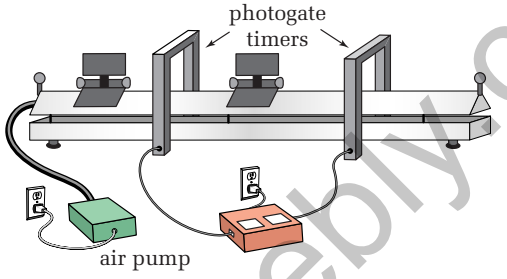
What are the characteristics of elastic and inelastic collisions?

Equipment

- air track (with source of compressed air)
- 2 gliders (identical, either middle- or large-sized)
- 2 photogate timers
- laboratory balance
- 4 glider bumper springs
- 2 Velcro™ bumpers (or a needle and a piece of wax)
- 2 velocity flags (10 cm) (or file cards cut to a 10 cm length)
- modelling clay

Procedure

1. Set up the air track and adjust the levelling screw to ensure that the track is horizontal. You can test whether the track is level by turning on the air pressure and placing a glider on the track. Hold the glider still and then release it. If the track is level, the glider will remain in place. If the glider gradually starts moving, the air track is not level.
2. Attach a velocity flag (or 10 cm card) and two bumper springs to each glider. If only one bumper spring is attached, the glider might not be properly balanced.
3. Position the photogates about one fourth the length of the track from each end, as shown in the diagram. Adjust the height of the photogates so that the velocity flags will pass through the gates smoothly but will trigger the gates.



4. Label one glider "A" and the other glider "B." Use the laboratory balance to determine accurately the mass of each glider.
5. With the air flowing, place glider A on the left end of the air track and glider B in the centre.
6. Perform a test run by pushing glider A so that it collides with glider B. Ensure that the photogates are placed properly so that the flags are not inside the gates when the gliders are in contact. Adjust the positions of the photogates, if necessary.
7. The first set of trials will be like the test run, with glider A on the left end of the track and glider B in the centre. Turn on the photogates and press the reset button. Push glider A and allow it to collide with glider B. Allow both gliders to pass through a photogate after the collision, then catch them before they bounce back and pass through a photogate again. Record the data in a table similar to the one shown on the next page. Since all of the motion will be in one dimension, only positive and negative signs will be needed to indicate direction. Vector notations will not be necessary.
   The displacement, Δd, is the distance that the gliders travelled while passing through the photogates. This displacement is the length of the flag. Time Δt_i is the time that a glider spent in the photogate before the

collision, while Δt_f is the time the glider took to pass through the photogate after the collision. Calculate velocity, v, from the displacement and the time interval. Be sure to include positive and negative signs.

	Glider A (mass = ?)				
Trial	$\Delta\vec{d}$	Δt_i	$\vec{v}_i$	Δt_f	$\vec{v}_f$
1					
2					
3					

	Glider B (mass = ?)		
Trial	$\Delta\vec{d}$	Δt_f	$\vec{v}_f$
1			
2			
3			

8. Increase the mass of glider B by attaching some modelling clay to it. Be sure that the clay is evenly distributed along the glider. If the glider tips to the side or to the front or back, the motion will not be smooth. Determine the mass of glider B. Repeat step 7 for the two gliders, which are now of unequal mass.

9. Exchange the gliders and their labels. That is, the glider with the extra mass is on the left and becomes glider "A." The glider with no extra mass should be in the centre and labelled "B." Repeat step 7 for the new arrangement of gliders.

10. Remove the clay from the glider. Place one glider at each end of the track. Practise starting both gliders at the same time, so that they collide near the middle of the track. The collision must not take place while either glider is in a photogate. When you have demonstrated that you can carry out the collision correctly, perform three trials and record the data in a table similar to the ones shown here. This table will need two additional columns — one for the initial time for glider B and a second for the initial velocity of glider B.

11. Remove the bumper springs from one end of each glider and attach the Velcro™ bumpers. (If you do not have Velcro™ bumpers, you can attach a large needle to one glider and a piece of wax to the other. Test to ensure that the needle will hit the wax when the gliders collide.)

12. With the Velcro™ bumpers attached, perform three sets of trials similar to those in steps 7, 8, and 9. You might need to perform trial runs and adjust the position of the photogates so that both gliders can pass through the photogate before reaching the right-hand end of the air track. Record the data in tables similar to those you used previously.

Analyze and Conclude

1. For each glider in each trial, calculate the initial momentum (before the collision) and the final momentum (after the collision).

2. For each trial, calculate the total momentum of both gliders before the collision and the total momentum of both after the collision.

3. For each trial, compare the momentum before and after the collision. Describe how well the collisions demonstrated conservation of momentum.

4. In any case for which momentum did not seem to be conserved, provide possible explanations for errors.

5. Calculate the kinetic energy of each glider in each trial. Then calculate the total kinetic energy of both gliders before and after the collision for each trial.

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6. Compare the kinetic energies before and after the collisions and decide which collisions were elastic and which were inelastic. Due to measurement errors, do not expect the kinetic energies to be identical before and after a collision. Decide if the values appear to be close enough that the differences could be attributed to measurement errors.

7. Examine the nature of the collisions that you considered to be elastic and those that you classed as inelastic. Look for a trend that would permit you to predict whether a collision would be elastic or inelastic. Discuss your conclusions with the rest of class. How well did your conclusions agree with those of other class members?

Apply and Extend

8. If you have access to an air table, a strobe light, and a Polaroid™ camera, you can observe and collect data for collisions in two dimensions. (If you do not have the equipment, your teacher might be able to provide you with simulated photographs.) Using the laboratory balance, determine the mass of each of two pucks. If the pucks have nearly the same mass, add some mass to one of them, using modelling clay.

CAUTION People with certain medical conditions, such as epilepsy, can experience seizures if exposed to strobe lighting.

9. With the air pressure on, place a puck in the centre of the table. Direct the strobe light onto the table and set up the camera so that it is above the table and pointing down. Turn on the strobe light and set the camera for a long exposure time. At the moment that one partner pushes the other puck toward the stationary puck in the centre of the table, the other partner should take a picture. Take enough photographs to provide each pair of partners with a photograph.

10. Determine the scale of the photograph by determining the ratio of the size of the air table to its apparent size in the photograph. Measure two or three distances before and after the collision. Correct the distances by using the scale that you determined. Measure the angles that the pucks took after the collision in relation to the original direction.

11. Using the rate at which the strobe light was flashing, determine the time between flashes. Calculate the velocity, momentum, and kinetic energy of each puck before and after the collision.

12. Compare the total momentum before and after the collision and comment on how well the motion seemed to obey the law of conservation of momentum.

13. Compare the total kinetic energies before and after the collision. Decide whether the collisions were elastic or nearly so.

14. Compare your results from your one-dimensional data and two-dimensional data, and comment on any differences that you noticed.

15. Review the results you obtained in Investigation 4-A, Newton's Cradle. Do you think the collisions were elastic or inelastic? Explain why.

16. Support your answer to question 15 by performing trial calculations. Assume that the spheres in Newton's cradle each has a mass of 0.200 kg and that, when one sphere collided with the row, it was moving at 0.100 m/s. Imagine that, when one sphere collided with the row, two spheres bounced up from the opposite end. Calculate the velocity that the two spheres would have to have in order to conserve momentum. Calculate the kinetic energy before and after. Use these calculations to explain why you did not observe certain patterns in the motion of Newton's cradle.

SAMPLE PROBLEM

Classifying a Collision

A 0.0520 kg golf ball is moving east with a velocity of 2.10 m/s when it collides, head on, with a 0.155 kg billiard ball. If the golf ball rolls directly backward with a velocity of –1.04 m/s, was the collision elastic?

Conceptualize the Problem

- *Momentum* is always conserved in a collision.
- If the collision is *elastic*, kinetic energy must also be conserved.
- The motion is in one dimension, so only positive and negative signs are necessary to indicate directions.

Identify the Goal

Is the total kinetic energy of the system before the collision, E_{kg}, equal to the total kinetic of the system after the collision, $E'_{kg} + E'_{kb}$?

Identify the Variables and Constants

Known		Implied	Unknown	
$m_g = 0.0520$ kg	$v_g = +2.10 \frac{m}{s}$	$v_b = 0.0 \frac{m}{s}$	v'_b	E'_{kg}
$m_b = 0.155$ kg	$v'_g = -1.04 \frac{m}{s}$		E_{kg}	E'_{kb}

Develop a Strategy

Since momentum is always conserved, use the law of conservation of momentum to find the velocity of the billiard ball after the collision.

$$m_g v_g + m_b v_b = m_g v'_g + m_b v'_b$$
$$m_g v_g + 0.0 - m_g v'_g = m_b v'_b$$
$$v'_b = \frac{m_g v_g - m_g v'_g}{m_b}$$
$$v'_b = \frac{(0.0520\text{ kg})(2.10\ \frac{m}{s}) - (0.0520\text{ kg})(-1.04\ \frac{m}{s})}{0.155\text{ kg}}$$
$$v'_b = 1.0534\ \frac{m}{s}$$

Calculate the kinetic energy of the golf ball before the collision.

$$E_{kg} = \frac{1}{2} m_g v_g^2$$
$$E_{kg} = \frac{1}{2}(0.0520\text{ kg})\left(2.10\ \frac{m}{s}\right)^2$$
$$E_{kg} = 0.114\ 66\text{ J}$$

Calculate the sum of the kinetic energies of the balls after the collision.

$$E'_{kg} = \frac{1}{2} m_g v'^2_g$$
$$E'_{kg} = \frac{1}{2}(0.0520\text{ kg})\left(-1.04\ \frac{m}{s}\right)^2$$
$$E'_{kg} = 0.028\ 12\text{ J}$$
$$E'_{kb} = \frac{1}{2} m_b v'^2_b$$
$$E'_{kb} = \frac{1}{2}(0.155\text{ kg})\left(1.0534\ \frac{m}{s}\right)^2$$
$$E'_{kb} = 0.086\ 00\text{ J}$$
$$E'_{kg} + E'_{kb} = 0.028\ 12\text{ J} + 0.085\ 99\text{ J}$$
$$E'_{kg} + E'_{kb} = 0.114\ 12\text{ J}$$

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The kinetic energies before and after the collision are the same to the third decimal place. Therefore, the collision was probably elastic.

Validate the Solution

Although the kinetic energies before and after the collision differ in the fourth decimal place, the difference is less than 1%. Since the data contained only three significant digits, this difference could easily be due to the precision of the measurement. Therefore, it is fair to say that the collision was elastic.

PRACTICE PROBLEMS

16. A billiard ball of mass 0.155 kg moves with a velocity of 12.5 m/s toward a stationary billiard ball of identical mass and strikes it with a glancing blow. The first billiard ball moves off at an angle of 29.7° clockwise from its original direction, with a velocity of 9.56 m/s. Determine whether the collision was elastic.

17. Car A, with a mass of 1735 kg, was travelling north at 45.5 km/h and Car B, with a mass of 2540 kg, was travelling west at 37.7 km/h when they collided at an intersection. If the cars stuck together after the collision, what was their combined momentum? Was the collision elastic or inelastic?

Elastic Collisions

By now, you have probably concluded that when objects collide, become deformed, and stick together, the collision is inelastic. Physicists say that such a collision is *completely inelastic*. Conversely, when hard objects such as billiard balls collide, bounce off each other, and return to their original shape, they have undergone elastic collisions. Very few collisions are perfectly elastic, but in many cases, the loss of kinetic energy is so small that it can be neglected.

Since both kinetic energy and momentum are conserved in perfectly elastic collisions, as many as four independent equations can be used to solve problems. Since you have two equations, you can solve for up to four unknown quantities. When combining these equations, however, the math becomes quite complex for all cases except head-on collisions, for which all motion is in one dimension.

An analysis of head-on collisions yields some very informative results, however. For example, if you know the velocities of the two masses before a collision, you can determine what the velocities will be after the collision. The following derivation applies to a mass, m_1, that is rolling toward a stationary mass, m_2. Follow the steps to find the velocities of the two objects after the collision in terms of their masses and the velocity of the first mass before the collision. Since the motion in head-on collisions is in one dimension, vector notations will not be used.

LANGUAGE LINK

In science, the word "elastic" does not mean "easily stretched." In fact, it can mean exactly the opposite. For example, glass is very elastic, up to its breaking point. Also, "elastic" is the opposite of "plastic." Find the correct meanings of the words "elastic" and "plastic" and then explain why "elastic" is an appropriate term to apply to collisions in which kinetic energy is conserved.

- Write the equations for the conservation of momentum and kinetic energy for a perfectly elastic collision, inserting zero for the velocity of the second mass before the collision.

$$m_1v_1 + 0 = m_1v_1' + m_2v_2'$$
$$\frac{1}{2}m_1v_1^2 + 0 = \frac{1}{2}m_1v_1'^2 + \frac{1}{2}m_2v_2'^2$$

- Multiply by 2 both sides of the equation for conservation of kinetic energy.

$$m_1v_1^2 = m_1v_1'^2 + m_2v_2'^2$$

- Algebraically rearrange both equations so that terms describing mass 1 are on the left-hand side of the equations and terms describing mass 2 are on the right-hand side.

$$m_1v_1 - m_1v_1' = m_2v_2'$$
$$m_1v_1^2 - m_1v_1'^2 = m_2v_2'^2$$

- Factor m_1 out of the left-hand side of both equations.

$$m_1(v_1 - v_1') = m_2v_2'$$
$$m_1(v_1^2 - v_1'^2) = m_2v_2'^2$$

- Divide the first equation by the second equation.

$$\frac{\cancel{m_1}(v_1 - v_1')}{\cancel{m_1}(v_1^2 - v_1'^2)} = \frac{\cancel{m_2}v_2'}{\cancel{m_2}v_2'^2}$$

- Notice that the masses cancel. Expand the expression in the denominator on the left. Notice that it is the difference of perfect squares.

$$\frac{\cancel{(v_1 - v_1')}}{\cancel{(v_1 - v_1')}(v_1 + v_1')} = \frac{\cancel{v_2'}}{v_2'^{\cancel{2}}}$$

- Simplify. Solve the equation for v_2' by inverting. Also, solve the equation for v_1'.

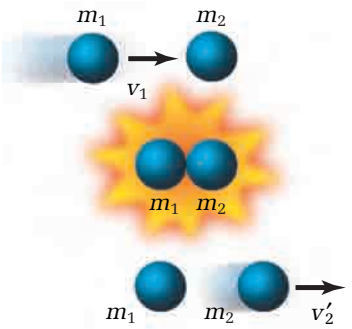
$$\frac{1}{(v_1 + v_1')} = \frac{1}{v_2'}$$
$$v_2' = v_1 + v_1'$$
$$v_1' = v_2' - v_1$$

- Develop two separate equations by substituting the values for v_1' and v_2' above into the equation for conservation of momentum, $m_1v_1 - m_1v_1' = m_2v_2'$. Expand and rearrange the equations and then solve for v_1' (left) and v_2' (right).

$$m_1v_1 - m_1v_1' = m_2v_2'$$
$$m_1v_1 - m_1v_1' = m_2(v_1 + v_1')$$
$$m_1v_1 - m_1v_1' = m_2v_1 + m_2v_1'$$
$$m_1v_1' + m_2v_1' = m_1v_1 - m_2v_1$$
$$v_1'(m_1 + m_2) = v_1(m_1 - m_2)$$
$$v_1' = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)v_1$$

$$m_1v_1 - m_1v_1' = m_2v_2'$$
$$m_1v_1 - m_1(v_2' - v_1) = m_2v_2'$$
$$m_1v_1 - m_1v_2' + m_1v_1 = m_2v_2'$$
$$2m_1v_1 = m_1v_2' + m_2v_2'$$
$$2m_1v_1 = (m_1 + m_2)v_2'$$
$$v_2' = \left(\frac{2m_1}{m_1 + m_2}\right)v_1$$

The two equations derived above allow you to find the velocities of two masses after a head-on collision in which a moving mass collides with a stationary mass. Without doing any calculations, however, you can draw some general conclusions. First, consider the case in which the two masses are identical.



When one moving mass collides head on with an identical stationary mass, the first mass stops. The second mass then moves with a velocity identical to the original velocity of the first mass.

Case 1: $m_1 = m_2$

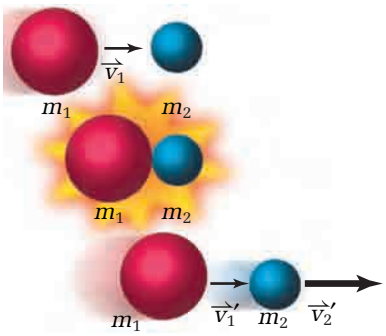
Since the masses are equal, call them both "m." Substitute m into the two equations for the velocities of the two masses after the collision. Then, mathematically simplify the equations.

$$v_1' = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) v_1 \qquad v_2' = \left(\frac{2m_1}{m_1 + m_2}\right) v_1$$

$$v_1' = \left(\frac{m - m}{m + m}\right) v_1 \qquad v_2' = \left(\frac{2m}{m + m}\right) v_1$$

$$v_1' = \left(\frac{0}{m + m}\right) v_1 \qquad v_2' = \left(\frac{\cancel{2m}}{\cancel{2m}}\right) v_1$$

$$v_1' = 0 \qquad v_2' = v_1$$



When one moving mass collides head on with a much smaller stationary mass, the first mass continues at nearly the same speed. The second mass then moves with a velocity that is approximately twice the original velocity of the first mass.

Case 2: $m_1 >>> m_2$

Since mass 1 is much larger than mass 2, you can almost ignore the mass of the second object in your calculations. You can therefore make the following approximations.

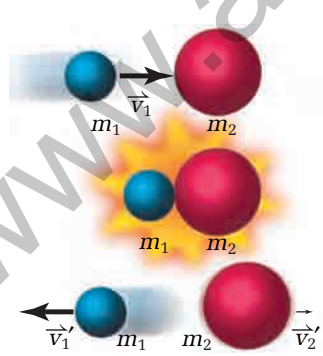
$$m_1 - m_2 \cong m_1 \quad \text{and} \quad m_1 + m_2 \cong m_1$$

Substitute these approximations into the two equations for the velocities of the two masses after the collision. Then, mathematically simplify the equations.

$$v_1' = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) v_1 \qquad v_2' = \left(\frac{2m_1}{m_1 + m_2}\right) v_1$$

$$v_1' \cong \left(\frac{\cancel{m_1}}{\cancel{m_1}}\right) v_1 \qquad v_2' \cong \left(\frac{2\cancel{m_1}}{\cancel{m_1}}\right) v_1$$

$$v_1' \cong v_1 \qquad v_2' \cong 2v_1$$



Case 3: $m_1 <<< m_2$

Since mass 1 is much smaller than mass 2, you can ignore the mass of the first object in your calculations. You can therefore make the following approximations.

$$m_1 - m_2 \cong -m_2 \quad \text{and} \quad m_1 + m_2 \cong m_2 \quad \text{and} \quad m_1 \cong 0$$

Substitute these approximations into the two equations for the velocities of the two masses after the collision. Then mathematically simplify the equations.

$$v_1' = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) v_1 \qquad v_2' = \left(\frac{2m_1}{m_1 + m_2}\right) v_1$$

$$v_1' \cong \left(\frac{-\cancel{m_2}}{\cancel{m_2}}\right) v_1 \qquad v_2' \cong \left(\frac{0}{m_2}\right) v_1$$

$$v_1' \cong -v_1 \qquad v_2' \cong 0$$

When one moving mass collides head on with a much larger stationary mass, the first mass bounces backward with a velocity opposite in direction and almost the same in magnitude as its original velocity. The motion of the second mass is almost imperceptible.

Conceptual Problems

- Using the special cases of elastic collisions, qualitatively explain what would happen in each of the following situations.
  - **(a)** A bowling ball collides head on with a single bowling pin.
  - **(b)** A golf ball hits a tree.
  - **(c)** A marble collides head on with another marble that is not moving.
- Cars, trucks, and motorcycles do not undergo elastic collisions, but the general trend of the motion is similar to the motion of objects involved in elastic collisions. Describe, in very general terms, what would happen in each of the following cases. In each case, assume that the vehicles did not become attached to each other.
  - **(a)** A very small car runs into the back of a parked tractor-trailer.
  - **(b)** A mid-sized car runs into the back of another mid-sized car that has stopped at a traffic light.
  - **(c)** A pickup truck runs into a parked motorcycle.

ELECTRONIC LEARNING PARTNER

Study the effects of the variable elasticity of a bouncing ball by using the interactive activity in your Electronic Learning Partner.

Inelastic Collisions

When you are working with inelastic collisions, you can apply only the law of conservation of momentum to the motion of the objects at the instant of the collision. Depending on the situation, however, you might be able to apply the laws of conservation of energy to motion just before or just after the collision. For example, a ballistic pendulum can be used to measure the velocity of a projectile such as a bullet, as illustrated in Figure 4.13. When the bullet collides with the wooden block of the ballistic pendulum, it becomes embedded in the wood, making the collision completely inelastic.

After the collision, you can apply the law of conservation of mechanical energy to the motion of the pendulum. The kinetic energy of the pendulum at the instant after the collision is converted into potential energy of the pendulum bob. By measuring the height to which the pendulum rises, you can calculate the velocity of the bullet just before it hit the pendulum, as shown in the following sample problem.

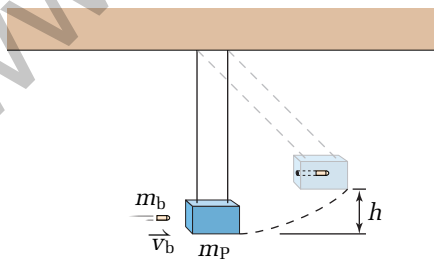
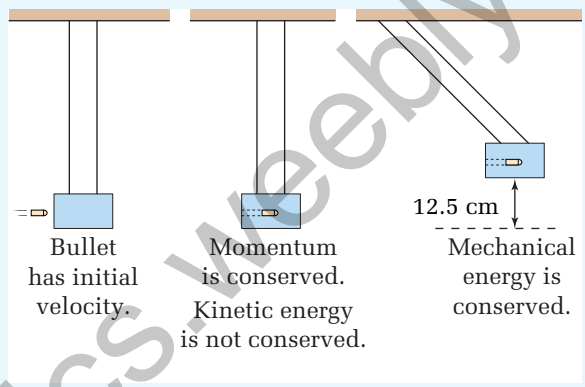


Figure 4.13 A ballistic pendulum is designed to have as little friction as possible. Therefore, you can assume that, at the top of its swing, the gravitational potential energy of the pendulum bob is equal to the kinetic energy of the pendulum bob at the lowest point of its motion.

Energy Conservation Before and After a Collision

1. **A forensic expert needed to find the velocity of a bullet fired from a gun in order to predict the trajectory of a bullet. He fired a 5.50 g bullet into a ballistic pendulum with a bob that had of mass 1.75 kg. The pendulum swung to a height of 12.5 cm above its rest position before dropping back down. What was the velocity of the bullet just before it hit and became embedded in the pendulum bob?**

Conceptualize the Problem



- Sketch the positions of the bullet and pendulum bob just before the collision, just after the collision, and with the pendulum at its highest point.
- When the bullet hit the pendulum, *momentum* was *conserved*.
- If you can find the *velocity* of the combined bullet and pendulum bob after the collision, you can use conservation of momentum to find the *velocity* of the bullet before the collision.
- The collision was completely inelastic so *kinetic energy* was *not* conserved.
- However, you can assume that the friction of the pendulum is negligible, so *mechanical energy* of the pendulum was *conserved*.
- The *gravitational potential energy* of the combined masses at the highest point of the pendulum is equal to the *kinetic energy* of the combined masses at the lowest point of the pendulum.
- If you know the kinetic energy of the combined masses just after the collision, you can find the *velocity* of the masses just *after* the collision.
- Use the subscripts "b" for the bullet and "p" for the pendulum.

Identify the Goal

The velocity, v_b, of the bullet just before it hit the ballistic pendulum

Identify the Variables and Constants

Known		Implied	Unknown	
$m_b = 5.50$ g	$\Delta h = 12.5$ cm	$g = 9.81 \frac{m}{s^2}$	$\vec{v}_b$	E_g
$m_p = 1.75$ kg			$\vec{v}_p$	E_k

Develop a Strategy

To find the velocity of the combined masses of the bullet and pendulum bob just after the collision, use the relationship that describes the conservation of mechanical energy of the pendulum.

$$E_{k(bottom)} = E_{g(top)}$$

Substitute the expressions for kinetic energy and gravitational potential energy that you learned in previous physics courses. Solve for velocity. Convert all units to SI units.

$$\frac{1}{2}\cancel{m}v_{\text{bottom}}^2 = \cancel{m}g\Delta h$$

$$v_{\text{bottom}}^2 = 2g\Delta h$$

$$v_{\text{bottom}} = \sqrt{2g\Delta h}$$

$$v_{\text{bottom}} = \sqrt{2\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)(12.5\ \cancel{\text{cm}})\left(\frac{1\ \text{m}}{100\ \cancel{\text{cm}}}\right)}$$

$$v_{\text{bottom}} = \sqrt{2.4525\ \frac{\text{m}^2}{\text{s}^2}}$$

$$v_{\text{bottom}} = \pm 1.566\ \frac{\text{m}}{\text{s}}$$

Define the direction of the bullet as positive during and immediately after the collision.

Apply the conservation of momentum to find the velocity of the bullet before the collision. Convert all units to SI units.

$$m_{\text{b}}\vec{v}_{\text{b}} + m_{\text{p}}\vec{v}_{\text{p}} = m_{\text{b}}\vec{v}'_{\text{b}} + m_{\text{p}}\vec{v}'_{\text{p}}$$

$$m_{\text{b}}\vec{v}_{\text{b}} + 0 = (m_{\text{b}} + m_{\text{p}})\vec{v}'_{\text{b/p}}$$

$$\vec{v}_{\text{b}} = \frac{(m_{\text{b}} + m_{\text{p}})v'_{\text{b/p}}}{m_{\text{b}}}$$

$$\vec{v}_{\text{b}} = \frac{\left[5.50\ \cancel{\text{g}}\left(\frac{1\ \text{kg}}{1000\ \cancel{\text{g}}}\right) + 1.75\ \text{kg}\right]1.566\ \frac{\text{m}}{\text{s}}}{5.50\ \text{g}\left(\frac{1\ \text{kg}}{1000\ \cancel{\text{g}}}\right)}$$

$$\vec{v}_{\text{b}} = \frac{(1.7555\ \cancel{\text{kg}})1.566\ \frac{\text{m}}{\text{s}}}{0.005\ 50\ \cancel{\text{kg}}}$$

$$\vec{v}_{\text{b}} = 499.8387\ \frac{\text{m}}{\text{s}}$$

$$\vec{v}_{\text{b}} \cong 5.00 \times 10^2\ \frac{\text{m}}{\text{s}}\ \text{[in positive direction]}$$

The velocity of the bullet just before the collision was about 500 m/s in the positive direction.

Validate the Solution

In both calculations, the units cancelled to give metres per second, which is correct for velocity. The velocity of 500 m/s is a reasonable velocity for a bullet.

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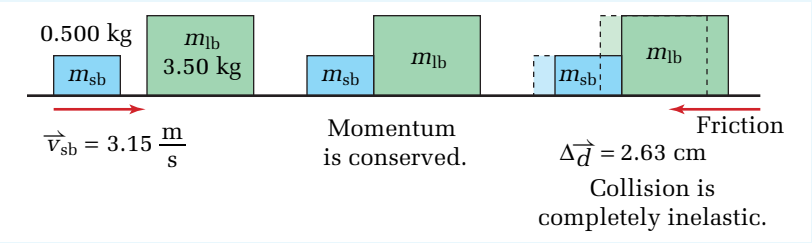
2. A block of wood with a mass of 0.500 kg slides across the floor toward a 3.50 kg block of wood. Just before the collision, the small block is travelling at 3.15 m/s. Because some nails are sticking out of the blocks, the blocks stick together when they collide. Scratch marks on the floor show that they slid 2.63 cm before coming to a stop. What is the coefficient of friction between the wooden blocks and the floor?

Conceptualize the Problem

- Sketch the blocks just before, at the moment of, and after the collision, when they came to a stop.
- *Momentum* is conserved during the collision.



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- Since the blocks stuck together, the collision was *completely inelastic*, so *kinetic energy* was *not* conserved. Some kinetic energy was lost to sound, heat, and deformation of the wood during the collision.
- Some *kinetic energy* remained after the collision.
- The force of *friction* did *work* on the moving blocks, converting the remaining kinetic energy into heat.
- Due to the *law of conservation of energy*, you know that the *work* done by the force of *friction* was equal to the *kinetic energy* of the blocks at the instant after the collision.
- Since the motion is in one direction, use a plus sign to symbolize direction.
- Use the subscripts "sb" for the small block, "lb" for the large block, and "cb" for connected blocks.

Identify the Goal

The coefficient of friction, μ, between the wooden blocks and the floor

Identify the Variables and Constants

Known		Implied	Unknown		
$m_{sb} = 0.500\ \text{kg}$	$\vec{v}_{sb} = 3.15\ \frac{\text{m}}{\text{s}}$	$g = 9.81\ \frac{\text{m}}{\text{s}^2}$	μ	W	$\vec{F}_N$
$m_{lb} = 3.50\ \text{kg}$	$\Delta\vec{d} = 2.63\ \text{cm}$	$\vec{v}_{lb} = 0.00\ \frac{\text{m}}{\text{s}}$	$\vec{F}_f$	E_k	$\vec{v}'_{cb}$

Develop a Strategy

Apply the law of conservation of energy to find the velocity of the connected blocks of wood after the collision.

$$m_{sb}\vec{v}_{sb} + m_{lb}\vec{v}_{lb} = m_{sb}\vec{v}'_{sb} + m_{lb}\vec{v}'_{lb}$$

$$m_{sb}\vec{v}_{sb} + 0 = (m_{sb} + m_{lb})\vec{v}'_{cb}$$

$$\vec{v}'_{cb} = \frac{m_{sb}\vec{v}_{sb}}{m_{sb} + m_{lb}}$$

$$\vec{v}'_{cb} = \frac{(0.500\ \text{kg})(3.15\ \frac{\text{m}}{\text{s}})}{0.500\ \text{kg} + 3.50\ \text{kg}}$$

$$\vec{v}'_{cb} = \frac{1.575\ \cancel{\text{kg}}\frac{\text{m}}{\text{s}}}{4.00\ \cancel{\text{kg}}}$$

$$\vec{v}'_{cb} = 0.393\ 75\ \frac{\text{m}}{\text{s}}\ [\text{to the right}]$$

Due to the law of conservation of energy, the work done on the blocks by the force of friction is equal to the kinetic energy of the connected blocks after the collision.

$$W_{(\text{to stop blocks})} = E_k\ (\text{after collision})$$

Substitute the expressions for work and kinetic energy into the equations.

$$F_{||}\Delta d = \frac{1}{2}mv^2$$

Friction is the force doing the work, and it is always parallel to the direction of motion. Substitute the formula for the force of friction.

$$F_f\Delta d = \frac{1}{2}mv^2$$

$$\mu F_N\Delta d = \frac{1}{2}mv^2$$

Since the blocks are moving horizontally, the normal force is the weight of the blocks. Substitute the weight into the expression and solve for the coefficient of friction.

$$\mu mg\Delta d = \frac{1}{2}mv^2$$

$$\mu = \frac{\frac{1}{2}\cancel{m}v^2}{\cancel{m}g\Delta d}$$

$$\mu = \frac{v^2}{2g\Delta d}$$

$$\mu = \frac{(0.393\ 75\ \frac{\text{m}}{\text{s}})^2}{2(9.81\ \frac{\text{m}}{\text{s}^2})(2.63\ \cancel{\text{cm}})(\frac{1\ \text{m}}{100\ \cancel{\text{cm}}})}$$

$$\mu = \frac{0.15\ 504\ \cancel{\frac{\text{m}^2}{\text{s}^2}}}{0.5160\ \cancel{\frac{\text{m}^2}{\text{s}^2}}}$$

$$\mu = 0.300\ 46$$

$$\mu \cong 0.300$$

The coefficient of friction between the blocks and the floor is 0.300.

Validate the Solution

All of the units cancel, which is correct because the coefficient of friction is unitless. The value of 0.300 is quite reasonable for a coefficient of friction between wood and another similar surface.

PRACTICE PROBLEMS

18. A 12.5 g bullet is shot into a ballistic pendulum that has a mass of 2.37 kg. The pendulum rises a distance of 9.55 cm above its resting position. What was the speed of the bullet?

19. A student flings a 23 g ball of putty at a 225 g cart sitting on a slanted air track that is 1.5 m long. The track is slanted at an angle of 25° with the horizontal. If the putty is travelling at 4.2 m/s when it hits the cart, will the cart reach the end of the track before it stops and slides back down? Support your answer with calculations.

20. A car with a mass of 1875 kg is travelling along a country road when the driver sees a deer dart out onto the road. The driver slams on the brakes and manages to stop before hitting the deer. The driver of a second car (mass of 2135 kg) is driving too close and does not see the deer. When the driver realizes that the car ahead is stopping, he hits the brakes but is unable to stop. The cars lock together and skid another 4.58 m. All of the motion is along a straight line. If the coefficient of friction between the dry concrete and rubber tires is 0.750, what was the speed of the second car when it hit the stopped car?

21. You and some classmates read that the record for the speed of a pitched baseball is 46.0 m/s. You wanted to know how fast your school's star baseball pitcher could throw. Not having a radar gun, you used the concepts you learned in physics class. You made a pendulum with a rope and a small box lined with a thick layer of soft clay, so that the baseball would stick to the inside of the box. You drew a large protractor on a piece of paper and placed it at the top, so that one student could read the maximum angle of the rope when the pendulum swung up. The rope was 0.955 m long, the box with clay had a mass of 5.64 kg, and the baseball had a mass of 0.350 kg. Your star pitcher pitched a fastball into the box and the student reading

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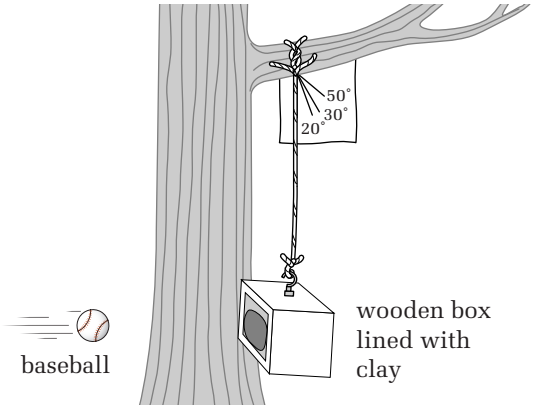
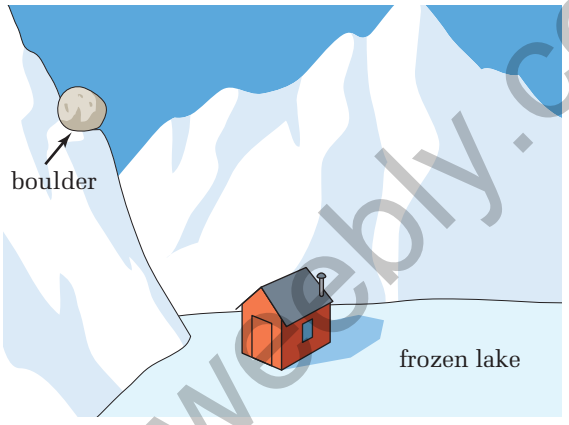
the angle recorded a value of 20.0° from the resting, vertical position. How fast did your star pitcher pitch the ball?



22. A 55.6 kg boulder sat on the side of a mountain beside a lake. The boulder was 14.6 m above the surface of the lake. One winter night, the boulder rolled down the mountain, directly into a 204 kg ice-fishing shack that was sitting on the frozen lake. What was the velocity of the boulder and shack at the instant that they began to slide across the ice? If the coefficient of friction between the shack and the rough ice was 0.392, how far did the shack and boulder slide?



4.3 Section Review

1. K/U What is the difference between an elastic collision and an inelastic collision?

2. C Describe an example of an elastic collision and an example of an inelastic collision that were not discussed in the text.

3. C Given a set of data for a collision, describe a step-by-step procedure that you could use to determine whether the collision was elastic.

4. I The results of the head-on collision in which the moving mass was much larger than the stationary mass ($m_1 >>> m_2$) showed that (a) that the velocity of mass 1 after the collision was almost the same as it had been before the collision and (b) that mass 2, which was stationary before the collision, attained a velocity nearly double that of mass 1 after the collision. Explain how it is possible for kinetic energy ($\frac{1}{2}mv^2$) to be conserved in such a collision, when there was a negligible change in the velocity of mass 1 and a large increase in the velocity of mass 2.

5. MC Imagine that you have a very powerful water pistol. Describe in detail an experiment that you could perform, including the measurements that you would make, to determine the velocity of the water as it leaves the pistol.

CAREERS IN PHYSICS

The Invisible Universe

Go outside on a cloudless night and look up. You might see the Moon, a few planets, and many stars. The universe stretches before you, but your eyes are not taking in the full picture. Astronomer Dr. Samar Safi-Harb and her colleagues see a very different universe by using instruments that detect X rays and several other wavelengths of electromagnetic radiation that are invisible to the human eye.

Dr. Safi-Harb is an assistant professor with the Department of Physics and Astronomy at the University of Manitoba. She uses the instruments aboard satellites such as NASA's Chandra X-ray Observatory to research the death throes of super-massive stars. When a super-massive star runs out of nuclear energy, it collapses under its own weight and its outer layers burst into space in a violent explosion called a "supernova." In some cases, the mass left behind compacts into a neutron star. This astounding type of star is so dense that all of its matter fits into a volume no larger than that of a city. A neutron star, along with its strong magnetic field, spins incredibly fast — up to several dozen times per second!

Dr. Samar Safi-Harb

A neutron star is a remarkable source of electromagnetic radiation. As its magnetic field spins through space, it creates an electric field that generates powerful beams of electromagnetic waves, ranging from radio waves to gamma rays. If the beams sweeps past Earth, astronomers detect them as pulses, like a lighthouse beacon flashing past. Such neutron stars are called "pulsars."

The Crab Nebula is one of Dr. Safi-Harb's favourite objects in the sky. It is the remains of a star that went supernova in 1054 A.D. The Crab Nebula is energized by fast-moving particles emitted from its central pulsar. "It looks different at different wavelengths," she explains. "The radio image reveals a nebula a few light-years across that harbours low-energy electrons. The diffuse optical nebula shines by intermediate energy particles, showing a web of filaments that trace the debris of the explosion. The X-ray image reveals a smaller nebula — the central powerhouse — containing very energetic particles. Its jets, rings, and wisp-like structures unveil the way pulsars dump energy into their surroundings."

In part, the ground-breaking work of Jocelyn Bell, the discoverer of pulsars, inspired Dr. Safi-Harb to follow this line of research. Although an astronomer, she has a doctorate in physics from the University of Wisconsin, Madison. Few universities today have astronomy programs that stand alone from physics.

Going Further

1. Earth is orbited by a wide array of satellites that explore the sky at high-energy and low-energy wavelengths. Research two or more of these satellites and describe how images taken by them enhance our understanding of the universe.
2. When two objects in space approach or recede from one another at great speed, light emitted from either object appears altered by the time it reaches the other object. Research and describe what astronomers mean when they talk about a "red shift" or a "blue shift" in light.

WEB LINK

www.mcgrawhill.ca/links/physics12

Radio, infrared, optical, X-ray, and gamma-ray images of our galaxy, the Milky Way, can be found on the Internet. You can also learn more about Dr. Safi-Harb's work and see images of several of her favourite objects in space by going to the above Internet site and clicking on **Web Links**.

CHAPTER 4 Review

REFLECTING ON CHAPTER 4

- Newton expressed his second law in terms of momentum: $\vec{F} = \frac{\Delta\vec{p}}{\Delta t}$.
- Rearrangement of Newton's form of the second law yields the quantities of impulse, $\vec{F}\Delta t$, and a change in momentum, $\Delta\vec{p}$, and shows that impulse is equal to the change in momentum: $\vec{F}\Delta t = \Delta\vec{p}$. This expression is called the "impulse-momentum theorem."
- Momentum is mass times velocity $\vec{p} = m\vec{v}$.
- The concept of impulse plays a significant role in the design of safety systems. By extending the time, Δt, of a collision, you can reduce the amount of force, $\vec{F}$, exerted.
- By applying Newton's third law, you can show that momentum is conserved in a collision.
- The momentum of an isolated system is conserved.
- Recoil is the interaction of two objects that are in contact with each other and exert a force on each other. Momentum is conserved during recoil.
- Kinetic energy is conserved in elastic collisions.
- Kinetic energy is *not* conserved in inelastic collisions.

Knowledge/Understanding

1. Write Newton's second law in terms of momentum. Show how this expression of Newton's law leads to the definition of impulse and to the impulse-momentum theorem.
2. Give an example of a situation in which it is easier to measure data that you can use to calculate a change in momentum than it is to determine forces and time intervals.
3. In many professional auto races, stacks of old tires are placed in front of walls that are close to turns in the racetrack. Explain in detail, using the concept of impulse, why the tires are stacked there.
4. Define "internal force" and "external force" and explain how these terms relate to the law of conservation of momentum.
5. At the instant after a car crash, the force of friction acts on the cars, causing a change in their momentum. Explain how you can apply the law of conservation of momentum to car crashes.
6. When two objects recoil, they start at rest and then push against each other and begin to move. Initially, since their velocities are zero, they have no momentum. When they begin to move, they have momentum. Explain how momentum can be conserved during recoil, when the objects start with no momentum and then acquire momentum.
7. An object that is moving in a circle has angular momentum. Explain why the amount of angular momentum depends on the object's distance from the centre of rotation.
8. Assume that you are provided with data for a collision between two masses that undergo a collision. Describe, step by step, how you would determine whether the collision was elastic or inelastic.
9. Under what conditions is a collision *completely inelastic*?
10. The equations $v_1' = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) v_1$ and $v_2' = \left(\frac{2m_1}{m_1 + m_2}\right) v_1$ were developed for perfectly elastic head-on collisions between two masses, when mass 1 was moving and mass 2 was stationary before the collision. Use these equations to show that, after the collision, the first mass will come to a complete stop and the second mass will move away with a velocity identical to the original velocity of the first mass.

11. Explain how a forensic expert can determine the velocity of a bullet by using a ballistic pendulum.

Inquiry

12. The International Tennis Federation (ITF) approaches you, a physics student, complaining that too many games in tournaments are being won on the strength of either player's serve. They ask you to examine ways to slow down the serve and thus make the game more interesting to watch. Devise a series of experiments to test the ball, the type of racquet used, and the surface of the courts, from which you could make recommendations to the ITF about how to improve the game of tennis.
13. Design a small wooden cart, with several raw eggs as passengers. Incorporate elements into your design to ensure that the passengers suffer no injury if the cart was involved in a collision while travelling at 5.0 m/s. If possible, test your design.
14. Design and carry out an experiment in which an object initially had gravitational potential energy that is then converted into kinetic energy. The object then collides with another object that is stationary. Include in your design a method for testing whether mechanical energy is conserved in the first part of the experiment. If possible, test your design.

Communication

15. A car and a bicycle are travelling with the same velocity. Which vehicle has greater momentum? Explain your reasoning.
16. It is a calm day on a lake and you and a friend are on a sailboat. Your friend suggests attaching a fan to the sailboat and blowing air into the sails to propel the sailboat ahead. Explain whether this would work.
17. Imagine you are standing, at rest, in the middle of a pond on *perfectly frictionless* ice. Explain what will happen when you try to walk back to shore. Describe a possible method that you could use to start moving. Would this method allow you to reach shore? Explain.
18. You and a friend arrive at the scene of a car crash. The cars were both severely mangled. Your friend is appalled at the damage to the cars and says that cars ought to be made to be sturdier. Explain to your friend why this reaction to the crash is unwarranted.
19. Start with expressions that apply the impulse-momentum theorem to two objects and use Newton's third law to derive the conservation of momentum for a collision between the two objects. Explain and justify every substitution and mathematical step in detail.
20. Write the units for impulse and for momentum. Show that these combinations of units are equivalent and explain your reasoning in detail.
21. Movie stunt people can fall from great heights and land safely on giant air bags. Using the principles of conservation of momentum and impulse, explain how this is possible.
22. Why is a "follow-through" important in sports in, for example, hitting, kicking, or throwing a ball?
23. A boy jumps from a boat onto a dock. Explain why he would have to jump with more energy than he would need if he was to jump the same distance from one dock to another.
24. In soccer, goalkeepers need to jump slightly forward to avoid being knocked into the goal by a fast-moving soccer ball as they jump up to catch it. If the ball and goalkeeper are momentarily at rest after the catch, what must have been the relative momenta of the goalkeeper and ball just before the catch?

Making Connections

25. Many automobiles are now equipped with air bags that are designed to prevent injuries to passengers if the vehicle is involved in a collision. Research the properties of air bags. In terms of impulse, how do they work? How quickly do they inflate? Do they remain inflated? What force do they exert on the driver or passenger? What are the safety concerns of using air bags?

26. A patient lies flat on a table that is supported by air bearings so that it is, effectively, a frictionless platform. As the patient lies there, the table moves slightly, due to the pulsating motion of the blood from her heart. By examining the subtle motion of the table, a ballistocardiograph is obtained, which is used to diagnose certain potential deficiencies of the patient's heart. Research the details of how this device works and the information that can be obtained from the data.

27. Particle physicists investigate the properties of elementary particles by examining the tracks these particles make during collisions in "bubble chambers." Examine some bubble chamber photographs (check the Internet) and research the information that can be obtained. Include a discussion of how the law of conservation of linear momentum is applied.

Problems for Understanding

28. Determine the momentum of a 5.0 kg bowling ball rolling with a velocity of 3.5 m/s[N] toward a set of bowling pins.

29. What is the mass of a car that is travelling with a velocity of 28 m/s[W] and a momentum of 4.2×10^4 kg · m/s[W]?

30. The momentum of a 55.0 kg in-line skater is 66.0 kg m/s[S]. What is his velocity?

31. How fast would a 5.0×10^{-3} kg golf ball have to travel to have the same momentum as a 5.0 kg bowling ball that is rolling at 6.0 m/s[forward]?

32. Calculate the impulse for the following interactions.
    (a) A person knocks at the door with an average force of 9.1 N[E] over a time interval of 2.5×10^{-3} s.
    (b) A wooden mallet strikes a large iron gong with an average force of 4.2 N[S] over a time interval of 8.6×10^{-3} s.

33. A volleyball player spikes the ball with an impulse of 8.8 kg · m/s over a duration of 2.3×10^{-3} s. What was the average applied force?

34. If a tennis racquet exerts an average force of 55 N and an impulse of 2.0 N · s on a tennis ball, what is the duration of the contact?

35. (a) What is the impulse of a 0.300 kg hockey puck slapshot that strikes the goal post at a velocity of 44 m/s[N] and rebounds straight back with a velocity of 9.2 m/s[S]?
    (b) If the average force of the interaction was -2.5×10^3 N, what was the duration of the interaction?

36. A 2.5 kg curling stone is moving down the ice at 3.5 m/s[W]. What force would be needed to stop the stone in a time of 3.5×10^{-4} s?

37. At an automobile test facility, a car with a 75.0 kg crash-test dummy is travelling 28 m/s[forward] when it hits a wall. Calculate the force that the seat belt exerts on the dummy on impact. Assume that the car and dummy travel about 1.0 m as the car comes to rest and that the acceleration is constant during the crash.

38. A 0.0120 kg bullet is fired horizontally into a stationary 5.00 kg block of wood and becomes embedded in the wood. After the impact, the block and bullet begin to move with an initial velocity of 0.320 m/s[E]. What was the velocity of the bullet just before it hit the wood?

39. A 48.0 kg skateboarder kicks his 7.0 kg board ahead with a velocity of 2.6 m/s[E]. If he runs with a velocity of 3.2 m/s[E] and jumps onto the skateboard, what is the velocity of the skateboard and skateboarder immediately after he jumps on the board?

40. Astrid, who has a mass of 37.0 kg, steps off a stationary 8.0 kg toboggan onto the snow. If her forward velocity is 0.50 m/s, what is the recoil velocity of the toboggan? (Assume that the snow is level and the friction is negligible.)

41. A 60.0 t submarine, initially travelling forward at 1.5 m/s, fires a 5.0×10^2 kg torpedo straight ahead with a velocity of 21 m/s in relation to the submarine. What is the velocity of the submarine immediately after it fires the torpedo?

42. Suppose that a 75.0 kg goalkeeper catches a 0.40 kg ball that is moving at 32 m/s. With what forward velocity must the goalkeeper jump when she catches the ball so that the goalkeeper and the ball have a horizontal velocity of zero?

43. In billiards, the 0.165 kg cue ball is hit toward the 0.155 kg eight ball, which is stationary. The cue ball travels at 6.2 m/s forward and, after impact, rolls away at an angle of 40.0° counterclockwise from its initial direction, with a velocity of 3.7 m/s. What are the velocity and direction of the eight ball?

44. Consider a nuclear reaction in which a neutron travelling 1.0×10^7 m/s in the $+x$ direction collides with a proton travelling 5.0×10^6 m/s in the $+y$ direction. They combine at impact to form a new particle called a "deuteron." What is the magnitude and direction of the deuteron velocity? Assume for simplicity that the proton and neutron have the same mass.

45. A ball of mass m_1 strikes a stationary ball of mass m_2 in a head-on, elastic collision.
    (a) Show that the final velocities of the two balls have the form
    $$v_1' = \frac{m_1 - m_2}{m_1 + m_2} v_1 \qquad v_2' = \frac{2m_1}{m_1 + m_2} v_1$$
    (b) Examine three cases for the masses
    $m_1 <<< m_2 \qquad m_1 \cong m_2 \qquad m_1 >>> m_2$
    (c) Comment on the results.

46. In a demonstration, two identical 0.0520 kg golf balls collide head on. If the initial velocity of one ball is 1.25 m/s[N] and the other is 0.860 m/s[S], what is the final velocity of each ball?

47. A 750 g red ball travelling 0.30 m/s[E] approaches a 550 g blue ball travelling 0.50 m/s[W]. They suffer a glancing collision. The red ball moves away at 0.15 m/s[E30.0°S] and the blue ball moves away in a north-westerly direction.
    (a) What is the final velocity of the blue ball?
    (b) What percentage of the total kinetic energy is lost in the collision?

48. You and a colleague are on a spacewalk, repairing your spacecraft that has stalled in deep space. Your 60.0 kg colleague, initially at rest, asks you to throw her a hammer, which has a mass of 3.0 kg. You throw it to her with a velocity of 4.5 m/s[forward].
    (a) What is her velocity after catching the hammer?
    (b) What impulse does the hammer exert on her?
    (c) What percentage of kinetic energy is lost in the collision?

CHAPTER 5

Conservation of Energy

CHAPTER CONTENTS

PREREQUISITE CONCEPTS AND SKILLS

- Concept of work
- Kinetic energy
- Potential energy
- Thermal energy and heat

With a "whump," the fireworks shell is lofted upward into the darkness. As the shell rises, it slows; kinetic energy transforms into gravitational potential energy. Then, the shell explodes and chemical potential energy rapidly converts into heat, light, and sound. The darkness gives way to brilliant colour and loud bangs startle the crowd below.

There is a balance in all of these transformations and effects. Energy gained in one form comes at the expense of another. This is the law of conservation of energy.

In this chapter, you will examine these energy transformations and balances and investigate a few of their applications.

MULTI LAB

Energy Transformations in Springs

TARGET SKILLS

- Performing and recording
- Analyzing and interpreting
- Communicating results

A Spring Pendulum

Hang a spring (at least 50 cm long) from a rigid support and attach a mass to the lower end of the spring. Ensure that the mass is heavy enough to extend the spring noticeably without overstretching it. The hanging mass should not come close to the desktop. Pull the mass to the side and release it, allowing the spring to swing from side to side. Observe the motion of the mass and the spring.

Analyze and Conclude

1. What types of periodic (repeating) motion did you observe?
2. When the amplitude of one type of vibration was at a maximum, what happened to the amplitude of the other type of vibration?
3. When the spring was stretched to a greater length than it was when the mass was at rest, was the mass moving rapidly or slowly?
4. What was being transferred between **(a)** the different types of vibration, and **(b)** the mass and the spring?
5. The law of conservation of energy states that the total energy of an isolated system remains constant. How was that law illustrated by the action of the spring and the mass in this case?
6. What eventually happened to the motion of the spring and the mass? Suggest why this occurred.
7. If the system regularly switched back and forth between the two patterns of motion, was the time taken for the change consistent?

Slinky Motion

Place a Slinky™ toy several steps up from the bottom of a set of stairs, with the axis of the spring vertical. Take the top coil and arc it over and down to touch the next lower step. Release the Slinky™ and observe its motion.

Analyze and Conclude

1. What is the condition of the coils of the spring when energy is stored in the spring?
2. At what stage or stages in the action of the spring is kinetic energy being converted into elastic potential energy in the spring?
3. At what stage or stages in the action of the spring is elastic potential energy in the spring being converted into kinetic energy?
4. Is there any instant during the motion of the spring when both the kinetic energy and the elastic potential energy are at a maximum? Would you expect this to be possible? Give a reason for your answer.
5. Any system loses energy due to friction, which converts mechanical energy into thermal energy. Why then does the spring continue going down the stairs? From where is it getting its energy?

5.1 Work and the Transformation of Energy

SECTION EXPECTATIONS

- Define and describe concepts and units related to energy forms and conservation.
- Analyze and explain common situations using the work-energy theorem.
- Investigate the law of conservation of energy experimentally.

KEY TERMS

- energy
- work
- work-kinetic energy theorem
- work-energy theorem
- conservation of mechanical energy

In the introduction to this chapter, you read about some different types of energy transfers and transformations. You might recall from previous science courses that the two mechanisms by which energy is transferred from one system to another are work and heat. In fact, **energy** is often defined as the ability to do work. In this section, you will focus on work, extending your knowledge and your ability to make predictions about work and solve problems involving work as the transfer of mechanical energy from one system to another.

Characterizing Work

What is work? How do you know if one object or system is doing work on another? If work is being done, how much work is done? In physics, these questions are easier to answer than in everyday life. If an object or system, such as your body, exerts a force on an object and that force causes the object's position to change, you are doing **work** on the object.

The most direct way to express work mathematically is with the equation $W = F\Delta d \cos\theta$, where F is the magnitude of the force doing work on an object, Δd is the magnitude of the displacement caused by the force, and θ is the angle between the vectors for force and displacement. Notice that the force, F, and displacement, Δd, do not have vector notations. The reason for the omission of the vector symbols is that work, W, is a scalar quantity and is the scalar product of the vectors $\vec{F}$ and $\Delta\vec{d}$. Since the product of the vectors is a scalar quantity, the directions of the force and displacement do not determine a final direction of their product. To understand why $\cos\theta$ is included in the equation, study Figure 5.1.

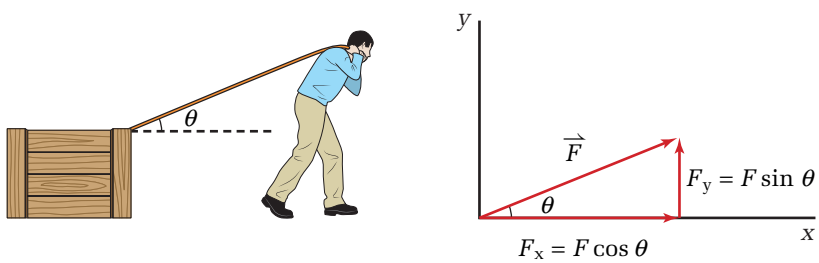


Figure 5.1 The only component of the force acting on an object that does work is the component that is parallel to the direction of the displacement.

In Figure 5.1, you see a person pulling a crate with a force that is at an angle, θ, relative to the direction of the motion. Only part of that force is actually doing work on the crate. In the diagram beside the sketch, you can see that the x-component (horizontal) of the force has a magnitude $F \cos \theta$. This component is in the direction of the motion and is the only component that is doing work. The y-component (vertical) is perpendicular to the direction of the motion and does no work on the crate.

Figure 5.2 shows four special cases that will clarify the question of whether work is being done by a force. In part (A), a person is pushing a cart with a force ($\vec{F}$) that is in the same direction as the motion of the cart. The angle between the force and the displacement is zero, so $\cos \theta = \cos 0 = 1$ and the work is $W = F\Delta d$. When the force and the displacement are in the same direction, the entire force is doing work. In this case, the cart is speeding up, so its kinetic energy is increasing. The work that the person is doing on the cart is transferring energy to the cart, so positive work is being done on the cart.

In part (B) of Figure 5.2, the cart has kinetic energy and is moving forward. The person is pulling on the cart to slow it down. Notice that the direction of the force that the person is exerting on the cart is opposite to the direction of the motion. The angle θ is 180°, so $\cos \theta = -1$ and, therefore, $W = -F\Delta d$. Just as the results indicate, the person is doing negative work on the cart by slowing it down and reducing its kinetic energy.

MATH LINK

In mathematics, a scalar product is also called a "dot product" and the equation for work is written as $W = \vec{F} \cdot \Delta\vec{d}$ The magnitude of a dot product is always the product of the magnitudes of the two vectors and the cosine of the angle between them.

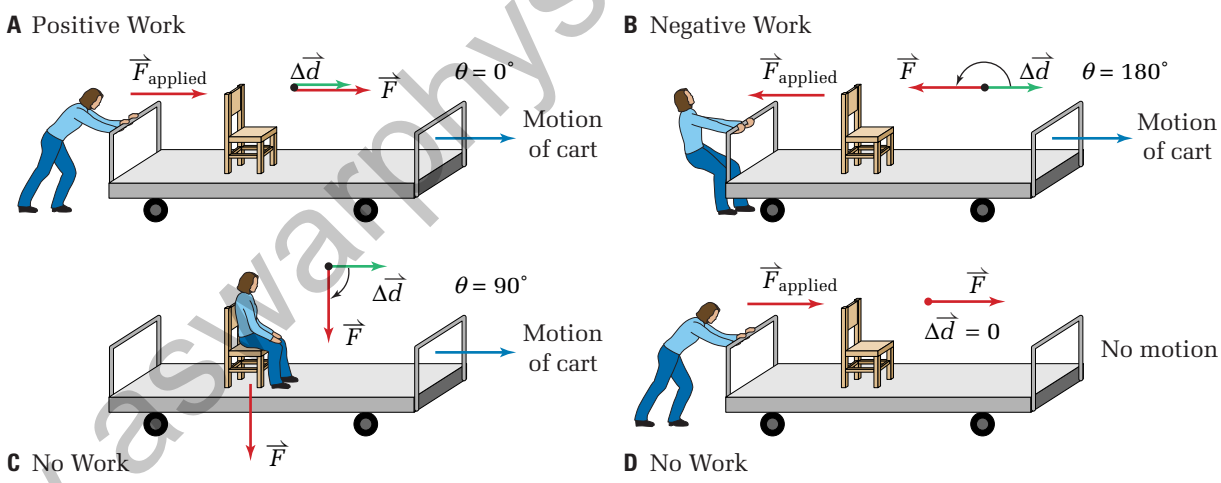


Figure 5.2 In order to do work, the force must be acting in a direction parallel to the displacement.

In part (C) of Figure 5.2, the person is sitting on the cart, exerting a downward force on it. The angle θ is 90°, so $\cos \theta = \cos 90° = 0$ and the work is $W = F\Delta d(0) = 0$. Even though the cart is moving, the force that the person is exerting is not doing work, because it is not directly affecting the horizontal motion of the cart. Notice that if you use the equation for work properly, the term $\cos \theta$ will tell you whether the work is positive, negative, or zero.

Finally, in part (D), the person is pushing on the cart, but the cart is stuck and will not move. Even though the person is exerting a force on the cart, the person is not doing work on the cart, because the displacement is zero.

DEFINING WORK

Work is the product of the force, the displacement, and the cosine of the angle between the force and displacement vectors.

$$W = F\Delta d \cos\theta$$

Quantity	Symbol	SI unit
work	W	J (joules)
force	F	N (newtons)
displacement	Δd	m (metres)
angle between force and displacement	θ	degrees (The cosine of an angle is a number and has no units.)

Unit Analysis

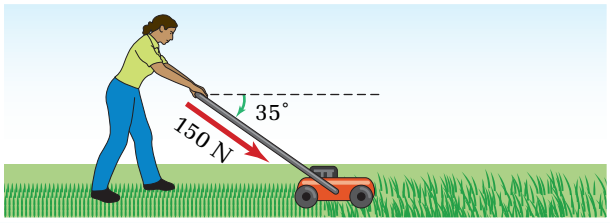
(joule) = (newton)(metre) $\quad$ J = N · m

A newton · metre is equivalent to a joule.

SAMPLE PROBLEM

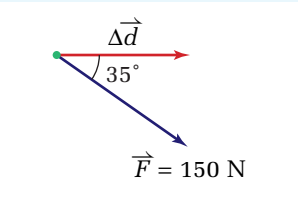
Working on the Lawn

A woman pushes a lawnmower with a force of 150 N at an angle of 35° down from the horizontal. The lawn is 10.0 m wide and requires 15 complete trips across and back. How much work does she do?



Conceptualize the Problem

- Draw a sketch to show the relationship between the force and the motion.
- A *force* is acting at an *angle* to the *direction of motion*.
- Since a *component* of the force is in the *direction of the motion*, the force is *doing work* on the lawnmower.
- *Work* done can be determined from the general work equation.



Identify the Goal

The work, W, done by the woman on the lawnmower

Identify the Variables and Constants

Known	Unknown
$\vec{F} = 150$ N	Δd
$\theta = 35^\circ$	W
width of lawn = 10.0 m	
15 trips	

Develop a Strategy

Determine the total distance over which the force acted.

$$\Delta d = 2(10.0 \text{ m})(15)$$
$$\Delta d = 300 \text{ m}$$

Use the general work equation.

$$W = F\Delta d \cos\theta$$

Substitute and solve.

$$W = (150 \text{ N})(300 \text{ m}) \cos 35^\circ$$
$$W = 36\ 861.8 \text{ J}$$
$$W \cong 3.7 \times 10^4 \text{ J}$$

The work done by the woman on the lawnmower is 3.7×10^4 J.

Validate the Solution

If the force had been horizontal, then the work done would have been 300 m × 150 N, which equals 45 000 J. Because the force is at an angle to the direction of motion, the work done is less than this value.

PRACTICE PROBLEMS

1. A man pulls with a force of 100.0 N at an angle of 25° up from the horizontal on a sled that is moving horizontally. If the sled moves a distance of 200.0 m, how much work does the man do on the sled?
2. A tow truck does 42.0 MJ of work on a car while pulling it with a force of 3.50 kN exerted upward at 10.0° to the horizontal. If the car moves horizontally, how far was it towed?
3. A kite moves 14.0 m horizontally while pulled by a string. If the string did 60.0 J of work on the kite while exerting a force of 8.2 N, what angle did the string make with the vertical?

Work and Kinetic Energy

In the examples you just examined, you determined the amount of work that was done on several objects. Now, consider the form of the energy that is given to an object on which work is done and the relationship to the work that was done. First, look at a situation in which all of the work done on a cart transfers only kinetic energy to the cart. Imagine that a cart is rolling horizontally to the right with a speed of v_i when a force is exerted on it in the direction of motion, as shown in Figure 5.3. The force acts

over a displacement of $\Delta\vec{d}$. Since all of the motion is horizontal, there will be no changes in gravitational potential energy. Assume also that friction is negligible. Study Figure 5.3 and then examine the steps that follow the illustration.

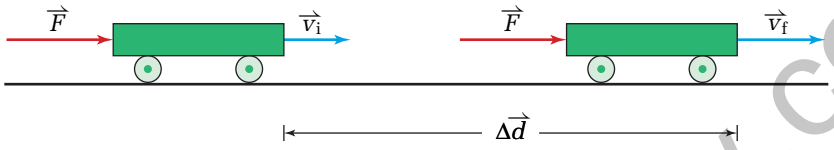


Figure 5.3 All of the work being done on the cart is increasing the cart's kinetic energy.

- Write the equation for work. From Figure 5.3, you can see that the angle between the force vector and the displacement vector is zero. Use this information to simplify the equation.

$$W = F\Delta d \cos\theta$$
$$W = F\Delta d \cos 0^\circ$$
$$\cos 0^\circ = 1$$
$$W = F\Delta d$$

- Recall the expression for force from Newton's second law. Substitute the expression for F into the equation for work. Since work is a scalar quantity, do not use vector symbols.

$$F = ma$$
$$W = ma\Delta d$$

- Write the kinematic equation that relates initial velocity, final velocity, displacement, and acceleration. Solve that equation for displacement.

$$v_f^2 = v_i^2 + 2a\Delta d$$
$$\Delta d = \frac{v_f^2 - v_i^2}{2a}$$

- Substitute the expression for displacement into the equation for work. Simplify the expression.

$$W = ma\left(\frac{v_f^2 - v_i^2}{2a}\right)$$
$$W = \frac{m(v_f^2 - v_i^2)}{2}$$

- Expand the equation.

$$W = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

- Recognize the expression $\frac{1}{2}mv^2$ as kinetic energy.

$$W = E_{kf} - E_{ki}$$
$$W = \Delta E_k$$

The work done on the cart is equal to the change in the kinetic energy of the cart. You can now generalize and state that, when work is done on an object in which the force and displacement are horizontal and friction is negligible, the work done is equal to the change in the kinetic energy of the object. The expression $W = \Delta E_k$ is often called the **work-kinetic energy theorem**.

SAMPLE PROBLEM

The Hammer and Nail

You drive a nail horizontally into a wall, using a 0.448 kg hammerhead. If the hammerhead is moving horizontally at 5.5 m/s and in one blow drives the nail into the wall a distance of 3.4 cm, determine the average force acting on

(a) the hammerhead

(b) the nail

Conceptualize the Problem

- The hammer possesses *kinetic energy*.
- The *backward force* exerted by the nail on the hammer *removes* all of the *kinetic energy*.
- The *magnitude of the force* exerted by the hammer on the nail equals the *magnitude of the force* exerted by the nail on the hammer, according to Newton's third law of motion.

Identify the Goal

(a) The force acting on the hammer, $\vec{F}_h$, by the nail

(b) The force applied to the nail, $\vec{F}_n$, by the hammer

Identify the Variables and Constants

Known	Implied	Unknown
$m = 0.448$ kg	$\vec{v}_f = 0\ \frac{m}{s}$	$\vec{F}_h$
$\vec{v}_i = 5.5\ \frac{m}{s}$[forward]		$\vec{F}_n$
$\Delta\vec{d} = 0.034$ m[forward]		

PROBLEM TIP

When solving problems involving work and energy, be sure to express all quantities in SI units. For example, a speed of 80 km/h must be converted into 22.2 m/s, and a mass of 25 g must be expressed as 0.025 kg.

Develop a Strategy

With only horizontal motion, work done equals the change in kinetic energy.

$$|\vec{F}_h||\Delta\vec{d}| = \Delta E_k$$

$$|\vec{F}_h||\Delta\vec{d}| = \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2$$

$$|\vec{F}_h| = \frac{\tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2}{\Delta d}$$

$$|\vec{F}_h| = \frac{\tfrac{1}{2}(0.448\text{ kg})\left(0\ \tfrac{m}{s}\right)^2 - \tfrac{1}{2}(0.448\text{ kg})\left(5.5\ \tfrac{m}{s}\right)^2}{0.034\text{ m}}$$

$$\vec{F}_h = -199.2941\text{ N}$$

$$\vec{F}_h \cong -2.0 \times 10^2\text{ N}$$

(a) The average force exerted on the hammer by the nail was 2.0×10^2 N[backward].

Apply Newton's third law to the forces between the hammer and the nail.

$$\vec{F}_n = -\vec{F}_h$$

$$\vec{F}_n \cong -(-2.0 \times 10^2\text{ N})$$

$$\vec{F}_n \cong 2.0 \times 10^2\text{ N}$$

(b) The force exerted on the nail by the hammer was 2.0×10^2 N[forward].

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Validate the Solution

Since kinetic energy must be transferred out of the hammerhead, the force on the hammer must be in the opposite direction to its motion and so the force must be negative.

PRACTICE PROBLEMS

4. A car with a mass of 2.00×10^3 kg is travelling at 22.2 m/s (80 km/h) when the driver applies the brakes. If the force of static friction between the tires and the road is 8.00×10^3 N[backward], determine the stopping distance of the car. Use the concepts of work and energy in solving this problem.

5. A 1.00×10^2 g arrow is fired horizontally from a bow. If the average applied force on the arrow is 150.0 N and it acts over a displacement of 40.0 cm, with what speed will the arrow leave the bow? Use the concepts of work and energy in solving this problem.

6. A 12.0 kg sled is sliding at 8.0 m/s over ice when it encounters a patch of snow. If it comes to rest in 1.5 m, determine the magnitude of the average force acting on the sled.

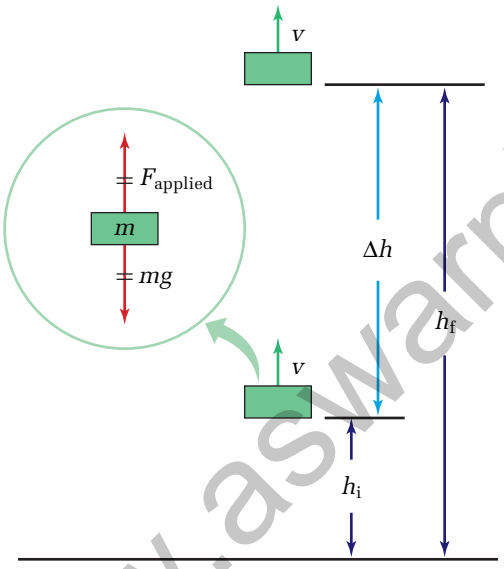

Figure 5.4 The upward force that you apply to the mass is equal in magnitude and opposite in direction to the force of gravity. Therefore, the velocity of the mass is constant and only its height changes.

Work and Gravitational Potential Energy

Consider, now, a contrasting situation — the motion of the object on which work is being done and the force that does the work are vertical and there is no change in the object's velocity. For example, imagine that you are lifting a mass at constant speed, so there is no change in its kinetic energy. The only energy that the mass will gain will be due to its position in the gravitational field — gravitational potential energy (E_g). The relationship between the quantities is shown in Figure 5.4.

Because neither the speed nor the direction of the mass is changing, it is not accelerating. If the acceleration of the mass is zero, then the net force must be zero and therefore the magnitude of upward applied force must equal the magnitude of the downward force of gravity.

$$F_{applied} = F$$

$$F_{applied} = mg$$

Examine the following steps to derive the relationship between work done by a vertical force and gravitational potential energy.

- Write the equation for work. From Figure 5.4, you can see that the angle between the force vector and the displacement vector is zero. Use this information to simplify the equation.

$$W = F\Delta d \cos\theta$$
$$W = F\Delta d \cos 0^\circ$$
$$\cos 0^\circ = 1$$
$$W = F\Delta d$$

- To find the amount of work done by the applied force, substitute the applied force into the equation.

$$W = F\Delta d$$
$$W = F_{\text{applied}}\Delta d$$
$$F_{\text{applied}} = mg$$
$$W = mg\Delta d$$

- From Figure 5.4, you can see that the displacement is the difference of the initial and final heights. Substitute this value into the equation for work.

$$\Delta d = h_f - h_i$$
$$W = mg(h_f - h_i)$$

- Expand the equation.

$$W = mgh_f - mgh_i$$

- Recognize the expression mgh as gravitational potential energy and substitute E_g into the equation for work.

$$W = E_{gf} - E_{gi}$$
$$W = \Delta E_g$$

When velocity does not change but the object's position changes in height, the work done on an object is equal to the change in the gravitational potential energy of the object.

SAMPLE PROBLEM

A Rescue at Sea

A gas-powered winch on a rescue helicopter does 4.20×10^3 J of work while lifting a 50.0 kg swimmer at a constant speed up from the ocean. Through what height was the swimmer lifted?

Conceptualize the Problem

- The *speed* was *constant*, so there was *no change* in *kinetic energy*.
- Assume that the *work done* by the winch equals the gain in *gravitational potential energy* of the swimmer.

Identify the Goal

The height, Δh, through which the swimmer was lifted

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Identify the Variables and Constants

Known	Implied	Unknown
$W = 4.20 \times 10^3$ J	$g = 9.81 \frac{\text{m}}{\text{s}^2}$	Δh
$m = 50.0$ kg		

Develop a Strategy

With no change in kinetic energy, work done equals the change in gravitational potential energy. Substitute and solve.

$$W = \Delta E_g$$

$$W = mg\Delta h$$

$$\Delta h = \frac{W}{mg}$$

$$\Delta h = \frac{4.20 \times 10^3 \text{ J}}{(50.0 \text{ kg})\left(9.81 \frac{\text{m}}{\text{s}^2}\right)}$$

$$\Delta h = 8.562\ 69 \text{ m}$$

$$\Delta h \cong 8.56\ m$$

The height through which the swimmer was lifted was 8.56 m.

Validate the Solution

$1 \text{ J} = \frac{1 \text{ kg} \cdot \text{m}^2}{\text{s}^2}$, so the answer has units of $\frac{\frac{1 \cancel{\text{kg}} \cdot \text{m}^{\cancel{2}}}{\cancel{\text{s}^2}}}{\cancel{\text{kg}} \cdot \frac{\text{m}}{\cancel{\text{s}^2}}} = \text{m}$.

PRACTICE PROBLEMS

7. A motorized crane did 40.4 kJ of work when slowly lifting a pile driver to a height of 8.00 m. What was the mass of the pile driver?

8. A 4.00×10^2 kg elevator car rose at a constant speed past several floors. If the motor did 58.8 kJ of work, through what height did the car rise?

9. A battery-powered scoop used by a Mars lander lifted a 54 g rock through a height of 24 cm. If $g_{\text{Mars}} = 3.8 \text{ m/s}^2$, how much work was done by the scoop?

The Work-Energy Theorem and Conservation of Energy

Very few processes are as limited as the two situations that you have just considered — changes in kinetic energy only or in potential energy only. Real processes usually involve more than one form of energy. However, you can combine the two cases that you just considered. For example, if an applied force does work on an object so that both its kinetic energy and its various forms of potential energy change, then the work done by that force equals the total change in both the kinetic energy and the potential energies: $W = \Delta E_k + \Delta E_p$. The relationship shown in this equation is known as the **work-energy theorem**.

Picture an object or a system of objects on which no work is being done by some outside agency. In other words, no energy is being added to the system and no energy is being removed from the system. This is called an "isolated system." A swinging pendulum could be such a system until someone comes along and gives it a shove. A roller coaster in which the car is running freely up and down slopes and around curves is isolated if wind effects and friction are ignored. The flight of an arrow away from its position in a stretched bow can be treated as an isolated system if air friction effects and wind are again ignored.

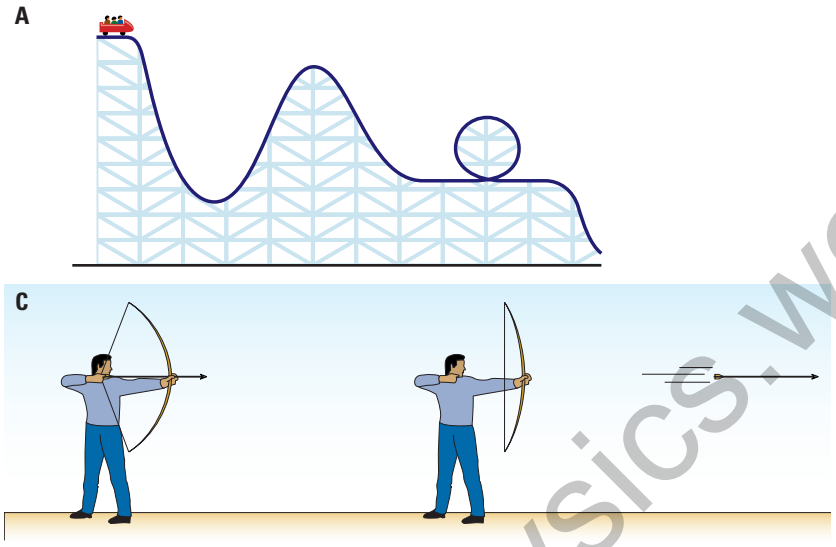


Figure 5.5 Our universe is possibly the only truly isolated system. However, in many applications, such as the ones shown here, you will work on the assumption that no energy enters from the outside and none is lost to the outside.

If a system is isolated in that no outside work is done on it, you can use the work-energy theorem to derive another important relationship, as shown in the following steps.

- Write the work-energy theorem. $W = \Delta E_k + \Delta E_p$
- If the system is isolated, $W = 0$. Substitute zero into the equation above. $0 = \Delta E_k + \Delta E_p$ $\Delta E_k + \Delta E_p = 0$
- Expand the expression and use primes to represent the energies after the process is complete. $(E'_k - E_k) + (E'_p - E_p) = 0$
- Rearrange the equation so that the initial energies are on the left-hand side of the equals sign and all of the final energies are on the right-hand side. $E_k + E_p = E'_k + E'_p$

PHYSICS FILE

The work-energy theorem links two apparently different types of quantities. On the left-hand side is a concept that is extremely concrete in that it deals with readily measured quantities of force and distance. On the right-hand side is the extremely abstract concept of energy. In fact, the right-hand side deals *only* with energy changes and never in absolute amounts of energy.

ELECTRONIC LEARNING PARTNER

Use the interactive pendulum simulation in your Electronic Learning Partner to enhance your understanding of energy transformations.

The last statement in the derivation is known as the law of **conservation of mechanical energy**. The equation $\Delta E_k + \Delta E_p = 0$ says that the change of total mechanical energy in an isolated system is zero. This does not mean, however, that no processes are occurring within the system. This last statement implies that kinetic energy of an object in the system can be transformed into potential energy, or the reverse can happen. In addition, one object in the system might transfer energy to another object in the system. Many processes can occur in an isolated system.

THE LAW OF CONSERVATION OF MECHANICAL ENERGY

The law of conservation of mechanical energy states that the sum of the kinetic and potential energies before a process occurs in an isolated system is equal to the sum of the kinetic and potential energies of the system after the process is complete.

$$E_k + E_p = E'_k + E'_p$$

Quantity	Symbol	SI unit
kinetic energy before the process occurred	E_k	J (joules)
potential energy before the process occurred	E_p	J (joules)
kinetic energy after the process was completed	E'_k	J (joules)
potential energy after the process was completed	E'_p	J (joules)

PROBEWARE

If your school has probeware equipment, visit ***www.mcgrawhill.ca/links/physics12*** and follow the links for an in-depth activity on energy and momentum in collisions.

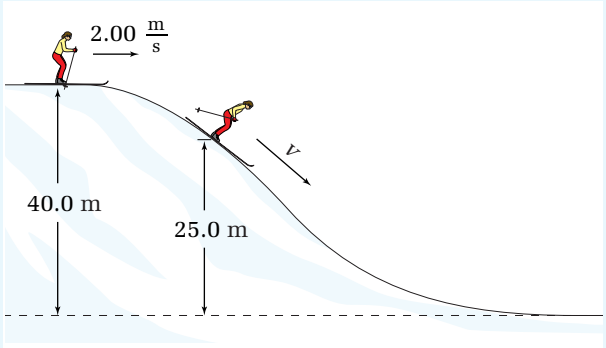
SAMPLE PROBLEM

Conservation of Energy on the Ski Slopes

A skier is gliding along with a speed of 2.00 m/s at the top of a ski hill, 40.0 m high, as shown in the diagram. The skier then begins to slide down the icy (frictionless) hill.

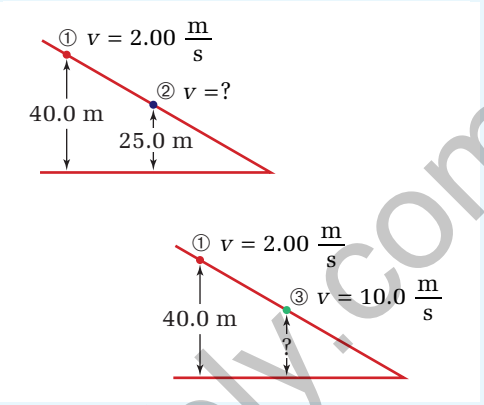
(a) What will be the skier's speed at a height of 25.0 m?

(b) At what height will the skier have a speed of 10.0 m/s?



Conceptualize the Problem

- Sketch the two parts of the problem separately. Label the initial conditions (top of the hill) "1." Label the position when $h = 25$ m as "2." Label the position at which the skier is travelling at 10.0 m/s as "3."
- Use subscripts 1 and 2 to indicate the initial and final conditions in part (a) and use subscripts 1 and 3 to indicate the initial and final conditions in part (b).
- Define the *system* as the *skier and the slope*.
- Assume that the system of skier and slope is *isolated*.
- The law of *conservation of energy* can be applied.



Identify the Goal

(a) the speed, v_2, at a height of 25.0 m

(b) the height, h_3, at which the skier's speed will be 10.0 m/s

Identify the Variables and Constants

Known	Implied	Unknown
$v_1 = 2.00 \frac{m}{s}$	$g = 9.81 \frac{m}{s^2}$	v_2
$h_1 = 40.0$ m		h_3
$v_3 = 10.0 \frac{m}{s}$		
$h_2 = 25.0$ m		

Develop a Strategy

State the law of conservation of mechanical energy.

$$E'_k + E'_p = E_k + E_p$$

Expand by replacing E with the expression that defined the type of energy.

$$\frac{1}{2}mv_2^2 + mgh_2 = \frac{1}{2}mv_1^2 + mgh_1$$

Divide through by m.

$$\frac{1}{2}v_2^2 = \frac{1}{2}v_1^2 + gh_1 - gh_2$$

Simplify and rearrange the equation to solve for v_2.

$$v_2^2 = v_1^2 + 2g(h_1 - h_2)$$

$$v_2 = \sqrt{v_1^2 + 2g(h_1 - h_2)}$$

Substitute and solve.

$$v_2 = \sqrt{\left(2.00 \frac{m}{s}\right)^2 + 2\left(9.81 \frac{m}{s^2}\right)(40.0 \text{ m} - 25.0 \text{ m})}$$

$$v_2 = \sqrt{298.3 \frac{m^2}{s^2}}$$

Speed does not involve direction, so choose the positive root since speed can never be negative.

$$v_2 = \pm 17.271 \frac{m}{s}$$

$$v_2 \cong 17.3 \frac{m}{s}$$

(a) The speed is 17.3 m/s at a height of 25.0 m.

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Write the expanded version of the conservation of mechanical energy. Rearrange and solve for height.

$$\frac{1}{2}mv_3^2 + mgh_3 = \frac{1}{2}mv_1^2 + mgh_1$$

$$gh_3 = \frac{1}{2}v_1^2 + gh_1 - \frac{1}{2}v_3^2$$

$$h_3 = \frac{\frac{1}{2}v_1^2 + gh_1 - \frac{1}{2}v_3^2}{g}$$

Substitute numerical values and solve.

$$h_3 = \frac{\frac{1}{2}\left(2.00\ \frac{\text{m}}{\text{s}}\right)^2 + \left(9.81\ \frac{\text{m}}{\text{s}^2}\right)(40.0\ \text{m}) - \frac{1}{2}\left(10.0\ \frac{\text{m}}{\text{s}}\right)^2}{9.81\ \frac{\text{m}}{\text{s}^2}}$$

$$h_3 = 35.107\ \text{m}$$

$$h_3 \cong 35.1\ \text{m}$$

(b) The height must be 35.1 m when the speed is 10.0 m/s.

Validate the Solution

(a) The units for the right-hand side of the equation for v_2 are

$$\left[\left(\frac{\text{m}}{\text{s}}\right)^2 + \left(\frac{\text{m}}{\text{s}^2}\right)\text{m}\right]^{\frac{1}{2}} = \frac{\text{m}}{\text{s}}.$$

These are the correct units for the speed. In addition, since the skier is going downhill, the final speed must be larger than the initial speed of 2.00 m/s.

(b) The units on the right-hand side of the equation for h_3 are

$$\frac{\left(\frac{\text{m}}{\text{s}}\right)^2 + \left(\frac{\text{m}}{\text{s}^2}\right)(\text{m}) - \left(\frac{\text{m}}{\text{s}}\right)^2}{\frac{\text{m}}{\text{s}^2}} = \text{m}.$$

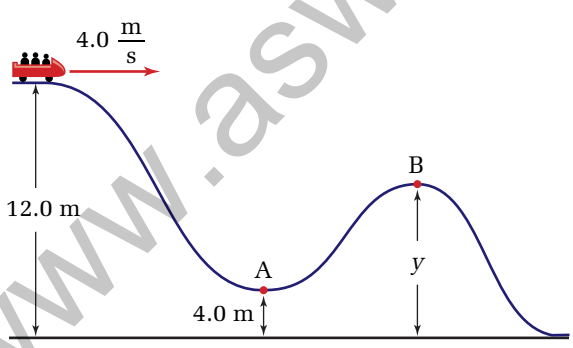
Since the speed in part (b) is less than the speed in part (a), the skier should be higher on the hill than in part (a).

PROBLEM TIP

Students are often tempted to apply the equations for linear motion to the solution of these problems. However, the paths are not always linear, so the equations might not apply. One of the great advantages of using conservation of energy is that you generally do not need to know the exact path between two points or vector directions. You need to know the conditions at only those two points.

PRACTICE PROBLEMS

Use the following diagram for practice problems 10, 11, and 12.



10. A car on a roller coaster is moving along a level section 12.0 m high at 4.0 m/s when it begins to roll down a slope, as shown in the diagram. Determine the speed of the car at point A.

11. What is the height of point B in the roller coaster track if the speed of the car at that point is 10.0 m/s?

12. What is the height of y if the speed of the roller coaster at B is 12.5 m/s?

INVESTIGATION 5-A

Testing the Law of Conservation of Energy

TARGET SKILLS

- Predicting
- Performing and recording
- Analyzing and interpreting

When scientists set out to test an hypothesis or challenge a law, they often use the hypothesis or law to make a prediction and then test it. In this investigation, you will make a prediction based on the law of conservation of energy.

Problem

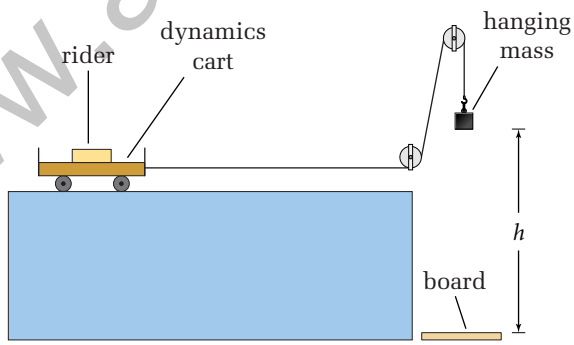
To perform a test of the law of conservation of energy

Equipment

- dynamics cart
- balance capable of measuring a mass of 1 to 2 kg
- 2 pulleys
- retort stand and clamps for the pulleys
- board (or similar material) to protect the floor from dropped masses
- metric measuring tape
- selection of masses, including several that can be suspended on a string
- stopwatch or photogate timers
- string about 4 m long

Procedure

1. Determine the mass of the dynamics cart.
2. Select a mass to be the rider and a mass to be the hanging mass. Decide on the height of the hanging mass above the board.
3. Set up the apparatus on a long desktop, as shown in the diagram.



4. Using the law of conservation of energy, calculate the expected speed of the cart and the hanging mass just before the mass strikes the board. If the cart is to be released from rest, determine the average speed of the system and then the predicted time interval for the hanging mass to drop to the board.
5. With the entire apparatus in place, hold the cart still. Release the cart and measure the time taken for the hanging mass to reach the board. Be sure to catch the cart before it hits the lower pulley.
6. Repeat the measurement several times and average the results.
7. Perform several trial runs with a different pair of masses for the cart and the hanging mass.

Analyze and Conclude

1. Prepare a table to show all of your data, as well as your calculations for the final speed, average speed, and the time interval.
2. What is the percent difference between the time as predicted by the law of conservation of energy and the measured average time?
3. Based on the precision of the timing devices, what range of experimental error would you expect in this investigation? How does this range compare with the percent difference determined in question 2?
4. Do the results of this investigation support the law of conservation of energy?

Apply and Extend

5. Which part of this investigation caused the greatest difficulty? Provide suggestions for overcoming this difficulty.
6. List some of the sources of error in this investigation and suggest how these errors might be reduced.

Work and Energy Change with a Variable Force

Until now, you have assumed that the force in the expression for work was constant. Quite often, however, you must deal with situations in which the force varies with the position, as shown in part (A) of Figure 5.6.

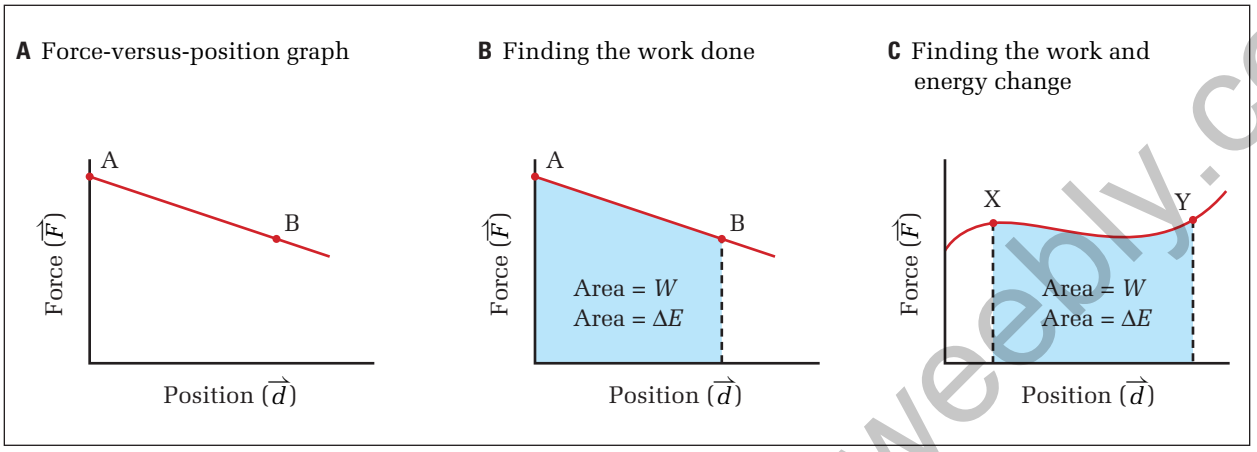


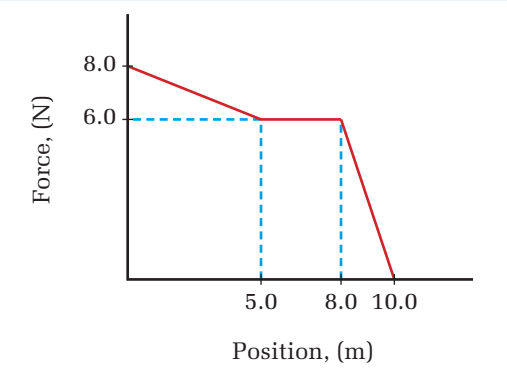
Figure 5.6 The work done, or the change in energy, is equal to the area under the graph

Since work is defined as the product of force times the displacement over which the force acts, then work must be equivalent to the graphical area under a graph of force versus position (displacement is a change in position). Such an area is illustrated in parts (B) and (C) of Figure 5.6. In the simplest cases, the graphical area forms a figure for which the area can be readily determined, as shown in the following sample problem.

SAMPLE PROBLEM

Work Done by a Variable Force

The graph shows the variation of applied force with position. Determine the work done by the force and the total energy change due to that force.



Conceptualize the Problem

- The problem involves a graph of *force versus position*.
- The graphical *area under the curve* provides the *work* done.
- *Work* is equal to the total *change in energy*.

Identify the Goal

- The work, W, done by the applied force
- The total energy change ΔE_{total}, due to the force

Identify the Variables and Constants

Known	Unknown
graph of force versus position	W
	ΔE_{total}

Develop a Strategy

Divide the graphical area under the curve up into recognizable shapes.

Determine the area of each region.

The region from the origin to 5.0 m is a trapezoid.

Area of trapezoid

A = (average of parallel sides)(distance between them)

$$A = \left(\frac{8.0\ N + 6.0\ N}{2}\right)5.0\ m$$

$$A = 35\ N \cdot m$$

$$A = 35\ J$$

The region from 5.0 m to 8.0 m is a rectangle.

Area of rectangle = (base)(height)

$$A = (3.0\ m)(6.0\ N)$$

$$A = 18\ N \cdot m$$

$$A = 18\ J$$

The region from 8.0 m to 10.0 m is a triangle.

Area of triangle = $\frac{1}{2}$(base)(height)

$$A = \frac{1}{2}(2.0\ m)(6.0\ N)$$

$$A = 6.0\ N \cdot m$$

$$A = 6.0\ J$$

Find the total area.

Total area = 35 J + 18 J + 6.0 J

Total area = 59 J

$$W = \Delta E_{total}$$

$$\Delta E_{total} = 59\ J$$

The work done, and therefore the total change in energy, is 59 J.

Validate the Solution

From examination of the graph, an average force would seem to be approximately 5 N and acts over 10 m. An approximate value for the work done would be 50 N. The answer is not far from this value.

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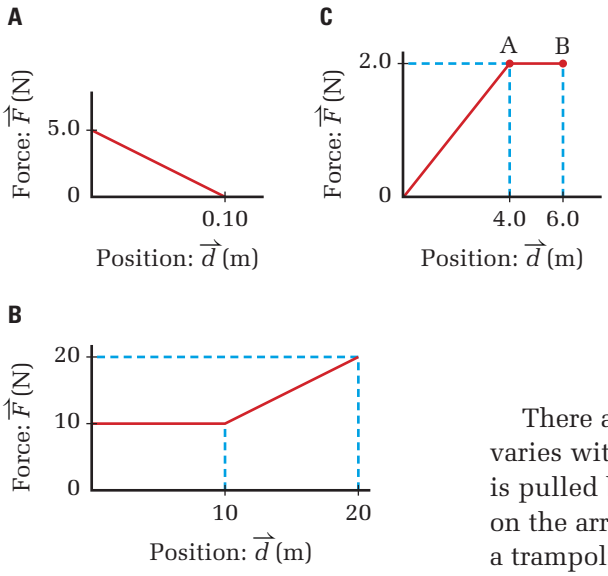
PRACTICE PROBLEMS



13. Calculate the work done by the force depicted in part (A) of the diagram.

14. Determine the magnitude of the energy change produced by the force illustrated in part (B) of the diagram.

15. The force shown in part (C) of the diagram acts horizontally on a 2.0 kg cart, initially at rest on a level surface. Determine the speed of the cart at point A and at point B.

There are many examples in which the applied force varies with displacement. The more an archery bow string is pulled back, the greater the force that the string can exert on the arrow. Force increases with the amount of stretch in a trampoline. Springs are interesting in that they can exert forces when they are stretched or compressed, and the amount of force depends on the amount of extension or compression. You will study this type of relationship in the next section of this chapter.

5.1 Section Review

1. K/U Give examples that were not used in the text to show that no work is done by an applied force that is perpendicular to the direction of the motion of the object.

2. I If the force of friction is constant, prove that the stopping distance of a car on a level road varies directly with the square of the initial speed.

3. K/U When developing the equation for gravitational potential energy, why was it necessary to assume that the mass was rising at a constant speed?

4. I Prove the expressions for gravitational potential energy and kinetic energy have units that are equivalent to the newton · metre.

5. MC The absolute temperature of a gas is a measure of the average kinetic energy of the gas atoms or molecules. What happens to the average speed of these particles when the absolute temperature of the gas is doubled?

6. C

(a) What is meant by the term "isolated system"?

(b) Describe an example of a system that could be considered as being isolated.

(c) Explain why this system is probably not completely isolated.

7. MC A variable force, $\vec{F}$, acts through a displacement, Δd. The magnitude of the force is proportional to the displacement, so $F = k\Delta d$, where k is constant.

(a) Sketch a graph of this force against position up to position x.

(b) According to the equation, what is the value of the force at x?

(c) Determine an expression for the work done by this force in terms of k and x.

5.2 Hooke's Law and Periodic Motion

The diver approaches the end of the board, bounces a couple of times, then arcs out into the air in a graceful dive. The diving board plays an important role in his action. The diver uses chemical energy to jump, gaining kinetic energy. His kinetic energy transforms into gravitational potential energy and then back into kinetic energy. When he returns to the board, slows, and stops, his kinetic energy does not transform into gravitational potential energy. In what form is the energy stored?

Figure 5.7 A diving board transforms a form of potential energy into kinetic energy of the diver.

SECTION EXPECTATIONS

- Analyze and explain common situations using the work-energy theorem.
- State Hooke's law and analyze it in quantitative terms.
- Define and describe the concepts and units related to elastic potential energy.
- Apply Hooke's law and the conservation of energy to periodic motion.

KEY TERMS

- elastic potential energy
- Hooke's law
- spring constant
- restoring force
- periodic motion

Describing Elastic Potential Energy

The diving board in Figure 5.7 is behaving much like a spring. When the diver lands on the board after jumping, the diving board exerts a force on him, doing work on him that reduces his kinetic energy to zero. At the same time, the diver is exerting a force on the diving board, doing work on the board and causing it to bend. It its bent condition, the diving board is storing energy called **elastic potential energy**. Because the diving board is elastic, it returns to its original form, and in doing so, it transfers its elastic potential energy back into kinetic energy.

Springs are commonly used, much like the diving board, to absorb energy, store it as elastic potential energy, then release it in the form of kinetic energy. A bicycle seat has a spring that reduces the jarring effects on the rider of bumps in the road. Springs in a mattress provide a flexible support that allows the surface to match the contours of the sleeper. Pressure is applied evenly over the lower surface of the sleeper, rather than being concentrated at a few points.

In this section, you will examine elastic potential energy in the form of stretched and compressed springs.

Figure 5.8 In what ways do these springs behave the same?

INVESTIGATION 5-B

Testing Hooke's Law

TARGET SKILLS

- Performing and recording
- Analyzing and interpreting
- Communicating results

Problem

What relationship exists between the force applied to a spring and its extension?

Equipment

- retort stand and C-clamp
- weight hanger and accompanying set of masses
- coil spring
- ring clamp
- metre stick

CAUTION Wear protective eye goggles during this investigation.

Procedure

1. Clamp the retort stand firmly to the desk.
2. Attach the ring clamp close to the top of the retort stand.
3. Hang the spring by one end from the ring clamp.
4. Prepare a data table with the headings: Mass on hanger, m(kg); Applied force, F(N); Height of hanger above desk, h(m); and Extension of spring, x(m).
5. Attach the weight holder and measure its distance above the desktop. Record this value in the first row of the table. This value will be your equilibrium value, h_o, at which you will assign the value of zero to the extension of the spring, x. Put these values in the first line of your table.
6. To create an applied force, add a mass to the weight holder. Wait for the spring to come to rest and measure the height of the weight holder above the desk. Record these values in the table.
7. Complete the second row in the table by calculating the value of the applied force (weight of the mass) and the extension of the spring ($x = h_o - h$).
8. Continue by adding more masses until you have at least five sets of data. Make sure that you do not overextend the spring.

Analyze and Conclude

1. Draw a graph of the applied force versus the extension of the spring. **Note:** Normally, you would put the independent variable (in this case, the applied force) on the x-axis and the dependent variable (in this case, the extension of the spring) on the y-axis. However, the mathematics will be simplified in this case by reversing the position of the variables.
2. Draw a smooth curve through the data points.
3. Describe the curve and write the equation for the curve.
4. State the relationship between the applied force and the extension. This relationship is known as "Hooke's law."
5. When the spring is at rest, what is the relationship between the applied force and the force exerted on the mass by the spring? This force is usually referred to as the "restoring force." Restate the spring relationship in terms of the restoring force of the spring.
6. By finding the area under the graph between the origin and the point of maximum extension, determine the amount of energy stored in the spring.
7. Write an equation for the energy stored in the spring when the slope of the graph is k and the extension is x.
8. Devise and carry out an experiment to determine whether a similar relationship exists for the bending of a metre stick. Obtain your teacher's approval before carrying out the experiment.

Hooke's Law

Investigation 5-B illustrated **Hooke's law**, which states that the amount of extension or compression of a spring varies directly with the applied force. A graphical illustration of this law for an extended spring is shown in Figure 5.9.

Since the data produce a straight line, the equation can be written in the form $y = mx + b$, where m is the slope and b is the y-intercept. The slope of the line describing the properties of a spring, called the **spring constant**, is symbolized by k and has units of newtons/metre. Each spring has its own constant that describes the amount of force that is necessary to stretch (or compress) the spring a given amount. In your investigation, you were directed to assign the reference or zero position of your spring as the position of the spring with no applied force. As a result, x was zero when F was zero. This choice is the accepted convention for working with springs, and it makes the y-intercept equal to zero because the line on the graph passes through the origin. This relationship leads to the mathematical form of Hooke's law (which is summarized in the following box): $F_a = kx$, where F_a is the magnitude of the applied force, x is the magnitude of the extension or compression, and k is the spring constant.

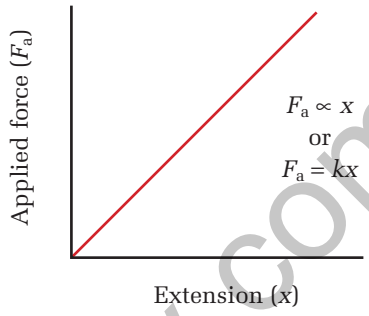


Figure 5.9 The applied force varies directly with the extension of a spring.

HOOKE'S LAW

The applied force is directly proportional to the extension or compression of a spring.

$$F_a = kx$$

Quantity	Symbol	SI unit
applied force	F_a	N (newtons)
spring constant	k	$\frac{N}{m}$ (newtons per metre)
amount of extension or compression of the spring	x	m (metres)

Unit Analysis

$$\text{newtons} = \left(\frac{\text{newtons}}{\text{metre}}\right)(\text{metre}) \qquad N = \frac{N}{\cancel{m}}\cancel{m} = N$$

According to Newton's third law of motion, the force exerted by the object that is applying the force to the spring is equal and opposite to the force that the spring exerts on that object. The force exerted by the spring is called the **restoring force**. Often, Hooke's law is written in terms of the restoring force of the

PHYSICS FILE

The spring constant is closely related to a quantity called the "modulus of elasticity." This is defined as the stress on the object divided by the strain. Stress is defined as the applied force divided by the cross-sectional area, and the strain is the amount of extension or compression per unit length. This quantity is used to predict how structural components, from aircraft wings to steel beams, will behave when under a given load.

spring: $F_s = -kx$. The negative sign shows that the restoring force is always opposite to the direction of the extension or compression of the spring.

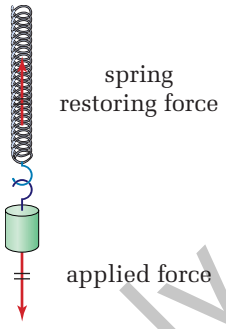


Figure 5.10 The restoring force always opposes the applied force and acts in the direction of the equilibrium position of the spring.

SAMPLE PROBLEM

Hooke's Law in an Archery Bow

A typical compound archery bow requires a force of 133 N to hold an arrow at "full draw" (pulled back 71 cm). Assuming that the bow obeys Hooke's law, what is its spring constant?

Conceptualize the Problem

- When an archer draws a bow, the *applied force* does *work* on the bow, giving it *elastic potential energy*.
- *Hooke's law* applies to this problem.

Identify the Goal

The spring constant, k, of the bow

Identify the Variables and Constants

Known	Unknown
$F_a = 133$ N	k
$x = 71$ cm	

Develop a Strategy

Use Hooke's law (applied force form). $F_a = kx$

Solve for the spring constant. $k = \frac{F_a}{x}$

Substitute numerical values and solve.

$$k = \frac{133\ \text{N}}{0.71\ \text{m}}$$

$$k = 187.32\ \frac{\text{N}}{\text{m}}$$

$$k \cong 1.9 \times 10^2\ \frac{\text{N}}{\text{m}}$$

The spring constant of the bow is about $1.9 \times 10^2\ \frac{\text{N}}{\text{m}}$.

Validate the Solution

When units are carried through the calculation, the final quantity has units of N/m, which are correct for the spring constant.

PRACTICE PROBLEMS

16. A spring scale is marked from 0 to 50 N. The scale is 9.5 cm long. What is the spring constant of the spring in the scale?

17. A slingshot has an elastic cord tied to a Y-shaped frame. The cord has a spring constant of 1.10×10^3 N/m. A force of 455 N is applied to the cord.

(a) How far does the cord stretch?

(b) What is the restoring force from the spring?

18. The spring in a typical Hooke's law apparatus has a force constant of 1.50 N/m and a maximum extension of 10.0 cm. What is the largest mass that can be placed on the spring without damaging it?

Calculating Elastic Potential Energy

The graph of Hooke's law in Figure 5.9 not only gives information about the forces and extensions for a spring (or any elastic substance), you can also use it to determine the quantity of potential energy stored in the spring. As discussed previously, you can find the amount of work done or energy change by calculating the area under a force-versus-position graph. The Hooke's law graph is such a graph, since extension or compression is simply a displacement. The area under the graph, therefore, is equal to the amount of potential energy stored in the spring, as illustrated in Figure 5.11.

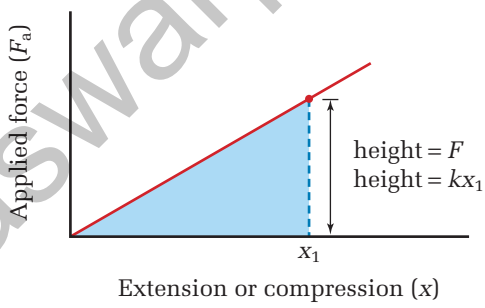
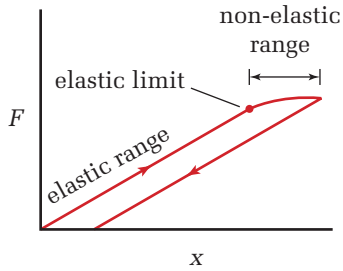


Figure 5.11 The triangular area under the Hooke's law graph gives you the amount of elastic potential energy stored in the spring at any amount of extension.

PHYSICS FILE

A *perfectly elastic* material will return precisely to its original form after being deformed, such as stretching a spring. No real material is perfectly elastic. Each material has an elastic limit, and when stretched to that limit, will not return to its original shape. The graph below shows that when something reaches its elastic limit, the restoring force does not increase as rapidly as it did in its elastic range.



PHYSICS FILE

Robert Hooke (1635–1703) was one of the most renowned scientists of his time. His studies in elasticity, which resulted in the law being named after him, allowed him to design better balance springs for watches. He also contributed to our understanding of optics and heat. In 1663, he was elected as a Fellow of the Royal Society in London. His studies ranged from the microscopic — he observed and named the cells in cork and investigated the crystal structure of snowflakes — to astronomy — his diagrams of Mars allowed others to measure its rate of rotation. He also proposed the inverse square law for planetary motion. Newton used this relationship in his law of universal gravitation. Hooke felt that he had not been given sufficient credit by Newton for his contribution, and the two men remained antagonistic for the rest of Hooke's life.

As you can see in Figure 5.11, the area under the curve of applied force versus extension of a spring is a triangle. You can use the geometry of the graph to derive an equation for the elastic potential energy stored in a spring.

- Write the equation for the area of a triangle. $A = \frac{1}{2}(\text{base})(\text{height})$
- The elastic potential energy stored in a spring is the area under the curve. $E_e = A$ $\quad E_e = \frac{1}{2}(\text{base})(\text{height})$
- The base of the triangle is the amount of extension of the spring, x_1. $\text{base} = x_1$
- The height of the triangle is the force at an extension of x_1. $\text{height} = F(x_1)$ $\quad F(x_1) = kx_1$ $\quad \text{height} = kx_1$
- Substitute the values into the expression for elastic potential energy. $E_e = \frac{1}{2}(x_1)(kx_1)$ $\quad E_e = \frac{1}{2}kx_1^2$
- The expression is valid for any value of x. $E_e = \frac{1}{2}kx^2$

The equation you just derived applies to any perfectly elastic system and is summarized in the box below.

ELASTIC POTENTIAL ENERGY

The elastic potential energy of a perfectly elastic material is one half the product of the spring constant and the square of the length of extension or compression.

$$E_e = \frac{1}{2}kx^2$$

Quantity	Symbol	SI unit
elastic potential energy	E_e	J (joules)
spring constant	k	$\frac{\text{N}}{\text{m}}$ (newtons per metre)
length of extension or compression	x	m (metres)

Unit Analysis

$$\text{joule} = \frac{\text{newton}}{\text{metre}}\,\text{metre}^2 \qquad \text{J} = \left(\frac{\text{N}}{\cancel{\text{m}}}\right)\text{m}^{\cancel{2}} = \text{N}\cdot\text{m} = \text{J}$$

SAMPLE PROBLEM

Elastic Potential Energy of a Spring

A spring with spring constant of 75 N/m is resting on a table.

(a) If the spring is compressed a distance of 28 cm, what is the increase in its potential energy?

(b) What force must be applied to hold the spring in this position?

Conceptualize the Problem

- There is *no change* in the *gravitational potential energy* of the spring.
- The *elastic potential energy* of the spring *increases* as it is compressed.
- *Hooke's law* and the definition of *elastic potential energy* apply to this problem.

Identify the Goal

The elastic potential energy, E_e, stored in the spring
The applied force, F_a, required to compress the spring

Identify the Variables and Constants

Known	Unknown
$k = 75\ \frac{\text{N}}{\text{m}}$	E_e
$x = 0.28\ \text{m}$	F_a

Develop a Strategy

Apply the equation for elastic potential energy.
Substitute and solve.

$$E_e = \frac{1}{2}kx^2$$
$$E_e = \frac{1}{2}\left(75\ \frac{\text{N}}{\text{m}}\right)(0.28\ \text{m})^2$$
$$E_e = 2.94\ \text{J}$$
$$E_e \cong 2.9\ \text{J}$$

(a) The potential energy of the spring increases by 2.9 J when it is compressed by 28 cm.

Use Hooke's law to calculate the force at 28 cm compression.

$$F_a = kx$$
$$F_a = \left(75\ \frac{\text{N}}{\text{m}}\right)(0.28\ \text{m})$$
$$F_a = 21\ \text{N}$$

(b) A force of 21 N is required to hold the spring in this position.

Validate the Solution

Round the given information to 80 N and 0.3 m and do mental multiplication. The resulting estimated change in elastic potential energy is 3.6 J and the estimated applied force is 24 N. The exact answers are reasonably close to these estimated values. In addition, a unit analysis of the first part yields an answer in N · m or joules, while the second answer is in newtons.

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PRACTICE PROBLEMS

19. An object is hung from a vertical spring, extending it by 24 cm. If the spring constant is 35 N/m, what is the potential energy of the stretched spring?

20. An unruly student pulls an elastic band that has a spring constant of 48 N/m, producing a 2.2 J increase in its potential energy. How far did the student stretch the elastic band?

21. A force of 18 N compresses a spring by 15 cm. By how much does the spring's potential energy change?

PHYSICS FILE

Gravitational potential energy is energy that an object possesses due to its position in a gravitational field. Elastic potential energy is a bit harder to picture. However, when a material is stretched or twisted, its atoms move relative to each other. Since the atoms are held together by electric forces, elastic potential energy is related to the position of an object, such as an atom in an electric field.

Restoring Force and Periodic Motion

The restoring force exerted by a spring always points toward the equilibrium or rest position for that spring. When the spring is extended, the restoring force pulls it back toward its equilibrium position. When the spring is compressed, the restoring force pushes it outward. The nature of this force makes it possible for a spring to undergo a back-and-forth, or oscillating, motion called **periodic motion**. If the restoring force obeys Hooke's law precisely, the periodic motion is called *simple harmonic motion*.

Periodic motion is closely associated with wave motion, a topic that you have studied in previous physics courses. You might recall that a vibrating or oscillating object often creates a wave. You can make the comparison by imagining that you attached a pen to the end of a spring and allowed it to rest on a long sheet of paper. If you extended the spring and then released it, the pen would oscillate back and forth, drawing a line on the paper. If you pulled the paper under the pen at a steady rate while the pen was in motion, you would create an image like the one in Figure 5.12. You probably recognize the figure as having the same shape as the waves that you studied. Many of the terms that you learned in connection with waves also apply to periodic motion.

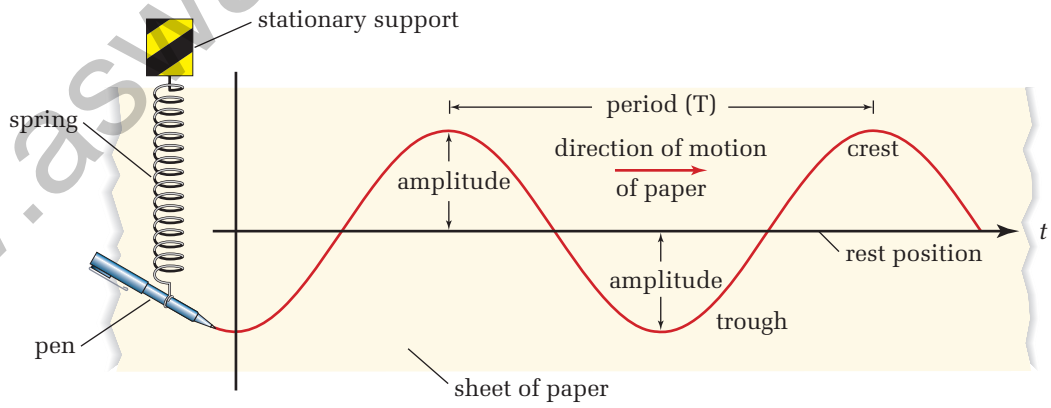


Figure 5.12 This wave shows how the position of the mass at the end of a spring changes with time. Mathematically, this graph is called a sine wave.

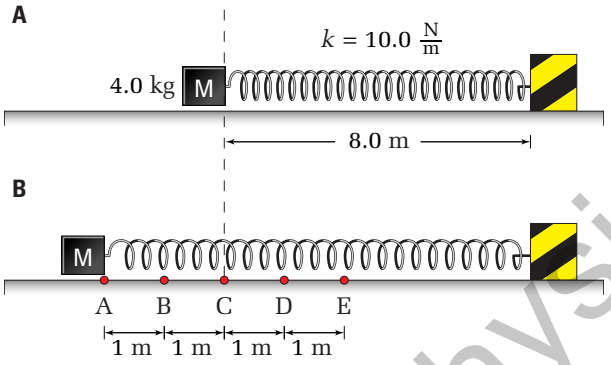
INVESTIGATION 5-C

Analyzing Periodic Motion

TARGET SKILLS

- **Predicting**
- **Analyzing and interpreting**
- **Communicating results**

Imagine that a spring is lying on a frictionless surface. One end is fastened to an immovable object and a 4.0 kg mass is attached to the free end, as shown in part (A) of the diagram. Then, the 4.0 kg mass is pulled 2.0 m to the left and held in place. You will follow the mass from the time it is released until it has travelled from Point A to point E, and then back to point A again, by determining the values of several of its variables at each labelled point. Let the positive direction be to the right and negative to the left.



Problem

Why does a spring continue to vibrate when stretched and then released?

Procedure

1. Prepare a table with the headings: Point, Extension or compression x(m), Restoring force F(N), Acceleration a(m/s^2), Elastic potential energy E_e(J), Kinetic energy E_k(J), Total energy E_t(J), Velocity $\vec{v}$ (m/s), and displacement of the mass Δd(m). Provide nine rows under the headings.
2. Under the heading "Point," list the letters A through E in the first five rows. In the following four rows, to represent the return trip, list D through A, with a prime on each letter. In the column headed "Extension or compression," write the displacement, x, between the given point and the equilibrium position, C.
3. Use the mathematical relationships that you have learned in this chapter and previous chapters to calculate all of the other values in the table. Be sure to include positive and negative directions in your calculations, where appropriate. (Hint: Note that the mass was released *from rest* at point A.)

Analyze and Conclude

1. At which points in the cycle does the mass have **(a)** the greatest acceleration and **(b)** the greatest speed?
2. At which points in the cycle does the spring **(a)** exert the greatest restoring force, **(b)** possess the greatest amount of elastic potential energy, and **(c)** possess the least amount of elastic potential energy?
3. Why does the mass reverse direction at point E and then at point A′?
4. If there is no friction, what will happen to the motion of the mass and spring? Give reasons for your answer.
5. Plot both of the following relationships on one graph: acceleration versus position and velocity versus position. Use different colours and scales on the vertical axis for each of the plots.
6. Explain the relationships between velocity and acceleration at those points where velocity is zero and where acceleration is zero.
7. On one graph, again using different colours, plot elastic potential energy versus position, kinetic energy versus position, and total energy versus position. Discuss the significance of the relationships among these graphs.

PHYSICS FILE

The previous investigation asked you to imagine a frictionless surface. Physics courses are littered with frictionless surfaces and massless springs, strings, and ropes. Making these assumptions simplifies calculations and helps to focus on the essential ideas. Once the process becomes one of engineering and design, however, friction and masses of components become extremely important and cannot be ignored.

Periodic motion always requires a restoring force that depends on a displacement from a rest or equilibrium position in order to keep on repeating. In the case of a vibrating spring, this restoring force is provided by the attractive forces between atoms in the spring. In a resonating air column, such as in a sounding trumpet or clarinet, the restoring force is provided by collisions with other air molecules. In an oscillating pendulum, such as a mass swinging on a length of string, the horizontal component of the tension (T) in the string returns the mass to the centre.

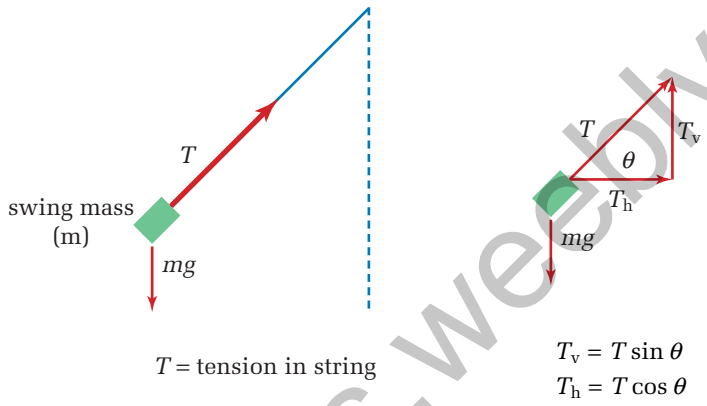


Figure 5.13 The horizontal component of the tension always acts in the direction of the equilibrium position of the pendulum. At the equilibrium position, the horizontal component of the tension is zero.

Periodic (or nearly periodic) motion is seen everywhere. Playground swings and teeter-totters exhibit periodic motion. Sound waves, water waves, and earthquake waves involve periodic motion. The electromagnetic spectrum from the radio waves that carry signals to our radios and televisions, to the gamma radiation emitted from radioactive materials, all embody periodic motion.

Figure 5.14 Waves on the ocean involve both transverse and longitudinal vibrations.

INVESTIGATION 5-D

Another Test of the Law of Conservation of Energy

TARGET SKILLS

- Predicting
- Performing and recording
- Analyzing and interpreting

In Investigation 5-A, you attempted to test the law of conservation of energy by making a prediction involving the transfer of energy from gravitational potential energy into kinetic energy. In this investigation, you will examine the transfer from elastic potential energy into kinetic energy. You will then use the predicted value of kinetic energy to determine the launch velocity of a projectile and, therefore, its range.

Problem

Does the law of conservation of energy make valid predictions when energy is converted from elastic potential energy into kinetic energy?

Equipment

- balance
- retort stand
- ramp or small, smooth board
- set of masses with a mass holder
- protractor
- metre stick or metric tape measure
- utility clamp
- small spring
- small cardboard box
- masking tape

CAUTION Safety goggles must be worn during this activity.

Procedure

Work in small groups for the investigation.

1. Measure the mass of the spring.
2. Using the equipment, determine the spring constant for the spring.
3. Set up the ramp on a desk, or make a ramp by resting one end of the board on a stack of books. Measure the angle that the ramp makes with the desktop. Make sure that there is a long stretch of clear space in front of the ramp.
4. Decide on the amount of extension that you intend to use with the spring and then determine the corresponding elastic potential energy stored in the spring at that extension.
5. Set up the spring by hooking one end over the upper edge of the ramp. Then, pull it backward to extend it the selected distance and release it. Use the law of conservation of energy to determine the velocity with which the spring will leave the ramp.
6. Use the velocity and the height of the end of the ramp to determine the point at which the spring will hit the floor (or the wall).
7. Place the cardboard box at that predicted point and perform the launch.

Analyze and Conclude

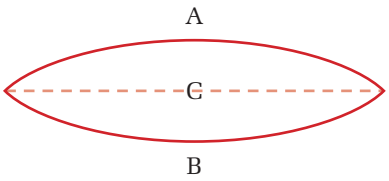
1. Provide a summary of your force-extension measurements for the spring.
2. Show your calculation of the spring constant.
3. What extension did the group choose? Show your calculation of the elastic potential energy stored in the spring.
4. Show your calculation of the
   (a) velocity of the spring as it leaves the ramp
   (b) range of the projectile (the spring)
5. How close did the spring come to its predicted landing point?
6. Describe the energy changes that occurred during the launch and flight of the spring.
7. Does this investigation further confirm the law of conservation of energy?

Apply and Extend

8. Spring-loaded dart guns with dart safety tips are available as toys. Decide how you could determine the spring constant and hence the maximum range of the projectile (the dart). If possible, repeat this investigation using one of these toy guns. You might recall from earlier studies that the maximum range occurs when the dart is launched at 45° to the horizontal.

5.2 Section Review

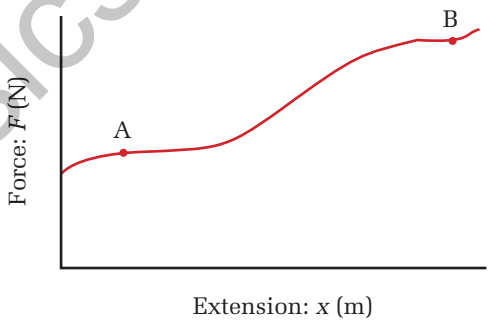
1. K/U Explain how each of the following behave like a spring.
   (a) a pole used in pole-vaulting
   (b) the strings in a tennis racquet
   (c) the string on a bow
2. I Prove that the expression for elastic potential energy has units equivalent to the joule.
3. MC In what way is a spring similar to a chemical bond?
4. MC List three other forms of periodic motion not mentioned in the section.
5. K/U A guitar string is vibrating horizontally, as shown in the diagram. It vibrates between positions A and B, passing through the equilibrium or rest position C. In which positions is the string vibrating with the following?



   (a) greatest speed
   (b) least speed
   (c) greatest kinetic energy
   (d) greatest elastic potential energy

   Give reasons for your choices.
6. MC There are four basic forces in our universe.
   - the weak nuclear force (between particles in the nucleus)
   - the strong nuclear force (between particles in the nucleus)
   - electromagnetic force (between charged particles)
   - gravitational force (between masses)

   Which force is responsible for the potential energy stored in the following?
   (a) a battery
   (b) the water behind a dam
   (c) a stretched spring
   (d) a mound of snow at the top of a slope just before an avalanche
7. I Describe an investigation to determine the force-extension characteristics of an archery bow.
8. C Prepare a diagram to demonstrate the relationships between the gravitational potential energy, and the kinetic energy of the swinging bob in a pendulum.
9. I Given the following graph of applied force against extension, describe a technique for determining the amount of potential energy stored in the object between points A and B.



UNIT PROJECT PREP

Once you understand both periodic motion and the conditions necessary to generate it, you will find that periodic motion frequently appears in both natural and manufactured systems.

- Brainstorm to identify systems that experience periodic motion.
- Attempt to formulate an argument supporting an intrinsic link between understanding the periodic transformation of energy and environmentalism.

5.3 Energy Transformations

The law of conservation of energy is one of the most useful tools in physics. Since work and energy are scalar quantities, directions are not involved, as they are in momentum. As a result, vector diagrams are not needed, and angles do not have to be calculated. In any given event, the problem is usually to identify the types of energy involved and to ensure that the total energy in all its different forms at the end of the event equals the total at the beginning.

The analysis is often easiest when the motion occurs in a horizontal plane. No change in gravitational potential energy is involved. The following sample problem illustrates this feature.

SECTION EXPECTATIONS

- Analyze situations involving the concepts of mechanical energy, thermal energy, and its transfer.
- Analyze situations involving the concept of conservation of energy.

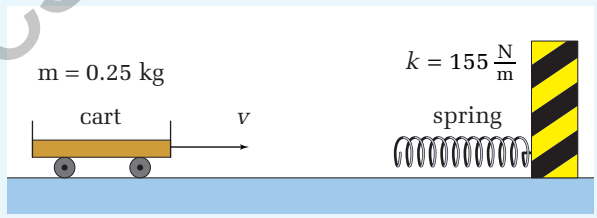
KEY TERMS

- conservative force
- non-conservative force

SAMPLE PROBLEM

Horizontal Elastic Collisions

A low-friction cart with a mass of 0.25 kg travels along a horizontal track and collides head on with a spring that has a spring constant of 155 N/m. If the spring was compressed by 6.0 cm, how fast was the cart initially travelling?



Conceptualize the Problem

- The cart is *moving* so it has *kinetic energy*.
- The spring does *negative work* on the cart, lowering its *kinetic energy*.
- The cart does *work* on the spring, giving it *elastic potential energy*.
- The *height* of the cart does *not change*, so there is no change in *gravitational potential energy*.
- The term *low friction* tells you to neglect the energy lost to work done by friction.
- The law of conservation of energy applies to this problem.

Identify the Goal

The initial speed, v, of the cart

Identify the Variables and Constants

Known		Unknown
$m = 0.25$ kg	$x = 0.060$ m	v
$k = 155 \frac{N}{m}$		

continued ▶

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Develop a Strategy

Write the law of conservation of energy, including the energy quantities associated with the interaction.

$$E'_k + E'_e = E_k + E_e$$

Initially, the spring was not compressed, so the initial elastic potential energy was zero.

$$E_e = 0\ \text{J}$$

After the interaction, the cart stopped, so the kinetic energy was zero.

$$E'_k = 0\ \text{J}$$

Substitute the values for energy listed above.

$$0\ \text{J} + E'_e = E_k + 0\ \text{J}$$
$$E_k = E'_e$$

Expand by substituting the expressions for the energies.

$$\frac{1}{2}mv^2 = \frac{1}{2}kx^2$$

Solve for the initial velocity.

$$v = \sqrt{\frac{kx^2}{m}}$$

Substitute numerical values and solve.

$$v = \sqrt{\frac{(155\ \frac{\text{N}}{\text{m}})(0.060\ \text{m})^2}{0.25\ \text{kg}}}$$
$$v = 1.493\ 99\ \frac{\text{m}}{\text{s}}$$
$$v \cong 1.5\ \frac{\text{m}}{\text{s}}$$

The cart was travelling at approximately 1.5 m/s before the collision.

Validate the Solution

Unit analysis of the equation $v = \sqrt{\frac{kx^2}{m}}$ shows that it is equivalent to m/s, the standard units for velocity. $\sqrt{\frac{\frac{\text{N}}{\cancel{\text{m}}}\text{m}^{\cancel{2}}}{\text{kg}}} = \sqrt{\frac{\frac{\cancel{\text{kg}}\cdot\text{m}}{\text{s}^2}\text{m}}{\cancel{\text{kg}}}} = \sqrt{\frac{\text{m}^2}{\text{s}^2}} = \frac{\text{m}}{\text{s}}$.

A velocity of 1.5 m/s is reasonable for a lab cart.

PRACTICE PROBLEMS

22. A 1.2 kg dynamics cart is rolling to the right along a horizontal lab desk at 3.6 m/s, when it collides head on with a spring bumper that has a spring constant of 2.00×10^2 N/m.

(a) Determine the maximum compression of the spring.

(b) Determine the speed of the cart at the moment that the spring was compressed by 0.10 m.

(c) Determine the acceleration of the cart at the moment that the spring was compressed 0.10 m.

23. A circus car with a clown has a total mass of 150 kg. It is coasting at 6.0 m/s, when it hits a large spring head on. If it is brought to a stop by the time the spring is compressed 2.0 m, what is the spring constant of the spring?

The analysis becomes a bit more complicated when the motion is vertical, since there are now changes in gravitational potential energy along with elastic potential energy and kinetic energy.

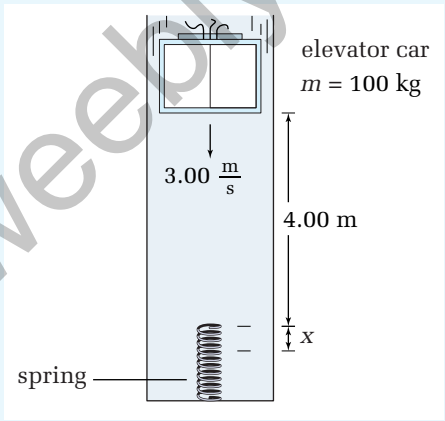
PHYSICS FILE

The energies discussed here are commonly found in mechanical systems with springs and pulleys. As a result, kinetic energy, gravitational potential energy, and elastic potential energy are commonly referred to as "mechanical energy."

SAMPLE PROBLEM

Vertical Elastic Collisions

A freight elevator car with a total mass of 100.0 kg is moving downward at 3.00 m/s, when the cable snaps. The car falls 4.00 m onto a huge spring with a spring constant of 8.000×10^3 N/m. By how much will the spring be compressed when the car reaches zero velocity?



Conceptualize the Problem

- Initially, the car is in *motion* and therefore has *kinetic energy*. It also has *gravitational potential energy*.
- As the car begins to *fall*, the *gravitational potential energy* transforms into *kinetic energy*. When the elevator hits the spring, the elevator *slows*, losing *kinetic energy*, and the spring compresses, gaining *elastic potential energy*.
- When the elevator comes to a complete *stop*, it has *no kinetic* or *gravitational potential energy*. All of the energy is now stored in the spring in the form of *elastic potential energy*.
- Since all of the motion is in a downward direction, define "down" as the positive direction for this problem.

Identify the Goal

The compression of the spring, x, when the car comes to rest

Identify the Variables and Constants

Known	Implied	Unknown
$m_{car} = 100.0$ kg	$g = 9.81 \frac{m}{s^2}$	x
$v = 3.00 \frac{m}{s}$ [down]		
$k = 8.000 \times 10^3 \frac{N}{m}$		
$h_{(above\ spring)} = 4.00$ m		

Develop a Strategy

Write the law of conservation of energy for the forms of energy involved in the problem.

$$E'_g + E'_e + E'_k = E_g + E_e + E_k$$

continued ▶

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Choose the lowest level of the elevator (maximum compression of the spring) as the reference level for gravitational potential energy.

$$E_g' = 0 \text{ J}$$

The car comes to a rest at the lowest point.

$$E_k' = 0 \text{ J}$$

Initially, the spring is not compressed.

$$E_e = 0 \text{ J}$$

Substitute these initial and final conditions into the equation for conservation of energy and simplify.

$$0 \text{ J} + E_e' + 0 \text{ J} = E_g + 0 \text{ J} + E_k$$
$$E_e' = E_g + E_k$$

Expand by substituting the expressions for the various forms of energy.

$$\tfrac{1}{2}kx^2 = mg\Delta h + \tfrac{1}{2}mv^2$$

The change in height for the gravitational potential is 4.00 m, plus the compression of the spring, x. Substitute this expression into the equation.

$$\tfrac{1}{2}kx^2 = mg(4.00 + x) + \tfrac{1}{2}mv^2$$

Since the equation yields a quadratic equation, you cannot solve for x. Substitute in the numerical values and rearrange so that the right-hand side is zero.

$$\tfrac{1}{2}\left(8.00 \times 10^3 \ \frac{\text{N}}{\text{m}}\right)x^2 = (100.0 \text{ kg})\left(9.81 \ \frac{\text{m}}{\text{s}^2}\right)(4.00 + x) \text{ m} + \tfrac{1}{2}(100.0 \text{ kg})\left(3.00 \ \frac{\text{m}}{\text{s}}\right)^2$$

$$4.00 \times 10^3 x^2 - 981x - 3924 - 450 = 0$$

$$4.00 \times 10^3 x^2 - 981x - 4374 = 0$$

Use the quadratic formula to find the value of x.

$$x = \frac{981 \pm \sqrt{(-981)^2 - 4(4.00 \times 10^3)(-4374)}}{2(4.00 \times 10^3)}$$

$$x = 1.1756 \text{ m} \quad \text{or} \quad -0.930\ 25 \text{ m}$$

$$x \cong 1.18 \text{ m}$$

Compression cannot be negative (or the spring would be stretching), so choose the positive value. The spring was compressed 1.18 m.

Validate the Solution

The units on the left-hand side of the final equation are $\frac{\text{N}}{\text{m}} \cdot \text{m}^2 = \frac{\frac{\text{kg} \cdot \text{m}}{\text{s}^2}}{\text{m}} \cdot \text{m}^2 = \frac{\text{kg} \cdot \text{m}^2}{\text{s}^2}$.

On the right-hand side of the equation, the units are $\text{kg} \cdot \frac{\text{m}}{\text{s}^2} \cdot \text{m} = \frac{\text{kg} \cdot \text{m}^2}{\text{s}^2}$.

Both sides of the equation have the same units, so you can have confidence in the equation. The answer is also in a range that would be expected with actual springs.

Note: The negative root in this problem is interesting in that it does have meaning. If the car had somehow latched onto the spring during the collision, the negative value would represent the maximum extension of the spring if the car had bounced up from the bottom due to the upward push of the spring.

24. A 70.0 kg person steps through the window of a burning building and drops to a rescue net held 8.00 m below. If the surface of the net is 1.40 m above the ground, what must be the value of the spring constant for the net so that the person just touches the ground when the net stretches downward?

25. A 6.0 kg block is falling toward a spring located 1.80 m below. If it has a speed of 4.0 m/s at that instant, what will be the maximum compression of the spring? The spring constant is 2.000×10^3 N/m.

26. In a "head dip" bungee jump from a bridge over a river, the bungee cord is fastened to the jumper's ankles. The jumper then steps off and falls toward the river until the cord becomes taut. At that point, the cord begins to slow the jumper's descent, until his head just touches the water. The bridge is 22.0 m above the river. The unstretched length of the cord is 12.2 m. The jumper is 1.80 m tall and has a mass of 60.0 kg. Determine the

(a) required value of the spring constant for this jump to be successful

(b) acceleration of the jumper at the bottom of the descent

Conservative and Non-Conservative Forces

Until now, you have been asked to assume that objects could move without friction. A pendulum would keep swinging repeatedly with the same amplitude, continuously converting energy between kinetic and gravitational potential forms of energy. A skier could slide down a hill, converting gravitational potential energy into kinetic energy and then, faced with an upward slope, could keep on going, converting the kinetic energy back into potential energy until the original height was reached.

The forces with which you have been dealing are referred to as **conservative forces**. This means that the amount of work that they do on a moving object does not depend on the path taken by that object. In the absence of friction, the boulder in Figure 5.15 will reach the bottom of the hill with the same kinetic energy and speed whether it dropped off the cliff on the left or slid down the slope on the right.

COURSE CHALLENGE

Energy Transformations

Light energy is transformed into stored chemical energy each time you take a photograph. The operation of infrared cameras, ultrasound images, and video cameras also relies on various energy transformations. Refer to page 604 for suggestions on relating energy transformations to your *Course Challenge*.

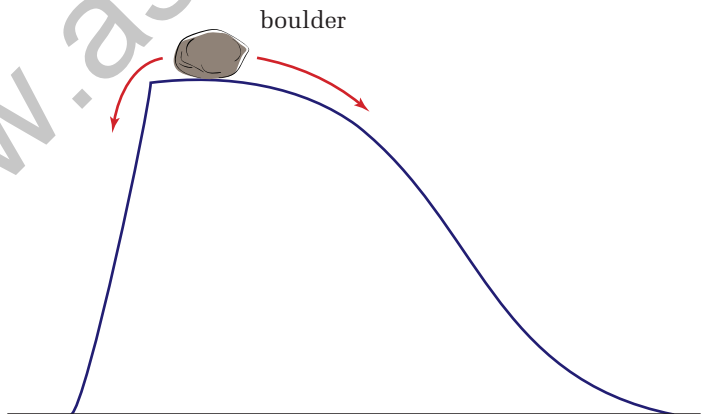


Figure 5.15 Gravity is a conservative force. If the boulder was dropped over the edge of the cliff, all of the gravitational potential energy would be converted into kinetic energy. Friction is not a conservative force. If the boulder slides down the hill, the kinetic energy at the bottom will not be as great as it would if the boulder fell straight down.

ELECTRONIC LEARNING PARTNER

To enhance your understanding of energy transformation, refer to your Electronic Learning Partner.

Friction is a **non-conservative force**. The amount of work done by a non-conservative force depends on the path taken by the force and the object. For example, the amount of energy transferred to the snow in Figure 5.16 depends on the path taken by the skier. The skier going straight down the slope should reach the bottom with a greater speed than the skier who is tracking back and forth across the slope.

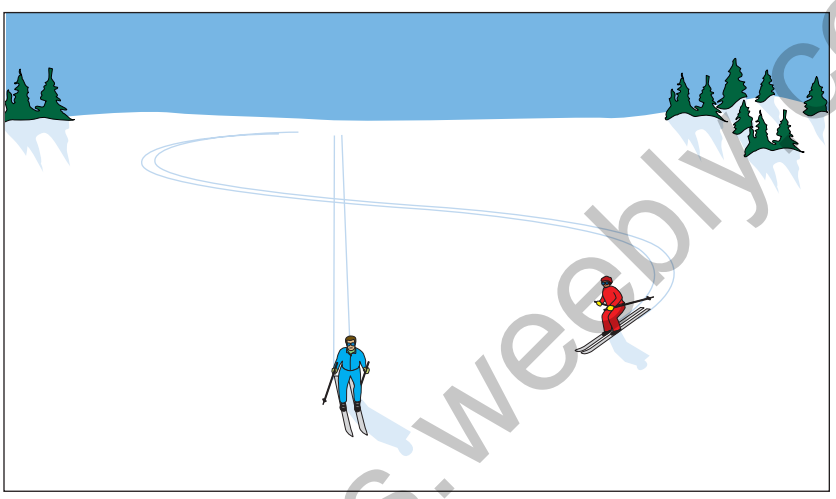


Figure 5.16 Although friction between the skis and the snow is small, friction nevertheless does some work on the skiers, slowing their velocity a little. The work done by friction is greater along the longer of the paths.

Friction causes the skier to do work on the environment. The snow heats up slightly and is moved around. For the skier, this is negative work — the skier is losing energy and cannot regain it as useful kinetic or potential energy. The sum of the skier's kinetic and gravitational potential energy at the end of the run will be less than it was at the start.

Wind pressure is another example of a non-conservative force. If the skier had the wind coming from behind, the wind (the environment) could be doing work on the skier. This would be positive work. The sum of the skier's kinetic and gravitational potential energies could increase beyond the initial total. However, the amount of energy transferred by the wind would depend to a large extent on the path of the skier, so the wind would be a non-conservative force.

When dealing with non-conservative forces, the law of conservation of energy still applies. However, you must account for the energy exchanged between the moving object and its environment. One approach to this type of situation is to define the system as the skier and the local environment; that is, the skier, wind, and snow become the system. The following sample problem illustrates this concept.

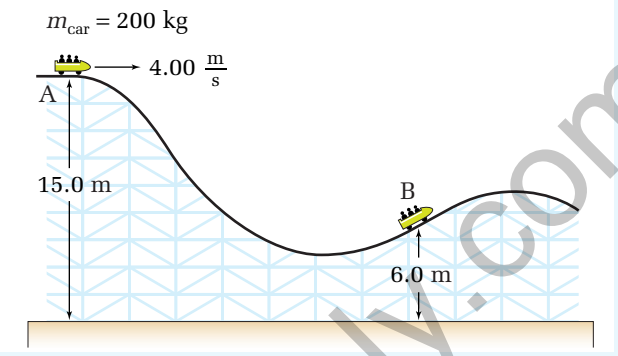
PHYSICS FILE

Quite often, you might not want your forces to be conservative. Without friction, many of your clothes would simply fall apart into strands as you moved. In addition, keep-fit programs would have to be greatly modified. A person who rides an exercise bicycle to lose mass (through chemical reactions that provide the energy) does not want the energy back. It simply is dissipated as sound and heat. Likewise, the weight lifter who does work to lift a bar bell does not expect to receive that energy back when the bar bell is lowered.

SAMPLE PROBLEM

Energy Conversions on a Roller Coaster

A roller-coaster car with a mass of 200.0 kg (including the riders) is moving to the right at a speed of 4.00 m/s at point A in the diagram. This point is 15.00 m above the ground. The car then heads down the slope toward point B, which is 6.00 m above the ground. If 3.40×10^3 J of heat energy are produced through friction between points A and B, determine the speed of the car at point B.



Conceptualize the Problem

- As the roller-coaster car *moves* down the track, most of the *gravitational potential energy* is converted into *kinetic energy*, but some is lost as *heat* due to *friction*.
- The law of conservation of total energy applies.
- *Heat energy* must be included as a *final energy*.

Identify the Goal

The speed of the car at point B, v_B

Identify the Variables and Constants

Known		Implied	Unknown
$h_A = 15.00$ m	$E_{heat} = 3.40 \times 10^3$ J	$g = 9.81 \frac{m}{s^2}$	v_B
$h_B = 6.00$ m	$m = 200.0$ kg		
$v_A = 4.00 \frac{m}{s}$			

Develop a Strategy

Write the law of conservation of energy, including heat as a final energy form.

$$E'_k + E'_g + E_{heat} = E_k + E_g$$

Expand by substituting the expressions for the forms of energy. Solve for the speed of the car at point B

$$\frac{1}{2}mv_B^2 + mgh_B + E_{heat} = \frac{1}{2}mv_A^2 + mgh_A$$

$$\frac{1}{2}mv_B^2 = -mgh_B - E_{heat} + \frac{1}{2}mv_A^2 + mgh_A$$

$$v_B^2 = \frac{2(-mgh_B - E_{heat} + \frac{1}{2}mv_A^2 + mgh_A)}{m}$$

$$v_B = \sqrt{\frac{2(-mgh_B - E_{heat} + \frac{1}{2}m(v_A)^2 + mgh_A)}{m}}$$

$$v_B = \sqrt{\frac{2\left[-\left(1.1772 \times 10^4 \frac{kg \cdot m^2}{s^2}\right) - (3.40 \times 10^3 \text{ J}) + \left(1.6 \times 10^3 \text{ kg}\frac{m^2}{s^2}\right) + \left(2.943 \times 10^4 \frac{kg \cdot m^2}{s^2}\right)\right]}{200.0 \text{ kg}}}$$

$$v_B = \sqrt{\frac{3.1716 \times 10^4 \frac{\cancel{kg} \cdot m^2}{s^2}}{200.0 \cancel{kg}}}$$

continued ▶

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$$v_B = \sqrt{1.5858 \times 10^2 \ \frac{m^2}{s^2}}$$

$$v_B = 1.2593 \times 10^1 \ \frac{m}{s}$$

$$v_B \cong 12.6 \ \frac{m}{s}$$

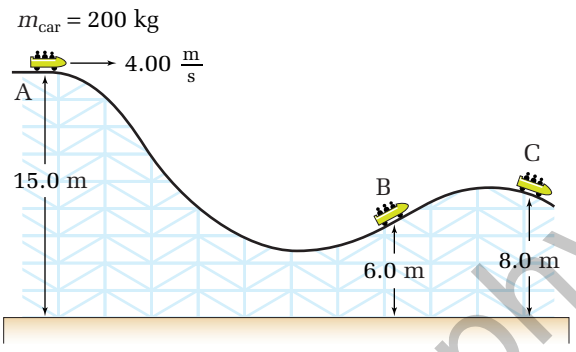
The speed of the car at point B will be 12.6 m/s.

Validate the Solution

The speed at point B is expected to be larger than its speed at point A.

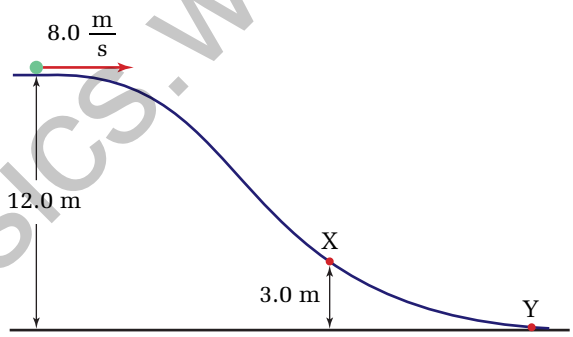
PRACTICE PROBLEMS

27. Determine the speed of the roller-coaster car in the sample problem at point C if point C is 8.0 m above the ground and another 4.00×10^2 J of heat energy are dissipated by friction between points B and C.



28. A sled at the top of a snowy hill is moving forward at 8.0 m/s, as shown in the diagram. The height of the hill is 12.0 m. The total mass of the sled and rider is 70.0 kg.

Determine the speed of the sled at point X, which is 3.0 m above the base of the hill, if the sled does 1.22×10^3 J of work on the snow on the way to point X.



29. If the sled in the previous question reaches the base of the hill with a speed of 15.6 m/s, how much work was done by the snow on the sled between points X and Y?

PROBEWARE

If your school has probeware equipment, visit ***www.mcgrawhill.ca/links/physics12*** and follow the links for an in-depth activity on energy, Hooke's law, and simple harmonic motion.

In solving these problems, you have assumed that the value for the acceleration due to gravity (g) is constant at 9.81 m/s^2. You probably recall reading that this value is valid only for a small region close to Earth's surface. In Chapter 3, you learned that, as you go to the higher altitudes, the acceleration due to gravity decreases. You worked with forces of gravity at any distance from Earth, other planets, and even stars. You learned how to calculate the radii of orbits and orbital speed of satellites.

In the next chapter, you will focus on the energy requirements for sending a satellite into orbit and even for escaping Earth's gravitational pull entirely. You will also learn the importance of the conservation of momentum in navigating through space.

INVESTIGATION 5-E

Mechanical and Thermal Energy

TARGET SKILLS

- Performing and recording
- Analyzing and interpreting
- Identifying variables

Before 1800, physicists and chemists did not know that a relationship existed between mechanical energy and heat. Count Rumford (Benjamin Thompson: 1753–1814) was the first to observe such a relationship, followed by Julius Robert Mayer (1814–1878). Rumford and Mayer made some very important discoveries. Mayer was unable to express himself clearly in writing, however, so his discoveries were overlooked. Eventually, James Prescott Joule (1818–1889) was credited with the determination of the mechanical equivalence of heat. In this investigation you will perform experiments similar to those of Mayer and Joule.

Problem

How much heat is produced when a mass of lead pellets is repeatedly lifted and dropped through a known distance?

Equipment

- balance
- thermometer (°C)
- lead shot
- cardboard or plastic tube with a small hole in the side, close to one end; the ends must be able to be closed
- metre stick
- small amount of masking or duct tape

Procedure

1. Determine the mass of the lead shot.
2. Place the lead shot into the tube and close up the tube. Let the tube sit upright on a desk for several minutes to allow the tube and its contents to come to room temperature. Make sure that the hole is close to the bottom of the tube.
3. Insert the thermometer or temperature probe through the hole in the tube and nestle the end in the lead shot. Measure and record the temperature.
4. Close the hole.
5. Measure the length of the tube.
6. Repeatedly invert the tube for several minutes, waiting only to allow the lead shot to fall to the bottom on each inversion. Keep track of the number of inversions.
7. Finish the inversions with the hole near the bottom of the tube. Remove the tape and measure the temperature of the lead shot. Record the final temperature.

Analyze and Conclude

1. What were the initial and final temperatures of the lead shot? What was the total mass of the lead shot?
2. Determine the quantity of heat gained by the lead shot (the specific heat capacity of lead is 128 J/kg · C°).
3. Determine the total distance through which the lead shot was lifted by the inversions and calculate the total gain in gravitational potential energy of the lead.
4. Determine the percentage of the gravitational potential energy that was converted into heat.
5. If the conversion into heat does not account for all of the gravitational potential energy gained by the lead shot, where else might some of the energy have gone?

Apply and Extend

6. How could this investigation be improved? Try to design a better apparatus and, if possible, carry out the investigation again.
7. Do research and write a summary of the work of Rumford, Mayer, and Joule on the mechanical equivalence of heat.

Follow Your Dreams

One of the great honours in physics is to have a physical law or constant named after you — Newton's laws, Planck's constant, the Heisenberg uncertainty principle. Now, the name of Canadian physicist Dr. Ian Keith Affleck can be added to this list. "Affleck-Dine Baryogenesis" is the name given to a physical mechanism that might have played an important role in the early universe in creating one of the classes of particles that now make up all of the matter that exists today.

For Dr. Affleck, who was born in Vancouver and grew up both there and in Ottawa, understanding nature has always been one of his great interests. "I became rather fascinated at a fairly young age with the idea that deep things about the universe could be understood by using mathematics," he explains. At high school in Ottawa, he was inspired by the intellectual enthusiasm of his physics teachers, and in university decided on a career in theoretical physics. Ironically, at the time, he "was not very optimistic about actually being able to make a career from my interests."

Dr. Ian Affleck

One of the great questions plaguing theoretical physicists is nothing less than the age-old philosophical question: Why are we here? It is believed that at the time of the Big Bang, there were nearly equal amounts of matter and antimatter. Since matter and antimatter annihilate each other, if the amounts of each were *exactly* equal, there would be nothing left after particle annihilation. So, there had to be some excess of matter over antimatter, and it had to be just the right amount of excess to yield the universe and the physics that exist today. Dr. Affleck, together with fellow theoretician Michael Dine, proposed a possible explanation — Affleck-Dine Baryogenesis. Verifying this principle is now an active part of physics research all over the world.

Dr. Affleck has since gone on to bring his mathematical talents to more immediate problems. Specifically, he has been hard at work adapting the mathematics he helped develop for an understanding of the universe to problems of understanding how and why high-temperature superconductors work. "I saw some opportunities to apply the same sort of mathematical ideas more directly," he says. Just as there is a problem with the pairing of particles and antiparticles in the early universe, there appears to be a pairing mechanism at work in the behaviour of high-temperature superconductors, so that it is possible to gain a deeper understanding of these materials through mathematics originally devised for more abstract research.

Such creative thinking has earned the physicist a number of awards, including the Rutherford Medal and the Governor General's Medal. He is also the recipient of many honorary degrees. His advice to aspiring physicists is simple: "They should follow up on what they find interesting, and not be afraid to follow their dreams."

Going Further

1. The astronomer Carl Sagan used to say, "You never know where inspiration will come from." One of Dr. Affleck's great achievements was to adapt what seemed like very abstract and very specific physical theory to a more concrete problem. This is not the first time this has happened in the history of physics; look into a few of the popular books on physics and see if you can find some other examples. (Hint: You can start with Carl Sagan.)

2. Much of Dr. Affleck's recent work has had to do with superconductivity. Superconductors have applications in medicine, engineering, and elsewhere. Research two different present-day applications related to superconductivity.

3. What are some of the difficulties with the superconductors now in use? Report and discuss with the class. Design a poster or a media presentation to present your findings.

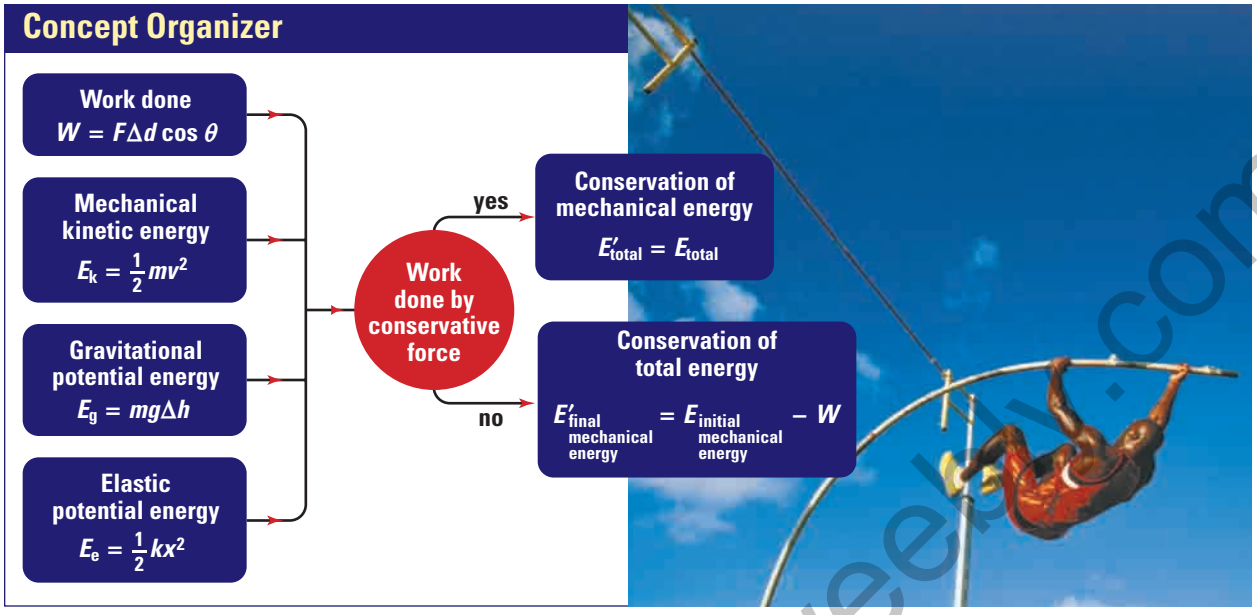


Figure 5.17 How many energy transformations are taking place in the photograph?

5.3 Section Review

1. K/U
   (a) Why is the application of the law of conservation of energy often much easier than the application of the law of conservation of momentum?
   (b) What conditions can increase the difficulty of applying the law of conservation of energy?
2. K/U Which types of energy are generally referred to as mechanical energy?
3. C Using examples not found in the textbook, describe and explain an example in which the forces are
   (a) conservative
   (b) non-conservative
4. I A student is sliding down a frictionless water slide at an amusement park.
   (a) Sketch a graph of gravitational potential energy against height for the descent. (No numbers are required on the axes.)
   (b) On the same axes, sketch a graph of the total energy of the student against height for the descent.
   (c) On the same axes, sketch a graph of the kinetic energy of the student against height for the descent.
5. MC
   (a) In an amusement park there is a ride on which children sit in a simulated log while it slides rapidly down a water-covered slope. At the bottom, the log slams into a trough of water, which slows it down. Why did the ride designers not simply have the log slam into a large spring?
   (b) Steel or plastic barrels are located along highways to cushion the impact if a car skids into a bridge abutment. These barrels are often filled with energy-absorbing material. Why are these barrels used instead of large springs to bring the cars to a stop?

CHAPTER 5 Review

REFLECTING ON CHAPTER 5

- Work is defined as the product of force times displacement. In general, if the force acts at an angle (θ) to the displacement, the work done by the force is given by $W = F\Delta d \cos\theta$.
- The work done by an applied force equals the change in energy produced by that applied force.
- Kinetic energy is expressed as $E_k = \frac{1}{2}mv^2$.
- Gravitational potential energy for positions near Earth's surface is expressed as $E_g = mg\Delta h$.
- An isolated system is one that neither gains energy from its environment nor loses energy to its environment.
- The law of conservation of energy states that, in an isolated system, the total energy is conserved, but can be transformed from one form to another.
- For an ideal spring, the restoring force is proportional to the amount of extension or compression of the spring. This is expressed as $F = -kx$, where k is the spring constant. The applied force that causes the spring to stretch (or compress) is equal in magnitude and opposite in direction to the restoring force: $F_a = kx$
- The amount of elastic potential energy stored in a spring is equal to the area under the force-extension (or compression) graph for the spring. It can be calculated from $E_e = \frac{1}{2}kx^2$.
- Applied forces are conservative if the amount of work that they do on an object as it moves between two points is independent of the path of the object between those points.
- Applied forces are non-conservative if the amount of work that they do on an object as it moves between two points is dependent on the path of the object between those points.
- Work done by an object on its environment is negative work and decreases the total energy of that object. Work done on an object by its environment is positive work and increases the total energy of the object.

Knowledge/Understanding

1. Explain what happens to the total mechanical energy over a period of time for open systems, closed systems, and isolated systems.
2. Write a general equation that relates the change in mechanical energy in systems to the amount of work done on it and the amount of heat lost by it.
3. You wind up the spring of a toy car and then release it so that it travels up a ramp. Describe all of the energy transformations that take place.
4. Compare how the everyday notion of work as "exerting energy to complete a task" differs from the physical definition of work.
5. Explain the sign convention for designating whether work is being done by an object on its environment or whether the environment is doing work on an object.
6. **(a)** Explain whether work done by a frictional force on an object can be positive.
   (b) Explain when the work done by the restoring force of a spring on a mass is considered to be positive and when it is considered to be negative with respect to the mechanical energy of the mass.
   (c) Discuss your answers to the above questions in terms of conservative and non-conservative forces.
7. Define and give an example of periodic motion.

Inquiry

8. Imagine taking a spring of 10 coils and cutting it in half. Will each smaller spring have a smaller, larger, or the same spring constant as the larger spring? (Hint: Consider the force required to compress the large and small springs by the same amount.)

9. A basic clock consists of an oscillator and a mechanism that is "turned" by the oscillator (to count the oscillations). Design a clock based on a simple pendulum or other oscillating device, using readily available materials. If possible, construct the clock and determine its accuracy. Even if you are not successful in constructing a functional clock, outline the technological challenges that you encountered.

10. The transformation of energy between kinetic and potential forms in an ideal simple harmonic oscillator can be modelled mathematically by writing a total mechanical energy equation for specified points during its motion. Consider a spring attached to a wall at floor level. A block of wood is attached to the other end of the spring so that the block can oscillate across the floor in a horizontal plane. Assume that the floor is frictionless. Set a frame of reference for the spring so that the equilibrium position of the system is $x = 0$ and the maximum displacements of the block is $x = -A$ and $x = +A$. Set the block of wood in motion by pulling it back to position +A.
   - **(a)** Write expressions for the total energy of the system at points –A, +A, and zero.
   - **(b)** At which of the three above points is the kinetic energy at its maximum and at its minimum?
   - **(c)** At which of the three points is the velocity at its maximum and at its minimum?
   - **(d)** Sketch a graph of energy versus position with individual curves for the elastic potential energy and the kinetic energy of the block as it oscillates. What is the geometrical shape of each curve?
   - **(e)** Sketch a velocity-versus-position graph.

11. Bowling balls need to be returned promptly from the end of the alley so that they can be used again. Sketch a ball-return system that requires no external energy source. Explain the energy transformations involved in the operation of your system. Identify the conservative and non-conservative forces that need to be taken into consideration. What features does the design include to minimize wear and tear on the bowling balls, despite their large mass?

Communication

12. Imagine that you are moving a negatively charged sphere toward a Van de Graaff generator. As you bring the sphere closer, does the energy of the system increase or decrease? Explain your reasoning.

13. Each of three stones is displaced to a vertical height of *h*. Stone R is placed on the top of a ramp, stone P is at the end of a taut pendulum string, and stone G is simply held above the ground. Do each of these stones have the same gravitational potential energy?
   - **(a)** If frictional forces are neglected, will each stone have the same kinetic energy at the instant before it reaches the bottom of its path? Explain your reasoning.
   - **(b)** If you consider likely frictional forces, will each stone have the same kinetic energy at the instant before it reaches the bottom of its path?
   - **(c)** Use the above two examples to differentiate between conservative and non-conservative forces.

14. A child descends a slide in the playground. Write expressions to show the total mechanical energy of the child at the top, halfway down, and at the bottom of the slide. Write a mathematical expression that relates the energy total at the three positions.

Making Connections

15. When stretched or compressed, a spring stores potential energy. Make a list of other common devices that store potential energy when temporarily deformed.

16. Research and write a brief report about how chemists use the concept of ideal springs to model the action of the bonds holding atoms together in molecules.

17. Car bumper systems are designed to absorb the impact of slow-speed collisions in such a way that the vehicles involved sustain no permanent damage. Prepare a presentation on how a bumper system works, including an explanation of the energy transformations involved.

Problems for Understanding

18. A 0.80 kg block of wood has an initial velocity of 0.25 m/s as it begins to slide across a table. The block comes to rest over a distance of 0.72 m.
    (a) What is the average frictional force on the block?
    (b) How much work is done on the block by friction?
    (c) How much work is done on the table by the block?
19. A 1.5 kg book falls 1.12 m from a table to the floor.
    (a) How much work did the gravitational force do on it?
    (b) How much gravitational potential energy did it lose?
20. A 175 kg cart is pushed along level ground for 18 m, with a force of 425 N, and then released.
    (a) How much work did the applied force do on the cart?
    (b) If a frictional force of 53 N was acting on the cart while it was being pushed, how much work did the frictional force do on the cart?
    (c) Determine how fast the cart was travelling when it was released.
    (d) Determine how far the cart will travel after it is released.
21. A man is pushing a 75 kg crate at constant velocity a distance of 12 m across a warehouse. He is pushing with a force of 225 N at an angle of 15° down from the horizontal. The coefficient of friction between the crate and the floor is 0.24. How much work did the man do on the crate?
22. A boy, starting from rest, does 2750 J of work to propel himself on a scooter across level ground. The combined mass of the boy and scooter is 68 kg. Assume friction can be neglected.
    (a) How fast is he travelling?
    (b) What is his kinetic energy?
    (c) If he then coasts up a hill, to what vertical height does he rise before stopping?
23. While coasting on level ground on a bicycle, you notice that your speed decreased from 12 m/s to 7.5 m/s over a distance of 50.0 m. If your mass combined with the bicycle's mass is 65 kg, calculate the average force that opposes your motion.
24. A 0.50 kg air puck is accelerated from rest with a force of 12.0 N. If the force acts over 45 cm and the surface is frictionless, how fast is the puck going when it is released?
25. If 25 N are required to compress a spring 5.5 cm, what is the spring constant of the spring?
26. (a) What is the change in elastic potential energy of a spring that has a spring constant of 120 N/m if it is compressed by 8.0 cm?
    (b) What force is required to compress the spring by 8.0 cm?
27. A 0.500 kg mass resting on a frictionless surface is attached to a horizontal spring with a spring constant of 45 N/m. When you are not looking, your lab partner pulls the mass to one side and then releases it. When it passes the equilibrium position, its speed is 3.375 m/s. How far from the equilibrium position did your lab partner pull the mass before releasing it?
28. A mass m_1 is hung on a spring and stretches the spring by $x = 10.0$ cm. What is the spring constant in terms of the variables?
29. A dart gun has a spring with a constant of 74 N/m. An 18 g dart is loaded into the gun, compressing the spring from a resting length of 10.0 cm to a compressed length of 3.5 cm. If the spring transfers 75% of its energy to the dart after the gun is fired, how fast is the dart travelling when it leaves the gun?

30. A 12 g metal bullet (specific heat capacity: c = 669 J/kg˚C) is moving at 92 m/s when it penetrates a block of wood. If 65% of the work done by the stopping forces goes into heating the metal, how much will the bullet's temperature rise in the process?

31. Consider a waterfall that is 120 m high. How much warmer is the water at the bottom of the waterfall than at the top? (The specific heat of water is 4186 J/kg ˚C.)

32. A spring with a constant of 555 N/m is attached horizontally to a wall at floor level. A 1.50 kg wooden block is pushed against it, compressing the spring by 12 cm, and then released.
    (a) How fast will the block be travelling at the instant it leaves the spring? (Assume that friction can be ignored and that the mass of the spring is so small that its kinetic energy can be ignored.)
    (b) If the block of wood travels 75 cm after being released and then comes to rest, what friction force opposes its motion?

33. A simple pendulum swings freely and rises at the end of its swing to a position 8.5 cm above its lowest point. What is its speed at its lowest point?

34. A 50.0 g pen has a retractable tip controlled by a button on the other end and an internal spring that has a constant of 1200 N/m. Suppose you hold the pen vertically on a table with the tip pointing up. Clicking the button into the table compresses the spring 0.50 cm. When the pen is released, how fast will it rise from the table? To what vertical height will it rise? (Assume for simplicity that the mass of the pen in concentrated in the button.)

35. A spring with a spring constant of 950 N/m is compressed 0.20 m. What speed can it give to a 1.5 kg ball when it is released?

36. A basketball player dunks the ball and momentarily hangs from the rim of the basket. Assume that the player can be considered as a 95.0 kg point mass at a height of 2.0 m above the floor. If the basket rim has a spring constant of 7.4×10^3 N/m, by how much does the player displace the rim from the horizontal position?

37. A 35 kg child is jumping on a pogo stick. If the spring has a spring constant of 4945 N/m and it is compressed 25 cm, how high will the child bounce? (Assume that the mass of the pogo stick is negligible.)

38. A spring with a spring constant of 450 N/m hangs vertically. You attach a 2.2 kg block to it and allow the mass to fall. What is the maximum distance the block will fall before it begins moving upward?

39. A 48.0 kg in-line skater begins with a speed of 2.2 m/s. Friction also does –150 J of work on her. If her final speed is 5.9 m/s,
    (a) determine the change (final – initial) in her gravitational potential energy.
    (b) By how much, and in which direction (up or down), has her height changed?

CHAPTER 6

Energy and Motion in Space

CHAPTER CONTENTS

PREREQUISITE CONCEPTS AND SKILLS

- Newton's law of universal gravitation
- Potential energy
- Centripetal force
- Kinetic energy

Master jugglers can keep as many as eight plates or seven flaming torches airborne and under perfect control at the same time. Amazing muscle and hand-eye co-ordination enables the launching of each object with precisely the right kinetic energy. Opposing Earth's gravitational attraction, this energy allows the object to free fall for a precise interval, returning to the height of the juggler's hand at just the right time and location to be caught and passed to the other hand for another toss.

Launching a missile or an Earth satellite is much like juggling. Work done against gravitational forces partially overcomes Earth's attraction and allows the object to follow a planned trajectory or to be inserted into a previously defined orbit. With even more initial energy, a space probe can eventually escape from Earth's orbit — or even from the solar system entirely. Successful launches depend on calculating, modelling, and simulating the energies needed to attain orbits or trajectories with specific shapes and sizes.

Your investigations of impulse, momentum, work, and energy have given you many of the mathematical tools needed to analyze energy and motion in space. In this chapter, you will refine your concept of gravitational potential energy, find out how much work must be done to boost an object away from a planet's surface, and investigate the energy of satellites in orbit.

QUICK LAB

Escape from a Planetoid

TARGET SKILLS

- **Analyzing and interpreting**
- **Communicating results**

Imagine that you are stationed on a spherical planetoid (a small planet-like object) somewhere in space. The planetoid has a mass of 1.0×10^{22} kg and a radius of 1.0×10^{6} m. You want to send a small 6.0 kg canister off into space so that it will escape the gravity of the planetoid and not fall back to the surface. You can accomplish this task by estimating the amount of work that must be done to lift the canister to 10 times the radius of the planet. You cannot use the formula $W = F\Delta d \cos\theta$, because the force changes with the distance from the centre of the planet. Therefore, you will need to use a graphical method, such as the one described in the following steps.

- Prepare a table with two headings: Distance from the centre of the planetoid (d), and Gravitational force (N). In the first column, write the following distances: 1.0×10^{6} m, 2.0×10^{6} m, 3.0×10^{6} m, and so on, up to 10.0×10^{6} m. (Notice that these values are multiples of the radius of the planetoid, where 1.0×10^{6} m represents the surface of the planetoid.)
- Calculate the force of gravity on the 6.0 kg canister for each of these distances.
- Plot the graph of gravitational force (y-axis) against distance from the centre of the planetoid. Since your graph is of force versus position, the area under the graph represents the amount of work required to move the canister to a separation of 10 radii (9 radii from the surface). Graphically determine the area under the curve and, thus, the amount of work done.

  Note: There are several ways to find the area under the graph. One is to determine the graphical area represented by each square and then count the number of squares under the curve. Where the curve actually crosses a square, include the square if half or more of it is under the curve. Another method is to divide the area up into different regions and approximate their areas by using figures such as trapezoids and triangles.

Analyze and Conclude

1. If the canister has been lifted a distance of 10 radii and remains there, what type of energy does the area under the curve represent?
2. To launch the canister so that it will be able to travel straight out to a separation of 10 radii, how much kinetic energy must it be given at the start? From this kinetic energy, determine the required speed that would allow the canister to reach this separation.
3. The gravitational force that is trying to pull the canister back is extremely small at a separation of 10 radii. With only slightly more speed, the canister would never return to the planetoid, so the speed that you found is essentially the escape speed for the planetoid. What is the escape speed for this planetoid?

Apply and Extend

4. Considering the energies involved, does the canister have to be thrown straight up at its escape speed for it to be able to escape? Give reasons for your answer.

6.1 Energy for Lift-Off

SECTION EXPECTATIONS

- Calculate the generalized gravitational potential energy for an isolated system involving two objects, based on the law of universal gravitation.
- Determine the escape speed for a given celestial object.
- Develop appropriate scientific models for natural phenomena.

KEY TERMS

- escape energy
- binding energy
- escape speed

Did you know that it takes almost 10 t of fuel for a large passenger jet to take off? It is hard to even imagine the amount of energy required for a rocket or space shuttle to lift off. How do the engineers and scientists determine these values?

Figure 6.1 The energy to hurl this spacecraft into orbit comes from the chemical potential energy of the fuel.

MISCONCEPTION

Gravity and Orbiting Spacecraft

Many people believe that gravity does not act on orbiting spacecraft. In fact, a satellite such as the International Space Station *Freedom* still has about 80% of its initial weight. The impression of weightlessness comes from the fact that the weight is being used to hold the space station in its orbit. If there was no weight, it would simply continue to move off into space.

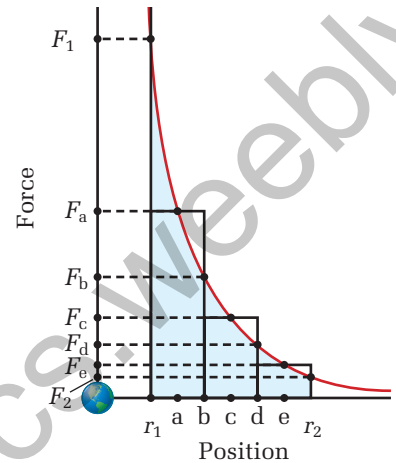
Work for Lift-Off

One way to determine the amount of energy needed to carry out a particular task is to determine the amount of work that you would have to do. When a spacecraft is lifting off from Earth, the force against which it must do work is the force of gravity.

In Chapter 3, Planetary and Satellite Dynamics, you learned that the equation for the gravitational force is $F_g = G\frac{m_1 m_2}{r^2}$. When working with a planet and a small object, physicists often use M for the planet and m for the small object. You can then write the equation as $F_g = G\frac{Mm}{r^2}$. In the Quick Lab, Escape from a Planetoid, you used this expression for force and multiples of the radius of

the planetoid for position, and then estimated the area under the curve of force versus position to estimate the amount of work needed to escape from the planetoid. However, if you were an engineer working for the space program, you would want a much more accurate value before you launched a spacecraft. In the following derivation, you will develop a general expression for the area under the curve of F_g versus r from position r_1 to r_2. This area will be the amount of work needed to raise an object such as a spacecraft of mass m from a distance r_1 to a distance r_2 from the centre of a planet of mass M.

- Draw a graph of gravitational force versus position, where the origin of the graph lies at the centre of the planet.
- Choose points r_1 and r_2. Divide the axis between r_1 and r_2 into six equal spaces and label the end point "a" through "e."
- Draw three rectangles with heights F_a, F_c, and F_e.



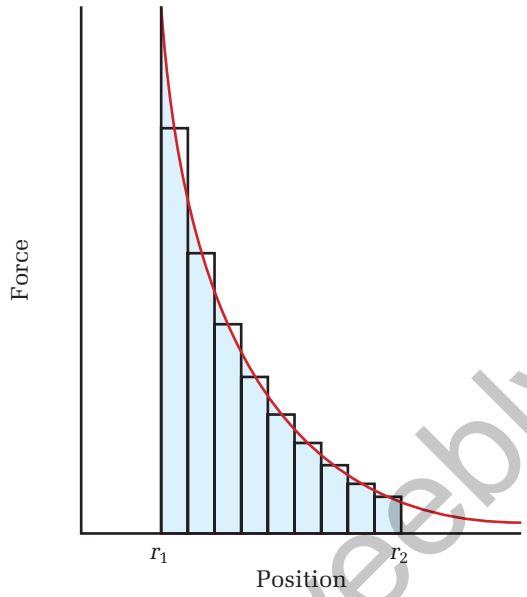
- A first rough estimate of the total work done to move m from r_1 to r_2 will be the sum of the areas of the rectangles.

$$W_{total} = W_e + W_c + W_a$$
$$W_{total} = F_a(b - r_1) + F_c(d - b) + F_e(r_2 - d)$$

- You could simplify this equation if you could express the forces in terms of the points on the curve at the ends of the rectangles, instead of the centre. For example, how can you express F_a in terms of F_1 and F_b? Clearly, F_a is not the average or arithmetic mean of F_1 and F_b, because the curve is an exponential curve. However, it can be accurately expressed as the *geometric* mean, which is expressed as $\sqrt{F_1F_b}$. Substitute the geometric mean of each value for force into the equation for work. Notice that in the last step, all intermediate terms have cancelled each other and only the first and last terms remain.

$$W_{total} = \sqrt{F_1F_b}(b - r_1) + \sqrt{F_bF_d}(d - b) + \sqrt{F_dF_2}(r_2 - d)$$
$$W_{total} = \sqrt{\frac{GMm}{r_1^2}\cdot\frac{GMm}{b^2}}(b - r_1) + \sqrt{\frac{GMm}{b^2}\cdot\frac{GMm}{d^2}}(d - b) + \sqrt{\frac{GMm}{d^2}\cdot\frac{GMm}{r_2^2}}(r_2 - d)$$
$$W_{total} = \frac{GMm}{r_1b}(b - r_1) + \frac{GMm}{bd}(d - b) + \frac{GMm}{dr_2}(r_2 - d)$$
$$W_{total} = GMm\left(\frac{b - r_1}{r_1b} + \frac{d - b}{bd} + \frac{r_2 - d}{dr_2}\right)$$
$$W_{total} = GMm\left(\frac{1}{r_1} - \frac{1}{b} + \frac{1}{b} - \frac{1}{d} + \frac{1}{d} - \frac{1}{r_2}\right)$$
$$W_{total} = GMm\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

- At first consideration, this result would appear to be a rough estimate. However, consider the fact that you could make as many rectangles as you want. Examination of the figure on the right shows that as the number of rectangles increases, the sum of their areas becomes very close to the true area under the curve. If you drew an infinite number of rectangles, your result would be precise. Now, analyze the last two mathematical steps above. No matter how many rectangles you drew, all of the intermediate terms would cancel and the result would be exactly the same as the result above. In this case, the result above is not an approximation but is, in fact, exact.



Escape Energy and Speed

MATH LINK

The arithmetic mean of two values, m and n, is $\frac{m+n}{2}$. The geometric mean is $\sqrt{mn}$.

You can now use the equation that you just derived — $W_{\text{total}} = GMm\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$ — to determine the amount of energy needed by a spacecraft to escape from Earth's gravitational pull. Let r_1 be Earth's radius so that the spacecraft will be sitting on the ground. Let r_2 be so far out into space that the force of gravity is negligible. Notice that as r_2 becomes exceedingly large, $\frac{1}{r_2}$ approaches zero, so the equation for the amount of work that must be done to free the spacecraft from the surface of the planet is $W_{\text{to escape}} = GMm/r_1$.

Work represents the change in energy that, in this case, is the amount of energy that a spacecraft would need to escape Earth's gravity. When a spacecraft blasts off from Earth, that amount of energy is provided as kinetic energy through the thrust of the engines. If the spacecraft is to escape Earth, therefore, it must be provided with at least GMm/r_1 J of kinetic energy, which probably come from GMm/r_1 J of chemical potential energy in the fuel. For this reason, the quantity GMm/r_1 is known as the **escape energy** for the spacecraft. If a spacecraft has any less energy, you could say that it is *bound* by Earth's gravity. Therefore, you can think of the value GMm/r_1 as the **binding energy** of the spacecraft to Earth.

Typically, when a spacecraft lifts off, rockets fire, the craft lifts off, and the rockets continue to fire, accelerating the spacecraft as it rises. However, you can often obtain important information by considering the extreme case. For example, if all of the escape energy must be provided as initial kinetic energy at the moment of lift-off, what would be the spacecraft's initial speed?

PHYSICS FILE

Escape speed is often referred to as "escape velocity." However, since the direction in which the escaping object is headed has no effect on its ability to escape (unless it is headed into the ground), the correct term is "escape speed."

- The initial kinetic energy of the spacecraft would have to be equal to the escape energy. Let r_p be the radius of the planet.

$$\frac{1}{2}mv^2 = \frac{GMm}{r_p}$$

- Solve for v.

$$v^2 = \frac{2GM\cancel{m}}{\cancel{m}r_p}$$

$$v = \sqrt{\frac{2GM}{r_p}}$$

This equation gives the **escape speed**, the minimum speed at the surface that will allow an object to leave a planet and not return. Notice that the speed does not depend on the mass of the escaping object.

ESCAPE SPEED

The escape speed of an object from the surface of a planet is the square root of two times the product of the universal gravitational constant and the mass of the planet divided by the radius of the planet.

$$v = \sqrt{\frac{2GM}{r_p}}$$

Quantity	Symbol	SI unit
escape speed	v	$\frac{\text{m}}{\text{s}}$ (metres per second)
mass of planet	M	kg (kilograms)
radius of planet	r_p	m (metres)
universal gravitational constant	G	$\frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}$ (newton metres squared per kilograms squared)

Unit Analysis

$$\frac{\text{m}}{\text{s}} = \sqrt{\frac{\frac{\text{N}\cdot\text{m}^{\cancel{2}}}{\text{kg}^{\cancel{2}}}\cdot\cancel{\text{kg}}}{\cancel{\text{m}}}} = \sqrt{\frac{\frac{\cancel{\text{kg}}\cdot\text{m}}{\text{s}^2}\cdot\text{m}}{\cancel{\text{kg}}}} = \sqrt{\frac{\text{m}^2}{\text{s}^2}} = \frac{\text{m}}{\text{s}}$$

SAMPLE PROBLEM

Escaping from Earth

Determine the escape energy and escape speed for a 1.60×10^4 kg rocket leaving the surface of Earth.

continued ►

continued from previous page

Conceptualize the Problem

- *Escape speed* is the speed at which a spacecraft would have to be lifting off Earth's surface in order to *escape Earth's gravity* with no additional input of energy.
- You can find the *radius* and *mass* of *Earth* in Appendix B, Physical Constants and Data.

Identify the Goal

The escape energy, E_{escape}, and escape speed, v_{escape}, for a rocket from Earth

Identify the Variables and Constants

Known	Implied	Unknown
$m_{rocket} = 1.60 \times 10^4$ kg	$G = 6.673 \times 10^{-11} \frac{N \cdot m^2}{kg^2}$	E_{escape}
	$r_{Earth} = 6.38 \times 10^6$ m	v_{escape}
	$m_{Earth} = 5.98 \times 10^{24}$ kg	

Develop a Strategy

State the equation for escape energy. Substitute and solve.

$$E_{escape} = \frac{GM_{Earth}m_{object}}{r_{Earth}}$$

$$E_{escape} = \frac{\left(6.673 \times 10^{-11} \frac{N \cdot m^{\cancel{2}}}{\cancel{kg^2}}\right)(5.98 \times 10^{24}\ \cancel{kg})(1.60 \times 10^4\ \cancel{kg})}{6.38 \times 10^6\ \cancel{m}}$$

$$E_{escape} = 1.00 \times 10^{12}\ J$$

State the equation for escape speed. Substitute and solve.

$$v_{escape} = \sqrt{\frac{2GM_{Earth}}{r_{Earth}}}$$

$$v_{escape} = \sqrt{\frac{2\left(6.673 \times 10^{-11} \frac{N \cdot m^2}{kg^2}\right)(5.98 \times 10^{24}\ kg)}{6.38 \times 10^6\ m}}$$

$$v_{escape} = 1.1184 \times 10^4\ \frac{m}{s}$$

$$v_{escape} \cong 1.12 \times 10^4\ \frac{m}{s}$$

The escape energy for this rocket is 1.00×10^{12} J and its escape speed is 1.12×10^4 m/s or 11.2 km/s.

Validate the Solution

A unit analysis escape energy shows $\frac{\frac{N \cdot m^2}{kg^2} \cdot kg \cdot kg}{m} = \frac{N \cdot m^{\cancel{2}} \cdot \cancel{kg^2}}{\cancel{kg^2} \cdot \cancel{m}} = N \cdot m = J$

which is correct for energy. A unit analysis for escape speed shows

$\sqrt{\frac{\frac{N \cdot m^2}{kg^2} \cdot kg}{m}} = \sqrt{\frac{N \cdot m^{\cancel{2}} \cdot \cancel{kg}}{\cancel{m} \cdot kg^{\cancel{2}}}} = \sqrt{\frac{\cancel{kg} \cdot m}{s^2} \cdot \frac{m}{\cancel{kg}}} = \sqrt{\frac{m^2}{s^2}} = \frac{m}{s}$ which is correct

for speed. A value of a few km/s agrees with the types of speeds observed during rocket lift-offs.

PRACTICE PROBLEMS

1. Determine the escape energy and escape speed for an asteroid with a mass of 1.00×10^{22} kg and a radius of 1.00×10^{6} m. How closely does your answer for escape energy compare to the value obtained by finding the area under the force-separation graph in the Quick Lab at the beginning of this chapter?

2. Calculate the escape energy and escape speed for a 15 g stone from Mars. Such stones have been blasted off the surface of Mars by meteor impacts and have fallen to Earth, where they are found preserved in the snow and ice of the Antarctic. The mass of Mars is 6.42×10^{23} kg and the radius of Mars is 3.38×10^{6} m.

3. The *Pioneer 10* spacecraft, shown in the photo, was the first to journey beyond Jupiter and is now well past Pluto. To escape from the solar system, how fast did *Pioneer 10* have to be travelling as it passed the orbit of Jupiter? Assume that the mass of the solar system is essentially concentrated in the Sun. The mass of the Sun is 1.99×10^{30} kg and the radius of Jupiter's orbit is 7.78×10^{11} m.

6.1 Section Review

1. K/U State the equations for escape energy and escape speed. Indicate the meaning of each factor and the appropriate units for each factor.

2. I Prove from basic energy equations that the escape speed for an object from the surface of a planet is independent of the mass of the object.

3. K/U Explain the meaning of (a) escape energy, (b) escape speed, and (c) binding energy.

4. C Sketch graphs to show how the escape speed from a planet varies with
   (a) the mass of the planet for constant planetary radius
   (b) the radius of the planet for constant planetary mass
   (c) the mass of the escaping object from a given planet

5. C What factors would make the actual energy that must be provided in the form of fuel greater than the escape energy? Explain the role of each factor.

6. MC Look up the meaning of the term "bond energy"as it applies to bonds between the atoms in a diatomic molecule. How does the concept of bond energy relate to the concept of escape energy?

UNIT PROJECT PREP

Space-based energy schemes have for a long time been promoted as the environmentally friendly way to provide energy of the future. Understanding the physics concepts of low Earth orbit provides you with a method of judging each scheme's feasibility.

- List environmental factors involved in getting into Earth orbit.
- How do you envision space travel in the near future?
- Do you believe that environmental or other factors will motivate more space-based power initiatives?

6.2 Energy of Orbiting Satellites

SECTION EXPECTATIONS

- Analyze the factors affecting the motion of isolated celestial objects, and calculate the gravitational potential energy for such a system.
- Analyze isolated planetary and satellite motion and describe it in terms of the forms of energy and energy transformations that occur.
- Calculate the kinetic and gravitational potential energy of a satellite that is in a stable circular orbit around a planet.

KEY TERMS

- circular orbit
- total orbital energy

The rockets that launched *Voyager 1* and *Voyager 2,* were designed to escape Earth's gravity and send them into space to search the solar system. The *Voyager* craft have found such things as new moons orbiting Jupiter, Saturn, Uranus, and Neptune. They have also discovered volcanoes on Io and rings around Jupiter. However, the majority of satellites are launched into Earth's orbit and will remain captive in Earth's gravitational field, destined to circle the planet year after year and perform tasks of immediate importance to people on Earth.

Satellites in Earth Orbit

Some of these satellites monitor the weather, the growth and health of crops, the temperature of the oceans, the presence of ice floes, and the status of the ozone layer. Others actively scan Earth's surface with radar to enhance our knowledge of the geography and geology of our planet. These days, many people routinely use satellites to tell them where they are (for example, the Global Positioning System) and to provide them with mobile communication and seemingly limitless television entertainment.

Other satellites look outward, monitoring regions of the electromagnetic spectrum that cannot pass easily through Earth's atmosphere. In doing so, they tell us about our own solar system, as well as other solar systems, stars, and galaxies located many light-years away from us.

The largest artificial satellite in Earth's orbit is the International Space Station. During its construction and lifetime, it has been serviced from other temporary satellites, the space shuttles. For these shuttles to rendezvous successfully with the space station, teams of scientists, engineers, and technicians must solve problems involving the orbital motions and energies that are the subject of this section.

Most satellites are in either a **circular orbit** or a near-circular orbit. In Chapter 3, you learned how the force of gravity acts as a centripetal force, holding each satellite in its own unique orbit. You learned how to calculate the orbital speeds of the satellites that orbit at specific radii. In this section, you will focus on the energies of these satellites.

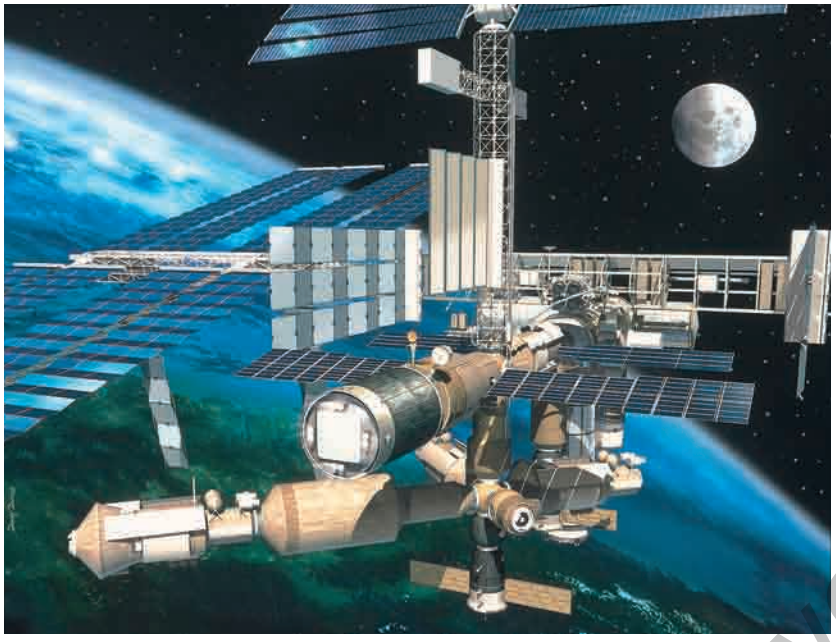


Figure 6.2 A thorough understanding of orbital mechanics is necessary for a successful rendezvous with the space station or with any other satellite.

Orbital Energies

The orbital energy of a satellite consists of two components: its kinetic energy and its gravitational potential energy. To a great extent, the Earth-satellite system can be treated as an isolated system. Subtle effects, such as the pressure of light, the solar wind, and collisions with the few atmospheric molecules that exist at that distance from the surface, can change the energy of the system. In fact, without the occasional boost from a thruster, the orbits of all satellites will decay. However, this generally takes decades. These effects are so tiny over the short run that you will neglect them in the following topics.

- Write the relationship that represents a planet's gravity providing a centripetal force.

$$\frac{mv^2}{r} = \frac{GMm}{r^2}$$

- Multiply both sides of the equation by r.

$$\frac{mv^2\cancel{r}}{\cancel{r}} = \frac{GMm}{r^{\cancel{2}}}\cancel{r}$$

$$mv^2 = \frac{GMm}{r}$$

- Multiply both sides of the equation by $\frac{1}{2}$.

$$\frac{1}{2}mv^2 = \frac{GMm}{2r}$$

- Since $\frac{1}{2}mv^2$ is the kinetic energy of any object of mass m, you can substitute E_k for the expression.

$$E_k = \frac{GMm}{2r}$$

The kinetic energy of an orbiting satellite of mass m is $E_k = \frac{GMm}{2r}$.

Gravitational Potential Energy

In Chapter 5, Conservation of Energy, you demonstrated that the change in the gravitational potential energy of an object was equal to the work done in raising the object from one height to another. That relationship ($W = mg\Delta h$) was the special case, where any change in height was very close to Earth's surface. Since you are now dealing with objects being launched into space, you cannot use the special case. You must consider the change in the force of gravity as the distance from Earth increases. Fortunately, however, you have already developed an expression for the amount of work required to lift an object from a distance r_1 to a distance r_2 from Earth's centre. Therefore, the result of your derivation is equal to the change in the gravitational potential energy between those two positions.

$$\Delta E_g = GMm\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

As you know, you must choose a reference point for all forms of potential energy. Earth's surface in no longer an appropriate reference, because you are measuring distances from Earth's centre to deep into space. Physicists have accepted the convention of assigning the reference or zero point for gravitational potential energy as an infinite distance from the centre of the planet or other celestial body that is exerting the gravitational force on the object of mass m. This is appropriate because at an infinite distance, the gravitational force goes to zero. You can now state that the gravitational potential energy of an object at a distance r_2 from Earth's centre is the amount of work required to move an object from an infinite distance, r_1, to r_2.

$$E_g = GMm\left(\frac{1}{\infty} - \frac{1}{r_2}\right)$$

$$E_g = GMm\left(0 - \frac{1}{r_2}\right)$$

$$E_g = -\frac{GMm}{r_2}$$

Since there is only one distance (r_2) in the equation, it is often written without a subscript. Notice, also, that the value is negative. This is simply a result of the arbitrary choice of an infinite distance for the reference position. You will discover as you work with the concept that it is a fortunate choice.

GRAVITATIONAL POTENTIAL ENERGY

The gravitational potential energy of an object is the negative of the product of the universal gravitational constant, the mass of the planet or celestial body, and the mass of the object, divided by the distance from the centre of the planet or celestial body.

$$E_g = -\frac{GMm}{r}$$

Quantity	Symbol	SI unit
gravitational potential energy	E_g	J (joules)
universal gravitational constant	G	$\frac{N \cdot m^2}{kg^2}$ (newton metres squared per kilograms squared)
mass of the planet or celestial body	M	kg (kilograms)
mass of the object	m	kg (kilograms)
distance from centre of planet or celestial body	r	m (metres)

Unit Analysis

$$\text{joule} = \frac{\frac{\text{newton} \cdot \text{metre}^2}{\text{kilogram}^2} \cdot \text{kilogram} \cdot \text{kilogram}}{\text{metre}}$$

$$J = \frac{\frac{N \cdot m^{\cancel{2}}}{\cancel{kg^2}} \cdot \cancel{kg} \cdot \cancel{kg}}{\cancel{m}} = N \cdot m = J$$

Note: Use of this equation implies that the reference or zero position is an infinite distance from the planet or celestial body.

It might seem odd that the potential energy is always negative. Since changes in energy are always of interest, however, these changes will be the same, regardless of the location of the zero level.

To illustrate this concept, consider the houses in the Loire Valley in France that are carved out of the face of limestone cliffs as shown in Figure 4.3(B). To the person on the cobblestone street, everyone on floors A, B, and C in Figure 6.3(A) would have positive gravitational energy, due to their height above the street. However, to a person on floor B, those on floor A are at a negative height, and so have negative gravitational potential energy relative to them. At the same time, the person on floor B would consider

that people on floor C would have a positive gravitational potential energy because they are higher up the cliff.

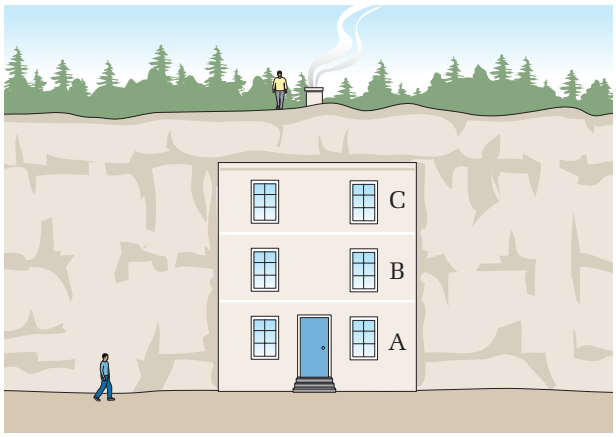


Figure 6.3 Some houses in the Loire Valley are carved out of limestone cliffs.

Naturally, the person standing on the roof beside the chimney would consider that everyone in the house had negative gravitational potential energy. All of the residents would agree, however, on the amount of work that it took to carry a chair up from floor A to floor C, so the energy change would remain the same, regardless of the observer's level. At the same time, a book dropped from a window in floor B would hit the ground with the same kinetic energy, regardless of the location of the zero level for gravitational potential energy.

Figure 6.4 is a graph of the gravitational potential energy of a 1.0 kg object as it moves away from Earth's surface. Since work must be done on that object to increase the separation, the object is often referred to as being in a gravitational potential energy "well."

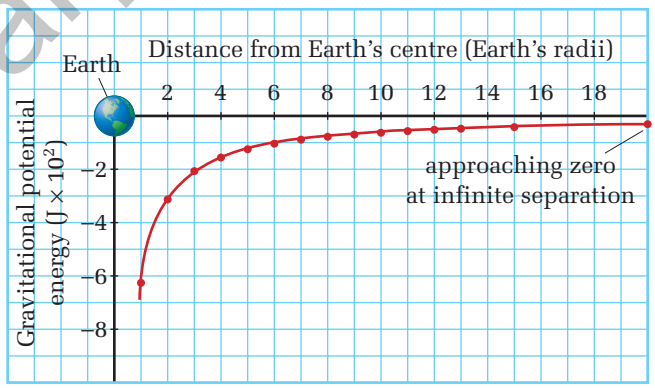


Figure 6.4 Since work must be done on the 1.0 kg object to move it away from Earth, although the gravitational potential energy is always negative, it is increasing (becoming less negative) as it retreats farther and farther from Earth.

In summary, the energies of orbiting objects can be expressed as

$$E_g = -\frac{GMm}{r}$$

$$E_k = \frac{GMm}{2r}$$

Adding, you obtain

$$E_{total} = E_k + E_g$$

$$E_{total} = \frac{GMm}{2r} + \left(-\frac{GMm}{r}\right)$$

$$E_{total} = -\frac{GMm}{2r}$$

The last equation, the **total orbital energy,** involves only the mechanical energies — gravitational potential energy and kinetic energy. Other forms of energy, such as thermal energy, are not considered unless the satellite comes down in flames through the atmosphere.

You can obtain key information by determining whether the total orbital energy of an object is positive, zero, or negative. First, consider the conditions under which an object would have zero total orbital energy around a central object, such as a planet or star.

If an object is so far from Earth that gravity cannot pull it back, its gravitational potential energy is zero. If the object is motionless at that point, its kinetic energy is also zero, which gives a total energy of zero. The total orbital energy could also be zero if the magnitude of the kinetic and potential energies were equal. Under these conditions, the kinetic energy would be just great enough to carry the object to a distance at which gravity could no longer pull it back. It would then have no kinetic energy left and it would be motionless. By a similar analysis, you could draw all of the following conclusions.

- If the total of the kinetic and gravitational potential energies of an object is zero, it can just escape from the central object.
- If the total of the kinetic and gravitational potential energies of an object is greater than zero, it can escape from the central object and keep on going.
- If the total of the kinetic and gravitational potential energies of an object is less than zero, it cannot escape from the central object. It is said to be bound to the object.

The extra energy needed to free the object is called the binding energy. Since the object will be free with a total energy of zero, the binding energy is always the negative of the total energy: $E_{binding} = -E_{total}$.

TECHNOLOGY LINK

Fire a cannon ball into space? In the early 1960s, Project HARP (High Altitude Research Project) did just that. Scientists at McGill University in Montréal welded two U.S. Navy cannon barrels together into a "super-gun" that fired 91 kg instrumentation packages to a height of more than 145 km from a launch site on the Caribbean island of Barbados.

HARP cannon

Photo courtesy from the web sites: http://www.phy6.org/stargaze/Smartlet.htm and http://www-istp.gsfc.nasa.gov/stargaze/Smartlet.htm taken by Peter Millman.

The orbital energy equations also have some informative simple relationships among themselves, as listed below.

- The magnitudes of the kinetic energy, the total energy, and the binding energy of an orbiting object are the same $|E_k| = |E_t| = |E_{binding}|$
- The magnitude of the gravitational potential energy is twice that of the other energies. $|E_g| = 2|E_k| = 2|E_t| = 2|E_{binding}|$
- If a satellite is in an orbit close to the planet, the radius of the orbit is essentially the same as the radius of the planet: $r_{orbit} \cong r_{planet} \cong r$. $E_k = \frac{GMm}{2r}$
- At the planet's surface, the energy needed to break free was seen in Section 6.1 to be $E_{binding} = \frac{GMm}{2r}$

By comparing the last two equations, you can see that the satellite in a circular orbit close to the planet already has half of the energy it needs to completely escape from that planet. The following problems will help you to develop a deeper understanding of orbital energies.

SAMPLE PROBLEMS

Space Problems

1. **On March 6, 2001, the Mir space station was deliberately crashed into Earth. At the time, its mass was 1.39×10^3 kg and its altitude was 220 km.**
   (a) Prior to the crash, what was its binding energy to Earth?
   (b) How much energy was released in the crash? Assume that its orbit was circular.

Conceptualize the Problem

- When Mir was in Earth orbit, it had *kinetic* and *gravitational potential energy*, both of which are determined by its *orbital radius*.
- Mir's *binding energy* is the *negative* of its *total energy*.
- After the crash, Mir had *zero kinetic energy*.

- The law of *conservation of energy* applies; therefore, the energy released in the crash is the *difference* between the *total energy in orbit* and the *total energy* when resting on *Earth's surface*.

Identify the Goals

The binding energy, $E_{binding}$, of the Mir space station to Earth

The energy released during the crash of the Mir space station

Identify the Variables and Constants

Known	Implied	Unknown
$m = 1.39 \times 10^3$ kg (Mir)	$G = 6.673 \times 10^{-11} \frac{N \cdot m^2}{kg^2}$	E_{total} in orbit
$h = 2.20 \times 10^5$ m	$M = 5.978 \times 10^{24}$ kg (Earth)	$E_{binding}$ in orbit
	$r_{Earth} = 6.378 \times 10^6$ m	E_g on ground
		ΔE_{total}

Develop a Strategy

Determine the orbital radius.

$$r_{orbit} = r_{earth} + h_{from\ Earth}$$

$$r_{orbit} = 6.378 \times 10^6 \text{ m} + 2.20 \times 10^5 \text{ m}$$

$$r_{orbit} = 6.598 \times 10^6 \text{ m}$$

Calculate the total orbital energy before the crash.

$$E_t = -\frac{GMm}{2r}$$

$$E_t = -\frac{\left(6.673 \times 10^{-11} \frac{N \cdot m^2}{kg^2}\right)(5.978 \times 10^{24} \text{ kg})(1.39 \times 10^3 \text{ kg})}{2(6.598 \times 10^6 \text{ m})}$$

$$E_t = -4.2019 \times 10^{10} \text{ J}$$

Binding energy is the negative of the total energy.

$$E_{binding} = +4.2019 \times 10^{10} \text{ J}$$

$$E_{binding} \cong +4.20 \times 10^{10} \text{ J}$$

(a) The binding energy of the Mir space station in orbit was 4.20×10^{10} J.

Calculate the mechanical energy after the crash. Note that when Mir was on Earth's surface, its kinetic energy was zero.

$$E_k = 0 \text{ J}$$

$$E_g = -\frac{GMm}{r}$$

$$E_g = -\frac{\left(6.673 \times 10^{-11} \frac{N \cdot m^2}{kg^2}\right)(5.978 \times 10^{24} \text{ kg})(1.39 \times 10^3 \text{ kg})}{6.378 \times 10^6 \text{ m}}$$

$$E_g = -8.6938 \times 10^{10} \text{ J}$$

$$E_{total} = E_k + E_g$$

$$E_{total} = 0 \text{ J} - 8.6938 \times 10^{10} \text{ J}$$

$$E_{total} = -8.6938 \times 10^{10} \text{ J}$$

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Determine the difference in total energy before and after the crash.

$$\Delta E = E'_{total} - E_{total}$$
$$\Delta E = -8.6938 \times 10^{10}\ \text{J} - (-4.2019 \times 10^{10}\ \text{J})$$
$$\Delta E = -4.4919 \times 10^{10}\ \text{J}$$
$$\Delta E \cong -4.49 \times 10^{10}\ \text{J}$$

(b) When Mir crashed, 4.49×10^{10} J of energy were released into the environment.

Validate the Solution

The final answer is negative, which indicates a decrease in the energy of the system or a loss of energy to the environment.

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2. A 4025 kg spacecraft (including the astronauts) is in a circular orbit 256 km above the lunar surface. Determine

(a) the kinetic energy of the spacecraft

(b) the total orbital energy of the spacecraft

(c) the binding energy of the spacecraft

(d) the speed required for escape

Conceptualize the Problem

- The spacecraft is in a circular orbit around the Moon, so the *Moon* is the *central* body.
- The spacecraft is *moving*, so it has *kinetic energy*.
- The spacecraft is in *orbit*, so it has *gravitational potential energy*.
- *Binding energy* is the amount of energy necessary to escape the *gravitational* pull of the *central body*.
- To escape a central body, a spacecraft must *increase* its *kinetic energy* until the *total energy* is *zero*.
- If you know the *kinetic energy*, you can find *speed*.

Identify the Goals

(a) The kinetic energy of the spacecraft, E_k

(b) The total orbital energy of the spacecraft, E_t

(c) The binding energy of the spacecraft, $E_{binding}$

(d) The speed required for escape, v_{escape}

Identify the Variables and Constants

Known	Implied	Unknown
$m = 4025$ kg	$G = 6.673 \times 10^{-11}\ \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}$	r_{orbit}
$h = 256$ km	$r_{Moon} = 1.738 \times 10^6$ m	E_k
	$M_{Moon} = 7.36 \times 10^{22}$ kg	E_t
		$E_{binding}$
		v_{escape}

Develop a Strategy

Determine the orbital radius of the spacecraft.

$$r_{orbit} = r_{Moon} + h$$

$$r_{orbit} = 1.738 \times 10^6 \text{ m} + 0.256 \times 10^6 \text{ m}$$

$$r_{orbit} = 1.994 \times 10^6 \text{ m}$$

Calculate the orbital kinetic energy.

$$E_k = \frac{GMm}{2r}$$

$$E_k = \frac{\left(6.673 \times 10^{-11} \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)(7.36 \times 10^{22} \text{ kg})(4025 \text{ kg})}{2(1.994 \times 10^6 \text{ m})}$$

$$E_k = 4.9569 \times 10^9 \text{ J}$$

$$E_k \cong 4.96 \times 10^9 \text{ J}$$

(a) The kinetic energy of the spacecraft is 4.96×10^9 J.

Total orbital energy is the negative of the kinetic energy.

$$E_t = -E_k$$

$$E_t = -4.96 \times 10^9 \text{ J}$$

(b) The total energy of the spacecraft is -4.96×10^9 J.

The binding energy is the negative of the total energy

$$E_{binding} = -E_t$$

$$E_{binding} = -(-4.952 \times 10^9 \text{ J})$$

$$E_{binding} = 4.96 \times 10^9 \text{ J}$$

(c) The binding energy of the spacecraft is 4.96×10^9 J.

The binding energy must come through additional kinetic energy

$$E_k' = E_k + E_{binding}$$

$$E_k' = 4.9569 \times 10^9 \text{ J} + 4.9569 \times 10^9 \text{ J}$$

$$E_k' = 9.9138 \times 10^9 \text{ J}$$

Find the speed from the kinetic energy.

$$\frac{1}{2}mv^2 = E_k$$

$$v = \sqrt{\frac{2E_k}{m}}$$

$$v = \sqrt{\frac{2(9.9138 \times 10^9 \text{ J})}{4025 \text{ kg}}}$$

$$v = 2.2195 \times 10^3 \frac{\text{m}}{\text{s}}$$

$$v \cong 2.22 \times 10^3 \frac{\text{m}}{\text{s}}$$

(d) The escape speed for the spacecraft is 2.22×10^3 m/s.

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Validate the Solution

The escape speed for the spacecraft could have been determined from the equation $v_{escape} = \sqrt{\frac{2GM}{r}}$. Carrying out this calculation yields the same escape speed. Since it agrees with the results of the energy calculations, it acts as a check on the first three answers.

PRACTICE PROBLEMS

4. A 55 kg satellite is in a circular orbit around Earth with an orbital radius of 7.4×10^6 m. Determine the satellite's

(a) kinetic energy

(b) gravitational potential energy

(c) total energy

(d) binding energy

5. A 125 kg satellite in a circular orbit around Earth has a potential energy of -6.64×10^9 J. Determine the satellite's

(a) kinetic energy

(b) orbital speed

(c) orbital radius

6. A 562 kg satellite is in a circular orbit around Mars. Data: $r_{Mars} = 3.375 \times 10^6$ m; $r_{orbit} = 4.000 \times 10^6$ m; $M_{Mars} = 6.420 \times 10^{23}$ kg

(a) If the satellite is allowed to crash on Mars, how much energy will be released to the Martian environment?

(b) List several of the forms that the released energy might take.

7. From the orbital kinetic energy of the lunar spacecraft in the second sample problem, determine its orbital speed. What increase beyond that speed was required for escape from the Moon?

8. A 60.0 kg space probe is in a circular orbit around Europa, a moon of Jupiter. If the orbital radius is 2.00×10^6 m and the mass of Europa is 4.87×10^{22} kg, determine the

(a) kinetic energy of the probe and its orbital speed

(b) gravitational potential energy of the probe

(c) total orbital energy of the probe

(d) binding energy of the probe

(e) *additional* speed that the probe must gain in order to break free of Europa

9. A 1.00×10^2 kg space probe is in a circular orbit, 25 km above the surface of Titan, a moon of Saturn. If the radius of Titan is 2575 km and its mass is 1.346×10^{23} kg, determine the

(a) orbital kinetic energy and speed of the space probe

(b) gravitational potential energy of the space probe

(c) total orbital energy of the space probe

(d) binding energy of the space probe

(e) *additional* speed required for the space probe to break free from Titan

10. Material has been observed in a circular orbit around a black hole some five thousand light-years away from Earth. Spectroscopic analysis of the material indicates that it is orbiting with a speed of 3.1×10^7 m/s. If the radius of the orbit is 9.8×10^5 m, determine the mass of the black hole.

6.2 Section Review

1. C Explain why the determination of orbital speed does not require knowledge of the satellite's mass, while determination of orbital energies does require knowledge of the satellite's mass.

2. K/U Explain why the binding energy of a satellite is the negative of its total orbital energy. Why does this relationship not depend on the satellite being in a circular orbit?

3. C Draw a concept organizer to show the links between the general equations for work and energy and the orbital energy equations. Indicate in the organizer which equations are joined together to produce the new equation.

4. I A satellite with an orbital speed of v_{orbit} is in a circular orbit around a planet. Prove that the speed for a satellite to escape from orbit and completely leave the planet is given by $v_{escape} = \sqrt{2}(v_{orbit})$.

5. MC The magnitude of the attractive force between an electron and a proton is given by $F = \frac{kq_e q_p}{r^2}$, where q_e is the magnitude of the charge on the electron, q_p is the magnitude of the charge on the proton, r is the separation between them, and k is a constant that plays the same role as G. If the mass of the electron is represented by m_e, derive an equation for the orbital speed of the electron.

Go To Mars With Newton

In his well-known and universally acclaimed book, *The Principia*, Sir Isaac Newton proposed a way to launch an object into orbit. His method was to place a cannon on a mountaintop and fire cannon balls parallel to Earth's surface. By using more gunpowder each time, the cannon ball would fly farther before falling to the ground. Newton imagined increasing the gunpowder until the cannon ball took off with such a great initial speed that it fell all the way around Earth — the cannon ball went into orbit and became a cannon ball satellite. In fact, if the cannon ball was able to make it half-way around the world, it would continue to orbit.

Today, scientists do not use cannons and gunpowder to launch rockets, but Newton's method and his laws still apply. Launching a rocket into orbit simply requires that the trajectory of its "free fall" carry it around the globe. While the rocket continually falls toward Earth's surface, the surface itself continually curves away from the rocket's path. Orbital motion is much like a perpetual game of tag between the satellite and the planet's surface. If the speed of the satellite drops below the orbital speed, then the satellite's trajectory will lead it to an impact on the surface.

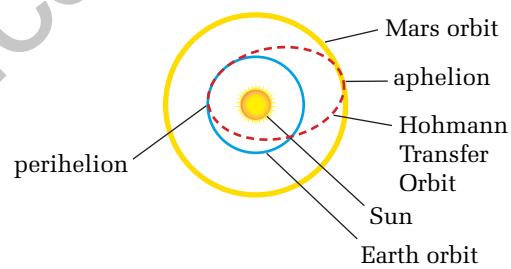
The speed required for an orbit close to Earth is around 28 000 km/h. Part of this can be provided by launching the rocket in an easterly direction from a location near the equator. As Earth revolves, its surface at the equator moves eastward at about 1675 km/h, thus providing a free, although relatively small, boost.

Lift-off involves the ignition of fuel and oxidizer in a reaction which is much like a controlled explosion. Hot, high pressure gas is formed as the product of the burning. When the high-speed molecules of the gas collide with the walls of the combustion chamber, they exert forces on the walls of the chamber. Because of an opening at one end of the chamber, the net force exerted by the gas molecules on the chamber is in the forward direction. By Newton's third law of motion, the chamber walls exert an equal and opposite force on the gas molecules, causing them to stream backward out of the nozzle at the end of the chamber. The nozzle controls the direction and rate of flow of the exhaust gases and so provides control of the direction and magnitude of the thrust. Once the thrust becomes greater than the weight of the rocket, the rocket begins to accelerate upward.

Once the fuel in the first stage has been consumed, that stage can be separated from the rest of the rocket and a second stage compartment is ignited. By letting the first compartment drop away, the rocket has less mass that needs to be propelled. Applying Newton's second law, which states that the acceleration is proportional to the ratio of the applied force and the mass, for the same amount of thrust, the rocket will accelerate more quickly.

Sending a Rocket to Mars

Contrary to popular opinion, the best time to send a spacecraft to Mars is not when Mars and Earth are closest in their orbits around the Sun. Instead, the launch opportunity occurs when the spacecraft can be fired tangentially from Earth's orbit, travelling along an elliptical orbit around the Sun, and arrive at Mars about 259 days later travelling tangentially to the orbit of Mars. The elliptical orbit is called a Hohmann transfer orbit. The Earth–Sun distance represents the closest point to the Sun (the perihelion) and the Mars–Sun distance represents the farthest point from the Sun (the aphelion). Energy is saved in two ways: The first is due to the use of Earth's orbital speed as the starting speed for the spacecraft. In fact, the spacecraft only needs an additional 3 km/s above the orbital speed of Earth around the Sun. The second comes from the fact that the average radius of the orbit of the ellipse is less than the orbital radius of Mars. If the spacecraft was not captured by the gravitational field of Mars, the spacecraft would continue along the ellipse and fall back toward its perihelion. It is thus necessary to launch the spacecraft such that its arrival time at aphelion coincides with the arrival of Mars at the same location. Such launch opportunities come only every 25 to 26 months.



In general then, the Hohmann transfer orbit only requires a burst of thrust when the spacecraft leaves Earth orbit and a second burst to allow it to settle into an orbit around Mars. Further manoeuvring would be required if the spacecraft was then going to land on the surface of the Red Planet. On its return, the spacecraft would drop away from Mars and follow the second half of the Hohmann transfer orbit back to Earth.

Making Connections

1. Describe how Newton's laws apply to a rocket at lift-off, in orbit, and landing.
2. **(a)** What differences would you need to consider to send a rocket to Venus instead of to Mars?
   (b) Draw a Hohmann transfer orbit for a rocket travelling to Venus.

6.3 Energy and Momentum in Space

SECTION EXPECTATIONS

- Apply quantitatively the law of conservation of linear momentum.
- Analyze the factors affecting the motion of isolated celestial objects.

KEY TERMS

- combustion chamber
- exhaust velocity
- thrust
- reaction mass
- gravitational assist or gravitational slingshot

Collisions in space are among the more interesting celestial events. The collision of comet Shoemaker-Levy 9 in July of 1994 created great excitement for both astronomers and the general public. Other collisions in the past have greatly affected Earth. The demise of the dinosaurs, along with about 70% of all other species, is attributed to a collision between Earth and an asteroid some 65 million years ago. Remains of such a collision can be seen on the sea floor of the Gulf of Mexico near the coast of the Yucatan Peninsula.

More recently, on June 30, 1908, an object with a mass of about one hundred thousand tonnes slammed into Earth's atmosphere above Siberia, not far from the Tunguska River. The explosion, which occurred about eight kilometres above the ground, flattened one hundred thousand square kilometres of forest, killing all of the wildlife in the area. Since the region is remote, no humans are thought to have perished.

Figure 6.5 The dark blemishes on the face of Jupiter after the collision with the comet Shoemaker-Levy 9 mark the impact locations of fragments of the comet.

Not all celestial collisions are devastating. Present theories about the formation of our solar system suggest that planets were formed as a result of collisions of smaller rocky objects. If the collisions were energetic enough, they would have generated enough thermal energy to fuse the rocks together into a larger mass.

QUICK LAB

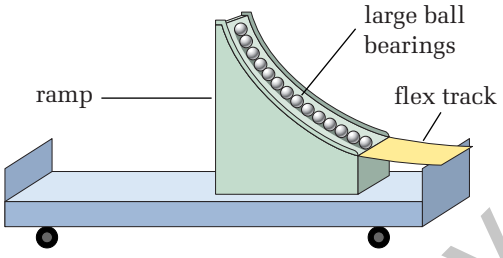
The Reaction Engine

TARGET SKILLS
- Hypothesizing
- Performing and recording
- Analyzing and interpreting

The Reaction Engine

According to Newton's first law of motion, an object requires a net force to push out in order to produce a change in speed or direction. If a rocket is out in space, what is available to provide this push? This activity should give you some ideas.

Set up a light dynamics cart with a ramp which could be made from Hot Wheels™ track as shown in the diagram. The ramp should be as high as possible and curved at the base so that ball bearings will be ejected horizontally from the back of the cart.



Arrange a track for the cart by clamping or taping metre sticks to the demonstration desk or tape them to the floor.

Place as many large ball bearings as possible on the ramp and hold them in place. Release the ball bearings and observe the motion of the ball bearings and the cart.

Analyze and Conclude

1. Describe the motion of the cart and the ball bearings. Did the last ball bearings move as quickly along the desk or floor as the first ones did? Did any actually end up moving in the direction of the cart?
2. Using Newton's laws of motion, explain why the cart accelerated.
3. What was the source of the energy that was transformed into the kinetic energy of the cart and the ball bearings?
4. The ramp with the ball bearings and a rocket are examples of reaction engines. Explain why the term is appropriate. What is the reaction mass in each case?

Apply and Extend

5. Research the topic of magnetohydrodynamic propulsion and prepare a brief report in diagram form. Where is this process mainly used?

Propulsion in Space

Newton's third law of motion states that if you exert a backward force on an object, that object will exert a forward force on you. In Chapter 4, Momentum and Impulse, you learned how Newton's third law led to the law of conservation of momentum. This concept is the basis for all motion and manoeuvring of astronauts and rockets in space. In fact, a spacecraft could be propelled by having an astronaut stand at the rear of the spacecraft and throw objects backward. This process is an example of recoil. As the astronaut pushed the objects backward, they would push just as hard forward on the astronaut.

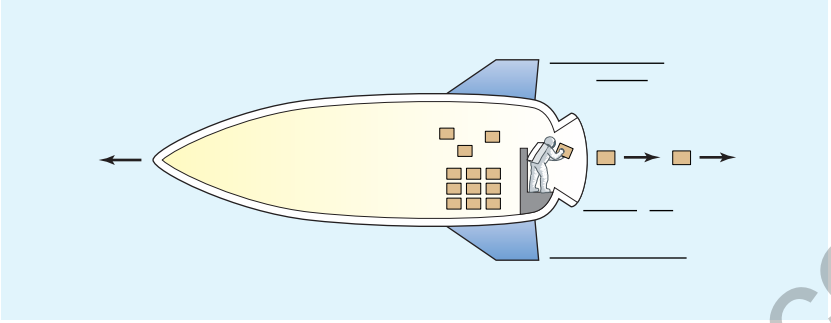
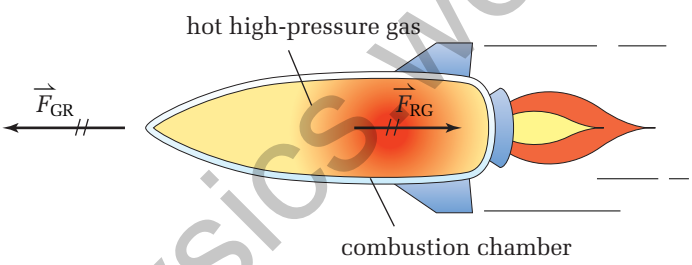


Figure 6.6 Recoil, a result of the conservation of momentum, is the basis of rocket propulsion. The concept is the same as the motion of an ice skater throwing a rock — a problem that you solved in Chapter 4.

Although this is the general principle on which rocket engines operate, most rely on hot, high-pressure gas to provide the reaction mass. The burning of the gas takes place in a **combustion chamber**, as shown in Figure 6.7. The walls of the combustion chamber exert a backward force on the gas, causing it to stream out backward. The gas in turn exerts a force on the walls of the combustion chamber, pushing it and the rocket forward.



$\vec{F}_{GR}$ = force exerted by the gas on the rocket (thrust) [forward]
$\vec{F}_{RG}$ = force exerted by the rocket on the gas [backward]
$\vec{F}_{GR} = -\vec{F}_{RG}$

Figure 6.7 Newton's third law explains how the exhaust of gases creates thrust for a rocket.

The relationship between the motion of the gas and the forces on the gas can be found by applying the impulse momentum theorem: $\vec{F}\Delta t = m\Delta\vec{v}$. When physicists and engineers apply this theorem to rocket exhaust gases, they usually rearrange it as follows.

$$\vec{F}_{(\text{on gas})}\Delta t = m_{\text{gas}}\Delta\vec{v}_{\text{gas}}$$

$$\vec{F}_{(\text{on gas})} = \frac{m_{\text{gas}}\Delta\vec{v}_{\text{gas}}}{\Delta t}$$

$$\vec{F}_{(\text{on gas})} = \left(\frac{m_{\text{gas}}}{\Delta t}\right)\Delta\vec{v}_{\text{gas}}$$

In rocket technology, the term $\left(\frac{m_{\text{gas}}}{\Delta t}\right)$ is important because it represents the rate of flow of a given mass of exhaust in kilograms of gas per second. Because of the law of conservation of mass, it also represents the burn rate of the fuel and oxidizer combined. Since the gas is initially at rest in the combustion chamber, the $\Delta\vec{v}_{\text{gas}}$ represents the backward velocity of the gas relative to the combustion chamber of the rocket. This is also known as the

exhaust velocity. For most chemical propellants, the exhaust velocity ranges from 2 km/s to 5 km/s.

If you know the rate of combustion and the velocity of the exhaust gases, you can calculate the force with which the rocket pushes on the gas: $\vec{F}_{(\text{on gas})} = \left(\frac{m_{\text{gas}}}{\Delta t}\right)\Delta\vec{v}_{\text{gas}}$. According to Newton's third law of motion, this also represents the force with which the gas pushes on the rocket. This force is known as the **thrust** (action force, in Newton's third law). The gas experiences the reaction force and its mass is referred to as **reaction mass**.

The idea that the rocket exerts a force on the gas might seem strange, but when molecules of gas strike the walls, they exert a force on the walls. At the same time, the walls exert a backward force on the molecules of gas, causing them to recoil. The two forces are equal in magnitude, but opposite in direction.

PHYSICS FILE

As early as 1232 A.D., the Chinese were using gunpowder as a propulsive agent for arrows and incendiary bombs.

SAMPLE PROBLEM

Rocket Propulsion

A rocket engine consumes 50.0 kg of hydrogen and 400.0 kg of oxygen during a 5.00 s burn.

(a) If the exhaust speed of the gas is 3.54 km/s, determine the thrust of the engine

(b) If the rocket has a mass of 1.5×10^4 kg, calculate the acceleration of the rocket if no other forces are acting.

Conceptualize the Problem

- Because the hot gases *move* rapidly out of the combustion chamber, they have *momentum*.
- The *total momentum* of the gases plus rocket must be *conserved*; therefore, the *momentum of the rocket* must be *equal* in magnitude and *opposite* in direction to the gases.
- If you know the *change in momentum* of the rocket and the *time interval* over which that change occurs, you can determine the *force* on the rocket.
- From the *force* and the *mass* of the rocket, you can find its *acceleration*.

Identify the Goal

(a) The thrust, $\vec{F}_{\text{gas on rocket}}$, of the engine

(b) The acceleration, $\vec{a}_{\text{rocket}}$, of the rocket

Identify the Variables and Constants

Known

$m_{\text{hydrogen}} = 50.0$ kg

$m_{\text{oxygen}} = 400.0$ kg

$t = 5.00$ s

$\vec{v}_{\text{exhaust}} = 3.54 \times 10^3 \ \frac{\text{m}}{\text{s}}$ [back]

$m_{\text{rocket}} = 1.5 \times 10^4$ kg

Unknown

$m_{\text{exhaust gas}}$

$\vec{F}_{\text{gas on rocket}}$

$\vec{a}_{\text{rocket}}$

continued ▶

continued from previous page

Develop a Strategy

Find the total mass of the exhaust gases.

$$m_{\text{exhaust gas}} = m_{\text{hydrogen}} + m_{\text{oxygen}}$$
$$m_{\text{exhaust gas}} = 50.0\text{ kg} + 400.0\text{ kg}$$
$$m_{\text{exhaust gas}} = 450.0\text{ kg}$$

Find the flow rate of the exhaust gas.

$$\text{Flow rate of the exhaust gas} = \frac{m_{\text{exhaust gas}}}{\Delta t}$$
$$\text{Flow rate of the exhaust gas} = \frac{450.0\text{ kg}}{5.00\text{ s}}$$
$$\text{Flow rate of the exhaust gas} = 90.0\ \frac{\text{kg}}{\text{s}}$$

Use impulse equals change in momentum to determine the force on the gas.

$$\vec{F}\Delta t = m\Delta\vec{v}$$
$$\vec{F} = \frac{m\Delta\vec{v}}{\Delta t}$$

Since the gases started from rest relative to the combustion chamber, $\Delta\vec{v} = \vec{v}_{\text{exhaust}}$.

$$\vec{F} = \left(\frac{m}{\Delta t}\right)\Delta\vec{v}$$
$$\vec{F} = \left(90.0\ \frac{\text{kg}}{\text{s}}\right)\left(3.54 \times 10^3\ \frac{\text{m}}{\text{s}}\right)[\text{back}]$$
$$\vec{F} = 3.186 \times 10^5\text{ N[back]}$$
$$\vec{F} \cong 3.19 \times 10^5\text{ N[back]}$$

Use Newton's third law to determine the force on the rocket (combustion chamber).

$$\vec{F}_{(\text{gas on rocket})} = -\vec{F}_{(\text{rocket on gas})}$$
$$\vec{F}_{(\text{gas on rocket})} = -(3.186 \times 10^5\text{ N[back]}$$
$$\vec{F}_{(\text{gas on rocket})} = 3.186 \times 10^5\text{ N[forward]}$$

(a) The thrust on the rocket is 3.19×10^5 N[forward].

Use Newton's second law to calculate the acceleration of the rocket.

$$\vec{F} = m\vec{a}$$
$$\vec{a} = \frac{\vec{F}}{m}$$
$$\vec{a} = \frac{3.186 \times 10^5\text{ N[forward]}}{1.5 \times 10^4\text{ kg}}$$
$$\vec{a} = 21.24\ \frac{\text{m}}{\text{s}^2}[\text{forward}]$$
$$\vec{a} \cong 21.2\ \frac{\text{m}}{\text{s}^2}[\text{forward}]$$

(b) The acceleration of the rocket is $21.2\ \frac{\text{m}}{\text{s}^2}$[forward].

Validate the Solution

The magnitude of the change in momentum for the rocket must equal the magnitude of the change in momentum for the gas, so

$$m_{\text{rocket}}\Delta v_{\text{rocket}} = m_{\text{gas}}\Delta v_{\text{gas}}$$
$$\Delta v_{\text{rocket}} = \frac{m_{\text{gas}}\Delta v_{\text{gas}}}{m_{\text{rocket}}}$$

The change in velocity is inversely proportional to the masses, so you would expect that the velocity of the rocket would be much less than the velocity of the gases. This is in agreement with the calculated value of the acceleration.

PRACTICE PROBLEMS

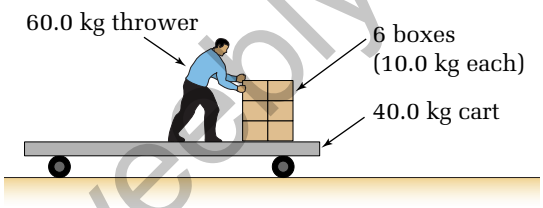
11. Determine the thrust produced if 1.50×10^3 kg of gas exit the combustion chamber each second, with a speed of 4.00×10^3 m/s.

12. What must be the burn rate in kilograms per second if gas with an exhaust speed of 4.15×10^3 m/s is to exert a thrust of 20.8 MN?

13. As an analogy for a reaction engine, imagine that a 60.0 kg person is standing on a 40.0 kg cart, as shown in the diagram. Also on the cart are six boxes, each with a mass of 10.0 kg. The cart is initially at rest. The person then throws the boxes backward, one at a time at 5.0 m/s *relative to the cart*.

(a) Determine the velocity of the cart after each throw, until you have the final velocity of the cart. Keep in mind that the mass on the cart decreases with each throw.

(b) Would the final velocity of the cart be different if the person had thrown all of the boxes at once with a velocity of 5.0 m/s[backward]? If there is a difference, give reasons for it.



The process of burning fuel to provide reaction mass is not the only way to generate a thrust. One extremely efficient method involves an ion engine, such as the one shown in Figure 6.8. In an engine such as this, gas atoms are ionized and the resulting positive ions are driven backward by electrostatic repulsion. The thrust is quite low, but it can act steadily month after month, gradually increasing the velocity of the spacecraft.

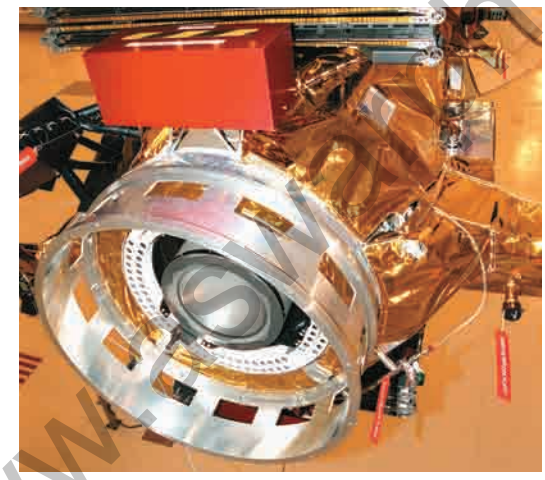


Figure 6.8
Experimental ion engine

TECHNOLOGY LINK

The *Deep Space 1* probe, launched on October 15, 1998, was the first spacecraft to use an ion engine. Xenon atoms are ionized and then repelled electrostatically, emerging from the spacecraft at speeds of up to 28 km/s and producing a maximum thrust of 90 mN. The spacecraft has enough propellant to operate continuously for 605 days.

Gravitational Assist

Sometimes, free energy seems to be gained for a spacecraft through a manoeuvre known as a **gravitational assist** or a **gravitational slingshot**. The process involves directing a spacecraft to swing around a planet, while keeping far from the atmosphere of the

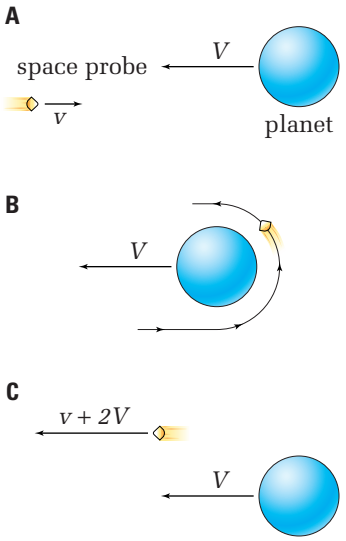


Figure 6.9 A celestial slingshot

planet. The interaction represents an extremely elastic collision, even through the objects do not actually meet. Figure 6.9 illustrates the process.

Earlier studies of elastic collisions showed that the speed of approach of colliding objects is equal to the speed with which they separate. In this case, the spacecraft is approaching the planet with a relative speed of $(v + V)$, where v is the speed of the spacecraft and V is the orbital speed of the planet. If the collision is elastic, the speed with which the spacecraft moves away from the planet must also be $(v + V)$. Since the planet itself is moving at speed V, the spacecraft must be moving at $V + (v + V)$ or $v + 2V$. As a result, if the spacecraft arcs around the planet and returns parallel to its initial path, it will gain a speed of $2V$, which is twice the orbital speed of the planet.

A similar effect can be seen on Earth. If a tiny Superball™ is held just above a more massive ball (such as a lacrosse ball) and they are dropped together, the Superball™ will rebound at high speed from the collision. The effect is shown in Figure 6.10.

In part (A) of the diagram, both balls are falling. Since they are close together, their speeds are about the same. In part (B), the large ball has hit the ground and is about to bounce upward. If that collision is elastic, it will rebound with the same speed it had just before hitting the ground, as shown in (C).

The two balls are now approaching each other, closing the gap between them at a speed of $2v$. If their collision is elastic, the speed with which they separate must also be $2v$. Because of the huge difference in their masses, the large ball is only slightly slowed down in the collision, and so is still effectively travelling at speed v. The small ball will therefore rebound with a speed that is $2v$ greater than the larger ball's speed. In other words, it will have a speed of $3v$.

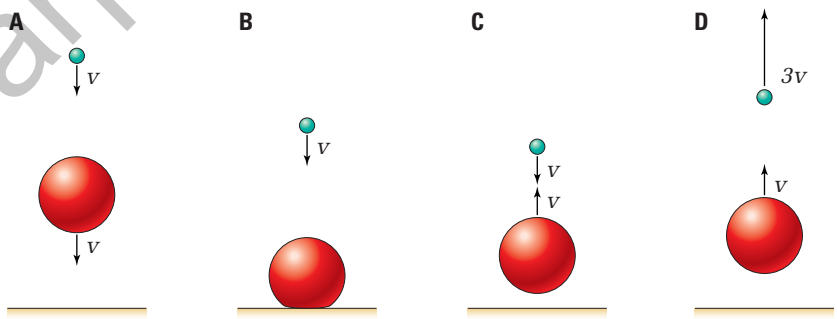


Figure 6.10 In elastic collisions, the speed of approach of colliding objects is equal to the speed with which they separate, as demonstrated by this experiment with a Superball™ and a lacrosse ball.

Because kinetic energy varies with the square of the speed, tripling the speed of the Superball™ will multiply its kinetic energy by a factor of nine. As a result, it will bounce to a height that is nine times its initial height.

This Superball™ discussion assumes that the collision is completely elastic. If there is some energy loss, the ball will not rise as high as predicted. The following investigation looks at just how elastic this collision actually is.

INVESTIGATION 6-A

Superball™ Boost

TARGET SKILLS

- Predicting
- Performing and recording
- Analyzing and interpreting

In the discussion in the text, it was predicted that if a tiny Superball™ is held above a far more massive ball and the two are dropped at the same time, the Superball™ should triple its speed. This assumes that the collision is completely elastic and that the large ball does not significantly slow down during the collision. In this investigation, you will determine how valid those assumptions are.

Problem

How does the speed with which a Superball™ leaves a collision with a more massive ball compare with the theoretical speed?

Equipment

- Superball™
- more massive ball, such as a lacrosse ball
- metric measuring tape

CAUTION Wear a face shield if you are conducting this experiment. The other students must wear safety goggles.

Procedure

1. Hold the Superball™ just above the larger ball and at a height of 0.50 m from the floor.
2. Drop the two together so that the Superball™ will land on top of the larger ball.
3. If the Superball™ bounces straight upward, observe how close the ball comes to the ceiling.
4. Adjust the drop height until the upward-bouncing Superball™ just touches the ceiling.
5. Measure and record the drop height, the diameter of the lacrosse ball, and the height of the ceiling.

Analyze and Conclude

1. Determine the actual drop distance for the Superball™ by subtracting the diameter of the lacrosse ball from the initial height of the Superball™ above the floor. (This assumes that the lacrosse ball has not risen significantly before colliding with the Superball™.)
2. Calculate the speed of the Superball™ just before it collided with the lacrosse ball.
3. Determine the actual height through which the Superball™ rose to reach the ceiling.
4. From the maximum height that the Superball™ attained, determine its actual speed just after the collision with the lacrosse ball.
5. What was the theoretical speed of the Superball™ after the collision?
6. How well does the measured speed compare with the theoretical speed? Express your answer as a percentage.
7. Discuss possible reasons for the difference between the actual speed and the theoretical speed.
8. A comparison of the actual height of the bounce to the theoretical height gives a direct comparison between the amount of kinetic energy the ball received and the theoretical amount of kinetic energy. Express the actual kinetic energy as a percentage of the theoretical kinetic energy. How efficient was this process in transferring energy to the Superball™?

Apply and Extend

9. Provide several suggestions for improving the precision of this investigation.

6.3 Section Review

1. K/U Describe how Newton's third law of motion relates to propulsion in space.

2. K/U Show why the mass rate of flow and exhaust velocity are both involved in the development of thrust.

3. K/U

   (a) During the slingshot procedure for increasing the speed of a space probe, what happens to the orbital speed of the planet? Give reasons for your answer.

   (b) Should you be concerned about this? Justify your answer.

4. MC Which planet is most likely to provide the best "slingshot" effect, Jupiter or Mercury? Give reasons for your choice.

5. C Two identical rocks with equal masses and equal speeds collide head-on in space and stick together.

   (a) Explain why there will be no motion of the clump after the collision.

   (b) If all of the initial kinetic energy is changed into thermal energy in the collision, which situation will create the greater amount of thermal energy?

   - doubling the masses of the rocks, but leaving the speeds the same
   - doubling the speeds of the rocks, while leaving the masses the same

   Give reasons for your choice.

   (c) Is it possible that one of those two situations will result in no change in the temperature increase during the collision? Justify your answer.

6. I By means of a series of diagrams, predict the speed at which a Superball™ would bounce if it was falling on top of a much more massive ball, which was in turn falling on top of an extremely massive ball.

7. MC One method of propulsion that does not involve the ejection of reaction mass is the use of a "solar sail." This device consists essentially of a thin film that could cover an area equal to the size of several football fields. It would be stored during lift-off and unfurled out in space. Light from the Sun (or from huge lasers on Earth) would exert pressure on the sail. At the distance that Earth is from the Sun, the pressure of sunlight would be about 3.5 N/km^2. How realistic is this concept for space travel? What are its advantages and disadvantages?

CHAPTER 6 Review

REFLECTING ON CHAPTER 6

- The work done in moving a mass (m) from a separation of r_1 to a separation of r_2 is given by
$$W = \frac{GMm}{r_1} - \frac{GMm}{r_2}$$
$$\text{or } W = GMm\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$
where M is the mass of the larger object.
- The work that must be done to move a mass m from a separation of r_1 to infinite separation is given by $W_{\text{to escape}} = \frac{GMm}{r_1}$.
- If the escape energy from a planet is to be provided through kinetic energy, the kinetic energy can be expressed as $\frac{1}{2}mv^2 = \frac{GMm}{r_p}$, where r_p is the radius of the planet. This leads to the escape speed being $v = \sqrt{\frac{2GM}{r_p}}$.
- The energies associated with circular orbits are $E_k = \frac{GMm}{2r}$, $E_g = -\frac{GMm}{r}$ and $E_{total} = -\frac{GMm}{2r}$.
- The gravitational potential energy of an object in the vicinity of a planet is negative, since work must be done on the object to remove it completely from the gravitational influence of the planet. The object is said to be in a gravitational potential energy well.
- For an object to be able to escape from the gravitational field of a planet, its total energy must be equal to or greater than zero. The extra energy that must be added to an object to bring its energy up to zero (thus allowing it to escape) is called the "binding energy" and is given by $E_{binding} = -E_{total}$.
- The magnitude of the thrust exerted by the gas being ejected from a rocket can be determined from the equation $F_{thrust} = \left(\frac{m_{gas}}{\Delta t}\right)\Delta v_{gas}$. This gas is known as "reaction mass."
- Gravitational assist is a technique by which a spacecraft gains kinetic energy from a planet around which it swings.

Knowledge/Understanding

1. **(a)** What must be true about the total orbital energy of any planet? Give reasons for your answer.
   (b) What does it mean if some comets have total energies less than zero and others have total energies greater than zero?
2. Why does it take more energy to send a satellite into polar orbit (which follows Earth's longitudinal lines and passes over the North and South Poles) than into an equatorial orbit (which follows the equator eastward over Earth)?
3. For a satellite in circular orbit above Earth, state how the following properties depend on radius: **(a)** period; **(b)** kinetic energy; **(c)** speed.

Inquiry

4. Investigate the sizes of black holes by doing some simple escape velocity calculations.

   As postulated by Einstein's theory of general relativity, black holes are objects with such a strong gravitational field that their escape velocity exceeds the speed of light. Hence, nothing — not even light — can escape from a black hole. They are thought to be caused by massive stars that collapse in on themselves.
   (a) What would be the escape velocity if, without losing any mass, Earth shrank to the following percentages of its present radius?
   - $\frac{1}{100}$
   - $\frac{1}{10000}$
   - $\frac{1}{1 \times 10^6}$

   (b) To how small a size would Earth have to collapse for its escape velocity to equal the speed of light?
   (c) Although a full treatment of black holes requires general relativity, the radius (called the "Schwarzchild radius") can be calculated by using Newtonian theory: by setting $v_{escape} = c$ and solving for the radius. Suppose the core of a star 8.0 times more

massive than the Sun exhausted its fuel and collapsed. What size of black hole would form? (The Sun's mass is 1.99×10^{30} kg.)

(d) The centres of galaxies, including our own Milky Way, might contain black holes with masses of a million times the Sun's mass, or more. Calculate the size of a black hole with a mass of 1.99×10^{36} kg. Compare this to the size of the Sun's radius (6.96×10^{8} m).

5. Consider a space shuttle in circular orbit around Earth. If the commander briefly fires a forward-pointing thruster so that the speed of the shuttle abruptly decreases, what would be the resulting effects on the kinetic energy, the total mechanical energy, the radius of the orbit, and the orbital period? Sketch the new orbit. Explain whether the new orbit will take the shuttle to the same point at which the thrusters were fired.

Communication

6. Explain how a satellite should be launched so that its orbit takes it over every point on Earth as Earth rotates.

7. What is the reason for choosing the zero of gravitational potential energy at infinity rather than, for example, at Earth's surface?

8. If the Sun shrank to the size of a black hole without losing any mass, what would happen to Earth's orbit?

9. Discuss whether you and your friends on the surface of Earth can be considered to be satellites orbiting Earth at a distance of 1.0 Earth radii.

10. **(a)** Explain whether the gravitational force of the Sun ever does work on a planet in a circular orbit.

(b) Does the gravitational force of the Sun ever do work on a comet in an elliptical orbit?

11. Suppose you launched a projectile in Halifax and it landed in Vancouver. If you assumed that the mass of Earth was concentrated at the centre, could you consider the projectile to be in temporary orbit around the centre of Earth? Explain your reasoning. Sketch the trajectory of the projectile over Earth. What would the complete orbit look like?

Making Connections

12. Investigate some of the details of rocket launches for interplanetary probes. What percentage of the probe's mass at lift-off is fuel? How much of the fuel is consumed at lift-off? If an expendable rocket is used to launch the probe, what options are currently available? How much does a launch cost? Summarize your findings in a report.

13. The United States recently announced plans to send an astronaut to Mars and return him or her safely to Earth. Such space travel is very costly compared to sending remote controlled probes, which can often collect as much information. Investigate the issues involved in interplanetary space travel and stage a debate in your class to examine them.

Problems for Understanding

14. Calculate the binding energy that a 50.0 kg classmate has while on the surface of Earth.

15. **(a)** What is the change in gravitational potential energy of a 6200 kg satellite that lifts off from Earth's surface into a circular orbit of altitude 2500 km?

(b) What percent error is introduced by assuming a constant value of g and calculating the change in gravitational potential energy from $mg\Delta h$?

16. The small ellipticity of Earth's orbit causes Earth's distance from the Sun to vary from 1.47×10^{11} m to 1.52×10^{11} m, with the average distance being 1.49×10^{11} m. The Sun's mass is 1.99×10^{30} kg. What is the change in Earth's gravitational potential energy as it moves from its smallest distance to its greatest distance from the Sun?

17. The mass of Mars is 0.107 times Earth's mass and its radius is 0.532 times Earth's radius. How does the escape velocity on Mars compare to the escape velocity on Earth?

18. The Sun's mass is 1.99×10^{30} kg and its radius is 6.96×10^{8} m.

(a) Calculate the escape velocity from the Sun's surface.
(b) Calculate the escape velocity at the distance of Earth's orbit.
(c) Calculate the escape velocity at the distance of Pluto's orbit, 5.9×10^{12} m.

19. A rocket is launched vertically from Earth's surface with a velocity of 3.4 km/s. How high does it go **(a)** from Earth's centre and **(b)** from Earth's surface?

20. Calculate whether a major league baseball pitcher, who can pitch a fastball at 160 km/h (44 m/s), can pitch it right off of any of the following.
(a) Saturn's moon, Mimas, which has a mass of 3.8×10^{19} kg and a radius of 195 km.
(b) Jupiter's moon, Himalia, which has a mass of 9.5×10^{18} kg and a radius of 93 km.
(c) Mars' moon, Phobos, which has a mass of 1.1×10^{16} kg and a radius of 11 km.
(d) a neutron star, with 2.5 times the Sun's mass crammed into a radius of 8.0 km.

21. Consider an object at rest 1.00×10^2 Earth radii from Earth. With what speed will it hit the surface of Earth? Compare this to Earth's escape speed.

22. To exit from the solar system, the *Pioneer* spacecraft used a gravitational assist from Jupiter, which increased its kinetic energy at the expense of Jupiter's kinetic energy. If the spacecraft did not have this assist, how far out in the solar system would it travel? When it left Earth's vicinity, the spacecraft's speed, relative to the Sun, was 38 km/s.

23. A 650 kg satellite is to be placed into synchronous orbit around Earth.
(a) Calculate the gravitational potential energy of the satellite on Earth's surface.
(b) Calculate the total energy of the satellite while it is in its synchronous orbit with a radius of 4.22×10^7 m.
(c) What amount of work must be done on the satellite to raise it into synchronous orbit?
(d) Suppose that from its orbit you wanted to give the satellite enough energy to escape from Earth. How much energy would be required?

24. The atmosphere can exert a small air-drag force on satellites in low orbits and cause these orbits to decay.
(a) Despite an increased air-drag force as the orbit decays, the speed of the satellite increases. Show this by calculating the speed of a satellite when its altitude is 200 km (2.00×10^5 m) and when its altitude is 100 km (1.00×10^5 m).
(b) If the satellite's mass is 500 kg (5.00×10^2), show that the mechanical energy decreases, despite the increase in the satellite's kinetic energy.

25. One of the interesting things about a collapsed, compact object like a neutron star or a black hole is that, theoretically, a spacecraft could be sent close to it without suffering the effects of intense radiation. Consider a neutron star with a mass of 2.0 times the Sun's mass and a radius of 10.0 km. (**Note:** Think about the density that this implies!)
(a) Calculate the velocity of a spacecraft orbiting 500.0 km above the neutron star. (Note that at closer orbits, the effects of high gravity would need to be considered and the familiar formula used here would not apply.) What is this speed as a fraction of the speed of light?
(b) What is the spacecraft's period?

26. A rocket stands vertically on a launch pad and fires its engines. Gas is ejected at a rate of 1200 kg/s and the molecules have a speed of 40.0 km/s. What is the maximum weight the rocket can have if this thrust is to lift it slowly off the launch pad?

27. A 3.5×10^5 kg rocket stands vertically on a launch pad.
(a) Calculate the minimum thrust required to cause the rocket to rise from the launch pad.
(b) The rocket engines eject fuel at a rate of 28.0 kg/s. What is the velocity of the gas as it leaves the engine? Neglect the small mass change in the rocket due to the ejected fuel.

UNIT 2 PROJECT

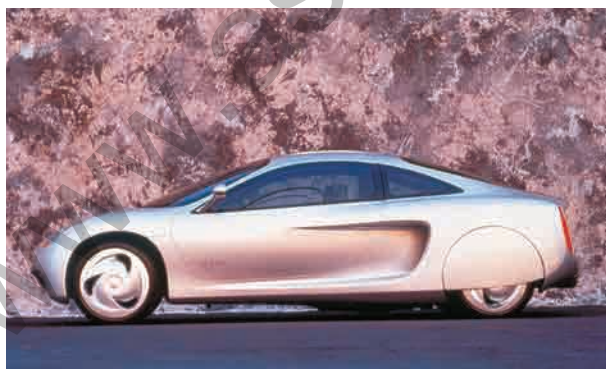
Just a Theory?

Background

Evidence of the deficiencies of existing transportation technologies is everywhere. Limited and increasingly expensive fuel supplies, noise and air pollution, and congested roads and highways are all indications that the existing methods of moving people and goods are less than ideal.

In response, more sustainable, environmentally responsible transportation methods are slowly emerging. Lightweight, low-drag body designs are commonplace. Computerization has improved the efficiency of gasoline engines and has led to the development of "hybrid" vehicles with both gasoline and electric engines. Vehicles powered by fuel cells and improved rechargeable batteries are in limited production. The space shuttle provides a far more efficient method of placing payloads in orbit than do single-use rockets.

Improved transportation technologies are too important to be left to lucky guesses or inspired tinkering. Research projects that apply basic scientific principles to guide and evaluate the development of new vehicles and their components are critically needed. Far from being "just theory," physics principles related to momentum and energy conversion and conservation are key to developing the environmentally friendly, sustainable technologies of the future.



Hybrid cars combine power-generating technologies.

This is a recumbent or "reclining" bicycle.

Scooters offer a transportation alternative.

Challenge

Research the information and prepare a presentation that illustrates how the scientific theories and principles studied in this unit can be used to develop environmentally responsible transportation alternatives. The presentation must include aspects of your study of momentum, energy, and energy transformations. The presentation is to be designed to provide an intelligent adult audience of non-scientists with an understanding of how scientific theories impact everyday life.

Project Criteria

A. As a class, develop clear, specific criteria for the presentation. Decide on acceptable methods of presentation, sourcing of information, time limits, and time lines for the project.

B. In small groups, brainstorm examples of transportation technologies and related scientific principles. The examples can be very specific, such as a particular type of fuel cell, or quite general, such as an innovative bicycle frame design. Select topics so that your completed presentation will include information that you studied in all three chapters of the unit.

Action Plan

1. Decide on a theme for your presentation, so that each of your examples contributes to the development of your overall thesis.
2. Prepare a one- or two-page background outline to summarize your research, including properly referenced sources.
3. Develop a questionnaire, quiz, or rating scale for your audience so that you can gather feedback on your presentation.
4. Develop and present your project.
5. Prepare a written evaluation of your project that includes a summary of audience feedback and ideas for improving the presentation.

ASSESSMENT

After you complete this project

- **assess the clarity of your background summaries about each topic. Can others read your report and formulate specific questions about the topic?**
- **assess the effectiveness of your argument and examples. How well did it persuade audience members of the need for responsible choices of transportation technology and the value of studying basic scientific principles?**
- **assess the impact of your group's presentation as a whole. How well were you able to link the separate examples into a unified, coherent presentation?**

Evaluate

1. What information sources did your group find most useful in this project? How did you ensure that your presentation was free of plagiarism?
2. To what extent has working on this project increased your awareness of alternatives for responsible, sustainable transportation technologies? Explain how your transportation choices in everyday life model these values.
3. Is providing information enough to change people's behaviour? What else can be done so that manufacturers and consumers are encouraged to make responsible choices regarding transportation technology?
4. Suppose a friend questioned the value of studying basic scientific principles by saying, "They are just a bunch of generalizations and mathematical tricks. We need to concentrate on practical ways of solving real problems"? What examples from this project would you use to demonstrate the importance of studying scientific theory?

UNIT 2 Review

Knowledge/Understanding

Multiple Choice

In your notebook, choose the most correct answer for each of the following questions. Outline your reasons for your choice.

1. A hockey puck and a curling stone are at rest on a sheet of ice. If you apply equal impulses to each of them with a hockey stick
   - **(a)** they will have the same acceleration
   - **(b)** the forces applied were equal
   - **(c)** they apply equal reaction forces to the hockey stick
   - **(d)** they will have the same velocity, but different momenta
   - **(e)** they will have the same momentum but different velocities
2. Ball B is moving and collides with a stationary ball A. After the collision, ball B bounces backwards with a velocity of nearly the same magnitude as it had before the collision. Ball A rolls forward very slowly. What is the relationship between the masses of the ball.
   - **(a)** $m_A = m_B/2$
   - **(b)** $m_A = 2\ m_B$
   - **(c)** $m_A = 4\ m_B$
   - **(d)** $m_A = m_B/4$
   - **(e)** $m_A = m_B$
3. You throw a rock straight up into the air. While it rises and falls, its kinetic energy
   - **(a)** remains constant
   - **(b)** increases steadily
   - **(c)** changes direction only
   - **(d)** decreases then increases
   - **(e)** increases then decreases
4. Starting from rest at the top of a hill, a bicyclist pedals furiously on the way down. The kinetic energy of the bicycle and rider at the bottom will be equal to
   - **(a)** lost potential energy
   - **(b)** work done
   - **(c)** work done plus lost potential energy
   - **(d)** work done plus kinetic energy plus potential energy
   - **(e)** zero
5. An astronaut in an orbiting spacecraft is said to be weightless because
   - **(a)** no force of gravity is exerted on the astronaut
   - **(b)** the spacecraft exerts a force opposite to Earth's gravity and acts to suspend the astronaut
   - **(c)** the astronaut and the spacecraft are both in free fall
   - **(d)** the astronaut wears a special gravity-resistant spacesuit
   - **(e)** there is no air resistance in the region where the astronaut is orbiting
6. A rocket launched with a velocity equal to the escape velocity of a planet has
   - **(a)** positive total energy
   - **(b)** negative total energy
   - **(c)** zero total energy
   - **(d)** a total energy that depends on its distance from the planet
   - **(e)** a constantly changing total energy

Short Answer

7. If you throw a ball against a wall, which of the three impulses is the greatest: throw, bounce, or catch?
8. How is it possible for an object to obtain a larger impulse from a smaller force than from a larger force?
9. **(a)** Describe the differences between solving problems for elastic and inelastic collisions.
   - **(b)** How can you tell whether a collision is elastic or not?
   - **(c)** What happens to the kinetic energy of each object in an elastic collision?
10. Distinguish between an open system, a closed system, and an isolated system.
11. Explain why a water hose recoils when the water is turned on.
12. Explain why the first hill of a roller-coaster ride must be the highest hill.
13. **(a)** Under what conditions will a marble of mass m_1 and a rock of mass $3m_1$ have the same gravitational potential energy?
    - **(b)** Under what conditions will a moving marble of mass m_1 and a moving rock of mass $3m_1$ have the same kinetic energy?
14. Write a general equation for the amount of mechanical energy in a system and include

expressions for as many different forms of potential energy as you can locate.

15. A physics wizard is sitting still, puzzling over a homework question. Provide an argument that she is not doing work in the physics sense. Provide a second argument that she *is* doing work in the physics sense.

16. Consider two bodies, A and B, moving in the same direction with the same kinetic energy. A has a mass twice that of B. If the same retarding force is applied to each, how will the stopping distances of the bodies compare?

17. **(a)** Under what circumstances does the work done on a system equal its change in kinetic energy only?

(b) Under what circumstances does the work done on a system equal the change in gravitational potential energy only?

(c) Under what circumstances does the change in kinetic energy of a system equal the change in gravitational potential energy?

18. Use the law of conservation of energy to discuss how the speed of an object changes while in an elliptical orbit.

Inquiry

19. The total momentum vector of a projectile is tangential to its path. This vector changes in magnitude and direction because of the action of an internal force (gravity).

(a) Sketch the path of a projectile and draw momentum vectors at several points along the path.

(b) The equation $\vec{F} = \Delta\vec{p}/\Delta t$ indicates that a change in momentum is evidence of a net force. Draw vectors that show the change in momentum at several points on the path and thus indicate the direction of the net force. (Neglect air resistance.) Discuss your result.

20. The law of conservation of energy can be written in the form $\Delta E = W + Q$, where ΔE is the change in energy in a system, W is the amount of useful work done, and Q is the amount of heat produced. In this form, it is called the "first law of thermodynamics." For centuries, crafty inventors have tried to violate the law by designing perpetual-motion machines. Such a machine would provide more energy as output than was input. The Canadian Patent Office has shown its faith in the law by refusing to grant patents for such machines based on design only. The inventor must submit a working model. Research and report on some designs for perpetual-motion machines. Include a sketch and use the first law of thermodynamics to discuss why the machine will not work.

21. Design a pogo stick for a child. Designate the age range of the child you hope will enjoy the stick and calculate the required spring constant. Determine other parameters, such as the length of the stick, the size of the spring, and the range of distances that the child will be able to depress the spring. Include a sketch of your design.

22. Insight into simple harmonic motion can be gained by contrasting it with non-simple harmonic motion.

(a) Consider the simple harmonic motion of an object oscillating on a spring. Does the velocity of the object change smoothly or abruptly when the object changes direction? Sketch a graph of the displacement of the object versus time.

(b) On the same graph, indicate how the spring force changes with time. Are the restoring force and displacement ever zero at the same time?

(c) Now consider the motion of a highly elastic rubber ball bouncing up and down on an elastic steel plate, always returning to the same height from which it fell. Set a frame of reference so that you can describe the ball's motion. Does the ball's velocity change smoothly or abruptly at its peak altitude and during impact? Sketch a graph of the displacement of the ball versus time.

(d) Draw a free-body diagram of the forces acting on the ball when it is in the air. What is the net force on the ball during contact with the steel plate? Are the net force and

displacement ever zero at the same time? On the same graph as (c), sketch how the net force on the ball changes with time.

(e) Contrast the two graphs in terms of the motions they represent.

23. In this question, you will use the kinetic theory of gases to probe the compositions of the atmospheres of four bodies in the solar system. The kinetic theory of gases can be used to relate the average kinetic energy of the molecules in a gas to the temperature, T, of the gas, $E_{k(average)} = \frac{1}{2} m v_{average}^2 = \frac{3}{2} kT$, where m is the mass of the molecule and k is the Boltzmann constant.

(a) Calculate the escape velocities of Jupiter, Mars, Earth, and the Moon.

(b) In a table of atomic masses, look up the masses for the following molecules: hydrogen (H_2), helium (He), water vapour (H_2O), methane (CH_4), oxygen (O_2), nitrogen (N_2), and carbon dioxide (CO_2).

(c) Calculate the average speed of each of the above gases of molecules at a temperature of 300.0 K.

(d) Some models of velocity distributions of gases indicate that over the lifetime of the solar system (approximately 5 billion years), a gas will escape from a planet unless its average speed times 10.0 ($v_{average} \times 100$) is less than the escape velocity of the planet. Use this to determine which gases should be present in each of the atmospheres in (a). (The velocity distribution of a gas is described as a Maxwellian velocity distribution — look up this term for further information.)

(e) Compare your results to observations.

(f) Summarize and discuss your results.

24. The orbit of a satellite is often used to determine the mass of the planet or star that it is orbiting. How can the mass of a satellite be determined?

25. Explain whether you could put a satellite in an orbit that kept it stationary over the North or South Pole.

26. Imagine that you found a very unusual spring that did not obey Hooke's law. In fact, you performed experiments on the spring and discovered that the restoring force was proportional to the square distance that the spring was stretched or compressed from its equilibrium or $F = -kx^2$.

(a) Describe an experiment that you might have done to find the expression for the restoring force.

Communication

27. To bunt a baseball effectively, at the instant the ball strikes the bat, the batter moves the bat in the same direction as the moving baseball. Explain what effect this action has.

28. You drop a dish from the table. Explain whether the impulse will be less if the dish lands on a carpet instead of a bare floor.

29. Explain whether it is possible to exert a force and yet not cause a change in kinetic energy.

30. You blow up a balloon and release the open end, causing the balloon to fly around the room as the air is rapidly exhausted. What exerts the force that causes the balloon to accelerate?

31. A jet engine intakes air in the front and mixes it with fuel. The mixture burns and is exhausted from the rear of the engine. Use the concept of momentum to explain how this process results in a force on the airplane that is directed forward.

32. Explain the difference between g and G.

Making Connections

33. When Robert Goddard first proposed sending a rocket to the Moon early in the twentieth century, he was ridiculed in the newspapers. People thought that the rocket would have nothing to push against in the vacuum of space and therefore could not move.

(a) How does a rocket move?

(b) Contrast the rocket's motion with the motion produced by a propeller or a wheel.

(c) A rocket can be considered to represent a case of the inverse of an inelastic collision. Explain this statement.

(d) Develop three analogies that could help explain rocket motion.

(e) To test his idea, Goddard set up a pistol in a bell jar from which the air had been evacuated and fired a blank cartridge. What do you think happened?

34. Analyze any appliance or technical device in terms of its component parts and the energy it consumes. Trace the path of this energy in detail backward through its various forms. How many steps does it typically take before you get to the Sun as the ultimate source of energy?

35. A Foucault pendulum can be used to demonstrate that Earth is rotating. Explain how this is possible. What differences would you notice if you used the pendulum at the North Pole, at Earth's equator, and at latitudes between these two points?

36. Before nuclear energy was postulated as the source of energy for the Sun, other energy-generation processes were considered. At the end of the nineteenth century, one promising method was proposed by Lord Kelvin. It was based on the perfect gas law: If a gas is compressed, it heats up. Heating the gas causes it to radiate energy away, so the gas can be further compressed. The process, gravitational contraction, is now thought to heat protostars (newly forming stars) before they begin nuclear fusion in their cores. Research this process and describe in detail how it could heat a star. How is gravitational potential energy converted into heat? What lifetime did this process predict for the Sun? Also, discuss how Darwin's theory of evolution led astronomers to believe that the lifetime for the Sun predicted by this process was too short.

Problems for Understanding

37. A 1400 kg car travels north at 25 m/s. What is its momentum?

38. What impulse is needed to stop the following?
    (a) 150 g baseball travelling at 44 m/s
    (b) 5.0 kg bowling ball travelling at 8.0 m/s
    (c) 1200 kg car rolling forward at 2.5 m/s

39. A 0.80 kg ball travelling at 12 m/s[N] strikes a wall and rebounds at 9.5 m/s[S]. The impact lasts 0.065 s.
    (a) What was the initial momentum of the ball?
    (b) What was the change of momentum of the ball?
    (c) What was the impulse on the wall?
    (d) What was the average force acting on the wall?
    (e) What was the average force acting on the ball?

40. A tennis player smashes a serve so that the racquet is in contact with the ball for 0.055 s, giving it an impulse of 2.5 N · s.
    (a) What average force was applied during this time?
    (b) Assume that the vertical motion of the ball can be ignored. If the ball's mass is 0.060 kg, what will be the ball's horizontal velocity?

41. A hockey player gives a stationary 175 g hockey puck an impulse of 6.3 N · s. At what velocity will the puck move toward the goal?

42. A 550 kg car travelling at 24.0 m/s[E] collides head-on with a 680 kg pickup truck. Both vehicles come to a complete stop on impact.
    (a) What is the momentum of the car before the collision?
    (b) What is the change in the car's momentum?
    (c) What is the change in the truck's momentum?
    (d) What is the velocity of the truck before the collision?

43. A rocket is travelling 160 m/s[forward] in outer space. It has a mass of 750 kg, which includes 130 kg of fuel. Burning all of the fuel produces an impulse of 41 600 N · s. What is the new velocity of the rocket?

44. A 19.0 kg curling stone for Team Ontario travels at 3.0 m/s[N] down the centre line of the ice toward an opponent's stone that is at rest. It strikes the opponent's stone and rolls off to the side with a velocity of 1.8 m/s[N22°W]. The opponent's stone moves in a northeasterly direction. What is the final velocity (magnitude and direction) of the opponent's stone?

45. Two balls collide on a horizontal, frictionless table. Ball A has a mass of 0.175 kg and is travelling at 1.20 m/s[E40°S]. Ball B has a mass of 0.225 kg and is travelling at 0.68 m/s[E]. The velocity of ball B after the collision is 0.93 m/s[E37°S].

(a) What is the velocity (magnitude and direction) of ball A after the collision?
(b) What percentage of kinetic energy is lost in the collision?

46. An 8.0 kg stone falls off a 10.0 m cliff.
(a) How much work is done on it by the gravitational force?
(b) How much gravitational potential energy does it lose?

47. Each minute, approximately 5×10^8 kg of water flow over Niagara Falls. The average height of the falls is 65 m.
(a) What is the gravitational potential energy of the water flow?
(b) How much power (in W or J/s) can this water flow generate?

48. A 0.250 kg ball is thrown straight upward with an initial velocity of 38 m/s. If air friction is ignored, calculate the
(a) height of the ball when its speed is 12 m/s
(b) height to which the ball rises before falling
(c) How would your answers to (a) and (b) change if you repeated the exercise with a ball twice as massive?

49. You are in a 1400 kg car, coasting down a 25° slope. When the car's speed is 15 m/s, you apply the brakes. If the car is to stop after travelling 75 m, what constant force (parallel to the road) must be applied?

50. An archery string has a spring constant of 1.9×10^2 N/m. By how much does its elastic potential energy increase if it is stretched
(a) 5.0 cm and (b) 71.0 cm?

51. You exert 72 N to compress a spring with a spring constant of 225 N/m a certain distance.
(a) What distance is the spring displaced?
(b) What is the elastic potential energy of the displaced spring?

52. A 2.50 kg mass is attached to one end of a spring on a horizontal, frictionless surface. The other end of the spring is attached to one end of a spring is attached to a solid wall. The spring has a spring constant of 75.0 N/m. The spring is stretched to 25.0 cm from its equilibrium point and released.
(a) What is the total energy of the mass-spring system?
(b) What is the velocity of the mass when it passes the equilibrium position?
(c) What is the elastic potential energy stored in the spring when the mass passes a point that is 15.0 cm from its equilibrium position?
(d) What is the velocity of the spring when it passes a point that is 15.0 cm from its equilibrium position?

53. A 275 g ball is resting on top of a spring that is mounted to the floor. You exert a force of 325 N on the ball and it compresses the spring 44.5 cm. If you release the ball from that position, how high, above the equilibrium position of the spring-ball system will the ball rise?

54. A 186 kg cart is released at the top of a hill.
(a) How much gravitational potential energy is lost after it descends through a vertical height of 8.0 m?
(b) If the amount of friction acting on the cart is negligible, determine the kinetic energy and the speed of the cart after it has descended through a vertical height of 8.0 m.
(c) Explain what variables you would need to know in order to calculate the kinetic energy and the speed of the cart for the same conditions if the frictional forces were significant. Assume some reasonable values and make calculations for the kinetic energy and the speed of the cart influenced by friction.

55. A small 95 g toy consists of a piece of plastic attached to a spring with a spring constant of 365 N/m. You compress the spring against the floor through a displacement of 5.5 cm, then release the toy. How fast is it travelling when it rises to a height of 10.0 cm?

56. Suppose a 1.5 kg block of wood is slid along a floor and it compresses a spring that is attached horizontally to a wall. The spring constant is 555 N/m and the block of wood is travelling 9.0 m/s when it hits the spring. Assume that the floor is frictionless and the spring is ideal.
(a) By how much does the block of wood compress the spring?

(b) If the block of wood attaches to the spring so that the system then oscillates back and forth, what will be the amplitude of the oscillation?

57. A spring with a spring constant of 120 N/m is stretched 5.0 cm from its rest position.
- **(a)** Calculate the average force applied.
- **(b)** Calculate the work done.
- **(c)** If the spring is then stretched from its 5.0 cm position to 8.0 cm, calculate the work done.
- **(d)** Sketch a graph of the applied force versus the spring displacement to show the extension of the spring. Explain how you can determine the amount of work done by analyzing the graph.

58. A 32.0 kg child descends a slide 4.00 m high. She reaches the bottom with a speed of 2.40 m/s. Was the mechanical energy conserved? Explain your reasoning and identify the energy transformations involved.

59. A 2.5 kg wooden block slides from rest down an inclined plane that makes an angle of 30° with the horizontal.
- **(a)** If the plane is frictionless, what is the speed of the block after slipping a distance of 2.0 m?
- **(b)** If the plane has a coefficient of kinetic friction of 0.20, what is the speed of the block after slipping a distance of 2.0 m?

60. **(a)** Given Earth's radius (6.38×10^6 m) and mass (5.98×10^{24} kg), calculate the escape velocity from Earth's surface.
- **(b)** What is the escape velocity for a satellite orbiting Earth a distance of 2.00 Earth radii from Earth's centre?
- **(c)** How far away do you have to travel from Earth so that the escape velocity at that point is 1% of the escape velocity at Earth's surface? Answer in metres and in Earth radii.

61. A projectile fired vertically from Earth with an initial velocity v reaches a maximum height of 4800 km. Neglecting air friction, what was its initial velocity?

62. An amateur astronomer discovers two new comets with his backyard telescope. If one comet is moving at 38 km/s as it crosses Earth's orbit on its way toward the Sun and the other at 47 km/s, calculate whether each orbit is bound or not.

63. You want to launch a satellite into a circular orbit at an altitude of 16 000 km (above Earth's surface). What orbital speed will it have? What launch speed will be required?

64. In a joint international effort, two rockets are launched from Earth's surface. One has an initial velocity of 13 km/s and the other 19 km/s. How fast is each moving when it crosses the Moon's orbit (3.84×10^8 m)?

65. A 460 kg satellite is launched into a circular orbit and attains an orbital altitude of 850 km above Earth's surface. Calculate the
- **(a)** kinetic energy of the satellite
- **(b)** total energy of the satellite
- **(c)** period of the satellite
- **(d)** binding energy of the satellite
- **(e)** additional energy and speed required for the satellite to escape

66. **(a)** Calculate the gravitational potential energy of the Earth-Moon system. (Assume that their mean separation is 3.84×10^8 m.)
- **(b)** Calculate the gravitational potential energy of the Earth-Sun system. (Assume that their mean separation is 1.49×10^{11} m.)

67. Proposals for dealing with radioactive waste include shooting it into the Sun. Consider a waste container that is simply dropped from rest in the vicinity of Earth's orbit. With what speed will it hit the Sun?

COURSE CHALLENGE

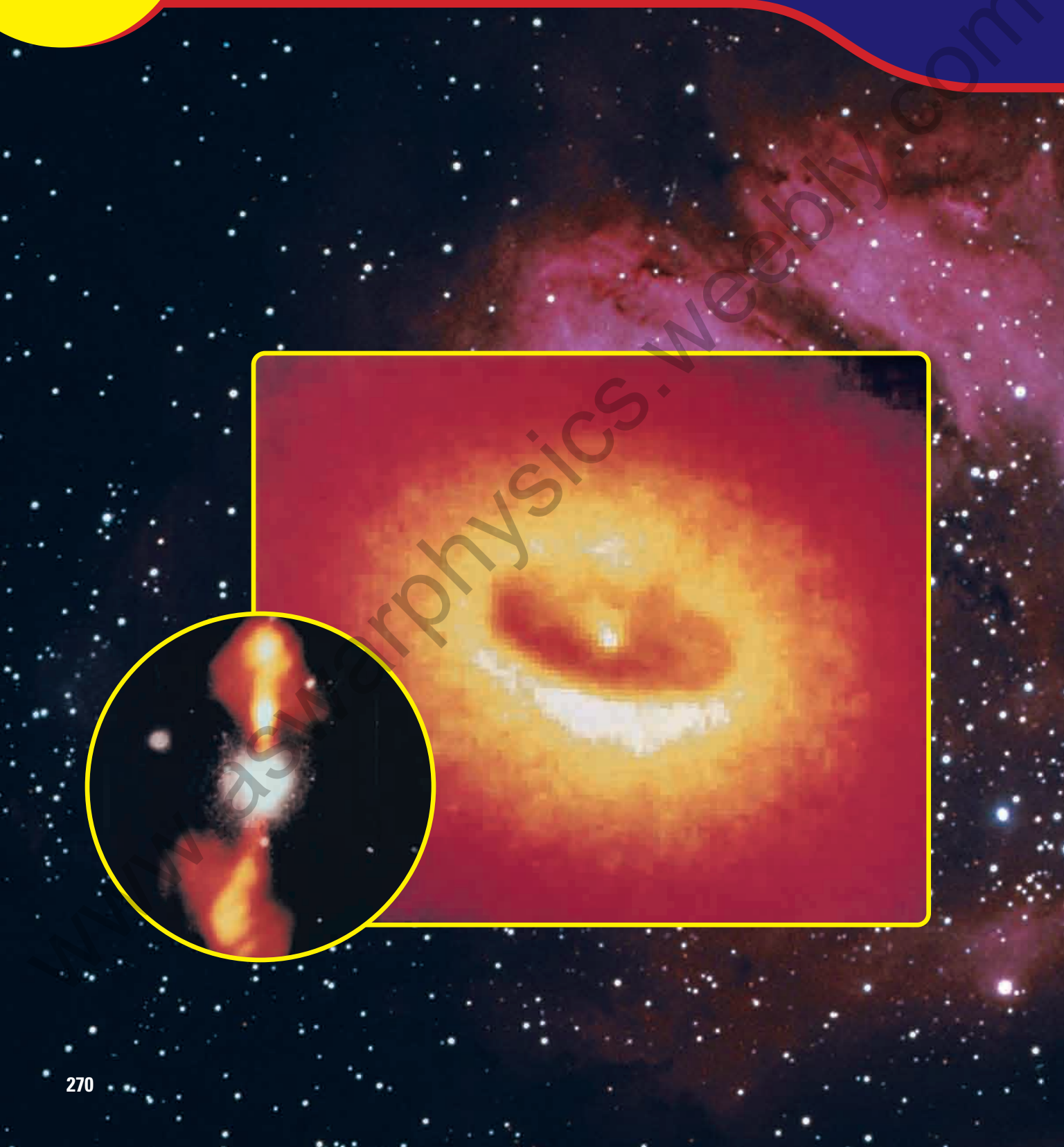
Scanning Technologies: Today and Tomorrow

Consider the following as you continue to build your Course Challenge research portfolio.

- Add important concepts, equations, interesting and disputed facts, and diagrams from this unit.
- Review the information you have gathered in preparation for the end-of-course presentation. Consider any new findings to see if you want to change the focus of your project.
- Scan magazines, newspapers, and the Internet for interesting information to enhance your project.

UNIT 3

Electric, Gravitational, and Magnetic Fields



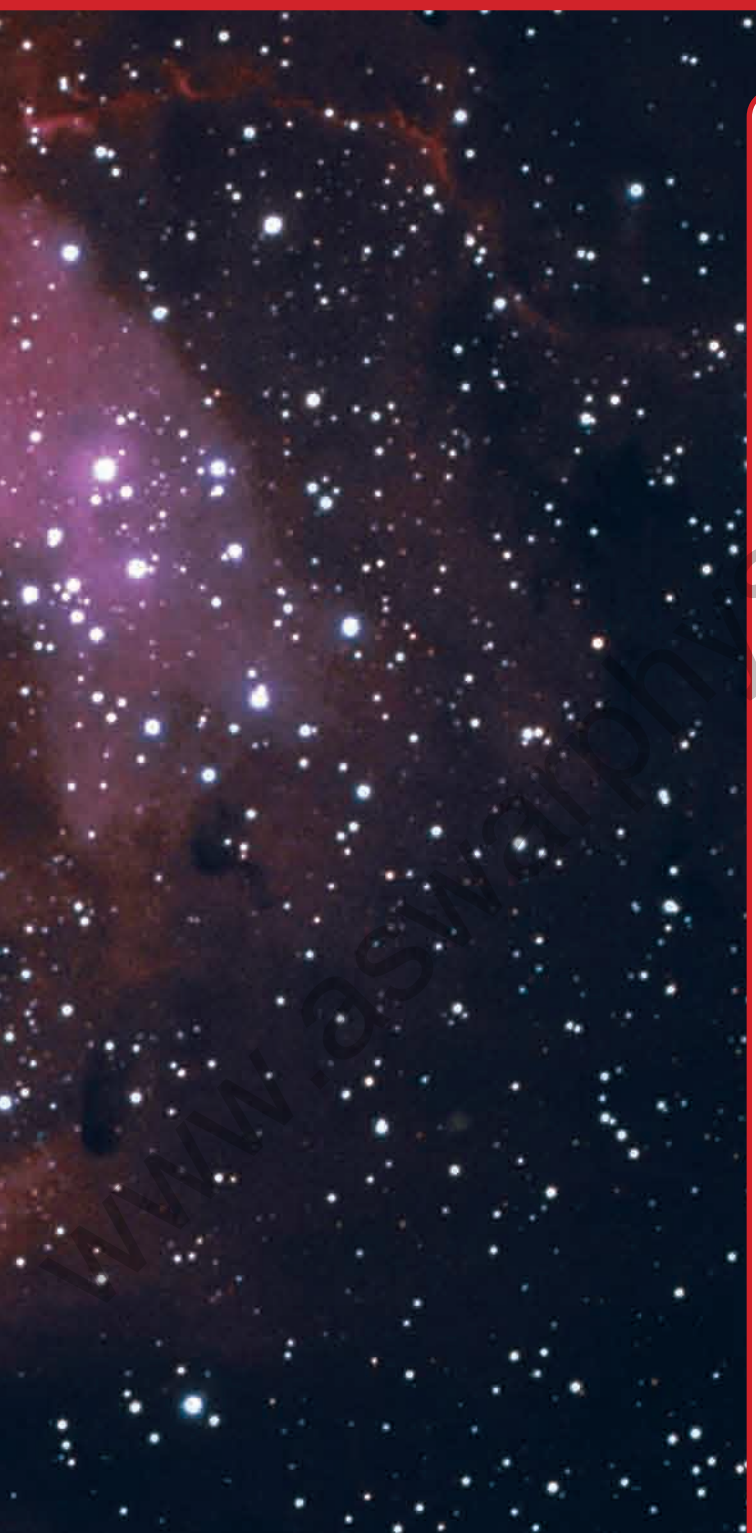
OVERALL EXPECTATIONS

DEMONSTRATE an understanding of the principles and laws related to electric, gravitational, and magnetic forces and fields.

INVESTIGATE and analyze electric, gravitational, and magnetic fields.

EVALUATE the impact of technological developments related to the concept of fields.

UNIT CONTENTS



What is it about "black holes" that stretches the imagination to the limit? Is it that black holes, such as the artist's conception here, defy reason because both matter and energy seemingly disappear into nothingness?

A major part of understanding the black hole phenomenon lies in the characteristics of fields, regions of space over which a force seemingly acts at a distance. You are already familiar with everyday forces that act in this manner — gravity, magnetism, and electricity. Based on straightforward laboratory studies, you can begin to answer such questions as: "How are these fields formed? How are they related to each other?

Recent research indicates, for example, that black holes are points with almost infinite density. The gravitational field generated by this concentration of mass is so strong that not only objects but even light passing within range can never escape.

This unit provides an examination of the properties of electric, gravitational, and magnetic fields. As our understanding of fields increases, so to do the technological applications that use fields. You will study the fundamental properties of fields, how civilization has harnessed this knowledge, and consider possible directions for future research.

UNIT PROJECT PREP

Refer to pages 370–371. In this unit project you will prepare a report and a debate on particle accelerators and relevant research.

- How can you use electric and magnetic fields to accelerate charged particles to very high speeds?
- What are the costs and benefits to society of the research into particle accelerators and the application of the knowledge gained?

CHAPTER 7

Fields and Forces

CHAPTER CONTENTS

PREREQUISITE CONCEPTS AND SKILLS

- Magnetic fields
- Law of universal gravitation placement

You cannot see electric energy, but the electric eel in the photograph can. It is not really an eel — it is actually a knife fish, or *Electrophorous electricus* — but it *is* electric. This fish can detect and generate an electric potential difference. Nearly half of the knife fish's body consists of specialized muscle cells that function like a series of electric cells. This living "battery" can generate an electric potential difference of up to 600 V. The electric shock caused by the knife fish can kill some small prey and often stuns large prey, which the knife fish then devours.

The pits along the side of the knife fish's head and body, called the "lateral line system," are specialized to detect electric fields. The knife fish uses its ability to generate and detect electric energy to navigate, detect enemies, kill or stun prey, and possibly even communicate with other knife fish. If water is polluted, it modifies the electric field generated by the knife fish. A university in France is studying the possibility of using the knife fish to monitor water quality. As you can see, there are even some areas of research in biology that require a basic understanding of physics.

In this chapter, you will learn more about electric energy and fields and compare them with gravitational and magnetic energy and fields.

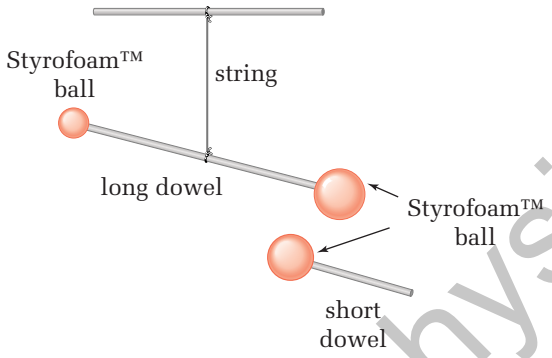
QUICK LAB

A Torsion Balance

TARGET SKILLS

- Hypothesizing
- Identifying variables
- Performing and recording

The torsion balance was an important tool in early studies of both gravitational force and the electrostatic force. As you know, Henry Cavendish was able to determine the universal gravitational constant, *G,* using a torsion balance. Charles Coulomb, unaware of Cavendish's balance, developed a very similar balance, which he used to develop the law now known as Coulomb's law. This lab will help you to understand the principles of the torsion balance, as well as to develop an appreciation of those who used it.



Attach a string (approximately 1.0 m long) to the centre of a thin wooden dowel (approximately 80 cm long) and suspend it from a retort stand or the ceiling. Wrap four Styrofoam™ balls with aluminum foil. Push one of the balls onto each end of the dowel. Make two probes by pushing one ball onto the end of each of two shorter-length wooden dowels. Charge one of the balls on the longer dowel on the suspended balance and also charge the balls on one of the probes. (Use either an ebonite rod and wool or an electrostatic generator to charge the balls.) Now, hold the charged "probe" ball in the vicinity of the "balance" ball and allow the system to reach equilibrium, with the torsion balance turned a small amount.

Experiment with different-sized charges by holding a charged probe in a fixed position near the "balance" ball, and observe the equilibrium position of the balance. Then, touch the charged probe ball to the uncharged probe ball to reduce (by approximately one half) the quantity of charge it carries. Then hold the probe ball in exactly the same position as before and observe the position of the balance.

Experiment with different types of string, a heavier dowel, protection from air currents, and any other variables that you think might affect the performance of the balance.

Analyze and Conclude

1. Describe the performance of the torsion balance.
2. How did the response of the balance change when you reduced the amount of charge on the probe?
3. How would you calibrate your balance if you wanted to obtain quantitative data?
4. What type of string and weight of dowel seemed to perform best?
5. Comment on the use of a torsion balance as a precision tool by early physicists.

WEB LINK

www.mcgrawhill.ca/links/physics12

To see an illustration of Charles Coulomb's torsion balance, go the the above Internet site and click on **Web Links**.

7.1 Laws of Force

SECTION EXPECTATIONS

- Define and describe the concepts related to electric, gravitational, and magnetic fields.
- Analyze and compare Coulomb's law and Newton's law of universal gravitation.
- Apply quantitatively Coulomb's law and Newton's law of universal gravitation.
- Collect, analyze, and interpret data from experiments on charged particles.

KEY TERMS

- inverse square law
- electrostatic force
- torsion balance
- Coulomb's law
- Coulomb's constant

In science courses over the past several years, you have gained experience in applying the laws of motion of Sir Isaac Newton (1642–1727) and analyzing the motion of many types of objects. The two forces that you encounter most frequently are the forces of gravity and friction. In many cases, you have also dealt with an applied force, in which one object or person exerted a force on another. In this unit, you will focus on the nature of the forces themselves.

Gravity and the Inverse Square Law

Several astronomers and other scientists before Newton developed the concept that the force of gravity obeyed an **inverse square law**. In other words, the magnitude of the force of gravity between two masses is proportional to the inverse of the square of the distance separating their centres: $F \propto \frac{1}{r^2}$. It was Newton, though, who verified the relationship.

Figure 7.1 The centripetal force that keeps the Moon in its orbit is the gravitational force between Earth and the Moon.

Newton reasoned that, since the Moon is revolving around Earth with nearly circular motion, the gravitational force between Earth and the Moon must be providing the centripetal force. His reasoning was similar to the following.

- Write the equation for centripetal acceleration.

$$a_c = \frac{v^2}{r}$$

- Write the equation for speed.

$$v = \frac{\Delta d}{\Delta t}$$

- The Moon travels the circumference of an orbit in one period. Therefore, its speed is

$$\Delta d = 2\pi r = 2\pi(3.84 \times 10^8 \text{ m}) = 2.41 \times 10^9 \text{ m}$$

$$T = 2.36 \times 10^6 \text{ s}$$

$$v = \frac{2\pi r}{T} = \frac{2.41 \times 10^9 \text{ m}}{2.36 \times 10^6 \text{ s}} = 1.02 \times 10^3 \frac{\text{m}}{\text{s}}$$

- The centripetal acceleration of the Moon is therefore

$$a_c = \frac{v^2}{r} = \frac{\left(1.02 \times 10^3 \ \frac{\text{m}}{\text{s}}\right)^2}{3.84 \times 10^8 \ \text{m}} = 2.71 \times 10^{-3} \ \frac{\text{m}}{\text{s}^2}$$

- If the force of gravity decreases with the square of the distance between the centre of Earth and the centre of the Moon, then the acceleration due to gravity should also decrease. Write the inverse square relationships and divide the first by the second.

$$a_{g(\text{Moon})} \propto \frac{1}{r^2_{\text{E-Moon}}} \qquad a_{g(\text{Moon})} = \frac{GM_E}{r^2_{\text{E-Moon}}}$$

$$g \propto \frac{1}{r^2_E} \qquad g = \frac{GM_E}{r^2_E}$$

$$\frac{a_{g(\text{Moon})}}{g} = \frac{\frac{1}{r^2_{\text{E-Moon}}}}{\frac{1}{r^2_E}} = \frac{r^2_E}{r^2_{\text{E-Moon}}}$$

- In the ratio above, solve for the acceleration due to gravity at the location of the Moon. Insert the value of g and the distances.

$$a_{c(\text{Moon})} = \frac{gr^2_E}{r^2_{\text{E-Moon}}}$$

$$a_{c(\text{Moon})} = \frac{\left(9.81 \ \frac{\text{m}}{\text{s}^2}\right)(6.38 \times 10^6 \ \text{m})^2}{(3.84 \times 10^8 \ \text{m})^2}$$

$$a_{c(\text{Moon})} = 2.71 \times 10^{-3} \ \frac{\text{m}}{\text{s}^2}$$

The values of acceleration due to gravity that were calculated in two completely different ways are in full agreement. The centripetal acceleration of the Moon in orbit is exactly what you would expect it to be if that acceleration was provided by the force of gravity and if the force of gravity obeyed an inverse square law.

The force of gravity exerts its influence over very long distances and is the same in all directions, suggesting that the influence extends outward like a spherical surface. The equation relating the surface area of a sphere to its radius is $A = 4\pi r^2$, or the area of a sphere increases as the square of the radius. You can relate the influence of the force of gravity with a portion of a spherical surface, A, at a distance r, as shown in Figure 7.2. When the distance doubles to $2r$, the area increases by 2^2, or four. When the distance increases to $4r$, the area of the sphere increases by 4^2, or 16. The influence of the force of gravity appears to be spreading out over the surface area of a sphere. How does this property of the force of gravity compare to the electromagnetic force?

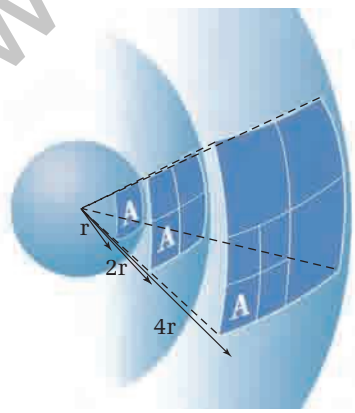
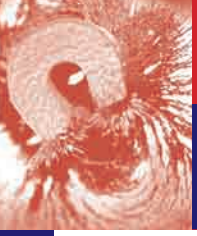


Figure 7.2 The intensity of physical phenomena that obey inverse square laws can be compared to the spreading out of the surface of a sphere.

COURSE CHALLENGE

Contact versus Non-Contact

Action at a distance — something that might have seemed magical to you as a child — lies at the heart of several cutting-edge technologies. Refer to page 604 of this textbook for suggestions about non-contact interactions for your Course Challenge project.



INVESTIGATION 7-A

The Nature of the Electrostatic Force

TARGET SKILLS
- Hypothesizing
- Performing and recording
- Analyzing and interpreting

In this investigation, you will use pith balls to quantitatively analyze the electrostatic force of repulsion.

Problem

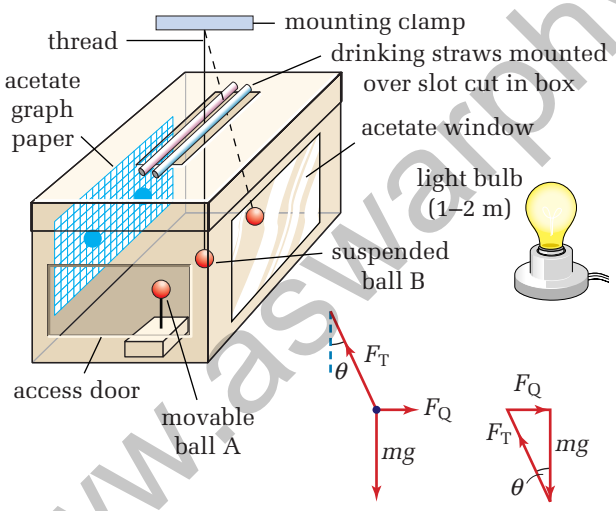
What is the relationship between electrostatic force and the distance of separation between two charged pith balls?

Equipment

- electronic balance
- clear straight filament lamp
- razor knife
- pith ball on thread
- pith ball mounted on wooden base
- acetate graph paper
- clear acetate sheet
- ebonite and fur
- cardboard shoe box
- two drinking straws

Procedure

CAUTION Be careful when using any sharp cutting object.



1. Cut rectangular holes in the front, rear, and side of the box and a slit on top, as shown.
2. Mount the clear acetate in the front hole and the acetate graph sheet in the rear. Mount the drinking straws on either side of the slit on top.
3. Poke the free end of the thread attached to pith ball B up between the drinking straws and mount on a clamp above. Ensure that the thread hangs vertically.
4. Place the pith ball with the wooden base (A) inside the box. Record the rest positions of both pith balls on the acetate grid.
5. Rub the ebonite with fur and reach in and charge both pith balls. Adjust the height of the mount of pith ball B so that it is level with pith ball A. Record the position of both pith balls.
6. Move pith ball A toward pith ball B several times. Adjust the mount of pith ball B each time to keep B level with A. For each trial, read and record the positions of both pith balls.
7. Measure the mass of a large number of balls and take an average to find the mass of one.

Analyze and Conclude

1. For each trial, use the rest positions and the final positions of the pith balls to determine the distance between A and B.
2. For each trial, use the lateral displacement of B, relative to its original rest position, to determine the electrostatic force acting on B. (Prove for yourself that $F_Q = mg \tan \theta$.)
3. Draw a graph with the electrostatic force on the vertical axis and the distance of separation between the charges on the horizontal axis. What does your graph suggest about the relationship between the electrostatic force and the distance of separation?
4. Calculate $1/r^2$ for each of your trials and plot a new graph of F versus $1/r^2$. Does your new graph provide evidence to back up the prediction you made in your original analysis? Discuss.

Electromagnetic Force

The exact nature of frictional forces and applied forces that are due to the electromagnetic force is very complex. How would anyone obtain fundamental information about such complex forces? Physicists start with the simplest cases of such forces, analyze these cases, and then extend them to more and more complex situations. The simplest case of an electromagnetic force is the electrostatic force between two stationary point charges.

Several scientists, including Daniel Bernoulli, Joseph Priestly, and Henry Cavendish, had proposed that the **electrostatic force** obeyed an inverse square relationship, based on a comparison with Newton's inverse square law of universal gravitation.

Coulomb's Experiment

French scientist Charles Augustin Coulomb (1736–1806) carried out experiments in 1785 similar to the investigation that you have just completed. Coulomb had previously developed a **torsion balance** for measuring the twisting forces in metal wires. He used a similar apparatus, shown in Figure 7.3, to analyze the forces between two charged pith balls.

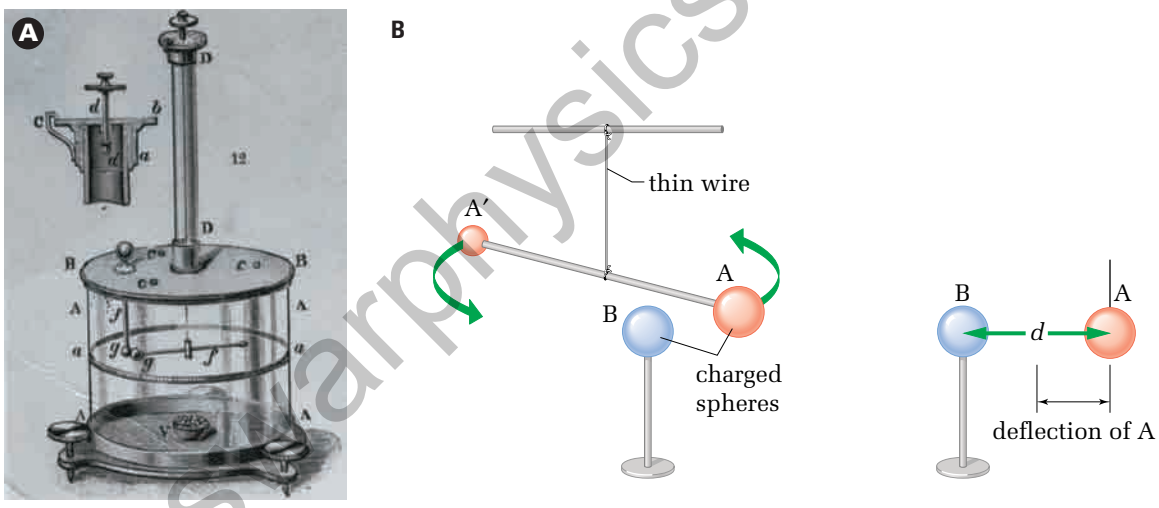


Figure 7.3 Coulomb's torsion balance **(A)** is simplified in **(B)**. Coulomb measured the force required to twist the thread a given angle. He then used this value to determine the force between the two pith balls.

Coulomb charged the two pith balls equally, placed them at precisely measured distances apart. Observing the angle of deflection, he was able to determine the force acting between them for each distance of separation. He found that the electric force, F, varied inversely with the square of the distance between the centres of the pith balls $\left(F_Q \propto \frac{1}{r^2}\right)$.

To investigate the dependence of the force on the magnitude of the charge on the pith balls, Coulomb began with two identically charged pith balls and measured the force between them. He then touched a pith ball with a third identical but uncharged pith ball to reduce the amount of charge on the ball by half. He found that

PHYSICS FILE

You can develop a sense of the meaning of the Coulomb constant by considering two charges that are carrying exactly one unit of charge, a coulomb, and located one metre apart. Substituting ones into Coulomb's law, you would discover that these two charges exert a force of 9.00×10^9 N on each other. This amount of force could lift about 50 000 railroad cars or 2 million elephants. Clearly, one coulomb is an exceedingly large amount of charge. Typical laboratory charges would be much smaller — in the order of μC or millionths of a coulomb.

the force was now only one half the previous value. After several similar modifications of the charges, Coulomb concluded that the electric force varied directly with the magnitude of the charge on each pith ball ($F_Q \propto q_1 q_2$). The two proportion statements can be combined as one $\left(F_Q \propto \frac{q_1 q_2}{r^2}\right)$ and expressed fully as **Coulomb's law**.

Any proportionality can be written as an equality by including a proportionality constant. Although the value of the constant was not known until long after Coulomb's law was accepted, it is now known to be $8.99 \times 10^9\ \mathrm{N \cdot m^2/C^2}$, in SI units.

The value of the proportionality constant in a vacuum is denoted k and known as the **Coulomb constant**. In fact, air is so close to "free space" — the early expression for a vacuum — that any effect on the value of the constant is beyond the number of significant digits that you will be using. For practical purposes, the Coulomb constant is often rounded to $9.00 \times 10^9\ \mathrm{N \cdot m^2/C^2}$.

Coulomb's law can now be written as $F_Q = k\frac{q_1 q_2}{r^2}$. The direction of the force is always along the line between the two point charges. Between charges of like sign, the force is repulsive; between charges of unlike sign, the force is attractive.

COULOMB'S LAW

The electrostatic force between two point charges, q_1 and q_2, distance r apart, is directly proportional to the magnitudes of the charges and inversely proportional to the square of the distance between their centres.

$$F_Q = k\frac{q_1 q_2}{r^2}$$

Quantity	Symbol	SI unit
electrostatic force between charges	F_Q	N (newtons)
Coulomb's constant	k	$\frac{\mathrm{N \cdot m^2}}{\mathrm{C^2}}$ (newton · metres squared per coulomb squared)
electric charge on object 1	q_1	C (coulombs)
electric charge on object 2	q_2	C (coulombs)
distance between object centres	r	m (metres)

Unit Analysis

$$\text{newton} = \frac{(\text{newton})(\text{metre})^2}{(\text{coulomb})^2} \cdot \frac{(\text{coulomb})(\text{coulomb})}{(\text{metre})^2}$$

$$\frac{\mathrm{N} \cdot \cancel{\mathrm{m^2}}}{\cancel{\mathrm{C^2}}} \cdot \frac{\cancel{\mathrm{C}} \cdot \cancel{\mathrm{C}}}{\cancel{\mathrm{m^2}}} = \mathrm{N}$$

PHYSICS FILE

Note that not only does the proportionality constant have to validate the numerical relationship, it must also make the units match. Thus, the units for k are obtained by rearranging the Coulomb equation.

$$k = \frac{F \cdot d^2}{q_1 \cdot q_2}$$

$$= \frac{(\text{force})(\text{distance})^2}{(\text{charge})(\text{charge})}$$

$$= \frac{\mathrm{N \cdot m^2}}{\mathrm{C^2}}$$

Strictly speaking, the description of Coulomb's law given on the previous page is meant to apply to point charges. However, just as Newton was able to develop the mathematics (calculus) that proved that the mass of any spherical object can be considered to be concentrated at a point at the centre of the sphere for all locations outside the sphere, so it might also be proven that if charge is uniformly distributed over the surface of a sphere, then the value of the charge can be considered to be acting at the centre for all locations outside the sphere.

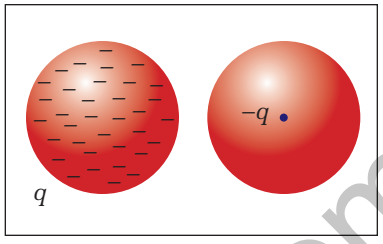


Figure 7.4 A uniformly charged sphere acts as if all of its charge is concentrated at its centre.

SAMPLE PROBLEM

Applying Coulomb's Law

A small sphere, carrying a charge of −8.0 μC, exerts an attractive force of 0.50 N on another sphere carrying a charge with a magnitude of 5.0 μC.

(a) What is the sign of the second charge?

(b) What is the distance of separation of the centres of the spheres?

Conceptualize the Problem

- *Charged spheres* appear to be the same as *point charges* relative to any point *outside* of the sphere.
- The *force*, *charge*, and *distance* are related by *Coulomb's law*.

Identify the Goal

The sign, ±, and separation distance, r, of the charges

Identify the Variables and Constants

Known		Implied	Unknown
$q_1 = -8.0 \times 10^{-6}$ C	$F = 0.50$ N	$k = 9.0 \times 10^9 \ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}$	r
$\lvert q_2 \rvert = 5.0 \times 10^{-6}$ C			

Develop a Strategy

Since the spheres are uniformly charged, they can be considered to be points and Coulomb's law can be applied.

$$F = k\frac{q_1 q_2}{r^2}$$

$$r^2 = \frac{kq_1q_2}{F}$$

$$r = \pm\sqrt{\frac{kq_1q_2}{F}}$$

Only the positive root is chosen to represent the distance in this situation

$$r = \sqrt{\frac{\left(9.0 \times 10^9 \ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)(8.0 \times 10^{-6}\ \text{C})(5.0 \times 10^{-6}\ \text{C})}{5.0 \times 10^{-1}\ \text{N}}}$$

$$r = 0.84853 \text{ m}$$

$$r \cong 0.85 \text{ m}$$

(a) Since the force is attractive, the second charge must be positive.

(b) The distance between the centres of the charges is 0.85 m.

continued ►

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Validate the Solution

Charges in the microcoulomb range are expected to exert moderate forces on each other.

PRACTICE PROBLEMS

1. Calculate the electrostatic force between charges of $-2.4\ \mu C$ and $+5.3\ \mu C$, placed 58 cm apart in a vacuum.
2. The electrostatic force of attraction between charges of $+4.0\ \mu C$ and $-3.0\ \mu C$ is 1.7×10^{-1} N. What is the distance of separation of the charges?
3. Two identically charged objects exert a force on each other of 2.0×10^{-2} N when they are placed 34 cm apart. What is the magnitude of the charge on each object?
4. Two oppositely charged objects exert a force of attraction of 8.0 N on each other. What will be the new force of attraction if the objects are moved to a distance four times their original distance of separation?
5. Two identical objects have charges of $+6.0\ \mu C$ and $-2.0\ \mu C$, respectively. When placed a distance d apart, their force of attraction is 2.0 N. If the objects are touched together, then moved to a distance of separation of $2d$, what will be the new force between them?

QUICK LAB

Graphical Analysis of Coulomb's Law

TARGET SKILLS

- **Analyzing and interpreting**
- **Communicating results**

In this Quick Lab, you will use sample data to gain practice with the inverse square dependence of the electrostatic force between two point charges on the distance between them. Two equally charged, identical small spheres are placed at measured distances apart and the force between them is determined by using a torsion balance. Prepare a table similar to the one shown here, in which to record your data.

1. Draw a graph of force versus distance for this data. What is the shape of this graph?
2. Rearrange the distance data (use the third column in your table) and draw a graph that shows the relationship as a linear one (refer to Skill Set 4, Mathematical Modelling and Curve Straightening).
3. Measure the slope of the straight line.
4. Using the known value of Coulomb's constant ($k = 9 \times 10^9\ N \cdot m^2/C^2$), calculate the value of the original charge on the spheres.

Force ($\times 10^2$ N)	Distance between centres (cm)	
5.63	1.2	
2.50	1.8	
1.30	2.5	
0.791	3.2	
0.383	4.6	
0.225	6.0	

The Nature of Electric, Magnetic, and Gravitational Forces

All forces, including electrostatic forces, are vector quantities and obey the laws of vector addition. The equation describing Coulomb's law uses only scalar quantities, with the understanding that the direction of the force always lies along the line joining the centre of the two charges. However, when one charge experiences a force from more than one other charge, the direction must be resolved.

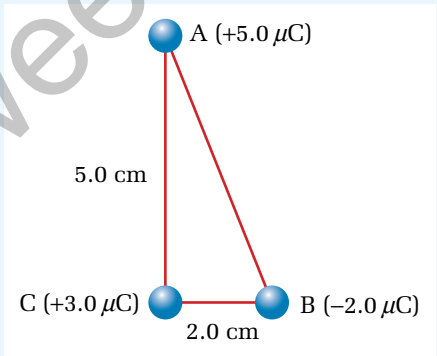
ELECTRONIC LEARNING PARTNER

Go to your Physics 12 Electronic Learning Partner to enhance your knowledge of Coulomb's law.

SAMPLE PROBLEM

Multiple Charges

Three charges, A (+5.0 μC), B (–2.0 μC), and C (+3.0 μC), are arranged at the corners of a right triangle as shown. What is the net force on charge C?



Conceptualize the Problem

- *Charges* A and B both exert a *force* on C.
- Although A and B exert forces on each other, these forces have no effect on the forces that they exert on C.
- The *net force* on charge C is the *vector sum* of the *two forces* exerted by charges A and B.
- The *forces* exerted by A and B are related to the magnitude of the *charges* and the *distance* between the charges, according to *Coulomb's law*.

Identify the Goal

The net force, $\vec{F}_{net}$, on charge C

Identify the Variables and Constants

Known

$q_A = +5.0 \times 10^{-6}$ C
$q_B = -2.0 \times 10^{-6}$ C
$q_C = +3.0 \times 10^{-6}$ C
$r_{AC} = 5.0 \times 10^{-2}$ m
$r_{BC} = 2.0 \times 10^{-2}$ m

Implied

$k = 9.0 \times 10^9 \ \frac{N \cdot m^2}{C^2}$

Unknown

$\vec{F}_{net}$

Develop a Strategy

Use Coulomb's law to find the magnitude of the forces acting on C.

Let F_{AC} represent the magnitude of the force of charge A on charge C.

$$F_{AC} = k\frac{q_A q_C}{r^2}$$

$$F_{AC} = \left(9.0 \times 10^9 \ \frac{N \cdot m^2}{C^2}\right)\frac{(5.0 \times 10^{-6}\ C)(3.0 \times 10^{-6}\ C)}{(0.050\ m)^2}$$

$$F_{AC} = 54\ N$$

continued ▶

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Let F_{BC} represent the magnitude of the force of charge B on charge C (attraction).

$$F_{BC} = k\frac{q_B q_C}{r^2}$$

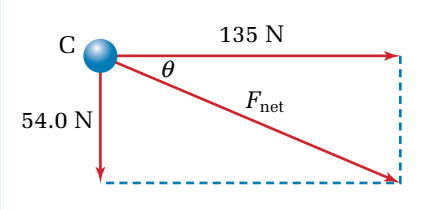
$$F_{BC} = \left(9.0 \times 10^9 \ \frac{\text{N} \cdot \text{m}^2}{\text{C}^2}\right)\frac{(2.0 \times 10^{-6}\ \text{C})(3.0 \times 10^{-6}\ \text{C})}{(0.020\ \text{m})^2}$$

$$F_{BC} = 135\ \text{N}$$

Since charges A and C are both positive, the force will be repulsive and point directly downward on C.

Since B and C are oppositely charged, the force will be attractive and will point directly to the right of C.

Draw a diagram of the forces on charge C.



Use the Pythagorean theorem to calculate the magnitude of F_{net}.

$$F_{net}^2 = (135\ \text{N})^2 + (54.0\ \text{N})^2$$

$$F_{net}^2 = 21\ 141\ \text{N}^2$$

$$F_{net} = 145.39\ \text{N}$$

$$F_{net} \cong 1.5 \times 10^2\ \text{N}$$

Use the definition of the tangent function to find the angle, θ.

$$\tan\theta = \frac{54.0}{135}$$

$$\tan\theta = 0.40$$

$$\theta = \tan^{-1} 0.40$$

$$\theta = 21.8^\circ$$

$$\theta \cong 22^\circ$$

The net force on charge C is 1.5×10^2 N at an angle of 22° clockwise from the horizontal.

Validate the Solution

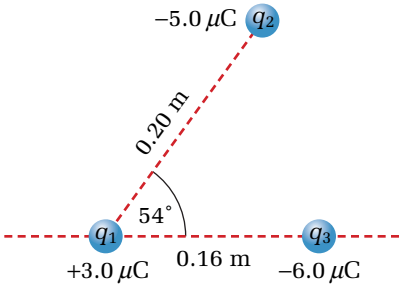
The magnitude and direction of the net force are consistent with the orientation of the three charges.

PRACTICE PROBLEMS

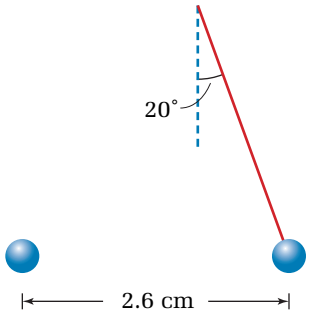
6. A single isolated proton is fixed on a surface. Where must another proton be located in relation to the first in order that the electrostatic force of repulsion would just support its weight?

7. Three charged objects are located at the vertices of a right triangle. Charge A (+5.0 μC) has Cartesian coordinates (0,4); charge B (–5.0 μC) is at the origin; charge C (+4.0 μC) has coordinates (5,0), where the coordinates are in metres. What is the net force on each charge?

8. The diagram shows three charges situated in a plane. What is the net electrostatic force on q_1?



9. The diagram below shows two pith balls, equally charged and each with a mass of 1.5 g. While one ball is suspended by a thread, the other is brought close to it and a state of equilibrium is reached. In that situation, the two balls are separated by 2.6 cm and the thread attached to the suspended ball makes an angle of 20° with the vertical. Calculate the charge on each of the pith balls.



10. Two 2.0 g spheres are attached to each end of a silk thread 1.20 m long. The spheres are given identical charges and the midpoint of the thread is then suspended from a point on the ceiling. The spheres come to rest in equilibrium, with their centres 15 cm apart. What is the magnitude of the charge on each sphere?

Although magnetic forces and electrostatic forces are related and both fit into the category of electromagnetic forces, the strength of a magnetic force cannot be defined in the same way as electrostatic and gravitational forces. The reason for the difference is that magnetic monopoles do not exist or, at least, have never been detected, in spite of the efforts of physicists. Where there is a north pole, you will also find a south pole. Nevertheless, Coulomb was able to approximate isolated magnetic monopoles by measuring the forces between the poles of very long, thin magnets.

If one pole of a long, thin bar magnet is placed in the vicinity of one pole of another long, thin bar magnet, Coulomb's magnetic force law states: The magnetic force F between one pole of magnetic strength p_1 and another pole of magnetic strength p_2 is inversely proportional to the square of the distance r between them, or $F \propto \frac{p_1 p_2}{r^2}$. It is not possible, however, to find a proportionality constant, because it is not possible to define a unit for p, a magnetic monopole.

You have seen that the three different types of forces — electrostatic, gravitational, and magnetic — all exhibit some form of an inverse square distance relationship. Are there any significant differences that you should note?

Probably the greatest difference between gravitational and electromagnetic forces is the strength. Gravitational forces are much weaker than electrostatic and magnetic forces. For example, you do not see uncharged pith balls, nor demagnetized iron bars, moving toward each other under the action of their mutual gravitational attraction.

In summary, the similarities and differences among electrostatic, gravitational, and magnetic forces are listed in Table 7.1.

Table 7.1 Differences among Electrostatic, Gravitational, and Magnetic Forces

Electrostatic force	Gravitational force	Magnetic force
■ can be attractive or repulsive ■ demonstrates an inverse square relationship in terms of distance ■ depends directly on the unit property (charge) ■ law easily verified using point charges (or equivalent charged spheres)	■ can only be attractive ■ demonstrates an inverse square relationship in terms of distance ■ depends directly on the unit property (mass) ■ law easily verified using point masses (or solid spheres) ■ magnitude of the force is much weaker than electrostatic or magnetic force	■ can be attractive or repulsive ■ demonstrates an inverse square relationship in terms of distance (between isolated poles) ■ depends directly on the unit property (pole strength) ■ law cannot be verified using magnetic monopoles as they do not exist independently (must be simulated using long, thin magnets or thin, magnetized wire)

7.1 Section Review

1. K/U What is meant by the statement that Coulomb "quantified" the electric force?
2. K/U In what way did Coulomb determine the dependence of the electrostatic force on different variables?
3. C Explain the similarities and differences between the Coulomb experiment for charge and the Cavendish experiment on mass.
4. C Explain how, in one sense, Coulomb's law is treated as a scalar relationship, but on the other hand, its vector properties must always be considered.
5. K/U State some similarities and some differences between the gravitational force and the electrostatic force.
6. MC Research the role of electrostatic charge in technology and write a brief report on your findings. Examples could include photocopiers and spray-painting equipment.
7. I By what factor would the electrostatic force between two charges change under the following conditions?
   (a) The distance is tripled.
   (b) Each of the charges is halved.
   (c) Both of the above changes are made.

7.2 Describing Fields

In Figure 7.5 (A), a woodcutter exerts a splitting force on a log by direct contact between the axe and the log. In contrast, in Figure 7.5 (B), a charged comb is exerting a force on charged pith balls without coming into contact with the balls. This electrostatic force between the comb and pith balls is an example of an action-at-a-distance force. You have just been studying the characteristics of the three common action-at-a-distance forces: gravitational, electric, and magnetic forces. The phrase **action at a distance** describes some of the characteristics of these forces, but does not really explain how these results are achieved. The critical question now is: How is each mass or charge or magnet "aware" of the other?

Figure 7.5 **(A)** A woodcutter chopping a log is an example of a contact force. **(B)** When a charged comb exerts a force on charged pith balls, the force is acting at a distance and is a non-contact force.

The question of how an object can exert a force on another object without making contact with the object was addressed by Michael Faraday (1792–1867), who proposed the concept of a field. This field concept became quite popular and was extended to explain the gravitational forces between masses.

The fundamental concept is that a **field** is a property of space. An object influences the space around it, setting up either an electric, gravitational, or magnetic field. The object producing the field is called the "source" of the field. This field in turn exerts a force on other objects located within it. This concept is consistent with the inverse square law, which implies that an object influences the space around it.

SECTION EXPECTATIONS

- Define and describe the concepts related to electric, gravitational, and magnetic fields.
- Compare the properties of electric, gravitational, and magnetic fields.
- Analyze the electric field and the electric forces produced by point charges.
- Sketch simple field patterns using field lines.

KEY TERMS

- action at a distance
- field
- test charge
- electric field intensity
- electric field
- gravitational field intensity
- magnetic field intensity
- electric field line
- gravitational field line
- magnetic field line

HISTORY LINK

In 1600, William Gilbert (1540–1603) hypothesized that the rubbing of certain materials, such as amber, removes a fluid or "humour" from the material and releases an "effluvium" into the surroundings. He proposed that the effluvium made contact with other materials and caused the force now known as the "electrostatic force." As you continue to study this section, decide whether Gilbert was on the right track.

Defining Field Intensity

Figure 7.6 illustrates the generation of an electric field by a charge, q_1. The density of the shading designates the strength of the field. If a second charge, q_2, is introduced into the field at point P, for example, it is the *field* that interacts with q_2. Because this is a local interaction, it is not necessary to explain how forces can act between objects separated by any distance.

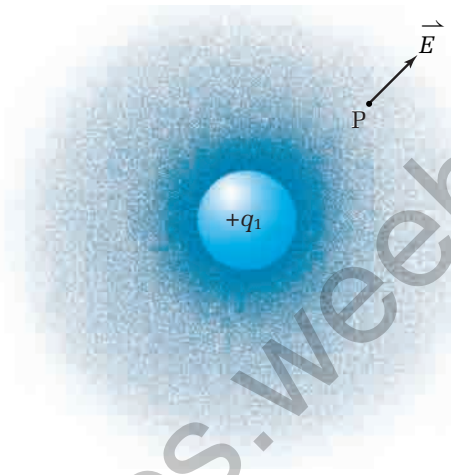


Figure 7.6 Charge q_1 influences the space around it by generating an electric field. The density of the shading indicates the strength of the field.

To describe the field around a charge, q, it is convenient to use the concept of a test charge. By definition, a **test charge** is a point charge with a magnitude so much smaller than the source charge that any field generated by the test charge itself is negligible in relation to the field generated by the source charge. You can place the test charge, q_t, at any point within the field generated by q, and then take the following steps.

- Write Coulomb's law to describe the force between the source charge, q, and the test charge, q_t. $$F = k\frac{qq_t}{r^2}$$
- Divide both sides of the equation by q_t. $$\frac{F}{q_t} = k\frac{q}{r^2}$$

The term on the right-hand side of the equation contains only the source charge and the distance that q_t is from the source charge. Since it is independent of anything that might be located at q_t, it provides a convenient way to describe the condition of space at q_t. Now the term on the left-hand side of this equation is defined as the magnitude of the **electric field intensity**, $\vec{E}$, which is commonly called the **electric field**.

DEFINITION OF ELECTRIC FIELD INTENSITY

The electric field intensity at a point is the quotient of the electric force on a charge and the magnitude of the charge located at the point.

$$\vec{E} = \frac{\vec{F}_Q}{q}$$

Quantity	Symbol	SI unit
electric field intensity	$\vec{E}$	$\frac{N}{C}$ (newtons per coulomb)
electric force	$\vec{F}_Q$	N (newtons)
electric charge	q	C (coulombs)

Unit Analysis

$$\frac{\text{newtons}}{\text{coulomb}} = \frac{N}{C}$$

Note: Electric field intensity has no unit of its own.

Since force is a vector quantity, so also is an electric field. An electric force can be attractive or repulsive, so physicists have accepted the convention that the direction of the electric field vector at any point is given by the direction of the force that would be exerted on a *positive* charge located at that point. Using this concept, you can illustrate an electric field by drawing force vectors at a variety of points in the field. As shown in Figure 7.7, the length of the vector represents the magnitude of the field at the tail of the vector, and the direction of the vector represents the direction of the field at that point.

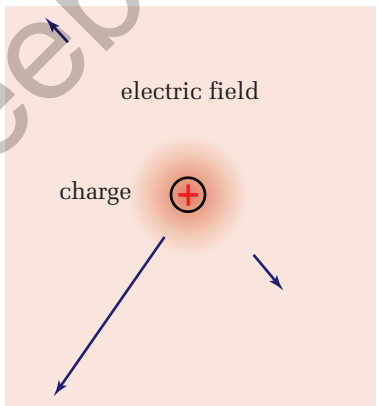


Figure 7.7 Vector arrows can be used to represent the magnitude and direction of the electric field around a charge at various locations.

SAMPLE PROBLEM

Calculating Electric Field Intensity

A positive test charge, $q_t = +2.0 \times 10^{-9}$ C, is placed in an electric field and experiences a force of $\vec{F} = 4.0 \times 10^{-9}$ N[W].

(a) What is the electric field intensity at the location of the test charge?

(b) Predict the force that would be experienced by a charge of $q = +9.0 \times 10^{-6}$ C if it replaced the test charge, q_t.

Conceptualize the Problem

- The *electric field intensity* is related to the *force* and the test *charge*.
- If you know the *electric field intensity* at a point in space, you can determine the *force* on any *charge* that is placed at that point without knowing anything about the source of the field.

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Identify the Goal

The electric field, $\vec{E}$, at a given point in space

The force, $\vec{F}$, on the new charge located at the same point

Identify the Variables

Known	Unknown
$\vec{F}_{q_t} = 4.0 \times 10^{-9}$ N[W]	$\vec{E}$
$q_t = 2.0 \times 10^{-9}$ C	$\vec{F}$
$q = 9.0 \times 10^{-6}$ C	

Develop a Strategy

Find the electric field intensity by using the equation that defines electric field.

$$\vec{E} = \frac{\vec{F}}{q_t}$$

$$\vec{E} = \frac{4.0 \times 10^{-9}\ \text{N[W]}}{2.0 \times 10^{-9}\ \text{C}}$$

$$\vec{E} = 2.0\ \frac{\text{N}}{\text{C}}\text{[W]}$$

(a) The electric field intensity is $\vec{E} = 2.0\ \frac{\text{N}}{\text{C}}\text{[W]}$.

Rearrange the equation for electric field to solve for the new force.

$$\vec{E} = \frac{\vec{F}}{q}$$

$$\vec{F} = q\vec{E}$$

Substitute the numerical values and solve.

$$\vec{F} = (9.0 \times 10^{-6}\ \text{C})\left(2.0\ \frac{\text{N}}{\text{C}}\text{[W]}\right)$$

$$\vec{F} = 18 \times 10^{-6}\ \text{N[W]}$$

$$\vec{F} = 1.8 \times 10^{-5}\ \text{N[W]}$$

(b) The force on the 9.0×10^{-6} C charge is $\vec{F} = 1.8 \times 10^{-5}$ N[W].

Validate the Solution

You would expect the electric field to have units N/C and be pointing west. The magnitude of the field seems to be reasonable in relation to the charge and force.

Since the second charge is larger than the first, you would expect the second force to be larger than the first. Charges in the microcoulomb range are considered to be average charges that occur in electrostatic experiments.

PRACTICE PROBLEMS

11. A positive charge of 3.2×10^{-5} C experiences a force of 4.8 N to the right when placed in an electric field. What is the magnitude and direction of the electric field at the location of the charge?

12. An electric field points due east with a magnitude of 3.80×10^{3} N/C at a particular location. If a charge of $-5.0\ \mu$C is placed at this location, what will be the magnitude and the direction of the electric force that it experiences?

13. A negative charge of 2.8×10^{-6} C experiences an electrostatic force of 0.070 N to the right. What is the magnitude and direction of the electric field at the location of the charge?

14. A small charged sphere is placed at a point in an electric field that points due west and has a magnitude of 1.60×10^{4} N/C. If the sphere experiences an electrostatic force of 6.4 N east, what is the magnitude and sign of its charge?

A discussion similar to that for the electric field intensity can be made for gravitational field intensity. A mass, such as Earth, can exert a gravitational force on a test mass placed in its vicinity. The ratio of the gravitational force to the test mass depends only on the source and the location in the field. This ratio is called the **gravitational field intensity**, for which the symbol is $\vec{g}$.

DEFINITION OF GRAVITATIONAL FIELD INTENSITY

The gravitational field intensity at a point is the quotient of the gravitational force and the magnitude of the test mass.

$$\vec{g} = \frac{\vec{F}_g}{m}$$

Quantity	Symbol	SI unit
gravitational field intensity	$\vec{g}$	$\frac{\text{N}}{\text{kg}}$ (newtons per kilogram)
gravitational force	$\vec{F}_g$	N (newtons)
mass	m	kg (kilograms)

Unit Analysis

$$\frac{\text{newtons}}{\text{kilogram}} = \frac{\text{N}}{\text{kg}}$$

In the past, you have used the symbol g to represent the acceleration due to gravity at Earth's surface. If you analyze the equation that described gravitational field intensity in the box above, you will see that it can be rearranged to give $\vec{F} = m\vec{g}$, which is the same as the equation for the weight of an object at Earth's surface. So, in fact, the g that you have been using is the same as the gravitational field intensity at Earth's surface.

Conceptual Problem

- Show that the units for g, m/s^2, are equivalent to the units for gravitational field intensity, or N/kg.

SAMPLE PROBLEM

Calculating Gravitational Field Intensity

A mass of 4.60 kg is placed 6.37×10^6 m from the centre of a planet and experiences a gravitational force of attraction of 45.1 N.

(a) Calculate the gravitational field intensity at this location.

(b) Discuss the significance of your answer.

Conceptualize the Problem

- The definition of *gravitational field intensity* is the gravitational *force* per unit *mass*.

Identify the Goal

The gravitational field intensity, $\vec{g}$, at this location

Identify the Variables

Known	Unknown
$\lvert\vec{F}\rvert = 45.1$ N	$\vec{g}$
$m = 4.60$ kg	
$r = 6.37 \times 10^6$ m	

Develop a Strategy

Find the gravitational field intensity by using the equation for field intensity and the given variables.

$$\vec{g} = \frac{\vec{F}}{m}$$

$$\lvert\vec{g}\rvert = \frac{45.1 \text{ N}}{4.60 \text{ kg}}$$

$$\vec{g} = 9.80 \frac{\text{N}}{\text{kg}} \text{[in the direction of the force]}$$

(a) The gravitational field intensity at this location is 9.80 N/kg.

Look for recognizable characteristics, then investigate other data.

The value of the field intensity is identical to that of Earth's near its surface.

The distance given is actually the average radius of Earth.

(b) The location seems to be at the surface of Earth, although another alternative is that it could be *above* the surface of a planet with gravitational field intensity at its surface that is greater than that of Earth.

Validate the Solution

The units are correct for gravitational field. The values for both distance and field intensity provide more validation, because they are identical to the values for the surface of Earth. However, this does not preclude the possibility of the object being above another planet.

PRACTICE PROBLEMS

15. What is the gravitational field intensity at the surface of Mars if a 2.0 kg object experiences a gravitational force of 7.5 N?

16. The gravitational field intensity on the surface of Jupiter is 26 N/kg. What gravitational force would a 2.0 kg object experience on Jupiter?

17. The planet Saturn has a gravitational field intensity at its surface of 10.4 N/kg. What is the mass of an object that weighs 36.0 N on the surface of Saturn?

18. What would be the gravitational field intensity at a location exactly one Saturn radius above the surface of Saturn?

19. What is the centripetal acceleration of a satellite orbiting Saturn at the location described in the previous problem?

The gravitational field can also be mapped in the region of a source mass by drawing the gravitational field vectors at corresponding points in the field. Similar to the electric field, the vector length represents the magnitude of the gravitational field and the direction of the vector represents the direction in which a gravitational force would be exerted on a test mass placed in the field.

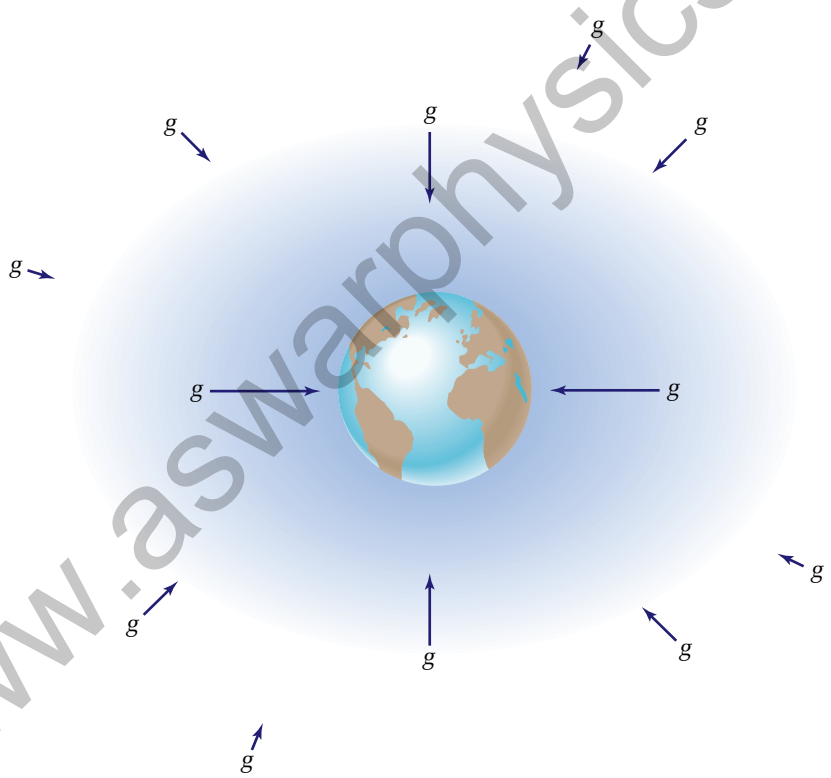
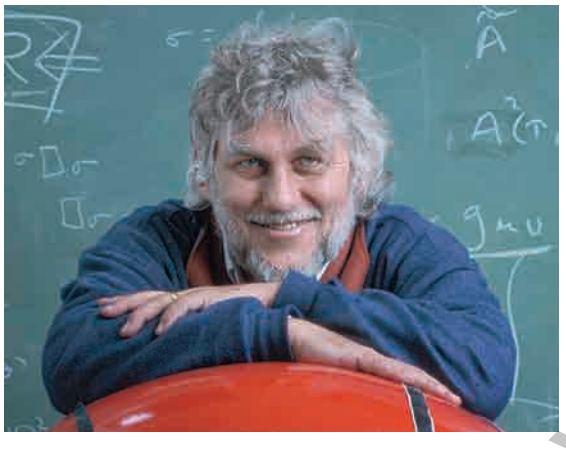


Figure 7.8 Earth's gravitational field can be represented by vectors, with the length of each being proportional to the field intensity at that point.

CANADIANS IN PHYSICS

Gravity: A Matter of Time

William George Unruh was born in Winnipeg, the son of a high school physics teacher. "As a boy I loved looking at the pictures in my father's physics textbooks," he recalls. "They aroused my curiosity about how the world works." He attended the University of Manitoba and then Princeton University, where he received his Ph.D. Today, he is a professor of physics and astronomy at the University of British Columbia and a Fellow of the Canadian Institute for Advanced Research.



Dr. Unruh with the beach ball that he sometimes uses to help explain the concept of gravity.

Dr. Unruh explains that, according to Albert Einstein, "The rate at which time flows can change from place to place, and it is this change in the flow of time that causes the phenomenon we usually refer to as gravity." Dr. Unruh's work focusses on understanding aspects of Einstein's theories. "For example," he says, "Since matter can influence time and matter influences gravity, which is just the variable flow of time, the very measuring instruments we use to measure time can change time. While this is not important in most situations, it becomes very important in trying to decide how the universe operates; for example, in understanding black holes." Dr. Unruh explains that, in black holes, the structures of space and time collaborate, creating regions through which even light cannot travel.

"All of physics is now described in terms of field theories," Dr. Unruh points out. "However, we also experience the world in terms of particles. Since fields exist everywhere at all times, part of my work has been trying to understand the particulate nature of fields. Probably my best known work is showing that the particle nature of fields depends on the observer's state of motion. If an observer is accelerated through a region that seems to be empty of particles to an observer at rest, that region will, to the accelerated observer, appear to be filled with a hot bath of particles. Thus, the existence or non-existence of particles in a field can depend on how the observer moves as he or she observes that field. The effect is extremely small, but it is there."

Another area in which Dr. Unruh works is gravity wave detection. A gravity wave might be called a "vibration of space and time." It is caused by the acceleration of masses; for example, of black holes around each other. The techniques that Dr. Unruh and others have developed will be important to the future refinement of gravity wave detectors now being built in the states of Louisiana and Washington, as well as elsewhere in the world.

Since magnetic monopoles are not known to exist, it is not practical to try to define magnetic field intensities in a way that is analogous to the definitions of electric and gravitational fields. The most practical way to describe magnetic field intensity at this point is to relate it to the effect of a magnetic field on a current-carrying wire, which you studied in previous science courses. The following steps show you how to relate the **magnetic field intensity**, B, to the force, $\vec{F}_B$, exerted by the magnetic field on a length, l, of wire carrying a current, I.

- Write the equation describing the force on a current-carrying conductor in a magnetic field when the direction of the current is perpendicular to the magnetic field.

$$\vec{F}_B = \vec{I}\vec{l}\vec{B}$$

- Rearrange the equation to solve for the magnetic field intensity.

$$\vec{B} = \frac{\vec{F}}{\vec{I}\vec{l}}$$

- The SI unit of magnetic field intensity is the tesla, T. Substitute SI units for the symbols in the equation above.

$$\mathrm{T} = \frac{\mathrm{N}}{\mathrm{A} \cdot \mathrm{m}} \text{ or}$$

$$\text{tesla} = \frac{\text{newton}}{\text{ampere} \cdot \text{metre}}$$

The above relationship states that if each metre of a conductor that is carrying a current of one ampere experiences a force of one newton due to the presence of a magnetic field that is perpendicular to the direction of the current, the magnitude of the magnetic field is one tesla.

Fields near Point Sources

The definition and accompanying equation that you learned for electric field strength, $\vec{E} = \vec{F}_Q/q$, is a general definition. If you know the force on a charge due to an electric field, you can determine the electric field intensity without knowing anything about the source of the field. It is convenient, however, to develop equations that describe the electric field intensity for a few common, special cases, such as point charges.

- Write the equation describing the magnitude of the force on a test charge, q_t, that is a distance, r, from a point charge, q.

$$|\vec{F}_Q| = k\frac{qq_t}{r^2}$$

- Write the general definition for the electric field intensity.

$$\vec{E} = \frac{\vec{F}_Q}{q_t}$$

- Substitute the expression for force into the above equation and simplify.

$$|\vec{E}| = \frac{k\frac{q\cancel{q_t}}{r^2}}{\cancel{q_t}}$$

$$|\vec{E}| = k\frac{q}{r^2}$$

ELECTRIC FIELD INTENSITY NEAR A POINT CHARGE

The electric field intensity a distance away from a point charge is the product of Coulomb's constant and the charge, divided by the square of the distance from the charge. The direction of the field is radially outward from a positive point charge and radially inward toward a negative point charge.

$$|\vec{E}| = k\frac{q}{r^2}$$

Quantity	Symbol	SI unit
electric field intensity	$\vec{E}$	$\frac{\text{N}}{\text{C}}$ (newtons per coulomb)
Coulomb's constant	k	$\frac{\text{N}\cdot\text{m}^2}{\text{C}^2}$ (newton · metres squared per coulomb squared)
source charge	q	C (coulombs)
distance	r	m (metres)

Unit Analysis

$$\frac{\text{newton}\cdot\text{metre}^2}{\text{coulomb}^2}\cdot\frac{\text{coulomb}}{\text{metre}^2} = \frac{\text{newton}}{\text{coulomb}}$$

$$\frac{\text{N}\cdot\cancel{\text{m}^2}}{\text{C}^{\cancel{2}}}\cdot\frac{\cancel{\text{C}}}{\cancel{\text{m}^2}} = \frac{\text{N}}{\text{C}}$$

Note: This equation applies only to the field surrounding an isolated point charge.

SAMPLE PROBLEMS

Field Intensity near a Charged Sphere

1. **What is the electric field intensity at a point 30.0 cm from the centre of a small sphere that has a positive charge of 2.0×10^{-6} C?**

Conceptualize the Problem

- At any point outside of a *charged sphere*, the *electric field* is the same as it would be if the charge was *concentrated at a point* at the *centre* of the sphere.
- The *electric field* is related to the source *charge* and *distance*.
- The *direction* of the field is the direction in which a *positive charge would move* if it was placed at that point in the field.

Identify the Goal

The electric field intensity, $\vec{E}$

Identify the Variables and Constants

Known	Implied	Unknown
$q = +2.0 \times 10^{-6}$ C	$k = 9.0 \times 10^{9} \dfrac{\text{N} \cdot \text{m}^2}{\text{C}^2}$	$\vec{E}$
$r = 0.30$ m		

Develop a Strategy

Find the field intensity by using the equation for the special case of the field near a point charge.

$$|\vec{E}| = k\frac{q}{r^2}$$

Substitute the numerical values for charge and distance and solve.

$$|\vec{E}| = \left(9.0 \times 10^{9} \frac{\text{N} \cdot \text{m}^2}{\text{C}^2}\right)\left(\frac{2.0 \times 10^{-6}\ \text{C}}{(0.30\ \text{m})^2}\right)$$

The direction is radially outward from the positive charge.

$$|\vec{E}| = 2.0 \times 10^{5} \frac{\text{N}}{\text{C}}$$

The electric field intensity is 2.0×10^5 N/C in a direction pointing directly away from the source charge.

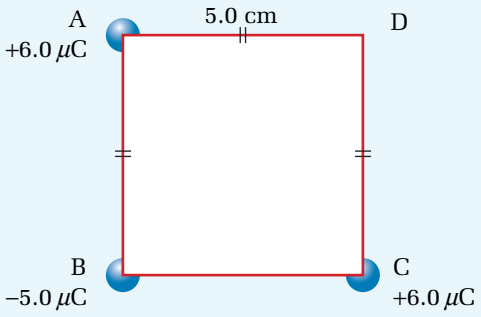
Validate the Solution

Close to a charge of "average" magnitude, the field is expected to be quite strong. Check that the units cancel to give N/C.

$$\frac{\text{N} \cdot \cancel{\text{m}^2}}{\text{C}^{\cancel{2}}} \cdot \frac{\cancel{\text{C}}}{\cancel{\text{m}^2}} = \frac{\text{N}}{\text{C}}$$

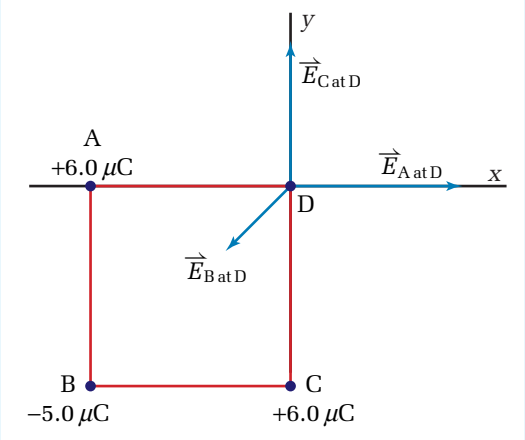
---

2. Three charges, A (+6.0 μC), B (–5.0 μC), and C (+6.0 μC), are located at the corners of a square with sides that are 5.0 cm long. What is the electric field intensity at point D?



Conceptualize the Problem

- Since field intensities are vectors they must also be *added vectorally*.
- The magnitude of the field vectors can be determined individually.
- Draw a *vector diagram* showing the *field intensity* vectors at point D and then superimpose an *x-y coordinate* system on the drawing, with the *origin* at point D.



Identify the Goal

The resultant electric field intensity, $\vec{E}$, at point D

Identify the Variables and Constants

Known	Implied	Unknown
$d_{AB} = d_{BC} = 5.0$ cm	$k = 9.0 \times 10^{9} \dfrac{\text{N} \cdot \text{m}^2}{\text{C}^2}$	$\vec{E}_D$
$q_A = +6.0\ \mu$C		d_{BD}
$q_B = -5.0\ \mu$C		
$q_c = +6.0\ \mu$C		

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Develop a Strategy

Calculate the diagonal of the square by using the Pythagorean theorem.

Since the result is a distance, the negative root has no meaning. Use the positive root.

$$d_{BC}^2 = (5.0\text{ cm})^2 + (5.0\text{ cm})^2$$
$$d_{BC}^2 = 50.0\text{ cm}^2$$
$$d_{BC} = \pm\sqrt{50.0\text{ cm}^2}$$
$$d_{BC} = \pm 7.07\text{ cm}$$
$$d_{BC} = 7.07\text{ cm}$$

Calculate the magnitude of the electric field intensity of each of the given charges at point D, using the equation for the special case of the field intensity near a point charge.

$$|\vec{E}| = k\frac{q}{r^2}$$
$$|\vec{E}_{\text{A at D}}| = \left(9.0 \times 10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\left(\frac{6.0 \times 10^{-6}\text{ C}}{(0.050\text{ m})^2}\right)$$
$$|\vec{E}_{\text{A at D}}| = 2.16 \times 10^7\ \frac{\text{N}}{\text{C}}$$
$$|\vec{E}_{\text{B at D}}| = \left(9.0 \times 10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\left(\frac{5.0 \times 10^{-6}\text{ C}}{(0.0707\text{ m})^2}\right)$$
$$|\vec{E}_{\text{B at D}}| = 9.00 \times 10^6\ \frac{\text{N}}{\text{C}}$$
$$|\vec{E}_{\text{C at D}}| = \left(9.0 \times 10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\left(\frac{6.0 \times 10^{-6}\text{ C}}{(0.050\text{ m})^2}\right)$$
$$|\vec{E}_{\text{C at D}}| = 2.16 \times 10^7\ \frac{\text{N}}{\text{C}}$$

Use the method of components to find the resultant electric field vector.

The angle between the x-axis and the vector for the field at point D due to charge B is 45°, because it points along the diagonal of a square.

x-components

$$E_{(\text{A at D})x} = 2.16 \times 10^7\ \frac{\text{N}}{\text{C}}$$
$$E_{(\text{B at D})x} = -\left(9.00 \times 10^6\ \frac{\text{N}}{\text{C}}\right)\cos 45^\circ$$
$$E_{(\text{B at D})x} = -6.36 \times 10^6\ \frac{\text{N}}{\text{C}}$$
$$E_{(\text{C at D})x} = 0$$
$$E_{(\text{net})x} = 1.524 \times 10^7\ \frac{\text{N}}{\text{C}}$$

y-components

$$E_{(\text{A at D})y} = 0$$
$$E_{(\text{B at D})y} = -\left(9.00 \times 10^6\ \frac{\text{N}}{\text{C}}\right)\sin 45^\circ$$
$$E_{(\text{B at D})y} = -6.36 \times 10^6\ \frac{\text{N}}{\text{C}}$$
$$E_{(\text{C at D})y} = 2.16 \times 10^7\ \frac{\text{N}}{\text{C}}$$
$$E_{(\text{net})y} = 1.524 \times 10^7\ \frac{\text{N}}{\text{C}}$$

Use the Pythagorean theorem to find the magnitude of the resultant vector.

$$|\vec{E}_{(\text{net})}|^2 = \left(E_{(\text{net})x}\right)^2 + \left(E_{(\text{net})y}\right)^2$$
$$|\vec{E}_{(\text{net})}|^2 = \left(1.524 \times 10^7\ \frac{\text{N}}{\text{C}}\right)^2 + \left(1.524 \times 10^7\ \frac{\text{N}}{\text{C}}\right)^2$$
$$|\vec{E}_{(\text{net})}|^2 = 4.6452 \times 10^{14}\ \left(\frac{\text{N}}{\text{C}}\right)^2$$
$$|\vec{E}_{(\text{net})}| = 2.1553 \times 10^7\ \frac{\text{N}}{\text{C}}$$
$$|\vec{E}_{(\text{net})}| \cong 2.2 \times 10^7\ \frac{\text{N}}{\text{C}}$$

Use the definition of the tangent function to find the direction of the electric field vector at point D.

$$\tan\theta = \frac{1.524 \times 10^7\ \frac{\text{N}}{\text{C}}}{1.524 \times 10^7\ \frac{\text{N}}{\text{C}}}$$
$$\tan\theta = 1.00$$
$$\tan\theta = \tan^{-1} 1.00$$
$$\tan\theta = 45^\circ$$

The electric field intensity at point D is 2.2×10^7 N/C at an angle of 45° counterclockwise from the positive x-axis.

Validate the Solution

Since two positive charges and one negative charge of similar magnitudes are creating the field, you would expect that the net field would be similar in magnitude to those created by the individual charges. The angle is 45° as predicted.

PRACTICE PROBLEMS

20. Calculate the electric field intensity at a point 18.0 cm from the centre of a small conducting sphere that has a charge of $-2.8\ \mu$C.

21. The electric field intensity at a point 0.20 m away from a point charge is 2.8×10^6 N/C, directed toward the charge. What is the magnitude and sign of the charge?

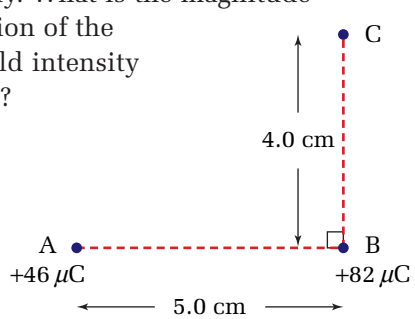
22. The electric field intensity at a point, P, near a spherical charge of 4.6×10^{-5} C, is 4.0×10^6 N/C. How far is point P from the centre of the charge?

23. How many electrons must be removed from a spherical conductor with a radius of 4.60 cm in order to make the electric field intensity just outside its surface 3.95×10^3 N/C?

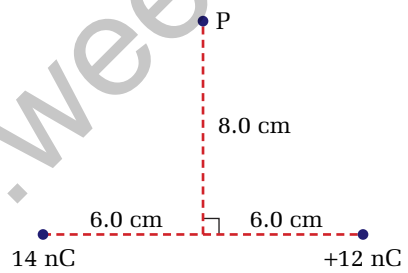
24. What is the electric field intensity at a point 15.2 cm from the centre of a sphere charged uniformly at $-3.8\ \mu$C?

25. A charge of $+7.4\ \mu$C establishes an electric field intensity at point M of 1.04×10^7 N/C. How far is point M from the centre of the charge?

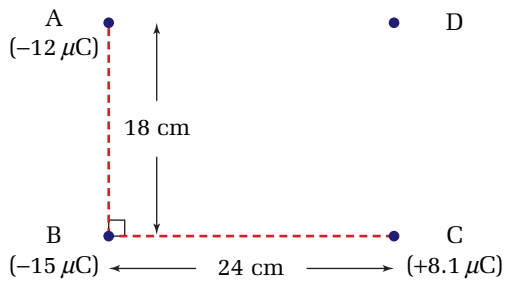
26. In the diagram, A and B represent small spherical charges of $+46\ \mu$C and $+82\ \mu$C, respectively. What is the magnitude and direction of the electric field intensity at point C ?



27. Determine the magnitude and direction of the electric field intensity at point P in the diagram.



28. The diagram shows three small charges at three corners of a rectangle. Calculate the magnitude and direction of the electric field intensity at the fourth corner, D.



29. Two point charges of $-40.0\ \mu$C and $+50.0\ \mu$C are placed 12.0 cm apart in air. What is the electric field intensity at a point midway between them?

30. Points A and B are 13.0 cm apart. A charge of $+8.0\ \mu$C is placed at A and another charge of $+5.0\ \mu$C is placed at B. Point P is located 5.0 cm from A and 12.0 cm from B. What is the magnitude and direction of the electric field intensity at P?

The approach taken above for electric fields can also be applied to gravitational fields. The following steps develop an expression for the gravitational field intensity near a point source. As stated previously, the field at any point outside of a spherical mass is the same as it would be if the mass was concentrated at a point at the centre of the sphere.

- Write the equation for the general definition of gravitational field intensity.

$$\vec{g} = \frac{\vec{F}_g}{m}$$

- Write the general equation for the gravitational force between two masses. Let m_1 be the source of a gravitational field and m_2 be any mass, m, in that field.

$$|\vec{F}_g| = G\frac{m_s m}{r^2}$$

- Substitute the expression for the force of gravity into the general expression for gravitational field intensity.

$$|\vec{g}| = \frac{G\frac{m_s \cancel{m}}{r^2}}{\cancel{m}}$$

$$|\vec{g}| = G\frac{m_s}{r^2}$$

GRAVITATIONAL FIELD INTENSITY NEAR A POINT MASS

The gravitational field intensity at a point a distance r from the centre of an object is the product of the universal gravitation constant and mass, divided by the square of the distance from the centre of the object. The direction of the gravitational field intensity is toward the centre of the object creating the field.

$$|\vec{g}| = G\frac{m_s}{r^2}$$

Quantity	Symbol	SI unit
gravitational field intensity	$\vec{g}$	$\frac{\text{N}}{\text{kg}}$ (newtons per kilogram)
universal gravitation constant	G	$\frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}$ (newton · metres squared per kilogram squared)
mass of source of field	m_s	kg (kilograms)
distance from centre of source	r	m (metres)

Unit Analysis

$$\left(\frac{\text{newton}\cdot\text{metre}^2}{\text{kilogram}^2}\right)\left(\frac{\text{kilogram}}{\text{metre}^2}\right) = \left(\frac{\text{newton}}{\text{kilogram}}\right)$$

$$\frac{\text{N}\cdot\cancel{\text{m}^2}}{\text{kg}^{\cancel{2}}} \times \frac{\cancel{\text{kg}}}{\cancel{\text{m}^2}} = \frac{\text{N}}{\text{kg}}$$

SAMPLE PROBLEM

Field Intensity near Earth

Calculate the gravitational field intensity at a height of 300.0 km from Earth's surface.

Conceptualize the Problem

- Since the point in question is *outside* of the *sphere* of Earth, the gravitational field there is the same as it would be if Earth's mass was concentrated at *a point at Earth's centre*. Therefore, the equation for the *gravitational field intensity* near a *point mass* applies.

Identify the Goal

The gravitational field intensity, $\vec{g}$

Identify the Variables and Constants

Known	Implied	Unknown
$h = 300.0\ \text{km}$	$G = 6.67 \times 10^{-11}\ \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}$	$\vec{g}$
	$r_E = 6.38 \times 10^6\ \text{m}$	
	$m_E = 5.98 \times 10^{24}\ \text{kg}$	

Develop a Strategy

Convert the height above Earth's surface into SI units and calculate the distance, r, from the centre of Earth.

$$h = 300.0\ \text{km} = 3.000 \times 10^5\ \text{m}$$
$$r = 3.000 \times 10^5\ \text{m} + 6.38 \times 10^6\ \text{m}$$
$$r = 6.68 \times 10^6\ \text{m}$$

Use the equation for the gravitational field intensity near a point source.

$$|\vec{g}| = G\frac{m_s}{r^2}$$

Substitute numerical values and solve.

$$|\vec{g}| = \frac{\left(6.67 \times 10^{-11}\ \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)(5.98 \times 10^{24}\ \text{kg})}{(6.68 \times 10^6\ \text{m})^2}$$

$$|\vec{g}| = 8.9387\ \frac{\text{N}}{\text{kg}}$$

$$|\vec{g}| \cong 8.94\ \frac{\text{N}}{\text{kg}}$$

The gravitational field intensity 300.0 km from the surface of Earth is 8.94 N/kg.

Validate the Solution

You would expect the gravitational field intensity to be less than 9.81 N/kg at a great distance from Earth's surface.

continued ►

continued from previous page

PRACTICE PROBLEMS

31. What is the gravitational field intensity at a distance of 8.4×10^7 m from the centre of Earth?

32. If the gravitational field intensity at the surface of Saturn is 26.0 N/kg and its mass is 5.67×10^{26} kg, what is its radius?

33. What is the acceleration due to gravity on the surface of Venus? ($m_{Venus} = 4.83 \times 10^{24}$ kg; $r_{Venus} = 6.31 \times 10^6$ m)

34. An astronaut drops a 3.60 kg object onto the surface of a planet. It takes 2.60 s to fall 1.86 m to the ground. If the planet is known to have a radius of 8.40×10^6 m, what is its mass?

35. What is the gravitational field intensity at a distance of 2.0 m from the centre of a spherical metal ball of mass 3.0 kg? (Calculate only the field due to the ball, not to Earth.)

36. Calculate the gravitational field intensity at a height of 560.0 m above the surface of the planet Venus. (See problem 33 for data.)

37. The planet Neptune has a gravitational field intensity of 10.3 N/kg at a height of 1.00×10^6 m above its surface. If the radius of Neptune is 2.48×10^7 m, what is its mass?

Field Lines

Electric Field Lines

You have learned that an electric field at a particular point can be represented by a vector arrow with a length that corresponds to the magnitude of the field intensity at a given point. The direction of the vector arrow indicates the direction of the electric field at that point.

If you wanted to visualize the entire field around an electric charge, however, you would need to draw a set of these vector arrows at many points in the space around the charge. This process would be very tedious and complicated, so an idea originally used by Michael Faraday has been adapted. Using this method, the vectors are replaced by a series of lines that follow the path that a tiny point charge would take if it was free to move in the electric field. These lines are called **electric field lines**. In the vicinity of a positive charge, such field lines would radiate straight out, just as a positive test charge would be pushed straight out.

The field lines are constructed so that, at every point on the line, the direction of the field is tangent to the line. The strength of the field is represented by the density of the lines. The farther apart these lines are, the weaker the field is. Figure 7.9 shows the electric field lines that represent the electric field in various charge arrangements.

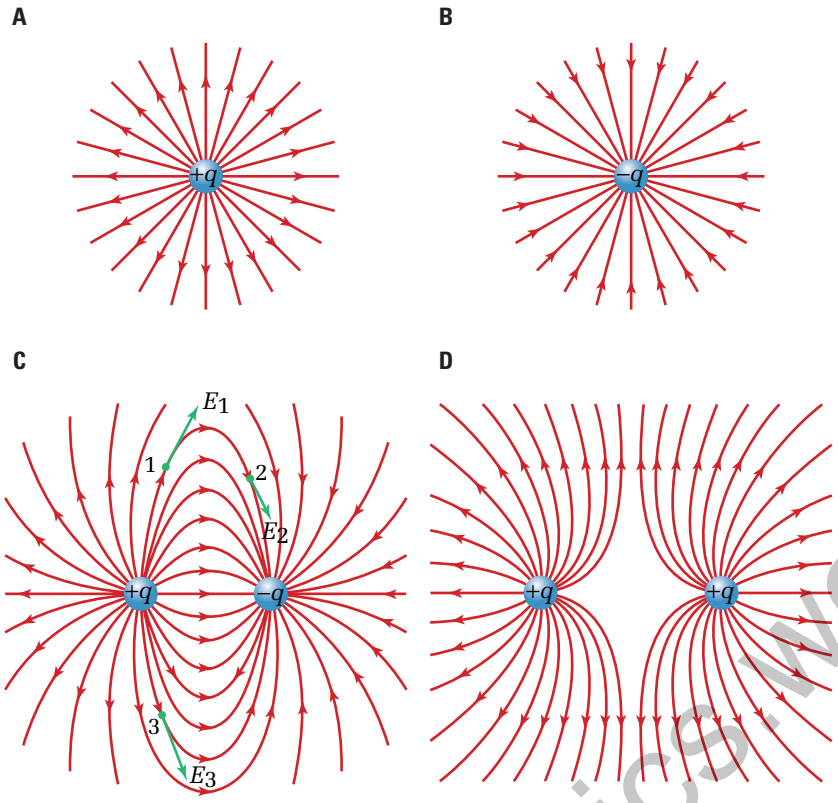


Figure 7.9 **(A)** The electric field lines from positive charge $+q$ are directed radially outward. **(B)** The electric field lines are directed radially inward toward negative point charge $-q$. **(C)** The electric field lines of an electric dipole are curved, and extend from the positive to the negative charge. At any point, such as 1, 2, or 3, the field created by the dipole is tangent to the line through the point. **(D)** The electric field lines for two identical positive point charges are shown. If both of the charges were negative, the directions of the lines would be reversed.

Note that when more than one electric source charge is present, the electric field vector at a point is the vector sum of the electric field attributable to each source charge separately. Since the field lines are often curved, this vector will be tangent to the field line at that point.

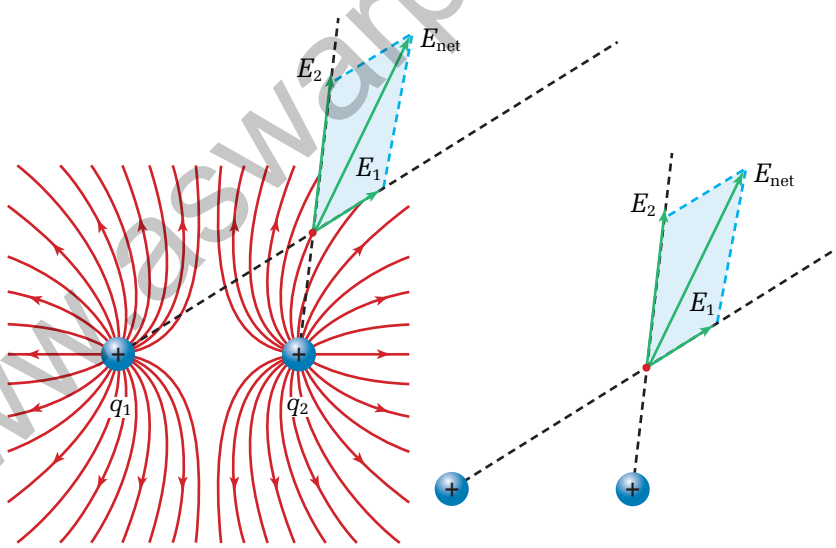


Figure 7.10 The electric field at a point near two positive charges

Gravitational Field Lines

Since the force of gravity is always attractive, the shape of **gravitational field lines** will resemble the electric field lines associated with a negative charge. Gravitational field lines will always point toward the centre of a spherical mass and arrive perpendicular to the surface.

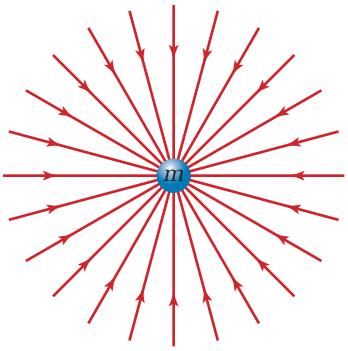


Figure 7.11 The gravitational field lines are directed radially inward toward a mass, *m*.

Conceptual Problems

- Can there be a gravitational field diagram similar to the electric field in Figure 7.9 (D)? Explain.
- Sketch the gravitational field lines due to the two identical masses shown in the diagram here.

Magnetic Field Lines

Since there are no isolated magnetic poles (magnetic monopoles), the **magnetic field lines** have to be drawn so that they are associated with both poles of the magnet (magnetic dipole). The direction of the magnetic field at a particular location is defined as the direction in which the N-pole of a compass would point when placed at that location. The magnetic field lines leave the N-pole of a magnet, enter the S-pole, and continue to form a closed loop inside the magnet. The number of magnetic field lines, called the "magnetic flux," passing through a particular unit area is directly proportional to the magnetic field intensity. Consequently, flux lines are more concentrated at the poles of a magnet, where the magnetic field is greatest.

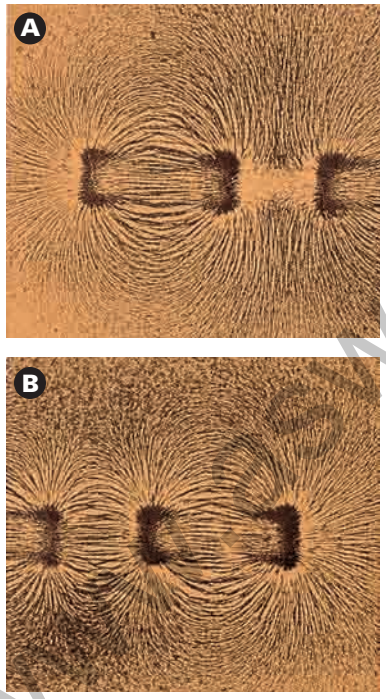


Figure 7.13 The field lines for **(A)** like poles and **(B)** unlike poles

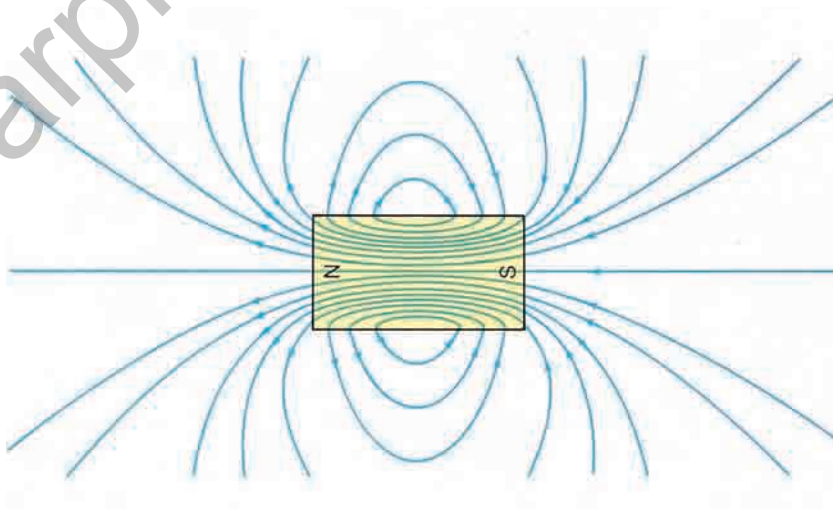


Figure 7.12 The magnetic field lines are closed loops leaving the N-pole of the magnet and entering the S-pole.

Conceptual Problems

- How does the magnetic dipole pattern compare with the electric field pattern of two opposite charges (an electric dipole)?
- What electrostatic evidence suggests that a water molecule is an electric dipole?
- What happens if you place a small bar magnet in a uniform magnetic field?
- What happens if you place a water molecule in a uniform electric field?

7.2 Section Review

1. I Place a strong bar magnet flat on a semi-rough surface, with the N-pole to the right. Place another bar magnet to the right of the first, but with its like N-pole to the left, suspended directly over the other N-pole. Adjust the top magnet until it balances. Now slide a piece of paper over the first magnet to hide it. Gently tap the suspended N-pole to start it vibrating vertically in space. What do your observations suggest about magnetic fields?
2. K/U What is the general definition for the electric field intensity at a distance r from a point charge q?
3. I Why is it not considered useful to define magnetic field intensity in the same way in which you defined the electric field intensity in question 2?
4. C Explain how you might calculate the gravitational field intensity at the various points along the path of a communication satellite orbiting Earth.
5. K/U In the vicinity of several point charges, how is the direction of the electric field intensity vector calculated?
6. C List four characteristics of electric field lines.

UNIT PROJECT PREP

A magnet held close to a refrigerator door is pulled toward the door. A ball rolls off a tabletop and is pulled toward the ground. Your hair sticks out in all directions after you remove a warm woollen cap. Each of these examples involve action at a distance. Forces are exerted without apparent contact.

- How does the use of fields help to explain action at a distance?
- Do descriptions of electric fields relate to descriptions of gravitational fields?
- Does an understanding of one type of field help with questions about another?

7.3 Fields and Potential Energy

SECTION EXPECTATIONS

- Define and describe the concepts and units related to electric and gravitational fields.
- Apply the concept of electric potential energy and compare the characteristics of electric potential energy with those of gravitational potential energy.

KEY TERMS

- electric potential difference
- equipotential surface

As a thundercloud billows, rising ice crystals collide with falling hailstones. The hail strips electrons from the rising ice and the top of the cloud becomes predominantly positive, while the bottom is mostly negative. Negative charges in the lower cloud repel negative charges on the ground, inducing a positive region, or "shadow," on Earth below. Electric fields build and a spark ignites a cloud-to-ground lightning flash through a potential difference of hundreds of millions of volts.

The lightning bolt featured in Figure 7.14 dramatically demonstrates that when a charge is placed in an electric field, it *will* move. The potential to move implies the existence of stored energy. In this chapter, you will focus on the energy stored in the gravitational and electric fields.

Figure 7.14 Tremendous amounts of electric energy are "stored" in the electric fields created by the separation of charge between thunderclouds and the ground. This energy is often released in the "explosion" of a lightning bolt.

Potential Energy

In Chapter 6, Energy and Motion in Space, you derived an equation for the gravitational potential energy of one mass due to the presence of a central mass. You started the derivation by determining the amount of work that you would have to do on the first mass to move it from a distance r_1 to a distance r_2 from a central mass. Then you learned that physicists have agreed on a reference

position that is assigned a value of zero gravitational potential energy. That distance is infinitely far from the central mass. In this application, an infinite distance means so far away that the magnitude of the force of gravity is negligible.

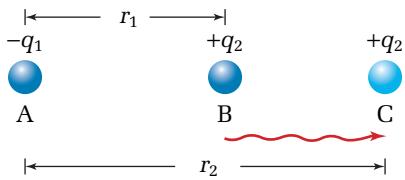


Figure 7.15 By doing work on charge q_2, you give it potential energy.

Physicists take the same approach in developing the concept of electric potential energy of a charge q_1 in the vicinity of another charge q_2 as shown in Figure 7.15. The change in electric potential energy of charge q_1 due to the presence of q_2, in moving q_1 from r_1 to r_2, is the work that you would have to do on the charge in moving it. In Figure 7.16, note the similarities in the equations for the force of gravity and the Coulomb force as well as the curves for force versus position.

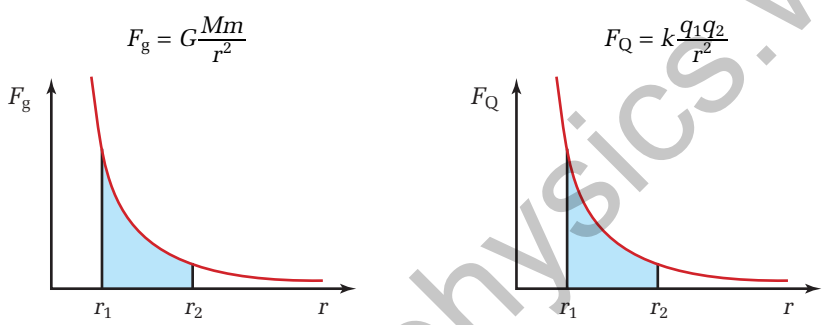


Figure 7.16 The Coulomb force and the force of gravity both follow inverse square relationships, so the curves of force versus position have exactly the same form.

Since the two equations and the two curves have identical mathematical forms, the result of the derivation of the change in the electric potential energy in moving a charge will be mathematically identical to the form of the change in the gravitational potential energy in moving a mass from position r_1 to position r_2.

$$\Delta E_g = \frac{GMm}{r_1} - \frac{GMm}{r_2} \qquad \Delta E_Q = \frac{kq_1q_2}{r_1} - \frac{kq_1q_2}{r_2}$$

The choice of a reference position for electric potential energy is the same as that for gravitational potential energy — an infinite distance — so far apart that the force between the two charges is negligible. Therefore, the equations for potential energy have the same mathematical form, with one small difference: There is no negative sign in the equation for the electric potential energy.

$$E_g = -\frac{GMm}{r} \qquad E_Q = \frac{kq_1q_2}{r}$$

The negative sign is absent from the equation for electric potential energy, because the energy might be negative or positive, depending on the sign of the charges. If the charges have opposite signs, the Coulomb force between them is attractive. Consequently, if one charge moves from infinity to a distance r from the second charge, it does work and therefore has less potential energy. Less than zero is negative. If the charges have the same sign, you must do work on one charge to move it from infinity to a distance r from the second charge, and therefore it has positive potential energy. If you include the sign of the charges when using the equation for electric potential energy, the final sign will tell you whether the potential is positive or negative.

- Two positive charges $E_Q = \frac{k(q_1)(q_2)}{r}$ $E_Q > 0$ Both q_1 and q_2 are positive, so the charges have positive potential energy when they are a distance r apart.
- Two negative charges $E_Q = \frac{k(q_1)(q_2)}{r}$ $E_Q > 0$ Both q_1 and q_2 are negative, so the charges have positive potential energy when they are a distance r apart.
- A positive and a negative charge. $E_Q = \frac{k(q_1)(q_2)}{r}$ $E_Q < 0$ The product q_1q_2 is negative, so the charges have negative potential energy when they are a distance r apart.

A second difference between electric potential energy and gravitational potential energy is that the two interacting charges might be similar in magnitude. Therefore, either charge could be considered the stationary or central charge, or the "movable" charge. You could therefore consider the two charges to be a system, and refer to the electric potential energy of the system that results from the proximity of the two charges.

SAMPLE PROBLEM

Electric Potential Energy

What is the electric potential energy stored between charges of +8.0 μC and +5.0 μC that are separated by 20.0 cm?

Conceptualize the Problem

- Two *charges* are close together and therefore they exert a *force* on each other.

- *Work* must be done on or to the charges in order to bring them close to each other.
- Since work was done on or by a charge, it has *electric potential energy*.

Identify the Goal

The electric potential energy, E_Q, stored between the charges

Identify the Variables and Constants

Known	Implied	Unknown
$q_1 = 8.0 \times 10^{-6}$ C	$k = 9.0 \times 10^9 \ \frac{\text{N} \cdot \text{m}^2}{\text{C}^2}$	E_Q
$q_2 = 5.0 \times 10^{-6}$ C		
$r = 0.200$ m		

Develop a Strategy

Write the equation for electric potential energy between two charges.

$$E_Q = k\frac{q_1 q_2}{r}$$

Substitute numerical values and solve.

$$E_Q = \frac{\left(9.0 \times 10^9 \ \frac{\text{N} \cdot \text{m}^2}{\text{C}^2}\right)(+8.0 \times 10^{-6}\ \text{C})(+5.0 \times 10^{-6}\ \text{C})}{0.200\ \text{m}}$$

$$E_Q = +1.8\ \text{J}$$

The electric potential energy stored in the field between the charges is +1.8 J.

Validate the Solution

Magnitudes seem to be consistent. The units cancel to give J: $\frac{\text{N} \cdot \text{m}^{\cancel{2}}}{\cancel{\text{C}^2}} \cdot \frac{\cancel{\text{C}} \cdot \cancel{\text{C}}}{\cancel{\text{m}}} = \text{N} \cdot \text{m} = \text{J}$. The sign is positive, indicating that the electric potential energy is positive. A positive sign is correct for like charges, because work was done *on* the charges to put them close each other.

PRACTICE PROBLEMS

38. Find the electric potential energy stored between charges of +2.6 μC and −3.2 μC placed 1.60 m apart.

39. Two identical charges of +2.0 μC are placed 10.0 cm apart in a vacuum. If they are released, what will be the final kinetic energy of each charged object (assuming that no other objects or fields interfere)?

40. How far apart must two charges of $+4.2 \times 10^{-4}$ C and -2.7×10^{-4} C be placed in order to have an electric potential energy with a magnitude of 2.0 J?

41. Two charges of equal magnitude, separated by a distance of 82.2 cm, have an electric potential energy of 2.64×10^2 J. What are the signs and magnitudes of the two charges?

CAREERS IN PHYSICS

Seeing Inside Storms

Blizzards can cause traffic accidents. Hurricanes can cause flooding. Tornadoes can destroy houses. Often, advance warning of these and other severe storms helps prevent deaths and reduce damage. For example, radio announcements can warn motorists to stay off roads, and municipal authorities can prepare to deal with possible flooding.

Giving advance warning is part of Dr. Paul Joe's work. Dr. Joe, a radar scientist and cloud physicist, is based at Environment Canada's radar site in King City, north of Toronto. Radar — short for *ra*dio *d*etection *a*nd *r*anging — involves transmitting pulses of electromagnetic waves from an antenna. When objects such as snowflakes or raindrops interrupt these pulses, part of their electromagnetic energy is reflected back. A receiver picks up the reflections, converting them into a visible form and indicating a storm's location and intensity.

Dr. Paul Joe, radar scientist and cloud physicist

Conventional radar cannot detect a storm's internal motions, however. This is why, in recent years, Environment Canada has been improving its radar sites across the country by adding Doppler capability. This improved radar technology applies the Doppler effect: If an object is moving toward the radar, the frequency of its reflected energy is increased from the frequency of the energy that the radar is transmitting. If an object is moving away from the radar, the frequency of its reflected energy is decreased.

"This is the same effect we notice with a subway train," Dr. Joe explains. "As it approaches, we hear a higher-pitched sound than when it leaves."

On Dr. Joe's radar screen, the frequency shifts are visualized using colours. In general, blue means an object is approaching; red means it is receding. But it's not that simple. Doppler images are complex and difficult for conventional weather forecasters to interpret, and Dr. Joe is working on ways to make them simpler. He also specializes in nowcasting — forecasting weather for the near future; for example, within an hour. As part of the 2000 Olympics, he went to Sydney, Australia, to join other scientists in demonstrating nowcasting technologies.

"I have it great," says Dr. Joe. "I love using what I've learned in mathematics, physics, and meteorology to decipher what Mother Nature is telling us and warning people about what she might do. Using the radar network, I can be everywhere chasing storms and seeing inside them in cyberspace."

Going Further

Dr. Joe's field, known in general as meteorology, includes radar science, cloud physics, climatology, and hydrometeorology. Research one of these fields and prepare a two-page report for presentation to the class.

WEB LINK

www.mcgrawhill.ca/links/physics12

The Canadian Hurricane Centre site maintained on the Internet by Environment Canada has a wide variety of information about hurricanes. Just go to the above Internet site and click on **Web Links**.

Electric Potential Difference

In previous physics courses, you learned that **electric potential difference** is the difference in the electric potential energy of a unit charge between two points in a circuit. You can broaden this definition to include any type of electric field, not just a field that is confined to an electric conductor. This concept allows you to describe the condition of a point in an electric field, relative to a reference point, without placing a charge at that point.

You have just derived an equation for the electric potential energy of a point charge, relative to infinity, a distance r from another point charge that can be considered as having created the field. For this case, you can find the electric potential difference between that point and infinity by considering the charge q_1 as the charge creating an electric field and q_2 as a unit charge.

- The definition of electric potential difference between a point and the reference point is

$$V = \frac{E_Q}{q_2}$$

- Substitute the expression for the difference in electric potential energy of charge q_2 between the reference at infinity and the distance r from the charge q_1 due to the presence of q_1.

$$V = \frac{\frac{kq_1q_2}{r}}{q_2}$$

- Since only one q, the charge creating the field, remains in the expression, there is no need for a subscript.

$$V = \frac{kq}{r}$$

ELECTRIC POTENTIAL DIFFERENCE DUE TO A POINT CHARGE

The electric potential difference, a scalar, between any point in the field surrounding a point charge and the reference point at infinity charge is the product of Coulomb's constant and the electric charge divided by the distance from the centre of the charge to the point.

$$V = k\frac{q}{r}$$

Quantity	Symbol	SI unit
electric potential difference	V	V (volts)
Coulomb's constant	k	$\frac{N \cdot m^2}{C^2}$ (newton metres squared per coulombs squared)
electric charge	q	C (coulombs)
distance	r	m (metres)

Unit Analysis

$$\frac{N \cdot m^{\cancel{2}} \cdot \cancel{C}}{C^{\cancel{2}} \cdot \cancel{m}} = \frac{N \cdot m}{C} = \frac{J}{C} = V$$

PHYSICS FILE

Physicists often use the phrase, potential at a point, when they are referring to the potential difference between that point and the reference point an infinite distance away. It is not incorrect to use the phrase as long as you understand its meaning.

Problems involving electric potential difference can be extended, as can those involving electric field, to situations in which several source charges create an electric field. Since electric potential is a scalar quantity, the electric potential difference created by each individual charge is first calculated, being careful to use the correct sign, and then these scalar quantities are added algebraically.

You can go one step further and describe the electric potential difference between two points, P_1 and P_2, within a field. To avoid confusion, this quantity is symbolized ΔV and the relationship is written as follows.

$$\Delta V = V_2 - V_1$$

Always keep in mind that V_1 and V_2 represent the electric potential difference between point 1 and infinity, and point 2 and infinity — a location so far away that the field is negligible. The following sample problems will help you to clarify these concepts in your mind.

SAMPLE PROBLEMS

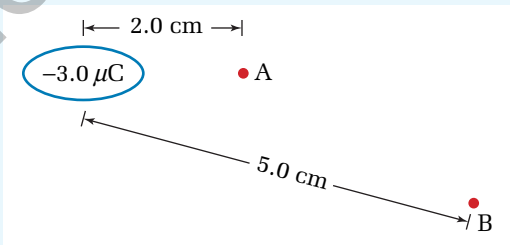
Calculations Involving Electric Potential Difference

1. A small sphere with a charge of –3.0 μC creates an electric field.

(a) Calculate the electric potential difference at point A, located 2.0 cm from the source charge, and at point B, located 5.0 cm from the same source charge.

(b) What is the potential difference between A and B?

(c) Which point is at the higher potential?



Conceptualize the Problem

- A *charged sphere* creates an *electric field*.
- At *any point* in the field, you can describe an *electric potential difference* between *that point* and a location an *infinite distance* away.
- Electric *potential difference* is a *scalar* quantity and depends only on the distance from the source charge and not the direction.
- The *potential difference* between *two points* is the *algebraic* difference between the individual potential differences of the points.

Identify the Goal

The electric potential difference, V, at each point
The electric potential difference, ΔV, between the two points
The point at a higher potential

Identify the Variables and Constants

Known	Implied	Unknown
$q = -3.0 \times 10^{-6}$ C	$k = 9.0 \times 10^{9} \dfrac{\text{N} \cdot \text{m}^2}{\text{C}^2}$	V_A
$d_A = 2.0 \times 10^{-2}$ m		V_B
$d_B = 5.0 \times 10^{-2}$ m		

Develop a Strategy

Use the equation for the electric potential difference at a point a distance r from a point charge.

Substitute numerical values and solve.

$$V_A = k\frac{q}{d_A}$$

$$V_A = \left(9.0 \times 10^{9} \frac{\text{N} \cdot \text{m}}{\text{C}^2}\right)\left(\frac{-3.0 \times 10^{-6}\ \text{C}}{2.0 \times 10^{-2}\ \text{m}}\right)$$

$$V_A = -1.35 \times 10^{6}\ \text{V}$$

$$V_A \cong -1.4 \times 10^{6}\ \text{V}$$

$$V_B = k\frac{q}{d_B}$$

$$V_B = \left(9.0 \times 10^{9} \frac{\text{N} \cdot \text{m}}{\text{C}^2}\right)\left(\frac{-3.0 \times 10^{-6}\ \text{C}}{5.0 \times 10^{-2}\ \text{m}}\right)$$

$$V_B = -5.4 \times 10^{5}\ \text{V}$$

(a) The electric potential difference is -1.4×10^{6} V at point A, and -5.4×10^{5} V at point B.

Use algebraic subtraction to determine the potential difference between the two points.

$$\Delta V = V_B - V_A$$

$$\Delta V = (-5.4 \times 10^{5}\ \text{V}) - (-1.35 \times 10^{6}\ \text{V})$$

$$\Delta V = 8.1 \times 10^{5}\ \text{V}$$

(b) The electric potential difference, ΔV, between points A and B is 8.1×10^{5} V.

Analyze the algebraic result and validate by considering the path of a positive test charge.

Algebraically, since $(V_B - V_A) > 0$, V_B is at the higher potential.

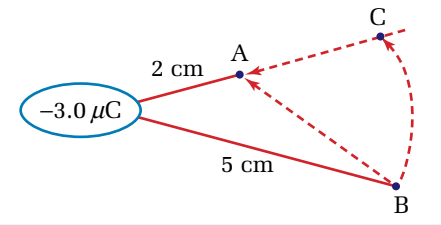
A positive test charge placed at point A would have to be dragged against the electric forces to get it to point B, which again places point B at the higher potential.

(c) Point B is at the higher potential.

Validate the Solution

The more distant point has a smaller magnitude potential, but its negative sign makes it a higher value. The analysis with a positive test charge validates the statement of higher potential.

Note: The answers were obtained in this sample problem by taking into account whether the two points were on the same radial line. The diagram shows two possible paths a test charge could take in moving from B to A. If the test charge followed the path BCA, no work would be done on it from B to C, because the force would be perpendicular to the path.



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The only segment of the path where work is done, and therefore the electric potential energy changed, is from C to A, parallel to the direction of the force acting.

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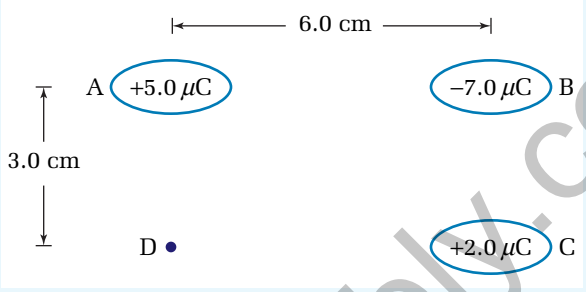
2. The diagram shows three charges, A (+5.0 μC), B (−7.0 μC), and C (+2.0 μC), placed at three corners of a rectangle. Point D is the fourth corner. What is the electric potential difference at point D?



Conceptualize the Problem

- There is an *electric potential difference* at point D, due to each of the separate charges.
- The separate potential values can be calculated and then *added algebraically*.

Identify the Goal

The electric potential difference, V, at point D

Identify the Variables and Constants

Known	Implied	Unknown
$q_A = 5.0\ \mu C$	$k = 9.0 \times 10^9\ \frac{N \cdot m^2}{C^2}$	$V_{at\,D}$
$q_B = -7.0\ \mu C$	$d_{CD} = 6.0$ cm	
$q_C = 2.0\ \mu C$		
$d_{AB} = 6.0$ cm		
$d_{AD} = 3.0$ cm		

Develop a Strategy

Calculate d_{BD}, using the Pythagorean theorem. Choose the positive value as a measure of the real distance.

$$d_{BD}^2 = (6.0\ \text{cm})^2 + (3.0\ \text{cm})^2$$
$$d_{BD}^2 = 45\ \text{cm}^2$$
$$d_{BD} = \pm\sqrt{45\ \text{cm}^2}$$
$$d_{BD} = \pm 6.7\ \text{cm}$$

Calculate the contribution of each charge to the potential difference at point D independently.

$$V_{A\,at\,D} = \left(9.0 \times 10^9\ \frac{N \cdot m^2}{C^2}\right)\left(\frac{+5.0 \times 10^{-6}\ C}{0.030\ m}\right) = 1.5 \times 10^6\ V$$
$$V_{B\,at\,D} = \left(9.0 \times 10^9\ \frac{N \cdot m^2}{C^2}\right)\left(\frac{-7.0 \times 10^{-6}\ C}{0.067\ m}\right) = -9.4 \times 10^5\ V$$
$$V_{C\,at\,D} = \left(9.0 \times 10^9\ \frac{N \cdot m^2}{C^2}\right)\left(\frac{+2.0 \times 10^{-6}\ C}{0.060\ m}\right) = 3.0 \times 10^5\ V$$

Calculate the net potential difference at point D by adding the separate potential differences algebraically.

$$V_{at\,D} = (1.5 \times 10^6\ V) + (-9.4 \times 10^5\ V) + (3.0 \times 10^5\ V)$$
$$V_{at\,D} = 8.6 \times 10^5\ V$$

The electric potential difference at point D is 8.6×10^5 V.

Validate the Solution

The electric potential difference contributed by A is expected to be stronger, due to its closer proximity and average charge.

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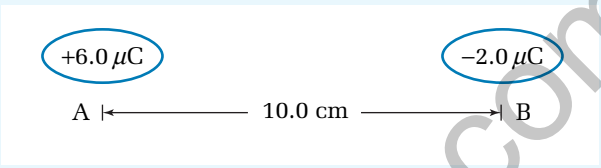
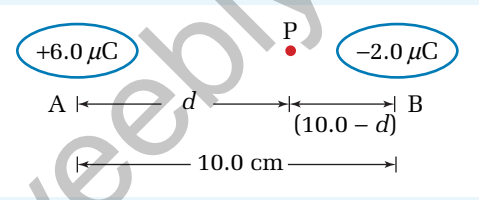
3. A charge of +6.0 μC at point A is separated 10.0 cm from a charge of −2.0 μC at point B. At what locations on the line that passes through the two charges will the total electric potential be zero?



Conceptualize the Problem



- The *total electric potential* due to the combination of charges is the *algebraic sum* of the electric potential due to *each point* alone.
- Draw a diagram and assess the likely position.
- Let the points be designated a distance d to the right of point A, and set the absolute magnitudes of the potential equal to each other. This allows for two algebraic scenarios.

Identify the Goal

The location of the point of zero total electric potential

Identify the Variables and Constants

Known	Implied	Unknown
$q_A = +6.00 \times 10^{-6}$ C	$k = 9.0 \times 10^9 \dfrac{\text{N}\cdot\text{m}^2}{\text{C}^2}$	d at zero total electric potential
$q_B = -2.00 \times 10^{-6}$ C		
$d_{AB} = 10.0 \times 10^{-2}$ m		

Develop a Strategy

For the potentials to cancel algebraically, the point cannot be to the left of point A, which would be closer to the larger positive charge and could not be balanced by the potential of the negative charge. That leaves two locations: one between points A and B, and one to the right of point B, where the smaller distance to the negative charge balances the smaller value of that charge.

$$|V_{\text{due to A}}| = |V_{\text{due to B}}|$$

Scenario 1

$$k\frac{q_A}{d} = k\frac{q_B}{(0.10 - d)}$$

$$q_A(0.10 - d) = q_B(d)$$

$$0.10q_A - q_A d = q_B d$$

$$0.10q_A = d(q_A + q_B)$$

$$d = \frac{0.10q_A}{q_A + q_B}$$

$$d = \frac{(0.10\text{ m})(6.0\ \mu\text{C})}{6.0\ \mu\text{C} + (-2.0\ \mu\text{C})}$$

$$d = \frac{0.60\text{ m}\cdot\cancel{\mu\text{C}}}{4.0\ \cancel{\mu\text{C}}}$$

$$d = 0.15\text{ m}$$

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Scenario 2

$$k\frac{q_A}{d} = -k\frac{q_B}{(0.10 - d)}$$

$$q_A(0.10 - d) = -q_B(d)$$

$$0.10q_A - q_A d = -q_B d$$

$$0.10q_A = d(q_A - q_B)$$

$$d = \frac{0.10q_A}{q_A - q_B}$$

$$d = \frac{(0.10\ \text{m})(6.0\ \mu\text{C})}{6.0\ \mu\text{C} - (-2.0\ \mu\text{C})}$$

$$d = \frac{0.60\ \text{m} \cdot \mu\text{C}}{8.0\ \mu\text{C}}$$

$$d = 0.075\ \text{m}$$

The points of zero potential are 7.5 cm to the right of point A and 5.0 cm to the right of point B. (Note: 15 cm to the right of A is the same as 5 cm to the right of B.)

Validate the Solution

The electric potentials due to point A at the two points are

$$(9.0 \times 10^9)\left(\frac{+6.0 \times 10^{-6}}{0.075}\right) = +7.2 \times 10^5\ \text{V and } (9.0 \times 10^9)\left(\frac{+6.0 \times 10^{-6}}{0.15}\right) = +3.6 \times 10^5\ \text{V}$$

The electric potentials due to point B at the two points are

$$(9.0 \times 10^9)\left(\frac{-2.0 \times 10^{-6}}{0.025}\right) = -7.2 \times 10^5\ \text{V and } (9.0 \times 10^9)\left(\frac{-2.0 \times 10^{-6}}{0.050}\right) = -3.6 \times 10^5\ \text{V}$$

In both locations, the potentials due to points A and B add algebraically to zero.

PRACTICE PROBLEMS

42. Find the electric field due to a point charge of 4.2×10^{-7} C at a point 2.8 cm from the charge.

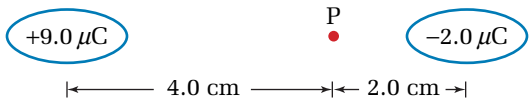
43. How far from a positive point source of 8.2 C will the electric potential difference be 5.0 V? (**Note:** 8.2 C is a very large charge!)

44. The electric potential difference due to a point charge is 4.8 V at a distance of 4.2 cm from the charge. What will be the electric potential energy of the system if a second charge of +6.0 μC is placed at that location?

45. The electric potential difference at a distance of 15 mm from a point charge is –2.8 V. What is the magnitude and sign of the charge?

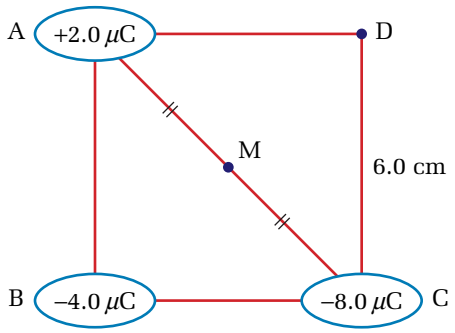
46. Point charges of +8.0 μC and –5.0 μC, respectively, are placed 10.0 cm apart in a vacuum. At what location along the line through them will the electric potential difference be zero?

47. What is the potential difference at point P situated between the charges +9.0 μC and –2.0 μC, as shown in the diagram.

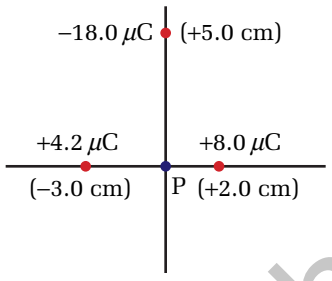


48. Point X has an electric potential difference of +4.8 V and point Y has a potential difference of –3.2 V. What is the electric potential difference, ΔV, between them?

49. Charges of +2.0 μC, –4.0 μC, and –8.0 μC are placed at three vertices of a square, as shown in the diagram. Calculate the electric potential difference at M, the midpoint of the diagonal AC.



50. The diagram shows three small charges located on the axes of a Cartesian coordinate system. Calculate the potential difference difference at point P.



51. Two charges are placed at the corners of a square. One charge, +4.0 μC, is fixed to one corner and another, –6.0 μC, is fixed to the opposite corner. What charge would need to be placed at the intersection of the diagonals of the square in order to make the potential difference zero at each of the two unoccupied corners?

52. Point A has an electric potential difference of +6.0 V. When a charge of 2.0 C is moved from point B to point A, 8.0 J of work are done on the charge. What was the electric potential difference of point B?

53. The potential difference between points X and Y is 12.0 V. If a charge of 1.0 C is released from the point of higher potential and allowed to move freely to the point of lower potential, how many joules of kinetic energy will it have?

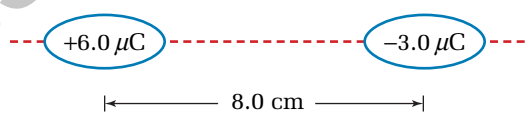
54. Identical charges of +2.0 μC are placed at the four vertices of a square of sides 10.0 cm. What is the potential difference between the point at the intersection of the diagonals and the midpoint of one of the sides of the square?

55. **(a)** If 6.2×10^{-4} J of work are required to move a charge of 3.2 nC (one nanocoulomb = 10^{-9} coulombs) from point B to point A in an electric field, what is the potential difference between A and B?

(b) How much work would have been required to move a 6.4 nC charge instead?

(c) Which point is at the higher electric potential? Explain.

56. Two different charges are placed 8.0 cm apart, as shown in the diagram. Calculate the location of the two positions along a line joining the two charges, where the electric potential is zero.

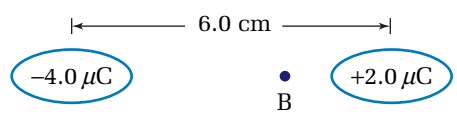


57. A charge of +8.2 nC is 10.0 cm to the left of a charge of –8.2 nC. Calculate the locations of three points, all of which are at zero electric potential.

58. A charge of –6.0 μC is located at the origin of a set of Cartesian coordinates. A charge of +8.0 μC. is 8.0 cm above it. What are the coordinates of the points at which the potential is zero?

59. A charge of +4.0 μC is 8.0 cm to the left of a point that has zero potential. Calculate three possible values for the magnitude and location of a second charge causing the potential to be zero.

60. Calculate the location of point B in the diagram below so that its electric potential is zero.



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Conceptual Problem

- In practice problem 56, do you think there could be locations (other than along a line joining the two charges) where the electric potential difference could be the same, but not zero? Explain.

Equipotential Surfaces

The quantities of gravitational potential energy, electric potential energy, and electric potential difference are all scalar quantities. Although it is rarely used, there is also a quantity called "gravitational potential difference," which is defined as gravitational potential energy per unit mass. It is expressed mathematically as $V_g = \frac{E_g}{m} = -\frac{GM}{r}$. Since these are scalar quantities, the direction from the charge or mass that is creating the field does not affect the values. If you connected all of the points that are equidistant from a point mass or an isolated point charge, they would have the same potential difference and they would be creating a spherical surface. Such a surface, illustrated in Figure 7.17, is called an **equipotential surface**.

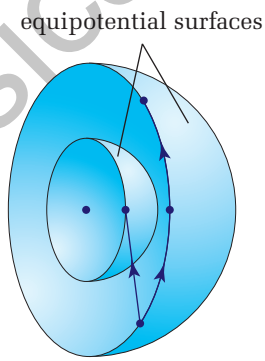


Figure 7.17 The spherical shells could represent equipotential surfaces either for a gravitational field around a point mass (or spherical mass) or for an electric field around an isolated point charge. In cross section, the equipotential spherical surfaces appear as concentric circles.

GEOGRAPHY LINK

The equipotential lines around a system of charges could be compared to the contour lines on topographical maps. Since these contour lines represent identical heights above sea level, they also represent points that have the same gravitational energy per unit mass, and so are equipotential lines.

$$E_g = mgh$$

$$\frac{E_g}{m} = gh$$

$$V_g \propto h$$

You will recall that the work done per unit charge in moving that charge from a potential V_1 to a potential V_2 is $\frac{W}{q} = V_2 - V_1$. Since, on an equipotential surface, $V_1 = V_2$, the work done must be zero. In other words, no work is required to move a charge or mass around on an equipotential surface, and the electric or gravitational force does no work on the charge or mass. Consequently, a field line must have no component along the equipotential surface. An equipotential surface must be perpendicular to the direction of the field lines at all points. Figure 7.18 shows the electric field lines and equipotential surfaces for pairs of point charges.

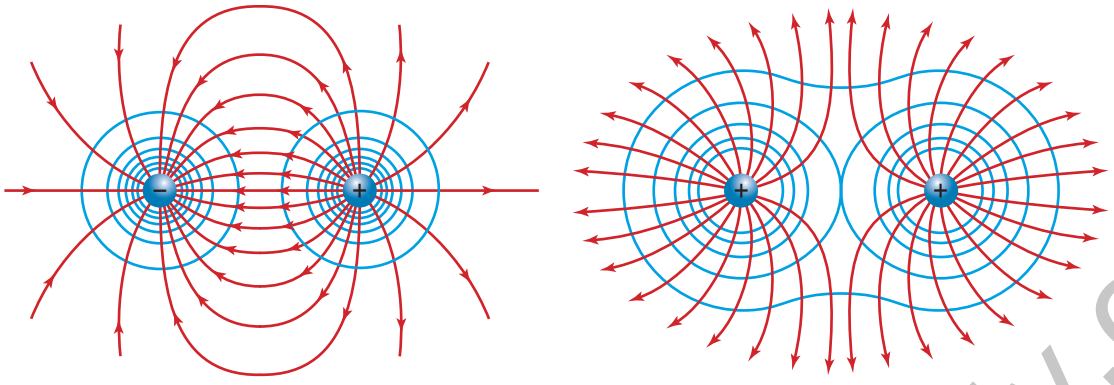


Figure 7.18 The field lines for these electric dipoles are shown in red and the cross section of the equipotential surfaces are in blue. Notice that field lines are always perpendicular to equipotential surfaces.

Conceptual Problem

- Could the barometric lines on a weather map be considered to be equipotential lines?

7.3 Section Review

1. K/U What are the differences in the data required to calculate the gravitational potential energy of a system and the electric potential energy?
2. K/U How does the amount of work done relate to the electric potential difference between two points in an electric field?
3. MC Research and briefly report on the use of electric potential differences in medical diagnostic techniques such as electrocardiograms.
4. MC Research and report on the role played by electric potential differences in the transmission of signals in the human nervous system.
5. C Can an equipotential surface in the vicinity of two like charges have a potential of zero? Explain the reason for your answer.
6. I Investigate Internet sites that use computer programs to draw the electric field lines near a variety of charge systems. Prepare a portfolio of various patterns.
7. K/U How could you draw in the equipotential surfaces associated with the patterns obtained in question 6?
8. I
   (a) Why do you think atomic physicists tend to speak of the electrons in atoms as having "binding energy"?
   (b) Investigate the use of the term "potential well" to describe the energy state of atoms.

CHAPTER 7 Review

REFLECTING ON CHAPTER 7

- The gravitational force and the Coulomb force both follow inverse square laws.
$$F_g = G\frac{m_1 m_2}{r^2} \qquad F_Q = k\frac{q_1 q_2}{r^2}$$
- The equations for gravitational force and Coulomb force were developed for point masses and point charges. However, if the masses or charges are perfect spheres, the laws apply at any point outside of the spheres.
- Since magnetic monopoles do not exist or have never been detected, magnetic forces cannot be described in the same form as gravitational and electrostatic forces. However, they appear to follow an inverse square relationship.
- Because charges, masses, and magnets do not have to be in contact to exert forces on each other, early physicists classified their interactions as action-at-a-distance forces.
- Michael Faraday developed the concept of a field in which masses, charges, and magnets influence the space around themselves in the form of a field. When a second mass, charge, or magnet is placed in the field created by the first, the field exerts a force on the object.
- The strength of an electric field on a point P is described as the electricity field intensity and is mathematically expressed as "force per unit charge," $\vec{E} = \frac{\vec{F}_Q}{q_t}$.
- The direction of an electric field at any point is the direction that a *positive* charge would move if it was placed at that point.
- The strength of a gravitational field at any point P is called the gravitational field intensity and is mathematically expressed as "force per unit mass," $\vec{g} = \frac{\vec{F}_g}{m_t}$.
- The direction of a gravitational field is always toward the mass creating the field.
- For the special case of a point charge q creating the field, the electric field intensity at a point P, a distance r from the charge, is given by $|\vec{E}| = k\frac{q}{r^2}$.
- For the special case of a point mass m creating the field, the gravitational field intensity at a point P, a distance r from the mass, is given by $\vec{g} = G\frac{m}{r^2}$.
- To find the electric field intensity in the vicinity of several point charges, find the field intensity due to each charge alone and then add them vectorially.
- Field lines are used to describe a field over a large area or volume. Field lines are drawn so that the intensity of the field is proportional to the density of the lines. The direction of a field at any point is the tangent to the field line at that point.
- A charge placed in an electric field or a mass in a gravitational field has potential energy.
- Potential energy of any type is not absolute, but relative to an arbitrary reference position or condition. The reference position for gravitational or electric potential energy in a field created by a point source is often chosen to be at an infinite distance from the point source.
- The electric potential energy of two point charges a distance r apart is given by
$$E_Q = k\frac{q_1 q_2}{r}$$
- The gravitational potential energy of two masses a distance r apart is given by
$$E_g = -G\frac{m_1 m_2}{r}$$
- Electric potential difference is defined as the potential energy per unit charge and is expressed mathematically as $V = \frac{E_Q}{q}$.
- For the special case of the electric potential difference in an electric field created by a point charge q, the electric potential difference is $V = k\frac{q}{r}$. This is the potential difference between a point the distance r from the charge and an infinite distance from the charge.

- Since work must be done on a charge to give it potential energy, the change in the potential between two points is the amount of work done on a unit charge that was moved between those two points, or $\Delta V = \frac{W}{q}$.
- In any field, there will be many points that have the same potential. When all of the points that are at the same potential are connected, an equipotential surface is formed.
- No work is done when a charge or mass moves over an equipotential surface.

Knowledge/Understanding

1. In your own words, define
   (a) electric charge
   (b) Coulomb's law
   (c) field
2. The field of an unknown charge is first mapped with a 1.0×10^{-8} C test charge, then repeated with a 2.0×10^{-8} C test charge.
   (a) Would the same forces be measured with the two test charges? Explain your answer.
   (b) Would the same fields be determined using the two test charges? Explain your answer.
3. Both positive and negative charges produce electric fields. Which direction, toward or away from itself, does the field point for each charge?
4. What is the difference between electric field intensity, electric potential difference, and electric potential energy?
5. What determines the magnitude and direction of an electric field at a particular point away from a source charge?
6. Is electric field strength a scalar quantity or a vector quantity? Is electric potential difference a scalar or vector quantity?
7. If the gravitational potential energy for an object at height h above the ground is given by $mg\Delta h$, what is the gravitational potential difference (similar in nature to the electric potential difference) between the two levels? What are the units of gravitational potential difference?
8. Units of electric field strength can be given in N/C or volts per metre, V/m. Show that these units are equivalent.

Inquiry

9. Consider a charge of +2.0 μC placed at the origin of an x–y-coordinate system and a charge of –4.0 μC placed 40.0 cm to the right. Where must a third charge be placed — between the charges, to the left of the origin, or beyond the second charge — to experience a net force of zero? Argue your case qualitatively without working out a solution. Consider both positive and negative charges.
10. (a) In a room, gravity exerts a downward pull on a ball held by a string. Sketch the gravitational field in the room.
    (b) Suppose a room has a floor that is uniformly charged and positive and a ceiling that carries an equal amount of negative charge. Neglecting gravity, how will a small, positively charged sphere held by a string behave? Sketch the electric field in the room.
    (c) Comment on any similarities and differences between the above situations.

Communication

11. (a) Sketch the electric field lines for a positive charge and a negative charge that are very far apart.
    (b) Show how the field lines change if the two charges are then brought close together.
12. Sketch the field lines for two point charges, 2Q and –Q, that are close together.
13. Explain why electric field lines never cross.
14. What is the gravitational field intensity at the centre of Earth?

Making Connections

15. Develop a feeling for the unit of the coulomb by examining some everyday situations. How much charge do you discharge by touching a doorknob after walking on a wool rug? How much charge does a comb accumulate when combing dry hair? How much charge does a lightning bolt discharge? What is the smallest charge that can be measured in the laboratory? The largest charge?

16. Make a list of the magnitudes of some electric fields found in everyday life, such as in household electric wiring, in radio waves, in the atmosphere, in sunlight, in a lightning bolt, and so on. Where can you find the weakest and greatest electric fields?

17. In November 2001, NASA launched the Gravity Recovery and Climate Experiment, or GRACE, involving a pair of satellites designed to monitor tiny variations in Earth's gravitational field. The two satellites follow the same orbit, one 220 km ahead of the other. As both satellites are in free fall, regions of slightly stronger gravity will affect the lead satellite first. By accurately measuring the changes in the distance between the satellites with microwaves, GRACE will be able to detect minute fluctuations in the gravitational field. Research the goals and preliminary findings of GRACE. In particular, examine how both ocean studies and meteorological studies will benefit from GRACE.

Problems for Understanding

18. What is the force of repulsion between two equal charges, each of 1 C, that are separated by a distance of 1 km?

19. Calculate the force between two free electrons separated by 0.10 nm.

20. The force of attraction between two charged Ping-Pong™ balls is 2.8×10^{-4} N. If the charges are +8.0 nC and −12.0 nC, how far apart are their centres?

21. Three point charges, A (+2.0 μC), B (+4.0 μC), and C (−6.0 μC), sit consecutively in a line. If A and B are separated by 1.0 m and B and C are separated by 1.0 m, what is the net force on each charge?

22. Three charges sit on the vertices of an equilateral triangle, the sides of which are 30.0 cm long. If the charges are A = +4.0 μC, B = +5.0 μC and C = +6.0 μC (clockwise from the top vertex), find the force on each charge.

23. In the Bohr model of the hydrogen atom, an electron orbits a proton at a radius of approximately 5.3×10^{-11} m. Compare the gravitational and the electrostatic forces between the proton and the electron.

24. Suppose the attractive force between Earth and the Moon, keeping the Moon in its orbit, was not gravitational but was, in fact, a Coulombic attraction. Predict the magnitude of the possible charges on Earth and the Moon that would cause an identical force of attraction.

25. What is the ratio of the electric force to the gravitational force between two electrons?

26. Calculate the charge (sign and magnitude) on a 0.30 g pith ball if it is supported in space by a downward field of 5.2×10^{-5} N/C.

27. A 3.0 g Ping Pong™ ball is suspended from a thread 35 cm long. When a comb is brought to the same height, the Ping Pong™ ball is repelled and the thread makes an angle of 10.0° with the vertical. What is the electric force exerted on the Ping Pong™ ball?

28. The gravitational field intensity at a height of 150 km (1.50×10^2 km) above the surface of Uranus is 8.71 N/kg. The radius of Uranus is 2.56×10^7 m.

(a) Calculate the mass of Uranus.

(b) Calculate the gravitational field intensity at the surface of Uranus.

(c) How much would a 100 kg (1.00×10^2 kg) person weigh on the surface of Uranus?

29. If a planet, P, has twice the mass of Earth and three times the radius of Earth, how would the gravitational field intensity at its surface compare to that of Earth?

30. The Bohr model of the hydrogen atom consists of an electron ($q_e = -e$) travelling in a circular orbit of radius 5.29×10^{-11} m around a proton ($q_p = +e$). The attraction between the two gives the electron the centripetal force required to stay in orbit. Calculate the
   (a) force between the two particles
   (b) speed of the electron
   (c) electric field the electron experiences
   (d) electric potential difference the electron experiences

31. What mass should an electron have if the gravitational and electric forces between two electrons were equal in magnitude? How many times greater than the accepted value of the electron mass is this?

32. A charge, $q_1 = +4$ nC, experiences a force of 3×10^{-5} N to the east when placed in an electric field. If the charge is replaced by another, $q_2 = -12$ nC, what will be the magnitude and direction of the force on the charge at that position?

33. If the electric potential energy between two charges of 1.5 μC and 6.0 μC is 0.16 J, what is their separation?

34. Two electric charges are located on a coordinate system as follows: $q_1 = +35$ μC at the origin (0,0) and $q_2 = -25$ μC at the point (3,0), where the coordinates are in units of metres. What is the electric field at the point (1,2)?

35. (a) What is the change in electric potential energy of a charge of −15 nC that moves in an electric field from an equipotential of +4 V to an equipotential of +9 V?
   (b) Does the charge gain energy or lose energy?

36. To move a charge of +180 nC from a position where the electric potential difference is +24 V to another position where the potential difference is +8 V, how much work must be done?

37. For breakfast, you toast two slices of bread. The toaster uses 31 000 J of energy, drawn from a 110 volt wall outlet. How much charge flows through the toaster?

38. A spherical Van de Graaff generator terminal (capable of building up a high voltage) has a radius of 15 cm.
   (a) Calculate the potential at the surface if the total charge on the terminal is 75 nC.
   (b) If you touch the generator with a hollow steel ball of radius 6.5 cm, are the spheres "equipotential" while in contact?
   (c) Calculate the charge on each sphere when they are separated.

39. Two identical charges, $q_1 = q_2 = 6.0$ μC, are separated by 1.0 m.
   (a) Calculate the electric field and electric potential difference at point P, midway between them.
   (b) Replace one of the charges with a charge of the same magnitude but opposite sign and repeat the calculation in (a).
   (c) Discuss your solutions.

40. Points X and Y are 30.0 mm and 58 mm away from a charge of +8.0 μC.
   (a) How much work must be done in moving a +2.0 μC charge from point Y to point X?
   (b) What is the potential difference between points X and Y?
   (c) Which point is at the higher potential?

41. Points R and S are 5.9 cm and 9.6 cm away from a charge of +6.8 μC.
   (a) What is the potential difference between the points R and S?
   (b) Which point is at the higher potential?

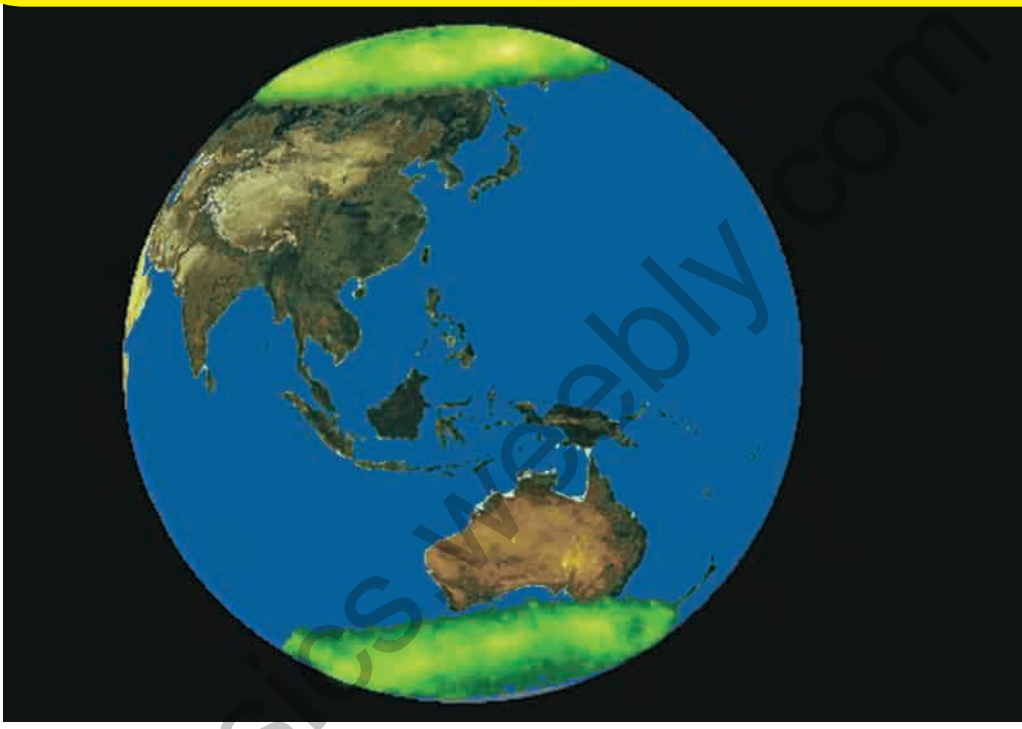
CHAPTER 8

Fields and Their Applications

CHAPTER CONTENTS

PREREQUISITE CONCEPTS AND SKILLS

- Newton's law of universal gravitation
- Electric potential difference
- Magnetic fields
- Moving charges in magnetic fields



The photograph above is the first image ever obtained of auroras at both the North Pole and the South Pole at the same time — a reminder that Earth's magnetic field protects all living organisms from frequent bombardment by high-energy, charged particles in the solar wind.

When the onslaught of charged particles enters Earth's magnetic field at an angle with the field, they curve away from Earth's surface. Many of the particles become trapped in the magnetic field and follow a helical path, circling back and forth in the field for long periods of time. These ions form the ionosphere. Only at the magnetic poles do the charged particles enter Earth's magnetic field parallel to the field lines and, therefore, are not diverted from their path. As these particles collide with oxygen and nitrogen molecules in the atmosphere, they excite the molecules, which then emit light as they return to their ground state.

Electric, magnetic, and gravitational fields exert a great influence on the structures in the universe. In this chapter, you will study how scientists and engineers are able to construct and manipulate some fields for practical purposes. The study of the behaviour of electric and magnetic fields has led to great progress in our understanding of the electromagnetic field and its enormous significance in, for example, telecommuncations.

MULTI LAB

Electric Fields

TARGET SKILLS
- Predicting
- Performing and recording
- Analyzing and interpreting

Cover It Up

Rub an ebonite rod with fur. Bring the rod close to the cap of a metal leaf electroscope, then remove the rod. Sit a small inverted metal can over the cap of the electroscope and repeat the experiment.

Analyze and Conclude

1. What is the reason for the difference in the results of the two experiments?
2. Suggest an explanation of the role of the metal can.

Swinging Pith Ball

Support two aluminum squares (about 10 cm square) in grooved wooden blocks and place them 3.0 cm apart. Charge a pith ball with an ebonite rod rubbed with fur and suspend the pith ball at roughly the midpoint between the plates. Now, ask your teacher to connect a Van de Graaff generator or other charging device across the plates, using alligator leads. After the plates have been charged, disconnect the alligator leads. Predict how changing the separation of the plates will affect the pith ball. Predict how changing the length of time of charging by the generator will affect the pith ball. Test your predictions. When moving the plates, do not touch the plates themselves. Touch only the wooden supports.

CAUTION Care must be exercised in the use of charge generators. Serious heart or nerve injury could occur through contact with large potential differences, depending on the resulting current.

Analyze and Conclude

1. What relationship did you observe between the separation of the plates and the behaviour of the pith ball?
2. What relationship did you observe between the time of charging and the behaviour of the pith ball?
3. Propose an explanation for the behaviour of the pith ball under the changing conditions.

8.1 Field Structure

SECTION EXPECTATIONS

- Analyze and illustrate the electric field produced by various charge arrangements and two oppositely charged parallel plates.
- Describe and explain the electric field that exists inside and on the surface of a charged conductor.
- Analyze and explain the properties of electric fields.

KEY TERMS

- charge density
- gradient
- potential gradient
- Stokes' Law
- Millikan's oil-drop experiment

In Chapter 7, Fields and Forces, you learned about electric fields and studied a few special cases of fields, such as the electric field around a single point charge and the combination of two point charges, either like or unlike. Much more complex fields exist, however, both natural and generated in the laboratory. For example, Figure 8.1 shows areas of equal potential around the human heart.

In this section, you will be studying the electric field and the corresponding field line patterns of a number of different-shaped, charged conductors. Regardless of how many individual charges are included in the configuration, the electric field vector at any point can be determined by calculating the sum of electric field vectors contributed by each charge influencing the field. For some configurations, however, this method would become very tedious and time-consuming, so physicists have developed techniques for a few special cases of fields. In addition, computer programs have been developed that can generate the field lines for different arrangements of charges. The user can create the distribution of charges and the computer will generate the associated electric field lines.

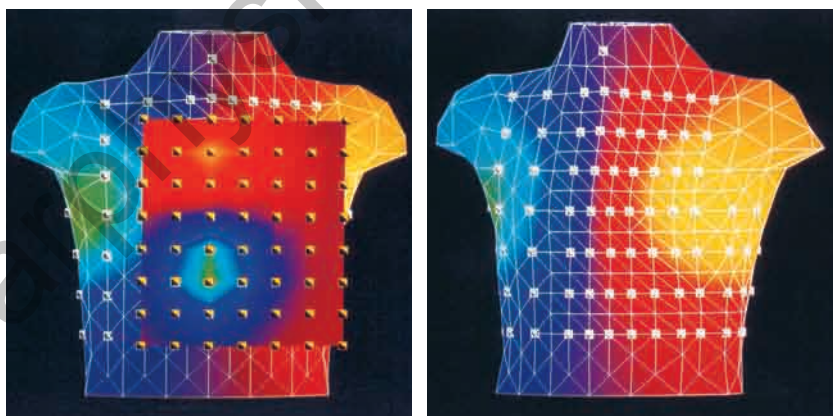


Figure 8.1 Electric and magnetic fields are a very real part of life. The photo on the right shows areas of equal electric potential difference, while the photo on the left shows areas of equal magnetic field intensity and direction around the human heart in varying shades of colour. The electric activities of the heart can provide a physician with important information about the health of a patient's heart.

WEB LINK

www.mcgrawhill.ca/links/physics12

If you would like to experiment with creating charge distributions and generating field lines, go to the above Internet site and click on **Web Links**.

Properties of an Electric Field Near a Conductor

Until now, you have been considering fields in the region of point charges. As you will see in the following Quick Lab, you can create some unusual fields with point charges. In real situations,

however, charged conductors take on a variety of shapes, but the same basic concepts about fields that apply to point charges also apply to conductors of all shapes. In fact, you can think of a conductor as a very large number of point charges lined up very close together. One important concept to remember when working with conductors is that electric field lines enter and leave a conductor perpendicular to the surface. Figure 8.2 shows why field lines cannot contact a conductor at any angle other than 90˚.

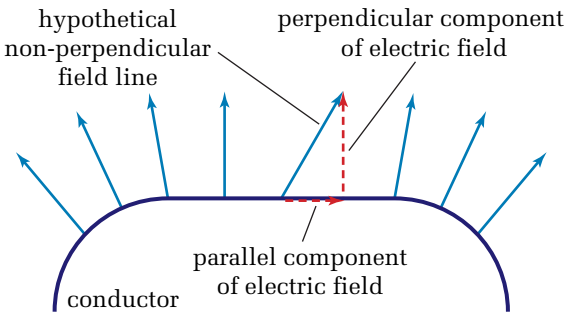


Figure 8.2 To indicate that an electric field line leaves a conductor at an angle implies that there is a component of the electric field that is parallel to the surface of the conductor. If this was the case, charges in the conductor would move until they had redistributed themselves in the conductor in a way that would change the field until there was no longer a parallel component.

QUICK LAB

Charge Arrays

TARGET SKILLS
- **Predicting**
- **Analyzing and interpreting**

In this Quick Lab, you will extend your knowledge into new and more complex charge arrangements. You will predict electric field lines and equipotential lines for several charge arrangements and then check your predictions.

For each of the following charge arrangements (arrays) located on the Cartesian coordinate plane, predict and sketch electric field line patterns and some equipotential lines. Use different colours for the field lines and for the equipotential lines.

(a) +1.0 C at (0,0)

(b) +1.0 C at (0,0) and an identical +1.0 C at (4,0)

(c) +1.0 C at (0,0) and –1.0 C at (4,0)

(d) +2.0 C at (0,0) and –1.0 C at (4,0)

(e) +3.0 C at (0,0) and –1.0 C at (4,0)

(f) +1.0 C at (0,0), +1.0 C at (4,0), and +1.0 C at (2,–4)

(g) +1.0 C at (0,0), +1.0 C at (4,0), and –1.0 C at (2,–4)

(h) +1.0 C at (0,0), +1.0 C at (4,0), –1.0 C at (4,–4), and +1.0 C at (0,–4)

Visit one of the Internet sites suggested by the Web Link on the previous page and simulate the charge arrays listed above. Observe the actual electric field lines and equipotentials that would be generated.

Analyze and Conclude

1. How well did your predicted patterns correspond to those generated by the simulation program?
2. How could you actually verify one specific value of the electric field intensity?

Consider the charged conductors in Figure 8.3 (A). Before you start to draw electric field lines, count the number of unit charges. Notice that the sphere has more negative charges than the plate has positive charges. So, more field lines will be ending on the sphere than leaving the plate. When you start to draw field lines, decide on the number entering and leaving each conductor so that the number of lines is proportional to the amount of charge on the conductor. Then, draw the beginning and end of each line perpendicular to the surface of the conductors, as shown in Figure 8.3 (B). Next, smoothly connect the lines so that those leaving the positive plate enter at adjacent lines ending on the negative sphere. The remaining lines on the sphere will spread out, but will not contact the positive plate. Finally, you can draw equipotential lines that are perpendicular to the electric field lines, as shown in Figure 8.3 (D).

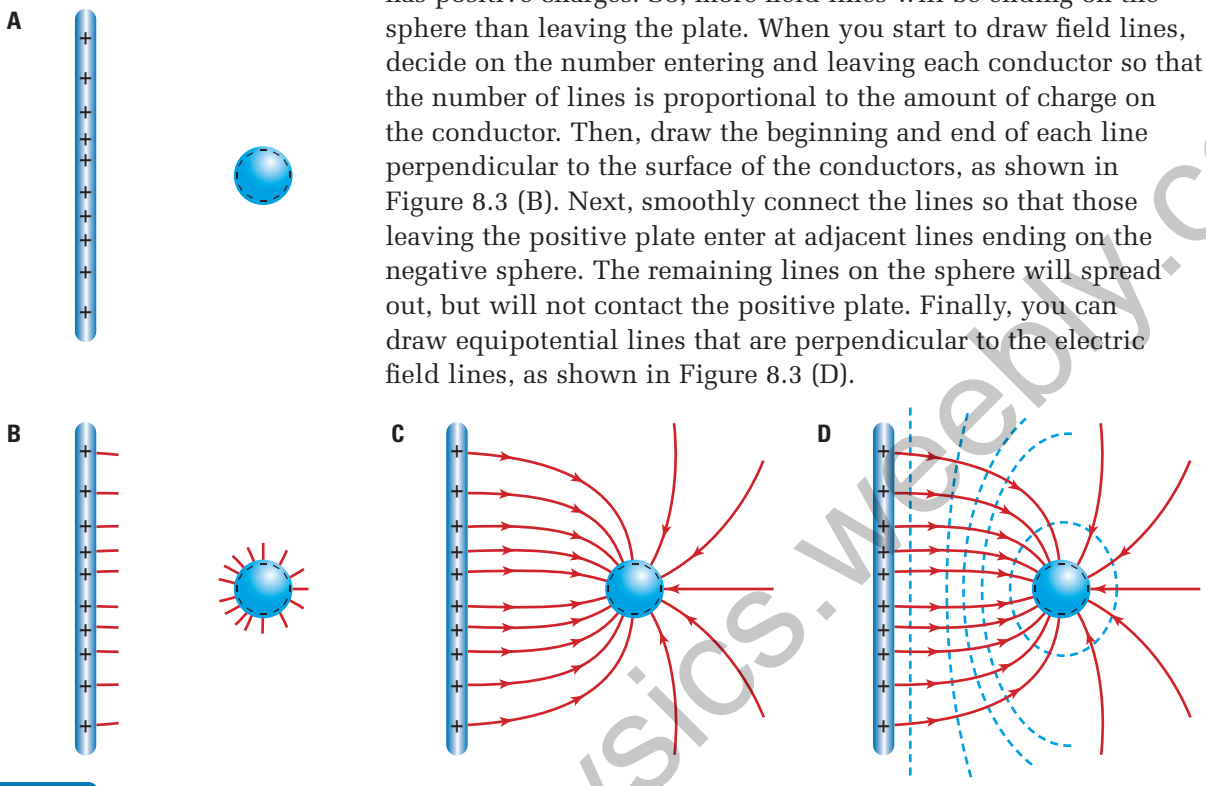
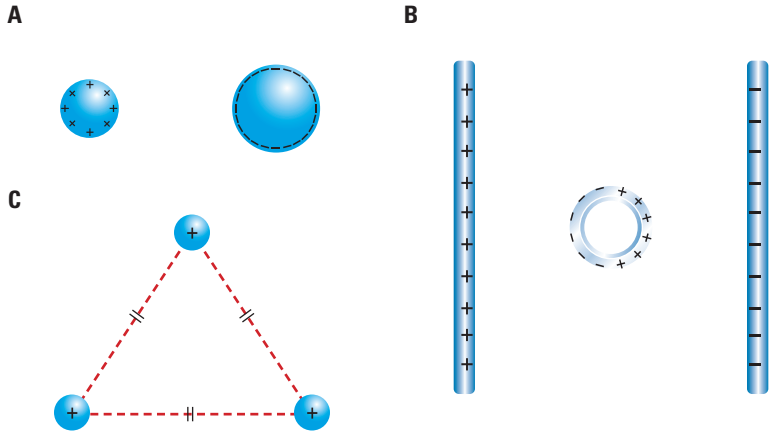


Figure 8.3 The positive plate attracts the negative charge on the sphere so that it is more dense on the side near the plate and less dense on the side away from the plate.

Conceptual Problem

- Copy each of the following diagrams (do not write on the diagrams in your textbook), showing various-shaped conductors, and draw in a representative sample of electric field lines and equipotential lines. Note that a uniform charge distribution has been assumed for each object except the cylinder in (B).



Parallel Plates

Charged parallel plates are a convenient way to create an electric field and therefore warrant in-depth examination. When two large, oppositely charged parallel plates are placed close together, the electric field between them is uniform, except for a certain spreading or "fringing" of the field at the edges of the plates, as shown in Figure 8.4.

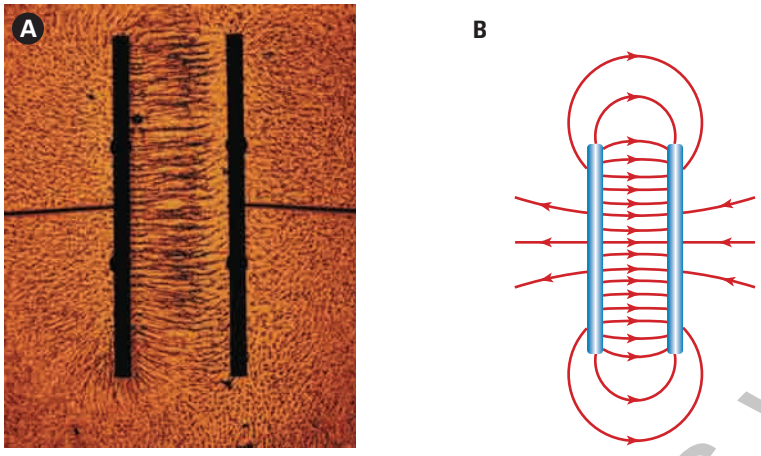


Figure 8.4 **(A)** When grass seeds are placed in an electric field between two parallel plates, they line up to reveal the shape of the electric field. **(B)** Using part (A) of this illustration as a model, a schematic diagram of an electric field is drawn.

The plates are too large to act like point charges, but the fact that the total charge on each plate is the sum of a large number of individual charges provides a way to explain the uniform field between the plates, as illustrated in Figure 8.5.

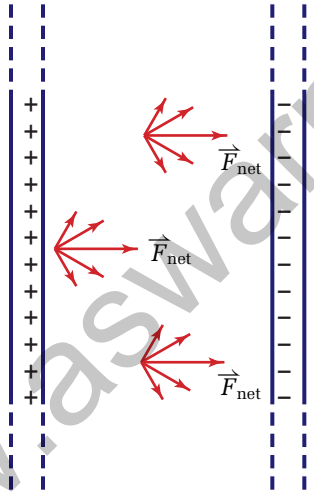


Figure 8.5 By symmetry, all of the force vectors add to produce a net force directly to the right.

A positive test charge placed at any point between the plates would experience a force from every positive charge on the left plate and every negative charge on the right plate. The magnitude of each of these forces would be determined by Coulomb's law, and the direction of each force would be along the line joining the test charge to each charge on the plates.

ELECTRONIC LEARNING PARTNER

To enhance your understanding of charges and fields, go to your Electronic Learning Partner.

PHYSICS FILE

It is important to remember that the parallel plates are mounted on insulators, isolated from any circuit. If they were charged by a battery, the battery was disconnected, so the same amount of charge would remain on the plates. If instead the plates remained connected to the battery and, for example, the area of the plates or the distance between them was changed, the battery would then adjust the charge on the plates and the field would change as well. Parallel plates connected within a circuit are called "capacitors" and their operation is beyond the scope of this course.

The net (resultant) force on the test charge would then be determined by the vector sum of all of the forces acting on it. Since the system is perfectly symmetrical, for every upward force, there would be a force of equal magnitude pointing down. The net force on the test charge would be a constant vector perpendicular to the plates, regardless of its location between the plates. The resulting field between two parallel plates can be summarized as follows.

- The electric field intensity is uniform at all points between the parallel plates, independent of position.
- The magnitude of the electric field intensity at any point between the plates is proportional to the **charge density** on the plates or, mathematically, $|E_Q| \propto \sigma$, where $\sigma = q/A$ (charge density = charge per unit area).
- The electric field intensity in the region outside the plates is very low (close to zero), except for the fringe effects at the edges of the plates.

SAMPLE PROBLEM

Parallel Plates

An identical pair of metal plates is mounted parallel on insulating stands 20 cm apart and equal amounts of opposite charges are placed on the plates. The electric field intensity at the midpoint between the plates is 400 N/C.

(a) What is the electric field intensity at a point 5.0 cm from the positive plate?

(b) If the same amount of charge was placed on plates that have twice the area and are 20 cm apart, what would be the electric field intensity at the point 5.0 cm from the positive plate?

(c) What would be the electric field intensity of the original plates if the distance of separation of the plates was doubled?

Conceptualize the Problem

- The *electric field* between isolated parallel plates is *uniform*.
- The *electric field* between isolated parallel plates depends on the *charge density* on the plates.

Identify the Goal

The magnitude of the electric field, $|E_Q|$, under three different conditions

Identify the Variables and Constants

Known	Unknown
$\lvert E_{Q(\text{initial})}\rvert = 400$ N/C	$\lvert E_{Q(\text{final})}\rvert$

Develop a Strategy

The magnitude of the electric field intensity is uniform between parallel plates, so it will be the same at every point.

$$|E_Q| = 400\ \frac{\text{N}}{\text{C}}$$

(a) The magnitude of the electric field intensity is 400 N/C at a point 5 cm from the positive plate.

The magnitude of the electric field intensity is inversely proportional to the area of the plates.

$$|E_{Q_1}| \propto \frac{q_1}{A_1} \quad \text{and} \quad |E_{Q_2}| \propto \frac{q_2}{A_2}$$

Divide.

$$\frac{|E_{Q_2}|}{|E_{Q_1}|} = \frac{\frac{q_2}{A_2}}{\frac{q_1}{A_1}}$$

Substitute.

$$q_2 = q_1 \quad \text{and} \quad A_2 = 2A_1$$

$$\frac{|E_{Q_2}|}{|E_{Q_1}|} = \frac{\frac{\cancel{q_1}}{2\cancel{A_1}}}{\frac{\cancel{q_1}}{\cancel{A_1}}}$$

$$|E_{Q_2}| = \frac{|E_{Q_1}|}{2}$$

$$|E_{Q_2}| = \frac{400\frac{\text{N}}{\text{C}}}{2}$$

$$|E_{Q_2}| = 200\ \frac{\text{N}}{\text{C}}$$

(b) The magnitude of the electric field intensity is 200 N/C when the area is doubled.

(c) The magnitude of the electric field intensity is 400 N/C (unchanged) when the distance is doubled, because electric field intensity is independent of the distance of separation.

Validate the Solution

Only the charge density affects the field intensity between the plates. Therefore, changing the area of the plates and consequently reducing the charge density is the only change that will affect the value of the field intensity.

PRACTICE PROBLEMS

1. A pair of metal plates, mounted 1.0 cm apart on insulators, is charged oppositely. A test charge of +2.0 μC placed at the midpoint, M, between the plates experiences a force of 6.0×10^{-4} N[W].

(a) What is the electric field intensity at M?

(b) What is the electric field intensity at a point 2.0 mm from the negative plate?

(c) What is the electric field intensity at a point 1.0 mm from the positive plate?

(d) What are two possible ways in which you could double the strength of the electric field?

2. The electric field intensity at the midpoint, M, between two oppositely charged (isolated) parallel plates, 12.0 mm apart, is 5.0×10^3 N/C[E].

(a) What is the electric field intensity at a point 3.0 mm from the negative plate?

(b) If the plate separation is changed to 6.0 mm and the area of the plates is changed, the electric field intensity is found to be 2.0×10^4 N/C[E]. What was the change made to the area of the plates?

continued ▶

continued from previous page

3. A test charge of +5.0 μC experiences a force of 2.0×10^3 N[S] when placed at the midpoint of two oppositely charged parallel plates. Assuming that the plates are electrically isolated and have a distance of separation of 8.0 mm, what will be the force experienced by a different charge of –2.0 μC, located 2.0 mm from the negative plate?

Parallel Plates and Potential Difference

In Chapter 7, you learned that the potential difference between two points in an electric field is the work required to move a unit charge from one point to the other. What generalizations can you make about potential difference between two parallel plates?

Consider a test charge, q, placed against the negative plate of a pair of parallel plates. You can derive an expression for the potential difference between the plates by considering the work done on a test charge when moving it from the negative plate to the positive plate.

- Write the equation for the amount of work you would have to do to move the charge a displacement, Δd. To eliminate the $\cos\theta$, work only with the component of displacement that is parallel to the force and therefore to the electric field.

$$W = F\Delta d \cos\theta$$
$$W = F\Delta d \text{ (parallel to field)}$$

- Write the expression for the force on a charge in an electric field.

$$\vec{F} = q\vec{E}_Q$$

- Substitute the expression for force into the equation for work. (**Note:** The vector notation and absolute value symbol will be used with the electric field intensity to avoid confusion with electric potential energy.)

$$W = q|\vec{E}_Q|\Delta d$$

- Divide both sides of the equation by q.

$$\frac{W}{q} = |\vec{E}_Q|\Delta d$$

- The definition of electric potential difference is work per unit charge.

$$\Delta V = \frac{W}{q}$$
$$\Delta V = |\vec{E}_Q|\Delta d$$

- Another useful equation results when you divide both sides of the equation by displacement.

$$|\vec{E}_Q| = \frac{\Delta V}{\Delta d}$$

ELECTRIC FIELD AND POTENTIAL DIFFERENCE

The magnitude of the electric field intensity in the region between two points in a uniform electric field is the quotient of the electric potential difference between the points and the component of the displacement between the points that is parallel to the field.

$$|\vec{E}_Q| = \frac{\Delta V}{\Delta d}$$

Quantity	Symbol	SI unit
electric field intensity	$\vec{E}_Q$	$\frac{N}{C}$ (newtons per coulomb)
electric potential difference	ΔV	V (volts)
component of displacement between points, parallel to field	$\Delta\vec{d}$	m (metres)

Unit Analysis

$$\frac{V}{m} = \frac{J/C}{m} = \frac{N \cdot \cancel{m}}{C \cdot \cancel{m}} = \frac{N}{C}$$

Note: When doing a unit analysis, it is very useful to remember that a volt per metre is equivalent to a newton per coulomb.

Potential Gradient

In general, a gradient is similar to a rate. While a rate is a change in some quantity relative to a time interval, a **gradient** is a change in some quantity relative to a change in position, or displacement; therefore, the expression $\frac{\Delta V}{\Delta d}$ is known as the **potential gradient**.

As you move from one plate to the other, the electric potential difference changes linearly, since $\Delta V = |\vec{E}|\Delta d$ and $\vec{E}$ is constant. So, if the potential difference across the plates is 12 V, the potential difference at a point one third of the distance from the negative plate will be 4.0 V. The potential difference is higher close to the positive plate. In other words, the potential difference increases in a direction opposite to the direction of the electric field.

Physicists commonly refer to the "potential at a point" in an electric field. As you know, there are no absolute potentials, only potential differences. Therefore, the phrase "potential at a point" means the potential difference between that point and a reference point. In the case of parallel plates, the reference point is always the negatively charged plate.

SAMPLE PROBLEM

Field and Potential

Two parallel plates 5.0 cm apart are oppositely charged. The electric potential difference across the plates is 80.0 V.

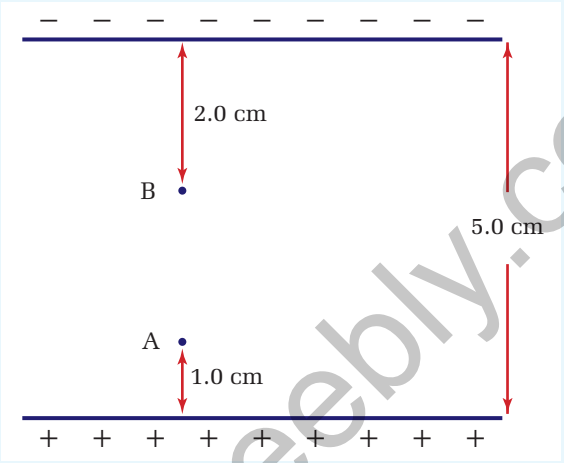
(a) What is the electric field intensity between the plates?

(b) What is the potential difference at point A?

(c) What is the potential difference at point B?

(d) What is the potential difference between points A and B?

(e) What force would be experienced by a small 2.0 μC charge placed at point A?



Conceptualize the Problem

- The *electric field* between *parallel plates* is *uniform*.
- Identify the lower plate as positive.
- The *electric field intensity* is related to the *potential difference* and the *distance of separation*.

Identify the Goal

The electric field intensity, $|\vec{E}_Q|$, between the plates

The potential difference at point A, V_A, and point B, V_B

The potential difference between points A and B, ΔV_{AB}

The electric force, $\vec{F}_Q$, on a charge placed at point A

Identify the Variables and Constants

Known	Unknown	
$\Delta d = 5.0$ cm	$\vec{E}_Q$	ΔV_{AB}
$\Delta V = 80.0$ V	V_A	$\vec{F}_Q$
Points A and B	V_B	
$q = 2.0\ \mu$C		

Develop a Strategy

The electric field is related to the potential difference and the distance of separation.

$$|\vec{E}_Q| = \frac{\Delta V}{\Delta d}$$

$$|\vec{E}_Q| = \frac{80.0\ \text{V}}{5.0 \times 10^{-2}\ \text{m}}$$

$$\vec{E}_Q = 1.6 \times 10^3\ \frac{\text{N}}{\text{C}}$$

directed from the positive to the negative plate

(a) The electric field intensity is 1.6×10^3 N/C away from the positive plate.

Use the equation that relates the electric potential difference to the electric field intensity. (**Note:** Point A is 4.0 cm from the negative plate.)

$$V_A = |\vec{E}_Q|\Delta d$$
$$V_A = \left(1.6 \times 10^3 \frac{V}{\cancel{m}}\right)(0.040\ \cancel{m})$$
$$V_A = 64\ V$$

(b) The potential difference at point A is 64 V.

Use the equation that relates the electric potential difference to the electric field intensity.

$$V_B = |\vec{E}_Q|\Delta d$$
$$V_B = \left(1.6 \times 10^3 \frac{V}{\cancel{m}}\right)(0.020\ \cancel{m})$$
$$V_B = 32\ V$$

(c) The potential difference at point B is 32 V.

Point A is at the higher potential, because it is closer to the positive plate.

$$\Delta V = V_A - V_B$$
$$\Delta V = 64\ V - 32\ V$$
$$\Delta V = 32\ V$$

(d) The potential difference between points A and B is 32 V.

The electric force is related to the field and charge.

$$\vec{F}_Q = q\vec{E}_Q$$
$$\vec{F}_Q = (2.0 \times 10^{-6}\ \cancel{C})\left(1.6 \times 10^3 \frac{N}{\cancel{C}}\right)$$
$$\vec{F}_Q = 3.2 \times 10^{-3}\ N[\text{away from positive plate}]$$

(e) The force experienced by the small charge at point A is 3.2×10^{-3} N, away from the positive plate.

Validate the Solution

The values are reasonable in terms of the given data. The units are logical.

PRACTICE PROBLEMS

4. Calculate the electric field intensity between two parallel plates, 4.2 cm apart, which have a potential difference across them of 60.0 V.

5. The potential difference between two points 8.0 mm apart in the field between two parallel plates is 24 V.

(a) What is the electric field intensity between the plates?

(b) The plates themselves are 2.0 cm apart. What is the electric potential difference between them?

6. When an 80.0 V battery is connected across a pair of parallel plates, the electric field intensity between the plates is 360.0 N/C.

(a) What is the distance of separation of the plates?

(b) What force will be experienced by a charge of $-4.0\ \mu C$ placed at the midpoint between the plates?

(c) Calculate the force experienced by the charge in part (b) if it is located one quarter of the way from the positive plate.

7. What electric potential difference must be applied across two parallel metal plates 8.0 cm apart so that the electric field intensity between them will be 3.2×10^2 N/C?

8. The potential gradient between two parallel plates 2.0 cm apart is 2.0×10^3 V/m.

(a) What is the potential difference between the plates?

(b) What is the electric field intensity between the plates?

QUICK LAB

Quantum Pennies

TARGET SKILLS
- Hypothesizing
- Analyzing and interpreting

This Quick Lab will give you some insight into the approach that Robert Andrews Millikan (1868–1953) used to determine the charge on the electron. Instead of charge, however, you will determine the mass of a penny.

Your teacher has prepared a class set of small black film canisters that contain various numbers of identical pennies. You and your classmates will use an electronic balance to determine the mass of each film canister and its contents. Carry out this procedure and post the class results.

Analyze and Conclude

1. Draw a bar graph, with the canister number on the horizontal axis and the mass of that canister on the vertical axis.
2. Calculate and record the increments between each mass value and all other mass values.
3. Is there any minimum increment of which all other increments are a multiple?
4. What do you predict to be the mass of a penny? Check your prediction by direct measurement.

Apply and Extend

5. As a class, open the canisters and randomly add or remove pennies. Again, measure the mass, and repeat the analysis.

Millikan's Oil-Drop Experiment: Charge on the Electron

A very important series of experiments dependent on the uniform electric field between a pair of parallel plates was performed during the years 1909 to 1913 by Millikan. The results of these experiments, together with his contributions to research on the photoelectric effect (see Chapter 12), led to his Nobel Prize in Physics in 1923. Millikan was able not only to verify the existence of a fundamental electric charge — the electron — but also to provide the precise value of the charge carried by the electron. This had a tremendous impact on the further development of the theory of the structure of matter.

The experimental procedures used by Millikan were actually a modification of earlier techniques used by J.J. Thomson (1856–1940). A pair of parallel plates was very finely ground to smoothness and a tiny hole was drilled in the top plate. An atomizer was mounted above the plates and used to spray tiny droplets of oil into the region above the plates. These oil droplets acquired an electric charge as they were sprayed, presumably from friction. The whole apparatus was kept in a constant-temperature enclosure, and the region between the plates was illuminated with an arc lamp.

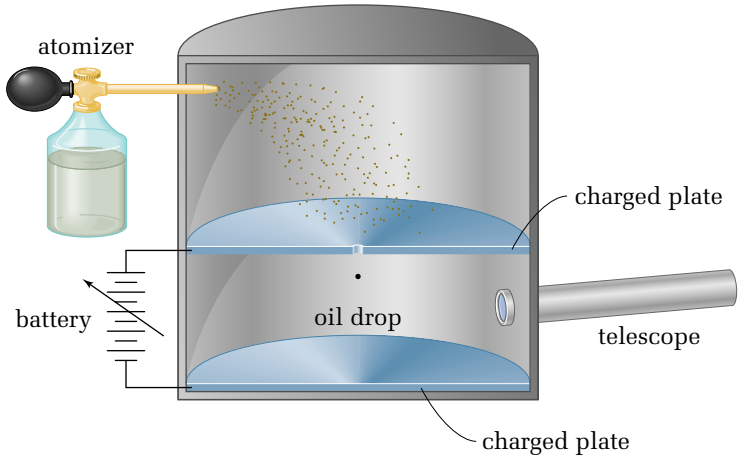


Figure 8.6 Not shown in this simplified cross-section of Millikan's apparatus is a source of X rays that could ionize the air in the electric field.

A droplet that fell through the hole and into the region between the plates could be viewed through a short telescope. The droplet would very quickly acquire terminal velocity as it fell under the influence of the force of gravity and the resistance (viscosity) of the air. This terminal velocity, v_0, could be measured by timing the drop as it fell between the lines on a graduated eyepiece. (In Millikan's apparatus, the distance between the cross hairs was 0.010 cm). A potential difference (3000 to 8000 V) was then applied across the plates (separated 1.600 cm) by means of a variable battery.

Usually the oil drops had attained a negative charge, so the potential difference would be applied so that the top plate was made positive. In that way, a negatively charged oil drop could be made to reach an upward terminal velocity under the action of the applied electric field, its effective weight, and air friction. This second terminal velocity, v_1, was also recorded.

Millikan then directed X rays to the region between the plates. The X rays ionized the air molecules between the plates and caused the charge on the oil drop to change as it either gained or lost electrons. Again, the procedure of determining the two terminal velocities, one with and one without the applied electric field, could be repeated many times with differently charged oil drops.

Millikan observed that the terminal velocity of the charged oil drops, which depended on the charge itself, varied from trial to trial. Over a very large number of trials, however, the velocity values could be grouped into categories, all of which represented an integral multiple of the lowest observed value. This led him to conclude that the charge on the oil drops themselves could be quantified as integral multiples of one fundamental value. Millikan then applied a mathematical analysis to determine that value. The following is a simplified version of Millikan's analysis.

- When the oil drop travelled with a uniform velocity, the upward electric force was equal in magnitude to the downward gravitational force.

$$\vec{F}_Q = \vec{F}_g$$

- Write the expressions for the electric force in terms of the electric field intensity and for the gravitational force.

$$\vec{E}_Q = \frac{\vec{F}_Q}{q} \qquad \vec{F}_g = mg$$

$$\vec{F}_Q = q\vec{E}_Q$$

- Substitute the expressions into the first equation.

$$q\vec{E}_Q = m\vec{g}$$

- Express the electric field intensity in terms of the potential difference across the plates.

$$E_Q = \frac{\Delta V}{\Delta d}$$

- Substitute the expression for the electric field intensity.

$$\frac{qV}{\Delta d} = mg$$

- Solve for the charge, q.

$$q = \frac{mg\Delta d}{V}$$

All of the variables in this equation for q could be measured easily, except the mass. Millikan then turned to the work of Sir George Gabriel Stokes (1819–1903), a British mathematician and physicist who had helped to develop the laws of hydrostatics. Based on a particle's rate of fall as it falls through a viscous medium, **Stokes' law** can be used to calculate the particle's mass.

Millikan measured the terminal velocity of each oil drop with the battery turned off, that is, the rate of fall under the influence of gravity and the resistance of the fluid (air) only. When he substituted the mass of each oil drop into the equation for the charge q, above, he found that the magnitude of the charge on each oil drop was always an integral multiple of a fundamental value. He assumed that this particular fundamental charge was actually the charge on the electron, and the multiple values arose from the oil drops having two, three, or more excess or deficit electrons.

The electronic charge computed from many trials of Millikan's method is found to be $e = 1.6065 \times 10^{-19}$ C, which agrees well with values determined by other methods. The currently accepted value for the charge on the electron is $e = 1.602 \times 10^{-19}$ C. Knowing the charge, e, on an electron, it has become common practice to express the charge on an object in terms of the number, n, of the excess or deficit of electrons on the object, or $q = ne$.

SAMPLE PROBLEM

Millikan Experiment

Two horizontal plates in a Millikan-like apparatus are placed 16.0 mm apart. An oil drop of mass 3.00×10^{-15} kg remains at rest between the plates when a potential difference of 420.0 V is applied across the plates, the upper plate being positive. Calculate the

(a) net charge on the oil drop

(b) sign of the charge on the oil drop

(c) number of excess or deficit electrons on the oil drop

Conceptualize the Problem

- The oil drop is held in place by its own *weight* (down) and the *electric force* (up).
- The *electric force* depends on the *electric field* value between the plates.

Identify the Goal

The magnitude of the charge, q, on the oil drop; its sign, ±; and electron number, n, of excess or deficit electrons

Identify the Variables and Constants

Known	Implied	Unknown
$d = 16.0$ mm	$g = 9.81 \frac{\text{m}}{\text{s}^2}$	q
$m = 3.00 \times 10^{-15}$ kg	$e = 1.602 \times 10^{-19}$ C	n
$V = 420.0$ V		

Develop a Strategy

The electric force will be equal in magnitude to the force of gravity.

$$F_Q = F_g$$
$$qE_Q = mg$$
$$q\frac{V}{\Delta d} = mg$$
$$q = \frac{mg\Delta d}{V}$$
$$q = \frac{(3.00 \times 10^{-15}\text{ kg})\left(9.81 \frac{\text{m}}{\text{s}^2}\right)(1.60 \times 10^{-2}\text{ m})}{420.0\text{ V}}$$
$$q = 1.1211 \times 10^{-18}\text{ C}$$
$$q \cong 1.12 \times 10^{-18}\text{ C}$$

(a) The charge on the oil drop is -1.12×10^{-18} C.

The drop was suspended.

Since the electric force was in a direction opposite to the gravitational force, it had to be "up." The upper plate was positive, so the charge had to be negative.

(b) The net charge was negative.

continued ►

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The net charge is related to the charge on the electron.

$$n = \frac{q}{e}$$
$$n = \frac{1.12 \times 10^{-18}\,\text{C}}{1.60 \times 10^{-19}\,\text{C}}$$
$$n = 7.00$$

(c) There is an excess of seven electrons on the oil drop, causing its net negative charge.

Validate the Solution

The values are consistent with the size of the oil drop, the plate separation, and the potential difference. The units in the calculation are consistent.

$$\frac{\cancel{\text{kg}} \cdot \frac{\text{N}}{\cancel{\text{kg}}} \cdot \text{m}}{\text{V}} = \frac{\text{N} \cdot \text{m}}{\frac{\text{J}}{\text{C}}} = \frac{\cancel{\text{J}}}{\frac{\cancel{\text{J}}}{\text{C}}} = \text{C}$$

PRACTICE PROBLEMS

9. Two large horizontal parallel plates are separated by 2.00 cm. An oil drop, mass 4.02×10^{-15} kg, is held balanced between the plates when a potential difference of 820.0 V is applied across the plates, with the upper plate being negative.

(a) What is the charge on the drop?

(b) What is the number of excess or deficit electrons on the oil drop?

10. A small latex sphere experiences an electric force of 3.6×10^{-14} N when suspended halfway between a pair of large metal plates, which are separated by 48.0 mm. There is just enough electric force to balance the force of gravity on the sphere.

(a) What is the mass of the sphere?

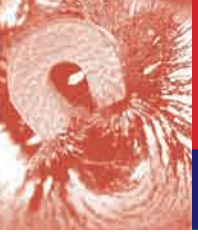
(b) What is the potential difference between the plates, given that the charge on the sphere is 4.8×10^{-19} C?

11. The density of the oil used to form droplets in the Millikan experiment is 9.20×10^{2} kg/m^3 and the radius of a typical oil droplet is 2.00 μm. When the horizontal plates are placed 18.0 mm apart, an oil drop, later determined to have an excess of three electrons, is held in equilibrium. What potential difference must have been applied across the plates?

UNIT PROJECT PREP

Understanding the costs and benefits of any issue often begins by (a) gathering useful facts and (b) identifying personal bias.

- Identify your bias. Do you believe that fundamental research is worthwhile if it does not have any obvious applications?
- Do you believe that research resulting in greater understanding will some day, perhaps decades later, be put to use in an application?
- How would you complete the statement "Knowledge for the sake of knowledge ... "?



INVESTIGATION 8-A

Millikan's Oil-Drop Experiment

TARGET SKILLS
- Performing and recording
- Analyzing and interpreting

In this investigation, you will demonstrate that electric charge exists as a quantized entity, using apparatus that allows you to apply a potential difference across parallel plates as you observe the movement of latex spheres.

Problem

Does charge exist in fundamental units and can you find evidence of differently charged objects?

Equipment

- Millikan apparatus for use with latex spheres
- supply of latex spheres
- stopwatch

CAUTION Be careful not to touch open terminals that are connected to a high potential difference.

Procedure

1. Follow the manufacturer's instructions to adjust and focus the light source and also to connect the plates to the source of potential difference. Your aim is to make repeated measurements of the velocity of a sphere under the action of gravity alone (v_0, down) and also under the action of both the gravitational force and the electric force (v_1, up).
2. Examine the position and function of the voltage switch. In the *off* position no electric field will be applied and the sphere will fall under the action of the force of gravity alone. In the *on* position a potential difference will be applied across the plates, with the top plate being positive, and the sphere will rise as the electric force is greater than the force of gravity.
3. Place the switch in the *off* position and squeeze some latex spheres into the region between the plates. (You may need to practise observing the spheres before you actually start timing them. They will appear as tiny illuminated dots.) Follow the manufacturer's instructions for determining direction. The telescope usually inverts the field of view, so the force of gravity is then "up," although some manufacturers have included an extra lens to compensate.
4. Using the voltage switch, clear the field of fast-moving dots. They carry a large charge and are hard to measure. Choose one of the slowly falling spheres and measure its time of travel as it falls, under the action of the force of gravity. Observe the motion for several grid marks in the field of view. (Remember it might be falling "up" in your apparatus.) Without losing the sphere, change the switch so that the sphere rises under the action of the electric field and again measure the time of travel over the grid marks. Before the sphere disappears from the field of view, place the switch in the *off* position and again measure the time of travel over the grid marks. You will need a laboratory partner to record the results so that you can keep your eye on the selected sphere.
5. Repeat your observations for a different sphere from a new batch, and continue making observations for at least 20 different spheres. (Alternate with your lab partner to allow your eyes to rest!)

Analyze and Conclude

1. Calculate the velocity of the spheres for every trial, using an arbitrary unit for distance. For example, if one sphere moved 8.0 gridlines in 3.1 seconds, record its velocity as

$$v = \frac{\Delta d}{\Delta t} = \frac{8.0 \text{ grid lines}}{3.1 \text{ s}} = 2.6 \frac{\text{grid lines}}{\text{s}}.$$

2. Record the velocity of the sphere in two different ways: v_0 to represent the velocity of the sphere under the force of gravity alone, and v_1 to represent the velocity when

the electric force up is greater than the gravitational force on the sphere. Record your data in a table similar to the one below.

Sphere	v_0	v_1	$v_0 + v_1$

3. Complete the calculations for each column in the table.
4. Since the value of $v_0 + v_1$ represents the strength of the electric force alone, acting on the sphere, it can also be considered to represent the electric charges on the sphere. Draw a bar graph with the quantity $v_0 + v_1$ on the vertical axis and "Trial number" evenly distributed on the horizontal axis.
5. Does your bar graph offer any evidence that electric charge exists as an integral multiple of a fundamental charge? Are you able to state the number of fundamental charges that are excess or deficit on your spheres? Explain your reasoning.

8.1 Section Review

1. K/U
   (a) Draw the electric field pattern for a +4 μC charge and a –4 μC charge separated by 4.0 cm. Include four equipotential lines.
   (b) Repeat part (a) for a –16 μC and a –4 μC charge.
2. I If you have access to the Internet, use the sites listed in your Electronic Learning Partner to verify your answers.
3. K/U
   (a) List four properties of electric field lines.
   (b) List two properties of equipotential surfaces.
4. MC Research and report on the use of electric fields in technology and medicine (for example, laser printers, electrocardiograms).
5. C With your classmates, prepare a dramatic skit to simulate Millikan and his colleagues preparing and performing his oil-drop experiment.
6. K/U A pair of parallel plates is placed 2.4 cm apart and a potential difference of 800.0 V is connected across them.
   (a) What is the electric field intensity at the midpoint between the plates?
   (b) What is the electric potential difference at that point?
   (c) What is the electric field intensity at a point 1.0 cm from the positive plate?
   (d) What is the electric potential difference at that point?
7. I A pair of horizontal metal plates are situated in a vacuum and separated by a distance of 1.8 cm. What potential difference would need to be connected across the plates in order to hold a single electron suspended at rest between them?

8.2 Conductors and Fields

You will now examine how conductors are used in the transport of electric current and electromagnetic signals. An electric field can be established not only in the spatial region around point charges or in the air gap between parallel plates, but also in the metal conducting wires that enable electric current to be transmitted. Shielded coaxial wires can also be used as a "guide" to transport electromagnetic waves to a convenient location (such as your television receiver) with minimal loss of strength. You will learn more about electromagnetic waves in Unit 4, but for now, you can at least gain a qualitative idea of how they can be transported efficiently, with minimal loss of energy.

SECTION EXPECTATIONS

- Define and describe the concepts related to electric fields.
- Describe and explain the electric field that exists inside and on the surface of a charged conductor.
- Demonstrate how an understanding of electric fields can be applied to control the electric field around a conductor.

KEY TERM

- Faraday cage

Conducting Wires

In previous science courses, you worked with conducting wires and circuits. You learned that if you placed an electric potential difference across the ends of the conductor, a current would flow. You have just learned that an electric potential difference creates an electric field and that charges in an electric field experience electric forces. Now you can examine conductors in more detail.

In previous studies, you learned that the copper atoms have heavy positive nuclei and a cloud of negative electrons surrounding it. An isolated atom has electrons filling up the lower energy "shells," but there are also a few electrons outside of these complete shells. These outer electrons can move relatively easily if they are replaced with another electron from another copper atom. The electrons are then free to move through the metal, colliding randomly with the stationary positive nuclei.

If a battery is connected to the ends of a metal wire, it will create an electric field inside the wire and parallel to its axis. Consequently, the free electrons will move in a direction opposite to the direction of the field, as shown in Figure 8.7.

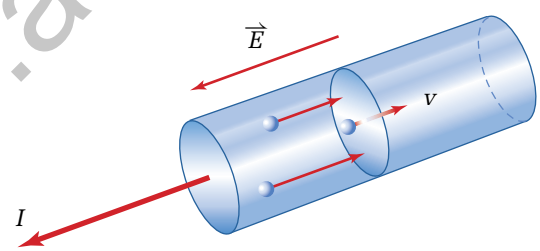


Figure 8.7 In a conductor, electrons move opposite to the direction of the electric field, because the direction of the field is defined as the direction in which a positive charge would move.

Hollow Conductors: Faraday's Ice-Pail Experiment

When you carried out the Cover It Up activity in the Multi-Lab at the beginning of this chapter, you probably noticed that when you placed the can over the sphere on the electroscope, it eliminated the effect that you originally observed when you brought the charged rod close to the electroscope sphere. You probably did not realize that you were performing an experiment very similar to one of the most famous experiments in the history of the study of electric fields — Michael Faraday's ice-pail experiment. Faraday devised this experiment to show that electric charge will reside only on the *outside* of a hollow conductor. The experiment is outlined schematically in Figure 8.8 and described in the steps that follow.

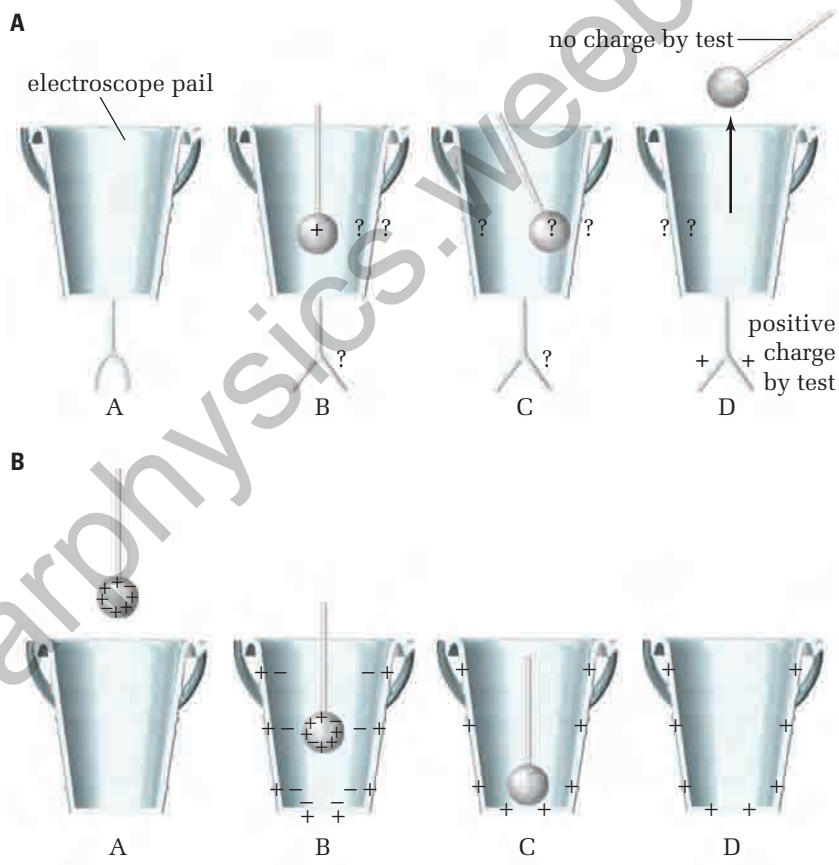


Figure 8.8 **(A)** The questions raised by Faraday's ice-pail experiment; **(B)** the answers

Faraday's Ice-Pail Experiment

- A hollow metal can (Faraday happened to use an ice pail), insulated from its surroundings, was connected to an uncharged electroscope.

- A positively charged metal ball was lowered into the pail by its insulated handle. The electroscope leaves diverged and stayed at a fixed divergence. When the metal ball was moved around inside the ice pail, the electroscope leaves stayed at a fixed angle of divergence.
- The metal ball was allowed to touch the inside of the ice pail. The angle of divergence of the leaves of the electroscope remained the same.
- The metal ball was then removed from the ice pail and the ball and the leaves were tested for charge. The ball was found to be uncharged, and the leaves were charged positively.

From his experiment, Faraday deduced the following.

- The positive ball had induced a negative charge on the inside wall of the pail and a positive charge on the outside wall.
- The induced charge was of the same magnitude as the charge on the ball, since the charges on the ball and the inside wall of the pail cancelled each other.
- The induced charges on the inside and outside walls of the pail were of equal magnitude, since the angle of divergence of the leaves did not change throughout the experiment.

Two general properties were illustrated by this experiment.

1. The formation of one charge is always accompanied by the formation of an equal, but opposite, charge.
2. The net charge in the interior of a hollow conductor is zero; all excess charge is found on the outside.

The latter property led to the general conclusion that an external electric field will not affect the inside of a hollow conductor. In fact, it will be shielded. Faraday pursued this with a further demonstration in which he built a very large metal cage, mounted on insulators, and then entered the cage to perform electrostatic experiments, while a very high electric field was generated all around him.

This electric screening was the basis of the Cover It Up activity in the Multi-Lab at the beginning of the chapter. If you charge an electroscope and then place a cage (or inverted can) over the sphere of the electroscope, you will shield it from external electric fields. If you bring a charged rod close to the cage, the leaves of the electroscope are unaffected. This principle has become a popular method for screening sensitive electric circuit elements by placing them in some form of metal cage. Today, anything that is used to shield a region from an external electric field is called a **Faraday cage**.

WEB LINK

www.mcgrawhill.ca/links/physics12

For more information about Michael Faraday, both scientific and personal, go to the above Internet site and click on **Web Links**.

QUICK LAB

Faraday Cage

TARGET SKILLS

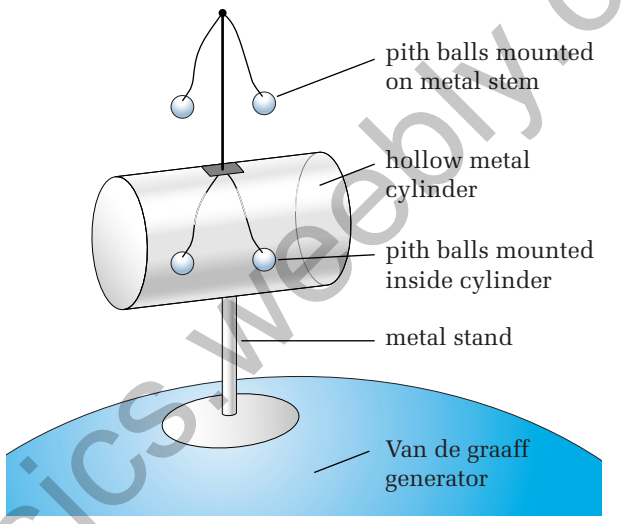
- **Hypothesizing and predicting**
- **Analyzing and interpreting**

CAUTION Van de Graaff generators generate very high potential differences that might cause harm to some individuals.

You can demonstrate the shielding effect of a hollow metal cylinder by using the apparatus shown in the diagram. Either use tape to attach the metal base of the hollow cylinder-pith ball apparatus to the top of the sphere on the Van de Graaff generator, or use an electric lead to connect the two. Turn on the generator and allow it to run for a few seconds.

Analyze and Conclude

1. How does the behaviour of the pith balls inside the hollow cylinder differ from the behaviour of the pith balls mounted outside the hollow cylinder?
2. What does this experiment demonstrate about the electric field inside a hollow conductor?



Coaxial Cable

When electromagnetic waves, such as television signals, are transmitted to the home, either through or beyond the atmosphere, they are captured by a receiver (antenna) and then delivered to your television as an electric signal.

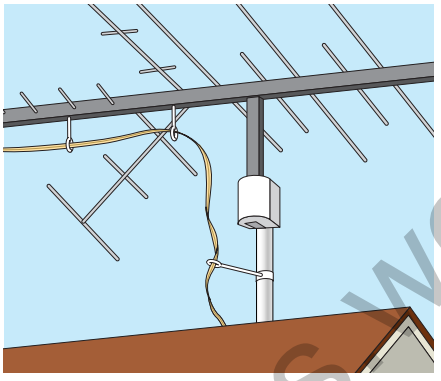
Early antenna cables consisted of a flat, twin-lead wire, with two braided wires (through which the signal was conducted) mounted in a flat, plastic insulating band. This type of wire has become less common, as it is very susceptible to interference from unwanted electromagnetic signals, such as those arising from sunspot activity, lightning storms, or even just local extraneous transmissions, such as those from power tools.

An improvement on the twin-lead wire was the shielded twin lead, in which the braided wires were each wrapped in foam insulation. The pair of wires was then wrapped in foil sheathing to provide shielding and then in an outer layer of plastic insulation.

The most efficient and popular signal-conducting wire today is the coaxial cable, consisting of concentric rings: an inner conducting wire, sometimes stranded (stereo) but usually solid

(television), a sheath of foam insulation, a second sheath of braided (or solid foil) conducting wire, and an outer sheath of plastic insulation. The actual mathematics and physics of the transport of a signal along a coaxial cable is quite complex, but for the purposes of this section, it is sufficient to say that the two wires transporting the signal are the inner core wire and the outer braided wire. The latter provides a form of Faraday shielding from external interference.

Conceptual Problems



- When twin-lead wire is used to carry the television signal from the antenna to a television receiver, the directions require that the lead be twisted and not installed straight. What would be the purpose of this instruction?
- Why must the twin-lead wire be held away from the metal antenna mast, using insulating clamps?
- A homeowner knew some physics and decided to run a coaxial cable through the house inside the metal heating ducts.
  - **(a)** What would be one advantage of this procedure?
  - **(b)** State one disadvantage of this method.
- Before the advent of transistors, old superheterodyne "wireless" receivers used "radio valves" in the amplification circuit. Why were these valves often enclosed in metal cylinders?
- When transistors became the basic component of electric circuits, did they also need shielding? Research your answer.

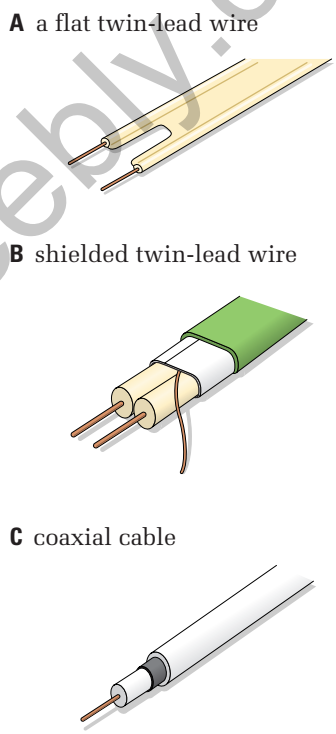


Figure 8.9 Common types of signal conducting wires.

8.2 Section Review

1. **K/U** What evidence supports the practice of enclosing electronic components in metal shells?
2. **MC** Some people who felt that TV antennas looked unsightly hid them in the attics of their houses. Discuss how the type of roof and siding material has relevance to this practice.
3. **MC** Some people feel that it is relatively safe to take shelter in a car during a lightning storm, because the rubber tires will provide insulation. However, a lightning strike that has travelled several kilometres is not going to be discouraged from jumping the last few centimetres. In what way does a car offer protection from lightning?

Levitation: How Does It Work?

Levitating an object should be easy. All you need is a repulsive force strong enough to counteract Earth's gravity. So why not use an electric charge or a magnet to create the repulsive force? Scientists have been thinking about this idea for years. In fact, the first proposal to use magnetism to levitate vehicles was made in 1912, just one year after the discovery of superconductivity.

Superconductors can conduct electricity with no resistance at all. In normal conductors, moving electrons collide with atoms, a process that resists the flow of current and causes the conductor to heat up. Superconductors can carry large currents without heating up, which means that they can be used to create powerful electromagnets. Once the current is introduced into the superconducting wire, it can flow indefinitely, without dissipation, because there is no resistance. So, over time, the created magnetic field will not lose strength.

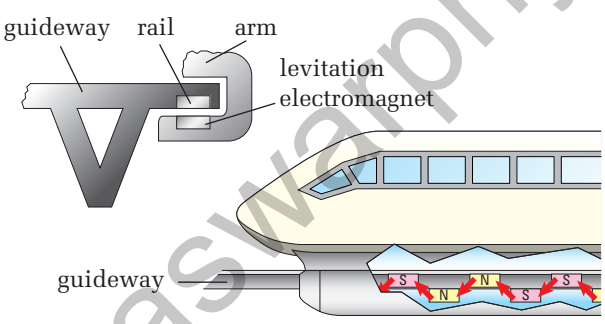
A Surprising Effect

What is the connection between superconductors and levitation? Superconductors have an additional surprising property when placed in a magnetic field. When a normal conductor is placed in a magnetic field, the magnetic field lines go right through the conductor — as if it was not there. When a superconductor is placed in a magnetic field, it expels the magnetic field from its interior and causes levitation. You can clearly see the levitation factor at work in a magnetically levitated (maglev) train — the train rides (or levitates) a few centimetres above the track (guideway).

Exciting Technology

Since the train rides above the guideway, what about propulsion? How does that happen? One design for a magnetically levitated train that is currently being built in Japan uses a push-pull system. Electromagnets are placed in the bottom of the train and along the track. The current is set so that the electromagnets along the track have opposite polarity to those on the train. It's possible, then, to have an unlike pole just ahead of each electromagnet on the train and a like pole just behind. The electromagnetic force pulls the unlike poles together and pushes the like poles apart.

After the train has moved forward enough to line up all of the electromagnets on the track and the train, the track electromagnets are briefly switched off. When they are switched on again, they have the opposite polarity, so that each electromagnet on the train is now pushed and pulled by those on the track. It's an intuitively basic design. The train's speed can be adjusted by timing the switches of the polarity of the track electromagnets. Slowing or braking the train is similarly accomplished.



Levitation electromagnets drawn up toward the rail in the guideway levitate Japan's magnetically levitated train.

Because the train rides a few centimetres above the track, there is no friction due to moving parts; this allows maglev trains to achieve much greater speeds than conventional trains. Test models in Japan and Germany have recorded speeds of 400 to 500 km/h. However, maglevs are expensive to build and operate. They have no wheels, so they cannot run on existing tracks and conventional trains cannot run on maglev tracks —— so entirely new tracks must be built. Still, several countries are investigating how best to use this exciting new technology.

Levitation and Diamagnetic Materials

Levitation can also be achieved using certain types of materials, which are known as "diamagnetic" and are not superconductive. Diamagnetic materials are normally non-magnetic materials that become magnetized in a direction opposite to an applied magnetic field. Recently, physicists in Holland used a magnetic field of about 10 T to levitate a variety of seemingly non-magnetic materials, including a hazelnut, a strawberry, a drop of water, and a live frog. According to the researchers, the frog showed no ill effects from its adventure in levitation.

Frogs helped in levitation experiments and experienced no ill effects.

Then there's the other obvious question: Can people be levitated? In principle, the answer is yes, but in practice, the answer for now is no. Existing magnets are capable of levitating objects a few centimetres in diameter. Levitating a person would require an enormous electromagnet operating at 40 T, with about 1 GW of continuous power consumption. That's the same amount of power required to light 10 million 100 W light bulbs! Until more efficient ways can be found to make strong electromagnets, or people can be turned into frogs, people will be forced to walk with their feet on the ground.

Making Connections

1. Conduct a feasibility study for a maglev train to operate between two large cities.
2. Discuss the importance of space-based research and how diamagnetic levitation can be used to augment it.

8.3 Applications of Magnetic and Electric Fields

SECTION EXPECTATIONS

- Define and describe the concepts related to electric, gravitational, and magnetic fields.
- Predict the forces acting on a moving charge and on a current-carrying conductor in a uniform magnetic field.
- Determine the resulting motion of charged particles by collecting quantitative data from experiments or computer simulations.
- Describe instances where developments in technology resulted in advancement of scientific theories.

KEY TERMS

- particle accelerator
- mass spectrometer
- cyclotron
- synchrocyclotron
- betatron
- linear accelerator
- synchrotron

In previous science courses, you have probably read that the mass of an electron is 9.1094×10^{-31} kg and that the mass of a proton is 1.6726×10^{-27} kg. Did you ever wonder how it was possible for anyone to measure masses that small — especially to five significant digits — when there are no balances that can measure masses that small?

Atomic masses are determined by mass spectrometers, which are instruments that are based on the behaviour of moving charges in magnetic fields. The same principle causes motors to turn and prevents high-speed ions in the solar wind from bombarding Earth — except at the North and South Poles, as you read in the chapter introduction. In this section, you will learn more about moving charges in magnetic fields and many of the technologies based on this principle.

In Grade 11 physics, you were introduced to the force acting on a current-carrying conductor in a magnetic field and its application, the motor principle. The force acting on a conductor is actually due to the flow of charge through it and, in fact, the force acting on the charge is quite independent of the conductor through which the charge travels.

When a beam of charged particles is fired into a magnetic field, the following properties are observed.

- The beam will not be deflected if the direction of travel of the charges is parallel to the magnetic field.
- Maximum deflection occurs when the beam is aimed *perpendicular* to the direction of the magnetic field.
- The magnetic deflecting force is always perpendicular to *both* the direction of travel of the charge *and* the magnetic field.
- The magnitude of the magnetic deflecting force is directly proportional to the magnitude of the charge on each particle: $F_M \propto q$.
- There is no magnetic force on a stationary charge.
- The magnitude of the force is directly proportional to the speed of the charged particles: $F_M \propto v$.
- The magnitude of the force is directly proportional to the magnetic field intensity: $F_M \propto B$.
- The magnitude of the force depends on the sine of the angle between the direction of motion of the charge and the applied magnetic field: $F_M \propto \sin\theta$.

These proportional relationships can be summarized by one joint proportion statement.

$$F \propto qvB \sin \theta$$
$$F = kqvB \sin \theta$$

The definition of the unit for the magnetic field intensity, B, was chosen to make the value of the constant k equal to unity, so $F = qvB \sin \theta$. If you solve the equation for B, you can see the units that are equivalent to the unit for the magnetic field intensity.

$$B = \frac{F}{kqv \sin \theta}$$

The unit, one tesla (T), was chosen as the strength of the magnetic field. A charge of one coulomb, travelling with a speed of one metre per second perpendicular to the magnetic field ($\theta = 90^\circ$ and $\sin \theta = 1$) experiences a force of one newton. By substituting units into the equation for B and letting $k = 1$ and $\sin 90^\circ = 1$, you can find the equivalent of one tesla.

$$\text{tesla} = \frac{\text{newton}}{\text{coulomb}\,\frac{\text{metre}}{\text{second}}}$$

$$\text{T} = \frac{\text{N}}{\text{C}\frac{\text{m}}{\text{s}}} = \frac{\text{N} \cdot \text{s}}{\text{C} \cdot \text{m}}$$

The direction of the magnetic force on the charge q follows a right-hand rule. If you arrange your right hand so that the fingers are pointing in the direction of the magnetic field, $\vec{B}$, and the thumb is pointing in the direction of motion of a *positively* charged particle, q, then the palm of the hand points in the direction of the magnetic force, $\vec{F}_M$, acting on the particle.

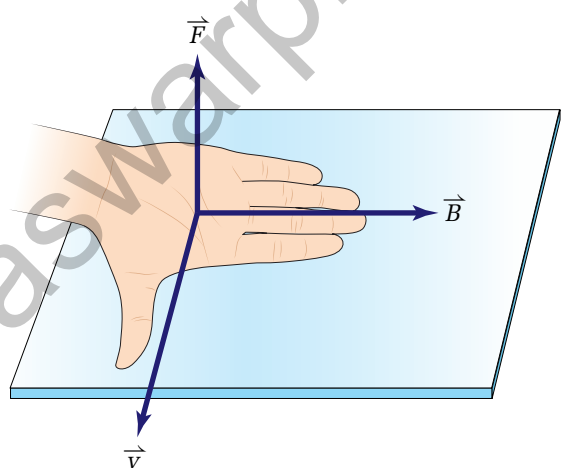


Figure 8.10 The direction of the magnetic force on the charge q follows a right-hand rule.

MATH LINK

You have probably noticed that the equation for the force on a moving charge in a magnetic field is the product of two vectors that yields another vector. In mathematics, this type of equation is called a "vector product" or "cross product" and is written $\vec{F} = q\vec{v} \times \vec{B}$. The magnitude of a vector product is equal to the product of the magnitudes of the two vectors times the sine of the angle between the vectors. The direction of the vector product is perpendicular to the plane defined by the vectors that are multiplied together.

ELECTRONIC LEARNING PARTNER

Refer to your Electronic Learning Partner to enhance your understanding of magnetic fields.

FORCE ON A MOVING CHARGE IN A MAGNETIC FIELD

The magnitude of the magnetic force exerted on a moving charge is the product of the magnitudes of the charge, the velocity, the magnetic field intensity, and the sine of the angle between the velocity and magnetic field vectors.

$$F_M = qvB \sin \theta$$

Quantity	Symbol	SI unit
magnetic force on a moving charged particle	F_M	N (newtons)
electric charge on the particle	q	C (coulombs)
magnitude of the velocity of the particle (speed)	v	$\frac{m}{s}$ (metres per second)
magnetic field intensity	B	T (teslas)
angle between the velocity vector and the magnetic field vector	θ	degree (The sine of an angle is a number and has no units.)

Unit Analysis

$$\text{newton} = \text{coulomb}\left(\frac{\text{metre}}{\text{second}}\right)\text{tesla}$$

$$\text{N} = \text{C}\left(\frac{\text{m}}{\text{s}}\right)\text{T} = \frac{\text{C}\cdot\text{m}\cdot\text{T}}{\text{s}} = \text{N}$$

$\vec{F}$ is perpendicular to the plane containing $\vec{v}$ and $\vec{B}$. Since one coulomb per second is defined as an ampere ($1\ \frac{\text{C}}{\text{s}} = 1\ \text{A}$), the tesla is often defined as $T = \frac{\text{N}}{\text{A}\cdot\text{m}}$.

Since the vectors $\vec{F}$, $\vec{v}$, and $\vec{B}$ are never in the same plane, physicists have accepted a convention for drawing magnetic fields. As shown in Figure 8.11, a magnetic field that is perpendicular to the plane of the page is drawn as crosses or dots. The crosses represent a field directed into the page and the dots represent a field coming out of the page.

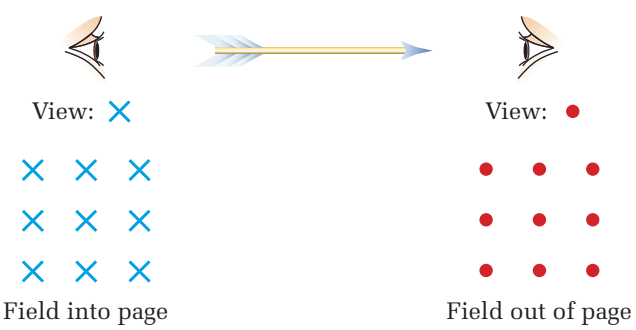


Figure 8.11 To remember the convention for drawing magnetic fields, think of the dot as the point of an arrow coming toward you. Think of the cross as the tail of the arrow going away from you.

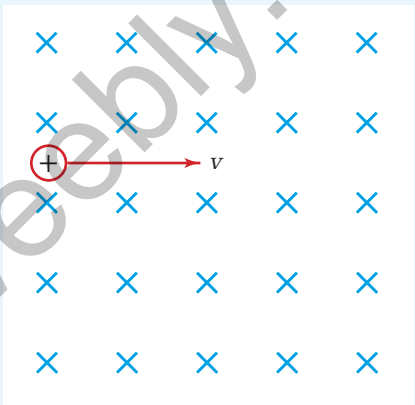
SAMPLE PROBLEM

Force on a Moving Charge

A particle carrying a charge of +2.50 μC enters a magnetic field travelling at 3.40×10^5 m/s to the right of the page. If a uniform magnetic field is pointing directly into the page and has a strength of 0.500 T, what is the magnitude and direction of the force acting on the charge as it just enters the magnetic field?

Conceptualize the Problem

- Make a sketch of the problem.
- The *charged particle* is *moving* through a *magnetic field*; therefore, it experiences a *force*.
- The *force* is always *perpendicular* to both the direction of the *velocity* and of the *magnetic field*.



Identify the Goal

The magnetic force, F_M, on the charged particle

Identify the Variables and Constants

Known	Implied	Unknown
$q = 2.50\,\mu\text{C}$	$\theta = 90^\circ$	F_M
$v = 3.40 \times 10^5\ \frac{\text{m}}{\text{s}}$		
$B = 0.500\ \text{T}$		

Develop a Strategy

Use the equation that relates the force on a charge in a magnetic field to the charge, velocity, and magnetic field intensity. Substitute numerical values and solve.

$$F_M = qvB\sin\theta$$

$$F_M = (2.50 \times 10^{-6}\ \text{C})\left(3.40 \times 10^5\ \frac{\text{m}}{\text{s}}\right)(0.500\ \text{T})(\sin 90^\circ)$$

$$F_M = 0.425\ \text{N}$$

Use the right-hand rule to determine the direction.

- Thumb represents travel of charge to the right.
- Fingers represent direction of magnetic field into the page.
- Palm represents direction of magnetic force on charge toward the top of the page.

The magnetic force on the moving charge is 4.25×10^{-1} N toward the top of the page.

Validate the Solution

A small charge combined with a high speed reasonably would produce the force calculated.

$$\cancel{C} \cdot \frac{\cancel{m}}{\cancel{s}} \cdot \frac{\text{N} \cdot \cancel{s}}{\cancel{C} \cdot \cancel{m}} = \text{N}$$

continued ►

continued from previous page

PRACTICE PROBLEMS

12. An alpha particle, charge $+3.2 \times 10^{-19}$ C, enters a magnetic field of magnitude 0.18 T with a velocity of 2.4×10^{6} m/s to the right. If the magnetic field is directed up out of the page, what is the magnitude and direction of the magnetic force on the alpha particle?

13. A proton is projected into a magnetic field of 0.5 T directed into the page. If the proton is travelling at 3.4×10^{5} m/s in a direction [up 28° right], what is the magnitude and direction of the magnetic force on the proton?

14. An electron travelling at 6.00×10^{5} m/s enters a magnetic field of 0.800 T. If the electron experiences a magnetic force of magnitude 3.84×10^{-14} N, what was the original direction of the electron's velocity relative to the magnetic field?

15. A particle having a mass of 0.200 g has a positive charge of magnitude 4.00×10^{-6} C. If the particle is fired horizontally at 5.0×10^{4} m/s[E], what is the magnitude and direction of the magnetic field that will keep the particle moving in a horizontal direction as it passes through the field?

16. A +4.0 μC charge is projected along the positive x-axis with a speed of 3.0×10^{5} m/s. If the charge experiences a force of 5.0×10^{-3} N in the direction of the negative y-axis, what must be the magnitude and direction of the magnetic field?

The magnetic force experienced by a charged particle moving freely through a perpendicular magnetic field can be compared to the force exerted on a current-carrying conductor that also is perpendicular to the magnetic field. The net force on a conductor of length l will be the total of the individual forces acting on each charge.

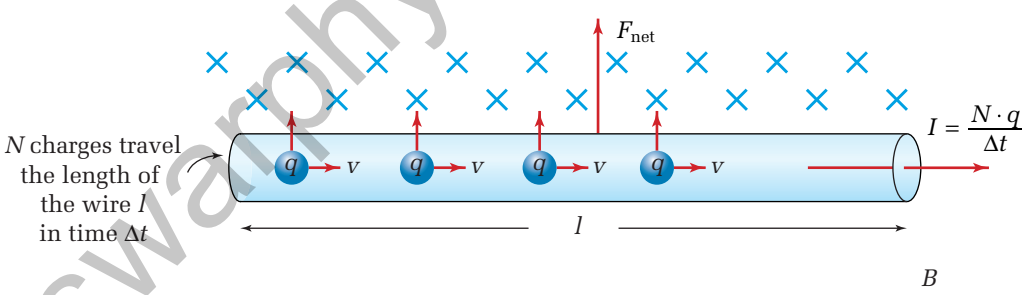


Figure 8.12 When the charges that are moving through a magnetic field are confined to a wire, the magnetic force appears to act on the wire.

If N charges, each of magnitude q, travel the distance equal to the length of the wire l in a time interval Δt, the velocity will be $l/\Delta t$. The net force will be as follows.

$$F_{net} = N \cdot qvB \sin\theta$$

$$F_{net} = N \cdot q \cdot \frac{l}{\Delta t} \cdot B \sin\theta$$

$$F_{net} = \left(\frac{N \cdot q}{\Delta t}\right) \cdot l \cdot B \sin\theta$$

$\frac{N \cdot q}{\Delta t}$ is the total charge per unit time, the current.

FORCE ON A CURRENT-CARRYING CONDUCTOR IN A MAGNETIC FIELD

The magnitude of force on a conductor carrying a current in a magnetic field is the product of the magnetic field intensity, the length of the conductor, the current in the conductor, and the sine of the angle that the electric current makes with the magnetic field vector.

$$F_M = IlB \sin \theta$$

Quantity	Symbol	SI unit
magnetic force on a current-carrying conductor	F_M	N (newtons)
electric current in the conductor	I	A (amperes)
length of the conductor	l	m (metres)
magnetic field intensity	B	T (teslas)
angle between the conductor and the magnetic field vector	θ	degree (The sine of an angle is a number and has no units.)

Unit Analysis

$$\mathrm{T \cdot A \cdot m} = \frac{\mathrm{N} \cdot \cancel{\mathrm{s}}}{\cancel{\mathrm{C}} \cdot \cancel{\mathrm{m}}} \cdot \frac{\cancel{\mathrm{C}}}{\cancel{\mathrm{s}}} \cdot \cancel{\mathrm{m}} = \mathrm{N}$$

Although many of the quantities in the equation for the magnetic force on a current-carrying wire are vectors, the equation can be used only to determine the magnitude of the force, so the vector notation has not been used. Directions must be determined by the relevant right-hand rules. $\vec{F}_M$ is perpendicular to the plane containing $\vec{v}$ and $\vec{B}$. The right-hand rule for the direction of the force is shown in Figure 8.13.

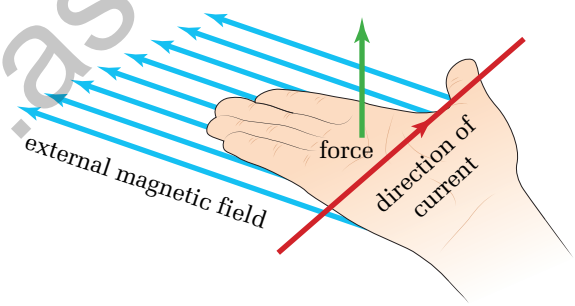


Figure 8.13 The thumb points in the direction of the current, the fingers point in the direction of the magnetic field vector, and the palm of the hand indicates the direction of the force on the conductor.

SAMPLE PROBLEM

Force on a Current-Carrying Conductor

A wire segment of length 40.0 cm, carrying a current of 12.0 A, crosses a magnetic field of 0.75 T[up] at an angle of [up 40° right]. What magnetic force is exerted on the wire?

Conceptualize the Problem

- *Charges* in the wire are *moving* through a *magnetic field*.
- *Moving charges* in a *magnetic field* experience a *force*.
- The *magnetic force* is related directly to the *magnetic field intensity*, the *electric current*, the *length* of the wire segment, and the *angle* between the wire and the magnetic field.

Identify the Goal

The magnetic force, F_M, on the wire segment

Identify the Variables and Constants

Known	Unknown
$l = 40.0\,\text{cm}$	F_M
$I = 12.0\,\text{A}$	
$B = 0.75\,\text{T}$	
$\theta = 40^\circ$ between B and I	

Develop a Strategy

Find the force using the relevant equation that relates force, magnetic field, current, and length of wire that is in the field.

$$F = IlB\sin\theta$$
$$F = (12.0\ A)(0.40\ m)(0.75\ \text{T})(\sin 40^\circ)$$
$$F = 2.3140\ \text{N}$$
$$F \cong 2.3\ \text{N}$$

Determine the direction using the right-hand rule; only the [right] component of the current, perpendicular to the magnetic field direction, contributes to the magnetic force.

- Thumb of right hand points right
- Fingers point up toward top of page

The force will be out of the page, according to the right-hand rule.

- Palm will be facing up out of the page

The force of the magnetic field on the conductor is 2.3 N[out of the page].

Validate the Solution

The force seems to be consistent with the magnetic field and current values. The direction is consistent with the right-hand rule.

$$\text{T}\cdot\text{A}\cdot\text{m} = \frac{\text{N}}{\cancel{\text{A}}\cdot\cancel{\text{m}}}\cdot\cancel{\text{A}}\cdot\cancel{\text{m}} = \text{N}$$

17. A wire 82.0 m long runs perpendicular to a magnetic field of strength 0.20 T. If a current of 18 A flows in the wire, what is the magnitude of the force of the magnetic field on the wire?

18. A wire 65 cm long carries a current of 20.0 A, running east through a uniform magnetic field. If the wire experiences a force of 1.2 N[N], what is the magnitude and direction of the magnetic field?

19. A segment of conducting wire runs perpendicular to a magnetic field of 2.2×10^{-2} T. When the wire carries a current of 15 A, it experiences a force of 0.60 N. What is the length of the wire segment?

20. (a) What current would need to flow east along the equator through a wire 5.0 m long, which weighs 0.20 N, if the magnetic field of Earth is to hold the wire up against the force of gravity? (Assume that Earth's horizontal magnetic field intensity at this location is 6.2×10^{-5} T.)

(b) Discuss the practicality of this result.

Circular Motion Caused by a Magnetic Field

When a charge enters a magnetic field at right angles, the resulting magnetic force on the particle is perpendicular to both the velocity vector and the magnetic field vector. Consequently, there is no component of the force in the direction of motion and the speed will not change. As the charge is deflected by the force, it still remains perpendicular to the magnetic field. This means that it will always experience a constant magnitude of force *perpendicular* to its motion. This is the standard requirement for circular motion at constant speed. The magnetic force is providing the centripetal force on the particle.

$$qvB = \frac{mv^2}{r}$$

$$r = \frac{mv^{\cancel{2}}}{q\cancel{v}B}$$

$$r = \frac{mv}{qB}$$

Motion Due to Both Electric and Magnetic Fields

You have now studied ways in which electric and magnetic fields can exert forces on a charged particle. The following are examples in which both types of field affect the motion of a particle.

Simple Particle Accelerator

A simple **particle accelerator** consists of a particle source, a pair of parallel plates, and an accelerating potential difference. The particle source can be simply a spark gap that causes the surrounding gas molecule to become ionized, that is, separate into positive and negative particles. These "ions" then enter the region

COURSE CHALLENGE

Field Energy

A book falls from your desk, a movie plays on a television screen, and a homing pigeon can find its way home, all because of the energy within a field. Refer to page 604 of this textbook for suggestions relating field energy to your Course Challenge.

between the parallel plates and are accelerated by the potential difference between the plates. A hole in the opposite plate allows the particles to continue into the region beyond the plates. For this reason the apparatus is sometimes called a "particle gun," or in the case of electrons, an "electron gun." As a result, the kinetic energy of the emerging particles can be expressed in terms of the work done on them between the parallel plates: $\frac{1}{2}mv^2 = qV$.

Velocity Selector

A velocity selector is a device quite often associated with the parallel plate particle accelerator. A beam of particles having different velocities, as a result of carrying different charges, is "filtered" so that only those particles with the same velocity continue. The apparatus consists of a crossed (perpendicular) electric and magnetic field. A positively charged particle, for example, would experience an upward force due to the magnetic field and a downward force due to the electric field. If the two forces are equal, the particle will travel straight through the velocity selector.

You can determine the velocity of particles that will pass directly through the velocity selector by taking the following steps.

- Set the electric and magnetic forces equal to each other. $F_M = F_Q$
- Write the expressions for the two forces. $F_M = qvB$, $F_Q = qE$
- Substitute the expressions for the values of the forces into the first equation. $qvB = qE$
- Solve for the velocity. $v = \frac{\cancel{q}E}{\cancel{q}B}$, $v = \frac{E}{B}$

Only charged particles with a velocity that matches the ratio of the electric field intensity to the magnetic field intensity will continue to travel in a straight line. Particles with other speeds will be deflected up or down and absorbed by the surrounding material.

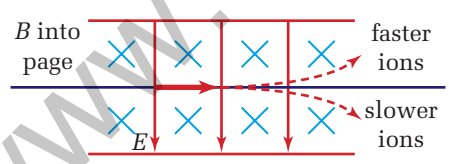


Figure 8.14 Only charges having one specific velocity will travel in a straight line. All others will be diverted up or down.

Mass Spectrometer

The **mass spectrometer** is an instrument that can separate particles of different mass and, in fact, measure that mass. The first stage of a mass spectrometer is a velocity selector. Then, ions of the selected speed enter a magnetic field in a direction perpendicular to the field. While in the magnetic field, the ions experience a magnetic force that is always perpendicular to the direction of their motion.

You will recognize this type of force as a centripetal force. You can see how the mass spectrometer separates particles of different masses by analyzing the following steps.

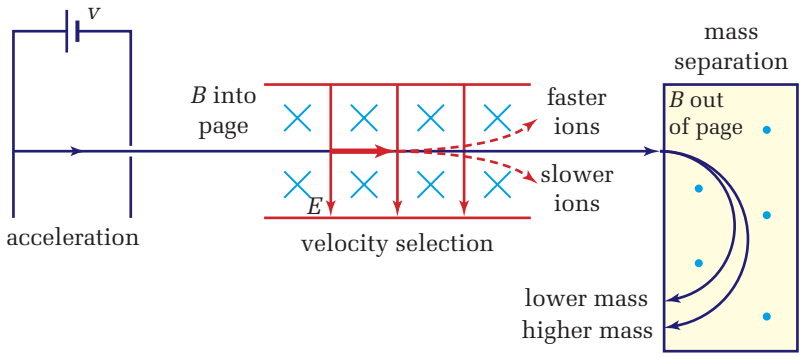


Figure 8.15 The path of positive ions through the acceleration, velocity selection, and mass separation process

- The magnetic field supplies the centripetal force. $F_C = F_M$
- Substitute the expressions for centripetal and magnetic forces. $\frac{mv^2}{r} = qvB$
- Solve for m. $m = \frac{rq\cancel{v}B}{v^{\cancel{2}}}$

$$m = \frac{rqB}{v}$$

The velocity of the particles is known because it was selected before the particles entered the magnetic field. The charge is known due to the method of creating ions that entered the velocity selector. The instrument measures the radius of the circular path. The only unknown quantity is the mass.

By observing the radius for particles of known charge, the mass can be determined. This is particularly useful for determining the relative proportions of "isotopes," atoms that have the same number of protons but different numbers of neutrons.

PHYSICS FILE

Portable mass spectrometers are used at airports and in other areas where security is a priority, in an attempt to detect particles associated with materials used in manufacturing explosives.

SAMPLE PROBLEM

Mass Spectrometer

A positive ion, having a charge of 3.20×10^{-19} C, enters at the extreme left of the parallel plate assembly associated with the velocity selector and mass spectrometer shown in Figure 8.15.

(a) If the potential difference across the simple accelerator is 1.20×10^3 V, what is the kinetic energy of the particle as it leaves through the hole in the right plate?

continued ▶

continued from previous page

(b) The parallel plates of the velocity selector are separated by 12.0 mm and have an electric potential difference across them of 360.0 V. If a magnetic field of strength 0.100 T is applied at right angles to the electric field, what is the speed of the particles that will be "selected" to pass on to the mass spectrometer?

(c) When these particles then enter the mass spectrometer, which shares a magnetic field with the velocity selector, the radius of the resulting circular path followed by the particles is 6.26 cm. What is the mass of the charged particles?

(d) What is the nature of the particles?

Conceptualize the Problem

- When the *charged* particles enter the *electric field*, the field does *work* on the particles, giving them *kinetic energy*.
- When the *moving* particles pass through the crossed *electric* and *magnetic* fields, only those of *one specific velocity* pass through undeflected.
- When the selected particles enter the *magnetic field*, the magnetic force provides a *centripetal force*.

Identify the Goal

(a) The kinetic energy, E_k, of the particle

(b) The speed, v, of the particles that will be "selected"

(c) The mass, m, of the charged particles

(d) The nature of the particles

Identify the Variables and Constants

Known		Unknown
$q = 3.20 \times 10^{-19}$ C	$\Delta d = 12.0$ mm	E_k
$V_1 = 1.20 \times 10^3$ V	$B = 0.100$ T	v
$V_S = 360.0$ V	$r = 6.26$ cm	m

Develop a Strategy

The energy of a charged particle is related to the accelerating potential difference.

$$E_k = qV$$
$$E_k = (3.20 \times 10^{-19}\ \text{C})(1.2 \times 10^3\ \text{V})$$
$$E_k = 3.84 \times 10^{-16}\ \text{J}$$

(a) The kinetic energy of the particle was 3.84×10^{-16} J.

The selected velocity is related to the electric and magnetic fields. The electric field is related to the potential difference and the distance of separation of the plates.

$$E = \frac{V}{\Delta d} = \frac{360.0\ \text{V}}{1.20 \times 10^{-2}\ \text{m}} = 3.00 \times 10^4\ \frac{\text{N}}{\text{C}}$$

$$v = \frac{E}{B} = \frac{3.00 \times 10^4\ \frac{\text{N}}{\text{C}}}{0.100\ \text{T}}$$

$$v = 3.00 \times 10^5\ \text{m/s}$$

(b) The speed of the particles was 3.00×10^5 m/s.

The mass of the particle is related to the charge, magnetic field, radius of path, and speed.

$$m = \frac{qBr}{v}$$

$$m = \frac{(3.20 \times 10^{-19}\ \text{C})(0.100\ \text{T})(6.26 \times 10^{-2}\ \text{m})}{3.00 \times 10^{5}\ \frac{\text{m}}{\text{s}}}$$

$$m = 6.68 \times 10^{-27}\ \text{kg}$$

(c) The mass of the particles was 6.68×10^{-27} kg.

The charge is two times the charge on a proton. The mass is four times the mass of a proton. The particle seems to be an alpha particle, which is the positive nucleus of a helium atom.

(d) The particles seem to be alpha particles.

Validate the Solution.

The mass is what you would expect for a small atom. The units cancel to give kg, which is correct.

$$\frac{\text{C}\cdot\text{T}\cdot\text{m}}{\frac{\text{m}}{\text{s}}} = \cancel{\text{C}}\cdot\frac{\text{N}\cdot\text{s}}{\cancel{\text{C}}\cdot\cancel{\text{m}}}\cdot\cancel{\text{m}}\cdot\frac{\text{s}}{\text{m}} = \frac{\text{N}\cdot\text{s}^2}{\text{m}} = \frac{\text{kg}\cdot\cancel{\text{m}}}{\cancel{\text{s}^2}}\cdot\frac{\cancel{\text{s}^2}}{\cancel{\text{m}}} = \text{kg}$$

PRACTICE PROBLEMS

21. A proton is accelerated across parallel plates, through a potential difference of 180.0 V. Calculate

(a) the final kinetic energy of the proton

(b) the final velocity of the proton, assuming its mass is 1.67×10^{-27} kg

22. A particle of mass 1.2 g and charge +3.0 μC is held suspended against the force of gravity between a parallel pair of plates that are 15.0 mm apart.

(a) In which direction does the electric field vector point?

(b) What is the magnitude of the electric potential difference connected across the plates?

23. An isotope of hydrogen having a proton and a neutron in its nucleus is ionized and the resulting positive ion (deuteron) travels in a circular path of radius 36.0 cm in a perpendicular magnetic field of strength 0.80 T.

(a) Calculate the speed of the deuteron.

(b) What was the accelerating potential that gave the deuteron this speed?

24. An electron of mass 9.11×10^{-31} kg travels perpendicularly through a magnetic field of strength 6.8×10^{-5} T at a speed of 3.4×10^{5} m/s. What is the radius of the path of the electron?

25. What is the speed of a beam of electrons if in passing through a 0.80 T magnetic field they remain undeflected, due to a balancing electric field of 5.4×10^{3} N/C?

26. An isotope of hydrogen passes, without deflection, through a velocity selector that has an electric field of 2.40×10^{5} N/C and a magnetic field of 0.400 T. It then enters a mass spectrometer that has an applied magnetic field of 0.494 T and consequently describes a circular path with a radius of 3.80 cm.

(a) What is the mass of the particle?

(b) Which isotope of hydrogen is it?

INVESTIGATION 8-B

Measuring a Magnetic Field

TARGET SKILLS

- Performing and recording
- Analyzing and interpreting
- Communicating results

In this investigation, you will use a current balance to determine the strength of the magnetic field at the central axis of a solenoid.

Problem

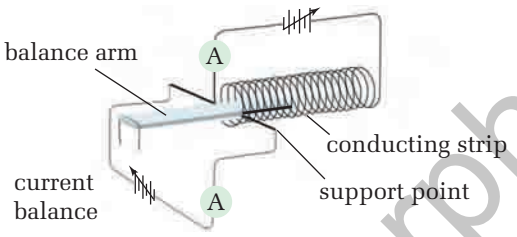
How can you measure magnetic field intensity with a current balance?

Equipment

- current balance and solenoid
- 2 variable power supplies (12 V DC)
- 2 DC ammeters
- electronic balance
- scissors
- string

Procedure

1. Set up the current balance-solenoid apparatus, as shown in the diagram.



2. With the power off, adjust the balance arm so that it is horizontal.
3. Turn on the power to the coil and balance arm. Adjust the polarity so that the conducting arm inside the solenoid is forced downward.

CAUTION The current in both the arm and solenoid can create enough heat to cause a burn.

4. Set and record the current in the solenoid to the upper range of its values. Set and record the current through the balance arm to the high end of its range, forcing down the balance arm inside the solenoid.
5. Loop a length of string over the outside end of the balance arm and, using scissors, adjust its length until the balance arm is horizontal.
6. Without changing any settings, turn off the current to both sources. Determine the mass of the string.
7. Keeping the solenoid current constant, repeat the experiment five more times, using a smaller balance current. Record the value of the balance current and the mass of the string each time.
8. Carefully measure
   (a) the lengths of the solenoid and the current arm
   (b) the number of turns in the solenoid
   (c) the distance of the suspension point of the current balance to each of its ends (lever arms)

Analyze and Conclude

1. For each trial, use the mass of the string and the principle of levers to calculate the force acting down on the current arm. Record your data.
2. Draw a graph with the force acting on the current arm versus current in the current arm.
3. Describe the relationship between I and F when the magnetic field is kept constant?
4. Measure the slope of your graph. Use your data to determine the magnetic field, B, inside the solenoid.

Apply and Extend

5. Using your data and the equation below, calculate the strength of the magnetic field.

$$B = \mu_0 \frac{N \cdot I_S}{l}$$

$\mu_0 = 1.257 \times 10^{-6}\ \mathrm{T \cdot m/A}$, N is the number of turns in the solenoid, l is the length of the solenoid, and I_S the current flowing in the solenoid wire.

6. How did your two values for the magnetic field in the solenoid compare? What might cause them to differ?

Particle Accelerators

In the early part of the twentieth century, the development of the theory of the structure of the atom and its nucleus depended to a large degree either on the spontaneous disintegration of radioactive nuclei or on observations made when the products of those spontaneous disintegrations were directed at other nuclei. The particles emitted during natural disintegrations, however, such as the α-particles used by Rutherford in his experiments, provided only limited opportunity to observe nuclear reactions during bombardment. The particles were limited in energy and were emitted randomly in all directions, so they were difficult to harness in sufficient quantities to provide reliable results.

To overcome the difficulties of availability and reliability, particle accelerators were developed that were capable of emitting high-speed, subatomic-sized particles (protons, electrons) in sufficient numbers. Particle accelerators today are capable of accelerating charged particles to energies close to one million million electron volts, or 1000 GeV. This in turn has allowed physicists to investigate the fundamental composition of matter even more deeply, with the result that more and more fundamental particles are known to exist and complex models of the structure of matter have been developed. You will learn more about these models in Unit 5.

The Cockcroft-Walton Proton Accelerator

The first particle accelerator for use in nuclear research was built in 1932 by J.D. Cockcroft and E.T.S. Walton, students of Ernest Rutherford at the Cambridge Laboratory in England. In this accelerator, protons were introduced into the top of an evacuated glass tube and accelerated by using a potential difference between electrically charged metal cylinders. Since it is not possible to maintain a potential difference much more than 200 000 V between electrodes in an evacuated tube, Cockcroft and Walton used special multi-stage accelerator tubes, with each stage powered by a unique charging circuit. The protons accelerated by this arrangement approached energies of 1 MeV.

At the bottom of the glass tube, they placed a lithium target and consequently observed the first nuclear transformation caused by artificially accelerated particles. The bombardment of the lithium atoms with protons resulted in the formation of helium nuclei. For their work, Cockcroft and Walton were awarded the Nobel Prize in Physics in 1951.

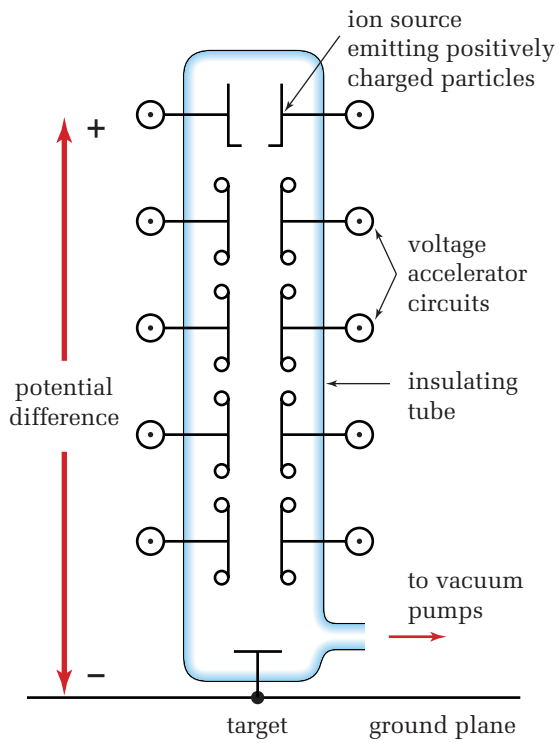


Figure 8.16 One type of multi-stage accelerator tube

The Cyclotron

To avoid the problems associated with very high voltages, Ernest O. Lawrence and his colleagues at the University of California at Berkeley designed an accelerator based on a circular path that subjected the charged particles to a large number of small increases in potential. This was achieved by the use of a pair of evacuated hollow semicircular chambers (called "dees," because they are shaped like the letter D). The charged particles are injected into the chambers at the centre. This device is called a **cyclotron**.

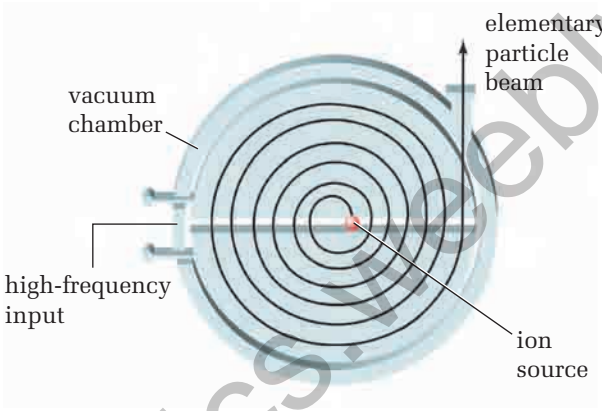


Figure 8.17 A cyclotron

The dees are positioned between the poles of an electromagnet that provides a uniform magnetic field perpendicular to the path of the charged particle inside the chamber, thus causing its circular motion. A potential difference is applied between the two chambers, so that as the charged particle crosses from one chamber to the next, it will be accelerated by the potential difference. The particle will speed up and, as a result, the radius of its path will increase. In order for the particle to speed up when it crosses the gap between the dees again, the direction of the potential difference must be reversed. This alternating potential difference is kept in phase with the frequency of orbit of the charged particle so that it will always speed up when it crosses the gap between the chambers. Consequently the particle will spiral outward until it reaches the outer edge of the dee, where a magnetic field is applied to deflect the particle out through a gate and onto a target. The first cyclotron built in 1931 produced ions of energy 80 keV, but by the latter part of that decade, energies of 30 MeV were quite common.

As you will learn in Unit 5, when particles reach speeds close to the speed of light, relativistic effects become prominent. In the case of the cyclotron, the mass of the particle increases to such an extent that it becomes necessary to synchronize the alternating potential difference with the time of travel of the particle.

The Synchrocyclotron

In the **synchrocyclotron**, an adaptation of the cyclotron, the frequency of the accelerating electric field, applied between the dees, is adjusted to allow for the relativistic increase in mass of the particles. Since the change in frequency required takes approximately 10 ms, the ions are delivered in small bursts, rather than continuously. This results in the intensity of the ion beam being lower than the conventional cyclotron. This is compensated for by using larger magnets, although cost then becomes a limiting factor.

The Betatron

The principle of the cyclotron has been adapted to allow for the acceleration of electrons. Since electrons were historically called "beta particles," the accelerator is called a **betatron**. Instead of allowing the electrons to spiral outward, a magnetic field applied along the central axis of an evacuated doughnut is uniformly increased. This increasing magnetic field induces an electric field that causes the electron to speed up but retain the same radius, inside the doughnut.

The Linear Accelerator (LINAC)

New **linear accelerators** differ from earlier machines, such as the Cockcroft-Walton accelerator, in that they use electric fields alternating at radio frequencies to accelerate the particles, rather than high voltages.

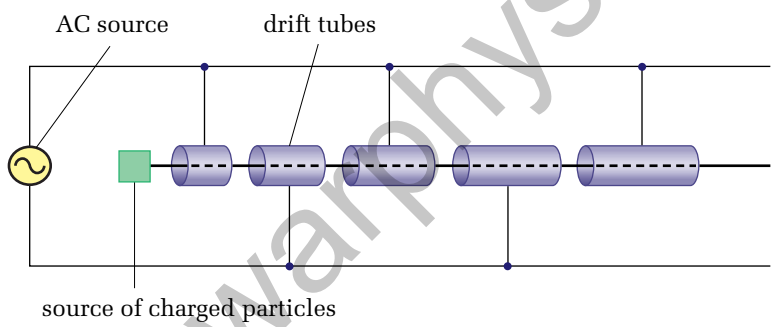


Figure 8.18 Schematic of a linear accelerator

The acceleration tube consists of many individual drift tubes that are charged alternately positive and negative. When a positive particle enters the tube, if the first drift tube is negative, it will attract the particle. Inside the tube, there is no electric field, so the particle "drifts" through at constant speed. If the electric field is reversed as the particle leaves the first tube, it will accelerate toward the second drift tube and enter it at a higher speed. This second tube is longer and the particle will leave it just as the potential reverses and it will be attracted to the third drift tube. Hence, the particle is accelerated between a long series of drift tubes. The Stanford Linear Accelerator Centre linear accelerator is

3.2 km long, contains 240 drift tubes, and is designed to accelerate electrons to energies above 20 GeV.

Synchrotron

A very efficient way to accelerate protons is to combine the features of the cyclotron and the linear accelerator. Such a device is the **synchrotron**.

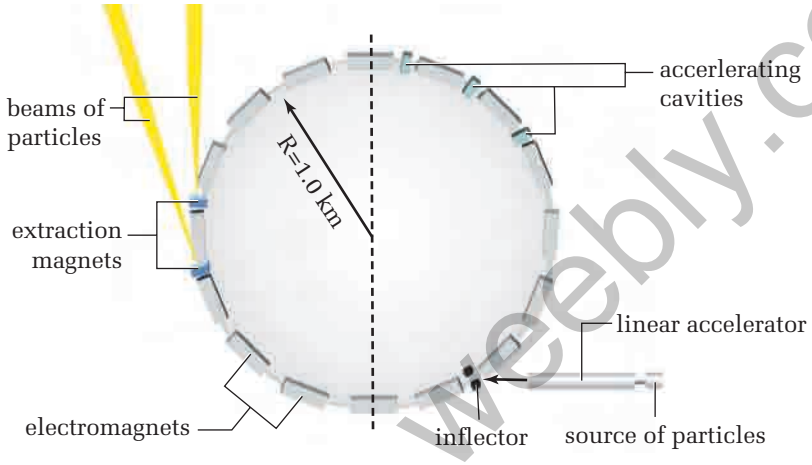


Figure 8.19 A synchrotron

Since a magnetic field is required only to maintain the circular orbit, rather than use one large central magnet, a series of ring magnets surrounding a doughnut-shaped vacuum tank is used, making the synchrotron much more economical. At repeated locations along the circular path, high-frequency accelerating cavities (much like short linear accelerators) are inserted to accelerate the protons. This combined technique produces protons of enormous energy that can in turn be directed at other targets and the resulting fundamental particles can be investigated. In 1954, Lawrence, the designer of the cyclotron, developed a synchrotron that produced protons with energies in the range of 6.2 billion electron volts. It was therefore called the "bevatron." (Today, it is identified as 6.2 GeV.) These protons were in turn used to discover the antiproton.

Other renowned synchrotron installations include the 1.0 TeV Tevatron at Fermilab (the Fermi National Accelerator Laboratory) in Illinois and the 400 GeV at CERN (European Council for Nuclear Research) near Geneva, Switzerland.

The Tokamak Fusion Test Reactor

An international group, International Thermonuclear Experimental Reactor (ITER), which includes Canada, is attempting to develop efficient nuclear fusion reactors, in which two isotopes of hydrogen (deuterium and tritium) collide with such high energy that they "fuse" to produce a helium nucleus, and at the same time release enormous amounts of energy. Fusion can occur only at

temperatures equivalent to the centre of stars, about 10^8 °C. At these temperatures, the fusion reactants actually break down into individual positive nuclei and negative electrons. This ionized gas is called a "plasma." It is because these ions are charged that it has been found both possible, and necessary, to confine them within a toroidal (doughnut-shaped) magnetic bottle, since no material bottle can exist at such high temperatures for its containment.

WEB LINK

www.mcgrawhill.ca/links/physics12

For an award-winning photograph taken inside a Tokamak reactor, go to the above Internet site and click on **Web Links**.

Figure 8.20
The Tokamak fusion test reactor

This magnetic confinement seems to have the greatest potential and its popular design is based on the Tokamak system, developed in the former U.S.S.R. ("Tokamak" is an acronym for the Russian translation of "toroidal magnetic chamber.")

8.3 Section Review

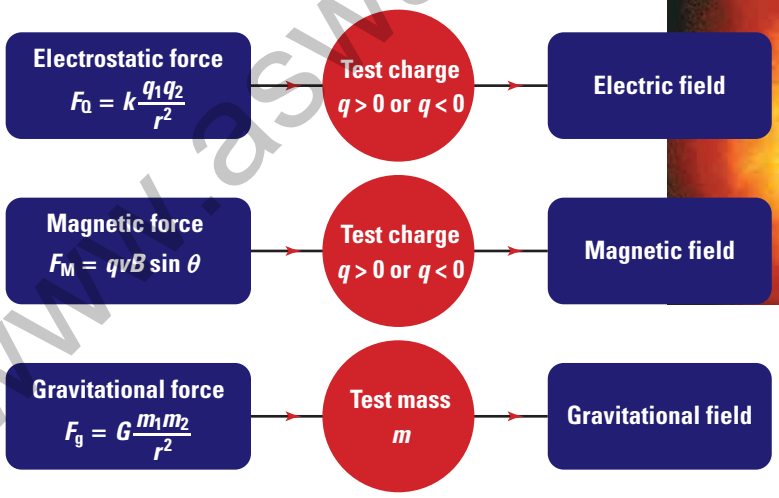
1. K/U
   (a) Under what conditions will a charged particle be subject to the maximum possible deflecting force when entering a magnetic field?
   (b) Under what conditions would the deflection be minimal?
2. K/U In what way is the force acting on a conductor carrying a current in a magnetic field similar to the deflecting forces described in question 1?
3. C Prepare a report or other presentation describing the many applications of the deflection of a charge by a magnetic field. Give a detailed account of the social significance of one of these applications.
4. K/U Explain how a particle accelerator and velocity selector complement the operation of a mass spectrometer.
5. MC In 2001, Canada and Japan were competing for the right to build a Tokamak-style fusion reactor. Canada's plan is to locate the reactor in Clarington, Ontario, adjacent to the Darlington nuclear plant. Research Canada's bid and make a presentation in which you
   (a) outline the reasons Canada's ITER team had for wanting to build the reactor
   (b) explain why that particular location was chosen
   (c) give your own opinions on the merit of the plan

CHAPTER 8 Review

REFLECTING ON CHAPTER 8

- The electric field pattern for a collection of charges can be generated by considering the field vectors due to each individual charge.
- Electric field lines leave a positive charge and/or enter a negative charge and are always perpendicular to the surface of a conductor.
- The number of field lines is proportional to the magnitude of the net charge.
- Equipotential surfaces are always perpendicular to the electric field lines
- The electric field is uniform between two oppositely charged parallel plates placed close together.
- The magnitude of the electric field between parallel plates is proportional to the charge density on the plates.
- The magnitude of the electric field intensity between parallel plates is given by the equation $\vec{E} = \frac{\Delta V}{\Delta d}$.
- The potential gradient describes the linear change in electric potential difference at positions between the plates.
- Electric fields can be used to transfer kinetic energy to charged particles, so that $qV = \frac{1}{2}mv^2$.
- An electric field can also be used to balance the force of gravity on a charged particle.
- The electron volt is an alternative unit for the energy of a charged particle.
- The charge on the electron was determined by Robert Millikan by measuring the terminal velocities of oil drops placed between a parallel plate apparatus.
- The drift velocity of the electrons in a conductor carrying a current is very slow, compared with the speed of the current itself.
- The Faraday ice-pail experiment demonstrated that the net charge inside a hollow conductor is zero; all charge resides on the outer surface.
- Faraday shielding is a useful way of preventing external electromagnetic interference in circuit components.
- The magnetic force on a charged particle travelling in a magnetic field is $F_M = qvB\sin\theta$ and its direction is described by a right-hand rule.

Concept Organizer



Solar flares are a result of the build up and then release of magnetic energy. Electrons, protons and nuclei are accelerated into the solar atmosphere. An amount of energy equivalent to millions of 100 Mt bombs is released.

- The magnetic force on a conductor carrying a current in a magnetic field is $F_M = IlB \sin \theta$ and its direction is described by a right-hand rule.
- A magnetic field can be used to cause the circular motion of a charged particle, so that $qvB = \frac{mv^2}{r}$.
- The velocity of a charged particle can be determined by electric and magnetic fields.
- The motion of a charged particle under the action of electric and/or magnetic fields forms the basis for applications such as cyclotrons, synchrocyclotrons, and mass spectrometers, among others.
- The containment of a plasma in a Tokamak fusion reactor is achieved through magnetic fields.

Knowledge/Understanding

1. Answer the following questions about Millikan's oil-drop experiment.
   (a) Describe the main features of the experiment.
   (b) What were the results of the experiment and their significance?
   (c) Draw free-body diagrams of an oil drop that is between two horizontal, parallel electrically charged plates under three conditions: the oil drop is stationary, the oil drop is falling toward the bottom plate, the oil drop is drifting upward.
   (d) What was the effect of Millikan's use of X rays in his experiment?
   (e) Explain why the plates in the experiment need to be horizontal.
2. Imagine that you are probing the field around a charge of unknown magnitude and sign. At a distance r from the unknown charge, you place a test charge of q_1. You then substitute q_1 with a second test charge, q_2, that has twice the charge of $q_1 (q_2 = 2q_1)$.
   (a) Compare the forces that would act on the two test charges.
   (b) Compare the electric field that would affect the two test charges.
3. State mathematically and describe in words the definition of a tesla.
4. Can the magnetic force change the energy of a moving charged particle? That is, can the magnetic force do work on the particle?
5. (a) What is the function of the alternating potential difference in a cyclotron? The potential difference is applied across which two parts of the cyclotron? Why does the potential difference have to alternate in polarity?
   (b) What is the function of the magnetic field in a cyclotron? Is the magnetic field constant or alternating in direction?
6. Mass spectrometers are used to determine the masses of positively charged atoms and molecules.
   (a) Draw a concept map of the physics principles on which mass spectrometers were developed.
   (b) Explain the function of a velocity selector when it is used in conjunction with a mass spectrometer.
7. What types of studies were conducted to probe atomic structure prior to the development of particle accelerators? What were the limitations of such studies for developing an understanding of the micro-world?
8. Contrast the designs of particle accelerators that accelerate particles linearly and those that accelerate particles in circular paths. What are the advantages and disadvantages of each design?

Inquiry

9. Research and make a model of one type of particle accelerator that is being used currently. Your model should include all critical components and show their relationship with each other. Write a report to describe the physics involved in the accelerator's operation.

10. Suppose a cyclotron that normally accelerates protons is now to be used with alpha particles. What changes will have to be made to maintain synchronism?

11. Write a proposal for a new experimental facility to study the structure of the atom. Evaluate different particle accelerators and make a case for why you want to use a particular design. Include a cost analysis in your proposal. What are the most expensive components?

Communication

12. Use the rules of electric field line formation to explain why the lines around a negatively charged sphere are uniformly spaced and directed radially inward.

13. Outline, using vector diagrams, why the electric field at any point between two parallel plates is uniform and independent of the distance between the plates.

14. Consider a large, positively charged sphere. Two positively charged objects, A and B, are the same distance away from the sphere. Object A has a charge three times as large as that of object B. Which property will be the same for the two objects, the electric potential energy or the electric potential difference? Which property will be three times as large for object A compared to object B, the electric potential energy or the electric potential difference?

15. A proton passes through a magnetic field without being deflected. What can be said about the direction of the magnetic field in the region? Draw a sketch to illustrate your reasoning.

16. An electron is moving vertically upward when it encounters a magnetic field directed to the west. In what direction is the force on the electron?

17. Consider two parallel current-carrying wires. If the currents are in the same direction, will the force between the wires be attractive or repulsive? If the currents are in opposite directions, will the force between the wires be attractive or repulsive? Draw sketches to illustrate your answers.

18. A simple particle accelerator consists of three components. Make a sketch that identifies each component and its function. Why must ions be used instead of neutral particles?

Making Connections

19. A television uses a cathode ray tube to direct a beam of electrons toward a screen.
    (a) Draw a schematic diagram of a television picture tube as seen from the side and explain how electric and magnetic fields are used to accelerate and deflect the electrons.
    (b) Although electrons do not orbit in the magnetic field of a television cathode ray tube, their trajectory does follow a definable circular arc. On your diagram, label where this circular arc is located and explain how the radius of the arc can be used to determine the size of the picture tube.

20. The origins of naturally occurring magnetic fields are still poorly understood. Outline theories that explain the origin of Earth's magnetic field, the Sun's magnetic field, and the magnetic field of the Milky Way galaxy. Explain how these theories can be tested.

21. Although the cause of Earth's magnetic field is uncertain, it is known to be unstable. Analysis of rock strata in Earth's crust suggests Earth's magnetic field has reversed itself several times over the past five million years. How is this analysis done? What is the current thinking on why this occurs?

Problems for Understanding

22. The electric field intensity between two large, charged parallel plates is 400 N/C. If the plates are 5.0 cm apart, what is the electric potential difference between them?

23. Two parallel charged metal plates are separated by 8.0 cm. Identify four points along a line between the plates, A, B, C, and D, located at the following distances from the negatively charged plate: 0.0 cm, 2.0 cm, 4.0 cm, and 6.0 cm. The electric potential difference at point B, V_B, is measured to be 40.0 V.

(a) What is the electric potential difference across the plates?
(b) What is the electric potential difference at points A, C, and D?
(c) What is the potential difference between points A and B, B and C, and A and D?
(d) What is the electric field strength between the plates?
(e) A 1.0 μC test charge is placed first at point B, then at point C. What force does it experience at each point?
(f) Repeat (e) above for a 2.0 μC test charge.

24. In a Millikan oil-drop experiment, an oil drop of unknown charge is suspended motionless when the electric field is 3500 N/C. If the upper plate is positive and the drop weighs 2.8×10^{-15} N, determine **(a)** the charge on the oil drop and **(b)** the number of excess or deficit electrons on the oil drop.

25. A pith ball has a charge of –5.0 nC. How many excess electrons are on the pith ball?

26. A 10.5 cm wire carries a current of 5.0 A. What is the magnitude of the magnetic force acting on the wire if the wire is perpendicular to a uniform magnetic field of 1.2 T?

27. A small body moving perpendicular to a magnetic field of 0.25 T carries a charge of 6.5 μC. If it experiences a sideways force of 0.52 N, how fast is it travelling?

28. Consider a horizontal, straight 2.0 m wire carrying a 22 A current that runs from west to east. If the wire is in Earth's magnetic field, which points north with a magnitude of 4.0×10^{-5} T, calculate
(a) the magnetic force on the wire
(b) the maximum mass of the wire that would be supported by Earth's magnetic field

29. A velocity selector consists of an electric field of 20 000 V/m (2.0×10^4 V/m) perpendicular to a magnetic field of magnitude 0.040 T. A beam of ions, having passed through a velocity selector, is passed into a mass spectrometer that has the same magnetic field. Under these conditions, the radii of curvature of the path of singly charged lithium ions is found to be 78 cm. Calculate the mass of the lithium ions.

30. The period of a charged particle's circular orbit in a uniform magnetic field can be calculated from the radius of its orbit and its tangential velocity. Interestingly, both the period and its inverse, the frequency, are independent of the particle's speed and the radius of its orbit. Consider two electrons moving perpendicular to a 0.40 T magnetic field. One has a speed of 1.0×10^7 m/s and the other has a speed of 2.0×10^7 m/s.
(a) Calculate the radii of the orbits of the two electrons.
(b) Calculate their periods.
(c) Calculate their frequencies.
(d) Comment on the above results.

31. Suppose an electron and a proton are each injected perpendicularly into a uniform magnetic field with equal kinetic energies.
(a) Compare the periods of their orbits.
(b) Compare the radii of their orbits.

32. Charged particles from the Sun can be trapped by the magnetic field that surrounds Earth. If the particles enter the atmosphere, they can excite atoms in the air, resulting in the phenomenon of auroras. Consider a proton with a speed of 1.2×10^7 m/s that approaches Earth perpendicular to Earth's magnetic field. It is trapped and spirals down a magnetic field line.
(a) If the magnetic field strength at the altitude where the proton is captured is 2.0×10^{-5} T, calculate the frequency and radius of curvature of the proton's orbital motion.
(b) Repeat (a) for a proton that comes in at half the speed of the first proton.

33. An electron moves with a velocity of 5.0×10^6 m/s in a horizontal plane perpendicular to a horizontal magnetic field. It experiences a magnetic force that just balances the gravitational force on the electron.
(a) Calculate the strength of the magnetic field.
(b) If the electron is travelling north, what is the magnetic field direction?

Costs and Benefits of Physics Research

Background

Throughout history, societies have expended tremendous amounts of money and other resources on the accumulation of scientific knowledge. Often, at the time of expenditure, the direct value in monetary or other terms was not readily evident, so the debate always arises as to whether the costs of scientific research and related high-tech applications outweigh their benefits.

This unit contained an overview of a number of different particle accelerators. In some cases, the device, such as a mass spectrometer, is used to identify the elements contained in a substance. In other cases, such as the Conseil Européen pour la Recherche Nucléaire and Fermi National Accelerator Laboratory accelerators, the device accelerates charged particles to a speed at which not only do their own properties change, but their collisions with other particles create the formation of yet new and different particles. The high-energy collisions made possible by particle accelerators lead to new understandings of the structure of matter.

History has shown that the more society learns about the structure and behaviour of matter, the more this knowledge can be used to improve society's standard of living. On the other hand, the costs of such endeavours are not all monetary. Often related to research and development are side effects that affect the environment and the health and freedoms of a society.

Challenge

Build a class consensus on the costs and benefits of continuing public support for using particle accelerators in research and development in particular, and for physics research in general.

The U.S. Department of Energy's Fermi National Accelerator Laboratory.

Plan and Present

A. As a class, research and compile a list of particle accelerators that are currently in use, either for pure scientific research or for a particular technological application. Identify a select list of accelerators for further study. Divide the class into groups, assigning one accelerator to each group. Each group is to write a report on the accelerator's function, its associated costs, and its potential benefits. While developing this report, in preparation for the class debate described below, class members should decide on which side of the debate they want to participate.

B. Set up a class debate on the costs and benefits to society of the public funding of research using particle accelerators in particular, and on physics research in general.

ASSESSMENT

After you complete this project

- **assess the clarity of your report in explaining the costs and benefits of particle accelerators for research and development**
- **assess the success of your team in convincing the audience of your perspective during the debate**
- **assess the ability of the class to come to a consensus on a rational position on the costs and benefits of physics research**

Action Plan

1. Establish an evaluation method by preparing
   - a class rubric for evaluating individual group reports
   - a rubric for evaluating the class debate
2. Establish groups and then
   - as a class, brainstorm and conduct preliminary research into the types of particle accelerators currently in use
   - establish small working groups to investigate a representative number of particle accelerators
3. For the assigned accelerator, each small group will gather data on
   - the location and size of the accelerator
   - the physics principles about which the accelerator is designed to further knowledge
   - the monetary cost of building and operating the accelerator
   - the source of its funding
   - the type of particle accelerated
   - the final energy of the particle
   - the type of research that can be accomplished only by using these high-energy particles
   - any monetary return (profit) from applications of the accelerator
   - possible future (direct or indirect) benefits derived from the knowledge gained as a result of the research made possible by use of the accelerator
   - the environmental and societal impact of the use of the accelerator or of the knowledge gained
4. Prepare a report that summarizes the information gathered by the group.
5. Delegate responsibilities for publishing the report.
6. Prepare for the debate by
   - setting up two class teams that will debate on the costs/benefits of physics research; the debating teams will include both debaters and technical advisers from each of the small working groups
   - selecting a neutral person to act as moderator

   Each debating team will
   - analyze the small groups' reports to determine the costs and benefits of accelerators
   - assign roles for the debate (e.g., organizing and compiling material, preparing notes, developing arguments, serving as debaters)
   - rehearse the debate
7. Publish and present the small group reports.
8. Conduct a class debate.

Evaluate

1. Small group publications and class debate: Use the rubric prepared in step 1 of the Action Plan to evaluate the publications and class debate.
2. After the debate, through class discussion, attempt to establish a class position on the costs and benefits of the public funding of particle accelerators, and of science funding in general.

UNIT 3 Review

Knowledge/Understanding

Multiple Choice

In your notebook, choose the most correct answer for each of the following questions. Outline your reasons for your choice.

1. Two parallel oppositely charged metal plates have an electric field between them. The magnitude is
   (a) greatest near the positive plate
   (b) greatest near the negative plate
   (c) zero
   (d) uniform throughout the region
2. The magnitude of the electric field at a point in space is equal to the
   (a) force a charge of 1 C would experience there
   (b) force a negative charge would experience there
   (c) force a positive charge would experience there
   (d) potential difference there
   (e) electric charge there
3. The force on a proton in an electric field of 100 N/C (1.0×10^2 N/C) is
   (a) 1.6×10^{-17} N
   (b) 1.6×10^{-19} N
   (c) 1.6×10^{-21} N
   (d) 6.2×10^{20} N
4. Magnetic fields do not interact with
   (a) stationary permanent magnets
   (b) moving permanent magnets
   (c) stationary electric charges
   (d) moving electric charges
   (e) none of the above
5. An electron moves horizontally to the east through a magnetic field that is downward. The force on the electron is toward the
   (a) N **(c)** E
   (b) S **(d)** W
6. A current is flowing west along a power line. Neglecting Earth's magnetic field, the direction of the magnetic field above it is
   (a) N **(c)** E
   (b) S **(d)** W
7. The electric and magnetic forces in a velocity selector are directed
   (a) at 90° to each other
   (b) parallel to each other, in the same direction
   (c) opposite to each other

Short Answer

8. Do electric field lines point in the direction of increasing or decreasing electric potential?
9. Why do electric field lines come out of positive charges and enter negative charges?
10. What similarities and differences are there between electric potential energy and gravitational potential energy?
11. In a 10 000 V power line, how many units of energy is carried by each unit of charge making up the current?
12. How is the principle of superposition used in problems of determining the field value due to multiple charges?
13. Explain why there is no parallel component to the electric field on the surface of conductors.
14. **(a)** The direction of motion of a positively charged particle, the direction of the magnetic field, and the direction of the magnetic force on the particle are mutually perpendicular. Draw a sketch of this situation and describe the right-hand rule that models the relationship among these directions.
    (b) The direction of a current in a conductor, the direction of the magnetic field, and the direction of the force on the conductor are mutually perpendicular. Draw a sketch of this situation and describe the right-hand rule that models the relationship among these directions.
15. **(a)** Describe the characteristics of the force required to create and maintain circular motion at constant speed.
    (b) Discuss examples that illustrate how each of the following fields can provide such a force on an object or charged particle and cause circular motion: gravitational field, electric field, and magnetic field.

16. Why is it more difficult to provide a simple equation for the strength of a magnetic force than it is for the strength of a gravitational force, the universal law of gravitation, or the strength of an electrostatic force, Coulomb's law?

17. Consider an electric field around an irregularly shaped, positively charged object. Draw a sketch of this situation by placing the charged object at the origin of a Cartesian coordinate system. Make labelled drawings to illustrate your written answers to the following questions.
    (a) In which direction will the field push a small positive test charge?
    (b) Where does the positive test charge have its greatest electric potential energy?
    (c) In which direction will the field push a small negative charge?
    (d) Where does the negative charge have the greatest magnitude of its electric potential energy?

18. (a) Describe the main features of coaxial cable.
    (b) Explain why coaxial cables were designed to replace flat, twin-lead wire.

19. Explain whether it is possible to determine the charge and mass of a charged particle by separate electric or magnetic forces, that is, individually and not simultaneously.

Inquiry

20. Describe an experiment in which you could determine whether the charges on a proton and electron were the same in magnitude.

21. Devise an experiment that verifies Coulomb's law. Show that the electric force should be proportional to the product of the charges and show that the electric force should be proportional to the inverse square of the distance.

22. You place a neutral object between a pair of parallel charged plates. Will it experience a net force? Will it rotate?

23. The following table shows some results that Millikan obtained during his oil-drop experiment. In this trial, the distance over which the oil drop was measured (the distance between the cross hairs in the eyepiece) was always 1.0220 cm. The second column shows the time of travel under the action of gravity alone, and the third column shows the time for an oil drop to rise when the electric field was turned on.
    (a) Calculate the velocity that corresponds to each trial.
    (b) Group the common velocities.
    (c) Analyze the velocities in a manner similar to Millikan's and show the evidence for a fundamental charge.

Trial	Fall (seconds)	Rise (seconds)
1	51.13	30.55
2	51.25	21.86
3	51.19	50.72
4	51.32	148.63
5	51.53	147.46
6	51.69	50.29
7	51.55	50.25
8	51.54	50.39
9	51.98	49.70
10	51.64	146.41

24. Two identical pith balls, mass 1.26 g, have a charge of +4.00 nC. One ball (A) is attached to the end of a light rod made of insulating material; the other (B) is suspended from a fixed point by an insulated thread 80.0 cm long. When ball A is held at various horizontal distances from B, the angle between the thread and the vertical is measured. Determine whether the results support Coulomb's law.

Horizontal distance between A and B (cm)	Angular displacement of thread
0.50	25.0°
1.00	6.65°
1.50	2.97°
1.80	2.06°
2.10	1.51°
2.50	1.07°

Communication

25. The two statements "like poles repel" and "unlike poles attract" are throwbacks to the action-at-a-distance theory, in that they imply the two poles interact with each other directly. Rewrite these two statements to reflect a field theory perspective.

26. Use the concepts of the electric field and electric field lines to convince someone that like charges should repel each other.

27. Explain how it would be possible to measure Coulomb's constant.

28. Contrast the concepts of potential difference and difference of potential energy.

29. Use Newton's law of universal gravitation to explain why Earth is round.

30. Determine the direction of the unknown vector for each of the following situations. Consider north as the top of the page and sketch the directions of the magnetic field lines, the direction of the charged particle and the force that acts on it.

(a) an electron moving east, experiencing a force directed into the page

(b) a proton moving north in a magnetic field directed west

(c) an electron moving in a magnetic field directed into the page, experiencing a force to the south

(d) a proton experiencing a force to the east, moving north

(e) an electron, experiencing no force, moving in a magnetic field directed east

(f) a proton experiencing a force to the south as it travels west

31. Describe the significance to twentieth-century physics of the Millikan oil-drop experiment.

32. Consider a stream of protons moving parallel to a stream of electrons. Is the electric force between the streams attractive or repulsive? Is the magnetic force between the streams attractive or repulsive? What factor(s) determine which force will dominate?

33. **(a)** Sketch the electric field between two parallel charged plates. Label the orientation of the charges on the plates. Show the trajectory of a positive charge sent into the field in a direction perpendicular to the field. In which direction is the electric force on the particle? Is work done on the particle as it passes between the plates?

(b) Sketch the magnetic field between the north pole of one magnet and the south pole of a different magnet. Both are set up in such a way that the field will be uniform. Show the trajectory of a positive charge sent into the field in a direction perpendicular to the field. In which direction is the magnetic force on the particle? Is work done on the particle as it passes through the field?

34. A current runs from west to east in a horizontal wire. If Earth's magnetic field points due north at this location, what is the direction of the force on the current?

35. Explain how a current balance can be used to measure the intensity of the magnetic field along the axis of a solenoid.

36. Explain how a velocity selector is able to filter a beam of particles of different velocities so that only particles with the same velocity continue in a mass spectrometer.

Making Connections

37. **(a)** Use science journals, your library, and/or the Internet to determine how auroras are formed.

(b) Discuss the phenomenon in terms of electric, gravitational, and magnetic fields.

(c) The photograph opening Chapter 8 shows the aurora borealis and the aurora australis occurring simultaneously. Explain whether you think this is a unique occurrence or one that will recur.

38. The torsion balance played an essential role in Coulomb's work. Research the history of the use of the torsion balance in physics. How is a torsion pendulum different?

39. Research and report on how the concept of the field has evolved. Discuss Faraday's and Maxwell's contributions. Also, discuss the role of Einstein's general theory of relativity in our present view of gravitational fields.

40. Albert Einstein spent the last several years of his life trying to devise a unified field theory that would show that gravity and the electric and magnetic forces were different aspects of the same phenomenon. He did not succeed. In the 1960s, it was shown that electric and magnetic forces and the weak nuclear force are different aspects of the same force: the unified electroweak force. To date, no one has linked gravity or the strong nuclear force with the unified electroweak force. Research the unification of forces and explain why the problem is so difficult to solve.

41. The Sun's magnetic field is responsible for sunspots, small regions on the surface of the Sun that are cooler and have a much higher magnetic field concentration than their surroundings. The Sun's magnetic field is also responsible for producing solar flares and other solar activity. Prepare a report that summarizes the latest research on the Sun's magnetic field and the types of solar phenomena that are being examined. Incorporate into your report the findings provided by the orbiting solar satellite, the Solar and Heliospheric Observatory (SOHO).

42. Research the principle behind the defibrilator and the steps that have been made to ensure its presence on all major aircraft.

43. Research the structure of an electrostatic air cleaner and discuss the function of the charging electrode and the grid.

44. In what way is electrostatic force used in the electroplating process in automobile manufacturing?

45. Research and explain the part played by the electric field in
    (a) the xerographic process
    (b) laser printers
    (c) inkjet printers

46. Prepare a cost-benefit analysis of the use of the electric car.

47. "Electron guns" are used in television sets to propel electrons toward the screen. What techniques are then used to deflect the electron beam and "paint" a picture?

48. Discuss the role of electric potential difference in the following medical diagnostic techniques.
    (a) electroencephalography
    (b) electroretinography

Problems for Understanding

49. What is the total charge on 1.0 g of electrons?

50. What is the magnitude of the electric force between a proton and electron in a hydrogen atom if they are 52.9 pm apart?

51. A nucleus of argon has a charge of +18 e and a nucleus of krypton has a charge of +36 e, where e is the elementary charge, 1.60×10^{-19} C. If they are 8.0 nm apart, what force does one exert on the other?

52. Two small ball bearings sit 0.75 m apart on a table and carry identical charges. If each ball bearing experiences a force of 3.0 N, how large is the charge on each?

53. How many electrons must be removed from an isolated conducting sphere 12 cm in diameter to produce an electric field of intensity 1.5×10^{-3} N/C just outside its surface?

54. Two identical charges exert a force of 50.0 N[repulsion] on each other. Calculate the new force if

(a) one of the charges is changed to the exact opposite
(b) instead, the distance between the charges is tripled
(c) instead, one charge is doubled in magnitude and the other is reduced to one third of its magnitude
(d) all of the above changes are made

55. Two identical pith balls, each with a mass of 0.50 g, carry identical charges and are suspended from the same point by two threads of the same length, 25.0 cm. In their equilibrium position, the angle between the two threads at their suspension point is 60°. What are the charges on the balls?

56. Suppose you wanted to replace the gravitational force that holds the Moon in orbit around Earth by an equivalent electric force. Let the Moon have a net negative charge of $-q$ and Earth have a net positive charge of $+10\ q$. What value of q do you require to give the same magnitude force as gravity?

57. Earth carries a net charge of -4.3×10^5 C. When the force due to this charge acts on objects above Earth's surface, it behaves as though the charge was located at Earth's centre. How much charge would you have to place on a 1.0 g mass in order for the electric and gravitational forces on it to balance?

58. Suppose you want to bring two protons close enough together that the electric force between them will equal the weight of either at Earth's surface. How close must they be?

59. Calculate the repulsive force between two 60 kg people, 1.0 m apart, if each person were to have 1% more electrons than protons. (Assume for simplicity that a neutral human body has equal numbers of protons and neutrons.)

60. What will be the net force, considering both gravitational and electrostatic forces, between a deuterium ion and a tritium ion placed 5.0 cm apart?

61. What must be the charge on a pith ball of mass 3.2 g for it to remain suspended in space when placed in an electric field of 2.8×10^3 N/C[up]?

62. (a) Calculate the repulsive Coulomb force between two protons separated by 5×10^{-15} m in an atomic nucleus.
(b) How is it possible that such a force does not cause the nucleus to fly apart?

63. The electric potential difference between two large, charged parallel plates is 50 V. The plates are 2.5 cm apart. What is the electric field between them?

64. How many electrons make up a charge of 1.0 μC?

65. A 2.0 pC charge is located at point A on an imaginary spherical surface which is centred on a 4.0 μC point charge 2.8 cm away. How much work is required to move the 2.0 pC charge to the following two points?
(a) to point B, which is located on the same spherical surface an arc length 3.0 cm away
(b) to point C, which is located radially outward from A on another imaginary spherical surface of radius 4.2 cm
(c) What name could be used to describe these spherical surfaces?

66. Two horizontal plates used in an oil-drop experiment are 12 mm apart, with the upper plate being negative. An oil drop, with a mass of 6.53×10^{-14} kg, is suspended between the plates. The electric potential difference is 1.6×10^4 V. Calculate the
(a) total charge on the oil drop
(b) number of excess or deficit electrons on the oil drop
(c) electric potential difference required to suspend the oil drop if an electron is knocked off it by an X ray

67. A current of 2.0 A runs through a wire segment of 3.5 cm. If the wire is perpendicular to a uniform magnetic field and feels a magnetic force of 7.0×10^{-3} N, what is the magnitude of the magnetic field?

68. A small body of unknown charge, travelling 6.1×10^5 m/s, enters a 0.40 T magnetic field directed perpendicular to its motion.
(a) If the particle experiences a force of 9.0×10^{-4} N, what is the magnitude of the charge?

(b) If the object is sent into the magnetic field so that its velocity makes an angle of 30.0° with the magnetic field, by how much will the magnetic force be reduced?

69. Consider a proton that is travelling northward with a velocity of 5.8×10^6 m/s in a particle accelerator. It enters an east-directed magnetic field of 0.25 T.

(a) Calculate the magnetic force acting on the proton.

(b) What is the magnitude and direction of its acceleration?

70. A proton travelling at 2×10^7 m/s horizontally enters a magnetic field of strength 2.4×10^{-1} T, which is directed vertically downward. Calculate the consequent radius of orbit of the proton.

71. Prove that the radius of orbit of a particle in a mass spectrometer is equal to p/qB, where p is its momentum.

72. **(a)** An electron is fired into a 0.20 T magnetic field at right angles to the field. What will be its period if it goes into a circular orbit?

(b) If the electron is moving at 1.0×10^7 m/s, what is the radius of its orbit?

73. You want to create a beam of charged particles that have a speed of 1.5×10^6 m/s. You use a crossed electric and magnetic field and choose a magnet with a strength of 2.2×10^{-4} T. What must be the magnitude of the electric field?

74. A charged particle that is sent into a magnetic field at an angle will follow a helical path, the characteristics of which can be calculated from the particle's velocity parallel and perpendicular to the field. Consider a magnetic field of strength 0.26 T directed toward the east. A proton with a speed of 6.5×10^6 m/s is shot into the magnetic field in the direction [E30.0°N].

(a) Calculate the proton's velocity in the directions parallel and perpendicular to the magnetic field.

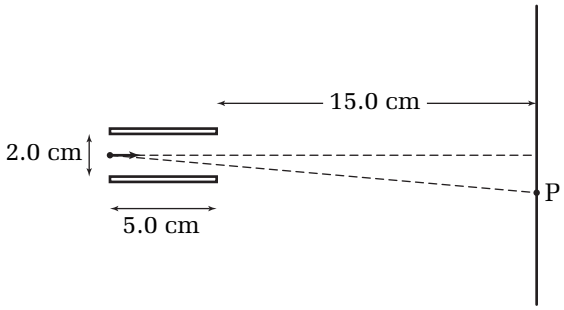
(b) Calculate the radius of the proton's orbit as it spirals around the magnetic field. (Hint: Which component of the velocity contributes to this motion?)

(c) How long will it take the proton to complete a singular circular orbit?

(d) During the time that it takes the proton to complete one orbit, how far will the proton travel toward the east? (Hint: Which component of the proton's velocity contributes to this motion?)

(e) Sketch the proton's path as seen from the side and as seen looking west into the magnetic field.

75. The diagram shows an electron entering the region between the plates of a cathode ray tube (the basic structure of a television tube). The electron has an initial velocity of 2.7×10^7 m/s horizontally and enters at the exact mid-axis of the plates. The electric field intensity between the plates is 2.80×10^4 N/C upward. How far below the axis of the plates will the electron strike the screen at point P?



COURSE CHALLENGE

Scanning Technologies: Today and Tomorrow

Plan for your end-of-course project by considering the following.

- Are you able to incorporate electric, gravitational, and magnetic fields into your analysis?
- Consider time and equipment requirements that might arise as you design project-related investigations.
- Examine the information that you have gathered to this point. Produce a detailed plan, including a time line, to guide you as you continue gathering information.

UNIT 4

The Wave Nature of Light

OVERALL EXPECTATIONS

DEMONSTRATE an understanding of the wave model of electromagnetic radiation.

PERFORM experiments relating to the wave model of light and applications of electromagnetic radiation.

ANALYZE light phenomena and explain how the wave model provides a basis for technological devices.

UNIT CONTENTS

Is there life beyond Earth? It seems inconceivable that life has formed only on this planet, yet there is no direct evidence that there is life outside our own solar system. If civilizations exist in space, might they be discovered by electromagnetic radiation monitoring from here on Earth?

The Search for Extraterrestrial Intelligence (SETI) program, a range of research projects dedicated to the search for intelligent life beyond Earth, is investigating this possibility. Using the world's largest radio telescope, located in Arecibo, Puerto Rico (shown in the photograph), the sky is scanned around the clock for non-natural electromagnetic signals. SETI research projects attempt to answer questions, such as: How many stars might have planets? And of those planets, how many have environments that could support life?

Developing an understanding of electromagnetic radiation has provided modern civilization with a powerful communication tool. This unit will introduce the theoretical framework that predicted the existence of electromagnetic waves, how these waves (including light) are produced and detected, their properties, and some applications in modern society.

UNIT PROJECT PREP

Refer to pages 454–455. In this unit project, you will have the opportunity to build and test an FM transmitter.

- How will an understanding of a wave model for electromagnetic radiation help you to understand FM transmission?
- What properties of electromagnetic waves will be easiest to verify using your transmitter?

CHAPTER 9

Wave Properties of Light

CHAPTER CONTENTS

PREREQUISITE CONCEPTS AND SKILLS

- Physical properties of waves
- Reflection and refraction of waves
- Superposition of waves
- Using a ripple tank

Peering through a telescope, you can see the "Red Planet," Mars, and, off in the distance, Jupiter's stripes. Earth looks like a blue marble and the gas giant Neptune appears to be crystal blue. This composite photograph reveals a richness of knowledge transmitted in the form of light that reaches Earth from the expanse of space.

What are the properties of light that allow it to travel millions of kilometres through deep space from the Sun, to the other planets, and back to our telescope, carrying information in the form of colour and intensity. Careful visual observation of solar system objects yields a great deal of knowledge.

Galileo used a telescope that today would be considered primitive to discover four of Jupiter's moons. His discovery solidified in his mind that Copernicus' concept of a Sun-centred solar system was correct, even though such a concept clashed with the scientific and religious theories of his time.

Less than 50 years later, a new debate raged, not about the solar system, but about the very nature of light, which streams from the Sun, illuminates Earth, and seems to light up a room instantly. The new debate struggled to compare light to something more common to everyday experience, attempting to classify this elusive form of energy as either a wave or a particle.

In this chapter, you will learn about the attempts to formulate and verify a model for light. You will discover that the techniques that established the wave model for light also led to some practical applications and research tools.

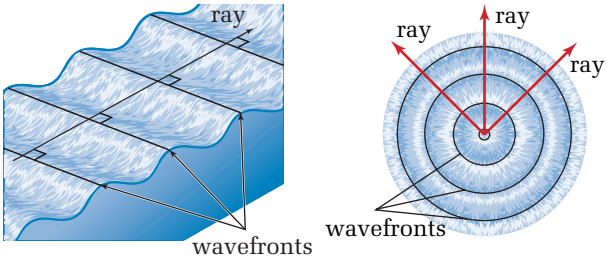
INVESTIGATION 9-A

Properties of Waves

TARGET SKILLS

- Performing and recording
- Analyzing and interpreting
- Identifying variables

For light to be classified as a wave, it must exhibit specific properties of waves. In this investigation, you will analyze an important property of water waves that must also be true of light — if light is, in fact, a wave.



Problem

Investigate how waves behave when they

(a) encounter a small barrier

(b) pass through a narrow slit

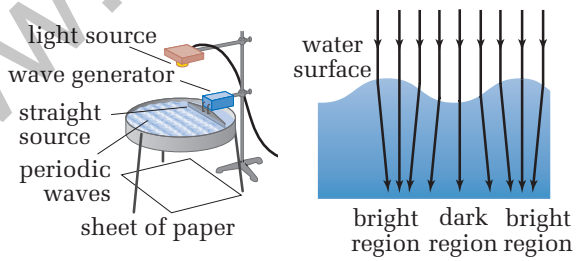
Equipment

- ripple tank
- wave generator
- 2 solid barriers (less than half the width of the tank)
- wooden dowel

CAUTION Care must be taken with any electrical equipment near ripple tanks. Firmly attach lights and wave generators to the tank or lab bench, and keep all electrical wiring away from the water.

Procedure

1. Assemble the ripple tank, light source, and wave generator as shown in the diagram. Add water and carefully level the tank so that the depth of the water is approximately 1.5 cm at all points in the ripple tank.



A Wave tank set-up **B** Water as a lens

2. Align the straight-wave generator so that parallel wavefronts travel perpendicularly from the dowel. Vary the frequency of the generator to find a wavelength that produces the clearest image on the paper below the tank. Use the light and dark regions cast on the paper to view the wave properties during the investigation.
3. Place a solid barrier in the tank that is about half the width of the tank. Send straight waves at the barrier and observe their behaviour. Sketch the appearance of the waves as they pass the edge of the barrier.
4. Vary the wavelength of the incident waves. Draw cases that exhibit maximum and minimum spreading around the edge of the barrier.
5. Place two solid barriers in the tank, leaving a narrow slit between them. Send straight waves toward the narrow opening and observe the nature of the waves that pass through it.
6. Systematically vary the width of the opening and then the wavelength to determine a general relationship between the amount of the spreading of the waves, wavelength, and the size of the opening.

Analyze and Conclude

1. Describe what happens to waves when they pass the edge of a solid barrier. Is the effect altered as wavelength is changed? If so, how?
2. Describe what happens when waves pass through a narrow opening between two solid barriers. What relationship between the wavelength and the width of the opening appears to be the most significant?

Apply and Extend

3. In your experience, does light exhibit any of the properties of waves that you have just studied? Provide examples.

9.1 Two Models for Light

SECTION EXPECTATION

- Define and explain the units and concepts related to the wave nature of light.

KEY TERMS

- dispersion
- diffraction
- Huygens' principle
- superposition of waves
- constructive interference
- destructive interference
- nodal point

You flip a switch as you walk into a room, flooding the room with light that instantly reaches every corner. Objects in the path of the light generate shadows, and yet the light reaches far enough under your bed to illuminate an old shirt. What allows light to seemingly be everywhere instantaneously, be blocked by objects, and yet be able to reflect and bounce into tiny nooks and crannies?

Physicists have been trying to develop a complete model of light for centuries. Is light best modelled as a particle or as a wave? These competing models for light originated in the late 1600s, proposed by physicists who were attempting to describe the propagation of light.

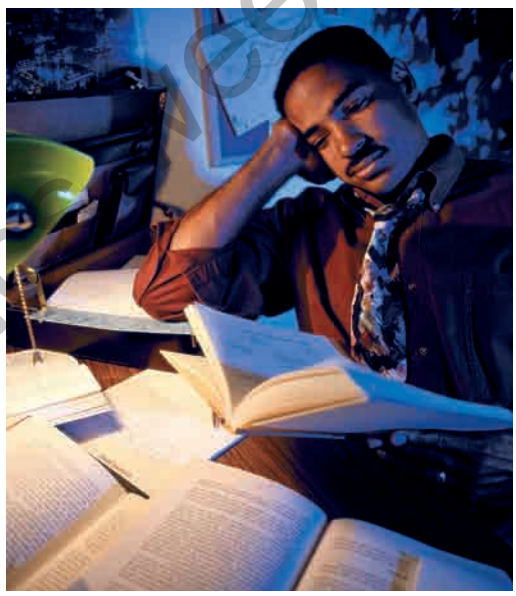


Figure 9.1 Even though the light casts shadows, you can still clearly see objects in those shadows. A little light seems to reach everywhere.

The basic properties of light that were understood in the 1600s (and that any acceptable model must be able to explain) were straight-line propagation, reflection, refraction, dispersion, and the ability of light to travel undisturbed across millions of kilometres of space. Which model — particle or wave — could best explain and predict these properties?

Newton's Corpuscular Model

Although scientists and philosophers had been hypothesizing about the nature of light for centuries, Sir Isaac Newton (1642–1727) was the first to formulate a detailed, systematic model of light. He published his "corpuscular" theory of light in 1704. Newton's proposed corpuscles were particles with exceedingly small masses that travelled in straight lines through space,

penetrated some media, and bounced off other solid surfaces. Newton could explain refraction (the bending of light when it travels from one medium to another) if the speed of the particles increased when entering a more-dense medium. Although speeding up in a dense medium does not seem logical, Newton explained it by proposing that an attractive interaction existed between the light particles and the medium.

Conceptual Problem

- Use the conservation of momentum to show that when a particle such as a billiard ball collides with a solid wall, it follows the law of reflection, which states that the angle of reflection is equal to the angle of incidence.

The **dispersion** of light, which is the separation of light into the colours of the spectrum when passing through a prism, had been observed by scientists before Newton proposed his theory of light. Newton himself had demonstrated that white light was actually a composite of all of the colours of the rainbow by showing that the colours could be combined by a second prism and produce white light.

Figure 9.2 Using a refracting prism, white light can be separated into a spectrum of colour.

According to Newton's corpuscular model, each colour had a different mass. Violet light was refracted the most and therefore must have the least amount of mass, making it easiest to divert from its original path. Blue light was more massive than violet and therefore refracted less. Following this argument, Newton assumed that red light particles were the most massive of all of the visible colours.

Another question that Newton's corpuscular theory was able to answer was: What occupies the space between Earth and the Sun? If light was, in fact, small particles of insignificant mass, the particles would be able to travel millions of kilometres to Earth from the Sun. Possibly, the most important reason that Newton did not consider the wave model for light was the apparent lack of

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diffraction — the spreading of a wave after encountering a barrier. Italian scientist Francesco Grimaldi (1606–1680) provided evidence that light does undergo diffraction by demonstrating that light passing through a small opening in a barrier produced a spot of light on a distant screen that was larger than strict ray diagrams predicted. The edges of the bright spot also appeared fuzzy: The region of light faded into dark, rather than being crisply divided into two regions. Newton and the proponents of the corpuscular theory of light discounted the effects, citing that the amount of diffraction seemed to be too small to be of consequence.

Newton was not entirely convinced of the correctness of his own corpuscular model for light and was surprised that some of his proponents approved of it so strongly. Nevertheless, until stronger evidence of wave-like properties was obtained, Newton would not accept a wave model for light.

Huygen's Wave Model

Christiaan Huygens (1629–1695) refined and expanded the wave model of light, originally proposed by Robert Hooke (1635–1703). Hooke rejected a particle model partly because two beams of light can pass through each other without scattering each other, as particles do. One problem with the wave model was the ability of light to travel through the apparently empty space of the universe.

During early discussions about the nature of light, scientists knew that mechanical waves required a medium through which to propagate. Various properties of the medium would undergo periodic changes from a maximum, to an equilibrium, to a minimum, and back through the cycle again. For example, in a water wave, the particles of water actually move between a maximum and minimum height. In sound waves, the pressure of the medium increases and decreases. What medium could be carrying light energy? Since no medium was known to exist throughout space, scientists proposed that an as yet undetected medium called "ether" existed to carry light waves.

Huygens developed a priniciple that is still helpful in analyzing and predicting the behaviour of waves. He compared the propagation of light to the travelling disturbance observed when a pebble is dropped into a pond of still water. The disturbance, or wave pulse, moves outward in concentric circles from the pebble's point of impact. Huygens realized that the waves travelling outward from the centre continue to travel even after the pebble has struck the pond's bottom. The waves are effectively travelling without a source.

Extending this example, he postulated that disturbances existing at each point on a wavefront could be a source for disturbances along the wavefront an instant later. Figure 9.3 illustrates Huygens' thinking for straight and circular waves.

PHYSICS FILE

Robert Hooke contributed to science in several fields, from cellular biology to the physics of light. Hooke developed the wave theory of light, while Newton proposed the corpuscular or particle theory of light. Hooke and Newton had argued before, because Hooke had developed an early version of Newton's gravitation equation and felt that Newton had not given him enough credit. Both men realized, however, that more evidence would be necessary before either of their models would be accepted.

Huygens' principle states: Every point on an advancing wavefront can be considered to be a source of secondary waves called "wavelets." The new position of the wavefront is the envelope of the wavelets emitted from all points of the wavefront in its previous position.

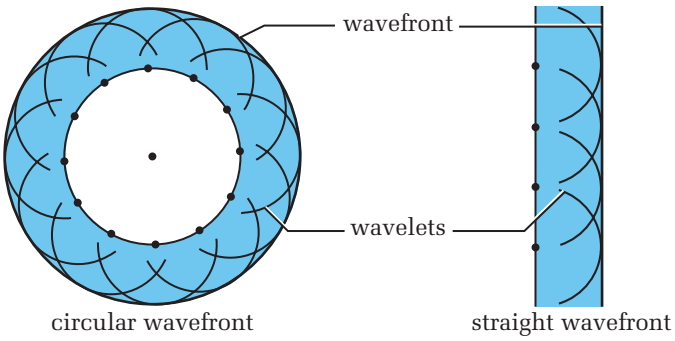

Figure 9.3 Each point of a wavefront can be considered to be a source of a secondary wave, called a "wavelet."

Conceptual Problem

- Carefully mark a small dot every 0.5 cm over a distance of 8.0 cm on a blank sheet of white paper. Beginning with the first dot, use a 25 cent coin to draw semicircles on every fourth dot. Ensure that each semicircle arc is drawn with the leading edge always closest to the top of the page, intersecting a single dot as shown. Draw more arcs, one for each dot.

  (a) Does the leading edge of the sum of the arcs form a more complete wavefront?

  (b) Would an infinite number of wavelets form a continuous wavefront? What is happening behind the wavefront to cause a single wavefront to form?

Either by applying Huygens' principle or by observing visible waves such as water waves, you can show that waves propagate in straight lines while moving unobstructed through a single medium, and that they reflect off solid or opaque barriers. Huygens also showed that if the velocity of light decreases when it passes from a less-dense to a more-dense medium, it will bend or refract in such a way that the angle of refraction is smaller than the angle of incidence. A slight difference in the speed of the various colours of light in a given medium could also explain dispersion.

Huygens' wave model accurately predicted the behaviour of light as strongly or even more strongly than did Newton's model in terms of rectilinear propagation, reflection, refraction, and